EVALUATION AND PREDICTION OF WORLD RECORDS AND ULTIMATE PERFORMANCE IN TRACK AND FIELD

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ABSTRACT

The study deals with mathematical models as they apply to predict sports performances with track and field events. The purposes of this study were the following: 1) to identify the best applied mathematical model based on their assumptions, strengths and weaknesses, and the outcome predictions among the models using a comprehensive updated data set; 2) use a comprehensive updated data set and the chosen best fitting model to predict future performances for males and females in selected track and field events, and determine whether women will outperform men and if so, when; 3) develop a new random sampling model to predict the world record and ultimate performances based on the assumption (testable) that the performance has already reached an asymptotic level and the best performance population will be stable in the next 50 or more years.

BMDP-1R and BMDP-3R software were used to fit the linear and nonlinear models and produce statistics to assist in identifying the best fitting model. A FORTRAN 77 Monte Carlo simulation program was written to do the simulation utilizing values derived from extreme value theory for the men's 1500m event. The world prediction results obtained from the random sampling model were then compared with Glick's theoretical expected number of world records in a given period.

The results showed that: 1) the best performance per year data are the most appropriate data in track and field for model development, and the exponential model relating running time and historical year with the best performance data is the most valid deterministic model for prediction of world records and the ultimate performance; 2) the differences between women's and men's performances in track and field will keep diminishing, however, women are not predicted to catch up to the men in the chosen events in this study; 3) a greater performance improvement is expected in the near future for those events in which the performances still exhibit a linear trend (e.g.,10000m, and High jump); 4) under the assumption that the average ultimate performance has been reached in the men's 1500m event, the random sampling model is an effective method to predict the new world records for this event; 5) according to the random sampling model the waiting time between world records becomes progressively longer with every newly established world record. A world record beyond the limit of 205 seconds for the men's 1500m event could take up to one hundred years or more.

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CHAPTER I

INTRODUCTION

1.1 Introduction

Since the beginning of this century there has been a consistent interest in the analysis of track and field performance and, in particular, in predicting world records and ultimate performance which relate distance and running time. Studies of the nature and extent of track and field performance (particularly running events) have been conducted within a number of disciplines and interest areas: statistics, physiology, biomechanics, and athletics. Physiologists (Hill, 1925; Peronnet & Thibault, 1989) have utilized a metabolic-based model to provide a physiological explanation of the time-distance relationship and to predict record performance. In the biomechanics area the principles of Newton's Laws have been applied to develop a model predicting ultimate performance (Keller, 1973; Senator, 1982), and Ward-Smith (1985) combined this approach with the metabolic energy expenditure approach to develop his thermodynamic model. Coaches and athletes in the sport of track and field have a long history of predicting future world records and ultimate performance based solely on their experiences and intuition (Hamilton, 1934). However, the most common approach, and the one which will be followed in this study, is that used by mathematicians, statisticians, and psychometricians (e.g., Chatterjee & Chatterjee, 1982; Deakin, 1967; Schutz & McBryde, 1983). These individuals, and many others, have developed several different models to explain past performances and predict future ones. A comparison of all these approaches reveals that the variability among the predictions is large, as different investigators have utilized different mathematical models, different data sets, and different assumptions.

All models examined in this study, except the linear models, have a clear assumption that human beings have physiological limits in running performance and that maximum or ultimate performance is rapidly approaching in some events. Linear models, on the other hand, assume that no such limit exists, and are based on the premise that through skillful coaching, individualized medical attention, and various scientific developments (e.g., steroids) humans can keep improving their best performances forever. This assumption results in linear models having very different evaluation and prediction values from all other models.

A possible source of bias in most of the reported studies is the use of world records as the only data to evaluate and predict performances. A problem with such data is that they are discontinuous, as records progress in step functions of varying sizes (Schutz & McBryde,1983). This results in unstable parameter estimation, with the estimates being sensitive to the number of years since the last record was set. It is expected that more stable and accurate estimates will be possible by using yearly best performances. The attraction of linear models is that they do fit world record data well (R^2s>.90) in some events and have intuitive appeal to some researchers (e.g., Ballerini & Resnick, 1985; Whipp & Ward, 1992). However, in virtually every track and field event, for both men and women, a nonlinear model may give a superior fit.

The validity of methods used to predict world records and ultimate performance depends on the following factors: (1) the form of the fitted curve, (2) the independent variable chosen, (3) the raw data employed, (4) the method of curve-fitting used (Deakin, 1967), and (5) the soundness of the underlying substantive assumptions. Examination of the methods used to date suggests that none of them are entirely satisfactory.

Another area of interest is the comparison of male and female past and future performances. Since women's track and field events were introduced to the Olympic Games in 1928 there has been an increasing interest in evaluating and predicting women's performances and comparing their performances with men's. Whether female performances will surpass male performances in the future is always a fascinating topic in athletics and science and recently there have been predictions that women runners may surpass their male counterparts in a few years (Dyer, 1977; Ullyot, 1978; Whipp & Ward, 1992). However, some researchers have argued that the results of a linear extrapolation of future world records on the basis of the past progression is questionable. In addition, sufficient performance data for women were not available when some of these studies were done in the 1960s and 1970s. Because of its short history, many women's events did not exhibit a tendency towards asymptotic levels in the 1970's and early 1980's. However, by 1992 evidence of a nonlinear component in women's track records is beginning to emerge, as thus it may be now feasible to compare female and male performances with a common model.

It seems probable that there are certain physiological limits which may prevent females and males from improving their performance much beyond today's records. In some events the rates of improvement have become smaller and smaller in the last two decades; for example, in the 1500 meters the current women's world record was set in 1980 and the men's world record has been broken only once since 1985. It could be hypothesized that human beings have reached this physiological limitation in some events, and any further records merely reflect "outliers" from a random sampling model. All models used to date have been deterministic in that they predict a single specific time or distance for each point in the time line. It is proposed here that a possible alternate model is one in which it is assumed that ultimate "true score" performance has been achieved, and further improvements are the result of sampling fluctuations.

1.2 Purpose

1.2.1 Best Model Identification

The first purpose of this study is to: 1) compare the previously applied mathematical models on the basis of their assumptions, strengths and weaknesses, 2) compile a comprehensive updated data set on track and field events since 1900, 3) compare outcome predictions among the models, and 4) identify the best model.

1.2.2 Female and Male's Performance Comparison

The second purpose of this study is to use a comprehensive updated set of data and the best fitting model (from above) to predict future performances for males and females in selected track and field events, and to determine whether and when women will outperform men.

1.2.3 New Model Development

The third purpose of this study is to develop and test a new model to predict future performances. The distinguishing characteristic of the model is the (testable) assumption that performance has already reached asymptotic levels. Assuming that the best performance population will be stable for the next 50 or more years, this new "random sampling" model will predict the new world records and expected waiting times for each new world record.

CHAPTER II

LITERATURE REVIEW

Attempts to predict world records and ultimate performance in track and field commenced at the beginning of the century. In 1906, Kennelly examined the relationship between velocity and distance for various track events on a log-log scale and showed that a linear relationship between velocity and distance was stable over all events. Given that the world record at that time was 4:15.6 for the mile run, Kennelly predicted that the ultimate performance would be 3:58.1 for the mile, 8:39.4 for the two-mile and 13:39.6 for the three-mile run. However, his record prediction for the 10-mile event was almost two minutes slower than the record at that time (Meade, 1966). Since then, the evaluation and prediction of world records and ultimate performances in track and field have been extensively investigated by mathematicians, psychometricians, physiologists, and biomechanists (e.g., Chatterjee & Chatterjee 1982; Deakin, 1967; Glick, 1978; Hill, 1925; Keller, 1973; Peronnent & Thibault, 1989; Schutz & Mcbryde, 1983; Senator, 1982; Ward-Smith, 1985). The prediction and evaluation of female and male future performances were also fascinating topics for sociologists, coaches and athletes in this century. (e.g., Dyer, 1977, 1984; Hamilton, 1934; Meade, 1966).

Extensive developments in mathematics and statistics have made it possible for psychometricians, physiologists, and biomechanists to develop more accurate and more efficient models for evaluation and prediction purposes over the last 50 years.

2.1 Mathematics and Statistics in Relation to World Records and Ultimate Performance Prediction

Theoretically, there are two kinds of observations of chronological sequences from the real world; random fluctuations over time (e.g., weather temperatures over the years), and average trends over time (e.g., most sports performances over years). The literature on world records and ultimate observations in track and field indicates that running performances are treated as representing the second category, and the analyses are therefore based upon linear and nonlinear regression theory. The deterministic prediction per year is based on a linear model or a nonlinear model. Data representing the first type of observations lead to analyses utilizing extreme value theory. World record predictions with a confidence interval can be provided by a random sampling model. However, world records in track and field have always been assumed to exhibit trends (improvement) and the extreme value theory has usually not been applied to such data.

To distinguish which category the observations belong to, Foster and Stuart (1954) developed a formal procedure using the sum or the difference of record high and record low observations to test the randomness of sequential data. The test examines whether a trend exits in the chronological data and if there is a trend in the variance.

For data which do not exhibit a trend, Glick's procedure (1978) provides the expectation for the number of future records within a given time period and the expected waiting time between records. This procedure is useful to evaluate extreme weather in meteorology and to decide a strategy for destructive tests in product testing. In sports the primary interest is what the next record will be and what period of time it will take for a new world record to be established. However, Glick's procedure does not provide the expected values for a given random sequence of data.

Because the rates of improvement have become progressively smaller in the 1980s and the average ultimate performance could have been reached in some track and field events, the performances can be treated as the first category up to the 1990s. Extreme value theory can be used and a random sampling model can be developed for the world record predictions and evaluations in track and field based on the assumption that average ultimate performance has been reached.

For observations which have a trend over time (e.g., most sport performances), linear and nonlinear regression theory have been used to investigate record performance and to evaluate characteristics of the trend. The behavior in estimation of linear regression models has been extensively developed in the literature, and nonlinear regression theory has provided another basis for the deterministic predictive world record and ultimate performance in track and field. Nonlinear regression theory and its applications were described in detail by Bates and Watts (1988) and a unified practical approach was addressed by Ratkowsky (1983). Because the track and field performances were usually treated as the second category of observations in the literature, linear models were used by many researchers to predict the world records and the ultimate performances in track and field (e.g., Dyer, 1977, Whipp and Ward, 1992). However, some researchers think that the assumptions for linear models are not reasonable, hence nonlinear models were suggested because human beings can not improve their performance forever (e.g., Mognoni, Lafortuna, Russo, & Minetti, 1982, Schutz & McBryde, 1983). The details for these models found in the literature will be discussed in the following sections.

2.2 Prediction of World records and Ultimate Performances

Under the assumption that the performances in track and field have a clear trend, linear models and nonlinear models have been used for evaluating and predicting world records in track and field since the beginning of this century.

2.2.1 Linear Models

Linear models have been used to fit two different types of data for the purposes of evaluation and prediction; (1) running performance over multiple events in a specific year, (2) world record or best performance over years for a single event.

Using the first type of data, Kennelly (1906) found that when running speed and running time for a specific year were plotted with respect to distance on logarithmic paper, the points fell on approximately straight lines. His record predictions were not accurate even at that time (e.g., 3:58.1 for the mile, 8:39.4 for the two mile and 13:39.6 for the three mile). However, Kennelly's contribution must be valued for it was the first which studied records statistically, and the first to fit the empirical data available to a mathematical formula. Similar to Kennelly's study, Lindsey (1975) used a linear model to fit the logarithm of the running time and the logarithm of the distance for world record data in the year 1974. Running events from the 100m to the 10000m were examined. The factors that affect the speeds in different distances were discussed, but no world record predictions were made. This type of model is inappropriate for the record predicting purpose, primarily because; (1) the log-log linear function doesn't fit the data well, and (2) the prediction requires two stages of model fitting, thus compounding errors in estimation of ultimate performance.

Using the second type of data, Ryder, Carr, and Herget (1976) examined the speed world record improvements from 1900 to 1970. The performances examined were all distances of running events from the 100m to the marathon. The speed in meters per minute was plotted over years. It appeared that the rate of improvement in speeds was linear and varied slightly with distance run (from about 0.6 meters/min/year for the 100m to 0.9 meters/min/year for the marathon). They concluded that although there must be a physiological limit to the speed at which a human can run, "it certainly has not yet materialized at any distances", and that the barriers holding back further improvements are mainly psychological. No justification or explanation of this psychological concept was offered and they utilized world records as the only data. The problem with such world record data is that they are not continuous. This results in an unstable parameter estimation, with the estimates being sensitive to the number of years since the last record was set. Using only a linear model, this study ignored the fact that the improvement rate slowed down over the 1960s and 1970s in some events.

Other researchers (e.g., Dyer, 1977; Whipp & Ward, 1992) used the second type of data to fit linear models for both men and women. The world record predictions and the performance comparisons between men and women for various running events were investigated. The weakness of these studies is the same as that found in Ryder, Carr, and Herget's study. The results from these studies will be discussed in a later section.

Record sequences from linear model have been studied by some statisticians in last two decades as well (Ballerini & Resnick, 1985; Ballerini, 1987; and Smith, 1988). However, they focused on the investigation of the error distributions based on the linear model. Because they based their world record and ultimate performance predictions on the linear model, the problems with these studies are still the same as the linear models mentioned above.

2.2.2 Nonlinear Models

As with linear models, nonlinear models have been used by many researchers to predict and evaluate performances in track and field. Exponential models and polynomial models have been the most common.

Lucy (1958) used a nonlinear model to predict the mile ultimate performance. This is the earliest reported attempt which is purely predictive in nature (Schutz et al., 1983). The model is similar to the exponential model:

$$T(n) = b_0 + b_1 a^n$$

where b_0, b_1, and a are the constants to be determined, and n is the time in years. Only nine years of best performance data in the mile run were used to predict the ultimate performance, because Lucy claimed that due to the interruptive effect of World War II one should not use data prior to 1950. However, this resulted in insufficient data for his model and affected the accuracy of predictions for the mile run. The primary contribution of this study was the application of a nonlinear model to predict the world record and ultimate predictions.

Schutz, Carr and Halliwell (1975) used an exponential function to predict the best performances in the 100m to 10000m running events and the four jumping events. The function was fitted by three yearly best performance data sets; (1) all years from 1886, (2) post-World War I only, and (3) post-World War II only. They found that no one set of data yielded consistently good predictions for all events at that time. Schutz and McBryde (1983) studied the exponential model, the linear model, the power model (Lietzke, 1954), and the Chatterjee's model (Chatterjee & Chatterjee, 1982). The exponential model with the best performance data since the beginning of this century yielded the most consistent projections. Their contribution to the literature is that they used best performance data rather than world record data for the model fitting and the record predictions. However, in almost all cases, the predictions for women's events were unrealistic due to the lack of data at that time. They suggested that "some years must be allowed to elapse before women's performance become amenable to mathematical analysis." (p513).

Chatterjee and Chatterjee (1982) related time and year for each of the 100m, 200m, 400m, and 800m events, with an exponential model. Their data base was comprised of Olympic winning times (1900 to 1976) for these four events. The unique contribution of their work is the two-way analysis of variance model. Their independent variables were historical year and running distance with the dependent variable being time in seconds. The entire data were then analyzed as a two-way fully crossed factorial model, to examine the effect on running times of race distance, and the year in which the Olympic Games were held. The 'year effects' estimate improvements in techniques and training methods over the years. The effect of the altitude of the meet on reported times was also studied. The running times for the events in the 1980 Olympics were predicted using models which included all the effects and were compared with the observed times. A set of predictions for the 1984 Olympics was provided, but insufficient data (17 Olympic performances) affected the validity of the model. Schutz and McBryde (1983) examined the Chatterjee's model and stated that the model did not yield acceptable predictions, and substitution of empirical values into the equations did not yield the values reported by Chatterjee and Chatterjee.

World records for running, swimming, and ice-skating, over various distances, were analyzed by Mognoni et al., (1982) using a polynomial model. The dependent variable was speed and the independent variable was historical year. A mean period of about 66 years for the 18 male events and of about 50 years for the 14 female events was studied. As of June of 1981, they found that tendency towards an asymptotic speed was not yet a general phenomenon, however, the rate of record growth was slowing down in some of the selected events. Even though the polynomial function was used, very few events required a fourth power component in the equation. The discontinuous world record data may be one of the reasons that affected the validity of the model. As mentioned above this results in an unstable parameter estimation, with the estimates being sensitive to the number of years since the last record was set. By using updated best performance data we can compare the validity of the polynomial model with the exponential model.

The literature shows that different researchers used different data in their studies. World record data and best performance per year data, and a number of different models have been used for world record and ultimate performance predictions. A problem with world record data is that they are not continuous. As mentioned above it affects the stability of parameter estimation. In addition, since the 1970s and the early 1980s, a nonlinear component has emerged in most track and field events, thus invalidating the utilization of linear models for performance predictions.

2.3 Comparisons Between Female and Male Performance in Track and Field

In the last few decades female and male performance comparisons and evaluations in track and field have been a topic of considerable interest to researchers and track and field enthusiasts. The questions usually asked are whether female performances in track and field will equal or surpass male performances and when this may occur.

There were some early attempts to evaluate past performances of women and speculate on their future accomplishments in 1960s. In 1963, Craig stated that "as a consequence of the narrow experience, it is difficult to comment on the present women's running records except to note that the mark for 400m which is 53.4 could be 52.8 seconds (Craig, p17, 1963)." Frucht and Jokl (1964) compared women's performance with men's performance on the long jump using world record data up to 1964. Linear extrapolation was used to show that since 1948 the rate of ascent of the women's curve was greater than that for men.

An analysis by Hodgkins and Skubic (1968) led to predictions of women's performances for various track and field events. The improvement percentage of women's performances between 1928 to 1955 and 1956 to 1965 were compared. They found that the improvements in the field events greatly exceeded the improvements in the running events. The rate of improvement indicated that in the women's running events the curve was beginning to flatten.

They showed that while linear improvement would result in a 27 percent increase, the actual improvement between 1956 to 1965 was 18 percent. It indicated that although it is clear that women were continuing to better their performance, their progress showed signs of becoming more gradual. Based on the assumption that the trend of past performances can be approximated by an exponential curve, predictions were made for the 1968 Olympics and for the 1975 world records. Mathematical procedures to derive parameter estimates were not used for the predicting purpose in this study. The linear or exponential trend was decided arbitrarily by the researchers.

The linear model fitting functions have been used since the 1970s. Ullyot (1978) plotted women's marathon performances from 1967 to 1977 (velocity vs historical year). Near future predictions seems valid up to 1981 but long term predictions were not valid (Schutz & McBryde, 1983). The mean percentage differences of track events per year between female and male were fitted in a linear model by Dyer in 1977. He concluded that if the changes between 1948 and 1976 were maintained, average female performance would equal that of males at some time during the next century for all chosen events in his study. Both women's and men's world records expressed as mean running velocity versus historical year were fitted in a linear model by Whipp and Ward (1992). They predicted that women will be running at the same velocities as men before the year 2050 in all the chosen events for the study, and will exceed men in the marathon by 1998. However, as stated previously in this paper, it is most unlikely that a linear improvement will continue forever. A closer examination of women's best running times over the past 15 years clearly indicates a plateauing of performances, similar to that observed for men over the past 15-25 years. Schutz and McBryde (1983) are the only researchers who used an exponential model with best performance data to predict and evaluate women performances in track and field. As the very rapid improvement in performances before the 1980s and insufficient data for some events, long term predictions of women's performances were unrealistic at that time. Ten years have passed since their study was published, and the women's athletic results are now more amenable to mathematical analysis.

From the literature, we can see that the linear model with world record data is the one most often used for the prediction and evaluation of women's performances in track and field. However, by 1992 the prediction and the evaluation of women's performances based on a linear model are not valid. There are several reasons to suspect the result from these linear models. Firstly, because of the short history for women participating in the sport, the dramatic improvement rates had not levelled off before the 1980s, and the sufficiently long data baseline was not established. Secondly, discontinued world record data set could result in unstable parameter estimation, with the estimates being sensitive to the number of years since the last record was set. Thirdly, soundness of the assumptions for the linear model is speculative.

In terms of the deterministic new world record predictions and comparisons for women and men's performances in track and field, the nonlinear models could be the better choice opposed to linear models. Because different researchers utilized different nonlinear models and different data sets, some work should be done to evaluate the validity of the models and identify the best model with the valid data set.

Nonlinear models and linear models can not predict any performance after the performance reaches the asymptotic level. A new technique has to be employed for the situation in which human beings have reached their average limitation. It is suggested that extreme value theory could be used to develop a valid model for this situation. Even though the first extreme value theory book was published in 1958 (Gumbel, 1958), it was usually categorized under the topic of order statistics. Extreme value theory is applicable to predictions in meteorological, biological, engineering, and athletic studies, although it is seldom used in the latter. The application to sport may have been inappropriate some years ago when athletic data still exhibited at trend of improvement. However, in the last 15-20 years there is evidence of plateauing. Based on the assumption that human beings have already reached their average limit to improve their best performance in some events the extreme value theory could be used to predict future performances and world records in track and field.

CHAPTER III

METHODS AND PROCEDURES

3.1 Data Collection Procedures

Seven events (100m, 400m, 1500m, 5000m, 10000m, Marathon and High-jump) were analyzed in this study. These events were selected because the competitive conditions are identical for both sexes, and most of the events have been contested by both males and females in most international competitions since 1900 (men) or 1927 (women). Three types of data were collected for the study: men and women's world records since the beginning of this century; best performances per year since the beginning of this century; and the top 50 performances per year (only for the men's 1500m) from 1980 to 1992. Schutz and McBryde suggested in their study (1983) that an ideal data base would be one which consists of the best times or distances of the top performers per year. This would permit a detailed analysis of the distributions of elite performance, rather than just the single most elite yearly performances. Best performance per year and top-50 performance data were obtained from selected issues of Track and Field News. The world records for all the events were obtained from the official world record list (1992) as approved by the International Amateur Athletic Federation (IAAF) (Megede & Hymans, 1992), the recognized governing body of the sport which formulates uniform rules and ratifies world records.

3.2 Mathematical Models of Running Performance

A large number of different methods have been utilized to model track and field records. Brief descriptions of the models used in the literature are given below.

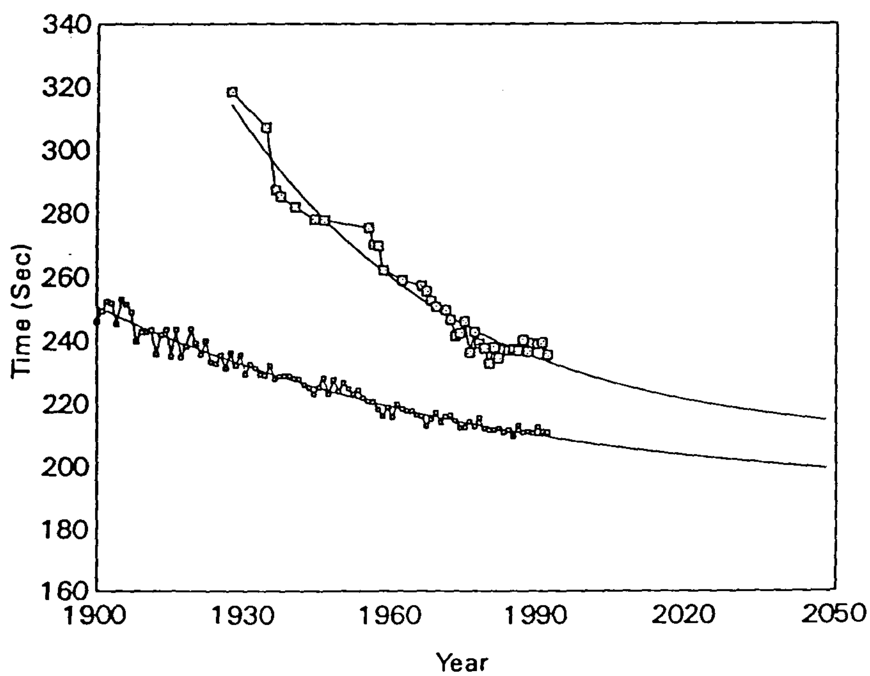
3.2.1 Historical Date and Time Relationships

In these models a time-year or velocity-year relationship is developed separately for each event.

A_1: An exponential model relating running time and historical date for a specific event.

$$T(n) = b_0 + b_1 e^{-b_2 n}, \qquad (3\text{-}1)$$

Figure 3-1. Historical Date and Time Relationship (exponential model)

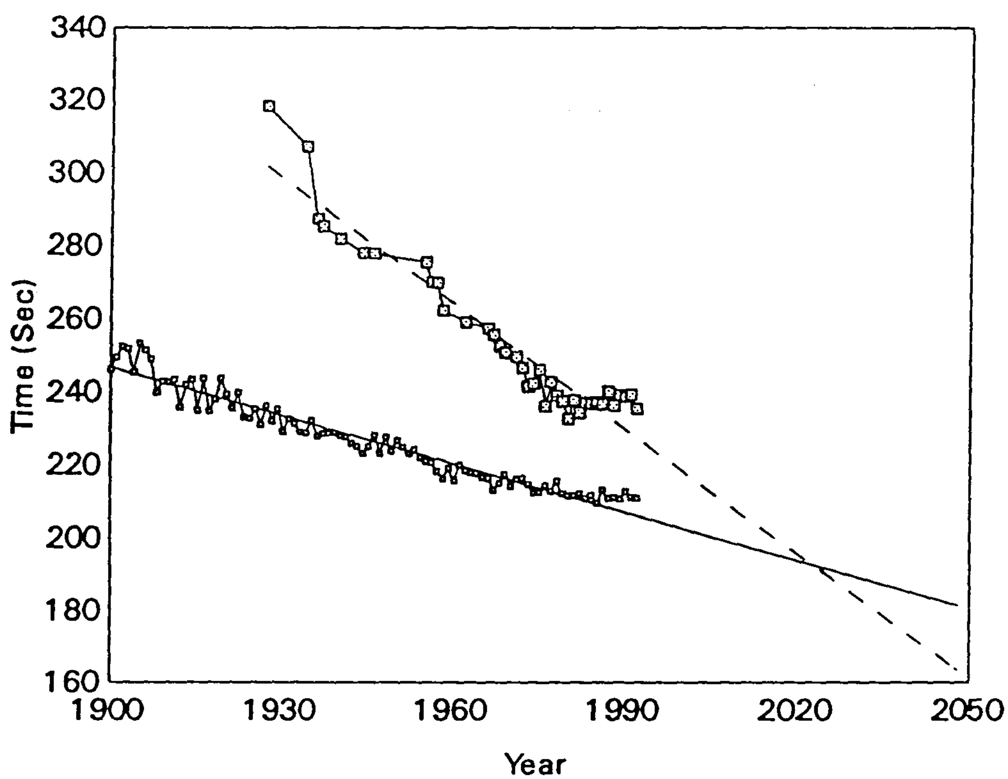


where T(n) is the predicted time in year n, b_0 is the estimated asymptotic value of T(n) as n approaches infinity, and b_1 and b_2 are calculated parameters which govern the shape of the curves (Chatterjee & Chatterjee, 1982; Lucy, 1958; Schutz & McBryde, 1983).

A_2: A linear model between running time or velocity and historical date for a specific event.

$$T(n) = b_0 + b_1 n, \tag{3-2}$$

Figure 3-2. Historical Date and Time Relationship (linear model)

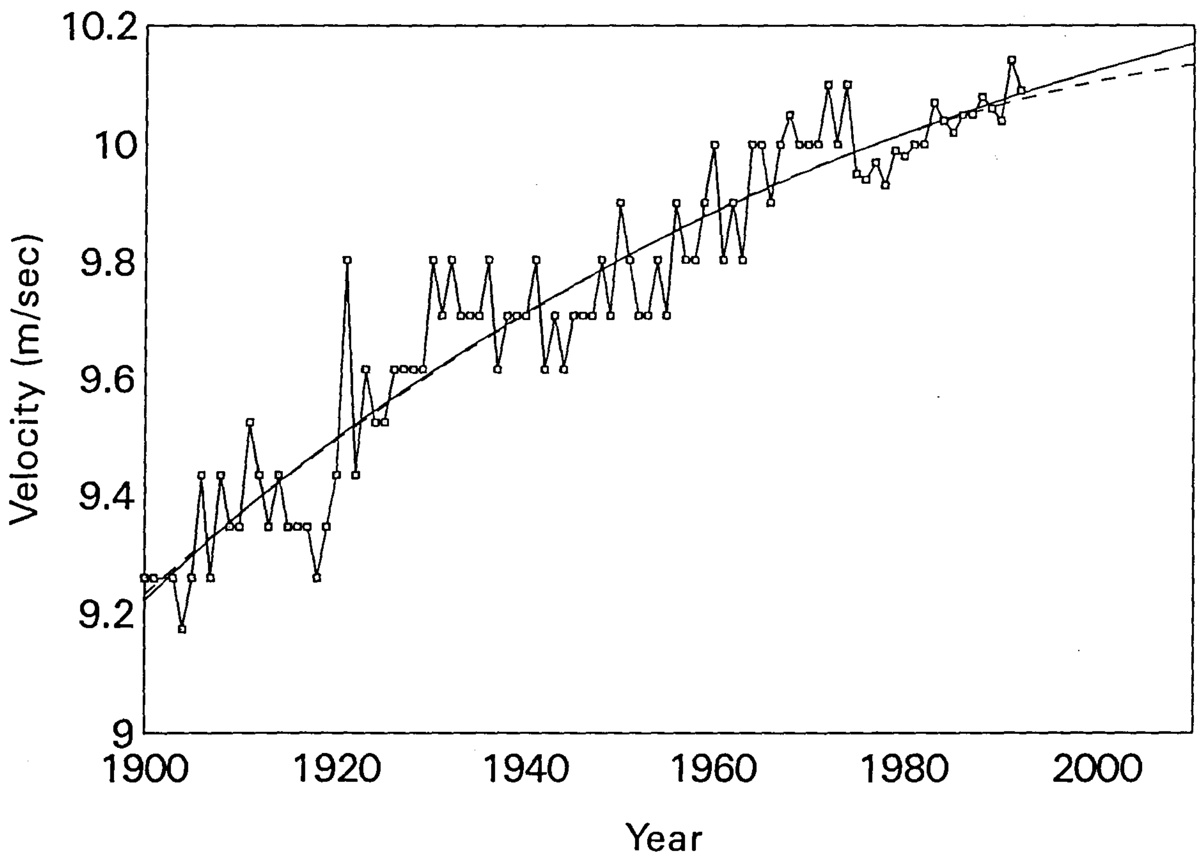


where T(n) is running time for a specific event, n is the historical year, and b_0 and b_1 are calculated parameters which govern the intercept and slope (e.g., Ballerini & Resnick, 1985, 1987; Whipp & Ward, 1992).

A_3: A polynomial exponential model relating velocity and historical time for a specific event.

$$V(n) = b_0 + b_1 n + b_2 n^2 + \ldots\ldots + b_m n^m, \qquad (3\text{-}3)$$

Figure 3-3. Velocity and Historical Time for A Specific event (polynomial model)



where V(n) is velocity, n is the historical year, and $b_0, b_1, \ldots b_m$ are calculated parameters which determine the shape of the curve (Mognoni, Lafortuna, Russo & Minetti, 1982).

3.2.2 Time vs Distance

In these models the time or velocity vs distance relationship is developed separately for specific years. By comparing the different yearly relationships, future projections are made via a second-level modelling of the derived parameters.

B_1: An exponential model relating mean running velocity and running time or distance for all events in a specific year.

$$V(d) = P_3 - P_1 e^{-P_2 d} \tag{3-4}$$

where V(d) is the mean velocity for a specific year, d is running time or distance, e is the base of the natural logarithm, and P_3, P_1 and P_2 are calculated parameters reflecting the shape of the curve (Furusawa, Hill, & Parkinson, 1927; Ward-Smith, 1985).

B_2: A power function model relating running time and distance for all events in a specific year.

$$T(d) = P_1 d^{P_2} \tag{3-5}$$

where T(d) is running time, d is distance, and P_1 and P_2 are constants (Lloyd, 1966; Riegel, 1981). This model is equivalent to a linear relationship between time and distance on a logarithmic scale (e.g,; Kennelly, 1906).

B_3: A linear-log function model between velocity and time (or related distance) for all events in a specific year.

$$V = P_1 f(\log(t)) \tag{3-6}$$

where V is velocity, t is time, and f( ) is a function which is varied for different researchers. Craig (1963) used log-linear paper to draw the relationship between running time for different distances and velocities. Hill (1925) drew the running speed and distance on a log-linear paper. Craig and Hill did not give a function in their studies. Francis' function (1943) is

$$\log(d)-1.5)(V-3.2)=6.081.$$

3.2.3 Physiological Based Models

In these models the parameters or components of the model have a direct physiological representation. They are similar to the models described in the section 3.2.2 above in that each equation relates time and distance over the full range of events (100m to marathon) for a single year. Projections are made as described in the section 3.2.2 above, or by setting theoretical limits on one or more components of physiological function.

C_1: Polynomial Exponential Model (between velocity and running time for a specific year).

$$V(t) = b_1e^{-k1t} + b_2e^{-k2t} + b_3e^{-k3t} + b_4e^{-k4t} + b_5e^{-k5t} \qquad (3\text{-}7)$$

where V(t) is velocity, t is time in seconds, e the natural logarithm, k_i a rate constant, and b_i the velocity constant in yards per sec (Henry, 1954b). The five components of this model represent the energy loss, alactate O_2 debt, lactate O_2 debt, glycogen depletion factor, and fat.

C_2: Peronnet and Thibault's Model

The model (1989) developed describes the average power output P_t (w/kg) sustained over T (the natural logarithm of race duration).

$$P_t=(S/T(1 - e^{-T/k_2})) + 1/T\ {}^{T}BMR + B(1 - e^{-t/k_1})dt \tag{3-8}$$

B (w/kg) is the difference between peak and BMR, BMR (w/kg) is basal metabolic rate, k_1 is a time constant for the kinetics of aerobic metabolism at the beginning of exercise, k2 is the time constant for the kinetics of anaerobic metabolism at the beginning of exercise, S is the energy from anaerobic metabolism actually available to the runner over T, and T is the race duration (seconds).

Although other models were examined, the models A1, A2, A3 and B1 (d is defined as the historical date in this study) are the only ones that resulted in consistently plausible results. Thus the methodological explanations which follow pertain only to these four models. All the four are similar in that the model is applied separately for each event on the data set of time (or some other performance measure such as velocity, distance) and historical date. That is, there is a pair of scores for each year (e.g., 1900 to 1992). In this study the validity of these physiological models was not tested, nor were those based upon the time (seconds) -- distance (100m to 42.2 km) relationship for a given year. The latter type of models have been used by a number of researchers (e.g., Lloyd, 1966; Riegel, 1981), but were deemed inappropriate, primarily because; (1) the assumptions that the times for all events in a given year confirm to some linear or exponential functions, and that this function is constant over historical time, are questionable, and (2) the prediction requires two stages of model fitting, thus compounding errors in estimation of ultimate performance. Also, not being especially knowledgeable in physiology, I did not attempt to test any physiological models such as those proposed by Henry (1954b) or Peronnet and Thibault. However, I did compare our predictions with those of Peronnet and Thibault (1989) wherever possible.

3.3 Fitting Procedures And Model Comparisons

The first two data sets (world record and best performance) were fitted by all four models for each of the seven events. The least squares estimates of the linear function parameters were calculated using the software BMDP-P1R. The least square estimates of the parameters in linear models are unbiased and have the property of being minimum variance estimators. The least squares estimates of the nonlinear function parameters were calculated using the software BMDP-P3R, and, unlike a least square estimate in the linear model, the estimates in the nonlinear model are only asymptotically minimum unbiased estimators. However, if a good model is chosen, the asymptotic properties can be closely approximated (Ratkowsky, 1983). A1-A3 and B1 models were fitted from: 1) start year to 1992, 2) start year to 1970 for men, and 3) start year to 1980 for women, with "start year" being a function of data availability (in most cases this was 1900 for the men and 1927 for the women). The latter two data sets will allow for the comparison of predicted and actual performances, thus providing information for model validation. The second criteria for model validation is R^2 which gives the proportion of variance accounted for by the models. For linear models R^2 is computed by the P1R program and for nonlinear models a comparable R^2 can be calculated by using the residual mean square and standard deviation of the dependent variable (running time or running speed). The third criteria used to validate models is the standard error of the estimated parameters. The last criteria is face validity of the asymptote given by the models. Based on these criteria we determined a best model for each event and one best model common to all events. Predictions of performances in the year 2050 and ultimate performances were evaluated for each model, and the best model then used to evaluate and compare the rates of improvement and future predictions for females and males for each event.

3.4 New Model Development

3.4.1 Assumptions

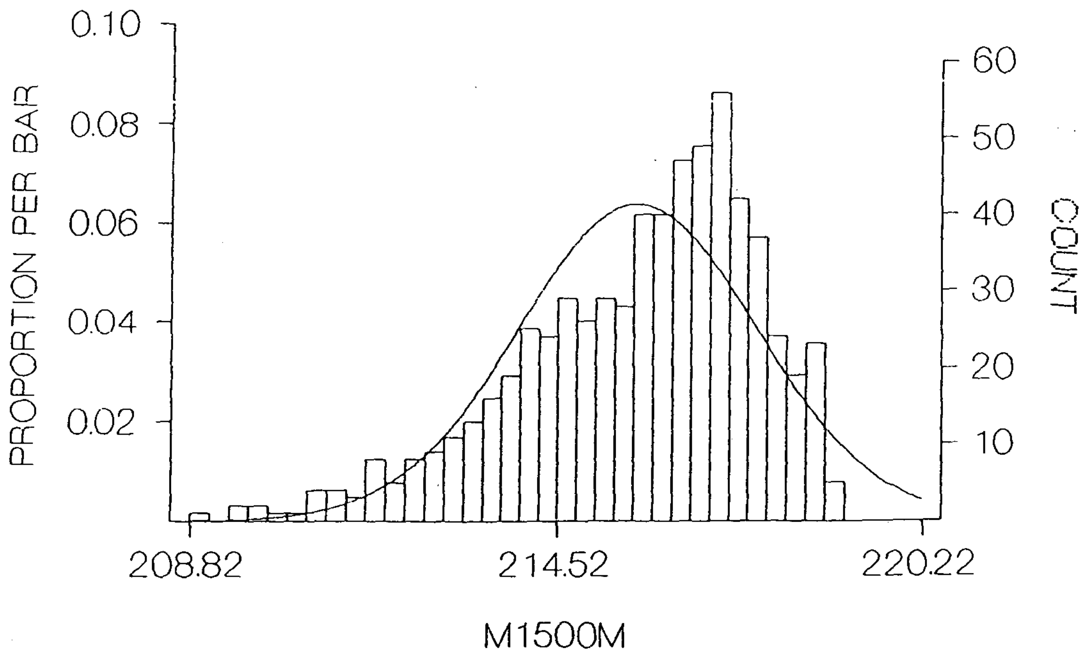
A new model called the Random Sampling Model was developed in this study. The basic assumption underlying this model is that the average ultimate performance has been reached, and the true score distribution of the top performances is now stabilized and is expected to be constant for the next 50 years or more for some events. However, world records can still be set as the theoretical distribution is asymptotic and "outliers" or "extreme values" will occur at random intervals over time. The men's 1500m was the event which appeared to meet this assumption of stability and thus it was used for model development and predictions.

3.4.2 The Model

Schutz and McBryde (1983) suggested that the ideal data base would be one which consists of the best times of the top-50 performances per year for the detailed analysis of the distributions of elite performance. As a first step Foster and Stuart's procedures (1954) were used to test the assumption of stability of the men's 1500m yearly best running performance (1980-1992). The results indicated no significant departure from randomness ($Z=0.96$, $p>.16$), thus suggesting that a valid random sampling model could be applied to this data set. A number of approaches were utilized in an attempt to determine the most appropriate distribution for this model. The best performance of the top-50 performers (running time) per year in the men's 1500m from 1980 to 1992 were therefore used as the population distribution of best performances for the men's 1500m (650 data points). The distribution test showed that the

the distribution of the top-50 performance data is clearly a negatively skewed distribution (Sk=-.78, see Figure 3-5).

Figure 3-5: Distribution of The Top-50 Performances (1980-1992)



Thus a random sampling model based on the normal distribution would not be valid. A number of transformations were attempted in order to reduce the skewness. It was found that the 6th power transformation function

$$Y=(X-180)^6/10^6 \qquad (3-9)$$

was most effective (Sk=-0.19), and the transformed distribution could be modelled as a normal distribution. Following the transformation from the negatively skewed distribution to a near normal distribution, the random number generator can be used to produce values for the simulation within this population. The value can, theoretically at least, be retransformed back to the negatively skewed distribution. The problem, however, is that an increase in extreme values within the transformed normal distribution is not transformed to a linear increase in the extreme values in the skewed distribution. In other words, the transformation to a skewed distribution from a normal distribution inflates the values of the skewed distribution at extreme values in the normal distribution.

Finally, extreme value theory was used in an attempt to develop a suitable random sampling model for the men's 1500m. Since the tail values in the top-50 performance distribution are the ordered extreme values, the basic two parameter function

$$f(t)=\lambda e^{-\lambda(ti-a)}/p \qquad (t_i<a) \qquad (3\text{-}10)$$

from extreme value theory was used to fit the tail distribution of the top-50 performances (Weissman, 1978; Boos, 1984). In this function the parameter *a* (the cut point) is the maximum tail value of the ordered extreme performances used for the fitting purpose; parameter λ is given from the function $\lambda^{-1}=\Sigma(a-t_i)/n$; p is the percentage of the ordered extreme performances chosen in the 50-top performance distribution; and t_i (i=1, 2, ...n) is each performance in the chosen group of the ordered extreme performances. The criteria to determine the best fit of an exponential distribution to the two parameter function is the equality of the mean value and standard deviation of ($a-t_1$, $a-t_2$,......,$a-t_{n-1}$) should be the same or very close.

Several different cut points were used and compared to determine the value which best defined an exponential tail of the observed data. According to this criteria, minimizing (mean-Sd), the goodness fit curve showed that 80 best ordered performances was the best group to be fitted by the basic extreme value theory function (Table 3-1, Figure 3-6). The derived parameters were:

a=213.46 Sec.; p=0.123; n=80; $^{-1}$=1.251.

based on the distribution of 1500m times (N=650, Mean=215.76, Sd=1.88, Sk=-0.78, Min=208.82, Max=218.91).

Figure 3-6 The Equality of the Mean and Sd

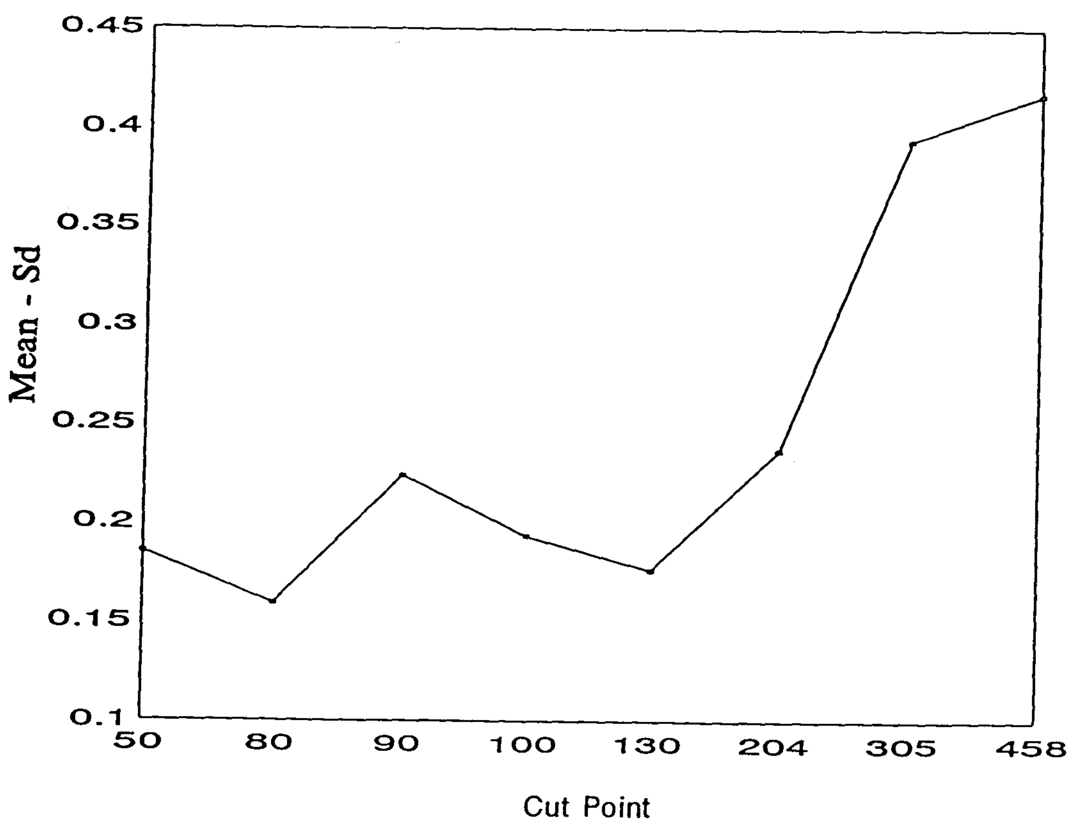


Table 3-1 The Equality of the Mean and Sd for Different Cut-points

Cut-Point	Mean	Sd	(Mean-Sd)	(Sd/Mean)
50	1.167	1.982	0.185	0.84
60	1.484	1.022	0.462	0.69
80	**1.251**	**1.092**	**0.159**	**0.873**
90	1.340	1.116	0.224	0.83
100	1.330	1.137	0.193	0.85
130	1.370	1.194	0.176	0.871
204	1.525	1.288	0.237	0.84
305	1.839	1.445	0.394	0.79
458	2.049	1.632	0.417	0.80

A FORTRAN 77 Monte Carlo simulation program was written to do the simulation based on the density of the best extreme value theory fitting model and top-50 performance data to estimate future world records and waiting times of each new record. The dependent variable of interest for each simulation was the value of each world record and the waiting time (number of years) for it to occur. Based on the empirical distribution of the top-50 best performances, top-50 performances per year for each year in the future were randomly generated (an uniform distribution subroutine and an exponential distribution subroutine are available in the UBC NORMAL generator in MTS system). The program generated a number X (0.00 to 1.00) from the uniform generator and then generated another value XX from the exponential generator whenever the uniform generated number was smaller than p (80/650=0.123). The function

$$T(i)=a-XX/\lambda \tag{3-11}$$

was used to obtain the simulating performance T(i) (a=213.46, λ^{-1}=1.251 in this study). The program compared the generated best performance of each year with the previous world record, identified any new world records and recorded them in a file. The simulation generated 50 best performances per year, and continued until a specific number of world records (three in this study) had been generated. This process was replicated 1200 times. The 1200 simulations yielded distributions for the next three world records, as well as the waiting time distributions for each of these world records. Based on these distributions, the expected world records and waiting times were then established, and confidence intervals were computed.

Using probability theory, Glick (1978) developed an analytic expectations table giving the frequencies of record breaking and waiting times from a true random record sequences. For a given time period the theoretical number of new records can be calculated from Glick's procedure (1978). The randomness of the random sampling model was checked by comparing the simulation results with analytic expectations of true random record breaking sequences using Glick's procedure.

CHAPTER IV

RESULTS AND DISCUSSION

4.1 The Data: General Trends

Performance data (world records, and best performance per year) were collected for this study (Figure 4-1 to 4-7) beginning in the 1900s for men and in the 1920s for women (most events). Table 4-1 presents the men's and women's initial performances, the recent world records and the percent improvements of the world records since the initial year for the seven events (100m, 400m, 1500m, 5000m, 10000m, Marathon and High Jump). Over the years both sexes have notably improved their performances in the seven events, but women have done so by a much greater extent in most events. For instance, the men's performances in the 100m and the 400m have improved nine percent since 1900, whereas, the women's performances in these two events have improved 18 and 27 percent, respectively, since 1921. Women's improvements are two and three times greater than that of the men's improvements in these two events. Women have improved their Marathon performance significantly by 35 percent in 30 years and men improved their Marathon performance 28 percent in 80 years. The greatest improvement was women's high-jump at 70 percent since the initial year 1900. The improvement of the women's 5000m performance was less than that of the men's, but women have a very short history of running this event (Figure 4-4). The performance data of women's 5000m are not stable and any inference from these data is not reliable at this time.

The question now is whether the next fifty years or so will see continual improvements for both men and women. As was mentioned in the early chapters some researchers predicted that humans will keep improving their performances in future and women will catch up to men in next century for most track events. However, close scrutiny of the raw data (Figure 4-1 to 4-7) revealed that there has clearly been a levelling off of performance in most events in the last 10 years for both men and women.

Table 4-1. Men and Women Performances and World Records

	Initial Data		Best Perf.		World Record		Improv.
	Date	Perform	1980	1992	Date	Perform	%
100m							
M	1900	10.80	10.02	9.91	1992	9.86	**9**
W	1921	12.80	10.93	10.79	1988	10.49	**18**
400m							
M	1900	47.80	44.60	43.50	1988	43.29	**9**
W	1921	1:05.00	48.88	48.82	1985	47.60	**27**
1500m							
M	1900	4:06.20	3:31.40	3:28.82	1992	3:28.82	**15**
W	1927	5:18.20	3:52.47	3:55.30	1980	3:52.47	**27**
5000m							
M	1900	15:02.00	13:16.40	13:00.93	1987	12:58.39	**14**
W	1969	15:53.60	15:30.60	14:44.15	1986	14:37.33	**8**
10000m							
M	1903	34:13.80	27:29.20	27:14.26	1989	27:08.06	**21**
W	1967	38:06.40	32:57.20	31.06.02	1986	30:13.74	**21**
Marathon							
M	1908	2:55:18	2:09:01	2:08:14	1988	2:06:50	**28**
W	1963	3:37:07	2:25:42	2:23:43	1985	2:21:06	**35**
High-Jump							
M	1900	1.92	2.36	2.38	1989	2.44	**27**
W	1900	1.23	1.98	2.07	1987	2.09	**70**

Figure 4-1. 100m Raw Data - Best Performance per Year

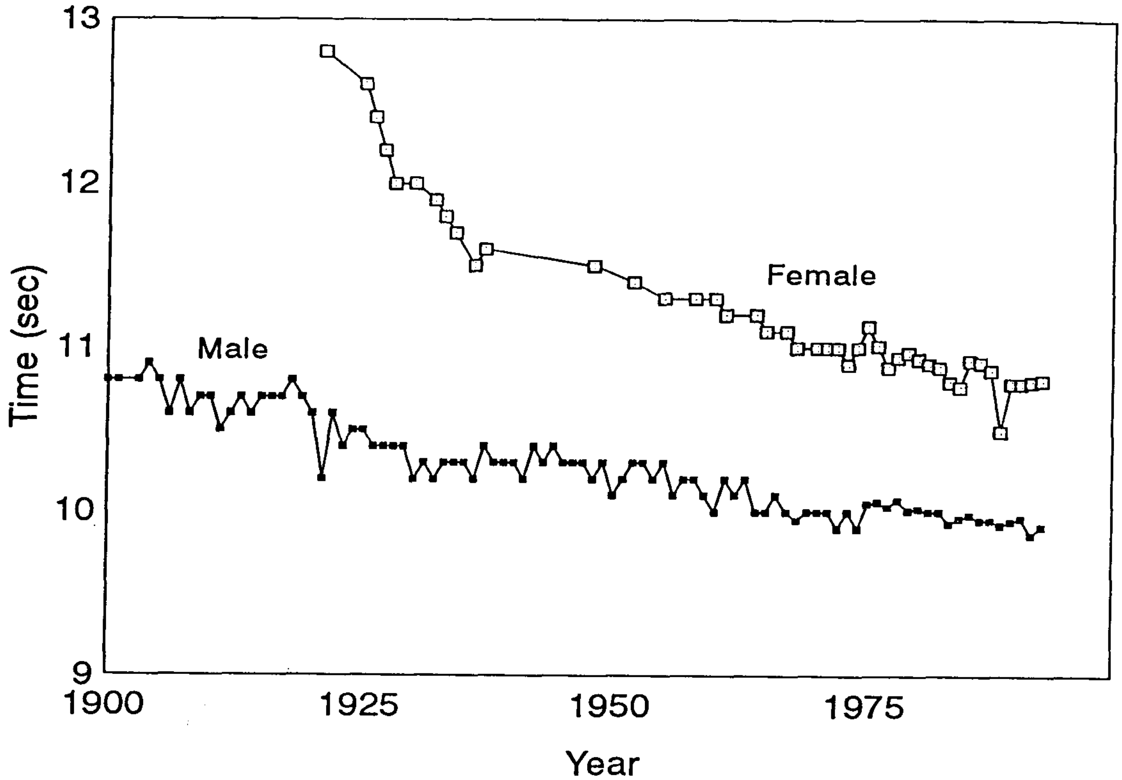


Figure 4-2. 400m Raw Data - Best Performance per Year

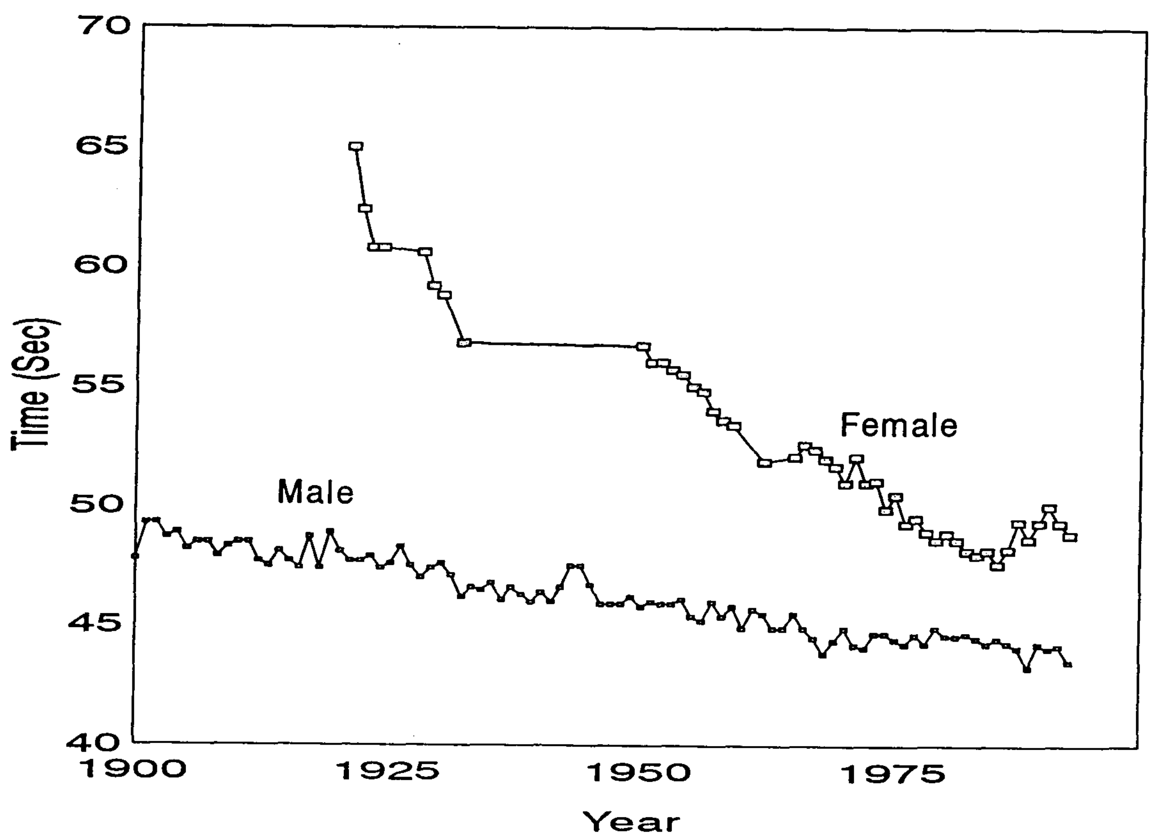


Figure 4-3. 1500m Raw Data - Best Performance per Year

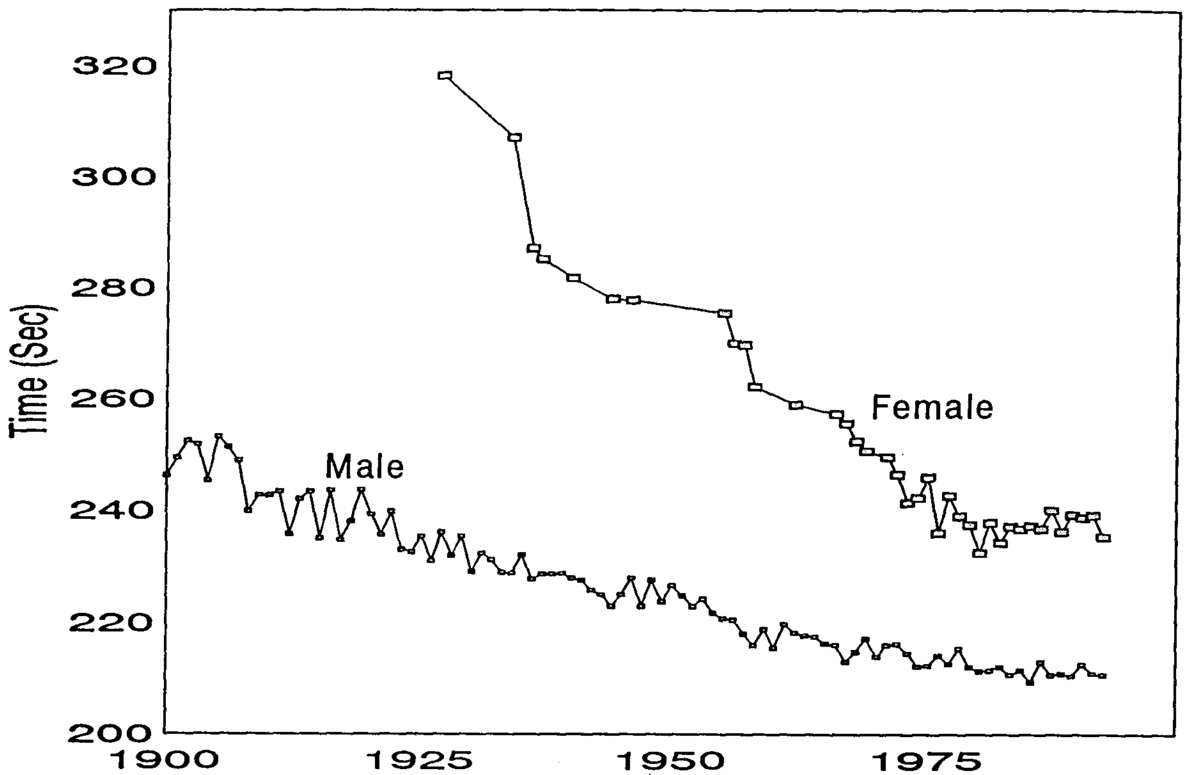


Figure 4-4. 5000m Raw Data - Best Performance per Year

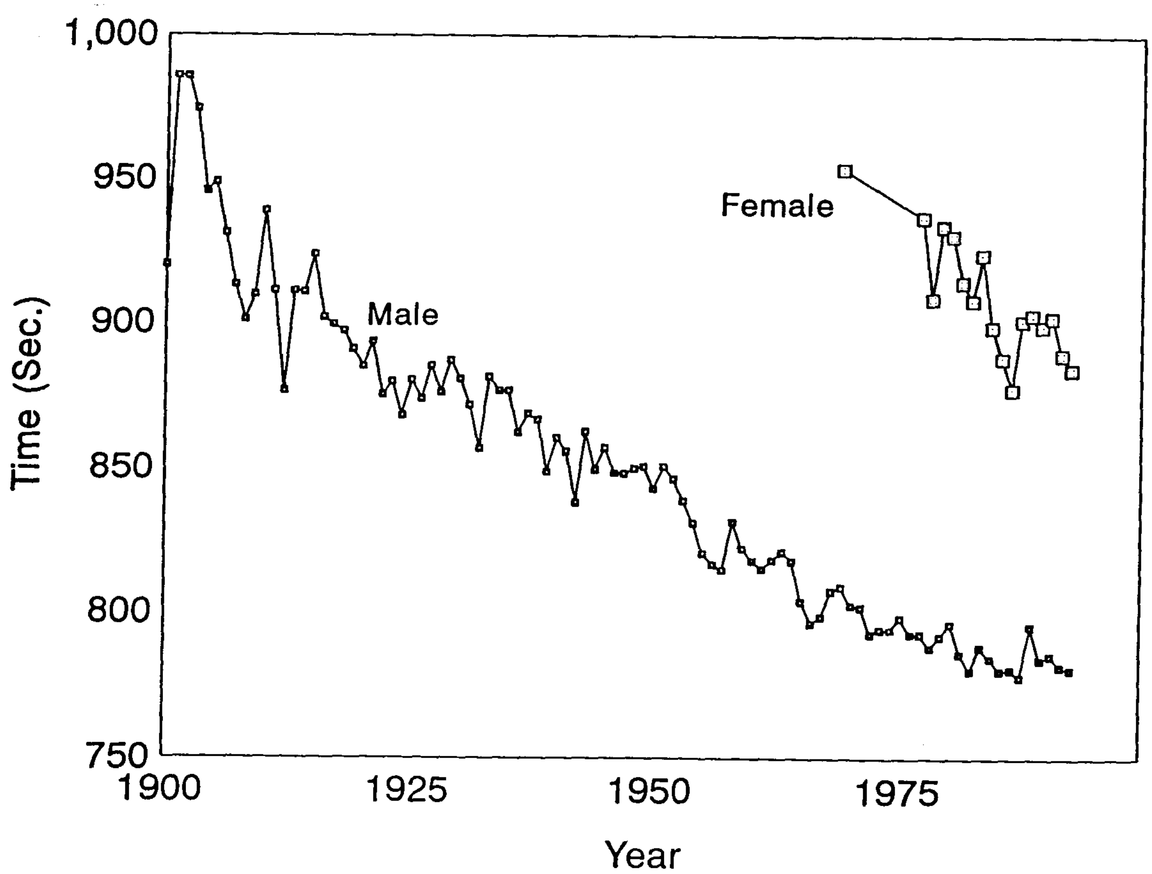


Figure 4-5. 10000m Raw Data - Best Performance per Year

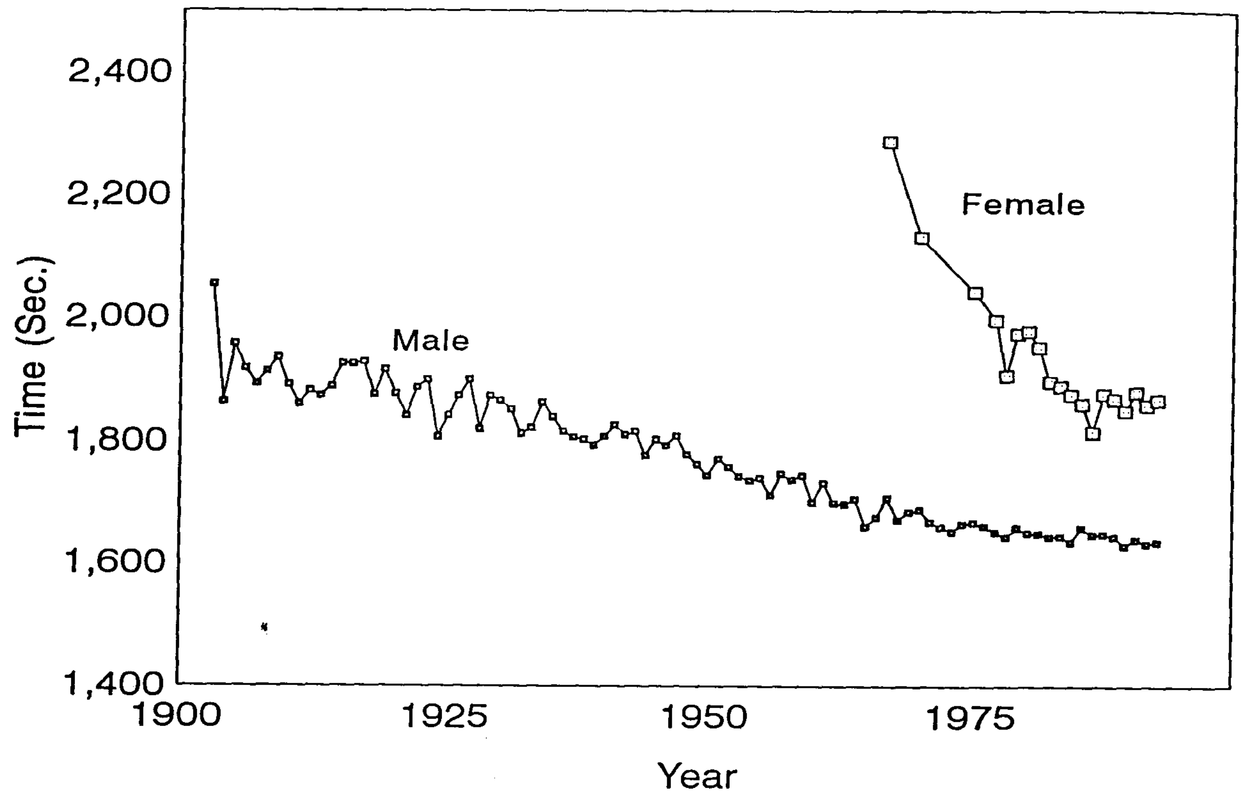


Figure 4-6. Marathon Raw Data - Best Performance per Year

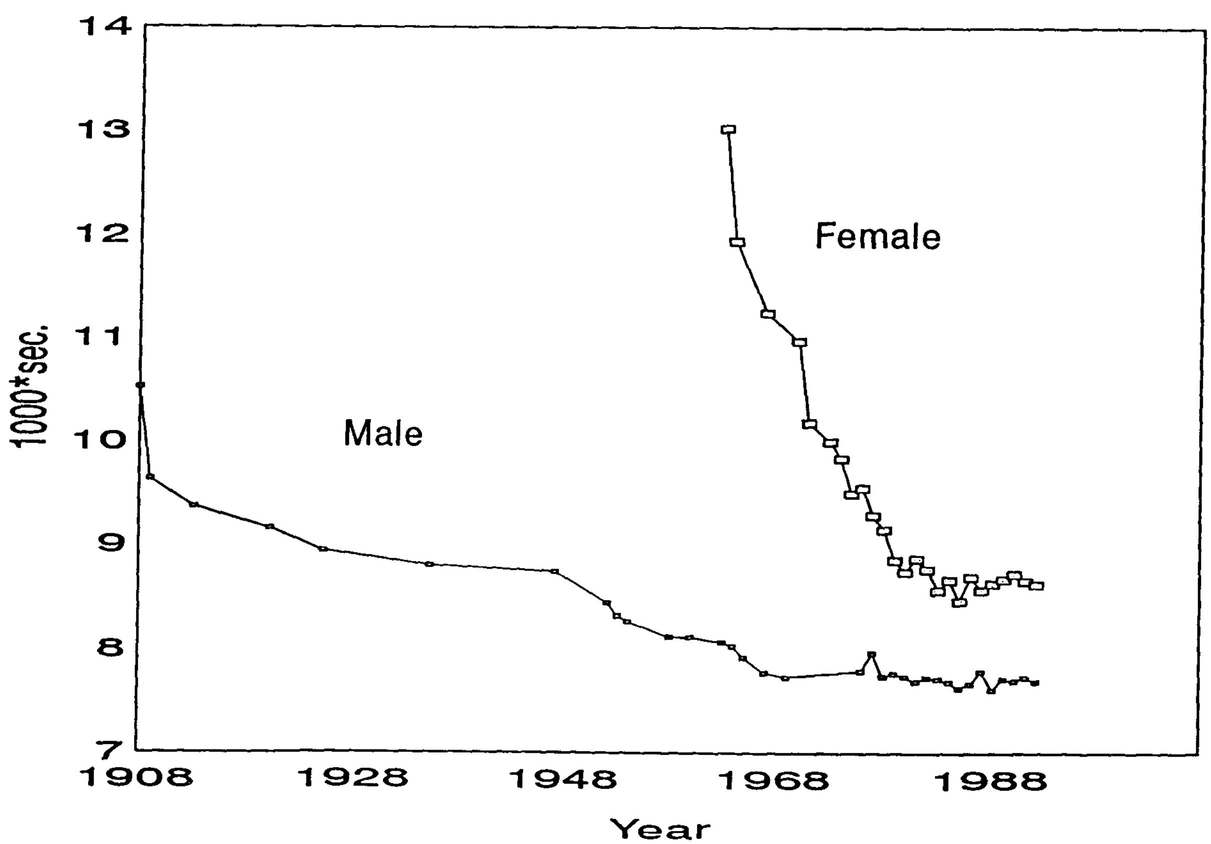
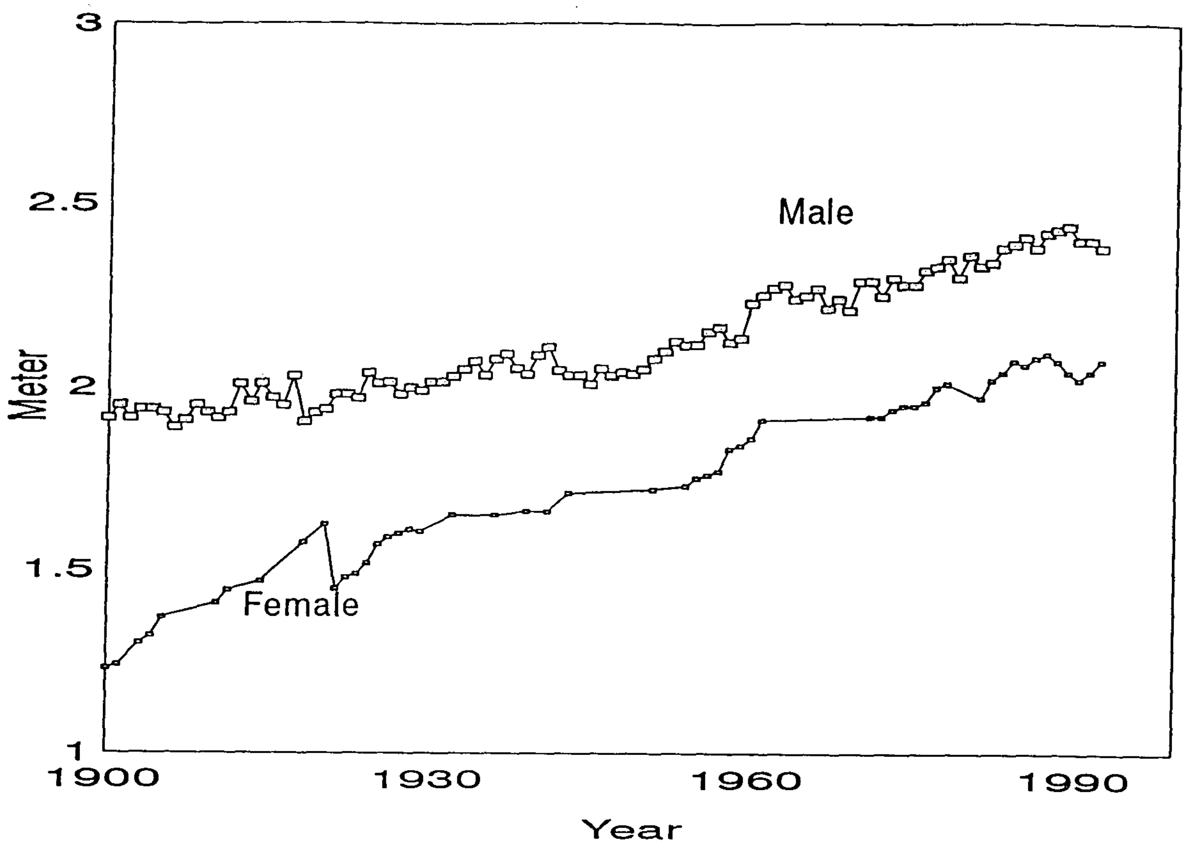


Figure 4-7. High Jump Raw Data - Best Performance per Year



The raw data graphs suggest that men and women may not continue to improve their performances at the same rate in the future, and that statistical models may be used to evaluate and predict the men's and women's future performances.

4.2 Model Fitting Results

Four deterministic models and three data sets were examined in this study. A general overview of the model fitting results will be discussed in the next section. A detailed interpretation and discussion of the results for the 1500m as well as a brief discussion of the results for the other events will be presented in the following sections.

4.2.1 Overview of the Results

Models. Table 4-2 and 4-3 show the evaluation of the model fitting for three women's events (100m, 1500m, Marathon) and five men's events (100m, 400m, 1500m, 5000m, Marathon). According to the criteria for choosing the best model, the predictions of the best model from two comparison data sets should be close to actual performances, the best model should have highest R^2 which indicates the greatest proportion of variance accounted for by the model, the smallest standard error which indicates the highest level of confidence in the accuracy of the parameter estimates, and the soundness of an asymptote.

Table 4-2. Overview of Model Fittings (Women)

		Model							
		Linear		Exponent (T)		Exponent (V)		Polynomial	
Event	Data	*Fit	*Pred	Fit	Pred	Fit	Pred	Fit	Pred
100m	BP(80)	B	C	A	A	A	A	A	A
	**BP(92)	B	C	A	A	A	A	A	A
	BP(92)	B	C	A	A	A	A	A	A
1500m	WR	A	C	A	B	A	C	A	C
	BP(80)	A	C	A	B	A	B	A	C
	BP(92)	A	C	A	A	A	A	A	A'
Marathon									
	BP(80)	A	C	A	A	A	A	A	A
	BP(92)	C	C	A	A	A	A	A	A

*Fit (fitting):
A: good $R^2>.90$
B: acceptable $.85<=R^2<=.89$
C: poor $R^2<.85$

*Pred (ultimate prediction):
A: reasonable ultimate value
B: ultimate value exist
C: no ultimate value

**BP(92): outlier out.

Table 4-3. Overview of Model Fittings (Men)

		Model							
		Linear		Exponent (T)		Exponent (V)		Polynomial	
Event	Data	*Fit	*Pred	Fit	Pred	Fit	Pred	Fit	Pred
100m	BP(70)	C	C	C	A	C	B	C	A
	BP(92)	B	C	A	A	A	A	A	A
400m	WR	A	C	A	C	A	B	A	C
	BP(70)	A	C	A	B	A	C	A	C
	BP(92)	A	C	A	B	A	B	A	B
1500m	WR	A	C	A	A	A	B	A	A
	BP(70)	A	C	A	A	A	B	A	A
	BP(92)	A	C	A	A	A	B	A	A
5000m	WR	A	C	A	B	A	B	A	B
	BP(70)	A	C	A	A	A	B	A	A
	BP(92)	A	C	A	A	A	A	A	A
Marathon									
	BP(70)	A	C	A	A	A	B	A	B
	BP(92)	B	C	A	A	A	A	A	A

*Fit (fitting):
A: good R^2>=.90
B: acceptable .85<=R^2<=.89
C: poor R^2<.85

*Pred (ultimate prediction):
A: reasonable ultimate value
B: ultimate value exist
C: no ultimate value

Generally, the nonlinear models have higher R^2s than the linear models, indicating that a greater proportion of variance is accounted for by the nonlinear models than the linear models. Among the nonlinear models, the R^2s did not show substantive differences and good prediction values for 1992 were provided by all three of these models. However, as expected, the predictions of the polynomial model are not acceptable for ultimate limits.

The exponential model for speed and the exponential model for running time have similar accuracy in the prediction of known records, but the exponential model for speed usually exhibits liberal ultimate predictions. The results also show that the linear models always have less accuracy for predicting 1992 performances with the estimates being consistently over estimated (see detailed discussion by event).

Four women's events (400m, 5000m, 10000m, and high jump) and two men's events (10000m, and high jump) were not included in the evaluation table. The best performance data of women's 400m were not stable in the last decade. The large deviations between predictions and actual values in the last 10 years made the model predictions and the asymptote unrealistic. Because of women's very late participation in the 5000m and 10000m events not enough data are available in these events to obtain any reasonable model fits. Since very little leveling off has occurred in the women's High jump, asymptotic models did not converge to satisfactory solutions. The nonlinear model did not yield a reasonable fit for the men's 10000m and High jump, suggesting that the men's 10000m and High jump is still on a linear trend and that the long term predictions are not reasonable for these events at this time.

Data Sets. In terms of the data set comparisons, using the best performance data $BP_{(92)}$ it was possible to obtain parameter estimates with smaller standard errors as compared to using world record and best performance comparison data ($BP_{(70)}$ for men and $BP_{(80)}$ for women). In some events (i.e., men's 100m, 400m, 1500m, and 5000m) the predictions of comparison data are close to that of $BP_{(92)}$ data in the exponential model fitting. This indicates that a plateauing component emerged before 1970 and confirms the validity of the exponential model. The large difference between the predictions of the comparison data and the predictions of the $BP_{(92)}$ data in the best fitting (exponential) model (i,e., women's 1500m and Marathon) indicates that a plateauing of the performances only showed in the last decade for these events. This is discussed in more detail under each event.

One could conclude that the best performance $BP_{(92)}$ data are the most appropriate data to be used to evaluate and predict the future world records and ultimate performances in track and field for the deterministic models. The world record data (WR), although often used in the literature, can make parameter estimation much more unstable than the best performance data, and discontinuing world records with historical date can make the model fitting procedure unreliable.

Best Model/Data. The models developed for the prediction and evaluation purposes are actually more appropriately referred to as a "model/data set" combination, rather than a "model", since a specific set of observed data in conjunction with a specified model determines the model's behavior (Ratkowsky, 1983). This study showed that among the deterministic models chosen in the study, the exponential model relating running time and historical year with the best performance data is the best choice to evaluate and predict the world record and the ultimate performance in the future. The predicted ultimate performance for each event is shown in Table 4-4. According to the ultimate values given by the best fitting model, both men and women have the capacity to improve their performances in the near future. The details are discussed under each event.

Comparison Between Women's and Men's Performance. In terms of women's and men's performance comparison the exponential model shows that women were improving faster than men before the 1980s. However, a nonlinear component in most women's events began to emerge around 1970. Figures 4-8 to 4-14 show the best fitting model comparisons between men's and women's performances. The predictions from the best model (exponential model) indicate that the difference

Figure 4-8. Female and Male 100m Best Model Comparison

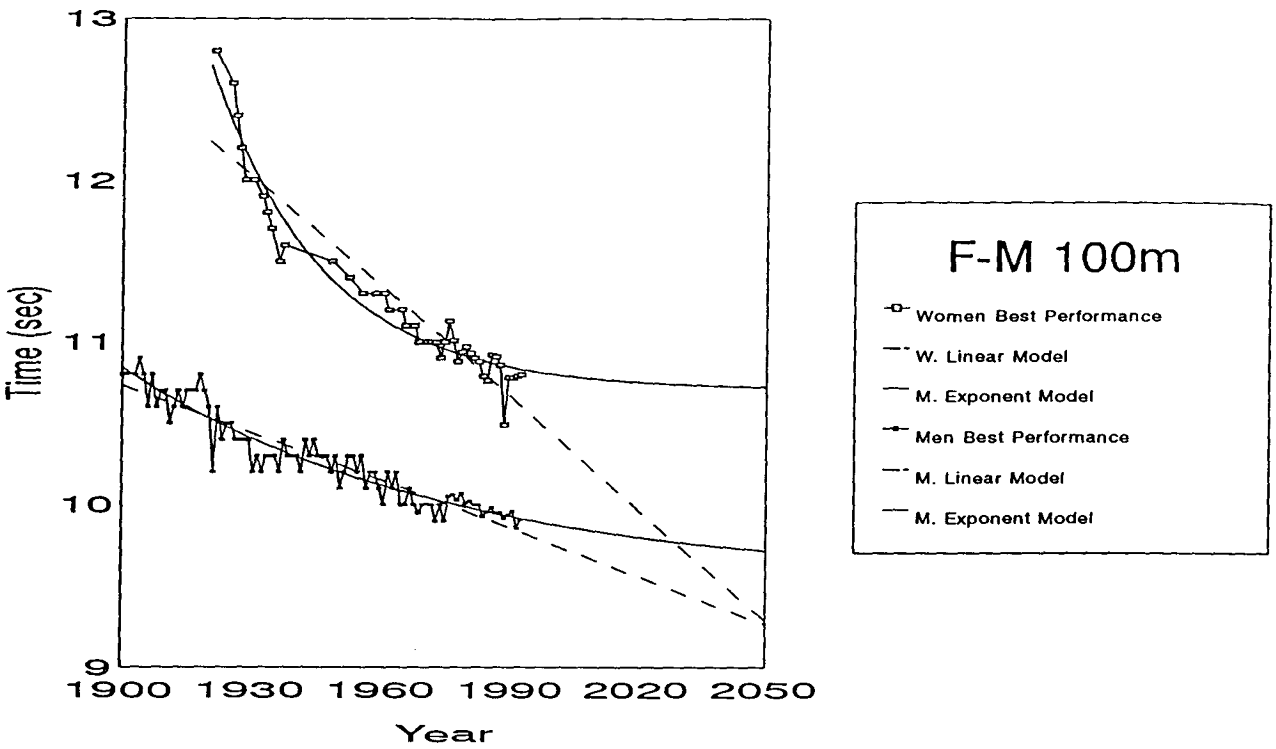


Figure 4-9. Female and Male 400m Best Model Comparison

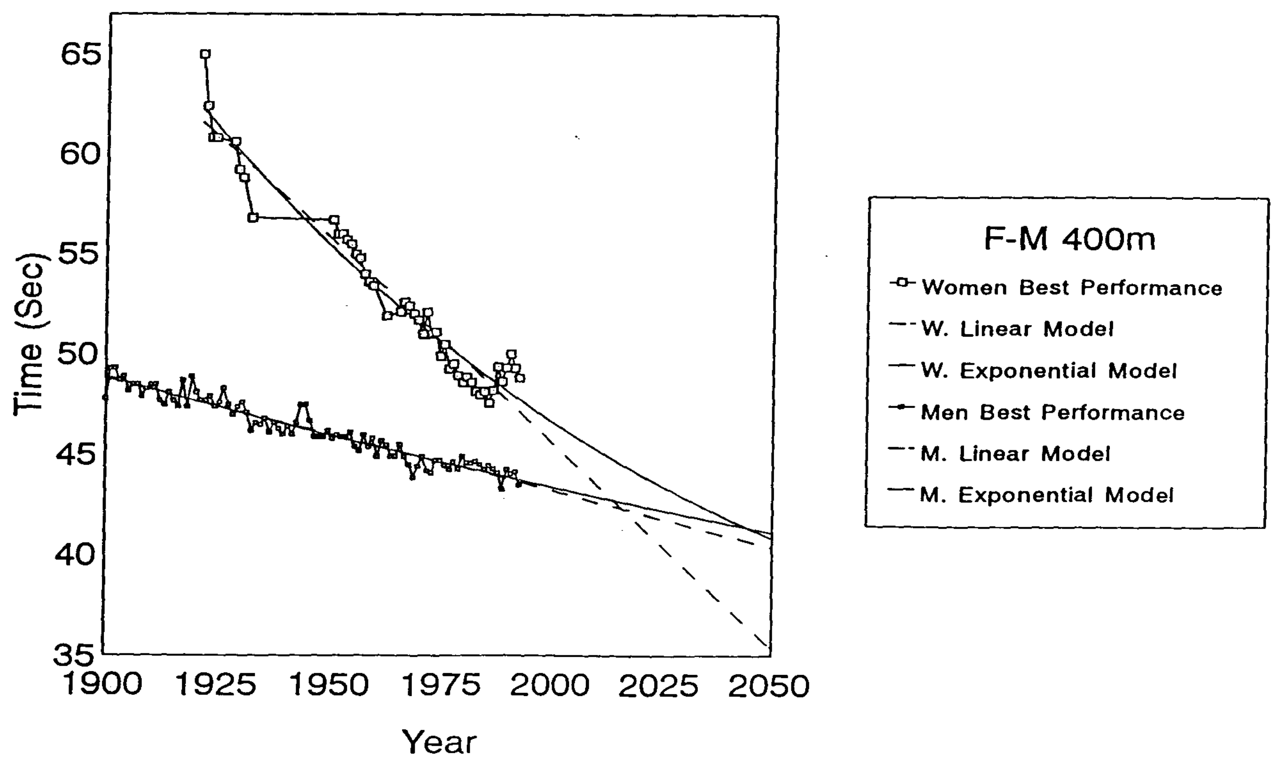


Figure 4-10. Female and Male 1500m Best Model Comparison

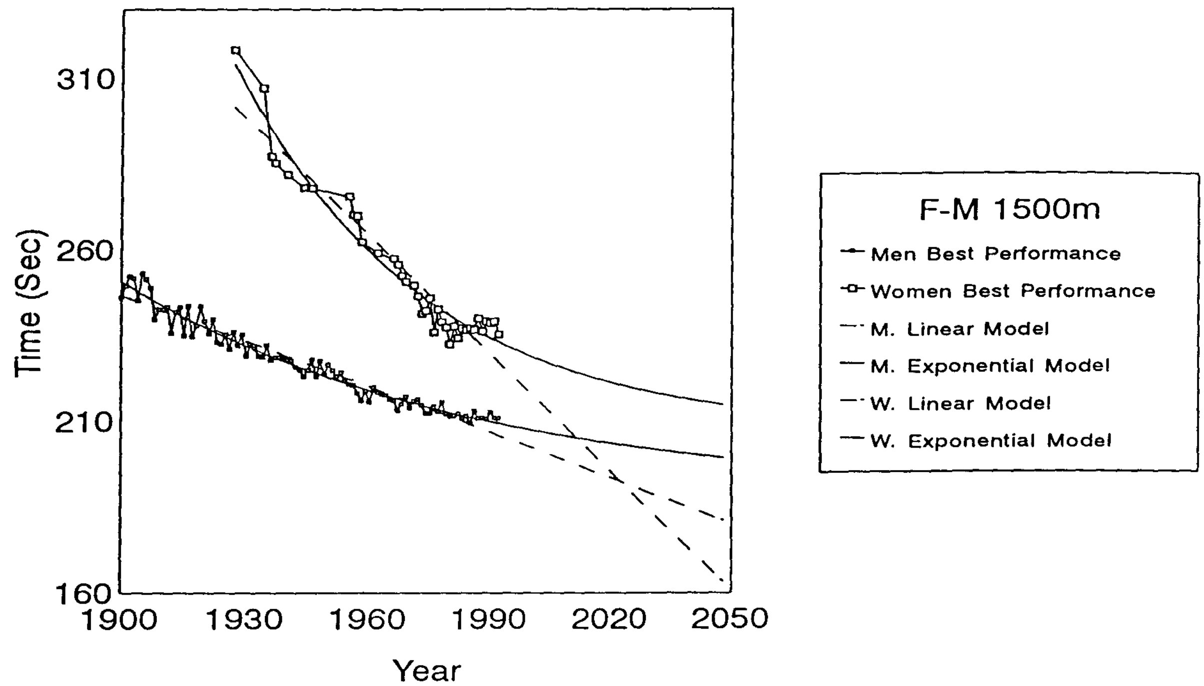


Figure 4-11. Female and Male 5000m Best Model Comparison

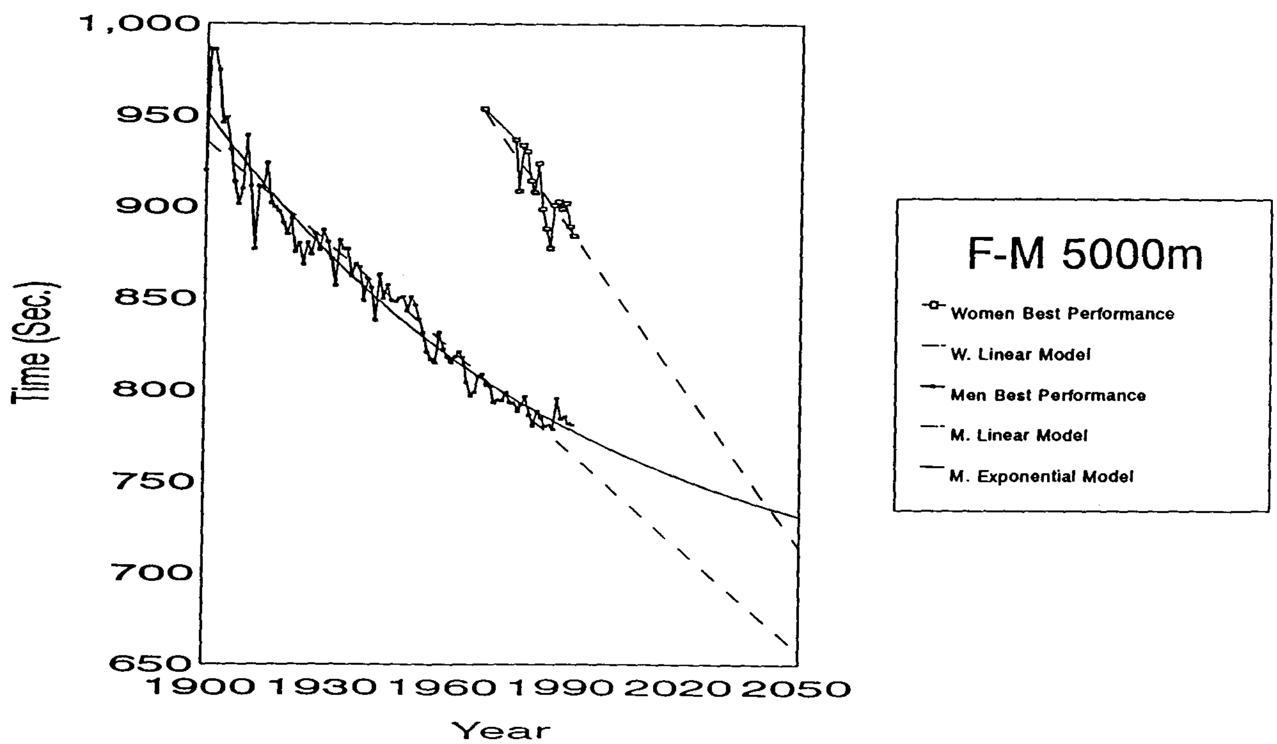


Figure 4-12. Female and Male 10000m Best Model Comparison

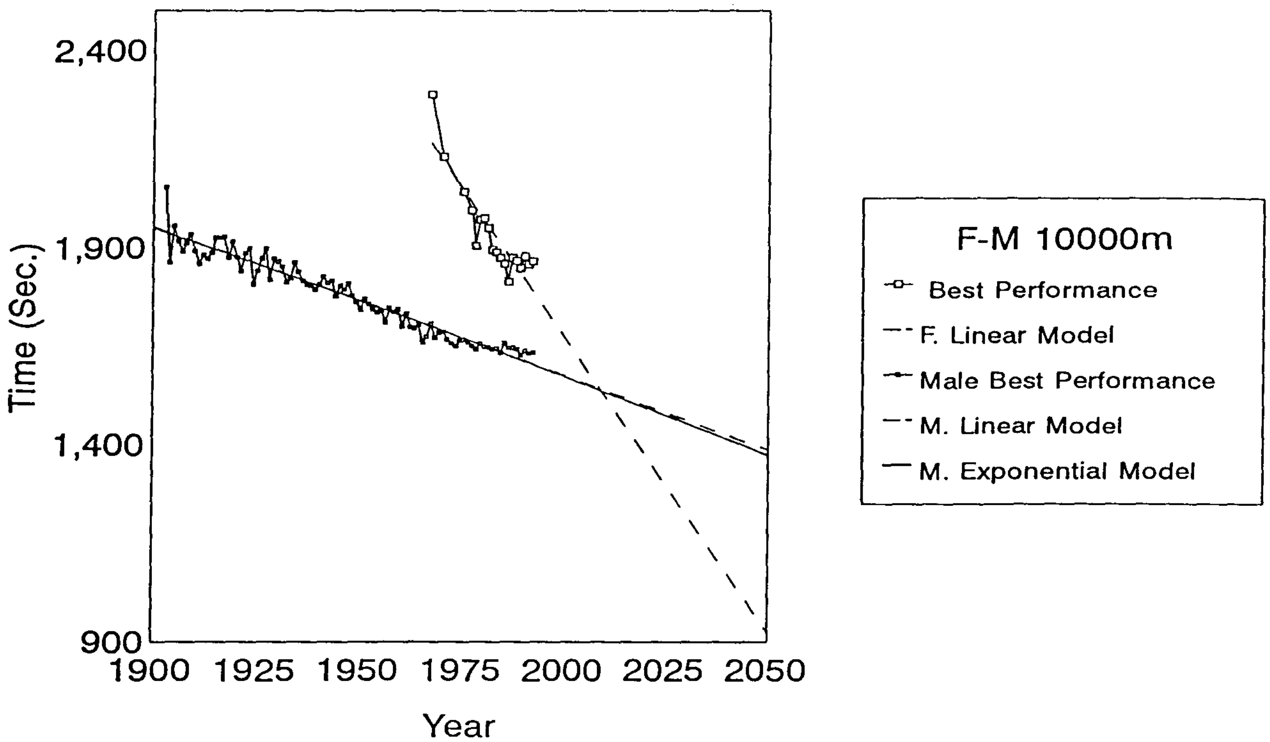


Figure 4-13. Female and Male Marathon Best Model Comparison

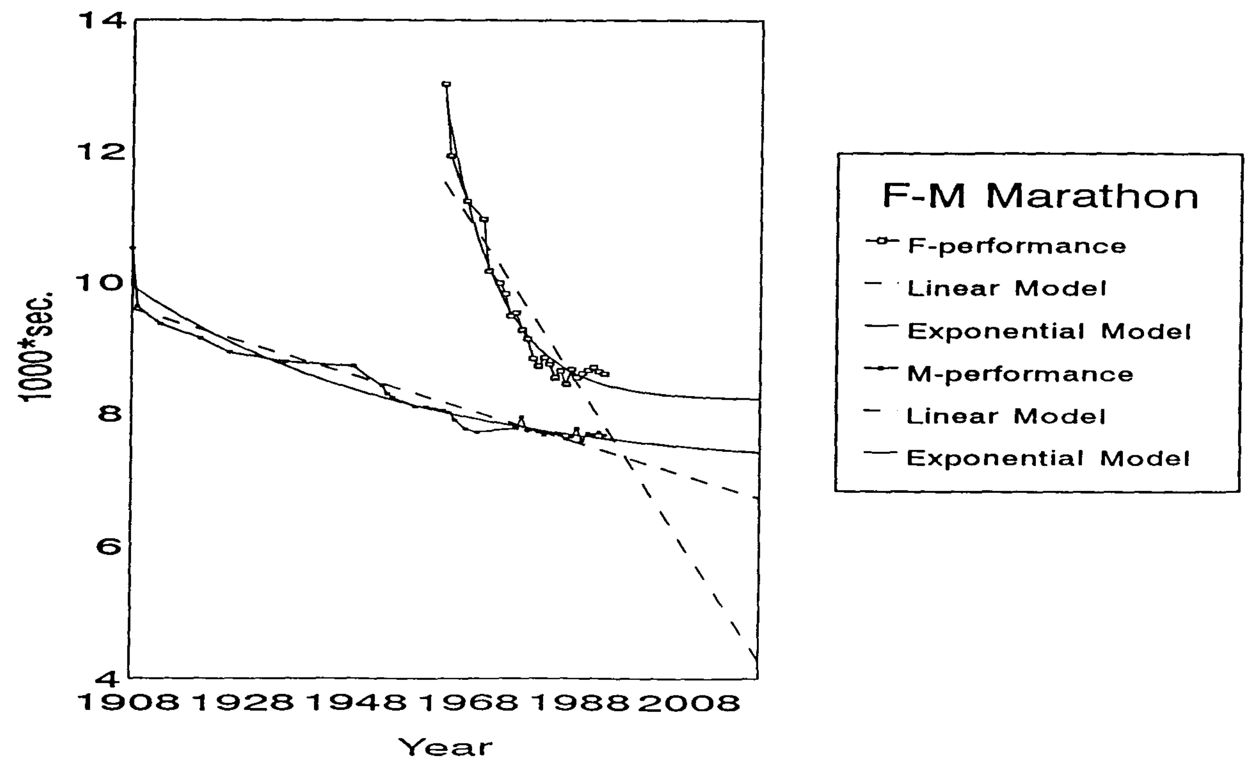
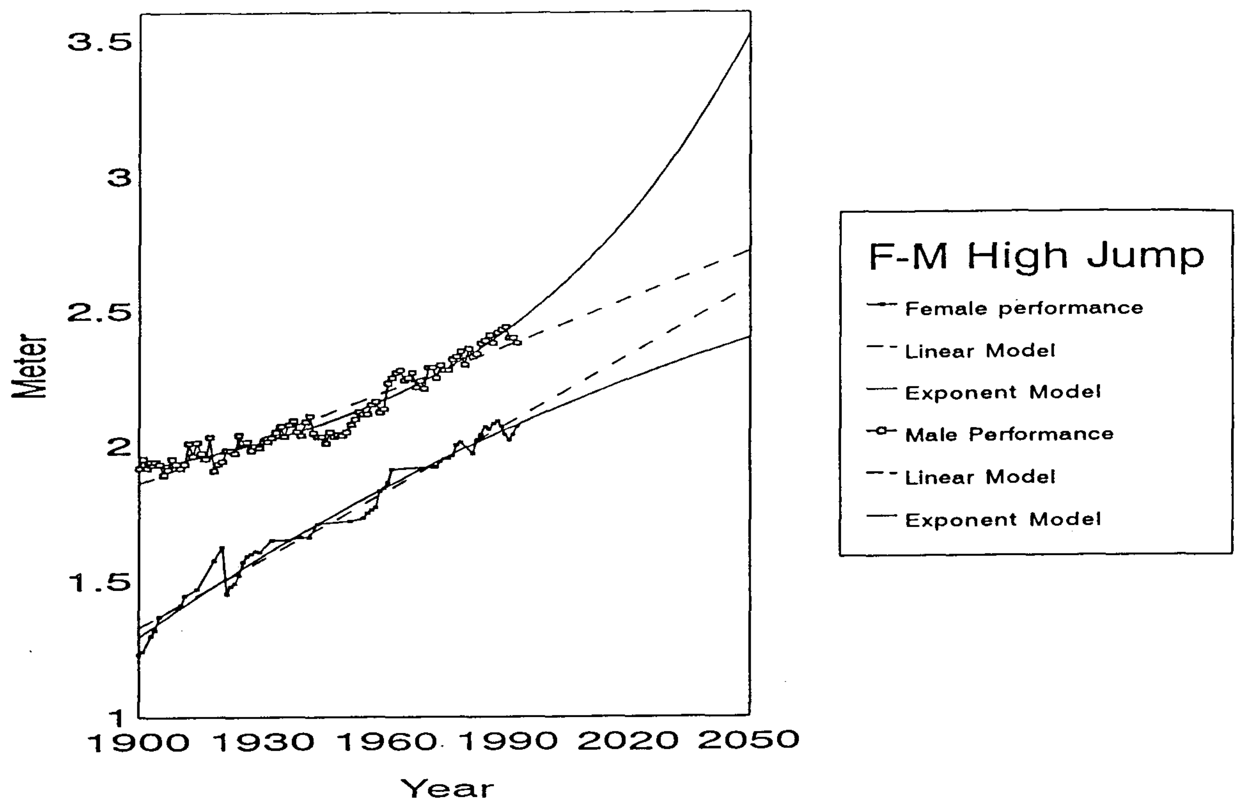


Figure 4-14. Female and Male High Jump Best Model Comparison



between women's and men's performance in the 100m, 1500m, and Marathon will keep decreasing in the near future. However, the best fitting model suggests that women are not predicted to catch up to men in these events.

Table 4-4. Mathematical Projections of Seven Events

Event		Record (1993)	Prediction (2050)	Ultimate Value	SE.
100m (sec)	Men	9.86	9.72	9.56	0.16
	Women	10.49	10.72	10.71	0.08
1500m (min:sec)	Men	3:29.46	3:19.11	3:09.04	5.49
	Women	3:52.47	3:34.42	3:26.96	11.09
Marathon (hr:m:s)	Men	2:06:50.00	2:01:44.37	1:59:25.21	240.1
	Women	2:21:06.00	2:17:12.81	2:17:12.26	143.1
400m (s)	Men	43.29	41.15	26.86	17.46
5000m (m:s)	Men	12:58.39	12:11.42	11:20.06	33.29
H-Jump (meter)	Women	2.09	2.40	3.24	0.54

Because of insufficient data in the women's 5000m and 10000m valid comparisons between women's and men's performances are not possible for these events.

The only equality between the women's and men's performances provided by the best fitting model was for the 400m where equality between the women's and men's performance was predicted by the year 2050. However, women's performance data were not stable in the last decade, and any long term predictions may not be accurate at this time. Before a good prediction can be made, more years of data are needed.

This study indicates that the difference between men's and women's track and field performances will continue to decrease in the future. However, the exponential model shows that the women's performances are not predicted to catch up to the men at any future time (Figure 4-8 to 4-14). As Schutz has stated; "To expect women to catch men on the track may simply be an unrealistic expectation and could be dangerous. With this unrealistic expectation comes the perception that if women don't perform as well as men, they are failures. That's obviously not the case."(Hefter,1993). Detailed comparisons are discussed under each event.

4.2.2 The 1500m: A Detailed Analysis

Table 4-5 and Table 4-6 present the results of the 1500m based on four models applied to each of the following data sets: 1) WR data - the world records were selected from 1900 (men) and 1927 (women) to the present world record year (1985 for men, and 1980 for women); 2) $BP_{(70)}$ data: the best performance each year was chosen up until 1970 for men to compare with W-R data and $BP_{(92)}$ data; 3) $BP_{(80)}$ data: the best performance each year was chosen up until 1980 for women to compare with W-R data and $BP_{(92)}$ data; and 4) $BP_{(92)}$ data: the best performance each year was chosen up until 1992 for both men and women (see 3.2.1 for functions).

Table 4-5. Model Comparison (Men's 1500m)

	Linear Model	Exponent Model	Exponent Model (velocity)	Polynomial Model (velocity)
R^2 WR data	0.97	0.98	0.98	0.98
BP data (92)	0.94	0.96	0.96	0.96
BP data (70)	0.93	0.94	0.94	0.94
$P_1 \pm$ SE WR data	-0.41 0.01	72.88 14.67	-3.63 1.72	0.0142 0.0012
BP data (92)	-0.44 0.01	61.80 5.00	-2.25 0.31	0.0177 0.0010
BP data (70)	-0.50 0.02	70.36 16.49	-3.38 1.86	0.0164 0.0017
$P_2 \pm$ SE WR data	-	-0.0078 0.0027	-0.0040 0.0022	-0.000024 0.000014
BP data (92)	-	-0.0121 0.0017	-0.0082 0.0017	-0.000053 0.000011
BP data (70)	-	-0.0100 0.0034	-0.0049 0.0032	-0.000033 0.000023
$P_3 \pm$ SE WR data	-	172.17 15.25	9.75 1.73	6.12 0.0242
BP data (92)	-	189.04 5.49	8.23 0.33	5.98 0.0202
BP data (70)	-	180.12 17.14	9.37 1.88	5.99 0.0250
Prediction				
(1992) WR	204.99	209.57	207.44	207.45
BP (92)	206.02	209.34	209.27	209.41
BP (70)	202.10	208.14	207.56	207.66
(2050) WR	181.16	194.62	193.58	194.36
BP (92)	180.37	199.11	198.18	201.42
BP (70)	172.95	195.80	178.80	194.53
Ultimate Perf.				
WR		172.17	153.83	182.21
BP (92)		189.04	182.22	201.00
BP (70)		180.12	160.05	186.81

*WR (1985) 209.46 *BP (1992) 210.75

Table 4-6. Model Comparison (Women's 1500m)

	Linear Model	Exponent Model	Exponent Model (velocity)	Polynomial Model (velocity)
R^2 WR data	0.94	0.94	0.95	0.95
BP data (92)	0.92	0.95	0.94	0.95
BP data (80)	0.95	0.95	0.95	0.96
$P_1 \pm$ SE WR data	-1.37 0.0784	325.70 348.66	1.09 1.1621	0.0131 0.0124
BP data (92)	-1.14 0.0561	192.36 11.01	-4.15 0.3713	0.0469 0.0071
BP data (80)	-1.35 0.0623	276.07 160.83	-101.83 0.0000	0.0170 0.0104
$P_2 \pm$ SE WR data	-	-0.0057 0.0088	0.013 0.0080	0.00014 0.0001
BP data (92)	-	-0.0217 0.0045	-0.013 0.0047	-0.00018 0.00006
BP data (80)	-	-0.0073 0.0070	-0.00003 0.00001	0.00010 0.00009
$P_3 \pm$ SE WR data	-	30.39 367.76	3.36 1.3679	4.43 0.3176
BP data (92)	-	206.96 11.09	7.66 0.6565	3.62 0.2096
BP data (80)	-	82.94 177.46	105.88 0.0777	4.35 0.2721
Prediction				
(1992) WR	218.05	222.52	216.83	234.88
BP (92)	227.42	233.16	233.04	233.60
BP (80)	219.36	237.03	237.46	221.72
(2050) WR	138.32	168.14	138.11	156.65
BP (92)	161.11	214.42	211.46	153.49
BP (80)	141.27	175.45	181.74	163.58
Ultimate Perf.				
WR	-	30.39	-	-
BP (92)	-	206.96	195.83	153.49
BP (70)	-	82.94	14.17	-

*WR (1980) 232.47 *BP (1992) 235.30

Men's Performance. Compared with the other two data sets the smallest standard error of the parameter estimate was exhibited by the $BP_{(92)}$ data within any model. For instance, the 99 percent confidence interval of P_1 is 61.8 $\pm$ 15.00 using the $BP_{(92)}$ data, 70.36 $\pm$ 49.47 using the $BP_{(70)}$ data, and 72.88 $\pm$ 44.01 using the WR data in the exponential model. The $BP_{(92)}$ data shows the similar best confidence intervals for P_2, P_3 as well (Table 4-5). The results indicate that no matter which model was used the $BP_{(92)}$ data is always the best choice for the fitting procedure based on the smallest standard error of the parameter estimates.

The raw data showed that a clear plateauing component emerged in the 1970s (see Figure 4-3), and the deviation between predicted values and actual performances have become progressively larger with linear models since 1980 (Figure 4-10). Using the best fitting data set ($BP_{(92)}$ data) the nonlinear models provide a better fit than the linear model. The variance accounted for by the linear model is 94 percent, whereas 96 percent is accounted for by the nonlinear models. A nonlinear model is thus more suitable for fitting the men's 1500m performance and it is suggested that ultimate performances can be predicted by such nonlinear models. The same goodness of fit has been shown with all three nonlinear models (R^2=.96) using $BP_{(92)}$ data, however, the polynomial model is clearly inappropriate as it will not accurately fit asymptotic data. For the polynomial model the ultimate performance is reached by the year 2077, and after that the predictions are not acceptable because of a predicted decreased velocity in subsequent performance. This problem arises because of the squared term in the polynomial function which is preceded by the negative sign. Compared with the exponential model for running time, the exponential model with velocity has the same validity and goodness of fit but forecasts liberal values and ultimate performance prediction (see Table 4-5). In comparison with other models, the exponential model for running time with the $BP_{(92)}$ data seems to be the best model for data fitting and prediction purposes in the men's 1500m. According to the best fitting model, the men's ultimate performance in the 1500m is 3:09.04.

Last January, Track and Field News published a men's 1500m new world record. Comparing the new world record 3:28.82 with 3:29.34 $\pm$ 2.46 (the prediction of the best performance at 1992 from the exponential model with running time) we can see that this new record falls within the 95% confidence interval of the predicted value. It also supports the validity of the best fitting model.

Women's Performance. Women's track and field athletics began as an international sport in the 1920s and the initial year of data collection for the women's 1500m was 1927. Because the data did not show the clear plateauing of performances before the year 1970 (Figure 4-3), nonlinear model could not be used to get reasonable predictions with the best performance data leading up to 1970. However, a levelling off component clearly emerged in 1970s, and the nonlinear models may now be used to give a superior fit. The best performance comparison data from 1927 to 1980 (instead of to 1970) are used to get enough eligible data points for the nonlinear model fitting and the validity comparison of the nonlinear models. Since the world records are discontinuous and the current world record has stood for more than 10 years in women's 1500m, the attraction of the linear model is that it does fit the world record data quite well (R^2=.94). However, the linear model with the best performance data up to 1992 ($BP_{(92)}$) did not yield a good fit in comparison to the exponential model (see Table 4-6), due to the levelling off component which emerged in last decade. Similar to the men's results, the best performance data up to 1992 ($BP_{(92)}$) provides the smallest standard error of the parameter estimates. For example, the parameter estimate P_1 of the exponential model is 192.36 with standard error 11.01 using $BP_{(92)}$ data, 325.70 with standard error 348.66 using world record data, 276.07 with standard error 160.83 using $BP_{(80)}$. Using world record data and best performance data up to 1980 ($BP_{(80)}$) reasonable ultimate performance predictions could not be obtained from any model. The reasonable ultimate performance prediction (206.96 sec.) can only be achieved by the exponential model with the $BP_{(92)}$ data (see Table 4-6). The greatest R^2, the smallest standard error of the parameter estimates, and the reasonable prediction value for the women's 1500m make the exponential model for running time with the $BP_{(92)}$ data the best choice for the fitting and predicting purpose in the women's 1500m.

According to the ultimate values given by the exponential model both women and men have some room to improve their performances. The difference between men's and women's performances for the 1500m event will be smaller in future. However, the exponential model showed that the women's ultimate performance is 3:26.96 and men's ultimate performance is 3:09.04. The best fitting model suggests that women therefore are not predicted to catch the men in the 1500m event, although the 17.92 second difference between women's and men's ultimate performance predictions is somewhat smaller than the current difference of 23.65 seconds in their world records.

4.2.3 Overview of Other Events

100m. Since the number of the world record data points are too few to achieve acceptable model fits, only best performance data were used for the model comparisons in this event. The results show that the exponential model with the running time $BP_{(92)}$ is the best model for this event (Figure 4-8, Appendix A-1, and Appendix A-2). The 100m ultimate performance is predicted by the best fitting model at 9.53 seconds for men and 10.71 seconds for women.

A anomaly exists with the world record (10.49 seconds) in the women's 100m, where the current world record is 0.22

seconds lower than the ultimate performance value predicted by the best fitting model. Since the present world record can be considered an outlier and a levelling-off trend of performance has clearly been shown, the best performance data without the present world record were fitted using the four deterministic models for the validity comparisons. The results indicate that the exponential model with the best performance data has good stability because inclusion or exclusion of the present world record did not affect the stability of the parameter estimates or the accuracy of the predictions. The validity comparison showed that the exponential model with the best performance data ($BP_{(92)}$) provides the best fit for the women's 100m even though there is an anomaly in the data and the reason for this anomaly in this event is not clear. Obviously, this model is not valid for the women's 100m data at this time.

400m. All nonlinear models yield approximately the same fit for the men's performance in this event (Appendix A-3). The asymptote predicted by the exponential model suggests that the plateauing component just emerged in recent years because the ultimate performance prediction seems unreasonable for this event at the present time (26.86 from the exponential model) and the linear model also has a good fit (R^2=0.96). It can be seen (Figure 4-9, Appendix A-3) that the rate of improvement, although linear, is very slow. The projected world record of 41.15 seconds for the year 2050 may be valid, given the current record of 43.30 seconds. The projected "ultimate" value would not be reached for approximately 3000 years. Some years should be allowed to elapse for long term world record and ultimate performance predictions for the men's 400m event.

The women's 400m data are not stable in last 10 years, and the deviation between the predicted and actual performances are large. Although the equality between the women's and men's performance is predicted in the 2040s by the exponential model, which is the only equality between sexes shown by the exponential model in this study, the result is not reliable due to the reason mentioned above.

5000m and 10000m. The exponential model with running time $BP_{(92)}$ gives reasonable predictions for the men's 5000m (see Figure 4-11 and Appendix A-4). The men's ultimate performance in this event is predicted at 11:20.06. According to the best fitting model, men still have two more minutes by which to improve their performance in this event.

A linear model still exhibits a good fit for the men's 10000m data whereas the nonlinear model can not properly fit the event at this time. Close scrutiny of the raw data (Figure 4-12) reveals that there has not been a clear levelling off of performance within the last six to seven years. Again, some years must elapse in order to obtain the proper nonlinear model fit.

Since very few female athletes ran these two events before the 1960s, the available data are insufficient for a valid model fitting. The comparisons between the women's performance and the men's performance for these two events are not possible at this time.

Marathon. Marathon is a popular event run for men and women. However, women's participation in this event did not start until the 1960s. The data were collected since 1908 for men and 1963 for women. Similar to the 1500m, according to the criteria for selecting the best model, the exponential model with running time $BP_{(92)}$ is the best choice for both men's and women's data fitting and predicting purposes (see Figure 4-13, Appendix A-5, and Appendix A-6). Women had a very high improvement rate and a linear trend was exhibited before the 1980s. Some researchers predicted that women would catch up to the men in 1990 (e.g., Dyer, 1982), however, a clear levelling off of women's performance emerged in the 1980s. The linear model is therefore not suitable to fit women's performance data in 1992 and the nonlinear models exhibit the better fits (Figure 4-13). Comparing the goodness of fit among the models we found that in the women's Marathon the R^2 for the linear model is only 0.80 while the exponential model with running time $BP_{(92)}$ provides a good fit (R^2=.97). The nonlinear model showed a better fit (R^2=0.94 from exponential model) than the linear model (R^2=0.89) for the men's performance as well. Using the best fitting model from this study we predict that the ultimate performance for men in this event is 1:59:25.21 and 2:17:12.26 for women, which suggests a 17 minutes difference is still expected between men and women's ultimate performance. The results show that the ultimate difference is larger than the current difference (15 minutes), which indicate that men have more room to improve their performance than women in future since the women's performances have been close to their ultimate performance. Some physiologists predicted that because of physiological differences, women's performances would soon approach, or even exceed, men's performance in the longer events which require greater endurance than the shorter events. Results from this study suggest that while women's times do approach men's since the 1960s in Marathon it does not seem reasonable to expect women to equal or surpass them in this long distance event.

High Jump. The men's best performance in the high jump is still on a linear trend. The nonlinear model can not exhibit a reasonable fit at this time (Figure 4-14). In the near future performances may be predicted by the linear model but long term performances and the ultimate performance can not be predicted by the linear model.

With best performance data $BP_{(92)}$ the linear model and the exponential model both show a good fit for the women's high jump. The ultimate performance prediction from the exponential model in the women's high jump is 3.24 meters.

Since the nonlinear component has just emerged in last few years and the linear model still provides a good fit to these data, the ultimate prediction is very large and may be quite unrealistic. Some more years should be allowed to elapse before the reasonable ultimate prediction can be made.

4.3 Random Sampling Model Result

The top-50 performances since 1980 (650 scores in total) from men's 1500m were used to develop the random sampling model (see Table 4-7).

Table 4-7. Descriptive Statistics of Top-50 Performance Data (1980-1992)

Year	Mean (sec)	Sd.	Min. Value	Max. Value
1980	216.35	2.06	211.36	218.71
1981	216.71	2.00	211.57	218.91
1982	216.75	2.03	212.12	218.80
1983	215.78	1.93	210.77	218.18
1984	215.54	1.40	211.54	217.10
1985	215.52	2.45	209.46	218.25
1986	215.32	1.80	209.77	217.75
1987	215.19	1.57	210.69	217.40
1988	215.56	1.48	210.95	217.29
1989	215.75	1.74	210.55	217.92
1990	215.52	1.50	212.60	217.52
1991	215.50	1.68	211.00	217.73
1992	215.49	1.96	208.82	217.76

The empirical distribution of the top-50 data was shown in a previous chapter (Figure 3-5). The assumption is that the best performances per year since 1980 do not have an improving trend and the true score distribution of the top-50 performances will be constant for the next 50 or more years. The Foster and Stuart's d-test (1954) was used to test randomness of the best performances since 1980:

$$d=\Sigma\, d_r=2, \qquad V(d)=2\,\Sigma\,(1/r)=4.36,$$

$$S_d=(V(d))^{1/2}=2.088, \quad Z_d=d/S_d=0.96, \ (p>.16).$$

where d is the statistic of the d-test, V(d) is the variance of d, d_r is coded as 1 if the r^{th} year's performance is an upper record, and it is coded -1 if the r^{th} year's performance is a lower record. The result of the d-test indicated that the men's 1500m best performances do not have an improvement trend in the last 13 years (even with the new world record established in October 1992 being included). Since the test suggests that the men's 1500m performances have reached an average ultimate performance, the deterministic models may not be appropriate for predicting future world records, and the top-50 data are eligible to be used for the development of the random sampling model based on extreme value theory.

As mentioned in the previous chapter, the equality of mean and standard deviation of $(a-t_1,\ a-t_2,\ \ldots\ldots\ a-t_{n-1})$ is the criteria used to determine the best fit of an exponential distribution to the basic extreme value theory function. The 80 best ordered performances in 650 total performances were identified as the best group to be fitted by the function (Table 3-1, Figure 3-6). The FORTRAN 77 Monte Carlo simulation program generated top-50 performances per year for each year in the future and identified the performances belong to the ordered exponential distribution (the probability of the performance in the ordered exponential distribution is p=80/650=0.123). For each year, the identified best performance was then compared with the previous world record to determine the new world record. The next three world records and the waiting times for the men's 1500m were estimated in each simulation. One thousand and two hundred simulations were conducted by the FORTRAN 77 Monte Carlo simulation program using MTS system. The distribution of the next three new world records were established. As expected, the results showed that the world record distributions with the waiting time distributions are negatively skewed (Figure 4-15 to 4-17). Table 4-8 shows the descriptive statistics for the random sampling model.

The random sampling model established the expected world records and the waiting times with confidence intervals for means and medians. Because of the high skewness of the expected world record and waiting time distributions the mean is not appropriate for world record predictions, thus the medians are used to predict the world records in this study. According to the results from the random sampling model the first world record would be 207.86 seconds ($\pm$0.04) with a waiting time of 10 years, the second world record would be 206.73 seconds ($\pm$0.067) with a waiting

Figure 4-15 The Distribution of the First World Record

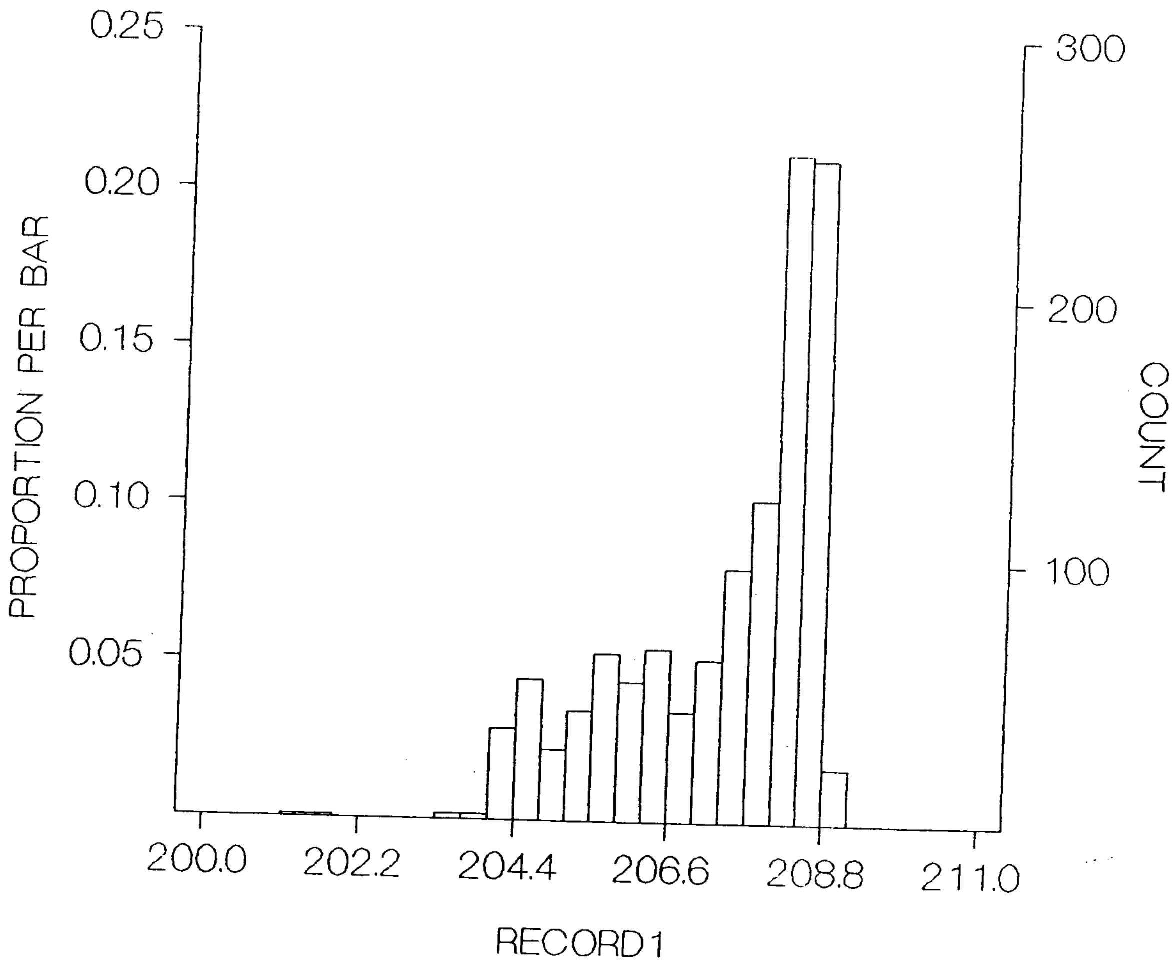


Figure 4-16 The Distribution of the Second World Record

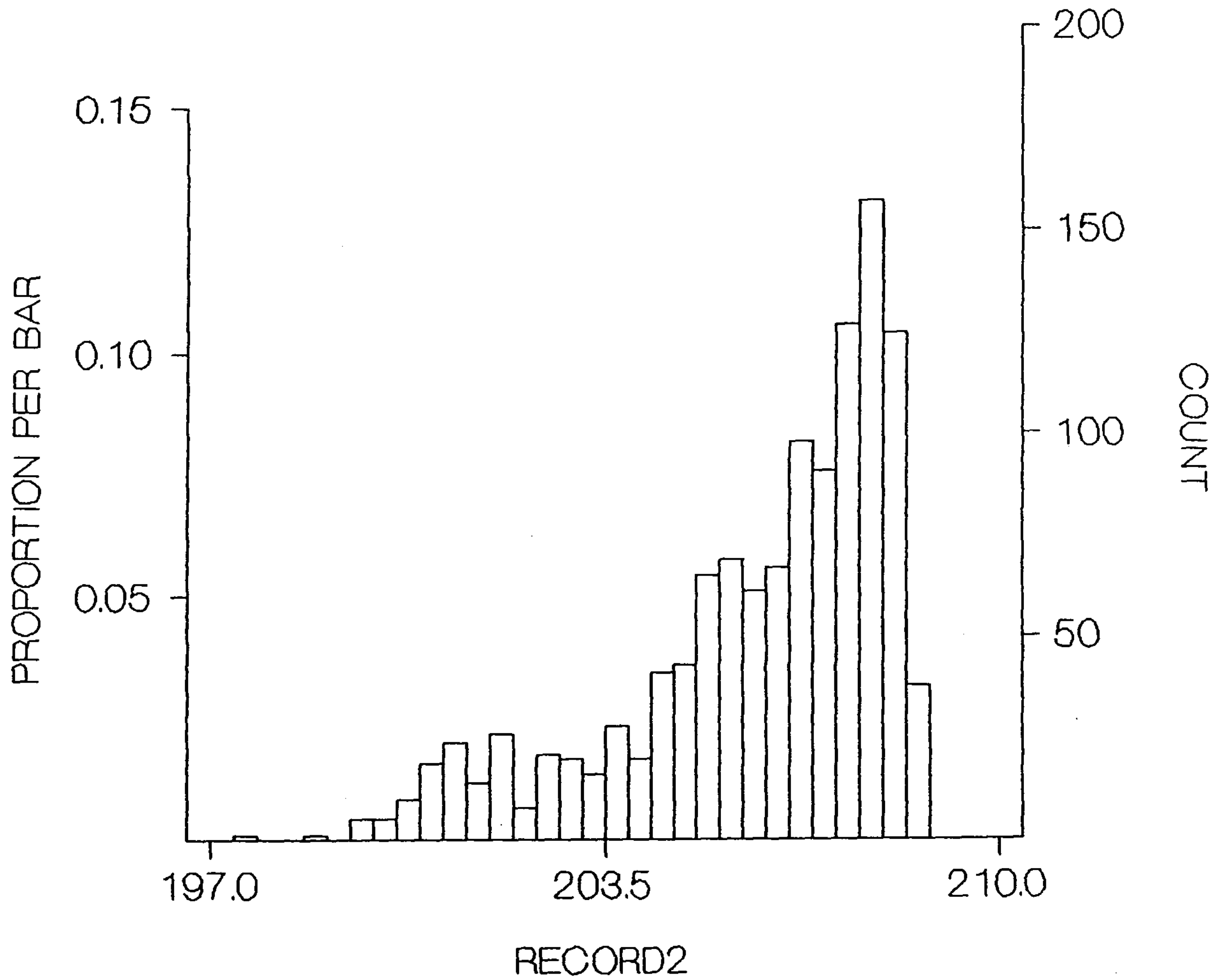
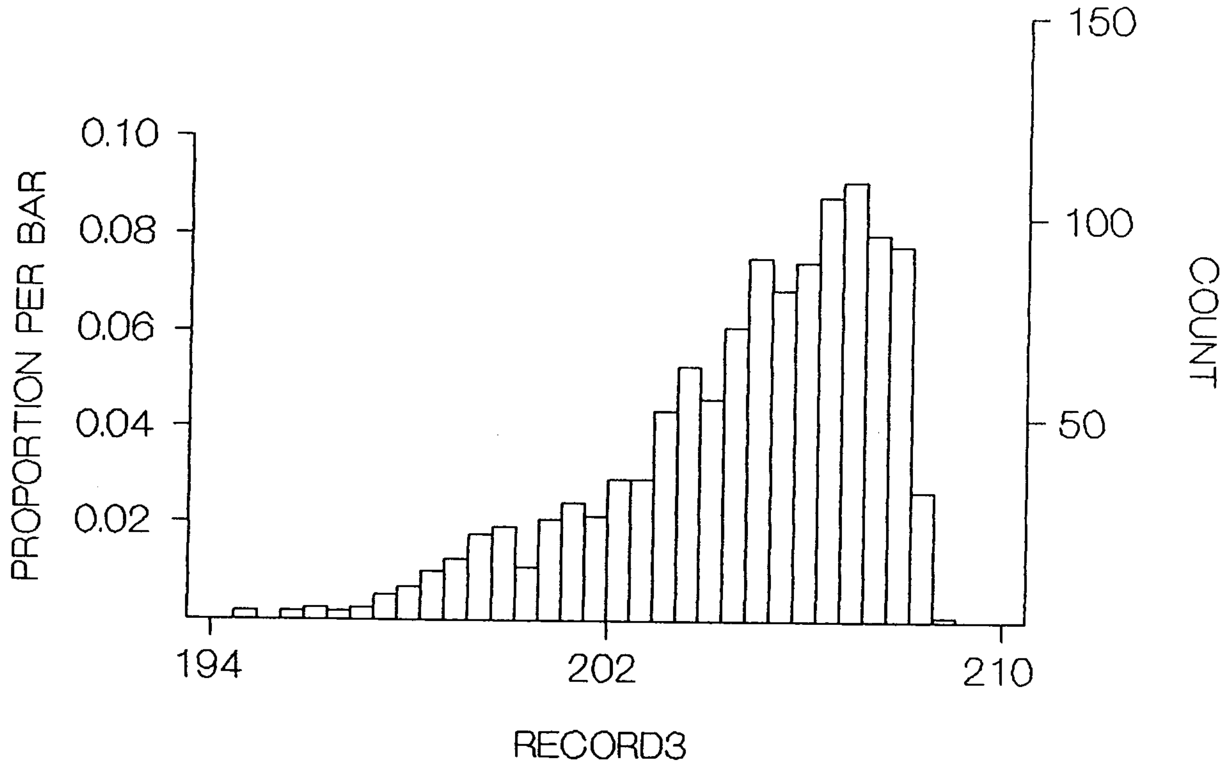


Figure 4-17 The Distribution of the Third World Record



time 24 year ($\pm$0.58), and the third world record would be 205.33 seconds ($\pm$0.081) with a waiting time 64 years ($\pm$3.75). Glick (1978) developed an analytic expectation table giving the number of record values in a random sequence of n independent and identically distributed observations (Table 4-9).

Table 4-8. Descriptive Statistics of the New World Record (sec.) and the Waiting Times (years)

	Mean	St.Er	Median	St.Er	Mode	Skew.
$Record_1$	207.30	0.040	**207.86**	**0.055**	208.44	-0.90
Wait T_1	9.75	0.590	**7.00**	**0.001**	1.00	17.03
Year	2002		1999			
$Record_2$	206.01	0.064	**206.73**	**0.067**	206.67	-1.08
Wait T_2	78.65	9.747	**24.00**	**0.577**	9.00	28.26
Year	2071		2016			
$Record_3$	204.73	0.077	**205.33**	**0.081**	201.05	-0.95
Wait T_3	970.03	93.07	**63.50**	**3.753**	23.00	4.58
Year	2962		2056			

Table 4-9. Theoretic Expecting Number of Records From a Random Sequence (Glick,1978)

Year	10	20	30	40	50	60	65
No. Record	2.93	3.60	3.99	4.28	4.50	4.68	4.76
Sd.	1.17	1.41	1.54	1.63	1.70	1.75	1.77

The random sampling model predicted that one world record will be established in 10 years, two world records in 24 years, and three world records in 64 years, while the theoretic predictions (and the 95th confidence interval) according to Glick's procedure are the following: three records ($\pm$ 2) in ten years, four records ($\pm$ 3) in 30 years, and five records ($\pm$ 4) in 65 years. Comparing the results with the analytic expectations of a true random record breaking sequence, using Glick's table we can see that the waiting time intervals of the world record predictions from the random sampling model are less than expected, but are always in the theoretical expected range. This statistically supports the validity of the random sampling model. Since Glick's expectations have quite a large variation the empirical validity of the random sampling model can only be accurately verified by future world records.

It is interesting to note the differences in predictions between the deterministic model and Random sampling model (see Table 4-10). As expected, the deterministic model exhibited more liberal predictions. A difference of 0.16 seconds between predictions was obtained from the two models for the first world record, 2.5 seconds for the second record, and 6.93 seconds for the third record. Since the Foster and Stuart's d-test suggested that the men's 1500m performances have reached an average limit, the results of random sampling model are believed to be more reasonable than those obtained from the deterministic models. According to the results from the random sampling model the waiting time between world records is going to be progressively longer as each new world record is established. It seems that the world record beyond the limit of 205 seconds could take one hundred more years for the men's 1500 meters. New world records will be getting more difficult to establish in the future for those events in which the performance plateau has been clearly shown, such as for the men's 1500m data.

Table 4-10. Monte Carlo Results (men's 1500m)

	Exponential Model	Random Sampling Model
	(sec) (min:sec)	
1999	207.70 (3:27.70)	207.86 (3:27.86)
2016	204.23 (3:24.23)	206.73 (3:26.73)
2056	198.40 (3:18.40)	205.33 (3:25.33)

4.4 Projection of World Records and Ultimate Performances

In this section the predicted results of the best deterministic fitting model (exponential model) and the random sampling model are compared with the predictions of other researchers. The validity of the models for the predictions will also be discussed.

Table 4-11 and Table 4-12 show the predictions in six events based on different models. Lloyd's predictions were utilized from the B_2 model on the basis of the improvement in the "oxygen debt" and "maximal usage of oxygen". The Peronnet's and Morton's predictions were based on the physiological model C_2 (see the previous chapter for the functions of B_2 and C_2 models) which allows the estimation of the characteristics of the metabolic energy-yielding processes (A, MAP, and E). The projections of Ryder et al. (1976) were based on the apparently linear relationship between the average velocity over a given distance, and the chronological year between 1925 and 1975. Dyer's (1982) and Stefani's (1977) predictions were based on the linear improvement rate of the performances over chronological years. Schutz and McBryde's predictions (1983) and the projections of the best fitting model in this study were utilized from model A_1 (exponential model), but the best fitting model used the updated best performance data to 1992.

Close scrutiny of the projections from Table 4-11 and Table 4-12 suggests that Dyer's and Ryder's predictions were more optimistic than that of other researchers, even for near future predictions. The reason for the liberal predictions of these two studies is that a linear based model was used in their studies, however, the assumption of a simple linear increase in velocity over a given distance over years is not appropriate to present the true improvement trend of track events. In addition, the second-level modelling from the type B model resulted in less accuracy of the predictions for this model.

The physiological based models (Morton, 1983; Peronnet et al., 1989) tried to describe the running performance over a wide range of events and estimate the characteristics of the metabolic energy-yielding processes. For the predictions, the physiological based model may also be slightly more optimistic than the best fitting model which is based on the actual trends of the performances over years. For instance, Peronnet's (1989) ultimate prediction of the men's 1500m was 3.89 seconds fast than the best fitting model prediction (Table 4-11).

Schutz and and McBryde's (1983) model and the best fitting model both are based on the three parameter exponential function, but the best fitting model of this study used the updated best performance data. The results show that the predictions from Schutz's model are slightly liberal, since the performances in the last ten years showed continuous levelling off and the improvement rate kept decreasing in track and field. The slightly liberal predictions from Schutz and McBryde's study in 1983 are reasonable. However, all the future predictions from the deterministic fitting models may be slightly optimistic due to the continuing expected decrease in the future rate.

Based on the assumption that the men's 1500m performances have reached the average ultimate limit, the predictions of the random sampling model are more conservative than that of the best fitting model (Table 4-11). Since we believe that human beings can not keep improving their performances forever, and the best performance data showed no improvement trend since 1980, the random sampling model is believed to be more valid than the best fitting model for the men's 1500m predictions.

Comparing other researcher's predictions, the 400m event is the only event in which lower values are predicted by the best fitting model. However, the predictions of the best fitting model for the 400m are believed not accurate due to the instability of the best performance data (see detail discussion in the previous chapter).

Table 4-11. Predictions in History (Men)

	100	400	1500	3000	5000	Marathon
			(Year 2000)			
Stefani(77)	9.72	42.65	**3:33.08**		12:44.07	
Schutz(83)	9.85	43.20	**3:27.00**		12:45.00	
Peronnet(89)	9.74	43.44	**3:25.45**	7:22.54	12:42.72	2:05:23.72
B.P. Model	9.88	43.37	**3:27.47**		12:49.53	2:06:04.56
R.S. Model			**3:27.86**(1999)			
			(Year 2028)			
Ryder(76)	9.34	41.32	**3:14.70**	6:54.10	11:51.90	1:53:13.00
Peronnet(89)	9.57	42.12	**3:17.45**	7:03.91	12:09.39	1:59:36.08
B.P. Model	9.78	42.09	**3:22.18**		12:25.63	2:03:06.49
R.S. Model			**3:26.73**(2016)			
			(Ultimate Prediction)			
Morton(83)	9.15	39.33	**3:04.15**	6:16.91	11:22.87	1:52:14.47
Peronnet(89)	9.37	39.60	**3:04.27**	6:24.81	11:11.61	1:48:25.25
B.P. Model	9.56		**3:09.04**		11.20.06	1:59:25.21
R.S. Model			**3:25.33**(2056)			

Table 4-12. Predictions in History (Women)

	100	400	1500	3000	50000	Marathon
			(Year 2000)			
Lloyd(66)	10.77	45.49	**3:42.13**		13:52.84	2:14:36.80
Dyer(82)	10.00	44.00	**3:22.20**		12:43.00	2:05:00.00
Schutz(83)		44.91				
Peronnet(89)	10.66	46.85	**3:47.93**	8:11.98	14:19.33	2:18:43.34
B.P. Model	10.80	46.60	**3:48.99**			2:18:50.91
			(Year 2028)			
Peronnet(89)	10.46	45.34	**3:38.91**	7:50.61	13:41.56	2:12:19.55
B.P. Model	10.74	43.09	**3:38.97**			2:17:17.66
			(Ultimate Prediction)			
Peronnet(89)	10.15	44.71	**3:26.95**	7:11.42	12:33.36	2:00:33.22
B.P. Model	10.71		**3:26.96**			2:17:12.26

CHAPTER V

SUMMARY AND CONCLUSIONS

The purposes of this study were: 1) to compare the previously applied mathematical deterministic models on the basis of their assumptions, strengths and weaknesses and identify the best deterministic model for selected track and field events; 2) to use a comprehensive updated data set and the best fitting model to predict future performances for males and females in selected track and field events, and to determine whether and when women will outperform men; and 3) to develop and test a new model to predict future performances under the assumption that the average performance in a specific event has reached an asymptotic level.

In order to identify the best deterministic model, the updated best performance data and the world record data for seven events were collected and four mathematical fitting models were examined. The software BMDP-P1R and BMDP-P3R were used to fit the data sets and calculate the least squares estimates for the linear and the nonlinear functions. The validity of the four deterministic models was compared.

A new model called the Random Sampling Model was developed for the men's 1500m in this study. Top-50 performances per year were collected since 1980 for the random sampling model development, and Foster and Stuart's d-test showed that the men's 1500m performances had no progressive trend over this 12 year period. A FORTRAN 77 Monte Carlo simulation program was written to do the simulation utilizing values derived from extreme value theory. The world record prediction results from the random sampling model were compared with Glick's theoretical expected number of world records in a given period.

The following conclusions have been drawn from the results attained in this study.

With respect to the best deterministic model:

1) The validity of the deterministic models for the predictions in track and field is sensitive to the chosen data. The best performance per year data is the most appropriate data in track and field for the model development.

2) For the events in which the performance data are stable, the exponential model relating running time and historical year with the best performance data are the most valid in evaluation and prediction of track and field world records and ultimate performance.

3) According to the best fitting model (exponential model), both men and women have the capacity to improve their performances in the near future. The difference between women's and men's performances in track and field will keep diminishing, however, women are not predicted to catch up to the men in the chosen events in this study.

4) A greater performance improvement is expected in the near future for those events in which the performances still exhibit on a linear trend (e.g.,10000m and high jump).

With respect to the Random Sampling Model:

1) Under the assumption that the average ultimate performance has been reached in the men's 1500m event, the random sampling model is an effective method to predict the new world records for this event.

2) According to the random sampling model the waiting time between world records is progressively longer with every newly established world record. A world record beyond the limit of 205 seconds could take up to one hundred or more years for the men's 1500m event.

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A-1. Model Comparisons (Men's 100m)

	Linear Model	Exponent Model	Exponent Model (velocity)	Polynomial Model (velocity)
R^2 BP data (92)	0.87	0.90	0.90	0.90
BP data (70)	0.83	0.84	0.83	0.84
$P_1 \pm$ SE BP data (92)	-0.0098 0.000390	1.28 0.1398	-1.295 0.1803	0.014 0.0013
BP data (70)	-0.0112 0.000606	1.31 0.3431	17.189 0.0225	0.014 0.0022
$P_2 \pm$ SE BP data (92)	-	-0.0139 0.0031	-0.012 0.0031	-0.000054 0.000014
BP data (70)	-	-0.0134 0.0058	0.000589 0.000032	-0.000051 0.000030
$P_3 \pm$ SE BP data (92)	-	9.56 0.1586	10.52 0.1987	9.23 0.0267
BP data (70)	-	9.52 0.3688	-7.9086 0.0000	9.23 0.0337
Prediction				
(1992) BP (92)	9.83	9.92	9.91	9.92
BP (80)	9.74	10.04	9.77	10.08
(2050) BP (92)	9.27	9.72	9.71	9.84
BP (80)	9.09	9.70	9.20	9.81
Ultimate Perf.				
BP (92)	-	9.56	9.51	9.84
BP (80)	-	9.53	-	9.81

*WR (1991) 9.86 *BP (1992) 9.91

A-2. Model Comparisons (Women's 100m)

	Linear Model	Exponent Model	Exponent Model (velocity)	Polynomial Model (velocity)
R^2 BP**	0.89	0.96	0.96	0.95
BP data (92)	0.89	0.95	0.95	0.94
BP data (80)	0.88	0.96	0.96	0.94
$P_1 \pm$ SE BP**	-0.0224 0.0012	4.65 0.52	-2.91 0.2419	0.040 0.0041
BP data (92)	-0.0227 0.0012	4.36 0.49	-2.78 0.2183	0.038 0.0045
BP data (80)	-0.0250 0.0017	6.22 1.01	-3.64 0.0068	0.051 0.0068
$P_2 \pm$ SE BP**	-	-0.043 0.0051	-0.033 0.0046	-0.000207 0.000036
BP data (92)	-	-0.039 0.0055	-0.029 0.0051	-0.000185 0.000040
BP data (80)	-	-0.058 0.0070	-0.048 0.0068	-0.000313 0.000066
$P_3 \pm$ SE BP**	-	10.77 0.0588	9.36 0.0671	7.260 0.1063
BP data (92)	-	10.71 0.0767	9.44 0.0969	7.304 0.1183
BP data (80)	-	10.95 0.0503	9.18 0.0524	7.049 0.1557
Prediction				
BP**	10.62	10.86	10.83	10.83
(1992) BP	10.61	10.83	10.81	10.79
BP (92)	10.50	10.98	10.99	10.98
(2050) BP**	9.32	10.78	10.71	10.83
BP (92)	9.29	10.72	10.63	10.76
BP (80)	9.05	10.95	10.90	10.98
Ultimate Perf.				
BP**	-	10.77	10.68	10.83
BP (92)	-	10.71	10.59	10.76
BP (80)	-	10.95	10.89	10.98

*BP (1988) 10.49 BP (1992) 10.80 BP** last record out

A-3. Model Comparisons (Men's 400m)

	Linear Model	Exponent Model	Exponent Model (velocity)	Polynomial Model (velocity)
R^2 WR data	0.96	0.96	0.95	0.96
BP data (92)	0.92	0.92	0.92	0.92
BP data (70)	0.90	0.90	0.90	0.90
$P_1 \pm$ SE WR data	-0.055 0.0028	112.05 0.0000	-18.44 0.0000	0.0071 0.0018
BP data (92)	-0.056 0.0018	22.06 17.36	-12.00 52.35	0.0110 0.0013
BP data (70)	-0.059 0.0022	-25.83 69.92	2.93 3.7679	0.0094 0.0016
$P_2 \pm$ SE WR data	-	-0.0005 0.000027	-0.000588 0.000034	0.000042 0.000020
BP data (92)	-	-0.0029 0.002613	-0.0009 0.0041	-0.000005 0.000014
BP data (70)	-	0.0021 0.005155	0.0033 0.0037	0.000019 0.000019
$P_3 \pm$ SE WR data	-	-63.82 0.1386	26.72 0.0287	8.33 0.0348
BP data (92)	-	26.86 17.4627	20.17 52.038	8.17 0.0256
BP data (70)	-	74.68 70.0799	5.26 3.79	8.19 0.0276
Prediction				
(1992) WR	43.12	43.14	43.26	42.85
BP (92)	43.56	43.76	43.77	43.78
BP (80)	43.47	43.44	43.43	43.42
(2050) WR	39.91	40.05	40.66	38.71
BP (92)	40.41	41.15	41.21	41.22
BP (80)	40.06	39.47	39.88	39.91
Ultimate Perf.				
WR	-	-	14.97	-
BP (92)	-	26.86	19.63	28.22
BP (80)	-	-	-	-

*WR (1988) 43.30 *BP (1992) 43.50

A-4. Model Comparisons (Men's 5000m)

	Linear Model	Exponent Model	Exponent Model (velocity)	Polynomial Model (velocity)
R^2 WR data	0.98	0.98	0.98	0.98
BP data (92)	0.93	0.94	0.95	0.95
BP data (70)	0.92	0.93	0.94	0.94
$P_1 \pm$ SE WR data	-1.62 0.052	386.40 148.04	-7.52 11.05	0.012 0.0013
BP data (92)	-1.86 0.054	271.44 30.73	-2.82 0.70	0.016 0.0012
BP data (70)	-1.98 0.066	279.00 50.91	-3.65 1.97	0.015 0.0015
$P_2 \pm$ SE WR data	-	-0.0053 0.0026	-0.0016 0.0026	-0.000010 0.000014
BP data (92)	-	-0.0111 0.0022	-0.0059 0.0020	-0.000037 0.000012
BP data (70)	-	-0.0106 0.0031	-0.0043 0.0028	-0.000026 0.000018
$P_3 \pm$ SE WR data	-	532.14 150.70	12.96 11.07	5.44 0.0263
BP data (92)	-	680.06 33.29	8.06 0.71	5.26 0.0235
BP data (70)	-	672.04 53.88	8.92 1.99	5.27 0.0262
Prediction				
(1992) WR	764.09	769.80	769.86	770.30
BP (92)	764.85	777.84	776.75	776.94
BP (80)	757.40	791.29	773.73	773.61
(2050) WR	670.40	707.08	706.08	707.65
BP (92)	657.13	731.42	723.66	730.10
BP (80)	642.83	728.72	713.81	715.62
Ultimate Perf.				
WR	-	532.14	385.79	514.07
BP (92)	-	680.06	620.10	712.48
BP (80)	-	672.04	560.47	663.92

*WR (1987) 778.39 *BP (1992) 780.93

A-5. Model Comparisons (Men's Marathon)

		Linear Model	Exponent Model	Exponent Model (velocity)	Polynomial Model (velocity)
R^2	BP data (92)	0.89	0.94	0.95	0.95
	BP data (70)	0.90	0.92	0.93	0.98
$P_1 \pm$ SE	BP data (92)	-25.54 1.56	3288.97 175.07	-2.08 0.22	0.025 0.0027
	BP data (70)	-29.51 2.17	3365.15 431.22	-2.51 0.95	0.125 0.1155
$P_2 \pm$ SE	BP data (92)	-	-0.02 0.0042	-0.014 0.0036	-0.0001 0.000025
	BP data (70)	-	-0.02 0.0075	-0.010 0.0062	-0.0003 0.0008
$P_3 \pm$ SE	BP data (92)	-	7165.21 240.09	6.10 0.2711	4.057 0.0639
	BP data (70)	-	7053.05 571.43	6.58 1.0178	-3.451 4.1112
Prediction					
(1992)	BP (92)	7451.94	7637.93	7626.34	7639.93
	BP (70)	7237.13	7612.30	7558.85	7341.55
(2050)	BP (92)	5970.52	7304.37	7210.35	7503.19
	BP (70)	5525.47	7233.45	7006.10	4587.60
Ultimate Perf.					
	BP (92)	-	7165.21	6908.39	7503.19
	BP (70)	-	7053.05	6410.37	3845.22

*BP(1992) 7694.00

A-6. Model Comparisons (Women's Marathon)

		Linear Model	Exponent Model	Exponent Model (velocity)	Polynomial Model (velocity)
R^2	BP data (92)	0.80	0.97	0.96	0.97
	BP data (80)	0.95	0.97	0.98	0.93
$P_1 \pm$ SE	BP data (92)	-127.49 13.35	3164789.0 2064904.0	-279.13 187.92	0.4380 0.0348
	BP data (80)	10175.14 1279.81	225072.6 270836.95	-20.93 38.7574	0.0231 0.0044
$P_2 \pm$ SE	BP data (92)	-	-0.104 0.0105	-0.0789 0.0111	-0.0025 0.0002
	BP data (80)	-	-0.057 0.0228	-0.0066 0.02305	-0.000076 0.000049
$P_3 \pm$ SE	BP data (92)	-	8232.26 143.09	5.18 0.1045	-14.55 1.3503
	BP data (80)	-	6445.62 1595.05	17.15 45.6780	4.09 0.0855
Prediction					
(1992)	BP (92)	7841.11	8458.51	8461.24	8591.74
	BP (80)	5853.79	7639.72	7340.17	7566.00
(2050)	BP (92)	446.88	8232.81	8144.52	8591.74
	BP (80)		6489.54	4498.82	7209.62
Ultimate Perf.					
	BP (92)	-	8232.26	8141.36	8591.74
	BP (80)	-	6445.62	2457.79	7209.15

*BP (1992) 8623.00

Appendix B

Fitting Model Function

B-1. 100m

B-1-1. Linear Model

Men: $T=10.73223-0.009756Y$
Women: $T=12.69524-0.0227Y$

B-1-2. Exponential Model (T)

Men: $T=9.560003+1.28318e^{-0.013925Y}$
Women: $T=10.71041+4.357061e^{-0.038894Y}$

B-1-3. Exponential Model (V)

Men: $V=10.517246-1.295419e^{-0.01195Y}$
Women: $V=9.440208-2.776764e^{-0.029394Y}$

B-1-4. Polynomial Model

Men: $V=9.232171+0.014145Y-0.000054Y^2$
Women: $V=7.304+0.038351Y-0.000185Y^2$

B-2. 400m

B-2-1. Linear Model

Men: $T=48.80621-0.056Y$
Women: $T=65.85229-0.2031$

B-2-2. Exponential Model (T)

Men: $T=26.860173+22.062962e^{-0.002897Y}$
Women: $T=29.574607+38.888183e^{-0.008258Y}$

B-2-3. Exponential Model (V)

Men: $V=20.172319-11.998686e^{-0.000911Y}$
Women: $V=23.588149-17.796922e^{-0.001701Y}$

B-2-4. Polynomial Model

Men: $V=8.173189+0.010954Y-0.000005Y^2$
Women: $V=5.787178+0.030391Y-0.000026Y^2$

B-3. 1500m

B-3-1. Linear Model

Men: $T=246.70363-0.4422Y$
Women: $T=332.60034-1.1433Y$

B-3-2. Exponential Model (T)

Men: $T=189.039335+61.799477e^{-0.012097Y}$
Women: $T=206.961693+192.35536e^{-0.021671Y}$

B-3-3. Exponential Model (V)

Men: $V=8.231904-2.254443e^{-0.008159Y}$
Women: $V=7.659728-4.152243e^{-0.013285Y}$

B-3-4. Polynomial Model

Men: $V=5.981493+0.01772Y-0.000053Y^2$
Women: $V=3.622446+0.046948Y-0.00018Y^2$

B-4. 5000m

B-4-1. Linear Model

Men: $T=935.73511-1.8574Y$
Women: $T=1154.40332-2.9351Y$

B-4-2. Exponential Model (T)

Men: $T=680.062065+271.440679e^{-0.011099Y}$

B-4-3. Exponential Model (V)

Men: $V=8.063197-2.802138e^{-0.005919Y}$

B-4-4. Polynomial Model

Men: $V=5.265+0.016106Y-0.000037Y^2$

B-5. 10000m

B-5-1. Linear Model

Men: $T=1952.13623-3.7623Y$
Women: $T=3166.53271-14.9505Y$

B-5-2. Exponential Model (T)

Men: $T=6942.275084-4992.331191e^{0.000728Y}$

B-5-3. Exponential Model (V)

Men: $V=-6.318446+11.416464e^{0.000999Y}$

B-5-4. Polynomial Model

Men: $V=5.101201+0.011213Y+0.000008Y^2$

B-6. Marathon

B-6-1. Linear Model

Men: $T=9801.77344-25.5417Y$
Women: $T=19569.89844-127.4868Y$

B-6-2. Exponential Model (T)

Men: $T=7165.212649+3288.970562e^{-0.021085Y}$
Women: $T=8232.262759+3164789.018712e^{-0.10376Y}$

B-6-3. Exponential Model (V)

Men: $V=6.102581-2.076421e^{-0.013966Y}$
Women: $V=5.178376-279.133995e^{-0.07894Y}$

B-6-4. Polynomial Model

Men: $V=4.057203+0.025365Y-0.000103Y^2$
Women: $V=-14.554973+0.437993Y-0.002464Y^2$

B-7. High Jump

B-7-1. Linear Model

Men: $H=1.85847+0.005727Y$
Women: $H=1.32772+0.008406Y$

B-7-2. Exponential Model

Women: $H=3.239284-1.946985e^{-0.005604Y}$

B-7-3. Polynomial Model

Women: $H=1.296493+0.0105Y-0.000022Y^2$

Appendix C

FORTRAN 77 Monte Carlo Simulation Program

```
 CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
 C     FORTRAN PROGRAM FOR THE RADOM SAMPLING MODEL      C
 C     FILE:  THESIS.FOR                                 C
 C     ZMM:   The new world record.                      C
 C     RECD:  The new world record array.                C
 C     MK:    The times of world record broken.          C
 C     K2:    The year range for the simulation.         C
 C     K3(i): The world record year.                     C
 CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

       DIMENSION  Y(1:50),K3(1:10),WMIN(1:8),RECD(1:8)
 C     OPEN (UNIT=10, FILE='p',STATUS='OLD',
 C    * ACCESS='SEQUENTIAL')
 C     OPEN (UNIT=3, FILE='M15NEWD3',STATUS='OLD',
 C    * ACCESS='SEQUENTIAL')
       S=SCLOCK(9.)
       DO 100 II=1,20
       K1=1
       K2=0
       KK=2
       K4=0
       MK=0
       DO 15 J=1,10
         K3(J)=0
          WMIN(J)=0
         RECD(J)=0
  15   CONTINUE
       WMIN(1)=208.82
  5    IF (MK .NE. 3) THEN
         K2=K2+1
       DO 20 I=1,50
       S=SCLOCK(9.)
          X=RAND(S)
         IF (X .GT. 0.123) THEN
            GO TO 20
         ENDIF
      S=SCLOCK(9.)
         XX=RANDE(S)
           Y(I)=213.46-XX*1.251
 C          PRINT*, I
 C          read (3,*) Y(I)
            WMI=Y(I)
           ZMM=WMIN(K1)
          IF (WMI .LT. ZMM) THEN
            K1=K1+1
            K4=K4+1
 C          PRINT*, K4
            WMIN(K1)=Y(I)
            ZMM=WMIN(K1)
 C          PRINT*, ZMM
```

```
            KK1=K1-1
            RECD(KK1)=ZMM
C           PRINT*, ZMM
C          ENDIF
C
C  FOR ONE YEAR TWO OR MORE RECORDS
C
            IF (K4 .GE. 2 ) THEN
             IF (MK .EQ. 4) THEN
                GO TO 5
             ENDIF
             MK=MK+1
             K3(MK)=K2
C            PRINT*, 'MK', MK
C            PRINT*, 'K2', K2
             KK=KK+1
            ENDIF
C           IF (K4 .EQ. 3) THEN
C            MK=MK+1
C            K3(MK)=K2
C           ENDIF
C
C FOR ONE YEAR ONE RECORD
C
C          PRINT*, 'KK', KK
C          PRINT*, 'K1', K1
          IF (KK .EQ. K1 .AND. K4 .EQ. 1) THEN
            IF (MK .GT. 3) THEN
              GO TO 5
            ENDIF
            MK=K1-1
            K3(MK)=K2
C           PRINT*, K2
            KK=KK+1
          ENDIF
       ENDIF
 20    CONTINUE
            K4=0
       GO TO 5
       ENDIF
C      PRINT*,MK,K2,K3(1),K3(2),K3(3),k3(4)
            WRITE (10,35) (RECD(K),K3(K),K=1,3)
 35         FORMAT (3(F8.4,1X,I5,1X))
C           WRITE (10,40) (K3(MK), MK=1,3)
C40         FORMAT (T30,3(I4,2X))
C      rewind(3)
 100   CONTINUE
       STOP
       END
```

Appendix D

BMDP Model Fitting Command File in MTS System

D-1. Linear Model

```
/PROBLEM TITLE IS ' LINEAR MODEL'.
/INPUT VARIABLES ARE 2.
        FILE='-D'.
        FORMAT IS'(T4,F2.0,1x,F7.2)'.
/VARIABLE NAMES ARE YEAR, TIME.
/REGRESS DEPENDENT IS TIME.
         INDEPENDENT IS YEAR.
/PRINT COVARIANCE.
/PLOT  RESIDUAL.
/END
```

D-2. Exponential Model (T)

```
/PROBLEM  TITLE IS 'EXPONENTIAL MODEL (T)'.
/INPUT   VARIABLES=2.
         FILE='-d'.
         FORMAT='(T4, F2.0,1x,F7.2)'.
/VARIABLE NAMES ARE YEAR, TIME.
/REGRESS DEPENDENT IS TIME.
         INDEPENDENT IS YEAR.
         NUMBER IS 1. PARAMETERS ARE 3.
/PARAMETER INITIAL ARE 5, 0.0001, 40.
/PLOT VARIABLE IS YEAR. RESIDUAL.
/END
```

D-3. Exponential Model (V)

```
/PROBLEM  TITLE IS ' EXPONENTIAL MODEL (V)'.
/INPUT    VARIABLES=2.
          FILE='-D'.
          FORMAT='(T4, F2.0,1x,f7.2)'.
/VARIABLE NAMES ARE YEAR, TIME, SPEED. add=1.
/TRANS    SPEED=400/TIME.
/REGRESS DEPENDENT IS SPEED.
          INDEPENDENT IS YEAR.
          NUMBER IS 1.  PARAMETERS ARE 3.
/PARAMETER INITIAL ARE -15, -0.001, 20.
/PLOT VARIABLE IS YEAR. RESIDUAL.
/END
```

D-4. Polynomial Model

```
/PROBLEM  TITLE IS 'POLYNOMIAL MODEL'.
/INPUT    VARIABLES=2.  FILE='-D'.
          FORMAT='(T4, F2.0,1x,F7.2)'.
/VARIABLE NAMES ARE YEAR, TIME, SPEED. add=1.
/TRANS    SPEED=400/TIME.
/REGRESS DEPENDENT IS SPEED.
          INDEPENDENT IS YEAR. PARAMETERS ARE 3.
/PARAMETER INITIAL ARE 0.1, 0.05, 8.
/FUN      D1=P1*YEAR. D2=P2*(YEAR**2).
          F=D1+D2+P3.
/PLOT VARIABLE IS YEAR. RESIDUAL.
/END
```