AN INTEGRATED SYSTEM FOR THE ESTIMATION OF TREE TAPER AND VOLUME

by

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ABSTRACT

A new taper equation is presented,

$\log d = b_0 + b_1 \log D + b_2 \log l + b_3 \log H$ where d is the diameter inside bark in inches at any given l in feet,D is the diameter breast height outside bark in inches, l is the distance from the tip of the tree in feet,H is the total height of the tree in feet and b_0,b_1,b_2 and b_3 are the regression coefficients.

Two methods of deriving a compatible system of tree taper and volume equations are discussed.One method involves conversion of the logarithmic taper equation into a logarithmic volume equation.The other involves the derivation of the logarithmic taper equation from an existing logarithmic volume equation to provide compatability in volume estimation and at the same time ensure as a good fit as possible for the estimation of upper bole diameters (taper).

Tests for precision and bias of volume estimates, carried out on the British Columbia Forest Service taper curves and logarithmic volume equations,indicate that the latter approach is preferable to the former.

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INTRODUCTION

"We must develop a mathematical tree volume expression which can be efficiently programmed for generally available electronic computing equipment to yield tree and stand volumes from inputs of tree diameter outside bark and total height (form estimates optional) and for any demanded stump height and top diameter".

(Honer and Sayn-Wittgenstein,1963)

This thesis demonstrates that a mathematical stem profile equation which can be integrated to volume can meet these requirements.When Y is the diameter inside bark at any given height in feet and X is the height above the ground in feet,then volume can be calculated by revolving the equation of the stem curve about the X-axis and integrating for values of X from base to tip. Merchantable volume to any standard of utilization can be calculated by using the appropriate X-values. Section diameter can be estimated at any height or the section height for any diameter.The maximum volume available in certain sizes and qualities and the loss of wood by breakage and defect can be determined precisely.

Taper functions,suitable for all these purposes, have been proposed by many mensurationists.However all these previous studies are similar in that a taper equation is calculated from the data to give the best fit for taper and therefrom volume is calculated. This often results in a good estimate of taper but a less than satisfactory estimate for volume.

In most practical cases a local or regional volume equation already exists,has been used widely for a long time and will certainly continue to be used in the future. It is clear that for such a situation a taper equation has to be derived which on the one hand gives the best possible fit for taper but on the other hand is compatible[1] with the existing volume equation.

This study deals with both approaches.The new taper equation presented is a logarithmic one.Two methods are derived,illustrated and tested:

a)Derivation of a compatible logarithmic volume equation from an existing logarithmic taper equation.

b)Derivation of a compatible logarithmic taper equation from an existing logarithmic volume equation.

Both techniques are tested on the British Columbia Forest Service(B.C.F.S.) taper curves (B.C.F.S.,1968) and on the logarithmic volume equations (Browne,1962).
The standard errors of estimate (SE_E) are compared with those given by Kozak,Munro and Smith (1969b,TableI)

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[1]Compatible means here that both equations (for taper and volume) give identical results for total volume.

which are based on the same taper curves.

Some attempt is made to find a useful relationship between the taper equation coefficients and some tree characteristics such as total height,diameter breast height (dbh) and the ratio of both.

LITERATURE REVIEW

In basic "Forest Mensuration" textbooks (Chapman and Meyer,1949;Bruce and Schumacher,1950;Spurr,1952; Meyer,1953;Loetsch and Haller,1964;Prodan,1965;Avery, 1967) almost no comments are made about the desirability of developing compatible taper and volume equations which would be useful for the estimation of merchantable volume to any standard of utilization.

Petterson (1927) suggested the use of a logarithmic curve for the main stem.Taper of the different form-classes would be given by different parts of this curve.A tangential function was used by Heijbel (1928) to describe the main part of the stem.Different equations were used for the top profile and the stem below 10% of the height.

Volume and taper were more or less combined in one system by the Girard form-class tables (Mesavage and Girard,1946).

According to Spurr (1952) a possible solution for merchantable cubic-feet tree volume tables is to calculate different equations for stump and top volumes and then to subtract these volumes from total volume. Another solution could be to calculate the regression between merchantable volume and total volume.
Meyer (1953) stated that the construction of a taper

curve (or equation) for a certain species or group of species is still a difficult task.

Graphical techniques were used by Duff and Burstall (1955) to develop taper and volume tables showing merchantable volumes for each ten-foot-height class within each dbh and total height class. Volume tables and taper tables were first prepared independently. To make them compatible the taper data were adjusted to fit the independently calculated volumes and the diameters were made to agree with the already known volumes.

Speidel (1957) used graphical techniques to relate the percentage of total volume to the percentage of total tree height.

It was shown by Newnham (1958) that a quadratic parabola gave a good fit to a large part of the bole shape.

Three models were developed by Honer (1964;1965a, b;1967) to express the distribution of volume over the tree stem:

a) $v / V = b_0 + b_1\, h / H + b_2\, h^2 / H^2$

b) $v / V = b_0' + b_1'\, d^2 / D^2 + b_2'(d^2 / D^2)^2$

c) $v / V = b_0'' + b_1''\, d / D + b_2''(1 - h / H)^2$

where v is the volume below the merchantable limit, V is the total volume, h is the merchantable height from the base, d is the merchantable diameter and D is the dbh.

These models describe well the distribution of volume over the tree stem and can be used to estimate volume to any standard of utilization when applied to an estimate of total volume.They cannot be used to estimate diameter at a given height or height of a certain diameter.

Tarif tables,like those of Turnbull and Hoyer (1965), will not be discussed here because they don't give a compatible system of volume and taper.

Heger (1965) reported a trial of Hohenadl's approach on lodgepole pine[2] grown in Alberta.Stanek (1966) illustrated the method for lodgepole pine and Engelmann spruce in British Columbia.

New tree-measurement concepts were introduced by Grosenbaugh (1954;1966).

Some work has been carried out on tree taper curves using multivariate methods (Fries,1965;Fries and Matern, 1965).However,after comparison of multivariate and other methods for analysis of tree taper,Kozak and Smith (1966) concluded that the use of simpler methods is best.

While many authors have made it clear (Kozak,Munro and Smith,1969a;Munro,1970) that no practical advantage can be gained from any measurement of form

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[2]The common tree names used throughout this thesis are given with the corresponding Latin names in Appendix 1.

in addition to dbh and total height, Schmid, Roiko-Jokela, Mingard and Zobeiry (1971) have shown that the measurement of dbh, total height and diameter at 6-9 meters height is the best method of volume determination.

Some important taper functions are worthy of more detailed review. A Swedish civil engineer, Höjer (1903), was the first to propose a mathematical equation to describe the stem profile:

$$d / D = c_1 \ln ((c_2 + l\ 100 / H) / c_2)$$

where d was the diameter inside bark at any given distance from the tip, D was the dbh inside bark, l was the distance from the tip, H the total tree height above breast height and c_1 and c_2 were the constants to be defined for each form-class.

Jonson (Claughton-Wallin, 1918) described this mathematical formula as completely conforming with nature when applied to spruce of all form-classes, but stated that in some stands, which had been grown from imported seeds, overestimations occurred in the upper sections. The diameter at any height of the stem being known there is no difficulty in estimating volume. A volume table can also be calculated by deriving a volume equation from the taper equation by integration:

$$V = D^2\ H\ 0.005454\ c_1^2\ (K\ (\ln K\ (\ln K - 2) + 2\ ) - 2)$$

(for proof see Appendix 2)

where $K = 1 + 100 / c_2$

In order to obtain better results,Jonson (1910;1911; 1926-27) introduced a new constant which he called a "biological constant":

$$d / D = c_1 \ln ((c_2 + 1\ 100 / H - c_3)/ c_2)$$

where c_3 was the new constant.Equations were computed for each form-class.With the introduction of this "biological constant" an inconsistency was introduced because this taper equation didn't give a result for a portion equal to c_3 on the upper stem.A volume equation can be derived in the same way as for the formula of Höjer.

The taper equations of Jonson and Höjer are in fact composite taper equations.They are compiled independently of tree species.The form-class which had to be known was usually measured or estimated by the "form point" approach.Claughton-Wallin and Vicker (1920) reported about this that the difficulty is to estimate the form-class of a standing tree or the average form-class of a stand but they believed that a little practise would overcome this.

Wickenden (1921) claimed that the form quotient of any type of forest does not vary much even for large regions.Wright (1923) believed however that there was a considerable variation in the form of individual trees in a stand of timber.

As a result of his investigations on many species, Behre (1923;1927;1935) presented a new equation for the stem curve which seemed to be more consistent with nature:

$$d / D = (l / H) / (b_0 + b_1 l / H)$$

where the symbols have the same meaning as in the Höjer's equation. The coefficients b_0 and b_1 can be calculated by fitting the regression line:

$$(l / H) / (d / D) = b_0 + b_1 (l / H)$$

this function is identical to the equation:

$$(D / d) = b_0' + b_1' (H / l)$$

Behre's taper equation, when integrated to volume, yields the following compatible volume equation:

$$V = D^2 H \, 0.005454 (1 / b_1^3) (1 - b_0^2 + 2 b_0 \ln b_0)$$

(for proof see Appendix 3)

Matte (1949) described the stem profile above breast height by the function:

$$d^2 / D^2 = b_0 l^2 / H^2 + b_1 l^3 / H^3 + b_2 l^4 / H^4$$

where the symbols have the same meaning as in the equation of Höjer. It is worthwile to mention that the taper equation coefficients are partially defined by a condition about volume.

The following volume equation can be derived by integration:

$$V = 0.005454 D^2 H (b_0 / 3 + b_1 / 4 + b_2 / 5)$$

(for proof see Appendix 4)

b_0 and b_1 were found to be related to dbh and total height.

A quite similar equation was tested by Osumi (1959)

$$d / D = b_0 \, l / H + b_1 \, l^2 / H^2 + b_2 \, l^3 / H^3$$

from which also a volume equation can be derived.

The taper equation preferred by Giurgiu (1963) was a 15th degree polynomial:

$$d / D = \text{15th degree polynomial of } l / H$$

where D was the diameter inside bark at .1 of total height and was further expressed as a function of dbh outside bark and total height.This function can also be integrated to volume.

Prodan (1965) found the following taper function satisfactory:

$$d / D = (h / H)^2 / (b_0 + b_1 \, h / H + b_2 \, h^2 / H^2)$$

where h is the height above the ground.
With respect to the taper equation of Osumi,he stressed that a 4th degree polynomial with intercept would be much better.

As an extension of the methods used by Matte, Osumi and Giurgiu,an integrated system of taper and volume equation for red alder was provided by Bruce, Curtis and Vancoevering (1968):

$$\begin{aligned} d^2 / D^2 = b_0 X^{3/2} &+ (X^{3/2} - X^3) (b_1 D + b_2 H) \\ &+ (X^{3/2} - X^{32}) (b_3 H D + b_4 H^{1/2}) \\ &+ (X^{3/2} - X^{40}) (b_5 H^2) \end{aligned}$$

where X is 1 / (H - 4.5) and D is dbh outside bark. Very high powers of X were required to describe the butt swell.The authors expected that the use of some measure of form would improve the fit of this taper equation.In their opinion,the principal difficulties encountered by Höjer,Jonson,Behre and others were due to oversimplified equations which did not satisfactory describe the butt swell and tip.

After Munro (1968) found that upper stem diameters could be estimated with reasonable SE from a function involving dbh, h / H and h^2 / H^2 ,the following taper equation was proposed by Kozak,Munro and Smith (1969a,b):

$$d^2 / D^2 = b_0 + b_1 h / H + b_2 h^2 / H^2$$

where D is the dbh outside bark in inches and h is the height above the ground in feet.The least squares solution was conditioned by imposing the restraint:

$$b_0 + b_1 + b_2 = 0$$

For spruce and redcedar additional conditions were necessary to prevent negative diameters near the top. These taper functions were computed for 23 species or speciesgroups from B.C.F.S. taper curves (B.C.F.S., 1968) to facilitate efficient analysis with modern electronic computers.Several tests on these equations (Kozak, Munro and Smith,1969a;Smith and Kozak,1971) suggested a stable estimating system.It appeared as if little real advantage resulted from the use of more complex powers, like those used by Bruce,Curtis and Vancoevering (1968), to estimate tree taper.These taper equations were,later on,converted into volume equations and point sampling factors (Demaersohalk,1971).

Awareness of the desirability of development of comprehensive systems for estimation of net merchantable volumes of trees by log size and utilization classes is growing. The need has been felt first in operations research analyses of logging systems in Sweden and in studies to develop improved methods of inventory in Austria. However, no publications incorporating the features described herein have come to the author's attention.

No review will be given about the different tree form theories (nutritional, mechanistic, water conductive, hormonal and pipe model). Interesting discussions about the different alternatives were given by Gray (1956), Newnham (1958), Larson (1963), Heger (1965) and Shinozaki *et al.* (1965).

DERIVATIONS OF THE EQUATIONS AND TESTS

The new taper equation

The logarithmic taper equation tested in this study is:

$$\log d = b_0 + b_1 \log D + b_2 \log l + b_3 \log H \qquad (1)$$

where d is the diameter inside bark in inches at any given l in feet, D is the dbh outside bark in inches, l is the distance from the tip of the tree in feet, H is the total height of the tree in feet and b_0, b_1, b_2 and b_3 are the regression coefficients.

The same taper equation can be expressed in other ways:

$$d = 10^{b_0} D^{b_1} l^{b_2} H^{b_3} \qquad (2)$$

or

$$d^w / D^v = K l^y / H^z \qquad (3)$$

where $w = 1.$ $\qquad z = -b_3$

$v = b_1$ $\qquad K = 10^{b_0}$

$y = b_2$

Just as the logarithmic volume equation

$$V = 10^a D^b H^c$$

is the unconditioned form (with respect to the powers of D and H) of the combined variable volume equation

$V = b_0 D^2 H^1$ (without intercept)

where the power of D is conditioned to 2 and the power of H to 1, this taper equation (the square of formula 3) is the unconditioned form of the well known general formula for the profile of certain solids of revolution (cone, paraboloid and neiloid):

$$d^2 / D^2 = (1 / H)^v$$

where the powers of d and 1 are conditioned to be equal to respectively the powers of D and H.

This taper equation is very simple. No conditioning is necessary to ensure that the estimated diameter at the top is zero and that no negative estimates of diameter occur. From formula 2 it can be seen easily that d can never be negative and becomes zero when 1 is zero (at the tip of the tree).

Formula 2 can be used to estimate diameter inside bark at any selected distance (1) from the tip. Distance to any specific top diameter (d) can be estimated by transformation of the basic equation to the form:

$$1 = (10^{-b_0} d D^{-b_1} H^{-b_3})^{1/b_2} \quad (4)$$

(for proof see Appendix 5)

The logarithmic taper equation can be derived in two basically different ways:

a) The taper equation can be fitted on taper data by the least squares method. This function can easily be converted subsequently to a compatible logarithmic volume equation.

b)The taper equation can be derived from an existing logarithmic volume equation when some data about taper are available.This taper equation will be compatible with the existing volume equation.

Both ways will be explained and tested on the B.C.F.S. taper curves (B.C.F.S.,1968) and logarithmic volume equations (Browne,1962).

Fitting the taper equation on the British Columbia Forest Service taper curves

The logarithmic taper function (formula 1) was computed for 23 species or speciesgroups on the B.C.F.S. taper curves (B.C.F.S.,1968) by the least squares method.Diameters inside bark had been taken from each taper curve at the height of 1ft.,4.5ft. and at deciles of total height and punched on computer cards for Kozak,Munro and Smith (1969_b).In the calculations, dbh outside bark was used as the measure of diameter inside bark at 1ft. height[3].
The assumptions of the regression analysis were tested by plotting for each species log d over log D,log l, and log H.For every species and for every variable,there was almost a perfect straight line relationship between dependent and independent variable.Variances were homogeneous.Because relative standard errors are sometimes greatly affected by the size of the mean and comparisons in terms of real d's were desired,the following approximation was used:

$$SE_E = ((\ S\ (\ da - de\ )^2)\ /\ (\ n - m - 1\ ))^{1/2} \qquad (5)$$

where da is the actual diameter inside bark,de is the estimated diameter inside bark,n is the number of

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[3]Except for mature coastal Douglas-fir,which gave a much better fit without adjusting.

observations used for the least squares fit,m is the number of independent variables and S is the sum.
The regression constants of the taper equation and the SE_E's are summarized in table I and the average bias of diameter inside bark at different heights is given in table II.

The SE_E's ranged from .245 to 2.431 inches.An absolute frequency distribution of the SE_E's is given in table IX.Large SE_E's,however,do not necessarily indicate a poor fit,but more likely represent a wider range of taper curves (Hejjas,1967).

All the species follow almost the same trend of average bias.There is usually an underestimation at the base of the tree,an overestimation from .1 until .4 or .5 of the total height,a slight underestimation from .4 or .5 until .8 of the total height and a small overestimation at the top.For eleven species the average bias at any height is less than one inch.

TABLE I

Summary of Taper Equations Fitted on the British Columbia Forest Service Taper Curves

Species group	R[4]	M[5]	b_0	b_1	b_2	b_3	SE_E (inches)	No[6]
			Equation coefficients					
Alder	C	M	-0.071459	0.802730	0.795203	-0.637059	0.587	92
Aspen	I	M	0.014506	0.944389	0.766655	-0.742576	0.448	53
Balsam	C	M	0.369223	1.064119	0.656080	-0.878118	0.813	85
"	I	M	0.025923	0.925263	0.729963	-0.697525	0.420	85
Birch	I	M	0.051560	0.979513	0.899947	-0.913041	0.245	55
Cedar	C	M	0.448191	0.968076	0.812954	-1.032109	2.431	114
"	C	I	0.195945	0.759688	0.824254	-0.783356	1.128	134
"	I	M	0.379992	1.011860	0.799019	-1.012500	1.379	127
Cotton-wood	CI	M	-0.262843	0.865273	0.827023	-0.613233	0.755	92
Douglas fir	C	M	0.204389	0.984578	0.701165	-0.821202	1.431	114
"	C	I	0.092707	0.826471	0.680451	-0.637352	1.032	174
"	I	M	0.004827	0.892425	0.741884	-0.690821	1.251	160
Hemlock	C	M	0.299130	1.016430	0.746148	-0.908821	0.871	118
"	C	I	0.065941	0.857932	0.829013	-0.776866	0.691	128
"	I	M	0.036873	0.999704	0.716169	-0.736237	0.740	104
Lodgep. pine	CI	M	0.472702	1.044069	0.634633	-0.909768	0.774	65
Larch	I	M	-0.012680	0.843926	0.696431	-0.618374	1.230	148
Maple	C	M	-0.010447	0.863337	0.909104	-0.822074	0.316	48
Spruce	C	M	0.294001	0.978388	0.783387	-0.912494	2.376	378
"	I	M	0.100700	0.915903	0.742631	-0.744977	0.526	93
White pine	CI	M	0.690044	1.215400	0.707159	-1.185360	1.159	81
Yellow cedar	CI	M	0.130260	0.891170	0.762970	-0.762321	0.574	50
Yellow pine	CI	M	0.044221	1.148219	0.674247	-0.810415	1.205	124

[4] C is Coast, I is Interior

[5] M is Mature, I is Immature

[6] Number of taper lines scaled from the B.C.F.S. taper curves

TABLE II

Test of the Taper Equations Fitted on the British Columbia Forest Service Taper Curves

Species group	R	M[7]	1'	4.5'	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
			Average bias (in inches) of diameter inside bark at											
Alder	C	M	0.06	0.34	0.42	0.17	-0.14	-0.36	-0.45	-0.39	-0.22	0.01	0.23	0.0
Aspen	I	M	-0.12	0.44	0.55	0.34	0.07	-0.14	-0.29	-0.32	-0.24	-0.06	0.15	0.0
Balsam	C	M	-0.43	0.17	0.86	0.61	0.14	-0.23	-0.45	-0.45	-0.28	-0.06	0.11	0.0
"	I	M	-0.31	0.35	0.51	0.29	0.02	-0.17	-0.25	-0.23	-0.16	-0.05	0.08	0.0
Birch	I	M	-0.03	0.25	0.29	0.09	-0.06	-0.14	-0.16	-0.14	-0.08	-0.01	0.04	0.0
Cedar	C	M	-4.21	-3.33	0.48	1.60	1.22	0.73	0.37	0.20	0.10	0.01	-0.08	0.0
"	C	I	-1.32	-1.10	0.19	0.75	0.55	0.24	0.01	-0.11	-0.13	-0.04	0.10	0.0
"	I	M	-2.47	-1.45	0.75	1.30	0.92	0.42	0.03	-0.17	-0.20	-0.11	0.03	0.0
Cottonwood	CI	M	-0.43	0.94	1.04	0.51	0.00	-0.38	-0.57	-0.56	-0.36	-0.07	0.18	0.0
Douglas-fir	C	M	-3.24	0.13	1.05	1.27	0.81	0.23	-0.25	-0.46	-0.36	-0.13	0.15	0.0
"	C	I	-2.01	-0.01	0.42	0.54	0.41	0.19	-0.02	-0.16	-0.17	-0.04	0.16	0.0
"	I	M	-2.74	0.66	1.48	1.22	0.46	-0.23	-0.67	-0.71	-0.45	-0.03	0.35	0.0
Hemlock	C	M	-1.01	0.11	0.95	0.74	0.24	-0.18	-0.43	-0.48	-0.32	-0.06	0.19	0.0
"	C	I	-0.48	0.01	0.60	0.45	0.10	-0.18	-0.34	-0.35	-0.18	0.03	0.21	0.0
"	I	M	-0.33	0.74	1.07	0.76	0.32	-0.12	-0.45	-0.57	-0.51	-0.24	0.14	0.0
Lodgepole pine	CI	M	-0.12	0.06	0.37	0.50	0.29	0.07	-0.12	-0.28	-0.35	-0.26	-0.10	0.0
Larch	I	M	-2.73	0.31	1.27	1.34	0.82	0.16	-0.37	-0.63	-0.56	-0.23	0.28	0.0
Maple	C	M	-0.16	-0.05	0.16	0.16	0.03	-0.05	-0.11	-0.11	-0.07	-0.01	0.05	0.0
Spruce	C	M	-3.25	-2.77	1.66	2.11	1.14	0.24	-0.35	-0.55	-0.39	0.01	0.13	0.0
"	I	M	-0.78	-0.18	0.48	0.56	0.26	-0.05	-0.23	-0.25	-0.16	-0.02	0.14	0.0
White pine	CI	M	-0.42	0.21	0.85	0.65	0.24	-0.14	-0.39	-0.45	-0.35	-0.19	-0.16	0.0
Yellow cedar	CI	M	-0.28	-0.23	0.52	0.73	0.44	0.04	-0.34	-0.54	-0.44	-0.09	0.19	0.0
Yellow pine	CI	M	-1.13	1.34	1.47	0.99	0.37	-0.18	-0.57	-0.67	-0.53	-0.29	0.02	0.0

[7] see fn. 4 and 5 in table I

Derivation of a compatible logarithmic volume equation from the logarithmic taper equation

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The logarithmic taper equation can be converted into a compatible logarithmic volume equation:

$$\log V = a + b \log D + c \log H \qquad (6)$$

where $a = \log (0.005454\ 10^{2b_0} / (2 b_2 + 1))$

$b = 2 b_1$

$c = 2 b_2 + 2 b_3 + 1$

where b_0, b_1, b_2 and b_3 are the coefficients from the logarithmic taper equation.

(for proof see Appendix 6)

This volume equation is the formula to be used to estimate total volume of the tree in cubic-feet.An alternative form of this equation is:

$$V = 10^a D^b H^c \qquad (7)$$

To estimate volumes of logs between specific distances from the tip of the tree,the following equation has to be used:

$$V = K D^v H^y (l_1^z - l_2^z) \qquad (8)$$

where $v = 2 b_1$

$y = 2 b_3$

$z = 2 b_2 + 1$

$K = 0.005454\ 10^{2b_0} / (2 b_2 + 1)$

and l_1 and l_2 are respectively the lower and upper distance from the tip of the tree.

(for proof see Appendix 7)

If the limit sizes of the log are given as diameters inside bark,the same formula 8 can be used after corresponding distances from the tip of the tree have been calculated with formula 4.

Derivation of a compatible logarithmic volume equation from the logarithmic taper equation fitted on the British Columbia Forest Service taper curves

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The logarithmic volume function (6) was derived from the taper equations in table I for the 23 B.C. species-groups and are summarized in table III.Because of the fact that the B.C.F.S. logarithmic volume equations and taper curves are based on the same sample trees (B.C.F.S., 1968),we would expect that the volume equations,derived from the taper functions,would be similar to the B.C.F.S. logarithmic volume equations.Although it is true for certain species,for others there are some rather large deviations.This suggests that a good taper equation is no guarantee for a good volume equation if only the precision of this taper function is indicated by the SE_E on diameter.A SE_E of 1 inch,for example,has no meaning for volume when one knows nothing about the bias.The effect of bias varies considerably with the position on the tree and with the size of the tree.Therefore the best check of a taper table,which is to be used to calculate volume,is a check of a volume table derived therefrom,as was recognized by Bruce and Schumacher (1950). The fact that in the B.C.F.S. logarithmic volume equations the sum of squares of the residuals of the logarithm of volume is minimized,while in the logarithmic taper equation the sum of squares of the residuals of

TABLE III

Summary of the Logarithmic Volume Equations Derived from the Logarithmic Taper Equations, Fitted on the British Columbia Forest Service Taper Curves

Species group	R	M[8]	Equation coefficients a	b	c
Alder	C	M	-2.819557	1.605459	1.316288
Aspen	I	M	-2.637947	1.888778	1.048159
Balsam	C	M	-1.888843	2.128239	0.555924
"	I	M	-2.602346	1.850526	1.064877
Birch	I	M	-2.607292	1.959025	0.973812
Cedar	C	M	-1.786168	1.936152	0.561689
"	C	I	-2.294382	1.519376	1.081796
"	I	M	-1.917933	2.023720	0.573038
Cottonwood	CI	M	-3.212865	1.730546	1.427580
Douglas-fir	C	M	-2.235126	1.969155	0.759926
"	C	I	-2.450935	1.652942	1.086198
"	I	M	-2.648728	1.784849	1.102126
Hemlock	C	M	-2.061610	2.032860	0.674654
"	C	I	-2.555949	1.715863	1.104294
"	I	M	-2.575549	1.999408	0.959865
Lodgepole pine	CI	M	-1.673752	2.088139	0.449730
Larch	I	M	-2.667549	1.687852	1.156114
Maple	C	M	-2.734138	1.726673	1.174060
Spruce	C	M	-2.084657	1.956776	0.741786
"	I	M	-2.457243	1.831805	0.995308
White pine	CI	M	-1.265978	2.430799	0.043597
Yellow cedar	CI	M	-2.405174	1.782339	1.001298
Yellow pine	CI	M	-2.545619	2.296438	0.727664

[8] see fn. 4 and 5 in table I.

the logarithm of diameter is minimized is one of the reasons for these apparent contradictions.Another reason can be the fact that the basic data used for the calculations of the B.C.F.S. logarithmic volume equations included for certain species deformed trees,for example forked trees for redcedar (Browne,1962),while the B.C.F.S. taper curves are probably only based on normal trees.Table IV gives an example of three species, two where the similarity is high and one where the deviations are rather large.

TABLE IV

Comparison of the British Columbia Forest Service Logarithmic Volume Equations with the Volume Equations from Table III

Species group(R M)[9] Differences as a percentage of B.C.F.S. volume

Spruce (I M)

dbh (inch.)	Total height (feet) 20	40	60	80	100	120	140	160	180	200
10	+5.41	+2.62	+1.02	-0.10	-0.96	-1.66				
20		+1.95	+0.36	-0.75	-1.61	-2.30	-2.88			
30			-0.02	-1.13	-1.98	-2.67	-3.25	-3.75		
40				-1.40	-2.25	-2.94	-3.15	-4.01	-4.45	
50					-2.45	-3.14	-3.72	-4.21	-4.65	-5.04
60					-2.62	-3.31	-3.88	-4.38	-4.81	-5.20

Hemlock (I M)

dbh (inch.)	Total height (feet) 20	40	60	80	100	120	140	160	180	200
10	+0.80	-0.39	-1.08	-1.56	-1.94					
20		+1.68	+0.98	+0.48	+0.10	-0.21	-0.48			
30			+2.20	+1.70	+1.31	+1.00	+0.73	+0.50	+0.30	
40				+2.57	+2.18	+1.86	+1.59	+1.36	+1.16	+0.97
50					+2.86	+2.54	+2.27	+2.04	+1.83	+1.64
60					+3.42	+3.10	+2.82	+2.59	+2.38	+2.20

White pine (CI M)

dbh (inch.)	Total height (feet) 20	40	60	80	100	120	140	160	180	200
10		+79.69	+22.21	-7.04	-24.81					
20				+37.39	+11.12	-6.56	-19.30			
30					+39.65	+17.42	+1.42	-10.68		
40						+38.09	+19.26	+5.04	-6.08	
50							+35.24	+19.12	+6.50	-3.65
60							+49.88	+32.01	+18.02	+6.77

[9] see fn. 4 and 5 in table I.

Derivation of a compatible logarithmic taper equation from a logarithmic volume equation

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Any logarithmic volume equation

$$\log V = a + b \log D + c \log H$$

can be converted into a logarithmic taper equation

$$\log d = b_0 + b_1 \log D + b_2 \log l + b_3 \log H$$

where $b_0 = \log ((\ 4\ 144\ 10^a\ p\ c\ /\ 3.1416\ )^{1/2})$

$b_1 = b / 2$

$b_2 = (\ p\ c - 1\ ) / 2$

$b_3 = (\ 1 - p\ )\ c / 2$

where a,b and c are the coefficients from the logarithmic volume equation.The value of p which is not yet defined, has to be chosen so as to minimize the SE_E of diameter. Therefore some data about taper are needed.

This taper equation,when integrated to total volume, will for any value of p yield exactly the same volume as given by the logarithmic volume equation from which it is derived.

(for proof see Appendix 8)

Derivation of compatible logarithmic taper equations from the British Columbia Forest Service logarithmic volume equations

The B.C.F.S. logarithmic volume equations (Browne, 1962) are for 23 species or speciesgroups converted to compatible logarithmic taper equations by selecting the value of p so as to minimize the SE_E of diameter on the B.C.F.S. taper curves (B.C.F.S.,1968).
Figure 1 shows for mature coastal Douglas-fir the SE_E of diameter as a function of the value of p.The value for which the SE_E is minimized is the optimum value to be adopted for p in deriving the taper equation from the volume equation.

A summary of the taper equation coefficients, the optimum p values as well as the SE_E's is given in table V.These taper equations give by integrating the same total volume as given by the B.C.F.S. logarithmic volume equations.The optimum value of p ranged from 2.03 to 2.85 and had a mean value of 2.32.
The average bias of diameter inside bark at the different heights is given in table VI.For most of the species, these taper equations have the same pattern of under- and overestimation as in table II.For some species, however,such as alder,birch,immature coastal Western hemlock and maple,there is a slight overestimation along the entire stem of the tree.This again can be due to the

Figure 1. The standard error of estimate as a function of the value of p in deriving the logarithmic taper equation from the logarithmic volume equation (for mature coastal Douglas-fir).

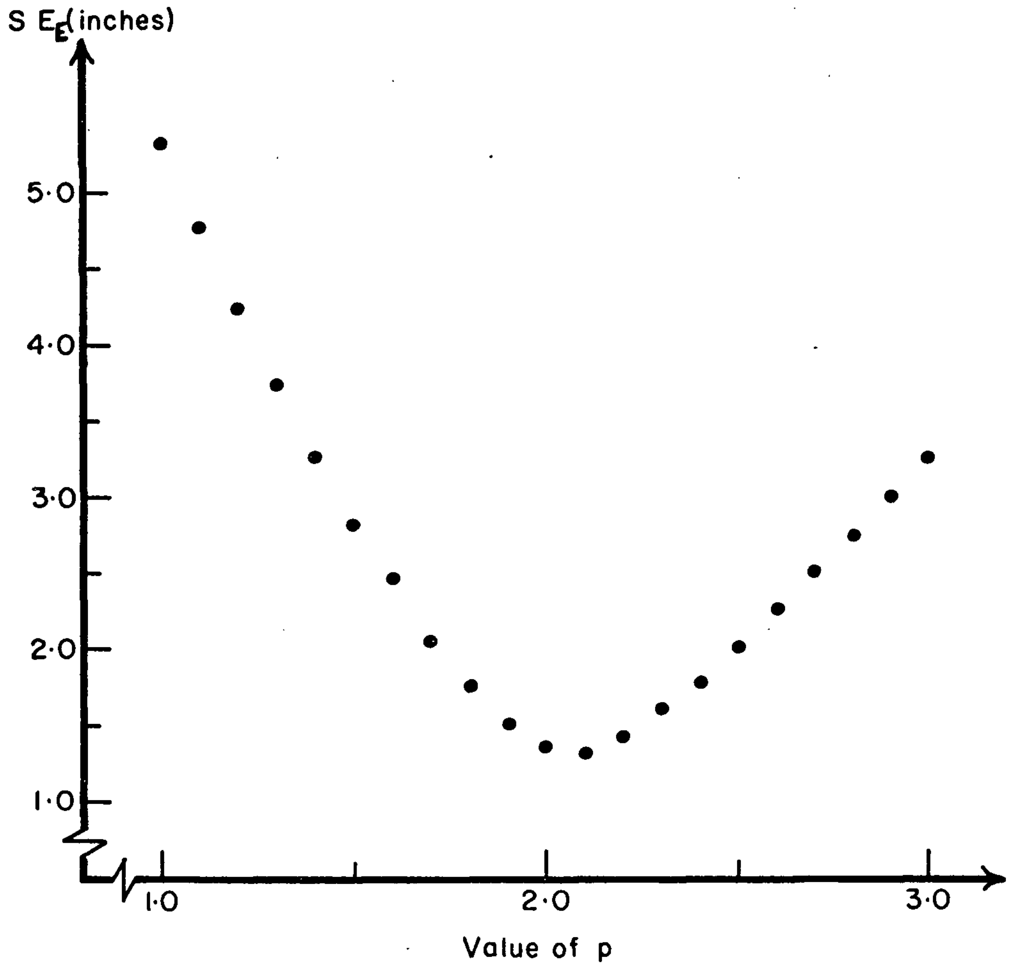

TABLE V

Summary of the Taper Equations Derived from the British Columbia Forest Service Logarithmic Volume Equations

Species group	R	M[10]	b_0	b_1	b_2	b_3	SE_E (inches)	optimum p value
			Equation coefficients					
Alder	C	M	-0.007438	0.960308	0.740496	-0.703485	0.746	2.31
Aspen	I	M	0.004808	0.973017	0.704131	-0.691734	0.532	2.35
Balsam	C	M	0.023528	0.903387	0.643924	-0.596591	0.752	2.09
"	I	M	0.072540	0.932481	0.710907	-0.708456	0.421	2.41
Birch	I	M	-0.035405	0.955840	0.826483	-0.773782	0.505	2.40
Cedar	C	M	0.177694	0.841150	0.981589	-0.961733	2.002	2.85
"	C	I	0.130780	0.860380	0.875467	-0.850480	1.264	2.62
"	I	M	0.120068	0.850996	0.881813	-0.848294	1.340	2.59
Cotton-wood	CI	M	-0.137444	0.901986	0.776018	-0.656591	0.684	2.06
Douglas fir	C	M	-0.026595	0.829506	0.743543	-0.645686	1.348	2.08
"	C	I	-0.000998	0.869962	0.735172	-0.668579	0.987	2.18
"	I	M	-0.034045	0.869709	0.765145	-0.682128	1.381	2.17
Hemlock	C	M	-0.002533	0.895115	0.742983	-0.680547	0.795	2.21
"	C	I	-0.014590	0.921340	0.786591	-0.724761	1.182	2.29
"	I	M	0.027231	0.984855	0.652863	-0.664361	0.610	2.36
Lodgep. pine	CI	M	-0.004542	0.923752	0.602058	-0.559172	0.760	2.03
Larch	I	M	0.006838	0.923561	0.684947	-0.662943	1.281	2.27
Maple	C	M	-0.033635	0.942906	0.876421	-0.816900	0.483	2.46
Spruce	C	M	-0.002832	0.877085	0.850855	-0.768590	2.322	2.32
"	I	M	0.061738	0.920613	0.756372	-0.739346	0.522	2.43
White pine	CI	M	0.076789	0.933643	0.673334	-0.676158	0.938	2.36
Yellow cedar	CI	M	0.101402	0.870522	0.738370	-0.709152	0.587	2.34
Yellow pine	CI	M	-0.057496	0.954739	0.623679	-0.580839	1.063	2.07

[10] see fn. 4 and 5 in table I.

TABLE VI

Test of the Taper Equations Derived from the British Columbia

Forest Service Logarithmic Volume Equations

Species group	R	M[11]	Average bias (in inches) of diameter inside bark at 1'	4.5'	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
Alder	C	M	0.48	0.77	0.88	0.67	0.39	0.19	0.12	0.17	0.32	0.50	0.61	0.0
Aspen	I	M	-0.13	0.47	0.60	0.45	0.25	0.08	-0.01	0.00	0.09	0.27	0.44	0.0
Balsam	C	M	-0.76	-0.15	0.57	0.37	-0.05	-0.38	-0.54	-0.50	-0.28	-0.03	0.17	0.0
"	I	M	-0.44	0.24	0.42	0.24	0.01	-0.14	-0.20	-0.15	-0.05	0.08	0.20	0.0
Birch	I	M	0.26	0.58	0.63	0.48	0.37	0.32	0.32	0.35	0.39	0.40	0.35	0.0
Cedar	C	M	-1.90	-1.21	2.18	2.68	1.72	0.71	-0.11	-0.65	-1.02	-1.22	-1.16	0.0
"	C	I	-0.13	0.01	1.21	1.59	1.22	0.74	0.35	0.08	-0.07	-0.09	-0.01	0.0
"	I	M	-1.98	-1.03	1.03	1.35	0.77	0.07	-0.47	-0.79	-0.91	-0.83	-0.57	0.0
Cottonwood	CI	M	-1.09	0.33	0.50	0.10	-0.28	-0.53	-0.61	-0.50	-0.22	0.13	0.39	0.0
Douglas-fir	C	M	-2.96	0.38	1.22	1.33	0.77	0.09	-0.48	-0.76	-0.73	-0.52	-0.21	0.0
"	C	I	-1.58	0.38	0.74	0.74	0.49	0.16	-0.15	-0.37	-0.46	-0.38	-0.17	0.0
"	I	M	-3.12	0.28	1.09	0.80	0.03	-0.66	-1.10	-1.14	-0.85	-0.39	0.07	0.0
Hemlock	C	M	-1.10	0.04	0.88	0.68	0.19	-0.21	-0.46	-0.50	-0.32	-0.05	0.21	0.0
"	C	I	0.31	0.81	1.39	1.25	0.89	0.59	0.40	0.34	0.45	0.56	0.59	0.0
"	I	M	-0.87	0.25	0.68	0.52	0.23	-0.07	-0.26	-0.26	-0.09	0.24	0.62	0.0
Lodgepole pine	CI	M	-0.39	-0.17	0.15	0.34	0.19	0.02	-0.12	-0.23	-0.24	-0.12	0.06	0.0
Larch	I	M	-2.33	0.70	1.66	1.72	1.18	0.51	-0.03	-0.31	-0.28	0.01	0.46	0.0
Maple	C	M	0.24	0.36	0.56	0.56	0.43	0.33	0.26	0.23	0.23	0.24	0.22	0.0
Spruce	C	M	-2.69	-2.26	1.93	2.09	0.85	-0.29	-1.09	-1.46	-1.40	-1.01	-0.74	0.0
"	I	M	-0.55	0.04	0.67	0.71	0.37	0.02	-0.20	-0.25	-0.19	-0.07	0.08	0.0
White pine	CI	M	-0.74	-0.07	0.61	0.49	0.15	-0.15	-0.34	-0.33	-0.17	0.02	0.06	0.0
Yellow cedar	CI	M	-0.27	-0.20	0.56	0.81	0.56	0.19	-0.18	-0.36	-0.25	0.10	0.36	0.0
Yellow pine	CI	M	-1.89	0.66	0.85	0.52	0.05	-0.35	-0.60	-0.56	-0.31	0.03	0.39	0.0

[11] see fn. 4 and 5 in table I

above mentioned inconsistency for certain species between the B.C.F.S. logarithmic volume equations and the taper curves.Therefore,it is doubtful if the results for coast and interior redcedar,yellow cedar and the deciduous species,for which forking of the stem is a common abnormality,can be used as such.

An example for mature coastal Douglas-fir in table VII shows how the bias is distributed over the various height classes within the same species.Except for the three smallest height classes,the over-all SE_E is a fairly good representative for all the height classes.

Table VIII gives a summary of the SE_E's of table I, table V and those of Kozak,Munro and Smith (1969b,table I). For the logarithmic taper equations,derived from the B.C.F.S. logarithmic volume equations,the SE_E's ranged from .421 to 2.322 inches.Table IX gives the absolute frequency distribution of the SE_E's for each case.
An absolute frequency distribution of the differences in SE_E's is shown in table X.
Because they include the errors inherent in both bark and wood,these SE_E's are relatively small compared with the SE_E's of section double bark thickness,estimated from diameter outside bark,total height,section height and section height as a percentage of tree height,ranging from .111 to .842 inches (Smith and Kozak,1967).
The SE_E of the logarithmic taper equation,fitted on the taper curves or derived from the logarithmic volume

TABLE VII

Distribution of the Bias over the Different Height Classes within the Same Species (for Mature Coastal Douglas-fir)

Height class (feet)	Average bias (in inches) of diameter inside bark at												SE_E (inches)
	1'	4.5'	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H	
50	-0.20	0.65	0.61	0.47	0.19	-0.14	-0.42	-0.68	-0.85	-0.83	-0.59	0.0	0.610
60	-0.26	0.71	0.77	0.65	0.29	-0.03	-0.28	-0.47	-0.59	-0.61	-0.39	0.0	0.560
70	-0.04	0.73	0.82	0.56	0.16	-0.14	-0.26	-0.28	-0.21	0.00	0.21	0.0	0.458
80	-1.07	0.47	0.85	0.95	0.48	-0.15	-0.73	-1.11	-1.24	-1.24	-0.88	0.0	1.168
90	-1.83	0.70	1.23	1.21	0.63	-0.08	-0.73	-1.23	-1.65	-1.80	-1.42	0.0	1.348
100	-2.12	0.71	1.32	1.28	0.65	0.04	-0.44	-0.69	-0.90	-0.90	-0.49	0.0	1.109
110	-2.69	0.59	1.47	1.29	0.65	0.04	-0.48	-0.82	-0.82	-0.49	-0.13	0.0	1.345
120	-3.60	0.50	1.46	1.34	0.66	-0.10	-0.76	-1.08	-1.13	-1.11	-0.99	0.0	1.449
130	-4.13	0.46	1.42	1.37	0.74	-0.03	-0.61	-0.76	-0.60	-0.42	-0.24	0.0	1.452
140	-4.55	0.08	1.62	1.52	1.00	0.20	-0.39	-0.61	-0.53	-0.40	-0.21	0.0	1.564
150	-5.03	0.13	1.44	1.57	1.02	0.32	-0.12	-0.31	-0.30	-0.28	-0.13	0.0	1.667
160	-3.98	0.10	1.43	1.63	1.07	0.48	-0.07	-0.48	-0.40	-0.07	0.27	0.0	1.426
170	-5.10	-0.06	1.27	1.77	1.08	0.38	-0.13	-0.44	-0.24	0.30	0.64	0.0	1.701
180	-4.46	0.16	1.88	1.70	1.03	0.29	-0.37	-0.54	-0.07	0.45	0.70	0.0	1.613
190	-3.57	0.09	-0.21	1.48	1.01	0.16	-0.77	-1.09	-0.63	-0.10	0.28	0.0	1.658
200	-3.63	0.23	1.69	1.94	1.24	0.21	-0.61	-0.78	-0.40	0.23	0.81	0.0	1.502
Total	-2.96	0.38	1.22	1.33	0.77	0.09	-0.48	-0.76	-0.73	-0.52	-0.21	0.0	1.348

TABLE VIII

Comparison of Standard Errors of Estimate of Several Methods

Species group	R	M[12]	$SE_E(1)$[13]	$SE_E(2)$[14]	$SE_E(3)$[15]
Alder	C	M	0.84	0.587	0.746
Aspen	I	M	0.59	0.448	0.532
Balsam	C	M	0.90	0.813	0.752
"	I	M	0.58	0.420	0.421
Birch	I	M	0.32	0.245	0.505
Cedar	C	M	2.13	2.431	2.002
"	C	I	1.61	1.128	1.264
"	I	M	1.30	1.379	1.340
Cottonwood	CI	M	0.84	0.755	0.684
Douglas-fir	C	M	1.54	1.431	1.348
"	C	I	1.35	1.032	0.987
"	I	M	1.33	1.251	1.381
Hemlock	C	M	0.98	0.871	0.795
"	C	I	1.16	0.691	1.182
"	I	M	0.73	0.740	0.610
Lodgepole pine	CI	M	0.72	0.774	0.760
Larch	I	M	1.33	1.230	1.281
Maple	C	M	0.41	0.316	0.483
Spruce	C	M	2.34	2.376	2.322
"	I	M	0.71	0.526	0.522
White pine	CI	M	1.01	1.159	0.938
Yellow cedar	CI	M	0.78	0.574	0.587
Yellow pine	CI	M	1.02	1.205	1.063

[12] see fn. 4 and 5 in table I.
[13] from table I of Kozak, Munro and Smith (1969b)
[14] equations fitted on the taper curves (table I)
[15] equations derived from the volume equations (table V)

TABLE IX

Absolute Frequency Distribution of the

Standard Errors of Estimate

SE_E (inches)	Number of speciesgroups (1)	(2)	(3)[16]
.0 < - ≤ .5	2	4	2
.5 < - ≤1.0	10	9	12
1.0 < - ≤1.5	7	8	7
1.5 < - ≤2.0	2	-	-
2.0 < - ≤2.5	2	2	2

[16]see fn. 13,14 and15 in table VIII.

TABLE X

Absolute Frequency Distribution of the Differences

in Standard Errors of Estimate

Difference (inches)	Number of speciesgroups (2) BT (1)	(1) BT (2)	(3) BT (1)	(1) BT (3)	(3) BT (2)	(2)[17] BT[18] (3)
.0 — .1	6	4	5	6	9	4
.1 — .2	6	2	9	1	3	4
.2 — .3	2	1	-	-	-	1
.3 — .4	2	-	2	-	-	-
.4 — .5	-	-	-	-	1	1

[17]see fn. 13,14 and 15 in table VIII.

[18]BT means Better Than.

equation,is for sixteen speciesgroups out of twenty-three smaller than the SE_E given by Kozak,Munro and Smith (1969b). The SE_E of the taper function derived from the volume equation is for thirteen speciesgroups smaller,but for ten speciesgroups larger than the SE_E of the taper equation fitted on the taper curves.

IMPROVEMENT OF THE ACCURACY AND THE PRECISION OF THE LOGARITHMIC TAPER EQUATION

A well known technique for improving the accuracy and precision of a taper equation consists of relating the taper equation coefficients to some known tree characteristics. In most inventory work only dbh outside bark and total height are measured. Although the number of possible relationships to investigate is large, a close look was taken only into some very simple approaches.

To improve the taper equation fitted on the taper curves, the relationships between the taper equation coefficients (b_0, b_1, b_2 and b_3) and dbh, total height and the ratio of both were investigated. The correlation was very poor (apparently a second or third degree polynomial) or non-existent, except for b_2 where a good relationship with the ratio of total height over dbh was always present. However, this relationship did not have the same pattern for the different species. A trial of a taper equation model in which each coefficient was expressed as a second degree polynomial function of the ratio of total height over dbh was successful only for five species, decreasing the SE_E by from .1 to .3 inches.

To improve the taper equation derived from the volume equation the relationship between the optimum value of p and dbh,total height and the ratio of both was investigated.This was attempted for mature coastal Douglas-fir.A good correlation was found between the optimum value of p and total height (see figure 2). It is expected that using this relationship between the optimum value of p and total height,instead of only the over-all optimum value,can improve the precision and accuracy.But even if the optimum value of p for each total height class can be predicted from total height without error,the decrease in SE_E will only be moderately important for the smallest height classes (see table XI). The over-all SE_E will likely change only little.

Figure 2· Optimum value of p as a function of total height (for mature coastal Douglas-fir)·

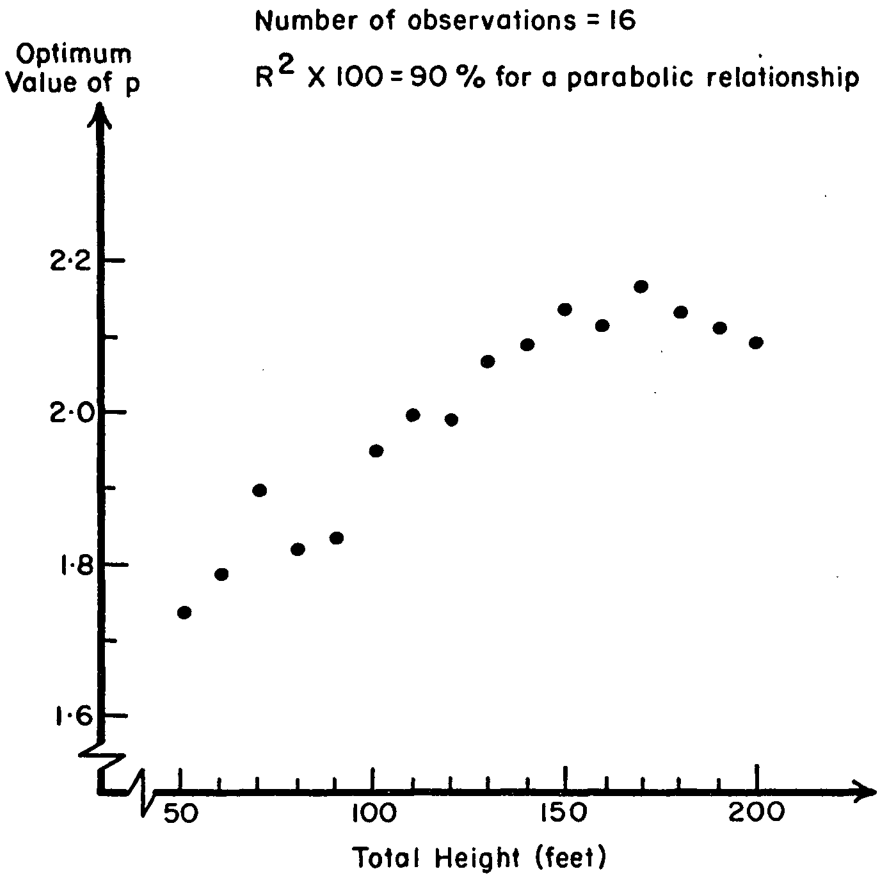

TABLE XI

Maximum Decrease in Standard Error of Estimate to Be Expected from Using the Relationship between the Optimum Value of p and Total Height (for Mature Coastal Douglas-fir)

Height class (feet)	Optimum value of p	SE_E (inches) (1)[19]	(2)[20]
50	1.75	0.610	0.350
60	1.80	0.560	0.365
70	1.92	0.458	0.383
80	1.84	1.168	0.957
90	1.85	1.348	1.117
100	1.97	1.109	1.046
110	2.01	1.345	1.321
120	2.01	1.449	1.409
130	2.09	1.452	1.441
140	2.11	1.564	1.550
150	2.15	1.667	1.626
160	2.13	1.426	1.396
170	2.18	1.701	1.618
180	2.15	1.613	1.562
190	2.13	1.658	1.633
200	2.12	1.502	1.481

[19] using the over-all optimum value of p

[20] using for each total height class the appropriate optimum value of p

DISCUSSION, SUMMARY AND SUGGESTIONS

This proposed system of taper and volume functions derived from each other and compatible with each other can meet the requirements stated by Honer and Sayn-Wittgenstein (1963).

The taper function fitted on the taper curves as well as the equation derived from the volume equation describes well the stem profile of the most important species of British Columbia. However, it should be realized that a taper equation fitted on diameter data gives no guarantee of a good volume equation. Tests on tree measurements should be carried out in this field. Whenever a taper equation is fitted on data, the function should be tested both for diameter and volume to know for both the precision and the accuracy of the system. This was recognized by Bruce and Schumacher (1950) and done by Duff and Burstall (1955) in the application of graphical techniques.
Giving more weight to large diameters would ensure a better fit for volume, probably resulting in a better fit at the butt of the tree but a worse fit higher on the stem. This would make the equation less suitable for prediction of section diameters or heights. Instead of weighting, the dependent variable could be taken as d^2

and the calculation of the taper equation could be done by a non-linear least squares procedure.But this again would probably have the same disadvantages as weighting.

The trials to improve the taper equation by relating the taper equation coefficients to some known tree characteristics were not comprehensive enough to draw final conclusions.The preliminary investigations in that direction were discouraging.

Preference is given to the system in which the taper equation is derived from the logarithmic volume equation.In this way the best fit is achieved for volume and the fit for diameter is optimized by the choice of the optimum value of p.Moreover,it is the only possible way to create a truly compatible system of taper and volume in those instances where a logarithmic volume equation already exists and probably will continue to be used in the future.

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APPENDIX 1

Common Names and Latin Names of the Tree Species [21]

1. Red Alder (Alnus rubra Bong.).
2. Trembling Aspen (Populus tremuloides Michx.).
3. Coast Balsam Species (Abies amabilis (Dougl.) Forbes and A.grandis (Dougl.) Lindl.).
4. Interior Balsam Species (Abies lasiocarpa (Hook.) Nutt. and A.grandis).
5. White Birch Species (Betula papyrifera varieties).
6. Western Red Cedar (Thuja plicata Donn).
7. Black Cottonwood (Populus trichocarpa Torr. and Gray).
8. Douglas Fir (Pseudotsuga menziesii (Mirb.) Franco).
9. Western Hemlock (Tsuga heterophylla (Raf.) Sarg.).
10. Lodgepole Pine (Pinus contorta Dougl.).
11. Western Larch (Larix occidentalis Nutt.).
12. Broadleaf Maple (Acer macrophyllum Pursh).
13. Coastal Spruce (Picea sitchensis (Bong.) Carr.).
14. Interior Spruce Species (Picea glauca (Moench) Voss, P.Engelmanni Parry,and P.mariana (Mill.) B.S.P.).
15. Western White Pine (Pinus monticola Dougl.).
16. Yellow Cedar (Chamaecyparis nootkatensis (D.Don) Spach).
17. Western Yellow Pine (Pinus ponderosa Laws.).

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[21] Based on Appendix I from Browne (1962).

APPENDIX 2

Derivation of a Volume Equation from the Taper Equation of Höjer

$$V = D^2H\ 0.005454 \int_0^1 (d^2 / D^2)\ d\ (1 / H) \tag{1}$$

$$d^2 / D^2 = c_1^2\ (\ln((c_2 + 1\ 100 / H) / c_2))^2 \tag{2}$$

substituting (2) in (1)

$$V = D^2H\ 0.005454 \int_0^1 (c_1^2(\ln((c_2 + 100\ 1 / H) / c_2))^2)\ d\ (1 / H) =$$

$$= D^2H\ 0.005454\ c_1^2 \left[(1 + \frac{1\ 100}{H\ c_2})(\ln(1 + \frac{1\ 100}{H\ c_2}))^2 - 2\ (1 + \frac{1\ 100}{H\ c_2})(\ln(1 + \frac{1\ 100}{H\ c_2})) + 2\ (1 + \frac{1\ 100}{H\ c_2}) \right]_1^0 =$$

$$= D^2H\ 0.005454\ c_1^2 \left((1 + \frac{100}{c_2})(\ln(1 + \frac{100}{c_2}))^2 - 2\ (1 + \frac{100}{c_2})(\ln(1 + \frac{100}{c_2})) + 2\ (1 + \frac{100}{c_2}) - 2 \right) =$$

$$= D^2H\ 0.005454\ c_1^2\ (K(\ln K(\ln K - 2) + 2) - 2)$$

where $K = 1 + \dfrac{100}{c_2}$

APPENDIX 3

Derivation of a Volume Equation from the Taper Equation of Behre

$$V = D^2H\ 0.005454 \int_0^1 (d^2 / D^2)\ d\ (1 / H) \tag{1}$$

$$d^2 / D^2 = (1 / H)^2/(b_0 + b_1\ 1 / H)^2 \tag{2}$$

substituting (2) in (1)

$$V = D^2H\ 0.005454 \int_0^1 ((1 / H)^2/(b_0 + b_1\ 1 / H)^2)\ d\ (1 / H) =$$

$$= D^2H\ 0.005454 \frac{1}{b_1^3} \left[(b_0 + b_1\ 1 / H) - 2\ b_0 \ln (b_0 + b_1\ 1 / H) - \frac{b_0^2}{(b_0 + b_1\ 1 / H)} \right]_1^0 =$$

$$= D^2H\ 0.005454 \frac{1}{b_1^3} (b_0 + b_1 - 2\ b_0 \ln (b_0 + b_1) - b_0^2 /(b_0 + b_1) + 2\ b_0 \ln b_0)$$

If $b_o + b_1 = 1$ what was usually the case,according to Behre (1927) then

$$V = D^2H\ 0.005454 \frac{1}{b_1^3} (1 - b_0^2 + 2\ b_0 \ln b_0)$$

APPENDIX 4

Derivation of a Volume Equation from the Taper Equation of Matte

$$V = 0.005454 \int_0^H (d^2)\; d(1) \tag{1}$$

$$d^2 = b_0\, D^2\, (1^2 / H^2) + b_1\, D^2\, (1^3 / H^3) + b_2\, D^2\, (1^4 / H^4) \tag{2}$$

substituting (2) in (1)

$$V = 0.005454\, D^2 \int_0^H (b_0\, (1^2 / H^2) + b_1\, (1^3 / H^3) + b_2\, (1^4 / H^4))\; d\,(1) =$$

$$= 0.005454\, D^2 \left[\frac{b_0\, 1^3}{3\, H^2} + \frac{b_1\, 1^4}{4\, H^3} + \frac{b_2\, 1^5}{5\, H^4} \right]_0^H =$$

$$= 0.005454\, D^2\, H\, (b_0 / 3 + b_1 / 4 + b_2 / 5)$$

APPENDIX 5

Derivation of the Height Equation from the Logarithmic Taper Equation

The taper equation (formula 2) is:

$$d = 10^{b0}\ D^{b1}\ l^{b2}\ H^{b3}$$

thus

$$l^{b2} = d\ /\ (10^{b0}\ D^{b1}\ H^{b3})$$

$$l = (d\ /\ (10^{b0}\ D^{b1}\ H^{b3}))^{1/b2}$$

or

$$l = (10^{-b0}\ d\ D^{-b1}\ H^{-b3})^{1/b2}$$

APPENDIX 6

Derivation of a Logarithmic Volume Equation from the Logarithmic Taper Equation

$$V = 0.005454 \int_0^H (d^2)\, d\,(l) \qquad (1)$$

$$d^2 = 10^{2b_0}\, D^{2b_1}\, l^{2b_2}\, H^{2b_3} \qquad (2)$$

substituting (2) in (1)

$$V = 0.005454\ 10^{2b_0} \int_0^H (D^{2b_1}\, l^{2b_2}\, H^{2b_3})\, d\,(l) =$$

$$= 0.005454\ 10^{2b_0} \left[\frac{D^{2b_1}\, l^{2b_2+1}\, H^{2b_3}}{(2\ b_2 + 1)} \right]_H^0 =$$

$$= \frac{0.005454\ 10^{2b_0}}{(2\ b_2 + 1)}\, D^{2b_1}\, H^{2b_2 + 2b_3 + 1}$$

after taking the logarithm

$$\log V = a + b \log D + c \log H$$

$$\text{where } a = \log \left(\frac{0.005454\ 10^{2b_0}}{(2\ b_2 + 1)} \right)$$

$$b = 2\ b_1$$

$$c = 2\ b_2 + 2\ b_3 + 1$$

APPENDIX 7

Derivation of the Formula to Estimate Volumes of Logs between Specific Distances from the Tip of the Tree, from the Logarithmic Taper Equation

When the lower and upper distance from the tip of the tree are respectively l_1 and l_2, then the volume of the log between these two distances is:

$$V = 0.005454 \int_{l_2}^{l_1} (d^2)\, d\,(l) =$$

$$= 0.005454\ 10^{2b_0} \int_{l_2}^{l_1} (D^{2b_1}\ l^{2b_2}\ H^{2b_3})\, d\,(l) =$$

$$= \frac{0.005454\ 10^{2b_0}}{(2\ b_2 + 1)} \left[D^{2b_1}\ l^{2b_2+1}\ H^{2b_3} \right]_{l_1}^{l_2} =$$

$$= \frac{0.005454\ 10^{2b_0}}{(2\ b_2 + 1)} D^{2b_1}\ H^{2b_3}\ (l_1^{2b_2+1} - l_2^{2b_2+1}) =$$

$$= K\ D^v\ H^y\ (l_1^z - l_2^z)$$

where $K = \dfrac{0.005454\ 10^{2b_0}}{(2\ b_2 + 1)}$ $\qquad y = 2\ b_3$

$v = 2\ b_1$ $\qquad z = 2\ b_2 + 1$

APPENDIX 8

Derivation of a Compatible Logarithmic Taper Equation from a Logarithmic Volume Equation

The proof is given that the taper equation

$$\log d = b_0 + b_1 \log D + b_2 \log l + b_3 \log H$$

where $b_0 = \log \left(\frac{10^a \; p \; c}{0.005454} \right)^{1/2}$ $\qquad b_2 = (p\,c - 1)/\,2$

$b_1 = b\,/\,2$ $\qquad b_3 = (1 - p)c\,/\,2$

yields for any value of p,after integrating,the same volume as given by the logarithmic volume equation

$$\log V = a + b \log D + c \log H$$

Proof:

$$V = 0.005454 \int_0^H (d^2)\; d\;(l) \qquad (1)$$

from the taper equation,d^2 is defined as

$$d^2 = \frac{10^a \; p \; c}{0.005454} \, D^b \, l^{(p\,c\,-\,1)} \, H^{(1\,-\,p)\,c} \qquad (2)$$

substituting (2) in (1)

$$V = 0.005454 \int_0^H \left(\frac{10^a \; p \; c}{0.005454} \; D^b \; l^{(p\;c\;-\;1)} \; H^{(1\;-\;p)\;c} \right) d\;(l) =$$

$$= \frac{0.005454 \; 10^a \; p \; c}{0.005454 \; p \; c} \left[D^b \; l^{p\;c} \; H^{(1\;-\;p)\;c} \right]_H^0 =$$

$$= 10^a \; D^b \; H^{p\;c} \; H^{c\;-\;p\;c} =$$

$$= 10^a \; D^b \; H^c$$

after taking the logarithm

$$\log V = a + b \log D + c \log H$$

this completes the proof.