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SOME METHODS OF SAMPLING TRIANGLE BASED PROBABILITY POLYGONS FOR FORESTRY APPLICATIONS

by

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ABSTRACT

There is interest in forest sampling methods which have the ability to provide reliable estimates of volume without incurring unreasonable costs. Fraser (1977), to this end, described an individual tree variable probability sampling method which selects sample trees with probabilities based on the areas of polygons derived from triangles. A comparison of some alternative methods of sampling these polygons confirms Fraser's work and demonstrates that the method proposed by him probably has the greatest potential for practical forest sampling.

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SOME METHODS OF SAMPLING TRIANGLE BASED PROBABILITY POLYGONS FOR FORESTRY APPLICATIONS

INTRODUCTION

Of the various levels of forest inventory, the operational cruise requires the most precise estimate of volume since it is usually on the basis of this estimate that investment decisions are made. There is interest, therefore, in sampling techniques which provide such estimates without incurring greatly increased costs. With the availability of dendrometers and inexpensive data processing more attention has been given to methods whose sample units are individual trees rather than groups of trees. Grosenbaugh (1967) demonstrated that selecting single trees with probabilities related to their size is more efficient, statistically, than point or plot sampling. Jack (1967) and Fraser (1977) described individual tree methods which select sample trees with probabilities based on the areas of polygons. These area-based methods have the advantage of not requiring a visit to every tree of the target population, as do methods which select trees from a list or which are based on ocular estimates of tree size. Hence area-based methods are better suited to the measurement of large tracts of timber. Furthermore, Fraser's method, which is based on the location of triangles whose vertices are points on the ground defined by tree stems, is relatively easy to apply in the field and provides additional information on stand density and tree spatial distribution.

The purpose of this study is to investigate some alternative methods related to the method outlined by Fraser in order to:

a) independently confirm his work,
b) determine if any improvements in statistical efficiency can be provided by these alternatives and
c) provide insights for further work.

It is hoped that this investigation will help to further the development of the use of triangle based probability polygons in forest measurements.

CHAPTER ONE

LITERATURE REVIEW

In the context of allocating areas to individual trees, Brown (1965) constructed polygons whose sides were perpendicular bisectors of line segments joining tree stem positions. This resulted in a set of polygons (historically named either Dirichlet cells or Voronoi polygons), one per tree, which had no gaps or overlaps. Brown called the area of a polygon "Area Potentially Available" (APA) to the tree which it contained and used it as a measure of point density. He demonstrated that using the APA concept one could detect correlation between basal area and tree density more readily than with the conventional fixed radius plot method of determining tree density. He also indicated the utility of APA as a competition index.

Jack (1967) employed the polygons described by Brown in the development of a single tree sampling technique. In this method a tree is selected as part of a sample when a uniformly distributed random coordinate point falls within its polygon. Thus, trees are selected with probability proportional to their APA. Jack concluded from preliminary trials that this sampling method will give results having acceptable limits of accuracy at lower cost than other methods which require visiting every tree, where the sampled area is reasonably large. In addition, he gives results of using APA in the prediction of tree volume increment, showing that a slight improvement in prediction can be made with the inclusion of APA.

In the interests of obtaining better correlation between APA and tree size, Adlard (1974) proposed adjusting polygon sides such that they no longer bisected the line segments between trees but instead divided the segments at a point weighted by tree size. This resulted in allocating more APA to larger stems.

Fraser and van den Driessche (1971) discussed describing the line segments which join points in a plane to form a network of non-overlapping triangles. Such networks have consistent traits, that is, a population of N points yields 2N triangles with 3N common sides. Also, a single point has an expected value of six sides radiating from it. Thus, sampling triangles for average triangle area enables one to estimate population density and total population size. In addition, variances of triangle areas and triangle side lengths can be used to indicate regularity and degree of clumping of points. Construction of triangle sets is facilitated with the selection of least diagonal neighbour (LDN) pairs of points. A pair of points are LDN's provided that no other point occurs on the line segment between the pair and that the line segment cannot be intersected by a shorter line segment between any other pair of points. Except for a few special cases, forming triangles from pairs of points defined this way will result in a unique set of triangles.

Fraser (1977) advanced the use of LDN triangle networks constructed among tree stem positions with the development of a single tree variable probability sampling method based on such networks. By allocating a portion of the area of a triangle to each of its vertex trees according to some proportioning scheme one can construct polygons around trees. Having located the LDN triangle in which a sample point falls, one calculates the probabilities of selection of the three vertex trees (based on the chosen proportioning scheme) and then selects a tree by list sampling with variable probabilities. Fraser compared two formulae for proportioning triangle areas, one being a direct proportioning according to tree size and the other being a geometric proportioning which "conceptually" divides the triangle into three quadrilaterals resulting from partitioning triangle sides according to tree size and joining the partitioning point to the opposite triangle vertex. As such, one need not think in terms of physical polygons but only in terms of probabilites. For each formula of area partitioning he applied four different measures of tree size or proportioning weights. These were 1 (or equal weights), tree diameter at breast height (D), D^2 (or basal area), and $D^2\alpha$ (or portion of basal area found in a triangle) where α is the angle measure of the triangle vertex. It should be noted that field measurements require only conventional tree volume measures plus triangle side distances. Angles, areas, and proportions are calculated later. Fraser found that the geometric proportioning formula using the $D^2\alpha$ weight resulted in the most precise estimate of volume.

Fraser also pointed out that work on APA polygons can be related to triangle based polygons. For example, three trees are vertices of a Delauney triangle provided no other points occur on or within the circumcircle of the triangle. The centre of the circle is found by the intersection of perpendicular bisectors of triangle sides. Rogers (1964) proves that the polygons formed by these bisectors are the same as the Voronoi polygons. Thus, the polygons used by Brown and Jack may also be described as triangle based polygons where the vertex trees are weighted equally and the triangles are partitioned by the polygons formed by the perpendicular bisectors of their sides. Fraser suggests that these Voronoi polygons are statistically inefficient in the context of sampling for tree volumes. While Adlard's modification might improve this efficiency there would be considerable difficulty in implementing practical field procedures in order to establish the related Delauney triangles.

CHAPTER TWO

METHOD OF ANALYSIS

THE BASIC METHOD

Fraser (1977) outlined a sampling method utilizing polygons formed from triangles constructed among LDN trees (Figure 1). This method considers polygons as variable sized plots, each containing one whole tree, and is hereafter called the Basic Method. For a sample size of n (i= 1 to n), if:

y_i = volume of sample $tree_i$,

a_i = area of polygon containing sample $tree_i$,

A = total area covered by target population,

z_i = probability of selecting $tree_i$ = a_i/A, then the estimate of total volume (probability proportional to estimated size) is:

$$\text{Yppes} = \frac{1}{n} \sum_{i=1}^{n} \frac{y_i}{z_i} = A \left[\frac{1}{n} \sum_{i=1}^{n} \frac{y_i}{a_i} \right] \qquad (1)$$

and its variance is:

$$V(\text{Yppes}) = \frac{A^2}{n} \left[\frac{1}{(n-1)} \sum_{i=1}^{n} \left(\frac{y_i}{a_i} - \frac{1}{n} \sum_{i=1}^{n} \frac{y_i}{a_i} \right)^2 \right] \qquad (2)$$

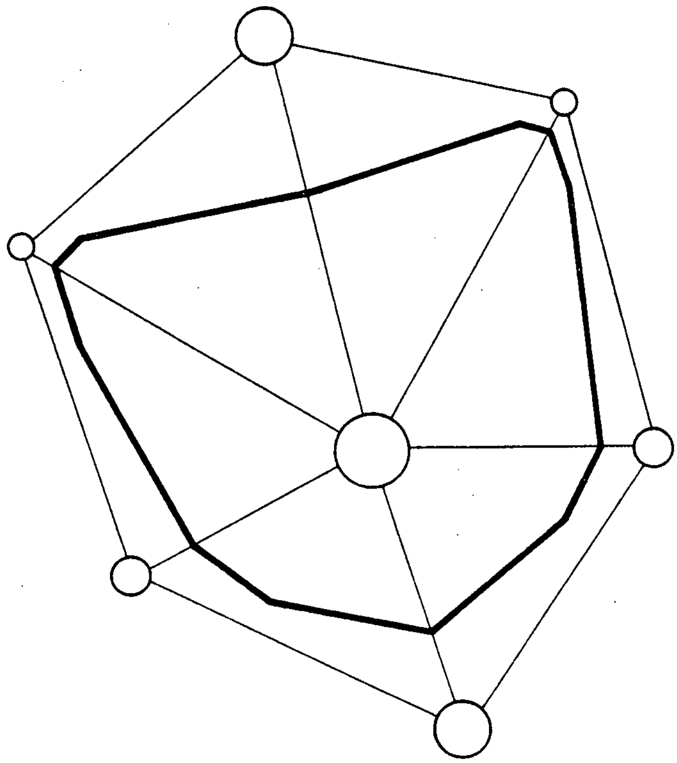

Figure 1 Stem map showing least diagonal neighbours and polygon of the type used in the Basic Method

The field procedure for sampling using this type of polygon is as follows:

i) Locate a sample point and establish the LDN triangle among trees in which it falls.

ii) Take the necessary measurements for determining proportioning weights from each vertex tree of the triangle. Using the desired weighting and proportioning formula calculate the probabilities of selecting each tree: P_1, P_2, and P_3. Generate a uniformly distributed random number between 0 and 1. Select tree 1, if the random number is less than or equal to P_1; select tree 2 if the random number is greater than P_1 but less than or equal to $P_1 + P_2$; otherwise select tree 3.

iii) Measure selected tree for volume.

iv) Locate the remaining LDN trees of the selected tree and record their weighting measures and side lengths of the triangles which they form. These measurements are necessary for the calculation of polygon areas. Note that angle measurements are not required as they are also calculated from side distances.

In summary this method requires the measurement of:

1 tree measured for volume (assumed to include measures for proportioning weights),

6 (average) trees measured for proportioning weights,

12 (average) distance measures between LDN trees (this is based on the fact that a point in a triangle network has an expected value of 6 sides radiating from it),

1 probability calculation in the field in order to select volume tree.

Four alternative methods related to the Basic Method are now proposed. In these methods the same formulae for Yppes and V(Yppes) apply, however, the calculation of the y_i and a_i and the field procedures differ. In particular, three of the methods dispense with any probability calculation in the field. In the following discussion, in order to keep notation simple, the same variable names are kept throughout, even though their meaning may change slightly from method to method. It is felt that this will be more easily understood than having a completely different set of variable names for each method.

METHOD 1

This consists of a sample unit of only a part of one tree of a field selected LDN triangle (Figure 2). The selected tree is chosen with probability proportional to its quadrilateral area. This method considers quadrilaterals as variable sized plots, each containing the fractional part of the tree which falls within the quadrilateral. In this case, if

α = size of triangle vertex angle at which volume tree is located,

v = volume of volume tree,

then

y_i = volume of portion of volume tree contained in triangle

$$= \frac{v\alpha}{2\pi}$$

a_i = area of quadrilateral containing volume tree.

Sampling using this method requires per sample unit:

1 tree measured for volume (includes a weighting measure),

2 trees measured for proportioning weights,

3 distance measures betweeen LDN trees (weighting and distance measures are used for calculation of quadrilateral area),

1 probability calculation in the field for selection of volume tree.

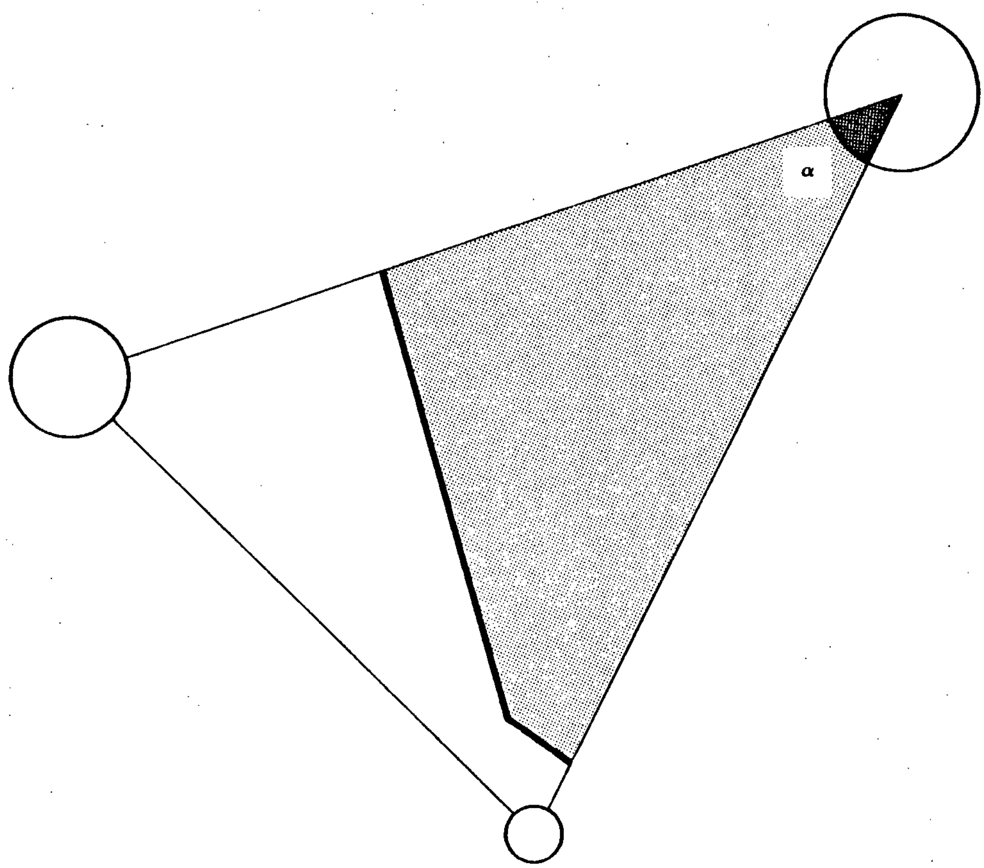

Figure 2 Stem map showing a least diagonal neighbour triangle and quadrilateral of the type used in Method 1

METHOD 2

This is the simplest case of measuring three trees of a field selected triangle (Figure 3). Once a triangle is selected, its three vertex trees are automatically measured. This method considers triangles as variable sized plots, each containing the fractional parts of the three trees which fall within the triangle. If, for the triangle vertices j = 1 to 3 :

α_j = measure of triangle vertex angle_j,

v_j = volume of tree at vertex_j,

then

y_i = sum of volumes of portions of vertex trees contained in triangle

$$= \frac{v_1\alpha_1}{2\pi} + \frac{v_2\alpha_2}{2\pi} + \frac{v_3\alpha_3}{2\pi}$$

$$= \frac{1}{2\pi} \sum_{j=1}^{3} v_j\alpha_j$$

a_i = triangle area.

This method requires per sample unit:

3 trees measured for volume,

3 distance measures between trees (used in calculation of triangle angles and area),

no proportioning weight measures,

no probability calculation in the field.

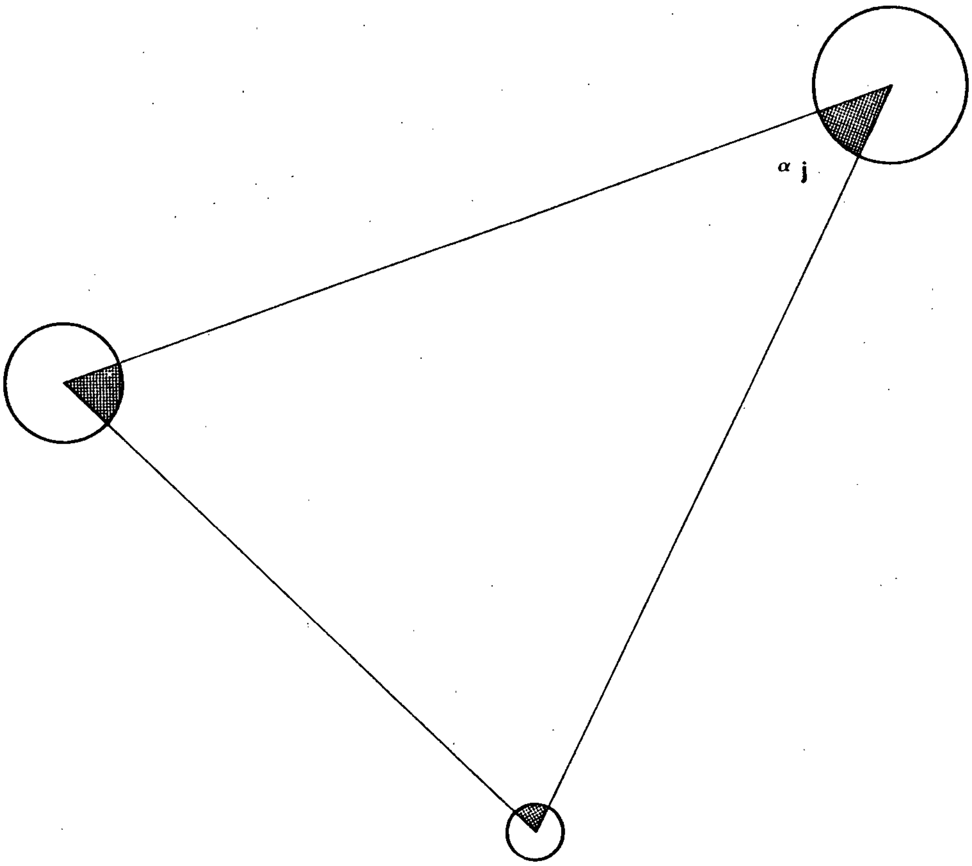

Figure 3 Stem map showing a least diagonal neighbour triangle of the type used in Method 2

METHOD 3

This is another three tree case using a field selected triangle much like Method 2. The distances to the 9 (average) LDN neighbours of the three vertex trees are measured in addition to those measurements required in Method 2 (Figure 4). This method considers triangles as variable sized plots, each containing weighted fractional portions of the three trees at its vertices. The fractional portion of a tree is weighted by the ratio of the selected triangle area to the sum of the areas of all triangles common to that tree. Thus it is not equal to the fractional volume portion falling within the triangle as in Method 2. For the selected triangle vertices j = 1 to 3 let

PL_j = area of the large polygon which is the sum of the areas of all LDN triangles having $tree_j$ at a vertex,

v_j = volume of tree at selected triangle $vertex_j$,

t = area of selected triangle.

The selected triangle has allocated to its area "t" a portion of each tree volume v_j proportional to t/PL_j.

Therefore,

$$y_i = \frac{v_1 t}{PL_1} + \frac{v_2 t}{PL_2} + \frac{v_3 t}{PL_3}$$

$$= t \sum_{j=1}^{3} \frac{v_j}{PL_j}$$

$$a_i = t.$$

This method requires per triangle sampled:

3 trees measured for volume,

24 (average) distance measures between LDN trees,

no proportioning weight measures,

no probability calculations in the field.

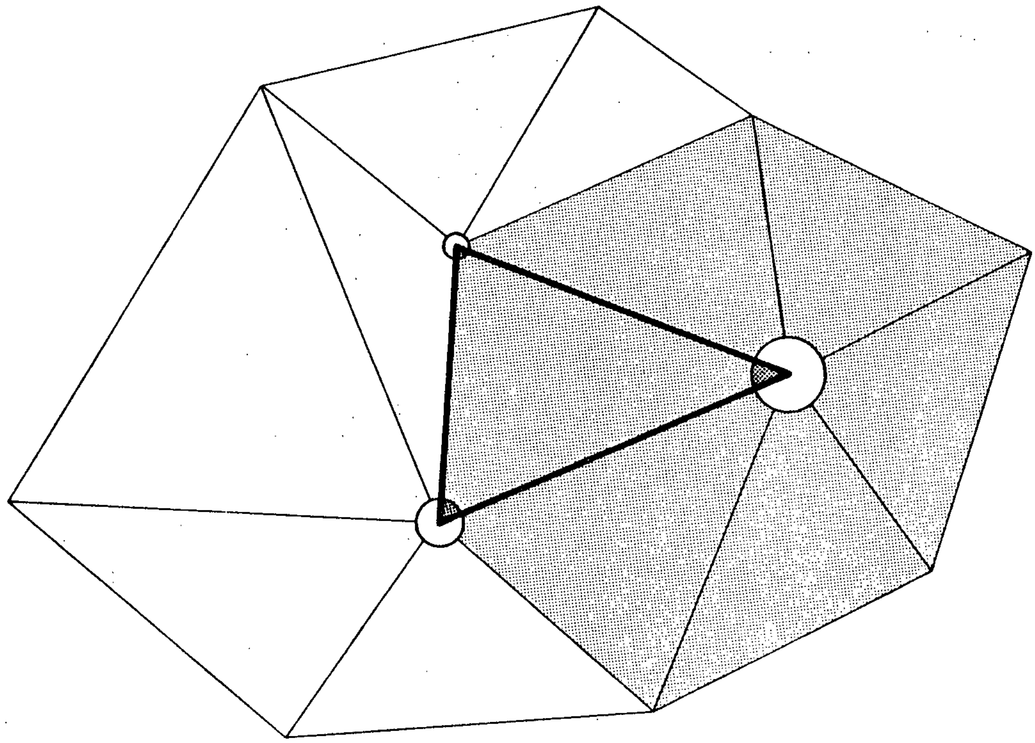

Figure 4 Stem map showing least diagonal neighbours and one shaded large polygon of the type used in Method 3

METHOD 4

This is also a three tree sample of a field selected triangle as in Method 3 except that, in addition, the weighting measures of the nine (average) surrounding LDN trees are taken, and the weights and distances are used to calculate polygon areas in the same manner as in Fraser's Basic Method (Figure 5). This method considers triangles as variable sized plots, each containing weighted fractional portions of the three trees at its vertices as with Method 3. However, unlike Method 3, the fractional portion of a tree is weighted by the ratio of the area of its quadrilateral in the selected triangle to the sum of the areas of all its quadrilaterals. Thus, for the selected triangle vertices j = 1 to 3 let

PS_j = area of the small polygon which is the sum of all quadrilaterals having tree_j at a vertex,

q_j = area of the quadrilateral, in the selected triangle, having tree_j at a vertex,

v_j = volume of tree_j.

A portion of each tree volume v_j proportional to q_j/PS_j is allocated to the selected triangle area. So,

$$y_i = \frac{v_1 q_1}{PS_1} + \frac{v_2 q_2}{PS_2} + \frac{v_3 q_3}{PS_3}$$

$$= \sum_{j=1}^{3} \frac{v_j q_j}{PS_j}$$

a_i = area of selected triangle.

This method requires per triangle sampled:

3 trees measured for volume (includes weighting measures),

24 (average) distance measures between LDN trees,

9 (average) trees measured for proportioning weights (weighting and distance

measures are necessary for

calculation of quadrilateral areas),

no probability calculations in the field.

Tables 1 and 2 summarize the important features of each of the methods.

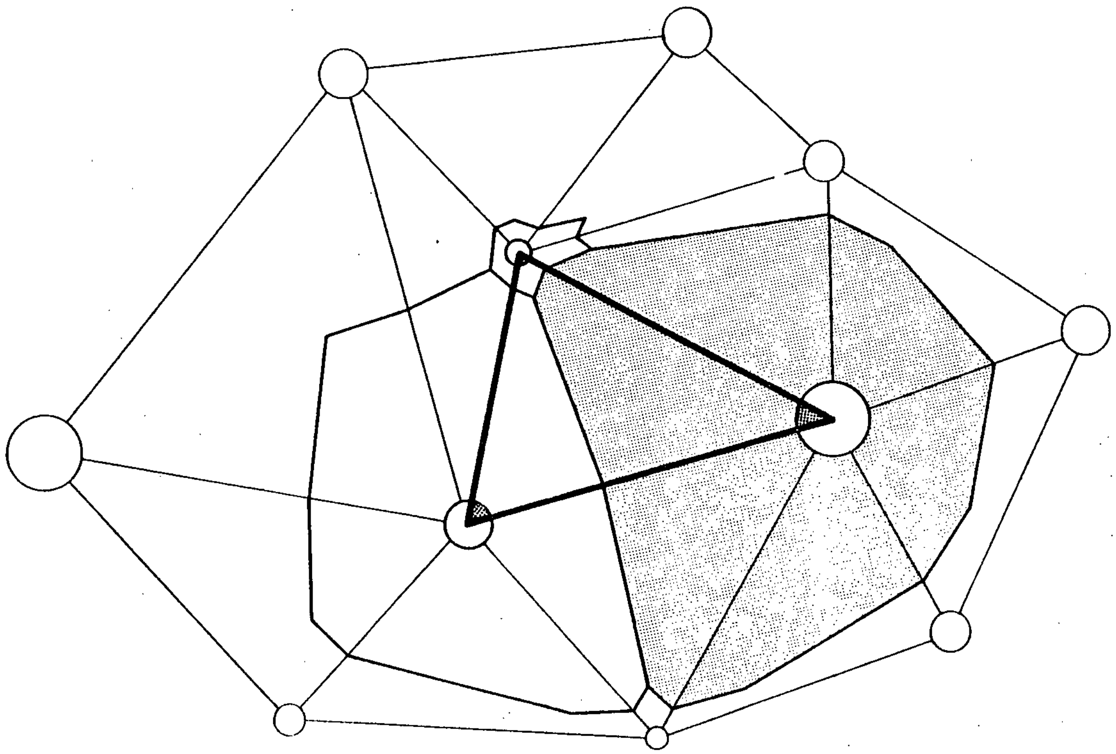

Figure 5 Stem map showing least diagonal neighbours and one shaded small polygon of the type used in Method 4

Table 1 Summary of the compilation of the y_i and a_i for each method (used in formulae (1) and (2))

Method	y_i	a_i
Basic	sample tree volume	area of polygon
Method 1	volume of tree portion contained in triangle	area of quadrilateral containing tree
Method 2	sum of volumes of tree portions contained in triangle	area of selected triangle
Method 3	sum of: $\frac{(\text{triangle area})\ (\text{tree volume})}{\text{large vertex polygon area}}$	area of selected triangle
Method 4	sum of: $\frac{(\text{quadrilateral area})\ (\text{tree volume})}{\text{small vertex polygon area}}$	area of selected triangle

Table 2 Summary of measures required per sample for each method

Method	Number of volume measures	Average number of weighting measures	Average number of distance measures	Probability calculation required in field
Basic	1	6	12	yes
Method 1	1	2	3	yes
Method 2	3	none	3	no
Method 3	3	none	24	no
Method 4	3	9	24	no

METHOD OF COMPARISON

Four data sets were analyzed in this study. These are the identical data sets used by Fraser, being stem map and diameter information for four forest types. (The data for a fifth type used by Fraser had been misplaced and could not be reconstructed). The type symbols and species composition are:

- HB - mature western hemlock (*Tsuga heterophylla* (Raf.) Sarg.) and balsam (*Abies amabilis* Dougl.) Forbes),
- HC - mature western hemlock and western red cedar (*Thuja plicata* Donn),
- HCB - mature western hemlock, western red cedar and balsam (*Abies lasiocarpa* (Hook.) Nutt.),
- FPy - mature Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco) and yellow pine (*Pinus ponderosa* Laws)

Information describing these stands is found in Table 3. Figures 6, 7, 8, and 9 are frequency histograms of areas of quadrilaterals, triangles, small polygons, and large polygons for each stand. Note that all distributions are similar, being skewed right. These histograms give no information as to spatial arrangement. As Fraser noted the FPy stand appears to be highly aggregated in spatial pattern while the other three stands show random pattern.

Table 3 Description of stands tested

	STAND TYPE			
	HC	HB	HCB	FPy
Area (m^2)	3520	2378	3176	3567
Number of trees	94	106	76	51
Dbh, min (cm)	25	22	18	25
max (cm)	93	64	92	73
Height min (m)	18	19	15	29
max (m)	44	40	43	47
Volume per tree (m^3)	2.80	1.35	2.48	2.76
Coeff of variation, percent	69.6	59.4	75.7	81.0
Volume per 200m^2 plot (m^3)	14.97	12.07	11.89	7.90
Coeff of variation percent	43.4	32.3	68.1	76.1
Trees per plot (average)	5.34	8.90	4.80	2.86

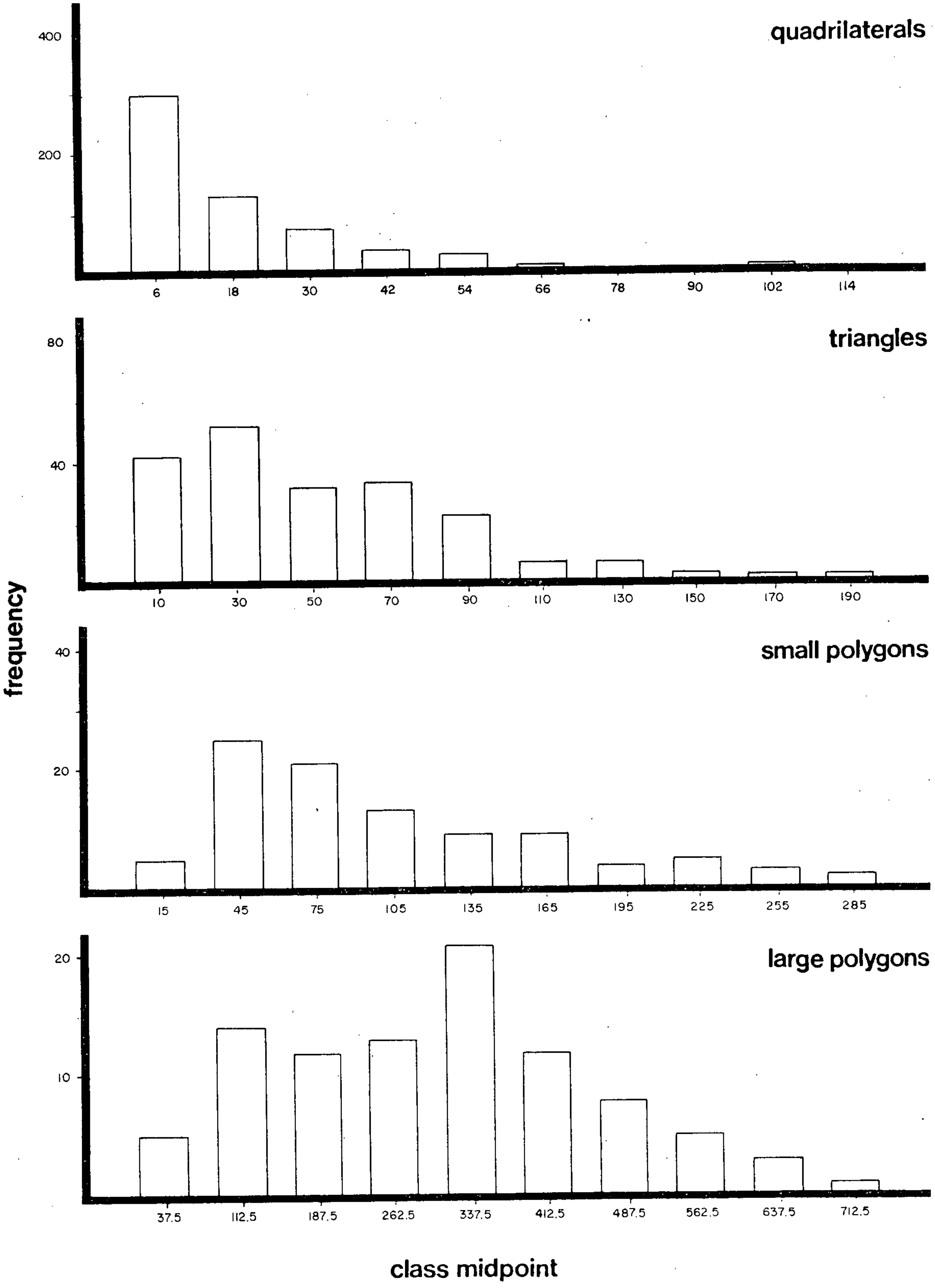

Figure 6 Frequency histograms of areas of quadrilaterals, triangles, small polygons, and large polygons for stand HC

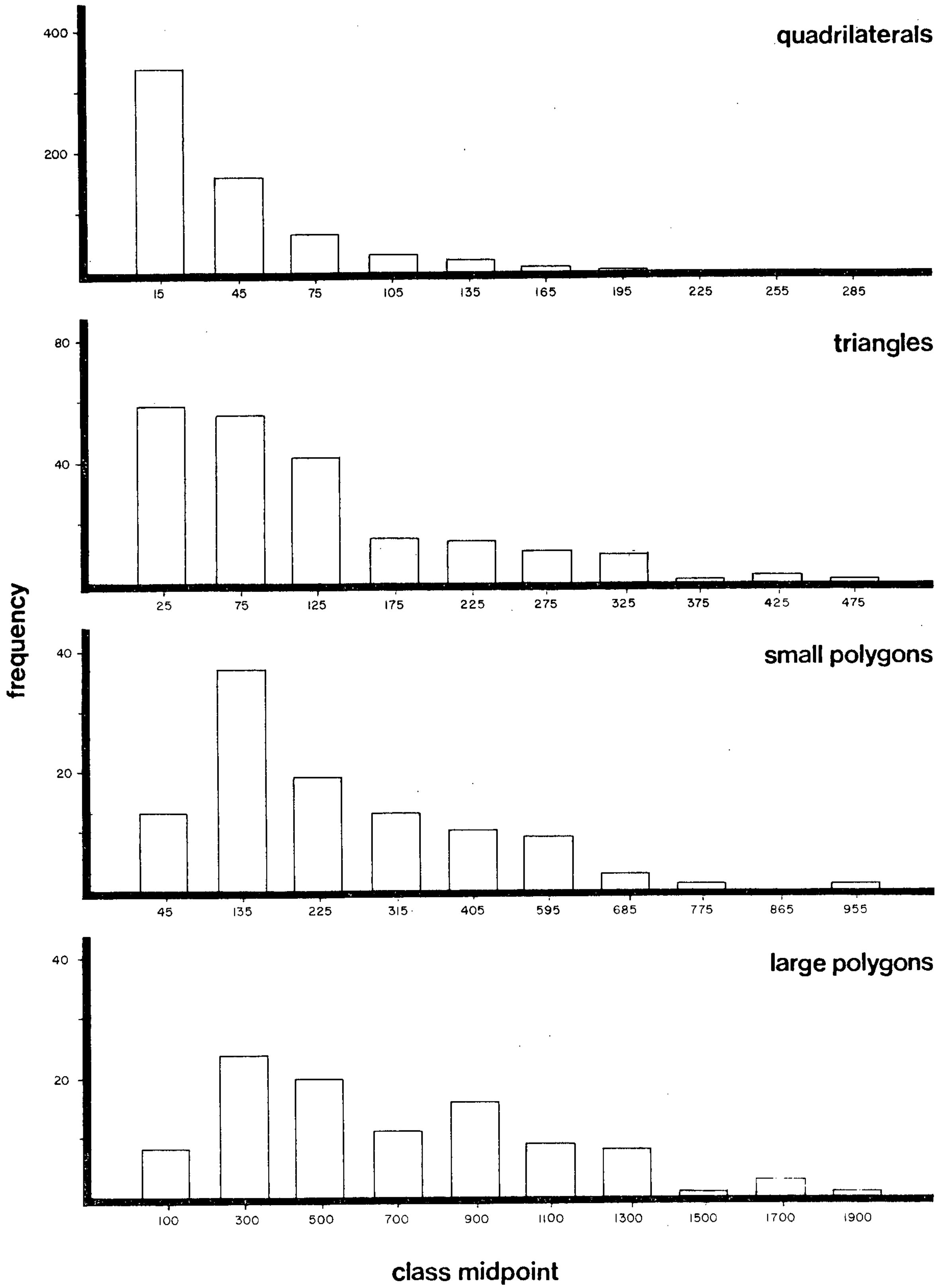

Figure 7 Frequency histograms of areas of quadrilaterals, triangles, small polygons, and large polygons for stand HB

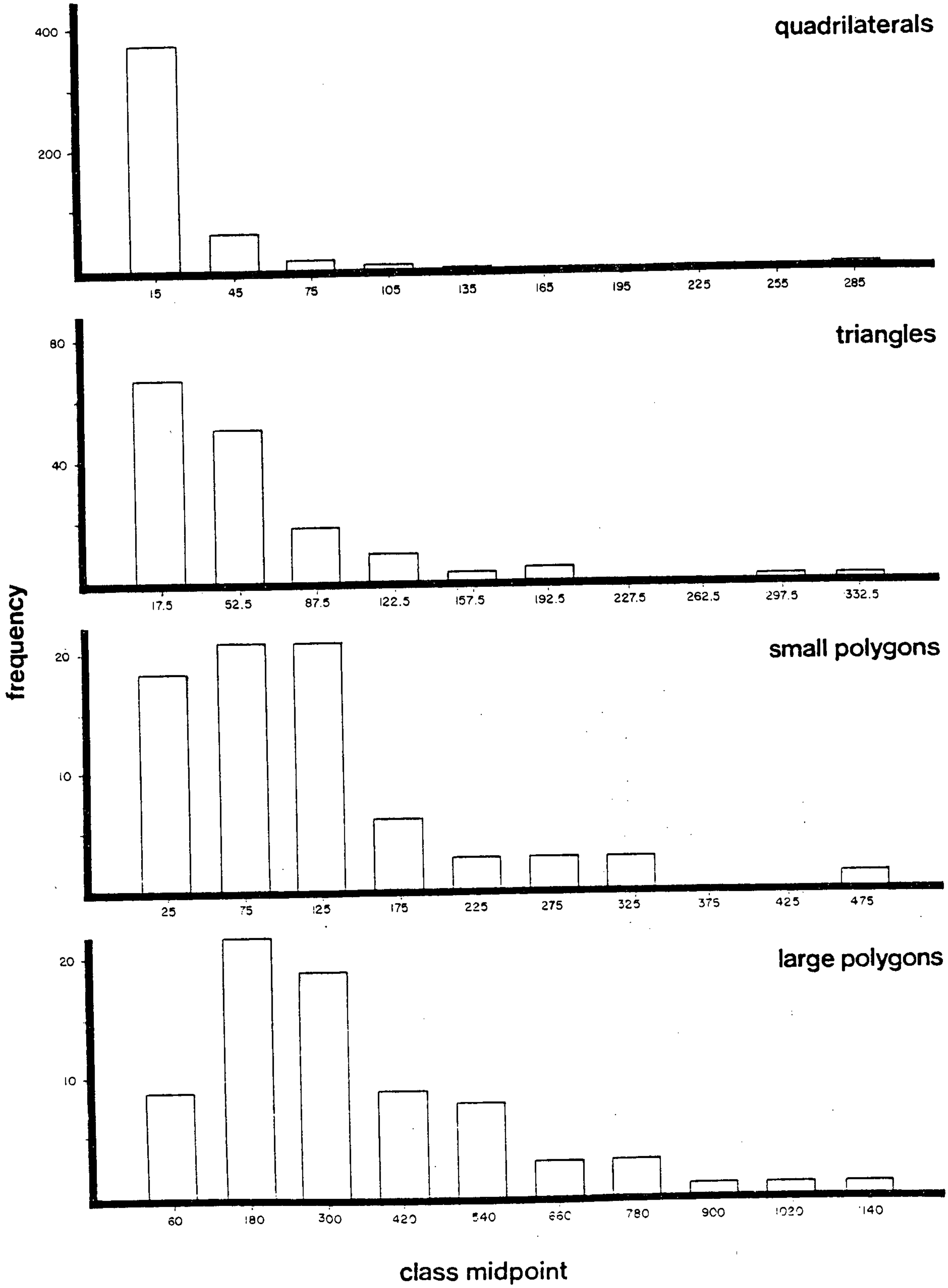

Figure 8 Frequency histograms of areas of quadrilaterals, triangles, small polygons, and large polygons for stand HCB

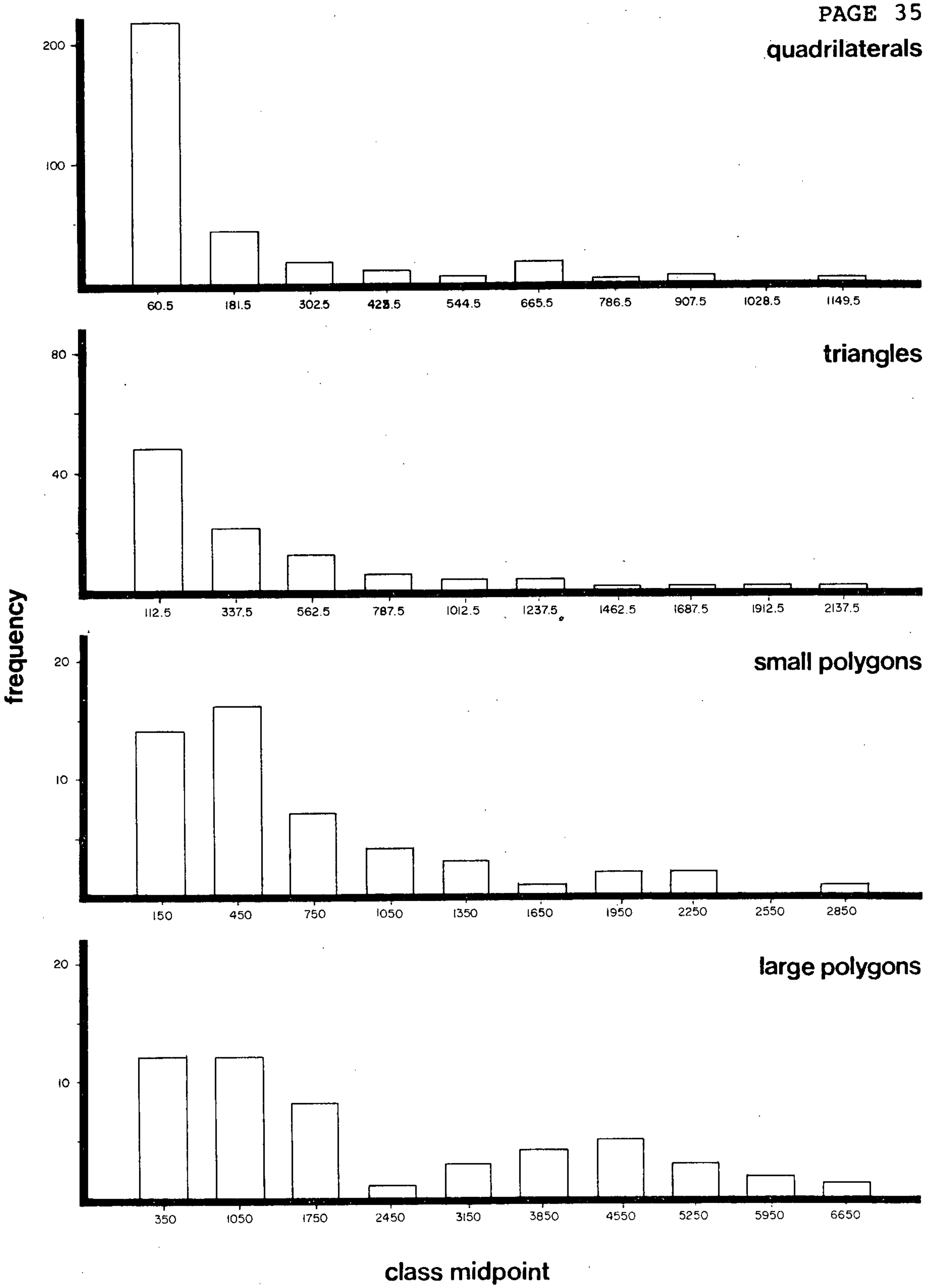

Figure 9 Frequency histograms of quadrilaterals, triangles, small polygons, and large polygons for stand FPy

FORTRAN programs were written to enable the comparison of the methods described herein. These programs identified LDN pairs; solved their resultant triangles; partitioned the triangle areas into their polygon portions; and calculated the desired statistics.

The population forms of equations (1) and (2), that is, summed over all possible samples, were applied to these data. The triangle partitioning formula used was the geometric proportioning formula which calculates proportions (P_1, P_2, and P_3) of a triangle area in each quadrilateral. Thus

$$P_1 = \frac{w_1^2 \, (w_1 w_2 + 2w_2 w_3 + w_1 w_3)}{W(w_1 + w_2)\,(w_1 + w_3)}$$

where

$$W = w_1 w_2 + w_1 w_3 + w_2 w_3 ,$$

and w_1, w_2 and w_3 are tree weights. P_2 and P_3 are calculated similarly following symmetry in subscripts. The proportioning weight used was the $D^2\alpha$ weight described by Fraser. In addition, a $D^2H\alpha$ weight, with H being tree height, was tested for the FPy, HC, and HCB data sets, since height information was available for these types. It was hoped that this might give a partitioning of total area A into polygon areas which is more highly correlated with tree volumes.

Comparisions were made using values of C where

$$C = 100\sqrt{\frac{\frac{1}{(n-1)} \sum_{i=1}^{n} \left(\frac{y_i}{a_i} - \frac{1}{n} \sum_{i=1}^{n} \frac{y_i}{a_i} \right)^2}{\frac{1}{n} \sum_{i=1}^{n} \frac{y_i}{a_i}}}$$

and is the coefficient of variation of the estimate of mean volume per unit area. As noted by Fraser, since the sample size required to achieve a given level of precision is directly proportional to C^2, reductions in C of 1 or 2 percent are important from the standpoint of improving sampling efficiency provided that they can be obtained by change in partitioning formula with no other change in costs.

In order to test sampling efficiency where cost items change (that is where measurements and calculations change) relative variance ratios were calculated from these coefficients of variation:

$$\text{Relative variance ratio} = \frac{C^2 \text{ for Method X}}{C^2 \text{ for Basic Method}}$$

in order to relate each method to the Basic Method. These relative variance ratios give the relative numbers of samples required for equal precision. That is, if n is used to denote sample size, then:

$$\frac{n \text{ for Method X}}{n \text{ for Basic Method}} = \frac{C^2 \text{ for Method X}}{C^2 \text{ for Basic Method}}$$

Thus, one can calculate the average number of measurements required to gain equal precision for each method relative to the Basic Method by multiplying the relative variance ratios by the average number of measurements given in Table 2.

CHAPTER THREE

RESULTS AND DISCUSSION

Computed population total volumes and areas for the four stand types were identical to those results obtained by Fraser. Since the two studies used different methods to construct LDN triangle networks (Fraser found his LDN pairs manually; the present study was performed using LDN pairs identified by a FORTRAN program) slightly different triangle sets were obtained. The results for either study are still, of course, meaningful since each triangle set was consistently applied throughout each analysis; hence the relative differences of values of C will not change. In the current study, the results for the Basic Method were judged to be similar enough to those results of Fraser (in terms of their absolute values and their behaviour from type to type) as to verify their correctness. Indeed, when the FPy type was tested with the tree weights of 1, D, and D^2 that Fraser had applied, the same trend in the values of C was observed.

Table 4 gives the values of C for the four types using the $D^2\alpha$ weight. The results are consistent, showing decreasing values in order of Method 1, Method 2, the Basic Method, Method 3, and Method 4. It can immediately be seen that improved statistical performance is obtained with Methods 3 and 4 but not with Methods 1 and 2.

Table 4 Coefficients of variation (C) for each method using the $D^2\alpha$ weight

Method	HC	HB	HCB	FPy
	Stand Type			
Basic	62.4	65.5	79.7	114.6
1	130.6	153.1	136.6	213.8
2	128.0	150.9	135.2	211.7
3	61.0	60.6	79.2	103.2
4	51.2	53.7	69.8	97.4

Table 5 gives the values of the relative variance ratios for each method compared to the Basic Method for each stand. As noted previously, these are the ratios of samples required to obtain the same precision as the Basic Method. Thus, for example, it would require 4.38 times more Method 1 samples than Basic Method samples in order to obtain equal precision with the HC stand type. To see what these ratios mean in terms of the average number of measurements required to obtain the same precision as with the Basic Method one needs only to multiply the values of Table 5 with the average number of measurements required in each method (from Table 2). These values are shown in Table 6.

Table 5 Variance ratios for each method relative to the Basic Method

Method	Stand Type: HC	HB	HCB	FPy
Basic	1.00	1.00	1.00	1.00
1	4.38	5.46	2.94	3.48
2	4.21	5.31	2.88	3.41
3	.96	.86	.99	.81
4	.67	.67	.77	.72

Table 6 Comparison of average number of measurements required for each method to obtain the same precision as the Basic Method

Method	Stand Type	Number of volume measures	Average number of weighting measures	Average number of distance measures	Probability calculation required in field
Basic	All	1	6	12	yes
1	HC	4.38	8.76	13.14	
	HB	5.46	10.92	16.38	yes
	HCB	2.94	5.88	8.82	
	FPy	3.48	6.96	10.44	
2	HC	12.63	0	12.63	
	HB	15.93	0	15.93	no
	HCB	8.64	0	8.64	
	FPy	10.23	0	10.23	
3	HC	2.88	0	23.04	
	HB	2.58	0	20.64	no
	HCB	2.97	0	23.76	
	FPy	2.43	0	19.44	
4	HC	2.01	6.03	16.08	
	HB	2.01	6.03	16.08	no
	HCB	2.31	6.93	18.48	
	FPy	2.16	6.48	17.28	

For the data tested here, it is apparent that none of the new methods offers any advantage over the Basic Method. More specifically, Method 1 requires a probability calculation in the field -- in addition to weighting measures -- and requires three to five times as many volume measures as the Basic Method. (Volume measures typically are the most costly as they usually include a diameter measure, a height measure, quality assessment, and sometimes diameter measures up the stem.) Method 2 has the advantage of not requiring a probability calculation in the field nor does it require any weighting measures; however it does require eight to sixteen times as many volume measures. Method 3 also dispenses with the probability calculation in the field, and does not require any weighting measures; however, it needs two to three times as many volume measures and almost twice as many distance measures in order to obtain the same precision as the Basic Method. Method 4 does not require the probability calculation in the field, but it does require weighting measures (about the same number as does the Basic Method) and about twice as many volume measures.

Table 7 gives the values of C for the Basic Method and Methods 1 and 4 using the $D^2H\alpha$ weight (Methods 2 and 3, of course, show no change since they make no use of the proportioning weights). There are, once again, the same trends as before, that is, the values decrease in the order of Method 1, the Basic Method, and Method 4. However, when these values are compared with those of the $D^2\alpha$ weight of Table 4 no consistent trends are observed. With Method 1 values actually increase or remain the same, going from a $D^2\alpha$ to a $D^2H\alpha$ weight. With Method 4 and the Basic Method values decrease slightly. These results certainly do not encourage the measurement of height for weighting purposes, at least in a volume sampling context; especially since height measurements are so time consuming. It might be worthwhile, however, to test a $D^2H\alpha$ weight for partitioning triangle areas to form polygons which correlate well with volume increment.

These results confirm Fraser's work and, in addition, demonstrate that some improvement in statistical efficiency may be obtained with two of the methods proposed here. It seems unlikely, though, that any of the four new methods offers any improvements in sampling costs. Another conclusion to be drawn from this is that sampling methods which use volumes of parts of trees, rather than of the whole tree, introduce more variation and hence require more samples to obtain equal precision. Therefore, costs of making such samples would have to be reduced in order to make them practically applicable. Thus it appears that those methods which use whole tree volumes might provide the greatest potential for any future sampling work.

Table 7 Coefficients of variation (C) for each method using the $D^2H\alpha$ weight

Method	Stand Type		
	HC	HCB	FPy
Basic	59.3	78.6	113.5
1	132.1	139.6	213.8
4	49.8	69.0	96.7

CHAPTER FOUR

SUGGESTIONS FOR FURTHER WORK

Obviously, the most important need is to obtain field experience in order to assess the cost of sampling triangle based probability polygons relative to conventional methods. Such trials need to be rigorously tested with experienced field crews in order to obtain the most realistic results.

Somewhat related to this is a need to test for sensitivity of the volume estimate to measurement errors in data collection. Stochastic simulation would probably be the best approach to accomplish this.

In the Basic Method and Methods 1 and 4 the volume estimates make no direct use of tree volumes which might be derived from the additional weighting measures taken for the purpose of calculating polygon areas. Estimates of volume may be improved by using these measures and in addition, diameter distributions may be derived. (The assumption here is that diameters are part of the weighting measurements taken.) This might result in Methods 1 and 4 becoming more palatable sampling alternatives.

Currently, the Research Branch of the British Columbia Forest Service is testing the identification of plus trees using competition indices as a basis for the assessment of stress. Trees exhibiting desirable traits may be doing so simply due to a social position which is relatively free of stress, whereas trees exhibiting the same traits while being subjected to high stress may be of exceptional genetic stock. Thus competition is an important quantity to be determined. The advantage of using triangle probability polygons to estimate competition is the ease of field measurement. Awkward and expensive stem mapping procedures are not necessary, nor is there the possibility of measuring too few or too many trees. The indentification of LDN pairs always results in the optimum number of trees. In addition, there is the potential to test various partitioning and weighting alternatives to obtain those which best suit the needs of this work.

Measures of density and pattern are important for thinning and spacing work. Reliable assessment of stands prior to entry for thinning or spacing can be useful for determining the necessity of such treatments and for defining contract specifications. Likewise, similar assessment after entry can be used to check satisfactory completion for approval of forestry costs or prior to contract pay-off. Simple stems per hectare estimates do not suffice; they give no information as to degree of clumping. Using the statistics of average triangle area and variance of this estimate proposed by Fraser and van den Driessche (1971), one has indicators of density and spatial distribution. This is achieved simply through the sampling of individual LDN triangles and measuring side distances. Stauffer (1979) has advanced this aspect of Fraser and van den Driessche's work and has estimated the distributions of these spatial indicators so that confidence intervals may be calculated for them. With these indicators and the ability to test their statistical significance the next step is to gain experience through simulation and from the field in order to develop an interpretation of what their magnitudes mean in practical terms. Development of field technique is also required. Note that an offshoot of this application would be to sample for volume information (with little extra measurement required) for the purposes of estimating volume of wood removed or for testing growth response through periodic measurements. Information about diameter distributions is also useful from the point of view of spacing contracts; therefore, investigation of the production of diameter distributions from simple diameter measures of triangle vertex trees would be useful.

Regeneration surveys would also enjoy a similar application of triangle spatial indicators; however, it is difficult to visualise a practical field technique to apply to small seedlings. Results, though, could be much more reliable than the use of stocked quadrats.

Another area requiring more study is the problem of identifying LDN pairs from stem map data. Shamos and Hoey (1975) provide a good summary of algorithims for joining points according to various criteria. Included are algorithms for construction of triangles and Voronoi polygons. They point out that using a linear programming approach (i.e., the Simplex Method) in two variables with N constraints (where N represents number of points) results in computation time increasing as N^2, while polygons can be constructed, in two dimensions, using geometric techniques which result in a computation time increasing as N log N. Developing these algorithms to produce LDN triangle networks, or their resultant polygons directly would greatly improve computational efficiency and facilitate simulation studies involving spatial pattern problems.

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