SYSTEMS FOR THE SELECTION

OF

TRULY RANDOM SAMPLES FROM TREE POPULATIONS

AND

THE EXTENSION OF VARIABLE PLOT SAMPLING TO THE THIRD DIMENSION

by

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ABSTRACT

Means of drawing truly random samples from populations of trees distributed non-randomly in a plane are practically unknown. Only the technique of numbering all items and drawing from a list is commonly suggested. Two other techniques are developed, reducing plot size and selecting from a cluster with probability (1/M) where M is larger than the cluster size. The exact bias from some other selection schemes is shown by the construction of "preference maps". Methods of weighting the selection by tree height, diameter, basal area, gross volume, vertical cross-sectional area and combinations of diameter and basal area are described. None of them require actual measurement of the tree parameters. Mechanical devices and field techniques are described which simplify field application. The use of projected angles, such as are used in Variable Plot Sampling is central to most of these methods.

Critical Height Sampling Theory is developed as a generalization of Variable Plot Sampling. The field problem is simply to measure the height to where a sighted tree is "borderline" with a relaskop. The average sum of these "critical heights" at a point multiplied by the Basal Area Factor of a prism gives a direct estimate of stand volume without the aid of volume tables or tree measurements. Approximation techniques which have the geometrical effect of changing the expanded tree shape are described. The statistical advantages of using the

system were not found to be large, and the problems of measuring the critical height on nearby trees was severe. In general use there appears to be no advantage over standard techniques of Variable Plot Sampling, however in situations where no volume tables exist it may have application, and the problem of steep measurements angles to nearby trees can be overcome by using an optical caliper. The system can also overcome the problem of "ongrowth" for permanent sample plots.

Donald D. Munro

TABLE OF CONTENTS

Page

LIST OF TABLES

LIST OF FIGURES

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INTRODUCTION

This thesis will develop two basic areas of interest. The first part will examine the problem of drawing a truly random sample of trees distributed in a non-random pattern in a plane. It is difficult to believe that such systems have not been developed in the past. Other than the classical method requiring labelling of all trees in the population the author has found little mention of methods to accomplish this. Two major methods of assuring a random sample will be given, and one of them (the elimination method) will be used to select trees with probability proportional to a number of different parameters for that tree. Selection proportional to diameter, height, basal area, gross volume and weighted combinations of diameter and basal area will be of particular interest. The biases inherent in incorrect attempts to gather random samples will be examined and methods of specifying the magnitude of such biases will be developed in the form of "preference maps" constructed from standard stem location maps. In several cases mechanical aides will be devised for field work in selecting samples with particular weightings.

The second part of the thesis will develop the theory of Critical Height Sampling for the inventory of tree stands without the use of volume tables. The basic concept is to extend the theory of Bitterlichs' Variable Plot Sampling to the third dimension, so that not only the tree basal area, but the entire tree volume is expanded by a constant and then directly sampled. A large scale field trial will not be attempted, but the field work will be described and some field experience will be gained to anticipate problems in application and to develop alternative measurement schemes to solve them. Approximation techniques will be discussed and some computer simulation will be used to determine the relative variability of the system in comparison to standard Fixed Plot or Variable Plot techniques. Applications and timber types for which the system is particularly suited will be discussed. Emphasis throughout the thesis will be on the geometrical reasoning involved, since an understanding of this will greatly aid attempts to adapt these methods to local conditions.

Reasons to Sample Individual Trees

Selection of trees for sampling, as opposed to complete enumeration, is now standard practice in forest inventory. In most cases, the trees are selected as a cluster, and the value of interest is a sum of individual values in that cluster. Statistics are then applied to that sum as a single observation and usually expanded on an area basis. In their classic application both fixed plot sampling and variable plot sampling are examples of this process of selecting a cluster of trees. The cluster has four major advantages, one is statistical and the other three procedural.

The statistical advantage is that the sum of a cluster is often less variable than individual tree values, both because it incorporates several observations which may tend to "average out" under even random spacing arrangements, and because there is a tendency for trees to react to each others presence. This "competition effect" tends to cause higher variation between individuals, but lower variation among the groups, since as one tree increases its growth it is often at the expense of its neighbors. The process of competing for a <u>shared</u> amount of available light, water and nutrients serves to retain a constant effect on a group of trees even while increasing individual differences. This reasoning certainly applies to growth, if not to total volume. We are thus led to the classic situation of variation within (rather than between) clusters which gives cluster sampling its statistical advantage.

The first procedural advantage is that once a plot center is established it is usually small additional effort to determine several trees for sampling at the same time. With the expense involved in travel time, particularly in random sampling, it often decreases total project cost to measure a cluster even when variability between and within clusters would not indicate a statistical advantage.

The second procedural advantage is that most sampling systems are based on the concept of volume measured on a land area basis. It often is easy to determine the total area involved in an inventory, but rather hard to determine the total number of trees, hence a system based on volume in an individual tree requires a more ingenious approach to sampling. Such approaches as Triangle Sampling by Fraser (1977), and the same concept applied earlier by Jack (1967) are examples of using a land area base for individual trees, but they lack widespread acceptance.

The third advantage to selection of a cluster is that simple rules for the unbiased selection of a cluster of trees are easy to develop and readily available, while unbiased selection rules for individual trees are difficult to find or simply unavailable. Selection of individuals by even so simple a criterion as frequency is difficult indeed. The following quote is from Pielou (1977).

> In order to choose a random individual from which to measure the distance to its nearest neighbor, the only satisfactory method is to put numbered tags on all the plants in the population and then to consult a random numbers table to decide which of the tagged plants are to be included in the sample. In doing this, we acquire willy-nilly a complete count of the population from which its density automatically follows. There is another method of picking random plants, but it, too, requires that the size of the total population be known. If a sample of size n, say, is wanted from a population of size N, the probability that any given plant in the population will belong to the sample is $p=n/N$. We must then take each population member in turn and decide by some random process whose probability of "success" is p whether that member is to be admitted to the sample. Even if we are willing to guess the magnitude of N intuitively and assign to p a value that will give a sample of approximately the desired size, it is still necessary to subject every population member to a "trial" in order to decide whether it should be included in the sample; as the successive trials are performed, a complete census of the population is automatically obtained.

The sampling of tree clusters based on a fixed area has three main disadvantages. First, there may be little statistical advantage in doing so. In an area where the growth of a cluster may be very consistent the total volume may be quite different. Dollar values, particularly where different species are involved, are likely to be even more variable. Stratification, either before or after data collection, may be only partly helpful in reducing this variance.

Secondly, as trees are measured with greater accuracy and for multiple characteristics, the cost difference between establishing the sample point and measuring the tree diminishes, and it is less reasonable to measure many trees "as long as you are there anyway". The concern for accurate net volumes in lump sum sales has led the United States Bureau of Land Management (BLM) to "Fall, Buck and Scale" cruising (Johnson, 1972). This kind of destructive, intensive measurement can only be justified when sample sizes are as small as statistically feasible. "Extra" trees can no longer be measured just because it will simplify the selection process to measure a cluster rather than an individual.

Third, the total land area involved may be more difficult to determine than the number of trees. It may be complexly defined, intricate in pattern or there may be difficulty physically measuring the border. In addition there may be "boundary effects" for trees physically near the border. This has resulted in a great number of papers, for instance (Martin et al., 1977; Beers, 1966; Beers, 1976; Barrett, 1964). It might be an advantage, where possible, to avoid rather than solve these problems.

Reasons to Sample Randomly

Random sampling does not always mean selection of individuals with equal probability, although it is often discussed in this manner. It does mean that an individual item has a particular probability of being included in a sample, and usually that the selection of the individual does not affect the further probability of selecting any additional member of the population for the sample. A more rigorous definition of random sampling can be found in Brunk (1965). We will consider, for the following discussion, only equal probability selection with replacement because it is simple and illustrates the main points of the following topics.

In equal probability selection with replacement each permutation of observations has an equal probability of occurrence. This usually means that during the sampling process each observation has an equal chance of selection at any time. This is desirable, since it permits unexpected termination of the sampling process without affecting the randomness of the smaller sample.

This type of random selection is presumed for most estimates of population variance. Many sampling schemes can be considered of this type by suitable definitions of what shall be considered "an observation". Two main points are of interest.

First, such a sample yields an unbiased estimate of the mean (μ) and variance (σ^2) of the population. Unbiasedness is still considered by many to be a very desirable feature for an estimator. There are other sampling schemes which also yield unbiased estimates of the mean.

One way such estimates are easily produced is by simply assuring that each observation has an equal probability of being sampled. Systematic sampling can often have this effect, and is usually meant to.

Unbiasedness is not hard to produce in an estimator, nor is it universally accepted as a desirable feature. The smaller expected mean square error $E\left[(\bar{x} - \mu)^2\right]$ which can be produced by Baysian estimation, many robust methods and other techniques, is often gained by accepting very small biases. The problem is to assure the researcher that these biases can indeed be expected to be small. The major arguments for the concept of unbiasedness are given by Brunk (1965).

> 1. To state that an estimator is unbiased is to state that there is a measure of central tendency, the mean, of the distribution of the estimator, which is equal to the population parameter. This is simply the definition of unbiasedness. An equally appealing property, however, from this point of view, might, for example, be that the median (page 348) of the estimator be equal to the population parameter.
> 2. For many unbiased estimators one can conclude, by applying the law of large numbers, that when the sample size is large the estimator is likely to be near the population parameter. However, this is the property of consistency, discussed below; and the argument here is not primarily in favor of unbiasedness, but in favor of consistency. For example, the unbiased estimator, $\left[\frac{n}{n-1}\right]s^2$, of the population variance has this property; but so also does the sample variance s^2 itself.
> 3. An important advantage from the point of view of the development of the theory of statistical inference is that in many respects unbiased estimators are simpler to deal with. The linear properties of the expectation are particularly convenient in dealing with unbiased estimators. If, for example

> θ is a parameter having $\hat{\theta}_1$ and $\hat{\theta}_2$ as unbiased estimators in two different experiments, every weighted mean $\alpha\hat{\theta}_1 + \beta\hat{\theta}_2$ with $\alpha + \beta = 1$ is also an unbiased estimator of θ. We note, however, that non-linear transformations do not in general preserve unbiasedness. For example, if $\hat{\theta}$ is an unbiased estimator of θ, then $\hat{\theta}^2$ is not an unbiased estimator of θ^2.
> One point of view is that the class of all possible estimators of a particular parameter is unmanageably large. A way of approaching the problem is to restrict attention to an important subclass, such as the class of unbiased estimators.

From the point of view of most practical research we can dismiss the random sample from further consideration if unbiasedness is the only criteria of interest. The use of the random sample in much of sampling theory is not necessary, but rather a device for simplifying the mathematics.

The second main feature of a random sample is the known standard deviation of the mean ($\sigma_{\bar{x}}$) or "sampling error" as it is often called in the biological literature. Random sampling does not by any means minimize this sampling error. It is well known that systematic sampling often has a smaller actual sampling error. By forcing the observations throughout a non-random population dispersion a systematic sample will often obtain higher variance within the sample and subsequently a lower variance between samples. This actual increase in precision is usually accompanied by an <u>apparent</u> decrease when the sample variance is computed as if the sample were random. Intelligent direction of the systematic grids can add further precision, leading to suggestions that sampling be done "at right angles to the drainage pattern" (Husch, *et al.*, 1972) and similar advice in many texts.

Fisher (1936a) covers the systematic sample thoroughly. His criticism is mainly in two parts. First, such a systematic allocation can be quite variable (even with a so-called "random start") when the systematic pattern is in phase with a periodically arranged population, and will then also have an apparent decrease in sampling error. The variability would actually be worse than a random sample while it would seem to be better. If the sampler also has control over the placement of the systematic grid it may actually be quite biased. Attempts to estimate the actual error by successive differences (Meyer, 1956), multiple systematic surveys (Shine, 1960) and more advanced methods have not been entirely successful.

Second, because of the uncertain sampling error, confidence intervals and tests of hypotheses cannot be made with known probabilities. While it is true that most confidence intervals and tests with systematic samples will be conservative - that is the probability (α) of falsely rejecting a true null hypothesis is even smaller than stated - this is not guaranteed. Fisher's examination of Mendel's work (Fisher, 1936b) would not be correct when the true sampling error was smaller than assumed. In this case the wrong sampling error would have increased the probability that Fisher would accuse Mendel of tampering with the data or of using a different method than Mendel had stated. For these reasons, Fisher came down solidly against non-random arrangements for most purposes.

The behavior of the sampling error ($\sigma_{\bar{x}}$) of a normal population is well known, readily available and thoroughly documented. Research on the behavior of non-normal parent populations seems to indicate that even rather small sample sizes give distributions of means which are roughly normal. If the population size frequency distribution is known (but not its spatial distribution) a random sample of observations is still usually assumed before theoretical calculations can be made about the behavior of the sample mean. It is this known behavior of the sample mean which makes the random sample so popular with statisticians. Other methods, even if known to have smaller sampling error are more open to the kind of criticism that researchers would rather avoid, and it seems likely that they will continue to spend the additional time and effort to do so. It therefore seems desirable to produce methods of obtaining such random samples.

We shall primarily be interested in the case where a population of trees is dispersed on a tract of land with unknown spatial distribution. The problem is to draw a random sample of individuals. This is, in many respects, no different than selecting a plant at random on rangeland, a geologic specimen from an area, or a seaweed on the ocean floor. When not taking advantage of the circular cross-section of the tree stem or other special features, these methods will have application in a number of disciplines.

Sampling Without Replacement

When each observation is allowed to enter the sample only once, there is a decrease in the sampling error expressable by means of the "finite population correction factor". In many cases, sampling without replacement is desirable, but when the finite population correction cannot be computed accurately, and because of the reasons mentioned in the last section, it may be desirable to sample with replacement. The methods capable of sampling with replacement are easily modified for sampling without replacement, but the reverse is not always true. When sampling without replacement there is a decrease in the number of permutations allowed as a sample. The decrease results in the definition of sampling without replacement giving equal probability to all combinations of samples. It seems to the author to be more general to use the phrase "permutations of observations" with the understanding that the number of allowable permutations is implied when dealing with sampling without replacement. This shall be done throughout the thesis. The methods developed in this thesis will be applicable to sampling with or without replacement.

Weighting Selection Probabilities

Individuals in a population are often selected for sampling with a probability proportional to one of their characteristics. It reduces field work to be able to make such a selection without actually measuring that characteristic. There are several reasons to make weighted selections. First, it may be desirable to have more precise answers regarding some size classes of the items in the population. Often larger trees are more valuable than smaller trees. Second, larger sample sizes are often required for some classes of items because they are more variable. Third, it is sometimes mathematically easier to sample by weighting the selection probability (and give the measurements equal treatment thereafter) than to sample with equal probability and weight all the subsequent calculations. Finally, there is the statistical advantage that the final result can be less variable when selection probabilities are varied. A very general equation for the estimation of a population total can be written:

$$\frac{V_i}{p\left\{S_i\right\}} = T$$

where: V_i = the value measured on item i from the population

T = the estimated population total for the type of population value measured

$p\left\{S_i\right\}$ = the probability of sampling item i from the population.

The variance of the total is proportional to the variability of this ratio. How can the variability be reduced? Clearly this can be done by sampling with a probability proportional to the measured value of each item. One or more of these reasons often applies in sampling forest stands, therefore considerable effort will be made to derive means of selection for random samples proportional to several tree characteristics, and to devise ways of selecting trees without the actual measurement of those characteristics. Sampling methods will first be derived for sampling with equal probability for each individual and some of these techniques will then be adapted for sampling with other probabilities.

Much of the first part of the thesis is concerned with "bias" and how to avoid it. More specifically the concern is with "selection bias" where objects may in fact be selected with a probability much different than the one intended by the sampler. This in turn will generally result in a bias in any parameter estimated from the data gathered on those objects. In a few cases suggestions will be made for changes to estimating equations which will compensate for selection bias, but the emphasis will be on selecting trees with the intended probabilities.

SAMPLING SELECTION SYSTEMS
PROPORTIONAL TO VARIOUS PROBABILITY WEIGHTINGS

Frequency Weighting

The common system for selecting members of a population is one based on equal frequency, where each of the N members of a population is measured with the same probability. In addition to this requirement a random sample would also give equal selection probability to each permutation of observations.

Surprisingly little advice can be found on the problem of selecting a random sample of trees from a forest area. The only common system suggested for drawing such a sample is first to label all of the N individuals, to draw a random number from 1 to N, and to find and measure that individual. The process is repeated to select a sample of desired size. The effort involved in this process usually eliminates it from serious practical consideration.

"Nearest Tree" Methods

A common method in practice is to first find a random point on the tract to be sampled. This is simply accomplished by the intersection of two random coordinates. A tree "near" this point is then chosen subjectively, or the tree nearest to the point is chosen by measurement.

This "nearest tree" idea will be examined in some detail throughout the thesis. The bias in the first system cannot be calculated, but that of the second system is easy to examine. It can most easily be studied by the use of polygons constructed around each tree. These polygons are called Thiessen diagrams (Jack, 1967), "Voroni polygons" and "Dirichlet cells" (Fraser, 1977), "Plant Polygons" (Mead, 1966), "Area Potentially Available" (Brown, 1965) or "Occupancy Polygons" (Overton et al., 1973). Their use in forestry is illustrated by such publications as Jack (1967), Overton et al. (1973) and Brown (1965).

The construction of a Dirichlet cell around a tree incloses all points to which the tree is closer than any other tree in the population. The size of the cell is dependant only on the spacing of the trees in the population. Construction of the cell is defined concisely by Jack (1967) as follows:

> ".....the smallest polygon that can be obtained by erecting perpendicular bisectors to the horizontal lines joining the center of the tree to the centers of its neighbors at breast height of the tree center....."

The main features of the cells are that they cover the entire tract without overlap or gap, that they are easy to construct; and that they are not affected by the shape or size of the cell constructed around other trees, which insures the same cell regardless of the order of construction. Jack (1967) gives a number of useful equations for determining which trees are critical to the construction of the cell and for calculating the cell area. He also mentions a computer program for automatically doing this from field data giving the bearing and distances to surrounding trees. Newnham and Maloley (1970) provide a computer routine for computing cell areas from stem maps. The cells are not difficult to construct on large scale stem maps, since the main process involved is line bisection. Figure 1 shows the general process involved, and examples of the cells around 3 trees.

Such diagrams, noting the area in which a tree may be chosen for sampling will be called "preference maps" for that selection system. It is obvious that the probability of choosing the tree nearest to a random point is directly proportional to the area of the Dirichlet cell around the tree. The exact probability of choosing the tree i would be:

$$p\left\{S_i\right\} = \left(\frac{DC_i}{T}\right) \qquad (1.1)$$

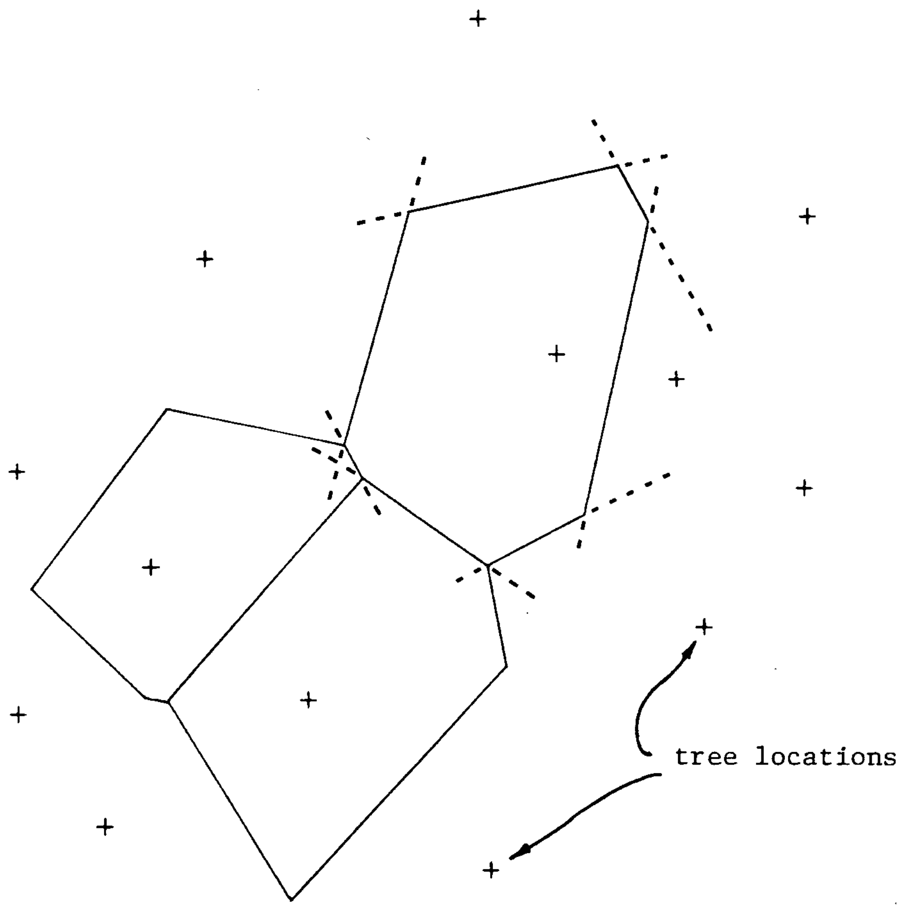
where: $p\left\{S_i\right\}$ = The probability of selecting tree i for sampling under the selection system.

DC_i = The area of the Dirichlet cell around tree i

T = The total area of the tract where sampling is conducted.

All symbols in this work will be defined when first used, and also listed in Appendix 1 for easy reference.

Figure 1. Construction of Dirichlet cells around stem mapped trees on a tract to be sampled.



There is every reason to believe, on the basis of experience, that large trees are spaced at wider intervals than small trees in the same conditions. At the very least we know that different forest areas of the same size have different numbers of trees. It is therefore apparent that trees from both areas <u>cannot</u> have the same probability of selection by the nearest tree method, and that the bias is very probably in favor of larger trees, since these often have wider spacing. The exact bias can be computed from a stem map on which the Dirichlet cells have been drawn.

Since the nearest tree system does not select trees with equal frequency, is there a simple system which does?? The use of any size <u>fixed plot</u> will assure that every tree has exactly the same probability of selection. This is probably the easiest way to select a sample where each tree has an equal chance of selection. This, unfortunately, does not allow us to pick a random individual as easily. Often such a cluster of trees picked by a plot will serve the sampling purpose but occasionally a subsample is required or a truly random selection of individuals is desired. A truly random sample requires that every permutation of observations be equally likely, so selection of more than one tree from a plot would be non-random even though it might be unbiased.

If clusters with positive covariance among trees are selected, but treated as a random sample of individuals, the sampling error will be underestimated. The reverse situation can also occur, as stated earlier.

Bias in Subsampling From a Cluster

One common approach to subsampling is to choose a tree by a uniform random number R between 1 and np, where np is the number of trees found in a plot. The tree corresponding to the random number is chosen for sampling. If only one tree in each fixed plot is chosen in this way (or any other), there is an obvious bias in favor of trees of sparse distribution. The probability of sampling an individual on a particular plot is (1/np). For an individual tree in a given plot this can rise to a maximum of 1 or decrease to a minimum of $1/n_{max}$, where n_{max} is the greatest number of trees found on any plot in the study area. The expectation of an individual for sampling can be calculated from a preference map constructed in the following way:

1. Construct a plot around each of the N trees on the area with the size, shape and orientation desired.
2. Determine the number of trees which share each compartment formed by overlaps of the plots, and the area of each. In complex situations this will best be done by digitizer or planim ter, in simpler cases perhaps by equation.
3. Calculate the expectation of sampling tree i from a randomly located single point by the formula:

$$p\left\{S_i\right\} = \frac{1.0}{T}\left[\sum_{k=1}^{t_i}\left(\frac{a_k}{n_k}\right)\right] \tag{2.1}$$

where: t_i = total number of compartments formed by overlap of tree i with other plots

a_k = area of compartment k

n_k = number of trees sharing compartment k

T = total area of tract sampled.

The probability that tree i has been selected given that some tree has been selected (hereafter called relative probability) is:

$$Pr\left\{S_i\right\} = \frac{p\left\{S_i\right\}}{\sum_{i=1}^{N} p\left\{S_i\right\}} = \frac{T * p\left\{S_i\right\}}{\sum_{k=1}^{t}\left(a_k\right)} \tag{2.2}$$

where: t = the total number of compartments on the preference map.

Figure 2 illustrates the method and calculations using circular fixed area plots of 25 m^2.

The probability that some tree will be sampled with a randomly chosen point is:

$$p\left\{S_s\right\} = \frac{\sum a_k}{T} = \frac{79.5\ m^2}{200\ m^2} = 39.75\% \tag{2.3}$$

Figure 2. Illustration of the computation of sampling probability when a single tree is chosen randomly from all those on a fixed plot.

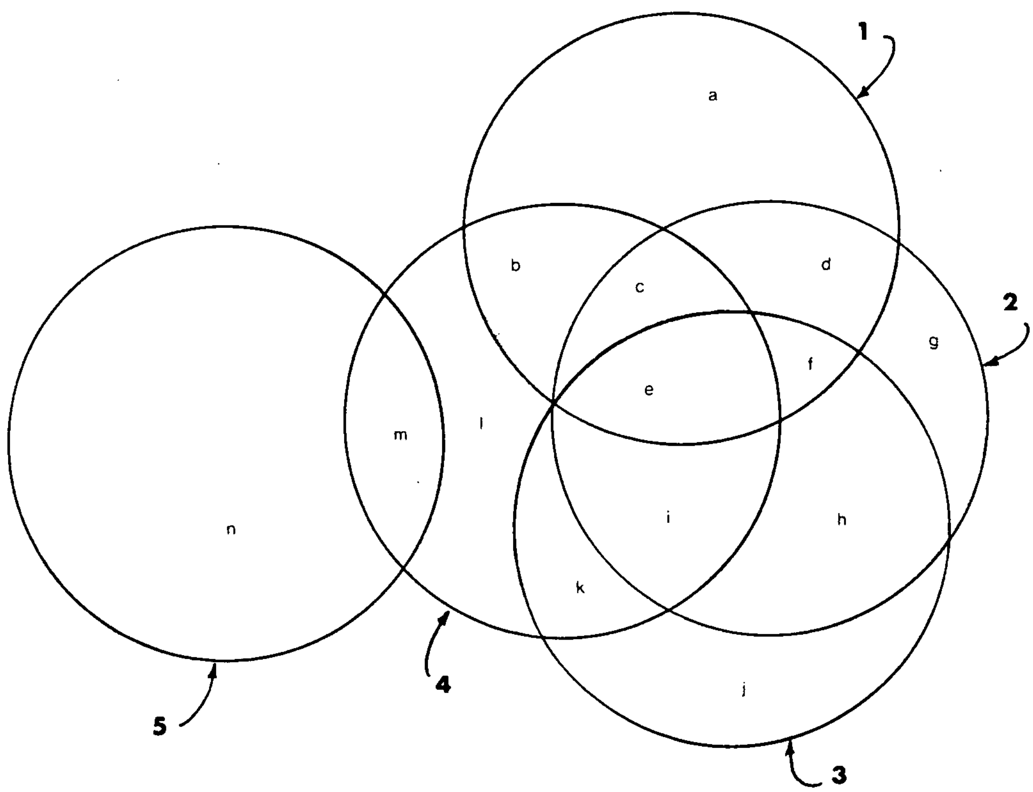


Table 1. Computations involved for the example illustrated in Figure 2.

Compartment from diagram	tree 1	2	3	4	5	a_k	n_k	$\left[\frac{a_k}{n_k}\right]$
a	*					12.0	1	12.0
b	*			*		4.0	2	2.0
c	*	*		*		2.6	3	0.87
d	*	*				2.7	2	1.35
e	*	*	*	*		3.1	4	0.78
f	*	*	*			0.6	3	0.2
g		*				2.9	1	2.9
h		*	*			8.0	2	2.0
i		*	*	*		5.1	3	1.7
j			*			6.2	1	6.2
k			*	*		2.0	2	1.0
l				*		5.3	1	5.3
m				*	*	2.9	2	1.45
n					*	22.1	1	22.1
						79.5		

Tree	$\sum[a_k/n_k]$	Relative Probability	Selection Bias
1	17.2	.22	+10%
2	11.8	.15	-25%
3	13.88	.17	-15%
4	13.1	.16	-20%
5	23.52	.30	+50%
Totals:	79.50	1.0	

This is simply the relative area covered by the plot of one or more trees.

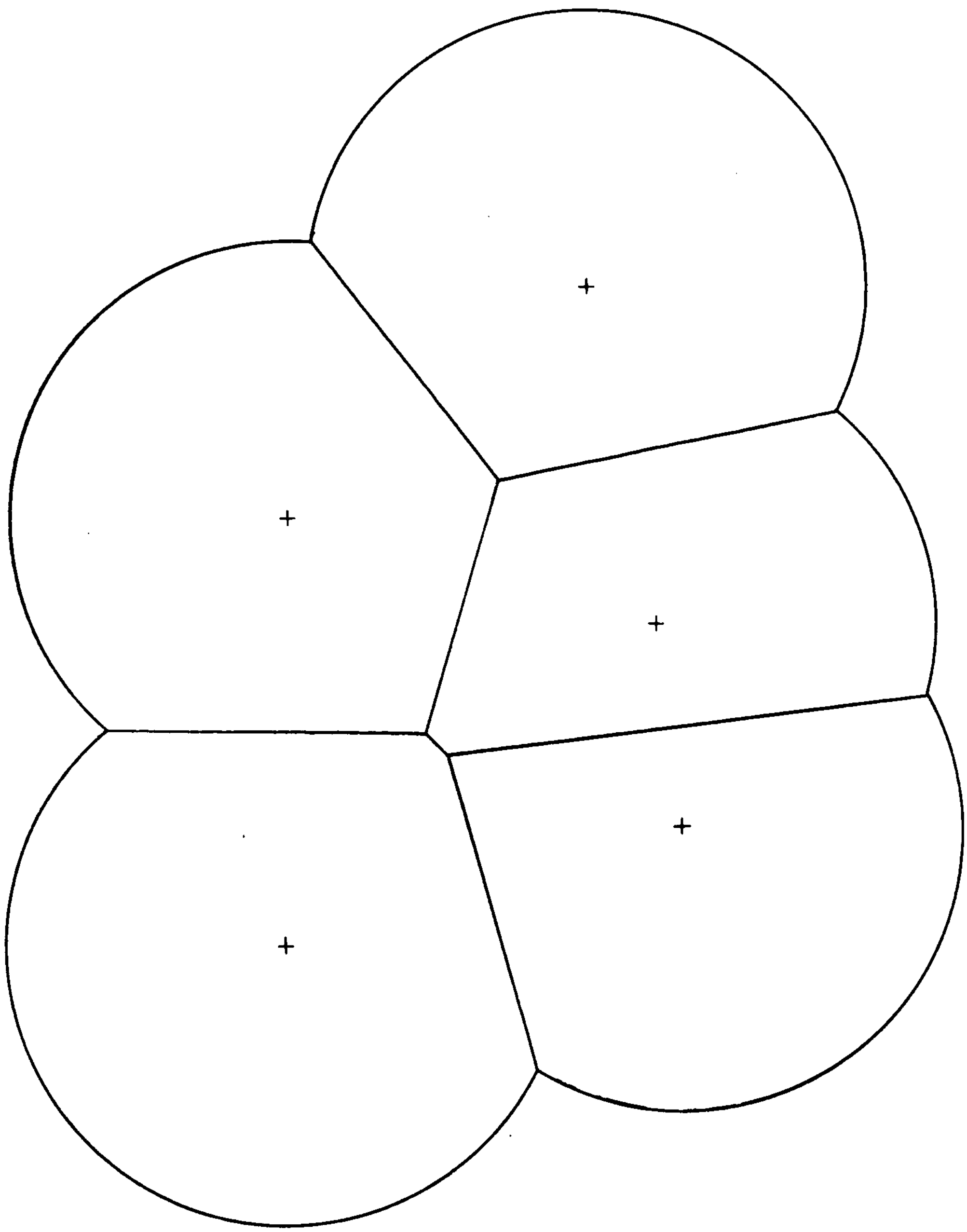
As a practical matter, if one tree must be randomly chosen from each cluster then that observation should be weighted proportional to the size of the cluster. This will remove any bias from estimation based on the sample mean.

Selection of the Closest Tree in Each Cluster

The equations are the same once the preference map has been drawn. There are fewer compartments but construction is more tedious. Figure 3 shows results. When plots overlap the bisector is drawn. This is particularly easy, since most of the bisector construction is provided already by the overlap of the circles. The nearest bisectors reduce the size of the plot as in a Dirichlet cell. Relative reduction is most severe in clumps of trees, decreasing the selection probability of trees in dense spacings. As the size of plots increases the bisectors are increasingly important, with the Dirichlet cells being the limiting distribution.

Using the term md_i to denote the area of these modified Dirichlet cells, the probabilities of sampling are given below.

Figure 3. Preference map when selection is based on the closest tree within a fixed plot.



For selecting an individual tree:

$$p \left\{ S_i \right\} = \left(\frac{md_i}{T} \right) \tag{3.1}$$

relative probability of selecting a tree:

$$Pr \left\{ S_i \right\} = \frac{p \left\{ S_i \right\}}{\sum_{i=1}^{N} p \left\{ S_i \right\}} \tag{3.2}$$

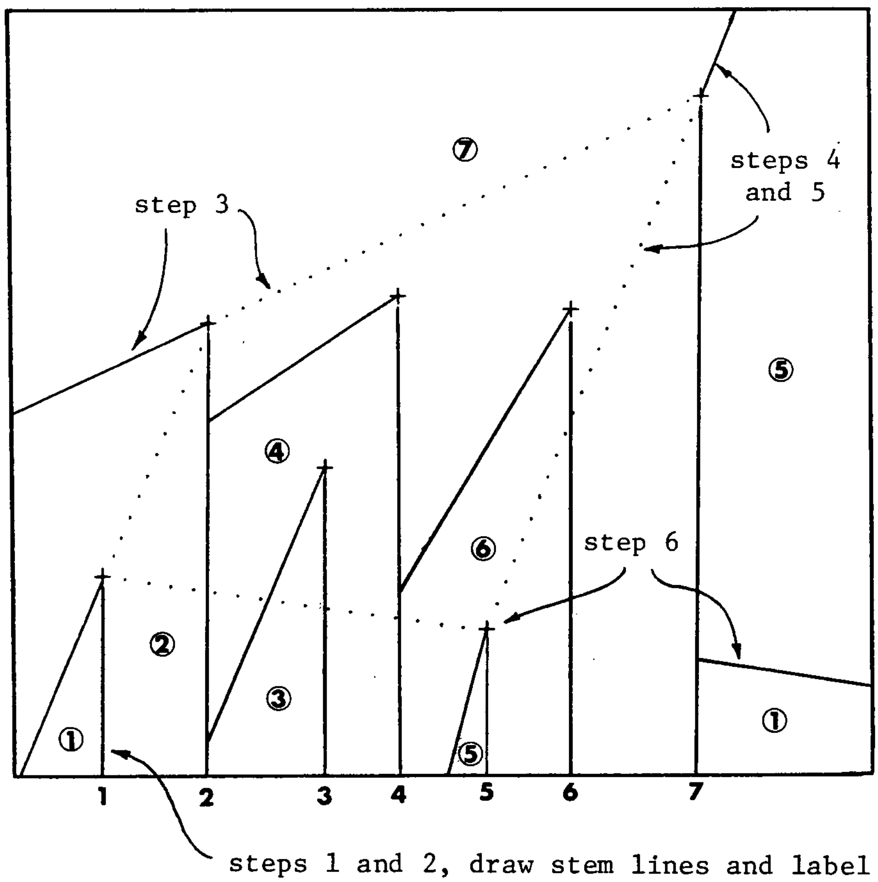
probability of selecting some tree at a random point:

$$p \left\{ S_s \right\} = \frac{\sum_{i=1}^{N} md_i}{T} \tag{3.3}$$

The "Azimuth Method"

The system, sometimes used in forest inventory, is to establish a sample point, then choose for subsampling the first tree whose center is encountered in a clockwise (or counter-clockwise) direction from the plot center. The starting azimuth can be fixed (such as always starting from North) or randomly chosen. A preference map can be constructed for any given starting direction. Figure 4 shows the construction of such a map. Examples of each of the following steps are noted. A North starting azimuth and clockwise rotation are assumed for this example.

Figure 4. Construction of preference map based on the azimuth method.



Construction steps are:

1. Construct a line (called the "stem line") from each tree South to the area border.
2. Starting at the West border label the stem lines 1-N from West to East.
3. Starting with tree 1 ($tree_i$) and continuing sequentially, sweep clockwise from North until another tree is encountered. Draw a line from $tree_i$ opposite the tree encountered until the border or another stem line is reached.

Special steps must be taken after tree N-1.

4. Consider only the final tree N. From $tree_N$ sweep clockwise from South until a tree is encountered.
5. Draw a line from the stem line of $tree_N$ to the border in a direction opposite the tree encountered.
6. Consider the tree last encountered. From it sweep clockwise from South until another tree is encountered, and repeat step 5. Continue this until the line so constructed strikes the area border west of the $tree_N$ stem line.

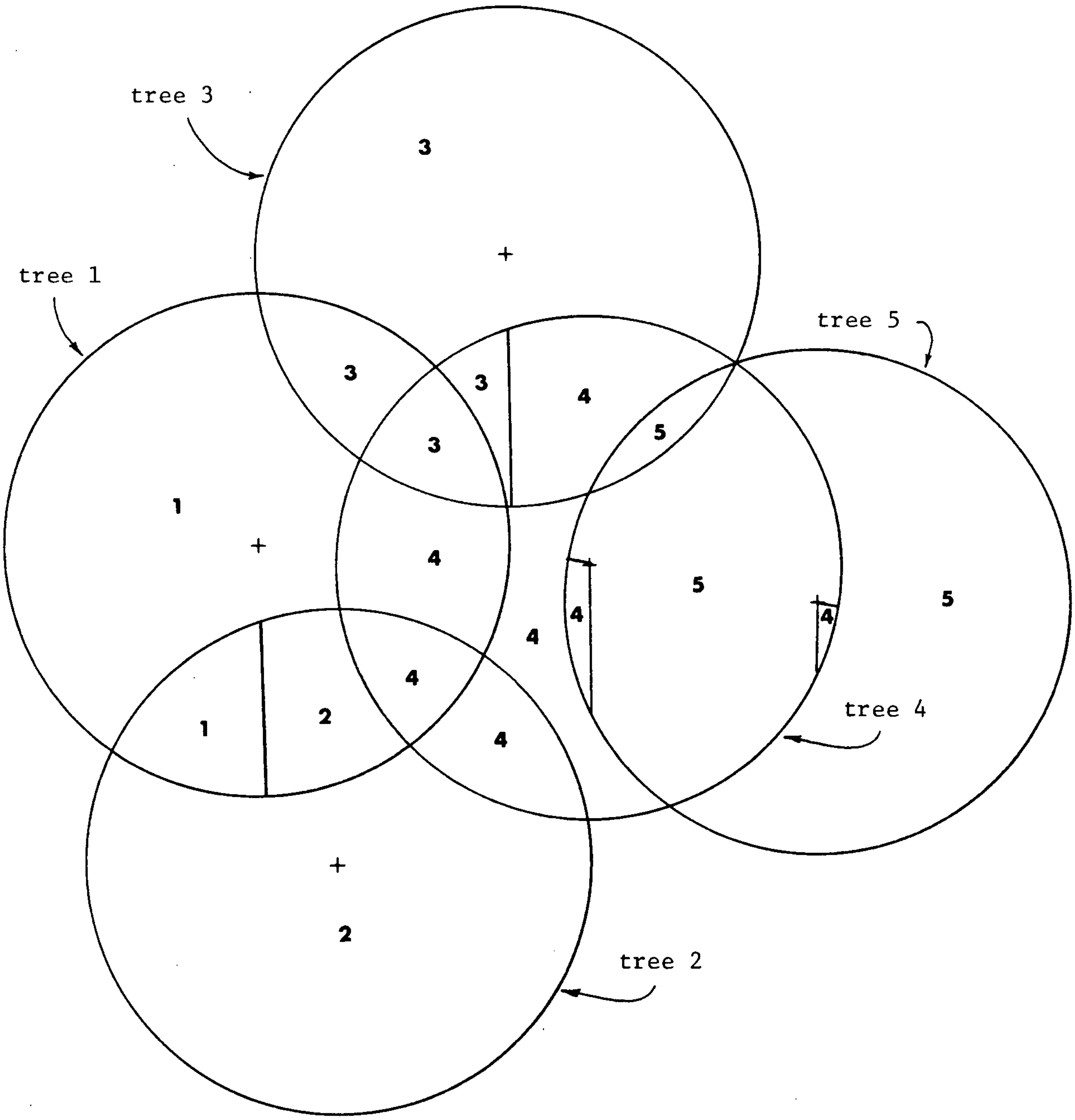
Polygons in which a particular tree will be chosen are noted on the diagram by circled tree number. Note that slight additions may occur due to areas near the border east of $tree_N$, but otherwise they are in one part.

The straight lines of such a map simplifies its programming for computer plotting. The area of the polygons is easily computed from intersection points using standard surveying computations.

When fixed plots are used, and the first tree from an azimuth is selected as a subsample, the construction is slightly more difficult. A map is drawn as in Figure 2, indicating the polygons in which a particular subset of trees may be chosen. For each of these subsets a further construction is made as described in Figure 4 but only considering the trees of that subset, and drawn across the polygon being considered. This is not too difficult, since few of the lines considered will cross each polygon. The separate areas are then labelled indicating which tree will be selected in each case. Figure 5 shows an example of such a construction.

Three points are worth mentioning when construction is done by hand. As a first step construct only the stem lines which are in polygons occupied by two or more trees. Second, label the obvious situations first, particularly cases where only one tree is eligible. Third, as you move from one polygon to the next the number of trees to be considered normally changes by one. This helps to assure that no trees are ignored. Preference maps of this sort are very difficult to draw, except in large scale. The most common errors will probably be due to steps 4, 5, and 6. Once the compartments have been designated on the preference map selection probabilities are calculated as follows:

Figure 5. Azimuth method applied with fixed plots.



$$p\left\{S_i\right\} = \frac{\sum_{c=1}^{z_i} a_{ci}}{T} \qquad (4.1)$$

where: a_{ci} = the area of a compartment on the preference map in which tree i will be selected.

z_i = the number of all the compartments allocated to a particular tree.

Relative probability of sampling a tree is:

$$Pr\left\{S_i\right\} = \frac{p\left\{S_i\right\}}{\sum_{i=1}^{N} p\left\{S_i\right\}} \qquad (4.2)$$

Probability of choosing at least one tree from a random point is:

$$p\left\{S_s\right\} = \frac{\sum_{c=1}^{z_p} a_c}{T} \qquad (4.3)$$

where: z_p = the total number of compartments in the area being sampled.

Methods for the Elimination of Bias

Much attention has been given to the distribution of individuals in plots of various size under random spacing. Matern (1971) discusses such a distribution. In fact, we know that tree distributions in the plane are not usually random. The situation was nicely described by Warren (1972) as follows:

> Trees are, of course, not points. The diameter of a tree, conventionally measured at breast height (4 ft. 6 in. or 1.3 m) is generally not negligible with respect to the distance between trees. Further, competition between trees ultimately produces an area about each tree within which no other tree can exist. These restrictions are often conveniently neglected in theoretical studies (notably the writer's); exceptions are the theoretical derivation of Matern (1960, p.47) and the simulation studies of Newnham (1968) and Newnham and Maloley (1970).

The idea that there is a truncation of actual distributions at some minimum plot size, as well as an actual upper limit for the number of trees in a given plot size, gives rise to two systems for selecting random samples. These will be called the "plot reduction method" and the "elimination method".

Plot Reduction Method

Since the problem causing bias is the overlap of plots, they can be reduced in size to a diameter equal to the smallest distance between members of the population. This means maximum diameter in the case of non-circular plots. The probability of being chosen is now equal for all members of the population and the choice of not more than one member is assured with each plot. In addition, observations selected are independent, insuring a truly random sample where any permutation of objects is equally likely. The probability of finding a particular sample tree is given by:

$$p\left\{S_i\right\} = \left(\frac{ap}{T}\right) \tag{5.1}$$

where: ap = the area of the fixed plot used.

The probability of sampling a tree with a random point is:

$$p\left\{S_s\right\} = \frac{\sum_{i=1}^{N} ap}{T} = \left(\frac{N * ap}{T}\right) \tag{5.2}$$

There are several practical ways of equalizing the spatial distribution of objects to be sampled, thereby increasing plot size and minimizing the probability of a vacant plot. It is not the tree itself which must be selected, but something which can be associated with the tree, and detectable on the plot. Even though tree stems may be quite close together the upper parts of the tree will usually be more evenly distributed. It will probably be of advantage to use the tops of trees or perhaps the edge of the crown (say the center of the north edge) to determine which tree to sample. This more even distribution will lead to more efficient sampling. In addition, such methods are useful on aerial photographs which have many advantages, particularly their high potential for automation.

The Elimination Technique

Reduction of plot size can become quite severe under conditions of high clumping, this leads to inefficiency in sampling due to unoccupied plots. The elimination technique may be more effective in this case. A plot of size ap is chosen, and the maximum number of trees which could fall into such a plot is symbolized by Mp. For any plot the trees included are numbered from 1 to np and a random number R is chosen between 1 and M_p. If R corresponds to the number of one of the trees in the plot that tree is chosen, otherwise a new plot is established. This system again gives a random sample of individuals. The probability of selecting a single tree is calculated by:

$$p\left\{S_i\right\} = \left(\frac{ap}{T}\right)\left(\frac{1}{Mp}\right) \qquad (6.1)$$

which is a constant. The relative probability is simply 1/N.

If the expected maximum number of trees is exceeded, the system will be slightly biased. The actual probability for a particular tree can then be computed by the standard formula <u>except</u> in those compartments where np > Mp. Where the plots are not large and Mp conservatively estimated this should happen infrequently. The relative frequency of any tree will not then be 1/N, but:

$$\Pr\left\{S_i\right\} = \frac{p\left\{S_i\right\}}{\sum_{i=1}^{N} p\left\{S_i\right\}} \tag{6.2}$$

where the maximum of the values ($1/np$, $1/M_p$) has been used in equation 6.1 to calculate $p\left\{S_i\right\}$.

Increasing Efficiency

The way to minimize the occurrence of unoccupied plots is to maximize the product of two probabilities:

$$p\left\{\text{sampling a tree}\right\} = p\left\{\text{one or more trees in plot}\right\} * p\left\{\text{selecting one of those trees}\right\}$$

more formally:

$$p\left\{S_s\right\} = \sum_{np=1}^{Mp} \left[\left(p\left\{S_{np}\right\}\right)\left(\frac{np}{Mp}\right)\right] \tag{6.3}$$

where: $p\left\{np > Mp\right\} = 0$

$p\left\{S_{np}\right\}$ is the probability of selecting a plot with np trees present for possible selection.

The probabilities of selecting particular numbers of trees might be calculated from theoretical considerations or by field tests.

The elimination procedure is a very general one, and can be used to choose a final sample from the equal probability first stage selection with plots. Suppose one is interested in choosing a random sample of trees from a population with probability proportional to $DBH_i^{2.6}$. One way to make the selection is to choose a subset with equal probability using a plot, then select from the manageably short list of $DBH_i^{2.6}$ by means of a random number. The random number would be chosen between 1 and the maximum expected value for $\sum_{i=1}^{np} DBH_i^{2.6}$. Grosenbaugh's system for 3P sampling is similar, except that he deals with the entire population rather than a subset.

The discussions in this thesis will generally regard the occasion when a sample is not drawn at a random point as a waste of time and effort. This is not strictly true, since the occurrence of vacant plots might be used for post-stratification to increase the precision of the estimator or for other uses. In the example of choosing a tree proportional to $DBH_i^{2.6}$ the lists of DBH on plots where no sample was chosen might be used as additional information for estimating the parameter of interest, much like Grosenbaugh's use of the "adjusted" rather than the "unadjusted" estimator (Grosenbaugh, 1976). Since efficiency under these considerations becomes heavily and complexly involved with the choice of estimator, a simpler criterion will be applied. The criteria of efficiency is the probability of selecting a random sample with minimum field work, which generally means selecting a tree at each random sample point.

Systems which screen the population through two or more stages can be constructed in a variety of ways. Consider the equal probability selection scheme. At the first stage we could select a cluster of trees, each selection being proportional to tree basal area (or any other criteria which would simplify field selection). At the second stage this manageable subset could select an individual proportional to (1/basal area) by the elimination system. The product of the two probabilities would be:

$$p \left\{S_i\right\} = \left(\frac{BA_i}{C_1 * T}\right) * \left(\frac{C_2}{BA_i}\right) = \left(\frac{C_2}{C_1 * T}\right) \qquad (6.4)$$

where: BA_i = the basal area of tree i

C_1, C_2 = constants depending on the exact selection system used.

C_1 depends on the 1st stage selection probability (probably on the critical angle discussed later).

C_2 depends on the largest possible value for $\sum_{i=1}^{n} \left(\frac{1}{BA_i}\right)$.

The product is a constant, regardless of tree basal area. The relative probability of selection is therefore 1/N. Such a multiple stage selection system may be of advantage where simple mechanical means can be contrived to select trees with compensating probabilities at one or more stages. The variability of these schemes would be of primary interest, and would have to be robust under a variety of spatial distributions. Simulation of stem maps would probably be the best device for assuring this property.

Because of the field simplicity of selections based on fixed distances or angles, and list selection from short lists, the elimination method will be of primary interest.

Diameter or Circumference Weighting

Selection Proportional to Diameter Alone

This problem was essentially solved by Strand (1958) with the introduction of horizontal line sampling. Trees are sighted at right angles along transects through the sample area. A tree can be selected when the tree stem subtends the angle projected perpendicular to the line. Figure 6 shows an example. A tree will be picked if the line crosses an unseen plot surrounding the tree and proportional to its basal area. The use of the angle gauge determines when you are in the plot, the magnitude of the angle determines the absolute size of the unseen plot. The dashed lines in Figure 6 indicate the unseen borders of the plots around each tree. To simplify field work the trees are sometimes sighted to only one side of the transect. Changes to the equations used are simple. The assumption in the following discussions is that trees are sighted to both sides of the transect.

The probability of selection obviously is proportional to the diameter of the unseen plot around each tree. Figure 7 illustrates the principle. The angle, when small, acts in the same way as calipering

Figure 6. Horizontal line sampling, basic idea of the selection rule.

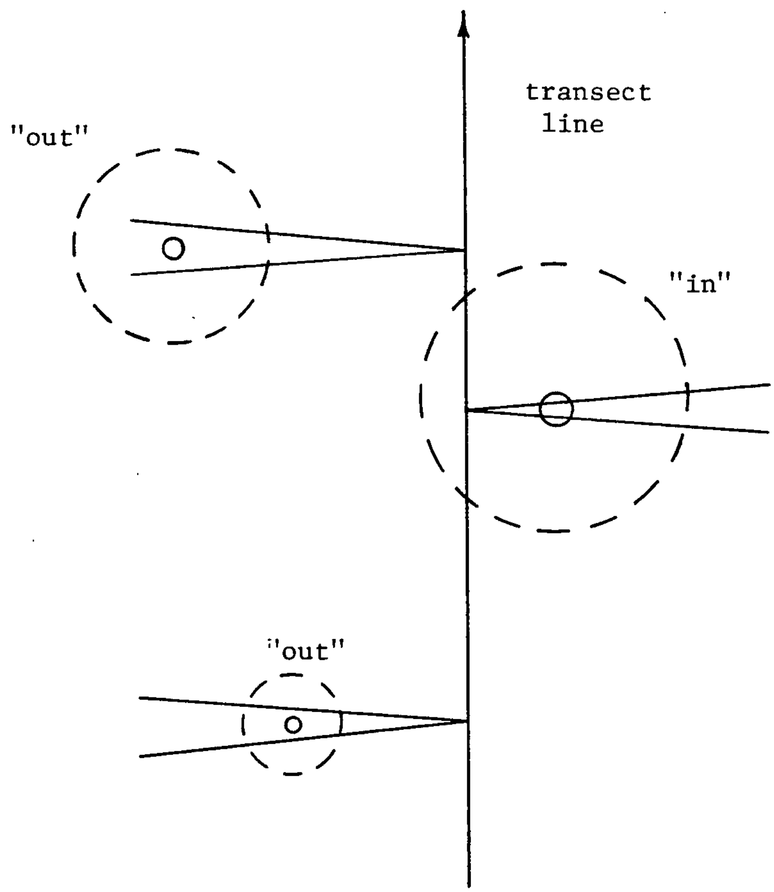
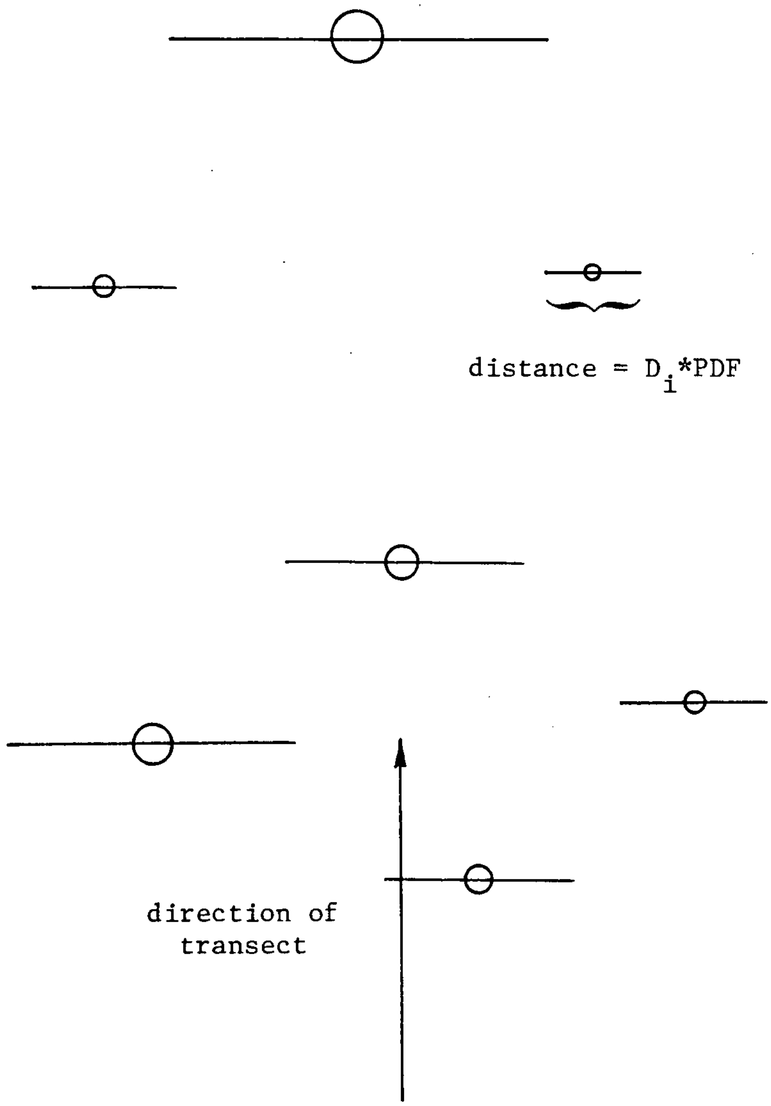


Figure 7. Illustration of selection probability with line sampling.

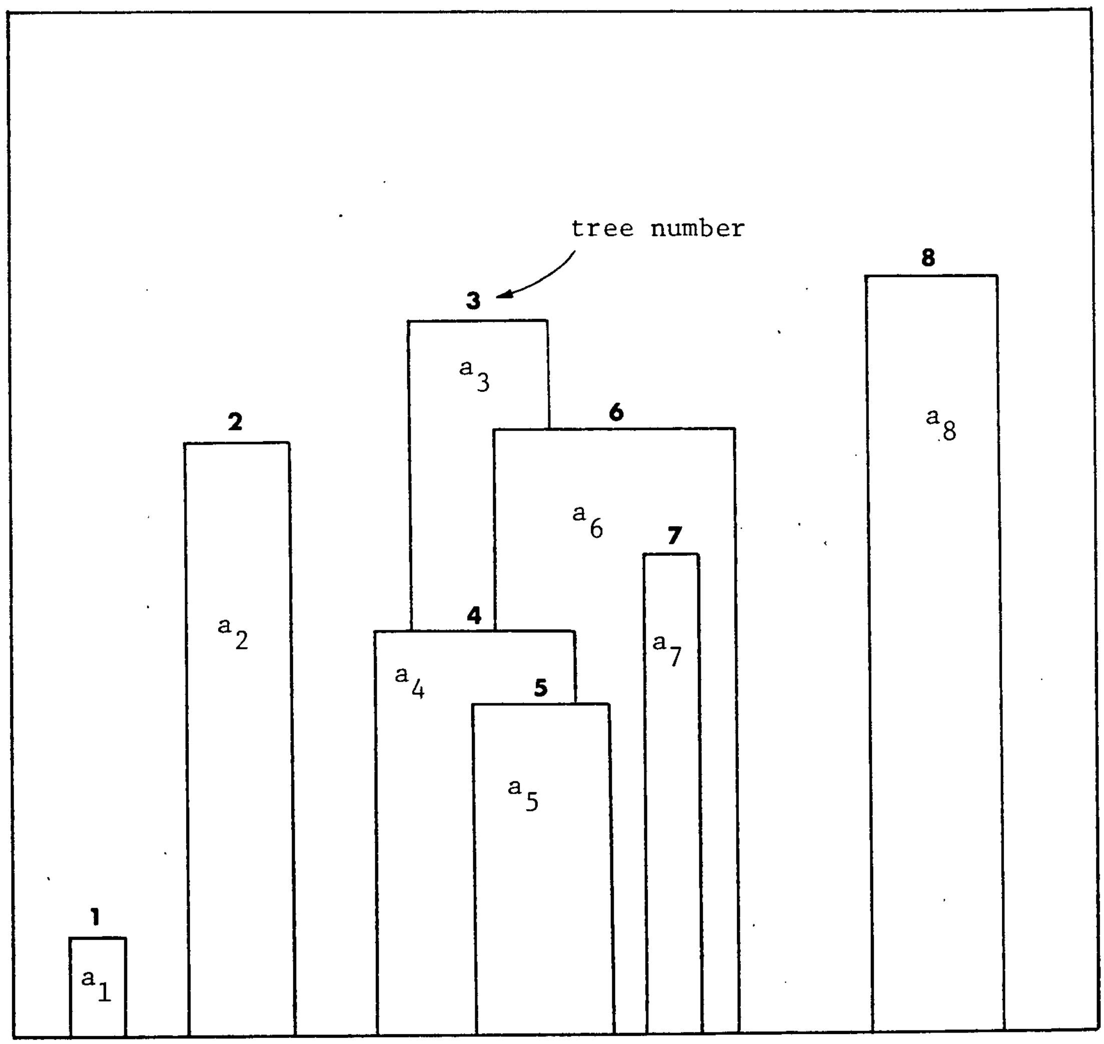


a tree for diameter (D_i) and multiplying by a constant which we shall call the Plot Diameter Factor (PDF). Under the assumption that trees are "convex outward" in cross-section (i.e. flat or curved outward so that a string wrapped around it would always touch the surface) the average of all possible caliper measurements is equal to the perimeter divided by π . The horizontal line sample therefore selects trees proportional to the perimeter or average diameter. Grosenbaugh (1958) has pointed out that large angles can cause certain biases, since the angle gauge no longer nearly resembles the caliper measurement. This is of little concern for most practical work which uses angles in the range of .03 to .08 radians (roughly 1.7 to 4.5 degrees).

In the case where angles must be large for some reason, or trees cannot be considered convex outward there is always the option of sampling with a plot, then choosing from a list of cumulative diameters or perimeters by the elimination method. Two variants of horizontal line methods are biased, and the bias can be calculated by the construction of preference maps.

With the first method a transect is started from a random point and the first tree "in" with the angle gauge is chosen for sampling. A preference map can be drawn only where the direction of the line is assumed. A line centered at the tree, and of length (PDF * D_i) is constructed (see Figure 8). D_i is the diameter of the tree at the sighting point of the angle gauge.

Figure 8. Construction of line sample preference map.



The PDF is a constant relating tree diameter to the diameter of an unseen plot surrounding the tree. The value depends upon the "critical angle" (θ) which is used.

$$\mathrm{PDF} = \left(\frac{\text{plot diameter}}{\text{tree diameter}} \right)$$

This tree will be chosen for sampling whenever a point is chosen along its band. The bands run opposite the direction of the sampling transects until intercepted by another tree or the area border. The proportional area of these bands (ab_i) determine the probability of sampling the tree. Specifically:

Probability of sampling a particular tree:

$$p\left\{S_i\right\} = \left(\frac{ab_i}{T}\right) \tag{7.1}$$

Relative probability is:

$$Pr\left\{S_i\right\} = \frac{p\left\{S_i\right\}}{\sum_{i=1}^{N} p\left\{S_i\right\}} \tag{7.2}$$

The probability of sampling a tree along a transect beginning at a random point is:

$$p\left\{S_s\right\} = \frac{\sum_{i=1}^{N} ab_i}{T} \tag{7.3}$$

If the transect lines are of fixed length (L_b) there is only a slight modification. All bands of the preference map are truncated at length L_b from the tree. All equations above will then apply to the revised diagram. There is an obvious bias toward solitary trees and those on one edge of clumps. As the diameter of the plots increases we approach a limiting situation where only the distance to the next tree in the population (in the direction of the transect) is the determining factor.

These diagrams lend themselves easily to computer plotting. The straight lines, truncated only by interception of another tree or the area border, are simply calculated and drawn, and areas are easily computed. When the transect is run other than North-South, it is easy to translate all the coordinates and act as though the transects were North-South.

A second method is to extend a line from a random point in the tract, and sample the first tree intercepted. This is essentially the same as the previous system, but interception along a simple compass line is used instead of using an angle gauge. The bands of the preference map will be longer and narrower, and a mathematical approach will probably be needed because of the small scale involved. The width of the band is simply the diameter of the tree.

Selection Proportional to Diameter and a Constant (C_s)

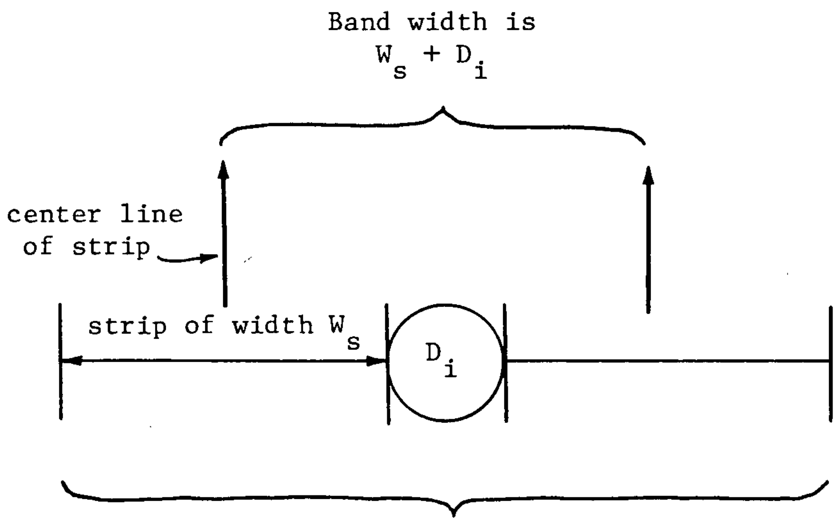
A different variation is to sample the first tree encountered by running a strip of specified width (W_s) centered on a transect

If the tree is included when any part of it is within the strip sampling is proportional to diameter plus a constant. When the tree is sampled only if the tree center is included this takes the form of sampling with a rectangular plot of varying length. The preference map for the first method can be easily constructed. The band width is simply $D_i + W_s$, centered at the tree. Further construction is the same. Figure 9 shows the basic idea. The elimination technique can be used, with a constant length strip, to obtain a random sample. When selection is to be proportional to $D_i - C_s$ only a small change is required. In this case, the tree is chosen only when it is entirely within the strip. Figure 10 shows the construction of the preference band of such a tree.

Problems of Scale and Field Use

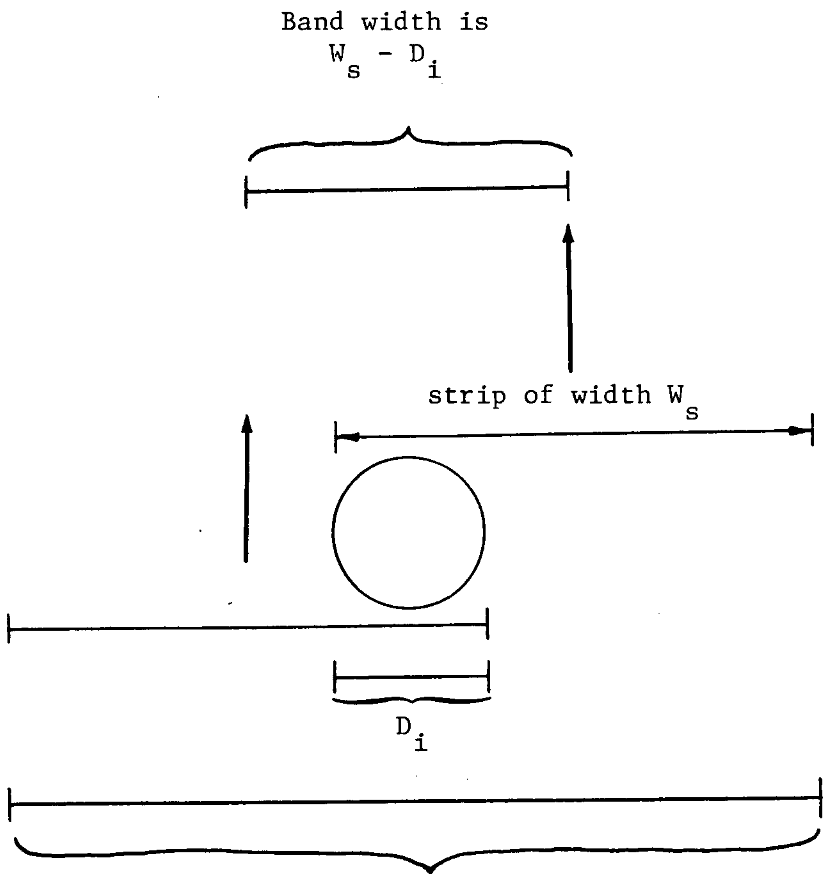
If the ratio of strip width to diameter is too large or too small the diameter can be "expanded" or "reduced" to make it easier to do the sampling. Reduction can be done by calipering and using $(1/x) * (D_i)$ as the diameter. Alternately an angle gauge can be used to get a proportional reduction on the principles of the Biltmore Stick. Unfortunately, such a system is very dependent on a round cross-section. Figure 11 shows the principle.

Figure 9. Band width for the preference map when sampling is proportional to $D_i + C_s$.



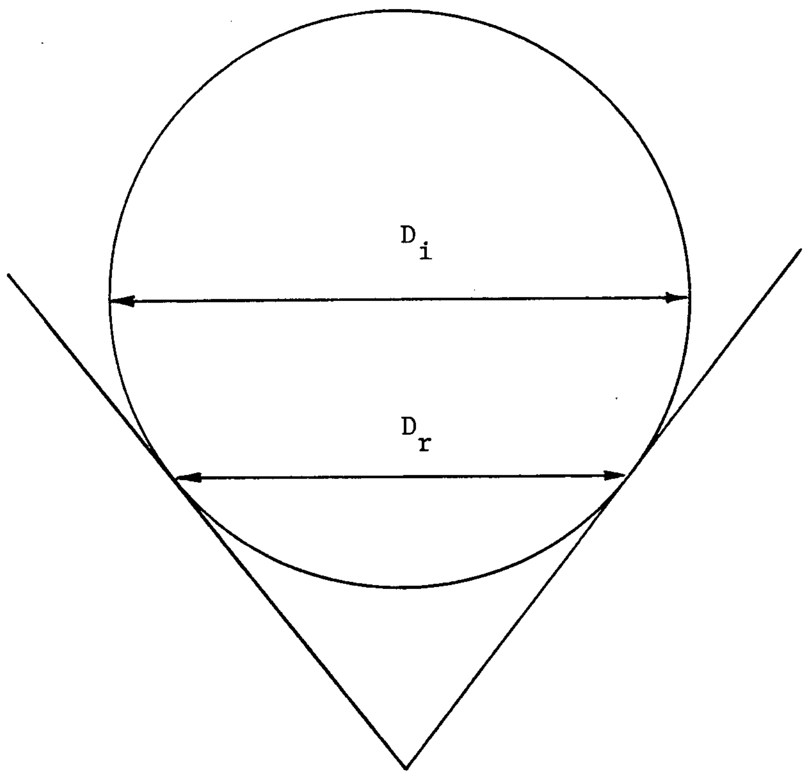
The tree will be chosen whenever the strip passes between either of these limiting situations. The distance between the center lines then determines the probability of selecting that tree.

Figure 10. Band width for the preference map when sampling is proportional to $D_i - C_s$.



The tree is chosen when the strip is between these two extremes. The distance between the centerlines determines the probability of selection.

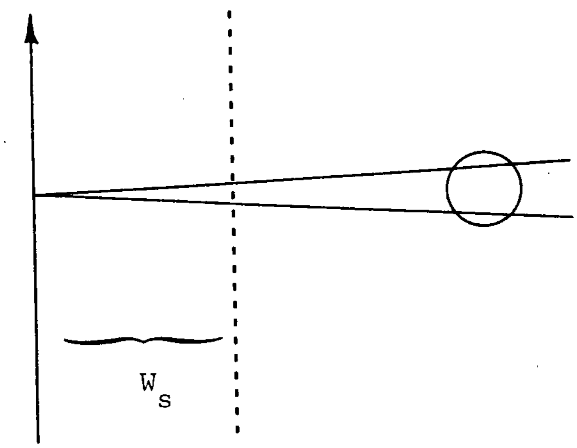
Figure 11. Using an angle gauge to establish the reduced diameter of a tree.



The angle of the instrument determines the proportion of D_i to D_r. The distance between the contact points is then used as the reduced diameter of the tree.

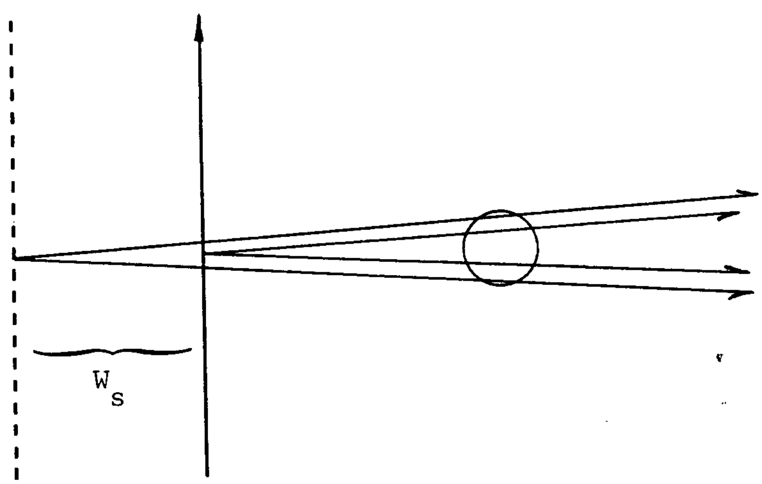
Proportional expansion of the tree diameter is easy to establish with the angle gauge. Often the use of the expanded diameter makes the constant term a more reasonable distance to measure. Figures 12 and 13 show two ways to select, in the field, sample trees proportional to $D_i - C_s$. Similarly Figure 14 shows a system for selection proportional to $D_i + C_s$. A later discussion will outline the use of the Wheeler Pentaprism to automatically account for the strip width.

Figure 12. Field systems to select trees proportional to D_i minus C_s.



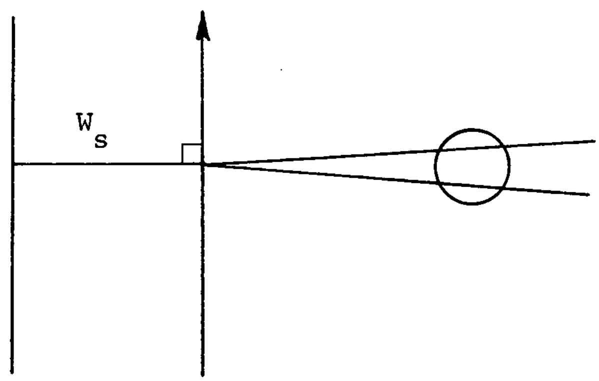
Select "in" trees as long as the tree center is <u>not</u> within a distance W_s from the transect.

Figure 13.



If the tree is "in" along the transect move back a distance W_s and sight again. Sample the tree only if it is "in" at both points.

Figure 14. Field system to select tree proportional to D_i plus C_s.



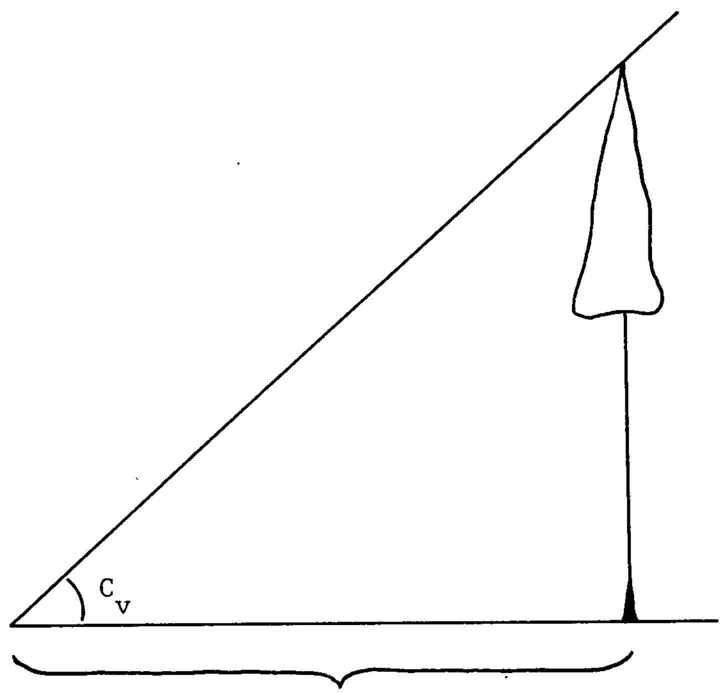
Select the tree if it is "in" with the prism to one side of the transect <u>or</u> within the distance W_s from the transect on the other side.

Tree Height Weighting

This problem is solved by using vertical angles in much the same way that a horizontal angle was used on tree diameters. A tree is to be sampled when a transect passes within a distance proportional to tree height. A simple method, in flat terrain, is to sample the tree when it falls within a particular vertical angle (C_V). Figure 15 shows the general idea. The angle is being used to establish the proportion of height to distance. Hirata (1962) describes this method to sample forest areas for volume. In sloping terrain this angle selection method still selects proportional to height, but the exact proportion is no longer given by the tangent of the angle. In such cases, the difference between two tangents is used, read from a suitably calibrated instrument such as the Sunto Clinometer. Figure 16 shows the principle involved.

The preference map is constructed in exactly the same way as with horizontal line sampling. The system applies equally well to some segment of the tree height such as distance to the first limb, merchantable height, crown length, etc. The reasoning concerning the elimination method and application of a constant term are the same as with horizontal line sampling. The band width is determined by height rather than diameter, but otherwise there is no difference in the probabilities or preference maps.

Figure 15. Use of the angle gauge on flat ground for vertical sampling.



Sampling distance is calculated by:

$$\text{Distance} = \text{cotan } C_v * \text{Height}$$

The effect of leaning trees is discussed by Loetsch et al. (1973). If the tree is not easily seen along a perpendicular from the transect it can be tested from any other point which maintains the same distance as from the line to the tree.

An Adaptation of Line-Intersect Sampling to Standing Trees

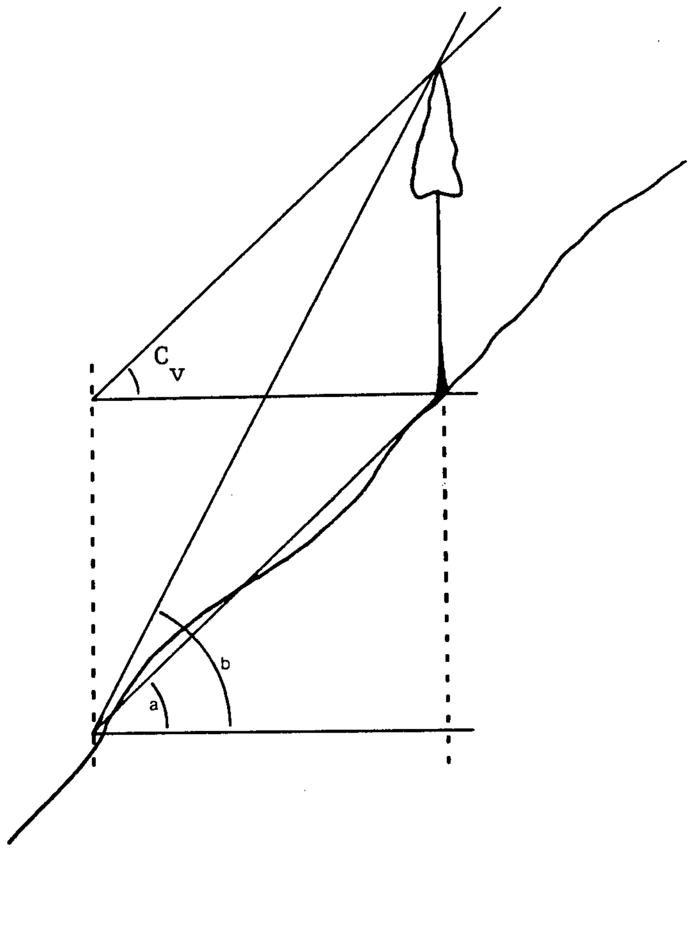
If all the trees on a tract were felled perpendicular to a random transect of length L, the tract volume could be simply estimated as a special case of line-intersect sampling. The estimate, from a single transect, of total tract volume would be:

$$V = \left[\frac{\sum_{i=1}^{n} CA_i}{L} \right] * T$$

where: CA_i is the cross-sectional area of a stem crossed by the transect.

This is the average depth of tree cross-sections along the transect multiplied by the tract area. The trees will not in fact be conveniently laid across the transect, but could this situation be simulated in some way? The key point is that the diameter to measure is the one which is the same distance up the standing tree stem as the horizontal distance from the transect to the tree. That point can easily be found on level ground by looking up the tree at a $[\pi/4]$ radian (45°) vertical angle. On sloped ground a "% scale" as illustrated in Figure 16 can easily be used. We thus have a simple way of using line-intersect sampling theory on standing trees.

Figure 16. Use of two measurements on sloping ground.



(tan b - tan a) = tan C_v, giving correct horizontal sampling distance in sloped areas.

If $\pi/4$ radians is an inconvenient angle for some reason, the estimating equation can be slightly changed. The distance over which a tree will be sighted with a vertical angle C_v is:

$$\text{distance} = \text{tree height} \ * \ \text{cotan } C_v$$

Since the tree is "stretched" out over a longer distance, the estimate must be correspondingly decreased. The final formula being:

$$V = \left[\frac{\sum_{i=1}^{n} BA_i}{L} \right] * T * \tan C_v$$

A recent work by Minowa (1976) in the Japanese language appears to have been based on the same idea. Although a complete translation is not available his formula is clearly equivalent to the one just derived. Minowa gives it as follows:

$$V = \left(\frac{T \quad \pi}{4 * L * \cot \beta} \right) * \sum d^2$$

From the article cited and subsequent work (Minowa, 1978), it seems obvious that Minowa has not recognized that this part of his work is a special case of line-intersect sampling. This is an important step, since this form of sampling has accumulated a considerable amount of literature, field testing and acceptance by field foresters.

Basal Area Weighting

Since the development of the sampling method of Bitterlich (1948), usually called horizontal point sampling or variable plot sampling, a great deal of forest inventory has been done with selection of trees proportional to basal area. Since a simple count of trees which are "in" with an angle gauge only provides an estimate of basal area, several trees are usually subsampled for volume characteristics.

This is done mainly to establish the "Volume to Basal Area Ratio" (VBAR). This ratio is usually much less variable than the number of "in" trees. Only a few of these trees should be measured for maximum sampling efficiency. The cost of <u>measuring</u> trees, compared to simply counting those which subtend the angle used, is quite high. To minimize cost perhaps only one in twenty trees should be measured. The problem is to choose the volume sample trees at least unbiasedly if not at random.

One attempt to evade the relative cost problem is to construct "Diameter-Height Curves" showing the tree heights for each diameter class. The diameters of all "in" trees are then measured, at little expense, and heights are read from the curve. It is open to question whether the extra effort might not be invested more profitably in taking more plots for tree count only. Once such a Diameter-Height Curve is established it constitutes a bias in subsequent measurements which should be accounted for statistically.

The process by which subsamples are chosen will be examined in some detail in regards to possible biases and non-randomness.

Basic Ideas of Point Sampling

In point sampling a tree is counted when the stem subtends an angle established with some form of angle gauge. The angle radiates from a single point on the land area being sampled. There is therefore a circle around each tree of diameter (D_i * PDF) where that tree will be counted "in" with the angle gauge. Figure 17 shows an example. The illustration is very much like Figure 2, except that the circles have an area proportional to the basal area of the stem. The probability of picking a tree with an angle gauge from a random point is obviously proportional to the basal area of the stem. More formally the probabilities are:

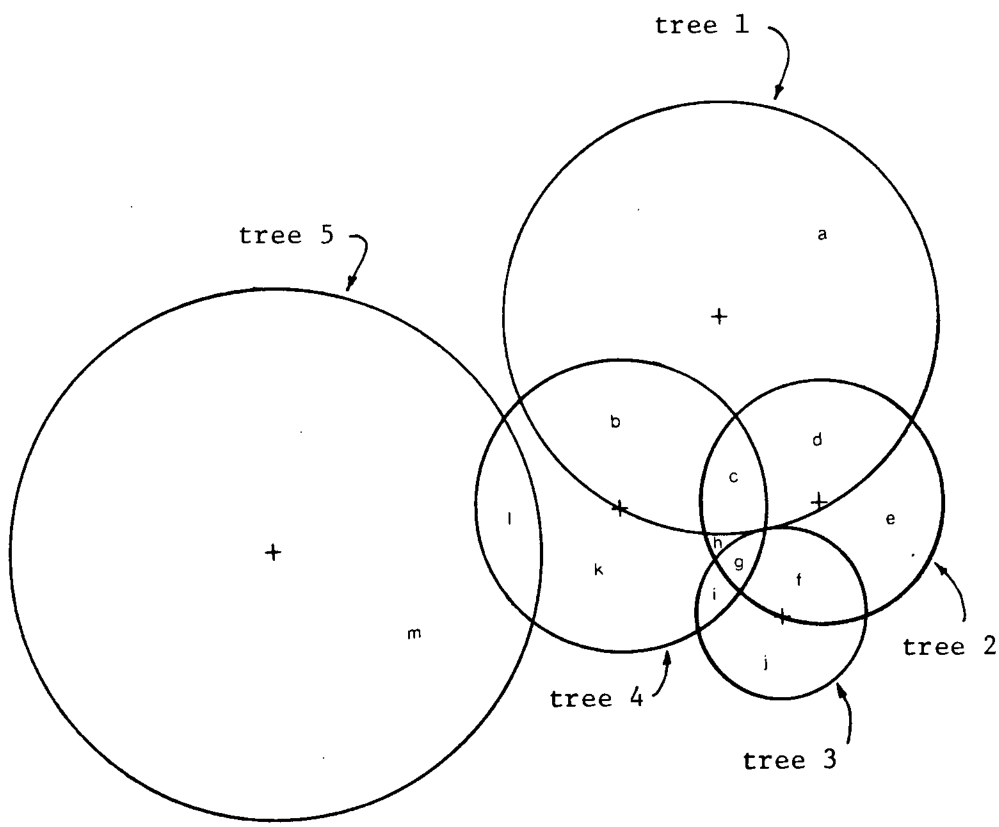
For picking an individual tree:

$$p\left\{S_i\right\} = \left[\frac{BA_i * PDF^2}{T}\right] \tag{8.1}$$

Relative probability of picking a tree:

$$Pr\left\{S_i\right\} = \frac{p\left\{S_i\right\}}{\sum_{i=1}^{N} p\left\{S_i\right\}} = \frac{BA_i}{\sum_{i=1}^{N} BA_i} \tag{8.2}$$

Figure 17. Plots of variable size in horizontal point sampling.



Slightly more difficult are the probabilities of sampling when one of the trees is randomly chosen from a cluster which are all "in" from a particular point. Choosing clusters of trees makes a sample non-random, but selecting one tree from each cluster can seldom fail to be biased as well.

Bias From Selection of a Single Individual From Every Cluster

Selection of one individual has all the biases discussed earlier with fixed plots. It favors clusters with small numbers of trees, but this clustering effect is changed because smaller trees have smaller plots. In addition, when the selection is not random, further biases can exist. Each of the simple systems can be explored by preference maps.

Bias From Random Selection From Each Group

After the mapping of plots around each of the trees a group of compartments is formed. For each of these t compartments the number of trees involved (n_k) and its area (a_k) are determined. The expectations for sampling are then calculated as follows:

Probability of sampling a particular tree:

$$p\left\{S_i\right\} = \frac{1.0}{T}\left[\sum_{k=1}^{t_i}\left(\frac{a_k}{n_k}\right)\right] \tag{2.1}$$

Relative probability of sampling a tree:

$$Pr\left\{S_i\right\} = \frac{p\left\{S_i\right\}}{\sum_{i=1}^{N} p\left\{S_i\right\}} \tag{2.2}$$

These are simply the equations used earlier with fixed plots. The difference is that the area of the plots is now (BA_i * PDF^2) rather than fixed. To complete the example, the calculations are given for Figure 17 in Table 2.

In general, there is a decrease in individual probability as more plots overlap, indicating a bias again toward sparsely distributed trees. The average number of trees counted from a random point is a function of the basal area of the trees, not of their numbers. Where basal area is evenly distributed bias is reduced. Stratification will help to reduce the bias when it is done to equalize basal area. The overlap of plots will have a greater effect on smaller trees, where the overlap is a greater proportion of total area, therefore the mixture of size classes favors selection of larger trees.

Table 2. Calculations for the example shown in Figure 17.

Compartment	tree 1	2	3	4	5	a_k	n_k	$\left[\frac{a_k}{n_k}\right]$
a	*					15.5	1	15.5
b	*			*		4.83	2	2.42
c	*	*		*		1.33	3	0.44
d	*	*				3.33	2	1.67
e		*				3.00	1	3.00
f		*	*			1.83	2	0.92
g		*	*	*		0.5	3	0.17
h		*		*		0.17	2	0.08
i			*	*		0.5	2	0.25
j			*			2.17	1	2.17
k				*		4.83	1	4.83
l				*	*	1.83	2	0.92
m					*	43.17	1	43.17
						83.00		

Tree	$\sum[a_k/n_k]$	Relative Probability actual	theoretical	Selection Bias
1	20.02	.2412	25/99*	-4.5%
2	6.28	.0757	10/99	-25%
3	3.51	.0423	5/99	-16%
4	9.11	.1098	14/99	-22.5%
5	44.08	.5311	45/99	+17%
	83.00 m^2			

* sum of plot areas = 99 m^2, tree 1 has a plot of 25 m^2.

Nearest Tree Method

When the nearest tree which is "in" with an angle gauge is chosen for sampling the preference map is basically the same as it was with fixed plots, but the size of the plot around the tree varies with tree size. The area formed by the overlap of two plots is allocated to the smaller one unless the bisector falls within the overlap. See Figures 18c to 18e.

As shown in Figure 18, the initial increase in selection probability for two plots is in favor of the smaller (See 18b). The bias in relative selection probability eventually equalizes as plot centers approach each other, and favor the larger plot only with very close spacing as in Figure 18d and 18e. As will now be shown, a random spacing favors the smaller tree.

Consider the plots of two trees (a) is the smaller and (b) is the larger. See Figure 19. We will choose one plot, and its associated tree, for sampling when we are within that plot and that plot center is nearest the random point P. Consider what will happen when a and b overlay the random point. b will never be chosen when a and the outer ring of b (b_o) occur at the point, since the center of b would always be further away. Since b_i is the same size as a it has an equal chance of being closest to P when a and b_i occur together. The conditional probability of choosing b when a and b overlay, P is therefore reduced. b has a 50% chance of selection only if b_i and a overlay P, and no chance otherwise.

Figure 18. Constructions with "nearest tree" selection and variable plots.

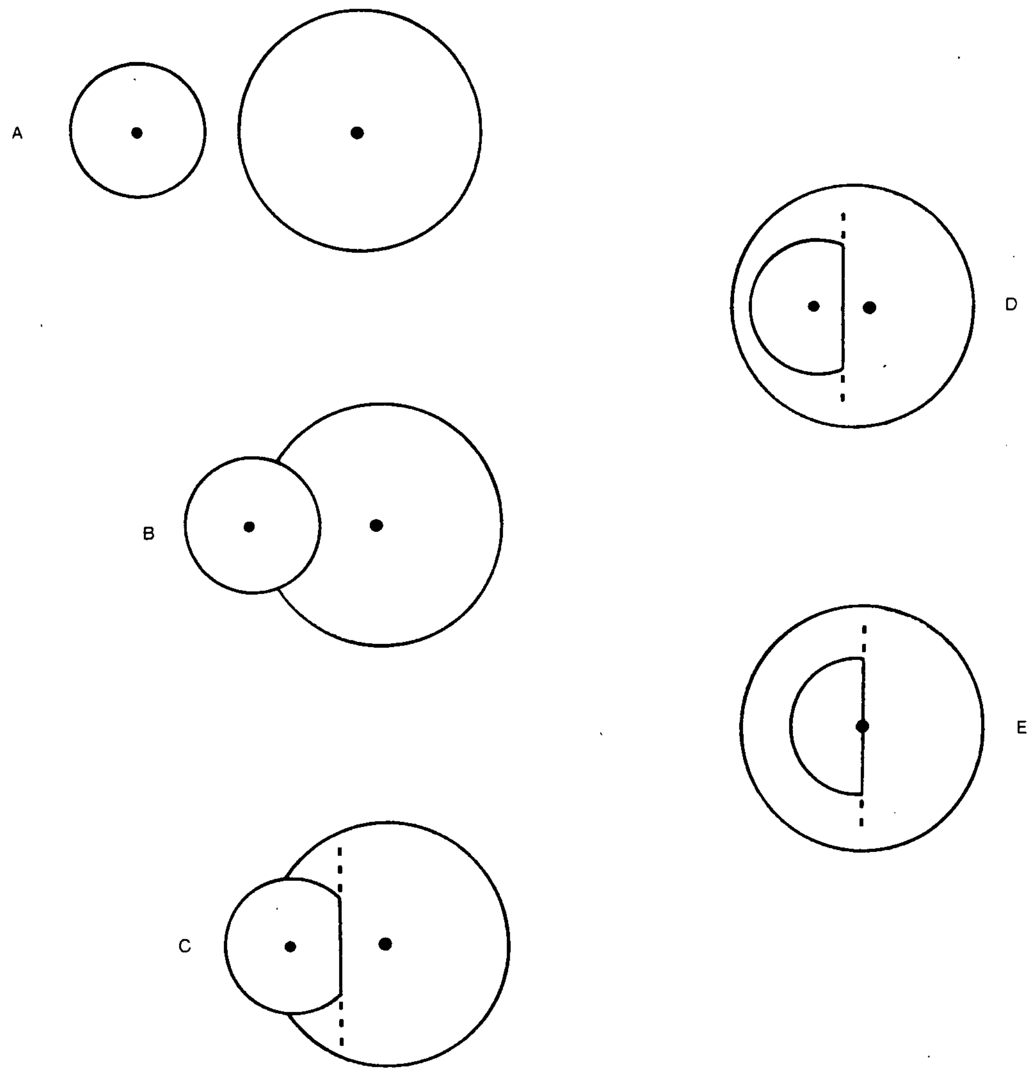
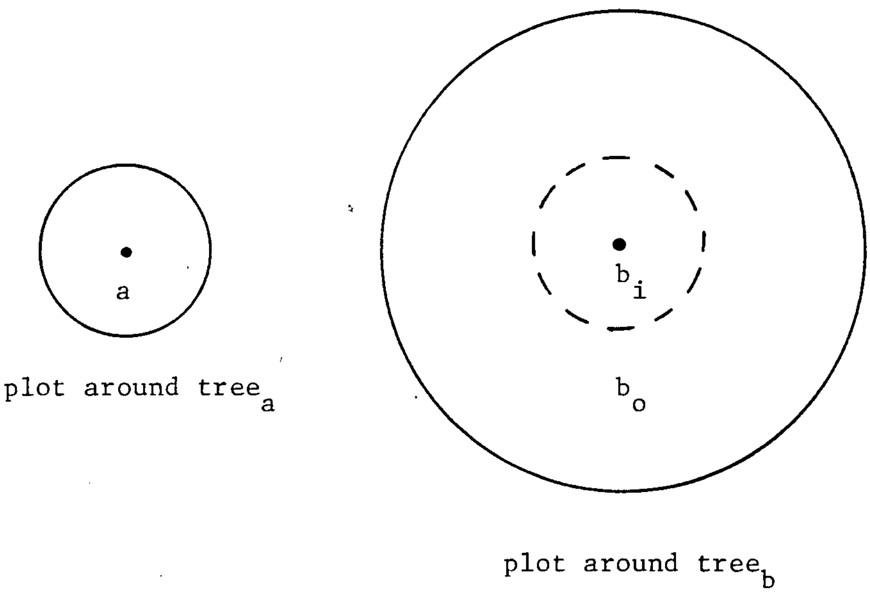


Figure 19. Illustration of terms used in proof of bias toward smaller trees.



area b_i = area a

area b = area b_i + area b_o

Ignoring edge effects, the exact probabilities are noted below.

A = area of plot a

B = area of plot b

B_o = area of outer ring of plot b as in Figure 19, it is the same as B-A.

B_i = area of the inner circle of plot b, it is the same size as plot a.

Probability that a occurs alone, hence a will always be chosen.

$$p\left\{a, \overline{b}\right\} = \left(\frac{A}{T}\right)\left(1 - \frac{B}{T}\right) \tag{9.1}$$

Probability that b occurs alone, hence b will always be chosen.

$$p\left\{b, \overline{a}\right\} = \left(\frac{B}{T}\right)\left(1 - \frac{A}{T}\right) \tag{9.2}$$

Probability that a and b occur together.

$$p\left\{a, b_i\right\} = \left[\frac{A}{T}\right]*\left[\frac{B_i}{T}\right]=\left[\frac{A}{T} * \frac{A}{T}\right]=\left[\frac{A}{T} * \frac{B}{T} * \frac{A}{B}\right] \tag{9.3}$$

since $B_i = A = B * \left[\frac{A}{B}\right]$

Conditional probability of choosing a and b given that a and b_i occur together:

$$p\{a\,|\,a,b_i\} = \tfrac{1}{2} \tag{9.4}$$

$$p\{b\,|\,a,b_i\} = \tfrac{1}{2} \tag{9.5}$$

Probability that a and b_o occur together, hence a is always chosen.

$$p\{a,b_o\} = \left(\frac{A}{T}\right)\left(\frac{B-A}{T}\right) \tag{9.6}$$

The effect of the relative probabilities is not difficult to derive formally using these equations in combination.

The probability of choosing a is:

$$p\{S_a\} = p\{a,\bar{b}\} + \frac{1}{2}\left[p\{a,b_i\}\right] + p\{a,b_o\} \tag{9.7}$$

$$= \left[\frac{A}{T}\left(1 - \frac{B}{T}\right)\right] + \left[\tfrac{1}{2}\ \frac{A}{T}\left[\frac{B_i}{T}\right]\right] + \left[\frac{A}{T}\left(\frac{B-A}{T}\right)\right]$$

$$= \frac{A}{T}\left[\left(1 - \frac{B}{T}\right) + \tfrac{1}{2}\ \frac{A}{T} + \frac{B-A}{T}\right]$$

$$= \frac{A}{T}\left[\frac{(T-B) + (\tfrac{1}{2}A) + (B-A)}{T}\right]$$

$$= \frac{A}{T}\left[\frac{T - \tfrac{1}{2}A}{T}\right] = \frac{A}{T}\left[1 - \frac{\tfrac{1}{2}A}{T}\right] \tag{9.8}$$

The probability of sampling a is $\left[A/T\ (1 - R_a)\right]$, where the term R_a stands for the proportional reduction due to both plot overlap and the selection system. We can do the same thing for tree b.

$$p\left\{S_b\right\} = p\left\{b,\bar{a}\right\} + \frac{1}{2}\left[p\left\{a,b_i\right\}\right] \qquad (9.9)$$

$$= \left[\frac{B}{T}\left(1-\frac{A}{T}\right)\right] + \left[\tfrac{1}{2}\,\frac{A}{T}\left(\frac{B_i}{T}\right)\right]$$

$$= \left[\frac{B}{T}\left(1-\frac{A}{T}\right)\right] + \left[\frac{\tfrac{1}{2}A}{T}\;\frac{B}{T}\;\frac{A}{B}\right]$$

$$= \frac{B}{T}\left[\left(1-\frac{A}{T}\right) + \left[\frac{\tfrac{1}{2}A}{T}\;\frac{A}{B}\right]\right]$$

$$= \frac{B}{T}\left[1-\left(\left[\frac{A}{T}\right.-\left[\frac{\tfrac{1}{2}A}{T}\;\frac{A}{B}\right]\right)\right] \qquad (9.10)$$

We now have the form $\left[B/T \quad (1 - R_b)\right]$

If R_b is larger than R_a there is a greater proportional reduction in the large plot than in the small one. $p\{b\}$ is proportionally reduced if and only if:

$$R_b > R_a$$

iff

$$\left(\frac{A}{T}\right) - \left[\frac{\tfrac{1}{2}A}{T}\;\frac{A}{B}\right] > \frac{\tfrac{1}{2}A}{T}$$

iff

$$\left(2-\frac{A}{B}\right)\frac{\tfrac{1}{2}A}{T} > \frac{\tfrac{1}{2}A}{T}$$

iff

$$\left(2-\frac{A}{B}\right) > 1$$

iff

$$\frac{A}{B} < 1$$

which is always the case when A is smaller than B.

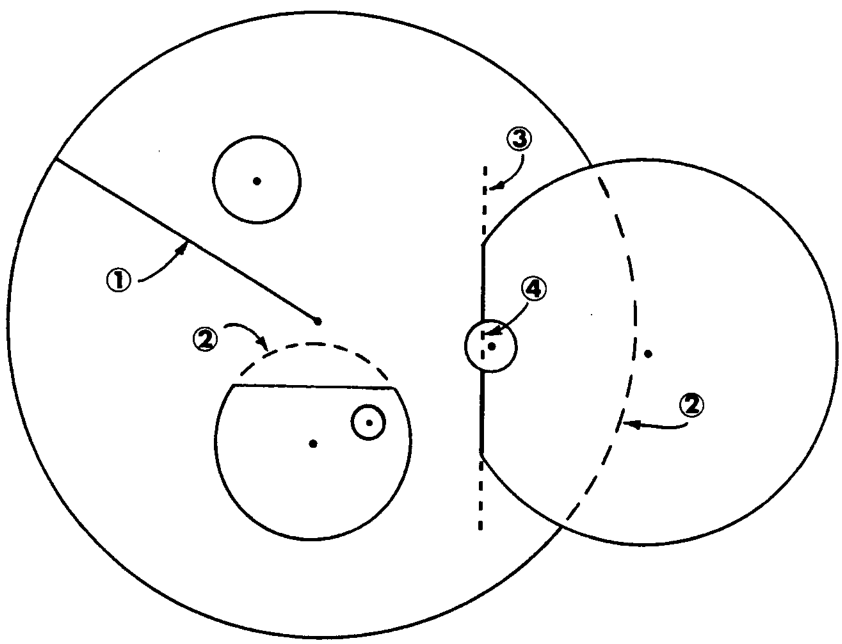
It is admittedly of minor interest what will happen under random spacing, since we know that trees are not distributed in this manner. As the angle gauge changes to produce larger plots the preference map uses more bisectors to describe the areas, eventually forming Dirichlet cells as the limiting case.

The computer plotting of the preference maps can be simplified by some of the following rules about lines. The circled numbers on Figure 20 refer to use of the rules noted below.

1) There is a distance, called the "plot radius" for each tree i, computed by $PR_i = (D_i * PDF/2)$
2) A point on the circle of radius PR_i surrounding the tree is <u>not</u> drawn when such a point is within the plot radius of the smaller tree <u>or</u> closer to another tree than a bisector with that tree.
3) Bisectors between trees are not drawn when the distance from the point on the bisector to the smaller tree is less than the distance PR_i of the smaller tree.
4) Bisectors are not drawn at points which are within the plot radius of a third tree <u>and</u> are closer to that third tree than to either of the trees used for the bisector.

Using these rules, parts of the preference map can be drawn as each tree is considered. The cell around an individual tree will not usually be completed until several other trees are also plotted. This

Figure 20. Illustration of construction rules for the nearest tree preference map.



is particularly true of larger trees. A more complete preference map is shown in Figure 21. Probabilities of sampling are exactly the same as in Equations 3.1 to 3.3. They are listed again here for convenience.

Probability of selecting an individual tree:

$$p\left\{S_i\right\} = \left(\frac{md_i}{T}\right) \tag{3.1}$$

Relative probability of sampling:

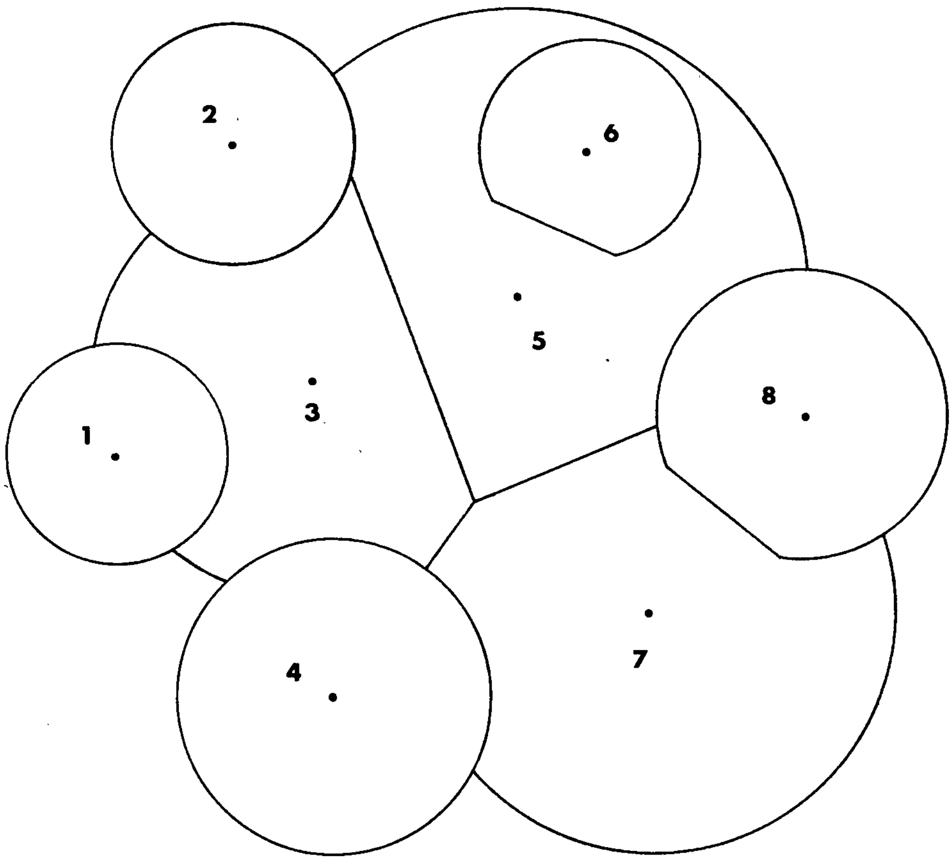
$$Pr\left\{S_i\right\} = \frac{p\left\{S_i\right\}}{\sum_{i=1}^{N} p\left\{S_i\right\}} \tag{3.2}$$

Probability of selecting a tree at some random point:

$$p\left\{S_s\right\} = \frac{\sum_{i=1}^{N} md_i}{T} \tag{3.3}$$

In summary, the system will have the following general properties. The actual biases toward small trees will probably be even larger than with random sampling, since trees tend to overlap the borders of other trees rather than to be quite near them as might occur under random spacing. The biases are at any rate in favor of small trees and trees which are sparsely distributed, and to a larger degree than with random selection of one tree per group. An increased mixture

Figure 21. Completed preference map, nearest tree method with variable plots.



of size classes will increase the bias toward smaller trees, as will greater variation in size classes.

The plot reduction method would be most difficult to apply with variable plots, and the elimination method would be the best way to obtain a random sample. In this case, the "in" trees are numbered sequentially, then a random number would be drawn between 1 and Mp where Mp is the estimated maximum number of "in" trees at any point. A tree is only chosen if its number corresponds to the random number, otherwise a new point is selected and the process is repeated. In this case, the probability of sampling a tree is:

$$p\left\{S_i\right\} = \left[\frac{BA_i}{\sum_{i=1}^{N} BA_i} * \left(\frac{1}{Mp}\right)\right] \qquad (10.1)$$

Relative probability is:

$$Pr\left\{S_i\right\} = \left[\frac{BA_i}{\sum_{i=1}^{N} BA_i}\right] \qquad (10.2)$$

The probability of choosing some sample at a random point is:

$$p\left\{S_s\right\} = \sum_{i=1}^{Mp} \left(p\left\{S_{np}\right\} \left[\frac{np}{Mp}\right] \right) \qquad (10.3)$$

The last equation uses the preference map, and is identical to Equation 6.3.

The Azimuth Method With Variable Plots

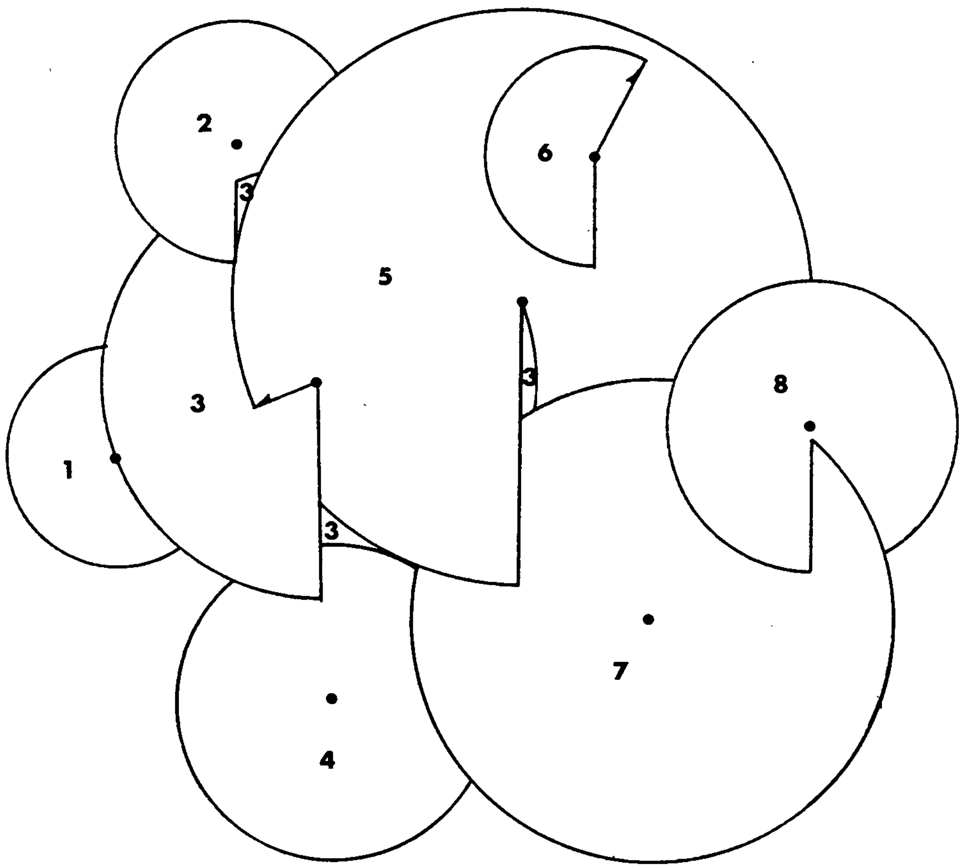
This is just a modification for the system based on fixed plots. After the different sized plots have been constructed the compartments are further subdivided based only on the trees involved with that subcompartment. Equations 5.1, 5.2 and 5.3 are then used. Bias is increased toward small trees which are well spaced or on the border of clumps. Figure 22 shows the preference map with extra lines removed. It is based on the same spacing and sizes as Figure 21.

Non-Random But Unbiased Methods For Subsampling

Selection of clusters of all "in" trees at a point assures a probability based strictly on basal area. Individual trees can also be picked with a fixed probability in 2 ways. First, each time a tree is "in" it is picked with a probability (say 1/10) with a randomizing device. This allows more permutations of samples than systematically choosing every tenth tree, although either method would be unbiased. Second, if the approximate number of trees which will be counted on all points is known, then random integers between 1 and this sum can be chosen to select the subsample. The only advantage to such a system is that it allows sampling with replacement. The first method is probably best on the basis of operational convenience and the fact that the sampler does not know when the next sample will occur.

Figure 22. Completed preference map for the azimuth system and variable plots.

coded numbers indicate the tree chosen within that cell.



Choice of one sample tree at each point, particularly without stratification, is to be avoided. If random selection within clusters must be done for some reason, weighting the sample proportional to tree count can remove the bias.

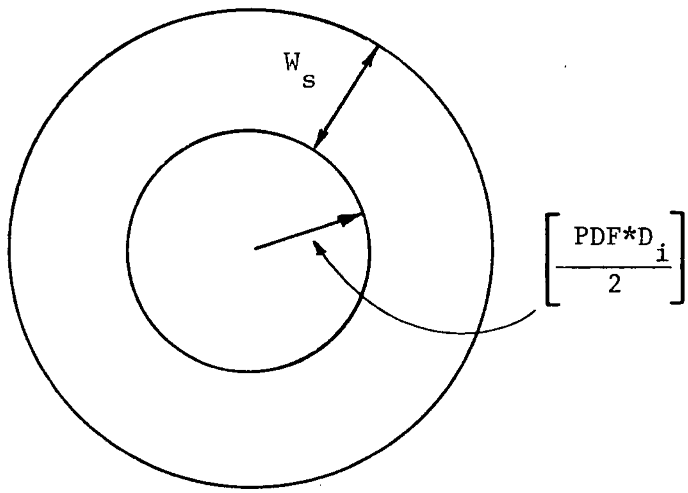
Height Squared Weighting

This is an extension of horizontal point sampling, but uses tree height rather than tree diameter. Developed by Hirata (1955) for sampling the height of forest stands, the technique uses a vertical angle and sights all trees from a point randomly chosen on the area. The size of the circular plot on the preference map is determined by the vertical angle and tree height, otherwise the equations are identical to horizontal point sampling. The reasoning concerning biases and spacing is the same but with diameter replaced by height.

Combining Diameter Squared and Diameter Weightings

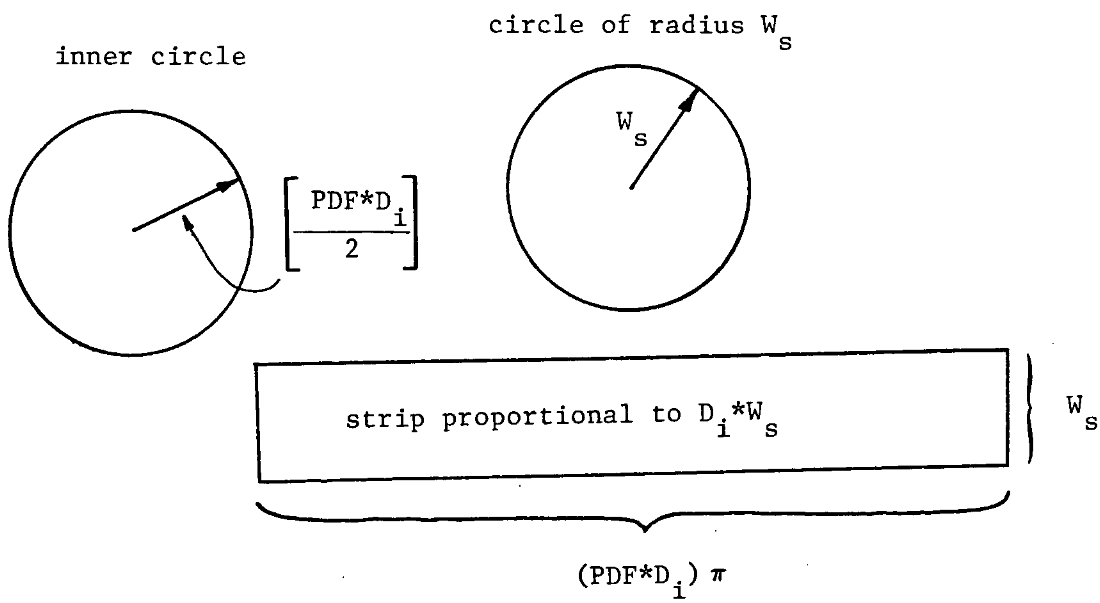
Let us first consider the problem of sampling proportional to $aD_i^2 + bD_i$. The perimeter of a plot is proportional to diameter, and so, very nearly, is a strip around the plot. Figure 23 illustrates the following line of reasoning.

Figure 23. Plot area composed of 3 simple figures.



Plot area = area of inner circle + area of ring.

This area is equivalent to the 3 simple plane figures shown below.



$$\text{Area of inside circle} = \left[\frac{PDF * D_i}{2}\right]^2 \pi = \left[\frac{PDF^2\ \pi}{4}\right] D_i^2 \qquad (10.4)$$

$$\text{Outside ring area} = \left[W_s * PDF * D_i * \pi\right] + \left[W_s^{\ 2}\ \pi\right] \qquad (10.5)$$

Total area = inside plot + strip area + small circle area

$$= \left[\frac{PDF^2 * \pi * D_i^{\ 2}}{4}\right] + \left[W_s * D_i * \pi * PDF\right] + \left[W_s^{\ 2}\pi\right] \qquad (10.6)$$

Here we have solved the problem except for the final complicating term $W_s^{\ 2}\pi$, the area of a circle of radius W_s. The easiest way out of this is to remove it from the center of the plot. We then have a plot which has a void in the center (of radius W_s) as shown in Figure 24.

The area of this plot is $aD_i^{\ 2} + bD_i$. The angle gauge used to pick the plot determines a, while b is determined by the strip width. An example may help to clarify the procedure.

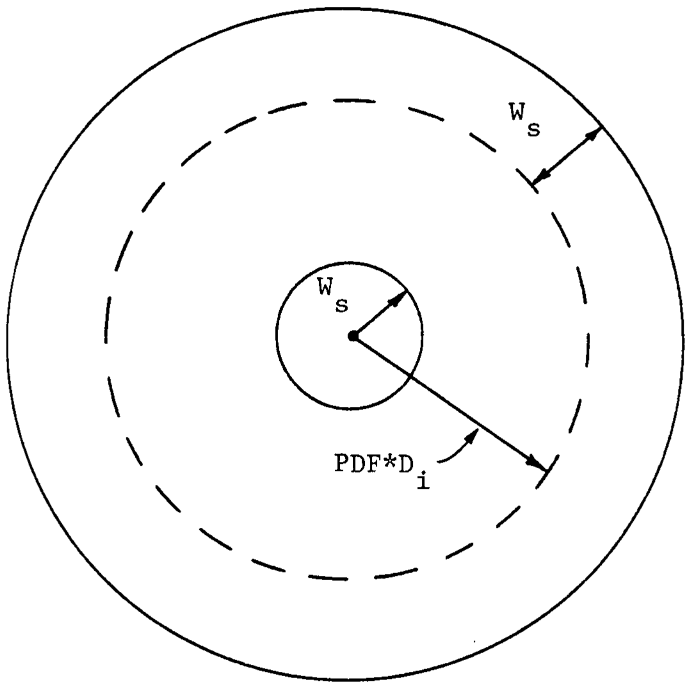
We wish to sample trees proportional to the equation:

$$P\left\{S_i\right\} \propto 7.8\ D_i^{\ 2} + 3\ D_i$$

We also wish to use a strip width of 2 meters for convenience in the field. As already shown:

$$\left[\frac{PDF^2 * \pi}{4}\right] D_i^{\ 2} + \left[PDF * W_s * \pi * D_i\right] = \text{plot area}$$

Figure 24. Plot of an area proportional to aD_i^2 plus bD_i.



$$\text{Plot area} = \left[\frac{PDF^2\,\pi}{4}\right] D_i \;+\; (PDF*\pi*W_s)\; D_i$$

The problem is to choose PDF. By definition:

$$\left[\frac{PDF^2 * \pi}{4}\right] D_i^2 = aD_i^2 = 7.8\ D_i^2$$

and

$$\left[PDF * W_s * \pi * D_i\right] = bD_i = 3\ D_i$$

by cancelling the D_i term from both sides we obtain:

$$\left[\frac{PDF^2 * \pi}{4}\right] = a$$

and

$$\left[PDF * W_s * \pi\right] = b$$ and multiplying by a/b:

$$\left[(a/b) * PDF * W_s * \pi\right] = a$$

therefore

$$\left[(a/b) * PDF * W_s * \pi\right] = \left[PDF^2 * \pi * 1/4\right] = a$$

cancelling terms we get the general equation:

$$\left[(a/b) * W_s * 4\right] = PDF$$

inserting the example values we have:

$$PDF = \left[(7.8/3) * 2 * 4\right] = 20.8$$

The angle needed to produce this relationship is

$$\Theta = 2 * \arcsin\left(\frac{1}{\text{PDF}}\right)$$

Figure 25 shows the basic geometry needed to derive this formula.

To check the results we compute the following examples.

Tree 1, diameter = 20 cm, PDF = 20.8, strip width = 2 meters.

$$7.8\ (.2)^2 + 3\ (.2) = .912$$

$$\text{plot area} = \left[\frac{20.8\ \pi}{4}\right]\ (.2)^2 + (20.8 * \pi * 2)\ \ (.2)$$

$$= \ 39.73$$

39.73/.912= 43.563

Tree 2, diameter = 1 meter

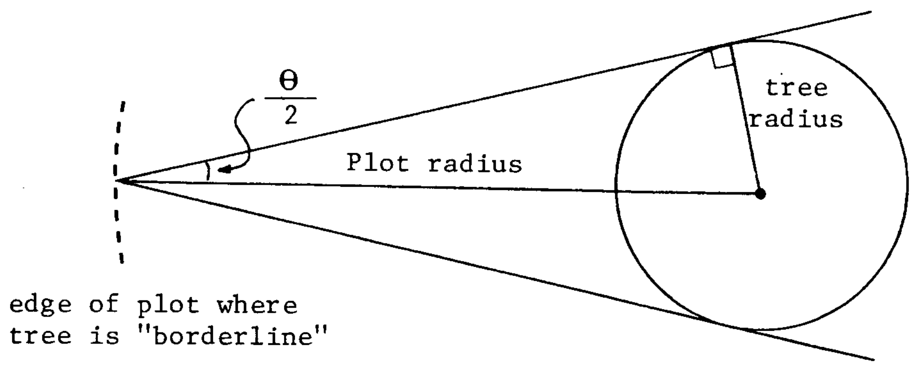
$$7.8\ (1)^2 + 3\ (1) = 10.8$$

$$\text{plot area} = \left[\frac{20.8\ \pi}{4}\right]\ (1)^2 \quad + \quad (20.8 * \pi * 2)\ (1)$$

$$= \ 470.485$$

470.485 / 10.8 = 43.563

Figure 25. Geometry used to determine the angle Θ .



$$\left(\frac{\text{tree radius}}{\text{plot radius}}\right) = \left(\frac{\text{tree diameter}}{\text{plot diameter}}\right) = \left(\frac{1}{\text{PDF}}\right) = \sin\frac{\Theta}{2}$$

$$\left(\frac{1}{20.8}\right) = \sin\frac{\Theta}{2}$$

hence: $\frac{\Theta}{2}$ = .0480 radians (2.755°)

Θ = .0962 radians (5.511°)

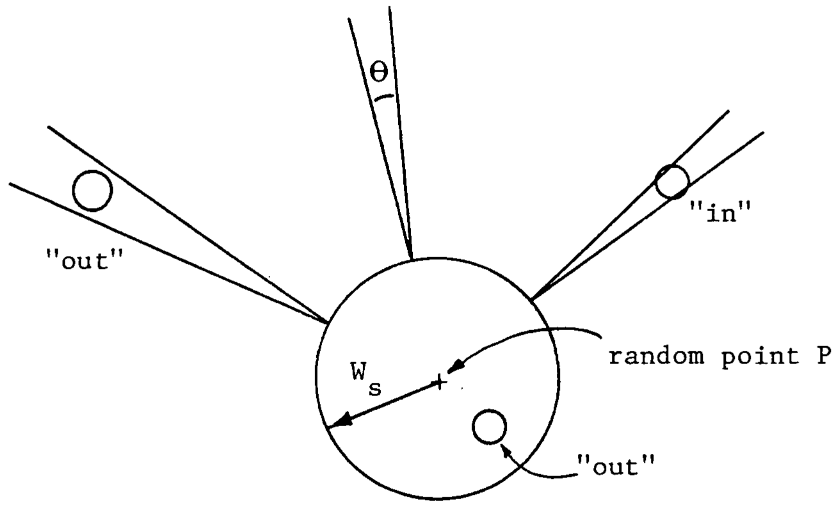
The actual plot is 43.563 times as large as the number given by the formula $(7.8D_i^2 + 3D_i)$ due to the specified strip width of 2 meters. If it were desirable we could reduce both the linear dimensions of the Plot Diameter Factor and the strip width by a factor of $\sqrt{43.563}$ to make the plot area in square meters have the same magnitude.

Once the strip width and angle gauge have been established the field work is straight forward. First, a random point P is selected. Second, all trees around P are sighted with the angle gauge at a distance W_s from P. Figure 26 shows the field selection scheme. A random selection can then be made from trees by the elimination method.

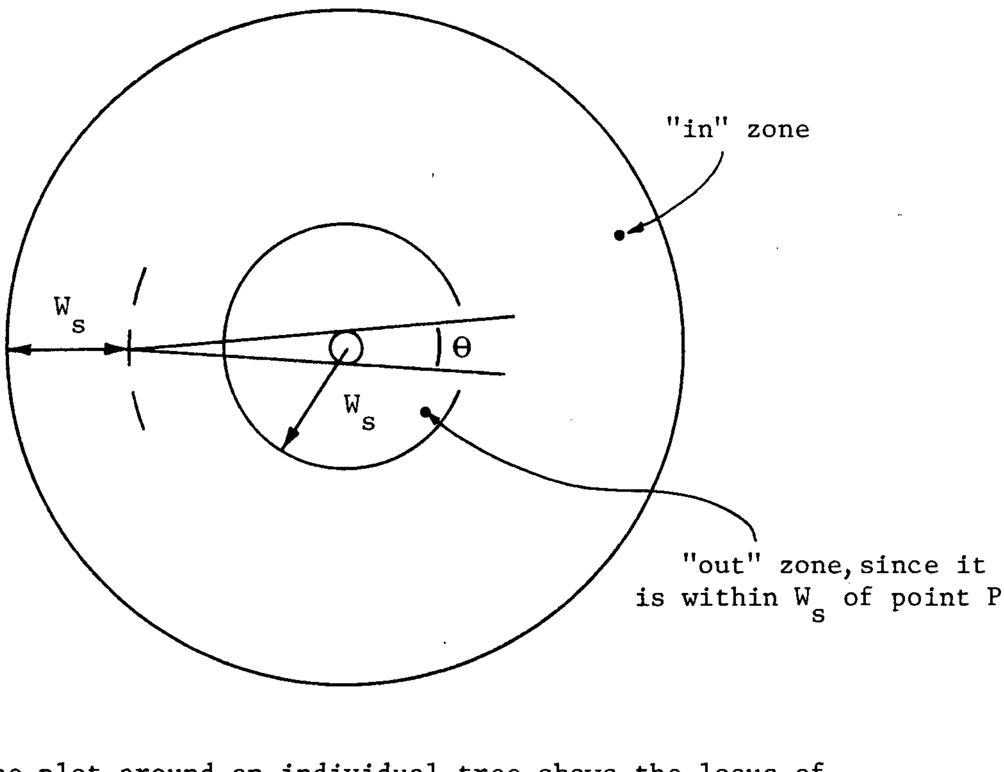
A recent article by Schreuder (1978) uses just such a selection scheme in a method called "Count Sampling". Although Schreuder does not consider it "a practical system" it can be considerably simplified and improved for field application by modifying the Wheeler Pentaprism as described in a following section.

Selection of trees with a probability proportional to $\left[aD_i^2 - bD_i\right]$ requires a slight modification. The area $W_s^2 \pi$ must be <u>added</u> to the plot, and this can be done by adding it to the center of the plot and giving the tree <u>two</u> chances of selection when a random point falls in this area. A strip is "removed" from the basic plot by sighting across the point from a distance W_s. Figure 27 shows the selection rule.

Figure 26. Selection proportional to aD_i^2 plus bD_i.

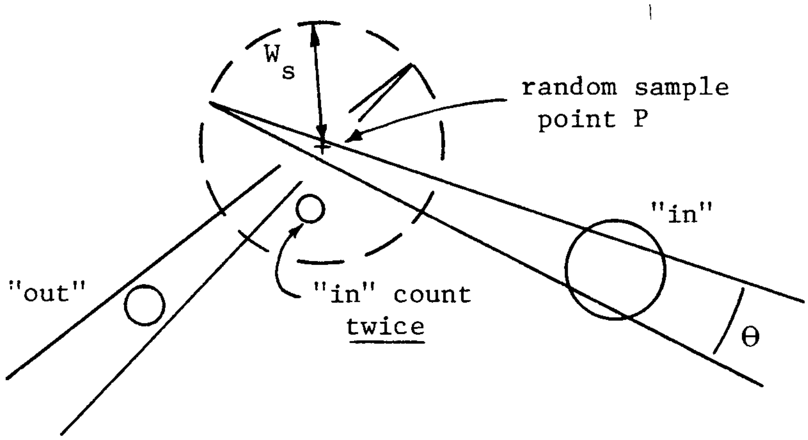


To select trees from point P, view all trees with an angle gauge at distance W_s from P.

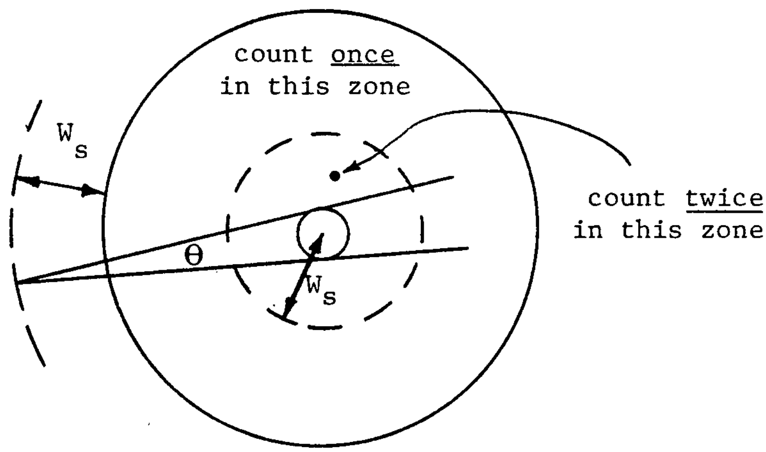


The plot around an individual tree shows the locus of all points where the tree is "in" with the selection rule described above.

Figure 27. Principles of selection proportional to aD_i^2 minus bD_i.



To select trees from point P view tree across the point with a prism from distance W_s.



This diagram shows the plot around each tree. The tree is counted once at all points within the solid circle or twice when within the dotted circle.

Mechanical Devices to Aid Selection

The selection method shown in Figure 26 could be simulated without moving from point P by mechanical device which functions much like a split-image rangefinder. Figure 28 shows the geometry of the device and the views to be expected through the eyepiece. Sampling is proportional to $aD_i^2 + bD_i$.

a) tree is within the distance W_s of point P.

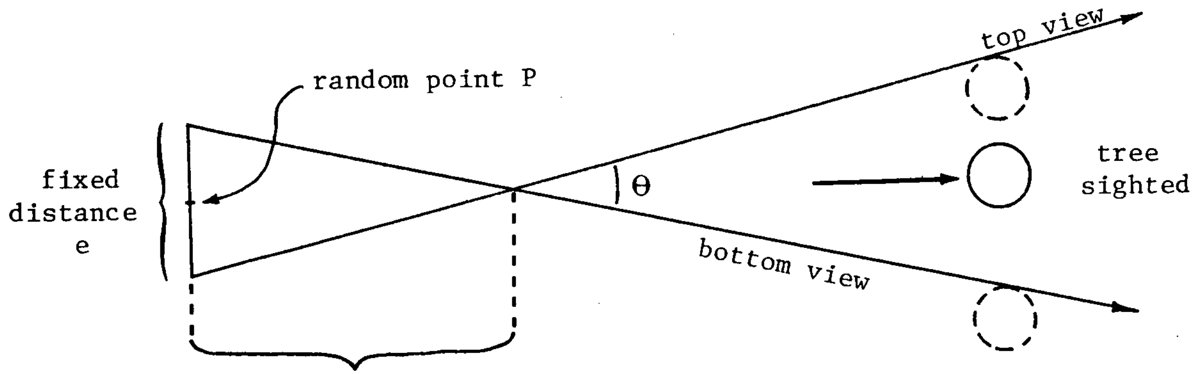
b) tree is outside W_s, but "in" with the angle gauge.

c) tree is "out" with the angle gauge.

Figure 29 shows the same basic idea used for sampling proportional to $aD_i^2 - bD_i$.

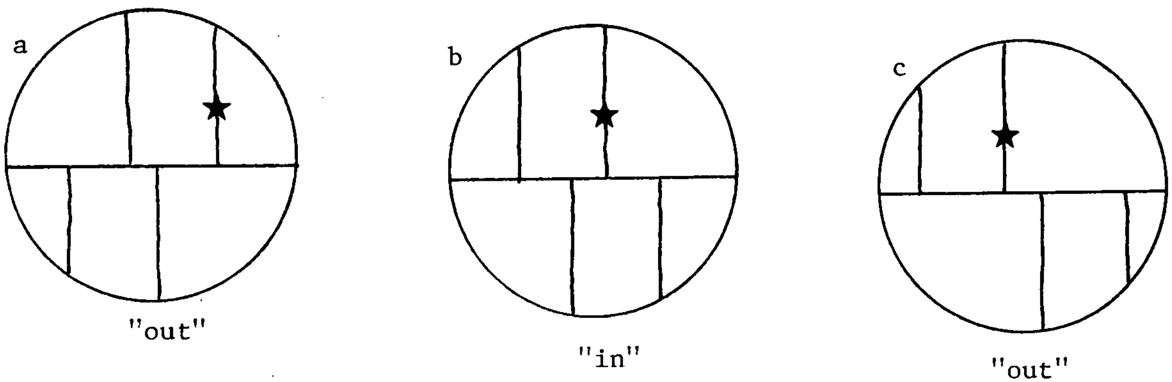
Such a device could be built by simply attaching prisms in front of the two lenses of a Wheeler Pentaprism which is available commercially. This allows easy determination without leaving the sample point as long as the tree can be seen. Fairly wide border strips could be accommodated in this way. Using an angle of .0262 radians (1.5^o) a 100 cm base would allow a 38 meter width for W_s. Alignment, measurement, and slope problems would make such a strip infeasible under field conditions by other methods. Line of sight will remain a major problem.

Figure 28. Device principles for sampling proportional to aD_i^2 plus bD_i.



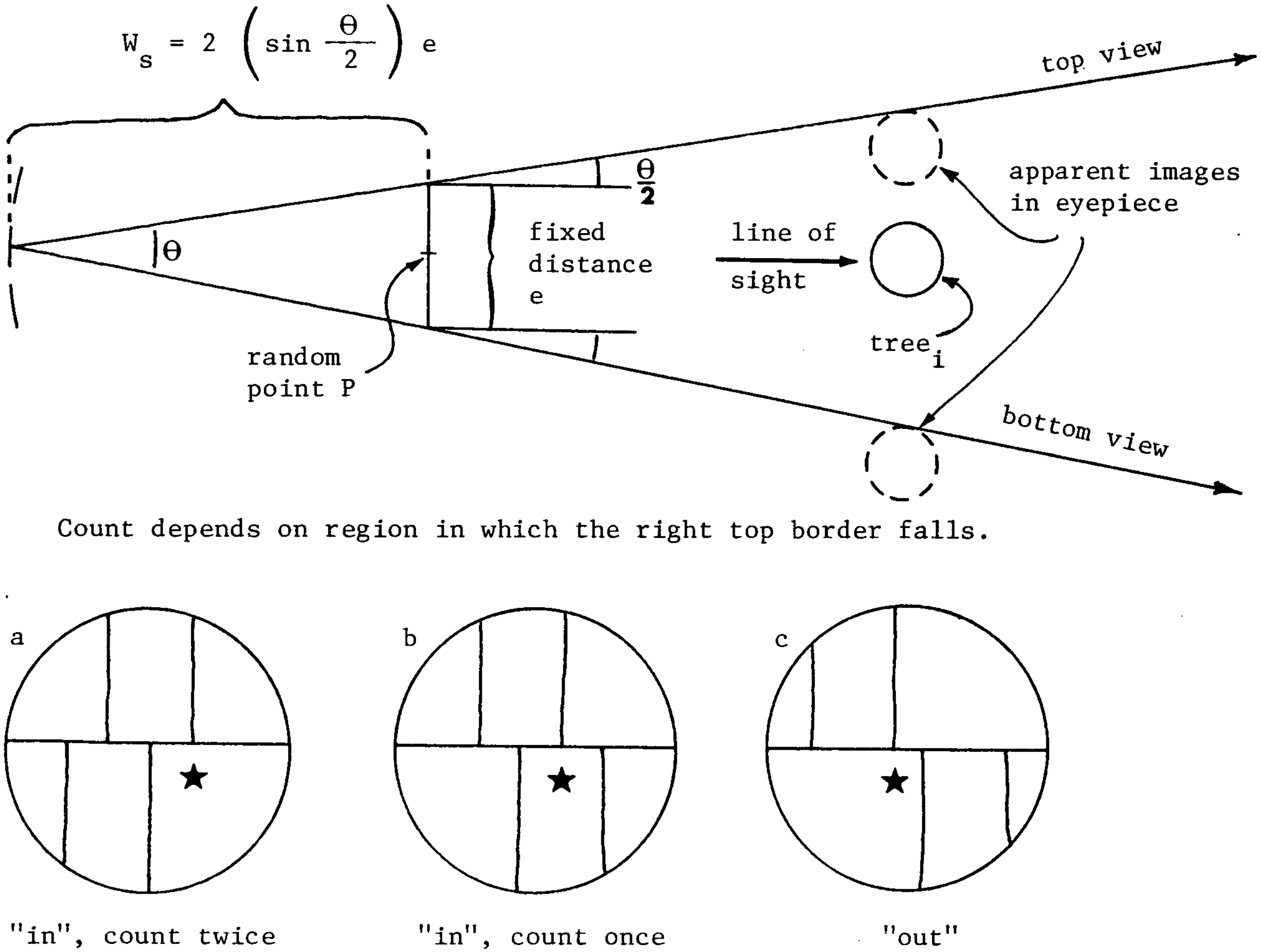
$$W_s = 2\left(\sin\frac{\theta}{2}\right)e$$

Views of the tree through the split image device.



Tree is "in" when top right edge is over bottom image.

Figure 29. Device principles for sampling proportional to aD_i^2 minus bD_i.



Adding a Constant, Selection Proportional to $aD_i^2 \pm bD_i + c$

Adding this further restriction to the previous methods forms a cumbersome system. The only reasonable field method would be to apply the methods outlined for $aD^2 + bD$ to a π radian (180^o) sweep, while using a fixed plot to select trees through the opposite π radians. Figures 30 and 31 show the basic idea. It would appear desirable to generalize the rather awkward method and allow selection with any plot size computed by an equation using only diameter as a variable. One method, using only standard cruising prisms, requires only that the diameter be measured, or adequately estimated. A prism is then rotated to form an angle which will establish the plot size around the tree. For accurate work, particularly with wider angles, two prisms, each of which establish ½ the initial critical angle (Θ_d) should be counter-rotated. The geometrical theory for this adaptation is given by Beers (1964). For simplicity the case of a single prism will be used for the following discussion.

A Generalized Instrument

For each diameter tree the plot size is listed. Using the formula below Θ_d is established for each diameter tree.

$$(\sin \tfrac{1}{2} \Theta_d)^2 = \left[\frac{\text{tree basal area}}{\text{plot area}}\right] = \left[\frac{\text{tree diameter}}{\text{plot diameter}}\right]^2$$

Principles of selection proportional to $aD^2 \pm bD + c$.

Figure 30.

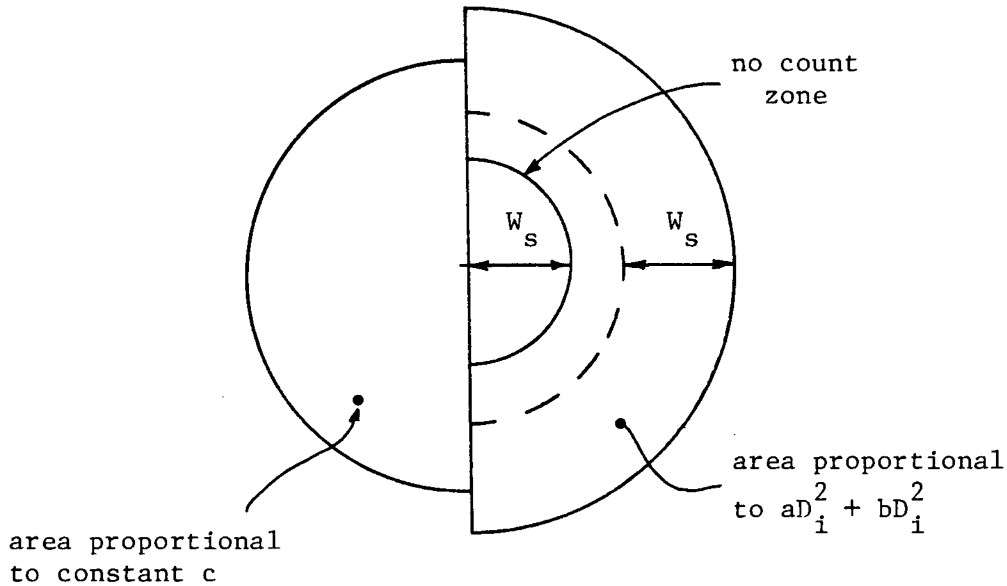
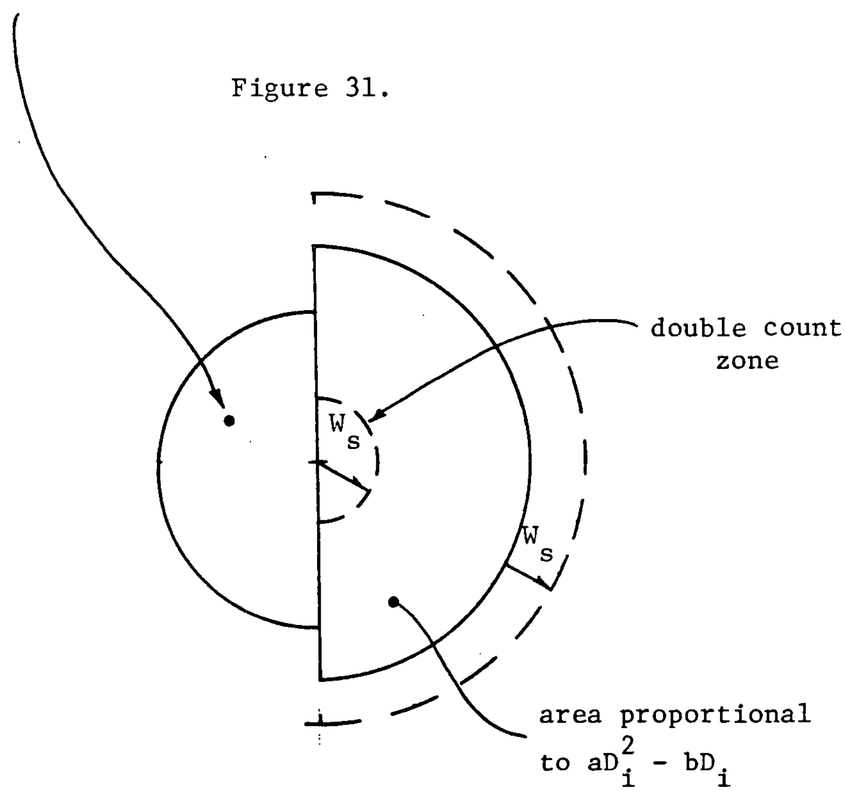


Figure 31.



A prism of deflective angle Θ_{max} is used where Θ_{max} is always larger than Θ_d. A round prism is best, preferably with a hole in the middle for easy mounting. Θ_d can be formed by rotation of the prism around its center by an angle (A_r).

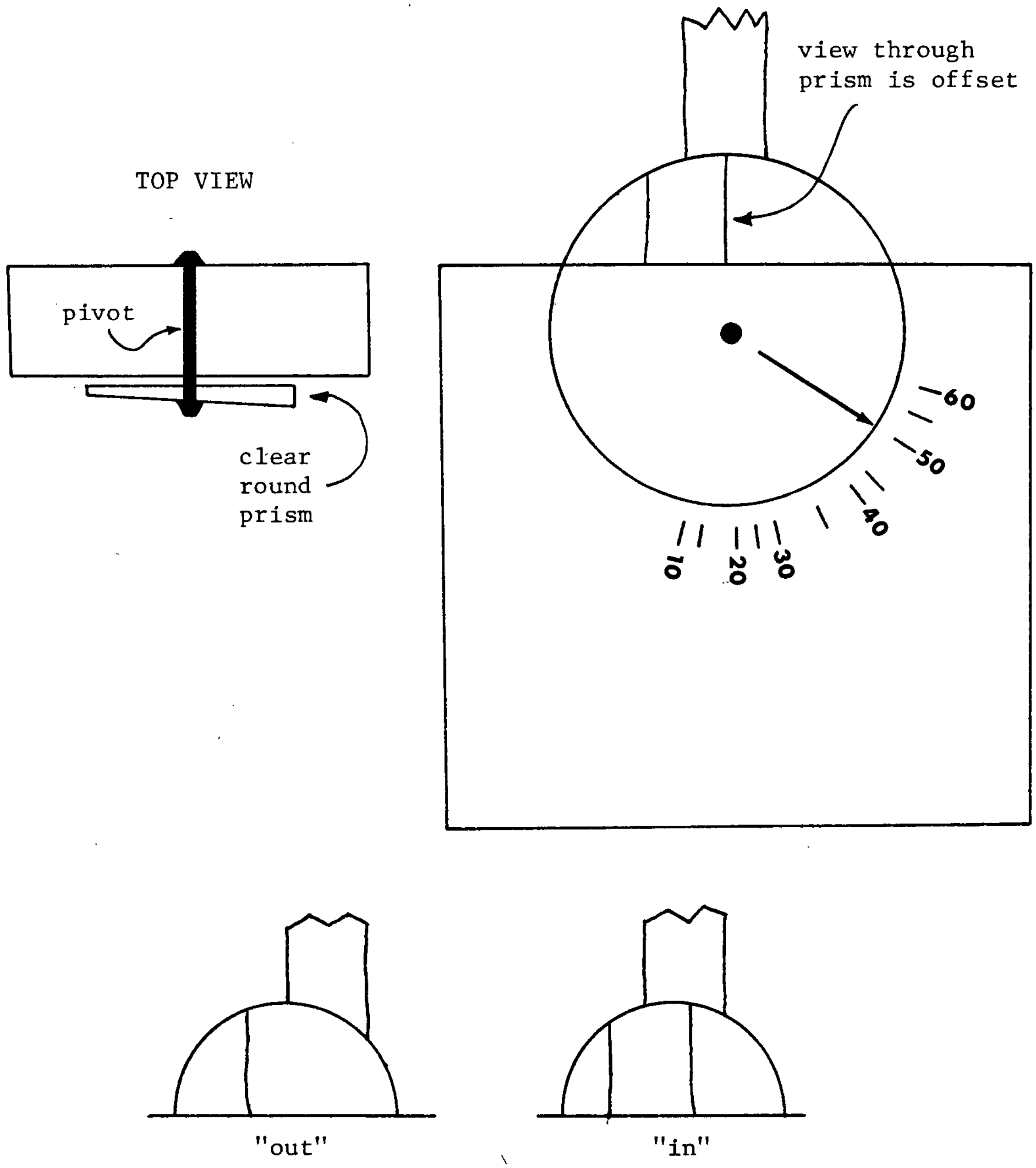
$$\Theta_d = \Theta_{max} * \cos(A_r) \tag{11.1}$$

The prism is mounted and the angle of rotation is marked with the tree diameter. Figure 32 shows such a mechanism set for checking a 50 cm tree. The decision of whether a tree is "in" or "out" is the same as in conventional uses of the prism as an angle gauge.

Such a selection system has two advantages. First, size of plot can be derived by any means, as long as it can be indexed only by diameter. Second, the plot is a solid circle centered at the tree. Such areas around trees are often described when dealing with problems such as competition, rooting zones, moisture depletion etc.

When interpolation cannot be done a programmable calculator can easily be used to establish the rotation angle. If the differences in the values for Θ_d are small the adjustment may be difficult. When the angles of rotation are nearly alike the bias of setting the instrument may be large. In such a case two prisms should be used. One fixed prism establishes the basic angle (A_{main}), while the other rotatable thinner one (A_{plus}) corrects it by larger angles of rotation. The final angle Θ_d is then given by:

Figure 32. Prism device for selection of trees by plots indexed by diameter alone.



$$\Theta_d = A_{main} + (\cos A_r) * (A_{plus}) \qquad (11.2)$$

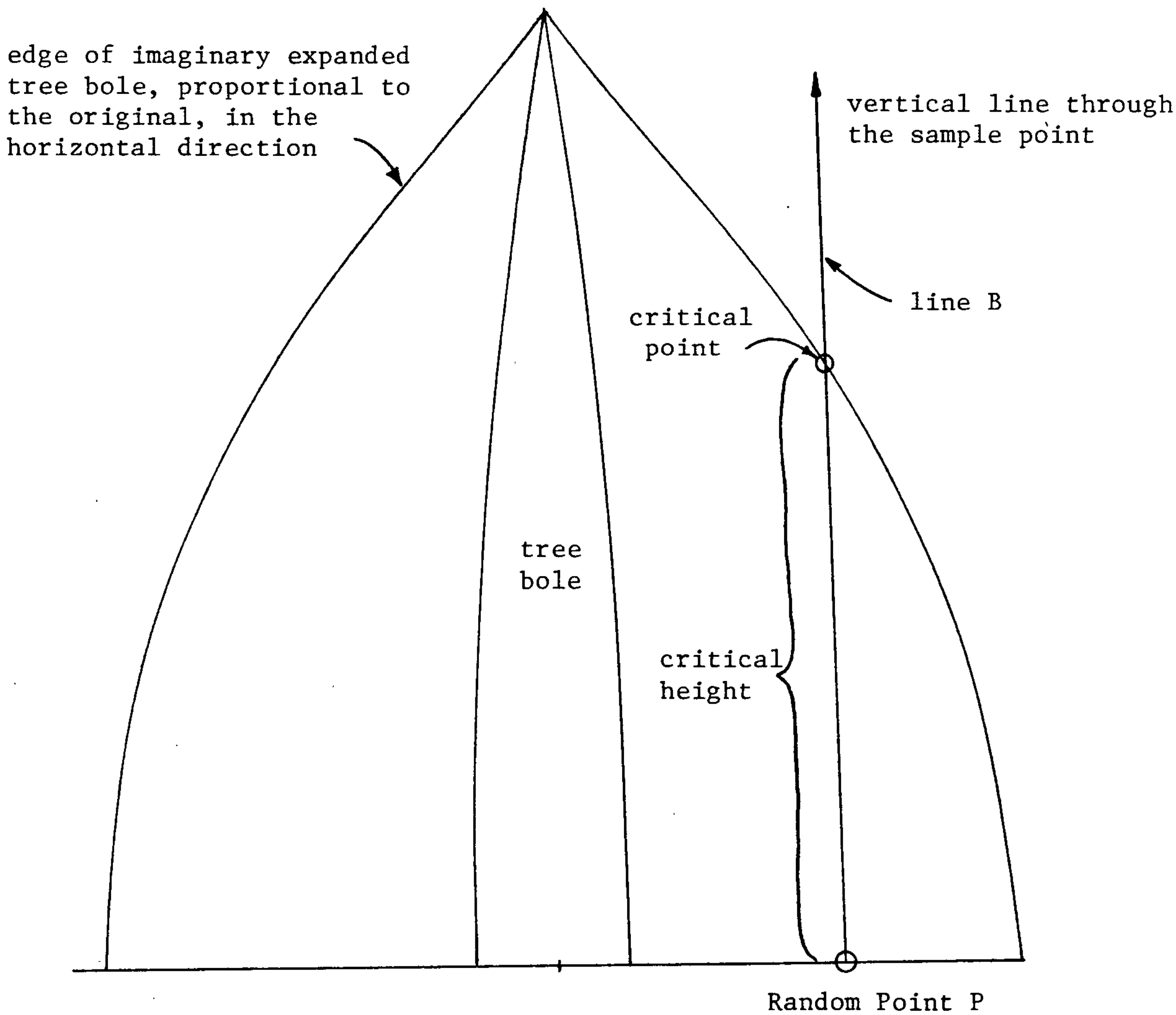
This method will increase the range of rotation of the smaller prism and simplify the problem of setting the instrument accurately.

Gross Volume Weighting

Basics of the Critical Height Method

When an angle gauge using a critical angle Θ is used to sight along a tree stem the effect is to "expand" the diameter by a constant all along that stem. This gives a simple method to expand the stem of a tree. The basal area, and hence the volume is expanded by the same constant at each point along the stem. Average height of the expanded stem is obviously the same as the original tree, and proportional to tree volume. The length of the vertical line from a random sample point level with the base of the tree to the point where it leaves the expanded tree is a sample of the average tree volume (see Figure 33). This distance, called the "critical height" will allow us to select trees proportional to total volume. The tree need only be selected on the basis of its critical height. This basic idea was discovered by M. Kitamura in 1962, but no English translations of the work were available and it was virtually unknown in North America. In 1974 the author independently discovered the principle, from a slightly more general viewpoint, and called it "Penetration Sampling" (Iles, 1974). A more detailed history will be given in the second part of the thesis.

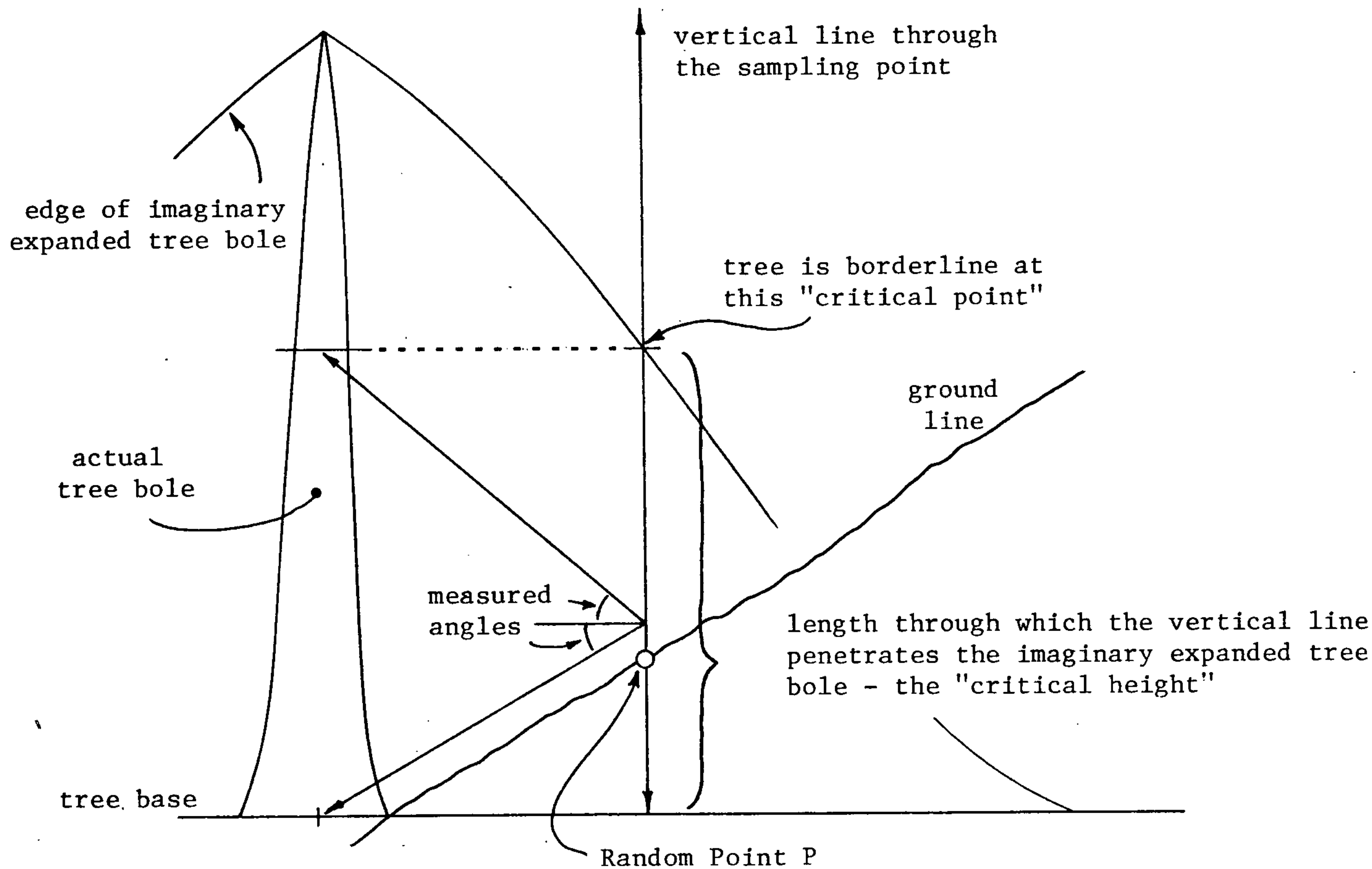
Figure 33. The "expanded tree", critical point and critical height.



One of the simplest ways to test what part of a vertical line through point P lies within the imaginary expanded tree bole is to move up the vertical line (B) with an angle gauge while sighting the tree bole along the horizontal plane. A prism, relaskop or any other angle gauge would serve the purpose. If the tree image is "in" when sighting horizontally towards the tree for a total of 12 meters, then the vertical line passes through the imaginary expanded tree stem for a distance of 12 meters.

For the system to be practical the physical problem of moving up and down the vertical line with the angle gauge must be circumvented. The solution to this problem is fortunately quite simple. It is commonly known that the relaskop (or newer telerelaskop) has an "automatic slope correction". The geometric form of that correction does not seem to be as well known. When an observer views a tree diameter at any vertical angle the automatic correction is made by slightly decreasing the critical angle θ , which is then being projected along a slope distance. The view through the instrument is exactly the same as if the observer was floating vertically above or below the sample point and was sighting horizontally at the tree with the original critical angle. This form of automatic correction then makes it possible to theoretically "levitate" above or below the sample point along a vertical axis and "observe the tree horizontally". These are the exact requirements of the system. We now have a practical method for determining the point at which the vertical line leaves the unseen expanded tree bole (see Figure 34).

Figure 34.   Illustration of some of the basic concepts of Critical Height determination.



To get the distance desired only 3 measurements are needed.

1) The vertical angle to the critical point (the point where it becomes "borderline"). This is the point where the vertical line leaves the expanded tree bole.
2) The vertical angle to the base of the tree, or the point below which cubic volume is not to be considered. It is computationally convenient if these angles are measured in "%".
3) The slope distance to the base of the tree. This should be measured at the same angle of slope as the second reading, and can be done with an optical system if desired.

The second vertical angle and the slope distance can be used to calculate the horizontal distance to the tree. Both vertical angles along with this horizontal distance will give the critical height. These measurements can be made without leaving the sample point if a range finding system is used for slope distance.

At any random point P we can get the critical heights of all the trees. Since the length of these critical heights are proportional to tree volume the elimination method may be used to choose a single tree. This allows selection of a random individual or a random sample proportional to total volume and without prior assumptions about tree shape or any use of volume tables.

When E is the expansion factor of the angle gauge used and calculated by:

$$E = \left[\frac{1}{\left(\sin\frac{\theta}{2}\right)^2}\right] = \left[\frac{\text{plot area}}{\text{tree basal area}}\right] \tag{12.1}$$

$$= \left[\frac{\text{expanded tree volume}}{\text{actual tree volume}}\right] = \left[\frac{E(V_i)}{V_i}\right]$$

and using M_{sum} as the maximum expected sum of critical heights at a point, the probabilities with the elimination method are:

$$p\left\{S_i\right\} = \left[\begin{matrix}\text{probability of sighting}\\ \text{the tree with the prism}\end{matrix}\right] * \left[\begin{matrix}\text{expected critical}\\ \text{height of tree}\end{matrix}\right] * \left[\frac{1}{M_{sum}}\right]$$

$$p\left\{S_i\right\} = \left[\frac{BA_i * E}{T}\right]\left[\frac{V_i}{BA_i}\right]\left[\frac{1}{M_{sum}}\right] = \left[\frac{E * V_i}{T}\right]\left[\frac{1}{M_{sum}}\right] \tag{12.2}$$

The relative probability is:

$$Pr\left\{S_i\right\} = \frac{p\left\{S_i\right\}}{\left[\sum_{i=1}^{N} p\left\{S_i\right\}\right]} = \frac{V_i}{\left[\sum_{i=1}^{N} V_i\right]} \tag{12.3}$$

The probability of choosing some tree at a random point is given by:

$$p\left\{S_s\right\} = \frac{\left[\frac{\sum_{i=1}^{N}\left(E * V_i\right)}{T}\right]}{M_{sum}} = \left[\frac{\text{average sum of critical heights}}{M_{sum}}\right] \tag{12.4}$$

Random Selection From a Cluster Proportional to Critical Height of the Tree, a Biased Method

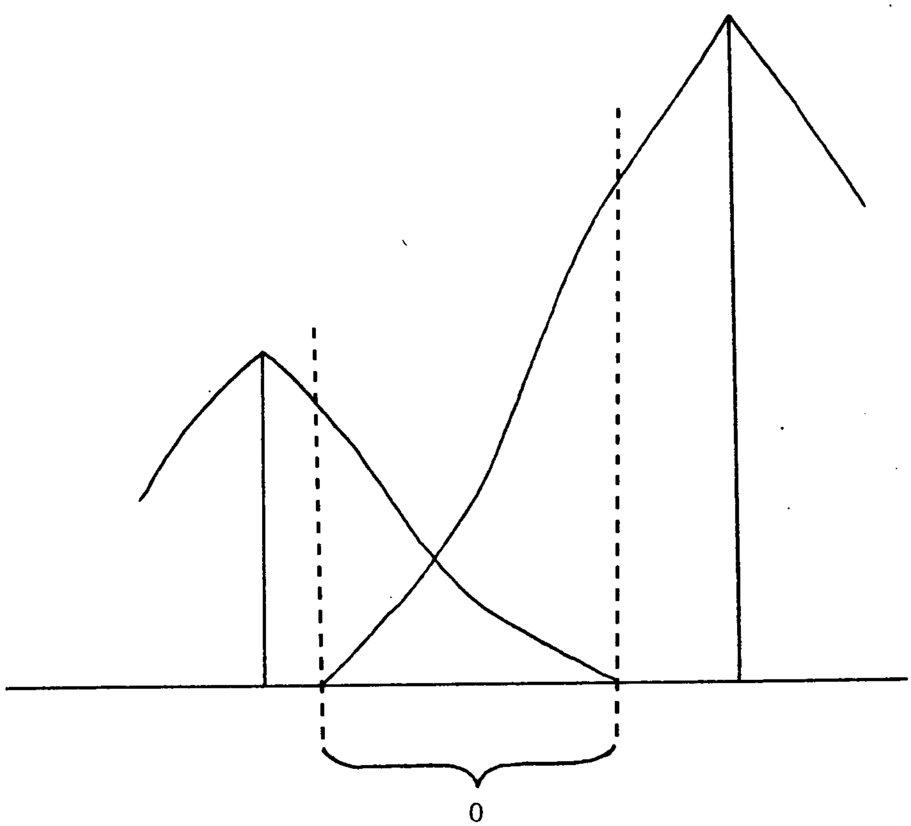
If one tree is chosen using the sum of critical heights at a point (S_c) and drawing a random number uniformly from the interval $1-S_c$, the problem becomes very difficult. The overlap of trees is now in 3 dimensions. Figure 35 shows a side view for 2 trees only. In the area indicated by "0" there is a probability of selection proportional to the ratio of the critical heights of the expanded trees. This ratio must be integrated over the land surface where the tree boles intersect in order to compute the selection probabilities. When 6-10 trees all overlap, even with the same stem form, an exact solution is too difficult. The only practical solution is by numerical approximation methods. Using a grid of xp points across the land area, the estimated probability of selection is then given by:

$$p\left\{S_i\right\} \approx \left[\frac{\sum_{i=1}^{xp}\left(\frac{CH_i}{S_c}\right)}{xp}\right] \quad \text{where } S_c > 0 \qquad (13.1)$$

where: CH_i is the critical height of tree i,

$$Pr\left\{S_i\right\} = \left[\frac{p\left\{S_i\right\}}{\sum_{i=1}^{N} p\left\{S_i\right\}}\right] \qquad (13.2)$$

Figure 35. Selection probabilities proportional to critical height.



The probability of selection of a tree at a random point is like that of horizontal point sampling. The maximum diameter is used to construct a plot around each tree. The proportion of land area covered by at least one plot gives the desired probability.

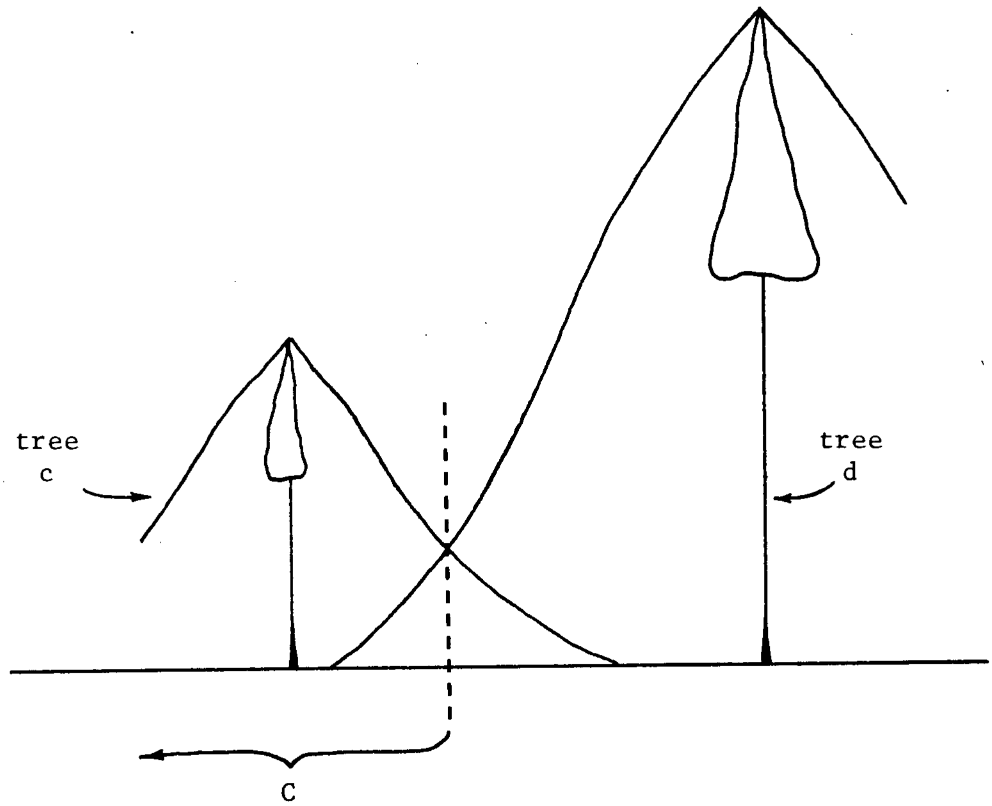
One bias in this case is toward trees which are widely spaced relative to their basal area.

Selection of the Tree With Largest Critical Height, a Second Biased Method

This again is a mathematical problem best solved by numerical approximation. Here selection is based on the maximum critical height. See Figure 36. The area C will favor selection of tree c, since at all points to the left of the dotted line the critical height of a is larger. Unfortunately, this line cannot be located as easily as with other systems we have examined.

One way to establish the line by computer is to plot perimeter points on circles of size (stump D_i * PDF) except where two or more such circles overlap. When two circles overlap only the intersection points will be drawn, for diameters at intervals up the stem, until they no longer overlap and providing that no other circle overlaps this intersection. The cells around trees so produced will be the points at which that expanded tree is the highest. A better alternative, where software plotter facilities are available, is to plot the surface of each of the trees from a top view with hidden lines removed. This will produce the same kind of mapping. Using the area of these cells (C_{ch}) the probabilities are:

Figure 36. Selection of tree with largest critical height.



$$p\left\{S_i\right\} = \left(\frac{C_{ch_i}}{T}\right) \tag{14.1}$$

$$Pr\left\{S_i\right\} = \left[\frac{C_{ch_i}}{\sum_{i=1}^{N} C_{ch_i}}\right] \tag{14.2}$$

The probability of choosing a tree at a random point is:

$$p\left\{S_s\right\} = \left[\frac{\sum_{i=1}^{N} C_{ch_i}}{T}\right] \tag{14.3}$$

Taller trees have increased probability of selection as do trees of better form and those which are well distributed relative to their basal area. As the expansion factor increases the bias towards taller trees increases because more smaller trees are completely enclosed in the larger expanded tree.

The system is applicable to any set of objects which have the same cross-sectional shape but different magnitudes. To calculate total volume of the objects the relationship of cross-sectional area to plot area will have to be known, and this is particularly simple with objects circular in cross-section. While this method can be used on sections of trees (merchantable height, height below first limb, knot free sections, etc.) it measures outside the bark and cannot deduct for breakage or rot, except perhaps by the samplers estimation of % reduction as in most systems. The use of the system to sample for total volume will be discussed shortly.

Vertical Cross-Sectional Weighting

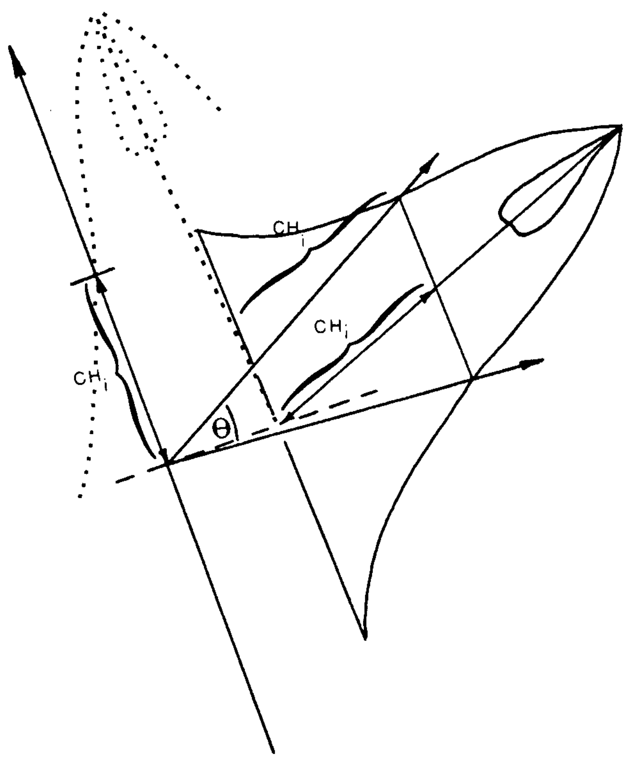
We have established a method of sampling proportional to horizontal cross-sectional area with horizontal point sampling. Critical height line sampling can be used to sample proportional to vertical cross-sectional area of a tree. Consider the vertical cross-section of a tree parallel to a random transect through the woods as in Figure 37.

The average critical height along random transects is proportional to the cross-sectional area. To choose a random tree first sum the critical heights along a transect then use the elimination method.

Cylinder Volume Weighting

Tree volume can be calculated by $V_i = (\pi/4)D_i^2H_iF_i$. F_i relates the volume of the tree to the volume of a cylinder of the same diameter and height ($\pi/4 * D_i^2 * H_i$). When F_i is the same for all trees, sampling can be done by a two stage selection system. First, the trees are picked proportional to basal area by the use of an angle gauge, then they are measured for total height (an easier task than measuring critical height) and finally selected from a short list of total heights. This produces a selection proportional to cylinder

Figure 37. Selection proportional to vertical cross-sectional area of the stem.



The critical height calculated is the same as the distance run across the profile <u>if</u> it were laid down at right angles to the transect and centered at the tree.

volume. Grosenbaugh (1974) uses a system like this one for what he calls the "Point-3P" sampling method. Unfortunately, he still has the problem of actually measuring tree volume on all the trees at some of the points. The critical height method has other advantages besides freedom from volume tables, and they will be discussed shortly.

CRITICAL HEIGHT SAMPLING, HISTORICAL DEVELOPMENT AND LITERATURE REVIEW

In one of Bitterlich's early articles concerning variable plot sampling (Bitterlich, 1956), he "indicated that the volume contributed by each tree in an angle count sample is related to its 'critical height' " (Bitterlich, 1976). It is strange that Bitterlich himself did not find the exact form of that relationship. Perhaps he was hampered by a view which was restricted to the two-dimensional plane in which tree cross-sections were being "expanded" by his use of the angle gauge. At any rate his system developed into a two phase sample. The first phase consisted of estimating stand basal area by counting trees chosen with an angle gauge. The second phase was sampling for the "Volume to Basal Area Ratio" (VBAR) which established the cubic volume relating to each square unit of basal area. This is usually done by selecting sample trees and dividing their volume by their basal area. These estimates of the volume to basal area ratio are then averaged, weighting individual ratios if necessary. The tract volume is then estimated by

$$\begin{pmatrix}\text{tract volume}\\ \text{per hectare}\end{pmatrix} = \begin{pmatrix}\text{basal area}\\ \text{per hectare}\end{pmatrix} * \begin{pmatrix}\text{solid volume per}\\ \text{unit of basal area}\end{pmatrix}$$

In 1962, Masami Kitamura delivered a paper laying out the basic technique of critical height sampling (Kitamura, 1962). The basic geometry has been described in a previous chapter. The crucial idea was that a vertical line passing through the forest could be used to sample directly for VBAR. Two years later, he published a major paper concerning the theory of the system (Kitamura, 1964). Another article (Kitamura, 1968) discussed indirect methods of critical height measurement. Bitterlich reported the development of Kitamuras system in 1971 (Bitterlich, 1971) and a brief summary of the system was included in the directions for the wide scale relaskop (Finlayson, 1969).

In 1973, the author independently derived the system as a special case of a more general system of random lines penetrating a volume of space in the forest. The method was called "Penetration Sampling" and was later developed as a class project (Iles, 1974). A search of the literature at that time revealed only one translated paper (Kitamura, 1968), but it contained a diagram and one formula which were sufficient to establish the similarity of my own work to that of Kitamura.

An article by Loetsch in the IUFRO proceedings from Nancy, France (Nash _et al._, 1973) listed field tests of the critical height system as one of the components of forest inventory which needed research. Also in 1973, Bitterlich wrote an article for the first meeting of the International Association of Survey Statisticians at the 39th Session of the International Statistical Institute in Vienna. It described the use of the angle gauge in implementing the critical height sampling system (Bitterlich, 1973). The article does not seem to have been printed with the other papers presented at that meeting. In 1973 the system was also mentioned briefly in a book on forest inventory (Loetsch, 1973). These last two works contained the first discussions of the method in the English language.

Due to the lack of other English translations the existence of the system was virtually unknown in North America. By 1974, Thomas W. Beers, an authority on the variable plot technique, had still not heard of the method (Beers, 1974). As late as September 1976, a plea for information about the system in the newsletter INFO 75 by Mike Bonnor (Bonnor, 1975) brought no responses except from this author.

In 1976, the first English journal article on the system appeared in the Commonwealth Forest Review (Bitterlich, 1976). The article was adapted from material published in Allgemeine Forstzeitung (Bitterlich, 1975) and printed for the information of users of the relaskop and telerelaskop by the manufacturer of that instrument (Bitterlich, W. and W. Finlayson, 1975).

In 1977, Kitamura developed a variation of a process by Minowa (Kitamura, 1977). Minowa's basic system was described earlier in connection with using a vertical angle to do line-intersect sampling. Kitamura's new method and further descriptions of similarities of Kitamura's and Minowa's systems were outlined in an article at the IUFRO conference in Freiburg (Kitamura, 1978). At present, no articles have appeared in North American publications and knowledge of the existence of the system is still unusual.

ADVANTAGES AND APPLICATIONS

The basic theory of the critical height system (hereafter often abbreviated as CH system) is not difficult. The fundamental insight is that the solid content of the forest can be sampled by passing random vertical lines through the tract area and sampling the "depth" of wood encountered. The variance of the estimate is obviously based on the variation found at each of these points where the vertical line penetrates the stand. To decrease the variation the stems can be "expanded" and the distance that a vertical line penetrates the expanded stem (the critical height) can be determined in the field as described earlier. There is no reason, other than operational convenience, for the sampling lines to be oriented vertically. A 45 degree angle to the ground would probably be less variable.

The first advantage to critical height sampling is that it is a direct sample for stem volume. The bias involved in the use of volume tables is not included in the volume estimate. This is particularly important where the top diameter is highly variable, and cannot be predicted from taper equations. This is often the case in hardwood species, especially where log grade is an important factor. The length to which individual standing trees will be cut simply cannot be predicted, but is easily judged while looking at the particular tree.

The system is sensitive to actual tree form, and does not require any assumptions. This makes it useful for situations where standards of utilization change frequently or where no research has been available on a species. When the critical height is divided by total tree height it provides a weighted sampling of the cylindrical form factor. A weighting proportional to basal area is automatically implemented since trees are selected with an angle gauge.

A second advantage, dependent on the local utilization standards, species and spacing, may be in the variance of the system. In the standard variable plot system, with all trees measured for VBAR, a tree is either included or excluded from a cluster, and therefore that tree's VBAR is added to the total or completely ignored. The result is a "step function" over the area. Figure 38 illustrates the sum of VBARs for the three trees on a sample area. The preference map for horizontal point sampling establishes the size and shape of these cells. The sum of the VBARs in each of these cells determines the variance of the volume estimate for the tract. This is shown by the form of the volume estimate from a single variable plot.

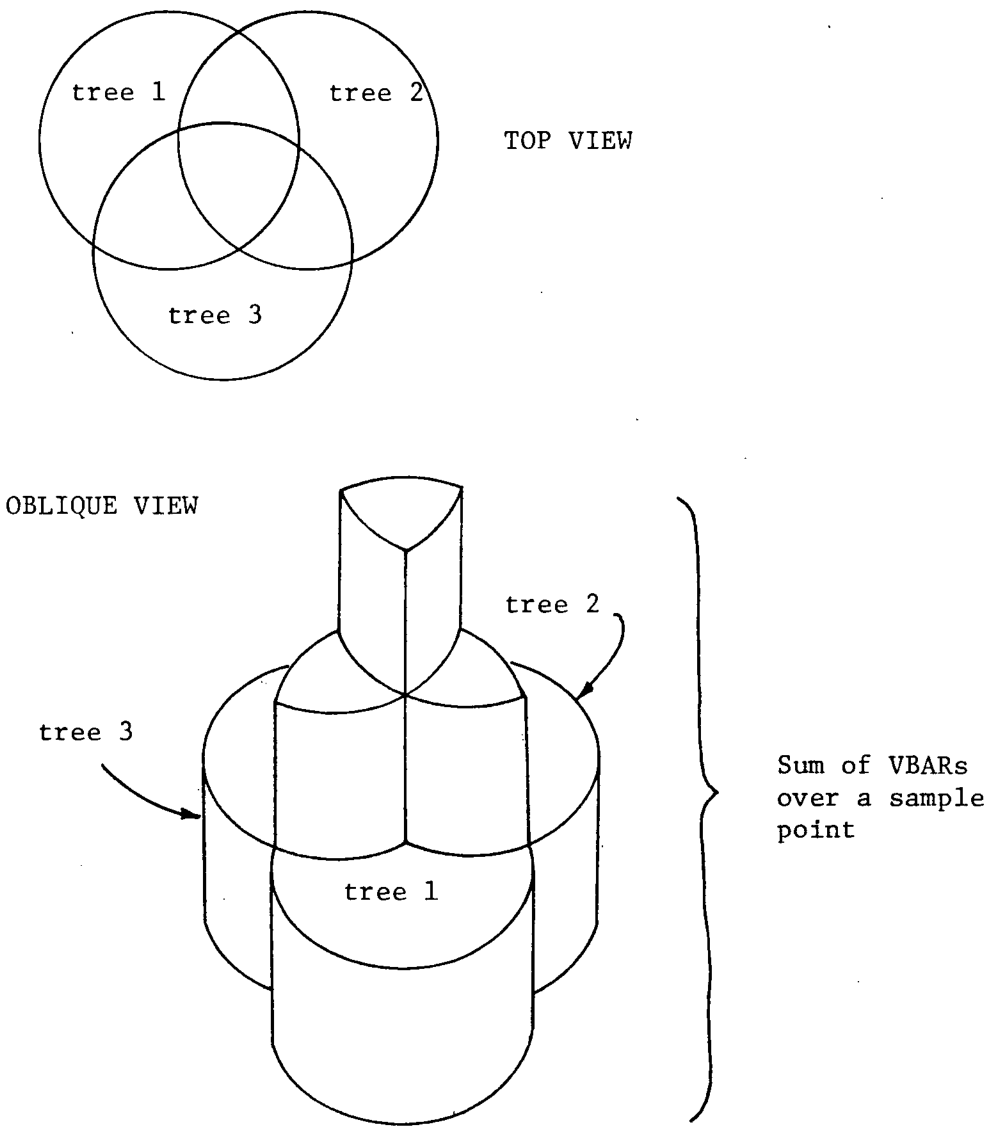
$$\begin{bmatrix}\text{estimated volume}\\ \text{per hectare}\end{bmatrix} = \text{BAF} \; * \; \left[\sum_{i=1}^{I} \text{VBAR}_i\right]$$

where: BAF = the Basal Area Factor of the angle gauge used.

I = the number of "in" trees at a point.

This $\sum$ VBAR term changes discretely. The same reasoning applies to fixed plots using tree volume rather than VBAR. In critical height sampling each observation is an estimate of the VBAR of the tree. The overlapping heights accumulate in a continuous manner.

Figure 38. The step function formed by the sum of VBARs of overlapping expanded trees in standard variable plot sampling.



See Figure 39. As trees tend to regularly space themselves this continuous function may be less variable than other sampling systems, particularly the fixed plot methods. This situation will be explored later in the thesis by simulation techniques.

A more important consequence of the smoother continuous distribution is the effect on permanent forest inventory plots. One of the serious problems in the use of the variable plot technique for continuous forest inventory is "ongrowth", where a tree which was previously "out" grows sufficiently to be included at a subsequent measurement. If the tree is quite large the increase in the sum of VBARs for the plot can be quite important. This consequence of the step function, allowing trees to "jump" into a plot, can be avoided with critical height sampling. With the CH technique the tree overlaps the point on the second measurement, but contributes only a small value to the sum of the critical heights. Figure 40 demonstrates the effect. When mortality is considered the CH method may have a greater <u>or</u> smaller effect, since the critical height can be larger or smaller than the tree VBAR.

APPROXIMATION METHODS

There are three basic problems to consider in the field application of CH sampling. First, the critical point may not be visible from the sample point, generally because of foliage. Second, the angle of measurement may be so steep that it makes measurement difficult.

Figure 39. Side view showing sum of critical heights as a smooth continuous function.

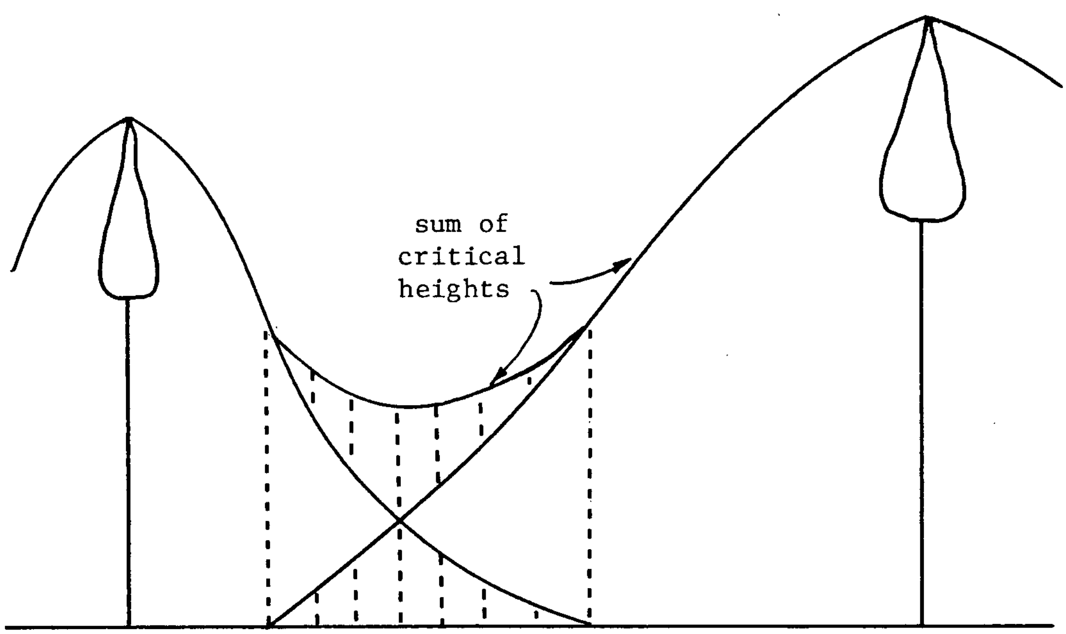
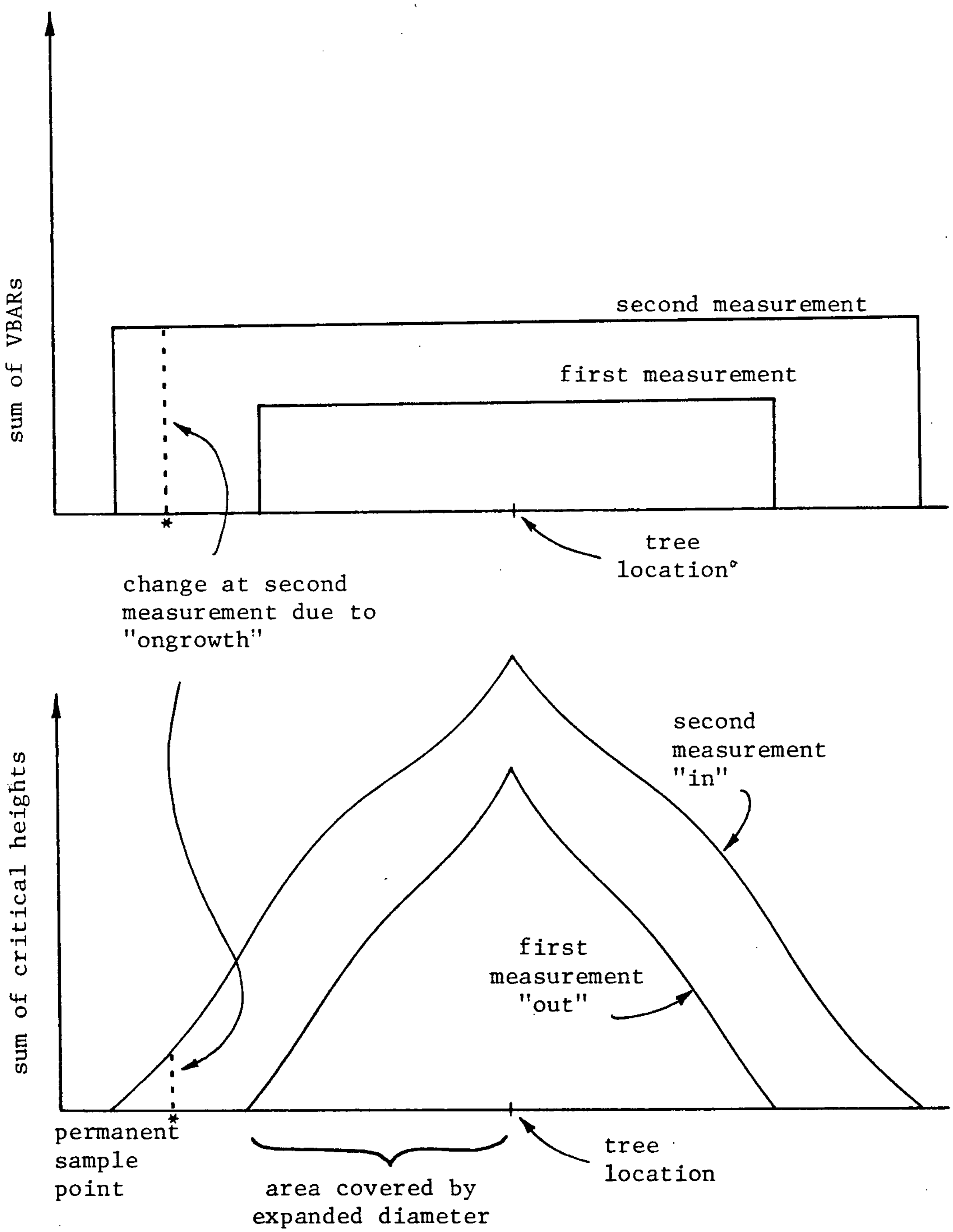


Figure 40. The effect of "ongrowth" in permanent sample points with variable plot vs. critical height sampling systems.



Third, the instrument simply may not be sufficiently accurate in locating the critical point even when it is clearly visible at a reasonable angle.

The probability of falling into the crown section of the tree is easy to compute if the diameter at the base of the crown is adequately known. The critical point will occur in the crown when the sample point falls within the expanded area of the diameter of the crown base. The proportion of times the critical point falls within the crown is given by

$$\left[\frac{D_{bc}}{D_L}\right]^2$$

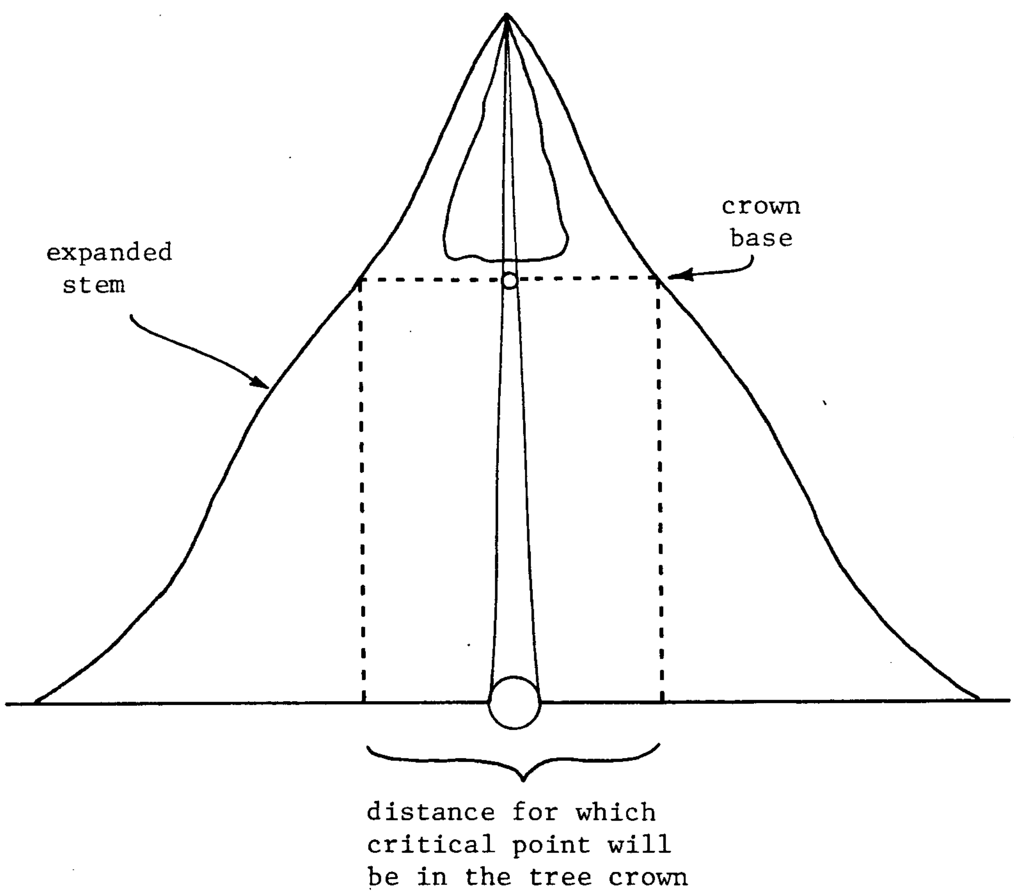
where: D_{bc} = the diameter at the base of the crown.

D_L = the diameter of the base of the stem.

Figure 41 illustrates the principle.

Some method must be devised for calculating CH when the critical point is not visible. Perhaps the most immediate solution is an approximate interpolation scheme. Using nearby visible points and the number of bar widths on the relaskop one can interpolate to find the critical point. This is of course a possibly biased approach, but the bias should be small in the upper parts of the crown where taper is rapid. An unbiased sample is still maintained for the lower bole.

Figure 41. The proportion of occasions the critical point will be in the crown.



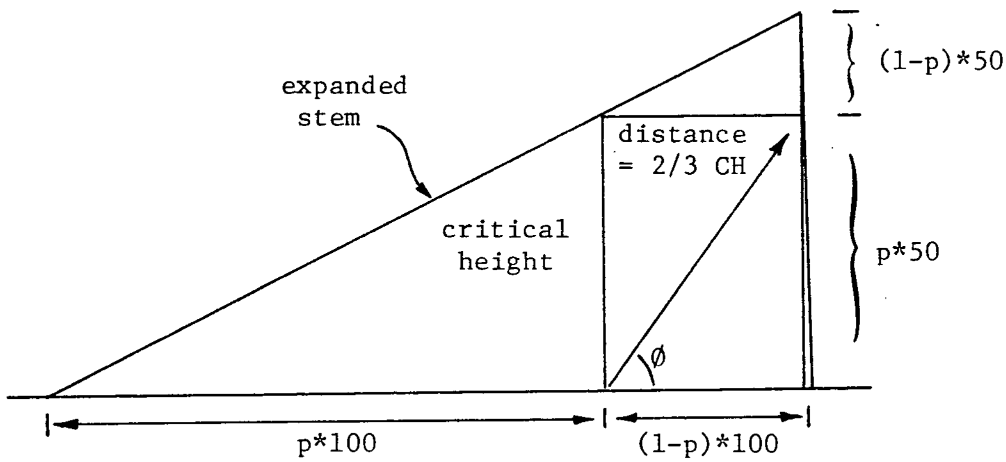
A second approach would be to use a taper equation with distance to the tree, tree height and DBH to calculate where the critical height should fall. It might be wise to check nearby visible sections to place limits on the range of possible values, particularly in the case where frequent broken tops are encountered in the stand. Such an "indirect" method was suggested by Bitterlich (1976) and informally by Beers (1974) and Bonnor (1975). Any bias in such a procedure will only affect those measurements where the critical point is obscured.

The second field problem occurs even when the critical point is below the crown. The angle of measurement to the critical point will probably be considered too steep when the tangent of the angle exceeds 1.5 (about .98 radians or 56 degrees). This happens in an area around the tree where the expanded tree radius is no less than 2/3 of the height to that point. This will depend on the taper of the tree and the angle gauge used.

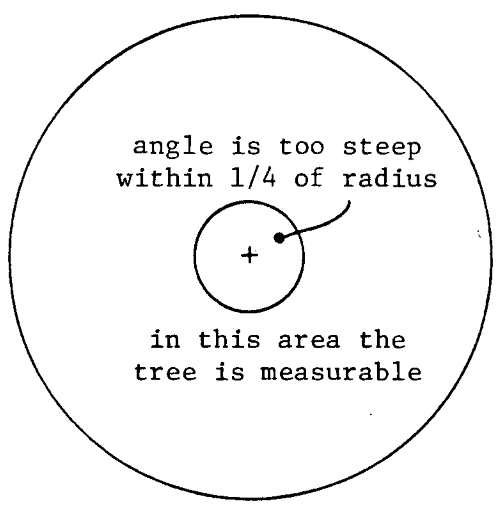
As an example consider a cone where the height is 50 times the base radius and the plot diameter factor (PDF) is 100. This leads to the geometry shown in Figure 42. The distance we wish to find is B, from the tree to a point where the angle to the critical point has tangent 1.5. The proportion of critical height to total height is the same as the proportion of distance from the plot boundary to the tree. This relationship is simple because of the conical shape. Since we wish the ratio of critical height to distance from B to the tree to be 3/2 it is possible to solve for p.

Figure 42. Calculation of the probability that the critical point will be at too steep an angle for accurate measurement.

Side View



Top View



$$\left[\frac{3}{2}\right] = \left[\frac{\text{critical height}}{\text{distance B}}\right] = \left[\frac{p * 50}{(1-p) * 100}\right] = \left[\frac{p}{(1-p)2}\right]$$

$$p = 3\ (1-p)$$

$$p = 3 - 3p$$

$$4p = 3$$

$$p = (3/4) \qquad \text{hence } B = 1/4 * 100 = 25$$

The general form of the equation for p, assuming a conical tree form, is:

$$p = \left[\frac{1}{1 + F}\right] \tag{15.1}$$

where: $F = \left[\frac{\text{(maximum acceptable tangent) * (plot diameter factor)}}{\text{(tree height to radius ratio)}}\right]$

The proportion of times a tree crown will not be measurable will then be $(1-p)^2$.

The result is unaffected by the height of the tree, although it is directly changed by the plot diameter factor and the relationship between the tree height and tree radius. This example calculation tells us that one sixteenth of the time the critical point could not be observed on a tree. This proportion can rise sharply as the expanded stems grow taller and narrower. At a ratio of 50:1 for tree height and 30:1 for plot diameter factor (a more reasonable ratio in practice) p becomes .474, giving an intercept in the crown about 28% of the time.

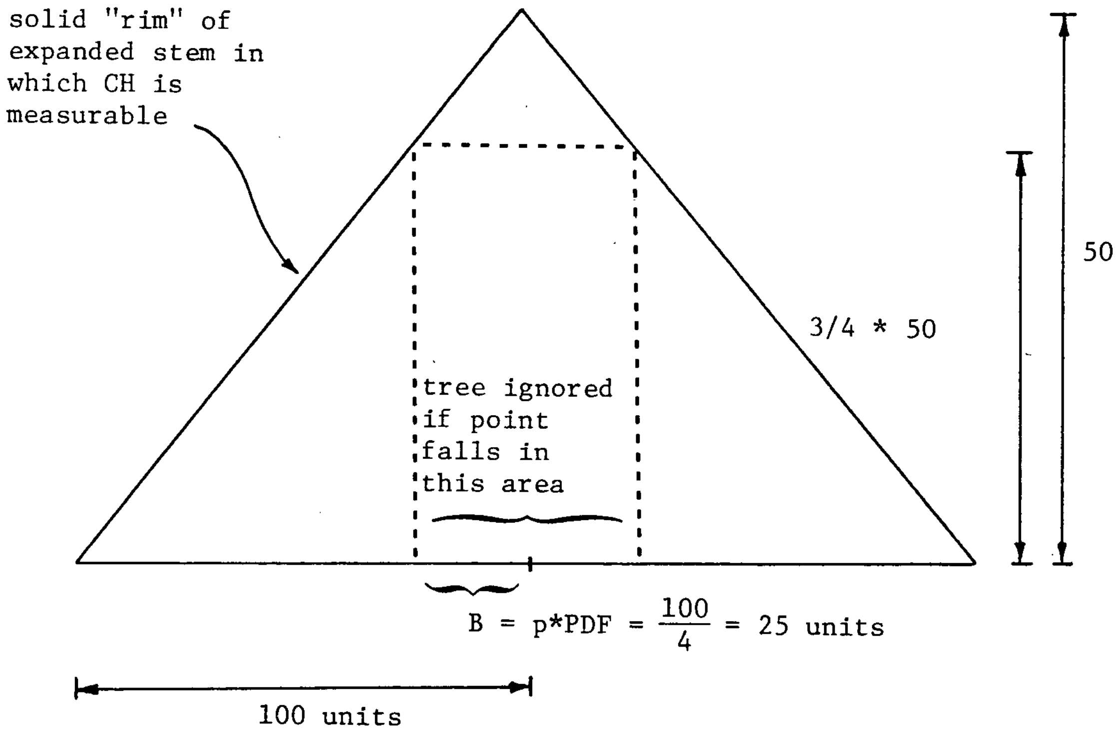
One method, when the angle to the critical point is too large, is to move away from the tree X times the distance to the tree and use an angle which has (1/X) the tangent of the original. For example, with the wide scale relaskop normally using 2 bars we could double the distance and use one bar width to find the critical point. We could also move 4 times as far away and use 1/2 bar to find the same point. This would probably not be too difficult since the distances would be small, but it might be best to avoid the whole issue by not sampling trees in these cases.

The Rim Method

To avoid these cases we must assume some kind of tree form. We could simply ignore the center parts of the expanded stem, sampling only the "rim" which remained and where the angle to the critical point was not too steep. Let us again assume a cone shape with a 50:1 height to radius ratio (HRR) and a 100:1 plot diameter factor (PDF). Figure 43 illustrates this example.

In the field all trees which covered more than 4 times the usual angle would be ignored. These are the cases when the sampler is inside the central area. The problem here is to specify the change in expected critical height caused by not measuring critical height within that section of the expanded stem. In practice this means specifying tree form and solving mathematically, or sampling for the proportion. The correction term (C_{rm}) which would afterwards be applied to calculate the <u>full tree</u> average critical height would be:

Figure 43. Example of calculations when only the "rim" of the expanded stem is sampled.



(1) cylinder volume $= 25^2 * \pi * 37.5 = 73{,}631.1$ units^3

(2) small cone volume (top) $= \frac{25^2 * \pi * 12.5}{3} = 8{,}181.2$

(3) entire expanded stem $= \frac{100^2 * \pi * 50}{3} = 523{,}598.8$

(4) "rim" volume = (3)-(2)-(1) =441,786.5 or 84.4% of full cone

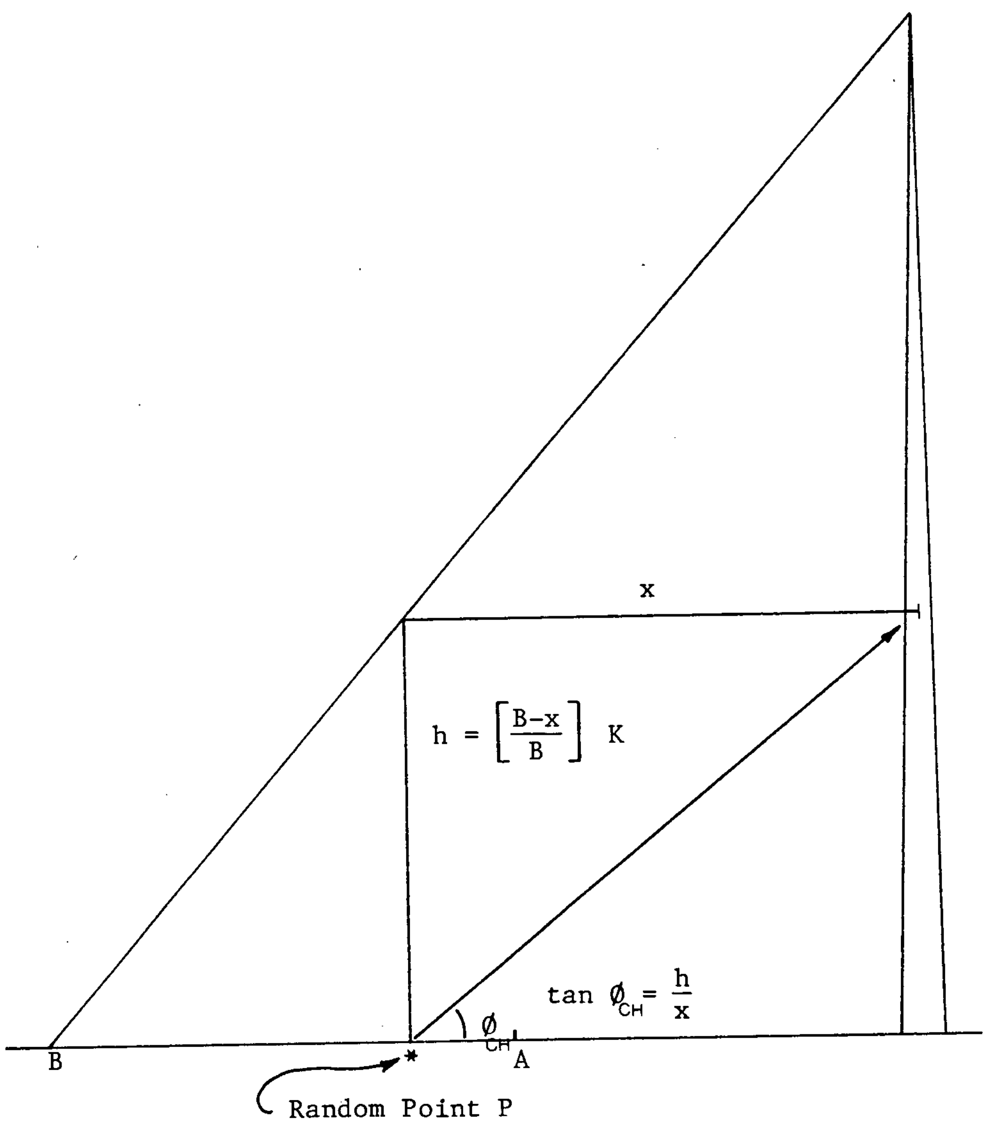
$$C_{rm} = \left[\frac{\text{cone volume}}{\text{rim volume}}\right]$$

Approximating Critical Height With the Rim Method

From the viewpoint of a field worker the ideal sampling system would require no measurements at all. At most perhaps one would like to make very simple counts. An example would be tree counts with an angle gauge. It is also very little trouble simply to read a vertical angle to a point on a tree, since instruments which do this quickly and easily are commercially available. Providing that the variability of any sampling method using virtually no measurements were low enough that method would be very desirable. The problem usually reduces to one of instrumentation, and often to the specific problems of correction for slope or distance. If a system requires measurement of some sort one seeks to base it on the simplest measurements for field work. Kitamura has attempted to solve this problem (Kitamura, 1968) but the translation is very difficult to follow. The following line of reasoning was developed by the author and is somewhat different, but will be much easier to understand.

Let us assume a cone shaped tree of radius B=1 and some height K. We note that only the height and form affect the Volume to Basal Area Ratio, so we can work with any diameter we desire. See Figure 44. We will measure the critical height of the tree whenever

Figure 44. Illustration of the terms used to develop an estimating system for critical height.



we fall within the range A to B, so we will be measuring critical height in the "rim" area just discussed. We wish to develop a method to estimate the average critical height of the tree using only the tangent of the angle (ϕ_{CH}) and tree diameter. The following approach can be used, referring to Figure 44. We would like to have a system for estimating critical height in the form:

$$CH_e = B*Q*(\tan \phi_{CH}) \tag{16.01}$$

and

$$\overline{CH} = B*Q*\overline{T} \tag{16.02}$$

where:

Q = a constant, as yet unknown

$\tan \phi_{CH}$ = the tangent of the angle from the base of the tree to the critical height when viewed from the sample point.

$\overline{T}$ = the average tangent for a tree.

We begin by finding the average tangent ($\overline{T}$):

$$\overline{T} = \int_A^B \left[\tan \phi_{CH}\right] * \left[\begin{matrix}\text{probability of x and} \\ \text{hence of } \tan \phi_{CH} \text{ itself}\end{matrix}\right] dx \tag{16.03}$$

$$= \int_A^B \left[\tan \phi_{CH}\right] * \left[\begin{matrix}\text{probability density} \\ \text{function of x}\end{matrix}\right] dx \tag{16.04}$$

The cumulative density function for x, given that x is within distance B, is:

$$\mathrm{CDFN} = \left(\frac{x}{B}\right)^2 = \frac{x^2}{B^2} \qquad (16.05)$$

hence the probability density function is:

$$\mathrm{PDFN} = \left[\frac{2x}{B^2}\right] \qquad (16.06)$$

Using T as the tangent from a random point x we solve the following formula:

$$\overline{T} = \int_A^B \hat{T} \left[\frac{2x}{B^2}\right] dx \qquad (16.07)$$

$$\overline{T} = \int_A^B \left[\frac{K\left[\frac{x-B}{B}\right]}{x}\right] \left[\frac{2x}{B^2}\right] dx \qquad (16.08)$$

cancelling x and removing $2K/B^3$ yields:

$$\overline{T} = \left[\frac{2K}{B^3}\right] \int_A^B (x-B)\ dx \qquad (16.09)$$

$$\overline{T} = \left[\frac{2K}{B^3}\right] \left[\tfrac{1}{2}(x-B)^2 \Big|_{B/2}^{B} \right] \tag{16.10}$$

$$\overline{T} = 0 - \left[\frac{2K}{B^3}\right] \left[\tfrac{1}{2} \left(\frac{B}{2} - B\right)^2 \right] \tag{16.11}$$

$$\overline{T} = \left[- \frac{2K}{B^3}\right] \left[- \frac{1}{8} \quad B^2\right] \tag{16.12}$$

$$\overline{T} = \left[\frac{1}{4} \quad \frac{K}{B}\right] \tag{16.13}$$

Knowing that the volume in the modified cone is half that of the original cone we can now solve for Q in the form we wish to have our final estimator:

$$\overline{CH} = \left[\frac{\text{Volume}}{\text{Basal area}}\right] = \left[\frac{1/2 * 1/3 \text{ K } B^2 \pi}{B^2 \pi}\right] = \left[\frac{1}{6} \text{ K}\right] \tag{16.14}$$

Combining this result with equation 16.02 we have:

$$\left[\frac{1}{6} \text{ K}\right] = \overline{CH} = B*Q*\overline{T} = B*Q*\left[\frac{1}{4} \quad \frac{K}{B}\right] \tag{16.15}$$

Therefore the value of Q is:

$$Q = \frac{2}{3}$$

The estimator for the critical height will then be:

$$CH_e = (\frac{2}{3} B) \tan \phi_{CH} \quad (16.16)$$

$$= \frac{2}{3} \left(D_i * \frac{PDF}{2} \right) \tan \phi_{CH} \quad (16.17)$$

A second way to prove this would be as follows:

$$\overline{CH} = \left[\frac{\text{Volume}}{\text{Basal area}} \right]$$

expressing the volume of a solid of revolution by the Theorem of Pappus we get:

$$\overline{CH} = \frac{2 \ \pi \ C_g \ S}{B^2 \ \pi} \quad (16.18)$$

where: C_g = the center of gravity of the cross-section of the expanded tree stem curve.

S = the cross-sectional area of the stem from A to B

$$\overline{CH} = \frac{2 \ (\frac{2}{3} B) \ \pi \int_A^B (x) \tan \phi_{CH} \, dx}{B^2 \ \pi} \quad (16.19)$$

shifting terms gives:

$$\overline{CH} = \left[\frac{2}{B^2}\right]\left(\frac{2}{3}B\right)\int_A^B (x) \tan \phi_{CH}\, dx \qquad (16.20)$$

$$= \left(\frac{2}{3}B\right)\int_A^B x \left[\frac{2}{B^2}\right] \tan \phi_{CH}\, dx \qquad (16.21)$$

At this point, we can recognize that the term in square brackets is the probability of a particular value of the tangent occurring (the probability density function of x). When sampling randomly in the plane we would be choosing the tangent with that probability and under a random sampling process we can drop that term. This leaves:

$$\overline{CH} = \left(\frac{2}{3}B\right)\int_A^B \tan \phi_{CH}\, dx \qquad (16.22)$$

which gives the same result as before. It is of interest because the method is very general, and applies to <u>any</u> cross-section (stem curve) that is of interest. Thus for any stem curve we may use the approximating formula for critical height as follows:

$$CH_e = C_g * \tan \phi_{CH} \qquad (16.23)$$

Where C_g is the center of gravity of the expanded stem curve on one side of the vertical axis.

Kitamura (1968) develops a method similar to the rim method and also uses a similar estimator, but there are differences in application. Instead of allowing the hollow center his system requires the sampler to back up from the tree until he is a certain proportional distance from the stem, and then measure the angle to the critical point. In effect, he would be "filling" the otherwise hollow section with a constant. This is a great deal of trouble in the field.

All this appears to be much ado about very little indeed. In adopting the estimation scheme we lose one of the major advantages of critical height sampling - the unbiased sampling procedure sensitive to tree form. If we are willing to assume some tree form why not just measure (or estimate) total height and get VBAR directly?? Certainly if one goes to all the trouble of making Kitamura's scheme work it is more effort than simply measuring the distance to the tree. Indeed the whole business seems to be an awkward contrivance simply to avoid the one horizontal measurement. Still, there may be an advantage which Kitamura has overlooked.

Consider the two procedures for estimating critical height from a random point.

(a) critical height = distance to tree * tan ϕ_{CH}

(b) critical height = a constant * tan ϕ_{CH}

Method (a) is the direct method and measures the height of an imaginary shell around the tree. The estimator therefore has the same distribution and statistical characteristics as the tree bole itself. The mean, variance and density functions are proportional to the stem form.

Method (b) on the other hand has a distribution which is not the same as the tree bole. In effect we have created an "expanded tree" with the same volume (or known proportion thereof) but with an entirely different "shape". The distribution of the estimator of tree volume physically surrounding the tree will overlap differently with the estimator of nearby trees. By manipulating the form of the estimator we can then change the variance of the sum of critical heights which depends on tree spacing. It may be that in forest stands, or perhaps in the measurement of objects in other fields of study, such manipulation of the overlapping shapes could significantly reduce the sampling variance.

FIELD APPLICATION

The field work for the critical height system has been described from a theoretical point of view. As with any sampling system there will be adjustments necessary for practical field application. Several plots were established in a Douglas-fir stand near the University of British Columbia to identify problems in application and possible solutions.

The most striking problem in application is with trees which are close to the sample point. With nearby trees a number of measurement problems become serious. Tree lean can be a large source of error. Although the critical point will still be accurately located the critical height measurement will often be unreliable. The maximum intercept bias is possible, as discussed by Grosenbaugh (1963). Correction for this type of bias can be made following his suggestions. The critical point of nearby trees tends to be in the crown, and obscured by foliage or branches.

On the other hand, taper is rapid in the top section of the tree, and there is a great advantage to the depth of field for distinguishing between the subject tree and the background. The depth of field advantage seems noticeable up to about 11 meters distance from the tree. While it is possible to move away from the tree in multiples of the distance between the tree and the point center this was found to be awkward for very short distances. It is clear that either some technique to bypass the nearby trees or some other method of critical height measurement is needed.

One alternate method is to calculate the diameter at the critical point (from tree diameter and distance) then locate that point and its critical height using an optical fork like the Wheeler Pentaprism. This optical fork can be used from any point where the stem is visible. The method bypasses problems of steep measurement angles and considerably reduces the difficulty of seeing the tree stem.

Taper equations can be used, but doing so waives the main advantage of a system designed to be sensitive to actual tree form. Taper function use would only be advisable if it helped to maintain another possible advantage of the system - lower variability due to the distribution characteristics of the tree stems. This would certainly be indicated on permanent growth plots for instance. Lacking any proof that the use of critical height sampling will reduce sampling variance, it would seem advisable to use the same taper functions in a normal variable plot sampling procedure.

The "rim method" discussed previously is one way to ignore the nearby trees altogether, but the sampler must carefully keep track of "in" trees, especially if the Basal Area Factor is reduced in order to increase the tree count at each point. This method seems to be the most promising field adaptation even though it too requires an assumption about tree shape.

Not all problems are removed by sighting trees in the lower bole. The lower section of the stem has less taper, is more often obscured by brush and often has a bad background for sighting with the relaskop. In addition the stem in this area is more likely to be elliptical and rough on the surface. The base of trees is often impossible to see, although a flashlight held at stump height will help a great deal in brush. A more practical method might be to sight the lower reading on a collapsable fiberglass pole and directly add the distance later in the computations.

Relaskop Use

Several hints about relaskops may be useful. Keep both eyes open when using the relaskop. The use of binocular vision decreases the difficulty of a poor background on the tree stem. Moving very slightly from side to side will often reveal an adequate outline of an upper stem even when the crown is rather dense. It is often less trouble to read the degree scale in the relaskop and convert afterwards to percent. The percent scale is frequently hard to read and abruptly changes scale without adequate labelling. It is helpful if the BAF is chosen such that an odd number of bars is used. This way they can both be white _or_ black against the stem profile as may best suit the background to the tree.

Adjustment to the sunshade can result in an almost transparent image of the relaskop scale, which helps in locating the critical point. The intensity of the scale can be varied by moving a thumb in front of the forward circular window of the relaskop. When this window is covered the scale nearly disappears. Causing the scale to "blink" by moving a forefinger on and off the window is sometimes helpful in finding the critical point, particularly in low light conditions.

Two modifications to the relaskop were useful. A threaded insert available at most camera stores will change the metric European tripod thread at the base to a standard English system thread for easy tripod mounting. The second modification involved removing the side panel of the relaskop to expose the wheel bearing the measuring scale.

The negative side of it was marked with a red transparency pen used in overhead projectors. Shallow negative angles were then very apparent when the scale turned bright red. It is necessary to insure that the marking pen is not the permanent type, so that errors can be corrected.

To explore the precision of measuring a tree under field conditions critical height was determined repeatedly on a 60 cm Douglas-fir from the distances 12, 16, 20 and 24 meters. The background and crown condition of the tree was typical for a Douglas-fir stand, but the tree was chosen so that no brush would interfere with sightings on the lower bole. The points on the tree where it was obviously "in" and "out" were also recorded at the same time the critical point was estimated. The results are shown in Figure 45 for hand held relaskop readings and Figure 46 for readings with a tripod mounted relaskop. The increased precision is obvious with the tripod. Every error of 1 meter in critical height implies an error of (1 m cubic meter * BAF) in volume per hectare. In this case, the BAF was 1.0.

The greater precision of the tripod mounting was impressive during the field work. Differences in critical height still appeared, and tended to occur in clumps just as they did with the hand held instrument, however the cause was easily determined. The relaskop scale was so sensitive that it was picking up the bumps and overgrown knots on the tree stem. If anything the relaskop was <u>too</u> sensitive, even without magnification. Often the critical point occurred at two or three places, and whether you moved up or down the tree determined

Figure 45. Critical height as measured by <u>hand held</u> relaskop.

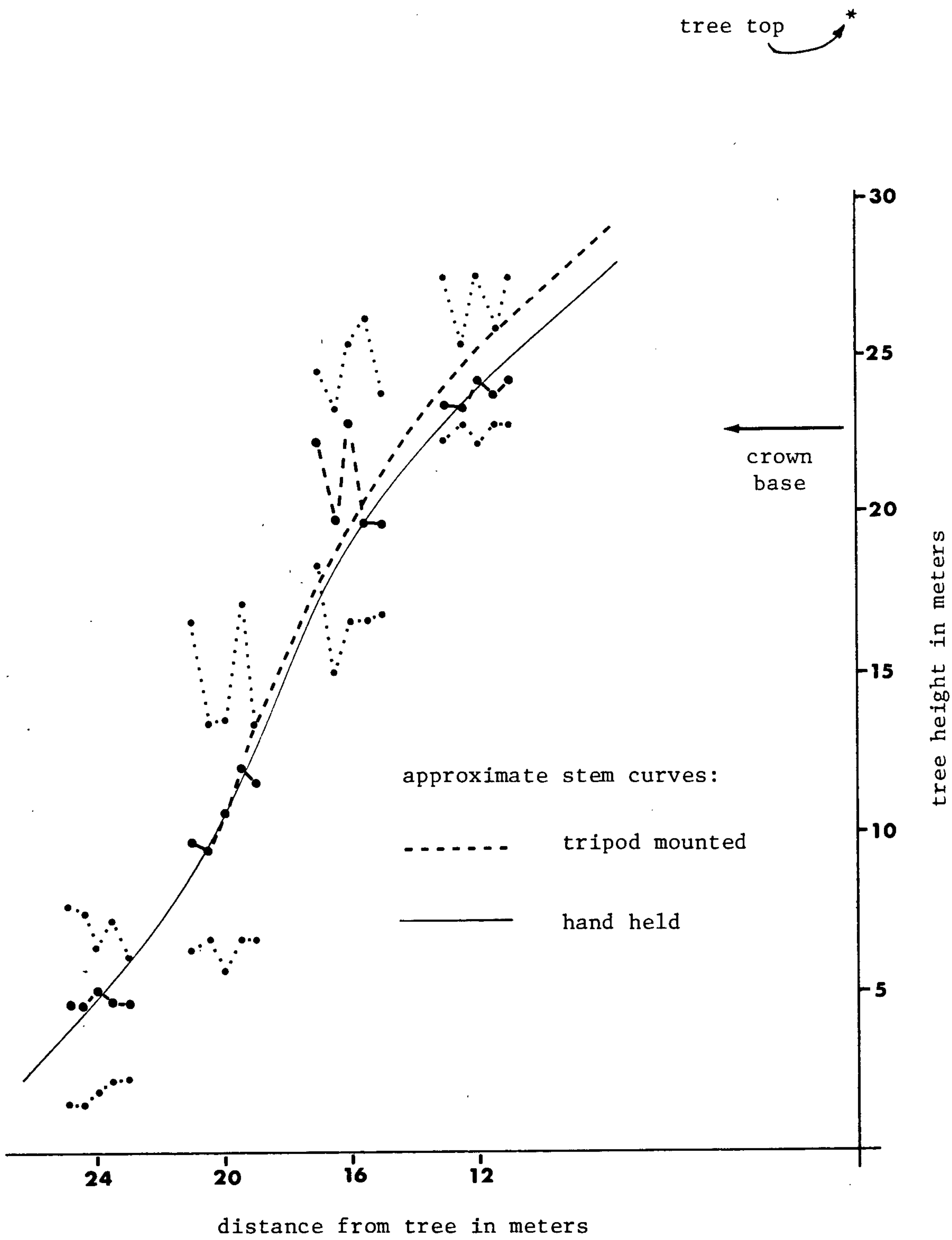
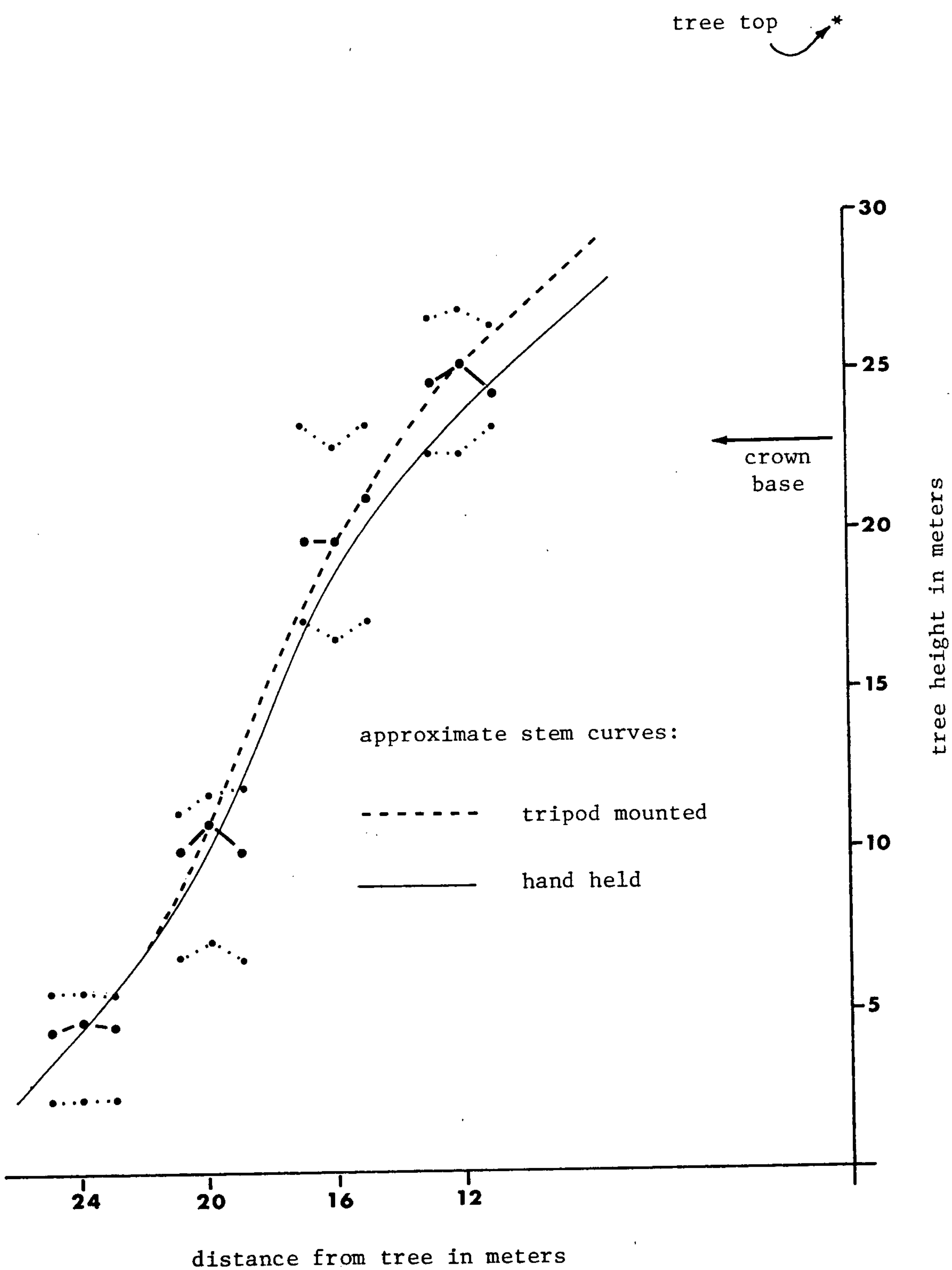


Figure 46. Critical height as measured by tripod mounted relaskop.



which one was first noticed. This problem was not as serious with species such as hemlock and cedar where taper was either smoother or more rapid.

In general, there is no problem determining critical height to acceptable accuracy providing that the tangent of the angle is not beyond about 1.50 and the line of view is clear. If no assumptions can be made about tree form it is recommended that the diameter of the critical point be calculated and then located using an optical caliper.

Log Grading

The grading of logs with critical height sampling is the same as in standard variable plot cruising. In place of the use of a VBAR for each grade stating the cubic volume of wood per unit area in a particular grade we have a critical height for each grade. The portion of the stem between the critical point and tree base is divided in a series of critical heights attributed to the grades of those sections. The volume in a grade at each sample point is then computed by:

$$\text{Volume}_{\text{grade}} \text{ per ha} = \sum CH_{\text{grade}} * BAF$$

These estimates are averaged over all points in the cruise. With the critical height method the amount of sampling in a grade is proportional to the volume in the grade. In standard variable plot cruising the sampling in a grade is proportional to the basal area of the trees containing that grade.

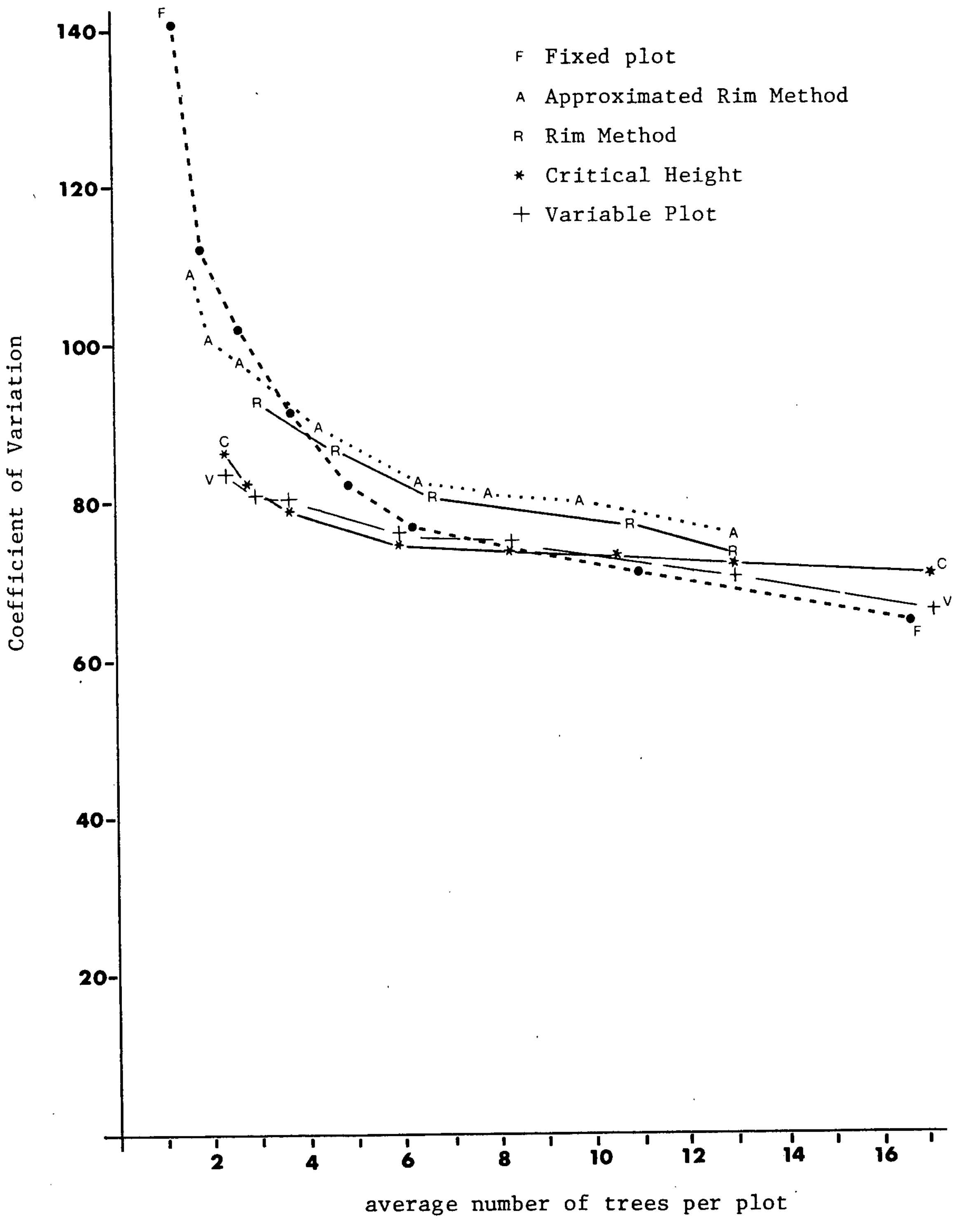
VARIABILITY OF THE SYSTEM

The variability of critical height sampling was briefly explored using a simulation study on an actual stand of Douglas-fir trees. The stand used was established in approximately 1860, and contained 192 trees ranging in diameter from 14 to 160 cm and in height from 15 to 47 meters. The median tree was approximately .4 cubic meters. A conical form was assumed for all trees. The stand was clumped and more variable than usual.

Repeated simulations were done over 200 random points throughout the area. BAF and plot size were varied on each run and variance of the total volume estimate was calculated. The results are shown in Figure 47, recorded by the average number of trees measured in each test. A few additional runs with different random points and larger sample sizes were made to verify that these results were representative.

There appears to be no statistical advantage in the critical height method. The variability of both critical height sampling or standard variable plot sampling are the same for practical purposes. The advantage of CH sampling is that it is an unbiased estimate of stand volume. The disadvantage is that the trees near the sampling point are difficult to measure. The approximation to the rim method using the percent angle and tree diameter proved to have a coefficient of variation about 10% higher than the first two methods. The loss

Figure 47. Coefficient of Variation for 5 sampling methods.



in efficiency does not seem warranted simply to eliminate the measurement of the distance to the tree. The standard rim method, simply ignoring trees which were more than twice the critical angle at the base, was more competitive with the standard critical height technique and also eliminated the problem of measuring nearby trees. Fixed plot sampling was competitive as long as the average number of trees measured was kept above 6 trees per plot. The range of 6-10 trees per point seems to be most efficient for sampling purposes in stands of this type.

CONCLUSIONS

In the final analysis the use of the critical height method will depend on the importance of the bias in volume tables and the difficulty of measuring the nearby trees for critical height. The most promising method for measuring these difficult trees seems to be the use of the Wheeler Pentaprism. Application of the system will probably be limited to cases where volume tables are very unreliable due to variability of the merchantable top, continuous forest inventory where "ongrowth" is a problem, and use of the method to select trees randomly with probability proportional to gross volume.

LITERATURE CITED

Barrett, J.P. Correction for edge effect bias in point sampling. Forest Science, Volume 10, pages 52-55.

Beers, T.W. and C.I. Miller. 1964. The Purdue point-sampling block. Journal of Forestry, Volume 54, pages 267-272.

Beers, T.W. 1966. The Direct Correction for Boundary-line Slopover in Horizontal Point Sampling. Research Progress Report 224, February 1966, Purdue University Agricultural Station, Lafayette, Indiana.

Beers, T.W. 1974. Private Communication.

Beers, T.W. 1976. Practical Boundary Overlap Correction. Journal Paper 6175, Purdue University Agricultural Station, Lafayette, Indiana.

Bitterlich, W. 1948. Die Winkelzahlprobe. Allgemeine Forst- und Holzwirtschaftliche Zeitung, 59(1/2), pages 4-5.

Bitterlich, W. 1956. Fortschritte der Relaskopmessung. Holz-Kurier Volume 11, Number 1.

Bitterlich, W. 1971. Fortschritte der Relaskopie: Die Volumstichprobe. Holz-Kurier 26(17), 8.

Bitterlich, W. 1973. The Angle-Count Sampling Technique. International Statistical Institute, Vienna, Austria.

Bitterlich, W. 1975. Volumstichprobe aus indirekt bestimmten Deckpunkthohen. Allgemeine Forstzeitung 86(4), 113-5.

Bitterlich, W. and W. Finlayson. 1975. Tele-Relaskop. Feinmechanische-Optische Betriebsgesellschaft, Salzburg.

Bitterlich, W. 1976. Volume sampling using indirectly estimated critical heights. Commonwealth Forest Review, Volume 55, Number 4, pages 319-330.

Bonnor, M. 1975. Private communication.

Brown, G.S. 1965. Point Density in Stems Per Acre. New Zealand Forest Service, Forest Research Institute, Forest Research Note Number 38.

Brunk, H.D. 1965. An introduction to Mathematical Statistics, 2nd Edition, Blaisdell Publishing Company, 429 pages.

Finlayson, W. 1969. The Relascope. Feinmechanische-Optische Betriebsgesellschaft, Salzburg.

Fisher, R.A. 1936a. The Design of Experiments. Oliver and Boyd Ltd.

Fisher, R.A. 1936b. Has Mendels work been rediscovered? Annals of Science, Volume 1, pages 115-137.

Fraser, A.R. 1977. Triangle Based Probability Polygons for Forest Sampling. Forest Science, Volume 23, Number 1.

Grosenbaugh, L.R. 1958. Point-Sampling and Line-Sampling: Probability Theory, Geometric Implication, Synthesis. Southern Forest Experiment Station, Occasional Paper 160.

Grosenbaugh, L.R. 1963. Optical Dendrometers for out-of-reach Diameters: A conspectus and some new theory. Forest Science Monograph, Number 4, 47 pages.

Grosenbaugh, L.R. 1974. STX 3-3-73: Tree Content and Value Estimation Using Various Sample Designs, Dendrometry Methods, and V-S-L Conversion Coefficients. USDA Forest Service Research Paper SE-117.

Grosenbaugh, L.R. 1976. Approximate Sampling Variance of Adjusted 3P Estimates. Forest Science, Volume 22, Number 2, pages 173-176.

Hirata, T. 1955. Height Estimation Through Bitterlichs Method, Vertical Angle Count Sampling. Japanese Journal of Forestry, Volume 37, pages 479-480.

Hirata, T. 1962. Studies on methods for the estimation of the volume and increment of a forest by angle count sampling. Bull. Tokyo Univ. For., Volume 56, pages 1-76.

Husch, B., C.I. Miller and T.W. Beers. 1972. Forest Mensuration, 2nd Edition, Ronald Press Company, New York, 410 pages.

Iles, K. 1974. Penetration Sampling - An Extension of the Bitterlich System to the Third Dimension. Oregon State University. Unpublished manuscript.

Jack, W.H. 1967. Single tree sampling in even-aged plantations for survey and experimentation. Proc. 14th Congress Int. Union For. Res. Organ., Munich, 1967, Pt. VI, Sect. 25, pages 379-403.

Johnson, F.A. 1972. Fall, Buck and Scale Cruising. Journal of Forestry, Volume 70, Number 9, pages 566-568.

Kitamura, M. 1962. Zur Bestandesmassenermittlung durch die Deckpunkthöhensumme. Report of the 73th meeting of the Japanese Forestry Society, pp. 64-67.

Kitamura, M. 1964. Theoretical Studies on the Estimation of Stand Volume Through the Sum of Deckpunkthöhen. Bulletin of the Yamagata University (Agricultural Science), Vol. 4, No. 4, pp. 365-403.

Kitamura, M. 1968. Einfaches Verfahren zur Bestandesmassenermittlung durch die Deckpunkthöhensumme. Journal of the Japanese Forestry Society, Vol. 50, No. 11, pp. 331-335.

Kitamura, M. 1977. Eine Methode zur Bestandesmassenermittlung mit dem vertikalen kritischen Winkel. Journal of the Japanese Forestry Society, Vol. 59, No. 3, pp. 101-103.

Kitamura, M. 1978. Moglichkeiten der bestandesmassenermittlung mit dem horizontalen oder verticalen kritischen winkel. I.U.F.R.O. proceedings, Nancy, France.

Loetsch, F., F. Zohrer and K.E. Haller. 1973. Forest Inventory, Volume II. Munchen, Basel, Wien. 469 pages.

Martin, G.L., A.R. Ek and R.A. Monserud. 1977. Control of plot edge bias in forest stand growth simulation models. Can. Jour. For. Res., Volume 7, pages 100-105.

Matern, B. 1971. Doubly Stochastic Poisson Processes in the plane. Statistical Ecology, Volume 1, Patil, Pielou and Waters, eds. University Park: Penn State University Press, pages 195-214.

Mead, R. 1966. A relationship between individual plant-spacing and yield. Annals of Botany, N.S. Volume 30, Number 118, pages 301-309.

Meyer, H.A. 1956. The calculation of the sampling error of a cruise from the mean square of the successive differences. Journal of Forestry, Volume 54, page 341.

Nash, A.J., T. Cunia and K. Kuusela. 1973. Proceedings of I.U.F.R.O. Subject Group S4.02, Nancy, France. University of Missouri, School of Forestry, Fisheries and Wildlife.

Minowa, M. 1976. Stand Volume Estimation Through Upper-Stem Diameter. Journal of the Japanese Forestry Society, Volume 58, Number 3, pages 112-115.

Minowa, M. 1978. Precision of a New Method with Vertical Angle for the Estimation of Stand Volume (II), Line Sampling. Journal of the Japanese Forestry Society, Volume 60, Number 5, pages 186-190.

Newnham, R.M. and G.T. Maloley. 1970. The generation of hypothetical forest stands for use in simulation studies. Forest Management Institute Information Report. FMR-X-26, Canadian Forestry Service. 75 pages.

Overton, W.S., D.P. Lavender and R.K. Herman. 1973. Estimation of biomass and nutrient capital in stands of old-growth Douglas-fir. I.U.F.R.O. Biomass Studies, pages 89-104, College of Life Sciences and Agriculture, University of Maine, Orono, Main. 532 pages.

Pielou, E.C. 1977. Mathematical Ecology. John Wiley and Sons. 385 pages.

Schreuder, Hans T. 1979. Count Sampling in Forestry. Forest Science, Volume 24, pages 267-272.

Shiue, C.J. 1960. Systematic sampling with multiple random starts. Forest Science, 1960, Volume 6, pages 42-50.

Strand, L. 1958. Sampling for volume along a line. Norske Skogforsøksv. Number 51.

Warren, W.G. 1972. Stochastic Point Processes in Forestry. Stochastic Point Processes, John Wiley and Sons, Inc. pages 801-814.

APPENDIX I

LIST OF SYMBOLS AND TERMS

a_c	area of a compartment formed on a preference map by overlapping of plots, as well as cell, indicating the first tree from a given azimuth.
a_k	area of a subcompartment formed by the overlapping of plots.
A_r	The angle of rotation used with a prism.
ab_i	area of a band associated with a particular tree.
ap	area of a fixed plot used in the sampling process.
BA_i	The basal area of tree i.
BAF	Basal Area Factor.
C_{ch}	Area of a cell around a tree in which that tree's critical height is greater than any other tree.
C_g	Center of gravity for a portion of a stem curve.
C_{rm}	Correction term to calculate full average critical height from the critical height estimated by the "rim method".
C_s	a general constant, the exact value of which depends on the details of the sampling scheme.
C_v	a vertical angle used to select a tree for possible sampling.
CA_i	cross-sectional area of a part of a tree stem crossed by a transect.
$\overline{CH}$	average critical height.
CH_e	Estimated critical height.
CH_i	Critical height of a particular tree.
CDFN	Cumulative density function.
D_{bc}	Diameter at the base of the tree crown.
D_i	Diameter at some point on tree i.

D_L	Diameter at the lowest sighting point on the tree, presumed to be the largest diameter as well.
DBH_i	Diameter at breast height (1.3 meters) on tree i.
DC_i	Area of a Dirichlet cell around tree i.
E	Expansion factor of an angle gauge. Equal to (plot area/tree basal area).
F	A factor used in calculating p.
I	The number of "in" trees at a point
K	Arbitrary height of a cone.
L_b	length of lines used in line sampling or one of its variations.
md_i	area of a modified Dirichlet cell.
Mp	largest number of trees selected as a cluster.
M_{sum}	The maximum expected sum of critical heights.
n	sample size
N	Number of possible observations in the population. Usually the number of trees in an area.
n_k	number of trees involved in a compartment.
n_{max}	The largest number of trees selected in any cluster throughout the sample area.
np	number of trees present in a cluster chosen by fixed or variable plots.
p	A proportion of the distance to the edge of the plot from the tree located at the center.
$p\left\{S_i\right\}$	probability of sampling tree i.
$p\left\{S_{np}\right\}$	probability of sampling a cluster of np trees.
$p\left\{S_s\right\}$	probability of sampling a tree after establishing a random point on the tract.
P	a random point on the area to be sampled.

$Pr\{S_i\}$	relative probability of sampling tree i compared to any other tree on the tract.
PDF	A constant relating tree diameter to the diameter of an unseen plot surrounding the tree.
PDFN	Probability density function.
Q	Proportion of plot radius, to be multiplied by tan $\emptyset_{CH}$ in approximating "rim method".
R	a uniform random number between 1 and some specified upper limit.
S	cross-sectional area of part of the stem profile.
S_c	sum of critical heights at a sample point.
t	total number of subcompartments formed by overlapping plots or strips.
T	Total area of the tract of land on which sampling is conducted.
$\overline{T}$	Average tangent throughout the plot area.
$\hat{T}$	tangent of the angle to the critical point from a random point within the plot radius.
t_i	Total number of compartments formed by overlap of plot of tree i with other plots.
tan $\emptyset_{CH}$	the tangent of the angle to the critical point. Equal to (CH/distance to tree).
$tree_i$	a particular tree from the population.
$\hat{V}$	The estimated volume with line-intersect sampling.
V_i	The volume of a particular tree i.
VBAR	Volume to Basal Area ratio.
W_s	width of strip used in selecting a tree with a particular sampling system. Sometimes the distance between two transects.
xp	The number of points on a grid placed on the tract to be sampled.

z_i	number of compartments on preference map favoring selection of tree i.
z_p	total number of compartments on a preference map.
Θ	The angle used to select a tree with variable plot sampling.
$\emptyset$	Vertical angle to the critical point in critical height sampling.