ECONOMICS OF MULTIPLE-USE FOREST MANAGEMENT: SPATIAL CONSIDERATIONS

by

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ABSTRACT

Historically, Canada's productive forests have been assumed to be reserved for timber use while nonproductive lands have been reserved for other uses. However, demands on Canada's forestlands are becoming increasingly more diverse and traditional timber harvesting practices are now being scrutinized with regard to their consistency with these new demands. The response by provincial and federal policy makers has been a movement towards the concept of multiple-use forest management. However, due to the numerous meanings of the concept, policy makers and practitioners are finding it difficult to implement this new direction.

Many of the issues surrounding forest management for multiple use are spatial in nature. Problems include where (location) to manage for single or multiple goods and services, and what scale (size) to choose for management units. The spatial issue, the issue of where, is of great importance in multiple-use forest management, because location is central to the long standing debate in forestry as to whether certain forest areas should be allocated to specialized or general multiple-use management.

This dissertation focuses on the problem of modeling the issue of space in an economic model of multiple-use forestry. The study first involves modeling the problem of managing a two-stand forest over a two-period time horizon with and without intensive timber management and then solving a three-stand forest for several case studies by numerical simulation.

The analytical and simulation results suggest that relative prices, the discount rate, forest productivity, nontimber productivity, and interdependencies between forest stands

are all important determinants of the optimal harvesting and inventory solutions. Within a multiple-stand forest, areas are managed similarly if complementarity exists between stands and differently if substitutability exists between the stands in producing nontimber values, ceteris paribus. The results support both zoning for intensive timber management and integrated resource management everywhere. Thus, there is no a priori optimal management paradigm in forestry. However, intensive timber zones are supported under particular circumstances. Furthermore, the result suggest that forest policy tools, such as forest practices laws and forest land-use zoning, need to be flexible over time and space to promote and achieve efficient resource allocation.

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DEDICATION

This work is dedicated to my Grandfather, Ted Shaw.

CHAPTER 1

BACKGROUND AND SUMMARY

1.1 Introduction

Canada is a country with vast tracts of forestland. The total land area of Canada is estimated to be 997.1 million hectares (ha.) of which 417.6 million ha. is classified as forestland. Some 235.6 million ha. is considered productive and available (non reserved) for timber uses. Of the 235.6 million ha., an estimated 144.5 million ha. is accessible for exploitation (Compendium of Canadian Forestry Statistics (CCFS) 1996). The majority of this forestland is classified as natural forest, with the majority considered "mature" or "over-mature" forest (46.2 per cent of the area and 68.4 per cent of total standing timber volume). The ownership of the forestland base is predominately public. Of the 235.6 million ha. of non-reserved productive land, 188.7 million is provincially owned, 24.3 million ha. is privately owned, 19.3 is owned by Canada's territories, 3 million is federally owned and 0.3 million is of unclassified ownership (CCFS 1996).

In spite of the vast natural resource base in Canada, issues surrounding the use of Canadian forests and the impact on the environment are pointed. Historically, Canada's productive forests have been assumed reserved for timber use. Currently, only 9 million ha. of the 144.5 million ha. of accessible commercial timber land is excluded from commercial timber use. A further 90 million ha. of the potential timber land base (235.6 million ha.) and approximately 270 million ha. of the total forest base (418 million ha.) is wilderness (CCFS 1996). However, public awareness of the importance of forest resources on the health and welfare of people now challenges this assumption and current distribution of land use. Demands on Canada's forestlands are becoming increasingly more diverse while traditional timber harvesting practices are now being scrutinized with regard to their consistency with these new demands. The response by provincial and federal policy makers, academics and the forest industry has been a movement towards the concept of multiple-use forest management (e.g., Booth et al. 1993; MacDonald 1999). However, due to the numerous held meanings of this concept, policy makers and practitioners are finding it difficult to find a new direction.

Hyde and Newman (1991) highlight the obvious problem with forests; forests can produce multiple products and services for a wide range of competing demands. These demands may be social or private, a function of distance, and can vary among people of different incomes, while goods can be complements or substitutes in consumption. There are also technological issues in producing forest goods and services. These include: many products can be produced jointly while others are completely incompatible with one another, minimum and maximum scales of production differ between goods and services, time, and lastly, forest stocks are both an output and an input in production. Informational issues such as a poor understanding of the relative values of alternative forest uses and biological processes complicates the development and application of multiple-use forest management. Therefore, market failure and resultant misallocation of resources are the result of the very nature of forest resources.

One policy response to market failure in forestry is the promotion of the concept of multiple-use forestry. However, the complex nature of forests makes the practice and understanding of multiple-use forestry difficult. As a result, the many issues surrounding the concept of multiple-use forestry have led to numerous definitions of the concept and the development and practice of various forms of multiple-use forest management around the world.

Two public policy tools currently used or proposed to promote and enhance efficient use of forest resources are forest practices laws (programs) and forest land-use zoning. There are numerous examples of forest practices and programs in the world (see Brown et al. 1993; Cook 1998). In many cases, these programs/laws impose command-and-control regulations on forest users without any consideration for physical or socioeconomic differences among regions. The second tool, forest land-use zoning, is less common but widely debated as a useful planning tool to achieve the goals inherent with multiple-use forestry. For example, in British Columbia the topic is actively debated (see Sahajananthan et al. 1996; Binkley 1997; Rayner 1998) while it is being considered on a global scale by The Council of Foreign Relations in conjunction with the World Bank and the World Wildlife Fund (see http://greatrestoration.rockerfeller.edu/). A common concern surrounding the use of land-use regulation is that it is inflexible, and, therefore, will prevent optimal resource allocation in the future. A common question is will these policy tools be successful in promoting and achieving efficient forest resource allocation?

Many of the issues surrounding forest management for multiple uses are spatial in nature. Such problems include determining where to manage for single or multiple goods and services, and at what scale of management unit. The spatial issue, the issue of where, is of great importance in multiple-use forest management. The issue of location is central to the long-standing debate in forestry as to whether certain forest areas should be allocated to specialized or general multiple-use management.

Economic modeling of the issue of where to have specialized or multiple-use forest management has contributed greatly to the intuitive understanding of the management problem. The application of economic theory to the multiple-use forest problem from the view of production and capital theory, as evidenced by the pioneering work of Gregory (1955) and more recent contributions of Bowes and Krutilla (1985) and Swallow et al. (1997), has greatly increased our understanding of the importance of economic variables in determining forest management decisions, particularly concerning where to harvest timber. However, the two approaches suggest two very different management models. Production theory suggests that areas be permanently allocated to a particular management emphasis (specialized or multiple use), while recent modeling work using capital theory (Bowes and Krutilla 1985; Swallow and Wear 1993; Swallow et al. 1997) suggests areas should be periodically allocated to particular management units but eventually may need to be reallocated. On the other hand, Rose (1999) suggests that under the condition of a significant fixed harvesting costs, management is fixed over time.

Each theoretic approach captures an incomplete picture of reality. Production models partially explain why particular areas are managed for single uses and suggest a permanent allocation, while new capital theory partially explains how management efforts move spatially over time reflecting the dynamic nature of forests. Neither modeling approach, with the exception of Rose (1999), is capable of determining the conditions under which we expect one multiple-use pattern to be favoured over another.

In short, current economic modeling of the multiple-use problem falls short of capturing the richness of spatial issues in a comprehensive framework. As Clawson (1978) noted over 20 years ago,

> An interplay between spatially differentiated and temporally differentiated areas in a moderately large forest managed under a multiple use philosophy can create many productive situations and relationships, to the benefit of all forest outputs and of all forest users. (p.308)

1.2 Problem and Purpose of the Study

The focus of this dissertation is on the problem of modeling the issue of space in an economic model of multiple-use forestry. The study involves two parts: first, modeling the management of a two-stand forest for more than one value over a two-period time horizon to maximize net present value; second, extending the model to three stands and multiple time periods.

The general two-period model is solved analytically to obtain insights into the optimal short-run and long-run harvest rules and to generate predictive comparative statics results. The analytical model is then extended to a multiple-period model and solved using a dynamic programming algorithm. Three cases are presented to highlight the importance of space in the problem of multiple-use forest management. One case introduces pecuniary interdependencies and non-convexities as done by Swallow and Wear (1993). A second case introduces technological interdependencies, while a third introduces asymmetries by introducing a third good. In all cases, the problem is how much harvest and how much inventory to hold on each stand over time, accounting for linkages between stands or forest-level asymmetries, to maximize the net present value of the forest over a finite time horizon.

The two-stand analytical model is sufficiently abstract to cover different issues of scale and can capture different ownership contexts. Such abstraction permits analysis at the forest-level or multi-forest scale encompassing various ownership regimes. The model is extended to include management effort and can be further extended to allow for explicit modeling of owner preferences, uncertainty and risk, specific nontimber values, and can be used to analyze various policy instruments such as taxes and regulations.

This study extends the two-period model of harvest-inventory to include explicitly intensive silvicultural management, to allow for three stands, to capture biological population dynamics, to explore management for more than two goods, and to analyze pecuniary and technological externalities.

The results support the conclusion that forest policy tools need to account for spatial issues in forestry and be flexible to change if they are to be effective instruments to promote multiple-use forestry and achieve forest resource efficiency. The results also support the conclusion that land-use specialization for timber production is economically efficient under particular conditions.

1.3 Key Assumptions and Limitations of Analysis

Throughout the dissertation the term nontimber value is used very loosely. Nontimber value may mean a recreational value such as hunting that is not difficult to value in reality. Or, nontimber can mean biodiversity or a culturally modified tree that are very difficult to value. In all cases, nontimber values are assumed to be well defined and can be valued. Furthermore, nontimber values are assumed to be a function of stand or forest inventory. For example, a nontimber value may increase, decrease or remain unchanged with a change in inventory.

In this dissertation, perfect information is assumed. The reality is that nonpriced goods, public goods, multiple users and multiple and overlapping property rights, limited scientific knowledge, information and other transaction costs all exist and affect optimal forest management plans (see Wang and van Kooten 2000).

The numerical results for the two-period model suggest that, to achieve the maximum economic value from a forest via multiple-use management, requires considerable information. In particular, information is needed on the marginal value of all goods, peoples' rate of time preference, knowledge of timber harvesting and other forest management technologies, forest ecology, and, finally, the physical inventories of flora and fauna. However, the reality is that many outputs are public goods or are nonpriced, there are multiple users of the forest, and little is known about forest ecosystems or how forest management practices can produce different sets of goods and services.

The existence of public goods and nonpriced goods in forestry is an important issue in multiple-use forest management. Their presence makes it difficult to know with any certainty the relative marginal values of nontimber goods. The fact that timber markets do not exist in many forest regions around the world, coupled with the lack of information on nontimber values, adds even greater uncertainty and confusion to the practice of multiple-use forestry. This issue is not only germane to public forestlands but also to private lands as public goods can and do occur on both.

Another important issue that potentially hinders the successful practice of multiple-use forestry is the presence of multiple users. With multiple users there is the possibility of differing demands on the same forest as well as differing rates of time preference. The introduction of different demands adds confusion in a manner not unlike nonpriced goods and public goods, as it raises the issue of what weights or relative values to assign to each good in each area. In addition, the issue of multiple users introduces the possibility of multiple time preferences that can further compound the confusion surrounding the practice of multiple-use forest management. The existence of different rates of time preference raises several ethical and empirical questions. Whose preferences count? What is the social discount rate? How are risk preferences and uncertainty included in its determination? These questions are difficult to answer but are critical for the practice of multiple-use forestry if it is to be a successful management practice in achieving efficient resource use and socially optimal outcomes.

A final issue important for the successful practice of multiple-use forestry relates to forest ecosystem and management knowledge. In reality, little is known about the actual function of forest ecosystems or how various forest management practices can be used to produce different quantities of goods and services. The lack of understanding of forest ecology or forest management technology adds greater uncertainty to the practice of multiple-use forestry.

Therefore, modeling institutions, transactions costs and the valuation of nonmarket goods and services, to understand better their importance in determining the optimal management of forestlands for multiple uses, are areas of important research but beyond the scope of this dissertation.

1.4 Outline of the Thesis

Chapter two discusses the various concepts of multiple-use and the issues involved in the theoretical modeling and practice of multiple-use forest management. Chapter three presents a review of earlier studies with emphasis on how modeling approaches include space and time. In Chapter four, the multiple-use problem is analyzed with a two-period three-stand economic model of multiple-use forestry. In Chapter five a dynamic programming algorithm is used to solve various case studies. Conclusions are presented in Chapter six.

CHAPTER 2

SPATIAL ISSUES IN MULTIPLE-USE FORESTRY

In this chapter concepts of multiple-use forestry are discussed and the importance of spatial considerations in the practice of multiple-use forestry are highlighted. In Section 2.1, I discuss the meanings of multiple-use forestry and provide an overview of the important issues in its practice. In Section 2.2, I discuss spatial issues involved in multiple-use forest management. Conclusions are presented in Section 2.3.

2.1 Meanings of Multiple-use Forestry

In practice, multiple-use forestry can have several forms and be applicable in many contexts. In Canada, multiple-use forestry is synonymous with the practice of managing for several competing uses on public forestlands, where 94 percent of forestland is publicly owned. This is also the case in the Province of British Columbia (Hoberg and Schwichtenberg 1999). However, the practice of managing forestlands for more than one use is prolific across many ownership forms, across cultures and across many spatial scales.[1] Multiple-use forestry is practiced on community forests (Allan and Frank 1994; Duinker et al 1994), on timbered range lands (Anderson 1994; Standiford

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[1]See Hytönen (1995) for a compilation of multiple-use forestry in the Nordic countries. Stridsberg (1984) gives a historic account of multiple-use forest in Sweden while Ammer (1992) discusses multiple-use forestry on commercial forests in Germany. Ito and Nakumura (1994) discuss recent changes in forestland use in Japan from single-use forestry to current multiple-use forest planning with particular attention to the issues of spatial and temporal scales. Yasumura and Nagata (1998) provide an account of multiple-use forest management operations on 13 districts of the national forests of Taiwan. Clawson (1978) provides a history of multiple use on U.S.national forests, which has been a guiding principle since at least 1905.

and Howitt 1992), in the form of agroforestry (see Carne and Prinsely 1992 for definitions), in urban forests (Konijnendijk 1997), on industrial forests, on non-industrial private forests (Binkley 1981), on second-growth, mature natural forest, and on intensive plantations in tropical and temperate regions (Rimoldi 1999).

One of the most pointed and frequently occurring debates in modern forestry surrounds the choice of land management philosophy to employ to best satisfy societal demands and ensure resource-use efficiency (Binkley 1997; Burton 1994; Kutay 1977; Sahajananthan et al 1996; Alverson et al 1994; Walker 1974; Benson 1988, 1990; Haas et al 1987; Sedjo 1990; Bird 1990; Behan 1990; Wilson 1978; Juday 1978; Dancik 1990; Reed 1990; Conrad and Sales 1993). Two basic notions of multiple-use land management are debated:

1. Intensive multiple-use forestry attempts to produce the maximum feasible and preferred outputs from the entire land base.
2. Extensive multiple-use forestry attempts to produce the maximum feasible and preferred outputs from every part of the land under management.

These two perspectives are further refined to include: 1) integrated resource management which is synonymous with extensive multiple use; 2) a mosaic of single uses over the land base (a form of intensive multiple use); 3) a single-use intensive timber zone surrounded by various forms of multiple use, further surrounded by wilderness areas, or the so-called triad approach (Teeguarden 1975; Binkley 1997); 4) management for a dominant use and all other uses, often referred to as dominant-use zoning (DUZ), (Alverson et al 1994); and 5) the management of many uses sensitive to temporal and spatial scales. The application of the concept of multiple-use forestry is made difficult due to all the alternative notions and management models (Clawson 1978).[2]

In this thesis, I define multiple-use forestry as:

> *Multiple-use forest management is the practice of forestry that exploits and augments flora and fauna stocks within defined forested areas together with man-made capital and labor inputs to produce more than one good or service over various time scales.*

This definition is sufficiently general to include natural and intensively managed forest areas, any geographical resolution, and any form of ownership. The definition also acknowledges durable, reproducible capital inputs, such as roads, buildings, bridges, water reservoirs, fencing, waste disposal systems and management effort, as an input that becomes capitalized in the natural resource stocks. Management effort can involve fire protection and efforts such as education programs on wildlife. However, the definition may be too strict as it excludes agricultural areas that through afforestation, become neither strictly agricultural areas nor forest areas and urban areas that are a mosaic of farmland, residential and commercial lands, and forested lands. This omission highlights the issue of spatial scale and location. It is clear that multiple-use forestry is simply a subset of the greater concept of the multiple use of land (Randall and Castle 1985; Barbier and Burgess 1997).

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[2]Clawson (1978, p.308) argues, "greater possibilities for innovative and imaginative management exist when both time and space are considered variable, than if an effort was made to manage every forest acre every year for multiple use." This argument suggests efficient resource management involves moving effort and focus across the forest area (space) over time.

2.2 Spatial Issues in Multiple-Use Forestry

Central to the notion of multiple-use forestry, and the debates surrounding the practice of it, are the issues of spatial scale and location.[3] In fact, the issues of spatial scale and location are well recognized in land-use planning for traditional timber management. However, now in British Columbia and other jurisdictions, demands for a greater range of goods and services from the forests are increasing. Consequently, spatial issues of scale and location are of great importance surrounding land-use planning, ownership and control of forests, forest management regulations, timber pricing policies and other forest policies that attempt to produce efficiently the mix of goods and services that satisfy societal demands.

Historically, traditional timber management has utilized two distinct land units for the purpose of regulating the cut of timber from a land area (Smith 1986, p.22). In forest management the basic land unit is a forest. A forest is a collection of stands that is administered as an integrated unit. The basic management objective from this land unit is a sustained, annual yield of products, typically commercial logs.

In the practice of silviculture the basic land unit is called a stand. The size of a stand is arbitrary, in that it depends on many subjective factors (Smith 1986, p.22). While forest management determines the annual cut from the entire forest, silvicultural principles govern the timing and manner in which individual stands are treated so that

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[3]Shaw (1985, p.193) discusses the issue of scale and location with respect to determining biological reserves to protect species. The suggestion of reserves implies that certain land areas be subject to a permanent land management focus over time. Bishop et al. (1995) challenge this presumption and make the claim that reserves may not be forever. If this is true, the question becomes, how long does a reserve remain under a particular management focus until it is reallocated? Spatial scale and location are also central to the ecological concept of island biogeography.

forest management goals are achieved (Smith 1986). Although the concepts of a stand and a forest are important for traditional timber management and silviculture, they are restrictive in their application to multiple-use forestry management and the associated silvicultural concepts.[4]

The traditional management concepts of a stand and a forest are less useful in the practice of multiple-use forest management. This is due, in great part, to multiple scale issues commonly involved in multiple-use forestry and the multiple dimensions of a standing forest. Traditionally, forest management and silviculture focused on the harvesting and regeneration of trees for wood. The scale issues involved in timber management relate to the ability to organize economically and plan the harvest schedule and the adoption of a silviculture regime to ensure proper regeneration. Due to differences in species, geography, value and available technology, the working size of forest and stand ensure there is no size that fits all. The lack of a standard stand and forest units for forest planning in multiple-use forestry is also certain. The capacity of forests to be managed for more products and services challenges the traditional concept of a forést.[5] In particular, the analysis is complicated as different forest growth processes and ecological functions occur at different spatial and temporal scales.

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[4]In Canada, where conscious practice of multiple-use forestry is relatively new, multiple-use forestry management is likened to gardening while the traditional forest model of converting natural old-growth forests to second-growth is likened to farming (or creation thereof) (Kryzanowski, 1999). Extending this analogy to the multiple-use debate leads to a set of questions such as these: When should farming be preferred to gardening? When should one manage for more than one crop? If gardening is best, what form should this gardening take? Should gardening involve only market goods or should it involve the provision of non-market goods? Should gardening and farming practices be conducted adjacent to each other? Why?

[5]The increased use of landscape management in Forestry indicates a departure from solely relying on the concepts of a forest and stand management, as a landscape need not solely include forest but also rock outcrops, streams, bogs and other nonforested land

As Samuel Dana (1943) notes, "a forest is not a continuous area of trees but is composed of meadows, bogs, marshes, non restocking burns, range lands, barren rock, streams and lakes." Consequently, a vast array of values is generated from a forest, including timber, forage, wildlife, watershed protection, climatic services, biodiversity, recreational opportunities and viewscapes. These values naturally vary in quality and quantity within and between forests due to climate, topography and soil. Standing forests are an inventory of timber and other products, a collection of habitats for various species, and an input into a broader geographical landscape. Standing forests are changing due to natural growth processes and due to natural impacts such as fire, wind, disease and pest, and due to man-made impacts. Standing forests also occupy space. These facts complicate the practice of multiple-use forestry and modeling of the problem.

The multiple-use problem is succinctly described by Dana (1943),

> What he (the forester) may not know is how to evaluate the various possible products and services of a given area fairly and intelligently from the point of view both of timber management, wildlife management, range management, watershed management, and recreation management that will result in the optimum production of different values. This is the nob of the problem of multiple-use.

The problem of evaluation is not solely a technical problem. The problem is also one of valuation, monetary or otherwise that stems from market failure. Unfortunately, the relative values, present and future, of few forest goods and services are known with certainty. This is due to the absence of markets or presence of poorly developed markets. The lack of formal markets means that timber prices do not reflect true opportunity costs. Further, the lack of formal markets, as well as the presence of public goods, means that

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areas. For example, see Wallin et al. (1994).

other formal and informal markets are required to direct the exploitation, protection and management of all forest resources. Without markets, how are values revealed to the persons responsible to manage the resources in a manner that is consistent with the current and future demands of society?

The quantity and quality of production of many forest goods and services is largely a function of the size, location and nearness to other land areas. Clearly different flora and fauna have different natural habitat ranges and, as such, the choice of management scale will not be identical to timber management. Aesthetic and recreational values will also be affected by the location and the size of the forestland in addition to its other features. It is also the case that management activities in one part of the forest affect the production or value of other amenities elsewhere in the forest.

An interesting aspect of forests is that most, if not all, the values generated from them are dependent on the physical forest itself. More precisely, the values generated from forests are dependent on the physical characteristics of the forest environment and its perceived quality. An environment consists of the whole complex of factors (soil, climate, and living things) that influence the form and the ability of a plant or animal or ecological community to survive. Thus the use of forests not only affects the form of the environment but can degrade its capacity to support various forms of life. In fact, many nontimber values in one environment are dependent on the state of or changes in adjacent forest environments such as salmon resources and animals such as the Vancouver Island marmot.[6] Therefore, all decisions regarding the use and nonuse of the forest resources

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[6]Personal correspondence with Susan Glenn, Dept. of Forest Science, Faculty of Forestry, University of British Columbia.

involve impacts on the forest environment that in turn impact on the values derived from forests.

There are two types of interdependencies between areas in a forest (or between forests). One type of interdependence is a pecuniary interdependence where the value of a good or service in one area of the forest is affected by the total availability of the good across the entire forest(s). This is pecuniary (monetary) interdependence as actions in one area of the forest affect the valuation of the physical production in another area. A type of pecuniary externality can exist if there is more than one independent land manager involved in the management of each area of the forest unit.[7] This form of externality does not lead to inefficient resource use but can lead to management decisions across a planning area that differ than when each forest area is considered independent of another; interdependencies are ignored perhaps due to ignorance. For a pecuniary interdependence to exist, the marginal value of the good in question needs to be non-constant. This may be the case when either 1) the good is unique or 2) there are no or few alternative sources of supply. For example, if harvest of trees for lumber at location *A* drives down the lumber price, the decision to harvest at location B is affected.

A second type of interdependency between areas in a forest is a technological interdependence. A technological interdependence is when a change in the condition in one area of the forest can affect the production capabilities in another location of the forest. There are several examples of technological externalities. One example is weed species (or pests or disease) infestation from adjacent forest areas surrounding a clear-cut. For example, often it is common in tropical forest systems to have hundreds of tree

[7]An externality exists if there are any benefits or costs derived from consumption or production that are not taken into consideration by the decision maker(s).

species of which only a few have commercial value (or ecological value). After a clear-cut many noncommercial tree species take hold quickly, which retards the regeneration of more valuable commercial species. Another example relates to the notion of metapopulations in ecology. Changes in population densities among various sub-populations of a species can lead to migration. Therefore, harvesting of animals or changes in environmental conditions in one area of a forest can affect relative population densities and lead to migration (changes in production) (Sanchirico and Wilen, 1999). Another example has to do with the impact that logging in one area of a forest has on the visual quality of another area of a forest in relation to the overall view of the landscape. Another example is when each stand involves an interaction with an off-site value such as timber harvesting and its impact on riparian values in an adjacent stream or downstream water quality. A final example involves species that utilize two different habitat types in two different geographical locations, such as with migratory birds. In each case, the presence of the technological interdependency suggests that management decisions will differ from those when areas are technologically independent. It also suggests that separate ownership of the forest areas can result in inefficient resource allocation, even when markets exist for all goods.

A final issue of space in multiple-use forestry relates to the natural stand heterogeneity of a a forest. Biogeographical factors, such as distance, species type, presence of water bodies or courses, and aesthetic factors such as colours and shapes, determine the physical and perceived differences within or between a forest(s). These factors are important in determining if a forest area complements, substitutes or is independent of other forest areas in providing a particular good or service.[8] The presence of asymmetries also suggest that harvesting impacts (or other physical impacts) in different areas of the forest will impact the same off-site value differently.[9]

Figure 2.1 illustrates three common spatial aspects of multiple-use forestry. Figure 1a depicts different cases of interdependence between stands within a forest. Figure 2.1a-i depicts a forest that consists of one stand and clearly demonstrates that no spatial relationship is involved with one stand. Figures 2.1a-ii and -iii demonstrate two possible spatial relationships between two stands. Figure 2.1a-ii illustrates an asymmetric relation where one stand is somehow affected by conditions on the other. This captures the classic externality problem of timber harvesting and its impact on riparian values in an adjacent area of the forest. On the other hand, Figure 2.1a-iii illustrates a symmetric relationship between two stands, as both stands are somehow affected by conditions on the other stand. This captures the case where a stand value is dependent on forest level conditions, for example a wildlife value. The remaining diagrams of Figures 2.1a illustrate different spatial interdependencies between three stands. Figure 2.1a-iv depicts a linear, symmetric interdependency between three stands while Figure 2.1a-v depicts a symmetric interdependence between each pair combination of the three-stand forest. Figure 2.1a-iv might represent the case where aesthetic values on a stand are dependent on the visual quality of adjacent stands in the forest. Figure 2.1a-v might represent a

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[8]Examples include forested parks, biological reserves and timberlands adjacent to manufacturing facilities. With respect to timber areas with location advantage see Ledyard and Moses (1976) and Gray et al. (1997a). Economic conditions or physical conditions can change exogenously, which can change location comparative advantage.

[9]For example, imagine a forest that varies in slope and soil stability. It stands to reason that a manager, who is concerned with on-site and off-site values, will classify areas in the forest (spatially define areas) and determine a level and system of harvesting on each area that 'maximizes' the total economic value.

wildlife value that is produced and consumed on each stand within the forest. Figures 2.1a-vi and 2.1a-vii each depict different forms of asymmetric relationships between stands. Figure 2.1a-vi represents the case where the value on the first and second stands depend symmetrically on each other while the value on the third stand is affected asymmetrically by the first stand. Finally, Figure 2.1a-vii depicts the case where the first stand has a stand value dependent on total forest conditions while the second and third stands have values that are only dependent on subsections of the forest.

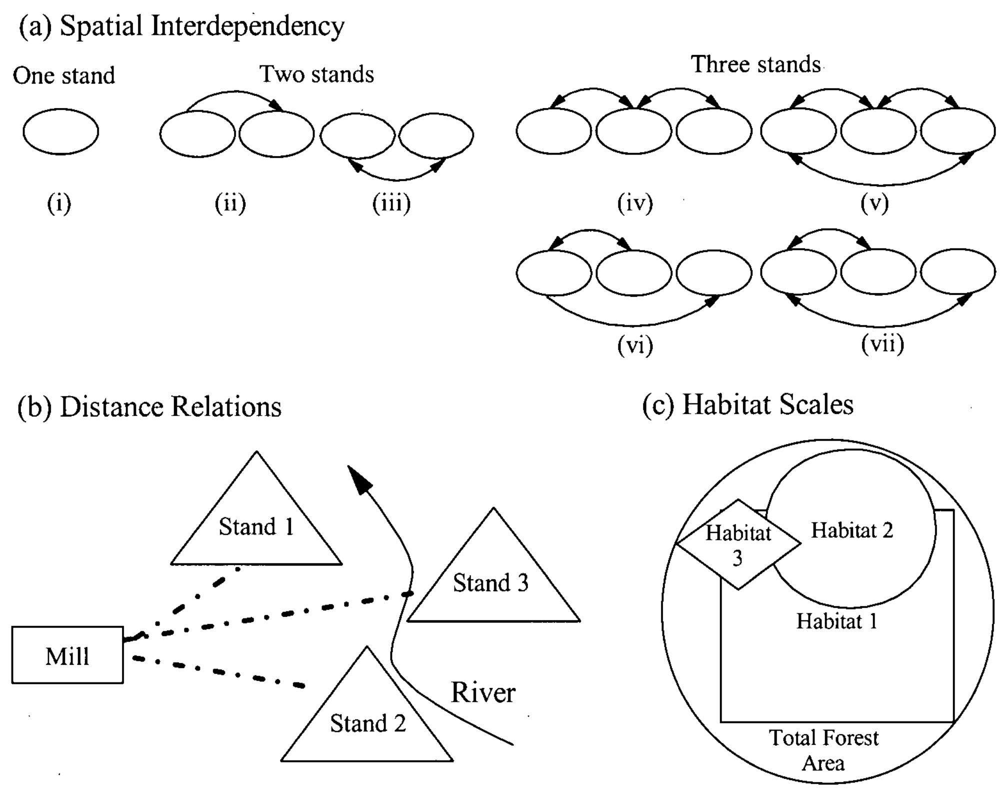

Figure 2.1 Spatial Aspects Relevant to Multiple-use Forest Management

Figure 2.1b demonstrates that there are many possible distance relationships that are important to consider. Such distance relationships include: distance of timber to a mill, distance of harvesting to sensitive areas or water courses and bodies, and distance between recreation opportunities and consumers. Finally, Figure 2.1c captures the issue of scale and its importance for habitat management and timber harvesting operations. In can be the case that habitats are overlapping and are of different scale, as is depicted in Figure 2.1c, which suggests that management effort will vary over the forest landscape.

2.3 Conclusions

There are many spatial issues in multiple-use forest management. The existence of unique areas, the presence of interdependence between areas in a forest, the allocation of existing demand and production, heterogeneous physical features of the land, differing habitat scales, and different ownership types imply that the management of the forest resources and land involves location and spatial scale questions. All these issues are important considerations for the day-to-day management and long-term planning and management of forest resources for multiple use that is efficient and economically sustainable. How are these spatial issues incorporated into current economic models of forestry? In the next Chapter, the current literature is reviewed.

CHAPTER 3

MULTIPLE-USE FORESTRY: A REVIEW OF EARLIER LITERATURE

In this chapter, a review of the multiple-use forestry literature is presented. The literature is organized by the criteria space and time. The literature review is focused on the theoretical literature with empirical and numerical studies covered in less detail. I review the static multi-product firm framework in Section 3.1. This part of the review includes single- and multiple-stand models. The following two sections present two different approaches for including time into a forest management model. In Section 3.2, the review extends to optimal rotation models, or even-aged models, of forest management. This section is organized by timber-only and multiple-use models and further by single- and multiple-stand models. In Section 3.3, uneven-aged models and discrete two-period models of forest management are reviewed. This section is again organized by the number of goods and by the number of stands. Section 3.4 summarizes the survey to clarify the problem and orients this investigation in relation to previous literature.

3.1 Static Models of Multiple-Use Forest Management

Single-Stand, Multi-Product Firm

Gregory (1955) was apparently the first to address the multiple-use forest management problem using an analytical framework of the multi-product firm. Gregory's use of a multi-product framework was not unlike Hopkin's (1954) application of the model to the economics of range management.[1] Subsequent uses of the multi-product framework include Pearse (1969), Walters (1977), Bowes and Krutilla (1982,1989), and Vincent and Binkley (1993). This framework assumes there is a forestland area of given attributes, such as volume of standing timber, age class distribution, site qualities, and nontimber attributes. A single stand or a homogeneous forest area is assumed in all works except those of Bowes and Krutilla (1982, 1989) and Vincent and Binkley (1993). Bowes and Krutilla implicitly extend the framework to involve more than one forest area while Vincent and Binkley (1993) explicitly treat two stands in their analysis. Each application of the model assumes competitive input and output markets and known prices, although these are not crucial assumptions they do simplify the models. The objective in all papers is to determine the level of outputs and inputs to maximize the value derived from the land.

Central to the multi-product framework is the notion of a joint production function. The production function is expressed as, $T(Q, X) = 0$, where Q represents a vector of outputs, and X represents a vector of inputs. This function expresses the maximum attainable level of an output, given a set of inputs, while maintaining feasible production of other outputs. Conversely, the function expresses the minimum amount of an input needed to produce a specific quantity of an output, given levels of other outputs and other inputs. A cost function, which represents the minimum costs to obtain a given level of output, also captures the technological relation between inputs and outputs expressed by the production function. Cost concepts, such as marginal and average costs,

[1]It is interesting to note, Hopkin (1954) was well aware of the spatial aspects of livestock management over multiple range areas, though his multiple-use model of range management was non spatial.

may be derived from the cost function that can further be used to explain the notions of economies of scope and scale. In the analysis, the production and cost functions are interchangeable given the assumptions stated above. The perspective that is most useful for analysis depends on the question addressed.

Technology is defined to be non joint if and only if the overall cost function can be expressed as the sum of independent cost functions for each product. If production is non joint, costs can be unambiguously be assigned to each type of output. Put another way, the marginal cost of a particular output is unaffected by changes in the production level of the other products. However, when production is joint, the cost function is not the sum of independent cost functions and costs cannot be uniquely assigned to each output. This simple economic fact has been the source of much confusion and debate in forestry.[2]

The idea of jointness is extended in the multiple-use forestry literature by defining the effect that one output has on the cost of producing an extra unit of another output. If, at a particular output level, an increase in the production of good *A* causes a decrease in the marginal cost of output *B*, then the goods are defined as local complements. If, on the other hand, a marginal increase in the production of good *A* causes an increase in the marginal cost of *B*, then the goods are local substitutes (or competitive products). If no change in marginal cost occurs then the goods are defined as locally independent.

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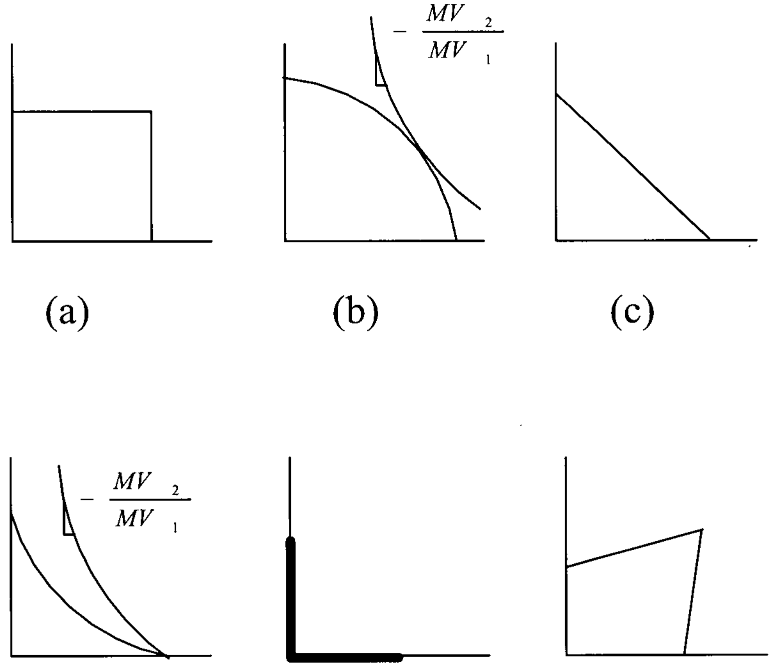
[2]Although there is no economic justification for allocating fixed costs between jointly produced goods, there are financial and budgeting reasons for doing so. See Schuster (1988) and Rideout and Wagner (1988) for discussions on allocating costs between goods in multiple-use forestry.

The degree of "competition" between outputs is reflected in the shape of the cost function and the production possibility frontier (PPF). Figure 3.1 depicts the degree of competitiveness between pairs of outputs with the use of the PPF.



(a) (b) (c)

(d) (e) (f)

Figure 3.1 Production Possibility Frontiers

Figure 3.1a represents two goods, that are independent of one another, as the level of production of one good does not affect the production of another good. A commonly used example, is the management of watershed quality and recreation. Figures 3.1b, 3.1c and 3.1d all represent pairs of goods that are competitive, as the increase in production of one good leads to a decrease in the production of the other good. The tradeoff between each good is decreasing in 3.1b, constant in 3.1c, and increasing in 3.1d. Figure 3.1e represents a pair of goods that reflect an all or nothing production choice set; two goods that are completely incompatible such as timber harvesting and wilderness, by definition of wilderness. Figure 3.1f presents a pair of goods, that are complementary, as an increase in one good leads to an increase in the production of the other good.

The depiction in Figure 3.1 is common in forest economics textbooks and central to studies using the one-stand framework (Gregory 1955; Pearse 1969, 1990). As noted by Gregory (1955) and Pearse (1969), if the PPF is concave to the origin (Figure 3.1d), the optimal management regime is to produce (manage) a single output from the resource. This conclusion derives from the result that maximum value occurs when the slope of the PPF is equated with the ratio of marginal values of the goods. Compare Figures 3.1b and 3.1d. In Figure 3b the slope of the PPF is equal to the ratio of marginal values, $-\frac{MV_2}{MV_1}$, at a combination of production that includes production of both goods, where MV_i is the marginal value of good i. However, when the PPF is convex to the origin, such as in Figure 3.1d, the tangency occurs at a point where only one good is produced. The general conclusion is that single-use management is economically superior to multiple-use management when goods are highly competitive. The weakness with the analysis is the focus on one stand, which obscures the spatial dimensions of the problem.

Bowes and Krutilla (1982, 1989), and Vincent and Binkley (1993), extend the framework to more stands. Bowes and Krutilla (1982, 1989) show that the concept of jointness and non jointness are global measures, as they refer to the cost function over all feasible levels of output. On the other hand, concepts of substitution, complementarity and independence are local measures of competition between forest goods, as they all refer to a specific level of output. Therefore, when considering the decision to focus management on single value or multiple values, we must consider the scale of production over the entire forest. Only if the local measurement concepts hold over all output levels are they identical to the global measurement of jointness. Bowes and Krutilla (1982) argue that the simple graphical analysis of Figure 3.1 is misleading as it does not consider expanding the scale of production to more forested land. If expansion to new areas are considered, the actual PPF may resemble Figure 3.1b not 3.1d. Implicit to this conclusion is the assumption that potentially competitive uses can be spatially separated to weaken the negative impacts between uses. This argument also implies that there exists more than one stand or forest area relevant to the area of analysis.

Multiple-Stand, Multi-Product Model

Extending the multi-product framework from one stand to two or more stands leads to more results. In these models, stands are not spatially defined; stands can be adjacent or located in very different areas within a broader landscape. The first extension involves a forest with stands of different forest production. Holding all things constant, two forest areas of different natural capacities to produce forest outputs have a different shaped PPF.[3] In Figure 3.2, stand *B* is relatively better than stand *A* for producing nontimber outputs. This is reflected in the shape of the production possibility frontiers. The points where the relative value of nontimber to timber is equal to the slope of the PPF for stand *A* and *B* are labeled X_A and X_B, respectively. For a given level of expenditure on both sites, more timber is produced from site *A* than *B*. The common conclusions drawn from this analysis are that forests of differing quality are likely to be managed differently and the degree of management specialization depends on the degree of site quality differences across the forest landscape. However, Bowes and Krutilla

[3]Forest level isocost curves, that reflect the production of different combinations of outputs at equal costs, can be used for the analysis. See Bowes and Krutilla (1989).

(1989) caution that, although stand differences favor specialization of management effort, the final determination of whether combined production or spatial specialization of substitute products is least costly depends on the degree of diseconomies of jointness of combined production and the diseconomies of scale of specialized production (Bowes and Krutilla 1989, p.69).

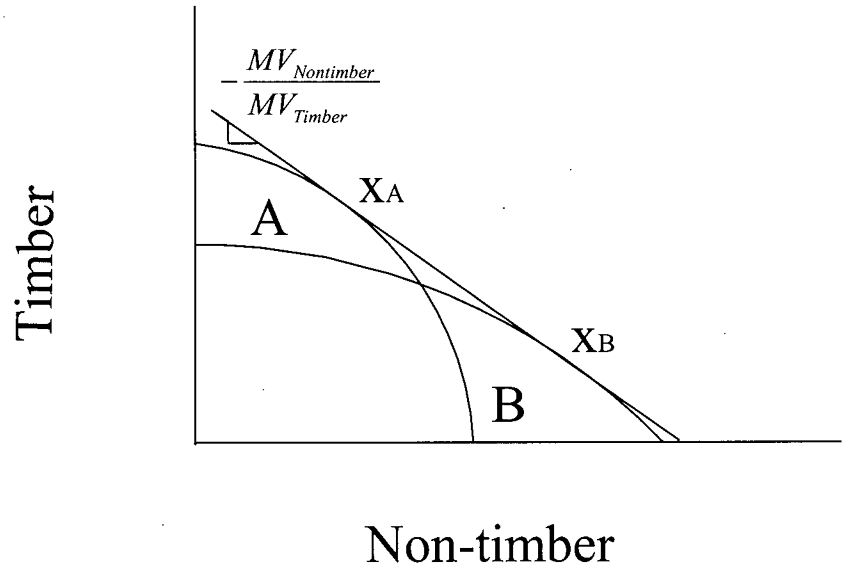


Figure 3.2 Specialization and Site Quality

Economies of jointness imply combined production of outputs on a hectare of forestland is less costly than single-use production on separated land units. Diseconomies of jointness, on the other hand, imply single-use production on each land area is cost minimizing. The difference between joint and separated production costs becomes more significant as competition between goods increases. However, even if diseconomies of jointness are present this is not always sufficient justification for specialized land use, because of diseconomies of scale. Economies of scale of a multi-product firm is not to be confused with economies of scope that is analogous to economies of jointness. Economies of scope refers to the benefits associated with the production of a greater set of goods from the same firm. Economies of scope has become a more general notion of multiple goods production than economies of jointness. Economies of jointness is a form of economies of scope and specifically relates to production cost benefits. Tirole (1988) offers a definition of economies of scope that is identical to joint production. New definitions of economies of scope expand the definition from solely production costs to include the benefits of marketing a mix of goods through one firm. These benefits, although not specifically defined, are associated with informational costs and transaction costs. See Clarke (1985) for a summary of some of the sources of economies of scope and see Dana (1993) and Iossa (1999) for contemporary arguments for informational economies of scope.

Economies of scale in the multi-product case is an extension of the single-product case. In the single-product case economies of scale refer to the proportional effect on the production of an output from a proportional change in inputs. A measure of returns to scale for the multi-product case is the proportionate increase in costs arising from a marginally small equi-proportionate increase in all outputs.[4] This is a local measure. If at a given Q a marginal equi-proportionate change in outputs leads to a less than (more, equal) proportionate change in costs, then there is said to be increasing (decreasing,

[4]Goetz (1992) provides the following formulation of multi-product economies of scale (MPSE) for 2 goods, MPSE= $1 - \sum_{i=1}^{2} \beta \ln C(Y) / \beta \ln Y_i$, where $C(Y)$ is total cost and Y_i is the production of good i. When MPSE>0 {<0} there are multi-product scale economies (diseconomies).

constant) returns to scale. As mentioned already, diseconomies of jointness are not sufficient for specialization. The opportunity cost of expanding production to twice the land area needs to be considered. Therefore, single-use production on separate land areas is economically optimal if the diseconomies of jointness exceed the diseconomies of scale.

Specialization of land may also be efficient when marginal values vary across different stands or forest areas across the forest landscape. This is easily shown with Figure 3. Imagine that the PPFs are identical for the two stands in Figure 3. Now consider that the timber value is different on each stand due to the transport and road costs to access the timber on each site. As timber prices are different on each site, the ratio of marginal values will be different at each site, ceteris paribus. As a consequence, the optimal production mix on each stand is different in general. However, if prices and quality (the PPFs) are different it is possible that production levels are identical on each stand.

Vincent and Binkley (1993) present a refined argument of economies of scope that favors specialized production across a forest landscape. The analysis involved the multi-product firm framework and two identical stands. The authors demonstrate, that due to differences in the responsiveness of the production of outputs to increased intensive management, it may be efficient to specialize. Helfand and Whitney (1994) indicate, in a note on Vincent and Binkley (1993), that the differences in responsiveness to management effort are due to diseconomies of scope in production. Varying responsiveness to management effort implies that by dividing a fixed unit of management effort unequally between two identical sites leads to changes in the optimal production on each management site. They then show that the output which management favors unambiguously increases while the "non favored" output change is ambiguous. However, they did show that given relative prices and specialization through management effort that total aggregated value from the forest unambiguously rises. The conclusions reached by Vincent and Binkley are strengthened when sites differ as the magnitude of change on the production possibility frontier is greater. This condition lends further support to the rationale for greater land use specialization, although they note the extent to specialization is constrained by the degree of diminishing returns to management effort. They further comment that the presence of externalities between stands may be such that again general multiple-use management may prove superior. The principle source of weakness in their analysis is the lack of reference as to where the stands are located relative to one another. A second weakness is that no explanation as to the source of the economies or diseconomies of scope is provided by Vincent and Binkley (1993), nor by Helfand and Whitney (1994).

Summary of Multi-Product Models

In summary, the multi-product framework determines four conditions that address the question of when to produce a single output. The four conditions are: 1) differences in site productivity, 2) diseconomies of jointness (scope), 3) economies of scale, and 4) variation of marginal values across the forest (Bowes and Krutilla, 1989). The first three are all cost related while the fourth is related to accessibility differences across sites in the forest landscape; net price of a resource varies with access and transportation costs. The multi-product framework is a useful model of multiple-use forestry. However, as it is a static framework it fails to capture the richness of the multiple-use problem, because a one-stand model obscures the issues of location and space and none account for changes in values due to time. The multiple-stand application of the multi-product framework addresses some elements of the spatial issues. The works of Vincent and Binkley (1993), and Bowes and Krutilla (1982, 1989), attempt to account for scale by expanding production to more than one stand. Unfortunately, the arguments involving (dis)economies of scale are obscure at best with regard to the location of the stands, although the framework permits different prices across the landscape that account for location of the stands. Lastly, the framework is a 'black box' approach as sources of economies of scope and scale, which are important to the arguments for and against land-use specialization, are not explained.

3.2 Temporal Models of Multiple-Use Forest Management

In this section, the economic models of forest management that incorporate time are reviewed. Before doing so, I review two basic biological growth functions that are central to the literature in Section 3.2.1. The functions are a sigmoid shaped yield function of an even-aged stand and a biomass growth function of an uneven-aged stand. The literature is grouped into two categories depending on the biologic growth functions assumed to better organize the literature Section 3.2.2 includes models focused on even-aged stand management. These models assume the sigmoid yield function and are commonly referred to as optimal-rotation models. Section 3.2.3 includes uneven-aged stand management models and two-period forest models. These models assume a biomass growth function. Each category of work is further organized by one-stand versus multiple-stand models, and by timber versus multiple-use models.

3.2.1 Biological Models of Stand Production

There are two basic timber management methods, even-aged and uneven-aged. Management methods can be classified as one or the other depending on the proportion of the trees removed during timber harvest and by the defined scale of the stand. The distinction between the two essentially rests on what we define as a stand and a forest. In a standard silvicultural text (Smith 1986), a stand is considered even-aged when the difference in age between the oldest and youngest trees does not exceed 20 percent of the length of rotation. However, the determination of the rotation age is not explained. An uneven-aged stand contains at least three age classes intermingled on the same area. Stands with two age classes are considered an intermediate category. An uneven-aged stand can be further categorized as regular and irregular. "Irregular uneven-aged [stands] do not contain sufficient age classes necessary to ensure that trees arrive at a rotation age at short intervals indefinitely" (Smith 1986, p.18). As stated earlier, the size and determination of a stand will depend on economics and biology. Therefore, the choice between each perspective of growth will also depend on the underlying biology, harvesting and silvicultural knowledge, and input and output prices, which in turn, are likely a function of ownership characteristics. Theoretically either approach is acceptable, however to better organize the literature models are categorized as even-aged or uneven-aged models

Even-Aged Stands

Some tree species do not regenerate if not provided sufficient light (shade intolerant) after disturbances such as harvest or fire. Often these species are best suited to even-aged management techniques such as clear cutting, where nearly all tree vegetation is removed from a stand at the same moment in time. Species that are shade tolerant and benefit from an overstory may be best managed with uneven-aged harvesting techniques. The choice between systems will depend on economic concerns (input cost and output prices) and physical factors such as climate, soil, tree (stand) growth, and risks of pests, disease and fire.

The underlying biology assumed in even-aged models is described by the familiar sigmoid yield function. Figure 3.3 illustrates the even-aged production function. Stand volume begins to increase very fast once the stand has established a significant root system. In later years, the increase in stand volume starts to slow. Eventually the annual growth of the stand approaches zero and begins to decline, as the decay of wood volume exceeds new growth. The growth process of the stand over its life results in the sigmoid shape. The model explicitly assumes that the growth of the stand is from existing members and not from recruitment (Berck 1976). Consequently, stand volume and growth is best described by the age of a stand.

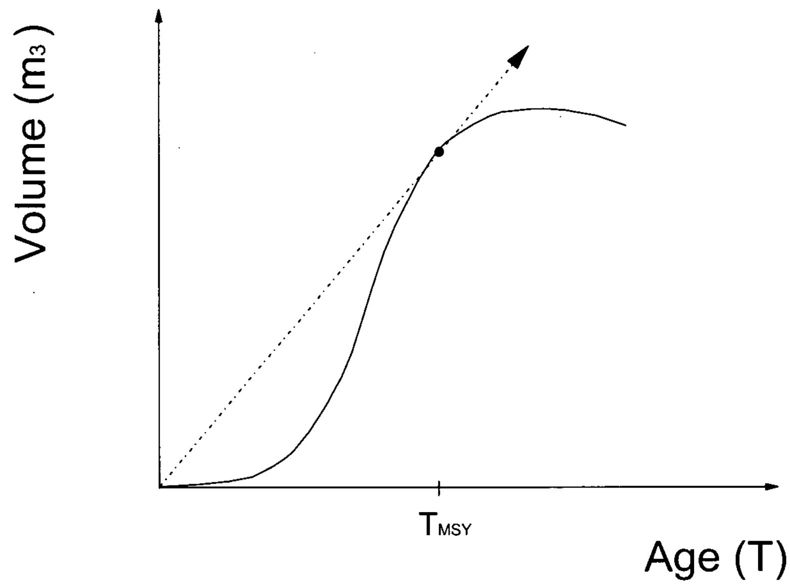


Figure 3.3 Logistic Timber Function

Uneven-Aged Stands

Uneven-aged management models describe tree growth as a function of the stock of biomass. The basic difference between this model and the even-aged model is that it incorporates the assumption that growth occurs from current members of the stand and from new recruits (Berck 1976). Therefore, the model represents a population of a species (or mix of species) of different ages. Commonly, this model is said to describe uneven-aged stand management that involves selection cutting systems within a defined stand. However, this is not necessarily the case. The claim rests on the scale of the stand and the underlying biology of the species. As noted already, the definition of a stand is very subjective and depends on related biological, technological and economic factors. Therefore, it is possible that a feasible stand scale can incorporate all types of silvicultural technologies, including clear cutting, thinning and selection cuts, and be well represented by the biomass growth function (Ovaskainen 1992). A common form of the biomass growth function is depicted in Figure 3.4.

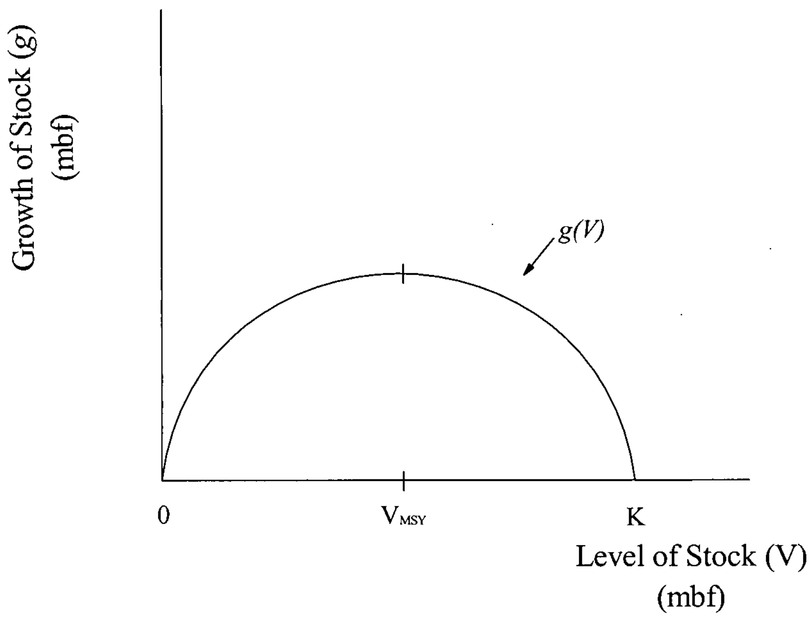


Figure 3.4 Biomass Growth Function

The vertex of Figure 3.4 corresponds to the maximum volume that can be harvested from the stand per period in perpetuity. The stock that corresponds to the maximum sustained yield is marked as V_{MSY}. This stock level has the greatest growth in biomass from one point in time to another implying that marginal growth is zero, $g'(V)=0$. At this point, the increase in biomass from recruitment and growth of immature trees is equal to the loss of biomass from the removal of older trees. All stock levels less than the V_{MSY} have lower growth but have higher marginal growth, $g'(V)>0$. Higher marginal growth occurs as growth of young trees is greater than the loss of growth from the harvest of the older trees. For stock levels greater than V_{MSY} the marginal growth is negative, $g'(V)<0$. Negative rates of growth occur as the trees are permitted to reach ages greater than the maximum sustained yield age in Figure 3.4. Therefore, growth decreases as less recruitment occurs and younger tree growth is hampered by the presence of older and larger trees.

One form of the biomass growth function is to implicitly incorporate the assumptions of the sigmoid yield function in Figure 3.3. The biomass growth function extends the concepts underlying the sigmoid function to be a collection of individual or groups of trees of a particular age. Two different approaches to modeling the problem of timber management in economics have been developed to reflect the two management strategies.

3.2.2 Even-Aged Management Models

Single Timber Stand

Timber resources have been recognized as a typical example of the point-input, point-output class of investment problems in capital theory. The basic problem is to determine the timing of harvest that maximizes some measure of net return. This model has been the basis of the economics of timber management and assumes an even-aged stand management strategy where all like trees in a definable geographical space are harvested at one point in time (clear cut). It is also assumed that forestland is used for successive timber crops in perpetuity.

There have been many formulations of the problem described above. It is now recognized by forest and mainstream economists that the correct formulation of the problem is the so-called Faustmann formulation, named after the German mathematician

Martin Faustmann.[5] However, Scorgie and Kennedy (1996) find that the first correct formulation was not Faustmann (1849) but by a British agriculturist William Marshall in 1790. Lofgren (1983) credits Max Robert Pressler (1860) and Bertil Ohlin (1921) as the first to correctly express the optimality conditions of the Marshall-Faustmann problem.

The basic model assumes a timber owner chooses the harvest age of a stand to maximize returns to the fixed factor of production, land. The value is known as the bare land value or soil expectation (SE). The stumpage price and discount rate are determined by market equilibrium and are taken as given, as are costs. The model is static in the sense that prices and the discount rate are assumed constant over time. So, if the best use of the land now is for timber production, it will also be timber production in the future and the optimal rotation will be constant through time. The problem is to

$$\max \pi(T) = \frac{pV(T)e^{-rT} - C}{1 - e^{-rT}} \tag{3.1}$$

where T is the harvest age, p is the stumpage price, C is a fixed cost incurred at the beginning of each rotation, $V(T)$ is the volume yield function for a stand of trees (volume of timber per unit area), and r is a real discount rate. The necessary condition for the optimal solution is

$$PV'(T) = rPV(T) + r\pi^\star \tag{3.2}$$

given that $V'' < 0$, the sufficient condition for a maximum. Equation 3.2 indicates, that the owner postpones harvest until the value of the incremental growth of the stand (LHS) is greater than the opportunity cost of the timber and the land (RHS). The opportunity cost of the timber is the foregone interest earnings on the income from current harvest,

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[5]See Bentley and Teeguarden (1965) and Samuelson (1976) for a discussion of the Faustmann formulation and alternative formulations.

rPV(T), while the opportunity cost of the land is the foregone interest earnings on the value of the bare land (successive use of the land for timber), $r\pi\ (T)$. The inclusion of rent, opportunity cost of land, is an important determinant of the optimal rotation and differentiates the Faustmann model from the one rotation or Fisher model.[6]

The model can be used to predict changes in the optimal rotation when the exogenous parameters are changed. A one-time increase in the stumpage price or discount rate leads to a decrease in the optimal rotation. An increase in the fixed cost, *C*, increases the rotation length as owners reduce the present value cost of *C*. The basic model has been used to predict the impact of policy instruments such as taxes and regulation and can be extended to include management effort (Montgomery and Adams 1995).

Single Multiple-Use Stand

The even-aged management approach to managing a single stand for multiple uses is a direct extension of Faustmann (1849). The problem of selecting the optimum harvest of a stand of trees that provides timber and nontimber services was first formulated by Hartman (1976) and then soon after by Nguyen (1979) and Strang (1983). Hartman is a direct extension of Faustmann, while Nguyen incorporates a minimum stocking level while maintaining the spirit of the Faustmann calculus. The economic multiple-use management problem reduces to finding the rotation time in years that maximizes net

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[6]The Fisher model considers the problem of when to cut one crop of trees Samuelson (1976). This problem is identical to the textbook wine storage problem. The optimal cutting rule is identical to Equation 3.2 with the omission of the last term on the RHS. The current single even-aged stand is cut when the growth in volume is equal to the discount rate.

present value through successive cutting and through the flow of nontimber services from the standing forest.

Hartman (1976) assumes that the flow of net benefits from the nontimber services can be expressed as a function of stand age. Hartman (1976) assumes the value of the nontimber services flowing from a standing forest of age t is denoted as $E(t)$ and that $E'(t) > 0$, $E''(t) < 0$. The stumpage value of timber in a forest of age t is $F(t)$. It is further assumed that the timber stand is of even age and the site begins in a state of bare ground and remains in forest use forever. There are no planting costs or other outlays other than harvesting costs which are reflected in the function $F(t)$ and are assumed constant through time. All future receipts are discounted to present dollars at a known and invariable competitive real discount rate r.

The problem for the stand manager is

$$\max_{T} \pi = \frac{F(t)e^{-rT} + \int_0^T E(x)e^{-rx}dx}{1 - e^{-rt}} \tag{3.3}$$

where T is the rotation age.[7] The first-order condition (FOC) necessary for a optimum value of $\pi^\star$ is

$$F'(T) + E(T) = rF(T) + r\pi^\star \tag{3.4}$$

The $T^\star$ that solves Equation 3.4 is when the increase in value from a marginal delay in the harvest date equals the opportunity cost of the delay. As $E(t)$ can take on many possible shapes there can be many solutions to the FOC representing maxima or minima. Further, it is possible that there may not exist a $T^\star$ that equates equation 3.4, implying that the optimum decision is never to cut the standing forest. The general conclusion of this paper

[7]Nautiyal and Fowler (1980) consider the optimal rotation with price of timber as a function of stand volume, $P = P(V(t))$.

is that the inclusion of amenities into the formulation can lead to a rotation shorter, longer, or identical to the Faustmann rotation (Hartman, 1976). The difference between the Hartman and the Faustmann rotations will depend on the nontimber function. If the nontimber value from a particular output is such that it outweighs all other values it will determine the optimal rotation age.

Bowes and Krutilla (1985) and Strang (1983) point out an important consideration in the Hartman analysis. If there are a variety of services present there is no a priori reason to expect that the total benefit function is monotonically increasing or decreasing with stand age. This suggests that there can be various local maxima and minima and marginal analysis alone is not sufficient to ensure resource efficiency. The policy implication is that tax or subsidy devices may not achieve efficient resource allocation (Swallow et al. 1990).

Bowes and Krutilla (1985) also point out that, unlike in the Faustmann model, starting inventory in the Hartman framework is important in the determination of the rotation age. If the starting inventory is greater than the Faustmann rotation it may be optimal to never harvest (Bowes and Krutilla 1985). This result is contrary to Nguyen (1979) who finds that inclusion of a minimum stocking level ensures that the optimal rotation is always equal or less than the biological rotation (MSY).[8] The inclusion of a minimum stocking level in Nguyen (1979) mitigates the temporary scarcity of nontimber values after clear-cut harvest. Further, Snyder and Bhattacharyya (1990) show that

[8]The biological rotation is the rotation age that maximizes the annual flow of timber volume from a given forest area in perpetuity. This rotation is commonly referred to as the maximum sustained yield rotation (MSY) or technical or physical rotation.

inclusion of annual and periodic nontimber management costs changes the optimal Hartman rotation.

Comparative statics results of the Hartman analysis are not as straightforward as with the Faustmann analysis. An increase in stumpage price leads to a decrease (increase) in the optimal rotation if the Hartman rotation is more than (less than) the Faustmann rotation. A proportionate increase (decrease) in $E(x)$ for each x leads to a longer (shorter) rotation if the Hartman rotation is longer (shorter) than the timber-only rotation. Disproportionate increases in $E(x)$ for each x or proportionate changes in stumpage price and $E(x)$ and changes in the discount rate are ambiguous (Bowes and Krutilla 1985, p.540).

Empirical work by Calish et al (1978) found that the inclusion of nontimber values do change the optimal rotation.[9] They find that the optimal joint rotation can vary greatly from the timber only rotation when nontimber values are high. They also find that nontimber values contribute a large proportion to total stand value. In their study they find that very high values need to be considered before the solution approaches the biological rotation. Englin and Klan (1990) extend the Calish study to investigate the impact of various taxes on the Faustmann rotation and resulting mix of nontimber values provided. They also extend the Calish study by including a different stand type in the

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[9]Calish et al. (1978) note that many of the stand-level production functions they use (non-game wildlife, deer, elk) only are meaningful in the context of a regulated forest as the nontimber productivity of a particular age-class depends on the whole mix of age classes. For example, elk need older (large) stands for cover to survive. If stands in a regulated forest where cut on a 5 year rotation the forest could not sustain an elk population unless there was cover provided in a nearby forest area. This type of stand interdependence was modeled by Bowes and Krutilla (1985) and is reviewed below.

analysis. They conclude that taxes can have a marked impact on the private provision of various non-marketed and public goods.

Multiple Timber Stands

Extending the Faustmann model to multiple stands or a forest leads to a very simple optimal management strategy. The optimal strategy for an owner whose sole objective is to maximize the net present value from timber when costs and prices are fixed is to cut a stand when it reaches the optimal rotation age. Therefore, if a forest is composed of n identical stands of equal age, the entire forest is cut down at the same time. This strategy is optimal when there are no constraints or interdependencies among the stands. From the point of view of timber supply, the supply of timber depends on the initial age composition of the forest. A forest of uniform age will generate a periodic supply of timber over time. The so-called normal forest in the forestry literature, where there are n-age classes of equal size and the oldest age class is equal to the optimal rotation so that stands of equal size ranging from ages of 1 years to the optimal rotation age N remain, only occurs by chance, except for plantation forests. Thus an even flow timber supply over time is highly unlikely. Changes in prices, costs or the discount rate lead to the same behavior at the forest level as at the stand level. Consequently, an unexpected price increase leads to a shorter rotation and more stands are chosen to be cut, which leads to an increase in short-term timber supply. An increase in the discount rate leads to the same behavior and impact on current harvest from the forest. In this model of forest management, location is not important to the model. Stands are cut when they reach a particular age and where this occurs is not important.

If we drop the assumption that stands are not independent, or include new constraints or transportation cost, the results of the model change. Sometimes the stumpage price received is a function of volume harvested. This is the consequence of a downward-sloping price function for logs or an increasing harvest cost function (see Nautiyal and Fowler 1980). In either case, the stumpage function is no longer constant but varies with total volume of wood harvested in any period of time. Further, stand-level harvest decisions are dependent on harvest decisions on other stands, as the harvest on one stand affects the stumpage price on other stands. The result is that harvests are smoothed out over time (Montgomery and Adams 1995).

Ledyard and Moses (1976) present an optimal forest land-use model with transportation costs. The model is built on the Faustmann calculus. The owner of land must choose a rotation age, T, and a level of management effort, Q, so as to maximize the present discounted value of profits. In making these decisions the owner must also consider the distance between his land (log supply) and the mill (log demand). The principle conclusion of Ledyard and Moses is that the further away the land is from the mill, the less management is employed and longer the rotation. It follows from this result that the supply per land area increases with distance from the mill, if and only if optimal economic rotation is less than the MSY rotation. It also follows that land units sufficiently far from the mill with negative land rent will not be used for timber production and will remain in their natural state. Gray et al. (1997) reach similar conclusions to Ledyard and Moses when road development and maintenance costs are included in the calculus. Gray et al. argue that infrastructure costs (building and maintaining road networks) increase non linearly with distance from the mill. This implies more intensive timber management close to a mill.[10]

Interdependent, Multiple-Use Stands

The Faustmann calculus has been extended to a multiple stand level by Bowes and Krutilla (1985), Paredes and Brodie (1989), Swallow and Wear (1993), Swallow et al. (1997), and Rose (1999).

Bowes and Krutilla (1985) build on the work of Hartman by extending the Faustmann calculus to a forest level. Here stand nontimber values are interrelated and thus affect forest level planning. They focus on finding the optimal scheduling of harvests on all stands in the forest planning unit. The planning unit is made up of many stands of varying ages. Total value includes the value from timber stumpage sales and from the services of the land and its stock of vegetation. These values are assumed to be known and constant over time, as are the productivity of the sites. The nontimber values relate to the overall conditions of the forest unit, which is described by the mix of ages across the set of timber stands. Therefore, changing the proportion of the forest hectares held in each class alters the flow of nontimber values. This further implies that there is a nonlinear dependence of multiple-use values on the mix of stand ages unlike the single-stand analysis of Hartman (1976).

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[10]The conclusions of these models are intuitive, and they can only explain part of the picture with regard to the spatial allocation of resource management. Central to these papers is the location of a mill. Why was the mill built where it is? Is a mill located in a particular area due to proximity to existing transport infrastructure, productive timber land, energy sources, labor supply? These questions are central to the fields of land economics, regional economics and development, and urban and rural geography.

The Bowes and Krutilla (1985) problem involves selecting a sequence of harvests and stocks over time so as to maximize the net present value from all current and future flows of harvest and nontimber services. This is done sequentially. In each period, the manager chooses the pattern of harvests across the forest that maximizes the net flow of value plus the appreciation in the asset value of the land unit and its stock. For each age class, the landowner selects a treatment (harvest or growth) that provides the highest marginal return. The assumed management strategy is even-aged management. Under an optimal treatment schedule, the rate of return from a marginal unit in stock is equal to the return available on alternative market investments.

The Bowes and Krutilla (1985) approach to modeling the problem differs from the Hartman model. Unlike Hartman (1976), timber price and nontimber flow values are endogenously determined and depend upon the selected harvest pattern from the whole area.[11] This is a consequence of the marginal values being dependent on the stock of standing timber in each age class, i.e., the mix of age class proportions. The principle finding is that land values do not need to be constant from one rotation to another, as conditions change within the whole forest. This suggests that the rotation length on each stand can differ through time. Although forest stands are interdependent, the model does not involve spatial harvest decisions.

Paredes and Brodie (1989) take a slightly different approach to the forest-level, multiple-use problem than Bowes and Krutilla (1985). They model a multiple-stand timber problem with imposed nontimber constraints. The model explicitly accounts for

[11] The timber harvest value is represented by the function $F(H)$ which represents a willingness-to-pay based on the demand for timber. It is assumed to be constant over time. The marginal value of $F(H)$ is the net price at time t that would be offered for timber in a competitive market.

each stand in the forest and implicitly for the spatial location of the stands within the forest. They show that if all information is known and included in the calculus, stand-level decisions will result in overall forest level optimization. However, the model is not well suited to comparative statics analysis and so provides few conclusions or insights.

A series of papers, beginning with Swallow and Wear (1993), and followed by Swallow et al. (1997) and Rose (1999), narrow the multiple-stand, multiple-use problem to two adjacent stands. The general model extends to any number of stands but the actual numerical and analytical analysis in all three papers is restricted to two stands. The first paper by Swallow and Wear (1993) introduces the basic notion of stand interdependence. The authors explain the hypothetical situation of a stand owner who manages her stand for timber and forage values. The owner is aware of the exogenous production of forage on an adjacent stand. Swallow et al. (1997) extend the situation to one landowner who manages both stands for timber and forage, while Rose (1999) adds a fixed harvesting cost in the analysis. These papers extend the single-stand, Hartman-type model to account for spatial interdependence between stands. Swallow and Wear (1993) claim that their model is complementary to Bowes and Krutilla (1985) as they also recognize that unharvested portions of the total forest may still impact a harvest decision. However, Swallow and Wear (1993) correctly observe that Bowes and Krutilla's (1985) treatment of the problem does not specify which stands in the forest of age class *j* should be subject to treatment *i* at time *t* (i.e., it is void of spatial considerations).

Swallow and Wear (1993) argue that the interaction of benefits between adjacent stands may affect optimal management decisions. The approach the authors take to illustrate their thesis is loosely analogous to an industry with many firms. The external interactions among firms are evaluated from the perspective of a single one. In the forestry context, the forest is the industry and the individual stands are firms.

The analysis builds on Hartman's (1976) simple model. The model incorporates into the Hartman formulation a measure of the conditions of nearby stands, while retaining a focal stand. Their analysis starts with the focal stand at bare ground. The problem is to maximize the net present value of timber and nontimber values through an infinite planning horizon. The problem involves two stands, the focal stand and a single neighboring stand. The owner of the neighboring stand ignores the conditions on the focal stand while the focal stand owner is aware of what happens on the neighboring stand. It is assumed that the owner of each stand manages her stand for timber plus any contribution that nontimber output makes to their own utility. The model is essentially a sequential decision process that is represented by a dynamic program linking the sequential optimization problems through the impacts on the neighboring stand. Taking each rotation in succession, the manager attempts to optimize the contributions from the current rotation plus the benefits from the future rotations.

The nontimber benefits drive the results. For example, let the nontimber benefits from the focal stand and the nontimber benefits on the neighboring stand be substitutes. If benefits rise with stand age (e.g., aesthetics), a harvest event on the neighboring stand causes an upward shift in the flow of nontimber benefits on the focal stand. Specifically, harvest decreases the supply of nontimber services on the neighboring stand, which causes the marginal value of amenities on the focal stand to rise. This event increases the opportunity cost of harvesting on the focal stand causing a delay in harvesting. The net effect is to lengthen the current rotation age. Thus there is an effect on harvest timing and timber and nontimber supply from the focal stand. In conclusion, shifts in the nontimber benefits quantify the effects of nearby harvesting by changing the relative scarcity of nontimber services in the geographic areas surrounding the focal stand. In effect, harvesting a nearby stand has an impact on the value of the focal stand.

The weakness of Swallow and Wear (1993) is that only decisions are made on the focal stand and thus the model remains non spatial.

Swallow et al. (1997) extend Swallow and Wear (1993) to the case of two interdependent stands controlled by one decision maker. Thus, the externality problem that arises in Swallow and Wear (1993) is internalized. This formulation of the problem leads to similar solutions to Swallow and Wear (1993). However, now the model prescribes the optimal form of action on each stand so that the "forest" value is maximized. In general, they find from their simulation results that the timing of harvest on each stand tends to be staggered over time and that rotation lengths are non constant. The limitation of this model is that it addresses the spatial interdependence between only two adjacent stands. In fact, the Swallow et al. (1997) case study does not correspond to their original hypothetical example of three stands and fails to capture the richness of spatial problems associated with more than 2 stands and 2 goods.[12]

Rose (1999) extends the two-stand forage-timber model to include fixed harvesting costs. Rose recreates the simulations of Swallow et al. (1997) with the

[12]Their hypothetical case is when a manager is producing deer and timber products. Deer need both forage area and a calving or shelter area. If for some reason an ecological barrier such as an impassable river or timber harvesting separates these two habitats, the ecosystem becomes less supportive of the deer population. Thus the spatial arrangement of these three stands (or land uses) affects benefits received by people who value wildlife for viewing, hunting or existence.

inclusion of fixed harvesting, finding that, if the fixed harvesting costs become very large, specialization vanishes and the two stands are treated identically. This is a specific example of the effect of scale economies; as fixed costs rise the benefits from specialization become less important than the benefits from a greater scale of timber production. This finding raises the question: under what economic circumstances is specialization of production favored over integrated production of joint goods?

Another recent model to address the multiple-use problem is that of Tahvonen and Salo (1999). These authors take the approach of a combining utility maximization with the optimal rotation framework of Faustmann. This approach, they argue, is consistent with the non-industrial private timber owner who has other means of income and values her forest for in-situ nontimber values. This approach leads to non-constant rotation ages due to linkages between stands via the budget constraint (wealth) and in-situ values. The model is extended to multiple stands in the fashion of Bowes and Krutilla (1985). The important conclusions from this work are that private land owners may not harvest indefinitely and that the inclusion of an in-situ nontimber can lead to forests with increasing heterogeneity of age-class structures, and thus nonconstant flow of timber to mills.

Summary of Even-Aged Models

The even-aged models, as applied to the multiple-use forestry problem, have generated many interesting theoretical results. The single-stand model of Hartman (1976) suggests that the inclusion of nontimber values, which are age-dependent, can alter the optimal rotation. The multiple-stand models, such as the forest level model of Bowes and

Krutilla (1985) and the two-stand model of Swallow and Wear (1983), further suggest that the optimal rotation on a given stand need not be constant over time. This last conclusion suggests simple stand-level rules of thumb are inadequate when nontimber values depend on the overall conditions of the forest. However, this is no longer the case if interdependencies are included in making the stand harvest decision (Paredes and Brodie 1989).

The shortcomings of the even-aged models are, first, that even-aged management practices such as clear cutting are assumed, and second, that interdependencies between stands are vague and thus the spatial dimensions of the multiple-use problem are not fully captured. For example, the forest level models of Bowes and Krutilla (1985), or Tahvonen and Salo (1999), consider the problem from the perspective of managing age-class distributions within a forest. Such a modeling simplification fails to offer any insight into spatial decisions; which stands in an age-class are harvested and which are not? Likewise, the two-stand adjacency models, starting with Swallow and Wear (1993), are too narrow in their focus. By simplifying the problem to only two stands again evades the richness of spatial decisions prevalent in multiple-use forestry. It seems that at least three stands are required to begin modeling the possible variety of situations that are faced in multiple-use management.

3.2.3 Uneven-Aged Forest Management and Two-Period Models

Single Timber Stand

In this section we review models that relax the assumption of even-aged management. If we assume uneven-aged management, then the timber management problem is analogous to an investment-disinvestment problem similar to other renewable resource problems such as fisheries and game animals. However, as property rights over forest resources are usually well defined, there is no open access problem as with fisheries and wild game management. The assumed objective of uneven-aged management is to choose the volume of timber to remove each period to maximize the present value of future net revenue. The choice is constrained by the initial stock of trees and the growth function. .

The problem can be caste in continuous or discrete time. The continuous time problem is solved using optimal control while the discrete time model is commonly caste in a two-period framework.

The simplest version of the continuous time model is to start with a homogeneous stock of trees, X, which grows according to a simple growth function $g(V)$. We assume that the growth rate is strictly decreasing in the level of growing stock. This function is represented in Figure 3.4. A natural equilibrium of the population stock occurs when growth equals mortality, V_{max}. The maximum sustained yield of harvest from the population occurs at V_{MSY}; the greatest attainable harvest from the population such that the stock maintains a constant level (growth of biomass equals mortality and harvest). The mathematical properties of the growth function are $g' > 0$ for $0 < V \leq V_{MSY}$ and $g' < 0$ for $V > V_{MSY}$, with $g'' < 0 \ \forall V$.

Montgomery and Adams (1985) review the formulation of the uneven-aged timber management problem as an optimal control problem. The owner must choose a volume of harvest in each period to maximize the present value of all future earnings. This decision is constrained by the initial stock and by the growth function. The optimal decision rule for the owner is to hold a stock of trees as long as the long-run rate of return on the additional stock (g_s) exceeds the rate of return on competing assets (discount rate minus any real price increase in stumpage prices). If the stumpage price is constant, then the owner will hold a level of stock where the marginal return is equal to the discount rate. If the price is rising at a rate equal to the discount rate, the optimal management strategy is to maintain a stock consistent with the maximum sustained yield (biological maximum). The interpretation of these results are like those of the even-aged stand. The owner delays the timber harvest as long as the rate of return on timber investments exceeds the rate of return on alternative assets.

Unlike the Faustmann model, only changes in the rate of price growth and discount rate will affect optimal stocking decisions. Higher rates of price growth yield higher levels of stock as the opportunity cost of holding the stock fall. Lower discount rates also increase the optimal stock. Changes in the initial stock do not affect the steady state conditions but will affect the approach to the steady state. This is intuitive but can not be shown with this "bang-bang" formulation of the problem as the optimal stock is solved for instantaneously.

The basic optimal control problem can be extended to heterogeneous stands and nontimber values. Adams and Ek (1974) and Haight et al. (1985) extend the uneven-aged formulation to account for a heterogeneous stand (distribution of trees of different sizes and/or species). The model can also be extended to include nontimber values by assuming that these vary with the level of stock. The inclusion of nontimber values leads to an increase in the steady state stock if nontimber values increase at a decreasing rate with increasing stock (Montgomery and Adams 1995).

Two-Period Models

The basic results derived from the optimal control problem above can be derived from a simple two-period model of timber harvesting (Ovaskainen 1992). The advantage of a two-period framework is that it generates short-term and long-term results. This is unlike steady-state frameworks that do not account for path dynamics. The framework is also very flexible and easily adaptable to fit new problems.

The two-period framework has been extensively utilized in the forestry literature to examine timber harvesting behavior (Ovaskainen 1992; Binkley 1981; Max and Lehman 1988; Koskela and Ollikainen 1999) and public policy issues such as taxation (Ovaskainen 1992; Amacher and Brazee 1997; Amacher 1999; Koskela and Ollikainen 1997) and harvest regulation (Binkley 1980). The two-period framework is well suited to include nontimber values, management effort and multiple stands into the calculus.

The bulk of the work that utilizes the two-period framework is concerned with harvesting behavior, in particular, harvesting behavior on private lands. This approach is essentially a forestry application of household production theory.[13] As such, the basic ingredients for these problems are a utility function, a budget constraint, initial endowments of wealth, and a forest production function. Persons receive utility from consumption, which is financed from initial wealth, nontimber income and timber income. The problem for the owner is to maximize utility from consumption over time subject to the intertemporal budget constraint. That is the owner decides how much to consume (harvest) and save (inventory) in each period. A decision to consume involves

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[13]Binkley (1981) was likely the first to apply household production theory to timber harvesting behavior.

spending money income and harvesting timber. The remaining stock of timber after each harvest grows according to a known biomass growth function.

This basic framework has been used to analyze changes in harvesting and management decisions from changes in the discount rate, harvesting costs, stumpage prices, the imposition of harvest and forest taxes, and harvesting regulations.

Max and Lehman (1988), Ovaskainen (1992), Koskela and Ollikainen (1997), Koskela and Ollikainen (1999) extend the basic framework to include nontimber values. All of these works have focused on one stand except for Koskela and Ollikainen (1999).

Multiple-Use One Stand

The basic two-period timber-inventory model is Ovaskainen (1992). Ovaskainen considers the problem where a private landowner must consider how much to cut from an endowment of trees in each period so as to maximize utility subject to her budget constraint. The owner derives utility from consumption goods and from nontimber benefits from the standing forest. Nontimber benefits are assumed to be monotonically increasing in inventory.

The basic harvesting rule when only timber is considered is

$$\frac{p^2}{p^1}(1+g_v)=(1+r) \tag{3.5}$$

where p^1 and p^2 are first- and second-period timber prices, r is the real rate of discount and g_v is the change in growth from a change in standing inventory.[14] Equation 3.5 indicates that the owner harvests the initial endowment of timber until the marginal

[14]Ovaskainen (1992) only considered management effort in the timber problem.

incremental growth in value is equated with the incremental growth in value from alternative capital investments.

When both timber and nontimber values are considered, the harvesting rule is

$$\frac{p^2}{p^1}[1+g_v^L]=(1+r)-\frac{v_v(V_{L1})}{p^1\beta u_c(c_s)} \tag{3.6}$$

where u_c is the marginal utility from consumption, v_v is the marginal utility from nontimber forest benefits, and β is a discount factor ($\beta = (1+\rho)^{-1}$ where ρ is a subjective rate of time preference). The harvest rule now incorporates the foregone marginal value derived from the standing forest in terms of timber values.

Comparative statics results are obtained from this model. An increase in first-period price, discount rate or initial inventory results in an ambiguous change in harvest while an increase in second period price or exogenous initial income leads to a decrease in first-period harvest.

Multiple-Use Two Stands

Koskela and Ollikainen (1999) extend the two-period utility maximization model to two stands. One stand is assumed to be private and the second stand is assumed to be public land. The authors derive the optimal public harvest when nontimber benefits are public goods, while nontimber benefits on private lands are public or private goods. The authors derive different optimal harvesting rules under different assumptions of the interdependence between the two stands in providing nontimber benefits. They assume that the stands can be complements, independent, or substitutes in the provision of the nontimber good.

3.3 Summary and Conclusions

Current literature is summarized in Table 3.1. The literature is categorized as to the modeling framework and then ranked by its application to multiple-use forestry, inclusion of time, and application of spatial issues. Current models of multiple-use management do not successfully capture the spatial aspects of forestry. The recent optimal rotation models do offer interesting insights when non-convexities exist, but this framework does not extend well beyond two stands. It is very difficult to obtain analytical results and, thus, economic insights. Furthermore, no model explicitly considers three or more stands. It is clear that to capture many spatial issues of forestry, a minimum of three stands will need to be considered.

The two-period framework, like the optimal rotation framework, has been extended to two stands. However, unlike the rotation model, the model does generate testable analytical results. This framework is less complicated than the optimal rotation frameworks, generates results similar to optimal control models, and is very flexible.

There are drawbacks with the two-period framework. First, the framework does not explicitly consider the opportunity cost of land. However, Ovaskainen (1992) suggests that the growth function can be constructed to incorporate age-class structure so as to account for the opportunity cost of land. Second, the framework does not explicitly consider stand age or age-class distribution, which are surely important determinants of stand growth and nontimber benefits. Again this can be potentially overcome by making growth a function of age and accounting for age-class structure. However, this introduces greater complexity at the cost of loosing economic insight.

The two-period model has several advantages. The model generates short-term and long-term harvesting results. The framework can be extended to include risk and uncertainty and can be used to investigate the various policy instruments designed to change harvesting behavior.

Table 3.1 Summary of Current Literature

	Multiple Use	Time	Quasi Space[a]	Explicit Space	Stands Linked	2-stands (adjacent)	N-stands
Multi-Product Framework							
Bowes and Krutilla (1982)	✓		✓				✓
Gregory (1955)	✓						
Pearse (1969)	✓						
Vincent and Binkley (1993)	✓		✓			✓	
Walters (1977)	✓						
Optimal Rotation Framework							
Bowes and Krutilla (1985)	✓	✓	✓		✓		✓
Faustmann (1849)		✓					
Hartman (1976)	✓	✓					
Leyland and Moses (1976)		✓		✓			✓
Paredes and Brodie (1989)	✓	✓		✓			✓
Rose (1999)	✓	✓		✓	✓	✓	
Swallow et al. (1997)	✓	✓		✓	✓	✓	
Swallow and Wear (1993)	✓	✓	✓		✓	✓	
Tahvonen and Salo (1999)	✓	✓	✓		✓		✓
Two Period Framework							
Amacher (1999)	✓	✓	✓		✓	✓	
Amacher and Brazee (1997)	✓	✓					
Binkley (1981)	✓	✓					
Koskela and Ollikainen (1997)	✓	✓					
Koskela and Ollikainen (1999)	✓	✓		✓	✓	✓	
Max and Lehman (1988)	✓	✓					
Ovaskainen (1992)	✓	✓					
Other Forestry Models							
Gray et al. (1997)				✓			✓

Note:
[a]Stands are not spatially defined; stands can be adjacent or detached

In the remainder of this dissertation, I extend the two-period harvest-inventory framework to a three-stand forest where net present value is maximized. This reduces the complexity of the model and permits greater economic insights. The model is presented in the next chapter. It is then used to demonstrate how interdependencies between stands and location differences affect optimal harvesting patterns on a multiple-use forest.

CHAPTER 4

TWO-PERIOD MULTIPLE-USE MODEL

In this chapter, a two-period framework is used to model multiple-use forestry. Section 4.1, presents a three-stand profit-maximization model. It contains the basic assumptions of the model, how it differs from previous models, and illustrate different forms of the model to capture different spatial components. I then present special cases of the three-stand model beginning in Section 4.2 with a one-stand timber-only model and a one-stand multiple-use model. Next, in Section 4.3, I present various two-stand versions of the multiple-use model. In Section 4.4, the two-stand model is extended to include management effort and the issue of intensive timber management zones is considered. I revisit the three-stand problem in Section 4.5 with insights gained from the previous sections. Section 4.6 presents conclusions drawn from the model.

4.1 Three Stands

The multiple-use problem for the forestlands manager is to maximize the net present value of a three-stand forest over a two-period planning horizon. The manager accomplishes this by choosing a level of harvest from each stand in each period to maximize net present forest benefits, $B(.)$. The three stands are denoted by L-left, M-middle, and R-right and the two periods are referenced as 1 and 2. The use of L, M and R emphasizes the spatial arrangement and location of the stands.[1]

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[1]Stands can be adjacent or differentiated in space by distance functions. For example, the model can easily include transport costs of timber or people (travel time to recreation site) or distance of stands from another land form, such as a river or lake. An obvious point, but worth mentioning, distance functions require points of reference. Either distance

Total net present benefits, $B(h)$, are the sum of the net present value of timber benefits, $F(h)$, and the net present value of nontimber benefits, $E(V)$. Timber benefits, $F(h)$, are simply the volume harvested, h, in each period multiplied by the stumpage price, p, i.e., there are no pecuniary interactions. Stumpage price can vary across all stands in the forest and between periods. This is the simplest means of differentiating one stand from another. To keep the model simple, we initially assume that the timber price is equal across the three stands but can differ between periods.

We assume there is an initial inventory of timber at the beginning of the time horizon. We assume that after the first-period harvest the stand grows according to a known growth function. The stand growth function, $g(V)$, is a function of the inventory left after the first harvest. The growth function has the properties of $g' \geq 0$ for $V \leq V^{MSY}$, $g'<0$ for $V>V^{MSY}$, and $g''<0$ for all V, where V^{MSY} is the inventory that corresponds to the maximum sustained yield (MSY).[2] We allow the growth function to be different on each stand.

The nontimber benefits, $E(V)$, are assumed to be a function of inventory, V. Although this is an unrealistic assumption, as it ignores nontimber benefits independent of stand inventory, dependent on spatial distribution of age-classes or dependent on stand age, it is not without merit. In fact, nontimber benefits are a function of many variables of which the standing inventory is a major factor.[3] In the general case, nontimber benefits

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refers to how the stands differ from each other or how they differ from another point of reference, such as a mill or river.

[2]Ovaskainen (1992) reviewed function forms for the growth function. The most commonly assumed form is the logistics yield function shown in Figure 3.4 (p.36).

[3]Nontimber benefits are likely to be a function of stand inventory, which is a function of stocking density, species type and mix, and age, non-tree attributes of the land such as topography and geographical location, and non-land attributes such as numerous ecological factors.

on each stand are a function of its own inventory and the inventory on other stands in the forest. There can also be more than one nontimber good or service on each stand.

The relationship between a particular stand's nontimber value with other stands is central to the model. The nontimber values derived from a given stand in the forest can vary from being independent to completely dependent on other stands in the forest. In general, nontimber benefits are a function of the inventory on the *L*, *M* and *R* stands, $E(V^L, V^M, V^R)$. More specifically, the forest-level nontimber benefits function can be written as the sum of stand-level nontimber benefits functions or as a general forest-level function. Whatever the specific form of the nontimber function, we assume always that the function is smooth and continuous and strictly concave in inventory, V. Explicitly, we assume that $E_{V_sV_s}<0$ for all $s \in S$, where $S=\{L,M,R\}$, and that the Hessian matrix of second-order partial derivatives is negative definite (Chiang 1984).[4]

Figure 4.1 illustrates the inventory profile of a three-stand forest over two periods. In time zero, we start with an initial inventory of timber on each stand. At this instant, a harvest decision is made on each stand leaving a new stock of timber. The interval between harvests can be of any time period, e.g., one year, five years, or more.[5] Prior to the second harvest, the inventory is augmented by forest growth. The growth function accounts for the interval between harvests. For example, if the interval between harvests

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[4]The assumption of smooth, continuous and strict concavity ensures that the solution is a unique global maximum. The assumption can be relaxed but a unique global maximum cannot be guaranteed. The greatest benefit of the concavity assumption is that the summation of concave functions is also a concave function (Beavis and Dobbs 1990). This is not true with the sum of quasi-concave functions. For this reason, we assume that the functions are concave.

[5]The time period between harvests is exogenous to the model. A more complete model would determine the optimal cutting cycle for an uneven-aged forest, perhaps in a Faustmann-type model similar to Chang (1981).

is five years the growth function accounts for five years of accumulated growth from the last harvest. During the interval between harvests a constant flow of nontimber value is derived from the inventory. We assume that the timber and present value of nontimber benefits are realized at the time of harvest to simplify the model.[6] At the beginning of period 2, a second harvest from the second-period inventory (inventory after first harvest plus growth) is realized leaving a new inventory to produce nontimber benefits. Second-period timber revenues and nontimber benefits are discounted by $1/(1+r)$, where r is a real rate of interest.

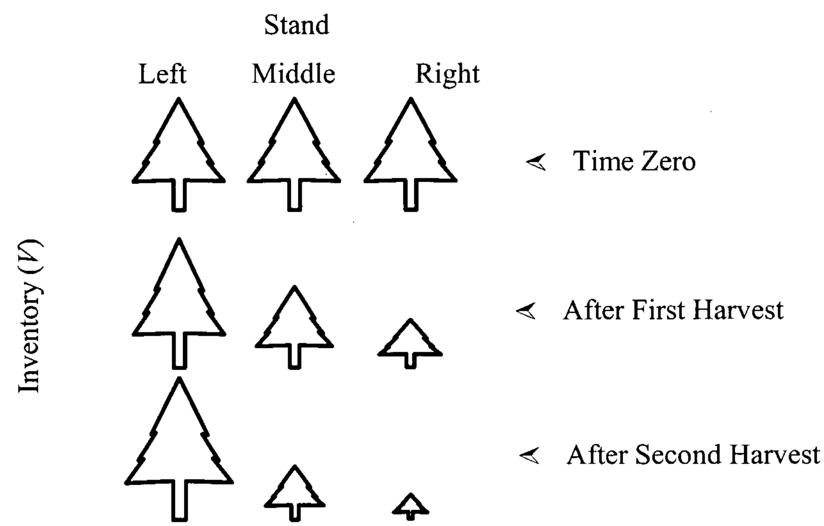

Figure 4.1 Inventory Profile of a Three-stand Forest over Two Periods

Formally, the problem is to

$$\max_{h^{L1},h^{M1},h^{R1},h^{L2},h^{M2},h^{R2}} B = p^1(h^{L1}+h^{M1}+h^{R1}) + E(V^{L1},V^{M1},V^{R1}) + \frac{p^2(h^{M2}+h^{M2}+h^{R2})}{(1+r)} + \frac{E(V^{L2},V^{M2},V^{R2})}{(1+r)} \tag{4.1}$$

[6]For example, if the harvest interval is five years then a flow of annual nontimber benefits are accounted for over this term. A more exact estimation of benefits entails continuously accounting for annual forest growth and the change in nontimber benefits over the term.

subject to the first-period harvesting constraints on each stand,

$$h^{L1} \leq X^{L} \tag{4.2a}$$

$$h^{M1} \leq X^{M} \tag{4.2b}$$

$$h^{R1} \leq X^{R} \tag{4.2c}$$

and second-period harvesting constraints

$$h^{L2} \leqslant X^{L} - h^{L1} + g^{L}(X^{L} - h^{L1}) \tag{4.2d}$$

$$h^{M2} \leqslant X^{M} - h^{M1} + g^{M}(X^{M} - h^{M1}) \tag{4.2e}$$

$$h^{R2} \leqslant X^{R} - h^{R1} + g^{R}(X^{R} - h^{R1}) \tag{4.2f}$$

where p^1 and p^2 are timber prices in period one and two, and X^L, X^M, and X^R are the initial inventories.[7] The stock accounting identity for period one on stand s is

$$V^{s1} = X^{s} - h^{s1} \tag{4.3a}$$

while the inventory in period two on stand s is

$$V^{s2} = X^{s} - h^{s1} + g^{s}(X^{s} - h^{s1}) - h^{s2} \tag{4.3b}$$

The two-period model presented here differs from earlier two-period forest models as more than two stands are included. This formulation permits a more general representation of the multiple-use problem than earlier work and the introduction of a greater variety of spatial problems.

The three-stand model can capture many different asymmetries. Figure 2.1 (p.17) depicts some of the spatial issues that can be captured by this model. Distance relationships (Figure 2.1b) can be incorporated into the model by introducing different stumpage prices or nontimber values on each stand. Specific spatial considerations, such

[7]Superscripts are used to refer to the stand and period, e.g., h^{L1} refers to the harvest on stand L in period 1 and p^1 refers to the stumpage price in period 1.

as stand interdependencies and habitat scale depicted in Figure 2.1a and 2.1c, can be incorporated by explicitly expressing stand- or forest-level nontimber value functions.

Different forms of interdependency are captured by expressing each stand function as a function of one or more stands. For example, the case illustrated in Figure 2.1a-vi can be expressed as

$$\begin{aligned} \max_{h^{L1},h^{M1},h^{R1},h^{L2},h^{M2},h^{R2}} B = p^{1}(h^{L1} + h^{L1} + h^{R1}) + E^{L}(V^{L1}, V^{M1}) + E^{M}(V^{L1}, V^{M1}) \\ + E^{R}(V^{L1}, V^{R1}) + \frac{p^{2}(h^{L2} + h^{M2} + h^{R2})}{(1+r)} + \frac{E^{L}(V^{L2}, V^{M2})}{(1+r)} \\ + \frac{E^{M}(V^{L2}, V^{M2})}{(1+r)} + \frac{E^{R}(V^{L2}, V^{R2})}{(1+r)} \end{aligned} \tag{4.4}$$

The stand-level nontimber benefits on the left and middle stands, E^{L} and E^{M}, are both functions of the inventory on the left and middle stands indicating a symmetric interdependency between the two stands. Nontimber benefits on the right stand, E^{R}, are a function of the inventories on the left and right stands, indicating a one-way interdependence between the left stand and right stands. The exact asymmetric relationship between the right and left stand is not expressed in Equation 4.4. A separable formulation of the nontimber benefit function on the right stand might be

$$E^{R}(V^{L}, V^{R}) = E^{R}(V^{L}) + E^{R}(V^{R}) \tag{4.5}$$

Equation 4.5 expresses right-stand nontimber benefits as the sum of nontimber benefits generated from conditions on the right and left stands. The second term can be positive or negative to indicate the nature of dependence between the stands. If the stands are independently owned then this term measures the level of external cost or benefit associated with a level of inventory on the right stand. Such an interpretation captures external costs associated with the rate or level of harvest on nontimber values such as visual quality, water quality, fish and biodiversity. This form of interdependence is the classic case considered in environmental economics.

Equation 4.1 can also be rewritten to express more than one nontimber good and habitat scale. Consider the case of two nontimber goods, E and W. Assume that good E is best expressed at a stand level while good W is best expressed as a forest-level benefit, but production occurs only on two stands in the forest. This case might be represented by the following problem:

$$\max_{h^{L1},h^{M1},h^{R1},h^{L2},h^{M2},h^{R2}} B = p^1(h^{L1}+h^{M1}+h^{R1}) + E^L(V^{L1},V^{M1}) + E^M(V^{L1},V^{M1},V^{R1}) + E^R(V^{M1},V^{R1}) + W(V^{L1},V^{M1}) + \frac{p^2(h^{L2}+h^{M2}+h^{R2})}{(1+r)} + \frac{E^L(V^{L2},V^{M2})}{(1+r)} + \frac{E^M(V^{L2},V^{M2},V^{R2})}{(1+r)} + \frac{E^R(V^{M2},V^{R2})}{(1+r)} + \frac{W(V^{L2},V^{M2})}{(1+r)} \qquad (4.6)$$

Equation 4.6 states that the nontimber good E is measured as the sum of benefits on each stand. The stand-level E-benefits on each stand depend on the inventory on each stand and the adjacent stand. The nontimber good W depends on the level of inventory on the left and middle stand and is measured as a forest-level benefit. For example, W may be a wildlife value. The breeding and rearing habitat range (production) of the species occurs over the left and middle stand but individuals move over the entire forest. Thus, the benefit realized from use or nonuse of the species occurs over the entire forest. The left and middle stand conditions are inputs into production while all three stands are sources of supply. E, on the other hand, is a nontimber benefit that is produced and realized on each stand, but the level of production depends on adjacent stand conditions. A possible example is recreational benefits. Recreational benefits derived from enjoying a view of a forested valley are realized on each stand but are affected by conditions of the visible landscape of adjacent stands.

Consider problem 4.6 in more detail. We can rewrite the problem by substituting the accounting equations 4.3a and 4.3b into Equation 4.6:

$$
\begin{aligned}
\max_{h^{L1},h^{M1},h^{R1},h^{L2},h^{M2},h^{R2}} B = {} & p^1(h^{L1}+h^{M1}+h^{R1}) + E^L(X^L-h^{L1},X^M-h^{M1}) + E^R(X^M-h^{M1},X^R-h^{R1}) \\
& + E^M(X^L-h^{L1},X^M-h^{M1},X^R-h^{R1}) + W(X^L-h^{L1},X^M-h^{M1}) + \frac{p^2(h^{L2}+h^{M2}+h^{R2})}{(1+r)} \\
& + \frac{E^L(X^L-h^{L1}+g^L(X^L-h^{L1})-h^{L2},X^M-h^{M1}+g^M(X^M-h^{M1})-h^{M2})}{(1+r)} \\
& + \frac{E^M\begin{pmatrix} X^L-h^{L1}+g^L(X^L-h^{L1})-h^{L2},X^M-h^{M1}+g^M(X^M-h^{M1})-h^{M2}, \\ X^R-h^{R1}+g^R(X^R-h^{R1})-h^{R2} \end{pmatrix}}{(1+r)} \\
& + \frac{E^R(X^M-h^{M1}+g^M(X^M-h^{M1})-h^{M2},X^R-h^{R1}+g^R(X^R-h^{R1})-h^{R2})}{(1+r)} \\
& + \frac{W(X^L-h^{L1}+g^L(X^L-h^{L1})-h^{L2},X^M-h^{M1}+g^M(X^M-h^{M1})-h^{M2})}{(1+r)}
\end{aligned}
\tag{4.7}
$$

If we assume that the harvesting constraints are satisfied, the first-order conditions (f.o.c.) of the unconstrained problem are[8]

$$
\frac{\partial B}{\partial h^{L1}} = p^1 - E^L_{L1} - E^M_{L1} - W_{L1} - \frac{E^L_{L2}(1+g^L_v)}{(1+r)} - \frac{E^M_{L2}(1+g^L_v)}{(1+r)} - \frac{W_{L2}(1+g^L_v)}{(1+r)} = 0 \tag{4.8a}
$$

$$
\begin{aligned}
\frac{\partial B}{\partial h^{M1}} = {} & p^1 - E^L_{M1} - E^M_{M1} - E^R_{M1} - W_{M1} - \frac{E^R_{M2}(1+g^M_v)}{(1+r)} - \frac{E^M_{M2}(1+g^M_v)}{(1+r)} \\
& - \frac{E^R_{M2}(1+g^M_v)}{(1+r)} - \frac{W_{M2}(1+g^M_v)}{(1+r)} = 0
\end{aligned}
\tag{4.8b}
$$

---

[8]Subscripts on the nontimber functions refer to the partial derivative with respect to the stand inventory and period. For example, $W_{L2} = \frac{\partial W}{\partial V^{L2}} = \left(\frac{\partial W}{\partial V^{L2}}\right)\left(\frac{\partial V^{L2}}{\partial h^{L2}}\right)$ refers to the change in nontimber benefits on stand L from a change in the second period inventory via harvest and, $g^R_v = \frac{\partial g^R}{\partial V^R} = \left(\frac{\partial g^R}{\partial V^R}\right)\left(\frac{\partial V^R}{\partial h^R}\right)$ is the change in growth on R from a change in inventory via harvest. The superscript on the nontimber and growth functions refer to the stand while the absence of a superscript on a nontimber function implies a forest-level function.

$$\frac{\partial B}{\partial h^{R1}} = p^1 - E_{R1}^L - E_{R1}^M - \frac{E_{R2}^L(1+g_v^R)}{(1+r)} - \frac{E_{R2}^M(1+g_v^R)}{(1+r)} = 0 \tag{4.8c}$$

$$\frac{\partial B}{\partial h^{L2}} = \frac{p^2}{(1+r)} - \frac{E_{L2}^L}{(1+r)} - \frac{E_{L2}^M}{(1+r)} - \frac{W_{L2}}{(1+r)} = 0 \tag{4.8d}$$

$$\frac{\partial B}{\partial h^{M2}} = \frac{p^2}{(1+r)} - \frac{E_{M2}^L}{(1+r)} - \frac{E_{M2}^M}{(1+r)} - \frac{E_{M2}^R}{(1+r)} - \frac{W_{M2}}{(1+r)} = 0 \tag{4.8e}$$

$$\frac{\partial B}{\partial h^{R2}} = \frac{p^2}{(1+r)} - \frac{E_{R2}^M}{(1+r)} - \frac{E_{R2}^R}{(1+r)} = 0 \tag{4.8f}$$

Rearranging Equations 8d-8f and substituting them into Equations 8a-8c gives the first-period harvest decision rules:

$$p^1 = E_{L1}^L + E_{L1}^M + W_{L1} + \frac{p^2}{(1+r)} \tag{4.9a}$$

$$p^1 = E_{M1}^L + E_{M1}^M + E_{M1}^R + W_{M1} + \frac{p^2}{(1+r)} \tag{4.9b}$$

$$p^1 = E_{R1}^L + E_{R1}^M + \frac{p^2}{(1+r)} \tag{4.9c}$$

The decision rule on each stand is different but has a common structure. The harvest rule is to cut until the marginal timber benefit is equal to the foregone marginal nontimber benefits, including the marginal impacts on interdependent stands, plus the foregone marginal timber revenues in the second period. If functions in Equation 4.6 are known and exactly specified, the problem is solved by simultaneously solving the system of equations, 4.8a-4.8e.

From the implicit function theorem, comparative statics results can be obtained from the general model. First, from our assumption that the functions are all smooth, continuous and strictly concave, a solution exists which solves the system of first-order conditions. The solutions to the problem can implicitly be written as functions of the parameters of the model, $h^{sn^\star} = h^{sn^\star}(p^1, p^2, r, X^L, X^M, X^R)$ for all s and n. Second, we substitute the optimal solutions into the first-order conditions to form six identities that we totally differentiate to obtain the Jacobian matrix of second-order partial derivatives. Finally, changes in the optimal harvests from changes in the parameters, comparative statics results, can be obtained by applying Cramer's rule.

Due to the size of the model, the comparative statics results can not be derived.[9] Instead of working through the full model, we reduce the problem to one and two stands to obtain comparative statics results and gain insights into the more complicated three-stand problem.

Though the comparative statics results are difficult to obtain, envelope results are easily obtained. We substitute the implicit harvest solutions back into the objective function, Equations 4.8. We then totally differentiate Equations 4.8 to obtain the following envelope results

$$\frac{dB}{dp^1} = h^{L1\star} + h^{M1\star} + h^{R1\star} \geq 0 \tag{4.10a}$$

$$\frac{dB}{dp^2} = \frac{h^{L1\star} + h^{M1\star} + h^{R1\star}}{1+r} \geq 0 \tag{4.10b}$$

$$\frac{dB}{dX^L} = p^1 > 0 \tag{4.10c}$$

$$\frac{dB}{dX^M} = p^1 > 0 \tag{4.10d}$$

$$\frac{dB}{dX^R} = p^1 > 0 \tag{4.10e}$$

[9]A mathematical program such as *Maple V* can be used to solve the problem. However, due to the size of the problem, the solutions are so complicated and difficult that they are not reported.

$$\frac{dB}{dr} = -\frac{p^2(h^{L2\star} + h^{M2\star} + h^{R2\star})}{(1+r)^2} - \frac{E^L + E^M + E^R + W}{(1+r)^2} < 0 \tag{4.10f}$$

Each envelope result measures the total change in the objective function from changes in a model parameter. An increase in prices or initial inventories will both increase net present value, while an increase in the interest rate decreases the net present value of the forest. Equations 4.10a and 4.10b are standard neoclassical results for a profit-maximizing firm and are commonly referred to as Hotelling's lemma (see Varian, 1984).

The general three-stand model can collapse into smaller problems by assuming independence between all or any of the stands. These special cases are considered below. The insights gained from these sub-models provide better understanding of the more complicated three-stand problems.

4.2 One Stand

If we assume that all three stands are independent in the production of timber and nontimber benefits, the model collapses into three single-stand problems. Consider first the case when only timber is valued and then the case when timber and nontimber amenities are both valued.

4.2.1 Timber Only[10]

Consider the problem of maximizing the net present value from timber revenues over two periods. We assume timber benefits are independent between stands and thus

[10]The analysis is very similar to Ovaskainen (1992). Ovaskainen's (1992) model is a utility maximization model and management effort is explicitly modeled. The exclusion of utility and management effort greatly simplifies the analysis. Management effort is introduced later in the chapter.

only one stand is analyzed; hence, we drop the stand identifier in this subsection. We assume that prices in each period are known and inventory grows according to the growth function, $g(V)$.

The simple two-period timber problem is

$$\max_{h^1,h^2} F(h^1,h^2) = p^1h^1 + \frac{p^2h^2}{(1+r)} \tag{4.11}$$

subject to the harvesting constraints for period 1,

$$h^1 \leq X \tag{4.12a}$$

and for period 2,

$$h^2 \leqslant X + g(X - h^1) - h^1 \tag{4.12b}$$

where r is a real rate of interest and X is an initial endowment of inventory.

This model is not dissimilar from the basic two-period model of exploiting a nonrenewable resource, such as a mine or a ground water reservoir, and is similar to Binkley (1980).[11] The notable exception between this model and the nonrenewable model is the natural growth of the physical stock of timber over time and Binkley (1980) uses a constant rate of growth.

The problem is a constrained optimization problem and can be solved by rewriting the model as a Lagrangian. However, as there is no value in the standing forest at the end of the planning horizon, the entire forest must be cut down within the two periods. Specifically, the harvest constraints reduce to the simple constraint

$$h^1 + h^2 = X + g(X - h^1) \tag{4.13}$$

---

[11]See Hartwick and Olewiler (1998) for good exposition of the simple two-period model and the history of the optimal extraction model for a nonrenewable resource.

Therefore, the sum of harvests in each period must equal the initial stock of timber plus any growth of the forest.

We solve Equation 4.13 for the second period harvest, h^2, and substitute this into the objective function, Equation 4.11; hence, we drop the period identifier on harvest for the rest of this subsection. The unconstrained problem is

$$\max_h F(h) = p^1 h + \frac{p^2(X - h + g(X - h))}{(1 + r)} \tag{4.14}$$

The first-order condition sufficient for a maximum is

$$p^1 = p^2 \frac{(1 + g_v)}{(1 + r)} \tag{4.15}$$

The necessary second-order condition for a maximum, $F_{vv}=g_{vv}<0$, is satisfied.

The harvest rule in Equation 4.15, is similar to the optimal decision rule for exploiting a nonrenewable resource, $p^1 = \frac{p^2}{1+r}$. This model can be extended to an industry or country (small open economy or closed economy) and is a fundamental building block for natural resource accounting for renewable and nonrenewable resources (see National Research Council 1999). The simple intertemporal harvest rule is to harvest until present marginal value in each period is equated. We can rearrange Equation 15 to form

$$\frac{p^2}{p^1}(1 + g_v) = (1 + r) \tag{16}$$

which states the optimal harvest occurs when the rate of return on the remaining inventory is equal to the rate of return on competing assets.[12] This decision rule is identical to the rule derived by the utility maximization two-period model of Ovaskainen

[12]If we consider g a constant then the optimal harvesting rule is to harvest in period one if $r \geq g$ or harvest in period two if $r < g$.

(1992). Incremental growth in value has two components; the remaining stock grows in value from changes in timber prices between periods and from growth in the physical stock.[13]

The first-period harvest is a measure of the short-run timber supply as stands that exceed the opportunity cost of capital are cut. Note that the arbitrary decision of ending the model at two periods biases the second period decision. Therefore, we do not concern ourselves with the second-period harvest and inventory solutions. That said, the terminal period does not bias the first-period harvest and inventory results (Ovaskainen 1992). Ovaskainen (1992) shows that, in a three-period model, the second period harvest is analogous to the long-run steady state harvest derived from optimal control models and the first-period harvest is short-run harvest.

It is obvious from the first-order condition that when r is zero and stumpage price constant the optimal inventory corresponds to the maximum sustained yield stock (g_v=0). Therefore, for a positive interest rate the optimal inventory will be less than the maximum sustained yield inventory. This is a standard result with the exception of Binkley (1987).[14]

Comparative statics results are derived by first, substituting the optimal harvest into the first-order condition to form the identity

$$p^1 - \frac{1 + g_v(X - h^\star(p^1, p^2, r, X))}{1 + r} p^2 \equiv 0 \tag{4.17}$$

---

[13]In natural resource accounting, the change in value from a change in price over time is referred to as the capital gain/loss of holding the capital stock (growing stock) (National Research Council 1999).

[14]Binkley (1987), using the Faustmann calculus, showed that the optimal rotation can be longer than MSY for very fast growing species.

and then, differentiating with respect to the parameters. The comparative statics results are

$$\frac{\partial h}{\partial p^1} = -\frac{1+r}{p^2 g_{vv}} > 0 \tag{4.18a}$$

$$\frac{\partial h}{\partial p^2} = \frac{(1+g_v)}{p^2 g_{vv}} < 0 \tag{4.18b}$$

$$\frac{\partial h}{\partial r} = -\frac{1+g_v}{(1+r)g_{vv}} > 0 \tag{4.18c}$$

$$\frac{\partial h}{\partial X} = 1 \tag{4.18d}$$

The first-period harvest increases when the first-period price, interest rate and initial inventory increase and decreases when the second-period price increases. Note that solving for the optimal harvest simultaneously solves for the optimal growing inventory, $V^\star = X - h^\star$. Therefore, all the comparative statics results with respect to the optimal growing inventory are readily obtained.

$$\frac{\partial V}{\partial p^1} = \frac{1+r}{p^2 g_{vv}} < 0 \tag{4.18a}$$

$$\frac{\partial V}{\partial p^2} = \frac{-(1+g_v)}{p^2 g_{vv}} > 0 \tag{4.18b}$$

$$\frac{\partial V}{\partial r} = \frac{1+g_v}{(1+r)g_{vv}} < 0 \tag{4.18c}$$

$$\frac{\partial V}{\partial X} = 0 \tag{4.18d}$$

Simple envelope results are derived by substituting the implicit harvest solution into the objective function and totally differentiating with respect to the parameters.

$$\frac{dF}{dp^1} = h^\star \geq 0 \tag{4.19a}$$

$$\frac{dF}{dp^2} = \frac{X - h^\star + g(X - h^\star)}{1+r} = \frac{h^\star}{1+r} \geq 0 \tag{4.19b}$$

$$\frac{dF}{dX}=\frac{p^2(1+g_v)}{1+r}=p^1>0 \tag{4.19c}$$

$$\frac{dF}{dr}=-\frac{p^2(X-h^\star+g(X-h^\star))}{1+r}=-\frac{p^2h^\star}{1+r}\leq 0 \tag{4.19d}$$

The results are similar to those of the more complicated three-stand multiple-good problem.

4.2.2 Timber and Nontimber Good

Here we extend the simple two-period model to include a nontimber good, again focusing on one stand so stand notation is dropped for convenience. Assume that the standing forest produces a nontimber benefit, $E(V)$. Recall that the function is assumed to be smooth, continuous and concave in inventory, $E_{VV}(V)<0$. Assume that the nontimber benefits earned between decision periods are realized at the same instant as the timber revenues. The problem is now to

$$\max_{h_1,h_2} B(h_1,h)=p^1h^1+E(X-h^1)+\frac{p^2h^2}{(1+r)}+\frac{E(X-h^1+g(X-h^1)-h^2)}{(1+r)} \tag{4.20}$$

subject to the harvesting constraints,

$$h^1\leq X \tag{4.21a}$$

$$h^2\leqslant X-h^1+g(X-h^1) \tag{4.22b}$$

Unlike the timber-only problem, we cannot assume that the entire forest is cut during the time horizon as the standing forest has value. Only if the maximum nontimber benefit occurs at zero does the previous assumption hold.

If we assume that the harvesting constraints are satisfied, the first-order of sufficient conditions for a maximum are[15]

[15]The first-period harvesting constraint is binding when: one, the maximum nontimber benefits occur at zero inventory and the relative price of timber to nontimber is less than

$$\frac{\partial B}{\partial h^1} = p^1 - E_1 - E_2 \frac{(1+g_v)}{(1+r)} = 0 \tag{4.23a}$$

$$\frac{\partial B}{\partial h^2} = \frac{p^2}{1+r} - \frac{E_2}{1+r} = 0 \tag{4.23b}$$

As the nontimber function is strictly concave and the forest production function is also strictly concave, the objective function is assuredly strictly concave. The necessary conditions for a maximum, $B_{h1h1} \leq 0$, $B_{h2h2} \leq 0$ and $B_{h1h1}B_{h2h2}\text{-}2B_{h1h2} \geq 0$, are also satisfied.

From Equation 4.23a, we see that the optimal first-period harvest occurs when the current marginal timber benefits are equated with the discounted marginal nontimber benefits of periods one and two. Equation 4.23b indicates that the optimal second-period inventory and harvest equate the marginal NPV timber benefits with the marginal NPV nontimber benefits.

Substituting Equation 23b into Equation 23a reveals the first-period harvest rule,

$$p^1 = E_1 + p^2 \frac{(1+g_v)}{(1+r)} \tag{4.24}$$

We now compare this rule with the timber-only rule (Equation 4.15). The inclusion of nontimber values affects the equilibrium level of inventory and the first-period harvest. The marginal nontimber benefits can favor a first- or second-period harvest. This is best understood by considering a special case. If timber prices in each period are assumed to be zero, the optimal harvest (ignoring harvesting costs) is simply where nontimber benefits are maximized. Note that, in this case, the decision rule is independent of the rate of interest and the forest growth. Now consider the form of the nontimber function.

1, or the discount rate is infinite. Throughout it is assumed that neither of these cases occur. A second corner solution is also possible; the entire stand is left uncut. This is likely to occur when the nontimber benefits are monotonically increasing and are much greater than the timber benefits. We entertain this possibility briefly but again assume an interior solution.

There are two extreme possibilities. First, maximum nontimber values occur at zero inventory. Second, maximum nontimber values occur at the carrying capacity of the stand, K (see Figure 3.4, p.36).

Now, reintroduce positive timber prices. When timber prices are positive and maximum nontimber values occur at zero inventory, the first-period harvest (inventory) is lower (greater) than without timber values. When nontimber benefits reach a maximum at K, the first-period harvest (inventory) is greater (lower) than without timber values. In both cases, the optimal harvest (inventory) moves towards the timber-only harvest (inventory). Therefore, the joint-production harvest is greater (lower) than the timber-only harvest (inventory) if the nontimber value reaches a maximum at an inventory less (greater) than the optimal timber-only inventory. For example, the inclusion of water values may lead to a greater harvest (lower inventory) while spotted owl values may lead to a lower harvest (higher inventory) than the timber-only harvest (inventory). This conclusion expands on Ovaskainen (1992) and is consistent with Hartman (1976) and Strang (1983).[16]

Equation 4.24 can be rearranged to form

$$\frac{p^2}{p^1}(1+g_v)=(1+r)(1-\frac{E_1}{p^1}) \tag{4.25}$$

to provide an alternative interpretation of the harvesting rule that is similar to Ovaskainen (1992).[17] The manager harvests from the initial inventory until the marginal rate of return

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[16]Ovaskainen (1992) only considered monotonically increasing nontimber benefits.

[17]Ovaskainen (1992) derives the first period harvest rule $\frac{P^2}{P^1}[1+g_v]=(1+r)-\frac{v_v(V^1)}{P^1\beta u_c(c_s)}$ from a two-period utility maximization model, where u_c is the marginal utility from consumption, v_v is the marginal utility from nontimber forest benefits, and $\beta$is a discount factor that reflects individual time preferences. Letting β equal $1/(1+r)$, V_v=E^1 and dropping marginal utility from the formulation results in

on the growing inventory, in terms of timber value, is equated with the marginal rate of return on other assets, adjusted by the marginal value of nontimber amenities from the standing forest. From Equation 4.25, it is possible that the optimal first-period inventory is greater than the maximum sustained yield inventory. This occurs when nontimber values reach a maximum at an inventory greater than the MSY inventory and the marginal nontimber benefits are greater than the first period timber price for $V > V^{MSY}$, where V^{MSY} is the maximum sustained yield inventory.

Figure 4.2 illustrates the two extreme nontimber cases described above. Figure 4.2a illustrates the case where maximum nontimber benefits occur at zero inventory while Figure 4.2b illustrates the case where the maximum occurs at or beyond the carrying capacity of the stand, K. The upper panels illustrate the growth function and nontimber benefits for each case.

By setting $p^1 = p^2$ in Equation 4.25,

$$(1+g_v) = (1+r)(1-\frac{E_1}{p^1}) \tag{4.26}$$

The left-hand (LHS) and right-hand (RHS) sides of Equation 4.26 are illustrated in the lower panels for the two different cases. In the lower panel of Figure 4.2a, $E_1 < 0$ for $V>0$ and thus results in an upward sloping line starting at $1+r$. Therefore, for $r \geq 0$ the optimal inventory for joint production of nontimber amenities plus timber, V^{NT}, is less than the optimal timber inventory, V^{T}, and the MSY inventory. Conversely, the case illustrated in Figure 4.2b has $E_1 \geq 0$ for all V and therefore, the RHS of Equation 4.26 lies below $1+r$ for all V. Consequently, V^{NT} lies to the right of the optimal timber inventory and can be greater than the MSY inventory. For $r = 0$ and $p^1 = p^2 = 1$, the joint

expression identical to Equation 4.25 above.

timber and nontimber solution is equal to MSY only if, maximum nontimber benefits coincide with the MSY inventory. For timber prices greater than one, the joint-production inventory can coincide with the MSY inventory when the maximum nontimber benefits occur at a higher inventory. In conclusion, the joint-production inventory is greater or less than the MSY and the optimal timber inventories.

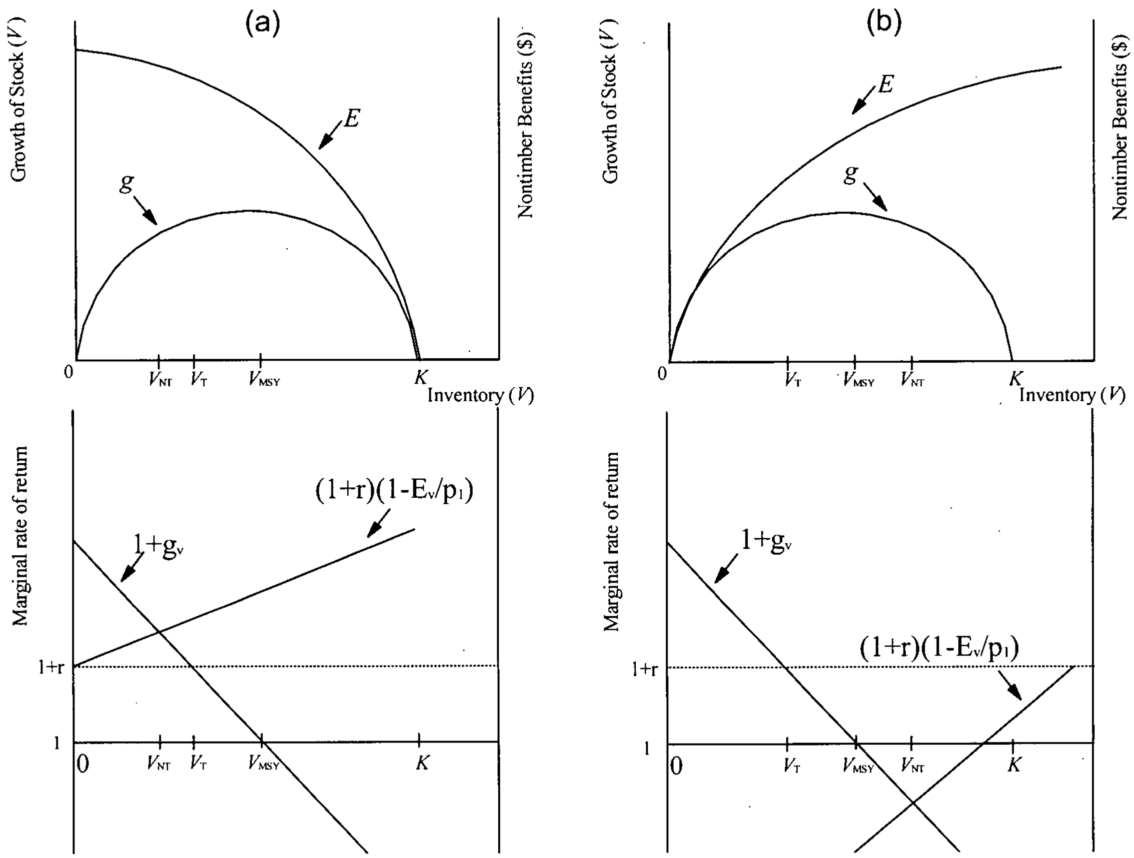

Figure 4.2 Optimal Inventory with Joint Production

Again, we derive comparative statics results by substituting the implicit solution of the first and second period harvests into the first-order conditions to form the identities

$$p^1 - E_1(X - h^{1\star}(a)) - E_2[X - h^{1\star}(a) + g(X - h^{1\star}(a))]\frac{(1 + g_v(X - h^{1\star}(a)))}{(1 + r)} \equiv 0 \qquad (4.27a)$$

$$\frac{p^2}{1+r} - \frac{E_2[X-h^{1\star}(a)+g(X-h^{1\star}(a))]}{1+r} \equiv 0 \tag{4.27b}$$

where α contains X, p^1, p^2, and r.

The comparative statics results are obtained by totally differentiating Equations 27a and 27b with respect to α and solving the system of equations using Cramer's rule.

$$\frac{\partial h^{1\star}}{\partial p^1} = -\frac{E_{22}}{(1+r)}\frac{1}{D} > 0 \tag{4.28a}$$

$$\frac{\partial h^{1\star}}{\partial p^2} = \frac{(1+g_v)E_{22}}{(1+r)^2}\frac{1}{D} < 0 \tag{4.28b}$$

$$\frac{\partial h^{1\star}}{\partial r} = -\frac{p^2(1+g_v)E_{22}}{(1+r)^3}\frac{1}{D} > 0 \tag{4.28c}$$

$$\frac{\partial h^{1\star}}{\partial X} = 1 \tag{4.28d}$$

where $D = \dfrac{E_{11}E_{22}(1+r)+g_{vv}E_2E_{22}}{(1+r)^2} > 0$ is the Jacobian determinant whose sign follows from the concavity assumptions of g and E and the first-order conditions. The comparative statics results are similar to the timber-only model with signs identical to the timber-only model.

The envelope results are identical in sign to the timber-only model and the three-stand model. The results are

$$\frac{dB}{dp^1} = h^{1\star} \geq 0 \tag{4.29a}$$

$$\frac{dB}{dp^2} = \frac{h^{2\star}}{1+r} \geq 0 \tag{4.29b}$$

$$\frac{dB}{dX} = E_1 + \frac{E_2(1+g_v)}{1+r} = p^1 > 0 \tag{4.29c}$$

$$\frac{dB^L}{dr} = -\frac{(p_2h^{2\star}+E)}{(1+r)^2} < 0 \tag{4.29d}$$

4.3 Two-Stand Two-Good Models

4.3.1 Two Stands - One Influencing the Other

The simplest extension of the one-stand multiple-use model is to include an exogenous second stand. Here it is assumed that the objective is to maximize the net present value of benefits on one stand, denoted by L for the left stand. Assume that the level of nontimber benefits on the left stand depends on the exogenous conditions of an adjacent stand.[18] The adjacent stand is denoted by R for the right stand. This problem is conceptually identical to the problem considered by Swallow and Wear (1993):

$$\max_{h^{L1}h^{L2}} B^L = p^1 h^{L1} + E((X^L - h^{L1}); \bar{V}^{R1}) + \frac{p^2 h^{L2}}{(1+r)} + \frac{E^L((X^L - h^{L1} + g^L(X^L - h^{L1}) - h^{L2}); \bar{V}^{R2})}{(1+r)} \tag{4.30}$$

subject to the harvesting constraints,

$$h^{L1} \leq X^L \tag{4.31a}$$

$$h^{L2} \leqslant X^L - h^{L1} + g^L(X^L - h^{L1}) \tag{4.31b}$$

$\bar{V}^{Rn}$ is the exogenous level of inventory on the right stand in period n. The first-order conditions are

$$\frac{\partial B^L}{\partial h^{L1}} = p^1 - E^L_{L1}(V^{L1}; \bar{V}^{R1}) - E^L_{L2}(V^{L2}; \bar{V}^{R2}) \frac{(1+g^L_v)}{(1+r)} = 0 \tag{4.32a}$$

$$\frac{\partial B^L}{\partial h^{L2}} = \frac{p^2}{1+r} - \frac{E^L_{L2}(V^{L2}; \bar{V}^{R2})}{1+r} = 0 \tag{4.32b}$$

Equations 4.32a and 4.32b are identical to the first-order conditions for the one-stand problem except the solution now depends on $\bar{V}^{Rn}$. The difference between the optimal

[18]The stands need not be adjacent. Consider the case of migratory birds. The stands can represent two different forest habitats, summer habitat and winter habitat.

harvesting solution on an independent stand and a stand dependent on an adjacent stand will depend on the nature of interdependence between the two stands.

The comparative statics results are derived by substituting the implicit harvest solution into the first-order conditions to form the identities

$$\begin{aligned} &p^1 - E^L_{L1}(X^L - h^{L1\star}(a); V^{R1}) \\ &- E^L_{L2}(X^L - h^{L1\star}(a) + g^L(X^L - h^{L1\star}(a)) - h^{L2\star}(a); \bar{V}^{R2}) \frac{(1 + g^L_v(X^L - h^{L1\star}(a)))}{(1+r)} \equiv 0 \end{aligned} \tag{4.33a}$$

$$\frac{p^2}{1+r} - \frac{E^L_{L2}(X^L - h^{L1\star}(a) + g^L(X^L - h^{L1\star}(a)) - h^{L2\star}(a); \bar{V}^{R2})}{1+r} \equiv 0 \tag{4.33b}$$

The comparative statics results for p^1, p^2, r and X^L are identical to the one-stand case and are not repeated. Differentiate Equations 4.33 with respect to $\bar{V}^{Rn}$ and solve the system of equations to obtain

$$\frac{\partial h^{L1\star}}{\partial V^{R1}} = \frac{E_{L2L2}E_{L1R1}}{(1+r)} \frac{1}{D} \gtreqless if\, E_{L1R1} \lesseqgtr 0 \tag{4.34a}$$

$$\frac{\partial h^{L1\star}}{\partial V^{R2}} = 0 \tag{4.34b}$$

Thus, an increase in the inventory on the right stand leads to an increase (decrease) of the harvest on the left stand when the cross-partial derivative, E^L_{L1R1}, is negative (positive). Where the cross-partial is zero there is no change in the optimal harvest.

Formally, the cross-partial derivative measures the change in the marginal nontimber benefits on the left stand from a marginal change of the inventory on the right stand. The cross-partial derivative can be interpreted as a measure of substitution or complementarity between the left and right stands in producing nontimber benefits on the left stand. This is generically referred to as the indirect effect, while the second-order partial derivative is referred to as the direct effect.[19]

Now consider a specific case. Assume an increase in right-stand inventory leads to an increase of available habitat and subsequent increase of wildlife numbers on the right stand. Migration of wildlife species from the right stand to the left stand can lead to greater or lesser total wildlife values and decrease (substitute) or increase (complement) the marginal value of wildlife on the left stand. If the increase of wildlife on the left stand (via the right stand) increases total value of wildlife and does not affect the marginal value, then the left-stand harvesting decisions are unchanged. Stands are independent and the indirect effect is zero. If the marginal value of wildlife is affected by the increase in wildlife from the right stand, then the left-stand manager will find it profitable to change the level of harvest and inventory. In this case, if the indirect effect is negative (stands are substitutes), harvest is increased. If the indirect effect is positive (stands are complementary), harvest is decreased. The optimal response depends on the nature of interdependence between the stands.

The magnitude of increase in harvest depends on both the indirect effect, E^L_{L1R1}, and the second-period direct effect, E^L_{L2L2}. Therefore, the first-period harvest decision is forward looking as it explicitly considers the impact of current harvest decisions on future nontimber benefits.

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[19]Koskela and Ollikainen (1999) interpret the cross-partial as Auspitz-Lieben-Edgeworth-Pareto (ALEP) measure of substitution, independence and complementarity. This is used in the context of the interdependence between public and private forests in providing a public nontimber good (recreation). The interdependence is via the marginal valuation of nontimber benefits. The nature of the interdependence between stands is discussed further in Chapter 5.

The envelope results for p^1, p^2, r and X^L are identical to the one-stand model and are not repeated. The envelope result for a change in first-period inventory on the right stand are

$$\frac{dB^L}{d\bar{V}_{R1}} = E^L_{R1} \lessgtr 0 \tag{4.35}$$

An increase of right-stand inventory can increase or decrease the forest benefits enjoyed by the owner of the left stand. For example, recall Equation 4.5. The change in inventory (or harvest) on the right can cause a negative impact on left's nontimber benefits. For example, larger inventory on the right provides greater habitat for predator species (wolves) which then compete with prey species on the left (deer). Therefore, higher inventory on the right involves a negative production externality.

On the other hand, and probably more common, changes in inventory can positively affect the level of nontimber benefits on the left stand. This does not imply that stands are complements but simply suggests that having more inventory on each stand increases overall nontimber benefits on the left stand. This is likely the case with values that monotonically increase with the total forest inventory, e.g., recreation values, water quality, aquatic values such as fish, and many wildlife species.

To reiterate, optimal harvesting decisions on the left stand can be affected by changing conditions on adjacent stands. Swallow and Wear (1993) also find that changes on one stand may change marginal conditions on the other stand.

4.3.2 Two Stands: Asymmetric Nontimber Benefits on One Stand

An alternative two-stand problem is where two stands are controlled by one decision maker. We maintain the assumption that the level of inventory on the right stand affects the production of nontimber benefits on the left stand. We can either view this problem as one of asymmetric or symmetric interdependence. An asymmetric case might be represented by Equation 4.5. That is an asymmetry in forest level nontimber production is assumed. This is the problem depicted in Figure 2.1a-ii. The externality is internalized. However, we can also assume that each stand is an input and positively affects the production of the nontimber goods. In this case, a symmetric interdependence between the two stands is assumed. In either case, net present forest value is maximized. Now the problem becomes:

$$\begin{aligned}\max_{h^{L1},h^{R1},h^{L2},h^{R2}} B &= p^1(h^{L1}+h^{R1}) + E^L(X^L-h^{L1},X^R-h^{R1}) + \frac{p^2(h^{L2}+h^{R2})}{(1+r)} \\ &+ \frac{E^L[X^L-h^{L1}+g^L(X^L-h^{L1})-h^{L2},X^R-h^{R1}+g^R(X^R-h^{R1})-h^{R2}]}{(1+r)}\end{aligned} \tag{4.36}$$

subject to the normal harvesting constraints on each stand (Equations 4.2).

The interpretations of the results for an asymmetric or symmetric interdependence are very different. As noted above, an asymmetric interdependence is analogous to the production link between two firms assumed in environmental economics. In this case, it is assumed that the right stand is negatively or positively affecting the production or consumption of nontimber amenities on the left stand. It is assumed that activities on the left stand do not affect timber production on the right stand. On the other hand, a symmetric link between two stands assumes a different set of circumstances. If indeed the nontimber benefit is a forest-level benefit, and the manager controls both stands, there can be a technical link between the stands. This may be the case where both stands contribute to habitat (points of production) but the wildlife value is enjoyed only on the left stand (point of consumption).

Assume that the nontimber functions on each stand are concave. In particular, assume that the nontimber benefits function for the left stand has the properties: $E^L_{LnLn} < 0, E^L_{RnRn} < 0, E^L_{RnLn} = E^L_{LnRn} \lesseqqgtr 0$ and $E^L_{LnLn}E^L_{RnRn} - E^L_{RnLn}E^L_{LnRn} > 0$ for all $n=\{1,2\}$. Further, assume that conditions on the right stand can directly affect, negatively or positively, the production of nontimber benefits on the left stand, specifically $E^L_{Rn} \lessgtr 0$ for all n.

Assuming the harvest constraints are satisfied, the first-order conditions are

$$\frac{\partial B}{\partial h^{L1}} = p^1 - E^L_{L1} - E^L_{L2}\frac{(1+g^L_v)}{(1+r)} = 0 \tag{4.37a}$$

$$\frac{\partial B}{\partial h^{R1}} = p^1 - E^L_{R1} - E^L_{R2}\frac{(1+g^R_v)}{(1+r)} = 0 \tag{4.37b}$$

$$\frac{\partial B}{\partial h^{L2}} = \frac{p^2}{1+r} - \frac{E^L_{L2}}{1+r} = 0 \tag{4.37c}$$

$$\frac{\partial B}{\partial h^{R2}} = \frac{p^2}{1+r} - \frac{E^L_{R2}}{1+r} = 0 \tag{4.37d}$$

Substituting Equations 4.37c and 4.37d into 4.37a and 4.37b gives the first-period harvesting rules for each stand. The harvesting rule for the left stand is

$$p^1 = E^L_{L1} + p^2\frac{(1+g^L_v)}{(1+r)} \tag{4.38a}$$

which is identical in form to the single-stand problem (Equation 4.24). The harvesting rule for the right stand is

$$p^1 = E^L_{R1} + p^2 \frac{(1+g^R_v)}{(1+r)} \tag{4.38b}$$

which is different from the single-stand problem. The optimal decision rule on the right stand explicitly factors in its effect on left-stand nontimber benefits. The interdependence is internalized. The right stand harvesting rule says that, for optimal management, harvest until marginal first-period timber benefits are equated with marginal second-period timber benefits and left-stand marginal nontimber benefits (costs). Although the harvesting rule on the left stand is unchanged, the optimal harvest on the left stand depends on the conditions on the right stand.

The optimal harvest on each stand depends on the relationship between the two stands and whether the interdependence is symmetric or asymmetric. Consider the decision rule on the right stand. If the two stands are completely independent then the term E^L_{R1} disappears from Equation 4.38b and the optimal harvest is identical to the independent single-stand timber-only model, ceteris paribus. If $E^L_{R1} > 0$ and $E^L_{R1R1} = 0 \; \forall V^R$, so that there is an asymmetric link between stands, then net first-period price is reduced relative to the second period. Therefore, the first-period harvest, in the presence of an external cost (benefit), is less (more) than in the independent case. In the case of an asymmetric link between stands, the harvest is less or more than the independent-stand multiple-use case. It is also the case that harvest levels will be different on each stand, even if stands are identical in timber and nontimber productivity.

Now consider symmetric interdependence between stands. If stands contribute equally to nontimber benefits (both contribute equally to the production of wildlife), then harvest on each stand is identical. If stands are different in nontimber, or forest productivity or contribute unequally to its production, then stands are treated differently.

The comparative statics results for this model are found in Appendix 1 and summarized in Table 2. Except for the comparative statics results for changes of the initial inventory (see Appendix 1), the comparative statics results depend on the cross-partial derivatives.

Table 4.1 Comparative statics Results for Two Stands: Asymmetric Nontimber Benefits

Comparative statics	Sign[a]	Conditions and Comments
(a) $\frac{\partial h^{L1*}}{\partial p^1}$	+	$E^L_{L1R1} \geq 0$ or $\lvert(1+r)E^L_{R1R1} + g^R_{vv}E^L_{R2}\rvert \geq \lvert(1+r)E^L_{L1R1}\rvert$
	-	$\lvert(1+r)E^L_{R1R1} + g^R_{vv}E^L_{R2}\rvert < \lvert(1+r)E^L_{L1R1}\rvert$
(b) $\frac{\partial h^{R1*}}{\partial p^1}$	+	$E^L_{R1L1} \geq 0$ or $\lvert(1+r)E^L_{L1L1} + g^L_{vv}E^L_{L2}\rvert \geq \lvert(1+r)E^L_{R1L1}\rvert$
	-	$\lvert(1+r)E^L_{L1L1} + g^L_{vv}E^L_{L2}\rvert < \lvert(1+r)E^L_{R1L1}\rvert$
(c) $\frac{\partial h^{L1*}}{\partial p^{R1}}$	+	$E^L_{L1R1} \geq 0$
	-	$E^L_{L1R1} < 0$
(d) $\frac{\partial h^{R1*}}{\partial p^{R1}}$	+	
(e) $\frac{\partial h^{L1*}}{\partial p^2}$	-	$E^L_{L1R1} \geq 0$ or $\lvert(1+g^L_v)((1+r)E^L_{R1R1} + g^R_{vv}E^L_{R2})\rvert > \lvert(1+r)(1+g^R_v)E^L_{L1R1}\rvert$
	+	$\lvert(1+g^L_v)((1+r)E^L_{R1R1} + g^R_{vv}E^L_{R2})\rvert \leq \lvert(1+r)(1+g^R_v)E^L_{L1R1}\rvert$
(f) $\frac{\partial h^{R1*}}{\partial p^2}$	-	$E^L_{R1L1} \geq 0$ or $\lvert(1+g^R_v)((1+r)E^L_{L1L1} + g^L_{vv}E^L_{L2})\rvert > \lvert(1+r)(1+g^L_v)E^L_{R1L1}\rvert$
	+	$\lvert(1+g^R_v)((1+r)E^L_{L1L1} + g^L_{vv}E^L_{L2})\rvert \leq \lvert(1+r)(1+g^L_v)E^L_{R1L1}\rvert$
(g) $\frac{\partial h^{L1*}}{\partial p^{R2}}$	-	$E^L_{L1R1} > 0$
	+	$E^L_{L1R1} \leq 0$
(h) $\frac{\partial h^{R1*}}{\partial p^{R2}}$	-	
(i) $\frac{\partial h^{L1*}}{\partial r}$	+	conditions are similar to (e) see Equation A1i in Appendix 1
	-	
(j) $\frac{\partial h^{R1*}}{\partial r}$	+	conditions are similar to (f) see Equation A1j in Appendix 1
	-	

[a]A positive sign (+) includes zero for presentation purposes.

Only if stands are complements or independent can the comparative statics results be unambiguously signed. If $E^L_{L1R1} = E^L_{R1L1} \geq 0$, the comparative statics results are the same as the one-stand results; an increase in p^1 or r leads to an increase in the first-period harvest and decrease in inventory, while the opposite occurs with an increase in p^2. If

$E^L_{L1R1} = E^L_{R1L1} < 0$, there is the possibility that the signs of the comparative statics results are reversed. The result depends on the magnitude of the indirect effect relative to the direct effect. For the comparative statics results to be reversed, the indirect effect must be greater than the direct effect. However, this is a necessary but not sufficient condition due to the presence of other terms that share the same sign as the direct effect. Therefore, only under special circumstances do the comparative statics results take the reverse sign.

It is possible to infer the sign of the comparative statics results under particular conditions. From Young's theorem, $E^L_{L1R1} = E^L_{R1L1}$. Further, from strict concavity of the nontimber function, $E^L_{L1L1}E^L_{R1R1} - E^L_{L1R1}E^L_{R1L1} > 0$. Therefore, it is possible that E^L_{R1R1} and E^L_{L1L1} are both greater than E^L_{L1R1}, or only one is greater than E^L_{L1R1}. When both E^L_{R1R1} and E^L_{L1L1} are greater than E^L_{L1R1}, the signs of the comparative statics results are unambiguous for p^1 but remain ambiguous for p^2 and r. If either E^L_{R1R1} or E^L_{L1L1} are less than E^L_{L1R1}, the comparative statics results can not be signed. Therefore, if $E^L_{L1L1} > E^L_{L1R1} > E^L_{R1R1}$ and stands have identical growth functions, then an increase in first-period price increases the harvest on the right stand, while the harvest on the left stand increases by a lesser amount or may even decrease. Therefore, if two stands are substitutes in providing the nontimber value, a backward bending short-run timber supply is theoretically possible.

If the change in the rate of growth on the right stand (g^R_{vv}) is less than the left stand, then the right-stand harvest increases by even more relative to the left-stand harvest. Therefore, if the left stand is more productive in nontimber amenities and timber output, an increase in p^1 leads to less of a change in harvest on the left stand than the right stand. This suggests that the more productive left stand specializes in the production of nontimber production amenities. If the nontimber benefit is associated with inventory levels below the timber-only solution, the left (right) stand's first-period harvest is higher (lower) than the independent multiple-use solution. The converse occurs if the nontimber benefit is associated with inventory levels greater than the timber-only inventory. The degree of specialization will depend on the contribution each stand makes to nontimber benefits, the difference in timber productivity, and the magnitude of the first-order, E^{L}_{R1}, and second-order interaction, E^{L}_{L1R1}, between the stands.

If we consider solely a change in first-period timber price on the right stand, the comparative statics results are less ambiguous. This may occur when road access is improved in only one area of the forest resulting in increased stumpage prices on stands serviced by the road. It can also occur when a mill is built closer to one stand than the other. Results (c) and (d) in Table 4.1 report a change in left and right stand harvests from a change in the timber price on only the right stand. An increase in p^{R1} leads to an increase in right harvest and increases (decreases) left harvest if the stands are complements (substitutes). The opposite results occur if the right-stand timber price increases in the second period ((g) and (h) in Table 4.1). This suggests that physical location and interdependence between stands can lead to very different treatment of stands when stands are substitutes. However, the opposite is true with stands that are complements. If interdependence between stands is ignored, different timber prices can lead to very different harvest levels on the two stands. However, the results suggest that stands that are complementary and affect each other positively will tend to be treated similarly even with different timber prices.

The cross-partial derivative can be negative, positive or zero in equilibrium. For this reason, the comparative statics results can not be interpreted as general results as they depend on the forest growth function and the nontimber benefit functions. The results only hold at the point of equilibrium. Only if stands are globally independent or complementary do the comparative statics results hold as general results with identical stands. If stands are global substitutes, the comparative statics results are ambiguous with the exception of the first-period price.

Two general conclusions can be drawn from the comparative statics results. If stands are identical in timber and nontimber production and there is a symmetrical relationship between the two stands, harvests are also identical and the comparative statics results are identical in sign and magnitude, $\frac{\partial h^{L1}}{\partial p^1} = \frac{\partial h^{R1}}{\partial p^1} > 0, \frac{\partial h^{L1}}{\partial p^2} = \frac{\partial h^{R1}}{\partial p^2} < 0, \frac{\partial h^{L1}}{\partial r} = \frac{\partial h^{R1}}{\partial r} > 0$. On the other hand, if the stands have different growth functions, contribute differently to the production of nontimber benefits, or there is an asymmetry, such as negative (positive) first-order impact ($E^L_{Rn} \gtrless 0$), then harvests differ on each stand and the comparative statics results are different in magnitude and in some cases in sign.

The envelope results are as follows:

$$\frac{dB}{dp^1} = h^{L1*} + h^{R1*} \geq 0 \tag{4.39a}$$

$$\frac{dB}{dp^2} = \frac{h^{L2*} + h^{R2*}}{1+r} \geq 0 \tag{4.39b}$$

$$\frac{dB}{dr} = -\frac{h^{L2*} + h^{R2*}}{(1+r)^2} - \frac{E^L}{(1+r)^2} < 0 \tag{4.39c}$$

$$\frac{dB}{dX^L} = E^L_{L1} + \frac{E^L_{L2}(1+g^L_v)}{(1+r)} = p^1 > 0 \tag{4.39d}$$

$$\frac{dB}{dX^R} = E^L_{R1} + \frac{E^L_{R2}(1+g^R_v)}{(1+r)} = p^1 > 0 \tag{4.39e}$$

which are similar to the other problems.

4.3.3 Two Stands with Symmetric Nontimber Benefits

A final possibility with two-stands involves the case where conditions on each stand affect the production of nontimber benefits on the other stand symmetrically. This problem is conceptually similar to the problem posed by Swallow et al. (1997) and is depicted in Figure 1a-iii. It involves linear interdependence between the two stands. The symmetric stands problem is:

$$\begin{aligned}\max_{h^{Ln},h^{Rn}} B = {} & p^1(h^{L1}+h^{R1}) + E^L(X^L-h^{L1},X^R-h^{R1}) + E^R(X^L-h^{L1},X^R-h^{R1}) + \\ & \frac{p^2(h^{L2}+h^{R2})}{(1+r)} + \frac{E^L(X^L-h^{L1}+g^L(X^L-h^{L1})-h^{L2},X^R-h^{R1}+g^R(X^R-h^{R1})-h^{R2})}{(1+r)} \\ & + \frac{E^R(X^L-h^{L1}+g^L(X^L-h^{L1})-h^{L2},X^R-h^{R1}+g^R(X^R-h^{R1})-h^{R2})}{(1+r)}\end{aligned} \tag{4.40}$$

subject to the usual harvest constraints given by Equations 4.2.

Equation 4.40 can express two types of problems. E in Equation 4.40 might represent a nontimber value that is realized and produced on each stand or represent two different goods, E^L and E^R, that depend on both stands but realized on only one stand. Whichever case is considered, the calculus is the same for each problem.

Assuming an interior solution, the first-order conditions are

$$\frac{\partial B}{\partial h^{L1}} = p^1 - E^L_{L1} - E^R_{L1} - (E^L_{L2} + E^R_{L2})\frac{(1+g^L_v)}{(1+r)} = 0 \tag{4.41a}$$

$$\frac{\partial B}{\partial h^{R1}} = p^1 - E^L_{R1} - E^R_{R1} - (E^L_{R2} + E^R_{R2})\frac{(1+g^R_v)}{(1+r)} = 0 \tag{4.41b}$$

$$\frac{\partial B}{\partial h^{L2}} = \frac{p^2}{1+r} - \frac{(E_{L2}^L + E_{L2}^R)}{1+r} = 0 \tag{4.41c}$$

$$\frac{\partial B}{\partial h^{R2}} = \frac{p^2}{1+r} - \frac{(E_{R2}^L + E_{R2}^R)}{1+r} = 0 \tag{4.41d}$$

Substitute Equations 41c-d into Equation 41a-b to obtain the first-period harvest rules

$$p^1 = E_{L1}^L + E_{L1}^R + p^2 \frac{(1+g_v^L)}{(1+r)} \tag{4.42a}$$

$$p^1 = E_{R1}^L + E_{R1}^R + p^2 \frac{(1+g_v^R)}{(1+r)} = 0 \tag{4.42b}$$

The optimal harvesting rule on each stand fully accounts for impacts on the nontimber benefits on the other stand. If the stand-level nontimber benefits represent the same good then the harvest rules have a similar interpretation as in the previous problem. However, if these functions represent two distinct values then the harvest rule has a different interpretation. Equation 4.42b states that the optimal harvest occurs where the sum of first-period marginal benefits, which include right-stand timber and nontimber benefits and left-stand nontimber benefits, equates second-period marginal (timber) benefits. Equation 4.42a is interpreted similarly.

The comparative statics results are presented in Appendix 1 and summarized in Table 4.2. They are similar to those of the previous problem, but they are more difficult to interpret. The only unambiguous results are where both stands are complements or independent, $E_{LR}^R, E_{LR}^R \geq 0$. Other results are inferred under particular conditions. If stands are substitutes and have identical timber growth then the signs of the comparative statics results are unambiguous if $E_{SS}^s > E_{LR}^s$ for stand *s;* otherwise the sign is ambiguous. These conditions are not dissimilar to the previous problem.

If we only consider a change in right-stand timber price, new insights are gained. An increase in the first-period (second-period) price leads to an increase (decrease) in first-period (second-period) harvest on the right stand. Result c in Table 4.2 supports the inference that an increase of p^{R1} leads to an increase in harvest if both stands are complements, $E_{LR}^{L} + E_{LR}^{R} \geq 0$. If stands are substitutes in providing nontimber benefits then an increase in p^{R1} leads to a decrease in harvest and increase in inventory. If in fact there are two nontimber benefits and stands are complementary in providing one good and substitutes in providing the other, then the results are ambiguous. It will depend on the relative value of the two nontimber benefits.

Multiple stand-level nontimber functions or more than two distinct nontimber values, which is often the case in practice, increases the complexity of the problem considerably. If stands are independent or complementary in the production of the nontimber benefits, it is possible to derive comparative statics results; otherwise, the results are ambiguous.

Table 4.2 Comparative statics Results for Two Stands with Symmetric Nontimber Benefits

Comparative statics	Sign [a]	Conditions and Comments
(a) $\frac{\partial h^{L1*}}{\partial p_1}$	+	$E^L_{L1R1}, E^R_{L1R1} \ge 0$ or $\lvert(1+r)(E^L_{R1R1}+E^R_{R1R1})+g^R_{vv}(E^L_{R2}+E^R_{R2})\rvert \ge \lvert(1+r)(E^L_{L1R1}+E^R_{L1R1})\rvert$
	-	$\lvert(1+r)(E^L_{R1R1}+E^R_{R1R1})+g^R_{vv}(E^L_{R2}+E^R_{R2})\rvert \le \lvert(1+r)(E^L_{L1R1}+E^R_{L1R1})\rvert$
(b) $\frac{\partial h^{R1*}}{\partial p_1}$	+	$E^L_{R1L1}, E^R_{R1L1} \ge 0$ or $\lvert(1+r)(E^L_{L1L1}+E^R_{L1L1})+g^L_{vv}(E^L_{L2}+E^R_{L2})\rvert \ge \lvert(1+r)(E^L_{R1L1}+E^R_{R1L1})\rvert$
	-	$\lvert(1+r)(E^L_{L1L1}+E^R_{L1L1})+g^L_{vv}(E^L_{L2}+E^R_{L2})\rvert < \lvert(1+r)(E^L_{R1L1}+E^R_{R1L1})\rvert$
(c) $\frac{\partial h^{L1*}}{\partial p^{R1}}$	+	$E^L_{L1R1}, E^R_{L1R1} \ge 0$ or $E^L_{L1R1}+E^R_{L1R1} \ge 0$
	-	$E^L_{L1R1}, E^R_{L1R1} < 0$ or $E^L_{L1R1}+E^R_{L1R1} < 0$
(d) $\frac{\partial h^{R1*}}{\partial p^{R1}}$	+	
(e) $\frac{\partial h^{L1*}}{\partial p_2}$	-	$E^L_{L1R1}, E^R_{L1R1} \ge 0$ or $\lvert(1+g^L_v)((1+r)(E^L_{R1R1}+E^R_{R1R1})+g^R_{vv}(E^L_{R2}+E^R_{R2}))\rvert$ $> \lvert(1+g^R_v)(1+r)(E^L_{L1R1}+E^R_{L1R1})\rvert$
	+	$\lvert(1+g^L_v)((1+r)(E^L_{R1R1}+E^R_{R1R1})+g^R_{vv}(E^L_{R2}+E^R_{R2}))\rvert \le \lvert(1+g^R_v)(1+r)(E^L_{L1R1}+E^R_{L1R1})\rvert$
(f) $\frac{\partial h^{R1*}}{\partial p_2}$	-	$E^L_{R1L1}, E^R_{R1L1} \ge 0$ or $\lvert(1+g^R_v)((1+r)(E^L_{L1L1}+E^R_{L1L1})+g^L_{vv}(E^L_{L2}+E^R_{L2}))\rvert$ $> \lvert(1+r)(1+g^L_v)(E^L_{R1L1}+E^R_{R1L1})\rvert$
	+	$\lvert(1+g^R_v)((1+r)(E^L_{R1R1}+E^R_{R1R1})+g^L_{vv}(E^L_{L2}+E^R_{L2}))\rvert \le \lvert(1+r)(1+g^L_v)(E^L_{R1L1}+E^R_{R1L1})\rvert$
(g) $\frac{\partial h^{L1*}}{\partial p^{R2}}$	-	$E^L_{L1R1}, E^R_{L1R1} > 0$ or $E^L_{L1R1}+E^R_{L1R1} > 0$
	+	$E^L_{L1R1}, E^R_{L1R1} \le 0$ or $E^L_{L1R1}+E^R_{L1R1} \le 0$
(h) $\frac{\partial h^{R1*}}{\partial p^{R2}}$	-	
(i) $\frac{\partial h^{L1*}}{\partial r}$	+	conditions are similar to (e) see Equation A2i in Appendix 1
	-	
(j) $\frac{\partial h^{R1*}}{\partial r}$	+	conditions are similar to (f) see Equation A2j in Appendix 1
	-	

[a] A positive sign (+) includes zero for presentation purposes .

4.4 Extension to Management Effort

Here the two-stand model is extended to include management. Management effort is any silvicultural activity beyond timber harvesting such as planting, fertilization, brushing, site preparation and drainage. First, we consider two stands with two owners and then we consider two stands owned by one owner. The purpose is to determine the conditions that support the notion of single-use management areas.

Consider the problem of managing two stands for joint products where timber management on one stand is a choice variable. Assume the stand growth function, g, is a function of growing inventory, V, and the level of management effort, Q, specifically $g^s = (V^s, Q^s)$ for stand s. We assume that $g^s_Q > 0$ and $g^s_{QQ} < 0$ for all levels of Q. We continue to hold the assumption that $g^s_v \geq 0$ for $V \leq V^{MSY}$ and $g^s_v < 0$ for $V > V^{MSY}$ and $g^s_{vv} < 0$ for all V. We assume g is a strictly concave function with $g^s_{vv}g^s_{QQ} - g^s_{vQ}g^s_{Qv} > 0$.

Figure 4.3 depicts the inventory-management growth function. In both figures, Q_0 refers to without management and Q_1 refers to with management. Figure 4.3a depicts how management effort causes stand growth to increase everywhere, which also increases the maximum sustainable yield, the MSY inventory level and the carrying capacity of the site. Figure 4.3b depicts how management effort increases growth everywhere and reduces the MSY inventory, but does not affect the carrying capacity of the stand. Ovaskainen (1992) discusses the introduction of management into the growth function, emphasizing the sign of the cross-partial derivative. Ovaskainen notes that, in general, $g_{sv} \gtreqless 0$, but he proposes other signs of g_{sv} to address various forms of silvicultural prescriptions. For example, he proposes that $g_{sv} > 0$ for fertilization and drainage of the site (Figure 4.3a). These management activities increase the overall productivity of the stand. On the other hand, $g_{sv} < 0$ is consistent with management activities such as site preparation, brushing or planting of larger seedlings (Figure 4.3b). These activities do not increase site productivity but shorten the time for the stand to establish and, consequently, to reach maximum growth and carrying capacity of the stand.

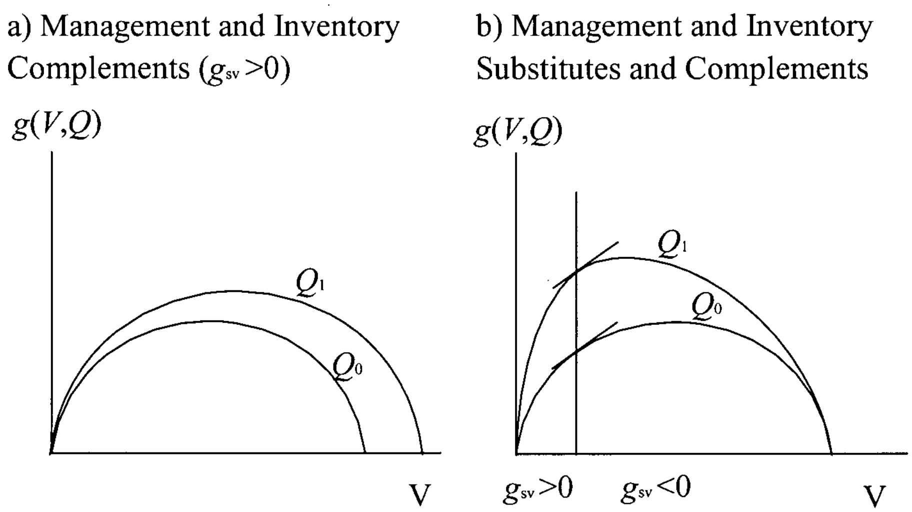

Figure 4.3 Growth Function with Management (Source: Ovaskainen 1992)

The simplest two-stand case is where the conditions on one stand, the right stand say, are considered exogenous to the decision maker. Then the problem for the manager of the left stand can be represented as:

$$\max_{h^{L1},h^{L2}} B^L = p^1 h^{L1} + E^L((X^L - h^{L1}); \bar{V}^{R1}) - wQ^L + \frac{p^2 h^{L2}}{(1+r)} + \frac{E^L[(X^L - h^{L1} + g^L(X^L - h^{L1}, Q^L) - h^{L2}); \bar{V}^{R2}]}{(1+r)} \tag{4.43}$$

subject to the harvesting constraints,

$$h^{L1} \le X^L \tag{4.44a}$$

$$h^{L2} \leqslant X^L - h^{L1} + g^L(X^L - h^{L1}, Q) \tag{4.44b}$$

$\bar{V}^{Rn}$ is the exogenous level of inventory on the right stand in period n and w is the per unit cost of management effort. The first-order conditions are

$$\frac{\partial B^L}{\partial h^{L1}} = p^1 - E^L_{L1}(V^{L1}, \bar{V}^{R1}) - E^L_{L2}(V^{L2}, \bar{V}^{R2})\frac{(1+g^L_V)}{(1+r)} = 0 \tag{4.45a}$$

$$\frac{\partial B^L}{\partial h^{L2}} = \frac{p^2}{1+r} - \frac{E^L_{L2}(V^{L2}, \bar{V}^{R2})}{1+r} = 0 \tag{4.45b}$$

$$\frac{\partial B^L}{\partial Q} = \frac{E^L_{L2}(V^{L2}, \bar{V}^{R2})}{1+r} g^L_Q - w_1 = 0 \tag{4.45c}$$

We substitute 4.45b into 4.45a and 4.45c into 4.45b to form the first-period harvesting and management decision rules:

$$p^1 = p^2 \frac{(1+g^L_V)}{(1+r)} + E^L_{L1} \tag{4.46a}$$

$$w_1 = \frac{p^2}{(1+r)} g_s \tag{4.46b}$$

The first-period harvest rule, Equation 4.46a, is unchanged from the without management problem. The management decision rule, Equation 4.46b, is identical to that obtained by Ovaskainen (1992), who only considered timber values in a two-period utility maximization problem. The rule simply states invest in timber management until the marginal present value of benefits are equated with the marginal cost of management effort. The marginal present value of benefits is equal to the marginal growth from management times the discounted second-period timber price.

The comparative statics results are presented in Appendix 1 and summarized in Table 4.3. The comparative statics results for the first-period harvest are now more ambiguous than in the model without management effort due to the presence of g^L_{Qv}. Only where $g^L_{Qv} \geq 0$ are the results unambiguous and consistent with the without-management problem. The cross-partial derivative is absent of the results for the first-period price and right-stand inventory and are thus easier to interpret.

Table 4.3 Results for left stand with management and exogenous right stand

Comparative statics	Sign[a]	Conditions and Comments
(a) $\frac{\partial h^{L1*}}{\partial p^1}$	+	
(b) $\frac{\partial h^{L1*}}{\partial p^2}$	-	$g_{vQ}^L \geq 0$ or $\lvert g_{QQ}^L(1+g_v^L)\rvert > \lvert g_Q^L g_{vQ}^L\rvert$
	+	$\lvert g_{QQ}^L(1+g_v^L)\rvert \leq \lvert g_Q^L g_{vQ}^L\rvert$
(c) $\frac{\partial h^{L1*}}{\partial r}$	+	conditions are similar to (b) see Equation A3c in Appendix 1
	-	
(d) $\frac{\partial h^{L1*}}{\partial V^R}$	+	$E_{L1R1}^L \leq 0$
	-	$E_{L1R1}^L > 0$
(e) $\frac{\partial h^{L1*}}{\partial w}$	-	$g_{vQ}^L < 0$
	+	$g_{vQ}^L \geq 0$
(f) $\frac{\partial Q^{L*}}{\partial p^1}$	-	$g_{vQ}^L > 0$
	+	$g_{vQ}^L \leq 0$
(g) $\frac{\partial Q^{L*}}{\partial p^2}$	+	$g_{vQ}^L \geq 0$ or $\{\quad\} \leq 0$ see Equation A3h in Appendix 1
	-	$\{\quad\} > 0$
(h) $\frac{\partial Q^{L*}}{\partial r}$	-	conditions are similar to (g) see Equation A3i in Appendix 1
	+	
(i) $\frac{\partial Q^{L*}}{\partial V^R}$	+	E_{L1R1}^L and $g_{Qv}^L > 0; E_{L1R1}^L$ and $g_{Qv}^L < 0; E_{L1R1}^L$ or $g_{Qv}^L = 0$
	-	$E_{L1R1}^L > 0$ and $g_{Qv}^L < 0$ or $E_{L1R1}^L < 0$ and $g_{Qv}^L > 0$
(j) $\frac{\partial Q^{L*}}{\partial w}$	-	

[a]A positive sign (+) includes zero for presentation purposes.

A new set of results is obtained for the changes in the optimal level of management on the left stand from changes in the model parameters. If $g_{Qv}^{L} > 0$, management effort increases with increases in second-period price and right-stand inventory if $E_{L1R1}^{L} \geq 0$ (complements) and decreases with first-period price, rate of interest and right-stand inventory if $E_{L1R1}^{L} < 0$ (substitutes). If $g_{Qv}^{L} < 0$, management effort increases with an increase in first-period price and an increase in right-hand inventory if $E_{L1R1}^{L} < 0$ (substitutes) and decreases with an increase in right-hand inventory if otherwise (complements). Management effort decreases with an increase in management cost. How the optimal harvest changes with changes in second-period price and rate of interest is ambiguous when $g_{Qv}^{L} < 0$.

The results support the idea of intensively managed timber areas. Assume that the inventory on the right stand increases. From Table 4.3, if stands are substitutes, harvest and management are increased on the left stand where management effort (e.g., planting) can efficiently substitute for inventory. Therefore, if timber markets support the production of early-aged trees and stands are substitutes in providing nontimber values, which require older trees and larger inventory conditions, then dedicating one forest stand to intensive timber production and the other to nontimber production may be economically efficient. On the other hand, the opportunity for specialization is narrow as there are more conditions which favor no intensification of management or if so, it is done to increase inventory to complement the adjacent forest stand.

The above model can be extended to a solely-owned, two-stand forest with timber management on one stand. Assume management is feasible only on the right stand. This is assumed so as to keep the problem as simple as possible. The problem for the manager of the two stands is

$$\max_{h^{s1},h^{s2}} B^L = p^1 h^{L1} + E^L(X^L - h^{L1}, X^R - h^{R1}) - w_1 Q^R + \frac{p^2 h^{L2}}{(1+r)}$$
$$+ \frac{E^L(X^L - h^{L1} + g^L(X^L - h^{L1}) - h^{L2}, X^R - h^{R1} + g^R(X^R - h^{R1}, Q^R) - h^{R2})}{(1+r)} \tag{4.47}$$

The first-order conditions are:

$$\frac{\partial B^L}{\partial h^{L1}} = p^1 - E^L_{L1} - E^L_{L2}\frac{(1+g^L_v)}{(1+r)} = 0 \tag{4.48a}$$

$$\frac{\partial B^L}{\partial h^{R1}} = p^1 - E^L_{R1} - E^L_{R2}\frac{(1+g^R_v)}{(1+r)} = 0 \tag{4.48b}$$

$$\frac{\partial B^L}{\partial h^{L2}} = \frac{p^2}{1+r} - \frac{E^L_{L2}}{1+r} = 0 \tag{4.48c}$$

$$\frac{\partial B^L}{\partial h^{R2}} = \frac{p^2}{1+r} - \frac{E^L_{R2}}{1+r} = 0 \tag{4.48d}$$

$$\frac{\partial B^L}{\partial Q^R} = \frac{E^L_{R2}}{1+r} g^R_Q - w = 0 \tag{4.48e}$$

We substitute 4.48c into 4.48a, 4.48d into 4.48b and 4.48e into 4.48d to form the first-period harvesting and management decision rules.

$$p^1 = E^L_{L1} + p^2\frac{(1+g^L_v)}{(1+r)} \tag{4.49a}$$

$$p^1 = E^L_{R1} + p^2\frac{(1+g^R_v)}{(1+r)} \tag{4.49b}$$

$$w = \frac{p^2}{(1+r)} g^R_Q \tag{4.49c}$$

The comparative statics results are very complicated and largely ambiguous when the cross-partial derivative of the nontimber function is non zero. A summary of the results is presented in Table 4.4 while the full results are found in Appendix 1.

The comparative statics result for first-period harvests are similar to the two-stand model without management although they are more complicated. Where $E^{L}_{L1R1} \geq 0$, first-period price has the same result as the previous 2-stand model. The results for second-period price, rate of interest and management costs all result in the same signs as the without management model if $E^{L}_{L1R1} > 0$ and $g^{R}_{Qv} > 0$.

A change in the price on only the right stand, p^{R1} or p^{R2}, generates interesting comparative statics results. Consider the case of $g^{R}_{Qv} < 0$ and $E^{L}_{L1R1} < 0$. From (c) and (l) in Table 4.4, an increase p^{R1} leads to an increase in first-period right harvest, a decrease in left harvest and an increase in management effort on the right stand. In other words, if the stands substitute for one another in the provision of the nontimber good and timber prices rise on the right stand, the right stand is intensively managed for timber while the harvest on the left stand decreases. However, where $g^{R}_{Qv} < 0$ and $E^{L}_{L1R1} \geq 0$ is the case, harvest is increased on both stands and management is also increased on the right stand. Where $g^{R}_{Qv} > 0$ and $E^{L}_{L1R1} \geq 0$, left-stand harvest increases and management decreases. If $g^{R}_{Qv} > 0$ and $E^{L}_{L1R1} < 0$, both left-stand harvest and right-stand management effort decrease. The comparative statics results for an increase in p^{R2} are ambiguous except when $g^{R}_{Qv} > 0$ and $E^{L}_{L1R1} \geq 0$. Where $g^{R}_{Qv} > 0$ and $E^{L}_{L1R1} \geq 0$, first-period left- and right-stand harvest decrease and right stand management effort increases.

Table 4.4 Results for Two Stands with Management on One

Comparative statics	Sign[a]	Conditions and Comments
(a) $\frac{\partial h^{L1*}}{\partial p^1}$	+	$E^L_{L1R1} \geq 0$ or $\{\ \} \leq 0$ see Equation A4a in Appendix 1
	-	$\{\ \} > 0$
(b) $\frac{\partial h^{R1*}}{\partial p^1}$		conditions similar to (a) see Equation A4b
(c) $\frac{\partial h^{L1*}}{\partial p^{R1}}$	+	$E^L_{L1R1} \geq 0$
	-	$E^L_{L1R1} < 0$
(d) $\frac{\partial h^{R1*}}{\partial p^{R1}}$	+	
(e) $\frac{\partial h^{L1*}}{\partial p_2}$	-	g^R_{vQ} and $E^L_{L1R1} \geq 0$; $\{\ \} < 0$ see Equation A4e
	+	$\{\ \} \geq 0$
(f) $\frac{\partial h^{R1*}}{\partial p^2}$		conditions similar to (e) see Equation A4f
(g) $\frac{\partial h^{L1*}}{\partial p^{R2}}$	-	$g^R_{vQ} \geq 0$ or $\{\ \} < 0$ and $E^L_{L1R1} > 0$; $\{\ \} > 0$ and $E^L_{L1R1} < 0$ see Equation A4g
	+	$E^L_{L1R1} \geq 0$ and $\{\ \} \geq 0$; $E^L_{L1R1} \leq 0$ and $\{\ \} \leq 0$
(h) $\frac{\partial h^{R1*}}{\partial p^{R2}}$	-	$g^R_{vQ} \geq 0$; $\{\ \} < 0$ see Equation A4h
	+	$\{\ \} \geq 0$
(i) $\frac{\partial h^{L1*}}{\partial r}$	+	g^R_{vQ} and $E^L_{L1R1} \geq 0$; $\{\ \} \leq 0$ see Equation A4i
	-	$\{\ \} > 0$
(j) $\frac{\partial h^{R1*}}{\partial r}$		conditions similar to (i) see Equation A4j
(k) $\frac{\partial h^{L1*}}{\partial w}$	+	E^L_{L1R1} and $g^R_{vQ} \lesseqgtr 0$; E^L_{L1R1} or $g^R_{vQ} = 0$
	-	$E^L_{L1R1} > 0$ and $g^R_{vQ} < 0$; $E^L_{L1R1} < 0$ and $g^R_{vQ} > 0$
(l) $\frac{\partial h^{R1*}}{\partial w}$	+	$g^R_{vQ} \geq 0$
	-	$g^R_{vQ} < 0$
(m) $\frac{\partial Q^{R*}}{\partial p^1}$	+	$E^L_{L1R1} \geq 0$ and $g^R_{Qv} > 0$; g^R_{Qv} and $\{\ \} < 0$ see Equation A4q
	-	$E^L_{L1R1} > 0$ and $g^R_{Qv} < 0$; $g^R_{Qv} > 0$ and $\{\ \} < 0$
(n) $\frac{\partial Q^{R*}}{\partial p^{R1}}$	+	$g^R_{Qv} \leq 0$
	-	$g^R_{Qv} > 0$
(o) $\frac{\partial Q^{R*}}{\partial p^2}$	+	E^L_{L1R1} and $g^R_{Qv} \geq 0$; $\{\ \} \leq 0$ see Equation A4s
	-	$\{\ \} > 0$
(p) $\frac{\partial Q^{R*}}{\partial p^{R2}}$	+	$g^R_{vQ} \geq 0$; $\{\ \} \leq 0$ see Equation A4t
	-	$\{\ \} > 0$
(q) $\frac{\partial Q^{R*}}{\partial r}$	+	E^L_{L1R1} and $g^R_{vQ} \geq 0$; $[\ ] \geq 0$ see Equation A4u
	-	$[\ ] < 0$
(r) $\frac{\partial Q^{R*}}{\partial w}$	-	

[a]A positive sign (+) includes zero for presentation purposes.

Intensive Timber Management Areas

The notion that specific areas of the forest be intensively managed for timber is supported by the above analysis. The analysis suggests that, if intensive timber management is economically feasible, then intensive timber management is supported in areas where nearby forests (stands) substitute for reduced nontimber benefits on the intensively managed stand and if intensive timber management leads to faster growth (Figure 4.3b). This is likely where nontimber benefits are favored at low inventory levels (young ages), stands are homogeneous, and stands respond well to management activities such as planting, brushing and site preparation. In such areas, stands outside the intensive timber zone will have adjusted inventories so as to generate higher nontimber benefits. The adjustment in management will depend on whether nontimber benefits require low or high inventory conditions.

In areas where stands are complementary in providing nontimber benefits, the optimal intensive timber management takes the form of activities that improve site productivity, such as fertilization or improved drainage. In these areas, stands outside the intensive timber zone will also have adjusted inventories to generate higher nontimber benefits.

Therefore, intensive timber management areas are economically efficient if conditions support increased management effort (future returns justify expenditures) and are consistent with the interdependencies between stands in the forest. Therefore, the question of whether intensive timber management areas are economically efficient is an empirical question as it is a theoretical possibility.

4.5 Three Stands Again

Consider the management of three stands again. Recall that as there are many possible outcomes with three stands that unambiguous interpretation of the comparative statics analysis is impossible . However, we can make some speculative suggestions of the comparative statics results based on the results in the preceding sections. Consider again the case first examined in Equation 4.6. Recall that the left and middle stands produce the nontimber benefit W. Also, all stands produce nontimber benefit E and each stand is dependent on adjacent stand conditions. To infer a solution requires knowing the interdependency between the left and middle stands in producing W and the interdependencies between the three stands in producing E. It is also necessary to know the relative values of the nontimber goods and if they favor low or high inventory conditions. Assume that the left and middle stands are complementary in producing W and that this nontimber benefit reaches a maximum at high inventory levels. Also assume that all stands are substitutes in producing E and that E reaches a maximum at low inventory levels. Finally, assume that all stands are identical in productivity and timber price is equal on each stand. On the basis of these assumptions, the lowest levels of harvest (highest inventory) will occur on the left stand and the highest (lowest inventory) on the right stand. The middle stand is likely to have a harvest level (inventory) somewhere between the two other stands. The resultant forest is, therefore, likely to have a heterogeneous stand structure across the forest landscape.

Combining the insights for the three-stand case with those of the management model, it is possible to speculate further. Imagine a three-stand forest where the left and middle stands are substitutes, the middle and right stands are complementary, and the left and right stands are independent. Also assume that the timber price is higher on the middle stand, due to its location vis à vis a road. The situation is depicted in Figure 4.4.

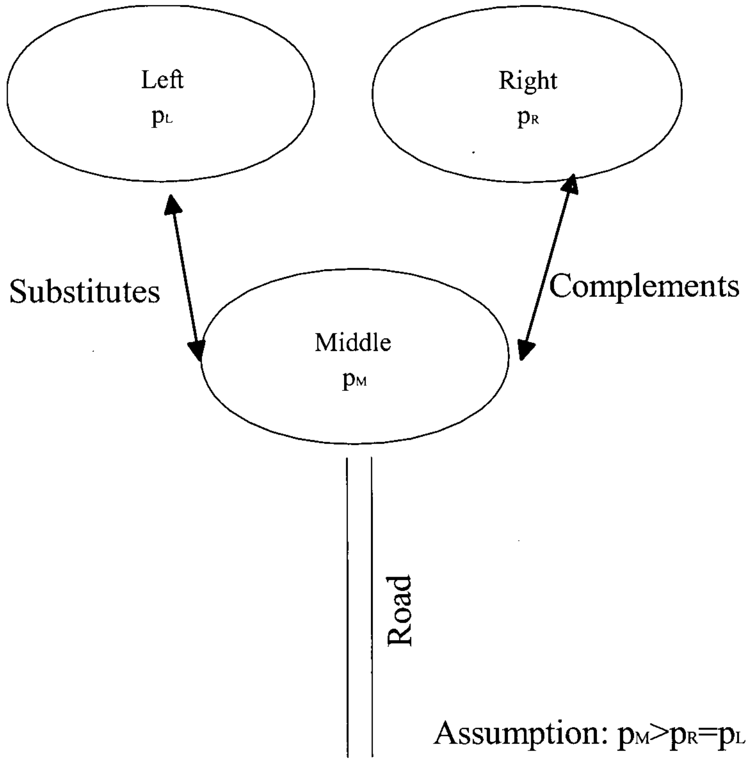

Figure 4.4 Two Stands are Complementary and Two Stands are Substitutes

Assume that tree planting is only marginally economical and that nontimber benefits occur at high inventory levels. Therefore, if stands are treated independently, the optimal harvest is lower than the timber-only harvest. Now consider an increase of the timber price on the middle stand. An increase in p^M makes tree planting more economical and management on the middle stand increases. The price increase also encourages an increase in harvest and lower inventory on the middle stand. The optimal response on the left stand is to decrease harvest and increase inventory. The optimal response on the right stand is to increase harvest and decrease inventory. The end result is variable management effort across the forest and a heterogeneous forest structure.

4.6 Conclusions

The two-period, net present value, harvest-inventory model produces results similar to those of the utility maximization two-period models for one and two stands. The addition of a third stand increases the complexity of the problem greatly. However, in general, the presence of complementarity and substitutability between stands in a forest encourages some areas of the multiple-stand forest to be managed similarly and others to be managed differently, perhaps for specialized uses.

Several results were generated from the case studies.

1. If stands are identical and have perfectly symmetrical interdependence, then all stands are treated identically.
2. Stands that are complementary to one another require similar treatment.
3. If stands are different, first-period harvest levels and inventory are different.
4. The solution on each stands depends on the form of interdependence between stands, the timber price on each stand, forest productivity and productivity of amenities.
5. Heterogeneous stands or asymmetric interdependence between stands in a forest support differentiated treatment of stands.
6. Land-use specialization for intensive timber management may be economically superior in some circumstances. In particular, land-use specialization is likely to be superior on stands that support intensive timber management activities that substitute for inventory, such as planting and brushing, and the production of nontimber benefits on nearby stands substitute for the lost nontimber benefits on the intensively managed stand.

In the next chapter, numerical simulations of various hypothetical cases of the three-stand model are provided to gain a better understanding of the solutions to a multiple-use problem with stand interdependence.

CHAPTER 5

SIMULATIONS WITH THREE STANDS

In this chapter, a set of dynamic programming problems are solved to demonstrate the implications of multiple values on the spatial management of a forested area for the case of three spatially-differentiated stands. A brief discussion of the possible forms of nontimber functions is provided in Section 5.1, while the forage-timber problem first considered by Swallow and Wear (1993) to demonstrate the relevance of non-convex nontimber functions is recast in Section 5.2. The implication of more than two goods, in particular a forest-level nontimber benefit function with technological interdependence among three stands is considered in Section 5.3. Conclusions are presented in section 5.4.

5.1 Modeling Nontimber Benefits

Central to the problem of modeling multiple-use forest management is the form of the nontimber benefits function. Presumably there are two components to a nontimber function: a marginal value function (or price function), and a technical production function. The nontimber benefits function can be expressed as $E(V^i, V^j) = f(N(.))N(.)\ \forall$i,j, where $f(.)$ is a marginal value function and $N(.)$ is a nontimber production function. The nontimber function may express stand-level benefits that depend on the conditions of surrounding stands or may express forest-level benefits.

The marginal value function assigns a value to the last unit of output produced/consumed. The marginal value function can be a constant or declining function of quantity, both of which are common in consumer theory. Mathematically, the marginal value function for nontimber amenities, $f(N)$, has the properties that $f'(N) \leq 0$ and $f''(N) \leq 0$. A constant value is consistent with the notion of a price taker in the market; supply from the forest is small relative to the market or there are many alternative sources of supply or substitutes. A declining marginal value function is likely where there are few substitutes goods or the good is unique.

The marginal value function needs to be consistent with the circumstances considered. Consider deer, for example, whose marginal value may reflect their value for food, leather, fur and medicinal ingredients or simply recreational value associated with hunting or viewing. The marginal value function for a particular supply area (forest) may be constant if deer are a homogeneous good with many substitutes and supply is small compared to the overall market for deer. Conversely, if the nontimber good produced in the forest region is unique then the marginal value changes with the amount produced from the forest. This may be the case for big game animals, such as Grizzly bear.

The function describing nontimber production, $N(\ )$, is likely to be of great research interest. Unlike the marginal value function, that is either constant or downward sloping, the production function can take on many more forms. Knowing the relationship between each stand in the forest and the production of the nontimber good is of critical importance in modeling the multiple-use problem. The simplest case is where each stand is physically independent from each other. The production of nontimber amenities for technologically independent stands can be expressed as,

$$N = N^L(V^L) + N^M(V^M) + N^R(V^R) \tag{5.1}$$

The nontimber production is assumed to be separable, with conditions on one stand in the forest not affecting the technological production of nontimber goods and services on the other stand.[1] However, it may be the case that stand-level production of a nontimber good is dependent on the conditions of other stands in the forest. Therefore, forest-level production is more accurately expressed as

$$N = N^s(V^L, V^M, V^R),\ s=\{L,M,R\} \tag{5.2}$$

It is not clear how to capture a specific interdependence between two or more stands. As illustrated in Chapter 4, many forms of asymmetry and interdependence can be modeled. In the next section, the case where two stands are linked via a marginal forage value function is examined. We then extend the analysis to three stands and other nontimber goods.

5.2 Timber and Forage Problem

In this section, a forage-timber problem is considered using a forage function first explored by Swallow and Wear (1993). The problem links two stands in a forest via a marginal value function for forage. The link is a pecuniary interdependence between the two stands and results in a nonconvex nontimber benefits expression. Here, the problem is cast in the simple two-period framework of Chapter 4 and the solution is explored using 3-dimensional graphics of the objective function.

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[1]A common nontimber example in the optimal rotation literature is the production of deer (Calish et al 1978, Swallow and Wear 1993, Swallow et al 1997). Deer require forage and shelter areas. To model this problem within a Hartman framework, a partial analysis is assumed. The production of deer on a particular stand is conditional on the availability of adequate habitat on surrounding stands. Therefore, the stand-level deer production function measures the level of deer benefits as the stand conditions change over time, holding conditions on surrounding stands constant. Under these assumptions, nontimber production from a collection of a few stands is simply the sum of production from each stand. This problem is explored in detail below.

Originally Swallow et al (1997) found the optimal sequence of harvest rotations on two adjacent stands that maximize the net present value of timber and forage values over a infinite period of time. This extended Swallow and Wear (1993) who considered two interdependent stands with separate ownership/control. The timber and nontimber functions used in each paper are the same. The authors solved these applied problems using a dynamic programming algorithm, where the decision in each period is to clear cut one or both stands given the histories of each stand. This formulation of the problem is in keeping with the Faustmann tradition and even-aged forest management.

Now the forage-timber joint-production problem of a two-stand forest is recast within the discrete harvest-inventory framework of Chapter 4. This involves converting the forage function used by Swallow and Wear (1993) to be a function of inventory instead of stand age.

The problem involves choosing the amount of timber to harvest from each stand at each decision period to maximize net present value over a finite time horizon. The choice of harvest is tantamount to choosing the level of inventory to leave between periods; the decision maker chooses the level to consume/harvest and to save/inventory in each decision period. Holding a level of inventory between harvests accomplishes two things: inventory grows according to some known production relationship allowing greater future timber consumption, and inventory conditions provide habitat and forage conditions for deer.

Forage Value Function

A forage production function that relates forage to stand age can be transformed to a function of inventory by assuming a correspondence between stand age and inventory.

If we assume a one-to-one correspondence between stand inventory and merchantable volume, the functions utilized by Swallow and Wear (1993) can be converted into a function of inventory and used in the two-period model of Chapter 4.

Swallow and Wear (1993) use the following specification for stand-level forage production

$$f(t^{sn}) = \beta_{0s} t^s e^{-\beta_{1s} t^s} \tag{5.3}$$

where $f(t)$ estimates forage production from stand s in animal-unit-months (AUM) per year for a stand of age t^s. Different values of β_0 generate different levels of production. High production is associated β_0=0.0770 and low production with β_0=0.0616; β_1=0.085.

The timber function utilized by Swallow and Wear (1993) was derived from data for the Lolo National Forest in western Montana. Timber production is assumed to be represented by a logistic growth function

$$V^s(t^{sn}) = \frac{K^s}{(1 + e^{a_s - \theta_s t^{sn}})} \tag{5.4}$$

where V^s is volume (thousands of board-feet, mbf) per acre on stand s for trees of age t^{sn} and K^s is the carrying capacity of land on stand s. The estimated values are: K=15.055,

α=6.1824 and θ=(0.0801, 0.06408) for high and low production sites. Figure 5.1 illustrates the timber production function for both high and low quality sites.

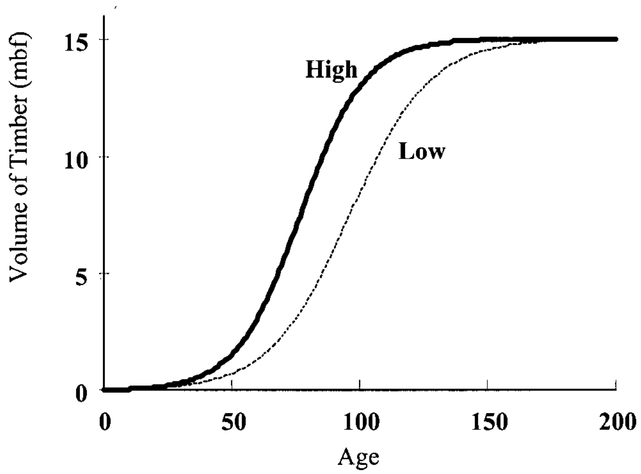

Figure 5.1 Timber Yield for High and Low Productivity Stands

As the logistics function is monotonic its inverse function exists and is

$$t^s(V) = \frac{a - \ln\left(\frac{K}{V} - 1\right)}{\theta} \tag{5.5}$$

Substituting Equation 5.5 into Equation 5.3 yields

$$f^s(V^s) = \beta_0\left(\frac{a - \ln\left(\frac{K^s}{V^s} - 1\right)}{\theta}\right)e^{-\beta_1\left(\frac{a-\ln\left(\frac{K^s}{V^s}-1\right)}{\theta}\right)} \tag{5.6}$$

for stand s, which can be rewritten as

$$f^s(V^s) = (a - b\ln(\tfrac{K^s}{V^s} - 1))e^{-c+d\ln\left(\frac{K^s}{V^s}-1\right)} \tag{5.7}$$

where a, b, c and d take the values reported in Table 5.1 for high and low sites. Only low timber and low forage, and high timber and high forage, are considered in the analysis. High and low site values reported in Table 5.1 refer to a stand that has both high or low productivity parameter values for Equations 5.5 and 5.3. There are two other parameter sets that correspond to stands having low productivity in only one good. These parameter sets are not necessary to demonstrate the issue of non convexities and so are omitted.

Table 5.1: Parameter Values for Forage Production Function

	Parameter Values			
Site Productivity	*a*	*b*	*c*	*d*
High	5.94	0.96	6.56	1.06
Low	5.94	0.96	8.20	1.33

The marginal value of forage is assumed to have the following form (Swallow and Wear 1993):

$$a(t^L, t^R) = f_0 e^{-\psi[f^L(t^L)+f^R(t^R)]} \tag{5.8}$$

so that the marginal forage value is an inverse function of the total quantity of forage produced from the two-stand forest. The upper limit on the value of forage is f_0 and ψ is an adjustment parameter, where f_0=\$30/AUM and ψ=2. This produces a range of marginal values between \$8 and \$29 per AUM.

Substituting Equation 5.7 for stand *L* and *R* into Equation 5.8 gives:

$$a(V^L, V^R) = f_0 e^{-\psi[f^L(V^L)+f^R(V^R)]}, \tag{5.9}$$

so that marginal forage value is a function of the inventory on the two stands. Swallow and Wear (1993) calculated forest-level forage benefits by multiplying Equation 5.8, the marginal valuation of forage, by the sum total of stand forage production, Equation 5.3:

$$E(t^L, t^R) = [f^L(t^L) + f^R(t^R)] \cdot f_0 e^{-\psi[f^L(t^L)+f^R(t^R)]} \tag{5.10}$$

Equation 5.10 is illustrated in Figure 5.2 for two stands with high forage productivity. Note that maximum forage benefits occur at young stand ages, there are multiple optima, and the function has non-convex regions.

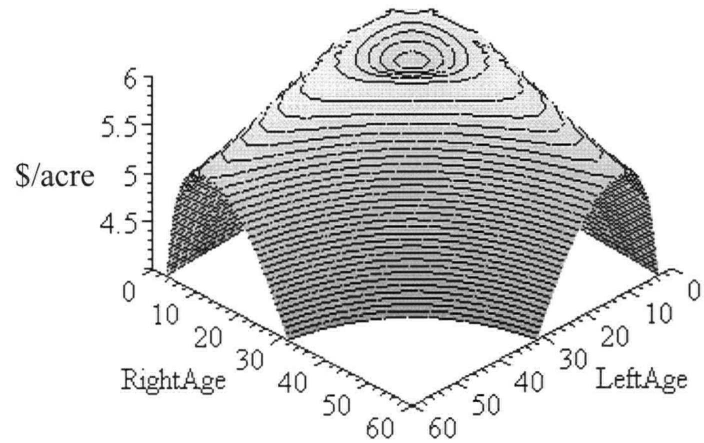

Figure 5.2 Forage Benefits for Two High Forage Stands - Age

The forage-benefits-inventory function is obtained likewise by multiplying Equation 5.9 by the sum of forage production from stands L and R, Equation 5.7,

$$E(V^L, V^R) = [f^L(V^L) + f^R(V^R)] \cdot f_0 e^{-\psi[f^L(V^L)+f^R(V^R)]} \qquad (5.11)$$

Three-dimensional graphs obtained from Equation 5.11 for various parameter values are depicted in Figure 5.3. Figures 5.3a and 5.3b illustrate how Equation 5.11 changes with productivity on both stands and Figure 5.3c demonstrates how it changes with forage productivity (note how the graph is slightly skewed towards the right inventory axis). Although Equation 5.11 was obtained with some heroic assumptions it is not too dissimilar from Equation 5.10. Comparing Figure 5.3a (high forage) with Figure 5.2 (high forage), one sees that multiple optima occur at low inventory and low stand ages.[2]

[2]At very low inventory levels (young ages) the function exhibits a donut shape. This demonstrates that there are many combinations of inventories (or ages) that produce the

Equation 5.11 also leads to multiple optima and nonconvex regions.

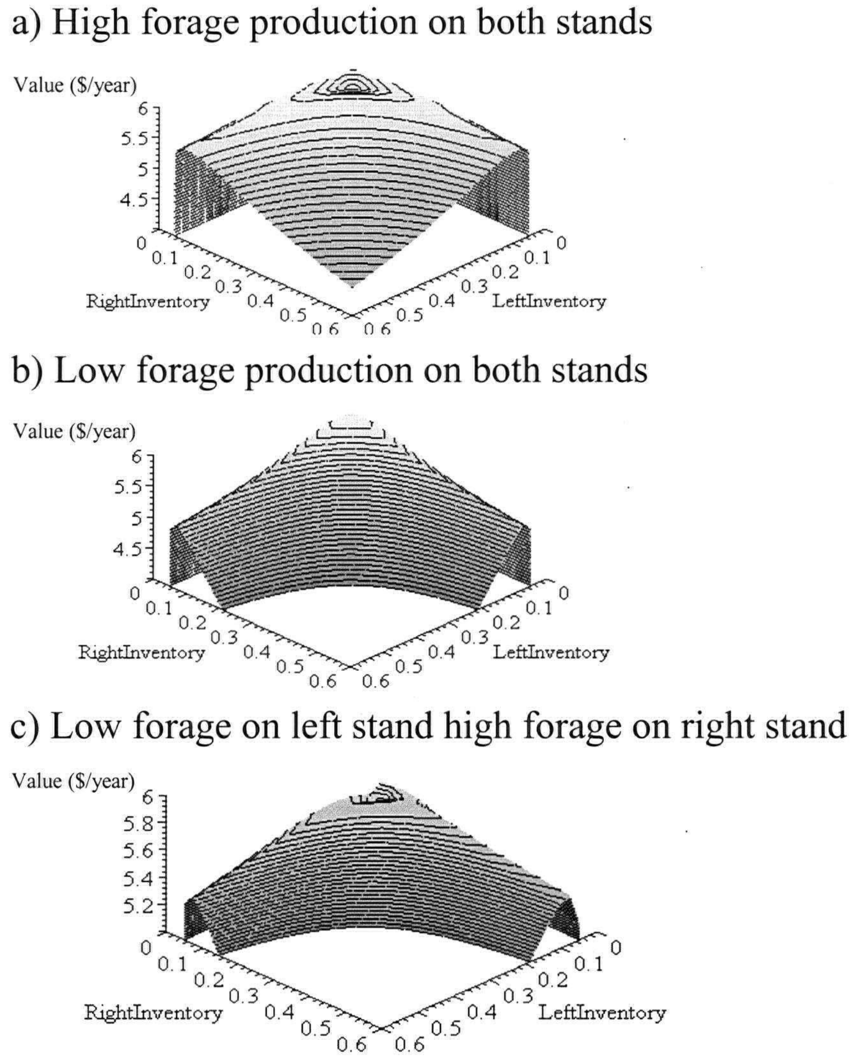

Figure 5.3 Forage Benefits for Two Stands - Inventory

Timber Growth Function

A simple quadratic growth function is assumed for the analysis. This form was suggested by Ovaskainen (1992) and is based on the logistics yield function. It is used

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maximum level of forage benefits from the forest. In fact, maximum forage benefits for two high productivity stands occur where a combination of 0.5 AUM are produced from both stands. Therefore any combination of inventory levels (or ages) that produce a sum total of 0.5 AUM produce the maximum value of forage benefits (maximum forest forage benefits are $5.52/acre).

simply to demonstrate the relevance of nonconvex, nontimber functions, rather than to validate previous studies. The quadratic growth function is

$$g^s(V^s) = \gamma_s V^s - \eta_s V^{s^2} \tag{5.12}$$

where γ is the maximum incremental growth attainable on the stand and η is a growth adjustment parameter. Incremental growth is

$$\frac{dg^s(V^s)}{dV^s} = \gamma_s - 2\eta_s V^s \tag{5.13}$$

As a first approximation of the parameter values, some of the information from the logistics growth function used by Swallow and Wear (1993) is borrowed, and it is assumed that there is a one-to-one correspondence between inventory and merchantable volume.

Figure 5.4 illustrates the annual growth or current annual increment (CAI) and mean annual increment (MAI) for a high quality site (a) and low quality site (b) derived from Equation 5.4. The age that maximizes the maximum flow of timber volume over time from successive clear cuts, the maximum sustained yield (MSY) rotation, occurs when the MAI is equal to the CAI (average growth is maximized). The MSY occurs at an age of 102 years for high sites and 127 years for low sites. The volume that corresponds with these rotation ages, for high and low sites, is a volume of 13.24 mbf. We will assume that 13.24 mbf corresponds with the maximum value (maximum growth) of Equation 5.12.

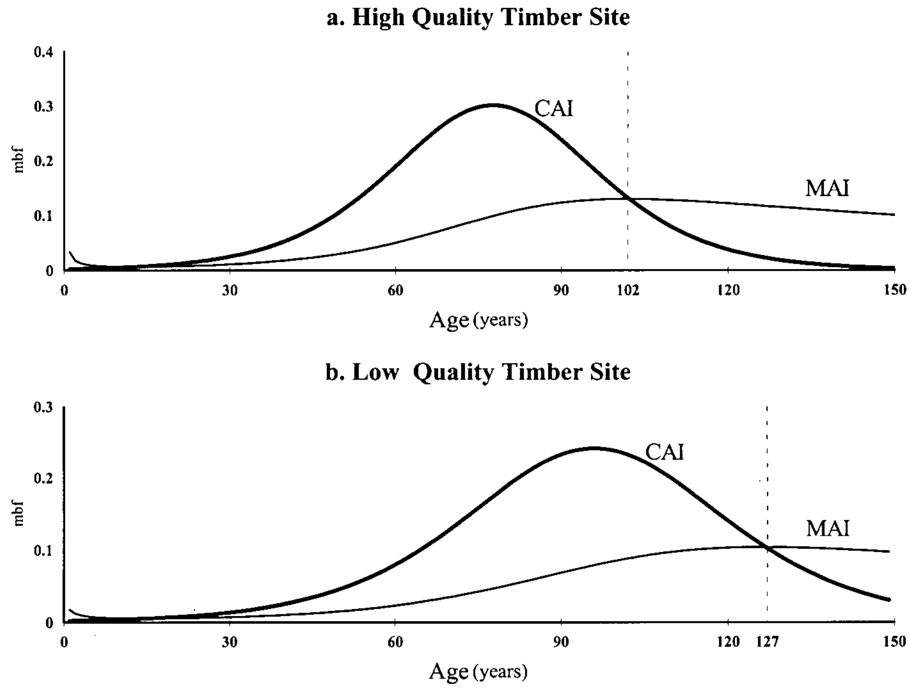

Figure 5.4 Current and Mean Annual Increment of High and Low Timber Stands

Assume that γ takes a value equal to θ in Equation 4 for one year of growth, and continue to assume that harvest decisions are every 5 years. To account for 5 years of growth, assume growth is constant between harvest periods and simply multiply θ by a factor of 5. Therefore, γ=0.4 and γ=0.32 for high and low growth stands, respectively. The slope of Equation 5.13, 2η, is readily obtained as we know two coordinates, (γ,0) and (0,13.24). Therefore, η=0.015 and η=0.012 for high and low stands, respectively. The parameter values for Equation 5.12 are summarized in Table 5.2. Figure 5.5 illustrates the quadratic growth function and incremental growth for high and low parameter values of Equations 5.12 and 5.13.

Table 5.2 Parameter Values for Quadratic Growth Function

Site productivity	Intercept (γ)	Slope (η)
High	0.4	-0.015
Low	0.32	-0.012

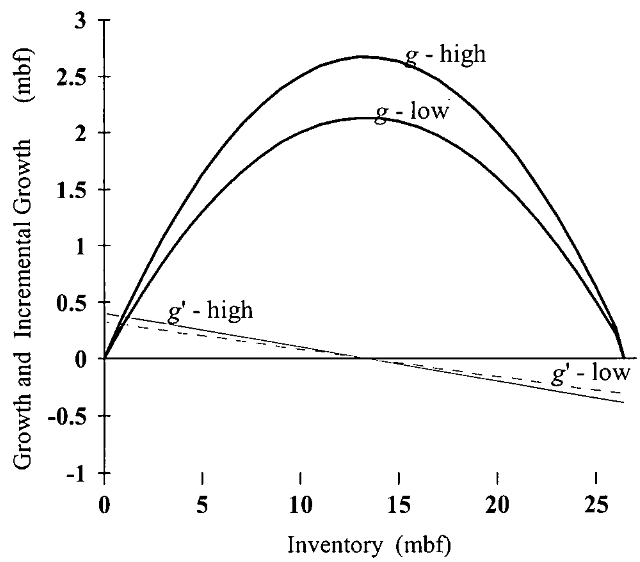

Figure 5.5 Growth and Incremental Growth for High and Low Stands

Forest-Level Objective Function

Before solving the multiple-period dynamic programming algorithm, considerable insight can be gained as to the form of the objective function and probable solutions with the use of graphical illustrations. Substituting Equations 5.11 and 5.12 into Equation 4.1, and assuming that all timber is cut by the end of the second period, yields

$$\max_{h^{L1},h^{R1}} B = p^1(h^{L1} + h^{R1}) + [f^L(X^L - h^{L1}) + f^R(X^R - h^{R1})] \cdot f_0 e^{-\psi(f^L(X^L-h^{L1})+f^R(X^R-h^{R1}))}$$

$$+ \frac{p^2\begin{pmatrix} X^L - h^{L1} + (\gamma^L(X^L - h^{L1}) - \eta^L(X^L - h^{L1})) \\ +X^R - h^{R1} + (\gamma^R(X^R - h^{R1}) - \eta^R(X^R - h^{R1})) \end{pmatrix}}{(1+r)^5} \tag{5.14}$$

Three-dimensional contour plots of Equation 14 are presented in Figure 5.6. In each plot the inventory endowment is 12 mbf and p^1 and p^2 are \$80/mbf.

For the graphical analysis in Figure 5.6 and subsequent dynamic programming simulations, harvest decisions are 5 years apart. This interval is somewhat arbitrary but is necessary to reduce the computational burden of the multiple-period dynamic programming problem. Therefore, to approximate the net present value of forage benefits, forage benefits are assumed to be constant between harvests. Between harvests, inventory contributes to the production of annual forage benefits according to Equation 5.9. The present value of an annual series that terminates in 5 years is $Z_0 = a\frac{(1+r)^5 - 1}{r(1+r)^5}$, where a is an annual value and r is an annual discount rate. Therefore, we multiple Equation 5.11 by Z_0 to approximate the NPV of five years of forage benefits.

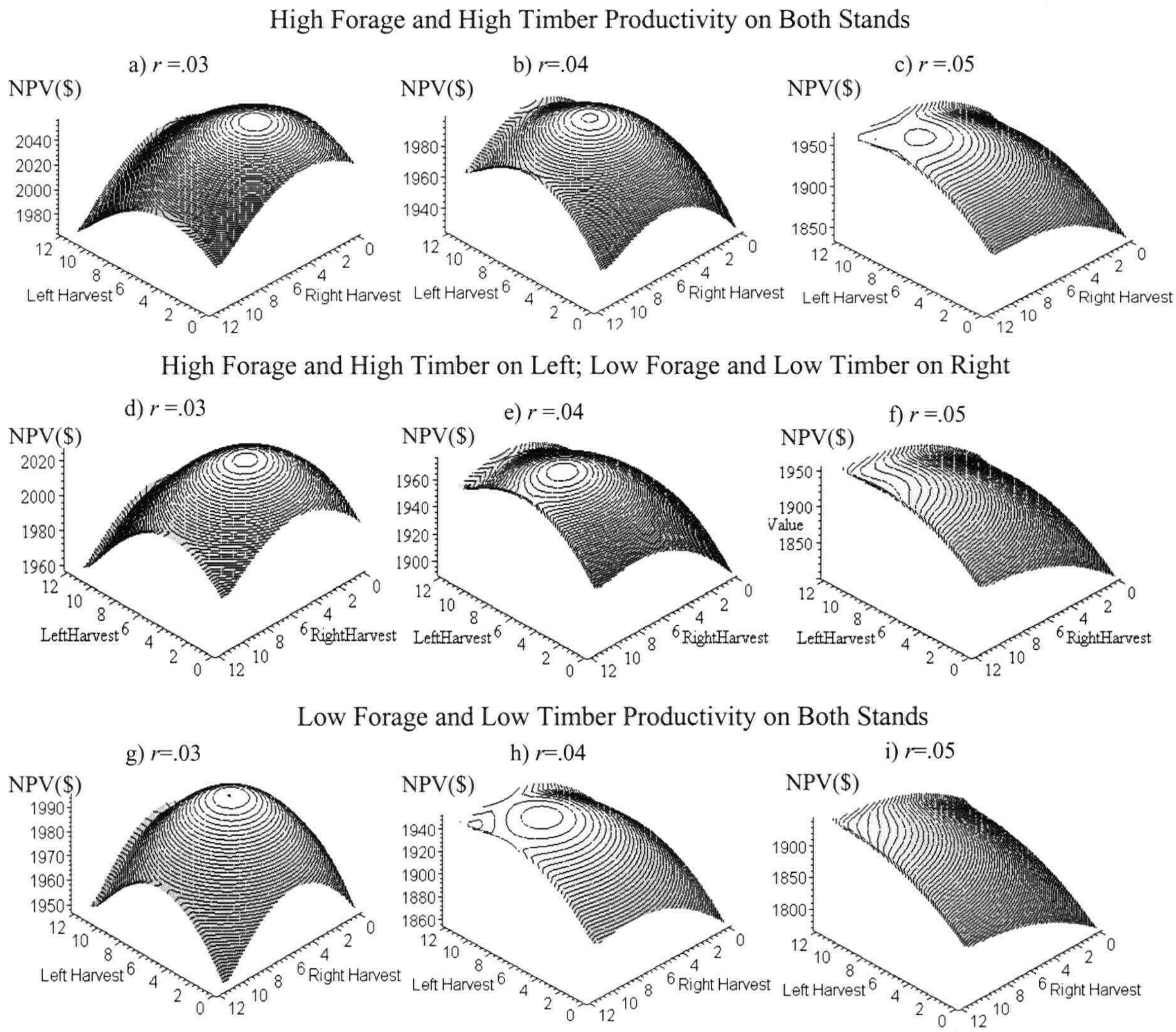

Figure 5.6 Forest Benefits for Two Stands

Figures 5.6a-c are for high forage and timber parameter values and different discount rates, r=(0.03, 0.04, 0.05). Figures 5.6d-f illustrate the objective function for high forage and timber production on the left stand and low forage and timber production the right stand. Figures 5.6g-i illustrate the objective function for the case of low forage and timber parameter values on each stand.

The graphs in Figure 5.6 suggest several conclusions.

1. The form of the objective function depends greatly on the assumed discount rate. A global maximum results for low discount rates, while higher discount rates are associated with multiple optima.

2. There are two types of optima. There is an interior optimum and two border optima. Where the stands are identical the interior optimum is symmetrical. The border optima are not corner solutions but in fact one stand has very low inventory and the other high inventory. That is, one stand specializes in nontimber benefits and the other in timber benefits.

3. The global optimum tends towards an asymmetric solution as the discount rate is increased. This last result is consistent with findings in Chapter 4. Recall that the optimal response to an increase in the discount rate is to increase harvest (see Equations 4.18c and 4.28c for one stand). As the inventory is reduced, the relative value of forage benefits to timber benefits increases as high forage values occur at low inventory levels. Therefore, as the discount rate rises, nontimber benefits become more significant. Given the shape of the forage function (Figure 5.3), it is not surprising that an asymmetric optimum becomes superior to a symmetric optimum as the discount rate is increased.

4. The value of the function decreases with an increase in the discount rate and with a reduction in the productivity of the stand in timber and forage.

5. Lastly, and most importantly, the graphical representation in Figure 5.6 supports the conclusion that the optimal harvest on identical stands can be different.[3]

As the graphical presentation is limited to two harvest periods and binding second-period harvest constraints, the conclusions are tenuous. Next we attempt to

[3]Swallow and Wear's (1993) analysis found alternating harvests on two stands (an asymmetric solution) with a real discount rate of 4 percent.

confirm the conclusions suggested from the graphical analysis of Equation 14 with the use of a multiple-period dynamic programming algorithm.

5.3 Dynamic Program

Here we Equation 5.14 is extended to multiple periods. The benefit of a multiple-period model is that harvesting constraints for each period can remain as inequalities and permit final period inventories.

The model considered has 20 harvest decision periods that occur every 5 years over a 100 year time horizon. The overall objective is to choose a timber harvest in each period on each stand to maximize the net present sum of benefits over the planning horizon. Specifically, a level of harvest h^s on each stand in each period n for n=1...20 is chosen to generate periodic timber benefits, price of timber times total harvest, plus annual nontimber benefits. Harvest revenues and nontimber benefits are realized at the start of each decision period.

The optimization problem in period n is

$$\max_{h^{Ln},h^{Rn}} B = \frac{p^n(h^{Ln}+h^{Rn})}{(1+r)^{5(n-1)}} + Z_0 \frac{[f^L(V^{Ln})+f^R(V^{Rn})]\cdot f_0 e^{-\psi(f^L(V^{Ln})+f^R(V^{Rn}))}}{(1+r)^{5(n-1)}} \tag{5.15}$$

At each decision node, a harvest of all to nothing can be removed from each stand. A constant nontimber benefit is realized for 5 years and is discounted to the beginning of the period. Forest renewal or growth on each stand is not realized until the beginning of the next harvest period. At the end of each period the inventory is renewed to a new starting inventory equal to last period inventory plus growth. The inventory renewal equation

$$X_{n+1}^s = X_n^s + g^s(X_n^s) \tag{5.16}$$

describes the state of the forest from one period to the next, where X_n^s is the stock or inventory in period n and $g(\ )$ is inventory growth between harvesting periods. The growth function is described by Equation 5.12 and the parameter values are presented in Table 5.2. In each period, the total amount harvested can not exceed the level of inventory available $h^{sn} \le X_n^s$.

The multiple-period dynamic program is solved using GAMS and its nonlinear solver MINOS. The basic dynamic programming algorithm for three stands is presented in Appendix 2.

5.4 Results for Forage-Timber Problem

The dynamic programming algorithm was solved for various scenarios. The scenarios were created by varying the number of stands from one to three, timber and forage productivity, the price of timber, the maximum forage value and the discount rate. The results from the scenarios are presented in Tables 5.3-5.6. Discount rates of 3, 4, 5, and 6 percent are used, with stumpage prices of \$80/mbf and \$84/mbf, and values of \$30/AUM and \$60/AUM for forage, f_0. The base-case values are r=.04, $p_1 = p_2$ =80, and f_0=30. In all scenarios, the starting inventory on each stand, X^s, is 12 mbf.

Independent stands

First- and second-period harvest and inventory solutions are presented for independent stands (or one stand) are presented in Table 5.3. The first and second period harvest and inventory are reported along with the NPV for the entire 100-year time horizon. The first-period results can be interpreted as short-term while the second-period are long run or steady state as the solutions are unchanged for subsequent periods. The second column indicates the values considered while the third column indicates the quality of timber and nontimber production. The fourth column indicates the assumed discount rate.

Table 5.3 Timber Harvest and Inventory (mbf) for One Stand

Case	Values	Stand Conditions	r	Period 1 h^{L1}	Period 1 V^{L1}	Period 2 h^{L2}	Period 2 V^{L2}	NPV
	Timber Only							
a)		high	3	3.98	8.02	2.24	8.02	$1,415
b)		high	4	5.88	6.11	1.88	6.11	$1,162
c)		high	5	7.88	4.12	1.39	4.12	$1,033
d)		low	3	5.3	6.7	1.61	6.7	$1,213
e)		low	4	7.7	4.31	1.16	4.31	$1,040
f)		low	5	10.18	1.82	0.54	1.82	$971
	Forage Only							
g)		high	4	11.92	0.08	0.03	0.08	$126
h)		low	4	11.93	0.07	0.03	0.07	$115
	Forage and Timber							
i)		high	3	4.1	7.91	2.23	7.91	$1,423
j)		high	4	6.1	5.91	1.84	5.91	$1,170
k)		high	5	11.91	0.09	0.03	0.09	$1,065
l)		low	3	5.36	6.64	1.6	6.64	$1,214
m)		low	4	11.93	0.07	0.02	0.07	$1,077
n)		low	5	11.93	0.06	0.02	0.06	$1,053

Cases a-f in Table 5.3 are for timber values only. It is clear that the timber only first-period harvest (inventory) increases (decreases) with an increase in the discount rate and a reduction in timber productivity. Second-period harvest decreases with an increase in r and a decrease in $g(.)$. It is also clear that an increase in r and a decrease in $g(.)$ reduces NPV. Similar results hold for the forage and timber problem (Cases i-n).

Cases g and h in Table 5.3 consider only high and low forage benefits. A decrease in forage productivity decreases NPV.

Cases i-n combine forage and timber benefits. Cases i-k consider high forage and timber production on the stand while Cases l-n consider low forage and timber production. It is clear that joint-production leads to greater stand benefits. The inclusion of nontimber benefits increases first-period harvest and decreases the optimal growing inventory. This is consistent with the findings in Chapter 4. Further the inclusion of nontimber benefits decreases the level of subsequent harvests. This result is not available from the two-period model of Chapter 4.

Forage-Timber Problem with Two Stands

Now consider the forage/timber problem with two stands, presented in Tables 5.4 and 5.5.

Table 5.4 Timber Harvest and Inventory Two Stand Timber-Forage Problem

	Conditions			Period 1				Period 2				
Case	Left	Right	*r*	h^{L1}	h^{R1}	V^{L1}	V^{R1}	h^{L2}	h^{R2}	V^{L2}	V^{R2}	NPV
a)	high	high	3	4.09	4.09	7.91	7.91	2.23	2.23	7.91	7.91	$2,842
b)	high	high	4	6.08	6.08	5.92	5.92	1.84	1.84	5.92	5.92	$2,339
c)	high	high	5	7.94	11.91	4.06	0.09	1.38	0.03	4.06	0.09	$2,099
d)	high	low	3	4.09	5.36	7.91	6.64	2.23	1.6	7.91	6.64	$2,635
e)	high	low	4	5.94	11.93	6.06	0.07	1.87	0.02	6.06	0.07	$2,241
f)	high	low	5	7.98	11.93	4.02	0.07	1.37	0.02	4.02	0.06	$2,090
g)	low	low	3	5.36	5.36	6.64	6.64	1.6	1.6	6.64	6.64	$2,426
h)	low	low	4	7.34	11.93	4.26	0.07	1.15	0.02	4.26	0.07	$2,118
i)	low	low	5	11.9	11.9	0.1	0.1	0.03	0.03	0.1	0.1	$2,031

Three sets of results are presented in Table 5.4. Cases a-c consider two identical stands with high forage and timber productivity for three different discount rates. Cases d-f consider one stand with high forage and timber and another with low forage and timber for three different discount rates. Cases g-i consider two identical stands with low forage and low timber productivity for three different discount rates.

As in the one stand case, an increase in the discount rate and a decrease in productivity decreases NPV, increases first-period harvest and decreases inventory and subsequent harvests levels.

Stands also have equal harvests and inventory for some cases, while harvests and inventory levels are different for other cases. This result confirms what was suggested by the graphical illustrations of the objective function over two periods with no ending inventory.

From Cases a-c an asymmetric solution occurs for a discount rate greater than 4 percent while in Cases d-i an asymmetric solution occurs for discount rates greater than 3 percent. An asymmetric result also occurs when stands are different. Therefor, an asymmetric solution occurs where stands are different or where discount rates are high relative to growth rates and stands have low nontimber productivity.

Where stands are identical, there is no spatial decision, even though the optimal harvest on each stand can be different. The reason is that NPV is invariant to which stand has a low harvest and which a high harvest. This is not the case with physically different stands. In this case, a spatial decision is important; the stand that has low productivity specializes in nontimber production, while the more productive stand is used for timber production. Therefore, a spatial decision is necessary to achieve the maximum NPV.

Table 5.5 Timber Harvest and Inventory of Two High Quality Stands for Select Cases of the Forage-Timber Problem

		Period 1				Period 2				Period 3				
Case		h^{L1}	h^{R1}	V^{L1}	V^{R1}	h^{L2}	h^{R2}	V^{L2}	V^{R2}	h^{L3}	h^{R3}	V^{L3}	V^{R3}	NPV
a)	p^1=84	7.86	7.86	4.14	4.14	1.37	1.37	5.54	5.54	1.84	1.84	5.92	5.92	$2,389
b)	p^2=84	4.07	4.07	7.93	7.93	4.23	4.23	5.93	5.93	1.85	1.85	5.93	5.93	$2,414
c)	f_0=60	5.95	11.91	6.05	0.08	1.87	0.03	6.05	0.08	1.87	0.03	6.05	0.08	$2,381

In Table 5.5, three more cases are reported that demonstrate the effect of changes in the price of timber and the maximum marginal forage value on the optimal solutions.

In each case the discount rate is assumed to be 4 percent. Case a in Table 5.5 considers an unexpected increase in the first-period timber price. The result is an increase in first-period harvest, and decreases in inventory and second-period harvest. Inventory and harvests reach the steady state levels reported in Table 5.4 by the third period. Case b in Table 5.5 constitutes a known increase in the timber price in the second and subsequent periods. The result is a decrease in first-period harvest, increase in first-period inventory, a large increase in second-period harvest and then a marginal increase in the steady-state harvest and inventory level reported in Table 5.4. Finally, case c in Table 5.5 considers an increase in the maximum marginal forage value, f_0. The results reported are for a value of \$60/AUM however, but this was not the only value investigated as f_0 was increased by increments of 10 until an asymmetric solution resulted. This demonstrates that, similar to the discount rate, the relative value of forage is an important determinant of the optimal harvest.

How do the solutions on each stand differ in the two stand endogenous model from the one stand solutions? For cases with interior solutions (symmetrical solutions for identical stands) we see by comparing Tables 5.3 and 5.4 it is clear that solutions are very similar. For asymmetric stands, the solutions are very different from the one stand solutions. Indeed, with asymmetry, one stand should specialize in production of timber and the other in nontimber amenities.

Forage-Timber Problem for Three Stands

Finally, consider three stands. The results for two cases for the forage/timber problem with three interdependent stands and two different discount rates are reported in

Table 5.6. Cases a and b in Table 5.6 show that the inclusion of a third stand marginally changes the solutions reported in Table 5.4. The direction of change is to decrease first-period harvest and increase inventory. By induction, this suggests that the influence of nontimber benefits on the harvest solutions decreases as the number of stands increases. Heroically, we can conclude that the non-convex features of the forage benefits function become a non issue as the forest scale reaches some critical level. This is a tenuous conclusion. As Bowes and Krutilla (1982) noted, the analysis does not consider the opportunity cost of expanding operations to a larger scale. That said, if harvesting and access costs are constant and there is no alternative use for the land the conclusion is plausible.

Table 5.6 Timber Harvests and Inventories for Three High Quality Stands for Different Nontimber Values

	Forest Values				Period 1						Period 2						
Case	Left	Middle	Right	r	h^{L1}	h^{M1}	h^{R1}	V^{L1}	V^{M1}	V^{R1}	h^{L2}	h^{M2}	h^{R2}	V^{L2}	V^{M2}	V^{R2}	NPV
a)	E	E	E	4	6.07	6.07	6.07	5.93	5.93	5.93	1.85	1.85	1.85	5.93	5.93	5.93	$3,508
b)	E	E	E	5	7.93	7.93	11.91	4.07	4.07	0.09	1.38	1.38	0.03	4.07	4.07	0.09	$3,134
c)	C	C	C	4	5.56	4.04	5.56	6.44	7.96	6.44	1.95	2.23	1.95	6.44	7.96	6.44	$3,690
d)	C	C	C	5	7.51	5.97	7.51	4.49	6.03	4.49	1.49	1.87	1.49	4.49	6.03	4.49	$3,222
e)	W	W	W	4	5.1	5.1	5.1	6.9	6.9	6.9	2.04	2.04	2.04	6.90	6.90	6.90	$4,109
f)	W	W	W	5	6.74	6.74	6.74	5.24	5.24	5.24	1.68	1.68	1.68	5.24	5.24	5.24	$3,557
g)	E,W	E,W	E,W	5	6.95	6.95	6.95	5.05	5.05	5.05	2.02	2.02	2.02	5.05	5.05	5.05	$3,581
h)	E,W	E,W	E,W	6	11.9	8.53	8.53	0.1	3.48	3.48	0.04	1.21	1.21	0.10	3.48	3.48	$3,293
i)	C,E	C,E	C,E	5	7.81	6.16	7.81	4.19	5.84	4.19	1.41	1.83	1.41	4.19	5.84	4.19	$3,250
j)	C	C,E	E	4	5.55	3.85	6.08	6.45	8.15	5.92	1.96	2.26	1.84	6.45	8.15	5.92	$3,710
k)	C	C,E	E	5	7.49	5.82	8.3	4.51	6.18	3.7	1.5	1.9	1.28	4.51	6.18	3.70	$3,246
l)	C	C,E	E	6	9.45	7.69	11.92	2.55	4.31	0.08	0.92	1.45	0.03	2.55	4.31	0.08	$3,067

5.5 Addition of More Nontimber Goods

The model is extended to the case of two nontimber values. Consider a late seral stage or high inventory good, such as recreation or wildlife value, that is best suited to high density or older-aged stand structure. The function employed here is from Swallow et al (1990), who suggest that the stand-level production of spotted owl, red-cockaded woodpecker, squirrels and scenic views can be captured by the following function

$$W = \frac{w}{1 + e^{(c_0 - c_1 t^s)}}, \tag{5.17}$$

where w is the maximum level of production, c_0 and c_1 are parameter values, and t^s is stand age. Assuming that there is a one-to-one correspondence between stand age and stand inventory, the stand-level nontimber function is

$$W = \frac{w}{1 + e^{(c_0 - c_1 V^s)}} \tag{5.18}$$

In the absence of estimates for the parameters, values are assigned so that maximum values are greater than maximum forage values and reach a maximum at inventory levels greater than the timber-only inventory reported in Table 5.3. Let w=1, c_0=0.2, and c_1=0.3, and assume that each unit of output has a constant marginal value of \$10 (stands are not assumed to be interdependent). These parameter values generate stand-level nontimber production and benefit functions illustrated in Figure 5.7. The benefit function is quasi-concave (maximum occurs in a concave region) with maximum nontimber benefits and production occurring around a stand volume of 20 mbf, which exceeds the MSY timber volume.

Cases e and f in Table 5.6 consider the case of timber and nontimber benefits, W, for two different discount rates. Inclusion of this benefit decreases the first-period harvest and increases inventory and subsequent harvest levels. This is consistent with the findings in Chapter 4, which predict that the optimal joint-production inventory exceeds the timber-only inventory when the maximum nontimber value occurs at a high inventory level. Note that all stands are treated the same as stand-level production and marginal value are independent of production on other stands.

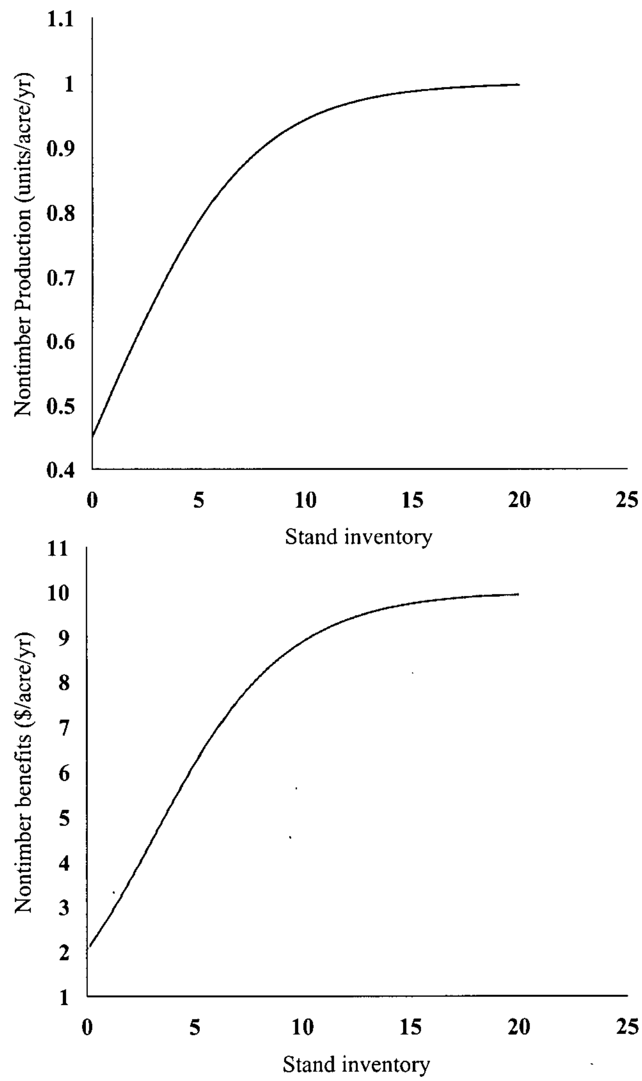

Figure 5.7 Wildlife Production Function

The case of timber and nontimber values *E* and *W* are presented as Cases g and h in Table 5.6. Two discount rates are used, 5 and 6 percent. By comparing g and a in Table 5.4, it is clear that the inclusion of *W* changes the stands from being treated asymmetrically to symmetrically. The asymmetric treatment of the stands now only occurs at high discount rates (case h). This suggests that the presence of a third nontimber value, that occurs at high levels of inventory, may dominate nontimber values that occur at low inventory levels. Therefore, the influence of non-convex nontimber functions on the optimal harvesting solution is dependent on the weight of this value relative to other values. This conclusion is consistent with the findings from the optimal rotation model (Bowes and Krutilla 1989).

A third nontimber function considered here is a forest-level benefit expressed by a Cobb-Douglas function:

$$C = K(V^L)^{\lambda}(V^M)^{\mu}(V^R)^{\rho} \tag{5.19}$$

where K, λ, μ, and ρ are parameters. In Equation 5.19, a nontimber benefit (C) is dependent on conditions on all three stands in the forest. Assume that C is concave so that K, λ, μ, $\rho \geq 0$ and $\lambda+\mu+\rho<1$; $C_{V^sV^s} < 0 \; \forall s$ and $C_{V^sV^m} > 0 \; \forall m \neq s$; and the Hessian matrix, D, is negative definite. The stands are complementary in producing C. Assuming that only the left and middle stands contribute to the production of the nontimber benefit C, Equation 5.19 can be rewritten as:

$$C = K(V^L)^{\lambda}(V^M)^{\mu} \tag{5.20}$$

Assume that K=1.5, λ=0.1, μ=(0.7,0.8) and ρ=(0,0.1), and that the per unit price of nontimber good C is \$1. Again, the chosen parameter values are ad hoc and have no scientific foundation. However, the function demonstrates the issue of technical externalities absent from previous work in this area. Equation 5.20 is depicted in Figure 5.8 for μ=0.8 for three exogenous inventory levels on the middle stand. As the inventory level is increased on the middle stand, nontimber benefits increase for all left-stand inventory levels. Also note that, a doubling of inventory on the middle stand from 3 mbf to 6 mbf to 12 mbf does not double nontimber benefits, i.e., there are diminishing returns to scale.

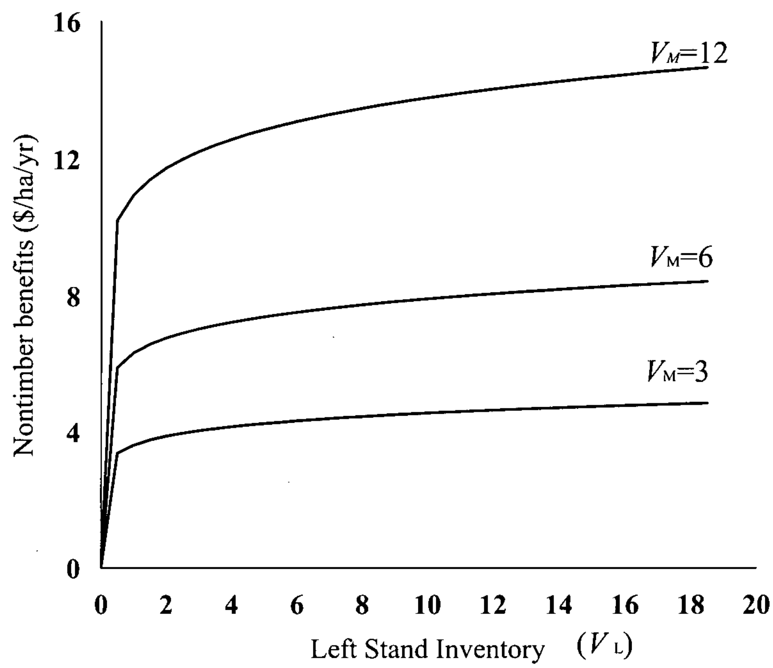

Figure 5.8 Cobb-Douglas Forest-Level Nontimber Function

The inclusion of the nontimber benefits C into Equation 5.15 is presented in Table 5.6 (Cases c and d and i through to l). Cases c and d exclude forage benefits (E), which are included in case i. Cases j through to l consider the scenario where the left and middle stands contribute to the nontimber benefit C and the middle and right stands contribute to forage benefits. These cases demonstrate that:

1) stands technologically related (not solely related by the marginal value function) will always be treated differently if the stands contribute differently to the provision of the nontimber benefit; and

2) the presence of more than two goods on the three stands and the spatial allocation of nontimber benefits can generate very different optimal harvest patterns across the forest.

5.6 Conclusions

The numerical simulations demonstrate the influence of non-convexities in a manner similar to previous studies (Swallow and Wear 1993; Swallow et al 1997; Rose 1999). However, unlike these studies, the simulations suggest non-convexities need not be important determinants of management strategies. Many factors are important in determining how closely or differently stands are treated. These factors include nontimber productivity, relative values, the discount rate and timber productivity.

Truly spatial problems occur when more than two stands and two goods are considered. The optimal solutions for such spatial problems diverge greatly from the solutions derived from one- and two-stand analyses. This suggests that ignoring the explicit location of stands, spatial scale of habitats, and interdependence among stands involves an opportunity cost. The results also suggest very different management strategies can arise in different forest regions even when the technical production relationships and objectives are identical. The converse is also suggested. It is possible that similar management strategies are followed in different jurisdictions even if the physical forest, prices and discount rates are different.

CHAPTER 6

CONCLUSIONS

In this dissertation, the two-stand model of multiple-use forestry was simplified and generalized. Many of the results from previous two-period models were shown to be special cases of this general model. For instance, when stands were assumed independent results of previous one-stand models were produced. Furthermore, the importance of stand interdependence was more readily derived from this simplified model than previous models. In particular, it was found that the presence of nontimber benefits in a two-stand forest can lead to short-term backward bending timber supply on one of the stands. The two-stand model was also extended to consider silvicultural management intensity. It was found that allocating land to single-use management for timber production is efficient if both capital and technology can substitute for land in timber production and forest management can be adjusted in other forest areas to achieve the optimal mix of timber and nontimber values. However, the results in general support the conclusion that there is no one optimal land management regime; the optimal land management regime depends on the circumstances considered. Next, a three-stand model was specified, and showed, through numerical simulations, that the results differ qualitatively from the two-stand case. The differences arise out of the possibility of asymmetries and non-adjacencies in the three-stand case. Finally, it was found that the discount rate, timber productivity, nontimber productivity, and relative values are all important factors in determining whether it is optimal for some forest stands to specialize in the production of a single forest value or many values.

The results from this work have important policy and forest planning implications. The work supports the argument that forest regulations and taxes used to promote or protect nontimber values need to be sensitive to the spatial location of stands, the natural physical differences of stands, and stand interdependencies. Therefore, forest policy tools and initiatives that promote or support the principle of socially optimal use of forest resources, such as forest practices laws and land-use zoning frameworks, need to vary from one forest region to another to account for spatial considerations; in general, there is no one dominant land management paradigm.

In many jurisdictions of the world, including British Columbia, governments have introduced various laws that attempt to regulate timber harvesting practices on public and private land as a means to promoting multiple-use forest management and achieving forest resource use efficiency. In some cases, laws are stringent blanket approaches, such as in British Columbia (see Cook 1998), that do not do well to account for linkages and interdependencies between forest areas, regional differences in marginal values, or differences in forest ecosystems, while others are flexible, such as in Sweden (see van Kooten et al. 1999). The results from the model indicate that initiatives which do not account for spatial differences and are inflexible over time will fail to achieve the policy goal of efficient resource use. It could be argued that stringent blanket approaches are a first effort towards the socially optimum when faced with little or no information. However, as asserted by Brown et al (1993), forest practices laws and programs need to adapt over time as information becomes known so as to better account for the spatial and temporal aspects of forestry, and achieve better social outcomes. If forest practice laws fail to do so, they will certainly fail to achieve resource use efficiency.

Another key policy issue in multiple-use forestry is the question of zoning forests for single-uses versus integrated resource management everywhere. The analysis presented in this dissertation provides support for both positions, depending on the particulars of the situation studied. However, one case of interest and importance obtains when it is possible to substitute capital and technology for land in timber production. This is the case with management intensity discussed in Section 4.4. Under these circumstances, and the additional circumstances that other forest areas can be managed to provide the optimal mix of forest values, the results clearly support the idea of intensive use zones.

Proponents of zoning often have argued that zoning will better promote multiple-use forestry and achieve the social optimal outcome by reducing resource use conflicts, and thereby reduce transaction costs, better focus management priorities, and provide greater security essential for forest management investments. However, opponents of zoning argue that it prevents future land-use changes and, therefore, simply trades off short-term savings for larger long-term costs. As stated, the analysis presented in this thesis supports the use of zones under specific circumstances. However, the results also indicate that the optimal zones can change with changes in values over time. Therefore, the issue of institutional inflexibility with regard to zones is important and relevant.

To ensure that forest land is allocated to and managed for its most efficient use over time, be it for a single or multiple uses, a transparent and well designed compensation policy is likely an important and complementary policy. A compensation policy is in essence a form of insurance against the complete loss of property in the event that forest land is reallocated among users for alternative uses. A compensation policy therefore affects the risks associated with undertaking intensive silvicultural activities in well defined intensive timber zones on public forests or on private lands. In designing a compensation policy, policy makers must be aware of the "tradeoffs" involved. A compensation policy can create incentives that are not in the public's interest. For example, a full compensation policy could encourage too much investment while a zero compensation policy might encourage too little. A full compensation policy can lead to too much investment due to moral hazard. Moral hazard is when a company does not consider the possibility that the current land area might best be used for another purpose in the future. As a consequence, the company might invest too much into timber management, for example, if it does not consider the full risk of the land being reallocated to another use. On the other hand, a zero compensation policy may not provide enough security for companies when government policies are rapidly changing or vague. On the other hand, if compensation is set at a high level it could constrain a Government's ability to reallocate land to its highest valued use. Therefore, a compensation policy needs to consider the tradeoff between the Government's ability to reallocate resources and encouraging the socially optimal level of investment.

Compensation policy essentially affects property rights, in particular it affects the security of property. Other government policies as well as social and cultural institutions will also affect property rights and thus the incentives to manage and exploit forest resources. Understanding how institutions affect incentives and how these can be designed to change with time is an area of important research in multiple use forestry.

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Appendix 1 Comparative statics Results

The comparative statics results for problem 4.36, the simple two-stand problem with nontimber benefits on the left stand, are as follows:

$$\frac{\partial h^{L1*}}{\partial p^1} = \frac{H_1\{-(1+r)E^L_{R1R1} - g^R_{vv}E^L_{R2} + (1+r)E^L_{L1R1}\}}{(1+r)^3}\frac{1}{D_1} \begin{array}{l} \geq 0 \text{ if } E^L_{L1R1} \geq 0 \text{ or } \{\ \} \geq 0 \\ < 0 \text{ if } \{\ \} < 0 \end{array} \quad \text{(A1a)}$$

$$\frac{\partial h^{R1*}}{\partial p^1} = \frac{H_1\{-(1+r)E^L_{L1L1} - g^L_{vv}E^L_{L2} + (1+r)E^L_{R1L1}\}}{(1+r)^3}\frac{1}{D_1} \begin{array}{l} \geq 0 \text{ if } E^L_{R1L1} \geq 0 \text{ or } \{\ \} \geq 0 \\ < 0 \text{ if } \{\ \} < 0 \end{array} \quad \text{(A1b)}$$

$$\frac{\partial h^{L1*}}{\partial p^{R1}} = \frac{H_1 E^L_{L1R1}}{(1+r)^2}\frac{1}{D_1} \begin{array}{l} \geq 0 \ \text{ if } E^L_{L1R1} \geq 0 \\ < 0 \ \text{ if } E^L_{L1R1} < 0 \end{array} \quad \text{(A1c)}$$

$$\frac{\partial h^{R1*}}{\partial p^{R1}} = -\frac{H_1((1+r)E^L_{L1L1} + g^L_{vv}E^L_{L2})}{(1+r)^3}\frac{1}{D_1} > 0 \quad \text{(A1d)}$$

$$\frac{\partial h^{L1*}}{\partial p^2} = \frac{H_1\left\{\begin{array}{c}(1+r)(1+g^L_v)E^L_{R1R1} + (1+g^L_v)g^R_{vv}E^L_{R2} \\ -(1+r)(1+g^R_v)E^L_{L1R1}\end{array}\right\}}{(1+r)^4}\frac{1}{D_1} \begin{array}{l} < 0 \text{ if } E^L_{L1R1} \geq 0 \text{ or } \{\ \} > 0 \\ \geq 0 \text{ if } \{\ \} \leq 0 \end{array} \quad \text{(A1e)}$$

$$\frac{\partial h^{R1*}}{\partial p^2} = -\frac{H_1\left(\begin{array}{c}-(1+r)(1+g^R_v)E^L_{L1L1} - (1+g^R_v)g^L_{vv}E^L_{L2} \\ +(1+r)(1+g^L_v)E^L_{R1L1}\end{array}\right)}{(1+r)^4} \begin{array}{l} < 0 \text{ if } E^L_{R1L1} \geq 0 \text{ or } \{\ \} > 0 \\ \geq 0 \text{ if } \{\ \} \leq 0 \end{array} \quad \text{(A1f)}$$

$$\frac{\partial h^{L1*}}{\partial p^{R2}} = -\frac{H_1(1+g^R_v)E^L_{L1R1}}{(1+r)^3}\frac{1}{D_1} \begin{array}{l} < 0 \ \text{ if } E^L_{L1R1} > 0 \\ \geq 0 \ \text{ if } E^L_{L1R1} \leq 0 \end{array} \quad \text{(A1g)}$$

$$\frac{\partial h^{R1*}}{\partial p^{R2}} = \frac{H_1(1+g^R_v)(rE^L_{L1L1} + g^L_{vv}E^L_{L2})}{(1+r)^4}\frac{1}{D_1} < 0 \quad \text{(A1h)}$$

$$\frac{\partial h^{L1*}}{\partial r} = -\frac{H_1 p^2\left\{\begin{array}{c}(1+r)(1+g^L_v)E^L_{R1R1} + (1+g^L_v)g^R_{vv}E^L_{R2} \\ -(1+r)(1+g^R_v)E^L_{L1R1}\end{array}\right\}}{(1+r)^5}\frac{1}{D_1} \begin{array}{l} \geq 0 \text{ if } E^L_{L1R1} \geq 0 \text{ or } \{\ \} \geq 0 \\ < 0 \text{ if } \{\ \} < 0 \end{array} \quad \text{(A1i)}$$

$$\frac{\partial h^{R1*}}{\partial r} = -\frac{H_1 p^2\left\{\begin{array}{c}(1+r)(1+g^R_v)E^L_{L1L1} + (1+g^R_v)g^L_{vv}E^L_{L2} \\ -(1+r)(1+g^L_v)E^L_{R1L1}\end{array}\right\}}{(1+r)^5}\frac{1}{D_1} \begin{array}{l} \geq 0 \text{ if } E^L_{R1L1} \geq 0 \text{ or } \{\ \} \geq 0 \\ < 0 \text{ if } \{\ \} < 0 \end{array} \quad \text{(A1j)}$$

$$\frac{\partial h^{L1*}}{\partial X^L} = 1 \quad \text{(A1k)}$$

$$\frac{\partial h^{R1*}}{\partial X^L} = 0 \quad \text{(A1l)}$$

$$\frac{\partial h^{L1*}}{\partial X^R} = 0 \quad \text{(A1m)}$$

$$\frac{\partial h^{R1*}}{\partial X^R} = 1 \tag{A1n}$$

where, $D_1 = \dfrac{H((r^2+2r)(E^L_{L1L1}E^L_{R1R1} - E^L_{L1R1}E^L_{R1L1}) + (1+r)(E^L_{R2}g^R_{vv}E^L_{L1L1} + E^L_{L2}g^L_{vv}E^L_{R1R1}))}{(1+r)^4} > 0$ and

$H_1 = (E^L_{L2L2}E^L_{R2R2} - E^L_{L2R2}E^L_{R2L2}) > 0$ is a common factor.

The comparative statics results for problem 4.40, the two-stand problem with nontimber benefits on both stands, are as follows:

$$\frac{\partial h^{L1*}}{\partial p^1} = \frac{H_2\begin{pmatrix} -(1+r)(E^L_{R1R1}+E^R_{R1R1}) - g^R_{vv}(E^L_{R2}+E^R_{R2}) \\ +(1+r)(E^L_{L1R1}+E^R_{L1R1}) \end{pmatrix}}{(1+r)^3}\frac{1}{D_2} \begin{matrix} \geq 0 \text{ if } E^L_{L1R1}, E^R_{L1R1} \geq 0 \text{ or } \{\ \} \geq 0 \\ < 0 \text{ if } \{\ \} < 0 \end{matrix} \tag{A2a}$$

$$\frac{\partial h^{R1*}}{\partial p^1} = -\frac{H_2\begin{pmatrix} (1+r)(E^L_{L1L1}+E^R_{L1L1}) + g^L_{vv}(E^L_{L2}+E^R_{L2}) \\ -(1+r)(E^L_{R1L1}+E^R_{R1L1}) \end{pmatrix}}{(1+r)^3}\frac{1}{D_2} \begin{matrix} \geq 0 \text{ if } E^L_{R1L1}, E^R_{R1L1} \geq 0 \text{ or } \{\ \} \geq 0 \\ < 0 \text{ if } \{\ \} < 0 \end{matrix} \tag{A2b}$$

$$\frac{\partial h^{L1*}}{\partial p^{R1}} = \frac{H_2(E^L_{L1R1}+E^R_{L1R1})}{(1+r)^2}\frac{1}{D_2} \begin{matrix} \geq 0 \text{ if } E^L_{L1R1}, E^R_{L1R1} \geq 0 \text{ or } E^L_{L1R1}+E^R_{L1R1} \geq 0 \\ < 0 \text{ if } E^L_{L1R1}, E^R_{L1R1} < 0 \text{ or } E^L_{L1R1}+E^R_{L1R1} < 0 \end{matrix} \tag{A2c}$$

$$\frac{\partial h^{R1*}}{\partial p^{R1}} = -\frac{H_2((1+r)(E^L_{L1L1}+E^R_{L1L1}) + g^L_{vv}(E^L_{L2}+E^R_{L2}))}{(1+r)^3}\frac{1}{D_2} > 0 \tag{A2d}$$

$$\frac{\partial h^{L1*}}{\partial p^2} = \frac{H_2\begin{pmatrix} (1+g^L_v)\begin{pmatrix} (1+r)(E^L_{R1R1}+E^R_{R1R1}) + \\ g^R_{vv}(E^L_{R2}+E^R_{R2}) \end{pmatrix} \\ -(1+r)(1+g^R_v)(E^L_{L1R1}+E^R_{L1R1}) \end{pmatrix}}{(1+r)^4}\frac{1}{D_2} \begin{matrix} < 0 \text{ if } E^L_{L1R1}, E^R_{L1R1} \geq 0 \text{ or } \{\ \} > 0 \\ \geq 0 \text{ if } \{\ \} \leq 0 \end{matrix} \tag{A2e}$$

$$\frac{\partial h^{R1*}}{\partial p^2} = \frac{H_2\begin{pmatrix} (1+g^R_v)\begin{pmatrix} (1+r)(E^L_{L1L1}+E^R_{L1L1}) + \\ g^L_{vv}(E^L_{L2}+E^R_{L2}) \end{pmatrix} \\ -(1+r)(1+g^L_v)(E^L_{R1L1}+E^R_{R1L1}) \end{pmatrix}}{(1+r)^4}\frac{1}{D_2} \begin{matrix} < 0 \text{ if } E^L_{R1L1}, E^R_{R1L1} \geq 0 \text{ or } \{\ \} > 0 \\ \geq 0 \text{ if } \{\ \} \leq 0 \end{matrix} \tag{A2f}$$

$$\frac{\partial h^{L1*}}{\partial p^{R2}} = -\frac{H_2(1+g^R_v)(E^L_{L1R1}+E^R_{L1R1})}{(1+r)^4}\frac{1}{D_2} \begin{matrix} < 0 \text{ if } E^L_{L1R1}, E^R_{L1R1} > 0 \text{ or } E^L_{L1R1}+E^R_{L1R1} > 0 \\ \geq 0 \text{ if } E^L_{L1R1}, E^R_{L1R1} \leq 0 \text{ or } E^L_{L1R1}+E^R_{L1R1} \leq 0 \end{matrix} \tag{A2g}$$

$$\frac{\partial h^{R1*}}{\partial p^{R2}} = \frac{H_2((1+r)(E^L_{L1L1}+E^R_{L1L1}) + g^L_{vv}(E^L_{L2}+E^R_{L2}))(1+g^R_v)}{(1+r)^4}\frac{1}{D_2} < 0 \tag{A2h}$$

$$\frac{\partial h^{L1*}}{\partial r} = -\frac{H_2 p^2 \left(\begin{array}{c} (1+g_v^L)\left(\begin{array}{c} (1+r)(E_{R1R1}^L + E_{R1R1}^R) \\ +g_{vv}^R(E_{R2}^L + E_{R2}^R) \end{array} \right) \\ -(1+r)(1+g_v^R)(E_{L1R1}^L + E_{L1R1}^R) \end{array} \right)}{(1+r)^5} \frac{1}{D_2} \begin{array}{l} \geq 0 \text{ if } E_{L1R1}^L, E_{L1R1}^R \geq 0 \text{ or } \{\ \} \leq 0 \\ < 0 \text{ if } \{\ \} > 0 \end{array} \tag{A2i}$$

$$\frac{\partial h^{R1*}}{\partial r} = -\frac{H_2 p^2 \left(\begin{array}{c} (1+g_v^R)\left(\begin{array}{c} (1+r)(E_{L1L1}^L + E_{L1L1}^R) \\ +g_{vv}^L(E_{L2}^L + E_{L2}^R) \end{array} \right) \\ -(1+r)(1+g_v^L)(E_{R1L1}^L + E_{R1L1}^R) \end{array} \right)}{(1+r)^5} \frac{1}{D_2} \begin{array}{l} \geq 0 \text{ if } E_{R1L1}^L, E_{R1L1}^R \geq 0 \text{ or } \{\ \} \leq 0 \\ < 0 \text{ if } \{\ \} > 0 \end{array} \tag{A2j}$$

$$\frac{\partial h^{L1*}}{\partial X^L} = 1 \tag{A2k}$$

$$\frac{\partial h^{R1*}}{\partial X^L} = 0 \tag{A2l}$$

$$\frac{\partial h^{L1*}}{\partial X^R} = 0 \tag{A2m}$$

$$\frac{\partial h^{R1*}}{\partial X^R} = 1 \tag{A2n}$$

where

$$D_2 = \frac{H_2 \left(\begin{array}{c} (1+r^2+2r)H_2 + \\ (1+r)(E_{R2}^L + E_{R2}^R)(E_{L1L1}^L + E_{L1L1}^R)(g_{vv}^R + g_{vv}^L) + \\ g_{vv}^L g_{vv}^R (E_{L2}^L E_{R2}^L + E_{L2}^L E_{R2}^R + E_{L2}^R E_{R2}^L + E_{L2}^R E_{R2}^R) \end{array} \right)}{(1+r)^4} > 0$$ is the Jacobian determinant and

$$H_2 = \left(\begin{array}{c} E_{L2L2}^L E_{R2R2}^L + E_{L2L2}^L E_{R2R2}^R + E_{L2L2}^R E_{R2R2}^L + E_{L2L2}^R E_{R2R2}^R - E_{L2R2}^L E_{R2L2}^L \\ -E_{L2R2}^L E_{R2L2}^R - E_{L2R2}^R E_{R2L2}^L - E_{L2R2}^R E_{R2L2}^R \end{array} \right) > 0.$$

The comparative statics results for one stand with management and nontimber benefits, Equation 4.43, are:

$$\frac{\partial h^{L1*}}{\partial p^1} = -\frac{E_{L2L2}^L g_{ss}^L E_{L2}^L}{(1+r)^2} \frac{1}{D_3} > 0 \tag{A3a}$$

$$\frac{\partial h^{L1*}}{\partial p^2} = \frac{E_{L2L2}^L \left(g_{QQ}^L + g_v^L g_{QQ}^L - g_Q^L g_{vQ}^L \right) E_{L2}^L}{(1+r)^3} \frac{1}{D_3} \begin{array}{l} < 0 \text{ if } g_{vQ}^L \geq 0 \text{ or } \left| g_{QQ}^L (1+g_v^L) \right| > \left| g_Q^L g_{vQ}^L \right| \\ \geq 0 \text{ if } \left| g_{QQ}^L (1+g_v^L) \right| \leq \left| g_Q^L g_{vQ}^L \right| \end{array} \tag{A3b}$$

$$\frac{\partial h^{L1*}}{\partial r} = -\frac{E_{L2L2}^L p_2 E_{L2}^L \left(g_{QQ}^L + g_v^L g_{QQ}^L - g_Q^L g_{vQ}^L \right)}{(1+r)^4} \frac{1}{D_3} \begin{array}{l} \geq 0 \text{ if } g_{vQ}^L \geq 0 \text{ or } \left| g_{QQ}^L (1+g_v^L) \right| \geq \left| g_Q^L g_{vQ}^L \right| \\ < 0 \text{ if } \left| g_{QQ}^L (1+g_v^L) \right| < \left| g_Q^L g_{vQ}^L \right| \end{array} \tag{A3c}$$

$$\frac{\partial h^{L1*}}{\partial X^L} = 1 \tag{A3d}$$

$$\frac{\partial h^{L1*}}{\partial \bar{V}^R} = \frac{E^L_{L2L2}E^L_{L2}g^L_{QQ}E^L_{L1R1}}{(1+r)^2}\frac{1}{D_3} \begin{matrix} \geq 0 \text{ if } E^L_{L1R1} \leq 0 \\ < 0 \text{ if } E^L_{L1R1} > 0 \end{matrix} \tag{A3e}$$

$$\frac{\partial h^{L1*}}{\partial w_1} = \frac{E^L_{L2L2}E^L_{L2}g^L_{vQ}}{(1+r)^2}\frac{1}{D_3} \begin{matrix} < 0 \text{ if } g^L_{vQ} < 0 \\ \geq 0 \text{ if } g^L_{vQ} \geq 0 \end{matrix} \tag{A3f}$$

$$\frac{\partial Q^{L*}}{\partial p^1} = -\frac{E^L_{L2L2}E^L_{L2}g^L_{Qv}}{(1+r)^2}\frac{1}{D_3} \begin{matrix} < 0 \text{ if } g^L_{Qv} > 0 \\ \geq 0 \text{ if } g^L_{Qv} \leq 0 \end{matrix} \tag{A3g}$$

$$\frac{\partial Q^{L*}}{\partial p^2} = -\frac{E^L_{L2L2}\left\{(1+r)g^L_Q E^L_{L1L1} + E^L_{L2}\left(g^L_Q g^L_{vv} - (1+g^L_v)g^L_{Qv}\right)\right\}}{(1+r)^3}\frac{1}{D_3} \begin{matrix} \geq 0 \text{ if } g^L_{Qv} \geq 0 \text{ or } \{\quad\} \leq 0 \\ < 0 \text{ if } \{\quad\} > 0 \end{matrix} \tag{A3h}$$

$$\frac{\partial Q^{L*}}{\partial r} = \frac{E^L_{L2L2}\left((1+r)g^L_Q E^L_{L1L1} + E^L_{L2}\left(g^L_Q g^L_{vv} - (1+g^L_v)g^L_{Qv}\right)\right)}{(1+r)^3}\frac{1}{D_3} \begin{matrix} < 0 \text{ if } g^L_{Qv} \geq 0 \text{ or } \{\quad\} < 0 \\ \geq 0 \text{ if } \{\quad\} \geq 0 \end{matrix} \tag{A3i}$$

$$\frac{\partial Q^{L*}}{\partial X^L} = 0 \tag{A3j}$$

$$\frac{\partial Q^{L*}}{\partial \bar{V}^R} = \frac{E^L_{L2L2}E^L_{L1R1}E^L_{L2}g^L_{Qv}}{(1+r)^2}\frac{1}{D_3} \begin{matrix} \geq 0 \text{ if } E^L_{L1R1} \text{ and } g^L_{Qv} > 0; E^L_{L1R1} \text{ and } g^L_{Qv} < 0; E^L_{L1R1} \text{ or } g^L_{Qv} = 0 \\ < 0 \text{ if } E^L_{L1R1} > 0 \text{ and } g^L_{Qv} < 0 \text{ or } E^L_{L1R1} < 0 \text{ and } g^L_{Qv} > 0 \end{matrix} \tag{A3k}$$

$$\frac{\partial Q^{L*}}{\partial w} = \frac{E^L_{L2L2}\left((1+r)E^L_{L1L1} + E^L_{L2}g^L_{QQ}\right)}{(1+r)^2}\frac{1}{D_3} < 0 \tag{A3l}$$

where

$$D_3 = \frac{E^L_{L2L2}E^L_{L2}\left((1+r)g^L_{QQ}E^L_{L1L1} + E^L_{L2}\left(g^L_{QQ}g^L_{vv} - g^L_{Qv}g^L_{Qv}\right)\right)}{(1+r)^3} < 0.$$

The comparative statics results for two stands with timber management on the right stand and nontimber benefits on the left stand, Equation 4.47, are:

$$\frac{\partial h^{L1*}}{\partial p^1} = \frac{H_4 E^L_{R2}\left\{g^R_{QQ}(1+r)(-E^L_{R1R1} + E^L_{L1R1}) - G_4 E^L_{R2}\right\}}{(1+r)^4}\frac{1}{D_4} \begin{matrix} \geq 0 \text{ if } E^L_{L1R1} \geq 0; \{\quad\} \leq 0 \\ < 0 \text{ if } \{\quad\} > 0 \end{matrix} \tag{A4a}$$

$$\frac{\partial h^{R1*}}{\partial p^1} = \frac{H_4 E^L_{R2}g^R_{QQ}\{(E^L_{R1L1} - E^L_{L1L1})(1+r) - g^L_{vv}E^L_{L2}\}}{(1+r)^4}\frac{1}{D_4} \begin{matrix} \geq 0 \text{ if } E^L_{R1L1} \geq 0; \{\quad\} \leq 0 \\ < 0 \text{ if } \{\quad\} > 0 \end{matrix} \tag{A4b}$$

$$\frac{\partial h^{L1*}}{\partial p^{R1}} = \frac{H_4 E^L_{R2}g^R_{QQ}E^L_{L1R1}}{(1+r)^3}\frac{1}{D_4} \begin{matrix} \geq 0 \text{ if } E^L_{L1R1} \geq 0 \\ < 0 \text{ if } E^L_{L1R1} < 0 \end{matrix} \tag{A4c}$$

$$\frac{\partial h^{R1*}}{\partial p^{R1}} = -\frac{E^L_{R2} g^R_{QQ} H_4 (E^L_{L1L1}(1+r) + g^L_{vv} E^L_{L2})}{(1+r)^4} \frac{1}{D_4} > 0 \tag{A4d}$$

$$\frac{\partial h^{L1*}}{\partial p^2} = -\frac{E^L_{R2} H_4 \left\{ \begin{array}{c} \begin{pmatrix} -E^L_{R1R1}(1+g^L_v) \\ +E^L_{L1R1}(1+g^R_v) \end{pmatrix} g^R_{QQ}(1+r) \\ -G_4 E^L_{R2}(1+g^L_v) - g^R_{Q} g^R_{vQ} E^L_{L1R1}(1+r) \end{array} \right\}}{(1+r)^5} \frac{1}{D_4} \begin{array}{l} < 0 \text{ if } g^R_{vQ} \text{ and } E^L_{L1R1} \ge 0; \{\quad\} < 0 \\ \ge 0 \text{ if } \{\quad\} \ge 0 \end{array} \tag{A4e}$$

$$\frac{\partial h^{R1*}}{\partial p^2} = \frac{\begin{array}{c} E^L_{R2} H_4 \\ -\left\{ \begin{array}{l} (-E^L_{L1L1}(1+g^R_v) + E^L_{R1L1}(1+g^L_v)) g^R_{QQ}(1+r) \\ -g^R_{QQ} g^L_{vv} E^L_{L2}(1+g^R_v) + \\ g^R_{Q} g^R_{vQ} (E^L_{L1L1}(1+r) + g^L_{vv} E^L_{L2}) \end{array} \right\} \end{array}}{(1+r)^5} \frac{1}{D_4} \begin{array}{l} < 0 \text{ if } g^R_{vQ} \text{ and } E^L_{L1R1} \ge 0; \{\quad\} < 0 \\ \ge 0 \text{ if } \{\quad\} \ge 0 \end{array} \tag{A4f}$$

$$\frac{\partial h^{L1*}}{\partial p^{R2}} = -\frac{\begin{array}{c} E^L_{R2} H_4 E^L_{L1R1} \\ \left\{ \begin{array}{c} g^R_{QQ}(1+g^R_v) \\ -g^R_{Q} g^R_{vQ} \end{array} \right\} \end{array}}{(1+r)^5} \frac{1}{D_4} \begin{array}{l} < 0 \text{ if } g^R_{vQ} \ge 0 \text{ or } \{\quad\} < 0 \text{ and } E^L_{L1R1} > 0; \{\quad\} > 0 \text{ and } E^L_{L1R1} < 0 \\ \ge 0 \text{ if } \{\quad\} \ge 0 \text{ and } E^L_{L1R1} \ge 0; \{\quad\} \le 0 \text{ and } E^L_{L1R1} \le 0 \end{array} \tag{A4g}$$

$$\frac{\partial h^{R1*}}{\partial p^{R2}} = \frac{E^L_{R2} H_4 \left\{ g^R_{QQ}(1+g^R_v) - g^R_{Q} g^R_{vQ} \right\} (E^L_{L1L1}(1+r) + g^L_{vv} E^L_{L2})}{(1+r)^5} \frac{1}{D_4} \begin{array}{l} < 0 \text{ if } g^R_{vQ} \ge 0; \{\quad\} < 0 \\ \ge 0 \text{ if } \{\quad\} \ge 0 \end{array} \tag{A4h}$$

$$\frac{\partial h^{L1*}}{\partial r} = \frac{\begin{array}{c} E^L_{R2} H_4 \\ \left\{ \begin{array}{c} \begin{bmatrix} -E^L_{R1R1}(1+g^L_v) \\ +E^L_{L1R1}(1+g^R_v) \end{bmatrix} p_2 g^R_{QQ}(1+r) \\ -G_4 p_2 E^L_{R2}(1+g^L_v) - E^L_{R2} g^R_{vQ} E^L_{L1R1}(1+r) \end{array} \right\} \end{array}}{(1+r)^5} \frac{1}{D_4} \begin{array}{l} \ge 0 \text{ if } g^R_{vQ} \text{ and } E^L_{L1R1} \ge 0; \{\quad\} \le 0 \\ < 0 \text{ if } \{\quad\} > 0 \end{array} \tag{A4i}$$

$$\frac{\partial h^{R1*}}{\partial r} = \frac{E^L_{R2} H_4 \left\{ \begin{array}{c} \begin{pmatrix} -E^L_{L1L1}(1+g^R_v) \\ +E^L_{R1L1}(1+g^L_v) \end{pmatrix} p_2 g^R_{QQ}(1+r) \\ -\left(g^R_{QQ} g^L_{vv} p_2 (1+g^R_v) - g^R_{vv} g^R_{vQ} E^L_{R2} \right) E^L_{L2} \\ -E^L_{R2} g^R_{vQ} E^L_{L1L1}(1+r) \end{array} \right\}}{(1+r)^5} \frac{1}{D_4} \begin{array}{l} \ge 0 \text{ if } g^R_{vQ} \text{ and } E^L_{R1L1} \ge 0; \{\quad\} \le 0 \\ < 0 \text{ if } \{\quad\} > 0 \end{array} \tag{A4j}$$

$$\frac{\partial h^{L1*}}{\partial X^L} = 1 \tag{A4k}$$

$$\frac{\partial h^{L1*}}{\partial X^R} = 0 \tag{A4l}$$

$$\frac{\partial h^{R1*}}{\partial X^L} = 0 \tag{A4m}$$

$$\frac{\partial h^{R1*}}{\partial X^R} = 1 \tag{A4n}$$

$$\frac{\partial h^{L1*}}{\partial w}=-\frac{E^L_{R2}g^R_{vQ}H_4E^L_{L1R1}}{(1+r)^3}\frac{1}{D_4}\begin{array}{l}\geq 0 \text{ if } E^L_{L1R1} \text{ and } g^R_{vQ}\lesseqgtr 0; \text{either } E^L_{L1R1} \text{ or } g^R_{vQ}=0\\ <0 \text{ if } E^L_{L1R1}>0 \text{ and } g^R_{vQ}<0; E^L_{L1R1}<0 \text{ and } g^R_{vQ}>0\end{array} \tag{A4o}$$

$$\frac{\partial h^{R1*}}{\partial w_1}=\frac{E^L_{R2}g^R_{vQ}H_4(E^L_{L1L1}(1+r)+g^L_{vv}E^L_{L2})}{(1+r)^4}\frac{1}{D_4}\begin{array}{l}\geq 0 \text{ if } g_{vQ}\geq 0\\ <0 \text{ if } g_{vQ}<0\end{array} \tag{A4p}$$

$$\frac{\partial Q^{R*}}{\partial p^1}=\frac{E^L_{R2}g^R_{Qv}H_4\left\{\begin{array}{c}(1+r)\\(-E^L_{L1L1}+E^L_{R1L1})\\-g^L_{vv}E^L_{L2}\end{array}\right\}}{(1+r)^4}\frac{1}{D_4}\begin{array}{l}<0 \text{ if } E^L_{L1R1}\geq 0 \text{ and } g^R_{Qv}>0; g^R_{Qv}<0 \text{ and } \{\ \}<0\\ \geq 0 \text{ if } E^L_{L1R1} \text{ or } \{\ \}\geq 0 \text{ and } g^R_{Qv}\leq 0; g^R_{Qv}\geq 0 \text{ and } \{\ \}\leq 0\end{array} \tag{A4q}$$

$$\frac{\partial Q^{R*}}{\partial p^{R1}}=-\frac{H_4E^L_{R2}g^R_{Qv}(E^L_{L1L1}(1+r)+g^L_{vv}E^L_{L2})}{(1+r)^4}\frac{1}{D_4}\begin{array}{l}\geq 0 \text{ if } g^R_{Qv}\leq 0\\ <0 \text{ if } g^R_{Qv}>0\end{array} \tag{A4r}$$

$$\frac{\partial Q^{R*}}{\partial p^2}=\frac{H_4\left\{\begin{array}{c}\left(\begin{array}{c}E^L_{L1L1}(1+g^R_v)\\-E^L_{R1L1}(1+g^L_v)\end{array}\right)(1+r)E^L_{R2}g^R_{vQ}\\-(1+r)g^R_Q(E^L_{R1R1}E^L_{L2}g^L_{vv}+E^L_{L1L1}E^L_{R2}g^R_{vv})\\+g^L_{vv}E^L_{R2}E^L_{L2}\left(g^R_{vQ}(1+g^R_v)-g^R_Qg^R_{vv}\right)\\-K_4(1+2r+r^2)\end{array}\right\}}{(1+r)^5}\frac{1}{D_4}\begin{array}{l}\geq 0 \text{ if } E^L_{L1R1} \text{ and } g^R_{vQ}\geq 0 \text{ or } \{\ \}\leq 0\\ <0 \text{ if } \{\ \}>0\end{array} \tag{A4s}$$

$$\frac{\partial Q^{R*}}{\partial p^{R2}}=\frac{H_4\left\{\begin{array}{c}-K_4(1+2r+r^2)\\-(1+r)g^R_Q(E^L_{R1R1}E^L_{L2}g^L_{vv}+E^L_{L1L1}E^L_{R2}g^R_{vv})\\+E^L_{L1L1}(1+g^R_v)(1+r)E^L_{R2}g^R_{vQ}\\+g^L_{vv}E^L_{R2}E^L_{L2}\left(g^R_{vQ}g^R_v-g^R_Qg^R_{vv}\right)\end{array}\right\}}{(1+r)^5}\frac{1}{D_4}\begin{array}{l}\geq 0 \text{ if } g^R_{vQ}\geq 0 \text{ or } \{\ \}\leq 0\\ <0 \text{ if } \{\ \}>0\end{array} \tag{A4t}$$

$$\frac{\partial Q^{R*}}{\partial r}=-\frac{p^2H_4\left[\begin{array}{c}(1+r)(E^L_{R1R1}g^L_{vv}+E^L_{L1L1}g^R_{vv})p^2\\+g^L_{vv}(p^2)^2g^R_{vv}+K_4(1+2r+r^2)\\+\left\{\begin{array}{c}\left(\begin{array}{c}E^L_{R1L1}(1+g^L_v)-\\E^L_{L1L1}(1+g^R_v)\end{array}\right)(1+r)\\-g^L_{vv}p_2(1+g^R_v)\end{array}\right\}\\p_2g^R_{vQ}\end{array}\right]}{(1+r)^6}\frac{1}{D_4}\begin{array}{l}\geq 0 \text{ if } g^R_{vQ} \text{ and } E^L_{R1L1}\geq 0; [\ ]\geq 0\\ <0 \text{ if } [\ ]<0\end{array} \tag{A4u}$$

This last expression was simplified by exploiting the f.o.c.s and setting $p^2=E^L_{L2}=E^L_{R2}$. This expression is correct for equal timber prices on each stand. If prices are different the expression changes but the result does not.

$$\frac{\partial Q^{R*}}{\partial X^{L}} = 0 \tag{A4v}$$

$$\frac{\partial Q^{R*}}{\partial X^{R}} = 0 \tag{A4w}$$

$$\frac{\partial Q^{R*}}{\partial w} = \frac{H_4[(1+2r+r^2)K_4 + (1+r)(E^L_{L1L1}g^R_{vv}E^L_{R2} + E^L_{R1R1}g^L_{vv}E^L_{L2}) + E^L_{R2}g^R_{vv}g^L_{vv}E^L_{L2}]}{(1+r)^4}\frac{1}{D_4} < 0 \tag{A4x}$$

where

$D_4 = \dfrac{-E^L_{R2}H_4[-(1+2r+r^2)g^R_{ss}K_4 - (1+r)E^L_{R1R1}E^L_{L2}g^R_{ss}g^L_{vv} - (1+r)E^L_{L1L1}E^L_{R2}G_4 - g^L_{vv}E^L_{L2}E^L_{R2}G_4]}{(1+r)^5} < 0$ is

the Jacobian determinant, $H_4 = E^L_{L2L2}E^L_{R2R2} - E^L_{L2R2}E^L_{R2L2} > 0$, $K_4 = E^L_{L1L1}E^L_{R1R1} - E^L_{L1R1}E^L_{R1L1} > 0$

and $G_4 = g^R_{QQ}g^R_{vv} - g^R_{Qv}g^R_{vQ} > 0$.

Appendix 2 GAMS/Minos Program

```
$Title  Discrete Harvest-Inventory Multiple-use Model
$Ontext
      This is a discrete harvest-inventory multiple-use model.
         •Three Stands and 5 years between each harvest decision
         •Forage benefits, timber benefits, 'recreation benefits', Cobb-Douglas benefit
         function
$Offtext
$OFFSYMXREF OFFSYMLIST

SETS

      T        optimization time 100 years  /P1*P19,TERMINAL/
      TI(T)    initial decision period
      TB(T)    all decision periods but last
      TBF(T)   all but first decision period
      TL(T)    last or terminal decision period;

      TI(T)  = YES$(ORD(T) EQ 1);
      TB(T)  = YES$(ORD(T) LT CARD(T));
      TL(T)  = NOT TB(T);
      TBF(T)= NOT TI(T);

SCALARS
$Ontext
Parameters for forage benefits
$Offtext
        AL     first part forage production  left       /5.943 /
        BL     second part forage production left       /0.961 /
        CL     third part forage production  left       /6.561 /
        DL     fourth part forage production left       /1.061 /
        AM     first part forage production  middle     /5.943 /
        BM     second part forage production middle     /0.961 /
        CM     third part forage production  middle     /6.561 /
        DM     fourth part forage production middle     /1.061 /
        AR     first part forage production  right      /5.943 /
        BR     second part forage production right      /0.961 /
        CR     third part forage production  right      /6.561 /
        DR     fourth part forage production right      /1.061 /
    F      maximum value of forage                      / 30   /
    G      value adjustment for forage                  / 2    /
    K      timber carrying capacity                     / 15.055 /
$Ontext
```

Parameters for wildlife benefits

```
$Offtext
   WAL      maximum wildlife production  left      / 1    /
   WAM      maximum wildlife production  middle    / 1    /
   WAR      maximum wildlife production  right     / 1    /
   WB       max increment of wildlife production   / .2   /
   WC       change in incremental wildlife         / .3   /
   WF       max value of wildlife                  / 10   /
   WG       value adjustment for wildlife          / 1    /
$Ontext
```

Parameters for Cobb-Douglas benefits function

```
$Offtext
   CDK      cobb-douglas constant                        / 1.5  /
   CDL      cobb-douglas exponent on left stand          / 0.1  /
   CDM      cobb-douglas exponent on middle stand        / 0.8  /
   CDR      cobb-douglas exponent on right stand         / 0    /
$Ontext
```

Parameters for stand growth function

```
$Offtext
   G1L  first constant on quadratic growth left          /.40   /
   G2L  second constant on quadratic growth left         /.015  /
   G1M  first constant on quadratic growth middle        /.40   /
   G2M  second constant on quadratic growth middle       /.015  /
   G1R  first constant on quadratic growth right         /.40   /
   G2R  second constant on quadratic growth right        /.015  /
$Ontext
```

Economic parameters

```
$Offtext
   PB     base stumpage value of timber                  / 80   /
   PL     low stumpage value of timber                   / 80   /
   RHO    discount rate                                  /.04   /

$Ontext
```

Initial and terminal conditions

```
$Offtext

   KTL    terminal stock on left stand         /.5    /
```

```
KTM    terminal stock on middle stand        /.5    /
KTR    terminal stock on right stand         /.5    /
KIL    initial stock on left stand           / 12   /
KIM    initial stock on middle stand         / 12   /
KIR    initial stock on right stand          / 12   /

DELT   discount factor for 5 year annual series  ;

DELT = (((1+RHO)**5)-1)/(RHO*(1+RHO)**5) ;


PARAMETER
  DELTAP(T)  5-year periodic discount rate;

  DELTAP(T)=(1+RHO)**((-ORD(T)+1)*5);


VARIABLES
  P(T)      price in period t
  HL(T)     harvest from left stand
  HM(T)     harvest from middle stand
  HR(T)     harvest from right stand
  KL(T)     inventory on left stand
  KM(T)     inventory on middle stand
  KR(T)     inventory on right stand
  FPL(T)    forage production on left stand
  FPM(T)    forage production on middle stand
  FPR(T)    forage production on right stand
  FV(T)     forage value
  WPL(T)    wildlife production on left stand
  WPM(T)    wildlife production on middle stand
  WPR(T)    wildlife production on right stand
  WV(T)     wildlife value
  CDV(T)    cobb-douglas value
  TV(T)     timber value
  REV(T)    revenue in each period
  Z         total value;
POSITIVE VARIABLE HL,HM,HR,KL,KM,KR;

EQUATIONS
  PRICEI(T)    stumpage function for first period
  PRICE (T)    stumpage function all but periods but first
  INVL(T)      inventory accounting equation on left stand
  INVM(T)      inventory accounting equation on middle stand
  INVR(T)      inventory accounting equation on right stand
```

```
   INVLI(T)      initial inventory accounting equation on left stand
   INVMI(T)      initial inventory accounting equation on middle stand
   INVRI(T)      initial inventory accounting equation on right stand
   CONLI(T)      initial harvest constraint on left stand
   CONMI(T)      initial harvest constraint on middle stand
   CONRI(T)      initial harvest constraint on right stand
   CONL(T)       harvest constraint on left stand
   CONM(T)       harvest constraint on middle stand
   CONR(T)       harvest constraint on right stand
   QFPL(T)       forage production on left stand
   QFPM(T)       forage production on middle stand
   QFPR(T)       forage production on right stand
   QFV(T)        periodic forage value
   QWPL(T)       wildlife production on left stand
   QWPM(T)       wildlife production on middle stand
   QWPR(T)       wildlife production on right stand
   QWV(T)        periodic wildlife value
   QCDV(T)       cobb-douglas value
   QTV(T)        periodic timber value
   APROFIT(T)    periodic objective function
   TPROFIT       total objective function;
$Double
PRICEI("p1")..  P("P1")    =E=  PB;
PRICE(TBF)..    P(TBF)     =E=  PL;
INVLI("p1")..   KL("p1")   =E=  KIL - HL("p1");
INVMI("p1")..   KM("p1")   =E=  KIM - HM("p1");
INVRI("p1")..   KR("p1")   =E=  KIR - HR("p1");
INVL(T+1)..     KL(T+1)    =E=  KL(T) + (G1L*KL(T) - G2L*KL(T)**2) - HL(T+1);
INVM(T+1)..     KM(T+1)    =E=  KM(T) + (G1M*KM(T) - G2M*KM(T)**2) -
HM(T+1);
INVR(T+1)..     KR(T+1)    =E=  KR(T) + (G1R*KR(T) - G2R*KR(T)**2) - HR(T+1);
CONLI(TI)..     HL(TI)     =L=  KIL;
CONMI(TI)..     HM(TI)     =L=  KIM;
CONRI(TI)..     HR(TI)     =L=  KIR;
CONL(T)..       HL(T+1)    =L=  KL(T) + (G1L*KL(T) - G2L*KL(T)**2);
CONM(T)..       HM(T+1)    =L=  KM(T) + (G1M*KM(T) - G2M*KM(T)**2);
CONR(T)..       HR(T+1)    =L=  KR(T) + (G1R*KR(T) - G2R*KR(T)**2);
QFPL(T)..       FPL(T)     =E=  (AL - BL*LOG(K/KL(T)-1))*EXP(-CL +
                                     DL*LOG(K/KL(T)-1));
QFPM(T)..       FPM(T)     =E= (AM - BM*LOG(K/KM(T)-1))*EXP(-CM +
                                    DM*LOG(K/KM(T)-1));
QFPR(T)..       FPR(T)     =E= (AR - BR*LOG(K/KR(T)-1))*EXP(-CR +
                                    DR*LOG(K/KR(T)-1));
QFV(T)..        FV(T)      =E= DELT*F*EXP(-G*(FPL(T)+FPM(T)+FPR(T)))*
                                    (FPL(T)+FPM(T)+FPR(T));
```

```
QWPL(T)..       WPL(T)      =E=  WAL/(1+exp(WB-WC*KL(T)));
QWPM(T)..       WPM(T)      =E=  WAM/(1+exp(WB-WC*KM(T)));
QWPR(T)..       WPR(T)      =E=  WAR/(1+exp(WB-WC*KR(T)));
QWV(T)..        WV(T)       =E=  DELT*WF*(WPL(T)+WPM(T)+WPR(T));
QCDV(T)..       CDV(T)      =E=  DELT*CDK*(KL(T)**CDL)*
                                   (KM(T)**CDM)*(KR(T)**CDR);
QTV(T)..        TV(T)       =E=  P(T)*(HL(T) + HM(T) + HR(T));
APROFIT(T)..    REV(T)      =E=  (FV(T) +WV(T)+CDV(T)+ TV(T))*DELTAP(T);
TPROFIT..       Z           =E=  SUM(T,REV(T));
$SINGLE
*
*Fixed initial conditions such as lower and upper bounds on inventory and
*starting point for possible first period harvest levels
*
KL.lo(T)=KTL;KM.LO(t)=KTM;KR.LO(T)=KTR;
KL.UP(T)=15;KM.UP(T)=15;KR.UP(T)=15;
HL.L(T)=6; HM.L(T)=5; HR.L(T)=10;
*
*Command statement
*
MODEL BRYAN1 /ALL/;SOLVE BRYAN1 USING NLP MAXIMIZING Z;

DISPLAY KL.L, KM.L, KR.L, HL.L, HM.L, HR.L;
```