STRUCTURAL PROPERTY RELATIONSHIPS FOR CANADIAN DIMENSION LUMBER

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ABSTRACT

Lumber property data from Canadian In-Grade Program for visually-graded dimension lumber are used to model lumber property relationships. The lumber properties studied are modulus of elasticity (MOE), modulus of rupture (MOR), ultimate tensile stress parallel to the grain (UTS), and ultimate compression stress parallel to the grain (UCS) for Douglas-fir, Hem-Fir and Spruce-Pine-Fir species groups. Structural property relationships based on three different approaches using Canadian dimension lumber have been modeled.

The nonlinear models were adopted for the general stiffness-strength property relationships. The fitted regression models for the general stiffness-strength property relationships then were used to model the strength property relationships.

Band-width method was used to derive the relationships between modulus of elasticity and lower exclusion limits of strength values. The fitted models then were used to model the strength property relationships. The resulted models represent the relationship between two strength properties at the lower exclusion limit for lumber selected on the basis of modulus of elasticity.

The strength property relationships derived using equal-rank method agree with that derived using general stiffness-strength property relationships. Therefore, for lumber selected on the basis of modulus of elasticity, the models derived using equal-rank method yield an average or mean trend for the estimated properties.

The results of the analysis show that there exist good relationships between lumber strength properties. The strength property ratios for Canadian dimension lumber show significant species dependency particularly at the higher strength level. Property relationships trends are consistent across the species and methods of analyses.

The property ratio models are intended to provide property estimates of characteristic values for untested properties. The property ratios for Canadian dimension lumber are significantly higher than that proposed by the American Society for Testing and Materials (ASTM) standard D 1990.

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LIST OF ABBREVIATIONS

ASTM	American Society for Testing and Materials
B.C.	British Columbia
cdf	cumulative distribution function
COFI	Council of Forest Industries of British Columbia
CSA	Canadian Standard Association
CWC	Canadian Wood Council
D-fir	Douglas-fir-Larch species group
EQRA	Equal-rank assumption
f-E	Allowable MOR and MOE for MSR lumber
FPL	Forest Products Laboratory
FPRS	Forest Products Research Society
GEN.REL	General relationship
H-Fir	Hem-Fir species group
K-S	Kolmogorov-Smirnov
ksi	kip per square inch
LR	Likelihood Ratio
M	Moisture content
MOE	Modulus of Elasticity
MOR	Modulus of Rupture
MPa	Mega Pascal
MSR	Machine Stress Rated
MSRD	Maximum Strength Reducing Defect

NLGA	National Lumber Grades Authority
N/mm^2	Newton per millimeter square
No.2	Number 2 grade
psi	pound per square inch
SAS	SAS System software for data analysis
S-P-F	Spruce-Pine-Fir species group
SPS	Special Product Standard
SS	Select Structural
SSE	Sum of Square Error
UDL	Uniformly Distributed Load
UCS	Ultimate Compressive Stress
U.K.	United Kingdom
UTS	Ultimate Tensile Stress
US	United States
WWPA	Western Wood Products Association

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1. INTRODUCTION

1.1. Definition

Knowledge of material properties is essential in the design of structures. The designer must know the mechanical properties of a member under one or more applied load(s). Every structural material, including wood, has its own certain physical and mechanical properties. Unlike man-made materials such as concrete and steel, wood is a natural fiber composite whose properties are influenced by nature. Moreover, wood exhibits different strength values for different grain directions. Therefore, wood can be described as an unisotropic material. Moreover, in the same direction e.g., parallel to the grain, wood also exhibits significant difference between tension and compression properties.

Based on the type of loading, there are three basic strength properties of wood which are crucial to timber design engineers, i.e., bending strength, tensile strength parallel to the grain and compression strength parallel to the grain. These are the strength properties obtained at maximum loads. For design and standard purposes, and in this thesis, the terms modulus of rupture (MOR), ultimate tensile stress (UTS), and ultimate compression stress (UCS) are assigned to these three basic strength properties, respectively. Besides those three basic strength properties, the elastic property or stiffness is also needed in the design application. The measure of the elastic property is called modulus of elasticity (MOE) and is determined from a static bending test. This property is used primarily for determining the deflection of beams. The existing linkage between any two of these four mechanical properties is defined as property relationship. As outlined in the American Society for

Testing and Materials (ASTM) standard ASTM D 1990, the property relationships are intended to produce conservative estimates of characteristic values for untested properties (ASTM 1991). In this standard, the characteristic value is defined as the population mean, median or tolerance limit value estimated from the test data after it has been adjusted to standardized conditions of temperature, moisture content, and characteristic size.

Wood, for structural use, is available in a number of size categories such as lumber and timber. Lumber is a general term which includes dimension lumber, timber, decking, boards and finished lumber used as siding and flooring (CWC 1988a). Lumber properties, herein, are determined from dimension lumber. Dimension lumber is defined as surfaced softwood lumber of thickness from 38 to 102 mm and is intended for use as framing members such as joists, planks, rafters, studs and small posts or beams (CWC 1988a). Dimension lumber from the manufacturer is specified by species, grade and size.

1.2. Stress-Grade Lumber

Pieces of lumber of similar mechanical properties are placed in classes called stress grades (FPL 1990). There are two type of stress-grading methods, i.e., visual grading and mechanical grading. Thus, there are two types of lumber, i.e., visually graded lumber and mechanically graded lumber. The purpose of grading is to provide material suited for the intended uses such as housing construction, etc. In Canada, the Standard Grading Rules for Canadian Lumber are published by Canadian Lumber Grades Authority (NLGA) and the rules for stress grades are intended to provide a reliable measure for determining the strength value of lumber (CWC 1991).

1.2.1. Visually Graded Lumber

Visual grading is based on the premise that mechanical properties of lumber differ from mechanical properties of wood due to visual growth characteristics such as density, slope of grain, presence of knots etc. that affect the properties (FPL 1990). There are two methods for deriving mechanical properties for visually graded lumber, i.e., small clear specimens procedure outlined in the ASTM standard ASTM D 143 "Standard Method of Testing Small Clear Specimens of Timber" and structural size (In-grade) procedure outlined in the ASTM standards ASTM D 198 "Standard Method of Static Tests of Timbers in Structural Sizes" and ASTM D 4761 "Standard Test Methods for Mechanical Properties of Lumber and Wood-Base Structural Material" (ASTM 1991).

The property values that the design engineer uses in his design calculation are known as allowable properties. The allowable engineering design properties must be either inferred or measured nondestructively (FPL 1990). Generally, the allowable properties depend upon the particular sorting criteria and on additional factors that are independent of the sorting criteria (FPL 1990).

For small clear procedure, sorting criteria are handled with strength ratios for strength properties and with quality factors for modulus of elasticity as outlined in the ASTM standard ASTM D 245 "Standard Practice for Establishing Structural Grades and Related Allowable

Properties for Visually Graded Lumber" (ASTM 1991). To account for variability in clear wood properties, the near minimum values, 5 % exclusion limit, and the mean value are used for strength properties and modulus of elasticity, respectively, as outlined in the ASTM standard ASTM D 2555 "Standard Test Methods for Establishing Clear Wood Strength Values" (ASTM 1991). In Canada, small clear test data for all commercially important species are available in Forest Technical Report 21 (Jessome 1977). The similar test data for commercially important North American softwood and hardwood species are summarized in the ASTM D 2555 (ASTM 1991).

The small clear procedure is less preferable because the design values do not necessarily represent the true strength characteristics of structural lumber as used in service. Hence, testing full-size member is believed to provide a better representation of the strength behaviour of structural lumber in structures. This full-size testing, called In-grade testing, has been conducted in Canada and US where visually-graded structural lumber, collected over a wide geographic range within the respective countries, have been tested to destruction (FPRS 1989). This research became known as the In-Grade Testing Program and was initiated to verify the existing allowable design properties for softwood lumber and to provide a basis for more accurately estimating the mechanical properties of lumber for use in reliability-based engineering design codes and standards (FPRS 1989). The standard practice for establishing allowable properties from visually-graded dimension lumber that resulted from Canadian and United States (US) In-Grade Program is outlined in the ASTM standard ASTM D 1990 "Standard Practice for Establishing Allowable Properties for Visually-Graded Dimension Lumber from In-Grade Tests of Full-Size Specimens" (ASTM 1991). In this standard, the characteristic values for strength properties are taken as the nonparametric 5-th percentile point estimates of the test data. For MOE, the characteristic values for each grade are the mean, and the lower 5 % tolerance limit (or other measure of dispersion).

In Canada, the result of the In-grade testing on dimension lumber conducted by the Canadian Wood Council (CWC) has been incorporated in the 1989-edition of the Canadian Standard Association (CSA) CAN/CSA-086.1-M89 "Engineering Design in Wood (Limit State Design)" in which the specified strength values for MOR, UTS, UCS, shear strength and MOE were derived based on the reliability-based design principles (CWC 1990).

1.2.2 Mechanically Graded Lumber

Mechanically grading, called machine stress rating (MSR), uses a machine to sort lumber based on flatwise bending MOE of the piece. The piece is then given the grade mark including f-E classification indicating allowable stress values for MOR and MOE (CWC 1988b). MSR lumber is also required to meet certain visual requirements on defects such as edge-knot size, checks, etc. (NLGA 1987).

Generally, for MSR lumber, the allowable stress for tension and compression properties are developed from the relationships with allowable bending stress rather than being estimated directly by the nondestructive parameter, MOE (FPL 1990). Grades and their mechanical property requirements for MSR lumber produced in Canada are described in National Lumber Grades Authority (NLGA) Special Product Standard 2 (SPS 2), " Machine Stress-Rated Lumber " (NLGA 1987). Evaluations were conducted on MSR lumber in order to evaluate the tension and bending property requirements for selected grades of mechanically graded lumber (Barrett and Lau 1992). These evaluations confirmed that the property specifications for MSR lumber given in SPS 2 are attained for tension and bending for Spruce-Pine-Fir. Allowable properties for 38 mm thick machine stress-rated (MSR) lumber are published in the CWC Datafile WP-5 " Machine Stress-Rated Lumber " (CWC 1988b). In this publication, the ratio of allowable tensile stress parallel to grain to bending stress follows a sliding scale ranging from 0.39 for 900f-1.2E grade to 0.8 for the 2400f-2.0E grade, whereas the ratio of allowable compression stress to bending stress is a constant factor 0.8 for each grade.

1.3. Objectives

As mentioned above, the relationship between MOR and MOE has been used to predict bending strength (MOR) for MSR lumber. Hence, the prediction of UTS and UCS from bending MOE is of significant interest in the recent investigations of structural lumber. This is one of the reasons for the need of the stiffness-strength property relationships. Moreover, since the assignments of UTS and UCS are based on the relationships with MOR, studies on the relationships of UTS and UCS to MOR are crucial for the evaluation of the property assignments.

For the evaluation of the property assignments, it is clear that MOE can be obtained in a nondestructive fashion, therefore the deterministic approach to the material property evaluation is possible only for MOE. Since the determination of MOR, UTS and UCS on full-size lumber requires a destructive testing, one can only measure one strength property on a single member. As a result, there are three pairs of stiffness-strength property relationships that are of interest and which must be collected from tests, there are MOE-MOR, MOE-UTS and MOE-UCS.

As for MSR lumber, the property relationships for visually graded lumber are often used to estimate properties for which test data are unavailable. The In-grade data offer an opportunity to establish conservative property estimates for untested properties when only one property is tested. Thus it allows for the development of models (property relationships) to estimate untested properties so that the amount of testing and cost to establish property values for untested species, grades, or sizes could be greatly reduced (ASTM D 1990-ASTM 1991). Moreover, the need for the property relationships is also prompted by the use of property ratios for standardized property classification systems (stress class system) in international standards (Green and Kretschmann 1990). In general, a better knowledge of lumber property relationships could contribute to the development of more standardized grading systems and improved property assignment in wood design standards (Barrett and Griffin 1989, Green and Kretschmann 1991)

Strength property relationships, derived using the equal-rank method to analyze the US In-grade and Canadian Spruce-Pine-Fir data, have been reported by Green and Kretschmann (1991). The strength property relationships, derived using the same method on combined data of Canadian and US Douglas-fir, Hem-Fir, Southern Pine and Spruce-Pine-Fir (North-American In-grade data) for the estimates of untested properties are described in the ASTM D 1990 (ASTM 1991).

According to the Council of Forest Industries of British Columbia (COFI), Canada is the largest exporter of softwood lumber in the world with 50 % of the world market supply in

1991 (COFI 1993). COFI also reported that British Columbia (B.C.) alone accounts for 34 % of world exports of softwood lumber with the largest export customer being the United States which received 44.4 % of all wood products shipments from B.C. in 1992. Nearly 30 % of the total value of $7.2 billion of the shipments of solid wood products in 1992 from B.C. were used within Canada (COFI 1993). Because Canada uses its own lumber and is a major exporter, it is important to develop property relationships for Canadian species.

Toward the international standards, the results from North American In-grade data (Barrett and Griffin 1989, Green and Kretschmann 1989, Green and Kretschmann 1990, 1991) were compared with property ratios assumed in Eurocode 5 (Fewell and Glos 1989). The relationships adopted in Eurocode 5 were adopted from the work of Curry and Fewell (1977). Curry and Fewell (1977) used an MOE based band-width approach to derive property relationships. Therefore, it is important to examine the equal rank method used in North America and band-width method used by Curry and Fewell (1977) so that the results can be compared. In other words, it is important to evaluate how the results are affected by different methods of analysis.

Using the data base from CWC Full-Size Lumber Properties Program (Canadian In-Grade Program) which contains the three pairs of the stiffness-strength property values for given grades, sizes, and species, the objectives of this thesis are:

1. To develop the relationships between lumber properties for the estimates of characteristic values for untested properties,
2. To evaluate the characteristics of property relationships developed by using different methods,

3. To provide the information on the mechanical property relationships based on Canadian dimension lumber.

It should be noted that even though the data used in this study are visually graded materials, the results are relevant to MSR lumber. This is because basically the materials are the same; only the grading methods are different.

The reader is referred to Barrett and Griffin (1989) for the information on strength property relationships for each grade, size and species, test configuration effects relative to those assumed in Eurocode 5.

2. CURRENT MECHANICAL PROPERTY RELATIONSHIPS FOR DIMENSION LUMBER

For better design and standards, many small scale investigations as well as larger scale investigations have been conducted to search for reliable property relationships for structural lumber. The review of the existing studies and reports on the mechanical property relationships will emphasize the existing stiffness-strength property relationships and strength property relationships for dimension lumber.

As mentioned before, the relationship between MOE and MOR has been used as the basis for MSR grading. Thus, the relationships between MOE and the other strength properties are also important since MOE can be measured directly. Because the variability in strength as a function of MOE is high, the information from studies with large data bases is very important.

The development of the strength property relationships was prompted by the need for estimating characteristic values of untested properties. The development of the strength property relationships were also driven by the need for a better design property assignments for MSR lumber. The information from the test results on full-size lumber from other sources, therefore, are needed in the evaluation and verification of models for the property relationships derived from studies of Canadian visually graded lumber.

2.1. Relationships between MOE and Strength Properties

Since a good relationship exists between MOE and MOR, further studies have shown that compression and tension parallel to grain also are related to MOE (Hoyle 1968). Test results on stiffness-strength property relationships dating back to the years before 1966 were summarized and presented by Hoyle (1968). He proposed the linear empirical models, MOR = 0.00513 MOE - 2265, UCS = 0.00285 MOE + 480, and UTS = 0.00346 MOE - 1850 measured in pound per square inch (psi) for bending, compression parallel to the grain and tension parallel to the grain, respectively, as an average for all North American species.

Relationships between MOE and strength properties, generally, are modelled using a simple linear regression equation $MOR = \beta_0 + \beta_1 (MOE)$, where β_0 is the intercept and β_1 is the slope of the regression line (Hoyle 1968, Curry and Tory 1976, FPL 1977). For the subsequent evaluation, the value of the coefficient of determination r^2 will be emphasized. The coefficient of determination (r^2), which is the square of coefficient of correlation, is the measure of the goodness-of-fit of the model. This is a ratio that describes the relative amount of variation of the dependent variable that has been explained by the regression line.

In relating MOE to a particular strength property, however, no unique value of strength exists. Because MOE can be measured in a nondestructive fashion, MOE can be used to study the relationship between two strength properties. Thus, it is important to evaluate the recent stiffness-strength property relationships for the comparative study with that of In-grade results from Canadian species.

2.1.1. Relationships between MOE and MOR

For structural lumber, MOE is measured from a static bending test, therefore, it is called flexural modulus of elasticity. This MOE is found to be a good indicator of flexural strength or MOR. This relationship is the foundation of MSR grading (Hoyle 1961, Kramer 1964, Sunley and Hudson 1964).

The coefficient of determination (r^2) for the relationships between MOE and MOR ranges from 0.32 to 0.76 for Douglas-fir, Hemlock, Spruce and Pine as calculated from the report by Hoyle (1968). Curry and Tory (1976) reported the relation MOR = 3.576×10^{-3} MOE - 1.66 (N/mm^2) with $r^2 = 0.67$ for European redwood and whitewood. Linear regression results derived from In-grade tests of US species (Green and Kretschmann 1991) are shown in Table 1.

A perfect straight-line fit will have an r^2 value of one, and as the r^2 value decreases from one, the proportion of the total variation in MOR which is explained by the regression with MOE decreases. The higher value of r^2, indicates that MOE is a good indicator variable for MOR. As an example, for Hem-Fir in Table 1, 52 % of the total variation of the MOR values is accounted for or explained by the linear relationship with MOE.

In MSR grading, the lower exclusion limit (usually 5 %) of bending strength is used rather than the mean value in order to account for variability along the linear regression line (Kramer 1964, Hoyle 1968, FPL 1977). According to Bodig and Jayne (1982), for some cases the variance is not constant along the linear regression line, therefore, the lower 5 % exclusion limit is not represented by straight line parallel to the regression line. It will be shown in the subsequent analysis that the band-width method introduced by Curry and Tory

(1976) indirectly takes into account the variability along the regression line in determining the lower exclusion limit. Another method to overcome this problem is to treat the standard error of the estimate as a function of MOE (Woste et al. 1979).

Because the relationship between strength property and MOE needs not always to be linear, other alternative procedures and models have been used to relate strength and elastic properties of lumber. O'Halloran et al. (1972) reported that the nonlinear model, $MOR = \beta_1 (MOE)^{\beta_2}$, gives a better fit on the scatter plot of MOE versus MOR particularly for the data at the extreme ends of MOE range and this model seems more realistic for the lower bound of MOR results for Lodgepole pine dimension lumber. Curry and Tory (1976) also reported that the minimum or lower exclusion limit for the relationship between MOE and MOR is fitted best by this nonlinear model for European species redwood and whitewood and Canadian hemlock.

2.1.2. Relationships between MOE and UTS

The simple linear regression model is also adopted for the relationship between MOE and UTS. The coefficient of determination for the relationships between MOE and UTS as calculated from the report by Hoyle (1968) was 0.55, 0.56 and 0.66 for Douglas-fir, white fir and hemlock, respectively.

By testing full-size structural lumber of two species namely Swedish redwood and whitewood of three sizes 38 x 100, 150, and 200 mm, Curry and Fewell (1977) proposed the model UTS = 0.00242 MOE - 1.51 (N/mm^2) with $r^2 = 0.59$.

The linear regression models for the relationships between MOE and UTS for the US In-grade data reported by Green and Kretschmann (1991) are presented in Table 2. For each species, the r^2 is smaller than that observed for the relationship between MOE and MOR.

Curry and Fewell (1977) also proposed the nonlinear model, $UTS = \beta_1 (MOE)^{\beta_2}$ for the relationship between MOE and the lower exclusion limit of UTS.

2.1.3. Relationships between MOE and UCS

Like the relationship between MOE and UTS, the general relationship between MOE and UCS is represented typically by simple linear regression equation. The coefficient of determination for this relationship, as calculated from the report by Hoyle (1968), was 0.61, 0.71 and 0.45 for Douglas-fir, Grand fir and Southern pine, respectively. Curry and Fewell (1977) reported the model $UCS = 0.00148\ MOE + 10.41$ (N/mm^2) for Polish redwood (combined size 38 x 100 and 50 x 100 mm) with the $r^2 = 0.58$.

Based on tests of 2-inch Southern pine dimension lumber, Doyle and Markwardt (1966) proposed the model $UCS = 0.0001767\ MOE_c + 1881$ (1,000 psi) for all grades and sizes with coefficient of determination $r^2 = 0.45$. MOE_c is the modulus of elasticity in compression parallel to the grain entered in million psi units. They also found that the compression modulus of elasticity is closely comparable to the flexural modulus of elasticity both flatwise and edgewise.

The relationship between MOE and the lower exclusion limit of UCS has the same nonlinear form as the relationship between MOE and UTS proposed by Curry and Fewell (1977).

2.2. Strength Property Relationships

The development of strength property relationships to estimate untested properties was prompted by the need for multiple property assignments (ASTM D 1990-ASTM 1991). In practice, however, bending and tension or bending and compression may occur simultaneously at a cross section. Therefore, different members require different types of assigned stresses. As a consequence, the multiple allowable properties have to be assigned to the product since the end use of the product is unknown in the outset.

Measurement of the strength properties of lumber generally involve a destructive test. Normally, only one failure mode can be evaluated from a single piece of lumber. Because only a single failure mode can be obtained, it is not possible to measure MOR, UTS and UCS at the same member cross-section. In other words, because one can not break a piece of lumber twice, the relationships between two strength properties, particularly for MOR, UTS and UCS, can only be described in probabilistic terms.

Strength property assignments for visually graded lumber based on the full-size testing procedure, practically, does not require property relationships for the tested properties. The assignments of bending strength, tensile strength and compression strength parallel to the grain are based on the 5 % lower exclusion limit (nonparametric fifth percentile estimate) as outlined in the ASTM D 1990 (ASTM 1991).

For MSR lumber, different rules apply. In machine grading, one can select lumber with specified minimum MOR or minimum MOE. The assignments of the other properties such as UTS and UCS were established as fixed proportions of bending stress. Historically, WWPA (1965) assigned both UTS and UCS as 80 % of allowable MOR. The work by Littleford (1967) showed that this allowable tensile stress was over estimated. The historical overview on the evolution of tensile stress assignment dating back to the years until 1979 was reported by Galligan et al. (1979). According to their report, in 1969, a sliding tension property scale factor, ranging from 0.39 to 0.8 of bending strength, was used to calculate the allowable tensile stress parallel to the grain for MSR lumber. Until now, similar scale factors are maintained for the UTS/MOR ratios, while the UCS/MOR ratios are maintained at approximately 80 % of the allowable bending stress for all grades as proposed by NLGA (1987) and CWC (1988b).

Due to the need for the improvement of the grading and standards, several studies have been carried out to model the property relationships for lumber (Curry and Fewell 1977, Johnson and Galligan 1983, Green et al. 1984, Bartlett and Lwin 1984, Evans et al. 1984, Green and Kretshmann 1991). There are several methods introduced by these investigators for determining the property relationships.

Curry and Fewell (1977) made use of the relationships between MOE and strength properties to establish the strength property relationships. Using European redwood and white wood, they showed that the ratio of the near minimum value (1 % and 5 % lower exclusion levels) of UTS to MOR is approximately 0.60 in the design property range. Whereas the ratio of UCS to MOR is represented by:

$$R_{C/B} = 4.93\ MOR^{-0.54} \tag{1}$$

where $R_{C/B}$ is the ratio of UCS to MOR and MOR is entered in N/mm^2 (MPa). From this equation, it is clear that the UCS/MOR property ratio decreases as bending strength increases.

The property relationships, established on the basis of the equality of the percentile rank, have been proposed by Barrett and Griffin (1989) and Green and Kretschmann (1989) for Canadian In-grade data and US In-grade data, respectively. Using North-American In-grade data, Green and Kretschmann (1991) further developed the models for the strength property relationships. Their results were incorporated in the ASTM standard D 1990 (ASTM 1991) for North-American In-grade data.

Green and Kretschmann (1991) proposed a constant factor 0.56 for the ratio of UTS to MOR for MOR values below 48.3 MPa (7 ksi), for all grades, species and sizes, whereas a conservative factor of 0.45 is adopted in the ASTM standard ASTM D 1990 (ASTM 1991) for data adjusted to nominal 2 by 8 size and 15 % average moisture content. For the ratio of UCS to MOR, Green and Kretschmann (1991) reported (for a 2 by 8 size, all species and grades) a constant factor 0.596 for MOR > 49.6 MPa (7.2 ksi) and below this limit the UCS/MOR ratio is given by:

$$R_{C/B} = 1.745 - 0.320\ MOR + 0.0223\ MOR^2 \tag{2}$$

where MOR is entered in ksi. The same model, except for the intercept is 1.55 and 0.22 for the quadratic term, is adopted in the ASTM standard ASTM D 1990 (ASTM 1991) for all values of MOR adjusted to 2 by 8 and 15 % moisture content.

Green and McDonald (1993) reported that the ratio model:

$$R_{C/B} = 0.338 + \frac{2.061}{MOR} \quad (3)$$

for MOR ≥ 2.835 ksi (19.55 MPa) and the constant 1.06 if MOR is less than this limit for data adjusted to 15 % moisture content was adopted for MSR softwood lumber by the American Lumber Standards Committee, Board of Review.

Green and Kretschmann (1991) reported that (for a 2 by 8 size, all species and grades) for the property relationship set on the basis of tensile strength then the ratio of UCS to UTS is a constant factor 0.837 for UTS > 38.6 MPa (5.6 ksi) and below this limit is given by:

$$R_{C/T} = 2.724 - 0.678\ UTS + 0.0608\ UTS^2 \quad (4)$$

where $R_{C/T}$ is the ratio of UCS to UTS and UTS is entered in ksi. However, this ratio is said to vary somewhat with species, lumber size and grade. A conservative model:

$$R_{C/T} = 2.4 - 0.7\ UTS + 0.065\ UTS^2 \quad (5)$$

where UTS is entered in ksi, is adopted in the ASTM standard D 1990 based on UTS values adjusted to 2 by 8 size and 15 % moisture content (ASTM 1991). The ratio of MOR to UTS set on the basis of UTS is taken to be constant factor 1.2 in this standard for the same adjustment conditions.

The ASTM standard D 1990 (ASTM 1991) recommends that when both UTS and MOR data are available, the most conservative should be used for calculating UCS.

3. THE CANADIAN IN-GRADE DATA BASE FOR DIMENSION LUMBER

One of the objectives of the Canadian In-Grade Program is to determine the mechanical properties of dimension lumber (CWC 1988c). With the introduction of the Limit State Design version of National Standard of Canadian CAN/CSA-086.1 in 1984, the CSA Committee responsible for the Code Engineering Design in Wood adopted the philosophy that design properties for structural wood products should be based on full-size structural tests (CWC 1990).

3.1. Data Source

As mentioned in the CSA Commentary to the 1989-edition of CAN/CSA-086.1-M89 by J. D. Barrett (CWC 1990), the Canadian Wood Council, through its Lumber Properties Steering Committee, conducted a lumber properties research program for bending, tension and compression parallel to grain strength properties for 38 mm (nominal 2 inch) dimension lumber of all commercially important species groups. In this program, short term bending stiffness properties and bending, tension and compression parallel to grain strengths were evaluated in accordance with the ASTM standard ASTM D 4761.

The test data provided by Canadian Wood Council for this project will be used to establish lumber property relationships for Canadian dimension lumber.

3.2. Sampling and Test Methods

The detailed description of the sampling, testing, moisture-content adjustment procedures are given in the report from the Council of Forest Industries of British Columbia by Fouquet and Barrett (1989). The summary on the mechanical property data is presented in the report by Canadian Wood Council (CWC 1988c). The following discussion provides a brief summary of key elements of the testing procedure for the mechanical properties (CWC 1988c).

The experiments were conducted at the Vancouver laboratory of Forintek Canada Corp. The specimens were conditioned to approximately 15 % moisture content prior to bending, tension parallel to the grain, and compression parallel to the grain tests. Moisture content of each specimen was measured at the time of testing.

Prior to destructive testing the flatwise MOE profile was measured using a Cook-Bolinders mechanical grading system. Edgewise bending MOE of each sample was measured using a third point loading system. The bending test was conducted on a 17 to 1 ratio of test span (L) to member width (W) with the tension edge and the maximum strength reducing defect (MSRD) randomly assigned in the test span. MOR, UTS, and UCS values were determined using the maximum load and the actual dimensions of the specimen. The gauge lengths of the test specimens are summarized in Table 3 (Barrett and Griffin 1989).

The MOR, UTS and UCS data are available from three species combinations, i.e., Douglas-fir-Larch, Hem-Fir and Spruce-Pine-Fir (hereafter, abbreviated to D-fir, H-Fir and S-P-F, respectively), two grades, (select structural (SS) and number 2 (No.2)) and 3 sizes (2x4, 2x8 and 2x10).

4. PROPERTY ADJUSTMENTS FOR PROPERTY RELATIONSHIP STUDIES

Physical properties of wood such as moisture content, density or specific gravity influence the mechanical properties of wood. Generally, moisture content of wood is controllable in the experiment. It is well known that moisture content influences both strength and stiffness of wood especially for bending and compression (Madsen 1992). For dimension lumber, the testing procedure requires the test samples to be conditioned to the target moisture content prior to testing. However, moisture content at the time of test will vary in a narrow range.

Testing the full size structural lumber has shown that the strength decreases with an increase in member size. The early works on size effects by Bohannan (1966), Barrett (1974), and Kunesh and Johnson (1974) have shown that size has a very significant effect on strength properties of lumber. Because of anisotropic nature of wood, Madsen and Buchanan (1986) suggested that different size parameters should be used to quantify for different size effects in member width and length.

4.1. Moisture Content Adjustment

Because strength and stiffness of wood are influenced by moisture content, the individual test results were adjusted to a common moisture content in order to reduce the bias due to moisture content variations. Following the ASTM standard D 1990-90 (ASTM 1990)

MOE of each piece of lumber was adjusted to 15 % moisture content level using the In-grade formula:

$$MOE_2 = MOE_1 \left\{ \frac{1.8566 - 0.023722\,M_2}{1.8566 - 0.023722\,M_1} \right\} \tag{6}$$

where:

MOE_1 = MOE (psi) at moisture content level 1,

M_1 = Moisture content level 1(decimal),

M_2 = Moisture content level 2 (decimal).

This MOE value then was adjusted for uniformly distributed load (UDL) 21:1 span-to-depth ratio according to ASTM D 2915-90 (ASTM 1990).

MOR and UCS data (as tested) were adjusted to 15 % moisture content level using the Linear Surface Model (Barrett and Lau 1991a, 1991b) as follows:

$$P_{15} = \frac{\{1 - D_1(15 - M_1)\} - \sqrt{\{(15 - M_1)D_1 - 1\}^2 - 4P_1D_2(15 - M_1)}}{2D_2(15 - M_1)} \tag{7}$$

where:

P_1 = property value (ksi) at moisture content M_1,

P_2 = property value (ksi) at moisture content 15 %,

D_1 = -0.95689 for MOR, and -2.36662 for UCS,

D_2 = 0.2033 ksi^{-1} for MOR, and -0.215548 ksi^{-1} for UCS.

M_1 = actual moisture content.

Moisture content adjustments were not made for the tension data in this study.

4.2. Size Adjustment

Table 3 shows that the test gauge length for bending, tension and compression specimens vary by specimen widths. Bending specimens were tested at constant span-to-depth ratio 17:1. Since the test gauge lengths for tension and compression specimens are different from bending specimens, the UTS and UCS test values were adjusted to the test length of the bending specimens (i.e., the length is 17 x member width). Then, to account for width effects all test results were adjusted to a common width of 7.25 inches and a length of 17 x 7.25 inches. The size adjustment procedure is similar to the standard procedure in the ASTM D 1990 (ASTM 1991). The property adjustment formula is as follows:

$$P_2 = P_1\left(\frac{L_1}{17W_1}\right)^{S_L}\left(\frac{W_1}{7.25}\right)^{S_R} \tag{8}$$

where:

P_1 = Property measured at length L_1 and width W_1,

P_2 = Property adjusted to nominal 7.25 inch at length = 17 W_1,

S_L = Size effect factor for length,

S_R = Size effect factor for a member with a constant span to depth ratio 17:1.

The factors, S_L and S_R are 0.17 and 0.4 respectively, for bending and tension members and 0.1 and 0.21 respectively, for compression members (Barrett, Lam and Lau 1992).

After adjusting the 2x4, 2x8 and 2x10 data to nominal size 38 x 184 mm (1.5 x 7.25 inch), preliminary analysis (not presented here), showed that there were no differences in the mean values among the adjusted size groups. Therefore, for the subsequent analysis, the data for the adjusted sizes were pooled. The same preliminary analysis showed that there were significant differences in the mean values between the two grades, SS and No.2. Nevertheless, in order to have the results represent both low and high strength materials, select structural and No. 2 grades were pooled for the analysis on property relationships. In addition, the unit measurement of the data were converted from Imperial to Metric (SI) units after all the necessary adjustments have been performed.

The histograms of the pooled MOE and strength property data (adjusted size and moisture content) are presented in Fig. 1. In this figure, the histograms for MOE and the strength properties are presented on a common scale in order to allow direct comparisons of the shape of the distribution for each case. The height of the histogram is equal to the relative frequency divided by base length called density scale (Devore 1991). Therefore, the ordinate is equal to the probability density function since the area under all of the histograms is equal to unity.

Graphical presentations of 2x4, 2x8 and 2x10 (15 % moisture content) data before and after size adjustments (Eq. (8)) are given in Fig. 2. The graph shows that the size adjustment quite successfully eliminated the effects of size variations especially for the lower and medium strength levels.

5. DATA ANALYSIS

The models for strength and stiffness-strength property relationships were derived using regression analysis and other modelling techniques. Regression analysis is defined as a statistical tool for evaluating the relationship of one or more independent variables to a single continuous dependent variable (Kleinbaum et al. 1988). Whereas, modelling refers to the development of mathematical expressions that describe in some sense the behaviour of a random variable of interest (Rawlings 1988). Regression analysis is applied for several reasons such as finding the quantitative formula or equation to describe the dependent variable as a function of the independent variable(s) or determining the best mathematical model for describing the relationship between a dependent variable and one or more independent variables (Kleinbaum et al. 1988). In the subsequent analysis, of the relationships between MOE and the strength property, MOE is considered as the independent variable whereas the strength property, i.e., MOR, UTS, or UCS, is considered as the dependent variable. Likewise, when assessing the strength property relationship between two properties, one will be considered as the independent and the other as the dependent variable depending on the intended application of the model.

There are two main steps in the subsequent analysis. First, determining the empirical models for the relationships and second, finding the best model by means of the regression analysis. There are many models that could be chosen to represent the relations between properties. Some of the models can be developed based on assumptions about the underlying wood property distributions. In other cases the choice of the model is based on experience or previous work.

5.1. General Relationships between MOE and Strength Properties

The general relationship is defined as the relationship between MOE and the corresponding strength property obtained by fitting the regression model to the full data set. After the necessary adjustments have been carried out and the combination of the two grades, the regression analyses were performed for the general relationships between each strength property (i.e., MOR, UTS, and UCS) and MOE.

Two regression models, such as a linear and a nonlinear, were used to characterize the relationships between each strength property and MOE. As mentioned before, the simple linear regression model has been used extensively to model the relationships between MOE and the strength properties. Green and Kretschmann (1991) also used this model for their studies of the US In-grade data. With the assumption of additive error, the linear model is of the form (Neter and Wasserman 1990, Rawlings 1988):

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i \tag{9}$$

where:

- Y_i = Strength property in the ith trial,
- X_i = MOE in the ith trial (assumed to be a set of known constants),
- β_0 and β_1 = Parameters,
- ε_i = Independent random error (assumed normally distributed with mean = 0 and variance = σ^2),
- i = 1,2,...,n.

The following are the nonlinear models that were evaluated:

$$Y_i = \beta_0 + \beta_1 X_i^{\beta_2} + \varepsilon_i \tag{10}$$

$$Y_i = \beta_1 X_i^{\beta_2} + \varepsilon_i \qquad (11)$$

$$Y_i = \beta_1 X_i^{\beta_2} \beta_3^{X_i} + \varepsilon_i \qquad (12)$$

where:

Y_i	=	Strength property in the *i*th trial,
X_i	=	MOE in the *i*th trial,
β_0-β_3	=	Parameters,
ε_i	=	Random error (assumed independently and identically distributed with mean = 0 and variance = σ^2) (Judge et al. 1985),
i	=	1,2,...,n.

The regression analyses were carried out using SAS package Release 6.03 (SAS 1988). The starting points for the parameters in the nonlinear model of Eq. (10) were taken from the results of Eq. (9). Whereas, for Eqs. (11) and (12), the starting points were estimated from the result of fitting a linear regression to the logarithmic transformation of the models. All of the nonlinear models were fitted using the Gauss-Newton iterative method (SAS 1988). Tables 4, 5 and 6 show the results of these two regression analyses for all species.

The following steps were taken in order to determine the best model for each relationship in general:

1. Test if $\beta_0 = 0$ in Eq. (9); if yes, then the model without intercept should be used; use Eq. (9) otherwise,
2. Test if $\beta_2 = 1$ in Eq. (10); if yes, then Eq. (9) is adequate; use Eq.(10) otherwise,
3. For $\beta_2 \neq 1$ in Eq. (10), test if $\beta_0 = 0$ in Eq. (10); if yes, then Eq. (11) is adequate; use Eq. (10) otherwise,

4. Test if $\beta_3 = 1$ in Eq. (12); if yes, then Eq. (11) is adequate; use Eq. (12) otherwise,

5 If Eq. (10) and Eq. (12) are sufficient models, compare Eq. (10) with Eq. (12) in terms of the lowest Sum of Square Error (SSE),

The hypothesis test for the parameter is carried out using 2-tailed t-test (Gallant 1987):

$$|t| = \left[\frac{\hat{\theta} - \theta}{s(\hat{\theta})}\right] \tag{13}$$

where:

$\hat{\theta}$ = Estimated parameter,

θ = Known parameter,

$s(\hat{\theta})$ = Standard error of the estimate.

The null-hypothesis was rejected when $|t| > t_{(1-\alpha/2,\ n-p)}$ where n is the sample size and p is the number of the parameter in the model. Since n is large, for $\alpha = 0.05$, then $t = 1.96$.

It can be seen from Tables 4, 5, and 6 (in the last column) that the $|t|$ value revealed that the intercept in Eq. (9) was significantly different from 0 for each species and relationship. Therefore, the intercept in Eq. (9) is necessary at the level of significance $\alpha = 0.05$.

Parameter β_2 in Eq. (10) was not significantly different from 1 for MOE-MOR relationships for Douglas-fir and Hem-Fir and MOE-UTS relationship for Douglas-fir, therefore Eq (9) is adequate to represent these relationships. However, this parameter was significantly different from 1 for MOE-MOR relationship for S-P-F, MOE-UTS for Hem-Fir and S-P-F and MOE-UCS for all species. It indicates that the nonlinear rather than the linear model should be used.

For the case where parameter β_2 in Eq. (10) was significantly different from 1, Tables 4, 5 and 6 show that both MOE-MOR and MOE-UTS relationships are sufficiently represented by Eq. (11) since the intercept (β_0) in Eq. (10) for each relationship was not significantly different from 0. This results were also justified by parameter β_3 in Eq. (12) which in most cases was not significantly different from 1. Only for the MOE-UCS relationship (Table 6) Eq. (10) is shown to best represent the data since parameter β_0 was significantly different from 0 in all cases.

As can be seen from Table 6 where the parameter test showed that Eq. (10) is generally adequate; Eq. (12) also gives good results for representing the data. Eq. (12), in many cases, yielded a comparable SSE when the parameter β_3 was significantly different from 1. In this case, the proposed model should be chosen from these two equations based on the SSE and the simplicity of the model. For this reason, Eq. (10) is preferable than Eq. (12).

In summary, the above analysis showed that generally MOE-MOR, MOE-UTS and MOE-UCS relationships were represented by, respectively, Eq. (9), Eq. (11) and Eq. (10). The complexity of Eq. (12) did not seem to be justified given the small improvement in the model performance.

In the subsequent analysis on modelling of strength properties, a single model for the stiffness-strength relationships was adopted. Although, models like Eqs. (10) and (11) will perform nearly equal, Eq. (10) was adopted for the modelling purposes. The regression models (Eq. (10)) for MOE-MOR, MOE-UTS, and MOE-UCS for each species are depicted in Figs. 3, 4 and 5.

It should be noted that, statistically, there are disadvantages in assuming a nonlinear model rather than the linear one for the MOE-strength relationships when the linear model is adequate. If the model is linear and all the necessary assumptions concerning regression procedures are met, then the least-squares estimators of the parameters in Eq. (9) are optimal since they are minimum variance unbiased estimators. However, when the model is nonlinear, there are no such best estimators of the parameters, i.e., none of these properties are possessed by the least-squares estimators (Myers 1990). Nevertheless, if the error terms ε_i are normally distributed, then the least squares estimator is the maximum likelihood estimator and, under these conditions, the estimators posses asymptotic properties, i.e., the sample size must be large to approach the unbiasness and minimum variance (Myers 1990, Seber and Wild 1989).

Based on the underlying objective on the development of the property relationships of lumber, however, the purpose of using regression analysis is merely as a tool for obtaining the empirical model relating any two properties. Therefore, the above limitations on the nonlinear model are beyond the scope of this study.

In this section four related regression models were evaluated for representing the relationship between strength properties and MOE. The analysis showed that a nonlinear model is best suited for representing the results considered in this study. A single nonlinear model was adopted for relating structural properties of lumber. This model will degenerate to the power-type models used in the United Kingdom (UK) (Curry and Fewell 1977) when the intercept is zero, and yields the common linear regression model if the power term on the independent variable is not significant.

5.2. Strength Property Relationships

As mentioned before, for a given piece of lumber only one type of failure mode is available from destructive measurement, thus the relationship between two strength properties can only be established indirectly. Several attempts have been made to relate one type of strength property (e.g., MOR) to the other (e.g., UTS or UCS). Johnson and Galligan (1983) introduced the method for estimating the concomitance or cofunction of lumber strength properties. With their method, the choice can be made either with or without considering the relationships of MOE and knot size to a particular strength property. Similar work, but without the information of knot size, has been presented by Bartlett and Lwin (1984). Other methods of estimating the correlation or degree of concomitance between lumber strength properties without using nondestructive information such as MOE and knot size have been developed by Green et al. (1984) and Evans et al. (1984). However, all of these methods require proof loading of the materials, i.e., testing every board in the sample population up to a pre-set load in one failure mode followed by testing the survivors in the second mode. In this case, the correlation between two failure modes or strength properties depends on the choice of the proof load or cut-off point using in the proof loading.

Relationships between structural properties have been estimated using MOE-based regression method and the so-called equal rank method. The MOE-based regression method was used by Curry and Fewell (1977) and Green and Kretschmann (1991) to develop relations between UTS and MOR. As has been discussed in the review for the strength property relationships, the equal-rank method was used by Barrett and Griffin (1989) and Green and Kretschmann (1989). It is the simpler method which allows the relationship between two strength properties to be established on the basis of their empirical cumulative distribution functions.

The applications of these methods rely on different underlying fundamental assumptions about the relationship between property distributions which cannot be verified experimentally.

To date there are no published studies comparing the property relationship results derived using these two different approaches. In this study the In-grade data base will be analyzed to establish property relationships using both the MOE-based regression method and the equal-rank assumption method.

5.2.1. Equal-Rank Method

The equal-rank method is an extension of standard analysis procedures involving comparison of mean or percentile property results obtained from different strength property evaluations.

Suppose, the strength data are obtained for a particular product and the cumulative distribution functions are constructed for property A and B as shown in Fig. 6a. Then for any selected cumulative probability level (F) the property levels P_A and P_B can be estimated from the data or computed from a fitted cumulative distribution function. The property P_A can be plotted as a function of P_B for a range of cumulative probability levels as shown schematically in Fig. 6b.

Alternatively, property ratio P_A/P_B can be derived from data or the cumulative distribution function and presented as a function of P_A or P_B as appropriate.

The concept of comparing property values at selected cumulative probability levels (or selected rank in the case of ranked data set) is called the equal-rank (equal probability) method.

The results obtained from an equal-rank analysis will vary depending on the specific methods adopted in the study. If the cumulative distributions for property A and B are known, then the property relationships can be derived directly from the cumulative distribution functions. The procedure can be illustrated using the Weibull cumulative distribution function.

Weibull distribution has been widely used to represent the distribution of lumber strength data (Evans et al. 1989). The cumulative distribution function (cdf) for a 2- and 3-parameter Weibull distribution can be written as follows (Barrett 1974, Bodig and Jayne 1982), respectively:

$$F_w(x; m, k) = 1 - EXP\left[-\psi V\left(\frac{X}{m}\right)^k\right] \tag{14}$$

$$F_w(x; x_0, m, k) = 1 - EXP\left[-\psi V\left(\frac{X - x_0}{m}\right)^k\right] \tag{15}$$

where:

X = The strength of a given piece,

V = Volume of the given piece,

ψ = A constant depending on the type of loading and the shape parameter,

k = Shape parameter,

m = Scale parameter,

x_0 = Location parameter.

Eqs. (14) and (15) represent the probability of failure of a member having the strength property X. If X is the bending strength (MOR) then, Eq. (15) can be rewritten with subscripts b to indicate bending strength parameters as follows:

$$F_W(\text{MOR}; x_{0b}, m_b, k_b) = 1 - EXP\left[-\psi V\left(\frac{\text{MOR} - x_{0b}}{m_b}\right)^{k_b}\right] \tag{16}$$

Similarly, for tensile strength and compression strength the Weibull cdf's are given by, respectively:

$$F_W(\text{UTS}; x_{0t}, m_t, k_t) = 1 - EXP\left[-V\left(\frac{\text{UTS} - x_{0t}}{m_t}\right)^{k_t}\right] \tag{17}$$

$$F_W(\text{UCS}; x_{0c}, m_c, k_c) = 1 - EXP\left[-V\left(\frac{\text{UCS} - x_{0c}}{m_c}\right)^{k_c}\right] \tag{18}$$

where $\psi = 1$ for tension and compression strength parallel to the grain which have uniform stress distributions (Barrett 1974).

Provided that the member has the same size and probability of failure (F_W) then, the relationship between UTS and MOR can be derived from Eqs. (16) and (17) as follows:

$$\left(\frac{\text{UTS} - x_{0t}}{m_t}\right)^{k_t} = \left(\frac{\text{MOR} - x_{0b}}{m_b}\right)^{k_b} \psi$$

$$\text{UTS} = x_{0t} + \frac{m_t}{m_b^{\left(k_b/k_t\right)}} \psi^{\left(1/k_t\right)} \left(\text{MOR} - x_{0b}\right)^{\left(k_b/k_t\right)} \tag{19}$$

Thus, the relationship between UTS and MOR can be derived if the scale (m), shape (k), location (x_0) parameters, and the constant ψ of the bending and tension strength distribution are known. Likewise, the relationship between UCS and MOR can be derived from Eqs. (16) and (18).

Examining Eq. (19), it is apparent that the relationship between strength properties can be written in a simplified form similar to that for regression analysis presented earlier. The simplified forms for the 2- and 3-parameter Weibull distributions can be expressed as follows:

$$Y = \beta_1 (X)^{\beta_2} \tag{20}$$

$$Y = \beta_0 + \beta_1 (X - \beta_2)^{\beta_3} \tag{21}$$

where:

Y	=	UTS or UCS,
X	=	MOR,
β_0 - β_3	=	Parameters.

Relationships between property values have been derived using the equal-rank assumption and Weibull cumulative distribution functions to represent the property distributions. The derived relationships are similar in form to the nonlinear regression model (Eqs. (11) and (12)). In fact, the equal-rank assumption applied to a 2-parameter Weibull cdf leads to exactly the same form of property model as Eq. (11).

The 3-parameter Weibull cdf leads to a nonlinear model very similar in form to Eq. (12). Thus, the equal rank concept has provided a basis for deriving a property relationship model which is consistent with test data if the individual data sets are adequately represented by Weibull cumulative distribution functions. Since Weibull models are widely used to represent strength data, the models given in Eqs. (20) and (21) were chosen for subsequent property relationship studies.

The application of Eqs. (20) and (21) in property relationship analysis can be illustrated by considering UTS and MOR relations. Because the ranking was carried out to each grade and size before pooling of the adjusted grades and sizes, the direct approach for model parameter as per Eqs. (20) and (21) cannot be obtained. Therefore, the models of Eqs. (20) and (21) were fitted to UTS-MOR data using regression techniques (see Fig. 7).

For each species and property, the data for select structural and No.2 grades, 2x4, 2x8 and 2x10 sizes were analyzed as follows:

1. Adjust the strength data to 15 % moisture content using Eq. (7),
2. Rank the data in ascending order,
3. Estimate the non-parametric strength values for the corresponding percentile levels 0.02, 0.05, 0.1,..., 0.95, 0.98,
4. Adjust all data to a nominal size 38 x 184 mm (1.5 x 7.25 inch) and length L = 3128 mm using Eq. (8),
5. Fit the regression model of Eqs. (20), (21) and (12) to the combined SS and No.2 grade data.

Eq. (12) was also fitted to the data even though it does not have the basic assumption about the property distributions as for Eqs. (20) and (21). Nevertheless, this model is preferred, as an alternative model to Eq. (21), if the argument of zero value for the dependent variable for given zero value for the independent variable is to be maintained.

All equations are fitted using the Gauss-Newton iterative procedure in SAS package for nonlinear regression. The starting points for Eq. (21) were found by firstly fitting a 3-parameter Weibull distribution to the strength data (full adjusted data set) to estimate parameters β_0 and β_2, and secondly by fitting the regression to the reduced model after knowing β_0 and β_2.

The Sum of Square Error of Eq. (21) was tested against that of Eq. (20) for the significance of parameters β_0 and β_2 using Likelihood Ratio (LR) test (Judge et al. 1985) below:

$$\lambda_{LR} = n\left\{\ln s(b^*) - \ln s(b)\right\} \tag{22}$$

where:

$s(b^*)$ = Sum of Square Error of Eq. (20),

$s(b)$ = Sum of Square Error of Eq. (21),

n = Sample size.

The null hypothesis that parameters β_0 and β_2 (3-parameter Weibull) have significant effect in reducing the error variance is rejected if the statistic λ_{LR} exceeds $\chi^2_{(\alpha,\nu)}$ for a pre-specified level of significant α. For $\alpha = 0.05$ and the number of the restricted parameters $\nu = 2$, then $\chi^2_{(0.05,2)} = 5.991$. If $\lambda_{LR} < 5.991$, then Eq. (20) is adequate to represent the relationship. However, the test shows that λ_{LR} exceeds 5.991 except for the relationship between UTS and UCS for all species as shown in the last column (LR) of Table 7 and Table 8.

The results of the fitted regression of Eqs. (12), (20) and (21) are presented in Table 7 and Table 8 for all species and property relationships.

5.2.2. Band-Width Method

Due to the need for establishing property relationships for MSR lumber, Curry and Tory (1976) introduced a method based on analysis of strength and MOE data which involves calculating strength properties for MOE subgroups or bands. In MSR grading, for a given grade, the assigned bending stress is calculated for those pieces with an MOE in a pre-selected MOE range. According to Curry and Tory (1976), this bending stress depends on the width of the grade, i.e., the difference between the boundary value of MOE for one grade and the next higher grade, and also on the location of the grade within the full range of MOE values for the species. Their main objective was to evaluate the effects of these two factors on the assignment of bending stress. Fig. 11a is a sketch that illustrates their method. In the study reported herein, this method is called band-width method.

The main objective here is to find the relationship between MOE and the lower percentile of the strength properties corresponding to the 1-st and 5-th percentile strength levels. Permissible stresses for machine stress-graded timber are constructed by taking the 5 % exclusion limit for MOR as shown by the hypothetical 5 % exclusion line in Fig. 11a. This line is chosen to insure that 95 % of all the strength values will be above this line. However, according to Curry and Tory (1976), this linear lower exclusion line can lead to zero or negative values of a strength property associated with a non-zero value of MOE and often no account is taken of the effect of the range of MOE values included in a particular grade on the corresponding 5-th percentile of strength values. Following their method, this problem is solved by sub-dividing the strength data into bands of MOE values in order to determine the lower exclusion levels of the strength properties. This procedure is similar to the derivation of design properties for MSR lumber.

It can be seen from Fig. 11a that by setting the boundaries of MOE at points A and B, then the strength and MOE data that fall inside this boundary or band will have a certain frequency distributions as illustrated by the bell-shape curves on their margins. The MOE values at points A and B are known at the outset once a range has been chosen as the representation of a grade increment. This boundary acts as a "window" in which the distribution of MOE and the strength data have to be determined as shown in Fig. 11b. This window can be moved in a certain step-length or increment while a distribution model can be fitted to the strength and MOE data in order to determine the point estimates.

The method described above involves the following steps:

1. Select the width of the band (window) for MOE and the step-length (increment),
2. Start moving the window from zero MOE one step-length at a time,
3. For each increment in point 2, rank the strength data in ascending order and fit the appropriate distribution to estimate the values corresponding to the 1-st, 5-th and 50-th percentile levels. Fit the appropriate distribution to the MOE data that fall in the same window to estimate the value corresponding to the 50-th percentile level.

Following the method by Curry and Tory (1976), the band-widths chosen in step 1 are 500, 690, 1,000, 1,500, 2,000 and 2,500 MPa. The 690 MPa band-width was added to correspond to the grade-increment 10^5 psi used in North America. The step-length is 100 MPa irrespective of the band-width. Because the estimated values in the lower tail of the strength distribution, particularly towards the extremes of MOE where the number of data points are scarce, could be influenced by the initial location of the band or window, the initial location was set to zero and was advanced by increments (step-length) of 100 MPa along the MOE axis in order to minimize this effect. The number of data points in step 3 is arbitrarily chosen for which the minimum 30 data points is believed to be sufficient for the estimation.

In order to determine the relationship between MOE and the minimum value of the strength, one has to define what MOE value inside the window should be related to the 1-st, 5-th and 50-th percentile values of the strength data for a given band-width. There is no warranty of an exact value of MOE for these percentile values of the strength. Curry and Tory (1976) related minimum MOE for a given band-width to each corresponding 1-st and 5-th percentile strength values similar to what occurs in bending stress assignment in MSR lumber. Their results show higher strength values for wider MOE band-width.

By comparing band or window A-B with A-C in Fig. 11a in terms of their corresponding 50-th percentile values (denoted by m_1 and m_2), it is clear that wider MOE band-width will have higher estimated strength value for the same minimum value of MOE. The regression line in this figure shows that, for a given MOE, the expected value of the strength will lie on it provided that this is the best fitted line. Also the horizontal and vertical lines corresponding to the mean values of the entire MOE and strength data, respectively, will intersect each other exactly on the regression line. In analogy, the 50-th percentile values of the strength distribution also will lie close to the regression line regardless of the band-width since any band-width must have the same mean trend values for the same data set. The preliminary analysis using the minimum values of MOE as the matching pair for the 50-th percentile strength value for the band showed that there were no close results for the mean trends (50-th percentile) of strength as a function of MOE across the band-widths. The wider band-width produced higher 50-th percentile strength values for the same minimum MOE value. Therefore, there was a shifting effect due to the width of the band. Thus, in order to maintain the same predicted mean trend (50-th percentile) of the strength distribution across the band-widths, the 50-th percentile of MOE was chosen as the corresponding point for the 50-th percentile of the strength distribution as illustrated in Fig. 11b.

The strength data that fall inside the window were fitted using a 3-parameter Weibull distribution to estimate the strength values at the selected percentile levels, i.e., 1-st, 5-th and 50-th percentile. Whereas, the Johnson's S_b distribution was fitted to the MOE data in the same window in order to estimate the 50-th percentile value of MOE.

The MOE data for the given window is fitted using Johnson's S_b distribution with known lower and upper bounds. These lower and upper bounds are known as the boundaries for the band or window. The Johnson's S_b distribution is given by (Johnson 1949):

$$f(x;\gamma,\eta,\lambda,\varepsilon) = \frac{1}{\sqrt{2\pi}}\frac{\eta\lambda}{(x-\varepsilon)(\lambda+\varepsilon-x)} EXP\left\{-0.5(z)^2\right\} \tag{23}$$

for which,

$$z=\gamma+\eta\, ln\left(\frac{x-\varepsilon}{\varepsilon+\lambda-x}\right) \qquad (\varepsilon \le x \le \varepsilon+\lambda)$$

where:

z	=	Standard normal distribution,
x	=	MOE,
ε	=	MOE minimum for a given band-width,
λ	=	The width of the band or window,
γ and η	=	Parameters.

For $y_i = ln\left(\frac{x_i - \varepsilon}{\varepsilon + \lambda - x_i}\right)$ $(i = 1,2,\ldots,n)$ then, the Maximum Likelihood estimates for γ and η are given, respectively, by:

$$\hat{\gamma} = -\frac{\bar{y}}{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2}} \quad \text{, and} \quad \hat{\eta} = \frac{1}{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2}} \tag{24}$$

where: $\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}$.

If the relative measure of the skewness and kurtosis fall in the S_b region (see Shapiro and Gross (1981)) then the distribution is Johnson's S_b distribution. The relative measure of the skewness ($\sqrt{b_1}$) and kurtosis (b_2) of MOE inside the window are calculated, respectively, as follows:

$$\sqrt{b_1} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3}{\sqrt{\left(\frac{1}{(n-1)}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)^3}} \quad \text{, and} \quad b_2 = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^4}{\left(\frac{1}{(n-1)}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)^2} \tag{25}$$

where: $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$

For known lower and upper bounds, the MOE at any percentile level α can be calculated by:

$$x_\alpha = \frac{x_m \left\{ EXP\left(\frac{z_\alpha - \hat{\gamma}}{\hat{\eta}}\right)\right\} + x_o}{1 + EXP\left(\frac{z_\alpha - \hat{\gamma}}{\hat{\eta}}\right)} \tag{26}$$

where:

x_m = MOE maximum,

x_0 = MOE minimum.

In order to verify if the Johnson's S_b distribution fits the MOE data, the goodness-of-fit test was performed. Although there are six band-widths, the test on a single band-width is assumed adequate for this purpose. The result of the goodness-of-fit test on the MOE distribution for a 1,000 MPa band-width is depicted in Table 9. This table shows that the relative measures of skewness and kurtosis, b_1 and b_2, fall in the S_b region (see Shapiro and Groos 1981). Moreover, the Kolmogorov-Smirnov (K-S) test (Neave and Worthington 1988) shows that the hypothesis of the distribution originating from Johnson's S_b distribution can not be rejected for all cases at $\alpha = 0.05$ significant level as can be seen on the last three columns in Table 9 where the K-S critical value is less than the tabulated values. Fig. 12 shows the histogram and probability density function of Johnson's S_b distribution for MOE data from the Douglas-fir MOE-MOR relationship. It is clear from this figure that Johnson's S_b distribution represents the MOE data better than a Normal distribution.

Due the large data base, the 3-parameter Weibull distribution is employed without any comparison with other distributions through goodness-of-fit analysis. However, the 3-parameter Weibull has been widely used to represent the strength distribution of dimension lumber (Pellicane and Bodig 1981, Taylor and Bender 1988, Heatwole et al. 1991). Curry and Tory (1976) reported that 3-parameter Weibull gave the best fit compared to normal, lognormal, and 2-parameter Weibull distributions for the strength data inside the boundary or band. Pellicane (1985) conducted the study on the goodness-of-fit of normal, lognormal, 3-parameter Weibull, and Johnson's S_b distributions on dimension lumber data and concluded that Johnson's S_b distribution provided the best fit. However, he also reported that at the lower 5 % level no distribution seemed to be substantially superior to the others.

If the distribution of the sample is assumed from the 3-parameter Weibull distribution, then its cumulative distribution is given by (Bury 1975):

$$F_W(x; x_0, m, k) = 1 - EXP\left[-\left(\frac{x - x_0}{m}\right)^k\right] \tag{27}$$

and the property value at any percentile level α can be calculated by:

$$x_\alpha = x_0 + m\left\{-ln(1 - F_\alpha)^{1/k}\right\} \tag{28}$$

where:

x_0 = Location parameter,

k = Shape parameter,

m = Scale parameter.

As for MOE data, the result of the goodness-of-fit test of Weibull parameter on the strength data for a 1,000 MPa band-width are depicted in Table 9 for all species. The Kolmogorov-Smirnov test shows the hypothesis that the sample is from the 3-parameter Weibull distribution can not be rejected at the level of significant $\alpha = 0.05$ for all cases as shown in this table. The cumulative distribution function of the 3-parameter Weibull distribution for the Douglas-fir MOR is depicted in Fig. 13.

Finally, the 1-st , 5-th and 50-th percentile point estimates of the strength are then plotted against the corresponding 50-th percentile MOE as shown in Figs. 14, 15 and 16 for the MOE band-width of 1,000 MPa. Following Curry and Tory (1976), the minimum values or the lower tail of the strength distribution for the given band-width was fitted using the regression method to model the relationship between this minimum strength level and MOE. It should be noted that the data relating MOE to strength are correlated in cases where overlapping band-width occur. However, the overlapping bands provide more data which smooth out the trend and provide a better regression result for which the bigger sample size is required. Therefore, the sampling effects should be ignored.

The plot of the strength versus MOE shows nonlinear trend. The relationships between MOE and the lower tail of the strength distribution then were represented using Eqs. (10) and (11). A preliminary analysis showed that Eq. (10) resulted in significant SSE reduction compared to that given by Eq. (11) for every case. Therefore only the results for Eq. (10) are presented. The fitted model of Eq. (10) (1-st and 5-th percentile) for a 1,000 MPa band-width for each species and property relationship is depicted in Figs. 3, 4 and 5 so that it can be compared with the full data set. The results of the regression analyses for all band-widths are presented in Tables 10, 11, and 12 for all species, property relationships and percentile levels.

It has been shown that the 50-th percentile point of MOE is the corresponding point for the 50-th percentile of the strength for each window position. For consistency, the 1-st and 5-th percentile points of the strength were related to the 50-th percentile of MOE. Since the 50-th percentile values of MOE rather than the minimum value for the band is used, the procedure is different from the one introduced by Curry and Tory (1976). Nevertheless, the minimum value of MOE for the grade can be used to calculate the assigned strength values once the MOE-strength relationship has been obtained. The purpose of using the 50-th percentile MOE value for the band is to eliminate the shifting effects which would be introduced by changing the MOE reference point. The band-width selected may, however, affect the lower exclusion limits (1-st and 5-th percentile) of strength values since a wider MOE band will include a wider range of strength values.

Since each band-width has been given the same weight, such as using the 50-th percentile value of MOE for the band, the relationship between MOE and the 50-th percentile strength should be the same for all band-widths. This can be seen from Figs. 17, 18, and 19 where the predicted lines for the 50-th percentile generally overlap each other. The fitted models for the general relationships between strength and MOE (abbreviated GEN.REL) are depicted in these figures as well. It can be seen that, for each species, this general relationship agrees very well with the fitted models for the 50-th percentile strength for all band-widths. Because the fitted model for the 50-th percentile strength from each band-width gives similar results, it shows that the expected mean trend (50-th percentile) for the strength value is less affected by the band-width by changing the MOE reference points. The expected values for the lower 1-st and 5-th percentiles strength levels are affected by band-width. This is because wider band-width will give wider strength distribution for the same value of the 50-th percentile MOE. The effect of band-width, called grade increment factor, is depicted in Figs. 20, 21 and 22. This is a factor by which the minimum strength value, determined on the basis of a 1,000 MPa band-width, should be multiplied if the actual band-width is different from this value.

The band-width method can be used to derive the relationship between strength and MOE. The strength property relationships can be derived at a selected MOE level. The results depend on the model chosen for the MOE-strength relationships. The relationship between UTS and MOR, for instance, can be derived analytically if the MOE-strength models relating MOE to UTS and MOR, respectively, both have the forms that allow MOE to be eliminated in relating MOR to UTS. Models with this property have been selected. Other models, for example Eq. (12), are not suitable because they do not allow MOE to be eliminated to derive a strength property relationship.

Another drawback from band-width method is that the narrow band-width would not cover the extremes (the lower and upper ranges) of the entire MOE data for the species with sufficient data points. In other words, the number of data points in the extremes are not large enough to guarantee a good estimation and as a consequence, the extremes will be excluded in the moving band-width process. The larger the width of the band the more likely to cover the extremes of MOE data for the species.

6. RELATIONSHIPS BETWEEN MOE AND STRENGTH PROPERTIES

The regression analysis shows that the general relationship between MOE and MOR is defined adequately by the simple linear model of Eq. (9). However, the relationships between MOE and UTS or between MOE and UCS are nonlinear and are best defined by, the nonlinear model of Eqs. (11) and (10), respectively.

For modelling the strength property relationship based on the MOE-strength relationships, the proper MOE-strength models should be chosen. By using the linear model for the MOE-strength relationships to formulate UTS/MOR ratio as a function of MOR, Green and Kretschmann (1991) found that UTS/MOR ratios did not agree with the results from the equal-rank analysis at the lower MOR levels. Thus, the following discussions will be focused on the establishment of the MOE-strength relationships in order to develop the strength property relationships.

6.1. Relationships between MOE and MOR

The general relationship between MOE and MOR is defined, commonly, by the simple linear model of Eq. (9). The coefficients of determination, r^2, obtained for the In-grade data are 0.58, 0.47 and 0.60 for Douglas-fir, Hem-Fir and S-P-F, respectively. These values fall in the range reported by Hoyle (1968) and they are a slightly larger than the values reported by Green and Kretschmann (1991) for the US In-grade data set. However, the nonlinear model

Eq. (10) gives the lowest Sum of Square Error. In order to be consistent for the subsequent analysis on the strength property relationships, the model of Eq. (10) was chosen to represent the general relationship between MOE and MOR. The resulting regression model is shown graphically in Fig. 3 for each species. All the graphs are presented on the same scale so that the comparisons among the species can easily be made.

For the band-width analysis, Eq. (10) is found to be a better model for the relationships between MOE and MOR at the lower 1 % and 5 % exclusion levels. The model parameters for all band-widths and species are presented in Table 10. The predicted lines, for a 1,000 MPa band-width, are depicted in Fig. 3 for every species. It can be seen from the graph for each species that the predicted lines both for the 1-st and 5-th percentile generally follow the lower trend (margin) of the MOR scatter.

At the lower MOE levels, it seems that the 5-th percentile relation from band-width analysis is not conservative compared to the 5 % exclusion line from simple linear model. This can be seen from the result for Douglas-fir in Fig. 23. In this figure, the nonlinear 5-th percentile trend from the band-width analysis gives higher MOR values at the lower MOE levels. However, the 5-th percentile trend from band-width analysis gives lower MOR values at higher MOE levels compared to that of linear 5 % exclusion limit. The 5-th percentile trend from band-width analysis reflects the pattern of the changes of variance for each level of MOE. This figure shows that at MOE below about 7,000 MPa, 5 % exclusion line from linear model has negative values. This is one of the reasons why Curry and Tory (1976) introduced the band-width method. They found the power model of Eq. (11) to be a better model for relating minimum values of MOR to MOE. Unlike their model, the model of Eq.

(10) can result in negative values of MOR, however Fig. 24 shows that the 5-th percentile line from the nonlinear model still gives positive values for MOR even for low MOE values.

The general relationships or mean trends for MOE-MOR vary little by species (Fig. 24). This trend is similar to that found in the US In-grade data (Green and Kretschmann 1991) which was intended to test if the single regression line can represent all species. For this, the proposed Eq. (10) was fitted to the pooled data for all species (Ratkowsky 1983). The F-test in Table 13 shows that the hypothesis of single regression line for all species was rejected at the confidence level $\alpha = 0.05$. Fig. 24 shows that there are minor differences in the 5-th percentile level of MOR regression from band-width analysis.

6.2. Relationships between MOE and UTS

The regression analysis shows that the general relationship between MOE and UTS for Hem-Fir and S-P-F is in a nonlinear form. Statistically, Eq. (11) is adequate for representing the general relationship between MOE and UTS. However, Eq. (10) is chosen to represent this relationship in order to be consistent with the subsequent analysis on the strength property relationships. Furthermore, it gives the lowest SSE. The general relationships between MOE and UTS are shown graphically in Fig. 4 for each species.

The general relationship between MOE and UTS is species dependent as shown by Fig. 25. For the same MOE, S-P-F has the highest value of UTS especially at higher MOE levels. The test for a single regression line for all species in Table 13 showed no evidence for rejecting different regression lines for different species. In other words, the regression lines were not similar.

As for the MOE-MOR relationship, the band-width analysis also shows that Eq. (10) is a better model for MOE-UTS relationship at the lower 1 % and 5 % exclusion levels. The model parameters are presented in Table 11 for all band-widths and species. The models , for a 1,000 MPa band-width, are depicted in Fig. 4. It can be seen from the figure for each species that the predicted lines for the lower 1 % and 5 % exclusion levels successfully follow the lower margin of the UTS scatter.

The MOE-UTS relationships for the 5 % lower exclusion level also reveal differences by species particularly at high MOE levels as shown by Fig. 25.

6.3. Relationships between MOE and UCS

The regression analysis shows that MOE-UCS has a strong nonlinear relationship. The statistical test on the parameters and SSE showed that Eq. (10) is a better model for relating MOE and UCS. This model is shown graphically in Fig. 5 for each species.

The band-width analysis also showed that Eq. (10) is the best model for relating MOE and UCS at the lower 1 % and 5 % exclusion levels. The results of the regression analysis are presented in Table 12 for all band-widths and species. Again, the equations, for a 1,000 MPa band-width, are depicted in Fig. 5 for each species. The figures reveal that the fitted lines follow the lower boundary of the UCS scatter for each species.

The mean trend in MOE-UCS relationships are similar for Douglas-fir and Hem-Fir as shown in Fig. 26. S-P-F shows lower UCS value for the same given MOE compared to Douglas-fir and Hem-Fir. Again, the test for a single regression line for all species in Table 13 showed that the hypothesis of single regression line for all species was rejected at confidence level $\alpha = 0.05$.

For the lower 5 % exclusion level, all species show little variation in the higher MOE levels.

7. RELATIONSHIPS BETWEEN STRENGTH PROPERTIES

The relationship between two strength properties can be described in the form of the ratio of one property to the other. Since MSR predicts bending strength, UTS and UCS are assigned as a proportion or ratio to MOR. Based on the failure mode of lumber in bending, it will be shown that it is reasonable to assign MOR as a proportion of UTS.

Using the results from equal-rank method, MOE-strength based general relationships and band-width method, the following analysis will be focused on the development and evaluation of the property ratio relationships based on the Canadian In-grade data. The result from equal-rank method will be discussed first followed by MOE-strength general relationship and band-width method so that the evaluation and comparison among these methods can be made.

7.1. Strength Property Ratios Based on MOR

There is no fundamental reason, based on the mechanical behaviour of lumber, to expect that the ultimate tensile strength and compression strength parallel to the grain depend on MOR. Since the invention of mechanical grading (MSR), it was found that there is a high correlation between flexural strength (MOR) and the flexural stiffness (MOE). However, experimental studies have shown that strength properties generally increase as MOE increases. Therefore, for design and grading purposes, tensile strength and compression strength parallel to the grain are often expressed as a function of MOR. The assignments of UTS and UCS are expressed as the ratio or percentage of MOR and MOR can be predicted from flexural stiffness (MOE).

7.1.1. Property Ratio Models from Equal-Rank Analysis

The equal-rank method allows the strength property relationships to be formulated by assuming a 3-parameter Weibull distribution for each property which yields an expression in the form of Eq. (21). The property ratio based on MOR is constructed by dividing the model in Table 7 by MOR. For example, the ratio of UTS to MOR as a function of MOR for Douglas-fir is as follows:

$$R_{T/B} = \frac{6.6678}{MOR} + 0.1373\ \frac{(MOR - 6.0215)^{1.3274}}{MOR} \qquad (29)$$

Fig. 7 contains the plots of this equation and the similar results for Hem-Fir and S-P-F with the scatter plots of data points.

The clear picture of the property ratio as the function of MOR across the species can be seen in Fig. 27. It is clear from this figure that UTS/MOR ratio is species dependent especially for higher bending strength. Below 48.3 MPa, the ratio of UTS to MOR, for each species, agrees well with the average value of 0.56 for all grades, species and sizes as proposed by Green and Kretschmann (1991), and is slightly lower than the 0.60 as proposed by Curry and Fewell (1977). It is clear that the constant factor 0.45 proposed in ASTM D 1990 (ASTM 1991) is conservative for these species.

The ratio of UCS to MOR is also depicted in Fig. 27 so that the comparison between UTS and UCS as a function of MOR can be made. Like the UTS/MOR ratio, the UCS/MOR ratio is also constructed by dividing the model results in Table 7 by MOR. The property ratio for Douglas-fir is given by:

$$R_{C/B} = \frac{15.5812}{MOR} + 0.6317\,\frac{(MOR - 6.0215)^{0.926}}{MOR} \tag{30}$$

The plots of this equation and similar results for Hem-Fir and S-P-F are depicted in Fig. 8.

Fig. 27 also contains the models for UCS/MOR ratios reported by Curry and Fewell (1977), Green and Kretschmann (1991) and proposed by ASTM D 1990 (ASTM 1991). The UCS/MOR ratio for S-P-F is very close to that reported by Curry and Fewell (1977) and Green and Kretschmann (1991). It is clear from Fig. 27 that the proposed model from ASTM D 1990 (ASTM 1991) is more conservative than that of S-P-F whose UTS/MOR ratio is the lowest among that of the three species groups.

The differences in the property relationships are inevitable since there are some differences in the property adjustments, size, test span and the method of testing used by the various authors (see Curry and Fewell 1977, Green and Kretschmann 1991 and ASTM D 1990-ASTM 1991). It should be noted that the proposed property ratios in ASTM D 1990 (ASTM 1991) were accumulated from the North American (US and Canada) In-grade data base, whereas, Green and Kretschmann (1991) used only the US In-grade data and Canadian S-P-F data to derive the property ratio models.

The strength property relationships based on MOR show that the relationship between UTS and MOR is not as good as that found between UCS and MOR. This finding was also reported by Green and Kretschmann (1991). For each species, UTS-MOR relationship shows higher variance than that shown by UCS-MOR relationship as can be seen in Table 7.

The mean trend for the ratio of UCS to MOR for clear wood is included in Fig. 27. This trend is formulated by fitting Eq. (10) to the mean values of UCS and MOR data for species included in Douglas-fir, Hem-Fir and S-P-F groups (Jessome 1977). The UCS and MOR data were not adjusted for size effects, therefore the absolute values of the property ratio is not directly comparable with the structural lumber results. The clear wood samples are defect-free wood, therefore, one would expect the stronger commercial lumber to have the property ratio trends close to that of clear wood. Fig. 27 shows that, for UCS/MOR ratios, all species have the tendency to reach a lower limit as predicted by clear wood property data.

7.1.2. Property Ratio Models from General Relationships

Strength-MOE equations can be used to formulate the relationship between two strength properties. For instance, if Eq. (10) is employed, the general relationship between UTS and MOR can be formulated as follows:

$$UTS = \beta_{0t} + \beta_{1t}\,(MOE)^{\beta_{2t}}, \quad \text{thus} \quad MOE = \left(\frac{UTS - \beta_{0t}}{\beta_{1t}}\right)^{1/\beta_{2t}}$$

$$MOR = \beta_{0b} + \beta_{1b}\,(MOE)^{\beta_{2b}}, \quad \text{thus} \quad MOE = \left(\frac{MOR - \beta_{0b}}{\beta_{1b}}\right)^{1/\beta_{2b}}$$

solving for UTS gives:

$$\left(\frac{UTS - \beta_{0t}}{\beta_{1t}}\right)^{1/\beta_{2t}} = \left(\frac{MOR - \beta_{0b}}{\beta_{1b}}\right)^{1/\beta_{2b}}$$

$$UTS = \beta_{0t} + \left(\frac{\beta_{1t}}{(\beta_{1b})^{(\beta_{2t}/\beta_{2b})}}\right)(MOR - \beta_{0b})^{(\beta_{2t}/\beta_{2b})}$$

or more generally,

$$UTS = \beta_0 + \beta_1 (MOR - \beta_2)^{\beta_3} \tag{31}$$

In other words, for a piece of lumber with a certain MOE, the mean trend relationship between bending strength and tensile strength is expected to follow a trend given by Eq. (31). It is clear that the parameters for this equation are obtained directly from MOE-UTS and MOE-MOR equations and are presented in Table 14 for each species. The property ratio is derived by dividing this equation by MOR. The plots of these property ratios (abbreviated GEN.REL.) are illustrated in Fig. 28 for every species in order to compare them with the results from the band-width analysis.

7.1.3. Property Ratio Models from Band-Width Analysis

Following the same procedure as for the property ratios from general relationship (Eq. (31)) the results from band-width analysis can be used to formulate the relation between UTS and MOR. The model parameters for all band-widths and percentile levels are presented in Table 15 for all species. Fig. 28 contains the scatter plots for the 5-th and 50-th percentile strength levels for a 1,000 MPa band-width. The scatter plot of UTS/MOR as a function of MOR, for example, is constructed by plotting UTS/MOR ratio against MOR for the same percentile level inside the same band-width.

The predicted models derived from the MOE-strength general relationship and from the result of equal-rank analysis (abbreviated GEN.REL and EQRA, respectively) are depicted in Fig. 28. It is clear from this figure that the property ratios derived using these two different methods are similar and they agree very well with the scatter of the median (50-th percentile) from band-width analysis. At low MOR levels the relationships based on equal-rank analysis yield property ratios close to the 5-th percentile data derived from the band-width analysis. Therefore, it shows that the equal-rank method gives results which are consistent with the predictions of the band-width analysis at low MOR levels.

There are slight differences between GEN.REL and EQRA in Fig. 28, especially at the lower and higher levels of MOR data. These deviations at the extremes can be explained by looking at how each model was constructed. The property ratio resulted from MOE-strength general relationship is formulated by solving two regression equations i.e., two MOE-strength equations. In this case, the fitted regression line depends on the range of MOE, that is, the distance between the minimum and maximum. The number of data points are small in the extremes. On the other hand, the equal-rank procedure is independent of MOE, namely, it depends on the range of strength data, such as, the distance between the minimum and maximum. Then, by examining the scatter plot of MOE versus strength (Figs. 3,4 and 5), it is clear that the lower strength values are not always in the lower MOE range and, likewise, the higher strength values are not always in the higher MOE range. MOE range is the horizontal distance whereas the strength range is the vertical distance. Therefore, if the increase of the strength values does not follow the increase of MOE, then there will be differences in the extremes. That is because, if the MOE-strength relationship has a perfect relationship, which means that the plot of strength versus MOE is just a straight line, the two methods will provide the same results.

In Fig. 28, the property ratio resulted from MOE-strength based general relationships gives close results to the 50-th percentile (median) data from band-width analysis. There are slight differences in the extremes most likely due to cut-off points in the extremes for band-width analysis and the distribution of strength values along the regression line for the MOE-strength general relationships.

Design stresses for lumber are obtained from estimates of minimum values for each grade, traditionally at the 5 % lower exclusion level. Visually, the difference between this lower exclusion level and the mean trend on the property ratios is small if presented on the strength basis especially for UCS/MOR ratios as illustrated in Fig. 28 for each species.

In Fig. 28, for band-width method, it is obvious that UTS/MOR and UCS/MOR ratio data points at 5 % lower exclusion limit (5-th percentile) do not go up to the higher MOR levels as that at 50-th percentile. Hence, the fitted lines for the property ratios, in this case at 5-th percentile, were limited to the maximum values permitted by the data. For example, in the case of Douglas-fir, the maximum data values of MOR at 5-th percentile is about 50 MPa.

The difference between the 5 % lower exclusion level and the mean trend (50-th percentile) for the property ratio is significant if presented on the basis of MOE as shown graphically in Fig. 29 for each species. Each data point for the scatter plot of UTS/MOR as a function of MOE, for example, is constructed by plotting the ratio of UTS to MOR against the 50-th percentile MOE data point inside the same band or window. This MOE data point can be chosen from either the MOE-MOR or the MOE-UTS relationship. Either one of these MOE data points can be used only if the value from the MOE-MOR is not different from the MOE-UTS for the same band or window. Fig. 30 was generated to check if there is any significant difference between these two MOE values for the same band or window. Each graph in this figure shows that there is a perfect correlation between these two 50-th percentile values of MOE.

In Fig. 29, each predicted line for the 5 % lower exclusion level is just the ratio between the predicted UTS and the predicted MOR both as the function of MOE as shown by the following equation:

$$\frac{UTS}{MOR} = R_{T/B} = \frac{\beta_{0t} + \beta_{1t}(MOE)^{\beta_{2t}}}{\beta_{0b} + \beta_{1b}(MOE)^{\beta_{2b}}} \tag{32}$$

where the subscripts t and b represent UTS and MOR, respectively. This equation can be calculated when MOE is given.

The same predicted line (abbreviated GEN.REL) but from the general relationship between strength and MOE is also plotted in Fig. 29. This line agrees very well with the scatter of the 50-th percentile data points. This agreement indicates that the 50-th percentile strength values fall around the regression line in the band-width analysis discussed in Fig. 11a.

In Fig. 29, the UCS/MOR ratios as a function of MOE shows a wider difference between the 5-th and 50-th percentiles than that shown by the UTS/MOR ratios especially for Douglas-fir and Hem-Fir.

7.2. Property Ratios Based on UTS

There are at least two common failure modes found in a bending member. The member compression zone may exhibit compression failure whereas the tension zone always exhibits tension failure parallel to the grain at ultimate bending capacity. In the direction parallel to the grain, defect-free wood, tested in compression will exhibit a linear stress-strain relationship up to the proportional limit after which yielding will take place. Whereas, in tension, the stress-strain relationships is almost linear up to failure. The test results show that clear wood, in the direction parallel to the grain, is much stronger in tension than in compression (Maholtra and Bazan 1980, Anderson 1981). According to Schniewind (1962), tensile strength is approximately two to three times as great as compression strength.

Lumber most often contain some defects such as knots, depending on the assigned grade. Unlike defect-free or clear wood, low strength beams will exhibit tension failure before the beam reaches its proportional limit stress in compression zone leaving the linear stress-strain distribution (Ramos 1961). Higher grade beam, mostly defect-free, will reach its proportional limit stress in compression zone. Further loading will cause yielding or buckling in this zone and the neutral axis will shift toward the tension zone resulting in higher stress in that zone. By increasing the load, the beam will fail in tension at its maximum load capacity (Maholtra and Bazan 1980).

It is clear now that the ultimate bending strength (MOR) is governed by the strength of the tension zone of the beam. In other words, one would expect the tension failure even for higher grade material. This behaviour of bending member provides the foundation for expressing bending strength as a function of tensile strength.

Unlike bending strength and tensile strength, compression strength and tensile strength are fundamentally different properties. However, since bending strength is related to tensile strength, it seems appropriate to express compression strength as a function of tensile strength.

7.2.1. Property Ratio Models from Equal-Rank Analysis

The model parameters for the property relationships based on UTS resulting from the equal-rank analysis are presented in Table 8. Following the approach for deriving the property ratios based on MOR, the property ratios based on UTS are formulated by dividing the models in Table 8 by UTS. For instance, the ratio of MOR to UTS for Douglas-fir is as follows:

$$R_{B/T} = \frac{6.0215}{UTS} + 4.7865\,\frac{(UTS - 6.6180)^{0.7299}}{UTS} \quad (33)$$

This equation and the similar results for Hem-Fir and S-P-F are depicted in Fig. 9. The scatter plots follow the pattern shown in the report by Green and Kretschmann (1991).

The ratio of UCS to UTS based on UTS, for example, for Douglas-fir is as follows:

$$R_{C/T} = \frac{14.5265}{UTS} + 2.5256\,\frac{(UTS - 5.7852)^{0.703}}{UTS} \quad (34)$$

This equation and similar results for Hem-Fir and S-P-F are illustrated in Fig. 10.

The property ratios based on UTS across the species are illustrated in Fig. 31. The proposed model by Green and Kretschmann (1991) and ASTM D 1990 (ASTM 1991) for UCS/UTS ratio are depicted as well. The UCS/UTS ratio from S-P-F is very close to that reported by Green and Kretschmann (1991). It is clear from this figure that the UCS/UTS ratio suggested by ASTM D 1990 (ASTM 1991) is lower than that given by each species. The constant factor 1.2 for MOR/UTS ratio suggested by ASTM D 1990 (ASTM 1991) is more conservative than that given by each species especially in the middle range of UTS values.

Fig. 31 shows that the MOR/UTS ratios are species dependent, as well as the UCS/UTS ratios. UCS/UTS ratios show similar pattern across the species. For the same given UTS value, the UCS/UTS ratio from one species to the other almost can be factored by a constant.

It is expected to have a good relationship between MOR and UTS as a function of UTS, however, the relationship between MOR and UTS is not as good as that found between UCS and UTS. This finding was also reported by Green and Kretschmann (1991) for the U.S In-grade data. For every species, MOR-UTS relationship shows higher variance than that shown by UCS-UTS relationship as can be seen in Table 8.

7.2.2 Property Ratio Models from General Relationships

Strength property relationships based on UTS resulted from solving their general relationships with MOE can be constructed in the same way as that shown for the property relationships based on MOR. It can be shown as per Eq. 31, that MOR as a function of UTS, for example, is as follows:

$$MOR = \beta_0 + \beta_1 \left(UTS - \beta_2\right)^{\beta_3} \tag{35}$$

The model parameters for this type of equation are presented in Table 14 for all species. Again, this means that given a piece of lumber with certain MOE value, the mean trend relationship between bending and tensile strength, the former as a function of the latter, is given by this equation. The property ratio is taken by dividing the model in Table 14 by UTS.

This property ratio (abbreviated GEN.REL.) for each species is depicted in Fig. 32 for the comparison with that of band-width analysis.

7.2.3. Property Ratio Models from Band-Width Analysis

The results from band-width analysis for the property relationships based on UTS are presented in Table 16 for all band-widths, percentile levels and species. The scatter plots for the 5-th and 50-th percentile values for a 1,000 MPa band-width are illustrated in Fig. 32 for each species. The scatter plots are constructed the same way as that shown for the property relationships based on MOR.

The predicted models derived from MOE-strength general relationship and from equal-rank analysis (denoted by GEN.REL and EQRA, respectively) are depicted in Fig. 32 as well. Since they both agree with the scatter of the 50-th percentile data points, again it is proven that the equal-rank procedure gives about the mean trend for the property relationships. Similar to the property ratios based on MOR in Fig. 28, Fig. 32 shows that at low UTS levels, the relationships based on equal-rank analysis yield property ratios close to the 5-th percentile data derived from band-width analysis.

Fig. 32 shows that UCS/UTS ratios give very consistent results compared to that of MOR/UTS ratios because the UCS/UTS ratios have less scatter than that of MOR/UTS ratios.

As for the property ratios based on MOR, in Fig. 32, the fitted lines for the 5-th percentile strength level were limited to the maximum values permitted by the data.

The property ratios as a function of MOE are depicted in Fig. 33. It can be seen that the effect of taking the lower exclusion level (5-th percentile) is very significant; i.e., there is a large difference in property values between 5-th and 50-th percentile levels. The scatter plots of MOR/UTS as a function of MOE, for example, is constructed by plotting the MOR/UTS ratio against the 50-th percentile value of MOE (from MOE-UTS relationship) inside the same band or window. The predicted line for the 5-th percentile level is the ratio between the predicted MOR and the predicted UTS both as the function of MOE as shown for Eq. (32), but in reverse order.

The property ratios presented on the basis of MOE also indicate that the median (50-th percentile) values of the strength fall around the regression line postulated in the band-width method (see Fig. 11a) because the plot of the property ratios from the general relationship between strength and MOE agrees with the scatter of the 50-th percentile data points.

8. APPLICATION OF THE PROPERTY RELATIONSHIPS

8.1. MOE-Strength Property Relationships

Modulus of elasticity can be measured directly, therefore having known the MOE, one can predict the desired strength values using the empirical models developed in this study. Since the design value for structural application of lumber is traditionally based on the lower 5 % exclusion limit, the analysis has been emphasized on finding this limit by means of the band-width method.

It has been shown that the equation for the 5 % lower exclusion level from band-width method is a better model for estimating the 5 % lower exclusion limit than the traditional 5 % lower exclusion line from the linear model which is adequate but conservative (see Fig. 23). The nonlinear model is more realistic for the lower exclusion level. However, the model for the 5 % exclusion level from band-width analysis depends on the boundary or the width of the band used in the analysis. Nevertheless, the results show that the width of the band has a little effect on the estimation of this 5 % lower exclusion value. The band-width effects (called grade increment factors) determined on the basis of a 1,000 MPa band-width are less than 8 % for all band-widths.

Because the width of the band is unknown at the outset, a conservative approach would be to use 1,000 MPa as the standard. It is wide enough to cover the number of data points of the strength data, even on the extremes of MOE range, for estimating the lower 5-th percentile point estimates. It can be seen from Figs. 14, 15 and 16 by comparing to the full data range in Figs. 3, 4 and 5 that the 1,000 MPa band-width successfully covers the extremes of MOE values for the species.

The relationships between MOE and the predicted MOR, UTS and UCS for each species group can be seen from Fig. 34. For the same given MOE, generally each species shows that MOR is the highest one follows by UCS and lastly UTS, however, the differences are not in the same degree. Therefore, the property ratios vary with MOE as shown by the figures for the property ratios as a function of MOE (see Fig. 35).

It has been mentioned before that there is a strong relationship between flexural strength (MOR) and flexural stiffness (MOE) which is used in MSR grading system. As an analogy, tensile strength has to be related to its tensile modulus of elasticity and likewise compressive strength has to be related to its modulus of elasticity parallel to the grain. However, Doyle and Markwardt (1966) reported that MOE in compression is about equal to the flexural MOE. Götz et al. (1989) stated that those three different elastic properties are practically equal below the proportional limit. If the flexural MOE alone is believed to be a reliable indicator of strength properties, then by measuring MOE one can calculate any desired strength property for the property assignments using the proposed models in this study.

8.2. Strength Property Relationships

If MOR is available, like in MSR practice where MOR is predicted from measured MOE, the models from equal-rank analysis can be used to calculate the assignments of tensile strength and compressive strength. In this case, these two strength properties are based on the assumption that each of them has the same probability of failure (equal rank) with MOR but are independent of the nondestructive parameter MOE. Similar results will be obtained by using the models derived from the general relationships between MOE and strength properties.

Because the models from the equal-rank method and MOE-strength based general relationship provide the mean trend or average values for the predicted strength values, the model for the 5 % lower exclusion level from the result of band-width method should be used for lumber selected on the basis of MOE. The comparisons between 5 % lower exclusion level (5-th percentile) and the mean trend for the property ratios based on MOR are depicted in Fig. 35. From this figure, it can be seen that Douglas-fir and Hem-Fir show higher values of property ratios for 5 % lower exclusion level for higher MOR levels and vice versa for lower MOR levels. However, S-P-F shows lower values of UTS/MOR ratios for 5 % lower exclusion level for all levels of MOR and about equal for UCS/MOR ratios for MOR $\geq$ 20 MPa. The maximum MOR values in Fig. 35 are set to that of maximum 5-th percentile values derived in the band-width analysis (see Fig. 14).

The difference, called percentile-level effect, between using the mean trend and 5 % lower exclusion level, as the percentage of the values of the 5 % lower exclusion level is shown in Fig. 36 for every species. Depending on the species and strength levels, the difference can be up to 30 % higher or lower if one would use the model from the MOE-strength based general relationship or the equal-rank analysis result rather than the 5-th percentile model from band-width analysis.

As proposed in ASTM D 1990 (ASTM 1991), UTS can be used for the estimates of untested properties. Moreover, according to Green and Kretschmann (1991), there has been an interest in determining the UTS for MSR lumber in quality control programs. If the property relationships are based on UTS, then the models for the property relationships based on UTS proposed in this study can be implemented.

The comparisons between 5 % lower exclusion level (5-th percentile) and the mean trend for the property ratios based on UTS are depicted in Fig. 37. The MOR/UTS ratios in this figure basically are the inverse of UTS/MOR ratios in Fig. 35. In Fig. 37, the UCS/UTS ratios show a trend of decreasing function with the increase in UTS similar to UCS/MOR ratios in Fig. 35. For the 5 % lower exclusion level in Fig. 37, Douglas-fir and S-P-F show higher ratios for higher levels of UTS, whereas Hem-Fir shows higher ratios at the lower levels of UTS and slightly lower ratios for higher levels of UTS. The maximum UTS values in Fig. 37 are set to that of maximum 5-th percentile strength values derived in the band-width analysis (see Fig. 17).

The percentile-level effects are shown in Fig. 38. Depending on the species and strength values, the difference can be up to 25 % higher or lower if one would use the model from the general relationship or the equal-rank analysis result rather than the 5-th percentile model from band-width analysis.

The empirical models for the stiffness-strength property relationships and for the strength property relationships are summarized in Table 17 for the estimation of untested properties.

Table 17 shows that there are three equations expressing strength as a function of MOE, i.e., Strength = f(MOE), two equations expressing strength as a function of MOR, i.e., UTS = f(MOR) and UCS = f(MOR), and two equations expressing strength as a function of UTS, i.e., MOR = f(UTS) and UCS = f(UTS).

Because MOR can be expressed as a function of UTS and vice versa, it should be noted that their relationships resulted from equal-rank analysis were from different fitted regressions. For Eq. 21, however, one can simply works out to find the inverse function of the given equation, therefore one fitted regression is adequate. However, Fig. 39 shows that the effects (called error) of using the result of two different fitted regressions are less than 5 % (as the percentage of MOR) for MOR $\geq$ 20 MPa if one would use the inverse function rather than the fitted regression. For the models resulting from the MOE-strength general relationships and the band-width analysis, the error is zero since MOR as a function of UTS is exactly the inverse function for UTS as the function of MOR.

9. CONCLUSIONS

The relationships between modulus of elasticity and the strength properties have been modeled. The traditional linear relationship has been found to still be a good model for the mean trend or general relationship between MOE and MOR. It also shows that the general relationships between MOE and MOR vary little by species especially for Hem-Fir and S-P-F but there is no statistical justification for using a single regression equation for all species. However, the relationship between MOE and the minimum values of MOR (5 % lower exclusion level) determined using band-width method shows slightly different values for different species. The differences depend on the variability of MOR for the species.

The general relationships between MOE and UTS, as well as, UCS are found to be well represented by the nonlinear models. The relationship between MOE and UTS shows significant species effects especially for higher MOE values. S-P-F shows the highest tensile strength values follows by Hem-Fir among the three species groups. Unlike the relationships between MOE and UTS, the relationships between MOE and UCS, especially for the lower 5 % exclusion level, show significant species effect at the lower MOE levels.

The modified band-width method, such as, by changing the reference point of MOE to the 50-th percentile values determined using Johnson's S_b distribution for the band, in this study shows that the effects of the width of the band on the estimation of the 5 % lower exclusion limit is insignificant. For all band-widths, from 500 MPa to 2,500 MPa, the differences are less than 8 % determined on the basis of a 1,000 MPa band-width.

Strength property relationships formulated on the basis of the relationship between modulus of elasticity and strength properties have been modeled. The relationships have also been modeled based on the equality of the probability of failure (equal rank) for the strength properties. There exists good relationships between lumber strength properties.

The models, both resulting from the equal-rank analysis and general MOE-strength based relationships, show similar results. Therefore, it can be concluded that the equal-rank method yields the mean trend or average values for the strength property relationships. The band-width method justifies this conclusion by showing that the models from equal-rank analysis give very close results to the strength value at the median or 50-th percentile level.

The difference (called percentile-level effects) between using the model from equal-rank method and the model for the 5 % lower exclusion limit from band-width method can be up to 30 % as the percentage of the values given by the model for 5 % lower exclusion limit from band-width method. This means that the model from equal-rank analysis will give about 30 % lower or higher values if one would use this model rather than the model for 5 % lower exclusion limit from band-width analysis. The effects of percentile level on the property ratios are very significant if presented on the basis of modulus of elasticity especially for the UCS/MOR and UCS/UTS ratios.

The mean trend of the strength property ratios for the Canadian In-grade data show strong species dependency. The mean trends of UCS/MOR and UCS/UTS ratios for Douglas-fir and Hem-Fir are higher than those reported by Green and Kretschmann (1991) as well as those suggested by ASTM D 1990 (ASTM 1991). In this case, only S-P-F show closer results with that reported by Green and Kretschmann (1991) for these two property ratios.

UTS/MOR ratios for MOR below 48.3 MPa for all species agree very well with the ones reported by Curry and Fewell (1977), and Green and Kretschmann (1991) and are slightly higher than the ones suggested by ASTM D 1990 (ASTM 1991).

MOR/UTS ratio for every species shows much higher value than that suggested by ASTM D 1990 (ASTM 1991).

As per ASTM D 1990 (ASTM 1991), the models developed in this study can be used for the estimates of characteristic values for untested properties of Canadian dimension lumber.

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APPENDIX A: (Table 1 to Table 17)

Table 1. Relationships between MOR and MOE for the US In-grade data (Green and Kretschmann 1992)*

Species groups	Intercept	Slope	r (r^2)
Southern pine	0.012	4.249	0.72 (0.521)
Douglas-fir-Larch	-0.394	4.341	0.73 (0.538)
Hem-fir	-0.175	4.299	0.72 (0.52)

* MOR is in ksi, MOE is in million psi, data adjusted to 2 by 8 size 15% moisture content.

Table 2. Relationships between UTS and MOE for the US In-grade data (Green and Kretschmann 1992)*

Species group	Intercept	Slope	r (r^2)
Southern pine	-1.258	3.420	0.67 (0.442)
Douglas-fir-Larch	-0.515	2.878	0.64 (0.405)
Hem-fir	-0.867	3.363	0.65 (0.421)

* MOR is in ksi, MOE is in million psi, data adjusted to 2 by 8 size 15% moisture content.

Table 3. Width, test span and gauge length for bending, tension and compression specimens (Barrett and Griffin 1989).

Width (mm)	Test span	Gauge length	
	Bending (mm)	Tension (mm)	Compression (mm)
89	1510	2640	2440
184	3130	3680	3660
235	3990	3680	4270

Table 4. General relationships between MOE and MOR

Species	Sample Size	Model *	SSE. Variance	Parameter	Estimate	Asymptotic Std. Error **	Asymptotic 95% Confidence Interval Lower**	Upper**	r^2	|t|
D-FIR	2229	1	333788.89	B0	-13.2271	1.0925	-15.3684	-11.0858	0.58	12.11
			149.88	B1	4.9333	0.0887	4.7594	5.1071		
		2	333687.11	B0	-18.9570	7.9839	-34.6140	-3.3000		2.37
			149.90	B1	6.7651	2.7241	1.4230	12.1072		
				B2	0.9111	0.1119	0.6917	1.1304		0.80
		3	335383.60	B1	1.9385	0.1203	1.7025	2.1744		
			150.60	B2	1.2705	0.0238	1.2238	1.3172		
		4	332818.02	B1	0.6539	0.1800	0.3009	1.0069		
			149.50	B2	1.9694	0.1739	1.6285	2.3104		
				B3	0.9484	0.0124	0.9242	0.9727		4.16
H-FIR	2295	1	289313.94	B0	-9.0746	1.2159	-11.4578	-6.6915	0.47	7.46
			126.17	B1	4.7233	0.1045	4.5186	4.9281		
		2	289306.92	B0	-11.2593	10.0043	-30.8782	8.3595		1.13
			126.23	B1	5.4160	3.2559	-0.9690	11.8011		
				B2	0.9604	0.1728	0.6215	1.2993		0.23
		3	289565.56	B1	2.4078	0.1667	2.0809	2.7347		
			126.28	B2	1.1993	0.0276	1.1452	1.2533		
		4	289131.70	B1	1.2738	0.4492	0.3928	2.1547		
			126.15	B2	1.6338	0.2376	1.1679	2.0997		
				B3	0.9644	0.0190	0.9271	1.0016		1.87
S-P-F	3192	1	186720.03	B0	-8.0930	0.6778	-9.4216	-6.7645	0.60	11.94
			58.53	B1	4.5865	0.0668	4.4556	4.7174		
		2	186174.67	B0	2.2346	2.7475	-3.1525	7.6217		0.81
			58.38	B1	1.7435	0.5534	0.6584	2.8286		
				B2	1.3057	0.1033	1.1032	1.5082		2.96
		3	186210.51	B1	2.2216	0.0988	2.0279	2.4152		
			58.37	B2	1.2279	0.0187	1.1913	1.2646		
		4	186201.92	B1	2.2847	0.4470	1.5082	3.2613		
			58.39	B2	1.1746	0.1386	0.9028	1.4463		
				B3	1.0051	0.0132	0.9792	1.0311		0.39

* Model 1 = Eq. (9)
Model 2 = Eq. (10)
Model 3 = Eq. (11)
Model 4 = Eq. (12)
** Standard error and confidence interval for model 1

Table 5. General relationships between MOE and UTS

Species	Sample Size	Model*	SSE. Variance	Parameter	Estimate	Asymptotic Std. Error **	Asymptotic 95% Confidence Interval Lower**	Upper**	r^2	|t|
D-FIR	2232	1	165237.78	B0	-7.0968	0.7628	-8.5920	-5.6016	0.47	9.30
			74.10	B1	2.7366	0.0617	2.6157	2.8575		
		2	165170.06	B0	-3.4928	3.5865	-10.5261	3.5405		0.97
			74.10	B1	1.7929	0.8178	0.1890	3.3967		
				B2	1.1218	0.1329	0.8611	1.3824		0.92
		3	165264.51	B1	1.1052	0.0828	0.9429	1.2676		
			74.11	B2	1.2631	0.0287	1.2069	1.3193		
		4	165031.94	B1	0.6530	0.2008	0.2592	1.0467		
			74.04	B2	1.5954	0.1905	1.2218	1.9690		
				B3	0.9759	0.0135	0.9494	1.0024		1.79
H-FIR	2245	1	162757.68	B0	-12.5818	0.8693	-14.2856	-10.8779	0.49	14.47
			72.56	B1	3.3752	0.0732	3.2318	3.5187		
		2	161370.87	B0	2.1844	2.3207	-2.3666	6.7354		0.94
			71.98	B1	0.4624	0.2119	0.0468	0.8780		
				B2	1.6090	0.1462	1.3223	1.8957		4.17
		3	161431.53	B1	0.6859	0.0567	0.5748	0.7971		
			71.97	B2	1.4853	0.0322	1.4221	1.5485		
		4	161431.46	B1	0.6785	0.2376	0.2126	1.1445		
			72.00	B2	1.4923	0.2238	1.0534	1.9312		
				B3	0.9995	0.0172	0.9658	1.0331		0.03
S-P-F	2694	1	138818.92	B0	-10.2807	0.7084	-11.6692	-8.8921	0.46	14.51
			51.57	B1	3.2747	0.0686	3.1402	3.4092		
		2	137800.40	B0	1.6290	1.9654	-2.2249	5.4830		0.83
			51.21	B1	0.5228	0.2293	0.0732	0.9723		
				B2	1.5923	0.1477	1.3027	1.8819		4.01
		3	137836.25	B1	0.7301	0.0567	0.6189	0.8413		
			51.20	B2	1.4817	0.0322	1.4185	1.5449		
		4	137819.37	B1	0.5974	0.2070	0.1914	1.0034		
			51.21	B2	1.6278	0.2492	1.1392	2.1164		
				B3	0.9867	0.0225	0.9426	1.0308		0.59

* Model 1 = Eq. (9)
Model 2 = Eq. (10)
Model 3 = Eq. (11)
Model 4 = Eq. (12)
** : Standard error and confidence interval for model 1

Table 6. General relationships between MOE and UCS

Species	Sample Size	Model*	SSE. Variance	Parameter	Estimate	Asymptotic Std. Error **	Asymptotic 95% Confidence Interval		r^2	|t|
							Lower**	Upper**		
D-FIR	2237	1	61664.17	B0	7.1133	0.4584	6.2148	8.0117	0.63	15.52
			27.59	B1	2.3376	0.0380	2.2632	2.4120		
		2	61279.88	B0	14.8683	1.5448	11.8389	17.8977		9.63
			27.43	B1	0.6300	0.2143	0.2098	1.0501		
				B2	1.3908	0.1045	1.1858	1.5958		3.74
		3	62172.20	B1	4.8072	0.1594	4.4946	5.1198		
			27.82	B2	0.8026	0.0131	0.7770	0.8282		
		4	61269.17	B1	9.8312	1.2377	7.4039	12.2584		
			27.43	B2	0.3253	0.0823	0.1639	0.4867		
				B3	1.0388	0.0067	1.0256	1.0520		5.79
H-FIR	2289	1	65535.61	B0	3.3500	0.5583	2.2557	4.4443	0.58	6.00
			28.66	B1	2.6642	0.0479	2.5703	2.7581		
		2	65053.05	B0	13.4110	1.7873	9.9060	16.9159		7.50
			28.46	B1	0.5112	0.2091	0.1011	0.9213		
				B2	1.5065	0.1304	1.2508	1.7622		3.89
		3	65742.84	B1	3.6891	0.1530	3.3892	3.9891		
			28.75	B2	0.9098	0.0166	0.8772	0.9423		
		4	65155.16	B1	8.2866	1.4399	5.4630	11.1103		
			28.50	B2	0.3538	0.1176	0.1232	0.5845		
				B3	1.0479	0.0103	1.0277	1.0681		4.65
S-P-F	2602	1	37603.70	B0	3.5721	0.3771	2.8329	4.3113	0.61	9.47
			14.46	B1	2.3522	0.0371	2.2794	2.4250		
		2	37322.98	B0	11.0819	1.2064	8.7163	13.4475		9.17
			14.36	B1	0.5112	0.1726	0.1727	0.8496		
				B2	1.4895	0.1126	1.2687	1.7103		4.35
		3	37778.29	B1	3.6305	0.1207	3.3938	3.8673		
			14.53	B2	0.8737	0.0141	0.8461	0.9014		
		4	37338.29	B1	7.3907	0.9559	5.5161	9.2652		
			14.37	B2	0.3298	0.0970	0.1395	0.5201		
				B3	1.0546	0.0099	1.0351	1.0740		5.52

* Model 1 = Eq. (9)
Model 2 = Eq. (10)
Model 3 = Eq. (11)
Model 4 = Eq. (12)
** : Standard error and confidence interval for model 1

Table 7. Strength property relationships based on MOR (Equal-Rank analysis)

Relation	Species	Sample Size	Model*	SSE Variance	Parameter	Estimate	Asymptotic Std. Error	Asymptotic 95 % Confidence Interval Lower	Upper	LR
UTS-MOR	D-FIR	126	1	799.72	B0	6.6678	9.6509	-12.4373	25.7729	17.46
				6.56	B1	0.1373	0.2152	-0.2887	0.5633	
					B2	6.0215	30.5591	-54.4740	66.5170	
					B3	1.3274	0.2914	0.7505	1.9043	
			2	918.58	B1	0.3481	0.0362	0.2764	0.4198	
				7.41	B2	1.1226	0.0253	1.0724	1.1727	
			3	831.64	B1	1.0067	0.2982	0.4165	1.5970	
				6.76	B2	0.7609	0.0991	0.5647	0.9572	
					B3	1.0067	0.0018	1.0031	1.0102	
	H-FIR	126	1	845.48	B0	8.4913	4.5178	-0.4521	17.4347	23.29
				6.93	B1	0.0548	0.0989	-0.1409	0.2505	
					B2	8.8262	18.4406	-27.6791	45.3316	
					B3	1.5938	0.3506	0.8998	2.2879	
			2	1017.14	B1	0.1641	0.0207	0.1230	0.2051	
				5.20	B2	1.3296	0.0312	1.2678	1.3915	
			3	904.92	B1	0.9063	0.3776	0.1590	1.6537	
				7.36	B2	0.7426	0.1419	0.4617	1.0234	
					B3	1.0113	0.0028	1.0059	1.0168	
	S-P-F	126	1	351.94	B0	6.2877	4.4008	-2.4242	14.9997	27.11
				2.89	B1	0.0409	0.0877	-0.1326	0.2145	
					B2	4.5935	18.2172	-31.4696	40.6565	
					B3	1.6981	0.4250	0.8568	2.5395	
			2	436.43	B1	0.1437	0.0144	0.1153	0.1722	
				3.52	B2	1.3912	0.0260	1.3397	1.4428	
			3	364.38	B1	0.8143	0.2766	0.2668	1.3618	
				2.96	B2	0.7506	0.1243	0.5046	0.9966	
					B3	1.0153	0.0030	1.0094	1.0212	

Table 7. Continued

Relation	Species	Sample Size	Model*	SSE Variance	Parameter	Estimate	Asymptotic Std. Error	Asymptotic 95 % Confidence Interval		LR
								Lower	Upper	
UCS-MOR	D-FIR	126	1	654.07	B0	15.5812	69.5356	-122.0725	153.2349	19.59
				5.36	B1	0.6317	1.1308	-1.6070	2.8703	
					B2	6.0215	130.1654	-251.6560	263.6990	
					B3	0.9260	0.3203	0.2920	1.5599	
			2	764.12	B1	3.9738	0.2433	3.4922	4.4554	
				6.16	B2	0.5717	0.0154	0.5411	0.6022	
			3	636.90	B1	8.3015	1.2976	5.7330	10.8700	
				5.17	B2	0.3051	0.0550	0.1963	0.4138	
					B3	1.0057	0.0011	1.0034	1.0079	
	H-FIR	126	1	428.16	B0	16.6192	21.6001	-26.1406	59.3791	33.44
				3.51	B1	0.2146	0.4384	-0.6533	1.0826	
					B2	6.0996	63.0865	-118.7875	130.9867	
					B3	1.1910	0.3802	0.4383	1.9438	
			2	558.31	B1	2.6262	0.1685	2.2927	2.9597	
				4.50	B2	0.6746	0.0163	0.6423	0.7069	
			3	423.89	B1	7.7605	1.3654	5.0577	10.4632	
				3.45	B2	0.2825	0.0624	0.1589	0.4060	
					B3	1.0086	0.0014	1.0059	1.0113	
	S-P-F	126	1	89.01	B0	14.0910	8.1171	-1.9778	30.1598	90.77
				0.73	B1	0.1953	0.2323	-0.2646	0.6552	
					B2	7.0520	25.7198	-43.8632	57.9672	
					B3	1.2201	0.2339	0.7571	1.6832	
			2	182.94	B1	2.2987	0.1092	2.0825	2.5149	
				1.48	B2	0.6831	0.0127	0.6579	0.7083	
			3	81.10	B1	8.2709	0.8575	6.5735	9.9683	
				0.66	B2	0.1856	0.0395	0.1074	0.2639	
					B3	1.0133	0.0011	1.0112	1.0154	

* Model 1 = Eq. (21)
Model 2 = Eq. (20)
Model 3 = Eq. (12)

Table 8. Strength property relationships based on UTS (Equal-Rank analysis)

Relation	Species	Sample Size	Model*	SSE Variance	Parameter	Estimate	Asymptotic Std. Error	Asymptotic 95 % Confidence Interval Lower	Upper	LR
MOR-UTS	D-FIR	126	1	1745.89	B0	6.0215	5.1755	-4.2241	16.2671	27.12
				14.31	B1	4.7865	1.3074	2.1983	7.3746	
					B2	6.6180	0.9814	4.6752	8.5609	
					B3	0.7299	0.0597	0.6117	0.8481	
			2	2165.20	B1	2.8489	0.1794	2.4938	3.2039	
				17.46	B2	0.8584	0.0178	0.8231	0.8937	
			3	1578.75	B1	0.9419	1.3284	0.0742	1.1815	
				12.84	B2	1.3284	0.0742	1.1815	1.4754	
					B3	0.9851	0.0023	0.9806	0.9896	
	H-FIR	126	1	1168.50	B0	6.0996	8.8200	-11.3605	23.5597	44.65
				9.58	B1	7.5577	3.1746	1.2732	13.8421	
					B2	8.0101	1.3776	5.2830	10.7372	
					B3	0.5786	0.0827	0.4148	0.7424	
			2	1665.43	B1	4.0596	0.2296	3.6052	4.5140	
				13.43	B2	0.7385	0.0161	0.7066	0.7703	
			3	1259.74	B1	1.5656	0.2623	1.0463	2.0849	
				10.24	B2	1.1460	0.0692	1.0090	1.2830	
					B3	0.9869	0.0021	0.9826	0.9911	
	S-P-F	126	1	436.89	B0	7.0520	6.6974	-6.2063	20.3103	39.80
				3.58	B1	5.1927	1.9231	1.3858	8.9996	
					B2	5.9813	1.5774	2.8586	9.1041	
					B3	0.6394	0.0757	0.4895	0.7892	
			2	599.16	B1	4.0447	0.1651	3.7181	4.3714	
				4.83	B2	0.7168	0.0122	0.6926	0.7410	
			3	384.86	B1	1.8607	0.1931	1.4785	2.2429	
				3.13	B2	1.0724	0.0458	0.9818	1.1630	
					B3	0.9865	0.0017	0.9832	0.9898	

Table 8. Continued

Relation	Species	Sample Size	Model*	SSE Variance	Parameter	Estimate	Asymptotic Std. Error	Asymptotic 95 % Confidence Interval		LR
								Lower	Upper	
UCS-UTS	D-FIR	126	1	311.32	B0	14.5265	6.9739	0.7208	28.3322	2.24
				2.55	B1	2.5256	1.3053	-0.0584	5.1096	
					B2	5.7852	3.7598	-1.6577	13.2281	
					B3	0.7030	0.1025	0.5002	0.9058	
			2	316.91	B1	6.1348	0.1853	5.7679	6.5016	
				2.56	B2	0.5394	0.0088	0.5219	0.5570	
			3	312.44	B1	6.8239	0.5868	5.6623	7.9855	
				2.54	B2	0.4924	0.0367	0.4198	0.5650	
					B3	1.0016	0.0012	0.9992	1.0041	
	H-FIR	126	1	167.44	B0	16.7095	2.5017	11.7570	21.6619	5.08
				1.37	B1	2.2696	0.5838	1.1139	3.4252	
					B2	8.2496	1.2126	5.8491	10.6500	
					B3	0.7076	0.0542	0.6002	0.8149	
			2	174.33	B1	5.8757	0.1385	5.6020	6.1498	
				1.41	B2	0.5397	0.0068	0.5261	0.5532	
			3	173.72	B1	6.1461	0.4453	5.2649	7.0274	
				1.41	B2	0.5200	0.0307	0.4594	0.5807	
					B3	1.0007	0.0010	0.9987	1.0027	
	S-P-F	126	1	144.10	B0	14.0910	0.7706	12.5656	15.6164	5.18
				1.18	B1	1.6714	0.2846	1.1079	2.2348	
					B2	6.5370	0.1597	6.2210	6.8530	
					B3	0.7436	0.0408	0.6628	0.8244	
			2	150.15	B1	5.5494	0.1515	5.2496	5.8492	
				1.21	B2	0.5123	0.0084	0.4958	0.5288	
			3	144.61	B1	6.4628	0.4836	5.5055	7.4202	
				1.18	B2	0.4405	0.0339	0.3734	0.5077	
					B3	1.0029	0.0013	1.0003	1.0055	

* Model 1 = Eq. (21)
Model 1 = Eq. (20)
Model 1 = Eq. (12)

Table 9. Goodness-of-fit analysis (1,000 MPa Band-Width)

Species	Property	MOE	MOE	Sample	MOE	Rel. skew.	Rel. kurt.	Sb parameter		Weibull parameter			K-S test critical values		
		min.	max.	size	50-th pctl.	B_1	B_2	Gamma	Eta	Shape	Scale	Location	Sb	3-p W	Table
D-FIR	MOE-MOR	6	7	39	6.6514	0.2227	2.0888	-0.3821	0.6113	1.2746	10.4470	10.6637	0.0795	0.0898	0.2130
		7	8	110	7.6429	0.1888	1.8490	-0.3136	0.5335	1.6350	16.4439	10.0000	0.0497	0.0634	0.1297
		8	9	173	8.5506	0.0329	1.7625	-0.1008	0.4961	2.1281	22.1725	8.4533	0.0641	0.0595	0.1034
		9	10	263	9.5184	0.0026	1.7512	-0.0426	0.5796	2.1874	25.1663	11.3565	0.0399	0.0400	0.0839
		10	11	323	10.4989	0.0083	1.8167	0.0023	0.5306	2.6662	31.4652	9.4399	0.0651	0.0279	0.0757
		11	12	288	11.4747	0.0137	1.8055	0.0559	0.5523	3.0908	38.2943	9.0170	0.0443	0.0563	0.0801
		12	13	260	12.4810	0.0026	1.7596	0.0439	0.5779	4.3004	53.9730	0.0000	0.0573	0.0553	0.0843
		13	14	222	13.4556	0.0482	1.8986	0.1035	0.5807	3.6504	43.4080	16.2513	0.0502	0.0434	0.0913
		14	15	189	14.4696	0.0062	1.8656	0.0683	0.5613	5.0218	63.2577	0.0000	0.0798	0.0430	0.0989
		15	16	139	15.4913	0.0042	1.7732	0.0202	0.5825	4.4568	65.0164	4.2392	0.0454	0.0472	0.1154
		16	17	82	16.5467	0.0001	1.8435	-0.0935	0.4990	3.8732	58.0020	14.6376	0.1035	0.0681	0.1502
		17	18	58	17.4414	0.1275	1.5633	0.1337	0.5680	5.4871	79.6668	0.0000	0.1382	0.0934	0.1786
		18	19	43	18.4822	0.0061	1.7713	0.0333	0.4690	4.7198	82.8478	0.0000	0.1243	0.0704	0.2074
	MOE-UTS	6	7	34	6.5916	0.0587	1.5833	-0.1731	0.4668	1.4907	7.7049	5.5362	0.0613	0.0711	0.2270
		7	8	97	7.6554	0.1313	2.0339	-0.3928	0.6111	2.3665	11.4052	4.0346	0.0788	0.1065	0.1381
		8	9	159	8.6413	0.0807	1.8274	-0.3041	0.5232	1.9176	13.4757	4.2398	0.0740	0.1112	0.1079
		9	10	268	9.5348	0.0330	1.8497	-0.0762	0.5468	1.9136	13.0209	7.2283	0.0571	0.0589	0.0831
		10	11	322	10.5106	0.0005	1.7495	-0.0257	0.6083	1.9427	14.3969	7.6699	0.0265	0.0396	0.0758
		11	12	326	11.5398	0.0018	1.8228	-0.0861	0.5393	1.8878	18.9379	8.3051	0.0565	0.0557	0.0753
		12	13	271	12.4769	0.0165	1.7664	0.0553	0.5973	2.2069	20.8978	8.7902	0.0327	0.0513	0.0826
		13	14	212	13.4596	0.0201	1.8889	0.0838	0.5169	2.2124	23.0119	9.3342	0.0628	0.0531	0.0934
		14	15	185	14.5159	0.0004	1.8437	-0.0386	0.6077	3.1392	34.3803	2.4073	0.0766	0.0672	0.1000
		15	16	117	15.4925	0.0033	2.1907	0.0183	0.6129	2.4292	27.1956	11.7731	0.0932	0.0473	0.1257
		16	17	93	16.5296	0.0000	1.5896	-0.0651	0.5489	2.0211	21.6176	16.5401	0.0604	0.0447	0.1410
		17	18	47	17.4072	0.0109	1.8326	0.1830	0.4870	2.3671	26.9972	16.7774	0.1362	0.0858	0.1984
		18	19	33	18.3804	0.0898	1.6622	0.3488	0.7149	1.9284	25.3481	22.0217	0.0935	0.0733	0.2310

Table 9. Continued

Species	Property	MOE min.	MOE max.	Sample size	MOE 50-th pctl.	Rel. skew. B_1	Rel. kurt. B_2	Sb parameter Gamma	Sb parameter Eta	Weibull parameter Shape	Weibull parameter Scale	Weibull parameter Loc.	K-S test critical values Sb	K-S test critical values 3-p W	K-S test critical values Table
D-FIR	MOE-UCS	6	7	49	6.7018	0.1519	1.8616	-0.3675	0.4294	4.1735	12.3385	11.7543	0.1073	0.0765	0.1943
		7	8	120	7.6233	0.1187	1.9124	-0.3163	0.6279	2.5363	12.4827	13.9577	0.0386	0.0662	0.1242
		8	9	214	8.5070	0.0003	1.8910	-0.0182	0.6474	5.3821	20.6278	8.1830	0.0417	0.0571	0.0930
		9	10	278	9.5228	0.0044	1.8884	-0.0548	0.5990	4.4308	20.1583	11.1606	0.0439	0.0510	0.0816
		10	11	342	10.4955	0.0003	1.7876	0.0092	0.5122	7.7701	32.7514	0.3530	0.0453	0.0610	0.0735
		11	12	314	11.4887	0.0076	1.8155	0.0276	0.6078	6.4883	31.3847	4.6746	0.0435	0.0532	0.0767
		12	13	233	12.5150	0.0053	1.8122	-0.0361	0.6007	4.7227	24.3131	13.7191	0.0508	0.0767	0.0891
		13	14	198	13.5230	0.0080	1.7531	-0.0570	0.6180	3.1862	20.9622	19.3460	0.0341	0.0771	0.0967
		14	15	144	14.5028	0.0003	1.7491	-0.0066	0.5894	4.9296	33.0048	10.7609	0.0408	0.0746	0.1133
		15	16	134	15.5057	0.0046	1.7518	-0.0120	0.5283	4.5563	31.3886	14.5376	0.0820	0.0813	0.1175
		16	17	80	16.4850	0.0001	1.7290	0.0320	0.5334	3.2011	20.4940	27.6479	0.0738	0.0699	0.1521
		17	18	63	17.4133	0.0256	1.9840	0.1965	0.5608	2.5873	16.1954	32.8940	0.1423	0.0877	0.1713
H-FIR	MOE-MOR	7	8	100	7.6045	0.1054	2.0455	-0.2517	0.5932	2.2542	22.5885	7.0218	0.0787	0.0675	0.1360
		8	9	186	8.5875	0.0917	1.9926	-0.2114	0.5978	2.6648	27.6578	6.8377	0.0437	0.0355	0.0997
		9	10	325	9.5687	0.0439	1.9251	-0.1681	0.6078	2.8127	29.8325	8.9331	0.0362	0.0253	0.0754
		10	11	389	10.5018	0.0002	1.8655	-0.0042	0.5766	3.1148	34.0683	9.5795	0.0440	0.0460	0.0690
		11	12	387	11.5308	0.0083	1.8491	-0.0687	0.5563	4.2298	46.5812	3.1323	0.0505	0.0493	0.0691
		12	13	317	12.4946	0.0121	1.7790	0.0114	0.5311	3.6566	44.2984	10.2780	0.0451	0.0414	0.0764
		13	14	232	13.4492	0.0235	1.8628	0.1259	0.6178	5.6681	59.2831	0.0000	0.0415	0.0734	0.0893
		14	15	168	14.3979	0.0235	1.7738	0.2314	0.5585	5.1835	64.5463	0.0000	0.0612	0.0919	0.1049
		15	16	98	15.4029	0.0317	1.8189	0.2373	0.6033	6.4892	69.2675	0.0000	0.0652	0.0615	0.1374
		16	17	49	16.5314	0.0116	1.9155	-0.0675	0.5370	5.0769	72.3666	0.0000	0.1050	0.1052	0.1943
	MOE-UTS	7	8	105	7.5346	0.0208	2.0049	-0.0823	0.5933	2.5442	11.7797	4.2299	0.0873	0.0910	0.1327
		8	9	193	8.5629	0.0115	1.7457	-0.1459	0.5771	2.0672	13.2853	5.1780	0.0490	0.0935	0.0979
		9	10	273	9.5438	0.0668	1.8570	-0.1000	0.5700	2.3205	14.9675	6.5117	0.0676	0.0633	0.0823
		10	11	328	10.5279	0.0073	1.7680	-0.0610	0.5469	2.2661	17.3938	5.8856	0.0389	0.0587	0.0751
		11	12	384	11.5160	0.0038	1.6989	-0.0340	0.5312	2.4859	24.2345	4.2084	0.0308	0.0741	0.0694
		12	13	324	12.4902	0.0014	1.8708	0.0227	0.5815	2.5908	27.3884	5.5305	0.0488	0.0373	0.0756
		13	14	244	13.4739	0.0077	1.8539	0.0560	0.5349	2.6570	28.3762	7.5208	0.0513	0.0446	0.0871
		14	15	152	14.4155	0.0514	1.9454	0.2080	0.6097	2.2715	23.9767	13.2310	0.0602	0.0325	0.1103
		15	16	112	15.3955	0.0084	1.7604	0.2525	0.5949	4.9179	45.7075	0.0000	0.0766	0.0613	0.1285
		16	17	53	16.4115	0.0008	1.7091	0.1848	0.5167	3.8960	34.1306	15.0052	0.1203	0.1173	0.1868

Table 9. Continued

Species	Property	MOE min.	MOE max.	Sample size	MOE 50-th pctl.	Rel. skew. B_1	Rel. kurt. B_2	Sb parameter Gamma	Sb parameter Eta	Weibull parameter Shape	Weibull parameter Scale	Weibull parameter Loc.	K-S test critical values Sb	K-S test critical values 3-p W	K-S test critical values Table
H-FIR	MOE-UCS	7	8	100	7.6040	0.0625	2.0663	-0.2561	0.6066	2.5840	12.3490	13.2671	0.0857	0.0792	0.1360
		8	9	196	8.5445	0.0038	1.7549	-0.0942	0.5275	3.3178	14.3463	13.0638	0.0637	0.0297	0.0971
		9	10	326	9.5309	0.0000	1.7660	-0.0738	0.5972	3.3004	14.8310	15.1400	0.0374	0.0433	0.0753
		10	11	389	10.5382	0.0165	1.8647	-0.0868	0.5672	3.8227	18.5069	14.0778	0.0433	0.0321	0.0690
		11	12	360	11.4862	0.0043	1.7857	0.0303	0.5466	3.6777	20.4712	15.5242	0.0460	0.0401	0.0717
		12	13	304	12.4304	0.0939	2.0269	0.1725	0.6151	3.3177	21.1213	17.3862	0.0648	0.0485	0.0780
		13	14	260	13.5314	0.0005	1.8451	-0.0798	0.6354	3.3629	19.9759	21.3896	0.0532	0.0491	0.0843
		14	15	161	14.4418	0.0011	1.7397	0.1336	0.5715	4.0280	26.5986	17.9206	0.0564	0.0592	0.1072
		15	16	83	15.4032	0.0366	2.1677	0.2228	0.5681	3.9321	28.3484	19.0699	0.1042	0.1083	0.1493
		16	17	45	16.4166	0.0470	1.7901	0.2495	0.7408	2.9680	22.2168	27.3957	0.0655	0.0885	0.2027
S-P-F	MOE-MOR	5	6	57	5.6400	0.2498	1.9295	-0.3582	0.6225	1.8799	10.9541	9.4932	0.0864	0.0641	0.1801
		6	7	153	6.6083	0.0890	1.7549	-0.2332	0.5298	1.7387	12.4261	11.8676	0.0568	0.0475	0.1099
		7	8	332	7.5648	0.0442	1.9219	-0.1517	0.5824	2.6037	19.4357	8.7473	0.0503	0.0355	0.0746
		8	9	489	8.5242	0.0032	1.7839	-0.0552	0.5698	3.7197	27.7671	5.6948	0.0345	0.0248	0.0615
		9	10	596	9.4750	0.0015	1.8219	0.0565	0.5635	4.2513	31.6334	6.3399	0.0534	0.0271	0.0557
		10	11	564	10.4732	0.0060	1.8710	0.0616	0.5742	4.9601	38.5306	4.4490	0.0461	0.0211	0.0573
		11	12	484	11.4357	0.0136	1.7921	0.1380	0.5338	4.7958	36.5218	11.0816	0.0405	0.0349	0.0618
		12	13	291	12.4357	0.0301	1.8098	0.1374	0.5309	4.9929	41.0497	11.4261	0.0448	0.0535	0.0797
		13	14	141	13.4020	0.1376	1.8113	0.2406	0.6057	3.9473	35.7802	21.1868	0.0525	0.0709	0.1145
		14	15	49	14.3675	0.1700	1.9819	0.2855	0.5256	6.7751	62.6777	0.0000	0.0822	0.1005	0.1943
	MOE-UTS	5	6	37	5.6496	0.0655	1.8869	-0.4101	0.6642	1.0000	5.2329	5.7209	0.0855	0.1016	0.2180
		6	7	111	6.6124	0.1814	2.1980	-0.2679	0.5855	1.2732	6.5403	6.6574	0.0809	0.0815	0.1291
		7	8	232	7.5575	0.0467	2.0274	-0.1360	0.5888	2.2111	11.1885	4.0716	0.0625	0.0875	0.0893
		8	9	384	8.5610	0.0303	1.8435	-0.1339	0.5460	2.3277	13.5103	5.0745	0.0501	0.0579	0.0694
		9	10	509	9.5225	0.0021	1.7871	-0.0510	0.5670	2.4858	17.3313	4.8343	0.0324	0.0543	0.0603
		10	11	544	10.4878	0.0034	1.7166	0.0259	0.5287	2.6488	21.2869	4.9810	0.0364	0.0582	0.0583
		11	12	403	11.4751	0.0059	1.8562	0.0567	0.5675	2.2486	19.4106	10.4089	0.0508	0.0298	0.0677
		12	13	251	12.5054	0.0007	1.7172	-0.0122	0.5651	2.7327	25.0856	7.8997	0.0368	0.0283	0.0858
		13	14	135	13.3918	0.1169	1.9461	0.2551	0.5800	3.5567	31.6897	8.0351	0.0508	0.0460	0.1171
		14	15	53	14.4568	0.0807	1.8894	0.1018	0.5878	3.5984	37.8502	3.0083	0.0930	0.0510	0.1868

Table 9. Continued

Species	Property	MOE min.	MOE max.	Sample size	MOE 50-th pctl.	Rel. skew. B_1	Rel. kurt. B_2	Sb parameter Gamma	Sb parameter Eta	Weibull parameter Shape	Weibull parameter Scale	Weibull parameter Loc.	K-S test critical values Sb	K-S test critical values 3-p W	K-S test critical values Table
S-P-F	MOE-UCS	5	6	39	5.7209	0.2059	1.9196	-0.5142	0.5417	1.6301	4.9395	13.5306	0.0906	0.0692	0.2100
		6	7	117	6.6003	0.0464	1.9107	-0.2504	0.6157	5.7208	18.3635	2.4134	0.0564	0.0535	0.1257
		7	8	258	7.5212	0.0169	1.8067	-0.0470	0.5550	4.7308	15.6762	6.9415	0.0665	0.0593	0.0847
		8	9	427	8.5649	0.0387	1.8529	-0.1448	0.5546	3.5754	13.1073	11.8432	0.0398	0.0708	0.0658
		9	10	495	9.4976	0.0013	1.8849	0.0056	0.5800	5.6706	20.7791	6.2837	0.0481	0.0577	0.0611
		10	11	480	10.4625	0.0089	1.8283	0.0886	0.5899	3.7373	14.1221	15.2608	0.0379	0.0520	0.0621
		11	12	380	11.4514	0.0286	1.8689	0.1090	0.5588	3.4713	13.5007	18.1641	0.0405	0.0490	0.0698
		12	13	222	12.4802	0.0083	1.7742	0.0452	0.5696	3.3410	17.0699	17.6197	0.0410	0.0604	0.0913
		13	14	106	13.4257	0.0452	1.8209	0.1649	0.5511	2.9839	15.8604	21.6010	0.0467	0.1053	0.1321
		14	15	44	14.3441	0.3980	2.0514	0.3750	0.5811	6.7083	31.7716	8.6972	0.1073	0.0725	0.2050

Table 10. Relationships between MOE and MOR (Band-Width analysis)

Species	Band-Width	Sample Size	PCTL	Parameter	Estimate	Asymptotic Std. Error	Asymptotic 95% Confidence Interval Lower	Upper
D-FIR	500	106	1-st	B0	2.571055	3.644842	-4.657675	9.799785
				B1	0.292945	0.317370	-0.336486	0.922377
				B2	1.652017	0.347744	0.962345	2.341689
			5-th	B0	-8.322920	5.308494	-18.851132	2.205293
				B1	1.943278	1.187438	-0.411740	4.298296
				B2	1.164390	0.182746	0.801955	1.526825
			50-th	B0	-37.485075	8.355181	-54.055711	-20.914440
				B1	13.409229	3.894428	5.685499	21.132958
				B2	0.736805	0.076054	0.585968	0.887641
	690	117	1-st	B0	1.469774	3.324246	-5.115580	8.055128
				B1	0.420124	0.361308	-0.295630	1.135878
				B2	1.526763	0.269673	0.992540	2.060986
			5-th	B0	-7.438555	4.344389	-16.044818	1.167709
				B1	1.782691	0.914939	-0.029809	3.595191
				B2	1.186753	0.152916	0.883826	1.489680
			50-th	B0	-28.467992	5.545797	-39.454254	-17.481731
				B1	9.427161	2.192234	5.084331	13.769991
				B2	0.832443	0.062975	0.707690	0.957196
	1000	128	1-st	B0	2.729538	2.605000	-2.426122	7.885197
				B1	0.298591	0.227350	-0.151367	0.748549
				B2	1.626803	0.239856	1.152093	2.101513
			5-th	B0	-6.181606	3.390445	-12.891772	0.528560
				B1	1.578368	0.667756	0.256785	2.899950
				B2	1.217588	0.126157	0.967906	1.467270
			50-th	B0	-30.079164	3.826081	-37.651514	-22.506814
				B1	10.227783	1.567528	7.125426	13.330140
				B2	0.808537	0.040983	0.727427	0.889648
	1500	133	1-st	B0	5.155194	1.424008	2.337941	7.972448
				B1	0.104857	0.060583	-0.015000	0.224714
				B2	1.965553	0.186808	1.595973	2.335133
			5-th	B0	-2.374785	2.049975	-6.430450	1.680880
				B1	0.854583	0.279221	0.302172	1.406994
				B2	1.407217	0.100284	1.208815	1.605620
			50-th	B0	-28.712142	3.008615	-34.664377	-22.759907
				B1	9.559147	1.192668	7.199577	11.918717
				B2	0.828437	0.033605	0.761954	0.894920
	2000	139	1-st	B0	6.266269	0.960674	4.366461	8.166077
				B1	0.050140	0.023823	0.003028	0.097251
				B2	2.213721	0.155768	1.905677	2.521765
			5-th	B0	-0.059217	1.407603	-2.842861	2.724427
				B1	0.521047	0.140149	0.243892	0.798203
				B2	1.565577	0.084184	1.399096	1.732058
			50-th	B0	-26.494196	2.437466	-31.314475	-21.673917
				B1	8.572840	0.919723	6.754015	10.391665
				B2	0.859810	0.029254	0.801958	0.917661

Table 10. Continued

Species	Band-Width	Sample Size	PCTL	Parameter	Estimate	Asymptotic Std. Error	Asymptotic 95% Confidence Interval	
							Lower	Upper
D-FIR	2500	145	1-st	B0	8.785936	0.970455	6.867513	10.704359
				B1	0.004979	0.003694	-0.002324	0.012282
				B2	3.009873	0.248480	2.518671	3.501075
			5-th	B0	3.250637	1.389436	0.503959	5.997314
				B1	0.200167	0.069915	0.061956	0.338377
				B2	1.882847	0.111933	1.661574	2.104120
			50-th	B0	-27.718570	2.323216	-32.311172	-23.125969
				B1	8.866346	0.880559	7.125631	10.607061
				B2	0.852897	0.026924	0.799672	0.906122
H-FIR	500	97	1-st	B0	5.988787	1.571110	2.869295	9.108279
				B1	0.042854	0.032474	-0.021623	0.107332
				B2	2.360331	0.258383	1.847304	2.873359
			5-th	B0	2.745504	1.968412	-1.162846	6.653854
				B1	0.299710	0.135087	0.031489	0.567930
				B2	1.787788	0.148065	1.493800	2.081776
			50-th	B0	-17.178204	5.038259	-27.181839	-7.174570
				B1	7.003413	1.833555	3.362828	10.643999
				B2	0.896915	0.073578	0.750822	1.043007
	690	101	1-st	B0	1.861866	2.312249	-2.726743	6.450476
				B1	0.275442	0.185206	-0.092097	0.642980
				B2	1.712137	0.219228	1.277084	2.147190
			5-th	B0	-2.006605	2.748602	-7.461149	3.447939
				B1	0.860304	0.380687	0.104839	1.615768
				B2	1.433515	0.139715	1.156254	1.710776
			50-th	B0	-16.114421	4.426557	-24.898831	-7.330011
				B1	6.790301	1.599185	3.616753	9.963849
				B2	0.902537	0.066329	0.770909	1.034165
	1000	102	1-st	B0	0.376822	2.229768	-4.047548	4.801193
				B1	0.419569	0.239108	-0.054876	0.894015
				B2	1.570087	0.183472	1.206037	1.934136
			5-th	B0	-3.830542	2.662910	-9.114365	1.453282
				B1	1.123835	0.436860	0.257004	1.990666
				B2	1.348737	0.121486	1.107680	1.589794
			50-th	B0	-14.912589	3.497610	-21.852646	-7.972531
				B1	6.286618	1.220784	3.864303	8.708932
				B2	0.926075	0.055224	0.816499	1.035652
	1500	108	1-st	B0	-2.495990	2.863008	-8.172842	3.180862
				B1	0.898215	0.505165	-0.103441	1.899872
				B2	1.312329	0.174849	0.965633	1.659024
			5-th	B0	-8.111302	3.377821	-14.808940	-1.413663
				B1	2.055066	0.795233	0.478255	3.631877
				B2	1.154753	0.116832	0.923096	1.386411
			50-th	B0	-16.293184	2.965933	-22.174120	-10.412248
				B1	6.706326	1.068714	4.587250	8.825402
				B2	0.908949	0.045107	0.819509	0.998388

Table 10. Continued

Species	Band-Width	Sample Size	PCTL	Parameter	Estimate	Asymptotic Std. Error	Asymptotic 95% Confidence Interval	
							Lower	Upper
H-FIR	2000	116	1-st	B0	-4.264076	3.143135	-10.491239	1.963086
				B1	1.336127	0.711390	-0.073275	2.745529
				B2	1.176650	0.161361	0.856963	1.496338
			5-th	B0	-12.317054	3.978593	-20.199421	-4.434686
				B1	3.255466	1.204902	0.868321	5.642612
				B2	1.011739	0.107944	0.797882	1.225597
			50-th	B0	-22.746187	3.058526	-28.805722	-16.686652
				B1	9.021110	1.257058	6.530634	11.511586
				B2	0.828661	0.038318	0.752746	0.904576
	2500	120	1-st	B0	-4.287888	2.713924	-9.662712	1.086937
				B1	1.280502	0.597156	0.097856	2.463148
				B2	1.189456	0.141520	0.909181	1.469730
			5-th	B0	-12.544163	3.638584	-19.750241	-5.338084
				B1	3.213171	1.087544	1.059331	5.367012
				B2	1.016835	0.098718	0.821329	1.212342
			50-th	B0	-24.009176	3.200023	-30.346702	-17.671650
				B1	9.340913	1.325164	6.716475	11.965351
				B2	0.821805	0.038855	0.744854	0.898756
S-P-F	500	86	1-st	B0	7.044558	1.529416	4.002598	10.086519
				B1	0.064656	0.051725	-0.038224	0.167537
				B2	2.282763	0.287593	1.710749	2.854776
			5-th	B0	3.422649	1.622465	0.195618	6.649680
				B1	0.418324	0.167998	0.084181	0.752467
				B2	1.699077	0.138253	1.424096	1.974058
			50-th	B0	-6.744349	1.479942	-9.687906	-3.800793
				B1	3.892515	0.465563	2.966525	4.818505
				B2	1.058035	0.037045	0.984353	1.131716
	690	91	1-st	B0	7.275396	1.089795	5.109647	9.441145
				B1	0.049557	0.030344	-0.010745	0.109859
				B2	2.379522	0.220925	1.940479	2.818565
			5-th	B0	4.090538	1.180774	1.743988	6.437089
				B1	0.332991	0.105473	0.123385	0.542597
				B2	1.779949	0.109858	1.561629	1.998269
			50-th	B0	-5.259637	0.953320	-7.154168	-3.365105
				B1	3.413180	0.281073	2.854603	3.971757
				B2	1.100349	0.025792	1.049094	1.151605
	1000	99	1-st	B0	7.673859	0.744568	6.195894	9.151823
				B1	0.029076	0.013496	0.002287	0.055865
				B2	2.581875	0.168033	2.248331	2.915419
			5-th	B0	5.108397	0.801766	3.516895	6.699899
				B1	0.221265	0.053743	0.114585	0.327946
				B2	1.928540	0.084983	1.759849	2.097230
			50-th	B0	-2.508248	0.746902	-3.990844	-1.025652
				B1	2.576767	0.189282	2.201044	2.952491
				B2	1.192948	0.023448	1.146405	1.239492

Table 10. Continued

Species	Band-Width	Sample Size	PCTL	Parameter	Estimate	Asymptotic Std. Error	Asymptotic 95% Confidence Interval	
							Lower	Upper
S-P-F	1500	107	1-st	B0	7.434986	0.557691	6.329056	8.540916
				B1	0.028694	0.009825	0.009211	0.048177
				B2	2.587710	0.123484	2.342835	2.832585
			5-th	B0	5.413194	0.600958	4.221465	6.604923
				B1	0.178988	0.034295	0.110979	0.246997
				B2	2.007319	0.067197	1.874065	2.140573
			50-th	B0	-1.569846	0.614023	-2.787485	-0.352206
				B1	2.242168	0.142843	1.958904	2.525432
				B2	1.242683	0.020495	1.202040	1.283325
	2000	112	1-st	B0	7.335125	0.490492	6.362979	8.307271
				B1	0.025825	0.007917	0.010134	0.041517
				B2	2.628222	0.110675	2.408867	2.847577
			5-th	B0	5.435238	0.503957	4.436405	6.434071
				B1	0.161641	0.026627	0.108867	0.214415
				B2	2.045633	0.057897	1.930882	2.160383
			50-th	B0	-1.679381	0.453394	-2.577999	-0.780764
				B1	2.217766	0.104328	2.010989	2.424544
				B2	1.248618	0.015147	1.218596	1.278640
	2500	117	1-st	B0	7.619745	0.451479	6.725363	8.514128
				B1	0.016478	0.005056	0.006462	0.026494
				B2	2.798143	0.111301	2.577655	3.018632
			5-th	B0	5.588123	0.448350	4.699938	6.476308
				B1	0.139342	0.021178	0.097388	0.181296
				B2	2.100166	0.053579	1.994026	2.206307
			50-th	B0	-1.683456	0.454811	-2.584440	-0.782472
				B1	2.171703	0.102874	1.967909	2.375497
				B2	1.257636	0.015275	1.227376	1.287896

Table 11. Relationships between MOE and UTS (Band-Width analysis)

Species	Band-Width	Sample Size	PCTL.	Parameter	Estimate	Asymptotic Std. Error	Asymptotic 95% Confidence Interval	
							Lower	Upper
D-FIR	500	101	1-st	B0	4.870822	0.885183	3.114194	6.627449
				B1	0.004279	0.005400	-0.006436	0.014995
				B2	2.876210	0.433478	2.015982	3.736437
			5-th	B0	3.091072	1.027859	1.051307	5.130837
				B1	0.080350	0.048643	-0.016180	0.176881
				B2	1.943165	0.199134	1.547987	2.338343
			50-th	B0	-12.817155	6.154476	-25.030582	-0.603727
				B1	4.682104	2.414845	-0.110105	9.474314
				B2	0.839767	0.140721	0.560508	1.119025
	690	109	1-st	B0	4.585838	0.740336	3.118041	6.053636
				B1	0.005732	0.005625	-0.005420	0.016885
				B2	2.768894	0.334240	2.106226	3.431562
			5-th	B0	3.435984	0.747549	1.953885	4.918083
				B1	0.060579	0.028746	0.003586	0.117572
				B2	2.036539	0.156472	1.726316	2.346763
			50-th	B0	-7.419808	3.652154	-14.660607	-0.179010
				B1	2.847667	1.133232	0.600909	5.094425
				B2	0.977321	0.113385	0.752523	1.202119
	1000	123	1-st	B0	4.891729	0.510018	3.881922	5.901536
				B1	0.002662	0.001912	-0.001125	0.006448
				B2	3.034269	0.242870	2.553400	3.515139
			5-th	B0	4.159771	0.434311	3.299859	5.019682
				B1	0.032058	0.009944	0.012369	0.051746
				B2	2.250622	0.102248	2.048176	2.453067
			50-th	B0	-3.166194	1.862004	-6.852861	0.520473
				B1	1.656225	0.426827	0.811131	2.501319
				B2	1.136985	0.075747	0.987009	1.286961
	1500	137	1-st	B0	4.258485	0.486880	3.295512	5.221457
				B1	0.008516	0.004937	-0.001248	0.018280
				B2	2.614979	0.193496	2.232275	2.997683
			5-th	B0	4.160421	0.282753	3.601180	4.719661
				B1	0.031882	0.006655	0.018721	0.045044
				B2	2.246680	0.068759	2.110685	2.382675
			50-th	B0	1.026110	1.014719	-0.980845	3.033065
				B1	0.786889	0.153050	0.484179	1.089599
				B2	1.368061	0.059702	1.249980	1.486142
	2000	147	1-st	B0	4.324355	0.415126	3.503819	5.144891
				B1	0.006558	0.003251	0.000132	0.012984
				B2	2.698128	0.164192	2.373587	3.022668
			5-th	B0	4.572160	0.242603	4.092632	5.051687
				B1	0.018948	0.003630	0.011773	0.026123
				B2	2.422681	0.062943	2.298269	2.547094
			50-th	B0	2.355579	0.777990	0.817810	3.893348
				B1	0.557161	0.091357	0.376586	0.737736
				B2	1.481541	0.050672	1.381384	1.581698

Table 11. Continued

Species	Band-Width	Sample Size	PCTL.	Parameter	Estimate	Asymptotic Std. Error	Asymptotic 95% Confidence Interval Lower	Upper
D-FIR	2500	156	1-st	B0	4.715891	0.352542	4.019406	5.412375
				B1	0.002497	0.001190	0.000146	0.004849
				B2	3.026342	0.157842	2.714508	3.338175
			5-th	B0	4.976790	0.222741	4.536741	5.416840
				B1	0.010362	0.001972	0.006466	0.014257
				B2	2.628880	0.062488	2.505427	2.752333
			50-th	B0	2.569025	0.696529	1.192956	3.945094
				B1	0.505602	0.074655	0.358112	0.653092
				B2	1.516380	0.045539	1.426413	1.606348
H-FIR	500	91	1-st	B0	6.704616	0.314761	6.079092	7.330139
				B1	0.000015	0.000016	-0.000017	0.000047
				B2	4.989596	0.397311	4.200020	5.779172
			5-th	B0	8.058023	0.265309	7.530775	8.585270
				B1	0.000038	0.000024	-0.000010	0.000086
				B2	4.755550	0.228398	4.301655	5.209445
			50-th	B0	8.658697	1.022116	6.627445	10.689948
				B1	0.042020	0.019239	0.003787	0.080253
				B2	2.428779	0.158193	2.114403	2.743154
	690	101	1-st	B0	6.812487	0.219617	6.376662	7.248312
				B1	0.000002	0.000001	-0.000001	0.000005
				B2	5.737293	0.257871	5.225554	6.249033
			5-th	B0	7.770990	0.202654	7.368827	8.173153
				B1	0.000058	0.000022	0.000014	0.000102
				B2	4.602322	0.134833	4.334749	4.869895
			50-th	B0	7.446784	0.887646	5.685270	9.208298
				B1	0.073958	0.023785	0.026756	0.121159
				B2	2.231843	0.108545	2.016437	2.447249
	1000	106	1-st	B0	6.416685	0.207420	6.005315	6.828056
				B1	0.000006	0.000004	-0.000001	0.000013
				B2	5.325898	0.208005	4.913367	5.738429
			5-th	B0	7.416872	0.178397	7.063063	7.770682
				B1	0.000103	0.000031	0.000042	0.000164
				B2	4.395493	0.105215	4.186823	4.604163
			50-th	B0	6.864771	0.769678	5.338286	8.391256
				B1	0.090226	0.023377	0.043863	0.136588
				B2	2.166317	0.086816	1.994137	2.338497
	1500	112	1-st	B0	5.946716	0.195834	5.558576	6.334856
				B1	0.000027	0.000013	0.000002	0.000052
				B2	4.770560	0.164973	4.443586	5.097534
			5-th	B0	6.963850	0.177353	6.612341	7.315360
				B1	0.000231	0.000060	0.000112	0.000350
				B2	4.103058	0.091313	3.922076	4.284039
			50-th	B0	6.195801	0.603487	4.999700	7.391901
				B1	0.112075	0.021117	0.070221	0.153929
				B2	2.095021	0.062697	1.970756	2.219285

Table 11. Continued

Species	Band-Width	Sample Size	PCTL.	Parameter	Estimate	Asymptotic Std. Error	Asymptotic 95% Confidence Interval	
							Lower	Upper
H-FIR	2000	118	1-st	B0	5.889656	0.158387	5.575921	6.203392
				B1	0.000025	0.000010	0.000006	0.000045
				B2	4.791856	0.138127	4.518252	5.065460
			5-th	B0	6.909466	0.145293	6.621665	7.197266
				B1	0.000212	0.000046	0.000120	0.000303
				B2	4.132754	0.076307	3.981603	4.283906
			50-th	B0	5.984979	0.451950	5.089746	6.880211
				B1	0.116007	0.016198	0.083921	0.148093
				B2	2.086596	0.046457	1.994573	2.178618
	2500	124	1-st	B0	5.763927	0.136301	5.494081	6.033773
				B1	0.000023	0.000007	0.000008	0.000037
				B2	4.827503	0.113866	4.602073	5.052932
			5-th	B0	6.643247	0.145050	6.356080	6.930415
				B1	0.000289	0.000058	0.000175	0.000404
				B2	4.022266	0.069714	3.884247	4.160286
			50-th	B0	5.476390	0.397496	4.689436	6.263344
				B1	0.133550	0.015352	0.103157	0.163943
				B2	2.041717	0.037989	1.966507	2.116926
S-P-F	500	85	1-st	B0	3.070890	1.808084	-0.525980	6.667759
				B1	0.058393	0.093789	-0.128184	0.244971
				B2	2.035668	0.563417	0.914847	3.156489
			5-th	B0	3.283926	1.178172	0.940157	5.627695
				B1	0.050529	0.042302	-0.033624	0.134681
				B2	2.221839	0.297146	1.630718	2.812960
			50-th	B0	1.920224	1.258342	-0.583028	4.423476
				B1	0.345769	0.112724	0.121523	0.570015
				B2	1.751353	0.111717	1.529111	1.973595
	690	90	1-st	B0	3.655284	1.013582	1.640670	5.669898
				B1	0.028510	0.033167	-0.037413	0.094434
				B2	2.282979	0.415683	1.456761	3.109197
			5-th	B0	3.361493	0.779641	1.811866	4.911120
				B1	0.045477	0.026921	-0.008031	0.098986
				B2	2.256109	0.211223	1.836279	2.675940
			50-th	B0	1.971697	0.937985	0.107342	3.836052
				B1	0.354180	0.087871	0.179526	0.528834
				B2	1.741275	0.085298	1.571735	1.910814
	1000	98	1-st	B0	4.206708	0.515611	3.183085	5.230331
				B1	0.008869	0.006784	-0.004600	0.022337
				B2	2.708079	0.275984	2.160178	3.255980
			5-th	B0	3.362588	0.518033	2.334157	4.391019
				B1	0.043980	0.017158	0.009917	0.078043
				B2	2.262104	0.138217	1.987707	2.536500
			50-th	B0	-0.145353	1.149766	-2.427939	2.137233
				B1	0.667303	0.160879	0.347916	0.986690
				B2	1.515688	0.080378	1.356116	1.675260

Table 11. Continued

Species	Band-Width	Sample Size	PCTL.	Parameter	Estimate	Asymptotic Std. Error	Asymptotic 95% Confidence Interval	
							Lower	Upper
S-P-F	1500	106	1-st	B0	4.155866	0.333148	3.495141	4.816590
				B1	0.005471	0.002940	-0.000359	0.011301
				B2	2.895961	0.194627	2.509962	3.281961
			5-th	B0	3.441043	0.330689	2.785195	4.096892
				B1	0.034250	0.009085	0.016232	0.052268
				B2	2.356515	0.094330	2.169432	2.543599
			50-th	B0	0.006193	1.009469	-1.995863	2.008249
				B1	0.672739	0.143737	0.387669	0.957809
				B2	1.510425	0.071228	1.369160	1.651690
	2000	111	1-st	B0	4.128789	0.277944	3.577851	4.679726
				B1	0.003945	0.001862	0.000253	0.007636
				B2	3.023383	0.171550	2.683338	3.363428
			5-th	B0	3.468405	0.267243	2.938681	3.998130
				B1	0.028816	0.006411	0.016108	0.041523
				B2	2.422322	0.079345	2.265046	2.579598
			50-th	B0	0.027042	0.901674	-1.760245	1.814328
				B1	0.655024	0.125726	0.405813	0.904236
				B2	1.521664	0.064073	1.394659	1.648669
	2500	115	1-st	B0	4.056976	0.256635	3.548482	4.565469
				B1	0.003454	0.001550	0.000383	0.006525
				B2	3.075922	0.163594	2.751779	3.400066
			5-th	B0	3.470156	0.271934	2.931349	4.008963
				B1	0.024516	0.005772	0.013079	0.035954
				B2	2.484889	0.084348	2.317764	2.652014
			50-th	B0	0.527670	0.782025	-1.021822	2.077163
				B1	0.558417	0.098344	0.363560	0.753275
				B2	1.580004	0.059294	1.462518	1.697489

Table 12. Relationships between MOE and UCS (Band-Width analysis)

Species	Band-Width	Sample Size	PCTL.	Parameter	Estimate	Asymtotic Std. Error	Asymtotic 95% Confidence Interval	
							Lower	Upper
D-FIR	500	103	1-st	B0	14.319708	0.829260	12.674470	15.964945
				B1	0.004141	0.004211	-0.004213	0.012496
				B2	2.982119	0.352566	2.282634	3.681603
			5-th	B0	14.280493	1.043247	12.210708	16.350278
				B1	0.067876	0.041551	-0.014561	0.150312
				B2	2.045655	0.204246	1.640434	2.450876
			50-th	B0	10.314038	1.330582	7.674184	12.953891
				B1	1.548303	0.330031	0.893527	2.203079
				B2	1.116504	0.063478	0.990564	1.242444
	690	115	1-st	B0	15.501505	0.435396	14.638817	16.364192
				B1	0.000627	0.000426	-0.000217	0.001471
				B2	3.633793	0.234888	3.168390	4.099197
			5-th	B0	15.609150	0.567982	14.483758	16.734542
				B1	0.026629	0.010855	0.005121	0.048136
				B2	2.358114	0.136498	2.087658	2.628569
			50-th	B0	7.309948	1.534782	4.268952	10.350945
				B1	2.457620	0.478160	1.510200	3.405039
				B2	0.977673	0.055473	0.867759	1.087586
	1000	121	1-st	B0	15.843633	0.281890	15.285409	16.401857
				B1	0.000272	0.000136	0.000002	0.000542
				B2	3.919065	0.173051	3.576375	4.261755
			5-th	B0	15.846048	0.382029	15.089520	16.602576
				B1	0.020385	0.006017	0.008470	0.032300
				B2	2.447311	0.099117	2.251031	2.643590
			50-th	B0	7.466283	1.079322	5.328916	9.603650
				B1	2.395523	0.333675	1.734750	3.056296
				B2	0.985765	0.039858	0.906834	1.064696
	1500	129	1-st	B0	15.981458	0.214433	15.557097	16.405819
				B1	0.000177	0.000072	0.000035	0.000319
				B2	4.062718	0.140080	3.785501	4.339935
			5-th	B0	15.909763	0.289500	15.336845	16.482680
				B1	0.017006	0.003984	0.009122	0.024890
				B2	2.509377	0.078748	2.353536	2.665218
			50-th	B0	8.271287	0.719965	6.846486	9.696088
				B1	2.093486	0.207995	1.681867	2.505104
				B2	1.027746	0.028774	0.970802	1.084690
	2000	141	1-st	B0	15.506693	0.214354	15.082847	15.930539
				B1	0.000510	0.000156	0.000201	0.000819
				B2	3.685973	0.103886	3.480558	3.891388
			5-th	B0	15.939766	0.231188	15.482632	16.396899
				B1	0.014869	0.002620	0.009689	0.020049
				B2	2.554945	0.058448	2.439374	2.670515
			50-th	B0	11.371619	0.607346	10.170701	12.572537
				B1	1.203962	0.124719	0.957353	1.450571
				B2	1.198673	0.030782	1.137806	1.259539

Table 12. Continued

Species	Band-Width	Sample Size	PCTL.	Parameter	Estimate	Asymtotic Std. Error	Asymtotic 95% Confidence Interval	
							Lower	Upper
D-FIR	2500	147	1-st	B0	15.540000	0.183461	15.177372	15.902627
				B1	0.000412	0.000113	0.000188	0.000635
				B2	3.755150	0.093038	3.571252	3.939049
			5-th	B0	15.957414	0.196076	15.569852	16.344976
				B1	0.013450	0.002068	0.009362	0.017538
				B2	2.587700	0.051053	2.486789	2.688610
			50-th	B0	11.304761	0.528300	10.260527	12.348995
				B1	1.184385	0.107100	0.972692	1.396077
				B2	1.206616	0.026915	1.153416	1.259817
H-FIR	500	87	1-st	B0	1.739825	6.859341	-11.900783	15.380434
				B1	1.943710	2.261295	-2.553142	6.440563
				B2	0.962590	0.336008	0.294397	1.630782
			5-th	B0	0.748564	4.097333	-7.399467	8.896594
				B1	2.440054	1.397499	-0.339039	5.219147
				B2	0.942342	0.164395	0.615424	1.269259
			50-th	B0	9.139132	1.467774	6.220290	12.057974
				B1	1.154010	0.290275	0.576764	1.731256
				B2	1.253008	0.077799	1.098295	1.407722
	690	96	1-st	B0	5.870451	3.084131	-0.254054	11.994957
				B1	0.793218	0.618814	-0.435629	2.022066
				B2	1.231997	0.238130	0.759116	1.704877
			5-th	B0	3.822761	2.222452	-0.590610	8.236133
				B1	1.455692	0.570398	0.322989	2.588394
				B2	1.096308	0.116259	0.865440	1.327176
			50-th	B0	9.680853	1.304746	7.089873	12.271833
				B1	1.064108	0.240658	0.586208	1.542009
				B2	1.277393	0.069603	1.139174	1.415612
	1000	104	1-st	B0	9.269859	1.282473	6.725763	11.813955
				B1	0.246164	0.126218	-0.004221	0.496549
				B2	1.603547	0.164845	1.276537	1.930557
			5-th	B0	5.137106	1.249322	2.658772	7.615440
				B1	1.103193	0.275752	0.556172	1.650214
				B2	1.180484	0.075448	1.030815	1.330154
			50-th	B0	9.219007	0.966494	7.301731	11.136284
				B1	1.187807	0.190908	0.809096	1.566519
				B2	1.240888	0.049079	1.143528	1.338247
	1500	112	1-st	B0	11.563760	0.699232	10.177896	12.949624
				B1	0.066061	0.027756	0.011049	0.121073
				B2	2.042838	0.139751	1.765853	2.319822
			5-th	B0	8.668061	0.798599	7.085252	10.250870
				B1	0.421313	0.098518	0.226052	0.616575
				B2	1.486325	0.073965	1.339727	1.632923
			50-th	B0	10.275347	0.569551	9.146507	11.404186
				B1	0.947618	0.097679	0.754020	1.141216
				B2	1.314820	0.031814	1.251766	1.377875

Table 12. Continued

Species	Band-Width	Sample Size	PCTL.	Parameter	Estimate	Asymtotic Std. Error	Asymtotic 95% Confidence Interval	
							Lower	Upper
H-FIR	2000	119	1-st	B0	12.093802	0.589258	10.926694	13.260911
				B1	0.037177	0.015161	0.007149	0.067206
				B2	2.244106	0.136998	1.972762	2.515451
			5-th	B0	9.752055	0.694314	8.376870	11.127241
				B1	0.261090	0.062249	0.137797	0.384383
				B2	1.647178	0.076734	1.495196	1.799161
			50-th	B0	10.399676	0.450416	9.507564	11.291788
				B1	0.886779	0.073734	0.740739	1.032819
				B2	1.339489	0.025747	1.288493	1.390485
	2500	124	1-st	B0	11.910396	0.538171	10.844935	12.975856
				B1	0.036814	0.013622	0.009846	0.063782
				B2	2.250211	0.124278	2.004168	2.496253
			5-th	B0	10.061189	0.596300	8.880646	11.241732
				B1	0.213822	0.046466	0.121830	0.305815
				B2	1.715485	0.070372	1.576164	1.854806
			50-th	B0	10.548875	0.432877	9.691875	11.405875
				B1	0.834697	0.067847	0.700376	0.969019
				B2	1.361444	0.025244	1.311467	1.411421
S-P-F	500	88	1-st	B0	6.645319	1.866935	2.933337	10.357302
				B1	0.236926	0.193866	-0.148534	0.622386
				B2	1.685655	0.279864	1.129208	2.242102
			5-th	B0	6.777005	1.535020	3.724961	9.829050
				B1	0.455337	0.233079	-0.008089	0.918762
				B2	1.477281	0.170795	1.137692	1.816870
			50-th	B0	9.423833	0.730945	7.970512	10.877154
				B1	0.759385	0.136117	0.488746	1.030024
				B2	1.361950	0.058798	1.245042	1.478857
	690	88	1-st	B0	6.327167	1.860141	2.628693	10.025642
				B1	0.244505	0.198684	-0.150534	0.639544
				B2	1.677959	0.278666	1.123893	2.232025
			5-th	B0	6.691020	1.426772	3.854204	9.527836
				B1	0.453349	0.218619	0.018673	0.888024
				B2	1.478863	0.161481	1.157794	1.799931
			50-th	B0	10.062145	0.531569	9.005238	11.119052
				B1	0.658968	0.092451	0.475150	0.842786
				B2	1.407212	0.046500	1.314756	1.499667
	1000	95	1-st	B0	4.246146	2.381444	-0.483632	8.975924
				B1	0.647732	0.476919	-0.299475	1.594940
				B2	1.322331	0.239865	0.845937	1.798725
			5-th	B0	6.386674	1.133229	4.135971	8.637377
				B1	0.538696	0.194697	0.152008	0.925384
				B2	1.409824	0.119374	1.172736	1.646912
			50-th	B0	11.555058	0.396663	10.767248	12.342868
				B1	0.413296	0.051300	0.311410	0.515182
				B2	1.567950	0.041863	1.484806	1.651094

Table 12. Continued

Species	Band-Width	Sample Size	PCTL.	Parameter	Estimate	Asymtotic Std. Error	Asymtotic 95% Confidence Interval	
							Lower	Upper
S-P-F	1500	103	1-st	B0	-1.800864	3.493775	-8.732456	5.130729
				B1	2.324027	1.348474	-0.351322	4.999377
				B2	0.922596	0.171717	0.581913	1.263280
			5-th	B0	4.637319	0.954450	2.743706	6.530931
				B1	0.831609	0.208983	0.416990	1.246228
				B2	1.268792	0.080922	1.108244	1.429339
			50-th	B0	11.908701	0.291163	11.331038	12.486363
				B1	0.354444	0.033746	0.287493	0.421395
				B2	1.624480	0.032223	1.560549	1.688410
	2000	109	1-st	B0	-0.780178	2.657068	-6.048108	4.487752
				B1	1.788484	0.905244	-0.006262	3.583230
				B2	1.005102	0.153841	0.700095	1.310108
			5-th	B0	3.841471	0.832707	2.190538	5.492405
				B1	0.934129	0.194218	0.549070	1.319189
				B2	1.235090	0.066634	1.102981	1.367199
			50-th	B0	12.004631	0.246442	11.516032	12.493230
				B1	0.330463	0.027288	0.276361	0.384564
				B2	1.652523	0.028064	1.596882	1.708164
	2500	117	1-st	B0	-3.257293	3.429507	-10.051169	3.536583
				B1	2.700002	1.409464	-0.092155	5.492159
				B2	0.878604	0.151763	0.577960	1.179247
			5-th	B0	2.852842	0.944509	0.981762	4.723921
				B1	1.149292	0.243521	0.666874	1.631709
				B2	1.167869	0.066669	1.035796	1.299941
			50-th	B0	11.421461	0.207555	11.010294	11.832628
				B1	0.390287	0.024871	0.341018	0.439556
				B2	1.598578	0.021411	1.556163	1.640994

Table 13. Analysis of variance table for the test on uniform regression line

	Description of test	Number of parameter	Degress of freedom	Sum of Square Error (SSE)	Mean Square Error (MSE)	F Ratio
A	Individual β_0, β_1, β_2					
	MOR	9	7707	809168.7	104.99	
	UTS	9	7162	464341.33	64.83	
	UCS	9	7119	163655.91	22.99	
B	Common β_0, β_1, β_2					
	MOR	3	7713	814177.4		
	UTS	3	7168	480368.05		
	UCS	3	7125	180365.87		
	Test of common β_0, β_1, β_2 (B-A)			Change in SSE		
	MOR		6	5008.7	834.78	7.95*
	UTS		6	16026.72	2671.12	41.2*
	UCS		6	16709.96	2784.99	121.14*

*significant at $\alpha = 0.05$

Table 14. Strength property relationships (General relationships)

Relationships	Species	Parameter			
		B0	B1	B2	B3
UTS-MOR	D-FIR	-3.4930	0.1702	-18.9570	1.2316
	H-FIR	2.1844	0.0272	-11.2590	1.6760
	S-P-F	1.6290	0.2655	2.2350	1.2193
UCS-MOR	D-FIR	14.8683	0.0340	-18.9570	1.5269
	H-FIR	13.4110	0.0361	-11.2590	1.5693
	S-P-F	11.0820	0.2711	2.2350	1.1412
MOR-UTS	D-FIR	-18.9570	4.2110	-3.4930	0.8119
	H-FIR	-11.2590	8.5855	2.1844	0.5966
	S-P-F	2.2350	2.9669	1.6290	0.8202
UCS-UTS	D-FIR	14.8683	0.3055	-3.4930	1.2398
	H-FIR	13.4110	1.0534	2.1844	0.9363
	S-P-F	11.0820	0.9377	1.6290	0.9359

Table 15. Strength property relatiosnhips based on MOR (Band-Width analysis)

Relations	Species	Band-Width	PCTL.	Parameter			
				B0	B1	B2	B3
UTS-MOR	D-FIR	500	1-st	4.870822	0.036285	2.571055	1.741029
			5-th	3.091072	0.026514	-8.322920	1.668826
			50-th	-12.817155	0.242936	-37.485075	1.139741
		690	1-st	4.585838	0.027628	1.469774	1.813572
			5-th	3.435984	0.022463	-7.438555	1.716060
			50-th	-7.419808	0.204422	-28.467992	1.174039
		1000	1-st	4.891729	0.025364	2.729538	1.865173
			5-th	4.159771	0.013790	-6.181606	1.848426
			50-th	-3.166194	0.062971	-30.079164	1.406224
		1500	1-st	4.258485	0.171094	5.155194	1.330404
			5-th	4.160421	0.040974	-2.374785	1.596540
			50-th	1.026110	0.018918	-28.712142	1.651376
		2000	1-st	4.324355	0.251784	6.266269	1.218820
			5-th	4.572160	0.051962	-0.059217	1.547469
			50-th	2.355579	0.013744	-26.494196	1.723103
		2500	1-st	4.715891	0.516400	8.785936	1.005472
			5-th	4.976790	0.097914	3.250637	1.396226
			50-th	2.569025	0.010442	-27.718570	1.777917
	H-FIR	500	1-st	6.704616	0.011470	5.988787	2.113939
			5-th	8.058023	0.000936	2.745504	2.660019
			50-th	8.658697	0.000216	-17.178204	2.707926
		690	1-st	6.812487	0.000146	1.861866	3.350955
			5-th	7.770990	0.000094	-2.006605	3.210516
			50-th	7.446784	0.000648	-16.114421	2.472854
		1000	1-st	6.416685	0.000114	0.376822	3.392104
			5-th	7.416872	0.000070	-3.830542	3.258970
			50-th	6.864771	0.001224	-14.912589	2.339245
		1500	1-st	5.946716	0.000040	-2.495990	3.635187
			5-th	6.963850	0.000018	-8.111302	3.553190
			50-th	6.195801	0.001395	-16.293184	2.304883
		2000	1-st	5.889656	0.000008	-4.264076	4.072455
			5-th	6.909466	0.000002	-12.317054	4.084801
			50-th	5.984979	0.000456	-22.746187	2.518033
		2500	1-st	5.763927	0.000008	-4.287888	4.058582
			5-th	6.643247	0.000003	-12.544163	3.955671
			50-th	5.476390	0.000519	-24.009176	2.484430

Table 15. Contoinued

Relations	Species	Band-Width	PCTL.	Parameter			
				B0	B1	B2	B3
UTS-MOR	S-P-F	500	1-st	3.070890	0.671439	7.044558	0.891756
			5-th	3.283926	0.157934	3.422649	1.307674
			50-th	1.920224	0.036457	-6.744349	1.655289
		690	1-st	3.655284	0.509281	7.275396	0.959427
			5-th	3.361493	0.183281	4.090538	1.267513
			50-th	1.971697	0.050759	-5.259637	1.582475
		1000	1-st	4.206708	0.362596	7.673859	1.048881
			5-th	3.362588	0.258016	5.108397	1.172962
			50-th	-0.145353	0.200464	-2.508248	1.270539
		1500	1-st	4.155866	0.291058	7.434986	1.119121
			5-th	3.441043	0.258115	5.413194	1.173962
			50-th	0.006193	0.252131	-1.569846	1.215455
		2000	1-st	4.128789	0.264669	7.335125	1.150353
			5-th	3.468405	0.249353	5.435238	1.184143
			50-th	0.027042	0.248140	-1.679381	1.218678
		2500	1-st	4.056976	0.315063	7.619745	1.099273
			5-th	3.470156	0.252445	5.588123	1.183187
			50-th	0.527670	0.210779	-1.683456	1.256328
UCS-MOR	D-FIR	500	1-st	14.319708	0.037991	2.571055	1.805138
			5-th	14.280493	0.021125	-8.322920	1.756847
			50-th	10.314038	0.030301	-37.485075	1.515333
		690	1-st	15.501505	0.004937	1.469774	2.380064
			5-th	15.609150	0.008442	-7.438555	1.987030
			50-th	7.309948	0.176255	-28.467992	1.174462
		1000	1-st	15.843633	0.005004	2.729538	2.409059
			5-th	15.846048	0.008146	-6.181606	2.009966
			50-th	7.466283	0.140695	-30.079164	1.219195
		1500	1-st	15.981458	0.018726	5.155194	2.066959
			5-th	15.909763	0.022506	-2.374785	1.783219
			50-th	8.271287	0.127226	-28.712142	1.240584
		2000	1-st	15.506693	0.074452	6.266269	1.665058
			5-th	15.939766	0.043085	-0.059217	1.631951
			50-th	11.371619	0.060219	-26.494196	1.394114
		2500	1-st	15.540000	0.307327	8.785936	1.247611
			5-th	15.957414	0.122705	3.250637	1.374355
			50-th	11.304761	0.054037	-27.718570	1.414726

Table 15. Continued

Relations	Species	Band-Width	PCTL.	Parameter			
				B0	B1	B2	B3
UCS-MOR	H-FIR	500	1-st	1.739825	7.023133	5.988787	0.407820
			5-th	0.748564	4.605004	2.745504	0.527099
			50-th	9.139132	0.076084	-17.178204	1.397021
		690	1-st	5.870451	2.005990	1.861866	0.719567
			5-th	3.822761	1.633224	-2.006605	0.764769
			50-th	9.680853	0.070727	-16.114421	1.415336
		1000	1-st	9.269859	0.597668	0.376822	1.021311
			5-th	5.137106	0.996034	-3.830542	0.875252
			50-th	9.219007	0.101138	-14.912589	1.339942
		1500	1-st	11.563760	0.078076	-2.495990	1.556651
			5-th	8.668061	0.166708	-8.111302	1.287136
			50-th	10.275347	0.060409	-16.293184	1.446529
		2000	1-st	12.093802	0.021392	-4.264076	1.907199
			5-th	9.752055	0.038214	-12.317054	1.628066
			50-th	10.399676	0.025333	-22.746187	1.616450
		2500	1-st	11.910396	0.023061	-4.287888	1.891799
			5-th	10.061189	0.029841	-12.544163	1.687083
			50-th	10.548875	0.020603	-24.009176	1.656651
	S-P-F	500	1-st	6.645319	1.790151	7.044558	0.738428
			5-th	6.777005	0.971432	3.422649	0.869461
			50-th	9.423833	0.132036	-6.744349	1.287245
		690	1-st	6.327167	2.034498	7.275396	0.705166
			5-th	6.691020	1.130361	4.090538	0.830846
			50-th	10.062145	0.137094	-5.259637	1.278877
		1000	1-st	4.246146	3.965616	7.673859	0.512159
			5-th	6.386674	1.622681	5.108397	0.731032
			50-th	11.555058	0.119115	-2.508248	1.314349
		1500	1-st	-1.800864	8.243011	7.434986	0.356530
			5-th	4.637319	2.467156	5.413194	0.632083
			50-th	11.908701	0.123351	-1.569846	1.307236
		2000	1-st	-0.780178	7.240354	7.335125	0.382426
			5-th	3.841471	2.807108	5.435238	0.603769
			50-th	12.004631	0.115162	-1.679381	1.323482
		2500	1-st	-3.257293	9.800470	7.619745	0.313995
			5-th	2.852842	3.438682	5.588123	0.556084
			50-th	11.421461	0.145639	-1.683456	1.271098

Table 16. Strength property relationships based on UTS (Band-Width analysis)

Relations	Species	Band-Width	PCTL.	Parameter			
				B0	B1	B2	B3
MOR-UTS	D-FIR	500	1-st	2.571055	6.718249	4.870822	0.574373
			5-th	-8.322920	8.804249	3.091072	0.599224
			50-th	-37.485075	3.460701	-12.817155	0.877392
		690	1-st	1.469774	7.234936	4.585838	0.551398
			5-th	-7.438555	9.133810	3.435984	0.582730
			50-th	-28.467992	3.866029	-7.419808	0.851760
		1000	1-st	2.729538	7.170769	4.891729	0.536143
			5-th	-6.181606	10.150768	4.159771	0.541001
			50-th	-30.079164	7.144313	-3.166194	0.711124
		1500	1-st	5.155194	3.769988	4.258485	0.751652
			5-th	-2.374785	7.397123	4.160421	0.626354
			50-th	-28.712142	11.052215	1.026110	0.605556
		2000	1-st	6.266269	3.100528	4.324355	0.820466
			5-th	-0.059217	6.759955	4.572160	0.646217
			50-th	-26.494196	12.037721	2.355579	0.580348
		2500	1-st	8.785936	1.929532	4.715891	0.994558
			5-th	3.250637	5.281693	4.976790	0.716217
			50-th	-27.718570	13.011857	2.569025	0.562456
	H-FIR	500	1-st	5.988787	8.278111	6.704616	0.473051
			5-th	2.745504	13.759848	8.058023	0.375937
			50-th	-17.178204	22.576131	8.658697	0.369286
		690	1-st	1.861866	13.955459	6.812487	0.298422
			5-th	-2.006605	17.965825	7.770990	0.311476
			50-th	-16.114421	19.465324	7.446784	0.404391
		1000	1-st	0.376822	14.535501	6.416685	0.294802
			5-th	-3.830542	18.804660	7.416872	0.306845
			50-th	-14.912589	17.579298	6.864771	0.427488
		1500	1-st	-2.495990	16.197808	5.946716	0.275089
			5-th	-8.111302	21.700072	6.963850	0.281437
			50-th	-16.293184	17.332725	6.195801	0.433861
		2000	1-st	-4.264076	18.017616	5.889656	0.245552
			5-th	-12.317054	25.828325	6.909466	0.244810
			50-th	-22.746187	21.222006	5.984979	0.397135
		2500	1-st	-4.287888	17.857618	5.763927	0.246391
			5-th	-12.544163	25.209186	6.643247	0.252802
			50-th	-24.009176	21.005026	5.476390	0.402507

Table 16. Continued

Relations	Species	Band-Width	PCTL.	Parameter			
				B0	B1	B2	B3
MOR-UTS	S-P-F	500	1-st	7.044558	1.563119	3.070890	1.121382
			5-th	3.422649	4.101453	3.283926	0.764716
			50-th	-6.744349	7.393681	1.920224	0.604124
		690	1-st	7.275396	2.020387	3.655284	1.042288
			5-th	4.090538	3.813819	3.361493	0.788946
			50-th	-5.259637	6.576796	1.971697	0.631922
		1000	1-st	7.673859	2.630537	4.206708	0.953397
			5-th	5.108397	3.173924	3.362588	0.852542
			50-th	-2.508248	3.542785	-0.145353	0.787067
		1500	1-st	7.434986	3.012769	4.155866	0.893558
			5-th	5.413194	3.169768	3.441043	0.851817
			50-th	-1.569846	3.106741	0.006193	0.822737
		2000	1-st	7.335125	3.175732	4.128789	0.869298
			5-th	5.435238	3.231359	3.468405	0.844493
			50-th	-1.679381	3.138252	0.027042	0.820561
		2500	1-st	7.619745	2.859588	4.056976	0.909692
			5-th	5.588123	3.200915	3.470156	0.845175
			50-th	-1.683456	3.453136	0.527670	0.795970
UCS-UTS	D-FIR	500	1-st	14.319708	1.183016	4.870822	1.036822
			5-th	14.280493	0.964902	3.091072	1.052744
			50-th	10.314038	0.198828	-12.817155	1.329541
		690	1-st	15.501505	0.548281	4.585838	1.312363
			5-th	15.609150	0.684377	3.435984	1.157902
			50-th	7.309948	0.862704	-7.419808	1.000360
		1000	1-st	15.843633	0.575961	4.891729	1.291601
			5-th	15.846048	0.858909	4.159771	1.087393
			50-th	7.466283	1.546764	-3.166194	0.866999
		1500	1-st	15.981458	0.290889	4.258485	1.553633
			5-th	15.909763	0.798036	4.160421	1.116927
			50-th	8.271287	2.506480	1.026110	0.751243
		2000	1-st	15.506693	0.489943	4.324355	1.366123
			5-th	15.939766	0.974428	4.572160	1.054594
			50-th	11.371619	1.932558	2.355579	0.809072
		2500	1-st	15.540000	0.697804	4.715891	1.240822
			5-th	15.957414	1.208398	4.976790	0.984335
			50-th	11.304761	2.037880	2.569025	0.795721

Table 16. Continued

Relations	Species	Band-Width	PCTL.	Parameter			
				B0	B1	B2	B3
UCS-UTS	H-FIR	500	1-st	1.739825	16.629585	6.704616	0.192919
			5-th	0.748564	18.339703	8.058023	0.198156
			50-th	9.139132	5.920668	8.658697	0.515901
		690	1-st	5.870451	13.367453	6.812487	0.214735
			5-th	3.822761	14.873319	7.770990	0.238208
			50-th	9.680853	4.724127	7.446784	0.572349
		1000	1-st	9.269859	9.197338	6.416685	0.301085
			5-th	5.137106	12.989039	7.416872	0.268567
			50-th	9.219007	4.711330	6.864771	0.572810
		1500	1-st	11.563760	5.959610	5.946716	0.428218
			5-th	8.668061	8.753135	6.963850	0.362248
			50-th	10.275347	3.742451	6.195801	0.627593
		2000	1-st	12.093802	5.310379	5.889656	0.468317
			5-th	9.752055	7.606896	6.909466	0.398567
			50-th	10.399676	3.534836	5.984979	0.641949
		2500	1-st	11.910396	5.383614	5.763927	0.466123
			5-th	10.061189	6.908103	6.643247	0.426497
			50-th	10.548875	3.195661	5.476390	0.666813
UCS-UTS	S-P-F	500	1-st	6.645319	2.489654	3.070890	0.828060
			5-th	6.777005	3.313889	3.283926	0.664891
			50-th	9.423833	1.734314	1.920224	0.777656
		690	1-st	6.327167	3.340716	3.655284	0.734987
			5-th	6.691020	3.437461	3.361493	0.655493
			50-th	10.062145	1.524612	1.971697	0.808150
		1000	1-st	4.246146	6.507892	4.206708	0.488291
			5-th	6.386674	3.774994	3.362588	0.623236
			50-th	11.555058	0.628052	-0.145353	1.034481
		1500	1-st	-1.800864	12.213822	4.155866	0.318580
			5-th	4.637319	5.115492	3.441043	0.538419
			50-th	11.908701	0.542876	0.006193	1.075511
		2000	1-st	-0.780178	11.263624	4.128789	0.332443
			5-th	3.841471	5.699152	3.468405	0.509879
			50-th	12.004631	0.523199	0.027042	1.085998
		2500	1-st	-3.257293	13.630885	4.056976	0.285639
			5-th	2.852842	6.566996	3.470156	0.469988
			50-th	11.421461	0.703720	0.527670	1.011756

Table 17. Summary of the property relationships for the property estimates

Given property	Predicted property		
MOE	MOR	UTS	UCS
MOR	-	UTS	UCS
UTS	MOR	-	UCS

APPENDIX B: (Figure 1 to Figure 39)

Figure 1. Histograms of MOE and strength properties

D-FIR

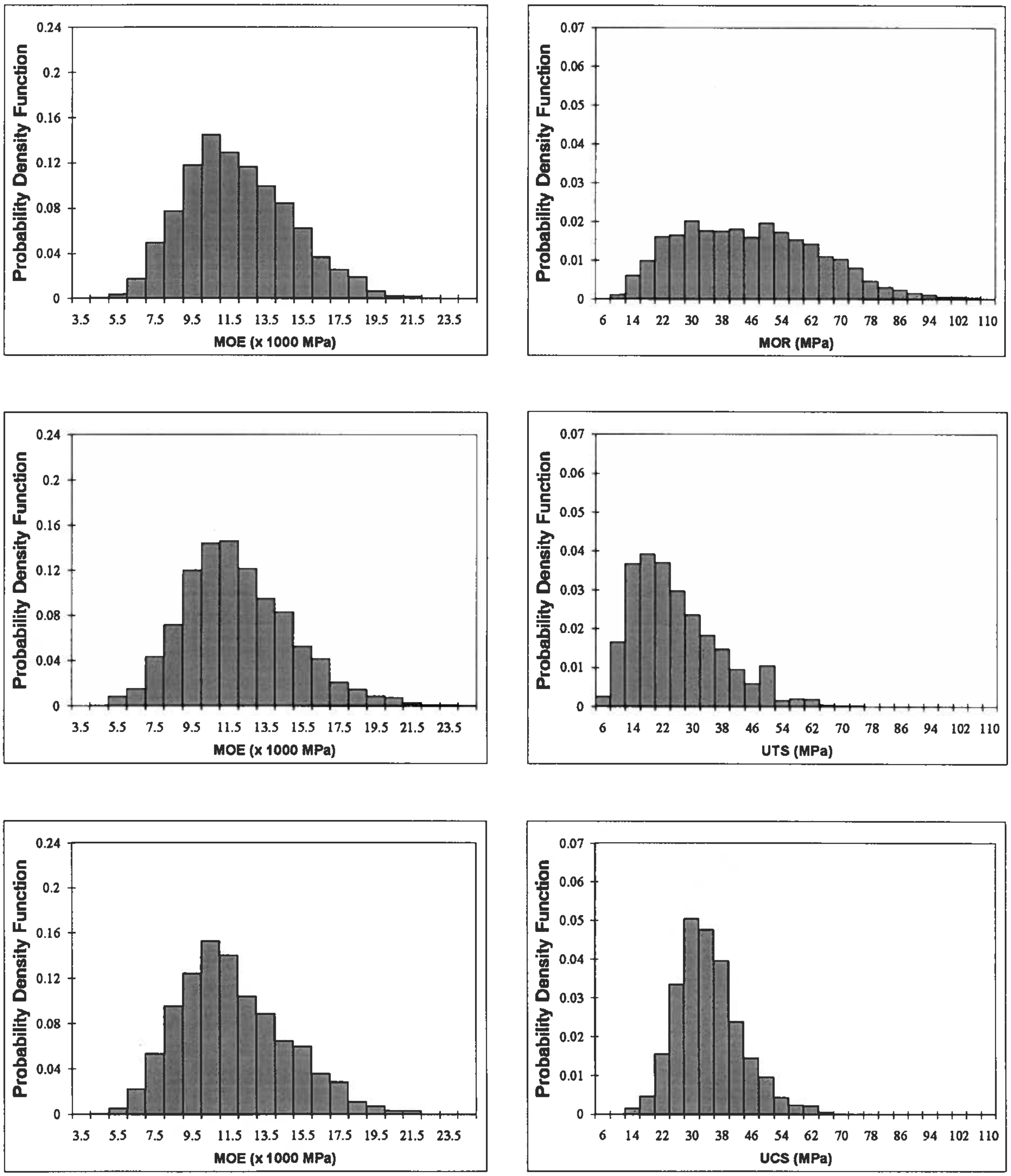


Figure 1. Continued

H-FIR

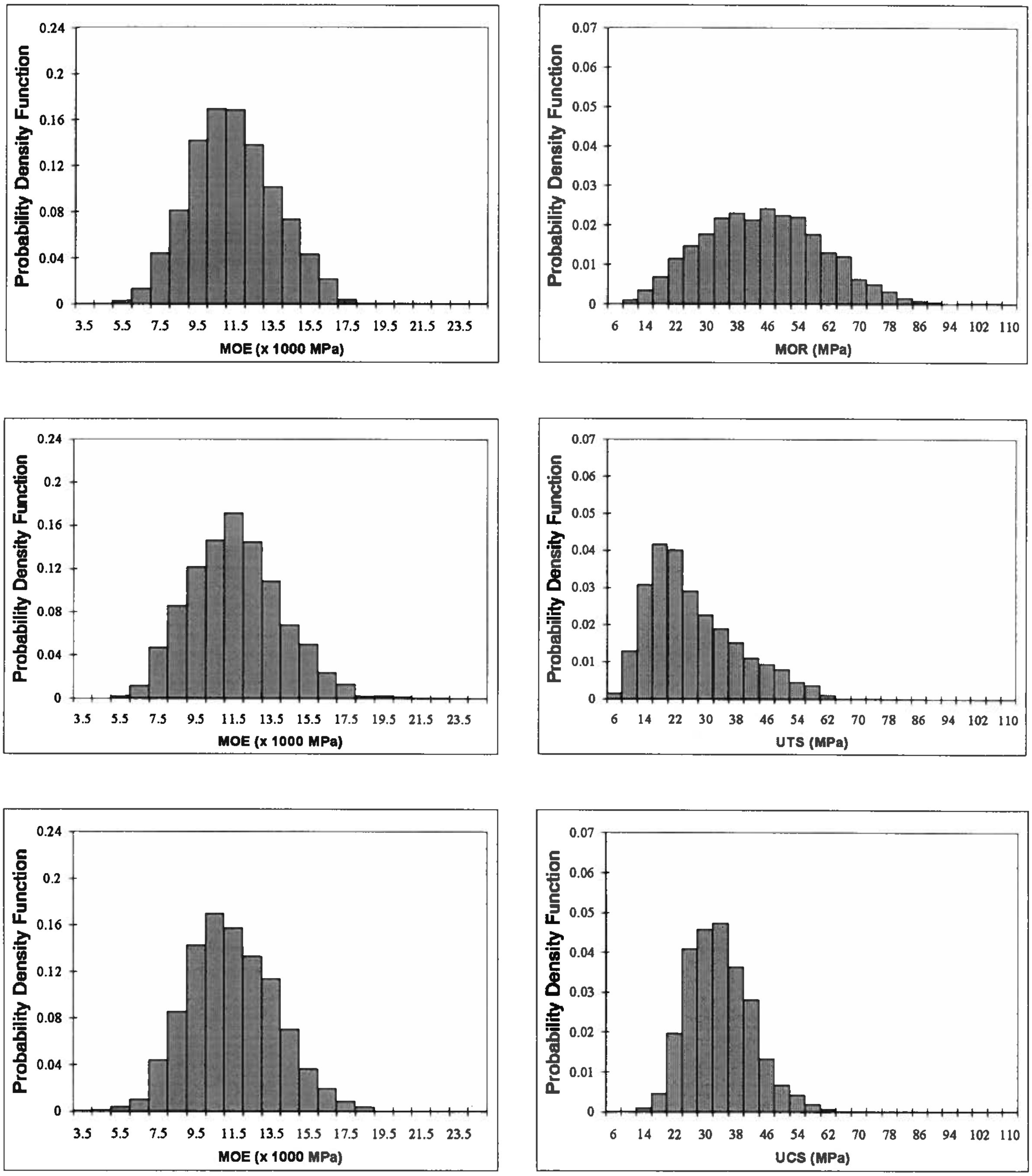


Figure 1. Continued

S-P-F

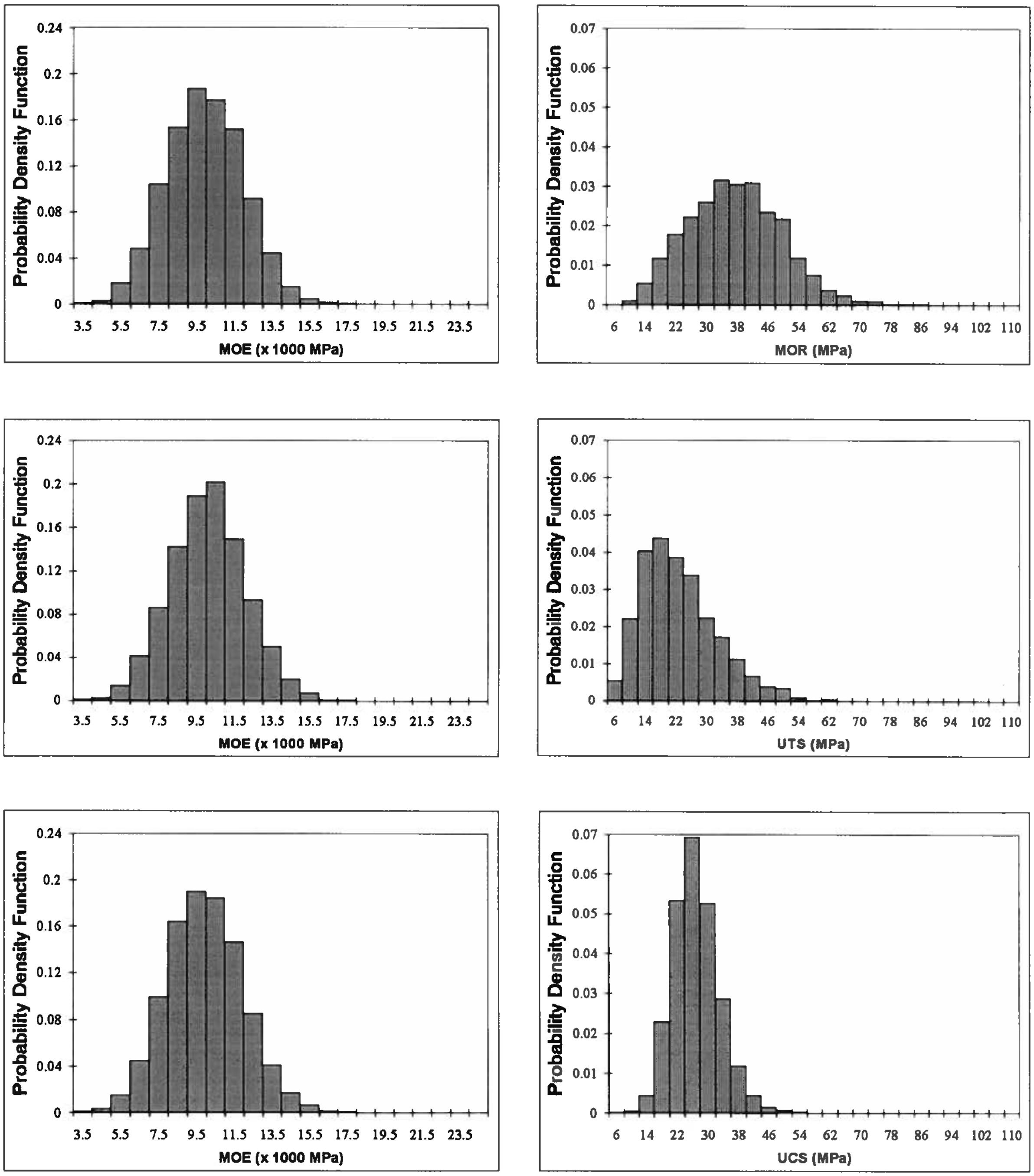


Figure 2. Graphical presentation of size adjustment on strength properties

Unadjusted D-FIR Adjusted

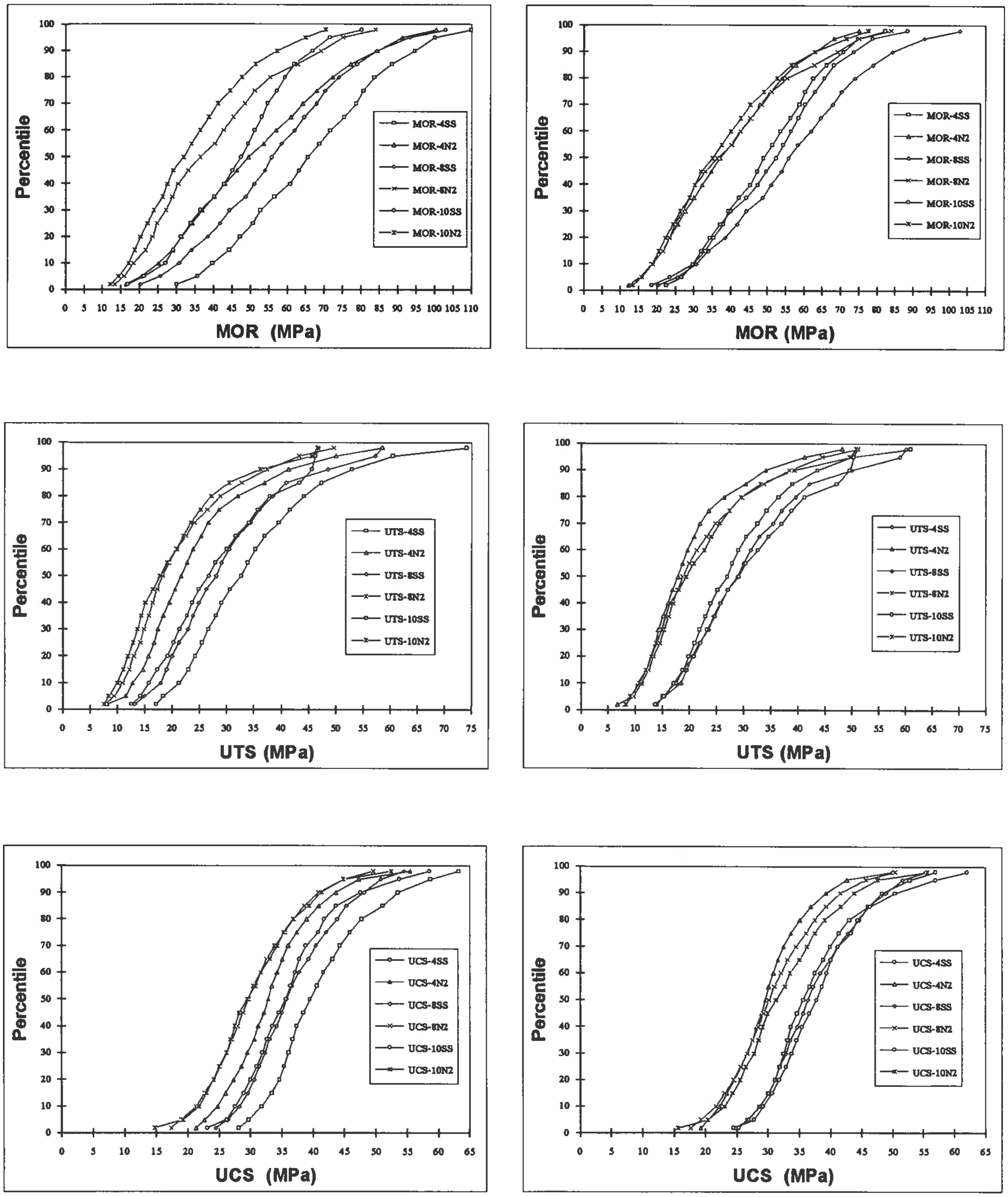
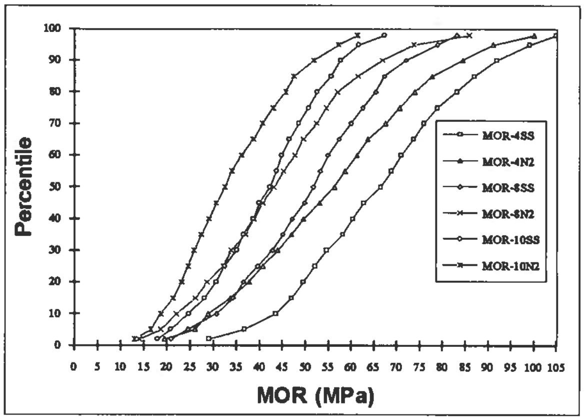
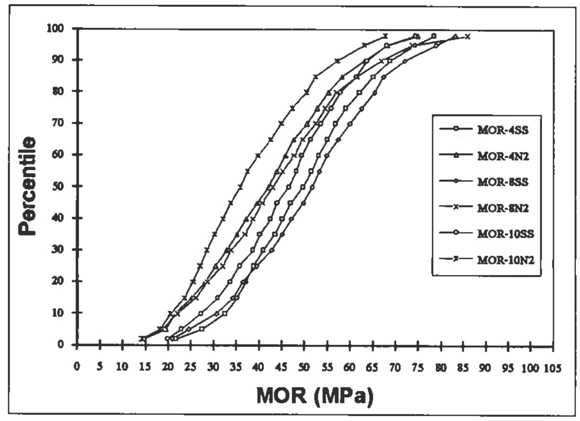
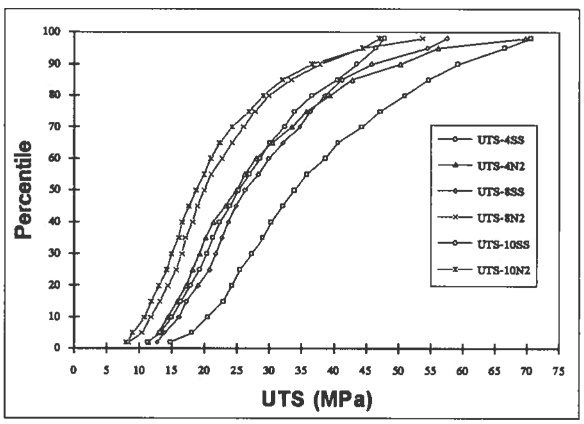


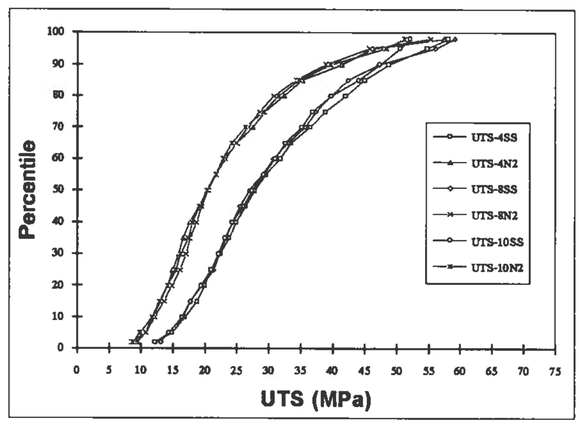
Figure 2. Continued

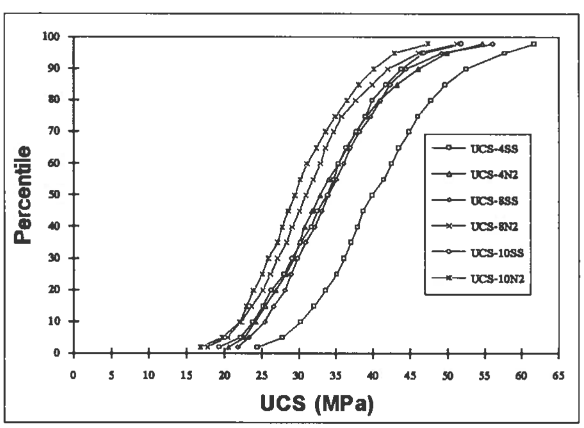
Unadjusted H-FIR Adjusted











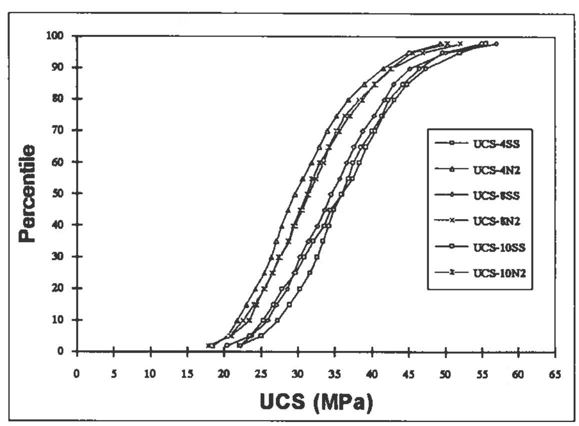
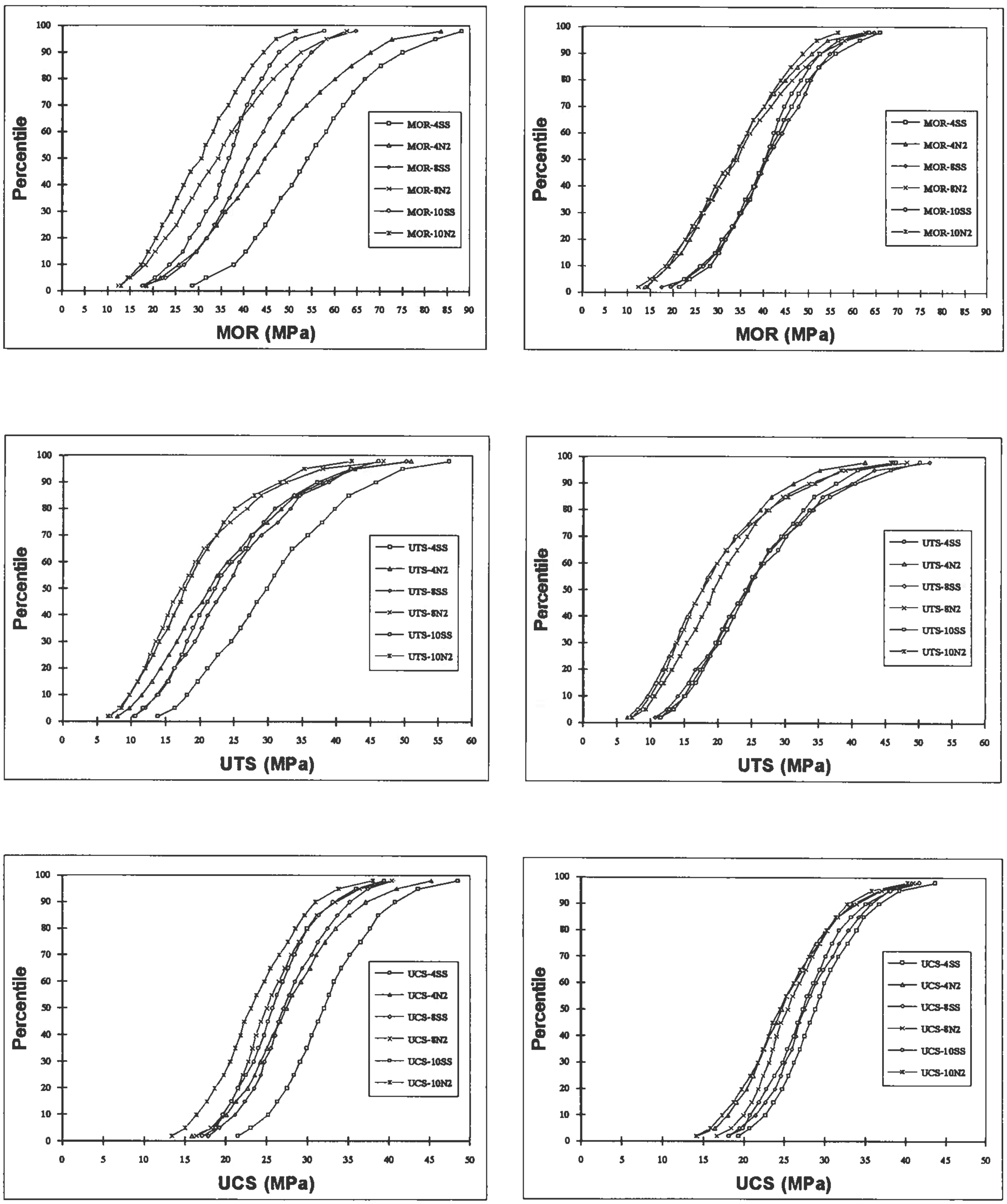


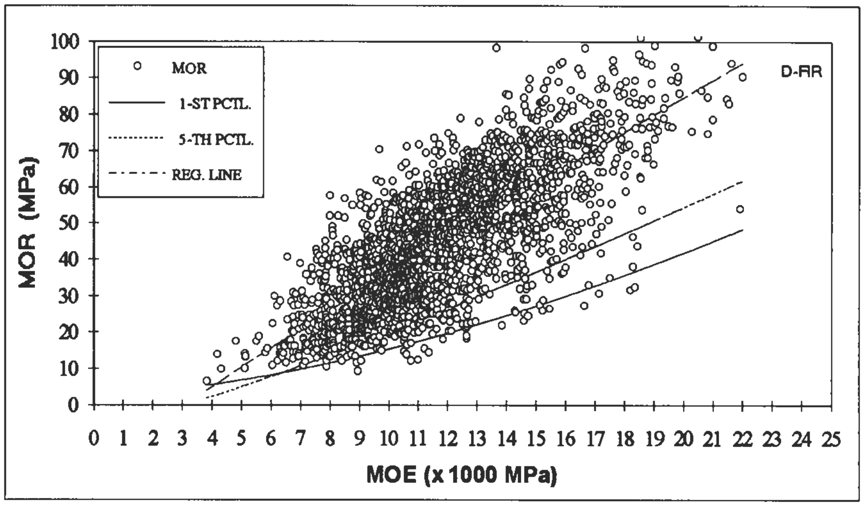
Figure 2. Continued

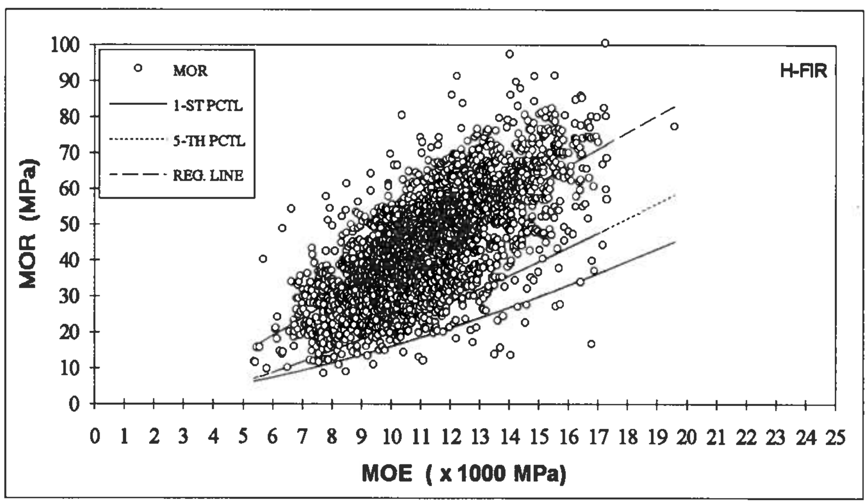
Unadjusted S-P-F Adjusted



4SS: 2 x 4 Select Structural
4N2: 2 x 4 No. 2

Figure 3. Relationships between MOE and MOR





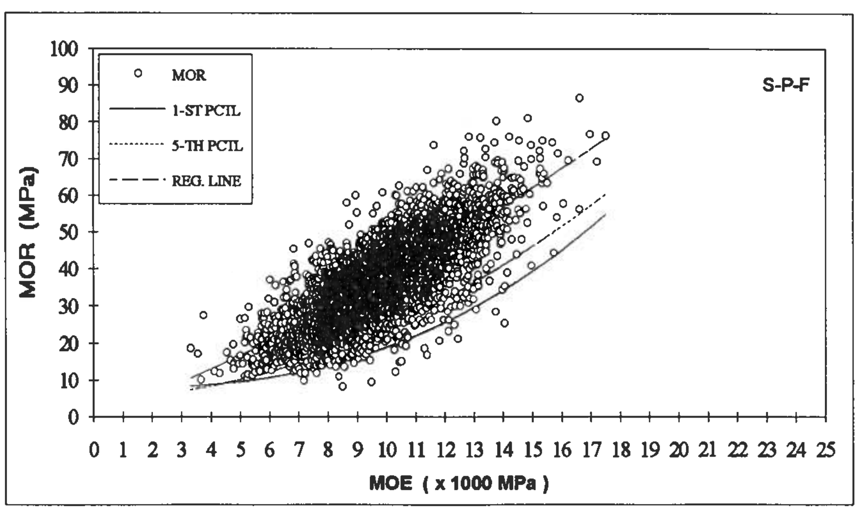
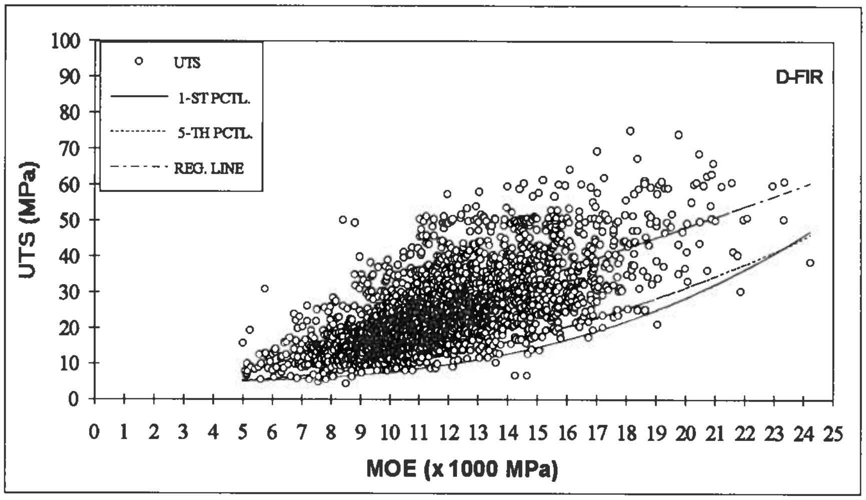
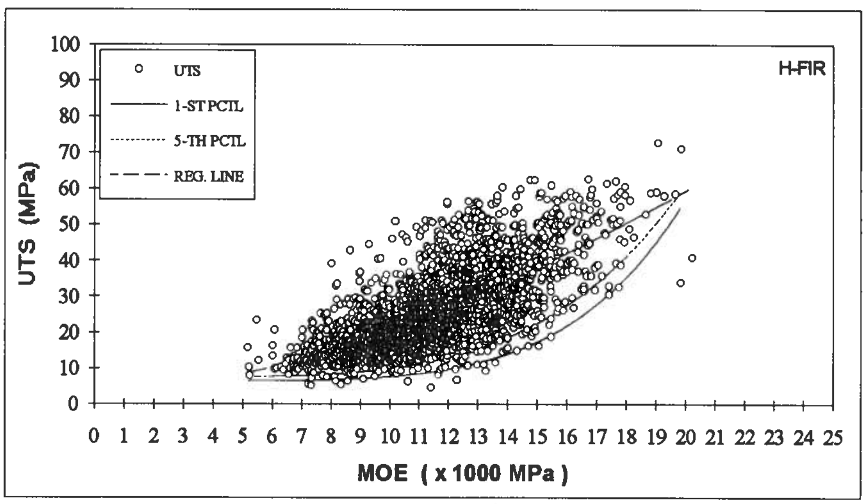


Figure 4. Relationships between MOE and UTS





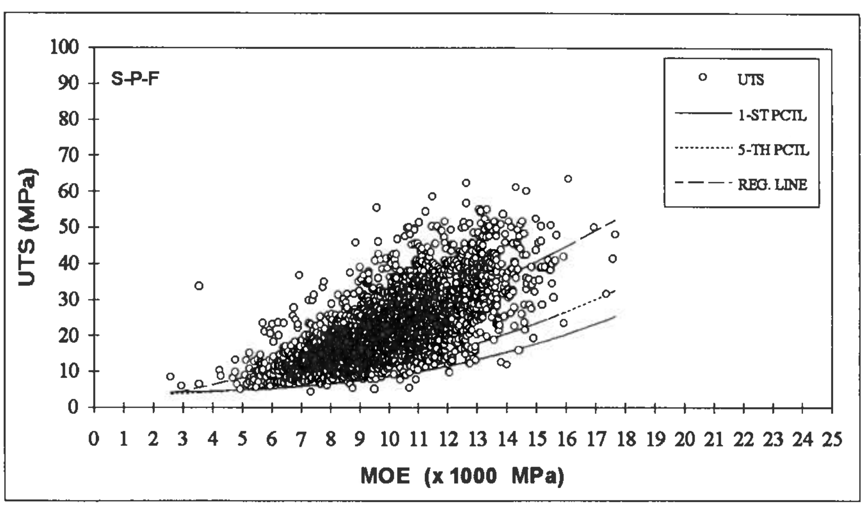
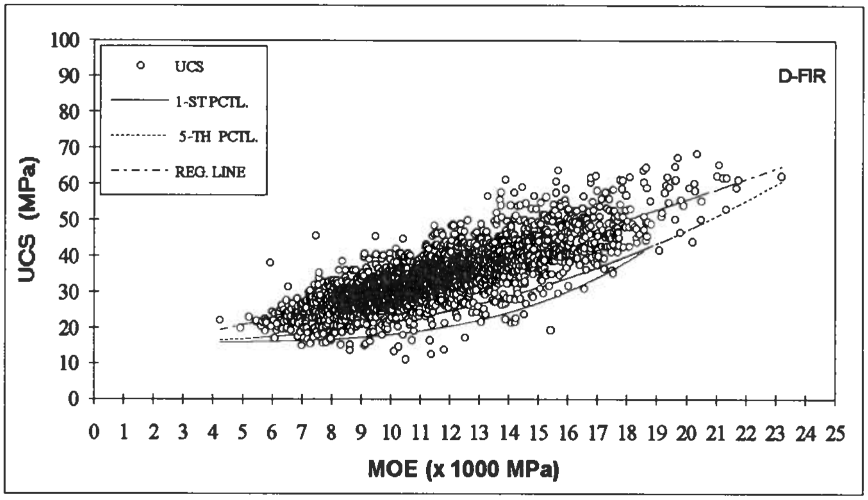
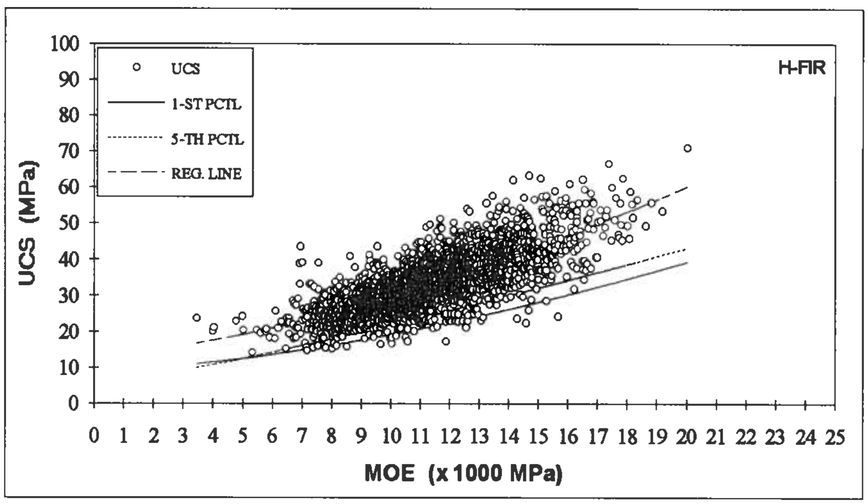
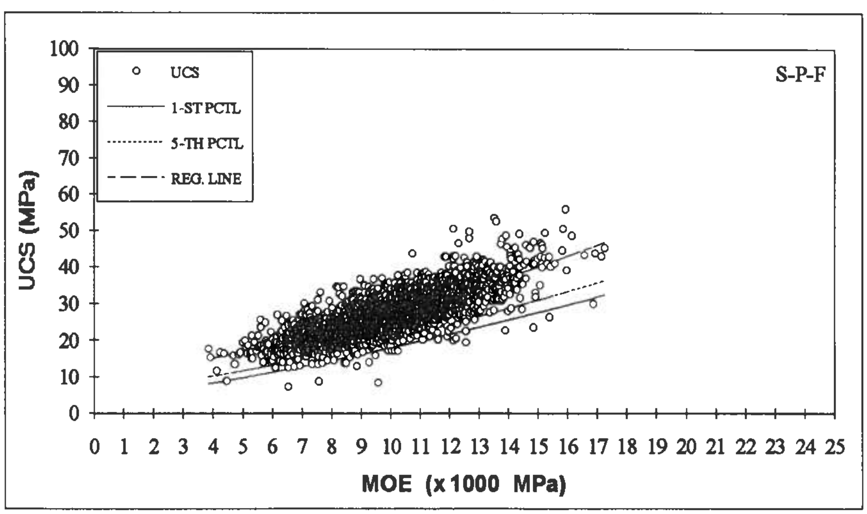


Figure 5. Relationships between MOE and UCS







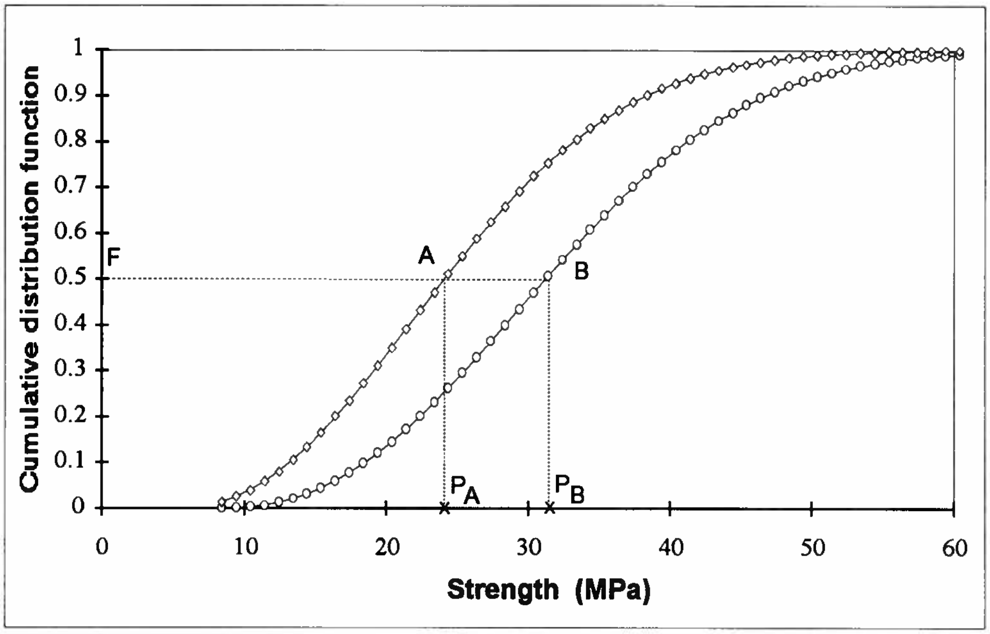


Figure 6a. Two strength properties at equivalent rank of percentile level

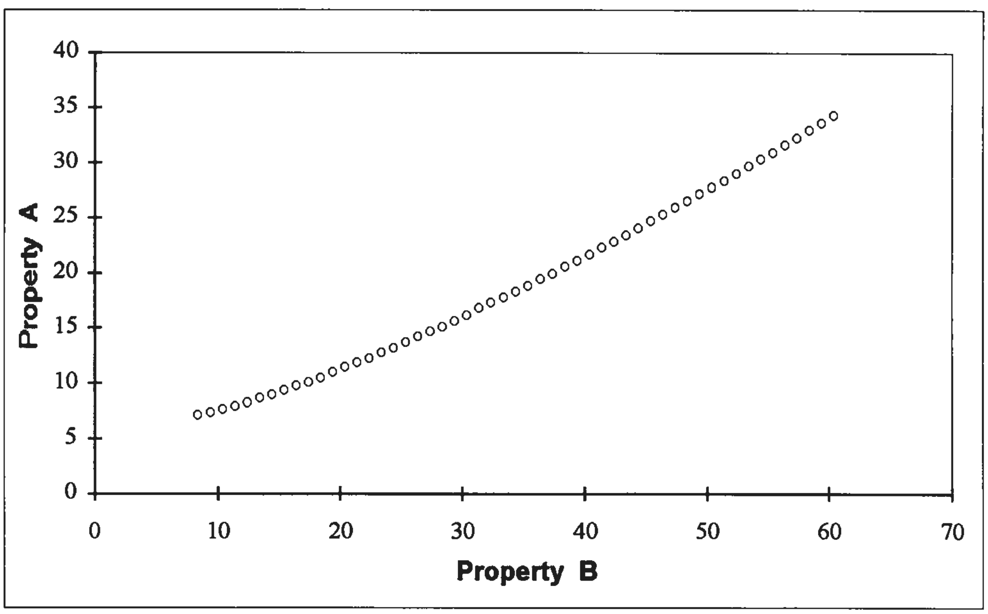


Figure 6b. Relationship between two strength properties at percentiles of equivalent rank

Figure 7. UTS and UTS/MOR as a function of MOR

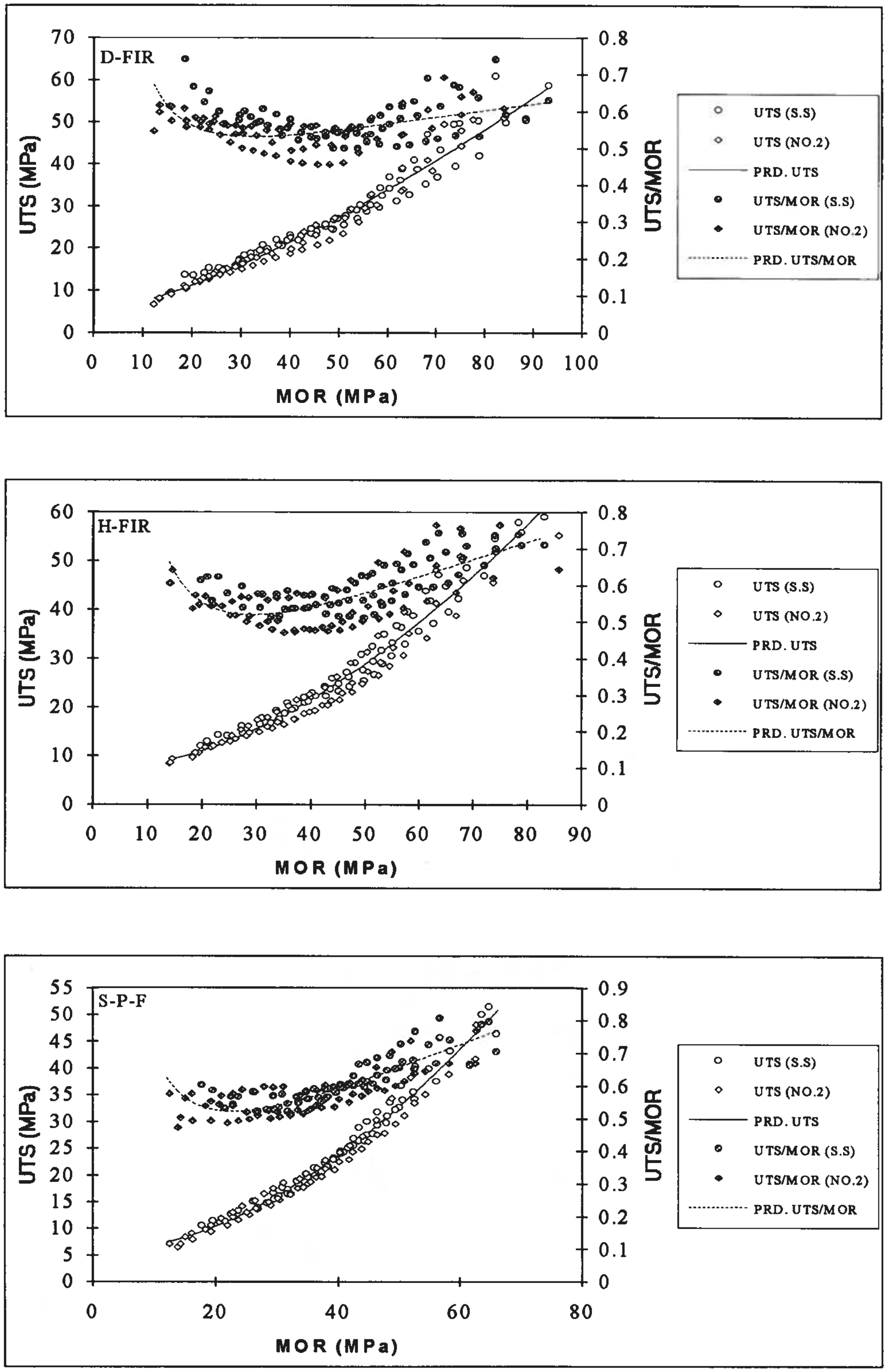


Figure 8. UCS and UCS/MOR as a function of MOR

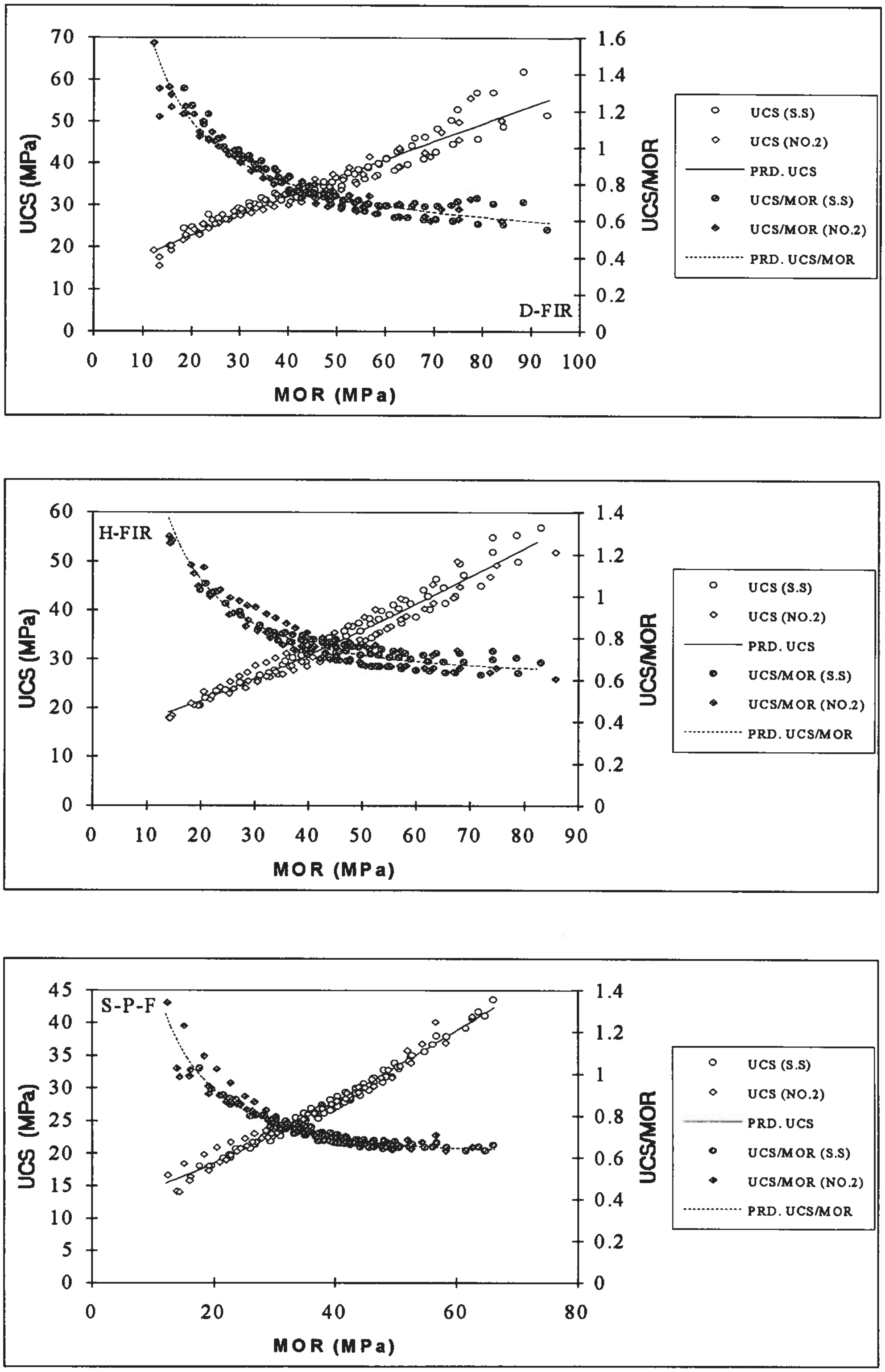
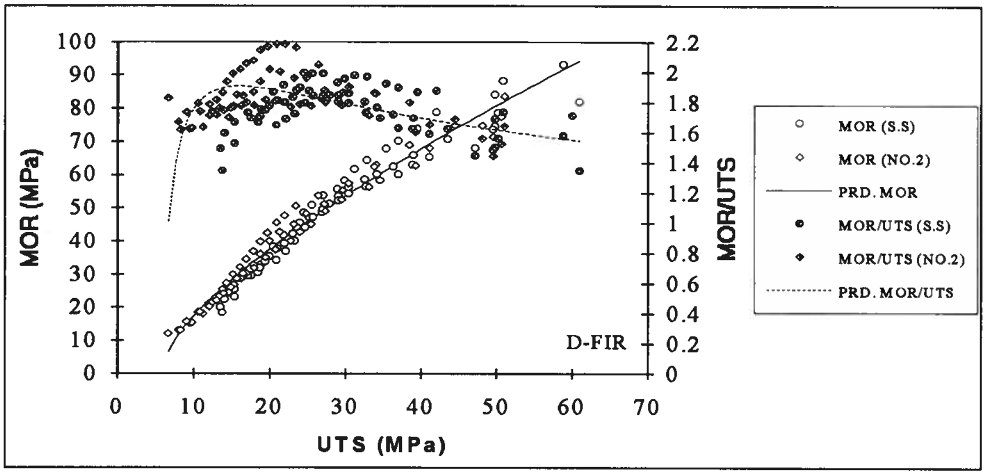
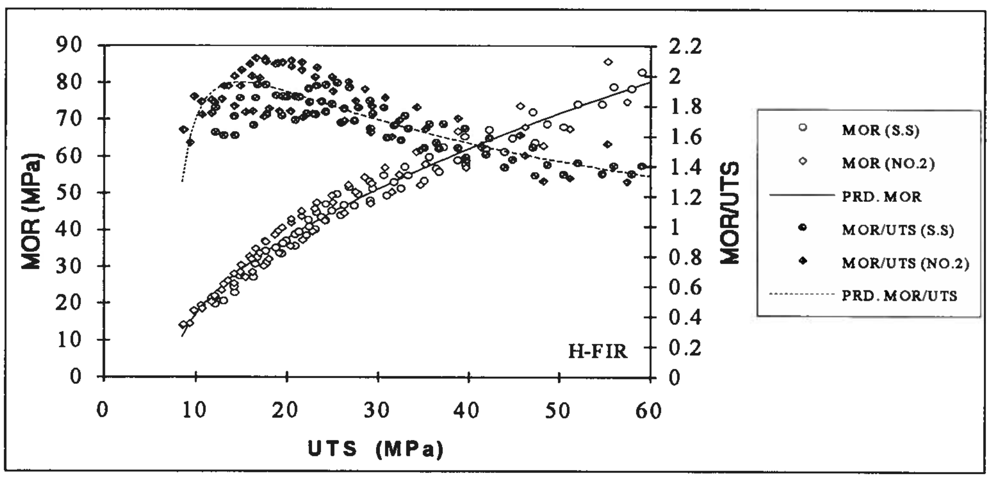


Figure 9. MOR and MOR/UTS as a function of UTS





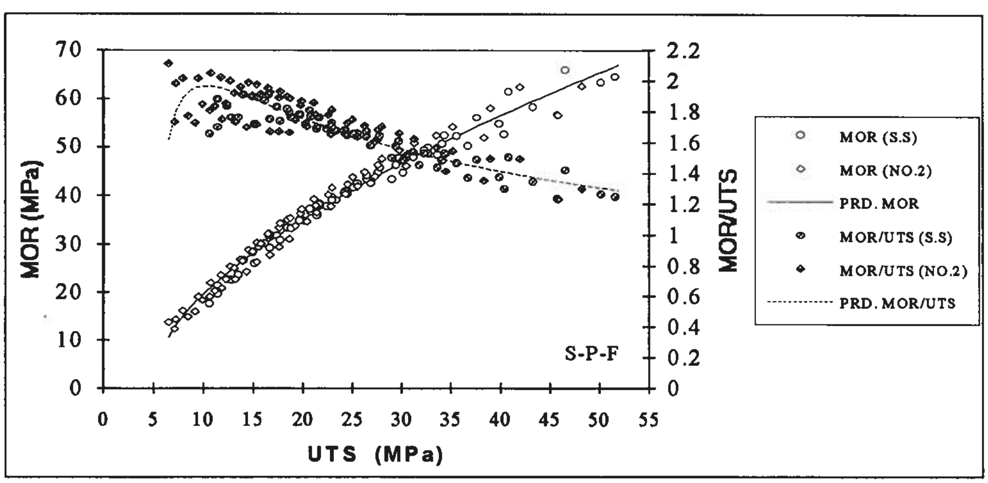
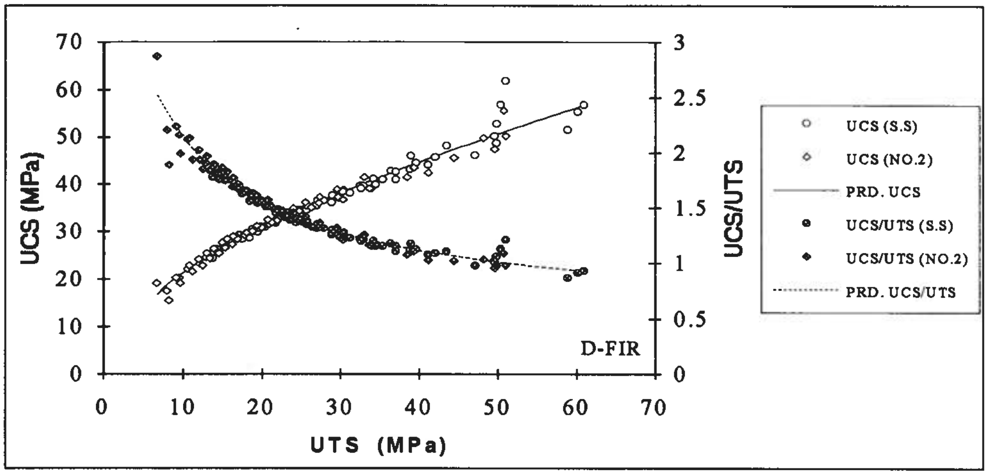
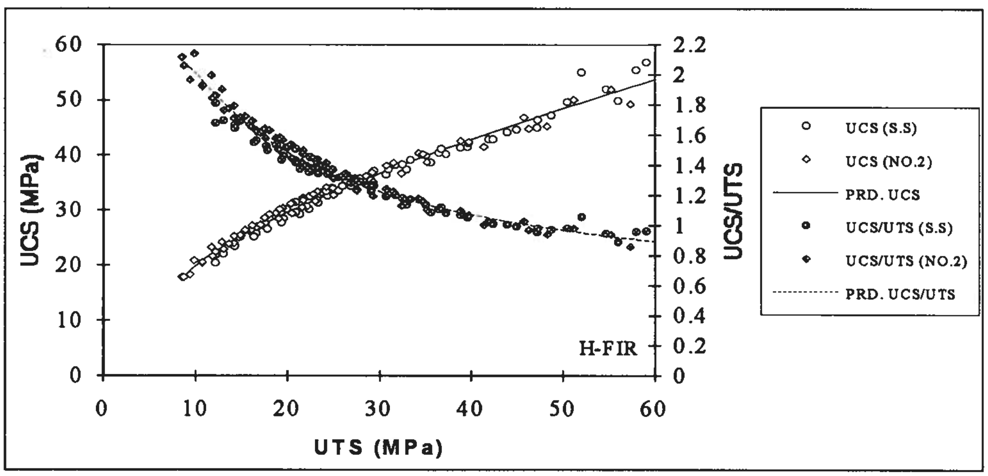
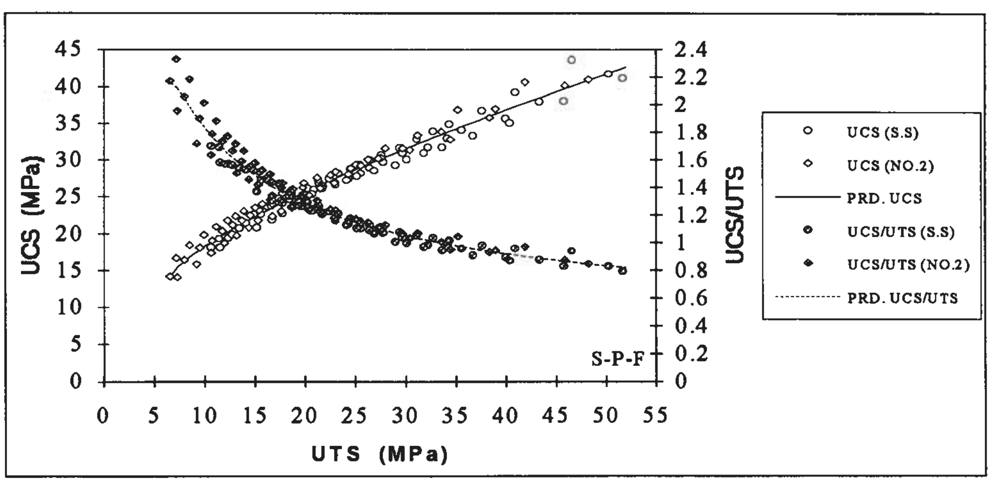


Figure 10. UCS and UCS/UTS as a function of UTS







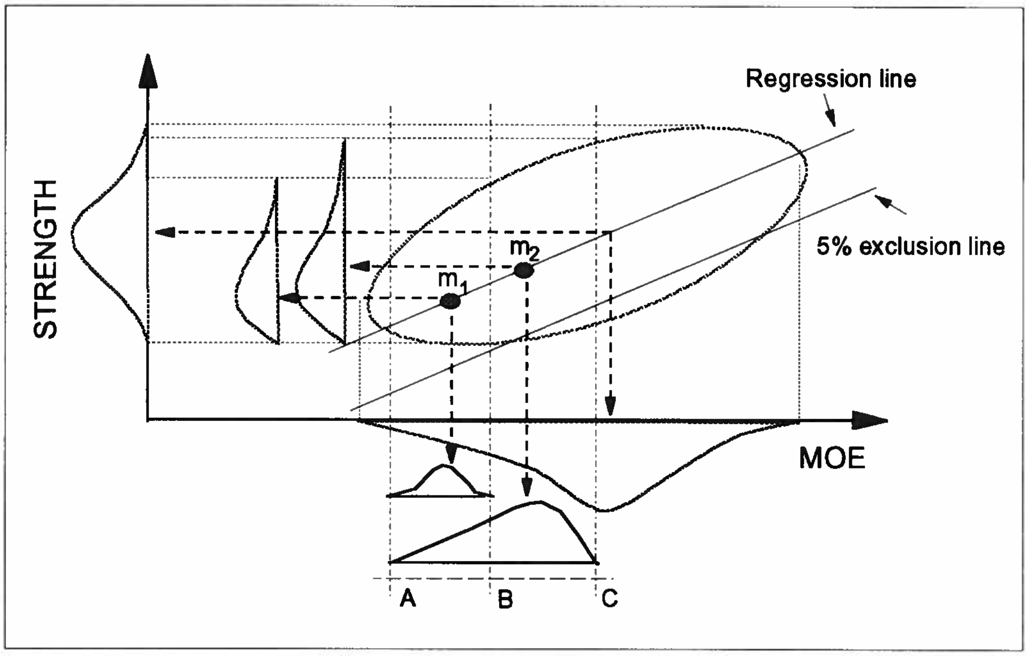


Figure 11a. Illustration of the band-width effects

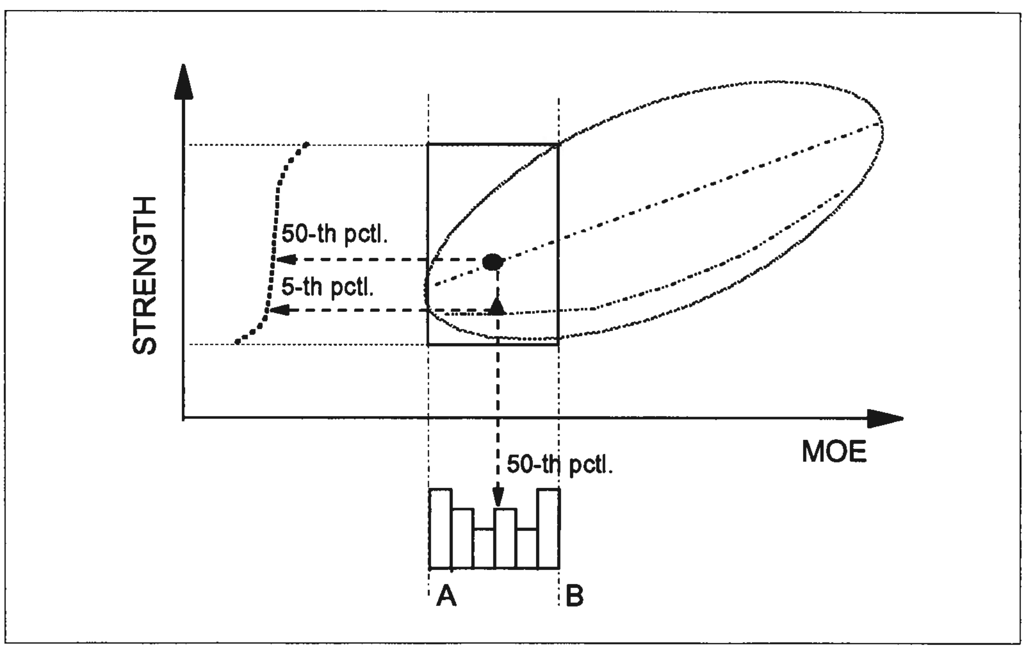


Figure 11b. Moving band-width (window) method

Figure 12. Johnson's Sb fit on MOE data (D-fir, Band-Width = 1,000 MPa)

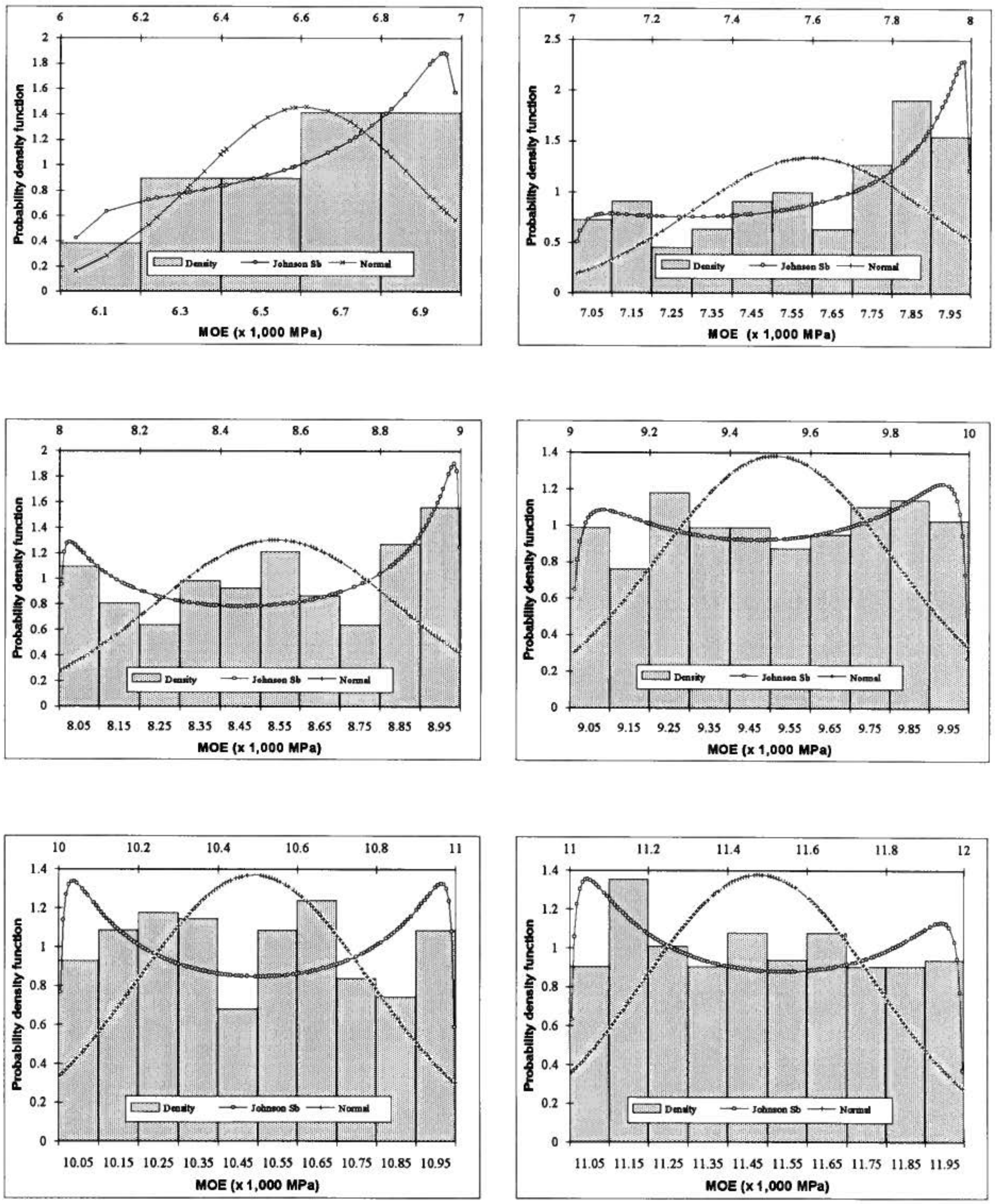


Figure 12. Continued

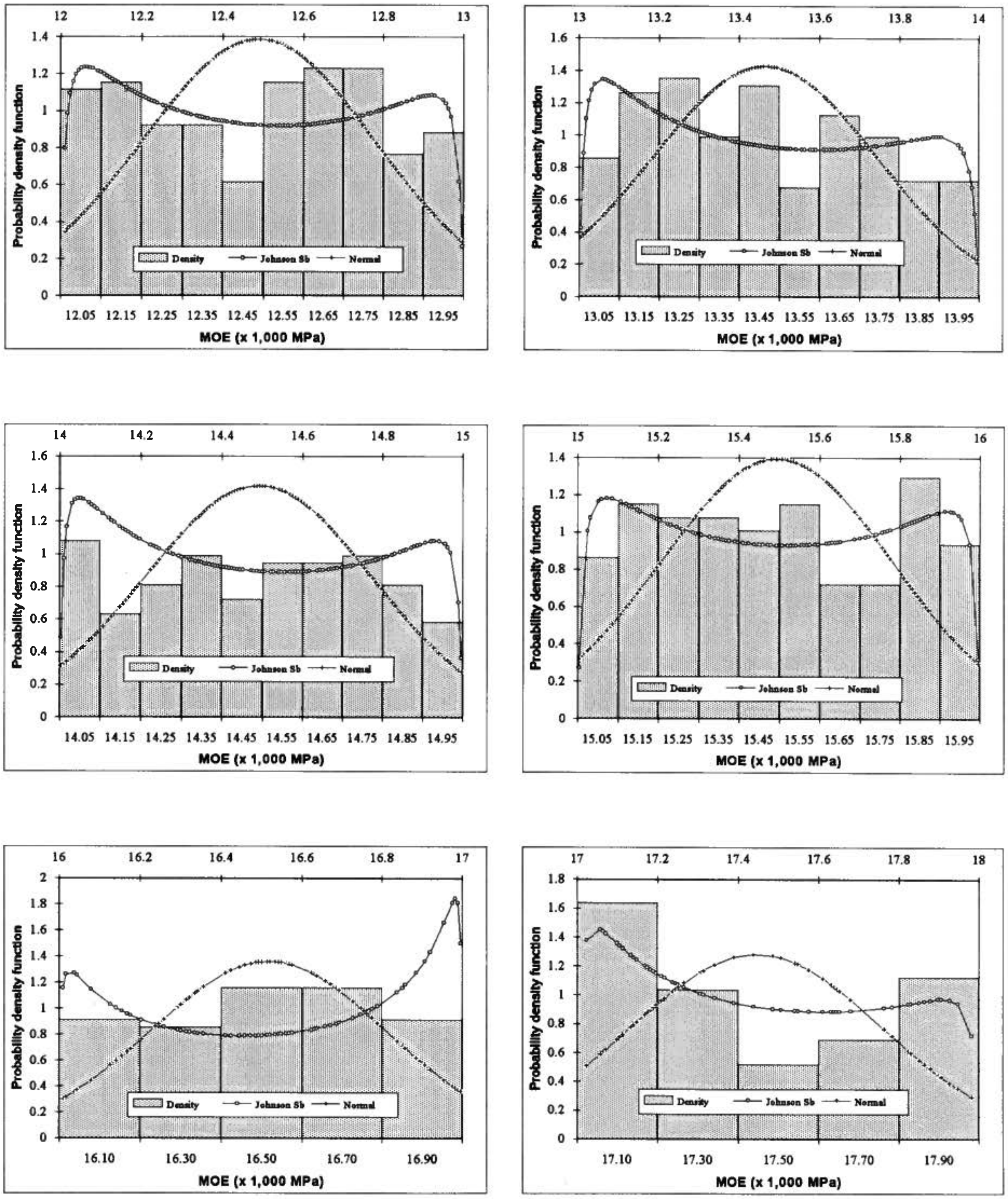


Figure 12. Continued

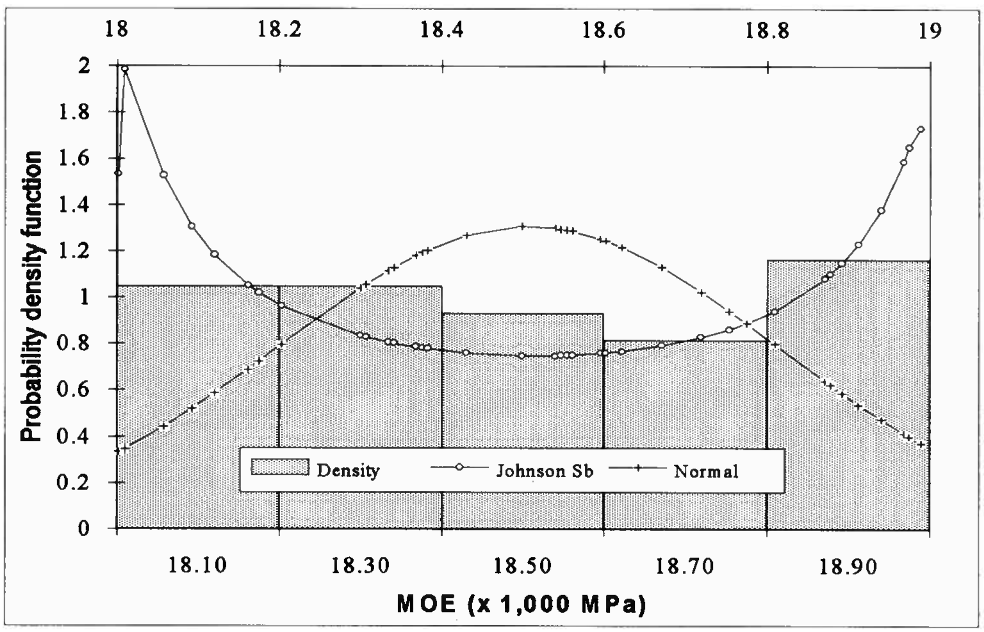
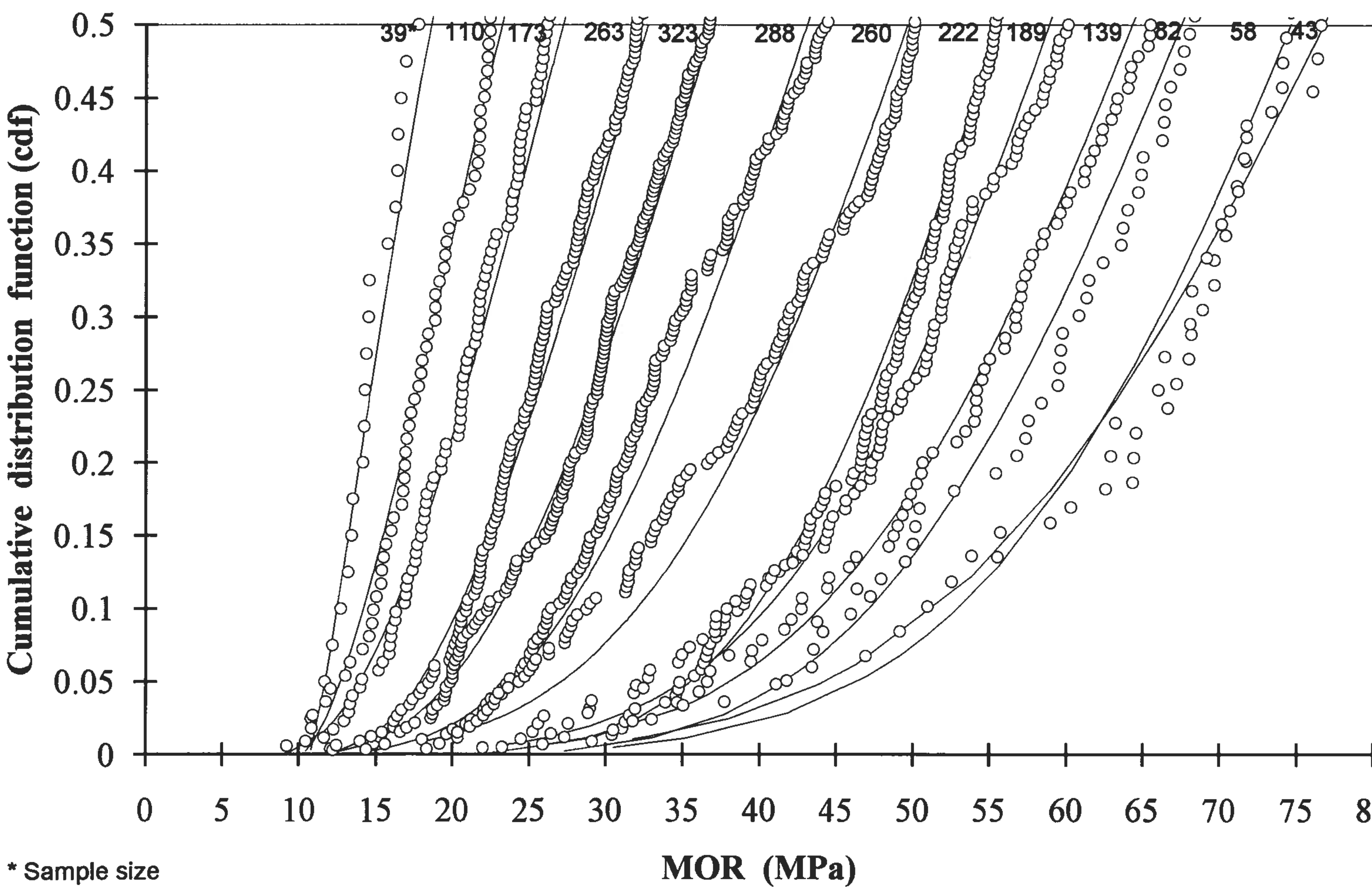
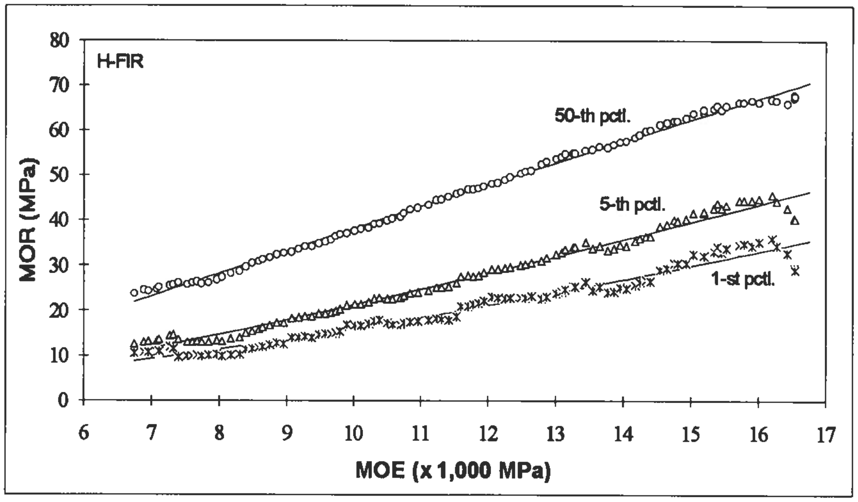


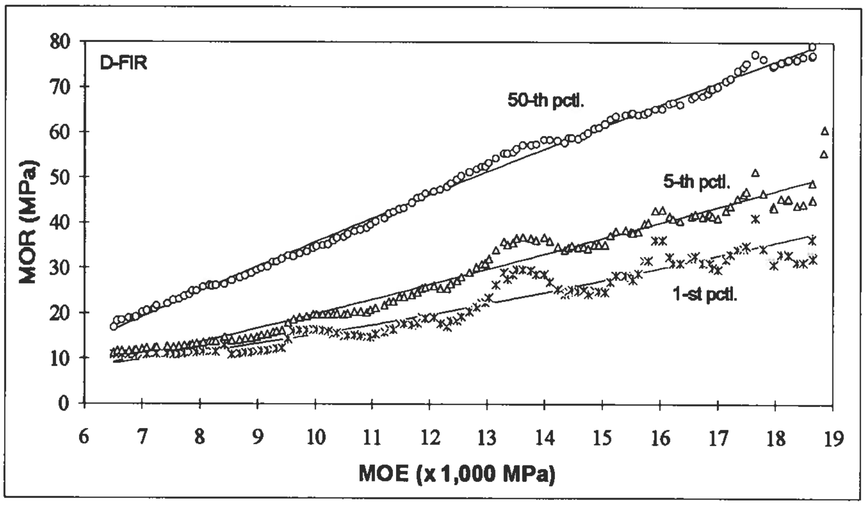
Figure 13. Three parameter Weibull fit on MOR data (D-fir, 1,000 MPa Band-Width)



* Sample size

Figure 14. Relationship between MOE and MOR (Band-Width = 1,000 MPa)





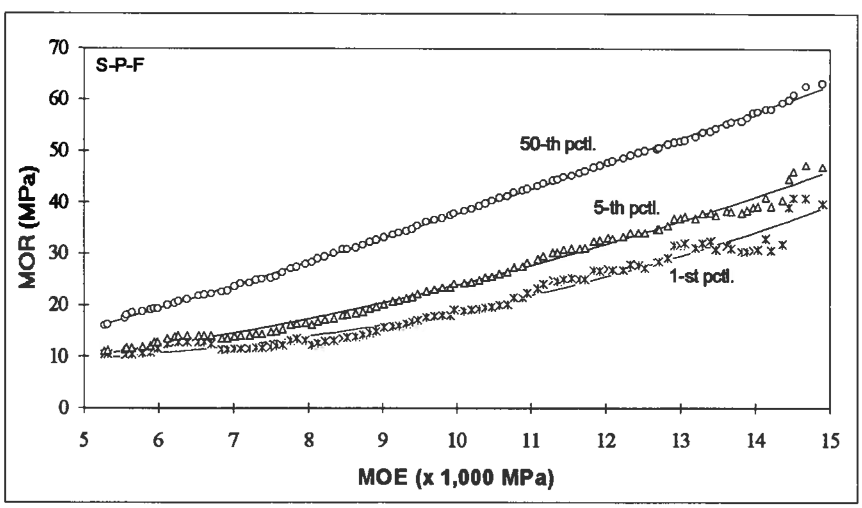
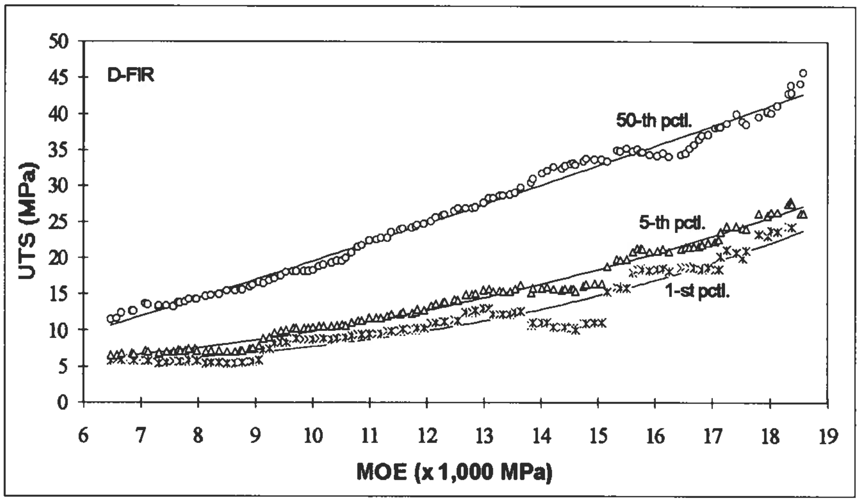
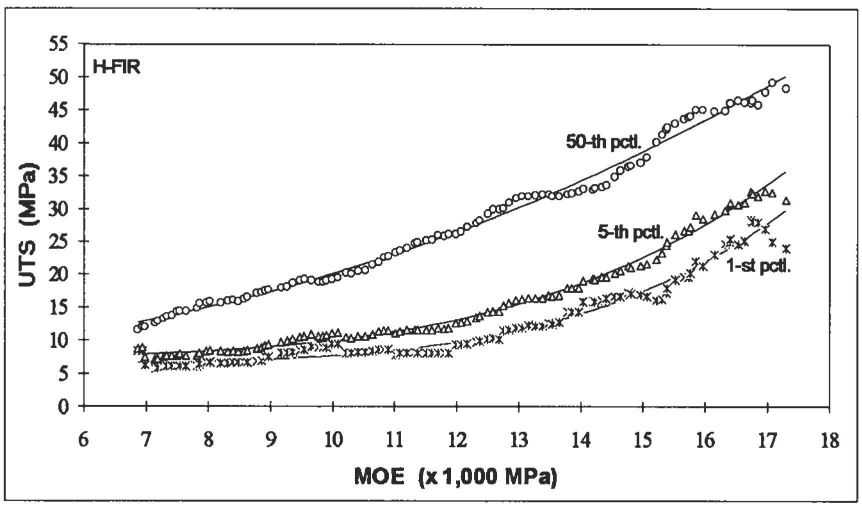


Figure 15. Relationship between MOE and UTS (Band-Width = 1,000 MPa)





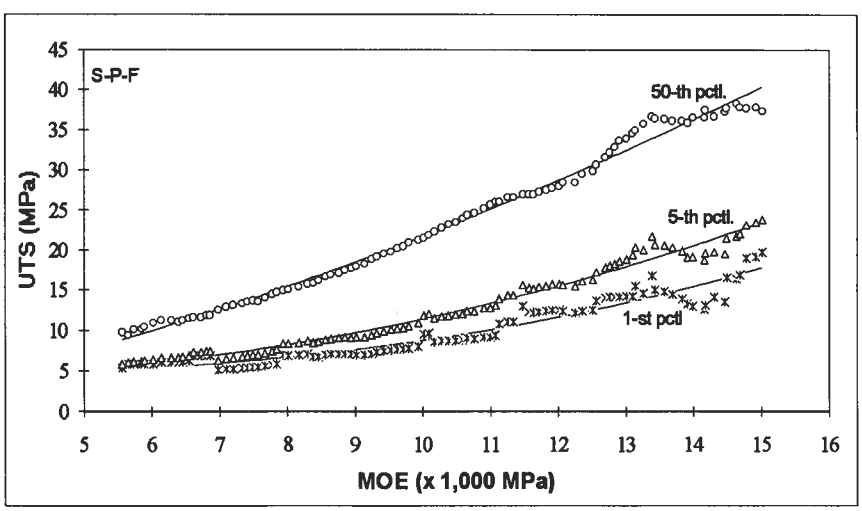
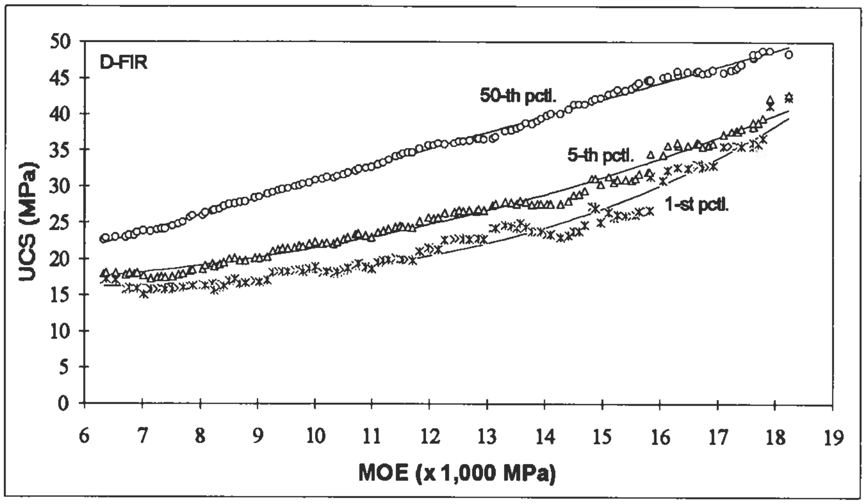
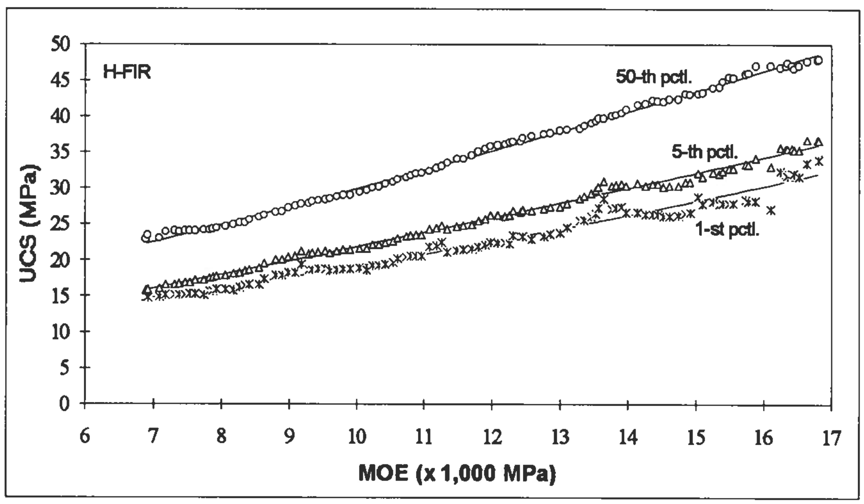


Figure 16. Relationship between MOE and UCS (Band-Width = 1,000 MPa)





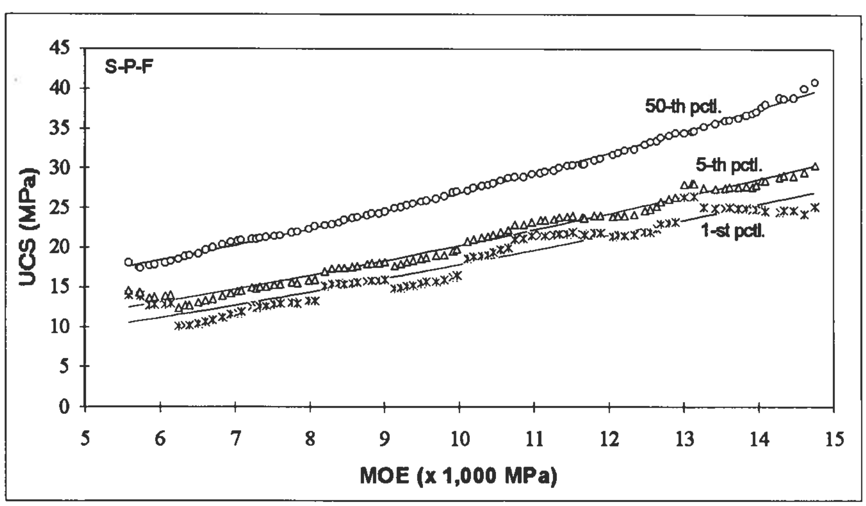
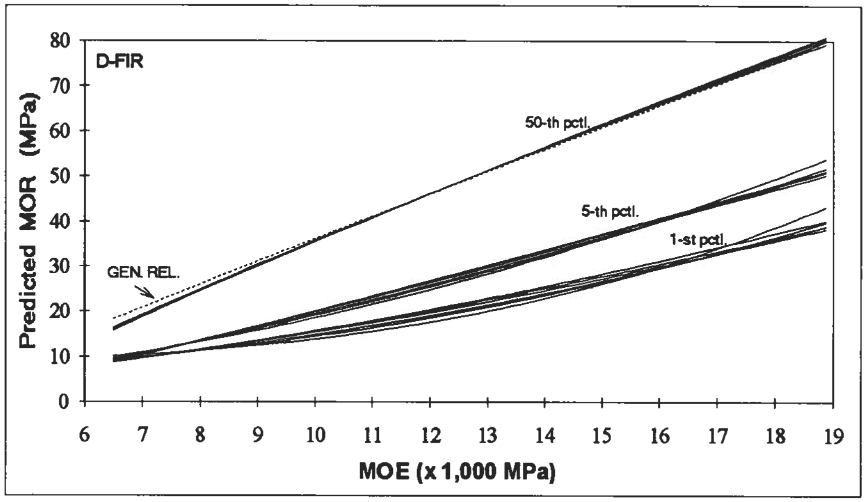
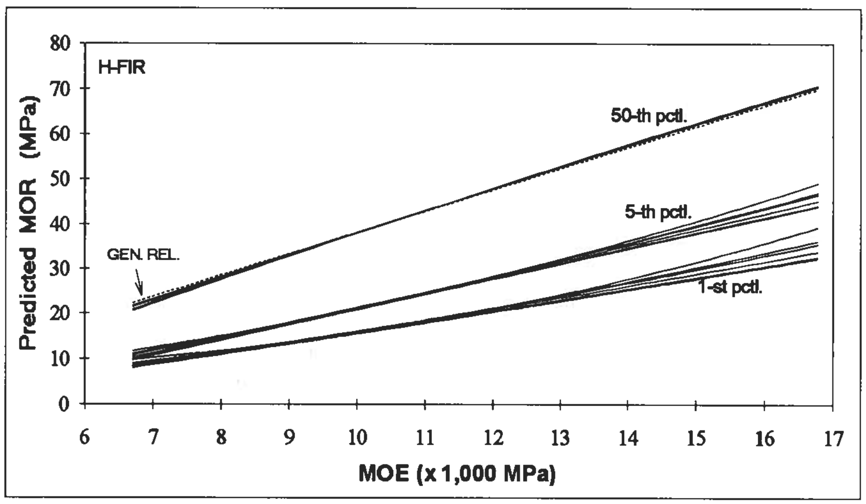


Figure 17. Predicted MOR for each band-width and percentile level





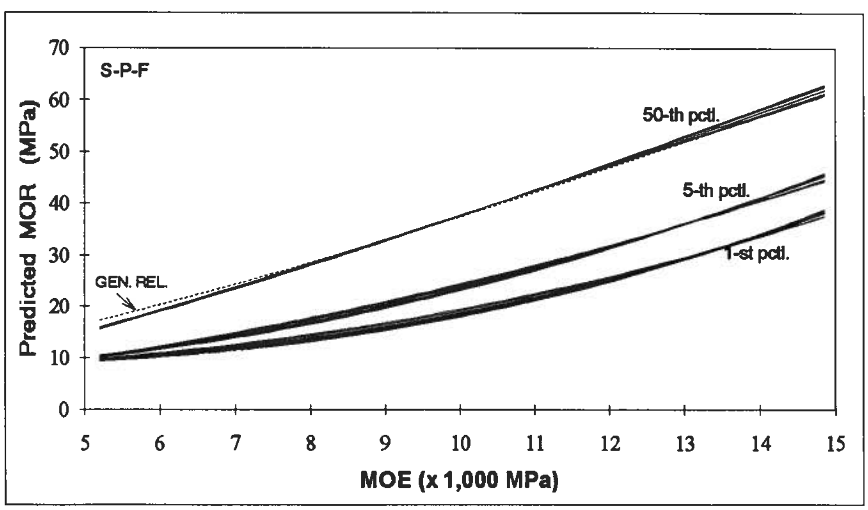
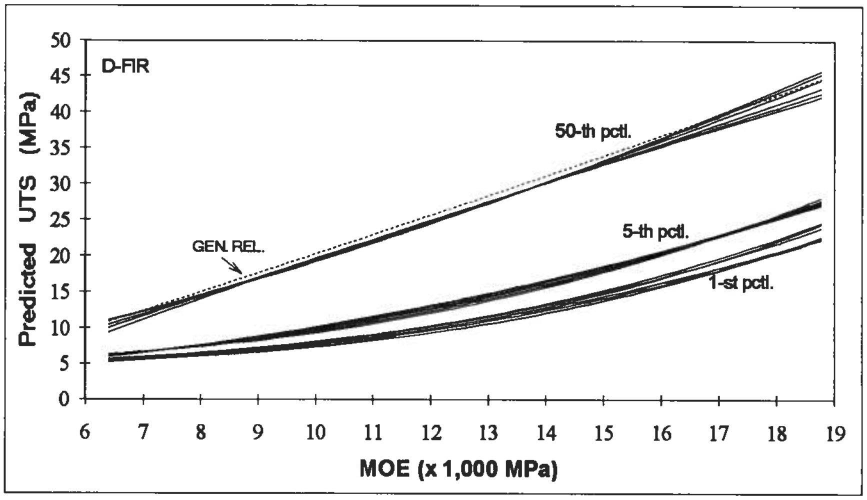
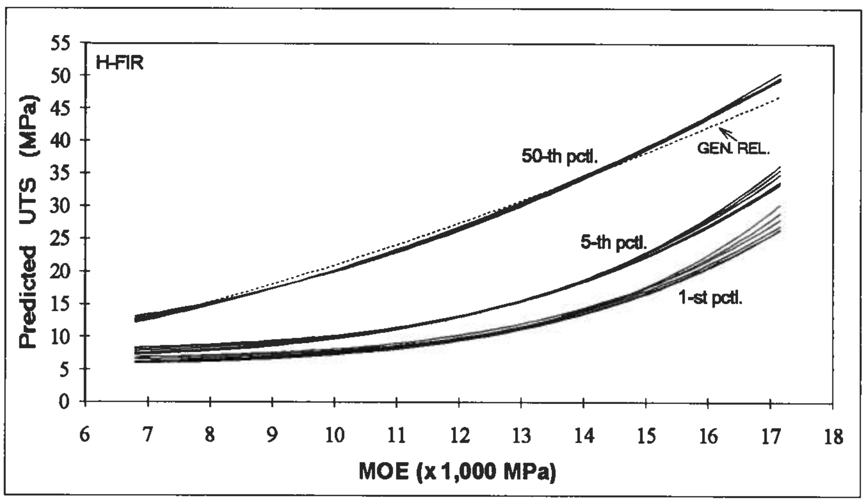


Figure 18. Predicted UTS for each band-width and percentile level





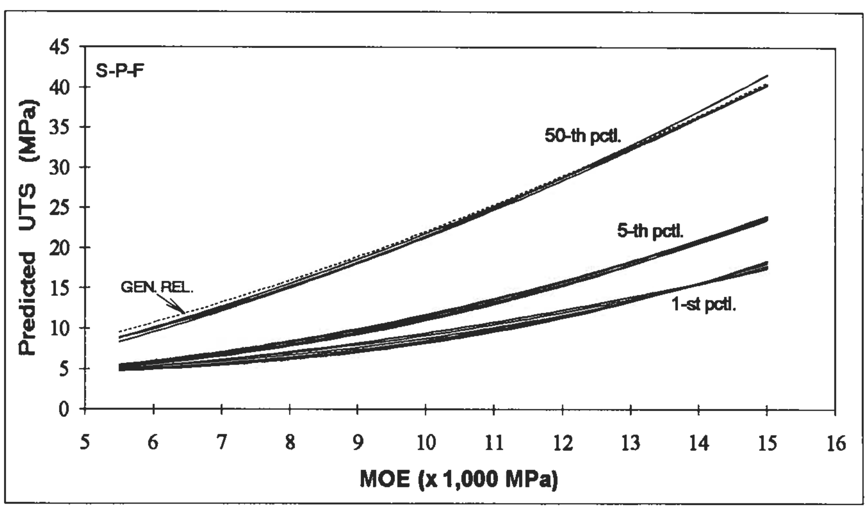
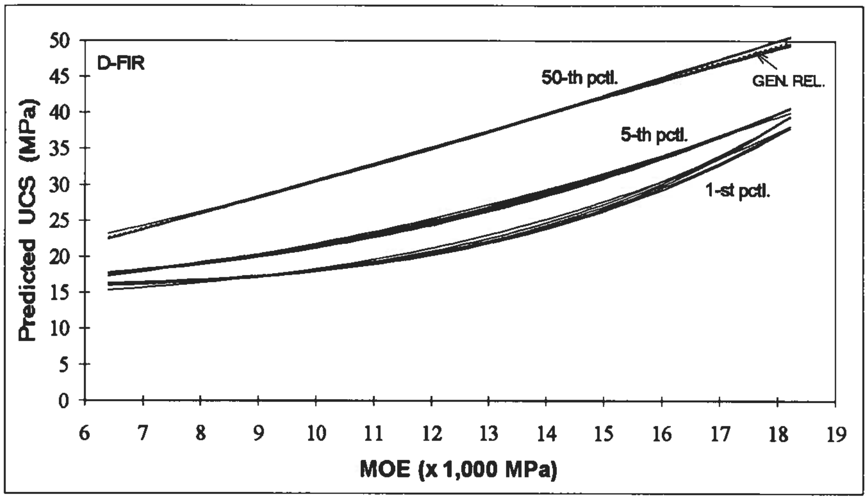
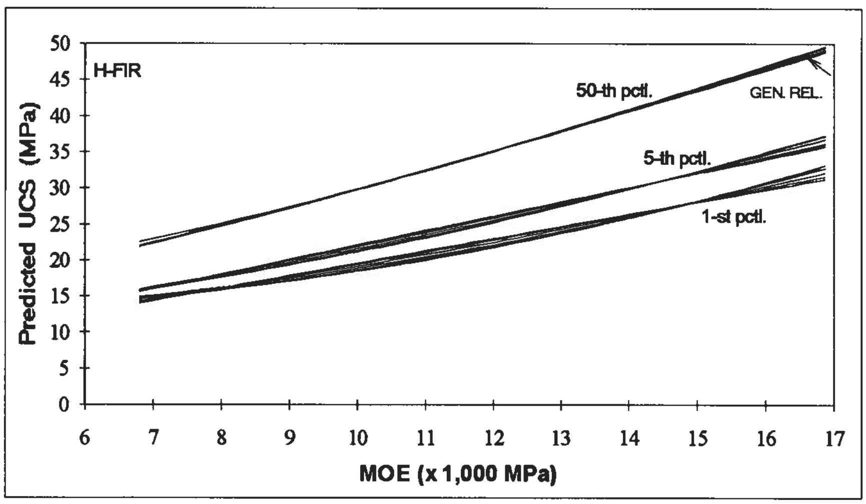


Figure 19. Predicted UCS for each band-width and percentile level





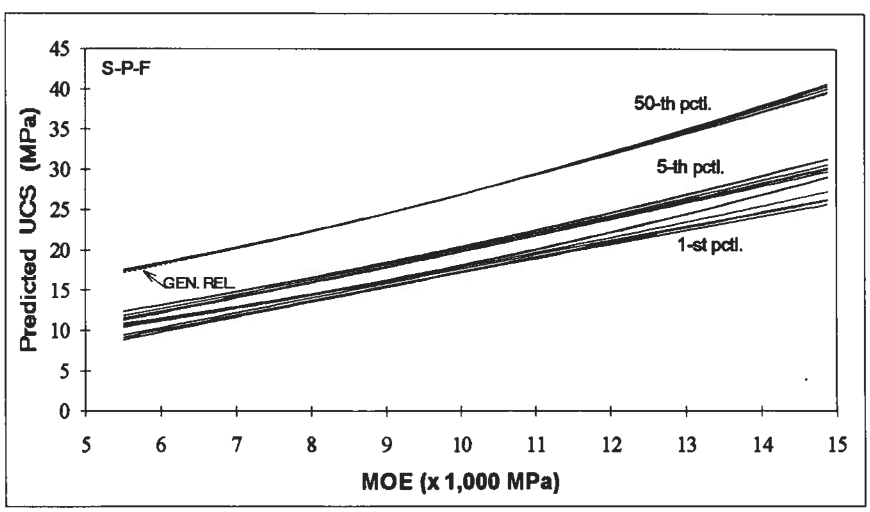
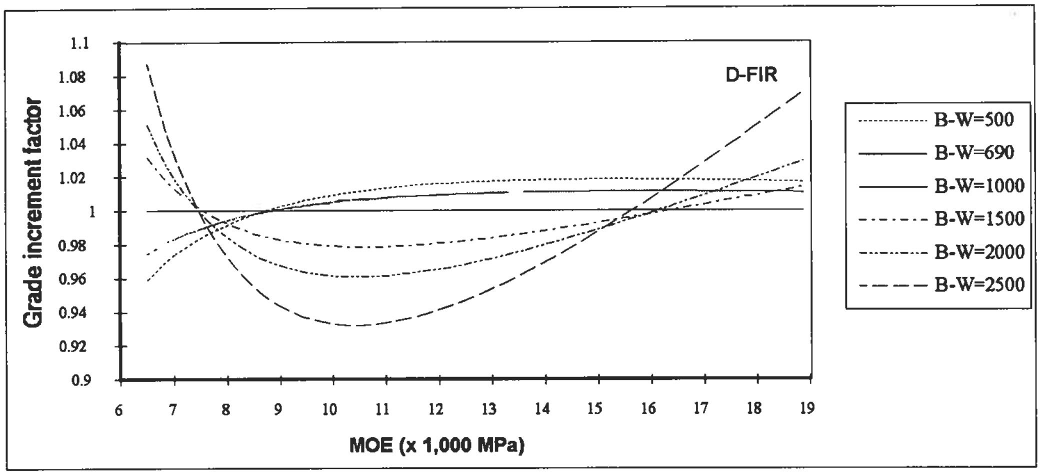
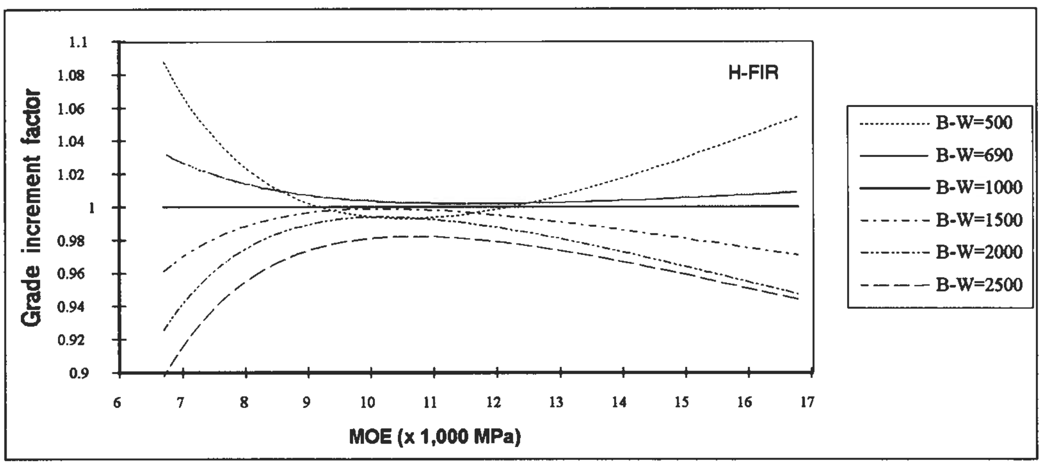


Figure 20. Grade increment factors for MOE-MOR (5-th percentile)





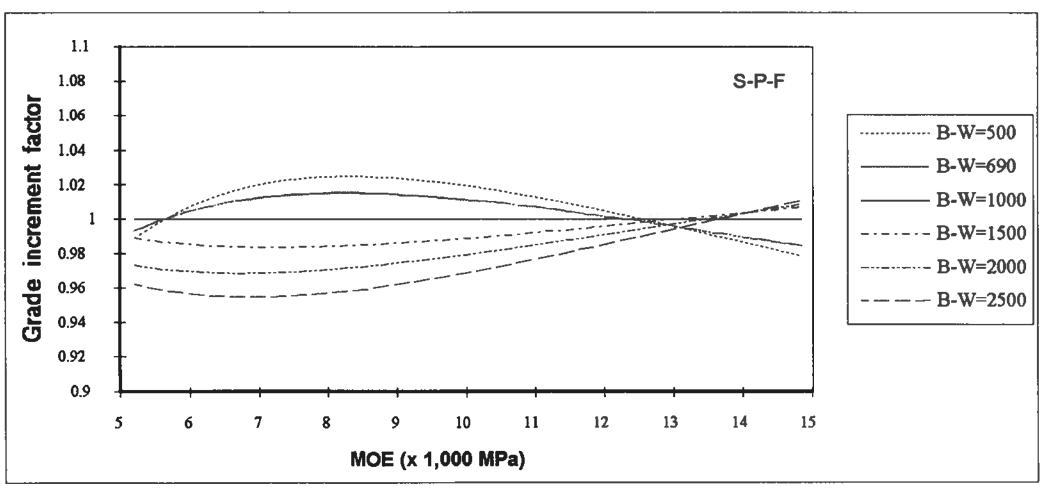


Figure 21. Grade increment factors for MOE-UTS (5-th percentile)

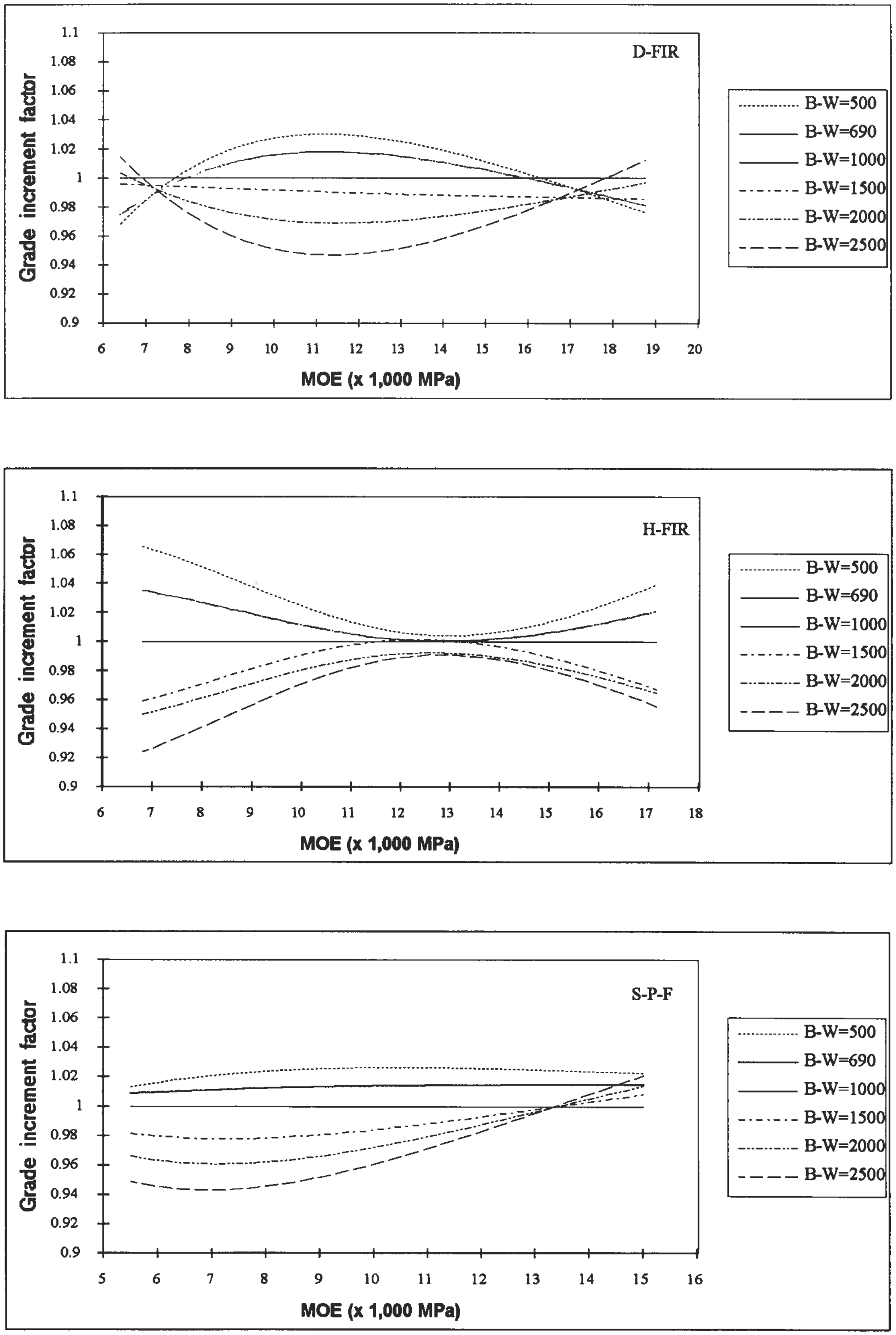
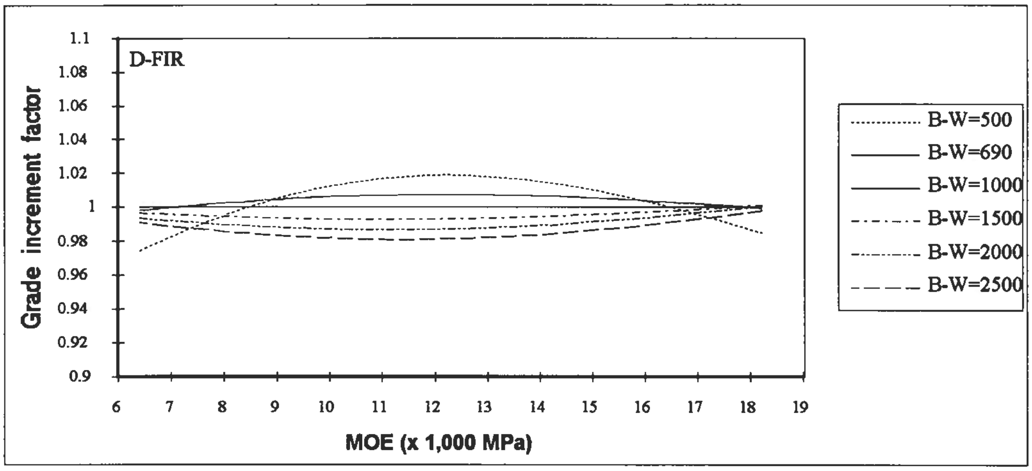
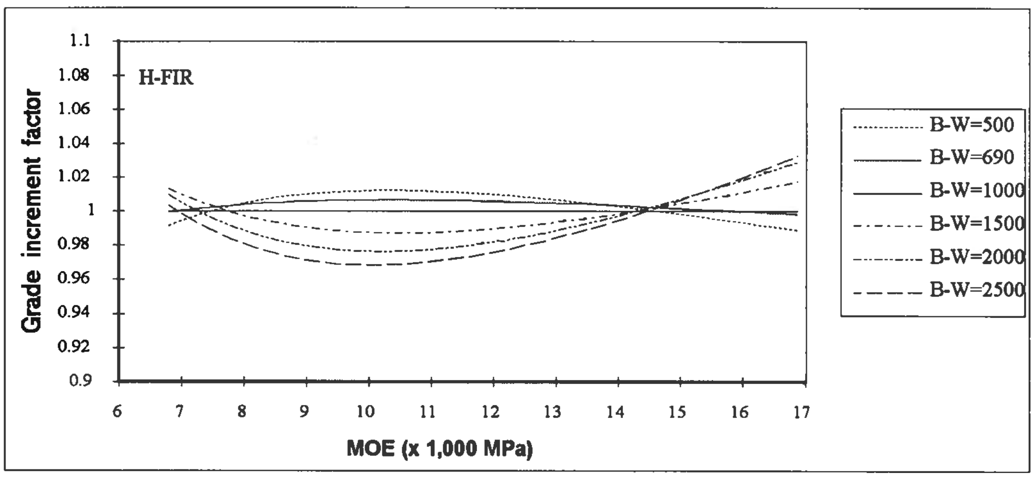
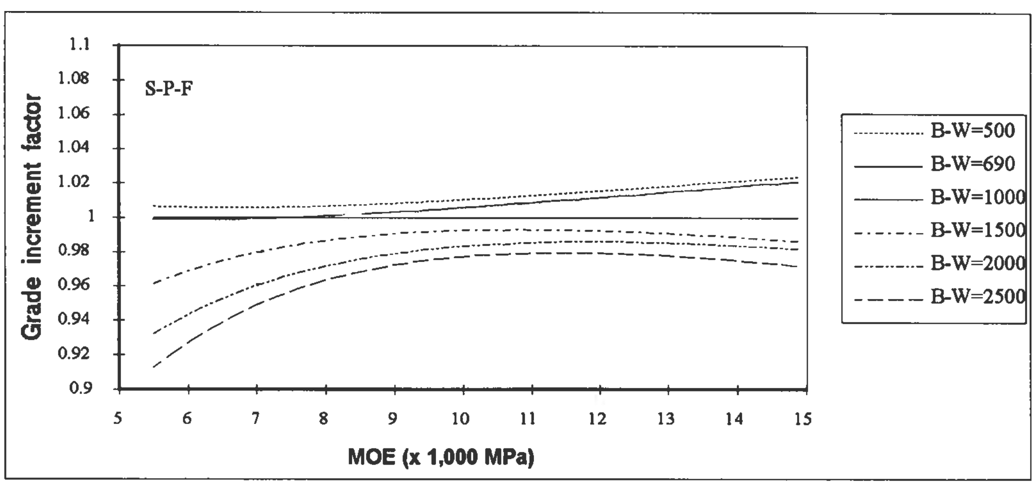


Figure 22. Grade increment factors for MOE-UCS (5-th percentile)







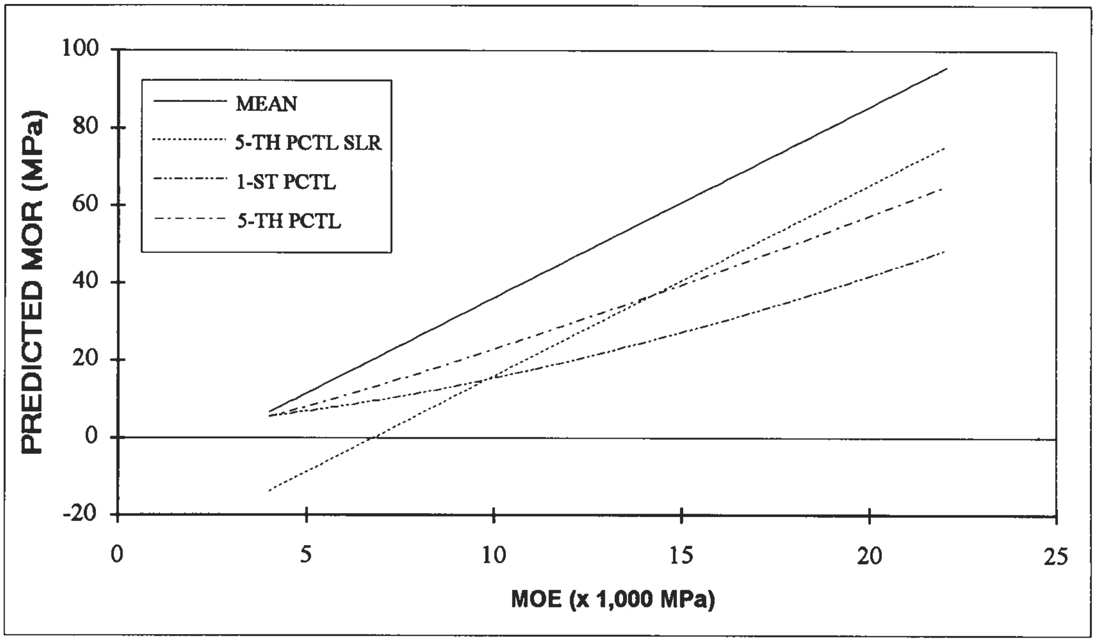


Fig. 23. Comparison of the 5 % exclusion lines between linear and non linear model

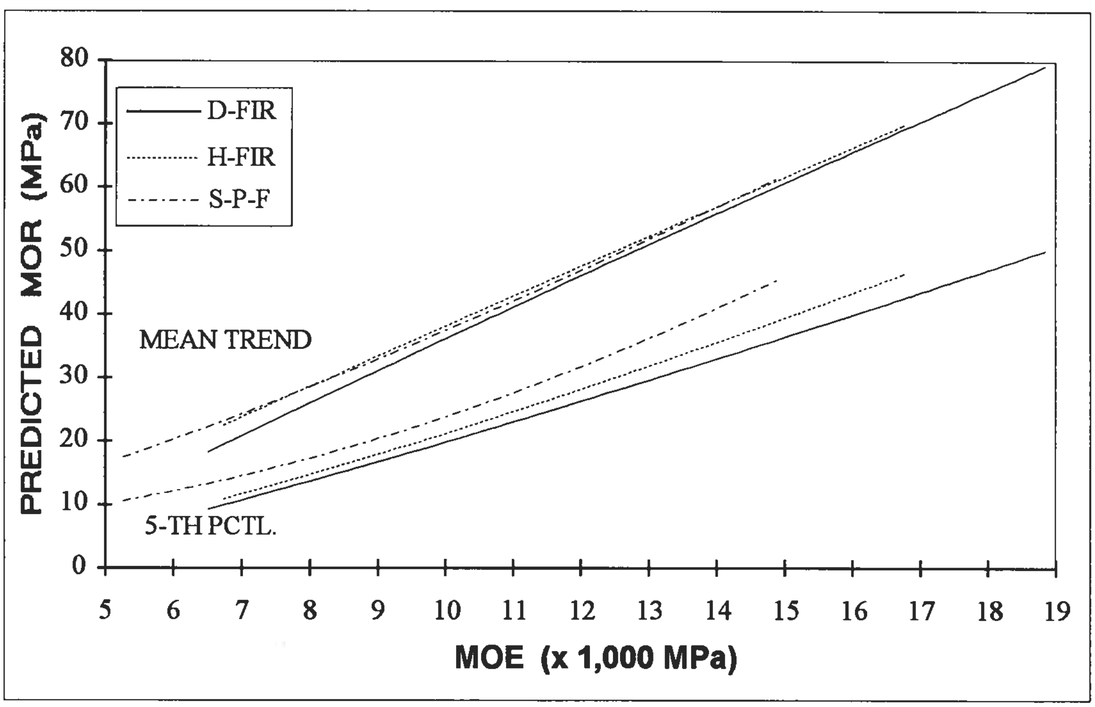


Figure 24. Relationships of MOE to the 5-th percentile and mean trend of MOR

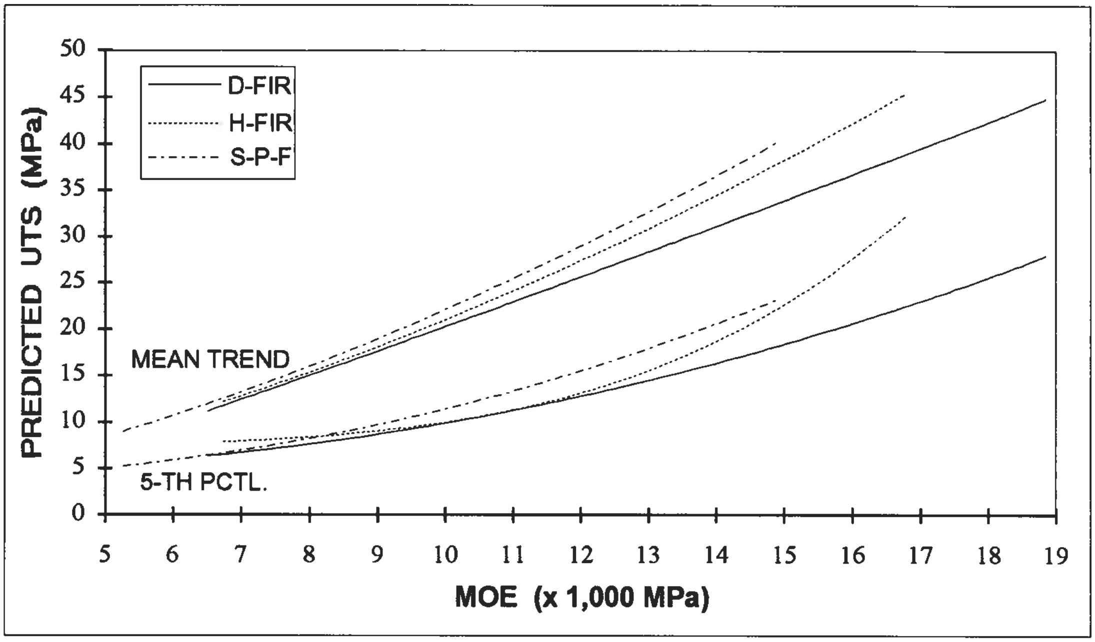


Figure 25. Relationships of MOE to the 5-th percentile and mean trend of UTS.

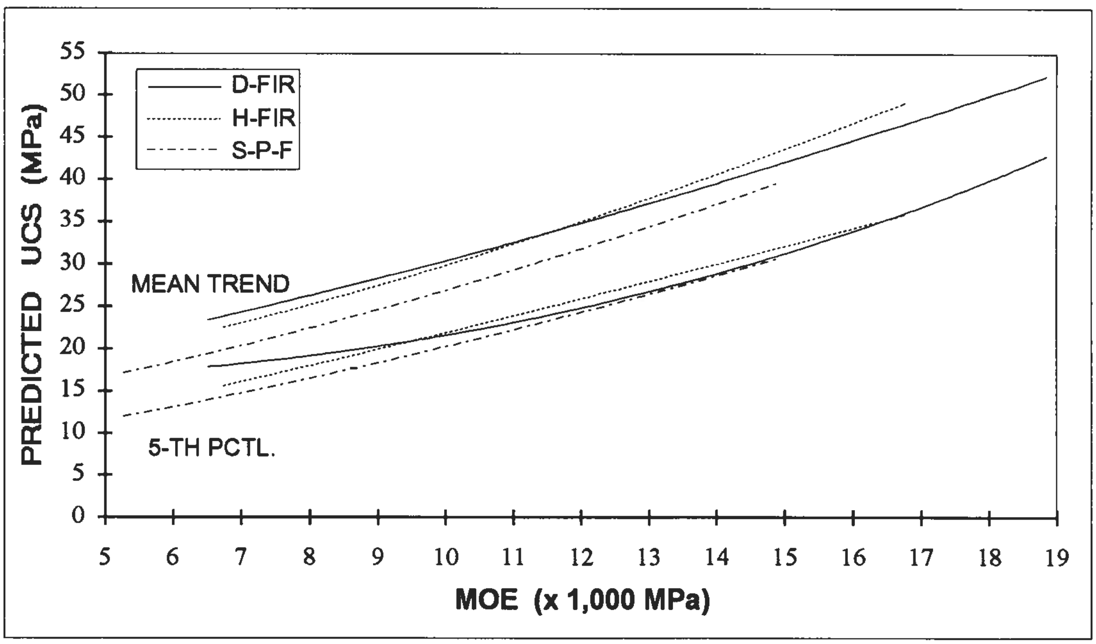
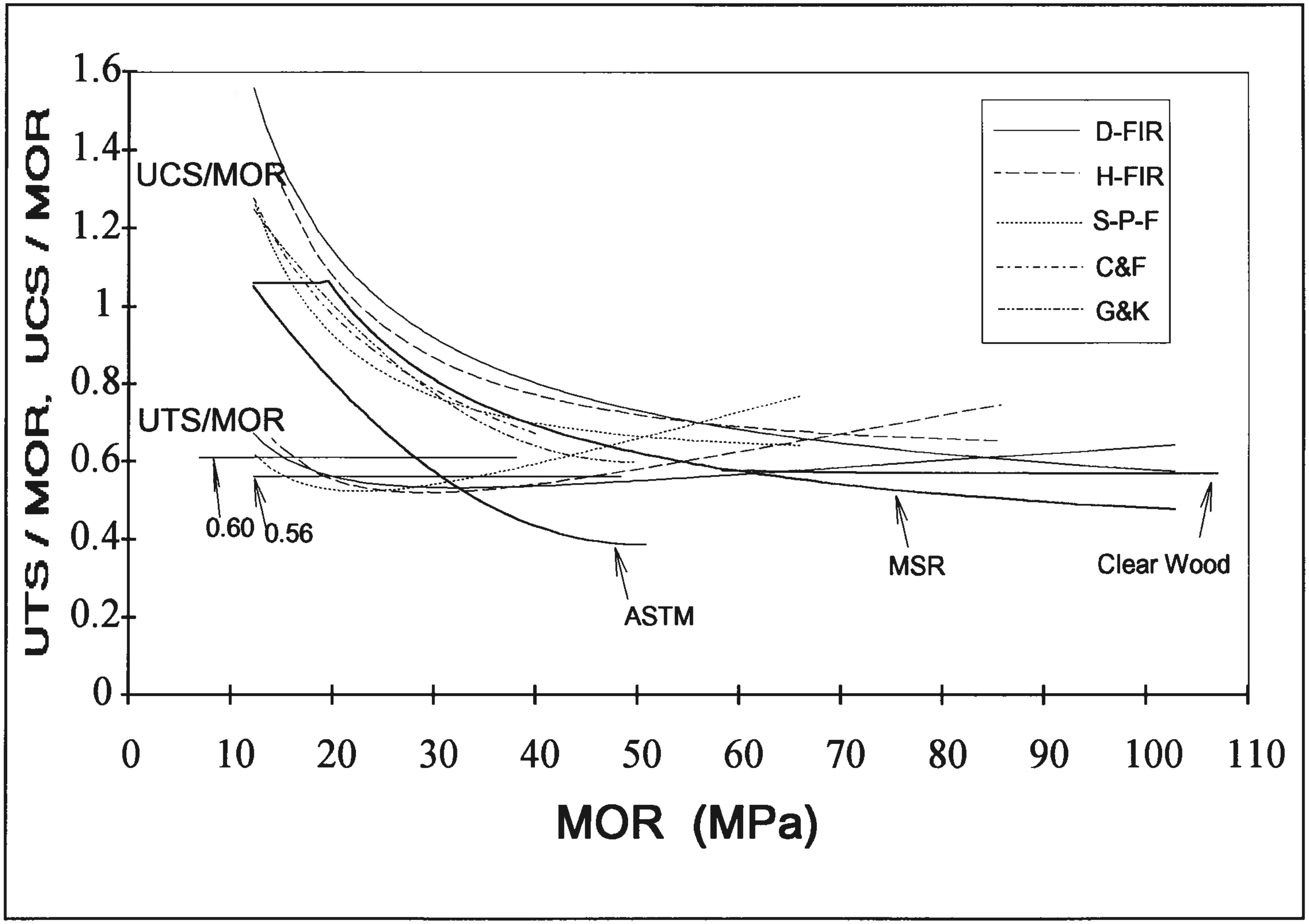


Figure 26. Relationships of MOE to the 5-th percentile and mean trend of UCS.

Figure 27. Strength property ratios based on MOR



C & F : Curry and Fewell (1977)
G & K : Green and Kretschmann (1991)
ASTM : ASTM D 1990 (ASTM 1991)

Figure 28. UTS/MOR and UCS/MOR as a function of MOR (Band-Width = 1,000 MPa)

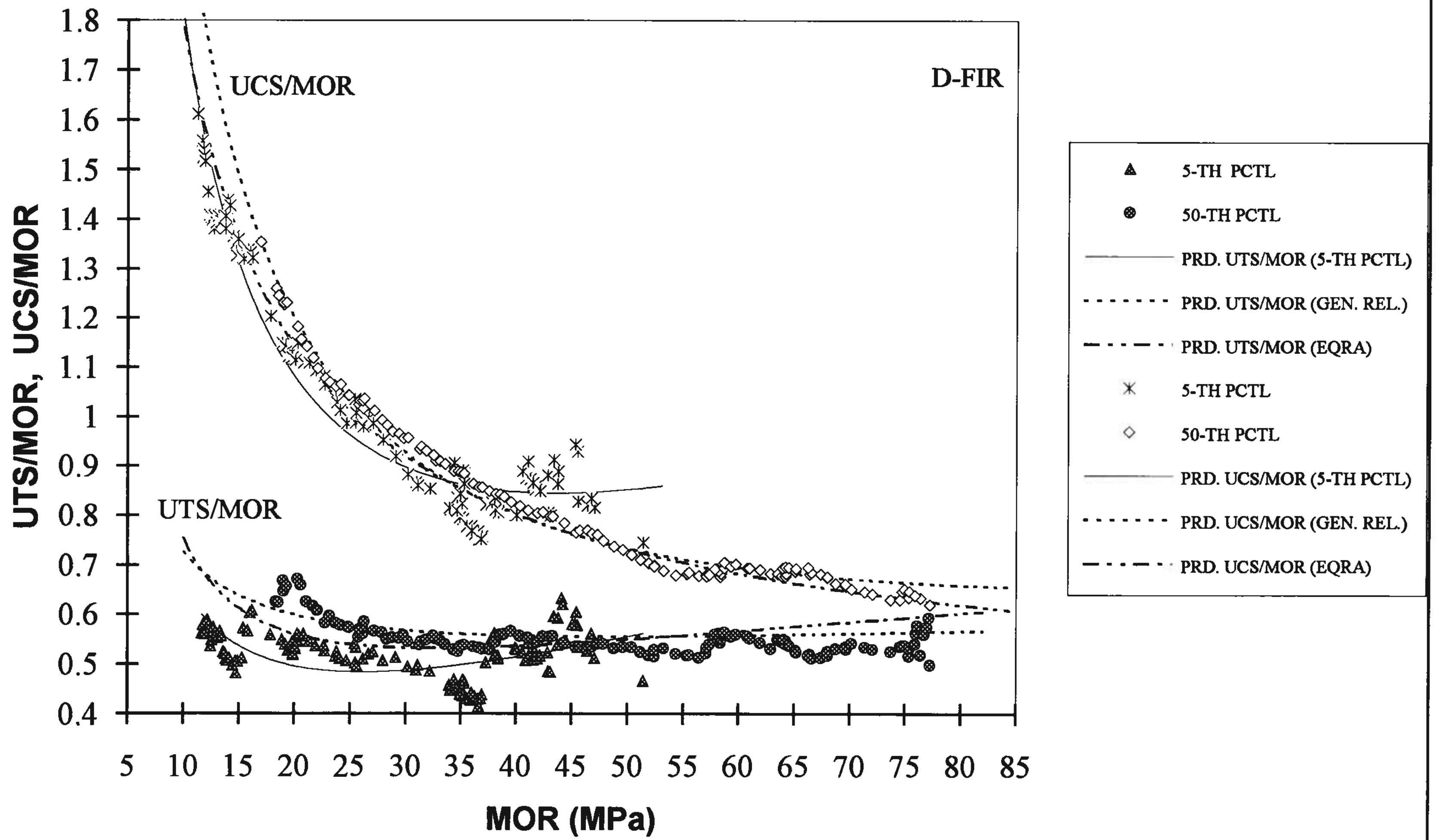


Figure 28. Continued

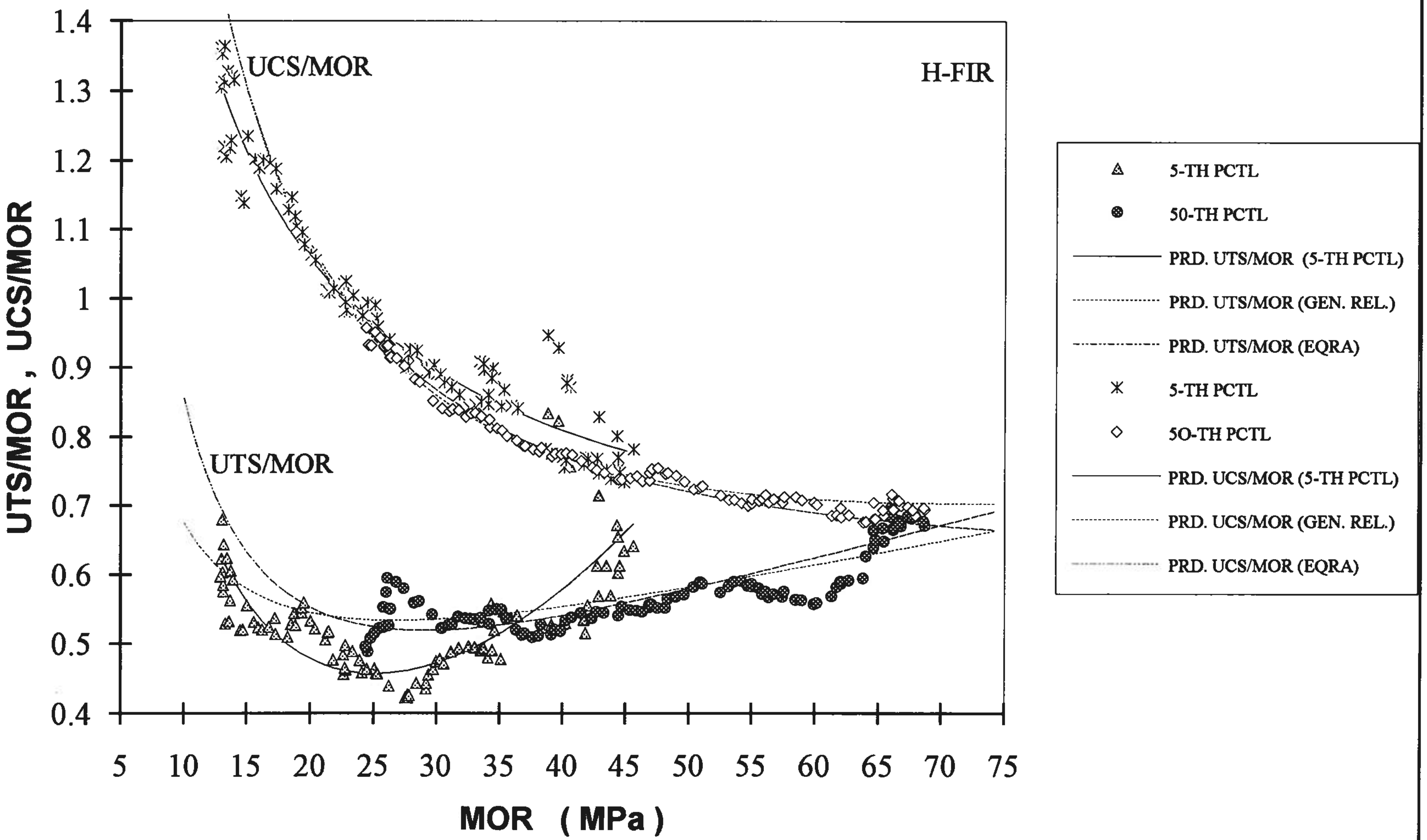


Figure 28. Continued

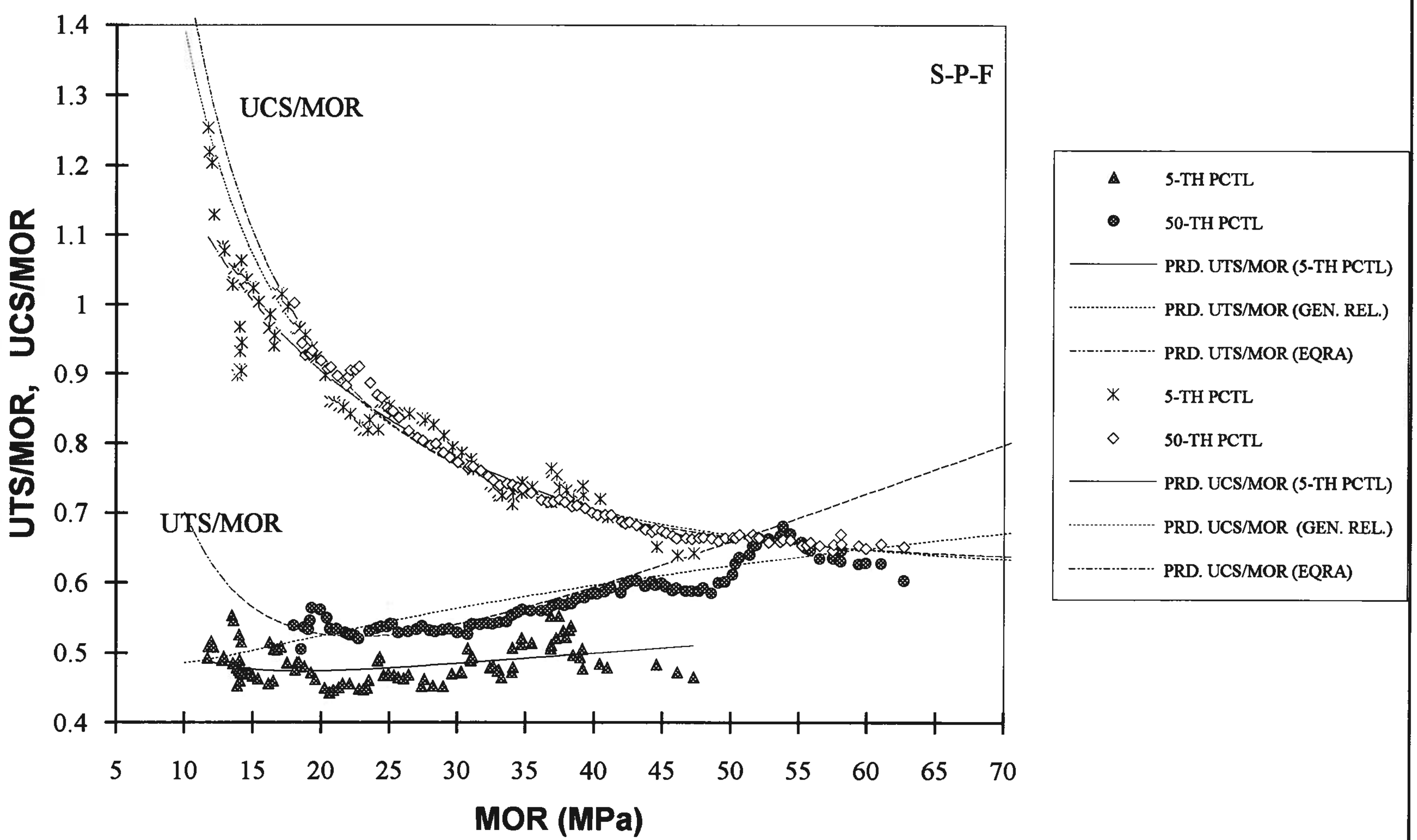


Figure 29. UTS/MOR and UCS/MOR as a function of MOE (Band-Width = 1,000 MPa)

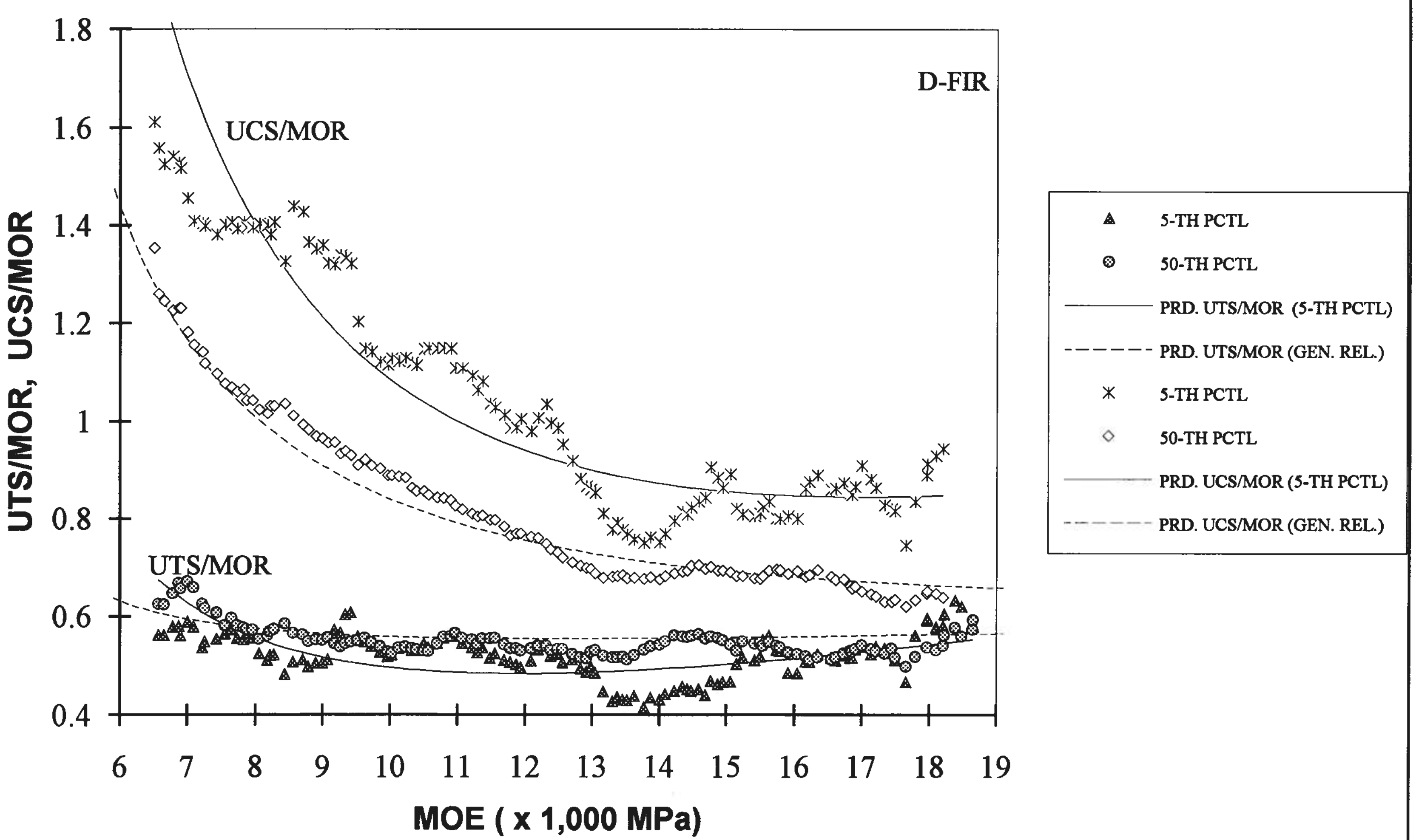


Figure 29. Continued

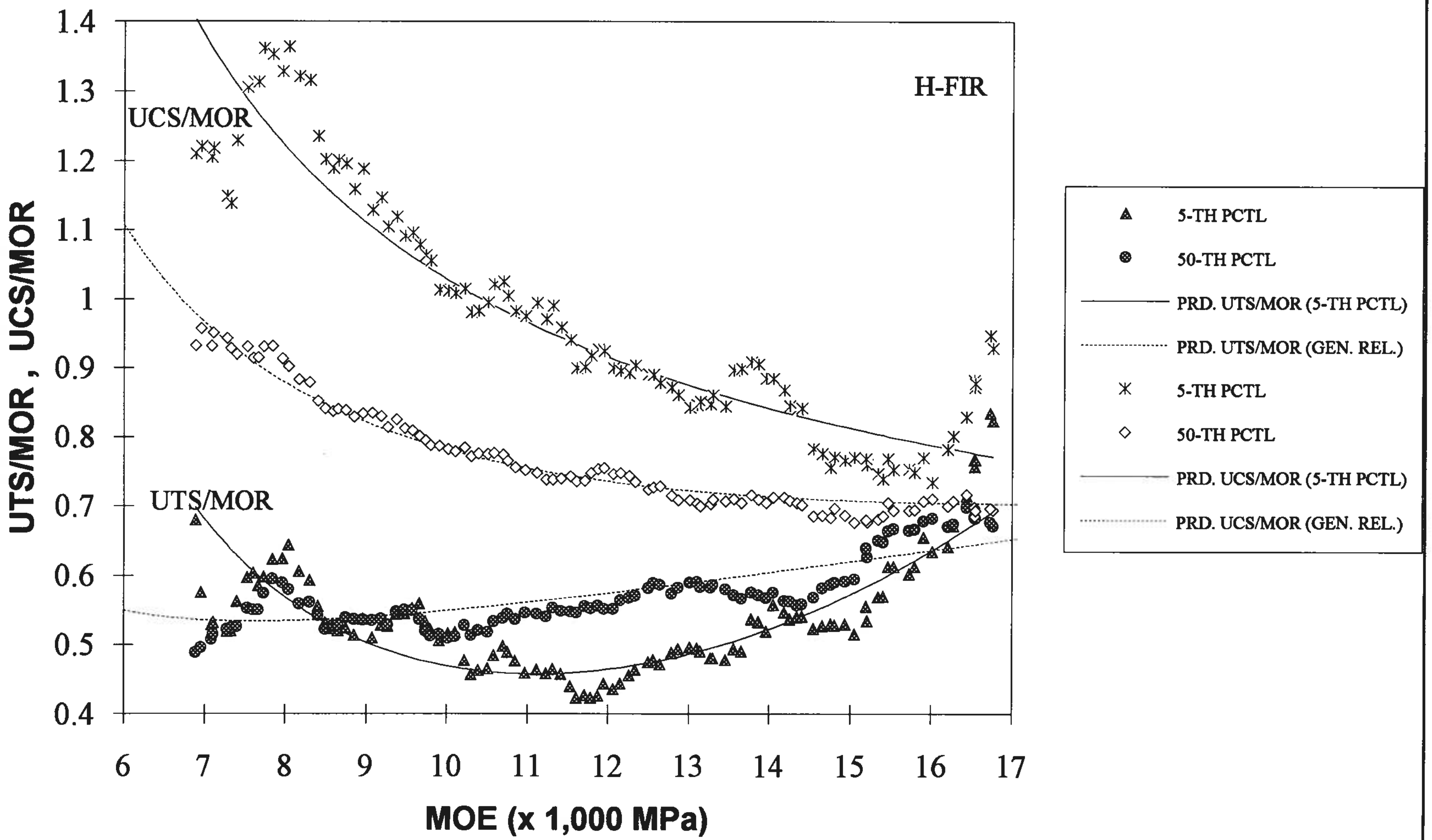


Figure 29. Continued

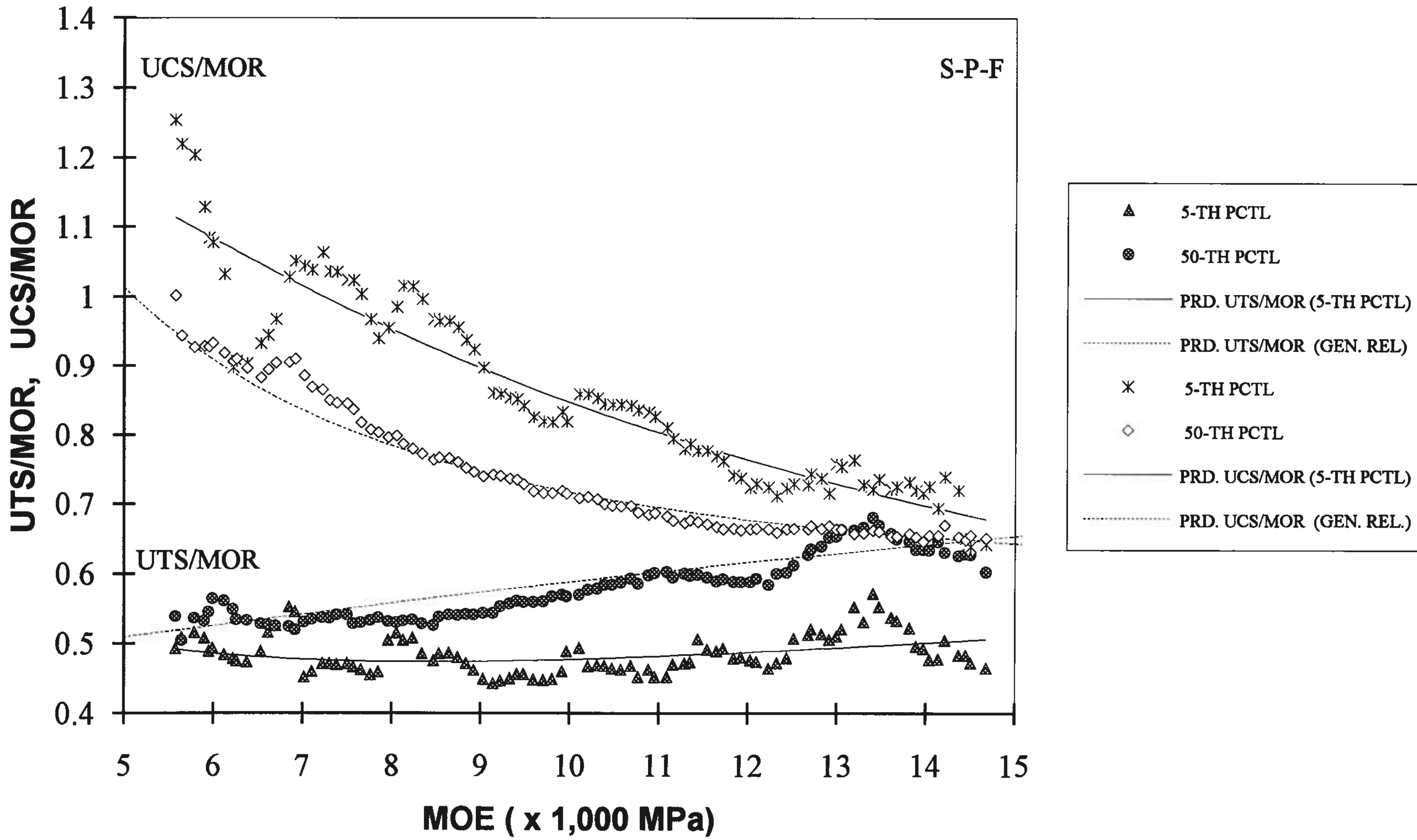
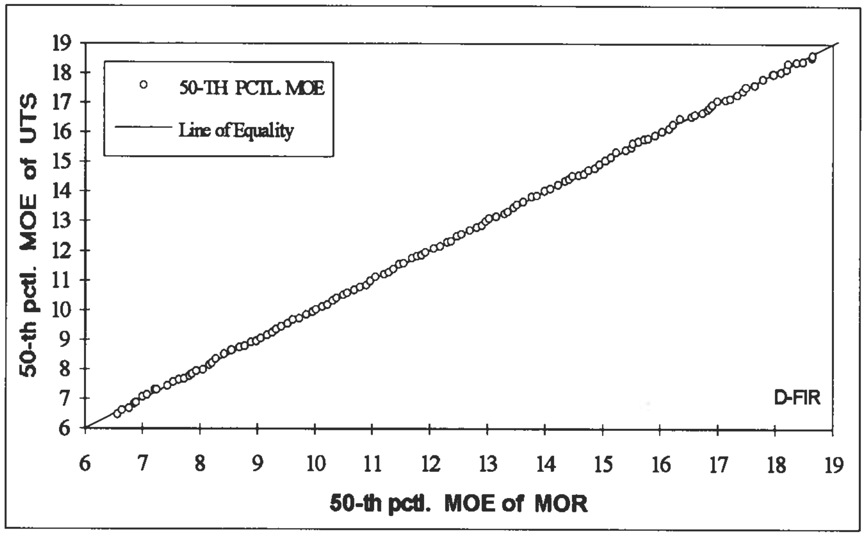
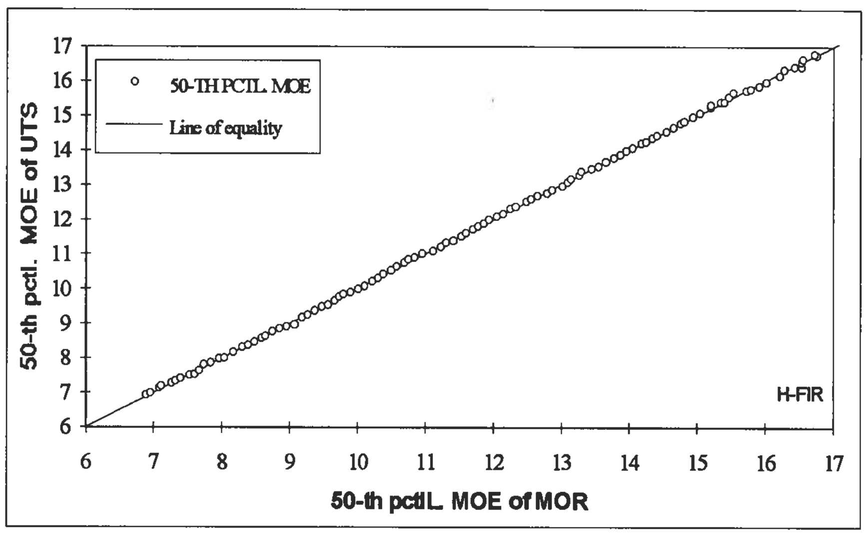


Figure 30. MOE of MOR vs. MOE of UTS at 50-th percentile level





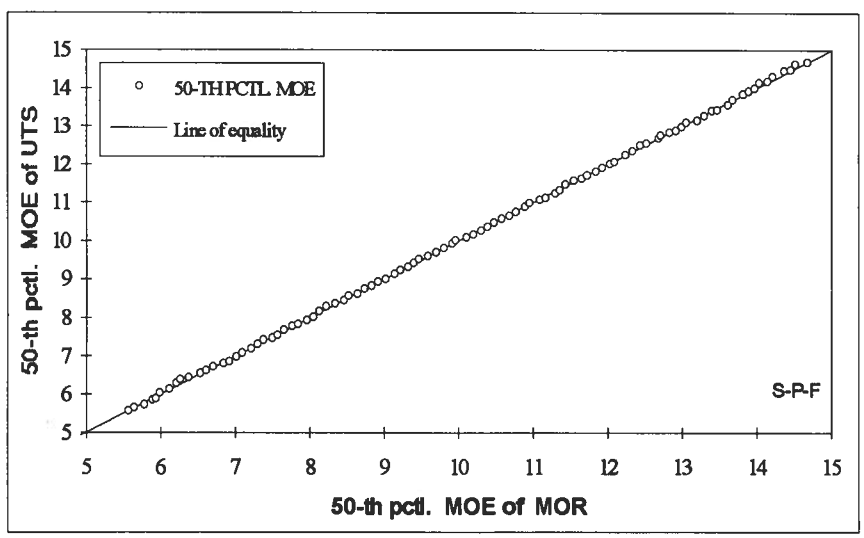
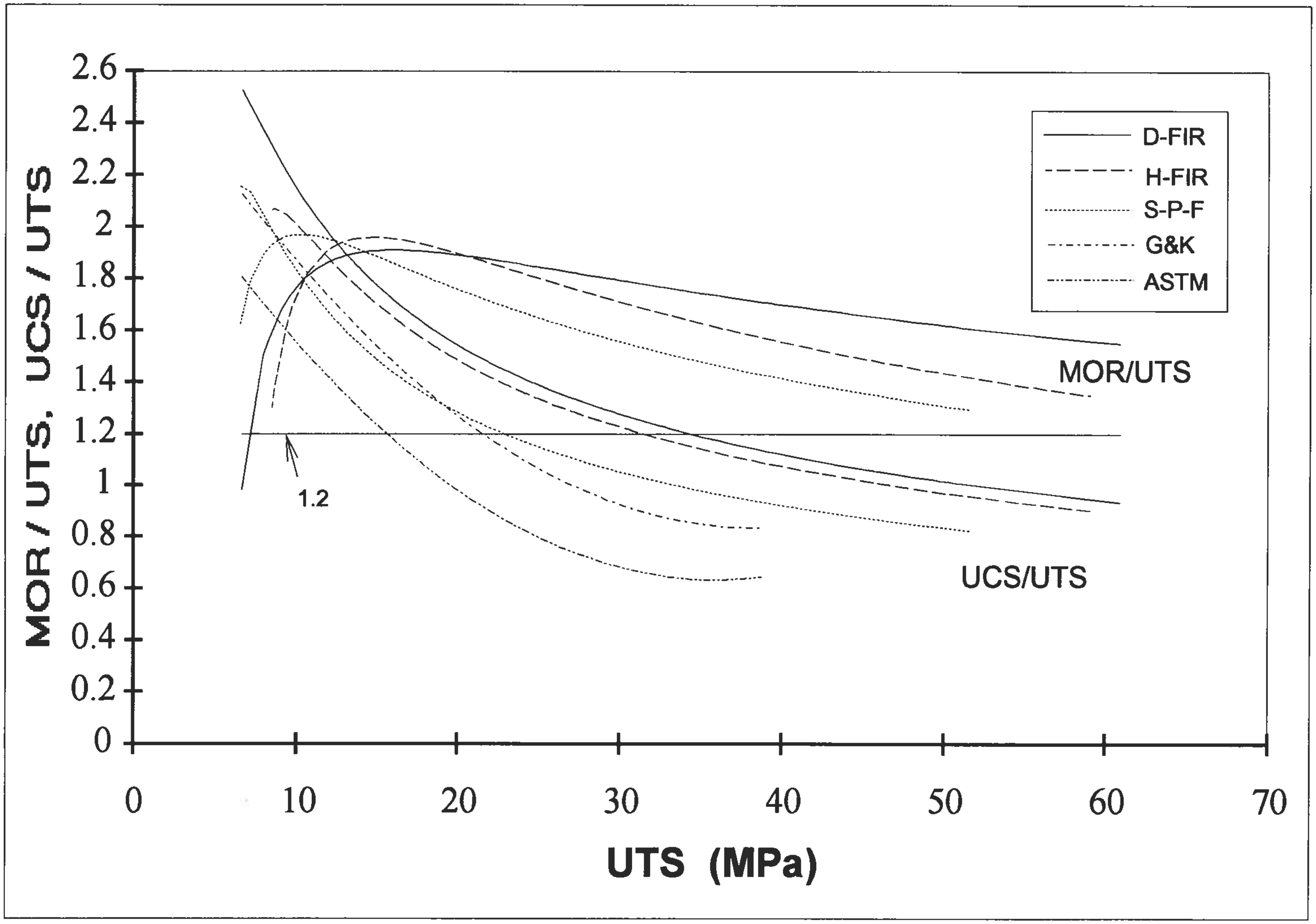


Figure 31. Strength property ratios based on UTS



G & K : Green & Kretschmann (1991)
ASTM : ASTM D 1990 (ASTM 1991)

Figure 32. MOR/UTS and UCS/UTS as a function of UTS (Band-Width = 1,000 MPa)

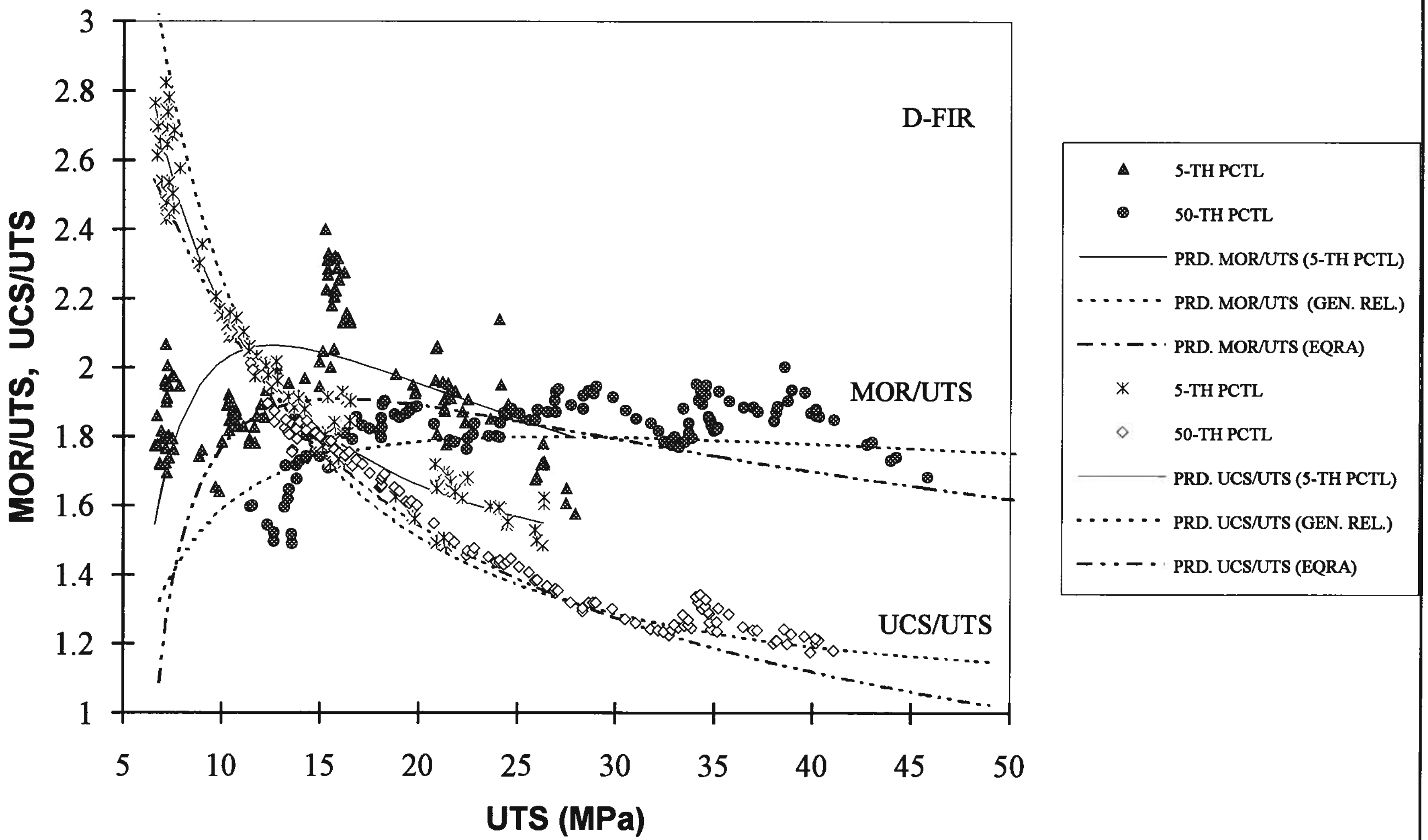


Figure 32. Continued

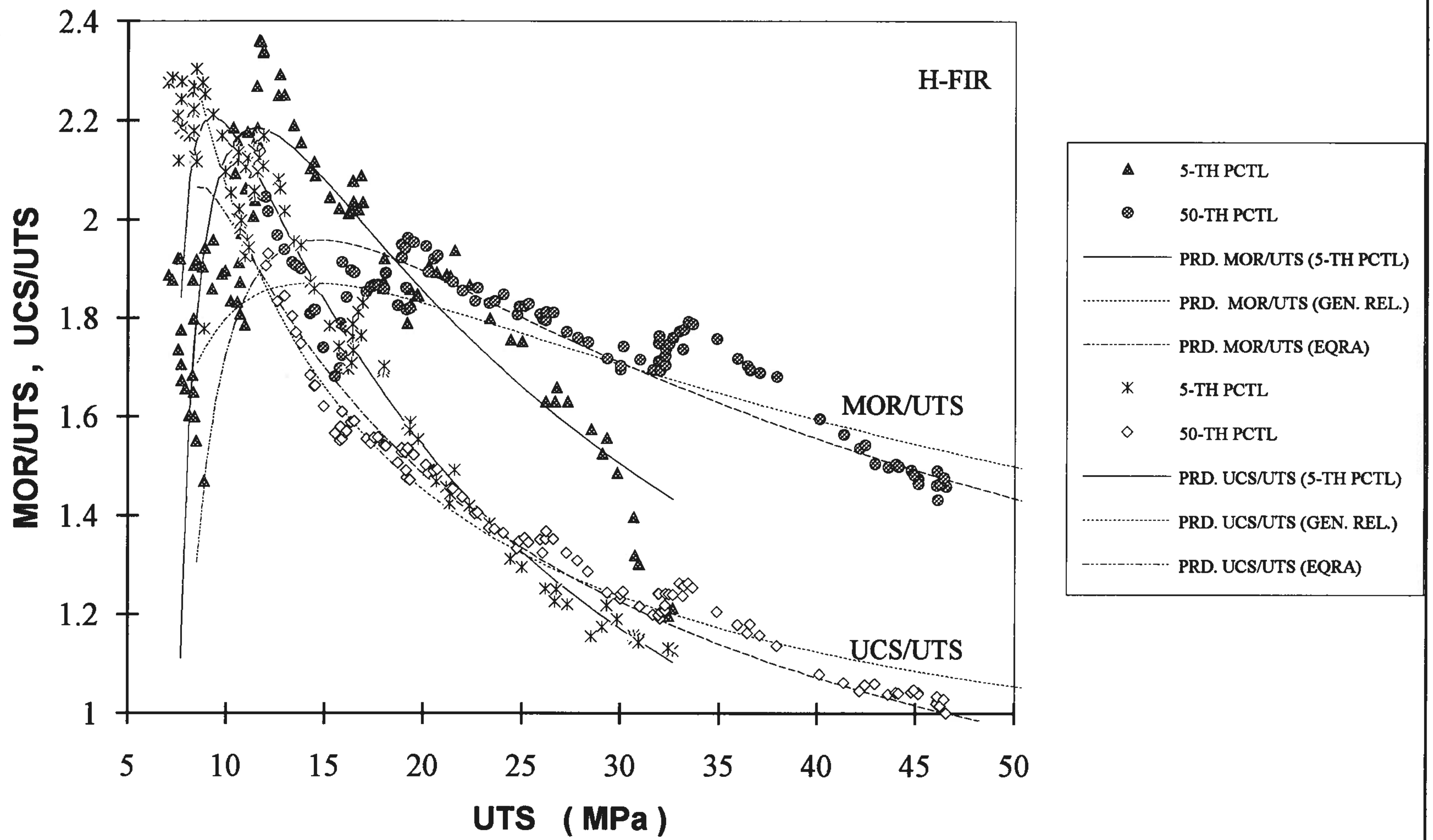


Figure 32. Continued

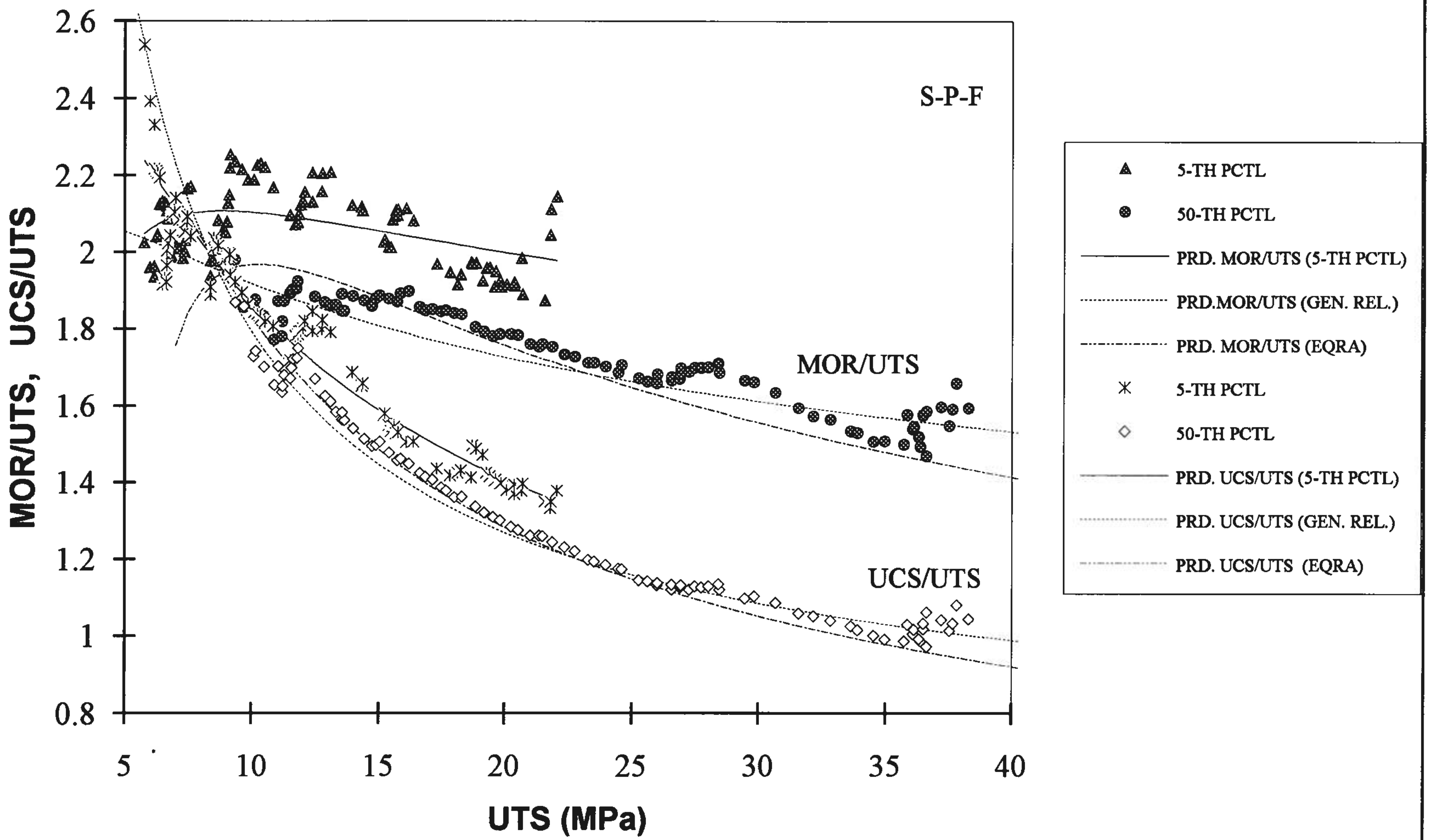


Figure 33. MOR/UTS and UCS/UTS as a function of MOE (Band-Width = 1,000 MPa)

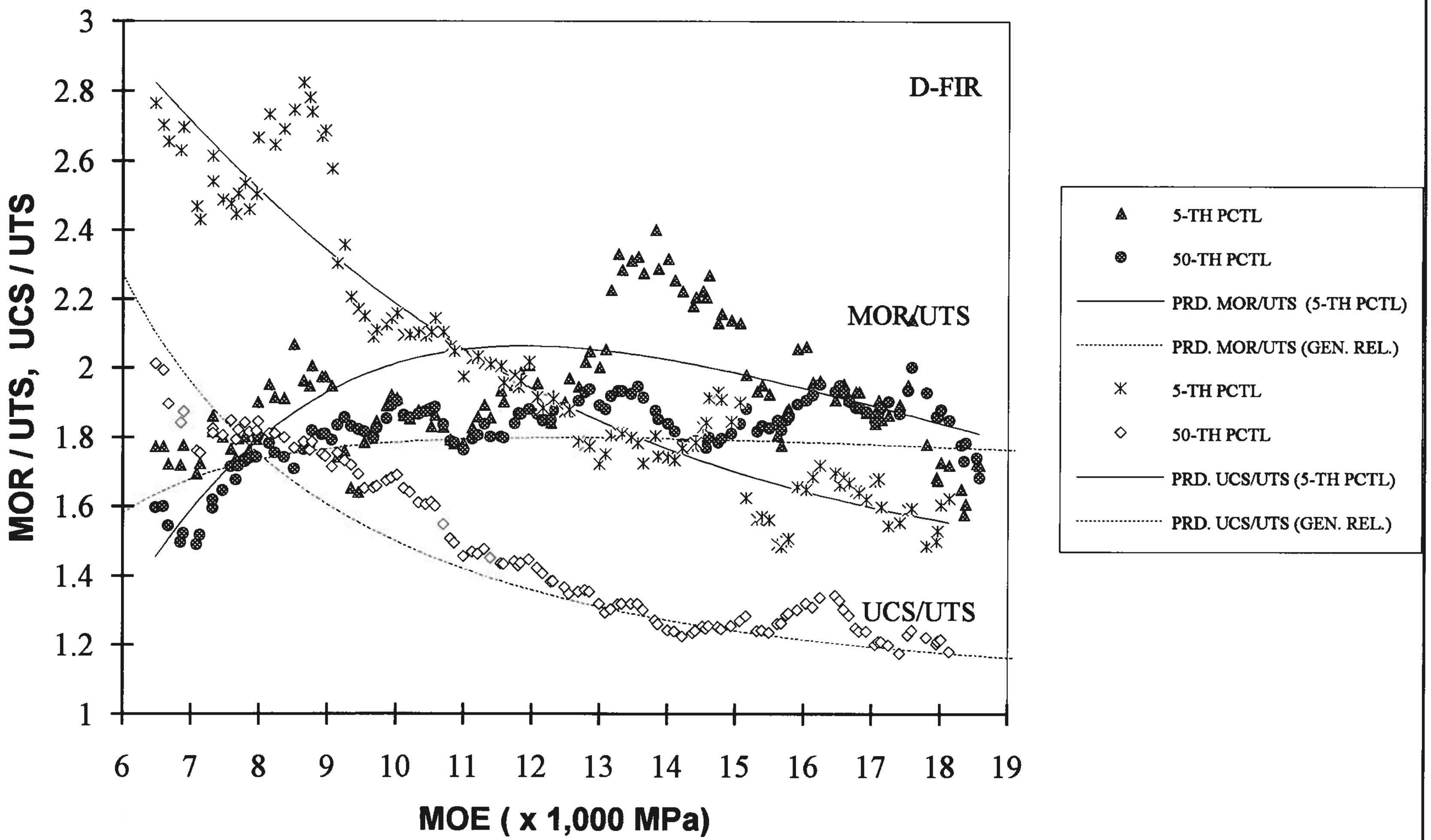


Figure 33. Continued

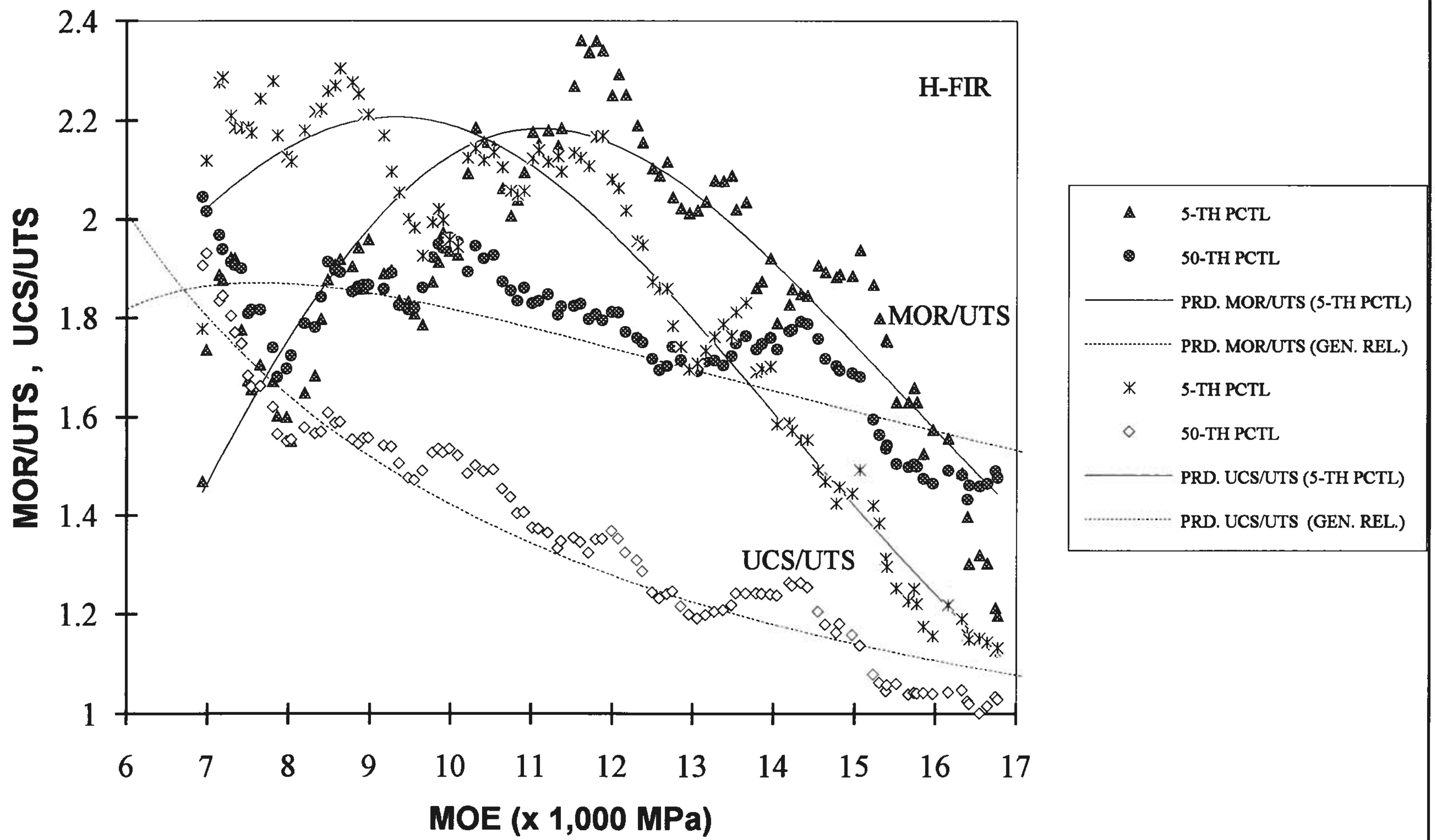


Figure 33. Continued

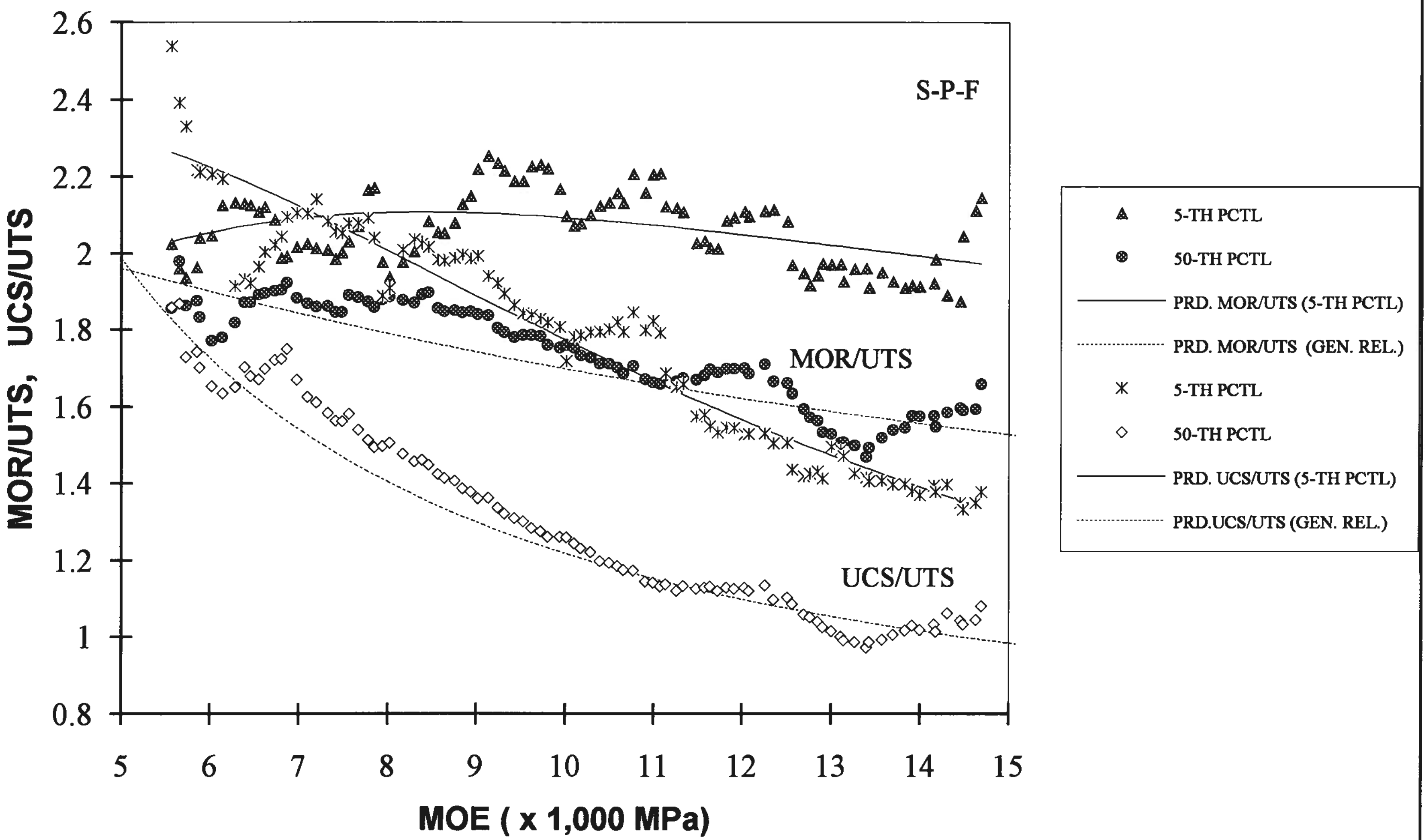
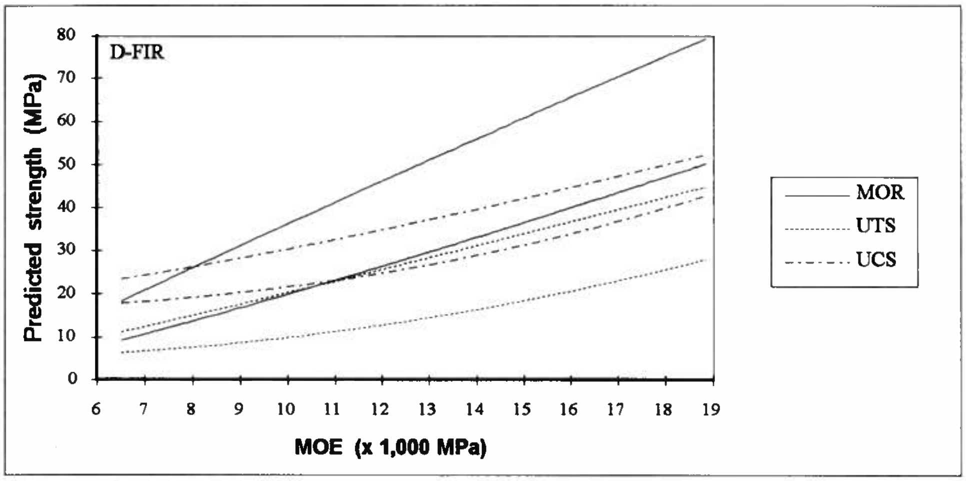
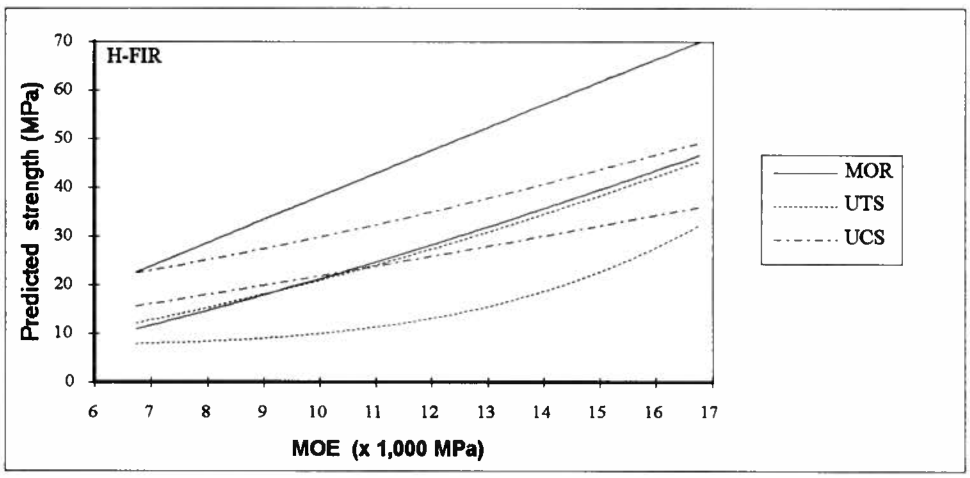


Figure 34. Relationships between MOE and the predicted strength (General Relationship and 5-th percentile)





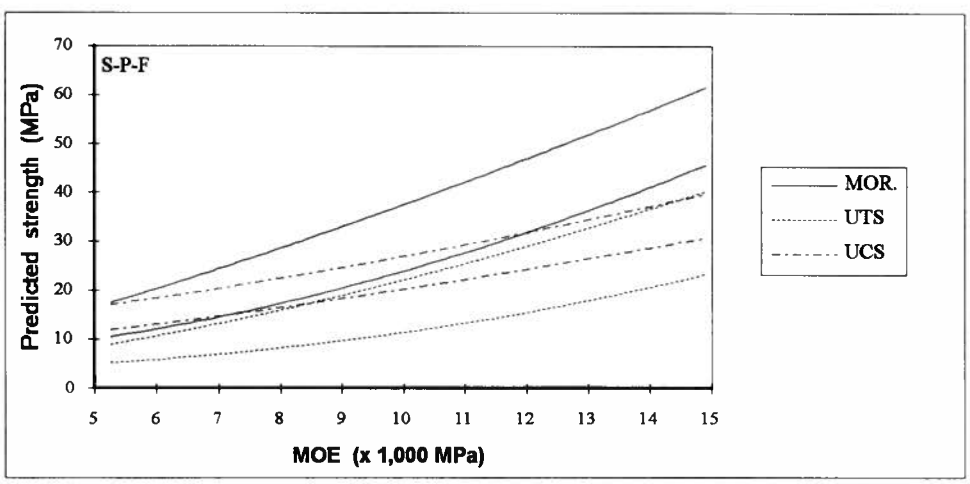
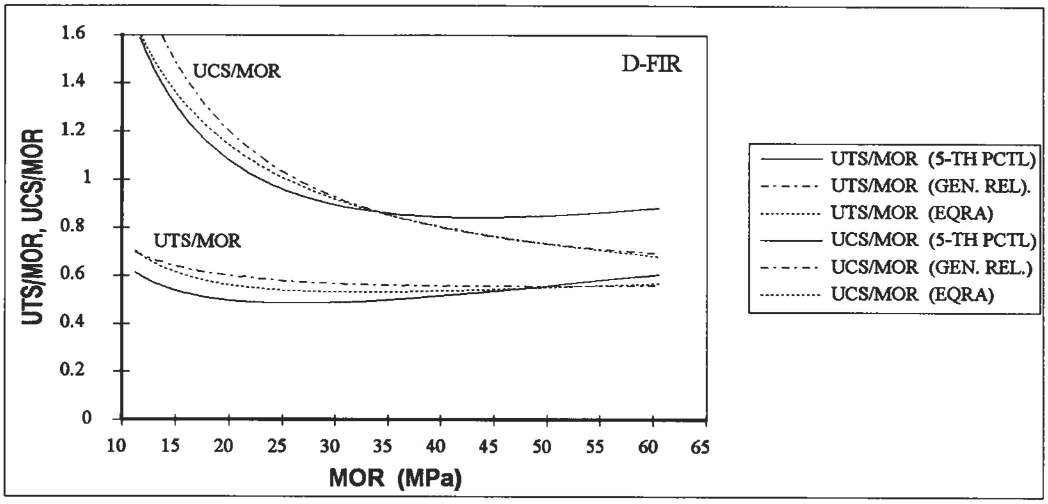
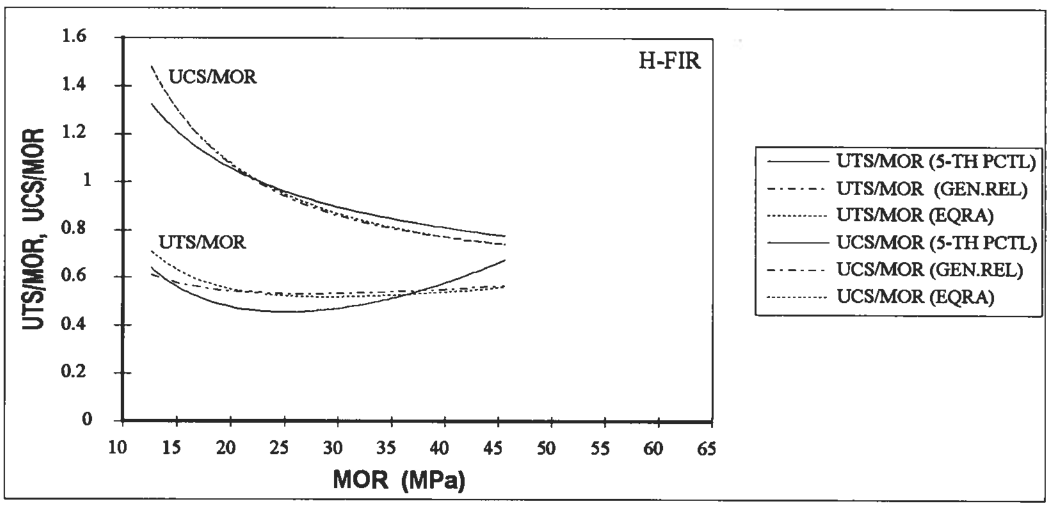


Figure 35. Comparison between 5-th percentile and mean trend for property ratios based on MOR





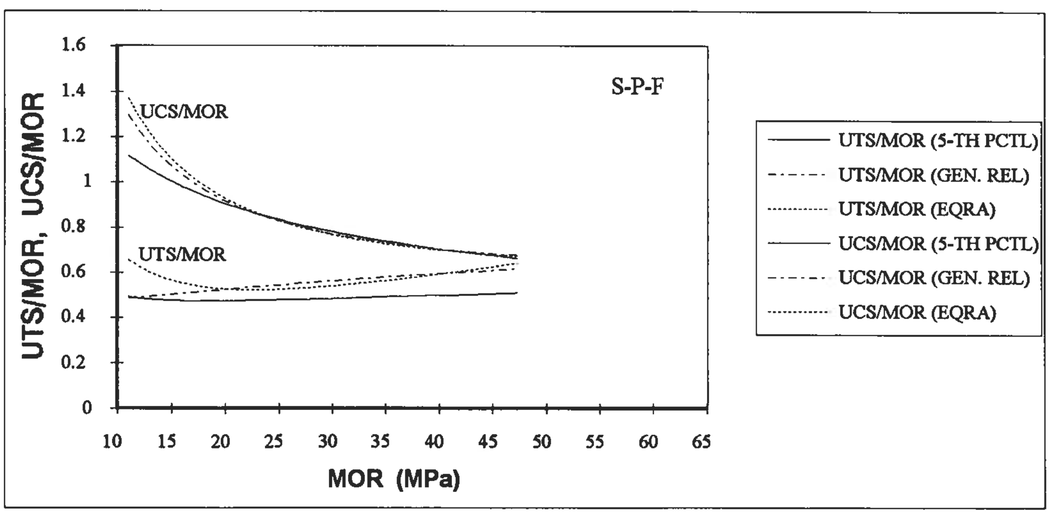
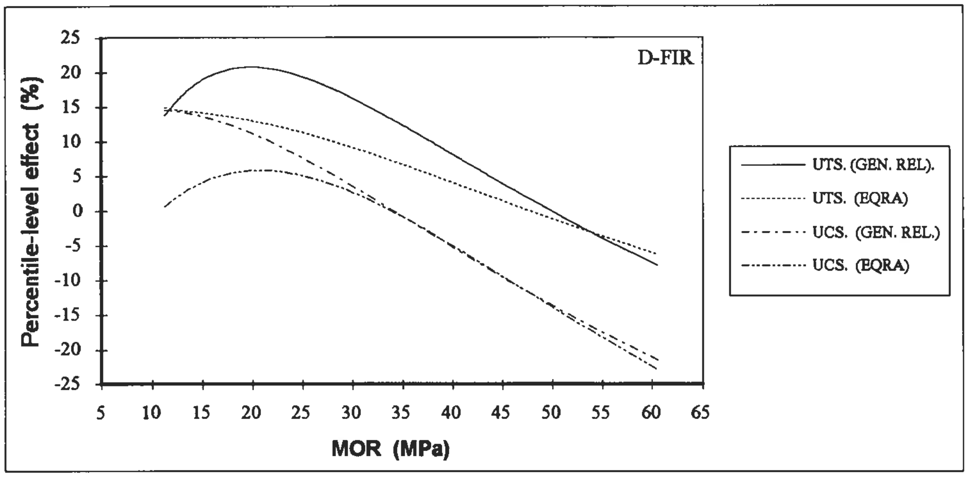
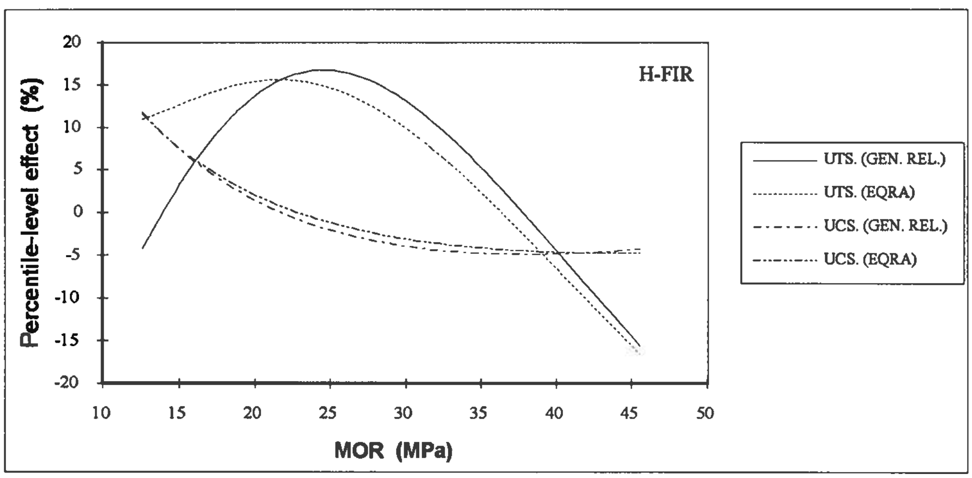


Figure 36. Percentile-level effects on predicted strength ratios based on MOR





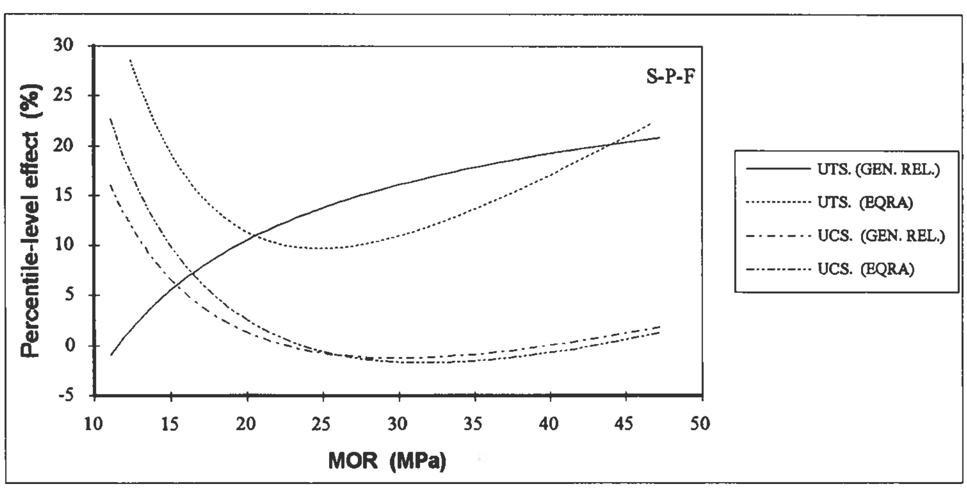
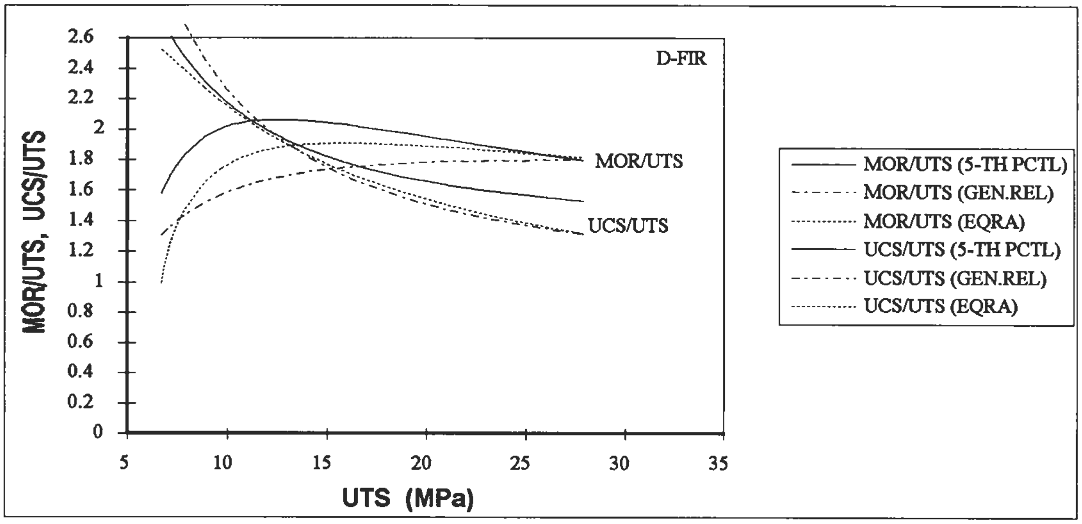
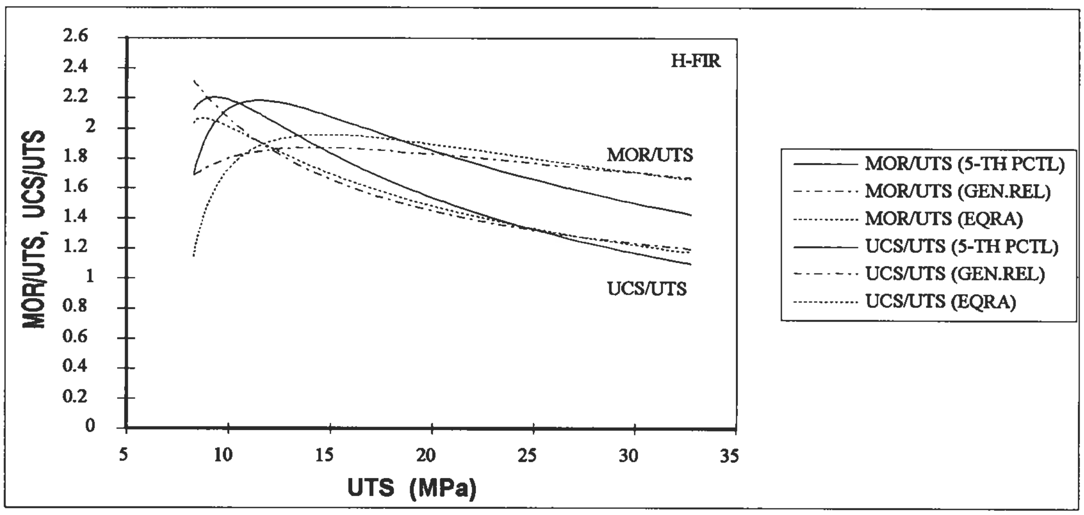


Figure 37. Comparison between 5-th percentile and mean trend for property ratios based on UTS





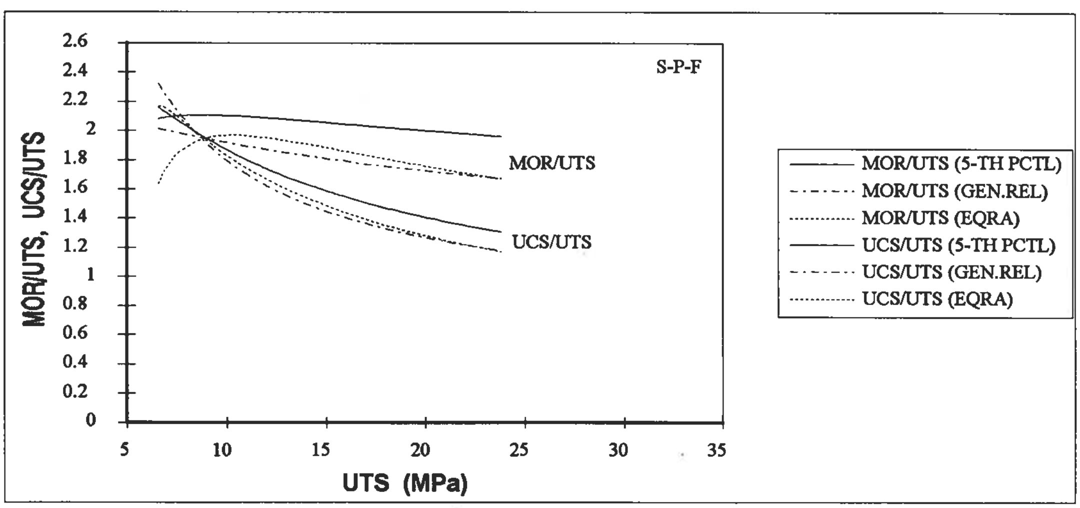
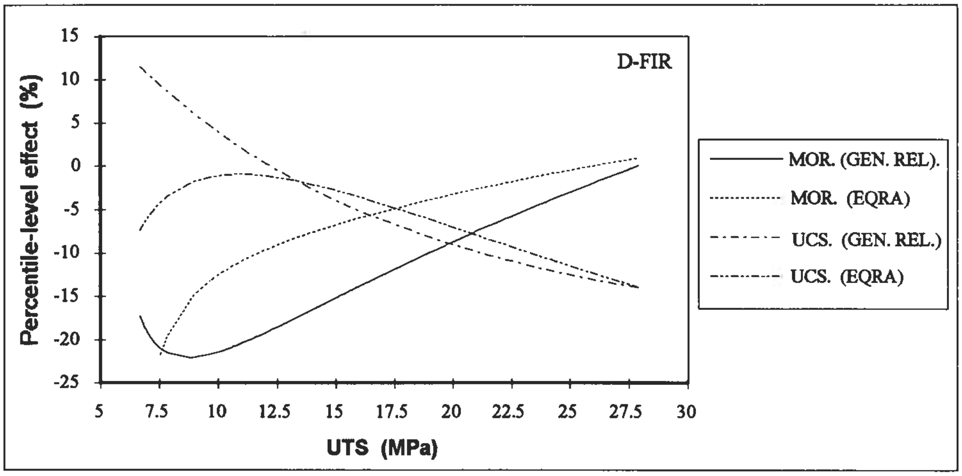
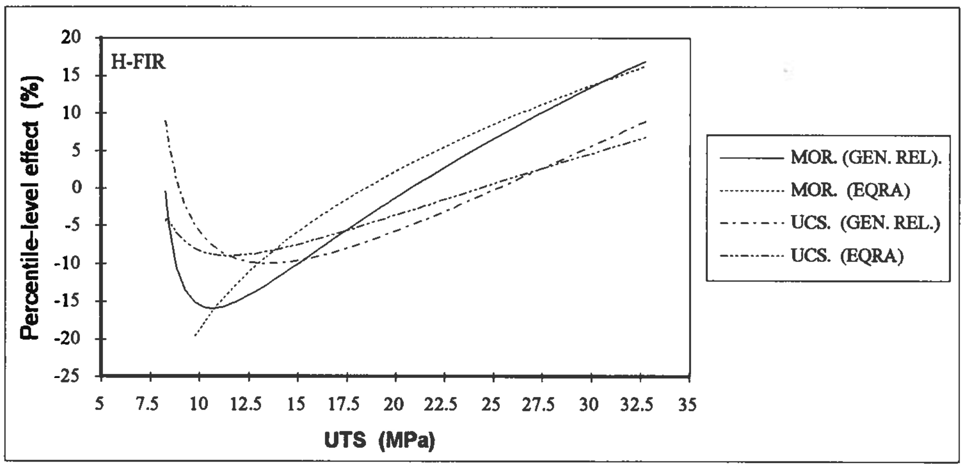


Figure 38. Percentile-level effects on predicted strength (based on UTS)





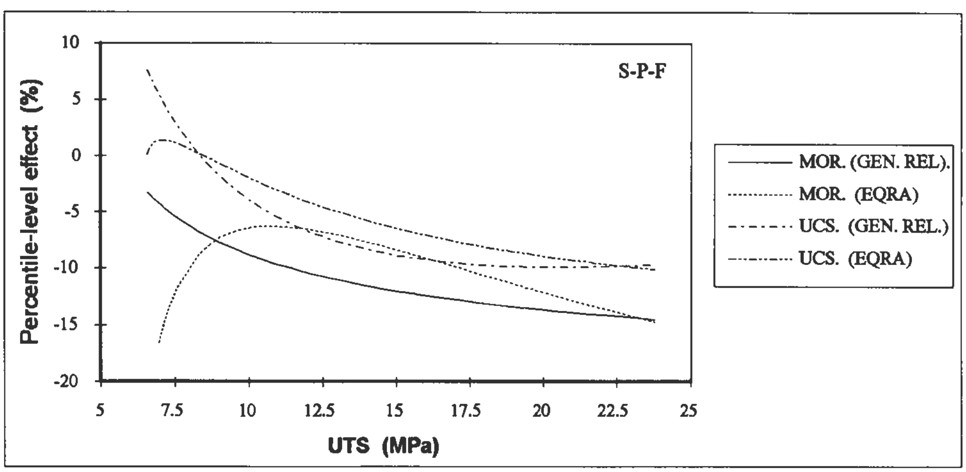


Figure 39. Error in the prediction of MOR as the percentage of the given MOR