PERFORMANCE OF LAMINATED VENEER WOOD PLATES
IN DECKING SYSTEMS

by
Frank Chung-Fat Lam
B.A.Sc. (Civil Engineering) University of British Columbia, 1982
M.A.Sc. (Civil Engineering) University of British Columbia, 1985

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY
in
THE FACULTY OF GRADUATE STUDIES
Department of Forestry

We accept this thesis as conforming
to the required Standard

The University of British Columbia
August, 1992
© Frank Chung-Fat Lam, 1992
In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the head of my department or by his or her representatives. It is understood that copying or publication of this thesis thesis for financial gain shall not be allowed within my written permission.

Faculty of Graduate Studies
Department of Forestry
The University of British Columbia
Vancouver, B.C.
Canada

Date: August 5, 1992
ABSTRACT

A new laminated veneer wood plate has been developed which is a specialty product intended for highly engineered end-use including flat bed truck and dry freight van trailer decking systems. The investigation described in this thesis represents the first known attempt to 1) develop a theoretical framework for evaluating the performance of laminate veneer panels in prototype dry freight van trailer decking systems, 2) develop a testing facility to generate an experimental database on these product through full scale testing, and 3) develop and model the fatigue behavior of this type of product through small specimen tests and damage accumulation laws.

A structural analysis model has been developed to predict the structural behavior of a prototype decking system. A comprehensive database on the mechanical properties of 3.2 and 2.5 mm ($\frac{1}{8}$ and $\frac{1}{10}$ inch) thick Douglas-fir veneers has been generated through experimental studies as input to the model. The database includes information on bending, tension, and compression strength properties, shear moduli of rigidity, ultrasonic transmission time, and connection stiffness. Analyses of variance has indicated that the mean strength properties of 3.2 and 2.5 mm thick veneers are significantly different for the parallel to grain direction but not significantly different for the perpendicular to grain direction at the 95% probability level. Statistical information and distributions parameters have been established for the various veneer strength properties so that simulations can be performed to model the strength properties of the veneers.

A trailer decking load simulator test facility has been developed so that full scale testing of prototype dry freight van trailer decking systems can be performed. The experimental program has been divided into two phases: 1) static test program and 2) cyclic test program. Four prototype decking systems have been considered. The static test program has generated information on the structural behavior of the panels in the prototype system. Experimental results agree well with predictions from the computer model. The cyclic test program has generated information on the performance of the prototype system under fatigue condition.
Information on the fatigue behavior of the panels in a system has been established from testing of small specimens in bending mode with the appropriate stress history. The relationship between the fatigue life and failure mode of the small specimen tests and the full size panels in system has been established. Damage accumulation laws have been developed from the small specimen tests results which provide a basis for evaluation of the fatigue behavior of the material in decking systems.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>ii</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>viii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>xi</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>xv</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2. BACKGROUND</td>
<td>7</td>
</tr>
<tr>
<td>2.1 Structural Analysis Models for Floor Systems</td>
<td>7</td>
</tr>
<tr>
<td>2.2 Material Properties</td>
<td>11</td>
</tr>
<tr>
<td>2.2.1 Elastic Properties of Laminated Veneer Wood Plates</td>
<td>11</td>
</tr>
<tr>
<td>2.2.2 Fatigue Data</td>
<td>12</td>
</tr>
<tr>
<td>2.3 Full Scale Testing of Dry Freight Van Trailer Decking Assembly</td>
<td>14</td>
</tr>
<tr>
<td>2.4 Modeling Fatigue Behavior in Wood Panel Products</td>
<td>15</td>
</tr>
<tr>
<td>3. STRUCTURAL ANALYSIS MODEL</td>
<td>18</td>
</tr>
<tr>
<td>3.1 Introduction</td>
<td>18</td>
</tr>
<tr>
<td>3.2 Strain Energy in the Cover</td>
<td>19</td>
</tr>
<tr>
<td>3.3 Strain Energy in the Supporting I-beams</td>
<td>22</td>
</tr>
<tr>
<td>3.4 Strain Energy in the Connectors</td>
<td>24</td>
</tr>
<tr>
<td>3.5 Potential Energy in the Applied Load</td>
<td>25</td>
</tr>
<tr>
<td>3.6 Finite Strip Formulation</td>
<td>26</td>
</tr>
<tr>
<td>3.6.1 Cover</td>
<td>27</td>
</tr>
<tr>
<td>3.6.2 Supporting I-beams</td>
<td>36</td>
</tr>
<tr>
<td>3.6.3 Connectors</td>
<td>36</td>
</tr>
<tr>
<td>3.6.4 Load Potential</td>
<td>37</td>
</tr>
</tbody>
</table>
# TABLE OF CONTENTS

3.7 Minimization of Energy in System 39

3.7.1 Cover 40

3.7.2 Supporting I-beams 43

3.7.3 Connectors 43

3.7.4 Load Potential 44

3.8 Numerical Solution of Global System of Equations 45

3.9 Shear Deflection of the Supporting I-beams 47

3.10 Bending Stresses and Rolling Shear Stresses in Cover 48

3.11 Computing Environment and Efficiency 57

3.12 Sample Problems 58

3.12.1 Program Input 58

3.12.2 Program Output 61

4. VENEER MECHANICAL PROPERTIES TESTING PROGRAM 81

4.1 Ultrasonic Veneer Testing Program 81

4.2 Mechanical Properties Test Program 82

4.2.1 Materials and Methods 82

4.2.1.1 Bending Tests 84

4.2.1.2 Tension Tests 88

4.2.1.3 Compression Tests 90

4.2.1.4 Shear Modulus of Rigidity Tests 94

4.2.1.5 Connector Load Slip Tests 95

4.2.2 Results 99

4.2.2.1 Veneer Strength Properties Statistics 99

4.2.2.2 Effects of Veneer Thickness and Number of Plies 100
# TABLE OF CONTENTS

4.2.2.3 Correlations of Veneer Strength Properties 127
4.2.2.4 Effectiveness of Ultrasonic Testing on Veneer Grading 133
4.2.2.5 Connector Stiffness 136

5. FULL SCALE TESTING OF DRY FREIGHT VAN TRAILER DECK ASSEMBLY 141
5.1 Testing Facility 141
5.2 Materials and Methods 149
5.2.1 Static Test Program 153
5.2.2 Cyclic Test Program 155
5.2.3 Short Term Small Specimen Bending Tests 156
5.3 Experimental Results 156
5.3.1 Static Test Program 156
5.3.1.1 Verification of Deck Analysis Model 158
5.3.2 Cyclic Test Program 172

6. CYCLIC TESTING OF SMALL SPECIMENS 187
6.1 Materials and Methods 187
6.1.1 Short Term Bending Tests 188
6.1.2 Cyclic Bending Tests 188
6.1.2.1 Applied Stress Levels 190
6.2 Experimental Results 192
6.2.1 Short Term Bending Tests 192
6.2.2 Cyclic Bending Tests 199

7. MODELING OF SMALL SPECIMEN FATIGUE PROPERTIES 208
7.1 Ramp Load Case 209
7.2 Piecewise Linear Representation of Stress History 210
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.3 Representation of Stress History by a Series of Stress Pulses</td>
<td>214</td>
</tr>
<tr>
<td>7.4 Model Calibration</td>
<td>217</td>
</tr>
<tr>
<td>7.4.1 Calibration Results</td>
<td>219</td>
</tr>
<tr>
<td>8. CONCLUSIONS</td>
<td>231</td>
</tr>
<tr>
<td>8.1 Summary and Conclusions</td>
<td>231</td>
</tr>
<tr>
<td>8.2 Future Research</td>
<td>233</td>
</tr>
<tr>
<td>9. REFERENCES</td>
<td>235</td>
</tr>
</tbody>
</table>
LIST OF TABLES

Table 1. Descriptions of the various case studies in the example problem. 62
Table 2. Summary deflection results of the supporting I-beams in the eight case studies. 63
Table 3. Summary bending stress results of the supporting I-beams in the eight case studies. 64
Table 4. Maximum deflections and stresses in the cover of the eight case studies. 65
Table 5. Locations of maximum stresses in the cover of the eight case studies. 67
Table 6. Test span for the bending specimens. 85
Table 7. Specimen depths for the compression tests. 91
Table 8. Specimen sizes for the shear modulus of rigidity tests. 94
Table 9. Classification of veneer test specimens. 99
Table 10. Statistical data on the veneer elastic moduli. 101
Table 11. Statistical data on the veneer strengths. 102
Table 12. Analysis of variance results on veneer bending strength properties. 110
Table 13. Analysis of variance results on veneer tension strength properties. 111
Table 14. Analysis of variance results on veneer compression strength properties. 112
Table 15. Analysis of variance results on veneer modulus of rigidity. 113
Table 16. Duncan’s multiple range test results for the various groups. 114
Table 17. Statistical data and distribution parameters of the veneer bending strength properties. 123
Table 18. Statistical data and distribution parameters of the veneer tension strength properties. 124
Table 19. Statistical data and distribution parameters of the veneer compression strength properties. 125
Table 20. Statistical data and distribution parameters of the veneer shear modulus of rigidity. 126
### LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 21</td>
<td>The dependent and independent variables considered in the various regression models of veneer strength properties.</td>
<td>128</td>
</tr>
<tr>
<td>Table 22</td>
<td>Results of various regression models of 3.2 mm veneer strength properties for the parallel to grain direction.</td>
<td>129</td>
</tr>
<tr>
<td>Table 23</td>
<td>Results of various regression models of 2.5 mm veneer strength properties for the parallel to grain direction.</td>
<td>130</td>
</tr>
<tr>
<td>Table 24</td>
<td>Results of various regression models of 3.2 mm veneer strength properties for the perpendicular to grain direction.</td>
<td>131</td>
</tr>
<tr>
<td>Table 25</td>
<td>Results of various regression models of 2.5 mm veneer strength properties for the perpendicular to grain direction.</td>
<td>132</td>
</tr>
<tr>
<td>Table 26</td>
<td>Statistical data on the veneer strength properties of the various subgroups.</td>
<td>134</td>
</tr>
<tr>
<td>Table 27</td>
<td>Statistical data on the veneer elastic properties of the various subgroups.</td>
<td>135</td>
</tr>
<tr>
<td>Table 28</td>
<td>Duncan's multiple range test results for the various groups.</td>
<td>137</td>
</tr>
<tr>
<td>Table 29</td>
<td>Summary of connection load slip tests.</td>
<td>139</td>
</tr>
<tr>
<td>Table 30</td>
<td>Analysis of variance results and Duncan multiple range test results on connector stiffness.</td>
<td>139</td>
</tr>
<tr>
<td>Table 31</td>
<td>Trailer decking load simulator calibration results.</td>
<td>146</td>
</tr>
<tr>
<td>Table 32</td>
<td>Peak midspan deflections under 40 kN front axle loading.</td>
<td>161</td>
</tr>
<tr>
<td>Table 33</td>
<td>Front axle load levels versus peak midspan prototype deformation.</td>
<td>162</td>
</tr>
<tr>
<td>Table 34</td>
<td>Veneer elastic moduli for DAP analyses.</td>
<td>164</td>
</tr>
<tr>
<td>Table 35</td>
<td>Input panel stiffness values for DAP analyses.</td>
<td>164</td>
</tr>
<tr>
<td>Table 36</td>
<td>Statistics of panel stiffness values for DAP analyses.</td>
<td>170</td>
</tr>
<tr>
<td>Table 37</td>
<td>Cyclic test results of full scale prototypes.</td>
<td>173</td>
</tr>
</tbody>
</table>
LIST OF TABLES

Table 38. Regression parameters for the $F_R$ and $N_f$ relationships of the four prototype assemblies. 174
Table 39. Results of regression approach to analysis of covariance of the $F_R$ and $N_f$ relationships. 177
Table 40. Results of regression approach to analysis of covariance of the $S_R$ and $N_f$ relationships. 180
Table 41. Short term static bending test results of various prototypes. 185
Table 42. Short term small specimen static bending test results of various prototypes. 194
Table 43. Analysis of variance results on parallel to grain bending strengths. 197
Table 44. Small specimen cyclic bending test results. 201
Table 45. The parameters describing piecewise linear segments of a nondimensional stress cycle. 211
Table 46. The mean and standard deviation of the model parameters for prototypes 1 and 2. 222
Table 47. Model predicted small specimen fatigue performance. 225
Table 48. Model predicted full scale panel fatigue performance. 227
LIST OF FIGURES

Figure 1  Veneer lay up of transDeck™. 4
Figure 2  A prototype dry freight van trailer decking system. 10
Figure 3  Finite strip representation of a section of decking assembly. 20
Figure 4  Degrees of freedom for a supporting I-beam. 23
Figure 5  Deformation profile of cover: case 7. 71
Figure 6  Bending stress profile σ_y in exterior ply of cover: case 7. 72
Figure 7  Bending stress profile σ_x in exterior ply of cover: case 7. 73
Figure 8  Rolling shear stress profile τ_yz in cross ply of cover: case 7. 74
Figure 9  Rolling shear stress profile τ_xz in cross ply of cover: case 7. 75
Figure 10 Deformation profile of cover: case 6. 76
Figure 11 Bending stress profile σ_y in exterior ply of cover: case 6. 77
Figure 12 Bending stress profile σ_x in exterior ply of cover: case 6. 78
Figure 13 Rolling shear stress profile τ_yz in cross ply of cover: case 6. 79
Figure 14 Rolling shear stress profile τ_xz in cross ply of cover: case 6. 80
Figure 15 Sonic transmission time cumulative probability distributions for 2.5 and 3.2 mm thick veneer. 83
Figure 16 The veneer bending test set up. 86
Figure 17 The veneer parallel to grain tension test set up. 89
Figure 18 The veneer compression test set up. 92
Figure 19 The veneer shear modulus of rigidity test set up. 96
Figure 20 The connector load slip test set up. 97
Figure 21 The cumulative probability distributions of veneer bending modulus of elasticity. 103
Figure 22 The cumulative probability distributions of veneer bending strength. 104
Figure 23 The cumulative probability distributions of veneer tension modulus of elasticity. 105
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 24</td>
<td>The cumulative probability distributions of veneer tension strength.</td>
<td>106</td>
</tr>
<tr>
<td>Figure 25</td>
<td>The cumulative probability distributions of veneer compression modulus of elasticity.</td>
<td>107</td>
</tr>
<tr>
<td>Figure 26</td>
<td>The cumulative probability distributions of veneer compression strength.</td>
<td>108</td>
</tr>
<tr>
<td>Figure 27</td>
<td>The cumulative probability distributions of veneer shear modulus of rigidity.</td>
<td>109</td>
</tr>
<tr>
<td>Figure 28</td>
<td>The cumulative probability distributions and the 3-parameter Weibull distributions of the veneer bending modulus of elasticity of each group.</td>
<td>116</td>
</tr>
<tr>
<td>Figure 29</td>
<td>The cumulative probability distributions and the 3-parameter Weibull distributions of the veneer bending strength of each group.</td>
<td>117</td>
</tr>
<tr>
<td>Figure 30</td>
<td>The cumulative probability distributions and the 3-parameter Weibull distributions of the veneer tension modulus of elasticity of each group.</td>
<td>118</td>
</tr>
<tr>
<td>Figure 31</td>
<td>The cumulative probability distributions and the 3-parameter Weibull distributions of the veneer tension strength of each group.</td>
<td>119</td>
</tr>
<tr>
<td>Figure 32</td>
<td>The cumulative probability distributions and the 3-parameter Weibull distributions of the veneer compression modulus of elasticity of each group.</td>
<td>120</td>
</tr>
<tr>
<td>Figure 33</td>
<td>The cumulative probability distributions and the 3-parameter Weibull distributions of the veneer compression strength of each group.</td>
<td>121</td>
</tr>
<tr>
<td>Figure 34</td>
<td>The cumulative probability distributions and the 3-parameter Weibull distributions of the veneer shear modulus of rigidity of each group.</td>
<td>122</td>
</tr>
<tr>
<td>Figure 35</td>
<td>Connector load deformation curves.</td>
<td>138</td>
</tr>
<tr>
<td>Figure 36</td>
<td>The trailer decking load simulator.</td>
<td>142</td>
</tr>
<tr>
<td>Figure 37</td>
<td>Relationship between the front to rear axles load ratios and wheel cart load cell readings.</td>
<td>147</td>
</tr>
<tr>
<td>Figure 38</td>
<td>Relationship between the front axle loadings and wheel cart load cell readings.</td>
<td>148</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

Figure 39  Prototype dry freight van trailer decking system used in the full scale test program. 150
Figure 40  Prototype dry freight van trailer decking system during a static test. 154
Figure 41  Midspan deformation profiles of the four prototype deck assemblies with the front
            axle located directly on Bay 9. 157
Figure 42  Midspan deformation profiles of the four prototype deck assemblies with the front
            axle centered between Bays 2 and 3. 159
Figure 43  Midspan deformation profiles of the four prototype deck assemblies with the front
            axle located directly on Bay 13. 160
Figure 44  Comparisons of model predictions and measured midspan deformation profiles of
            the four prototype deck assemblies with the front axle located directly on Bay 9. 166
Figure 45  Comparisons of model predictions and measured midspan deformation profiles of
            the four prototype deck assemblies with the front axle centered between Bays 2
            and 3. 167
Figure 46  Comparisons of model predictions and measured midspan deformation profiles of
            the four prototype deck assemblies with the front axle located directly on Bay 13. 168
Figure 47  Upper and lower bounds of measured midspan deformation profiles of prototype 1
            with the front axle located directly on Bay 9. 171
Figure 48  Performance of transDeck\textsuperscript{TM} under fatigue loading. 175
Figure 49  Normalized fatigue performance of transDeck\textsuperscript{TM} with regular veneer. 183
Figure 50  Normalized fatigue performance of transDeck\textsuperscript{TM} with special veneer. 184
Figure 51  Parallel to grain bending stress profiles for prototypes 1 and 2. 191
Figure 52  Nondimensional profiles versus time for prototypes 1 and 2. 193
Figure 53  Displacement controlled parallel to grain bending strength cumulative probability
            distributions. 195
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>54</td>
<td>Displacement controlled perpendicular to grain bending strength cumulative</td>
<td>196</td>
</tr>
<tr>
<td></td>
<td>probability distributions.</td>
<td></td>
</tr>
<tr>
<td>55</td>
<td>Load controlled parallel to grain bending strength cumulative probability</td>
<td>198</td>
</tr>
<tr>
<td></td>
<td>distributions.</td>
<td></td>
</tr>
<tr>
<td>56</td>
<td>Small Specimen during a cyclic bending test.</td>
<td>200</td>
</tr>
<tr>
<td>57</td>
<td>Typical rolling shear failure mode of a specimen under cyclic loading.</td>
<td>202</td>
</tr>
<tr>
<td>58</td>
<td>Fatigue data of small specimens in bending.</td>
<td>203</td>
</tr>
<tr>
<td>59</td>
<td>Comparisons of fatigue performance of small specimens and full size panels.</td>
<td>205</td>
</tr>
<tr>
<td>60</td>
<td>Representation of stress cycle with stress pulses.</td>
<td>215</td>
</tr>
<tr>
<td>61</td>
<td>Cumulative probability distribution of $N_f$ for Prototype 1.</td>
<td>220</td>
</tr>
<tr>
<td>62</td>
<td>Cumulative probability distribution of $N_f$ for Prototype 2.</td>
<td>221</td>
</tr>
<tr>
<td>63</td>
<td>Comparisons of model predicted and actual cumulative probability distribution</td>
<td>223</td>
</tr>
<tr>
<td></td>
<td>of $N_f$ for Prototype 1.</td>
<td></td>
</tr>
<tr>
<td>64</td>
<td>Comparisons of model predicted and actual cumulative probability distribution</td>
<td>224</td>
</tr>
<tr>
<td></td>
<td>of $N_f$ for Prototype 2.</td>
<td></td>
</tr>
<tr>
<td>65</td>
<td>Comparisons of model predicted and actual fatigue performance of small</td>
<td>228</td>
</tr>
<tr>
<td></td>
<td>specimens.</td>
<td></td>
</tr>
<tr>
<td>66</td>
<td>Comparisons of model predicted and actual fatigue performance of full size</td>
<td>229</td>
</tr>
<tr>
<td></td>
<td>panels.</td>
<td></td>
</tr>
</tbody>
</table>
I would like to thank Drs. J.D. Barrett and R.O. Foschi for their guidance during the research. Dr. S. Avramidis is thanked for serving on the supervisory committee.

Mr. R. J-M. Fouquet, the inventor of transDeck™, is thanked for sharing his knowledge on the product and his active role in setting up important linkages between this research project and the industry.

The following technical staff members of the Department of Harvesting and Wood Science are acknowledged for their invaluable contributions into the experimental program: Mr. G. Lee, Mr. J. Bernaldez, Mr. D. Trickett, Mr. R. Johnson, and Mr. B. Myronuk.

Finally, Ainsworth Lumber Company Ltd., Forestry Canada, Natural Science and Engineering Research Council and the Department of Harvesting and Wood Science are thanked for their support with finances, equipment, and materials.
1. INTRODUCTION

Laminated veneer wood products such as parallel laminated veneer lumber and plywood are common construction materials. Applications of parallel laminated veneer lumber include beams, flanges, headers, columns, and truss chord members. Plywood, on the other hand, is used mainly as sheathing for floors, roofs and walls, web members for wooden I beams or box beams, stress-skin panels, form work, and connection gussets. Laminated wood plates such as plywood play a very important role in typical wooden floor and roof systems. These structural systems behave as stiffened plates where the applied loads are carried by composite action. Interaction between the supporting framing members and the panel cover makes possible the sharing of loads amongst the supporting members. This is especially important when the stiffnesses of supporting members are nonuniform and when the ratio of stiffness between cover and framing members is not large.

In some design codes, the design stress computed for a single beam in a system can be multiplied by a system modification factor to account for system behavior. For example, Foschi et al. (1989) obtained an average system modification factor of 1.34 for bending members in joists supported floor and roof systems. Similarly Lam and Varoğlu (1988) obtained a system modification factor of 1.10 for tension members in parallel chord trusses supporting roof systems. Therefore in these structural systems where the parallel supporting members were spaced not more than 600 mm apart, significant increase in load carrying capacity in the system over the load carrying capacity of individual members can be expected. It is evident that engineered application of laminated wood plates in new markets, where the demand for quality and performance is high, should be explored to take advantage of the composite action and load sharing behavior of stiffened plate systems.

Decking systems for commercial trucks and trailers are required to perform in a demanding environment. In these systems high concentrated loads are repeatedly applied onto the decking by the wheels of lift trucks. Also the decking systems are exposed to weathering. The expected service life of
these systems is typically three years. Traditionally planks made from Apitong or Keruing species are widely used in commercial truck and trailer decking. Apitong is a commercial species designation for a group of over fifteen hardwood species grown in Malaysia with superior structural properties, wear and decay resistance; therefore, it is widely accepted as the preferred species group for commercial truck and trailer decking. Planks from certain North American hardwood and softwood species are also accepted in truck and trailer decking applications where weight optimization is of primary concern and the limited life expectancy of the softwood decking can be tolerated by the end user.

In a limited volume, traditional softwood plywood panels are also being used as flooring materials for trucks and trailers. Factors limiting wider acceptance of the plywood structural panels include: 1) inferior structural capabilities in truck and trailer decking application, 2) poor wear resistance, and 3) the inherent sensitivity of wooden panels to changes in moisture content. Increasing concern about the continued availability of Apitong hardwood resource in suitable quality, the irregularity of its supply, its relatively high cost and sensitivity to edge and surface damage due to impact, coupled with a need to reduce the dead weights for increased performance in vehicles have led to an opportunity for softwood laminated veneer wood plates for decking material provided its structural performance, wear resistance performance, and moisture resistance characteristics can conform with the end-use requirements.

Ainsworth Lumber Company Ltd. has recently developed a new innovative concept and technology for manufacturing a specialized value-added laminated veneer wood plate called transDeck™. It is intended for highly engineered end-uses such as the market for truck and trailer decking systems. A patent application was granted by the U.S. Patent Office on the manufacturing concept of this product (Fouquet, 1991). The background research for the development of transDeck™ was documented in two confidential research notes from Council of Forest Industries of British Columbia for Ainsworth Lumber Company Ltd. (Parasin and Nagy, 1989 and 1990).

TransDeck™ is made from either 2.5 mm (1/10 inch) or 3.2 mm (1/8 inch) or combinations of 2.5
mm and 3.2 mm thick veneers. The current study focuses on transDeck™ panels made from either 2.5 mm or 3.2 mm thick veneers. Such panels consist of 11 plies. A typical veneer lay up scheme of these transDeck™ panels is shown in Figure 1. The lay up is a 3-1-3-1-3 scheme where the exterior and center of the panel consist of 3-ply veneers with the grain running in the longitudinal direction of the panel. The two interior cross plies, each consisting of single ply veneer oriented 90° to the longitudinal direction, are sandwiched by the 3-ply veneers. This particular lay up was judged to be most appropriate for decking of truck and trailer systems based on results of the pilot research projects (Parasin and Nagy, 1989 and 1990).

The durability and moisture resistance requirements of the prototype product for truck and trailer decking systems were resolved by introducing proprietary surface treatments and sealants. The surfaces of prototype transDeck™ panels were covered by a proprietary paper product which was normally used in concrete forming panels. Since the propriety paper product performed very well in concrete forms, it was judged that it would perform equally well in decking systems for trucks and trailers. All edges were sealed by a moisture sealant which demonstrated only a 3% pick up of moisture (by weight) in a 24 hour soak test while untreated specimen showed over 20% water pick up (by weight). It was judged that transDeck™ would be able to meet or exceed the end-use requirements in terms of durability and moisture resistance for truck and trailer decking systems.

Structural performance is also a crucial requirement in truck and trailer decking applications. The trucking industry has developed performance standards based on loadings applied by lift trucks operating on a prototype dry freight van trailer decking assembly. A cyclic proof load test is specified as structural performance test by the Truck Trailer Manufacturers Association (TTMA) which is the industry association of the U.S. trailer manufacturers (Truck Trailer Manufacturers Association, 1989). In previous evaluations, both parallel-laminated veneer and plywood type constructions failed to satisfy these performance requirements. In parallel-laminated veneer products the perpendicular to grain strengths were too low whereas in the traditional alternate cross-band plywood type constructions the
Figure 1 Veneer lay up of transDeck™.
parallel to grain capacities were inadequate at the thicknesses required for the truck deck application.

Previous in-house research by Ainsworth Lumber Company Ltd. demonstrated that the structural requirements of TTMA were satisfied for certain specific lay-ups of Douglas-fir veneer. A theoretical framework for evaluating the performance of the product and experimental data required for evaluation of alternative constructions and species combination are lacking. This thesis addresses these issues to provide a basis for eventual optimization of the design and in-service performance of transDeck™.

Improving the knowledge of the static structural behavior of laminated wood plates requires the development and verification of an appropriate structural analysis model to correctly predict the structural behavior of a prototype decking system. Implementation of such a model requires as input data the appropriate elastic material properties of the panels, the elastic properties of the supporting steel frame, and the connector stiffness between the panels and the steel frame.

Model verification can be achieved by comparing model predictions to results from either experiments or another verified model. In this case since no other verified model exists, full scale testing of prototype dry freight van trailer decking system is required. The cyclic testing program generates important experimental data on the behavior of the panels in system and provides information to guide the design and construction of the panels. Since full scale testing is very expensive and time consuming, it is practical to perform only a limited number of tests. This is especially true when the fatigue behavior of the panels under cyclic load is of interest. In these cases, testing at a number of stress levels is usually needed to develop a reliable database relating the load level and number of cycles to panel failure. Therefore, an alternate approach is needed to develop the data on fatigue behavior of the panels in a system. Such data can be established from testing of small specimens in the bending mode with the appropriate span to depth ratio and stress history. The relationship between the fatigue life and failure mode of the small specimen tests and the panels in system can be established to provide a basis for evaluation of the fatigue behavior of the alternate panels in the deck system.
The overall objective of the study is to develop a framework to evaluate the performance of laminated wood plates in decking systems. The overall objective can be divided into the following subobjectives:

1) to develop a computer analysis program to evaluate the static behavior of the prototype dry freight van trailer decking systems (Decking Analysis Program);

2) to develop material properties data required as input for DAP for panels comprising of 2.5 mm and 3.2 mm thick Douglas fir veneer;

3) to experimentally study the structural behavior of prototype decking systems, with laminated veneer panels made from 2.5 mm and 3.2 mm thick Douglas fir veneers, under static loading and to verify DAP using the experimental results;

4) to experimentally study the structural behavior of prototype decking systems under cyclic loading;

5) to develop a database on the structural behavior of small specimens under cyclic bending loads.

6) to use a damage-accumulation model to predict the relationships between load level and time to failure for cyclically loaded panels and beams specimens.
2. BACKGROUND

2.1 Structural Analysis Models for Floors

Finite element and finite strip methods are well known numerical methods commonly used in structural analysis problems. Both numerical methods can be used in the analysis of floor systems. In the finite element method, the displacement field is approximated by polynomial functions and the governing equations are replaced by a set of algebraic equations which are obtained by discretizing the continuum into a finite number of elements. The finite element method is very versatile as complicated geometry, boundary conditions, and loadings can be readily considered. The solution accuracy typically depends on the number of elements and the type of element used in the analysis.

The finite strip method is suitable for analysis of structures with regular boundaries where the structure can be divided in strips. Here the displacement field is approximated by continuously differentiable analytical functions in one direction and polynomial functions in the other direction. It should be noted that the kinematic boundary conditions at the end of the strip must be satisfied by the continuously differentiable analytical functions a priori. The solution accuracy typically depends on the choice of the continuously differentiable analytical functions.

The major advantage of the finite strip method over the finite element method is that less input data, less core memory, and reduced execution times are required. The major disadvantage of the finite strip method compared to the finite element method is that it is less versatile.

General purpose finite element analysis programs are available to analyze orthogonally stiffened plate structures. These models typically ignore the slip between the cover and the supporting beams; therefore, only special cases of wooden floor systems can be analyzed.

Thompson, Goodman, and Vanderbilt (1975) developed a computer program, FEAFLO, based on the finite element method to analyze the structural behavior of wooden floor systems. FEAFLO
considered the floor as a system of crossing beams which consisted of a series of T-beams and sheathing strips running perpendicular to the T-beams. The web of each T-beam represented a joist in the floor system. The flange of a T-beam represented part of the cover which contained either one or two layers of sheathing. The crossing sheathing strips allowed the adjacent T-beams to interact which contributed to the load sharing behavior of the system. The T-beams also took into consideration the slip between the cover and the joists by including the load slip behavior of the nails during the formulation of the stiffness matrix. Finite elements were used to model the T-beams and sheathing strips with their deformation matched at the points of intersection. The model was verified against test results through a complicated calibration procedure. The calibration steps included the estimation of the effective width of the cover for the T-beam and the introduction of zones of low stiffness in the cover to compensate for the restricted degrees of freedom in the model. Extraction of cover stresses from this model was considered not appropriate; therefore, FEAFLO was not adopted in the current study.

Foschi (1982) introduced finite strip representation of wooden floor systems by developing a computer model, Floor Analysis Program (FAP), which considered the strain energy of individual components in a wooden floor system: the cover, the joists, and the nail connectors. The deformation of the floor system was assumed to be represented by Fourier series in the direction parallel to the joists and by finite element approximation in the direction perpendicular to the joist. Gaps in the cover were considered by removing the contribution of strain energy in the cover occupied by gaps. Foschi (1982) noted that the off-diagonal stiffness submatrices were coupled when gaps and/or discrete nailing patterns were considered. This effectively increased execution time and reduced the major advantage of applying the finite strip method over the finite element method to structural analysis of floor systems. Despite of the reduced efficiency Foschi (1982) successfully implemented FAP in a microcomputer environment with small core memory. Finally results show that the model was still very efficient and program predictions agreed well with experiment results.

Chen et al. (1990) developed the B-spline compound strip method to analyze stiffened plates
under transverse loading. In B-spline compound strip method, finite element method was first employed to partially discretize a problem to an ordinary differential equation. The Ritz-Galerkin method was then used to solve the equation with unequally spaced cubic B-spline functions. The use of unequally spaced cubic B-spline functions allowed an accurate description of the response in regions of high stress concentration which might result from wheel loads in the case of lift truck loading on dry freight van trailer decking systems. Also the boundary conditions at the end of each spline can be prescribed to represent the fixity in the structure. The model, however, has not dealt with semi-rigid connectors between the plate and the stiffeners nor gaps in the cover plate which are important features in dry freight van trailer decking systems.

From these models, FAP analysis concepts, based on the finite strip method, appear to be most suited for structural analysis of prototype dry freight van trailer decking system. However, as shown in Figure 2, the prototype decking system contains a gap along the center line C-C and the edges of the cover along lines A-A and B-B are not fully supported. If FAP was directly used to model the prototype decking system, the vertical deformation of the cover along the edges A-A and B-B (Figure 2) would be restricted by the use of Fourier Sine series to represent the vertical deformation of the cover in the direction parallel to the joist. Also the treatment of gaps in FAP would ensure that the vertical deformation of the cover across a gap to be continuous. However, due to the relatively large wheel loads which are concentrated in a small area, the different stiffness characteristics of the cover panel on each side of the mid-span gap (one panel on each side of the gap), and the possibility of unsymmetrical loading, it is expected that the gaps along the line C-C (Figure 2) will open under load and the estimated stresses in the panel may not be accurate. Therefore, modification of FAP is needed to properly model the behavior of the prototype dry freight van trailer decking system. Specifically, the development of DAP would require modifications in the formulation of the strain energy terms in FAP by including the additional degrees of freedom to represent the boundary conditions and the unsupported gaps along the mid-span of the prototype dry freight van trailer decking system and the
Figure 2 A prototype dry freight van trailer decking system.
unequal plate stiffnesses across the center line gap.

2.2 Material Properties

2.2.1 Elastic Properties of Laminated Veneer Wood Plates

DAP would require information on the elastic properties of transDeck™ as input. Necessary data include the bending and axial moduli of elasticity in the direction perpendicular and parallel to grain. The modulus of rigidity in the plane of the panel, Poisson's ratios, and load-slip characteristic of the connector between the panel and the support members must also be known.

Smith (1974) reported the strength properties of unsanded grades of Douglas-fir plywood. Parasin (1981) evaluated the mechanical properties of 3-ply 9.5 mm (\(\frac{3}{8}\) inch) and 5-ply 15.5 mm (\(\frac{5}{8}\) inch) western hemlock and amabilis fir sheathing grade plywood. The strength properties of 3-ply 9.5 mm (\(\frac{3}{8}\) inch) and 5-ply 15.5 mm (\(\frac{5}{8}\) inch) western white spruce sheathing grade plywood from B.C. and Alberta were also reported by Parasin (1983a, b). Although the data on the mechanical properties of plywood are substantial, they are based on testing of plywood with the regular veneer lay ups encountered in plywood constructions. Comprehensive database on the mechanical properties of transDeck™ was lacking.

If information on the veneer properties were available, a more general approach can be taken in which the elastic properties of a laminated veneer panel can be built up analytically using data on individual veneer properties. Such a model would provide a framework where the performance of the laminated veneer panel can be optimized based on veneer placement during lay up.

Limited experimental work on veneers strength properties has been reported. Kingston (1947) investigated the effect of grain direction on the tension strength and stiffness of Hoop pine veneer. McGowan (1974) tested Douglas fir veneers with and without knots to study the effect of prescribed defects on the tensile properties of Douglas fir plywood strips. Booth and Hettiarachchi (1990) conducted an experimental program to evaluate the strength and stiffness of 1.55 mm (\(\frac{1}{16}\) inch) thick
beech veneers in tension and compression. It is clear that limited information on veneer mechanical properties existed. More importantly information on veneer mechanical properties specific to the veneer source available to Ainsworth Lumber Company Ltd. to manufacture transDeck™ was lacking.

2.2.2 Fatigue Data

Limited information on the fatigue behavior of laminated veneer wood products was available. Kommers (1943) reported test data on solid Sitka spruce (593 specimens), solid Douglas-fir (424 specimens), 5-ply yellow birch plywood (102 specimens), and 5-ply yellow poplar plywood (51 specimens). Thermosetting phenol-formaldehyde Tego film glue were used as binding agent for the plywood. The specimen dimensions were 229 x 32 x 8 mm (9 x 1\frac{1}{4} x \frac{5}{16} inch) for the plywood and 229 x 32 x 5 mm (9 x 1\frac{1}{4} x \frac{3}{16} inch) for the solid wood. They were conditioned and tested as a cantilevered plank over a 152 mm (6 inch) span at 75 °F and 65% relative humidity. Completely reversed bending stress cycles (i.e., with zero mean stress per cycle) were applied to the plywood specimens. The solid wood specimens were stressed either with completely reversed bending stress cycles or repeated stress bending stress cycles (i.e., mean stress per cycle equals 0.5 the maximum stress). Maximum stress to number of cycles to failure curves (S-N curves) were developed. Kommers (1943) reported fatigue strengths of 27% and 36% of the mean static modulus of rupture for 50 million cycles of reversed stress, for plywood and solid wood, respectively. The research was unable to detect an endurance limit even at the fairly high load cycle of 50 million cycles.

Kommers (1944) performed fatigue tests on solid Sitka spruce (593 specimens) and Douglas-fir (424 specimens). The specimen dimensions were 229 x 32 x 5 mm (9 x 1\frac{1}{4} x \frac{3}{16} inch). The test conditions were similar to the first study by Kommers (1943) except completely reversed bending stress cycles were superimposed on a constant stress level. The results indicate that the S-N curves of solid wood were influenced by the shape of the applied stress cycle.

Since S-N curves were influenced by the shape of the applied stress cycles, testing should be
performed using applied stress cycles which are similar in shape to that experience by the material in
service. Development of such data may not be practical for lumber or composite products in general
since in-service load cases vary widely. However, this may be possible for a specialty product such as
transDeck™ where the loading histories in dry freight van trailer decking systems are expected to be
more predictable compared to the loadings on typical roof or floor systems in buildings.

McNatt (1970) studied the effect of rate, duration and repeated loading on the strength
properties of 6 mm (1/4 inch) thick tempered hardboard. S-N curves for interlaminar shear (25
specimens) and tension parallel to surface (27 specimens) for the material were reported. The fatigue
strength for 10 million stress cycles was found to be about 40% to 45% of the mean static strength in
tension and shear.

McNatt and Werren (1976) reported the fatigue properties of three particleboards in tension
(60 specimens) and interlaminar shear (61 specimens). The specimens were: 1) urea-bonded 16 mm (5/8
inch) thick southern pine particle board; 2) phenolic-bonded 16 mm (5/8 inch) thick Douglas-fir particle
board; 3) urea-bonded 13 mm (1/2 inch) thick Douglas-fir particle board. The specimens were conditioned
to equilibrium at 73 °F and 50% relative humidity. The resulting S-N curves for the three particles
boards indicated that the fatigue strengths for 10 million stress cycles were approximately 45% and
40% of the mean static tensile and interlaminar shear strengths, respectively.

A special case of the fatigue phenomenon in wood is known as static fatigue or load duration
effect on the strength properties of wood products. This phenomenon refers to the possibility of failure
of a member carrying a constant load sustained over time. Palka and Rovner (1990) studied load
duration behavior of commercially available 16 mm (5/8 inch) thick waferboard. McNatt and Laufenberg
(1991) studied load duration behavior of commercially available 16 mm (5/8 inch) thick plywood and
oriented strand board. In both studies, 300 x 1000 mm specimens were under third point loading with
simple support condition over a 900 mm span. Information on time to failure and creep were obtained
for a range of stress levels. In the study by Palka and Rovner (1990), a range of environmental
conditions were also considered.

2.3 Full Scale Testing of Dry Freight Van Trailer Decking Assembly

In an in-house research and development study, Ainsworth Lumber company Ltd. tested one prototype dry freight van trailer decking assembly under cyclic loading (Figure 2). This is a typical test prototype assembly which is recommended by TTMA for test decking material for dry freight van trailers. The dimension of the assembly was 4.9 m x 2.4 m (8 x 16 feet). The transDeck™ panels were connected to the supporting I-beams using 8 mm ($\frac{5}{16}$ inch) diameter bolts at a spacing of 102 mm (4 inches) on center. The supporting members were 102 mm (4 inches) deep I beams with a 57 mm ($2\frac{1}{4}$ inches) wide flange. The I beams had a mass distribution of 4.78 $\frac{kg}{m}$ ($3.2 \frac{lb}{ft}$ mass). These beams were spaced at 305 mm (12 inches) on center. The ends of each I beam were connected to two 158 mm (6.0 inches) deep supporting channels using four steel bolts with a diameter of 9 mm ($\frac{3}{8}$ inch) at each connection. The supporting channel in turn directly rested on the floor. At the supports, the I beam were restricted from vertical displacement but allowed to rotate; therefore, simple support conditions can be assumed for the I beams. The transDeck™ panels were made with 2.5 mm Douglas fir veneer. Prior to the manufacturing of the transDeck™, the veneers were visually selected such that high quality material was used. A lift truck carrying 76 kN (17,000 lb) with a front wheel to back wheel weight distribution of 9:1 was repeatedly driven over the system. The foot print of a front wheel was 229 x 229 mm (9 x 9 inches). Results from the test indicate rolling shear type failure in the cross ply after 5000 load cycles which met the minimum TTMA requirement of 3000 load cycles without failure.

Trailmobile Inc., a major truck trailer manufacturer in the United States, tested a similar prototype dry freight van trailer decking assembly under cyclic loading. The transDeck™ panels were connected to the supporting I-beams using 8 mm ($\frac{5}{16}$ inch)-18 torx drive, flat head, type G, phosphate and oil coated self tapping screws at a spacing of 102 mm (4 inches) on center. The supporting members were 2.59 m (102 inches) long. A wheel cart carrying 77 kN (17,222 lb) with a front wheel to
back wheel weight distribution of 9:1 was repeatedly pushed and pulled over the system by a lift truck. The footprint of a front wheel was 89 x 229 mm (3.5 x 9 inches). Results from the test indicate failure in the panels after only 14 load cycles.

Clearly, the limited research indicates great discrepancies in results. More testing is needed to gain a better understanding of the performance of the laminate veneer wood plates in trailer decking assemblies under repeated loading. For this type of research to proceed a trailer decking load simulator facility is needed.

2.4 Modeling Fatigue Behavior in Wood Panel Products

Since fatigue properties in wood panel products has not been a popular topic of research, there has been little attention paid to developing models to describe the general fatigue properties in panel products. Damage accumulation models have been used in the past to describe fatigue behavior in other materials. The most widely used model for fatigue in metal is based on the Palmgren-Miner approach. The model defines the cumulative damage, α, after the structural element experiences \( n_i \) cycles at a particular stress level as:

\[
\alpha = \sum_{i=1}^{k} \frac{n_i}{N_i} \tag{1}
\]

where \( N_i \) is the total number of cycles to failure. \( \alpha = 0 \) denotes undamaged state and \( \alpha = 1 \) denotes failure. The advantage of Palmgren-Miner approach is its ease of application; however, its disadvantages include a) not accounting for the order in which stresses are applied and b) ignoring the possible presence of a stress endurance limit.

In the recent past, the special case of static fatigue has received much attention and several researchers have proposed alternative damage accumulation laws to model the duration of load (creep-rupture) phenomenon in dimension lumber (Foschi and Barrett, 1978; Foschi and Yao, 1986a and b;
The Foschi and Barrett model takes the following form:

\[
\frac{d\alpha}{dt} = a \left\{ \frac{\tau(t)}{\tau_s} - \sigma_0 \right\}^b + \lambda \alpha(t)
\]  

[2]

where \(a\) and \(\lambda\) are independent lognormally distributed random variables, \(b\) and \(\sigma_0\) are constants, \(\tau(t)\) is the applied stress, and \(\tau_s\) is the short term strength.

This model was calibrated with duration of load data for dimension lumber in bending (Foschi and Barrett, 1978). However, Foschi and Yao (1986a and b) pointed out a situation when \(\alpha\) has been accumulating to a significant level and \(\tau(t)\) is infinitesimally greater than \(\tau_s\), the damage will still grow exponentially regardless of the level of applied stress. The model will therefore predict failure of substantially damaged specimen over time although little stress is applied after the initial damage.

Foschi and Yao (1986a and b) extended the original model (Foschi and Barrett, 1978) by making the damage dependent term a function of the stress level. The Foschi and Yao model takes the following form:

\[
\frac{d\alpha}{dt} = a \left\{ \tau(t) - \sigma_0 \tau_s \right\}^b + c \left\{ \tau(t) - \sigma_0 \tau_s \right\}^n \alpha(t)
\]  

[3]

where \(a\), \(b\), \(c\), \(n\) and \(\sigma_0\) are random model parameters which vary between members. \(\tau(t)\) is the applied stress and \(\tau_s\) is the short term strength. This model was satisfactorily calibrated using load duration test results of dimension lumber in bending, tension, and compression (Karacabeyli, 1987) and load duration test results of waferboard in bending (Palka and Rovner, 1990). The calibration procedure was presented in details by Foschi, Folz, and Yao, (1987).

The Gerhards and Link model is expressed as:
where \( a \) and \( b \) are model parameters. \( \tau(t) \) is the applied stress and \( \tau_s \) is the short term strength. Model calibration with load duration test results of dimension lumber was given by Gerhards and Link (1986) and Foschi et al., (1989). This model was also calibrated with load duration results of commercially produced plywood and oriented strandboard (McNatt and Laufenberg, 1991) and waferboard (Palka, Rovner and Deacon, 1991) where satisfactory agreements were obtained. Since the rate of damage in this model is independent of the previously sustained damage, the linearity in Gerhards and Link model does not permit a good fit to the nonlinear upward trend in probability of failure versus the logarithm of time to failure results for lumber (Foschi, Folz and Yao, 1989).

The major criticism of using damage accumulation laws to describe duration of behavior in lumber is that the model parameters lack physical meaning. Another approach to model the load duration phenomenon in wood, based on fracture mechanics theory for viscoelastic bodies, was proposed by Nielsen (1978, 1985). The model represents wood by a single crack with viscoelastic characteristics and considers the propagation of the crack under load through time. Duration of load results of dimension lumber were used to calibrate the model (Nielsen, 1985). Nielsen (1990) extended the model for fatigue behavior of wood products by considering crack closure. The model was calibrated using fatigue data of wood products (Nielsen, 1990). However, the use of Nielsen’s model is rather complicated because the rate of crack growth is controlled by a nonlinear first order equation; its solution in general requires numerical integration procedures which is not suited for simulation studies. Furthermore, Nielsen’s model also requires parameters which describe the viscoelastic behavior of the wood. However, these parameters cannot be obtained easily from experiments a priori; therefore, they are typically obtained indirectly through model calibration. So, although Nielsen’s model has more physical meaning attached to its parameters in comparison with other damage accumulation models, the parameters for both types of model are obtained through model calibration.
3.3.1 Introduction

The deformation and stress in an elastic body under load can be obtained from energy methods. Examples of energy methods include the principle of virtual work, the principle of minimum potential energy and the Ritz method. Energy methods are based on the fact that the governing equation of a deformed elastic body is derivable by minimizing the energy associated with the deformation and the loading through calculus of variation. Application of energy methods are particularly effective in cases where irregular shapes, nonuniform loads, variable cross sections, and anisotropic materials are encountered. In this study analysis based on discrete models are used.

This chapter describes the development of Deck Analysis Program (DAP) to evaluate the structural performance of the prototype dry freight van trailer deck system shown in Figure 2. Here, the basic theory for the governing equations of the deformation of a deck assembly is formulated using the principle of minimum potential energy. First, the strain energy terms in the basic components of the deck assembly, which include the cover plate, the supporting I-beams, and the connectors between the cover and the supporting I-beams, are developed. The load potential terms associated with the wheel loads from lift trucks are then derived. Application of the principle of minimum potential energy results in a system of equations relating the applied load and the unknown displacement field. The system of equations is then solved to obtain the deformations of the assembly. Based on the estimated deformations in the assembly, the bending and rolling shear stress profiles in the cover can be derived. The critical locations where stresses are maximum can therefore be identified.

Sample problems considering various case studies of a seventeen bay prototype dry freight van trailer deck assembly are modeled and solved using DAP to illustrate the required program inputs and the program outputs. The sensitivity of the DAP results to parameters such as connector stiffnesses and locations of wheel loads are studied to provide guidance to the planning of the experimental phase of
The development of DAP is based on a semi-analytical approach using the finite strip method where the unknown displacement field is expressed as combinations of Fourier and polynomial series. As mentioned in the literature survey, Foschi (1982) used finite strip method to develop FAP to study wooden floor systems. The major differences in the formulation between DAP and FAP are: 1) the assumed displacement field in DAP includes additional degrees of freedom to model the unsupported edges and the mid-span gap in the prototype dry freight van trailer deck assembly; and 2) unequal elastic properties for the two sides of the cover in the deck system which corresponds to the use of two individual panels (one on each side of the mid-span gap) in the deck assembly are considered. These additional issues are addressed so that dry freight van trailer decking systems sheathed with transDeck™ panels can be properly modeled to predict the structural responses such as deformations and stresses in the system.

3.2 Strain Energy in the Cover

Consider the finite strip shown in Figure 3, it represents one bay in the prototype trailer decking assembly shown in Figure 2. The strip contains a gap at midspan; i.e., \( x = \frac{L}{2} \). Also the edges of the strip at \( x = 0 \) and \( x = L \) which are not supported by the I beam are free to move in the vertical direction. The \( x \) and \( y \) axes shown in Figure 3 are the principal material axes for the cover. In the application of transDeck™ in dry freight van trailer decking systems, the face grain of the cover is typically oriented parallel to the \( y \)-direction and the grain direction of the cross-ply is typically oriented perpendicular to the \( y \)-direction (i.e., in the \( x \)-direction).

Assuming small deflections and orthotropic plate theory for a medium thick plate where bending and membrane forces are considered, the strain energy of the cover under load is expressed as:
Figure 3  Finite strip representation of a section of the decking assembly.
\[
U_c = \int_{-\frac{s}{2}}^{\frac{s}{2}} \int_0^L \left\{ \frac{K_{x1}}{2} \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + \frac{K_{y1}}{2} \left( \frac{\partial^2 w}{\partial y^2} \right)^2 + \frac{K_{z1}}{2} \left( \frac{\partial^2 w}{\partial x^2 \partial y} \right)^2 + 2K_{G1} \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \\
+ \frac{D_{x1}}{2} \left( \frac{\partial^2 u}{\partial x^2} \right)^2 + \frac{D_{y1}}{2} \left( \frac{\partial^2 v}{\partial y^2} \right)^2 + D_{v1} \left( \frac{\partial^2 u}{\partial x} \frac{\partial v}{\partial y} \right)^2 + \frac{D_{G1}}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right\} \, dx \, dy
\]

where \( u(x,y), v(x,y), w(x,y) \) represent the displacement field of the mid-plane of the cover in the \( x, y \) and \( z \) directions, respectively; \( s \) represents the width of a strip of the cover in the \( y \)-direction; and \( L \) represents the length of the cover in the \( x \)-direction. The plate stiffnesses, \( K_{xi}, K_{yi}, K_{vi}, K_{Gi}, D_{xi}, D_{yi}, D_{v1}, \) and \( D_{Gi} \) (\( i = 1 \) or \( 2 \)) are given by:

\[
K_{xi} = \frac{E_{xi}}{12(1 - \nu_{xy} \nu_{yx})} \frac{d^3}{x^3}; \quad K_{yi} = K_{xi}; \quad K_{vi} = \nu_{xy} K_{xi}; \quad K_{Gi} = G_i \frac{d^3}{12x};
\]

\[
D_{xi} = \frac{E_{xi}}{12(1 - \nu_{xy} \nu_{yx})} \frac{d}{x}; \quad D_{yi} = D_{xi}; \quad D_{v1} = \nu_{xy} D_{xi}; \quad D_{Gi} = G_i \frac{d}{x} \quad [6]
\]

where the subscript \([i]\) denotes plate stiffnesses and elastic properties of the \( i \)th section of the cover in which sections 1 and 2 represent the regions \( 0 < x \leq \frac{L}{2} \) and \( \frac{L}{2} < x < L \), respectively.

Here, the moduli of elasticity and rigidity of the cover are equivalent values for homogeneous material. \( E_{xi} \) is the modulus of elasticity in the \( x \)-direction; \( E_{yi} \) is the modulus of elasticity in the \( y \)-direction; \( G_i \) is the modulus of rigidity in the \( x-y \) plane of the \( i \)th section of the cover. Also \( \nu_{xy} \) and \( \nu_{yx} \) are Poisson’s ratios where the first subscript denotes the direction of transverse strain and the second subscript denotes the direction of applied stress; and \( d \) is the thickness of the plate.
3.3 Strain Energy in the Supporting I-beams

When the finite strip shown in Figure 3 is under load, the strain energy in the supporting I-beam resulting from axial deformation, lateral bending (in the y-direction), vertical bending (in the z-direction), and torsional deformation can be expressed in terms of the displacements $U(x)$, $V(x)$, $W(x)$, and $\theta(x)$. Here the displacements $U(x)$, $V(x)$, $W(x)$ represent deformations of the supporting I-beam at its geometric center, as shown in Figure 4 (point A), in the x, y and z directions, respectively. Finally $\theta(x)$ represents the rotation of the geometric center of the supporting I-beam about the x axis at point A.

The strain energy in the I-beam is given as:

$$U_I = \int_0^L \left( \frac{E A}{2} \left( \frac{dU}{dx} \right)^2 + \frac{E I_z}{2} \left( \frac{d^2V}{dx^2} \right)^2 + \frac{E I_y}{2} \left( \frac{d^2W}{dx^2} \right)^2 + \frac{G J}{2} \left( \frac{d\theta}{dx} \right)^2 \right) \, dx \tag{7}$$

where $E$ = the Young’s modulus; $G$ = the shear modulus; $A$ = the cross sectional area; $I_z$ = the moment of inertia about the z axis; $I_y$ = the moment of inertia about the y axis; and $J$ = the torsional moment of inertia of the I-beam, respectively. Given the specifications and physical dimensions of the supporting steel I-beams, the elastic moduli and the geometrical parameters can be easily evaluated.

In the prototype dry freight van trailer deck system, steel I-beams with $E \approx 200,000$ MPa ($29 \times 10^6$ psi) and $G \approx 77000$ MPa ($11 \times 10^6$ psi) are typically used. The moments of inertia $I_z$, $I_y$, $J$ and the cross sectional area $A$ can be expressed in terms of the thickness of the flange ($t_f$), the thickness of the web ($t_w$), and the depth ($h_I$) and the width ($b_I$) of the I-beam as:

$$I_z = \frac{2 t_f b_I^3 + t_w^3 (h_I - 2 t_f)}{12} \tag{8a}$$

$$I_y = \frac{b_I h_I^3 - (h_I - 2 t_f)^3 (b_I - t_w)}{12} \tag{8b}$$
Figure 4 Degrees of freedom for a supporting I-beam.
3.4 Strain Energy in the Connectors

The construction of the prototype dry freight van trailer deck system involves initial positioning of the cover panels over the supporting steel I-beams. Then the panels and the flange of the I-beam are pre-drilled with holes, 7 mm (\( \frac{5}{32} \) inch) in diameter, at the appropriate locations. Finally self-tapping screws, 8 mm (\( \frac{5}{16} \) inch) in diameter, are used to secure the panels to the I-beams to form a completed assembly.

The cover plate is therefore assumed to be semi-rigidly connected to the supporting I-beam through uniformly spaced connectors with linear load-slip relationship. The strain energy in the connectors resulting from the relative movement between the cover and the supporting I-joist can be expressed as:

\[
U_N = \frac{1}{2} \sum_{i=1}^{NA} \left( K_{xc} (\Delta u)_i^2 + K_{yc} (\Delta v)_i^2 + K_{\theta c} \phi_i^2 \right) \tag{9}
\]

where \((\Delta u)_i\) = the slip in the x-direction of the \(i^{th}\) connector; \((\Delta v)_i\) = the slip in the y-direction of the \(i^{th}\) connector; \(\phi_i\) = the relative rotation between the cover and the supporting I-beam at the location of the \(i^{th}\) connector; \(NA\) = the number of connectors per finite strip; \(K_{xc}\) = the connector stiffness in the x-direction; \(K_{yc}\) = the connector stiffness in the y-direction; and \(K_{\theta c}\) = the rotational stiffness of the connector.

Since the screw connectors are unlikely to withdraw, it is reasonable to assume that there is no connector withdrawal; therefore, \((\Delta u)_i\) and \((\Delta v)_i\) can be expressed in terms of the displacements of the cover and the supporting I-beam as:
(Δui) = u(x_i, 0) - \frac{d}{2} \frac{dW}{dx}(x_i) - \left( U(x_i) + \frac{h_f}{2} \frac{dW}{dx}(x_i) \right) \tag{10a}

(Δvi) = v(x_i, 0) - \frac{d}{2} \frac{∂w}{∂y}(x_i, 0) - \left( V(x_i) + \frac{h_f}{2} \frac{dθ}{dx}(x_i) \right) \tag{10b}

φ_i = \frac{∂w}{∂y}(x_i, 0) - θ(x_i) \tag{10c}

where x_i = the location of the i^{th} connector in the x-direction and d = the thickness of the cover.

The discrete connector pattern modeled in Equation 9 can be converted to an equivalent continuous connector by considering the slips (Δui), (Δvi), and φ_i as continuous functions such that U_N can be expressed as:

\[ U_N = \frac{1}{2} e \int_0^L \left( K_{xc} (Δu)^2 + K_{yc} (Δv)^2 + K_{θc} φ^2 \right) dx \] \tag{11}

where

\[ Δu = u(x, 0) - \frac{d}{2} \frac{dW}{dx} - \left( U + \frac{h_f}{2} \frac{dW}{dx} \right) \] \tag{12a}

\[ Δv = v(x, 0) - \frac{d}{2} \frac{∂w}{∂y}(x, 0) - \left( V + \frac{h_f}{2} θ \right) \] \tag{12b}

\[ φ = \frac{∂w}{∂y}(x, 0) - θ \] \tag{12c}

\[ e = \text{connector spacing.} \] \tag{12d}

3.5 Potential Energy in the Applied Load

The load potential can be expressed as:
\[ \Omega_L = \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{s}{2}}^{\frac{s}{2}} p(x, y) w(x, y) \, dx \, dy \]  

[13]

where \( p(x, y) \) corresponds to the loading onto a bay of the assembly due to a pair of wheels on a lift truck which can either be the pair of front wheels or the pair of rear wheels. Note that the pair of wheels is assumed to be aligned in the x-direction of the cover.

Assuming the loading of each wheel is uniformly distributed over its footprint, the loading \( p(x, y) \) is completely described by: 1) the magnitude of load on each wheel; 2) the dimensions of the wheel footprint; and 3) the location of the wheels on the deck which can be obtained from information on the center to center distance between the two wheels and the location of one of the wheels on the bay.

3.6 Finite Strip Formulation

Using a Fourier series expansion of the unknown functions in the x-direction and a one dimensional finite element approximation along the y-direction, the displacement field of the cover can be represented by \( w(x, y), u(x, y) \) and \( v(x, y) \), which are expressed as:

\[
w(x, y) = \sum_{n=1}^{N} F_{1n}(y) \sin\left(\frac{n\pi x}{L}\right) + B_n(y)\left(\frac{2x}{L}\right) + A_n(y)(1 - \frac{2x}{L}) \quad \text{for } 0 < x < \frac{L}{2} \tag{14a}
\]

\[
w(x, y) = \sum_{n=1}^{N} F_{1n}(y) \sin\left(\frac{n\pi x}{L}\right) + D_n(y)\left(\frac{2x}{L} - 1\right) + C_n(y)(2 - \frac{2x}{L}) \quad \text{for } \frac{L}{2} < x < L \tag{14b}
\]

\[
u(x, y) = \sum_{n=1}^{N} F_{2n}(y) \cos\left(\frac{n\pi x}{L}\right) \quad \tag{14c}
\]

\[
v(x, y) = \sum_{n=1}^{N} F_{3n}(y) \sin\left(\frac{n\pi x}{L}\right) \quad \tag{14d}
\]

where the functions \( F_{1n}(y), F_{2n}(y), A_n(y), B_n(y), C_n(y), D_n(y), F_{2n}(y), \) and \( F_{3n}(y) \) are one
dimensional finite element approximations in the form of fifth degree polynomials in the y-directions; \( N \) is the number of terms used in the Fourier series expansion. The chosen displacement fields of the cover in the \( x \) and \( y \) directions, \( u(x,y) \) and \( v(x,y) \), are described by continuous functions. The deformation of the cover in the \( y \)-direction at \( x=0 \) and \( x=L \) and the deformation of the cover in the \( x \)-direction at \( x=\frac{L}{2} \) are restricted; i.e., \( v(0,y)=0 \), \( v(L,y)=0 \) and \( u(\frac{L}{2},y)=0 \). Also the polynomial functions are chosen such that the edges of the cover (\( x=0 \) and \( x=L \)) are free to displace vertically except over the supporting I-beam which properly models the boundary conditions of the prototype decking system where the cover is indeed unsupported at the edges except over the I-beams.

The deformation in the supporting I-beam \( U(x) \), \( V(x) \), \( W(x) \), and \( \theta(x) \) can be expressed as:

\[
U(x) = \sum_{n=1}^{N} U_n \cos\left(\frac{n\pi x}{L}\right) \quad [15a]
\]

\[
V(x) = \sum_{n=1}^{N} V_n \sin\left(\frac{n\pi x}{L}\right) \quad [15b]
\]

\[
W(x) = \sum_{n=1}^{N} W_n \sin\left(\frac{n\pi x}{L}\right) \quad [15c]
\]

\[
\theta(x) = \sum_{n=1}^{N} \theta_n \sin\left(\frac{n\pi x}{L}\right) \quad [15d]
\]

Here the coefficients \( U_n \), \( V_n \), \( W_n \), and \( \theta_n \) are displacement amplitudes of the supporting I-beam which is assumed to be simply supported and restricted from lateral deformation and rotation at \( x=0 \) and \( x=L \).

3.6.1 Cover

Let us consider individual terms in the expression of the strain energy of the cover shown in Equation 5 as:
\[ U_c = U_c^{(1)} + U_c^{(2)} + U_c^{(3)} + U_c^{(4)} + U_c^{(5)} + U_c^{(6)} + U_c^{(7)} + U_c^{(8)} \]  

where

\[
U_c^{(1)} = \int_{-s}^{s} \left\{ \frac{L}{2} \left( \frac{\partial^2 w}{\partial x^2} \right)^2 \right\} dx + \int_{L/2}^{L} \left\{ \frac{L}{2} \left( \frac{\partial^2 w}{\partial x^2} \right)^2 \right\} dy
\]

\[
U_c^{(2)} = \int_{-s}^{s} \left\{ \frac{L}{2} \left( \frac{\partial^2 w}{\partial y^2} \right)^2 \right\} dx + \int_{L/2}^{L} \left\{ \frac{L}{2} \left( \frac{\partial^2 w}{\partial y^2} \right)^2 \right\} dy
\]

\[
U_c^{(3)} = \int_{-s}^{s} \left\{ \frac{L}{2} \left( \frac{\partial^2 w}{\partial x^2} \right) \left( \frac{\partial^2 w}{\partial y^2} \right) \right\} dx + \int_{L/2}^{L} \left\{ \frac{L}{2} \left( \frac{\partial^2 w}{\partial x^2} \right) \left( \frac{\partial^2 w}{\partial y^2} \right) \right\} dy
\]

\[
U_c^{(4)} = \int_{-s}^{s} \left\{ 2 \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \right\} dx + \int_{L/2}^{L} \left\{ 2 \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \right\} dy
\]

\[
U_c^{(5)} = \int_{-s}^{s} \left\{ \frac{L}{2} \left( \frac{\partial^2 u}{\partial x} \right)^2 \right\} dx + \int_{L/2}^{L} \left\{ \frac{L}{2} \left( \frac{\partial^2 u}{\partial x} \right)^2 \right\} dy
\]

\[
U_c^{(6)} = \int_{-s}^{s} \left\{ \frac{L}{2} \left( \frac{\partial^2 u}{\partial y} \right)^2 \right\} dx + \int_{L/2}^{L} \left\{ \frac{L}{2} \left( \frac{\partial^2 u}{\partial y} \right)^2 \right\} dy
\]

\[
U_c^{(7)} = \int_{-s}^{s} \left\{ \frac{L}{2} \left( \frac{\partial^2 u}{\partial x} \right) \left( \frac{\partial^2 v}{\partial y} \right) \right\} dx + \int_{L/2}^{L} \left\{ \frac{L}{2} \left( \frac{\partial^2 u}{\partial x} \right) \left( \frac{\partial^2 v}{\partial y} \right) \right\} dy
\]

\[
U_c^{(8)} = \int_{-s}^{s} \left\{ \frac{L}{2} \left( \frac{\partial^2 u}{\partial y} + \frac{\partial^2 v}{\partial x} \right)^2 \right\} dx + \int_{L/2}^{L} \left\{ \frac{L}{2} \left( \frac{\partial^2 u}{\partial y} + \frac{\partial^2 v}{\partial x} \right)^2 \right\} dy
\]
Let us define the functions \( R_{nm1} \) and \( R_{nm2} \) as:

\[
R_{nm1} = \frac{L}{2\pi} \left( \frac{\sin(m - n) \frac{n\pi}{2}}{(m - n)} - \frac{\sin(m + n) \frac{n\pi}{2}}{(m + n)} \right)
\]

\[
R_{nm2} = \frac{L}{2\pi} \left( \frac{\sin(m - n) \frac{n\pi}{2}}{(m - n)} + \frac{\sin(m + n) \frac{n\pi}{2}}{(m + n)} \right)
\]  

[18]

Also let the superscripts \([\cdot]\) and \([\cdot\cdot]\) be defined as the first and second derivative with respect to \( y \), respectively. Substituting Equations 14a to 14d into Equations 17a to 17h and performing the integration with respect to \( y \) yields individual terms in the strain energy of the cover, expressed in terms of the functions, \( F_{1n}(y) \), \( F_{in}(y) \), \( A_n(y) \), \( B_n(y) \), \( C_n(y) \), \( D_n(y) \), \( F_{2n}(y) \), and \( F_{3n}(y) \), as:

\[
U_c^{(1)} = \int_{-\frac{S}{2}}^{\frac{S}{2}} \sum_{n=1}^{N} \frac{n\pi^4}{8L^3} \left( K_{x1} F_{1n}^2 - K_{x2} F_{1n}^2 \right)
\]

\[
+ \sum_{n=1}^{N} \sum_{m=1}^{N} \frac{n^2m^2\pi^4}{2L^4} R_{nm1} (K_{x1} F_{1n}F_{1m} - K_{x2} F_{1n}F_{1m}) \, dy
\]  

[19a]

\[
U_c^{(2)} = \int_{-\frac{S}{2}}^{\frac{S}{2}} \sum_{n=1}^{N} \frac{I}{8} \left( K_{y1} F_{1n}''^2 + K_{y2} F_{1n}''^2 \right) + \sum_{n=1}^{N} \sum_{m=1}^{N} R_{nm1} \left( K_{y1} F_{1n}F_{1m}'' - K_{y2} F_{1n}F_{1m}'' \right)
\]

\[
+ \sum_{n=1}^{N} \sum_{m=1}^{N} \frac{I}{12} \left( K_{y1} (A_n'' B_m + B_n'' A_m) + K_{y2} (C_n'' D_m + D_n'' C_m) \right)
\]

\[
+ \frac{L}{n\pi} \left( K_{y1} F_{1n}'' \left( B_m \left( \frac{2}{n\pi} \sin \frac{n\pi}{2} - \cos \frac{n\pi}{2} \right) + A_m (1 - \frac{2}{n\pi} \sin \frac{n\pi}{2}) \right) \right.
\]

\[
+ K_{y2} F_{1n}'' \left( C_m \left( \cos \frac{n\pi}{2} + \frac{2}{n\pi} \sin \frac{n\pi}{2} \right) - D_m \left( \cos n\pi + \frac{2}{n\pi} \sin \frac{n\pi}{2} \right) \right) \) dy
\]  

[19b]

\[
U_c^{(3)} = -\int_{-\frac{S}{2}}^{\frac{S}{2}} \sum_{n=1}^{N} \frac{L}{4} \left( \frac{n\pi}{L} \right)^2 \left( K_{\nu1} F_{1n} F_{1n}'' + K_{\nu2} F_{1n} F_{1n}'' \right) + ...
\]
\[ + \sum_{n=1}^{N} \sum_{m=1}^{N} \left( \frac{n \pi}{L} \right)^2 R_{nm} \left( K_{1} F_{1n} F_{1m} - K_{2} F_{1n} F_{1m} \right) \]

\[ + \sum_{n=1}^{N} \sum_{m=1}^{N} \left( \frac{n \pi}{L} \right)^2 R_{nm} \left( K_{1} F_{1n} \left( B_{m} \left( \frac{2}{n \pi} \sin \frac{n \pi}{2} - \cos \frac{n \pi}{2} \right) + A_{m} \left( 1 - \frac{2}{n \pi} \sin \frac{n \pi}{2} \right) \right) \right. \]

\[ + K_{2} \cdot F_{1n} \left( C_{m} \left( \cos \frac{n \pi}{2} + \frac{2}{n \pi} \sin \frac{n \pi}{2} \right) - D_{m} \left( \cos n \pi + \frac{2}{n \pi} \sin \frac{n \pi}{2} \right) \right) \left( \frac{\pi}{2} \right) \right] \]

\[ U_{c}^{(4)} = 2 \int_{-\pi/2}^{\pi/2} \sum_{n=1}^{N} \left( \frac{n \pi}{L} \right)^2 \left( K_{G1} F_{1n}^2 + K_{G2} F_{1n}^2 \right) \]

\[ + \sum_{n=1}^{N} \sum_{m=1}^{N} \left( \frac{n \pi}{L} \right)^2 R_{nm} \left( K_{G1} F_{1n} F_{1m} - K_{G2} F_{1n} F_{1m} \right) \]

\[ + \sum_{n=1}^{N} \sum_{m=1}^{N} K_{G1} \left( A_{n} A_{m} + B_{n} B_{m} - B_{m} A_{n} - A_{n} B_{m} \right) \]

\[ + K_{G2} \left( C_{n} C_{m} + D_{n} D_{m} - D_{m} C_{n} - C_{n} D_{m} \right) \]

\[ + 2 \sin \frac{n \pi}{2} \left( K_{G1} F_{1n} (B_{m} - A_{m}) - K_{G2} F_{1n} (D_{m} - C_{m}) \right) \left( \frac{\pi}{2} \right) \left( \frac{\pi}{2} \right) \]

\[ U_{c}^{(5)} = \int_{-\pi/2}^{\pi/2} \sum_{n=1}^{N} \left( \frac{n \pi}{L} \right)^2 \left( \frac{D_{x1} + D_{x2}}{8L} \right) F_{2n}^2 + \sum_{n=1}^{N} \sum_{m=1}^{N} \left( \frac{n \pi}{L} \right)^2 R_{nm} \left( \frac{D_{x1} - D_{x2}}{2} \right) F_{2n} F_{2m} \left( \frac{\pi}{2} \right) \left( \frac{\pi}{2} \right) \]

\[ U_{c}^{(6)} = \int_{-\pi/2}^{\pi/2} \sum_{n=1}^{N} \left( \frac{n \pi}{L} \right)^2 \left( \frac{D_{y1} + D_{y2}}{8L} \right) F_{3n}^2 + \sum_{n=1}^{N} \sum_{m=1}^{N} \left( \frac{n \pi}{L} \right)^2 R_{nm} \left( \frac{D_{y1} - D_{y2}}{2} \right) F_{3n} F_{3m} \left( \frac{\pi}{2} \right) \left( \frac{\pi}{2} \right) \]

\[ U_{c}^{(7)} = \int_{-\pi/2}^{\pi/2} \sum_{n=1}^{N} \left( \frac{n \pi}{L} \right)^2 \left( \frac{D_{y1} + D_{y2}}{4} \right) F_{2n} F_{3m} \left( \frac{\pi}{2} \right) \left( \frac{\pi}{2} \right) + \sum_{n=1}^{N} \sum_{m=1}^{N} \left( \frac{n \pi}{L} \right)^2 R_{nm} \left( \frac{D_{y1} - D_{y2}}{2} \right) F_{2n} F_{3m} \left( \frac{\pi}{2} \right) \left( \frac{\pi}{2} \right) \]
\[ U_c^{(8)} = \sum_{n=1}^{N} \frac{L}{8} \left( D_{G1} + D_{G2} \right) \left( F_{2n}^2 + \left( \frac{n\pi}{L} \right)^2 F_{3n}^2 + \left( \frac{2n\pi}{L} \right) F_{2n} F_{3n} \right) \]

\[ + \sum_{n=1}^{N} \sum_{n=1}^{N} R_{nm} \frac{D_{G1} - D_{G2}}{2} \left( F_{2n} F_{2m} + nm \left( \frac{\pi}{L} \right)^2 F_{3n} F_{3m} + \frac{3\pi}{L} F_{2n} F_{3m} + \frac{\pi}{L} F_{3n} F_{2m} \right) dy \]

The unknown polynomials and coefficients can be expressed in terms of the unknown displacements and their derivatives for points along lines 0-0, 1-1, and 2-2, of the finite strip shown in Figure 3. The vectors of unknowns, \( \delta_n \) (n=1, 2, ..., N), associated with the \( n^{th} \) Fourier term of a finite strip are given by:

\[
\delta_n^T = \{ w_{1n}^I, w_{1n}^\cdot, w_{1n}^\cdot S, w_{2n}^I, w_{1n}^I S, w_{1n}^I S, w_{1n}^\cdot, w_{1n}^\cdot S, w_{1n}^I, w_{1n}^I S, w_{1n}^I S, w_{1n}^\cdot, w_{1n}^\cdot S, w_{1n}^I, w_{1n}^I S, w_{1n}^I S, w_{1n}^\cdot, w_{1n}^\cdot S, w_{1n}^I, w_{1n}^I S, w_{1n}^I S, w_{1n}^\cdot, w_{1n}^\cdot S, w_{1n}^I, w_{1n}^I S, w_{1n}^I S, w_{1n}^\cdot, w_{1n}^\cdot S, w_{1n}^I, w_{1n}^I S, u_{1n}^I, u_{1n}^I S, \}
\]

\( \quad n=1, 2, ..., N \)  

where the superscript \([T]\) denotes the transpose of a vector, the superscript \([\cdot]\) denotes the derivative with respect to \( y \), the superscript \([\cdot]\) denotes section 1 of the cover where \( 0 < x < \frac{L}{2} \), the superscript \([\cdot]\) denotes section 2 of the cover where \( \frac{L}{2} < x < L \), and the subscripts \([0], [1], \) and \([2]\) represent the points along the lines, 0-0, 1-1, and 2-2, respectively. The superscripts \([I], [II], [III], \) and \([IV]\) denote the corner positions on the cover of a finite strip as shown in Figure 3 such that superscripts \([I]\) and \([II]\) denote the positions along the lines 1-1 and 2-2 at \( x=0 \) and \( x=\frac{L}{2} \) of section 1 of the cover, respectively; and superscripts \([III]\) and \([IV]\) denote the positions along the lines 1-1 and 2-2 at \( x=\frac{L}{2} \) and \( x=L \) of section 2 of the cover, respectively. Note that each vector of unknowns, \( \delta_n \) (n=1, 2, ..., N), has 39 degrees of freedom.
The functions $F_{1n}(y)$, $F_{1n}^{\text{\_\_}}(y)$, $A_n(y)$, $B_n(y)$, $C_n(y)$, $D_n(y)$, $F_{2n}(y)$, and $F_{3n}(y)$ can be written in terms of a nondimensional variable $\xi = \frac{2y}{S}$, the shape function vectors, and the unknown displacement vectors $\delta_n$ ($n=1, 2, ..., N$) as:

\[
\begin{align*}
F_{1n}(\xi) &= M_0(\xi) \delta_n; & F_{1n}^{\text{\_\_}}(\xi) &= M_0^\prime(\xi) \delta_n; & A_n(\xi) &= M_a(\xi) \delta_n; \\
B_n(\xi) &= M_b(\xi) \delta_n; & C_n(\xi) &= M_c(\xi) \delta_n; & D_n(\xi) &= M_d(\xi) \delta_n; \\
F_{2n}(\xi) &= M_3(\xi) \delta_n; & F_{3n}(\xi) &= M_5(\xi) \delta_n
\end{align*}
\]

[21]

where the non-zero components of the shape function vectors, $M_0(\xi)$, $M_0^\prime(\xi)$, $M_a(\xi)$, $M_b(\xi)$, $M_c(\xi)$, $M_d(\xi)$, $M_3(\xi)$, and $M_5(\xi)$ are given as:

\[
\begin{align*}
M_0^{\prime}(1) &= M_0^{\prime}(3) = M_a(5) = M_b(7) = M_c(9) = M_d(11) = \xi^2 - \frac{5}{4} \xi^3 - \frac{1}{2} \xi^4 + \frac{3}{4} \xi^5 \\
M_0^{\prime}(2) &= M_0^{\prime}(4) = M_a(6) = M_b(8) = M_c(10) = M_d(12) = \frac{\xi^2 - \xi^3 - \xi^4 + \xi^5}{8} \\
M_0^{\prime}(19) &= M_0^{\prime}(19) = \frac{\xi - 2\xi^3 + \xi^5}{2} \\
M_0(20) &= M_0(20) = 1 - 2\xi^2 + \xi^4 \\
M_0^{\prime}(24) &= M_0^{\prime}(26) = M_a(28) = M_b(30) = M_c(32) = M_d(34) = \xi^2 + \frac{5}{4} \xi^3 - \frac{1}{2} \xi^4 - \frac{3}{4} \xi^5 \\
M_0^{\prime}(25) &= M_0^{\prime}(27) = M_a(29) = M_b(31) = M_c(33) = M_d(35) = \frac{-\xi^2 + \xi^3 + \xi^4 + \xi^5}{8} \\
M_3(13) &= M_5(15) = -\frac{3}{4} \xi + \xi^2 + \frac{1}{4} \xi^3 - \frac{1}{2} \xi^4 \\
M_3(14) &= M_5(16) = \frac{-\xi + \xi^2 + \xi^3 - \xi^4}{8} \\
M_3(17) &= M_5(18) = 1 - 2\xi^2 + \xi^4
\end{align*}
\]
The first and second derivatives of these functions with respect to \( \xi \) can be also expressed as:

\[
\frac{dF_1(n)}{d\xi} = M_1(\xi)^T \delta_n; \quad \frac{d^2F_1(n)}{d\xi^2} = M_2(\xi)^T \delta_n; \quad \frac{dF_1(n)}{d\xi} = M_1(\xi)^T \delta_n;
\]
\[
\frac{dF_2(n)}{d\xi} = M_2(\xi)^T \delta_n; \quad \frac{d^2F_2(n)}{d\xi^2} = M_3(\xi)^T \delta_n; \quad \frac{dF_2(n)}{d\xi} = M_2(\xi)^T \delta_n;
\]
\[
\frac{dF_3(n)}{d\xi} = M_3(\xi)^T \delta_n; \quad \frac{d^2F_3(n)}{d\xi^2} = M_4(\xi)^T \delta_n; \quad \frac{dF_3(n)}{d\xi} = M_3(\xi)^T \delta_n;
\]

where the components of vectors, \( M_1(\xi), M_2(\xi), M_3(\xi), M_4(\xi), M_5(\xi), M_6(\xi) \), and \( M_7(\xi), M_8(\xi), M_9(\xi), M_{10}(\xi), \) are given as:

\[
M_1(k) = \frac{dM_{0(k)}}{d\xi}; \quad M_2(k) = \frac{d^2M_{0(k)}}{d\xi^2}; \quad M_3(k) = \frac{dM_{0(k)}}{d\xi}; \quad M_4(k) = \frac{d^2M_{0(k)}}{d\xi^2};
\]
\[
M_5(k) = \frac{dM_{a(k)}}{d\xi}; \quad M_6(k) = \frac{dM_{b(k)}}{d\xi}; \quad M_7(k) = \frac{d^2M_{a(k)}}{d\xi^2}; \quad M_8(k) = \frac{d^2M_{b(k)}}{d\xi^2};
\]
\[
M_9(k) = \frac{dM_{c(k)}}{d\xi}; \quad M_10(k) = \frac{dM_{d(k)}}{d\xi}; \quad M_11(k) = \frac{dM_{c(k)}}{d\xi}; \quad M_12(k) = \frac{dM_{d(k)}}{d\xi};
\]
\[
M_13(k) = \frac{dM_{3(k)}}{d\xi}; \quad M_14(k) = \frac{dM_{5(k)}}{d\xi}; \quad M_15(k) = \frac{dM_{3(k)}}{d\xi}; \quad M_16(k) = \frac{dM_{5(k)}}{d\xi};
\]

for \( k = 1, 2, \ldots, 39 \)

The various components in the strain energy of the cover can be expressed in terms of the
unknown displacement vectors, $\delta_n$ (n = 1, 2, ..., N), and the shape functions, $M'(\xi)$, $M_0(\xi)$, $M(\xi)$, $M_1(\xi)$, $M_2(\xi)$, $M_1(\xi)$, $M_2(\xi)$, $M_{a1}(\xi)$, $M_{a2}(\xi)$, $M_{b1}(\xi)$, $M_{b2}(\xi)$, $M_{c1}(\xi)$, $M_{c2}(\xi)$, $M_{d1}(\xi)$, and $M_{d2}(\xi)$ as:

$$U_c^{(1)} = \frac{S}{16 L^3} \sum_{n=1}^{1} \int \sum_{n=1}^{N} (n\pi)^4 \delta_n^T \left( K_{x1} M_0 M_0^T + K_{x2} M_0 M_0^T \right) \delta_n$$

$$+ \sum_{n=1}^{N} \sum_{m=1}^{N} \frac{4 n^2 m^2 \pi^4}{L} R_{nm} \delta_n^T \left( K_{x1} M_0 M_0^T - K_{x2} M_0 M_0^T \right) \delta_m d\xi$$

$$U_c^{(2)} = \frac{(L/2)^3}{S} \int \sum_{n=1}^{N} \frac{1}{4} \delta_n^T \left( K_{y1} M_2 M_2^T + K_{y2} M_2 M_2^T \right) \delta_n$$

$$+ \sum_{n=1}^{N} \sum_{m=1}^{N} \frac{1}{L} R_{nm} \delta_n^T \left( K_{y1} M_2 M_2^T - K_{y2} M_2 M_2^T \right) \delta_m$$

$$+ \sum_{n=1}^{N} \sum_{m=1}^{N} \delta_n^T \left( K_{y1} (M_{a2} M_{a2}^T + M_{b2} M_{b2}^T + M_{b2} M_{a2}^T) \right)$$

$$+ K_{y2} \left( M_{c2} M_{c2}^T + M_{d2} M_{d2}^T + M_{d2} M_{c2}^T \right)$$

$$+ \frac{2}{n\pi^3} \left( K_{y1} M_2 \left( \frac{2}{n\pi} \sin \frac{n\pi}{2} - \cos \frac{n\pi}{2} \right) + M_{a2}^T \left( 1 - \frac{2}{n\pi} \sin \frac{n\pi}{2} \right) \right)$$

$$+ K_{y2} M_2 \left( M_{c2}^T \left( \frac{2}{n\pi} \sin \frac{n\pi}{2} + \frac{2}{n\pi} \sin \frac{n\pi}{2} \right) - M_{d2}^T \left( \cos n\pi + \frac{2}{n\pi} \sin \frac{n\pi}{2} \right) \right) \delta_n d\xi$$

$$U_c^{(3)} = -\frac{2}{S} \int \sum_{n=1}^{N} \frac{(\pi \xi)^2}{L} \delta_n^T \left( K_{\xi\xi1} M_0 M_2^T + K_{\xi\xi2} M_0 M_2^T \right) \delta_n + ...$$
\[
U_c^{(4)} = \frac{4}{S} \int \sum_{n=1}^{+1} \frac{I}{4} \left( \frac{\pi}{L} \right)^2 \delta_n^T \left( K_{G1} M_1 \cdot M_1^T + K_{G2} M_1^\prime \cdot M_1^\prime T \right) \delta_n
\]

\[
+ \sum_{n=1}^{N} \sum_{m=1}^{N} \left( \frac{\pi}{L} \right) \delta_n^T \left( K_{G1} M_1 \cdot M_1^T + K_{G2} M_1^\prime \cdot M_1^\prime T \right) \delta_m
\]

\[
+ \sum_{n=1}^{N} \sum_{m=1}^{N} \left( \frac{\pi}{L} \right) R_{nm1} \left( \delta_n^T \left( K_{G1} M_1 \cdot M_1^T + K_{G2} M_1^\prime \cdot M_1^\prime T \right) \delta_m
\]

\[
+ \frac{2}{S} \sum_{n=1}^{N} \sum_{m=1}^{N} \delta_n^T \left( K_{G1} (M_{a1}[M_{a1} - M_{bi}])^T + M_{bi} [M_{bi} - M_{a1}]^T \right)
\]

\[
+ K_{G2} (M_{c1} [M_{c1} - M_{di}])^T + M_{di} [M_{di} - M_{c1}]^T
\]

\[
+ 2 \sin \frac{n\pi}{2} (K_{G1} M_1^\prime [M_{bi} - M_{a1}]^T - K_{G2} M_1^\prime [M_{c1} - M_{di}]^T) \delta_m \, d\xi \quad [25d]
\]

\[
U_c^{(5)} = \frac{S}{2} \int \sum_{n=1}^{+1} \frac{1}{2} \left( \frac{D_{x1} + D_{x2}}{8L} \right) \delta_n^T M_3 M_3^T \delta_n
\]

\[
+ \sum_{n=1}^{N} \sum_{m=1}^{N} \left( \frac{\pi}{L} \right)^2 \delta_n^T \left( \frac{D_{x1} - D_{x2}}{2} \right) \delta_m^T M_3 M_3^T \delta_m \, d\xi \quad [25c]
\]

\[
U_c^{(6)} = \frac{2}{S} \int \sum_{n=1}^{+1} \frac{1}{8} \left( D_{y1} + D_{y2} \right) \delta_n^T M_6 M_6^T \delta_n + \sum_{n=1}^{N} \sum_{m=1}^{N} \left( \frac{D_{y1} - D_{y2}}{2} \right) \delta_n^T M_6 M_6^T \delta_m \, d\xi \quad [25f]
\]
\[ U_c(7) = - \int_{-1}^{+1} \sum_{n=1}^{N} \frac{n \pi}{4} \left( \frac{D_{\nu 1} + D_{\nu 2}}{4} \right) \delta_n^T M_3 M_6^T \delta_n \]

\[ + \sum_{n=1}^{N} \sum_{m=1}^{N} \frac{(n \pi L)}{R_{nm}} \left( D_{\nu 1} - D_{\nu 2} \right) \delta_n^T M_3 M_6^T \delta_m \, d\xi \]  

\[ U_c(8) = \frac{S}{2} \int_{-1}^{+1} \sum_{n=1}^{N} \frac{I}{8} \left( D_{G1} + D_{G2} \right) \delta_n^T \left( \frac{2}{\delta} \right)^2 M_4 M_4^T + \left( \frac{2n \pi}{L} \right)^2 M_5 M_5^T + \left( \frac{4n \pi}{S} \right) M_4 M_5^T \delta_n \]

\[ + \sum_{n=1}^{N} \sum_{m=1}^{N} \frac{R_{nm}}{2} \left( \frac{D_{G1} - D_{G2}}{2} \right) \delta_n^T \left( \frac{2}{\delta} \right)^2 M_4 M_4^T + \left( \frac{2n \pi}{L} \right)^2 M_5 M_5^T \delta_m \, d\xi \]  

3.6.2 Supporting I-beams

Similarly substituting Equations 15a to 15d into Equation 7 and performing the integration, the strain energy in the joist can be rewritten in terms of the coefficients \( U_n, V_n, W_n \) and \( \theta_n \) which are elements in the unknown displacement vectors \( \delta_n \) \((n=1, 2, \ldots, N)\) as:

\[ U_I = \frac{I}{4} \sum_{n=1}^{N} \left( \frac{A}{n \pi} \right)^4 \left( \frac{I_y}{n \pi} \right)^2 U_n^2 + I_y W_n^2 + I_x V_n^2 \right) + G J \left( \frac{2n \pi}{S} \right)^2 \right) (\theta_n)^2 \]  

3.6.3 Connectors

Furthermore substituting Equations 14a to 14d and Equations 15a to 15d into Equation 11 and performing the appropriate differentiation and integration, the strain energy in the connectors are expressed in terms of the coefficients \( U_n, U_{on}, W_n, V_n, V_{on}, w_{on}, S \) and \( \theta_n \) which are also elements in the unknown displacement vectors \( \delta_n \) \((n=1, 2, \ldots, N)\) as:
3.6.4 Load Potential

Two general load cases are considered in the treatment of the load potential: 1) two wheels with equal loading (either the front or back wheels in a lift truck) located on section 1 of the finite strip which represent the region \((0 < x \leq \frac{L}{2})\); and 2) two wheels with equal loading (either the front or back wheels in a lift truck) with one wheel located on section 1 and the second wheel located on section 2 of the finite strip where section 2 representing the region \(\frac{L}{2} < x \leq L\).

In the first load case the footprint and the associated loading from the two wheels are defined as:

\[
p(x,y) = \begin{cases} 
  p_0(i) & \text{if } (x_1 < x < x_2) \text{ or } (x_3 < x < x_4); \text{ and } (y_1 < y < y_2) \\
  0 & \text{otherwise}
\end{cases} \quad [28a]
\]

where \(p_0(i)\) represents the load per unit area from a wheel in which \((0 < x_1 < \frac{L}{2}); (0 < x_2 < \frac{L}{2}); (0 < x_3 < \frac{L}{2}); \text{ and } (0 < x_4 < \frac{L}{2})\).

Substituting Equations 14a, 28a, and 28b into Equation 13, the load potential for case 1 can therefore be expressed in terms of the functions \(F_{1n}, A_n, \text{ and } B_n\) as:

\[
\Omega_{L(1)} = p_0(i) \sum_{n=1}^{\frac{L}{\pi}} \left( F_{1n} \left( -\frac{L}{\pi n} \right) \left( \cos \frac{n\pi x_2}{L} - \cos \frac{n\pi x_1}{L} + \cos \frac{n\pi x_4}{L} - \cos \frac{n\pi x_3}{L} \right) \right)
\]

\[
+ A_n \left( x_2 - x_1 + x_4 - x_3 - \frac{x_2^2 - x_1^2 + x_4^2 - x_3^2}{L} \right) + ...
\]
The load potential for case 1 can also be expressed in terms of the unknown displacement vectors \( \delta_n \) 
\((n=1, 2, \ldots, N)\) as

\[
\Omega_{L(1)} = \frac{S}{2} P_0(1) \left( -\frac{L}{n\pi} \left( \cos \frac{n\pi x_2}{L} - \cos \frac{n\pi x_1}{L} + \cos \frac{n\pi x_4}{L} - \cos \frac{n\pi x_3}{L} \right) M_\gamma(\xi)^T + \left( \frac{x_2^2 - x_1^2 + x_4^2 - x_3^2}{L} \right) M_\gamma(\xi)^T \right. 
+ \left. \left( x_2 - x_1 + x_4 - x_3 - \frac{x_2^2 - x_1^2 + x_4^2 - x_3^2}{L} \right) M_\alpha(\xi)^T \delta_n \right) \tag{30}
\]

where

\[
M_\gamma(\xi) = \int \dot{\gamma}(\xi) \, d\xi; \quad M_\alpha(\xi) = \int \dot{\alpha}(\xi) \, d\xi; \quad M_\beta(\xi) = \int \dot{\beta}(\xi) \, d\xi \tag{31}
\]

In the second load case the footprint and the associated loading from the two wheels are defined as:

\[
p(x, y) = p_0(2) \quad \text{if} \quad (x_1 < x < x_2) \text{ or } (x_3 < x < x_4); \text{ and } (y_1 < y < y_2) \tag{32a}
\]

\[
p = 0 \quad \text{otherwise} \tag{32b}
\]

where \( p_0(2) \) represents the load per unit area from a wheel in which \((0 < x_1 < \frac{L}{2}); \ (0 < x_2 < \frac{L}{2}); \ (\frac{L}{2} < x_3 < L); \text{ and } (\frac{L}{2} < x_4 < L). \)

Substituting Equations 14a to 14b and Equations 32a to 32d into Equation 13, load potential for case 2 can also be expressed in terms of the functions \( F_{1n}^\gamma, F_{1n}^\alpha, A_n, B_n, C_n, \text{ and } D_n \) as:
\[ \Omega_{L(2)} = P_{O(2)} \int_{y_1}^{y_2} \sum_{n=1}^{N} F_{1n} \left( - \frac{L}{n\pi} \right) \left( \cos \frac{n\pi x_2}{L} - \cos \frac{n\pi x_1}{L} \right) + B_n \left( \frac{x_2^2 - x_1^2}{L} \right) + A_n \left( x_2 - x_1 - \frac{x_2^2 - x_1^2}{L} \right) \]

\[ + F_{1n} \left( - \frac{L}{n\pi} \right) \left( \cos \frac{n\pi x_4}{L} - \cos \frac{n\pi x_3}{L} \right) + D_n \left( \frac{x_4^2 - x_3^2}{L} - x_4 + x_3 \right) \]

\[ + 2 C_n \left( x_4 - x_3 - \frac{x_4^2 - x_3^2}{2L} \right) dy \]  \[\text{[33]}\]

Finally the load potential for case 2 can be expressed in terms of the unknown displacement vectors \( \delta_n \) \((n=1, 2, \ldots, N)\) as:

\[ \Omega_{L(2)} = \frac{S}{2} P_{O(2)} \sum_{n=1}^{N} \left( - \frac{L}{n\pi} \right) \left\{ \left( \cos \frac{n\pi x_2}{L} - \cos \frac{n\pi x_1}{L} \right) \mathbf{M}_7^T(\xi) + \left( \cos \frac{n\pi x_4}{L} - \cos \frac{n\pi x_3}{L} \right) \mathbf{M}_7^T(\xi) \right\} \]

\[ + \left( \frac{x_2^2 - x_1^2}{L} \right) \mathbf{M}_b(\xi)^T \left( x_2 - x_1 - \frac{x_2^2 - x_1^2}{L} \right) \mathbf{M}_a(\xi)^T \]

\[ + \left( \frac{x_4^2 - x_3^2}{L} - x_4 + x_3 \right) \mathbf{M}_d(\xi)^T + 2 \left( x_4 - x_3 - \frac{x_4^2 - x_3^2}{2L} \right) \mathbf{M}_c(\xi)^T \delta_n \]  \[\text{[34]}\]

where

\[
\begin{align*}
\mathbf{M}_7^T(\xi) &= \int M_0(\xi) \, d\xi; \quad \mathbf{M}_7^T(\xi) = \int M_4(\xi) \, d\xi; \quad \mathbf{M}_7^T(\xi) = \int M_4(\xi) \, d\xi \\
\mathbf{M}_7^T(\xi) &= \int M_1(\xi) \, d\xi; \quad \mathbf{M}_7^T(\xi) = \int M_1(\xi) \, d\xi; \quad \mathbf{M}_7^T(\xi) = \int M_1(\xi) \, d\xi \\
\end{align*}
\]  \[\text{[35]}\]

3.7 Minimization of Energy in the System

The total energy in a finite strip is given by:

\[ \Pi = U_c + U_1 + U_N - \Omega_L \]  \[\text{[36]}\]

which is a minimum when the first variation of \( \Pi \) with respect to the unknown displacement vectors \( \delta_k \)
\( k = 1, 2, \ldots, N \) is set to zero as:

\[
\delta_k[\Pi] = \delta_k[U_c] + \delta_k[U_1] + \delta_k[U_N] - \delta_k[\Omega_L] = 0 \quad \text{for } (k=1, 2, \ldots, N) \tag{37}
\]

### 3.7.1 Cover

Consider now the first variation of \( U_c \) with respect to \( \delta_k \) \((k=1, 2, \ldots, N)\) to determine the contribution from \( U_c \) into the system of equation. The matrix \( \delta_k[U_c] \) can be expressed as:

\[
\delta_k[U_c] = \sum_{j=1}^{8} \delta_k[U_c^{(j)}] \tag{38}
\]

where

\[
\begin{align*}
\delta_k[U_c^{(1)}] &= \frac{S}{8L^3} (k\pi)^4 \left\{ \frac{1}{4} \left( Kx_1 M_0^T M_0^T + Kx_2 M_0^T M_0^T - Kx_1 M_1^T M_1^T - Kx_2 M_1^T M_1^T \right) d\xi \delta_k \\
&\quad + \sum_{n \neq k}^{N} \left( \frac{1}{4} \right)^2 \frac{4}{L} R_{kn} \left( Kx_1 M_0^T M_0^T - Kx_2 M_0^T M_0^T \right) d\xi \delta_n \right\} \tag{39a} \\
\delta_k[U_c^{(2)}] &= \left( \frac{2T}{S^3} \right) \left\{ \frac{1}{4} \left( K_y M_2^T M_2^T + K_y M_1^T M_1^T \right) d\xi \delta_k \\
&\quad + \sum_{n \neq k}^{N} \frac{4}{L} R_{kn} \left( K_y M_1^T M_2^T - K_y M_2^T M_1^T \right) d\xi \delta_n \\
&\quad + \sum_{n=1}^{N} \left( \frac{2}{3} \left[ M_{a2} M_{a2}^T + M_{b2} M_{b2}^T \right] + \frac{1}{3} \left[ M_{b2} M_{a2}^T + M_{a2} M_{b2}^T \right] \right) \\
&\quad + K_y \left( \frac{2}{3} \left[ M_{c2} M_{c2}^T + M_{d2} M_{d2}^T \right] + \frac{1}{3} \left[ M_{d2} M_{c2}^T + M_{c2} M_{d2}^T \right] \right) + \ldots \right\}
\end{align*}
\]
\[
\frac{4}{k\pi} \left( K_{y_1} M_2^T \left( M_{b_2} \left( \frac{2}{k\pi} \sin \frac{k\pi}{2} \right) - \cos \frac{k\pi}{2} \right) + M_{a_2} T \left( 1 - \frac{2}{k\pi} \sin \frac{k\pi}{2} \right) \right) \\
+ K_{y_2} M_2^T \left( M_{c_2} T \left( \cos \frac{k\pi}{2} + \frac{2}{k\pi} \sin \frac{k\pi}{2} \right) - M_{d_2} T \left( \cos k\pi + \frac{2}{k\pi} \sin \frac{k\pi}{2} \right) \right) \\
+ \frac{4}{n\pi} \left( K_{y_1} \left( M_{b_2} \left( \frac{n\pi}{2} \sin \frac{n\pi}{2} \right) - \cos \frac{n\pi}{2} \right) + M_{a_2} \left( 1 - \frac{2}{n\pi} \sin \frac{n\pi}{2} \right) \right) M_2^T \\
+ K_{y_2} \left( M_{c_2} \left( \cos \frac{n\pi}{2} + \frac{2}{n\pi} \sin \frac{n\pi}{2} \right) - M_{d_2} \left( \cos n\pi + \frac{2}{n\pi} \sin \frac{n\pi}{2} \right) \right) d\xi \delta_k \]  

\[
\delta_k \left[ U_e^{(3)} \right] = - \frac{k^2 \pi^2}{2SL} \left\{ \int_{1}^{+1} \left( K_{v_1} [M_0^T M_2^T + M_2^T M_0^T] + K_{v_2} [M_0^T M_2^T + M_2^T M_0^T] \right) d\xi \right\} \delta_k \\
+ \sum_{n=1}^{N} \left( K_{v_1} \left[ M_0^T \left( M_{b_2} \left( \frac{2}{n\pi} \sin \frac{n\pi}{2} \right) - \cos \frac{n\pi}{2} \right) + M_{a_2} \left( 1 - \frac{2}{n\pi} \sin \frac{n\pi}{2} \right) \right] \right) M_0^T \\
+ K_{v_2} \left( M_{c_2} \left( \cos \frac{n\pi}{2} + \frac{2}{n\pi} \sin \frac{n\pi}{2} \right) - M_{d_2} \left( \cos n\pi + \frac{2}{n\pi} \sin \frac{n\pi}{2} \right) \right) M_0^T d\xi \delta_n \]  

\[
\delta_k \left[ U_e^{(4)} \right] = \frac{2k^2 \pi^2}{SL} \left\{ \int_{1}^{+1} \left( K_{G_1} M_1^T M_1^T + K_{G_2} M_1^T M_1^T \right) d\xi \right\} \delta_k \\
+ \sum_{n=1}^{N} \left( K_{G_1} \left[ M_1^T \left( M_{b_2} \left( \frac{n\pi}{2} \sin \frac{n\pi}{2} \right) - \cos \frac{n\pi}{2} \right) + M_{a_2} \left( 1 - \frac{2}{n\pi} \sin \frac{n\pi}{2} \right) \right] \right) M_1^T d\xi \delta_n + ...
\[\sum_{k=1}^{N} \left( \frac{8}{k^2 \pi^2} \right)^{+1} \left( K_{G1} \left( M_{a1} \left[ M_{a1} - M_{b1} \right]^T + M_{b1} \left[ M_{b1} - M_{a1} \right]^T \right) + \right. \]

\[+ K_{G2} \left( M_{c1} \left[ M_{c1} - M_{d1} \right]^T + M_{d1} \left[ M_{d1} - M_{c1} \right]^T \right) \]

\[+ \sin \left( \frac{k \pi}{2} \right) \left( K_{G1} \left[ M_{b1} - M_{a1} \right]^T - K_{G2} \left[ M_{c1} - M_{d1} \right]^T \right) \]

\[+ \sin \left( \frac{n \pi}{2} \right) \left( K_{G1} \left[ M_{b1} - M_{a1} \right]^T - K_{G2} \left[ M_{c1} - M_{d1} \right]^T \right) \]

\[= \frac{k^2 \pi^2 S}{\delta L} \left( (D_{x1} + D_{x2}) \int_{-1}^{+1} M_{3} M_{3}^T d\xi \right) + \left( \frac{4}{k \pi} \right) \left[ \delta_k \right] \]

\[\delta_k[U_c^{(6)}] = \frac{L}{2S} \left( (D_{y1} + D_{y2}) \int_{-1}^{+1} M_{6} M_{6}^T d\xi \right) + \left( \frac{4}{k \pi} \right) \left[ \delta_k \right] \]

\[\delta_k[U_c^{(7)}] = -\frac{\pi}{4} \left( k \left( D_{\nu1} + D_{\nu2} \right) \right) \int_{-1}^{+1} \left[ k M_{3} M_{6}^T + n M_{6} M_{3}^T \right] d\xi \]

\[\delta_k[U_c^{(8)}] = \frac{SL}{16} \left( (D_{G1} + D_{G2}) \int_{-1}^{+1} 2 \left( \frac{2}{S} \right)^2 M_{4} M_{4}^T + \left( \frac{k \pi}{L} \right)^2 M_{5} M_{5}^T \right) + \left( \frac{4k \pi}{S L} \right) \left[ M_{4} M_{5}^T + M_{5} M_{4}^T \right] d\xi \]

\[+ \left( \frac{4}{k \pi} \right) \left[ \delta_k \right] \]

\[\delta_k[U_c^{(9)}] = \frac{1}{2S} \left( (D_{\nu1} + D_{\nu2}) \right) \int_{-1}^{+1} \left[ k M_{3} M_{6}^T + n M_{6} M_{3}^T \right] d\xi \]

\[\delta_k[U_c^{(10)}] = \frac{1}{2S} \left( (D_{\nu1} + D_{\nu2}) \right) \int_{-1}^{+1} \left[ k M_{3} M_{6}^T + n M_{6} M_{3}^T \right] d\xi \]

\[\delta_k[U_c^{(11)}] = \frac{1}{2S} \left( (D_{\nu1} + D_{\nu2}) \right) \int_{-1}^{+1} \left[ k M_{3} M_{6}^T + n M_{6} M_{3}^T \right] d\xi \]

\[\delta_k[U_c^{(12)}] = \frac{1}{2S} \left( (D_{\nu1} + D_{\nu2}) \right) \int_{-1}^{+1} \left[ k M_{3} M_{6}^T + n M_{6} M_{3}^T \right] d\xi \]

\[\delta_k[U_c^{(13)}] = \frac{1}{2S} \left( (D_{\nu1} + D_{\nu2}) \right) \int_{-1}^{+1} \left[ k M_{3} M_{6}^T + n M_{6} M_{3}^T \right] d\xi \]
3.7.2 Supporting I-Beams

The contribution of the supporting I-beams to the system of equations is obtained by taking the first variation of $U_I$ with respect to the appropriate terms in the unknown displacement vectors $\delta_k$ ($k = 1, 2, ..., N$) which yields the following non-zero components of the matrices $\delta_k[U_I]$ ($k = 1, 2, ..., N$) as:

\[
\frac{\partial U_I}{\partial W_k} = \frac{E}{2} \frac{L}{k^4 \pi^4} I_y W_k \quad [40]
\]

\[
\frac{\partial U_I}{\partial V_k} = \frac{E}{2} \frac{L}{k^4 \pi^4} I_z V_k \quad [41]
\]

\[
\frac{\partial U_I}{\partial U_k} = \frac{E}{2} \frac{A}{k^2 \pi^2} U_k \quad [42]
\]

\[
\frac{\partial U_I}{\partial \theta_n S} = \frac{G}{2} \frac{J}{L} (k^2 \pi^2) \theta_n S \quad [43]
\]

3.7.3 Connectors

Similarly taking first variation of $U_N$ with respect to the appropriate terms in the unknown displacement vectors $\delta_k$ ($k = 1, 2, ..., N$), yields the contributions of the connectors to the system of equations as the following non-zero components of the matrices $\delta_k[U_N]$ ($k = 1, 2, ..., N$):

\[
\frac{\partial U_N}{\partial u_{0k}} = \frac{K_{xc}}{2} \frac{L}{e} \left( u_{0k} - U_k - \frac{(d+h_1) k \pi}{2 L} W_k \right) \quad [44a]
\]

\[
\frac{\partial U_N}{\partial u_{0k}} = \frac{K_{xc}}{2} \frac{L}{e} \left( u_{0k} - U_k - \frac{(d+h_1) k \pi}{2 L} W_k \right) \quad [44b]
\]

\[
\frac{\partial U_N}{\partial W_k} = - \frac{K_{xc}}{2} \frac{(d+h_1) k \pi}{4 e} \left( u_{0k} - U_k - \frac{(d+h_1) k \pi}{2 L} W_k \right) \quad [44c]
\]

\[
\frac{\partial U_N}{\partial v_{0k}} = \frac{K_{yc}}{2} \frac{L}{e} \left( v_{0k} - V_k - \frac{d}{2 S} w_{0k} S - \frac{h_f}{2 S} \theta_k S \right) \quad [44d]
\]
\[
\frac{\partial U_N}{\partial v_k} = -\frac{K_{ve} L}{2 e} (v_{0k} - V_k - \frac{d}{2 S} w_{0k} S - \frac{h_1}{2 S} \theta_k S)
\]

\[
\frac{\partial U_N}{\partial \theta_k} = -\frac{K_{ve} L h_1}{4 e S} (v_{0k} - V_k - \frac{d}{2 S} w_{0k} S - \frac{h_1}{2 S} \theta_k S) - \frac{K_{\theta e} L}{2 e S^2} (w_{0k} S - \theta_k S)
\]

\[
\frac{\partial U_N}{\partial w_{0k} S} = -\frac{K_{ve} L d}{4 e S} (v_{0k} - V_k - \frac{d}{2 S} w_{0k} S - \frac{h_1}{2 S} \theta_k S) + \frac{K_{\theta e} L}{2 e S^2} (w_{0k} S - \theta_k S)
\]

### 3.7.4 Load Potential

The first variation of \( \Omega_{L(j)} \) (\( j = 1 \) or \( 2 \)) with respect to \( \delta_k \) (\( k = 1, 2, ..., N \)) for the two cases of loading can be determined to obtain the contribution of \( \Omega_L \) into the system of equation. The vectors \( \delta_k [\Omega_{L(j)}] \) (\( j = 1 \) or \( 2 \)) can be expressed as:

\[
\delta_k [\Omega_{L(i)}] = \frac{S}{2} p_{o(i)} \left\{ \left( -\frac{L}{k \pi} \right) \left( \cos \frac{k \pi x_2}{L} - \cos \frac{k \pi x_1}{L} + \cos \frac{k \pi x_4}{L} - \cos \frac{k \pi x_3}{L} \right) M_7(\xi)^T + \left( \frac{x_2^2 - x_1^2 + x_4^2 - x_3^2}{L} \right) M_7(\xi)^T \right. \\
+ \left( x_2 - x_1 + x_4 - x_3 - \frac{x_2^2 - x_1^2 + x_4^2 - x_3^2}{L} \right) M_7(\xi)^T \right\}
\]

and

\[
\delta_k [\Omega_{L(2)}] = \frac{S}{2} p_{o(2)} \left\{ \left( -\frac{L}{k \pi} \right) \left( \cos \frac{k \pi x_2}{L} - \cos \frac{k \pi x_1}{L} \right) M_7(\xi)^T + \left( \cos \frac{k \pi x_4}{L} - \cos \frac{k \pi x_3}{L} \right) M_7(\xi)^T \right\} + \left( \frac{x_2^2 - x_1^2}{L} \right) M_7(\xi)^T + \left( \frac{x_2^2 - x_1^2}{L} \right) M_7(\xi)^T + \left( \frac{x_4^2 - x_3^2}{L} - x_4 + x_3 \right) M_7(\xi)^T + 2 \left( x_4 - x_3 - \frac{x_4^2 - x_3^2}{2L} \right) M_7(\xi)^T \right\}
\]
For a single bay system, Equation 37 can be expressed in matrix form as:

\[
\delta_k \left[ U_c \right] + \delta_k \left[ U_1 \right] + \delta_k \left[ U_N \right] = \delta_k \left[ \Omega_{L_i} \right]
\]

for \( k = 1, 2, \ldots, N \) \[47\]

or alternately as:

\[
\begin{bmatrix}
\mathbf{K}(1,1) & \mathbf{K}(1,2) & \ldots & \mathbf{K}(1,k) & \ldots & \mathbf{K}(1,N) \\
\mathbf{K}(2,1) & \mathbf{K}(2,2) & \ldots & \mathbf{K}(2,k) & \ldots & \mathbf{K}(2,N) \\
\vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\
\mathbf{K}(k,1) & \mathbf{K}(k,2) & \ldots & \mathbf{K}(k,k) & \ldots & \mathbf{K}(k,N) \\
\vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\
\mathbf{K}(N,1) & \mathbf{K}(N,2) & \ldots & \mathbf{K}(N,k) & \ldots & \mathbf{K}(N,N)
\end{bmatrix} \begin{bmatrix}
\delta_1 \\
\delta_2 \\
\vdots \\
\delta_k \\
\vdots \\
\delta_N
\end{bmatrix} = \begin{bmatrix}
\mathbf{R}_1 \\
\mathbf{R}_2 \\
\vdots \\
\mathbf{R}_k \\
\vdots \\
\mathbf{R}_N
\end{bmatrix}
\]

\[48\]

in which the load sub-vectors \( \mathbf{R}_k \ (k = 1, 2, \ldots, N) \) correspond to the first variation of the load potential \( \delta_k [\Omega_{L(j)}] \) \((j = 1 \text{ or } 2)\) for the two load cases; the stiffness sub-matrices \( \mathbf{K}(i,j) \ (i,j = 1, 2, \ldots, N) \) correspond to contributions from the first variation of the strain energies for the various Fourier terms. Note that \( \mathbf{K}(i,j) \) is a square 39 x 39 matrix and \( \mathbf{K}(i,j) = \mathbf{F}(j,i) \). Finally the integration needed in Equations 39a to 39h and Equations 31 and 35 can be performed by a six point Gaussian quadrature procedure.

3.8 Numerical Solution of Global System of Equations

For a deck systems with multiple bays, the total energy in the system is given by:

\[
\Pi_{\text{Total}} = \sum_{i=1}^{NJT} \left( U_c + U_1 + U_N - \Omega_{L_i} \right)
\]

\[49\]

in which \( NJT \) represents the number of bays and the index \([i]\) corresponds to a particular bay in the deck system. Defining \( \Delta_k \ (k = 1, 2, \ldots, N) \) as the global unknown displacement vectors, \( \Pi_{\text{Total}} \) is
minimized by taking the first variation of \( \Pi_{\text{Total}} \) with respect to the unknown global displacement vectors \( \Delta_k \) \((k=1, 2, \ldots, N)\) and setting it to zero as:

\[
\delta_k[\Pi_{\text{Total}}] = \sum_{i=1}^{N_JT} (\delta_k[U_1] + \delta_k[U_2] + \delta_k[U_N] - \delta_k[\Omega_L])_i = 0 \quad \text{for } (k=1, 2, \ldots, N) \quad [50]
\]

Therefore, the system of equation of individual bays can be coupled together by standard methods to form a global system of equations of the assembly where the degrees of freedom along lines 1-1 and 2-2 will be shared between adjacent member. The following thirty-two degrees of freedom per Fourier term are shared between adjoining finite strips in a deck systems:

\[
\delta_T^{\text{shared}} = \{\begin{align*}
\delta_1^I, \delta_2^I, \delta_3^I, \delta_4^I, \delta_5^I, \delta_6^I, \delta_7^I, \delta_8^I, \delta_9^I, \delta_10^I, \delta_11^I, \delta_12^I, \\
\delta_1^II, \delta_2^II, \delta_3^II, \delta_4^II, \delta_5^II, \delta_6^II, \delta_7^II, \delta_8^II, \delta_9^II, \delta_10^II, \delta_11^II, \delta_12^II,
\end{align*}\}

\[
(n=1, 2, \ldots, N) \quad [51]
\]

The global system of equations can be obtained when the appropriate terms are substituted into Equation 50. Defining \( K(i,j) \) \((i,j=1, 2, \ldots, N)\) as the global stiffness sub-matrices and \( R_i \) \((i=1, 2, \ldots, N)\) as the global load sub-vectors, the global system of equations can be expressed in matrix form as:

\[
\begin{align*}
K(1,1) & \quad K(1,2) & \quad \cdots & \quad K(1,k) & \quad \cdots & \quad K(1,N) \\
K(2,1) & \quad K(2,2) & \quad \cdots & \quad K(2,k) & \quad \cdots & \quad K(2,N) \\
\vdots & & & \vdots & & \vdots \\
K(k,1) & \quad K(k,2) & \quad \cdots & \quad K(k,k) & \quad \cdots & \quad K(k,N) \\
\vdots & & & \vdots & & \vdots \\
K(N,1) & \quad K(N,2) & \quad \cdots & \quad K(N,k) & \quad \cdots & \quad K(N,N)
\end{align*}
\begin{align*}
\Delta_1 \\
\Delta_2 \\
\vdots \\
\Delta_k \\
\vdots \\
\Delta_N
\end{align*}
\begin{align*}
= \\
R_1 \\
R_2 \\
\vdots \\
R_k \\
\vdots \\
R_N
\end{align*}
\quad [52]
As a result of sharing of the 32 degrees of freedom between adjoining bays in the deck system, \( R_i \) are \((23 \times NJT + 16) \times 1\) vectors and \( K(i,j) \) are \((23 \times NJT + 16) \times (23 \times NJT + 16)\) square matrices. Note that \( K(i,i) \) are symmetrical with a bandwidth of 39 but \( K(i,j) \) \((j \neq i)\) are unsymmetrical. The global stiffness matrix is symmetrical; i.e., \( K(i,i) = K(i,j) \).

Foschi (1982) demonstrated that such a system of equation can be efficiently solved by the Jacobi iterative procedure as follows:

\[
\Delta^i_k = K(k,k)^{-1}\left\{R_k - \sum_{n=1}^{N} K(k,n) \Delta^{i-1}_n \right\} \quad (k=1, 2, ..., N) \tag{53}
\]

where the superscript \( i \) indicates the \( i \)th iteration and the initial vectors are taken as:

\[
\Delta^0_k = K(k,k)^{-1}R_k \tag{54}
\]

The iteration is terminated when the following convergence criterion is met:

\[
\frac{\| \Delta^i_k - \Delta^{i-1}_k \|}{\| \Delta^{i-1}_k \|} < \epsilon \quad (k=1, 2, ..., N) \tag{55}
\]

where \( \| \cdot \| \) represents the norm of the vector and \( \epsilon \) is taken as a small number (say 0.005).

Given a particular loading pattern \( p(x,y) \), the solution of global system of equation yields \( \Delta_k \) \((k = 1, 2, ..., N)\); therefore, the deformed shape of the structure can be estimated which in turn will be used to predict the critical stresses in the cover.

3.9 Shear Deflection of the Supporting I-beams

When the span to depth ratio of the supporting I-beams is small, the influence of shear on the
deflection of the I-beams may become important. It can be shown that the shear deflection $W_{\text{shear}}$ of
the supporting I-beams satisfies the following condition approximately:

$$\frac{dW_{\text{shear}}}{dx} = \frac{\alpha_s}{G A} \left\{ -E I_y \frac{d^3 W}{dx^3} - \frac{K_x h_I}{2 e} \left( u(x,0) - U - \frac{h_I + d}{2} \frac{dW}{dx} \right) \right\}$$

where $\alpha_s$ is a shape factor which depends on the geometry of the cross section and the distribution of
the shear stresses. For an I-beam it can be shown that $\alpha$ can be approximated by:

$$\alpha_s = \frac{A}{A_y} \left( \frac{b}{2} + t_f \left( t_f - t_f^2 \right) + \frac{(h_I - 2 t_f)^2}{8} t_w \right)$$

Substituting Equations 14c, 15a, and 15c into Equation 56 and performing the integration
yields the shear deflection in each supporting I-beam as:

$$W_{\text{shear}} = \sum_{n=1}^{N} \frac{\alpha_s}{G A} \left\{ E I_y \left( \frac{n \pi}{L} \right)^2 + K_x h_I \frac{h_I + d}{4 e} \right\} W_n - \frac{K_x h_I L}{n \pi 2 e} \left( u_{on} - U_n \right) \sin \frac{n \pi}{L}$$

3.10 Bending Stresses and Rolling Shear Stresses in Cover

Two common failure modes of the cover in truck deck applications are bending failure and
rolling shear failure. Rolling shear failures result from shear forces which roll wood fibers within a layer
of veneer over each other. Consider again only one bay in a deck system, the solution of the global
system of equations yields $\Delta_k$ ($k=1, 2, ..., N$) from which $\delta_k$ ($k=1, 2, ..., N$) can be obtained. Assuming
the face grain is oriented perpendicular to the direction of the supporting I-beams, the bending stresses
of the exterior ply in the cover (at $z = \frac{d}{2}$) in the parallel to face grain direction can be estimated as:

$$\sigma_y(x, y) = \frac{E_y}{1 - \nu_{xy} \nu_{yx}} \left( \frac{\partial^2}{\partial y^2} + \nu_{xy} \frac{\partial u}{\partial x} - \frac{d}{2} \left( \frac{\partial^2 w}{\partial y^2} + \nu_{xy} \frac{\partial^2 w}{\partial x^2} \right) \right)$$
where $E_y$ denotes the modulus of elasticity of the exterior ply in the cover in the y-direction and $d$ denotes the thickness of the cover. Note that $E_y$ is assumed to be equaled for both sections 1 and 2 of the cover. Similarly, the bending stresses of the exterior ply in the cover (at $z = \frac{d}{2}$) in the perpendicular to face grain direction can be estimated as:

$$
\sigma_x(x, y) = \frac{E_x}{1 - \nu_{xy}^2} \left( \frac{\partial u}{\partial x} + \nu_{xy} \frac{\partial v}{\partial y} - \frac{d}{2} \left( \frac{\partial^2 w}{\partial x^2} + \nu_{xy} \frac{\partial^2 w}{\partial y^2} \right) \right)
$$

where $E_x$ denotes the modulus of elasticity of the exterior ply in the cover in the x-direction. Note that $E_x$ is also assumed to be equal for both sections 1 and 2 of the cover.

Consider the bending stresses in y-direction in one bay of the deck system, substituting the shape functions and the unknown displacement vectors into Equation 59 and performing the appropriate differentiation for the location $y = -\frac{S}{2}$, Equation 59 can be expressed in terms of elements in the unknown displacement vectors $\delta_n$ ($n=1, 2, ..., N$) for the location $y = -\frac{S}{2}$ as:

$$
\sigma_y(x, -\frac{S}{2}) = \frac{E_y}{1 - \nu_{xy}^2} \sum_{n=1}^{N} \left\{ \nu_{1n}' - \nu_{xy} \left( \frac{n\pi}{L} \right) u_{1n} - \frac{d}{2S^2} \left\{ -46 w_{1n} - 12 w_{2n} S - 16 w_{on} S 
+ 32 W_n + 14 w_{2n} - 2 w_{2n} S \right\} + \frac{d}{2} \nu_{xy} \left( \frac{n\pi}{L} \right)^2 w_{1n} \sin \left( \frac{n\pi x}{L} \right) \right\} - \frac{d}{2S^2} \left\{ -46 w_{1n} - 12 w_{1n} S + 14 w_{2n} - 2 w_{2n} S \right\} \left( 1 - \frac{2x}{L} \right) 
+ \left\{ -46 w_{1n} - 12 w_{1n} S + 14 w_{2n} - 2 w_{2n} S \right\} \left( \frac{2x}{L} \right) \right\} \quad \text{for } (0 < x < \frac{L}{2}) \quad [61a]
$$

and
\[
\sigma_y(x, -\frac{S}{2}) = \frac{E_y}{1 - \nu_{xy}^2} \sum_{n=1}^{N} \left\{ v_{1n}' - \nu_{xy} \left( \frac{n\pi}{L} \right) u_{1n} - \frac{d}{2S^2} \left( -46 w_{1n}^{\cdot\cdot} - 12 w_{2n}^{\cdot\cdot} ' S - 16 w_{0n} ' S \right) + 32 w_n + 14 w_{2n}^{\cdot\cdot} - 2 w_{2n}^{\cdot\cdot} ' S + \frac{d}{2} \nu_{xy} \left( \frac{n\pi}{L} \right)^2 w_{1n}^{\cdot\cdot}\right\} \sin \frac{n\pi x}{L} \\
- \frac{d}{2S^2} \left\{ \left( -46 w_{1n}^{I\cdot\cdot} - 12 w_{2n}^{I\cdot\cdot} ' S + 14 w_{2n}^{I\cdot\cdot} - 2 w_{2n}^{I\cdot\cdot} ' S \right) \left( 2 - \frac{2x}{L} \right) \right\} + \left( -46 w_{1n}^{IV\cdot\cdot} - 12 w_{1n}^{IV\cdot\cdot} ' S + 14 w_{2n}^{IV\cdot\cdot} - 2 w_{2n}^{IV\cdot\cdot} ' S \right) \left( \frac{2x}{L} - 1 \right) \right\} \text{ for } \left( \frac{L}{2} < x < L \right) \ [61b]
\]

Similarly substituting the shape functions and the unknown displacement vectors into Equation 59 and performing the appropriate differentiation for the location \( y=0 \), Equation 59 can be expressed in terms of elements in the unknown displacement vectors \( \delta_n \ (n=1, 2, ..., N) \) for the location \( y=0 \) as:

\[
\sigma_y(x, 0) = \frac{E_y}{1 - \nu_{xy}^2} \sum_{n=1}^{N} \left\{ \left( -\frac{3}{2} v_{1n} - \frac{1}{4} v_{1n} ' S + \frac{3}{2} v_{2n} - \frac{1}{4} v_{2n} ' S \right) \left( \frac{1}{S} - \nu_{xy} \left( \frac{n\pi}{L} \right) u_{0n} \right) - \frac{d}{2S^2} \left\{ 8 w_{1n} + w_{1n} ' S - 16 W_n + 8 w_{2n} - w_{2n} ' S \right\} + \frac{d}{2} \nu_{xy} \left( \frac{n\pi}{L} \right)^2 W_n \right\} \sin \frac{n\pi x}{L} \\
- \frac{d}{2S^2} \left\{ \left( 8 w_{1n}^{I\cdot\cdot} + w_{1n}^{I\cdot\cdot} ' S + 8 w_{2n}^{I\cdot\cdot} - w_{2n}^{I\cdot\cdot} ' S \right) \left( 1 - \frac{2x}{L} \right) \right\} + \left( 8 w_{1n}^{IV\cdot\cdot} + w_{1n}^{IV\cdot\cdot} ' S + 8 w_{2n}^{IV\cdot\cdot} - w_{2n}^{IV\cdot\cdot} ' S \right) \left( \frac{2x}{L} - 1 \right) \right\} \text{ for } \left( 0 < x < \frac{L}{2} \right) \ [62a]
\]

and

\[
\sigma_y(x, 0) = \frac{E_y}{1 - \nu_{xy}^2} \sum_{n=1}^{N} \left\{ \left( -\frac{3}{2} v_{1n} - \frac{1}{4} v_{1n} ' S + \frac{3}{2} v_{2n} - \frac{1}{4} v_{2n} ' S \right) \left( \frac{1}{S} - \nu_{xy} \left( \frac{n\pi}{L} \right) u_{0n} \right) + ... \right\}
\]
Here consider the bending stresses in x-direction in one bay of the deck system, substituting the shape functions and the unknown displacement vectors into Equation 60 and performing the appropriate differentiation for the location \( y = -\frac{S}{2} \). Equation 60 can be expressed in terms of elements in the unknown displacement vectors \( \delta_n \) (\( n=1, 2, ..., N \)) for the location \( y = -\frac{S}{2} \) as:

\[
\sigma_x(x, -\frac{S}{2}) = \frac{E_x}{1 - \nu_{xy} \nu_{yx}} \sum_{n=1}^{N} \left\{ \nu_{yx} v_{1n}' - \left( \frac{n\pi}{L} \right) u_{1n} - \frac{d}{2S^2} \nu_{yx} \left\{ -46 w_{1n}' - 12 w_{2n}' \right\} - 16 w_{0n}' \right\} \sin \left( \frac{n\pi x}{L} \right)
\]

\[
+ 32 W_n + 14 w_{2n}' - 2 w_{2n}' \right\} + \frac{d}{2} \left( \frac{n\pi}{L} \right)^2 w_{1n}' \sin \left( \frac{n\pi x}{L} \right)
\]

\[
- \frac{d}{2S^2} \nu_{yx} \left\{ -46 w_{1n}' - 12 w_{2n}' \right\} + 14 w_{2n}' - 2 w_{2n}' \right\} \left( \frac{2x}{L} - 1 \right)
\]

\[
+ \left( -46 w_{1n}' - 12 w_{2n}' \right\} + 14 w_{2n}' - 2 w_{2n}' \right\} \left( \frac{2x}{L} - 1 \right)
\]

for \( 0 < x < \frac{L}{2} \) \[63a\]
Similarly substituting the shape functions and the unknown displacement vectors into Equation 60 and performing the appropriate differentiation for the location $y=0$, Equation 60 can be expressed in terms of elements in the unknown displacement vectors $\delta_n$ ($n=1, 2, ..., N$) for the location $y=0$ as:

$$
\sigma_x(x, 0) = \frac{E_x}{1-\nu_{xy}\nu_{yx}} \sum_{n=1}^{N} \left\{ \nu_{yx} \left( -\frac{3}{2} v_{1n} - \frac{1}{4} v_{1n}' S + \frac{3}{2} v_{2n} - \frac{1}{4} v_{2n}' S \right) \frac{1}{S} - \left( \frac{n\pi}{L} \right) u_{0n} \right\} - \frac{d}{2S^2} \nu_{yx} \left\{ 8 w_{1n} + w_{1n}' S - 16 W_n + 8 w_{2n} - w_{2n}' S \right\} + \frac{d}{2} \left( \frac{n\pi}{L} \right)^2 W_n \sin \left( \frac{n\pi x}{L} \right) + \left( 8 w_{1n} + w_{1n}' S + 8 w_{2n} - w_{2n}' S \right) \left( \frac{2x}{L} - 1 \right) \right\} \quad \text{for } (0 < x < \frac{L}{2}) \quad [64a]
$$

and

$$
\sigma_x(x, 0) = \frac{E_x}{1-\nu_{xy}\nu_{yx}} \sum_{n=1}^{N} \left\{ \nu_{yx} \left( -\frac{3}{2} v_{1n} - \frac{1}{4} v_{1n}' S + \frac{3}{2} v_{2n} - \frac{1}{4} v_{2n}' S \right) \frac{1}{S} - \left( \frac{n\pi}{L} \right) u_{0n} \right\} - \frac{d}{2S^2} \nu_{yx} \left\{ 8 w_{1n} + w_{1n}' S - 16 W_n + 8 w_{2n} - w_{2n}' S \right\} + \frac{d}{2} \left( \frac{n\pi}{L} \right)^2 W_n \sin \left( \frac{n\pi x}{L} \right) + \left( 8 w_{1n} + w_{1n}' S + 8 w_{2n} - w_{2n}' S \right) \left( \frac{2x}{L} - 1 \right) \right\} \quad \text{for } (\frac{L}{2} < x < L) \quad [64b]
$$
From equilibrium, the rolling shear stresses at any ply in the cover in the direction parallel to face grain \((\tau_{yz})\) and in the direction perpendicular to face grain \((\tau_{xz})\) can be expressed as:

\[
\tau_{yz} = -\int \left( \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} \right) dz
\]

and

\[
\tau_{xz} = -\int \left( \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \right) dz
\]

where

\[
\sigma_y(x, y) = \frac{E_y}{1 - \nu_{xy} \nu_{yy}} \left( \frac{\partial v}{\partial y} + \nu_{xy} \frac{\partial u}{\partial x} - z \left( \frac{\partial^2 w}{\partial y^2} + \nu_{xy} \frac{\partial^2 w}{\partial x^2} \right) \right)
\]

\[
\sigma_x(x, y) = \frac{E_x}{1 - \nu_{xy} \nu_{yx}} \left( \frac{\partial u}{\partial x} + \nu_{yx} \frac{\partial v}{\partial y} - z \left( \frac{\partial^2 w}{\partial x^2} + \nu_{yx} \frac{\partial^2 w}{\partial y^2} \right) \right)
\]

\[
\tau_{xy} = G_c \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2 z \frac{\partial^2 w}{\partial x \partial y} \right)
\]

Note that \(E_x\) and \(G_c\) denotes the modulus of elasticity in the x-direction and the modulus of rigidity of the exterior ply in the cover, respectively. Here \(E_x\) and \(G_c\) are again assumed to be equal for both sections 1 and 2 of the cover.

Consider the transDeck\textsuperscript{TM} panel with the 3-1-3-1-3 lay up scheme shown in Figure 1, it is of interest to estimate the rolling shear stresses at the interface between the top three exterior parallel plies and the cross band. Assuming identical thickness \((t)\) for each ply and identical elastic properties for the three exterior parallel plies, the rolling shear stresses at the interface in the direction parallel and perpendicular to face grain can be expressed as:
\[
\tau_{xz} = - \left( \frac{E_y}{1 - \nu_{xy} \nu_{yx}} \left( \frac{\partial u}{\partial x} \right)^2 \right) + G_c \left( \frac{\partial v}{\partial y} \right) + \left( \frac{E_y}{1 - \nu_{xy} \nu_{yx}} \nu_{xy} + G_c \left( \frac{\partial u}{\partial x} \right) \right) \frac{3}{3} t \\
- \left( \frac{E_y}{1 - \nu_{xy} \nu_{yx}} \left( \frac{\partial w}{\partial y} \right)^2 \right) + \left( \frac{E_y}{1 - \nu_{xy} \nu_{yx}} \nu_{xy} + 2G_c \left( \frac{\partial w}{\partial x} \right) \frac{3}{3} t^2 \right.
\]

and

\[
\tau_{yz} = - \left( \frac{E_x}{1 - \nu_{xy} \nu_{yx}} \left( \frac{\partial u}{\partial x} \right)^2 \right) + G_c \left( \frac{\partial v}{\partial y} \right) + \left( \frac{E_x}{1 - \nu_{xy} \nu_{yx}} \nu_{xy} + G_c \left( \frac{\partial u}{\partial x} \right) \right) \frac{3}{3} t \\
- \left( \frac{E_x}{1 - \nu_{xy} \nu_{yx}} \left( \frac{\partial w}{\partial y} \right)^2 \right) + \left( \frac{E_x}{1 - \nu_{xy} \nu_{yx}} \nu_{xy} + 2G_c \left( \frac{\partial w}{\partial x} \right) \frac{3}{3} t^2 \right)
\]

Performing the integration yields the following:

\[
\tau_{yz} = - \left( \frac{E_y}{1 - \nu_{xy} \nu_{yx}} \left( \frac{\partial v}{\partial y} \right)^2 \right) + G_c \left( \frac{\partial v}{\partial y} \right) + \left( \frac{E_y}{1 - \nu_{xy} \nu_{yx}} \nu_{xy} + G_c \left( \frac{\partial u}{\partial x} \right) \right) \frac{3}{3} t \\
- \left( \frac{E_y}{1 - \nu_{xy} \nu_{yx}} \left( \frac{\partial w}{\partial y} \right)^2 \right) + \left( \frac{E_y}{1 - \nu_{xy} \nu_{yx}} \nu_{xy} + 2G_c \left( \frac{\partial w}{\partial x} \right) \frac{3}{3} t^2 \right)
\]

Substituting the shape functions and \( \delta_n \) into Equations 70 and performing the appropriate differentiation for the location \( y=0 \), the rolling shear stresses at the interface between the cross and parallel ply for the parallel to face grain direction can be expressed in terms of elements in \( \delta_n \) as:

[68]

[69]

[70]

[71]
\[ \tau_{yx}(x, 0) = - \sum_{n=1}^{\infty} \left\{ \left( \frac{E_y}{1 - \nu_{xy} \nu_{yx}} \right) \left( \frac{1}{S^2} \right) \left( 8 v_{1n} + v_{1n}^1 S - 16 v_{0n} + 8 v_{2n} - v_{2n}^1 S \right) - \frac{G_c (n \pi)^2}{L^2} v_{0n} \right\} \]

\[ = - \left( \left( \frac{E_y}{1 - \nu_{xy} \nu_{yx}} \right) \nu_{xy} + G_c \left( \frac{n \pi}{L} \right)^2 \left( -\frac{3}{2} u_{1n} - \frac{1}{4} u_{1n}^1 \right) S + \frac{3}{2} u_{2n} - \frac{1}{4} u_{2n}^1 \right) S \right\} \left( \frac{n \pi}{L} \right)^2 \left( S \right) \sin(S) \]

\[ - \left( \left( \frac{E_y}{1 - \nu_{xy} \nu_{yx}} \right) \left( \frac{1}{S^3} \right) \left( -60 w_{1n}^1 - 6 w_{1n}^1 S - 48 w_{0n}^1 S + 60 w_{2n}^1 - 6 w_{2n}^1 S \right) \right) \left( \frac{2 \pi}{L} \right) \left( \frac{2 \pi}{L} \right)^2 \left( S \right) \sin(S) \]

\[ + \left( \left( \frac{E_y}{1 - \nu_{xy} \nu_{yx}} \right) \left( \frac{1}{S^3} \right) \left( 60 w_{1n}^1 + 6 w_{1n}^1 S + 60 w_{2n}^1 - 6 w_{2n}^1 \right) \left( \frac{2 \pi}{L} \right) \left( \frac{2 \pi}{L} \right)^2 \left( S \right) \sin(S) \]

\[ \text{for} \quad (0 < x < \frac{L}{2}) \quad \text{[72a]} \]

and

\[ \tau_{yx}(x, 0) = - \sum_{n=1}^{\infty} \left\{ \left( \frac{E_y}{1 - \nu_{xy} \nu_{yx}} \right) \left( \frac{1}{S^2} \right) \left( 8 v_{1n} + v_{1n}^1 S - 16 v_{0n} + 8 v_{2n} - v_{2n}^1 S \right) - \frac{G_c (n \pi)^2}{L^2} v_{0n} \right\} \]

\[ = - \left( \left( \frac{E_y}{1 - \nu_{xy} \nu_{yx}} \right) \nu_{xy} + G_c \left( \frac{n \pi}{L} \right)^2 \left( -\frac{3}{2} u_{1n} - \frac{1}{4} u_{1n}^1 \right) S + \frac{3}{2} u_{2n} - \frac{1}{4} u_{2n}^1 \right) S \right\} \left( \frac{n \pi}{L} \right)^2 \left( S \right) \sin(S) \]

\[ - \left( \left( \frac{E_y}{1 - \nu_{xy} \nu_{yx}} \right) \left( \frac{1}{S^3} \right) \left( -60 w_{1n}^1 - 6 w_{1n}^1 S - 48 w_{0n}^1 S + 60 w_{2n}^1 - 6 w_{2n}^1 S \right) \right) \left( \frac{2 \pi}{L} \right) \left( \frac{2 \pi}{L} \right)^2 \left( S \right) \sin(S) \]

\[ + \left( \left( \frac{E_y}{1 - \nu_{xy} \nu_{yx}} \right) \left( \frac{1}{S^3} \right) \left( 60 w_{1n}^1 + 6 w_{1n}^1 S + 60 w_{2n}^1 - 6 w_{2n}^1 \right) \left( \frac{2 \pi}{L} \right) \left( \frac{2 \pi}{L} \right)^2 \left( S \right) \sin(S) \]

\[ \text{for} \quad (\frac{L}{2} < x < 0) \quad \text{[72b]} \]
Substituting the shape functions and $\delta_n$ into Equations 71 and performing the appropriate differentiation for the location $y=0$, the rolling shear stresses at the interface between the cross and parallel ply for the perpendicular to face grain direction can be expressed in terms of elements in $\delta_n$ as:

$$
\tau_{xz}(x, 0) = - \sum_{n=1}^{N} \left\{ \left( - \frac{E_x}{1 - \nu_{xy} \nu_{yx}} \right) \left( \frac{\pi}{L} \right)^2 u_{0n} + G_c \left( \frac{1}{S^2} \right) \left( 8 u_{1n} + u_{1n}' S - 16 u_{0n} + 8 u_{2n} - u_{2n}' S \right) 
+ \left( \frac{E_x}{1 - \nu_{xy} \nu_{yx}} \nu_{yx} + G_c \right) \left( \frac{\pi}{SL} \right) \left( -\frac{3}{2} v_{1n} - \frac{1}{4} v_{1n}' S + \frac{3}{2} v_{2n} - \frac{1}{4} v_{2n}' S \right) \right\} t
- \left( - \frac{E_x}{1 - \nu_{xy} \nu_{yx}} \nu_{yx} + G_c \right) \left( \frac{\pi}{L} \right)^3 W_n + \left( \frac{E_x}{1 - \nu_{xy} \nu_{yx}} \nu_{yx} + 2 G_c \right) \left( \frac{\pi}{LS^2} \right) \left( 8 w_{1n} + w_{1n}' S - 16 W_n + 8 w_{2n} - w_{2n}' S \right) \right\} 12 t^2 \cos \left( \frac{n \pi x}{L} \right)
$$

and

$$
\tau_{xz}(x, 0) = - \sum_{n=1}^{N} \left\{ \left( - \frac{E_x}{1 - \nu_{xy} \nu_{yx}} \right) \left( \frac{\pi}{L} \right)^2 u_{0n} + G_c \left( \frac{1}{S^2} \right) \left( 8 u_{1n} + u_{1n}' S - 16 u_{0n} + 8 u_{2n} - u_{2n}' S \right) 
+ \left( \frac{E_x}{1 - \nu_{xy} \nu_{yx}} \nu_{yx} + G_c \right) \left( \frac{\pi}{SL} \right) \left( -\frac{3}{2} v_{1n} - \frac{1}{4} v_{1n}' S + \frac{3}{2} v_{2n} - \frac{1}{4} v_{2n}' S \right) \right\} t
- \left( - \frac{E_x}{1 - \nu_{xy} \nu_{yx}} \nu_{yx} + G_c \right) \left( \frac{\pi}{L} \right)^3 W_n + \left( \frac{E_x}{1 - \nu_{xy} \nu_{yx}} \nu_{yx} + 2 G_c \right) \left( \frac{\pi}{LS^2} \right) \left( 8 w_{1n} + w_{1n}' S - 16 W_n + 8 w_{2n} - w_{2n}' S \right) \right\} 12 t^2 \cos \left( \frac{n \pi x}{L} \right) + ...
$$
\[ -12 t^2 \left( \left( \frac{E_x}{1-\nu_{xy}\nu_{yx}} \right) \nu_{yx} + 2 G_c \right) \left( \frac{2}{S^2 L} \right) \left( S \left( w^{IV}_{1n} + w^{IV}_{2n} - w^{III}_{1n} - w^{III}_{2n} \right) \right. \]

\[ + \left( w^{IV}_{1n} \cdot S - w^{IV}_{2n} \cdot S - w^{III}_{1n} \cdot S + w^{III}_{2n} \cdot S \right) \left( \frac{L}{2} < x < L \right) \]  

Finally considering each bay in a deck system, \( \delta_n (n = 1, 2, \ldots, N) \) can be obtained from the global displacement vectors \( \Delta_n (n = 1, 2, \ldots, N) \). Therefore, the bending stresses of the exterior ply in the cover in the parallel to face grain direction can be obtained directly from Equations 61a, 61b, 62a and 62b. Similarly, the bending stresses of the exterior ply in the cover in the perpendicular to face grain direction can be obtained directly from Equations 63a, 63b, 64a and 64b. Furthermore, the rolling shear stresses at the interface between the top three exterior parallel plies and the cross band in the direction parallel and perpendicular to face grain can also be obtained from Equations 72a, 72b, 73a and 73b.

Given a particular loading pattern \( p(x,y) \), the bending and rolling shear stress profile of the cover can be obtained. Furthermore, the location and magnitude of critical stresses in the cover can therefore be predicted.

3.11 Computing Environment and Efficiency

The DAP was coded in Fortran-77 Language and implemented in a 80386 personal computer environment with a 33 MHz processor speed and a minimum of 2 Mbytes of random access memory (RAM). A special Fortran Language compiler, Lahey F77L EM/32, was used so that all the available RAM in the minicomputer could be accessed by the program; standard Fortran compilers allowed only 640 kbytes of RAM to be accessed by the program which was insufficient for DAP. The program can consider up to seventeen supporting I-beams and 10 terms in the Fourier series.

Considering solution of a case with 17 supporting beams and 8 terms in the Fourier series, typically 15 to 20 minutes of computing time was required. The execution time depends on the choice of convergence requirement \( \epsilon \) and the coupling of the off-diagonal stiffness submatrices due to the
modeling of the midspan gap. During program execution typically 80 to 90 Jacobi iterations were required to satisfy the convergence requirements of ε = 0.005. Therefore, the execution time required for each iteration is reasonably short which is an indication of the efficiency of the finite strip formulation.

3.12 Sample Problems

The deck system shown in Figure 2 was considered in the example problems. It contained seventeen bays with dimensions of 2.44 x 4.88 m (8 x 16 feet) in plan. The deck was sheathed by 11-ply 35 mm (1⅜ inch) thick transDeck™ panels. The panels were supported by 2.44 m (8 feet) long steel I-beams spaced at 305 mm (12 inches) on center. Eight cases which included various combinations of: 1) two different wheel locations; 2) either four or six terms used in the Fourier Series; and 3) two different connector stiffnesses were studied.

3.12.1 Program Input

The dimensions of the transDeck™ panel were 1.22 x 2.44 m (4 x 8 feet) in plan. The face grain and the long axis of the panels were oriented parallel to the long axis of the deck. Each end of the deck was sheathed by two half sheets of panels, 1.22 x 1.22 m (4 x 4 feet) in dimension. The middle portion of the deck was sheathed by two full size panels, 1.22 x 2.44 m (4 x 8 feet) in dimension. Therefore, each deck contained 4 half sheets and 2 full sheet of panels in total. All panel edges perpendicular to the long axis of the deck were fully supported by I-beams. However, the panel edges parallel to the long axis of the deck were unsupported except over the I-beams.

The elastic properties of a transDeck™ panel can be estimated from the elastic properties of individual veneer. Assuming the thickness of each ply equals to t, the stiffnesses of the panel can be obtained from:
\[ K_X = \sum_{j=1}^{N_1} \frac{E_{x_1}^j I_x^j}{1 - \nu_{xy} \nu_{yx}} + \sum_{j=1}^{N_2} \frac{E_{y_1}^j I_y^j}{1 - \nu_{xy} \nu_{yx}} \]  
[74a]  

\[ K_Y = \sum_{j=1}^{N_1} \frac{E_{y_1}^j I_x^j}{1 - \nu_{xy} \nu_{yx}} + \sum_{j=1}^{N_2} \frac{E_{x_1}^j I_y^j}{1 - \nu_{xy} \nu_{yx}} \]  
[74b]  

\[ K_Y = \sum_{j=1}^{N_1} \frac{E_i^j \nu_{xy} I_x^j}{1 - \nu_{xy} \nu_{yx}} + \sum_{j=1}^{N_2} \frac{E_i^j \nu_{yx} I_y^j}{1 - \nu_{xy} \nu_{yx}} \]  
[74c]  

\[ K_G = \sum_{j=1}^{N_1} G^j I_x^j + \sum_{j=1}^{N_2} G^j I_y^j \]  
[74d]  

\[ D_X = \sum_{j=1}^{N_1} \frac{E_a^j t}{1 - \nu_{xy} \nu_{yx}} + \sum_{j=1}^{N_2} \frac{E_a^j t}{1 - \nu_{xy} \nu_{yx}} \]  
[74e]  

\[ D_Y = \sum_{j=1}^{N_1} \frac{E_a^j t}{1 - \nu_{xy} \nu_{yx}} + \sum_{j=1}^{N_2} \frac{E_a^j t}{1 - \nu_{xy} \nu_{yx}} \]  
[74f]  

\[ D_Y = \sum_{j=1}^{N_1} \frac{E_a^j \nu_{xy} t}{1 - \nu_{xy} \nu_{yx}} + \sum_{j=1}^{N_2} \frac{E_a^j \nu_{yx} t}{1 - \nu_{xy} \nu_{yx}} \]  
[74g]  

\[ D_G = \sum_{j=1}^{N_1} G^j t^j + \sum_{j=1}^{N_2} G^j t^j \]  
[74h]  

where \( N_1 \) and \( N_2 \) are the number of plies with the grain parallel to the x and y direction, respectively; \( I_x^j \) and \( I_y^j \) are the moment of inertia per unit of width, with respect to the center line of the panel, of the \( j^{th} \) ply with the grain parallel to the x and y direction, respectively; \( E_{x_1}^j \) and \( E_{y_1}^j \) are the axial moduli of elasticity of the \( j^{th} \) ply (\( j = 1 \) to \( N_1 \) or \( N_2 \)) with the grain parallel to the x and y direction, respectively; \( E_{x_f}^j \) and \( E_{y_f}^j \) are the flexure moduli of elasticity of the \( j^{th} \) ply (\( j = 1 \) to \( N_1 \) or \( N_2 \)) with the grain parallel to the x and y direction, respectively; \( G^j \) is the modulus of rigidity of the \( j^{th} \) ply (\( j = 1 \) to \( N_1 \) or \( N_2 \)) of the panel; and the \( \nu_{xy} \) and \( \nu_{yx} \) are the Poisson's ratio.

In these example problems, the individual veneers in a panel were assumed to have identical elastic properties. Here the axial moduli of elasticity of the veneer in the x and y directions were
assumed to equal 488.2 MPa (70811 psi) and 9828 MPa (1.425x10^6 psi), respectively. The flexure moduli of elasticity of the veneer in the x and y directions were assumed to equal 430.9 MPa (62500 psi) and 11566 MPa (1.678x10^6 psi), respectively. The moduli of rigidity of the veneer was assumed to equal 729.0 MPa (105738 psi). Furthermore, the Poisson's ratio \( \nu_{xy} \) and \( \nu_{yx} \) were taken as 0.02 and 0.4 respectively. Finally, the thickness of each veneer was taken as 3.2 mm (0.125 inch).

The stiffnesses of a panel in the cover were obtained from Equations 74a to 74h as:

\[
\begin{align*}
K_x &= 4.48 \text{ kN} \cdot \text{m} \quad (39618.1 \text{ lb} \cdot \text{in}); \\
K_y &= 38.48 \text{ kN} \cdot \text{m} \quad (340384.8 \text{ lb} \cdot \text{in}); \\
K_\nu &= 1.25 \text{ kN} \cdot \text{m} \quad (11042.6 \text{ lb} \cdot \text{in}); \\
K_G &= 2.59 \text{ kN} \cdot \text{m} \quad (22906.4 \text{ lb} \cdot \text{in}); \\
D_x &= 76.97 \frac{\text{MN}}{\text{m}} \quad (439554.3 \frac{\text{lb}}{\text{in}}); \\
D_y &= 286.22 \frac{\text{MN}}{\text{m}} \quad (1634468.0 \frac{\text{lb}}{\text{in}}); \\
D_\nu &= 25.44 \frac{\text{MN}}{\text{m}} \quad (145305.9 \frac{\text{lb}}{\text{in}}); \\
D_G &= 25.46 \frac{\text{MN}}{\text{m}} \quad (145389.5 \frac{\text{lb}}{\text{in}}).
\end{align*}
\]

The calculation of the stresses in the outer ply of the cover requires information on the \( E_y, E_x \) and \( G_c \) which were taken as 9828.0 MPa (1.425x10^6 psi), 488.2 MPa (70811 psi), and 729.0 MPa (105738 psi), respectively. The moduli of elasticity and rigidity of the supporting I-beams in the deck system were taken as 200,000 MPa (29x10^6 psi) and 77000 MPa (11x10^6 psi), respectively. Also the supporting I-beams were assumed to have a yield strength of 550 MPa (80000 psi). The thickness of the flange \( t_f \) and the web \( t_w \) equaled 3.2 mm (0.125 inch). The depth \( h_1 \) and the width \( b_1 \) of the I-beam were 102 mm (4 inches) and 57 mm (2.25 inches), respectively. The connectors between the cover and the supporting I-beams were considered uniformly spaced at 102 mm (4 inches) on center. The two cases of assumed connector stiffnesses were: 1) \( K_{x_c}, K_{y_c}, \text{ and } K_{\theta_c} \) equaled 1.75 \( \frac{\text{MN}}{\text{m}} \) (10000 \( \frac{\text{lb}}{\text{in}} \)); 2) \( K_{x_c}, K_{y_c}, \text{ and } K_{\theta_c} \) equaled 3.50 \( \frac{\text{MN}}{\text{m}} \) (20000 \( \frac{\text{lb}}{\text{in}} \)).

The loading on the deck was assumed to result from the wheels of a lift truck carrying a load
of 81.5 kN (18333 lb). The two front wheels were assumed to carry 90% of the total load, 73.4 kN (16500 lb). The two rear wheels therefore carried 8.2 kN (1833 lb).

The front and rear axles were spaced at a distance of 1.22 m (4 feet) apart. Both axles were oriented parallel to the direction of the supporting I-beams in the deck. The two different wheel locations considered were:

1) the front axle was assumed to be centrally positioned between bays 8 and 9 of the deck and the rear axle was therefore centrally positioned between bays 12 and 13 of the deck;

2) the front axle was assumed to be positioned on bay 9 of the deck and the rear axle was therefore positioned on bay 13 of the deck.

The foot print of each front wheel was assumed to be 203 x 89 mm (8 x 3.5 inches) which covered an area of 18064 mm$^2$ (28 inches$^2$). The rear wheels were smaller than the front wheels; each rear wheel had a foot print of 89 x 80 mm (3.5 x 3.15 inches) which covered an area of 7113 mm$^2$ (11 inches$^2$). The two front wheels were spaced at a distance of 965 mm (38 inches) apart and centered in the x-direction in the deck. Similarly, the rear wheels were centered in the x-direction of the deck and spaced at a distance of 749 mm (29.5 inches) apart.

3.12.2 Program output

Shown in Table 1 are the eight case studies which included the various combinations of: 1) two different wheel locations; 2) either four or six terms used in the Fourier Series; and 3) two different connector stiffnesses. Summary results from DAP on the maximum vertical deflection of the supporting I-beams in each bay for the eight cases are shown in Table 2. Similar summary results on the maximum bending stress in the supporting I-beams in each bay for the various cases are shown in Table 3.

The results show that a maximum supporting I-beam deflection, W(x), of 20 mm (0.799 inch) occurred in case 4 at midspan of I-beam number 9. A maximum bending stress of 363 MPa (52631 psi)
in the supporting I-beams occurred in midspan of I-beam number 9 in case 6. In each finite strip, the
deflections of the cover in the vertical direction, \( w(x,y) \), were monitored along the lines 0-0 and 1-1 as shown in Figure 3, with a grid size of 61 mm (2.4 inches) in the x-direction. Table 4 shows the maximum cover deflections for the eight cases. In cases 2, 4, 6, and 8, when the front axle was directly on top of I-beam Number 9, the maximum cover deflection was located at the midspan \( (x = 1.22 \text{ m}) \) of Bay 9. When the front axle was centrally located between I-beams 8 and 9 for cases 1, 3, 5, and 7, the maximum cover deflection was located at the midspan \( (x = 1.22 \text{ m}) \) and centered between Bays 8 and 9. Within the eight cases, a maximum deflection of 20 mm (0.783 inch) was found in case 4.

Table 1. Descriptions of the various case studies in the example problem.

<table>
<thead>
<tr>
<th>Case</th>
<th>No. of terms in Fourier Series</th>
<th>( K_x, K_y, ) and ( K_{\theta} ) (( \text{MN/m} ))</th>
<th>Front Axle Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>3.50</td>
<td>centered between bays 8 and 9</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>3.50</td>
<td>on bay 9</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>1.75</td>
<td>centered between bays 8 and 9</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>1.75</td>
<td>on bay 9</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>3.50</td>
<td>centered between bays 8 and 9</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>3.50</td>
<td>on bay 9</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>1.75</td>
<td>centered between bays 8 and 9</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>1.75</td>
<td>on bay 9</td>
</tr>
</tbody>
</table>

\( 1 \text{ lb/in} = 0.000175 \text{ MN/m} \)
Table 2. Summary deflection results of the supporting I-beams in the eight case studies.

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Beam No. 1</th>
<th>Beam No. 2</th>
<th>Beam No. 3</th>
<th>Beam No. 4</th>
<th>Beam No. 5</th>
<th>Beam No. 6</th>
<th>Beam No. 7</th>
<th>Beam No. 8</th>
<th>Beam No. 9</th>
<th>Beam No. 10</th>
<th>Beam No. 11</th>
<th>Beam No. 12</th>
<th>Beam No. 13</th>
<th>Beam No. 14</th>
<th>Beam No. 15</th>
<th>Beam No. 16</th>
<th>Beam No. 17</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.1</td>
<td>-0.4</td>
<td>-0.6</td>
<td>-0.4</td>
<td>1.0</td>
<td>4.7</td>
<td>11.0</td>
<td>17.8</td>
<td>18.0</td>
<td>11.7</td>
<td>6.1</td>
<td>3.3</td>
<td>1.8</td>
<td>0.7</td>
<td>0.1</td>
<td>-0.1</td>
<td>-0.2</td>
</tr>
<tr>
<td></td>
<td>-0.0</td>
<td>-0.3</td>
<td>-0.6</td>
<td>-0.5</td>
<td>0.1</td>
<td>2.5</td>
<td>7.5</td>
<td>14.7</td>
<td>19.2</td>
<td>15.1</td>
<td>8.5</td>
<td>4.4</td>
<td>2.5</td>
<td>1.2</td>
<td>0.4</td>
<td>-0.0</td>
<td>-0.3</td>
</tr>
<tr>
<td></td>
<td>-0.2</td>
<td>-0.5</td>
<td>-0.7</td>
<td>-0.7</td>
<td>1.1</td>
<td>5.0</td>
<td>11.8</td>
<td>18.8</td>
<td>19.0</td>
<td>12.5</td>
<td>6.6</td>
<td>3.5</td>
<td>1.9</td>
<td>0.7</td>
<td>0.1</td>
<td>-0.1</td>
<td>-0.2</td>
</tr>
<tr>
<td></td>
<td>-0.0</td>
<td>-0.4</td>
<td>-0.6</td>
<td>-0.7</td>
<td>0.1</td>
<td>2.7</td>
<td>8.1</td>
<td>15.6</td>
<td>20.3</td>
<td>16.0</td>
<td>9.2</td>
<td>4.7</td>
<td>4.7</td>
<td>1.3</td>
<td>0.4</td>
<td>-0.1</td>
<td>-0.3</td>
</tr>
<tr>
<td></td>
<td>-0.1</td>
<td>-0.3</td>
<td>-0.2</td>
<td>-0.3</td>
<td>1.0</td>
<td>2.7</td>
<td>11.1</td>
<td>17.7</td>
<td>19.0</td>
<td>15.1</td>
<td>6.6</td>
<td>3.5</td>
<td>1.9</td>
<td>0.7</td>
<td>0.1</td>
<td>-0.1</td>
<td>-0.2</td>
</tr>
<tr>
<td></td>
<td>-0.2</td>
<td>-0.0</td>
<td>-0.2</td>
<td>-0.2</td>
<td>0.1</td>
<td>4.7</td>
<td>7.5</td>
<td>14.7</td>
<td>18.9</td>
<td>16.0</td>
<td>9.2</td>
<td>4.7</td>
<td>2.6</td>
<td>1.3</td>
<td>0.4</td>
<td>-0.1</td>
<td>-0.3</td>
</tr>
</tbody>
</table>

1 inch = 25.4 mm
Table 3. Summary bending stress results of the supporting I-beams in the eight case studies.

<table>
<thead>
<tr>
<th>Case No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam No. 1</td>
<td>-2.41</td>
<td>-0.29</td>
<td>-3.18</td>
<td>-0.63</td>
<td>-2.42</td>
<td>-0.29</td>
<td>-3.19</td>
<td>-0.63</td>
</tr>
<tr>
<td>Beam No. 3</td>
<td>-12.72</td>
<td>-10.97</td>
<td>-13.78</td>
<td>-12.17</td>
<td>-12.68</td>
<td>-10.95</td>
<td>-13.75</td>
<td>-12.15</td>
</tr>
<tr>
<td>Beam No. 5</td>
<td>16.14</td>
<td>-0.40</td>
<td>18.65</td>
<td>0.72</td>
<td>16.32</td>
<td>-0.35</td>
<td>18.83</td>
<td>0.76</td>
</tr>
<tr>
<td>Beam No. 6</td>
<td>84.04</td>
<td>43.50</td>
<td>89.50</td>
<td>47.52</td>
<td>82.95</td>
<td>43.32</td>
<td>88.39</td>
<td>47.33</td>
</tr>
<tr>
<td>Beam No. 7</td>
<td>203.68</td>
<td>138.49</td>
<td>210.31</td>
<td>144.95</td>
<td>208.97</td>
<td>139.14</td>
<td>215.78</td>
<td>145.64</td>
</tr>
<tr>
<td>Beam No. 8</td>
<td>308.83</td>
<td>265.74</td>
<td>313.19</td>
<td>271.56</td>
<td>319.47</td>
<td>262.63</td>
<td>323.75</td>
<td>268.54</td>
</tr>
<tr>
<td>Beam No. 9</td>
<td>311.93</td>
<td>324.95</td>
<td>316.60</td>
<td>328.59</td>
<td>322.22</td>
<td>362.86</td>
<td>326.77</td>
<td>366.48</td>
</tr>
<tr>
<td>Beam No. 10</td>
<td>215.40</td>
<td>272.58</td>
<td>222.83</td>
<td>278.98</td>
<td>220.58</td>
<td>269.45</td>
<td>228.20</td>
<td>275.94</td>
</tr>
<tr>
<td>Beam No. 11</td>
<td>109.88</td>
<td>156.56</td>
<td>116.32</td>
<td>163.97</td>
<td>109.24</td>
<td>157.25</td>
<td>115.67</td>
<td>164.71</td>
</tr>
<tr>
<td>Beam No. 12</td>
<td>59.06</td>
<td>78.17</td>
<td>62.27</td>
<td>83.11</td>
<td>56.12</td>
<td>77.71</td>
<td>59.33</td>
<td>82.64</td>
</tr>
<tr>
<td>Beam No. 13</td>
<td>33.61</td>
<td>46.43</td>
<td>34.36</td>
<td>48.19</td>
<td>31.49</td>
<td>43.54</td>
<td>32.20</td>
<td>45.14</td>
</tr>
<tr>
<td>Beam No. 14</td>
<td>12.70</td>
<td>22.03</td>
<td>12.52</td>
<td>22.20</td>
<td>13.18</td>
<td>21.78</td>
<td>13.00</td>
<td>21.94</td>
</tr>
<tr>
<td>Beam No. 15</td>
<td>2.03</td>
<td>6.37</td>
<td>1.55</td>
<td>5.96</td>
<td>1.95</td>
<td>6.44</td>
<td>1.49</td>
<td>6.03</td>
</tr>
<tr>
<td>Beam No. 16</td>
<td>-2.27</td>
<td>-1.27</td>
<td>-2.81</td>
<td>-1.90</td>
<td>-2.26</td>
<td>-1.29</td>
<td>-2.79</td>
<td>-1.93</td>
</tr>
<tr>
<td>Beam No. 17</td>
<td>-4.04</td>
<td>-5.48</td>
<td>-4.50</td>
<td>-6.15</td>
<td>-4.05</td>
<td>-5.51</td>
<td>-4.52</td>
<td>-6.18</td>
</tr>
</tbody>
</table>

1 ksi = 6.89 MPa
Table 4. Maximum deflections and stresses in the cover of the eight case studies.

<table>
<thead>
<tr>
<th>Case</th>
<th>Maximum $w(x,y)$ (mm)</th>
<th>Maximum $\sigma_y(x,y)$ (MPa)</th>
<th>Maximum $\sigma_x(x,y)$ (MPa)</th>
<th>Maximum $\tau_{xz}(x,y)$ (MPa)</th>
<th>Maximum $\tau_{xx}(x,y)$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>18.7</td>
<td>32.46</td>
<td>0.63</td>
<td>0.84</td>
<td>0.04</td>
</tr>
<tr>
<td>2</td>
<td>18.9</td>
<td>21.17</td>
<td>0.32</td>
<td>0.32</td>
<td>0.04</td>
</tr>
<tr>
<td>3</td>
<td>19.7</td>
<td>32.80</td>
<td>0.74</td>
<td>0.84</td>
<td>0.03</td>
</tr>
<tr>
<td>4</td>
<td>19.9</td>
<td>21.60</td>
<td>0.45</td>
<td>0.33</td>
<td>0.03</td>
</tr>
<tr>
<td>5</td>
<td>18.4</td>
<td>36.20</td>
<td>0.85</td>
<td>0.76</td>
<td>0.05</td>
</tr>
<tr>
<td>6</td>
<td>18.7</td>
<td>20.57</td>
<td>0.36</td>
<td>0.31</td>
<td>0.04</td>
</tr>
<tr>
<td>7</td>
<td>19.4</td>
<td>36.57</td>
<td>0.96</td>
<td>0.77</td>
<td>0.04</td>
</tr>
<tr>
<td>8</td>
<td>19.8</td>
<td>20.99</td>
<td>0.48</td>
<td>0.32</td>
<td>0.03</td>
</tr>
<tr>
<td>9</td>
<td>19.4</td>
<td>43.70</td>
<td>1.21</td>
<td>0.81</td>
<td>0.05</td>
</tr>
<tr>
<td>10</td>
<td>19.6</td>
<td>44.96</td>
<td>1.37</td>
<td>0.88</td>
<td>0.06</td>
</tr>
</tbody>
</table>
The bending stresses in the exterior ply of the cover \( \sigma_\gamma(x,y) \) and \( \sigma_\chi(x,y) \) were monitored at the same locations as the cover deflections. Table 4 also shows the maximum cover bending stresses for the eight cases. Case 7 yielded the maximum cover bending stresses, \( \sigma_\gamma \) and \( \sigma_\chi \), of 36.57 MPa (5304 psi) and 1.21 MPa (175 psi), respectively. Table 5 shows the maximum cover bending stresses locations for the eight cases. In cases 2, 4, 6, and 8, when the front axle was directly on top of I-beam Number 9, the maximum \( \sigma_\gamma(x,y) \) were located in Bay 9 between the foot prints of the front wheels. In cases 1, 3, 5, and 7, when the front axle was centered between Bays 8 and 9, the maximum \( \sigma_\gamma \) values were located between Bays 8 and 9 under the foot prints of the front wheels. The maximum \( \sigma_\chi \) values were typically located under the foot prints of the front wheels except in Cases 2 and 4 where the maximum stresses were located between the foot prints of the front wheels.

The rolling shear stresses in the cross ply of the cover in the x and y direction, \( \tau_{xz} \) and \( \tau_{yz} \), were monitored over the supporting I-beams, with a grid size of 61 mm (2.4 inches) in the x-direction. Table 4 shows the maximum cover rolling shear stresses in the x- and y-direction for the eight cases. Within the eight cases, a maximum \( \tau_{yz} \) of 0.77 MPa (111.9 psi) was found in case 7 and a maximum \( \tau_{xz} \) of 0.05 MPa (7.2 psi) was found in case 5. Table 5 shows the locations of the maximum cover shear stresses for the eight cases. The maximum \( \tau_{yz} \) values were typically located between the foot prints of the front wheels except in Cases 5 and 7 where the maximum stresses were located under the foot prints of the front wheels. In all cases the maximum \( \tau_{xz} \) values were located near the edge of the panels at x=0 and 2.44 m.

The results from the case studies clearly show that the cover bending stress and the rolling shear stress in the x-direction were insignificant compared to the other stresses. Therefore, \( \sigma_\chi \) and \( \tau_{xz} \) will not be further considered in detail when evaluating the impact of connector stiffness, wheel location and number of Fourier terms used in the analysis.

The influence of connector stiffness, which varies from 3.50 to 1.75 MN/m, on the deflection and the stresses of the supporting I-beams and the cover seems to be small. In general, less than three
Table 5. Locations of maximum stresses in the cover of the eight case studies.

<table>
<thead>
<tr>
<th>Case</th>
<th>y-location</th>
<th>x-location</th>
<th>y-location</th>
<th>x-location</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Bending Stresses</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\sigma_y(x,y)$</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>centered between Bays 8 and 9</td>
<td>0.79 and 1.65 m</td>
<td>0.79 and 1.65 m</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Bay 9</td>
<td>1.22 m</td>
<td>1.16 and 1.28 m</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>centered between Bays 8 and 9</td>
<td>0.79 and 1.65 m</td>
<td>0.79 and 1.65 m</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Bay 9</td>
<td>1.16 and 1.28 m</td>
<td>1.16 and 1.28 m</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>centered between Bays 8 and 9</td>
<td>0.73 and 1.71 m</td>
<td>0.73 and 1.71 m</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Bay 9</td>
<td>1.16 and 1.28 m</td>
<td>0.79 and 1.65 m</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>centered between Bays 8 and 9</td>
<td>0.73 and 1.71 m</td>
<td>0.73 and 1.71 m</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Bay 9</td>
<td>0.98 and 1.46 m</td>
<td>0.79 and 1.65 m</td>
<td></td>
</tr>
</tbody>
</table>
percent difference in cover stresses, $\sigma_\gamma$ and $\tau_{yz}$, and and less than six percent difference in cover deflection can be observed between the cases with the two connector stiffnesses.

However the wheel location and the number of Fourier terms used in the analysis significantly influence the estimated stresses of the supporting I-beams and the cover. Rolling shear stresses and bending stresses in the cover in the y-direction, $\tau_{yz}$ and $\sigma_\gamma$, were much higher when the wheels were centrally positioned between the supporting I-beams compared to the cases when the wheels were directly over the supporting I-beams. Also bending stresses of the supporting I-beams, $\tau_{yz}$ and $\sigma_\gamma$, were higher by as much as 10% when the six rather than four terms in the Fourier series were used. The influence of the number of terms used in the Fourier series on the bending deflections of the supporting I-beams and the deflection of the cover was insignificant because less than two percent difference in response was observed between the cases with the four and six terms in the Fourier series.

Finally two more case studies were conducted to investigate the convergence of the deflections and stresses. Here similar to case 7, the connector stiffness was considered to be 1.75 $\frac{MN}{m}$ and the front wheels were considered to be centrally placed between Bays 8 and 9. However, in cases 9 and 10, eight and ten terms were used in the Fourier series, respectively. In case 9, the results indicate a maximum $W(x)$ and supporting I-beam bending stress of 18.9 mm (0.744 inch) and 332.5 MPa (48227 psi), respectively. In case 10, the results indicate a maximum $W(x)$ and supporting I-beam bending stress of 18.9 mm (0.744 inch) respectively. The maximum $w(x,y)$, $\sigma_\gamma(x,y)$, $\sigma_x(x,y)$, $\tau_{yz}(x,y)$, and $\tau_{xx}(x,y)$ estimates for these cases are shown in Table 4.

Comparing the results from cases 9 and 10, the absolute percentage difference in estimating maximum $W(x)$, supporting I-beam bending stress, $w(x,y)$, $\sigma_\gamma(x,y)$, $\sigma_x(x,y)$, $\tau_{yz}(x,y)$, and $\tau_{xx}(x,y)$ were 0.05%, 0.92%, 0.72%, 2.82%, 11.76%, 7.85%, and 14.46%, respectively. Large differences occur in the estimation of maximum $\sigma_x(x,y)$ and $\tau_{xx}(x,y)$ of the cover. The magnitude of these stresses were small compared to the stresses in the other directions such as the maximum $\sigma_\gamma(x,y)$ and $\tau_{yz}(x,y)$ of the cover. It is possible that a few more terms were needed for convergence of these stresses because of the
concentrated applied loads and the strong directional elastic properties of the cover. This is especially true when the loads were applied in between two bays rather than directly on top of the supporting I-beams. Since the deflections and the other major stresses in the system converged rapidly, eight terms in the Fourier series were deemed sufficient to achieve the desired accuracy for estimation of stresses and deflections.

Finally Figures 5 to 9 show the $w(x,y)$ profile, the $\sigma_x(x,y)$ profile, the $\sigma_y(x,y)$ profile, the $\tau_{yx}(x,y)$ profile and the $\tau_{xx}$ profile the of the cover for case 7. Figures 10 to 14 show the $w(x,y)$ profile, the $\sigma_x(x,y)$ profile, the $\sigma_y(x,y)$ profile, the $\tau_{yx}(x,y)$ profile and the $\tau_{xx}$ profile the of the cover for case 8. From Figures 5 and 10, it can be seen that the shapes of both cover deformation profiles were consistent with the peak deflections occurring directly under the front wheels. From Figures 6 to 9 and 11 to 14, it is seen that the stress profiles were complicated and dependent on the wheel location. When the front wheels were located in between the supporting I-beam, large stress concentrations occurred which could initiate damage in the cover. It can also be noted that the stresses at the gap ($x=\frac{1}{2}$) are non-zero. With a large number of Fourier terms, it is expected that the normal and shear stresses on the free edges at the gap will converge to zero. In terms of bending and rolling shear stresses in the $y$-direction, the critical locations were clearly directly under the front wheels and in midspan of the deck along the direction of the front axle, respectively. Considering a lift truck moving across the deck system, the stresses for any wheel position (i.e., the stress history) in these critical points can be estimated from DAP.

DAP is a general purpose deck analysis program which considers the deck as a plate stiffened by supporting beams. The versatility of DAP can be illustrated by the following examples:

1) Both the supporting beams and the cover plate can be made up of any material as long as the strength properties remain linear elastic within the load range of interest. Materials for cover may include composite wood products, fiber glass products, metal products, or composite wood and fiber glass products provided that their strength properties are known.
2) With statistical modeling of the cover strength properties, simulation and reliability studies can be performed to evaluate the performance of the decking.

3) Finally different shapes and dimensions of supporting beams and different types of connector can also be considered to improve the structural performance of the deck.

Although these studies are beyond the scope of the current program, it is clear that DAP can easily be used to consider other problems.
Figure 5 Deformation profile of cover: case 7.
Figure 6  Bending stress profile $\sigma_3$ in exterior ply of cover: case 7.
Figure 7  Bending stress profile $\sigma_x$ in exterior ply of cover: case 7.
Figure 8  Rolling shear stress profile $\tau_{yz}$ in cross ply of cover: case 7.
Figure 9 Rolling shear stress profile $\tau_{xx}$ in cross ply of cover: case 7.
Figure 10 Deformation profile of cover: case 6.
Figure 11  Bending stress profile $\sigma_\phi$ in exterior ply of cover: case 6.
Figure 12  Bending stress profile $\sigma_x$ in exterior ply of cover: case 6.
Figure 13 Rolling shear stress profile $\tau_{yz}$ in cross ply of cover: case 6.
Figure 14 Rolling shear stress profile $\tau_{xz}$ in cross ply of cover: case 6.
4. VENEER MECHANICAL PROPERTIES TESTING PROGRAM

The elastic properties of the cover are required by DAP as input. Here a database on the mechanical properties on 2.5 mm (1/10 inch) and 3.2 mm (1/8 inch) thick Douglas-fir veneer was developed. The veneers were sampled from all the sources available to Ainsworth Lumber Company Ltd. for transDeck™ manufacturing. Using this information, the elastic properties of laminated veneer panels can be estimated from Equations 74a to 74h.

Since it is difficult to test the mechanical properties of single ply veneer, 3-ply and 4-ply panels with the face grain in each veneer oriented in the longitudinal direction of the panels were considered to extrapolate single ply veneer mechanical properties. However, this approach required that the mechanical properties of the veneers within each 3-ply or 4-ply panel not be significantly different. Therefore, a non-destructive testing program based on ultrasonic techniques was initiated to sort the veneers into groups with similar mechanical properties prior to making the 3-ply and 4-ply panels.

The elastic properties of the 3-ply and 4-ply veneers in the directions parallel and perpendicular to grain were then obtained from bending, tension, and compression tests. Also a plate twisting test was performed to obtain the shear modulus of rigidity tests of the 3-ply and 4-ply veneers.

4.1 Ultrasonic Veneer Testing Program

Metriguard Inc. developed a continuous ultrasonic veneer tester, Metriguard Model 2600 veneer grader, to sort veneer for structural applications (Logan, 1987). This technology is based on the well known principle that sonic transmission velocity and the density of the transmission medium are strongly correlated. Since the density of veneer is also correlated to the mechanical properties of the veneer, it is reasonable to use the machine to grade the veneers for structural use. Currently, more than 10 machines are in service in North America.
Sonic propagation time in the longitudinal direction of each veneer sheet was monitored as each sheet of veneer was transversely passed through the veneer tester. The ratio between the number of signals received and transmitted by the transducers was also continuously monitored which indicated defects such as splits in the veneer.

Each sheet of veneer sampled was tested with the Metriguard veneer grader. For the two veneer thicknesses, the cumulative probability distributions of sonic transmission time are shown in Figure 15. Based on the cumulative probability distributions of sonic transmission time of the veneers, the 2.5 mm ($\frac{1}{8}$ inch) and 3.2 mm ($\frac{1}{10}$ inch) thick veneers were sorted into the following three groups:

1) group A contained veneers from lower 25\textsuperscript{th} percentile of the sonic transmission time distribution;
2) group B contained veneers in the middle 25\textsuperscript{th} to 75\textsuperscript{th} percentiles of the sonic transmission time distribution;
3) group C contained veneers from upper 75\textsuperscript{th} percentile of the sonic transmission time distribution.

Group C was expected to contain the worst material while group A was expected to contain the best quality veneer.

4.2 Mechanical Properties Test Program

4.2.1 Materials and Methods

In the veneer mechanical properties test program, a total of eighty 3-ply and eighty 4-ply, 1.2 x 2.4 m (4 x 8 feet), specimens were made. For each veneer thickness, in the 3-ply case, 15 sheets of veneer were randomly selected from group A to be made into 5 panels; 30 sheets of veneer were randomly selected from group B to be made into 10 panels; and 15 sheets of veneer were randomly selected from group C to be made into 5 panels. Similarly in the 4-ply cases, 20, 40 and 20 sheets of veneer were randomly selected from groups A, B and C, respectively. The veneers from groups A, B
Figure 15  Sonic transmission time cumulative probability distributions for 2.5 and 3.2 mm thick veneer.
and C were made into 5, 10 and 5 four-ply panels, respectively. The specimens were then conditioned at a temperature of 20 ± 3°C and relative humidity of 65 ± 5% for more than four weeks until equilibrium moisture content was reached.

There are no standard test methods to determine the mechanical properties of veneers. Therefore, the 3-ply and 4-ply veneers were tested in the directions parallel and perpendicular to grain under bending, tension, and compression by following as closely as possible ASTM D3043A, ASTM D3500B, and ASTM D3501B, respectively (ASTM 1990). The shear modulus of rigidity of the 3-ply and 4-ply veneers were also tested as closely as possible to ASTM D3044 (ASTM 1990).

4.2.1.1 Bending Tests:

Two specimens were obtained from each panel for the bending tests. The first specimen was cut with its long axis perpendicular to the grain and the second specimen was cut with its long axis parallel to the grain. The lengths of the specimens were chosen such that the minimum span to depth ratios of 48:1 and 24:1 were maintained when the veneers were oriented parallel to span and perpendicular to span, respectively. The test spans of various plies/thickness combinations of the specimens excluding the 25 mm (1 inch) of overhang from each end are given in Table 6. The specimen width was 51 mm (2 inches).

The specimens were simply supported at two ends with roller bearing plates and loaded under a center point load. A MTS model 810 hydraulic control close loop universal testing machine with a capacity of 222.4 kN (50000 lb) applied the load in a deflection control mode. An uniform rate of cross head motion of $1.37 \text{ mm/min. (0.054 inch/min.)}$ was used which resulted in specimen failure between 7 to 10 minutes of loading. Figure 16 shows a specimen being tested in bending.
Table 6. Test span for the bending specimens.

<table>
<thead>
<tr>
<th>No. of Pliests</th>
<th>Veneer Thickness (mm)</th>
<th>Orientation of Veneer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Parallel to span</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Test Span (mm)</td>
</tr>
<tr>
<td>3</td>
<td>3.2</td>
<td>457.2</td>
</tr>
<tr>
<td>4</td>
<td>3.2</td>
<td>609.6</td>
</tr>
<tr>
<td>3</td>
<td>2.5</td>
<td>368.3</td>
</tr>
<tr>
<td>4</td>
<td>2.5</td>
<td>495.3</td>
</tr>
</tbody>
</table>
Figure 16 The veneer bending test set up.
The specimen thicknesses at mid-span and at two points near each edge were measured, averaged, and recorded. The specimen width at mid-span was also recorded. A computer based data acquisition system and software were used to acquire the load versus deformation (cross head motion) data. A load cell with a 25 kN (5620 lb) capacity was used to monitor the loads.

The bending modulus of elasticity in the perpendicular and parallel to grain directions \( E_{f} \) and \( E_{y} \) were based on estimating the slope of the load deformation curve between two preset points on the linear portion of the load-deformation curve. Here, the two preset points were taken as the loads at five and twenty percent of the peak load, respectively. Linear regression of the data between the preset points yielded the slope of load-deformation curve. The bending moduli of elasticity were estimated as:

\[
E_{x_f} = \frac{P_x}{\Delta x_f} \frac{L^3}{48I} \tag{75a}
\]

\[
E_{y_f} = \frac{P_y}{\Delta y_f} \frac{L^3}{48I} \tag{75b}
\]

where \( I \) = the moment of inertia of the entire cross section, \( \frac{P_x}{\Delta x_f} \) = the slope of the linear portion of the load deflection curve in the perpendicular to grain bending tests, \( \frac{P_y}{\Delta y_f} \) = the slope of the linear portion of the load deflection curve in the parallel to grain bending tests, and \( L \) = the span.

The moduli of rupture in the perpendicular and parallel to grain directions, \( S_{xb} \) and \( S_{yb} \), for each specimen were obtained from the peak loads as:

\[
S_{xb} = \frac{P_{xf} L D}{8I} \tag{76a}
\]

\[
S_{yb} = \frac{P_{yf} L D}{8I} \tag{76b}
\]

where \( D \) = depth of the specimen, \( P_{xf} \) = the peak load in the perpendicular to grain bending tests, and
$P_{yf}$ = the peak load in the parallel to grain bending tests.

4.2.1.2 Tension Tests:

A specimen, 254 mm (10 inches) wide and 1219 mm (48 inches) long, was cut from each panel for the parallel to the grain tension tests. For the perpendicular to grain tension tests, a specimen, 50 mm (2 inches) wide and 406 mm (16 inches) long, was cut from each panel.

The parallel to grain tension tests used a 444.8 kN (100000 lb) capacity Metriguard hydraulic tension testing machine (model 412) with an uniform loading rate of $1.524 \text{ mm} / \text{min.}$ Specimen typically failed between 7 to 10 minutes of loading. The applied load was monitored by a 222.4 kN (50000 lb) capacity load cell. The two ends of a specimen were gripped by self aligned urethane friction pads which applied uniformly distributed loads along and across the cross section. The distance between the grips was 610 mm (24 inches). Figure 17 shows a specimen during the tension test.

In the perpendicular to grain tension tests, an Intron mechanical testing machine with a capacity of 48.9 kN (11000 lb) was used to apply the load in a deflection control mode. A uniform rate of cross head motion of $0.5 \text{ mm} / \text{min.}$ ($0.020 \text{ inch} / \text{min.}$) was used which resulted in specimen failure between 3 to 5 minutes of loading. The applied load was monitored by a 222.4 kN (1000 lb) capacity load cell. The two ends of a specimen were gripped by self aligned and tightened wedge type jaws. The distance between the grips was 305 mm (12 inches). The perpendicular to grain specimens were not necked because pilot tests indicated that failure zones were not in the gripping area.

Two linear variable differential transducers (LVDT) were mounted on the two opposite sides of each parallel and perpendicular to grain tension specimens. Gauge lengths of 152 and 178 mm (6 and 7 inches) were used in the perpendicular and parallel to grain tension tests, respectively. The two sets of LVDT readings were averaged to obtain the overall deflection. The specimen thickness and width at mid-span were measured and recorded. A computer based data acquisition system and software were used to acquire the load versus overall deformation information.
Figure 17  The veneer parallel to grain tension test set up.
The tension moduli of elasticity in the perpendicular and parallel to grain directions ($E_{xt}$ and $E_{yt}$) were based on estimating the slope of the load deformation curve between two preset points on the linear portion of the load-deformation curve. Here, the two preset points were taken as the loads at five and fifty percent of the peak load, respectively. Linear regression of the data between the preset point yielded the slope of the load-deformation curve. The tension moduli of elasticity in the perpendicular and parallel to grain directions were estimated as:

$$E_{xt} = \left( \frac{P_x}{\Delta x}_t \right) \frac{L}{A}$$

$$E_{yt} = \left( \frac{P_y}{\Delta y}_t \right) \frac{L}{A}$$

where $A$ = cross-sectional area, $\left( \frac{P_x}{\Delta x}_t \right)$ and $\left( \frac{P_y}{\Delta y}_t \right)$ = slope of the linear portion of the load deflection curve in the perpendicular and parallel to grain compression tests, respectively, and $L$ = gauge length.

The tensile strengths in the perpendicular and parallel to grain directions $S_{xt}$ and $S_{yt}$ of each specimen were obtained from the peak loads as:

$$S_{xt} = \frac{P_{x_t}}{A}$$

$$S_{yt} = \frac{P_{y_t}}{A}$$

where $P_{x_t}$ = the peak load in the perpendicular to grain tension tests, and $P_{y_t}$ = the peak load in the parallel to grain tension tests.

4.2.1.3 Compression Tests:

The compression test specimens were 191 mm (7.5 inches) wide and 381 mm (15 inches) long.
To eliminate buckling, a length to depth ratio of 10 was used; therefore, specimens cut from the same panels were glued face to back together using a polyvinyl acetate resin to form the final specimens. The specimens were cut slightly over sized and then trimmed to final size so that all adjacent edges were at right angles. The nominal specimen depths of the various ply/thickness combinations are given in Table 7.

Table 7. Specimen depths for the compression tests.

<table>
<thead>
<tr>
<th>No. of Plies</th>
<th>Veneer Thickness (mm)</th>
<th>Total No. of Plies</th>
<th>Specimen Thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3.2</td>
<td>15</td>
<td>46.7</td>
</tr>
<tr>
<td>4</td>
<td>3.2</td>
<td>16</td>
<td>50.8</td>
</tr>
<tr>
<td>3</td>
<td>2.5</td>
<td>18</td>
<td>45.7</td>
</tr>
<tr>
<td>4</td>
<td>2.5</td>
<td>16</td>
<td>40.6</td>
</tr>
</tbody>
</table>

In the perpendicular to grain compression tests, a MTS model 810 hydraulic universal testing machine with a capacity of 222.4 kN (50000 lb) was used to apply the load in a deflection control mode. A uniform rate of cross head motion of \(0.89 \text{ mm min.} \approx (0.035 \text{ inch min.})\) was used. Specimen typically failed between 7 to 10 minutes of loading. The applied load was monitored by a 222.4 kN (50000 lb) capacity load cell. Figure 18 shows a specimen being tested in compression.

Since the applied loads were considerable larger in the parallel to grain compression tests, a Baldwin hydraulic universal testing machine with a capacity of 1779.2 kN (400000 lb) was used. An approximate rate of cross head motion of \(0.07 \text{ mm min.} \approx (0.003 \text{ inch min.})\) was used which resulted in failure within 10 minute of loading. The applied load was monitored by a 1779.2 kN (400000 lb) capacity load cell.
Figure 18  The veneer compression test set up.
Two LVDT units were mounted on the two opposite faces of each parallel and perpendicular to grain compression specimen over a gauge length of 127 mm (5 inches). The two sets of LVDT readings were averaged to obtain the overall deflection. The specimen thickness and width at mid-span were measured and recorded. A computer based data acquisition system and software were used to acquire the load versus overall deformation information.

The compression moduli of elasticity in the perpendicular and parallel to grain directions \(E_{x_c}\) and \(E_{y_c}\) were based on estimating the slope of the load deformation curve between two preset points on the linear portion of the load-deformation curve. Here, the two preset points were taken as the loads at five and twenty percent of the peak load, respectively. Linear regression of the data between the preset point yielded the slope of the load-deformation curve. The compression moduli of elasticity in the perpendicular and parallel to grain directions were estimated as:

\[
E_{x_c} = \left( \frac{P_x}{\Delta x_c} \right) \frac{L}{A} \tag{80a}
\]

\[
E_{y_c} = \left( \frac{P_y}{\Delta y_c} \right) \frac{L}{A} \tag{80b}
\]

where \(A = \text{cross sectional area,} \) \(\frac{P_x}{\Delta x_c}\) and \(\frac{P_y}{\Delta y_c}\) = the slope of the linear portion of the load deflection curve in the perpendicular and parallel to grain compression tests, respectively, and \(L = \text{the gauge length.}\)

The compression strengths in the perpendicular and parallel to grain directions \(S_{x_c}\) and \(S_{y_c}\) of each specimen were obtained from the peak loads as:

\[
S_{x_c} = \frac{P_{x_c}}{A} \tag{81a}
\]

\[
S_{y_c} = \frac{P_{y_c}}{A} \tag{82a}
\]
where $P_{xc}$ and $P_{yc}$ = the peak load in the perpendicular and parallel to grain compression tests, respectively.

4.2.1.4 Shear Modulus of Rigidity Tests:

A single square specimen was taken from each panel for the shear modulus of rigidity tests. The specimen width and length were chosen as 40 times the nominal thickness. The specimen size for the various ply/thickness combinations are given in Table 8.

Table 8. Specimen sizes for the shear modulus of rigidity tests.

<table>
<thead>
<tr>
<th>No. of Plies</th>
<th>Veneer Thickness (mm)</th>
<th>Specimen Size (mm)</th>
<th>Loading Rate (mm/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3.2</td>
<td>381.0 x 381.0</td>
<td>4.57</td>
</tr>
<tr>
<td>4</td>
<td>3.2</td>
<td>508.0 x 508.0</td>
<td>6.10</td>
</tr>
<tr>
<td>3</td>
<td>2.5</td>
<td>304.8 x 304.8</td>
<td>3.66</td>
</tr>
<tr>
<td>4</td>
<td>2.5</td>
<td>406.4 x 406.4</td>
<td>4.88</td>
</tr>
</tbody>
</table>

The MTS model 810 hydraulic control close loop universal testing machine with a capacity of 222.4 kN (50000 lb) was used to apply the load in a deflection control mode. The specimen was supported on the opposite corners of one of its diagonals and loaded with a uniform rate of loading from the opposite corners of the other diagonal. Table 8 also shows the rates of cross head motion in the tests for the various ply/thickness combination.

A 25 kN (5620 lb) capacity load cell was used to monitor the loads. The deflection on quarter points on each diagonal relative to the center of the specimen was measured using an LVDT mounted
on a special yoke apparatus. Under this arrangement, the measured deflection was twice that of the deflection relative to the center of the specimen. The thicknesses at six locations in the specimen were measured, averaged, and recorded. A computer based data acquisition system and software were used to acquire the load versus deformation information. Figure 19 shows the experimental set up and a specimen during the shear modulus of rigidity test.

The specimen was loaded to a maximum load of 89 N (20 lb). To eliminate the effects of slight initial curvature, the test was repeated with the specimen rotated 90° about an axis through the center of the plate and perpendicular to the plane of the plies. The results from the two tests were averaged to obtain an overall shear modulus of rigidity for the specimen.

The slope of the load deformation curve was estimated from linear regression of the load-deformation data. The shear modulus of rigidity was estimated according to ASTM D3044 (ASTM 1990) as:

\[
G = \frac{3}{2} \frac{u^2}{h^3} \left( \frac{P}{\Delta} \right)_g
\]

where \( \left( \frac{P}{\Delta} \right)_g \) = slope of the load deflection curve, \( u \) = distance the panel center to the reference point for deflection measurement, and \( h \) = mean thickness of specimen.

4.2.1.5 Connector Load Slip Tests:

The load-slip relationship of the connector between the cover and the supporting member were tested using 11-ply transDeck™ specimens which were built from random sampling the veneers from the entire sonic propagation velocity distribution. A 305 mm (12 inch) long section of a typical dry freight van trailer deck supporting I-beam was sandwiched between two 152 x 254 mm (6 x 10 inch) transDeck™ specimens as shown in Figure 20. The moduli of elasticity and the yield strength of the I-beams were taken as 200,000 MPa (29x10^6 psi) and 550 MPa (80000 psi), respectively. The thickness of
Figure 19  The veneer shear modulus of rigidity test set up.
Figure 20 The connector load slip test set up.
the flange \((t_f)\) and the web \((t_w)\) equaled 3.2 mm (0.125 inch). The depth \((h_f)\) and the width \((b_f)\) of the I-beam were 102 mm (4 inches) and 57 mm (2.25 inches), respectively. Each transDeck™ panel was connected through one of the flanges of the I-beam using two 8 mm (\(\frac{5}{16}\) inch)-18 torx drive, flat head, type G, phosphate and oil coated self tapping screws. The two connectors were staggered with vertical and horizontal spacings of 102 mm (4 inch) and 38 mm (1.5 inch), respectively. Pilot holes with a diameter of 7 mm (\(\frac{9}{32}\) inch) were pre-drilled through the panels and the I-beams prior to applying the self tapping screws. A ratchet was used to tighten the connector until its head was flush with the panel surface. The technique to assemble the test specimen is consistent with the procedures used in the construction of commercial dry freight van trailer decking with the exception that air driven torque wrench which are not regulated for any torque value are usually used in the industry.

With each veneer thickness, five specimens with the face grain of the panel oriented parallel to the direction of the load and five specimens with face grain of the panel oriented perpendicular to the direction of the load were tested in compression to estimate the load slip characteristic of the connectors. A compression load of up to 14 kN (3147 lb) was applied using a MTS model 810 hydraulic control close loop universal testing machine with a capacity of 222.4 kN (50000 lb). The machine operated in a deflection control mode with an uniform rate of cross head motion of \(0.46 \text{ mm min}^{-1}\) (0.018 \text{ inch min}^{-1}).

Assuming each connector carries one quarter of the load in the assembly, the stiffness of each connector was estimated from the slope of the load deformation curve as:

\[
K_x = \frac{1}{4} \left( \frac{P_x}{\Delta_x} \right)_{\text{con}}
\]

\[
K_y = \frac{1}{4} \left( \frac{P_y}{\Delta_y} \right)_{\text{con}}
\]

where \(\left( \frac{P_x}{\Delta_x} \right)_{\text{con}}\) and \(\left( \frac{P_y}{\Delta_y} \right)_{\text{con}}\) = the slope of the linear portion of the load deflection curve in the
perpendicular and parallel to face grain connection tests, respectively.

The linear portion of the load-deformation curve was taken between two preset points of 2 and 7 kN (450 and 1574 lb), respectively. Linear regression of the data between the preset point yielded the slope of the load-deformation curve.

4.2.2 Results

4.2.2.1 Veneer Strength Properties Statistics:

From each modulus of elasticity test program, the specimens were classified by: the two directions of testing; the four combinations of number of plies/veneer thickness; and the three groups of ultrasonic test results. Shown in Table 9 is the classification system for the veneer test specimens.

Table 9. Classification of veneer test specimens.

<table>
<thead>
<tr>
<th>Group</th>
<th>No. of Plies</th>
<th>Veneer Thickness (mm)</th>
<th>Direction</th>
<th>Ultrasound Subgroup</th>
</tr>
</thead>
<tbody>
<tr>
<td>310A</td>
<td>3</td>
<td>2.5</td>
<td>Parallel</td>
<td>A1 B1 C1</td>
</tr>
<tr>
<td>410A</td>
<td>4</td>
<td>2.5</td>
<td>Parallel</td>
<td>A2 B2 C2</td>
</tr>
<tr>
<td>308A</td>
<td>3</td>
<td>3.2</td>
<td>Parallel</td>
<td>A3 B3 C3</td>
</tr>
<tr>
<td>408A</td>
<td>4</td>
<td>3.2</td>
<td>Parallel</td>
<td>A4 B4 C4</td>
</tr>
<tr>
<td>310E</td>
<td>3</td>
<td>2.5</td>
<td>Perpendicular</td>
<td>A5 B5 C5</td>
</tr>
<tr>
<td>410E</td>
<td>4</td>
<td>2.5</td>
<td>Perpendicular</td>
<td>A6 B6 C6</td>
</tr>
<tr>
<td>308E</td>
<td>3</td>
<td>3.2</td>
<td>Perpendicular</td>
<td>A7 B7 C7</td>
</tr>
<tr>
<td>408E</td>
<td>4</td>
<td>3.2</td>
<td>Perpendicular</td>
<td>A8 B8 C8</td>
</tr>
</tbody>
</table>
Statistical information on the bending, compression, tension and shear elastic moduli of each group is shown in Table 10. Similar statistical information on the bending, compression, and tension strength of each group is shown in Table 11. Figures 21 and 22 show the cumulative probability distributions of veneer bending modulus of elasticity and strength, respectively. The cumulative probability distributions of tension modulus of elasticity and strength of the veneers are also shown in Figures 23 to 24, respectively. Figures 25 and 26 show the cumulative probability distributions of veneer compression modulus of elasticity and strength, respectively. Finally, Figure 27 shows the cumulative probability distribution of modulus of rigidity of the veneers.

4.2.2.2 Effects of veneer thickness and number of plies:

For each strength property, regression approach to analysis of variance was performed on the 8 data groups and the 24 data subgroups. In each case, either the elastic modulus or the strength was treated as dependent variables. Indicators (0 or 1) were used as independent variables in the regression analysis to represent the various groups and subgroups.

Results of the analysis of variance for the veneer bending, tension, compression and shear modulus of elasticity strength properties are shown in Tables 12 to 15, respectively. The results show that there was a significant relationship between the dependent variable (either modulus of elasticity or strength) and the independent variables at 95% probability level. The results also show that both group effect and subgroup effect were significant at 95% probability level in all cases except for the modulus of rigidity results where the subgroup effect was found to be not significantly different.

With each strength property, the mean values between the various groups were compared using Duncan's multiple range test. Table 16 shows the results of the comparisons. First, consider the shear modulus of rigidity, the results indicate that the mean values of the 310 and 410 groups were not significantly different from each other and the mean values of the 410, 308, and 408 groups were not significantly different at the 95% probability level.
Table 10. Statistical data on the veneer elastic moduli.

<table>
<thead>
<tr>
<th>Group</th>
<th>310A</th>
<th>410A</th>
<th>308A</th>
<th>408A</th>
<th>310E</th>
<th>410E</th>
<th>308E</th>
<th>408E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bending Modulus of Elasticity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean (MPa)</td>
<td>12825</td>
<td>13091</td>
<td>11106</td>
<td>12026</td>
<td>424.3</td>
<td>435.3</td>
<td>408.6</td>
<td>455.4</td>
</tr>
<tr>
<td>STDV (MPa)</td>
<td>2337</td>
<td>2265</td>
<td>1712</td>
<td>2703</td>
<td>135.9</td>
<td>99.0</td>
<td>81.7</td>
<td>161.9</td>
</tr>
<tr>
<td>Tension Modulus of Elasticity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean (MPa)</td>
<td>13677</td>
<td>13485</td>
<td>12378</td>
<td>11941</td>
<td>420.7</td>
<td>434.4</td>
<td>332.1</td>
<td>303.9</td>
</tr>
<tr>
<td>STDV (MPa)</td>
<td>1729</td>
<td>2494</td>
<td>1896</td>
<td>2291</td>
<td>94.1</td>
<td>104.9</td>
<td>60.8</td>
<td>105.8</td>
</tr>
<tr>
<td>Compression Modulus of Elasticity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean (MPa)</td>
<td>11864</td>
<td>11380</td>
<td>9536</td>
<td>10120</td>
<td>491.2</td>
<td>550.6</td>
<td>411.2</td>
<td>499.8</td>
</tr>
<tr>
<td>STDV (MPa)</td>
<td>3272</td>
<td>2770</td>
<td>3161</td>
<td>2049</td>
<td>79.0</td>
<td>82.3</td>
<td>44.9</td>
<td>106.2</td>
</tr>
<tr>
<td>Shear Modulus of Rigidity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean (MPa)</td>
<td>815.8</td>
<td>780.2</td>
<td>733.7</td>
<td>724.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>STDV (MPa)</td>
<td>119.2</td>
<td>82.0</td>
<td>87.2</td>
<td>98.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Count</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

Note: STDV = Standard deviation
Table 11. Statistical data on the veneer strengths.

<table>
<thead>
<tr>
<th>Group</th>
<th>310A</th>
<th>410A</th>
<th>308A</th>
<th>408A</th>
<th>310E</th>
<th>410E</th>
<th>308E</th>
<th>408E</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bending Strength</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean (MPa)</td>
<td>89.4</td>
<td>85.8</td>
<td>75.8</td>
<td>80.3</td>
<td>3.52</td>
<td>2.95</td>
<td>2.59</td>
<td>3.04</td>
</tr>
<tr>
<td>STDV (MPa)</td>
<td>16.6</td>
<td>18.7</td>
<td>9.9</td>
<td>15.5</td>
<td>0.80</td>
<td>0.72</td>
<td>0.75</td>
<td>0.72</td>
</tr>
<tr>
<td><strong>Tension Strength</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean (MPa)</td>
<td>42.3</td>
<td>41.6</td>
<td>36.8</td>
<td>35.8</td>
<td>0.72</td>
<td>0.91</td>
<td>0.67</td>
<td>0.44</td>
</tr>
<tr>
<td>STDV (MPa)</td>
<td>16.6</td>
<td>18.7</td>
<td>9.9</td>
<td>15.5</td>
<td>0.80</td>
<td>0.72</td>
<td>0.75</td>
<td>0.72</td>
</tr>
<tr>
<td><strong>Compression Strength</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean (MPa)</td>
<td>50.2</td>
<td>50.2</td>
<td>43.8</td>
<td>46.4</td>
<td>7.59</td>
<td>7.66</td>
<td>7.67</td>
<td>6.94</td>
</tr>
<tr>
<td>STDV (MPa)</td>
<td>5.2</td>
<td>5.4</td>
<td>7.3</td>
<td>6.1</td>
<td>1.20</td>
<td>1.02</td>
<td>1.05</td>
<td>0.53</td>
</tr>
<tr>
<td>Count</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>
Figure 21  The cumulative probability distributions of veneer bending modulus of elasticity.
Figure 22  The cumulative probability distributions of veneer bending strength.
Figure 23 The cumulative probability distributions of veneer tension modulus of elasticity.
Figure 24 The cumulative probability distributions of veneer tension strength.
Figure 25 The cumulative probability distributions of veneer compression modulus of elasticity.
Figure 26 The cumulative probability distributions of veneer compression strength.
Figure 27 The cumulative probability distributions of veneer shear modulus of rigidity.
Table 12. Analysis of variance results on veneer bending strength properties.

<table>
<thead>
<tr>
<th>Source</th>
<th>Degree of Freedom</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Probability &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Dependent Variable: Bending Modulus of Elasticity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>23</td>
<td>5877784086</td>
<td>255555830</td>
<td>210.84</td>
<td>0.0001</td>
</tr>
<tr>
<td>Group</td>
<td>7</td>
<td>5645685275</td>
<td>806526482</td>
<td>665.41</td>
<td>0.0001</td>
</tr>
<tr>
<td>Subgroup</td>
<td>16</td>
<td>232098711</td>
<td>14506169</td>
<td>11.97</td>
<td>0.0001</td>
</tr>
<tr>
<td>Error</td>
<td>136</td>
<td>164842145</td>
<td>1212075</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>159</td>
<td>6042626231</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Dependent Variable: Bending Strength</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>23</td>
<td>263143.5431</td>
<td>11411.0236</td>
<td>128.96</td>
<td>0.0001</td>
</tr>
<tr>
<td>Group</td>
<td>7</td>
<td>256879.1005</td>
<td>36697.0144</td>
<td>413.63</td>
<td>0.0001</td>
</tr>
<tr>
<td>Subgroup</td>
<td>16</td>
<td>6264.4426</td>
<td>391.5277</td>
<td>4.41</td>
<td>0.0001</td>
</tr>
<tr>
<td>Error</td>
<td>136</td>
<td>12065.7523</td>
<td>88.7188</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>159</td>
<td>275209.2954</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 13. Analysis of variance results on veneer tension strength properties.

<table>
<thead>
<tr>
<th>Source</th>
<th>Degree of Freedom</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Probability &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent Variable: Tension Modulus of Elasticity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>23</td>
<td>6523595987</td>
<td>283634608</td>
<td>349.33</td>
<td>0.0001</td>
</tr>
<tr>
<td>Group</td>
<td>7</td>
<td>6290282873</td>
<td>898611839</td>
<td>1106.74</td>
<td>0.0001</td>
</tr>
<tr>
<td>Subgroup</td>
<td>16</td>
<td>233313114</td>
<td>14582070</td>
<td>17.96</td>
<td>0.0001</td>
</tr>
<tr>
<td>Error</td>
<td>136</td>
<td>110424541</td>
<td>811945</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>159</td>
<td>6634020528</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| **Dependent Variable: Tension Strength** |                   |                |             |         |                 |
| Model           | 23                | 62054.16735    | 2698.00728  | 77.16   | 0.0001          |
| Group           | 7                 | 59779.55558    | 8539.93651  | 244.23  | 0.0001          |
| Subgroup        | 16                | 2274.61177     | 142.16324   | 4.07    | 0.0001          |
| Error           | 136               | 4755.44448     | 34.96650    |         |                 |
| **Total**       | 159               | 66809.61183    |             |         |                 |
Table 14. Analysis of variance results on veneer compression strength properties.

<table>
<thead>
<tr>
<th>Source</th>
<th>Degree of Freedom</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Probability &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent Variable: Compression Modulus of Elasticity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>23</td>
<td>4513900934</td>
<td>196256562</td>
<td>72.66</td>
<td>0.0001</td>
</tr>
<tr>
<td>Group</td>
<td>7</td>
<td>4261854575</td>
<td>608836368</td>
<td>225.42</td>
<td>0.0001</td>
</tr>
<tr>
<td>Subgroup</td>
<td>16</td>
<td>252046359</td>
<td>15752897</td>
<td>5.83</td>
<td>0.0001</td>
</tr>
<tr>
<td>Error</td>
<td>136</td>
<td>367321926</td>
<td>2700897</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>159</td>
<td>4881222860</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| **Dependent Variable: Compression Strength** |                   |                |             |         |                 |
| Model        | 23                | 66934.28599    | 2910.18635  | 367.84  | 0.0001          |
| Group        | 7                 | 65127.91607    | 9303.98801  | 1175.99 | 0.0001          |
| Subgroup     | 16                | 1806.36993     | 112.89812   | 14.27   | 0.0001          |
| Error        | 136               | 1075.97628     | 7.91159     |         |                 |
| Total        | 159               | 68010.26228    |             |         |                 |
Table 15. Analysis of variance results on veneer modulus of rigidity.

<table>
<thead>
<tr>
<th>Source</th>
<th>Degree of Freedom</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Probability &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent Variable: Modulus of rigidity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>11</td>
<td>226945.2864</td>
<td>20631.3897</td>
<td>2.31</td>
<td>0.0180</td>
</tr>
<tr>
<td>Group</td>
<td>3</td>
<td>108806.5482</td>
<td>36268.8494</td>
<td>4.06</td>
<td>0.0103</td>
</tr>
<tr>
<td>Subgroup</td>
<td>8</td>
<td>118138.7382</td>
<td>14767.3423</td>
<td>1.65</td>
<td>0.1265</td>
</tr>
<tr>
<td>Error</td>
<td>68</td>
<td>607837.0589</td>
<td>8938.7807</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>79</td>
<td>834782.3752</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 16. Duncan's multiple range test results for the various groups.

<table>
<thead>
<tr>
<th>Strength Properties</th>
<th>Groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bending Modulus of Elasticity</td>
<td>410A 310A 408A 308A 410E 310E 408E 308E</td>
</tr>
<tr>
<td>Bending Strength</td>
<td>310A 410A 408A 308A 310E 308E 410E 408E</td>
</tr>
<tr>
<td>Tension Modulus of Elasticity</td>
<td>310A 410A 308A 408A 410E 310E 308E 408E</td>
</tr>
<tr>
<td>Tension Strength</td>
<td>310A 410A 308A 408A 410E 310E 408E 308E</td>
</tr>
<tr>
<td>Compression Modulus of Elasticity</td>
<td>310A 410A 408A 308A 410E 408E 310E 308E</td>
</tr>
<tr>
<td>Compression Strength</td>
<td>310A 410A 308A 408A 410E 410E 310E 308E</td>
</tr>
<tr>
<td>Shear Modulus of Rigidity</td>
<td>310 410 408 308</td>
</tr>
</tbody>
</table>

Note: The mean values of the underlined groups were not significantly different at 95% probability level. The groups were arranged in descending order with respect to the mean value of each group.
When considering each strength property in the perpendicular to grain direction, the results indicate that the mean values of the various groups were not significantly different at the 95% probability level. For each strength property in the parallel to grain direction, the results also indicate that the mean values of the 310 and 410 groups were not significantly different at the 95% probability level. Finally, the mean values of the 308 and 408 groups were not significantly different at the 95% probability level for all strength properties except for the bending modulus of elasticity and compression strength in the parallel to grain direction.

Based on these results, for the shear modulus of rigidity data, the 310 and 410 groups were considered as a single group and the 308 and 408 groups were considered as another single group. With the other strength properties, the 4 groups in the perpendicular direction (410E, 310E, 408E, and 308E) were considered as a single group. Each strength property in the perpendicular to grain direction was therefore represented by a probability distribution where the effects of number of plies and veneer thickness were assumed insignificant. Similarly, the 310A and 410A groups in the parallel to grain direction were considered as a single group. Finally for practical reason, the 308A and 408A groups in the parallel to grain direction were also considered as another single group. Therefore, each strength property in the parallel to grain direction was represented by two probability distributions (one for each veneer thickness) where the effects of number of plies were assumed insignificant.

Ignoring the effects of the number of plies, the normal, 2-parameter Weibull, and 3-parameter Weibull probability distributions were fitted to individual data groups following the maximum likelihood estimation approach (Lawless, 1982). Both the normal and the 3-parameter Weibull distributions were visually judged to provide the good fit to the data. Figures 28 to 34 shows the cumulative probability distributions, the normal distributions and the 3-parameter Weibull distributions of each group for the various strength properties. The statistical information and the distribution parameters of the elastic moduli and strengths of the various groups are summarized in Tables 17 and 20.
Figure 28 The cumulative probability distributions and the 3-parameter Weibull distributions of the veneer bending modulus of elasticity of each group.
Figure 29  The cumulative probability distributions and the 3-parameter Weibull distributions of the veneer bending strength of each group.
Figure 30 The cumulative probability distributions and the 3-parameter Weibull distributions of the veneer tension modulus of elasticity of each group.
Figure 31 The cumulative probability distributions and the 3-parameter Weibull distributions of the veneer tension strength of each group.
Figure 32 The cumulative probability distributions and the 3-parameter Weibull distributions of the veneer compression modulus of elasticity of each group.
Figure 33 The cumulative probability distributions and the 3-parameter Weibull distributions of the veneer compression strength of each group.
Figure 34 The cumulative probability distributions and the 3-parameter Weibull distributions of the veneer shear modulus of rigidity of each group.
Table 17. Statistical data and distribution parameters of the veneer bending strength properties.

<table>
<thead>
<tr>
<th>Direction</th>
<th>Modulus of Elasticity</th>
<th>Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Parallel</td>
<td>Perpendicular</td>
</tr>
<tr>
<td>Thickness (mm)</td>
<td>2.5</td>
<td>3.2</td>
</tr>
<tr>
<td>Mean (MPa)</td>
<td>12957.9</td>
<td>11563.3</td>
</tr>
<tr>
<td>Median (MPa)</td>
<td>13130.8</td>
<td>11600.8</td>
</tr>
<tr>
<td>STDV (MPa)</td>
<td>2275.8</td>
<td>2280.7</td>
</tr>
<tr>
<td>Count</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>2-Parameter Weibull</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shape</td>
<td>5.604</td>
<td>5.295</td>
</tr>
<tr>
<td>Scale (MPa)</td>
<td>13905.7</td>
<td>12501.4</td>
</tr>
<tr>
<td>3-Parameter Weibull</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shape</td>
<td>3.092</td>
<td>2.645</td>
</tr>
<tr>
<td>Scale (MPa)</td>
<td>7464.7</td>
<td>6307.5</td>
</tr>
<tr>
<td>Location (MPa)</td>
<td>6246.1</td>
<td>5950.8</td>
</tr>
</tbody>
</table>
Table 18. Statistical data and distribution parameters of the veneer tension strength properties.

<table>
<thead>
<tr>
<th>Thickness (mm)</th>
<th>Modulus of Elasticity</th>
<th>Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Parallel</td>
<td>Perpendicular</td>
</tr>
<tr>
<td>2.5</td>
<td>13580.8</td>
<td>12159.4</td>
</tr>
<tr>
<td>3.2</td>
<td>13340.0</td>
<td>11792.7</td>
</tr>
<tr>
<td></td>
<td>2120.4</td>
<td>2087.8</td>
</tr>
<tr>
<td>Count</td>
<td>40</td>
<td>40</td>
</tr>
</tbody>
</table>

2-Parameter Weibull

<table>
<thead>
<tr>
<th></th>
<th>Scale (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (MPa)</td>
<td>14487.3</td>
</tr>
<tr>
<td>Median (MPa)</td>
<td>13034.7</td>
</tr>
<tr>
<td>STDV (MPa)</td>
<td>412.15</td>
</tr>
<tr>
<td>Location (MPa)</td>
<td>8307.1</td>
</tr>
</tbody>
</table>

3-Parameter Weibull

<table>
<thead>
<tr>
<th></th>
<th>Scale (MPa)</th>
<th>Location (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (MPa)</td>
<td>15920.6</td>
<td>6570.6</td>
</tr>
<tr>
<td>Median (MPa)</td>
<td>1575.7</td>
<td>337.75</td>
</tr>
<tr>
<td>STDV (MPa)</td>
<td>39.51</td>
<td>25.09</td>
</tr>
<tr>
<td>Location (MPa)</td>
<td>8307.1</td>
<td>6280.9</td>
</tr>
</tbody>
</table>

Strength 2.5 | 3.2 | Combined | 2.5 | 3.2 | Combined

<table>
<thead>
<tr>
<th></th>
<th>Scale (MPa)</th>
<th>Location (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (MPa)</td>
<td>15920.6</td>
<td>6570.6</td>
</tr>
<tr>
<td>Median (MPa)</td>
<td>1575.7</td>
<td>337.75</td>
</tr>
<tr>
<td>STDV (MPa)</td>
<td>39.51</td>
<td>25.09</td>
</tr>
<tr>
<td>Location (MPa)</td>
<td>8307.1</td>
<td>6280.9</td>
</tr>
</tbody>
</table>
Table 19. Statistical data and distribution parameters of the veneer compression strength properties.

<table>
<thead>
<tr>
<th>Direction</th>
<th>Modulus of Elasticity</th>
<th>Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Parallel</td>
<td>Perpendicular</td>
</tr>
<tr>
<td>Thickness (mm)</td>
<td>2.5</td>
<td>3.2</td>
</tr>
<tr>
<td>Mean (MPa)</td>
<td>11622.0</td>
<td>9827.6</td>
</tr>
<tr>
<td>Median (MPa)</td>
<td>11548.7</td>
<td>9484.7</td>
</tr>
<tr>
<td>STDV (MPa)</td>
<td>3002.5</td>
<td>2646.0</td>
</tr>
<tr>
<td>Count</td>
<td>40</td>
<td>40</td>
</tr>
</tbody>
</table>

2-Parameter Weibull

| Shape | 4.278 | 3.866 | 5.385 | 8.981 | 6.969 | 6.870 |
| Scale (MPa) | 12769.1 | 10830.3 | 527.52 | 52.57 | 48.06 | 7.755 |

3-Parameter Weibull

| Shape | 2.232 | 1.556 | 2.188 | 2.916 | 2.034 | 6.870 |
| Scale (MPa) | 7047.1 | 4526.6 | 219.21 | 16.21 | 14.74 | 7.755 |
| Location (MPa) | 5378.6 | 5745.6 | 294.05 | 35.65 | 32.03 | 0.000 |
Table 20. Statistical data and distribution parameters of the veneer shear modulus of rigidity.

<table>
<thead>
<tr>
<th>Thickness (mm)</th>
<th>2.5</th>
<th>3.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (MPa)</td>
<td>798.02</td>
<td>729.00</td>
</tr>
<tr>
<td>Median (MPa)</td>
<td>804.78</td>
<td>709.32</td>
</tr>
<tr>
<td>STDV (MPa)</td>
<td>102.58</td>
<td>91.86</td>
</tr>
<tr>
<td>Count</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>2-Parameter Weibull</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shape</td>
<td>7.983</td>
<td>7.424</td>
</tr>
<tr>
<td>Scale (MPa)</td>
<td>843.10</td>
<td>770.99</td>
</tr>
<tr>
<td>3-Parameter Weibull</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shape</td>
<td>2.486</td>
<td>1.281</td>
</tr>
<tr>
<td>Scale (MPa)</td>
<td>265.64</td>
<td>129.83</td>
</tr>
<tr>
<td>Location (MPa)</td>
<td>562.12</td>
<td>608.22</td>
</tr>
</tbody>
</table>
4.2.2.3 Correlations of veneer strength properties:

Regression analyses were performed to examine the correlation amongst the various strength properties of the 2.5 and 3.2 mm veneer groups in the parallel and perpendicular to grain directions. Table 21 shows the dependent and independent variables in the various regression models considered. Tables 22 and 23 show the regression results for the 3.2 and 2.5 mm veneer groups in the parallel to grain direction. Tables 24 and 25 show the regression results for the 3.2 and 2.5 mm veneer groups in the perpendicular to grain direction.

When considering the 3.2 mm and 2.5 mm thick veneer in the parallel to grain direction, results indicate significant relationships existed at the 95% probability level for bending, tension and compression modulus of elasticity versus the respective strengths. The coefficient of determination, $r^2$, for these relationships ranged from 0.26 to 0.69. For the 3.2 mm thick veneer in the parallel to grain direction, results also indicate significant relationships existed at the 95% probability level for compression versus tension modulus of elasticity ($r^2=0.36$) and compression versus tension strength ($r^2=0.44$). With the 2.5 mm thick veneer in the parallel to grain direction, results also indicate significant relationships existed at the 95% probability level for bending versus compression modulus of elasticity ($r^2=0.37$), bending versus tension modulus of elasticity ($r^2=0.53$), tension versus compression modulus of elasticity ($r^2=0.13$), bending versus compression strength ($r^2=0.55$), bending versus tension strength ($r^2=0.31$), and compression versus tension strength ($r^2=0.36$).

For the 3.2 mm thick veneer in the perpendicular to grain direction, significant relationships existed at the 95% probability level for bending modulus of elasticity versus strength, and compression modulus of elasticity versus strength with $r^2$ ranging from 0.17 to 0.21. With the 2.5 mm thick veneer in the perpendicular to grain direction, significant relationships existed at the 95% probability level for bending modulus of elasticity versus strength, tension modulus of elasticity versus strength, bending versus tension modulus of elasticity, compression versus tension modulus of elasticity, and compression modulus of elasticity versus shear modulus of rigidity with $r^2$ ranging from 0.12 to 0.33.
Table 21. The dependent and independent variables considered in the various regression models of veneer strength properties.

<table>
<thead>
<tr>
<th>Model</th>
<th>Dependent Variable</th>
<th>Independent Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bending Strength</td>
<td>Bending Modulus of Elasticity</td>
</tr>
<tr>
<td>2</td>
<td>Compression Strength</td>
<td>Compression Modulus of Elasticity</td>
</tr>
<tr>
<td>3</td>
<td>Tension Strength</td>
<td>Tension Modulus of Elasticity</td>
</tr>
<tr>
<td>4</td>
<td>Bending Modulus of Elasticity</td>
<td>Compression Modulus of Elasticity</td>
</tr>
<tr>
<td>5</td>
<td>Bending Modulus of Elasticity</td>
<td>Tension Modulus of Elasticity</td>
</tr>
<tr>
<td>6</td>
<td>Bending Modulus of Elasticity</td>
<td>Shear Modulus of Rigidity</td>
</tr>
<tr>
<td>7</td>
<td>Compression Modulus of Elasticity</td>
<td>Tension Modulus of Elasticity</td>
</tr>
<tr>
<td>8</td>
<td>Compression Modulus of Elasticity</td>
<td>Shear Modulus of Rigidity</td>
</tr>
<tr>
<td>9</td>
<td>Tension Modulus of Elasticity</td>
<td>Shear Modulus of Rigidity</td>
</tr>
<tr>
<td>10</td>
<td>Bending Strength</td>
<td>Compression Strength</td>
</tr>
<tr>
<td>11</td>
<td>Bending Strength</td>
<td>Tension Strength</td>
</tr>
<tr>
<td>12</td>
<td>Compression Strength</td>
<td>Tension Strength</td>
</tr>
</tbody>
</table>
Table 22. Results of various regression models of 3.2 mm veneer strength properties for the parallel to grain direction.

<table>
<thead>
<tr>
<th>Model Number</th>
<th>Number of Observations</th>
<th>F-value</th>
<th>Probability &gt; F</th>
<th>$r^2$</th>
<th>Intercept (MPa)</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40</td>
<td>58.966</td>
<td>0.0001</td>
<td>0.6081</td>
<td>26.496</td>
<td>0.004460</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>46.748</td>
<td>0.0001</td>
<td>0.5516</td>
<td>26.311</td>
<td>0.001911</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>34.562</td>
<td>0.0001</td>
<td>0.4763</td>
<td>-2.0244</td>
<td>0.003151</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>0.581</td>
<td>0.4507</td>
<td>0.0151</td>
<td>10524</td>
<td>0.105753</td>
</tr>
<tr>
<td>5</td>
<td>40</td>
<td>2.747</td>
<td>0.1057</td>
<td>0.674</td>
<td>8114.7</td>
<td>0.283616</td>
</tr>
<tr>
<td>6</td>
<td>40</td>
<td>1.015</td>
<td>0.3201</td>
<td>0.0260</td>
<td>14482</td>
<td>-4.004565</td>
</tr>
<tr>
<td>7</td>
<td>40</td>
<td>21.866</td>
<td>0.0001</td>
<td>0.3653</td>
<td>514.33</td>
<td>0.765934</td>
</tr>
<tr>
<td>8</td>
<td>40</td>
<td>0.931</td>
<td>0.3406</td>
<td>0.0239</td>
<td>13075</td>
<td>-4.454788</td>
</tr>
<tr>
<td>9</td>
<td>40</td>
<td>3.286</td>
<td>0.0778</td>
<td>0.0796</td>
<td>16834</td>
<td>-6.412149</td>
</tr>
<tr>
<td>10</td>
<td>40</td>
<td>3.954</td>
<td>0.0540</td>
<td>0.0942</td>
<td>51.552</td>
<td>0.588132</td>
</tr>
<tr>
<td>11</td>
<td>40</td>
<td>0.533</td>
<td>0.4700</td>
<td>0.0138</td>
<td>72.235</td>
<td>0.160914</td>
</tr>
<tr>
<td>12</td>
<td>40</td>
<td>29.283</td>
<td>0.0001</td>
<td>0.4352</td>
<td>27.992</td>
<td>0.471331</td>
</tr>
</tbody>
</table>
Table 23. Results of various regression models of 2.5 mm veneer strength properties for the parallel to grain direction.

<table>
<thead>
<tr>
<th>Model Number</th>
<th>F-value</th>
<th>Probability &gt; F</th>
<th>$r^2$</th>
<th>Intercept (MPa)</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>85.806</td>
<td>0.0001</td>
<td>0.6931</td>
<td>4.4843</td>
<td>0.006413</td>
</tr>
<tr>
<td>2</td>
<td>18.333</td>
<td>0.0001</td>
<td>0.3254</td>
<td>38.605</td>
<td>0.000995</td>
</tr>
<tr>
<td>3</td>
<td>13.670</td>
<td>0.0007</td>
<td>0.2646</td>
<td>10.778</td>
<td>0.002297</td>
</tr>
<tr>
<td>4</td>
<td>21.945</td>
<td>0.0001</td>
<td>0.3661</td>
<td>7628.0</td>
<td>0.458604</td>
</tr>
<tr>
<td>5</td>
<td>42.744</td>
<td>0.0001</td>
<td>0.5294</td>
<td>2352.6</td>
<td>0.780904</td>
</tr>
<tr>
<td>6</td>
<td>0.075</td>
<td>0.7853</td>
<td>0.0020</td>
<td>12171</td>
<td>0.986621</td>
</tr>
<tr>
<td>7</td>
<td>5.473</td>
<td>0.0247</td>
<td>0.1259</td>
<td>4798.9</td>
<td>0.502403</td>
</tr>
<tr>
<td>8</td>
<td>0.564</td>
<td>0.4574</td>
<td>0.0146</td>
<td>8797.8</td>
<td>3.539026</td>
</tr>
<tr>
<td>9</td>
<td>0.191</td>
<td>0.6644</td>
<td>0.0050</td>
<td>12414</td>
<td>1.462549</td>
</tr>
<tr>
<td>10</td>
<td>46.618</td>
<td>0.0001</td>
<td>0.5509</td>
<td>-37.050</td>
<td>2.482062</td>
</tr>
<tr>
<td>11</td>
<td>17.385</td>
<td>0.0002</td>
<td>0.3139</td>
<td>44.045</td>
<td>1.037059</td>
</tr>
<tr>
<td>12</td>
<td>21.634</td>
<td>0.0001</td>
<td>0.3628</td>
<td>36.187</td>
<td>0.333130</td>
</tr>
</tbody>
</table>
Table 24. Results of various regression models of 3.2 mm veneer strength properties for the perpendicular to grain direction.

<table>
<thead>
<tr>
<th>Model</th>
<th>Number of Observations</th>
<th>F-value</th>
<th>Probability &gt; F</th>
<th>$r^2$</th>
<th>Intercept (MPa)</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40</td>
<td>7.976</td>
<td>0.0075</td>
<td>0.1735</td>
<td>1.7531</td>
<td>0.002461</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>10.313</td>
<td>0.0027</td>
<td>0.2135</td>
<td>5.2580</td>
<td>0.004496</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>1.097</td>
<td>0.3015</td>
<td>0.0281</td>
<td>0.4313</td>
<td>0.000396</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>2.096</td>
<td>0.1559</td>
<td>0.0523</td>
<td>286.408</td>
<td>0.319608</td>
</tr>
<tr>
<td>5</td>
<td>40</td>
<td>0.289</td>
<td>0.5940</td>
<td>0.0075</td>
<td>390.791</td>
<td>0.129595</td>
</tr>
<tr>
<td>6</td>
<td>40</td>
<td>0.138</td>
<td>0.7127</td>
<td>0.0036</td>
<td>370.600</td>
<td>0.084220</td>
</tr>
<tr>
<td>7</td>
<td>40</td>
<td>2.156</td>
<td>0.1503</td>
<td>0.0537</td>
<td>376.911</td>
<td>0.247242</td>
</tr>
<tr>
<td>8</td>
<td>40</td>
<td>1.791</td>
<td>0.1888</td>
<td>0.0450</td>
<td>300.410</td>
<td>0.212777</td>
</tr>
<tr>
<td>9</td>
<td>40</td>
<td>0.242</td>
<td>0.6253</td>
<td>0.0063</td>
<td>372.514</td>
<td>-0.074835</td>
</tr>
<tr>
<td>10</td>
<td>40</td>
<td>0.198</td>
<td>0.6589</td>
<td>0.0052</td>
<td>2.3698</td>
<td>0.061103</td>
</tr>
<tr>
<td>11</td>
<td>40</td>
<td>1.220</td>
<td>0.2764</td>
<td>0.0311</td>
<td>3.1827</td>
<td>-0.657982</td>
</tr>
<tr>
<td>12</td>
<td>40</td>
<td>0.271</td>
<td>0.6060</td>
<td>0.0071</td>
<td>7.5118</td>
<td>-0.369602</td>
</tr>
</tbody>
</table>
Table 25. Results of various regression models of 2.5 mm veneer strength properties for the perpendicular to grain direction.

<table>
<thead>
<tr>
<th>Model Number</th>
<th>F-value</th>
<th>Probability &gt; F</th>
<th>$r^2$</th>
<th>Intercept (MPa)</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.185</td>
<td>0.0285</td>
<td>0.1201</td>
<td>2.2141</td>
<td>0.002370</td>
</tr>
<tr>
<td>2</td>
<td>1.594</td>
<td>0.2145</td>
<td>0.0402</td>
<td>5.4689</td>
<td>0.003660</td>
</tr>
<tr>
<td>3</td>
<td>18.354</td>
<td>0.0001</td>
<td>0.3257</td>
<td>0.0364</td>
<td>0.001829</td>
</tr>
<tr>
<td>4</td>
<td>1.278</td>
<td>0.2654</td>
<td>0.0325</td>
<td>300.124</td>
<td>0.248959</td>
</tr>
<tr>
<td>5</td>
<td>4.981</td>
<td>0.0316</td>
<td>0.1159</td>
<td>256.280</td>
<td>0.405862</td>
</tr>
<tr>
<td>6</td>
<td>1.286</td>
<td>0.2640</td>
<td>0.0327</td>
<td>264.419</td>
<td>0.207254</td>
</tr>
<tr>
<td>7</td>
<td>10.882</td>
<td>0.0021</td>
<td>0.2226</td>
<td>346.676</td>
<td>0.407531</td>
</tr>
<tr>
<td>8</td>
<td>10.125</td>
<td>0.0029</td>
<td>0.2104</td>
<td>217.089</td>
<td>0.380733</td>
</tr>
<tr>
<td>9</td>
<td>2.750</td>
<td>0.1055</td>
<td>0.0675</td>
<td>228.346</td>
<td>0.249642</td>
</tr>
<tr>
<td>10</td>
<td>0.081</td>
<td>0.7774</td>
<td>0.0021</td>
<td>3.4090</td>
<td>-0.023880</td>
</tr>
<tr>
<td>11</td>
<td>1.039</td>
<td>0.3146</td>
<td>0.0266</td>
<td>3.5725</td>
<td>-0.414925</td>
</tr>
<tr>
<td>12</td>
<td>0.213</td>
<td>0.6468</td>
<td>0.0056</td>
<td>7.0749</td>
<td>0.367330</td>
</tr>
</tbody>
</table>
Although these relationships were found to be statistically significant, in most cases their low $r^2$ values indicate the relationships were weak. This is especially true for the relationships between veneer elastic properties with the exception of the relationship between bending and tension moduli of elasticity for 2.5 mm thick veneer in the parallel to grain direction ($r^2=0.53$). For practical purposes, the elasticity properties of the veneers were considered as independent from each other. Therefore, based on the distribution parameters shown in Tables 17 to 20, each veneer elastic property can be independently generated in simulation which can be substituted into Equations 74a to 74h to evaluate the elastic properties of the transDeck™ panels.

The relationships between the various veneer strengths and elastic properties seem to be stronger than the relationships between the various veneer elastic properties as indicated by the higher $r^2$ values. Therefore, when simulations of veneer strength data are required, the correlation between veneer strength and elastic properties should be considered.

4.2.2.4 Effectiveness of Ultrasonic Testing on Veneer Grading:

Tables 26 and 27 show the statistical data on the strength and elastic properties of the various subgroups, respectively. With each parallel to grain strength property, the ratio between the mean strength value in subgroup A and the mean strength value of the combined groups A to C ranged from 1.07 to 1.14 for the 310 and 410 groups and 1.05 to 1.26 for the 308 and 408 groups. Similarly for each elastic property in the parallel to grain direction, the ratio between the mean elastic value of subgroup A and the mean elastic value of the combined groups A to C ranged from 1.11 to 1.21 for the 310 and 410 groups and 1.13 to 1.40 for the 308 and 408 groups. In the perpendicular to grain direction, the ratio between the mean strength or elastic property in subgroup A and the mean strength or elastic property of the combined groups A to C ranged from 0.67 to 1.11. Finally when considering the shear modulus of elasticity, the ratio between the mean value in subgroup A and the strength value of the combined groups A to C ranged from 0.93 to 1.05 for the 310, 410, 408, and 308 groups.
Table 26. Statistical data on the veneer strength properties of the various subgroups.

<table>
<thead>
<tr>
<th>Group</th>
<th>Subgroup</th>
<th>Count</th>
<th>Mean (MPa)</th>
<th>STDV (MPa)</th>
<th>Mean (MPa)</th>
<th>STDV (MPa)</th>
<th>Mean (MPa)</th>
<th>STDV (MPa)</th>
<th>Mean (MPa)</th>
<th>STDV (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>310</td>
<td>410</td>
<td>408</td>
<td>308</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_{yb}$</td>
<td>C</td>
<td>5</td>
<td>77.1</td>
<td>11.2</td>
<td>65.4</td>
<td>18.3</td>
<td>70.0</td>
<td>16.3</td>
<td>75.0</td>
<td>9.7</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>10</td>
<td>91.0</td>
<td>13.7</td>
<td>90.9</td>
<td>12.7</td>
<td>76.9</td>
<td>11.7</td>
<td>74.4</td>
<td>10.5</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>5</td>
<td>98.5</td>
<td>21.4</td>
<td>96.0</td>
<td>15.9</td>
<td>97.5</td>
<td>5.6</td>
<td>79.5</td>
<td>10.3</td>
</tr>
<tr>
<td>$S_{yc}$</td>
<td>C</td>
<td>5</td>
<td>46.5</td>
<td>0.9</td>
<td>44.0</td>
<td>4.9</td>
<td>38.9</td>
<td>3.3</td>
<td>37.7</td>
<td>1.9</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>10</td>
<td>50.1</td>
<td>3.8</td>
<td>51.5</td>
<td>3.6</td>
<td>46.1</td>
<td>2.0</td>
<td>41.7</td>
<td>3.8</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>5</td>
<td>53.9</td>
<td>7.9</td>
<td>53.7</td>
<td>4.2</td>
<td>54.6</td>
<td>2.4</td>
<td>54.1</td>
<td>5.4</td>
</tr>
<tr>
<td>$S_{yt}$</td>
<td>C</td>
<td>5</td>
<td>37.8</td>
<td>4.9</td>
<td>31.5</td>
<td>9.8</td>
<td>29.0</td>
<td>10.3</td>
<td>31.8</td>
<td>3.5</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>10</td>
<td>41.7</td>
<td>11.6</td>
<td>44.6</td>
<td>8.1</td>
<td>34.6</td>
<td>7.9</td>
<td>34.4</td>
<td>6.5</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>5</td>
<td>48.2</td>
<td>2.9</td>
<td>45.6</td>
<td>7.4</td>
<td>45.0</td>
<td>6.7</td>
<td>46.4</td>
<td>12.5</td>
</tr>
<tr>
<td>$S_{xb}$</td>
<td>C</td>
<td>5</td>
<td>3.11</td>
<td>0.47</td>
<td>3.17</td>
<td>0.35</td>
<td>2.70</td>
<td>0.88</td>
<td>2.67</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>10</td>
<td>3.69</td>
<td>0.90</td>
<td>2.81</td>
<td>0.83</td>
<td>2.62</td>
<td>0.83</td>
<td>3.16</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>5</td>
<td>3.57</td>
<td>0.84</td>
<td>3.01</td>
<td>0.80</td>
<td>2.42</td>
<td>0.52</td>
<td>3.19</td>
<td>0.75</td>
</tr>
<tr>
<td>$S_{xc}$</td>
<td>C</td>
<td>5</td>
<td>6.53</td>
<td>0.91</td>
<td>6.95</td>
<td>1.06</td>
<td>7.20</td>
<td>0.67</td>
<td>7.01</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>10</td>
<td>7.74</td>
<td>0.97</td>
<td>7.82</td>
<td>1.06</td>
<td>7.53</td>
<td>1.24</td>
<td>6.77</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>5</td>
<td>8.37</td>
<td>1.32</td>
<td>8.04</td>
<td>0.67</td>
<td>8.42</td>
<td>0.48</td>
<td>7.20</td>
<td>0.60</td>
</tr>
<tr>
<td>$S_{xt}$</td>
<td>C</td>
<td>5</td>
<td>0.93</td>
<td>0.32</td>
<td>1.10</td>
<td>0.39</td>
<td>0.82</td>
<td>0.08</td>
<td>0.46</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>10</td>
<td>0.68</td>
<td>0.25</td>
<td>0.89</td>
<td>0.28</td>
<td>0.71</td>
<td>0.21</td>
<td>0.41</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>5</td>
<td>0.60</td>
<td>0.26</td>
<td>0.76</td>
<td>0.29</td>
<td>0.45</td>
<td>0.14</td>
<td>0.49</td>
<td>0.17</td>
</tr>
</tbody>
</table>
Table 27. Statistical data on the veneer elastic properties of the various subgroups.

<table>
<thead>
<tr>
<th>Group</th>
<th>Subgroup Count</th>
<th>Mean (MPa)</th>
<th>STDV (MPa)</th>
<th>Mean (MPa)</th>
<th>STDV (MPa)</th>
<th>Mean (MPa)</th>
<th>STDV (MPa)</th>
<th>Mean (MPa)</th>
<th>STDV (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>310</td>
<td>410</td>
<td>408</td>
<td>308</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E_yb</td>
<td>C 5</td>
<td>10269</td>
<td>547</td>
<td>10543</td>
<td>2109</td>
<td>8876</td>
<td>1401</td>
<td>10392</td>
<td>1388</td>
</tr>
<tr>
<td></td>
<td>B 10</td>
<td>13228</td>
<td>809</td>
<td>13354</td>
<td>1547</td>
<td>11794</td>
<td>1005</td>
<td>10747</td>
<td>1501</td>
</tr>
<tr>
<td></td>
<td>A 5</td>
<td>14574</td>
<td>3432</td>
<td>15112</td>
<td>1071</td>
<td>15619</td>
<td>1344</td>
<td>12538</td>
<td>1844</td>
</tr>
<tr>
<td>E_yc</td>
<td>C 5</td>
<td>8750</td>
<td>1239</td>
<td>9921</td>
<td>2236</td>
<td>7963</td>
<td>750</td>
<td>7607</td>
<td>1748</td>
</tr>
<tr>
<td></td>
<td>B 10</td>
<td>12238</td>
<td>3263</td>
<td>10987</td>
<td>2835</td>
<td>10430</td>
<td>2006</td>
<td>8588</td>
<td>1765</td>
</tr>
<tr>
<td></td>
<td>A 5</td>
<td>14231</td>
<td>2426</td>
<td>13623</td>
<td>2908</td>
<td>11656</td>
<td>1144</td>
<td>13359</td>
<td>3499</td>
</tr>
<tr>
<td>E_yt</td>
<td>C 5</td>
<td>12262</td>
<td>1585</td>
<td>10567</td>
<td>1040</td>
<td>9285</td>
<td>791</td>
<td>10461</td>
<td>1665</td>
</tr>
<tr>
<td></td>
<td>B 10</td>
<td>13593</td>
<td>1330</td>
<td>13506</td>
<td>1084</td>
<td>11645</td>
<td>715</td>
<td>12349</td>
<td>1051</td>
</tr>
<tr>
<td></td>
<td>A 5</td>
<td>15260</td>
<td>1432</td>
<td>13630</td>
<td>2200</td>
<td>15187</td>
<td>980</td>
<td>14352</td>
<td>1536</td>
</tr>
<tr>
<td>E_xb</td>
<td>C 5</td>
<td>300.1</td>
<td>139.9</td>
<td>512.0</td>
<td>63.2</td>
<td>430.2</td>
<td>162.3</td>
<td>447.4</td>
<td>106.4</td>
</tr>
<tr>
<td></td>
<td>B 10</td>
<td>470.7</td>
<td>116.4</td>
<td>389.5</td>
<td>94.9</td>
<td>474.8</td>
<td>196.3</td>
<td>375.3</td>
<td>52.5</td>
</tr>
<tr>
<td></td>
<td>A 5</td>
<td>455.7</td>
<td>108.8</td>
<td>450.1</td>
<td>97.1</td>
<td>441.8</td>
<td>97.3</td>
<td>436.1</td>
<td>92.3</td>
</tr>
<tr>
<td>E_xc</td>
<td>C 5</td>
<td>468.4</td>
<td>71.2</td>
<td>559.9</td>
<td>74.9</td>
<td>533.7</td>
<td>73.8</td>
<td>401.0</td>
<td>67.9</td>
</tr>
<tr>
<td></td>
<td>B 10</td>
<td>519.3</td>
<td>79.3</td>
<td>573.4</td>
<td>88.4</td>
<td>514.0</td>
<td>133.5</td>
<td>429.1</td>
<td>27.9</td>
</tr>
<tr>
<td></td>
<td>A 5</td>
<td>457.7</td>
<td>80.3</td>
<td>495.9</td>
<td>61.8</td>
<td>437.5</td>
<td>35.2</td>
<td>385.7</td>
<td>38.5</td>
</tr>
<tr>
<td>E_xt</td>
<td>C 5</td>
<td>442.4</td>
<td>140.5</td>
<td>491.8</td>
<td>57.3</td>
<td>241.5</td>
<td>40.4</td>
<td>309.4</td>
<td>28.2</td>
</tr>
<tr>
<td></td>
<td>B 10</td>
<td>423.6</td>
<td>78.4</td>
<td>420.0</td>
<td>103.1</td>
<td>350.6</td>
<td>114.7</td>
<td>330.3</td>
<td>77.8</td>
</tr>
<tr>
<td></td>
<td>A 5</td>
<td>393.2</td>
<td>83.1</td>
<td>405.7</td>
<td>139.3</td>
<td>272.6</td>
<td>102.0</td>
<td>358.1</td>
<td>40.7</td>
</tr>
<tr>
<td>G</td>
<td>C 5</td>
<td>833.1</td>
<td>135.6</td>
<td>795.9</td>
<td>111.4</td>
<td>842.6</td>
<td>119.8</td>
<td>749.4</td>
<td>106.4</td>
</tr>
<tr>
<td></td>
<td>B 10</td>
<td>833.0</td>
<td>124.3</td>
<td>773.7</td>
<td>90.8</td>
<td>673.7</td>
<td>49.9</td>
<td>728.4</td>
<td>87.5</td>
</tr>
<tr>
<td></td>
<td>A 5</td>
<td>764.2</td>
<td>99.6</td>
<td>777.7</td>
<td>23.9</td>
<td>707.3</td>
<td>41.6</td>
<td>728.5</td>
<td>84.5</td>
</tr>
</tbody>
</table>
For each strength property, the mean values between the 24 subgroups were compared using Duncan's multiple range test. Table 28 shows the results of the comparisons amongst the three subgroups within each of the eight groups for each strength property. In the shear modulus of rigidity data, the results indicate that the mean values of the three subgroups within each group were not significantly different at the 95% probability level except in the 408 group where subgroup C4 was significantly different. With the other strength properties in the parallel to grain direction, the mean values in subgroup C were typically lower than those in subgroups A and B. Also subgroup A had the highest mean value except in 1 out of 24 cases. The ultrasonic procedure seems to be most effective in sorting the veneer for the parallel to grain tension modulus of elasticity as the results indicate that the mean values in all three subgroups within each group were significantly different at the 95% probability level. In the other strength properties, the distinction amongst the three subgroups were not as clear but subgroup C was typically identified as significantly different from the subgroups A and B at the 95% probability level except in 8 out of 24 cases. In the perpendicular to grain direction, the strength properties in the subgroups seem to be confounded. Therefore, it seems that the ultrasonic test procedure was able to sort the veneers into strength groups for the parallel to grain direction but could not deliver the same level of performance for strength properties in the perpendicular to grain direction.

4.2.2.5 Connector Stiffness:

Figure 35 shows the load deformation curves of four typical test specimens. Within the range of loading, the load slip curve seems relatively linear for the cases where the face grain was parallel to the direction of loading. When the face grain was perpendicular to the direction of loading, both the connector stiffness and the linearity of the load slip curves were clearly reduced. The connection load slip test results are summarized in Table 29. Analysis of variance and Duncan multiple range test were performed on the four test groups. Table 30 shows the results which indicate that the four groups were statistically different at the 95% probability level. Also the mean stiffness values of the 10A and 08A
Table 28. Duncan's multiple range test results for the various groups.

<table>
<thead>
<tr>
<th>Strength Properties</th>
<th>Subgroups</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bending Modulus of Elasticity</td>
<td>A1 B1 C1; A2 B2 C2; A3 B3 C3; A4 B4 C4;</td>
</tr>
<tr>
<td></td>
<td>B5 A5 C5; B6 A6 C6; C7 A7 B7; B8 A8 C8;</td>
</tr>
<tr>
<td>Bending Strength</td>
<td>A1 B1 C1; A2 B2 C2; A3 B3 C3; A4 B4 C4;</td>
</tr>
<tr>
<td></td>
<td>B5 A5 C5; C6 A6 B6; A7 B7 C7; B8 A8 C8;</td>
</tr>
<tr>
<td>Tension Modulus of Elasticity</td>
<td>A1 B1 C1; A2 B2 C2; A3 B3 C3; B4 A4 C4;</td>
</tr>
<tr>
<td></td>
<td>C5 B5 A5; C6 B6 A6; A7 B7 C7; B8 A8 C8;</td>
</tr>
<tr>
<td>Tension Strength</td>
<td>A1 B1 C1; A2 B2 C2; A3 B3 C3; A4 B4 C4;</td>
</tr>
<tr>
<td></td>
<td>C5 B5 A5; C6 B6 A6; A7 B7 C7; B8 C8 A8;</td>
</tr>
<tr>
<td>Compression Modulus of Elasticity</td>
<td>A1 B1 C1; A2 B2 C2; A3 B3 C3; A4 B4 C4;</td>
</tr>
<tr>
<td></td>
<td>B5 C5 A5; C6 B6 A6; B7 C7 A7; C8 B8 A8;</td>
</tr>
<tr>
<td>Compression Strength</td>
<td>A1 B1 C1; A2 B2 C2; A3 B3 C3; A4 B4 C4;</td>
</tr>
<tr>
<td></td>
<td>A5 B5 C5; A6 B6 C6; A7 C7 B7; A8 B8 C8;</td>
</tr>
<tr>
<td>Shear Modulus of Rigidity</td>
<td>C1 B1 A1; C2 A2 B2; C3 A3 B3; C4 A4 B4;</td>
</tr>
</tbody>
</table>

Note: Mean values of the underlined groups were not significantly different at 95% probability level.

The groups were arranged in descending order with respect to the mean value of each group.
Figure 35 Connectors load deformation curves.
Table 29. Summary of connection load slip tests.

<table>
<thead>
<tr>
<th>Group</th>
<th>Sample</th>
<th>Veneer</th>
<th>Specimen</th>
<th>Face Grain</th>
<th>Mean stiffness per connector (kN)</th>
<th>STDV stiffness per connector (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Size</td>
<td>Thickness</td>
<td>Thickness</td>
<td>Orientation</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(mm)</td>
<td>(mm)</td>
<td>(mm)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10A</td>
<td>5</td>
<td>2.5</td>
<td>28</td>
<td>parallel</td>
<td>4.157</td>
<td>0.614</td>
</tr>
<tr>
<td>08A</td>
<td>5</td>
<td>3.2</td>
<td>35</td>
<td>parallel</td>
<td>4.109</td>
<td>0.470</td>
</tr>
<tr>
<td>10E</td>
<td>5</td>
<td>2.5</td>
<td>28</td>
<td>Perpendicular</td>
<td>3.124</td>
<td>0.667</td>
</tr>
<tr>
<td>08E</td>
<td>5</td>
<td>3.2</td>
<td>35</td>
<td>Perpendicular</td>
<td>2.691</td>
<td>0.421</td>
</tr>
</tbody>
</table>

Table 30. Analysis of variance results and Duncan multiple range test results on connector stiffness.

<table>
<thead>
<tr>
<th>Source</th>
<th>Degree of Freedom</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Probability &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dependent Variable: Connector Stiffness</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>3</td>
<td>7.97518539</td>
<td>2.65839513</td>
<td>8.72</td>
<td>0.0012</td>
</tr>
<tr>
<td>Error</td>
<td>16</td>
<td>4.87620091</td>
<td>0.30476256</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>19</td>
<td>12.85138630</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Duncan multiple range test results: Group 08A 10A 08E 10E
groups were not statistically different at the 95% probability level. Similarly the mean stiffness values of the 10E and 08E groups were not statistically different at the 95% probability level. However the mean stiffness values between the parallel and perpendicular to face grain groups were statistically different.
5. FULL SCALE TESTING OF DRY FREIGHT VAN TRAILER DECK ASSEMBLY

This chapter describes the development of a Trailer Decking Load Simulator (TDLS) to test full scale deck assemblies. The structural behavior of full scale prototypes was evaluated using the TDLS in static and fatigue modes and the test results were used to verify DAP.

5.1 Testing Facility

Shown in Figure 36 is the TDLS designed and constructed to perform full scale testing of the prototype dry freight van trailer decking systems. The facility can test a 2.44 x 4.88 m (8 x 16 feet) dry freight van trailer deck assembly under either static or cyclic loading of up to 177.9 kN (40000 lb) which can simulate the range of loading expected from lift trucks on the dry freight van trailer decking. The TDLS consisted of four main components: a structural frame, a wheel cart apparatus, a vertical loading apparatus to apply simulated lift truck loading, and a lateral drive apparatus to cyclically pull the wheel cart along the long axis of the deck assembly. All forces were internally taken by the steel members in the structural frame which was designed with the criterion that the top beams deflect not more than 1 mm (0.045 inch) when a maximum load of 177.9 kN (40000 lb) was applied at midspan. The requirement was intended to allow proper tracking of the wheel cart apparatus when reacting against the top beams.

The loading systems were pneumatically driven by three compressors arranged in parallel: 1) a Powerrex 10 HP compressor Model CT 10312H, 2) a Ingersoll-Rand 10 HP compressor Model 71T2, and 3) a Devilbiss 5 HP compressor Model TAP5050. The system can deliver a maximum flow rate of $0.205 \text{ m}^3/\text{min.}$ ($7.23 \text{ ft}^3/\text{min.}$) for a pressure range of 861.8 to 1378.9 kPa (125 to 200 psi). A system of air valves and regulators was installed to control the pressure and flow rate for the various components in the loading system.

The vertical loading apparatus consisted of four Firestone two ply bellows air stoke actuators
Figure 36  The trailer decking load simulator.
Model IT19G-5. The capacity of each actuator was 57.8 kN (13000 lb) at 827.3 kPa (120 psi). The actuators required a flow rate of 0.17 m$^3$/min. (6 feet$^3$/min.) at 689 kPa (100 psi). The operating height of the actuators was 178 to 254 mm (7 to 10 inches); therefore, a stroke range of 76.2 mm (3 inches) was achieved. Within this stroke range, the actuator delivered a constant load of less than 1% difference in load at 689 kPa (100 psi). Therefore, in the cyclic tests of the decking assembly a constant load was applied to the deck while it underwent a range of deformation. A Interface Precision-Universal load cell (Model 1220) was installed between the vertical loading apparatus and the wheel cart to monitor the applied loads on the decking. The capacity of the load cell was 222 kN (50000 lb) and nonlinearity was expected when measuring loads at ±0.05% of full range. When pulling the wheel cart apparatus cyclically across the deck, extraneous lateral forces of up to 17.5 kN (3927 lb) might be applied to the load cell. The load cell can resist up to 88.8 kN (20000 lb) of extraneous lateral forces without damage to the electrical or mechanical components. These extraneous lateral forces can cause a maximum error of 0.1% in measuring the vertical loads.

The lateral drive apparatus consisted of a Greenco air cable cylinder Model CD50 108A-FTP. The cylinder had a diameter of 127 mm (5 inches) and a maximum stroke of 2.74 m (9 feet). It required a flow rate 0.71 m$^3$/min. (25 feet$^3$/min.) at a maximum air pressure of 1379 kPa (200 psi). The lateral driving system was designed to apply a maximum 5,000 cycles within 12 hours. A cycle is defined as the returned travel of the wheel cart front axle along the length of a 2.44 m (8 feet) long decking panel. Magnetic sensors were installed to detect the location of the wheel cart and to trigger the valving system for reversal of the airflow to change the direction of wheel cart. A cable extending from each end of the cylinder was connected to each end of the wheel cart. Each cable was pre-tensioned through a spring system with a maximum load 4.448 kN (1000 lb) to prevent fatigue of the cable system.

A wheel cart was designed and built such that the ratio between the front and rear wheel loadings was closed to 9:1. The wheel cart was connected to the loading apparatus through two 38 mm (1.5 inch) diameters pins. The pins were positioned such that the bottom part of the wheel cart
apparatus rotated freely about these moment free connections thus equal loading on both the front wheels was expected and equal loading on both the rear wheels was also expected after the wheel cart apparatus were properly aligned. The wheels were made by Industrial Tires Limited. The outside diameter, the width, and the inside diameter of the front wheel were 533, 229, and 381 mm (21, 9, and 15 inches), respectively. The front wheels were made from polyurethane with a load capacity of 53.4 kN (12000 lb) per wheel. The foot print of each front wheel was 203 x 89 mm (8 x 3.5 inches) which covered an area of 18064 mm$^2$ (28 inches$^2$). The outside diameter, the width, and the inside diameter of each rear wheel were 407, 127, and 267 mm (16, 5, 10.5 inches), respectively. It was made from SN rubber with a load capacity of 13744 kN (3090 lb) per wheel. Each rear wheel had a foot print of 89 x 80 mm (3.5 x 3.15 inches) which covered an area of 7113 mm$^2$ (11 inches$^2$).

The front and rear axles were spaced at a distance of 1.22 m (4 feet) apart. Both axles were oriented parallel to the direction of the supporting I-beams in the deck. The two front wheels were spaced at a distance of 965 mm (38 inches) apart and centered in the x-direction in the deck. Similarly, the rear wheels were centered in the x-direction of the deck and spaced at a distance of 749 mm (29.5 inches) apart.

The wheel cart also contained a system of guide wheels to ensure proper alignment and tracking of the wheel cart. A guide bushing apparatus was also installed to prevent shearing of the air actuators when pulling the cart laterally while allowing the proper transfer of vertical loads between the air actuators and the wheels.

Prior to testing, the loading mechanism was calibrated by supporting the wheel cart on three load cells while applying a load of up to 88.9 kN (20,000 lb) on the system. This calibration procedure was designed to establish: 1) the relationship of the load cell readings within the wheel cart against wheel loads; 2) the relationship between load cell readings within the wheel cart and pressure gauge readings; 3) the distribution of loads between the front and rear axles; and 4) the dead weight of the system. A special mounting device was built to attach two 45 kN (10,000 lb) capacity load cells
directly under the front axle. The rear wheels were supported temporarily by a steel channel which in
turn rested on a 222.4 kN (50,000 lb) capacity load cell. The dead weight of the system was measured
first. The air actuators were then activated to increase the total load in the system from its dead
weight to 88.9 kN (20,000 lb). The loading mechanism was fine tuned by adjusting the location of two
load transfer brackets so that a front to back wheel load distribution of approximately 9:1 was achieved
during loading of 80 kN (18,000 lb).

Results of the calibration are shown in Table 31. The results show the wheel cart assembly had
a dead weight of 14.63 kN (3289 lb) with a front to rear axles dead load distribution of approximately
7 to 3. Since the applied loads were positioned closed to the front axle, the front to rear axles load
distribution would shift as the load from the air actuators was increased. Shown in Figure 37 is the
relationship between the front to rear axles load ratio and the wheel cart load cell readings. Results
from regression analysis using a combined hyperbola and second order polynomial model is described as
follows:

\[ R_A = 0.2088 - 0.0032 \, TD + 0.000023 \, TD^2 + \frac{0.3815}{TD} \]  \[85\]

where \( R_A \) represents the front to rear axles load ratio and \( TD \) represents the wheel cart load cell
readings in kN. The coefficient of determination \( (r^2) \) of this relationship is 0.9999.

Figure 38 shows the relationship between the front axle loadings and wheel cart load cell
readings. This relationship can be described by a simple linear regression model as:

\[ F_R = 3.374 + 1.11688 \, TD \]  \[86\]

where \( F_R \) represents the front axle loadings in kN. The coefficient of determination \( (r^2) \) of this
relationship is 0.9999.
Table 31. Trailer decking load simulator calibration results.

<table>
<thead>
<tr>
<th>Pressure Gauge Readings (kPa)</th>
<th>Wheel Cart Load Cell Readings (kN)</th>
<th>Rear Axle Loading (kN)</th>
<th>Front Axle Loading (kN)</th>
<th>Total Load (kN)</th>
<th>Front to Rear Axles Load Ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.883</td>
<td>3.610</td>
<td>11.020</td>
<td>14.631</td>
<td>24.68</td>
</tr>
<tr>
<td>34.5</td>
<td>13.549</td>
<td>4.402</td>
<td>18.243</td>
<td>22.646</td>
<td>19.44</td>
</tr>
<tr>
<td>69.0</td>
<td>19.288</td>
<td>5.288</td>
<td>24.961</td>
<td>30.249</td>
<td>17.48</td>
</tr>
<tr>
<td>103.4</td>
<td>23.824</td>
<td>5.800</td>
<td>30.048</td>
<td>35.848</td>
<td>16.18</td>
</tr>
<tr>
<td>137.9</td>
<td>31.647</td>
<td>6.359</td>
<td>38.719</td>
<td>45.078</td>
<td>14.11</td>
</tr>
<tr>
<td>172.4</td>
<td>38.219</td>
<td>6.918</td>
<td>46.043</td>
<td>52.962</td>
<td>13.06</td>
</tr>
<tr>
<td>206.9</td>
<td>45.255</td>
<td>7.431</td>
<td>54.398</td>
<td>61.829</td>
<td>12.02</td>
</tr>
<tr>
<td>234.5</td>
<td>50.994</td>
<td>7.803</td>
<td>60.255</td>
<td>68.059</td>
<td>11.47</td>
</tr>
<tr>
<td>248.3</td>
<td>53.864</td>
<td>8.083</td>
<td>63.475</td>
<td>71.557</td>
<td>11.30</td>
</tr>
<tr>
<td>275.9</td>
<td>59.928</td>
<td>8.595</td>
<td>70.013</td>
<td>78.609</td>
<td>10.93</td>
</tr>
<tr>
<td>289.7</td>
<td>62.659</td>
<td>8.782</td>
<td>73.407</td>
<td>82.189</td>
<td>10.68</td>
</tr>
<tr>
<td>303.4</td>
<td>65.343</td>
<td>9.014</td>
<td>76.350</td>
<td>85.364</td>
<td>10.56</td>
</tr>
<tr>
<td>317.2</td>
<td>68.768</td>
<td>9.341</td>
<td>80.207</td>
<td>89.548</td>
<td>10.43</td>
</tr>
</tbody>
</table>
Figure 37  Relationship between the front to rear axles load ratios and wheel cart load cell readings.
Figure 38 Relationship between the front axle loadings and wheel cart load cell readings.
Clearly, \( R_A \) and \( F_R \) were successfully represented by the models shown in Equations 85 and 86, respectively. Therefore, from the wheel cart load cell readings, \( R_A \) and \( F_R \) can be evaluated to accurately estimate the loadings on both the front and rear wheels.

5.2 Materials and Methods

In the full scale test program, prototype deck assemblies were constructed on the structural steel frame and loaded by the TDLS apparatus. Each prototype deck assembly consisted of a seventeen bay steel deck frame, two full sheets of 1.22 x 2.44 m (4 x 8 feet) transDeck\textsuperscript{TM} panels and four half sheets of 1.22 x 1.22 m (4 x 4 feet) transDeck\textsuperscript{TM} panels. The transDeck\textsuperscript{TM} panels were mounted on the steel frame, 2.44 x 4.98 m (96 x 196 inches) in plan, with a pattern shown in Figure 39. The face grain of the panels was parallel to the long axis of the steel frame.

The steel frame was made with two C shape standard steel channels (C150x12) and 17 high strength light weight steel I-beams. The C channels were 152 mm (6 inches) deep and 4.98 m (196 inches) long. The weight and length of the steel I-beams were 46.7 \( \frac{N}{m} \) (3.2 \( \frac{lb}{in} \)) and 2432 mm (95.75 inches), respectively. The specification moduli of elasticity and yield strength of the I-beams were 200,000 MPa (29x10\textsuperscript{6} psi) and 550 MPa (80000 psi), respectively. The thickness of the flange \( (t_f) \) and the web \( (t_w) \) equaled 3.2 mm (0.125 inch). The depth \( (h_f) \) and the width \( (b_f) \) of the I-beam were 108 mm (4\( \frac{1}{4} \) inches) and 57 mm (2.25 inches), respectively. A steel connection plate, 3.2 mm (0.125 inch) thick, was welded to each end of a I-beam. The steel I-beams were spaced a distance of 305 mm (12 inches) on center and each end of a I-beam was bolted through the connection plate to the web of the channel with four symmetrically placed 9 mm (\( \frac{3}{8} \) inch) diameter bolts. The horizontal and vertical bolt spacings were 32 x 57 mm (1.25 and 2.25 inches), respectively. The I-beams were mounted to the C channels such that the top flange of each I-beam was leveled with the top flanges of the two channels. Each C channel rested on a 305 mm (12 inches) deep steel I-beam member in the TDLS structural steel frame. The C channels were not clamped to the structural steel frame and were free to rotate about
Figure 39  Prototype dry freight van trailer decking system used in the full scale test program.
their respective axes along the long direction when the prototype deck system was under load.

TransDeck™ panels were connected through the flanges of the I-beams using 8 mm (5/16 inch)-18 torx drive, flat head, type G, phosphate and oil coated self tapping screws. Two staggered rows of connectors were used in each I-beam. In each row, a total of twelve connectors were uniformly spaced at 203 mm (8 inch). The end distances of the connectors in each row were 51 and 152 mm (2 and 6 inches) from the two ends of a I-beam, respectively. The two rows of staggered connectors were centrally spaced a distance of 38 mm (1.5 inch) about the web. The stagger between the two rows of connectors was 102 mm (4 inch). Since it was not feasible to replace the steel I-beams after each test, repeated installation of self tapping screws through the I-beams was difficult without damaging the I-beam and altering the stiffness characteristics of the connection. A bolt and nut connection system was therefore used in the experimental program rather than relying solely on the self tapping screws. Slightly oversized pilot holes with a diameter of 8 mm (5/16 inch) rather than 7 mm (3/32 inch) were pre-drilled through the panels and the I-beams. The self tapping screws were inserted into the panels and through the flanges of the steel I-beams. Using the self tapping screw as a bolt, a nut was then applied onto each connector from the bottom of the deck. An air driven torque wrench without torque regulation was used to fasten the connector until its head was flush with the panel surface.

A special connection system was recommended by Ainsworth Lumber Company Ltd. for joining TransDeck™ panels along the perpendicular to face grain direction. In the prototype assembly, two such joints existed along Bays 5 and 13 which were the junctions between the full and half size panels. A connection steel band, 5 mm (3/16 inch) thick, 76 mm (3 inches) wide and 2.4 m (8 feet) long, was installed at the each junction. A 5 x 38 x 1220 mm (T6 x 1.5 x 48 inches) section was machined using a router from the top surface along one edge of each TransDeck™ panel where the full and half size panels were butted against each other. The steel band was fitted into this slot so that the top surfaces of the steel band and the original panel were flush. In each junction, two rows of connectors, 8 mm (5/16 inch)-18 torx drive self tapping screws, were uniformly spaced at 102 mm (4 inches) and
applied through the steel connection band, the panels and the I-beam. The end distance of the connectors was 51 mm (2 inches) from the each end of a I-beam. The two rows of connectors were centrally spaced a distance of 32 mm (1.25 inch) about the web. Standard pre-drilling and fastening procedures were followed during application of the self tapping screws and nut system.

The following 4 prototype dry freight van trailer decking assemblies built from 11-ply Douglas-fir veneer transDeck™ panels were considered in the full scale testing program:

1) built from regular 11-ply transDeck™ panels with 3.2 mm thick veneer;
2) built from regular 11-ply transDeck™ panels with 2.5 mm thick veneer;
3) built from special 11-ply transDeck™ panels with 3.2 mm thick veneer;
4) built from special 11-ply transDeck™ panels with 2.5 mm thick veneer.

All the veneers for the panels in the full scale testing program were graded using the Metriguard Model 2600 veneer grader. Similar to the veneer testing program, the veneers were divided into three groups based on their propagation time: group A (best quality material); group B (medium quality material); and group C (worst quality material). The regular 11-ply transDeck™ panels were made from veneers randomly and proportionally sampled from the three groups. In the special 11-ply transDeck™ panels, three exterior plies on each face were randomly selected from group A (the best quality veneer) while the interior veneers were randomly sampled from groups B and C. Seven replicates were considered in each of the two prototype decking assembly with regular transDeck™ panels. Only four replicates were available in each of the two prototype decking assembly with special transDeck™ panels.

Seventeen Duncan Model 606 linear motion position sensors were mounted under the decking system to monitor the vertical deformations in the system. Nine of the seventeen transducers were placed under the midspan of the supporting I-beams in Bays 5 to 13 of the prototype decking system. The other eight transducers monitored the vertical deformation at midspan of the transDeck™ panels. Each of these eight transducers were centrally mounted between the adjacent supporting I-beams and
at midspan in Bays 5 to 13. The transducers had a maximum travel of 152 mm (6 inches) and nonlinearity was expected when measuring deformations at ± 0.12% of full range. An eighteen channel (17 transducers and 1 load cell) data acquisition system was set up to amplify, condition and record the signals. Each channel recorded at a frequency of 300 readings per second.

5.2.1 Static test program

In each static test, the loading mechanism simulating the action of a lift truck with a front axle loading of up to 40 kN (9,000 lb) was used. The wheel cart was first lifted off the panels to obtain readings on the displacement transducers corresponding to zero load. The wheel cart was positioned at the following three locations in turn in the prototype seventeen bay decking system:

1) the front axle directly on top of Bay 9;
2) the front axle centered between Bays 6-7;
3) the front axle directly on top of the connection steel band in Bay 13.

At each of these loading positions, both axles in the wheel cart were centered across the width of the prototype deck system. When the wheel cart was positioned at the first loading position and the air bag actuators were pressurized with the loading in the front axle increased from its dead weight to 40 kN (9,000 lb). During the loading sequence, the load deformation characteristics of the prototype decking system were monitored and recorded using the eighteen channel data acquisition system. The wheel cart was then repositioned at the second loading position with the loading and data acquisition sequences repeated. This procedure was repeated until all three loading positions were considered. From the wheel cart load cell readings, the loading in each axle was estimated using the load distribution information collected during loading mechanism calibrations (Equations 85 and 86). The deformation profiles of the prototype decking system under various static load levels for a particular wheel cart location were therefore obtained. Figure 40 shows the prototype deck assembly during a typical static loading test.
Figure 40 Prototype dry freight van trailer decking system during a static test.
5.2.2 Cyclic test program

Since the number of available specimens in each prototype was limited, it was decided to characterize the fatigue behavior of the prototype test assemblies by obtaining information on a range of number of cycles to failure by testing the replicates at various load levels. Typically, the desired range of number of cycles to failure was between 10 to 3000 cycles. Testing at too low a load level can lead to an unpractically large number of cycles to failure. Contrary, testing at too high a load level may result in catastrophic failure under static load. With a new product such as transDeck™, limited information existed on the fatigue behavior; therefore, the choice of the load level for the first specimen was based on 60 to 70% of the estimated static load capacity of the prototype assembly. The load levels for subsequent tests were chosen based on the relationship between load level and number of cycles to failure information from previous tests.

After each set of static tests, the front axle of the wheel cart apparatus was positioned directly on Bay 9 the air bag actuators were pressurized while the load cell within the wheel assembly was monitored until the desired load level was reached. The applied load was maintained by closing the valves for the air bag actuators. Then the wheel cart apparatus was cycled along the long axis of the prototype decking assembly with near constant front and rear axle loadings. A half cycle of wheel cart travel is defined by the front wheels moving from the two half size panels over the connection steel band onto the adjacent full size panels and traveling 2.44 m (96 inches) along the long axis of the prototype assembly to the other end of the full size panels and moving off over the connection steel band onto the adjacent half size panels. The front wheels traversed a total distance of 2.54 m (100 inches) during each half cycle. Typically, the period of wheel cart travel was 18 seconds per cycle. The wheel cart was cycled until transDeck™ panels failure was encountered. A counter device was installed to monitor the total number of cycles of motion. The load and deformation of the prototype decking system during each cycle of movement were monitored by the eighteen channels data acquisition system. The maximum value in each channel monitored during a cycle was stored. The decking was
considered failed when punch through type failure was visually detected from the top surface. At this stage, the prototype assembly would have accumulated sufficient damage that system collapse would be imminent.

5.2.3 Short Term Small Specimen Bending Tests

One 102 x 406 mm (4 x 16 inches) specimen, with face grain oriented parallel to its long axis, was cut from each full size test panel evaluated in the cyclic test program. The specimen was taken from a bay where failure occurred at a location near the side of the prototype decking assembly where the estimated stresses experienced by the specimen during cyclic testing were minimal. The short-term bending strengths of these specimens were then tested under a center point load with simply support conditions at two ends obtained using roller bearing plates. A test span of 305 mm (12 inches) was chosen such that the span to depth ratio resembled the prototype decking assembly. A MTS model 810 hydraulic control close loop universal testing machine with a capacity of 222.4 kN (50000 lb) was used to apply the load in a deflection control mode. An uniform rate of cross head motion of 1.37 mm/ min. was used which resulted in specimen failure between 3 to 5 minutes of loading.

The specimen dimensions at mid-span were measured. A computer based data acquisition system was used to acquire the load versus deformation (cross head motion) data. A load cell with a 25 kN (5620 lb) capacity was used to monitor the loads. The time to failure information was also recorded. The bending strengths, based on the measured peak loads, were estimated according to Equations 76a and 76b, respectively.

5.3 Experimental Results

5.3.1 Static Test Program

Shown in Figure 41 are the experimentally measured midspan deformation profiles of the four prototype deck assemblies with the front axle located directly on Bay 9. The experimentally measured
Figure 41  Midspan deformation profiles of the four prototype deck assemblies with the front axle located directly on Bay 9.
midspan deformation profiles of the four prototype deck assemblies with the front axle centrally located between Bays 2 and 3 are shown in Figure 42. Finally shown in Figure 43 are the experimentally measured midspan deformation profiles of the four prototype deck assemblies with the front axle located directly on top of the steel band in Bay 13. For each loading position, the average and standard deviation of the peak midspan deflection in each prototype assembly under a front axle loading of 40 kN (8992 lb) are also shown in Table 32. The consistency of the deformation profiles indicates that the variability in the elastic properties within each prototype assembly was small. An increase in peak midspan deformation was noted when wheel cart apparatus was located at loading position 3 (front axle directly on top of the steel band). This increase was due to the discontinuity at Bays 5 and 13 where the half and full panels were connected to the steel I beams through the steel band. Apparently that the steel band could not completely compensate for the discontinuity at the joint by restricting panel rotation about its neutral axis over the supporting I beam; therefore, reduced assembly stiffness at the special connection was expected. The relationship between front axle load levels and peak midspan deformation is shown in Table 33 for the four prototype assemblies. The coefficient of determination values for the various cases range from 0.921 to 0.999 which indicate that linear load-deformation relationships under static front axle loading of up to 40 kN (8992 lb).

5.3.1.1 Verification of Deck Analysis Model:

The four prototype test assemblies were modeled using the DAP. The program predicted responses were compared to the experimental results for model verification. The modeling of the four prototype test assemblies required as input the elastic properties of the veneers in the transDeck™ panels. The axial and flexural moduli of elasticity in the x and y directions (\(E_x\), \(E_y\)) and the moduli of rigidity (G) of the veneers were obtained from the results of the veneer mechanical test program.

In prototypes 1 and 2, the individual veneers within a panel were assumed to have identical
Figure 42  Midspan deformation profiles of the four prototype deck assemblies with the front axle centered between Bays 2 and 3.
Figure 43  Midspan deformation profiles of the four prototype deck assemblies with the front axle located directly on Bay 13.
Table 32. Peak midspan deflections under 40 kN front axle loading.

<table>
<thead>
<tr>
<th>Prototype Loading</th>
<th>Peak Midspan Deflection</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Position</td>
<td>Experimental Results</td>
<td>Model Predictions</td>
<td>Prediction</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average (mm)</td>
<td>STDV (mm)</td>
<td>N</td>
<td>(mm)</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>10.393</td>
<td>0.233</td>
<td>7</td>
<td>10.147</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>10.175</td>
<td>0.073</td>
<td>7</td>
<td>10.037</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>11.691</td>
<td>0.744</td>
<td>7</td>
<td>10.267</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>11.938</td>
<td>0.103</td>
<td>7</td>
<td>11.871</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>11.418</td>
<td>0.364</td>
<td>7</td>
<td>11.672</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>12.283</td>
<td>0.345</td>
<td>7</td>
<td>11.931</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>9.914</td>
<td>0.052</td>
<td>4</td>
<td>9.770</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>9.879</td>
<td>0.031</td>
<td>4</td>
<td>9.689</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>10.931</td>
<td>0.107</td>
<td>4</td>
<td>9.918</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>11.713</td>
<td>0.213</td>
<td>4</td>
<td>11.559</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>11.070</td>
<td>0.075</td>
<td>4</td>
<td>11.385</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>12.690</td>
<td>0.468</td>
<td>4</td>
<td>11.635</td>
</tr>
</tbody>
</table>
Table 33. Front axle load levels versus peak midspan prototype deformation.

<table>
<thead>
<tr>
<th>Prototype Loading Position</th>
<th>Load (kN) = a Deformation (mm) + b</th>
<th>a (kN)</th>
<th>b (mm)</th>
<th>r²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 1</td>
<td>3.98</td>
<td>-1.21</td>
<td>0.996</td>
<td></td>
</tr>
<tr>
<td>2 3</td>
<td>4.08</td>
<td>-1.37</td>
<td>0.999</td>
<td></td>
</tr>
<tr>
<td>3 1</td>
<td>3.35</td>
<td>1.03</td>
<td>0.973</td>
<td></td>
</tr>
<tr>
<td>4 2</td>
<td>3.48</td>
<td>-1.36</td>
<td>0.999</td>
<td></td>
</tr>
<tr>
<td>2 2</td>
<td>3.63</td>
<td>-1.26</td>
<td>0.993</td>
<td></td>
</tr>
<tr>
<td>3 3</td>
<td>3.27</td>
<td>0.33</td>
<td>0.987</td>
<td></td>
</tr>
<tr>
<td>3 1</td>
<td>3.73</td>
<td>-0.11</td>
<td>0.994</td>
<td></td>
</tr>
<tr>
<td>2 2</td>
<td>4.38</td>
<td>-3.09</td>
<td>0.996</td>
<td></td>
</tr>
<tr>
<td>3 3</td>
<td>4.34</td>
<td>-2.90</td>
<td>0.999</td>
<td></td>
</tr>
<tr>
<td>4 1</td>
<td>3.56</td>
<td>-1.55</td>
<td>0.998</td>
<td></td>
</tr>
<tr>
<td>2 2</td>
<td>3.84</td>
<td>-2.19</td>
<td>0.998</td>
<td></td>
</tr>
<tr>
<td>3 3</td>
<td>3.11</td>
<td>1.22</td>
<td>0.921</td>
<td></td>
</tr>
</tbody>
</table>
elastic properties. Here, \( E_{y_f} \) and \( E_{x_f} \) were obtained from the veneer bending test results. \( E_{y_a} \) and \( E_{x_a} \) were obtained from the veneer compression test results rather than the veneer tensile test results. This is because accurate measurement of veneer tensile elastic properties in the perpendicular to grain direction was difficult. For the parallel to grain direction, the mean elastic moduli of the 3-plies and 4-plies specimens were averaged for each veneer thickness. For the perpendicular to grain direction, the mean elastic moduli of the 3-plies and 4-plies specimens with various veneer thickness were averaged. The modulus of rigidity properties (\( G \)) were obtained from the veneer shear modulus of rigidity test results where the mean rigidity moduli of the 3-plies and 4-plies specimens were averaged for each veneer thickness.

In prototypes 3 and 4, the three exterior plies on the face of each panel were made with veneer from subgroup A (best quality veneer) whereas the interior plies were made from subgroups B and C; therefore, the elastic properties of the exterior and interior plies were different. Using the mean elastic properties results of the various subgroups shown in Table 27, \( E_{y_f} \), \( E_{x_f} \), \( E_{y_a} \), \( E_{x_a} \), and \( G \) for the interior and exterior plies of panels in prototypes 3 and 4 were obtained. Table 34 shows the input veneer elastic properties for the four prototype assemblies. The Poisson’s ratio \( \nu_{xy} \) and \( \nu_{yx} \) of the veneers were taken as 0.02 and 0.4 respectively. The veneer thickness for prototypes 1 and 3 were taken as 3.2 mm (0.125 inch) and for prototypes 2 and 4 were taken as 2.5 mm (0.1 inch). The stiffnesses of a panel in the cover were obtained from Equations 74a to 74h for the various prototypes and the results are shown in Table 35.

A seventeen bay deck system was modeled. The moduli of elasticity and rigidity of the supporting I-beams in the deck system were taken as 200,000 MPa (29x10^6 psi) and 77,000 MPa (11x10^6 psi), respectively. Also the supporting I-beams were assumed to have a yield strength of 550 MPa (80,000 psi). The thickness of the flange (\( t_f \)) and the web (\( t_w \)) equaled 3.2 mm (0.125 inch). The depth (\( h_f \)) and the width (\( b_f \)) of the I-beam were 102 mm (4 inches) and 57 mm (2.25 inches), respectively. The connectors between the cover and the supporting I-beams were considered uniformly
Table 34. Veneer elastic moduli for DAP analyses.

<table>
<thead>
<tr>
<th>Prototype</th>
<th>$E_{yf}$ (MPa)</th>
<th>$E_{ya}$ (MPa)</th>
<th>$E_{xf}$ (MPa)</th>
<th>$E_{xa}$ (MPa)</th>
<th>G (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exterior</td>
<td>11566</td>
<td>11566</td>
<td>9828</td>
<td>9828</td>
<td>430.9</td>
</tr>
<tr>
<td>Interior</td>
<td>11566</td>
<td>11566</td>
<td>9828</td>
<td>9828</td>
<td>488.2</td>
</tr>
<tr>
<td>Exterior</td>
<td>12958</td>
<td>12958</td>
<td>11622</td>
<td>11622</td>
<td>430.9</td>
</tr>
<tr>
<td>Interior</td>
<td>12958</td>
<td>12958</td>
<td>11622</td>
<td>11622</td>
<td>488.2</td>
</tr>
<tr>
<td>Exterior</td>
<td>14079</td>
<td>10681</td>
<td>12508</td>
<td>8932</td>
<td>430.9</td>
</tr>
<tr>
<td>Interior</td>
<td>14079</td>
<td>10681</td>
<td>12508</td>
<td>8932</td>
<td>488.2</td>
</tr>
<tr>
<td>Exterior</td>
<td>14843</td>
<td>12246</td>
<td>13927</td>
<td>10653</td>
<td>430.9</td>
</tr>
<tr>
<td>Interior</td>
<td>14843</td>
<td>12246</td>
<td>13927</td>
<td>10653</td>
<td>488.2</td>
</tr>
</tbody>
</table>

Table 35. Input panel stiffness values for DAP analyses.

<table>
<thead>
<tr>
<th>Prototype</th>
<th>$K_x$ (kN·m)</th>
<th>$K_y$ (kN·m)</th>
<th>$K_{\nu}$ (kN·m)</th>
<th>$K_G$ (MN·m)</th>
<th>$D_x$ (MN·m)</th>
<th>$D_y$ (MN·m)</th>
<th>$D_{\nu}$ (MN·m)</th>
<th>$D_G$ (MN·m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.48</td>
<td>38.46</td>
<td>1.25</td>
<td>2.59</td>
<td>76.97</td>
<td>286.22</td>
<td>25.45</td>
<td>25.46</td>
</tr>
<tr>
<td>2</td>
<td>2.48</td>
<td>22.05</td>
<td>0.71</td>
<td>1.43</td>
<td>70.77</td>
<td>270.32</td>
<td>24.03</td>
<td>22.04</td>
</tr>
<tr>
<td>3</td>
<td>4.24</td>
<td>46.54</td>
<td>1.15</td>
<td>2.59</td>
<td>71.24</td>
<td>329.07</td>
<td>23.15</td>
<td>25.46</td>
</tr>
<tr>
<td>4</td>
<td>2.38</td>
<td>25.16</td>
<td>0.68</td>
<td>1.43</td>
<td>65.80</td>
<td>298.29</td>
<td>22.05</td>
<td>22.04</td>
</tr>
</tbody>
</table>

spaced at 102 mm (4 inches) on center. The connector stiffnesses $K_{yc}$, $K_{xc}$, and $K_{\theta c}$ were assumed to equal $4.133 \frac{MN}{m}$ (23600 lb/in.), $2.908 \frac{MN}{m}$ (16600 lb/in.), and $3.502 \frac{MN}{m}$ (20000 lb/in.), respectively. The $K_{yc}$ and $K_{xc}$ values were obtained by taking the average of the mean stiffness values of groups 10A and 08A and groups 10E and 08E from Table 29, respectively.

Four levels of front axle wheel loads were considered: 40 kN (8992 lb), 30 kN (6775 lb), 20 kN (4496 lb), and 10 kN (2248 lb). In the prototype test assemblies the front and rear axle load
distribution depended on the load level and Equation 85 was used to estimate the load distribution and the loading on the rear wheel for the various load levels. The front and rear axles, oriented parallel to the direction of the supporting I-beams, were spaced at a distance of 1.22 m (4 feet) apart. Three wheel locations corresponding to the static tests were also considered. The foot print of each front wheel was assumed to be 203 x 89 mm (8 x 3.5 inches) which covered an area of 18064 mm² (28 inches²). Each rear wheel has a foot print of 89 x 80 mm (3.5 x 3.15 inches) which covered an area of 7113 mm² (11 inches²). The two front wheels were spaced at a distance of 965 mm (38 inches) apart and centered in the x-direction in the deck. Similarly, the rear wheels were centered in the x-direction of the deck and spaced at a distance of 749 mm (29.5 inches) apart.

Shown in Figure 44 are the comparisons of the model predicted response and the experimentally measured midspan deformation profiles of the four prototype deck assemblies with the front axle located directly on Bay 9. Similar comparisons between model predicted and experimentally measured midspan deformation profiles of the four prototype deck assemblies with the front axle centrally located between Bays 2 and 3 are shown in Figure 45. Finally shown in Figures 46 are the comparisons of model predictions and experimentally measured midspan deformation profiles of the four prototype deck assemblies with the front axle located directly on top of the steel band in Bay 13.

Good agreement between model predictions and experimentally measured response was obtained. DAP could predict the system response for a region within 3 Bays of the front axle reasonably well. When considering the system response at a distance from the front wheel load, the level of agreement between model predictions and measured responses reduces. However, at these locations, small deformations were found; therefore, the lack of model agreement was considered relatively insignificant.

Also shown in Table 32 are the model predicted peak midspan deflection under a load of 40 kN (8992 lb) for each loading position. At 40 kN (8992 lb) front axle loading, a maximum prediction error of 2.76 % was found for loading positions 1 and 2. A maximum prediction error of 12.18 % was found
Figure 44 Comparisons of model predictions and measured midspan deformation profiles of the four prototype deck assemblies with the front axle located directly on Bay 9.
Figure 45 Comparisons of model predictions and measured midspan deformation profiles of the four prototype deck assemblies with the front axle centered between Bays 2 and 3.
Figure 46  Comparisons of model predictions and measured midspan deformation profiles of the four prototype deck assemblies with the front axle located directly on Bay 13.
for loading position 3. This error can be attributed to the inability of DAP to consider the 
discontinuity and the special connection steel bands in Bays 5 and 9. Based on the low prediction errors 
in estimating peak deflections and reasonable agreement between predicted and measured deformation 
profiles, DAP was considered successfully verified.

Using the input parameters for the four prototypes, DAP was used to estimate the maximum 
bending stress of the exterior ply in the panel (σ_Y) considering \( \frac{1}{2} \) cycle of wheel cart travel with a front 
axle load level of 73.39 kN (16500 lb) and a front to rear axles load ratio of 9:1. The maximum σ_Y was 
found to occur at under the foot print of the front wheels when the front axle was centrally located 
between Bays 5 and 6. The maximum σ_Y was found to be 43.22 MPa, 67.15 MPa, 52.55 MPa, and 
76.05 MPa for prototypes 1 to 4, respectively.

The variability in the measured deformation profiles can be examined by considering the 
variability in the veneer elastic properties shown in Table 17, 19 and 20. For each prototype, the elastic 
moduli (\( E_{y_r} \), \( E_{y_a} \), \( E_{x_f} \), \( E_{x_a} \), and G) for 11000 sheets of veneer were randomly simulated assuming 
normal distributions and perfect correlation in the rank of the various elastic moduli within each sheet 
of veneer. Panel stiffness values for 1000 replicates of each prototype were computed according to 
Equations 74a to 74h with their statistics shown in Table 36. The mean panel stiffness values from the 
simulation study agreed with the input panel stiffness values for DAP shown in Table 35.

Two sets of upper and lower bound panel stiffness values were obtained as: the mean panel 
stiffness ± 2 x the standard deviation of panel stiffness. Each of the two sets of panel stiffness values 
was used as input into DAP to approximate the upper and lower bounds of the deformation profiles. 
Figure 47 shows the deformation bounds for prototype 1 with the front axle located directly on Bay 9. 
The measured and DAP predicted deformation profiles were within the estimated deformation bounds. 
Similar results were obtained for other prototypes and other loading conditions.
Table 36. Statistics of panel stiffness values for DAP analyses.

<table>
<thead>
<tr>
<th>Prototype</th>
<th>$K_x$ (kN·m)</th>
<th>$K_y$ (kN·m)</th>
<th>$K_\nu$ (kN·m)</th>
<th>$K_G$ (kN·m)</th>
<th>$D_x$ (MN·m)</th>
<th>$D_y$ (MN·m)</th>
<th>$D_\nu$ (MN·m)</th>
<th>$D_G$ (MN·m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Values</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4.48</td>
<td>38.34</td>
<td>1.25</td>
<td>2.58</td>
<td>77.31</td>
<td>285.62</td>
<td>25.59</td>
<td>25.23</td>
</tr>
<tr>
<td>2</td>
<td>2.48</td>
<td>22.00</td>
<td>0.72</td>
<td>1.43</td>
<td>71.07</td>
<td>269.77</td>
<td>24.16</td>
<td>21.84</td>
</tr>
<tr>
<td>3</td>
<td>4.25</td>
<td>46.42</td>
<td>1.16</td>
<td>2.58</td>
<td>71.52</td>
<td>328.29</td>
<td>23.27</td>
<td>25.23</td>
</tr>
<tr>
<td>4</td>
<td>2.39</td>
<td>25.09</td>
<td>0.68</td>
<td>1.43</td>
<td>66.08</td>
<td>297.59</td>
<td>22.16</td>
<td>21.84</td>
</tr>
<tr>
<td>Standard Deviations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.45</td>
<td>3.20</td>
<td>0.17</td>
<td>0.13</td>
<td>12.05</td>
<td>25.54</td>
<td>4.82</td>
<td>3.28</td>
</tr>
<tr>
<td>2</td>
<td>0.23</td>
<td>1.63</td>
<td>0.09</td>
<td>0.07</td>
<td>10.93</td>
<td>23.19</td>
<td>4.37</td>
<td>2.93</td>
</tr>
<tr>
<td>3</td>
<td>0.39</td>
<td>3.26</td>
<td>0.14</td>
<td>0.13</td>
<td>10.58</td>
<td>24.56</td>
<td>4.23</td>
<td>3.28</td>
</tr>
<tr>
<td>4</td>
<td>0.23</td>
<td>1.80</td>
<td>0.08</td>
<td>0.07</td>
<td>10.09</td>
<td>22.92</td>
<td>4.04</td>
<td>2.93</td>
</tr>
</tbody>
</table>
Figure 47 Upper and lower bounds of measured midspan deformation profiles of prototype 1 with the front axle located directly on Bay 9.
5.3.2 Cyclic Test Program

Table 37 shows the applied front axle load level ($F_R$) and the number of cycles to failure ($N_f$) information for the four prototypes. Since it was difficult to maintain a completely constant load level during testing, the mean and standard deviation of the applied load were calculated and shown in Table 37. The results indicate that the applied loads were near constant during each test. A maximum standard deviation of 1.065 kN can be found in specimen 4 of prototype 2 which was caused by technical problems in the cable cylinders.

The failure locations in the panels are also shown in Table 37. It can be noted that the failure occurred only under the foot print of the front wheels. Over 50% of the time, failure occurs in between either Bays 5 and 6 or Bays 12 and 13 in the prototype systems. These locations were adjacent to the two steel connection bands in Bays 5 and 13 where discontinuity in the panel existed. Therefore, these frequent failure location should be considered as critical zones in the prototype systems.

The final failure mode was punch through type failure visible from upper surface of the panel. The stiffness of the prototype system was significantly reduced after punch through type failures occurred. Therefore, at this stage the tests were terminated before the final collapse of the system where permanent damage to the steel beams might result. However, prior to punch through type failure being visibly detected, a series of secondary failure modes such as tension parallel and perpendicular to grain failures in the exterior plies and rolling shear type failure in the interior plies were detected. Although the stiffness of the prototype system was reduced when secondary failures occurred, the overall load carrying capacity of the system remained available.

The midspan deformation information monitored by the seventeen transducers during cyclic testing showed a progressive increase of midspan deformation as the deck undergoes creep and accumulates damage under the cyclic load. The ratio between the initial and final midspan deformation recorded by each transducer ($R_d$) was calculated. The maximum $R_d$ in each specimen is shown in Table 37 which ranged from 1.233 to 1.066. This information was originally intended to signal the
Table 37. Cyclic test results of full scale prototypes.

<table>
<thead>
<tr>
<th>Prototype Specimen Number</th>
<th>FR Mean (kN)</th>
<th>FR STDV (kN)</th>
<th>Nf Mean</th>
<th>Maximum Rd (kN)</th>
<th>Failure Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>72.248</td>
<td>0.904</td>
<td>10</td>
<td>1.127</td>
<td>5 &amp; 6</td>
</tr>
<tr>
<td>2</td>
<td>70.249</td>
<td>0.596</td>
<td>10</td>
<td>1.104</td>
<td>12 &amp; 13</td>
</tr>
<tr>
<td>3</td>
<td>55.160</td>
<td>0.480</td>
<td>265</td>
<td>1.123</td>
<td>8 &amp; 9, 10 &amp; 11</td>
</tr>
<tr>
<td>4</td>
<td>55.473</td>
<td>0.122</td>
<td>313</td>
<td>1.208</td>
<td>7 &amp; 8, 11 &amp; 12</td>
</tr>
<tr>
<td>5</td>
<td>44.346</td>
<td>0.182</td>
<td>1517</td>
<td>1.233</td>
<td>9 &amp; 10, 11 &amp; 13</td>
</tr>
<tr>
<td>6</td>
<td>42.137</td>
<td>0.102</td>
<td>1434</td>
<td>1.145</td>
<td>7 &amp; 8</td>
</tr>
<tr>
<td>7</td>
<td>39.918</td>
<td>0.098</td>
<td>5436</td>
<td>1.160</td>
<td>5 &amp; 6</td>
</tr>
<tr>
<td>2</td>
<td>66.793</td>
<td>0.734</td>
<td>3</td>
<td>1.100</td>
<td>6 &amp; 8</td>
</tr>
<tr>
<td>3</td>
<td>60.404</td>
<td>0.290</td>
<td>10</td>
<td>1.066</td>
<td>7 &amp; 8, 12 &amp; 13</td>
</tr>
<tr>
<td>4</td>
<td>55.499</td>
<td>0.161</td>
<td>47</td>
<td>1.098</td>
<td>12 &amp; 13</td>
</tr>
<tr>
<td>5</td>
<td>49.317</td>
<td>1.065</td>
<td>92</td>
<td>1.169</td>
<td>12 &amp; 13</td>
</tr>
<tr>
<td>6</td>
<td>37.828</td>
<td>0.092</td>
<td>1226</td>
<td>1.146</td>
<td>12 &amp; 13</td>
</tr>
<tr>
<td>7</td>
<td>35.497</td>
<td>0.162</td>
<td>2536</td>
<td>1.119</td>
<td>7 &amp; 10</td>
</tr>
<tr>
<td>3</td>
<td>35.493</td>
<td>0.083</td>
<td>2198</td>
<td>1.119</td>
<td>11 &amp; 12</td>
</tr>
<tr>
<td>2</td>
<td>60.234</td>
<td>0.115</td>
<td>20</td>
<td>1.094</td>
<td>8 &amp; 9</td>
</tr>
<tr>
<td>3</td>
<td>55.267</td>
<td>0.064</td>
<td>86</td>
<td>1.082</td>
<td>5 &amp; 6</td>
</tr>
<tr>
<td>4</td>
<td>48.707</td>
<td>0.126</td>
<td>223</td>
<td>1.112</td>
<td>9 &amp; 10</td>
</tr>
<tr>
<td>5</td>
<td>44.419</td>
<td>0.139</td>
<td>214</td>
<td>1.128</td>
<td>9 &amp; 10</td>
</tr>
<tr>
<td>4</td>
<td>55.566</td>
<td>0.537</td>
<td>7</td>
<td>1.095</td>
<td>5 &amp; 6</td>
</tr>
<tr>
<td>2</td>
<td>55.270</td>
<td>0.118</td>
<td>40</td>
<td>1.089</td>
<td>12 &amp; 13</td>
</tr>
<tr>
<td>3</td>
<td>48.830</td>
<td>0.131</td>
<td>96</td>
<td>1.095</td>
<td>5 &amp; 6, 12 &amp; 13</td>
</tr>
<tr>
<td>4</td>
<td>44.296</td>
<td>0.076</td>
<td>180</td>
<td>1.107</td>
<td>6 &amp; 7, 8 &amp; 9, 11 &amp; 12</td>
</tr>
</tbody>
</table>
initiation of failure and provide a guide to the termination fatigue test. The relatively wide range in maximum \( R_d \) indicated the insensitivity of this parameter to the occurrence of final failure and made it unsuitable to be considered as a signal of the onset of final failure.

The relationships between \( F_R \) and \( N_f \) for the four prototype assemblies can be given by:

\[
F_R = b_{0i} + b_{1i} \log_{10} (N_f) \quad (i = 1,...,4) \quad [87]
\]

where \( b_{0i} \) and \( b_{1i} \) denote the slope and intercept of the relationships for the \( i \)th prototype, respectively.

Figure 48 shows a plot of the relationships between \( F_R \) and \( N_f \) (in \( \log_{10} \) scale) for the four prototype assemblies. Table 38 shows the \( b_{1i} \), \( b_{0i} \), and \( r^2 \) values for the various relationships. The \( r^2 \) values for the relationships between \( F_R \) and \( N_f \) (in \( \log_{10} \) scale) ranged from 0.75 to 0.99 which clearly indicated Equation 87 successfully represented the fatigue performance of transDeck™. It was noted that prototypes 3 and 4 had lower \( r^2 \) values compared to prototypes 1 and 2 which was attributed to the narrower range of load levels considered in the cyclic testing program for prototypes 3 and 4.

Table 38. Regression parameters for the \( F_R \) and \( N_f \) relationships of the four prototype assemblies.
Figure 48 Performance of transDeck™ under fatigue loading.
The static strength of a prototype assembly can be defined as the maximum front axle load level which the panel can carry for $\frac{1}{2}$ cycle of wheel cart travel. Shown in Table 38 are the estimated mean static capacity ($F_{R}$) and the estimated mean load capacity at 3000 cycles of wheel cart travel ($F_{R_{3000}}$) for each prototype using the relationships in Equation 87.

The relationships between $F_R$ and $N_f$ (in log$_{10}$ scale) for the four prototype assemblies can be tested using analysis of covariance to check whether their slopes can be considered statistically equal. Here, regression approach to analysis of covariance was used in which the full and reduced regression models for the relationships between $F_R$ and $N_f$ are respectively given by:

$$F_R = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_1 X_2 + b_4 Z + b_5 X_1 Z + b_6 X_2 Z$$ \[88\]

and

$$F_R = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_1 X_2 + b_4 Z$$ \[89\]

where $Z$ is the covariate which equals $(\log_{10} (N_f) - \log_{10} (N_0))$ and the indicator variables $X_1$ and $X_2$ for the various treatments were assigned the (-1, 0, 1) scheme as shown in Table 38.

The equality of slopes between treatments were checked using a partial F test that there was no interaction between the covariate and treatment. The partial F between the full and reduced models is given by:

$$\text{Partial F} = \frac{(SS_{R_f} - SS_{R_r}) \over DF_f}{2 SS_{E_f}} = 1.67$$ \[90\]

where $SS_{R_r}$ equals the sum of squares of regression of the reduced model; $SS_{R_f}$, $SS_{E_f}$ and $DF_f$ equals the sum of squares of regression, sum of squares of error and degree of freedom of error of the full model, respectively. The test statistics $F_c = F_{2, 15; 1-\alpha} = 3.68$ for $\alpha=0.05$. Since partial $F < F_c$, it was concluded that there was interaction between treatments and covariate; therefore, the slopes of the
relationship between $F_R$ and $N_f$ (in log$_{10}$ scale) between various treatment groups were considered statistically equal at the 95% probability level. The results are summarized in Table 39.

Table 39. Results of regression approach to analysis of covariance of the $F_R$ and $N_f$ relationships.

<table>
<thead>
<tr>
<th>Model in Equation</th>
<th>$b_0$</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>$b_4$</th>
<th>$b_5$</th>
<th>$b_6$</th>
<th>DF</th>
<th>SS</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>88</td>
<td>51.14</td>
<td>3.232</td>
<td>2.354</td>
<td>1.352</td>
<td>-10.798</td>
<td>-0.870</td>
<td>-0.716</td>
<td>15</td>
<td>2281.127</td>
<td>74.434</td>
</tr>
<tr>
<td>89</td>
<td>50.81</td>
<td>3.359</td>
<td>2.509</td>
<td>1.044</td>
<td>-11.267</td>
<td></td>
<td></td>
<td>17</td>
<td>2264.518</td>
<td>91.043</td>
</tr>
<tr>
<td>91</td>
<td>50.81</td>
<td>3.644</td>
<td>2.510</td>
<td></td>
<td>-11.272</td>
<td></td>
<td></td>
<td>18</td>
<td>2242.309</td>
<td>113.25</td>
</tr>
<tr>
<td>93</td>
<td>50.85</td>
<td>2.315</td>
<td>1.939</td>
<td>-10.680</td>
<td></td>
<td></td>
<td></td>
<td>18</td>
<td>2040.755</td>
<td>314.81</td>
</tr>
<tr>
<td>94</td>
<td>51.50</td>
<td>3.262</td>
<td></td>
<td>1.047</td>
<td>-10.621</td>
<td></td>
<td></td>
<td>18</td>
<td>2143.837</td>
<td>211.72</td>
</tr>
<tr>
<td>97</td>
<td>51.51</td>
<td>1.674</td>
<td>-0.060</td>
<td>1.091</td>
<td></td>
<td></td>
<td></td>
<td>18</td>
<td>109.7751</td>
<td>2245.8</td>
</tr>
</tbody>
</table>

Interaction between the veneer thickness effect and the effect of ultrasonic graded special veneers was checked using partial F test. Here the regression model given in Equation 89 was treated as the full model and the following regression model was treated as the reduced model:

$$F_R = b_0 + b_1 X_1 + b_2 X_2 + b_4 Z$$  \[91\]

The results are summarized in Table 39. The partial F between the full and reduced models is given by:

$$\text{Partial F} = \frac{(SS_{R_f} - SS_{R_r})}{SS_{E_f}} \frac{DF_f}{DF_r} = 4.15$$  \[92\]

The test statistics $F_c = F_{1, 17; 1-\alpha} = 4.45$ for $\alpha=0.05$. Since partial $F < F_c$, it was concluded that there
was no interaction between treatments.

The veneer thickness effect and the effect of ultrasonics graded special veneers were also checked using partial F tests. Again the regression model given in Equation 89 was treated as the full model and the following regression models were treated as the reduced models for veneer thickness effect and the effect of ultrasonic graded special veneers, respectively:

\[ F_R = b_0 + b_2 X_2 + b_3 X_1 X_2 + b_4 Z \]  
\[ \text{[93]} \]

and

\[ F_R = b_0 + b_1 X_1 + b_3 X_1 X_2 + b_4 Z \]  
\[ \text{[94]} \]

The results are summarized in Table 39. The partial F values between the full and reduced models for Equation 93 and 94 are respectively given by:

\[ \text{Partial } F = \frac{(SS_{R_f} - SS_{R_f})}{SS_{E_f}} \frac{DF_f}{DF_f} = 41.78 \]  
\[ \text{[95]} \]

\[ \text{Partial } F = \frac{(SS_{R_f} - SS_{R_f})}{SS_{E_f}} \frac{DF_f}{DF_f} = 22.53 \]  
\[ \text{[96]} \]

The test statistics \( F_c = F_{1, 17; 1-\alpha} = 4.45 \) for \( \alpha=0.05 \). Since partial \( F > F_c \) in both cases, it was concluded that both treatment effects were significant.

Finally the justification of using analysis of covariance was considered by checking the effect of covariate using a partial F test. Again the regression model given in Equation 89 was treated as the full model and the following regression model was treated as the reduced model for covariate effect:

\[ F_R = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_1 X_2 \]  
\[ \text{[97]} \]
The results are summarized in Table 39. The partial F value between the full and reduced models for Equation 97 is given by:

$$\text{Partial F} = \frac{(SSR_f - SSR_r) \cdot DF_f}{SS_E_f} = 402.35$$ [98]

The test statistics $F_c = F_{1, 17; 1-\alpha} = 4.45$ for $\alpha=0.05$. Since partial $F > F_c$, it was concluded that covariate effect was significant; therefore, analysis of covariance was justified.

The relationship between $F_R$ and $N_f$ (in log\(_{10}\) scale) for each prototype assembly was normalized with respect to its estimated static capacity $F_s$ to yield information on applied stress ratio ($S_R$) to number of cycles to failure. The relationships between $S_R$ and $N_f$ (in log\(_{10}\) scale) for the four prototype assemblies were tested using analysis of covariance to check whether their slopes can be considered statistically equal. Again regression approach to analysis of covariance was used in which the full and reduced regression models for the relationships between $S_R$ and $N_f$ are respectively given by:

$$S_R = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_1 X_2 + b_4 Z + b_5 X_1 Z + b_6 X_2 Z$$ [99]

and

$$S_R = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_1 X_2 + b_4 Z$$ [100]

The partial F between the full and reduced models is given by:

$$\text{Partial F} = \frac{(SSR_f - SSR_r) \cdot DF_f}{2 \cdot SS_E_f} = 0.43$$ [101]

The test statistics $F_c = F_{2, 15; 1-\alpha} = 3.68$ for $\alpha=0.05$. Since partial $F < F_c$, it can be concluded that there was interaction between treatments and covariate; therefore, the slopes of the relationship...
between $S_R$ and $N_f$ (in $\log_{10}$ scale) between various treatment groups were statistically equal at the 95% probability level. The results are summarized in Table 40.

Table 40. Results of regression approach to analysis of covariance of the $S_R$ and $N_f$ relationships.

<table>
<thead>
<tr>
<th>Model in Equation</th>
<th>$b_0$</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>$b_4$</th>
<th>$b_5$</th>
<th>$b_6$</th>
<th>DF$_f$</th>
<th>SS$_R$</th>
<th>SS$_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>99</td>
<td>0.697</td>
<td>-0.012</td>
<td>-0.040</td>
<td>0.018</td>
<td>-0.146</td>
<td>0.007</td>
<td>0.005</td>
<td>15</td>
<td>0.45709</td>
<td>0.01552</td>
</tr>
<tr>
<td>100</td>
<td>0.698</td>
<td>-0.012</td>
<td>-0.040</td>
<td>0.019</td>
<td>-0.143</td>
<td></td>
<td></td>
<td>17</td>
<td>0.45620</td>
<td>0.01641</td>
</tr>
<tr>
<td>102</td>
<td>0.699</td>
<td>-0.007</td>
<td>-0.041</td>
<td>0.019</td>
<td>-0.140</td>
<td></td>
<td></td>
<td>18</td>
<td>0.44906</td>
<td>0.02354</td>
</tr>
<tr>
<td>103</td>
<td>0.698</td>
<td>-0.040</td>
<td>0.016</td>
<td>-0.143</td>
<td></td>
<td></td>
<td></td>
<td>18</td>
<td>0.45337</td>
<td>0.01923</td>
</tr>
<tr>
<td>104</td>
<td>0.687</td>
<td>-0.012</td>
<td>0.021</td>
<td>-0.153</td>
<td></td>
<td></td>
<td></td>
<td>18</td>
<td>0.42566</td>
<td>0.04695</td>
</tr>
<tr>
<td>105</td>
<td>0.707</td>
<td>-0.011</td>
<td>-0.072</td>
<td>-0.002</td>
<td></td>
<td></td>
<td></td>
<td>18</td>
<td>0.11015</td>
<td>0.36245</td>
</tr>
</tbody>
</table>

Interaction between the veneer thickness effect and the effect of ultrasonic graded special veneers was checked by considering the regression model given in Equation 100 as the full model and the following regression model as the reduced model:

$$S_R = b_0 + b_1 X_1 + b_2 X_2 + b_4 Z$$  \[102\]

The results are summarized in Table 40. The partial $F$ between the full and reduced models was 7.39. The test statistics $F_c = F_{1, 17; 1-\alpha} = 4.45$ for $\alpha=0.05$. Since partial $F > F_c$, it can be concluded that there was interaction between treatments.

The veneer thickness effect and the effect of ultrasonic graded special veneers were also checked by considering the regression model given in Equation 100 as the full model and the following
regression models as the reduced models for veneer thickness effect and the effect of ultrasonic graded special veneers, respectively:

\[ \text{SR} = b_0 + b_2 X_2 + b_3 X_1 X_2 + b_4 Z \]  \hspace{1cm} [103]

and

\[ \text{SR} = b_0 + b_1 X_1 + b_3 X_1 X_2 + b_4 Z \]  \hspace{1cm} [104]

The results are summarized in Table 40. The partial F values between the full and reduced models for Equation 103 and 104 were 2.93 and 31.6, respectively. The test statistics \( F_c = F_{1, 17; 1-\alpha} = 4.45 \) for \( \alpha = 0.05 \). Since partial \( F < F_c \) for veneer thickness effect, it was considered not significant. However since partial \( F > F_c \) for ultrasonic effect, it was considered significant.

Finally the justification of using analysis of covariance was considered by treating the regression model given in Equation 100 as the full model and the following regression model as the reduced model for covariate effect:

\[ \text{SR} = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_1 X_2 \]  \hspace{1cm} [105]

The results are summarized in Table 40. The partial F value between the full and reduced models for Equation 105 is 358. The test statistics \( F_c = F_{1, 17; 1-\alpha} = 4.45 \) for \( \alpha = 0.05 \). Since partial \( F > F_c \), covariate effect was significant and analysis of covariance was justified.

The overall normalized relationship between \( \text{SR} \) and \( N_f \) (in log\(_{10}\) scale) for the various prototypes is given by:

\[ \text{SR} = 1.00009 - 0.14278 \log_{10}(N_f) - 0.01178 X_1 - 0.03991 X_2 + 0.018971 X_1 X_2 \]  \hspace{1cm} [106]
where the $r^2$ value is 0.965, the standard error of estimate is 0.031, and the degree of freedom is 17.

Since the effect of veneer thickness was not significant and the slopes of the normalized relationships of the various prototypes were not significantly different, 1) the data from prototypes 1 and 2 were combined to obtain the normalized relationships of the regular veneer and 2) the data from prototypes 3 and 4 were combined to obtain the normalized relationships of the special veneer.

The relationship between $S_R$ and $N_f$ using combined data from prototypes 1 and 2 is given by:

$$S_R = 0.956 - 0.14102 \log_{10}(N_f)$$ \hspace{1cm} [107]

where the $r^2$ value is 0.986, the standard error of estimate is 0.019, and the degree of freedom is 12.

The relationship between $S_R$ and $N_f$ using combined data from prototypes 3 and 4 is given by:

$$S_R = 1.017 - 0.13029 \log_{10}(N_f)$$ \hspace{1cm} [108]

where the $r^2$ value is 0.635, the standard error of estimate is 0.058, and the degree of freedom is 6.

Figures 49 and 50 show the relationships between $S_R$ and $N_f$ (in log_{10} scale) for the combined prototypes 1 and 2 data and combined prototypes 3 and 4 data, respectively. Based on Equations 107 and 108, the fatigue performance of transDeck$^{TM}$ can therefore be related directly to the static strength. The estimated applied stress ratios corresponding to 3000 cycles to failure were 0.466 and 0.564 for the regular and special veneer panels, respectively.

The short term small specimen parallel to grain bending test results using specimens cut from each full size test panel from the cyclic test program is presented in Table 41. The relatively low standard deviations clearly indicated the uniformity in bending strength of transDeck$^{TM}$. The difference of mean bending strengths amongst the four prototypes were consistent with the veneer mechanical test program results.
Figure 49 Normalized fatigue performance of transDeck™ with regular veneer.
Figure 50 Normalized fatigue performance of transDeck™ with special veneer.
Table 41. Short term static bending test results of various prototypes.

<table>
<thead>
<tr>
<th>Prototype</th>
<th>Number of Specimens</th>
<th>Bending Strength (MPa) Mean</th>
<th>STDV</th>
<th>$F_{s*}$ (kN)</th>
<th>$F_{R_s}$ (kN)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14</td>
<td>54.115</td>
<td>4.345</td>
<td>91.89</td>
<td>87.32</td>
<td>5.23</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
<td>65.437</td>
<td>6.384</td>
<td>71.52</td>
<td>75.13</td>
<td>-4.80</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>53.362</td>
<td>4.733</td>
<td>74.53</td>
<td>82.51</td>
<td>-9.67</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>73.374</td>
<td>6.235</td>
<td>70.81</td>
<td>66.17</td>
<td>7.01</td>
</tr>
</tbody>
</table>

For each transDeck$^\text{TM}$ prototype, the mean small specimen bending strength ($\bar{S}_{y_b}$) was compared to the DAP predicted maximum bending stress $\sigma_y^0$ for a front axle loading of 73.39 kN (section 5.3.1) to estimate the mean static capacity of the prototypes ($F_{s*}$) as:

$$F_{s*} = \frac{\bar{S}_{y_b}}{\sigma_y^0} \cdot 73.39 \text{ kN}$$

[109]

The predicted mean static capacity of the each prototype, $F_{s*}$, were further compared to the corresponding experimentally projected mean static capacity $F_{R_s}$. The absolute predicted errors ranged from 4.8% to 9.67% which was a further successful verification of DAP. The results of this analysis are shown in Table 41. Note that the small specimens used in the bending tests were obtained from the full size cyclic test specimens at the bays where failures occurred. It was assumed that size effect did not play a role when the small specimen bending test results were used to estimate the capacity of full scale assembly because the small specimens were sampled from the weakest bay within each panel.

For the development of a new prototype, pilot specimens can be made and tested to establish the small specimen short term bending strength. Since it is not possible to know the weakest location within each panel a priori, size effect models can be invoked to estimate the bending strength of the
full size panels from the small specimen test data. DAP can then be used to predicted the mean static capacity of the new prototype. From Equations 107 and 108, the applied stress ratios corresponding to 3000 cycles to failure were approximately 0.47 and 0.56 for the regular and special veneer panels, respectively. This factor can be applied to the estimated mean short term static capacity to rate the new product at 3000 cycles of loading. This simple scheme provides a practical method for gaining a preliminary assessment of the static and fatigue performance of new prototype transDeck™ constructions.
6. CYCLIC TESTING OF SMALL SPECIMENS

The full scale test program provided information on the performance of the prototype decking systems under static and cyclic loading. However, it is a very expensive and time consuming exercise; therefore, only a limited number of tests can practically be performed. A companion test program of small specimens is presented to develop the panel fatigue data. Here, static and cyclic bending tests have been performed with the appropriate span to depth ratio and stress history to determine the relationship between the fatigue life and failure mode of the small specimen and the full scale tests program results.

6.1 Materials and Methods

In this test program, a total of twenty 11-ply, 1.2 x 2.4 m (4 x 8 feet), transDeck™ panels with either 2.5 mm (1/10 inch) or 3.2 mm (1/8 inch) thick veneer were used. All the veneers for the panels were graded using the Metriguard Model 2600 veneer grader. Similar to the veneer testing program, the veneers were divided into three groups based on their propagation time: group A (best quality material); group B (medium quality material); and group C (worst quality material). The 11-ply transDeck™ panels were made from veneers randomly and proportionally sampled from the three groups; i.e., the specimens with 3.2 mm (1/8 inch) and 2.5 mm (1/10 inch) thick veneer corresponded to prototypes 1 and 2, respectively.

From each panel, eight 102 x 356 mm (4 x 14 inches) transDeck™ specimens were obtained. Seven of the eight specimens were oriented with face grain parallel to the long axis of the specimen. The other specimen was oriented with face grain perpendicular to long axis of the specimen. The specimens were then conditioned at a temperature of 20 ± 3°C and relative humidity of 65 ± 5% for more than four weeks where equilibrium was reached.
6.1.1 Short Term Bending Tests

From each panel, two specimens with face grain oriented parallel to specimen long axis and one specimen with face grain oriented perpendicular to specimen long axis were tested on the flat in static bending to establish the short term bending strengths. The specimens were simply supported at two ends with roller bearing plates and loaded under a center point load. A test span of 305 mm (12 inches) was used such that the span to depth ratio resembled dry freight van trailer deck applications. A MTS model 810 hydraulic control close loop universal testing machine with a 222.4 kN (50000 lb) capacity was used. From each panel, one specimen with face grain oriented parallel to specimen long axis and one specimen with face grain oriented perpendicular to specimen long axis were tested under deflection control mode. An uniform rate of cross head motion of 1.37 \( \frac{mm}{min} \) \((0.054 \frac{inch}{min})\) was used which resulted in specimen failure between 3 to 5 minutes of loading. The other specimen with face grain oriented parallel to long axis was tested in a load control mode. An uniform rate of loading of 44.44 \( \frac{kN}{s} \) \((9.99 \frac{kip}{s})\) was used which resulted in specimen failure between 0.5 and 1.0 second of loading.

The specimen thicknesses at mid-span and at two points near each edge were measured, averaged, and recorded. The specimen width at midspan was also recorded. A computer based data acquisition system and software were used to acquire the load versus deformation (cross head motion) data. A load cell with a 25 kN (5620 lb) capacity was used to monitor the loads. The time to failure information was also recorded. The bending strengths based on the measured peak loads in the perpendicular and parallel to grain directions were estimated from Equations 76a and 76b, respectively.

6.1.2 Cyclic Bending Tests

From each prototype, ten of twenty panels were randomly chosen for the small specimen cyclic tests. The five specimens from each panel, with face grain oriented parallel to the specimen long axis, were tested flatwise in cyclic bending to establish their fatigue characteristics at five different stress levels. The five stress levels were chosen such that the peak stress within each cycle was approximately
100%, 90%, 85%, 80%, and 75% of the mean short term parallel to grain bending strengths of the two prototypes which were obtained from the load controlled small specimen short term bending test results. This range of load levels was chosen so that the number of cycles to failure, in the lowest stress level (75%), was not prohibitively high so that testing program could be completed within a reasonable time frame.

Since fatigue data were influenced by the magnitude and frequency of loading; i.e., the shape of the applied stress cycles, the cyclic tests were performed using stress cycles which were expected to be similar in shape to that experienced by the material in service. As DAP results indicated, the panel at the critical location of the decking system (under the front wheel footprint centered between Bays 5 and 6) experienced stress reversal as the wheels traverse over the prototype deck assembly; therefore, a special fatigue bending test jig was designed and built to allow application of cyclic loading capable of inducing stress reversal in the specimen under simple support condition and center point loading.

The bending test jig consisted of a bearing system at each simple support and a loading head. At each simple support the specimen was free to move along its long axis and to rotate about the horizontal axis normal to its narrow face. Vertical movement of the specimen at the support was restricted to provide proper support when the loads were reversed. This was achieved by sandwiching the specimen between two bearing plates and bolting the plates together. The reactions were transmitted from the wood to the bearing plates through the bolt into the steel frame of the testing machine. A steel spacer block, slightly thicker than the specimen, was inserted between the plates during set up so that tightening the bolts did not damage the specimen. The loading head consisted of two channels in between which the specimen was sandwiched. The contact faces of the channels were rounded with a radius of curvature of 53 mm (2.06 inches). The channels were also bolted together with the proper spacer block to prevent specimen damage during set up and to ensure reversible loading. A MTS model 810 hydraulic control close loop universal testing machine with a capacity of 222.4 kN (50000 lb) was used to apply the load in a load control mode. The machine was programmed
to repeat a prescribed load cycle until specimen failure. The load history of each specimen was recorded using a sampling rate of 5 Hz. Knowing the period of each load cycle and the sampling rate, the number of cycles to failure was estimated.

6.1.2.1 Applied Stress Levels:

The computer program DAP was used to generate the parallel to face grain bending stress histories in the both prototypes 1 and 2 at the critical location of the decking system during one cycle of wheel cart travel. The front and rear axle loadings were 73.39 kN (16500 lb) and 8.15 kN (1833 lb), respectively. First the front and rear wheels were positioned on Bay 5 and Bay 1, respectively. For each type of panel, the DAP yielded the maximum parallel to face grain bending stress at the critical location (under the front wheel foot print at x=0.731 m and centered between Bays 5 and 6). Then the front and rear wheels were repositioned 152.4 mm (6 inches) along the long axis of the decking system from original position (front axle centered between Bay 6 and 7). DAP was executed to yield the maximum parallel to face grain bending stress at the critical location for the wheel cart second location. This procedure was repeated until the front and rear wheels were positioned on Bay 9 and Bay 5, respectively. The parallel to grain bending stress of the panel at the critical location of the decking system during the returning half cycle of the wheel cart travel was obtained directly from the DAP results for the first half cycle of the wheel cart travel. Since the period of wheel cart motion during the full scale cyclic test was 18 seconds, the critical parallel to grain bending stress histories of the two type of panels under a front and rear axle loadings of 73.39 kN and 8.15 kN were estimated.

Figure 51 shows one cycle of the parallel to grain bending stress history for each of the two prototypes at the critical location under a front and rear axle loadings of 73.39 kN and 8.15 kN,
Figure 51 Parallel to grain bending stress profiles for prototypes 1 and 2.
respectively. Each stress history was normalized with respect to its peak stress and yielded a nondimensional profile versus time. This was achieved by dividing each stress value by the peak stress within the cycle. Figure 52 shows the nondimensional profiles versus time for the two prototypes. Clearly two nondimensional profiles were very similar in shape; therefore, the average of the two profiles was judged to represent the prototypes well.

Given a target stress level, for example 85% of the mean short term parallel to grain bending strength of one of the prototypes, the average of the two nondimensional profiles was used to estimate the bending stress profile for the target stress level. From beam bending theory, the load profile for a simple supported beam under center point loading for the target stress level was also estimated. This load profile was programmed into the MTS controller to drive the fatigue test specimens. Different target stress levels were achieved by scaling the load level controls on the MTS controller. Since this was a load controlled experiment, the loading head would move in a particular direction until the prescribed load level was reached. Therefore, deflection and load limits was set up to keep the loading head from contacting the test bed after specimen failure.

6.2 Experimental Results

6.2.1 Short Term Bending Tests

Summary statistics from both the displacement and load controlled small specimen parallel to grain bending tests are shown in Table 42. Similar summary statistics from the displacement controlled small specimen perpendicular to grain bending tests are also shown in Table 42. The differences in mean parallel to grain bending strengths between the load controlled and displacement controlled tests were 3.67% and 4.05% for prototypes 1 and 2, respectively. The small difference indicates that within this range of loading rates the estimation of mean static bending strength was not significantly influenced by loading mode.
Figure 52 Nondimensional profiles versus time for prototypes 1 and 2.
Table 42. Short term small specimen static bending test results of various prototypes.

<table>
<thead>
<tr>
<th>Prototype</th>
<th>Number of Specimens</th>
<th>Displacement Controlled Bending Strength (MPa)</th>
<th>Load Controlled Bending Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean   STDV</td>
<td>Mean   STDV</td>
</tr>
<tr>
<td>Parallel to grain direction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>20</td>
<td>51.413  4.557</td>
<td>53.299  5.970</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>66.466  7.605</td>
<td>69.157  8.537</td>
</tr>
<tr>
<td>Perpendicular to grain direction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>20</td>
<td>14.633  2.936</td>
<td>–</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>20.176  3.082</td>
<td>–</td>
</tr>
</tbody>
</table>

Three failure modes were observed from the bending test specimens: 1) rolling shear failure at the cross ply (RS), 2) tension failure of the outer ply (B), and 3) both rolling shear failure at the cross ply and tension failure at the outer ply (RS+B). In the parallel to grain direction displacement controlled tests, the number of specimens with failure modes RS, B, and RS+B were 7, 7, and 6, respectively for prototype 1 and 13, 4, and 3, respectively for prototype 2. In the perpendicular to grain direction displacement controlled tests, only failure mode B was observed. In the parallel to grain direction load controlled tests, it was difficult to differentiate the various failure modes because typically the failed specimens were severely damaged.

Figure 53 shows the displacement controlled parallel to grain bending strength cumulative probability distributions for Prototypes 1 and 2. The perpendicular to grain bending strength cumulative probability distributions for Prototypes 1 and 2 are shown in Figure 54. The data clearly show that: 1) the bending strengths of both Prototypes were weak in the perpendicular to grain direction, 2) both Prototypes had relatively low variability, and 3) Prototype 2 had significantly
Figure 53  Displacement controlled parallel to grain bending strength cumulative probability distributions.
Figure 54  Displacement controlled perpendicular to grain bending strength cumulative probability distributions.
higher bending strengths when compared to Prototype 1. The information was consistent with results obtained in the veneer mechanical test program.

Figure 55 shows the load controlled parallel to grain bending strength cumulative probability distributions for Prototypes 1 and 2. Clearly the cumulative distributions from the load controlled tests compared closely with the distributions from the displacement controlled tests. Table 43 shows results of analyses of variance performed on the parallel to grain bending strength data. Results indicate that the difference between load and displacement controlled test data was not statistically significant at the 95% probability level. However, the difference between the bending strengths of Prototypes 1 and 2 was statistically significant at the 95% probability level. Finally, interaction effect between the mode of testing and the type of prototype was not statistically significant at the 95% probability level.

Table 43. Analysis of variance results on parallel to grain bending strengths.

<table>
<thead>
<tr>
<th>Source</th>
<th>Degree of Freedom</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Probability &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent Variable: Parallel to Grain Bending Modulus of Rupture</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>3</td>
<td>4844.265657</td>
<td>1614.755219</td>
<td>34.62</td>
<td>0.0001</td>
</tr>
<tr>
<td>Prototype</td>
<td>1</td>
<td>4738.175019</td>
<td>4738.175019</td>
<td>101.60</td>
<td>0.0001</td>
</tr>
<tr>
<td>Mode</td>
<td>1</td>
<td>102.897074</td>
<td>102.897074</td>
<td>2.21</td>
<td>0.1416</td>
</tr>
<tr>
<td>Prototype*Mode</td>
<td>1</td>
<td>3.193563</td>
<td>3.193562</td>
<td>0.07</td>
<td>0.7943</td>
</tr>
<tr>
<td>Error</td>
<td>75</td>
<td>3497.683176</td>
<td>46.635776</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>78</td>
<td>8341.9848833</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 55 Load controlled parallel to grain bending strength cumulative probability distributions.
Also shown in Figure 55 are the lognormal and two parameter Weibull distributions fitted to the parallel to grain bending strength data following the maximum likelihood estimation approach (Lawless, 1982). Visual inspection show both distributions fit the data well. The means of the lognormal parallel to grain bending strength distribution for Prototypes 1 and 2 were 52.325 MPa and 65.677 MPa, respectively. The standard deviations of the natural logarithm of parallel to grain bending strengths for Prototypes 1 and 2 were 0.0875 and 0.1062, respectively. The shape parameters of the two parameter Weibull distribution were 11.050 and 9.411 for Prototypes 1 and 2, respectively. The scale parameters of the two parameter Weibull distribution were 55.853 MPa and 72.825 MPa for Prototypes 1 and 2, respectively.

6.2.2 Cyclic Bending Tests

Figure 56 shows a small specimen under load during a cyclic bending test. Results from the cyclic test program are summarized in Table 44. Within each group, the variability in the number of cycles to failure \( N_f \) was found to be large. The large variability was consistent with fatigue performance of plywood (Kommers, 1943 and 1944). The peak loads in Groups A to E corresponded to bending stress levels of approximately 80%, 90%, 85%, 75% and 100%, respectively, of the mean bending strengths obtained from the load controlled bending tests. Figure 57 shows a typical rolling shear failure of the cross ply which was the predominant failure mode at the low peak load levels. At the high peak load levels, more tension failures at the outer ply were found.

The peak loads were normalized with the mean load controlled bending strengths to compute the bending stress level \( S_R \) for each group. Figure 58 shows a plot of the \( S_R \) versus \( N_f \) (in \( \log_{10} \) scale) relationships for the two prototypes. Here, the range of stress ratios was limited to accommodate a reasonable testing time. Direct regression analysis with such a narrow data range might not be appropriate. Instead the mean \( N_f \) (in \( \log_{10} \) scale) and the mean stress level (Sr) in each group were used to establish the relationship between stress level and mean fatigue life for each prototype.
Figure 56 Small Specimen during a cyclic bending test.
Table 44. Small specimen cyclic bending test results.

<table>
<thead>
<tr>
<th>Prototype</th>
<th>Number of Specimens</th>
<th>Group</th>
<th>Peak Load Mean (kN)</th>
<th>STDV Stress (kN)</th>
<th>Mean Stress to Failure (kN)</th>
<th>Number of Cycles</th>
<th>Number of Specimens</th>
<th>Failure Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>A</td>
<td>11.253</td>
<td>0.022</td>
<td>0.779</td>
<td>292.9</td>
<td>289.6</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>B</td>
<td>12.559</td>
<td>0.022</td>
<td>0.868</td>
<td>55.1</td>
<td>60.4</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>C</td>
<td>11.936</td>
<td>0.006</td>
<td>0.826</td>
<td>78.2</td>
<td>58.0</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>D</td>
<td>10.707</td>
<td>0.017</td>
<td>0.741</td>
<td>710.7</td>
<td>426.2</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>E</td>
<td>13.936</td>
<td>0.094</td>
<td>0.965</td>
<td>24.3</td>
<td>33.1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>A</td>
<td>9.083</td>
<td>0.047</td>
<td>0.757</td>
<td>470.8</td>
<td>332.3</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>B</td>
<td>10.291</td>
<td>0.044</td>
<td>0.858</td>
<td>85.9</td>
<td>129.1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>C</td>
<td>9.577</td>
<td>0.025</td>
<td>0.798</td>
<td>355.5</td>
<td>355.4</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>D</td>
<td>8.574</td>
<td>0.027</td>
<td>0.715</td>
<td>1775.8</td>
<td>1652.0</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>E</td>
<td>11.194</td>
<td>0.075</td>
<td>0.933</td>
<td>72.6</td>
<td>86.7</td>
<td>5</td>
</tr>
</tbody>
</table>
Figure 57  Typical rolling shear failure mode of a specimen under cyclic loading.
Figure 58 Fatigue data of small specimens in bending.
The relationship between \( S_{R_i} \) and \( N_f \) for the \( i^{th} \) prototype is given by:

\[
S_{R_i} = I_1 - S_i \log_{10} (N_f)
\]  

[110]

where \( I_1 \) and \( I_2 \) equal 1.128 and 1.161 respectively, \( S_1 \) and \( S_2 \) equal \(-0.14106\) and \(-0.14207\) respectively.

The \( r^2 \) values for Prototypes 1 and 2 were 0.912 and 0.900, respectively. The standard error of estimate values for Prototypes 1 and 2 were 0.030 and 0.031, respectively. Regression approach to analysis of covariance was also performed and partial F tests show:

1) the slopes \( S_1 \) and \( S_2 \) were not significantly different at 95% probability level;
2) the effect of prototypes was not significantly different at 95% probability level;
3) the effect of covariate was significant at 95% probability level which means the use of analysis of covariance was justified.

The results suggest that combining the fatigue performance of the small specimen of the two prototypes into a single relationship was justified.

Figure 59 compares the normalized small specimen fatigue data to the normalized fatigue performance of full size TransDeck\textsuperscript{TM} panels. Clearly the small specimens sustained a significantly higher stress level for a given \( N_f \) in comparison with the full size panels. This phenomenon could be explained by a size effect analysis. The full size test results were obtained by cycling a load over a test assembly which contained two sides with eight 304.8 mm (12 inches) wide bays. The span in the small specimen cyclic tests was only 304.8 mm (12 inches) long; therefore, the stressed volume in the full size panels of the test assembly was approximately 16 times that of the small specimen.

Using the Weibull's weakest link theory, the bending strength ratio between the small specimens and the full size specimens can be estimated as:

\[
\frac{S_{\text{small}}}{S_{\text{full size}}} = 16 k
\]  

[111]
Figure 59  Comparisons of fatigue performance of small specimens and full size panels.
where \( k \) is the shape parameter, and \( S_{\text{small}} \) and \( S_{\text{full size}} \) are the bending strengths of the small and full size specimens, respectively. From Figure 55 the small specimens bending strength data were fitted satisfactorily by a 2 parameter Weibull distribution with shape parameters of 11.050 and 9.411 for Prototypes 1 and 2, respectively. Therefore, using Equation 111, the parallel to grain bending strengths of the small specimens were adjusted to those of the full size panels. New stress ratios relating to the full size panels could therefore be established by normalizing the peak loads in the fatigue data with the mean size adjusted bending strengths. The relationship between the size adjusted \( S_{R_i} \) and \( N_f \) for the \( i \)th prototype is given by:

\[
S_{R_i} = I_1 - S_i \log_{10} (N_f) \tag{112}
\]

where \( I_1 \) and \( I_2 \) equal 0.878 and 0.865 respectively, \( S_1 \) and \( S_2 \) equal -0.10976 and -0.10581 respectively. The \( r^2 \) values for Prototypes 1 and 2 were 0.912 and 0.900, respectively. The standard error of estimate values for Prototypes 1 and 2 were 0.023 and 0.023, respectively.

Analysis of covariance was performed on the two prototypes of size adjusted \( S_R \) and \( N_f \) (in \( \log_{10} \) scale) data. The partial F test results show that the slopes of the relationship between \( S_R \) and \( N_f \) (in \( \log_{10} \) scale) of the two prototypes were statistically equal at the 95% probability level. Also the effect of prototype was not significant but the effect of covariate was significant at the 95% probability level. Therefore, the size adjusted relationships between \( S_R \) and \( N_f \) (in \( \log_{10} \) scale) for both prototypes were combined. The relationship between \( S_R \) and \( N_f \) projected for full size panels is given by:

\[
S_R = 0.874 - 0.10908 \log_{10} (N_f) \tag{113}
\]

where the \( r^2 \) value was 0.917, the standard error of estimate was 0.020, and the degree of freedom was eight.
Finally, the size adjusted $S_R$ and $N_f$ (in log$_{10}$ scale) data from the small specimen fatigue tests were combined with the $S_R$ and $N_f$ (in log$_{10}$ scale) data of prototypes 1 and 2 from the full-size specimen fatigue tests in a regression approach to analysis of covariance. The two treatment groups were 1) veneer thickness effect and 2) size adjustment procedure effect. The partial F test results show that the slopes of the relationship between $S_R$ and $N_f$ (in log$_{10}$ scale) between the size adjusted and the full size treatment groups was statistically equal at the 95% probability level. Also both the interaction effect between treatment groups and the effects of either treatment groups were not significant at 95% probability level. Finally, the effect of covariate was significant at 95% probability level.

Using the full-size relationship from Equation 107, the applied stress ratios corresponding to $\frac{1}{2}$ cycle and 3000 cycles to failure were approximately 1.00 and 0.47, respectively. Using the size-adjusted relationship from Equation 113, the applied stress ratios corresponding to $\frac{1}{2}$ cycle and 3000 cycles to failure were approximately 0.91 and 0.50, respectively. The reasonable agreement indicates the size adjustment procedure explains the differences in the fatigue information on full size panels and small specimens.
7. MODELING OF SMALL SPECIMEN FATIGUE PROPERTIES

The fatigue behavior of the small specimens in bending has been shown to be related to the fatigue behavior of the full size specimens through size effect adjustment procedure. It is desirable to identify and calibrate a damage accumulation model so that the relationship between load levels and number of cycles to failure can be predicted for small specimen in bending.

Based on the literature survey, the damage accumulation model developed by Foschi and Yao (1986a and b) was considered most appropriate for this investigation. The damage accumulation model takes the form:

\[
d\alpha/dt = a \{\tau(t) - \sigma_0 \tau_s\}^b + c \{\tau(t) - \sigma_0 \tau_s\}^n \alpha(t) \tag{114}
\]

where \(a, b, c, n, \) and \(\sigma_0\) are random model parameters which are constants for a given member but vary from member to member. \(\alpha\) is the damage where \(\alpha=0\) means no damage and \(\alpha \geq 1\) means failure. \(\tau(t)\) is the stress history experienced by the specimen. \(\tau_s\) is the short term strength of the specimens obtained from load controlled tests. \(\sigma_0\) is the threshold stress ratio where damage accumulates only if \(\tau(t) > \sigma_0 \tau_s\).

Let \(f_1 = \{\tau(t) - \sigma_0 \tau_s\}^b\) and \(f_2 = \{\tau(t) - \sigma_0 \tau_s\}^n\), Equation 114 can be rewritten as follows:

\[
\frac{d\alpha}{dt} e^{- \int c f_2 dt} = \left\{a f_1 + c f_2 \alpha(t)\right\} e^{- \int c f_2 dt} \tag{115}
\]

\[
\frac{d}{dt}\left\{\alpha e^{- \int c f_2 dt}\right\} = a f_1 e^{- \int c f_2 dt} \tag{116}
\]

Integrating Equation 116 yields:
Given a stress history $r(t)$, Equation 117 can be evaluated by performing the integration over the intervals when $r(t) - \sigma_0 r_s > 0$ to estimate the damage in the specimen if the model parameters are known.

7.1 Ramp Load Case

Consider now a ramp load test where the load increases at a constant rate $K_s$. Assuming the stress also increases at the same rate, the stress history can be expressed as $r(t) = K_s t$. This is the type of loading used to determine the load controlled parallel to grain bending strength of the small specimens. Since damage only occurs when $r(t) - \sigma_0 r_s > 0$, let us define a time $t_0$ at which $r(t_0) = \sigma_0 r_s$. Substituting the stress history from the ramp load tests into Equation 117 and integrating from $t=t_0$ to $t=T$ yields:

$$\alpha \exp\left\{ -\frac{c}{K_s(n+1)} [K_s t - \sigma_0 r_s]^{(n+1)} \right\} \bigg|_{t_0}^{T} = \int_{t_0}^{T} \alpha \exp\left\{ -\frac{c}{K_s(n+1)} [K_s t - \sigma_0 r_s]^{(n+1)} \right\} dt \quad [118]$$

Let us define the time to failure of a specimen during a short term ramp test as $T_s$. At $t=T_s$, $\alpha=1$ and $K_s T_s = \tau_s$ also at $t=t_0$, $\alpha=0$ and $K_s t_0 = \sigma_0$; therefore, Equation 118 becomes:

$$\alpha \exp\left\{ -\frac{c}{K_s(n+1)} [\tau_s - \sigma_0 r_s]^{(n+1)} \right\} \bigg|_{t_0}^{T_s} = \int_{t_0}^{T_s} \alpha \exp\left\{ -\frac{c}{K_s(n+1)} [K_s t - \sigma_0 r_s]^{(n+1)} \right\} dt \quad [119]$$

Foschi and Yao (1986a and b) showed that $K_s$ is typically large compared to the model parameter $c$; therefore, Equation 119 yields an approximate relation between the model parameters $a, b, \sigma_0$ and the
ramp rate $K_s$ as:

$$a = \frac{K_s (b+1)}{(\tau_s - \sigma_o \tau_s)^{b+1}}$$

This relationship suggests that the model parameter $a$ cannot be chosen independently from the other model parameters $b$, $\sigma_o$, the ramp rate $K_s$ and the short term strength $\tau_s$.

7.2 Piecewise Linear Representation of Stress History

Now consider one cycle of the nondimensional stress history experienced by the TransDeck$^{TM}$ small specimens during the fatigue tests shown in Figure 52. This stress history can be defined as composed of piecewise linear segments with equal intervals of 0.5625 second. The stress history in the $i^{th}$ interval is given by:

$$\tau(t) = \left[ I_i + S_i(t - t_{si} - t_j) \right] \tau_{max}$$

where $\tau_{max}$ is the peak applied stress within a cycle, $t_j$ is the starting time of the $j^{th}$ cycle, and $t_{si}$ and $t_{ei}$ are the starting and ending time of the $i^{th}$ segment, respectively. Table 45 shows the parameters $I_i$ and $S_i$ for the $i^{th}$ segment during the first cycle.

Let us now evaluate the accumulated damage of a specimen after the first piecewise linear segment in the first stress cycle where $t_j=0$ ($j=1$). Define $t_{01}$ as the time when $|\tau(t_{01})|$ just exceeds $\sigma_o \tau_s$ and note that $S_1>0$, the damage can be expressed as in terms of the model parameters as:

$$\alpha e^{f_3} \int_{t_{01}}^{t_{ei}} = \int_{t_{01}}^{t_{ei}} a \left[ I_i + S_i(t - t_{si}) \right] \tau_{max} - \sigma_o \tau_s \right] b e^{f_3} dt$$

(i=1)
Table 45. The parameters describing piecewise linear segments of a nondimensional stress cycle.

<table>
<thead>
<tr>
<th>Segment</th>
<th>Intercept</th>
<th>Slope</th>
<th>Segment</th>
<th>Intercept</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>I_i</td>
<td>S_i</td>
<td>i</td>
<td>I_i</td>
<td>S_i</td>
</tr>
<tr>
<td>1</td>
<td>0.260639</td>
<td>1.31419</td>
<td>17</td>
<td>0.034787</td>
<td>-0.00451</td>
</tr>
<tr>
<td>2</td>
<td>1.766239</td>
<td>-1.362</td>
<td>18</td>
<td>0.03042</td>
<td>-0.00405</td>
</tr>
<tr>
<td>3</td>
<td>0.599725</td>
<td>-0.3253</td>
<td>19</td>
<td>0.0129</td>
<td>-0.00232</td>
</tr>
<tr>
<td>4</td>
<td>0.329665</td>
<td>-0.16526</td>
<td>20</td>
<td>0.005047</td>
<td>-0.00159</td>
</tr>
<tr>
<td>5</td>
<td>0.104534</td>
<td>-0.0652</td>
<td>21</td>
<td>-0.06938</td>
<td>0.005025</td>
</tr>
<tr>
<td>6</td>
<td>-0.02553</td>
<td>-0.01896</td>
<td>22</td>
<td>-0.15386</td>
<td>0.012177</td>
</tr>
<tr>
<td>7</td>
<td>-0.16293</td>
<td>0.021748</td>
<td>23</td>
<td>-1.19411</td>
<td>0.096237</td>
</tr>
<tr>
<td>8</td>
<td>-0.38812</td>
<td>0.078941</td>
<td>24</td>
<td>1.979518</td>
<td>-0.14906</td>
</tr>
<tr>
<td>9</td>
<td>-0.70369</td>
<td>0.149067</td>
<td>25</td>
<td>1.032817</td>
<td>-0.07894</td>
</tr>
<tr>
<td>10</td>
<td>0.538161</td>
<td>-0.09623</td>
<td>26</td>
<td>0.228536</td>
<td>-0.02174</td>
</tr>
<tr>
<td>11</td>
<td>0.065321</td>
<td>-0.01217</td>
<td>27</td>
<td>-0.36684</td>
<td>0.018961</td>
</tr>
<tr>
<td>12</td>
<td>0.021069</td>
<td>-0.00502</td>
<td>28</td>
<td>-1.06921</td>
<td>0.065208</td>
</tr>
<tr>
<td>13</td>
<td>-0.02359</td>
<td>0.001591</td>
<td>29</td>
<td>-2.64513</td>
<td>0.165266</td>
</tr>
<tr>
<td>14</td>
<td>-0.02896</td>
<td>0.000235</td>
<td>30</td>
<td>-5.25571</td>
<td>0.325302</td>
</tr>
<tr>
<td>15</td>
<td>-0.04259</td>
<td>0.004056</td>
<td>31</td>
<td>-22.7534</td>
<td>1.362203</td>
</tr>
<tr>
<td>16</td>
<td>-0.04644</td>
<td>0.004512</td>
<td>32</td>
<td>23.92018</td>
<td>-1.31441</td>
</tr>
</tbody>
</table>
where

\[
f_3 = \left\{ \frac{-c}{S_i{(n+1)}_{\tau_{\text{max}}}} \left[ \left[ I_i + S_i(t - t_{si}) \right] \tau_{\text{max}} - \sigma_0 \tau_s \right]^{(n+1)} \right\}
\]

Since \( \alpha(t_{01}) = 0 \), the damage at the end of the first segment is:

\[
\alpha(t_{01}) = \frac{\int_{t_{01}}^{t_{e1}} a \left[ I_i + S_i(t - t_{si}) \right] \tau_{\text{max}} - \sigma_0 \tau_s \right]^{b} e^{f_3} dt}{\int_{S_i{(n+1)}_{\tau_{\text{max}}}} \left[ \left[ I_i + S_i(t - t_{si}) \right] \tau_{\text{max}} - \sigma_0 \tau_s \right]^{(n+1)} (i=1) \quad [123]}
\]

Consider now the second segment in the first cycle and define \( t_{02} \) as the time when \( |\tau(t_{02})| \) just exceeds \( \sigma_0 \tau_s \) and note that \( S_2 < 0 \), the damage accumulated during this segment can be expressed as in terms for the model parameters as:

\[
\alpha(t_{e2}) = \frac{\int_{t_{02}}^{t_{e2}} a \left[ I_i + S_i(t - t_{si}) \right] \tau_{\text{max}} - \sigma_0 \tau_s \right]^{b} e^{f_3} dt}{\int_{S_i{(n+1)}_{\tau_{\text{max}}}} \left[ \left[ I_i + S_i(t - t_{si}) \right] \tau_{\text{max}} - \sigma_0 \tau_s \right]^{(n+1)} (i=2) \quad [124]}
\]

where \( |\tau(t_{02})| \geq \sigma_0 \tau_s \) and \( t_{02} \leq t_{e2} \).

Since \( t_{02} = t_{e1} \), the total damage accumulated at the end of the second segment of the first cycle is:

\[
\alpha(t_{e2}) = \frac{\int_{t_{02}}^{t_{e2}} a \left[ I_i + S_i(t - t_{si}) \right] \tau_{\text{max}} - \sigma_0 \tau_s \right]^{b} e^{f_3} dt + \alpha(t_{e1}) e^{f_4}}{\int_{S_i{(n+1)}_{\tau_{\text{max}}}} \left[ \left[ I_i + S_i(t - t_{si}) \right] \tau_{\text{max}} - \sigma_0 \tau_s \right]^{(n+1)} (i=2) \quad [125]}
\]
where

\[ f_4 = \left\{ \frac{-c}{S_i(n+1)\tau_{\text{max}}} \left[ (t_i + S_i(t_{e1} - t_{s1}))\tau_{\text{max}} - \sigma_0\tau_{s} \right]^{(n+1)} \right\} \]

Repeating the above procedure for all the segments within the first cycle, the damage accumulated during the first cycle \( \alpha_1 \) can be evaluated.

Let the damage after \( I^{th} \) cycle be expressed in form of a recurrence relationship as:

\[ \alpha_I = \alpha_{I-1} K_0(I) + K_1(I) \] \[ \text{[126]} \]

Since each stress cycle is identical in shape, \( K_0(I) = K_0(J) = K_0 \) and \( K_1(I) = K_1(J) = K_1 \) where \( I \neq J \). Knowing \( \alpha_0 = 0 \), the two unknown coefficients \( K_0 \) and \( K_1 \) can be evaluated by considering the accumulated damage after the first two stress cycles; \( i.e., K_1 = \alpha_1 \) and \( K_0 = \left( \frac{\alpha_2}{K_1} - 1 \right) \). With \( K_0 \) and \( K_1 \) known, the recurrence relationship of Equation 126 can be evaluated to find the accumulated damage after the \( I+N \) cycle as:

\[ \alpha_{I+N} = \alpha_{I-1} K_0^N + K_1(K_0^{N-1} + K_0^{N-2} + \ldots + K_0 + 1) \] \[ \text{[127]} \]

when \( I=1, \alpha_0=0 \); therefore,

\[ \alpha_{1+N} = K_1(K_0^{N-1} + K_0^{N-2} + \ldots + K_0 + 1) \] \[ \text{[128]} \]

which can be rewritten as:

\[ \alpha_{1+N} = K_1 \left( \frac{1 - K_0^N}{1 - K_0} \right) \] \[ \text{[129]} \]
Let failure occur at N+1 cycle, where \( \alpha_{1+N}=1 \). Therefore, the number of cycles to failure \( N_f \) can be estimated from Equation 129 as:

\[
N_f = \frac{\log \left( \frac{K_1 + K_0 - 1}{K_1} \right)}{\log(K_0)} + 1
\]

Treating the stress history as composed of piecewise linear segments requires numerical integration for each segment when \( |\tau(t)| - \sigma_0 \tau_s > 0 \). If the model parameters of a specimen are known, the number of cycles to failure can be estimated. However, if the model parameters are unknown, it is necessary to calibrate the damage model to fatigue data using nonlinear function minimization procedures. This may be difficult when the stress history is assumed to be composed of piecewise linear segments because Equations 123 or 125 are very sensitive to the choice of model parameters. During the search for solution with a nonlinear function minimization procedure the parameters may fluctuate substantially which may lead to unstable solutions unless initial choice of model parameters are close to the solution. Therefore it is inappropriate to consider the stress history as composed of piecewise linear segments when model parameters are required to be calibrated to fatigue data. Alternately the stress history can be considered to be composed of a series of stress pulses.

7.3 Representation of Stress History by a Series of Stress Pulses

Consider several piecewise linear segments of a stress cycle shown in Figure 60. A piecewise linear segment can be subdivided into \( m \) sections each with a constant stress pulse of magnitude \( \tau_i \) \((i=1,.., m)\) and duration \( \Delta t \). The magnitude of the stress pulse \( \tau_i \) \((i=1,..,m)\) is taken as the average stress within the \( i \)th section and the duration \( \Delta t \) is taken as \( \frac{0.5625}{m} \) seconds.

The accumulated damage sustained by a specimen during the \( i \)th load pulse, from \( t = t_{i-1} \) to \( t_i \) when \( |\tau_i| \geq \sigma_0 \tau_s \), can be expressed as in terms of the model parameters as:
Figure 60 Representation of stress cycle with stress pulses.
\[ \alpha \left[ e^{f_6 t} \right]_{t_i}^{t_{i-1}} = \int_{t_{i-1}}^{t_i} a [\tau_1 - \sigma_o \tau_s] b e^f dt \]  

where

\[ f_6 = -c [\tau_1 - \sigma_o \tau_s]^n \]

Evaluation of Equation 131 yields:

\[ \alpha_i = \alpha_{i-1} e^{\left\{ -c \int_{t_{i-1}}^{t_i} [\tau_1 - \sigma_o \tau_s] \Delta t \right\} b} + \frac{a}{c} \left[ \tau_1 - \sigma_o \tau_s \right]^{(b-n)} \left( e^{\left\{ -c \int_{t_{i-1}}^{t_i} [\tau_1 - \sigma_o \tau_s] \Delta t \right\} - 1} \right) \]  

Equation 132 can be rewritten as:

\[ \alpha_i = \alpha_{i-1} k_0(i) + k_1(i) \quad \text{if } |\tau_i| > \sigma_o \tau_s \]  

where

\[ k_0(i) = e^{\left\{ -c \int_{t_{i-1}}^{t_i} [\tau_1 - \sigma_o \tau_s] \Delta t \right\}} \quad \text{and} \quad k_1(i) = \frac{a}{c} \left[ \tau_1 - \sigma_o \tau_s \right]^{(b-n)} \left( k_0(i) - 1 \right) \]

when \(|\tau_i| > \sigma_o \tau_s\)

and

\[ k_0(i) = 1 \quad \text{and} \quad k_1(i) = 0 \quad \text{when } |\tau_i| \leq \sigma_o \tau_s \]

For one stress cycle with \(k\) piecewise linear segments, the accumulated damage can be evaluated as:

\[ \alpha_1 = 0 \]

\[ \alpha_2 = \alpha_1 k_0(2) + k_1(2) \]

\[ \vdots \quad \text{[134]} \]

\[ \alpha_{m^k} = \alpha_{m^k-1} k_0(m^k) + k_1(m^k) \]

\[ = \alpha_1 m^k \prod_{j=2}^{m^k} k_0(j) + \sum_{p=2}^{m^k-1} k_1(p) \prod_{j=p+1}^{m^k} k_0(j) + k_1(m^k) \]
Let the subscript \( I \) denotes the stress cycle, the damage after \( I^{th} \) cycle is given by:

\[
\alpha_I = \alpha_{I-1} K_0 + K_1
\]

where

\[
K_0 = \prod_{j=2}^{m-k} k_0(j)
\]

and

\[
K_1 = \sum_{p=2}^{m-k-1} k_1(p) \prod_{j=p+1}^{m-k} k_0(j) + k_1(m-k)
\]

From Equation 130, the number of cycles to failure of a specimen can be estimated if the model parameters are known.

7.4 Model Calibration

The damage model was calibrated against the test data following the procedures presented by Foschi, Folz, and Yao, (1987). As shown in Figure 55, the lognormal distribution fits the short term strength data \( \tau_s \) well; therefore, \( \tau_s \) was assumed lognormally distributed. Each of the four independent model parameters (\( b, c, n, \sigma_o \)) were also assumed to be lognormally distributed. A nonlinear function minimization procedure using the quasi-Newton method was employed to estimate the mean and standard deviation of each independent lognormal distribution of the model parameters. Therefore, an vector \( \mathbf{X} \) with eight unknowns corresponding to the mean and the standard deviation of the four independent parameters, \( \mathbf{X} = \{ \overline{b}, \overline{c}, \overline{n}, \overline{\sigma_o}, \text{STD}_b, \text{STD}_c, \text{STD}_n, \text{STD}_{\sigma_o} \} \), was estimated by the minimization procedure. The procedure is summarized as follows:

1) Initial estimates of the mean and standard deviation of each independent lognormal distribution of the model parameters were provided;

2) Based on the initial distribution parameters, a random sample of 1000 number of cycles to failure
(N_f^s) was generated using Equations 130 and 135;

3) The cumulative probability distribution of the randomly generated sample of number of cycles to failure was obtained and compared to the experimental data by computing an objective function \( \Phi \) as:

\[
\Phi = \sum_{i=1}^{L} \left( 1.0 - \frac{N_{f_i}^s}{N_{f_i}^a} \right)^2
\]

where L is the number of probability levels considered, \( N_{f_i}^s \) is the simulated number of cycles to failure at the \( i^{th} \) probability level, and \( N_{f_i}^a \) is the actual number of cycles to failure at the same probability level obtained from the experimental results;

4) The gradient of the objective function with respect to unknown vector \( X_i \),

\[
\nabla \Phi = \left\{ \frac{\partial \Phi}{\partial X_1^i}, \ldots, \frac{\partial \Phi}{\partial X_8^i} \right\}^T
\]

was required by the minimization procedure and was numerically computed. First each unknown distribution parameter \( X_i \) (i=1,...,8) was perturbed in turn by a positive increment \( +\Delta X_i \) and the objective function was reevaluated by repeating step 2 and 3 using the same sequence of random numbers as \( \Phi^+ \). Then each unknown distribution parameter \( X_i \) (i=1,...,8) was perturbed in turn by a negative increment \( -\Delta X_i \) and the objective function is reevaluated by repeating step 2 and 3 using the same sequence of random numbers as \( \Phi^- \). In this study, the increment \( \Delta X_i \) was chosen as 0.001\( X_i \). The partial derivative of \( \Phi \) with respect to the unknown distribution parameters \( X_i \) (i=1,...,8) is given as:

\[
\frac{\partial \Phi}{\partial X_i} = \frac{\Phi^+ - \Phi^-}{2\Delta X_i} \quad (i=1,...,8)
\]

5) Equation 136 was minimized following the quasi-Newton method in which the initial choices of distribution parameters were modified automatically through an iteration procedure. An optimal solution was considered obtained when changes in \( X_i \) of magnitude \( \epsilon \Delta X_i \) did not reduce the objective function value for \( i = 1,...,8 \) where the convergence criterion of \( \epsilon = 0.001 \) was used.
The relatively small convergence target, step size and the reasonably large sample size were chosen to avoid wide fluctuations in the estimated model parameters when a new set of random seeds were considered.

7.4.1 Calibration Results

Figures 61 and 62 show the cumulative distributions of $N_f$ (in log$_{10}$ scale) of the small specimens under bending at five stress levels for prototypes 1 and 2, respectively. It can be observed that the distributions were highly skewed. Although some weak specimens failed during uploading, failure was still considered to have occurred at the first cycle. Note also that each load level had a limited sample size of 10 and the variability in the number of cycles to failure was large; therefore, model calibration with these two families of five cumulative distributions proved to be rather difficult.

When all five load levels were considered during model calibration, regardless of the initial model parameters, it seems that the calibrated results fitted the cumulative distributions with the higher mean bending stress level well but failed to provide good fit to the cumulative distributions with the lower mean bending stress level. The objective function in Equation 136 evaluated the square of the relative difference between the actual and the simulated $N_f$, which weighted the cumulative distributions with the lower $N_f$ in favor of the cumulative distributions with higher $N_f$. If the objective function in Equation 136 was formulated to evaluate the square of the absolute difference between the actual and the simulated $N_f$, the cumulative distributions with the higher $N_f$ would be weighted in favor over the cumulative distributions with lower $N_f$.

Rather than modifying the objective function, it was decided to calibrate the model only to the cumulative distribution with the lowest mean bending stress level in each prototype ($Sr=0.741$ and $Sr=0.715$ for prototypes 1 and 2, respectively). For each prototype, the calibrated model was then used to generate simulated $N_f$ for the five stress levels. The model was verified by comparing model predictions and experimental data at the various stress levels.
Figure 61 Cumulative probability distribution of $N_f$ for Prototype 1.
Figure 62 Cumulative probability distribution of $N_f$ for Prototype 2.
Table 46 shows the mean and standard deviation of the model parameters for prototypes 1 and 2. Also shown in Table 46 are the minimum $\Phi$ from each prototype obtained from the minimization procedure.

Figures 63 and 64 show the cumulative distributions of the simulated and actual $N_f$ (in log scale) of the small specimens under bending at five stress levels for prototypes 1 and 2, respectively. The matching between model predicted and actual number of cycles to failure was reasonable considering the limited sample size and the large variability in the number of cycles to failure.

Table 46. The mean and standard deviation of the model parameters for prototypes 1 and 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>STDV</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Prototype 1: Minimum $\Phi = 2.02488$</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>10.9536</td>
<td>2.5871 x 10^{-1}</td>
</tr>
<tr>
<td>c</td>
<td>1.25549 x 10^{-6}</td>
<td>1.81542 x 10^{-8}</td>
</tr>
<tr>
<td>n</td>
<td>5.57889 x 10^{-1}</td>
<td>2.27 x 10^{-2}</td>
</tr>
<tr>
<td>$\sigma_o$</td>
<td>9.47904 x 10^{-2}</td>
<td>6.29348 x 10^{-2}</td>
</tr>
<tr>
<td><strong>Prototype 2: Minimum $\Phi = 1.92129$</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>11.3832</td>
<td>4.30419 x 10^{-1}</td>
</tr>
<tr>
<td>c</td>
<td>6.445890 x 10^{-7}</td>
<td>1.84566 x 10^{-8}</td>
</tr>
<tr>
<td>n</td>
<td>4.34451 x 10^{-1}</td>
<td>2.01685 x 10^{-2}</td>
</tr>
<tr>
<td>$\sigma_o$</td>
<td>8.47986 x 10^{-2}</td>
<td>2.61574 x 10^{-2}</td>
</tr>
</tbody>
</table>

It can be observed that the both the simulated and the actual cumulative probability distributions were highly skewed. The fatigue performance of small specimen shown in Figure 59 was obtained from the mean number of cycles to failure test data at various mean stress levels. Since each
Figure 63 Comparisons of model predicted and actual cumulative probability
distribution of $N_f$ for Prototype 1.
Figure 64  Comparisons of model predicted and actual cumulative probability distribution of $N_f$ for Prototype 2.
load level contained only 10 data points, the mean and standard deviation of the number of cycles to failure reported in Table 43 were the average and standard deviation of the test data from the probability levels of approximately 0.07 to 0.93 assuming the rank of the \(i^{th}\) data point from a sample size of \(m\) was given by \(\frac{i-0.3}{m+0.4}\). The mean and standard deviation of the model predicted number of cycles to failure were also obtained from the probability levels of 0.07 to 0.93 to avoid the influence from the skewness in the cumulative distributions when making comparisons between model predictions and actual fatigue performance. Table 47 shows the model predicted fatigue performance of the two prototypes.

Table 47. Model predicted small specimen fatigue performance.

<table>
<thead>
<tr>
<th>Prototype</th>
<th>Stress Level (S_r)</th>
<th>(N_f) Mean</th>
<th>(N_f) STDV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.779</td>
<td>414.28</td>
<td>473.94</td>
</tr>
<tr>
<td></td>
<td>0.868</td>
<td>102.98</td>
<td>114.54</td>
</tr>
<tr>
<td></td>
<td>0.826</td>
<td>195.06</td>
<td>219.53</td>
</tr>
<tr>
<td></td>
<td>0.741</td>
<td>787.44</td>
<td>915.95</td>
</tr>
<tr>
<td></td>
<td>0.965</td>
<td>28.84</td>
<td>30.95</td>
</tr>
<tr>
<td>2</td>
<td>0.757</td>
<td>609.00</td>
<td>787.49</td>
</tr>
<tr>
<td></td>
<td>0.858</td>
<td>120.30</td>
<td>151.53</td>
</tr>
<tr>
<td></td>
<td>0.798</td>
<td>305.22</td>
<td>391.09</td>
</tr>
<tr>
<td></td>
<td>0.715</td>
<td>1299.26</td>
<td>1690.44</td>
</tr>
<tr>
<td></td>
<td>0.933</td>
<td>27.43</td>
<td>33.00</td>
</tr>
</tbody>
</table>

The relationship between the mean stress level \(S_r\) and the model predicted \(N_f\) for the \(i^{th}\) prototype is given by:
\[ S_{R_i} = I_i \cdot S_1 \cdot \log_{10} (N_f) \]  \[138\]

where \( I_1 \) and \( I_2 \) equal 1.187 and 1.126 respectively, \( S_1 \) and \( S_2 \) equal -0.15574 and -0.13187 respectively. The \( r^2 \) values of the model predicted fatigue performance for Prototypes 1 and 2 were 0.997 and 0.998, respectively. The standard error of estimate values of the model predicted fatigue performance for Prototypes 1 and 2 were 0.005 and 0.004, respectively.

Figure 65 compares the model predicted and actual fatigue performance of small specimen under cyclic loading. Clearly the model predicted and actual fatigue performance of small specimen under cyclic loading agreed reasonably well.

The size adjustment procedure given in Equation 111 was used to convert the short term strength (\( \tau_s \)) of small specimens in the damage model to that of the full size panels. In each prototype, five stress levels (0.9, 0.8, 0.7, 0.6 and 0.5) were chosen. For each stress level, the number of cycles to failure of 1000 replicates were randomly simulated. The model predicted mean number of cycles to failure were obtained from the probability levels of 0.07 to 0.93 to avoid the influence from the skewness in the cumulative distributions when making comparisons between model predictions and actual fatigue performance. Table 48 shows the model predicted fatigue performance of the two prototypes.

The relationship between the stress ratios \( S_{R_i} \) and \( N_f \) from the size adjusted damage model for the \( i^{th} \) prototype is given by:

\[ S_{R_i} = I_i \cdot S_1 \cdot \log_{10} (N_f) \]  \[139\]

where \( I_1 \), \( I_2 \), \( S_1 \) and \( S_2 \) equal 0.942, 0.908, -0.12794 and -0.13055, respectively. The \( r^2 \) values for Prototypes 1 and 2 were 0.983 and 0.975, respectively. The standard error of estimate values for Prototypes 1 and 2 were 0.024 and 0.029, respectively.
Table 48. Model predicted full scale panel fatigue performance.

<table>
<thead>
<tr>
<th>Prototype</th>
<th>Stress Level</th>
<th>Mean $N_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$S_{Rd}$</td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>3.3393</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>10.858</td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td>52.617</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>367.97</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>3974.3</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$S_{Rd}$</td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>2.0168</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>5.4156</td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td>24.920</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>175.39</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>1924.8</td>
<td></td>
</tr>
</tbody>
</table>

Finally, the size adjusted relationships between $S_{Rd}$ and $N_f$ (in log$_{10}$ scale) from the damage model for both prototypes were combined to yield the fatigue performance of full size panels as:

$$S_{Rd} = 0.921 - 0.12684 \log_{10} (N_f)$$  \[140\]

where the $r^2$ value was 0.961, the standard error of estimate was 0.031, and the degree of freedom was eight.

Figure 66 compares the damage model predicted fatigue performance of full size panels with the actual fatigue performance of full size prototype 1 and 2 panels. Again the model predictions and the full size test results under cyclic loading agreed reasonably well.
Figure 65  Comparisons of model predicted and actual fatigue performance of small specimens.
Figure 66  Comparisons of model predicted and actual fatigue performance of full size panels.
Using the full size relationship from Equation 107, the applied stress ratios corresponding to \( \frac{1}{2} \) cycle and 3000 cycles to failure were approximately 1.00 and 0.47, respectively. Using the size adjusted relationship from Equation 113, the applied stress ratios corresponding to \( \frac{1}{2} \) cycle and 3000 cycles to failure were approximately 0.91 and 0.50, respectively. Using the damage model and size adjusted relationship from Equation 140, the applied stress ratios corresponding to \( \frac{1}{2} \) cycle and 3000 cycles to failure were approximately 0.96 and 0.48, respectively. The reasonable agreement indicates the damage model and size adjustment procedure used to obtain fatigue information on full size panels from small specimen fatigue results was valid. It should be noted that damage model and size adjustment procedure may be generally applied to other material; however, the calibrated damage model parameters are restricted only to transDeck\textsuperscript{TM} panels.
8. CONCLUSIONS

8.1 Summary and Conclusions

The formulation of a structural analysis model (DAP) based on finite strip method was presented to predict the structural behavior of prototype dry freight van trailer decking systems. The assumed displacement field in DAP took into consideration of the degrees of freedom to model the unsupported edges and the mid-span gap in the prototype dry freight van trailer deck assembly. DAP also considered the unequal elastic properties for the two sides of the cover in the deck system which corresponded to the use of two individual panels (one on each side of the mid-span gap) in the deck assembly.

DAP treats the decking as a cover plate stiffened by supporting beams. The cover plate can be made up of any material as long as it behaves linearly within the load range of interest. Materials including composite wood products (e.g., TransDeck™, parallel stranded lumber, laminated veneer lumber), fiber glass products, metal products, or composite wood and fiber glass products can be considered if their elastic properties are known. In this study TransDeck™ panels were considered to provide the necessary database for program verification. Given as inputs the elastic properties of cover and the applied wheel load, DAP predicted the deformations in the system, the bending stresses in the supporting I-beams, the parallel and perpendicular to grain bending stresses in the extreme fiber of the panel, and the rolling shear stresses at the interior fiber of the panel.

A comprehensive database on the mechanical properties of 3.2 and 2.5 mm (\(\frac{1}{8}\) and \(\frac{1}{10}\) inch) thick Douglas-fir veneers was developed. The database included information on bending, tension, and compression strength properties, shear moduli of rigidity, ultrasonic transmission time, and connection stiffness. Analyses of variance indicated that the mean strength properties of 3.2 and 2.5 mm thick veneers were significantly different for the parallel to grain direction but not significantly different for the perpendicular to grain direction at the 95% probability level. Statistical information and
distributions parameters was established for the various veneer strength properties so that simulations could be performed to model the strength properties of the veneers.

A trailer decking load simulator test facility was developed to perform full scale testing of prototype dry freight van trailer decking systems. A static and a cyclic test program were performed on four prototype decking systems using TransDeck™ panels. In the static test program, load versus deformation relationships of the various prototypes were obtained. Results agreed well with DAP predictions. A maximum peak deformation prediction error of 12% was observed. In the cyclic test program, the fatigue performance of the four prototype decking systems were established. The static strengths of the prototypes were also projected from the data. A maximum error of 10% was found when comparing static strengths predicted by DAP with those projected from test data. It was concluded that 1) DAP was successfully verified by the full scale test program and 2) the first generation TransDeck™ did not meet the structural requirements for use in typical dry freight van trailer decking systems where a fatigue life of 3000 cycles under a front axle loading of 73 kN (16500 lb) was required.

The fatigue performance of full scale panels was normalized with respect to the static capacity and expressed in terms of stress ratio. The results indicate the applied stress ratio corresponding to 3000 cycles to failure was approximately 0.47 and 0.56 for the regular and special veneer panels. DAP could be used to estimate the mean short term static capacity of a second generation TransDeck™. The fatigue performance of the new product at 3000 cycles of loading could be estimated using an approximate stress ratio of 0.5.

A companion small specimen testing program was conducted to establish their fatigue performance in bending mode with the appropriate stress history. Failure modes of the small specimens and the full size panels agreed well. The fatigue performance of small specimens were adjusted to full size panels through a size adjustment procedure using the Weibull weakest link theory. The size adjusted small specimen fatigue data and the full size fatigue data agreed well.
A damage model which took into consideration the stress history was calibrated to the small specimen fatigue results. This damage model parameters are specific to the TransDeck™ although the method can be generalized to other decking material. Good agreement between damage model predictions and small specimen fatigue performance was obtained. Using Weibull weakest link theory, the damage model predicted small specimen fatigue performance was converted to full size panel fatigue performance. The size adjusted model predictions and the full size fatigue data agreed well.

8.2 Future Research

There is an obvious need to develop a product which can meet the structural requirements for use in typical dry freight van trailer decking systems where a fatigue life of 3000 cycles under a front axle loading of 73 kN (16,500 lb) is required. Based on the theoretical framework developed in this study, guidance can be provided to arrive at a product with increased capacity. For example, consider a second generation laminated veneer wood product, it is possible that 15 to 16 ply panels made with 2.5 mm thick veneers are needed. Fiber glass reinforcements in both the parallel and perpendicular to face grain directions may also be required. If sufficient fiber glass reinforcement in the perpendicular to grain direction is available, the cross ply should be eliminated to avoid the predominant rolling shear failure mode at the cross ply of the first generation product. Such a new product should be also tested using both full scale and small specimens so that proper technical information is available for product marketing. If the new product can be proven commercially viable, simulation studies and reliability studies should also be performed to better define the product performance.

The static and fatigue performance of currently acceptable decking system using hardwoods should be evaluated so that a baseline performance level can be established. Since the hardwood decks are not moisture protected, the effect of moisture on the static and fatigue performance should be examined.

DAP has been shown to accurately predict the response of dry freight van trailer prototype
decking system. Decking systems for flat bed trailers and containers have a different configurations where the supporting I-beams are in turn supported at approximately the third points. The simple support conditions assumed for dry freight van trailer decking systems are no long valid. Major work is required to formulate a new structural analysis program to address the decking systems for flat bed trailers and containers. Such a program may involve the B-spline compound strip analysis.


Foschi, R.O. and Yao, Z.C. 1986a. Duration of load effect and reliability based design (single member). In Proceeding of IUFRO Wood Engineering Group meeting, Florence, Italy.


Kommers, W.J. 1943. The fatigue behavior of wood and plywood subjected to repeated and reversed bending stresses. U.S. Forest Products Laboratory Report 1327. Madison, Wis.


Parasin, A.V. 1981. Strength properties of 9.5 mm - 3 ply and 15.5 mm 5 ply western hemlock and amabilis fir sheathing grade plywood. Council of Forest Industries of B.C. Report 121. Vancouver, B.C.

Parasin, A.V. 1983a. Strength properties of 9.5 mm - 3 ply and 15.5 mm 5 ply western white spruce sheathing grade plywood - B.C. Council of Forest Industries of B.C. Report 124. Vancouver, B.C.


