SAMPLING HEIGHTS OF SECOND GROWTH COASTAL DOUGLAS-FIR IN FIXED AREA PLOTS

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ABSTRACT

Knowledge of tree heights is important for classifying sites, projecting growth and yield and estimating stand volume. Tree height is expensive and time consuming to measure so samples should be taken in the most efficient way possible. The impact of different sample designs and sizes on the fitting of height-diameter equations and subsequent prediction of volume is explored in this thesis. Several different height-diameter equation forms were compared for estimating height in second growth Douglas-fir. After selecting the best equation, a variety of simple sampling designs and sizes were compared using this equation. It was found that a uniform design, which was based on sampling tree height uniformly from 3 diameter classes, gave good results for height estimation. A "large" design, which concentrated 50% of its samples in the largest diameter class, gave the best estimates for tree volume. In plots less than 50 years old, it was found that sampling more than 16 tree heights produced diminishing benefits in height and volume estimates.

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Dedicated to God's glory...

1. INTRODUCTION

The measurement of tree heights is an important factor in forest management in British Columbia (B.C.) for many reasons. Selected tree heights are often used for site classification and growth and yield projections. Despite being important, the measurement of tree heights can be an expensive and time consuming process (although, the use of a laser device could greatly reduce time and cost of obtaining measurements). This is because tree height, unlike tree diameter at breast height[1] (dbh), must be indirectly measured or estimated on trees more than several meters tall (Ker and Smith, 1957). As a result, it is common to measure all tree diameters in an area (such as a sample plot), but only a certain proportion of the tree heights. For example, stand volume is sometimes calculated using measurements of all diameters in a sample and estimates of height based on a sub-sample of those trees. The objective of this sub-sample is to obtain adequate information to represent the relationship between tree height and dbh at a reasonable cost (Ker and Smith, 1957). Height is then related to diameter using some form of a mathematical equation (a height-diameter curve) which allows prediction of height for every dbh within the sample. If stand volume is the objective of the sample, volume per tree may be obtained from a volume table or (more commonly today) a mathematical equation which uses tree diameter and the measured or estimated height given by the height-diameter curve (Curtis, 1967).

[1]In B.C., breast height is defined as 1.3 m above ground level, taken from the high side on sloped or uneven terrain.

The B.C. Ministry of Forests has recommended that for permanent sample plots (psp's), each plot should have the dbh measured on every tree within the plot boundaries, but have height measurements made only on a subsample of 15 trees plus top height trees (Forest Productivity Councils of B.C. 1990). Given these guidelines, it is important to know how the size and distribution of height samples affects both height and volume estimation.

Some of the effects and consequences that result from employing different height sample sizes and designs in second growth Douglas-fir (*Pseudotsuga Mensiesii* (Mirb.) Franco) on fixed area plots were explored in this thesis. More specifically, the effects of altering the number of heights sampled in a plot on height diameter curve construction and volume estimation were examined. Knowing how sensitive volume estimates are to the number of heights measured can help avoid under- or over-sampling tree heights in cases where volume estimation is an important goal of the sample. Also, different sampling designs were tested to determine the effects that sub-sample allocation has on volume estimation and height-diameter curve construction.

This thesis has been organized in the following format. First, a literature review of height-diameter equations and tree height sampling is presented. Next the methods used in fitting height-diameter equations are described along with the process of simulating the various sampling designs. Results of the equation fitting and sampling are presented in the next section. This is followed by a discussion of results and implications. Finally conclusions and recommendations are presented in the final chapter.

2.0 LITERATURE REVIEW

2.1 Sampling for Height-diameter curves

While much attention has been paid to the development of mathematical height-diameter models, much less attention has been given to determining the number of sample heights required and which trees are most suitable for sampling. The number of height samples required to provide an estimate of stand volume for a given level of precision will be governed by several factors including the number of species present, the variation in tree heights, and the degree of correlation of tree height with dbh (Ker and Smith 1957). The variation in tree heights is not a concern when that variation occurs among, rather than within, dbh classes. If there is little variation within dbh classes, height-diameter curves can be derived which give very good results. However, some variation often does occur within dbh classes. For a single tree species, this is generally due to the position attained by the tree within the crown canopy (Ker and Smith 1957).

It has been noted that it is not necessary (or even desirable) to sample all trees in a stand randomly when taking a height sub-sample, but to limit the sample to specific diameter classes, as long as the selection of samples within those diameter classes is not biased (Bruce and Schumacher 1950; Ker and Smith 1957). For example, Trorey (1932) recommended sampling from two dbh classes, one near the maximum diameter present and the other at one-half that diameter. Modifying Trorey's method, Alexander (1945) used sample heights ranging throughout all diameter classes on permanent sample plots. Ker and Smith (1955) found that good results could be obtained by sampling two large diameter trees and two trees approximately 30 percent of the dbh of those trees, when applying the parabolic height diameter equation.

In a study on height estimation for red pine (*Pinus resinosa* Ait.) and white spruce (*Picea glauca* (Moench) Voss), Stiell (1965) found somewhat different results than Ker and Smith (1955) using the same parabolic equation. In this study, many more sample heights were required to obtain acceptable results (two heights per diameter class across the diameter range). Even so, this was a small proportion of the total population.

The size and distribution of the sample can be important when regression techniques are used to estimate the height-diameter relationship. Commonly, regression estimation is inefficient because samples are concentrated in one area within the range of the independent variable(s) while other areas are under-represented. This generally occurs because sufficient effort has not been taken to plan the sample in accordance with the sample objectives. Those objectives should be clearly stated before sampling begins (Demaerschalk and Kozak 1974). When developing a predictive regression equation there are two basic objectives which the equation must satisfy (Demaerschalk and Kozak 1974):

1. The regression must be useful for a given range of the independent variable(s).
2. The regression line should satisfy a minimum precision requirement[2] at the lowest possible cost.

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[2]Demaerschalk and Kozak (1974) defined the minimum precision requirement as "the required maximum confidence interval of the mean of Y_i for different X_i values".

These two objectives will drive the selection of both sample size and sample distribution. The sample range of the independent variable should equal or exceed the range specified in objective one. In general, a uniform distribution should be used if there is any doubt about the form of the relationship. If the relationship is known to be a simple linear one, sampling only at the upper and lower extremes of the independent variable will provide the most efficient design; however, this design does not permit a test for lack of fit. The required sample size will then depend entirely on the minimum precision (objective 2) and the sample distribution chosen (Demaerschalk and Kozak 1974).

The theoretical study of optimizing the quantity and distribution of samples falls into the field of optimal design, of which much work has been published. Unfortunately, much of the work is theoretical, and little has been written to facilitate the application of the theory to practical problems (Penner 1989; Ziegel 1984). While many criteria can be used to define optimality, the most common for linear regression involve minimizing the generalized variance of the parameter estimates (called D-optimality) or minimizing the maximum variance of the predicted response over the design region (called G-optimality) (Atkinson 1982).

In traditional optimal design, it is assumed that the cost of sampling and the precision requirements are constant over the range of the independent variable(s), both of which may be untrue in a practical application (Penner 1989). For example, in biomass studies, the cost of sampling a small tree may be many times lower than the cost of sampling a large tree. Precision requirements may also vary throughout the range of interest as well. It is common in forestry to require estimates to be within a percentage of the true mean (e.g., volume estimated to ± 10 %). To deal with these situations, Penner (1989) developed a procedure to weigh the variance function[3] of traditional optimal design by cost and precision. This resulted in a weighted optimum design which minimized the cost of the sample for a given precision requirement. In her study she found that a uniform design, while not as cost effective as the weighted optimum design, gave excellent results and could be preferable to the weighted optimum design in some circumstances despite costing more.

2.2 Desirable Characteristics of a Height-Diameter Equation

According to Curtis (1967), the function which is used to model the height-diameter relationship should be reasonable, even when data are not adequate to fully define the shape of the curve. Curtis (1967) suggested that height diameter curves should be moderately flexible and possess the following characteristics:

1. a graph of the curve should show a slope that is positive, approaching zero as diameter (D) becomes large;
2. the y intercept of the curve should occur at breast height (1.3 m); and
3. the curve should be easily fitted by linear regression methods.

[3] In optimal design, the variance function measures the the gain in knowledge provided by an observation taken at a given x (Penner 1989).

Today, requirement 3 is unnecessary with advancements in nonlinear least squares solution packages. It may be necessary to use a sigmoidal curve to meet requirement 2 without distorting the curve. If small trees are absent (or are not important), then this requirement may not be necessary (Curtis 1967). In fact, if small trees are absent, it may be inappropriate to force the model through the origin. In any case, if requirement 2 is applied, it is important that the potential problems of using a fixed-intercept regression are recognized.

2.2.1 Problems With Restricted Regressions

As previously mentioned, it is very common to restrict height-diameter equations so that they pass through a fixed point on the *y*-axis (in this case, at 1.3 m height). It is less common to test whether this condition is valid.

Even when it is logical to do so, imposing restrictions on regression coefficients must be regarded as a very strong assumption and should be justified before accepting the conditioned model (Kozak 1973). More specifically, this condition should only be applied if three conditions are met (Kozak 1973):

1. there must be good reason to impose restrictions on the coefficients;
2. the basic assumptions of the regression analysis must be met after the restriction is imposed;
3. the sampling should be organized in such a way that the restriction is justified for the sample, not for the population only.

It has been recommend that a conditioned model, called the "hypothesis" model (Freese 1964) should only be accepted over the unconditioned or "maximum" model after it has been tested in at least one of the following ways (Kozak 1973):

a) the hypothesis that the residual sum of squares for the hypothesis model is not significantly different from the residual sum of squares for the maximum model is tested;

b) the hypothesis model is tested for lack of fit; or

c) the residuals are plotted over the independent variable or over the predicted y's.

2.3 Comparing Regression Curves

Often, when regression models are being fit to sample data, the question of comparing different regression curves arises. It is not always immediately clear which model best describes the relationship between the dependent and independent variables. In cases where the same dependent variable is used, the root mean square residual (or the standard error of estimate) is usually adequate to compare regression curves based on the same sample (Furnival 1961). However, it is very common in forestry to apply models in which the dependent variable has been transformed. It is meaningless to compare the standard error of estimate for these models with any which do not use the same dependent variable. For example, the standard error of estimate from the model $\log(\hat{H}) = a + b\log(D) + c\log^2(D)$ (where $\hat{H}$ represents total height in metres; D represents diameter at breast height outside bark in cm; and a, b and c are regression coefficients) cannot be compared with the standard error of estimate from the model $\hat{H} = a + bD + cD^2$. In order to deal with this problem, Furnival (1961) developed an index of fit ("I") based on relative likelihoods which "has the advantage of reflecting both the size of the residuals and possible departures from linearity, normality and homoscedasticity". While this index was originally conceived and developed for comparing different volume equations to be used in the construction of volume tables, it is also suitable for comparing height-diameter models and has been used for that purpose in the literature (e.g., Curtis 1967).

2.4 Curve Fitting Techniques

Height diameter curves were once plotted in a freehand, graphical style but are now generally fitted using mathematical techniques. While freehand curves were considered to be accurate enough for use in local volume tables, the advantages of a least squares solution has long been recognized (*i.e.*, statistical comparisons, construction of confidence intervals and repeatable results (Meyer 1936)).

Today, the least squares technique is commonly employed when fitting height diameter curves to experimental data. However, several different procedures have been used historically in British Columbia. Before electronic computers were readily available, such curves were often fit by hand (Ker and Smith 1955). Mathematical techniques other than least squares have also been employed. Trorey (1932) developed a simple equation that gave a close representation of the relationship between total height and diameter at breast height in the following form: $\hat{H} = a + bD + cD^2$, which he did *not* derive using least squares. Since the outside diameter (D) at 4.5 feet (breast height in imperial units, equals

1.37 m) is equal to zero, $a = 4.5$. b represents the initial height growth rate (feet increment in height per inch increment of diameter) and c, a negative value, represents the rate at which the initial rate decreases (in feet per inch). The constants b and c can be determined when the heights at any two diameter classes are known (generally one of the classes was taken from the middle of the diameter class range and one from the upper end of the range (Ker and Smith 1955)). Average values of height ($\overline{H}$) and dbh ($\overline{D}$) for each diameter class are determined and substituted into the original equation to form two equations with two unknowns (b and c). The equations are then simultaneously solved for the unknowns.

While this was a convenient and simple method for expressing the relationship between height and diameter, its accuracy was dependent upon the assumption that the relationship is actually parabolic and that the averages selected are truly representative of the population averages (Ker and Smith 1955). Because of these assumptions, Alexander (1945) used an approximation similar to Trorey's (1932), but included sample trees throughout the range of dbh classes. To solve for the parameters, he used a short-cut method of least squares described by Bruce and Schumacher (1950, pp. 199-200).

Three methods of approximating the least squares solution for a parabolic height-diameter equation were recommended by Ker and Smith (1955). They found that these methods yielded results very close to those obtained by least squares and were superior to other mathematical approximations of the least squares solution for different model forms.

Today, the proliferation of digital computers and the widespread availability of statistical software packages has rendered graphical techniques, or mathematical approximations of least squares regression, obsolete. However, other regression methods do exist. It may, for example, be desirable to perform least absolute values regression to minimize the influence of outliers. However, least squares (both linear and nonlinear) is most commonly employed to fit height-diameter equations.

The ease with which least squares solutions can be obtained is certainly a huge benefit for all types of modelling problems facing forest managers. Generally, it is possible for people with very modest statistical backgrounds to obtain linear least squares solutions and many packages offer reasonable documentation to assist in the interpretation of results. However, one area which has not been well documented is the tendency for some modern statistical packages to incorrectly calculate certain tests of significance when the regression model has been restricted[4]. Specifically, many popular statistical packages generate incorrect values for R^2, significance testing, and incorrectly calculate confidence intervals when performing least squares regression through the origin.

2.5 Common Height Diameter Equation Forms

Many different equation forms have been applied to model the height-diameter relationship over the years. Several forces have driven the change in model forms including evolving technology and advances in biological and mathematical theory. Table 1 (from Johnson

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[4]Kozak, A. 1991. Personal communication.

and Romero 1991) is a summary of height-diameter models which reflects the range of forms. Note that any model in Table 1 which restricts the intercept to 4.5 (4.5 feet is equal to breast height in imperial units) could be fit without that restriction, or could be fit using an intercept of 1.3 meters for metric units.

Table 1. Common height-diameter models (after Johnson and Romero 1991).

Model	Authors
1. $\hat{H} = 4.5 + e^{\left(b_0 + b_1/D\right)}$	Dimitrou (1978), Murphy and Farrar (1987), Wykoff *et al.* (1982)
2. $\hat{H} = 4.5 + b_0 e^{\left(-b_1/D\right)}$	Curtis (1967), Zakrzewski and Bella (1988)
3. $\hat{H} = b_0 + b_1\sqrt{D}$	Van Deusen and Biging (1985)
4. $\hat{H} = 4.5 + e^{\left(b_0 + b_1 (D)^{b_2}\right)}$	Wang and Hann (1988), West 1979
5. $\hat{H} = b_0 + b_1 e^{\left(b_2 (D)^{b_3}\right)}$	Johnson and Romero, 1991
6. $\hat{H} = b_0 + e^{\left(b_1 + b_2 (D)^{b_3} + b_4 BA\right)}$	Dolph (1989). Larsen and Hann (1987), Wang and Hann (1988)
7. $\hat{H} = 4.5 + b_0 D + b_1 D^2$	Curtis (1967), Ker and Smith (1955), Snowdon (1981), Trorey (1932)
8. $\hat{H} = 4.5 + b_0 e^{\left(-b_1 (D)^{b_2}\right)}$	Arney (1985)
9. $\hat{H} = 4.5 + D/b_0 + b_1 D$	Ker and Smith (1955), West (1979)
10. $\hat{H} = 4.5 + b_0\left(1.0 - e^{\left(-b_1 D\right)}\right)^{b_2}$	Ek *et al.* (1984), Farr *et. al.* (1989), Meyer (1940)

where: $\hat{H}$ = estimated tree height
e = base of natural logarithm ($e \approx 2.718282$)
D = dbh outside bark
b_0, b_1, b_2, b_3, b_4 = regression coefficients
BA = basal area

It can be seen that height diameter models may be linear or nonlinear in their parameters. In some cases (e.g., model 2) a nonlinear model can be transformed into an equivalent linear form (in this case, by taking logarithms of both sides of the equation). In this case, the model is described in the literature as being *intrinsically linear* [5] (Draper and Smith 1966).

2.5.1 Bias due to logarithmic transformations

It has been noted in the literature that logarithmic transformations of the independent variable result in systematic underestimation (Baskerville 1972, Flewelling and Pienaar 1981). There have been several proposed methods of dealing with this bias. Baskerville (1971) suggested that a correction taken from Brownlee (1967) would be appropriate for transforming predicted values from logarithmic to arithmetic (untransformed) units. This involved adding ½ of the sample variance of the logarithmic equation to the predicted values before transforming to arithmetic units. Snowdon (1990) recommended that a ratio of the arithmetic sample mean and the mean of the back-transformed predicted values from the regression be used to correct for bias.

Some models (e.g., model 6) are nonlinear in their parameters and can not be transformed to an equivalent linear model. These models are described as being *intrinsically nonlinear* (Draper and Smith 1966). For purposes of convenience, the term "nonlinear" will be used to describe only this class of model.

2.5.2 Features of Nonlinear Models

A nonlinear regression model can be described as possessing the following properties (Weisberg 1985):

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[5] In some cases, this class of models has been described as *nonintrinsically nonlinear* (Draper and Smith, 1966).

1. the function relating the dependent variable (response) to the independent variables (predictors) is a nonlinear function of the parameters;
2. unlike the linear model, there is no need for a direct correspondence between predictors and parameters;
3. parameterization is not unique, so many nonlinear regression models are equivalent; and
4. as in linear regression, the errors are assumed to be independent and normally distributed. Constant variance is also assumed, but this assumption can be relaxed using weighted least squares, as with linear regression.

Computing least squares estimates for nonlinear models can be a complex process which usually involves an iterative function minimization routine. There are many such routines, and it is not uncommon to use several different routines in the search for a least squares solution for a single nonlinear model (Weisberg 1985). Many of these routines require that the first (and sometimes second) derivative of the model be computed. All iterative routines require starting values (initial estimates of parameters).

The evaluation of nonlinear regression is not as well defined as it is for linear regression. Inferential statements lean very heavily on normality and are only accurate for large samples (Weisberg 1985). Estimates of standard errors produced by many computer packages can be seriously in error (Weisberg 1985; Wilkinson 1989).

Despite these difficulties and potential problems, nonlinear regression is desirable in many instances. For example, it may be completely inappropriate to model relationships that are nonlinear in nature with a linear function. Even some models which can be transformed to an equivalent linear form may be better handled with nonlinear techniques if the transformation results in a log-normal error distribution. Also, it may be safer to extrapolate nonlinear rather than linear functions (Payendah 1983). In some cases, it is possible to theoretically interpret the parameters of nonlinear models.

In any case, both linear and nonlinear models are used to describe the height-diameter relationship. The final choice of the model form used should ultimately depend on the purpose of the model, the quality and quantity of the available data, the ease with which it can be fit, and the overall quality of the fit.

3.0 METHODS AND ANALYSIS

3.1 Data Preparation

The data for this thesis were provided by Macmillan Bloedel, Ltd. and consisted of second growth Douglas-fir permanent sample plots for which all heights and diameters were measured. The plots were mostly 0.1 acres (.0405 ha) in area, although a few were slightly smaller at 0.0401 ha. In total there were 252 complete plots, many of which had been measured repeatedly. After receiving the data, a series of computer programs were written to facilitate analysis. First, header lines were stripped and delimiters added to enable easy importing to microcomputer statistical packages. Plots which contained less than 80 percent Douglas-fir (by stems) were removed, as were those which had been subjected to thinning. For those plots with more than one measurement, only the initial measurement was used to ensure that each tree appeared only once in the database.

The data were analyzed graphically to identify any obvious data points which would exert undo influence (*i.e.* outliers) on height diameter equations. Figure 1 shows the height-diameter relationship for all of the trees (5149 trees in total) and indicates that most of the points fall close together, with the exception of a few obvious outliers that were eliminated. Since there were no comments in the data set, it is not known if these trees were damaged, had broken tops, or were the result of measurement or data entry errors. The final data set (based on 5134 trees from 114 plots) is graphically presented in Figure 2.

Figure 1. Initial data set

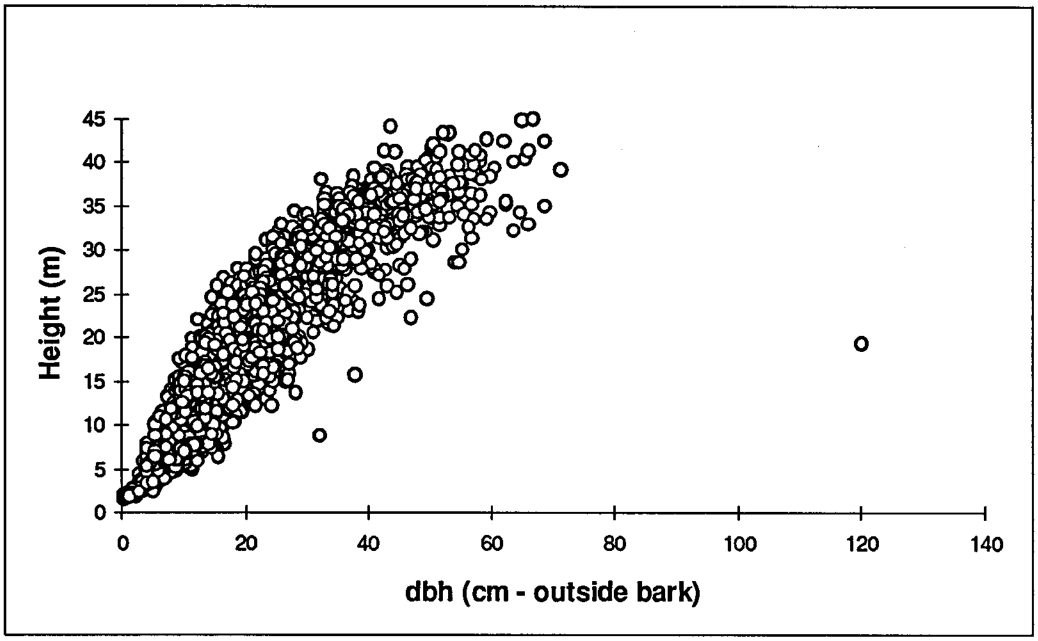
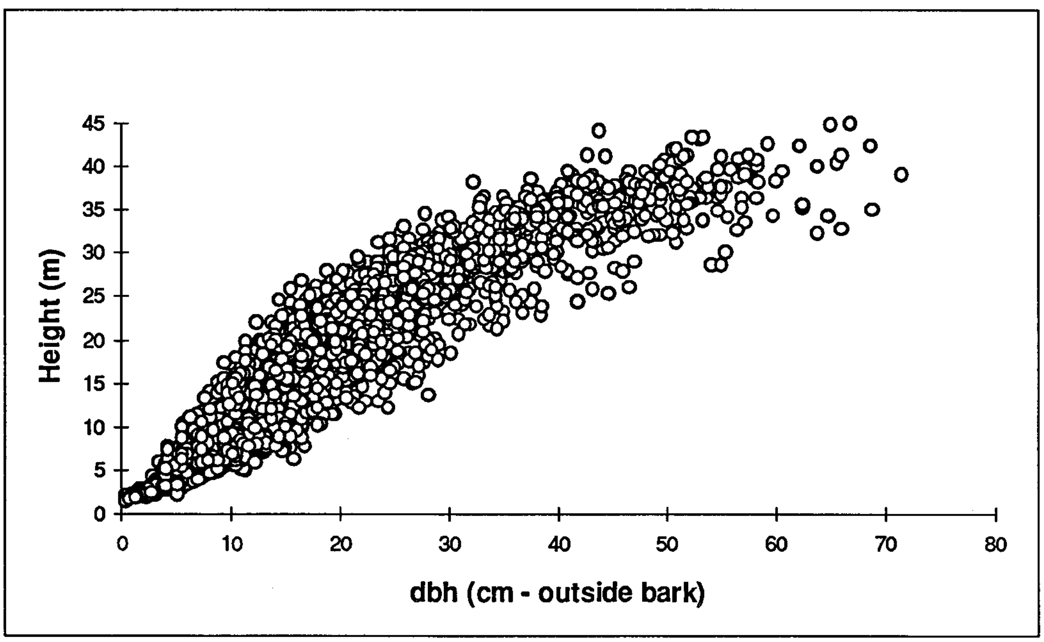


Figure 2. Final data set



3.2 Fitting Height Diameter Curves

Six different height-diameter models were fit to the final data set. These models were compared to determine the "best" model for height-diameter prediction. Of the six different models used, three involved transformations of the dependent variable (height) and three did not. While no rigorous criteria were used in selecting these six models from the many available in the literature, these were chosen because they would be relatively simple to use in the sample simulation. Nonlinear models would have presented difficulties in estimating regression coefficients. Models with a fixed intercept would have likely introduced serious bias in some of the samples. Table 2 shows the models used and the publisher of each model. In the case of models 3 and 6, the author (Curtis, 1967) suggested that the model be developed using either forward selection or backwards elimination.

The models were compared using Furnival's index of fit "I" (Furnival 1961) and by plotting the curves and the residuals. After comparing, a "best" model was selected for use in testing the sampling designs.

3.3 Simulating Sampling Designs

A variety of sampling designs were simulated. The plots were stratified into three site index classes and three age classes (Table 3). Site index (based on King 1966) was the height in metres at a reference age of 50 years (breast height age). An attempt was made to randomly select 5 plots in each class to reduce the quantity of data generated during the sample simulation and to provide representation across all age and site index classes. This proved impossible due to limitations in the data. As can be seen, some classes had no plots and some had less than 5 plots. It would have been preferable to have plots representing all age and site index classes as this would allow a better interpretation of the sampling requirements for stands of different age and site classes. The number of trees per plot varied for the selected plots, ranging from 42 to 124 trees.

Table 2. Height (H) diameter (D) models* and authors

Model	Equation	Author
1.	$\hat{H} = b_0 + b_1 D + b_2 D^2$	Curtis (1967), Strand (1959), Prodan (1965)
2.	$\hat{H} = b_0 + b_1 \log(D)$	Curtis (1967), Strand (1959), Prodan (1965)
3.	$\hat{H} = b_0 + b_1 D + b_2 \sqrt{D} + b_3 \frac{1}{\sqrt{D}} + b_4 \frac{1}{D}$	Curtis(1967) - Fitted in a stepwise fashion with non-significant terms deleted.
4.	$\widehat{\log(H)} = b_0 + b_1 \frac{1}{D}$	Curtis(1967), Zakrzewski and Bella (1988)
5.	$\widehat{\log(H)} = b_0 + b_1 \log(D)$	Curtis (1967)
6.	$\widehat{\log(H)} = b_0 + b_1 D + b_2 \sqrt{D} + b_3\left(\frac{1}{D}\right) + b_4\left(\frac{1}{\sqrt{D}}\right) + b_5\left(\frac{1}{D^2}\right)$	Curtis (1967) - Fitted in a stepwise fashion with non-significant terms deleted.

*In all cases, log refers to base 10 logarithm.

Table 3: Number of plots by age and site index classes

	Age Class		
Site Index Class	1. (10 - 30)	2. (31 - 50)	3. (> 50)
1. (< 25m)	-	4	5
2. (25 - 35m)	4	5	4
3. (> 35m)	5	3	-

After selecting the plots (randomly where possible) for the sample simulation, 5 different sampling designs were simulated for 6 different sample sizes. Each sample design and size was repeated 5 times for each plot. The sampling was simulated using a program called SampleSim (a copy can be obtained from the author on request). Individual plots were first extracted from the data set. Within each plot, trees were sorted by diameter. After sorting, each plot was stratified into three classes based on diameter size. If the plot could not be evenly divided into three classes, the extra trees (two at most) were randomly assigned to classes ensuring that class sizes never differed by more than one within a plot. Each plot was then "sampled". The program incremented the number of trees sampled in each plot - ranging from 8 sample trees to 28 in steps of 4 resulting in 6 sample sizes (n = the number of trees in the sample) for five different designs:

1. Random - all n trees were selected randomly from the plot.
2. Extremes - 1/2 of the n trees were sampled from the largest and 1/2 from the smallest diameter classes. None were taken from the middle.
3. Small - 1/2 of the n trees were taken from the smallest diameter class, 1/4 from the middle and 1/4 from the large diameter classes.
4. Uniform - The n trees were taken uniformly throughout all diameter classes.
5. Large - 1/4 of the n trees were taken from the smallest diameter class, 1/4 from the middle and 1/2 from the large diameter classes.

The program used a simple routine to randomly select trees without replacement from a diameter class. This ensured that there was no bias in selecting trees from the established diameter classes.

When a plot was sampled in any given design, several different arrays were created. The first held all the dbh measurements and three others held height values: one for all the height measurements; one for the sample heights (all elements in this array were set to zero at the start of each sample); and, one array of size n to hold measured heights for calculating regression coefficients. Sampling resulted in replacing some of the zeros in the sample height array with measured heights. These same values were used to estimate regression coefficients for the model selected as the best. The coefficients were then used to estimate height values for those remaining elements of the sample height array which had a value of zero.

Occasionally, the regression coefficients produced extremely unusual and unrealistic height estimations. This generally only occurred with small sample sizes and was characterized by a very large intercept and extremely large height estimations on very small trees. This condition is tested for in the program, and, if detected, another sample was taken using the design and sample size in question. While this introduced some bias into the results, it is very likely that operationally, samples such as these would either be rejected or fit with a different equation which did not give such poor estimates.

After sampling, the standard error of estimate was calculated for each regression equation and stored in an output file.

3.4 Height Estimate Comparisons

The sample heights were compared against the true heights and the following values were generated for each plot and sample and stored in the output file:

1. Mean deviation (bias). This is the average of the tree by tree differences between measured height and estimated height (equal to zero for measured trees). If there were no bias, mean deviation would be equal to zero.
2. Minimum deviation. This is the smallest (largest negative) difference between measured and estimated height.
3. Maximum deviation. This is the largest (positive) difference between measured and estimated height.
4. Standard deviation of differences. This is the standard deviation of the differences between measured and estimated height.
5. Mean absolute deviation (MAD). This is the average of the absolute value of the tree by tree differences between measured and estimated height (equal to zero for measured trees).

3.5 Volume Comparisons

Two volumes were estimated for each tree. The "true" volume, based on measured heights and diameters, and estimated volume, based on estimated heights and measured diameters. In the latter case, if a tree was sampled (*i.e.* height was measured), the measured height was used in the volume calculation (meaning that true and estimated volumes were equal for sample trees). The B.C. Ministry of Forests volume equation (Watts, 1983) was used to calculate tree volumes:

$$\hat{V} = 10^{-4.319071+1.81382\log(D)+1.04242\log(H)}$$

where: $\hat{V}$ = estimated volume (m^3);

D = diameter outside bark (cm);

H = total tree height (m); and

log = base 10 logarithm.

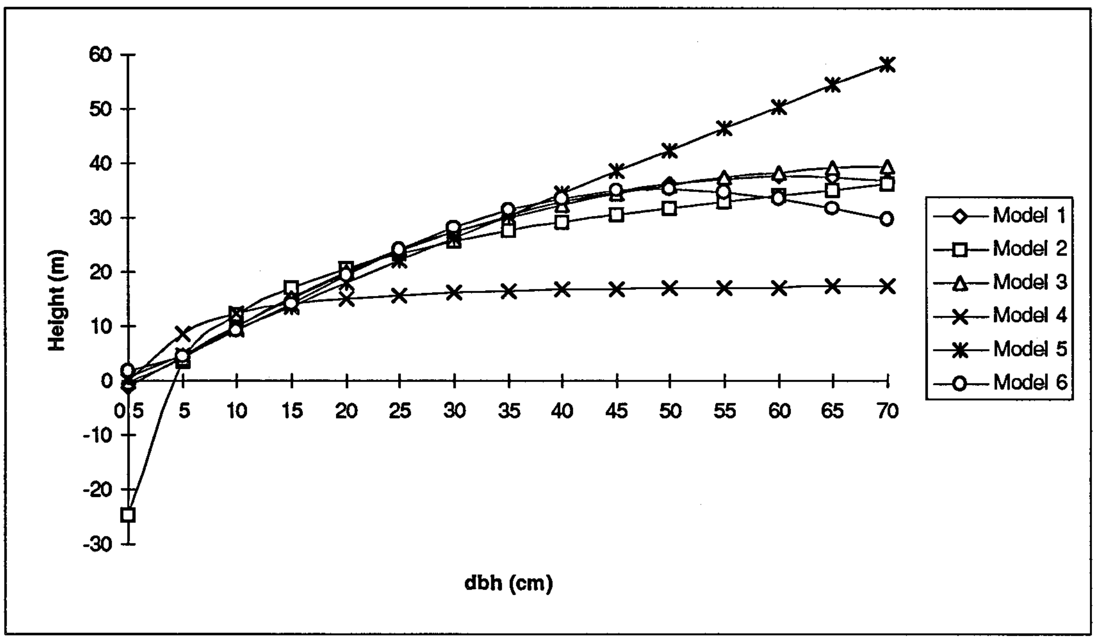
The estimated volumes were then compared against the true volumes using the same statistics used for height.

4.0 RESULTS

4.1 Height Diameter Curves

All models tested were significant at an $\alpha = 0.05$ level, although there were some very large differences among the shapes of the models. Figure 3 shows the shapes of the models across the dbh range used in fitting (from 0.5 cm to 71 cm). Models 1, 3, 5 and 6 performed similarly for diameters of up to about 35 cm. Model 2 predicted negative heights for any trees below about 4 cm in dbh. Model 4 predicted considerably smaller heights than the other models with dbh's larger than 15 cm. Model 6 predicted decreasing heights with dbh's larger than about 50 cm.

Figure 3: Six models compared



The fit statistics for the six models are summarized in Table 4. Normally, the square root of the mean square residual (or the standard error of estimate) can be used to determine the model which gives the best fit. However, neither this, nor the multiple R^2 values can be compared when the dependent variable has been transformed (Furnival, 1961). Therefore, Furnival's index of fit "I" was used as the basis for comparison.

Table 4: Height diameter models compared

Model	Equation	Multiple R^2*	Standard Error	I
1	$\hat{H} = -1.8057 + 1.287D - 0.0105D^2$	0.899	3.21092	3.2109
2	$\hat{H} = -16.2470 + 28.3340\log(D)$	0.822	4.26281	4.2628
3	$\hat{H} = -76.3726 - 1.1632D + 22.2295\sqrt{D} + 99.6243\left(\frac{1}{\sqrt{D}}\right) - 40.0196\left(\frac{1}{D}\right)$	0.901	3.18431	3.1843
4	$\widehat{\log(H)} = 1.2629 - 1.6941\left(\frac{1}{D}\right)$	0.427	0.2455	7.3832
5	$\widehat{\log(H)} = 0.0262 + 0.9425\log(D)$	0.884	0.11031	3.1313
6	$\widehat{\log(H)} = -0.444 - 0.0402D + 0.5651\sqrt{D} + 0.1543\left(\frac{1}{D}\right)$	0.902	0.10146	2.880

* Standard errors for models 4, 5 and 6 are in logarithmic (base 10) units. The sum of squares values used to calculate R^2 for models 4, 5 and 6 are in logarithmic (base 10) units.

Using this method to compare height-diameter curves, Curtis (1967) found that there were surprisingly few differences between curves he tested. He concluded that "almost any reasonable and moderately flexible curve will give similar values of I". For the most part this was the case here, with models 2, and especially 4, being exceptions. As a result, determining the "best" model also involved other criteria such the ease with which the relationship can be fit, and, very importantly, the result of analyzing graphs of the residual plots. After determining the index of fit for each model, residual plots (residual values versus predicted values) were generated to see if any of the models displayed obvious lack of fit (Figures 4 to 9). This process of plotting residuals also proved to be a valuable step in comparing equations.

Figure 4: Residual Plot for Model 1

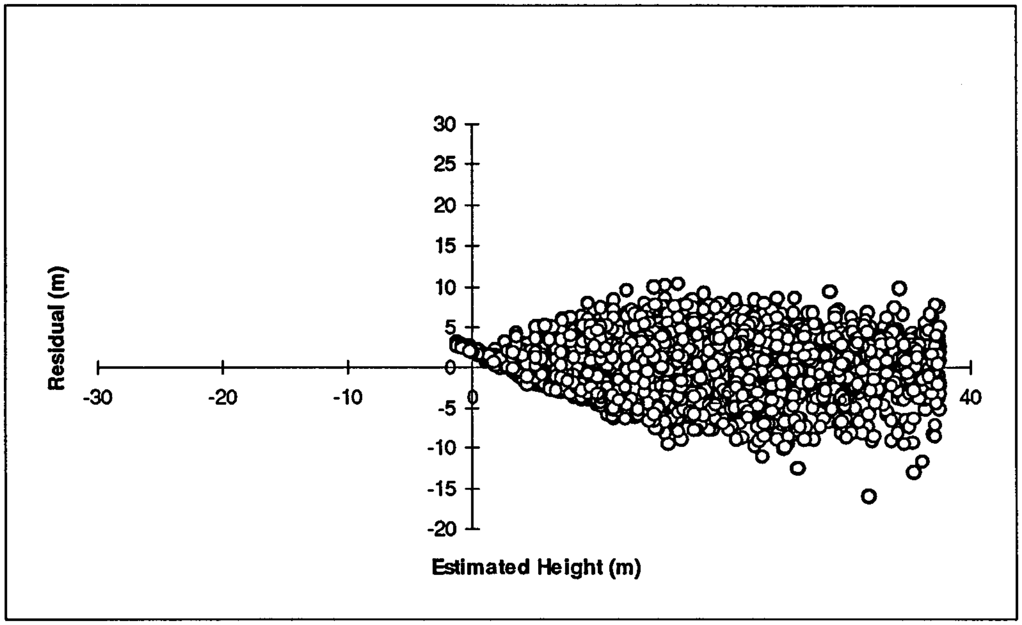


Figure 5. Residual Plot for Model 2

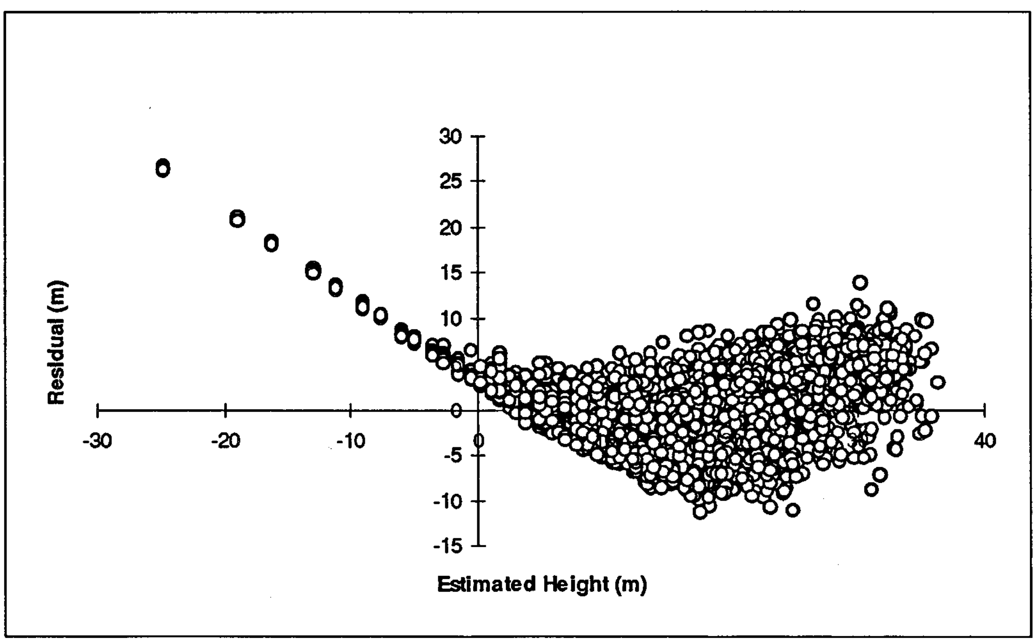


Figure 6. Residual Plot for Model 3

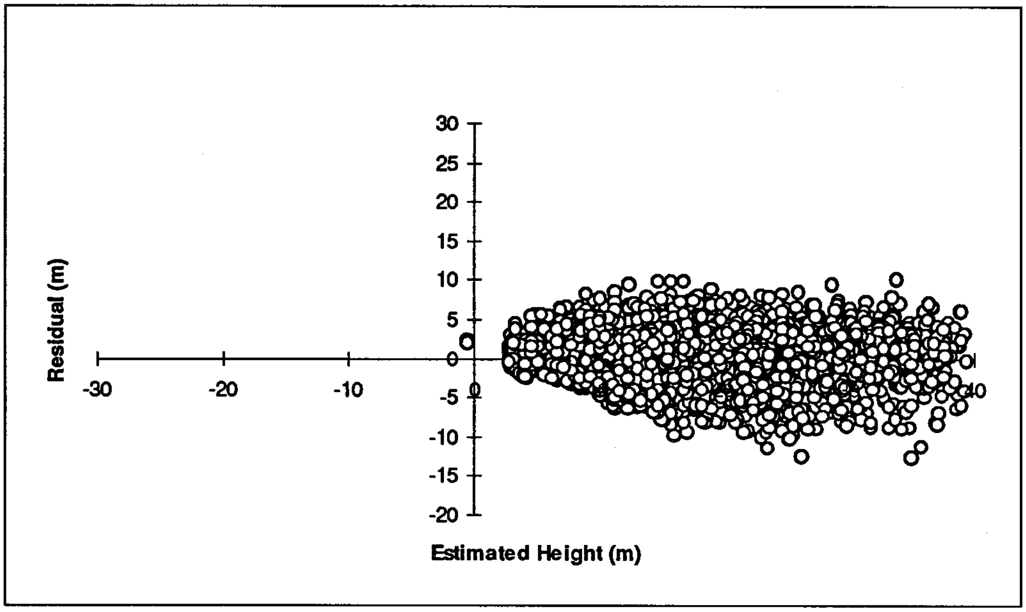


Figure 7. Residual Plot for Model 4

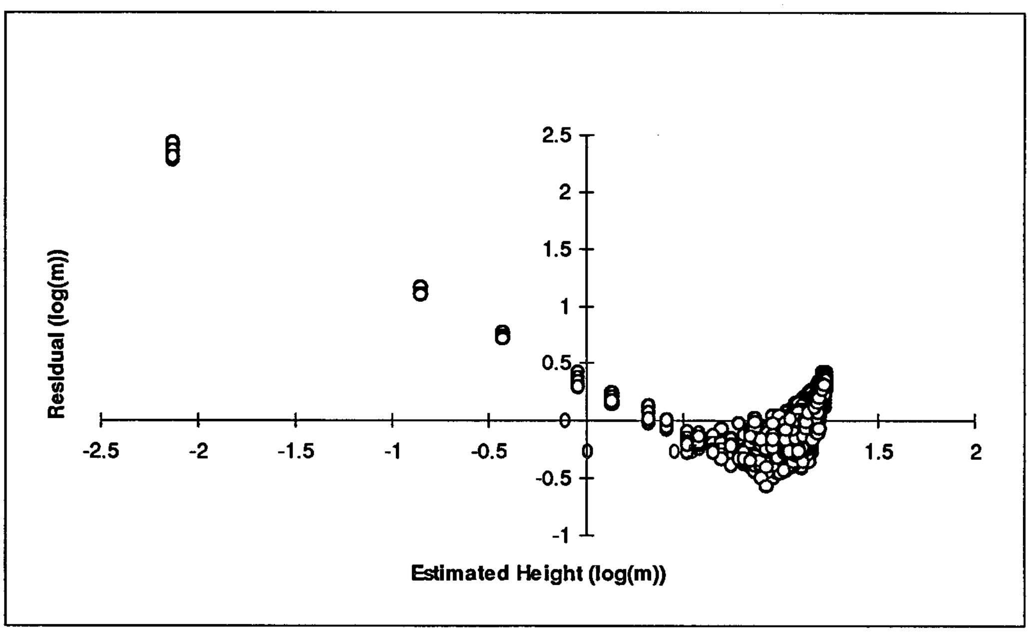


Figure 8. Residual Plot for Model 5

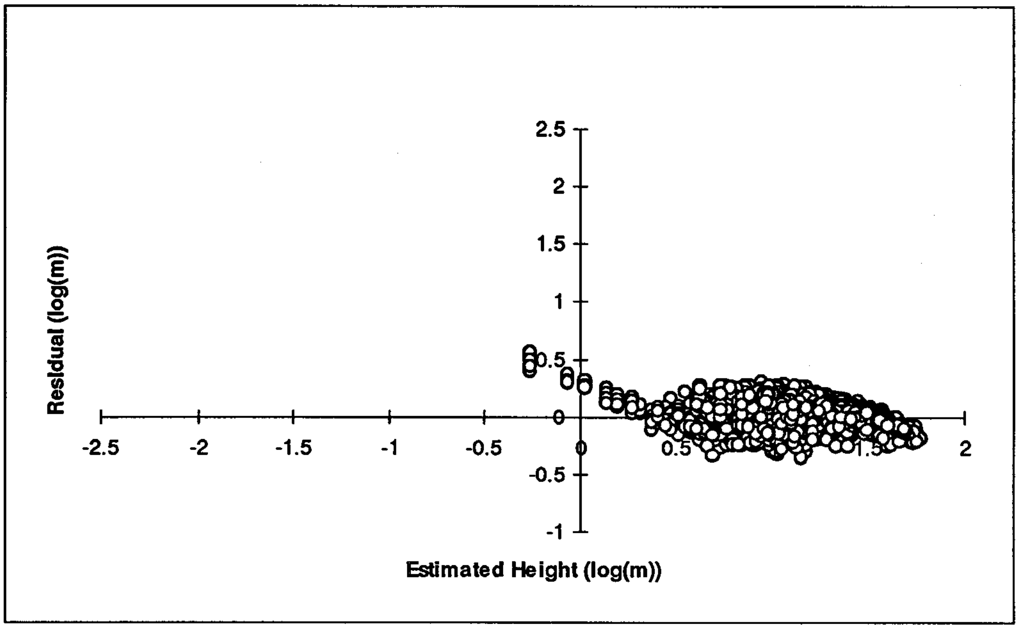
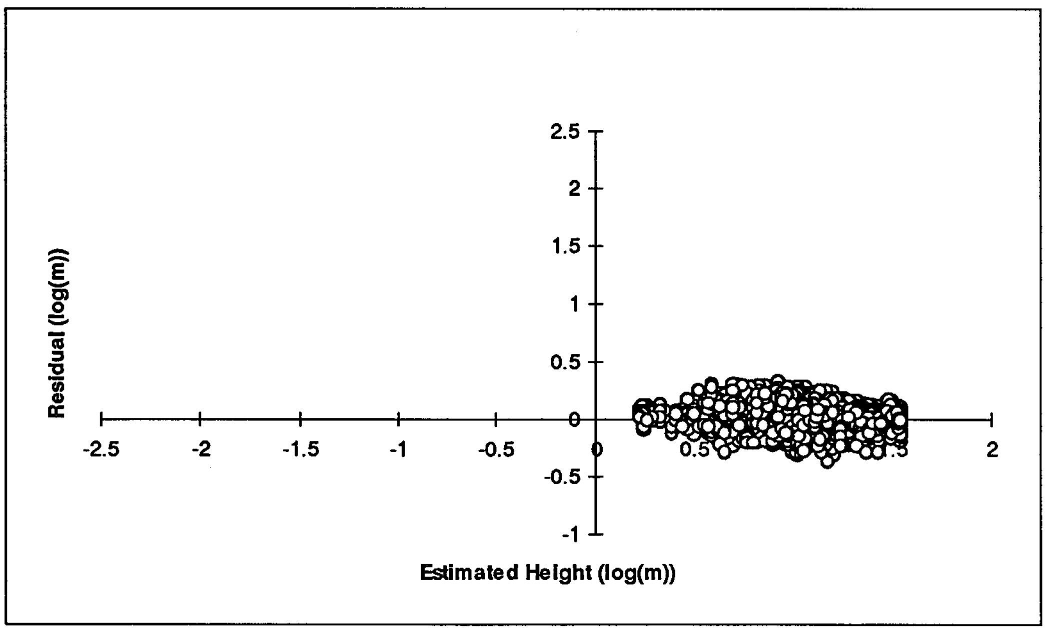


Figure 9. Residual Plot for Model 6



It can be seen from the residual plots that some models fit the data much better than others. Models 1, 3 and 6 showed a reasonable distribution of residuals while models 2, 4 and 5 displayed a lack of fit. Any of models 1, 3 or 6 would probably be satisfactory.

When the input data were sorted by dbh, it was apparent that diameters were repeated, some many times. This is an ideal situation to apply a lack of fit test. With repeated measurements, the sum of squares residual can be partitioned into two new sum of squares values: pure error and lack of fit. A simple F test can be applied to test for lack of fit. In the absence of repeated measurements, it is possible to apply an approximate lack of fit test based on clustering the data (Daniel and Wood 1981). However, this test is very sensitive to the clustering algorithm used and will give different results with different clustering methods (Weisberg 1985).

A lack of fit test was applied to models 1, 3 and 6 to facilitate the process of choosing the best model. Appendix 1 shows the coefficients and analysis of variance for all models, with the lack of fit test applied for models 1, 3 and 6.

Both models 1 and 3 tested significantly for lack of fit at an $\alpha = 0.01$ level. This was not apparent in the residual plots, but the huge number of points may have obscured some trends. The lack of fit test was not significant for model 6. Because of this, and the fact that model 6 had the lowest I value (i.e., the best fit index) and a reasonable distribution of residuals, it was selected as the "best" model.

4.2 Sampling Designs

4.2.1 Height estimation

To clarify results, the sampling simulations were summarized graphically, first comparing the effects on height estimation and then the effects on volume (refer to Appendix 2 for the tabular results of sampling).

4.2.1.1 Mean deviation in height

Figures 10 through 16 show the mean deviations in height by age-site classes, averaged for all plots in each class and for the 5 sample repetitions.

Figure 10. Mean deviation in height for AGE1SI2

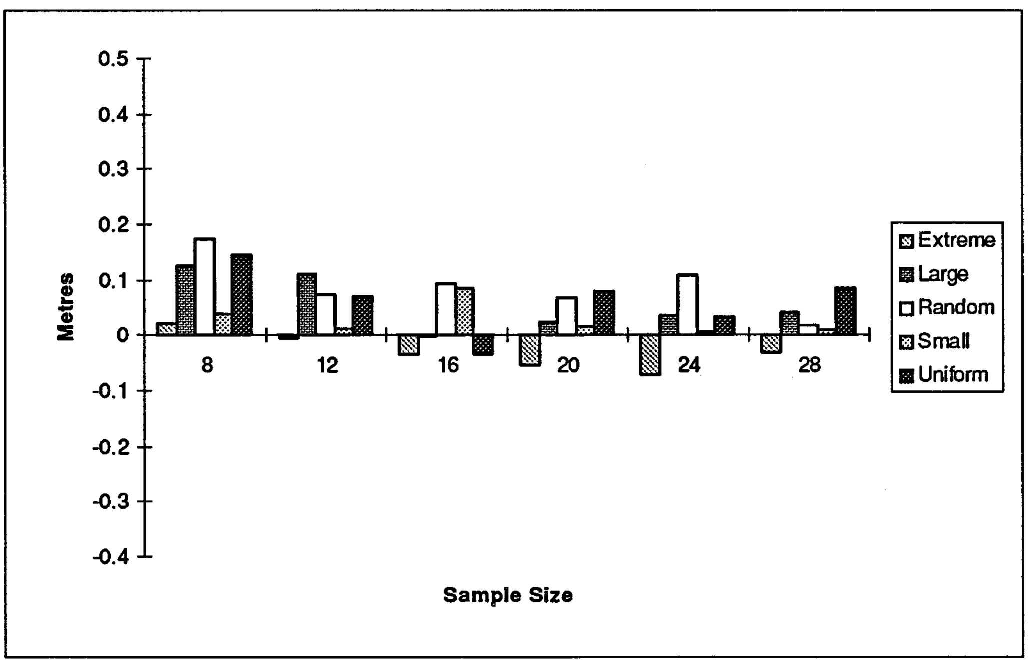


Figure 11. Mean deviation in height for AGE1SI3

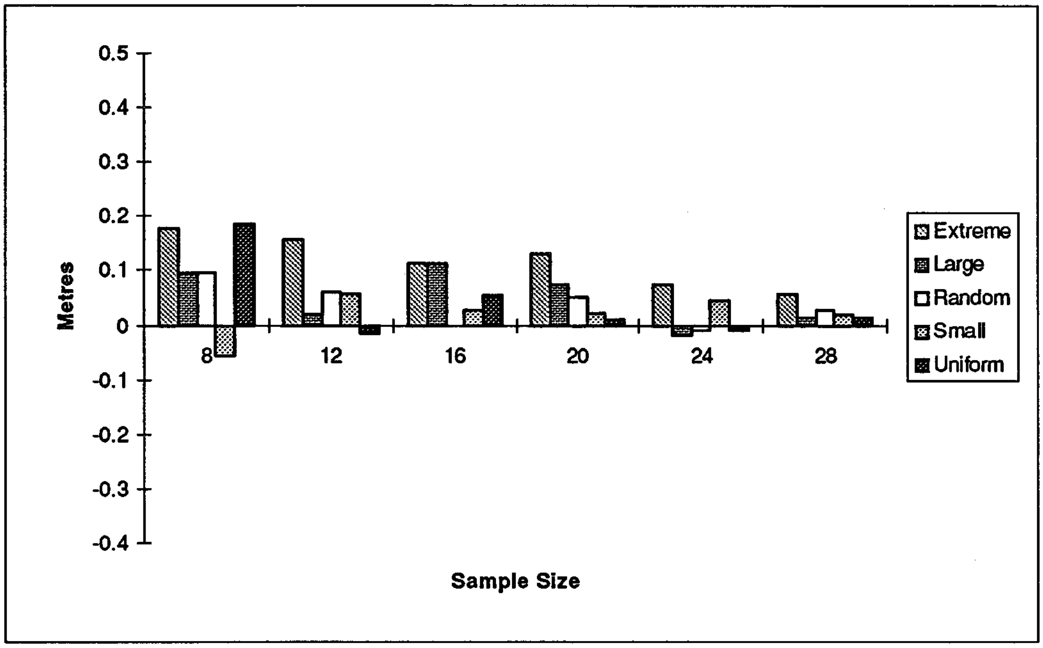


Figure 12. Mean deviation in height for AGE2SI1

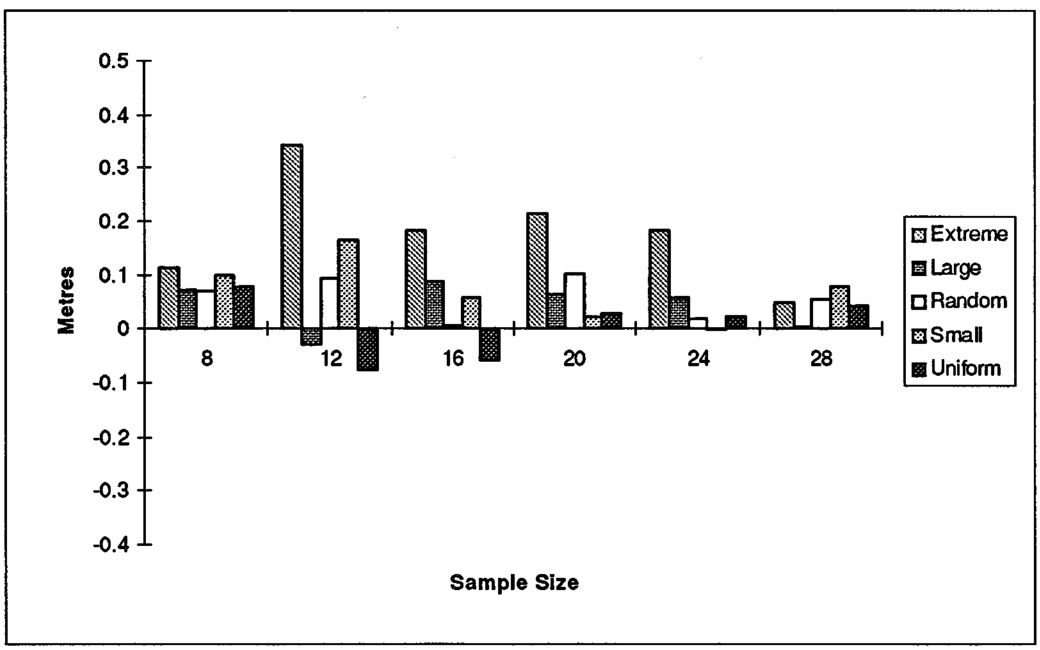


Figure 13. Mean deviation in height for AGE2SI2

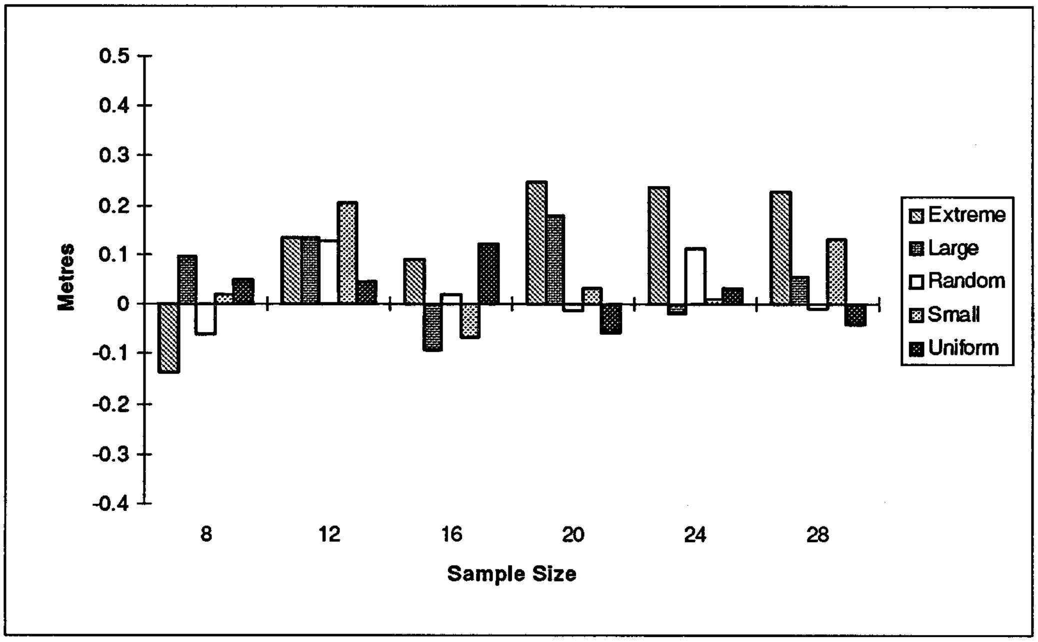


Figure 14. Mean deviation in height for AGE2SI3

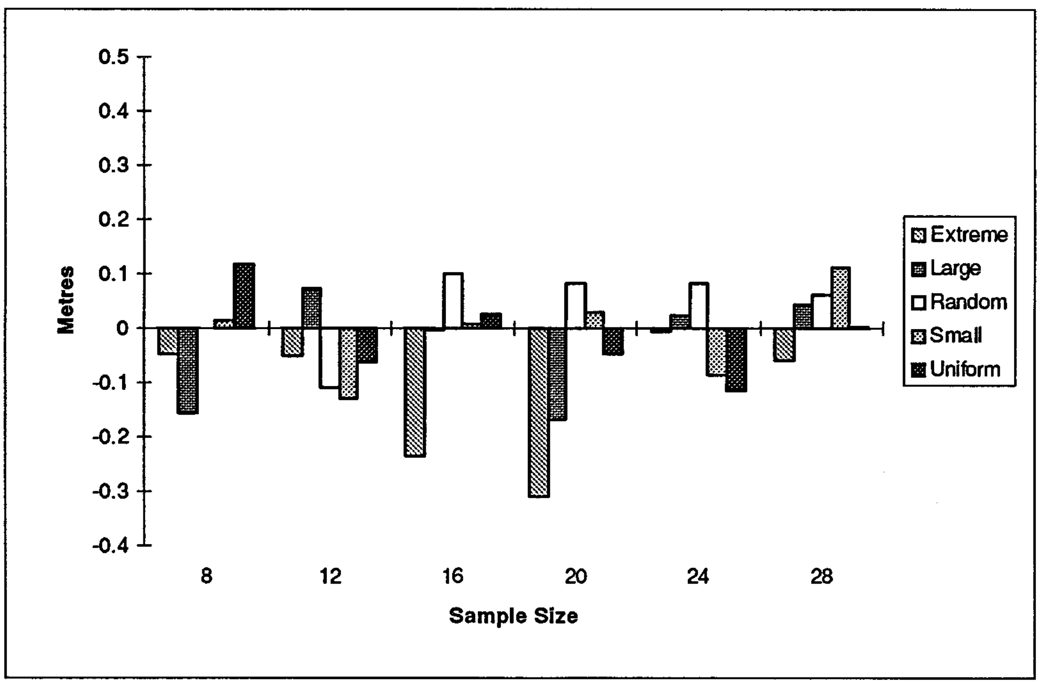


Figure 15. Mean deviation in height for AGE3SI1

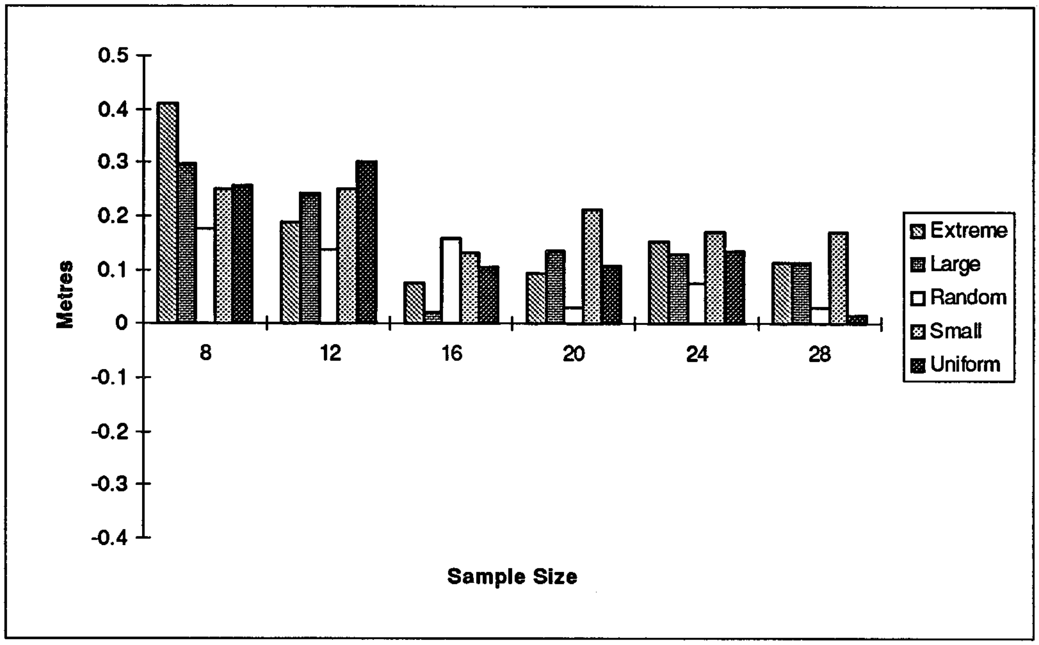
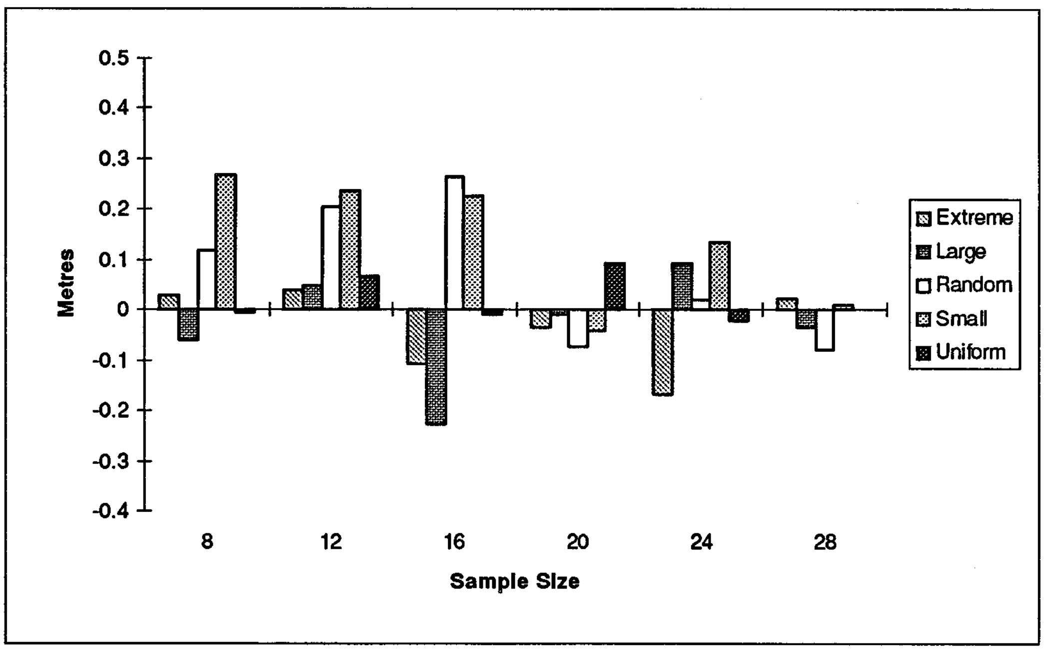


Figure 16. Mean deviation in height for AGE3SI2



These charts show that the mean deviation in height varied considerably with the different age class - site index groupings. Specifically, the older plots tended to show greater mean deviation than the younger plots, especially with small sample sizes. In age class 1, there was not much change with different sample sizes. In age class 3 - site class 1, the mean deviation was reduced when sampling 16 or more trees. In most cases, the extreme design produced quite large mean deviations. The uniform design usually displayed small mean deviations. In general, mean deviation was usually positive indicating that tree height was most often under-estimated.

4.2.1.2 Average maximum and minimum deviation in height

Figures 17 to 23 show the average maximum deviation in height for the age class - site index groupings. The values in the charts represent the average of the largest positive deviations for a given age - site index class. Figures 24 to 30 show the average minimum deviation in height for the age - site index groupings. These values represent the average of the largest negative deviations for a given age - site index class.

Figure 17. Average maximum deviation in height for AGE1SI2

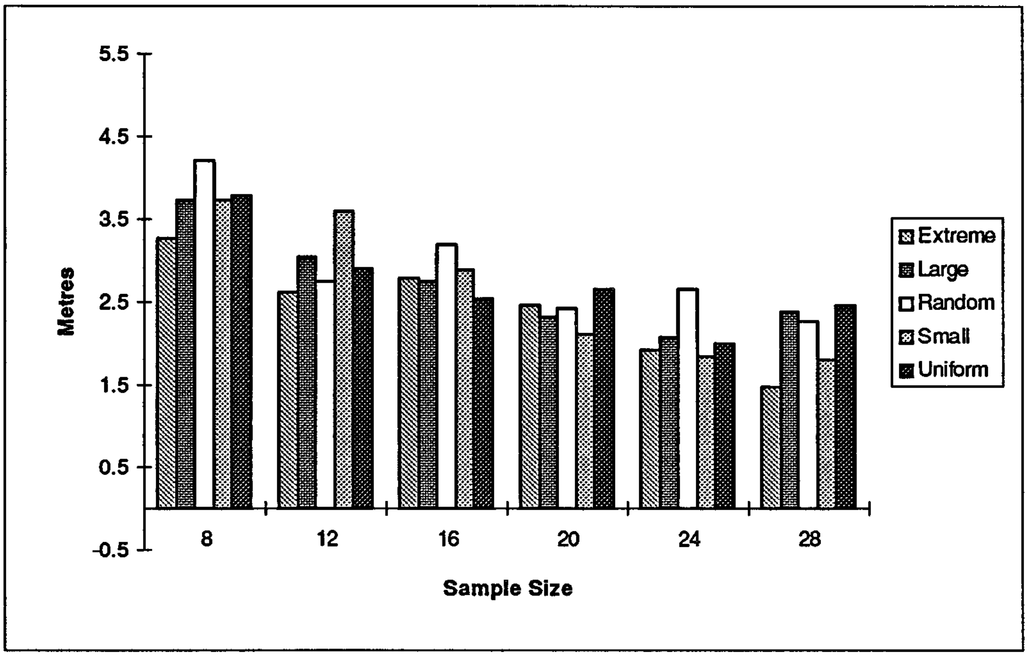


Figure 18. Average maximum deviation in height for AGE1SI3

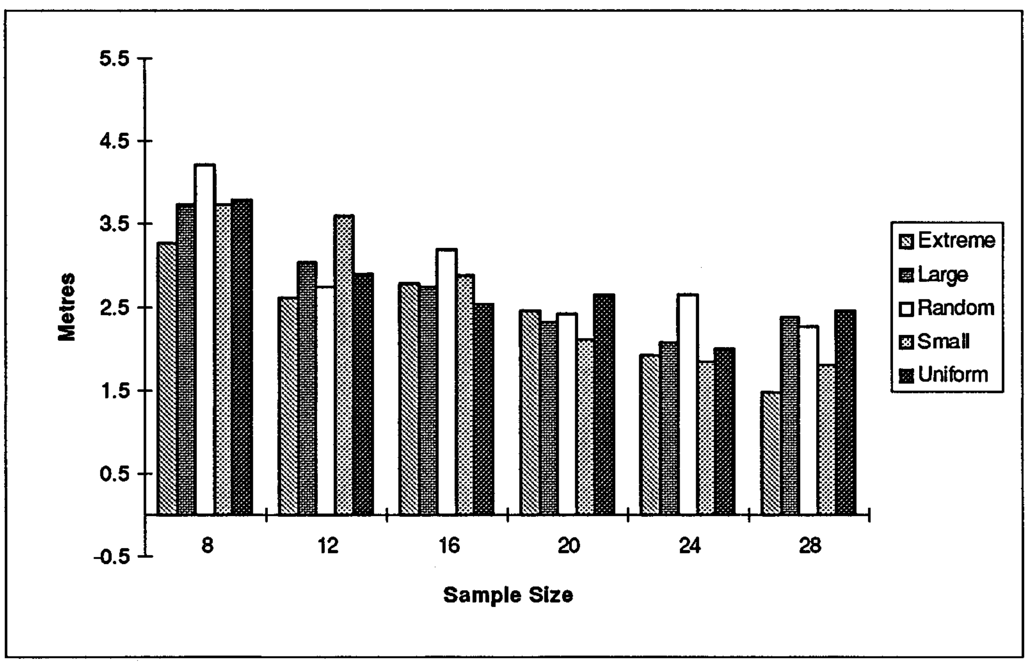


Figure 19. Average maximum deviation in height for AGE2SI1

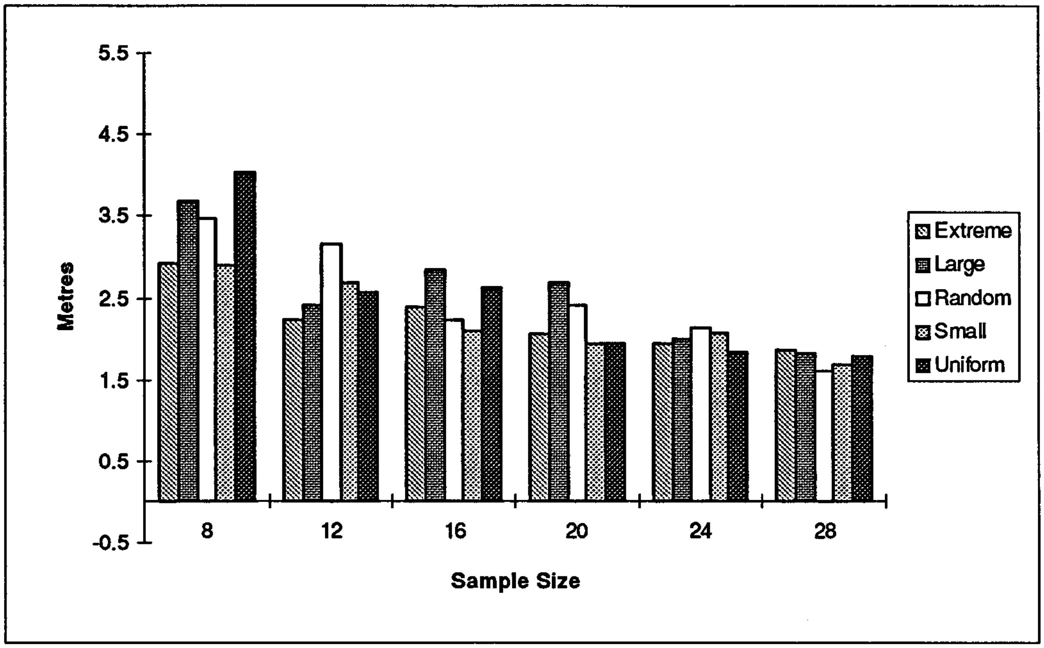


Figure 20. Average maximum deviation in height for AGE2SI2

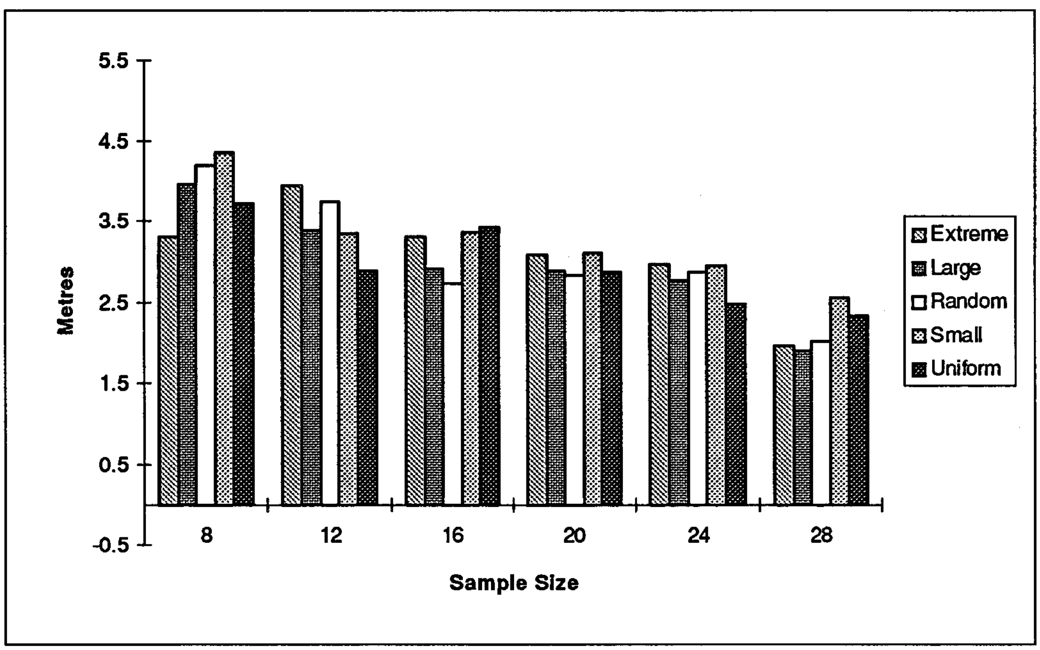


Figure 21. Average maximum deviation in height for AGE2SI3

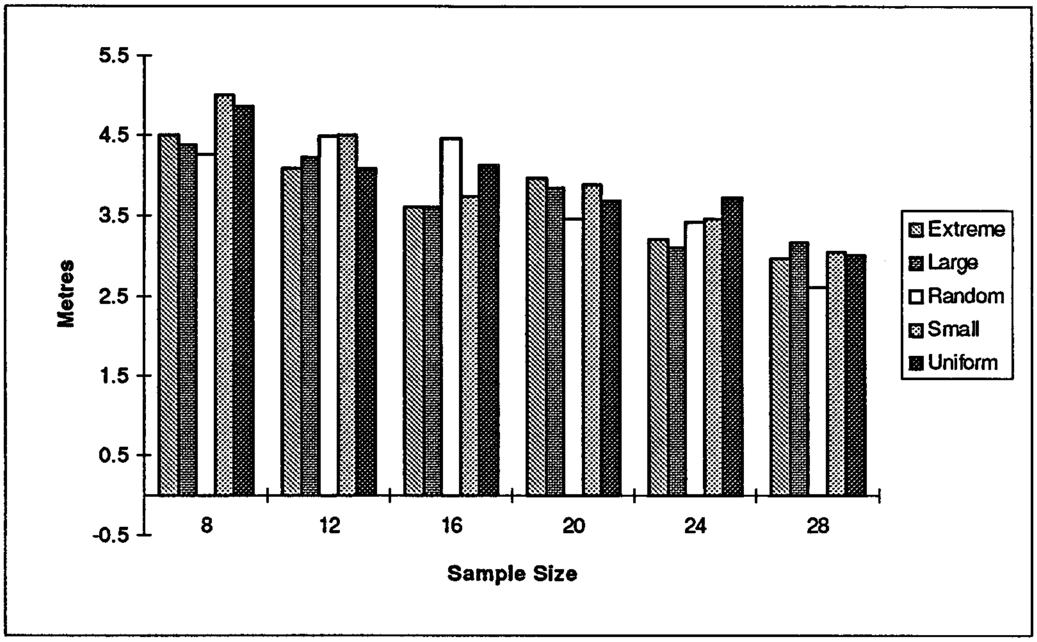


Figure 22. Average maximum deviation in height for AGE3SI1

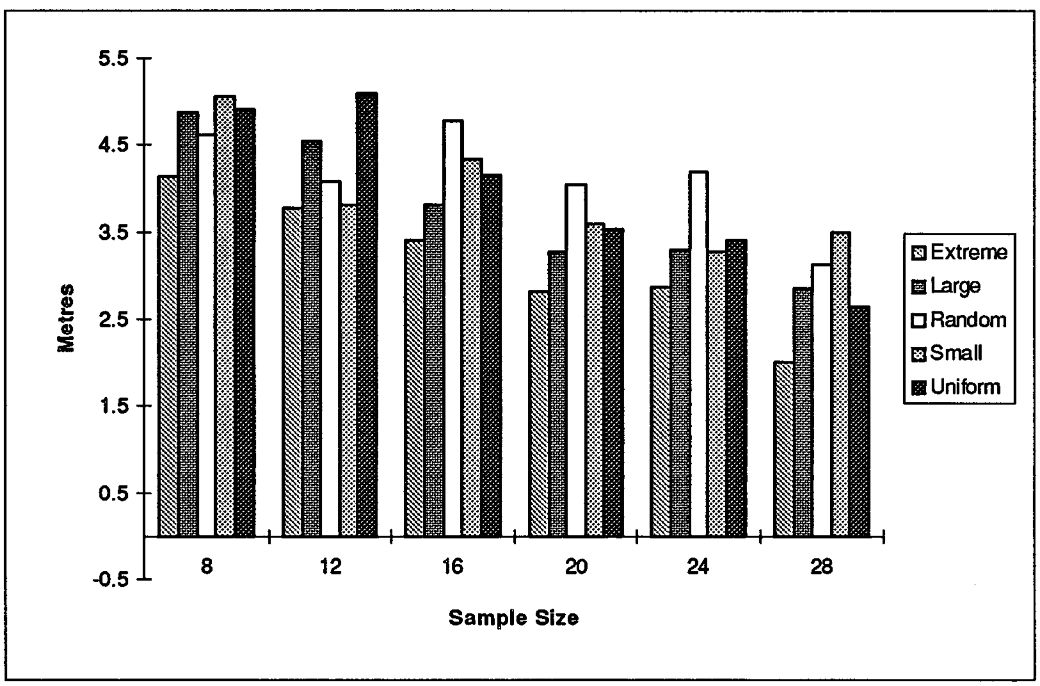
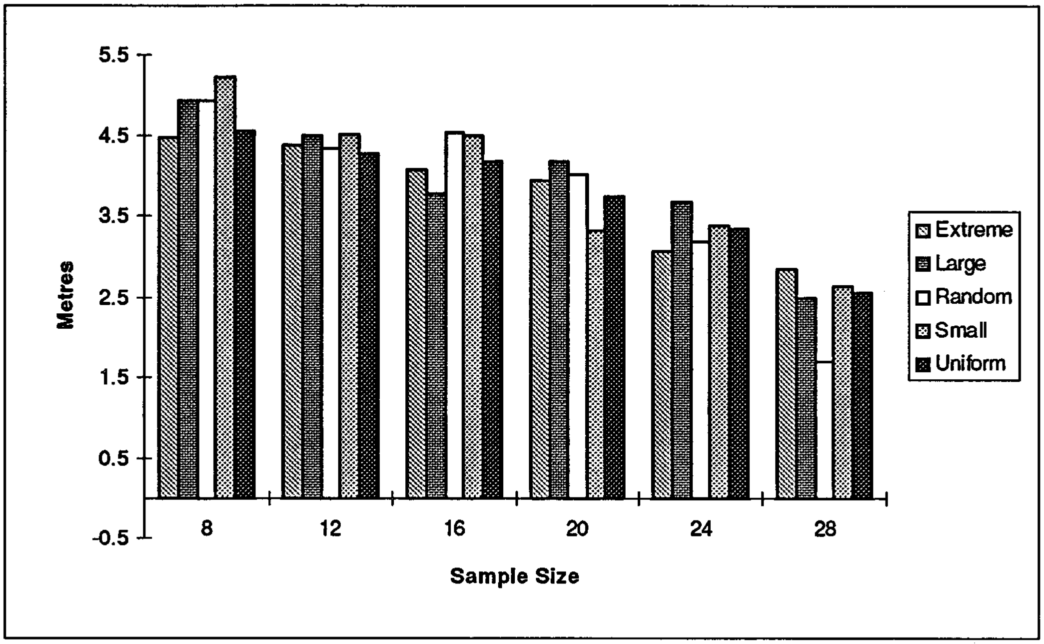


Figure 23. Average maximum deviation in height for AGE3SI2



The average maximum deviations showed progressive improvements with larger sample sizes for all age-site classes. In age class 1, these improvements tended to be very small, and the gain was not large when sampling more than 16 or 20 trees. In age class 2 - site classes 1 and 2, improvements were small when sampling more than 16 trees. The remaining age-site classes displayed improvements with each increase in sample size. The differences among age classes were not large, although age class 1 generally displayed lower average maximum deviations. Trends among sample designs were not obvious, although the random design was sometimes poor while the extreme design performed well for many samples

Figure 24. Average minimum deviation in height for AGE1SI2

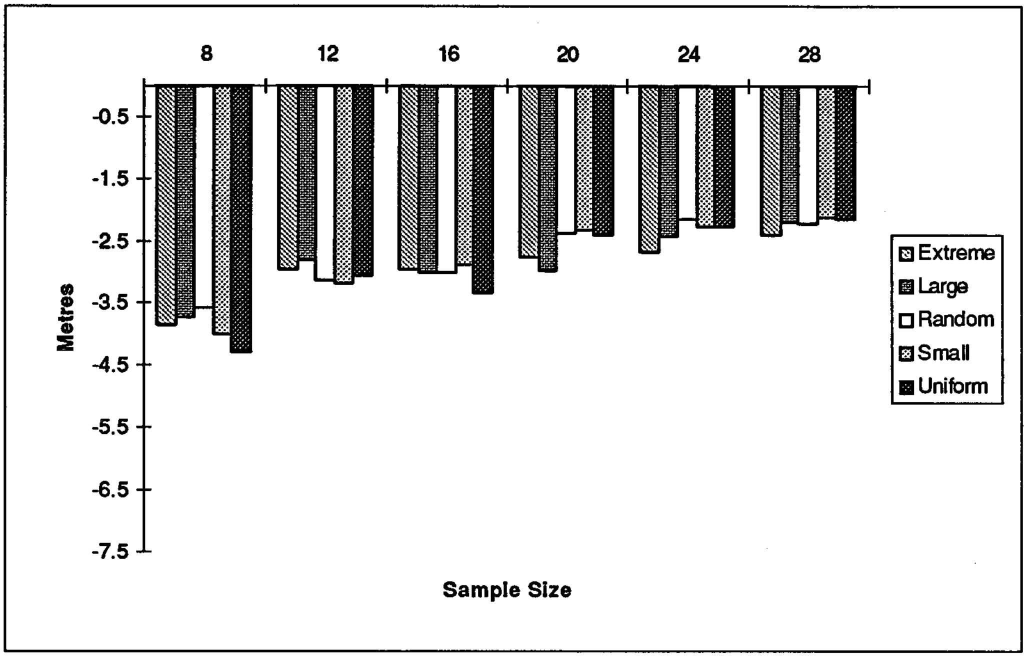


Figure 25. Average minimum deviation in height for AGE1SI3

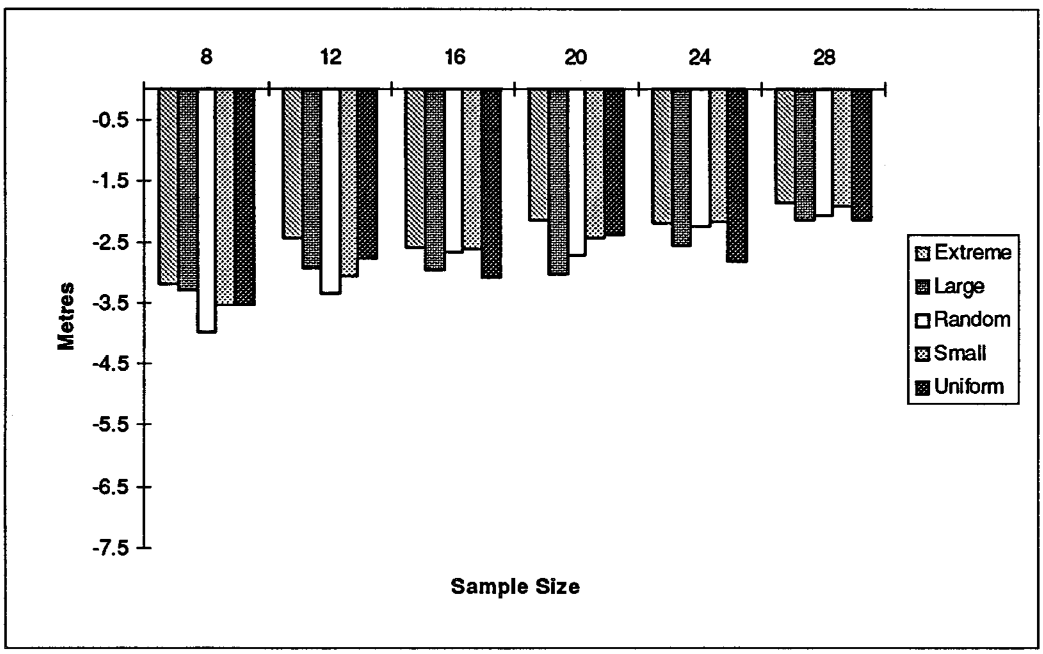


Figure 26. Average minimum deviation in height for AGE2SI1

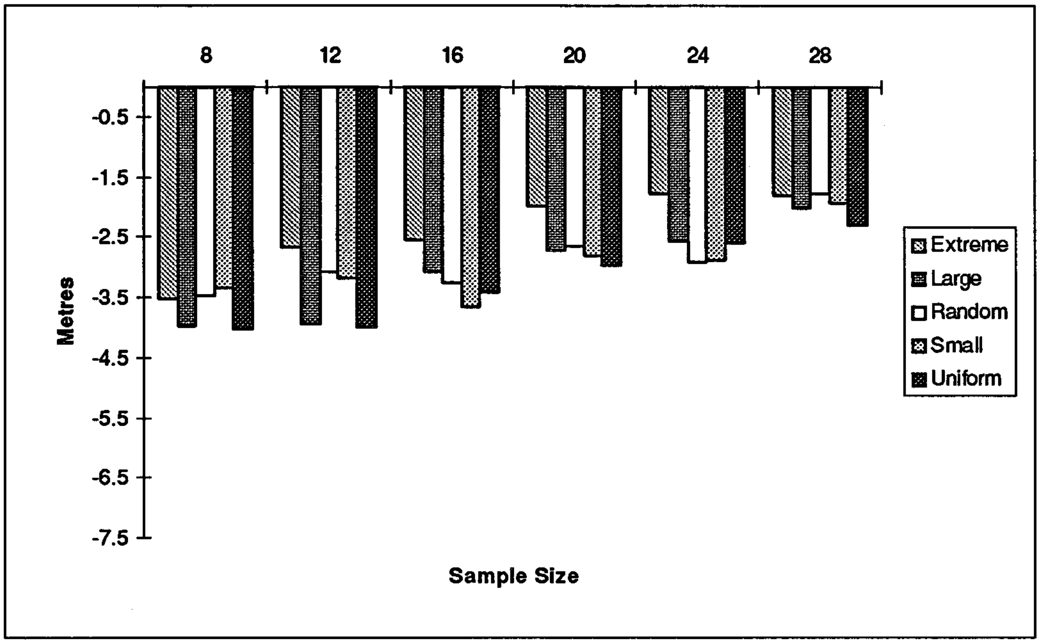


Figure 27. Average minimum deviation in height for AGE2SI2

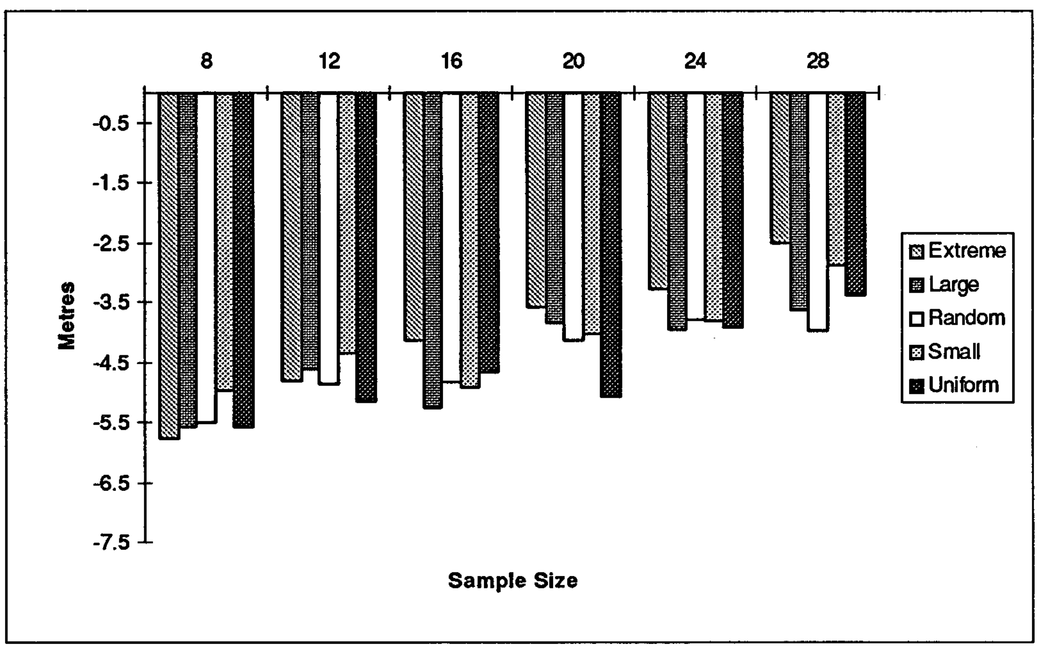


Figure 28. Average minimum deviation in height for AGE2SI3

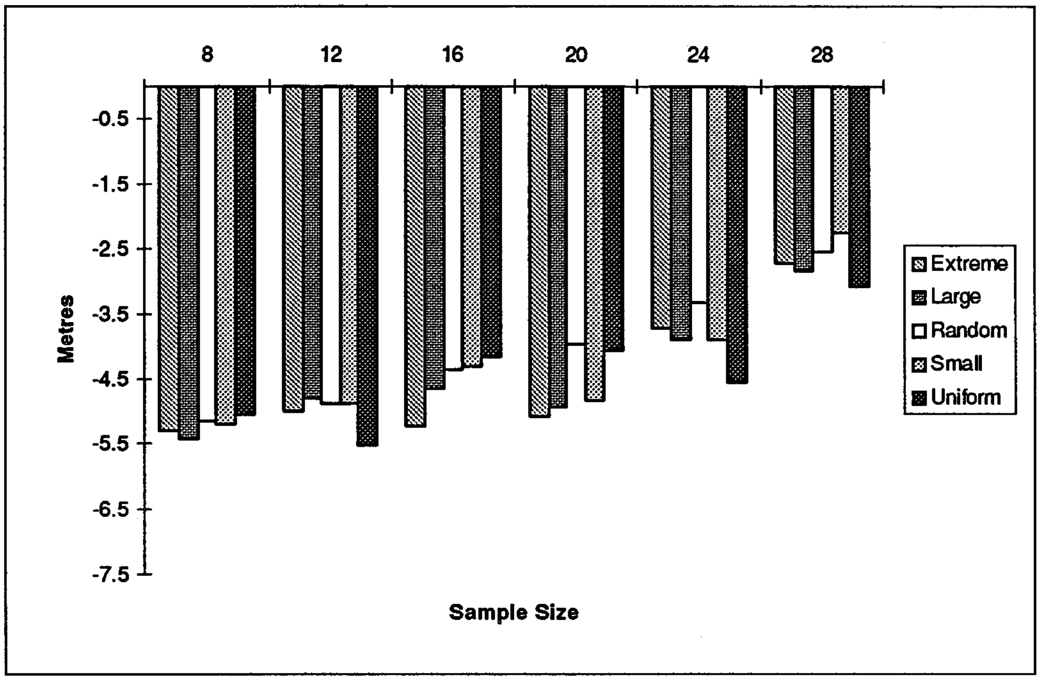


Figure 29. Average minimum deviation in height for AGE3SI1

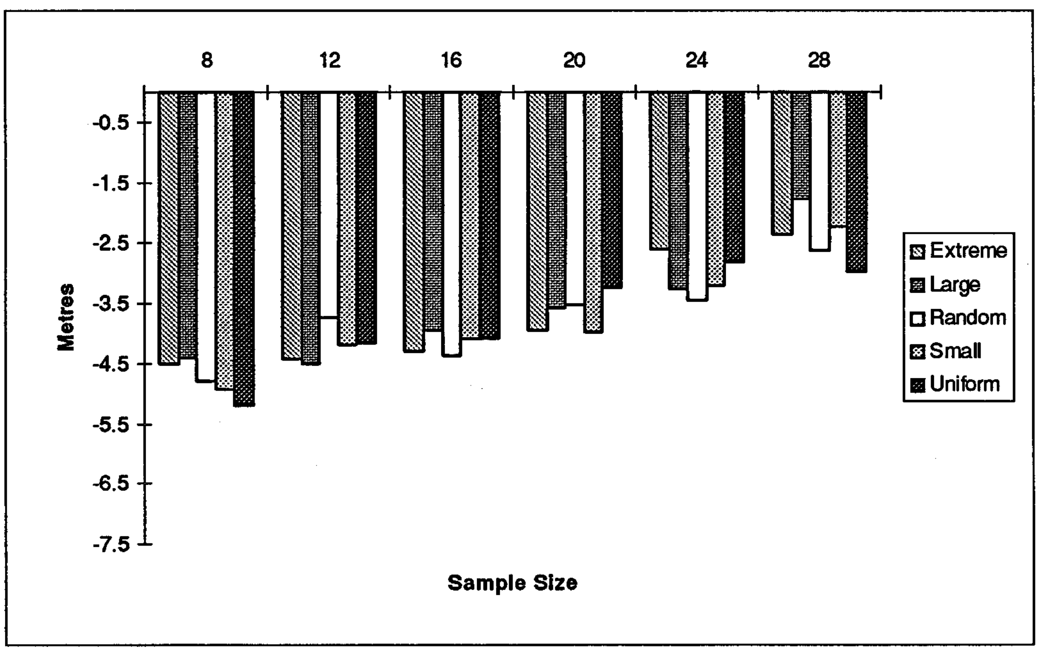
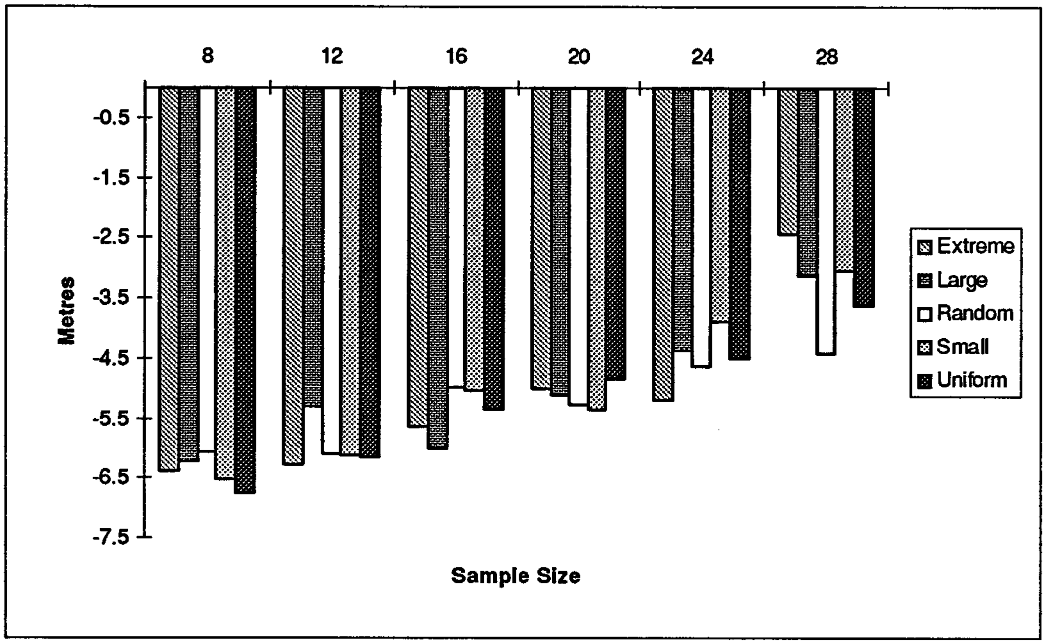


Figure 30. Average minimum deviation in height for AGE3SI2



The average minimum deviations displayed some differences between age - site index classes, but not as pronounced as those of the mean deviations. In age class 1, improvements were quite small when sampling more than 12 trees. In other age-site classes, improvements occurred with each increase in sample size, but those improvements were often very small. Age class 3 - site class 2 displayed the poorest results. In most cases, the difference among sample designs was quite small.

4.2.1.3 Average standard deviation of height differences

Figures 28 to 33 show the average standard deviation of height differences. These values are the standard deviations of the differences between measured and estimated tree heights averaged over each plot and repetition in each age - site index class for each sample size and design.

Figure 31. Average standard deviation of height differences for AGE1SI2

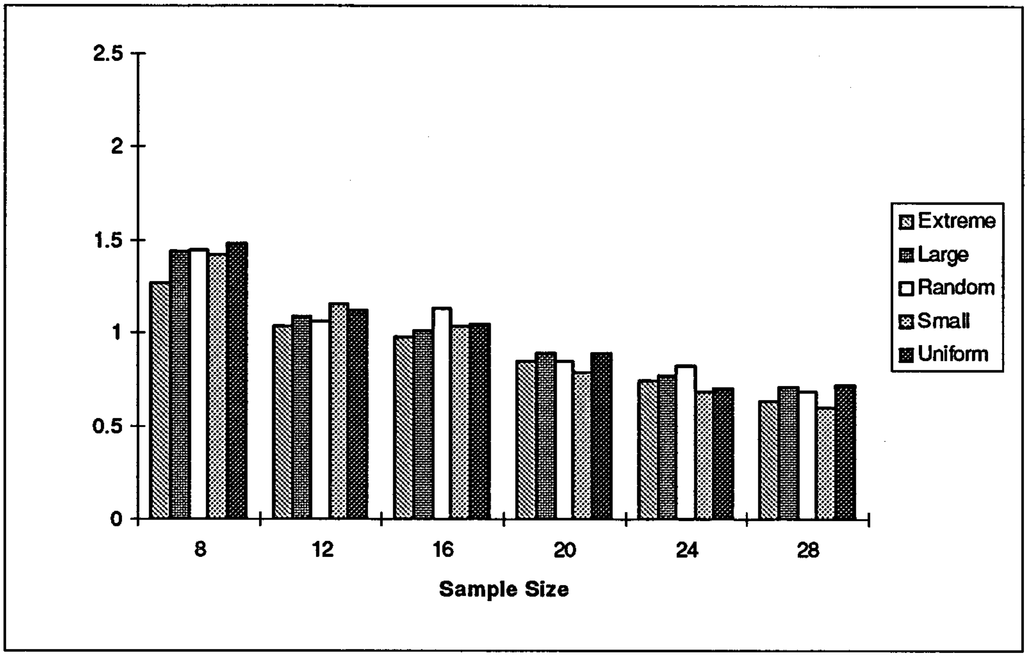


Figure 32. Average standard deviation of height differences for AGE1SI3

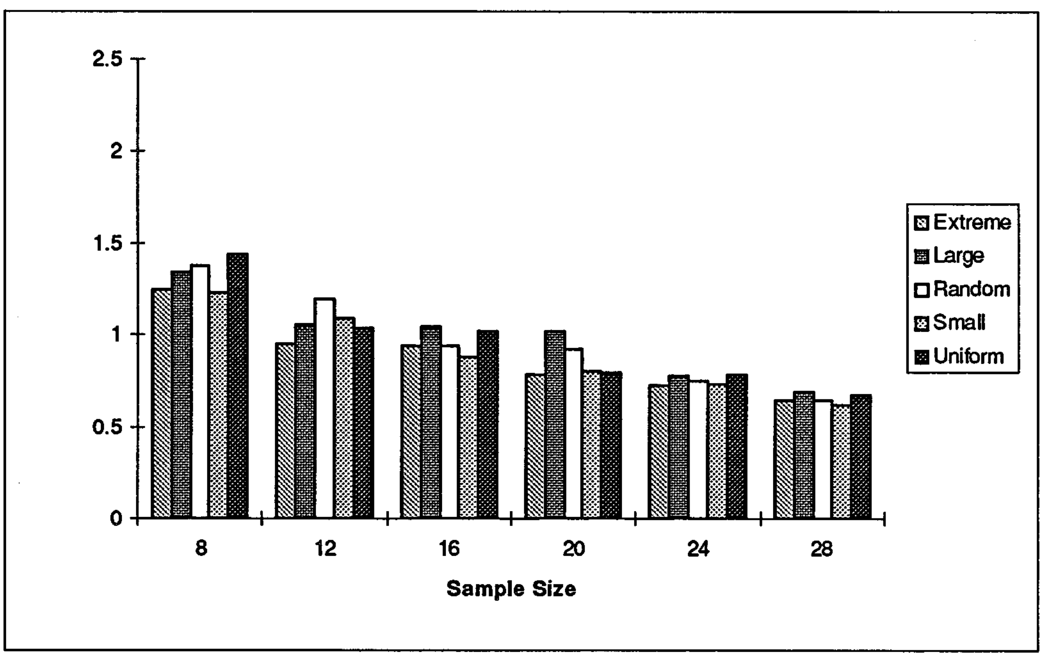


Figure 33. Average standard deviation of height differences for AGE2SI1

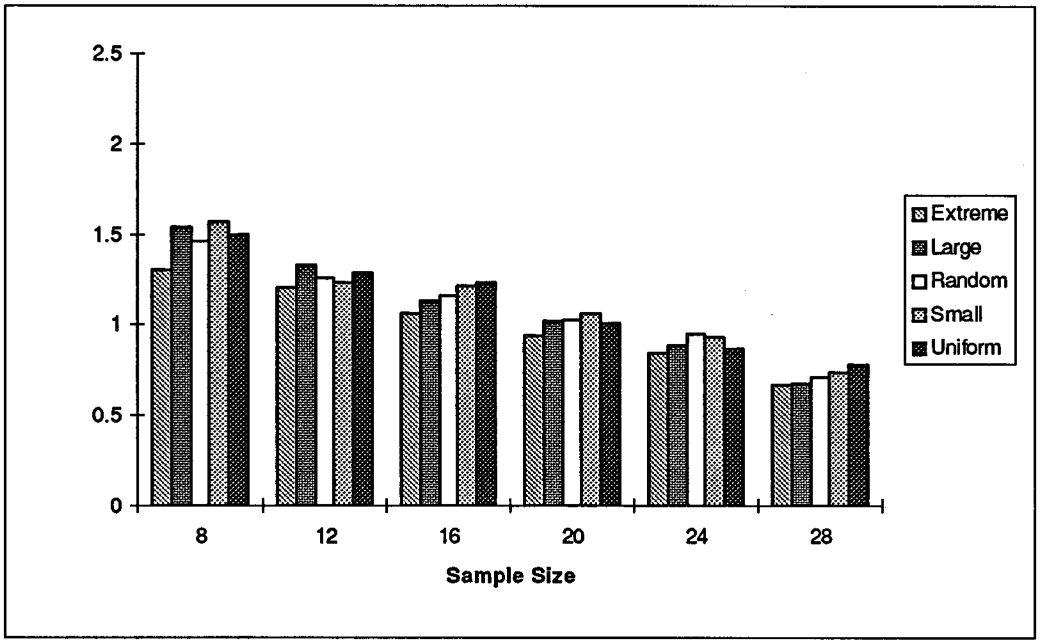


Figure 34. Average standard deviation of height differences for AGE2SI2

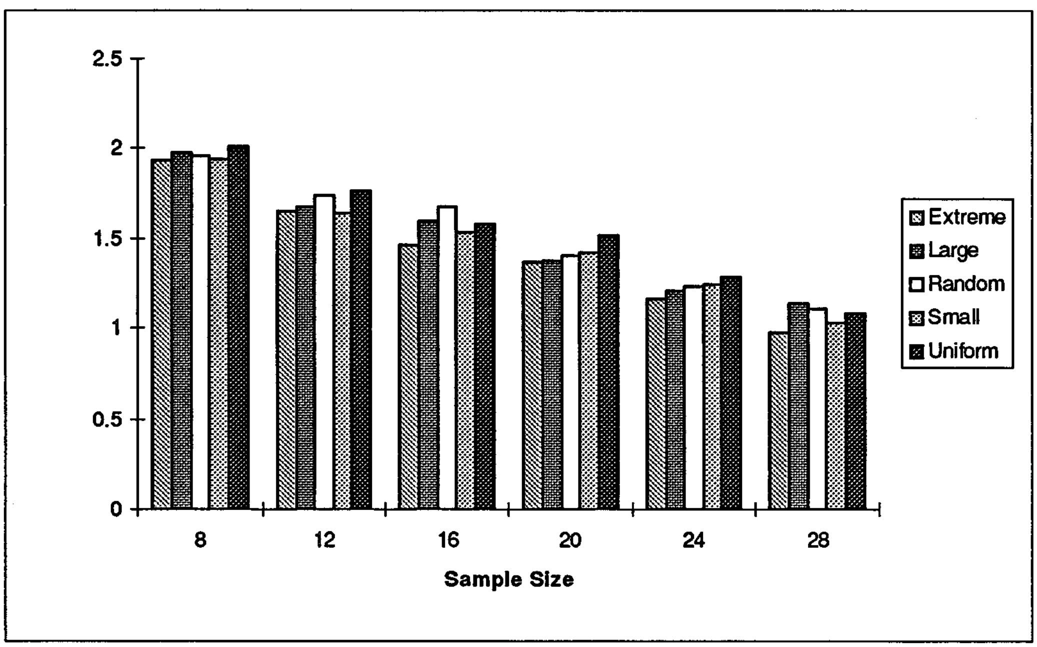


Figure 35. Average standard deviation of height differences for AGE2SI3

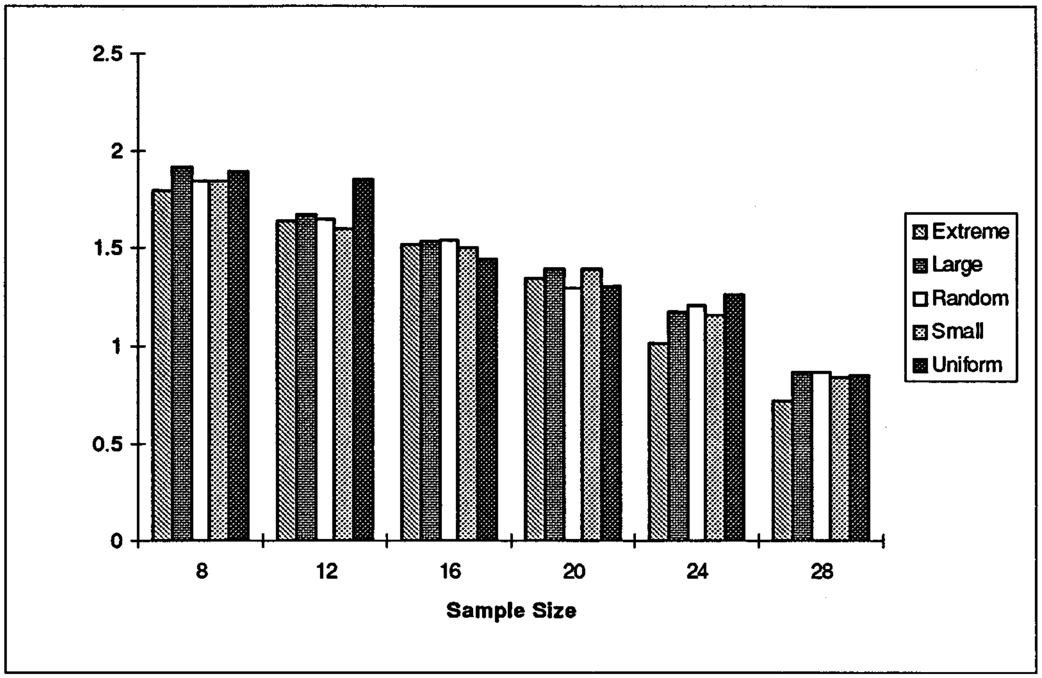


Figure 36. Average standard deviation of height differences for AGE3SI1

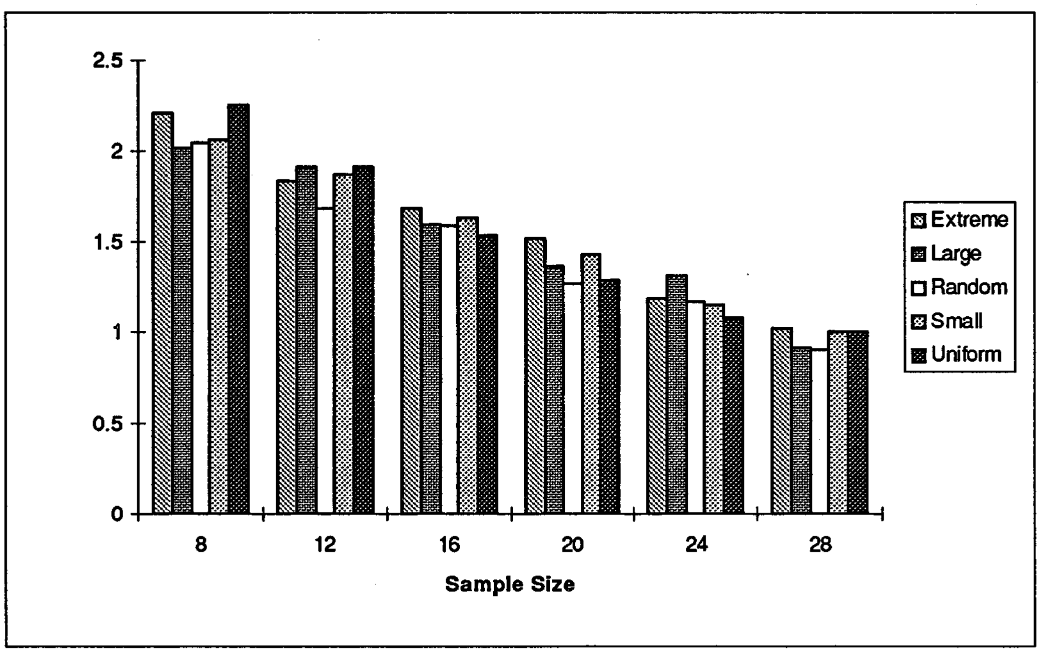
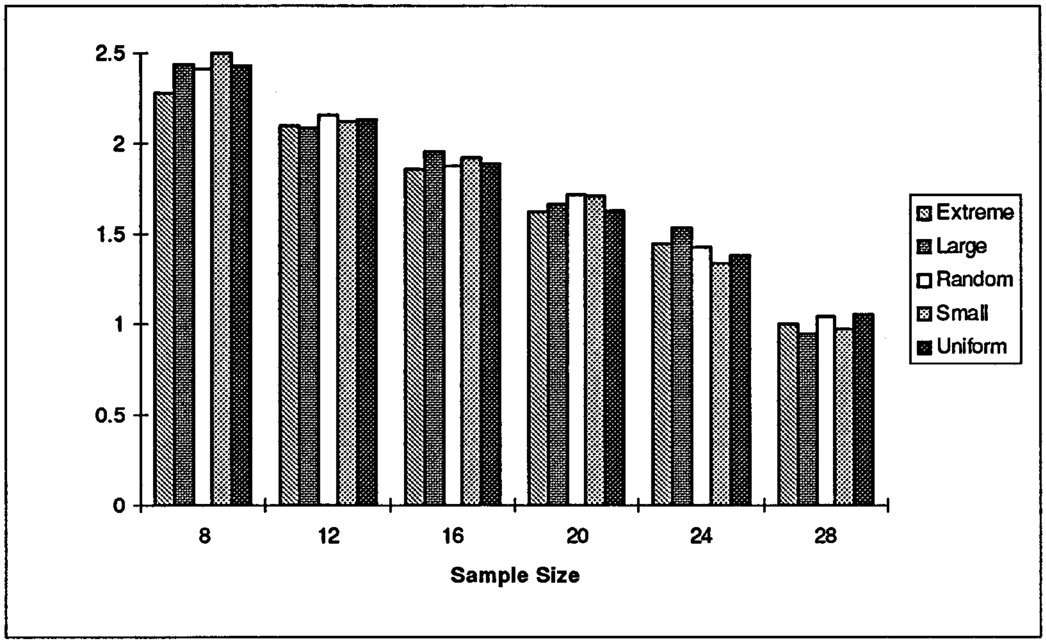


Figure 37. Average standard deviation of height differences for AGE3SI2



The average standard deviation of height differences displayed little variation among different sampling designs, although the extreme design generally gave good results. However, the variation that occurred among age - site index classes was very pronounced. Age class 1 and age class 2 - site index 1 had lower average standard deviations than the other classes at all sample sizes. In age class 1, the largest improvement occurred when sample size was increased to 12. The remaining age-site classes showed fairly steady improvements with each increase in sample size, although in age class 2 - site class 1 those improvements were quite small.

4.2.1.4 Mean absolute deviation in height

Figures 38 through 44 show the mean absolute deviation in height, averaged for each age - site class for sample sizes 8 through to 28.

Figure 38. Mean absolute deviation in height for AGE1SI2

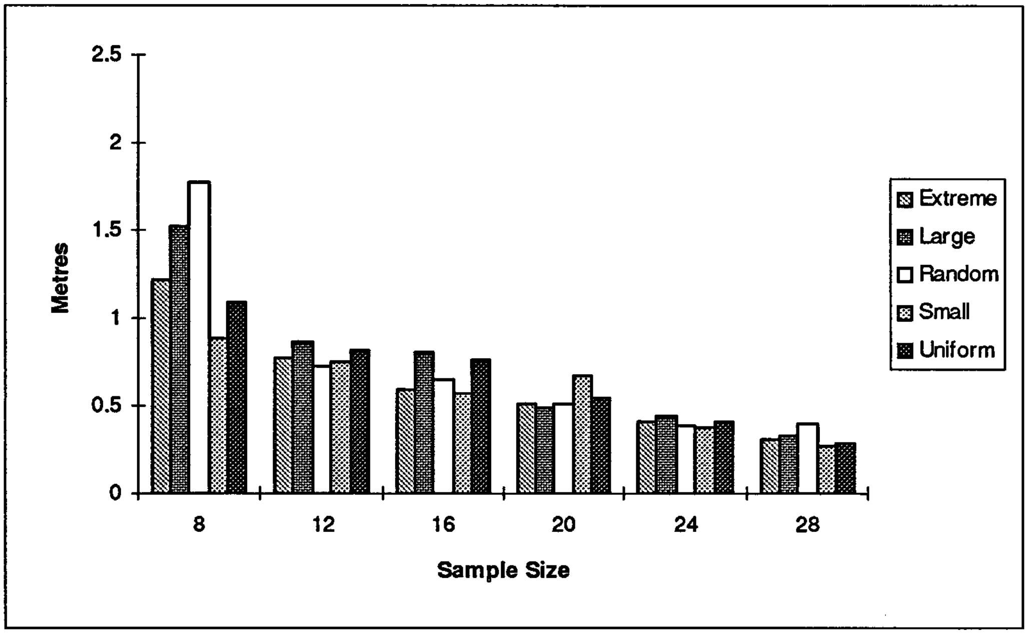


Figure 39. Mean absolute deviation in height for AGE1SI3

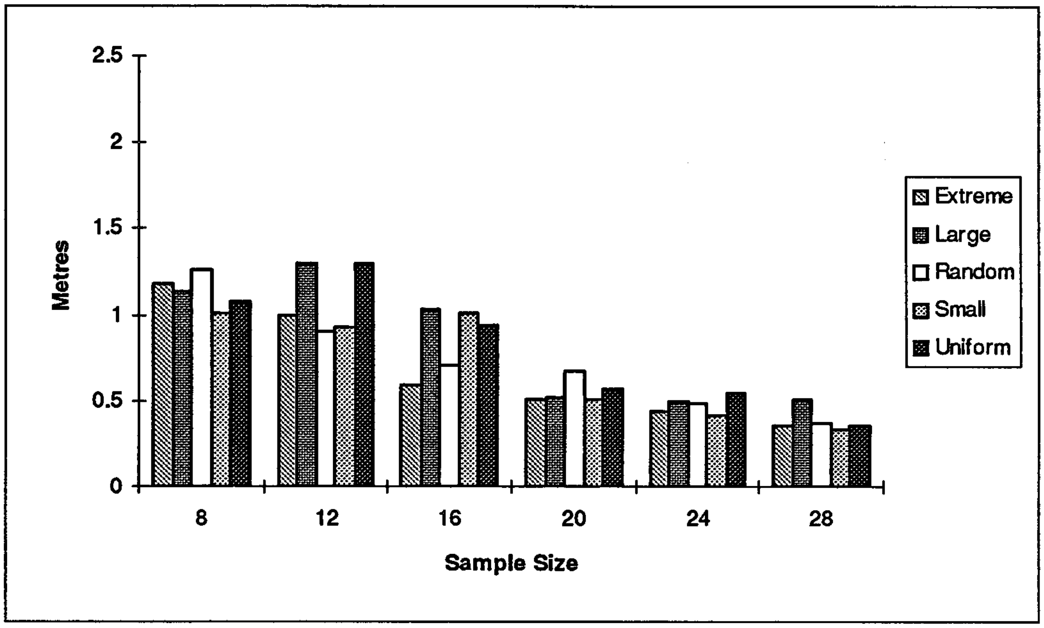


Figure 40. Mean absolute deviation in height for AGE2SI1

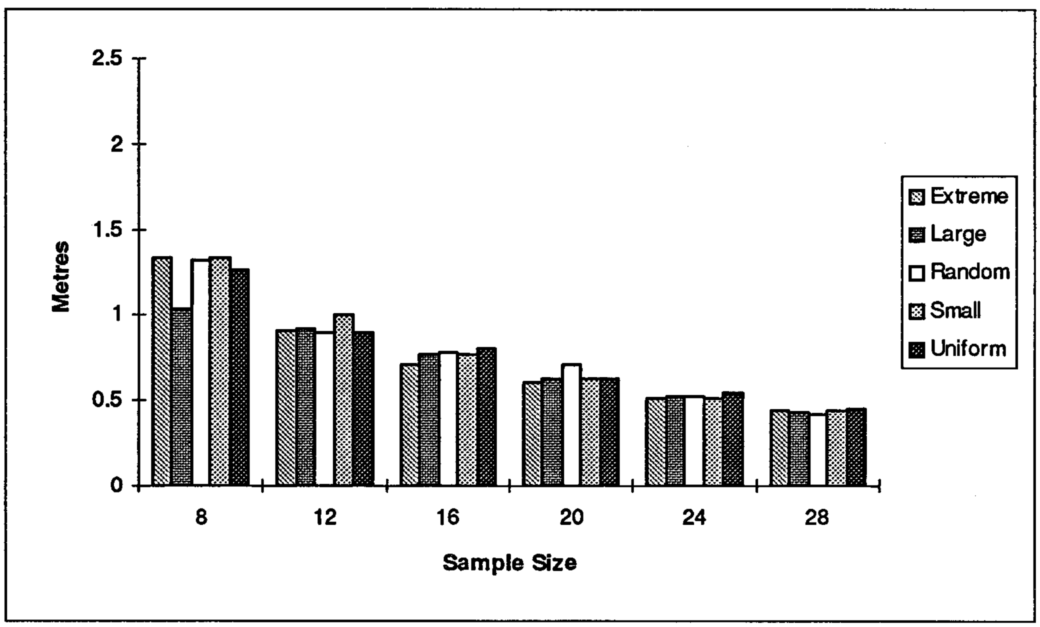


Figure 41. Mean absolute deviation in height for AGE2SI2

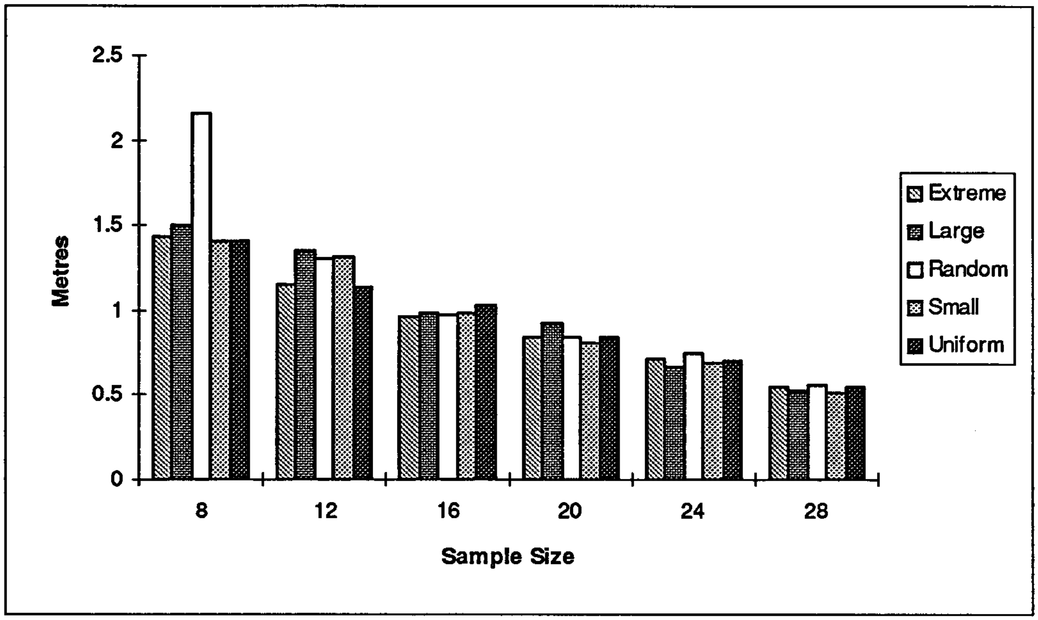


Figure 42. Mean absolute deviation in height for AGE2SI3

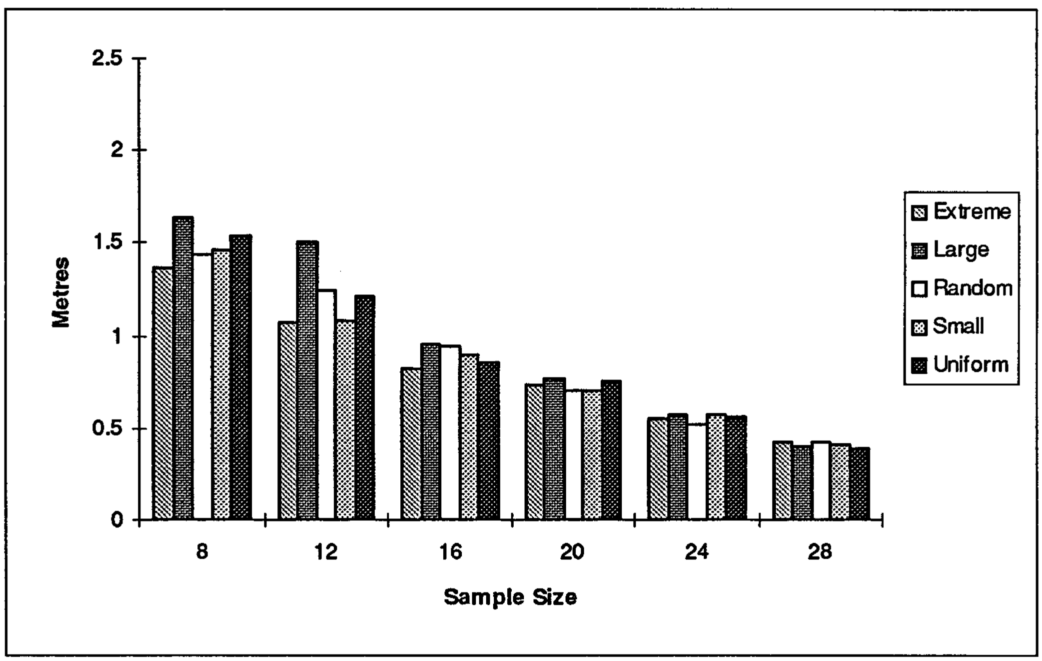


Figure 43. Mean absolute deviation in height for AGE3SI1

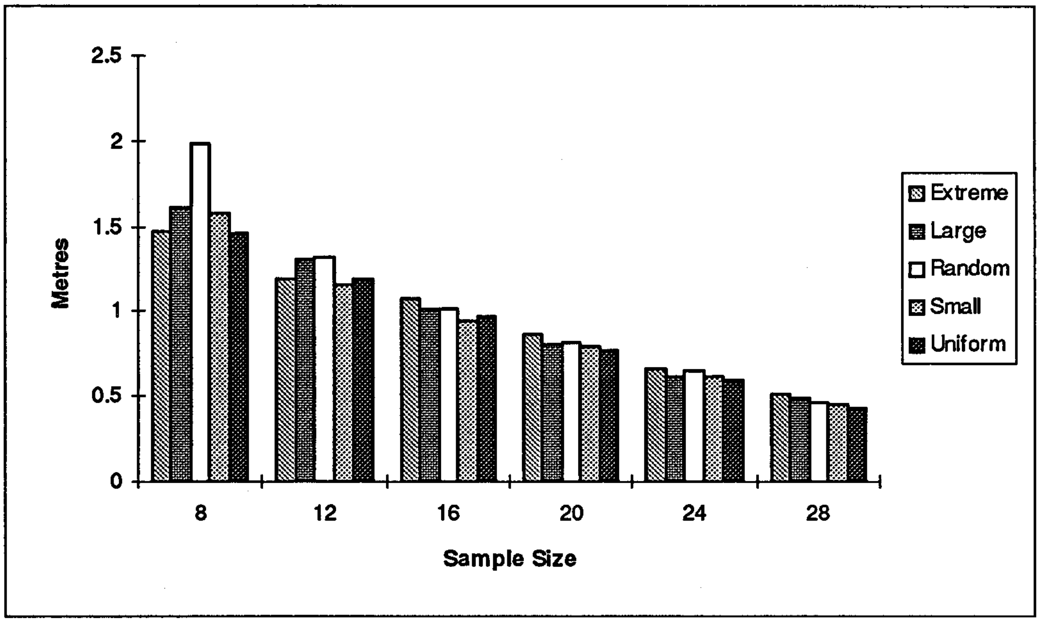
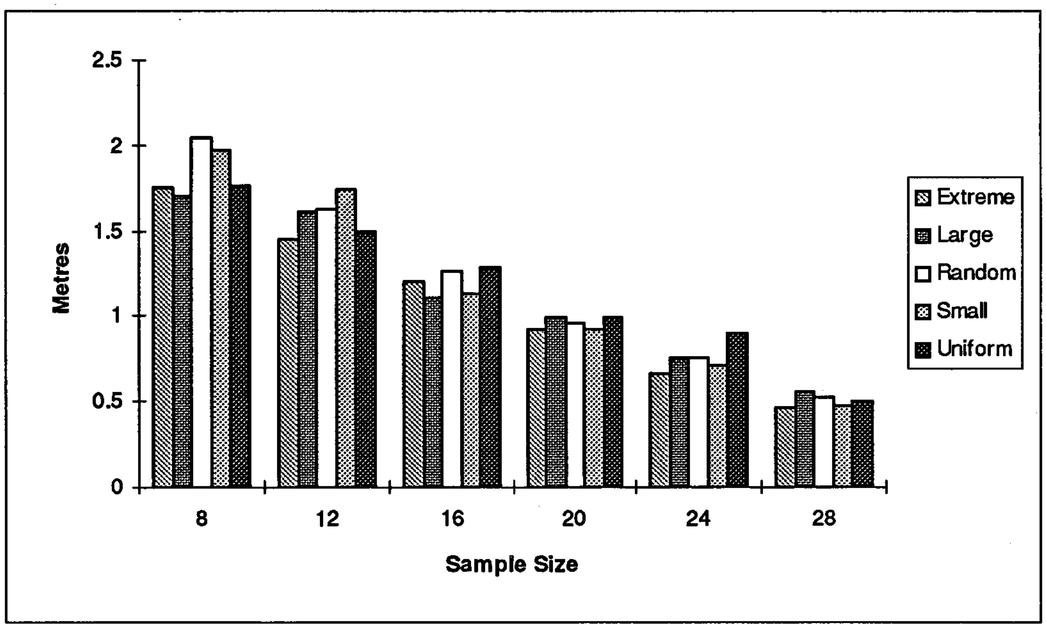


Figure 44. Mean absolute deviation in height for AGE3SI2



The differences among age-site classes were not very pronounced for mean absolute deviations. In general, all age classes improved with each increment in sample size, although in most cases the largest improvements occurred at or before reaching a sample size of 16. Age class 3 was an exception to this and displayed steady improvements with each increase in sample size. Differences among sampling designs were generally not very clear, except at a sample size of 8 where the random design displayed considerably larger mean absolute deviations for some age-site classes. Also, the extreme design generally performed well in age class 1.

4.2.2 Volume estimation

4.2.2.1 Mean deviation in volume

Figures 45 through 51 show the mean deviations in volume by age-site classes, averaged for all plots in each class and for the 5 sample repetitions.

Figure 45. Mean deviation in volume for AGE1SI2

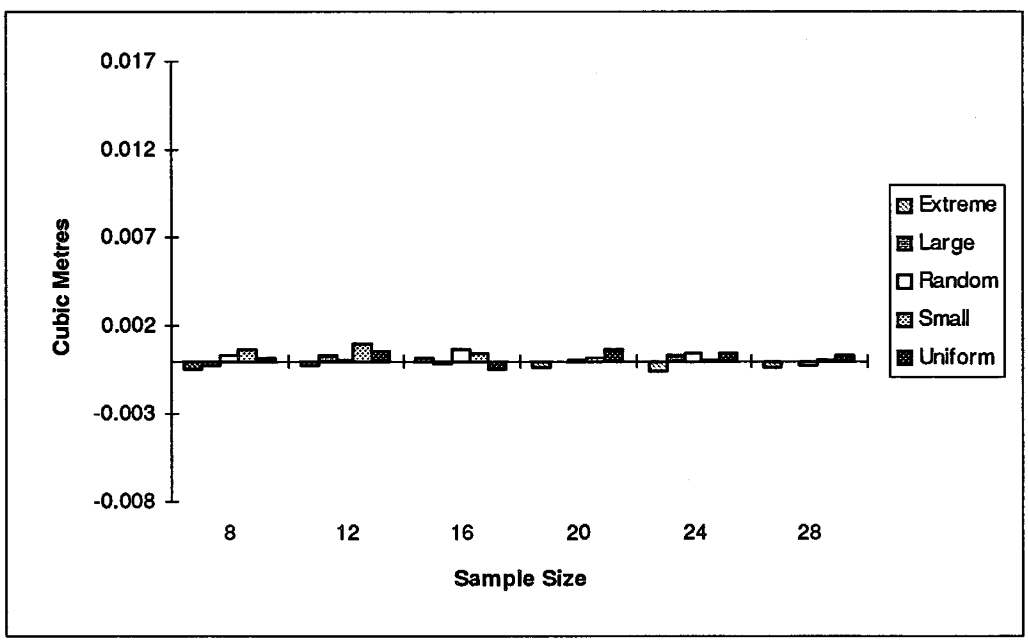


Figure 46. Mean deviation in volume for AGE1SI3

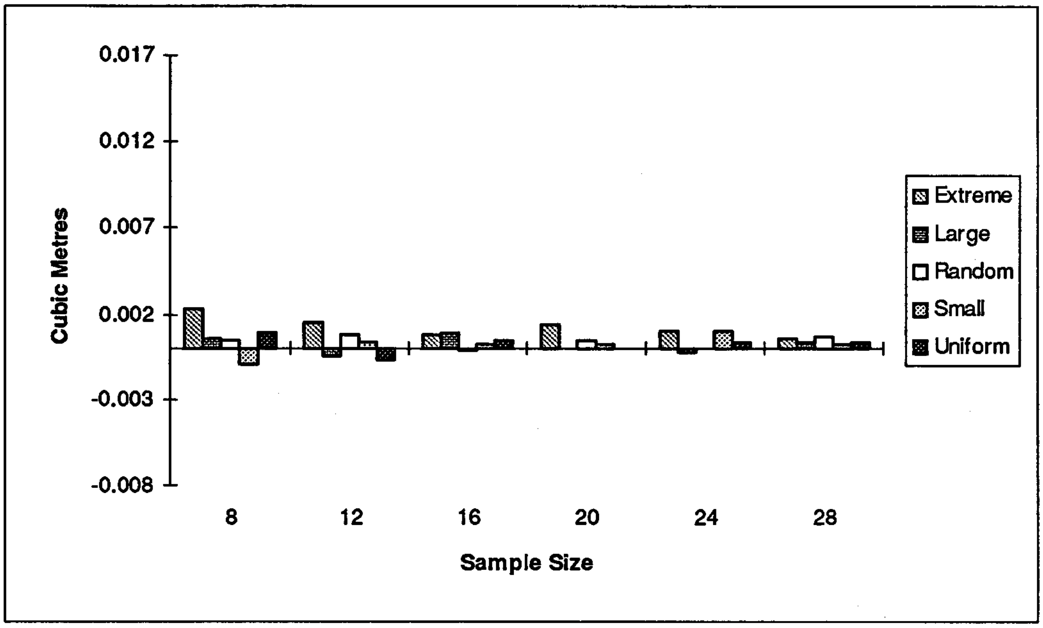


Figure 47. Mean deviation in volume for AGE2SI1

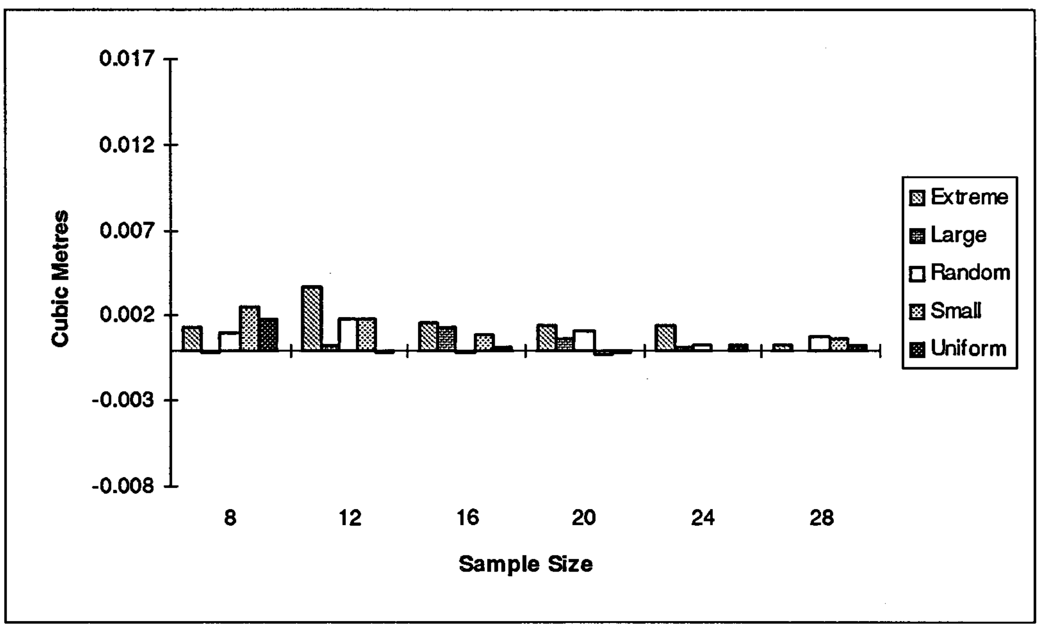


Figure 48. Mean deviation in volume for AGE2SI2

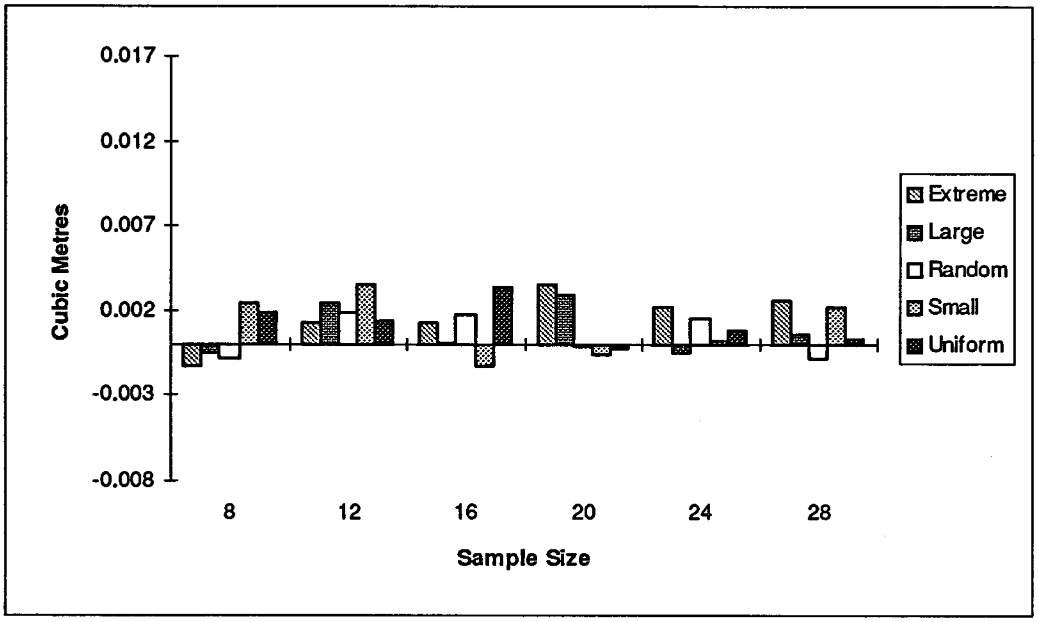


Figure 49. Mean deviation in volume for AGE2SI3

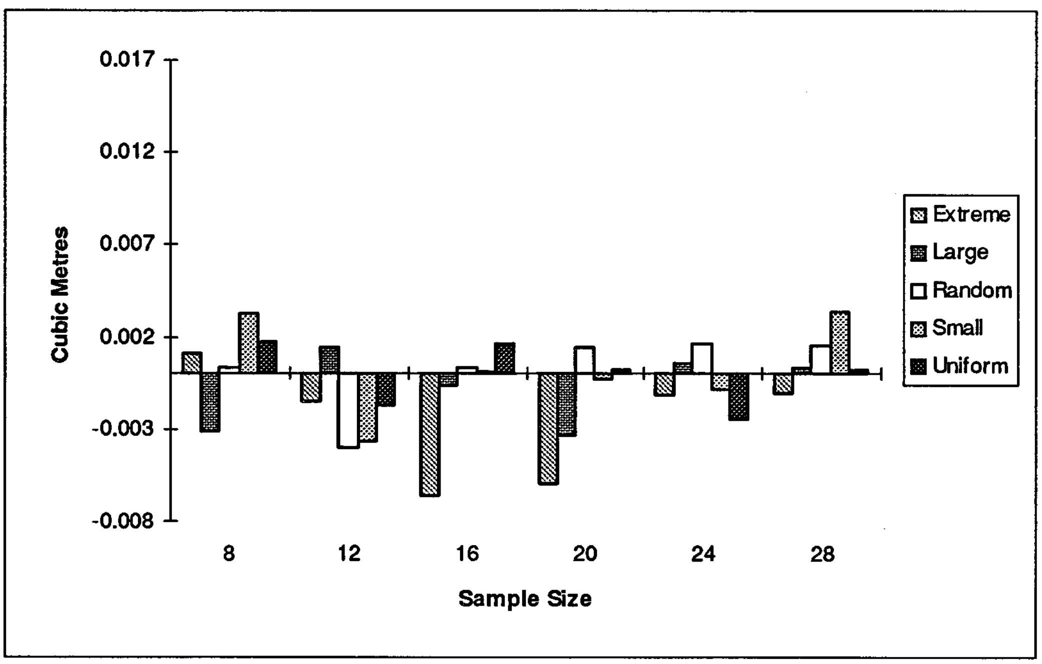


Figure 50. Mean deviation in volume for AGE3SI1

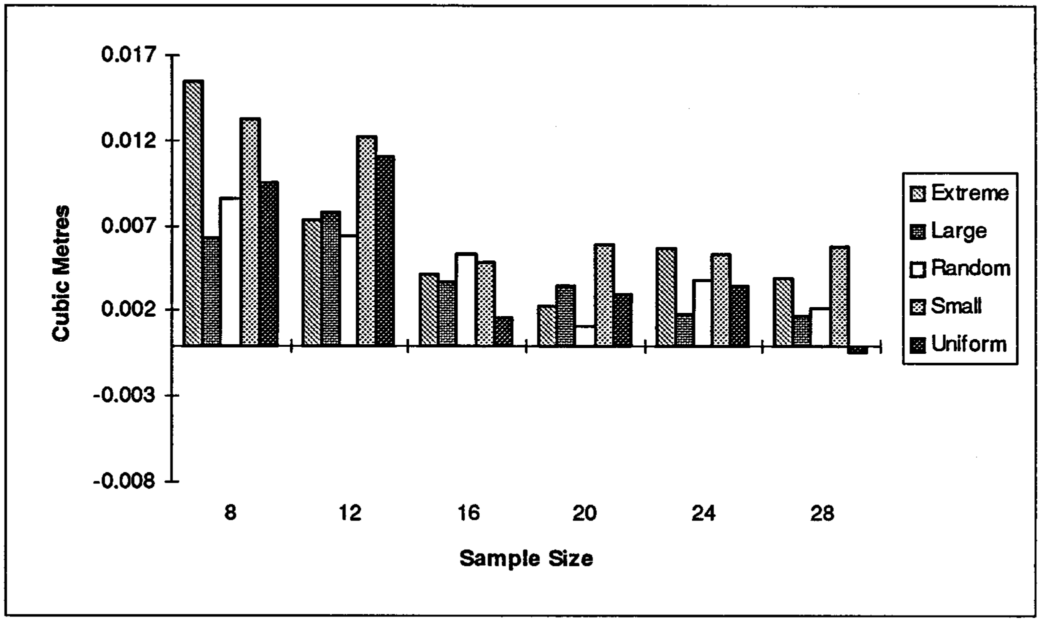
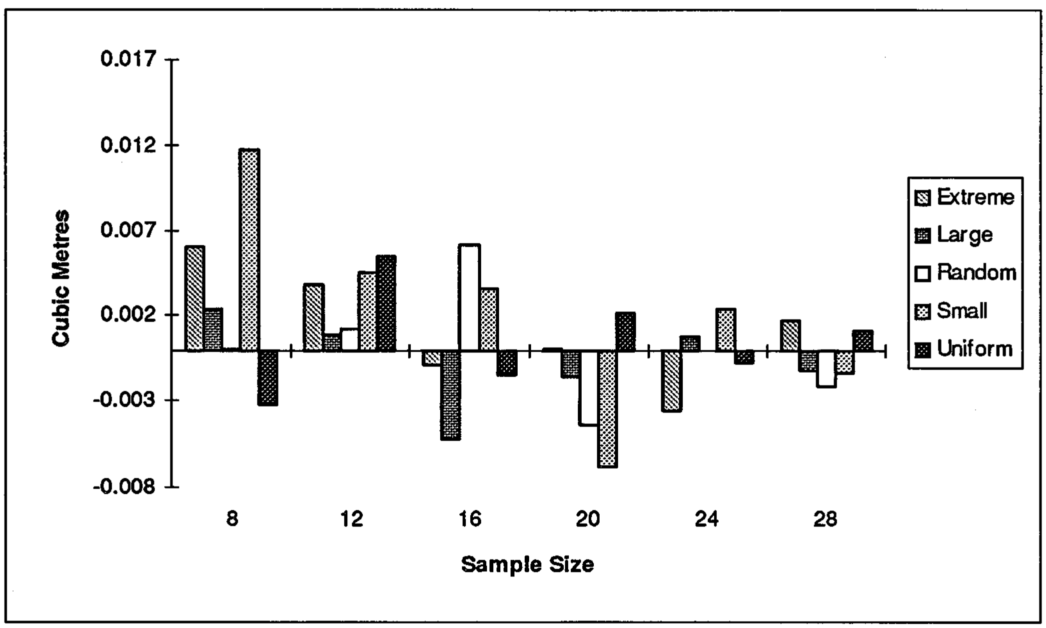


Figure 51. Mean deviation in volume for AGE3SI2



Some trends in the mean deviation in volume are apparent. First, volume was underestimated, on average, although some over-estimation did occur. The mean deviations were much larger in age class 3 (especially in site index class 1) than in the other age classes. Overall, the large and uniform designs gave good results.

4.2.2.2 Average maximum and minimum deviation in volume

Figures 52 to 58 show the average maximum deviation in volume for the age class - site index groupings. The values in the charts represent the average of the largest positive deviations for a given age - site index class. Figures 59 to 65 show the average minimum deviation in volume for the age - site index groupings. These values represent the average of the largest negative deviations for a given age - site index class.

Figure 52. Average maximum deviation in volume for AGE1SI2

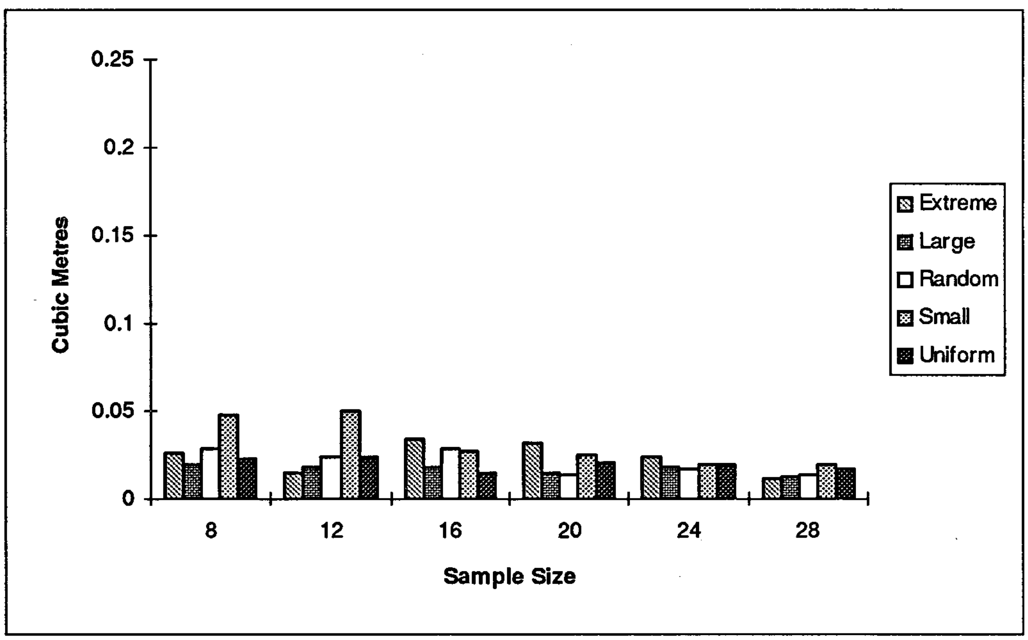


Figure 53. Average maximum deviation in volume for AGE1SI3

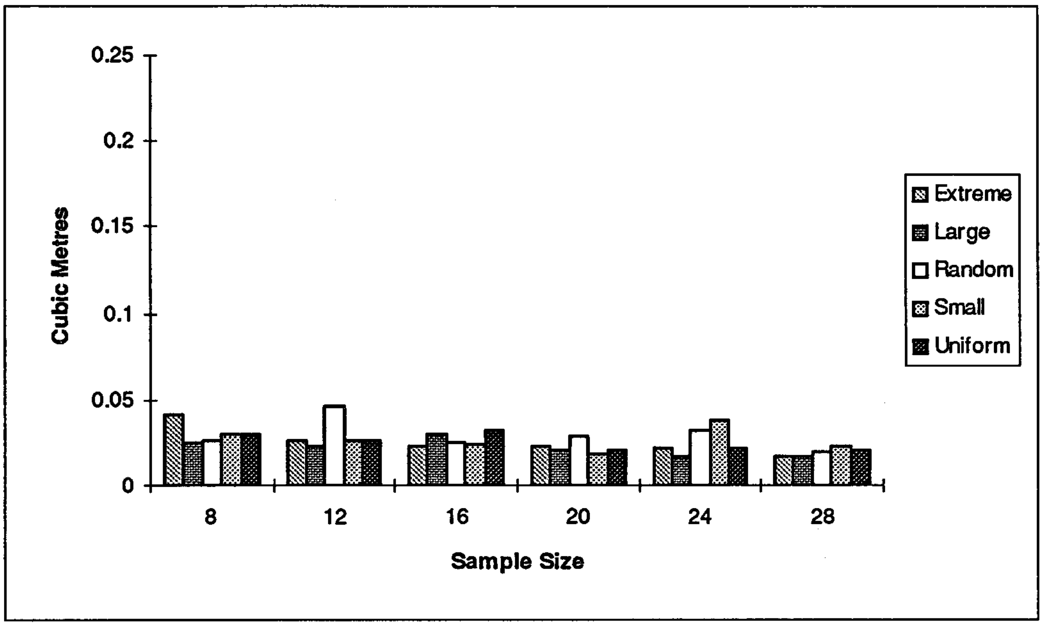


Figure 54. Average maximum deviation in volume for AGE2SI1

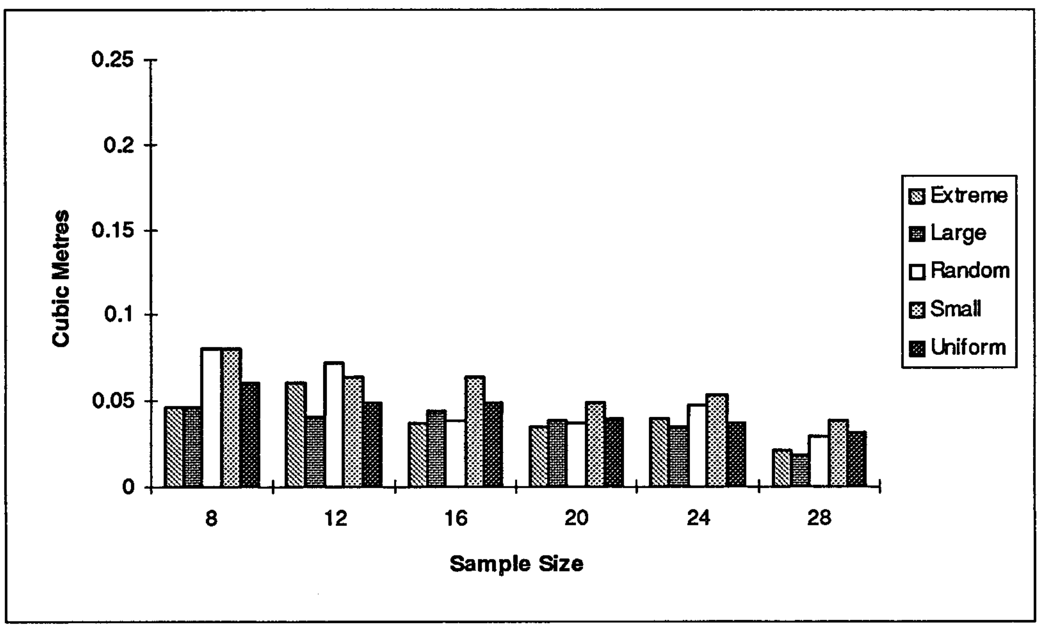


Figure 55. Average maximum deviation in volume for AGE2SI2

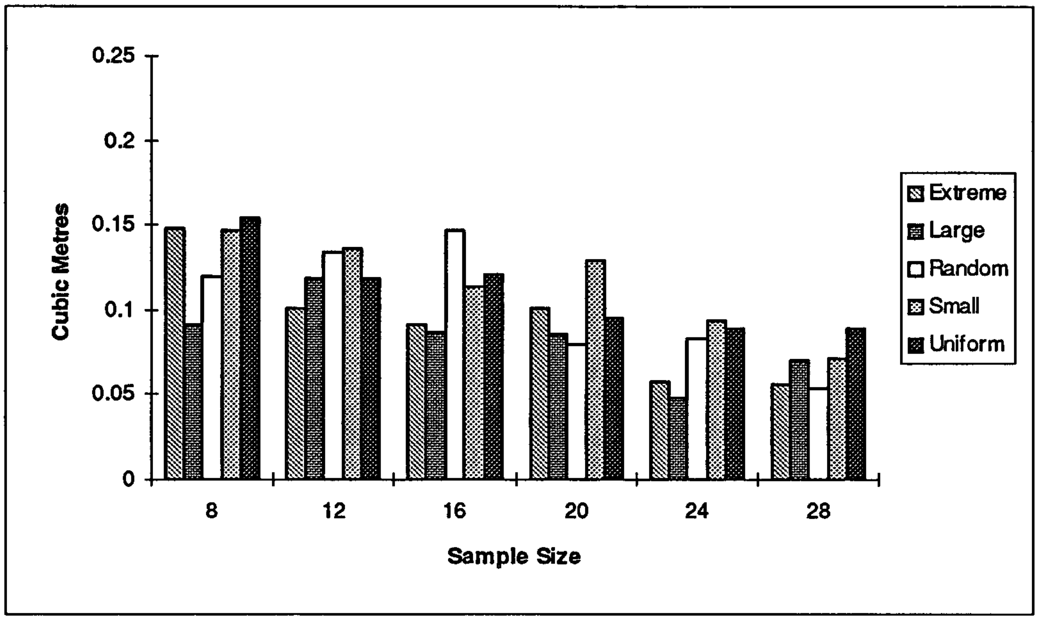


Figure 56. Average maximum deviation in volume for AGE2SI3

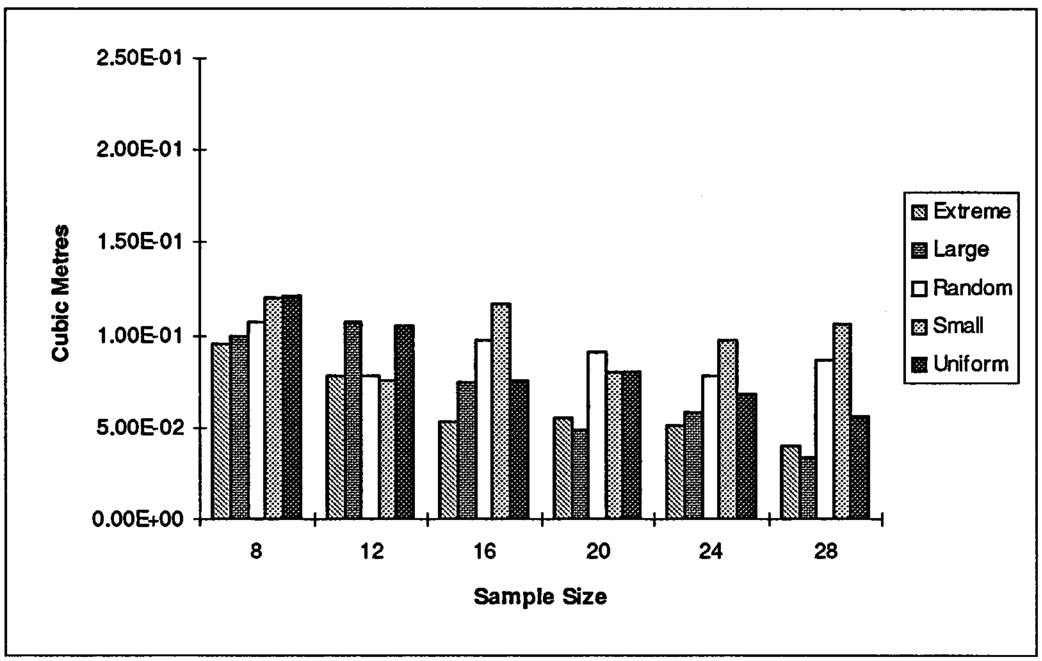


Figure 57. Average maximum deviation in volume for AGE3SI1

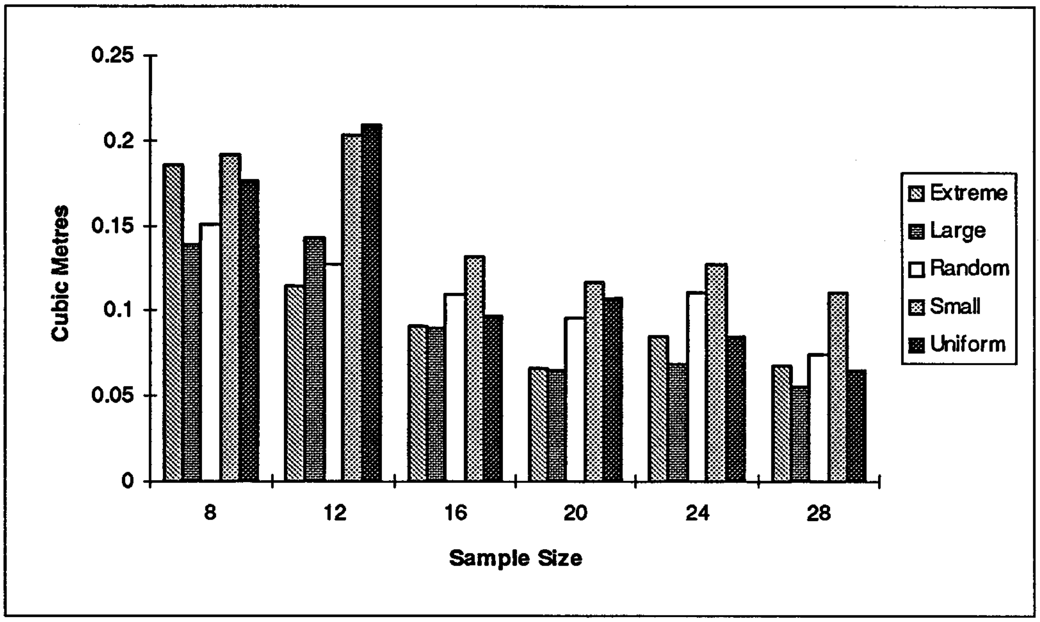
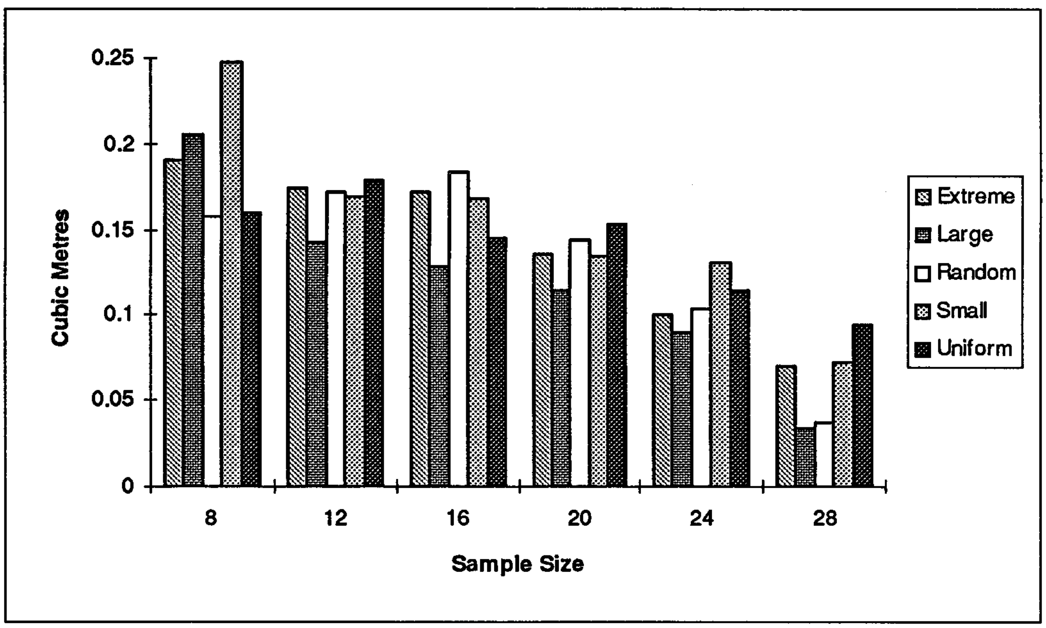


Figure 58. Average maximum deviation in volume for AGE3SI2



Age class 1 and class AGE2SI1 displayed lower average maximum deviation in volume than the other classes. These classes showed no discernible improvement with larger sample sizes. Age class 3 - site index 2 gave displayed the poorest results, and showed improvements with larger sample sizes. In age class 3 - site index 1 results improved when sample size was increased to 16. In most cases, the large design performed well, while the small design usually gave very poor results .

Figure 59. Average minimum deviation in volume for AGE1SI2

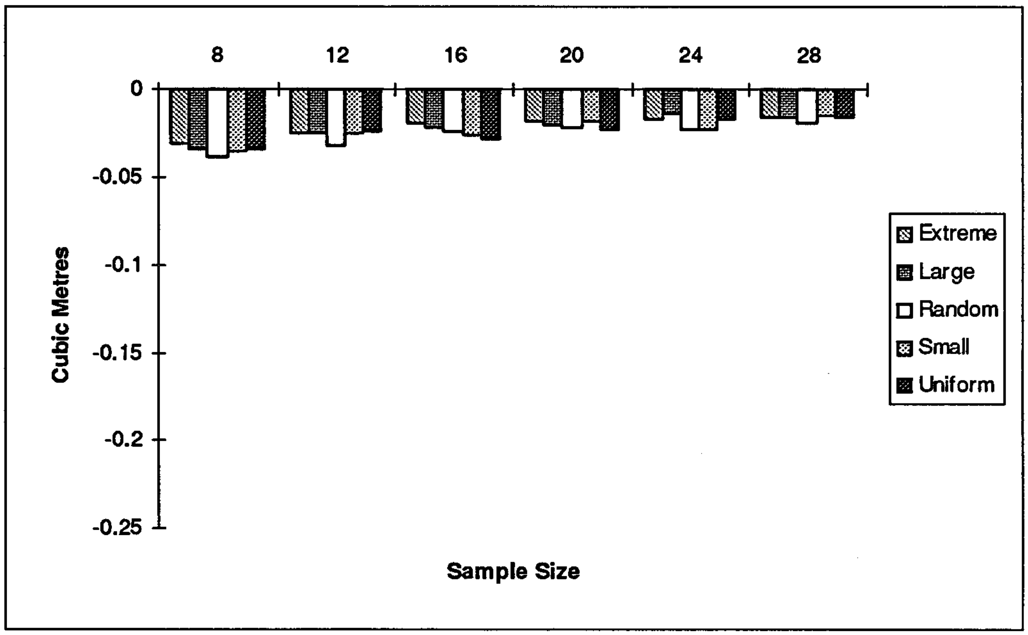


Figure 60. Average minimum deviation in volume for AGE1SI3

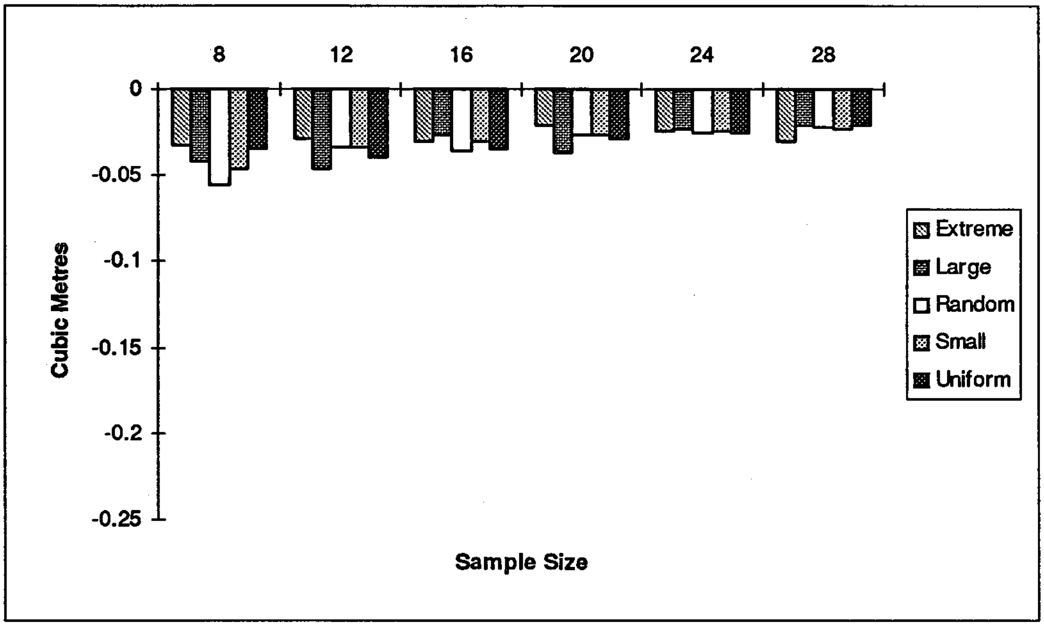


Figure 61. Average minimum deviation in volume for AGE2SI1

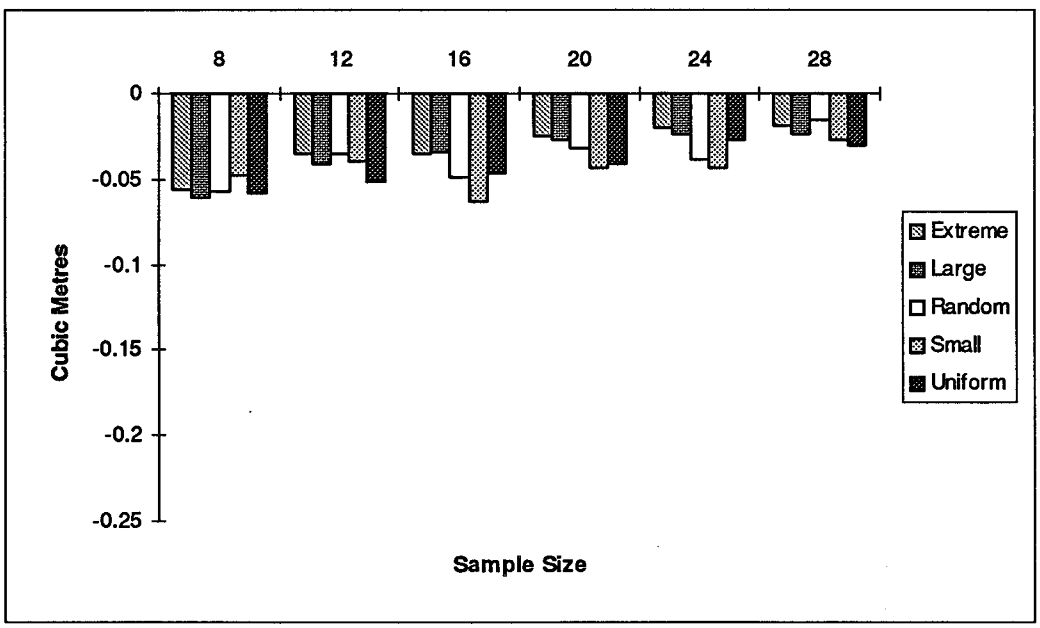


Figure 62. Average minimum deviation in volume for AGE2SI2

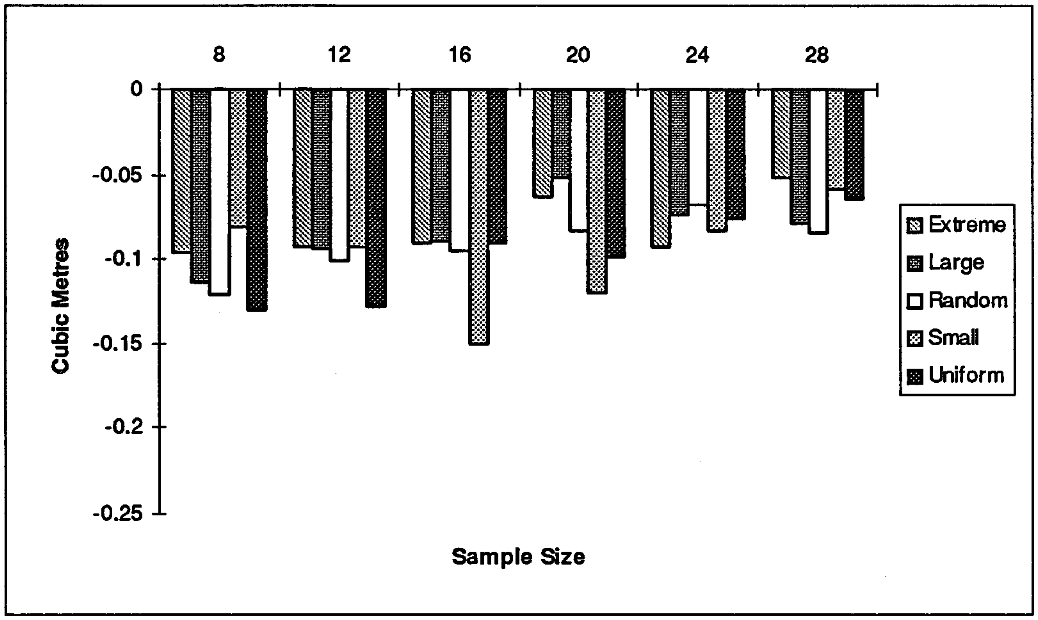


Figure 63. Average minimum deviation in volume for AGE2SI3

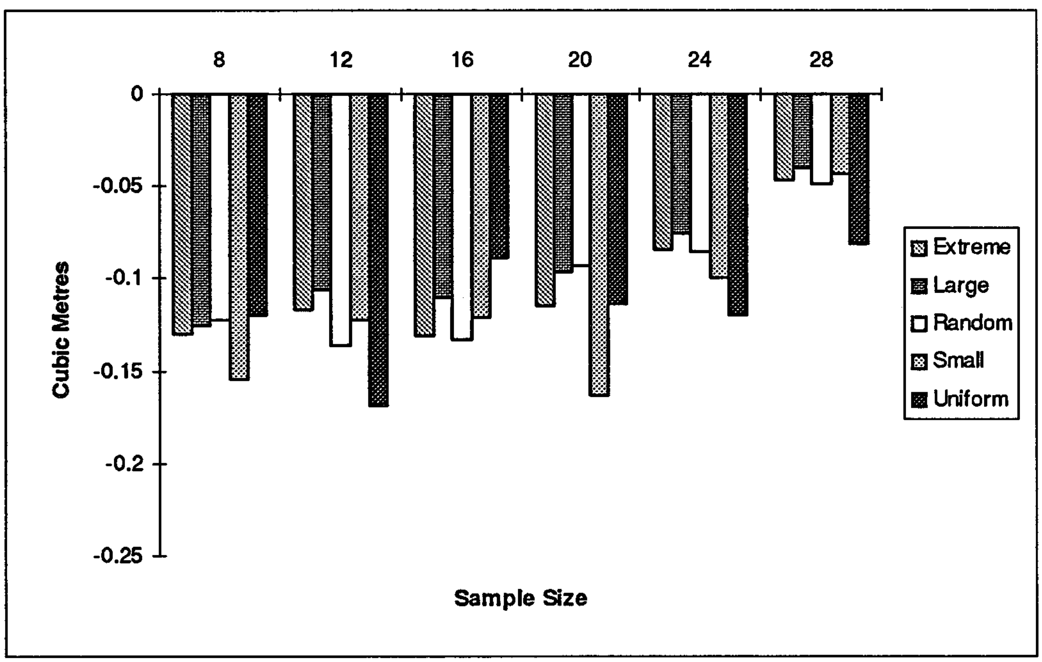


Figure 64. Average minimum deviation in volume for AGE3SI1

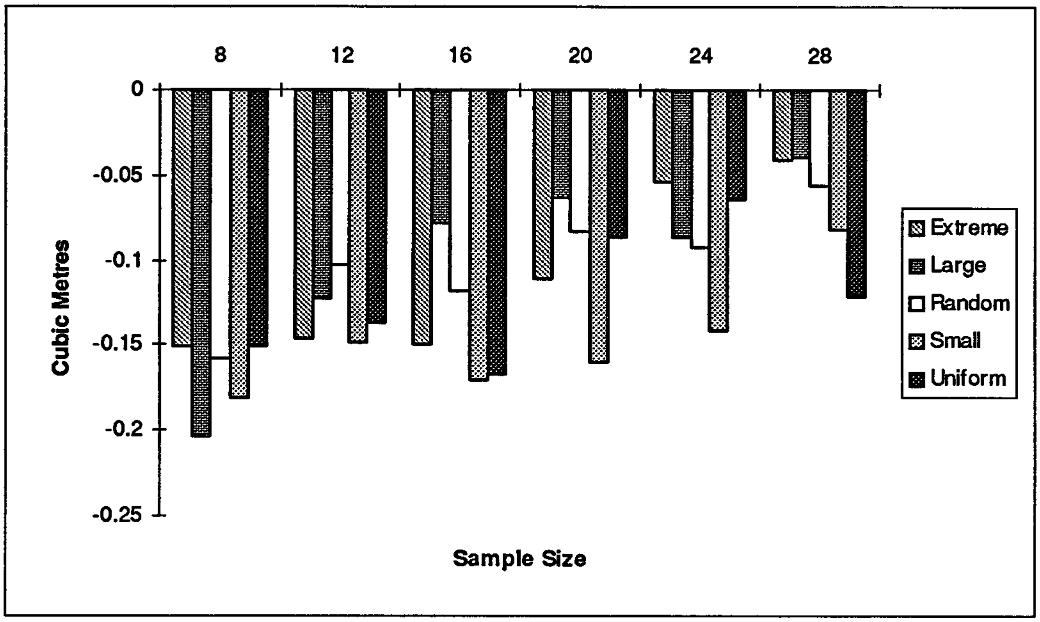
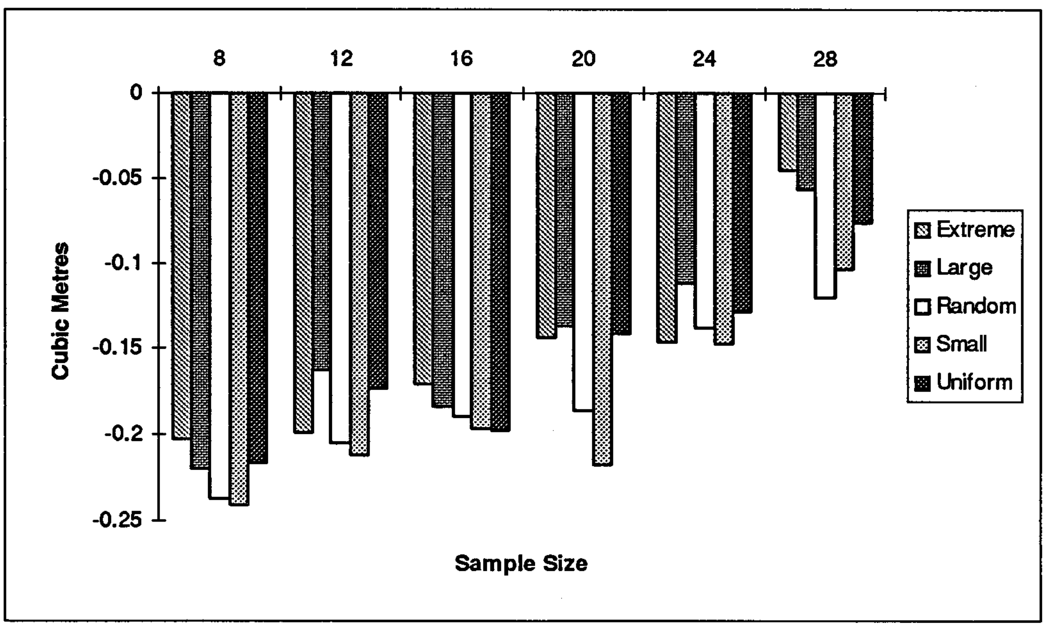


Figure 65. Average minimum deviation in volume for AGE3SI2



As with the average maximum deviation, the average minimum deviation in volume showed that the older age classes produced the worst results. In general, the large design performed well, though trends were not always clear. The small design often gave very poor results in age class 3. In age class 1 and the AGE2SI1 class there were no obvious trends among sample designs. In classes AGE2SI2, AGE2SI3 and AGE3SI1 there was little improvement in the large design when taking more than 16 samples. In class AGE3SI2 results improved with each increase in sample size.

4.2.2.3 Average standard deviation of volume differences

Figures 66 to 72 present the average standard deviation of volume differences. These values are the standard deviations of the differences between measured and estimated tree volumes averaged over each plot and repetition in each age - site index class for each sample size and design.

Figure 66. Average standard deviation of volume differences for AGE1SI2

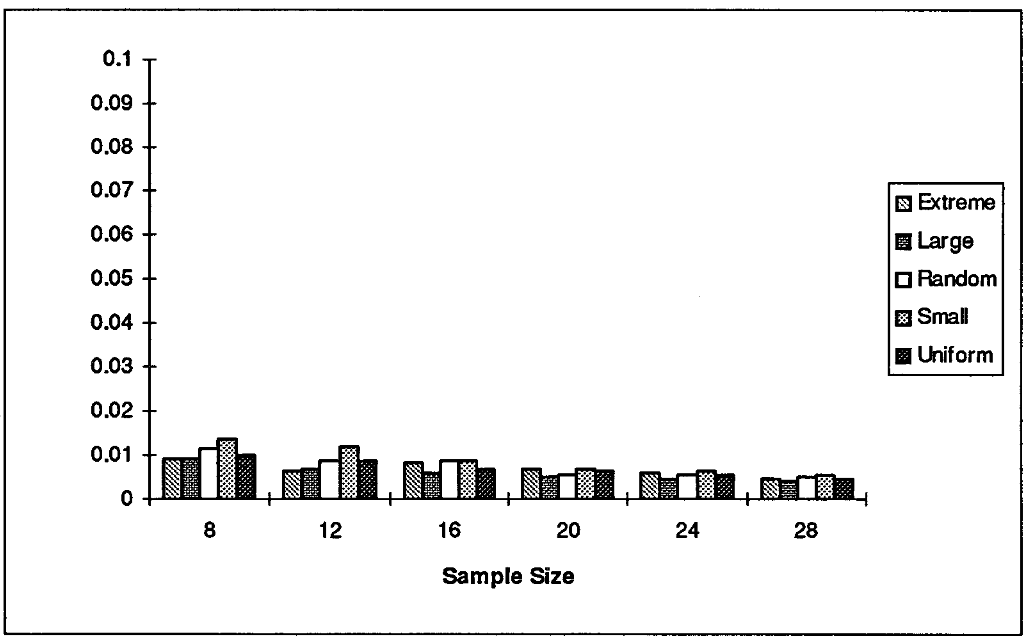


Figure 67. Average standard deviation of volume differences for AGE1SI3

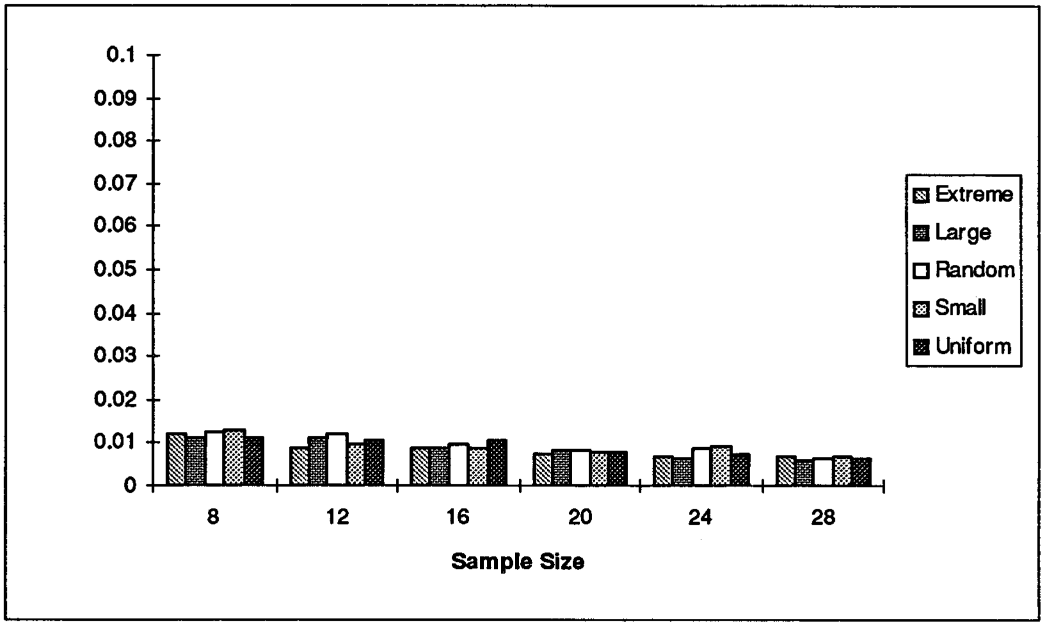


Figure 68. Average standard deviation of volume differences for AGE2SI1

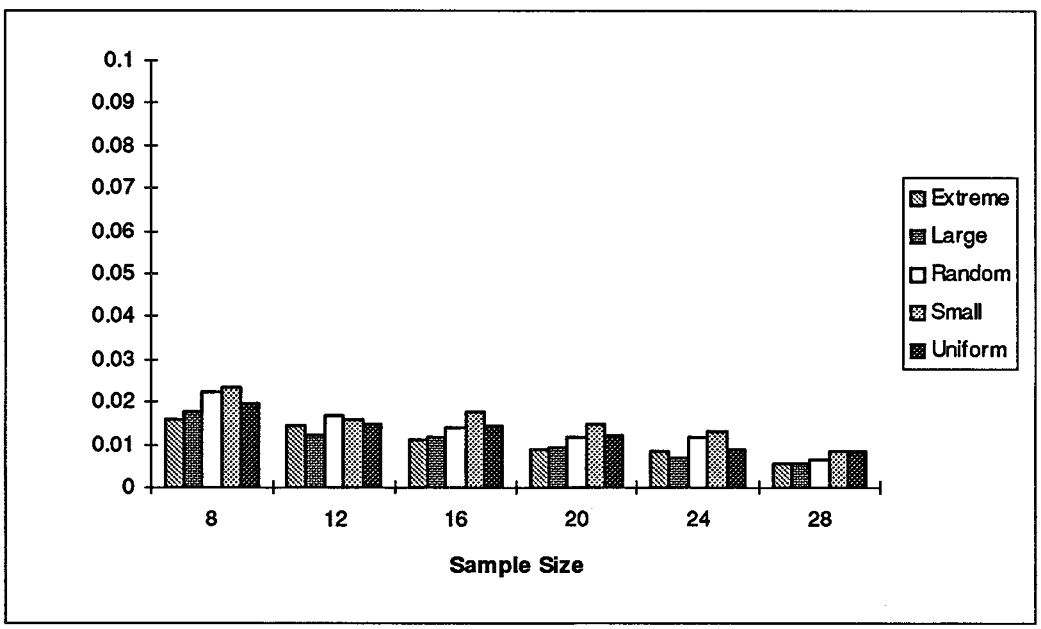


Figure 69. Average standard deviation of volume differences for AGE2SI2

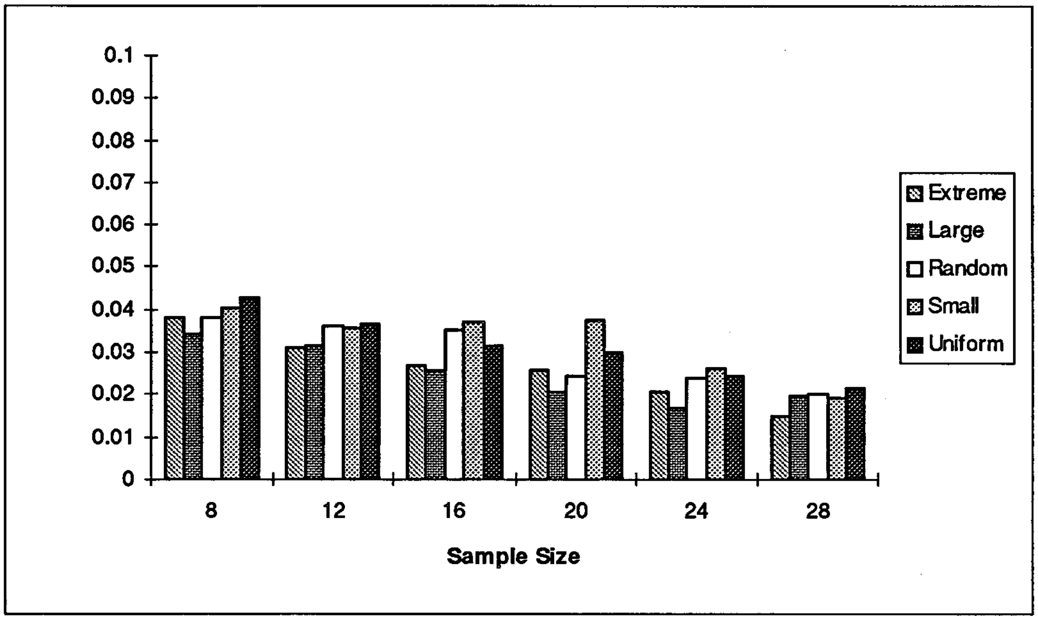


Figure 70. Average standard deviation of volume differences for AGE2SI3

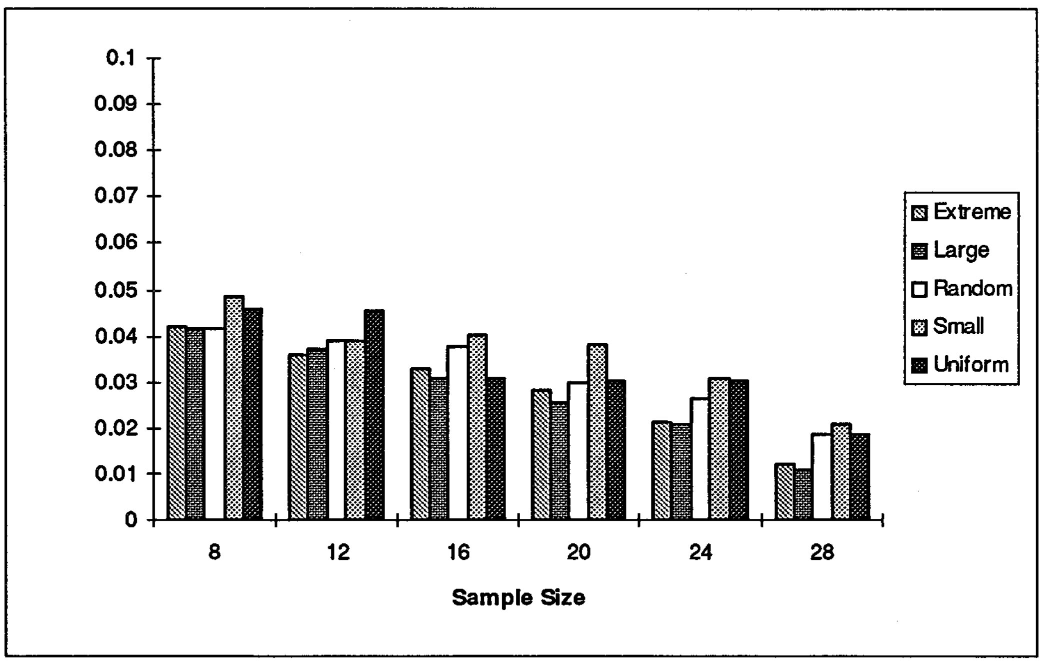


Figure 71. Average standard deviation of volume differences for AGE3SI1

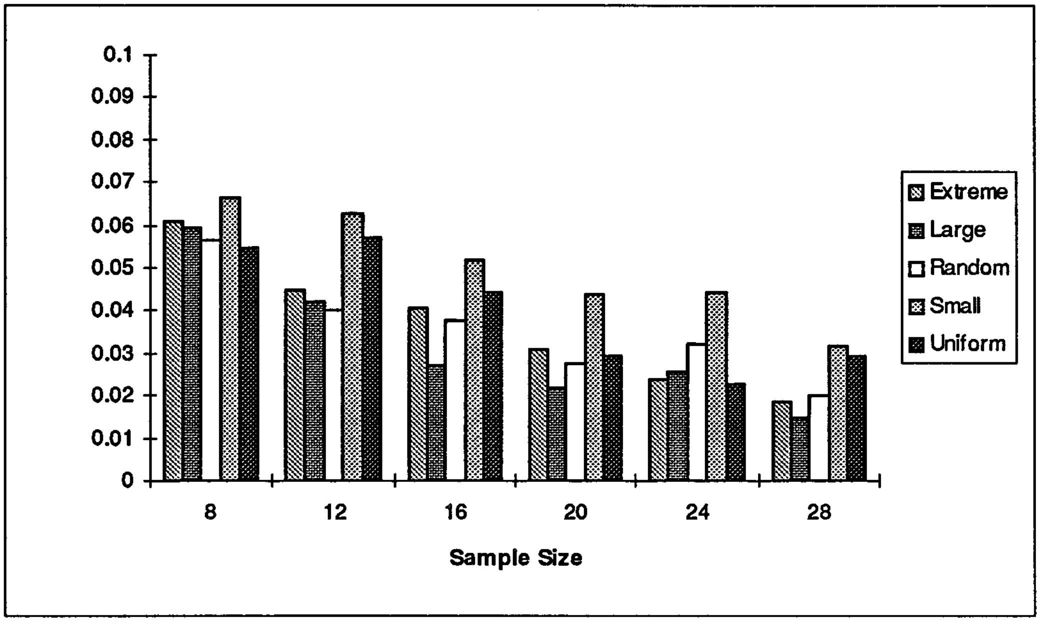
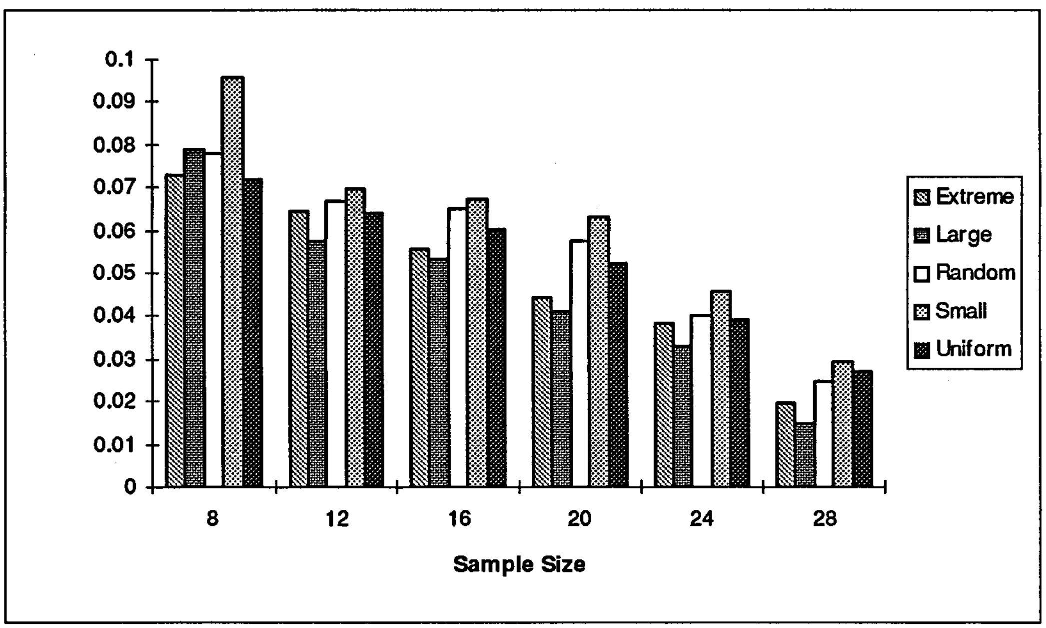


Figure 72. Average standard deviation of volume differences for AGE3SI2



The average standard deviation of volume differences showed some trends that were not apparent in the average standard deviation of height differences. In age class 3, there were noticeable differences among sample designs, with the small design consistently showing the largest average standard deviation. In general, the large design had the lowest average standard deviation in this age class. Age class 3 displayed a constant reduction in average standard deviation when sample size was increased. In age class 3 - site class 1 the most pronounced improvements occurred when sample size was increased to 16. In age class 2, there were no large differences among the different sample designs. In this age class, site index 1 had a lower average standard deviation than site classes 2 and 3. Age class 1 showed little difference among sampling designs and little reduction with increased sample size. There was no clear difference between site index classes for this age group.

4.2.2.4 Mean absolute deviation in volume

Figures 64 to 69 show the mean absolute deviation in volume, averaged for each age class-site index class grouping for sample sizes 8 through to 28

Figure 73. Mean absolute deviation in volume for AGE1SI2

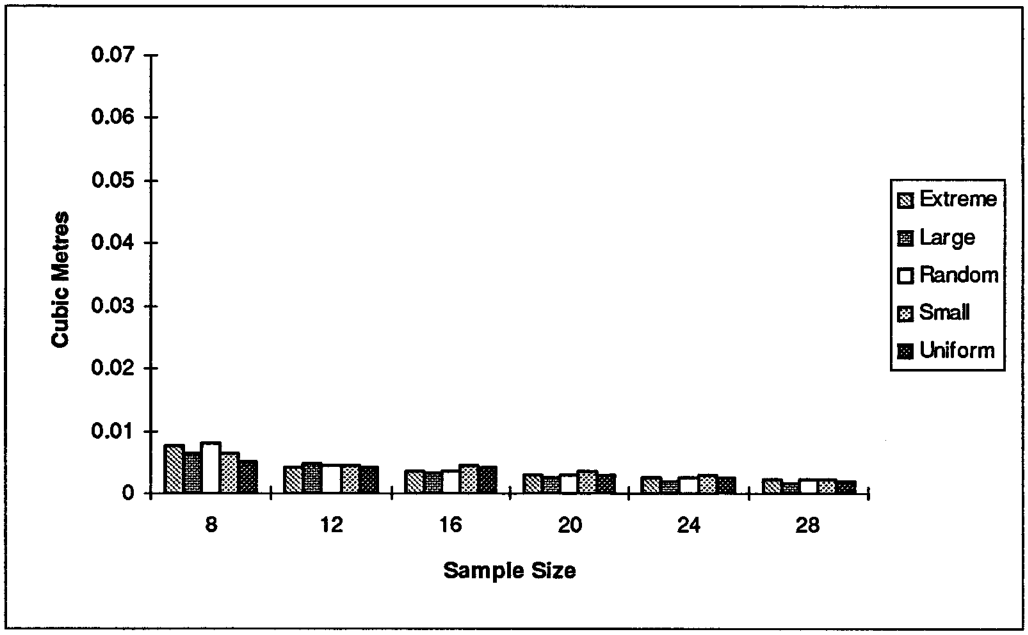


Figure 74. Mean absolute deviation in volume for AGE1SI3

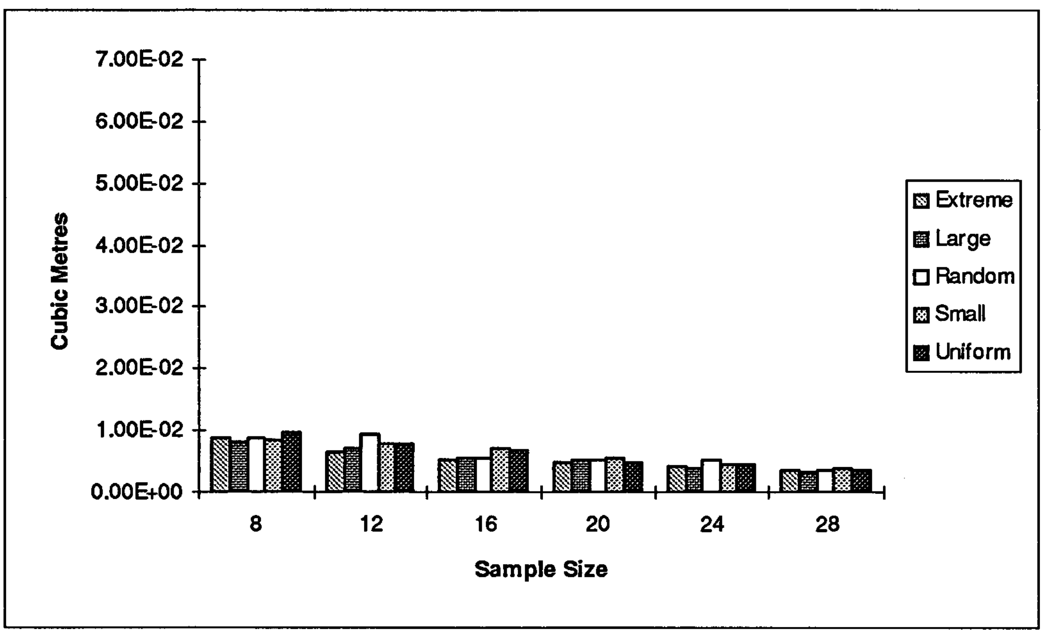


Figure 75. Mean absolute deviation in volume for AGE2SI1

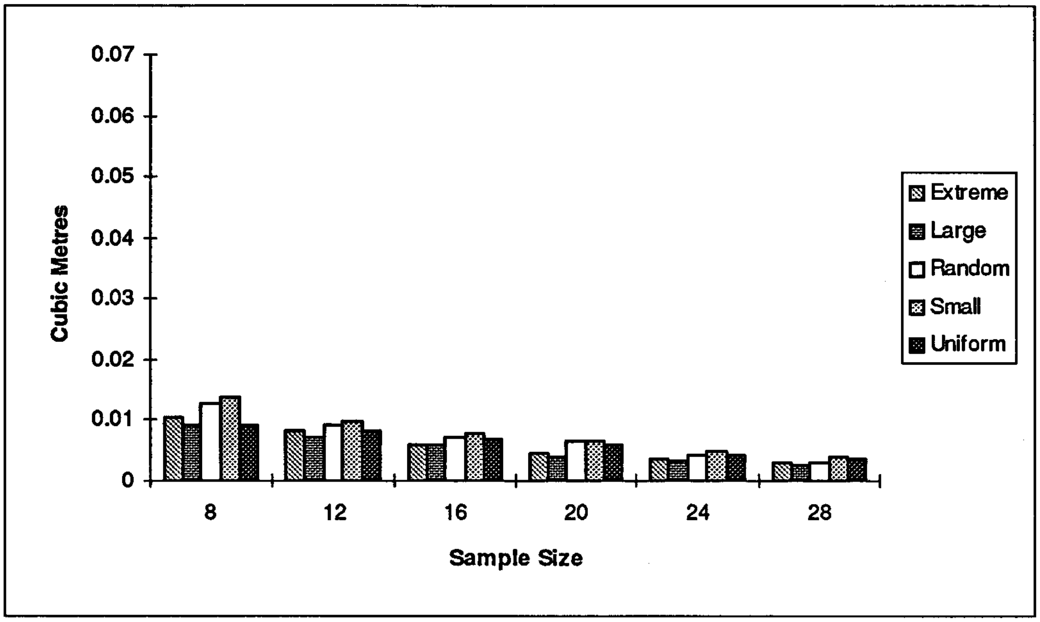


Figure 76. Mean absolute deviation in volume for AGE2SI2

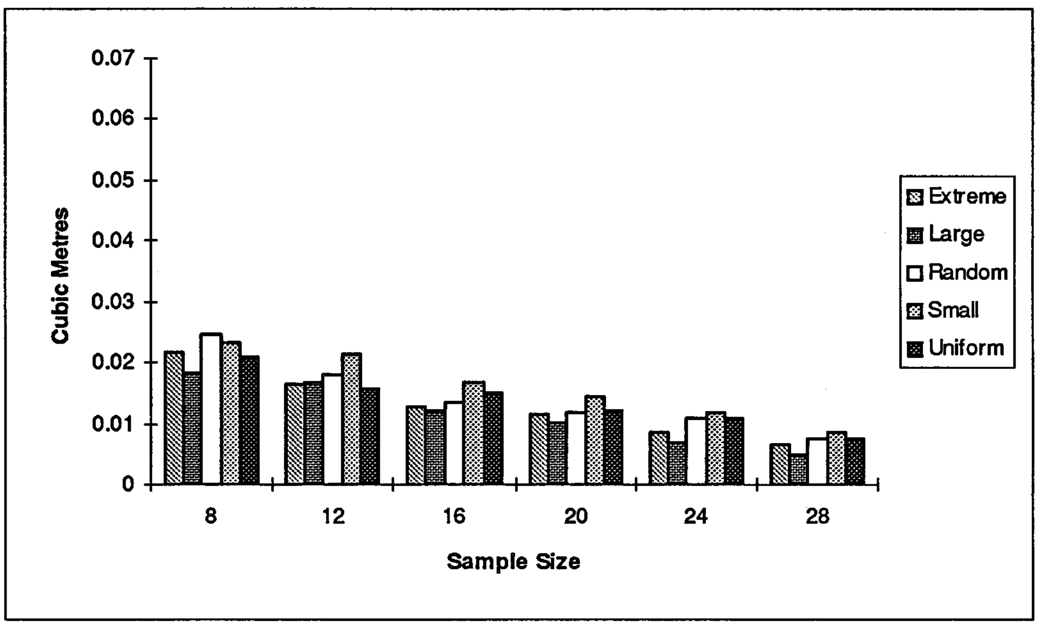


Figure 77. Mean absolute deviation in volume for AGE2SI3

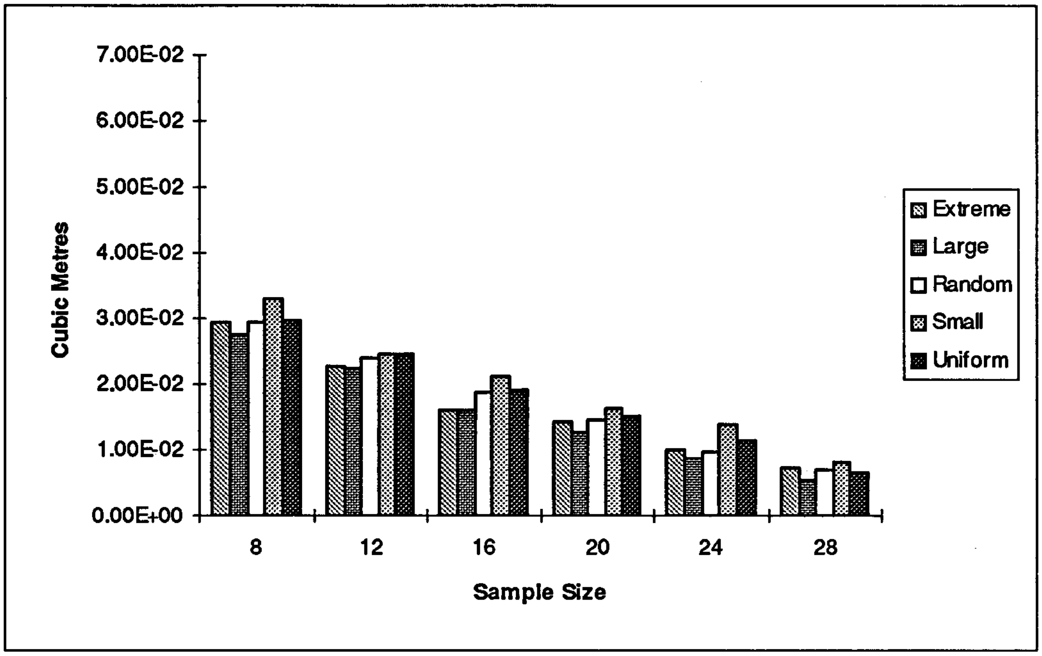


Figure 78. Mean absolute deviation in volume for AGE3SI1

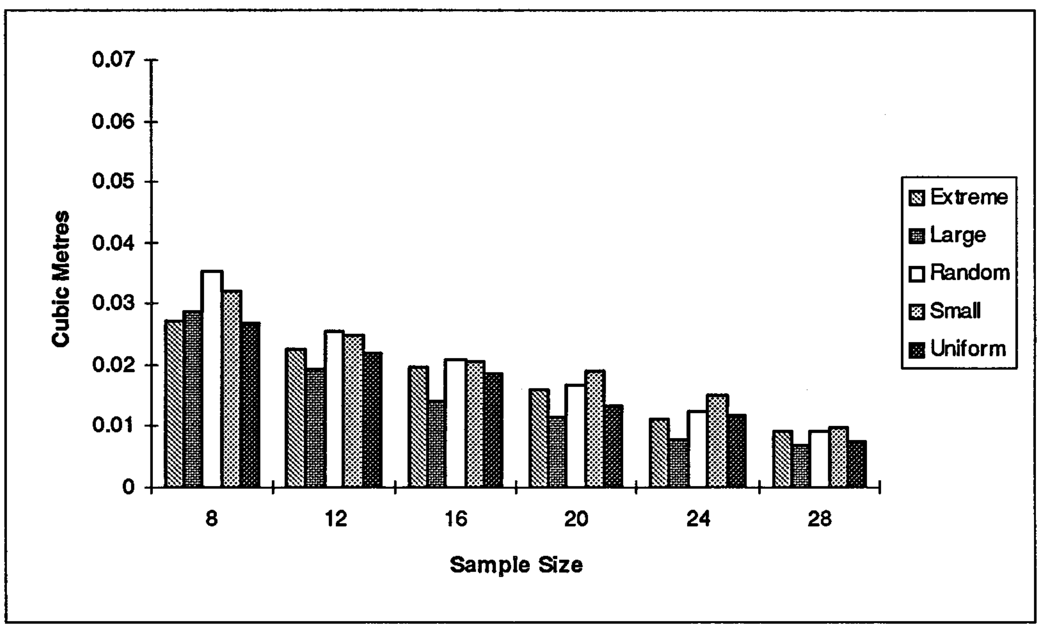
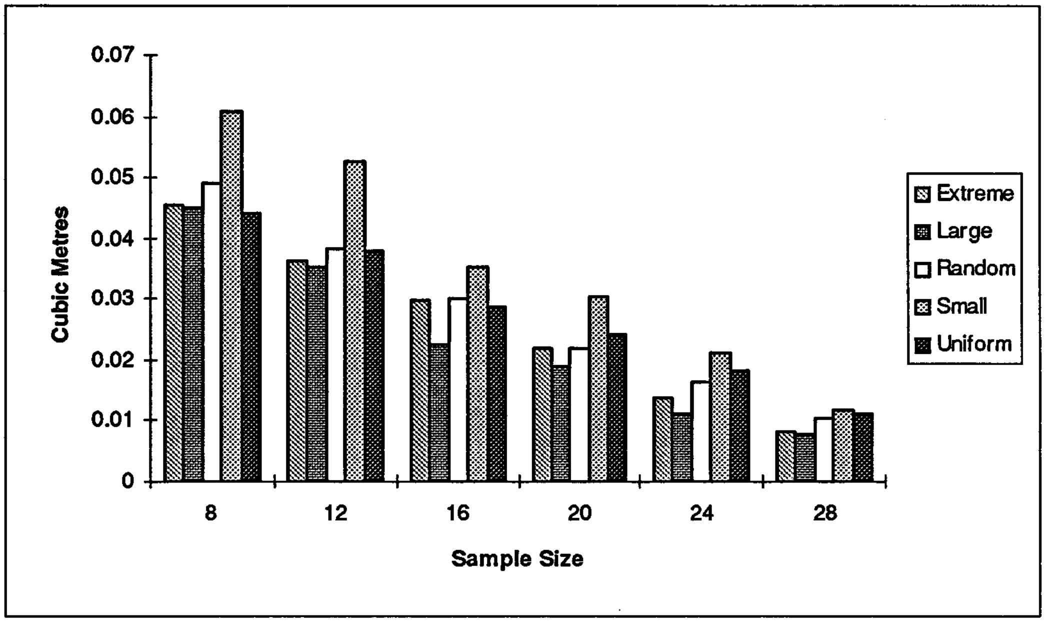


Figure 79. Mean absolute deviation in volume for AGE3SI2



Some trends were evident in the mean absolute deviations in volume. In younger age classes, the difference among sample designs was quite small, even with small sample sizes. In age class 3 site class 2 the small design displayed very poor results for sample sizes 8 and 12. The large design was almost always the best, and the extreme design also performed well. In age class 1, improvements were minimal with sample sizes larger than 12, but age classes 2 and 3 improved with each increase in sample size, with age class 3 displaying the largest relative improvement.

4.3 Ranking of sampling designs

Outputs for six criteria were ranked for each age-site class (Tables 32 to 37 in appendix 2). The rankings were then totalled across the site-age classes, then these totals ranked again to show the relative position of each sample design for each sample size (Table 5).

Table 5. Summarized rankings of sample designs*

Size	Sample	MDHT	MDHTMAX	MDHTMIN	MADHT	MDVOL	MDVOLMAX	MDVOLMIN	MADVOL
8	Extreme	5	1	3	3	4	3	2	3
8	Large	4	2	2	4	2	1	3	1
8	Random	2	3	1	5	1	2	5	5
8	Small	1	4	4	2	5	5	4	4
8	Uniform	3	5	5	1	3	4	1	2
12	Extreme	2	1	2	1	3	1	1	1
12	Large	3	4	1	5	1	2	2	2
12	Random	4	3	3	3	2	3	3	4
12	Small	5	5	4	4	5	4	4	5
12	Uniform	1	2	5	2	4	5	5	3
16	Extreme	5	2	1	1	3	2	2	2
16	Large	1	1	5	4	4	1	1	1
16	Random	4	5	3	3	5	5	4	4
16	Small	2	4	2	2	1	4	5	5
16	Uniform	3	3	4	5	2	3	3	3
20	Extreme	5	5	1	2	5	2	2	2
20	Large	4	4	5	5	2	1	1	1
20	Random	2	1	2	4	3	3	3	4
20	Small	1	3	4	1	4	4	5	5
20	Uniform	3	2	3	3	1	5	4	3
24	Extreme	5	1	1	2	5	2	2	2
24	Large	2	4	4	3	1	1	1	1
24	Random	1	5	3	4	3	3	3	3
24	Small	4	3	2	1	2	5	5	5
24	Uniform	3	2	5	5	4	4	4	4
28	Extreme	5	2	2	1	4	2	1	3
28	Large	2	4	3	2	2	1	2	1
28	Random	3	1	4	3	5	3	4	4
28	Small	4	5	1	4	3	5	3	5
28	Uniform	1	3	5	5	1	4	5	2

*Where:
MDHT is the average mean deviation in height;
MDHTMAX is the average of the largest positive mean deviations in height;
MDHTMIN is the average of the largest negative mean deviations in height;
MADHT is the average of the mean absolute deviations in height;
MDVOL is the average mean deviation in tree volume;
MDVOLMAX is the average of the largest positive mean deviations in volume;
MDVOLMIN is the average of the largest negative mean deviations in volume; and,
MADVOL is the average of the mean absolute deviations in volume.

Average standard deviations in height and volume generally highlighted differences among age-site classes rather than differences among sample designs and were, therefore, not included in this ranking. Likewise, minimum and maximum deviations in height and volume were generated, but not presented graphically or ranked (values are presented in tables 20 and 21, and tables 26 and 27 in appendix 2). These values represented the largest minimum and maximum deviations in height and volume for a single tree estimate. In general, there was little difference among designs and little improvement with larger sample sizes.

To further summarize overall design performance, the rankings from table 5 were summed across sample size to give the relative positions of the designs for each of the criteria (Table 6).

Table 6. Overall rankings of sample designs

Sample	MDHT	MDHTMAX	MDHTMIN	MADHT	MDVOL	MDVOLMAX	MDVOLMIN	MADVOL
Extreme	5	1	1	1	5	2	2	2
Large	2	2	4	5	1	1	1	1
Random	3	3	3	4	3	3	4	4
Small	4	5	2	2	4	5	5	5
Uniform	1	4	5	3	2	4	3	3

5.0 DISCUSSION

5.1 Model Selection

In selecting a model to describe the height-diameter relationship, many different model forms are available for consideration. The model selected in this thesis used a logarithmic transformation of the dependent variable (height). As described in chapter 2, this will result in systematic underestimation, and the results appeared to confirm this. Several methods to correct for this underestimation are available, but none were used in this study. Since the main objective was to compare sample sizes and designs, and the same model was used for the testing of each design, any bias introduced by logarithmic transformation was deemed unimportant.

Another problem that became visible with the selected model was its prediction of decreasing heights for dbh's greater than about 50 cm when it was fit to the entire database. This was likely due to the huge number of small trees in the database, and the very small number of large trees (greater than 50 cm). This condition was not duplicated exactly in the sample simulation, but it may explain in part why the small sample design performed poorly in estimating volume.

The model used in this thesis was linear in its parameters, and was, therefore, fit using standard linear least squares techniques. There are several nonlinear models available that could have been used. There are both advantages and disadvantages to using a nonlinear model, several of these were outlined in chapter 2. For this study, some disadvantages made the use of nonlinear models impractical. In particular, fitting nonlinear least squares is considerably more difficult than linear. The computer program written in this study fits regression coefficients for each plot 150 times: 5 sample designs by 6 sample sizes by 5 repetitions. Besides the greater difficulty of coding the algorithms for the nonlinear estimates, the time required to carry out the sample simulations would have been very large due to the iterative nature of nonlinear least squares. It also would have presented extraordinary difficulties if some samples wouldn't converge, and it would have been very difficult to determine if convergence was at a local minima. In contrast, writing the algorithms for the estimates of the selected model was relatively simple, and program execution was relatively quick.

As stated in chapter 2, it is extremely common to fit height diameter models restricted to an intercept of 1.3 meters (breast height). This is logical, but presented a potential difficulty in fitting the various samples. Because the selected model was used for a large variety of sample sizes taken from plots of different ages, the introduction of such a condition would very likely introduce serious bias into at least some of the samples. It was deemed more appropriate to use a model without any restrictions.

5.2 Sampling designs

Although the rankings presented in Tables 5 and 6 are useful to view the relative quality of the various sampling designs, it is important to recognize that much useful information is not presented. For example, the magnitude of the deviations is completely hidden. It is not clear if one design is only marginally better than another, or if the differences are quite large. However, when considered along with the deviations presented graphically in chapter 4, some useful observations can be made.

5.2.1 Height estimation

In estimating tree height, the uniform design performed well in MDHT and MADHT, especially with small sample sizes (8 - 12). As sample sizes increased to 16 and beyond, the magnitude of differences in MADHT among designs were generally quite small. The uniform design did not perform as well as the other designs in the MDHTMIN and MDHTMAX categories, but these categories often didn't display clear trends among sample designs. The extreme design performed well for most height categories, although it produced some large deviations in MDHT. This would probably be due to the extreme design producing biases that are mostly positive or negative for a given sample. The MADHT results for the extreme design were similar to the uniform design. Despite its relatively high ranking in Table 6, the small design offered little improvement over random sampling, except in MADHT with a sample size of 8. Random sampling gave very poor results in MADHT with a sample size of 8. The large design performed similarly to the small design, but generated better results in MDHT.

In general, the deviations increased with age and site quality (although MADHT did not vary much across classes). This implies increased height variation in older plots, and increased height variation with higher site quality.

When evaluating the effects of increasing sample sizes, it becomes clear that different age-site index classes will require different sample sizes to achieve the same results. In age class 1, and age class 2 site index 1, there was little to be gained by sampling beyond 16, even though these classes had more trees per plot than the other classes (average trees per plot for AGE1SI2 was 67, 58 for AGE1SI3, and 73 for AGE2SI1). Other age classes showed improvements with successive increases in sample size. Even in these classes, the most dramatic improvements occurred at or before reaching a sample size of 20. The average number of trees per plot for these classes were quite similar, with values ranging from 50 to 56.

5.2.2 Volume estimation

In estimating volume, the large design was clearly the best. The large design underestimated volume by the smallest amount as evidenced by the lowest overall rankings in MDVOL and MDVOLMAX, and had the lowest MADVOL in almost every class and sample size.

There are at least two reasons for the success of the large design. First, there is likely greater variation among the larger dbh trees in most of the plots. The large design increases the number of samples taken from the more variable stratum. The second reason has to do with the shape of the chosen height-diameter model. If large samples are omitted when fitting the regression, larger trees can be seriously underestimated because the model may reach its maximum early and begin to decrease. The estimates for the largest trees will be extrapolations beyond the range of the data, resulting in serious underestimation. This problem also existed for the height estimates. However, errors in large trees have a much greater effect on volume estimation than on height estimation, because volume is a cubic measure while height is in linear units.

Overall, the extreme design performed well in estimating MDVOLMAX, MDVOLMIN and MADVOL, although it did show some large underestimations in MDVOL. As with the large design, the good results were likely due to the increased sampling of larger, more variable trees.

The small design was very poor in estimating volume for all criteria. This is likely due in part to the shape of the model. The uniform design performed well in MDVOL, but was not much better than the random design in MDVOLMAX and was poorer in MDVOLMIN. However, it did perform better in MADVOL than the random design.

With volume estimates, the average standard deviation and the maximum mean deviation demonstrated generally good results at sample size 16 for plots in age classes 1 and 2. In age class 1, sampling beyond 12 showed very little improvement in MADVOL. In age class 3, a sample size of 16 or 20 usually gave good results for these criteria and larger sample sizes generally produced diminishing benefits.

5.2.4 Application in the field

Overall, the uniform design performed well in estimating height. The uniform design would improve if there were more dbh classes as this would ensure more even sampling across the dbh range. Three classes were chosen for this study because it could be easily and quickly applied in the field.

There may be several ways to apply any of the designs from this study in the field. The sampling design of choice could be applied in a strict fashion, or in a more flexible manner. For example, a strict implementation of the uniform design would begin with the measuring and recording of all dbh's in the plot. On a separate tally sheet the trees would be transcribed from the original sheet, in order of ascending dbh size. The trees would then be divided into strata, and trees selected for sampling from the tally sheet. If damage and pathological comments are recorded while measuring dbh's there would be no danger of selecting damaged trees as sample trees. If a data recorder or a field computer is being used, it may be possible to program the sorting and stratifying capability into the system. If the plot has been previously measured, and it is not required to remeasure heights taken previously (or, if tree heights have not yet been taken), the stratification could be performed in the office using the dbh's from the previous measurement.

This application does have some drawbacks. Mainly, it would add to the time and cost of the sample, especially in plots with a large number of trees. It may be extremely difficult and not at all cost effective to sort by dbh in the field if there are a large number of trees and the dbh range is small. In many cases, this application may not be practical unless measurements have been recorded on a field computer or data recorder with sorting and stratifying capabilities, or, if the stratification can be performed in the office based on previous dbh measurements.

The advantage of a strict application like this is the ability to select a very uniform sample. If the effort is already being put into sorting and stratifying, it may also not add much cost to increase the number of strata and further improve the sample.

A flexible application would not require a formal stratification, but simply require that the field crew make an effort to identify trees to be sampled for height while measuring dbh's and noting these trees on the tally sheet or data recorder. If the target number of samples is 15 trees, the crew would make an effort to mark 5 suitable trees that, in their estimation, fall into each of the three classes of small, medium and large. This need not be carried out in a strict sense, as long as an effort is made to get a reasonably uniform distribution. After measuring all dbh's, the crew could examine their selections on the field sheet to determine if a reasonable range has been selected. If necessary, some minor adjustments could be made in the selections and the trees could then be sampled for height.

This flexible approach has the advantage of being quick, easily implemented, and not adding significantly to the cost of a normal sample. However, it will not likely achieve the results that a strict implementation would. Mistakes could easily be made and trees incorrectly stratified. It is quite likely that field crew experience could play a critical role in the success of this flexible application.

5.2.5 Measurement errors and costs

While the cost of measuring tree heights is not likely to vary widely with changes in tree size, it is quite likely that the measurement of large trees will be somewhat more costly than small trees. If trees are small enough to be measured with a height pole then the costs of measurement will be much lower.

Many other factors can affect the cost of sampling. In very dense stands it may be difficult to see tree tops and bottoms, adding to the time, and therefore, the cost, of the sample. Stands of mixed species may also be more costly to measure than relatively pure stands, especially if there are indistinct canopy layers that interfere with crown visibility. Higher site stands will tend to have greater density and larger trees, and will therefore be more expensive to measure than lower site stands. Older stands will have larger trees and will exhibit greater variability than younger stands, but they may be less dense.

Some of the factors that affect costs will also affect measurement error. In general, there will be larger errors associated with measuring larger trees, especially if measurement is performed with a clinometer. This means that old stands, and stands of higher site quality may require more samples than younger, lower site stands. Anything which affects visibility in the stand can add to errors in measurement.

6.0 CONCLUSIONS AND RECOMMENDATIONS

Given the importance and relative high cost of tree height measurement it is desirable to sample in such a way that achieves satisfactory precision at the lowest possible cost. This thesis has explored several different inexpensive ways of designing a sample, and simulated the effects of those designs at different sample sizes in fixed area plots of second growth Douglas-fir. The results of this study indicate that the uniform design is a good design for estimating tree height, and that it shows improvement over purely random sampling. The problems which became evident in volume estimation were likely caused in part by the shape of the curve. Much of this could likely be alleviated either by choosing a model which does not have a peak, or by ensuring that sampling does not exclude the largest trees in the plot, and that the remaining samples are not concentrated in small diameter classes. If one of the objectives of the sample is to determine site index, it is likely that one or two of the largest diameter trees will be sampled. The simple addition of these largest dbh trees would likely improve the uniform sample by preventing the curve from reaching its maximum too early, and would therefore minimize the level of underestimation in large trees.

In this study it was found that there was often a diminishing benefit to using sample sizes larger than 16 for both height estimation and volume estimation for young plots (from 10 to 50 years old) of Douglas-fir. If plots are young and relatively uniform, sampling more than 16 trees will probably be wasteful unless precision requirement are high. For plots older than 50 years, the benefits of sampling more trees was greater because the variation was larger than in the younger plots, and it probably would be desirable to sample closer to 20 trees. Given these results, the current B.C. ministry of forests recommendation of 15 trees plus top height trees is likely sufficient for Douglas-fir.

Deciding on an appropriate sample size in practice will have to include the costs of sampling. Since a formal stratification may be expensive to implement, a flexible implementation of the uniform design with a sample size of 15 plus 1 or 2 top height trees should give good results in both height and volume estimation at reasonable cost.

Given the pressures to manage forests more intensively, it is essential that sampling of any kind be as efficient as possible. This study has suggested ways in which height sampling could be carried out in a more efficient manner than the commonly applied random sample and quantified the impact of different designs. It would be desirable to further this work with a larger range of age and site index classes, and to look at other tree species whose characteristics and sampling requirements may be different than those of Douglas-fir. It would also be useful to explore the benefits and costs of using more than 3 strata as this would almost certainly improve the precision of the results. Given the rising pressures and demands on forest management, the issue of efficiency in sampling will be very costly to ignore.

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APPENDIX 1 - ANALYSIS OF VARIANCE FOR HEIGHT-DIAMETER MODELS

Model 1: $H = b_0 + b_1 DBH + b_2 DBH^2$

Table 7. Model 1 coefficients

VARIABLE	COEFFICIENT	STD ERROR	STD COEF	TOLERANCE	T	P(2 TAIL)
CONSTANT	-1.80566	0.12178	0.00000	.	-.15E+02	0.00000
DBH	1.28700	0.01182	1.57537	0.09398	.11E+03	0.00000
DBH2	-0.01052	0.00022	-0.68352	0.09398	-.47E+02	0.00000

Table 8. Model 1 Analysis of Variance

SOURCE	SUM-OF-SQUARES	DF	MEAN-SQUARE	F-RATIO	P
REGRESSION	.471331E+06	2	.235665E+06	22857.97452	0.00000
RESIDUAL	52900.55624	5131	10.30999		
LACK OF FIT	4340.417	245	17.71599	1.782539	0.00000
PURE ERROR	48560.14	4886	9.938629		

Model 2: $H = b_0 + b_1 \log(DBH)$

Table 9. Model 2 coefficients

VARIABLE	COEFFICIENT	STD ERROR	STD COEF	TOLERANCE	T	P(2 TAIL)
CONSTANT	-16.24695	0.21618	0.00000	.	-.75E+02	0.00000
LOGDBH	28.33397	0.18398	0.90670	1.00000	.15E+03	0.00000

Table 10. Model 2 Analysis of Variance

SOURCE	SUM-OF-SQUARES	DF	MEAN-SQUARE	F-RATIO	P
REGRESSION	.430975E+06	1	.430975E+06	23716.96767	0.00000
RESIDUAL	93256.58198	5132	18.17159		

Model 3: $H = b_0 + b_1 DBH + b_2 \sqrt{DBH} + b_3 \left(\frac{1}{\sqrt{DBH}} \right) + b_4 \left(\frac{1}{DBH} \right)$

Table 11. Model 3 Coefficients

VARIABLE	COEFFICIENT	STD ERROR	STD COEF	TOLERANCE	T	P(2 TAIL)
CONSTANT	-76.37259	3.34736	0.00000	.	-.23E+02	0.00000
DBH	-1.16316	0.05666	-1.42378	0.00402	-.21E+02	0.00000
SQRDBH	22.22949	0.75625	3.09126	0.00175	.29E+02	0.00000
INVDSQ	99.62434	5.64473	1.23466	0.00395	.18E+02	0.00000
INVDBH	-40.01958	2.87550	-0.49485	0.01530	-.14E+02	0.00000

Table 12. Model 3 Analysis of Variance

SOURCE	SUM-OF-SQUARES	DF	MEAN-SQUARE	F-RATIO	P
REGRESSION	.472224E+06	4	.118056E+06	11642.80974	0.00000
RESIDUAL	52007.17594	5129	10.13983		
LACK OF FIT	3447.037	243	14.18534	1.427293	0.00002
PURE ERROR	48560.14	4886	9.938629		

Model 4: $\log(H) = b_0 + b_1 \frac{1}{DBH}$

Table 13. Model 4 coefficients

VARIABLE	COEFFICIENT	STD ERROR	STD COEF	TOLERANCE	T	P(2 TAIL)
CONSTANT	1.26293	0.00442	0.00000	.	.29E+03	0.00000
INVDBH	-1.69409	0.02742	-0.65307	1.00000	-.62E+02	0.00000

Table 14. Model 4 Analysis of Variance

SOURCE	SUM-OF-SQUARES	DF	MEAN-SQUARE	F-RATIO	P
REGRESSION	230.03811	1	230.03811	3816.65593	0.00000
RESIDUAL	309.31674	5132	0.06027		

Model 5: $\log(H) = b_0 + b_1 \log(DBH)$

Table 15. Model 5 coefficients

VARIABLE	COEFFICIENT	STD ERROR	STD COEF	TOLERANCE	T	P(2 TAIL)
CONSTANT	0.02620	0.00559	0.00000	.	4.68300	0.00000
LOGDBH	0.94254	0.00476	0.94033	1.00000	.20E+03	0.00000

Table 16. Model 5 Analysis of Variance

SOURCE	SUM-OF-SQUARES	DF	MEAN-SQUARE	F-RATIO	P
REGRESSION	476.90788	1	476.90788	39193.12285	0.00000
RESIDUAL	62.44696	5132	0.01217		

Model 6: $\log(H) = b_0 + b_1 DBH + b_2\sqrt{DBH} + b_3\left(\frac{1}{DBH}\right)$

Table 17. Model 6 Coefficients

VARIABLE	COEFFICIENT	STD ERROR	STD COEF	TOLERANCE	T	P(2 TAIL)
CONSTANT	-0.44400	0.01684	0.00000	.	-.26E+02	0.00000
DBH	-0.04017	0.00075	-1.53284	0.02356	-.54E+02	0.00000
SQRDBH	0.56508	0.00713	2.44986	0.01995	.79E+02	0.00000
INVDBH	0.15434	0.01788	0.05950	0.40160	8.63072	0.00000

Table 18. Model 6 Analysis of Variance

SOURCE	SUM-OF-SQUARES	DF	MEAN-SQUARE	F-RATIO	P
REGRESSION	486.54562	3	162.18187	15754.69032	0.00000
RESIDUAL	52.80923	5130	0.01029		
LACK OF FIT	2.705902	244	0.01109	1.081457	0.18934
PURE ERROR	50.10333	4886	0.010254		

APPENDIX 2 - RESULTS OF SAMPLING

Tables 19 to 31 summarize the results of the sampling simulation. In each table, sample refers to the sampling distribution method where:

E = Extreme design

L = Large design

R = Random design

S = Small design

U = uniform design

Size refers to the sample size while the remaining column headings refer to the age - site index classes.

Table 18. Average mean deviation in height

Size	Sample	AGE1SI2	AGE1SI3	AGE2SI1	AGE2SI2	AGE2SI3	AGE3SI1	AGE3SI2
8	E	0.02132	0.17692	0.11477	-0.13642	-0.04609	0.41061	0.02861
8	L	0.12576	0.09416	0.07129	0.09620	-0.15591	0.29823	-0.06099
8	R	0.17445	0.09512	0.06793	-0.06120	-0.00084	0.17595	0.11868
8	S	0.03963	-0.05761	0.09971	0.01804	0.01487	0.25017	0.26676
8	U	0.14397	0.18552	0.07852	0.04839	0.11634	0.25503	-0.00614
12	E	-0.00358	0.15712	0.34184	0.13520	-0.05020	0.18857	0.03978
12	L	0.11185	0.01921	-0.02911	0.13351	0.07496	0.24263	0.04900
12	R	0.07390	0.06101	0.09326	0.12986	-0.10990	0.13810	0.20454
12	S	0.01270	0.05692	0.16562	0.20670	-0.12986	0.25034	0.23522
12	U	0.07002	-0.01508	-0.07659	0.04545	-0.06065	0.30031	0.06850
16	E	-0.03497	0.11398	0.18141	0.09119	-0.23597	0.07641	-0.10884
16	L	-0.00183	0.11256	0.08652	-0.09369	-0.00227	0.02321	-0.22905
16	R	0.09427	-0.00004	0.00661	0.02000	0.10101	0.15858	0.26369
16	S	0.08559	0.02887	0.05617	-0.06769	0.00766	0.13197	0.22523
16	U	-0.03398	0.05432	-0.05890	0.12188	0.02680	0.10347	-0.00749
20	E	-0.05517	0.13032	0.21421	0.24570	-0.30802	0.09383	-0.03285
20	L	0.02368	0.07437	0.06266	0.17864	-0.16696	0.13425	-0.00865
20	R	0.06754	0.05121	0.10236	-0.01314	0.08109	0.02980	-0.07261

Size	Sample	AGE1SI2	AGE1SI3	AGE2SI1	AGE2SI2	AGE2SI3	AGE3SI1	AGE3SI2
20	S	0.01611	0.02169	0.02192	0.03367	0.03077	0.21259	-0.04195
20	U	0.07802	0.00944	0.02754	-0.05678	-0.04578	0.10918	0.09263
24	E	-0.07173	0.07535	0.18234	0.23637	-0.00463	0.15280	-0.16639
24	L	0.03425	-0.01769	0.05857	-0.01908	0.02229	0.12954	0.09404
24	R	0.10698	-0.01067	0.01907	0.11093	0.08176	0.07488	0.01840
24	S	0.00562	0.04461	-0.00327	0.01046	-0.08621	0.17058	0.13322
24	U	0.03177	-0.01058	0.02199	0.03083	-0.11399	0.13386	-0.02077
28	E	-0.03123	0.05760	0.04853	0.22619	-0.05807	0.11505	0.02431
28	L	0.04157	0.01290	0.00378	0.05415	0.04332	0.11360	-0.03303
28	R	0.01700	0.02691	0.05564	-0.01019	0.06079	0.03031	-0.07818
28	S	0.00890	0.01803	0.07856	0.13267	0.11159	0.17081	0.00900
28	U	0.08422	0.01322	0.04349	-0.04025	0.00210	0.01610	0.00162

Table 19. Average maximum deviation in height

Size	Sample	AGE1SI2	AGE1SI3	AGE2SI1	AGE2SI2	AGE2SI3	AGE3SI1	AGE3SI2
8	E	3.25977	2.92441	3.29473	4.50977	4.13801	5.24299	4.46698
8	L	3.73699	3.68121	3.95492	4.38898	4.87098	4.60011	4.92505
8	R	4.21282	3.47162	4.19169	4.26385	4.61506	4.65826	4.93468
8	S	3.73393	2.90848	4.34794	5.01013	5.05476	5.09534	5.22667
8	U	3.77906	4.03527	3.72053	4.85712	4.90446	5.27529	4.54480
12	E	2.61917	2.22879	3.94471	4.07892	3.76823	4.34138	4.36644
12	L	3.04627	2.41767	3.38391	4.22174	4.55155	4.26147	4.48593
12	R	2.74408	3.15831	3.72894	4.48078	4.09106	3.93256	4.32527
12	S	3.60150	2.69531	3.34798	4.49816	3.80662	4.79726	4.51149
12	U	2.89515	2.56279	2.89439	4.08777	5.09879	5.30105	4.27180
16	E	2.79313	2.38715	3.29332	3.61558	3.40128	3.95664	4.07704
16	L	2.74055	2.84540	2.91365	3.61521	3.80088	3.83749	3.77010
16	R	3.18494	2.24203	2.72873	4.45434	4.78900	3.97234	4.53479
16	S	2.89147	2.09895	3.35851	3.75426	4.34733	4.20186	4.48213
16	U	2.54119	2.62785	3.42923	4.11621	4.16074	3.99704	4.17972
20	E	2.46414	2.06385	3.08227	3.96848	2.81322	3.58145	3.92655
20	L	2.32749	2.68093	2.89132	3.83604	3.27436	3.50842	4.17053
20	R	2.41414	2.41199	2.82663	3.46112	4.04160	3.45032	4.02030
20	S	2.12059	1.93743	3.09740	3.88169	3.57765	3.89258	3.32460
20	U	2.65902	1.94683	2.85932	3.68759	3.52779	3.81758	3.72863
24	E	1.91695	1.95115	2.96908	3.20231	2.86618	3.68308	3.05505
24	L	2.08574	1.99647	2.77332	3.10017	3.28430	3.71241	3.68362
24	R	2.65262	2.13737	2.87571	3.42276	4.19591	3.13726	3.18621
24	S	1.84797	2.08764	2.94904	3.46326	3.27137	3.35316	3.38944
24	U	2.00323	1.84991	2.46639	3.72922	3.40955	3.45049	3.33806
28	E	1.47456	1.85787	1.95535	2.97585	2.00289	3.23150	2.84613
28	L	2.37560	1.81950	1.89611	3.16350	2.84463	3.12797	2.48708
28	R	2.27322	1.60421	2.00834	2.60570	3.13087	2.68098	1.70306

Size	Sample	AGE1SI2	AGE1SI3	AGE2SI1	AGE2SI2	AGE2SI3	AGE3SI1	AGE3SI2
28	S	1.81506	1.69152	2.54066	3.04338	3.49166	3.32997	2.62904
28	U	2.46803	1.78232	2.32194	2.99851	2.65508	2.53690	2.55828

Table 20. Average minimum deviation in height

Size	Sample	AGE1SI2	AGE1SI3	AGE2SI1	AGE2SI2	AGE2SI3	AGE3SI1	AGE3SI2
8	E	-3.86460	-3.17650	-3.53454	-5.76553	-5.28387	-4.51590	-6.39588
8	L	-3.72141	-3.28301	-3.96924	-5.57000	-5.41690	-4.40032	-6.23129
8	R	-3.57749	-3.97145	-3.47334	-5.48943	-5.15804	-4.79764	-6.06278
8	S	-4.01926	-3.52828	-3.33813	-4.95862	-5.20577	-4.94017	-6.52815
8	U	-4.28121	-3.52202	-4.02877	-5.58101	-5.05188	-5.18658	-6.76975
12	E	-2.96492	-2.43414	-2.66876	-4.80114	-4.99656	-4.42716	-6.27827
12	L	-2.80080	-2.92437	-3.94687	-4.62135	-4.81127	-4.50441	-5.30959
12	R	-3.13976	-3.35396	-3.06790	-4.85583	-4.87794	-3.72514	-6.09554
12	S	-3.18269	-3.04853	-3.18263	-4.36269	-4.87126	-4.18547	-6.11902
12	U	-3.05336	-2.77411	-3.99520	-5.15383	-5.52138	-4.15665	-6.14780
16	E	-2.95332	-2.57674	-2.53627	-4.14027	-5.21295	-4.29690	-5.65267
16	L	-3.00672	-2.95018	-3.06412	-5.25934	-4.65090	-3.95386	-6.01774
16	R	-3.00380	-2.67808	-3.26978	-4.82006	-4.36875	-4.37990	-4.97999
16	S	-2.87173	-2.61160	-3.65379	-4.90995	-4.32290	-4.09187	-5.04560
16	U	-3.34357	-3.08853	-3.40954	-4.67012	-4.15662	-4.08393	-5.36278
20	E	-2.75113	-2.13755	-1.97912	-3.56790	-5.08421	-3.93632	-5.00226
20	L	-2.97621	-3.04223	-2.72419	-3.83142	-4.93435	-3.57786	-5.10241
20	R	-2.36009	-2.71702	-2.64149	-4.14873	-3.97383	-3.53537	-5.27198
20	S	-2.30365	-2.44169	-2.79995	-4.04129	-4.82308	-3.96977	-5.36238
20	U	-2.39643	-2.38566	-2.96554	-5.07853	-4.07713	-3.24350	-4.84853
24	E	-2.67301	-2.18342	-1.76379	-3.29136	-3.73278	-2.59819	-5.19975
24	L	-2.41376	-2.57047	-2.56068	-3.94597	-3.90249	-3.26492	-4.38572
24	R	-2.14129	-2.25981	-2.91801	-3.77955	-3.33079	-3.43584	-4.64081
24	S	-2.25093	-2.17264	-2.88525	-3.82642	-3.89548	-3.20160	-3.89885
24	U	-2.25666	-2.81101	-2.60850	-3.93510	-4.56043	-2.80067	-4.50497
28	E	-2.37925	-1.84358	-1.79514	-2.49960	-2.71286	-2.36547	-2.44942
28	L	-2.17781	-2.14381	-2.00901	-3.62586	-2.83833	-1.76471	-3.11698
28	R	-2.21157	-2.05162	-1.77034	-3.96623	-2.55278	-2.63397	-4.43269
28	S	-2.10283	-1.89700	-1.93322	-2.89014	-2.24838	-2.23738	-3.05406
28	U	-2.14202	-2.13157	-2.30860	-3.38219	-3.08381	-2.96111	-3.64177

Table 21. Average mean absolute deviation in height

Size	Sample	AGE1SI2	AGE1SI3	AGE2SI1	AGE2SI2	AGE2SI3	AGE3SI1	AGE3SI2
8	Extreme	1.21687	1.17635	1.32674	1.43259	1.36905	1.47645	1.75011
8	Large	1.51757	1.12957	1.03763	1.50684	1.63440	1.61720	1.71108
8	Random	1.77791	1.26312	1.32165	2.15701	1.43962	1.98202	2.03929
8	Small	0.88884	1.01131	1.32760	1.40990	1.45794	1.57626	1.97424
8	Uniform	1.09082	1.08184	1.26700	1.41388	1.53475	1.46052	1.76723

Size	Sample	AGE1SI2	AGE1SI3	AGE2SI1	AGE2SI2	AGE2SI3	AGE3SI1	AGE3SI2
12	Extreme	0.77609	0.99964	0.91218	1.14691	1.07163	1.19666	1.45169
12	Large	0.86249	1.29749	0.91917	1.35163	1.50797	1.30909	1.61294
12	Random	0.72496	0.90717	0.90502	1.30302	1.24380	1.31663	1.61904
12	Small	0.75017	0.92806	1.00504	1.31898	1.08097	1.15322	1.73933
12	Uniform	0.81756	1.29494	0.89716	1.13430	1.21698	1.18762	1.49119
16	Extreme	0.59436	0.58643	0.71020	0.96205	0.82291	1.07587	1.20837
16	Large	0.81080	1.03493	0.77478	0.98364	0.95371	1.01261	1.11491
16	Random	0.64597	0.70346	0.78248	0.97799	0.93710	1.01128	1.25650
16	Small	0.57171	1.00239	0.77143	0.99106	0.89984	0.94236	1.13237
16	Uniform	0.76476	0.93585	0.80422	1.02984	0.85486	0.97051	1.27965
20	Extreme	0.50802	0.51282	0.61058	0.84991	0.73664	0.86793	0.92114
20	Large	0.49005	1.04504	0.63420	0.92580	0.76572	0.80762	0.99862
20	Random	0.51095	0.66660	0.70886	0.83965	0.70554	0.81966	0.96066
20	Small	0.66484	0.50939	0.63666	0.81530	0.70057	0.79245	0.92720
20	Uniform	0.54013	0.56707	0.63258	0.84709	0.76066	0.77238	0.99781
24	Extreme	0.40716	0.43970	0.51289	0.71803	0.55004	0.66570	0.66272
24	Large	0.44664	0.49933	0.52021	0.66603	0.57721	0.61941	0.75971
24	Random	0.38952	1.21056	0.52923	0.74811	0.51531	0.65202	0.76065
24	Small	0.37059	0.42060	0.51739	0.68941	0.57594	0.61963	0.71718
24	Uniform	0.41415	0.54738	0.54598	0.70626	0.55973	0.59740	0.89817
28	Extreme	0.30932	0.35788	0.43814	0.55418	0.41768	0.51780	0.47029
28	Large	0.32578	0.50491	0.42977	0.52720	0.40428	0.48994	0.55839
28	Random	0.40177	0.37511	0.42431	0.56107	0.41808	0.46473	0.52255
28	Small	0.27581	0.33442	0.43877	0.51808	0.40859	0.45917	0.47894
28	Uniform	0.27942	0.35337	0.45149	0.55710	0.39424	0.42916	0.50696

Table 22. Average standard deviation of height differences

Size	Sample	AGE1SI2	AGE1SI3	AGE2SI1	AGE2SI2	AGE2SI3	AGE3SI1	AGE3SI2
8	E	1.26946	1.24888	1.30616	1.93566	1.80140	2.20894	2.27763
8	L	1.43999	1.34380	1.54073	1.97959	1.92343	2.01389	2.43949
8	R	1.44391	1.37308	1.45965	1.95628	1.84577	2.04221	2.41502
8	S	1.42175	1.22605	1.56407	1.93733	1.84319	2.05720	2.49971
8	U	1.47895	1.43942	1.49589	2.00871	1.89251	2.25400	2.42905
12	E	1.03349	0.95260	1.20945	1.65205	1.64120	1.83434	2.09656
12	L	1.09067	1.05013	1.33072	1.67977	1.67475	1.91281	2.08978
12	R	1.06015	1.19652	1.25796	1.73355	1.64766	1.68217	2.15650
12	S	1.15249	1.08742	1.22962	1.63881	1.60367	1.86745	2.12035
12	U	1.11829	1.03638	1.28922	1.76568	1.85749	1.91341	2.13361
16	E	0.97862	0.94172	1.06566	1.46299	1.51881	1.68291	1.86048
16	L	1.00994	1.04139	1.13530	1.59426	1.53298	1.59538	1.95231
16	R	1.13075	0.94108	1.16052	1.67232	1.54627	1.58178	1.87341
16	S	1.03366	0.87571	1.21725	1.53533	1.50177	1.62539	1.91620
16	U	1.04031	1.01770	1.23310	1.58090	1.44617	1.53169	1.88024
20	E	0.85005	0.78368	0.93842	1.36704	1.35119	1.51062	1.61560
20	L	0.89390	1.01759	1.02071	1.37005	1.39398	1.36262	1.66805

Size	Sample	AGE1SI2	AGE1SI3	AGE2SI1	AGE2SI2	AGE2SI3	AGE3SI1	AGE3SI2
20	R	0.84931	0.92295	1.03007	1.40288	1.30193	1.26513	1.71558
20	S	0.78788	0.79934	1.06332	1.42101	1.39555	1.42437	1.70565
20	U	0.89102	0.79574	1.01456	1.51235	1.30958	1.28784	1.63052
24	E	0.74616	0.72579	0.84129	1.16564	1.01664	1.18915	1.43971
24	L	0.77406	0.77941	0.89339	1.20899	1.17783	1.30914	1.53490
24	R	0.81939	0.75030	0.95255	1.23242	1.20706	1.17154	1.42358
24	S	0.68544	0.72737	0.93452	1.24294	1.16369	1.15129	1.34029
24	U	0.70598	0.78365	0.87443	1.28707	1.26268	1.08572	1.37993
28	E	0.63698	0.64125	0.67181	0.97099	0.71974	1.02501	1.00172
28	L	0.70841	0.68819	0.68161	1.13330	0.86236	0.91805	0.95409
28	R	0.68477	0.64293	0.71083	1.10486	0.86972	0.90653	1.04362
28	S	0.60006	0.61769	0.74147	1.02622	0.83886	1.00294	0.97642
28	U	0.72188	0.67074	0.78587	1.07738	0.84704	1.00520	1.05568

Table 23. Maximum deviation in height

Size	Sample	AGE1SI2	AGE1SI3	AGE2SI1	AGE2SI2	AGE2SI3	AGE3SI1	AGE3SI2
8	E	6.97870	6.23726	5.82258	8.72489	6.80243	9.88509	9.25374
8	L	8.78882	8.44242	8.13925	8.45071	8.39832	8.10823	9.58774
8	R	8.40106	9.83470	8.08259	7.33867	8.25253	8.12190	9.98997
8	S	9.21462	6.52908	9.76098	9.38904	7.89020	9.83525	8.75845
8	U	7.56749	8.72119	7.40698	9.39540	8.66276	9.10073	9.38567
12	E	7.11171	4.60496	8.88221	9.85276	5.82330	9.92266	7.53601
12	L	7.99431	5.31233	7.59155	9.18446	5.98672	8.34806	7.60134
12	R	6.77848	9.97526	9.55594	9.53652	5.66622	7.71248	7.34671
12	S	9.55534	8.54698	7.22131	8.46754	7.79184	9.67592	9.02018
12	U	7.93189	6.09726	4.92527	7.83960	7.89510	9.49975	6.82832
16	E	9.39645	5.19778	6.95456	6.32282	5.55144	8.98994	7.85649
16	L	7.38715	7.41852	6.09249	8.96876	5.53410	7.20071	9.47306
16	R	7.04353	5.82380	6.71519	9.66014	9.94361	7.05352	8.02210
16	S	9.75141	3.84658	8.59228	8.83304	9.98082	9.87158	8.44479
16	U	7.37128	6.14714	9.80810	7.06968	5.58212	7.84079	8.04379
20	E	5.05689	3.86932	5.66431	9.23966	5.50805	9.29513	6.95014
20	L	4.23243	8.54827	5.96412	8.06878	5.94780	8.15113	7.36875
20	R	5.01083	7.81731	6.06427	6.44606	7.58257	6.84056	8.91251
20	S	6.31416	4.29984	6.08038	9.79156	5.19253	8.06997	6.73421
20	U	6.40300	3.81059	8.02989	9.90486	5.69408	7.30908	6.04455
24	E	4.79342	3.92641	4.87198	5.01234	5.34229	8.41505	5.28619
24	L	4.91149	5.35887	6.87697	4.98735	4.57833	7.98737	9.19569
24	R	7.20855	9.57271	8.32039	6.30534	5.85622	7.17229	6.77404
24	S	3.78047	6.66393	7.72426	6.73575	6.11461	7.77025	5.98725
24	U	3.40763	4.40804	6.79885	6.81625	5.55581	7.63125	5.77963
28	E	3.93649	3.48821	3.13119	5.81943	5.18154	7.88561	5.78900
28	L	4.88971	6.46664	3.83030	7.75277	5.52777	7.62432	6.36125
28	R	4.04763	3.56913	4.84953	5.56489	6.13515	7.57923	5.97264
28	S	5.15947	3.40619	5.67194	4.51246	9.41762	7.58135	5.64743

Size	Sample	AGE1SI2	AGE1SI3	AGE2SI1	AGE2SI2	AGE2SI3	AGE3SI1	AGE3SI2
28	U	6.41589	4.41690	5.91453	8.98437	7.43092	7.08949	7.17184

Table 24. Minimum deviation in height

Size	Sample	AGE1SI2	AGE1SI3	AGE2SI1	AGE2SI2	AGE2SI3	AGE3SI1	AGE3SI2
8	E	-9.13838	-9.90684	-7.54092	-9.79867	-8.31010	-9.50333	-9.89526
8	L	-7.97744	-8.34651	-8.68960	-9.06183	-8.05737	-8.65429	-9.67357
8	R	-8.86607	-9.71342	-5.44407	-9.53654	-9.15936	-9.75261	-8.91976
8	S	-7.19362	-8.35578	-7.10947	-9.90311	-9.67540	-9.99862	-9.36474
8	U	-9.80928	-9.57769	-8.49960	-9.37838	-8.64384	-9.57405	-9.92746
12	E	-5.70985	-5.25701	-4.32431	-8.42876	-6.98480	-8.56483	-8.41200
12	L	-5.69083	-7.70506	-8.36924	-9.03925	-7.68806	-9.09987	-9.06308
12	R	-6.93995	-6.77775	-4.75739	-8.65475	-7.80993	-6.52657	-9.59041
12	S	-6.34440	-6.93437	-6.10195	-9.15033	-7.69417	-8.13212	-9.52373
12	U	-6.79423	-5.74627	-9.71750	-9.45383	-8.48240	-8.57126	-9.25014
16	E	-6.87813	-8.99086	-4.31903	-8.75814	-7.87160	-8.48988	-8.67036
16	L	-7.20796	-8.93433	-5.95763	-9.73409	-8.34920	-9.58919	-9.63966
16	R	-5.67239	-6.20433	-7.46283	-8.60953	-7.75975	-9.78070	-9.67262
16	S	-5.59860	-5.95299	-7.29884	-9.25175	-7.31988	-8.85680	-8.99169
16	U	-6.42720	-8.99156	-7.25401	-8.91977	-7.98699	-7.12489	-9.18485
20	E	-6.44482	-4.71466	-3.70965	-8.49960	-8.63562	-8.31846	-9.00390
20	L	-7.36406	-9.23976	-5.17299	-8.22242	-8.35769	-9.27230	-9.47571
20	R	-5.23313	-7.36517	-4.07359	-8.71971	-7.14514	-4.91174	-9.85714
20	S	-5.93961	-7.85964	-4.96424	-9.92212	-9.64337	-8.56585	-9.45443
20	U	-5.53374	-5.75360	-4.55003	-8.88191	-8.02414	-8.01462	-9.26192
24	E	-5.71673	-5.25520	-3.56080	-9.25230	-7.74400	-7.74255	-6.69419
24	L	-6.92274	-5.75892	-3.82585	-9.25619	-9.23877	-8.10010	-9.37523
24	R	-4.78170	-5.58043	-5.49378	-8.12283	-9.19624	-7.55549	-9.16435
24	S	-5.54150	-5.46296	-6.25759	-8.01595	-7.59922	-7.90229	-6.65543
24	U	-5.15592	-7.75570	-4.90204	-8.66024	-7.91389	-5.76521	-9.53883
28	E	-5.60568	-5.00193	-3.48074	-5.35087	-7.78066	-4.70290	-5.80312
28	L	-5.41101	-5.81885	-3.73864	-8.46724	-9.44050	-4.37136	-8.74710
28	R	-5.18210	-5.19776	-3.87461	-8.26387	-6.98652	-7.62760	-8.28888
28	S	-5.48249	-5.42208	-3.34845	-6.67200	-6.78807	-7.33385	-6.45534
28	U	-4.92921	-6.70079	-5.20005	-8.16251	-7.32831	-7.92488	-9.50034

Table 25. Average mean deviation in volume

Size	Sample	AGE1SI2	AGE1SI3	AGE2SI1	AGE2SI2	AGE2SI3	AGE3SI1	AGE3SI2
8	E	-0.00051	0.00232	0.00136	-0.00136	0.00114	0.01547	0.00600
8	L	-0.00026	0.00051	-0.00021	-0.00047	-0.00318	0.00629	0.00237
8	R	0.00029	0.00049	0.00098	-0.00088	0.00032	0.00862	0.00006
8	S	0.00062	-0.00092	0.00248	0.00247	0.00327	0.01332	0.01177
8	U	0.00021	0.00095	0.00180	0.00186	0.00177	0.00948	-0.00316
12	E	-0.00025	0.00154	0.00368	0.00127	-0.00152	0.00736	0.00384

Size	Sample	AGE1SI2	AGE1SI3	AGE2SI1	AGE2SI2	AGE2SI3	AGE3SI1	AGE3SI2
12	L	0.00026	-0.00046	0.00030	0.00243	0.00144	0.00774	0.00089
12	R	0.00005	0.00081	0.00184	0.00186	-0.00396	0.00633	0.00123
12	S	0.00093	0.00032	0.00186	0.00350	-0.00372	0.01224	0.00447
12	U	0.00054	-0.00074	-0.00022	0.00145	-0.00171	0.01109	0.00544
16	E	0.00019	0.00084	0.00154	0.00122	-0.00656	0.00415	-0.00088
16	L	-0.00019	0.00090	0.00137	0.00004	-0.00061	0.00362	-0.00514
16	R	0.00069	-0.00016	-0.00021	0.00173	0.00032	0.00527	0.00614
16	S	0.00039	0.00017	0.00092	-0.00131	0.00013	0.00486	0.00357
16	U	-0.00052	0.00042	0.00017	0.00336	0.00158	0.00160	-0.00148
20	E	-0.00036	0.00142	0.00149	0.00353	-0.00591	0.00227	0.00009
20	L	-0.00008	0.00001	0.00060	0.00287	-0.00339	0.00339	-0.00156
20	R	0.00010	0.00049	0.00115	-0.00010	0.00140	0.00114	-0.00432
20	S	0.00021	0.00021	-0.00029	-0.00056	-0.00029	0.00585	-0.00684
20	U	0.00060	-0.00007	-0.00014	-0.00023	0.00021	0.00300	0.00217
24	E	-0.00065	0.00099	0.00151	0.00226	-0.00120	0.00563	-0.00351
24	L	0.00030	-0.00026	0.00022	-0.00044	0.00050	0.00178	0.00074
24	R	0.00037	0.00004	0.00035	0.00151	0.00160	0.00377	-0.00006
24	S	0.00004	0.00106	-0.00008	0.00025	-0.00083	0.00535	0.00238
24	U	0.00041	0.00030	0.00028	0.00086	-0.00253	0.00349	-0.00070
28	E	-0.00042	0.00053	0.00034	0.00254	-0.00102	0.00394	0.00169
28	L	-0.00001	0.00036	-0.00006	0.00055	0.00031	0.00174	-0.00124
28	R	-0.00029	0.00067	0.00072	-0.00079	0.00155	0.00214	-0.00216
28	S	0.00009	0.00017	0.00064	0.00223	0.00333	0.00573	-0.00139
28	U	0.00032	0.00028	0.00027	0.00033	0.00018	-0.00040	0.00113

Table 26. Average maximum deviation in volume

Size	Sample	AGE1SI2	AGE1SI3	AGE2SI1	AGE2SI2	AGE2SI3	AGE3SI1	AGE3SI2
8	E	0.02571	0.04140	0.04679	0.14752	0.09523	0.18562	0.19024
8	L	0.01933	0.02571	0.04727	0.09132	0.09960	0.13850	0.20596
8	R	0.02836	0.02647	0.08072	0.11989	0.10730	0.15046	0.15801
8	S	0.04719	0.03040	0.08016	0.14691	0.12063	0.19176	0.24737
8	U	0.02326	0.03003	0.06105	0.15343	0.12129	0.17698	0.16031
12	E	0.01443	0.02616	0.06095	0.10123	0.07746	0.11467	0.17367
12	L	0.01796	0.02271	0.04056	0.11807	0.10768	0.14211	0.14295
12	R	0.02337	0.04633	0.07219	0.13333	0.07773	0.12714	0.17225
12	S	0.05031	0.02629	0.06470	0.13611	0.07596	0.20330	0.16990
12	U	0.02359	0.02605	0.04961	0.11889	0.10482	0.20920	0.17880
16	E	0.03436	0.02296	0.03717	0.09165	0.05352	0.09122	0.17177
16	L	0.01870	0.03026	0.04431	0.08702	0.07422	0.08940	0.12876
16	R	0.02882	0.02562	0.03896	0.14650	0.09755	0.10964	0.18357
16	S	0.02754	0.02426	0.06374	0.11394	0.11664	0.13247	0.16880
16	U	0.01446	0.03245	0.04877	0.12064	0.07615	0.09677	0.14467
20	E	0.03178	0.02292	0.03488	0.10061	0.05547	0.06651	0.13585
20	L	0.01447	0.02103	0.03867	0.08601	0.04917	0.06507	0.11488
20	R	0.01389	0.02925	0.03745	0.07936	0.09066	0.09622	0.14418

Size	Sample	AGE1SI2	AGE1SI3	AGE2SI1	AGE2SI2	AGE2SI3	AGE3SI1	AGE3SI2
20	S	0.02495	0.01904	0.04866	0.12868	0.08023	0.11656	0.13421
20	U	0.02055	0.02042	0.04001	0.09471	0.08062	0.10780	0.15285
24	E	0.02330	0.02214	0.03962	0.05701	0.05059	0.08496	0.10021
24	L	0.01873	0.01713	0.03494	0.04815	0.05865	0.06857	0.08992
24	R	0.01666	0.03259	0.04798	0.08355	0.07813	0.11150	0.10398
24	S	0.01923	0.03807	0.05373	0.09332	0.09712	0.12715	0.13142
24	U	0.01929	0.02251	0.03763	0.08916	0.06862	0.08546	0.11436
28	E	0.01114	0.01772	0.02123	0.05663	0.04026	0.06792	0.07033
28	L	0.01250	0.01708	0.01927	0.06993	0.03366	0.05629	0.03369
28	R	0.01337	0.01977	0.02970	0.05399	0.08651	0.07473	0.03787
28	S	0.01956	0.02334	0.03870	0.07122	0.10636	0.11142	0.07196
28	U	0.01740	0.02036	0.03169	0.08881	0.05591	0.06592	0.09439

Table 27. Average minimum deviation in volume

Size	Sample	AGE1SI2	AGE1SI3	AGE2SI1	AGE2SI2	AGE2SI3	AGE3SI1	AGE3SI2
8	E	-0.03069	-0.03198	-0.05653	-0.09615	-0.12974	-0.15032	-0.20175
8	L	-0.03400	-0.04192	-0.06067	-0.11370	-0.12510	-0.20369	-0.22005
8	R	-0.03880	-0.05533	-0.05704	-0.12066	-0.12231	-0.15778	-0.23753
8	S	-0.03525	-0.04687	-0.04836	-0.08155	-0.15519	-0.18110	-0.24069
8	U	-0.03386	-0.03448	-0.05834	-0.12970	-0.12020	-0.15060	-0.21556
12	E	-0.02505	-0.02933	-0.03466	-0.09290	-0.11699	-0.14591	-0.19812
12	L	-0.02451	-0.04671	-0.04095	-0.09440	-0.10585	-0.12238	-0.16207
12	R	-0.03176	-0.03380	-0.03472	-0.10113	-0.13663	-0.10285	-0.20450
12	S	-0.02551	-0.03411	-0.03936	-0.09301	-0.12182	-0.14799	-0.21160
12	U	-0.02401	-0.03930	-0.05142	-0.12788	-0.16877	-0.13652	-0.17344
16	E	-0.01932	-0.02989	-0.03546	-0.09000	-0.13094	-0.14955	-0.17026
16	L	-0.02133	-0.02620	-0.03438	-0.08945	-0.11025	-0.07846	-0.18311
16	R	-0.02403	-0.03542	-0.04924	-0.09463	-0.13330	-0.11751	-0.18959
16	S	-0.02596	-0.02999	-0.06306	-0.14973	-0.12138	-0.17023	-0.19674
16	U	-0.02830	-0.03486	-0.04688	-0.09054	-0.08911	-0.16720	-0.19740
20	E	-0.01804	-0.02128	-0.02397	-0.06388	-0.11445	-0.11105	-0.14312
20	L	-0.02038	-0.03690	-0.02669	-0.05125	-0.09637	-0.06300	-0.13615
20	R	-0.02207	-0.02606	-0.03160	-0.08304	-0.09258	-0.08299	-0.18566
20	S	-0.01763	-0.02673	-0.04325	-0.11919	-0.16300	-0.15996	-0.21676
20	U	-0.02229	-0.02951	-0.04111	-0.09893	-0.11333	-0.08600	-0.14140
24	E	-0.01665	-0.02449	-0.02001	-0.09330	-0.08423	-0.05359	-0.14639
24	L	-0.01319	-0.02353	-0.02319	-0.07361	-0.07588	-0.08656	-0.11169
24	R	-0.02228	-0.02552	-0.03902	-0.06830	-0.08579	-0.09261	-0.13777
24	S	-0.02231	-0.02462	-0.04323	-0.08280	-0.09922	-0.14150	-0.14728
24	U	-0.01686	-0.02564	-0.02702	-0.07662	-0.12031	-0.06441	-0.12800
28	E	-0.01568	-0.03062	-0.01925	-0.05205	-0.04640	-0.04108	-0.04500
28	L	-0.01621	-0.02049	-0.02330	-0.07914	-0.03960	-0.03939	-0.05697
28	R	-0.01955	-0.02146	-0.01540	-0.08425	-0.04845	-0.05592	-0.12065
28	S	-0.01527	-0.02289	-0.02686	-0.05898	-0.04289	-0.08141	-0.10380
28	U	-0.01620	-0.02054	-0.03041	-0.06480	-0.08073	-0.12109	-0.07688

Table 28. Average mean absolute deviation in volume

Size	Sample	AGE1SI2	AGE1SI3	AGE2SI1	AGE2SI2	AGE2SI3	AGE3SI1	AGE3SI2
8	Extreme	0.00759	0.00870	0.01031	0.02167	0.02930	0.02718	0.04552
8	Large	0.00627	0.00818	0.00909	0.01852	0.02747	0.02877	0.04502
8	Random	0.00809	0.00861	0.01262	0.02469	0.02938	0.03522	0.04914
8	Small	0.00623	0.00847	0.01388	0.02337	0.03301	0.03204	0.06069
8	Uniform	0.00501	0.00986	0.00914	0.02116	0.02979	0.02696	0.04417
12	Extreme	0.00424	0.00660	0.00805	0.01635	0.02276	0.02243	0.03630
12	Large	0.00470	0.00700	0.00707	0.01661	0.02240	0.01923	0.03522
12	Random	0.00454	0.00955	0.00912	0.01794	0.02391	0.02544	0.03834
12	Small	0.00453	0.00763	0.00969	0.02125	0.02468	0.02494	0.05276
12	Uniform	0.00429	0.00768	0.00816	0.01590	0.02458	0.02190	0.03797
16	Extreme	0.00351	0.00526	0.00588	0.01275	0.01598	0.01974	0.02980
16	Large	0.00306	0.00552	0.00596	0.01206	0.01615	0.01414	0.02252
16	Random	0.00343	0.00560	0.00729	0.01358	0.01875	0.02079	0.03019
16	Small	0.00439	0.00699	0.00779	0.01668	0.02136	0.02056	0.03548
16	Uniform	0.00402	0.00675	0.00671	0.01523	0.01911	0.01849	0.02879
20	Extreme	0.00299	0.00480	0.00472	0.01153	0.01425	0.01588	0.02188
20	Large	0.00252	0.00508	0.00393	0.01009	0.01270	0.01131	0.01888
20	Random	0.00285	0.00511	0.00668	0.01196	0.01443	0.01659	0.02207
20	Small	0.00351	0.00540	0.00647	0.01433	0.01648	0.01904	0.03053
20	Uniform	0.00288	0.00497	0.00575	0.01213	0.01526	0.01333	0.02407
24	Extreme	0.00241	0.00424	0.00368	0.00865	0.01007	0.01096	0.01386
24	Large	0.00200	0.00393	0.00327	0.00692	0.00867	0.00789	0.01122
24	Random	0.00253	0.00525	0.00413	0.01093	0.00965	0.01237	0.01634
24	Small	0.00292	0.00453	0.00482	0.01178	0.01389	0.01500	0.02128
24	Uniform	0.00244	0.00458	0.00433	0.01070	0.01140	0.01181	0.01830
28	Extreme	0.00226	0.00354	0.00303	0.00652	0.00722	0.00916	0.00818
28	Large	0.00156	0.00321	0.00259	0.00479	0.00540	0.00682	0.00773
28	Random	0.00224	0.00358	0.00291	0.00758	0.00703	0.00900	0.01040
28	Small	0.00237	0.00375	0.00389	0.00856	0.00822	0.00984	0.01180
28	Uniform	0.00178	0.00346	0.00352	0.00757	0.00680	0.00761	0.01102

Table 29. Average standard deviation of volume differences

Size	Sample	AGE1SI2	AGE1SI3	AGE2SI1	AGE2SI2	AGE2SI3	AGE3SI1	AGE3SI2
8	E	0.00903	0.01216	0.01608	0.03825	0.04213	0.06062	0.07281
8	L	0.00919	0.01097	0.01753	0.03407	0.04149	0.05919	0.07875
8	R	0.01151	0.01252	0.02232	0.03782	0.04160	0.05643	0.07794
8	S	0.01357	0.01291	0.02332	0.04021	0.04870	0.06625	0.09595
8	U	0.00978	0.01117	0.01950	0.04285	0.04573	0.05449	0.07175
12	E	0.00650	0.00898	0.01444	0.03100	0.03586	0.04481	0.06456
12	L	0.00672	0.01105	0.01215	0.03159	0.03725	0.04221	0.05751
12	R	0.00863	0.01199	0.01687	0.03600	0.03895	0.04008	0.06675

Size	Sample	AGE1SI2	AGE1SI3	AGE2SI1	AGE2SI2	AGE2SI3	AGE3SI1	AGE3SI2
12	S	0.01181	0.00978	0.01572	0.03589	0.03888	0.06257	0.06965
12	U	0.00841	0.01068	0.01512	0.03666	0.04543	0.05691	0.06416
16	E	0.00815	0.00861	0.01113	0.02653	0.03297	0.04055	0.05560
16	L	0.00601	0.00889	0.01148	0.02605	0.03088	0.02704	0.05339
16	R	0.00848	0.00958	0.01392	0.03510	0.03781	0.03780	0.06504
16	S	0.00844	0.00872	0.01796	0.03731	0.04037	0.05180	0.06720
16	U	0.00678	0.01074	0.01460	0.03168	0.03055	0.04451	0.06041
20	E	0.00693	0.00743	0.00878	0.02587	0.02827	0.03084	0.04462
20	L	0.00517	0.00851	0.00938	0.02046	0.02573	0.02197	0.04094
20	R	0.00558	0.00847	0.01164	0.02443	0.02974	0.02772	0.05731
20	S	0.00693	0.00775	0.01474	0.03751	0.03800	0.04389	0.06305
20	U	0.00643	0.00772	0.01216	0.02991	0.03034	0.02953	0.05244
24	E	0.00576	0.00717	0.00826	0.02070	0.02135	0.02398	0.03832
24	L	0.00460	0.00635	0.00724	0.01695	0.02077	0.02564	0.03323
24	R	0.00554	0.00872	0.01183	0.02374	0.02639	0.03225	0.04019
24	S	0.00618	0.00936	0.01301	0.02626	0.03063	0.04423	0.04561
24	U	0.00525	0.00732	0.00909	0.02456	0.03017	0.02273	0.03948
28	E	0.00439	0.00689	0.00548	0.01512	0.01231	0.01886	0.01949
28	L	0.00397	0.00582	0.00562	0.01955	0.01079	0.01499	0.01488
28	R	0.00489	0.00630	0.00655	0.01998	0.01847	0.01998	0.02473
28	S	0.00541	0.00710	0.00863	0.01928	0.02069	0.03170	0.02939
28	U	0.00471	0.00634	0.00828	0.02181	0.01848	0.02965	0.02726

Table 30. Maximum deviation in volume

Size	Sample	AGE1SI2	AGE1SI3	AGE2SI1	AGE2SI2	AGE2SI3	AGE3SI1	AGE3SI2
8	E	0.12977	0.14797	0.12391	0.53165	0.26007	0.46045	0.39928
8	L	0.10051	0.09497	0.11307	0.50929	0.22008	0.85092	0.49556
8	R	0.17920	0.09770	0.28724	0.34835	0.23098	0.65941	0.36947
8	S	0.21502	0.10474	0.26224	0.56528	0.29322	0.68795	0.50469
8	U	0.11411	0.13976	0.22076	0.57200	0.33221	0.45087	0.32431
12	E	0.07667	0.07227	0.18811	0.59290	0.21002	0.35101	0.30432
12	L	0.07718	0.08739	0.08827	0.43518	0.21219	0.33272	0.33313
12	R	0.12546	0.33254	0.20214	0.58048	0.16332	0.52137	0.32317
12	S	0.22286	0.07745	0.15336	0.51030	0.24045	0.80918	0.36461
12	U	0.09250	0.16056	0.11324	0.47824	0.24811	0.79168	0.37708
16	E	0.21920	0.08691	0.14775	0.38187	0.11912	0.25334	0.30516
16	L	0.09053	0.30414	0.12961	0.54022	0.16979	0.25230	0.32167
16	R	0.12958	0.14203	0.12529	0.58143	0.38076	0.32601	0.37602
16	S	0.11281	0.06730	0.19583	0.53212	0.38217	0.60193	0.36579
16	U	0.04745	0.23968	0.18059	0.43168	0.19673	0.51952	0.42146
20	E	0.11860	0.08102	0.12057	0.55637	0.16749	0.19234	0.29676
20	L	0.07390	0.07260	0.09907	0.48647	0.10622	0.22456	0.30265
20	R	0.03294	0.24292	0.12583	0.32378	0.29113	0.58589	0.31779
20	S	0.14787	0.05605	0.12588	0.58925	0.19139	0.31303	0.27209
20	U	0.14489	0.06659	0.17031	0.44817	0.19162	0.49970	0.32468

Size	Sample	AGE1SI2	AGE1SI3	AGE2SI1	AGE2SI2	AGE2SI3	AGE3SI1	AGE3SI2
24	E	0.11246	0.08221	0.10382	0.18389	0.08668	0.19339	0.24845
24	L	0.11521	0.06260	0.14612	0.19566	0.12707	0.22795	0.23222
24	R	0.07324	0.39155	0.17638	0.38082	0.22526	0.46862	0.20449
24	S	0.08879	0.27342	0.16391	0.40723	0.23513	0.81566	0.30756
24	U	0.08007	0.13524	0.14448	0.41148	0.15869	0.30511	0.31979
28	E	0.09244	0.06694	0.06502	0.35587	0.16932	0.17470	0.31249
28	L	0.07942	0.06516	0.08174	0.46757	0.16932	0.20556	0.20546
28	R	0.08175	0.07161	0.10335	0.33634	0.23592	0.45355	0.24687
28	S	0.12099	0.07409	0.12073	0.21765	0.36084	0.72825	0.31249
28	U	0.09174	0.06725	0.12585	0.54115	0.28536	0.31618	0.31249

Table 31. Minimum deviation in volume

Size	Sample	AGE1SI2	AGE1SI3	AGE2SI1	AGE2SI2	AGE2SI3	AGE3SI1	AGE3SI2
8	E	-0.06464	-0.12557	-0.17491	-0.21177	-0.35906	-0.64221	-0.35924
8	L	-0.17213	-0.33775	-0.20175	-0.42155	-0.25371	-0.79080	-0.37708
8	R	-0.19147	-0.33098	-0.18941	-0.42272	-0.22756	-0.70280	-0.40836
8	S	-0.08720	-0.27411	-0.17313	-0.16140	-0.42927	-0.75806	-1.00267
8	U	-0.16604	-0.10182	-0.19731	-0.57926	-0.31985	-0.63205	-0.47856
12	E	-0.06844	-0.17254	-0.09126	-0.36472	-0.28722	-0.90852	-0.33938
12	L	-0.08835	-0.31993	-0.18097	-0.35399	-0.19433	-0.78210	-0.33918
12	R	-0.14960	-0.18783	-0.10714	-0.39519	-0.21454	-0.55037	-0.37549
12	S	-0.05716	-0.17559	-0.11014	-0.55951	-0.23106	-0.86241	-0.42415
12	U	-0.07216	-0.14508	-0.11819	-0.36412	-0.37605	-0.60881	-0.32248
16	E	-0.05692	-0.14050	-0.10490	-0.29545	-0.21092	-0.41480	-0.33774
16	L	-0.07201	-0.08147	-0.13800	-0.37176	-0.24986	-0.20501	-0.43006
16	R	-0.06406	-0.19681	-0.16124	-0.38335	-0.31752	-0.64062	-0.36932
16	S	-0.07153	-0.13594	-0.16926	-0.57367	-0.21437	-0.74793	-0.36674
16	U	-0.07307	-0.13877	-0.16822	-0.49338	-0.15211	-0.54465	-0.82103
20	E	-0.05330	-0.08891	-0.08989	-0.45131	-0.17782	-0.62179	-0.31775
20	L	-0.07216	-0.26445	-0.11254	-0.10338	-0.18730	-0.44078	-0.31930
20	R	-0.06611	-0.11196	-0.09892	-0.22014	-0.21072	-0.19420	-0.60419
20	S	-0.05408	-0.08230	-0.12065	-0.58189	-0.42784	-0.72324	-0.47993
20	U	-0.07737	-0.17491	-0.11054	-0.52059	-0.35563	-0.38070	-0.37795
24	E	-0.04723	-0.18630	-0.07305	-0.57370	-0.15933	-0.36771	-0.33360
24	L	-0.04556	-0.08132	-0.04009	-0.30503	-0.16193	-0.68372	-0.33455
24	R	-0.06803	-0.07976	-0.10290	-0.37823	-0.40789	-0.60538	-0.33581
24	S	-0.06557	-0.08199	-0.14499	-0.14239	-0.16704	-0.66695	-0.33166
24	U	-0.06585	-0.08444	-0.05797	-0.22961	-0.31716	-0.27345	-0.27527
28	E	-0.04631	-0.16439	-0.03725	-0.33077	-0.16009	-0.10289	-0.24964
28	L	-0.06747	-0.13201	-0.08049	-0.52471	-0.14487	-0.19822	-0.27725
28	R	-0.06559	-0.07248	-0.04014	-0.41485	-0.30879	-0.28689	-0.33343
28	S	-0.04528	-0.09429	-0.07012	-0.14464	-0.25678	-0.61876	-0.31298
28	U	-0.05499	-0.08453	-0.11211	-0.14362	-0.32465	-0.61901	-0.33343

Tables 34 to 39 show sample design rankings for each age-site class and sample size. The following criteria is ranked:

the average mean deviation in height (MDHT);

the average of the largest positive mean deviations in height (MDHTMAX);

the average of the largest negative mean deviations in height (MDHTMIN);

the average mean absolute deviation in height

the average mean deviation in tree volume (MDVOL);

the average of the largest positive mean deviations in volume (MDVOLMAX);

the average mean absolute deviation in volume; and,

the average of the largest negative mean deviations in volume (MDVOLMIN).

Table 32. Sample rankings for sample size 8

Class	Sample	MDHT	MDHTMAX	MDHTMIN	MADHT	MDVOL	MDVOLMAX	MDVOLMIN	MADVOL
AGE1SI2	Extreme	1	1	3	3	4	3	2	4
	Large	3	3	2	4	2	1	4	3
	Random	5	5	1	5	3	4	5	5
	Small	2	2	4	1	5	5	3	2
	Uniform	4	4	5	2	1	2	1	1
AGE1SI3	Extreme	4	2	1	4	5	5	2	4
	Large	2	4	2	3	2	1	4	1
	Random	3	3	5	5	1	2	5	3
	Small	1	1	4	1	3	4	3	2
	Uniform	5	5	3	2	4	3	1	5
AGE2SI1	Extreme	5	1	3	4	3	1	4	3
	Large	2	3	4	1	1	2	1	1
	Random	1	4	2	3	2	5	3	4
	Small	4	5	1	5	5	4	5	5
	Uniform	3	2	5	2	4	3	2	2
AGE2SI2	Extreme	5	3	5	3	3	4	5	3
	Large	4	2	3	4	1	1	1	1
	Random	3	1	2	5	2	2	4	5
	Small	1	5	1	1	5	3	3	4
	Uniform	2	4	4	2	4	5	2	2
AGE2SI3	Extreme	3	1	4	1	2	1	1	2
	Large	5	3	5	5	4	2	5	1

Class	Sample	MDHT	MDHTMAX	MDHTMIN	MADHT	MDVOL	MDVOLMAX	MDVOLMIN	MADVOL
	Random	1	2	2	2	1	3	2	3
	Small	2	5	3	3	5	4	3	5
	Uniform	4	4	1	4	3	5	4	4
AGE3SI1	Extreme	5	4	2	2	5	4	2	2
	Large	4	1	1	4	1	1	4	3
	Random	1	2	3	5	2	2	3	5
	Small	2	3	4	3	4	5	5	4
	Uniform	3	5	5	1	3	3	1	1
AGE3SI2	Extreme	2	1	3	2	4	3	2	3
	Large	3	3	2	1	2	4	5	2
	Random	4	4	1	5	1	1	4	4
	Small	5	5	4	4	5	5	3	5
	Uniform	1	2	5	3	3	2	1	1

Table 33. Sample rankings for sample size 12

Class	Sample	MDHT	MDHTMAX	MDHTMIN	MADHT	MDVOL	MDVOLMAX	MDVOLMIN	MADVOL
AGE1SI2	Extreme	1	1	2	3	2	1	3	1
	Large	5	4	1	5	3	2	2	5
	Random	4	2	4	1	1	3	5	4
	Small	2	5	5	2	5	5	4	3
	Uniform	3	3	3	4	4	4	1	2
AGE1SI3	Extreme	5	1	1	3	5	3	1	1
	Large	2	2	3	5	2	1	5	2
	Random	4	5	5	1	4	5	2	5
	Small	3	4	4	2	1	4	3	3
	Uniform	1	3	2	4	3	2	4	4
AGE2SI1	Extreme	5	5	1	3	5	3	1	2
	Large	1	3	4	4	2	1	4	1
	Random	3	4	2	2	3	5	2	4
	Small	4	2	3	5	4	4	3	5
	Uniform	2	1	5	1	1	2	5	3
AGE2SI2	Extreme	4	1	3	2	1	1	1	2
	Large	3	3	2	5	4	2	3	3
	Random	2	4	4	3	3	4	4	4
	Small	5	5	1	4	5	5	2	5
	Uniform	1	2	5	1	2	3	5	1
AGE2SI3	Extreme	1	1	4	1	2	2	2	2
	Large	3	4	1	5	1	5	1	1
	Random	4	3	3	4	5	3	4	3
	Small	5	2	2	2	4	1	3	5
	Uniform	2	5	5	3	3	4	5	4
AGE3SI1	Extreme	2	3	4	3	2	1	4	3
	Large	3	2	5	4	3	3	2	1
	Random	1	1	1	5	1	2	1	5
	Small	4	4	3	1	5	4	5	4

Class	Sample	MDHT	MDHTMAX	MDHTMIN	MADHT	MDVOL	MDVOLMAX	MDVOLMIN	MADVOL
	Uniform	5	5	2	2	4	5	3	2
AGE3SI2	Extreme	1	3	5	1	3	4	3	2
	Large	2	4	1	3	1	1	1	1
	Random	4	2	2	4	2	3	4	4
	Small	5	5	3	5	4	2	5	5
	Uniform	3	1	4	2	5	5	2	3

Table 34. Sample rankings for sample size 16

Class	Sample	MDHT	MDHTMAX	MDHTMIN	MADHT	MDVOL	MDVOLMAX	MDVOLMIN	MADVOL
AGE1SI2	Extreme	3	3	2	2	1	5	1	3
	Large	1	2	4	5	2	2	2	1
	Random	5	5	3	3	5	4	3	2
	Small	4	4	1	1	3	3	4	5
	Uniform	2	1	5	4	4	1	5	4
AGE1SI3	Extreme	5	3	1	1	4	1	2	1
	Large	4	5	4	5	5	4	1	2
	Random	1	2	3	2	1	3	5	3
	Small	2	1	2	4	2	2	3	5
	Uniform	3	4	5	3	3	5	4	4
AGE2SI1	Extreme	5	3	1	1	5	1	2	1
	Large	4	2	2	3	4	3	1	2
	Random	1	1	3	4	2	2	4	4
	Small	2	4	5	2	3	5	5	5
	Uniform	3	5	4	5	1	4	3	3
AGE2SI2	Extreme	3	2	1	1	2	2	2	2
	Large	4	1	5	3	1	1	1	1
	Random	1	5	3	2	4	5	4	3
	Small	2	3	4	4	3	3	5	5
	Uniform	5	4	2	5	5	4	3	4
AGE2SI3	Extreme	5	1	5	1	5	1	4	1
	Large	1	2	4	5	3	2	2	2
	Random	4	5	3	4	2	4	5	3
	Small	2	4	2	3	1	5	3	5
	Uniform	3	3	1	2	4	3	1	4
AGE3SI1	Extreme	2	2	4	5	3	2	3	3
	Large	1	1	1	4	2	1	1	1
	Random	5	3	5	3	5	4	2	5
	Small	4	5	3	1	4	5	5	4
	Uniform	3	4	2	2	1	3	4	2
AGE3SI2	Extreme	2	2	4	3	1	4	1	3
	Large	4	1	5	1	4	1	2	1
	Random	5	5	1	4	5	5	3	4
	Small	3	4	2	2	3	3	4	5
	Uniform	1	3	3	5	2	2	5	2

Table 35. Sample rankings for sample size 20

Class	Sample	MDHT	MDHTMAX	MDHTMIN	MADHT	MDVOL	MDVOLMAX	MDVOLMIN	MADVOL
AGE1SI2	Extreme	3	4	4	2	4	5	2	4
	Large	2	2	5	1	1	2	3	1
	Random	4	3	2	3	2	1	4	2
	Small	1	1	1	5	3	4	1	5
	Uniform	5	5	3	4	5	3	5	3
AGE1SI3	Extreme	5	3	1	2	5	4	1	1
	Large	4	5	5	5	1	3	5	3
	Random	3	4	4	4	4	5	2	4
	Small	2	1	3	1	3	1	3	5
	Uniform	1	2	2	3	2	2	4	2
AGE2SI1	Extreme	5	4	1	1	5	1	1	2
	Large	3	3	3	3	3	3	2	1
	Random	4	1	2	5	4	2	3	5
	Small	1	5	4	4	2	5	5	4
	Uniform	2	2	5	2	1	4	4	3
AGE2SI2	Extreme	5	5	1	4	5	4	2	2
	Large	4	3	2	5	4	2	1	1
	Random	1	1	4	2	1	1	3	3
	Small	2	4	3	1	3	5	5	5
	Uniform	3	2	5	3	2	3	4	4
AGE2SI3	Extreme	5	1	5	3	5	2	4	2
	Large	4	2	4	5	4	1	2	1
	Random	3	5	1	2	3	5	1	3
	Small	1	4	3	1	2	3	5	5
	Uniform	2	3	2	4	1	4	3	4
AGE3SI1	Extreme	2	3	4	5	2	2	4	3
	Large	4	2	3	3	4	1	1	1
	Random	1	1	2	4	1	3	2	4
	Small	5	5	5	2	5	5	5	5
	Uniform	3	4	1	1	3	4	3	2
AGE3SI2	Extreme	2	3	2	1	1	3	3	2
	Large	1	5	3	5	2	1	1	1
	Random	4	4	4	3	4	4	4	3
	Small	3	1	5	2	5	2	5	5
	Uniform	5	2	1	4	3	5	2	4

Table 36. Sample rankings for sample size 24

Class	Sample	MDHT	MDHTMAX	MDHTMIN	MADHT	MDVOL	MDVOLMAX	MDVOLMIN	MADVOL
AGE1SI2	Extreme	4	2	5	3	5	5	2	2
	Large	3	4	4	5	2	2	1	1
	Random	5	5	1	2	3	1	4	4

Class	Sample	MDHT	MDHTMAX	MDHTMIN	MADHT	MDVOL	MDVOLMAX	MDVOLMIN	MADVOL
	Small	1	1	2	1	1	3	5	5
	Uniform	2	3	3	4	4	4	3	3
AGE1SI3	Extreme	5	2	2	2	4	2	2	2
	Large	3	3	4	3	2	1	1	1
	Random	2	5	3	5	1	4	4	5
	Small	4	4	1	1	5	5	3	3
	Uniform	1	1	5	4	3	3	5	4
AGE2SI1	Extreme	5	5	1	1	5	3	1	2
	Large	4	2	2	3	2	1	2	1
	Random	2	3	5	4	4	4	4	3
	Small	1	4	4	2	1	5	5	5
	Uniform	3	1	3	5	3	2	3	4
AGE2SI2	Extreme	5	2	1	4	5	2	5	2
	Large	2	1	5	1	2	1	2	1
	Random	4	3	2	5	4	3	1	4
	Small	1	4	3	2	1	5	4	5
	Uniform	3	5	4	3	3	4	3	3
AGE2SI3	Extreme	1	1	2	2	3	1	2	3
	Large	2	3	4	5	1	2	1	1
	Random	3	5	1	1	4	4	3	2
	Small	4	2	3	4	2	5	4	5
	Uniform	5	4	5	3	5	3	5	4
AGE3SI1	Extreme	4	4	1	5	5	2	1	2
	Large	2	5	4	2	1	1	3	1
	Random	1	1	5	4	3	4	4	4
	Small	5	2	3	3	4	5	5	5
	Uniform	3	3	2	1	2	3	2	3
AGE3SI2	Extreme	5	1	5	1	5	2	4	2
	Large	3	5	2	3	3	1	1	1
	Random	1	2	4	4	1	3	3	3
	Small	4	4	1	2	4	5	5	5
	Uniform	2	3	3	5	2	4	2	4

Table 37. Sample rankings for sample size 28

Class	Sample	MDHT	MDHTMAX	MDHTMIN	MADHT	MDVOL	MDVOLMAX	MDVOLMIN	MADVOL
AGE1SI2	Extreme	3	1	5	1	5	1	2	4
	Large	4	4	3	2	1	2	4	1
	Random	2	3	4	3	3	3	5	3
	Small	1	2	1	4	2	5	1	5
	Uniform	5	5	2	5	4	4	3	2
AGE1SI3	Extreme	5	5	1	1	4	2	5	3
	Large	1	4	5	2	3	1	1	1
	Random	4	1	3	3	5	3	3	4
	Small	3	2	2	5	1	5	4	5
	Uniform	2	3	4	4	2	4	2	2

Class	Sample	MDHT	MDHTMAX	MDHTMIN	MADHT	MDVOL	MDVOLMAX	MDVOLMIN	MADVOL
AGE2SI1	Extreme	3	2	2	4	3	2	2	3
	Large	1	1	4	5	1	1	3	1
	Random	4	3	1	3	5	3	1	2
	Small	5	5	3	2	4	5	4	5
	Uniform	2	4	5	1	2	4	5	4
AGE2SI2	Extreme	5	2	1	1	5	2	1	2
	Large	3	5	4	4	2	3	4	1
	Random	1	1	5	3	3	1	5	4
	Small	4	4	2	2	4	4	2	5
	Uniform	2	3	3	5	1	5	3	3
AGE2SI3	Extreme	3	1	3	3	3	2	3	4
	Large	2	3	4	1	2	1	1	1
	Random	4	4	2	4	4	4	4	3
	Small	5	5	1	2	5	5	2	5
	Uniform	1	2	5	5	1	3	5	2
AGE3SI1	Extreme	4	4	3	2	4	3	2	4
	Large	3	3	1	1	2	1	1	1
	Random	2	2	4	5	3	4	3	3
	Small	5	5	2	4	5	5	4	5
	Uniform	1	1	5	3	1	2	5	2
AGE3SI2	Extreme	3	5	1	2	4	3	1	2
	Large	4	2	3	3	2	1	2	1
	Random	5	1	5	1	5	2	5	3
	Small	2	4	2	5	3	4	4	5
	Uniform	1	3	4	4	1	5	3	4