A GENERAL THEORY ON COMMON POINT INTERSECT SAMPLING WITH SPECIAL APPLICATION TO DOWNED WOODY PARTICLES

BY

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ABSTRACT

A general sampling theory referred to as common point intersect sampling is developed and assessed. This new technique is specifically applied to the problem of estimating parameters of populations of downed woody particles of interest in fire research.

The performance of the common point intersect sampling method is compared to that of the well-established line intercept technique with respect to two lesser (less than 3 inches in diameter) downed woody particles populations. Results of these tests indicate that proper application of the new sampling system can yield total volume estimates of approximately 15 per cent precision with savings of up to 40 per cent of the total sampling time required by the line intercept technique.

The common point intersect sampling method is demonstrated to be a useful approach to solving the problem of obtaining estimates for numerous attributes of populations of downed woody particles. General formulas are also provided which facilitate the application of common point intersect sampling to the task of obtaining parameters of standing timber such as crown area and average crown diameter from aerial photographs.

The common point intersect technique is shown to be a fast and accurate means of sampling forest material. The new sampling system has been applied rigourously in only one problem area. The general nature of the common point intersect system suggests, however, that it has many other applications in a multiplicity of scientific disciplines.

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A special note of thanks is due my parents and family for their continuing moral support during the writing phase of the thesis.

CHAPTER I

Introduction

This thesis is designed with two primary objectives in mind. The first is to present a general technique for non-destructively obtaining quantitative estimates for attributes of any community of objects. The second is to apply this technique to an important social problem area.

Since the author is currently employed as a fire research officer in the Canadian Forestry Service, the problem area selected is forest fire oriented, namely precise measurement of downed woody particles populations. A woody particle is considered 'downed' if it has been detached from its source and lies within six feet of the forest floor (Brown, 1974). Attention is focused primarily on lesser downed woody particles populations from which two constituent populations are chosen for analysis.

Before considering the scope and methods of the thesis, it seems very appropriate to consider whether it is worthwhile or not to develop a new general quantitative sampling technique. Historically, general sampling schemes have been developed from specific sampling methods designed to satisfy very specific needs in well-defined professional disciplines. The different physical and mathematical constraints imposed by each separate discipline have made this established pattern of procedure a necessary rather than optimum one. There currently being no urgent demand for new quantitative sampling techniques in any of the fire-oriented phytosociological biomes, it appears that at present a new technique would be viewed with interest only on an academic plane. Any new sampling method should be significantly superior to the most successful existing one when both are applied to a specific problem. Once this superiority is demonstrated the utility of the new technique would be established operationally. This thesis demonstrates the superiority of the new sampling technique in at least one specific problem area.

At this time it is relevant that a comment be made concerning the problem area chosen for analysis. The significance of populations of downed woody particles as areas of concern for forest harvesters, land managers and fire scientists as well as numerous other professionals is not to be logically disputed (Bailey, 1969; Beaufait and Hardy, in prep; Deeming, 1972). The amounts, weights, and distributions of larger downed woody materials are of much concern when considering problems such as the assessment of logging waste, behaviour of wildlife, and probability of successful natural or artificial regeneration (Davis, 1959; Wagener and Offord, 1972). Also the volumes, weights, surface areas and distributions thereof for the smaller downed woody materials play a key role in rating the fire hazard within a particular region (Deeming, 1972; Beaufait and Hardy, in prep; Brown and Roussopoulos, in prep.). However, it could be argued that it seems silly to sample these downed woody particles directly. A more logical approach would be to sample the forest characteristics of interest when the woody materials are secured to the standing trees. Such initial values would then be combined with mathematical relationships which describe the effects of a given set of disturbances on the forest to arrive at estimates for the desired parameters of the downed woody particles populations. Although past efforts to apply this approach may have failed (Beaufait and Hardy, in prep.), the author recognizes this as a viable approach to the problem of quantifying parameters of downed woody particles populations. Nevertheless, until a technique is created which can successfully apply this systems approach to the downed woody materials complex, interim physical methods will have to be used which means at least temporarily that downed woody particles will have to be sampled. In this thesis, a new general sampling technique is applied to populations of downed woody materials. Normally when a new concept of sampling is applied to a problem area it is because the old ones are in some way unsatisfactory. It is not immediately apparent that this is true in the case of downed woody materials. For example both 0.1 acre plots and long transect lines have been successfully used to measure the larger fuels (Bailey, 1969; Howard and Ward, 1972). Also short transect lines have been effectively used to measure the smaller fuels (Beaufait, Marsden and Norum, 1974; Brown, in prep; Brown and Roussopoules; in prep.). None of the above methods have disadvantages which seriously impair their applications. The justification of applying a new general sampling technique to downed woody fuels rests upon the insight of the author. He has worked in populations of downed woody materials for seven years and has tried numerous versions of currently used sampling systems in many fuel complexes. He believes that the new general sampling concept is not only feasible when applied to downed woody fuels but also has a good chance of being significantly more efficient than all other previously applied direct sampling systems.

The scope and methods of the thesis are relevant to both of its major components which are the general theory and the case study. The scope of the general theory is very broad; it applies to any group of objects whose attributes of interest can be described by functions of suitably well-behaved mathematical expressions. The methods used in developing the theory are basic theorems and principles of advanced calculus.

The case study applies primarily to lesser downed woody particles. It is agreed that a woody particle which intersects a transect line will be described as lesser (greater) only if its width at the initial point of intersection is less (greater) than or equal to 3 inches. Lesser woody particles are selected for detailed scrutiny because past studies have shown that these components are most likely to be consumed by the majority of broadcast fires (Steele and Beaufait, 1969; Brown, in prep). It should be noted that, although needles satisfy the above agreed definition of lesser woody materials, they will not be considered directly in the case study. Needles play an important role in the ignition process and hence in the initial stages of fire growth (Beaufait, 1965). But in general the extreme difficulty of measuring or counting needles even over relatively short distances necessitates the use of indirect sampling techniques. Examples of such techniques are regression estimates from lesser downed woody fuel data (Brown, 1970) and tables of desired needle attributes for specified fuel types (Fahnestock and Chandler, 1960; Brown, 1970). Regardless of which indirect sampling scheme is selected, the main objective of the case study can be completely met simply by considering the problem of obtaining estimates for the parameters of interest with respect to all subsets of the lesser downed woody fuels population excluding needles. It should be kept in mind that the main objective of the case study is to evaluate the new general sampling technique by comparing it to the most successful existing one when the two are applied to two lesser downed woody fuels populations.

There are two basic methods employed in the development of the case study. The first is the generation of mathematical formulae from the new general sampling concept. These formulae will serve to estimate the desired properties of the downed woody fuels populations. Care must be taken that the assumptions made in arriving at the explicit estimates reflect common field situations. Attention must also be paid to the levels of accuracy and precision which the estimates should meet, cost and time constraints under which the new and existing techniques may be forced to operate, and the physical and mental tolerance levels of average field inventory personnel. The last item in this list is an especially important one. Quantitative sampling of lesser downed woody fuels in logging residue for example is tedious and requires painstaking work. However, at present it is highly recommended that this task be undertaken if reliable objective predictions or assessments of fire behavior and impact are to be made (Beaufait, Marsden, and Norum, 1974). If the quantitative sampling process used is incompatible with normal levels of physical and mental tolerance, the non-sampling errors (Husch, Miller, and Beers, 1972; p.201) introduced through improper or careless measurements may well have a major effect upon the precision of the derived formulae. The formulae derived from the general sampling concept will be determined using functional analysis, analysis of variance, events modelling, numerical analysis and parametric statistical hypothesis testing. The second method used in the development of the case study is the application of the new sampling process to two actual field situations and a comparison of this new technique to the most successful existing one with respect to these same two lesser downed woody fuels populations. The fuel complexes selected for examination are two areas of fresh (less than 1 year old) logging residue (slash). These areas are chosen because during the fire season many untreated slash areas become highly flammable. This means that objective quantitative projections of fire danger in slash are needed. Working in slash then maximizes the relevance and usefulness of the field exercise. The field comparison is made by inspecting pairs of total sampling times required to obtain pairs of population parameter estimates, where the members of each pair of estimates have both common units and a common allowable sampling error (Husch, Miller, and Beers, 1972) for a common percentage of the time. With respect to each subset of the lesser downed woody fuels population, the total sampling time is the sum of the total fuels inspection time plus the total travel time between sample units. Each resultant total sampling cost is directly proportional to its corresponding total sampling time. It is easy to see then that the total sampling times provide ranking indices for the two sampling techniques. This procedure is well established and has already been used in studies involved with logging residue (Bailey, 1969; Howard and Ward, 1972).

Having presented the general layout of this study, interest is now focussed upon fulfilling the two objectives cited at the beginning of the thesis.

CHAPTER II

Development of the common point intersect concept

Brief review of past general sampling techniques

Before attempting to achieve the first objective of the thesis which is to present a general non-destructive technique for quantitatively estimating population parameters or attributes for any community of objects, it seems logical to look briefly at some of the more successful general sampling schemes already in existence. The two selected on the basis of degree of flexibility and extent of proven usefulness are transect system sampling (T S S) and line intersect sampling (L I S).

T S S designs are simply applications of the solution offered by the 18th century French naturalist Buffon to the needle problem (Bradley, 1972). It is important to note that the solution given by Buffon and others (Segebaden, 1964; De Vries, 1973) is based on the assumption that the objects of interest are randomly distributed throughout the area of concern. The concept involved in this technique is quite ingenious and should be briefly described. Consider an area occupied by objects which are randomly distributed. If a network of transects is superimposed on this area those objects which both intersect any transect in the network and possess the attributes of interest are tallied. The total number of relevant intersections is then inserted into an appropriate formula derived from Buffon's solution to the needle problem. In this way an estimate is obtained for the desired population parameter. Approximate general formulas for the standard error of the estimate obtained, the transect system spacing required to achieve a specified precision level, and the number of sample points required to meet a desired degree of precision are all given by Bradley (1972). In passing it should be mentioned that the general

T S S theory can be extended to permit its application in any collection of objects whose placement and angular orientation distributions can be quantified.

T S S has already been applied successfully to the problem of determining cross-country transport distances for real road nets (Segebaden, 1964), and also to the problem of estimating road lengths (Bradley, 1972). Other applications can be made. For example the total length and volume of greater logging residue could be estimated with TSS. Also information about stand characteristics and lengths of streams could be obtained through the use of TSS. All of the above applications of TSS are greatly simplified by use of aerial photographs.

LIS is closely related to TSS and in fact may even be regarded as a special case of TSS, where the network of transects has been reduced to a single transect. Several derivations of LIS formulas are available (Canfield, 1941; Warren and Olsen, 1964; Van Wagner, 1968; Brown, 1971). The most general discussion of the LIS concept is that given by De Vries (1973, p.4-7). De Vries' formulation rests primarily upon two assumptions. The first is that the objects of interest can be viewed logically as line segments or shapes of moderate curvature and the second is that the placement and angular orientation distributions of the population of objects (identified as line segments or shapes of moderate curvature, whichever is appropriate) can be described quantitatively. It is possible to extend De Vries' argument to a more abstract plane making it independent of the two above assumptions.

LIS is a very popular technique having been applied to range vegetation (Canfield, 1941), greater logging residue (Warren and Olsen, 1964; Van Wagner, 1968; Howard and Ward, 1972), and lesser logging residue (Beaufait, Marsden, and Norum 1974; Brown, in prep.). Suggestions have also been made regarding its possible application in other forestry-related problems, such as the estimation of standing timber parameters (De Vries, 1973).

Before proceeding to the new proposed general sampling concept, one point should be stressed. It is not the intention of the author to imply through the introduction of a new technique that transect system sampling and line intersect sampling are in any way inadequate. More research conducted over a wide range of sampling problems is required in order to make a rigorous, unbiased comparison of the three approaches.

Presentation of the common point intersect theory

General discussion

Before developing the common point intersect theory mathematically, a non-mathematical introduction to the general theory will be made. This introduction will help non-mathematicians grasp a basic understanding of the general theory.

Suppose there exists a population of objects. To use some forestry examples, these objects could be standing trees, downed stems or branches, borer beetles, fructifications of wood destroying fungi, etc. Each population can be described in terms of variables which will be called parameters. The parameters of concern are listed by the investigator, say $p_1,\ldots,p_k$. But the investigator may be interested in more than just estimating $p_1,\ldots,p_k$. He may want to estimate some combination of these parameters, say $p = u(p_1,\ldots,p_k)$ where u is some function. The investigator does not know the value of any of the parameters, but he has defined $p_1,\ldots,p_k$ and so he knows what they mean. He does not know the value of p, but he has defined u and through the meaning attached to each parameter, he knows what p means. So suppose he wants to estimate p. He may do this by introducing a random variable X derived from the meaning of p. A random variable is simply a function which maps outcomes of some experiment E into real numbers. In this case E is the process of selecting locations for sampling units of a common size and shape within the population of objects. At this point the investigator knows what X means but he does not have a practical way to evaluate X. To be more specific, suppose p is the mean number of particles per unit area. Implicit in the definition of p is the random variable Y where Y represents the number of particles per unit area. Hence set $X = Y_{s1,s2}$ where $Y_{s1,s2}$ is a random variable mapping each sampling unit (all of common size s_1 and shape s_2) into the mean number of particles per unit area with respect to that sampling unit.

p is a variable consisting of some combination of $p_1,\ldots,p_k$ where $p_1,\ldots,p_k$ are parameters. This means almost by definition that they can be easily referenced to random variables since in effect they are descriptors of the distributions of random variables. A consideration of the random variables implicit within the meanings of $p_1,\ldots,p_k$ combined with the definition of p leads to the definition of X.

The investigator knows what X(S) means but as yet he does not know how to evaluate X at any S in a practical way. The definition of X(S) combined with the definition of p results in a point estimate $(T(X(S_1),\ldots,X(S_n)))$ of p, where $S_1,\ldots,S_n$ is a set of sampling units. T is a function which maps sets of real numbers into real numbers. T is not unique unless specific properties are required for the point estimator of p. Using standard statistical procedures, it can be assumed that a point estimator of p has been found which satisfies all required properties.

There are two major remaining items to discuss before developing a rigorous mathematical derivation of the general theory. These items involve an arbitrary sampling unit(S) and the random variable X. The sampling unit S is restricted to being a right circular cylinder because of mathematical considerations regarding X. Without going into statistical details, it suffices to say here that basically the size and location of S are arbitrary. The last important item is how to evaluate X at S in a practical way. The concepts involved here are so intrinsically linked to mathematical considerations that a meaningful non-mathematical discussion cannot be made.

Keeping the above introduction in mind, a mathematical presentation of the general theory is now made.

Consider any community (C) of objects temporarily fixed in space. Let u be a function which associates to each K-tuple $(p_1,\ldots,p_k)$ the unique value $u(p_1,\ldots,p_k)=p$, where $p_1,\ldots,p_k$ are parameters of unknown values describing C. A general technique will now be provided for determining quantitative knowledge about p with respect to C.

Having specified u the meaning of p(the image of $(p_1,\ldots,p_k)$ under u) is understood. From the meaning attached to p, it is possible to introduce a random variable X defined on the sample space of outcomes of an experiment E which consists of selecting locations for sampling units of a common size and shape within C. The origin of X is in no way mysterious. X is simply a function consisting of a combination of the attributes constituting p; this function is referenced to the sample space of outcomes of E whose infinite union comprises C. For example if p is a mean quantity per unit area, X is introduced as the obvious functional extension of p defined on the sample space of outcomes of E, i.e. X is a function which maps each sampling unit into a mean quantity per unit area with respect to that sampling unit. Through X and the meaning attached to p, a point estimate for p $(T(X(S_1)),\ldots,X(S_n))$ can be expressed explicitly in terms of $X(S_i)$ and known constants, $i\varepsilon\{1,\ldots,n\}$, where $X(S_i)$ is the value of X yielded by the $i^{\underline{th}}$ independent repetition of E, $i\varepsilon\{1,\ldots,n\}$ for a given set $\{S_i\}_{i=1}^{n}$. Of course there is no unique point estimator for p. The form of $T(X_1,\ldots,X_n)$ (where X_i is a random variable defined on the outcome of the $i^{\underline{th}}$ independent repetition of E, $i\varepsilon\{1,\ldots,n\}$) depends upon the choice of u. It also depends upon any properties which are desirable for

$T(X_1,\ldots,X_n)$ to have. Some examples of desirable properties are unbiasedness, small mean-square error, closeness and consistency (Ehrenfeld and Littauer, 1964). It is fair to assume that all properties required by the point estimator have been considered by the investigator and that $T(X(S_1),\ldots,X(S_n))$ has been expressed uniquely in terms of $X(S_i)$ and known constants, $i\varepsilon\{1,\ldots,n\}$ for a fixed function u.

Quantitative knowledge about the probabilistic location of p will be obtained once the size, shape and location of S_i, $i\varepsilon\{1,\ldots,n\}$ has been selected and once $X(S_i)$, $i\varepsilon\{1,\ldots,n\}$ has been expressed in terms of variables which can be measured in the field. This quantitative knowledge about p is obtained simply by combining the value $T(X(S_1),\ldots,X(S_n))$ for a given set of $\{S_i\}_{i=1}^{n}$ with the distribution of $T(X_1,\ldots,X_n)$ acquired by either using existing statistical theorems or else applying goodness-of-fit tests to $\{T(S(S_1^{(j)}),\ldots X(S_n^{(j)}))\}_{j=1}^{No}$ for some sufficiently large No and using standard statistical procedures.

First attention is paid to selecting the size, shape and location of S_i, $i=1,\ldots,n$. Consider the i^{th} trial for any $i\varepsilon\{1,\ldots,n\}$ in a set of n independent repetitions of E. The i^{th} sampling unit (S_i) is chosen to be a right circular cylinder of radius (r) and height (h_i) selected from an infinite population of units in the shape of right circular cylinders, each unit containing a portion of C. Naturally the variability of X is partially dependent upon the size and shape of the population elements. Right circular cylinders of common radius are chosen to facilitate the mathematics and r is chosen so that the variability of X will be small. Logically there should be optimum radius (r_o) above which the variability of X does not decrease significantly. If preliminary samples cannot be obtained here to help select (r_o), the radius used may have to be chosen largely on a basis of personal experience and intuition. The height of the ith sampling unit (S_i) is simply the maximum height of all objects of interest intersecting the ith open right circular cylinder of radius (r). Finally the location of S_i is a function of any important physical, time or statistical constraints under which E may be forced to operate.

Now attention is paid to expressing $X(S_i), i=1,\ldots,n$ in terms of variables which can be measured in the field. Consider again the ith sampling unit S_i whose basal center point (c_i), radius (r) and height (h_i) are all known. Here r can be regarded as either a radius under investigation or an optimum radius selected from either a preliminary sampling analysis or a subjective decision-making process. Project a line segment along the base of S_i from c_i to some fixed perimeter point (p_o) on S_i. Let this line segment (L_o) intersecting p_o define a unique zero angle. Then sweeping a line segment (L) of length r around the basal perimeter of S_i keeping one of its end points fixed at c_i, it can be seen that each location of L defines a unique angle $\theta \varepsilon [0, 2\pi]$, and hence the base of a rectangle R_θ, of width r and height h_i, $\forall \theta \varepsilon [0, 2\pi]$ (see Figure 2).

The objective is to express $X(S_i)$ in terms of variables which can be easily evaluated. From the meaning of X and from known properties of C, it is possible to attach meaning to a function F_i: $(0, 2\pi] \rightarrow \mathbb{R}^+ \cup \{0\}$ where $F_i(2\pi)=KX(S_i)$ for some constant K. From this understanding of F_i, construct a bounded function (f_i) defined on $[0, 2\pi]$ whose set of discontinuities (Lang, 1968; p.50) on $[0, 2\pi]$ has Lebesgue outer measure zero, (Taylor, 1965; p.191), such that:

$$(1) \qquad F_i(\theta) = \int_0^\theta f_i(t)\,dt, \ \forall \theta \varepsilon (0, 2\pi]$$

The right-hand side of (1.) is well-defined and exists (Speigel, 1963; p.81). Now if $f_i(t)$ can be readily expressed in terms of measurable variables, the only task left is to evaluate the right-hand side of (1.). However if it is not possible to express $f_i(t)$ in terms of measurable variables, it becomes necessary to approximate $g_i(t)=f_i(t)/k$ by $\hat{g}_i(t)$, $\forall t\varepsilon[0,\theta]$ $\forall\theta\varepsilon[0,2\pi]$, where $\hat{g}_i$ has both the bounded and 'almost everywhere' continuity properties (Spiegel, 1969; p.33) of f_i, and where $\hat{g}_i(t)$ can be expressed in terms of measurable variables. Then it is seen that $X(S_i)$ can be expressed as a function of measurable variables by evaluating the right-hand side of (1.) at $\theta=2\pi$ with $\hat{g}_i(t)$ replacing $f_i(t)$, $\forall t\varepsilon[0,2\pi]$.

The final remaining problem in producing $X(S_i)$ is to evaluate the right-hand side of (1.) at $\theta=2\pi$ with $\hat{\hat{g}}_i(t)$ replacing f_i. Unfortunately this may not be a trivial task. Very often the measurable variables used in defining $g_i(t)$ cannot be easily expressed in terms of t mathematically. In other cases the measurable variables of interest are not directly integrable. If the latter case arises, $\hat{g}_i$ is approximated by a suitable directly integrable function h_i and through integration of h_i over $[0,\theta=2\pi]$, $X(S_i)$ is evaluated. In the former case however either numerical or Gaussian integration methods (Scheid, 1968) must be used. In this case, $X(S_i)$ is approximated by:

$$\sum_{\omega=1}^{N} C_\omega \hat{g}_i(t_\omega), \text{ where } N,\ C_\omega, \text{ and } t_\omega,\ \omega=1,\ldots,N$$

are determined from the choice of a particular integration method applied at a particular level of sampling intensity. The choice of the method depends upon required precision subject to specific time and cost constraints.

From previous comments, quantitative knowledge about the probabilistic location of p is now theoretically obtainable. The general sampling technique described above will hereafter be referred to as common point intersect sampling (CPIS), since each sampling rectangle R_θ is generated from a common point, namely the center of the circle comprising the base of a particular cylindrical sampling unit. To clarify the logic behind the general CPIS theory a flowchart (Figure 1) depicting the major concepts involved is provided. Now before proceeding to some specific applications of CPIS, a few remarks regarding this new concept should be made. There seems to be two very serious drawbacks to CPIS. One of these is that knowledge about the distribution of $T(X_1,\ldots,X_n)$ is required. It is true that some general behaviour of the distribution of $T(X_1,\ldots,X_n)$ should be known if a parametric statistical confidence interval (Ehrenfeld and Littauer, 1974; p.364) is to be constructed about p. If this information is not available and furthermore cannot be obtained from prior preliminary sampling or related sampling due to time or cost constraints, distribution-free or non-parametric statistical methods can still be used to construct either a meaningful hypothesis test for p or a rough confidence interval for p. If the information yielded by the non-parametric investigation is not sufficiently precise to be very helpful, general statistical techniques can be used to estimate the standard deviation $D(X_1,\ldots,X_n)$ of $T(X_1,\ldots,X_n)$. Then given a set of independent trials of E, $D(X(S_1),\ldots,X(S_n))/T(X(S_1),\ldots,X(S_n))$ gives a measure which expresses the magnitude of the average variation of $T(X_1,\ldots,X_n)$ relative to the size of $T(X(S_1),\ldots,X(S_n))$. This ratio can serve as a tentative indicator of how successful $T(X(S_1),\ldots,X(S_n))$ can be expected to be and as such can be temporarily used in place of information regarding the distribution of $T(X_1,\ldots,X_n)$. In other words an absence of knowledge regarding the distribution of $T(X_1,\ldots,X_n)$ has only the effect

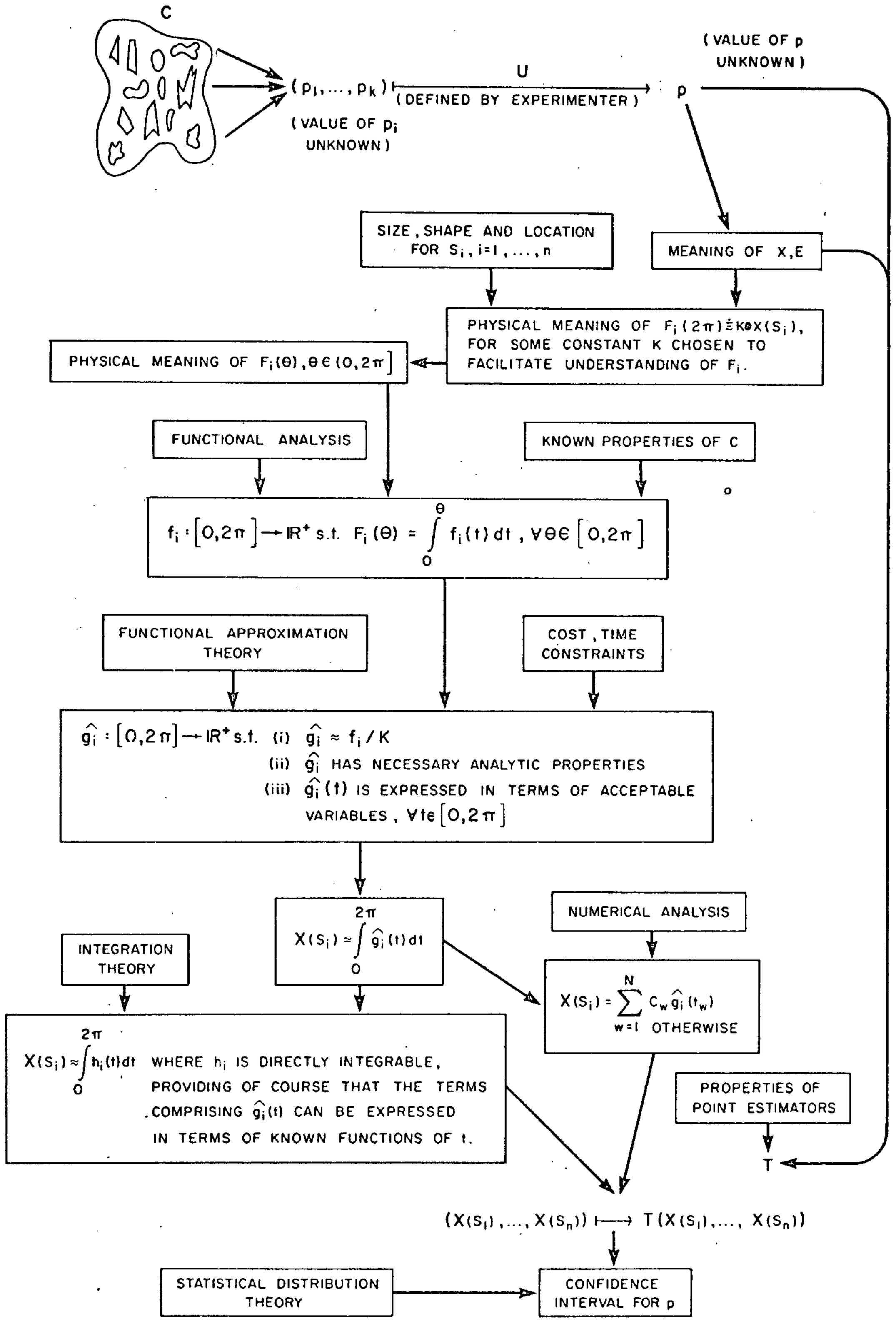


Figure 1. Flowchart depicting the development of the general common point intersect sampling concept.

of reducing the power of CPIS; it does not prevent CPIS from being a valid sampling method. CPIS seems to have a second serious drawback, namely the problem of obtaining f_i from an understanding of F_i. If the meaning of X is clear, then with proper selection of K, there should be no problem in attaching meaning to F_i. All that need be remembered when trying to achieve this understanding is that $F_i(\theta)$ for each $\theta \varepsilon [0,2\pi]$ simply considers objects in a portion of the ith sampling unit S_i. This means that as θ approaches 2π, $F_i(\theta)$ approaches $K \cdot X(S_i)$ in the most natural way, namely through increasing portions of S_i defined by the sweep of θ towards 2π. Note that K is chosen to convert $X(S_i)$ into a variable which is easier to work with. It remains then to consider the question of deriving f_i from both an acquired understanding of F_i and a knowledge of what is meant by a Reimann integral (Widder, 1947, p.149). It should be said now that in general there is no optimum approach to use in deriving f_i from F_i through (1). In practice this problem is usually very easy to solve. Since there is no preferable procedure to follow when obtaining f_i, the investigator must at this point rely largely on his own experience and ingenuity. It is the belief of the author that the best insight into the process of actually getting f_i is given through example. Hence attention is now turned to the application of CPIS to downed woody fuel particles.

Application to downed woody particles

Two experiments (E_1 and E_2) will be conducted upon a community (C) of downed woody fuel particles. It will be shown that the results of these experiments will ultimately yield estimators for a number of specific parameters of interest. Consider a community (C) of downed woody particles. Let $u(p1,\ldots,pk)=p1$, where $p1,\ldots,pk$ are parameters of unknown values describing C and p1 is the mean number of particles per unit area. Divide C into M subsets such that if the sampling rectangle (R_θ) intersects a fuel particle (q), q is said to belong to the $i^{\underline{th}}$ subset, $i\varepsilon\{1,\ldots,M\}$ providing that the width of q taken at its initial point of intersection lies between d_i and D_i, for some specified d_i, D_i where $d_i < D_i$, $i\varepsilon\{1,\ldots,M\}$. Notice here that q is implicitly assumed to have a well-defined length and hence a well-defined central axis with respect to which the widths of q are measured.

$p1^{(i)}$ is taken to be the mean number of fuel particles per unit area with respect to the population of sampling units comprising the $i^{\underline{th}}$ subset of C. From the meaning attached to $p1^{(i)}$, it is possible to introduce a random variable ($X1^{(i)}$) defined on the sample space of outcomes of an experiment $E_1^{(i)}$ where $E_1^{(i)}$ consists of selecting a sampling unit with respect to the i^{th} subset of C. From the general theory $X1^{(i)}$ is simply a function which maps each sampling unit into the average number of fuel particles per unit area (in the $i^{\underline{th}}$ subset of C) with respect to that sampling unit. Through $X1^{(i)}$ and the meaning of $p1^{(i)}$ a point estimate for $p1^{(i)}$ $(T1(X1^{(i)}(S1_1^{(i)}),\ldots,X1^{(i)}(S1_n^{(i)})))$ can be expressed in terms of $X1^{(i)}(S1_j^{(i)}), j\varepsilon\{1,\ldots,n\}$

and known constants for a given set $\{S1_j^{(i)}\}_{j=1}^{n}$:

$$T1(X1^{(i)}(S1_1^{(i)}),\ldots,X1^{(i)}(S1_n^{(i)})) = \sum_{j=1}^{n} X1^{(i)}(S1_j^{(i)})/n \equiv \overline{X1}^{(i)}$$

The parent estimator of $\overline{X1}^{(i)}$ $(\overline{XP1}^{(i)} \equiv \sum_{j=1}^{n} X1_j^{(i)}/n)$ is an unbiased estimator of $p_1^{(i)}$ and by applying the Central Limit Theorem (Ehrenfeld and Littauer, 1964; p.187) it can be shown that $\overline{XP1}^{(i)}$ is also a consistent estimator of $p1^{(i)}$. The Central Limit Theorem can also be applied together with the definition of $X1^{(i)}$ to demonstrate that:

(2) $\dfrac{\overline{XP1}^{(i)} - p1^{(i)}}{SD1^{(i)}\ \sqrt{n}}$, where (3) $(SD1^{(i)})^2 = \dfrac{1}{n-1} \sum_{j=1}^{n} (X1_j^{(i)} - XP1^{(i)})^2$

has approximately Student's t distribution with (n-1) degrees of freedom (Ehrenfeld and Littauer, 1964; p 189). It is important to realize that (2) has Student's t distribution with (n-1) degrees of freedom in most cases even when n is small. This is true because $X1_j^{(i)}$ is an average taken over a large sampling area $(\pi(r^{(i)})^2 >> 25)$ and hence by the Central Limit Theorem has approximately a normal distribution. If C contains large continuous areas $(>>\pi(r^{(i)})^2)$ differing drastically in fuel particle frequency, not only will the normality of $X1_j^{(i)}$ be probably violated but also $(SD1^{(i)})^2$ will probably take on very high values. To counteract these problems, C should be stratified wherever feasibly possible into regions of different fuel particle frequencies with respect to the ith subset of C and the theory of stratified random sampling (Freese, 1962; p.28) applied. It follows that for a given set of $\{S1_j^{(i)}\}_{j=1}^{n}$, (all lying within one area containing no large continuous sub-areas differing drastically in fuel particle frequencies), a confidence interval for $p1^{(i)}$ is $\overline{X1}^{(i)} \pm t_{1-\alpha/2;n-1}\ sd1^{(i)}/\sqrt{n}$ (Ehrenfeld and Littauer, 1964; p. 271) where $(1-\alpha)$ = level of statistical inference and $t_{1-\alpha/2;n-1}$ = value of Student's t distribution with (n-1) degrees of freedom at the $(1-\alpha/2)$ level. It remains to select the size, shape and location of $S1_j^{(i)}$ and to express $X1^{(i)}(S1_j^{(i)})$ in terms of measurable variables, $\forall i\varepsilon\{1,\ldots,M\}$, $\forall j\varepsilon\{1,\ldots,n\}$. This will now be done. Choose the sampling units in the $i^{\underline{th}}$ subset of C to be right circular cylinders of common radius $r1^{(i)}$. Due to time constraints $r1^{(i)}$ will be determined subjectively. Choose $r1^{(i)}$ sufficiently large such that the variability of $X1^{(i)}$ is expected to be small. The height $h1_j^{(i)}$ of the ith sampling unit $S1_j^{(i)}$ is well-defined from the general CPIS theory. A systematic plot sampling design with an equidistant grid pattern (Husch, Miller and Beers, 1972; p.233) is used to select the location of $S1_j^{(i)}$, $j\varepsilon\{1,\ldots,n\}$, due to its ease in application (Husch, Miller and Beers, 1972; p.228). It should be noted that the use of systematic sampling does introduce a problem in that now the n repetitions of $E_1^{(i)}$ are no longer completely independent. This means that $(SD1^{(i)})^2$ as defined in (3.) will not validly represent the sample variance (Husch, Miller and Beers, 1972; p. 229). In fact $(SD1^{(i)})^2$ tends to overestimate the sample variance (Osborne, 1942). A supposedly more representative expression for the sample variance for equidistant grid patterns is given by using successive difference formulas (Loetsch and Haller, 1964). However, unless the spacing between sampling units becomes coincidental with the pattern of population variation, the improvement offered by these successive difference formulas becomes negligible (Husch, Miller, Beers, 1972; p. 229). It will be assumed that the sampling unit locations have been selected so that no such coincidence occurs, making (3.) valid. In practice this is almost always done. If this manipulation process proves awkward or expensive with respect to a particular downed woody fuels population, alternate successive difference formulas (Husch, Miller and Beers, 1971; p.236) may be used to obtain a theoretically more realistic expression $(CD1^{(i)})^2$ for the sample variance. The remainder of the argument can then be applied with $(CD1^{(i)})^2$ replacing $(SD1^{(i)})^2$.

In order to determine $X1^{(i)}(S1_j^{(i)})$ in an appropriate form it is necessary to select a multiplicative constant k1 which will transform $X1^{(i)}(S1_j^{(i)})$ into a variable which can be related to more easily. Since particles are being considered with respect to their intersections along transect lines, it seems natural to select k1 as one unit length. Then $K1 \cdot X1^{(i)}(S1_j^{(i)})$ becomes the average number of particles per unit length with respect to $S1_j^{(i)}$ provided of course that the units of length are chosen sufficiently small. This relates better to the sampling design than does average number of particles per unit area. $F1_j^{(i)}$ is a function which associates to each $\theta \varepsilon [0,2\pi]$ a non-negative real number $F1_j^{(i)}(\theta)$ representative of the average number of particles per unit length of transect with respect to the area in $S1_j^{(i)}$ defined by a radial sweep from zero to θ.

In constructing $f1_j^{(i)}$, it seems logical as a first attempt to try $\sigma_j^{(i)}$ where $\sigma_j^{(i)}$ is a function which associates to each $t \varepsilon [0,2\pi]$ a non-negative real number $\sigma_j^{(i)}(t)$ representative of the number of particles which intersect the transect located at t, $0 \leq t \leq \theta$. This function is bounded on $[0,2\pi]$ and in fact is a step function by definition which ensures its 'almost everywhere' continuity on $[0,2\pi]$. Now from elementary calculus it is well known that:

$$(4) \quad \bar{\sigma}_j^{(i)}(\theta) = \frac{1}{\theta} \int_0^\theta \sigma_j^{(i)}(t)\,dt, \; 0<\theta\leq 2\pi$$

where $\bar{\sigma}_j^{(i)}(\theta)$ is the average value of the function $\sigma_j^{(i)}$ taken over all values of t ranging from 0 to $\theta>0$. It now becomes obvious for fixed

$\theta\varepsilon(0,2\pi]$, to set $f1_j^{(i)}=\sigma_j^{(i)}/(\theta \cdot r1^{(i)})$ since $F1_j^{(i)} = \overline{\sigma}_j^{(i)}/r1^{(i)}$. Note that for fixed $\theta\varepsilon(0,2\pi)$, $f1_j^{(i)}(t)$ is already expressed in terms of measurable variables, for each $t \ \varepsilon[0,\theta]$, namely number of relevant particle intersections at t. The final problem then in producing $X1^{(i)}(S1_j^{(i)})$ is to evaluate the right-hand side of (4) at $\theta=2\pi$ with $\sigma_j^{(i)}/r1^{(i)}$ replacing $\sigma_j^{(i)}$. This will not be done here for one simple reason. Knowledge about numbers of downed woody particles is not currently required as a significant direct input for evaluation or prediction of fire behaviour and impact. It should be stated again that throughout this thesis the applications of the CPIS general theory will be focussed on obtaining those parameters which relate most significantly to fire-oriented activities. It is true that results of E_1 can be combined with results of E_2 to yield estimators for a number of downed woody particle parameters. In fact this will be done later for completeness. But the results of E_1 do not relate significantly to fire evaluation, and so an explicit expression which estimates the right-hand side of (4.) will not be derived.

E_1 serves here primarily as a working example to demonstrate that the process undertaken to get a meaningful "handle" on the general CPIS theory is not a mysterious one. Each step taken in working out this first application of the general theory has been logical and very straight-forward. It is of interest to notice that nowhere in the argument have any assumptions been made regarding the distribution of the particles within the CPIS units themselves. Attention is now turned to E_2.

The general layout of $E_2^{(i)}$ is very similar to that of $E_1^{(i)}$, and reference will be periodically made to E_1 throughout the discussion of E_2. Here $u(p1,\ldots,pk) = p2$, where p2 is defined as the mean total volume of particles per unit area. Analogous to $E_1^{(i)}$, $E_2^{(i)}$ is an experiment consisting of selecting a sampling unit within C with respect to the ith subset of C. Also, analogous to $X1^{(i)}$, $X2^{(i)}$ is simply a function which maps each sampling unit into the average volume of fuel particles per unit area with respect to that sampling unit. The definitions of $T2(X1^{(i)}(S2_1^{(i)}),\ldots,X1^{(i)}(S2_n^{(i)}))$ and $(SD2^{(i)})^2$ are both obvious from $E_1^{(i)}$. All remarks concerning $\overline{X1}^{(i)}$ and $\dfrac{\overline{XP1}^{(i)} - p1^{(i)}}{SD1^{(i)}/\sqrt{n}}$ apply equally to $\overline{X2}^{(i)}$ and $\dfrac{\overline{XP2}^{(i)} - p2^{(i)}}{SD2^{(i)}/\sqrt{n}}$, respectively. The comments regarding both selection and justification of the sampling units used in $E_1^{(i)}$ also apply equally to $E_2^{(i)}$.

It remains then only to determine $X2^{(i)}(S2_j^{(i)})$ in an appropriate form. Since $X2^{(i)}(S2_j^{(i)})$ is an average volume per unit area it makes good sense to select K2 as $\pi(r2^{(i)})^2$. Then $K2 \cdot X2^{(i)}(S2_j^{(i)})$ becomes a total volume of particles which is more easily related to the sampling design. An understanding of $F2_j^{(i)}$ is now possible and $F2_j^{(i)}$ is analogous in meaning of $F1_j^{(i)}$ with the obvious difference that $F2_j^{(i)}(\theta)$ represents a total volume of particles with respect to the area in $S2_j^{(i)}$ defined by a radial sweep from 0 to $\theta>0$.

The construction of $f2_j^{(i)}$ is derived from a basic understanding of $F2_j^{(i)}$ and the meaning of a Riemann integral. $F2_j^{(i)}$ is a function which when evaluated at fixed θ yields a total volume. It is well known that when a cross-sectional area is swept through an arc, a volume is generated. (see Figure 2). This volume is a function of the distance separating the centre of the sampling unit and the cross-sectional region. Thus $f2_j^{(i)}$ will involve both the shape of the fuel particles in the ith subset of C and a separation distance factor. More specifically, downed

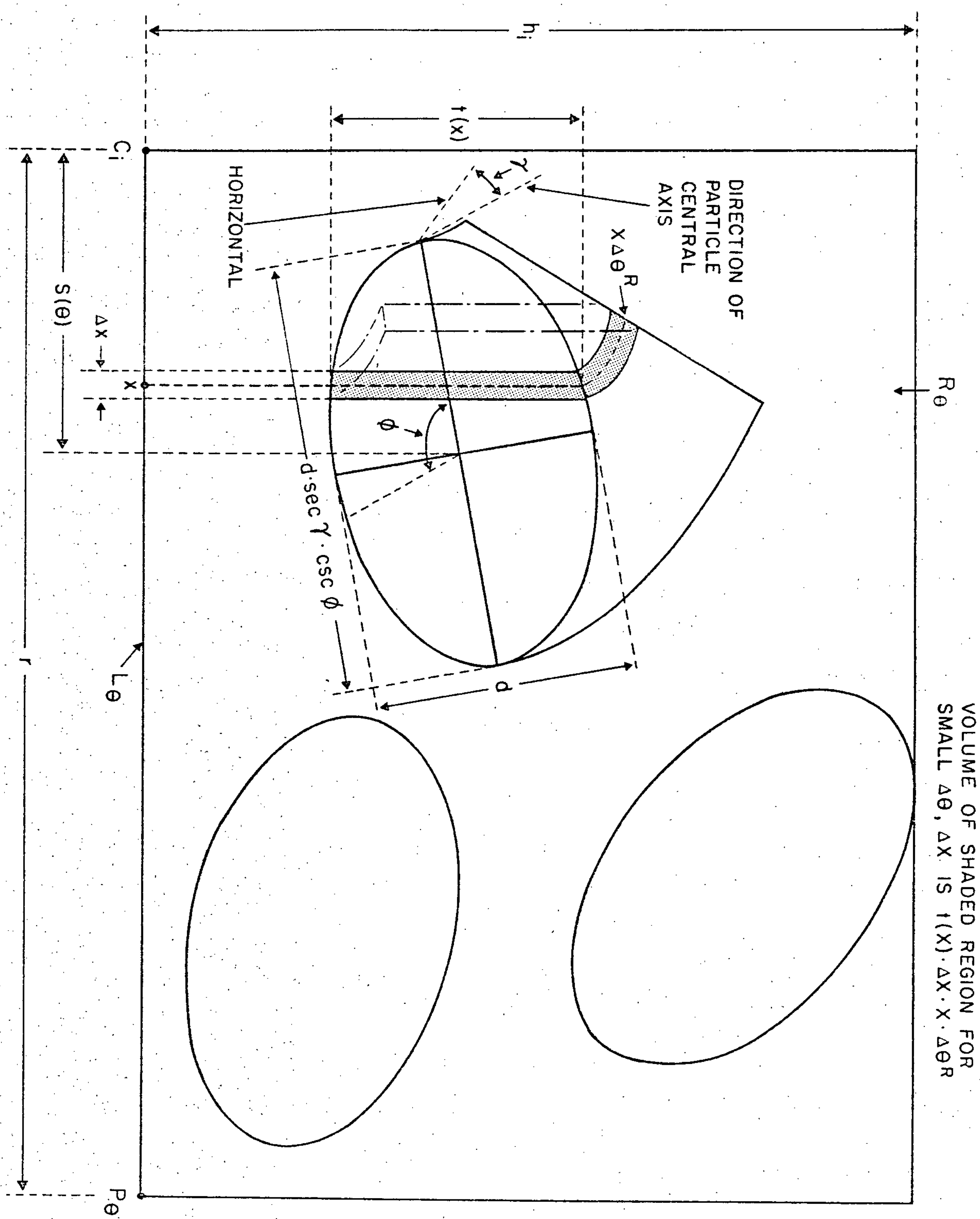


Figure 2. Typical cross-section of sampling unit S_i with special application to downed woody cylinders.

woody particles can be classified reasonably well into five geometric divisions: frustums of cylinders, parallelepipeds, cones, parabaloids and neiloids (Husch, Miller and Beers, 1972; p.120). The presence of particle boundary taper as possessed by cones or particle boundary concavity as possessed by parabaloids and neiloids provides a significant complication to both the theoretical and practical aspects of quantifying downed woody fuel parameters by working with the fuels themselves. If the initial points of intersection are consistently used to compute the fuel particle widths, and if the concavity present is not too severe, it should be feasible to classify the last three troublesome divisions under frustums of cylinders. It is of importance to mention that in practice the lack of symmetry if any of a given cross-sectional region is usually small. But due to time constraints no quantitative analysis was undertaken to support the above classification grouping made by using the above technique of width measurement. Hence further studies are required to define a completely valid set of conditions under which frustums of cones, parabaloids and neiloids may be considered cylinders with respect to geometric cross-sectional form. Continuing then, it is seen that it is temporarily fair to assume that all cross-sectional regions are whole ellipses, truncated ellipses, or rectangles. Without loss of generality only the first two of these regions will be considered. In practice the total number of parallelepipeds generally comprise a very small proportion of a downed woody fuels population. If parallelepipeds are of particular interest, an almost identical and in fact simpler argument to that offered below can be made by applying the concepts below to rectangles as opposed to ellipses. Now from the understanding gained from the above remarks, the values of $F2_j^{(i)}$ are obtained by summing up the volumes generated by rotating either whole or truncated ellipses through small arcs. But if $\theta_1 \varepsilon [\theta_0, \theta_2]$, where $(\theta_2 - \theta_0)$ is small, and if the fuel particle cross-section at θ_1 is a whole ellipse, then from elementary calculus, an excellent estimate for the particle volume generated by rotating this cross-section at θ_1 through $[\theta_0, \theta_2]$ is:

$$(5) \quad \left\{\frac{\pi}{4} \quad \overline{(d^{(i)})^2} \cdot S(\theta_1) \cdot \overline{(\sec(\gamma_1))} \cdot CSC(\phi(\theta_1))\right\} \cdot \left[(\theta_2 - \theta_o)\right]$$

where $\overline{(d^{(i)})}$ = an estimate for the quadratic mean particle diameter (dbar$^{(i)}$) (Brown, 1973) with respect to the ith subset of C, $i\varepsilon\{1,\ldots,M\}$.

$s(\theta_1)$ = the horizontal distance between the centre of the elliptical cross-section at θ_1 and the basal center point of the sampling unit, where the horizontal is defined parallel to the orientation of the base of the sampling unit.

$\overline{(\sec(\gamma_1))}$ = an estimate for the mean secant of the particle tilt (γ_1) with respect to the ith subset of C, $i\varepsilon\{1,\ldots,M\}$.

$CSC(\phi(\theta_1))$ = the cosecant of the angle of intersection $(\phi(\theta_1))$ between the transect at θ_1 and the central axis of the fuel particle, $0<\phi(\theta_1)\leq\pi/2, \quad \forall\theta_1\varepsilon[\theta_0,\theta_2]$.

See Appendix I for a complete proof that the volume obtained by rotating an elliptical cross-section of θ_1 through $[\theta_0^R, \theta_2^R]$ is given by (5). Through summation over relevant particles, it follows from the above that a logical first attempt for $f2_j^{(i)}$ is $v_j^{(i)}$ where

$$(6) \quad v_j^{(i)}(t) = \pi/4 \ \overline{(d^{(i)})^2} \ \overline{(\sec(\gamma i))} \sum_{k=1}^{m_j^{(i)}(t)} \left[S_{k,j}^{(i)}(t) \cdot CSC(\phi_{k,j}^{(i)}(t))\right],$$

where $m_j^{(i)}(t)$ is the number of particles which intersect the transect at t, and where the kth fuel particle intersects the transect at t, $\forall k\varepsilon\{1,\ldots,m_j^{(i)}(t)\}$. In $E_2^{(i)}$, a particle intersection is defined to occur only when both the particle central axis and at least one particle edge intercept the transect.

Since csc $(\phi_{k,j}^{(i)}(t))$ is bounded on $[0,2\pi]$ by $\sqrt{1+\left(\frac{2a^{(i)}}{\overline{d}^{(i)}}\right)^2}$ where $a^{(i)}$

is the maximum length of all particles in the ith subset of C, $i\varepsilon\{1,\ldots,M\}$, it follows that $v_j^{(i)}$ is also bounded on $[0,2\pi]$. $s_{k,j}^{(i)}$ and $\phi_{k,j}^{(i)}$ are sectionally continuous (Spiegel, 1963; p.26) on $[0,2\pi]$. Also the cosecant function is continuous on $(0,\pi/2)$. Hence the composite function $CSC\circ\phi_{k,j}^{(i)}$ is sectionally continuous on $[0,2\pi]$ (Lang, 1968; p.51). Thus $v_j^{(i)}$ satisfies all the required properties stated in the general CPIS theory. Setting $f2_j^{(i)}=v_j^{(i)}$ reveals:

$$(7)\quad F2_j^{(i)}(\theta) = \int_0^{\theta} v_j^{(i)}(t)dt,\ \forall\theta\varepsilon(0,2\pi]$$

It follows that:

$$X2^{(i)}(S2_j^{(i)}) = F2_j^{(i)}(2\pi)/\pi(r2^{(i)})^2 = \int_0^{2\pi} \frac{v_j^{(i)}(t)}{\pi(r2^{(i)})^2}\, dt = \int_0^{2\pi} g2_j^{(i)}(t)dt, \text{ where}$$

$$(8.)\ g2_j^{(i)}(t) \equiv v_j^{(i)}(t)/\pi(r2^{(i)})^2\ ,\ \forall t\varepsilon[0,2\pi]$$

The next step to be taken in order to express $X2^{(i)}(S2_j^{(i)})$ explicitly in terms of measurable variables is a consideration of the variables comprising $g2_j^{(i)}(t)$ as defined in (8) and (6). An inspection of $g2_j^{(i)}(t)$ reveals that it is expressed in terms of $S_{k,j}^{(i)}(t)$ and $\phi_{k,j}^{(i)}(t)$, $k\varepsilon\{1,\ldots,m_j^{(i)}(t)\}$, both of which are difficult to measure directly. In fact both are totally impractical to measure directly with respect to lesser downed woody particles. Hence from this point on, it will be necessary to consider the lesser downed particles and the greater downed particles separately.

First consider $i\varepsilon\{1,\ldots,M\}$ where $D_i \leq 3$ inches. From (6.),

$$(9)\quad g2_j^{(i)}(t) = \frac{\pi}{4}\ \overline{(d^{(i)})^2}\ \overline{(\sec(\gamma_i))} \sum_{k=1}^{m_j^{(i)}(t)} \left[S_{k,j}^{(i)}(t)\cdot CSC(\phi_{k,j}^{(i)}(t))\right] \Big/ \pi(r2^{(i)})^2$$

$$= \begin{cases} \frac{\pi}{4}\overline{(d^{(i)})^2}\ \overline{(\sec(\gamma i))}\ m_j^{(i)}(t) \sum_{k=1}^{m_j^{(i)}(t)} \left[S_{k,j}^{(i)}(t)\cdot\csc(\phi_{k,j}^{(i)}(t))\right] \Big/ \\ \qquad (m_j^{(i)}(t)\cdot\pi(r2^{(i)})^2),\ \text{if } m_j^{(i)}(t) \neq 0 \\ 0,\ \text{if } m_j^{(i)}(t) = 0 \end{cases}$$

From the above, the selection of $\hat{g2}_j^{(i)}(t)$ is obvious.

$$(10)\quad \hat{g2}_j^{(i)}(t) = \begin{cases} \frac{\pi}{4}\ \overline{(d^{(i)})^2}\,\overline{(\sec(\gamma i))}\cdot K^{(i)}\cdot m_j^{(i)}(t),\ \text{if } m_j^{(i)}(t) \neq 0 \\ 0,\ \text{if } m_j^{(i)}(t) = 0 \end{cases}$$

$$= \frac{\pi}{4}\ \overline{(d^{(i)})^2}\ \overline{(\sec(\gamma_i))}\cdot K^{(i)}\cdot m_j^{(i)}(t),\ \text{where } K^{(i)} \text{ is an appropriate constant}$$

$\hat{g2}_j^{(i)}$ is a bounded step function on $[0, 2\pi]$ and $\hat{g2}_j^{(i)}(t)$ is expressed exclusively in terms of measurable variables and $K^{(i)}$, $\forall j\varepsilon\{1,\ldots,n\}$. Hence it remains to produce $K^{(i)}$. It may seem somewhat optimistic to assume that for each $i\varepsilon\{1,\ldots,M\}$, $D_i \leq 3$ inches, there exists a unique constant $K^{(i)}$ independent of both j and t permitting $\hat{g2}_j^{(i)}(t)$ to be very close to $g2_j^{(i)}(t)$. However if for some j and t, $\hat{g2}_j^{(i)}$ is a poor approximation for $\hat{g2}_j^{(i)}(t)$, it does not really matter providing that the integrals of the two functions from 0 to 2π are reasonably close.

$K^{(i)}$ was explicitly determined for i = 1,2,3 where

$$(d_1,D_1] = (0,\tfrac{1}{4}'']$$
$$(d_2,D_2] = (\tfrac{1}{4}'',1'']$$
$$(d_3,D_3] = (1'',3'']$$

These three subsets were chosen for two reasons. The first is that the intervals are sufficiently small so as to facilitate the effective use of quadratic mean diameters (as required in (10.)) and hence accommodate the computation of accurate lesser particle volume estimates. The second reason for this choice is that the above three subsets correspond respectively to 1,10 and 100 hour average moisture time lag divisions for a number of common woody forest materials, (Fosberg, 1970). For each $i \in \{1,2,3\}$, $K^{(i)}$ was determined using simultaneous runs of a one-way classification random components analysis of variance model (ANOVA1) with unequal numbers of observations in the cells (Ehrenfeld and Littauer, 1964; p.399) and a two-way classification random components analysis of variance model with unequal numbers of observations in the cells (Ehrenfeld and Littauer, 1964; p.432). A description and computerized version of the first of these two models is offered in Appendix 2. Since the second of these models is virtually identical in design to the first model, no computerized version of it is required; however, a supplementary description of the two-way model is included in Appendix 2.

The two-way model was constructed using particle distribution and particle frequency or loading as the two influencing factors. The primary purpose of this model was to determine for each $i \in \{1,2,3\}$ the ranges of the two above influencing factors under which it was possible to assert the existence of a unique constant $K^{(i)}$ reasonably independent of both j and t which would permit the integral of $\hat{g2}_j^{(i)}$ to be close to the integral of $g2_j^{(i)}$ for each $j \in \{1,\ldots,n\}$, and for each $i \in \{1,2,3\}$. A thorough description of the processes involved in the two-way model is given in Appendix 2. It suffices to say here that the results of the two-way model indicated that a satisfactory unique constant $K^{(i)}$ could exist only when the fuel particle distribution (with respect to the $i^{\underline{th}}$ subset of C) was held fixed within the CPIS units being sampled, $\forall i \in \{1,2,3\}$.

Explicit determination of $K^{(i)}$ for each $i \in \{1,2,3\}$ was performed in the one-way model as described in Appendix 2. In order to produce

$K^{(i)}$, it was necessary to select a suitable sampling radius $r2^{(i)}$ for each $i\varepsilon\{1,2,3\}$. Due to time constraints, selection of $r2^{(i)}$ was made subjectively. More precisely,

$$r2^{(1)} = 8.5 \text{ feet (sampling area of 1/192 acre)}$$

$$r2^{(2)} = 16.5 \text{ feet (sampling area of 1.51 acre)}$$

$$r2^{(3)} = 28.5 \text{ feet (sampling area of 1/12 acre)}$$

It is noted that some experienced investigators may be more adept than others in selecting an appropriate $r2^{(i)}$. Hence to eliminate as much guesswork as possible in the selection of $r2^{(i)}$, an approximate formula for determining $r2^{(i)}$ as a function of D_i is presented below:

(11) $\text{Log } r2^{(i)} \approx 1.22 + 0.47 \log D_i$, where D_i is in inches and $r2^{(i)}$ is in feet, $\forall i\varepsilon\{1,\ldots,M\}$

(11.) is presented here merely as a rule of thumb. It serves primarily as a guide for the inexperienced investigator and as a reference for the experienced one when no preliminary data to help select $r2^{(i)}$ is available.

It is of interest to note that the analysis of the one-way model revealed that for randomly distributed fuel particles within the CPIS units being sampled the random variable defined by:

$$(12) \quad \left\{ \sum_{k=1}^{m_j^{(i)}} \left[S_{k,j}^{(i)} \cdot \text{CSC}(\phi_{k,j}^{(i)}) \right] \Big/ \left(m_j^{(i)} \cdot \pi (r2^{(i)})^2\right) - K^{(i)} \right\} \Bigg/ \left\{ \sum_{k=1}^{m_j^{(i)}} \left[S_{k,j}^{(i)} \cdot \text{CSC}(\phi_{k,j}^{(i)}) \right] \Big/ \left(m_j^{(i)} \cdot \pi (r2^{(i)})^2 \right\} \equiv R_j^{(i)},$$

i and j fixed

was found to be normally distributed with mean 0. It was also discovered that if j is ordered by loading, the variance of the above ratio generally decreased with increasing j, $\forall i \in \{1,2,3\}$. This means that the performance of $\hat{g2}_j^{(i)}$ as a percentage approximation of $\hat{g2}_j^{(i)}$ in the integral from 0 to 2π is satisfactory for all i and j. i and j may, if desired, be interpreted in terms of fuel particle size and loading, respectively. Note that $R_j^{(i)}$ is defined only on those values in $[0, 2\pi]$ such that $m_j^{(i)}(t) \neq 0$. It is of interest to note that the one-way model showed that the random variable defined by:

$$m_j^{(i)}, \text{ i and j fixed}$$

was found to have a coefficient of variation reasonably independent of i and j, $\forall i \in \{1,2,3\}$, $\forall j \in \{1,\ldots,n\}$. This means that on a percentage basis the variability of $\hat{g2}_j^{(i)}$ over $[0,2\pi]$ is virtually independent on both i and j. Combining the two above points of interest, it is seen that if numerical methods are required to evaluate the integral of $\hat{g2}_j^{(i)}$, one numerical technique applied at one level of sampling intensity will probably suffice for all i and j.

Numerous runs of the one-way model with randomly distributed fuel particles yielded values of $k^{(i)}$ within a small neighbourhood of the following:

$$(13) \quad \begin{cases} K^{(1)} = 2.83 \times 10^{-2} \text{ (feet)}^{-1} \\ K^{(2)} = 1.49 \times 10^{-2} \text{ (feet)}^{-1} \\ K^{(3)} = 8.56 \times 10^{-3} \text{ (feet)}^{-1} \end{cases}$$

$\hat{g2}_j^{(i)}(t)$ has now been explicitly determined in terms of measurable variables $\forall i \in \{1,2,3\}$, as previously defined, providing that the lesser particles considered are randomly distributed within the CPIS units. Attention is now turned to $i \in \{1,\ldots,M\}$ where $D_i > 3$ inches.

Suitable formulas for explicitly defined subsets of greater downed particles whose distributions are either random in the CPIS units or else can be quantified in the CPIS units can be obtained by a procedure almost identical to that used above for the lesser downed particles. Due to time constraints plus the fact that attention is to be focussed primarily upon the lesser fuels which as has been previously stated are most likely to be consumed by the majority of broadcast fires, formulas for the larger fuels are not derived. It is of interest to note that only a slight manipulation of (9.) will yield a formula for $g2_j^{(i)}(t)$ (in terms of measurable variables) which applies to greater downed particles and is independent of particle distribution. This formula is presented below:

$$(14) \quad g2_j^{(i)}(t) = \frac{\overline{d}^{(i)}}{8(r2^{(i)})^2} \sum_{k=1}^{mj^{(i)}(t)} \left[(P2_{k,j}^{(i)}(t))^2 - (P1_{k,j}^{(i)}(t))^2 \right] \text{, where}$$

$$(15) \quad P2_{k,j}^{(i)}(t) = (2S_{k,j}^{(i)}(t) + \overline{(d^{(i)})} \cdot \csc(\phi_{k,j}^{(i)}(t)))/2$$

$$(16) \quad P1_{k,j}^{(i)}(t) = (2S_{k,j}^{(i)}(t) - \overline{(d^{(i)})} \cdot \csc(\phi_{k,j}^{(i)}(t)))/2 \text{ and } \overline{(\sec(\gamma i)} = 1$$

Note that when $D_1 > 3$ inches, it is fair to assume that $\overline{(\sec(\gamma i))} = 1$ (Brown, 1973; p.3). Then from (15.) and (16.), it is clear that P1 and P2 can be interpreted as distances to the left and right end points of intersection, respectively for a whole ellipse, which are not difficult to measure for the greater woody particles. If a truncated ellipse is encountered (i.e. a particle end) it is necessary to extrapolate one edge of the particle to the transect in order to obtain the proper value providing of course that the particle central axis intercepts the transect. Hence (14.) may be used directly for greater downed fuels of any distribution and loading.

There is one remaining step to be made so that $X2^{(i)}(S2_j^{(i)})$ will be expressed in terms of measurable variables. This step is the evaluation of the integral of an appropriate function between 0 and 2π. Due to time constraints and the fact the thesis is designed to focus upon the lesser downed particles, evaluation of $X2^{(i)}(S2_j^{(i)})$ through the integral of an appropriate function will be done only for $i\varepsilon\{1,2,3\}$, as previously defined. The processes involved here are thoroughly described in Appendix 3, which includes a computerized version of a downed woody fuels model. A major result of this investigation was that the random variable defined by:

$$(17)\left\{\frac{V_j^{(i)}}{\pi(r2^{(i)})^2} - h2_j^{(i)}\right\}\Bigg/\left\{\frac{V_j^{(i)}}{\pi(r2^{(i)})^2}\right\},\ \text{i and j fixed, where:}$$

$$(18)\quad h2_j^{(i)}(t) = \sum_{w=2}^{12} C_\omega \cdot \hat{g}2_j^{(i)}\left(t + \frac{\omega\pi}{6}\right),\quad C_\omega = \begin{cases}4, & \text{if } \omega \text{ is odd}\\ 2, & \text{if } \omega \text{ is even}\end{cases}$$

and $V_j^{(i)}$ = the true total particles volume in the jth sample taken with respect to the ith subset of C, $i\varepsilon\{1,2,3\}$

was found to be normally distributed with mean 0 and very small variance independent of i and j, $\forall i\varepsilon\{1,2,3\}$ $\forall j\varepsilon\{1,\ldots,n\}$. The means that $V_j^{(i)}/\pi(r2^{(i)})^2$ can be very well approximated by an appropriate application of Simpson's rule (Schied, 1968; p.108) with twelve transects, $\forall i\varepsilon\{1,2,3\}$, $\forall j\varepsilon\{1,\ldots,n\}$. Selection of the placement for the first transect in a particular sampling unit does not significantly affect the performance of the estimate for the average particle volume per unit area with respect to that sampling unit. A typical performance of Simpson's rule with twelve transects as applied to subset i is given in Table 1 and displayed graphically in Figure (i+2) for $i\varepsilon\{1,2,3\}$. In summary then $X2^{(i)}$ $(S2_j^{(i)})$ can be very well approximated by:

Table 1. Actual and estimated computer-generated lesser downed woody particle volumes.

Actual Volumes			Estimated Volumes		
Class 1	Class 2	Class 3	Class 1	Class 2	Class 3
$((ft)^3/(ft)^2) X 10^{-2}$			$((ft)^3/(ft)^2) X 10^{-2}$		
0.03021	0.2129	1.317	0.02853	0.2178	1.413
0.03051	0.2171	1.345	0.02645	0.2087	1.257
0.03015	0.2162	1.322	0.03403	0.2141	1.295
0.02952	0.2070	1.309	0.03196	0.1986	1.204
0.03005	0.2167	1.365	0.02907	0.2004	1.381
0.02918	0.2138	1.305	0.02961	0.1913	1.375
0.0305	0.2133	1.326	0.03060	0.2297	1.680
0.03067	0.2032	1.294	0.02825	0.1977	1.268
0.04528	0.4686	2.665	0.04180	0.4759	2.916
0.04452	0.4681	2.614	0.04369	0.4686	2.905
0.04557	0.4757	2.543	0.04270	0.5299	2.611
0.04462	0.4766	2.658	0.04514	0.4438	2.622
0.04559	0.4709	2.658	0.04405	0.4704	3.023
0.04550	0.4855	2.590	0.03990	0.5042	2.852
0.04489	0.4711	2.653	0.03945	0.4695	2.793
0.04551	0.4688	2.663	0.04225	0.4512	2.889
0.05938	0.7496	3.997	0.06147	0.7815	4.045
0.05922	0.7501	3.897	0.05615	0.7706	3.815
0.06070	0.7336	3.899	0.06012	0.7019	3.810
0.05912	0.7465	3.918	0.05741	0.8017	4.125
0.05912	0.7465	3.918	0.05741	0.8017	4.125

con't.....

Actual Volumes			Estimated Volumes		
Class 1	Class 2	Class 3	Class 1	Class 2	Class 3
$((ft)^3/(ft)^2)X\ 10^{-2}$			$((ft)^3/(ft)^2X\ 10^{-2}$		
0.05993	0.7286	3.878	0.05678	0.7577	3.986
0.05934	0.7294	3.898	0.06048	0.6726	4.147
0.05988	0.7222	3.894	0.06184	0.7065	3.788
0.05967	0.7266	3.978	0.05886	0.7513	4.056
0.07372	0.9804	5.528	0.08106	1.052	5.399
0.07457	0.9973	5.251	0.07258	1.035	5.485
0.07457	1.012	5.166	0.07447	1.038	5.062
0.07359	0.9982	5.173	0.07068	1.045	5.217
0.07470	0.9934	5.152	0.07592	1.012	5.265
0.07550	0.9907	5.231	0.07285	0.9774	5.479
0.07477	1.009	5.358	0.07465	0.9783	5.784
0.07361	1.003	5.232	0.07330	0.9929	5.335
0.09020	1.240	6.539	0.08224	1.310	6.672
0.08955	1.276	6.593	0.08883	1.366	7.117
0.09011	1.266	6.488	0.08386	1.243	6.533
0.09021	1.257	6.491	0.08278	1.320	6.314
0.08909	1.261	6.487	0.08296	1.290	6.491
0.08826	1.274	6.688	0.09208	1.312	6.913
0.08996	1.263	6.617	0.09009	1.199	6.710
0.08936	1.283	6.387	0.09460	1.405	6.854

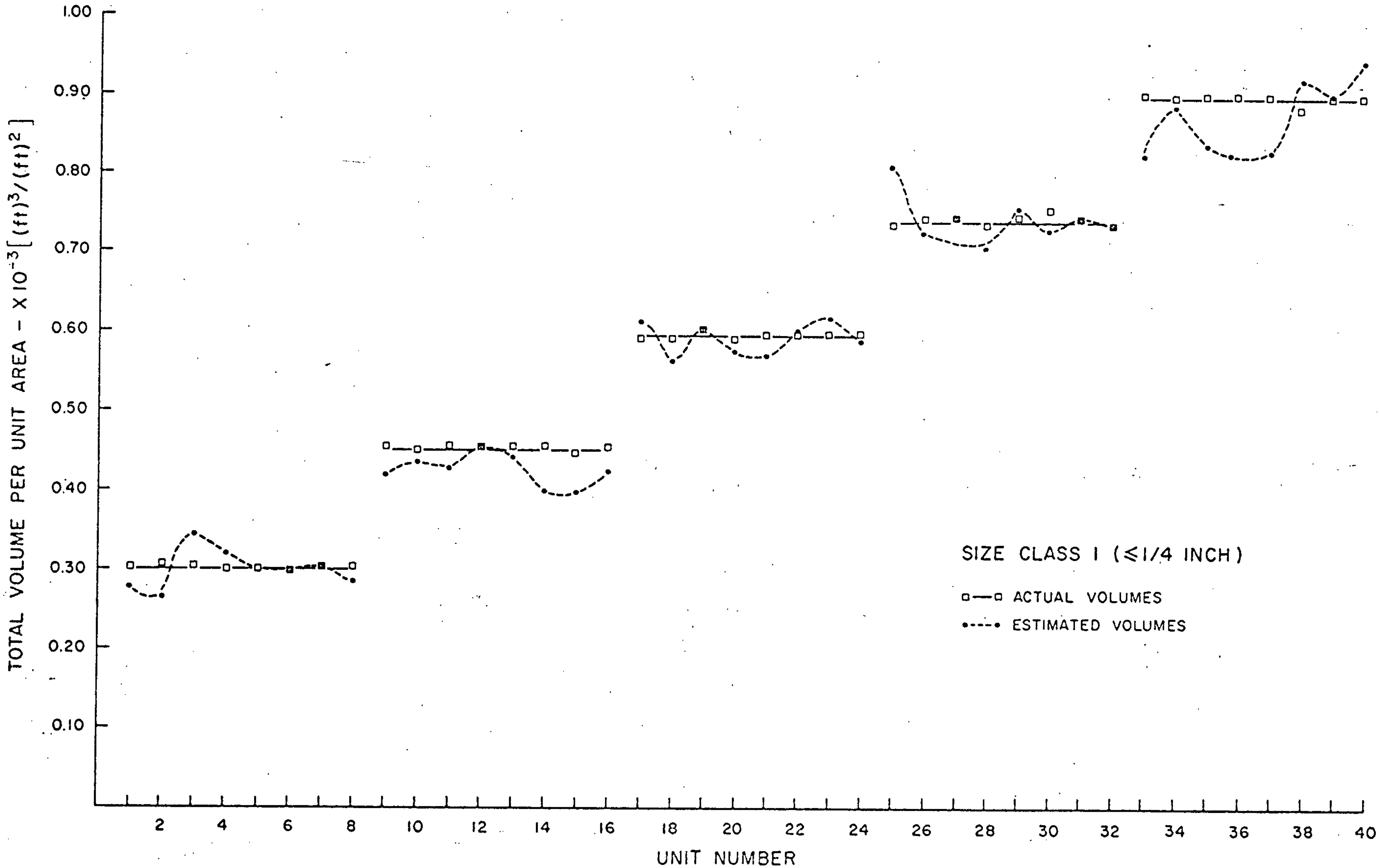


Figure 3. Curves representing actual and estimated computer-generated downed woody particle volumes with respect to size class 1 ($\leq \frac{1}{4}$ inch).

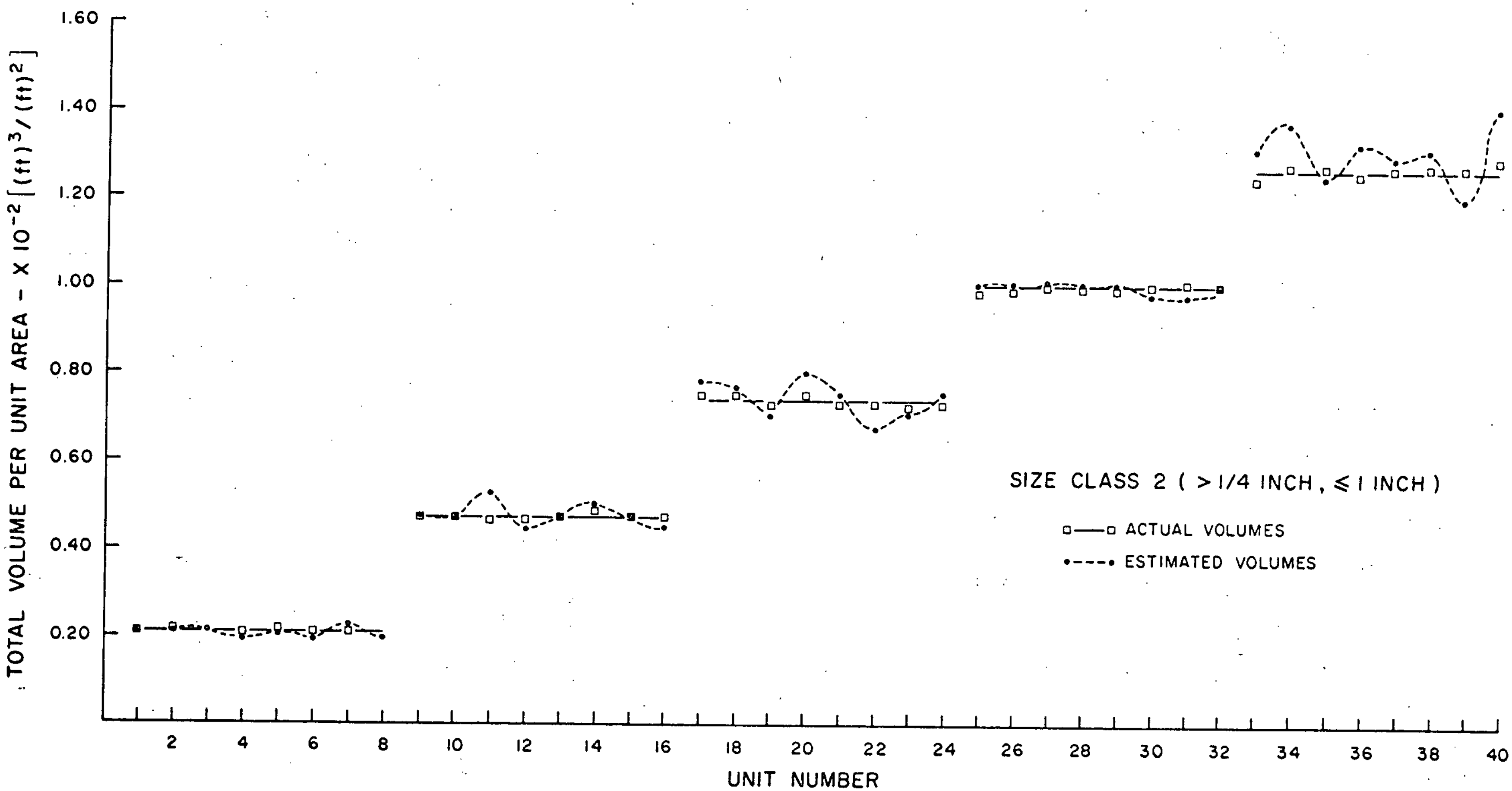


Figure 4. Curves representing actual and estimated computer-generated downed woody particle volumes with respect to size class 2 (> ¼ inch ≤ 1 inch)

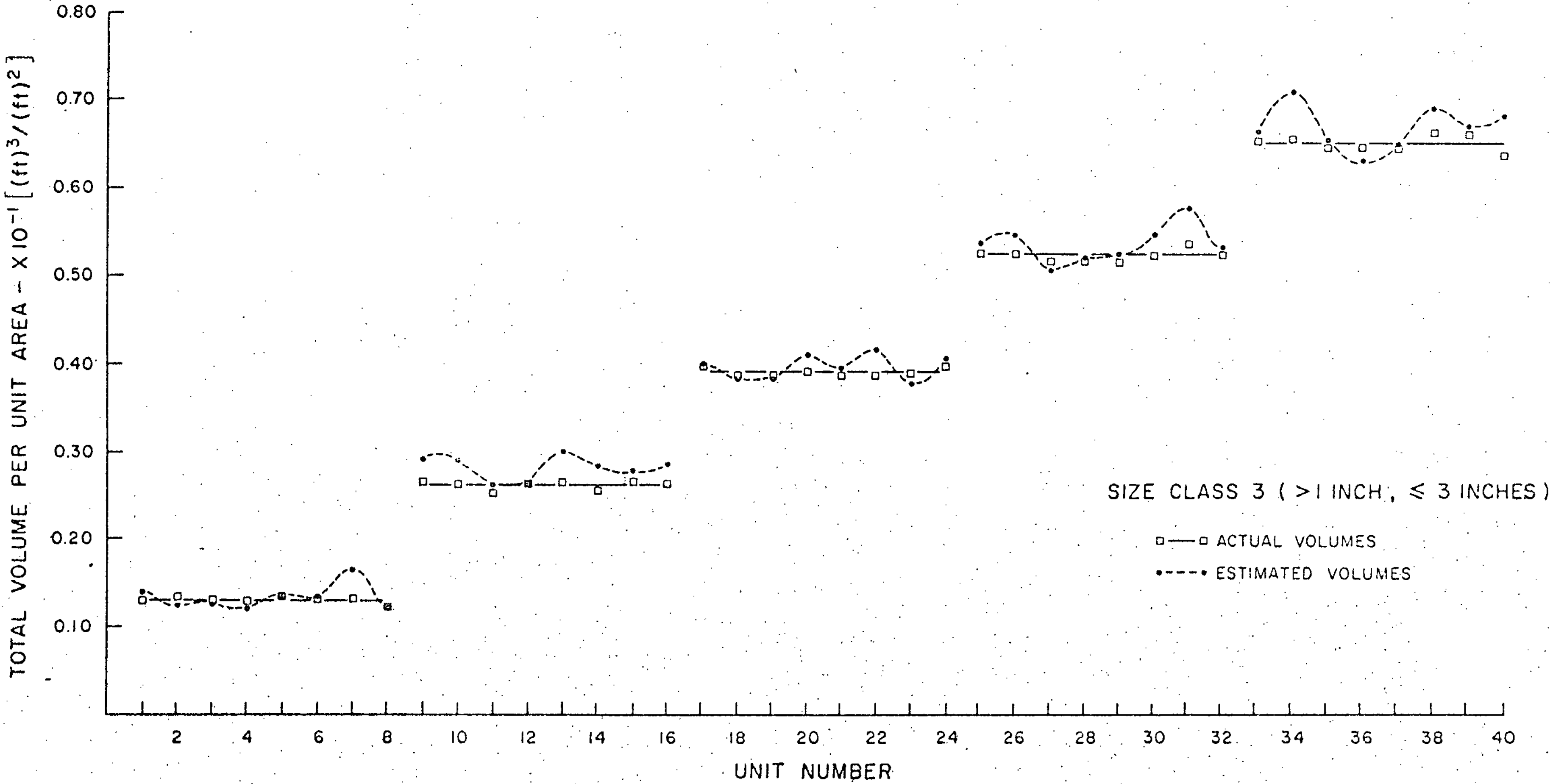


Figure 5. Curves representing actual and estimated computer-generated downed woody particle volumes with respect to size class 3 (> 1 inch, ≤ 3 inches)

$$(19)\quad \frac{\pi}{4}\,\overline{(d^{(i)})}\ \overline{(\sec(\gamma_i))}\cdot K^{(i)} \sum_{\omega=1}^{12} C_\omega \cdot m_j^{(i)}\left(t + \frac{\omega\pi}{6}\right),\ \forall i \varepsilon \{1,2,3\},\ \forall j \varepsilon \{1,\ldots,n\},$$

and where t is arbitrary in $[0,\ 2\pi]$ and

$$C_\omega = \begin{cases} 4, & \text{if } \omega \text{ is odd} \\ 2, & \text{if } \omega \text{ is even} \end{cases}$$

providing of course that the fuel particles of interest are randomly distributed within $S2_j^{(i)}$, $\forall i \varepsilon \{1,2,3\}$, $\forall j \varepsilon \{1,\ldots,n\}$.

Thus far, two estimators $\overline{XP1}^{(i)}$ and $\overline{XP2}^{(i)}$, have been explicitly determined as far as time constraints will permit. $\overline{XP1}^{(i)}$ is an estimator for $p1^{(i)}$, the mean number of fuel particles per unit area with respect to the population of cylinders of common radius $r1^{(i)}$. $\overline{XP2}^{(i)}$ is an estimator for $p2^{(i)}$, the mean volume of fuel particles per unit area with respect to the population of cylinders of common radius $r2^{(i)}$. Confidence intervals for $p1^{(i)}$ and $p2^{(i)}$ have also been produced and they too are as complete as time will permit. If $i \varepsilon \{1,2,3\}$, not only has the estimate $\overline{X2}^{(i)}$ been completely determined but also a complete confidence interval for $p2^{(i)}$ has been presented. The term 'complete' is used here in the sense that all terms comprising the confidence interval of interest can be precisely evaluated in any given downed woody particles population whose lesser components are randomly distributed within the CPIS units selected for sampling.

At the beginning of the section regarding applications to downed woody particles, it was asserted that the results of experiments E_1 and E_2 would yield estimators for a number of specific parameters of interest. This assertion will not be verified.

Let $r1^{(i)}=r2^{(i)}$, $\forall i \varepsilon \{1,\ldots,M\}$. Some common parameters of interest (De Vries, 1973) are: mean total volume per unit area ($p1^{(i)}$), mean number of particles per unit area ($p2^{(i)}$), mean total weight per unit area ($p3^{(i)}$), mean mid-sectional area per unit area ($p4^{(i)}$), mean total length per unit area ($p5^{(i)}$), mean total volume per particle ($p6^{(i)}$), mean mid-sectional area per particle ($p7^{(i)}$), mean mid-diameter per unit area ($p8^{(i)}$), mean length per particle ($p9^{(i)}$), mean mid-diameter per particle ($p10^{(i)}$), mean total surface area per unit area ($p11^{(i)}$), and mean total surface area per particle ($p12^{(i)}$). An estimator for $pk^{(i)}$, $k \neq 7,10$ can be easily expressed in terms of $X1_j^{(i)}$, $X2_j^{(i)}$ and suitable constants, $\forall i \in \{1,\ldots,M\}$. Notice that if k=7 an excellent estimator for $pk^{(i)}$ is $\pi(\mathrm{dbar}^{(i)})^2/4$, where an estimate for $\mathrm{dbar}^{(i)}$ is obtained from suitable tables (Brown, 1973). No confidence interval is really required for $pk^{(i)}$ providing the ith subset is sufficiently small. If a confidence interval is desired, it can be obtained through repeated observations of particle diameters in the ith subset of C. Similarly $p10^{(i)}$ can be very well estimated by $\mathrm{dbar}^{(i)}$, where a confidence interval, if desired, can also be obtained through repeated observations on particle diameters in the i^{th} subset of C. It is left to produce an estimator for $pk^{(i)}$, $k \neq 7,10$.

Estimators for $p1^{(i)}$, $p2^{(i)}$, $p3^{(i)}$, $p4^{(i)}$, $p5^{(i)}$, $p6^{(i)}$, $p8^{(i)}$, $p9^{(i)}$, $p11^{(i)}$ and $p12^{(i)}$ are in order $\overline{XP2}^{(i)}$, $\overline{XP1}^{(i)}$, $\overline{S}^{(i)}$, $\overline{XP2}^{(i)}$ where $\overline{S}^{(i)}$ is the mean specific gravity for particles in the ith subset, $\frac{\pi}{4}(\mathrm{dbar}^{(i)})^2 \cdot \overline{XP1}^{(i)}$, $\overline{XP2}^{(i)}/\frac{\pi}{4}(\mathrm{dbar}^{(i)})^2$), $\overline{XP3}^{(i)}$ ($\overline{XP3}^{(i)} \equiv \sum_{j=2}^{n} X3_j^{(i)}/n$, where $X3_j^{(i)} \equiv X2_j^{(i)}/X1_j^{(i)}$), $\mathrm{dbar}^{(i)} \cdot \overline{XP1}^{(i)}$, $\overline{XP3}^{(i)}/\frac{\pi}{4}\,(\mathrm{dbar}^{(i)})^2$, $\sigma^{(i)}\overline{XP2}^{(i)}$, and $\sigma^{(i)}\overline{XP3}^{(i)}$, where $\sigma^{(i)}$ is the average surface area to volume ratio for the ith subset (ie. $\sigma^{(i)} = 4/\mathrm{dbar}^{(i)}$). Confidence intervals for $pk^{(i)}$, $k \neq 6,9,12$ are obvious from an inspection of their estimators and from previous remarks made when discussing experiments E_1 and E_2. It should be observed that it is possible only to produce a sample coefficient of variation for $pk^{(i)}$, k=6,9,12 unless the distribution of $\overline{XP3}^{(i)}$ can be roughly determined. In passing it is of interest to note that the estimators of $p4^{(i)}$, $i \geq 3$, $p7^{(i)}$, $i \geq 3$, $p8^{(i)}$, $i \geq 3$ and $p10^{(i)}$, $i \geq 3$ can all be alternatively obtained (ie. without using tables for estimates of $dbar^{(i)}$) with a CPIS design simply by applying an argument almost identical to that given in experiment E_1. The only difference here would be of course that particle diameters and cross-sectional areas (for $i \geq 3$) as well as numbers of particles would be of interest.

It has now been shown that it is very possible to attach a meaningful "handle" to the general common point intersect sampling (CPIS) concept. The application of CPIS techniques to downed woody particles provides a vehicle whereby the general theory is mapped into an operational plane. But the value of CPIS procedures on an operational level has not as yet been demonstrated. It remains then to give the CPIS system an actual field test and to compare its performance to that of the most successfully established technique, line intersect sampling (LIS) (Brown, 1971). For reasons previously stated, the field test will be concerned only with lesser downed woody fuels sub-populations. Attention will now be turned to demonstrating the tentative superiority of CPIS over LIS with respect to at least two lesser downed woody particles populations located in areas of logging residue.

CHAPTER III

Field test to evaluate the common point intersect technique

General Discussion

In order to field test the CPIS technique and compare its performance to that of the LIS technique with respect to lesser downed woody particles, two areas of fresh (i.e. less than 1 year old) coastal logging residue were selected for reasons previously cited. The first area was a clearcut comprised of 165 acres of tractor logged residue (see Figure 6). It was situated approximately 8 miles west of Shawnigan Lake which lies 30 miles west of Victoria, British Columbia. The fuel types were predominantly coastal Douglas-fir, western hemlock and western red cedar. The second study area was a 180 acre clearcut of cable logged residue (see Figure 7). It was located approximately 8 miles southwest of Sooke Lake which lies 25 miles southwest of Victoria, British Columbia. The fuel types were similar to those of the first area. Both study areas possessed moderate volumes of residue and reasonably continuous terrains of moderate inclinations (i.e. less than 30° slope). The second study area possessed a noticeable ravine where lesser woody particles accumulations were evident.

Before the two sampling techniques were applied to these areas, each clearcut was examined in order to determine whether the downed particles in the ith subset $\forall i \varepsilon \{1,2,3\}$ were randomly distributed in the sampling space of cylinders of radius $r^{(i)}=r2^{(i)}$. Recall that the formula offered for $X2^{(i)}(S2_j^{(i)})$, $i\varepsilon\{1,2,3\}$, $j\varepsilon\{1,\ldots,n\}$ is valid only when the particles in the ith subset are randomly distributed in $S_j^{(i)}$, $i\varepsilon\{1,2,3\}$. Hence randomness in the first three subsets must be verified in the sampling units to be sampled at least on a tentative basis, before the CPIS technique can be applied to those subsets.

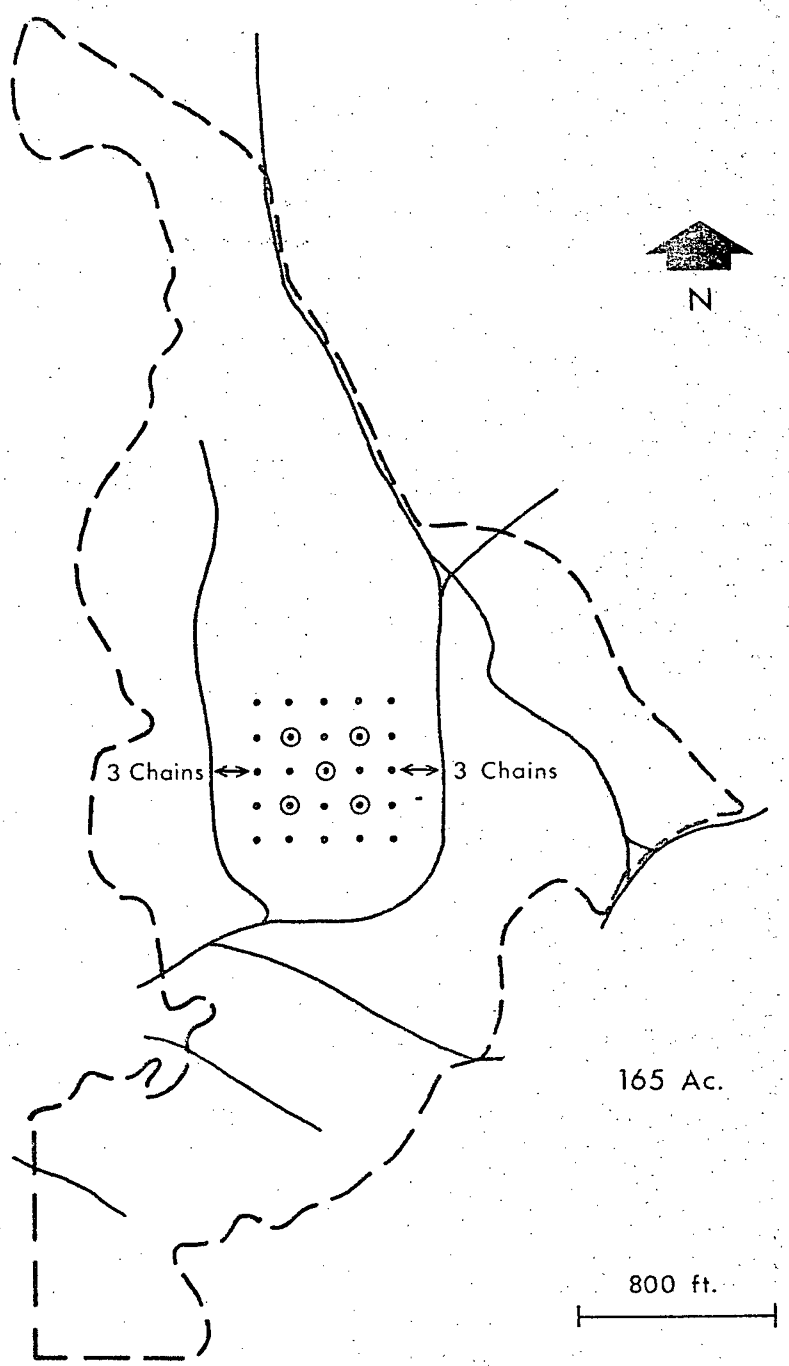


Figure 6. Map of Study Area 1 depicting placement of both the line intersect and common point intersect sampling units.

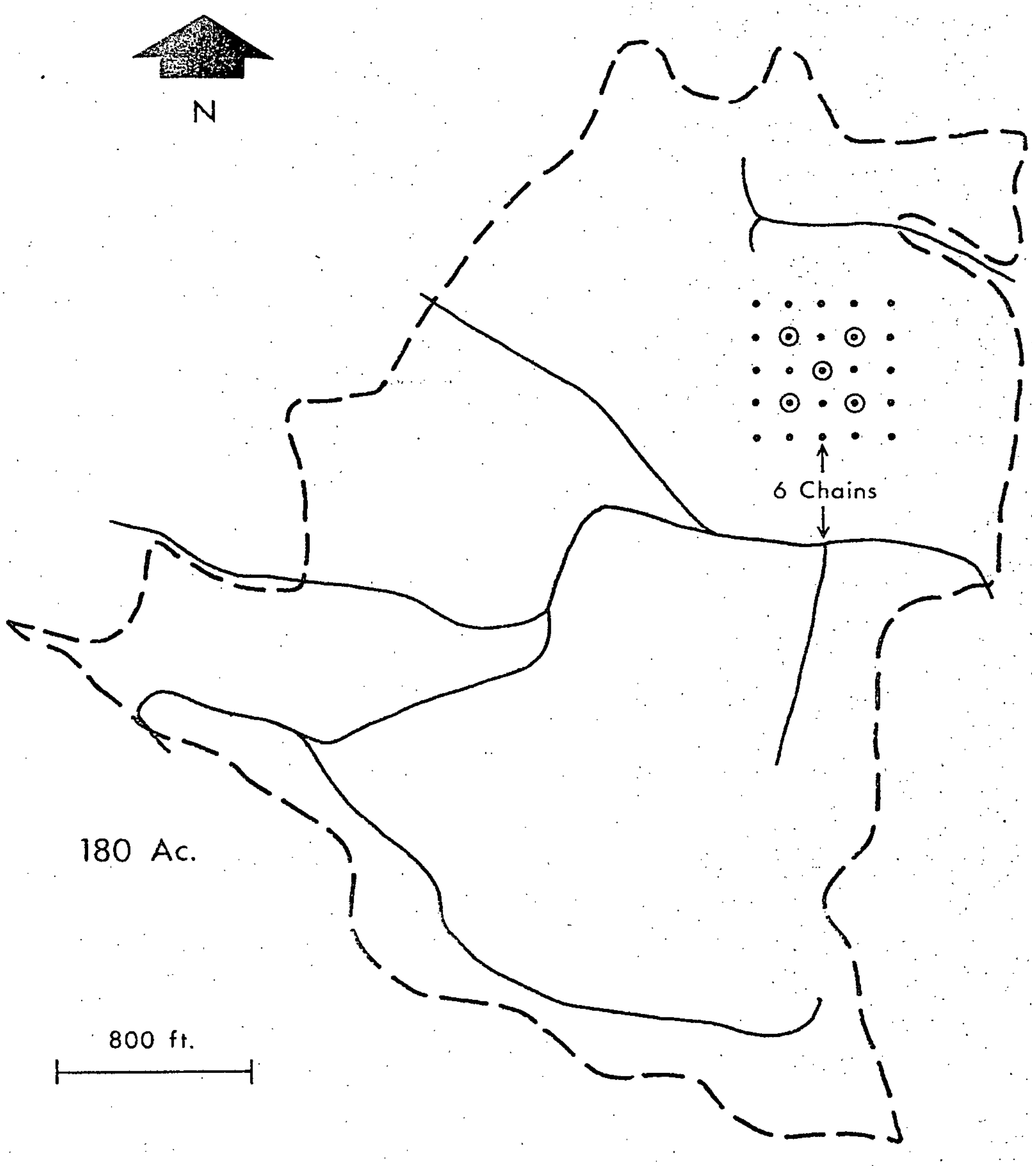


Figure 7. Map of Study Area 2 depicting placement of both the line intersect and common point intersect sampling units.

In order that the particles may be regarded as randomly distributed in the sampling units, they should satisfy two properties. The first is that the random variable defined by angular orientation of particle projection onto a planer section defined by adjacent ground level should have approximately a rectangular distribution on $[0, 2\pi]$ (Mize and Cox, 1968; p.50) with respect to each sampling unit in question. The second is that the random variable defined by distance from the geometric centre of the $i^{\underline{th}}$ sampling unit (of radius $r^{(i)}$) in question to the contained particle geometric centre-point should have approximately a rectangular distribution on $\left[0, r^{(i)}\right]$ $\forall i \varepsilon \{1,2,3\}$, for each sampling unit of radius $r^{(i)}$ selected.

The first property was tentatively verified for the ith subset, $\forall i \varepsilon \{1,2,3\}$ on each study area. Verification was made by constructing a χ^2 goodness-of-fit test at the 5 per cent significance level. On each area data for this test were collected by laying out 5 CPIS units (for each subset) systematically over the area to be sampled. Within each unit 20 transects of common length equal to the radius of the sampling unit were positioned systematically in a unidirectional pattern. Transect lengths for the first, second and third subsets were 8.5, 16.5 and 28.5 feet respectively. Note that these transects lengths are not critical to the test but that the unidirectional sampling design is. Note also that the CPIS units were not laid out over the entire area but rather only over the area to be sampled. This sampling region is selected to act as a representative fuel cell for the entire area. Size and location of the fuel cell are largely at the discretion of the investigator. It need only be said here that a 6 chain by 6 chain square was selected on each clearcut where no portion of either 3.6 acre fuel cell lay within 2.5 chains of any access road. Choice of the cells was based on the subjective decision that each 3.6 acre square proportionately reflected the major characteristics of interest on its particular area. Continuing then if angular orientation is truly random in the j^{th} sampling unit, it can be shown that the random variable $(\theta)_j$ defined by the intersection angle between the particle major axis and transect with respect to the jth unit has probability density function f_{θ_j} where:

$$(20)\quad f_{\theta_j}(u) = \begin{cases} \sin u, & u \varepsilon (0, \pi/2] \\ 0, & \text{elsewhere} \end{cases} \quad \forall j \varepsilon \{1, \ldots, 5\} \quad \text{(Van Wagner, 1968)}$$

Therefore under the assumption of randomness with respect to angular orientation, it is expected that the statistic:

$$\sum_{k=1}^{20} \left\{ (f_{o,k} - f_{e,k})^2 / f_{e,k} \right\}, \text{ where}$$

$f_{o,k}$ = observed frequency of angles in k^{th} partition

$f_{e,k}$ = expected frequency of angles in k^{th} partition, $\forall k \varepsilon \{1, \ldots, 20\}$

would have a χ^2 distribution with 19 degrees of freedom where $f_{e,k}$ is evaluated using f_θ; $\forall k \varepsilon \{1, \ldots, 20\}$. The scheme used here for defining angular class intervals is based on .05 proportions of area under the sine curve. Hence:

$$\sum_{k=1}^{20} \left\{ (f_{o,k} - f_{e,k})^2 / f_{e,k} \right\} = \frac{1}{f_e} \sum_{k=1}^{20} (f_{o,k} - f_e)^2, \text{ where } f_e = f_{e,k}, \quad \forall k \varepsilon \{1, \ldots, 20\} \text{ by design.}$$

All sampling units in the three subsets constructed for each area passed the χ^2 goodness-of-fit test at the .05 significance level. Observed class particle frequencies for 6 typical CPIS units are presented in tabular form in Table 2.

It is difficult to construct a practical test to either support or contradict the assertion that the random variable defined by distance from the geometric centre of the sampling unit (of radius $r^{(i)}$) in question to the contained particle geometric centre point has a rectangular

Table 2. Intersection angles of lesser downed woody particles in the study areas.

Class Intervals	Observed Frequency					
	Size Class 1		Size Class 2		Size Class 3	
	Area 1	Area 2	Area 1	Area 2	Area 1	Area 2
degrees	numbers					
0-18.20	124	94	31	18	10	18
18.21-25.83	156	73	48	29	6	25
25.84-31.80	125	96	36	25	18	27
31.81-36.87	120	100	34	35	8	29
36.88-41.40	122	79	28	19	14	17
41.41-45.58	155	101	32	40	9	26
45.59-49.45	121	95	30	28	17	14
49.46-53.13	156	75	44	33	10	14
53.14-56.63	152	101	34	22	19	27
56.64-60.00	125	98	47	27	14	13
60.01-63.27	146	102	33	34	11	29
63.28-66.42	154	103	45	28	15	17
66.43-69.52	136	79	48	21	17	30
69.53-72.53	144	78	34	23	8	18
72.54-75.52	123	80	27	33	13	25
75.53-78.47	132	71	47	36	15	18
78.48-81.37	151	95	42	35	14	26
81.38-84.25	140	98	36	35	8	28
84.26-87.13	160	81	50	38	19	19
87.14-90.00	122	69	45	24	9	28

distribution on $\left[0, r^{(i)}\right]$, $\forall i \in \{1,2,3\}$, for each sampling unit of radius $r^{(i)}$ selected. Recall that under the assumption of randomly distributed particles in the CPIS units the random variable defined by (12.) was found to have a normal distribution with mean 0. Now it has been shown that it is tentatively valid to assume that the random variable defined by angular orientation (as previously specified) has a rectangular distribution on $\left[0, 2\pi\right]$. Therefore it can be seen that if the above assertion regarding the random variable defined by distance (as previously specified) is true, the random variable defined by (12.) should have approximately a normal distribution with mean 0. Using this logic, a parametric statistical two-sided hypothesis test comparing a mean against 0 (where the population variance is assumed to be unknown) was set up at the .05 significance level for each study area. Five widely spaced CPIS units were selected systematically on each area $\forall i \in \{1,2,3\}$. Note that the comments made earlier justifying the use of systematic sampling designs apply equally here. Now within each unit, 24 evenly spaced transects were laid out and the image of t under $R_j^{(i)}$ (see (12.)) was found for each $t \in \{\frac{\pi i}{12}\}_{i=1}^{24}$, where t=0 was chosen randomly. This process led to the computation of five sample means and five sample variances for each $i \in \{1,2,3\}$ with respect to each study area. This data is presented in Table 3 in which 'orientation random variable' is to be identified with $R_j^{(i)}$. For each $j \in \{1,\ldots,5\}$ and for each $i \in \{1,2,3\}$ the jth sampling unit yielded a sample value which lay outside the rejection region constructed under the null hypothesis which is identified with the assumption that the random variable defined by distance from the geometric centre of the jth sampling unit (of radius $r^{(i)}$) to the contained particle geometric centre point has a rectangular

Table 3. Sample means and standard deviations of the orientation random variable for lesser downed woody particles in the study areas.

Unit Number	Sample Mean						Sample Standard Deviation					
	Class 1		Class 2		Class 3		Class 1		Class 2		Class 3	
	Area 1,	2	Area 1,	2	Area 1,	2	Area 1,	2	Area 1,	2	Area 1,	2
1	.072	.027	.096	.092	.0978	.108	.183	.169	.230	.228	.283	.302
2	.023	.005	.064	.021	.110	.124	.137	.133	.170	.159	.264	.296
3	.013	.049	.099	.068	.138	.116	.157	.195	.237	.221	.331	.285
4	.044	.039	.092	.080	.113	.118	.178	.106	.220	.194	.269	.282
5	.015	.059	.077	.025	.135	.137	.201	.206	.193	.206	.324	.325

distribution on $\left[0, r^{(i)}\right]$, $\forall i \varepsilon \{1,2,3\}$, $\forall j \varepsilon \{1,\ldots,5\}$.

Combining the results from the two above statistical tests, it was considered tentatively valid to apply the CPIS technique to the above CPIS units on each of the two study areas with respect to the first three downed woody particle subsets.

Field work undergone with respect to the line intersect technique

Within each fuel cell 25 line segments of common length 8.5 ft were systematically placed uniformly over the entire fuel cell using an equidistant grid pattern (see Figures 6 and 7). The orientation of each transect was determined randomly. Random transect orientation is not really necessary here. Note that it requires no more time to implement than does unidirectional transect orientation. Justification of the use of a systematic sampling design with equidistant grid pattern has been previously made. It remains to justify both the number and common length of the line segments used.

The use of 25 transects will be explained first. Consider the well-known formula for an infinite population of sampling units where the random variable of interest is normally distributed:

(21) $N = (CV)^2(t_{1-\alpha/2;\,N-1})^2/Z^2$

where N = number of sampling units

CV = coefficient of sample variation

$1-\alpha$ = level of statistical inference

$t_{1-\alpha/2;\,N-1}$ = Student's t value at the $(1-\alpha/2)$ level with (N-1) degrees of freedom

Z = degree of precision (expressed as a proportion of the sample mean)

An inspection of the behavior of the t distribution and application of the Central Limit Theorem reveal almost immediately that the minimum number of samples (N_{min}) required to meet precision level Z can be expressed as: (22) $N_{min} = 3.84(CV)^2/Z^2$, providing N_{min} is sufficiently large (i.e. $N_{min} \geq 25$) regardless of the distribution of the random variable of interest. It follows that if CV can be estimated from a preliminary sample, then N_{min} can be expressed roughly as a function of Z. With respect to downed woody particles, it usually is of interest to determine N_{min}

for Z within a small neighborhood of 0.15 (Howard and Ward, 1972; Brown, 1973). Hence for 15 percent precision, the following formula holds:

$$(23)\quad N_{min} \doteq 170.67 \left[(CV)_{ps}\right]^2, \text{ providing } N_{min} \text{ is sufficiently large (i.e. } N_{min} \geq 25)$$
$$= A \cdot \quad (CV)_{ps}{}^2$$

where $(CV)_{ps}$ = coefficient of preliminary sample variation

The validity of (23.) is crucial to the comparison of LIS and CPIS since (23.) indicates the minimum number of samples required to meet 15 percent precision at the 95% confidence level. It may be argued that (23.) leads to erroneous results unless the coefficient of sample variation stabilizes for sample sizes greater than the preliminary sample size. Unless the lesser downed woody particles within the LIS units are pathologically distributed, it is intuitively logical that the ratios of the sample means to the sample variances should stabilize beyond some minimum sample size usually quoted as 25 (Freund and Williams, 1958; Ehrenfeld and Littauer, 1964; Husch, Miller and Beers, 1972). No rigorous proof of this conjecture is offered in the literature because (23.) is traditionally regarded as only an approximation of N_{min}, not as a precise replacement. Unfortunately a thorough investigation of just how precise (23.) is when applied to downed woody fuels was not feasible. Therefore the validity of (23.) will have to rest temporarily upon statistical intuition. Note that a more precise knowledge of the adequacy of (23.) is not vital to the comparison between LIS and CPIS. For example if $(CV)_{ps}$ is smaller than CV with respect to a particular subset, N_{min} should be larger than the estimate given by (23.). Hence for comparison purposes (23.) works in favour of LIS in this case. Alternatively if $(CV)_{ps}$ is larger than CV with respect to a particular subset, the use of the oversized $(CV)_{ps}$ will mostly likely be offset by the fact that (A)in (23.) is at least 4% larger than it should be. Even if (23.) overestimates N_{min} for a particular subset, this would have to be very large to have a serious effect upon the sampling time required because the average measurement time per LIS point is relatively small. Thus (23.) is sufficiently credible for comparing LIS with CPIS. A preliminary sample size of 25 was chosen in order than (23.) be valid when applied to the comparison of LIS and CPIS. An explanation of the common transect length of 8.5 ft will now be given.

In previous studies, optimum lengths for LIS transects applied to lesser downed particles have been suggested as: 6.8 ft for material less than 3 inches in diameter at intersection (Brown, in prep), 6.56 ft for particles less than 1 inch in diameter at intersection and 9.84 ft for particles from 1 inch to about 4 inches in diameter at intersection (Brown, in prep). Although no statistical justification was given to support the selection of these lengths, they have proven satisfactory for previous investigators. The choice of minimum adequate transect length was difficult to make without a thorough analysis to evaluate the performance of the LIS technique when applied with different transect lengths. It was again necessary to resort to intuition and experience supported with all relevant information available in the literature. The transect length chosen was 8.5 ft which agrees very well with Brown, in prep. If the transect length was selected greater than or equal to 25 ft and the random variable of interest was an average per unit area, (23.) may be regarded as reasonably valid in many cases without the restriction that $N_{min} \geq 25$. Having completely specified and justified the lengths, number and locations of the LIS units on each area, attention is now turned to the data collected at each LIS unit and the calculations performed upon that data.

At each LIS unit, every lesser downed woody particle whose central axis (and at least one of whose edges) intersected the 8.5 ft transect segment was classified into one of the three previously defined subsets using a go-no-go gauge (Brown, in prep.; p. 5)(see Figure 8). Also the times taken to locate each LIS point and to perform necessary particle measurements at each point were recorded.

The device used to define each LIS transect segment consisted of two wooden stakes approximately 3 ft long, each of which was sharpened at one end for the purpose of easy insertion into the ground. Attached to each stake was a small metal ring with a wingnut to which was fastened one end of the 8.5 ft cord representing the transect. These wingnut devices permitted the LIS transect segment to be adjusted for fuel depth and slope. At each LIS point, the transect segment was oriented parallel to the adjacent ground level. This procedure is well-established for light to moderate slash (Brown, in prep).

Two sets of formulas were used to convert the preliminary LIS raw data into values required for the comparison of the LIS and CPIS techniques. The first set of formulas which converted particle intercept counts into average total volumes per unit area is presented below:

$$(24)\quad VOL_j^{(i)} = \left\{\pi^2 \overline{(d^{(i)})^2}\,\overline{(\sec(\gamma i))}/68\right\} K_j^{(i)}, \text{ (Brown, 1973),}$$

$$\forall i \varepsilon \{1,2,3\},\ \forall j \varepsilon \{1,\ldots,25\}, \text{ where}$$

$K_j^{(i)}$=number of particle intersections (as previously defined) in the jth LIS unit, $j \varepsilon \{1,\ldots,25\}$ with respect to the ith subset, $i \varepsilon \{1,2,3\}$.

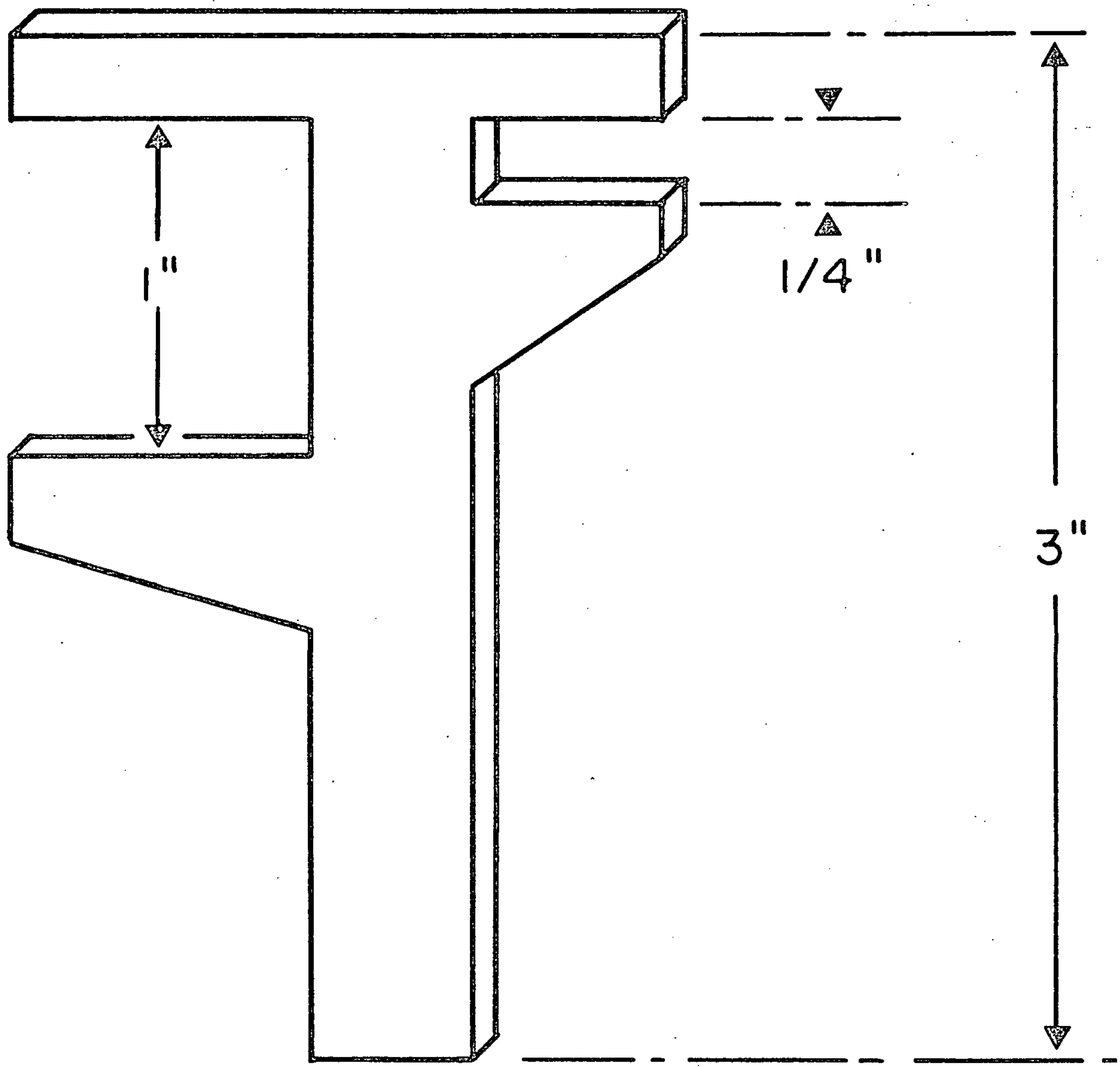


Figure 8. 'Go-no-go' gauge used to determine the diameter size class of each intersecting particle under 3" in diameter at initial point of intersection.

$VOL_j^{(i)}$ = average total volume per unit area in the jth LIS unit, $j\varepsilon\{1,\ldots,25\}$ with respect to the ith subset, $i\varepsilon\{1,2,3\}$, $((ft)^3/(ft)^2)$.

$\left\{\left\{K_j^{(i)}\right\}_{j=1}^{25}\right\}_{i=1}^{4}$ and $\left\{\left\{VOL_j^{(i)}\right\}_{j=1}^{25}\right\}_{i=1}^{4}$ (where $K_j^{(4)} = \sum_{i=1}^{3} K_j^{(i)}$, $\forall j\varepsilon\{1,\ldots,25\}$ and $VOL_j^{(4)}$ is similarly defined) obtained for each study area are presented in tabular form in Table 4. Values computed from (24.) were then used to calculate $\left\{(CV)_{ps}^{(i)}\right\}_{i=1}^{4}$ for each study area. The entries in this sequence were then inserted into (23.) to arrive at the values offered in Table 5. Note in Table 5 that $384\cdot(CV)^2$ is simply the right-hand side of (23.), evaluated at Z=.10. It is now evident that (24.) combined with (22.) (where CV may be identified with $(CV)_{ps}$) will yield the minimum number of LIS units required to obtain a population mean estimate with a specified degree of precision. The second set of formulas combines the results of (24.) and (22.) with the raw LIS sampling time data to obtain the minimum time required to obtain a population mean estimate with a specified degree of precision. This second set of formulas whose derivation is obvious is presented below:

$$(25)\quad T2^{(i)} = D^{(i)}\,\overline{(C^{(i)})} + U^{(i)}\left[\overline{P} + \overline{(M^{(i)})}\right],\ i\varepsilon\{1,2,3,4\}$$

where $T2^{(i)}$ = minimum total sampling time required (hours) using the LIS technique for the ith subset, $i\varepsilon\{1,2,3,4\}$.

$D^{(i)}$ = total distance walked to obtain particle measurements (chains) $= 6\,(U^{(i)}-1)/\sqrt[+]{U^{(i)} + \delta^{(i)}} + 1$, where

$U^{(i)}$ = the minimum number of LIS units required for the ith subset, $i\varepsilon\{1,2,3,4\}$

$\delta^{(i)} = \min\left\{Z\varepsilon\mathbb{Z}^+ : \sqrt[+]{U^{(i)} + Z}\ \varepsilon\,\mathbb{Z}^+\right\}$, where $\mathbb{Z}^+$ represents the set of all positive integers.

Table 4. Field particle intercept counts and corresponding volume estimates for the line intersect sampling units.

Unit Number	Fuel Component Count								Fuel Volume Estimate							
	Class 1		Class 2		Class 3		Class 4 (Total)		Class 1		Class 2		Class 3		Class 4 (Total)	
	Area 1, 2		Area 1, 2		Area 1, 2		Area 1, 2		Area 1, 2		Area 1, 2		Area 1, 2		Area 1, 2	
									$(feet)^3/(feet)^2 \times 10^{-3}$							
1	44	195	2	15	1	3	47	213	0.73	3.24	0.63	4.74	3.13	9.39	4.49	17.37
2	33	126	6	6	1	1	40	133	0.55	2.09	1.90	1.90	3.13	3.13	5.58	7.12
3	187	87	26	16	5	4	218	107	3.10	1.44	8.22	5.06	15.65	12.52	26.97	19.02
4	175	187	16	12	3	3	194	202	2.91	3.10	5.06	3.79	9.39	9.39	17.36	16.28
5	151	75	14	8	2	2	167	85	2.51	1.25	4.42	2.53	6.26	6.26	13.19	10.04
6	105	210	10	11	1	7	116	228	1.73	3.49	3.16	3.48	3.13	21.91	8.02	28.88
7	227	102	27	13	4	5	258	120	3.77	1.69	8.53	4.11	12.52	15.65	24.82	21.45
8	157	153	13	14	2	9	172	176	2.61	2.54	4.11	4.42	6.26	28.17	12.98	35.13
9	77	65	8	11	1	11	86	87	1.28	1.08	2.53	3.48	3.13	34.43	6.94	38.99
10	168	192	12	17	3	4	183	213	2.79	3.19	3.76	5.37	9.39	12.52	15.94	21.08
11	186	87	19	13	3	1	208	101	3.09	1.44	5.95	4.11	9.39	3.13	18.43	8.68
12	78	42	14	4	2	3	94	49	1.29	.70	4.38	1.26	6.26	9.39	11.93	11.35
13	66	59	5	20	1	4	72	83	1.10	.98	1.57	6.32	3.13	12.52	5.80	19.82
14	28	60	22	9	5	1	55	70	.46	1.00	6.89	2.84	15.65	3.13	23.00	6.97

Cont'd.....

Table 4 cont'd....

Unit Number	Fuel Components Count: Class 1		Class 2		Class 3		Class 4 (Total)		Fuel Volume Estimate, (feet)^3/(feet)^2 X 10^{-3}: Class 1		Class 2		Class 3		Class 4 (Total)	
	Area 1	Area 2	Area 1	Area 2	Area 1	Area 2	Area 1	Area 2	Area 1	Area 2	Area 1	Area 2	Area 1	Area 2	Area 1	Area 2
15	18	230	5	16	3	3	26	249	.30	3.82	1.57	5.06	9.39	9.39	11.26	18.27
16	51	57	9	16	2	1	62	74	.85	.95	2.82	5.06	6.26	3.13	9.93	9.14
17	68	122	14	29	6	13	88	164	1.13	2.03	4.38	9.16	18.78	40.69	24.29	51.88
18	193	93	40	21	9	4	242	118	3.20	1.54	12.52	6.64	28.17	12.52	43.89	20.70
19	237	115	21	20	4	4	262	139	3.93	1.91	6.57	6.32	12.52	12.52	23.02	20.75
20	218	90	23	20	4	10	245	120	3.62	1.49	7.20	6.32	12.52	31.30	23.34	39.11
21	130	98	19	22	3	10	152	130	2.16	1.63	5.95	6.95	9.39	31.30	17.50	39.88
22	75	130	4	15	0	6	79	151	1.25	2.16	1.25	4.74	0	18.78	2.50	25.68
23	7	92	4	21	0	5	11	118	.12	1.53	1.25	6.64	0	15.65	1.37	23.82
24	119	69	9	8	2	7	130	84	1.98	1.15	2.82	2.53	6.26	21.91	11.06	25.59
25	61	254	12	32	7	7	80	293	1.01	4.22	3.76	10.11	21.91	21.91	26.68	36.24

Table 5. Relevant statistics for the line intersect field sampling units.

Area	Class	Mean Volume	Standard Deviation	Coefficient of Variation (C.V.)	$170.67(CV)^2$	$384(CV)^2$
		$(ft)^3/(ft)^2$ X 10^{-3}	$(ft)^3/(ft)^2$ X 10^{-3}			
1	1	1.90	1.24	0.65	72	162
1	2	4.45	2.84	0.64	70	157
1	3	9.26	6.84	0.74	93	210
1	4	15.61	9.84	0.63	68	152
2	1	1.99	0.99	0.50	43	96
2	2	4.92	2.10	0.43	32	71
2	3	16.03	10.30	0.64	70	157
2	4	22.94	11.83	0.52	46	104

(This subformula is clear from the fact that a square grid pattern superimposed uniformly on a 3.6 acre fuel cell is used for each LIS design).

$\overline{(C^{(i)})}$ = average chain walking time per unit distance (hrs./chain) with respect to the ith subset, $i\varepsilon\{1,2,3,4\}$

$\bar{p}$ = average pin placement time per sampling unit (hrs./LIS unit)

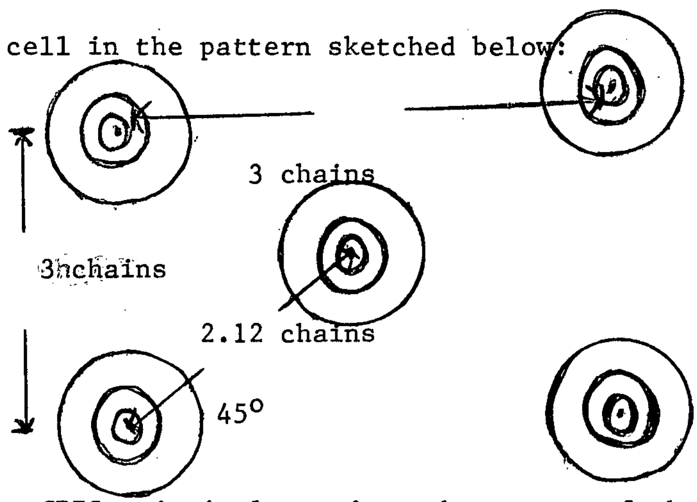
$\overline{(M^{(i)})}$ = average particle measurement time per LIS unit (hrs./LIS unit) with respect to the ith subset, $i\varepsilon\{1,2,3,4\}$.

Examples of minimum total LIS times required to obtain population mean estimates of specified precision levels are given in Table 8.

A detailed discussion of the LIS field work which was undergone has been presented. Also formulas converting the LIS field data into values which can be used to compare the LIS and CPIS techniques have been derived and discussed. Hence it is now appropriate to discuss the next topic of interest, namely the CPIS field work which was undertaken.

<u>Field work undergone with respect to the Common Point Intersect Sampling Technique</u>

Five sets of concentric circles were systematically placed within each 3.6 acre fuel cell in the pattern sketched below:



Note that the centre CPIS unit is located at the center of the fuel cell. Each set of circles consisted of three circles of radii 8.5, 16.5 and 28.5 ft., in which fuel measurements were made for the first, second and third subsets respectively. Inscribed within each of the above sets of circles were 12 radius transect segments, one every 30°, with the location of the first transect randomly selected. These radius segments were easily located using a compass and yardstick.

Five CPIS units were selected here only on a tentative basis. If computations using data from these preliminary CPIS units revealed that 5 samples were not sufficient to obtain a population mean estimate within a small neighbourhood of 15 percent precision, then more CPIS units would be collected using a different systematic CPIS design. The selection of radii for the CPIS units with respect to each subset has been previously justified. The use of 12 transects is required in order that the image of t under $h2_j^{(i)}$ (see (18.)) can be computed for some randomly selected $t \varepsilon [0,2\pi]$, $j \varepsilon \{1,\ldots,5\}$, $i \varepsilon \{1,2,3\}$. It is $h2_j^{(i)}(t)$ for the chosen t which will serve as the very good estimate of $V_j^{(i)}/\pi(r2^{(i)})^2$ (see (17.)). Having both specified and justified the size, shape and tentative number and locations of the CPIS units on each study area, attention will now be focussed upon the data collected at each CPIS unit and the calculations performed on that data.

Consider any of the 12 transects radiating from the center of the jth CPIS unit in which particle measurements are to be performed with respect to the ith subset. Every downed woody particle belonging to the ith subset whose central axis (and at least one of whose edges) intersected the transect was counted. Determination of whether or not a particular intersecting particle was a member of the ith subset was made using the go-no-go gauge previously described. This process was repeated for all 12 transects $\forall j \varepsilon \{1,\ldots,5\}$, $\forall i \varepsilon \{1,2,3\}$.

The tallies obtained are expressed in tabular form in Table 6. Also the times taken to locate each CPIS unit and to perform necessary particle measurements at each unit were recorded.

Each of the 12 transects inscribed within every CPIS unit was defined by a device almost identical to that used in the section on line intersect field sampling. The only difference here is the cord which now is 28.5 ft long and is marked with flagging tape at both 8.5 ft and 16.5 ft.

As in LIS, two sets of formulas were used to convert the preliminary CPIS raw data into values required for the comparison of the LIS and CPIS techniques. Analogous to the first set of formulas used in LIS, the first set of formulas used in CPIS converted particle intercept counts into average total volumes per unit area:

$$(26)\quad V_j^{(i)}/\pi(r2^{i})^2 = h2_j^{(i)}(t) = \sum_{\omega=1}^{12} C_\omega \hat{g}2_j^{(i)}\left(t + \frac{\omega\pi}{6}\right),\ t \text{ arbitrary in } [0,\ 2\pi]$$

and where

$V_j^{(i)}/\pi(r2^{(i)})^2$ = average total volume per unit area in the jth CPIS unit $j \varepsilon \{1,\ldots,5\}$, with respect to the i^{th} subset, $i \varepsilon \{1,2,3\}$ $((ft)^3/(ft)^2)$

$$C_\omega = \begin{cases} 4, & \text{if } \omega \text{ is odd} \\ 2, & \text{if } \omega \text{ is even} \end{cases}$$

and $$\hat{g}2_j^{(i)}(t) = \frac{\pi}{4} \cdot \overline{(d^{(i)})^2} \cdot \overline{(\sec(\gamma i))} \cdot K^{(i)} \cdot m_j^{(i)}(t),\ \forall t \varepsilon [0, 2\pi]$$

Note here that:

$$\left.\begin{aligned} \overline{(d^{(1)})} &= 0.0091 \text{ ft.} \\ \overline{(d^{(2)})} &= 0.0439 \text{ ft.} \\ \overline{(d^{(3)})} &= 0.1400 \text{ ft.} \end{aligned}\right\} \quad \text{(Brown, 1973)}$$

Table 6. Field particle intercept counts for the common point intersect sampling units.

Unit Number	Transect Number	Fuel Component Count							
		Size Class 1		Size Class 2		Size Class 3		Size Class 4 (Total)	
		Area 1	Area 2	Area 1	Area 2	Area 1	Area 2	Area 1	Area 2
1	1	41	158	26	32	11	21	78	211
1	2	34	110	25	20	12	16	71	146
1	3	47	69	17	24	4	12	68	105
1	4	67	81	27	31	16	13	110	125
1	5	75	49	87	23	21	20	183	92
1	6	91	57	24	39	14	15	129	111
1	7	75	97	13	17	8	20	96	134
1	8	106	92	22	34	13	21	141	147
1	9	79	94	34	27	8	14	121	135
1	10	151	83	28	17	12	18	191	118
1	11	135	147	32	18	12	19	179	184
1	12	68	128	27	32	13	20	108	180
2	11	189	93	28	41	7	19	224	153
2	2	78	38	54	27	8	12	140	77
2	3	61	39	32	11	7	20	100	70
2	4	68	26	12	20	11	15	91	61
2	5	144	49	17	19	7	22	168	90
2	6	101	28	21	29	11	21	133	78
2	7	79	47	23	33	11	19	113	99
2	8	137	62	16	25	12	13	165	100
2	9	343	81	56	28	24	20	423	129
2	10	118	64	50	34	9	18	177	116

con't.....

Table 6 con't.....

Unit Number	Transect Number	Fuel Component Count							
		Size Class 1		Size Class 2		Size Class 3		Size Class 4 (Total)	
		Area 1	Area 2	Area 1	Area 2	Area 1	Area 2	Area 1	Area 2
2	11	212	51	81	26	5	24	298	101
2	122	148	80	39	29	9	25	196	134
3	1	61	71	38	27	8	33	107	131
3	2	112	75	29	19	17	17	158	111
3	3	78	29	41	14	16	12	135	105
3	4	72	51	35	14	11	12	118	77
3	5	134	44	32	17	16	11	182	72
3	6	95	43	48	17	9	10	152	70
3	7	54	48	22	21	3	13	79	82
3	8	41	54	20	19	10	18	71	91
3	9	44	124	22	38	9	15	75	177
3	10	51	30	37	19	13	14	101	63
3	11	82	31	18	21	12	23	112	75
3	12	156	33	24	24	13	32	193	89
4	1	88	117	16	28	5	31	109	176
4	2	90	151	47	24	11	24	148	199
4	3	128	70	27	13	11	22	166	105
4	4	89	118	22	27	10	23	121	168
4	5	50	112	15	20	10	17	75	149
4	6	149	177	26	31	10	28	185	236
4	7	112	97	17	32	4	15	133	144
4	8	97	81	17	25	7	22	121	128

con't.....

Table 6 con't.....

Unit Number	Transect Number	Fuel Component Count							
		Size Class 1		Size Class 2		Size Class 3		Size Class 4 (Total)	
		Area 1	Area 2	Area 1	Area 2	Area 1	Area 2	Area 1	Area 2
4	9	42	31	21	26	11	14	74	71
4	10	554	122	26	26	8	12	88	160
4	11	131	59	49	29	5	20	185	108
4	12	138	134	22	43	6	19	166	196
5	1	177	109	30	28	18	9	225	146
5	2	130	81	21	21	7	14	158	116
5	3	208	28	45	10	11	12	264	50
5	4	225	23	38	20	7	17	270	60
5	5	255	29	32	21	10	20	297	70
5	6	442	41	36	21	10	23	488	85
5	7	310	45	36	18	19	16	365	79
5	8	185	24	39	7	11	5	235	36
5	9	135	54	22	37	13	31	170	122
5	10	135	61	23	34	14	22	172	117
5	11	138	38	20	22	8	24	166	84
5	12	95	91	24	26	10	25	129	142

and

$$\left.\begin{array}{l} \overline{(\sec(\gamma_1))} = 1.40 \\ \overline{(\sec(\gamma_2))} = 1.13 \\ (\sec(\gamma_3)) = 1.10 \end{array}\right\} \quad \text{(Brown, 1973)}$$

Also note that $K^{(i)}$ is evaluated in (13.), $\forall i \varepsilon \{1,2,3\}$ and that $m_j^{(i)}(t)$ is defined in (6.) $\forall i \varepsilon \{1,2,3\}$, $\forall j \varepsilon \{1,\ldots,5\}$ and $\forall t \varepsilon [0,2\pi]$.

$\left\{\left\{V_j^{(i)}/\pi(r2^{(i)})^2\right\}_{j=1}^{5}\right\}_{i=1}^{3}$ and $\left\{\sum_{i=1}^{3} V_j^{(i)}/\pi(r2^{(i)})^2\right\}_{j=1}^{5}$ were obtained for each study area and are presented in Table 7. (26.) was then combined with a special case of (21.) ($N \geq 3$) in order to obtain the minimum number of CPIS units corresponding to at least one sample mean of precision level as close to 15% as possible. Note that a sample size of two was not considered sufficient. This is done to reduce the probability that the spacing between sampling units coincides with any pattern of particles population variation not immediately apparent. Note also that (21.) is meaningful with respect to CPIS because the random variable of interest here is an average taken over a sufficiently large sampling area (see page (20.)). Now once a satisfactory number (N) of CPIS units has been obtained for some $\mathcal{Z}$ close to 15%, the second set of formulas combines the value obtained for N with the raw CPIS time data to obtain the minimum time required to obtain a population mean estimate with degree of precision ($\mathcal{Z}$). This second set of formulas (25.) is almost completely analogous to the second set of formulas (see (25.)) used in LIS. The only exception here is that now:

$$(27) \qquad D^{(i)} \left\{\begin{array}{l} = 5.50 \text{ chains if three samples are used} \\ = 7.24 \text{ chains if four samples are used} \\ = 11.12 \text{ chains if five samples are used} \end{array}\right.$$

Table 7. Field particle volume estimates for the common point intersect sampling units.

Unit Number	Fuel Volumes Estimate							
	Class 1		Class 2		Class 3		Class 4 (Total)	
	Area 1	Area 2	Area 1	Area 2	Area 1	Area 2	Area 1	Area 2
	$(feet)^3/(feet)^2 \times 10^{-3}$							
1	1.32	1.52	4.58	4.33	11.34	15.79	17.24	21.64
2	2.06	0.85	5.53	4.33	9.16	16.65	16.75	21.83
3	1.34	0.86	4.98	3.22	10.63	15.84	16.95	19.92
4	1.44	1.82	4.15	4.45	7.59	18.98	13.18	25.25
5	3.19	0.84	4.87	3.51	9.97	16.40	18.03	20.75

Examples of minimum total CPIS times required to obtain population mean estimates of specified precision levels are given in Table 8.

Before proceeding to the analysis of the field test data, it is appropriate to briefly summarize the process by which Table 8 was obtained. Consider any $i \varepsilon \{1,2,3,4\}$ with respect to each study area. First formula (26.) was used to derive $\left\{V_j^{(i)}/\pi(r2^{(i)})^2\right\}_{j=1}^{5}$, $i \neq 4$. This sequence or a suitable subsequence or a suitable summation thereof was then used in (21.) with $\alpha=0.05$ to derive an $N^{(i)} \geq 3$ such that at least one of its corresponding sample means had precision $(Z^{(i)})$ (see column 5 of Table 8) within some small neighborhood of 15%. The $N^{(i)}$ so obtained was inserted together with raw CPIS time data into (25.) (see (25.) and (27.)) to obtain $T1^{(i)}$ (see column 4 of Table 8), where $T1^{(i)}$ refers to the minimum total CPIS time required to obtain a population mean estimate of precision $Z^{(i)}$. Next formula (24.) was used to derive $\left\{VOL_j^{(i)}\right\}_{j=1}^{25}$. This sequence was used to obtain $(CV)_{ps}^{(i)}$ which was combined with $Z^{(i)}$ to obtain $N_{min}^{(i)}$ through the use of (22.) where $CV^{(i)}$ may be identified with $(CV)_{ps}^{(i)}$. Finally the $N_{min}^{(i)}$ so obtained was inserted with raw LIS time data into (25.) to obtain $T2^{(i)}$ which is defined analagous to $T1^{(i)}$ (see column 3 of Table 8). It is important to realize that this process cannot be simplified because there is no formula for the CPIS technique analagous to (22.) (where CV is identified with $(CV)_{ps}$) used for LIS. This is due primarily to the fact that the total sampling time for the minimum number of CPIS units required to obtain a population mean estimate with a specified degree of precision is far more sensitive to the coefficient of sample variation than is the sampling time for the corresponding minimum number of LIS units.

Table 8. Sampling time comparison of the line intersect and common point intersect methods.

Area	Class	Line Intersect Time	Common Point Intersect Time	Precision	Total Sampling Time Gain
		hours	hours	%	% of line intersect Sampling time
1	2	5.54	3.34	13.11	+39.71
1	3	2.46	2.04	18.41	+17.07
1	4	13.21	9.02	13.64	+31.72
1	2,3	6.28	5.09	13.11, 18.41	+18.95
2	2	1.47	2.64	17.56	-79.59
<u>2</u>	<u>2</u>	<u>2.24</u>	<u>2.01</u>	<u>14.50</u>	<u>+10.27</u>
2	3	3.01	1.84	14.14	+38.87
2	4	5.64	6.10	15.57	- 8.16
<u>2</u>	<u>4</u>	<u>7.63</u>	<u>4.58</u>	<u>10.28</u>	<u>+39.98</u>
2	2,3	3.70	4.30	17.56, 14.14	-16.22
<u>2</u>	<u>2,3</u>	<u>4.12</u>	<u>3.35</u>	<u>14.50</u>, <u>14.14</u>	<u>+18.69</u>

CHAPTER IV

Analysis of the field test data

Before interpreting the results listed in Tables 4 through 8, there are two points concerning the field test that warrant discussion. These are considered below:

The first of these two points relates to the fact that two statistical hypothesis tests are conducted on each area prior to application of the CPIS technique. These tests were performed in order to either verify or reject the necessary assumption that the downed particles in the ith subset were randomly distributed in the cylinders of radius $r^{(i)}$ to be sampled, $\forall i \varepsilon \{1,2,3\}$. As has been noted before, the data for these two hypothesis tests (see Tables 2 and 3) consistently give good credibility to the claim of particle placement and orientation randomness within the CPIS units sampled in each study area. Now this required condition of particle placement and orientation randomness within the sampling cylinders may seem like quite a severe restriction on the CPIS technique when applied to lesser downed fuels. It should be realized that randomness is required only with respect to the CPIS units being sampled and not with respect to either the entire fuel cell or all CPIS units within the fuel cell. This means that the CPIS formulas with $K^{(i)}$ as evaluated in (13.) may be applied to any population of lesser downed fuels with the one provision that randomness is present within the CPIS units being sampled. It should be mentioned here that the general theory on line intersect sampling (De Vries, 1973) utilizes randomness but this time it is the particles population that is considered to be randomly distributed. With respect to lesser downed fuels, LIS is somewhat more theoretically flexible than CPIS in that the general LIS theory utilizing randomness can be modified to virtually overcome bias due to nonrandom patterns of angular orientations of particle major axes (Van Wagner, 1968; De Vries, 1973). Unfortunately the price of this flexibility is that three times as many LIS units are required (De Vries, 1973).

The second point that warrants discussion is the fact that particle intercept counts were made on each LIS and each CPIS transect with respect to all three subsets. An alternative approach would have been to use Grosenbaugh's (1967) 3P subsampling procedures (Beaufait, Marsden and Norum, 1974). These 3P procedures were not applied to the second and third subsets because it was felt that here the reduction in sampling time offered by the 3P system was not sufficient to offset the statistical errors which these procedures introduced. Grosenbaugh's 3P subsampling techniques were not applied to the first subset because it was considered too difficult to make ocular estimates (required by the 3P system) of the numbers of twigs ((0-¼"]) intercepting most transects. This intersection count estimation process required by 3P subsampling was deemed too difficult for twigs because in many cases these finer particles were uniformly layered, making the number of twig interceptions not only impossible to estimate but also next to impossible to count. This difficulty in counting twig interceptions was a point of much concern. When layers of particles were encountered, it becomes necessary to disturb slightly the local particles in order to obtain a valid intersection count. In the fieldwork undergone in the study areas, this process was not found awkward unless the particles of concern were twigs in layers. Then the process became mentally exasperating. Through much painstaking effort, twig interception counts were obtained for both study areas (see Table 4 for LIS twig counts and Table 6 for CPIS twig counts). It was decided that if the five smallest preliminary CPIS units did not yield a twig population mean volume estimate with approximately 15% precision, no more CPIS units would be sampled, thus preventing a comparison of LIS and CPIS to be made for twigs. This decision was reached because the frequency of occurrence of twig layers was so great that it made the task of counting twig intersections next to futile. Since the twig data analysis (which utilized data from the five smallest CPIS units only) yielded mean volume estimates of about 40% precision for both study areas, no comparison was made of LIS and CPIS with respect to twigs. This fact is reflected in the absence of figures for the first size class in Table 8. It is suggested that if mean twig volume estimates are required in future investigations, regression equations expressing mean twig volumes per unit area as functions of mean particle volumes per unit area for at least the second and third subsets should be developed for the important fuel types. These regression relations could then be used in place of actual physical twig data. It is important to observe that when sampling in slash, the utilization standard implemented may have a significant effect on either the general form of regression model selected or the estimates for the regression coefficients used. Due to time constraints, suitable regression equations will not be developed here. Before analyzing the data in Tables 4 through 8, it should be stressed that the CPIS technique has not failed in its application with respect to twigs. The analysis with the available data has revealed that sampling twigs in the two study areas is not practical. In areas where it is practical to sample twigs and no regression equations for twig volumes are available, the CPIS technique can be applied providing that either a bigger sampling unit radius is chosen or alternatively a larger number of CPIS units are considered.

Tables 2 through 8 list both raw field data and important values computed from that field data. The values listed in Tables 2 and 3 have already been discussed. The data given in Tables 4, 6 and 7 are straight forward and require no further comment. In Table 5, it suffices to point out that the number of LIS units (see column 7) required to obtain a population mean volume estimate with 10% precision is more than double that required to obtain a population mean volume estimate with 15% precision (see column 6). It should be observed that this is a very high price to pay for an increase of 5% statistical accuracy. It remains only to consider Table 8 which deserves the most attention since it summarizes the performances of both the LIS and CPIS techniques when applied to the lesser downed fuels of the study areas.

An inspection of Table 8 reveals immediately that on the first study area the performance of CPIS was consistently superior to LIS. As previously mentioned the sampling times offered in Table 8 are representative of total sampling times required to obtain population mean volume estimates with specified degrees of precision (in a small neighbourhood of 15%) at the 95% confidence level. Notice that on area 1, the minimum time gain offered by CPIS was about 17% on 2.5 hours of LIS time. Also notice that on area 1 the maximum time gain offered by CPIS was about 40% on 5.5 hours of LIS time. These figures clearly reveal that on study area 1 the total sampling times required by LIS to obtain lesser fuel volume estimates of approximately 15% precision at the 95% confidence level can be significantly reduced through proper application of the CPIS concept. However the information presented in Table 8 for study area 2 is not so straight-forward. Recall that in the general discussion of the application of CPIS to downed woody particles, it was mentioned that statistical problems arose when the population of fuel particles contained large continuous areas differing drastically in fuel particle frequency. Recall also it was pointed out there that if such distinct areas occurred, it would be advisable to stratify the fuels population in a meaningful way and apply the theory of stratified sampling. Now it was remarked earlier in the general discussion of the field test that there was a noticeable ravine on study area 2 where lesser woody particle accumulations were evident. A closer inspection of this ravine revealed that the fuel particle frequencies of only the first and second diameter size classes (subsets) were dramatically high. Hence it became necessary to stratify the fuels populations of the first and second diameter size classes only. It is suggested here that a bias in fuel particle frequency of occurrence for the fuels of the first two size classes was present because it is probable that the distribution of fuel placement for the smallest particles becomes skewed when the direction of log pull interacts with unusual features such as significant continuous undulations or irregularities in the terrain. Going back to Table 8, notice that on study area 2 size classes 2, 4 and (2, 3) each have two sets of data associated with them. Notice also that one set is underlined and one is not. The underlined set refers to results of computing appropriate mean volume estimates with proportional stratification (Ehrenfeld and Littauer, 1964; p. 388); the non-underlined set refers to results without proportional stratification. The differences are highly significant, indicating that fuel stratification by fuel loading has a dramatic influence here especially upon the performance of CPIS. It should be remarked that the increases in the total LIS times after stratification are consistent with the corresponding decreases in precision levels. Fuel stratification has a favourable effect upon both LIS and CPIS. This effect is not exposed so obviously for LIS because LIS is sensitive to a decrease in precision level within a neighbourhood of 15%. Note that size class (2, 3) refers to the pair of size classes not their joint grouping into the size class, (¼", 3"). The inclusion of size class (2, 3) is justified by remarks made earlier to the effect that it was theoretically feasible to obtain volume estimates for size class 1 from volume estimates for both size classes 2 and 3 using regression analysis.

Proper application of CPIS techniques to the problem of obtaining mean downed woody volume estimates of approximately 15% precision at the 95% confidence level has consistently resulted in significant total LIS sampling time reductions with respect to both study areas considered. The time constraints imposed upon this investigation prevent a rigorous verification of the claim that CPIS is significantly superior to LIS with respect to all downed woody fuel applications. On the other hand the findings presented in Table 8 cannot be dismissed as simply interesting. These findings offer concrete confirmation that common point intersect sampling (CPIS) is a flexible and viable sampling technique which deserves much attention. At this point the data for the field test has been presented and analyzed. It is appropriate now to consider the overall significance of the thesis, and the inferences which can be made from the information the thesis has provided.

CHAPTER V

Conclusions

A general sampling technique referred to as 'Common Point Intersect Sampling' (CPIS) has been developed and discussed and tested operationally with encouraging results. Appropriate CPIS formulas were derived with respect to downed woody fuels. Using these formulas the performance of CPIS was compared to that of line intersect sampling (LIS) in two fresh cutover areas. Proper application of the CPIS technique yielded lesser fuel volume estimates of about 15% precision with savings of up to 40% of the total sampling time required by the LIS technique.

The general theory of CPIS as presented is extremely flexible. It can be applied to the problem of obtaining quantitative estimates for attributes of any community of objects temporarily fixed in space. It is imperative for the reader to realize that the basic concepts involved are seated in sound logic. To emphasize this point a consideration of all the seemingly negative aspects of CPIS will now be made.

The first apparent negative property of CPIS is that there is no cut and dried general formula for obtaining the common radius of the cylindrical sampling units used. This property is closely linked to the problems of selecting the appropriate number of transects to lay out in each CPIS unit and of specifying a sufficient number of CPIS units which will result in a satisfactory estimate. Although preliminary sampling holds some promise for solving these problems there is little doubt in the mind of the author that a truly satisfactory answer lies in the art of simulation modeling. Using a systems analysis approach, the system in question can be subjected to a multitude of sampling designs with statistical and cost criteria used as the basis for selecting the best set of outcomes. Time constraints forbid the construction of such a general systems model which would be interfaced with different general sampling techniques. In essence then what is being said is that a truly satisfactory answer to the general problem of selecting both the number of CPIS transects and the number and size of the CPIS units lies outside the scope of the thesis. Until such time as a general systems model is built, each investigator will be forced to either construct and experiment with events models similar to those presented in this thesis, conduct preliminary sampling, or else rely on his own experience and intuition in order to quantify precisely the CPIS system which will best serve him.

The second apparent negative property of CPIS is that it is not immediately clear how to get the function whose integral is to be used in obtaining an estimate for the parameter of interest. Of course it can be argued that the multitude of situations which can arise here is so great that it prevents the specification of a precise technique. But there is a related procedure embedded in the general theory and reinforced in the CPIS applications. More specifically, if the attribute is a one-dimensional average, try to relate it through the multiplicitive constant k to the image of a function whose component terms can be easily tallied either in the field or on a photograph. If successful, this image then leads directly to a form of the desired integrand using for example the concept of the average value of a function. The argument used for E_1 is a perfect example of this. If the attribute is two-dimensional, the integrand will probably be either one or two dimensional depending upon what characteristics can be measured. A good example here is that surface area is generated by rotating curvilinear length. Finally if the attribute is three dimensional, the integrand will probably be either one or two dimensional, again depending upon the inputs involved and any simplifying assumptions which are valid to use. A good example here is that volume is generated by rotating surface area. In practice the selection of the integrand is almost always simplified by the fact that only certain field variables can be measured to satisfactory precision. See Chapter VI for a further discussion of this problem.

The general common point intersect sampling concept has been applied to the problem of obtaining estimators for important parameters of downed woody fuels populations. A general approach has been presented which permits the complete specification of CPIS designs and formulas for computing total volume estimates of randomly distributed downed woody particles. This approach was followed in detail with respect to the lesser downed fuels, and was followed in general with respect to the greater downed fuels. A general formula which expressed CPIS unit radius as a function of maximum particle diameter of interest was presented for randomly distributed particles. Also a general CPIS formula independent of particle distribution was offered for computing total volumes of greater downed fuels.

These results will now be summarized by the following suggested step-by-step field procedure to be followed when total volume estimates of downed woody particles are of interest:

1) specify the set of diameter size classes of interest, $\left\{\left(d_i, D_i\right]\right\}_{i=1}^{M}$, $d_i \geq \frac{1}{4}$".

2) select a representative fuel cell in the area of concern. Avoid access roads and unusual topographic features such as rock

outcrops. If the fuel cell has no large continuous areas of unusually high or low fuel frequencies, apply steps 3 through 8 to the entire fuel cell. Otherwise stratify the fuel cell by loading and if desired apply steps 3 through 8 to each stratum. If this amount of detail is not required for each stratum, simply do steps 3 through 8 applying the theory of proportional statified sampling (Ehrenfeld and Littauer, 1964; p. 388). (Note that if this last process is conducted, the N sampling units are selected on a basis of appropriate representation in the fuel strata rather than on an arbitrary basis as implied by (5) and (6).

3) Lay out a systematic sampling design consisting of about seven common point intersect sampling units, each of common radius $r^{(M)}$ where:

$$\mathrm{Log}_{10} r^{(M)} \gtrsim 1.22 + 0.47 \log_{10} D_M, \quad D_M \text{ in inches}, \; r^{(M)} \text{ in feet}$$

Within each of these 7 large circles there are (M-1) concentric circles, where the radius of the ith circle is defined by:

$$\mathrm{Log}_{10} r^{(i)} \gtrsim 1.22 + 0.47 \log_{10} D_i, \quad D_i \text{ in inches}, \; r^{(i)} \text{ in feet}, \; \forall i \in \{1,\ldots,M-1\}$$

Make sure that the 7 circles are spread such that they cover the entire fuel cell uniformly.

4) Set i = 1.

5) Set N = 3, j = 1, and k = 0. Select N widely spaced large CPIS units from the 7 units laid out.

6) Consider the ith circle in the jth large CPIS unit. If the fuel particles in the ith diameter size class in the ith circle

are randomly distributed (assume this unless it is very obviously false), choose a radius transect (t) randomly using some set of random numbers. Each radius transect is defined by a sub-length of cord of length $r^{(M)}$ joining two or more wooden stakes, each sharpened at one end to facilitate easy insertion into the ground. Transects are laid out geodetically (i.e. parallel to the adjacent topographic surface) not horizontally. The cord should be attached to each wooden stake with a sliding ring equipped with a wingnut to permit the cord height to be raised or lowered according to fuel depth. Count the number of particles in the ith diameter size class $(d_i, D_i]$ which intersect this transect. Note that an intersection occurs only if both the particle central axis and at least one of the particle edges intersect the transect. Then rotate the radius transect through a 30° arc and repeat the above process. This procedure is carried out a total of 12 times to obtain the sequence $\{m_j^{(i)}(t + \frac{\pi\omega}{6})\}_{\omega=1}^{12}$, where $m_j^{(i)}(t + \frac{\omega\pi}{6})$, $\omega\varepsilon\{1,\ldots,12\}$ is the number of particles in the ith diameter size class which intersect the transect at $(\frac{\pi\omega}{6} + t)$ inscribed within the ith concentric circle in the jth large CPIS unit. This sequence is then inserted into (19.) to obtain the desired volume estimate for the ith circle in the jth CPIS unit. Note however that if the particles in the ith circle (where D_i<3") are not randomly distributed, no legitimate volume estimate can be obtained using (19.) with $K^{(i)}$ as evaluated in (13.), $\forall i\varepsilon\{1,2,3\}$. Now suppose that the fuel particles within the ith circle (where $D_i > 3"$) are not randomly distributed. Then repeat the sampling procedure described above for randomly distributed particles, with the exception that for each relevant intersection the distances to the left and right most points of the intersection (along the particle edge) are tallied and not the number of relevant particle intersections. As indicated previously if a truncated ellipse is encountered (i.e. a particle end), it is necessary to extrapolate one edge of the particle to the transect in order to obtain the proper value assuming of course that the particle central axis intersects the transect. The data so obtained are inserted into (14.) which is then used to obtain the desired volume estimate for the ith concentric circle ($3<i\leq M$) in the jth large CPIS unit. This estimate is described below:

$$\sum_{\omega=1}^{12} C_\omega g2_j^{(i)}\left(t + \frac{\omega\pi}{6}\right), \quad j\varepsilon\{1,\ldots,N\}$$
$$i\varepsilon\{4,\ldots,M\}, \; D_i>3",$$

$$C_\omega = \begin{cases} 4, & \text{if } \omega \text{ is odd} \\ 2, & \text{if } \omega \text{ is even} \end{cases}$$

7) Increment j by 1 and repeat (6.) until j=N+1+k.

8) Take the N volume estimates obtained above and compute:

$$\left(t_{0.975;\,N-1}\right)\cdot(CV)_i/\sqrt{N} \equiv (TS)_N^{(i)}$$

where $(CV)_i$ = coefficient of sample variation for the ith diameter size class.

$t_{0.975;\,N-1}$ = value from Student's t distribution with (N-1) degrees of freedom.

If $(TS)_N^{(i)} \leq 0.15$, the desired 95% confidence interval with 15% precision has been obtained. Hence increment i by 1 and repeat steps (5) through (8) until i=M+1. However if $(TS)_N^{(i)} > 0.15$, increment K by 1 repeat steps (6), (7) and (8) with j=N+K until either $(TS)_N^{(i)} \leq 0.15$ or j=8. Then increment i by 1 and repeat steps (5) through (8) until i=M+1. Note that (6), (7) and (8) are repeated with only one new added sampling unit. This means that it is necessary to select only one new CPIS unit each time (6), (7) and (8) are repeated for a particular $i \varepsilon \{1,\ldots,M\}$. It should be observed that the above guidelines are not applicable to particles which lie in the (0"-¼"] diameter size class. Upon investigation of these smaller particles it was discovered that in many downed woody fuel complexes, field problems (i.e. clumps or piles of particles interspersed with needle mats) were encountered making impractical the twig counting process or even the estimation of twig counts. It is recommended that for the (0"-¼"] size class, an interim suitable regression model be developed. This model would express particle volume in the (0"-¼"] size class as a function of particle volumes in the (¼"-1"] and (1"-3"] size classes and any other factors (e.g. type of disturbing influence and utilization standard for a harvesting operation) deemed important. Due to time constraints the above step-by-step procedure necessarily includes slight abuses of that portion of the general CPIS theory previously developed for greater downed particles. The above procedure can still validly serve as a sound tentative set of guidelines for all downed woody particles pending further studies of the performance of CPIS techniques with respect to greater downed particles.

It can validly be argued that there are many sampling problems of more economic importance than those involved with downed woody materials. In fact in Chapter VI suggestions are given as to how the common point intersect technique may be applied in other forestry-related areas. Perhaps the strength of CPIS should have been tested in some other forestry problem, such as the task of estimating important standing timber parameters. The fact that the CPIS concept was not applied there has been amply justified. It is the claim of the author that the problem of obtaining lesser downed fuel volume estimates is an extremely difficult one, and as such serves as an excellent test for evaluating any general sampling scheme. The common point intersect technique did more than pass this test on two study areas; when properly applied it proved itself to be significantly more time economical than line intersect sampling, which to date is the only economically realistic means of obtaining quantitative estimates for physical parameters with respect to lesser downed woody material. It could also be argued quite validly that the new sampling technique as presented is unappealing to the average potential user. This disenchantment stems almost entirely from the use of mathematics beyond the scope of the average potential user. More than likely he will be hesitant about using a sampling technique which he does not fully understand. Note that there is nothing complicated about picking locations for some points and imagining circles of a common specified diameter about those points. There is nothing complex about inscribing a few spokes at specified angles within these circles and counting particle intersections along each spoke and/or making a few simple measurements at each intersecting particle. It should not be difficult to see that these particle counts and/or intersections within a particular circle result in an estimate of some attribute of concern with respect to that circle. The only thing which the average potential user may find hard to follow is the higher mathematics used in converting the particle intercept counts and/or measurements to estimates of lengths, surface areas, volumes or whatever is of interest. The use of advanced mathematics here could be frowned upon because it complicates the theory. But it is because of the higher mathematics that the field work can be reduced to a minimal level. The average potential user is most interested in how much work he has to do and how much he has to spend to get what he wants. The common point intersect technique has been shown to make substantial reductions in both of these areas at least with respect to lesser downed woody materials. There is good promise that with the use of CPIS efficient solutions for other sampling problems can be found.

In the step-by-step field procedure offered on p.74-78 there are some formulas whose evaluation is periodically required in the field. The author does not expect the average potential user to take these guidelines verbatim into the field with him. But rather these guidelines are intended to form a basis for developing a field guidebook which would permit the user to look up in a table the volumes, etc. for his particular data. The level of resolution of these tables is primarily a function of the accuracy desired by most users. The average potential user is never expected to understand mathematical formulas. At this stage all the potential user need be aware of is that the common point intersect technique was tested on two dissimilar slash areas and was shown to require significantly shorter sampling times than the most successful sampling technique currently available, namely line intersect sampling. Field packages will come later.

The real power of common point intersect sampling lies in the fact that it is extremely versatile. It can be likened to cluster sampling and as such is most beneficial in those cases where the cost of selecting and locating a population element far exceeds the cost of determining the contribution which that element makes to the estimate for the attribute of interest (Freese, 1962; p.64). A great deal of effort has been made to convince the reader that common point intersect sampling is a sound and viable non-destructive technique which deserves the attention of investigators from almost every scientific discipline. It is almost redundant to say that more studies in many more sampling problem areas are required in order to thoroughly evaluate the worth of this new sampling concept. But this is still not a valid reason to ignore common point intersect sampling until its value has been conclusively determined. Through numerous arguments it has been indicated that applications of CPIS techniques are feasible now at least in the forestry-related areas of downed woody material and standing timber (see Chapter VI).

The two main objectives of this thesis were stated as the presentation of a new general sampling theory and the application of this theory to an important social problem area. But these objectives are only a means to an end, which is the full operational use of common point intersect sampling. The achievement of this end is what this thesis is all about.

CHAPTER VI

Practical applications of the common point intersect technique

Thus far the common point intersect sampling technique (CPIS) has been applied only to the problem of estimating parameters for populations of downed woody materials. But this is only one small problem for which CPIS offers a promising solution as this chapter will demonstrate. Before discussing alternative applications of CPIS consider first a simplified version of the sampling concept upon which CPIS is based. CPIS selects cylindrical sampling units enclosing elements of the population of interest. CPIS hopes to choose these units sufficiently large so that the variability among them with respect to the attribute of interest is small. Within each unit a number of line transects are inscribed. Regarding each sampling unit in two dimensions as a circle, each transect corresponds to a radial segment. Many measurements are made in each circle at places defined by the intersection points of the transects and relevant population elements. For a given circle these measurements are inserted into appropriate formulas which given an estimate of the attribute of interest with respect to that circle. When applying CPIS to the problem of estimating some physical property of interest, it is usually of benefit to ask the question 'What geometric figure when rotated about the centre of a circle will yield that physical property?'.

Consider first the problem of estimating the length of a stream or road network. The physical property of interest is length. So what geometric figure when rotated about the centre of a circle yields length? The answer is simply a point. So for a given CPIS unit let $X_i^{(j)}(\theta)$ be the distance along the base of that unit from the centre of the unit to the $j^{\underline{th}}$ point formed by the intersection of the transect at θ with the $i^{\underline{th}}$ stream or road intersecting the transect at θ. Let $m_i(\theta)$ be the number of such points formed by the intersection of the transect at θ with the $i^{\underline{th}}$ stream or road intersecting the transect at θ. Also let n(θ) be the number of stream or road intersections intersecting the transect at θ. Then

$$\int_0^{2\pi} \sum_{i=1}^{n(\theta)} \sum_{j=1}^{m_i(\theta)} X_i^{(j)}(\theta)d\theta$$ gives a measure of the total length of streams or roads within the CPIS unit of interest. Notice that the result is independent of stream or road distribution or frequency.

Consider next the problem of estimating large numbers of fructifications of wood-destroying fungi, large numbers of migrating wildlife or large numbers of host-attached plant parasites. The insight gained from the discussion of experiment E_1 should provide the key to these problems. For a fixed CPIS unit of radius r let n(θ) be the number of population elements intersecting the transect at θ. Then $\frac{1}{2\pi r}\int_0^{2\pi} n(\theta)d\theta$ gives a measure of the average number of elements per unit area within that CPIS unit providing of course that the units of r are sufficiently small. This result is also independent of distribution or frequency of the elements of concern. The problem of estimating crown area (Husch, Miller and Beers, 1972; p.106) and mean crown diameter (Husch, Miller, and Beers, 1972; p.49) with CPIS techniques will be considered next. Time constraints permit only a general investigation here and this is limited as above to the generation of the integrand in (1.) with respect to each of the above two stand parameters.

Focus upon the problem of estimating crown area using the CPIS concept. Suppose that aerial photographs are available which permit crown boundaries to be delineated. Let pl be a parameter representing the average crown area per unit area. Similar to experiment E_2 (see 'Application to downed woody fuels'), let k be πr^2. The $F_j(2\pi)$ represents a total crown area with respect to the area in the jth CPIS unit. Using notation as in the application of CPIS to downed woody particles, a function f_j is desired such that:

$$F_j(2\pi) = \int_0^{2\pi} f_j(t)dt, \text{ where } F_j(2\pi) = \pi r^2 X(S_j), \text{ and}$$

$X(S_j)$ = value taken on by X at S_j, $j \in \{1,\ldots,N\}$, where X again refers to a random variable defined on the sample space of outcomes of experiment E_2. Now in order to produce $f_j(t)$ in terms of variables which can be easily measured, it is necessary to make some assumptions regarding the regions enclosed by the crown boundaries. It is valid to assume that each region defined by a crown is a collection of connected sub-regions (Taylor, 1965; p.76). If desired, this can be taken to mean that each sub-region can be traced without ever having to lift the pencil. The attribute of concern here is a surface area which is two-dimensional. So what geometric figure when rotated about the centre of a circle yields surface area? The answer is a line segment. A basic understanding of calculus reveals that if a line segment of small length ΔX is located approximately a distance X away from the origin, a surface area of $(\Delta X)(X \cdot \Delta\theta)$ is generated by sweeping the line segment through $\Delta\theta$ radians. It follows trivially that:

$$f_j(t) = \tfrac{1}{2} \sum_{i=1}^{m_j(t)} \sum_{k=1}^{Pi,j} \left[e^2_{k,i,j}(t) - b^2_{k,i,j}(t) \right], \ \forall t \in [0,2\pi], \text{ where}$$

$m_j(t)$ = number of crowns intersecting the CPIS transect located at the t within the jth CPIS unit.

Pi,j = number of line segments formed by the intersection of the transect at t with the ith crown intersecting the transect at t,

$i\varepsilon\{1,\ldots,m_j(t)\}$, within the jth CPIS unit.

$e_{k,i,j}(t)$ = furthermost end-point of the k^{th} line segment formed by the intersection of the transect at t in the jth CPIS unit with the i^{th} crown intersecting the transect at t, $k\varepsilon\{1,\ldots,P_{i,j}\}$, $i\varepsilon\{1,\ldots,m_j(t)\}$

$b_{k,i,j}(t)$ = identical to $e_{k,i,j}(t)$ with the exception that $b_{k,i,j}(t)$ refers to the innermost end-point.

f_j is sectionally continuous on $[0,2\pi]$ and thus satisfies all required analytical properties. Note that the only inputs required by f_j are linear distances which are most often easily obtained from aerial photographs. It is important to realize that the CPIS formulas produced to obtain estimates of crown area are independent of tree distribution and also virtually independent of crown shape.

The problem of estimating mean crown diameter with CPIS techniques is completely analagous to experiment E_1 (See 'Application to downed woody particles'). The only assumptions required are those used directly above, namely that the aerial photographs used permit a meaningful delineation of crown boundaries, and that each crown may be regarded as a collection of connected sub-regions. Since crown diameters are usually well-defined except for some hardwoods (Sayn-Wittgenstein, 1960), they can be measured directly and used with an analogue to the average function value formula presented in experiment E_1 (See (4.)) to yield the desired estimate.

With CPIS techniques it is easily possible to obtain estimates for other stand parameters such as number of trees per unit area by species (where species can be identified), mean crown diameter per tree by species, mean height per tree by species (where tree height can be either estimated or else regressed from other measurable tree characteristics, mean tree volume per tree by species, etc. Note that these estimates can be inserted into suitable regression equations to produce estimates for component oven-dry weights of total-tree, bole-wood, bole-bark, total-crown and branches of different sizes (Kurucz, 1969).

Now consider the problem of estimating the surface areas of lakes, watersheds, large river systems, or areas occupied by different herbaceous and ligneous species. The application of CPIS to these problems is similar to the application of CPIS in estimating crown area for standing timber. So let $a_i^j(\theta)$ and $b_i^j(\theta)$ be the distances along the base of some CPIS unit from the centre of the base of the unit to the left and right hand end-points respectively of the $j^{\underline{th}}$ segment formed by the intersection of the transect at θ with the $i^{\underline{th}}$ population element intersecting the transect at θ. Let $n_i(\theta)$ be the number of line segments formed by the intersection of the transect at θ with the $i^{\underline{th}}$ population element intersecting the transect at θ. Also let $m(\theta)$ be the number of population elements intersecting the transect at θ. Then

$$\int_0^{2\pi}\left\{\sum_{i=1}^{m(\theta)}\sum_{j=1}^{n_i(\theta)}\int_{a_i^j(\theta)}^{b_i^j(\theta)} x\,dx\right\}d\theta$$ gives a measure of the total

surface area occupied by the population elements within that CPIS unit. Note that this result is independent of the distribution or frequency of the elements of concern. This result can also be used to measure rate of seasonal change or annual growth by comparing figures computed from photographs or direct measurements taken at appropriate times.

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APPENDIX I

Problem: To prove that the volume obtained by rotating an ellipse at θ_1 through a small arc $\left[\theta_o^R, \theta_2^R\right]$, $\theta_o \leq \theta_1 \leq \theta_2$, is given by (5).

Proof: Fix θ_1 in $\left[\theta_o^R, \theta_2^R\right]$. Let the ellipse be denoted by E1. Let the center-point of E1 by (S, Yo) and let the left and right hand end-points of E1 be X1 and X2 respectively. Finally let d, γ, and ϕ be defined as in (5) and assume that $\gamma > 0$.

E1 lies on a rectangle R (cross-section of some right-circular cylinder) inscribed within the population of concern. Let the base of R define an X^+ axis and the other edge of R intersecting the centre of the circular base of the sampling unit define a Y^+ axis. Let the major axis of E1 be inclined at an angle α, $-\frac{\pi}{2}^R < \alpha^R < \frac{\pi}{2}^R$, from the X^+ axis.

From simple geometric considerations it can be seen that the ellipse (E2) defined by:

$$\frac{(X-S)^2}{(d/2)^2} + \frac{(Y-Yo)^2}{\left(\frac{d \sec\gamma}{2}\right)^2} = 1$$

intersects E1 at (S, Yo$+ \frac{d\sec\gamma}{2}$). This gives one point on E1. Now consider the ellipse (E3) defined by:

$$\frac{(X - S)^2}{(d/2)^2} + \frac{(Y - Yo)^2}{\left(\frac{d\csc\gamma}{2}\right)^2} = 1 \text{ and the line defined by}$$

$$Y = (X-S)\cot\phi + Yo \text{ where } \phi = \sin^{-1}\left\{\frac{d}{(X2-X1)}\right\}, \quad 0 < \leq \pi/2$$

From elementary calculus the length of the line Y = X cot ϕ which lies to the right of (0,0) and which is enclosed by E3 is:

$$\frac{d\csc\phi}{2\sqrt{1 + \cot^2\phi\sin^2\gamma}}$$

Now let $((Xo+X_3) = X_4, Yo)$, $X_3 > \hat{0}$, be a point on E1. Then from geometric considerations, it is seen that a second point on E1 is:

$$\left(Xo + \frac{d\csc\phi}{2\sqrt{1+\cot^2\phi\sin^2\gamma}}, Yo \right)$$

These two points lead to the equation of E1 in the (X,Y) system. To see this, rotate the X and Y axes through α^R in either a clockwise or counter-clockwise direction, whichever is appropriate to obtain a new (X',Y') system. The equation of E1 in the (X',Y') system then becomes:

$$\frac{(X'-S')^2}{(a/2)^2} + \frac{(Y'-Yo')^2}{(d/2)^2} = 1, \text{ where}$$

$$(*) \qquad X' = Y\sin\alpha + X\cos\alpha$$

$$\text{and} \quad Y' = Y\cos\alpha - X\sin\alpha$$

Inserting the two points on E1 into (*) yields:

$$(**)\begin{cases} a = d\csc\phi\sec\gamma \\ \\ \text{and} \quad \cos\alpha = \dfrac{\cot\phi}{\sqrt{\csc^2\phi\sec^2\gamma-1}}, \text{ where } \sin\alpha \begin{cases} >0 \text{ if rotation is counterclockwise} \\ <0 \text{ if rotation is clockwise} \end{cases} \end{cases}$$

Note that these last two equations also hold if $\gamma = 0$. Consider a small vertical strip of the area enclosed by E1. Let this strip have width ΔX and height $t(X)$. The area of this strip is approximately $t(X)\cdot\Delta X$. When this strip is rotated about the X axis through a small arc of length $X(\Theta_2^R - \Theta_0^R) = X\cdot\Delta\Theta^R$, the resultant volume generated is approximately $t(X)\cdot\Delta X\cdot X\cdot\Delta\Theta^R$. Using the continuity of the height difference curve (maximum height less minimum height for each X) defined by E1, it follows that the volume generated by rotating E1 about the X axis through a small arc $\left[\Theta_0^R, \Theta_2^R\right]$ of measure $\Delta\Theta^R$ is:

$$\Delta\theta \int_{X1}^{X2} X\cdot t(X)\,dX$$

From (*) and the quadratic formula, it can be shown that:

$$\int_{X1}^{X2} X\cdot t(X)\,dX = \frac{1}{A}\int_{X1}^{X2} X \pm\sqrt{B^2 - 4AC}\,dX, \text{ where}$$

$$A = 4\csc^2\phi$$

$$B = 8(X-S)\ (1 - \csc^2\phi\cdot\sec^2\gamma)\ \cos\alpha \cdot \sin\alpha$$

$$C = 4(X-S)^2\ (1 + \cot^2\phi\sin^2\gamma)\ \sec^2\gamma - d^2\csc^2\phi\ \sec^2\gamma$$

Simplification using (**) results in

$$\pm\sqrt{B^2-4AC} = 4\csc\phi\sec\gamma \pm\sqrt{d^2\csc^2\phi - 4(X-S)^2}$$

Therefore

$$\int_{X1}^{X2} X\cdot t(X)\,dX = \frac{\sec\gamma}{\csc\phi}\int_{X1}^{X2} X\pm\sqrt{d^2\csc^2\phi-4(X-S)^2}\,dX$$

$$= \frac{\sec}{\csc\phi}\int_{X1-S}^{X2-S} (u+s)\pm\sqrt{d^2\csc^2\phi - 4u^2}\,du, \text{ where } u = X-s,\ s=S$$

$$= d\ \sec\gamma\int_{X1-S}^{X2-S} (u+s)\pm\sqrt{1 - \frac{u^2\sin^2\phi}{(d/2)^2}}\,du$$

$$= d\ \sec\gamma\int_{\pi}^{0} \left\{\frac{d\csc\phi\cos\beta}{2} + s\right\} |\sin\beta| \left\{\frac{-d\csc\phi\sin\beta}{2}\right\} d\beta,$$

$$\text{where } \beta = \cos^{-1}\left\{\frac{2u\sin}{d}\right\},\ 0 \leq \beta \leq \pi$$

$$= \frac{d^2\csc\ \phi\ \sec\gamma}{2}\int_{0}^{\pi}\left\{\frac{d\csc\phi\cos\beta}{2} + s\right\}\sin^2\beta\,d\beta$$

$$= \frac{d^2\csc\phi\sec\gamma}{2}\left\{\frac{d\csc\phi}{2}\int_{0}^{\pi}\cos\beta\sin^2\beta\,d\beta + S\int_{0}^{\pi}\sin^2\beta\,d\beta\right\}$$

$$= \frac{d^2 \cdot \csc\phi \cdot \sec\gamma \cdot s}{2} \int_0^{\pi} \sin^2\beta d\beta, \text{ since 1st integral is 0.}$$

$$= \frac{d^2 \cdot s \cdot \sec\gamma \cdot \csc\phi}{4} \left(\beta - \frac{\sin 2\beta}{2}\right)\Bigg]_{\beta=0}^{\beta=\pi}$$

$$= \frac{\pi}{4} d^2 \cdot s \cdot \sec\gamma \cdot \csc\phi$$

Therefore the volume generated by rotating E1 about the X axis through a small arc $\left[\theta_o^R, \theta_2^R\right]$ of measure $(\theta_2^R - \theta_o^R) = \Delta\theta^R$ is:

$$\frac{\pi}{4} \cdot d^2 \cdot s \cdot \sec\gamma \cdot \csc\phi \cdot \Delta\theta^R.$$

Since θ_1 was arbitrary in $\left[\theta_o^R, \theta_2^R\right]$, (5.) is established.

QED.

APPENDIX II

The computerized analysis of variance model (ANOVA 1) offered in this thesis is designed for a PDP-11 computer. It is primarily a simultaneous runs one-way classification random components model (Ehrenfeld and Littauer, 1964; p.391-399) with unequal numbers of observations in the cells. This program both generates the values of the response variable of interest and also does the standard analysis of variance computations. It also examines both the assumption that the terms comprising the T matrix (see ANOVA 1) are normally distributed with a common 0 mean and common variance and the assumption that the terms comprising the E matrix (see ANOVA 1) are normally distributed with a common 0 mean and common variance. Both these assumptions are required to be valid in order to correctly apply the standard parametric hypothesis test for homogeneity of 0 variance (Ehrenfeld and Littauer, 1964; p.396) with respect to the terms comprising the above mentioned T matrix.

This program was devised first to determine the conditions under which it was feasible to replace:

$$\sum_{k=1}^{m_{j1,j2}(t)} \left[S^{(i)}_{k,j1,j2}(t) \cdot \csc\left(\phi^{(i)}_{k,j1,j2}(t)\right) \right] \Big/ \left(m^{(i)}_{j1,j2}(t) \cdot \pi (r^{(i)})^2\right) \equiv S(i,j1,j2,t),$$

$$m^{(i)}_{j1,j2}(t) \neq 0,$$

where: t refers to transect location in $[0,\ 2\pi]$

j_1 refers to particle distribution

j_2 refers to particle loading

i refers to diameter size class

(see (5.) and (6.)) by a constant $K^{(i)}$ independent of j_1, j_2 and t such that not only the integral but also the shape of $\hat{g2}_j^{(i)}$ (see (10.)) over $[0, 2\pi]$ would be approximately the same as the integral and shape respectively of $g2_j^{(i)}$ (see (9.)) over $[0,2\pi]$. Having tentatively established these conditions, the program was then adjusted to evaluate $K^{(i)}$, for $i \in \{1,2,3\}$ (see (10.)). ANOVA 1 was necessary because for many downed woody particles it is not feasible to measure either distance to transect/particle intersection or angle of transect/particle intersection as required by (9.).

Determination of the conditions under which it was possible to replace $S(i,j_1,j_2,t)$ by $K^{(i)}$ was made by setting up ANOVA 1 for a two-way classification run (Ehrenfeld and Littauer, 1964; p.432-434). The two influencing factors were fuel particle loading and distribution. A wide range of lesser fuel particle loadings (0.75 particles/sq.ft-50.00 particles/ sq.ft.) and distributions was tested here under different fuel particle length distributions. Let $\overline{S}(i, j_1,j_2)$ be the mean value of S taken over all $t \in [0,2\pi]$, i, j_1, and j_2 fixed. Then an interesting result of this analysis was that $\overline{S}(i,j_1,j_2)$ was found to be reasonably independent (ie. within statistical limits) of j_2. No such independence could be found for j_1. This was found true $\forall i \in \{1,2,3\}$, $\forall j_1,j_2$ tested, and for every fuel particle length distribution tested. Hence due to time constraints it was decided to determine $K^{(i)}$, $i \in \{1,2,3\}$ for only the most common lesser fuel particles distribution, namely randomly distributed fuel particles with respect to the CPIS units.

Before proceeding to the one-way classification ANOVA 1 set-up, three relevant points should be briefly discussed. The first one relates to the fuel particle distributions which were used as levels of the second influencing factor in the two-way classification ANOVA set-up. A multiplicity of angular orientations and particle placements was used here. For example, particles were randomly oriented and randomly placed, randomly oriented and placed in clumps or strata, unidirectionally oriented and randomly placed, and unidirectionally oriented and placed in clumps of intensified fuel accumulations. The second point which merits consideration is the choice of fuel particle length distributions. Three distributions were tested here, each applied using a variety of length ranges. The first distribution was generated using a random number generator based on the Lehmer Multiplicative Congruential Method (Mize and Cox, 1968; p.68) where the general formula used is:

$$u_n \equiv X^n u_o \pmod{m}$$

This formula is embedded in a library function subprogram (RAN) on the PDP-11. RAN sets x equal to $2^{16}+3$ and m equal to 2^{32}; u_o is an arbitrarily chosen odd number. RAN has the unfortunate feature that the actual order of the cycle it generates is not easily determined. However RAN has been demonstrated on the PDP-11 to have actual orders of at least 2^{20} for several choices of u_o. Only these choices were used in generating the random number cycles for both ANOVA models. The maximum ranges used here for the first, second and third subsets were (¼"-2'), (1"-3'), and (6"-8') respectively. Both the second and third length distributions were generated using skewed versions of RAN. The second length distribution simply allotted one-half of the particles to the first one-third of each range and the second length distribution allotted one-half of the particles to the first two-thirds of each range. These length distributions were subjectively chosen to provide dramatically different particle length distributions in the modelling procedures. The third point which should be mentioned here is that there is a general purpose analysis of variance program designed for the PDP-11 called ANOVA which can accommodate up to 5 influencing factors, one hundred response variables, 50 transformation cards and 400 observation cells. Use of ANOVA would eliminate the need for a portion of the ANOVA 1 model. ANOVA is definitely a viable approach here but was not used because the author had written a large portion of ANOVA 1 prior to commencement of the thesis.

The one-way classification ANOVA 1 set-up derived $K^{(i)}$, $\forall i \varepsilon \{1,2,3\}$ for randomly distributed particles. Numerous runs were made here to test the validity of the two underlying assumptions (stated at the beginning of APPENDIX II) required for application of statistical parametric hypothesis tests. Using X^2 goodness-of-fit tests at the .05 significance level, both assumptions were found to be correct. The analysis performed by ANOVA 1 for the one-way classification revealed that the random variable defined by $R_j^{(i)}$, i and j fixed (see (12.)) was normally distributed with mean 0 (using a X^2 goodness-of-fit test at the .05 significance level), $\forall$ i, j tested. A secondary result of the analysis was that if j is ordered by loading for fixed i, the variance of $R_j^{(i)}$ generally decreased with increasing j for all i. These results may be interpreted to mean that with respect to randomly distributed particles, not only the integral of $\hat{g2}_j^{(i)}$ (see (10.)) but also the shape of $\hat{g2}_j^{(i)}$ over $[0,2\pi]$ is approximately the same as that of the integral and shape respectively of $g2_j^{(i)}$ (see (9.)) over $[0,2\pi]$ $\forall j$ tested and $\forall i \varepsilon \{1,2,3\}$ where:

$$K^{(1)} = 2.83 \ X10^{-2}(\text{feet})^{-1}$$

$$K^{(2)} = 1.49 \ X10^{-2}(\text{feet})^{-1}$$

$$K^{(3)} = 8.56 \ X10^{-3}(\text{feet})^{-1}$$

A further interesting result of ANOVA 1 for the one-way classification case is that the random variable defined by $m_j^{(i)}$, i and j fixed (see (6.)) has a coefficient of variation reasonably independent of i and j, $\forall i \varepsilon \{1,2,3\}$, $\forall j$ tested. This can be interpreted to mean that on a percentage basis the variability of $\hat{g2}_j^{(i)}$ over $[0,2\pi]$ is reasonably independent of i and j. Combining this observation with previous remarks, it follows that one numerical integration technique applied at one level of sampling intensity will probably produce the same degree of accuracy for the integral of $g2_j^{(i)}$ over $[0,2\pi]$, $\forall i \varepsilon \{1,2,3\}$, $\forall j$ as before.

Finally it is observed that the one-way classification ANOVA 1 set-up computes the non-parametric Spearman's rank-difference coefficient (Tate and Clelland, 1957; p.13) with respect to distance to particle central axis intersection and cosecant of particle central axis intersection. Results of this investigation showed that in 90% of all CPIS units with randomly distributed particles, the random variable represented by distance to particle central axis intersection and the random variable represented by the cosecant of particle central axis intersection were uncorrelated at the .05 significance level.

The one-way classification ANOVA 1 set-up presented below is very simple and straight-forward. It is supplemented with many comment statements explaining important procedures and definint important matrices to assist in the reader's understanding of what has been done.

APPENDIX III

From APPENDIX II it is now possible to express $\hat{g}2_j^{(i)}(t)$ as follows:

$$\hat{g}2_j^{(i)}(t) = A^{(i)} m_j^{(i)}(t), \qquad \forall i \varepsilon \{1,\dots,M\}$$
$$\forall j \varepsilon \{1,\dots,n\},\ \forall t \varepsilon [0,2\pi]$$

where $A^{(i)}$ can be explicitly determined $\forall i \varepsilon \{1,\dots,M\}$.

Since $X2^{(i)}(S2_j^{(i)}) \approx \int_0^{2\pi} \hat{g}2_j^{(i)}(t)\,dt, \quad \forall i \varepsilon \{1,\dots,M\}$
$\forall j \varepsilon \{1,\dots,n\}$ (see (8.))

it is clear that:

$$(28.) \quad X2^{(i)}(S2_j^{(i)}) \approx A^{(i)} \int_0^{2\pi} m_j^{(i)}(t)\,dt$$

where $m_j^{(i)}$ is referred to as the particle intercept counting function. Recall that $X2^{(i)}(S2_j^{(i)})$ refers to a total particle volume per unit area. Since the images of the particle intercept counting function cannot be easily expressed in terms of t mathematically, numerical techniques must be used to estimate the right-hand side of (28.). The main purpose of the downed woody fuel model is then to determine constants $C_\omega^{(i)}$, $P_\omega^{(i)}$ and $Q^{(i)}$ such that:

$$(29.) \quad A^{(i)} \int_0^{2\pi} m_j^{(i)}(t)\,dt \approx A^{(i)} \sum_{\omega=1}^{Q^{(i)}} C_\omega^{(i)} \cdot m_j^{(i)}(P_\omega^{(i)})$$

Due to time constraints $C^{(i)}$, $P^{(i)}$ and $Q^{(i)}$ are determined only for randomly distributed fuel particles with respect to $i \varepsilon \{1,2,3\}$ as previously defined.

In order to find $C^{(i)}$, $P^{(i)}$ and $Q^{(i)}$, $\forall i \varepsilon \{1,2,3\}$, three different integration techniques were proposed. Each was examined at five different levels of sampling intensity, namely 6, 8, 10, 12 and 14 transects. This range was chosen because preliminary investigations with randomly distributed matchsticks suggested that the desired level of sampling intensity lay somewhere between 6 and 14 transects. The three techniques tested were: Gaussian-Legendre (Scheid, 1968; p.126), Simpson's rule (Scheid, 1968; p.108) and the average function value method (Thomas, 1951; p.257). Thus in all 15 techniques were examined for each particle diameter size class. Two criteria were used as a basis for selection of the optimum integration formulas. The first criterion stipulated that the integration formula of interest was required to yield a volume estimate within 10% of the corresponding true volume for each of at least 90% of the circular sampling units simulated. The second criterion stated that the integration formula of interest must yield a volume estimate within 15% of the corresponding true volume for each of at least 97.5%, of the circular sampling units simulated. Obviously if more than one of these integration formulas satisfied these criteria for a particular diameter size class, selection was based on the level of sampling intensity required and ease in application and understanding. After conducting several runs of the downed woody fuels model for each diameter size class, the technique chosen using the above criteria was Simpson's rule with 12 transects. It is important to realize that this formula worked best for all particle diameter size classes tested. Hence it was found that:

$$\left.\begin{aligned} C_\omega^{(i)} \equiv C_\omega &= \begin{cases} 4, & \text{if w is odd} \\ 2, & \text{if w is even} \end{cases} \\ P_\omega^{(i)} &= t + \frac{\pi w}{6}, \text{ for arbitrary } t \ \boxed{0,2\pi} \\ Q^{(i)} &= 12 \end{aligned}\right\} , \forall i \epsilon \{1,2,3\}$$

Hence it may now be asserted that:

$$(30)\quad X2^{(i)}(S2_j^{(i)}) \approx A^{(i)} \sum_{\omega=1}^{12} C_\omega m_j^{(i)}\left(t + \frac{\pi w}{6}\right),\ t \text{ arbitrary in } [0,2\pi]$$
$$\forall i \varepsilon \{1,2,3\}$$
$$\forall j \varepsilon \{1,\ldots,n\}$$

It is important to realize that t is arbitrary in $[0,2\pi]$. The reason for t being allowed to be arbitrary in $[0,\ 2\pi]$ will now be examined in more detail. For fixed i and j as above, let $h2_j^{(i)}$ be a random variable defined by the following:

$$h2_j^{(i)}(t) = \sum_{\omega=1}^{12} C_\omega \hat{g}2_j^{(i)}\left(t + \frac{\pi\omega}{6}\right),\ \forall t \varepsilon [0,2\pi],\quad \text{where } C_\omega = \begin{cases} 4, & \text{if w is odd} \\ 2, & \text{if w is even} \end{cases}$$

Furthermore for fixed i and j as previously specified let $V_j^{(i)}$ be the true total particles volume in the jth sampling unit taken with respect to the ith particle diameter size class. Then the random variable defined by: $(V_j^{(i)}/\pi(r^{(i)})^2 - h2_j^{(i)})/(V_j^{(i)}/\pi(r^{(i)})^2)$, i and j fixed, was found to be normally distributed with mean 0 and small standard deviation (usually under 0.05) reasonably independent of i and j, $\forall$i and j, as previously specified. This means that (30.) holds independent of the choice of t. Although this result may seem obvious, it is nevertheless an important one to confirm, since it significantly simplifies the application of the common point intersect concept.

A typical run of the downed woody fuels model designed for a PDP-11 computer is presented below. This is only one run with one choice of three tested particle length distributions and one choice of many tested particle frequency ranges (see Appendix II). As previously indicated, this run is set up for randomly distributed particles. However it can easily be altered to accommodate non-randomly distributed particles. The downed woody fuel model is almost self-explanatory. Numerous comment statements have been inserted to assist the reader in his understanding of the computations and procedures involved.

```
$JOB FUEL[131,1]
DATE:-20-MAR-75
TIME:-13:02:48

$DEL *.*
PIP  V10-02
#*.*/DE

#
$EXECUTE
FORTRAN  V06.13
#SY:FUEL  ,LP:<BI:/ON/CK/GO
```

-115-

```
                C     THIS PROGRAM IS PRIMARILY A ONE-WAY CLASSIFICATION ANOVA RANDOM
                C     COMPONENTS MODEL SET UP FOR SIMULTANEOUS RUNS. THIS MEANS THAT
                C     WITH ONLY SLIGHT ALTERATIONS, IT CAN ALSO BE USED AS A TWO-WAY
                C     CLASSIFICATION ANOVA MODEL.
0001                  BYTE TIMER(8)
0002                  DIMENSION TLOCTS(24),NUM(5),TLOCT(24),COSL(24),SINL(24),TANL(24),
                     1CON1(24),CON2(24),RDCOEF(5,4),SUM(24),N(24),YIJK(5,8,2
                     24),YIJDOT(5,8),YIDDOT(5),YDJDOT(8),T(5,8),E(5,8,24),SVI1J1(5,8),SD
                     3I1J1(5,8),NSAMP(5,8),RANKDI(200),RANKCS(200),SCOLJ(8),SDCOLJ(8),SS
                     4B1WJ(8),FMB1WJ(8),SSW1WJ(8),FMW1WJ(8),TS1WJ1(8),VALIJK(5,8,24),VAL
                     5STA(5,8)
0003                  EQUIVALENCE (SDCOLJ,SCOLJ,CON2),(SSB1WJ,FMB1WJ,CON1),(SSW1WJ,FMW1W
                     1J,TLOCT),(SVI1J1,SDI1J1,TLOCTS),(VALIJK,YIJK)
0004                  KA=0
0005                  KB=0
                C     CV = COMMON RADIUS OF CIRCULAR SAMPLING UNITS.
0006                  READ(5,6) CV
0007            6     FORMAT(F5.2)
                C     K11 = TRANSECT NUMBER
0008                  DO 402 K11=1,24
0009                  TLOCTS(K11)=(K11-1)*6.283186/24.
0010            402   CONTINUE
                C     NUM = NUMBER OF POPULATION ELEMENTS EMBEDDED IN THE SAMPLING UNIT
                C             CONSIDERED.
0011                  READ(5,200) NUM
0012            200   FORMAT(5I5)
                C     TLMIN =  MINIMUM PARTICLE LENGTH
                C     TLMAX = MAXIMUM PARTICLE LENGTH
                C     SECB   = MEAN SECANT FOR PARTICLE TILT
                C     DBAR   = QUADRATIC MEAN DIAMETER
0013                  READ(5,201) TLMIN,TLMAX,SECB,DBAR
0014            201   FORMAT(4F8.6)
0015                  DEFINE FILE 1(200,2,U,JVAR)
0016                  DEFINE FILE 2(200,2,U,KVAR)
0017                  DEFINE FILE 3(960,1,U,IVAR)
0018                  IVAR=1
                C     I11 = FUEL LOADING NUMBER
                C     LOOP OVER FUEL LOADING
0019                  DO 114 I11=1,5
0020                  NFUELP=NUM(I11)
                C     LOOP OVER FUEL DISTRIBUTION
0021                  DO 116 J11=1,8
0022                  CALL TIME(TIMER)
0023                  WRITE(6,777) TIMER
0024            777   FORMAT(25X,'THE TIME IS',8A1)
0025                  DO 607 K11=1,24
0026                  SUM(K11)=0.
0027                  N(K11)=0
0028            607   CONTINUE
                C     TLOCT0 = LOCATION OF INITIAL TRANSECT
0029                  TLOCT0=6.283186*RAN(KA,KB)
0030                  JVAR=1
0031                  KVAR=1
0032                  ICOUN=0
                C     THIS SET OF NESTED DO'S (UP TO 115) FINDS THOSE PARTICLES (IN A
```

```
              C     GIVEN SAMPLING UNIT) WHICH INTERSECT EACH TRANSECT CONSIDERED.
              C     NEXT THE LOOP RECORDS THE DISTANCE (DINT) TO INTERSECTION AND
              C     COSECANT (CSC) OF THE ANGLE OF INTERSECTION FOR EACH PARTICLE.
              C     EACH PAIR IS THEN MULTIPLIED TOGETHER AND THE PRODUCTS ADDED UP
              C     WITH RESPECT TO EACH TRANSECT. THEN EACH RESULTANT PRODUCT (ONE
              C     FOR EACH TRANSECT IN THE GIVEN SAMPLING UNIT) IS DIVIDED BY THE
              C     NUMBER(N) OF PARTICLES INTERSECTING ITS CORRESPONDING TRANSECT.
              C     EACH AVERAGE (YIJK) SO OBTAINED COMPRISES ONE OBSERVATION FOR THE
              C     CELL(SAMPLING UNIT) BEING CONSIDERED. EACH CELL HAS NSAMP AVERAGES
              C     NFUELP = NUMBER OF FUEL PARTICLES IN CURRENT SAMPLING UNIT.
              C     LOOP OVER FUEL PARTICLES
0033                DO 115 M=1,NFUELP
0034                II=0
              C     LOOP OVER TRANSECTS
0035                DO 115 K11=1,24
0036                IF(K11.EQ.1) GO TO 141
0037                GO TO 130
0038          131   TLOCT(K11)=TLOCTS(K11)+TLOCT0
0039                COSL(K11)=COS(TLOCT(K11))
0040                SINL(K11)=SIN(TLOCT(K11))
0041          151   II=II+1
0042                IF(M.NE.1) GO TO 180
0043                CON1(II)=(CV/2.)*COSL(K11)
0044                CON2(II)=(CV/2.)*SINL(K11)
0045          180   IF(COSL(K11).EQ.0.) GO TO 550
0046                TANL(K11)=SINL(K11)/COSL(K11)
0047                IF(COST.EQ.0.) GO TO 555
0048                IF(TANT.EQ.TANL(K11)) GO TO 115
0049                XIN=YINT/(TANL(K11)-TANT)
0050                IF(XIN.GE.XMIN.AND.XIN.LE.XMAX) GO TO 565
0051                GO TO 115
0052          550   IF(COST.EQ.0.) GO TO 115
0053                IF(0..GE.XMIN.AND.0..LE.XMAX) GO TO 580
0054                GO TO 115
0055          555   YIN=TANL(K11)*XMIN
0056                IF(YIN.GE.YMIN.AND.YIN.LE.YMAX) GO TO 585
0057                GO TO 115
0058          580   IF(YINT.GE.AMIN1(0.,CON2(II)).AND.YINT.LE.AMAX1(0.,CON2(II))) GO
                   1TO 906
0059                GO TO 115
0060          585   IF(XMIN.GE.AMIN1(0.,CON1(II)).AND.XMIN.LE.AMAX1(0.,CON1(II))) GO
                   1TO 907
0061                GO TO 115
0062          565   IF(XIN.GE.AMIN1(0.,CON1(II)).AND.XIN.LE.AMAX1(0.,CON1(II))) GO TO
                   1605
0063                GO TO 115
0064          605   YIN=TANT*XIN+YINT
0065                GO TO 606
0066          906   YIN=YINT
0067          907   XIN=XMIN
0068          606   N(K11)=N(K11)+1
0069                DINT=SQRT(XIN**2+YIN**2)
0070                ICOUN=ICOUN+1
0071                IF(ICOUN.GT.200.OR.J11.GT.4) GO TO 952
0072                WRITE(1'JVAR) DINT
0073          952   ANGINT=ABS(TLOCT(K11)-THETA)
```

```
0074          XX=ABS(SIN(ANGINT))
        C     AN INTERSECTION IS SAID TO OCCUR ONLY IF BOTH THE PARTICLE CENTRAL
        C     AXIS AND AT LEAST ONE PARTICLE EDGE INTERSECT THE TRANSECT LINE.
        C     IT IS ASSUMED THAT THE INVERSE SINE OF THE RATIO OF FUEL PARTICLE
        C     DIAMETER TO TWICE THE FUEL PARTICLE LENGTH IS GREATER THAN OR
        C     EQUAL TO .00451 RADIANS (15.5 MINUTES).
0075          IF(XX.LT..00455) XX=.00455
0076          CSC=1./XX
0077          IF(ICOUN.GT.200.OR.J11.GT.4) GO TO 953
0078          WRITE(2'KVAR) CSC
0079    953   SUM(K11)=SUM(K11)+DINT*CSC
0080    115   CONTINUE
0081          ICOUNT=0
0082          SIJKOK=0.
0083          DO 900 K11=1,24
0084          IF(N(K11).EQ.0) GO TO 608
0085          YIJK(I11,J11,K11)=SUM(K11)/N(K11)
0086          GO TO 609
0087    608   ICOUNT=ICOUNT+1
0088          YIJK(I11,J11,K11)=0.
0089    609   SIJKOK=SIJKOK+YIJK(I11,J11,K11)
0090    900   CONTINUE
0091          NSAMP(I11,J11)=24-ICOUNT
        C     YIJDOT = MATRIX WHICH CONTAINS THE SAMPLE MEANS FOR THE CELLS.
0092          YIJDOT(I11,J11)=SIJKOK/NSAMP(I11,J11)
0093          IF(J11.GT.4) GO TO 998
0094          ID=JVAR-1
0095          WRITE(6,1002) I11,J11,ID
0096    1002  FORMAT(' ','THE NUMBER OF SAMPLE VALUES TAKEN INTO ACCOUNT IN THE
             1COMPUTATION OF RDCOEF'/' ','WITH RESPECT TO LOADING NUMBER',I5,'AN
             2D DISTRIBUTION NUMBER',I5,'=',I5)
        C     RDCOEF = SPEARMAN'S RANK-DIFFERENCE COEFFICIENT
        C     FROM HERE TO STATEMENT NUMBER 0145, THE PROGRAM SIMPLY COMPUTES
        C     RDCOEF.
0097          JVAR=1
0098          DO 908 KKK=1,ID
0099          READ(1'JVAR) RANKDI(KKK)
0100    908   CONTINUE
0101          INDIC=0
0102    626   ITALLY=1
0103          ISCORE=ID
0104    957   INOT=0
0105          IIN=1
0106          IF(ITALLY.EQ.1) GO TO 1417
0107          ITALES=ITALLY-1
0108    1416  CONTINUE
0109          DO 955 ITAL=1,ITALES
0110          IF(RANKDI(IIN).EQ.ITAL) GO TO 1411
0111    955   CONTINUE
0112          GO TO 1417
0113    1411  IIN=IIN+1
0114          GO TO 1416
0115    1417  CONTINUE
0116          DO 610 KKK=1,ID
0117          IF(ITALLY.EQ.1) GO TO 1418
0118          DO 1412 ITAL=1,ITALES
```

```
              IF(RANKDI(KKK).EQ.ITAL) GO TO 610
      1412    CONTINUE
      1418    CONTINUE
              IF(RANKDI(IIN).LE.RANKDI(KKK)) GO TO 610
              IIN=KKK
      610     CONTINUE
              SVDINT=RANKDI(IIN)
              RANKDI(IIN)=ITALLY
              DO 621 KKK=1,ID
              IF(RANKDI(KKK).NE.SVDINT) GO TO 621
              IF(KKK.EQ.IIN) GO TO 621
              RANKDI(KKK)=ITALLY
              INOT=INOT+1
      621     CONTINUE
              IF(INOT.EQ.0) GO TO 956
              WRITE(6,951) INOT,SVDINT,INDIC,ITALLY
      951     FORMAT(' ','NUMBER OF TIES HERE =',I5,E10.4,2I5)
      956     ITALLY=ITALLY+1
              ISCORE=ISCORE-(INOT+1)
              IF(ISCORE.GT.0) GO TO 957
              IF(INDIC.EQ.1) GO TO 623
              INDIC=INDIC+1
              JVAR=1
              KVAR=1
              DO 909 KKK=1,ID
              WRITE(1'JVAR) RANKDI(KKK)
              READ(2'KVAR) RANKDI(KKK)
      909     CONTINUE
              GO TO 626
      623     SDRAN2=0.
              JVAR=1
              DO 910 KKK=1,ID
              READ(1'JVAR) RANKCS(KKK)
      910     CONTINUE
              DO 911 KKK=1,ID
              SDRAN2=SDRAN2+(RANKCS(KKK)-RANKDI(KKK))**2
      911     CONTINUE
              RDCOEF(I11,J11)=1.-6.*SDRAN2/(((ID*1.)**2-1.)*1.*ID)
      998     DO 901 K11=1,24
              WRITE(3'IVAR) N(K11)
      901     CONTINUE
      116     CONTINUE
              SIJDOJ=0.
              DO 628 J11=1,8
              SIJDOJ=SIJDOJ+YIJDOT(I11,J11)
      628     CONTINUE
      C       YIDDOT = MATRIX WHICH CONTAINS THE OVERALL SAMPLE MEAN FOR EACH
      C                RUN.
              YIDDOT(I11)=SIJDOJ/8.
      114     CONTINUE
              NSSAMP=0
              DO 630 J11=1,8
              SYDJD=0.
              DO 629 I11=1,5
              SYDJD=SYDJD+YIJDOT(I11,J11)
      C       NSSAMP = THE TOTAL NUMBER OF OBSERVATIONS IN ALL CELLS FOR ALL
```

```
      C           SIMULTANEOUS RUNS.
            NSSAMP=NSSAMP+NSAMP(I11,J11)
      629   CONTINUE
      C     YDJDOT = MATRIX WHICH CONTAINS THE OVERALL SAMPLE MEAN TAKEN OVER
      C           CORRESPONDING CELLS IN ALL RUNS.
            YDJDOT(J11)=SYDJD/5.
      630   CONTINUE
            SYDDD=0.
            DO 631 I11=1,5
            SYDDD=SYDDD+YIDDOT(I11)
      631   CONTINUE
      C     YDDDOT = OVERALL SAMPLE MEAN TAKEN OVER ALL CELLS AND ALL RUNS.
            YDDDOT=SYDDD/5.
            SOVERP=0.
            DO 638 I11=1,5
            DO 638 J11=1,8
            SVI1J1(I11,J11)=0.
            DO 635 K11=1,24
      C     SOVERP = OVERALL SAMPLE STANDARD DEVIATION TAKEN OVER ALL CELLS
      C           AND ALL RUNS.
            SOVERP=SOVERP+(YIJK(I11,J11,K11)-YDDDOT)**2
      C     SVI1J1 = MATRIX WHICH CONTAINS THE SAMPLE STANDARD DEVIATION FOR
      C           EACH CELL.
            SVI1J1(I11,J11)=SVI1J1(I11,J11)+(YIJK(I11,J11,K11)-YIJDOT(I11,J11)
           1)**2
      C     E = MATRIX WHICH CONTAINS DIFFERENCES WITHIN CELLS FOR ALL RUNS.
            E(I11,J11,K11)=YIJK(I11,J11,K11)-YIJDOT(I11,J11)
      635   CONTINUE
      C     T = MATRIX WHICH CONTAINS DIFFERENCES BETWEEN CELLS FOR ALL RUNS.
            T(I11,J11)=YIJDOT(I11,J11)-YDJDOT(J11)
            SOVERP=SOVERP-((24-NSAMP(I11,J11))*YDDDOT**2)
            SVI1J1(I11,J11)=SVI1J1(I11,J11)-((24-NSAMP(I11,J11))*YIJDOT(I11,
           1J11)**2)
            SDI1J1(I11,J11)=SQRT(SVI1J1(I11,J11)/(NSAMP(I11,J11)-1))
      638   CONTINUE
            SSDFE=0.
            SSDFT=0.
            DO 8 I11=1,5
            XMEFT=0.
            SDFT=0.
            DO 13 K11=1,24
            XMEFE=0.
            SDFE=0.
            ICO=0
            DO 10 J11=1,8
            IF(YIJK(I11,J11,K11).EQ.0.) GO TO 22
            XMEFE=XMEFE+E(I11,J11,K11)
            GO TO 10
      22    ICO=ICO+1
      10    CONTINUE
            XMEFE=XMEFE/(8-ICO)
            DO 11 J11=1,8
            IF(YIJK(I11,J11,K11).EQ.0.) GO TO 11
            SDFE=SDFE+(E(I11,J11,K11)-XMEFE)**2
      11    CONTINUE
            SDFE=SDFE/(7-ICO)
```

```
0216            SSDFE=SSDFE+SDFE
0217      13    CONTINUE
0218            DO 9 J11=1,8
0219            XMEFT=XMEFT+T(I11,J11)
0220      9     CONTINUE
0221            XMEFT=XMEFT/8.
0222            DO 12 J11=1,8
0223            SDFT=SDFT+(T(I11,J11)-XMEFT)**2
0224      12    CONTINUE
0225            SDFT=SDFT/7.
0226            SSDFT=SSDFT+SDFT
0227      8     CONTINUE
0228            SDFE=SSDFE/120.
0229            SDFT=SSDFT/5.
0230            SDFE=SQRT(SDFE)
0231            SDFT=SQRT(SDFT)
0232            DO 17 J11=1,8
0233            DO 17 I11=1,5
0234            DO 18 K11=1,24
0235            E(I11,J11,K11)=E(I11,J11,K11)/SDFE
0236            IF(YIJK(I11,J11,K11).NE.0.) GO TO 18
0237            WRITE(6,15) I11,J11,K11,E(I11,J11,K11)
0238      15    FORMAT(' ','WE HAVE A 0 ENTRY AT',3I3,'HENCE IGNORE VALUE',E16.6)
0239      18    CONTINUE
0240            T(I11,J11)=T(I11,J11)/SDFT
0241      17    CONTINUE
0242            SDOVEP=SQRT(SOVERP/(NSSAMP-1))
0243            DO 640 J11=1,8
0244            SCOLJ(J11)=0.
0245            SNSAOI=0.
0246            SSB1WJ(J11)=0.
0247            SSW1WJ(J11)=0.
0248            DO 642 I11=1,5
0249            SNSAOI=SNSAOI+NSAMP(I11,J11)
          C     SSB1WJ = MATRIX WHICH CONTAINS THE SUM OF SQUARES BETWEEN CELLS
          C              FOR EACH RUN.
0250            SSB1WJ(J11)=SSB1WJ(J11)+NSAMP(I11,J11)*(YIJDOT(I11,J11)-
               1YDJDOT(J11))**2
          C     SSW1WJ = MATRIX WHICH CONTAINS THE SUM OF SQUARES WITHIN CELLS FOR
          C              EACH RUN.
0251            SSW1WJ(J11)=SSW1WJ(J11)+(NSAMP(I11,J11)-1)*SDI1J1(I11,J11)**2
0252            DO 942 K11=1,24
0253            SCOLJ(J11)=SCOLJ(J11)+(YIJK(I11,J11,K11)-YDJDOT(J11))**2
0254      942   CONTINUE
0255            SCOLJ(J11)=SCOLJ(J11)-((24-NSAMP(I11,J11))*YDJDOT(J11)**2)
0256      642   CONTINUE
          C     FMB1WJ = MATRIX WHICH CONTAINS THE MEAN SUM OF SQUARES BETWEEN
          C     CELLS FOR EACH RUN.
0257            FMB1WJ(J11)=SSB1WJ(J11)/4.
0258            SDCOLJ(J11)=SQRT(SCOLJ(J11)/(SNSAOI-1.))
          C     FMW1WJ = MATRIX WHICH CONTAINS THE MEAN SUM OF SQUARES WITHIN
          C              CELLS FOR EACH RUN.
0259            FMW1WJ(J11)=SSW1WJ(J11)/(SNSAOI-5.)
0260      640   CONTINUE
0261            DO 801 J11=1,8
0262            TS1WJ1(J11)=FMB1WJ(J11)/FMW1WJ(J11)
```

```
0263   801  CONTINUE
0264        GO TO 643
       C    STATEMENT NUMBER 0246 TO 0267 IS AN EXTENDED RANGE OF THE 115DO
       C    LOOP. THIS PORTION OF THE PROGRAM POSITIONS THE GEOMETRIC CENTER
       C    POINT OF EACH PARTICLE AND DEFINES THE ANGULAR ORIENTATION (THETA)
       C    AND LENGTH (TL) OF THE CENTRAL AXIS OF EACH PARTICLE.
       C    ALL OTHER TERMS IN THIS ANOVA MODEL ARE CLEAR FROM PREVIOUS
       C    COMMENTS AND THE FORMAT STATEMENTS BELOW.
0265   141  THETA=6.283186*RAN(KA,KB)
0266        XC=CV*RAN(KA,KB)-CV/2.
0267        YC=CV*RAN(KA,KB)-CV/2.
0268        COST=COS(THETA)
0269        SINT=SIN(THETA)
0270        TL=(TLMAX-TLMIN)*RAN(KA,KB)+TLMIN
0271        TLH=TL/SECB
0272        COST=TLH*COST/2.
0273        SINT=TLH*SINT/2.
0274        X1=XC+COST
0275        Y1=YC+SINT
0276        X2=XC-COST
0277        Y2=YC-SINT
0278        IF(COST.EQ.0.) GO TO 556
0279        TANT=SINT/COST
0280        YINT=YC-TANT*XC
0281   556  XMIN=AMIN1(X1,X2)
0282        XMAX=AMAX1(X1,X2)
0283        YMIN=AMIN1(Y1,Y2)
0284        YMAX=AMAX1(Y1,Y2)
0285   130  IF(M.EQ.1) GO TO 131
0286        GO TO 151
0287   643  IVAR=1
0288        DO 902 I11=1,5
0289        DO 902 J11=1,8
0290        SFORM=0.
0291        DO 903 K11=1,24
0292        READ(3'IVAR) N(K11)
0293        IF(YIJK(I11,J11,K11).EQ.0.) GO TO 878
0294        VALIJK(I11,J11,K11)=(YIJK(I11,J11,K11)-YDDDOT)/YIJK(I11,J11,K11)
0295        GO TO 879
0296   878  VALIJK(I11,J11,K11)=0.
0297   879  SFORM=SFORM+VALIJK(I11,J11,K11)
0298   903  CONTINUE
0299        SFORM=SFORM/NSAMP(I11,J11)
0300        SDIJK=0.
0301        DO 904 K11=1,24
0302        SDIJK=SDIJK+(VALIJK(I11,J11,K11)-SFORM)**2
0303   904  CONTINUE
0304        SDIJK=SDIJK-((24-NSAMP(I11,J11))*SFORM**2)
0305        SDIJK=SQRT(SDIJK/(NSAMP(I11,J11)-1))
0306        WRITE(6,1003) I11,J11,SFORM,SDIJK
0307  1003  FORMAT(' ','THE SAMPLE MEAN AND STANDARD DEVIATION FOR THE ERROR T
           1ERMS WITH RESPECT TO LOADING NUMBER',I5/,' ','AND DISTRIBUTION NUM
           2BER',I5,'ARE RESPECTIVELY',E16.6,'AND',E16.6)
0308        DO 905 K11=1,24
0309        VALIJK(I11,J11,K11)=VALIJK(I11,J11,K11)/SDIJK
0310   905  CONTINUE
```

-123-

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0311          VALSTA(I11,J11)=ABS(SFORM)/(SDIJK/SQRT(1.*NSAMP(I11,J11)))
0312          WRITE(6,688) I11,J11,(N(K11),K11=1,24)
0313    688   FORMAT(' ','LOADING=',I3,10X,'DISTRIBUTION=',I3/' ',24I5)
0314    902   CONTINUE
0315          WRITE(6,644)
0316          WRITE(6,645) ((NSAMP(I11,J11),J11=1,8),I11=1,5)
0317    644   FORMAT(' ','THE (I,J) ENTRY IN THE FOLLOWING 5*8 MATRIX REPRESENTS
             1THE NUMBER OF SAMPLES'/' ','IN CELL(I,J), WHERE I STANDS FOR FUEL
             2LOADING AND J FOR FUEL DISTRIBUTION.')
0318    645   FORMAT(' ',8(5X,I2))
0319          WRITE(6,646)
0320          WRITE(6,647) ((RDCOEF(I11,J11),J11=1,4),I11=1,5)
0321    646   FORMAT(' ','THE (I,J) ENTRY IN THE FOLLOWING 5*4 MATRIX REPRESENTS
             1SPEARMANS RANK DIFFERENCE COEFFICIENT'/' ','FOR CELL(I,J) WITH
             2RESPECT TO THE FOLLOWING TWO VARIABLES'/' ','(1.) DISTANCE TO
             3POINT OF INTERSECTION BETWEEN TNE TRANSECT AND FUEL PARTICLE
             4CENTRAL AXIS'/' ','(2.) COSECANT OF THE ANGLE OF INTERSECTION
             5IMPLICATED IN (1.).')
0322    647   FORMAT(' ',4(5X,E16.4))
0323          WRITE(6,648)
0324          WRITE(6,649) ((YIJDOT(I11,J11),J11=1,8),I11=1,5)
0325    648   FORMAT(' ','THE (I,J) ENTRY IN THE FOLLOWING 5*8 MATRIX REPRESENTS
             1THE SAMPLE MEAN IN CELL(I,J).')
0326    649   FORMAT(' ',8E16.4)
0327          WRITE(6,650)
0328          WRITE(6,651) (YIDDOT(I11),I11=1,5)
0329    650   FORMAT(' ','THE ITH ENTRY IN THE FOLLOWING COLUMN VECTOR
             1REPRESENTS THE SAMPLE MEAN'/' ','TAKEN OVER THE ITH ROW OF CELLS')
0330    651   FORMAT(' ',30X,E16.4)
0331          WRITE(6,652)
0332          WRITE(6,649) (YDJDOT(J11),J11=1,8)
0333    652   FORMAT(' ','THE JTH ENTRY IN THE FOLLOWING ROW VECTOR REPRESENTS
             1THE SAMPLE MEAN'/' ','TAKEN OVER THE JTH COLUMN OF CELLS.')
0334          WRITE(6,654) YDDDOT
0335    654   FORMAT(' ','THE OVERALL SAMPLE MEAN =',E16.4)
0336          WRITE(6,657) NSSAMP
0337    657   FORMAT(' ','TOTAL NUMBER OF COMPARTMENTS OR SAMPLES =',I5)
0338          WRITE(6,661)
0339          WRITE(6,649) (FMB1WJ(J11),J11=1,8)
0340    661   FORMAT(' ','THE JTH ENTRY IN THE FOLLOWING ROW VECTOR REPRESENTS M
             1SB'/' ','WITH RESPECT TO THE JTH COLUMN.')
0341          WRITE(6,662)
0342          WRITE(6,649) (FMW1WJ(J11),J11=1,8)
0343    662   FORMAT(' ','THE JTH ENTRY IN THE FOLLOWING ROW VECTOR REPRESENTS M
             1SW'/' ','WITH RESPECT TO THE JTH COLUMN.')
0344          WRITE(6,663)
0345          WRITE(6,649) (TS1WJ1(J11),J11=1,8)
0346    663   FORMAT(' ','THE JTH ENTRY IN THE FOLLOWING ROW VECTOR REPRESENTS T
             1HE RATIO OF'/' ','MSB TO MSW WITH RESPECT TO THE JTH COLUMN.')
0347          WRITE(6,664)
0348          WRITE(6,649) ((SDI1J1(I11,J11),J11=1,8),I11=1,5)
0349    664   FORMAT(' ','THE (I,J) ENTRY IN THE FOLLOWING 5*8 MATRIX REPRESENTS
             1THE SAMPLE STANDARD DEVIATION IN CELL(I,J).')
0350          WRITE(6,666)
0351          WRITE(6,649) (SDCOLJ(J11),J11=1,8)
0352    666   FORMAT(' ','THE JTH ENTRY IN THE FOLLOWING ROW VECTOR REPRESENTS T
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                  1HE SAMPLE STANDARD DEVIATION'/' ','TAKEN OVER THE JTH COLUMN OF CE
                  2LLS.')
0353              WRITE(6,667) SDOVEP
0354      667     FORMAT(' ',10X,'THE OVERALL SAMPLE STANDARD DEVIATION =',E16.4)
0355              WRITE(6,668)
0356              WRITE(6,669) (((VALIJK(I11,J11,K11),K11=1,24),J11=1,8),I11=1,5)
0357      668     FORMAT(' ','IN THE FOLLOWING ARRAY EACH PAIR OF ROWS CONTAINS 24 S
                  1AMPLE VALUES(12 VALUES PER ROW)'/' ','COMPUTED FOR A PARTICULAR SA
                  2MPLING UNIT. THE ARRAY CONSISTS OF FIVE GROUPS OF VALUES,EACH GROU
                  3P'/,' ','CONTAINING EIGHT SUBGROUPS, WHERE EACH SUBGROUP CONSISTS
                  4OF A PAIR OF ROWS. EACH GROUP'/' ','REFERS TO A FUEL LOADING CLASS
                  5, AND EACH SUBGROUP REFERS TO A FUEL DISTRIBUTION IN THAT CLASS.')
0358      669     FORMAT(' ',6E16.4)
0359              WRITE(6,670)
0360              WRITE(6,671) ((VALSTA(I11,J11),J11=1,8),I11=1,5)
0361      670     FORMAT(' ','THE (I,J) ENTRY IN THE FOLLOWING MATRIX REFERS TO A SA
                  1MPLE STATISTIC COMPUTED FOR CELL(I,J).'/' ','THIS ENTRY WILL BE CO
                  2MPARED TO DIFFERENT VALUES TAKEN ON BY THE T DISTRIBUTION WITH'/'
                  3',' (NSAMP(I11,J11)-1) DEGREES OF FREEDOM.')
0362      671     FORMAT(' ',8E16.4)
0363              WRITE(6,672) SDFT
0364      672     FORMAT(' ','IF A COMMON STANDARD DEVIATION DOES EXIST FOR THE T TE
                  1RMS,AN ESTIMATE FOR IT IS',E16.4)
0365              WRITE(6,673) SDFE
0366      673     FORMAT(' ','IF A COMMON STANDARD DEVIATION DOES EXIST FOR THE E TE
                  1RMS,AN ESTIMATE FOR IT IS',E16.4)
0367              WRITE(6,674)
0368              WRITE(6,675) ((T(I11,J11),J11=1,8),I11=1,5)
0369      674     FORMAT(' ', 'IF THE T TERMS IN THE ANOVA MODEL ARE NORMALLY DISTRI
                  1BUTED WITH 0 MEAN AND COMMON VARIANCE'/' ','WE WOULD EXPECT 95 PER
                  2CENT OF THE VALUES IN THE FOLLOWING MATRIX TO LIE BETWEEN -2.36 AN
                  3D 2.36'/' ','SINCE THE VALUES BELOW ARE EXPECTED TO LIE ROUGHLY ON
                  3 A T DISTRIBUTION WITH 7 DEGREES OF FREEDOM.')
0370      675     FORMAT(' ',8E16.4)
0371              WRITE(6,676)
0372              WRITE(6,677) (((E(I11,J11,K11),J11=1,8),K11=1,24),I11=1,5)
0373      676     FORMAT(' ','SIMILAR TO THE T MATRIX ABOVE WE EXPECT 95 PERCENT OF
                  1THE VALUES IN THE FOLLOWING E MATRIX'/' ','-FOR FIXED I11 AND J11-
                  2TO LIE BETWEEN T.025 AND T.975 WHERE T HAS (NSAMP(I11,J11)-1) DEGR
                  3EES OF FREEDOM.')
0374      677     FORMAT(' ',8E16.4)
0375              CALL EXIT
0376              END

$EOD

          ROUTINES CALLED:
          TIME  , RAN   , COS   , SIN   , AMIN1 , AMAX1 , SQRT
          ABS   , EXIT

          OPTIONS =/ON,/CK,/OP:1,/GO

          BLOCK     LENGTH
          MAIN.   11522  (055004)*

          **COMPILER ----- CORE**
            PHASE        USED  FREE
          DECLARATIVES 00622 14422
          EXECUTABLES  01969 13075
          ASSEMBLY     03399 16285
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      C     THIS PROGRAM IS A DOWNED WOODY PARTICLES MODEL. IT IS CURRENTLY
      C     SET UP FOR RANDOMLY DISTRIBUTED WOODY COMPONENTS. HOWEVER IT CAN
      C     EASILY BE ALTERED TO ACCOMMODATE NON-RANDOMLY DISTRIBUTED
      C     PARTICLES. THIS PROGRAM POSITIONS ALL FUEL PARTICLES WITHIN
      C     CIRCULAR SAMPLING UNITS. IT THEN EVALUATES THREE TECHNIQUES EACH
      C     APPLIED AT FIVE DIFFERENT LEVELS OF SAMPLING INTENSITY IN ORDER TO
      C     DETERMINE THE OPTIMUM SAMPLING TECHNIQUES TO USE FOR OBTAINING THE
      C     TOTAL WOODY RESIDUE VOLUMES FOR THE PARTICLE DIAMETER SIZE CLASSES
      C     OF INTEREST.
            BYTE TIMER(8)
            DIMENSION COEFF(14,5,3),TLOCTN(14,5),TLOCT3(14,5,3),NUM(5),TLOCT(1
           14,5,3),COSL(14,5,3),SINL(14,5,3),TANL(14,5,3),CON1(150),CON2(150)
            DIMENSION XIS1D(5),XIS1SD(5),XIVESD(5,3,5)
            REAL ISVMAP,ISVVAP,IVMAP,IVVAP,ISD,ID(40),ID8,ISD2,ISVEM,IVEM,ISVE
           1V,IVEV,IVEST(5,3,40),IS1PUA(8,5)
      C     TLMIN = MINIMUM PARTICLE LENGTH
      C     TLMAX = MAXIMUM PARTICLE LENGTH
      C     SEC8  = MEAN SECANT FOR PARTICLE TILT
      C     DBAR  = QUADRATIC MEAN DIAMETER
            REAL TLMIN,TLMAX,SEC8,DBAR
            DEFINE FILE 2(2,4,U,KVAR)
            KVAR=1
            KA=0
            KB=0
            ICR=1
            KKCR=1
            READ(5,25,END=100) ICODE
      25    FORMAT(I2)
            READ(2'KVAR) KA,KB
            READ(2'KVAR) ICR,KKCR
            WRITE(6,101) ICR,KKCR
      101   FORMAT(10X,'START-UP LOADING AT',I3,5X,'START-UP DISTRIBUTION AT',
           1I3)
      C     IB = SAMPLING INTENSITY
      100   DO 350 IB=1,5
            IBF=2*IB+4
      C     COEFF = MATRIX WHICH CONTAINS THE COEFFICIENTS FOR ALL FIFTEEN
      C                TESTED SAMPLING FORMULAS.
            READ(5,300) (COEFF(IC,IB,2),IC=1,IBF)
      350   CONTINUE
      300   FORMAT(10F8.6)
            DO 351 IB=1,5
            IBE=IB+2
      C     TLOCTN = MATRIX WHICH CONTAINS THE NORMALIZED LOCATIONS (BASED ON
      C                A FIXED 0 ANGLE) FOR ALL TRANSECTS USED TO OBTAIN
      C                PARTICLES DATA FOR FIVE SAMPLING FORMULAS OF INTEREST
      C                (IE. GAUSSIAN-LEGENDRE INTEGRATION FORMULAS)
            READ(5,301) (TLOCTN(IC,IB),IC=1,IBE)
            READ(5,300) (COEFF(IC,IB,1),IC=1,IBE)
      351   CONTINUE
      301   FORMAT(7F9.6)
            DEFINE FILE 3(600,2,U,IVAR)
            DEFINE FILE 1(40,2,U,JVAR)
            IVAR=(((ICR-1)*8+KKCR)*15)-14
            JVAR=(ICR-1)*8+KKCR
```

General fuel model

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      DO 404 IB=1,5
      IBF=2*IB+4
C     IC = NUMBER OF TRANSECTS
      DO 404 IC=1,IBF
      COEFF(IC,IB,3)=6.283186/IBF
404   CONTINUE
      DO 401 IB=1,5
      IBF=2*IB+4
      IND=IBF/2
      IBS=IBF/2+1
      DO 401 IC=IBS,IBF
      TLOCTN(IC,IB)=ABS(TLOCTN(IND,IB))
      COEFF(IC,IB,1)=COEFF(IND,IB,1)
      IND=IND-1
401   CONTINUE
C     IA = INTEGRATION TECHNIQUE(IE, GENERAL SAMPLING FORMULA)
      DO 402 IA=1,2
      DO 402 IB=1,5
      IBF=2*IB+4
      DO 402 IC=1,IBF
      IF(IA.EQ.1) GO TO 403
C     TLOCTS = MATRIX WHICH CONTAINS THE STANDARD LOCATIONS(IE, BASED ON
C                  FIXED 0 ANGLE) FOR ALL TRANSECTS USED TO OBTAIN PARTICLES
C                  DATA FOR ALL SAMPLING FORMULAS OF INTEREST.
      TLOCTS(IC,IB,IA)=(IC-1)*6.283186/IBF
      TLOCTS(IC,IB,IA+1)=TLOCTS(IC,IB,IA)
      GO TO 402
403   TLOCTS(IC,IB,IA)=3.141593*(TLOCTN(IC,IB)+1.)
402   CONTINUE
C     NUM = NUMBER OF POPULATION ELEMENTS EMBEDDED IN THE SAMPLING UNIT
C            CONSIDERED.
      READ(5,200) NUM
200   FORMAT(5I5)
      READ (5,201) TLMIN, TLMAX, SECB, DBAR
201   FORMAT(4F8.6)
      ICC=0
C     AVCON CORRESPONDS TO YODDOT IN THE ANOVA MODEL CONTAINED IN
C     THE PRECEDING ANOVA MODEL.
      READ(5,6) AVCON
6     FORMAT(F5.2)
C     CV = COMMON RADIUS OF CIRCULAR SAMPLING UNITS.
      READ(5,7) CV
7     FORMAT(F5.2)
C     CF = COEFFICIENT WHICH PRECEDES THE INTEGRAL OF THE PARTICLE
C          INTERCEPT COUNTING FUNCTION. CF IS A VEHICLE WHICH PERMITS
C          THE INTEGRAL OF A SIMPLE STEP-FUNCTION TO BE MAPPED INTO A
C          TOTAL PARTICLE VOLUME ESTIMATE.
      CF=DBAR**2*SECB*AVCON/(CV**2)
      DO 222 KKI=1,40
      DO 222 IA=1,3
      DO 222 IB=1,5
      IVEST(IB,IA,KKI)=0.
222   CONTINUE
      DO 223 I=1,5
      DO 223 KK=1,8
      IS1PUA(KK,I)=0.
```

```
      223  CONTINUE
      C    THIS SET OF NESTED DO'S (UP TO 116) FIRST FINDS THOSE PARTICLES
      C    (IN A PARTICULAR SAMPLING UNIT) WHICH INTERSECT EACH TRANSECT
      C    CONSIDERED FOR EACH OF THE FIFTEEN SAMPLING FORMULAS OF INTEREST.
      C    WITH THIS INFORMATION, THE SET OF DO'S COMPUTES FIFTEEN TOTAL
      C    VOLUME ESTIMATES FOR EACH OF THE FORTY SAMPLING UNITS SIMULATED
      C    WITH RESPECT TO A GIVEN PARTICLE DIAMETER SIZE CLASS. THESE VALUES
      C    ARE STORED IN IVEST. THEN THIS SET OF DO'S CONSIDERS EACH PARTICLE
      C    SIMULATED AND DETERMINES THAT PORTION OF IT WHICH INTERSECTS THE
      C    SAMPLING UNIT OF INTEREST. THIS PORTION IS THEN CONVERTED TO A
      C    PARTIAL PARTICLE VOLUME. THROUGH SUMMATION OVER RELEVANT
      C    PARTICLES, THIS LAST PROCESS RESULTS IN THE TRUE TOTAL PARTICLES
      C    VOLUME PER UNIT AREA (STORED IN IS1PUA) WITHIN EACH OF THE FORTY
      C    SAMPLING UNITS SIMULATED.
      C    LOOP OVER FUEL LOADING
           DO 116 I=ICR,5
           NP=NUM(I)
      C    LOOP OVER FUEL DISTRIBUTION
           DO 111 KK=KKCR,8
           CALL TIME(TIMER)
           WRITE(6,777) TIMER
      777  FORMAT(25X,'THE TIME IS',8A1)
           ICC=ICC+1
      C    TLOCT0 = LOCATION OF 0 ANGLE FOR THE CURRENT SAMPLING UNIT OF
      C              INTEREST.
           KVAR=1
           WRITE(2'KVAR) KA,KB
           WRITE(2'KVAR) I,KK
           WRITE(6,26) I,KK
      26   FORMAT(10X,'LOADING=',I5,10X,'DISTRIBUTION=',I5)
           TLOCT0=6.283186*RAN(KA,KB)
      C    LOOP OVER FUEL PARTICLES
           DO 111 M=1,NP
           II=0
      C    LOOP OVER INTEGRATION TECHNIQUES (IE. GENERAL SAMPLING FORMULAS)
           DO 115 IA=1,3
      C    LOOP OVER SAMPLING INTENSITIES
           DO 115 IB=1,5
           IBF=2*IB+4
      C    LOOP OVER TRANSECTS
           DO 115 IC=1,IBF
           IF(IC.EQ.1.AND.IB.EQ.1) GO TO 140
           GO TO 130
      131  TLOCT(IC,IB,IA)=TLOCTS(IC,IB,IA)+TLOCT0
           COSL(IC,IB,IA)=COS(TLOCT(IC,IB,IA))
           SINL(IC,IB,IA)=SIN(TLOCT(IC,IB,IA))
      151  II=II+1
           IF(M.NE.1) GO TO 180
           CON1(II)=CV/2.*COSL(IC,IB,IA)
           CON2(II)=CV/2.*SINL(IC,IB,IA)
      180  IF(COSL(IC,IB,IA).EQ.0.) GO TO 550
           TANL(IC,IB,IA)=SINL(IC,IB,IA)/COSL(IC,IB,IA)
           IF(COST.EQ.0.) GO TO 555
           IF(TANT.EQ.TANL(IC,IB,IA)) GO TO 560
           XIN=YINT/(TANL(IC,IB,IA)-TANT)
           IF(XIN.GE.XMIN.AND.XIN.LE.XMAX) GO TO 565
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      GO TO 987
550   IF(COST.EQ.0.) GO TO 575
      IF(0.GE.XMIN.AND.0.LE.XMAX) GO TO 580
      GO TO 987
555   YIN=TANL(IC,IB,IA)*XMIN
      IF(YIN.GE.YMIN.AND.YIN.LE.YMAX) GO TO 585
      GO TO 987
560   IF(YINT.EQ.0.) GO TO 590
      GO TO 987
575   IF(XMIN.NE.0.) GO TO 987
      IF((TLOCT(IC,IB,IA).GE.1.4835305.AND.TLOCT(IC,IB,IA).LE.1.6580625)
     1.AND.YMAX.GE.0.) GO TO 605
      IF((TLOCT(IC,IB,IA).GE.7.7667165.AND.TLOCT(IC,IB,IA).LE.7.9412485)
     1.AND.YMAX.GE.0.) GO TO 605
      IF((TLOCT(IC,IB,IA).GE.4.6251235.AND.TLOCT(IC,IB,IA).LE.4.7996555)
     1.AND.YMIN.LE.0.) GO TO 605
      IF((TLOCT(IC,IB,IA).GE.10.908309.AND.TLOCT(IC,IB,IA).LE.11.082841)
     1.AND.YMIN.LE.0.) GO TO 605
      GO TO 987
580   IF(YINT.GE.AMIN1(0.,CON2(II)).AND.YINT.LE.AMAX1(0.,CON2(II))) GO T
     1O 605
      GO TO 987
585   IF(XMIN.GE.AMIN1(0.,CON1(II)).AND.XMIN.LE.AMAX1(0.,CON1(II)))
     1GO TO 605
      GO TO 987
590   IF(XMIN.GE.AMIN1(0.,CON1(II)).AND.XMIN.LE.AMAX1(0.,CON1(II)))
     1GO TO 605
      IF(XMAX.GE.AMIN1(0.,CON1(II)).AND.XMAX.LE.AMAX1(0.,CON1(II)))
     1GO TO 605
      GO TO 987
565   IF(XIN.GE.AMIN1(0.,CON1(II)).AND.XIN.LE.AMAX1(0.,CON1(II))) GO
     1TO 605
      GO TO 987
605   ANGINT=ABS(TLOCT(IC,IB,IA)-THETA)
      XX=ABS(SIN(ANGINT))
      IF(XX.LT..00455) GO TO 987
      IVEST(IB,IA,ICC)=CF*COEFF(IC,IB,IA)+IVEST(IB,IA,ICC)
      IF(IC.NE.1) GO TO 987
      IF(IA.EQ.2) IVEST(IB,IA,ICC)=IVEST(IB,IA,ICC)+CF*COEFF(IC,IB,IA)
987   IF(M.NE.NP.OR.IC.NE.IBF) GO TO 115
      WRITE(3'IVAR) IVEST(IB,IA,ICC)
115   CONTINUE
      IF(DMAX.LE.CV/2.) GO TO 631
      IF(COST.EQ.0.) GO TO 606
      DISC=(2.*TANT*YINT)**2-4.*(1.+TANT**2)*(YINT**2-CV**2/4.)
      IF(DISC.LE.0.) GO TO 998
      XIN1=(-2.*TANT*YINT+SQRT(DISC))/(2.*(1.+TANT**2))
      XIN2=(-2.*TANT*YINT-SQRT(DISC))/(2.*(1.+TANT**2))
      IF(XIN1.GE.XMIN.AND.XIN1.LE.XMAX) GO TO 701
      IF(XIN2.GE.XMIN.AND.XIN2.LE.XMAX) GO TO 702
      GO TO 998
606   BPT2=CV**2/4.-XMIN**2
      IF(BPT2.EQ.0.) GO TO 998
      YIN1=SQRT(BPT2)
      YIN2=-YIN1
      IF(YIN1.GE.YMIN.AND.YIN1.LE.YMAX) GO TO 608
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0157          IF(YIN2.GE.YMIN.AND.YIN2.LE.YMAX) GO TO 609
0158          GO TO 998
0159    608   IF(YIN2.GE.YMIN.AND.YIN2.LE.YMAX) GO TO 611
0160    612   IF(X1**2+Y1**2.LT.X2**2+Y2**2) GO TO 610
0161          TLH=ABS(YIN1-Y2)
0162          GO TO 706
0163    610   TLH=ABS(YIN1-Y1)
0164          GO TO 706
0165    611   TLH=ABS(YIN1-YIN2)
0166          GO TO 706
0167    609   YIN1=YIN2
0168          GO TO 612
0169    701   YIN1=TANT*XIN1+YINT
0170          IF(XIN2.GE.XMIN.AND.XIN2.LE.XMAX) GO TO 703
0171    705   IF(X1**2+Y1**2.LT.X2**2+Y2**2) GO TO 704
0172          TLH=SQRT((XIN1-X2)**2+(YIN1-Y2)**2)
0173          GO TO 706
0174    704   TLH=SQRT((XIN1-X1)**2+(YIN1-Y1)**2)
0175          GO TO 706
0176    703   YIN2=TANT*XIN2+YINT
0177          TLH=SQRT((XIN1-XIN2)**2+(YIN1-YIN2)**2)
0178          GO TO 706
0179    702   YIN2=TANT*XIN2+YINT
0180          XIN1=XIN2
0181          YIN1=YIN2
0182          GO TO 705
        C     TL = TOTAL LENGTH OF THE PARTICLE CURRENTLY BEING CONSIDERED
0183    706   TL=TLH*SECB
0184    631   IS1PUA(KK,I)=IS1PUA(KK,I)+(DBAR**2)*TL/(CV**2)
0185    998   IF(M.NE.NP) GO TO 111
0186          WRITE(1'JVAR) IS1PUA(KK,I)
0187    111   CONTINUE
0188          KKCH=1
0189    116   CONTINUE
0190          GO TO 700
0191    140   IF(IA.EQ.1) GO TO 141
0192          GO TO 130
        C     STATEMENT NUMBER 0190 TO 0213 IS AN EXTENDED RANGE OF THE 116 DO
        C     LOOP. THIS PORTION OF THE PROGRAM POSITIONS THE GEOMETRIC CENTER
        C     POINT OF EACH PARTICLE AND DEFINES THE ANGULAR ORIENTATION (THETA)
        C     AND LENGTH (TL) OF THE CENTRAL AXIS OF EACH PARTICLE.
0193    141   THETA=6.283186*RAN(KA,KB)
0194          XC=CV*RAN(KA,KB)-CV/2.
0195          YC=CV*RAN(KA,KB)-CV/2.
0196          COST=COS(THETA)
0197          SINT=SIN(THETA)
0198          TL=(TLMAX-TLMIN)*RAN(KA,KB)+TLMIN
0199          TLH=TL/SECB
0200          COST=TLH*COST/2.
0201          SINT=TLH*SINT/2.
0202          X1=XC+COST
0203          Y1=YC+SINT
0204          X2=XC-COST
0205          Y2=YC-SINT
0206          IF(COST.EQ.0.) GO TO 556
0207          TANT=SINT/COST
```

```
              YINT=YC-TANT*XC
        556   XMIN=AMIN1(X1,X2)
              XMAX=AMAX1(X1,X2)
              YMIN=AMIN1(Y1,Y2)
              YMAX=AMAX1(Y1,Y2)
              DMAX2=AMAX1(X1**2+Y1**2,X2**2+Y2**2)
              DMAX=SQRT(DMAX2)
        130   IF(M.EQ.1) GO TO 131
              GO TO 151
        700   JVAR=1
              IVAR=1
              ICC=0
              ISVMAP=0.
              ISVVAP=0.
              DO 210 I=1,5
              DO 210 KK=1,8
              READ(1'JVAR) IS1PUA(KK,I)
              ISVMAP=ISVMAP+IS1PUA(KK,I)
        210   CONTINUE
              WRITE(6,6000) ((IS1PUA(KK,I),KK=1,8),I=1,5)
        6000  FORMAT(' ',8E16.4)
        C     IVMAP = MEAN TRUE TOTAL PARTICLES VOLUME PER UNIT AREA TAKEN OVER
        C             THE FORTY CIRCULAR SAMPLING UNITS SIMULATED.
              IVMAP=ISVMAP/40.
              DO 215 I=1,5
              DO 215 KK=1,8
              ISVVAP=ISVVAP+(IS1PUA(KK,I)-IVMAP)**2
        215   CONTINUE
        C     IVVAP = SAMPLE VARIANCE OF THE TRUE TOTAL PARTICLES VOLUME PER
        C             UNIT AREA TAKEN OVER ALL FORTY CIRCULAR SAMPLING UNITS
        C             SIMULATED.
              IVVAP=ISVVAP/39.
              DO 1 I=1,5
              DO 1 KK=1,8
              ICC=ICC+1
              DO 1 IA=1,3
              DO 1 IB=1,5
              READ(3'IVAR) IVEST(IB,IA,ICC)
        1     CONTINUE
              WRITE(6,6001) (((IVEST(IB,IA,ICC),IB=1,5),IA=1,3),ICC=1,40)
        6001  FORMAT(' ',5(2X,E16.4))
        C     LOOP OVER INTEGRATION TECHNIQUES (IE. GENERAL SAMPLING FORMULAS)
              DO 85 IA=1,3
        C     LOOP OVER SAMPLING INTENSITIES
              DO 90 IB=1,5
              ISD=0.
              IT=0
        C     LOOP OVER FUEL LOADING
              DO 95 I=1,5
        C     LOOP OVER FUEL DISTRIBUTION
              DO 95 KK=1,8
              IT=IT+1
              ID(IT)=IS1PUA(KK,I)-IVEST(IB,IA,IT)
        C     ID = MATRIX WHICH CONTAINS THE ERRORS INVOLVED IN USING THE
        C          CURRENT VOLUME ESTIMATING TECHNIQUE (ONE OF FIFTEEN TESTED)
        C     WITH RESPECT TO THE FORTY CIRCULAR SAMPLING UNITS SIMULATED. EACH
```

```
                C     ERROR HERE IS EXPRESSED AS A FRACTION OF THE CORRESPONDING TRUE
                C     VOLUME.
      0252            ID(IT)=ID(IT)/IS1PUA(KK,I)
      0253            ISD=ISD+ID(IT)
      0254      95    CONTINUE
                C     IDB = MEAN OF THE ID MATRIX UNDER CONSIDERATION.
      0255            IDB=ISD/40.
      0256            ISD2=0.
      0257            IT=0
      0258            DO 105 I=1,5
      0259            DO 105 KK=1,8
      0260            IT=IT+1
      0261            ISD2=ISD2+(ID(IT)-IDB)**2
      0262      105   CONTINUE
                C     TSD = STANDARD DEVIATION OF THE ID MATRIX UNDER CONSIDERATION
      0263            TSD=SQRT(1.*ISD2)/SQRT(39.)
                C     CVAR = COEFFICIENT OF VARIATION OF THE ID MATRIX UNDER
                C               CONSIDERATION.
      0264            CVAR=TSD/IDB
                C     CRVVM IS A SAMPLE STATISTIC TO BE USED IN THE STANDARD PARAMETRIC
                C     PAIRED COMPARISON TEST FOR MEANS.
      0265            CRVVM=ABS(IDB/(TSD/SQRT(40.)))
      0266            WRITE(6,6002) IDB,TSD,CRVVM,CVAR,IB,IA
      0267      6002  FORMAT(' ','IDB=',E16.4/' ','TSD=',E16.4/' ','CRVVM=',E16.4/' ','C
                     1VAR=',E16.4/' ','IB=',I3/' ','IA=',I3)
      0268            DO 20 ITT=1,40
                C     UNDER THE HYPOTHESIS THAT THE RANDOM VARIABLE DEFINED BY THE ERROR
                C     INVOLVED IN USING THE CURRENT VOLUME ESTIMATING TECHNIQUE IS
                C     NORMALLY DISTRIBUTED WITH MEAN 0, THE NEWLY DEFINED ID MATRIX
                C     CONTAINS VALUES OF A RANDOM VARIABLE HAVING A T DISTRIBUTION WITH
                C     39 DEGREES OF FREEDOM.
      0269            ID(ITT)=ID(ITT)/TSD
      0270      20    CONTINUE
      0271            WRITE(6,21) IB,IA
      0272            WRITE(6,19) (ID(ITT),ITT=1,40)
      0273      21    FORMAT(' ','THE FOLLOWING MATRIX CONTAINS VALUES OF A RANDOM VARIA
                     1BLE WHICH IS HYPOTHESIZED TO HAVE'/' ','A T DISTRIBUTION WITH 39 D
                     2EGREES OF FREEDOM. HENCE 95 PERCENT OF THESE VALUES SHOULD LIE'/'
                     3','BETWEEN T.025,39 AND T.975,39. HERE IB=',I5,'AND IA=',I5)
      0274      19    FORMAT(' ',8E16.6)
      0275      90    CONTINUE
      0276      86    DO 120 IB=1,5
      0277            ISVEM=0.
      0278            ICC=0
      0279            DO 230 I=1,5
      0280            DO 230 KK=1,8
      0281            ICC=ICC+1
      0282            ISVEM=ISVEM+IVEST(IB,IA,ICC)
      0283      230   CONTINUE
                C     IVEM = MEAN ESTIMATED TOTAL PARTICLES VOLUME PER UNIT AREA WITH
                C               RESPECT TO THE CURRENT ESTIMATING TECHNIQUE. THIS MEAN IS
                C               TAKEN OVER ALL FORTY SIMULATED CIRCULAR SAMPLING UNITS.
      0284            IVEM=ISVEM/40.
      0285            ISVEV=0.
      0286            ICC=0
      0287            DO 135 I=1,5
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          DO 135 KK=1,8
          ICC=ICC+1
          ISVEV=ISVEV+(IVEST(IB,IA,ICC)-IVEM)**2
    135   CONTINUE
C         IVEV = SAMPLE VARIANCE OF THE ESTIMATED TOTAL PARTICLES VOLUME PER
C                UNIT AREA WITH RESPECT TO THE CURRENT ESTIMATING TECHNIQUE.
C                THIS SAMPLE VARIANCE IS TAKEN OVER ALL FORTY SIMULATED
C                CIRCULAR SAMPLING UNITS.
          IVEV=ISVEV/39.
C         CRVVV = THE RATIO OF ESTIMATED VOLUME SAMPLE VARIANCE (FOR THE
C                 CURRENT ESTIMATING TECHNIQUE) TO THE TRUE VOLUME SAMPLE
C                 VARIANCE. UNDER THE HYPOTHESIS THAT THE POPULATION
C         VARIANCES ARE EQUAL, CRVVV HAS AN F DISTRIBUTION WITH (39,39)
C         DEGREES OF FREEDOM. CRVVV IS A SAMPLE STATISTIC TO BE USED IN THE
C         STANDARD PARAMETRIC TEST FOR COMPARING TWO VARIANCES WITH A TWO-
C         SIDED ALTERNATIVE. NOTE THAT WITH RESPECT TO ANY PARTICULAR
C         ESTIMATING TECHNIQUE, THIS TEST MUST BE PASSED IN ORDER FOR THE
C         CORRESPONDING PAIRED COMPARISON TEST FOR MEANS (SEE ABOVE) TO BE
C         CORRECTLY APPLIED.
          CRVVV=IVEV/(1.*IVVAP)
          WRITE(6,6003) IVEV,IVVAP,CRVVV,IB,IA
   6003   FORMAT(' ','IVEV=',E16.4/' ','IVVAP=',E16.4/' ','CRVVV=',E16.4/' '
         1,'IB=',I3/' ','IA=',I3)
C         IT IS IMPORTANT TO REALIZE THAT GENERATING A POPULATION WITH
C         SIMILAR MEAN AND VARIANCE TO THE POPULATION OF INTEREST IS NOT
C         SUFFICIENT, SINCE, IN PRACTICE, SAMPLE SIZES ARE SMALL, IT IS
C         HIGHLY DESIRABLE THAT THE ERROR TERM INVOLVED WITH THE SELECTED
C         ESTIMATING TECHNIQUE BE SMALL FOR EACH SAMPLING UNIT.
    120   CONTINUE
    85    CONTINUE
          CALL EXIT
          END
```

```
$EOD

ROUTINES CALLED:
ABS    , TIME   , RAN    , COS    , SIN    , AMIN1 , AMAX1
SQRT   , EXIT

OPTIONS =/ON,/CK,/OP:1

BLOCK      LENGTH
MAIN.   8889    (042562)*

**COMPILER ----- CORE**
   PHASE        USED  FREE
DECLARATIVES 00622 14418
EXECUTABLES  01823 13217
ASSEMBLY     03075 16605
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