c. 1

THE WEIBULL FUNCTION AS A DIAMETER DISTRIBUTION MODEL FOR MIXED STANDS OF DOUGLAS-FIR AND WESTERN HEMLOCK

by

SUSAN NANCY LITTLE

FOR. SCI., PENNSYLVANIA STATE UNIVERSITY, 1976

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE
in the Department
of
FORESTRY

We accept this thesis as conforming to the
required standard

THE UNIVERSITY OF BRITISH COLUMBIA
1980

In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the Head of my Department or by his representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Department of Forestry

The University of British Columbia
2075 Wesbrook Place
Vancouver, Canada
V6T 1W5

Date 9/30/80

ABSTRACT

The three-parameter Weibull function is a satisfactory model of the diameter distributions of mixed stands of western hemlock and Douglas-fir. Weibull distributions estimated by maximum likelihood (MLE) fit eighty of eighty three observed diameter distributions at the $\alpha = .20$ level of significance (Kolmogorov-Smirnov test). Weibull parameter predictors are derived by regressing stand characteristics of 42 stands against their MLE parameters. The Weibull diameter distributions predicted from stand age, mean diameter, mean height, site index and trees per acre fit 39 of 41 observed distributions in the test group at the $\alpha = .20$ level of significance. The results shown here compare favorably with those of other authors. The models given relating stand attributes to diameter distribution will prove useful in stand modeling and in updating forest inventories.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ACKNOWLEDGMENTS

The author is indebted to all individuals and agencies concerned with her studies and research.

The author thanks Dr. Donald D. Munro who suggested the problem and whose constant advice, criticism, and enthusiasm were essential to the completion of this project.

Data from the Regional Forest Nutrition Research Project were provided by Dr. William Atkinson and Mr. Al Becker of the University of Washington.

Weibull parameter estimating subroutines were provided by Dr. Robert L. Bailey, Weyerhaeuser Co., and Dr. William G. Warren, Canadian Forest Service.

Financial and computer support were supplied by the University of British Columbia and the United States Forest Service. The MacMillan Bloedel Ltd. Fellowship also gave financial assistance.

The author extends her thanks to Drs. Jules P. Demaerschalk, James Arney, William G. Warren and Mr. Michael Staley for their help and useful feedback in the early stages of research.

INTRODUCTION

The description of diameter (dbh) distributions poses a problem for the forest stand modeler. The distributional characteristics of the stand at any point in time may be required for the creation of stand tables, which delineate stand attributes such as volume or basal area per acre by diameter classes. Yet the computer storage of tree diameters may be cumbersome and costly in an already complicated model. Diameter distributions may be handled in either of two ways. The model may describe the initial stand in its entirety and "grow" individual trees or groups of trees over time (Mitchell, 1976; Leary, et al., 1977). In this case stand tables are readily available, as each tree or group of trees is stored with its diameter, height, crown ratio, etc.. The alternative approach is to generate the diameter distribution of the stand at the time it is needed via a mathematical function (Depta, 1974). The parameters of the function can be estimated from stand attributes, such as average diameter, trees per acre, and average height. Thus no individual tree data need be stored. Handling time is reduced, as stand tables can be generated directly by functions related to diameter classes.

Another use for functional diameter relationships is in the area of aerial photo interpretation. Average height and number of stems per acre can be interpreted from aerial photographs of forest stands. Stand age and site index may be obtained from past records. Once a functional relationship has been developed between these stand characteristics and diameter distribution, the photo interpreter can readily generate stand tables. This could be a valuable tool for periodically updating inventories on a large scale due to the simplicity of the method.

In this thesis, the Weibull distribution will be investigated as a model for the diameter distributions of mixed stands of second growth Pseudotsuga menziesii (Mirbel) Franco (Douglas-fir) and Tsuga heterophylla (Rafinesque) Sargent (western hemlock). The parameters, a, b, and c, of the Weibull function:

$$F(d) = 1 - \exp\left\{- \left(\frac{d-a}{b}\right)^c\right\}$$

where d represents diameter at breast height in inches; W(d), number of trees per acre having diameter d

will be estimated from the observed diameter distribution of each plot, O(d). The null hypothesis:

$$H_0: F(d) = O(d)$$

will be rejected if for one plot $\max | F(d) - O(d) |$ exceeds the Kolmolgorov-Smirnov critical value $d_\alpha(N)$. The acceptable probability level of committing a Type I error will be $\alpha = .20$ throughout this thesis.

If we cannot reject H_0, the parameters of F(d) will be regressed against the stand characteristics: arethmetic mean diameter, mean dominant and codominant height, stand age, number of trees per acre, and site index.

The distributions predicted from the resulting equations will be compared with F(d) and O(d) for goodness of fit.

LITERATURE REVIEW

Stand tables, which list stand attributes (such as number of trees per acre) by diameter class, have proven to be valuable for forest managers. The construction of these tables has evolved around the biometrician's ability to summarize frequency distributions in mathematical terms. Meyer (1930) used Charlier curves to describe diameter distributions. Bliss and Reinker (1964) fitted Meyer's diameter distributions of even-aged stands of Douglas-fir with lognormal curves. They found that these lognormal curves could be adequately defined by the mean diameter and the variance of the diameters of each distribution. Nelson (1964) characterized diameter distributions of loblolly pine plantations with the gamma distribution. Leak (1965) used negative exponential curves to describe diameter distributions of uneven-aged stands. A summary of growth functions applied in forest biometry was given by Prodan (1968).

Summarizing information by equations greatly reduces computer storage and calculation requirements. Depta (1974) developed a stand table generator for Weyerhaeuser Corp. Given sample data on basal area per acre, number of trees per acre, minimum, average, and maximum d.b.h., and average tree height, his model produced stand tables with diameter classes, stems per acre, basal area per acre, average total height, and cubic foot volume per acre for even- and uneven-aged stands. Each stand was summarized by a series of coefficients relating to equations for each stand attribute. Individual tree and size class statistics were not stored in the computer, but were generated only when they were needed for final output. The capacity to describe distributions via mathematical functions enabled this program to handle large inventories and to fill gaps in information. The Weibull distribution was introduced to forest biometry by Bailey and Dell (1973). This function has special appeal because of its ability to take on a variety of shapes and amounts of skewness. Since the parameters are directly related to shape (c) and scale (b), they should vary in a consistent manner with stand characteristics when applied to diameter distributions. Bailey and Dell gave an overview of parameter estimation methods and fitted four different diameter distributions with the Weibull.

Stauffer (1977) gave a mathematical derivation of the Weibull function in order to give a precise interpretation for the parameters. The three-parameter Weibull function is:

$$W(x) = \frac{b}{c}\left(\frac{x - a}{b}\right)^{c-1} \exp\left\{-\left(\frac{x - a}{b}\right)^{c}\right\}$$

a, the location parameter, corresponds to the minimum diameter; b, the scale parameter, is inversely proportional to the number of allocations (i.e., number of trees); and c, the shape parameter, is an indication of the degree of nonrandomness.

Bailey (1973) used maximum likelihood procedures to fit equations for 1095 _Pinus radiata_ D. Don diameter distributions. Ninety percent of these equations had an adequate fit at the $\alpha = .10$ level of significance by the

Kolmogorov-Smirnov test for goodness of fit. He then attempted to regress age, number of stems per acre, and average height on the characteristics of the percentiles, x_p:

$$x_p = b(-\ln(1-p))^{1/c}$$

where x is the d.b.h. class of the p-th percentile. He came up with these relationships for the 24-th and 93-d percentiles:

$$x_{.24} = B_0 + B_1 A + B_2/N + B_3 \log_{10}(H)$$

$$x_{.24} = C_0 + C_1 A^2 + C_2/N + C_3 \log_{10}(H)$$

From these he calculated the parameters via Dubey's (1967) percentile estimation method. (See below.) At the α = .05 level, 35% of these estimated curves had a satisfactory fit.

The Weibull distribution has since been used in some growth models. Clutter and Allison (1973) used the Weibull distribution for diameters in their growth and yield model for _Pinus radiata_. If the user does not specify a diameter distribution, the model will approximate the distribution using stand age and number of stems as done by Bailey (1973). The Weibull was used by Schreuder and Swank (1974) to summarize diameter, basal area, surface area, biomass, and crown profile distributions for _Pinus strobis_ and _Pinus taeda_. Rustagi (1977) modeled basal area distributions for even-aged Douglas-fir stands with the Weibull. Yang, Kozak and Smith (1978) described previous attempts to fit volume growth and increment distributions with mathematical functions. They then developed a modified Weibull function:

$$F(x) = A\left\{1 - \exp\left(\frac{-x}{\sigma}^{\gamma}\right)\right\}$$

the parameters of which can be estimated via nonlinear regression. This function had the best over-all fit for volume growth when compared with four other distributions found in the literature.

Clutter and Belcher (1978) used the Weibull function to estimate number of trees per acre by one inch classes for slash pine plantations. Parameters of the diameter distributions of 487 plots were estimated via Harter and Moore's (1965) maximum likelihood algorithm. Standard multiple regression was then used to regress stand attributes on the parameter estimates, resulting in the following relationships:

$$a = \gamma_0 + \gamma_1 A + \gamma_2 H_d + \gamma_3 \ln(H_d) \quad , \; r^2 = .107 \quad s_{y\cdot x} = 1.044 \; ;$$

$$b = \delta_0 + \delta_1 A + \delta_2 /N + \delta_3 \ln(H_d) \quad , \; r^2 = .357 \quad s_{y\cdot x} = 1.091 \; ;$$

$$c = \rho_0 + \rho_1 /A \quad , \; r^2 = .020 \quad s_{y\cdot x} = 1.091 \; ;$$

where A = stand age

H_d = mean height of dominants and codominants in feet and,

N = number of trees per acre.

The authors insisted that although the r-square values were low, these equations gave better results than using constant values for a, b, and c.

Observed mean diameter differed from predicted mean diameter by less than one inch in 99% of the observations (i.e., of the 487 plots that were used to derive the equations). Observed basal area differed by less than 25 ft^2 from predicted basal area in 97.4% of the cases.

Searching for a function more flexible than the Weibull, Hafley and Shreuder (1977) investigated the possibilities of Johnson's (1949) S_b distribution and its modifications. The four-parameter function:

$$f(x) = \frac{\delta}{\sqrt{2\pi}} \frac{\lambda}{(x-\varepsilon)(\varepsilon+\lambda-x)} \exp\left\{\frac{1}{2}\left[\gamma + \delta \ln\left(\frac{x-\varepsilon}{\varepsilon+\lambda-x}\right)\right]^2\right\}$$

$$\text{for } \delta,\ \lambda>0 \ ;\ -\infty<\gamma<\infty \ ;\ -\infty < \varepsilon < x < \varepsilon + \lambda < \infty$$

$$f(x) = 0 \text{ elsewhere.}$$

can take on a greater variety of shapes and is applicable to more cases than the Weibull. Schreuder and Hafley (1977) applied this curve to diameter-height relationships. They estimated parameters with maximum likelihood techniques, and achieved a satisfactory fit at the $\alpha = .05$ level with the Kolmogorov-Smirnov test. They did not compare the S_b directly with the Weibull function as a model of tree diameter distributions.

WEIBULL PARAMETER ESTIMATION

Weibull (1955) developed his equation in conjunction with his analysis of breaking strengths. It has been widely used in reliability and life testing analysis ever since. In life testing it is usually not feasible to test a large number of items, nor is it necessary to wait for all items to fail. Since most of the theoretical and applied work on Weibull parameter estimation has been done in the life testing field, the bulk of the algorithms for estimation are based on small, heavily censored samples.

Graphical estimation procedures were developed by Kao (1959).

Cohen (1965) defined the likelihood function for the Weibull distribution:

$$L(x_1, \ldots, x_n; b, c) = \prod_{i=1}^{n} \frac{c}{b} x_i^{c-1} \exp\left(\frac{-x_i^c}{b}\right).$$

Harter and Moore (1965) produced an iterative procedure for calculating maximum likelihood estimates. Warren (1976), in conjunction with his studies on breaking points of wood products, has further refined the maximum likelihood procedure. Seegrist and Arner (1978) discussed problems of maximum likelihood estimates when error components are correlated due to having repeated measurements on plots.

Maximum likelihood estimation can be time consuming and complicated. Point estimation, although often not as accurate, is easier to use than maximum likelihood. Dubey (1967) devised a percentile estimator for the two parameter Weibull distribution, namely:

$$c^* = \frac{\ln(-\ln(1-p_1)) - \ln(-\ln(1-p_2))}{\ln(y_{p1}) - \ln(y_{p2})}$$

$$b^* = \left(y_{p1}\right)^{c^*} / \ln(1-p_1)$$

where p_i is the i-th percentile and y_{pi} is the maximum value attributed to the percentile. He found that the percentiles which minimize the variance of the estimates are the 17-th and 97-th for c* and the 40-th and 82-d for b*. The percentile approach is appealing because it does not involve iterative calculations and is easily implemented.

Englehardt and Bain wrote profusely on point estimates of Weibull parameters: (Bain and Antle (1967), Englehardt and Bain (1973, 1974), Bain (1972), and Englehardt (1975)). In 1977, they published simplified estimators of the parameters for complete samples and tolerance bounds for those estimates.

Specifically:

for z = 1/c:

$$\hat{z} = \left[-\sum_{i=1}^{s} y_i + \left(\frac{s}{n-s}\right) \sum_{i=s+1}^{n} y_i \right] / nk_n,$$

where $s = [.84n]$ and k_n is a constant to remove bias.

$$k_n = E\left[-\sum_{i=1}^{s} y_i/z + \left(\frac{s}{n-s}\right) \sum_{i=s+1}^{n} y_i \;/\; z \right] /n, \qquad k_\infty = 1.5692$$

for $u = \ln b$:

$$\hat{u} = \sum_{i=1}^{n} y_i/n + \gamma \hat{z}$$

where γ is Euler's constant, 0.5772.

This procedure, like the percentile approach, is easy to use. Although the debiasing constant, k, is difficult to calculate, Englehardt and Bain have solved for values for k of all sample sizes under 60 and for infinitely large samples.

A complete discussion of the Weibull function, its history and applications, was given by Mann (1968). She discussed the applications and efficiency of several estimation procedures for censored and uncensored samples, as well as some methods of imposing confidence bounds on the various estimates. Mann, Shafer, and Singpurwalla (1974) published a text on statistical analysis of reliability and life testing data. They discussed the theory and mathematical derivation of estimation procedures and the applications thereof. They condensed the work of several previous authors, including d'Agostino (1971) (linear estimation) and Thomas and Wilson (1972) (point estimation). The tolerance bounds and confidence limits defined by Johns and Leiberman (1966), Thoman, Bain, and Antle (1969), Bogdanoff and Pierce (1973), and Lawless (1975) were also discussed. The text is an excellent reference for anyone interested in the use of the Weibull distribution.

DATA BASE

The data for this thesis came from the Regional Forest Nutrition Research Project (Univ. Wash., 1976) (a cooperative fertilization study with forest industry and several government resource agencies). The study was coordinated by the University of Washington under Dr. William Atkinson. Eighty three of the control plots in natural stands were selected for my study based on the following criteria:

1. Plot elevation between 0 and 2,500 ft.[1/] above sea level.
2. Douglas-fir site index (King, 50) total height of site trees 77-143 feet at 50 years from seed.
3. Age at breast height 12-42 yrs.
4. All plots located west of the Cascades.
5. Mixed stands of western hemlock and Douglas-fir.

Plot size ranged from 1/10 acre to 1/5 acre. Tree height was measured to the nearest foot and dbh to the 0.1 inch. Tables I and II list these plots along with some of their stand statistics.

1/ Because the data were all reported in English units, all measurements and results reported here are likewise in English units.

TABLE I: STAND DATA - ESTIMATION SET

Plot	Species	TPA	D	s_D	D_{min}	D_{max}	Stand age	Site index	Average height
11006	TOTAL	460	8.3	3.0	1.8	19.8	35	113	82
	Douglas-fir	447	8.5						
	W. Hemlock	13	2.0						
71037	TOTAL	205	14.3	3.4	7.6	21.8	39	133	112
	Douglas-fir	200	14.3						
	W. Hemlock	5	14.1						
81048	TOTAL	950	5.8	2.2	1.7	10.4	21	130	67
	Douglas-fir	950	5.8						
131076	TOTAL	1460	4.1	1.8	1.7	10.0	36	80	62
	Douglas-fir	1360	4.1						
	W. Hemlock	50	4.2						
	W. Redcedar	10	1.8						
	Other	40	2.5						
141082	TOTAL	470	8.8	3.6	2.0	16.1	52	118	108
	Douglas-fir	340	9.9						
	W. Hemlock	110	6.2						
	W. Redcedar	15	2.6						
	Red Alder	5	8.5						
141084	TOTAL	480	8.4	5.5	1.7	25.8	52	118	114
	Douglas-fir	215	12.7						
	W. Hemlock	145	5.4						
	W. Redcedar	95	2.9						
	Sitka Spruce	5	2.7						
	Red Alder	20	10.2						
161095	TOTAL	640	7.8	4.2	2.0	18.3	35	129	98
	Douglas-fir	334	10.7						
	W. Hemlock	280	4.9						
	W. Redcedar	13	4.2						
	Other	13	2.5						
201118	TOTAL	400	8.4	3.3	2.8	15.5	26	143	85
	Douglas-fir	393	8.5						
	Other	7	5.0						
211123	TOTAL	380	11.4	3.3	5.6	18.2	39	130	110
	Douglas-fir	373	11.4						
	Other	7	6.9						
211125	TOTAL	327	11.2	4.1	3.5	21.1	39	130	102
	Douglas-fir	320	11.4						
	Big leaf Maple	7	3.5						

D -- Mean diameter (inches)

s_D -- Standard deviation of diameters (inches)

D_{min} -- Minimum diameter (inches)

D_{max} -- Maximum diameter (inches)

TPA -- Trees per acre

Stand Age -- (years)

Site Index -- King, Douglas-fir (feet at 50 yrs)

Average Height -- Mean dominant, codominant ht. (feet)

TABLE I: STAND DATA - ESTIMATION SET

Plot	Species	TPA	D	s_D	D_{min}	D_{max}	Stand age	Site index	Average height
211128	TOTAL	407	9.6	3.8	2.1	18.8	45	124	105
	Douglas-fir	354	9.9						
	W. Hemlock	40	6.1						
	Red Alder	13	10.0						
221132	TOTAL	420	10.3	5.0	2.3	20.7	45	124	113
	Douglas-fir	293	12.5						
	W. Hemlock	107	5.6						
	Red Alder	20	3.6						
331193	TOTAL	880	4.2	2.3	1.6	12.2	19	123	51
	Douglas-fir	450	5.4						
	W. Hemlock	430	2.8						
341202	TOTAL	610	8.0	3.1	3.8	18.1	27	137	87
	Douglas-fir	580	8.1						
	W. Hemlock	30	7.2						
361215	TOTAL	305	10.2	5.7	1.7	21.9	39	138	110
	Douglas-fir	135	14.8						
	W. Hemlock	145	6.4						
	Red Alder	15	11.2						
	Other	10	1.7						
371221	TOTAL	320	11.3	3.7	3.0	19.7	42	127	109
	Douglas-fir	180	13.4						
	W. Hemlock	130	8.7						
	Red Alder	10	7.3						
411244	TOTAL	387	8.9	3.3	1.7	17.2	28	140	92
	Douglas-fir	354	9.6						
	W. Hemlock	13	2.3						
	Other	20	2.6						
431253	TOTAL	1610	5.1	2.2	1.7	11.2	37	98	67
	Douglas-fir	580	5.7						
	W. Hemlock	640	5.2						
	W. Redcedar	390	4.1						
451270	TOTAL	620	7.5	4.2	2.0	23.1	33	131	94
	Douglas-fir	167	12.3						
	W. Hemlock	313	6.2						
	W. Redcedar	80	4.7						
	Big leaf Maple	20	5.7						
	Red Alder	7	6.2						
	Other	33	4.7						

TABLE I: STAND DATA - ESTIMATION SET

Plot	Species	TPA	$\bar{D}$	s_D	D_{min}	D_{max}	Stand age	Site index	Average height
531313	TOTAL	493	7.5	2.0	3.8	13.0	21	125	65
	Douglas-fir	493	7.5						
531317	TOTAL	527	7.0	3.4	1.5	15.0	21	125	58
	Douglas-fir	447	7.9						
	Other	80	1.7						
541319	TOTAL	1100	4.6	2.3	1.6	12.0	35	106	72
	Douglas-fir	1100	4.6						
551327	TOTAL	640	7.4	2.9	2.0	13.4	37	94	77
	Douglas-fir	640	7.4						
571341	TOTAL	400	9.9	3.5	2.1	17.4	29	137	88
	Douglas-fir	333	10.6						
	W. Hemlock	20	6.7						
	W. Redcedar	27	4.5						
	Red Alder	13	7.4						
	Other	7	7.0						
601355	TOTAL	500	7.6	4.0	1.6	17.1	42	105	97
	Douglas-fir	400	9.0						
	Other	100	1.9						
601360	TOTAL	840	6.4	2.2	1.6	15.3	42	105	77
	Douglas-fir	820	6.5						
	Other	20	1.7						
681404	TOTAL	570	6.7	4.7	1.6	15.3	30	135	91
	Douglas-fir	340	9.9						
	Other	230	1.9						
761453	TOTAL	420	6.7	3.6	1.5	16.4	32	98	73
	Douglas-fir	360	7.0						
	Other	60	4.9						
771462	TOTAL	1280	3.7	1.8	1.6	8.9	15	77	43
	Douglas-fir	1270	3.7						
	Sitka Spruce	10	2.1						
791473	TOTAL	630	7.3	3.3	1.5	15.4	30	121	87
	Douglas-fir	420	8.5						
	W. Hemlock	40	3.6						
	W. Redcedar	50	1.9						
	Big leaf Maple	20	2.5						
	Red Alder	100	7.3						
811485	TOTAL	600	8.2	2.4	4.3	14.6	31	109	69
	Douglas-fir	600	8.2						

TABLE I: STAND DATA - ESTIMATION SET

Plot	Species	TPA	D	s_D	D_{min}	D_{max}	Stand age	Site index	Average height
821490	TOTAL	1110	5.2	2.6	1.7	12.8	21	122	60
	Douglas-fir	220	8.0						
	W. Hemlock	720	4.9						
	Red Alder	10	8.1						
	Other	160	2.4						
831494	TOTAL	1560	5.1	2.4	1.5	12.2	22	130	68
	Douglas-fir	520	6.5						
	W. Hemlock	950	4.5						
	Other	90	2.9						
951568	TOTAL	1340	4.5	1.7	1.5	9.1	19	100	44
	Douglas-fir	1340	4.5						
961576	TOTAL	280	11.3	3.4	3.0	18.6	39	112	94
	Douglas-fir	280	11.3						
971582	TOTAL	520	7.2	3.6	1.5	14.6	30	108	72
	Douglas-fir	420	8.3						
	Other	100	2.8						
981588	TOTAL	374	10.2	3.4	2.5	18.4	38	125	100
	Douglas-fir	347	10.7						
	Big leaf Maple	20	3.7						
	Other	7	2.5						
991591	TOTAL	380	6.8	5.4	1.6	28.5	34	121	79
	Douglas-fir	313	7.7						
	W. Hemlock	47	3.0						
	Other	20	1.9						
1011602	TOTAL	730	6.2	2.4	2.8	12.7	24	118	71
	Douglas-fir	730	6.2						
1011605	TOTAL	1060	4.9	2.0	1.2	9.4	24	118	59
	Douglas-fir	1060	4.9						
1101660	TOTAL	830	3.7	2.0	1.7	6.8	12	132	36
	Douglas-fir	400	4.2						
	W. Hemlock	340	2.3						
	W. Redcedar	10	1.7						
	Other	80	2.8						
1131675	TOTAL	1300	4.0	2.0	1.6	9.9	23	117	59
	Douglas-fir	1080	4.0						
	Other	220	4.0						

TABLE II: STAND DATA - PREDICTION SET

Plot	Species	TPA	D	s_D	D_{min}	D_{max}	Stand age	Site index	Average height
11004	TOTAL	407	9.3	2.9	4.3	15.2	35	113	81
	Douglas-fir	407	9.3						
51025	TOTAL	593	6.4	2.4	1.7	11.7	30	101	67
	Douglas-fir	553	6.6						
	W. Hemlock	40	4.1						
51027	TOTAL	626	7.0	2.4	2.2	17.0	30	101	65
	Douglas-fir	613	7.1						
	W. Hemlock	13	4.0						
71041	TOTAL	220	13.0	4.0	5.8	20.2	39	133	106
	Douglas-fir	205	13.4						
	W. Hemlock	15	7.4						
81043	TOTAL	1180	5.3	2.0	2.0	11.4	21	130	66
	Douglas-fir	1170	5.3						
	Other	10	2.4						
131074	TOTAL	1600	4.4	2.1	1.6	9.4	36	80	55
	Douglas-fir	1580	4.3						
	Other	20	5.1						
161091	TOTAL	659	7.7	3.7	2.2	16.8	35	129	98
	Douglas-fir	373	9.9						
	W. Hemlock	253	4.9						
	W. Redcedar	20	4.0						
	Other	13	5.3						
171098	TOTAL	1530	4.9	2.0	1.8	12.3	20	122	63
	Douglas-fir	980	5.4						
	W. Hemlock	550	4.1						
171100	TOTAL	2280	3.8	1.7	1.6	9.9	20	122	59
	Douglas-fir	1550	4.1						
	W. Hemlock	730	3.1						
191111	TOTAL	314	10.7	4.2	1.5	18.5	45	126	117
	Douglas-fir	280	11.7						
	W. Hemlock	7	2.3						
	W. Redcedar	27	2.2						
191114	TOTAL	267	11.7	3.5	7.0	20.0	45	126	117
	Douglas-fir	267	11.7						
201119	TOTAL	687	6.4	3.3	1.8	17.9	26	143	84
	Douglas-fir	680	6.4						
	Other	7	2.6						

D -- Mean diameter (inches)

s_D -- Standard deviation of diameters (inches)

D_{min} -- Minimum diameter (inches)

D_{max} -- Maximum diameter (inches)

TPA -- Trees per acre

Stand Age -- (years)

Site Index -- King, Douglas-fir (feet at 50 yrs)

Average Height -- Mean dominant, codominant ht. (feet)

TABLE II: STAND DATA - PREDICTION SET

Plot	Species	TPA	D	s_D	D_{min}	D_{max}	Stand age	Site index	Average height
331194	TOTAL	1150	4.7	1.9	1.8	8.9	19	123	52
	Douglas-fir	900	5.1						
	W. Hemlock	250	3.2						
341204	TOTAL	680	6.8	3.2	1.8	15.0	27	137	86
	Douglas-fir	480	7.9						
	W. Hemlock	160	3.7						
	Other	40	5.5						
361216	TOTAL	340	10.9	5.3	2.0	23.3	39	138	109
	Douglas-fir	185	14.2						
	W. Hemlock	155	7.0						
371222	TOTAL	430	10.3	4.4	3.1	18.9	42	127	108
	Douglas-fir	240	12.9						
	W. Hemlock	190	7.0						
411246	TOTAL	373	9.8	3.0	3.1	16.8	28	140	91
	Douglas-fir	360	10.0						
	Other	13	4.8						
431254	TOTAL	1050	6.0	3.0	1.5	13.5	37	98	83
	Douglas-fir	340	7.2						
	W. Hemlock	480	5.8						
	W. Redcedar	230	4.5						
451266	TOTAL	473	7.0	5.4	1.5	19.6	33	131	95
	Douglas-fir	193	12.8						
	W. Hemlock	100	4.2						
	W. Redcedar	73	3.0						
	Other	107	2.0						
541324	TOTAL	790	6.0	3.0	1.6	14.6	35	106	80
	Douglas-fir	790	6.0						
551330	TOTAL	930	5.8	2.2	2.4	13.3	37	94	73
	Douglas-fir	930	5.8						
571342	TOTAL	360	9.7	3.8	2.1	17.2	29	137	82
	Douglas-fir	327	9.8						
	W. Hemlock	33	9.3						
591353	TOTAL	813	6.3	3.5	1.4	17.1	34	99	72
	Douglas-fir	520	7.3						
	W. Hemlock	173	4.6						
	W. Redcedar	120	4.2						

TABLE II: STAND DATA - PREDICTION SET

Plot	Species	TPA	D	s_D	D_{min}	D_{max}	Stand age	Site index	Average height
591354	TOTAL	993	5.2	3.5	1.4	23.0	34	99	71
	Douglas-fir	633	6.2						
	W. Hemlock	73	2.7						
	W. Redcedar	280	3.6						
	Other	7	3.6						
681407	TOTAL	420	8.5	3.9	1.5	16.8	30	135	90
	Douglas-fir	350	9.9						
	Other	70	1.7						
691410	TOTAL	260	9.3	4.5	2.0	17.5	29	130	84
	Douglas-fir	247	9.6						
	Other	13	3.8						
691412	TOTAL	327	8.4	5.3	1.7	20.2	29	130	87
	Douglas-fir	220	11.4						
	W. Redcedar	20	2.2						
	Red Alder	7	9.7						
	Other	80	1.9						
761454	TOTAL	1774	4.0	2.0	1.6	11.4	32	98	61
	Douglas-fir	1687	3.8						
	Other	87	6.8						
771458	TOTAL	860	3.5	1.7	1.4	9.2	15	77	39
	Douglas-fir	790	3.6						
	Other	70	2.5						
781468	TOTAL	1013	5.7	2.4	2.2	16.7	42	108	85
	Douglas-fir	960	5.7						
	W. Hemlock	13	8.2						
	Red Alder	33	7.0						
	Other	7	2.3						
791472	TOTAL	690	6.8	3.1	1.6	13.7	30	121	85
	Douglas-fir	350	8.5						
	W. Hemlock	10	1.6						
	W. Redcedar	100	2.5						
	Big leaf Maple	140	5.5						
	Red Alder	90	7.1						
811481	TOTAL	550	8.5	3.1	1.8	17.5	31	109	75
	Douglas-fir	530	8.8						
	Other	20	2.0						

TABLE II: STAND DATA - PREDICTION SET

Plot	Species	TPA	D	s_D	D_{min}	D_{max}	Stand age	Site index	Average height
821492	TOTAL	880	6.7	2.7	2.3	12.1	21	122	59
	Douglas-fir	400	8.1						
	W. Hemlock	420	5.4						
	Red Alder	30	8.0						
	Other	30	2.7						
831498	TOTAL	1270	5.7	2.4	1.7	12.8	22	130	62
	Douglas-fir	610	7.2						
	W. Hemlock	660	4.4						
951565	TOTAL	1930	3.6	1.7	1.5	9.0	19	100	49
	Douglas-fir	1850	3.6						
	Red Alder	20	5.0						
	Other	60	2.1						
961572	TOTAL	380	9.8	3.3	1.9	15.4	39	112	93
	Douglas-fir	340	10.5						
	W. Hemlock	27	4.5						
	Other	13	3.1						
971580	TOTAL	470	8.5	3.0	3.4	14.8	30	108	74
	Douglas-fir	440	8.7						
	Big leaf Maple	30	5.5						
981584	TOTAL	327	11.3	2.9	2.3	17.0	38	125	94
	Douglas-fir	320	11.5						
	Other	7	2.3						
991592	TOTAL	599	6.9	4.4	1.5	16.5	34	121	79
	Douglas-fir	253	10.4						
	W. Hemlock	153	6.1						
	Sitka Spruce	33	6.0						
	Other	160	2.4						
1101658	TOTAL	670	3.6	1.9	1.7	7.7	12	132	34
	Douglas-fir	410	4.2						
	W. Hemlock	110	1.8						
	Red Alder	20	4.6						
	Other	130	1.9						
1131678	TOTAL	850	5.2	2.7	1.6	11.5	23	117	65
	Douglas-fir	550	5.7						
	W. Hemlock	20	3.5						
	Red Alder	100	4.9						
	Other	180	3.8						

METHODS

The data were split by DUPLEX, an algorithm developed by R. W. Kennard (Snee, 1977), into an estimation set and a prediction set. Each set contained a comparable range of sites, densities, and age groups. The estimation set was used to find a suitable relationship between stand characteristics and the Weibull diameter parameters a, b and c. The prediction set was used to test that relationship.

The parameters of the Weibull distribution for each plot were estimated from the given individual tree diameter data. This estimation relied on the availability of an adequate estimating routine. Three algorithms found in the literature were tested: Dubey's (1967) percentile method, and two maximum likelihood routines, Warren (1976) and Bailey (1973) (see Appendix I). Bailey's FITTER routine was selected because it gave a better fit than the percentile method and, unlike Warren's routine, it gave unbiased estimates.

After the Weibull parameters were estimated for the diameter distributions of all plots, stand characteristics from the data were regressed[1] on the estimated parameters of the estimation plots. The characteristics considered were: stand total age, mean dominant and codominant height,

---

1/ Regression done via BMD:2R routine.

arithmetic mean diameter, site index, number of stems ($\geq$0.1 inch d.b.h.) per acre, and the inverses and squares thereof. The best overall models were then used to predict the Weibull parameters for the prediction plots.

The resulting distributions were compared with the corresponding estimates from the observed diameter data. (See Appendix I). To be a useful model, the Weibull distribution should describe the diameter distributions of mixed species stands as well as, if not better than simpler models such as the normal distribution. Here, a regressed relationship was deemed satisfactory if all of the predicted distributions satisfied the Kolmogorov-Smirnov (K-S) goodness of fit test at the $\alpha = 0.20$ level (Massey, 1951). This level of significance was thought to be an acceptable compromise between the goodness of fit obtained for Weibull distributions on even-aged, single species stands (Bliss & Rienker, 1964; $\alpha = 0.15$; Bailey, 1973. 35% fit at $\alpha = 0.05$) and the fits obtainable with simpler one- and two-parameter functions.

RESULTS

The Weibull parameters a, b, and c were estimated by Bailey's FITTER routine for all 83 stands. The cumulative distributions defined by the maximum likelihood estimated parameters were compared with the observed diameter distributions. All of the estimated distributions passed the Kolmogorov-Smirnov (K-S) test at the $\alpha = 0.20$ level of significance; $d_{.2}(30) = .131$[1/] (Table III).

The stand characteristics mean diameter, mean dominant and codominant height, stand age, trees per acre, and site index were regressed against the estimated parameters of the Weibull diameter distributions of the estimation set. The best resulting predictors for the parameters based on correlation coefficients and F ratios are listed in Tables IV, V, and VI. The diameter distributions of the prediction set as defined by all combinations of a, b, and c predictors were compared with the distributions which were estimated from the observed diameter distributions via maximum likelihood. The only combinations to fail the K-S test at the $\alpha = 0.20$ level were those involving predictor a (8) or b (8).

1/ Lilliefors (1967) critical value for $d_{.2}(30)$ was used here because the parameters of the distribution were estimated from the sample.

TABLE III: Comparison of Maximum Likelihood Estimated Weibull Distribution with Original Diameter Distribution

Plot	a*1/	b*	c*	d 2/	d.2(N)3/
11004	4.0000	5.9216	1.8617	.068	.159
11006	1.0000	8.1952	2.4969	.056	.153
51025	1.0000	6.0071	2.3716	.062	.131
51027	1.5000	6.1935	2.4098	.063	.130
71037	6.5000	8.7014	2.3690	.053	.183
71041	3.0000	11.2241	2.6992	.072	.177
81043	1.5000	4.2696	1.9549	.041	.166
81048	1.0000	5.4440	2.3902	.061	.130
131074	1.0000	3.7726	1.6709	.095	.100
131076	1.0000	3.4661	1.7869	.093	.105
141082	0.5000	9.2849	2.4701	.058	.130
141084	1.0000	8.0054	1.3355	.071	.130
161091	1.5000	6.9514	1.7192	.054	.126
161095	1.5000	6.9613	1.4752	.041	.130
171098	1.5000	3.8235	1.7479	.046	.103
171100	1.0000	3.1728	1.8152	.080	.084
191111	0.0000	11.9021	2.7535	.094	.173
191114	6.5000	5.7082	1.4494	.087	.183
201118	2.0000	7.1932	1.9827	.051	.160
201119	1.5000	5.4706	1.5645	.046	.124
211123	5.0000	7.1396	1.9922	.063	.163
211125	2.5000	9.8310	2.2681	.092	.169
221128	1.5000	9.0550	2.2264	.055	.159
221132	1.0000	10.4275	1.9175	.076	.157
331193	1.0000	3.4860	1.4431	.081	.131
331194	1.5000	3.5633	1.7218	.089	.118
341202	3.5000	4.9760	1.4900	.049	.159
341204	1.5000	5.8880	1.6470	.051	.153
361215	1.0000	10.1425	1.5651	.063	.159
361216	0.0000	12.2880	2.1683	.075	.153
371221	0.0000	12.5558	3.2579	.056	.206
371222	2.0000	9.3328	1.9554	.051	.177
411244	0.0000	9.9678	2.8635	.066	.160
411246	0.0000	10.8782	3.6038	.057	.163
431253	1.0000	4.6327	1.9442	.046	.100
431254	1.0000	5.5194	1.6684	.047	.124
451266	1.0000	6.1243	1.0381	.098	.150
451270	1.5000	6.7211	1.5060	.095	.132
531313	3.5000	4.4572	2.0177	.075	.149
531317	0.0000	7.8732	2.1918	.083	.142
541319	1.0000	4.0349	1.5962	.055	.121
541324	1.0000	5.5978	1.7028	.043	.142

1/ a*, b*, c* are the estimated Weibull parameters

2/ d = max/Sn (x) - F_o (x)

3/ $d_{.2}(N)$ = Kolmogoror-Smirnov limit for d for fit at 20% level of significance

TABLE III (Continued)

Plot	a*[1]	b*	c*	d [2]	$d_{.2}$(N)[3]
551327	1.0000	7.1695	2.3550	.056	.157
551330	2.0000	4.2236	1.7714	.047	.132
571341	0.0000	10.9818	3.0761	.038	.160
571342	0.0000	10.9145	2.8211	.073	.166
591353	1.0000	5.8602	1.5572	.049	.155
591354	1.0000	4.5779	1.3143	.047	.104
601355	0.0000	8.4775	1.9472	.124	.169
601360	1.0000	6.0191	2.5392	.062	.139
681404	1.0000	5.8259	1.0608	.202	.163
681407	0.0000	9.5229	2.2623	.138	.183
691410	1.0000	9.2792	1.8733	.084	.190
691412	1.0000	7.8292	1.1820	.162	.169
761453	1.0000	6.2550	1.5107	.076	.159
761454	1.0000	3.3434	1.6222	.080	.078
771458	1.0000	2.7473	1.5137	.099	.137
771462	1.0000	3.0129	1.6326	.078	.112
781468	1.5000	4.7896	1.9342	.067	.103
791472	0.5000	7.0273	2.0676	.060	.153
791473	0.0000	8.1590	2.3065	.055	.159
811481	0.0000	9.4805	2.9009	.079	.163
811485	4.0000	4.7198	1.8031	.033	.160
821490	1.0000	4.6869	1.6929	.064	.121
821492	2.0000	5.1969	1.7361	.063	.131
831494	1.0000	4.6034	1.7963	.040	.102
831498	1.0000	5.3542	2.0770	.056	.112
951565	1.0000	2.9076	1.6261	.098	.091
951568	1.0000	3.9121	2.1489	.067	.110
961572	0.0000	10.9154	3.4388	.084	.163
961576	0.0000	12.4302	3.6758	.056	.183
971580	2.0000	7.3454	2.2804	.097	.173
971582	0.5000	7.5120	1.9103	.086	.166
981584	0.0000	12.3532	4.5658	.071	.169
981588	0.0000	11.3259	3.3332	.054	.163
991591	1.0000	6.1411	1.1413	.061	.163
991592	1.0000	6.3467	1.2426	.123	.131
1011602	2.5000	4.1552	1.6311	.093	.150
1011605	0.5000	4.9292	2.3247	.061	.123
1101658	1.0000	2.9.53	1.5847	.089	.157
1101660	1.0000	2.9956	1.7325	.107	.149
1131675	1.0000	3.3690	1.6392	.081	.111
1131678	1.0000	4.6670	1.6160	.113	.137

TABLE IV: a Equations

$$a(1) = 0.4737 + 448.4/TPA$$

$$a(2) = 3.086 - 0.6655(D) + 0.08021(D^2) - 27.58(A)/TPA$$

$$a(3) = -413.3 - 43.22(D) + 640.1/D - 574.1/TPA + 1.032(D^2) + 296.6(\ln D) - 21.83(A)/TPA$$

$$a(4) = -444.1 - 46.31(D) + 689.5/D - 1275.0/TPA + 1.093(D^2) + 318.9(\ln D) - 2.872(A)/SI$$

$$a(5) = -410.7 - 43.33(D) + 635.0/D + 0.00005556(SI^2) + 295.3(\ln D) + 1.048(D^2) - 14.86(HT)/TPA$$

$$a(6) = -466.0 - 47.99(D) + 724.6/D - 1347.0/TPA + 1.126(D^2) + 333.3(\ln D) - 2.544(HT)/SI$$

$$a(7) = -325.8 - 35.84(D) + 496.8/D + 0.8543(D^2) + 0.00003266(SI^2) + 238.6(\ln D) + 0.01514(TPA)/A$$

$$a(8) = 372.5 - 39.37(D) + 0.008466(SI) + 567.8/D - 0.0002378(A^2) + 0.9193(D^2) + 268.2(\ln D) + 0.06373(TPA)/HT$$

$$a(9) = -332.4 - 35.63(D) + 509.5/D - 0.0004968(A^2) + 0.8411(D^2) + 241.2(\ln D) + 0.0726(TPA)/SI$$

$$a(10) = -386.3 - 40.60(D) + 598.1/D - 0.0006213(A^2) + 0.9651(D^2) + 278.1(\ln D) - 7.318(SI)/TPA$$

A = stand age (total years)
D = arithmetic mean diameter (inches)
HT = ave. dominant & codominant height (feet)
SI = site index (King, Douglas fir, feet at 50 yrs)
TPA = trees per acre (dbh 0.1 in.)

TABLE V: b Equations

$$b(1) = 2.648 + 0.06581(HT) - 0.04242(TPA)/A$$

$$b(2) = 6.297 + 0.001277(A^2) + 0.00007376(SI^2) - 0.1870(TPA)/HT$$

$$b(3) = -0.2065 + 0.05548(HT) + 1.159(\ln A) - 0.2231(TPA)/SI$$

$$b(4) = 17.75 + 0.05619(HT) - 2.412(\ln TPA)$$

$$b(5) = 13.10 + 3.914(\ln A) - 2.600(\ln TPA) - 7.488(A)/HT$$

$$b(6) = 3.497 + 3.188(\ln HT) - 1.791(\ln TPA) + 14.72\ (A)/TPA$$

$$b(7) = 6.364 + 0.03952(A) - 29.30/A - 236.9/SI + 13.84(HT)/TPA$$

$$b(8) = 23.20 + 5.665(HT)/SI - 3.156(\ln TPA)$$

$$b(9) = 18.90 + 2.617(\ln A) - 3.276(\ln TPA)$$

$$b(10) = 4.011 + 0.0007134(A^2) + 4.451(\ln D) - 1.038(\ln TPA)$$

TABLE VI: c Equations

$$c(1) = 1.562 + 255.6/TPA$$

$$c(2) = 0.8321 + 0.2908(D) = 0.0004183(A^2) - 0.0002045(HT^2)$$

$$c(3) = 0.6100 + 0.4136(D) - 311.65/TPA + 0.0004033(A^2) - 0.0002179(HT^2)$$

$$c(4) = 165.4 + 16.77(D) - 253.8/D - 0.3649(D^2) - 0.0001321(HT^2) - 0.00003068(SI^2) - 115.9(\ln D)$$

$$c(5) = 38.96 - 0.1473(A) + 0.3883(D) + 0.4969(HT) + 0.002466(A^2) - 0.002059(HT^2) - 14.72(\ln HT) + 0.0674(TPA)/SI$$

$$c(6) = -0.1859 + 0.6313(D) + 0.1174(HT) + 10.28/D + 0.001338(A^2) - 0.0008732(HT^2) - 2.415(\ln A) - 2.940(SI)/TPA$$

Several combinations of predicting equations passed the K-S test at the α = 0.20 level with 100% fit of all plots. These superior combinations, (or distribution models), can be found in Table VIII with their respective d statistics. As a class, the combination of equations a(2) and b(10) was a superior model, having the lowest average d statistic and the least amount of variance in fit regardless of the c equation used. The best model was:

$$a(2)=3.086 - 0.6655(D) + 0.08021\ (D^2) - 27.58\ (A)/TPA$$

$$b(10) = 4.011 + 0.0007134\ (A^2) + 4.451\ (\ln D) - 1.038\ (\ln TPA)$$

$$c(4) = 165.4 + 16.77(D) - 253.8/D - .3649\ (D^2) - .0001321\ (HT^2) - .00003068\ (SI^2) - 115.9\ (\ln D)$$

where D = mean stand diameter (inches)

A = stand age (total years)

SI = site index (King, Douglas-fir, height at 50 yrs)

HT = mean dominant and codominant height (feet)

TPA = trees per acre

with an average d = .0587, s_d = .0317, and max d = .1441 .

TABLE VII: Regression Statistics

Equation	F(n,v)	R	SE_E (inches)
a(1)	3.896(1,40)	.2979	1.336
a(2)	6.717(3,38)	.5887	1.161
a(3)	5.737(6,35)	.7042	1.063
a(4)	5.480(6,35)	.6960	1.075
a(5)	5.674(6,35)	.7022	1.066
a(6)	5.736(6,35)	.7041	1.063
a(7)	3.852(6,35)	.6306	1.162
a(8)	3.432(7,34)	.6434	1.163
a(9)	4.088(6,35)	.6419	1.148
a(10)	4.820(6,35)	.6727	1.108
b(1)	37.58(2,39)	.8114	1.597
b(2)	22.62(3,38)	.8006	1.659
b(3)	26.95(3,38)	.8248	1.565
b(4)	51.29(2,39)	.8512	1.434
b(5)	33.97(3,38)	.8535	1.443
b(6)	34.45(3,38)	.8551	1.445
b(7)	20.57(4,37)	.8306	1.562
b(8)	48.64(2,39)	.8449	1.462
b(9)	48.42(2,39)	.8442	1.464
b(10)	39.18(3,38)	.8693	1.368
c(1)	7.719(1,40)	.4022	.5412
c(2)	8.863(3,38)	.6416	.4652
c(3)	7.485(4,37)	.6588	.4570
c(4)	5.827(6,35)	.7069	.4470
c(5)	5.068(7,34)	.7146	.4485
c(6)	5.039(7,34)	.7136	.4492

Table VIII -- Prediction Models which fit all MLE Distributions

EQUATION # a	b	c	d	s_d	max d
2	5	2	.0852	.0408	.1625
2	5	3	.0830	.0405	.1746
2	5	4	.0843	.0371	.1694
2	5	6	.0839	.0426	.1890
2	6	2	.0780	.0407	.1732
2	6	3	.0750	.0408	.1663
2	6	4	.0650	.0389	.1672
2	6	5	.0827	.0439	.1868
2	6	6	.0767	.0421	.1868
2	11	1	.0735	.0572	.1705
2	11	2	.0591	.0346	.1549
2	11	3	.0563	.0346	.1739
2	11	4	.0587	.0317	.1441
2	11	6	.0566	.0384	.1883
7	11	2	.0821	.0459	.1666
7	11	3	.0795	.0465	.1775
7	11	4	.0789	.0422	.1733
7	11	6	.0813	.0446	.1794

1. d = average K-S statistic over all prediction plots
   s_d = standard deviation of d
   max d = maximum value of d found over all plots.

A plot by plot comparison of the distributions predicted by the model:

$$F(d) = 1.0 - \exp\left\{-\left(\frac{x - a}{b}\right)^{c}\right\}$$

with the observed stand diameter distributions can be found in Table IX. The K-S statistic d was less than $d_{.2}(N)$ for 95% of the plots. According to the Kolmogorov - Smirnov test, the predicted Weibull diameter distributions are satisfactory models of the observed distributions. The quality of fit, as reflected by the K-S statistic d, was not dependent on the value of any one stand attribute (Figures 1 - 5). The model predicts consistantly over the range of mean diameter, height, site, stocking, and stand age exhibited in the prediction set. The figures in Appendix III show all three distributions (MLE, predicted, and observed) for 18 of the prediction set plots.

The predicted distributions did not fit the observed diameter distributions at the $\alpha = .20$ level of significance for plots 691412 and 991592. The trees in these plots divided easily into distinct species - size groups. Stand 691412 (fit at $\alpha = .01$) was 67% Douglas-fir (mean diameter 11.4 inches) and 33% cedar and other species (mean diameter 1.9 inches). Stand 991592 (fit at $\alpha = .10$) was 42% Douglas-fir (10.4 in. dbh) 31% hemlock and spruce (6.0 in. dbh) and 27% other species (2.4 in. dbh). The polymodal nature of these two diameter density distributions accounts for the lack of fit with the unimodal Weibull model.

Although the difference between the observed and predicted distributions was not significant at the α = .20 probability level, plots 71041, 361216, 451266, and 591354 showed a tendency in the model towards bias. The predicted distributions for these plots underestimated the number of trees in the small diameter classes and overestimated the number of trees in the middle diameter classes (Figures 6-9).

TABLE IX: MLE vs Predicted Distribution (a(2) b(10) c(4))
$d_{.20}(30) = .190$

Plot	a 1/	b 1/	c 1/	d_{MLE} 2/	d_O 3/
11004	4.0000	5.9126	1.8617	.0470	.0683
	1.4432	8.5643	2.7774		
51025	1.0000	6.0771	2.3716	.0594	.0651
	0.7155	6.2829	2.0695		
51027	1.5000	6.1935	2.4098	.0441	.1027
	1.0467	6.6419	2.2585		
71041	3.0000	11.2241	2.6992	.0301	.1006
	3.1164	10.9179	2.9267		
81043	1.5000	4.2696	1.9549	.0495	.0580
	1.3184	4.3939	1.7620		
131074	1.0000	3.7726	1.6709	.0881	.1825
	1.0847	3.8379	2.1205		
161091	1.5000	6.9514	1.7192	.0379	.0469
	1.2519	7.2286	1.5712		
171098	1.5000	3.8235	1.7479	.0327	.0388
	1.3898	3.7538	1.8461		
171100	1.0000	3.1728	1.8152	.1082	.0792
	1.4731	2.2186	1.4335		
191111	0.0000	11.9021	2.7535	.0578	.1295
	1.2053	10.0470	2.3373		
191114	6.5000	5.7082	1.4494	.1226	.1355
	1.6340	10.6077	2.6085		
201119	1.5000	5.4706	1.5645	.0580	.0949
	1.0683	5.9763	1.4188		
331194	1.5000	3.5633	1.7218	.0190	.0821
	1.2734	3.8310	1.9692		
341204	1.5000	5.8884	1.6467	.0441	.0800
	1.1693	6.2855	1.5151		
361216	0.0000	12.2880	2.1683	.1061	.1588
	2.2319	9.6907	2.5561		
371222	2.0000	9.3328	1.9554	.0700	.0988
	2.0727	9.3668	2.4634		
411246	0.0000	10.8782	3.6038	.0199	.0682
	2.2281	8.5962	2.6055		

1/ The first line contains the maximum likelihood estimate, the second line contains the predicted parameters.

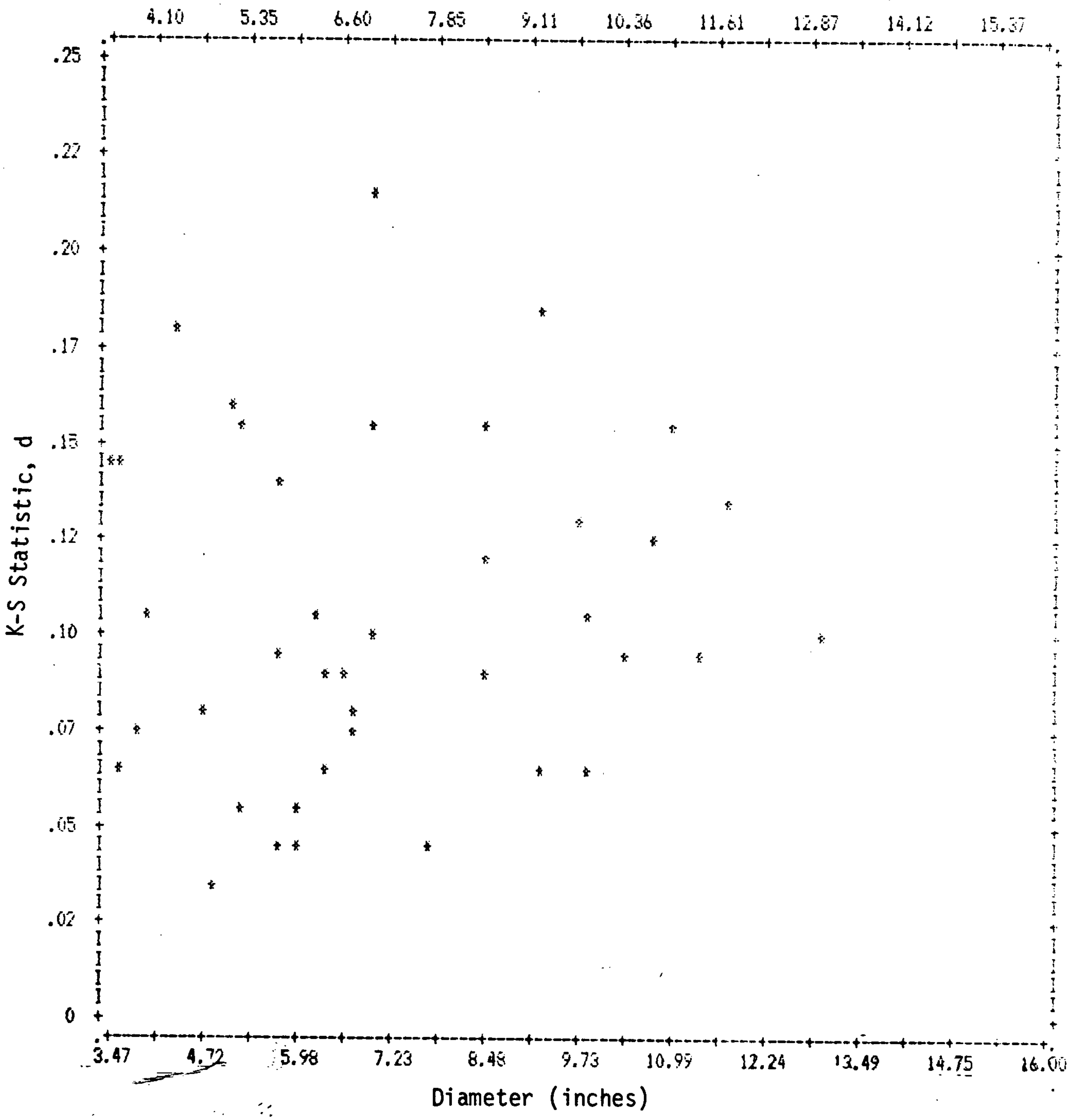
2/ K-S statistic for predicted distribution against MLE.

3/ K-S statistic for predicted distribution against observed.

TABLE IX: continued.

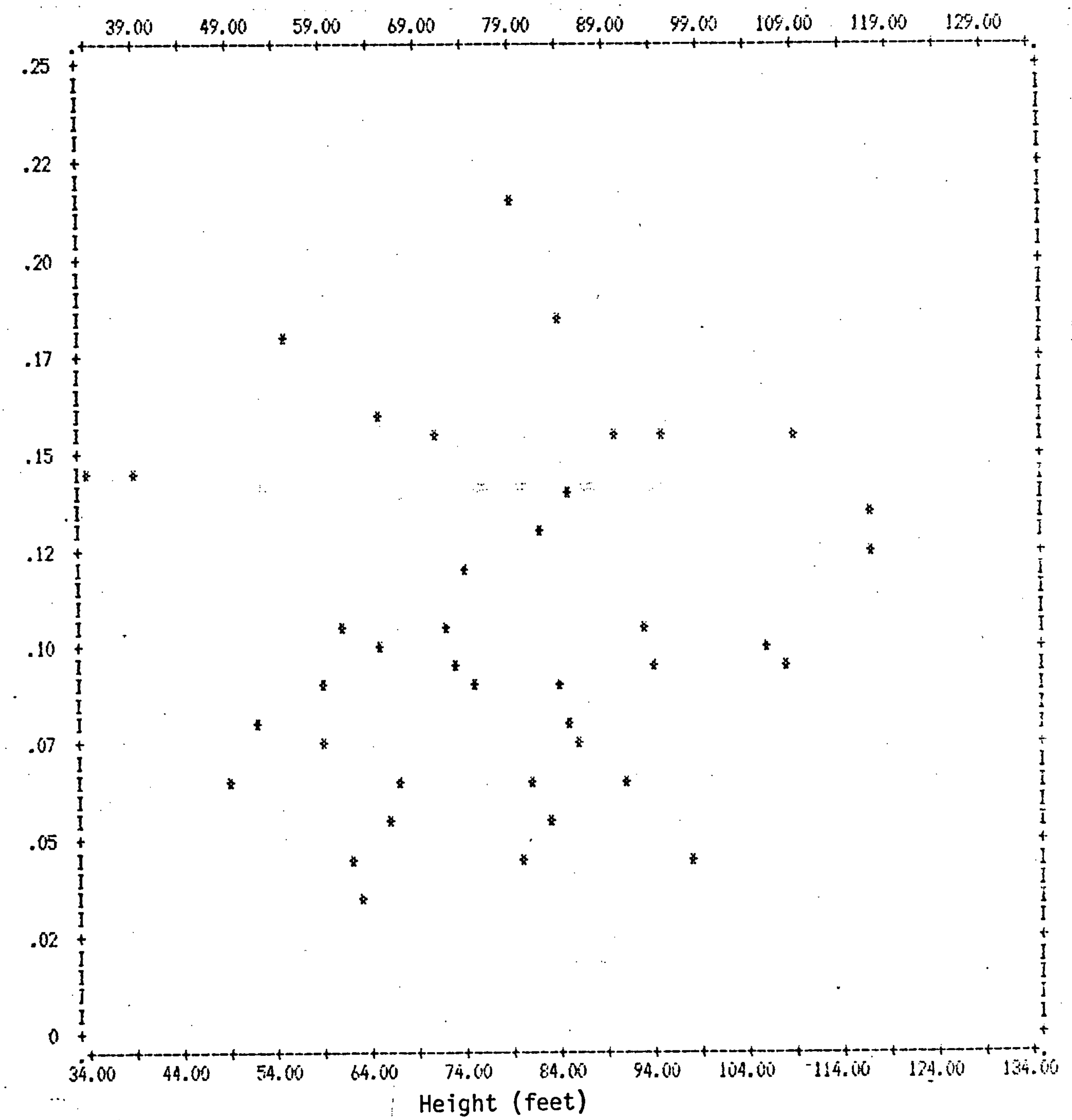
Plot	a	b	c	d_{MLE}	d_O
431254	1.0000	5.5479	1.6959	.0220	.0580
	1.0048	5.7322	1.7233		
451266	1.0000	6.1243	1.0381	.0637	.1600
	0.4506	7.0768	1.4097		
541324	1.0000	5.5978	1.7028	.0138	.0475
	0.7587	5.9345	1.7206		
551330	2.0000	4.2236	1.7714	.0727	.1000
	0.8161	5.6846	1.9221		
571342	0.0000	10.9145	2.8211	.0573	.1335
	1.9970	8.6355	2.7846		
591353	1.0000	5.8602	1.5572	.0628	.1090
	0.9106	6.0446	1.9707		
591354	1.0000	4.5779	1.3143	.1010	.1552
	0.8491	5.0046	1.8804		
681407	0.0000	9.5229	2.2623	.0332	.1571
	1.2825	7.9296	2.0860		
691410	1.0000	9.2792	1.8733	.1067	.1883
	0.7497	8.7600	2.5967		
691412	1.0000	7.8292	1.1820	.1399	.2809
	0.7406	8.1007	2.1373		
761454	1.0000	3.3434	1.6222	.0428	.1099
	1.2103	3.1180	1.7182		
771458	1.0000	2.7473	1.5137	.0578	.1485
	1.2616	2.6943	1.4734		
781468	1.5000	4.7896	1.9342	.0942	.1407
	0.7623	5.8533	1.5856		
791472	0.5000	7.0273	2.0676	.0361	.0810
	1.0497	6.3680	1.6580		
811481	0.0000	9.4805	2.9009	.0364	.0907
	1.6804	7.6802	2.5801		
821492	2.0000	5.1969	1.7361	.0305	.0946
	1.5498	5.7217	2.1237		
831498	1.0000	5.3542	2.0770	.0320	.0483
	1.4287	4.7089	1.8574		
951565	1.0000	2.9076	1.6261	.0806	.0686
	1.4584	2.1003	1.4339		
961572	0.0000	10.9154	3.4388	.0422	.1058
	1.4815	9.1113	2.7657		
971580	2.0000	7.3454	2.2804	.0192	.1235
	1.4864	7.8087	2.6171		
981584	0.0000	12.3532	4.5658	.0389	.0956
	2.6398	9.8388	3.1718		
991592	1.0000	6.3467	1.2426	.1000	.2172
	0.7664	6.8167	1.8280		
1101658	1.0000	2.9122	1.5748	.0780	.1499
	1.2042	3.1270	1.3974		
1131678	1.0000	4.6665	1.6158	.0462	.1632
	1.0407	4.6872	1.8621		

Figure 1. The relationship of goodness of fit to stand diameter.



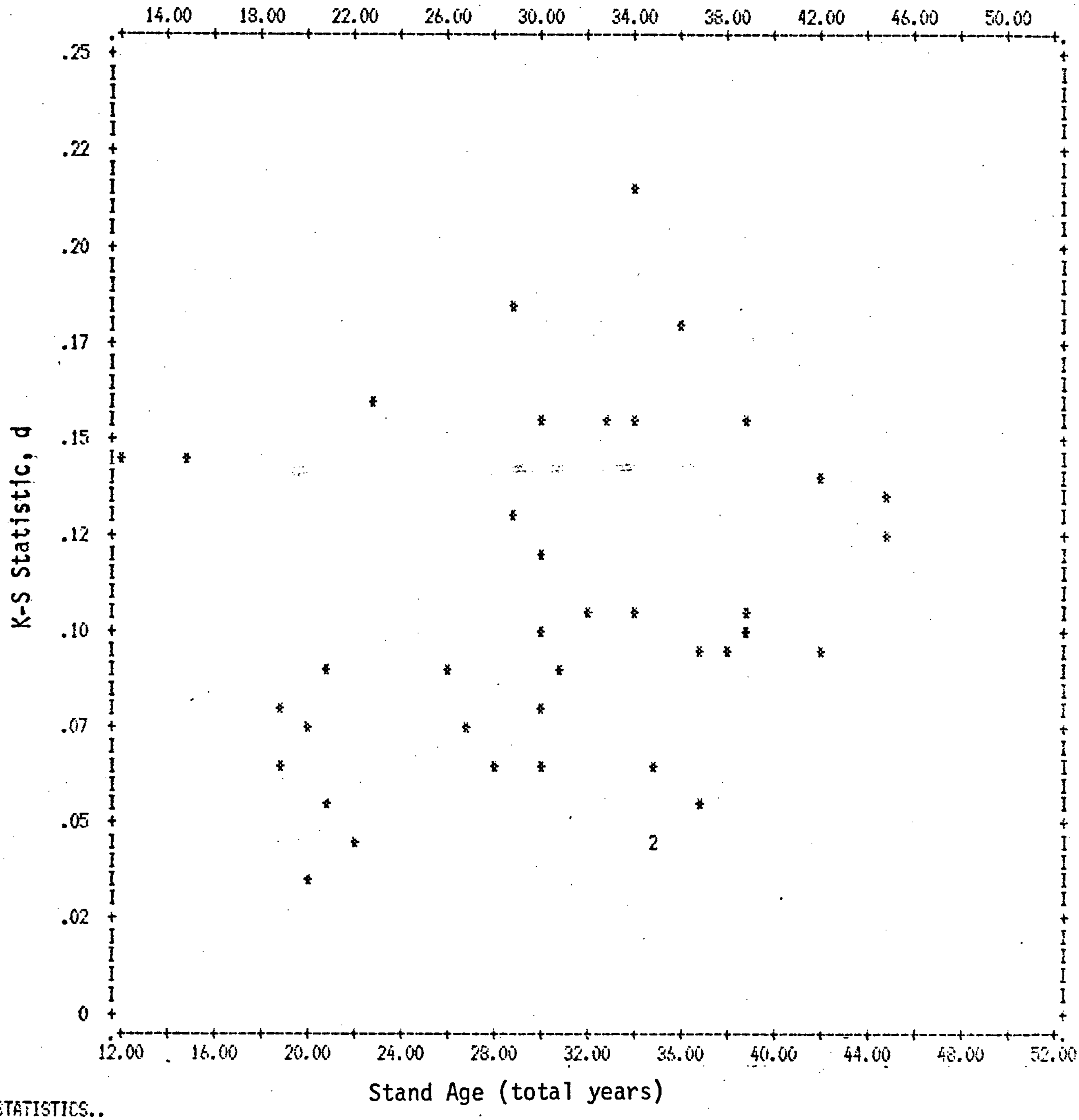
STATISTICS..
R SQUARED - .00848 SLOPE (B) - .00160 STD ERROR OF B - .00281

Figure 2. The relationship of goodness of fit to stand height.



STATISTICS..
R SQUARED - .00360 SLOPE (B) - .00013 STD ERROR OF B - .00036

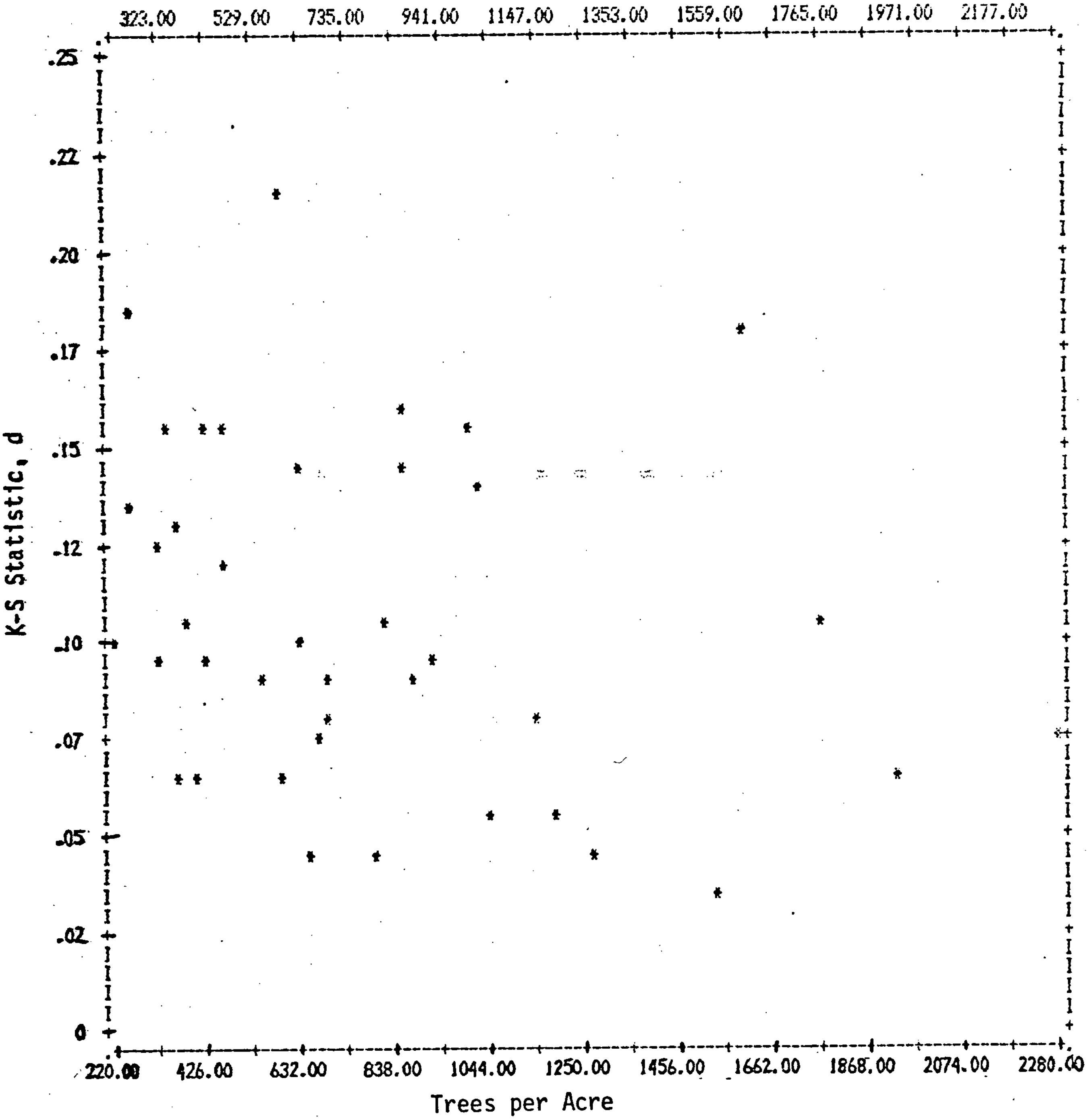
Figure 3. The relationship of goodness of fit to stand age.



STATISTICS..

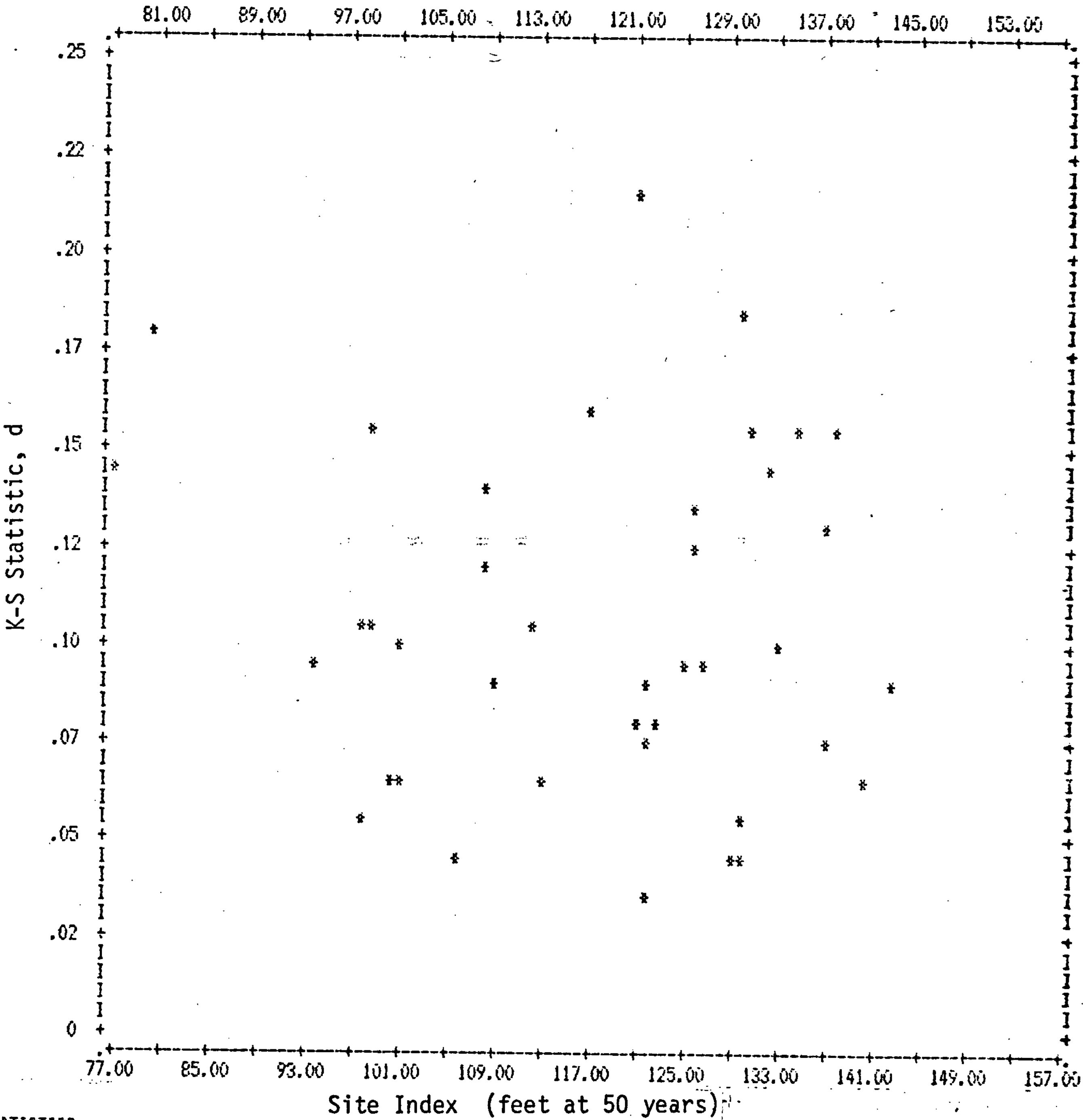
R SQUARED - .02950 SLOPE (B) - .00092 STD ERROR OF B - .00086

Figure 4. The relationship of goodness of fit to trees per acre.



STATISTICS..
R SQUARED - .07018 SLOPE (B) - -.00002 STD ERROR OF B - .00001

Figure 5. The relationship of goodness of fit to site index.



STATISTICS..
R SQUARED - .00299 SLOPE (B) - -.00015 STD ERROR OF B - .00043

Figure 6. Observed and predicted trees per acre by diameter class for plot 71041.

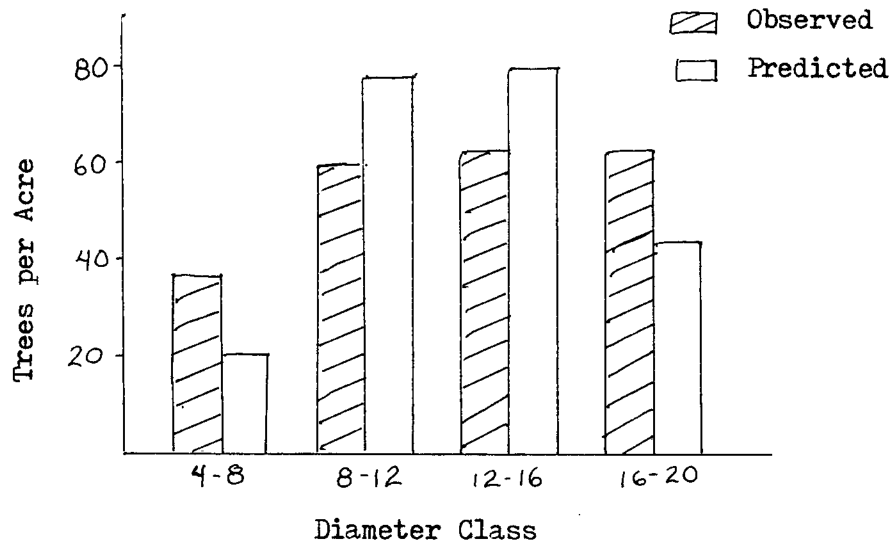


Figure 7. Observed and predicted trees per acre by diameter class for plot 361216.

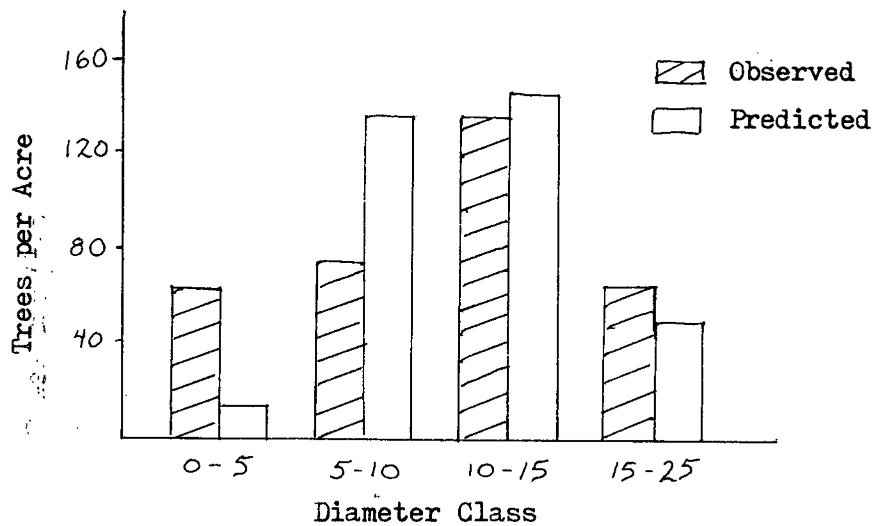


Figure 8. Observed and predicted trees per acre by diameter class for plot 451266

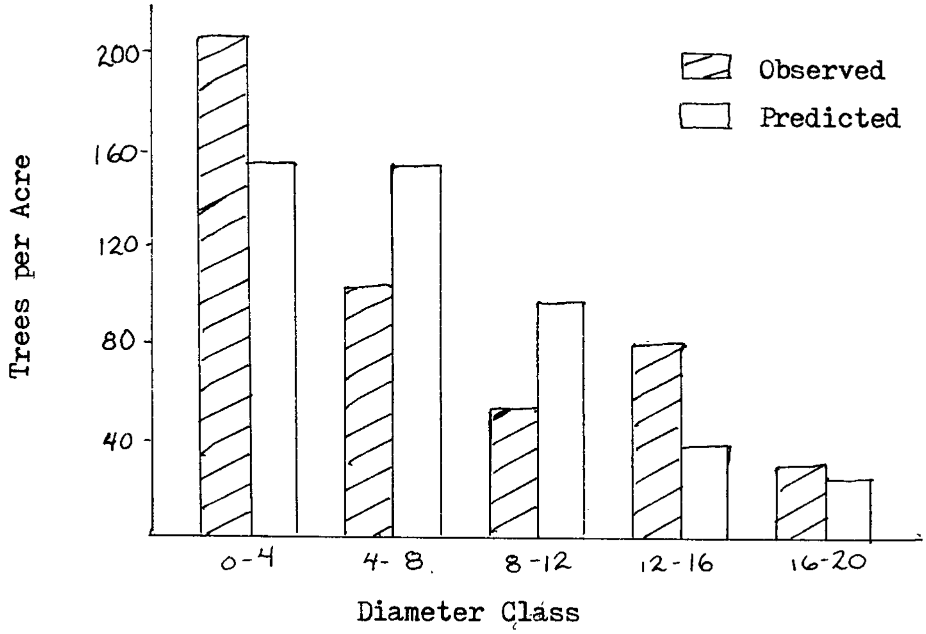
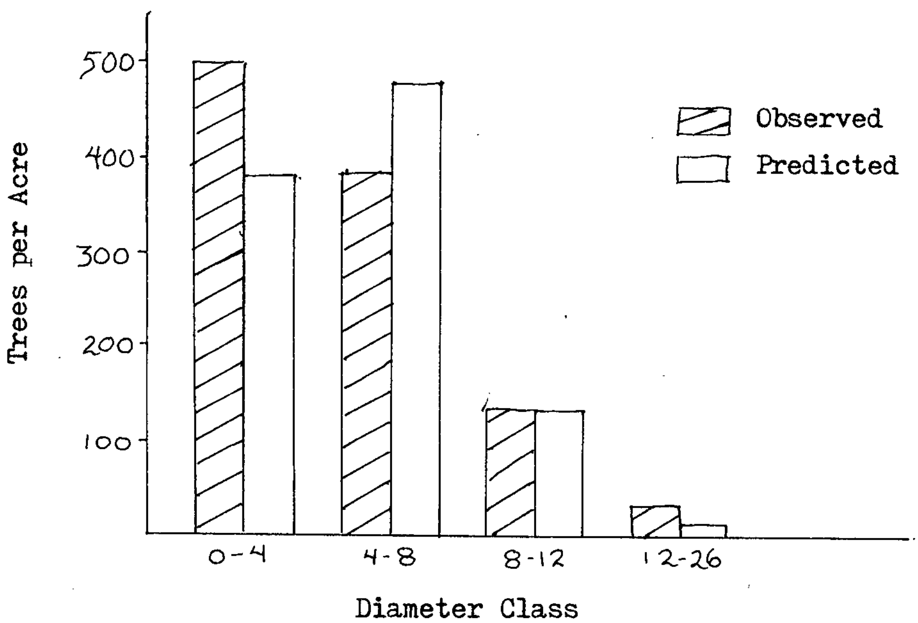


Figure 9. Observed and predicted trees per acre by diameter class for plot 591354.



DISCUSSION, APPLICATIONS, and FUTURE RESEARCH

All of the 83 MLE diameter distributions fit the observed distribution at the probability level α = .20. The null hypothesis, F(d) = O(d), cannot be rejected. The model cumulative diameter distribution

$$F(d) = 1.0 - \exp\left\{-\left(\frac{x - a}{b}\right)^{c}\right\},$$

where d = dbh

F(d) = cumulative frequency

a = f(mean diameter, stand age, trees per acre)

b = f(mean diameter, stand age, trees per acre)

c = f(mean diameter, height, site index)

successfully describes the MLE diameter distributions of the 41 prediction plots. This does not guarentee that the model is sound. When tested against the observed distributions, the model did not fit for two plots. In other cases, bias was observed towards underestimation of the number of small trees.

Previous applications of the Weibull to diameter distributions are difficult to compare due to the different statistical tests and the criteria for fit chosen. Shreuder and Swank (1974) compared four distributions by log likelihood (ln L) statistics. Although the Weibull has larger ln L than the other models for six of seven samples all of the ln L are extremely small ($\ln L \simeq e^{-4000}$). No criteria of fit was given by the authors to test the strength of the Weibull as a model for diameter distributions. Clutter and

Belcher (1978) gave the coefficients of determination for their prediction equations (a = f(age,height), r^2 = .107; b = f(age,height,trees per acre), r^2 = .357; c = f(age), r^2 = .200). They compared predicted and observed mean diameter and basal area per acre. This choice of test shows a concern for average stand descriptors rather than an interest in distributional qualities. Bailey (1973) predicted the percentiles of the two parameter Weibull from age, height, and trees per acre ($r^2 > .95$) and then obtained the Weibull parameters from the percentiles. Sixty five percent of the predicted distributions fit the observed diameter distributions at the $\alpha = .05$ level (K-S). The level of significance chosen by Bailey is not as exacting as the one used in the present study ($d_{.05}(N) = 1.36/\sqrt{N}$; $d_{.20}(N) = 1.07/\sqrt{N}$). Although the results of this study are not directly comparable with those of other authors, the fits demonstrated here for mixed species stands appear as good if not better than those found for plantations of pine.

The primary application for the Weibull diameter distribution model will be in computer simulation of forest stands. It is not necessary for the simulator to "grow" individual trees in order to maintain distributional information at each time interval. The model presented here treats diameter distribution as independent of stand history. If the simulator predicts trees per acre, mean diameter, and mean height for a stand given site index, the diameter distribution of the stand can be generated at any age. This greatly reduces the computation time and storage requirements of the simulator.

The model developed in this thesis describes the diameter distribution of the entire stand. Modeling the diameter distribution of individual species within a mixed species stand may eliminate the bias evident in the model presented in this thesis. Mean diameter, height, and trees per acre may be all that is needed to predict individual distributions. However, it may prove necessary to track the parameters of the Weibull over time for each species in the stand. These questions should be answered through future research.

The results reported here are based on data from untreated second growth stands. Silvicultural treatment is intended to have a positive impact on the diameter distribution of the stand. Fertilization may increase diameter growth through an increase in site quality. Thinning directly alters the distribution through selective removal of trees. Fertilization may induce a shift in the population mean diameter; thinning will tend to skew the diameter distribution. The Weibull function is flexible enough to handle such variation. Whether or not the model presented here can reflect the changes induced through silviculture, either by changing input values (site index, mean diameter, trees per acre), or by calibrating the coefficients will have to be answered by further research.

The diameter distribution model may be useful in inventory updating. Through aerial photograph interpretation, mean height, mean diameter, and trees per acre can be estimated. Site index and age may be obtained from past records. With these data, diameter distributions can be predicted for each stand. This will speed the process of inventory updating and reduce the number of costly field plots needed.

The Weibull function can model the diameter distributions of mixed stands of Douglas-fir and western hemlock. Its future will depend on the development of models which accurately predict small diameter classes and which predict separate distributions for individual species groups within a mixed stand.

LITERATURE CITED

Bailey, Robert L., 1973. Weibull model for Pinus radiata diameter distribution. In Statistics in Forestry Research, I.U.F.R.O. Proc. subgroup S6.02; Vancouver, B.C. pp. 51-19.

Bailey, Robert L. and T. R. Dell, 1973. Quantifying diameter distributions with the Weibull function. Forest Science 19:97-104.

Bain, Lee J. and Charles E. Antle, 1967. Estimation of parameters in the Weibull distribution. Technometrics 9:621-627.

Bliss, C. I. and K. A. Reinker, 1964. A lognormal approach to diameter distributions in even-aged stands. Forest Science 10:350-360.

Bogdanoff, David A. and Donald A. Pierce, 1973. Bayes fiducial inference for the Weibull distribution. Journal of American Statistical Association 68:659-664.

Clutter, Jerome L. and B. J. Allison, 1973. A growth and yield model for Pinus radiata in New Zealand. In Growth Models for Tree and Stand Simulation. Proc. I.U.F.R.O. working party S4, 01-4; Ed. J. Fries. Inst Skogsprod., Stockholm. Rapp, Uppsatser No. 30, pp. 136-160.

Clutter, Jerome L. and David M. Belcher, 1978. Yield of site prepared slash pine plantations in the lower coase plain of Georgia and Florida. In Growth Models for Long Term Forcasting of Timber Yields, Ed. Joran Fries, *et al*. Publ. FWS-1-78 School of Forest and Wildlife Resources, VPI and State Univ.; pp. 53-70.

Cohen, Clifford A., 1965. Maximum likelihood estimation in the Weibull distribution based on complete and on censored samples. Technometrics 13:171-182.

D'Agostino, Ralph B., 1971. Linear estimation of the Weibull parameters. Technometrics 13:171-182.

Depta, David J., 1974. Large in-place inventories based upon stand tables In Inventory Design and Analysis, Ed. W. E. Frayer, G. B. Hartman, D. R. Bower. Proc. Workshop of Inventory Working Group, S.A.F. pp. 275-288.

Downton, F., 1965. Linear estimates of parameters in the extreme value distribution. Technometrics 8:3-17.

Dubey, Satya D., 1967. Some percentile estimates for Weibull parameters. Technometrics 9:119-129.

Ek, A. R., J. N. Issos and R. L. Bailey, 1975. Solving for Weibull diameter distribution parameters to obtain specified mean diameters. Forest Science 21:290-292.

Englehardt, Max and Lee J. Bain, 1974. Some results on point estimation for the two parameter Weibull or extreme value distribution. Technometrics 16:49-56.

Englehardt, Max and Lee J. Bain, 1977. Simplified statistical procedures for the Weibull or extreme value distribution. Technometrics 19:323-331.

Finkelstein, J. M., 1976. Confidence bounds on the parameters of the Weibull process. Technometrics 18:115-117.

Freund, John E., 1971. Mathematical Statistics, Prentice Hall, Inc. Englewood Cliffs, N.J. pp. 125+

Hafley, W. L. and H. T. Schreuder, 1977. Statistical distributions for fitting diameter and height data in even-aged stands. Canadian Journal of Forest Research 7:481-487.

Harter, H. Leon and Albert H. Moore, 1965. Maximum likelihood estimation of the parameters of Gamma and Weibull populations from complete and from censored samples. Technometrics 7:639-643.

Johns, M. V. and G. J. Lieberman, 1966. An exact asymptotical efficient confidence bound for reliability in the case of the Weibull distribution. Technometrics 8:135-175.

Kao, J. H. K., 1959. A graphical estimation of mixed Weibull parameters in life testing of electron tubes. Technometrics 1:389-407.

Lawless, J. F., 1975. Construction of tolerance bounds for the extreme-value and Weibull distributions. Technometrics 17:255-261.

Leak, William B., 1965. The J-shaped probability distribution. Forest Science 11:405-411.

Leary, Rolph A., Jerold T. Hahn, and Roland G. Buchman, 1977. A generalized forest growth projection system - preliminary research findings. Phase I of Forest Resources Evaluation Program (FREP). Manuscript, North Central Forest Experiment Station, USDA (300 pp.)

Lilliefors, Hubert W., 1967. On the Kolmogorov-Smirnov test for normality with mean and variance unknown. American Statistical Assoc. Journal 62:399-402.

Mann, Nancy R., 1968. Point and interval estimation procedures for the two-parameter Weibull and extreme-value distributions. Technometrics 10(2):231-256.

Mann, Nancy R., Schafer, Ray E., and Singpurwalla, Nozer D.,1974. Methods of Statistical Analysis for Life Testing. John Wiley, New York.

Massey, Frank J., Jr., 1951. The Kolmogorov-Smirnov test for goodness of fit. American Statistical Assoc. Journal 46:68-78.

Meyer, Walter H., 1930. Diameter distribution series in even-aged forest stands.Yale Univ. Sch. For. Bull. 28, 105 pp.

Mitchell, Kenneth J., 1975. Dynamics and simulated yield of Douglas-fir. Forest Science Monograpoh No. 17, 39 pp.

Nelson, Thomas C., 1964. Diameter distribution and growth of loblolly pine. Forest Science 10:105-114.

Prodan, Michail, 1968. Forest Biometrics. Pergamon Press. N.Y. pp 39-56.

Rustagi, Krishna P., 1978. Predicting stand structure in even-aged stands. In Growth Models for Long Term Forecasting of Timber Yields, Ed. Joran Fries, et al. Publ. FWS-1-78. School of Forest and Wildlife Resources, VPI and State Univ.; pp. 193-208.

Schmidt, J. W. and R. E. Taylor, 1970. Simulation and Analysis of Industrial Systems. Richard D. Irwin, Inc., Homewood, Ill. 644 pp.

Schreuder, Hans T. and W. L. Hafley, 1977. A useful bivariate distribution for describing stand structure of tree heights and diameters. Biometrics 33:471-478.

Schreuder, Hans T. and Wayne T. Swank, 1974. Coniferous stands characterized with the Weibull distribution. Canadian Journal of Forest Research 4:518-523.

Seegrist, Donald W. and Stanford L. Arner, 1978. Statistical analysis of linear growth and yield models with correlated observations from permanent plots remeasured at fixed intervals. In Growth Models for Long Term Forecasting of Timber Yields. Ed. Joran Fries, et al. Publ. FWS-1-78 School of Forest and Wildlife Resources, VPI; pp. 209-223.

Snee, Ronald D., 1977. Validation of regression models: methods and examples. Technometrics 19:415-428.

Stauffer, 1977. A derivation for the Weibull Distribution. Univ. of B.C., Faculty of Forestry. Unpublished manuscript.

Thoman, D.R., L.J. Bain and C.E. Antle, 1969. Inferences on the parameters of the Weibull distribution. Technometrics 11:445-460.

Thomas, David R. and Wanda M. Wilson, 1972. Linear order statistic estimation for the two-parameter Weibull and extreme-value distributions from Type II progressively censored samples. Technometrics 14:679-692.

University of Washington, College of Forest Resources, 1976. Regional Forest Nutrition Research Project. Biennial Report 1974-1976. 67 pp.

Warren, William G., 1976. Maximum likelihood estimation of the three-parameter Weibull. Forintek, Vancouver, B.C., unpublished.

Weibull W., 1955. A statistical distribution function of wide applicability. Journal of Applied Mechanics 18:293-297.

Yang, R. C., Kozak, A. and J. H. G. Smith, 1978. The potential of Weibull-type functions as flexible growth curves. Canadian Journal of Forest Research 8:424-431.

APPENDIX I

The strength of the regression models depended on the accuracy of the inputs. It was necessary, therefore, to estimate the Weibull parameters of the individual plot diameter distributions from the tree diameter data as accurately as possible. The parameter estimation algorithms developed in the life-testing research were not considered, as they were developed for heavily censored samples, and in most cases were time consuming and inadequate when using complete samples of one hundred or more observations. (For a more thorough discussion, see Mann, et al., 1974.) The Englehardt and Bain (1977) algorithm for simplified point estimates was abandoned because the debiasing constant, k, would have to be calculated for each sample, thus rendering the technique more cumbersome than the more accurate maximum likelihood algorithms. Three estimation algorithms, Dubey's (1967) percentile method, Bailey's (1973) maximum likelihood estimator, and Warren's (1977) maximum likelihood method were considered for the estimation of the diameter distributions. These routines, DUBEY (Dubey), FITTER (Bailey), and WINWAR (Warren) were tested and compared in the following manner:

Using a random number generator, samples were taken from cumulative Weibull distributions having b and c parameters within the range of those expected for the data set. A number, y ($0 \leq y < 1$), corresponding in this case to a cumulative frequency, was randomly selected from a uniform distribution. From the Weibull formula it follows that

$$x = b(-\ln(1-y))^{1/c}.$$

Given y, b, and c, the diameter, x, was calculated. This was repeated until a sample of 100 diameters was obtained. (See Freund (1971) for more detail.)

The sample was then run through each of the three-parameter estimating routines, yielding three estimated diameter distributions. The Kolmogorov-Smirnov test for goodness of fit was used to compare these distributions with the original distribution. Critical values, $d_\alpha(N)$ were given by Massey 1951) such that

$$\Pr\left\{\max\left|S_N(x)-F_o(x)\right| > d\ (N)\right\} = \alpha$$

where $F_o(X)$ is the theoretical cumulative distribution

$S_N(X)$ is an observed cumulative distribution for a sample of N.

The difference, $d = \left|S_N(X) - F_o(X)\right|$ was calculated for twenty observations. The average maximum d, d, was calculiated for ten samples from each distribution. The d and corresponding standard deviations, s_d, are listed in Appendix II.

According to Chebyshev's theorem:

$$PR(\mu - k\sigma < x < \mu + k\sigma) \geq 1 - 1/k^2$$

If $k = 1/\sqrt{\alpha}$, $Pr\ (x < \mu + \sigma/\sqrt{\alpha}$. In this case, μ and σ were estimated by $\bar{d}$ and $s_{\bar{d}}$, respectively, so that

$$Pr(d < \bar{d} + s_{\bar{d}}\ -/\sqrt{\alpha}) \geq 1 - \alpha.$$

If for (1-α)% of the samples, $d < d_{\alpha}(N)$, the estimated distribution has a good fit at the α level of probability. In other words, if

$$\bar{d} + s_{\bar{d}}\ -/\sqrt{\alpha} < d_{\alpha}(N)$$

the estimated distribution fits the original distrubution at the level.

According to the results in Appendix II, all of the curves estimated by FITTER fit at the $\alpha = 0.01$ level, ($d_{.01}(20) = 0.356$). WINWAR estimates for distributions where $b = 10$ with $3.5 \leq c \leq 5.0$ fit at the $\alpha = .05$ level. The remaining WINWAR estimates fit at the $\alpha = 0.01$ level. The DUBEY estimates for ($b = 10$, $3.5 \leq c \leq 5.0$) did not fit at the $\alpha = 0.20$ level of probability. The remainder fit at the $\alpha = 0.05$ level. Bailey's FITTER routine was chosen for this study because it gave the best overall fit for distributions within the expected data range, and, unlike WINWAR, it is an unbiased estimator.

APPENDIX II: Comparison of Dubey, Warren (WINWAR), and Bailey (FITTER) estimates for b and c over a range of true values for b and c, with a = 1.0. For each combination of b and c, each estimator used the same random sample (N = 100) from the given population. Twenty observations from the estimated curve were used to calculate d. The Kolmogorov-Smirnov statistic, d, was averaged over ten estimations.

c --	2.0		2.5		3.0		3.5		4.0		4.5		5.0	
	d	s_d	d	s_d	d	s_d	d	s_d	d	s_d	d	s_d	d	s_d
b = 10.0														
DUBEY	.0806	.0337	.0906	.0392	.1034	.0424	.1179	.0440	.1334	.0450	.1497	.0458	.1664	.0456
WINWAR	.0334	.0232	.0330	.0233	.0330	.0234	.0394	.0324	.0390	.0319	.0457	.0340	.0451	.0336
FITTER	.0335	.0229	.0333	.0230	.0333	.0230	.0324	.0231	.0324	.0231	.0330	.2300	.0327	.0229
b = 20.0														
DUBEY	.0640	.0290	.0666	.0337	.0709	.0379	.0753	.0419	.0813	.0446	.0877	.0476	.0943	.0504
WINWAR	.0345	.0244	.0339	.0234	.0374	.0305	.0373	.0297	.0385	.0280	.0381	.0284	.0384	.0281
FITTER	.0351	.0242	.0346	.0239	.0363	.0268	.0363	.0267	.0365	.0268	.0363	.0270	.0363	.0271
b = 30.0														
DUBEY	.0410	.0296	.0418	.0327	.0435	.0353	.0462	.0377	.0498	.0393	.0545	.0396	.0597	.0402
WINWAR	.0381	.0278	.0377	.0270	.0388	.0279	.0317	.0287	.0378	.0298	.0383	.0306	.0388	.0299
FITTER	.0377	.0270	.0379	.0264	.0372	.0267	.0369	.0263	.0368	.0258	.0372	.0258	.0368	.0262
b = 40.0														
DUBEY	.0432	.0275	.0430	.0259	.0436	.0248	.0477	.0237	.0460	.0234	.0439	.0366	.0461	.0384
WINWAR	.0386	.0228	.0376	.0238	.0377	.0242	.0341	.0236	.0350	.0236	.0384	.0303	.0386	.0300
FITTER	.0371	.0236	.0363	.0241	.0362	.0243	.0327	.0241	.0329	.0212	.0369	.0261	.0371	.0261

APPENDIX III

Graphical illustration of the observed, estimated, and predicted diameter distributions of some typical stands in the prediction set.

Figure 1. Observed, estimated, and predicted diameter distributions of stand 11004

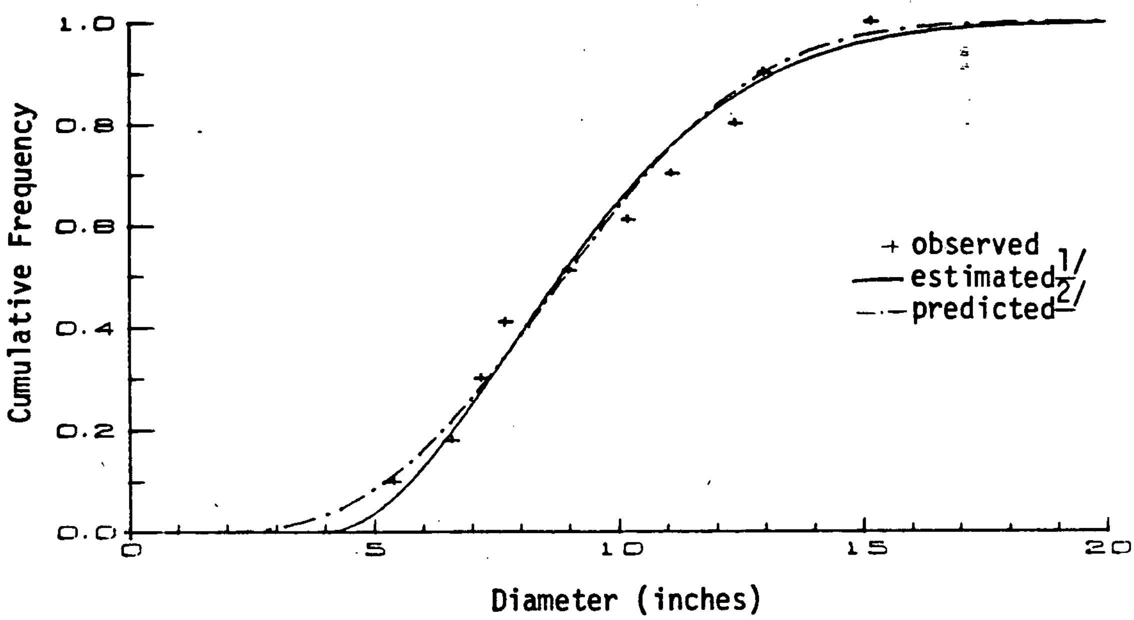
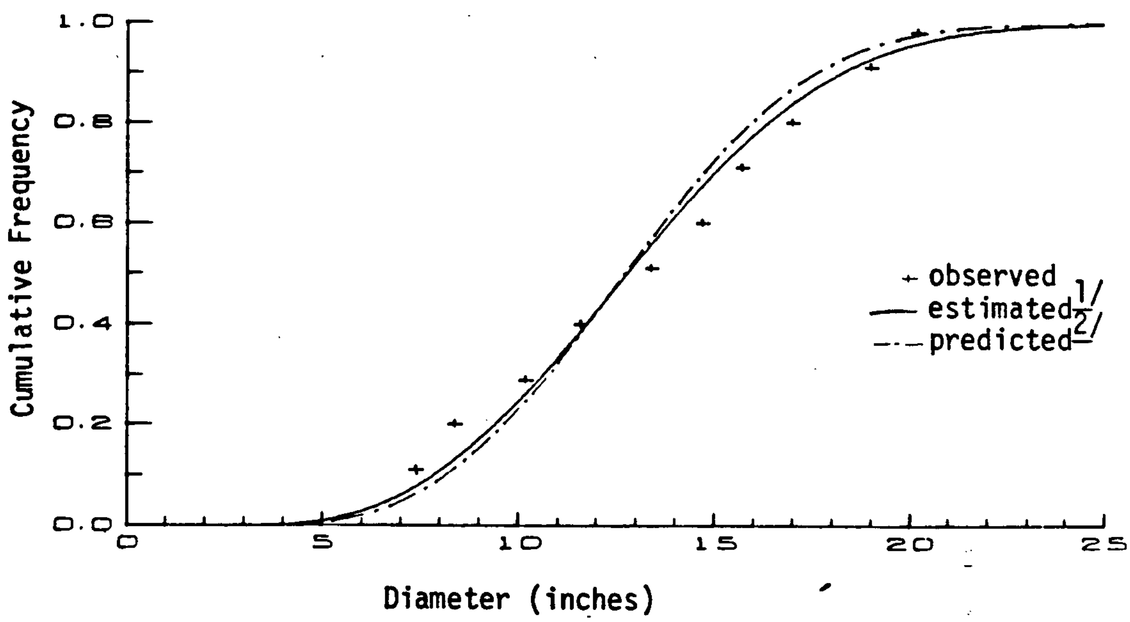


Figure 2. Observed, estimated, and predicted diameter distributions of stand 71041



1/ estimated from observed distribution by maximum likelihood
2/ predicted from stand characteristics by model a(2) b(10) c(4)

Figure 3. Observed, estimated, and predicted diameter distributions of stand 161091

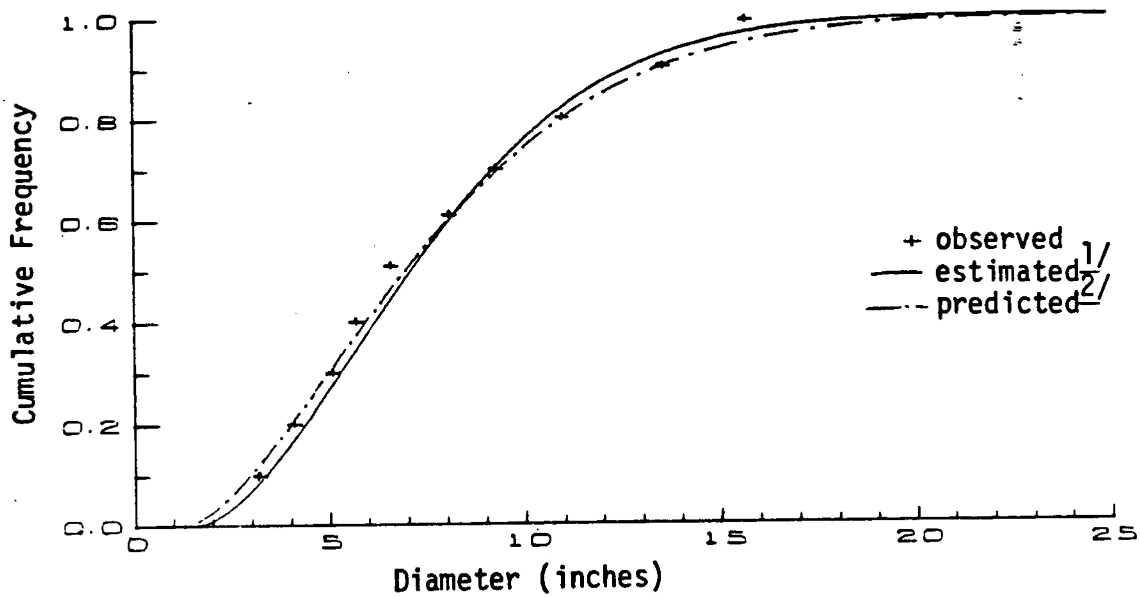
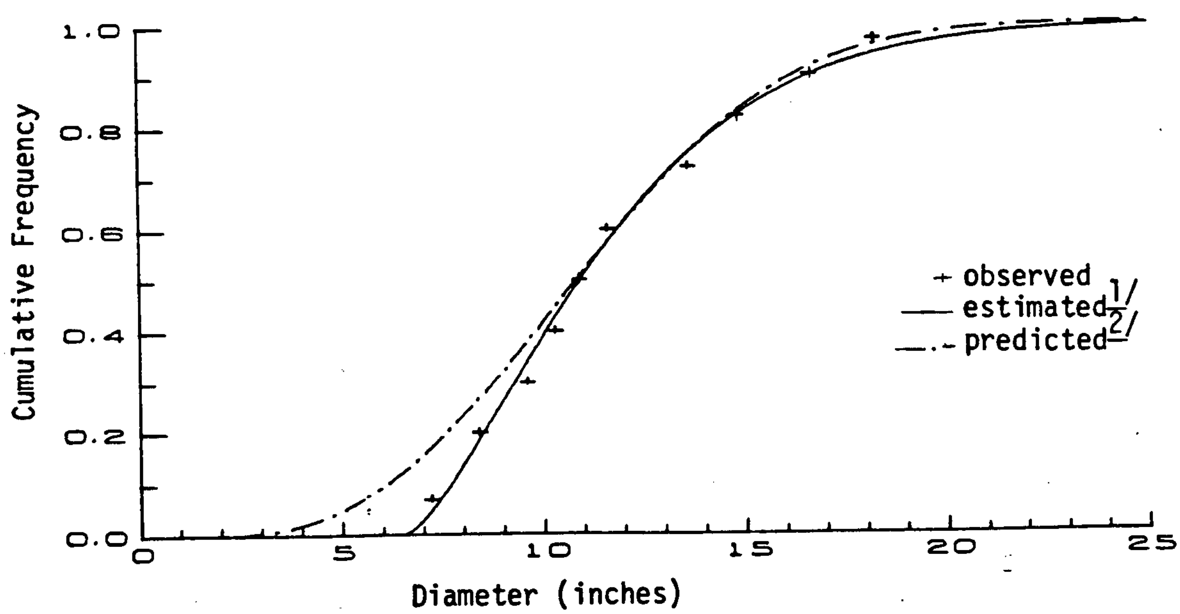


Figure 4. Observed, estimated, and predicted diameter distributions of stand 191114



1/ estimated from observed distribution by maximum likelihood
2/ predicted from stand characteristics by model a(2) b(10) c(4)

Figure 5. Observed, estimated, and predicted diameter distributions of stand 331194

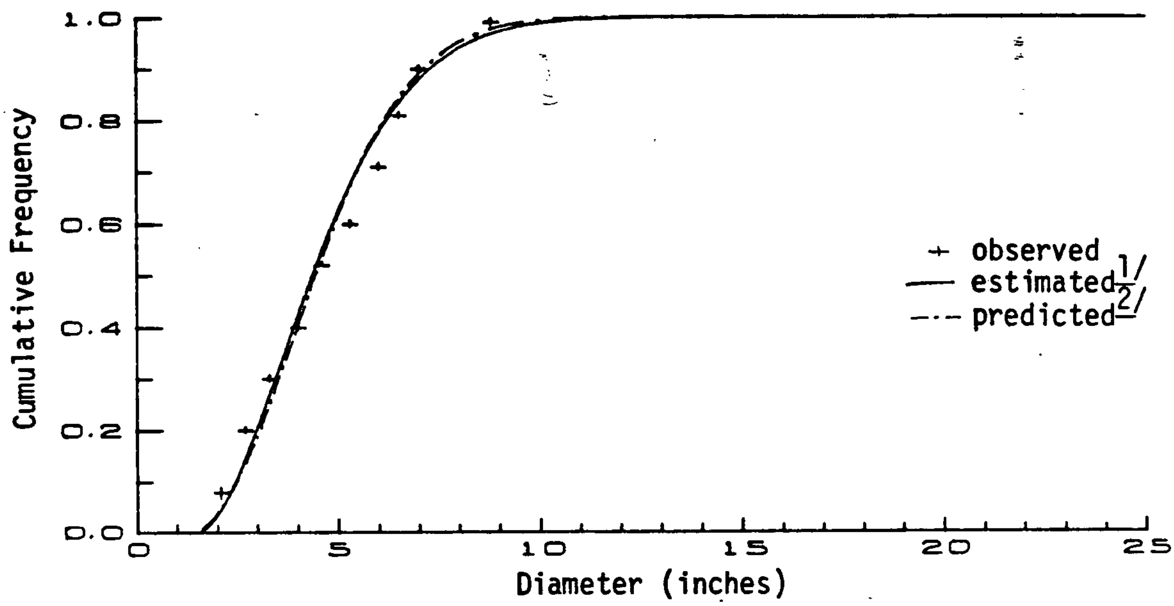
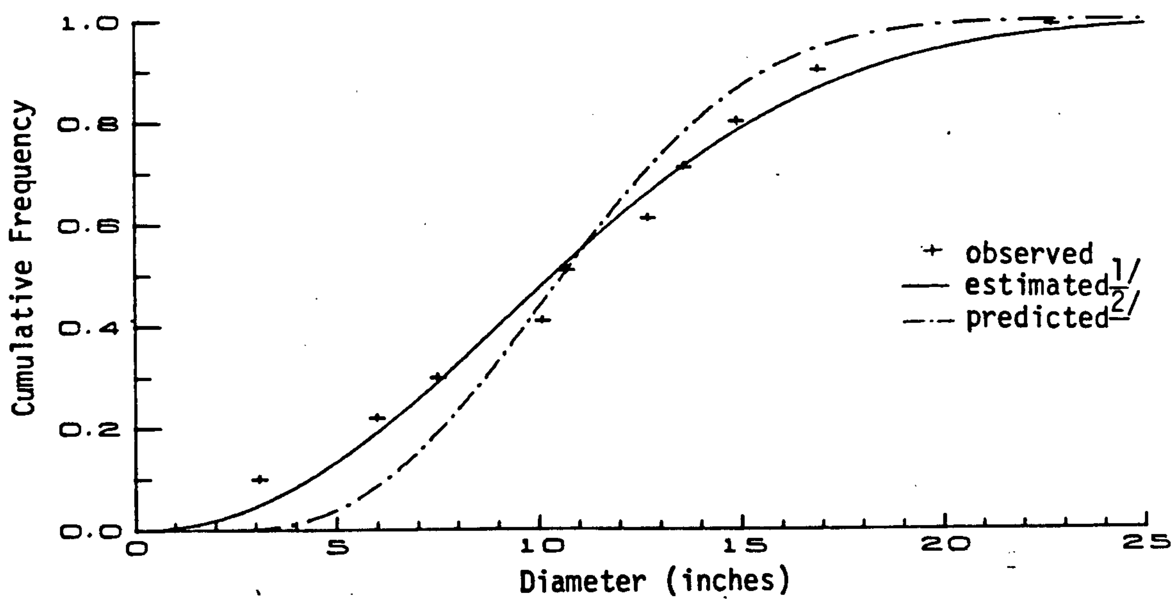


Figure 6. Observed, estimated, and predicted diameter distributions of stand 361216



1/ estimated from observed distribution by maximum likelihood
2/ predicted from stand characteristics by model a(2) b(10) c(4)

Figure 7. Observed, estimated, and predicted diameter distributions of stand 431254

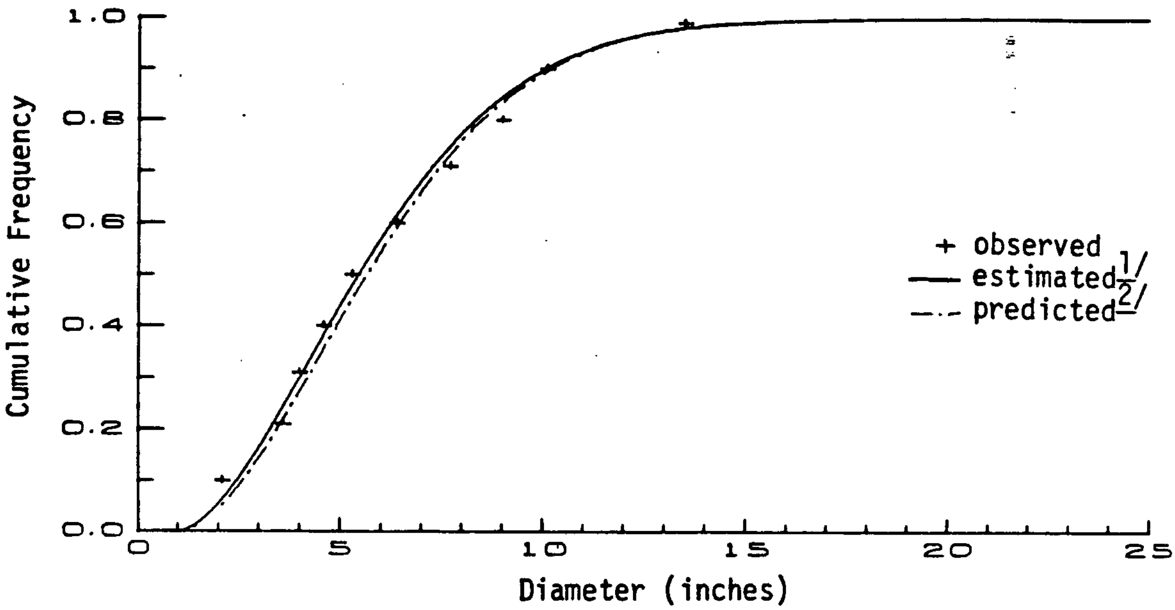
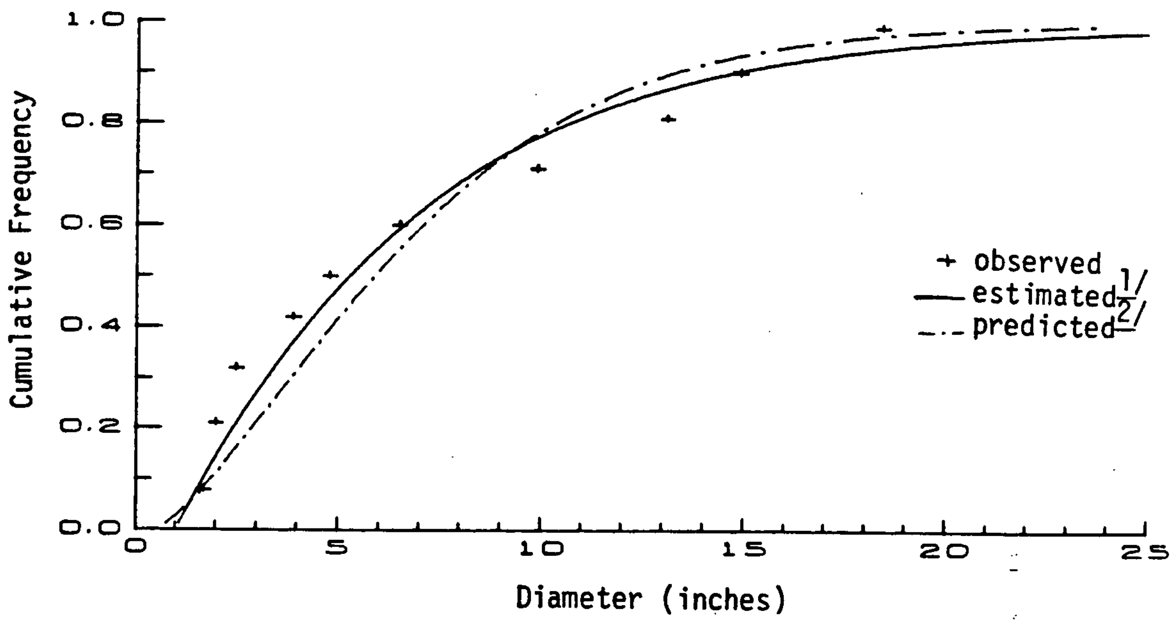


Figure 8. Observed, estimated, and predicted diameter distributions of stand 451266



1/ estimated from observed distribution by maximum likelihood
2/ predicted from stand characteristics by model a(2) b(10) c(4)

Figure 9. Observed, estimated, and predicted diameter distributions of stand 541324

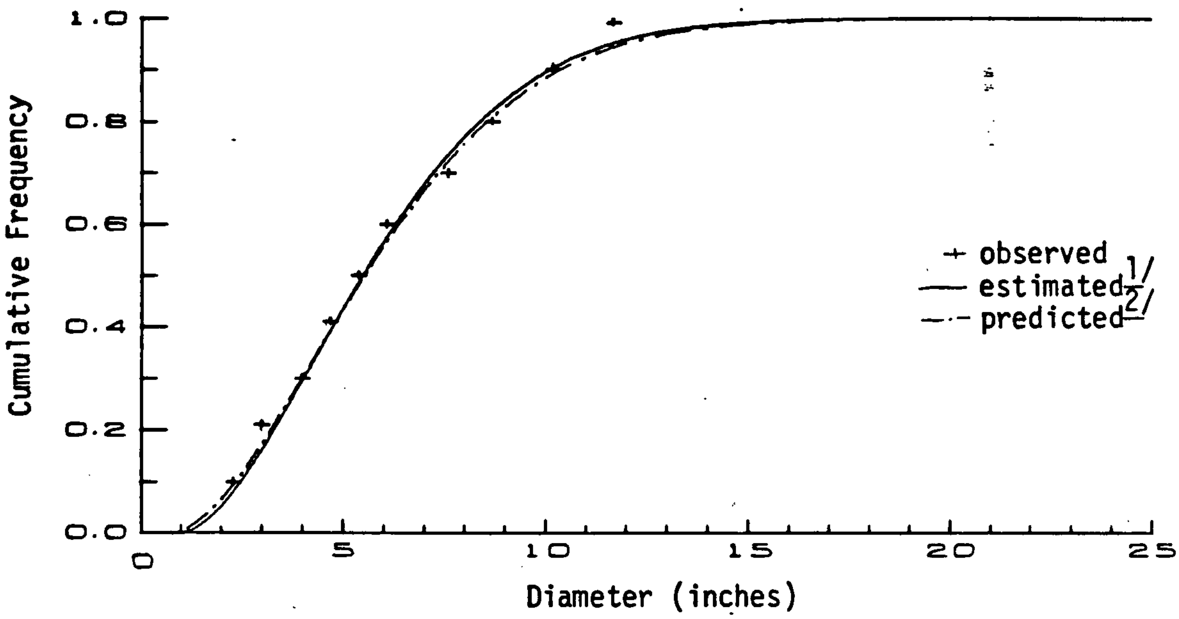
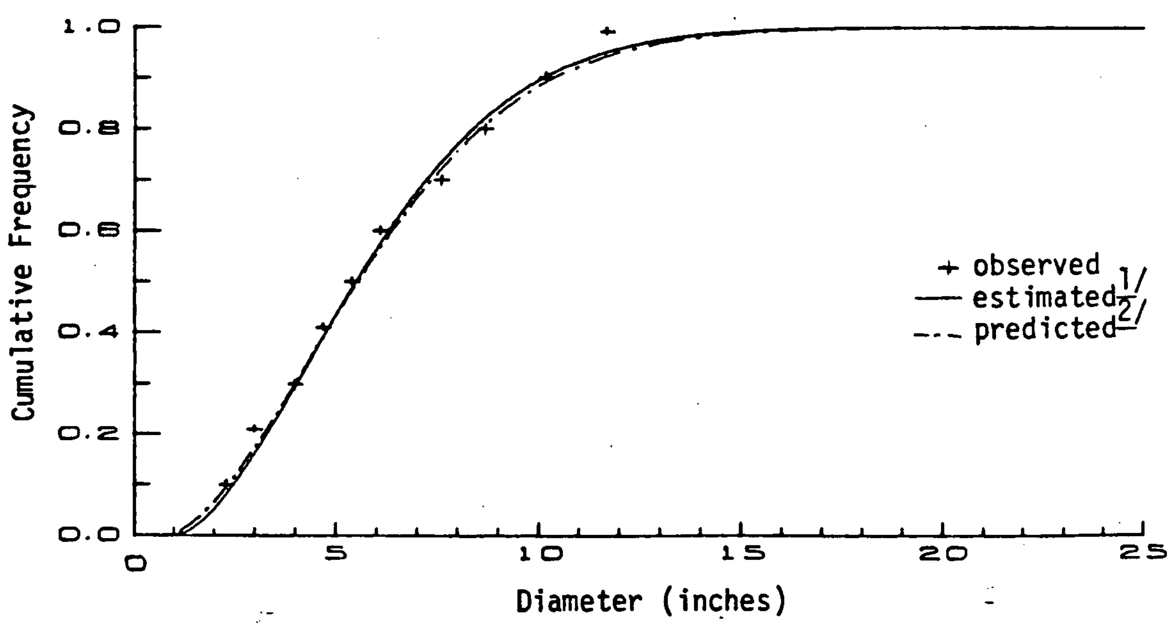


Figure 10. Observed, estimated, and predicted diameter distributions of stand 541324



1/ estimated from observed distribution by maximum likelihood
2/ predicted from stand characteristics by model a(2) b(10) c(4)

Figure 11. Observed, estimated, and predicted diameter distributions of stand 551330

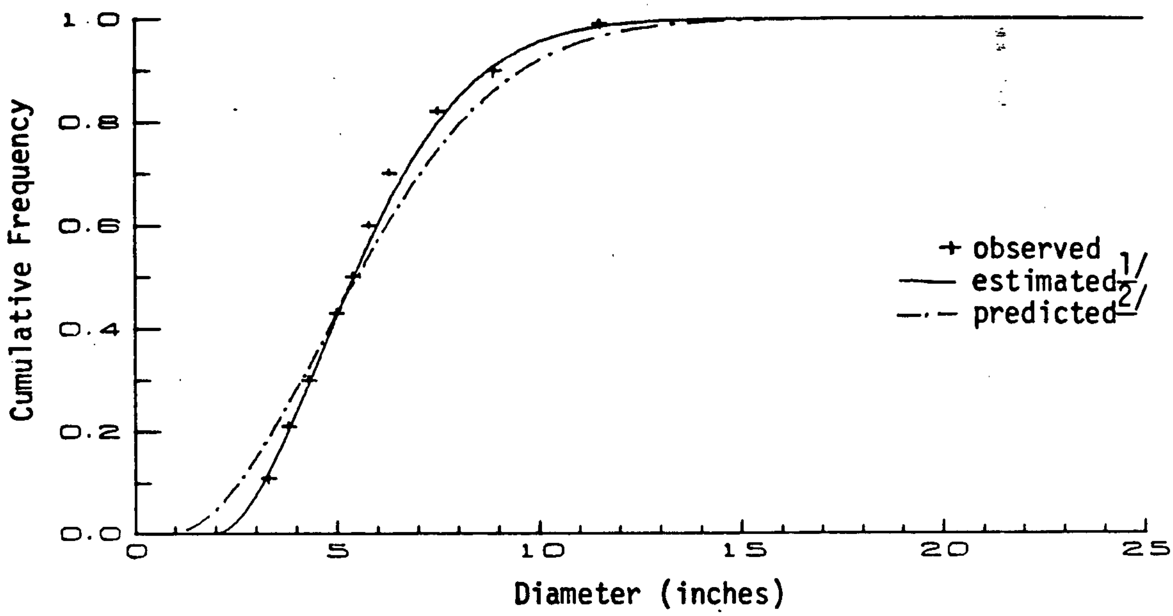
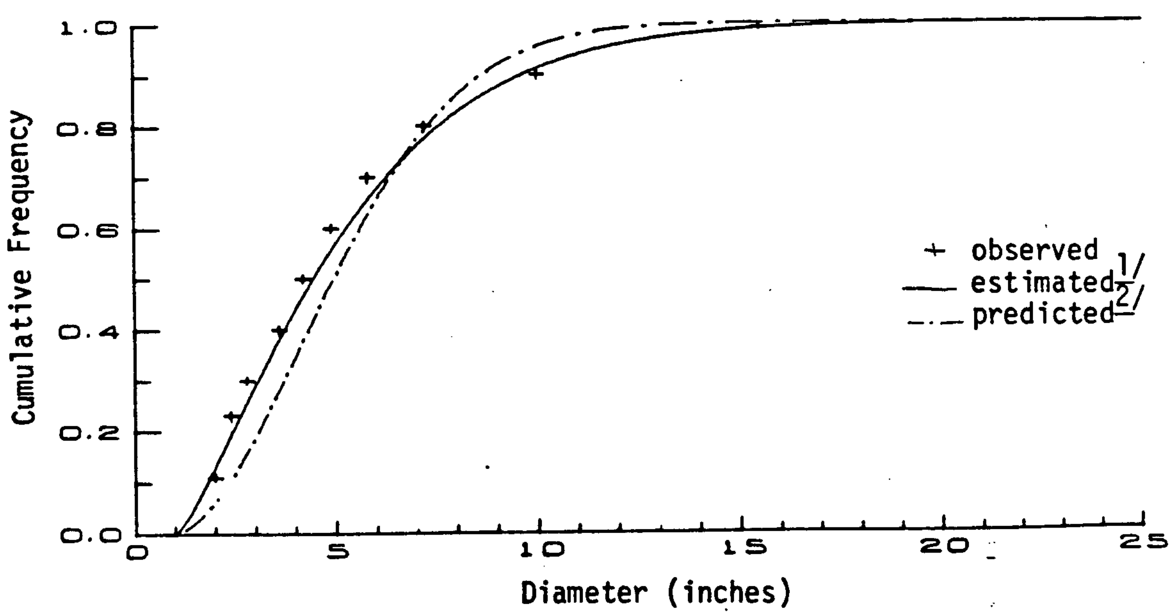


Figure 12. Observed, estimated, and predicted diameter distributions of stand 591354



1/ estimated from observed distribution by maximum likelihood
2/ predicted from stand characteristics by model a(2) b(10) c(4)

Figure 13. Observed, estimated, and predicted diameter distributions of stand 691410

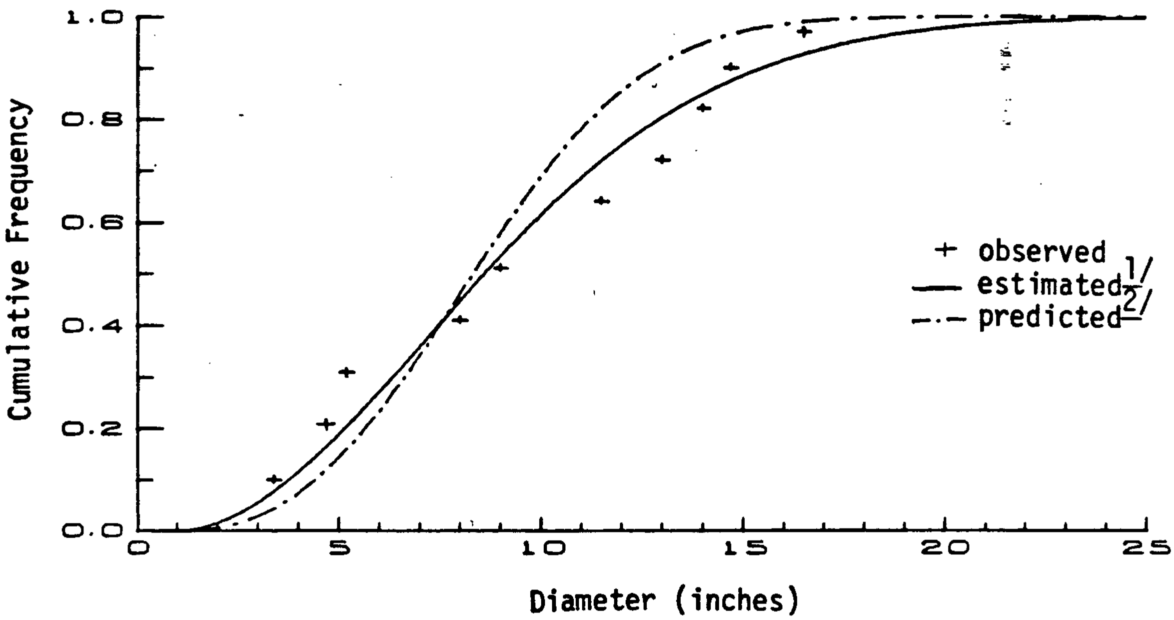
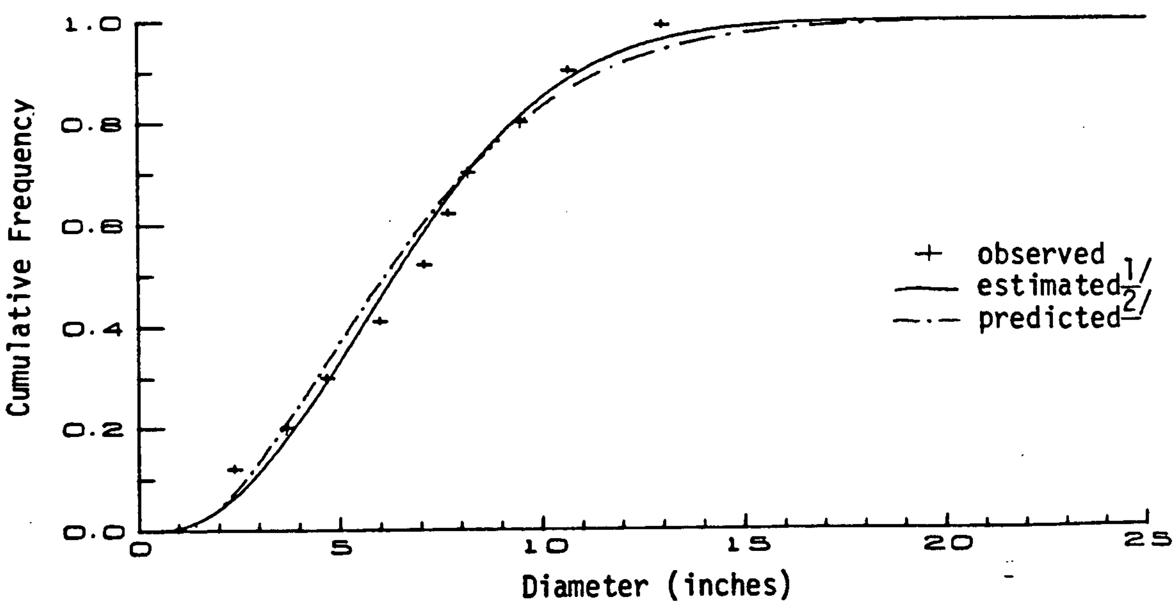


Figure 14. Observed, estimated, and predicted diameter distributions of stand 791472



1/ estimated from observed distribution by maximum likelihood
2/ predicted from stand characteristics by model a(2) b(10) c(4)

Figure 15. Observed, estimated, and predicted diameter distributions of stand 821492

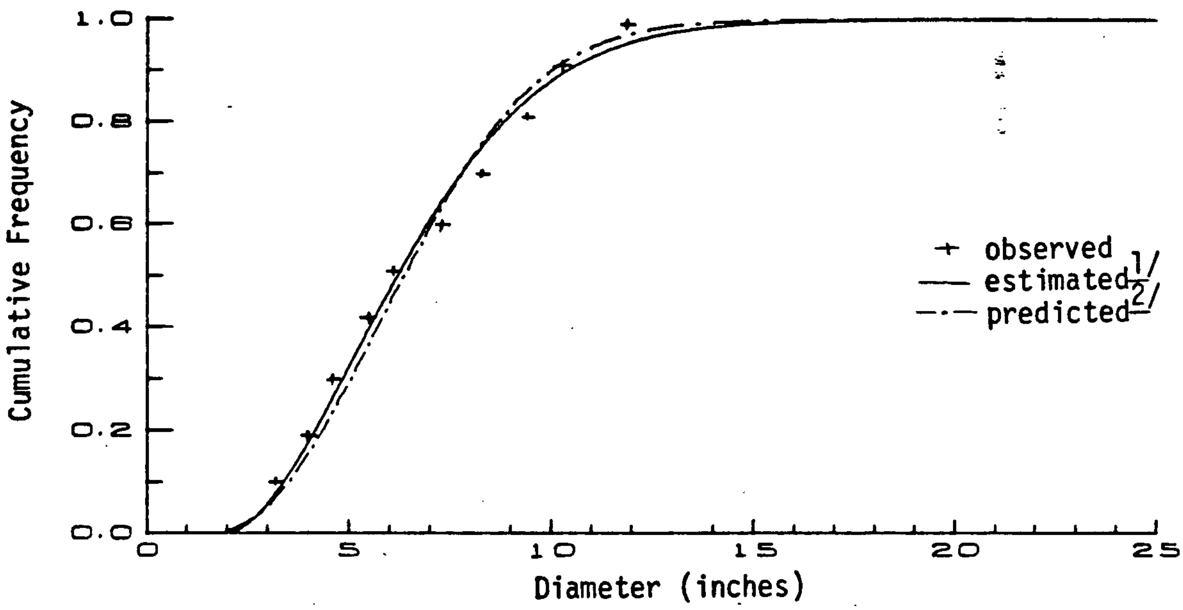
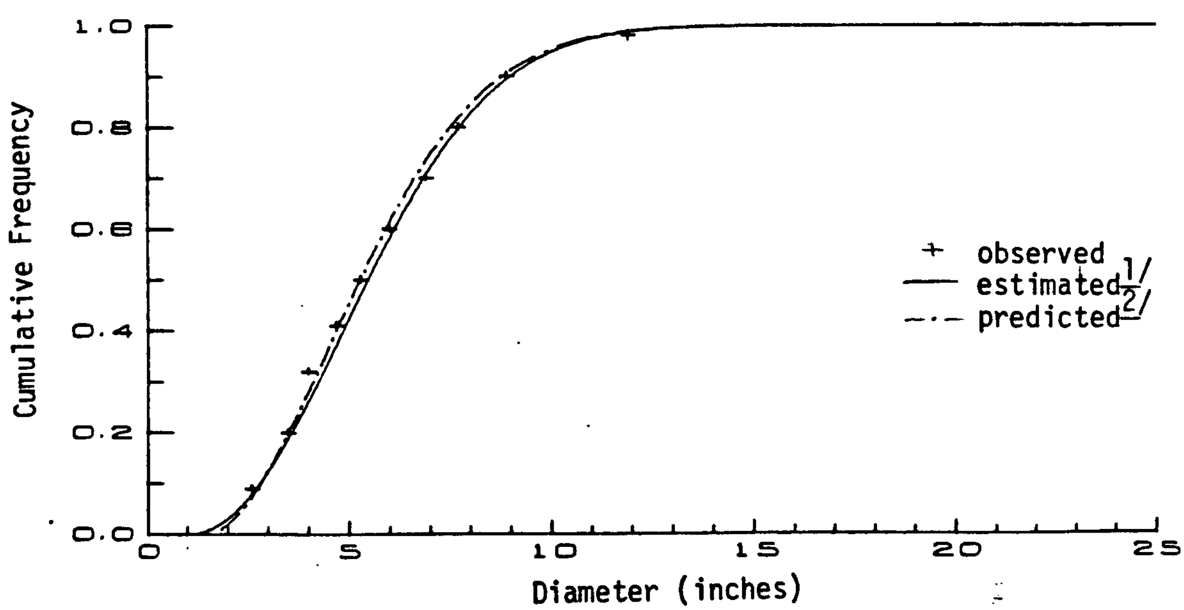


Figure 16. Observed, estimated, and predicted diameter distributions of stand 831498



1/ estimated from observed distribution by maximum likelihood
2/ predicted from stand characteristics by model a(2) b(10) c(4)

Figure 17. Observed, estimated, and predicted diameter distributions of stand 991592

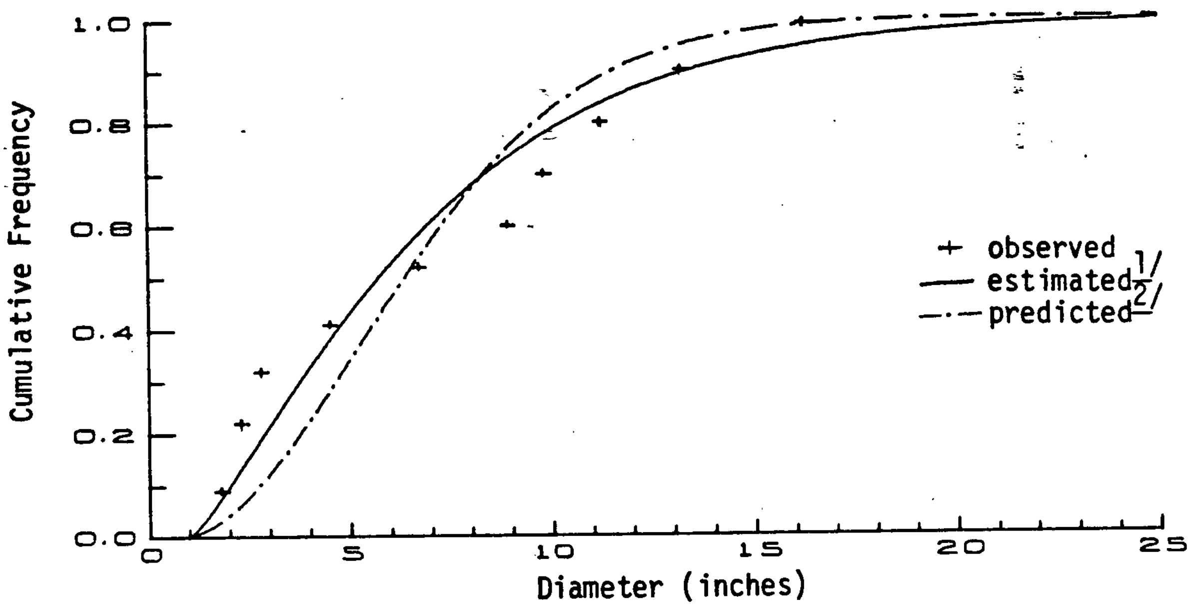
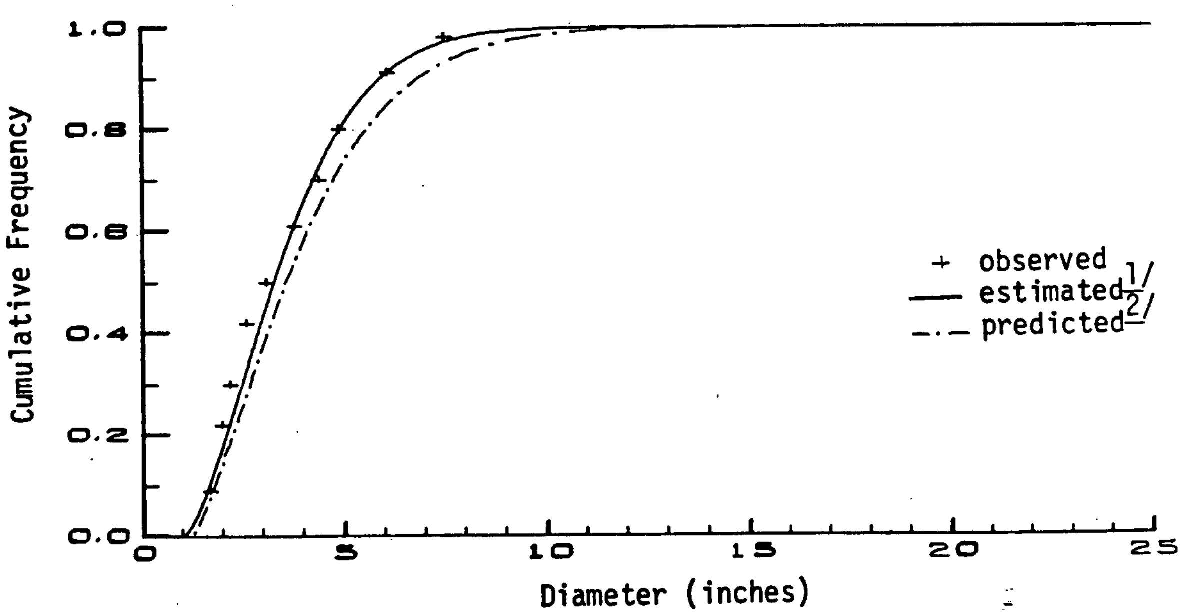


Figure 18. Observed, estimated, and predicted diameter distributions of stand 1101658



1/ estimated from observed distribution by maximum likelihood
2/ predicted from stand characteristics by model a(2) b(10) c(4)