STATISTICAL ESTIMATION AND PREDICTION OF AVALANCHE ACTIVITY FROM METEOROLOGICAL DATA

for the Rogers Pass area of British Columbia

by

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ABSTRACT

The prediction of avalanche activity, by observers in the field, is largely achieved along causal-intuitive lines, depending for its success upon the experience of the observer in his own particular area. Various attempts have been made in the past to quantify such procedures using predictive models based upon meteorological measurements. Modified forms of a multivariate statistical technique known as linear discriminant analysis, have been tried (Judson and Erickson (1973), Bois et al. (1974) and Bovis (1974)) with only partial success. The non-inclusion of time lag decay terms, autocorrelations in the data, insufficient variation in the dependent variable and sampling difficulties, combine to weaken the discriminant approach. These problems and the nature of the phenomenon suggest that a time series approach is required.

A completely flexible system of data storage, retrieval and computer analysis has been designed to facilitate the development of time series models for predicting avalanche activity from meteorological observations for the Rogers Pass area of British Columbia. These methods involve autoregressive integrated moving average (ARIMA) stochastic process description techniques, as well as transfer function and stochastic noise identification and estimation procedures. Such methods not only optimize the selection of the most appropriate intercorrelated independent variables for model development, but actually exploit these intercorrelations to considerable advantage.

A numerical weighting scheme was devised for the representation of avalanche activity in terms of terminus, size and moisture content codes

for each event. Various types of correlation analysis were performed on the data for the period, 1965-73, in which the relationship between avalanche activity and a comprehensive set of simple and complex meteorological variables was examined. Models were then developed for individual years and the entire period, using the three best weighting schemes for avalanche activity representation, and the most promising meteorological variables, as indicated by the results of the correlation analyses. Multiple correlation coefficients as high as 0.87, using a simple two-term model, based on a composite series, involving snowpack depth, water equivalent of new snow and humidity, have been obtained for individual years, and as high as 0.81, using a single six-term model consisting of only two composite meteorological series, for the entire period. Prediction profiles, plotted from these models, indicate that a high level of forecasting accuracy could be possible if such models are fitted to future years.

A simulated forecast was performed on data for the period, 1969-73, using a model developed for the period, 1965-69, with a multiple correlation coefficient of 0.83. A value of 0.76 was realized for the simulated forecast indicating a high degree of precision. During this study, great emphasis was placed on keeping the procedures general, rather than specific, so that, besides producing an accurate evaluation of the avalanche hazard at Rogers Pass, it would also be possible to successfully apply such methods to other areas which have an avalanche problem.

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Chapter I

INTRODUCTION

Geographical Considerations

The Rogers Pass, at an elevation of approximately 4350 ft., provides an important east-west route through the Selkirk Mountains of British Columbia, via the Trans Canada Highway and the Canadian Pacific Railway. It is also one of the most active avalanche areas in Western Canada. A combination of steep-sided mountains, a characteristic of the Selkirk Range, and heavy winter snowfalls, cause more than ninety major sites to affect the highway along a thirty mile length, from the east gate of Glacier National Park to just beyond the west boundary. The greatest concentration of these sites exists between two narrow defiles formed by Mts. Tupper and MacDonald, just east of the Pass, and Mts. Fidelity and Fortitude in the western section (see Appendix A). The terrain and climate of the area have been described by Schaerer (1962:2-5), who categorizes the Selkirks as the northern extension of the middle alpine zone after Roch, "characterized by heavy snowfalls of moist to dry snow, medium temperatures only occasionally below zero degrees Fahrenheit and strong wind action on the mountains." Schleiss (1970:115) recognizes three different climate sub-zones for the area, stating that, "the west side of the park is influenced by the Pacific weather systems, the east side by Arctic weather fronts and the clashing of both systems influences the weather in the central section." This rather complex meteorological situation necessitated the establishment of two major observatories, one at the Rogers Pass headquarters to monitor weather conditions for the eastern section, and the other on Mt. Fidelity, at an elevation of 6250 ft. to monitor the western section. These two observatories provide information on snowpack conditions on a continuous basis throughout the avalanche season, as an aid in the forecasting and control program. This information is supplemented by air temperature, wind velocity and direction, and humidity data, which is telemetered from two remote observatories, MacDonald West Shoulder (elevation 6500 ft.), located above the Rogers Pass, and Roundhill Station (elevation 6900 ft.), at Mt. Fidelity.

Avalanche Hazard Evaluation and Control

The Snow Research and Avalanche Warning Section (SRAWS), under the jurisdiction of Parks Canada and the leadership of the snow and avalanche analysts, V.G. and W.E. Schleiss, conducts an ongoing program of avalanche hazard evaluation and control for the Rogers Pass area. The operational objectives involve the maintenance of an optimum balance between minimum highway closure times and the safety of the public and parks personnel. This balance can only be achieved by the accurate evaluation of avalanche hazard, backed up by prompt action in the form of artillery control.

Potential for Avalanching. The avalanche hazard evaluation is based on an evaluation of the stability of the upper, often new snow layers and the lower layers within the snowpack, combined with an assessment of the amount of available snow for avalanching at each site.

Ideally, stability measurements should be made in the starting zone and avalanche track, but logistical difficulties, inaccessibility and danger to the observer prevent this. Ski-tests are performed, however, whenever possible, on short slopes at high elevations, which are representative of conditions in the slide paths. Such tests often reveal instability in the upper layers, when they fracture and move under the skier's weight, and hence provide a direct indication of instability.

More usually however, it is necessary to rely upon less direct structural measurements made at the study plot and indirect indicators in the form of meteorological observations. The presence of a weak layer in the new or partially settled snow may be detected and some form of strength test applied. The amount of new snowfall, air temperature and wind will also provide an indication of the stability of the upper layers.

The stability of the lower layers within the pack can be interpreted from current snow pit data, or interpolated from past data. The analyst will be aware of any deep-seated instabilities within the pack, for example, a persistent surface hoar layer which has been responsible for several avalanche cycles so far that winter.

Finally, to complete the evaluation, the analyst refers to his past records of avalanche activity to determine the availability of avalanchable snow for each particular site on an individual basis. As LaChapelle (1970:108) has observed, "The hazard evaluation is amenable to numerous refinements. For large avalanches falling over long paths, the volume of snow apt to reach the valley floor can be estimated by taking into account the amount of unstable snow in the middle and lower reaches of the path." For example, avalanches may recently have occurred at some sites resulting in the removal of the upper unstable layers and perhaps also the lower layers, if they were sufficiently unstable. Furthermore, at other sites, the lower layers may no longer be present, as a result of previous avalanching that occurred some time in the past. Therefore, a complete historical record of avalanche activity at each site, since the beginning of the season, is a necessary requirement for the determination of the amount of avalanchable snow likely to be available.

Hence, an accurate evaluation of the potential for avalanching relies upon three factors, as depicted in Figure 1, the stability of the upper layers, which will often be trigger snow, the stability of the lower layers, which may constitute the main mass of the avalanche released or set in motion by the trigger snow and the availability of snow for avalanching at each particular site.

Avalanche Hazard Evaluation. The evaluation of avalanche hazard relies heavily, but not exclusively, on the evaluation of the potential for avalanching, as defined above. Consideration must also be given to the possible effect of such avalanching on human life and property, which, in the case of the Rogers Pass, can be identified with the Trans Canada Highway. For example, the potential for avalanching on some sites may be extremely high, but these sites may not affect the highway, therefore the hazard to the highway would be low. In areas other than the Rogers Pass, hazard might perhaps be identified with respect to skiers in relation to ski areas or back country travel, in which case the hazard evaluation would be different.

Avalanche Hazard Forecast. Finally, the avalanche hazard evaluation can be combined with the weather forecast to produce an avalanche hazard forecast, which may be either short term or long term, depending on the nature of the weather forecast.

Figure 1 summarizes the important steps in the evaluation and forecasting procedures just described.

At the Rogers Pass, operational decisions with regard to highway closures are based upon the avalanche hazard evaluation. An avalanche hazard forecast, in the strict sense, is seldom, if ever, attempted due to the unreliability of mountain weather forecasting data. However, a current evaluation is all that is generally required as a basis for operational decisions. As LaChapelle (1970:107) points out, "The hazard evaluation seeks to ascertain current snow stability. It is the basis on which operational decisions (road closures, control measures, etc.,) are most often made. This is the most common function and the one which is usually labelled 'avalanche forecasting' in the loose sense."

Avalanche Control. Hence, the avalanche hazard evaluation may lead directly to a decision with regard to the possible closure of the highway, after which artillery control measures may be implemented. From various established gun positions alongside the highway, 105mm howitzer shellfire is directed at predesignated target areas, usually trigger zones which are generally situated above the main avalanche starting zones. These trigger zones often consist of small localized deposits of highly unstable snow, the release of which loads the lower slopes causing them to avalanche. Whenever possible, such 'stabilization shoots', as they are called, are implemented before large buildups of snow have occurred in the starting zones and slide paths, so that any avalanches which result do not reach unreasonable proportions, (such occurrences are referred to as 'artificial' avalanches as opposed to 'natural' avalanches which take place without human intervention). Minimizing the occurrence of large avalanches in this way not only decreases the hazard to the highway, but also reduces the times required for cleanup operations. Ideally however, controlled avalanches should be of a significant size, resulting in a substantial reduction of snow in the accumulation zones.

The major benefit of the avalanche control program lies not so much in the reduction of avalanche size, but in the fact that any artificial avalanches which occur as the result of stabilization procedures, do so under relatively safe conditions during periods of highway closure. Furthermore, a rigorous control program, executed during the height of the season, by preventing excessive buildup of snow in the avalanche paths, effectively cuts down the size and number of the more dangerous and unpredictable wet snow avalanches that take place in the spring. These spring avalanches are not amenable to artillery control due to the damping effect of wet snow which severely limits the propagation of the explosive energy through the snowpack.

Purpose of this Study. Returning to the problem of hazard evaluation, it is not always possible to identify periods of instability intuitively, and even those that are identified may be of short duration, if, for example, the snow is settling rapidly after a heavy snowfall. The time interval between the decision to perform artillery control and the firing of the first round may be such that the period of instability is missed.

This study addresses itself to the problem of statistical estimation and prediction of avalanche activity from meteorological data using analytical techniques. Mathematical models have been developed which describe the phenomenon in terms of the statistical behaviour of past data. It is hoped that such models will ultimately be used, by the avalanche analyst, as an important aid along with the other somewhat intuitive approaches, to enable him to more accurately evaluate the hazard situation, and identify and predict periods of instability with greater certainty.

Besides providing efficient working models, in the operational sense, the application of statistical methods to this complex problem should also eventually help to reveal the physical processes which govern the formation of avalanches.

Development of Models

A completely flexible system of data storage, retrieval and computer analysis has been designed to facilitate the development of simple or complex time series models involving auto-regressive integrated moving average (ARIMA) process description techniques, as defined by Box and Jenkins (1970), as well as transfer function and stochastic noise identification and estimation procedures. These methods not only facilitate the optimum selection of intercorrelated independent variables, but actually exploit these intercorrelations to considerable advantage.[1] A suite of computer programs was written in FORTRAN and thoroughly tested using avalanche and meteorological data for the period, 1965-73. The data were then systematically analysed and the best forecasting models developed, both for individual years and for the entire period. Multiple correlation coefficients as high as 0.87, using a simple two-term model have been obtained for individual years, and as high as 0.81, using a

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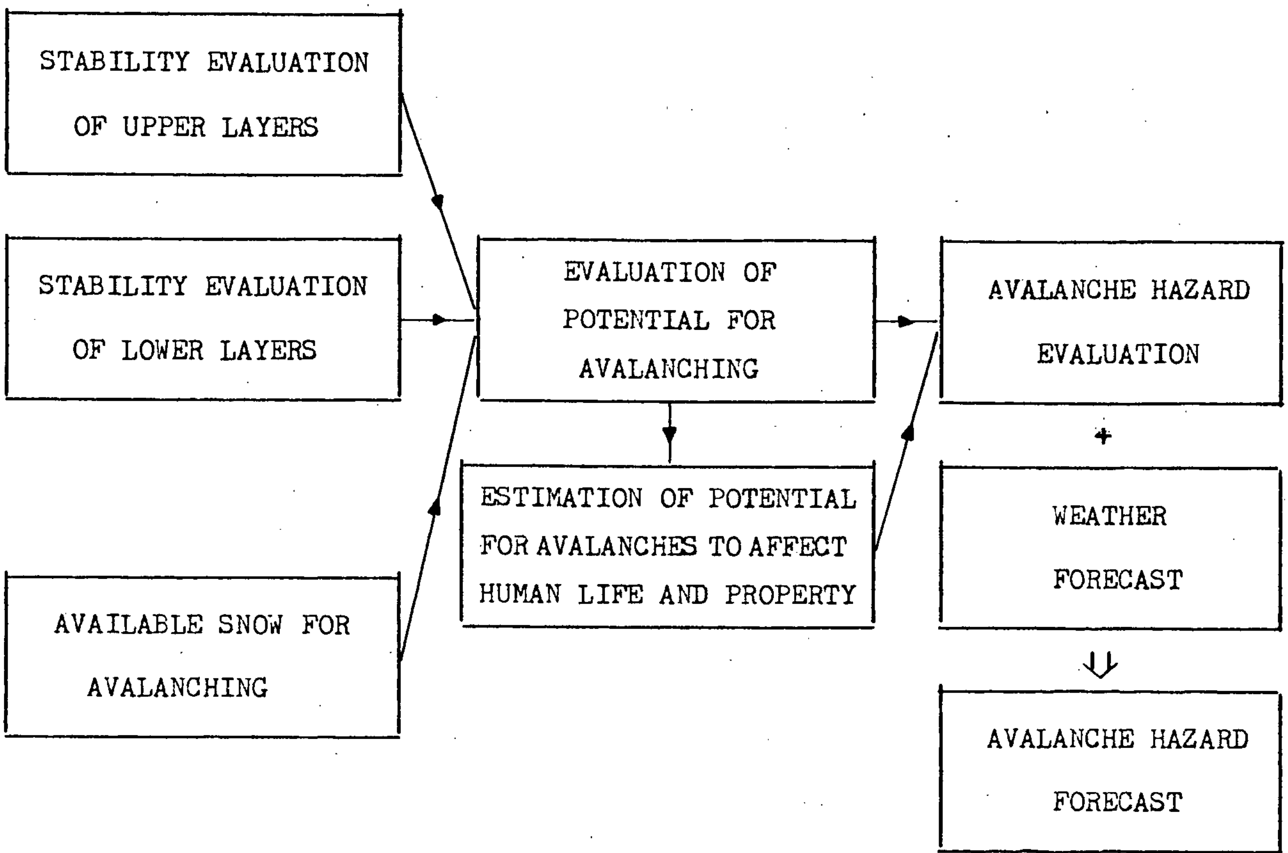
[1] The usual backwards, forwards, or stepwise selection procedures, employed in normal least squares regression and discriminant analysis, break down if strong intercorrelations exist among the independent variables (Draper and Smith,1966:163-195). As Judson and Erickson(1973), Bois, Obled and Good(1974), and Bovis et al.(1974) have discovered, such conventional approaches can lead to complicated but relatively weak models, peculiar to the particular data sets analysed, consisting of large numbers of interrelated unlagged meteorological terms, many of which are only just significant.

Figure 1

AVALANCHE HAZARD FORECAST



single six-term model, consisting of only two meteorological series of a composite nature for the entire period. Prediction profiles using these models have been plotted and a high degree of accuracy can be demonstrated. During this study, great emphasis was placed on keeping the procedures general, rather than specific, so that besides producing an accurate evaluation of the avalanche hazard at Rogers Pass, it would also be possible to successfully apply such methods to other areas which have an avalanche problem.

Chapter II

STATISTICAL EVALUATION OF AVALANCHE HAZARD:
A REVIEW

Delineation of the Most Significant Meteorological Factors

Atwater's Precipitation Intensity Term. Atwater (1952), was among the first to recognize the importance of precipitation intensity, P.I., measured on an hourly basis, as an "excellent indicator of avalanche hazard" (Atwater, 1952:17). Based on studies at three stations: Alta in Utah, Stevens Pass in Washington and Berthoud Pass in Colorado, he was able to devise the following 'rule of thumb', "P.I. continuously above 0.10 in. per hour, at wind velocities 15 mph or over and in the absence of sluff cycles equals a high degree of avalanche hazard whenever total precipitation is one inch" (Atwater, 1952:18).

Perla's Contributory Factors in Avalanche Hazard Evaluation. Perla (1970) investigated twenty years of storm and ramsonde profile data measured at Alta, Utah for the period 1950-69, considering only large avalanches on south facing slopes. After performing a contributory analysis, he found that, "the probability of an avalanche hazard varies considerably with precipitation and wind direction, only slightly with temperature change, and seems to have no definite relationship to wind speed and snow settlement"(Perla, 1970:418). Hence, while there is a concensus of opinion on the importance of precipitation, the role played by wind speed or direction is less clearly defined. However, the greater influence of wind direction compared to wind speed may simply be a consequence of the uniform orientation of the set of avalanche sites studied by Perla.

Judson's Univariate Analysis. Judson and Erickson (1973) conducted a univariate analysis similar to Perla's (1970) analysis of contributory factors in avalanche hazard evaluation. This analysis was performed on twenty-three avalanche paths, nineteen of which were controlled by explosives, located in the Central Rockies of Colorado's Front Range near Berthoud Pass, the Urad Mine and Loveland Pass.[2] Seven winters of data (1963-70) were used but the analysis was restricted to storm periods only. Simple linear regression analysis was applied using the number of avalanches from the twenty-three paths as the dependent variable and single weather factors or simple combinations of them as independent variables, in an attempt to identify the most significant terms. The four factors so identified were 24-hour water equivalent, 24-hour snowfall, maximum precipitation intensity and maximum precipitation intensity modified for excessive wind. "The factor best correlated with avalanche activity was the sum of the maximum precipitation intensities multiplied by a constant for excessive wind speed" (Judson and Erickson, 1973:2), which was termed the "storm index".

Recognition of Need for a Time Series Approach. Although the storm index seemed quite promising, Judson and Erickson (1973:4) saw the need for a time series approach, "The main drawback with the storm index is that the index is highest near the end of storms, even though

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[2] A separate analysis was performed on twenty-three uncontrolled avalanche paths which resulted in weaker correlations, implying that, "data from uncontrolled paths are difficult to interpret and are less reliable as forecast guides." (Judson and Erickson, 1973:4)

hazard may be decreasing because some avalanches have already fallen and the snow is stabilizing. A way of reducing the index toward the end of the storm (a decay function) is badly needed and is now under study."

Discriminant Analysis

The Linear Discriminant Analytical Procedure. Various attempts have been made, notably by Judson and Erickson (1973), Bois, Obled and Good (1974), and Bovis (1974), to produce forecasting models for avalanche occurrences using modified forms of a multivariate statistical technique known as linear discriminant analysis. This procedure, which is closely related to linear regression analysis, involves, in its simplest form, the assignment of 'cases' into one or other of two groups, using a linear discriminant function. The function consists of a linear combination of independent variables, multiplied by appropriate coefficients, which are least squares estimates, obtained by maximising the ratio of the between groups variance to the within groups variance (Rao, 1952).

After obtaining the function, the mean value of the discriminant for each group may be calculated by substituting the group mean values of each independent variable into the function. The difference between the two mean values of the discriminant is known as the generalized or Mahalanobis distance, and the average of the two multivariate group means, known as the discriminant index, serves as a criterion for the classification process. Significance tests may be performed on each independent variable and the Mahalanobis distance. A 'probability of misclassification' may be obtained by comparing the value of the Mahalanobis distance with a cumulative normal frequency distribution table of the normal deviate. The method is capable of extension into three or more group classifications, in which case two or more discriminant functions are required to be calculated.

Judson's Discriminant Analysis. After identifying their most significant variables, Judson and Erickson next performed a multivariate linear discriminant analysis using eight controlled sites on an individual basis, and data for the period, 1952-71. Group classifications were based on control results. Days were assigned to group 1 when control efforts produced a slide, or when a natural avalanche occurred, and to group 2, when control efforts failed to initiate an avalanche. Discriminant functions were developed for each site containing the following terms:

(1) a precipitation term made up of the sum of the maximum consecutive 3-hour precipitation intensities within each 6-hour period decayed over an interval. "The function is held at one for the first 2 days, reaches 0.5 on the 5th day, and levels off at 0.2 from the 9th day on." (Judson and Erickson, 1973:10),

(2) a temperature term consisting of the sum of the 6-hour negative temperature departures from 20°F,

(3) a wind term made up of the sum of the wind speeds greater than or equal to 15 mph resolved to an optimum direction for each path.

Probabilities of misclassification ranged from 21 to 30%.

Problems with Judson's Discriminant Analysis. Judson and Erickson's models are useful in that they indicate which of the meteorological factors are most significant. However, they are far too weak to be used in a real situation for avalanche hazard evaluation for the following reasons:

(1) Lagged Variables. The functions rely exclusively on current weather factors, although an attempt was made to introduce certain arbitrary decay terms to overcome this deficiency. Perla and Judson (1973)

have investigated the possibility of introducing fading memory terms, without arbitrary factors, into the discriminant analysis procedures. However, discriminant analysis does not readily lend itself to time series applications.

A stochastic transfer function time series approach, on the other hand, is far superior in that it automatically involves lagged values of precipitation, temperature and wind terms, the coefficients of which are least squares best estimates determined from the actual data.

(2) Intercorrelated Variables and Autocorrelated Data. Strong intercorrelations between the independent variables, a normal feature of weather data, are not handled well by conventional regression methods like discriminant analysis. Furthermore, the meteorological time series are usually quite strongly autocorrelated, or in other words, adjacent observations in time are not independent.

Such intercorrelations and the interdependence of observations adjacent in time are regarded as an undesirable feature of the data, in a conventional regression situation, resulting in the interference of normal variable selection procedures such as 'forwards selection' and 'backwards elimination'. This leads to models which do not necessarily contain the 'best' set of independent variables.

Time series analysis procedures, on the other hand are designed to operate on observations which are dependent and, "where the nature of this dependence is of interest in itself" (Box and Jenkins, 1970:vii). The time series approach exploits these intercorrelations to the fullest advantage, producing much more powerful models, containing an optimum selection of lagged and unlagged meteorological terms. Furthermore, such models, if developed for separate years, tend to display greater similarity than discriminant functions, which are often uniquely different. Model similarity between years is, of course, a desirable feature if the prediction of activity for future years is contemplated.

(3) Variation of the Dependent Variable. The assignment of all avalanche days, whether the level of activity is high or low, into one class is bound to lead to weak models. It is far better to treat avalanche activity as an ordinary dependent variable, allowing it to take on values corresponding to various levels of activity, thereby more accurately reflecting the changing meteorological conditions which give rise to the phenomenon.

(4) Data Imbalance between Avalanche Days and Non-Avalanche Days. A further undesirable feature of discriminant analysis, in its application to the avalanche forecasting problem, lies in the imbalance between avalanche and non-avalanche days. There are usually far more non-avalanche days, which results in discriminant functions which are biased in the direction of the non-avalanche group. Hence, a greater proportion of the avalanche days are misclassified than non-avalanche days. To overcome this difficulty, Judson and Erickson (1973) use a weighted average of the discriminant means for each group as their discriminant index. This somewhat artificial and unsatisfactory device causes the probabilities of misclassification for avalanche and non-avalanche days to be approximately equal, but does little to improve the overall classification scheme.

Bois et al. (1974), and later, Bovis(1974), try to overcome this 'zero imbalance' by a different device, which involves the selection of a random sample of non-avalanche days equal in number to the avalanche days.

However, tests using the Rogers Pass data have shown that randomly sampling non-avalanche days, in this fashion, gives rise to discriminant functions which are significantly different for the same block of avalanche data. Ten runs were made using data for the period, 1972-73, and avalanche occurrences at a single avalanche site called 'Portal'. A random sample of non-avalanche days, equal in number to the avalanche days, was selected for each run. After backwards elimination, using the same initial set of independent variables for each run, ten unique models were obtained, consisting of a minimum of two and a maximum of eight significant precipitation, temperature and wind terms, with probabilities of misclassification ranging from 7 to 24%. Thus, the models appeared to be a function of the particular set of non-avalanche days, even though the sets were chosen randomly. Hence, such a procedure must be viewed with a great deal of scepticism.

Bois' Discriminant Analysis. Bois, Obled and Good (1974) have analysed avalanche and meteorological data from the Parsenn area of Switzerland, for the period, 1961-70, restricting their analysis to natural occurrences only. They use a three-way discriminant analysis approach in an attempt to distinguish between wet snow avalanche days, dry snow avalanche days and non-avalanche days. A single event, on any site, serves to classify a day as an avalanche day. The ten-year sampling period was analysed on a monthly basis, for example, all Januaries in the ten-year sampling period were taken as the total population for that month. This procedure was adopted presumably on the assumption that similar conditions occur during the same month each year on a regular basis. This is not generally the case since some winters may be more advanced than others on a particular date each year.

As previously mentioned, Bois et al. (1974) select a random sample of non-avalanche days, approximately equal in number to the avalanche days, in order, not only to eliminate the 'zero-imbalance', but also because, "this eliminates serial correlation between successive days" (Bois et al., 1974:7). It has already been pointed out that meteorological and avalanche observations adjacent in time are generally not independent, that is to say, the series are autocorrelated. The autocorrelation functions of such series reveal a great deal about the processes involved and should certainly not be eliminated. Serial correlations should be exploited by use of proper time series procedures.

Bois et al. (1974) have documented the results of their analysis for March only, which indicate that,

(1) height of settled new snow summed over precipitation sequence,

(2) temperature at 1:00 P.M. on the previous day, plus 3°C,

(3) the number of precipitation sequences (longer than 2 days) since the beginning of the winter, are the three most important variables for dry snow avalanche classification, and,

(1) temperature at 1:00 P.M. on the previous day,

(2) the number of avalanche days in the test area per number of precipitation sequences, and,

(3) absorbed radiation flux, are the three most important variables for wet snow avalanche classification.

Probabilities of misclassification for both wet and dry avalanches for March using these variables were of the order of 19%.

Bovis' Discriminant Analysis. Bovis (1974) has performed a statistical analysis of avalanche events along station 152 (Highway 550) in the San Juan Mountains of southwestern Colorado for the 1972-73 and 1973-74 seasons. Bovis' approach is similar to that of Bois et al., in that he employs a linear discriminant analysis technique in order to discriminate between wet, dry and non-avalanche days. Random selection of a sample of non-avalanche days equal in number to the avalanche days is also used by Bovis.

Bovis has however introduced two important refinements. Firstly, avalanche events are stratified on the basis of magnitude for both the dry and wet seasons. Four magnitude classes of avalanche activity for the area are recognized. This is similar to a regression situation in which the dependent variable is allowed to take on any one of five values, including zero. As discussed previously, such a scheme, by decreasing the restrictions on the effective variation of the dependent variable is bound to result in stronger models.

However, stratification of avalanche events in this way does unfortunately result in a reduction in the sample sizes. As Bovis (1974: 71) points out, "stratification on the basis of magnitude provides a variable operational definition of an avalanche day, although it is constrained by considerations of sample size." If the sample sizes are too small, the discriminant analysis procedure breaks down. At least thirty cases are generally regarded as necessary to provide a good estimate of the group mean and variance. Hence, as Bovis has recognized, his data base is rather too small to produce reliable samples and hence discriminant functions from which any fundamental conclusions may be drawn.

Spurious terms appear in his models, for example, "although the importance of variable 2 in the table 16 comparisons can be related to slope loading, the interpretation of air temperature is less clear", (Bovis, 1974:81) and, "no physical significance can be attached readily to variable 8 (mean wind speed during preceeding 24 hours) in the three time integrations in table 17 since its average value is lower over the avalanche day group, indicating a higher wind-loading potential for non-avalanche days in this instance" (Bovis, 1974:85).

Of course, if avalanche activity is treated as a normal dependent variable and time series methods employed instead of discriminant analysis, no such sampling problem exists.

The second significant feature of Bovis' work is his use of meteorological and snowpack parameters, integrated over two, three or five days, as independent variables. This is certainly one way to introduce the effects of past conditions into the models, rather similar to Judson's arbitrary decay terms, except that, in Bovis' analysis, each term, integrated over the time interval, is equally weighted.

These attempts further serve to illustrate the need for a time series approach, in which the lagged variables appear as a necessary and elegant consequence of the procedures involved.

It is useful to compare Bovis' most significant variables with those of Bois et al. and Judson and Erickson, previously quoted. For dry slides and for the 1972-73 winter, Bovis found that,

(1) maximum 6-hour precipitation intensity in the 24-hour period,

(2) total precipitation over two, three or five days prior to the event, and,

(3) certain temperature terms, were the most important factors

for the unstratified events, and natural slides greater than or equal to magnitude 2. Overall probabilities of misclassification were of the order of 35%. Sample sizes for wet slides were too small to provide useful indicators of significant variables.

Summary and Conclusions

In summary, it is felt that the time series procedures about to be described in this study of avalanche activity as a function of meteorological parameters, are superior to the discriminant analysis techniques employed in the past, for the following reasons:

(1) Lagged variables (decay terms), representing the effects of previous precipitation amounts, temperatures, winds, etc., can be introduced into the models conveniently and elegantly, in the most efficient manner. Discriminant analysis does not lend itself to the introduction of such time series terms.

(2) Intercorrelated variables and autocorrelated data, a drawback in normal regression and discriminant analysis, can be exploited in the time series approach, to produce the 'best' models in an optimum sense.

(3) Avalanche activity is treated as a dependent variable, and allowed to take on values corresponding to various levels of activity, in unison with the independent variables. This results in much more powerful models.

(4) Problems related to small sample sizes of discriminant groups and the imbalance between avalanche and non-avalanche days are eliminated if a time series approach is employed.

Chapter III

AVALANCHE ACTIVITY AS A DEPENDENT VARIABLE

Field Observations of Avalanche Occurrences

Approximately one hundred active avalanche sites are recognized by the Snow Research and Avalanche Warning Section, and have been classified by name, number and mileage from the east boundary of Glacier National Park (see Appendix B).

Natural occurrences are recorded generally on a twice daily basis, often after the event, according to a prescribed format. The site name, date, and if possible, the time of occurrence is noted, along with the observer's estimate of size, terminus and moisture content, indicated by the designations in Table I. The continuous monitoring of such an intermittent phenomenon is often aggravated by limitations on the availability of man power, high hazard and poor visibility, particularly during periods of intense activity when observations are most needed. These problems, combined with the necessarily subjective nature of the measurements, set the limit on the overall accuracy of the data and ultimately determine the level of random noise in the prediction models. Artificial occurrences are noted during the stabilization shoot and can therefore be timed reliably when visibility is good. Size, terminus and moisture content are also recorded whenever possible.

Both for artificials and naturals, size is estimated relative to the actual size of the particular site, either from the visual appearance of the site and size of the deposit, in the case of naturals after the event, or from a visual impression of mass and energy, if the avalanche

Table I

Avalanche Activity Index Weighting Schemes

Designation	(12,12,6) SML	(12,4,2) SML	(12,2,2) ML	(12,3,1) SML	(12,1,1) SML	(1,1,1) ML
Terminus						
¼ or 1/3 path	1	1	1	1	1	1
½ path	2	2	2	2	2	1
2/3 or 3/4 path	3	3	3	3	3	1
End path, to fan, or gully	4	4	4	4	4	1
¼ fan	5	5	5	5	5	1
1/3 fan	6	6	6	6	6	1
½ fan	7	7	7	7	7	1
2/3 fan	8	8	8	8	8	1
3/4 fan, Old RR, or Bench	9	9	9	9	9	1
Over fan, or Mounds	10	10	10	10	10	1
Edge TCH	11	11	11	11	11	1
Over TCH	12	12	12	12	12	1
Size						
Small	1	1	0	1	1	0
Medium	6	2	1	2	1	1
Large	12	4	2	3	1	1
Moisture Content						
Dry	1	1	1	1	1	1
Damp	3	1.5	1.5	1	1	1
Wet	6	2	2	1	1	1

is actually observed, as is often the case with artificials. The terminus classification gives an indication of the farthest point reached by the avalanche, but does not include any information on the actual distance travelled from the starting zones. A low cloud base frequently obscures the starting zones, thereby preventing the point of origin or fracture line from being recorded. However, since individual sites consistently avalanche from the same rupture area, often at the base of cliffs, the terminus does provide a good indication of distance travelled.

The Avalanche Data File

The greatest overall accuracy that could reasonably be obtained from the records for natural event times was twice daily. Accordingly, therefore, both naturals and artificials were coded on a twice daily basis, along with size, terminus and moisture content, for the period, 1965-73. After sorting into two subsets of daily and twice daily observations, by site within date, the data was stored on a computer tape file, ready for analysis.

The Avalanche Activity Index

Definition and Physical Interpretation. The first problem prior to the application of statistical techniques is to devise a suitable index of avalanche activity which can be used as a dependent variable. In a pilot study, based on data for the winter of 1972-73, an "index of mass movement" was defined as the product of terminus, size, and moisture content for a particular event, after assigning arbitrary numerical codes of one to twelve for terminus, one to twelve for size, and one to six for moisture content, as outlined in Table I, column 1. The column heading

(12,12,6) SML will be explained later.

It is probable that this index is a good measure of avalanche activity, since it not only includes an indication of the size, and therefore the amount of snow picked up from the lower zones after the initial movement, but also an indication of the energy associated with the avalanche in terms of distance travelled.

However, both Schaerer and Shimizu have shown that the logarithm of mass may be a more useful measure of avalanche size than mass alone. Shimizu (1967), in fact, proposes and defines three measures of avalanche magnitude,

(1) Mass Magnitude--the logarithm of the mass of avalanched snow,

(2) Potential Magnitude--the logarithm of the product of mass and vertical distance moved by the avalanche; a measure of potential energy,

(3) Destructive Magnitude--the logarithm of the product of mass and the square of the sine of the slope angle divided by the square of a resistance coefficient; a measure of kinetic energy.

However, the somewhat subjective assignment of avalanches in the Rogers Pass area, into small, medium and large, by the field observer is probably already intrinsically logarithmic, since, as Schaerer (1971:2) points out, "it has also been found that an experienced observer, using visual observations only, would usually assign avalanches to the same class."

The index is therefore similar to the "Potential Magnitude" measure proposed by Shimizu.

Since the size of the avalanche is estimated relative to the site, the index does not provide an absolute estimate of the energy associated with the avalanche. Some researchers regard an estimate of the absolute size and energy of an avalanche as a more meaningful measure of avalanche activity. The schemes of both Schaerer (1971) and Shimizu (1967) are based on absolute sizes and Perla (1976) classifies avalanches according to their estimated destructive power on a scale of one to five, irrespective of the size of the site. However, such measurements are no less subjective than the relative size measurement, which may be easier to make. Besides, if instability is regarded as more important than absolute size in the assessment of hazard, relative sizes may provide a better measure, since a small site may never experience a large avalanche in the absolute sense, no matter how unstable the snow.

The question arises at this point as to whether the prediction of instability or absolute size of avalanches is the prime requisite. However, this question can be resolved after consideration of the main purpose of procedures developed from this study, which is to provide the avalanche control crew with an indication of the optimum time for the stabilization shoot. This surely coincides with the period of greatest instability and therefore, models should be developed to predict instability, rather than the absolute size of avalanches.

Furthermore, it is likely that a measure of instability is more closely related to meteorological processes, since absolute size depends to a large extent on the topography of the area. Therefore, it can be expected that greater success will be obtained in the development of prediction models, if a measure of instability based on relative avalanche size measurements is used. Such models should also be more generally applicable to other areas possessing different terrain characteristics, but similar meteorological conditions.

Computation. The avalanche activity index, when computed for individual events, can be summed for all sites, or a group of sites, on a daily or twice daily basis, resulting in values of avalanche activity which vary smoothly and continuously, and therefore lend themselves to the successful application of multiple regression and time series techniques. LaChapelle (1970:106) recognizes that the greatest potential of the "statistical approach" lies in this direction, since he states, "it is most useful when dealing with hazard probabilities over large areas, where individual avalanches fall effectively at random, but the patterns of their occurrence in time are related to snow and weather." Effects of random errors caused by individual site peculiarities and the subjective nature of the data are minimized. Not only does this index contain all three basic characteristics of the avalanche measured in the field, but also the relative contribution of each measurement can be altered by choosing a new set of weights.

Numerical Convention for Representation. Since a multitude of weighting schemes have been used in this study, it is necessary, at this point, to introduce a simple convention for their abbreviated representation. In this convention, the weights, as outlined in Table I, column 1, are referred to as (12,12,6) weights, the first figure indicating the maximum terminus code, the second figure, the maximum size code, and the third, the maximum moisture content. Values of unity are usually assigned to a quarter path, small, and dry, and the weights are evenly distributed between the other categories. Hence (12,4,2) weights indicate that terminus ranges from one to twelve, size ranges from one to four, and moisture content, from one to two, as shown in Table I, column 2. If small avalanches are omitted, unity is often assigned to mediums, as shown in Table I, column 3. The designation, SML, refers to small, medium and large avalanches. Besides the (12,12,6) weights used in the pilot study, (12,4,2) weights were tried with considerable success. Later, it will be shown that (12,3,1), (12,1,1), and (1,1,1) weights result in the best models. It should be noted that (12,1,1) weights indicate that terminus alone determines the value of the index, and (1,1,1) weights imply equal weighting for all classifications, and hence the index is simply a frequency count of the number of avalanches.

Chapter IV

METEOROLOGICAL FACTORS AS INDEPENDENT VARIABLES

The Meteorological File

Meteorological data from the Rogers Pass and Fidelity observatories for the period, 1965-73, consisting of twice daily observations, measured at approximately 0700 and 1600 hours, of snow accumulation, new snow depth, water equivalent, maximum and minimum air temperatures, wind speed and direction, cloud cover, shear test data, and humidity, were transcribed from field books and stored on computer tape. Snow profiles are included with a maximum frequency of two weeks, consisting of density, wetness, crystal size and type, hardness and temperature of each snow layer, as well as a ram penetrometer profile for the snowpack.

Table II contains a list of all the variables used in the analysis, together with their computer labels for future reference. The following is a summary of these variables, and the physical processes associated with them, which are thought to influence the level of avalanche activity.

Depth of Snowpack

SAC is the depth of the snowpack, obtained from snow stakes at the study plots, and is a direct measure of available snow for avalanching. According to Schaerer (1962:17), a certain minimum depth, in the order of seventy centimeters, is required for the Rogers Pass area, to cover the rocks and vegetation in the slide paths before the avalanche season is established. This figure agrees closely with Mellor's estimate for typical mountain terrain, in his discussion of the ten point system employed by the U.S. Forest Service (Mellor, 1968:148).

New Snowfall

SNO is the new snowfall measured from snow stakes at the study plots, and is the primary and obvious cause of direct action avalanching. According to Mellor (1968:149), "the depth of new snow gives a good measure of the quantity of snow likely to be released. As the depth of new snow increases above 1 ft. (30 cm.) or so, the probability of widespread avalanches of significant size tends to increase."

Schaerer (1971) has shown that there is considerable variation in precipitation with elevation for the Rogers Pass area. Thirty year maximum water equivalents were computed and they range from 0.9 m. at 1220 m. to 2.0 m. at 2200 m. for the east and 1.1 m. at 1200 m. to 2.4 m. at 2200 m. for the west. Conditions at the Mt. Fidelity observatory, because of its higher elevation, more closely approximate those in the starting zones, but since the Illecillewaet valley usually receives about twenty-five percent more snowfall than the Tupper area, forecasts based on Fidelity observations are more applicable to the western section.

The extent to which avalanching occurs, depends on the rate of stress build-up in relation to the rate of increase of strength by compactive creep, sintering and bond growth. According to Mellor (1968:149), an accumulation rate of one inch per hour or more, sustained for several hours is likely to produce major avalanching.

Precipitation

W.E is the precipitation or water equivalent, and is obtained as the product of snowfall and new snow density or from precipitation gauges. However, rainfall is also included in the data.

Precipitation is associated more strongly than snowfall with

Table II

Variables Treated as Independent

Labels Used In Computer Analysis	Description	Units
SAC	Depth of snowpack, (snow accumulation)	cm
SNO	Depth of new snow	cm
W.E	Precipitation, (water equivalent for new snow and/or rainfall)	mm
TMA	Maximum air temperature	°F
TMI	Minimum air temperature	°F
WNO	North wind component	mph
WWE	West wind component	mph
WSO	South wind component	mph
WEA	East wind component	mph
CLO	Cloud cover	1=25% of sky
HUM	Humidity (relative)	%
CD1	Critical depth to first weak layer	cm
SW1	Shear weight of snow above 1st weak layer, (pressure)	gm/cm^2
SS1	Shear strength of 1st weak layer (at zero normal stress)	gm/cm^2
CI1	First critical index (SW1/SS1)	
CD2	Critical depth of second weak layer	cm
SW2	Shear weight of snow above second weak layer	gm/cm^2
SS2	Shear strength of second weak layer	gm/cm^2
CI2	Second critical index (SW2/SS2)	
SET	Settlement (SAC1* + SNO - SAC)	cm
WIN**2	Wind speed squared	$(mph)^2$
WIN	Wind speed	mph
SNO*WIN	Product of new snow depth and wind speed	cm.mph
TMI*WIN	Product of minimum temperature and wind speed	°F.mph
SNO*TMI	Product of new snow depth and minimum air temperature	cm.°F
TGR	Minimum air temperature gradient (TMI - TMI1)	°F
TMI/SAC	Quotient of minimum air temperature and depth of snowpack	°F/cm
DEN	Density of new snow (W.E/SNO)	gm/cc
DEN*TMI	Product of new snow density and minimum temperature	(gm/cc).°F

Table II (continued)

Labels	Description	Units
DEN*WIN	Product of new snow density and wind speed	(gm/cc).mph
DEN*HUM	Product of new snow density and humidity	gm/cc
SNO*HUM	Product of new snow depth and humidity	cm
TMI*HUM	Product of minimum air temperature and humidity	°F
WIN*HUM	Product of wind speed and humidity	mph
NVTH	Product of new snow depth, wind speed, minimum air temperature and humidity	cm.mph.°F
W.E*WIN	Product of precipitation and wind speed	mm.mph
W.E*TMI	Product of precipitation and minimum temperature	mm.°F
W.E*HUM	Product of precipitation and humidity	mm
WVTH	Product of precipitation, wind speed, minimum air temperature and humidity	mm.mph.°F
SAC*W.E	Product of depth of snowpack and precipitation	cm.mm
SWH	Product of depth of snowpack, precipitation, and humidity	cm.mm
SWHT	Product of depth of snowpack, precipitation, humidity and minimum air temperature	cm.mm.°F
SWHTV	Product of depth of snowpack, precipitation, humidity, minimum temperature and wind speed	cm.mm.°F.mph

Note: AVAL is the label used to describe the dependent variable, avalanche activity index, used in the analysis.

*SAC1--the first lag of SAC.

avalanche occurrence and notably with the formation of slab avalanches[3] (USFS, 1961:34). This has been borne out by analysis as will be seen later.

Air Temperatures

Maximum and minimum air temperatures, TMA and TMI, are read from maximum and minimum thermometers at the study plots.

Upper air temperatures correlate with the type of snow which falls. Large intricate crystals occur at high temperatures, whereas small elementary crystals are most common at low temperatures. "Thus, air temperature is related to the type and density of the new snow, and hence to the initial mechanical properties" (Mellor, 1968:151).

The type and rate of metamorphism that occurs after the snow has fallen is also largely determined by air temperature. High temperatures induce equi-temperature (or destructive) metamorphism[4] and high rates of settlement, causing the snow to stabilize quickly. However, if temperatures rise above freezing during snowfall, the snow may turn to rain and melting may occur, creating a serious avalanche hazard. At the Rogers Pass, "rain following a snowfall in the avalanche rupture zones can start avalanches within one or two hours of its beginning" (Schaerer, 1962:18).

Low temperatures can induce temperature gradient (or constructive)

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[3] Two principal types of snow avalanche are widely recognized and referred to by the terms 'slab' or 'loose' (LaChapelle,1970b:8). Slab avalanches are usually characterized by a well defined fracture line and involve a mass of snow exhibiting some degree of internal cohesion. Loose avalanches generally start from a point and involve loose cohesionless snow.

[4] Transport of water molecules from convexities to concavities in the ice skeleton due to a vapour pressure difference, thereby producing smaller, more rounded crystalline grains and stronger inter-crystalline bonds.

metamorphism[5] causing increasing instability, and low rates of settlement resulting in a slow rate of stability gain. "Cold weather in January and February with a period of no snowfall for two or more weeks may cause considerable metamorphism of the snow at the surface. This snow layer has low cohesion and may fracture under the weight of new snow or during the snow-melt period", at the Rogers Pass (Schaerer, 1962:16).

Roch (1966:86-99) has also shown that the tensile strength, and Losev (1966:50), the shear strength, of given types of snow increases as temperature decreases, but as Mellor (1968:151) points out, "the probability of avalanche release tends to increase as temperature decreases over the usual range of sub-freezing temperatures".

Hence the overall effects of temperature are extremely complex and difficult to evaluate. Besides being significantly correlated with almost every other meteorological variable associated with avalanche activity, temperature undoubtedly has a non-linear relationship with the level of activity.

Wind

Wind speed and direction, measured by anemovane, is telemetered from the MacDonald West Shoulder and Roundhill stations to the Rogers Pass and Fidelity observatories, where it is recorded on anemographs. For the purpose of this analysis, it has been resolved into four rectangular components, WNO, WWE, WSO, and WEA, which can be treated as separate variables.

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[5] Transport of water molecules from warmer to colder grains due to a vapour pressure difference, thereby producing larger more angular crystals and a weakening of the ice skeleton.

Strong winds accompanying snowfalls often lead to a high level of direct action avalanching, by causing drifting in areas of low wind stress, such as gullies and lee slopes. For the Rogers Pass, "prolonged wind strengths of 15 mi/hr in the west area and 25 mi/hr for the centre and east area are critical" (Schleiss, 1970:117). The first figure is identical to Mellor's (1968:151), who states that, "significant wind transport and wind packing begins when the wind speed exceeds about 15 mph", for any avalanche prone area in general. The pattern of distribution and redistribution of snow is a complex function of wind speed, direction and topographical characteristics of the terrain.

Erosion zones may be more vulnerable to temperature gradient metamorphism if the pack is thin, possibly leading to greater avalanche hazard later in the season. Snow that is transported in the wind stream by saltation and turbulent suspension is fragmented and may be deposited in the form of wind slab[6] if the humidity is high enough.

Cloud Cover

Cloud cover, CLO, is recorded as the amount of overcast in approximate quarters. It is an important factor in determining the radiation balance of the snowpack, but is probably more directly correlated with storm periods than with avalanche activity.

Humidity

Relative humidity, HUM, is measured with hygrographs and psychrometers at the study plots, and is expressed as a percentage of

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6 Wind slab consists of snow grains held together by intergranular bonds. A gradation from soft slab to hard slab, depending on the degree of cohesion, is generally recognized.

saturation. According to Schleiss (1970:117), "data indicate that a relative humidity of 80 per cent and over, in combination with wind speeds of 15 mi/hr causes the formation of slab avalanches", at the Rogers Pass. Soft slab conditions, a characteristic feature of the Middle Alpine Zone (Mellor, 1968:154) are a common occurrence at the Rogers Pass and are undoubtedly responsible for a major portion of the avalanche activity. Seligman (1936:194-95) also recognizes the importance of humidity in the formation of wind slab, stating that a value of eighty-five per cent or over causes wind packing.

This situation is reflected in the prediction models, in which, as it will be shown later, humidity appears to play an important role.

The Shear Test

Shear test data, the most subjective of the study plot measurements employed by the Snow Research and Avalanche Warning Section at the Rogers Pass, consists of three structural observations designed to identify and estimate the strength in relation to loading of critical layers, frequently thin and fragile, in the new or partially settled snow of the upper section of the pack.

Such layering or stratification is a common cause of direct and delayed action avalanching at the Rogers Pass. These layers, the depths and weaknesses of which are a complex function of the antecedent meteorological conditions, often originate at the surface in the form of surface hoar, surface layers produced by temperature gradient metamorphism, rain, sun, melt, or wind crust. However, even a light sprinkling of loose powder snow on the old surface can result in the poor bonding of a new and heavier snowfall.

Shear plane depth, in centimeters, CD1, (and CD2, in the case of a second layer), is measured from the top of a sample block (approximately eighteen inches cube) resting on a thirty-five degree tilt table, down to the 'shear plane' after shear has been induced by a sharp 'tap' on the underside of the table. Shear weight, SW1, (SW2 for a second layer), is the weight of snow above the shear plane in grams per square centimeter, and shear strength, SS1, (SS2 for a second layer), at zero normal stress, is measured using a Roch $100cm^2$ frame just above the shear plane, and reduced to grams per square centimeter.

The ratio of shear strength to shear weight called the stability factor, the reciprocal of which is defined as the critical index, CI1, (CI2 for a second layer), in this study, obtained from the above measurements, is thought by Schleiss(1970:116) to be fundamentally related to the level of avalanche activity. In fact, a stability factor of 1.5 or less for the Rogers Pass area is considered critical. Also, if the shear plane depth is greater than twenty centimeters in combination with other factors, the hazard is likely to be high.

There is no doubt a strong relationship between the depth and weakness of critical layers within the snowpack and the level of avalanche activity, but there are a number of problems associated with the interpretation of such measurements. Firstly, the shear test is difficult to perform consistently and reliably, requiring the skill and practice of an experienced man. Secondly, the results of this test made at the study plot may not have a great deal of bearing on conditions in the fracture zone, unless such conditions are widespread and pronounced. As far as the statistical analysis used in this study is concerned, the measurements are too discontinuous and intermittent to produce reliable correlations.

Settlement

Settlement, SET, is calculated by adding the current new snow depth to the previous snow accumulation and subtracting the current value of snow accumulation, but can be calculated from a storm stake with perhaps greater reliability.

The rate of settlement or densification determines, to a large extent, the rate at which the snow is gaining strength. In general, the faster the snow settles, the faster it gains strength. However, the rate depends on temperature and the initial density of the snow. "Low density snow has little initial strength but settles rapidly; high density snow has high initial strength but densifies slowly, tending to gain strength more by sintering than by compaction"(Mellor, 1968:151).

Hence, settlement rate and avalanche activity have a complicated relationship which is aggravated by the fact that measurements of settlement are made at temperatures which may be quite different from those in the fracture zones.

Wind Terms

Wind speed squared, WIN**2, is directly related to the energy of the wind, which determines its carrying capacity for snow transport and its ability to create stress in the fracture zones. Wind speed, WIN, represents the scalar effect of wind.

Other Terms

SNO*WIN may be more highly correlated with slab formation than snowfall alone, besides being a measure of the amount of drifting during snowfall. TMI*WIN may provide a useful indication of the thermal conduction rate for spring avalanching. SNO*TMI is related to the type of snow crystal which falls and the initial structural properties of the snow on the ground.

Air Temperature Gradient

Minimum air temperature gradient, TGR, is the difference between the previous and current values of minimum temperature. A sudden or large temperature change may trigger avalanches, according to Losev, (1966:48) who states that, "avalanches related to an abrupt temperature drop are formed when the volume of the snow cover undergoes thermal contraction. This produces additional stresses within the snow layer so that avalanches are formed."

Temperature Gradient within Snowpack

TMI/SAC is the quotient of minimum air temperature and the depth of the snowpack and provides an indication of the average temperature gradient within the pack, since the temperature at the base of the pack is usually fairly constant and close to freezing point throughout the winter, provided that the pack is thick enough to supply sufficient insulation. Temperature gradient within the pack determines the rate and type of metamorphism. A gradient in excess of ten degrees Centigrade per meter can cause significant temperature gradient metamorphism and the formation of depth hoar.

Losev (1966:73), who has tried to quantify certain forecasting procedures in terms of analytical equations, principally concerned with establishing the time of onset of the avalanche activity period, assumes a direct proportionality between the stresses and temperature gradient within the snowpack, thus suggesting that there is a direct relationship between temperature gradient in the pack and instability.

Density

Density of new snow, DEN, expressed as the quotient of water equivalent and snowfall, W.E/SNO, is closely related to snow strength. Using a specially developed centrifugal or spin tester to measure tensile strength, Martinelli (1971:7-10) has demonstrated that snow strength increases rapidly with density over the range of samples tested. Since the initial snow density is largely determined by crystal type and mode of deposition, this parameter may be a good indicator of stability. However, the relationship between density and stability is probably non-linear, since, "it has been observed that when new snow density at a particular site departs widely from the mean density for that site, avalanches are likely" (Mellor, 1968:149). Schaerer (1962:19) has also noted that for the Rogers Pass, "new snow with specific gravities lower than 0.07 and higher than 0.10 are more likely to cause avalanches." Unusually low densities, 'wild snow', may indicate a lack of cohesion, and high densities, the presence of free water if temperatures are high, resulting in instability in both cases. High densities may also be associated with slab conditions.

Further Terms

Further terms have been included in the analysis in the hope that higher correlations with avalanche activity would be realized. From Table II, it can be seen that these terms consist of certain combinations of the primary variables already discussed, which might be more strongly related to instability than the simpler terms. DEN*TMI is associated with initial snowfall structure and free water content. DEN*WIN appertains to the type of snow deposit, a high value possibly indicating slab conditions. Humidity terms, DEN*HUM, SNO*HUM, TMI*HUM, and WIN*HUM may all be associated with slab formation, and NVTH is perhaps a composite wind slab term, in which snowfall is modified by wind speed, minimum air temperature and humidity. Water equivalent terms, W.E*WIN, W.E*TMI, W.E*HUM, and WVTH have been included, since in general, precipitation is more strongly correlated with instability than snowfall, as will be seen later.

The remaining variables and their evolution will be discussed in the chapter on correlation analyses, where it will be shown that two important composite terms emerge which correlate more highly with avalanche activity than any other previous factors.

Chapter V

CORRELATION ANALYSES

This phase of the study is concerned with the identification of appropriate starting and finishing dates for avalanche activity periods, suitable independent variables and the best activity index weighting schemes, as defined in Chapter III, to be used in the subsequent development of linear time series prediction models. Simple linear correlation coefficients[7] can be used to provide a good initial indication of the potential strength of such models.

In order to allow complete flexibility in the application of these procedures, a data selection program was written. This program takes the meteorological information for any ranges of dates requested, computes the appropriate avalanche activity indices for each day (or half day), and writes the entire record, including the index, onto a file ready for input to the analysis programs. Apart from allowing total freedom in the choice of dates, the program permits any combination of sites (or ranges of sites), types (natural or artificial) sizes and moisture contents,

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[7] Defined by the familiar equation,

$$r_{xy} = \frac{\sum_{1}^{N} (X - \bar{X})\ (Y - \bar{Y})}{\sqrt{\sum_{1}^{N} (X - \bar{X})^2 \ \sum_{1}^{N} (Y - \bar{Y})^2}},$$

where Y and X are the dependent and independent variables respectively, and N is the number of observations in the sample.

to be specified as criteria for including any one avalanche as part of the activity index.

Since the data does not actually consist of equi-spaced twelve hourly observations, but more closely resembles nine and fifteen hourly measurements, it was reduced to daily values for the purpose of the majority of the analyses. This was achieved by integrating avalanche activity, snowfall and precipitation, selecting minimum and maximum air temperatures, and averaging wind speeds, cloud cover, humidity and shear test data. As will be shown later, the use of twice daily observations is not justified, due to their high noise level.

Identification of Dates

As a first step in the investigation, starting and finishing dates were defined as the dates of the first and last avalanche occurrences for the eight avalanche seasons, as indicated in Table III. Division of each season into a first part, which is primarily snowfall dependent, and a second part, which is primarily temperature dependent, is important, for, as Schaerer (1962:7) points out, "there are two avalanche periods each year. In the first period, between early November and late February, avalanches are caused mainly by snowfalls, wind action, and rain in association with snowfalls. In the second period, between late March and mid-May, avalanches are caused mainly by warm weather and melting of the snow." Models should then be developed for each part, which best describe the two types of avalanching. Bois et al.(1974:5) and Bovis (1974:71) distinguish between dry and wet avalanches, which leads to a similar but not identical division of the data, since dry avalanches are not confined to the first nor wet avalanches to the second part of the winter. However, according to Bovis (1974:71), "dry avalanche and wet avalanche periods are defined by the transition (usually abrupt in the San Juan Mountains) from dry to wet slides."

The identification of suitable transition dates is by no means a simple procedure, since there is always a transition period between the two parts, during which both types of avalanching occur. However, it is possible, by visual inspection of the data correlations, to identify the interval over which avalanches tend to become more dependent on temperature. This somewhat subjective approach has been considerably improved by the introduction of a technique which can be referred to as 'incremental correlation analysis'. A subroutine which computes the correlation coefficients between the dependent and the individual independent variables, after each sequential data record is read and added to the file, has been written and incorporated into the main analytical program.

Table III

Starting and Finishing Dates

Total Season		
First Parts		Second Parts
Date of First Avalanche	Transition Date	Date of Last Avalanche
14/11/65	29/ 1/66	30/ 5/66
19/10/66	28/ 1/67	30/ 5/67
20/10/67	1/ 2/68	31/ 5/68
15/10/68	2/ 2/69	13/ 5/69
5/11/69	5/ 2/70	23/ 5/70
16/11/70	1/ 2/71	25/ 5/71
25/10/71	5/ 2/72	31/ 5/72
25/11/72	1/ 2/73	28/ 5/73

Hence, after a starting date has been established, the behaviour of correlations between avalanche activity and snowfall, for example, can be monitored as the winter progresses. For most winters, it is observed that such correlation coefficient values rise to a peak, near the end of January, or beginning of February, after which they drop off sharply, as snowfall becomes less significant than temperature. This procedure was applied to data for each winter using an avalanche activity index based on (12,4,2) weights for all sites and all avalanche events, with water equivalent, as well as snowfall as independent variables. Optimum dates for the separation of the data into first parts and second parts, were established for each winter, and are indicated in Table III.

Independent Variables

Employing these dates, a complete correlation analysis was performed using all the independent variables, described in Chapter IV, and meteorological data from the Rogers Pass observatory. Avalanche activity indices were computed using (12,4,2) weights, all sites, and all avalanche events, both artificial and natural (AN) and small, medium and large (SML). The results are indicated in Table IV, in which correlation coefficient values have been multiplied by one hundred for convenience in representation.

Total Seasons. The values in column (1) were obtained by combining total seasons, as defined in Table III, for the entire period, 1965-73. This sample consists of 1654 daily records and therefore, absolute values of the correlation coefficients in excess of 0.06 can be regarded as significant at the ninety-nine per cent level. It is immediately apparent that all the independent variables are significantly correlated with avalanche activity, with the exception of TMI/SAC. Temperature and density terms display only weak correlations, probably because of their non-linear association with avalanche events, as mentioned in Chapter IV. Wind terms are also not as significant as might be expected, possibly because wind observations suffer from a high level of noise, as will be discussed later. Precipitation terms, predictably, have the strongest correlations. It is important to note that W.E, that is, SNO*DEN, is more important than SNO alone, as was suggested in Chapter IV.

At this point, a peripheral study was made to determine to what extent the total avalanche activity for each year is correlated with the total amount of snowfall or water equivalent, for as Mellor (1968:157) points out, "it seems likely that avalanche activity will correlate closely with the amount of winter precipitation, although this is yet to be demonstrated." Using the (12,4,2) weights, and avalanche seasons defined by first and last occurrences, correlation coefficients of 0.92 and 0.97 were obtained for snowfall and water equivalent respectively, as shown in Table V. This not only indicates an extremely high level of correlation between total avalanche activity for each year and total snowfall, but it also demonstrates clearly that water equivalent is more important than new snow depth in determining avalanche activity. It is also interesting to compare the average annual snowfall of 1064 cm. (419 in.) for the period, 1965-73, and the maximum of 1530 cm. (602 in.), for the winter of 1966-67, measured at the Rogers Pass, with the average of 342 in. for the period, 1921-51, and the maximum of 680 in. for the winter of 1953-54, measured at Glacier (Schaerer, 1962:4).

Table IV

Correlation Analysis Rogers Pass Meteorological Data

Period 1965-73 Daily Observations

(12,4,2) Weights, SML, AN

	(1) AT*	(2) AF	(3) AS	(4) P AT	(5) P AF	(6) ET	(7) EF	(8) ES	(9) WT	(10) WF	(11) WS
SAC	24	33	21	24	27	23	31	20	22	29	19
SNO	46	54	41	-4	-4	45	53	43	33	39	30
W.E	54	62	50	0	0	54	62	54	45	53	39
TMA	8	22	-2	16	3	8	22	-3	7	17	-1
TMI	18	22	16	13	1	19	23	17	14	17	13
WNO	-6	-6	-7	-2	1	-6	-5	-8	-6	-6	-6
WWE	16	20	11	6	6	16	21	12	12	16	8
WSO	22	26	17	5	8	20	23	17	21	27	14
WEA	-7	-12	-1	0	-1	-9	-14	-4	-2	-8	5
CLO	19	23	18	-7	-7	21	24	20	13	16	12
HUM	19	15	24	0	1	19	14	24	16	12	21
CD1	25	31	21	-2	2	26	31	25	18	23	12
SW1	22	29	17	0	6	23	29	19	16	22	9
SS1	8	8	11	1	4	8	8	13	5	6	7
CI1	19	20	18	-1	-4	21	23	20	11	10	12
CD2	13	13	14	-3	-3	15	15	16	6	6	7
SW2	12	13	12	-3	-2	14	16	14	6	6	6
SS2	8	9	8	-3	-1	9	10	10	4	4	4
CI2	15	11	17	6	-5	15	14	16	12	4	16
SET	36	11	-1	6	-8	36	11	-2	30	9	2
WIN**2	26	31	22	8	12	24	27	22	25	32	19
WIN	20	24	17	6	9	18	21	16	21	26	16
SNO*WIN	41	45	40	3	4	41	43	42	35	40	30
TMI*WIN	27	33	22	14	9	26	31	21	24	29	20
SNO*TMI	44	46	42	3	2	45	48	45	33	36	31
TGR	14	15	8	3	-3	15	16	9	10	11	6
TMI/SAC	-4	-5	-6	-5	-4	-4	-5	-5	-4	-4	-6

Table IV continued

	(1) AT	(2) AF	(3) AS	(4) P AT	(5) P AF	(6) ET	(7) EF	(8) ES	(9) WT	(10) WF	(11) WS
DEN	22	21	24	0	-4	20	19	22	22	22	24
DEN*TMI	25	26	25	6	3	22	24	22	25	26	26
DEN*WIN	29	28	31	9	6	25	24	27	31	32	31
DEN*HUM	24	23	27	1	-3	22	20	24	24	23	27
SNO*HUM	45	52	42	-2	-3	46	53	45	34	40	31
TMI*HUM	24	23	28	13	2	25	23	29	19	18	23
WIN*HUM	25	27	25	7	9	23	23	24	25	28	22
NVTH	43	44	44	9	8	43	43	46	35	38	33
W.E*WIN	51	57	47	8	10	48	52	48	46	54	38
W.E*TMI	53	58	50	8	9	53	56	52	45	50	39
W.E*HUM	55	63	52	2	2	55	62	54	46	54	40
WVTH	52	54	54	16	16	50	50	54	48	51	44
SAC*W.E	59	70	51			58	67	52	51	62	41
SWH	61	71	53			60	68	53	52	64	42
SWHT	63	75	57			62	70	58	55	70	46
SWHTV	57	65	53			54	57	52	54	67	45

* A = all sites, E = eastern sites, W = western sites,
T = total seasons, F = first parts, S = second parts,
P = partial correlation coefficients.

First Parts. Column (2) of Table IV was obtained by combining first parts, as defined in Table III, for the entire period, 1965-73. This sample consists of 739 daily records resulting in a ninety-nine per cent significance level of 0.10 for simple correlation coefficients. The results suggest that models based on precipitation terms should achieve a high degree of predictive accuracy for the first part of the season, particularly if individual years are used.

Table V

Total Annual Avalanche Activity, (12,4,2) Weights, SML AN, Versus Total Annual Snowfall and Water Equivalent Using Rogers Pass Meteorological Data

YEAR	N	Σ AVAL	Σ SNO (cm.)	Σ W.E (mm.)
1965-66	198	24100	1074	897
1966-67	224	38700	1530	1260
1967-68	225	32300	982	975
1968-69	211	23030	903	792
1969-70	200	16610	780	656
1970-71	191	22900	1000	903
1971-72	220	37400	1475	1263
1972-73	185	16580	766	633
			R = 0.92	R = 0.97

N = Number of days in season from first avalanche to last avalanche, as per Table III.

Second Parts. Column (3) of Table IV was obtained by combining second parts, as defined in Table III, for the entire period, 1965-73. Consisting of 915 daily records, this sample results in a ninety-nine per cent level of significance for simple correlation coefficients of 0.09.

As expected, precipitation terms are less important than for first parts, but temperature terms, probably because of their non-linear effects, are also poorly correlated with avalanche activity. However, humidity and density terms appear to be slightly more important during second parts, but correlation values do not indicate a strong dependence. It seems likely that non-linear terms will have to be introduced into forecasting models designed specifically for second parts of the avalanche seasons, if an acceptable degree of accuracy is to be achieved. Suitable terms are under consideration and will be incorporated into future analyses.

Evolution of the Best Independent Variables. Recapitulating, W.E for total seasons and first parts, is by far the most important of the simple variables in its association with the level of avalanche activity, in terms of the avalanche activity index. However, there is a suggestion that W.E*HUM may be more significant than W.E alone, and perhaps a more complex composite term may display an even higher correlation. In order to test the validity of this proposition and also to identify any secondary variables which might be important after the variation accounted for by W.E has been subtracted out, partial correlation coefficients[8] were

[8] Defined by the following equation (Freese, 1964:104),

$$r_{y2.1} = \frac{r_{y2} - r_{y1}r_{21}}{\sqrt{(1 - r_{y1}^2)(1 - r_{21}^2)}},$$

where $r_{y2.1}$ is the correlation coefficient between y and x_2 after x_1,
r_{y2} is the correlation coefficient between y and x_2,
r_{y1} is the correlation coefficient between y and x_1, and,
r_{21} is the correlation coefficient between x_1 and x_2.

computed for total seasons and first parts. The results appear in columns (4) and (5) of Table IV, and clearly indicate that after W.E, SAC is the next most important term. A model containing water equivalent and snow accumulation should therefore be stronger than one containing water equivalent alone. However, there are good reasons why SAC cannot be introduced as a secondary variable after W.E,[9] but if SAC is used as a factor modifying W.E no such problem exists. Hence, SAC*W.E was introduced as a new variable in the analysis. Referring back to columns (1) and (2) of Table IV, it can be seen that correlation coefficients for SAC*W.E are 59 and 70 for total seasons and first parts, as opposed to 54 and 62 for W.E, indicating a substantial improvement.[10] This suggests that the amount of avalanche activity for a given quantity of precipitation increases with increasing snowpack depth. This improvement in the correlation could not merely be the result of the minimum snowpack depth criterion required for avalanching to start, as discussed in Chapter IV, since this depth has already been established on or near the starting dates which were used in this analysis. Therefore, the effect is undoubtedly 'real'. Furthermore, a similar effect has been reported in the literature. Losev (1966:75) quotes results obtained by V. Sh. Tsomaya and K. L. Abdushelishvili (1962) for a slope in the High Caucasus in the

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[9] Since the snow accumulation series is non-stationary and highly autocorrelated, it cannot be introduced separately into a time series model containing the water equivalent series, which is essentially stationary. First differences of snow accumulation are too highly correlated with water equivalent to result in a significant contribution, after the effect of water equivalent has been subtracted.

[10] The improvement is highly significant at the .999 level, as described by "Hotelling's t-Test". (Freese, 1964:108)

region of the Krestov Pass, which clearly demonstrate that the onset of avalanching requires progressively less precipitation as the snowpack increases in depth. The authors have empirically deduced the following equation:

$x_h = 55 - 2.8\sqrt{h}$, where x_h is the minimum precipitation, in millimeters, required for avalanching and h is the depth of old snow, in centimeters. Such a relationship strongly suggests that the amount of avalanche activity for a given quantity of precipitation increases with increasing snowpack depth. The physical interpretation of this result is that the snowpack, as it gets deeper, participates more and more in the avalanche activity, presumably as a consequence of an increase in the available amount of avalanchable snow. Of course, it should be pointed out that SAC measured at the study plot certainly does not represent directly the amount of accumulated snow in the avalanche paths, which may have already run several times so far during the winter. However, SAC, like all the other meteorological factors measured at the study plots, is an indicator of conditions in the slide paths. It is probable that the importance of SAC is indicative of a delayed action effect, which may represent the formation of soft slab conditions, particularly during the first parts of the seasons.

This proposition led to the development of the composite terms, SWH, SWHT, and SWHTV. Referring back to columns (1) and (2) of Table IV, it can be seen that correlation coefficient values for SWH and SWHT are 61 and 63 for total seasons, and 71 and 75 for first parts. Thus, humidity and temperature are also important modifying factors[11] probably associated

[11] SWH is significantly better than SAC*W.E at the .999 level, SWHT is significantly better than SWH at the .95 level, as described by "Hotelling's t-Test" (Freese, 1964:108).

with slab formation. The values for SWHTV are only 57 and 65, which suggests that wind is not an important modifying factor.

However, wind undoubtedly has a strong influence on slab formation, but this may be masked since its effect is definitely non-linear, diminishing as wind speeds exceed a critical level. "There also appears to be an upper critical wind level, not clearly defined, above which, snow tends to form wind pack rather than slab" (USFS, 1961:35). Very high winds may also cause too much erosion since, as Mellor (1968:151) points out, "in some locations very strong (full gale) winds may be less effective than moderately strong winds in loading up the release zones."

East-West Division of Data

The remaining columns (6) to (11) of Table IV contain correlation coefficient values for eastern and western avalanche sites, for, "at Rogers Pass there are two major climate areas, and the avalanche hazard for each should be evaluated separately. The two areas are: --The Tupper area on the east side of the Pass, --The Illecillewaet Valley on the west side of the Pass" (Schaerer, 1962:15). For this analysis, the division between eastern and western sites was established at mile 16.53, measured from the east boundary of Glacier National Park. Since the analysis is based on data from the Rogers Pass observatory, correlations for the eastern sites might be expected to be higher than those using the eastern and western sites combined. However, although the snowfall terms support this premise, water equivalent terms suggest the opposite. In other words, precipitation measured at the Rogers Pass is a good indicator of avalanche activity for the entire area. This argument is supported by Schaerer (1962:15) who states that, "observations during the two winters between

1957 and 1959 showed that the average total snowfall in the Tupper area was 80 percent of the snowfall measured in the Illecillewaet Valley. As less snowfall is required to cause avalanches on Mount Tupper, the avalanche hazard is usually about equal in both areas." Hence, it appears that Rogers Pass meteorological data is truly representative of the entire area, in terms of precipitation, as it relates to avalanche activity, and therefore, an east-west split may not by worthwhile.

Selection of the Best Weights for Avalanche Activity

The next phase of the study is concerned with the selection of the best set of weights to be used in the determination of the avalanche activity index. Various sets of weights were chosen, as outlined in Table VI, and a complete correlation analysis performed on all the independent variables using total seasons, first parts and second parts, for the period, 1965-73, as defined in Table III, all sites, artificial and natural avalanches, and meteorological data from the Rogers Pass observatory. Table VII contains a summary of the results of this study in terms of a reduced set of correlations. The variables, SNO, W.E, SWH, and SWHT were used as appropriate indicators of performance and the results quoted for totals, first parts and second parts.

Moisture Content. (12,4,1) SML weights result in generally better correlations than (12,4,2) SML weights, indicating that equal weights applied to the moisture content classification will lead to stronger models. The inability of moisture content to improve the index for either first parts or second parts, has been observed using various other sets of weights for terminus, size and moisture content. Therefore, it must be concluded that either moisture content is not likely to be an important

Table VI

Avalanche Activity Index Weighting Schemes Used in the Analysis

Designation	(12,4,2) SML	(12,4,1) SML	(12,36,1) SML	(12,12,1) SML	(12,12,1) ML	(12,3,1) SML
Terminus						
¼ or 1/3 path	1	1	1	1	1	1
½ path	2	2	2	2	2	2
2/3 or 3/4 path	3	3	3	3	3	3
End path, to fan, or gully	4	4	4	4	4	4
¼fan	5	5	5	5	5	5
1/3 fan	6	6	6	6	6	6
½ fan	7	7	7	7	7	7
2/3 fan	8	8	8	8	8	8
3/4 fan, Old RR, or Bench	9	9	9	9	9	9
Over fan, or Mounds	10	10	10	10	10	10
Edge TCH	11	11	11	11	11	11
Over TCH	12	12	12	12	12	12
Size						
Small	1	1	1	1	0	1
Medium	2	2	18	6	6	2
Large	4	4	36	12	12	3
Moisture Content						
Dry	1	1	1	1	1	1
Damp	1.5	1	1	1	1	1
Wet	2	1	1	1	1	1

Table VI continued

Designation	(12,1,1) SML	(12,1,1) ML	(1,4,1) SML	(1,3,1) SML	(1,2,1) ML	(1,1,1) SML	(1,1,1) ML
Terminus							
¼ or 1/3 path	1	1	1	1	1	1	1
½ path	2	2	1	1	1	1	1
2/3 or 3/4 path	3	3	1	1	1	1	1
End path, to fan, or gully	4	4	1	1	1	1	1
¼ fan	5	5	1	1	1	1	1
1/3 fan	6	6	1	1	1	1	1
½ fan	7	7	1	1	1	1	1
2/3 fan	8	8	1	1	1	1	1
3/4 fan, Old RR, or Bench	9	9	1	1	1	1	1
Over fan, or Mounds	10	10	1	1	1	1	1
Edge TCH	11	11	1	1	1	1	1
Over TCH	12	12	1	1	1	1	1
Size							
Small	1	0	1	1	0	1	0
Medium	1	1	2	2	1.5	1	1
Large	1	1	4	3	2	1	1
Moisture Content							
Dry	1	1	1	1	1	1	1
Damp	1	1	1	1	1	1	1
Wet	1	1	1	1	1	1	1

Table VII

Reduced Set of Correlations for Various Weighting Schemes

Rogers Pass Meteorological Data, Period 1965-73

Daily Observations, All Sites, AN

	SNO			W.E			SWH			SWHT		
	AT	AF	AS	AT	AF	AS	AT	AF	AS	AT	AF	AS
(12,4,2), SML	43	50	41	54	62	50	60	71	53	63	74	57
(12,4,1), SML	57	61	52	61	66	56	67	77	59	64	72	60
(12,36,1), SML	54	59	50	59	63	54	66	77	58	62	71	59
(12,12,1), SML	55	60	51	59	64	55	67	77	58	63	71	59
(12,12,1), ML	54	58	50	58	63	54	66	76	57	62	71	59
(12,3,1), SML	58	63	54	62	67	56	68	78	59	64	73	60
(12,1,1), SML	60	66	55	63	70	56	67	78	59	64	74	60
(12,1,1), ML	56	62	51	60	66	54	67	78	58	62	72	58
(1,4,1), SML	56	61	52	61	66	56	66	76	58	64	73	60
(1,3,1), SML	57	63	53	61	67	56	66	77	58	64	74	60
(1,2,1), ML	54	60	49	58	64	53	65	76	56	62	72	57
(1,1,1), SML	58	64	54	61	68	55	65	74	57	63	72	59
(1,1,1), ML	53	60	49	58	65	53	65	77	55	62	73	57
I(1,1,1), ML	55	62	49	58	65	53	64	75	55	61	71	56
I(1,3,1), SML	57	63	53	61	66	56	66	74	58	63	71	60

A = all sites,
T = total seasons,
F = first parts,
S = second parts,
I refers to individual site weights.

Correlation coefficients are multiplied by 100 for convenience in representation.

factor in assessing avalanche activity in terms of the meteorological variables used in this study, or the observation of moisture content is too subjective to be useful. Since avalanches may start dry but appear wet at the terminus, as a result of higher temperatures in the valley, picking up wet snow in the avalanche track, or pulverization, the former possibility is likely.

Terminus and Size. Progressive gains are realized as the index is changed from (12,36,1) SML, to (12,12,1) SML, to (12,4,1) SML, to (12,3,1) SML, to (12,1,1) SML. Thus the terminus classification provides a better indication of avalanche activity than size. This is borne out by a progressive loss as the index is altered from (1,1,1) SML, to (1,3,1) SML, to (1,4,1) SML. From an observational standpoint, terminus is certainly a less subjective and more precise estimate than size, but perhaps there is a more fundamental reason why terminus seems to be a better measure of avalanche activity. Distance travelled may be more indicative of instability in terms of meteorological factors than the size of the avalanche. In any case, a weighting scheme based on terminus alone, that is, (12,1,1) SML, gives the best results.

Small Avalanches. Dropping the small avalanches, as in (12,1,1) ML, weakens the index. Hence, it seems important to include the smalls, but their influence on the index is no doubt minimized by their generally small terminus codes.

Site Weightings. Individual site weightings, (see Appendix B) based on estimated site sizes, taken from highway and aerial photographs, were incorporated into the (1,1,1) ML and (1,3,1) SML indices. The first scheme effectively converts the data into an absolute one biased towards the sizes of the sites and the second into a more absolute measure of avalanche sizes. In both cases, the indices are weakened by the conversion.

The Best Weights. Besides the (12,1,1) SML scheme identified previously as the best, the (12,3,1) SML weights are of interest, since here, sizes are incorporated with the simple weights, 1, 2, and 3 for small, medium and large. Both the (12,1,1) SML and the (12,3,1) SML schemes are powerful, and in both, the small avalanches carry little weight. The (1,1,1) ML scheme is also of considerable interest, since this is merely a frequency count of the number of medium and large avalanches per day. If the smalls are included, as in the (1,1,1) SML scheme, the index is weakened. Therefore, smalls should be excluded if the index is based on frequency alone, as suggested by Schaerer.[12]

Hence, the three best weighting schemes selected for further analysis are the (12,1,1) SML, (12,3,1) SML and (1,1,1) ML schemes.

Revision of Dates

Transition dates for the first and second parts were revised using these weights and SNO, W.E, SWH and SWHT as independent variables, in a repeat of the incremental correlation analysis procedure, previously described. These dates are recorded in Table VIII, along with a new set of finishing dates for each of the avalanche seasons, also determined from the incremental correlation procedure. After rising to peak values which establish the transition dates, the correlations between avalanche activity and precipitation terms gradually taper off until the new finishing dates are reached, after which, even correlations for temperature terms suddenly plummet, indicating that the season is effectively over, although

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12 Studies at Rogers Pass, unpublished.

a few late spring avalanches have yet to take place. It is unwise to define the end of the season as the date of the last avalanche, for, as Schaerer points out, a large interval of negligible activity may separate the effective end of the season from this final event.[13]

Table VIII

Revised Starting and Finishing Dates

Total Season		
	First Parts	Second Parts
Date of First Avalanche	Transition Date	Revised Dates for End of Season
14/11/65	29/ 1/66	7/ 5/66
19/10/66	28/ 1/67	8/ 5/67
20/10/67	22/ 1/68*	30/ 4/68
15/10/68	2/ 2/69	24/ 4/69
5/11/69	5/ 2/70	5/ 5/70
16/11/70	11/ 2/71*	24/ 4/71
25/10/71	27/ 2/72*	23/ 5/72
25/11/72	1/ 2/73	14/ 5/73

* Indicates revised transition dates

The complete correlation analysis was repeated using these revised dates and the three best weighting schemes, for all sites, artificial and natural avalanches, and total seasons, first parts and second parts, for the entire period, 1965-73. Correlation coefficients of 67, 65 and 63 were realized for SNO, 71, 68 and 66 for W.E, 78, 79 and 77 for SWH, and 75, 74 and 74 for SWHT for (12,1,1) SML, (12,3,1) SML, and (1,1,1) ML, respectively, for first parts. Thus SWH is the strongest variable. Later, it will be shown that such high correlations lead to powerful models both

[13] Personal communication

for individual years, and for the eight year period. A single model is developed for the eight first parts of the entire period, 1965-73, with a multiple correlation coefficient of 0.81.

Artificial and Natural Avalanches

A complete correlation analysis was performed using natural avalanches only, for all sites, (1,1,1) ML weights, first parts, for the period, 1965-73, and Rogers Pass meteorological data. Correlations for SNO, W.E, SWH, and SWHT are recorded in Table IX. The results indicate that naturals alone are not as good as naturals and artificials combined, probably because artificials account for a high percentage of total avalanche activity for the Rogers Pass area, and therefore should not be excluded. Table X indicates the percentage contributions of artificial to total avalanche activity for all sites, in terms of (1,1,1) SML and (1,1,1) ML weights, for each individual year. Values range from 23 to 41 for (1,1,1) SML and 23 to 41 for (1,1,1) ML, and display a generally increasing trend from 1966-73, indicating that the control program has improved over the years. However, the percentages are anomalous for the year, 1965-66. It should be noted that the exclusion of the small avalanches does not alter these percentages significantly.

The stronger correlations between total avalanche activity and meteorological factors, for artificials and naturals combined, as opposed to naturals only, implies that the decision to shoot is usually made during optimum conditions for natural avalanching, which is an obvious consequence of the operational procedures involved. The control program may be improved if procedures can be developed to forecast instability prior to the onset of natural avalanche cycles. It is hoped that this study will ultimately lead to such procedures.

Table IX

Reduced Set of Correlations for Various Subsets of Avalanche Activity

Rogers Pass Meteorological Data, Period 1965-73, First Parts,

Daily Observations, (1,1,1) Weights, ML

	SNO	W.E	SWH	SWHT
All Sites, AN	63	66	77	74
All Sites, N	57	62	68	71
Tupper Gullies,* AN	62	63	74	68
MacDonald Gullies, AN	55	58	63	64
Lens, AN	50	53	58	53
Crossover, AN	46	48	51	52
Ross Peak, AN	13	15	13	14
All Sites, Storm Periods, AN	57	61	63	61

* Sites designated Tupper and MacDonald Gullies are listed in Appendix B.

Reduction of Sites

Two groups of sites, the Tupper gullies and the MacDonald gullies, (Appendix B), were examined using Rogers Pass data, (1,1,1) ML weights, artificials and naturals, for first parts and the period, 1965-73. Correlations for SNO, W.E, SWH and SWHT are indicated in Table IX. The Tupper gullies appear to be representative of the entire area in terms of these coefficients and powerful forecasting models could be based on these sites alone. The MacDonald gullies, however, display somewhat weaker correlation. Correlations for the individual sites known as Lens, Crossover and Ross Peak are also indicated in Table IX. While Lens gives rise to moderately

Table X

Total Annual Artificial and Natural Avalanche Activity

(1,1,1) Weights

Year	Σ AN SML	Σ A SML	ΣA/ΣAN SML Percent	Σ AN ML	Σ A ML	ΣA/ΣAN ML Percent
1965-66	372	131	35	238	96	40
1966-67	2953	715	24	1668	435	26
1967-68	1508	403	27	918	238	26
1968-69	1141	308	27	723	164	23
1969-70	936	214	23	567	130	23
1970-71	1159	410	35	638	240	38
1971-72	1815	749	41	1011	410	41
1972-73	1193	393	33	839	314	37

Σ AN is the total number of artificial and natural avalanches

Σ A is the total number of artificial avalanches

SML indicates that small, medium and large avalanches are included

ML indicates medium and large avalanches only.

high correlations, Crossover is less strong and Ross Peak is quite weak. Restricting the variation of the avalanche activity index in this way must inevitably result in such weak correlations, since there are many more non-avalanche days, during some of which, meteorological conditions may be quite favorable for avalanching at other sites.[14] There are insufficient occurrences at individual sites, even sites such as Lens, which is most active, to produce individual forecasting models of high precision. The development of models using all the sites is a much better procedure, leading to considerably more powerful models. As will be described later, the level of avalanche activity for the entire area can then be predicted, using these models, and the predictions decomposed into the most probable distribution of individual site activities.

Storm Periods

A peripheral analysis was conducted for individual storm periods, which were identified after careful scrutiny of the precipitation patterns for the eight avalanche seasons. Some winters consist of very clearly defined storm periods, whereas others may be made up of longer periods of intermittent snowfall. In such cases, the identification of specific storm periods is rather subjective. Nevertheless, forty-five storms (see Appendix C), were combined and correlation coefficients calculated. However, treatment of the data in this way is not only difficult to interpret, but also results in correlations which are in fact weaker than those for first parts as shown in Table IX.

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[14] The discriminant analysis techniques employed by Judson and Erickson(1973), Bois et al. (1974) and Bovis (1974) suffer from this major fault. Any attempt to assign all avalanche activity into a single class, or portions into a restricted number of sub-classes, results in high probabilities of misclassification.

Twice Daily Data

Finally, a complete correlation analysis was performed using twice daily data from the Rogers Pass observatory, in order to determine whether greater accuracy could be achieved. Avalanche activity indices were computed for all sites, all avalanches, and (12,1,1) weights, using data for the entire period, 1965-73, first parts only. Correlation coefficients of 62, 62, 68 and 67, were obtained for SNO, W.E, SWH and SWHT, as opposed to 67, 71, 78 and 75 for daily data. Hence, models based on twice daily data would be weaker than daily models, probably as a result of the relatively high level of noise in the twice daily data. As indicated previously, avalanche times of occurrence are not reliable enough to justify the use of twice daily observations, besides which, twice daily meteorological observations are not equispaced.

Individual Years

Using the three best sets of weights for avalanche activity, a complete correlation analysis was performed for the first parts of individual years as defined in Table VIII. The results are summarized in Table XI in terms of a reduced set of correlation coefficients for SNO, W.E, SAC*W.E, SWH, SWHT and SWHTV. As for the total first parts for the period, 1965-73, correlations for each individual year indicate that in general, (12,1,1) SML weights are better than (12,3,1) SML weights, which are in turn better than (1,1,1) ML weights. W.E is more highly correlated with avalanche activity than SNO for the years, 1966-67, 1967-68, 1968-69, 1970-71, and 1972-73 but, for the remaining years, it is somewhat weaker.

As discussed previously, W.E, that is, SNO*DEN, seems to be more important than SNO, in its association with avalanche activity, perhaps for the following reasons. Firstly, SNO*DEN is a more direct measure of 'shear weight' than SNO alone. Secondly, DEN contains the temperature effect, a high value of DEN possibly indicating high temperatures and perhaps free water. It is significant to note that, for the three years for which SNO correlations were stronger than W.E correlations, SWHT was also weaker than SWH, suggesting that temperature was not important as a modifying factor. Finally, DEN may also be a strong indicator of slab conditions. This is supported to some extent by the fact that, the years displaying an increase in correlation values for W.E compared to SNO, also tend to have a higher proportion of recorded slab avalanches.

The importance of SAC as a modifying factor for W.E is clearly shown in Table XI, in which correlation coefficient values for SAC*W.E indicate a very substantial improvement over values for W.E, for every individual year. This effect has already been discussed, and it was felt that SAC represents a 'delayed action effect', which could perhaps be associated with soft slab build-up. It is also worth noting that high values of SAC imply the participation of more snow layers in the avalanching and the availability of a greater number of potential sliding surfaces. Furthermore, SAC may contain a settlement effect, a high value of SAC indicating less settlement, and hence, greater instability.

Except for 1971-72 and 1972-73, correlation coefficient values for SWH are significantly better than those for SAC*W.E. Values for SWHT, on the other hand, are generally lower than those for SWH, except for the years, 1967-68 and 1970-71. The possible influence of humidity on the build-up of slab conditions has been fully described by Seligman (1936:195), who refers to the mechanism of slab formation as a condensation of water

Table XI

Reduced Set of Correlations for Individual Years

Rogers Pass Meteorological Data, First Parts,

Daily Observations, All Sites, AN

	SNO	W.E	SAC*W.E	SWH	SWHT	SWHTV
1965-66						
(12,1,1) SML	73	71	82	83	62	46
(12,3,1) SML	74	70	83	83	58	43
(1,1,1) ML	72	66	80	80	52	38
1966-67						
(12,1,1) SML	69	73	84	84	80	69
(12,3,1) SML	68	72	84	84	80	69
(1,1,1) ML	68	73	84	85	81	72
1967-68						
(12,1,1) SML	58	59	81	84	85	80
(12,3,1) SML	51	53	77	80	84	79
(1,1,1) ML	50	52	76	79	83	81
1968-69						
(12,1,1) SML	67	72	78	79	71	70
(12,3,1) SML	64	70	79	81	73	71
(1,1,1) ML	62	66	78	80	71	69
1969-70						
(12,1,1) SML	71	65	83	84	76	71
(12,3,1) SML	66	61	80	80	71	67
(1,1,1) ML	59	55	75	75	63	60
1970-71						
(12,1,1) SML	50	68	76	78	79	77
(12,3,1) SML	48	67	79	80	82	80
(1,1,1) ML	48	65	79	81	83	79
1971-72						
(12,1,1) SML	78	76	82	81	67	63
(12,3,1) SML	77	76	84	83	65	62
(1,1,1) ML	74	73	81	80	65	62

Table XI continued

	SNO	W.E	SAC*W.E	SWH	SWHT	SWHTV
1972-73						
(12,1,1) SML	76	78	83	82	79	75
(12,3,1) SML	73	76	82	81	81	71
(1,1,1) ML	66	70	74	73	80	64

vapor onto crystals or crystal fragments, brought together by moderate winds, and their subsequent cementing together. Avalanching often occurs at the Rogers Pass, along with humidities of 80% or over. Substantial levels of activity have also been observed in the data, for the first parts of the seasons, in association with humidities of over 85%, temperatures just above freezing, and some rainfall, measured at the Rogers Pass observatory. There are three such days in the first part of 1967-68, and another three for 1970-71, which undoubtedly give rise to the slightly higher correlation values for SWHT, as opposed to SWH. However, it is unlikely that a great deal of precipitation fell in the form of rain on the upper slopes, since temperatures were so close to freezing for these periods. Hence, although a portion of the humidity effect may be attributable to rainfall, indeed humidity may be regarded as a good indicator of rainfall, the major influence of humidity is probably related to slab formation, particularly since SWHT correlation values are lower than SWH values for the first parts of most of the winters.

The non-linear effect of wind has already been mentioned. However, even though the formation of slab conditions requires only moderate winds, it might be expected that, for some years at least, wind should play a more important role than the results of this analysis indicate. Wind terms may be more important than the data suggests, for the following reasons. There are several periods of missing observations, during which the anemovane was not operational, often because of icing problems. This frequently occurs during the height of snow storms, just when the measurements would be most significant. Since wind speed and direction varies radically from day to day, and indeed, even from hour to hour, it seemed only reasonable to substitute eight year averages for these missing values, rather than try to interpolate between measured values, particularly since the periods of missing wind speeds may be up to two weeks in length. Daily mean wind speed and directions were used in the analysis, but as Schaerer suggests,[15] maximum values should be more significant, since gusting speeds may be a better indicator of wind effects in the upper zones. Indeed, it may be necessary to use more frequent wind observations, perhaps three or six hourly, since this parameter, above all others, varies most radically.

Although the MacDonald West Shoulder wind station is ideally located, as far as point measurements are concerned, and may be entirely representative of the area as a whole, a network of stations, or even one other station, perhaps in the Hermit Meadows, would be quite advantageous in providing better control of these observations. It is hard to believe that one station can accurately describe the wind conditions on both sides of the Pass.

In conclusion, the results of this phase of the analysis indicate that, for the first parts of individual years, models based on SWH terms

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[15] Personal communication

should achieve a high level of precision. After presenting a brief outline of the theory of time series procedures in the next chapter, models will be developed in Chapter VII for the first parts of individual years, and for total first parts of the entire period, 1965-73, using (12,1,1) SML, (12,3,1) SML and (1,1,1) ML weights for the avalanche activity index, and SNO, W.E, SWH and SWHT as independent variables. Of course, models could be developed using any of the significant meteorological factors, such as are indicated in Table IV. For example, models based on wind terms or temperature terms could be designed. However, because all the meteorological factors are intercorrelated and since it is desirable to produce the strongest possible models, it is best to concentrate on those independent variables given above, which are most highly correlated with avalanche activity.

Chapter VI

TIME SERIES ANALYSIS

In this chapter, procedures used in the development of multilinear time series models, which best describe the processes governing the association between avalanche activity and the various meteorological factors, are discussed. Such procedures involve the determination of a suitable transfer function, or dynamic input-output relationship for the system, based on discrete observations, which are equispaced in time, after which the stochastic[16] noise component is identified and estimated in terms of an autoregressive, moving average or mixed process, as defined by Box and Jenkins (1970). "The stochastic models we employ are based on the idea, (Yule,1927), that a time series in which successive values are highly dependent can be usefully regarded as generated from a series of independent 'shocks', a_t. These shocks are random drawings from a fixed distribution, usually assumed Normal and having mean zero and variance σ_a^2 " (Box and Jenkins, 1970:8).

Some of the concepts involved in this approach will now be illustrated together with an outline of the specific procedures employed in this study. For a more complete and thorough exposition of the theory of linear time series processes, the reader is advised to consult Box and Jenkins.

Stochastic Processes

First Order Autoregressive Process. The first order autoregressive (Markov) process, AR(1), (Box and Jenkins, 1970:56) is of considerable

[16] A statistical phenomenon that evolves in time according to probabilistic laws is called a stochastic process.

practical importance and can be represented by the following equation,

$$z_t = \phi_1 z_{t-1} + a_t, \tag{1}$$

where z_t and z_{t-1} are values of the series (usually deviations from the mean), at times t and t-1 respectively, ϕ_1 is the first order autoregressive coefficient and a_t is the residual error or random 'white noise' term at time t. ϕ_1 must satisfy the condition, $-1 < \phi_1 < 1$, for the process to be stationary.[17] Rearranging equation (1),

$$(1 - \phi_1 B) z_t = a_t, \tag{2}$$

where B is the backward shift operator defined by,

$$B \cdot z_t = z_{t-1}.$$

Hence, dividing throughout in equation (2) by $(1 - \phi_1 B)$ gives,

$$z_t = (1 + \phi_1 B + \phi_1^2 B^2 + \ldots) a_t. \tag{3}$$

That is, z_t can be expressed in the form of an infinite moving average process (Box and Jenkins, 1970:10). The autocorrelation function,

$$\rho_k = \frac{\gamma_k}{\gamma_0} = \frac{\sum_{t=-\infty}^{\infty} z_t z_{t-k}}{\sum_{t=-\infty}^{\infty} z_t^2}, \qquad k = 1, 2, \ldots, \infty, \tag{4}$$

where γ_k is the covariance and γ_0 is the variance, is a powerful tool used in the identification and estimation procedures (Box and Jenkins, 1970:28).

Multiplying throughout in equation (1) by z_{t-k} results in,

$$z_{t-k} z_t = \phi_1 z_{t-k} z_{t-1} + z_{t-k} a_t, \tag{5}$$

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[17] A stationary process is said to be strictly stationary if its properties are unaffected by a change of time origin.

and taking expectations,

$$\gamma_k = \phi_1 \gamma_{k-1}, \quad k > 0. \tag{6}$$

Note that the expectation $E[z_{t-k} a_t]$ vanishes when $k > 0$, since z_{t-k} can only involve the shocks a_j up to time t-k, which are uncorrelated with a_t.

Dividing throughout in (6) by γ_0,

$$\rho_k = \phi_1 \rho_{k-1}, \quad k > 0, \tag{7}$$

which is the Yule-Walker equation for the first order autoregressive process. Setting $\rho_0 = 1$, equation (7) has the solution,

$$\rho_k = \phi_1^k, \quad k \geq 0. \tag{8}$$

Hence, the autocorrelation function decays exponentially to zero, when ϕ_1 is positive, but decays exponentially to zero and oscillates in sign when ϕ_1 is negative. In particular, it should be noted that,

$$\rho_1 = \phi_1. \tag{9}$$

In general, finite autoregressive processes of any order p, that is, AR(p) processes, have unique autocorrelation functions, and therefore, in principle, the characteristic features of these functions can be used to identify the processes from which they are generated. For finite time series, the autocorrelation function can be estimated from,

$$r_k = c_k/c_0, \tag{10}$$

$$c_k = \frac{1}{N} \sum_{t=1}^{N-k} z_t z_{t-k}, \quad k = 0, 1, 2, \ldots, K,$$

c_k and c_0 are the sample covariance and variance respectively, N is the number of observations in the sequence. r_k is called the sample autocorrelation function. "In practice, to obtain a useful estimate of the autocorrelation function, we would need at least fifty observations and the estimated autocorrelations r_k would be calculated for $k = 0, 1, 2, \ldots, K$, where K was not larger than say N/4" (Box and Jenkins, 1970:33).

General Autoregressive Process. The general autoregressive process of order p, that is, the AR(p) process can be written,

$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + \cdots + \phi_p z_{t-p} + a_t. \tag{11}$$

Multiplying throughout in (11) by z_{t-k} and taking expectations, gives,

$$\rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2} + \cdots + \phi_p \rho_{k-p}, \quad k > 0, \tag{12}$$

which is analogous to the difference equation satisfied by the process itself. Substituting, $k = 1, 2, \ldots, p$ in (12), the following set of linear equations for $\phi_1, \phi_2, \ldots, \phi_p$ in terms of $\rho_1, \rho_2, \ldots, \rho_p$ is obtained,

$$\begin{aligned}
\rho_1 &= \phi_1 + \phi_2 \rho_1 + \cdots + \phi_p \rho_{p-1}, \\
\rho_2 &= \phi_1 \rho_1 + \phi_2 + \cdots + \phi_p \rho_{p-2}, \\
&\vdots \\
\rho_p &= \phi_1 \rho_{p-1} + \phi_2 \rho_{p-2} + \cdots + \phi_p.
\end{aligned} \tag{13}$$

These are usually called the Yule-Walker equations from which estimates of the parameters can be obtained by replacing the theoretical autocorrelations, ρ_k, by the estimated autocorrelations r_k (Box and Jenkins, 1970:54-56). The equations are identical to the reduced 'normal' equations of multilinear regression analysis, which lead to the familiar least squares estimates .

Another useful tool in the identification process is the partial autocorrelation function, which is defined as the last autoregression coefficient obtained after successively fitting increasing orders of autoregressive process to the data. "For an autoregressive process of order p, the partial autocorrelation function, ϕ_{kk} will be nonzero for k less than or equal to p and zero for k greater than p" (Box and Jenkins, 1970:65). As a useful general rule in fitting autoregressive models of order p, the autocorrelation function of a stationary autoregressive process is infinite in extent and consists of a mixture of damped exponentials and damped sine waves, whereas the partial autocorrelation function is finite with a cutoff after lag p.

<u>First Order Moving Average Process</u>. The first order moving average process, MA(1), (Box and Jenkins, 1970:69) is represented by the following equation,

$$z_t = a_t - \theta_1 a_{t-1}, \tag{14}$$

which has the following alternative forms,

$$z_t = (1 - \theta_1 B)\, a_t, \text{ and,} \tag{15}$$

$$a_t = (1 + \theta_1 B + \theta_1^2 B^2 + \dots)\, z_t. \tag{16}$$

Hence, a_t can be expressed in the form of an infinite autoregressive process. Multiplying throughout in equation (14) by z_{t-k} results in,

$$z_{t-k}\, z_t = (a_{t-k} - \theta_1 a_{t-k-1})\,(a_t - \theta_1 a_{t-1}) \tag{17}$$

and on taking expectations,

$$\rho_k = \begin{cases} \dfrac{-\theta_1}{1 + \theta_1^2}, & k = 1, \\ 0, & k \geq 2. \end{cases} \tag{18}$$

Thus, in contrast to the AR(p) process, the autocorrelation function for the MA(q) process has a cutoff after lag q and the partial autocorrelation function tails off, and is dominated by a mixture of damped exponentials and damped sine waves.

Mixed Process. "To achieve greater flexibility in fitting of actual time series, it is sometimes advantageous to include both autoregressive and moving average terms in the model" (Box and Jenkins, 1970: 11). The first order mixed autoregressive-moving average process, ARMA(1,1) (Box and Jenkins, 1970 :76) is represented by the following equation,

$$z_t - \phi_1 z_{t-1} = a_t - \theta_1 a_{t-1}, \tag{19}$$

that is,

$$(1 - \phi_1 B) z_t = (1 - \theta_1 B) a_t. \tag{20}$$

Therefore,

$$z_t = (1 - \theta_1 B)(1 - \phi_1 B)^{-1} a_t, \tag{21}$$

$$a_t = (1 - \phi_1 B)(1 - \theta_1 B)^{-1} z_t. \tag{22}$$

Hence, both the autocorrelation and the partial autocorrelation functions are infinite in extent.

If the stochastic time series exhibits non-stationary behaviour, usually indicated by a slow and linear tapering of the autocorrelation function, it may be necessary to apply some degree of differencing to the data (Box and Jenkins, 1970:85-119).

Of particular interest in this respect is the first order autoregressive integrated moving average process, ARIMA(1,1,1), represented by the following equation,

$$w_t - \phi_1 w_{t-1} = a_t - \theta_1 a_{t-1}, \tag{23}$$

where,

$$w_t = \nabla z_t, \tag{24}$$

and ∇ is the difference operator defined by,

$$\nabla = 1 - B \tag{25}$$

that is,

$$\nabla z_t = z_t - z_{t-1}. \tag{26}$$

Transfer Function Representation

Let,

$$y_t = \nu(B)\ x_t, \tag{27}$$

be a linear representation of a deterministic process, known as a linear filter, where x_t and y_t are the independent variable (input series) and dependent variable (output series), respectively, and,

$$x_t = (X_t - \bar{X}),\ \text{and}$$

$$y_t = (Y_t - \bar{Y}).$$

The function,

$$\nu(B) = (\nu_0 + \nu_1\ B + \nu_2\ B^2 + \ldots), \tag{28}$$

is called the transfer function of the process. The weights, ν_0, ν_1, ν_2, ..., are called the impulse response function (Box and Jenkins, 1970:14). The linear filter is stable if the transfer function converges, that is, if the series is finite or infinite and convergent.

For 'real' data,

$$y_t = \nu(B)\ x_t + n_t, \tag{29}$$

where $\nu(B)$ is a deterministic transfer function and n_t is stochastic noise, with x_t and n_t assumed independent. x_t is also assumed to be a stochastic noise process (Box and Jenkins, 1970:371).

$$n_t = \psi(B)\ a_t, \tag{30}$$

where $\psi(B)$ is a stochastic transfer function, that is, n_t is the output of a linear filter whose input is the 'white' noise process, a_t. All ARIMA processes can be represented in this way.

Alternatively, (29) may take the form,

$$y_t = \delta^{-1}(B)\, w(B)\, x_t + \phi^{-1}(B)\, \theta(B)\, a_t. \tag{31}$$

The cross correlation function, as defined by the following equation,

$$\rho_{xy}(k) = \frac{\gamma_{xy}(k)}{\sigma_x \sigma_y},$$

$$= \frac{\sum_{t=-\infty}^{\infty} x_{t-k}\, y_t}{\sqrt{\sum_{t=-\infty}^{\infty} x_{t-k}^2 \sum_{t=-\infty}^{\infty} y_t^2}}, \qquad k = \pm 1, \pm 2, \ldots, \pm\infty, \tag{32}$$

where $\gamma_{xy}(k)$ is the covariance, and σ_x, σ_y are the standard deviations, is a powerful tool used in the identification and estimation procedures. For finite time series, the cross correlation function can be estimated from,

$$r_{xy}(k) = \frac{c_{xy}(k)}{s_x s_y}, \tag{33}$$

where,

$$c_{xy}(k) = \frac{1}{N} \sum_{t=1}^{N-k} x_{t-k}\, y_t, \qquad k = 0, 1, 2, \ldots, K.$$

$c_{xy}(k)$ is the covariance and s_x and s_y are sample standard deviations for x and y. N is the number of observations and $r_{xy}(k)$ is called the sample cross correlation coefficient.[18] "In practice, we would need at least 50 pairs of observations to obtain a useful estimate of the cross correlation

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[18] Unlike the autocorrelation function, the cross correlation function is generally asymmetrical. One half is 'physically realizable' and is known as the 'memory function', while the other half is 'not physically realizable' and is called the 'anticipation function'. Only the memory function is here defined.

function" (Box and Jenkins, 1970:374).

Suppose that,

$$y_t = \nu_0 \, x_t + \nu_1 \, x_{t-1} + \nu_2 \, x_{t-2} + \dots + n_t, \tag{34}$$

is a linear transfer function model, where y_t and x_t are the output and input series respectively, suitably differenced to induce stationarity, and ν_0, ν_1, ..., etc. is the impulse response function. "We assume that a degree of differencing, d, necessary to induce stationarity has been achieved when the estimated auto- and cross correlations $r_{xx}(k)$, $r_{yy}(k)$ and $r_{xy}(k)$ of $x_t = \nabla^d X_t$, and $y_t = \nabla^d Y_t$ damp out quickly. In practice, d is usually 0, 1, or 2" (Box and Jenkins, 1970:378). Multiplying throughout in equation (34) by x_{t-k}, gives,

$$x_{t-k} \, y_t = \nu_0 \, x_{t-k} \, x_t + \nu_1 \, x_{t-k} \, x_{t-1} + \dots + x_{t-k} \, n_t. \tag{35}$$

Assuming that x_{t-k} and n_t are uncorrelated for all $k \geq 0$, then, taking expectations,

$$\gamma_{xy}(k) = \nu_0 \, \gamma_{xx}(k) + \nu_1 \, \gamma_{xx}(k-1) + \dots, \quad k = 0, 1, 2, \dots, \tag{36}$$

or,

$$\rho_{xy}(k) = \frac{\sigma_x}{\sigma_y} \left[\nu_0 \, \rho_{xx}(k) + \nu_1 \, \rho_{xx}(k-1) + \dots \right], \quad k=0,1,2,\dots, \tag{37}$$

which are similar to the Yule-Walker equations. However, these equations, analogous to the reduced 'normal' equations of regression analysis, do not in general, provide efficient estimates of the transfer function coefficients. Considerable simplification in the identification procedure is achieved and more efficient estimates of the parameters obtained if the input and output series are 'prewhitened' prior to analysis (Box and Jenkins, 1970:379). The procedure is as follows. Given that,

$$y_t = \nu(B) \, x_t + n_t, \tag{38}$$

the series x_t is represented by the ARIMA model,

$$\phi_x(B)\ \theta_x^{-1}(B)\ x_t = \alpha_t, \tag{39}$$

which, to a close approximation, transforms the correlated input series, x_t, into the uncorrelated white noise series α_t. The same transformation is applied to y_t to obtain,

$$\beta_t = \phi_x(B)\ \theta_x^{-1}(B)\ y_t. \tag{40}$$

Hence, equation (38) can be written,

$$\beta_t = \nu(B)\ \alpha_t + \epsilon_t, \tag{41}$$

where ϵ_t is the transformed stochastic noise series,

$$\epsilon_t = \phi_x(B)\ \theta\ _x^{-1}(B)\ n_t. \tag{42}$$

Multiplying throughout in equation (41) by α_{t-k} and taking expectations, gives,

$$\gamma_{\alpha\beta}(k) = \nu_k\ \gamma_{\alpha\alpha}(0), \tag{43}$$

since α_t is a white noise process. Thus,

$$\nu_k = \frac{\gamma_{\alpha\beta}(k)}{\sigma_\alpha^2}, \tag{44}$$

or in terms of cross correlations,

$$\nu_k = \frac{\rho_{\alpha\beta}(k)\ \sigma_\beta}{\sigma_\alpha}, \quad k = 0, 1, 2, \ldots . \tag{45}$$

Hence, initial estimates for the transfer function coefficients ν_k, may be obtained directly from the sample cross correlation function, using equation (45) rewritten in terms of sample estimates, that is,

$$\hat{\nu}_k = \frac{r_{\alpha\beta}(k)\ s_\beta}{s_\alpha}, \quad k = 0, 1, 2, \ldots . \tag{46}$$

In practice, least squares estimates are obtained after prewhitening the input and output series, using an all combination approach, since prewhitening is always imperfect.

The stochastic noise process may now be identified and estimated, since,

$$\hat{n}_t = y_t - \hat{v}(B)\ x_t. \tag{47}$$

An ARIMA model is fitted to the estimated noise process of the form,

$$\hat{n}_t = \phi_{\hat{n}}^{-1}(B)\ \theta_{\hat{n}}(B)\ a_t, \tag{48}$$

giving the total model,

$$y_t = \nu(B)\ x_t + \phi_{\hat{n}}^{-1}(B)\ \theta_{\hat{n}}(B)\ a_t. \tag{49}$$

Since the transfer function and stochastic noise components are identified separately, estimates of the parameters are necessarily inefficient (Box and Jenkins, 1970:386). To obtain more efficient estimates after the identification procedures, the transfer function and noise models may be combined in a single least squares estimation.

If more than one input series is used, the prewhitening and transfer function estimation procedures are repeated for the second input series and an output series, formed by subtracting the transfer function for the first series from the original output series. Symbolically,

$$y_t - \nu(B)\ x_{1t} = u(B)\ x_{2t} + n_t, \tag{50}$$

replaces equation (38), where x_{1t} and x_{2t} are the two input series. This procedure can be repeated for any number of input series.

Computational Procedures

An extremely flexible operational system was devised for the development of optimum transfer function/stochastic noise models for avalanche forecasting. The main instrument in these procedures is a computer program, which was written specifically for this study, incorporating all the features described in the foregoing theory.

Using the program, the following steps are performed.

1) Autocorrelation functions for the input series, together with cross correlation functions between input and output are computed up to twenty lags, and examined for stationarity. In the case of avalanche activity, and most of the composite meteorological series, the correlation functions decayed sufficiently rapidly such that differencing was unnecessary. Tests have been carried out using first and second differences to see whether such treatment would lead to more powerful models. The contrary seems to be the case, since although R^2 - values[19] were increased, this was offset by an increase in the dispersion of the data, such that the residual errors were just as high as in the undifferenced data. Differencing also resulted in an increase in the stochastic noise component resulting in more complex models, an undesireable feature of the process.

2) Partial autocorrelation functions are computed for the input series up to twenty lags and suitable ARIMA models describing these processes identified and estimated.

3) The ARIMA model describing the first (primary) input series is used to transform this series into an approximate white noise process, and the same transformation applied to the output series.

4) The transfer function is then identified and least squares estimates obtained for the transformed input and output series using an all com-

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[19] Multiple correlation coefficient squared

bination approach up to five lags.

5) If a secondary input series[20] is contemplated, the transfer function obtained in step 4 is subtracted from the output series and steps 3 and 4 repeated using this secondary series.

6) Finally, the complete transfer function is subtracted from the output series to obtain the stochastic noise process.

7) The autocorrelation and partial autocorrelation functions for the stochastic noise process are computed up to twenty lags and a suitable ARIMA model describing the process identified and estimated.

8) The transfer function and noise models are then combined and efficient least squares estimates of the parameters obtained. Any insignificant terms are eliminated and the final, complete model re-estimated.

9) In this last step, tests of model adequacy are performed as described by Box and Jenkins (1970:392-5). Among other things, insignificant autocorrelation in the residuals is confirmed.

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[20] The selection of the secondary series is achieved by calculating partial correlation coefficients, as described in Chapter V, after the effect of the primary independent (meteorological) variable has been subtracted from the dependent (avalanche activity) variable.

Chapter VII

THE TIME SERIES MODELS

Employing the procedures outlined in Chapter VI, transfer function/stochastic noise functions were developed using the three best sets of weights for avalanche activity, as defined in Chapter V. These models are listed in Table XII, along with appropriate statistics indicating their strength. Models for individual years, besides the entire period, 1965-73, are quoted for first parts only, as defined in Table VIII. SNO, W.E, SWH, and SWHT, obtained from the Rogers Pass meteorological measurements, were used as input series and avalanche activity indices were computed for all sites, artificial and natural avalanches combined and (12,1,1) SML, (12,3,1) SML, and (1,1,1) ML weights. The models have been left in their transfer function/stochastic noise form so that their basic structure can be better illustrated. A final least squares estimation would be performed prior to their implementation as prediction models.

As anticipated from the results of the correlation analysis, given in Table XI, the highest R^2- values were realized with SWH models for individual years, with the exception of 1970-71, which has somewhat stronger SWHT models. To illustrate the procedures involved, by way of an example, the development of the SWH model for 1967-68 using (12,1,1) SML weights will now be shown. It should be noted that each series is reduced to deviations from its mean, prior to analysis.

SWH Model for 1967-68

1) Correlation functions decrease to insignificance at or before the third lag. Therefore, stationarity is assumed and no differencing is

applied. The number of observations, N = 95.

For the AVAL series, the corrected sum of squares,

$$SS_{TOT}(AVAL) = 1353670,$$

hence, the standard deviation,

$$SD(AVAL) = 120.0.$$

For the SWH series, the corrected sum of squares,

$$SS_{TOT}(SWH) = 4593.87,$$

and,

$$SD(SWH) = 6.991.$$

2) Inspection of the autocorrelation and partial autocorrelation functions suggests a (1,0,0) model for the SWH series. Hence,

$$\phi_1 = 0.3362,$$

that is,

$$SWH = 0.3362\ SWH1 + \alpha,^{*}$$

rearranging,

$$(1 - 0.3362\ B)\ SWH = \alpha.$$

3) Thus, the prewhitening operator is (1 - 0.3362 B)

$$\alpha = (1 - 0.3362\ B)\ SWH,$$

$$\beta = (1 - 0.3362\ B)\ AVAL.$$

4) The transfer function obtained by least squares is,

$$\beta = 13.02\ \alpha + 3.683\ \alpha_1.$$

5) Hence,

$$NSE = AVAL - 13.02\ SWH - 3.683\ SWH1,$$

is the noise series. For the NSE series, the corrected sum of squares,

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* For simplicity, the 't' subscript notation has been dropped. SWH_{t-1} has the abbreviated form SWH1, etc., and α_{t-1} has the abbreviated form α_1 etc.

$$SS_{TOT}(NSE) = 329660.$$

6) Inspection of the autocorrelation and partial autocorrelation functions indicates that the noise series is essentially random, that is, there is no stochastic noise component for this model.

7) Hence, the complete model may be written,

$$\widehat{AVAL} = 13.02\ SWH + 3.683\ SWH1,$$

and re-estimated more efficiently using least squares. The re-estimated model is,

$$\widehat{AVAL} = 13.08\ SWH + 4.010\ SWH1,$$

which has an R^2 of 0.756.

Figure 2 shows predicted values obtained with this model, plotted together with actual values of the avalanche activity index. Thus, a very close agreement has been achieved by use of this simple two-term model based on the independent variable SWH. Besides the unlagged term, SWH, the first lag term is highly significant and would have made a strong contribution to real-time forecasts of avalanche activity for the period. Since SWH1 would be precisely known at the time the forecast is made, the model does not rely exclusively on the weather forecast. Similar accuracies could be achieved for the other years, using the SWH models described in Table XII.

Models for Individual Years

A number of conclusions may be drawn after close examination of all the models depicted in Table XII. R^2- values for the models, consistently display a progressive increase for (1,1,1) ML weights, to (12,3,1) SML weights, up to (12,1,1) SML weights. SWH models are generally better than SWHT models, which are always better than W.E models. W.E models are more powerful than SNO models, except for the years, 1965-66, 1969-70, and 1971-72.

Models for the entire period, 1965-73, have more significant lagged terms than those for individual years, but R^2- values are lower as a result of the larger sample size. No models have terms which are higher than the third lag, a significant result. Models for each individual year appear to be structurally quite unique, but some similarities do exist. SNO models for 1966-67 and 1968-69 both consist of only one term, the unlagged SNO term, for which the coefficients are in close agreement. W.E models for these years are also similar. However, the SWH models differ slightly in structure, but the unlagged SWH coefficient values are almost identical for these two years, the same argument applying to the SWHT models. The SNO model for the 1972-73 year also consists of only one term, the unlagged SNO term, but the coefficient is higher than those for the two years just mentioned, indicating that equal amounts of precipitation produced more avalanching in 1972-73, than in 1966-67 or 1968-69, possibly a temperature effect. It is worthwhile to examine the models depicted in Table XII quite closely, and attempts can be made to group years together according to the class of model describing their avalanche activity.

SWH, W.E*TMI Model for 1965-73

However, for the practical forecasting of avalanches, it is necessary to have, at one's disposal, a single general model, which can be applied without having to assign the winter to a particular class. To this end, the SWH model for the entire period, 1965-73, using (12,1,1) SML weights, was developed and a secondary series, the W.E*TMI series, incorporated into the model in order to improve its forecasting accuracy.

Table XII

Time Series Models for Avalanche Activity, Daily Observations, Rogers Pass Meteorological Data, First Parts, All Sites, Artificial and Natural Avalanches

Year	Transfer Function SNO	SNO1	SNO2	SNO3	Stochastic Noise NSE1	NSE2	NSE3	Overall R^2	SD SE
1. SNO Models									
(12,1,1) SML									
1965-73	6.740	1.252	.6009		.1490	.0695	.0926	.494	103.3 73.6
1965-66	4.275	2.005			.2423			.636	79.3 48.2
1966-67	7.367							.485	126.1 90.5
1967-68	6.941	4.771			.2438	.2803		.508	120.0 85.2
1968-69	6.347							.451	78.4 58.1
1969-70	8.564				.2554			.527	80.0 55.3
1970-71	4.966		2.578		.2263			.289	114.2 96.8
1971-72	7.147	1.767						.620	117.5 72.5
1972-73	9.737							.566	114.3 75.3
(12,3,1) SML									
1965-73	12.87	2.365	1.347		.1843	.09096		.466	205.7 150.6
1965-66	8.546	3.377						.600	154.1 97.4
1966-67	14.47							.467	250.2 182.7
1967-68	13.27	10.41			.2721	.2913		.478	263.3 192.4

Table XII continued

Year	Transfer Function SNO	SNO1	SNO2	SNO3	Stochastic Noise NSE1	NSE2	NSE3	Overall R^2	SD SE
1968-69	12.98							.423	165.5 125.7
1969-70	14.82				.2684			.468	146.5 107.5
1970-71	8.922		4.947		.3095			.299	224.3 188.8
1971-72	14.20	3.819						.616	240.1 148.7
1972-73	15.42							.522	191.0 132.1
<u>(1,1,1) ML</u>									
1965-73	.5242	.0815	.0516	.0579	.1983			.438	8.60 6.45
1965-66	.3525	.1048						.550	6.34 4.25
1966-67	.6258							.462	10.80 7.93
1967-68	.5322	.4259			.2261	.2650		.441	10.60 7.99
1968-69	.5234							.398	6.88 5.34
1969-70	.6416				.2564			.386	6.84 5.39
1970-71	.3748				.4423			.321	9.72 8.06
1971-72	.5726	.1427						.566	10.10 6.63
1972-73	.4535		.1711					.499	6.61 4.68

Table XII continued

Year	Transfer Function W.E	W.E1	W.E2	W.E3	Stochastic Noise NSE1	NSE2	NSE3	Overall R^2	SD SE
<u>2. W.E Models</u>									
<u>(12,1,1) SML</u>									
1965-73	9.504	1.134			.1674	.0813		.533	103.3 70.7
1965-66	6.363	2.086			.3653			.601	79.3 50.4
1966-67	11.08							.540	126.1 85.5
1967-68	8.194	3.097			.2728	.2672		.512	120.0 84.8
1968-69	8.769							.512	78.4 54.8
1969-70	9.880				.2294			.453	80.0 59.5
1970-71	9.505							.456	114.2 84.3
1971-72	9.822							.577	117.5 76.4
1972-73	11.89							.611	114.3 71.2
<u>(12,3,1) SML</u>									
1965-73	18.33	2.059			.2006	.09365		.511	205.7 144.1
1965-66	12.61				.3288			.541	154.1 105.0
1966-67	21.71							.526	250.2 172.3
1967-68	15.89	6.401			.3089	.2729		.479	263.3 192.0

Table XII continued

Year	Transfer Function W.E	W.E1	W.E2	W.E3	Stochastic Noise NSE1	NSE2	NSE3	Overall R^2	SD SE
1968-69	18.07							.488	165.5 118.4
1969-70	17.23				.2477			.408	146.5 113.3
1970-71	18.05				.2582			.477	224.3 163.0
1971-72	19.81							.574	240.1 156.7
1972-73	19.37							.580	191.0 123.8
<u>(1,1,1) ML</u>									
1965-73	.7428	.06691		.06649	.2120			.479	8.60 6.21
1965-66	.4910				.2880			.475	6.34 4.62
1966-67	.9493							.538	10.80 7.36
1967-68	.6350	.2505			.2589	.2478		.428	10.60 8.07
1968-69	.7146							.442	6.88 5.14
1969-70	.7518				.2411			.343	6.84 5.57
1970-71	.7495				.3497			.486	9.72 7.01
1971-72	.8007							.529	10.07 6.91
1972-73	.6013							.489	6.61 4.73

Table XII continued

Year	SWH	SWH1	SWH2	SWH3	NSE1	NSE2	NSE3	Overall R^2	SD SE
	Transfer Function				Stochastic Noise				
3. SWH Models ($SWH/10^4$)									
(12,1,1) SML									
1965-73	8.307				.1136	.0850		.624	103.3 63.5
1965-66	6.122							.675	79.3 45.2
1966-67	9.770	-1.250			.2092			.729	126.1 65.9
1967-68	13.02	3.683						.756	120.0 59.2
1968-69	9.418							.619	78.4 48.4
1969-70	13.99				.2166			.722	80.0 42.4
1970-71	7.445							.596	114.2 72.7
1971-72	6.181		-1.440					.658	117.5 68.7
1972-73	11.77							.666	114.3 66.0
(12,3,1) SML									
1965-73	16.53				.1202	.09559		.633	205.7 124.9
1965-66	12.20							.683	154.1 86.7
1966-67	19.51	-2.702						.722	250.2 131.9
1967-68	26.42	8.359						.691	263.3 146.3

Table XII continued

Year	Transfer Function SWH	SWH1	SWH2	SWH3	Stochastic Noise NSE1	NSE2	NSE3	Overall R^2	SD SE
1968-69	20.17							.651	165.5 97.8
1969-70	24.78				.2765			.671	146.5 84.5
1970-71	14.63							.629	224.3 136.7
1971-72	12.73		-2.485					.690	240.1 133.6
1972-73	19.56							.650	191.0 112.9
<u>(1,1,1) ML</u>									
1965-73	.6777				.1227			.606	8.60 5.40
1965-66	.4747							.633	6.34 3.84
1966-67	.8493	-.1220			.2023			.741	10.80 5.54
1967-68	1.069	.3000						.669	10.60 6.09
1968-69	.8184							.633	6.88 4.17
1969-70	1.104				.2981			.596	6.84 4.37
1970-71	.6274							.637	9.72 5.86
1971-72	.5170		-.1248					.643	10.07 6.01
1972-73	.5920							.530	6.61 4.53

Table XII continued

Year	Transfer Function SWHT	SWHT1	SWHT2	SWHT3	Stochastic Noise NSE1	NSE2	NSE3	Overall R^2	SD SE
4. SWHT Models (SWHT/10^5)									
(12,1,1) SML									
1965-73	4.088				.1207	.0764	.0881	.580	103.3 67.0
1965-66	3.136				.4013			.470	79.3 58.1
1966-67	4.216	-.7841						.664	126.1 73.1
1967-68	5.727							.724	120.0 63.1
1968-69	4.479							.507	78.4 55.1
1969-70	6.322				.2656			.605	80.0 50.6
1970-71	3.286							.620	114.2 70.4
1971-72	3.204		-.7935		.1799			.479	117.5 85.2
1972-73	4.777							.621	114.3 70.3
(12,3,1) SML									
1965-73	8.011				.1314	.09506	.09460	.575	205.7 134.4
1965-66	5.844				.3752			.425	154.1 117.6
1966-67	8.352	-1.647						.659	250.2 146.1
1967-68	12.07	2.402						.732	263.3 136.3

Table XII continued

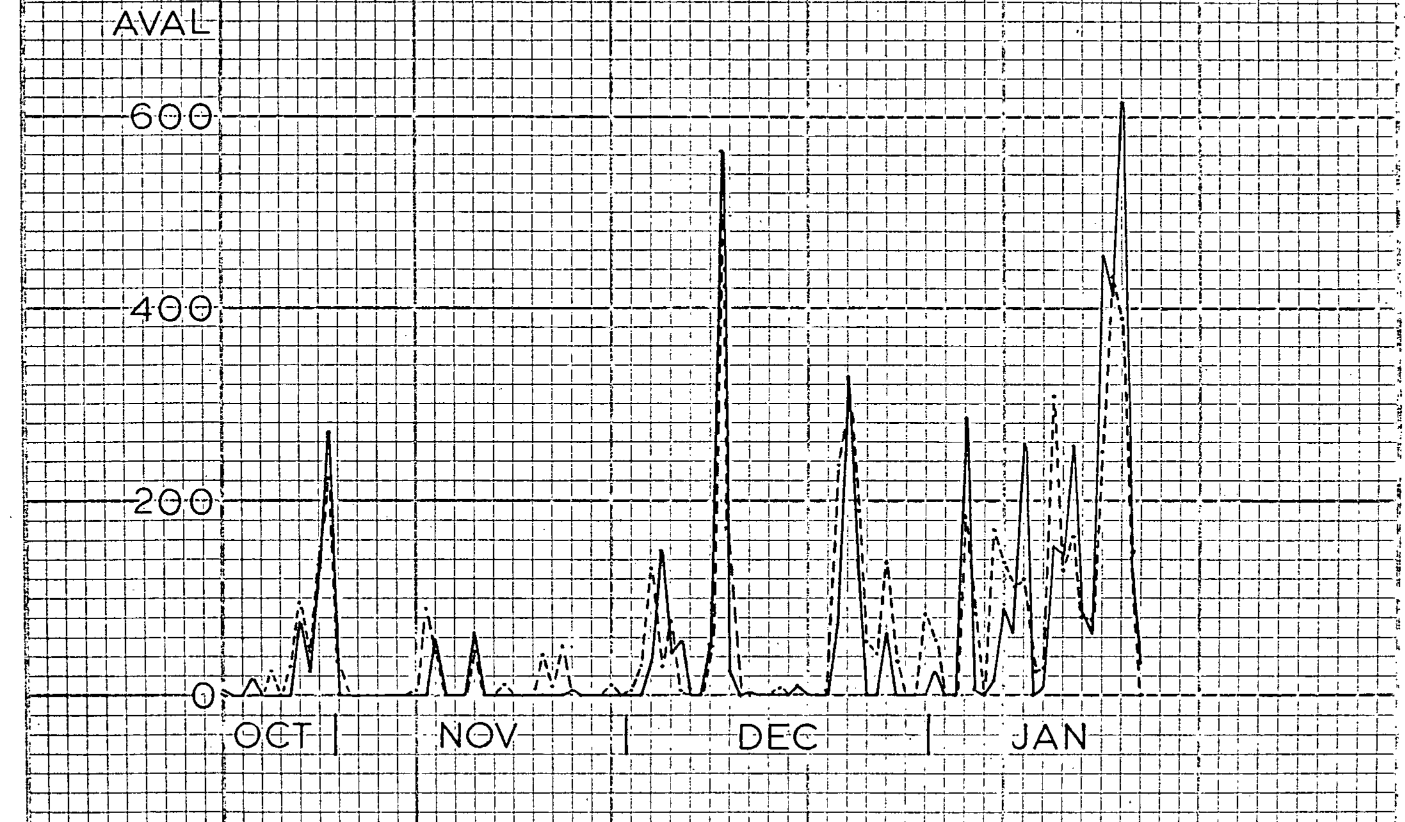
Year	Transfer Function SWHT	SWHT1	SWHT2	SWHT3	Stochastic Noise NSE1	NSE2	NSE3	Overall R^2	SD SE
1968-69	9.554							.526	165.5 114.0
1969-70	10.98				.3035			.537	146.5 100.4
1970-71	6.395							.670	224.3 128.9
1971-72	6.179				.1983			.441	240.1 180.3
1972-73	8.022							.651	191.0 112.8
(1,1,1) ML									
1965-73	.3323				.1283	.0746		.559	8.60 5.72
1965-66	.2107				.3237			.342	6.34 5.17
1966-67	.3678	-.0760						.679	10.80 6.14
1967-68	.4928							.691	10.60 5.87
1968-69	.3855							.503	6.88 4.85
1969-70	.4797				.3083			.448	6.84 5.11
1970-71	.2700							.678	9.72 5.52
1971-72	.2668		-.0758		.1978			.455	10.07 7.46
1972-73	.2555							.624	6.61 4.05

FIGURE 2

AVALANCHE ACTIVITY PREDICTION PROFILE* FOR 1967-68

based on the model

$$\widehat{AVAL} = 13{\cdot}08\,SWH + 4{\cdot}010\,SWH1$$



* actual values
predicted values

The following outlines the steps involved in obtaining this model.

1) Correlation functions decrease to insignificance at or before the sixth lag. Therefore, stationarity is assumed and no differencing is applied. Since auto, partial and cross correlation coefficients were calculated up to nine lags, data for individual years were separated by nine sets of zero values. This prevents any overlap of data between years from artificially influencing the values of these coefficients. Hence, the number of observations, N = 824. For the AVAL series, the corrected sum of squares,

$$SS_{TOT}(AVAL) = 8782950,$$

and hence, the standard deviation,

$$SD(AVAL) = 103.3.$$

For the SWH series, the corrected sum of squares,

$$SS_{TOT}(SWH) = 81708.7,$$

and

$$SD(SWH) = 9.964.$$

2) Inspection of autocorrelation and partial autocorrelation functions suggests a (2,0,0) model for the SWH series. Hence,

$$\phi_1 = 0.4096, \quad \phi_2 = 0.1047,$$

that is,

$$SWH = 0.4096\ SWH1 + 0.1047\ SWH2 + \alpha,^{*}$$

rearranging,

$$(1 - 0.4096\ B - 0.1047\ B^2)\ SWH = \alpha.$$

3) Thus, the prewhitening operator is $(1 - 0.4096\ B - 0.1047\ B^2)$

---

* As before, the 't' subscript notation has been dropped for simplicity.

$$\alpha = (1 - 0.4096\ B - 0.1047\ B^2)\ SWH,$$

and,

$$\beta = (1 - 0.4096\ B - 0.1047\ B^2)\ AVAL.$$

4) Transfer Function obtained by least squares is,

$$\beta = 8.307\ \alpha.$$

5) Hence,

$$NSE = AVAL - 8.307\ SWH,$$

is the noise series, For the noise series, the corrected sum of squares,

$$SS_{TOT}(NSE) = 3382620.$$

6) At this point, transfer function estimation can be terminated and the stochastic noise series identified and estimated, as is indicated in Table XII, p. 90.

Inspection of autocorrelation and partial autocorrelation functions suggests a (2,0,0) model for the NSE series. Hence,

$$\phi_1 = 0.1136, \quad \phi_2 = 0.0850,$$

that is,

$$NSE = 0.1136\ NSE1 + 0.0850\ NSE2 + a,$$

which can be rewritten,

$$(1 - 0.1136\ B - 0.0850\ B^2)\ NSE = a,$$

and the sum of squares residual,

$$SS_{RES}(a) = 3306470.$$

7) Hence, the complete model can be written,

$$AVAL = 8.307\ SWH + \frac{a}{(1 - 0.1136\ B - 0.0850\ B^2)},$$

as indicated in Table XII, or, by multiplying throughout by

$$(1 - 0.1136\ B - 0.0850\ B^2),$$

$$AVAL = 0.1136\ AVAL1 + 0.0850\ AVAL2 + 8.307\ SWH - 0.9437\ SWH1 - 0.7061\ SWH2 + a,$$

which can be re-estimated more efficiently using multiple regression techniques.

8) Partial correlation coefficients, calculated after the effect of SWH was subtracted out, suggest that W.E*TMI might be a good secondary variable.

9) For the W.E*TMI series, the corrected sum of squares,

$$SS_{TOT}(W.E*TMI) = 23408700,$$

and,

$$SD(W.E*TMI) = 168.7.$$

10) Inspection of autocorrelation and partial autocorrelation functions suggests a (1,0,0) model for the W.E*TMI series, Hence,

$$\phi_1 = 0.3336,$$

that is,

$$W.E*TMI = 0.3336\ W.E*TMI1 + \alpha.$$

Rearranging,

$$(1 - 0.3336\ B)\ W.E*TMI = \alpha.$$

11) Thus, the prewhitening operator is (1 - 0.3336 B).

$$\alpha = (1 - 0.3336\ B)\ W.E*TMI,$$

and, $$\beta = (1 - 0.3336\ B)(AVAL - 8.307\ SWH),$$

12) The transfer function obtained by least squares is,

$$\beta = 0.07211\ \alpha.$$

13) Hence,

$$NSE = AVAL - 8.307\ SWH - 0.07211\ W.E*TMI,$$

is the noise series. For the NSE series, the corrected sum of squares,

$$SS_{TOT}(NSE) = 3235040.$$

14) Inspection of autocorrelation and partial autocorrelation functions suggests a (2,0,0) model for the NSE series. Hence,

$$\phi_1 = 0.0941, \quad \phi_2 = 0.0954,$$

that is,

$$NSE = 0.0941\ NSE1 + 0.0954\ NSE2 + a,$$

which can be rewritten,

$$(1 - 0.0941\ B - 0.0954\ B^2)\ NSE = a,$$

and the sum of squares residual,

$$SS_{RES}(a) = 3171022.$$

15) Hence, the complete model can be written,

$$AVAL = 8.307SWH + 0.07211W.E*TMI + \frac{a}{(1 - 0.0941B - 0.0954B^2)},$$

or, multiplying throughout by $(1 - 0.0941\ B - 0.0954\ B^2)$,

$$AVAL = 0.0941\ AVAL1 + 0.0954\ AVAL2 + 8.307\ SWH - 0.782\ SWH1 - 0.792\ SWH2 + 0.07211\ W.E*TMI - 0.00679\ W.E*TMI1 - 0.00688\ W.E*TMI2 + a.$$

16) This equation was re-estimated more efficiently using multiple regression techniques, resulting in,

$$\widehat{AVAL} = 0.0824\ AVAL1 + 0.0819\ AVAL2 + 7.027\ SWH - 1.020\ SWH1 - 0.313\ SWH2 + 0.1220\ W.E*TMI + 0.0198\ W.E*TMI1 - 0.0365\ W.E*TMI2.$$

17) Since the SWH2 and W.E*TMI1 terms are insignificant, the model is re-estimated, giving,

$$\widehat{AVAL} = 0.0947\ AVAL1 + 0.0589\ AVAL2 + 6.850\ SWH - 0.937\ SWH1 + 0.1310\ W.E{*}TMI - 0.0350\ W.E{*}TMI2,$$

which has an R^2- value of 0.651 and a standard error of estimate, SE of 61.3.

18) Tests were applied to the residual errors to confirm model adequacy. The residuals were found to be uncorrelated and unbiased and the model therefore adequate.

Figure 3 shows predicted values obtained with this model, plotted along with actual values of the avalanche activity index, for the entire period, 1965-73, and an excellent agreement is demonstrated. This model can be used to predict avalanche activity for any winter which resembles the class of eight, defined by the period, 1965-73.

Confidence Limits

The variance of the predicted values of avalanche activity can be estimated for any set of values of the independent variables, using the following expression,

$$V(\hat{Y}) = s^2(\underline{X}_0' \underline{C}\ \underline{X}_0), \tag{51}$$

where $V(\hat{Y})$ is the variance of the predicted value $\hat{Y}$, s is the standard error of estimate for the regression, $\underline{X}_0$ is a set of $\underline{X}$ (in matrix notation) and $\underline{C} = (\underline{X}'\underline{X})^{-1}$ (Draper and Smith, 1966:121).

Thus $1 - \alpha$ confidence limits on the true mean value of Y at $\underline{X}_0$ are given by,

$$\hat{Y} \pm t\ (\ \nu,\ 1 - \tfrac{1}{2}\alpha)\ \ .\ s\sqrt{\underline{X}_0'\ \underline{C}\ \underline{X}_0} \tag{52}$$

where ν is the number of degrees of freedom upon which s is based.

FIGURE 3

AVALANCHE ACTIVITY PREDICTION PROFILES* FOR 1965-73

based on the single model

$$\widehat{AVAL} = \cdot 0947 AVAL1 + \cdot 0589 AVAL2 + 6 \cdot 85 SWH - \cdot 937 SWH1 + \cdot 131 W.E*TMI - \cdot 035 W.E*TMI2$$

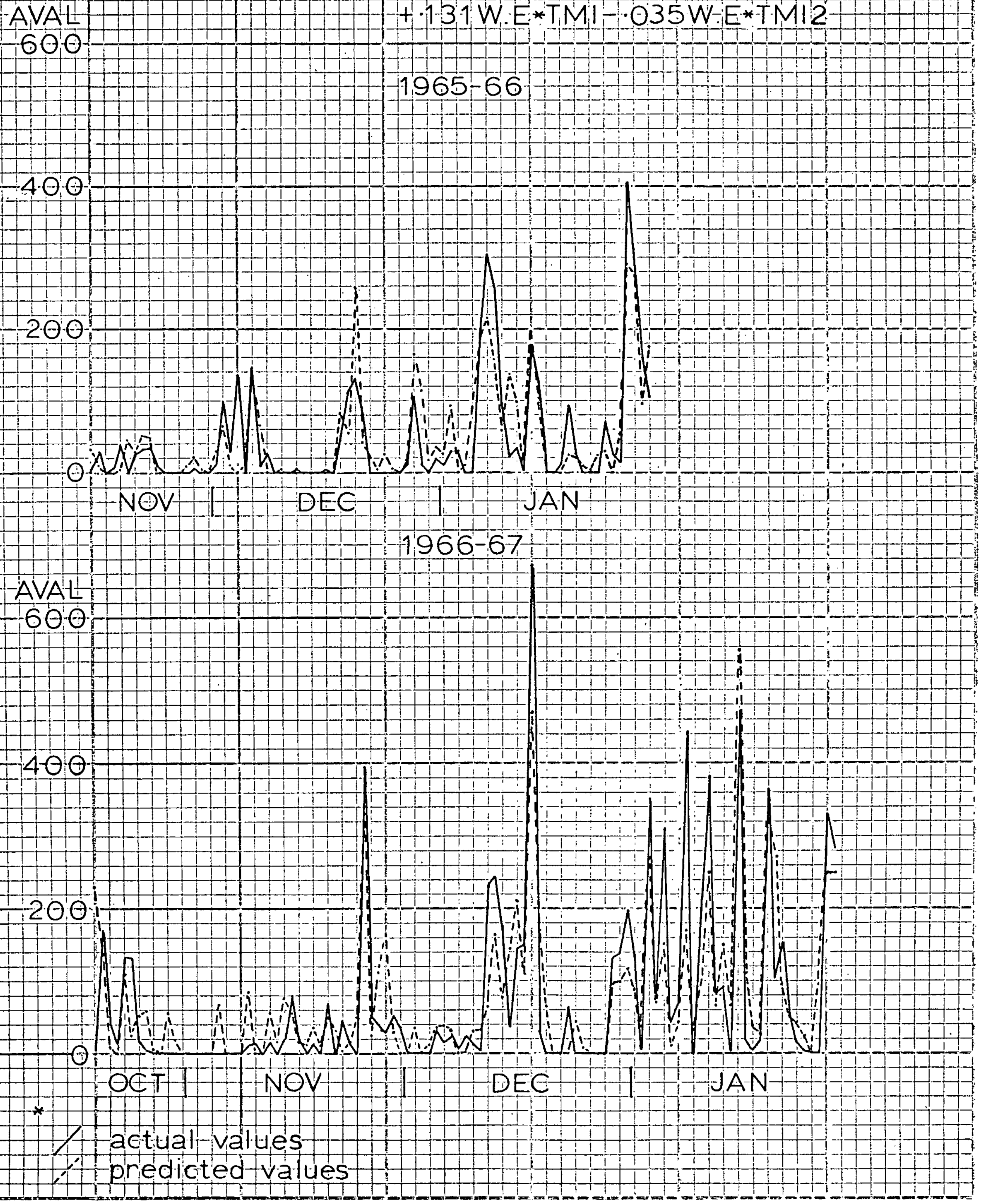


FIGURE 3 continued

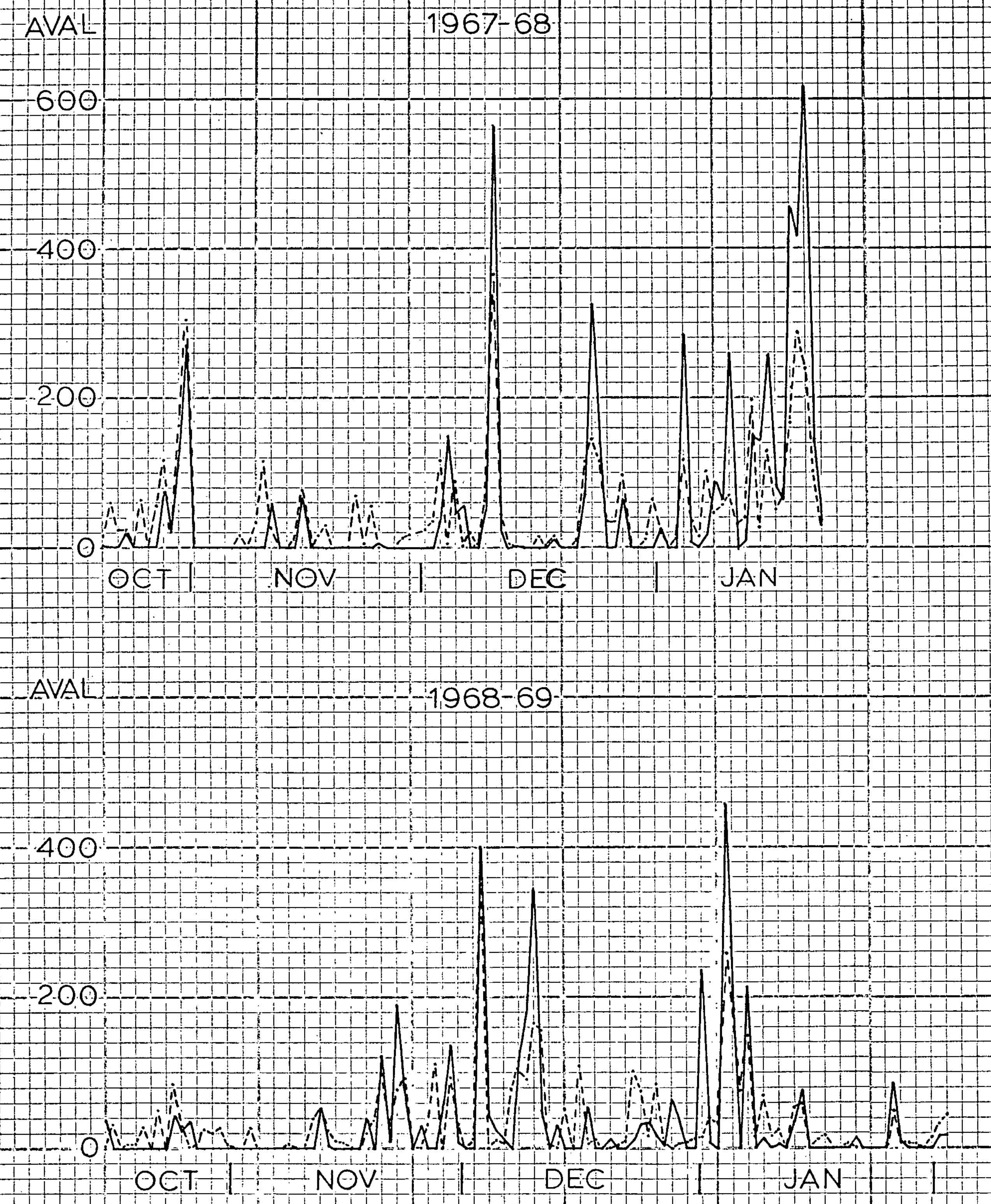
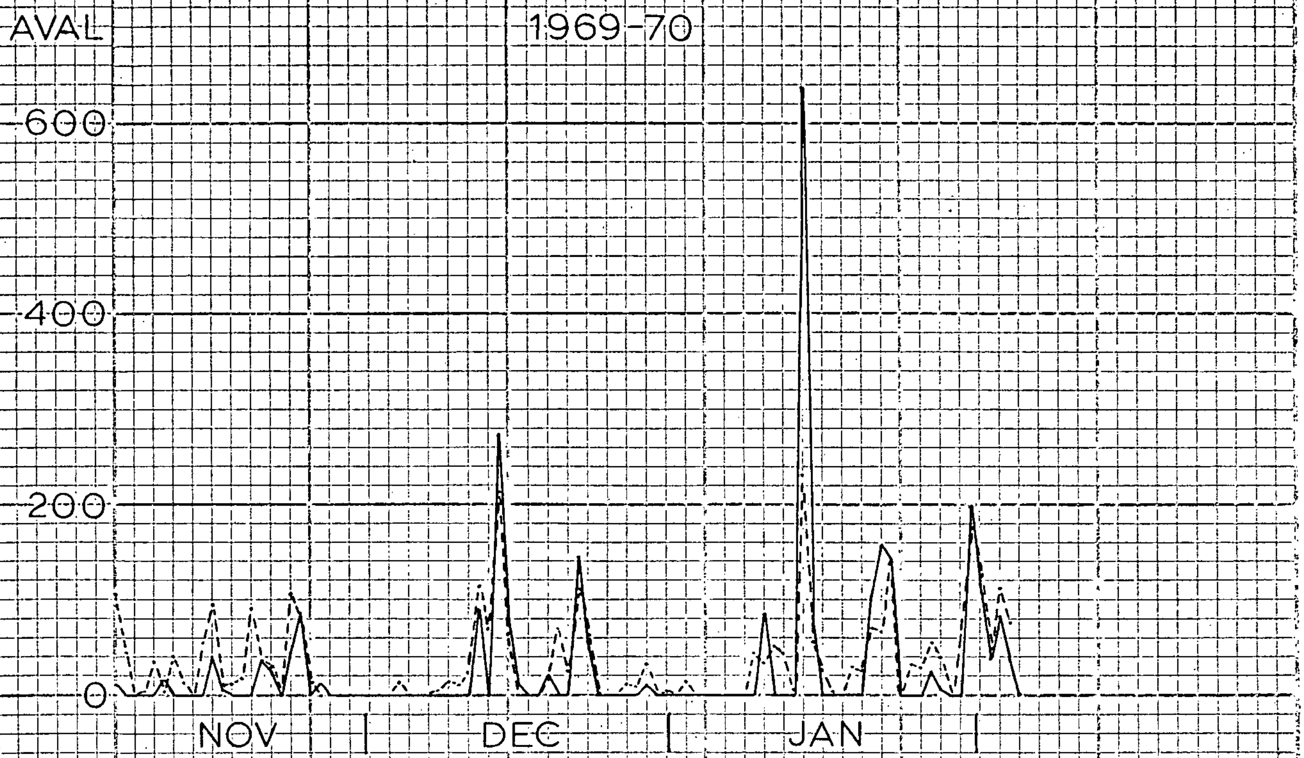


FIGURE 3 continued



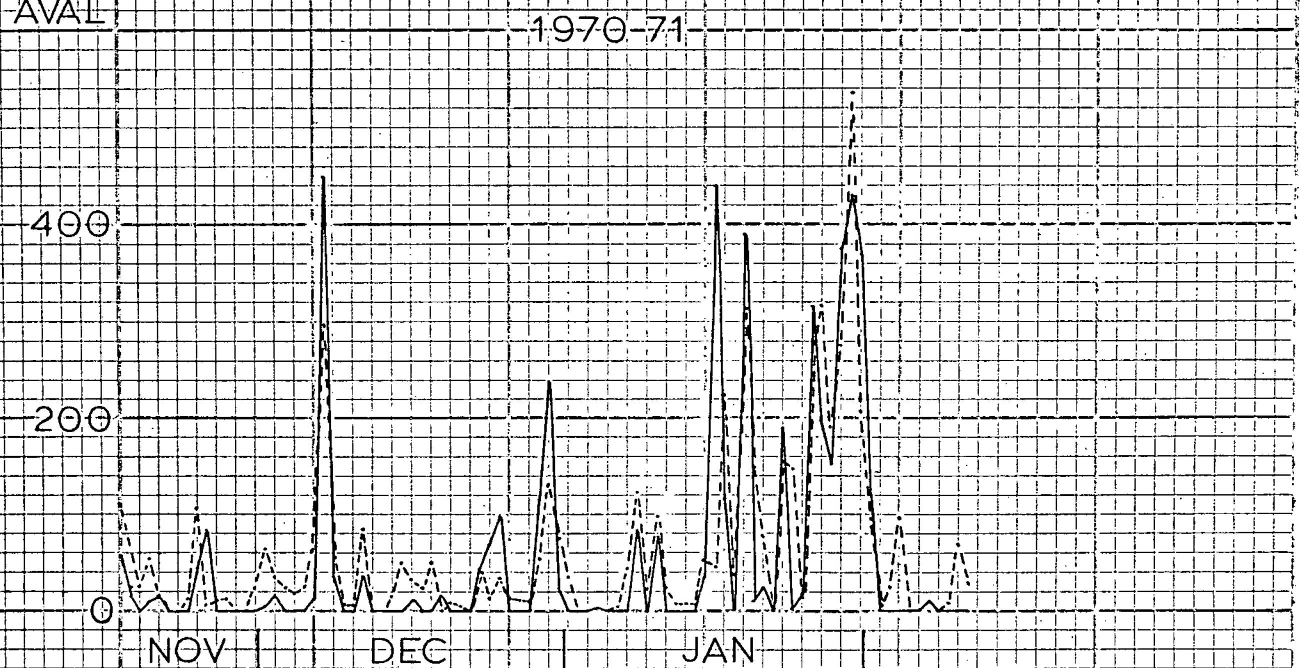
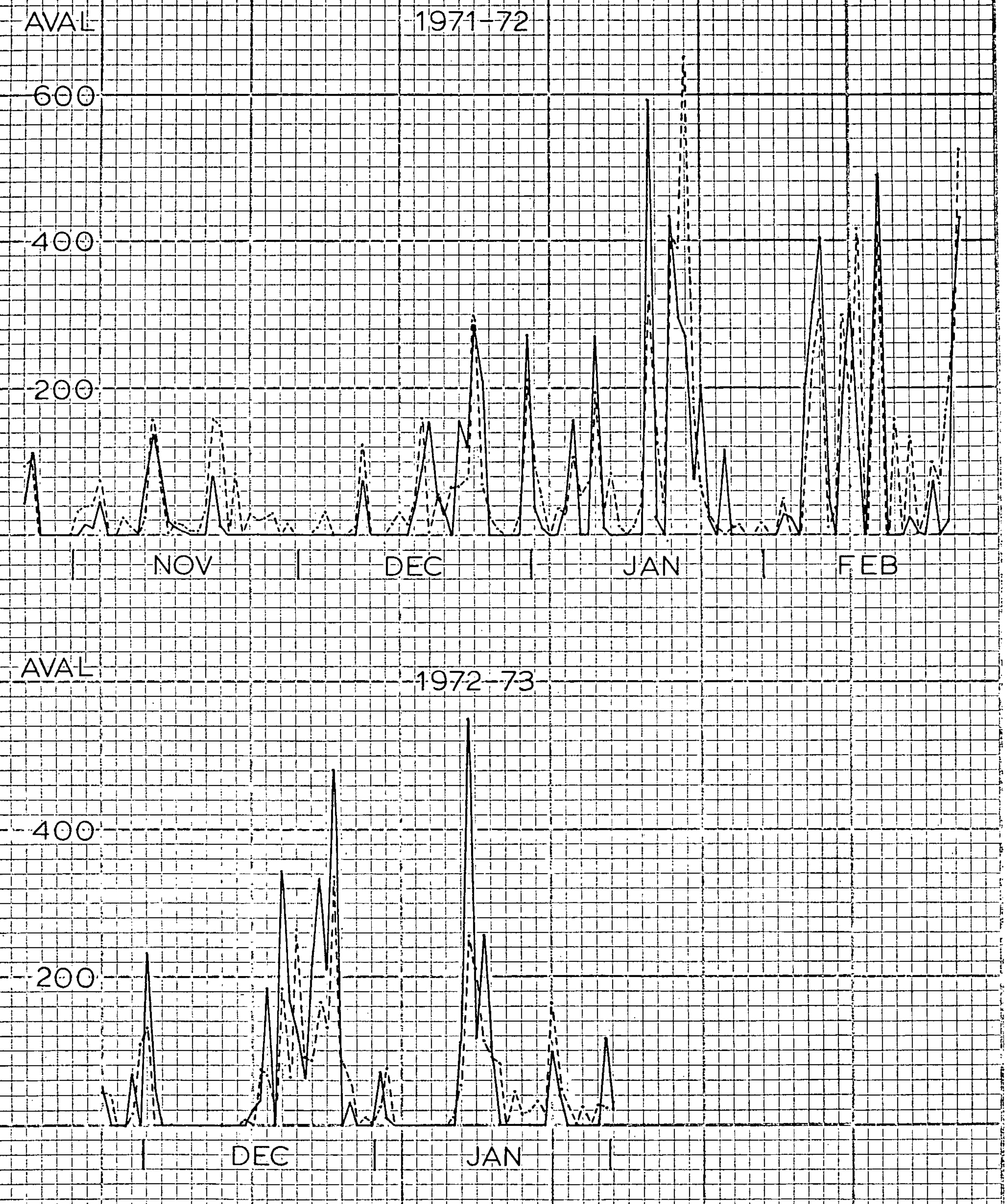


FIGURE 3 continued



For g future observations, confidence limits on the predicted values can be obtained from,

$$\hat{Y} \pm t\,(\,\nu,\ 1 - \tfrac{1}{2}\,\alpha)\ .\ s\,\sqrt{1/g + \underline{X}_0'\ \underline{C}\ \underline{X}_0} \tag{53}$$

(Draper and Smith, 1966:122).

Hence, estimates can be obtained of the reliability of the predictive model to forecast future values of the dependent variable. Assuming that the sample of observations from which the model was developed was large and truly representative of the population of past and future observations of avalanche activity and weather factors, the 95% confidence limits for a single future forecast are,

$$\hat{Y} \pm (1.96)\ .\ s\,\sqrt{1 + \underline{X}_0'\ \underline{C}\ \underline{X}_0}, \tag{54}$$

which has minimum limits of

$$\hat{Y} \pm (1.96)\ .\ s \tag{55}$$

at the mean values of all $\underline{X}_0$.

Thus, assuming complete knowledge of a set of future $\underline{X}_0$ values, that is a perfect weather forecast, a future forecast of avalanche activity will have a 95% probability of lying between the limits, $\hat{Y} \pm (1.96)\ .\ s$, if the set of $\underline{X}_0$ are mean values. These limits will of course become wider as the set of $\underline{X}_0$ departs from its mean values, as described by expression (54).

The standard error of estimate after fitting the SWH,W.E*TMI model for the period, 1965-73, was 61.3 (see page 99). Hence, 95% probability limits for future forecasts of avalanche activity for weather conditions which are not extreme, will be of the order of $\pm$ 120.

Simulated Forecast

The computation of confidence limits certainly provides a measure from which the forecasting capabilities of a model can be assessed. However, a more direct assessment may be obtained by dividing the data into two sets, one of which can be used for model development and the other for model testing in a simulated forecasting situation. This procedure is only satisfactory if the data can, in fact, be divided into two samples, which are each representative of the same population.

Using avalanche activity data based on (12,1,1) SML weights, all sites and artificial and natural avalanche events, together with Rogers Pass meteorological data, the following SWH,W.E*TMI model was developed for the four year period, 1965-69,

$$\widehat{AVAL} = 0.1341\ AVAL1 + 0.0543\ AVAL2 + 8.585\ SWH - 1.273\ SWH1 + 0.0700\ W.E{*}TMI,$$

which has an R^2- value of 0.683 and a standard error of estimate of 57.8. This model is quite similar to that developed for the entire period, 1965-73, described on pages 85 to 99, except that it has an even higher R^2-value. Thus, predictions based on the 1965-69 model for the period, 1965-69 would be even better than those depicted in Figure 3.

If the model developed for the period, 1965-69 is now applied to the four year period, 1969-73, in a simulated forecasting situation, some interesting results are obtained. As expected, the R^2- value for the forecasts is decreased somewhat from 0.683 to 0.579, but the forecasts are, nevertheless, still quite accurate. The standard error of estimate for these simulated forecasts is 68.8, indicating that predictions based on the 1965-69 model for the period, 1969-73, would not be much less accurate than those depicted in Figure 3, for the period 1969-73, using the 1965-73 model, which has a standard error of estimate of 61.3.

Hence, the two four-year samples for the periods, 1965-69 and 1969-73, are indeed quite similar. In fact, standard deviations for avalanche activity are 102 and 106 respectively, (SD for the period, 1965-73 is 103), and values range up to 672 and 637 in each case. Thus, in a real forecasting situation, the model developed for the period, 1965-69, could have been applied during the period, 1969-73, with considerable success, assuming that weather forecasts were sufficiently accurate.

Decomposition of Avalanche Activity

Since these SWH,W.E*TMI models are capable of producing reliable forecasts of avalanche activity in terms of the (12,1,1) SML activity index, it would be highly advantageous if such forecasts could be decomposed into forecasts of individual site activities. In order to facilitate this decomposition, a technique has been devised based on probability considerations. After dividing the range of avalanche activity into forty levels, the number of occurrences for each particular site at each level was computed, for the eight year period. These figures were then divided by the total number of times that level was realized, to obtain probabilities of occurrence for each site at each level. Hence, sites can be arranged in order of probability of occurrence for each level and tabulated. Such a table can be updated as more data becomes available. In an actual forecasting situation, the model provides a forecast of the activity level, after which forecasts of individual site activity can be obtained from this table. Of course, some very active sites will have high probabilities at almost every level, hence, this approach should be supplemented with a certain element of interpretative experience. For example, if a site has avalanched recently, then its probability may be somewhat diminished. Avalanche activity decomposition tables should be computed for each type of avalanche activity weighting scheme used.

Domain Analysis

Finally, a new concept is under investigation, whereby the time series techniques, which have been discussed, can be employed in the development of more accurate models based on observations which are unequally spaced in time. During periods of high precipitation and consequently high avalanche activity, observations should be, and often are more frequent. On the other hand, periods of low activity may result in more widely spaced observations. Therefore, if the data is transformed from the time domain into the snow domain, for example, in which observations are separated by equal snowfall increments, the theory can still be applied and the observations exploited to the greatest advantage. A domain transformation routine has been incorporated into the main analytical computer program and initial results seem promising. However, in order to obtain a significant improvement over the normal time series models, it will be necessary to make more frequent meteorological measurements and avalanche observations during storm periods, than have been made in the past.

Chapter VIII

CONCLUSIONS

It has been shown that avalanche activity, for the Rogers Pass area, expressed in terms of the avalanche activity index, can be accurately described in terms of certain composite meteorological variables, in the form of linear transfer function and stochastic time series processes. These composite meteorological variables are SWH (the product of snow accumulation, water equivalent and humidity) and SWHT (the product of snow accumulation, water equivalent, humidity and minimum air temperature). SWH and SWHT can be regarded as the most significant meteorological terms to evolve from this study in their relationship with avalanche activity for the Rogers Pass area of British Columbia.

The possible physical reasons for the presence of water equivalent of new snow, snow accumulation, humidity and minimum air temperature in these composite terms was discussed in some detail.

Water equivalent of new snow, the best of the simple meteorological variables, was felt to be more important than depth of new snow, according to the following reasoning. Water equivalent, the product of new snow depth and the density of the new snow, is a more direct measure of slope loading or 'shear weight' application than new snow depth alone. Furthermore, density contains a temperature effect, a high density often being related to high temperatures and the presence of free water. Finally, high densities may be an indication of developing 'slab' conditions in the upper zones.

The importance of the snow accumulation term, as a major factor modifying water equivalent, was thought to be the result of the greater participation, in the avalanching, of the deepening snowpack, presumably as a consequence of an increase in the available amount of avalanchable snow. There may also be a delayed action effect, due to snow accumulation, associated with the formation of 'soft slab' conditions. A further possible effect may be related to the influence of settlement rates on snow accumulation values, a high value indicating less settlement and hence, greater instability.

Relative humidity is most probably directly associated with 'soft slab' formation, which is thought to be the result of the condensation of atmospheric water vapour onto snow crystals or crystal fragments, brought together by moderate winds, and their subsequent cementing together. The appearance of minimum air temperature as a minor factor modifying the SWH term is probably also related to 'soft slab' conditions. Such conditions are a frequent cause of major avalanching at the Rogers Pass.

The three best avalanche activity weighting schemes were found to be the (12,1,1) SML, (12,3,1) SML and (1,1,1) ML in that order, indicating that terminus is a better measure than size and, that small avalanches are not statistically important.

Models were developed for individual years and for the entire period, 1965-73, of the study. Although the models obtained for individual years have somewhat higher R^2- values than those developed for the total period, in an actual forecasting situation, it would be difficult to know which one to apply. Variations in precipitation, temperature, and wind patterns lead to a different model for each year. Similarities do exist between some years, but it will be necessary to examine data for further years, before such differences and similarities can be thoroughly evaluated. Data for 1973-74 and 1974-75 will be analysed as soon as it becomes available.

A simulated forecast for the period, 1969-73, using a model developed from 1965-69 data produced accurate results. The models obtained for the total period, 1965-73, indicate a high degree of forecasting precision. These models can be directly applied to future winters, since they represent a type of averaging over eight seasons. If a future winter fits into this class of eight, accurate forecasts whould be possible. It should be noted however, that, in spite of the lagged terms in the models, the weather forecast is still and always will be an essential feature of the avalanche forecasting process. It is hoped that, ultimately, more reliable mountain weather forecasts will become available for the Rogers Pass area. Perhaps it will be possible to establish a more local weather forecasting system. As LaChapelle (1970:108) states, " the mountain weather forecast problem is an important one to solve, for many administrative decisions are based on the short-term hazard forecast."

These models can be improved, not only as further data becomes available, but also if more frequent measurements are made, particularly during storm periods. Avalanche events should be recorded as precisely as possible, for such records are undoubtedly the most important limiting factor in the development of accurate models. Wind measurements may be improved, perhaps by the establishment of another remote wind station, possibly on the north side of the Pass. Since humidity appears to be an important meteorological parameter in the formation of slab conditions, such measurements should be emphasized and more carefully monitored.

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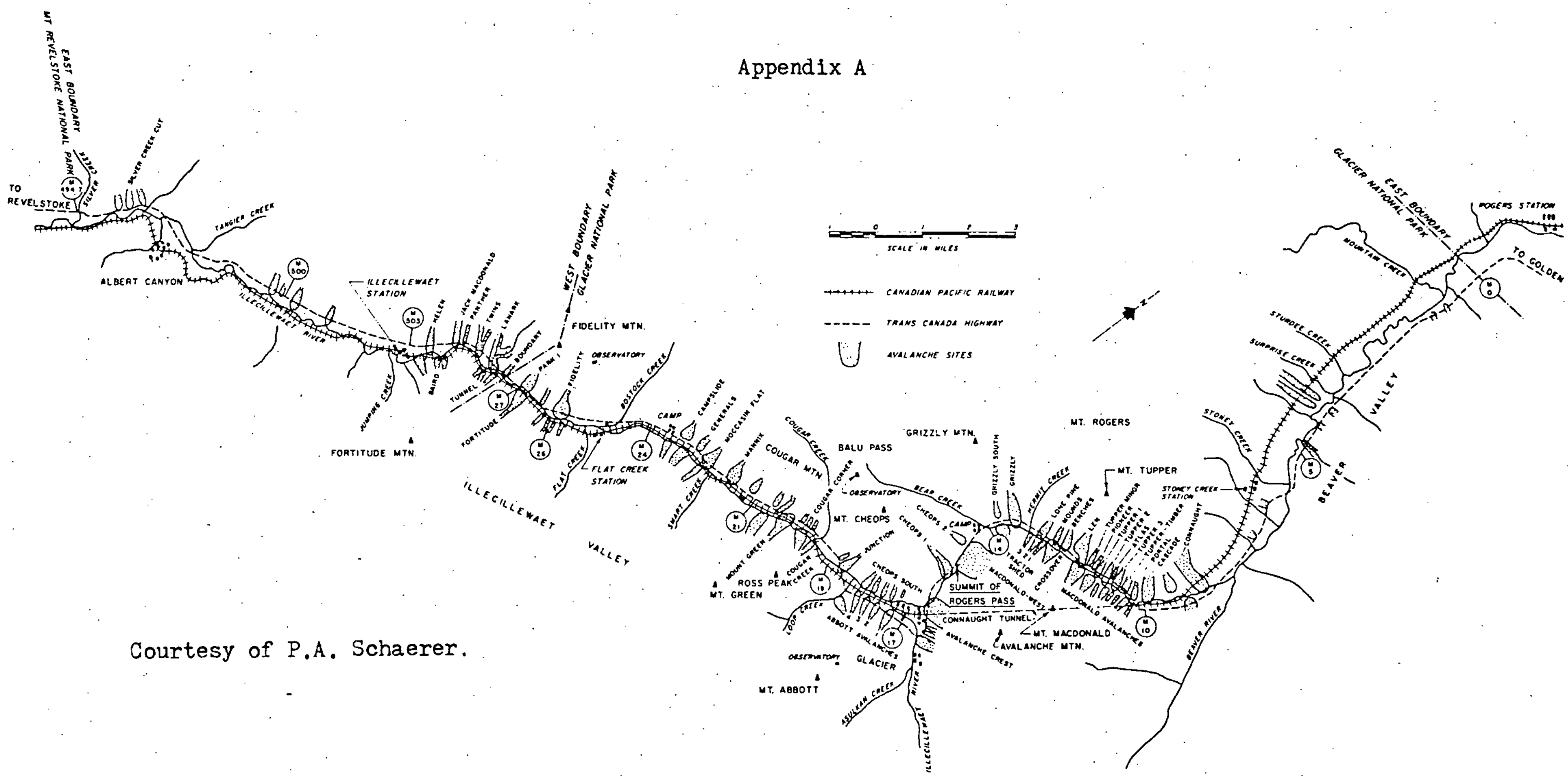
Appendix A



Courtesy of P.A. Schaerer.

AVALANCHE SITES ON THE TRANS CANADA HIGHWAY AT ROGERS PASS

Appendix B

THE AVALANCHE SITES

Name of Slide (1968)	Code	Mileage (from east)	Site Weight
Heather Hill	1	1.13	2
Water Tank Heather Hill	1a	1.45	2
Beaver East	2	4.48	2
Beaver West	3	4.68	2
Diverting Dam	4	4.90	2
Connaught	5*	9.00	8
Unnamed	6*	9.58	5
Stone Arch	7*	9.88	4
Portal South	7a**	10.07	2
MacDonald Gully No. 1	8**	10.18	2
MacDonald Gully No. 2	9**	10.23	2
Portal	10*	10.30	3
MacDonald Gully No. 3	11**	10.38	7
Tupper Timber	12*	10.50	2
MacDonald Gully No. 4	13**	10.68	6
MacDonald Gully No. 5	14**	10.78	4
Tupper No. 3	15*	10.80	3
MacDonald Gully No. 6	16**	10.88	6
Atlas	17*	10.90	3
MacDonald Gully No. 7	18**	10.98	2
Tupper No. 2	19*	11.05	10
MacDonald Gully No. 8	20**	11.18	3
Tupper No. 1	21*	11.30	5
Pioneer	22*	11.40	1
MacDonald Gully No. 9	23**	11.43	3
Tupper Cliffs	24*	11.50	1
Tupper Minor	25*	11.60	1
MacDonald Gully No. 10	26**	11.63	3
Lens	27*	11.70	17

MacDonald Gully No. 11	28**	11.75	3
MacDonald Gully No. 12	29**	11.83	3
Benches Unconfined	30*	12.00	2
Double Bench	31*	12.10	4
Single Bench	32*	12.30	8
Crossover	33**	12.35	14
Mounds	34*	12.40	3
Tractor Shed East	35	12.60	3
Lone Pine	36*	12.70	8
Tractor Shed West	37	12.80	2
Tractor Shed No. 3	38	13.30	3
Grizzly	39	13.40	6
Grizzly West	40	13.80	3
MacDonald West Shoulder No. 1	41	14.38	6
MacDonald West Shoulder No. 2	42	14.48	8
Cheops No. 2	43	14.50	3
MacDonald West Shoulder No.3	44	14.53	6
MacDonald West Shoulder No. 4	45	14.68	8
Cheops No. 1	46	15.20	8
Avalanche Crest No. 1	47	15.97	6
Avalanche Crest No. 2	48	16.08	6
Avalanche Crest No. 3	49	16.38	3
Avalanche Crest No. 4	50	16.53	6
Abbott Observatory	51	17.30	3
Abbott No. 1	52	17.55	3
Abbott No.2	53	17.63	3
Abbott No. 3	54	17.68	3
Abbott No. 4	55	17.75	3
Junction East	56	18.70	6
Junction West	57	19.00	3
Cougar Creek East	58	19.33	8
Cougar Creek West	59	19.73	8
Cougar Corner No. 4	60	19.78	1
Cougar Corner No. 3	61	19.85	1
Cougar Corner No. 2	62	19.93	1
Unnamed Cougar Corner No. 3	63	20.02	3

Unnamed Cougar Corner No. 2	64	20.08	6
Cougar Corner Kitten	65	20.23	5
Ross Peak	66	20.28	14
Cougar Corner No. 1	67	20.47	3
Unnamed Cougar Corner No. 1	68	20.68	3
Gunners No. 3	68a	20.90	6
R.R. Gunners	69	21.05	11
Gunners East	70	21.15	3
Gunners West	71	21.35	3
Unnamed Gunners	72	21.70	3
Mannix	73	21.90	6
Mannix West	74	22.20	3
Moccasin Flats	75	22.50	8
Generals	76	22.80	2
Smart Slide	77	23.20	4
Camp West	78	23.20	8
Fidelity	79	26.00	6
Park One	80	26.90	6
Fortitude	81	26.90	8
Boundary	82	27.40	1
Laurie	83	27.56	11
Lanark	84	27.63	11
Twins	85	27.88	12
Nellie's Jack MacDonald	87	28.75	3
Baird	87a	29.21	6
Downie No. 3	90	32.95	2

* Designates the Tupper Gullies as used in Chapter IV

** Designates the MacDonald Gullies as used in Chapter IV.

Appendix C

STORM PERIODS

Start	Finish	Start	Finish
27/11/65	08/12/65	19/01/70	04/02/70
18/12/65	14/01/66	14/02/70	19/02/70
18/01/66	19/02/66	04/03/70	09/03/70
06/03/66	20/03/66	21/03/70	24/03/70
16/10/66	25/10/66	04/04/70	10/04/70
12/11/66	21/12/66	30/11/70	08/12/70
29/12/66	21/02/67	23/12/70	01/01/71
06/03/67	27/03/67	06/01/71	01/02/71
20/10/67	31/10/67	09/02/71	16/02/71
01/12/67	11/12/67	22/02/71	27/02/71
21/12/67	20/01/68	07/03/71	12/03/71
29/01/68	07/02/68	23/03/71	03/04/71
18/02/68	24/02/68	13/12/71	25/12/71
12/03/68	19/03/68	30/12/71	26/01/72
26/03/68	30/03/68	06/02/72	10/03/72
18/11/68	23/11/68	04/04/72	09/04/72
26/11/68	18/12/68	23/11/72	02/12/72
21/12/68	17/01/69	13/12/72	05/01/73
31/01/69	13/02/69	11/01/73	25/01/73
15/03/69	23/03/69	30/01/73	05/02/73
08/12/69	15/12/69	10/02/73	22/02/73
19/12/69	24/12/69	09/03/73	23/03/73
09/01/70	15/01/70		