CONSTRUCTION, THEORY, AND APPLICATION OF NON-LINEAR TITANATE CONDENSERS

by

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DEPARTMENT OF PHYSICS

September 15, 1950.

Dr. L. W. Dunlap,
Librarian,
University of British Columbia.

Dear Dr. Dunlap:

This letter will certify that the thesis of Mr. John Peter Hobson has been carefully studied by the undersigned, and that the thesis meets the required standards and an abstract has been approved by the Department.

Yours sincerely,

A. M. Crooker
Acting-Head of the Department

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Associate Professor of Physics

AMC:lc

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18/50

ABSTRACT

A method for the making of non linear barium titanate condensers for audio frequencies is described. Preliminary measurements on these condensers are given.

An idealized theory for the behaviour of the non linear condensers in a carrier amplifier circuit is developed. A carrier amplifier built on this principle is described. A theoretically possible power amplification for this amplifier of 180 is derived. Experimental results obtained with the carrier amplifier are given. A power amplification of 70 was obtained.

Conclusions on the possible applications of non linear condensers are drawn.

ACKNOWLEDGEMENTS

The author is indebted to the National Research Council of Canada from whom he received a bursary for the year 1949-50.

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The author thanks Dr. A. Van der Ziel and Dr. A.J. Dekker of the Physics Department of the University of British Columbia for their kindness and assistance during the work.

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CONSTRUCTION, THEORY AND APPLICATION OF NON LINEAR TITANATE CONDENSERS

CHAPTER I

INTRODUCTION

A non linear circuit element is one whose inductance, resistance, transconductance or capacitance is not a constant but depends on the voltage applied. Such elements are all mixers, i.e. if two voltages are applied to them then certain currents flow which depend simultaneously on both voltages. This property of non linear circuit elements is widely used.

The magnetic amplifier which uses a non linear inductance has been discussed theoretically and experimentally in many papers (1,2,3,4).

The crystal rectifier or non linear resistance has been widely used and thoroughly investigated as a high frequency mixer (5,6).

Note: The references are typical but not complete.

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1. Lamm, A.U. "The Transductor, D.C. presaturated Reactor". Stockholm Esselte Aktiebolag 1943.

2. Boyajian, A. "Theory of D-C Excited Iron Core Reactors and Regulators". A.I.E.E.Trans, Vol.43, p.919, June 1924. Chicago,Ill.

3. Castellini, R.R. "The Magnetic Amplifier", Proc.I.R.E., Vol.38, No 2, pp 151-158. Feb. 1950 New York.

4. Greene, W.E. "Applications of Magnetic Amplifiers". Electronics, Sept. 1947.

5. Herold, E.W. "Frequency Mixing in Diodes". Proc.I.R.E., Vol.31, No 10, p 575, Oct.1943, New York.

6. Torrey, H.C. and Whitmer, C.A. "Crystal Rectifiers". M.I.T. Radiation Laboratory Series. McGraw-Hill, New York, 1948.

The non linear transconductance is universally used as a modulator.

The non linear capacitance has not however had wide use or theoretical treatment because non linear condensers with marked non linear properties have until recently been impossible to make. But recently it has been found that some of the compounds of titanium, notably barium titanate ($BaTiO_3$) are ferroelectric at room temperature, i.e. a graph of charge against voltage for a suitable condenser having $BaTiO_3$ as dielectric is not a straight line but is a hysteresis loop, similar in shape to a hysteresis loop of a ferromagnetic material. A condenser of this type is called a non linear condenser - its capacity is a function of the voltage across it.

The physical theory for the marked dielectric behaviour of some of the compounds of titanium has received much recent attention (7,8,9,10).

The research described in this thesis is on the use of non linear condensers with $BaTiO_3$ dielectrics, in circuits.

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7. Von Hippel, A., Breckenridge, R.G., Chesley, F.G., and Laszlo Tisza. "High Dielectric Constant Ceramics". Ind. and Eng. Chem. Vol.38. No 11. pp 1097-1109, Nov. 1946, Easton, Pa.

8. Jonker, G.H. and Van Santen, J.H. "Properties of Barium Titanate in Connection with its Crystal Structure". Science, Vol.109, No 2843, pp 632-635, June 1949.

9. Kay, H.F. and Rhodes, R.G. "Barium Titanate Crystals". Nat. Vol. 160 p 126. July 1947. London.

10. Wul, B.N. and Goldman, I.M., "Dielectric Constant of Barium Titanate as a Function of Strength of an Alternating Field," Compt. Rend. Acad. Sci. Vol.49, pp 177-180, Oct. 1945

Dr. A. Van der Ziel, the director of the work, has applied general mixer theory to non linear condensers in an audio carrier amplifier circuit(11) and in a high frequency (10 megacycles) mixer circuit (12). Very little published experimental work on non linear condensers in circuits was found. Donley (13) did some qualitative experiments on non linear condensers, with $BaTiO_3$ and $SrTiO_3$ dielectrics, of less than 100 mmfds in a frequency tripling circuit, a mixer at 20 megacycles, and a frequency modulator at 40 megacycles, but his results could not be used to check Van der Ziel's theory.

The primary object of this research was to build the circuits theoretically analysed by Van der Ziel in order to check experimentally his analysis in detail, and to revise it if necessary.

The secondary purpose was to evaluate the usefulness of non linear condensers in these and other circuits.

There follows a brief summary of the work done:

Non linear condensers suitable for use in audio frequency circuits were developed. First results showed that the condensers made were more suitable for experimental measurements in a slightly different amplifying circuit to that analysed by Van der Ziel. An

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11. Van der Ziel, A. Report to Defense Research Board of Canada. University of British Columbia. Jan. 1949.

12. Van der Ziel, A. "On the Mixing Properties of Non Linear Condensers", Jour. App. Phys. Vol.19, No 11, pp 999-1006, Nov. 1948. Lancaster, Pa.

13. Donley, H.L., "Effect of Field Strength on Dielectric Properties of Barium Strontium Titanate"., R.C.A. Rev. Vol. VIII, No 3, pp 539-553, Princeton, New Jersey, Sept. 1947.

adaption of his theory was made and the conclusions of the theory thus changed have been checked experimentally with good but not complete agreement. An account of the main audio frequency properties of these non linear condensers is now possible.

No measurements have been done on the high frequency mixer, although the circuit was roughly built, but the results of the work at low frequencies should greatly assist work at high frequencies.

CHAPTER II

MAKING OF NON LINEAR CONDENSERS FOR AUDIO FREQUENCIES

At the beginning of the work it was known that $BaTiO_3$ properly prepared has a marked hysteresis loop at 4800 volts per cm. (14) The problem was to use this knowledge to make a condenser of capacity large enough for audio circuits (.001 to .01 mfds), showing marked non linearity at voltages not exceeding 300v A.C., having a breakdown voltage at least above the voltage at which non linearity was marked, and having some means of removing the heat generated by hysteresis losses.

The preparation developed for condensers satisfying these requirements has three main stages:

1. Preparation of a Sample of Bulk Dielectric

The method used was very similar to that used by Von

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14. See Ref. 7.

Hippel and his co-workers (15) with a few minor changes.

Barium titanate in dry powder form (obtained from Titanium Alloy Manufacturing Co.) was pressed in a 1/2" diam. press at 60000 lbs/sq.in. to a thickness of about .5 mm. The discs were then placed on platinum foil in an alundum crucible and passed through the following temperature cycle in a high temperature furnace:

An increase of 100°C per hour for nearly 13-1/2 hours to a temperature of 1350°C.

A constant temperature of 1350°C for 6 hours.

A decrease in temperature at the cooling rate of the furnace. (The furnace took 36 hours to cool from 1350°C to room temperature).

After the sintering cycle the dielectric was a hard yellow brown, brittle disc which had shrunk about 20% of its original size. The furnace used (diag.1) was made at the beginning of the work and was controlled with a Wheelco Chronatrol Potentiotrol Model 23241, which controlled the power through a Superior Electric Co. Powerstat No 1156. A disc which may be cut for any 24 hour temperature cycle is the master control for the Potentiotrol which controls the temperature inside the furnace to about 5°C. It was found necessary to plot the relation between temperature at the sample and that given by the furnace thermocouple since these are not physically at the same place. A block diagram of the furnace and control is given in diagram 2.

2. Grinding the dielectric to 0.1 mm.

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15. See Ref. 7.

PLATE I

Facing Page 5

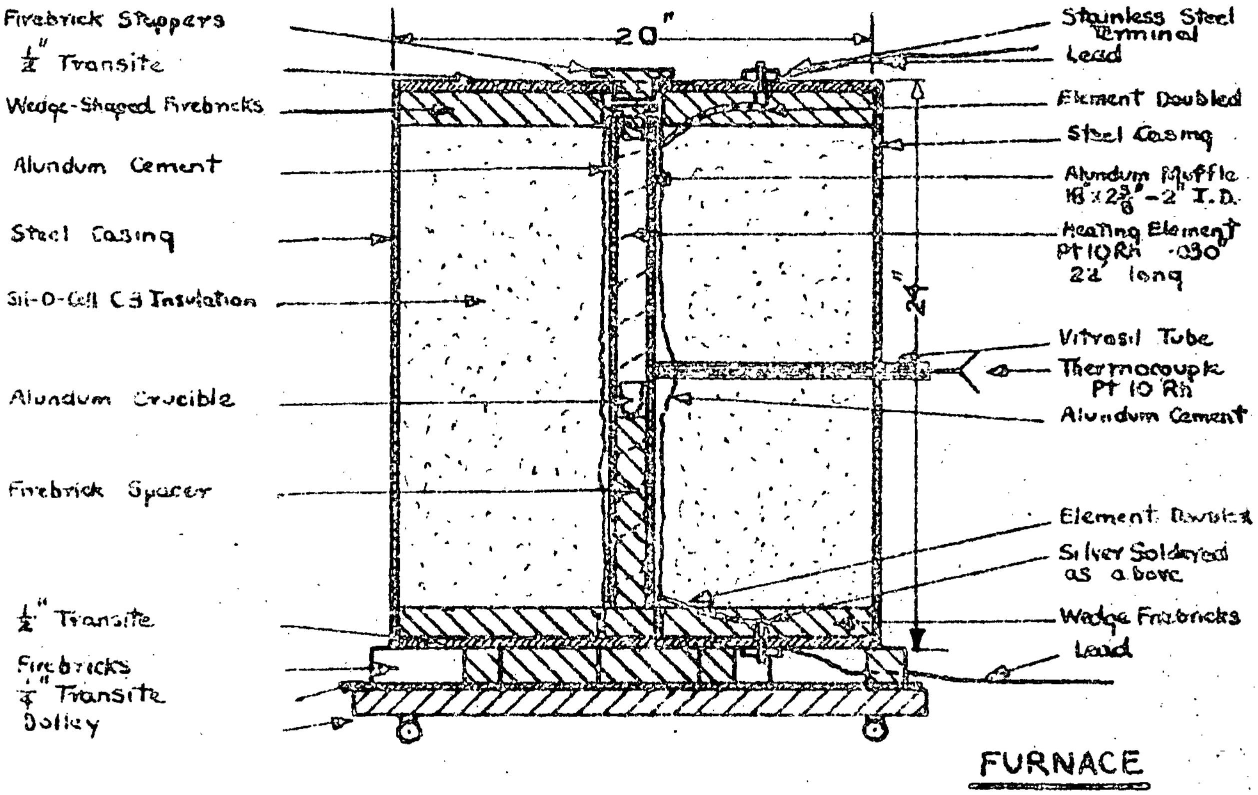

Diagram 1

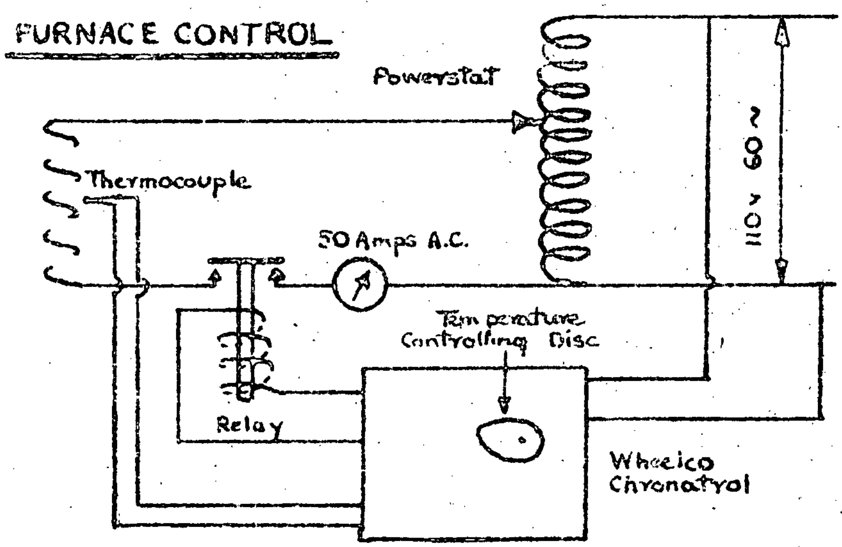

Diagram 2

To reduce the voltage necessary for marked non-linearity, to reduce the hysteresis losses, and to increase the capacity, as thin a dielectric as possible is required.

The technique used was borrowed from geologists who used it to prepare thin rock samples.

One side of the sintered disc was ground flat on a glass plate with carborundum and water and then polished on another glass plate with aluminum oxide and water. A thin film of beeswax was melted on the flat side of the disc and allowed to harden. Grinding of the flat side was repeated. Particles of carborundum became embedded in the wax which was forced into tiny depressions on the surface of the disc. Grinding was continued until all wax except that in the depressions had been removed, final removal of the wax being done on aluminum oxide. A coat of Dupont Conductive Coating No 4351 silver electrode paint was applied to the whole flat surface and allowed to dry. The disc was heated to 600°C, held there for 10 minutes, and allowed to cool. This operation fired the electrode to the dielectric. The wax technique was introduced to plug tiny holes in the dielectric to prevent flow of wet silver paint into them.

The disc was next mounted silvered side down with warm Canada balsam on a 1" x 2" glass microscope slide and the dielectric ground down and polished until thickness was .1mm. The waxing procedure was repeated and the upper electrode applied to the disc while still on the slide. This electrode was applied in 6 triangles (diag.3) from which 6 condensers were made. This made a semi-variable condenser from the whole disc and permitted a broken down section to be removed.

After the silver paint was dry the disc was carefully removed by heating from its mount, cooled, and dipped in benzene to remove the Canada balsam and the upper electrode fired as before.

Mechanically the disc could have been ground to .03 mm. and still handled without breakage but it was found that if the discs were thinner than .1 mm. electrical breakdown had either occurred before any voltage was applied or did occur at low voltages for a large percentage of the condensers made. This breakdown was attributed to tiny holes and flaws in the dielectric into which the electrode paint flowed or which in some other way caused breakdown. The waxing technique reduced the limiting thickness from .2 mm. to .1 mm.

In an effort to further reduce the limiting thickness electrodes of tin foil and gold were tried without success. A search of the literature yielded a paper by Howatt and co-workers (16) on the fabrication of thin ceramic sheets for capacitors. In this paper a method was given for making sheets of minimum thickness .15 mm. having a breakdown voltage for $BaTiO_3$ of about four times that found in the above condensers. Their method included in the original mix before sintering bonding materials to remove flaws in the final product. Their method however was complex and required skill. It was decided therefore to obtain whatever results could be obtained with the present technique before combining the two

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16. Howatt, G.N., Breckenridge, R.G., and Brownlow, J.M., "Fabrication of Thin Ceramic Sheets for Capacitors", Jour. Am. Cer. Soc., Vol.30, pp 237-242, 1947.

PLATE II

CONDENSER ELECTRODE

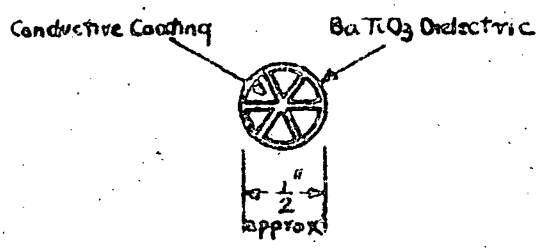


Diagram 3

CONDENSER MOUNT

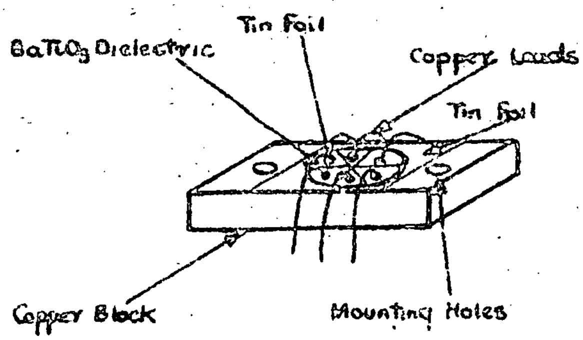


Diagram 4

CONDENSER STAND

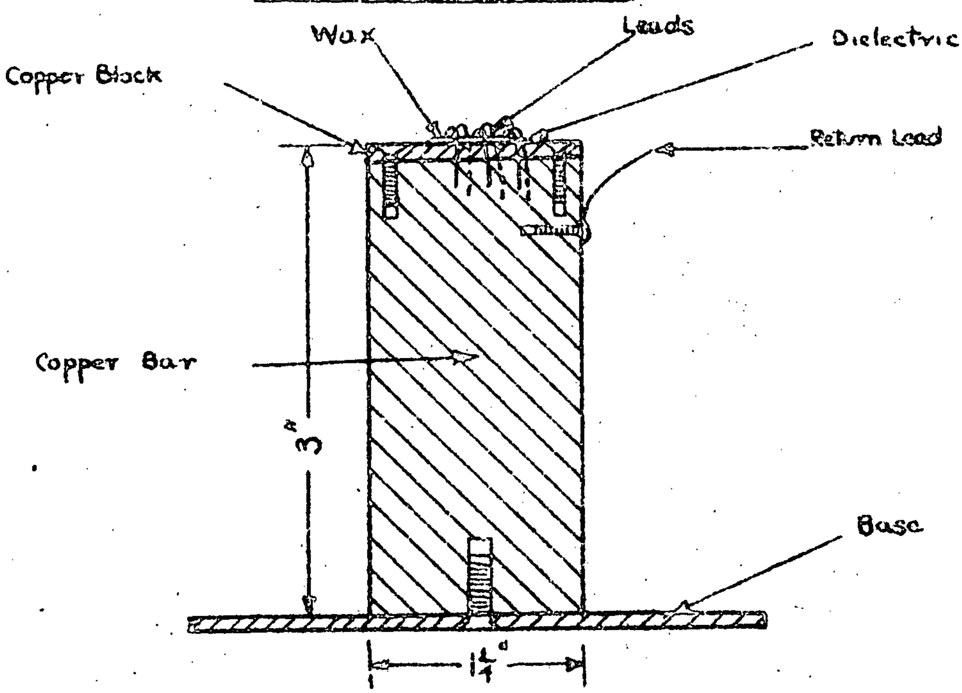


Diagram 5

techniques. The combination it is thought would realize the mechanical limit of .03 mm. for the above mentioned discs.

Another method tried to plug the holes was to separate by the thin disc without electrodes two liquids which formed a precipitate as they met in the holes of the dielectric. This method was used successfully with $CaCO_3$ but its breakdown voltage is not sufficiently high above that obtained by the waxing method. It is thought that this method could be developed if a material with a high breakdown value and which could be easily precipitated could be found.

3. Attachment of Leads and mounting of Condensers.

The condensers were mounted on a small copper block 1-1/8" x 1/2" x 3/16" with two countersunk holes 7/8" apart. A sheet of tin foil was placed on the block and the disc, the oxide on the silver coat having been removed with emery, placed on the tin foil with the divided electrode side up. A thin copper wire with a little tin foil wrapped on the end was placed on each upper electrode. The whole was heated until the tin melted and sealed the disc to the block and a lead to each upper electrode. Solder may not replace tin in this operation because solder tends to dissolve the silver electrode. Beeswax was melted to the upper electrode to seal the leads. (diag. 4 for details) When a condenser is to be used it is screwed to a solid copper (diag.5) which may be put in a beaker of water for cooling.

A summary of the properties of an average condenser of this type is now given:

Thickness .1 mm.

Area of dielectric used approximately .6 sq.cms.

Expected breakdown 250v A.C. R.M.S.

Recommended Max. Operating Volts 150v A.C. R.M.S.

Expected total capacity of 6 sections at low voltage .008 mfd.

Temperature rise caused by hysteresis heating very small on copper mount and base. If copper base is not used hysteresis heating causes temperature of dielectric to rise above 120°C (Curie Point for $BaTiO_3$) and all non linear properties are lost.

Voltage necessary for marked non linearity 80v A.C. R.M.S.

These properties satisfied all the requirements and it was decided to proceed with testing before an attempt to improve these condensers was made. It was originally planned to test condensers made of various titanate compositions but time has not permitted this to be done.

CHAPTER III

THE HYSTERESIS LOOP AND BASIC EQUATION OF THE NON LINEAR CONDENSER

1. The hysteresis loop.

The hysteresis loops of the condensers made were observed at different A.C. and D.C. voltages across the condensers in Sawyer and Tower's circuit (17) with a biassing addition (diag 6). Full

17. Sawyer, C.B. and Tower, C.H. "Rochelle Salt as a Dielectric" Phys. Rev. Vol.35 No 3 pp 269-273 Feb. 1930. Minneapolis, Minn.

sized tracings from the scope face are given in diagram 7 for a typical specimen at 60 c.p.s. 60 c.p.s. was used for convenience.

The approximate hysteresis loss for a given loop was calculated as follows:

Let V be the D.C. Voltage required across the vertical plate terminals to move the spot 1 inch.

Let H be the corresponding horizontal voltage.

It is assumed that all the applied voltage appears across N.L.C.

$\therefore$ 1" Vertical deflection means $VC = \frac{V}{10^7}$ coulombs on N.L.C.

1" Horizontal deflection means $\frac{H(R_1 + R_2)}{R_2} = 20$ H volts across N.L.C.

Now $V = 27$, $H = 54$

$\therefore$ 1 sq.in. of scope face corresponds to

$$\frac{(20H)(V)}{10^7} = \frac{(20)(54)(27)}{10^7} = \frac{2.9}{10^3} \text{ joules.}$$

Consider the loop at 150v A.C. 0 Bias.

Measured area = .252 sq.ins.

$\therefore$ Energy Loss per cycle $= (.252)\left(\frac{2.9}{10^3}\right)$ joules.

At 60 c.p.s. Power Loss $= (.252)\left(\frac{2.9}{10^3}\right)(60) = .045$ watts.

At 10000 c.p.s. Power Loss $= (.252)\left(\frac{2.9}{10^3}\right)(10000) = 7.5$ watts.

This latter calculation has assumed the hysteresis loop has the same area at 10000 c.p.s. as at 60 c.p.s. This was found to be nearly true (diag.8). Thus 7.5 watts is approximately the power to be dissipated in such a condenser if it is to be run at 150 v 10000 c.p.s. This power must be supplied by the carrier of the carrier amplifier.

PLATE III

HYSTERESIS CIRCUIT

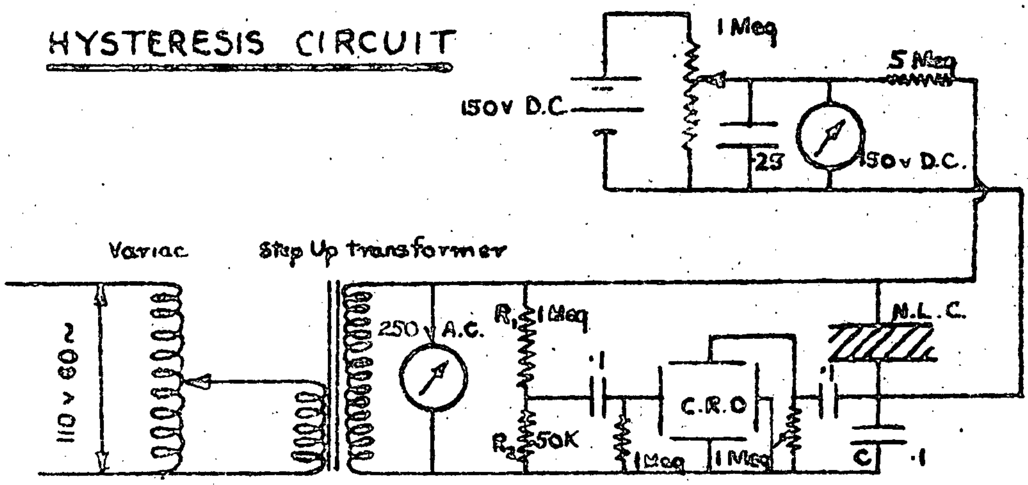


Diagram 6

HYSTERESIS LOOPS

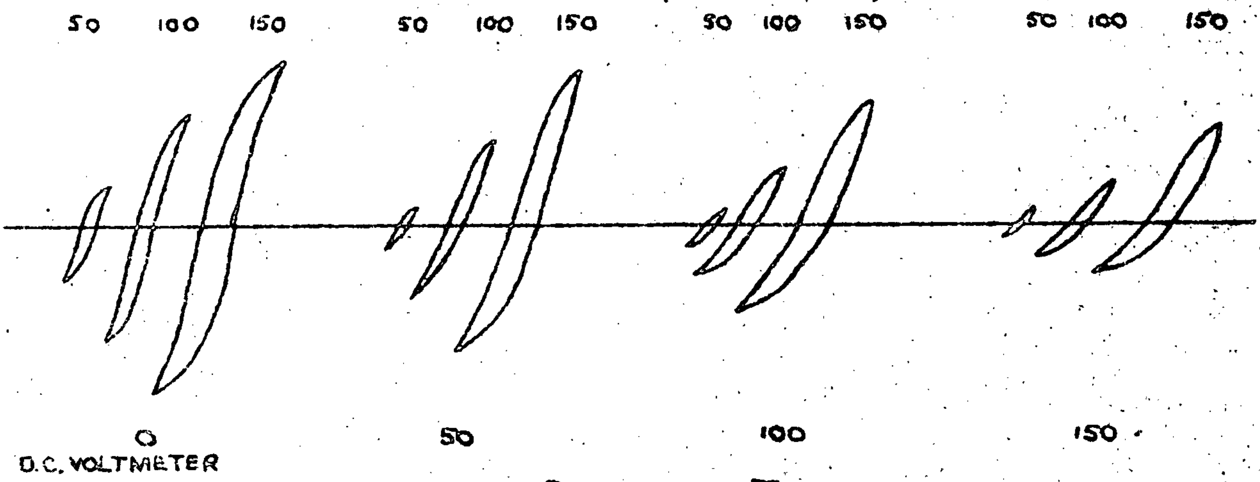


Diagram 7

VARIATION OF LOOP WITH FREQUENCY

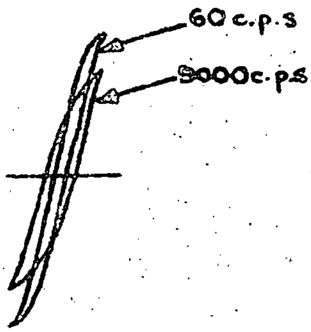


Diagram 8

2. Basic Equation of the Non Linear Condenser.

The problem is to place the information contained in the given hysteresis loops in an equation which can be used to calculate the performance of the condensers in circuits. The problem will be solved if an expression for the current flowing in a non linear condenser can be written as a function of the voltage across it. Without bias the hysteresis loops are symmetrical about the horizontal axis. Hence the current flowing if a sine wave of voltage is applied will contain only odd harmonics.

The non linear condenser without bias is represented by a condenser C^+ in parallel with a resistor R^+ (diag. 9).

C^+ has an equation:

$$Q = aV + bV^3 + cV^5 \quad \text{.............................(1)}$$

$$I_c = a\frac{dV}{dt} + 3bV^2\frac{dV}{dt} + 5cV^4\frac{dV}{dt}$$

R^+ has an equation:

$$I_R = dV + eV^3 + fV^5 \quad \text{.............................(2)}$$

where a,b,c,d,e,f ... are functions of the maximum value of A.C. Voltage (V_{max}) applied to the condenser. a,b,c are slightly dependent on frequency and d,e,f are strongly dependent on frequency.

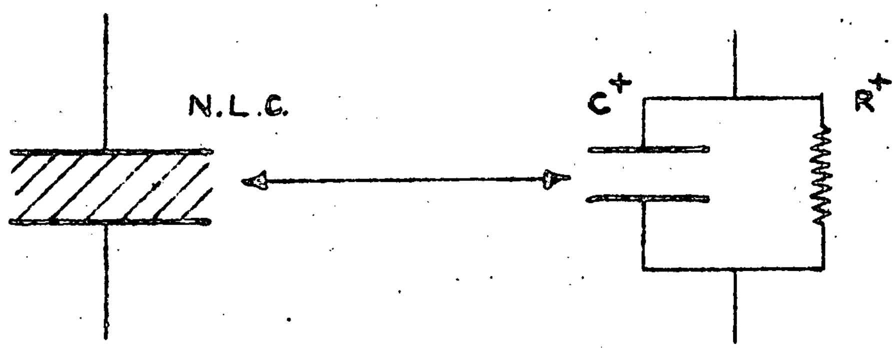
Thus total current into condensers is

$$I = a\frac{dV}{dt} + 3bV^2\frac{dV}{dt} + 5cV^4\frac{dV}{dt} + dV + eV^3 + fV^5 \text{....................(3)}$$

The coefficients a,b,c,d, etc. could be found by an expansion of (3) into fundamental and harmonic currents for a sine wave of applied voltage and a Fourier analysis of an observed current wave. The exact solution of the problem involves tedious experimental

PLATE IV

EQUIVALENT CIRCUIT OF NON-LINEAR CONDENSER WITHOUT BIAS



For C^+

$$I = a\left(\frac{dV}{dt}\right) + 3bV^2\left(\frac{dV}{dt}\right)\ldots\ldots$$

For R^+

$$I = dV + eV^3\ldots\ldots$$

Diagram 9

and theoretical work.

In the theory developed in the present research (3) has been approximated to:

$$I = a\frac{dV}{dt} + 3bV^2\frac{dV}{dt} \quad \text{.......................................} (4)$$

where a and b are functions of V_{max}.
The neglection of dV represents a serious quantitative error in the theory but not a serious qualitative error. The present work was concerned in the main with the qualitative results of (4).

Thus the model used of the non linear condenser consists of a non linear capacity with the equation:

$$Q = aV + bV^3$$

where b must be negative from the shape of the hysteresis loop.

If a bias is applied to the condenser even power terms are also introduced into (3) and even harmonic currents flow. If the shape of the hysteresis loop can be altered until it is almost rectangular as has been done with ferromagnetics, then with suitable bias the model of non linear condenser could be altered to a capacity with the equation:

$$Q = aV + bV^2$$

$$I = a\frac{dV}{dt} + 2bV\frac{dV}{dt} \quad \text{.................................} (5)$$

This was the model used by Van der Ziel in his two papers (18) and (19).

Since the condensers which had been made were more readily represented by (4) than by (5) even with large bias, it

18. See Ref. 11.

19. See Ref. 12.

was decided to work out Van der Ziel's theory using (4) rather than (5)and attempt to build a carrier amplifier based on (4). This theoretical and experimental work will be described later.

The measurements for the calculation of the coefficients a and b in (4) as functions of V_{max} at 9000 c.p.s. will next be given. This frequency was chosen as the carrier frequency rather than 10,000 c.p.s. because some of the parts for the carrier amplifier tuned more readily to 9000 c.p.s.

CHAPTER IV

DETERMINATION OF COEFFICIENTS IN BASIC EQUATION

1. Derivation of Equations for Coefficients.

The coefficients a and b could be found by the general method mentioned above but the method given below is somewhat simpler.

The neglection of condenser losses in (4) will not affect the validity of the calculations for a and b since these are concerned only with capacitive currents (provided d is constant).

If a voltage Vsin wt is applied to the non linear condenser the resulting current is:

$$I(t) = a\frac{dV(t)}{dt} + 3b\,(V(t))^2\frac{dV(t)}{dt}$$

$$= waV\cos wt + 3/4\ bwV^3\cos wt - 3/4\ bwV^3\cos 3wt \ldots (6)$$

Since only fundamental and 3rd harmonic currents are present a and b can be determined in terms of fundamental and 3rd harmonic currents in a suitable circuit. An ideal circuit would have a generator of Vsin wt with zero impedance to 3rd harmonic. This is difficult to make but an approximation to it (diag 10) was constructed. The equivalent circuit from which calculations are made is given in diagram 11.

Voltage applied to non linear condenser is:

$$V\sin wt + V_3 \sin(3wt + \phi)$$

where $V_3 \ll V$

Substitution of this voltage into

$$I(t) = a\frac{dV(t)}{dt} + 3b\,(V(t))^2\,\frac{dV(t)}{dt}$$

gives currents at fundamental and 3rd harmonic.

Main terms at fundamental frequency are:

$$I_1(t) = waV\cos wt + 3/4\ bw\ V^3\cos wt$$

Main terms at 3rd harmonic frequency are:

$$I_3(t) = 3aw\ V_3\cos(3wt + \phi) - \frac{3bwV^3}{4}\cos 3wt$$

In complex notation:

$$I_1 = j\,(waV + 3/4\ bw\ V^3) \quad \dots\dots (7)$$

$$I_3 = j\,(3aw\ V_3 - 3/4\ bw\ V^2\ V^+)$$

but $I_3 = \dfrac{-V_3}{R}$

$$\therefore\ \frac{-V_3}{R} = j(3aw\ V_3 - 3/4\ bw\ V^2\ V^+) \quad \dots\dots (8)$$

where $V = Ve^{jwt}$, $V_3 = V_3e^{j(3wt+\phi)}$, $V^+ = Ve^{j3wt}$

From (7) $|I_1| = waV + 3/4\ bwV^3 \quad \dots\dots (7^1)$

From (8) it can be shown that:

$$1 + 9a^2w^2R = \frac{9}{16}\,\frac{R^2b^2w^2V^6}{V_3^2} \quad \dots\dots (9)$$

(7^1) and (9) may now be solved simultaneously to give:

(Because $V_3 \ll V$ some expressions have been eliminated)

$$\left.\begin{aligned} a &= \frac{n \pm \sqrt{n^2 - 4mt}}{2m} \\ \text{where} \quad m &= w^2R^2V^2 \\ n &= 2I_1R^2wV \\ t &= I_1{}^2R^2 - V_3{}^2 \end{aligned}\right\} \quad \ldots\ldots (10)$$

$$\left.\begin{aligned} b &= \frac{-q \pm \sqrt{q^2 + 4pr}}{2p} \\ \text{where} \quad p &= \frac{9}{16} w^2R^2V^8 \\ q &= \frac{27}{2} wR^2V_3{}^2 I_1V^3 \\ r &= V_3{}^2V^2 + 9I_1{}^2 V_3{}^2R^2 \end{aligned}\right\} \quad \ldots\ldots (11)$$

(10) and (11) are the required expressions for a and b in terms of quantities whose measurement next will be described. When the measured quantities were substituted in these expressions one of the two possible values for a and b was eliminated because b had to be negative.

All values appearing in the above equations are peak values. Volts, Amps, Ohms were used.

Since the flow of fundamental current in the above calculations is independent of V_3 the fundamental current may be measured in a separate circuit to that shown above. This was done.

The following measurements were done on a typical condenser of capacity that desired for the carrier amplifier (.0045mfd).

2. Measurement of 3rd Harmonic Current flowing in a non linear condenser with varying Fundamental Voltage.

Diagram 10 gives the circuit used.

PLATE V

CIRCUIT FOR 3RD HARMONIC CURRENT MEASUREMENT

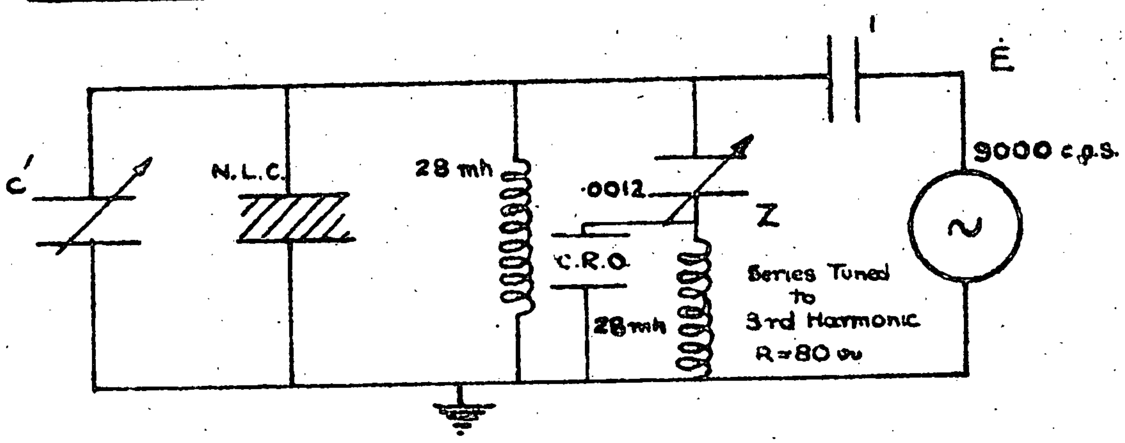


Diagram 10

EQUIVALENT CIRCUIT OF ABOVE AT 3RD HARMONIC FREQUENCY

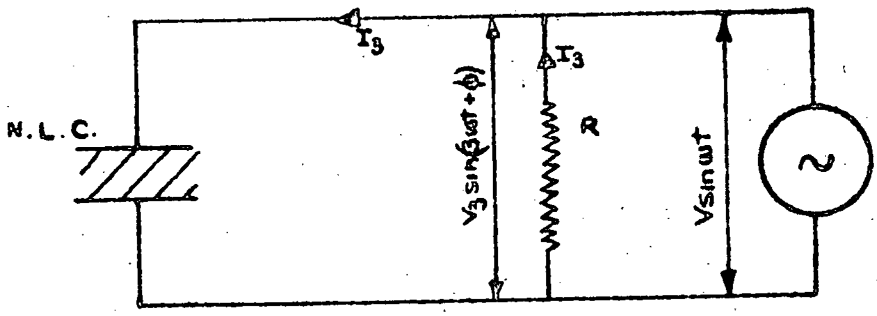


Diagram 11

TYPICAL WAVEFORM OBSERVED AT Z (DIAG 10)

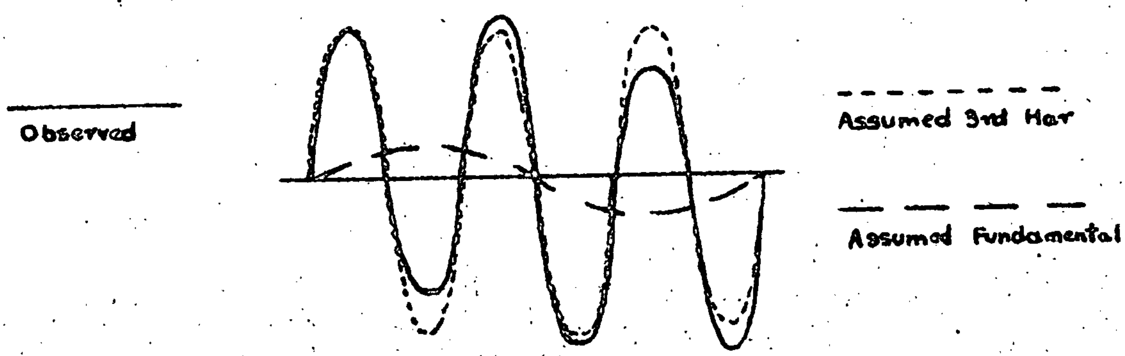


Diagram 12

The circuit was tuned to parallel resonance at fundamental frequency by c^1.

The voltage at Z was observed on a calibrated oscilloscope which was part of the tuned circuit, a correction made for the presence of the fundamental and the 3rd harmonic current (I_3) through the series tuned circuit calculated. A typical wave form observed at Z is shown in diagram 12 with the harmonic composition assigned to it. The peak deflection was observed and converted to volts. One half of the peak fundamental voltage calculated at Z was subtracted and the remaining voltage converted to R.M.S. value. Division by the reactance of the coil gave the required value of 3rd harmonic current.

Example: R.M.S. Volts at E at 9000 c.p.s. = 70

Oscilloscope Sensitivity = 44 Volts per inch.

Max Observed Deflection = 1/2 Total Deflection on Scope = 1.6 ins.

Peak Voltage = (1.6)(44) = 70.4 volts

Calculated fundamental peak voltage across coil

$$= \left(\frac{1}{8}\right)(70)(1.414) = 12.4 \text{ volts}$$

Peak Obs. - 1/2 peak fundamental = Peak 3rd Har. = 64.2 volts

$$\text{R.M.S. 3rd Har. Current} = \left(\frac{64.2}{1.414}\right)\left(\frac{1000}{WL}\right)\left(\frac{1}{3}\right)$$

= 9.6 m-as

The results for voltages at E from 0 to 150 are given with the results of the next sections in diagram 15.

3. Measurement of Fundamental current flowing in non linear condenser with varying Fundamental voltage.

Diagram 13 gives the circuit.

The Voltages at E and Y were observed on a Cossor double beam oscilloscope and the capacitive and resistance components of fundamental current flowing in the non linear condenser calculated from the vector diagram 14.

Example: R.M.S. Volts at E at 9000 c.p.s. = 70

θ from double beam oscilloscope = 27.5°

$V_y = 18.0$ v

$I = (V_y)(wC) = 30.6$ m-as

Let $V_1 = V_E - (V_y) \cos\theta = 70 - 18 \cos 27.5 = 54.0$

$V_2 = (V_y) \sin\theta = 18 \sin 27.5 = 8.3$

$\tan\phi = \frac{V_2}{V_1} = \frac{8.3}{54.0} = .1537$

$\phi = 8.7^\circ$

$\psi = 90 - \theta - \phi = 53.8^\circ$

Resistive Current $I_R = I\cos\psi = (30.6)(.59) = 18.1$ m-as

Capacitive Current $I_c = I\sin\psi = (30.6)(.807) = 24.7$ m-as

Voltage across condenser = $V_2 \operatorname{cosec}\phi = \frac{8.3}{.151} = 55$ volts

The results from these observations and calculations are given in diagram 16. The values of capacity and resistance for the non linear condenser are shown in diagram 17. It can be seen that the neglection of the condenser resistance will introduce a serious quantitative error in the theory. Further at low voltages this resistance is non linear and will introduce a further error.

PLATE VI

CIRCUIT FOR FUNDAMENTAL CURRENT MEASUREMENT

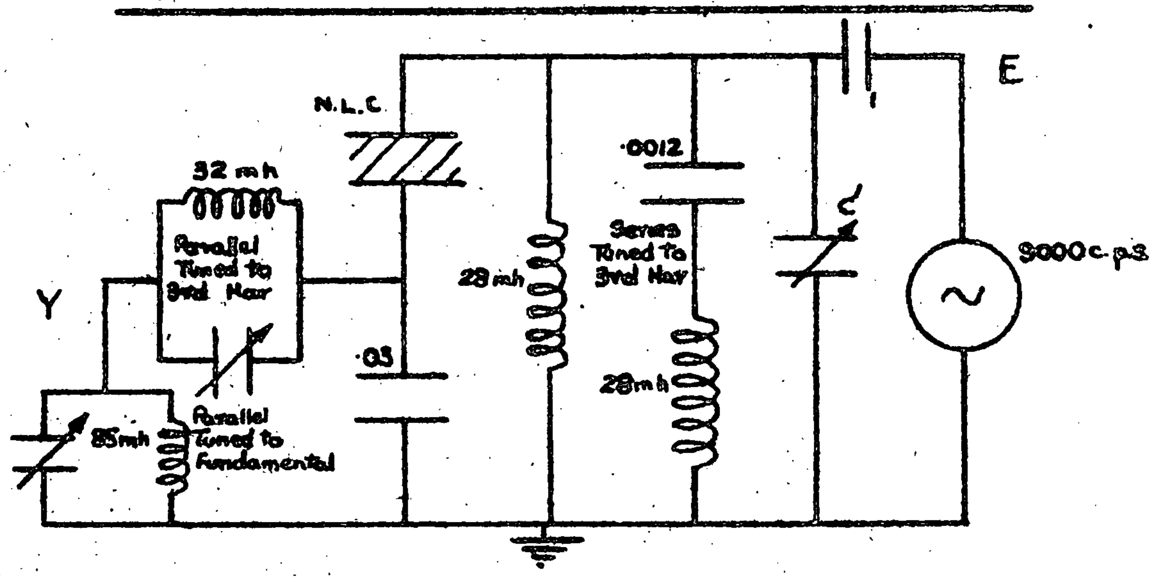


Diagram 13

VECTOR DIAGRAM FOR FUNDAMENTAL CURRENT MEASUREMENT

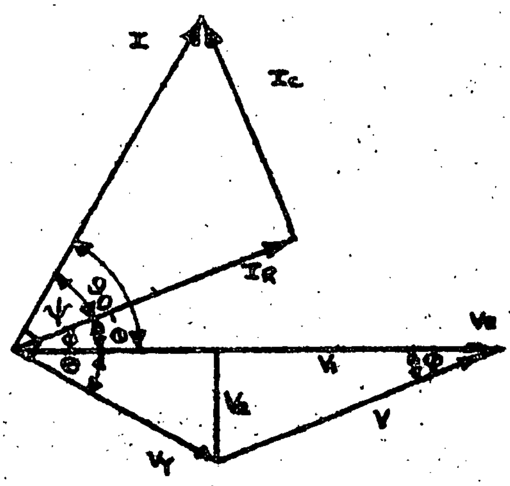


Diagram 14

4. Calculation of a and b.

The data for the calculation of a and b from formulas (10) and (11) has now been assembled.

The constants used are given here:

$$w = (5.65)(10^4)$$

$$w^2 = (3.2)(10^9)$$

$$R = 80$$

$$V_3 = I_3R$$

I_1, I_3, V, are taken from diagrams 15 and 16 and converted to peak values.

The values of a and b for voltages from 0 to 120 at 9000 c.p.s. are given in diagram 18.

Addition to Chapter IV.

The following two sections are added to Chapter IV because they are off the main line of theoretical and experimental results discussed in this thesis but are most closely connected to the discussion of Chapter IV and should not be omitted.

5. Audio Oscillator and Power Amplifier.

This unit was designed as the source of the carrier for the carrier amplifier. It has a push pull output capable of delivering 18 watts and is variable from 5000 c.p.s. to 15000 c.p.s. Since it is a standard oscillator and amplifier its design will not be discussed. Circuit is given in diagram 19. One half of the output was used for measurements described in sections 2 and 3 of this chapter.

6. Measurement of 2nd Harmonic current flowing in biassed non linear condensers.

PLATE VII

ALL GRAPHS FOR TYPICAL CONDENSER

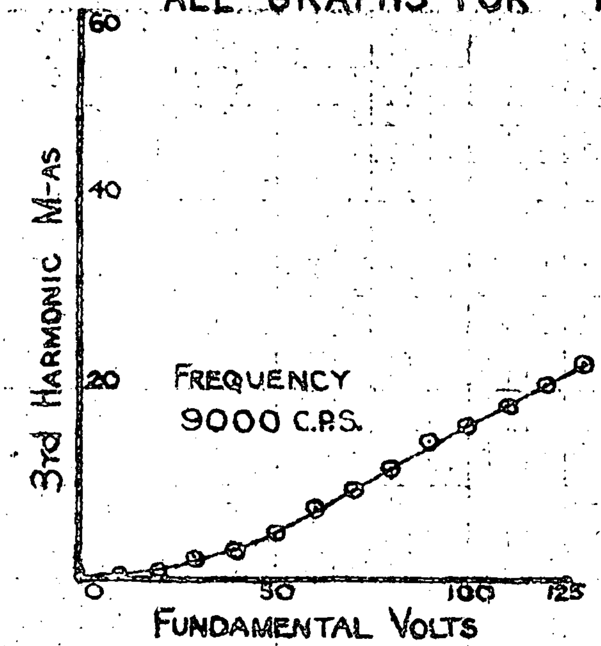


Diagram 15

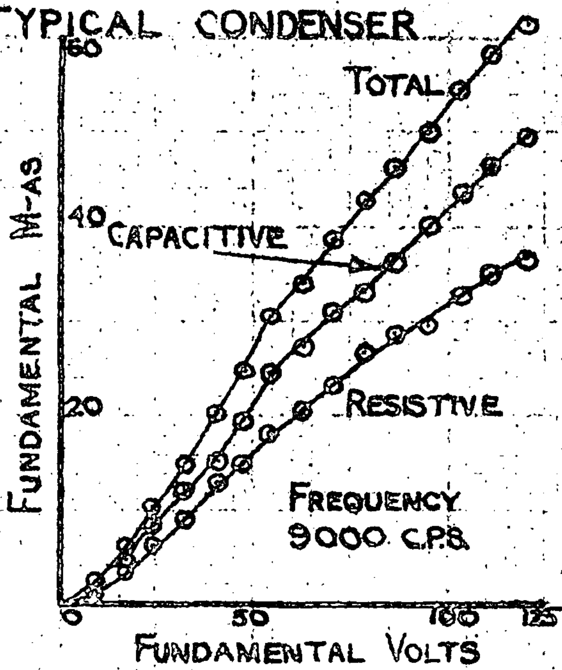


Diagram 16

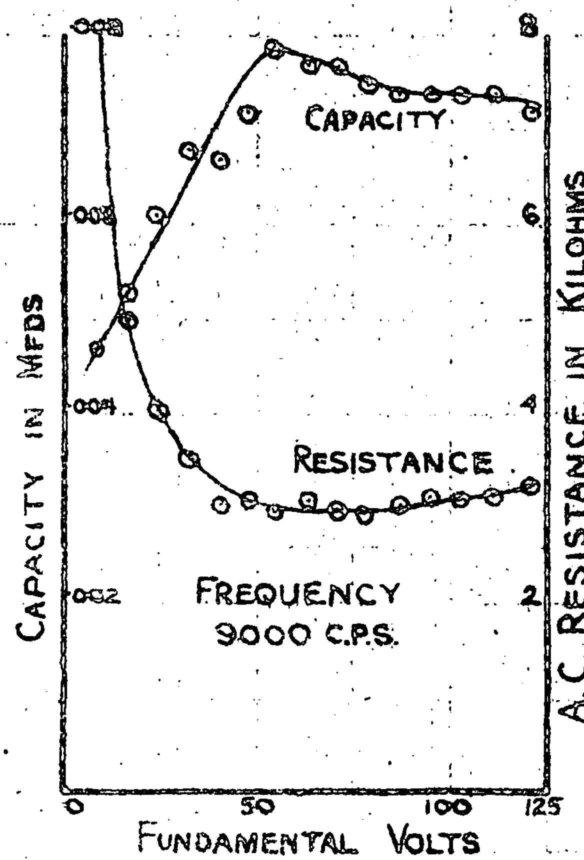


Diagram 17

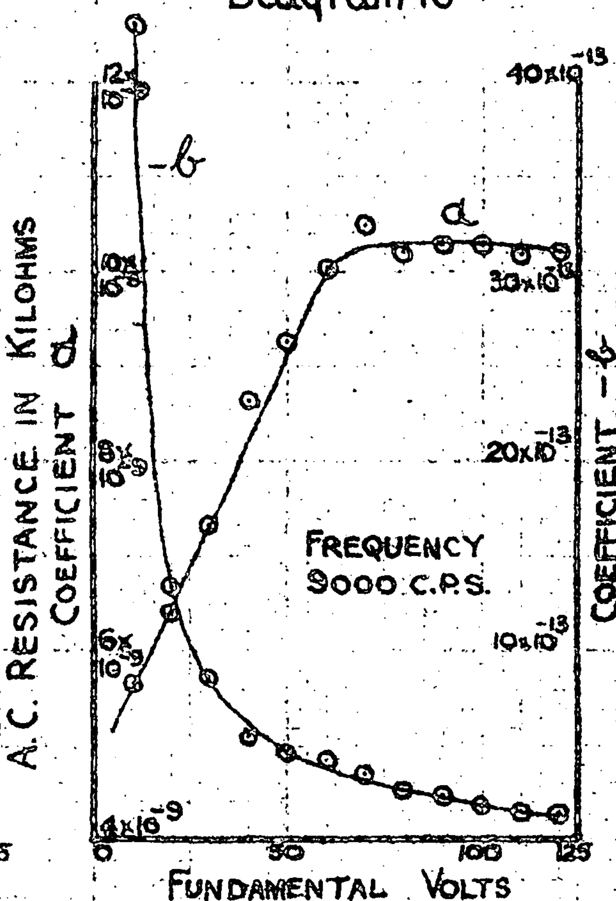


Diagram 18

After it had been decided to build an amplifier based on equation (4) this measurement assumed secondary importance since it would have been the decisive measurement of an amplifier based on equation (5).

Both sides of the push pull output of the audio amplifier were used in the circuit of diagram 20. The two condensers were matched as closely as was possible in the group of condensers available and placed in series from plate to plate of the 6L6S. Adjustment of R and C balanced the condensers with respect to the fundamental and the fundamental current flowing through series tuned circuit 1 could be reduced to zero. 3rd harmonic current flowing from X to ground could not also be matched with respect to the 3rd harmonic.

The voltage E to D was held fixed during the experiment at 200v R.M.S. at 9000 c.p.s. and the voltage B to ground observed for each value of bias on the condensers. The 2nd harmonic current from both condensers flowed through circuit 2 and could be calculated when the voltage at B was known. The circuit had to be retuned for each bias setting. The results are given in diagram 21.

The non coincidence of the curves for increasing and decreasing bias is not considered significant. It may be caused by slight permanent electrification. The small residual 2nd harmonic current at zero bias had a 90° phase shift to the current resulting from biassed condensers and was partly caused by detuning of the main circuit (this point is not yet understood) and partly by a small amount of D.C. rectification in the condensers.

PLATE VIII

VARIABLE FREQUENCY AUDIO OSCILLATOR AND AMPLIFIER

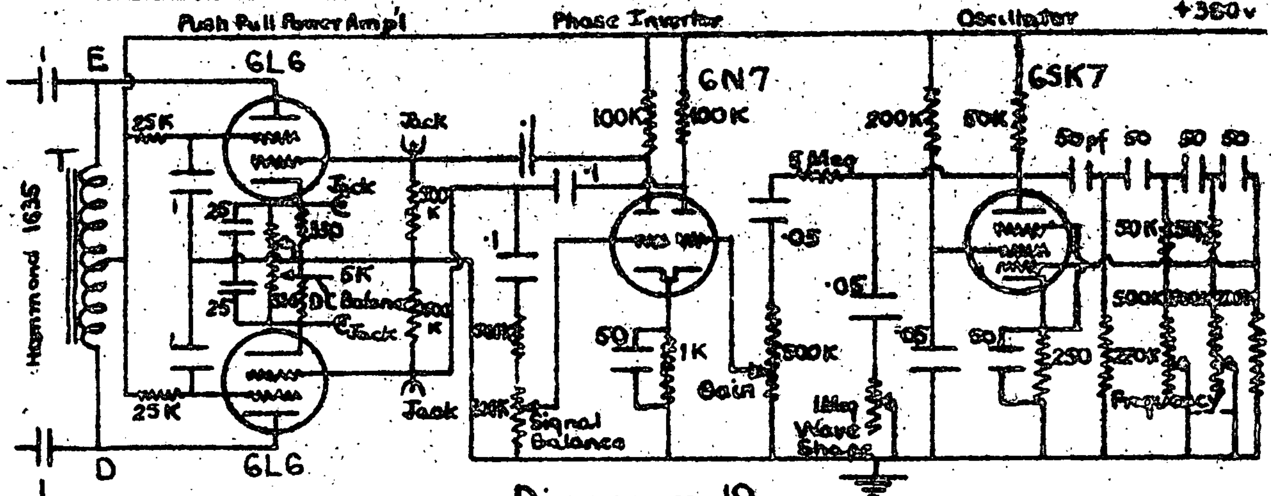


Diagram 19

CIRCUIT FOR 2ND HARMONIC CURRENT MEASUREMENT

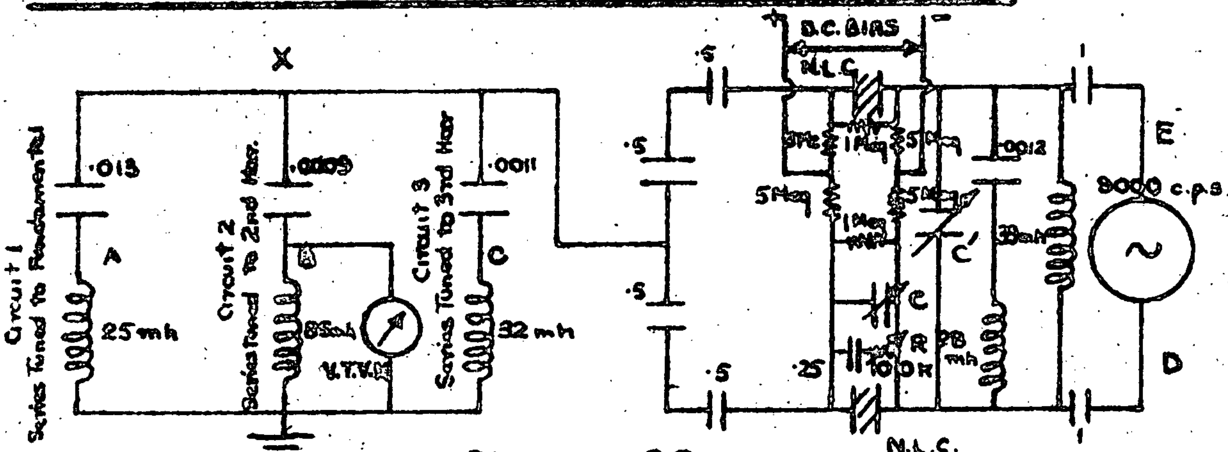


Diagram 20

2ND HARMONIC CURRENT VS D.C. BIAS VOLTAGE

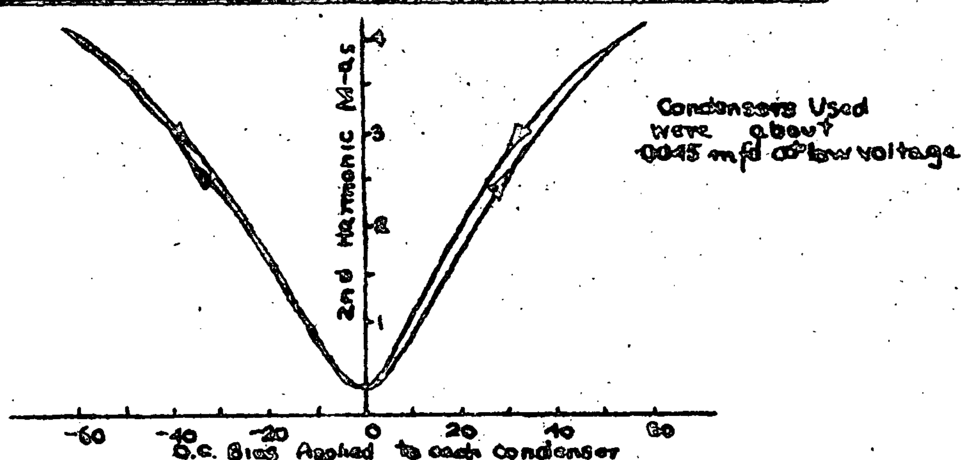


Diagram 21

The condensers were found to have a D.C. resistance of about 5 Megohms and to give 1 microamp of D.C. rectified current when 100v A.C. was across them at 9000 c.p.s. The residual 2nd harmonic current at zero bias was however small and was not further investigated.

CHAPTER V

THEORY OF A DIELECTRIC CARRIER AMPLIFIER

The theory given follows the pattern of Van der Ziel's analysis (20) but is applied to a condenser having:

$$Q = aV + bV^3$$

rather than $Q = aV + bV^2$

1. Principle of Amplification

A large voltage at high frequency (ang freq. w) and a small voltage at low frequency (ang freq. p) are applied to a non linear condenser of the type described above. Currents of frequency 2w + p and 2w - p flow. The potential power contained in these side bands is greater than the low frequency input power. The potential power amplification may be realized in a suitable circuit by the addition of a small current at angular frequency 2w at the correct phase and the detection of the resulting modulated signal. Some of the carrier power has been transferred to the low frequency signal.

2. Derivation of the currents flowing when two signals are applied to a non linear condenser.

---

20. See Ref.11.

Let V Sin wt and P Sin pt ($V \gg P$, $w \gg p$) be applied to a non linear condenser having

$$I(t) = a\frac{dV(t)}{dt} + 3b(V(t))^2\frac{dV(t)}{dt}$$

In this case $V(t) = V \sin wt + P \sin pt$

By substitution it can be shown that:

$$I(t) = awV\cos wt + apP\cos pt + 3b\Big[\frac{1}{4}wV^3\cos wt + \frac{wP^2V}{2}\cos wt - \frac{wV^3}{4}\cos 3wt - \frac{wV^2P}{2}\cos(2w+p)t$$

$$- \frac{pPV^2}{4}\cos(2w+p)t + \frac{wV^2P}{2}\cos(2w-p)t - \frac{pPV^2}{4}\cos(2w-p)t$$

$$- \frac{wP^2V}{4}\cos(w+2p)t - \frac{pVP^2}{2}\cos(w+2p)t - \frac{wP^2V}{2}\cos(w-2p)t$$

$$+ \frac{pVP^2}{2}\cos(w-2p)t + \frac{pPV^2}{2}\cos pt + \frac{pP^3}{4}\cos pt$$

$$- \frac{pP^3}{4}\cos 3pt\Big]$$

Of these the mixed currents of greatest size are:

$$3b\Big[-\frac{wV^2P}{2}\cos(2w+p)t - \frac{pPV^2}{4}\cos(2w+p)t$$

$$+ \frac{wV^2P}{2}\cos(2w-p)t - \frac{pPV^2}{4}\cos(2w-p)t\Big]$$

Since $w \gg p$ these become approximately:

$$3b\left(-\frac{wV^2P}{2}\cos(2w+p)t + \frac{wV^2P}{2}\cos(2w-p)t\right) \quad \ldots\ldots\ldots(12)$$

These are the currents upon which amplification is based. Since they are independent of the sign of V a balanced circuit may be designed where the high frequency is applied to the condensers in push pull, the low frequency in parallel, and these output currents fed into a load in parallel. The high frequency may thus be eliminated from the output circuit. The equivalent circuit of such a carrier amplifier is given in diagram 22.

PLATE IX

EQUIVALENT CIRCUIT OF CARRIER AMPLIFIER

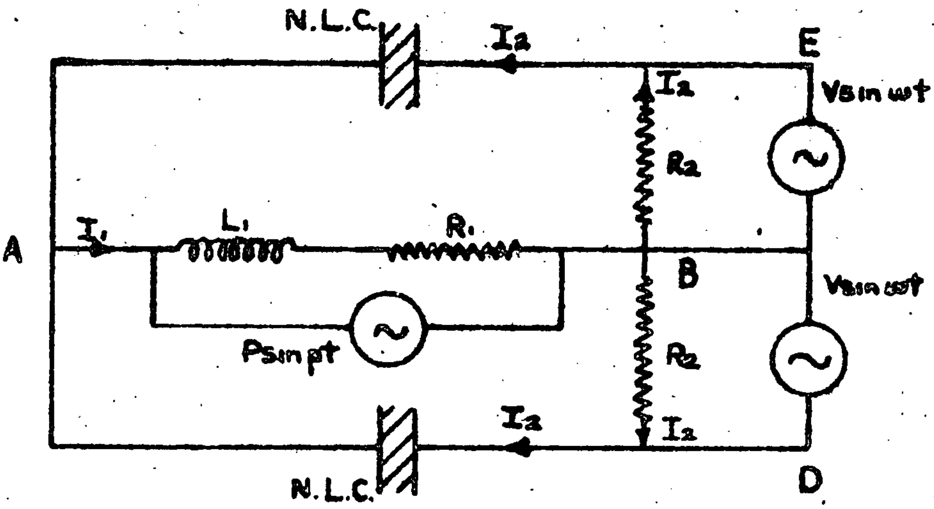


Diagram 22

CARRIER AMPLIFIER CIRCUIT

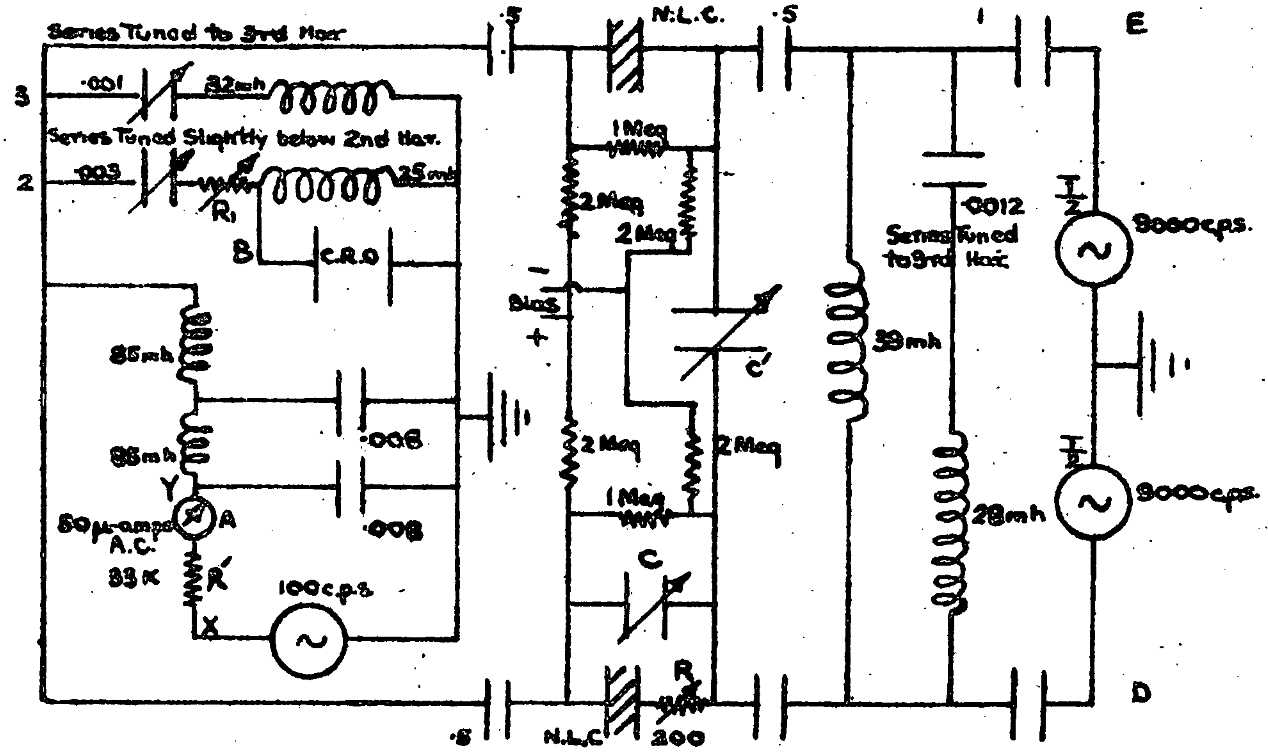


Diagram 23

3. Analysis of Carrier Amplifier Equivalent Circuit.

(1) Assumptions made in circuit:

(a) The non linear condensers are identical. Hence no fundamental or 3rd harmonic currents flow from A to B.

(b) The 2nd harmonic impedance between A and B is R_1+j2wL_1 and between B and E and B and D is R_2. The low frequency impedance between A and B is high and capacitive (it will be considered infinite) and between B and E and B and D is low (it will be considered zero)

(c) The low frequency generator presents a high impedance to 2nd harmonic and no side band currents flow into the low frequency generator.

(d) Both generators have zero internal impedance. Matching of practical generators will be considered later after the impedance seen by the ideal generators have been found.

(e) The 3rd harmonic voltage from D to E is neglected.

(2) Circuit Voltages.

Across either non linear condenser there are six main voltages. ($V \gg P$, $w \gg p$)

		complex form
1.	$V \sin wt$	$V = Ve^{jwt}$
2.	$P \sin(pt + \psi)$	$P = Pe^{j(pt + \psi)}$
3.	$V_1 \sin((2w + p)t + \phi)$	$V_1 = V_1 e^{j((2w + p)t + \phi)}$
4.	$V_1^1 \sin((2w - p)t + \phi^1)$	$V_1^1 = V_1^1 e^{j((2w - p)t + \phi^1)}$
5.	$V_2 \sin((2w + p)t + \theta)$	$V_2 = V_2 e^{j((2w + p)t + \theta)}$
6.	$V_2^1 \sin((2w - p)t + \theta^1)$	$V_2^1 = V_2^1 e((2w - p)t + \theta^1)$

(3) Derivation of Mixed Currents.

These are applied to a condenser having:

$$I = a\frac{dV}{dt} + 3bV^2\frac{dV}{dt}$$

From the first term the currents in complex form are:

$ja(wV + p^P + (2w + p)(V_1 + V_2) + (2w - p)(V_1{}^1 + V_2{}^1))$ ....(13)

From the second term the main currents in trigonometric form are:

$(V_1, V_2, V_1{}^1, V_2{}^1, << V)$

$$3b\Big[\frac{wV^3}{4}\cos wt - \frac{wV^3}{4}\cos 3wt$$

$$+ \frac{pPV^2}{2}\cos(pt + \psi) + \frac{wV^2P}{2}\cos((2w - p)t - \psi)$$

$$+ \frac{wV^2V_1}{2}\cos(pt + \phi) + \frac{wV^2V_1'}{2}\cos(pt - \phi^1) + \frac{wV^2V_2}{2}\cos(pt + \theta)$$

$$+ \frac{wV^2V_2{}^1}{2}\cos(pt - \theta^1) + \left(\frac{2w + p}{2}\right)V_1V^2\cos((2w+p)t+\phi) - \left(\frac{2w+p}{4}\right)V_1V^2\cos(pt$$

$$+ \left(\frac{2w - p}{2}\right)V_1{}^1V^2\cos((2w - p)t + \phi^1) - \left(\frac{2w - p}{4}\right)V_1{}^1V^2\cos(pt - \phi^1)$$

$$+ \left(\frac{2w + p}{2}\right)V_2V^2\cos((2w + p)t + \theta) - \left(\frac{2w + p}{4}\right)V_2V^2\cos(pt + \theta)$$

$$+ \left(\frac{2w - p}{2}\right)V_2{}^1V^2\cos((2w - p)t + \theta^1) - \left(\frac{2w - p}{4}\right)V_2{}^1V^2\cos(pt - \theta^1)\Big]$$

Collecting all currents of angular frequency p from both terms and using complex notation:

$I_p = japP + j\frac{3b_p V^2}{4}(2P + V_1{}^{1++} + V_2{}^{1++} - V_1{}^+ - V_2{}^+)$ ....(14)

where $V_1{}^+ = V_1e^{j(pt+\phi)}$

$V_2{}^+ = V_2e^{j(pt+\theta)}$

$V_1{}^{1++} = V_1{}^1e^{j(pt-\phi^1)}$

$V_2{}^{1++} = V_2{}^1e^{j(pt-\theta^1)}$

Similarly for angular frequency $2w+p$:

$I_{2w+p} = ja(2w + p)(V_1 + V_2) + j\frac{3}{2}bV^2(-wP + (2w + p)(V_1 + V_2))$...(15)

where $P+ = Pe^{j((2w+p)t+\psi)}$

For angular frequency $2w - p$

$$I_{2w-p} = ja(2w-p)(V_1^1+V_2^1) + j\frac{3}{2}bV^2(wP^{1++} + (2w-p)(V_1^1+V_2^1)) \quad \ldots\ldots(16)$$

where $P^{1++} = Pe^{j((2w-p)t-\psi)}$

(4) Derivation of output Power.

From circuit:

$$\left.\begin{aligned} V_1 &= -I_1(R_1 + 2jwL_1) \\ V_2 &= -I_2R_2 \\ V_1 &= -2I_2(R_1 + 2jwL_1) \end{aligned}\right\} \quad \ldots\ldots(17)$$

Since $I_1 = 2I_2$ because circuit is balanced

Substitution of (17) into (15) with the assumption $2w + p \doteq 2w$ yields the equation for I_2 at $2w + p$:

$$I_2 = -j\frac{3\ bw\ V^2\ P^+}{2\,(1 - 4w^2L_1(2a + 3bV^2) + jw(2R_1 + R_2)(2a+3bV^2)} \quad \ldots(18)$$

If the absolute value of I_2 is to be a maximum than:

$$1 - 4w^2L_1(2a + 3bV^2) = 0$$

$$2wL_1 = \frac{1}{2w(2a+3bV^2)} \quad \ldots\ldots(19)$$

If (19) be used in (18)

$$I_2 = -\frac{3\ b\ V^2\ P^+}{2(2R_1+R_2)(2a+3bV^2)} \quad \ldots\ldots(20)$$

Similarly for I_2 at $2w - p$ i.e. I_2^1

$$I_2^1 = \frac{3b\ V^2\ P^{1++}}{2(2R_1+R_2)(2a+3bV^2)} \quad \ldots\ldots(21)$$

The maximum possible power available from both side band frequencies for an input voltage P in R_1 and both R_2 can be calculated to be:

$$Po = \frac{9b^2 V^4 P^2}{2(2R_1+R_2)(2a+3bV^2)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

I_2 and I_2^1 have been converted to R.M.S. for this derivation.

(22) gives the maximum possible power available for reconversion to angular frequency P.

Of this power the amount available in R_1 is:

$$P = \frac{9b^2 P^2 V^4 R_1}{(2R_1+R_2)^2(2a+3bV^2)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

This is the practically useful output power.

(5) Derivation of Input Currents.

From (2) $\quad I_2 = -xP^+$

where $x = \dfrac{3bV^2}{2(2R_1+R_2)(2a+3bV^2)}$

From (21) $\quad I_2^1 = x\,P^{1++}$

Thus $\quad I_1 = 2I_2 = -2xP^+$

$\quad I_1^1 = 2I_2^1 = 2xP^{1++}$

From (17) $\quad V_2 = xR_2P^+$

$\quad V_2^1 = -x\,R_2P^{1++}$

$\quad V_1 = 2x(R_1 + 2jwL_1)P^+$

$\quad V_1^1 = -2x(R_1 + 2jwL_1)P^{1++}$

From this it can be shown that:

$$\left.\begin{aligned} V_2^+ &= xR_2P \\ V_2^{1++} &= -xR_2P \\ V_1^+ &= 2x(R_1+2jwL_1)P \\ V_1^{1++} &= -2x(R_1+2jwL_1)P \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (24)$$

Substitution of (24) into (14) yields:

$$Ip = P(j(ap+\tfrac{3}{2}\,bV^2p) - j\,\tfrac{3}{2}\,bpV^2x(2R_1 + R_2 + 4\,jwL_1))$$

This is the low frequency current supplied to one condenser only. For both condensers this must be multiplied by 2.

Substituting for x gives:

$$I_T = 2I_p = P\left(jp(2a+3bV^2) - \frac{j9b^2pV^4(2R_1+R_2+4jwL_1)}{2(2R_1+R_2)(2a+3bV^2)}\right)$$

When I_T is resolved into its real and imaginary components there results the low frequency resistive and capacitive input currents: ((19) is used)

$$I_R = \frac{9Pb^2pV^4}{2w(2R_1+R_2)(2a+3bV^2)^2} \quad \ldots\ldots(25)$$

$$I_c = P\left(jp(2a+3bV^2) - j\frac{9\, b^2\, p\, V^4}{2(2a+3bV^2)}\right) \quad \ldots\ldots(26)$$

(6) Derivation of Power Gain

From (25) the low frequency input power is:

$$P_I = \frac{9\, b^2\, P^2{}_p\, V^4}{4w(2R_1+R_2)(2a+3bV^2)^2} \quad \ldots\ldots(27)$$

Division of (22) by (27) gives the maximum possible power gain:

$$G = \frac{P_o}{P_I} = \frac{2w}{p} \quad \ldots\ldots(28)$$

(28) is the end result of the theory. It gives the maximum possible power gain under ideal conditions for this type of amplifier. It can be shown that the power gain but not power input or output is independent of the tuning condition (19) In the amplifier to be described in the next chapter $\frac{w}{p} = 90$

Thus maximum possible power gain = 180

(7) Discussion of the theory.

The theory as given yields the following conclusions:

If most of the output power is to be used in R_1 and not in R_2 then R_2 must be as small as possible relative to R_1. If a large output power is desired R_1 and R_2 must be small from (22) but for a given R_2 there is a value of R_1 where a maximum of power is used in R_1.

The low frequency input capacitive current given by (26) shows that it is not effected by the output circuit.

The low frequency input resistive current is strongly effected by the output circuit. As the output power increases so does the input power. Thus input circuit matching depends on the output circuit. The impedances seen by the low frequency generator are given by (25) and (26) and a practical generator must be matched to these.

The high frequency generator sees a capacitive impedance which can be found from the first term of (13) and the first term of the following expansion. It also sees a small resistive component since it must supply the power for the power gain.

4. Qualitative Effects of Inclusion of Condenser Losses in Amplifier Equivalent Circuit.

A complete analysis could be done with the non linear condensers in section 3 of this chapter shunted by appropriate resistors. Such an analysis is quantitatively more complete than that given above but does not bring forward results which cannot be predicted qualitatively by a simple discussion of the circuit. The theoretical picture is further complicated by the non linearity of the resistors at low voltages (diag 17).

The following general conclusions about the inclusion of condenser losses are drawn.

Since the currents flowing through the shunting resistors flow through L_1 the tuning condition (19) will be altered.

The low frequency signal input impedance will contain an additional resistive element representing the energy fed by the low frequency generator to hysteresis losses. If the greatest power gain is to be achieved this input power must be small relative to the input power of (27). Hence R_1 and therefore R_2 should be as small as possible but given R_2, R_1 must not fall below a certain value.

The high frequency input signal will have to supply considerably more power than indicated by the idealized theory. It must supply the hysteresis losses for the high frequency high voltage signal. These can be calculated from diagram 17.

Losses in condenser hysteresis at side band frequencies will also reduce the power gain achievable in practise.

The above theory and conclusions were checked in an amplifier which was a physical realization of diagram 22.

CHAPTER VI

EXPERIMENTAL RESULTS FROM CARRIER AMPLIFIER

1. Circuit.

The circuit was similar to that used for measurement of 2nd harmonic current and is given in diagram 23. As before the non linear condensers were balanced with respect to fundamental by R and C. Unless otherwise stated the following setting up procedure was used in tests on the amplifier:

The high frequency was tuned to 9000 c.p.s. and the low frequency to 100 c.p.s.

The circuit was tuned with c^1.

The condensers were matched until the voltage at Y was a minimum.

Circuit 2 was tuned until side band current was a maximum.

Since all these adjustments are not independent they were repeated until all held simultaneously.

About 1/2 hour warm-up time was required for conditions in the circuit to stabilize. Since non linear condensers are highly voltage sensitive fluctuations in line voltage were always observable but were not sufficient to necessitate use of voltage regulated supplies.

2. Measurement of Output Power.

The measurement of sideband current or output power was done as follows:

A calibrated oscilloscope was placed from B to ground. When the circuit was operating the side bands were observed. The condensers were biassed to a few volts until a little less than 100 per cent modulation existed.

(Theory quickly shows that biassing does produce 2nd harmonic in the correct phase to give a normal modulated signal.

If in the expansion $Q = f^{(V)}$ for a non linear condenser there is a term proportional to V^2 and V Sin wt is applied then the 2nd harmonic current has the phase Sin 2 wt. From equation (12) the sideband currents have the phase $\pm \cos(2w \mp p)^t$. This is the correct phase relation).

A sketch of a typical waveform at B is given in diagram 24. The distance x was measured and converted to volts. From the theory of an amplitude modulated signal one quarter of this voltage is due to one sideband. The current at one sideband is readily calculated by the division of this voltage by the impedance from B to ground. The power in one sideband is given by the square of the current times the resistance in series tuned circuit 2.

Example: 2" total deflection required 20v R.M.S. at 18000 c.p.s.

x measured = .83 ins.

wL = impedance B to gnd. = 2830 Ω

R_1 = 110 ohms.

Sideband current = $\frac{(.83)(20)(1000)}{(4)(2830)}$ = 1.47 m-as

Power in R_1 at 1 Sideband = $\left(\frac{1.47}{1000}\right)^2 (110)$ = 236 microwatts.

Power in R_1 at both sidebands = Power Output of Amplifier = 472 microwatts.

3. Measurement of Low Frequency input power.

For this measurement it was essential to remove all voltages except low frequency voltages at Y.

The voltages at X and Y and the microammeter reading

PLATE X

Facing Page 30

TYPICAL WAVEFORM AT AMPLIFIER OUTPUT
(B in diag 23)

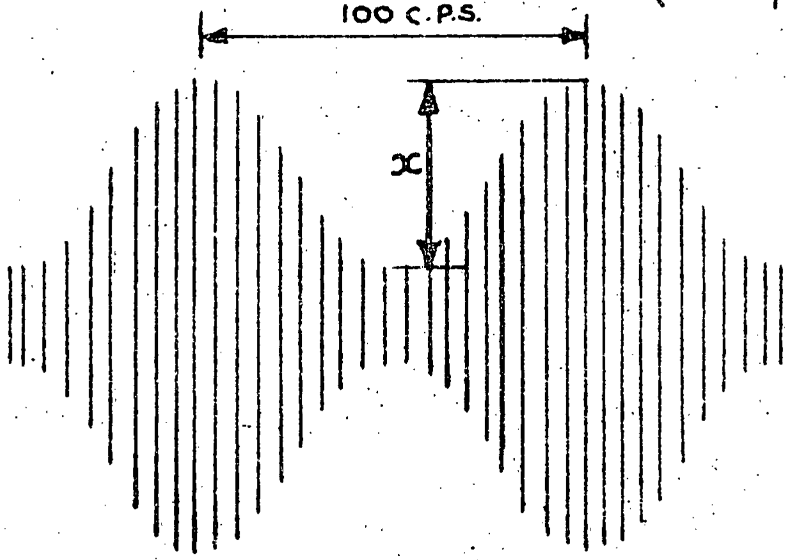


Diagram 24

VECTOR DIAGRAM FOR LOW FREQUENCY INPUT

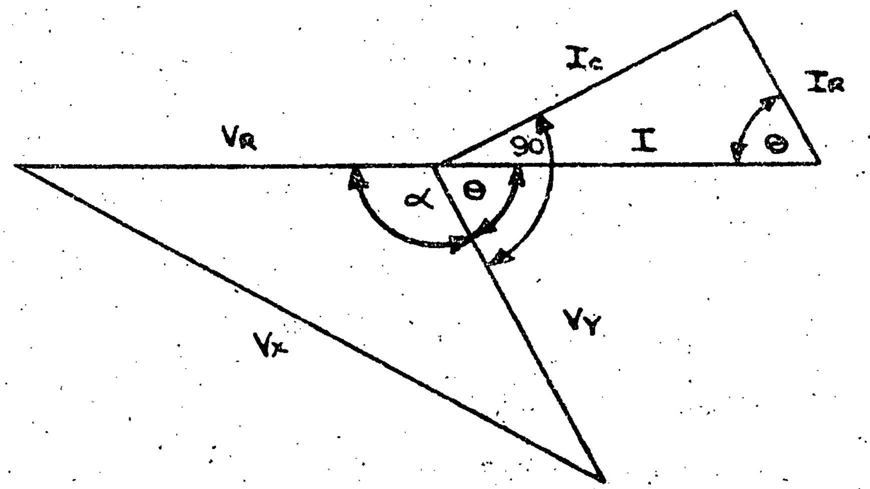


Diagram 25

were observed. V_R was the voltage drop across R^1. The current from X to Y as a function of V_R was carefully plotted independently. Hence the vector diagram 25 could be drawn since the lengths of all sides of the triangle were known and the angle θ calculated. This gave the phase relation between the voltage, between Y and ground and the current flowing into the circuit at Y. The capacitive and resistive components of the current could be found and the power input calculated. The capacitive and resistive currents flowing into the circuit without the non linear condensers were measured in the same way and subtracted from the results obtained with non linear condensers. The low frequency power and capacity currents flowing into the non linear condensers were thus found.

Example:

$V_x = 2.42$ volts

$V_y = 1.60$ volts at 100 c.p.s.

$A = 26.0$ microamps

$V_R = 1.09$ volts (from calibration curve).

$$S = \frac{1}{2}(Vx + Vy + V_R) = 2.555$$

$$S - V_y = .955$$

$$S - V_R = 1.465$$

$$V_y V_r = 1.745$$

$$\sin\frac{\alpha}{2} = \left(\frac{(S - V_y)(S - V_R)}{V_y V_R}\right)^{1/2} = \frac{(.955)(1.465)}{(1.745)} = .895$$

$\alpha = 127^\circ$ $\theta = 53$

Resistive Current = (26.0)(cos 53) = 15.6 microamps.

Capacitive Current = (26.0)(Sin 53) = 20.6 microamps.

Low frequency Power Input = (1.6)(15.6) = 25 microwatts.

From these the power, capacitive current and resistive current fed into the circuit without non linear condensers must be subtracted but the method of measurement of these is identical and is not given.

4. Variation of Output Sideband Current with Low Frequency Voltage.

The following were held constant:

High Frequency = 9000 c.p.s.

High Frequency Volts per condenser = 40 and 120

R_1 = 110 ohms.

Low Frequency 100 c.p.s.

The voltage at Y was varied and the output at B observed.

Results: Diagram 26.

Equation (26) predicts a linear variation. The observed curve is certainly very nearly linear with a tendency to be concave downward which increases at the lower value of high frequency voltage. This point was not investigated further since this bending might have been caused by a zero error in the voltmeter.

5. Variation of Output Sideband Current with Frequency of Low Frequency Input.

The following were held constant:

High Frequency = 9000 c.p.s.

High Frequency Volts per condenser = 40

R_1 = 110 ohms

Low Frequency Voltage = 1.5 volt

The low frequency was varied and the output at B observed.

Results: Diagram 27.

PLATE XI

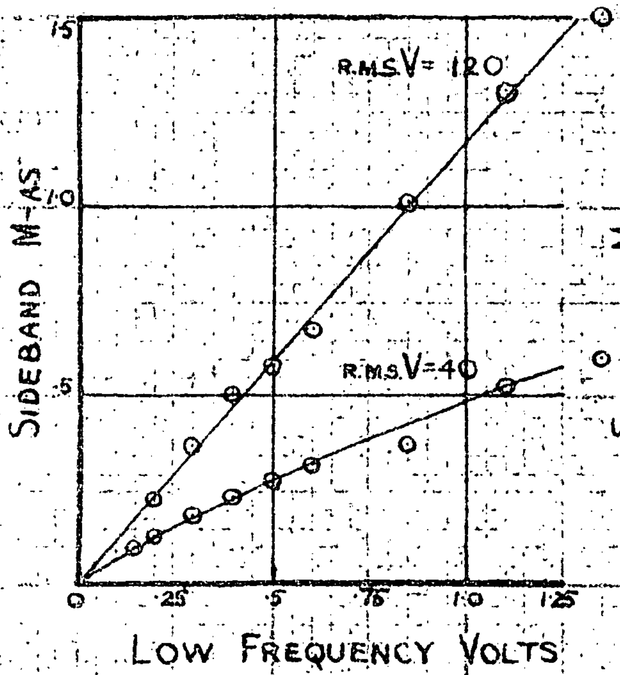


Diagram 26

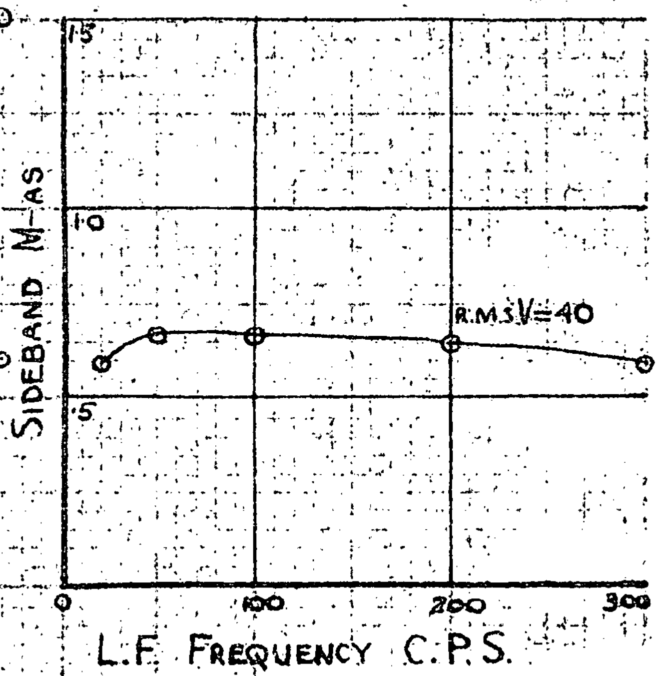


Diagram 27

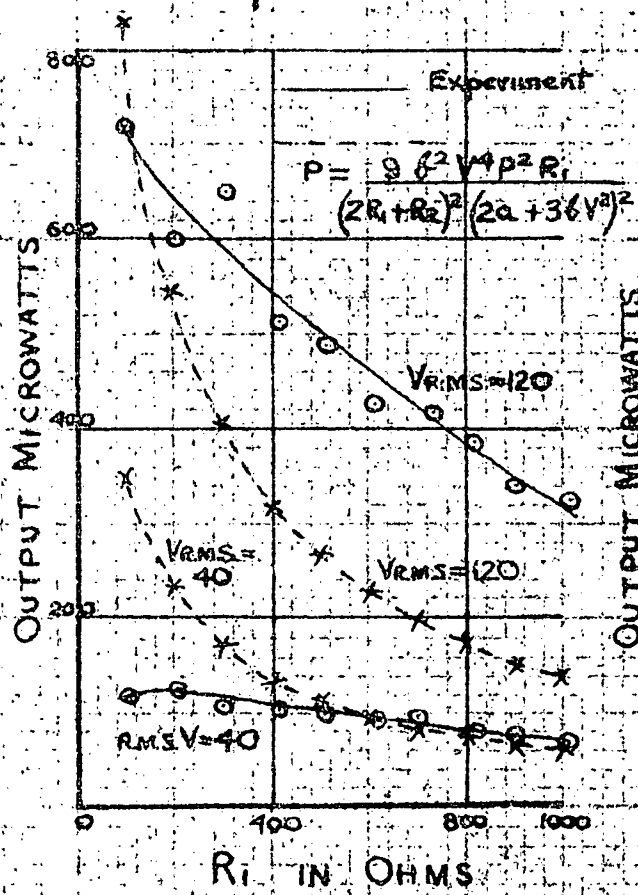


Diagram 28

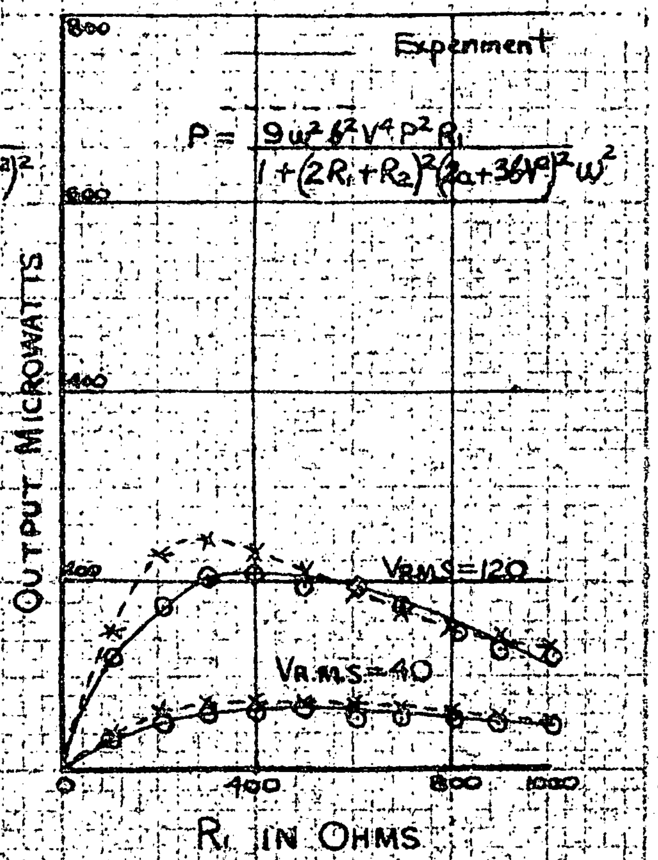


Diagram 29

Equation (26) predicts a horizontal straight line.

The curvature of the observed curve is more likely to have occurred from experimental than theoretical error. It was not investigated further.

6. Variation of Output Power with R_1.

The following were held constant:

High Frequency = 9000 c.p.s.

High Frequency Volts per condenser = 40 and 120.

Low Frequency = 100 c.p.s.

Low Frequency Volts = 1.5

R_2 = D.C. Resistance of one half of T = 70 ohms.

R_1 was varied and output at B observed.

Results: Diagram 28.

Equation (23) is also plotted and has used values of a and b given in diagram 18. The experiment was repeated for $L_1 = 0$ in which case (23) becomes:

$$P = \frac{9w^2 \, b^2 \, V^4 \, P^2 \, R_1}{1+(2R_1+R_2)^2(2a+3bV^2)^2 w^2} \qquad \text{.....................(29)}$$

Experiment and theory are given for this case in diagram 29.

In the latter case agreement between theory and practise is good but in the former agreement is poor. The absolute agreement between theory and practise within about 20% is not considered important since experimental error could possibly be this great. But in diagram 28 the errors greatly exceed this figure. No explanation is given for this divergence. It would seem incorrect to discard the theory entirely since agreement is good in diagram 29 and certainly the experimental curves are greatly different in diagram 28 to diagram 29 and the general qualitative features of the theoretical curves show the same differences. Nor will the consideration of condenser losses save equation (23) for this will lower rather than raise the theoretical curves.

It is thought that the next approach to this problem should be the careful alteration of L_1 and the plotting of output power as a function of R_1 as L_1 is changed from zero to the tuning condition of (19) which is considered to hold for diagram 28. The apparatus would have to be altered for this because at present L_1 is not easily controlled.

Since the factor between theory and practise from diagram 28 apparently depends on V, the variation of output power as a function of V for fixed R_1 was next done. $R_1 = 110$ was chosen since the power output was greatest for small resistances, this being the resistance of tuned circuit 2 without any added resistance.

7. Variation of Output Power with High Frequency Voltage.

The following were held constant:

High Frequency = 9000 c.p.s.

Low Frequency = 100 c.p.s.

Low Frequency volts = 1.5

R_1 = 110 ohms R_2 = 70 ohms.

The high frequency voltage was varied and the output at B observed.

Results: Diagram 30.

Equation (23) is again plotted.

The shape of the two curves agrees well except at low voltages. The divergence here may be assigned until extension of the theory to the neglection in the theory of the non linearity of the resistance component of the condensers at low voltages shown in diagram 17. This is the reason the low voltage theoretical points are not weighted as heavily in diagram 30.

Diagram 30 generally supports the validity of equation (23).

The low frequency input circuit was next investigated.

8. Variation of Low Frequency Input Currents with High Frequency Voltage.

The following were held constant:

High Frequency = 9000 c.p.s.

Low Frequency = 100 c.p.s.

Low Frequency Volts = 1.6

R_1 = 110 ohms R_2 = 70 ohms.

Results: Diagrams 31 and 32.

Equations (25) and (26) are plotted for comparison in diagrams 31 and 32 respectively.

The wide divergence between theory and experiment in diagram 31 was expected since the low frequency input resistive current will be very sensitive to the neglection of condenser losses. At low high-frequency voltages the theoretical points must again be lightly regarded. It is impossible to tell how much of the discrepancy would be eliminated by inclusion of condenser losses. The negative slope of the experimental curve at low high frequency volts is probably caused by non linearity

PLATE XII

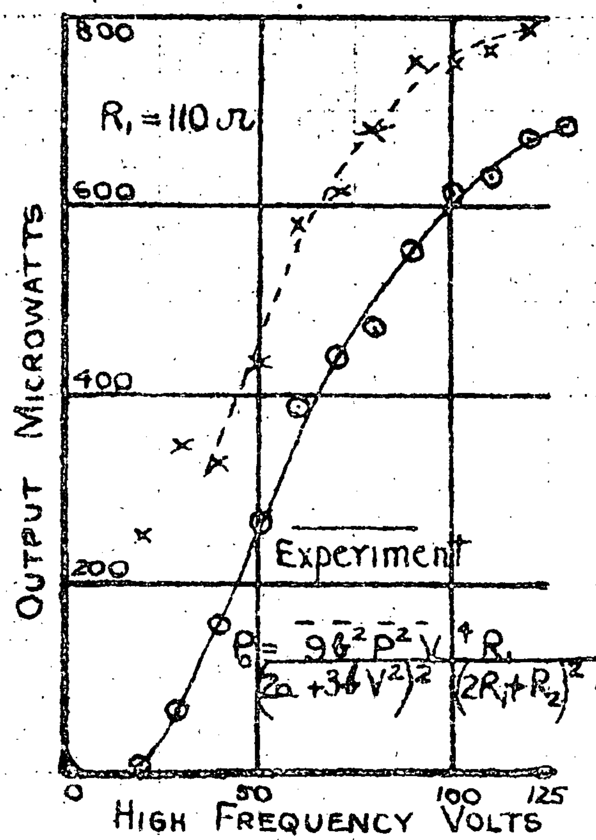


Diagram 30

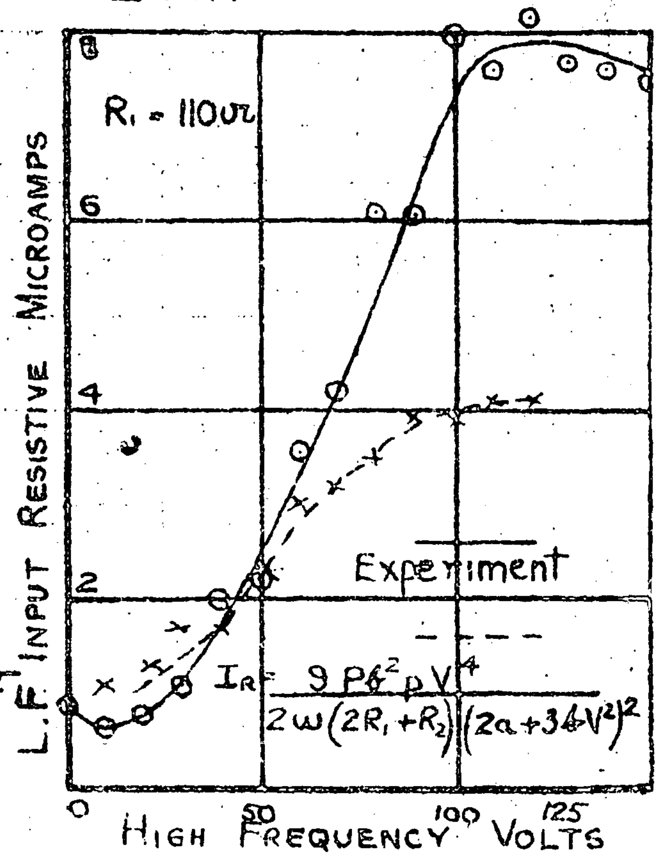


Diagram 31

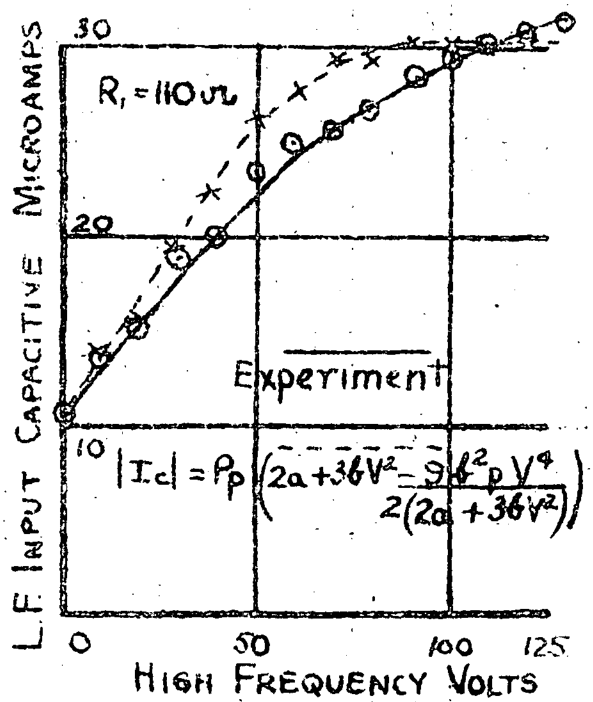


Diagram 32

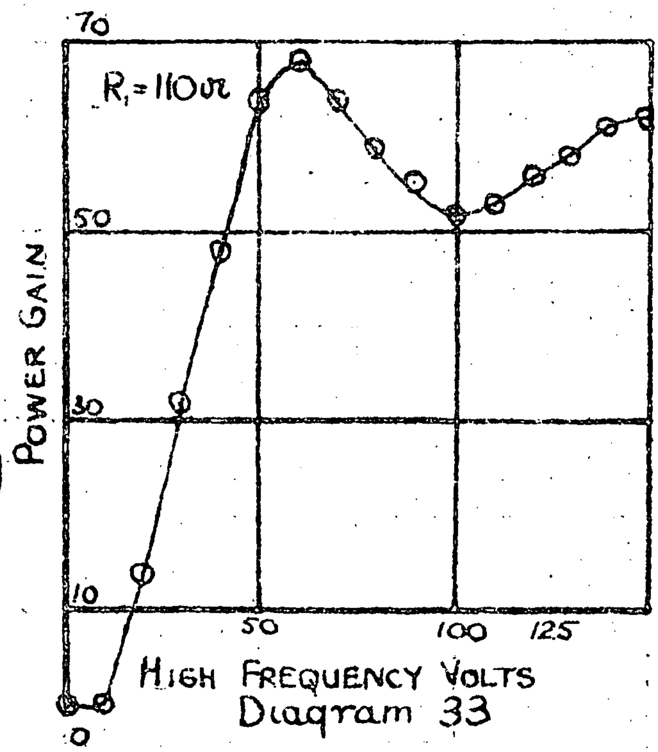


Diagram 33

of condenser resistance.

The agreement between theory and practise for the capacitive input current probably occurs because these currents in theory are almost independent of condenser losses.

9. Power Gain as a function of High Frequency Voltage.

The experimental curve of diagram 33 was obtained by passing a smooth curve through the input resistive current plots and through the output power plots (not given but similar to diagram 30), and obtaining the ratio of output to input power.

The corresponding theoretical curve is a horizontal line at height 135, $\left\{(180)\left(\frac{2R_1}{(R_2+2R_1)}\right)\right\}$, on the vertical axis (from no loss theory). The dropping of the experimental curve to zero at low volts was expected since the output power is negligible relative to condenser losses at low frequency, but the subsequent behaviour of the curve cannot have any theoretical explanation until an exhaustive analysis of non linear condensers has been done. However 70 of the theoretically available 135 is not considered unreasonable.

10. General Discussion of Results.

Although at least one experimental result (diag.28) cannot be explained by the theory even with a qualitative discussion of condenser losses, nevertheless the conclusions concerning the design of an amplifier, of the type described, drawn at the end of Chapter 5 have been confirmed by experiment. The magnitudes of the various input and output impedances must still be determined by experiment if exact magnitudes are necessary. However the general method given in this thesis predicts magnitudes to within about a factor of two.

It must be emphasized however that neither theory nor experiment has been exhaustive. Power gain has been checked at only one output load resistor, and the frequency of the high frequency voltage has been kept at all times at 9000 c.p.s. This was done because of the number oftuning changes necessary if this frequency was changed.

CHAPTER VII

CONCLUSIONS

1. Possibly the most important conclusion from the work done is that non linear condensers can in practise be used as power amplifiers as predicted by theory.
2. The greatest disadvantage of the condensers used in this research was their high losses. These losses make their theoretical analysis complex and reduce their usefulness.

The losses resulted in:

(a) Bulky cooling equipment. Because of the high temperature dependence of the characteristics of the non linear condenser constant temperatures are essential.

(b) Reduction of achievable power gain.

(c) A very low overall efficiency defined as total output power divided by total input power. This was of the order of .1%.

Two main lines of development could reduce these losses.

(a) The dielectric could be made thinner. Two possible methods were suggested in Chapter II, Sec.2. If a given capacity and a given field strength in the dielectric are required then the losses vary as the square of the dielectric thickness. A reduction of the present dielectric thickness by a factor of three which is considered possible for audio frequency condensers would reduce the losses by a factor of nine.

(b) The area of the hysteresis loop might be reduced by physical and chemical research. Combinations of materials might be superior to barium titanate.

3. As the condenser losses are reduced a more detailed check of the theory as given will be possible. In its present form it is sufficient for broad practical design of an amplifier. On the other hand the theory could be extended without great difficulty to include the condenser losses.

It should be mentioned that the extension of Van der Ziel's theoretical method was done independently by the author after Dr. Van der Ziel had left the University of British Columbia, and has not been completely checked by anyone.

4. Van der Ziel's analysis (21) using

$$Q = aV + bV^2$$

as the basic non linear condenser equation gave as a theoretically possible power amplification $\left(\frac{w}{P}\right)^2$ as against $2\left(\frac{w}{P}\right)$ derived in this thesis for a non linear condenser with the equation

$$Q = aV + bV^3$$

21. See Ref. 11.

In principle then the development of condensers obeying the former relation (rectangular hysteresis loops) would seem more promising. In the present condensers however the coefficient of V^2 in the expansion for Q is small (Chapter IV, Section 6), and the input power at low frequency to overcome feedback would be much smaller relative to the hysteresis component of input power than in the amplifier tested. However the greater possible gain might overcome this to give a greater overall than that achieved in this work. However disadvantages would be the necessity of about 100 volts of bias and the need to feed the low frequency input in push pull. Unfortunately time did not permit experiments on this type of amplifier.

5. The use of non linear condensers at high frequencies was not investigated. That they can be used at high frequencies was shown by Donley(22) in qualitative experiments at 20 megacycles and 40 megacycles. He used low voltages and a very thin chip as dielectric to give him about 100 m.m.fds. or less. He did not report overheating.

6. Since power amplification has been demonstrated possible with non linear condensers feed back may in principle be used to increase the amplification. It could be increased until low frequency oscillations were present provided a frequency control was supplied.

7. Dielectric amplifiers have all the advantages of magnetic amplifiers of ruggedness, and indefinite lifetime (this has not been verified), and the lack of heaters.

22. See Ref. 13.

In principle, in analogy with condensers and coils, it would seem dielectric could be made more efficient than magnetic amplifiers.

8. As a D.C. amplifier the dielectric amplifier might be most useful. The input power necessary to bias non linear condensers would be small since no hysteresis losses would be present. The theoretical gains are infinite for both types of dielectric amplifier mentioned in section 4 of this chapter, and a very high power gain might be achieved even with the present condensers which apparently have a relatively low D.C. resistance. The non linearity property might be used directly as an automatic control. The temperature dependence of the properties of non linear condensers and the jump to a linear condenser at the Curie temperature may also find application.

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