SPECTRAL DISTRIBUTION OF NOISE

by

William Arthur Bain

ABSTRACT

of

a thesis submitted in partial fulfilment of

the requirements for the degree of

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ABSTRACT

An apparatus for the measurement of noise as a function of frequency is described. This apparatus has been used to determine the spectral distribution of the excess noise caused by the flow of a d-c. current through a resistance. The samples used for the experiments were a zinc oxide semiconductor and two metal layer resistors. The frequency region investigated was from 10 kc. to 400 kc.

It was found that the excess noise in the ZnO semiconductor obeyed a $1/\nu^2$ law at room temperature while at lower temperatures (solid CO_2 and liquid nitrogen) it was proportional to $1/\nu$ at low frequencies and $1/\nu^2$ at high frequencies.

The excess noise in the metal layer resistors was proportional to $1/\nu$ at room temperatures while at 100°C there was a marked deviation from this law in the direction of a $1/\nu^2$ dependence for high frequencies.

The measurements show that the $1/\nu$ law gradually changes to a $1/\nu^2$ law at high frequencies in accordance with the theory proposed recently by Dr. A. van der Ziel. They also indicate that the correlation times involved are a function of temperature; the exact nature of this dependence has yet to be determined.

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CHAPTER I

INTRODUCTION

The random motion of electrons in a conductor causes small fluctuating potential differences to be developed across the terminals of the conductor. This is known as Thermal Noise and it can be described by a noise e.m.f. in series with R whose mean square value, for a frequency interval $\Delta\nu$, is given by the formula

$$\overline{e^2} = 4kTR\Delta\nu$$

where k is Boltzmann's constant, T is the absolute temperature in degrees Kelvin, and R is the resistance. The resistance R may depend on frequency in which case one must use the value of R for the particular frequency interval under investigation. This formula, due to Nyquist, has been carefully checked and is found to be true for frequencies up to the infra-red region of the spectrum. It has been proved theoretically from thermodynamical considerations (1),(2) and has been shown experimentally (3) that this noise is independent of the type of resistance whether it be carbon, composition, wire-wound, thin metal layer, or semi-conducting.

The formula does not always hold if a d-c current is passing through the resistance. In general it can be said that if a d-c current is passing through R there is an excess noise generated that increases with increasing current. Usually this increase is as the square of the current. One can describe this effect by introducing an additional noise e.m.f. $\overline{e_a^2}$ in series with R so that for a small frequency interval $\Delta\nu$

$$\overline{e_a^2} = I^2 f(\nu)\,\Delta\nu$$

where the function $f(\nu)$ describes the frequency dependence of this excess noise. It is usually of the form

$$f(\nu) = \frac{\text{constant}}{\nu}$$

for a fairly wide frequency range. For a wire-wound resistor we have

$$f(\nu) \equiv 0.$$

The object of this research is to measure $f(\nu)$ as a function of the frequency ν and to measure $\overline{e_a^2}$ as a function of current for various materials at different temperatures.

In order to measure $f(\nu)$ the noise in a certain frequency band of width $\Delta\nu$ (small in comparison to the frequency ν) is amplified and detected by a quadratic detector. If D_1 is the deflection of the meter for $I = 0$ and D_2 the deflection for a fixed current I then:

$$\frac{D_2}{D_1} = \frac{\overline{e^2} + \overline{e_a^2}}{\overline{e^2}} = 1 + \frac{I^2 f(\nu)}{4kTR.}$$

so that:

$$f(\nu) = \frac{4kTR}{I^2} \quad \frac{D_2 - D_1}{D_1.}$$

As soon as D_1 and D_2 have been measured for different frequencies $f(\nu)$ can be calculated as a function of frequency. If D_2 is much larger than D_1 one might use a calibrated attenuator to bring the reading of the output meter down to a reasonable value. If A is the attenuation factor then one has to substitute AD_2 for D_2 in the above equation.

CHAPTER II

APPARATUS

A block diagram of the apparatus used is shown in Figure I. The detailed wiring diagram of the pre-amplifier is shown in Figure II. The high tension and the filament current are supplied by a regulated power supply of conventional design. The frequency response of the pre-amplifier itself was found to be essentially flat between 1 kc. and 400 kc., the half-power frequencies being at 500 cycles and 500 kc. When the apparatus was first tested the noise resistance was found to be 3000 ohms. This has been reduced

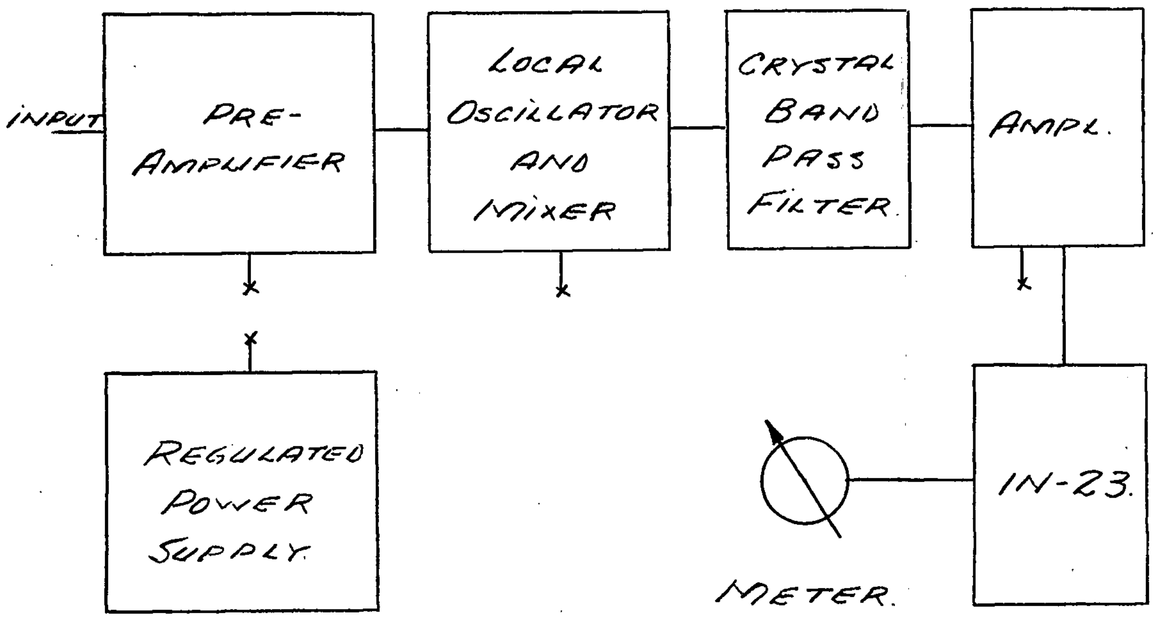


FIGURE I

to 800 ohms by using wire-wound resistors in the plate circuits of the first two stages. The maximum voltage gain of the pre-amplifier is 40,000.

The detailed wiring diagram of the local oscillator and mixer, crystal band-pass filter, and amplifier is shown in Figure III. The frequency range of the local oscillator is from 465 kc. to 1300 kc. The crystal band-pass filter is tuned at 456 kc. and has a width of 200 cycles. This allows us to examine the noise in a narrow band for all frequencies from about 10 kc. up to the point where the response of the circuit as a whole falls to a very low value. This upper limit of frequency is in the neighborhood of 400 kc.

A 1N-23 crystal diode is used as a quadratic detector. Its quadratic properties have been checked several

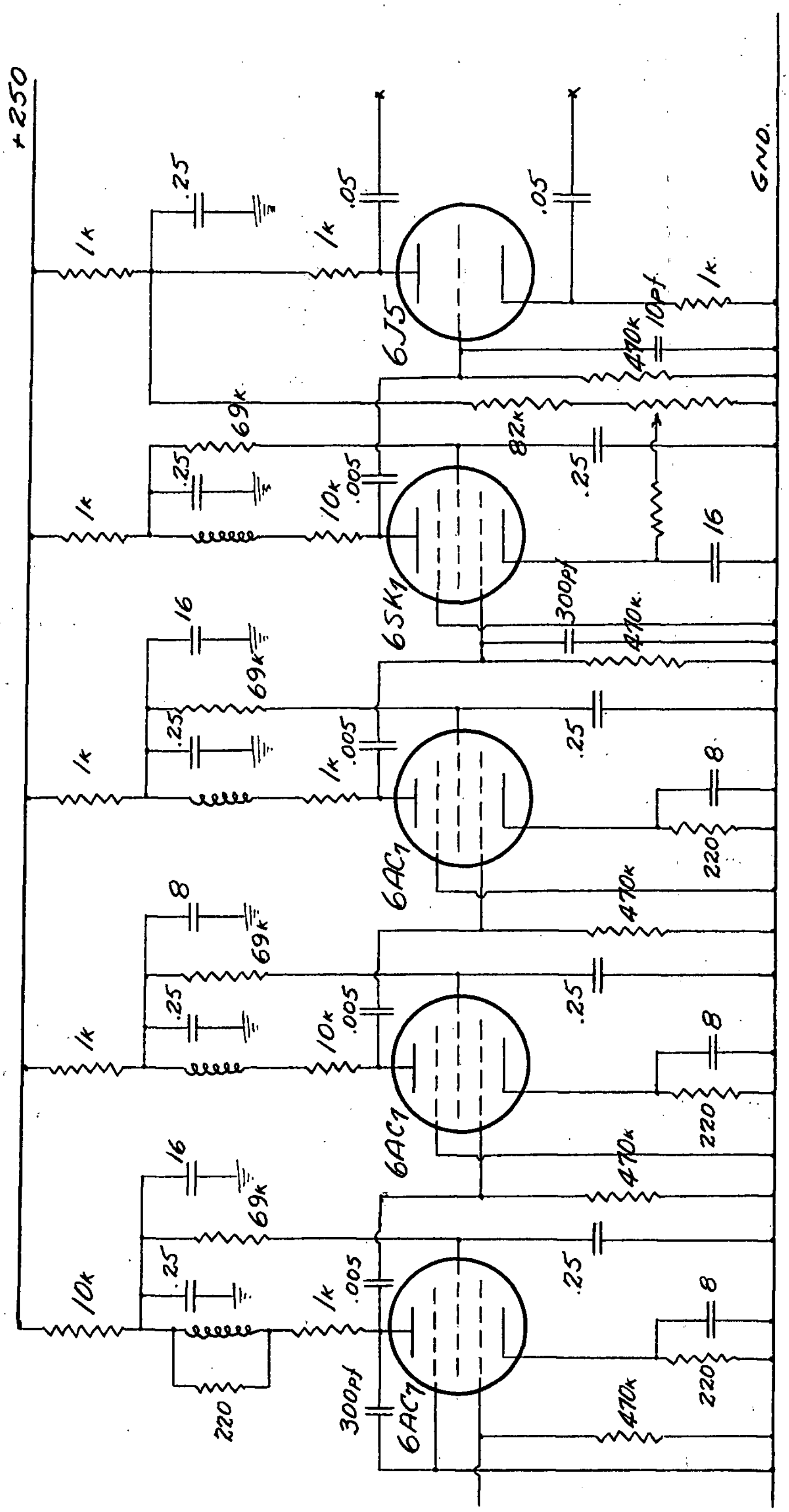


FIGURE II PRE-AMPLIFIER

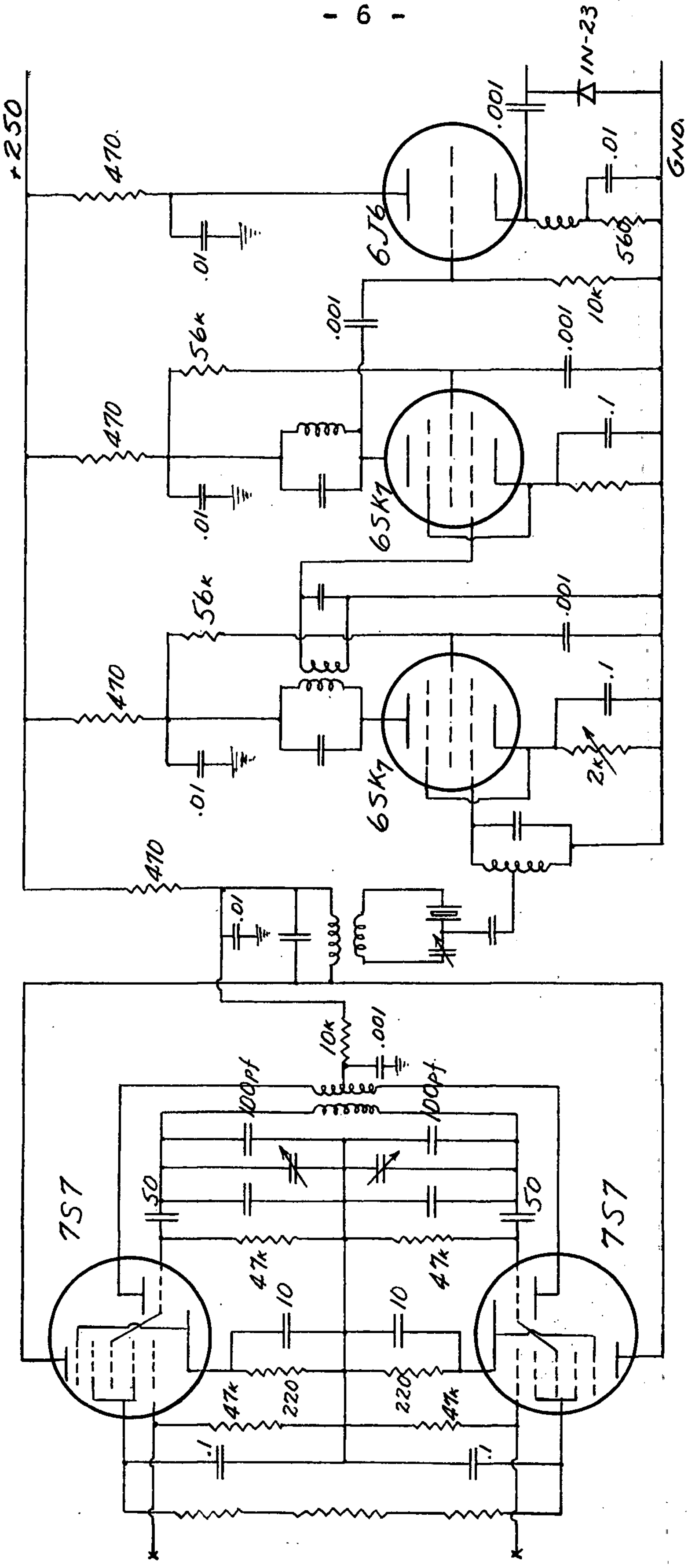

+250
470
.01
56k
.001
6J6
.001
1N-23
GND.
.01
560
10k
.001
6SK7
.1
6SK7
2k
470
.01
10k
.001
100pf
100pf
7S7
7S7
50
50
47k
47k
10
10
220
220
47k
47k
.1
.1

FIGURE III

time during the course of the research.

The output meter is a Rubicon galvanometer with a ten-centimeter scale and a sensitivity of 0.0056 microamperes per millimeter. (0.56 microamperes full scale)

A diagram of the circuit used for supplying the d-c. current to be passed through the sample is shown in Figure IV.

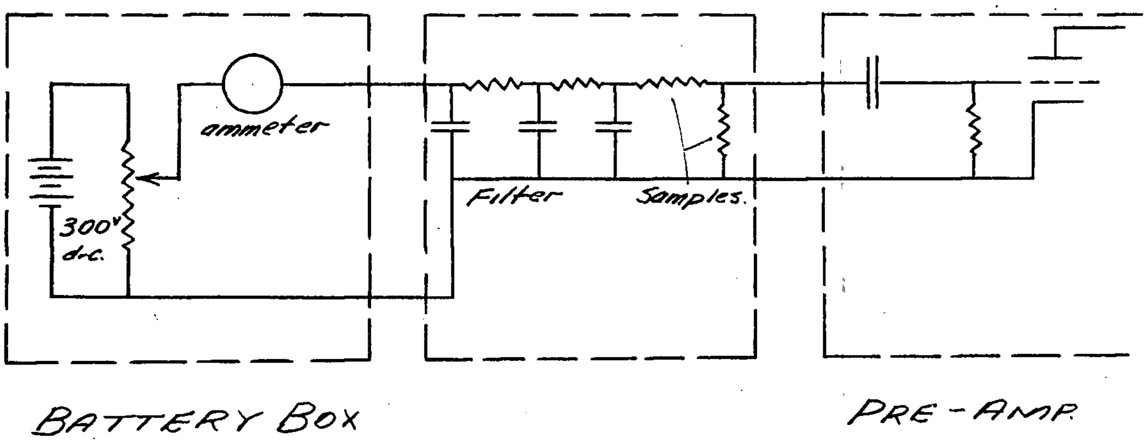


FIGURE IV

Seven 45-volt dry cells are used to supply the necessary d-c voltage. These batteries and the ammeter are enclosed in a grounded metal box. From this battery box the d-c. current is fed through a shielded cable to another grounded metal box where it is filtered and then allowed to pass through the sample under investigation. These precautions were taken to eliminate the possibility of feed-back through the power supply and to minimize the effect of any stray electrical disturbances in the vicinity. As a further precaution against these stray electric fields (due to fluorescent lights, etc.) all the research was done inside a cage made of two layers of chicken-wire mounted on a wooden frame. Both these layers are securely grounded and power is brought into the cage by means of an isolating transformer. Even with these precautions the apparatus has a tendency to be unstable at times. It is not known whether this is due to stray fields or to some faulty component in the apparatus itself. These periods of instability, however, are very infrequent and short-lived.

During normal operating conditions the apparatus is quite steady but shows a slow variation of gain so that over a period of say 30 minutes the reading of the galvanometer may change by as much as 10%. It was found that when the apparatus was left untouched for a period of many hours this drift never exceeded 10% and data could be reproduced from day to day within this limit.

The galvanometer has a rated time constant of three seconds. This was increased to approximately 30 seconds by inserting three stages of filtering in front of the galvanometer. Each stage consists of a 5000 ohm resistor in series with a 2000 microfarad condenser. This was done to damp out some very rapid fluctuations of the reading. The cause of these is believed to be due to the fact that resistors give out bursts of extra noise over and above those expected. It has been said that these bursts may be as large as 300% of the average value.

In order to show that the apparatus was working properly the following test was performed. Nyquist's formula:

$$\overline{e^2} = 4kTR\Delta\nu$$

or:

$$\overline{i^2} = \frac{4kT\Delta\nu}{R}$$

shows that the mean square value of the noise e.m.f. increases linearly with resistance. However, in any practical circuit this resistance is in parallel with both the input capacity and the grid resistance of the first stage of the amplifier. This grid resistor also produces a noise voltage.

Consider the circuits shown in Figure V and Figure VI where C is the input capacity, R_g is the grid resistance and R_1 is the resistance of the sample under consideration.

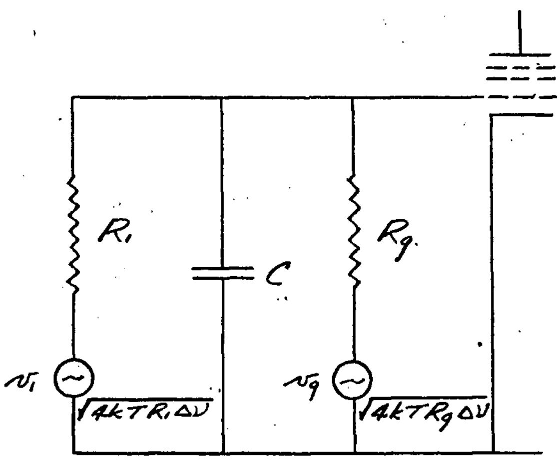


FIGURE V

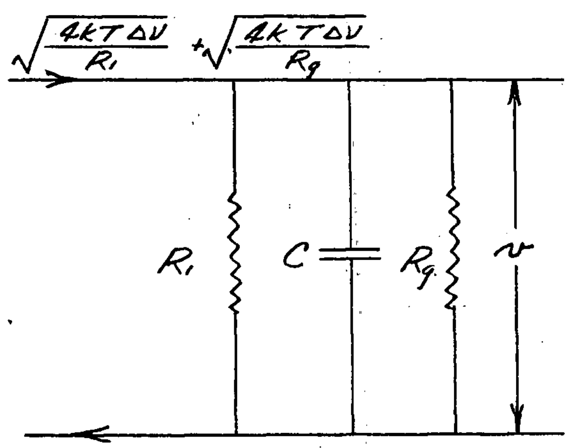


FIGURE VI

Figure V shows the actual circuit in the amplifier and Figure VI shows the equivalent circuit.

The voltage v developed between the grid and ground is given by the formula:

$$v = \left(\sqrt{\frac{4kT\Delta\nu}{R_1}} + \sqrt{\frac{4kT\Delta\nu}{R_g}}\right)\frac{1}{\frac{1}{R_1} + \frac{1}{R_g} + j\omega C} \qquad \underline{1.}$$

and the mean square value of the voltage is given by:

$$\overline{v^2} = \left(\frac{4kT\Delta\nu}{R_1} + \frac{4kT\Delta\nu}{R_g}\right)\frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_g}\right)^2 + \omega^2 C^2}$$

$$= 4kT\Delta\nu R_g \frac{\frac{R_1}{R_g}\left(1 + \frac{R_1}{R_g}\right)}{\left(1+\frac{R_1}{R_g}\right)^2 + \omega^2 C^2 R_g^2 \left(\frac{R_1}{R_g}\right)^2} \qquad \underline{2.}$$

$$= \frac{\beta x (1+x)}{(1+x)^2 + \omega^2 C^2 R_g^2 x^2}$$

where

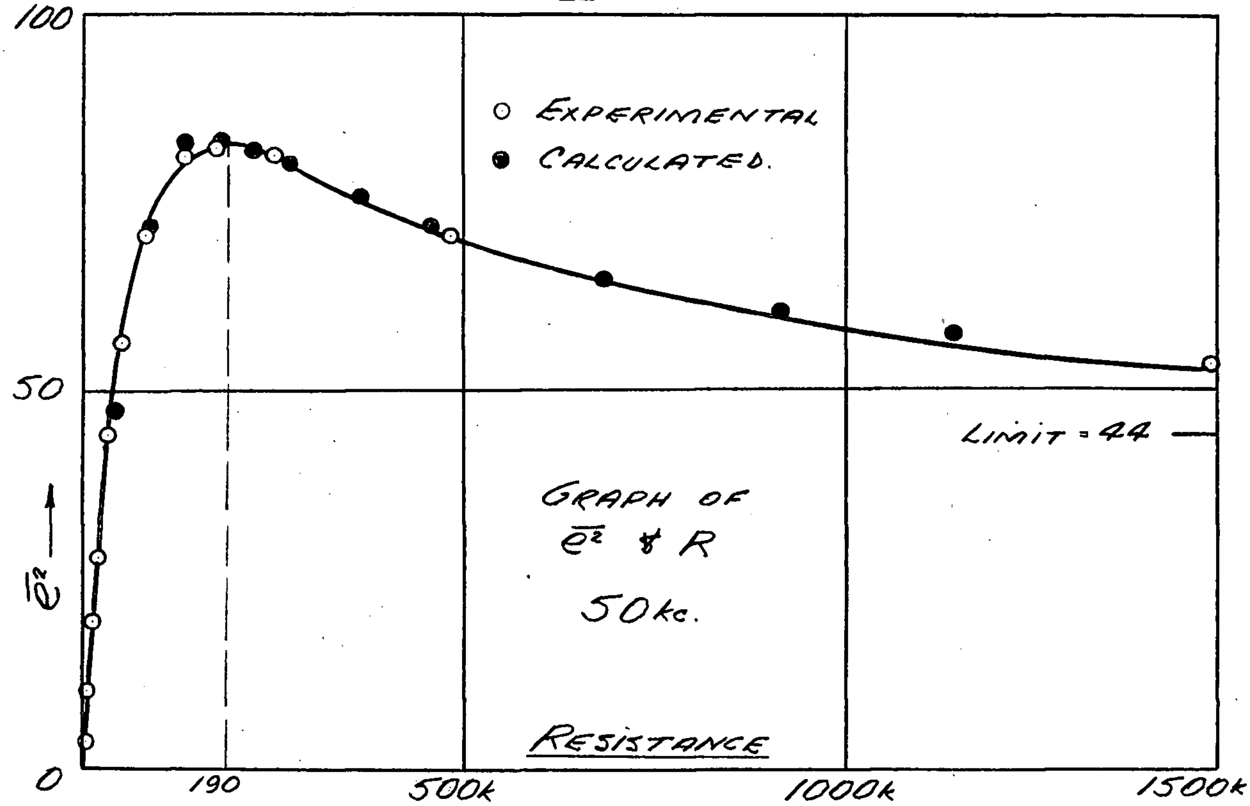
$$x = \frac{R_1}{R_g} \qquad \beta = 4kTR_g\Delta\nu \qquad \underline{3.}$$

This is the equation of a family of curves and, if $\omega C R_g > 1$, there is a maximum when:

$$x = \frac{1}{\omega C R_g - 1} \qquad \underline{4.}$$

This maximum value is:

$$\overline{v_m^2} = \frac{\beta}{2\omega C R_g} \qquad \underline{5}$$



Graph No. 1

If we let x approach infinity (i.e. leave the input open and R_1 becomes infinite) we have

$$\overline{v_\infty^2} = \frac{\beta}{1 + \omega^2 C^2 R_g^2} \qquad \underline{6.}$$

By plotting a graph of mean square voltage versus R_1 it is possible to obtain the maximum value of the curve and we can then solve equations 5 and 6 for the two unknowns β and C. This has been done and the results are shown on Graph No.1 for a frequency of 50,000 cycles. The R_g has a value of 460,000 ohms and maximum value of $\overline{v^2}$ is when R_1 = 190,000 ohms. Using this data we get a value of 24 micro-micro-farads for C which is quite reasonable for a circuit of this type.

Substituting the known values for C and β into equation 2 allows us to calculate a curve. This theoretical curve is also plotted on Graph No.1 and its close resemblance to the actual curve is a good indication that the apparatus is working properly.

CHAPTER III

SAMPLES

The metal layer resistors used were obtained from Continental Carbon Incorporated. They are ordinary 50,000-ohm precision resistors and consist of a thin metalized layer deposited on a ceramic material.

The zinc oxide semi-conductor was made in the laboratory as follows: zinc oxide powder was placed in a mould and subjected to a pressure of 10,000 pounds per square inch. The resulting block of material ($2'' \times \frac{3}{8}'' \times \frac{1}{8}''$) was heated to 1200°C at a rate of 100°C per hour and held there for twelve hours. It was allowed to cool over a period of twenty-four hours. After cooling silver contacts were painted on the ends. Platinum leads were joined to these contacts and the whole assembly was placed in a glass tube and sealed.

CHAPTER IV

THEORY

A d-c. current flowing through a resistor causes an extra noise which, for a small frequency interval $\Delta\nu$, is given by the formula:

$$\overline{e_a^2} = I^2 f(\nu)\, \Delta\nu \qquad 1.$$

where I is the d-c. current and $f(\nu)$ is a function describing the frequency dependence of this induced noise.

We can explain the I^2 dependence of the noise if we assume that the resistance shows spontaneous fluctuations in value. If the fluctuation in resistance is ΔR then an extra noise

$$\Delta E = I \Delta R \qquad 2.$$

is developed across the resistance R. The mean square value of this noise is

$$(\Delta E)^2 = I^2 (\Delta R)^2 \qquad 3.$$

To explain these fluctuations in resistance we turn to the definition of conductivity as given by the electron theory of matter. The conductivity σ is:

$$\sigma = \frac{ne^2 L}{2m}\left(\frac{1}{v}\right) \qquad 4.$$

where n is the number of free electrons per cubic centimeter, e the electronic charge, L the free path length of the electrons, and v the velocity of the electrons. The fluctuations of resistance may be due to a fluctuation in the number of free electrons per cubic centimeter. There is also the possibility that the free path length L or the velocity v of the electrons show fluctuations. To take all these possibilities into account we introduce a fluctuation in the conductivity as follows:

$$\Delta\sigma = \sigma - \sigma_0 \qquad \overline{\Delta\sigma} = 0 \qquad \underline{5.}$$

This gives

$$\Delta R = \left(\frac{1}{\sigma} - \frac{1}{\sigma_0}\right) = -\frac{\Delta\sigma}{\sigma\sigma_0} \doteq -\frac{\Delta\sigma}{\sigma_0^2} \qquad \underline{6.}$$

to a good approximation. We now have for the value of ΔE :

$$\Delta E = -\frac{I}{\sigma_0^2}\,\Delta\sigma \qquad \underline{7.}$$

We must carry out a Fourier analysis of ΔE and calculate $\overline{e_a^2}$.

Consider a fluctuating quantity X(t) which is known for the interval $0<t<t_0$ where t_0 is large but finite. If we assume that X(t) is a continuous function in the interval we may develop it into a complex Fourier series as follows:

$$X(t) = \sum_{n=-\infty}^{\infty} a_n e^{j\omega_n t} = a_0 + \sum_{n=1}^{\infty}\left(a_n e^{j\omega_n t} + a_{-n} e^{-j\omega_n t}\right) \qquad \underline{8.}$$

$$a_n = \frac{1}{t_0}\int_0^{t_0} X(t)\, e^{-j\omega_n t}\, dt$$

$$a_{-n} = \frac{1}{t_0}\int_0^{t_0} X(t)\, e^{j\omega_n t}\, dt \qquad \underline{9.}$$

$$\omega_n = \frac{2\pi n}{t_0}$$

$$a_0 = 0 \text{ because } \overline{X(t)} = 0$$

If a_n^* is the complex conjugate of a_n we see that $a_{-n} = a_n^*$

The Fourier component x_n of frequency ω_n is

$$x_n = a_n e^{j\omega_n t} + a_{-n} e^{-j\omega_n t} \qquad \underline{10.}$$

and its mean square value is

$$\overline{x_n^2} = \overline{a_n^2 e^{2j\omega_n t}} + \overline{2a_n a_{-n}} + \overline{a_{-n}^2 e^{-2j\omega_n t}}$$

$$= \overline{2a_n a_{-n}} \qquad \underline{11.}$$

$$= \overline{2a_n a_n^*}$$

because all terms containing the time to drop out when the average is taken. We now have to calculate the value of $\overline{2\, a_n\, a_n^*}$. From equation 9 we obtain:

$$\overline{2a_n a_n^*} = \overline{x_n^2} = \int_0^{t_0}\!\!\int_0^{t_0} \overline{X(u)X(v)}\, e^{-j\omega_n u + j\omega_n v}\, du\, dv \qquad \underline{12.}$$

We now change to a new variable w.

$$w = v - u \qquad 13.$$

Equation 12 becomes:

$$\overline{2a_n a_n^*} = \frac{2}{t_0^2}\int_0^{t_0} du \int_{-u}^{t_0-u} \overline{X(u)\,X(u+w)}\; e^{j\omega_n w}\, dw \qquad 14.$$

where $\overline{X(u)\;X(u+w)}$ is independent of u and is a function of w only. We also know that $\overline{X(u)\;X(u+w)}$ is symmetrical in w and equals zero for $|w| > \delta$ where δ is a measure of the correlation time. Therefore if $t_0 \gg \delta$ equation 14 may be rewritten as:

$$\overline{2a_n a_n^*} = \frac{2}{t_0^2}\int_0^{t_0} du \int_{-\delta}^{\delta} \overline{X(u)\,X(u+w)}\; e^{j\omega_n w}\, dw$$

$$= \frac{2}{t_0}\int_{-\infty}^{\infty} \overline{X(u)\,X(u+w)}\; e^{j\omega_n w}\, dw \qquad 15.$$

$$= \frac{4}{t_0}\int_0^{\infty} \overline{X(u)\,X(u+w)}\; e^{j\omega_n w}\, dw$$

And, if we put $\frac{1}{t_0} = \Delta\nu$, equation 15 becomes:

$$\overline{2a_n a_n^*} = 4\Delta\nu\int_0^{\infty} \overline{X(u)\,X(u+w)}\; e^{j\omega_n w}\, dw$$

$$= 4\Delta\nu\int_0^{\infty} \overline{X(u)\,X(u+w)}\; \cos\omega_n w\, dw. \qquad 16$$

since the imaginary term contributes nothing to the value of the integral. Equation 16 may be more conveniently written as:

$$\overline{2a_n a_n^*} = 4\Delta\nu\,\overline{X(u)^2}\int_0^\infty c(w)\cos\omega_n w\,dw \qquad \underline{17}$$

where:

$$c(w) = \frac{\overline{X(u)X(u+w)}}{\overline{X(u)^2}} \qquad \underline{18}$$

and is called the "normalized" correlation coefficient. We see that

$$c(w) = 1 \quad \text{for} \quad w = 0$$

$$c(-w) = c(w)$$

$$\text{and} \quad c(w) = 0 \quad \text{if} \quad |w| \gg \tau.$$

where τ is the correlation time of the fluctuations. When we consider the case of a fluctuating quantity which is caused by a large number of independent and random events τ is the duration of the event (e.g. the transit time of an electron in a radio tube). In the case of fluctuations involving decay problems τ is a measure of the average life of the decaying quantity.

We now have for the mean square voltage in a small frequency interval $\Delta\nu$

$$\overline{e_a^2} = \frac{4I^2\overline{(\Delta\sigma)^2}\,\Delta\nu}{\sigma_0^4}\int_0^\infty c(w)\cos\omega_n w\,dw$$

$$= I^2 f(\nu)\,\Delta\nu \qquad \underline{19}$$

The spectral distribution function $f(\nu)$ is completely determined by the correlation function c(w). for we find by applying a Fourier transform:

$$c(w) = \frac{1}{2\pi\,\overline{X(u)}^2} \int_0^\infty f(\omega_n) \cos \omega_n \, d\omega_n \qquad \underline{20.}$$

It is evident from this equation that it is not possible for $f(\omega)$ to be of the form

$$f(\omega) = \frac{\text{constant}}{w.} \qquad \underline{21.}$$

for $0 < \omega < \infty$. If we substitute 21 into 20 we obtain an integral that is divergent at $\omega = 0$ for all values of w and is also divergent at $\omega = \infty$ for w = o. To ensure the convergence of 20 for all frequencies we must impose the following restrictions:

a) $f(\nu)$ must vary slower than $1/\nu$ for very low frequencies.

b) $f(\nu)$ must vary faster than $1/\nu$ for very high frequencies.

Of course the $1/\nu$ law is satisfactory for intermediate frequencies.

It has usually been assumed that the correlation function represents an exponential decay of half-life τ. i.e.

$$c(w) = e^{-\frac{|w|}{\tau}} \qquad \underline{22.}$$

and hence, from equation 19 we obtain:

$$f(\nu) = \text{constant} \int_0^\infty e^{-\frac{|w|}{\tau}} \cos \omega w \, dw$$

$$= \text{constant} \frac{\tau}{1 + \omega^2 \tau^2} \qquad \underline{23.}$$

This states that $f(\nu)$ is independent of frequency at low frequencies and varies as $1/\nu^2$ at high frequencies. This relationship is in marked contrast to the experimental results where one usually finds that the noise varies as $1/\nu$ in a large frequency range.

A solution to this problem has been proposed recently by Dr. A. van der Ziel. (4) Instead of using a single correlation time τ we introduce a wide distribution of correlation times.

Let

$$dP = g(\tau) d\tau \qquad \left(\int_0^\infty g(\tau) d\tau = 1 \right) \qquad \underline{24.}$$

be the probability of a correlation time between τ and $\tau + d\tau$. This gives for $f(\nu)$ instead of 23:

$$f(\nu) = \text{constant} \int_0^\infty \frac{\tau}{1 + \omega^2 \tau^2} g(\tau) d\tau \qquad \underline{25.}$$

Of course by a proper choice of $g(\tau)$ one can always obtain agreement between theory and experiment even if one starts with the wrong correlation function. Therefore we can attribute physical meaning to the whole procedure only if there are sound arguments in favour of the distribution function chosen. It will be shown later that this is actually the case.

If we introduce the following normalized distribution function we obtain the $1/\nu$ law exactly. Let

$$g(\tau)d\tau = \left(\ln \frac{\tau_2}{\tau_1}\right)^{-1} \frac{d\tau}{\tau} \qquad \text{for } \tau_1 < \tau < \tau_2.$$

$$g(\tau)d\tau = 0 \qquad \text{for} \quad \tau < \tau_1 \text{ and } \tau > \tau_2 \tag{26}$$

and substitute it into equation 25. This gives

$$f(\nu) = \text{constant} \left(\ln \frac{\tau_2}{\tau_1}\right)^{-1} \int_{\tau_1}^{\tau_2} \frac{d\tau}{1 + \omega^2 \tau^2} \tag{27}$$

$$= \text{constant} \left(\ln \frac{\tau_2}{\tau_1}\right)^{-1} \frac{\arctan \omega\tau_2 - \arctan \omega\tau_1}{\omega}.$$

This gives a value of $f(\nu)$ that is independent of ν for very low frequencies, varies as $1/\nu$ for intermediate frequencies, and varies as $1/\nu^2$ for very high frequencies. One can extend the $1/\nu$ region asfar as is necessary by a proper choice of τ_1 and τ_2. We now have an expression for $f(\nu)$ that satisfies conditions a) and b) on Page 18.

The introduction of a distribution of correlation times is reasonable when we consider that in the theory of dielectric losses (which is also a problem of solid state physics (5)) the correlation time is given by

$$\tau = \tau_0 e^{E/kT} \tag{28}$$

where E is the activation energy. It is evident that a rather narrow distribution of E will give quite a large distribution in τ because kT is a small quantity at room temperature. Of course, it is not to be construed from these remarks that this is the solution to the problem of the spectral distribution of noise - equation 28 is introduced for the sole purpose of giving a physical meaning to a distribution of correlation times.

We might also turn to the field of Nuclear Physics where we have that the half-life of an excited state is given by an equation of the form

$$\tau = \tau_0 \, e^{-\frac{\epsilon}{kT}} \qquad \underline{29.}$$

where E is the excitation energy. Again, a narrow variation of E will give rise to a wide distribution in the values of τ.

Equations 28 and 29 both show that the values of τ (and thus the form of $f(\nu)$) might be dependent on temperature. This variation of the frequency dependence of the noise with temperature will, of course, be the governing factor in the choice of a suitable expression for τ.

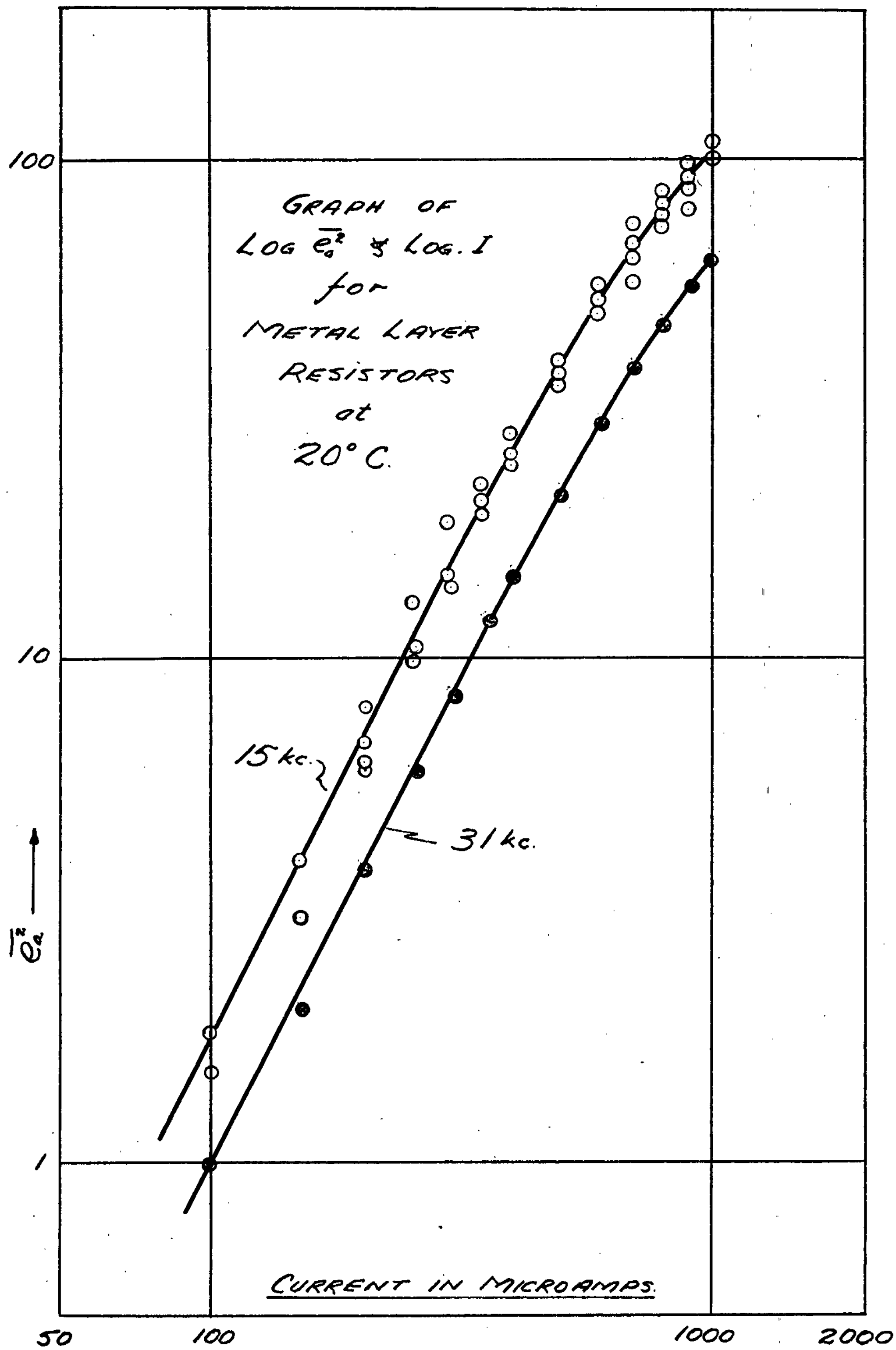
CHAPTER V

RESULTS

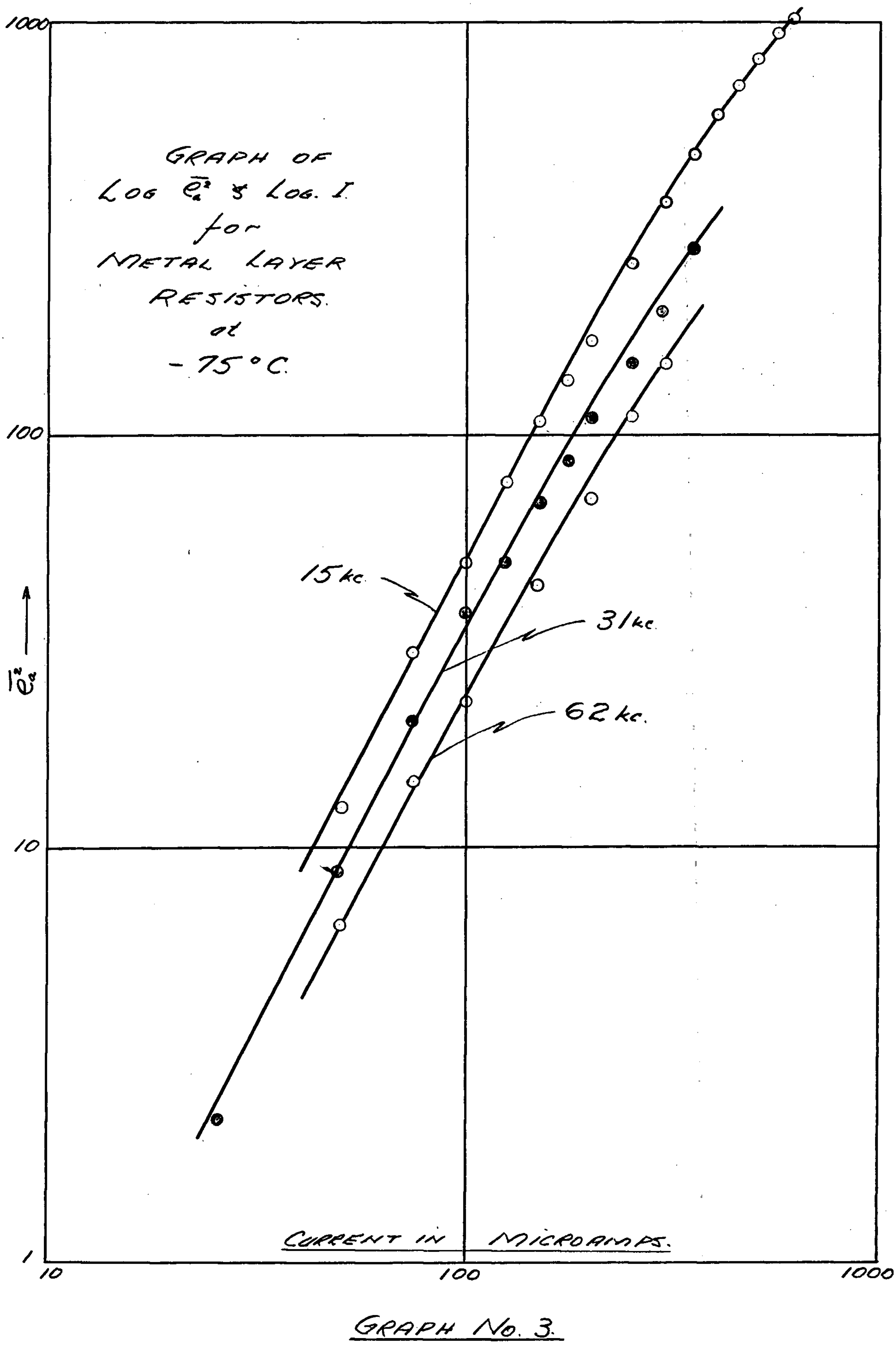
<u>Excess noise as a function of current at a constant frequency.</u>

Graph No.2 shows, on logarithmic coordinates, the excess noise in the metal layer resistors as a function of the d-c. current flowing. The readings were taken at room temperature. For low values of the current the noise increases as the square of the current while at higher currents it increases less rapidly with current. From the graph it can also be seen that the shapes of the curves are independent of frequency in the frequency range examined. Bernamont (6) found these same results for an even wider frequency range.

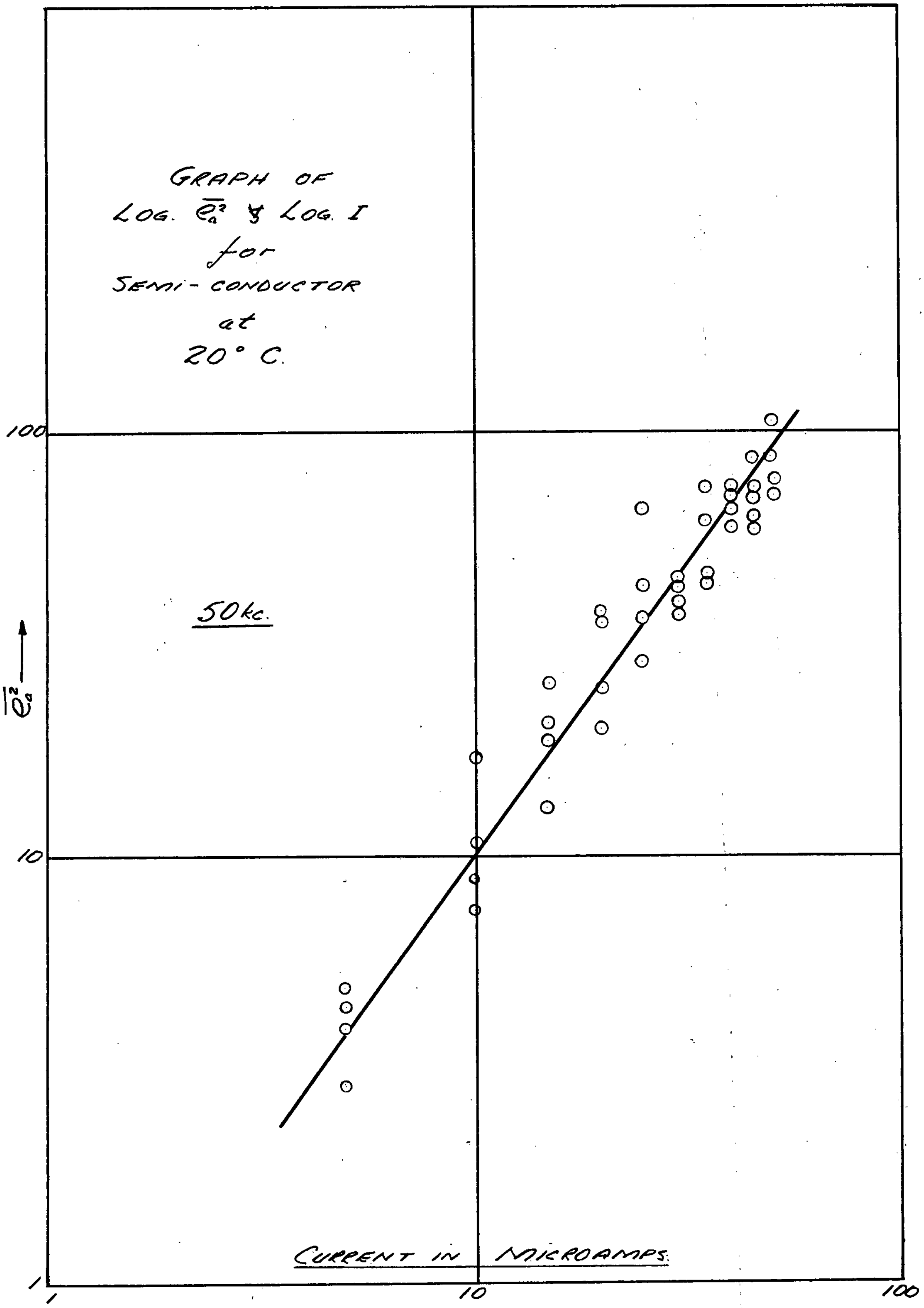
All the readings taken for the noise at 15 kc. are plotted on the graph. These readings were obtained over a period of several hours. For the sake of clarity only the average values of the readings are plotted for 31 kc. The noise values on this and all subsequent graphs are plotted in arbitrary units. All curves on the same graph are plotted to the same scale but there is no relation between the magnitudes of the noise on each separate graph. The actual units of noise are $(\text{volts})^2$ per cycle but here the noise is simply measured in millameters deflection of the galvanometer or multiples thereof. Since the apparatus is quadratic and since the band-pass is constant this unit of measurement is correct but, of course, the absolute magnitude is dependent


GRAPH OF
LOG $\overline{e_a^2}$ & LOG. I
for
METAL LAYER
RESISTORS
at
20° C.
100
10
1
15 kc.
31 kc.
$\overline{e_a^2}$
CURRENT IN MICROAMPS.
50
100
1000
2000

GRAPH No. 2


GRAPH OF
LOG $\bar{e}_a^2$ & LOG. I
for
METAL LAYER
RESISTORS
at
- 75°C
15kc.
31kc.
62kc.
$\bar{e}_a^2$
1000
100
10
1
10
100
1000
CURRENT IN MICROAMPS.

GRAPH No. 3.


GRAPH OF
LOG. $\overline{e_a^2}$ & LOG. I
for
SEMI-CONDUCTOR
at
20° C.
50kc.
100
10
1
$\overline{e_a^2}$
CURRENT IN MICROAMPS.
1
10
100

GRAPH No. 4

on the gain of the apparatus which can be varied over a wide range.

Graph No. 3 shows the excess noise in the same metal layer resistors for a temperature of about -75°C (solid carbon dioxide). Only the average points have been plotted. The resistance at this temperature is almost the same as at room temperature. Again we notice that the shapes of the curves are independent of frequency and that the slopes at low currents are approximately 2. We also see that the slopes decrease as the current rises.

Graph No. 4 is a plot of the excess noise in the semi-conductor as a function of d-c. current at room temperature. The resistance at this temperature is 600,000 ohms. The semi-conductor was connected in series with a one-megohm wire-wound resistor. This wire-wound resistor contributed nothing to the excess noise (since $f(\nu) \equiv 0$) but its presence was necessary to keep all the semi-conductor noise from being grounded through the filter. The slope of this curve is approximately 1.4 over the whole range of current. The noise for lower currents could not be determined with any degree of accuracy because of a large variation in the readings. It was found that the lower the currents used the more erratic were the readings. This fact was also noticed by Bernamont (6).

<u>Excess noise as a function of frequency for a resistor carrying a constant current.</u>

Graph No. 5 shows, on logarithmic coordinates, the excess noise in the metal layer resistors as a function of frequency for three different values of d-c. current. The readings are for room temperature. The curve for 0.25 milliampers only shows the average values. The slopes of all three curves are -1 within the experimental error. This indicates that the form of $f(\nu)$ is given by:

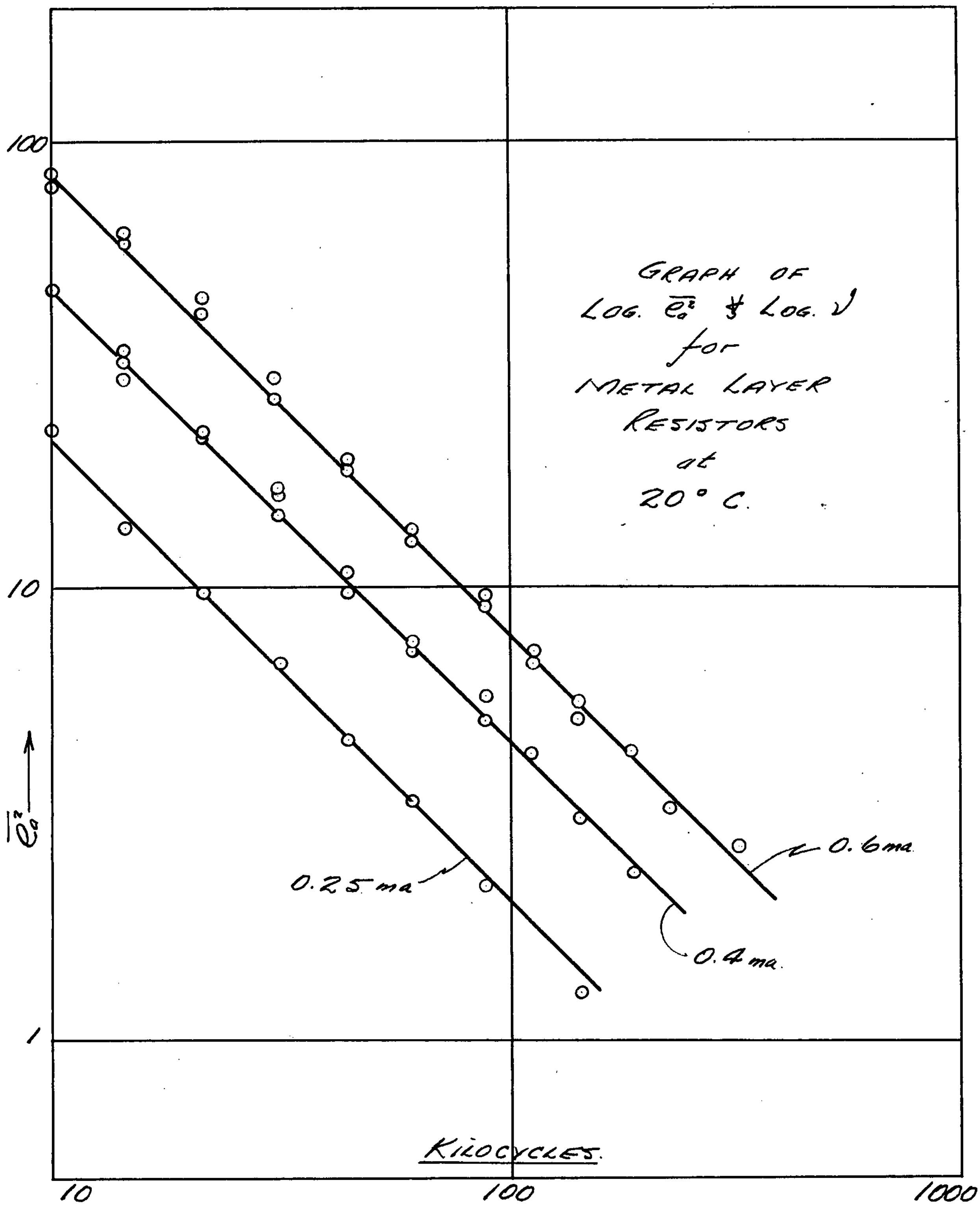
$$f(\nu) = \frac{constant}{\nu}$$

in this frequency range.

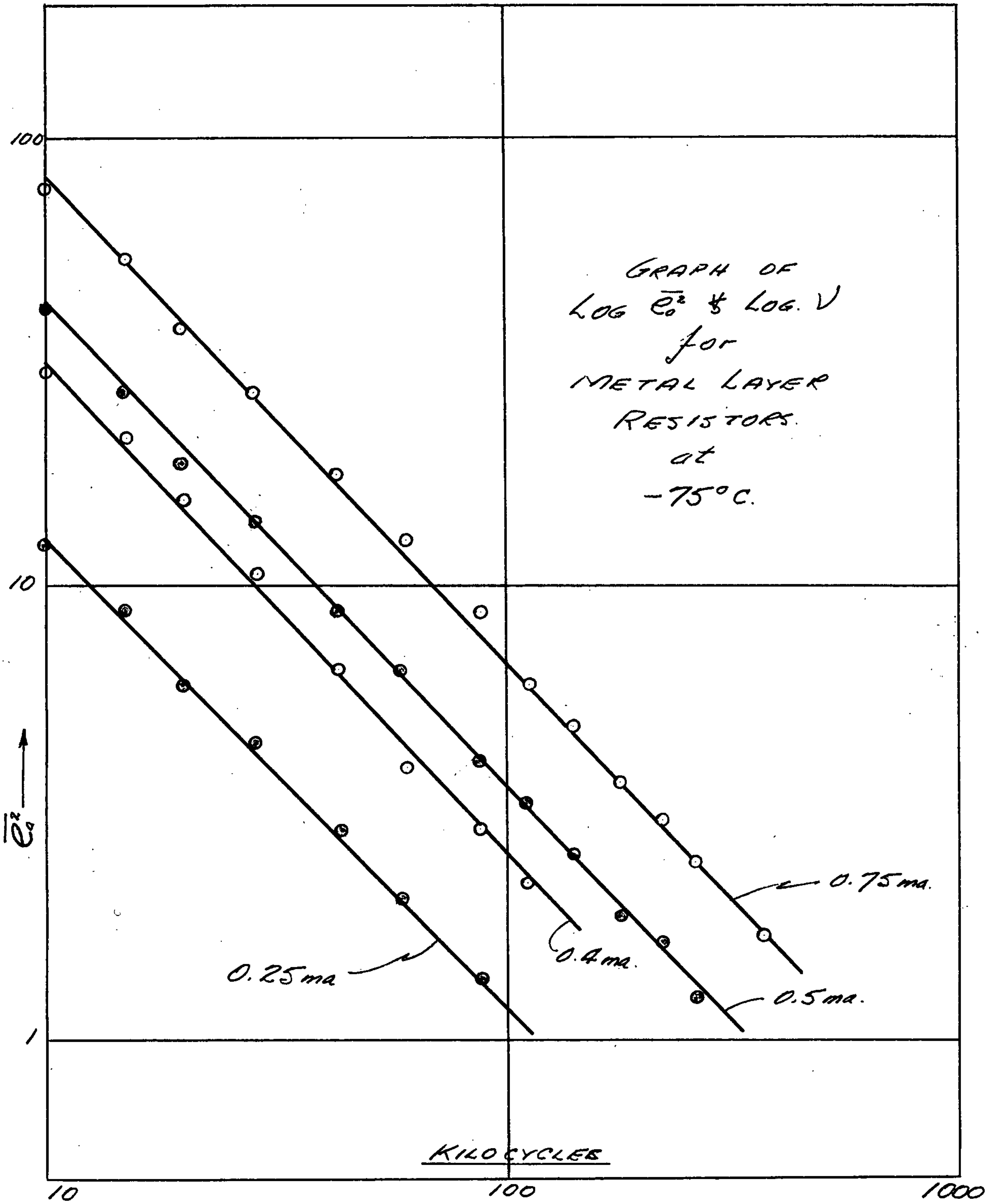
Graph No. 6 shows that the excess noise at -75°C is also inversly proportional to frequency in the range investigated.

However, Graph No. 7, taken at 100°C, shows a distinct departure from this law at higher frequencies. Up to a frequency of about 40 kc. the slope is -1 and above this frequency the slope attains a value of -1.6.

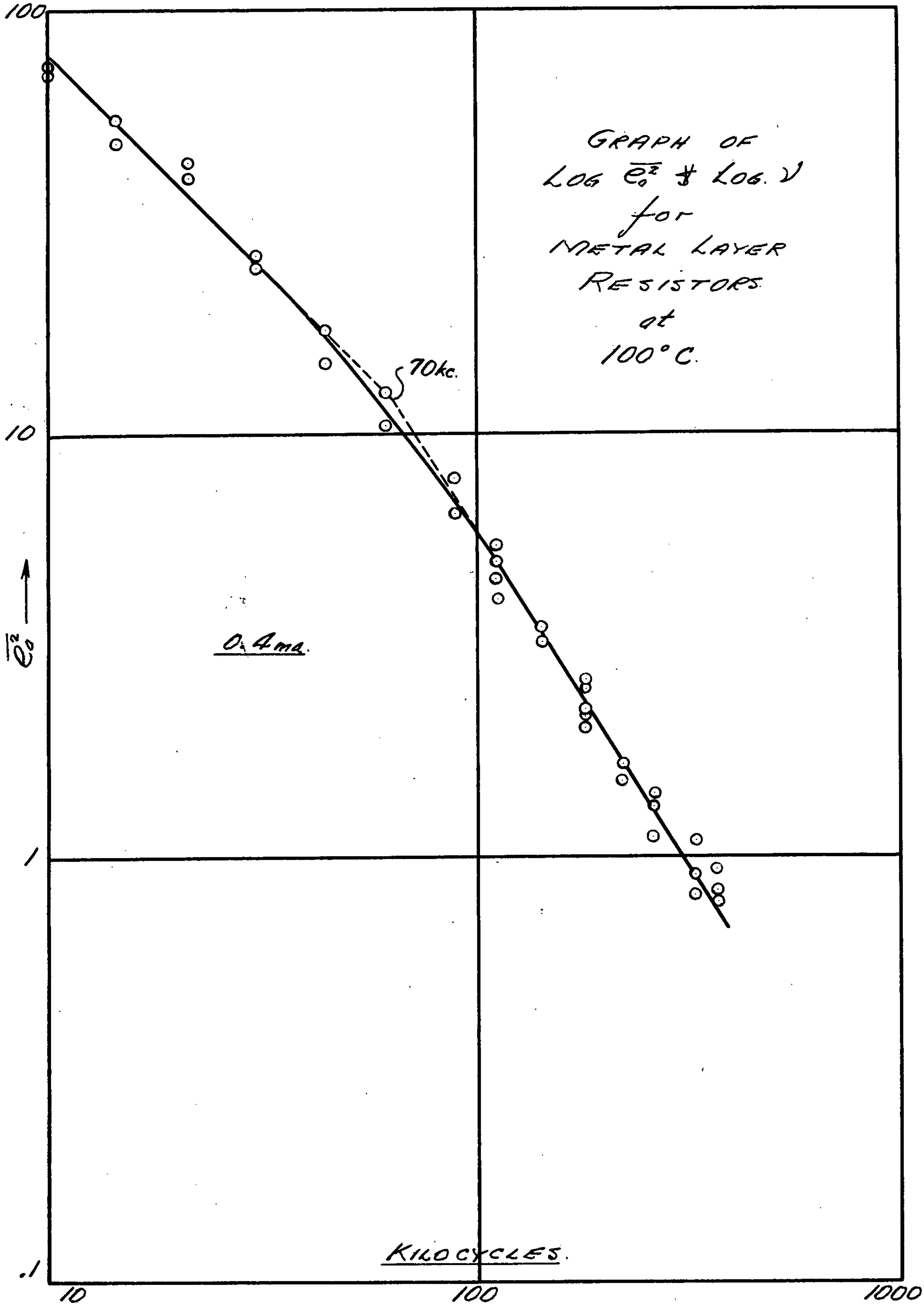
After the readings at 100°C had been taken the readings at room temperature were repeated and the results were the same as for the first trial. This indicates that heating and cooling the resistors had no permanent effect on the spectral distribution of the noise. The resistance at all three temperatures was almost constant - the variation being less than 10%. The response of the apparatus was plotted andfound to be the same at all three temperatures. All readings have been corrected for the decrease of gain at


GRAPH OF
LOG. $\overline{e_a^2}$ & LOG. ν
for
METAL LAYER
RESISTORS
at
20° C.
100
10
1
$\overline{e_a^2}$
0.6 ma
0.4 ma
0.25 ma
KILOCYCLES.
10
100
1000

GRAPH No. 5


GRAPH OF
LOG $\overline{e_o^2}$ & LOG. V
for
METAL LAYER
RESISTORS
at
$-75°C.$
100
10
1
$\overline{e_o^2}$
0.75 ma.
0.5 ma.
0.4 ma.
0.25 ma
KILOCYCLES
10
100
1000

GRAPH No. 6.


GRAPH OF
LOG $\overline{e_0^2}$ & LOG. ν
for
METAL LAYER
RESISTORS
at
100° C.
70kc.
0.4ma.
$\overline{e_0^2}$ →
KILOCYCLES.
100
10
1
.1
10
100
1000

GRAPH No. 7

higher frequencies.

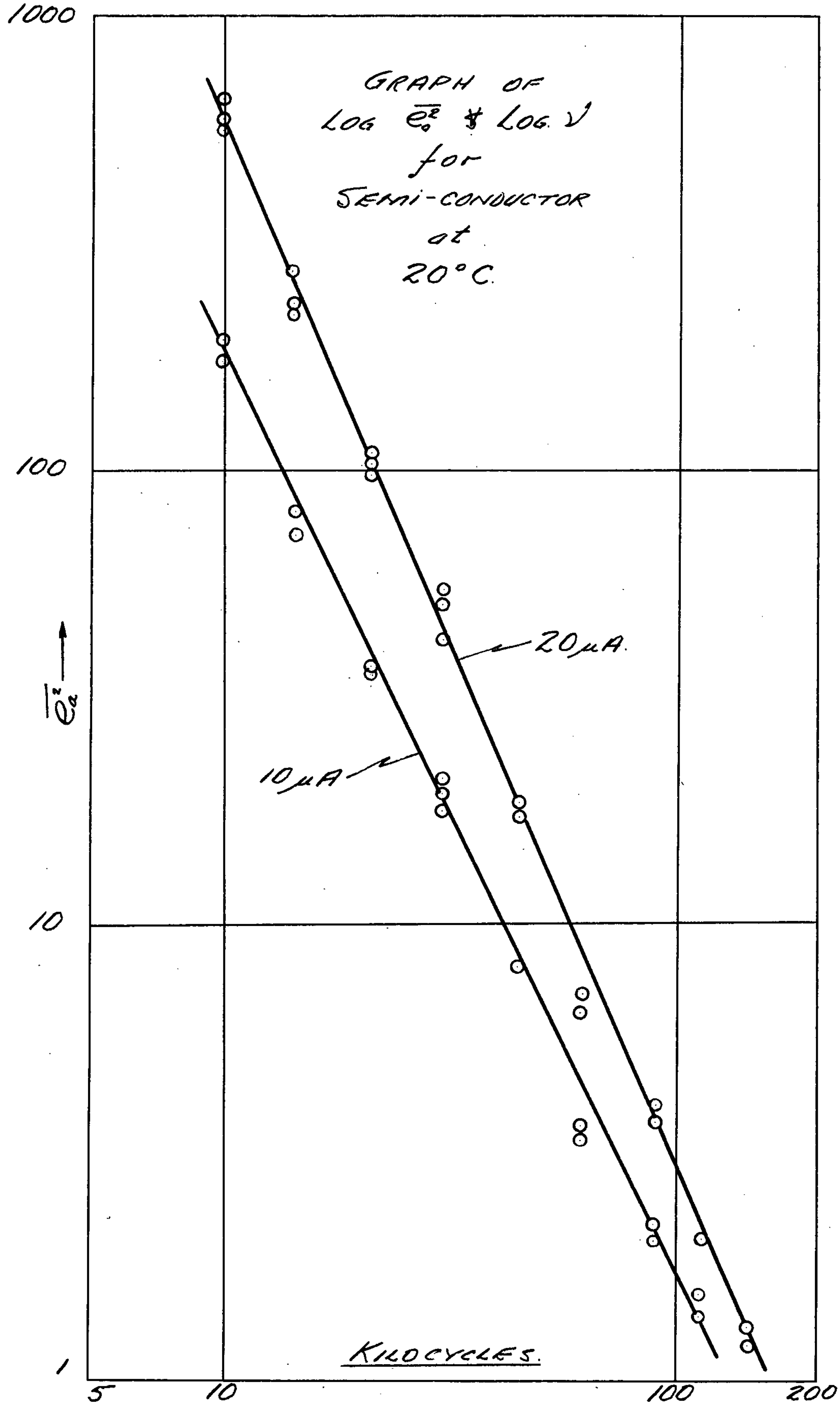
Graph No. 8 shows the excess noise in the semi-conductor as a function of frequency at a constant current and at room temperature. The slope of this curve is -2 over the whole frequency range which indicates that the form of $f(\nu)$ is:

$$f(\nu) = \frac{constant}{\nu^2}.$$

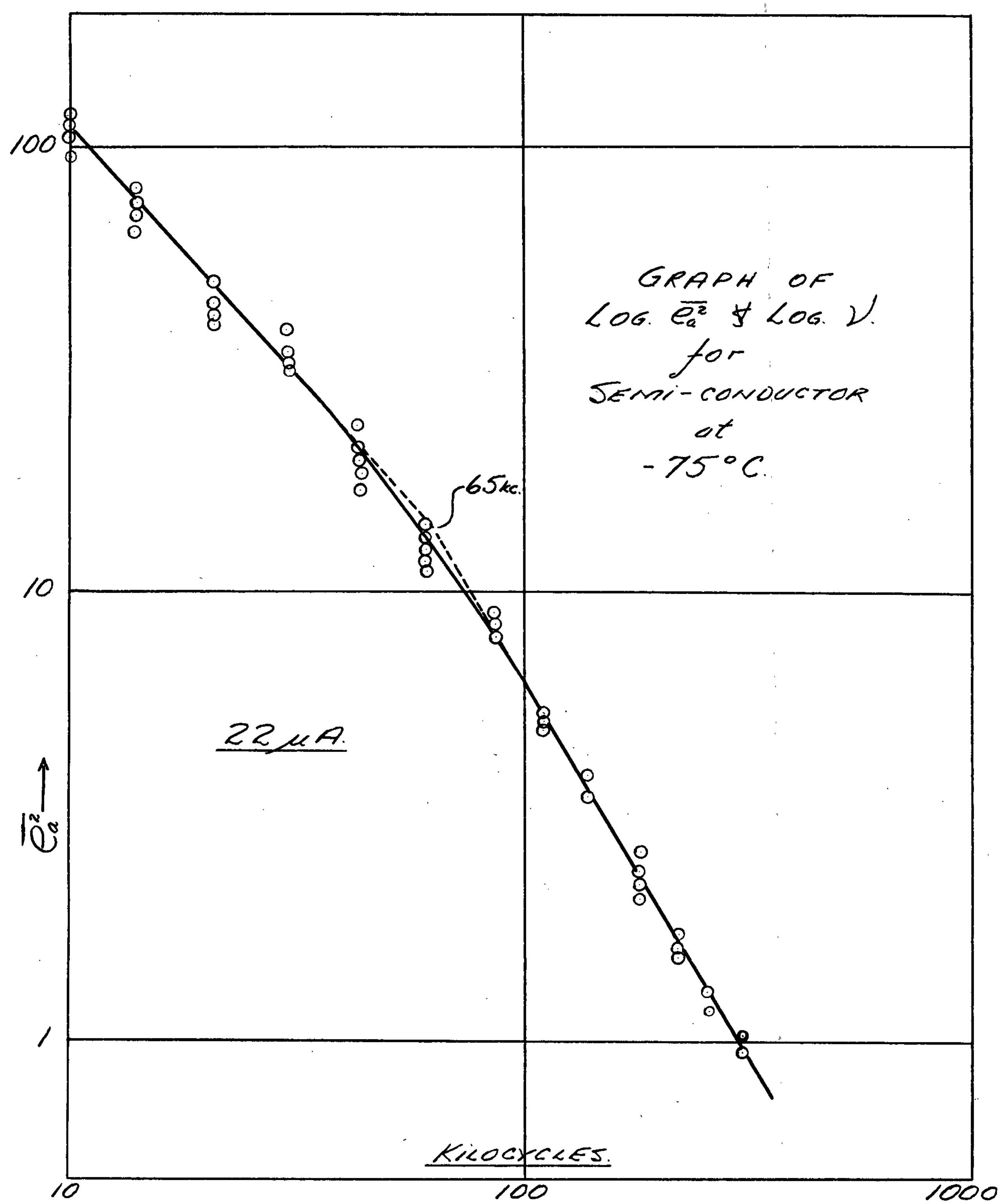
Graph No. 9, for a temperature of -75°C, shows that the noise at low frequencies is proportional to ν^{-1} and that the noise at high frequencies is proportional to ν^{-2}. By extending the two portions of this we see that they meet at a frequency of about 70 kc. This "transition frequency", as it might be called, cannot be accurately determined but it is the most convenient quantity to use when comparing graphs.

Graph No. 10, for a temperature of -186°C, also shows this transition of the noise dependence from ν^{-1} at low frequencies to ν^{-2} at high frequencies. It is also to be noted that the transition frequency at -186°C is almost the same as that for -75°C.

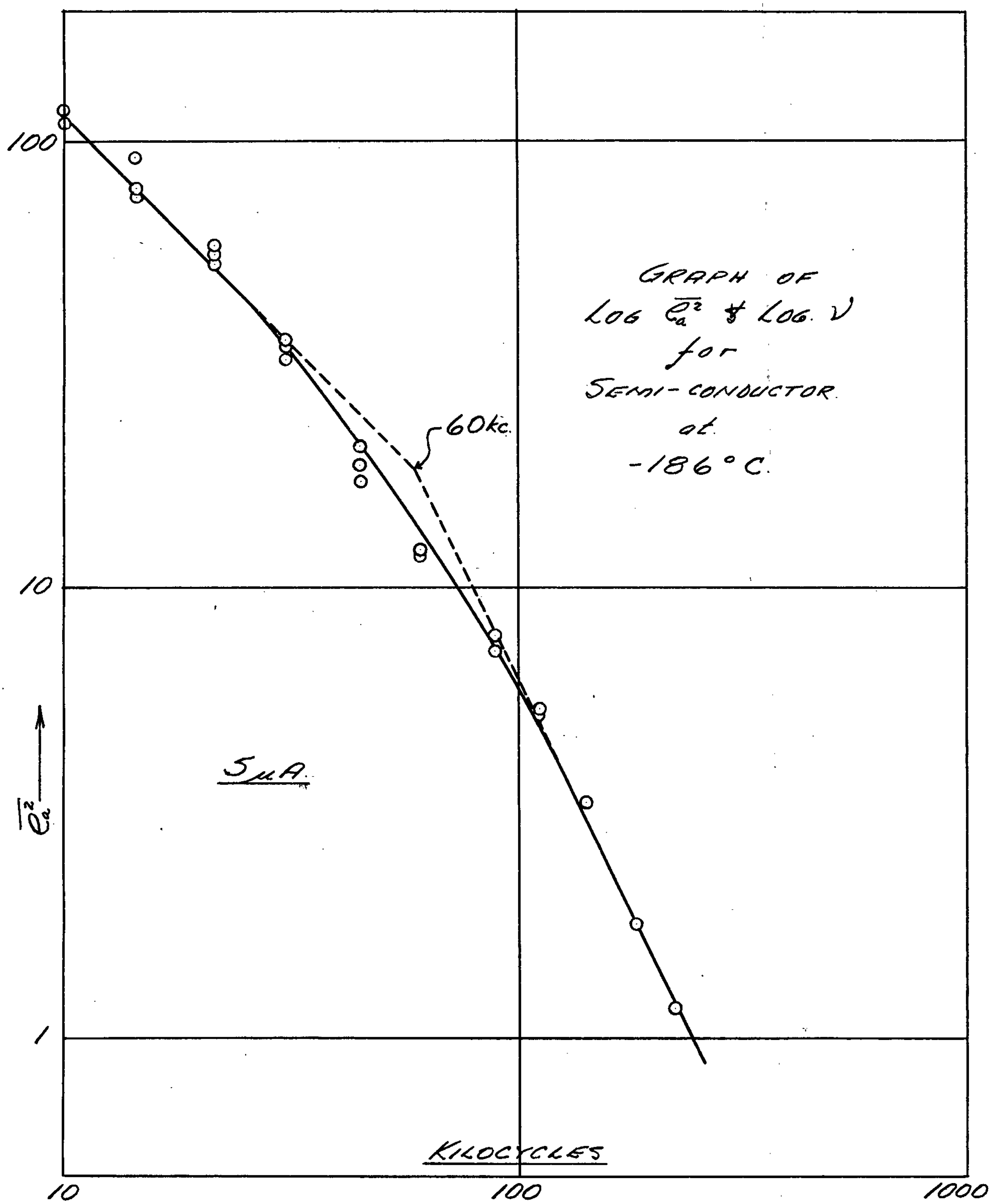
The resistance of the semi-conductor changes very rapidly with temperature. It is 0.6 megohms at room temperature, 15 megohms at -75°C, and 60 megohms at -186°C. The frequency response of the amplifier is greatly influenced by the input resistance and a response curve was plotted for each temperature. All readings on the graphs have been


GRAPH OF
LOG $\overline{e_a^2}$ & LOG. ν
for
SEMI-CONDUCTOR
at
20°C.
1000
100
10
1
$\overline{e_a^2}$
20μA.
10μA
KILOCYCLES.
5
10
100
200

GRAPH No. 8.


GRAPH OF
LOG. $\overline{e_a^2}$ & LOG. ν.
for
SEMI-CONDUCTOR
at
-75°C.
65kc.
22 μA.
$\overline{e_a^2}$
100
10
1
KILOCYCLES.
10
100
1000

GRAPH No. 9.


GRAPH OF
LOG $\overline{e_a^2}$ & LOG. ν
for
SEMI-CONDUCTOR
at
-186°C.
60kc.
5μA.
$\overline{e_a^2}$
KILOCYCLES
100
10
1
10
100
1000

GRAPH No. 10.

corrected using these response curves.

CHAPTER VI

CONCLUSIONS

The measurements indicate that the spectral distribution of noise shows a marked deviation from a $1/\nu$ law at high frequencies. This seems to indicate that it is permissable to introduce a distribution of correlation times as was done in the preceding theory (Chapter IV).

The dependence of the transition frequency upon temperature seems to indicate that the correlation times depend on temperature. Though further experimental data are needed we can at least draw some negative conclusions. Let us assume that the shape of the distribution function for τ does not depend upon temperature. The transition frequency at which the $1/\nu$ dependence changes to a $1/\nu^2$ dependence then determines the value of τ_1. The fact that the transition from $1/\nu$ to $\frac{1}{\nu^2}$ occurs at a lower frequency for higher temperatures then indicates that the correlation time τ_1 must increase with increasing temperature. This means that equation 28 of Chapter IV

$$\tau = \tau_0 e^{\frac{\epsilon}{kT}} \qquad 1.$$

does not explain the experimental result since it gives values for τ_1 that decrease with increasing temperature. On the other hand equation 29 of Chapter IV

$$\tau = \tau_0 e^{-\frac{E}{kT}} \qquad \underline{2.}$$

does give values for the correlation times that increase with increasing temperature. However, while this relation gives the right trend as a function of temperature, it does not give the right shape. Table I gives the predicted values of τ_1 at different temperatures for $E = 1.0eV$, $0.1eV$, and $0.01eV$.

TEMPERATURE IN DEGREES KELVIN	τ_1		
	$E = 1.0\ eV$	$E = 0.1\ eV$	$E = 0.01\ eV$
90	$10^{-53}\ \tau_0$	$10^{-5.3}\ \tau_0$	$10^{-.53}\ \tau_0$
200	$10^{-24}\ \tau_0$	$10^{-2.4}\ \tau_0$	$10^{-.24}\ \tau_0$
300	$10^{-16}\ \tau_0$	$10^{-1.6}\ \tau_0$	$10^{-.16}\ \tau_0$
600	$10^{-8}\ \tau_0$	$10^{-.8}\ \tau_0$	$10^{-.08}\ \tau_0$

TABLE I

Values of τ_1 at different temperatures and different values of E (according to equation 2).

For large values of E the dependence of τ_1 upon T is far too strong. A rather slow dependence of τ_1 upon T is obtained by assuming small values of E. However, even in that case equation 2 does not give the right dependence of τ_1 upon T. According to equation 2 τ_1 would decrease rapidly with decreasing T at very low temperatures and would be practically independent of T at higher temperatures whereas our experiments seem to indicate that τ_1 is independent of T at low temperatures and increases with increasing T at higher temperatures.

This means that neither equation 1 nor equation 2 represents the right dependence of τ upon temperature. More experimental data in a wider frequency range and for higher temperatures are needed before any definite conclusions can be made about this part of the problem.

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