PERFORMANCE DEGRADATION OF A TRANSMIT DIVERSITY SCHEME DUE TO CORRELATED FADING

by

TAO ZHENG

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Abstract

The error performance of the Alamouti simple transmit diversity (STD) scheme in the presence of time-selectivity and channel estimation errors has been previously studied. In this thesis, results are obtained for two other scenarios: (1) non time-selective, spatially correlated Rayleigh fading with imperfect channel estimation and (2) time-selective, spatially correlated Rayleigh fading with perfect channel estimation. Exact expressions for the conditional bit error rates given the estimated channel gains are derived and approximations for average bit error rates over correlated Rayleigh fading are obtained using matrix transformations.

It is found that STD performance generally degrades with increase in channel estimation errors, decrease in temporal correlation and increase in spatial correlation. The degradation is greatest with channel estimation errors, then time-selectivity and thirdly with spatial correlation.

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List of Acronyms and Symbols

Acronyms

BS	Base Station
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
DF	Decision Feedback
ML	Maximum Likelihood
MRC	Maximal Ratio Combining
MS	Mobile Station
OTD	Orthogonal Transmit Diversity
pdf	Probability Density Function
SNR	Signal to Noise Ratio
STD	Simple Transmit Diversity
STS	Space-Time Spreading
STTD	Space-Time Transmit Diversity
ZF	Zero Forcing

Symbols

λ	Wavelength
s_0, s_1	Transmitted bits
r_0, r_T	Received signals at time 0 and time T
G_i	Channel gain from transmit antenna i to the receive antenna
g_i	Sample of G_i
G_{ij}	Channel gain from transmit antenna i to the receive antenna at time j
g_{ij}	Sample of G_{ij}
H_i	Estimated channel gain from transmit antenna i to the receive antenna
h_i	Sample of H_i
H_{ij}	Estimated channel gain from transmit antenna i to the receive antenna at time j
h_{ij}	Sample of H_{ij}
Z_i	Channel estimation error on channel i
z_i	Sample of Z_i

n_0, n_T	Samples of noise at time 0 and T
$\tilde{s}_0$, $\tilde{s}_1$	Output of STD combiner
σ_G^2	Variance of the channel gain
σ_H^2	Variance of estimated channel gain
σ_Z^2	Variance of channel estimation error
σ_D^2	Variance of G_1 and G_2 given h_1 and h_2
σ_w^2	Variance of G_{1T} and G_{20} given g_{10} and g_{2T}
σ_N^2	Variance of noise N
ρ_e	Correlation coefficient of G_i and H_i
ρ_s	Spatial correlation coefficient
ρ_t	Time-selective correlation coefficient
ρ_d	Correlation coefficient of G_1 and G_2 given h_1 and h_2
ρ_w	Correlation coefficient of G_{1T} and G_{20} given g_{10} and g_{2T}
$\mathrm{Re}(\tilde{s}_i)$	Decision variable for decoding of s_i
P_e	Conditional error probability
P_f	Average error probability

$\mathbf{C_G}$	Covariance matrix of G
$\mathbf{C_H}$	Covariance matrix of H
$\mathbf{C}$	Covariance matrix of (H, G)
$\mathbf{X_H}$	Vector $(H_1,\ H_2)$
$p_{H_1,H_2}(h_1,h_2)$	Joint pdf of H_1, H_2
$\mathbf{X}$	Vector $(H_1,\ H_2,\ G_1,\ G_2)$
$p_{H_1,H_2,G_1,G_2}(h_1,h_2,g_1,g_2)$	Joint pdf of H_1, H_2, G_1 and G_2
$\mathbf{X}^\mathbf{T}$	Transpose of $\mathbf{X}$
λ_i	Eigenvalues of $\mathbf{C_H}$
$f_M(\mu)$	pdf of M
$\mathbf{T}$	Transformation matrix

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1 Introduction

The first and second generation (1G and 2G) cellular systems have enabled *wireless* voice communications. However, the data services in 1G and 2G systems are limited mostly to text messaging. Besides voice service, 3G systems are supposed to support higher data rates and make it possible to offer enhanced services such as web browsing, transmission of high quality images and videos, etc.

Due to the nature of the voice and text messaging services, the requirements for downlink and uplink capacities are similar. However, the battery life of mobile stations (MS) restricts the maximum power at which a MS can transmit and results in a poor uplink than downlink. As a result, many enhancements have been introduced on the base station (BS) side, such as increasing the BS receiver sensitivity or exploiting receive diversity at the BS to improve the uplink. In 3G systems, the services are more data-centric than voice-centric. Most of the services, such as web browsing, picture downloading and video downloading, require more downlink capacity than uplink capacity. However, in 3G systems, the uplink data throughput is higher than that of the downlink [1]. The data throughput on the uplink in macro cells is typically 1,040 kbps whereas that on the downlink is only 660 kbps. Therefore, improving the downlink capacity becomes more important in 3G systems.

Because of the power, size, weight and cost limitations on the MS side, improving receiver sensitivity or implementing receive diversity to improve the downlink may not be practical.

However, on the BS side, since receive diversity has already been widely deployed, there are usually two receive antennas installed on the BS side. We can achieve transmit diversity on the downlink by duplexing the downlink transmission to the receive antennas. Since a BS can serve hundreds to thousands of MSs, the use of transmit diversity at the BS is a more cost-effective solution to improving downlink quality.

Much research work including techniques such as time diversity, frequency diversity, polarization diversity [2], space-time coding [3], orthogonal transmit diversity [4], time switched transmit diversity [5], selective transmit diversity [5 , 6] and transmit adaptive array [4] has been carried out in order to achieve high-speed and reliable data transmission using transmit diversity. Some of these technologies have been proposed for 3G evolutions [7].

Previous works on transmit diversity can be classified into two categories: open loop diversity and closed loop diversity. Closed loop transmit diversity relies on feedback information from the MS while open loop transmit diversity does not use feedback information. Generally, the performance of closed loop transmit diversity is better, as the channel state information can be used to calculate optimal transmit weights, which makes it possible to maximize the desired received signal power at the desired MS and minimize the interference to other MSs. However, closed loop diversity requires the MS to send back channel information and this requires extra signalling overhead.

Open loop diversity does not have this requirement. It is a "one size fits all" approach.

The advantages of this kind of diversity are two-fold: signalling overhead is lower and the MS receiver complexity is relatively low. Some of the open loop transmit diversity techniques, such as orthogonal transmit diversity (OTD), space-time spreading (STS) and space-time transmit diversity (STTD), have already been adopted in 3G standards. STTD has been included in the 3GPP standard [8] while the other two methods, OTD and STS, are part of the 3GPP2 standard [9]. STS is a variation of STTD. In STTD, the symbols are transmitted over two time slots using a single Walsh code; whereas in STS, the symbols are transmitted over a single time slot using two Walsh codes [10]. In STTD, the symbols are transmitted using the simple transmit diversity (STD) scheme proposed by Alamouti [11].

STD is well-known for its simplicity in decoding. It has been the subject of many studies with some focused on the BER performance of STD in different channel conditions. These studies include the performance of STD with imperfect channel estimation, STD in time-selective Rayleigh-fading channels and STD in spatially correlated Rayleigh fading [12, 13]. They show that the performance of STD generally degrades as channel estimation errors, time-selectivity and spatial correlation increase. In [13], different detection strategies, such as maximum-likelihood (ML), decision-feedback (DF) and zero-forcing (ZF), are used to assess the BER performance assuming perfect channel estimation. The results show that the ML detector significantly outperforms the other two detectors.

In this thesis, we analyze the performance degradation of STD with the receiver structure in [1 1] in different channel conditions.

1.1 STD Scheme

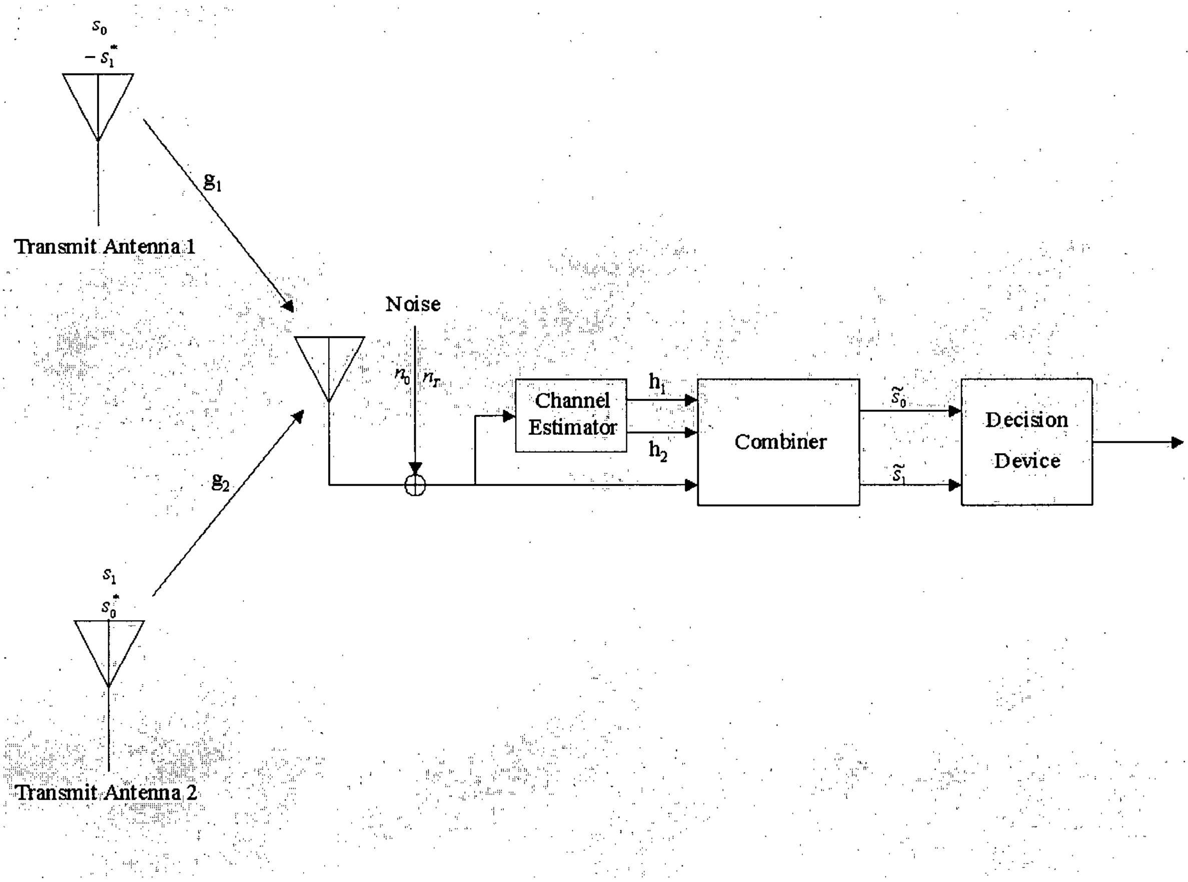

Fig. 1.1 STD scheme

In the STD scheme, two information bits s_0 and s_1 are sent simultaneously by transmit antennas 1 and 2 in two consecutive bit periods. It is assumed that the bandwidth of the signal is narrow compared to the channel coherence bandwidth and the channel coherence time is much larger than *T*, so that the channels can be considered as non frequency-selective and non time-selective [1 4]. In the first bit period, s_0 is sent by transmit antenna 1 and s_1 is sent by antenna 2; in the second bit period, $-s_1^*$ and s_0^* are sent by transmit antennas 1 and 2 respectively. The signals r_0 and r_T at the receive antenna in these two bit periods can be expressed as

$$\begin{bmatrix} r_0 \\ r_T^* \end{bmatrix} = \begin{bmatrix} g_1 & g_2 \\ g_2^* & -g_1^* \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + \begin{bmatrix} n_0 \\ n_T^* \end{bmatrix} \tag{1.1}$$

where g_1, g_2 are samples of the channel gains from the two transmit antennas to the receive antenna and n_0 and n_T are samples of thermal noise and interference at the receive antenna at time 0 and time *T*.

After both signals are received, the receiver combines the received signals using the estimated channel gains. It is assumed in [1 1] that the channel estimation is perfect, i.e., $h_1 = g_1$ and $h_2 = g_2$. Then the combiner can generate two combined signals $\tilde{s}_0$ and $\tilde{s}_1$ as

$$\begin{bmatrix} \tilde{s}_0 \\ \tilde{s}_1 \end{bmatrix} = \begin{bmatrix} g_1^* & g_2 \\ g_2^* & -g_1 \end{bmatrix} \begin{bmatrix} r_0 \\ r_T^* \end{bmatrix} \tag{1.2}$$

Substituting (1.1) into (1.2), we obtain

$$\begin{bmatrix} \tilde{s}_0 \\ \tilde{s}_1 \end{bmatrix} = \begin{bmatrix} |g_1|^2 + |g_2|^2 & 0 \\ 0 & |g_1|^2 + |g_2|^2 \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + \begin{bmatrix} g_1^* & g_2 \\ g_2^* & -g_1 \end{bmatrix} \begin{bmatrix} n_0 \\ n_T^* \end{bmatrix} \tag{1.3}$$

The combined signals $\tilde{s}_0$ and $\tilde{s}_1$ are then sent to a maximum likelihood detector to recover the original bits s_0 and s_1. It is shown in [1 1] that with perfect channel estimation, STD has the same BER performance as 2-branch MRC for a fixed value of the radiated power per transmit antenna.

1.2 Generalized STD Expression

In the original STD scheme, several assumptions are made: (1) the fading channels from the two transmit antennas to the receive antenna are spatially uncorrelated; (2) each channel is frequency flat and non time-selective; (3) the channel estimator provides perfect channel estimations. To better reflect conditions in a real system, the following model changes are introduced:

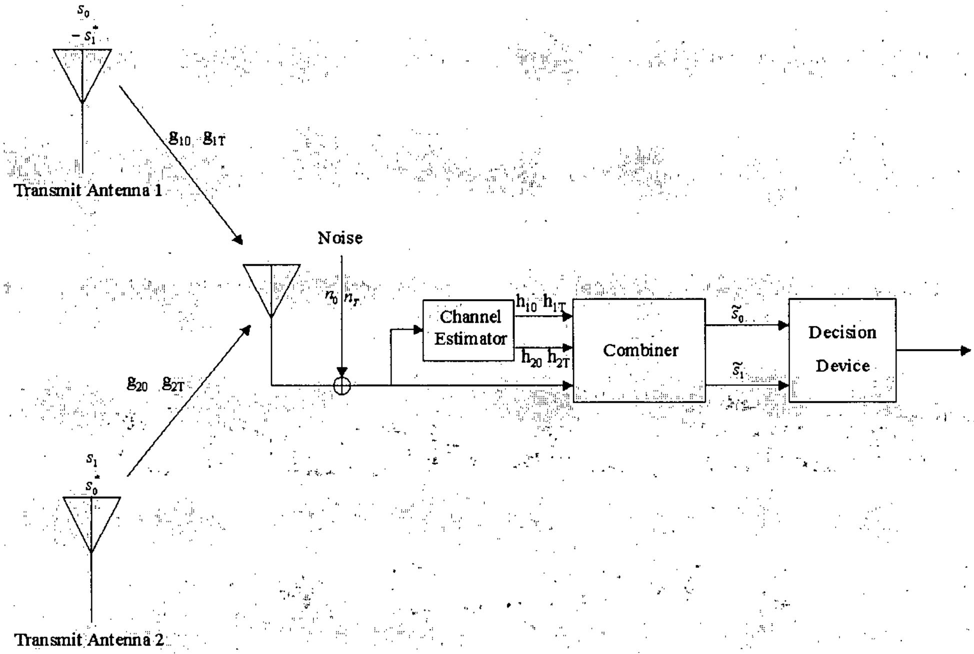

Fig. 1.2 STD in time-selective fading with imperfect channel estimation

- ✧ Due to time-selectivity, G_1 and G_2 may be different at time 0 and time *T*. To reflect this, they are denoted by G_{10}, G_{20}, G_{1T} and G_{2T} respectively.

- ✧ Due to spatial correlation, G_{10} and G_{20}, G_{1T} and G_{2T} may not be independent.

✧ The corresponding estimated channel gains are denoted by H_{10}, H_{20}, H_{1T} and H_{2T}.

Following the STD scheme, we can rewrite (1.1) and (1.2) as

$$\begin{bmatrix} r_0 \\ r_T^* \end{bmatrix} = \begin{bmatrix} g_{10} & g_{20} \\ g_{2T}^* & -g_{1T}^* \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + \begin{bmatrix} n_0 \\ n_T^* \end{bmatrix} \tag{1.4}$$

$$\begin{bmatrix} \tilde{s}_0 \\ \tilde{s}_1 \end{bmatrix} = \begin{bmatrix} h_{10}^* & h_{2T} \\ h_{20}^* & -h_{1T} \end{bmatrix} \begin{bmatrix} r_0 \\ r_T^* \end{bmatrix} \tag{1.5}$$

By substituting (1.4) into (1.5), we get the general expression for the signals at the output of the combiner as

$$\begin{bmatrix} \tilde{s}_0 \\ \tilde{s}_1 \end{bmatrix} = \begin{bmatrix} h_{10}^* g_{10} + h_{2T} g_{2T}^* & h_{10}^* g_{20} - h_{2T} g_{1T}^* \\ h_{20}^* g_{10} - h_{1T} g_{2T}^* & h_{20}^* g_{20} + h_{1T} g_{1T}^* \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + \begin{bmatrix} h_{10}^* & h_{2T} \\ h_{20}^* & -h_{1T} \end{bmatrix} \begin{bmatrix} n_0 \\ n_T^* \end{bmatrix} \tag{1.6}$$

Depending on various conditions, (1.6) can be simplified to different forms which can be used for signal detection and system performance evaluation.

In the case of non time-selective and perfect channel estimation, we can rewrite (1.6) as

$$\begin{bmatrix} \tilde{s}_0 \\ \tilde{s}_1 \end{bmatrix} = \begin{bmatrix} |g_1|^2 + |g_2|^2 & 0 \\ 0 & |g_1|^2 + |g_2|^2 \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + \begin{bmatrix} g_1^* & g_2 \\ g_2^* & -g_1 \end{bmatrix} \begin{bmatrix} n_0 \\ n_T^* \end{bmatrix} \tag{1.3}$$

which is the same expression as in [11]. The BER for spatially uncorrelated channels is given in [11] and the BER for spatially correlated channels is given in [12].

In the case of time-selective, spatially uncorrelated fading with imperfect channel estimation, the expression of the signals from the combiner is the same as (1.6). In this case, the actual and estimated channel gains of channel 1 are independent of those of channel 2. Although the inter-channel interference term $(h_{10}^{*} g_{20} - h_{2T} g_{1T}^{*}) s_1$ and $(h_{20}^{*} g_{10} - h_{1T} g_{2T}^{*}) s_0$ are non-zero in contrast to (1.3), each of the four product terms within the parentheses are products of two independent random variables. So, the BER performance can be evaluated exactly as in [1 2].

For STD in non time-selective, spatially correlated fading with imperfect channel estimation and STD in time-selective, spatially correlated fading with/without channel estimation errors, the BERs are more difficult to obtain because the terms within the parentheses are products of two *dependent* random variables. In this thesis, we study the performance for two cases: (1) STD in non time-selective and spatially correlated fading with channel estimation errors; (2) STD in time-selective and spatially correlated fading with perfect channel estimation.

2 STD in Non Time-selective, Spatially Correlated Fading with Imperfect Channel Estimation

This chapter evaluates the performance of STD with BPSK modulation in non time-selective, spatially correlated Rayleigh fading with imperfect channel estimation. The variance of any complex Gaussian random variable, i.e., X, will be defined as the variance of either its real or imaginary component, denoted as σ_X^2 in this thesis.

2.1 System Model

In the original STD scheme, the channel gains from two transmit antennas are assumed independent. Theoretically, the channels are spatially independent if the antenna spacing is greater than $\lambda/2$. However, in reality, an antenna spacing of 50λ and 100λ are necessary at the BS [15]. If the antennas are allocated too close to each other, the channels can no longer be considered independent. In this section, we discuss the performance of STD when the channels are spatially correlated.

Similar to the original STD scheme, we use two transmit antennas and one receive antenna, but here the two transmit antennas are very close to each other. We denote the gains of the two diversity paths as G_1 and G_2, which both are zero mean complex Gaussian random variables with variance σ_G^2. G_1 and G_2 are spatially correlated with correlation coefficient ρ_s. As to [16], ρ_s is defined as

$$\rho_s = \frac{E[G_1 G_2^*]}{\sqrt{E[|G_1|^2]E[|G_2|^2]}} \tag{2.1}$$

Consequently, the covariance matrix of G_1 and G_2, C_G can be expressed as

$$\mathbf{C_G} = \begin{bmatrix} \sigma_G^2 & \rho_s\sigma_G^2 \\ \rho_s\sigma_G^2 & \sigma_G^2 \end{bmatrix} \tag{2.2}$$

In the original STD scheme, the channel gains are known. However, in this model, the channel gains are unknown. They have to be estimated by channel estimator from the received signals. We denote the estimations of the channel gain G_1 and G_2 as H_1 and H_2, where H_1 and H_2 are zero mean complex Gaussian random variables. Each pair of H_i and G_i are correlated. Following [1 7], we define H_i as $h_i = g_i + z_i$, where Z_i is the channel estimation error. Z_i is a zero mean independent complex Gaussian random variable with variance σ_Z^2 and independent of G_i, i.e.,

$$E[Z_i Z_j^*] = E[G_i Z_i^*] = E[G_i Z_j^*] = 0 \quad (\text{i, j} = 1, 2) \tag{2.3}$$

It can be shown that the variance of H_i is $\sigma_H^2 = \sigma_G^2 + \sigma_Z^2$. The correlation coefficient between G_i and H_i is defined as ρ_e, where

$$\rho_e = \frac{E[G_i H_i^*]}{\sqrt{E[|G_i|^2]E[|H_i^*|^2]}}$$

$$= \frac{E[G_i(G_i + Z_i)^*]}{\sqrt{(2\sigma_G^2)(2\sigma_H^2)}}$$

$$= \sigma_G / \sigma_H \tag{2.4}$$

Therefore

$$\sigma_H^2 = \sigma_G^2 / \rho_e^2 \tag{2.5}$$

and

$$\sigma_Z^2 = (1/\rho_e^2 - 1)\sigma_G^2 \tag{2.6}$$

The covariance of H_1 and H_2 is expressed as

$$\mathrm{E}[H_1 H_2^*] = \mathrm{E}[(G_1 + Z_1)(G_2 + Z_2)^*] = 2\rho_s \sigma_G^2 \tag{2.7}$$

Now we can get the covariance matrix of H_1 and H_2 as

$$\mathbf{C_H} = \begin{bmatrix} \sigma_H^2 & \rho_s\sigma_G^2 \\ \rho_s\sigma_G^2 & \sigma_H^2 \end{bmatrix} = \begin{bmatrix} \dfrac{\sigma_G^2}{\rho_e^2} & \rho_s\sigma_G^2 \\ \rho_s\sigma_G^2 & \dfrac{\sigma_G^2}{\rho_e^2} \end{bmatrix} \tag{2.8}$$

Similarly, we can prove

$$\mathrm{E}[G_1 H_2^*] = \mathrm{E}[G_1 (G_2 + Z_2)^*] = 2\rho_s \sigma_G^2 \tag{2.9}$$

$$\mathrm{E}[G_2 H_1^*] = \mathrm{E}[G_2 (G_1 + Z_1)^*] = 2\rho_s \sigma_G^2 \tag{2.10}$$

Consequently, we can write the covariance matrix of H_1, H_2, G_1 and G_2 as

$$\mathbf{C} = \begin{bmatrix} \dfrac{\sigma_G^2}{\rho_e^2} & \rho_s\sigma_G^2 & \sigma_G^2 & \rho_s\sigma_G^2 \\ \rho_s\sigma_G^2 & \dfrac{\sigma_G^2}{\rho_e^2} & \rho_s\sigma_G^2 & \sigma_G^2 \\ \sigma_G^2 & \rho_s\sigma_G^2 & \sigma_G^2 & \rho_s\sigma_G^2 \\ \rho_s\sigma_G^2 & \sigma_G^2 & \rho_s\sigma_G^2 & \sigma_G^2 \end{bmatrix} \tag{2.11}$$

If we rewrite (1.16) according to this model, we have

$$\begin{bmatrix} \tilde{s}_0 \\ \tilde{s}_1 \end{bmatrix} = \begin{bmatrix} h_1^* g_1 + h_2 g_2^* & h_1^* g_2 - h_2 g_1^* \\ h_2^* g_1 - h_1 g_2^* & h_2^* g_2 + h_1 g_1^* \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + \begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix} \begin{bmatrix} n_0 \\ n_T^* \end{bmatrix} \tag{2.12}$$

2.2 Performance Analysis Based on Estimated Channel Gains

Since H_1 and H_2 are zero mean complex joint Gaussian random variables, based on their covariance matrix (2.8) we can write the joint pdf of H_1 and H_2 as [18]

$$p_{H_1,H_2}(h_1,h_2) = \frac{1}{2\pi(\det \mathbf{C_H})^{1/2}} \exp(-\frac{1}{2}\mathbf{X_H^T C_H^{-1} X_H}) \tag{2.13}$$

where $\mathbf{X_H^T} = [H_1 \quad H_2]$ is the transpose of $\mathbf{X_H}$, a 2×1 column vector of random variables H_1 and H_2.

Similarly, we can write the joint pdf of H_1, H_2, G_1 and G_2 as

$$p_{H_1,H_2,G_1,G_2}(h_1,h_2,g_1,g_2) = \frac{1}{(2\pi)^2(\det \mathbf{C})^{1/2}} \exp(-\frac{1}{2}\mathbf{X^T C^{-1} X}) \tag{2.14}$$

where $\mathbf{X^T} = [H_1 \quad H_2 \quad G_1 \quad G_2]$ is the transpose of $\mathbf{X}$, a 4×1 column vector of random variables H_1, H_2, G_1 and G_2.

Now that we know the joint pdf $p_{H_1,H_2,G_1,G_2}(h_1,h_2,g_1,g_2)$ and $p_{H_1,H_2}(h_1,h_2)$, we can write the joint pdf of G_1 and G_2 given $H_1 = h_1$ and $H_2 = h_2$ as [19]

$$p_{G_1,G_2|H_1,H_2}(g_g, g_2 \mid h_1,h_2) = \frac{p_{H_1,H_2,G_1,G_2}(h_1,h_2,g_1,g_2)}{p_{H_1,H_2}(h_1,h_2)}$$

$$= \frac{1}{2\pi\sigma_D^2\sqrt{1-\rho_d{}^2}}\exp\left[-\frac{(g_1-m_1)^2-2\rho_d(g_1-m_1)(g_2-m_2)+(g_2-m_2)^2}{2\sigma_D^2(1-\rho_d{}^2)}\right] \tag{2.15}$$

where

$$m_1 = \frac{\rho_e^2[(1-\rho_s^2\rho_e^2)h_1+\rho_s(1-\rho_e^2)h_2]}{1-\rho_s^2\rho_e^4} \tag{2.16}$$

$$m_2 = \frac{\rho_e^2[\rho_s(1-\rho_e^2)h_1+(1-\rho_s^2\rho_e^2)h_2]}{1-\rho_s^2\rho_e^4} \tag{2.17}$$

$$\sigma_D^2 = \frac{(1-\rho_e^2)(1-\rho_s^2\rho_e^2)}{1-\rho_s^2\rho_e^4} \tag{2.18}$$

$$\rho_d = \frac{\rho_s(1-\rho_e^2)}{1-\rho_s^2\rho_e^2} \tag{2.19}$$

This joint pdf is in the form of a bivariate Gaussian pdf given by [1 8]. Hence, given $H_1 = h_1$ and $H_2 = h_2$, G_1 and G_2 are correlated complex Gaussian random variables with means m_1 and m_2, variance σ_D^2 and correlation coefficient ρ_d.

Following the conditioned pdf, we can express g_1, g_2 by h_1, h_2 as

$$\begin{aligned} g_1 &= m_1 + d_1 = ah_1 + bh_2 + d_1 \\ g_2 &= m_2 + d_2 = bh_1 + ah_2 + d_2 \end{aligned} \tag{2.20}$$

where $a = \dfrac{\rho_e^2(1-\rho_s^2\rho_e^2)}{1-\rho_s^2\rho_e^4}$, $b = \dfrac{\rho_e^2\rho_s(1-\rho_e^2)}{1-\rho_s^2\rho_e^4}$ and D_1; D_2 are zero mean correlated complex Gaussian random variables with variance σ_D^2 and correlation coefficient ρ_d.

According to STD scheme, the signals received at time 0 and time T can be expressed as:

$$r_0 = g_1 s_0 + g_2 s_1 + n_0$$
$$r_T = g_2 s_0 - g_1 s_1 + n_T \tag{2.21}$$

where n_0 and n_T are samples of channel noises, which are zero mean independent complex Gaussian random variables with variance σ_N^2.

After receiving r_0 and r_T, the signal can be decoded based on the value of $\tilde{s}_0$ and $\tilde{s}_1$, where

$$\tilde{s}_0 = h_1^* r_0 + h_2 r_T^*$$
$$\tilde{s}_1 = h_2^* r_0 - h_1 r_T^* \tag{2.22}$$

When the real part of $\tilde{s}_0$ is greater than 1, $s_0 = 1$ is selected; otherwise, $s_0 = -1$ is selected. The same decision rule applies to the decoding of s_1.

By using (2.19) ~ (2.22), we can write $\tilde{s}_0$ as

$$\tilde{s}_0 = (as_0 + bs_1)|h_1|^2 + (as_0 - bs_1)|h_2|^2 + 2bs_0 h_1^* h_2$$
$$+ h_1^*(d_1 s_0 + d_2 s_1) + h_2(d_2^* s_0 - d_1^* s_1) + h_1^* n_0 + h_2 n_T^* \tag{2.23}$$

Since s_0 and s_1 are either +1 or –1 with equal probability, the chances of $s_1 = s_0$ and $s_1 = -s_0$ are equal. Therefore, we can calculate the BER of STD as

$$P_e = \frac{1}{2}(P_{e,s_1=s_0} + P_{e,s_1=-s_0}) \tag{2.24}$$

When $s_0 = s_1$, the decision variable from (2.23) can be expressed as

$$\mathrm{Re}(\tilde{s}_0) = [(a+b)|h_1|^2 + (a-b)|h_2|^2 + 2b\,\mathrm{Re}(h_1^* h_2)]s_0$$
$$+ \mathrm{Re}[h_1^*(d_1 + d_2)]s_0 + \mathrm{Re}[h_2(d_2^* - d_1^*)]s_0$$

$$+\operatorname{Re}[h_1^* n_0] + \operatorname{Re}[h_2 n_T^*] \tag{2.25}$$

When h_1 and h_2 are given, it is shown in Appendix A that $\operatorname{Re}[h_1^*(D_1 + D_2)]s_0$, $\operatorname{Re}[h_2(D_2^* - D_1^*)]s_0$, $\operatorname{Re}[h_1^* N_0]$ and $\operatorname{Re}[h_2 N_T^*]$ are zero mean independent Gaussian random variables with variances $2(1+\rho_d)\sigma_D^2|h_1|^2$, $2(1-\rho_d)\sigma_D^2|h_2|^2$, $\sigma_N^2|h_1|^2$ and $\sigma_N^2|h_2|^2$ respectively. Thus, $\operatorname{Re}(\tilde{s}_0)$ is the sum of $[(a+b)|h_1|^2 + (a-b)|h_2|^2 + 2b\operatorname{Re}(h_1^* h_2)]s_0$ and a zero mean independent Gaussian random variable with variance $[2(1+\rho_d)\sigma_D^2 + \sigma_N^2]|h_1|^2 + [2(1-\rho_d)\sigma_D^2 + \sigma_N^2]|h_2|^2$.

When $s_0 = s_1 = 1$, there is an error if the decision variable is less than 0. Thus, we can express the error probability as

$$P_{e,s_1=s_0=1} = Q\left(\frac{(a+b)|h_1|^2 + (a-b)|h_2|^2 + 2b\operatorname{Re}(h_1^* h_2)}{\sqrt{[2(1+\rho_d)\sigma_D^2 + \sigma_N^2]|h_1|^2 + [2(1-\rho_d)\sigma_D^2 + \sigma_N^2]|h_2|^2}}\right) \tag{2.26}$$

When $s_0 = s_1 = -1$, and if the decision variable is greater than 0, there is an error. So, the error probability can be expressed as

$$\begin{aligned} P_{e,s_1=s_0=-1} &= 1 - Q\left(\frac{-[(a+b)|h_1|^2 + (a-b)|h_2|^2 + 2b\operatorname{Re}(h_1^* h_2)]}{\sqrt{[2(1+\rho_d)\sigma_D^2 + \sigma_N^2]|h_1|^2 + [2(1-\rho_d)\sigma_D^2 + \sigma_N^2]|h_2|^2}}\right) \\ &= Q\left(\frac{(a+b)|h_1|^2 + (a-b)|h_2|^2 + 2b\operatorname{Re}(h_1^* h_2)}{\sqrt{[2(1+\rho_d)\sigma_D^2 + \sigma_N^2]|h_1|^2 + [2(1-\rho_d)\sigma_D^2 + \sigma_N^2]|h_2|^2}}\right) \end{aligned} \tag{2.27}$$

This is the same as (2.26).

Similarly, when $s_1 = -s_0$, we can get the decision variable as

$$\begin{aligned}\mathrm{Re}(\tilde{s}_0) &= [(a-b)|h_1|^2 + (a+b)|h_2|^2 + 2b\,\mathrm{Re}(h_1^* h_2)]s_0 \\ &\quad + \mathrm{Re}[h_1^*(d_1 - d_2)]s_0 + \mathrm{Re}[h_2(d_2^* + d_1^*)]s_0 \\ &\quad + \mathrm{Re}[h_1^* n_0] + \mathrm{Re}[h_2 n_T^*] \end{aligned} \tag{2.28}$$

and the corresponding error probability as

$$P_{e,s_1=-s_0} = Q\left(\frac{(a-b)|h_1|^2 + (a+b)|h_2|^2 + 2b\,\mathrm{Re}(h_1^* h_2)}{\sqrt{[2(1-\rho_d)\sigma_D^2 + \sigma_N^2]|h_1|^2 + [2(1+\rho_d)\sigma_D^2 + \sigma_N^2]|h_2|^2}}\right) \tag{2.29}$$

Therefore, given estimated channel gains h_1 and h_2, we can write the error probability as

$$\begin{aligned}P_e &= \frac{1}{2}\left[Q\left(\frac{(a+b)|h_1|^2 + (a-b)|h_2|^2 + 2b\,\mathrm{Re}(h_1^* h_2)}{\sqrt{[2(1+\rho_d)\sigma_D^2 + \sigma_N^2]|h_1|^2 + [2(1-\rho_d)\sigma_D^2 + \sigma_N^2]|h_2|^2}}\right)\right. \\ &\quad \left. + Q\left(\frac{(a-b)|h_1|^2 + (a+b)|h_2|^2 + 2b\,\mathrm{Re}(h_1^* h_2)}{\sqrt{[2(1-\rho_d)\sigma_D^2 + \sigma_N^2]|h_1|^2 + [2(1+\rho_d)\sigma_D^2 + \sigma_N^2]|h_2|^2}}\right)\right] \end{aligned} \tag{2.30}$$

By the same method, we can derive the conditional error probability for $\tilde{s}_1$. It is exactly the same expression as (2.30). Hence, for given h_1 and h_2, the conditional error probability of STD in non time-selective, spatially correlated fading with channel estimation errors can be expressed as (2.30). Whenever we collect a pair of estimated channel gains from the channel estimator, we can calculate the error probability by (2.30).

In case of $\rho_s = 0$, no spatially correlated fading, we can have $a = \rho_e^2$, $b = 0$, $\rho_d = 0$ and $\sigma_D^2 = (1-\rho_e^2)\sigma_G^2$. So (2.30) reduces to

$$P_e = Q\left(\rho_e^2\sqrt{\frac{|h_1|^2+|h_2|^2}{2\sigma_D^2+\sigma_N^2}}\right) \tag{2.31}$$

It is the same result as in [12] for STD in non time-selective, spatially uncorrelated fading with imperfect channel estimation.

In case of $\rho_e = 1$, with perfect channel estimation, we can have $a = 1$, $b = 0$, $\rho_d = 0$ and $\sigma_D^2 = 0$. Then (2.30) reduces to

$$P_e = Q\left(\sqrt{\frac{|h_1|^2+|h_2|^2}{\sigma_N^2}}\right) \tag{2.32}$$

which is the same result as in [12] for STD in non time-selective, spatially correlated fading with perfect channel estimation..

We can see from (2.31) and (2.32) that, given estimated channel gains h_1 and h_2, the introducing of the channel correlation will not affect the BER performance; however, the introducing of the channel estimation error will degrade the performance.

2.3 Approximation of Average BER Performance

Normally, given estimated channel gains $H_1 = h_1$ and $H_2 = h_2$, if we know the error probability P_e and the joint pdf of H_1 and H_2, $p_{H_1,H_2}(h_1,h_2)$ [20], we can evaluate the average error probability over the fading channels as

$$P_f = \int_0^{+\infty}\int_0^{+\infty} P_e \cdot p_{H_1,H_2}(h_1,h_2)\,dh_1\,dh_2 \tag{2.33}$$

Since the error probability expression P_e in (2.30) has the terms with $|h_1^2|$, $|h_2^2|$ and $\mathrm{Re}(h_1^* h_2)$ inside the Q-function and H_1 and H_2 are jointly Gaussian, this makes it difficult to calculate the overall BER performance. To simplify the expression, we use the transformation technique discussed in [2 1] to convert the two correlated Rayleigh fading channels into two independent Rayleigh fading channels, then use the new channels to evaluate the performance of the model.

Since we consider only BPSK modulation here, we can rewrite (1.1) as

$$\begin{bmatrix} r_0 \\ r_T \end{bmatrix} = \begin{bmatrix} g_1 & g_2 \\ g_2 & -g_1 \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + \begin{bmatrix} n_0 \\ n_T \end{bmatrix} \tag{2.34}$$

and write the estimated channel gains as

$$\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} g_1 + z_1 \\ g_2 + z_2 \end{bmatrix} \tag{2.35}$$

According to [2 1], if we define the transformation matrix $\mathbf{T}$ as

$$\mathbf{T} = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \tag{2.36}$$

then apply it to (2.34) and (2.35)

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} r_0 \\ r_T \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} g_1 & g_2 \\ g_2 & -g_1 \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} n_0 \\ n_T \end{bmatrix} \tag{2.37}$$

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} g_1 + z_1 \\ g_2 + z_2 \end{bmatrix} \tag{2.38}$$

After simplifying (2.37) and (2.38) to the same form as (2.34) and (2.35), we can write

$$\begin{bmatrix} r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} g_3 & g_4 \\ g_4 & -g_3 \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + \begin{bmatrix} n_3 \\ n_4 \end{bmatrix} \tag{2.39}$$

$$\begin{bmatrix} h_3 \\ h_2 \end{bmatrix} = \begin{bmatrix} g_3 + z_3 \\ g_4 + z_4 \end{bmatrix} \tag{2.40}$$

where fading channels 3 and 4 are new channels generated from the matrix transformation. Accordingly, R_3, R_4, G_3, G_4, N_3, N_4, H_3, H_4, Z_3 and Z_4 are new received signals, new channel gains, new noises, new estimated channel gains and new channel estimation errors. We can express them as

$$r_3 = \frac{\sqrt{2}}{2}(r_0 + r_T) \tag{2.41}$$

$$r_4 = \frac{\sqrt{2}}{2}(r_T - r_0) \tag{2.42}$$

$$g_3 = \frac{\sqrt{2}}{2}(g_1 + g_2) \tag{2.43}$$

$$g_4 = \frac{\sqrt{2}}{2}(g_2 - g_1) \tag{2.44}$$

$$n_3 = \frac{\sqrt{2}}{2}(n_0 + n_T) \tag{2.45}$$

$$n_4 = \frac{\sqrt{2}}{2}(n_T - n_0) \tag{2.46}$$

$$h_3 = \frac{\sqrt{2}}{2}(h_1 + h_2) \tag{2.47}$$

$$h_4 = \frac{\sqrt{2}}{2}(h_2 - h_1) \tag{2.48}$$

$$z_3 = \frac{\sqrt{2}}{2}(z_1 + z_2) \tag{2.49}$$

$$z_4 = \frac{\sqrt{2}}{2}(z_2 - z_1) \tag{2.50}$$

Since G_1, G_2, N_0, N_T, H_1, H_2, Z_1 and Z_2 are all zero mean Complex Gaussian random variables, it is clear that the sums of these random variables, G_3, G_4, N_3, N_4, H_3, H_4, Z_3 and Z_4 are also zero mean complex Gaussian random variables. It is shown in Appendix B that they are statistically independent with variances of $(1+\rho_s)\sigma_G^2$, $(1-\rho_s)\sigma_G^2$, σ_N^2, σ_N^2, $(\frac{1}{\rho_e^2}+\rho_s)\sigma_G^2$, $(\frac{1}{\rho_e^2}-\rho_s)\sigma_G^2$, σ_Z^2 and σ_Z^2 respectively. It is also shown in Appendix B that the correlation coefficient between new channel gains G_i and new estimated channel gains H_i are

$$\rho_{e3}=\rho_e\sqrt{\frac{1+\rho_s}{1+\rho_s\rho_e^2}} \tag{2.51}$$

$$\rho_{e4}=\rho_e\sqrt{\frac{1-\rho_s}{1-\rho_s\rho_e^2}} \tag{2.52}$$

Because the new channels are independent, the new channel gains G_i can be expressed exclusively by its channel estimations H_i. They can be written as

$$g_3=\rho_{e3}^2h_3+d_3 \tag{2.53}$$

$$g_4=\rho_{e4}^2h_4+d_4 \tag{2.54}$$

where D_3 and D_4 are zero mean independent complex Gaussian random variables with variances $\sigma_{D3}^2 = (1-\rho_{e3}^2)\sigma_{G3}^2$ and $\sigma_{D4}^2 = (1-\rho_{e4}^2)\sigma_{G4}^2$. Both are independent of H_3 and H_4.

Now we have converted two correlated channels G_1 and G_2 into two independent channels G_3 and G_4. We can use the same performance evaluation method to evaluate the performance of these new channels. By using the method for getting (2.23), we can prove the new combined signal $\tilde{s}_0$ is

$$\tilde{s}_0 = \rho_{e3}^2 s_0 |h_3|^2 + \rho_{e4}^2 s_0 |h_4|^2 + (\rho_{e4}^2 - \rho_{e3}^2) s_1 h_3^* h_4$$

$$+ h_3^*(d_3 s_0 + d_4 s_1) + h_4(d_4^* s_0 - d_3^* s_1) + h_3^* n_3 + h_4 n_4^* \qquad (2.55)$$

Then, as shown in Appendix C, the decision variable $\mathrm{Re}(\tilde{s}_0)$ is the sum of $\rho_{e3}^2 s_0 |h_3|^2 + \rho_{e4}^2 s_0 |h_4|^2 \pm (\rho_{e4}^2 - \rho_{e3}^2) s_1 \cdot \mathrm{Re}(h_3^* h_4)$ and a zero mean Gaussian random variable with variance $(|h_3|^2 + |h_4|^2)(\sigma_{D3}^2 + \sigma_{D4}^2 + \sigma_N^2) \pm 2\,\mathrm{Re}(h_3 h_4^*)(\sigma_{D4}^2 - \sigma_{D3}^2)$. We can write the conditional BER as

$$P_e = \frac{1}{2}\left[Q\left(\frac{\rho_{e3}^2 |h_3|^2 + \rho_{e4}^2 |h_4|^2 + (\rho_{e4}^2 - \rho_{e3}^2) \cdot \mathrm{Re}(h_3^* h_4)}{\sqrt{(|h_3|^2 + |h_4|^2)(\sigma_{D3}^2 + \sigma_{D4}^2 + \sigma_N^2) + 2\,\mathrm{Re}(h_3 h_4^*)(\sigma_{D4}^2 - \sigma_{D3}^2)}} \right) \right.$$

$$\left. + Q\left(\frac{\rho_{e3}^2 |h_3|^2 + \rho_{e4}^2 |h_4|^2 - (\rho_{e4}^2 - \rho_{e3}^2) \cdot \mathrm{Re}(h_3^* h_4)}{\sqrt{(|h_3|^2 + |h_4|^2)(\sigma_{D3}^2 + \sigma_{D4}^2 + \sigma_N^2) - 2\,\mathrm{Re}(h_3 h_4^*)(\sigma_{D4}^2 - \sigma_{D3}^2)}} \right) \right]$$

$$= \frac{1}{2}\left[Q\left(\frac{\rho_{e3}^2 r_3^2 + \rho_{e4}^2 r_4^2 + (\rho_{e4}^2 - \rho_{e3}^2) r_3 r_4 \cos(\theta_3 - \theta_4)}{\sqrt{(r_3^2 + r_4^2)(\sigma_{D3}^2 + \sigma_{D4}^2 + \sigma_N^2) + 2 r_3 r_4 \cos(\theta_3 - \theta_4)(\sigma_{D4}^2 - \sigma_{D3}^2)}} \right) \right.$$

$$\left. + Q\left(\frac{\rho_{e3}^2 r_3^2 + \rho_{e4}^2 r_4^2 - (\rho_{e4}^2 - \rho_{e3}^2) r_3 r_4 \cos(\theta_3 - \theta_4)}{\sqrt{(r_3^2 + r_4^2)(\sigma_{D3}^2 + \sigma_{D4}^2 + \sigma_N^2) - 2 r_3 r_4 \cos(\theta_3 - \theta_4)(\sigma_{D4}^2 - \sigma_{D3}^2)}} \right) \right] \qquad (2.56)$$

Here we rewrite the estimated channel gain as $h_i = r_i \exp(j\theta_i) = r_i \cos(\theta_i) + j r_i \sin(\theta_i)$, $i = 3, 4$, where r_i is Rayleigh distributed and θ_i is uniformly distributed in $(0, 2\pi)$. Because H_3 and H_4 are independent, we can write the pdf separately as

$$p_{R_3}(r_3) = \frac{r_3}{\sigma_{H_3}^2} \exp\left(-\frac{r_3^2}{2\sigma_{H_3}^2}\right)$$
$$p_{R_3}(r_3) = \frac{r_4}{\sigma_{H_4}^2} \exp\left(-\frac{r_4^2}{2\sigma_{H_4}^2}\right) \tag{2.57}$$

$$p_{\Theta_3}(\theta_3) = \frac{1}{2\pi}$$
$$p_{\Theta_4}(\theta_4) = \frac{1}{2\pi} \tag{2.58}$$

where

$$\sigma_{H_3}^2 = (\frac{1}{\rho_e^2} + \rho_s)\sigma_G^2$$
$$\sigma_{H_3}^2 = (\frac{1}{\rho_e^2} - \rho_s)\sigma_G^2 \tag{2.59}$$

Then we can write the average BER as

$$P_f = \int_0^{+\infty} \int_0^{+\infty} \int_0^{2\pi} \int_0^{2\pi} P_e \; p_{R_3}(r_3) p_{R_4}(r_4) p_{\Theta_3}(\theta_3) p_{\Theta_4}(\theta_4) d\theta_3 d\theta_3 dr_3 dr_4 \tag{2.60}$$

As θ_3 and θ_4 are independent, we use $\theta = \theta_3 - \theta_4$ to reduce the average BER as

$$P_f = \int_0^{+\infty} \int_0^{+\infty} \int_0^{2\pi} P_e \cdot \frac{1}{2\pi} p_{R_3}(r_3) p_{R_4}(r_4) d\theta \, dr_3 dr_4 \tag{2.61}$$

Although we eliminate one integral in (2.61) and H_3 and H_4 are independent compared to H_1 and H_2 in (2.33), we still have to perform a triple integral in (2.61). It would be better if we can eliminate all integrals and get a closed-form expression for P_f. Although the method described in [1 6] can diagonalize a Hermitian quadratic form in complex Gaussian variables and then get the corresponding pdf, it is shown in Appendix H that a closed-form expression is difficult to obtain.

We noticed that the STD scheme gives the best performance when two channels are spatially independent. As a result, in a real system, all efforts will be made to minimize the spatial correlation between channels to achieve the best performance. In that case, the performance of STD over small spatial correlation is more significant than that of big correlation.

In the case of small spatial correlation, that is, when $\rho_s << 1$, we have found that ρ_{e3} and ρ_{e4} are very close to ρ_e. Although the actual condition for ρ_{e3} and ρ_{e4} to equal ρ_e is $\rho_s = 0$ or $\rho_e = 1$, the difference between ρ_{ei} and ρ_e is very small when $\rho_s << 1$ or $\rho_e >> 0$. Therefore, to simplify the analysis, when the spatial correlation is very small, i.e., $\rho_s << 1$ or the channel estimation is almost perfect, i.e., $\rho_e >> 0$, we use approximations to complete the analysis. Therefore, we rewrite (2.51) and (2.52) as

$$\rho_{e3} = \rho_e \sqrt{\frac{1+\rho_s}{1+\rho_s\rho_e^2}} \approx \rho_e \tag{2.62}$$

$$\rho_{e4} = \rho_e \sqrt{\frac{1-\rho_s}{1-\rho_s\rho_e^2}} \approx \rho_e \tag{2.63}$$

and, (2.53) and (2.54) as

$$g_3 \approx \rho_e^2 h_3 + d_3 \tag{2.64}$$

$$g_4 \approx \rho_e^2 h_4 + d_4 \tag{2.65}$$

Consequently, the variances of H_1, H_2, D_3 and D_4 can be simplified to

$$\sigma_{H3}^2 \approx \frac{\sigma_{G3}^2}{\rho_e^2} \tag{2.66}$$

$$\sigma_{H4}^2 \approx \frac{\sigma_{G4}^2}{\rho_e^2} \tag{2.67}$$

$$\sigma_{D3}^2 \approx (1-\rho_e^2)\sigma_{G3}^2 \tag{2.68}$$

$$\sigma_{D4}^2 \approx (1-\rho_e^2)\sigma_{G4}^2 \tag{2.69}$$

By substituting (2.62) and (2.63) to (2.55), we can rewrite (2.55) as

$$\tilde{s}_0 \approx \rho_e^2 s_0(|h_3|^2+|h_4|^2)+h_3^*(d_3 s_0+d_4 s_1)+h_4(d_4^* s_0-d_3^* s_1)+h_3^* n_3+h_4 n_4^* \tag{2.70}$$

Now the decision variable $\mathrm{Re}(\tilde{s}_0)$ is the sum of $\rho_e^2(|h_3|^2+|h_4|^2)s_0$ and a zero mean Gaussian random variable with variance $(|h_3|^2+|h_4|^2)(2\sigma_D^2+\sigma_N^2)\mp 4\rho_s(1-\rho_e^2)\sigma_G^2\,\mathrm{Re}(h_3h_4^*)\approx(|h_3|^2+|h_4|^2)(2\sigma_D^2+\sigma_N^2)$. Thus, we can write the conditional error probability given h_3 and h_4 as

$$\begin{aligned} P_e &= Q\left(\frac{\rho_e^2(|h_3|^2+|h_4|^2)}{\sqrt{(|h_3|^2+|h_4|^2)(2\sigma_D^2+\sigma_N^2)}}\right) \\ &= Q\left(\sqrt{\frac{\rho_e^4(|h_3|^2+|h_4|^2)}{2\sigma_D^2+\sigma_N^2}}\right) \\ &= Q\left(\sqrt{2K(|h_3|^2+|h_4|^2)}\right) \\ &= Q\left(\sqrt{2\mu}\right) \end{aligned} \tag{2.71}$$

where

$$K = \frac{\rho_e^4}{2(2\sigma_D^2 + \sigma_N^2)} = \frac{\rho_e^4}{2[2(1-\rho_e^2)\sigma_G^2 + \sigma_N^2]} \tag{2.72}$$

$$\mu = K(|h_3|^2 + |h_4|^2) \tag{2.73}$$

We know that after approximation, H_3 and H_4 become zero mean independent complex Gaussian random variables with variances $2\sigma_{G3}^2 / \rho_e^2$ and $2\sigma_{G4}^2 / \rho_e^2$. We can write the covariance matrix of H_3 and H_4 as

$$C_{H34} = \begin{bmatrix} \frac{\sigma_{G3}^2}{\rho_e^2} & 0 \\ 0 & \frac{\sigma_{G4}^2}{\rho_e^2} \end{bmatrix} \tag{2.74}$$

If the pdf of $\mathrm{M} = K(|H_3|^2 + |H_4|^2)$ is $f_M(\mu)$, then from [1 6], its Laplace transform can be written as

$$P(s) = \prod_{i=3}^{4} \frac{1}{1 + s\Gamma_i} \tag{2.75}$$

where $\Gamma_i = 2K\lambda_i$, λ_i are the eigenvalues of (2.74) as

$$\lambda_3 = \frac{(1 + \rho_s \rho_e^2)\sigma_G^2}{\rho_e^2} \tag{2.76}$$

$$\lambda_4 = \frac{(1 - \rho_s \rho_e^2)\sigma_G^2}{\rho_e^2} \tag{2.77}$$

From [2 2] $f_M(\mu)$ can be written as

$$f_M(\mu) = \sum_{j=3}^{4} d_j \exp(s_j \mu), \qquad a \geq 0 \tag{2.78}$$

where d_j are the poles and s_j are the residues of (2.75). Then the average error probability can be calculated as

$$P_f = \int_0^{+\infty} Q(\sqrt{2\mu}) f_M(\mu) d\mu \tag{2.79}$$

By using [23, 24], we can reduce (2.79) to

$$P_f = \frac{1}{2(\Gamma_3 - \Gamma_4)}\left[\Gamma_3\left(1-\sqrt{\frac{\Gamma_3}{1+\Gamma_3}}\right) - \Gamma_4\left(1-\sqrt{\frac{\Gamma_4}{1+\Gamma_4}}\right)\right] \tag{2.80}$$

where

$$\Gamma_3 = 2K\lambda_3 = \frac{\rho_e^2(1+\rho_s\rho_e^2)\sigma_G^2}{2(1-\rho_e^2)\sigma_G^2 + \sigma_N^2} \tag{2.81}$$

$$\Gamma_4 = 2K\lambda_4 = \frac{\rho_e^2(1-\rho_s\rho_e^2)\sigma_G^2}{2(1-\rho_e^2)\sigma_G^2 + \sigma_N^2} \tag{2.82}$$

This is the approximate BER for STD with spatially correlated fading and channel estimation error when ρ_s<<1 or ρ_e>>0.

In the case of $\rho_e = 1$, (2.81) and (2.82) reduce to

$$\Gamma_3 = \frac{(1+\rho_s)\sigma_G^2}{\sigma_N^2} \tag{2.83}$$

$$\Gamma_4 = \frac{(1-\rho_s)\sigma_G^2}{\sigma_N^2} \tag{2.84}$$

The result is the same as in [12] for STD in non time-selective, spatially correlated fading with perfect channel estimation.

In the case of $\rho_s = 0$, (2.81) and (2.82) reduce to

$$\Gamma_3 = \Gamma_4 = \frac{\rho_e^2 \sigma_G^2}{2(1-\rho_e^2)\sigma_G^2 + \sigma_N^2} \tag{2.85}$$

We can write $f_M(\mu)$ as [16]

$$f_M(\mu) = \frac{\mu}{\Gamma_3^2} \exp(-\frac{\mu}{\Gamma_3}) \tag{2.86}$$

Then (2.79) can be reduced to

$$P_f = \frac{1}{4}\left(1 - \sqrt{\frac{\Gamma_3}{1+\Gamma_3}}\right)^2 \left(2 + \sqrt{\frac{\Gamma_3}{1+\Gamma_3}}\right) \tag{2.87}$$

This result is exactly the same as shown in [12] for STD in non time-selective, spatially uncorrelated fading with imperfect channel estimation.

2.4 Numerical Results

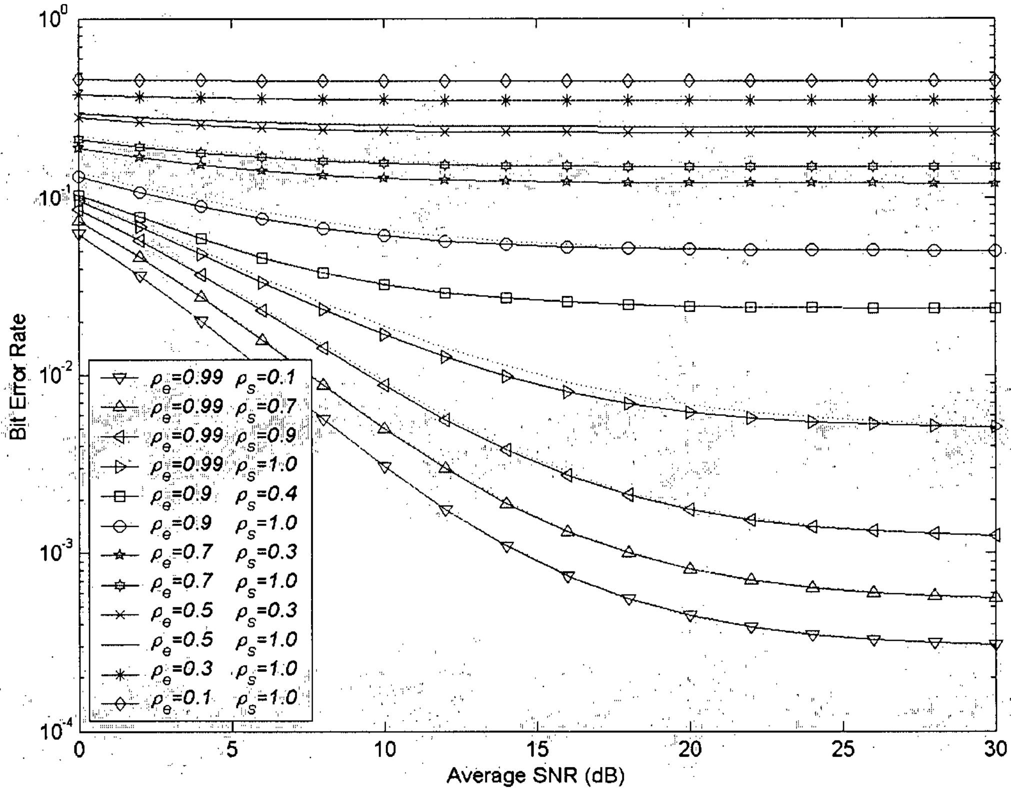

Fig. 2.1 Comparison of approximate analytic BER to simulation results (approximation: solid lines; simulation: dotted lines)

The approximate and simulated BER curves are plotted as a function of the average SNR, defined as the ratio of the variance of the channel gain to the variance of the additive Gaussian noise, i.e., σ_G^2/σ_N^2, for different (ρ_e, ρ_s) values. As expected from the analysis in Section 2.3, the approximate BER agrees very well with the values from simulation when ρ_s is close to 0 and ρ_e is close to 1. Moreover, the approximate and simulation results are close for all (ρ_e, ρ_s) values plotted. The largest percentage error is about 6% and occurs for (ρ_e = 0.9, ρ_s = 1). For SNR values greater than about 25 dB, the approximate and simulated values agree very closely for any (ρ_e, ρ_s) value.

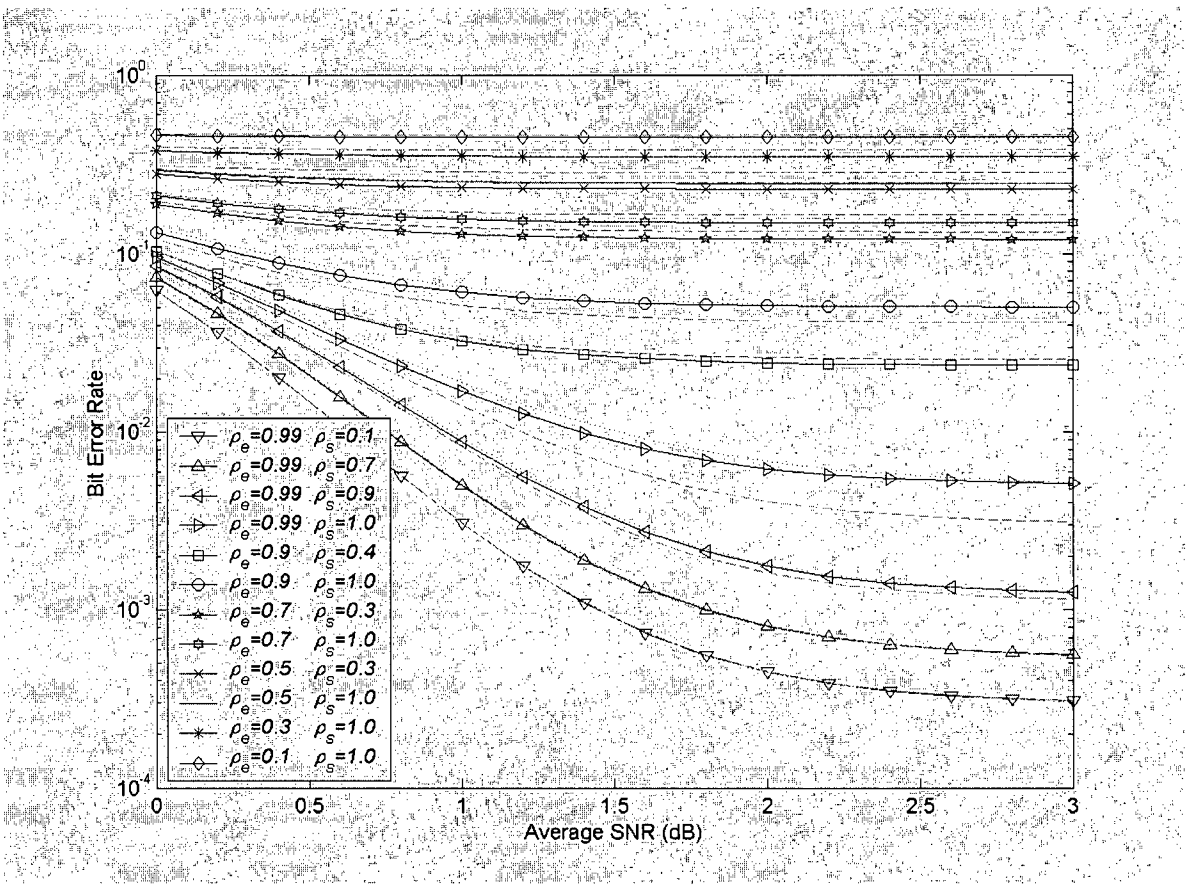

Fig. 2.2 Comparison of approximate analytic BER to results in [12]

(approximation: solid lines; simulation: dotted lines;

result from [12]: dashed lines)

Compared to the BER expression in [12], (2.80) gives the same result when $\rho_s = 0$ or $\rho_e = 1$. For other values of (ρ_e, ρ_s), it is shown in Fig. 2.2 that the method discussed in Section 2.3 gives more accurate results.

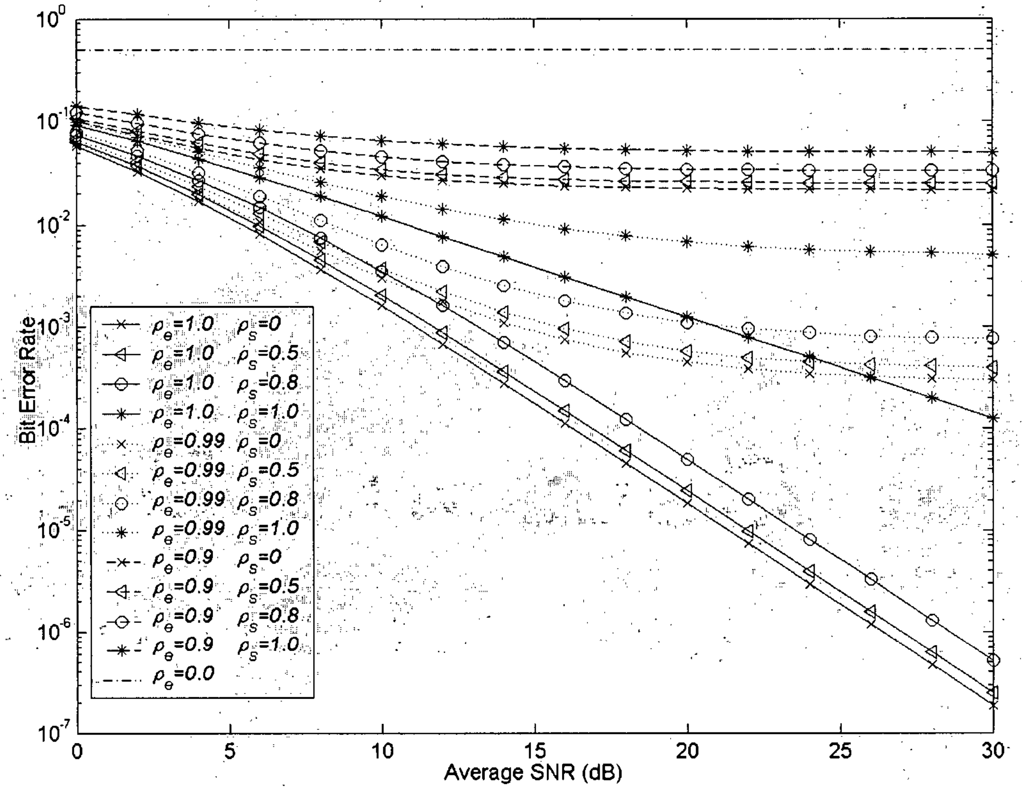

Fig. 2.3Simulated BER curves as a function of average SNR for different (ρ_e, ρ_s) values

In the case of spatially correlated fading with channel estimation error, the BER increases as ρ_s increases from 0 to 1 and as ρ_e decreases from 1 to 0.. The BER degradation for $\rho_e = 1-\Delta$ with ρ_s fixed is larger than for $\rho_s = \Delta$ with ρ_e fixed. As shown in Fig. 2.3, for $\rho_e = 1$, $\rho_s = 0$ and a target BER of 10^{-3}, there is about 0.6 dB degradation when ρ_s increases to 0.5 and about 2.1 dB degradation when ρ_s increases to 0.8. The degradation is about 3.2 dB when ρ_e decreases from 1.0 to 0.99; if ρ_e continues to decrease, i.e., to 0.9, the target BER cannot be attained. We can also see from Fig. 2.3 that for each ρ_e value, there is a performance floor which occurs at $\rho_s = 0$.

This is the best BER performance STD can achieve for a given ρ_e value. The exact expression for this BER floor is given by (2.87).

It can be observed from Fig. 2.3 that the spatial correlation influences the BER performance more as the channel estimation error increases. For a target BER of 10^{-3} and $\rho_e = 1$, the degradation is about 0.6 dB and 2.1 dB when ρ_s changes from 0 to 0.5 and 0.8 respectively. When $\rho_e = 0.99$, the degradations increase to 1.3 dB and 6.9 dB respectively.

The worst BER performance of STD occurs as ρ_e approaches 0. In such a case, the channel estimations become random and the BER approaches 0.5.

In the analysis above, for each BER curve, we assume that the channel estimation correlation coefficient ρ_e is fixed. However, the changes of SNR will affect the accuracy of the channel estimation. The influence of SNR to ρ_e varies with different channel estimation models. In a simple model described in Appendix I, we can see that $\rho_e = \dfrac{1}{\sqrt{1+\dfrac{1}{\mathrm{SNR}}}}$. By using this result together with the analytic results from Section 2.3, we can plot the BER curves for this model as a function of SNR where ρ_e changes with SNR.

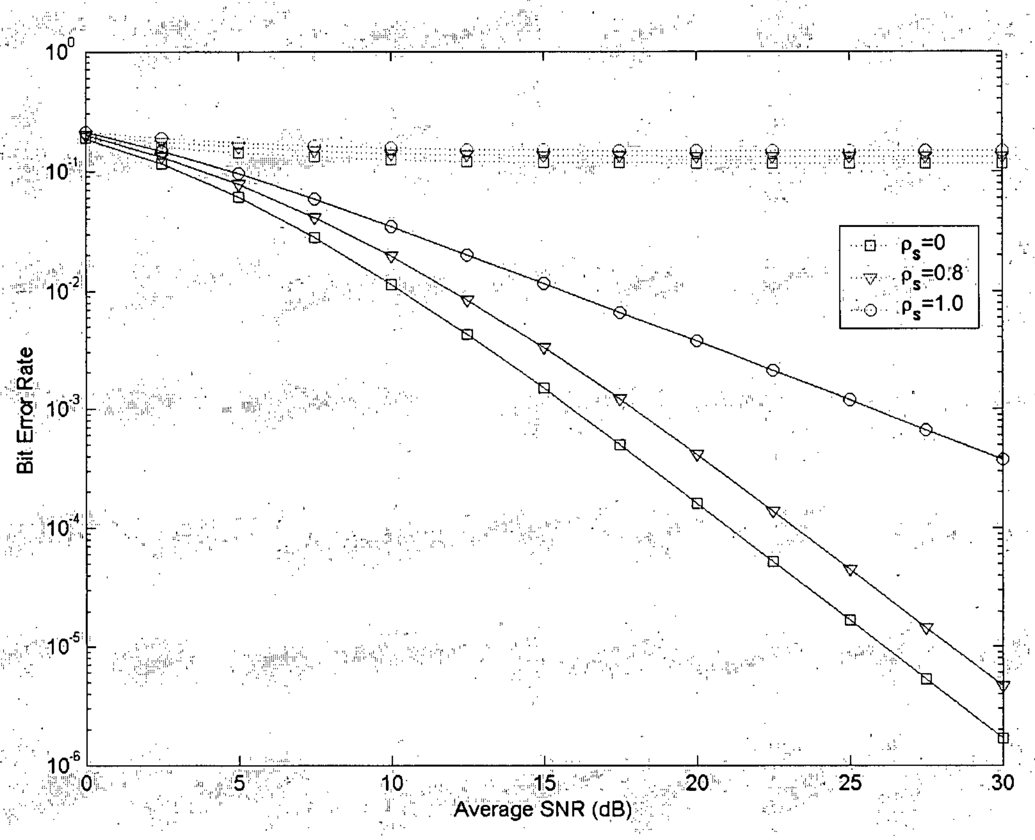

Fig. 2.4 Approximate analytic BER curves as a function of average SNR

($\rho_e = \dfrac{1}{\sqrt{2}}$: dotted lines; $\rho_e = \dfrac{1}{\sqrt{1+\dfrac{1}{\mathrm{SNR}}}}$: solid lines)

Fig. 2.4 shows with the increase of SNR form 0 dB, $\rho_e = \dfrac{1}{\sqrt{1+\dfrac{1}{\mathrm{SNR}}}}$ increases from $\dfrac{1}{\sqrt{2}}$ and the performance is continuously improving. The channel estimation becomes perfect when the SNR increases to infinity.

3 STD in Time-selective, Spatially Correlated Fading with Perfect Channel Estimation

In this chapter, we analyze the performance of STD in time-selective, spatially correlated fading channels with perfect channel estimation.

3.1 System Model

We combine the time-selective fading from [2 5] together with the spatially correlated fading mentioned before, but without channel estimation error in this model. For each spatial channel, the channel gain is constant over one symbol duration but can be changed in the successive symbol period. We denote the channel gains from two transmit antennas to the receive antenna as G_{10}, G_{1T}, G_{20} and G_{2T}. They are zero mean complex Gaussian random variables with the same variance σ_G^2. G_{10} and G_{1T}, and G_{20} and G_{2T} are spatially correlated with the correlation coefficient ρ_s. Meanwhile, G_{10} and G_{1T}, and G_{20} and G_{2T} are correlated with the time-selective correlation coefficient ρ_t. Appendix D shows that the correlation coefficient between G_{10} and G_{2T}, and between G_{20} and G_{1T} are $\rho_s \rho_t$. Thus, we can express the 2×2 covariance matrix of G_{10}, G_{20}, G_{1T} and G_{2T} as

$$\mathbf{C}_{\mathbf{G}_{4\times4}} = \begin{bmatrix} \sigma_G^2 & \rho_s\sigma_G^2 & \rho_t\sigma_G^2 & \rho_s\rho_t\sigma_G^2 \\ \rho_s\sigma_G^2 & \sigma_G^2 & \rho_s\rho_t\sigma_G^2 & \rho_t\sigma_G^2 \\ \rho_t\sigma_G^2 & \rho_s\rho_t\sigma_G^2 & \sigma_G^2 & \rho_s\sigma_G^2 \\ \rho_s\rho_t\sigma_G^2 & \rho_t\sigma_G^2 & \rho_s\sigma_G^2 & \sigma_G^2 \end{bmatrix} \tag{3.1}$$

We already know the covariance matrix of G_{10} and G_{20} is

$$\mathbf{C}_{\mathbf{G}_{2\times2}} = \begin{bmatrix} \sigma_G^2 & \rho_s\sigma_G^2 \\ \rho_s\sigma_G^2 & \sigma_G^2 \end{bmatrix} \tag{3.2}$$

By using the linear transformation [1 8] of G_{10}, G_{20} together with two new complex Gaussian random variables, we can express G_{1T} and G_{2T} by G_{10} and G_{20}. Appendix E shows that we can write them as

$$g_{1T} = \rho_t g_{10} + v_1 \tag{3.3}$$

$$g_{2T} = \rho_t g_{20} + v_2 \tag{3.4}$$

where V_1 and V_2 are zero mean correlated complex Gaussian random variables with variance $\sigma_V^2 = (1-\rho_t^2)\sigma_G^2$ and correlation coefficient ρ_s.

Following the STD scheme, the received signals at time 0 and time T can be written as

$$r_0 = g_{10}s_0 + g_{20}s_1 + n_0 \tag{3.5}$$

$$r_T = g_{2T}s_0 - g_{1T}s_1 + n_T \tag{3.6}$$

Then the combined signals $\tilde{s}_0$ and $\tilde{s}_1$ can be got from

$$\tilde{s}_0 = g_{10}^* r_0 + g_{2T} r_T^* \tag{3.7}$$

$$\tilde{s}_1 = g_{20}^* r_0 - g_{1T} r_T^* \tag{3.8}$$

Based on the values of the combined signals, we can use maximum-likelihood to decode the information bits s_0 and s_1. If the real part of $\tilde{s}_0$ is greater than 0, $s_0 = 1$ will be chosen; otherwise, $s_0 = -1$ will be selected. Similarly, the same decoding rule applies to recover $\tilde{s}_1$.

3.2 Performance Analysis

When there is no channel estimation error, the estimated channel gain H_{ij} is equal to the channel gain G_{ij}. Therefore, we can simplify (1.6) as

$$\begin{bmatrix} \tilde{s}_0 \\ \tilde{s}_1 \end{bmatrix} = \begin{bmatrix} |g_{10}|^2 + |g_{2T}|^2 & g_{10}^* g_{20} - g_{1T}^* g_{2T} \\ g_{10} g_{20}^* - g_{1T} g_{2T}^* & |g_{20}|^2 + |g_{1T}|^2 \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + \begin{bmatrix} g_{10}^* & g_{2T} \\ g_{20}^* & -g_{1T} \end{bmatrix} \begin{bmatrix} n_0 \\ n_T^* \end{bmatrix} \tag{3.9}$$

Now we consider the case for s_0 first. From Appendix E, we can represent G_{20} and G_{1T} by G_{10} and G_{2T} as

$$\begin{aligned} g_{1T} &= a g_{10} + b g_{2T} + w_1 \\ g_{20} &= b g_{10} + a g_{2T} + w_2 \end{aligned} \tag{3.10}$$

where

$$\begin{aligned} a &= \rho_t \left(\frac{1-\rho_s^2}{1-\rho_s^2 \rho_t^2} \right) \\ b &= \rho_s \left(\frac{1-\rho_t^2}{1-\rho_s^2 \rho_t^2} \right) \end{aligned} \tag{3.11}$$

W_1 and W_2 are zero mean correlated complex Gaussian random variables with variance $\sigma_W^2 = (1-\rho_t^2)\left(\frac{1-\rho_s^2}{1-\rho_s^2 \rho_t^2} \right)\sigma_G^2$ and correlation coefficient $\rho_w = -\rho_s \rho_t$.

By substituting (3.10), $\tilde{s}_0$ in (3.9) can be expressed as

$$\begin{aligned}\tilde{s}_0 &= (|g_{10}|^2 + |g_{2T}|^2)s_0 + (g_{10}^* g_{20} - g_{1T}^* g_{2T})s_1 + g_{10}^* n_0 + g_{2T} n_T^* \\ &= (|g_{10}|^2 + |g_{2T}|^2)s_0 + b(|g_{10}|^2 - |g_{2T}|^2)s_1 + (g_{10}^* w_2 - g_{2T} w_1^*)s_1 + g_{10}^* n_0 + g_{2T} n_T^*\end{aligned} \tag{3.12}$$

When $s_1 = s_0$, the decision variable $\mathrm{Re}(\tilde{s}_0)$ can be written as

$$\mathrm{Re}(\tilde{s}_0) = [(1+b)|g_{10}|^2 + (1-b)|g_{2T}|^2]s_0 + \mathrm{Re}(g_{10}^* w_2 - g_{2T} w_1^*)s_0 + \mathrm{Re}(g_{10}^* n_0 + g_{2T} n_T^*) \tag{3.13}$$

Appendix F shows that $\mathrm{Re}(g_{10}^* W_2 - g_{2T} W_1^*)$, $\mathrm{Re}(g_{10}^* N_0)$ and $\mathrm{Re}(g_{2T} N_T^*)$ are zero mean independent Gaussian random variables with variances $[|g_{10}|^2 + |g_{2T}|^2 - 2\rho_w \mathrm{Re}(g_{10} g_{2T}^*)]\sigma_W^2$, $|g_{10}|^2 \sigma_N^2$ and $|g_{2T}|^2 \sigma_N^2$ respectively. Thus $\mathrm{Re}(\tilde{s}_0)$ is the sum of $[(1+b)|g_{10}|^2 + (1-b)|g_{2T}|^2]s_0$ and an zero mean Gaussian random variable with variance $(|g_{10}|^2 + |g_{2T}|^2)(\sigma_W^2 + \sigma_N^2) - 2\rho_w \mathrm{Re}(g_{10} g_{2T}^*)\sigma_W^2$. Given the channel gains g_{10} and g_{2T}, the conditional error probability can be written as

$$P_{e,s_1=s_0} = Q\left(\frac{(1+b)|g_{10}|^2 + (1-b)|g_{2T}|^2}{\sqrt{(|g_{10}|^2 + |g_{2T}|^2)(\sigma_W^2 + \sigma_N^2) - 2\rho_w \mathrm{Re}(g_{10} g_{2T}^*)\sigma_W^2}}\right) \tag{3.14}$$

Similarly, when $s_1 = -s_0$, the conditional error probability can be written as

$$P_{e,s_1=-s_0} = Q\left(\frac{(1-b)|g_{10}|^2 + (1+b)|g_{2T}|^2}{\sqrt{(|g_{10}|^2 + |g_{2T}|^2)(\sigma_W^2 + \sigma_N^2) - 2\rho_w \mathrm{Re}(g_{10} g_{2T}^*)\sigma_W^2}}\right) \tag{3.15}$$

Because the chances of $s_1 = s_0$ and $s_1 = -s_0$ are equal, given g_{10} and g_{2T}, the conditional error bit of s_0 can be expressed as

$$P_{e,s_0} = \frac{1}{2} Q\left(\frac{(1+b)|g_{10}|^2 + (1-b)|g_{2T}|^2}{\sqrt{(|g_{10}|^2 + |g_{2T}|^2)(\sigma_W^2 + \sigma_N^2) - 2\rho_w \operatorname{Re}(g_{10} g_{2T}^*)\sigma_W^2}} \right)$$

$$+ \frac{1}{2} Q\left(\frac{(1-b)|g_{10}|^2 + (1+b)|g_{2T}|^2}{\sqrt{(|g_{10}|^2 + |g_{2T}|^2)(\sigma_W^2 + \sigma_N^2) - 2\rho_w \operatorname{Re}(g_{10} g_{2T}^*)\sigma_W^2}} \right) \quad (3.16)$$

Similarly, given g_{1T} and g_{20}, we can prove that the conditional error probability of s_1 is

$$P_{e,s_1} = \frac{1}{2} Q\left(\frac{(1+b)|g_{20}|^2 + (1-b)|g_{1T}|^2}{\sqrt{(|g_{20}|^2 + |g_{1T}|^2)(\sigma_W^2 + \sigma_N^2) - 2\rho_w \operatorname{Re}(g_{20} g_{1T}^*)\sigma_W^2}} \right)$$

$$+ \frac{1}{2} Q\left(\frac{(1-b)|g_{20}|^2 + (1+b)|g_{1T}|^2}{\sqrt{(|g_{20}|^2 + |g_{1T}|^2)(\sigma_W^2 + \sigma_N^2) - 2\rho_w \operatorname{Re}(g_{20} g_{1T}^*)\sigma_W^2}} \right) \quad (3.17)$$

When $\rho_s = 0$, we have $a = \rho_t$, $b = 0$, $\rho_w = 0$ and $\sigma_W^2 = (1-\rho_t^2)\sigma_G^2$. Thus, we can reduce (3.16) to

$$P_{e,s_0} = Q\left(\sqrt{\frac{|g_{10}|^2 + |g_{2T}|^2}{(1-\rho_t^2)\sigma_G^2 + \sigma_N^2}} \right) \quad (3.18)$$

This is the same result shown in [1 2] for STD in time-selective, spatially uncorrelated fading with perfect channel estimation.

When $\rho_t = 1$, we have $a = 1$, $b = 0$, $\rho_w = -\rho_s$ and $\sigma_W^2 = 0$. Then we can reduce (3.16) to

$$P_{e,s_0} = Q\left(\sqrt{\frac{|g_{10}|^2 + |g_{2T}|^2}{\sigma_N^2}}\right) \tag{3.19}$$

which is the same result shown in [12] for STD in non time-selective, spatially correlated fading with perfect channel estimation.

From (3.18) and (3.19) we can see that given the channel gains g_{10} and g_{2T}, introducing the spatial correlation will not increase the BER; however, introducing the time-selective correlation will degrade the BER performance.

3.3 Average Performance Approximation

In case of STD in time-selective, spatially correlated fading with perfect channel estimation, with BPSK modulation, (1.4) can be rewritten as

$$\begin{bmatrix} r_0 \\ r_T \end{bmatrix} = \begin{bmatrix} g_{10} & g_{20} \\ g_{2T} & -g_{1T} \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + \begin{bmatrix} n_0 \\ n_T \end{bmatrix} \tag{3.21}$$

As in section 2.3, if we use the transformation technique to simplify the analysis, we can apply the transformation matrix T in (2.36) to both sides of (3.21) and get the new received signals as

$$\begin{bmatrix} r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} g_{30} & g_{40} \\ g_{4T} & -g_{3T} \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + \begin{bmatrix} n_3 \\ n_4 \end{bmatrix} \tag{3.22}$$

where channel 3 and channel 4 are new channels generated from the matrix transformation of channel 1 and channel 2. Correspondingly, R_3, R_4, G_{30}, G_{40}, G_{3T}, G_{4T}, N_3 and N_4 are random variables for new received signals, new channel gains and new channel noises after the transformation. The samples of the new channel noises are the same as in (2.45) and (2.46). The samples of new channel gains are

$$\begin{bmatrix} g_{30} & g_{40} \\ g_{4T} & g_{3T} \end{bmatrix} = \frac{\sqrt{2}}{2}\begin{bmatrix} g_{10}+g_{2T} & g_{20}-g_{1T} \\ g_{2T}-g_{10} & g_{1T}+g_{20} \end{bmatrix} \tag{3.23}$$

As in Section 2.3, we can prove that all these new random variables are zero mean complex Gaussian random variables. The new channel noises N_3 and N_4 are statistically independent with the same variance σ_N^2. Appendix G shows that the gains of channel 3 and channel 4 are statistically independent. Therefore, the covariance of each channel can be expressed as

$$\mathbf{C}_{\mathbf{G}_3} = \begin{bmatrix} \sigma_{G_3}^2 & \rho_{t3}\sigma_{G_3}^2 \\ \rho_{t3}\sigma_{G_3}^2 & \sigma_{G_3}^2 \end{bmatrix} \tag{3.24}$$

$$\mathbf{C}_{\mathbf{G}_4} = \begin{bmatrix} \sigma_{G_4}^2 & \rho_{t4}\sigma_{G_4}^2 \\ \rho_{t4}\sigma_{G_4}^2 & \sigma_{G_4}^2 \end{bmatrix} \tag{3.25}$$

where

$$\sigma_{G_3}^2 = (1+\rho_s\rho_t)\sigma_G^2 \tag{3.26}$$

$$\sigma_{G4}^2 = (1-\rho_s\rho_t)\sigma_G^2 \tag{3.27}$$

$$\rho_{t3} = \frac{\rho_t+\rho_s}{1+\rho_s\rho_t} \tag{3.28}$$

$$\rho_{t4} = \frac{\rho_t - \rho_s}{1 - \rho_s \rho_t} \tag{3.29}$$

Hence the channel gains can be written as

$$g_{3T} = \rho_{t3} g_{30} + v_3 \tag{3.30}$$

$$g_{40} = \rho_{t4} g_{4T} + v_4 \tag{3.31}$$

where V_3 and V_4 are zero mean independent complex Gaussian random variables with variances

$$\sigma_{V3}^2 = (1 - \rho_{t3}^2)\sigma_{G3}^2 = \frac{(1-\rho_s^2)(1-\rho_t^2)}{1+\rho_s\rho_t}\sigma_G^2 \tag{3.32}$$

$$\sigma_{V4}^2 = (1 - \rho_{t4}^2)\sigma_{G4}^2 = \frac{(1-\rho_s^2)(1-\rho_t^2)}{1-\rho_s\rho_t}\sigma_G^2 \tag{3.33}$$

where V_3 and V_4 are independent of G_{30} and G_{4T}.

If we substitute (2.30) and (2.31) into (3.9), we get

$$\begin{aligned} \tilde{s}_0 &= (|g_{30}|^2 + |g_{4T}|^2)s_0 + (g_{30}^* g_{40} - g_{3T}^* g_{4T})s_1 + g_{30}^* n_3 + g_{4T} n_4^* \\ &= (|g_{30}|^2 + |g_{4T}|^2)s_0 + (\rho_{t4} - \rho_{t3}) g_{30}^* g_{4T} s_1 + (g_{30}^* v_4 - g_{4T} v_3^*)s_1 + g_{30}^* n_3 + g_{4T} n_4^* \end{aligned} \tag{3.34}$$

As in Section 2.3, if we look at the case of small spatial correlation, we can assume that ρ_{t3} and ρ_{t4} are very close to ρ_t. The condition for this assumption is that $\rho_s << 1$ and $\rho_t >> \rho_s$. By using $\rho_{t3} \approx \rho_{t4} \approx \rho_t$, we can rewrite (3.30) ~ (3.34) as

$$g_{3T} \approx \rho_t g_{30} + v_3 \tag{3.35}$$

$$g_{40} \approx \rho_t g_{4T} + v_4 \tag{3.36}$$

$$\sigma_{V3}^2 \approx \sigma_{V4}^2 \approx (1-\rho_t^2)\sigma_G^2 = \sigma_V^2 \tag{3.37}$$

$$\tilde{s}_0 \approx (|g_{30}|^2 + |g_{4T}|^2)s_0 + (g_{30}^* v_4 - g_{4T} v_3^*)s_1 + g_{30}^* n_3 + g_{4T} n_4^* \tag{3.38}$$

Because V_3, V_4, N_3 and N_4 are independent, given g_{30} and g_{4T}', the BER expressions from (3.38) are the same for all combinations of the BPSK signals s_0 and s_1. They can be expressed as

$$\begin{aligned} P_e &= Q\left(\frac{|g_{30}|^2 + |g_{4T}|^2}{\sqrt{(\sigma_V^2 + \sigma_N^2)(|g_{30}|^2 + |g_{4T}|^2)}}\right) \\ &= Q\left(\sqrt{\frac{|g_{30}|^2 + |g_{4T}|^2}{\sigma_V^2 + \sigma_N^2}}\right) \\ &= Q\left(\sqrt{2K(|g_{30}|^2 + |g_{4T}|^2)}\right) \\ &= Q\left(\sqrt{2\mu}\right) \end{aligned} \tag{3.39}$$

where

$$K = \frac{1}{2(\sigma_V^2 + \sigma_N^2)} = \frac{1}{2[(1-\rho_t^2)\sigma_G^2 + \sigma_N^2]} \tag{3.40}$$

$$\mu = K(|g_{30}|^2 + |g_{4T}|^2) \tag{3.41}$$

We know that G_{30} and G_{4T} are zero mean independent Gaussian random variables with variances $(1+\rho_s\rho_t)\sigma_G^2$ and $(1-\rho_s\rho_t)\sigma_G^2$. Thus, we can write the covariance matrix of

G_{30} and G_{4T} as

$$C_{G_{30,4T}} = \begin{bmatrix} (1+\rho_s\rho_t)\sigma_G^2 & 0 \\ 0 & (1-\rho_s\rho_t)\sigma_G^2 \end{bmatrix} \tag{3.42}$$

Its eigenvalues are

$$\lambda_3 = (1+\rho_s\rho_t)\sigma_G^2 \tag{3.43}$$

$$\lambda_4 = (1-\rho_s\rho_t)\sigma_G^2 \tag{3.44}$$

By using the same method in section 2.3, given $\rho_s << 1$ and $\rho_t >> \rho_s$, we can obtain the approximation of the average BER of STD in time-selective, spatially correlated fading with perfect channel estimation as

$$P_f = \frac{1}{2(\Gamma_3 - \Gamma_4)}\left[\Gamma_3\left(1-\sqrt{\frac{\Gamma_3}{1+\Gamma_3}}\right) - \Gamma_4\left(1-\sqrt{\frac{\Gamma_4}{1+\Gamma_4}}\right)\right] \tag{3.45}$$

where

$$\Gamma_3 = 2K\lambda_3 = \frac{(1+\rho_s\rho_t)\sigma_G^2}{(1-\rho_t^2)\sigma_G^2 + \sigma_N^2} \tag{3.46}$$

$$\Gamma_4 = 2K\lambda_4 = \frac{(1-\rho_s\rho_t)\sigma_G^2}{(1-\rho_t^2)\sigma_G^2 + \sigma_N^2} \tag{3.47}$$

When $\rho_t = 1$, (3.46) and (3.47) reduce to

$$\Gamma_3 = \frac{(1+\rho_s)\sigma_G^2}{\sigma_N^2} \tag{3.48}$$

$$\Gamma_4 = \frac{(1-\rho_s)\sigma_G^2}{\sigma_N^2} \tag{3.49}$$

The result is the same as in [1 2] for STD in non time-selective, spatially correlated fading with perfect channel estimation.

When $\rho_s = 0$, (3.46) and (3.47) reduce to

$$\Gamma_3 = \Gamma_4 = \frac{\sigma_G^2}{(1-\rho_t^2)\sigma_G^2 + \sigma_N^2} \tag{3.50}$$

As in Section 2.3, we can obtain the exact BER expression as

$$P_f = \frac{1}{4}\left(1 - \sqrt{\frac{\Gamma_3}{1+\Gamma_3}}\right)^2\left(2 + \sqrt{\frac{\Gamma_3}{1+\Gamma_3}}\right) \tag{3.51}$$

which is the same as shown in [1 2] for STD in time-selective, spatially uncorrelated fading with perfect channel estimation.

3.4 Numerical Results

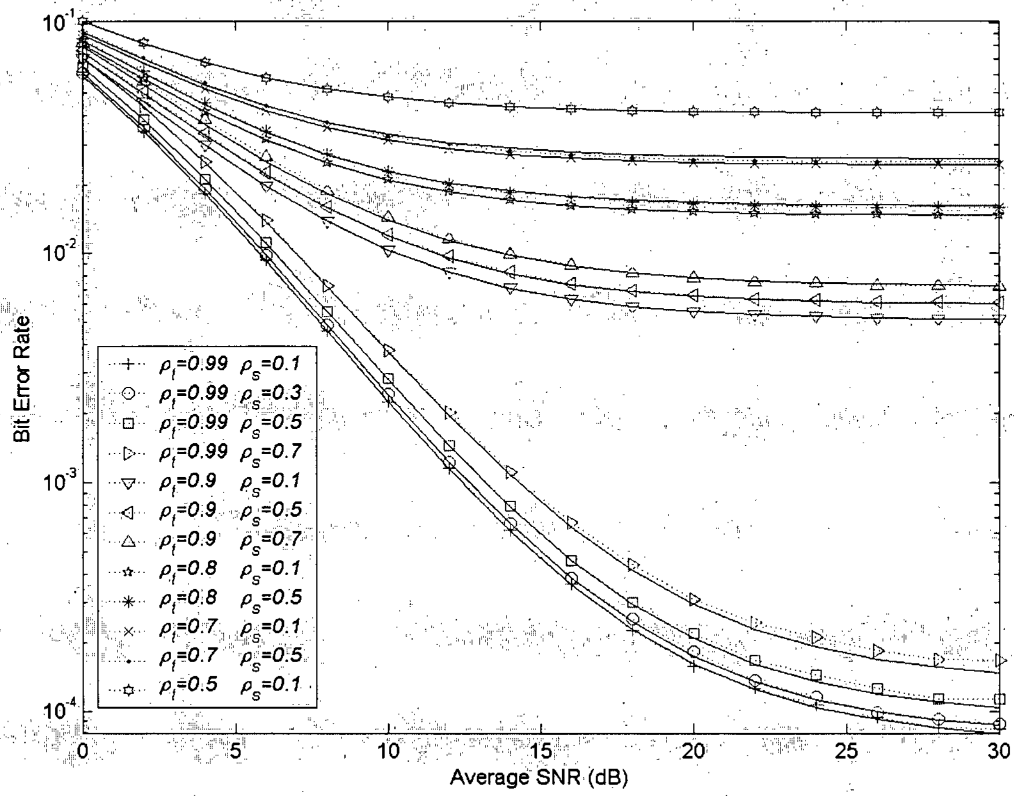

Fig. 3.1 Comparison of approximate analytic BER to simulation results

(Approximation: Solid Lines; Simulation: Dotted Lines)

The approximate and simulated BER curves are plotted in Fig. 3.1 as a function of the average SNR for different (ρ_t, ρ_s) values. As expected from the analysis in Section 3.3, the approximate BER agrees very well with the values from simulation when ρ_s is close to 0 and ρ_t is much greater than ρ_s. Moreover, the approximate and simulation results are close for $\rho_s < \frac{\rho_t}{3}$.

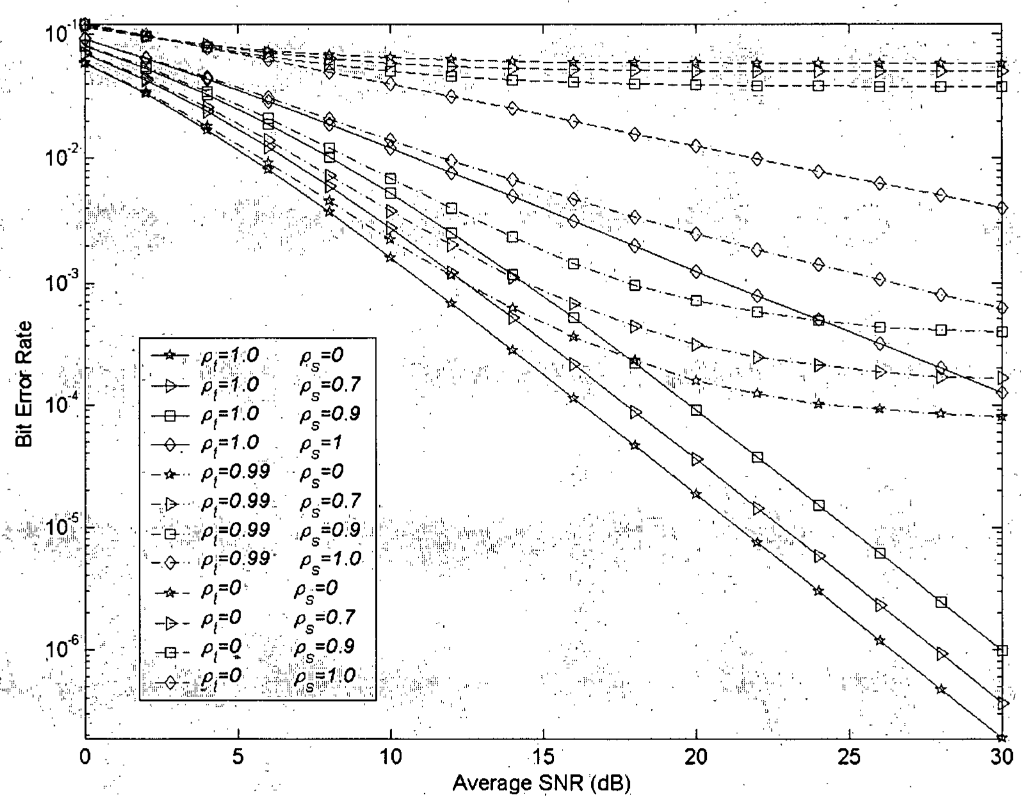

Fig. 3.2 Simulated BER curve as a function of average SNR for different (ρ_t, ρ_s) values

In the case of time-selective, spatially correlated fading with perfect channel estimation, for a fixed ρ_s, the BER increases as ρ_t decreases from 1 to 0.[1] When ρ_s is small, for a fixed ρ_t, the BER increases with ρ_s. The BER degradation for $\rho_t = 1-\Delta$ when ρ_s is fixed is larger than the degradation for $\rho_s = \Delta$ when ρ_t is fixed. As shown in Fig. 3.2, for a target BER of 10^{-3} for ρ_t = 1, there is about 1.3 dB degradation when ρ_s changes from 0 to 0.7 and about 10 dB degradation when ρ_s changes to 1. For the

[1] Although ρ_t changes from 1 to 0, when $\rho_t = 0$, the channel changes so fast that the gains are independent in two consecutive symbol periods. Thus, when ρ_t is small, the assumption that the channel is constant over one symbol interval is not valid.

same target BER for ρ_s = 0, when ρ_t changes from 1 to 0.99, there is about 1.3 dB degradation; when ρ_t changes to 0, the degradation is more than 25 dB.

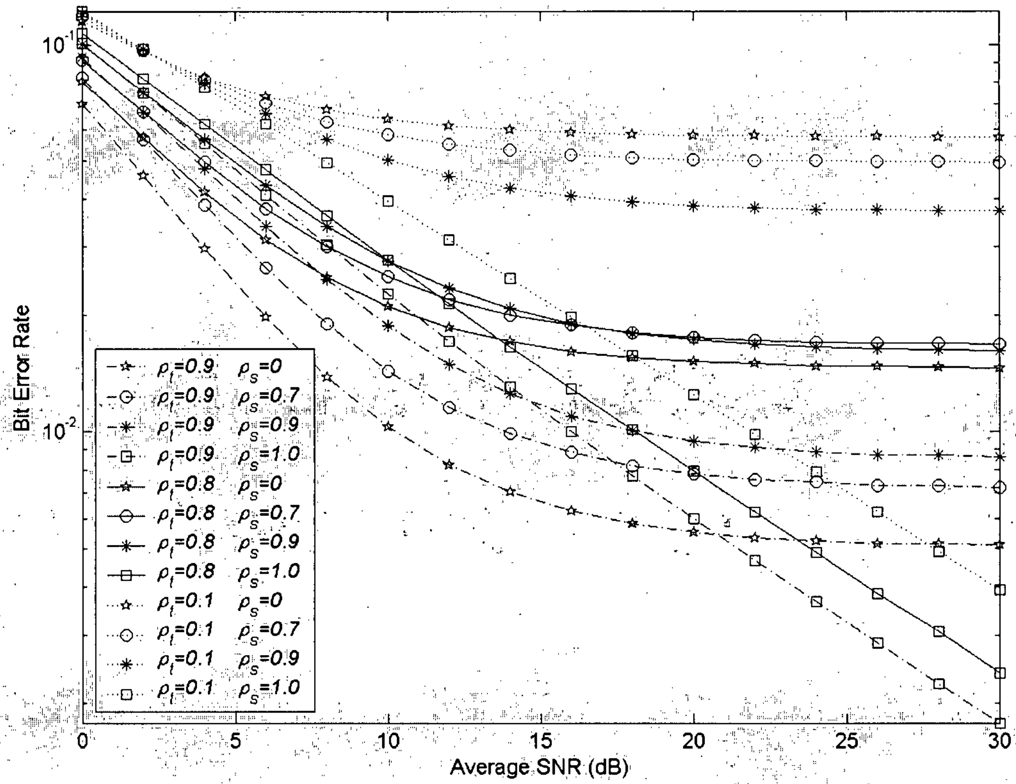

Fig. 3.3 Simulated BER curve as a function of average SNR for different (ρ_t, ρ_s) values (zoomed)

When ρ_s becomes bigger, for instance, ρ_s is equivalent to ρ_t or even bigger, from the simulated BER curve, we observe that the BER decreases with ρ_s. However, this occurs only when the average SNR exceeds certain thresholds. As shown in Fig. 3.3, for ρ_t = 0.9, when the SNR is small, the BER for ρ_s =1 is bigger than ρ_s = 0.9; when the SNR increases to about 14.5 dB, the BER for ρ_s = 1 is the same as ρ_s = 0.9. As the SNR continues increasing, the BER for ρ_s = 1 becomes smaller than the BER for ρ_s =

0.9 and eventually, becomes smaller than all other smaller ρ_s cases. If we look at the extreme case, i.e., $\rho_s = 1$, we can write (3.9) as

$$\begin{bmatrix} \tilde{s}_0 \\ \tilde{s}_1 \end{bmatrix} = \begin{bmatrix} |g_{10}|^2 + |g_{1T}|^2 & |g_{10}|^2 - |g_{1T}|^2 \\ |g_{10}|^2 - |g_{1T}|^2 & |g_{10}|^2 + |g_{1T}|^2 \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + \begin{bmatrix} g_{10}^* & g_{1T} \\ g_{10}^* & -g_{1T} \end{bmatrix} \begin{bmatrix} n_0 \\ n_T^* \end{bmatrix} \tag{3.52}$$

In (3.52), when SNR increases to infinity, the only interference is the inter-channel interference. Compared to (3.9), the signal to noise ratio changes from $\frac{(|g_{10}|^2 + |g_{1T}|^2)^2}{|g_{10}^* g_{20} - g_{1T}^* g_{2T}|^2}$ to $\frac{(|g_{10}|^2 + |g_{1T}|^2)^2}{(|g_{10}|^2 - |g_{1T}|^2)^2}$, which is always equal or greater than 1. This explains why for high SNR values, STD performance improves as ρ_s increases.

From the simulation results, we also observe that the SNR threshold for performance reversal decreases as ρ_t decreases. For instance, for $\rho_s = 1$, the SNR threshold for performance reversal is about 14.5 dB when $\rho_t = 0.9$; however, when $\rho_t = 0.1$, the threshold decreases to about 2.5 dB.

4 Conclusion

4.1 Main Thesis Contributions

This thesis presents a performance study of STD in non time-selective, spatially correlated fading with imperfect channel estimation and STD in time-selective, spatially correlated fading with perfect channel estimation.

- In the case of STD in non time-selective, spatially correlated fading with imperfect channel estimation, the error probability conditioned on the estimated channel gain is derived. A simple, approximate expression for the average BER over Rayleigh fading is given. A comparison with simulation results show that the approximate is quite accurate over a wide range of (ρ_e, ρ_s) values. The results also show that the channel estimation error has a bigger impact on STD performance than spatial correlation.

- In the case of STD in time-selective, spatially correlated fading with perfect channel estimation, the error probability conditioned on the channel gain is derived. A simple, approximate expression for the average BER over Rayleigh fading is given for (ρ_s << 1 and ρ_t >> ρ_s). It is found that time-selectivity has a bigger impact on STD performance than spatial correlation.

- From the results of STD in time-selective, spatially uncorrelated fading with channel estimation error in [1 2], it was found that the channel estimation error has a bigger impact on STD performance than time-selectivity. Combining this with our results, it can be deduced that STD performance is affected primarily by channel estimation errors, secondly by time-selectivity and thirdly by spatial correlation.

4.2 Topics for Further Study

- It would be useful to extend the derivation of the conditional error probability of STD to the general time-selective, spatially correlated fading with imperfect channel estimation scenario. Although it is possible to build the system model by introducing channel estimation error in Section 3.1 and using the same method to derive the conditional BER, the derivation of the joint pdf of G_{10}, G_{20}, G_{1T} and G_{2T} given H_{10}, H_{20}, H_{1T} and H_{2T} requires a Gaussian distribution that involves 8×8 and 4×4 covariance matrices to be solved and is thus awkward to deal with.

- A new combing scheme to cancel the inter-channel interference from the temporal and spatial correlation.

- An average BER expression over correlated Rayleigh fading and generalized fading, e.g. Ricean, Nakagami, etc..

Appendix A Derivation of the Means and Variances of Random Variables in (2.7)

In Chapter 2, the decision variable in (2.7) is expressed as

$$\begin{aligned}\mathrm{Re}\,(\tilde{s}_0) = {} & [(a+b)|h_1|^2+(a-b)|h_2|^2+2b\,\mathrm{Re}(h_1^* h_2)]s_0 \\ & +\mathrm{Re}[h_1^*(d_1+d_2)]s_0+\mathrm{Re}[h_2(d_2^*-d_1^*)]s_0 \\ & +\mathrm{Re}[h_1^* n_0]+\mathrm{Re}[h_2 n_T^*] \end{aligned} \tag{2.7}$$

When h_1 and h_2 are given, the first term in (2.7) $[(a+b)|h_1|^2+(a-b)|h_2|^2+2b\,\mathrm{Re}(h_1^* h_2)]s_0$ is determinate. For the rest, it is a sum of four random variables $\mathrm{Re}[h_1^*(D_1+D_2)]s_0$, $\mathrm{Re}[h_2(D_2^*-D_1^*)]s_0$, $\mathrm{Re}[h_1^* N_0]$ and $\mathrm{Re}[h_2 N_T^*]$.

It is shown below that the means of $h_1^*(D_1+D_2)$, $h_2(D_2^*-D_1^*)$, $h_1^* N_0$ and $h_2 N_T^*$ are zero.

$$E[h_1^*(D_1+D_2)]=h_1^* E[(D_1+D_2)]=h_1^*(E[D_1]+E[D_2])=0 \tag{A.1}$$

$$E[h_2(D_2^*-D_1^*)]=h_2 E[(D_2^*-D_1^*)]=0 \tag{A.2}$$

$$E[h_1^* N_0]=h_1^* E[N_0]=0 \tag{A.3}$$

$$E[h_2 N_T^*]=h_2 E[N_T^*]=0 \tag{A.4}$$

Thus, the mean of $\mathrm{Re}[h_1^*(D_1+D_2)]s_0$, $\mathrm{Re}[h_2(D_2^*-D_1^*)]s_0$, $\mathrm{Re}[h_1^* N_0]$ and $\mathrm{Re}[h_2 N_T^*]$ are zero too.

Next we prove that these random variables are independent of each other.

As we know, N_0 and N_T are statistically independent of any random variables, when h_1 and h_2 are fixed, $\mathrm{Re}[h_1^* N_0]$ and $\mathrm{Re}[h_2 N_T^*]$ are independent and they are also independent of $\mathrm{Re}[h_1^*(D_1+D_2)]s_0$ and $\mathrm{Re}[h_2(D_2^*-D_1^*)]s_0$. The variances of $\mathrm{Re}[h_1^* N_0]$ and $\mathrm{Re}[h_2 N_T^*]$ are

$$E\{\mathrm{Re}^2[h_1^* N_0]\} = |h_1^2|\sigma_N^2 \tag{A.5}$$

$$E\{\mathrm{Re}^2[h_2 N_T^*]\} = |h_2^2|\sigma_N^2 \tag{A.6}$$

Now we need to prove that $\mathrm{Re}[h_1^*(D_1+D_2)]s_0$ and $\mathrm{Re}[h_2(D_2^*-D_1^*)]s_0$ are independent. Or that $\mathrm{Re}[h_1^*(D_1+D_2)]$ and $\mathrm{Re}[h_2(D_2^*-D_1^*)]$ are independent, as s_0 is either +1 or –1.

Because D_1 and D_2 are zero mean correlated complex Gaussian random variables, we can express d_1 and d_2 as

$$d_1 = x_1 + \mathrm{j}\, y_1 \tag{A.7}$$

$$d_2 = x_2 + \mathrm{j}\, y_2 \tag{A.8}$$

where x_1, y_1, x_2 and y_2 are samples of zero mean correlated real Gaussian random variables with variances σ_D^2 and correlation coefficient ρ_d. The variances and covariances can be expressed as

$$E[X_i^2] = E[Y_i^2] = \sigma_D^2 \tag{A.9}$$

$$E[X_iY_j]=0 \qquad (i,j=1,2) \tag{A.10}$$

$$E[X_1X_2]=E[Y_1Y_2]=\rho_d\sigma_D^2 \tag{A.11}$$

Then we have

$$d_1+d_2=(x_1+x_2)+j(y_1+y_2) \tag{A.12}$$

$$d_2-d_1=(x_2-x_1)+j(y_2-y_1) \tag{A.13}$$

Similarly, if we denote h_i by its real and imaginary parts as $h_i = u_i + jw_i$, we can have

$$\mathrm{Re}[h_1^*(d_1+d_2)]=u_1(x_1+x_2)+w_1(y_1+y_2) \tag{A.14}$$

$$\mathrm{Re}[h^2(d_2-d_1)^*]=u_2(x_2-x_1)+w_2(y_2-y_1) \tag{A.15}$$

The covariance of $\mathrm{Re}[h_1^*(D_1+D_2)]$ and $\mathrm{Re}[h_2(D_2^*-D_1^*)]$ can be expressed as

$$E[\mathrm{Re}(h_1^*(D_1+D_2))\mathrm{Re}(h_2(D_2-D_1)^*)]$$

$$= E\{[u_1(X_1+X_2)+w_1(Y_1+Y_2)][u_2(X_2-X_1)+w_2(Y_2-Y_1)]\} = 0 \tag{A.16}$$

The result shows that $\mathrm{Re}[h_1^*(D_1+D_2)]$ and $\mathrm{Re}[h_2(D_2^*-D_1^*)]$ are independent. The variances of $\mathrm{Re}[h_1^*(D_1+D_2)]$ and $\mathrm{Re}[h_2(D_2^*-D_1^*)]$ can be expressed as

$$E\{\mathrm{Re}^2[h_1^*(D_1+D_2)]\}$$

$$= E[u_1^2(X_1^2+X_2^2+2X_1X_2)+w_1^2(Y_1^2+Y_2^2+2Y_1Y_2)+2u_1w_1(X_1+X_2)(Y_1+Y_2)]$$

$$= 2(1+\rho_d)(u_1^2+w_1^2)\sigma_D^2 = 2(1+\rho_d)\,|\,h_1^2\,|\,\sigma_D^2 \tag{A.17}$$

$$E\{\mathrm{Re}^2[h_2(D_2-D_1)^*]\}$$

$$= E[u_2^2(X_2^2 + X_1^2 - 2X_2X_1) + w_2^2(Y_2^2 + Y_1^2 - 2Y_2Y_1) + 2u_2w_1(X_2 - X_1)(Y_2 - Y_1)]$$

$$= 2(1-\rho_d)(u_2^2 + w_2^2)\sigma_D^2 = 2(1-\rho_d)\,|\,h_2^2\,|\,\sigma_D^2 \qquad \text{(A.18)}$$

Thus, we have proved that the decision variable $\mathrm{Re}(\tilde{s}_0)$ is a Gaussian random variable with mean $[(a+b)|h_1|^2 + (a-b)|h_2|^2 + 2b\,\mathrm{Re}(h_1^* h_2)]s_0$ and variance $[2(1+\rho_d)\,|\,h_1^2\,|\,\sigma_D^2 + 2(1-\rho_d)\,|\,h_2^2\,|\,\sigma_D^2]\ \ s_0 + (|\,h_1^2\,| + |\,h_2^2\,|)\sigma_N^2$.

Appendix B Derivation of the Variances and Correlation Coefficients of Matrix Transformed Spatially Correlated Fading

Originally, G_1 and G_2 are zero mean correlated complex Gaussian random variables with variance σ_G^2 and spatial correlation coefficient ρ_s. After matrix transformation, two new zero mean complex Gaussian random variables G_3 and G_4 are generated. From (2.41) and (2.42) we know that

$$g_3 = \frac{\sqrt{2}}{2}(g_1 + g_2) \tag{2.41}$$

$$g_4 = \frac{\sqrt{2}}{2}(g_2 - g_1) \tag{2.42}$$

We can calculate the covariance of G_3 and G_4 as

$$\begin{aligned} E[G_3 G_4^*] &= E\left[\frac{\sqrt{2}}{2}(G_1 + G_2)\frac{\sqrt{2}}{2}(G_2^* - G_1^*)\right] \\ &= \frac{1}{2}E\left[|G_2|^2 - |G_1|^2 + G_1 G_2^* - G_1^* G_2\right] = 0 \end{aligned} \tag{B.1}$$

The result shows that G_3 and G_4 are uncorrelated and statically independent. We can calculate their variances as

$$\sigma_{G_3}^2 = \frac{1}{2}E\left[G_3 G_3^*\right] = \frac{1}{2}E\left[\frac{\sqrt{2}}{2}(G_1 + G_2)\frac{\sqrt{2}}{2}(G_1^* + G_2^*)\right] = (1+\rho_s)\sigma_G^2 \tag{B.2}$$

$$\sigma_{G_4}^2 = \frac{1}{2}E\left[G_4G_4^*\right] = \frac{1}{2}E\left[\frac{\sqrt{2}}{2}(G_2 - G_1)\frac{\sqrt{2}}{2}(G_2^* - G_1^*)\right] = (1-\rho_s)\sigma_G^2 \qquad \text{(B.3)}$$

The original channel noises N_0 and N_T are zero mean independent complex Gaussian random variables with variance σ_N^2. Based on (2.43) (2.44)

$$n_3 = \frac{\sqrt{2}}{2}(n_0 + n_T) \qquad \text{(2.43)}$$

$$n_4 = \frac{\sqrt{2}}{2}(n_T - n_0) \qquad \text{(2.44)}$$

we can calculate the covariance of N_3 and N_4 as

$$E[N_3N_4^*] = E\left[\frac{\sqrt{2}}{2}(N_0 + N_T)\frac{\sqrt{2}}{2}(N_T^* - N_0^*)\right] = 0 \qquad \text{(B.4)}$$

They are also uncorrelated and statically independent. Their variances are

$$\sigma_{N_3}^2 = \frac{1}{2}E\left[N_3N_3^*\right] = \frac{1}{2}E\left[\frac{\sqrt{2}}{2}(N_0 + N_T)\frac{\sqrt{2}}{2}(N_0^* + N_T^*)\right] = \sigma_N^2 \qquad \text{(B.5)}$$

$$\sigma_{N_4}^2 = \frac{1}{2}E\left[N_4N_4^*\right] = \frac{1}{2}E\left[\frac{\sqrt{2}}{2}(N_T - N_0)\frac{\sqrt{2}}{2}(N_T^* - N_0^*)\right] = \sigma_N^2 \qquad \text{(B.6)}$$

Same way, we can prove that Z_3 and Z_4 are zero mean independent complex Gaussian random variables with variance σ_Z^2.

It can be shown that G_i and Z_i are independent.

$$E[G_iZ_i^*] = E\left[\frac{\sqrt{2}}{2}(G_2 \pm G_1)\frac{\sqrt{2}}{2}(Z_2^* \pm Z_1^*)\right] = 0 \qquad i = 3,4 \qquad \text{(B.7)}$$

From (2.38) we know that H_i is the sum of G_i and Z_i, $i = 3,4$

$$\begin{bmatrix} h_3 \\ h_2 \end{bmatrix} = \begin{bmatrix} g_3 + z_3 \\ g_4 + z_4 \end{bmatrix} \tag{2.38}$$

Therefore, the variance of H_i is the sum of the variances of G_i and Z_i. That is,

$$\sigma_{H_3}^2 = \sigma_{G_3}^2 + \sigma_{Z_3}^2 = (1+\rho_s)\sigma_G^2 + \left(\frac{1}{\rho_e^2} - 1\right)\sigma_G^2 = \frac{1+\rho_s\rho_e^2}{\rho_e^2}\sigma_G^2 \tag{B.8}$$

$$\sigma_{H_4}^2 = \sigma_{G_4}^2 + \sigma_{Z_4}^2 = (1-\rho_s)\sigma_G^2 + \left(\frac{1}{\rho_e^2} - 1\right)\sigma_G^2 = \frac{1-\rho_s\rho_e^2}{\rho_e^2}\sigma_G^2 \tag{B.9}$$

The covariance of G_i and H_i can be expressed as

$$E[G_3 H_3^*] = E[G_3(G_3^* + Z_3^*)] = 2\sigma_{G_3}^2 \tag{B.10}$$

$$E[G_4 H_4^*] = 2\sigma_{G_4}^2 \tag{B.11}$$

Now we can obtain the correlation coefficients of G_i and H_i as

$$\rho_{e3} = \frac{E[G_3 H_3^*]}{\sqrt{E[|G_3|^2]E[|H_3|^2]}} = \rho_e\sqrt{\frac{1+\rho_s}{1+\rho_s\rho_e^2}} \tag{B.12}$$

$$\rho_{e4} = \rho_e\sqrt{\frac{1-\rho_s}{1-\rho_s\rho_e^2}} \tag{B.13}$$

Appendix C Derivation of the Means and Variances of Random Variables in (2.55)

In Chapter 2, the combined signal $\tilde{s}_0$ in (2.55) is expressed as

$$\tilde{s}_0 = \rho_{e3}^2 s_0 |h_3|^2 + \rho_{e4}^2 s_0 |h_4|^2 + (\rho_{e4}^2 - \rho_{e3}^2) s_1 h_3^* h_4$$
$$+ h_3^* (d_3 s_0 + d_4 s_1) + h_4 (d_4^* s_0 - d_3^* s_1) + h_3^* n_3 + h_4 n_4^* \qquad (2.55)$$

The correspondent decision variable is $\mathrm{Re}(\tilde{s}_0)$. When h_1 and h_2 are given, the first three terms of $\mathrm{Re}(\tilde{s}_0)$ are determinate, as $\rho_{e3}^2 s_0 |h_3|^2 + \rho_{e4}^2 s_0 |h_4|^2 + (\rho_{e4}^2 - \rho_{e3}^2) s_1 \cdot \mathrm{Re}(h_3^* h_4)$. The rest part is variable; it is the real part of the sum of four random variables, expressed as $\mathrm{Re}[h_3^* (D_3 s_0 + D_4 s_1) + h_4 (D_4^* s_0 - D_3^* s_1) + h_3^* N_3 + h_4 N_4^*]$.

When $s_1 = s_0$, the variable part can be expressed as

$$\mathrm{Re}[h_3^* (D_3 + D_4) s_0 + h_4 (D_4^* - D_3^*) s_0 + h_3^* N_3 + h_4 N_4^*]$$
$$= \mathrm{Re}[(h_3^* - h_4^*) D_3^*] s_0 + \mathrm{Re}[(h_3^* + h_4^*) D_4] s_0 + \mathrm{Re}(h_3^* N_3) + \mathrm{Re}(h_4 N_4^*) \qquad \text{(C.1)}$$

Because D_3, D_4, N_3 and N_4 are zero mean independent complex Gaussian random variables, it is obvious that the mean of the variable part in (C.1) is zero and $\mathrm{Re}[(h_3^* - h_4^*) D_3] s_0$, $\mathrm{Re}[(h_3^* + h_4^*) D_4] s_0$, $\mathrm{Re}(h_3^* N_3)$ and $\mathrm{Re}(h_4 N_4^*)$ are statistically independent.

As in Appendix A, we write the samples of D_3 and D_4 as

$$d_3 = x_3 + \mathrm{j}\, y_3 \qquad \text{(C.2)}$$

$$d_4 = x_4 + j\, y_4 \tag{C.3}$$

where x_3, y_3, x_4, y_4 are samples of zero mean independent real Gaussian random variables with variances σ_{D3}^2 and σ_{D4}^2 respectively.

If we substitute h_i by its real and imaginary parts as $h_i = u_i + jw_i$, we have

$$\mathrm{Re}[(h_3^* - h_4^*)d_3] = \mathrm{Re}\{[(u_3 - u_4) - j(w_3 - w_4)](x_3 + jy_3)\}$$

$$= (u_3 - u_4)x_3 + (w_3 - w_4)y_3 \tag{C.4}$$

$$\mathrm{Re}[(h_3^* + h_4^*)d_4] = \mathrm{Re}\{[(u_3 + u_4) - j(w_3 + w_4)](x_4 + jy_4)\}$$

$$= (u_3 + u_4)x_4 + (w_3 + w_4)y_4 \tag{C.5}$$

The variances of $\mathrm{Re}[(h_3^* - h_4^*)D_3]s_0$ and $\mathrm{Re}[(h_3^* + h_4^*)D_4]s_0$ are

$$E\{\mathrm{Re}[(h_3^* - h_4^*)D_3]s_0\}^2 = E[(u_3 - u_4)^2 X_3^2 + (w_3 - w_4)^2 Y_3^2 + 2(u_3 - u_4)(w_3 - w_4)X_3Y_3]$$

$$= (u_3^2 + w_3^2 + u_4^2 + w_4^2 - 2u_3u_4 - 2w_3w_4)\sigma_{D3}^2$$

$$= [|h_3|^2 + |h_4|^2 - 2\,\mathrm{Re}(h_3h_4^*)]\sigma_{D3}^2 \tag{C.6}$$

$$E\{\mathrm{Re}[(h_3^* + h_4^*)D_4]s_0\}^2 = E[(u_3 + u_4)^2 X_4^2 + (w_3 + w_4)^2 Y_4^2 + 2(u_3 + u_4)(w_3 + w_4)X_4Y_4]$$

$$= (u_3^2 + w_3^2 + u_4^2 + w_4^2 + 2u_3u_4 + 2w_3w_4)\sigma_{D4}^2$$

$$= [|h_3|^2 + |h_4|^2 + 2\,\mathrm{Re}(h_3h_4^*)]\sigma_{D4}^2 \tag{C.7}$$

The variances of $\mathrm{Re}[h_3^* N_3]$ and $\mathrm{Re}[h_4 N_4^*]$ have already been shown in Appendix A, as $|\,h_3^2\,|\,\sigma_N^2$ and $|\,h_4^2\,|\,\sigma_N^2$ respectively.

So when $s_1 = s_0$, the decision variable $\mathrm{Re}(\tilde{s}_0)$ is a Gaussian random variable with mean $[\rho_{e3}^2|h_3|^2 + \rho_{e4}^2|h_4|^2 + (\rho_{e4}^2 - \rho_{e3}^2)\mathrm{Re}(h_3^* h_4)]s_0$ and variance $(|h_3|^2 + |h_4|^2)(\sigma_{D3}^2 + \sigma_{D4}^2 + \sigma_N^2) + 2\mathrm{Re}(h_3 h_4^*)(\sigma_{D4}^2 - \sigma_{D3}^2)$.

By using the same method, we can prove that when $s_1 = -s_0$, the decision variable $\mathrm{Re}(\tilde{s}_0)$ is a Gaussian random variable with mean $[\rho_{e3}^2|h_3|^2 + \rho_{e4}^2|h_4|^2 - (\rho_{e4}^2 - \rho_{e3}^2)\mathrm{Re}(h_3^* h_4)]s_0$ and variance $(|h_3|^2 + |h_4|^2)(\sigma_{D3}^2 + \sigma_{D4}^2 + \sigma_N^2) - 2\mathrm{Re}(h_3 h_4^*)(\sigma_{D4}^2 - \sigma_{D3}^2)$.

Appendix D Derivation of the Correlation Coefficient Between Time-selective, Spatially Correlated Channel Gains

In this model, the channel gain G_{10} is correlated with both G_{20} and G_{1T}. When setting up the model, we first generate G_{10}, then use G_{10} to generate G_{20} and G_{1T}. Therefore the expression of g_{20} and g_{1T} given g_{10} can be written as

$$g_{20} = \rho_s g_{10} + u_0 \tag{D.1}$$

$$g_{1T} = \rho_t g_{10} + v_1 \tag{D.2}$$

where U_0 and V_1 are zero mean independent complex Gaussian random variables with variances $\sigma_U^2 = (1-\rho_s^2)\sigma_G^2$ and $\sigma_V^2 = (1-\rho_t^2)\sigma_G^2$. U_0 and V_1 are independent of G_{10}.

Then the covariance of G_{20} and G_{1T} is given by

$$\begin{aligned} E[G_{20}G_{1T}^*] &= E(\rho_s G_{10} + U_0)(\rho_t G_{10}^* + V_1^*) \\ &= 2\rho_s\rho_t\sigma_G^2 \end{aligned} \tag{D.3}$$

Thus the correlation coefficients of G_{20} and G_{1T} can be expressed as

$$\frac{E[G_{20}G_{1T}^*]}{\sqrt{E[|G_{20}|^2]E[|G_{1T}|^2]}} = \frac{2\rho_s\rho_t\sigma_G^2}{\sqrt{2\sigma_G^2 \cdot 2\sigma_G^2}} = \rho_s\rho_t \tag{D.4}$$

Similarly, if we start with G_{20}, we can also get the correlation coefficient of G_{10} and

G_{2T} as $\rho_s \rho_t$.

Appendix E Linear Transformation of Jointly Gaussian Random Variables

From [1 8] we know that a linear transformation of a set of jointly Gaussian random variables results in another set of jointly Gaussian random variables. If we have a set of jointly Gaussian random variables, denoted as **X**, which is a $n \times 1$ column vector with $n \times 1$ mean vector $\mathbf{m_x}$ and $n \times n$ covariance matrix $\mathbf{C_X}$, by using

$$\mathbf{Y} = \mathbf{AX} \tag{E.1}$$

where **A** is a $n \times n$ non-singular matrix, we can transform **X** into a new set of jointly Gaussian random variables **Y**, which is a $n \times 1$ column vector with $n \times 1$ mean vector $\mathbf{m_y}$ and $n \times n$ covariance matrix $\mathbf{C_Y}$. Correspondingly, the transformation of the mean vector and the covariance matrix can be done by

$$\mathbf{m_y} = \mathbf{Am_x} \tag{E.2}$$

$$\mathbf{C_Y} = \mathbf{AC_XA^T} \tag{E.3}$$

Where $\mathbf{A^T}$ denotes the transpose of **A**.

Frist transformation case:

In case of representing G_{1T} and G_{2T} by G_{10} and G_{20}, a set of jointly Gaussian random variables is defined as

$$\mathbf{X^T} = [\, g_{10} \quad g_{20} \quad \varepsilon_1 \quad \varepsilon_2 \,] \tag{E.4}$$

where E_1 and E_2 are zero mean independent Gaussian random variables with variance σ_G^2. E_1 and E_2 are independent of G_{10} and G_{20}. By using the linear transformation, we can transform **X** into a new set of jointly Gaussian random variables

$$\mathbf{Y}^{\mathrm{T}} = [\, g_{10} \quad g_{20} \quad g_{1T} \quad g_{2T} \,] \tag{E.5}$$

with the desired means and variances.

Because the means of the random variables in our model are all zero, we only need to look after the transformation of covariance matrix in (E.3).

From the previous definition, we can write the covariance matrixes of **X**, **Y** as

$$\mathbf{C_X} = \begin{bmatrix} \sigma_G^2 & \rho_s\sigma_G^2 & 0 & 0 \\ \rho_s\sigma_G^2 & \sigma_G^2 & 0 & 0 \\ 0 & 0 & \sigma_G^2 & 0 \\ 0 & 0 & 0 & \sigma_G^2 \end{bmatrix} \tag{E.6}$$

$$\mathbf{C_Y} = \begin{bmatrix} \sigma_G^2 & \rho_s\sigma_G^2 & \rho_t\sigma_G^2 & \rho_s\rho_t\sigma_G^2 \\ \rho_s\sigma_G^2 & \sigma_G^2 & \rho_s\rho_t\sigma_G^2 & \rho_t\sigma_G^2 \\ \rho_t\sigma_G^2 & \rho_s\rho_t\sigma_G^2 & \sigma_G^2 & \rho_s\sigma_G^2 \\ \rho_s\rho_t\sigma_G^2 & \rho_t\sigma_G^2 & \rho_s\sigma_G^2 & \sigma_G^2 \end{bmatrix} \tag{E.7}$$

Using Maple®[2 6], we can find a **A** that complies with (E.3)~(E.5).

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \rho_t & 0 & \sqrt{1-\rho_t^2} & 0 \\ 0 & \rho_t & \rho_s\sqrt{1-\rho_t^2} & \sqrt{(1-\rho_s^2)(1-\rho_t^2)} \end{bmatrix} \tag{E.8}$$

If we use (E.8) to rewrite (E.1), we can get

$$g_{1T} = \rho_t g_{10} + \sqrt{1-\rho_t^2}\,\varepsilon_1 \tag{E.9}$$

$$g_{2T} = \rho_t g_{20} + \rho_s \sqrt{1-\rho_t^2}\,\varepsilon_1 + \sqrt{(1-\rho_s^2)(1-\rho_t^2)}\,\varepsilon_2 \qquad \text{(E.10)}$$

If we replace the two zero mean independent complex Gaussian random variables E_1 and E_2 by two correlated complex Gaussian random variables V_1 and V_2, we can rewrite the expression as

$$g_{1T} = \rho_t g_{10} + v_1 \qquad (3.3)$$

$$g_{2T} = \rho_t g_{20} + v_2 \qquad (3.4)$$

where V_1 and V_2 are zero mean complex Gaussian random variables with variance σ_V^2 $= (1-\rho_t^2)\sigma_G^2$ and correlation coefficient ρ_s. They are independent of G_{10} and G_{20}.

Second transformation case:

When representing G_{1T} and G_{20} by G_{10} and G_{2T}, we write **X**, **Y**, $\mathbf{C_X}$ and $\mathbf{C_Y}$ as

$$\mathbf{X}^{\mathrm{T}} = [\, g_{10} \quad g_{2T} \quad \varepsilon_1 \quad \varepsilon_2 \,] \qquad \text{(E.11)}$$

$$\mathbf{Y}^{\mathrm{T}} = [\, g_{10} \quad g_{2T} \quad g_{1T} \quad g_{20} \,] \qquad \text{(E.12)}$$

$$\mathbf{C_X} = \begin{bmatrix} \sigma_G^2 & \rho_s\rho_t\sigma_G^2 & 0 & 0 \\ \rho_s\rho_t\sigma_G^2 & \sigma_G^2 & 0 & 0 \\ 0 & 0 & \sigma_G^2 & 0 \\ 0 & 0 & 0 & \sigma_G^2 \end{bmatrix} \qquad \text{(E.13)}$$

$$\mathbf{C_Y} = \begin{bmatrix} \sigma_G^2 & \rho_s\rho_t\sigma_G^2 & \rho_t\sigma_G^2 & \rho_s\sigma_G^2 \\ \rho_s\rho_t\sigma_G^2 & \sigma_G^2 & \rho_s\sigma_G^2 & \rho_t\sigma_G^2 \\ \rho_t\sigma_G^2 & \rho_s\sigma_G^2 & \sigma_G^2 & \rho_s\rho_t\sigma_G^2 \\ \rho_s\sigma_G^2 & \rho_t\sigma_G^2 & \rho_s\rho_t\sigma_G^2 & \sigma_G^2 \end{bmatrix} \qquad \text{(E.14)}$$

and can get a **A** as

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \rho_t\left(\dfrac{1-\rho_s^2}{1-\rho_s^2\rho_t^2}\right) & \rho_s\left(\dfrac{1-\rho_t^2}{1-\rho_s^2\rho_t^2}\right) & \sqrt{\dfrac{(1-\rho_s^2)(1-\rho_t^2)}{1-\rho_s^2\rho_t^2}} & 0 \\ \rho_s\left(\dfrac{1-\rho_t^2}{1-\rho_s^2\rho_t^2}\right) & \rho_t\left(\dfrac{1-\rho_s^2}{1-\rho_s^2\rho_t^2}\right) & -\rho_s\rho_t\sqrt{\dfrac{(1-\rho_s^2)(1-\rho_t^2)}{1-\rho_s^2\rho_t^2}} & \sqrt{(1-\rho_s^2)(1-\rho_t^2)} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ a & b & c & 0 \\ b & a & d & e \end{bmatrix} \tag{E.15}$$

Thus, we can express G_{1T} and G_{20} as

$$g_{1T} = ag_{10} + bg_{2T} + c\varepsilon_1 \tag{E.16}$$

$$g_{20} = bg_{10} + ag_{2T} + d\varepsilon_1 + e\varepsilon_2 \tag{E.17}$$

Similarly, if we replace E_1 and E_2 by a new pair of correlated Gaussian random variables W_1 and W_2, we can rewrite (E.16) and (E.17) as

$$g_{1T} = ag_{10} + bg_{2T} + w_1 \tag{3.10}$$

$$g_{20} = bg_{10} + ag_{2T} + w_2 \tag{3.11}$$

where W_1 and W_2 are zero mean correlated complex Gaussian random variables with variance $\sigma_W^2 = c^2\sigma_G^2$ and correlation coefficient $\rho_w = -\rho_s\rho_t$.

Appendix F Derivation of the Mean and Variances of Random Variables in (3.13)

In (3.13) we need to evaluate the sum of four random variables, $\mathrm{Re}(g_{10}^{*}W_2)$, $\mathrm{Re}(g_{2T}W_1^{*})$, $\mathrm{Re}(g_{10}^{*}N_0)$ and $\mathrm{Re}(g_{2T}N_T^{*})$. We know that W_1, W_2, N_0 and N_T are zero mean Gaussian random variables. Therefore, their sum in (3.13) is also zero mean.

From Appendix A, we know that $\mathrm{Re}(g_{10}^{*}N_0)$ and $\mathrm{Re}(g_{2T}N_T^{*})$ are zero mean independent Gaussian random variables with variances $|g_{10}|^2\sigma_N^2$ and $|g_{2T}|^2\sigma_N^2$ respectively. They are independent of any other random variables.

For W_1 and W_2, we know that they are correlated. Therefore, we analyze them as one term $\mathrm{Re}(g_{10}^{*}W_2 - g_{2T}W_1^{*})$. We express the samples of W_1 and W_2 as

$$w_1 = x_1 + \mathrm{j}\, y_1 \tag{F.1}$$

$$w_2 = x_2 + \mathrm{j}\, y_2 \tag{F.2}$$

where x_1, y_1, x_2 and y_2 are samples of zero mean real Gaussian random variables with variance σ_W^2 and correlation coefficient ρ_w, i.e.,

$$E[X_i^2] = E[Y_i^2] = \sigma_W^2 \tag{F.3}$$

$$E[X_iY_j] = 0 \qquad i, j = 1,2 \tag{F.4}$$

$$E[X_1X_2] = E[Y_1Y_2] = \rho_w\sigma_W^2 \tag{F.5}$$

If we substitute g_{ij} by its real and imaginary parts as $g_{ij} = u_{ij} + jv_{ij}$, we have

$$\mathrm{Re}(g_{10}^{*}W_2 - g_{2T}W_1^{*}) = (u_{10}x_2 - u_{2T}x_1) + (v_{10}y_2 - v_{2T}y_1) \tag{F.6}$$

Then the variance of $\mathrm{Re}(g_{10}^{*}W_2 - g_{2T}W_1^{*})$ is

$$E[\mathrm{Re}^2(g_{10}^{*}W_2 - g_{2T}W_1^{*})]$$

$$= E[(u_{10}X_2 - u_{2T}X_1)^2 + (v_{10}Y_2 - v_{2T}Y_1)^2 + 2(u_{10}X_2 - u_{2T}X_1)(v_{10}Y_2 - v_{2T}Y_1)]$$

$$= (u_{10}^2 + u_{2T}^2 - 2u_{10}u_{2T}\rho_w)\sigma_W^2 + (v_{10}^2 + v_{2T}^2 - 2v_{10}v_{2T}\rho_w)\sigma_W^2$$

$$= [|g_{10}|^2 + |g_{2T}|^2 - 2\rho_w \mathrm{Re}(g_{10}g_{2T}^{*})]\sigma_W^2 \tag{F.7}$$

Appendix G Derivation of the Variances and Correlation Coefficients of Matrix Transformed Time-selective, Spatially Correlated Fading

Before transformation, G_{10}, G_{20}, G_{1T} and G_{2T} are zero mean complex Gaussian random variables with variance σ_G^2. Their covariance matrix is expressed in (3.1) as

$$\mathbf{C}_{\mathbf{G}_{4\times4}} = \begin{bmatrix} \sigma_G^2 & \rho_s\sigma_G^2 & \rho_t\sigma_G^2 & \rho_s\rho_t\sigma_G^2 \\ \rho_s\sigma_G^2 & \sigma_G^2 & \rho_s\rho_t\sigma_G^2 & \rho_t\sigma_G^2 \\ \rho_t\sigma_G^2 & \rho_s\rho_t\sigma_G^2 & \sigma_G^2 & \rho_s\sigma_G^2 \\ \rho_s\rho_t\sigma_G^2 & \rho_t\sigma_G^2 & \rho_s\sigma_G^2 & \sigma_G^2 \end{bmatrix} \tag{3.1}$$

After matrix transformation, four new zero mean complex Gaussian random variables G_{30}, G_{40}, G_{3T} and G_{4T} are created by (3.23)

$$\begin{bmatrix} g_{30} & g_{40} \\ g_{4T} & g_{3T} \end{bmatrix} = \frac{\sqrt{2}}{2}\begin{bmatrix} g_{10}+g_{2T} & g_{20}-g_{1T} \\ g_{2T}-g_{10} & g_{1T}+g_{20} \end{bmatrix} \tag{3.23}$$

The variance of G_{30} can be written as

$$\begin{aligned} \sigma_{G_{30}}^2 &= \frac{1}{2}E\left[G_{30}G_{30}^*\right] \\ &= \frac{1}{2}E\left[\frac{\sqrt{2}}{2}(G_{10}+G_{2T})\frac{\sqrt{2}}{2}(G_{10}^*+G_{2T}^*)\right] \\ &= (1+\rho_s\rho_t)\sigma_G^2 = \sigma_{G_3}^2 \end{aligned} \tag{G.1}$$

Similarly, the variances of G_{40}, G_{3T} and G_{4T} can be proved as σ_{G4}^2, $\sigma_{G_3}^2$ and σ_{G4}^2 respectively, where $\sigma_{G4}^2 = (1-\rho_s\rho_t)\sigma_G^2$.

The covariances of G_{30} and G_{40}, G_{30} and G_{3T} can be written as

$$E[G_{30}G_{40}^*] = E\left[\frac{\sqrt{2}}{2}(G_{10}+G_{2T})\frac{\sqrt{2}}{2}(G_{20}^*-G_{1T}^*)\right]$$

$$= E[G_{10}G_{20}^* - G_{1T}^*G_{2T} - G_{10}G_{1T}^* + G_{20}^*G_{2T}] = 0 \qquad \text{(G.2)}$$

$$E[G_{30}G_{3T}^*] = E\left[\frac{\sqrt{2}}{2}(G_{10}+G_{2T})\frac{\sqrt{2}}{2}(G_{1T}^*+G_{20}^*)\right]$$

$$= \frac{1}{2}E\left[G_{10}G_{20}^* + G_{1T}^*G_{2T} + G_{10}G_{1T}^* + G_{20}^*G_{2T}\right]$$

$$= \frac{1}{2}\left[4\rho_s\sigma_G^2 + 4\rho_t\sigma_G^2\right] = 2(\rho_t+\rho_s)\sigma_G^2 \qquad \text{(G.3)}$$

The correlation coefficient between G_{30} and G_{3T} is

$$\rho_{t3} = \frac{E[G_{30}G_{3T}^*]}{\sqrt{E[|G_{30}|^2]E[|G_{3T}|^2]}} = \frac{\rho_t+\rho_s}{1+\rho_s\rho_t} \qquad \text{(G.4)}$$

Similarly, we can prove the covariance and the correlation coefficient between G_{40} and G_{4T} as

$$E[G_{40}G_{4T}^*] = 2(\rho_t-\rho_s)\sigma_G^2 \qquad \text{(G.5)}$$

$$\rho_{t4} = \frac{\rho_t-\rho_s}{1-\rho_s\rho_t} \qquad \text{(G.6)}$$

Because the rest of the covariances between new channels are 0, the new covariance matrix can be written as

$$\mathbf{C}_{\mathbf{G}_{34}} = \begin{bmatrix} (1+\rho_s\rho_t)\sigma_G^2 & 0 & (\rho_t+\rho_s)\sigma_G^2 & 0 \\ 0 & (1-\rho_s\rho_t)\sigma_G^2 & 0 & (\rho_t-\rho_s)\sigma_G^2 \\ (\rho_t+\rho_s)\sigma_G^2 & 0 & (1+\rho_s\rho_t)\sigma_G^2 & 0 \\ 0 & (\rho_t-\rho_s)\sigma_G^2 & 0 & (1-\rho_s\rho_t)\sigma_G^2 \end{bmatrix} \tag{G.7}$$

Since channel 3 and channel 4 are uncorrelated, we can write (G.7) separately as

$$\mathbf{C}_{\mathbf{G}_3} = \begin{bmatrix} (1+\rho_s\rho_t)\sigma_G^2 & (\rho_t+\rho_s)\sigma_G^2 \\ (\rho_t+\rho_s)\sigma_G^2 & (1+\rho_s\rho_t)\sigma_G^2 \end{bmatrix} = \begin{bmatrix} \sigma_{G_3}^2 & \rho_{t3}\sigma_{G_3}^2 \\ \rho_{t3}\sigma_{G_3}^2 & \sigma_{G_3}^2 \end{bmatrix} \tag{G.8}$$

$$\mathbf{C}_{\mathbf{G}_4} = \begin{bmatrix} (1-\rho_s\rho_t)\sigma_G^2 & (\rho_t-\rho_s)\sigma_G^2 \\ (\rho_t-\rho_s)\sigma_G^2 & (1-\rho_s\rho_t)\sigma_G^2 \end{bmatrix} = \begin{bmatrix} \sigma_{G_4}^2 & \rho_{t4}\sigma_{G_4}^2 \\ \rho_{t4}\sigma_{G_4}^2 & \sigma_{G_4}^2 \end{bmatrix} \tag{G.9}$$

Appendix H Distribution of Hermitian Quadratic Form

Using (2.23), we can write the decision variable for s_0 as

$$\begin{aligned} f = \mathrm{Re}(\tilde{s}_0) &= (as_0 + bs_1)|h_1|^2 + (as_0 - bs_1)|h_2|^2 + bs_0(h_1^* h_2 + h_1 h_2^*) \\ &\quad + (h_1^* d_1 + h_1 d_1^*)s_0 + (h_1^* d_2 + h_1 d_2^*)s_1 + (h_2 d_2^* + h_2^* d_2)s_0 \\ &\quad - (h_2 d_1^* + h_2^* d_1)s_1 + (h_1^* + h_1)n_0 + (h_2 + h_2^*)n_T^* \\ &= \mathbf{Z}^{\mathbf{T}*}\,\mathbf{F}\,\mathbf{Z} \end{aligned} \tag{H.1}$$

where $\mathbf{Z} = [H_1\ H_2\ D_1\ D_2\ N_0\ N_T]^T$, a 6×1 column matrix of six jointly distributed complex Gaussian variables, with covariance matrix $\mathbf{R}$

$$\mathbf{R} = \begin{bmatrix} \frac{1}{\rho_e^2}\sigma_G^2 & \rho_s\sigma_G^2 & 0 & 0 & 0 & 0 \\ \rho_s\sigma_G^2 & \frac{1}{\rho_e^2}\sigma_G^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_D^2 & \rho_d\sigma_D^2 & 0 & 0 \\ 0 & 0 & \rho_d\sigma_D^2 & \sigma_D^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_N^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_N^2 \end{bmatrix} \tag{H.2}$$

and

$$\mathbf{F} = \begin{bmatrix} as_0 + bs_1 & bs_0 & \frac{1}{2}s_0 & \frac{1}{2}s_1 & \frac{1}{2} & 0 \\ bs_0 & as_0 - bs_1 & -\frac{1}{2}s_1 & \frac{1}{2}s_0 & 0 & \frac{1}{2} \\ \frac{1}{2}s_0 & -\frac{1}{2}s_1 & 0 & 0 & 0 & 0 \\ \frac{1}{2}s_1 & \frac{1}{2}s_0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{H.3}$$

It is clear that both **F** and **R** are Hermitian. Thus a unitary 6×6 matrix, **U**,

$$\mathbf{U}=\begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \tag{H.4}$$

can be formed with the 6 eigenvectors of **R** as its columns, such that

$$\mathbf{U}^{\mathbf{T}*}\,\mathbf{U}=\mathbf{I} \tag{H.5}$$

$$\mathbf{U}^{\mathbf{T}*}\,\mathbf{R}\,\mathbf{U}=\mathbf{\Lambda} \tag{H.6}$$

$$\mathbf{U}\,\mathbf{\Lambda}\,\mathbf{U}^{\mathbf{T}*}=\mathbf{R} \tag{H.7}$$

where **I** is the identity matrix and $\mathbf{\Lambda}$ is a diagonal matrix with the six eigenvalues of **R**.

$$\mathbf{\Lambda}=\begin{bmatrix} \frac{1+\rho_s\rho_e^2}{\rho_e^2}\sigma_G^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1-\rho_s\rho_e^2}{\rho_e^2}\sigma_G^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & (1+\rho_d)\sigma_D^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & (1-\rho_d)\sigma_D^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_N^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_N^2 \end{bmatrix} \tag{H.8}$$

There is an infinity of matrices that allow a factorization of $\mathbf{\Lambda}$ in the form

$$\mathbf{\Lambda}=\mathbf{\Psi}^{*}\,\mathbf{\Psi}^{\mathbf{T}}. \tag{H.9}$$

One such factorization is the "square-root" matrix.

$$\boldsymbol{\Psi} = \begin{bmatrix} \frac{\sqrt{1+\rho_s\rho_e^2}}{\rho_e}\sigma_G & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{1-\rho_s\rho_e^2}}{\rho_e}\sigma_G & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{1+\rho_d}\,\sigma_D & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{1-\rho_d}\,\sigma_D & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_N & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_N \end{bmatrix}. \tag{H.10}$$

Thus, **Z** can be transformed into **W**, a new set of Gaussian random variables, in which the random variables are statistically independent with covariance matrix **I**.

$$\mathbf{W} = \boldsymbol{\Psi}^{-1}\,\mathbf{U}^{\mathrm{T}}\,\mathbf{Z} = \begin{bmatrix} \frac{\rho_e}{\sqrt{1+\rho_s\rho_e^2}\,\sigma_G}\frac{H_1+H_2}{\sqrt{2}} \\ \frac{\rho_e}{\sqrt{1-\rho_s\rho_e^2}\,\sigma_G}\frac{H_2-H_1}{\sqrt{2}} \\ \frac{1}{\sqrt{1+\rho_d}\,\sigma_D}\frac{D_1+D_2}{\sqrt{2}} \\ \frac{1}{\sqrt{1-\rho_d}\,\sigma_D}\frac{D_2-D_1}{\sqrt{2}} \\ \frac{1}{\sigma_N}\frac{N_0+N_T}{\sqrt{2}} \\ \frac{1}{\sigma_N}\frac{N_T-N_0}{\sqrt{2}} \end{bmatrix} \tag{H.11}$$

The inverse of (H.11) is

$$\mathbf{Z} = \mathbf{U}^{*}\,\boldsymbol{\Psi}\,\mathbf{W}\,. \tag{H.12}$$

The quadratic form of (H.1) becomes

$$f = \mathbf{W}^{\mathrm{T}*}\,(\,\boldsymbol{\Psi}^{\mathrm{T}*}\,\mathbf{U}^{\mathrm{T}}\,\mathbf{F}\,\mathbf{U}^{*}\,\boldsymbol{\Psi}\,)\,\mathbf{W} = \mathbf{W}^{\mathrm{T}*}\,\mathbf{T}\,\mathbf{W} \tag{H.13}$$

where

$$\mathbf{T} = \mathbf{\Psi}^{\mathrm{T}*}\,\mathbf{U}^{\mathrm{T}}\,\mathbf{F}\,\mathbf{U}^{*}\,\mathbf{\Psi} = \begin{bmatrix} \alpha & \beta & \gamma & \delta & \varepsilon & 0 \\ \beta & \zeta & \eta & \theta & 0 & \iota \\ \gamma & \eta & 0 & 0 & 0 & 0 \\ \delta & \theta & 0 & 0 & 0 & 0 \\ \varepsilon & 0 & 0 & 0 & 0 & 0 \\ 0 & \iota & 0 & 0 & 0 & 0 \end{bmatrix} \tag{H.14}$$

$$\alpha = \frac{1+\rho_s\rho_e^2}{\rho_e^2}\sigma_G^2(a+b)s_0 \tag{H.15}$$

$$\beta = -\frac{\sqrt{1-\rho_s^2\rho_e^4}}{\rho_e^2}\sigma_G^2 b s_1 \tag{H.16}$$

$$\gamma = \frac{\sqrt{(1+\rho_s\rho_e^2)(1+\rho_d)}}{2\rho_e}\sigma_G\sigma_D s_0 \tag{H.17}$$

$$\delta = \frac{\sqrt{(1+\rho_s\rho_e^2)(1-\rho_d)}}{2\rho_e}\sigma_G\sigma_D s_1 \tag{H.18}$$

$$\varepsilon = \frac{\sqrt{1+\rho_s\rho_e^2}}{2\rho_e}\sigma_G\sigma_N \tag{H.19}$$

$$\zeta = \frac{1-\rho_s\rho_e^2}{\rho_e^2}\sigma_G^2(a-b)s_0 \tag{H.20}$$

$$\eta = -\frac{\sqrt{(1-\rho_s\rho_e^2)(1+\rho_d)}}{2\rho_e}\sigma_G\sigma_D s_1 \tag{H.21}$$

$$\theta = \frac{\sqrt{(1-\rho_s\rho_e^2)(1-\rho_d)}}{2\rho_e}\sigma_G\sigma_D s_0 \tag{H.22}$$

$$\iota = \frac{\sqrt{1-\rho_s\rho_e^2}}{2\rho_e}\sigma_G\sigma_N \tag{H.23}$$

Since **T** is also Hermitian, it can be diagonalized in a form

$$\mathbf{T} = \mathbf{S}\,\mathbf{\Phi}\,\mathbf{S}^{\mathrm{T}*} \tag{H.24}$$

where **S** is a unitary matrix of orthonormalized eigenvectors of **T**, and $\mathbf{\Phi}$ is the diagonal matrix of its eigenvalues, ϕ_i. Thus, one can introduce the transformation

$$\mathbf{X} = \mathbf{S}^{\mathbf{T}*}\,\mathbf{W} \tag{H.25}$$

in terms of which the quadratic form is diagonal,

$$f = \mathbf{X}^{\mathbf{T}*}\,\mathbf{\Phi}\,\mathbf{X} = \sum_{i=1}^{6} \phi_i \left|\chi_i\right|^2 . \tag{H.26}$$

and the covariance matrix of **X** is still **I**.

Since the Hermitian quadratic form f here is in a zero mean complex Gaussian process, the characteristic function of f, defined as a Fourier transform on its pdf, is

$$G_f(\xi) = \frac{1}{\det(\mathbf{I} - 2j\xi\,\mathbf{R}^*\,\mathbf{F})} \tag{H.27}$$

with its pdf given as the inverse

$$p_F(f) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp(-j\xi\, f)\, G_f(\xi) d\xi \tag{H.28}$$

We can derive $G_f(\xi)$ here, but due to its complexity, it is difficult to obtain the pdf by inverse Fourier transform.

Appendix I Derivation of Channel Estimation Correlation Coefficient as a Function of SNR

In a simple model, with a transmitted signal s, i.e., a BPSK signal, the received signal r can be represented by

$$r = gs + n \tag{I.1}$$

where G is the channel gain, which is a zero mean complex Gaussian random variable with variance σ_G^2 and N is the additive Gaussian noise, which is also a zero mean complex Gaussian random variable with variance σ_N^2.

If we use pilot symbols, we can obtain an estimate of g as

$$\hat{g} = \frac{r}{s} = g + \frac{n}{s} \tag{I.2}$$

Thus $\hat{G}$ is a zero mean complex Gaussian random variable with variance

$$\sigma_{\hat{G}}^2 = \sigma_G^2 + \sigma_N^2. \tag{I.3}$$

The correlation coefficient ρ_e between G and $\hat{G}$ can then be obtained as

$$\rho_e = \frac{E[\hat{G} \cdot G^*]}{\sqrt{E[|\hat{G}|^2] \cdot E[|G|^2]}} = \frac{E[(G+N) \cdot G^*]}{\sqrt{E[|\hat{G}|^2] \cdot E[|G|^2]}} = \frac{E[|G|^2]}{\sqrt{E[|\hat{G}|^2] \cdot E[|G|^2]}}$$

$$= \frac{2\sigma_G^2}{\sqrt{2(\sigma_G^2 + \sigma_N^2) \cdot 2\sigma_G^2}} = \frac{1}{\sqrt{1 + \sigma_N^2 / \sigma_G^2}} = \frac{1}{\sqrt{1 + 1/\mathrm{SNR}}} \tag{I.4}$$

where $\mathrm{SNR} = \sigma_G^2 / \sigma_N^2$.

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