NONREUNDANT ERROR CORRECTION OF $\pi/4$-SHIFT DQPSK SYSTEMS FOR MOBILE AND CELLULAR SYSTEM APPLICATIONS

by

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A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF APPLIED SCIENCE in

THE FACULTY OF GRADUATE STUDIES DEPARTMENT OF ELECTRICAL ENGINEERING

We accept this thesis as conforming to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

June 1991

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Date **Aug 20, 1991**
Abstract

The application of the nonredundant error correction (NEC) technique to the new North American and Japanese Digital Cellular modulation standard, the $\pi/4$-shift DQPSK modulation format, is proposed, analyzed and evaluated. Due to the nature of the mobile cellular communication channel, it is assumed that the $\pi/4$-shift DQPSK system is operated in a combined additive white Gaussian noise (AWGN) and cochannel interference (CCI) environment as well as in a frequency-nonselective fading environment.

The NEC techniques can be accommodated to the $\pi/4$-shift DQPSK by a modification of the NEC receivers for the DQPSK following which the performance of the NEC receivers with single-, double- and triple-error correction capability are theoretically analyzed and evaluated. The most elaborate system analyzed is the triple-error NEC receiver which employs four differential detectors with delay elements of one up to four symbol duration long and which requires the computation of 12 syndromes for the correction of error symbols.

For the CCI, the general model which includes $M$ statistical independent interferers also employing the $\pi/4$-shift DQPSK modulation format is adopted. The theoretical symbol error rate (SER) versus carrier-to-noise (C/N) ratio have been obtained having $M$ and the carrier-to-interference (C/I) ratio as parameters. These performance evaluation results indicate significant performance improvements over conventional differentially detected $\pi/4$-shift DQPSK systems without requiring any bandwidth expansion. For example, at a SER $= 10^{-4}$ and for C/I = 14 dB and $M = 6$, gains of more than 7 dB have been obtained. Compared with a coherent $\pi/4$-shift QPSK system operated in the same environment, this triple error correcting NEC is inferior by only 1.5 dB. Some of these theoretical results have also been verified by computer simulation. The gains offered by the NEC receivers have been found to increase as C/I decreases and/or $M$ increases. In addition to the performance improvements, significant error floor reductions (of at least one order of magnitude) have been observed.
For the fading channel, the theoretical error rate equation for the single-error correcting NEC receiver is newly derived. Since numerical evaluation of the derived equation is extremely time consuming, computer simulations were used to obtain the performance evaluation result of $\pi/4$-shift DQPSK system employing single- and double-error correcting NEC receivers. In general, the improvement are not as high as that in the CCI environment. For example, at a BER $= 10^{-2}$ and for a Rician fading channel with the $K$-factor of 1 dB with $B_D T = 6.29$, a performance gain of 6 dB is achieved. The gains offered by the NEC receivers increases as the $K$-factor decreases and/or when the $B_D T$ is large.

Since the NEC technique does not require any bandwidth or signal constellation expansion as do other coding schemes, it is a powerful and attractive technique to increase the capacity of digital communication systems operated in CCI controlled (frequency reuse) environment, such as the new all digital North American and Japanese mobile/cellular network. The significant improvement of the NEC receivers in a very fast fading environment suggests the NEC receivers can also be applied to communication applications in which the speed of the mobile unit is very high, for example, in aeronautical communication systems. Finally, it is noteworthy that the results obtained in this thesis for the $\pi/4$-shift DQPSK systems are directly applicable to the DQPSK systems in a linear channel.
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Acknowledgments

I would like to express my sincere gratitude to my supervisor Dr. P. T. Mathiopoulos for his proposition of this research topic, financial subsidization, guidance, and encouragement.

I am greatly indebted to Dr. S. Samejima of NTT, Japan, Dr. Y. Miyagaki of Okayama University of Science, Japan, Dr. J. Goldman of AT &T Bell Laboratory, U.S. and Dr. Makrakis for their useful suggestions and explanations.

Thanks are also extended to the help by Dimitri Bouras for generously lending his computer simulation programs, S.W. To and Brenden Wong for their useful discussions as well as many individuals during the course of this research.

Last, but certainly not least, I would like to thank my parents with my wholeheart for bringing me here. Their love, care, guidance, support, encouragement and everything goes beyond any thanks which can account for.
Chapter 1
Introduction

1.1 Analog Cellular Systems versus Digital Cellular Systems

Currently, cellular systems are analog with the speech or data being transmitted over the radio path as a frequency-modulated (FM) signal. Over the last couple of years, the tremendous demand of mobile cellular telephone services has created an urgent need for the increase of the available radio spectrum [1, 2]. In major cities, such as New York and Los Angeles, the present radio frequency allocation for analog cellular is being used up rapidly [1–3]. The exhaust of this radio spectrum is causing a lot of communication problems, such as high blockage rate and dropped calls [3]. Blockage rate is the rate of failure in call attempts resulting from the situation where all frequency channels are being occupied by other users. Dropped calls refer to the unexpected termination of phone calls. Many methods had been proposed to reduce the problems of high blockage rate and dropped calls. These methods include various channel assignment schemes, such as Dynamic Channel Assignment scheme [4] and Hybrid Channel Assignment [5] which were proposed for the current analog cellular systems, such as Advance Mobile Phone Services (AMPS) and Total Access Communication Systems (TACS) [2]. It is fair to say that the various proposed development and techniques have reached a state that further innovation of the existing analog cellular system can not keep up with the tremendous demands of mobile cellular telephone services [1–3]. As a result, the idea of replacing the analog cellular by the digital cellular system has emerged. The driving force behind the adoption of the digital cellular system comes from the current need for increased system capacity. For example, in the current analog cellular employing AMPS systems, the channel spacing, i.e., the RF bandwidth occupied by a radio channel, is 30 kHz per radio channel at a interchannel signaling rate of 10 kb/s [2, 6]. Channel spacing refers to the bandwidth occupied by a radio channel. On the other hand, the new North American digital cellular system has the same channel spacing, but the proposed
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digitized voice transmission rate has increased to 48 kb/s [6, 7]. Thus, the *spectral efficiency* is equal to 0.33 b/s/Hz\(^1\) in the analog cellular system, whereas, it is equal to 1.60 b/s/Hz for the digital cellular system. It can be seen that the digital cellular system is almost 5 times more spectral efficient than its analog counterpart.

In addition to the increased capacity, the digital cellular system enables operating companies to offer enhanced services and as a result to generate more revenue [1, 2]. For example, digital technology provides the flexibility to offer calling number identification, call tracing and many innovative call management features [3]. These and other services will eventually evolve into Integrated Service Digital Network (ISDN)-compatible services which can offer multi-media communications [8–10]. Currently, among the number of digital cellular systems being developed worldwide, the only ones which have been under intense development are the Pan-European Groupe Special Mobile (GSM) and the American Digital Cellular (ADC) network. Some of the most important specifications of the European and the North American digital cellular network have been briefly summarized in Table 1.1.

Table 1.1 Radio/Modem Illustrative Specifications for Pan-European GSM and US digital cellular [6, 7, 10, 11].

<table>
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<th>Description</th>
<th>GSM</th>
<th>ADC</th>
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<tr>
<td>RF Frequency Band [MHz]</td>
<td>890-915 (Base Transmit) 935-960(Mobile Transmit)</td>
<td>870-890 (Base Transmit) 825-845(Mobile Transmit)</td>
</tr>
<tr>
<td>Channel Spacing</td>
<td>200 kHz per 8 channels</td>
<td>30 kHz per 3 channels</td>
</tr>
<tr>
<td>Bit Rate [kb/s]</td>
<td>16</td>
<td>48</td>
</tr>
<tr>
<td>Access Method</td>
<td>TDMA</td>
<td>TDMA</td>
</tr>
<tr>
<td>Modulation</td>
<td>GMSK</td>
<td>(\pi/4)-shift DQPSK</td>
</tr>
<tr>
<td>Demodulation</td>
<td>Coherent</td>
<td>Coherent or non-coherent</td>
</tr>
</tbody>
</table>

An important difference between the current analog and the newly proposed digital cellular network is the multiple access method. In the current analog cellular systems, due to the nature

\(^1\) The *spectral efficiency* is determined by the transmission rate per channel bandwidth, and is equal to \((10 \text{ kb/s } \div 30 \text{ kHz}) = 0.33 \text{ b/s/Hz for the current analog cellular systems.}
of the continuous waveform, only Frequency Division Multiple Access (FDMA) can be used [2, 12]. On the other hand, in the new digital cellular systems, the Time Division Multiple Access (TDMA) would be employed. An illustration of the difference between the TDMA and the FDMA is shown in Fig. 1.1. The introduction of the digital cellular system brought with it several other advantages such as reduced power consumption and improved voice quality. For example, the digital paging transmissions can be made synchronous due to the TDMA. On the contrary, the receiver for the analog system must operate continuously due to the FDMA. Thus, the receiver for the new digital cellular system can be made to reduce the standby current drain to a small fraction of the consumption of its analog counterpart by being idle during periods when data transmission is not relevant [10]. The improved voice quality comes from the error control coding done on the digital voice.

![Figure 1.1 An illustration of (a) FDMA and (b) TDMA.](image)

Finally, it should be pointed out that for compatibility purposes, the modulation scheme for the digital transmission of the new digital cellular system has been pre-specified during the course of the system planning. Nevertheless, the demodulation/detection method, hence, the configuration of the receiver structure, is not specified [6]. This important decision leaves the door open for manufactures to choose the receiver structures which will achieve the best overall performance.
1.2 Selection of Digital Modulation Schemes for Digital Mobile Cellular System

In general, the digital modulation schemes can be divided into two categories, namely constant envelope schemes and linear modulation schemes which usually possesses a non-constant envelope. The constant envelope schemes belong to the family of the continuous phase modulation (CPM) [13], therefore, they intrinsically have constant envelope properties. Some well known constant envelope schemes include the minimum shift keying (MSK) [14], the Gaussian minimum shift keying (GMSK) [15], and the tamed frequency modulation (TFM) [16]. The linear modulation schemes include the $m$-ary quadrature amplitude modulation ($m$-QAM) and the phase shift keying ($m$-PSK) [2, 17, 18]. It should be noted that the 4-QAM scheme is identical to the 4-PSK scheme which is commonly known as the quadrature phase shift keying (QPSK). All of these modulation schemes have their advantages and disadvantages in terms of power efficiency, spectral efficiency and out-of-band spurious spectrum suppression. Power efficiency measures the energy required to transmit one information bit, and Out-of-band spurious spectrum refers to the portion of the transmitted signal spectrum which lies outside of the allowed bandwidth and causes adjacent channel interference. For instance, the $m$-QAM ($m \geq 8$) schemes are desirable for their spectral efficiency, whereas, it is not power efficient because extra energy is required for the additional bits per symbol [19]. In addition, serious out-of-band spurious spectrum would result when the $m$-QAM signal is passed through a non-linear power amplifier. On the other hand, the GMSK scheme is power efficient and has an excellent performance in terms of out-of-band spurious spectrum suppression. Nevertheless, compared with the $m$-QAM scheme, the GMSK scheme is not spectral efficient. There is always a trade-off between power efficiency, spectral efficiency, and immunity to non-linear amplification when selecting a suitable modulation scheme for a particular application.

In almost any mobile cellular communication systems, power efficient nonlinear amplifiers
are being used [17, 18, 20]. The constant envelope modulation schemes are popular, since they do not cause serious spectral spread even nonlinear amplifiers are used. The drawback of the constant envelope modulation schemes is that the bandwidth required to transmit the RF signals is considerably larger than the baseband bandwidth if multi-level signaling is used. In other words, the spectral efficiency is poor. As the demand of mobile cellular communication increases, the poor spectral efficiency of constant envelope modulation schemes can no longer be ignored. To improve the spectral efficiency of the mobile communication systems, linear modulation schemes have been considered [8, 9, 17, 18].

As discussed earlier, although the m-QAM (m≥8) exhibits good spectral efficiency characteristics, it is not power efficient. A good compromise between satisfactory spectral efficiency and power efficiency would be the QPSK modulation scheme. The conventional QPSK system has a maximum of 100% envelope fluctuation due to possible 180° phase reversals as shown in Fig. 1.2(a) [17]. If it is nonlinear amplified, serious adjacent channel interference would result [17, 20]. The offset QPSK (OQPSK), as shown Fig. 1.2(b) was proposed to reduce envelope fluctuation. Since the OQPSK takes no specific signal phases at any time, it can only be coherently detected. However, in mobile cellular communications, coherent detection may not be optimal due to multipath fading which causes random phase or frequency modulation of received signals. The OQPSK scheme would thus result in system degradation. It should be mentioned at this point that noncoherent detection has the advantages that it is simple to implement and exhibit a satisfactory performance in mobile communications in which fading is one of the most important serious interferences [17, 18, 21].

It is clear that a good modulation scheme for the mobile cellular application should have small envelope fluctuation and can be noncoherently detected. The π/4-shift DQPSK scheme described by Baker [22] back in 1962 is a compromise between the QPSK and the OQPSK. As illustrated in Fig. 1.2(c), the maximum phase change is 135°, hence the spectral spread is not
as large as the QPSK schemes when nonlinear amplified. In addition, it can be noncoherently detected, thereby increasing its robustness against fading. In light of these advantages, the $\pi/4$-shift DQPSK scheme was selected as the transmission standard for the new North American and Japanese all digital cellular mobile communication network [23].

![Diagram of signal space for QPSK, OQPSK, and $\pi/4$-shift DQPSK](image)

Figure 1.2 Signal space diagram for (a) QPSK, (b) OQPSK, and (c) $\pi/4$-shift DQPSK [17].

As discussed earlier, the use of nonlinear amplifiers would result in a spectral spread and hence reduce any spectrum efficiency gained through the use of linear modulation schemes. Nevertheless, there are many linearization and compensation techniques that can be used to improve this out-of-band spurious spectrum suppression [17, 24–27]. For instance, Akaiwa and Nagata [17] proposed the use of a transmitter configuration composed of a power-efficient nonlinear amplifier with a cartesian negative feedback control [17]. It was reported that nearly $-60$ dB out-of-band spurious spectrum suppression had been attended at $(f_c \pm 13 \text{ kHz})$, where $f_c$ is the carrier frequency. Since the linearization and compensation techniques have proven to perform reasonably well and satisfactory, it will thus be reasonable to assume that the communication channel considered in this thesis is linear.

1.3 Description of the $\pi/4$-shift DQPSK System

The state space signal diagram of an unfiltered $\pi/4$-shift DQPSK is shown in Fig. 1.3 [17]. As it can be seen, the transmitted signal points are selected in turn from two signal groups, the
squares (even numbered points \(\{0,2,4,6\}\)) and the circles (odd numbered points \(\{1,3,5,7\}\)). For every transition, which is one symbol duration long, the signal has to go from a circle to a square, or vice versa, but will never go back to its original group. Thus, the number of the possible relative phase shifts, i.e., differential phase shifts, between two successive symbols is limited to four, namely \(\pi/4, 3\pi/4, 5\pi/4,\) and \(7\pi/4\). Equivalently, this can be seen as the transmitted data symbols are being differentially encoded, hence, the name \(\pi/4\)-shift DQPSK, so that the carrier phase angle at the \(i\)-th symbol interval, \(\theta_i\), is given by

\[
\theta_i = \theta_{i-1} + \Delta \theta_i
\]

where \(\Delta \theta_i\) is the differential phase which takes values from the alphabet \(\{\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\}\) as stated previously.

![Figure 1.3 The state space diagram of an unfiltered \(\pi/4\)-shift DQPSK consisting of the even \(\{0,2,4,6\}\) and odd \(\{1,3,5,7\}\) points.](image)

It should be noted that although the \(\pi/4\)-shift DQPSK signal exhibits eight signal points in the signal space, the bandwidth required to transmit the signal is the same as that for the QPSK signal. This is due to the fact that similar to the QPSK signal, only 2 bits are enough to encode any signal point in the signal space. However, because of the 100% envelope fluctuation in the
DQPSK signal, serious spectral spread would result when the DQPSK signal is passed through a non-linear amplifier, thus resulting in a performance degradation. Nevertheless, in a linear channel the DQPSK and the $\pi/4$-shift DQPSK systems would give rise to the same performance.

1.4 Interferences in the Mobile/Cellular Communication Environment

In the mobile/cellular communication environment, there are two major interferences that could severely degrade the error rate performance, namely, (i) cochannel interference (CCI), and (ii) fading.

1.4.1 The Cochannel Interference (CCI) Environment

In mobile/cellular communication systems, frequency reuse is a core concept which can drastically increase the spectrum efficiency. Frequency reuse means the same frequency repeatedly used by different cells spanned in different geographic locations as shown in Fig. 1.4 [2]. It can be seen that a frequency channel with carrier frequency $f_1$ is being used in the central cell as well as in the surrounding six cells located at a reuse distance $D$ away from the central cell. The cochannels in these six cells would interfere with the central cell, if the signal coverage of each cell overlap with each other, therefore, CCI would result.

In general, the carrier-to-interference (C/I) ratio is defined by [2]

$$C/I = \sum_{k=1}^{K_f} \left( \frac{D_k}{R} \right)^\gamma$$

(1.2)

where $\gamma$ (typically equal to 4 [2]) is the propagation path-loss determined by the actual environment, $K_f$ is the number of cochannel cells in the first tier, $R$ is the cell size determined by the coverage area of the signal strength in each cell, and C/I is the receiver carrier-to-interference ratio at the desired mobile receiver. From Fig. 1.4, it can be observed that as the separation distance $D$ increases, the CCI between cells decreases. In turn, the frequency channel is less oftenly reused, which results in a reduction of the spectral efficiency. A compromise is thus required.
in order to optimize the spectrum efficiency while keeping the CCI to a minimum level. The requirement of C/I for minimum CCI operation in the current analog mobile cellular system was decided to be 18 dB [2]. It is clear that if the requirement of the value of C/I is less stringent, the frequency channels can be more oftenly reused, therefore the system's capacity can be enhanced.

\[ \text{Figure 1.4 Illustration of the frequency reuse concept, and the resulting possible CCI from the interfering cells.} \]

\textbf{1.4.2 The Multipath Fading Environment}

Buildings and houses can become natural scatterers when their sizes are equivalent over many wavelengths of a propagation frequency [28]. Therefore, in a typical mobile/cellular communication environment where the propagation frequency is in the vicinity of 850 MHz, these buildings and houses would create reflected waves, as illustrated in Fig. 1.5. Thus, as the mobile unit moves, for example, through a city, the received signal would compose of scattered and reflected waves coming from many directions. At the receiver these reflected waves cause a random fluctuation of the received signal amplitude as well as an uniformly distributed phase [28, 29]. Furthermore, due to the movement of the mobile unit, these fluctuations are time variant.
These phenomena are collectively given the name *multipath fading*, and the reflected waves are referred to as *multipath fading signal* (or in short *fading*).

![Diagram of multipath fading](image)

Figure 1.5 Illustration of the multipath fading in a mobile cellular environment.

If there is no line-of-sight (LOS), i.e., direct, path between the transmitter and the receiver, the fading signal would have Rayleigh characteristics [28, 29]. In this case, its envelope has the following probability density function (pdf)

\[
p(r) = \frac{r}{\sqrt{\bar{r}^2 / 2}} \exp \left( -\frac{r^2}{\bar{r}^2} \right)
\]

where \( r \) is the envelope of the fading signal and \( \bar{r}^2 \) is the average power. In many applications, including the mobile-satellite system, there is usually a direct path [30]. In that case, the fading would have Rician characteristics [28, 29] with the pdf given as

\[
p(r) = 2 \frac{r}{\bar{r}^2} \exp \left( -\frac{r^2 + a^2}{\bar{r}^2} \right) I_0 \left( \frac{r}{\sqrt{\bar{r}^2 / 2}} \cdot \frac{a}{\sqrt{\bar{r}^2 / 2}} \right)
\]

where \( r \) is the envelope of the fading signal, \( \bar{r}^2 \) is the average power, \( a \) is the amplitude of a direct wave, and \( I_0(\cdot) \) is the modified Bessel function of zeroth order.

Depending upon the speed of the mobile unit, the fading could be either fast or slow which, in turn, determines the rate of fluctuation of the received signal level. Finally, if the delay spread
Chapter 1. Introduction

of the reflected waves compared with the symbol duration is small, then the fading is referred to as flat or frequency non-selective fading. On the other hand, if the delay spread is large, the fading is frequency-selective [31].

1.5 The Nonredundant Error Correction (NEC) Technique

One way of improving the performance of the new digital cellular communication system using $\pi/4$-shift DQPSK signalling format is to employ the nonredundant error correction (NEC) technique. As it will be explained later on, the peculiarity of this coding technique is that it does not have any redundancy, hence, no additional bandwidth is required. This advantage makes the technique very attractive and useful for applications where bandwidth conservation is of paramount importance.

The NEC technique was first proposed in 1970 for the differential binary PSK (DBPSK) system by Chow and Ko [32]. They showed that the symbol detected by taking the difference between any two alternate signals can be used as a parity symbol for the symbol detected by a conventional differential detector. They also demonstrated that one error can be corrected with this arrangement. Since then, more studies have been carried out to improve a system performance by taking into consideration the difference in the carrier phase between two received signals separated by two or more symbol period. For example, Samejima et al. [33] illustrated that by employing differential detectors of order one up to $L$, the output sequence is a codeword sequence of a rate $1/L$ convolutional code which has an $(L-1)$ error correction capability. In the same reference, the NEC technique was applied to a $M$-ary Differential PSK (DPSK) signal operated in an AWGN environment. In addition, the NEC technique had been applied to the DQPSK system [34], and the Differential MSK (DMSK) [35–38]. In [35], the performance of DMSK with single error correcting NEC in an AWGN channel was investigated. An extension of this work, which includes double error correcting NEC as well as experimental performance
evaluation of DMSK in a bandlimited, with Intersymbol Interference (ISI), channel was recently published in [37], reporting gains of up to 2.5 dB. Nevertheless, the NEC technique has neither yet been applied to the $\pi/4$-shift DQPSK scheme, nor has been employed in channels where CCI and fading are the main sources of interferences.

1.6 Research Objectives of this Thesis

Based on the previous discussions, this thesis deals with improving the error rate performance of the $\pi/4$-shift DQPSK systems operated in the presence of the CCI and the fading environment by employing the NEC technique. In particular, the research contribution of this thesis can be summarized as follows.

1. The NEC technique is first applied to the $\pi/4$-shift DQPSK system. Novel differential detectors for the NEC receivers based on a proposed phase transformation concept are derived. The single-, double-, and triple-error correcting NEC techniques are analyzed and applied to the $\pi/4$-shift DQPSK modulation format.

2. The performance of the single-, double-, and triple-error correcting NEC receivers for the $\pi/4$-shift DQPSK scheme are analyzed and evaluated in a combined AWGN-CCI environment. The theoretical error rate performance for the double- and triple-correcting NEC systems are derived and evaluated. Computer simulation is used to verify some of the theoretical results.

3. The performance of the single- and double-error correcting NEC receivers is analyzed in a fading environment. The formula to evaluate the error probability of the single error correcting NEC receiver in a fading environment, along with the corresponding error rate performance equation is derived. Computer simulation are used to determine the performance of the two NEC receivers.
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1.7 Thesis Outline

Including this introduction chapter, the thesis consists of a total of 5 chapters and is organized as follows.

Chapter 2 presents an in-depth analysis of the NEC technique. After the introduction given in Section 2.1, a clear explanation will be given on how $L$ detectors give rise to a rate-$1/L$ convolutional code. In Section 2.3, the NEC technique for the $M$-PSK signals, associated with the equivalent syndrome feedback decoder, are described. The corresponding syndrome equations are derived. Various issues associated with this feedback decoding technique, such as infinite error propagation, decoding depth and constraint depth, are discussed. In Section 2.4, the application of the syndrome feedback decoding to the $\pi/4$-shift DQPSK signals is presented with particular attention given to the single-, double-, and the triple-error correcting NEC receivers. Syndrome equations, together with the detection patterns for all three receivers are also presented. The effects of infinite error propagation for all three NEC receivers are also analyzed. Finally, Section 2.5 presents the summary of the chapter.

Chapter 3 deals with the analysis and the evaluation of the performance of the $\pi/4$-shift DQPSK systems employing NEC receivers operated in a combined CCI-AWGN environment. After a brief introduction on CCI, a description of the digital communication system model employed in the analysis is presented in Section 3.2. In Section 3.3, the theoretical error rate performance of all three NEC receivers are analyzed, and the error rate equations of the NEC receivers are derived. In Section 3.4, the computer simulation model employed in verifying some of the theoretical results are described. Various numerical results complemented with results obtained by computer simulation can be found in Section 3.5. Interpretations of these results will be discussed in Section 3.6. The summary of this chapter is given in Section 3.7.

Chapter 4 concerns about the performance of the $\pi/4$-shift DQPSK systems employing NEC receiver in a fading environment. After a brief introduction on fading, the digital communication
Chapter 1. Introduction

system model together with the fading model employed are presented in Section 4.2. In Section 4.3, the error rate performance equation of the single error correcting NEC receiver in a fading environment is derived. The error probability for the specific error patterns of the NEC receiver in a fading environment is also derived. In Section 4.4, various numerical results obtained theoretically and by means of computer simulation are presented. Section 4.5 presents the interpretation of the obtained results. Section 4.6 summarizes the findings of this chapter.

Chapter 5 presents the conclusion of this thesis, along with the suggestion of some ideas for future research work.
Chapter 2
Nonredundant Error Correction of Differentially Encoded PSK systems

2.1 Introduction

Differential detection of differentially encoded phase shift keying (DPSK) signals is an attractive detection technique despite its inferior performance compared with coherent detection. Because of the need for more simple circuit configuration in many applications, such as in satellite communication and mobile cellular communication, differential detection could be a very promising demodulation technique if its error rate performance can be improved.

As mentioned in Chapter 1, one possible way to improve the performance of differentially detected system uses the nonredundant error correction (NEC) technique [34]. The main advantage of this technique is that error correction is achieved without using the additional bandwidth for redundant information that conventional coding schemes require. Hence, it is known as the nonredundant error correction technique. In 1970 Chow and Ko [32] showed that the output of a $k$th order differential detector* for a binary DPSK (DBPSK) signal is the product of $k$ successive outputs of the conventional differential detector† under a noise free condition. Based on the fact that differential phase error in two consecutive intervals causes an error the differential phase error in two or more alternate intervals does not necessarily cause an error, the output symbol from the second order differential detector was used as a reference to correct the error symbol from the first order differential detector [32]. They showed that one error can be corrected with this arrangement. Since then more studies have been carried out to improve the system performance by taking into consideration the difference in the carrier phase between two received signals separated by two or more time slots, and the NEC technique was applied to other modulation systems [33–37]. Schroeder and Sheehan [34] applied this idea to the DQPSK

---

* To be explained later, a $k$th order differential detector corresponds to a differential detector with $k$ symbol delay element.
† Conventional differential detector refers to the differential detector where differential phase between adjacent signals is detected.
system, and obtained a validity equation for doing the error correction. In [35], the performance of DMSK with single error correcting NEC in an AWGN channel was investigated. Samejima et al. [33] generalized this to an $M$-ary DPSK signal. In [36], the performance of the DMSK was experimentally evaluated in a combined AWGN and adjacent channel interference (ACI) channel. A recently published extension of this work [37] which includes double error correcting NEC as well as experimental performance evaluation of DMSK in a bandlimited (with Intersymbol Interference, ISI) channel reported gains of up to 2.5 dB.

In this chapter, the NEC technique will be reviewed and applied to the $\pi/4$-shift DQPSK signal. Section 2.2 presents the relationship between the received signals separated by two or more time slots with the convolutional encoding. Section 2.3 describes the application of the NEC technique to the DPSK system and various issues on this technique, such as infinite error propagation and decoding depth. Section 2.4 presents the application of the NEC technique to the $\pi/4$-shift DQPSK and discuss single-, double-, and triple-error correcting NEC technique. Section 2.5 presents the summary of this chapter.

2.2 Differential Detection from Convolutional Encoding Point of View

Chow and Ko [32] first pointed out that the output of a $k$th order differential detector for a DBPSK signal is the product of $k$ successive outputs of the conventional differential detector under noise free condition. It was reported that the symbol detected from the phase difference between two alternate intervals can be used as a reference for the symbol detected from the difference in phase between two consecutive intervals. The fact that the outputs of a conventional detector and a detector with a two-time-slot delay circuit corresponds to the data and the parity, respectively, of a single error-correcting self-orthogonal convolutional code was not documented until [35]. Since then, Samejima et al. [33] has generalized this "special relationship" to $m$-phase DPSK signal by considering from one up to $L$ differential detectors. With these $L$ differential detectors,
the output sequence is the codeword sequence of a rate-1/L convolutional code. It was however not clearly explained in either of these publications how these \(L\) differential detectors indeed give rise to a rate-1/L convolutional code. Various issues, such as ways of finding the test sequence for the free distance, error propagation and constraint length of the code, have not been discussed at all. In this section, the relationship of these \(L\) differential detectors with the convolutional code will be explained in detail. In addition, various properties of the code will also be discussed.

In differential detection of an \(m\)-phase DPSK system, a symbol is detected from the difference in phase between two consecutive time slots so that the carrier phase detected at the \(i\)th time slot can be expressed as

\[
\theta_i = \theta_{i-1} + \left(\frac{2\pi}{m}\right)a_i
\]  

(2.1)

where \(a_i\) is the transmitted data symbol of the \(i\)th time slot and takes values from the alphabet \(\{0, 1, 2, \ldots, m-1\}\). These values that the data symbol \(a_i\) would take are not the binary equivalent of its encoded data bits, i.e., they are not equivalent to Gray coding of the transmitted signals. As an illustration, the DQPSK signals with differential phase shifts of \(\{0, \pi/2, \pi, 3\pi/2\}\) having the signals constellation and the associated Gray-encoded bits are considered (see Fig. 2.1). The associated data symbol values are also shown in the figure. It can be noted that the zero degree phase signal point \(a_i\) has a value of zero with the rest of the signals points numbered in a counter-clockwise direction.
Chapter 2. Nonredundant Error Correction of Differentially Encoded PSK systems

Figure 2.1 DQPSK signals with Gray coding, and the associated numbers.

Following [33], in the absence of noise, the carrier phase \( \theta_i \) of an \( m \)-DPSK signal can be expressed by using the carrier phase \( \theta_{i-k} \) of the \((i-k)\)th time slot and the \( k \) successive transmitted data symbols from the \((i-k+1)\)th to the \( i \)th time slots, i.e.,

\[
\theta_i = \theta_{i-k} + \sum_{j=0}^{k-1} \left( \frac{2\pi}{m} \right) a_{i-j}. \tag{2.2}
\]

The phase difference between the present time slot and the previous \( k \) time slots can be obtained by rearranging (2.2) as

\[
\theta_i - \theta_{i-k} = \left[ (\frac{2\pi}{m}) \sum_{j=0}^{k-1} a_{i-j} \right] \mod -2\pi
\]

\[
= \left\{ \left[ \sum_{j=0}^{k-1} a_{i-j} \right] \mod m \right\} \left( \frac{2\pi}{m} \right) \mod -2\pi \tag{2.3}
\]

where \( \mod \) denotes the modulo arithmetic. In the absence of noise, the output symbol \( d_{k,i} \) detected from the difference in phase between the current signal and the signal delayed by \( k \) time slots is given as

\[
d_{k,i} = \left[ \sum_{j=0}^{k-1} a_{i-j} \right] \mod m. \tag{2.4}
\]

With the use of \( L \) detector, \( L \) output symbols are obtained in the \( i \)th time slot as shown in Fig. 2.2 [33]. The \( d_{k,i} \) in Fig. 2.2 corresponds to the output symbol of the \( k \)th order detector which
detects the phase difference between the current carrier phase and the carrier phase delayed by 
k time slots.

![Diagram of differential detectors for m-phase PSK signal](image)

Figure 2.2 $L$ detectors for $m$-phase PSK signal [33]. Note that $kT$ stands for $k$ time slot delay, and PC stands for phase comparator.

Expanding (2.4),

\[
d_{1,i} = a_i
\]

\[
d_{2,i} = [a_i + a_{i-1}] \mod m
\]

\[
d_{3,i} = [a_i + a_{i-1} + a_{i-2}] \mod m
\]

\[
\vdots
\]

\[
d_{k,i} = [a_i + a_{i-1} + a_{i-2} + \cdots + a_{i-k+1}] \mod m
\]

\[
\vdots
\]

\[
d_{L,i} = [a_i + a_{i-1} + a_{i-2} + \cdots + a_{i-k+1} + \cdots + a_{i-L+1}] \mod m.
\]
Since a systematic code is defined as a convolutional code which contains the original information symbol in the output symbol sequences [39], it is apparent from (2.5) that the $d_{1,i}$ corresponds to the information symbol of a systematic rate-1/$L$ convolutional code. The set $\{d_{k,i}\}$, $k$ from 2 to $L$, is analogous to the parity symbols of the code. The connections of the encoder for a rate-1/$L$ convolutional code can be observed from (2.5). For example, the first equation in (2.5) refers to the current information symbol whereas the second equation in (2.5) refers to the sum of the current and the previous information symbols. By the same token, the last equation in (2.5) refers to the sum of the current up to the previous $L$ information symbols. Based on these observations, the encoder is connected as shown in Fig. 2.3. It becomes clear that the $L$ detectors give rise to a systematic $(n,k,L)$ convolutional code with $n = L$, $k = 1$, and $\tilde{L} = (L-1)$ where $n$ is the number of encoded outputs, $k$ is the number of inputs, and $\tilde{L}$ is the encoder's memory. The constraint length of this code, defined as the number of stages in the encoder [40] equals $L$.

The generator matrix of our rate-1/$L$ convolutional code can be given as [41]

$$
G = \begin{bmatrix}
  g^{(1)}_0 & g^{(2)}_0 & \cdots & g^{(L)}_0 & 0 & \cdots & \cdots \\
  0 & g^{(1)}_0 & g^{(2)}_0 & \cdots & g^{(L)}_0 & g^{(1)}_{L-1} & g^{(2)}_{L-1} & \cdots & 0 \\
  \vdots & & \ddots & & \ddots & \ddots & \ddots & \ddots & \ddots \\
  111 \cdots 1 & 011 \cdots 1 & \cdots & 001 \cdots 1 & 000 \cdots 0 & \cdots & \cdots \\
  000 \cdots 0 & 111 \cdots 1 & 011 \cdots 1 & \cdots & 001 \cdots 1 & 000 \cdots 0 & \cdots & \cdots \\
  \vdots & & \ddots & & \ddots & \ddots & \ddots & \ddots & \ddots \\
  \end{bmatrix}
$$

(2.6)

where $g^{(k)}_j$, $k = 1 \sim L$ can be viewed as the connection of the $j$ shift register of the symbol $d_{k,i}$. 


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Figure 2.3 Encoder for a rate-1/L convolutional code with \( n = L, k = 1, \) and \( L = (L - 1). \)

The error correction capability of a convolutional code can be determined by its free distance which is defined as the minimum Hamming weight among the output codeword sequences generated by a transmitted data symbol sequence having a nonzero starting data symbol [41]. A transmitted data symbol sequence with a nonzero starting data symbol was given in [33] as

\[
a_i a_{i+1} a_{i+2} a_{i+3} a_{i+4} \cdots = f \bar{f} 000 \cdots
\]

where \( f \in \{1, 2, \ldots, m - 1\} \) and \( \bar{f} = (-f \mod m) = m - f. \)

The output codeword can be obtained by multiplying (2.7) by the generator matrix (2.6), i.e.,

\[
v = uG
\]

\[
= (f \bar{f} 000 \cdots)
\]

\[
\begin{pmatrix}
 111 \cdots 1 & 011 \cdots 1 & \cdots & 000 \cdots 1 & 000 \cdots 0 & \cdots & \cdots \\
 000 \cdots 0 & 111 \cdots 1 & 011 \cdots 1 & \cdots & 000 \cdots 1 & 000 \cdots 0 & \cdots & \cdots \\
 000 \cdots 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
 000 \cdots 0
\end{pmatrix}
\]

(2.8)

When the number of differential detectors is limited to \( L, \) the output codeword sequences shown in Table 2.1 are obtained.
Table 2.1 Output codeword sequence of $L$ differential detectors for $m$-phase DPSK signal.

<table>
<thead>
<tr>
<th>Time slot number</th>
<th>$i$</th>
<th>$i+1$</th>
<th>$i+2$</th>
<th>...</th>
<th>$i+k$</th>
<th>$i+L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st order detector</td>
<td>$f$</td>
<td>$f$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>...</td>
</tr>
<tr>
<td>2nd order detector</td>
<td>$f$</td>
<td>0</td>
<td>$f$</td>
<td>0</td>
<td>0</td>
<td>...</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k$th order detector</td>
<td>$f$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$f$</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L$th order detector</td>
<td>$f$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

In the $i$th time slot, all of the $L$ detectors output symbol $f$. In the $(i+k)$th time slot, only the $k$th order detector outputs nonzero symbol $f$. Observation over $(L+1)$ time slots, from the $i$th to the $(i+L)$th time slot, determines the Hamming weight as $2L$. The error correction capability $t$ of the code is given by [41]

$$t \leq \left\lfloor \frac{d_{\text{min}} - 1}{2} \right\rfloor$$  \hspace{1cm} (2.9)

where $\lfloor \cdot \rfloor$ denotes truncation and $d_{\text{min}}$ is the Hamming weight. When the observation interval is limited to $L$ time slots, the Hamming weight equals $(2L-1)$. From (2.9), this code has $(L-1)$ error correction capability [33].

2.3 The Application of the NEC Technique to the $m$-phase DPSK System

The fact that the $k$ differential detectors, $k$ from 1 to $L$, give rise to a systematic rate-$1/L$ code suggests that various convolutional decoding methods can be used to decode the received symbols. As mentioned earlier, Masamura et al. [35, 37], Samejima et al. [33], and more recently Weining [38] used the decoding method which was referred to as the NEC technique.
The NEC technique is, in fact, the well-known \textit{syndrome feedback decoding technique}, although it has not been explicitly mentioned in any of these papers.

The general block diagram of the DPSK demodulator given by Samejima \textit{et al.} [33] for \( m \)-phase PSK signal is shown in Fig. 2.4.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{demodulator_block_diagram.png}
\caption{Demodulator block diagram for \( m \)-phase PSK signal [33].}
\end{figure}

In the presence of noise, the received symbol will be equal to

\[ r_{k,i} = (d_{k,i} + e_{k,i}) \mod m \]

where \( d_{k,i} \) is the symbol of the \( k \)th order detector at the \( i \)th time slot under noiseless condition and \( e_{k,i} \) is the error symbol of the \( k \)th order detector at the \( i \)th time slot.

Since the NEC decoder inspects \((L-1)\) blocks of \( L \) consecutive syndromes and at the \((i+L-1)\)th time slot makes a decision on the symbol \( r_{1,i-L+1} \), the decoding depth of the code equals \((L-1)\) time units. The received symbols are stored in the \( L \)-stage shift registers, as are the \( L(L-1) \) syndrome symbols relevant to the decoding decision on the symbol \( r_{1,i-L+1} \). When the estimate of the error symbol \( e_{1,i-L+1} \) is zero, the information symbol \( r_{1,i-L+1} \) will be decoded as
received, and the contents of the syndrome registers remain unchanged. On the other hand, if the error estimate of $e_{1,i-L+1}$ is non-zero, it will be subtracted (mod-$m$ arithmetic) from the $r_{1,i-L+1}$, and from the syndrome registers containing the $e_{1,i-L+1}$ through feedback. The mapping from the observed syndrome sequence to the estimate of the $e_{1,i-L+1}$ can be implemented from a table-look-up procedure in which the predetermined syndrome patterns containing the most likely channel error-patterns are stored. The most likely channel error pattern refers to the pattern containing the smallest number of errors, i.e., the number of errors is less than or equal to the error correction capability of the system.

From Fig. 2.4, the set of syndromes $\{S_{k,i}\}$, $k$ from 1 to $(L-1)$, can be given by

$$S_{k,i} = \left[ \sum_{j=0}^{k} r_{1,i-j} - r_{k+1,i} \right] \mod m. \quad (2.11)$$

Substituting (2.10) in (2.11), the set of syndromes can be rewritten as

$$S_{k,i} = \left[ \sum_{j=0}^{k} (d_{1,i-j} + e_{1,i-j}) - (d_{k+1,i} + e_{k+1,i}) \right] \mod m$$

$$= \left[ \sum_{j=0}^{k} e_{1,i-j} - e_{k+1,i} \right] \mod m. \quad (2.12)$$

From (2.12), the syndromes are only a function of the error symbols, and will be equal to zero if there is no error. As long as the number of errors making up the syndromes is within the error correction capability of the system, all the syndrome patterns, i.e., $\{S_{k,i-j} \mid j = 0, 1, \ldots, L-1; k = 1, \ldots, L-1\}$ will be distinct. Following the fact that there are $L$ stage shift registers per syndrome set $\{S_{k,i-j}, j = 0, 1, \ldots, L-1\}$ and since each syndrome has a distinct error symbol $e_{k+1,i}$ (see equation 2.12), $L$ distinct error symbols exist in each syndrome set. As the number of syndrome sets is determined by the number of higher order differential detectors employed, a total of $(L-1)$ syndrome sets result. The $L$ error symbols from each syndrome set and another $L$ error symbols from the $L$-stage shift registers of the information symbols
result in a total of \([L(L - 1) + L] = L^2\) distinct error symbols among the \(L(L - 1)\) syndromes. The error symbols take values of \(\{1, 2, \ldots, m - 1\}\). The number of syndrome patterns \(N_s\) to be stored for error correction includes single error, double error, up to the error correction limitation of the NEC system, i.e., \((L-1)\) for the system under consideration here, and can be given by

\[
N_s = (m - 1) \left[ \left( \begin{array}{c} L^2 - 1 \\ 0 \end{array} \right) (m - 1)^0 + \left( \begin{array}{c} L^2 - 1 \\ 1 \end{array} \right) (m - 1)^1 + \cdots + \left( \begin{array}{c} L^2 - 1 \\ L - 2 \end{array} \right) (m - 1)^{L-2} \right] \\
= (m - 1) \sum_{j=0}^{L-2} \binom{L^2 - 1}{j} (m - 1)^j.
\]

\[(2.13)\]

Since it is most likely that the incorrect decisions are caused by signals going into its neighboring decision region [33], all of the error symbols are practically either +1 or \(-1(= -1 \mod m)\). The number of syndrome patterns \(N_s\) in (2.13) to be stored for error correction can therefore be reduced to [33]

\[
N_s = 2 \sum_{j=0}^{L-2} \binom{L^2 - 1}{j} 2^j. \tag{2.14}
\]

It is well known that infinite error propagation may occur with feedback decoding if the feedback decoder is not properly designed [40]. This is because when an incorrect estimate is fed back to the syndromes, it has the same effect as an additional transmission error, and can cause further decoding errors which would not otherwise occur. As a result, the decoder may continuously decode incorrectly forever [39]. There are various ways to combat this infinite error propagation effect, such as definite decoding [42]. It is, however, unnecessary to do this here, as the code has the automatic resynchronization properties whereby the effect of past errors (post-decoding errors) on the syndrome is automatically removed, thus halting any possible error propagation.

The automatic resynchronization property of the code can be explained by considering the general case of a rate-1/\(L\) code. As mentioned previously, the purpose of the feedback is to eliminate the effect of \(e_{1,i-L+1}\) from the syndromes containing \(e_{1,i-L+1}\), the first \((L-k)\)
syndromes of each set will therefore be unaffected by the post-decoding error. If there are no more transmission errors after the \((i+L-1)\)th time slot, zeros would enter the left most stages of the syndrome registers at the \((i+2L)\)th time slot which follows from the fact that the decoder contains \(L\) stage shift registers at the output of the first-order differential detector. When the syndrome patterns match any of the stored detection patterns, all the syndrome registers would be cleared immediately. On the other hand, if no match is found, the syndrome register contents would continue to be replaced by zeros since only zeros are being shifted in. Consequently, regardless of the contents of the syndrome registers are carrying, they would eventually be clear to zero. Further illustrations of this property on rate-1/2, 1/3, 1/4 codes will be presented in Section 2.4.

2.4 The Application of the NEC to the \(\pi/4\)-shift DQPSK System

Chapter 1 mentioned that every transition of the \(\pi/4\)-shift DQPSK signal would cause the signal going from a circle to a square or vice versa but will never go back to its original group. It is noteworthy that every even time of transition will bring the signal to its original group so that a decision problem is posed for the detectors with even time slots of delay. For instance, two successive transitions of \(\pi/4\) phase shift will lead to a total of \(\pi/2\) phase shift which is not allowed in the \(\pi/4\)-shift DQPSK signal. The differential detection of the \(\pi/4\)-shift DQPSK signal for the accommodation of the NEC technique is therefore necessary to be readdressed.

2.4.1 A Novel Differential Detection for the \(\pi/4\)-shift DQPSK system

Since the DQPSK signal takes values from the alphabet \(\{0, \pi/2, \pi, 3\pi/2\}\) whereas the \(\pi/4\)-shift DQPSK signal takes values from the alphabet \(\{\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\}\), shifting any \(\pi/4\)-shift DQPSK signal by an additional phase of \(\pi/4\) would bring the signal to the DQPSK signal constellation. It thus follows that the decision problem of the detector with even times slot of delay could be overcome by conceptually adding an extra \(\pi/4\) phase to every transition of
the $\pi/4$-shift DQPSK signal. The fact that the extra $\pi/4$ phase shift is not physically added to the transmitted or the received phase designate this process as the phase transformation concept. Next, this phase transformation concept will be illustrated in detail.

Suppose that the $\pi/4$-shift DQPSK signal is transmitted, in the absence of noise the received carrier phase $\theta_i$ of the $i$th time slot and the carrier phase $\theta_{i-k}$ of the $(i-k)$th time slot is given by

$$\theta_i = \theta_{i-k} + \left[ \sum_{j=0}^{k-1} \left( \frac{\pi}{4} \right) a_{i-j} \right] \mod 2\pi$$

(2.15)

where $a_{i-j}$ takes values of 1, 3, 5, or 7. With the phase transformation concept, an extra phase of $\pi/4$ is “added” to each received differential phase at the differential detectors as shown in Fig. 2.5.

```
Figure 2.5 Illustration of the transformation of the $\pi/4$-shift DQPSK signal to the DQPSK signal through the “addition” of an extra phase of $\pi/4$.
```

The new relationship between the carrier phases $\theta_i$ and $\theta_{i-k}$ is given by

$$\theta_i = \theta_{i-k} + \sum_{j=0}^{k-1} \left( \frac{\pi}{4} \right) a_{i-j} + \frac{\pi}{4} \mod 2\pi$$

(2.16)

$$= \theta_{i-k} + \frac{\pi}{4} \sum_{j=0}^{k-1} (a_{i-j} + 1) \mod 8.$$

By substituting values of 1, 3, 5 or 7 into $a_{i-j}$ of (2.16), it becomes apparent that the $\pi/4$-shift DQPSK signal is transformed into a DQPSK signal format. It can be noted that the “new” signal space diagrams of the $\pi/4$-shift DQPSK signal for the odd- and even-order detectors illustrated in Fig. 2.6 have the same signal space diagram as that of the DQPSK signal shown in Fig. 2.7.
It is clear how the \( \pi/4 \)-shift DQPSK signal can be "transformed" into a DQPSK signal by adding the extra \( \pi/4 \) phase shift. However, it now seems that the additional phase is physically added to the received differential phase. Next, it will be illustrated how this additional phase of \( \pi/4 \) can be incorporated into the decision threshold of the \( \pi/4 \)-shift DQPSK differential detectors without actually introducing the additional phase to the transmitted or the received phase.
First, the signal constellation in Fig. 2.7 can be assumed to result from having the extra phase of $\pi/4$ "added" to every differential phase. Since every differential phase will have an extra phase of $\pi/4$, a total phase shift of $k(\pi/4)$ results where $k$ denotes the number of delay elements. For the counteraction of this "added" phase of the $k$th order detector or to restore the original signal constellation shown in Fig. 2.6, the signal constellation shown in Fig. 2.7 should be rotated clockwise by an angle of $k(\pi/4)$. The decision thresholds together with the associated numbers assignment for the first- up to the eighth-order detector are shown in Fig. 2.8. It should be noted that the decision threshold repeated itself after the eighth-order detector. For example, an nineth-order detector will have the same decision thresholds and number assignments as the first-order detector. It is clear that using the decision threshold and the associated numbers assignment shown in Fig. 2.8, the $\pi/4$-shift DQPSK signal can be "transformed" into the DQPSK signal following which the NEC technique can be applied to the $\pi/4$-shift DQPSK signal without any additional phase shift.

Figure 2.7 The signal space diagram for the DQPSK signal with the associated numbering.
Figure 2.8 Decision region (shaded regions) for differentially detected $\pi/4$-shift DQPSK system with differential detectors of (a) $1T$, (b) $2T$, (c) $3T$, (d) $4T$, (e) $5T$, (f) $6T$, (g) $7T$, (h) $8T$ delays using the DQPSK transformation. (Continued . . .)
Figure 2.8 Decision region (shaded regions) for differentially detected $\pi/4$-shift DQPSK system with differential detectors of (a) $1T$, (b) $2T$, (c) $3T$, (d) $4T$, (e) $5T$, (f) $6T$, (g) $7T$, (h) $8T$ delays using the DQPSK transformation.

Based on the number assignment in Fig. 2.8, (2.16) can be rewritten as

$$\theta_i = \theta_{i-k} + \frac{\pi}{2} \sum_{j=0}^{k-1} (a_{i-j} + 1) \mod 4$$

(2.17)

where $a_{i-j}$ takes values of 0, 1, 2, or 3, and mod-4 arithmetic is required. The transformation of the phase difference and the corresponding assigned numbers for the detectors with even and odd times of delay are summarized in Table 2.2.

Since the value of each signal differs from that of its neighboring signals by either +1 or -1 while the signals are more likely to go incorrectly into the neighboring decision regions due to noise and interferences, the values of the error symbols would be either +1 or +3 ($= -1 \mod 4$).
Table 2.2 DQPSK transformation of the phase difference for the differential detectors with even and odd times of delay.

<table>
<thead>
<tr>
<th>Odd order differential detectors</th>
<th>Even order differential detectors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Order</strong></td>
<td><strong>Actual phase</strong></td>
</tr>
<tr>
<td>1T</td>
<td>(\pi/4)</td>
</tr>
<tr>
<td></td>
<td>3(\pi/4)</td>
</tr>
<tr>
<td></td>
<td>5(\pi/4)</td>
</tr>
<tr>
<td></td>
<td>7(\pi/4)</td>
</tr>
<tr>
<td>3T</td>
<td>(\pi/4)</td>
</tr>
<tr>
<td></td>
<td>3(\pi/4)</td>
</tr>
<tr>
<td></td>
<td>5(\pi/4)</td>
</tr>
<tr>
<td></td>
<td>7(\pi/4)</td>
</tr>
<tr>
<td></td>
<td>(\pi/4)</td>
</tr>
<tr>
<td></td>
<td>3(\pi/4)</td>
</tr>
<tr>
<td></td>
<td>5(\pi/4)</td>
</tr>
<tr>
<td></td>
<td>(\pi/4)</td>
</tr>
<tr>
<td></td>
<td>3(\pi/4)</td>
</tr>
<tr>
<td></td>
<td>5(\pi/4)</td>
</tr>
<tr>
<td></td>
<td>7(\pi/4)</td>
</tr>
</tbody>
</table>

Based on the observation that the \(\pi/4\)-shift DQPSK signal constellation (see Fig. 1.2) is similar to the 8-DPSK signal constellation, an alternative approach for the accommodation of the NEC technique to the \(\pi/4\)-shift DQPSK is to consider the signal as an "equivalent" 8-DPSK signal. It follows that the \(\pi/4\)-shift DQPSK system can be treated as if it is an 8-DPSK system and the mod-8 addition is required. One important difference between this "8-DPSK system" and the actual 8-DPSK system is that the former have the phase shift of \(\pi/4\), 3\(\pi/4\), 5\(\pi/4\) and 7\(\pi/4\) in the odd order detectors and 0, \(\pi/2\), \(\pi\) and 3\(\pi/2\) in the even order detectors whereas the latter has all these phase shift in every detector. The relationship between the actual phase shift
and the number assignments for the $\pi/4$-shift DQPSK employing the 8–DPSK transformation is in Table 2.3.

Table 2.3 8–DPSK transformation of the phase difference for the detectors with even and odd times of delay.

<table>
<thead>
<tr>
<th>Actual phase shift</th>
<th>Assigned number</th>
<th>Actual phase shift</th>
<th>Assigned number</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi/4$</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$3\pi/4$</td>
<td>3</td>
<td>$\pi/2$</td>
<td>2</td>
</tr>
<tr>
<td>$5\pi/4$</td>
<td>5</td>
<td>$\pi$</td>
<td>4</td>
</tr>
<tr>
<td>$7\pi/4$</td>
<td>7</td>
<td>$3\pi/2$</td>
<td>6</td>
</tr>
</tbody>
</table>

The decision regions for the even and odd order detectors in Fig. 2.9 show that each signal differs from its adjacent signal by a value of 2. Since mod-8 arithmetic is used, the values of the error symbols are either 2 or 6 ($= -2$ mod-8).

Figure 2.9 Decision region for differentially detected $\pi/4$–shift DQPSK system (a)odd-order detector, (b)even-order detector using the 8–DPSK transformation.

Although the performances of the mod-4 and the mod-8 approaches are identical, the first approach is more efficient from the hardware implementation point of view. It is because the mod-4 approach requires two bits to represent one symbol so the adders and the inverters are
more simple than those of the mod-8 approach which requires three bits. The mod-4 approach will be adopted for the rest of the analysis.

### 2.4.2 NEC with Single Error Correction Capability for $\pi/4$-shift DQPSK schemes

The block diagram of the single error correcting NEC system is illustrated in Fig. 2.10 [33].

![Block diagram of the single error correcting NEC receiver for a $\pi/4$-shift DQPSK system [33]. All inverters (INV) and adders are of mod-4.](image)

Similar to [33], a NEC $\pi/4$-shift DQPSK system with single error correction capability can be implemented by using one first-order differential detector and one second-order differential detector. The equations of the syndromes used in detecting the error symbol are given in [33] as

\[
S_{1,i} = (e_{1,i} + e_{1,i-1} - e_{2,i}) \mod 4
\]

\[
S_{1,i-1} = (e_{1,i-1} - e_{2,i-1}) \mod 4.
\]

In the above equation, the error symbol $e_{1,i-2}$ is assumed to be corrected in the previous time slot. The detection pattern summarized in Table 2.4 can be found by substituting "n" into $e_{1,i-1}$ of (2.18) where $n \in \{1, 3\}$ as mentioned in Section 2.4.1. It follows that the number of syndrome patterns to be stored for error correction equals 2 which can actually be determined by substituting $L=2$ in (2.14). Since $e_{1,i-1}$ is the only error symbol that exists in both syndromes simultaneously,
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the error estimate of this error symbol can be found by checking the coincidence of syndromes $S_{1,i}$ and $S_{1,i-1}$. When they are found to be nonzero and equal to each other, the error estimate of $e_{1,i-1}$ will be set to the syndrome value (see Table 2.4).

Table 2.4 Detection pattern for the error symbol $e_{1,i-1}$ of the single-error correcting NEC. $n \in \{1, 3\}$

<table>
<thead>
<tr>
<th>$S_{1,i}$</th>
<th>$S_{1,i-1}$</th>
<th>$e_{1,i-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n \neq 0$</td>
<td>$n \neq 0$</td>
<td>$n \neq 0$</td>
</tr>
</tbody>
</table>

Decoding of the symbol $r_{1,i-1}$ is done by subtracting the error estimate of $e_{1,i-1}$ from the symbol at the $(i+1)$th time interval. The error estimate will also be subtracted from the syndrome containing the error symbol $e_{1,i-1}$ so as to remove its effect on the syndrome in the following time interval. For example, when the error symbol $e_{1,i-1}$ is not removed from the syndrome $S_{1,i}$ in the $(i+1)$th time interval, this error will appear in the syndrome $S_{1,i-1}$ in the $(i+2)$th time interval. It can be noted from (2.18) that only the syndrome $S_{1,i}$ can possibly be affected by the post-decoding error through the feedback path. In the absence of any new transmission error beginning from the $(i+2)$th time interval, the syndrome $S_{1,i}$ will be equal to zero in the following time slot, i.e., the $(i+3)$th time interval. Eventually, all the syndrome register values would be replaced by zeros at the $(i+4)$th time interval. This clearly explains that this rate-1/2 convolutional code possesses the automatic resynchronization property.

2.4.3 NEC with Double Error Correction Capability for $\pi/4$-shift DQPSK schemes

For a NEC which can be correct up to two errors, one first-order differential detector, one second-order differential detector, and one third-order differential detector are employed [33]. For the $\pi/4$-shift DQPSK system under consideration here, the first-order and the third-order differential detectors correspond to an odd-order differential detector whereas the second-order differential detector corresponds to an even-order differential detector.
The block diagram for the double error correcting NEC system is illustrated in Fig. 2.11. A total of 6 syndromes which can be found by substituting $k$ from 1 to 2 into (2.12) are

\begin{align*}
S_{1,i} &= (e_{1,i} + e_{1,i-1} - e_{2,i}) \mod 4 \\
S_{1,i-1} &= (e_{1,i-1} + e_{1,i-2} - e_{2,i-1}) \mod 4 \\
S_{1,i-2} &= (e_{1,i-2} - e_{2,i-2}) \mod 4 \\
S_{2,i} &= (e_{1,i} + e_{1,i-1} + e_{1,i-2} - e_{3,i}) \mod 4 \\
S_{2,i-1} &= (e_{1,i-1} + e_{1,i-2} - e_{3,i-1}) \mod 4 \\
S_{2,i-2} &= (e_{1,i-2} - e_{3,i-2}) \mod 4.
\end{align*}

In these syndromes, $e_{1,i-3}$ and $e_{1,i-4}$ are assumed to be corrected through the feedback path from the output of the Pattern Detector as shown in Fig. 2.11. The error $e_{1,i-2}$ can be corrected if the total number of symbols in error is less than or equal to two. The number of detection patterns to be detected for error correction can be determined by substituting $L=3$ in (2.14), which in this case is equal to 17.

Figure 2.11 Block diagram of the double error correcting NEC receiver for a $\pi/4$-shift DQPSK system. All inverters (INV) and adders are of mod-4.
Chapter 2. Nonredundant Error Correction of Differentially Encoded PSK systems

Since $e_{1,i-2}$ is the only error symbol that has to be corrected whereas the double-error correcting NEC receiver can correct up to two errors, the detection patterns for the error symbol $e_{1,i-2}$ will include two cases:

i. only the $e_{1,i-2}$ is in error,

ii. the $e_{1,i-2}$ and one of the remaining error symbols in (2.19) are in error.

From the foregoing, the detection patterns were obtained by setting only the $e_{1,i-2}$ of (2.19) to "n" to account for the single error case where "n" $\in \{1,3\}$. To account for the double errors case, the $e_{1,i-2}$ of (2.19) would be set equal to "n" and one of the eight remaining error symbols, i.e., $e_{1,i-1}, e_{2,i-1}, e_{2,i-2}, e_{3,i}, e_{3,i-1}$ and $e_{3,i-2}$, equal to "m" where "m" $\in \{1,3\}$. It is noteworthy that when the "m" in these examples equals zero, it becomes only $e_{1,i-2}$ is in error, i.e., single error. The detection patterns summarized in Table 2.5 is a general summary of all 17 detection patterns which can be obtained by setting "n" to 1 or 3 together with "m" to zero for the single error case or with "m" to 1 or 3 for the double error case.

As an example of how the detection patterns in Table 2.5 are obtained, two cases are considered: (i) both $e_{1,i-2}$ and $e_{1,i}$ are in error and (ii) both $e_{1,i-2}$ and $e_{2,i}$ are in error. It is assumed that the other error symbols are zero, i.e., no error. For the first case, by the substitution of "n" in $e_{1,i-2}$ and "m" in $e_{1,i}$ of (2.19), the first row of Table 2.5 can be obtained. For the second case, the substitution of "n" in $e_{1,i-2}$ and "m" in $e_{2,i}$ of (2.19) result in the third row of Table 2.5.

When a received pattern are found matches to any one of the 17 stored patterns listed in Table 2.5, $e_{1,i-2}$ would be assigned a value of "n" otherwise it will be assigned the value of zero. This means that the total number of error symbols is either more than two, or there is no error. The output symbol $\tilde{r}_{1,i-2}$ is then corrected by subtracting the error estimate $e_{1,i-2}$ from the symbol $r_{1,i-2}$ at the $(i+2)$th time slot. The error estimate $e_{1,i-2}$ will also be subtracted from the syndromes $S_{1,i-1}, S_{2,i}$ and $S_{2,i-1}$ so that $e_{1,i-3}$ and $e_{1,i-4}$ would not appear at $S_{1,i-2}, S_{2,i-1}$ and $S_{2,i-2}$ in the $(i+3)$th and $(i+4)$th time slots.
It can be noted from (2.19) that only the syndromes $S_{1,i-2}, S_{2,i-1}$ and $S_{2,i-2}$ can possibly contain the post-decoding error through the feedback path. Assume that there is no more transmission error after the $(i+2)$th time interval. Beginning at the $(i+5)$th time interval, only zeros would enter the left most stages of the syndrome registers. If an incorrect estimate is fed back to the syndrome registers, the contents of the syndrome registers would immediately be clear to zero. On the other hand, when neither of the syndromes matches the stored detection pattern, error estimate of value zero will be fed back to the syndromes and the contents of the syndrome register remain unaffected. Since only zeros are shifting in, the syndrome registers would eventually be clear to zero. As a result, there will not be any infinitive error propagation.

<table>
<thead>
<tr>
<th>$S_{1,i}$</th>
<th>$S_{1,i-1}$</th>
<th>$S_{1,i-2}$</th>
<th>$S_{2,i}$</th>
<th>$S_{2,i-1}$</th>
<th>$S_{2,i-2}$</th>
<th>$\epsilon_{1,i-2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>$n$</td>
<td>$n$</td>
<td>$m+n$</td>
<td>$n$</td>
<td>$n$</td>
<td>$n$</td>
</tr>
<tr>
<td>$m$</td>
<td>$m+n$</td>
<td>$n$</td>
<td>$m+n$</td>
<td>$m+n$</td>
<td>$n$</td>
<td>$n$</td>
</tr>
<tr>
<td>$-m$</td>
<td>$n$</td>
<td>$n$</td>
<td>$n$</td>
<td>$m+n$</td>
<td>$n$</td>
<td>$n$</td>
</tr>
<tr>
<td>$0$</td>
<td>$n-m$</td>
<td>$n$</td>
<td>$n$</td>
<td>$n$</td>
<td>$n$</td>
<td>$n$</td>
</tr>
<tr>
<td>$0$</td>
<td>$n$</td>
<td>$n-m$</td>
<td>$n$</td>
<td>$n$</td>
<td>$n$</td>
<td>$n$</td>
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<td>$0$</td>
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<td>$n-m$</td>
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<td>$n$</td>
<td>$n$</td>
<td>$n-m$</td>
<td>$n$</td>
<td>$n$</td>
</tr>
</tbody>
</table>

### Table 2.5 Detection patterns for the error symbol $\epsilon_{1,i-2}$ of the second order NEC (see also Fig. 2.11). $n \in \{1,3\}$ and $m \in \{0,1,3\}$.

#### 2.4.4 NEC with Triple Error Correction Capability for $\pi/4$-shift DQPSK

Triple error correction can be achieved by using a total of four detectors: two even-order differential detectors (one second-order and one fourth-order) and two odd-order differential detectors (one first-order and one third-order). The block diagram of this NEC receiver for the $\pi/4$-shift DQPSK system (see Fig. 2.12). Because of its triple error correction capability, the obtained receiver structure is more complicated compared with the previously derived NEC receivers.
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The syndromes to be used for the triple error correction can be found by setting $k$ from 1 to 3 in (2.12). The total of 12 syndromes is given in the following equations

\[
S_{1,i} = (e_{1,i} + e_{1,i-1} - e_{2,i}) \mod 4
\]
\[
S_{1,i-1} = (e_{1,i-1} + e_{1,i-2} - e_{2,i-1}) \mod 4
\]
\[
S_{1,i-2} = (e_{1,i-2} + e_{1,i-3} - e_{2,i-2}) \mod 4
\]
\[
S_{1,i-3} = (e_{1,i-3} - e_{2,i-3}) \mod 4
\]
\[
S_{2,i} = (e_{1,i} + e_{1,i-1} + e_{1,i-2} - e_{3,i}) \mod 4
\]
\[
S_{2,i-1} = (e_{1,i-1} + e_{1,i-2} + e_{1,i-3} - e_{3,i-1}) \mod 4
\]
\[
S_{2,i-2} = (e_{1,i-2} + e_{1,i-3} - e_{3,i-2}) \mod 4
\]
\[
S_{2,i-3} = (e_{1,i-3} - e_{3,i-3}) \mod 4
\]
\[
S_{3,i} = (e_{1,i} + e_{1,i-1} + e_{1,i-2} + e_{1,i-3} - e_{4,i}) \mod 4
\]
\[
S_{3,i-1} = (e_{1,i-1} + e_{1,i-2} + e_{1,i-3} - e_{4,i-1}) \mod 4
\]
\[
S_{3,i-2} = (e_{1,i-2} + e_{1,i-3} - e_{4,i-2}) \mod 4
\]
\[
S_{3,i-3} = (e_{1,i-3} - e_{4,i-3}) \mod 4.
\] (2.20)

From these 12 syndromes, there are a total of 16 distinct error symbols. The error symbol $e_{1,i-3}$ can always be corrected if the total number of symbol errors is less than or equal to three. In these syndromes, the error symbols $e_{1,i-4}$, $e_{1,i-5}$ and $e_{1,i-6}$ are assumed to be corrected through the feedback path from the output of the Pattern Detection as shown in Fig. 2.12. The number of detection patterns to be stored in the Pattern Detector for error correction is 902 distinct detection patterns which can be obtained by putting $L=4$ in (2.14).
Chapter 2. Nonredundant Error Correction of Differentially Encoded PSK systems

Figure 2.12 Block diagram of the triple error correction NEC receiver for a π/4-shift DQPSK system. All inverters (INV) and adders are of mod-4.

Since only the error symbol \( e_{1,i-3} \) is of interest and the triple-error correcting NEC receiver can correct up to three errors, the detection patterns for the error symbol \( e_{1,i-3} \) will include three cases:

i. only the \( e_{1,i-3} \) is in error,

ii. the \( e_{1,i-3} \) and one of the remaining error symbols in (2.20) are in error,

iii. the \( e_{1,i-3} \) and two of the remaining error symbols in (2.20) are in error.

The detection patterns for the \( e_{1,i-3} \) can be found by setting only the \( e_{1,i-3} \) of (2.20) to "n" in order to account for the case of single error where "n" \( \in \{1,3\} \). For the case of double errors, the detection patterns can be found by setting \( e_{1,i-3} \) of (2.20) to "n" and one of the remaining 15 error symbols, i.e., \( e_{1,i}, e_{1,i-1}, e_{1,i-2}, e_{2,i}, e_{2,i-1}, e_{2,i-2}, e_{3,i}, e_{3,i-1}, e_{3,i-2}, e_{3,i-3}, e_{4,i}, e_{4,i-1}, e_{4,i-2} \) and \( e_{4,i-3} \), to "p" where "p" \( \in \{1,3\} \). Finally, for the case of triple errors, the detection patterns can be found by setting the \( e_{1,i-3} \) of (2.20) to "n", and two of the remaining 15 error symbols, i.e.,
Chapter 2. Nonredundant Error Correction of Differentially Encoded PSK systems

\[ e_{1,i}, e_{1,i-1}, e_{1,i-2}, e_{2,i}, e_{2,i-1}, e_{2,i-2}, e_{2,i-3}, e_{3,i}, e_{3,i-1}, e_{3,i-2}, e_{3,i-3}, e_{4,i}, e_{4,i-1}, e_{4,i-2} \text{ and } e_{4,i-3}, \]
to "p" and "q" where \( p, q \in \{1, 3\} \). A general summary of these detection patterns is given in Table 2.6. By setting "n" to 1 or 3, together with "t" and "s" to zero in order to account for the single error case; "t" or "s" to zero for the double error case; "t" and "s" to 1 or 3 for the triple error case. A total of 902 detection patterns can be obtained.

As an illustration of how the values in the table were obtained, two examples are considered:

(i) all three \( e_{1,i-3}, e_{1,i}, \text{ and } e_{2,i} \) are in error and (ii) all three \( e_{1,i-3}, e_{1,i}, \text{ and } e_{2,i-2} \) are in error. For the first example, the substitution of "n", "t" and "s" in \( e_{1,i-3}, e_{1,i}, \text{ and } e_{2,i} \) of (2.20), respectively, give rise to the third row of Table 2.6. For the second example, by the substitution of "n", "t" and "s" in \( e_{1,i-3}, e_{1,i}, \text{ and } e_{2,i-2} \) of (2.20), respectively, the fifth row of Table 2.6 is obtained.

If any of the 902 detection patterns are detected, the error symbol \( e_{1,i-3} \) is assigned the value of "n" otherwise it is assigned the value of zero. The output symbol \( \tilde{r}_{1,i-3} \) is corrected by subtracting the error estimate from the symbol \( r_{1,i-3} \) at the \( (i+3) \)th time slot. The error symbols \( e_{1,i-4}, e_{1,i-5} \) and \( e_{1,i-6} \) can be removed by subtracting the error estimate \( e_{1,i-3} \) from the syndromes \( S_{1,i-2}, S_{2,i-1}, S_{2,i-2}, S_{3,i}, S_{3,i-1} \text{ and } S_{3,i-2} \) so that the error symbols would not appear at the \( (i+4) \)th, \( (i+5) \)th and \( (i+6) \)th time slots.

It can be noted from (2.19) that only the syndromes \( S_{1,i-3}, S_{2,i-2}, S_{2,i-3}, S_{3,i-1}, S_{3,i-2} \) and \( S_{3,i-3} \) can possibly contain the post decoding error through the feedback path. Assume now there are no more transmission errors after the \( (i+3) \)th time interval. Beginning at the \( (i+7) \)th time interval, only zeros would enter the left most stages of the syndrome registers. Regardless of the contents in the syndrome registers at the \( (i+7) \)th time interval, they will eventually be clear to zero. For example, when any combination of the contents of the syndrome registers matches any of the detection patterns, the error estimate will be fed back to the syndromes, thereby clearing all the syndromes to zero. On the contrary, when neither of the syndrome patterns matches the
detection patterns, the error estimate would be set to zero, thus leaving all the syndrome contents unchanged. Since only zeros are shifting in, the syndrome registers would eventually be clear to zero. As a result, there will not be any infinitive error propagation.
Table 2.6 Detection Pattern for the error symbol $e_{1,i-3}$ of the triple error correcting NEC system. $n \in \{1, 3\}$, $l, s \in \{0, 1, 3\}.$

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<thead>
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<th>$S_{1,i-2}$</th>
<th>$S_{1,i-3}$</th>
<th>$S_{2,i}$</th>
<th>$S_{2,i-1}$</th>
<th>$S_{2,i-2}$</th>
<th>$S_{2,i-3}$</th>
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<th>$S_{3,i-1}$</th>
<th>$S_{3,i-2}$</th>
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Note: The table entries are placeholders for the actual values that would be filled in based on the detection pattern described in the text.
Table 2.6 (Cont.) Detection Pattern for the error symbol $e_{1,i-3}$ of the triple error correcting NEC system. $n \in \{1, 3\}$, $l, s \in \{0, 1, 3\}$.

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Table 2.6 (Cont.) Detection Pattern for the error symbol $e_{1,i-3}$ of the triple error correcting NEC system. $n \in \{1, 3\}$, $i, s \in \{0, 1, 3\}$.

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Table 2.6 (Cont.) Detection Pattern for the error symbol $e_{1, i-3}$
of the triple error correcting NEC system. $n \in \{1, 3\}$, $l, s \in \{0, 1, 3\}$.

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Chapter 2. Nonredundant Error Correction of Differentially Encoded PSK systems

2.5 Summary

In this chapter, the application of the NEC technique to differential detected $\pi/4$-shift DQPSK systems had been discussed. A modified NEC receiver structure based on a newly proposed transformation concept particularly suited for the $\pi/4$-shift DQPSK modulation format was proposed and analyzed. The NEC receivers with single, double, and triple correcting capability were considered. Finally, various properties of the NEC technique, such as the error propagation effect, decoding depth and constraint length had been discussed.
Chapter 3
Performance Analysis and Evaluation of $\pi/4$-shift DQPSK Systems
Employing NEC Technique Operated in a CCI-AWGN Environment*

3.1 Introduction

Co-channel Interference (CCI) refers to the degradation caused by an interfering waveform appearing within the signal bandwidth. These interfering waveforms bear the same carrier frequency as that of the desired signal and they could either be an unmodulated carrier or a modulated carrier of the same modulation type. The CCI limits the frequency reuse in different geographic locations and plays a dominant role in limiting the channel capacity and performance of mobile cellular communication systems. With the expected increase in congestion of frequency spectrum, the degradation caused by CCI is likely to increase in the future. The $\pi/4$-shift DQPSK signaling scheme has been adopted as the transmission standard for the new North American and Japanese all digital cellular radio [23]. It is thus important to devise ways to improve the error rate performance of the $\pi/4$-shift DQPSK systems which operated in CCI environment. Well-known techniques which are being employed to improve the performance of mobile communication systems under CCI include diversity receivers [43] and tilted antennas [44]. To the best of our knowledge, little attention has been given to the use of error control coding technique for combating CCI possibly due to the need of redundant bits. The transmission of redundant bits would result in an increase of signal bandwidth, hence a reduction of the spectral efficiency. The NEC technique is a very attractive choice since it does not require any bandwidth or signal constellation expansion as do other coding schemes.

This chapter deals with the analysis and evaluation of the performance of $\pi/4$-shift DQPSK systems employing NEC receivers operated in a combined CCI and AWGN environment. Section 3.2 presents a description of the model. In Section 3.3, the theoretical error rate performance

* The research material presented in this chapter has been accepted for publication in [45].
formulas of the NEC receivers in a CCI-AWGN environment are derived. In particular, the pdf of a phase error due to CCI-AWGN derived by Goldman [46] will be adopted. Section 3.4 describe the computer simulation model employed. Numerical results complemented with results obtained by computer simulation are in Section 3.5. Interpretations of these results are in Section 3.6. A summary of this chapter is given in Section 3.7.

3.2 System Model Description and Analysis

A general block diagram of the digital communication system under consideration illustrated in Fig. 3.1 consists of a π/4-shift DQPSK transmitter with an output \( x(t) \), additive cochannel interference(s), \( i(t) \), an AWGN source, \( n(t) \), a bandpass receiver filter (BPF) and a receiver with a structure based on NEC techniques. The \( n(t) \) and the \( i(t) \) are added at the input of the receiver, followed by passing through the bandpass receiver filter. The signal \( g_r(t) \) at the output of the receiver filter can be expressed as

\[
g_r(t) = x_r(t) + i_r(t) + n_r(t)
\]  

(3.1)

where \( x_r(t) \), \( i_r(t) \) and \( n_r(t) \) are the filtered bandpass signals of \( x(t) \), \( i(t) \), and \( n(t) \) respectively.
The block diagram of the transmitter model employed is illustrated in Fig. 3.2. The 2-bit information symbol, $\overline{a_i^2} = [a_i^1, a_i^2]$ ($a_i^1, a_i^2$ are independent and equiprobable bits), is transformed by the Signal Mapper (SM) into a differential phase $\Delta \theta_i$ which takes equiprobable values from the alphabet $\{\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\}$. The output of the Differential Encoder (DE) is the phase $\theta_i$ which takes values from the alphabet $\{0, \pi/4, 2\pi/4, ..., 7\pi/4\}$ and is given by

$$\theta_i = \theta_{i-1} \oplus \Delta \theta_i$$  \hspace{1cm} (3.2)

where $\oplus$ denotes mod-$2\pi$ addition. It can be noted that the phases $\theta_i$ represent the $\pi/4$-shift DQPSK signal illustrated in Fig. 1.3.

The signal at the output of the transmitter filter, $H_T(f)$, is given by

$$s(t) = \sum_{i=-\infty}^{\infty} e^{j\theta_i} h_T(t - iT)$$  \hspace{1cm} (3.3)

where $h_T(t)$ is the inverse Fourier transform of $H_T(f)$ and $T$ is the symbol duration. It will be assumed that $H_T(f)$ is the well-known square-root raised-cosine filter [31]. After the complex modulator, the transmitted $\pi/4$-shift DQPSK signal can be expressed as

$$x(t) = \text{Re}[s(t) e^{j2\pi f_c t}]$$  \hspace{1cm} (3.4)

where $f_c$ is the carrier frequency and $\text{Re}[\cdot]$ represents the real part of $[\cdot]$.

Finally, with the usual assumption that the transfer function of the receiver BPF is symmetric around $f_c$ and has square-root raised-cosine characteristics, $x_r(t)$ will be free of Inter-Symbol-Interference (ISI) at the ideal sampling instant [31].
The noise \( n(t) \) is presumed to originate thermally and is modelled as zero mean white Gaussian noise with double-sided power spectral density of \( \frac{N_0}{2} \). The noise after passing through the bandpass receiver filter can be written as

\[
n_r(t) = n_I(t) \cos(2\pi f_c t) - n_Q(t) \sin(2\pi f_c t)
\]

(3.5)

where \( n_I(t) \) and \( n_Q(t) \) are the in-phase and the quadrature phase low pass equivalents of \( n(t) \). Both \( n_I(t) \) and \( n_Q(t) \) are low-pass, independent, zero mean Gaussian random processes and have power equal to \( \sigma^2 = \frac{N_0}{2} \).

The CCI is modelled as a summation of several \( \pi/4 \)-shift DQPSK signals originating from other independent transmitters. At the transmitter end of an interferer, this interference can be expressed as

\[
i_k(t) = i_\Phi(t) \cos(2\pi f_c t + \phi_\Phi(t))
\]

(3.6)

where \( t \in [(k-1)T, kT] \). The phase angle \( \phi_\Phi = \phi_\Phi^{-1} + (\pi q)/4 \) for differential encoded interference transmitter, or \( \phi_\Phi = (\pi q)/4 \) for absolutely encoded interferer transmitter, where \( q \in \{1, 3, 5, 7\} \). Let \( \Phi_1(t) \) be the random process defined by \( \Phi_1(t) = \phi_\Phi(t) \). The transmitted interference is given by

\[
i_1(t) = R_i(t) \cos[2\pi f_c t + \Phi_1(t)].
\]

(3.7)

Since this interference is independently transmitted at any instant of time, the phase of the interference observed by the receiver at the sampling instant may take on any possible values between \([-\pi, \pi)\). The received interference can be represented as

\[
i_1(t) = \hat{R}_i(t) \cos[2\pi f_c t + \Gamma_1(t)]
\]

(3.8)

where \( R_i(t) \) and \( \Gamma_1(t) \) are stationary processes, with \( \Gamma_1(t) \) uniformly distributed between \([-\pi, \pi] \). These random processes constitute a circularly symmetric pair. A circular symmetric pair
(X, Y) is a pair of real random variables if and only if the joint probability density function of X and Y, \( f_{XY} \), exists and satisfies \( f_{XY} = g\left(\sqrt{x^2 + y^2}\right) \), for all x, y and for some function g. In the polar coordinate representation, where \( X = P \cos \Upsilon, Y = P \sin \Upsilon \) (\( P \geq 0, \Upsilon \in [-\pi, \pi] \)), this definition is equivalent to \( P \) and \( \Upsilon \) being independent and \( \Upsilon \) uniformly distributed on \([-\pi, \pi)\).

For the case of \( M \) independent cosinusoids interferers, the sum, \( i(t) \), of these interferers can be mathematically expressed as

\[
i(t) = \sum_{n=1}^{M} \hat{R}_n(t) \cos [2\pi f_c t + \Gamma_n(t)].
\]  

(3.9)

Following the fact that the sum of two independently circularly symmetric pair is circularly symmetric, the sum of the \( K \) interferences has circular symmetry [47]. More generally, \( i(t) \) can be expressed as

\[
i(t) = R(t) \cos [2\pi f_c t + \Gamma(t)]
\]  

(3.10)

where \( R(t) \) and \( \Gamma(t) \) are stationary processes.

The received signal, \( g_r(t) \), which is the sum of the transmitted signal, the interferer(s) and the noise can be written as

\[
g_r(t) = X(t) \cos [2\pi f_c t + Z(t)].
\]  

(3.11)

where \( X(t) \) and \( Z(t) \) denote the envelopes and the phase of \( g_r(t) \) respectively.

In this thesis, the interference will be modelled as the sum of \( M \) cosinusoids with constant amplitudes \( (a_1, a_2, \ldots, a_M) \) and independent phases \( (\phi_1, \phi_2, \ldots, \phi_M) \) which uniformly distributed in \([-\pi, \pi)\). In particular, the case when the total interference power is equally distributed among the interferers, i.e., \( a_1 = a_2 = \cdots = a_M \) is considered. As noted in [46, 48, 49], this configuration results in the highest probability of error at a given C/I ratio.
Chapter 3. Performance of π/4-shift DQPSK systems with NEC in CCI-AWGN Environment

Fig. 3.3 illustrates in vector form how CCI and noise corrupting the desired signal result in a phase error $\alpha$. The angle (or phase error) between the transmitted and the received signals is independent of which symbol was transmitted. With the reasonable assumptions that the noise and the interference components are independent from each other as well as that the samples of the components of noise and interference are independent from symbol to symbol \[46, 48, 50-52\], the pdf of $\alpha$, $f_A(\alpha)$ is given from \[46\] as

$$f_A(\alpha) = \frac{\exp(-\gamma)}{\pi} \sum_{n=0}^{\infty} \left[ \frac{\gamma^n \mu_{2n}}{(2n)!} \sum_{k=0}^{n} \binom{n}{k} H_{2n-2k} \left( \gamma^\frac{1}{2} \sin \alpha \right) H_{2k-2} \left( \gamma^\frac{1}{2} \cos \alpha \right) \right] \tag{3.12}$$

where $\gamma$ is the carrier-to-noise (C/N) ratio in linear scale and $\mu_{2n}$ is the $2n$ moment of $R \sin \Phi$ which in general is given by

$$\mu_{2n} = E[R^{2n} \sin^{2n} \Phi] = E[R^{2n}] E[\sin^{2n} \Phi] = E[R^{2n}] \frac{(2n)!}{2^{2n} (n!)^2} \tag{3.13}$$

where $E[R^{2n}]$ is the moment. The moments $E[R_j^{(2n)}]$ are given by \[47\]

$$E[R_j^{(2n)}] = a^{2n} C_j^{n-1} (1, 1, \cdots, 1)^T \tag{3.14}$$
where \( j \) is the number of interferers, \( T \) denotes the transpose of the matrix, and the \( C \) is an \((n+1) \times (n+1)\) matrix whose \((k,l)\)th element equals

\[
\binom{k-1}{l-1}^2
\]  

for \( k \geq l \) and is zero otherwise. The \( H \) given in (3.12) is the Hermite polynomial which is defined for \( n \geq 0 \) in [53] as

\[
H_n(x) = \sum_{j=0}^{[n/2]} (-1)^j \frac{n!}{r!(n-2j)!} (2x)^{n-2j}
\]  

where

\[
[n/2] = \begin{cases} 
\frac{n}{2}, & \text{if } n \text{ is even} \\
\frac{n-1}{2}, & \text{if } n \text{ is odd}.
\end{cases}
\]  

For \( n = -2 \), the Hermite polynomial is given by [46]

\[
H_{-2}(x) = \frac{1}{2} + \frac{\sqrt{\pi}}{2} x \exp(x^2) \text{erfc}(-x).
\]  

The \( C/I \) [in dB] is

\[
C/I = 10 \log_{10} \left( \frac{1}{Ma^2} \right)
\]  

and the \( C/N \) [in dB] is

\[
C/N = 10 \log_{10} \left( \frac{1}{2\sigma^2} \right)
\]  

As \( M \) increases, the statistics of the CCI will become Gaussian. In the limiting case in which \( M \) approaches infinity, the CCI will become a Gaussian random process with variance \( \sigma_I^2 = 0.5 \times 10^{-\frac{C/I_{\text{dB}}}{6}} \) by the Central Limit Theorem. In this case, the \( C/N \) [in dB] can be expressed as

\[
C/N = 10 \log_{10} \frac{1}{2(\sigma_N^2 + \sigma_I^2)}.
\]
3.3 Theoretical Analysis of the Performance of NEC Receivers in the CCI-AWGN Environment

It is worth noticing that the approach of evaluating the performance of the $\pi/4$-shift DQPSK system employing the NEC receivers in CCI-AWGN environment is similar to that in AWGN environment described in [33, 35, 37]. In these references, the pdf of a phase error due to AWGN [50] was used, whereas, the pdf due to CCI-AWGN [46] was adopted here. In other words, the only difference is the pdf used in evaluating the error probability.

In Chapter 2, it has been shown that the NEC receiver gives rise to a systematic rate-$1/L$ convolutional code with $(L-1)$ error correction capability. The error rate performance of the NEC receiver can be estimated by using input error patterns containing more errors than the error correction capability of the decoder. As each error pattern occurs with a different probability, a good approximation can be obtained by using those pattern with a higher occurrence probability which will be defined shortly.

As first noted in [50], in differential detection, each decision is based on a phase comparison between the present signal and the previous adjacent signal. The received signal over a symbol duration acts as a "signal" as well as a "reference". This concept can be extended to higher order differential detectors following which the relationship between them for cases of one up to four differential detectors can be illustrated in Table 3.1.

Table 3.1(a) shows that the signal arrived at the $i$th time interval acts as $\text{SIG}_{D_{i-1}}, \text{SIG}_{P_{i-1}}, \text{SIG}_{P_{i-2}}, \text{SIG}_{P_{i-3}}, \text{REF}_{D_{i}}, \text{REF}_{P_{i+1}}, \text{REF}_{P_{i+2}}$ and $\text{REF}_{P_{i+3}}$ simultaneously. Since the transmitted information is carried in the phase difference between two signals, the labels "SIG" and "REF" of the signal identity can be interchanged. Because each signal is involved in detecting two symbols in each detector, the signal $s(i)$ becomes the "reference" of the set of the $2L$ received signals, i.e., signals $s(i \pm j)$ with $j \in \{1, 2, \ldots, L\}$, for $L$ detectors. Based upon this observation, one incorrect reference at the $i$th time interval may possibly lead to $2L$ specific errors, namely
Table 3.1 Error symbols with relatively high probability of occurrence. (a) original, (b) after interchange of the phases "REF" and "SIG".

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<th>i−1</th>
<th>i</th>
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</table>
\( e_{j,i} \) and \( e_{j,i+j} \) where \( j \in \{1, 2, \cdots, L\} \). The error patterns making up of these error symbols are referred to as input error patterns with higher occurrence probability.

Since the decoding depth of this systematic rate-1/L convolutional code equals to \((L-1)\), the decision error(s) due to an incorrect reference at the \(i\)th time interval will not be corrected until the \((i+L-1)\)th time interval. It is noteworthy that, among the \(2L\) specific errors, the two error symbols which would be corrected are the ones coming from the first-order differential detector, i.e., \(e_{1,i}\) and \(e_{1,i+1}\) of the \(i\)th time interval. The \(2L\) specific errors, \(\{e_{j,i}, e_{j,i+j}\}\), of high occurrence probability observed at the \(i\)th time interval would appear as \(\{e_{j,i-(L-1)}, e_{j,i+j-(L-1)}\}\) at the decoding instant, i.e., the \((i+L-1)\)th time interval, of the \(e_{1,i}\) of the \(i\)th time interval. Finally, at the \((i+L)\)th time interval, the decoding instant of the \(e_{1,i+1}\) of the \(i\)th time interval, they would appear as \(\{e_{j,i-(L-1)-1}, e_{j,i+j-(L-1)-1}\}\).

Table 3.1(b) shows that for the single error correcting NEC receiver, one incorrect “reference” at \(i\)th time interval results in four potential errors: \(e_{1,i}, e_{1,i+1}, e_{2,i}\) and \(e_{2,i+2}\) [35] which are referred to as errors with high occurrence probability for the single-error correcting NEC receiver. With the use of similar arguments for the double error correcting NEC receivers, one incorrect “reference” at the \(i\)th time interval will lead to the following six potential errors: \(e_{1,i}, e_{1,i+1}, e_{2,i}, e_{2,i+2}, e_{3,i}\) and \(e_{3,i+3}\) (see [37] and Table 3.1) which are referred to as errors with high occurrence probability for the double-error correcting NEC receiver. Finally, Table 3.1 shows that by observing the relationship between the “signal” and the “reference” for the triple error correcting NEC receiver, the incorrect “reference” at the \(i\)th time interval results in eight potential errors: \(e_{1,i}, e_{1,i+1}, e_{2,i}, e_{2,i+2}, e_{3,i}, e_{3,i+3}, e_{4,i}\) and \(e_{4,i+4}\) which are referred to as errors with high occurrence probability for the triple error correcting NEC receivers.

The number of output errors resulted from the input error patterns with higher occurrence probability is summarized in Tables 3.2 - 3.4. As mentioned earlier, the input error patterns with higher occurrence probability referred to the patterns which contained the aforementioned error...
symbols, the number of error symbols in the these input error patterns is therefore limited to $2L$. In other words, the number of errors at the input can be either $1, 2, \ldots, 2L$. For example, for the single-error correcting NEC receiver, because there are four error symbols with high occurrence probabilities, the number of errors at the input can be either $1, 2, 3$ or $4$ which give rise to the numbers in the first column of Table 3.2. By the same token, because of the six error symbols with high occurrence probabilities, the number of errors at the input of the double-error correcting NEC receiver can be either $1, 2, \ldots, 6$ (see the first column of Table 3.3). Finally, the number of errors at the input of the triple-error correcting NEC receiver can be either $1, 2, \ldots, 8$ (see the first column of Table 3.4).

Since there are $2L$ error symbols with high occurrence probabilities, the number of possible combinations for $j$ errors is simply $C(2L, j)$ where $C$ denotes the combination and is defined as

$$C(n, k) = \frac{n!}{k!(n-k)!}.$$  \hspace{1cm} (3.22)

As each of the error symbol can take two values, namely 1 or 3, the total number of error patterns due to the combinations of $j$ errors would be given by

$$C(2L, j) \cdot 2^j.$$  \hspace{1cm} (3.23)

For example, the number of patterns for the two errors case in the single-error correcting NEC receiver is $C(4, 2) \cdot 2^2 = 24$ (see the sum of rows 2 - 4 of the third column of Table 3.4). Similarly, for the two errors case in the double-error correcting NEC receiver is $C(6, 2) \cdot 2^2 = 60$. On the other hand, for the two errors case in the triple-error correcting NEC receiver is $C(8, 2) \cdot 2^2 = 112$.

Following the fact that the signal of one time interval takes part in determining the symbols for each differential detector and since only the information symbol from the first-order differential detector are of interest, the number of remaining output errors at the NEC receiver output are 0, 1, or 2 only (see the second column of Table 3.2 - 3.4).
As an illustration of how the number of remaining output errors as a function of the input error patterns was determined, the single error correcting NEC receiver is considered. Since it has single error correction capability, the number of remaining output errors for the single error case is zero as indicated in the first row of Table 3.2. The cases of the double-, triple-, and quadruple cases require more explanations, since the number of remaining errors which can be 0, 1, or 2 depends on how the error symbols appeared. An extension of Table 3.2 which includes the number of remaining output errors for each input error pattern is illustrated in Table 3.5. For example, for the case of two input errors (see rows 2–4 of Table 3.2 and rows 1–6 of Table 3.5), the number of patterns corresponds to zero remaining output errors is 4 + 4 + 2 = 10, i.e., the sum of the number of patterns given in rows 1–2 and the upper portion of the third row of Table 3.5 (see also second row of Table 3.2).

Table 3.5 can be obtained as follows. For example, if both $e_{2,i}, e_{2,i+2}$ of the $i$th time interval are in error (see the first row of Table 3.5), they would appear as $e_{2,i-1}, e_{2,i+1}$ in the syndromes at the decoding instant, i.e., the $(i+1)$th time interval. It can be recalled from (2.18) that the syndromes are

$$S_{1,i} = (e_{1,i} + e_{1,i-1} - e_{2,i}) \mod 4$$
$$S_{1,i-1} = (e_{1,i-1} - e_{2,i-1}) \mod 4.$$  \hspace{1cm} (3.24)

Substitution of "n" in $e_{2,i-1}$ of (3.24), the syndromes $\{S_{1,i}, S_{1,i-1}\} = \{0, -n\}$ are obtained. Since these syndromes are distinct from the stored detection pattern in Table 2.4, the error estimate of $e_{1,i-1}$ would be set to zero which is indeed the case. The number of remaining output error at the $(i+1)$th time interval will therefore be equal to zero. In the following time interval, i.e., $(i+2)$th time interval, the two error symbols would appear as $e_{2,i-2}, e_{2,i}$ in the syndromes, respectively.

It can be noted that the error symbol $e_{2,i-2}$ is not in the syndromes since it has been shifted out from the syndrome registers. The substitution of "n" in $e_{2,i}$ of (3.24) gives the syndromes $\{S_{1,i}, S_{1,i-1}\} = \{-n, 0\}$. As before, these syndromes are not matched to the stored detection patterns in Table 2.4. As a result, at this $(i+2)$th time interval, the error estimate of $e_{1,i-1}$ would
again be set to zero which is indeed the case. The total number of remaining output errors will therefore be equal to zero as indicated in the first row of Table 3.5.

As another example, the case of both $e_{1,i+1}$ and $e_{2,i}$ of the $i$th time interval in error is considered (see the third row of Table 3.5). At the decoding instant, i.e., the $(i+1)$th time interval, these two symbols would appear as $e_{1,i}$ and $e_{2,i-1}$. The substitution of "$n$" and "$m$" in $e_{1,i}$ and $e_{2,i-1}$ of (3.24) results in the syndromes $\{S_{1,i}, S_{1,i-1}\} = \{n, -m\}$ where $n, m \in \{1, 3\}$. In this case, when "$m$" equals "$-n$", the error estimate of $e_{1,i-1}$, which should be equal to zero, would be set to "$n". This incorrect error estimate would be fed back to the syndrome $S_{1,i}$ and appear as a post-decoding error in the syndrome $S_{1,i-1}$ in the following time interval. In addition, this incorrect estimate will be subtracted from the output symbol so that one remaining output error, i.e., decoding error, results. In the following time interval, the $e_{1,i}$ and $e_{2,i-1}$ of the previous time interval appear as $e_{1,i-1}$ and $e_{2,i-2}$. By the substitution of "$n$" in $e_{1,i-1}$ of (3.24), together with that post-decoding error, the syndromes $\{S_{1,i}, S_{1,i-1}\} = \{n, n - n\} = \{n, 0\}$ are obtained. Since these syndromes do not match with the stored detection patterns in Table 2.4, the error estimate of $e_{1,i-1}$ which should have been equal to "$n" would be assigned a value of zero. From the light of this observation, when $e_{1,i+1}$ and $e_{2,i}$ are in error at the $i$th time interval with "$m" equals "$-n"", two remaining output errors result (see the lower portion of the third row of Table 3.5). On the other hand, if "$m" is equal to "$n", the syndromes from (3.24) at the $(i+1)$th time interval would be equal to $\{S_{1,i}, S_{1,i-1}\} = \{n, -n\}$. Since they are distinct from the stored detection patterns in Table 2.4, the error estimate of $e_{1,i-1}$ would be set to zero. Since the $e_{1,i-1}$ is, in fact, equal to zero and no decoding error would result. In the next time interval, i.e., the $(i+2)$th time interval, the error symbols $e_{1,i}$ and $e_{2,i-1}$ of the previous time interval would appear as $e_{1,i-1}$ and $e_{2,i-2}$. Substituting "$n" in $e_{1,i-1}$ of (3.24) gives the syndromes $\{S_{1,i}, S_{1,i-1}\} = \{n, n\}$. Compared with the stored detection patterns in Table 2.4, the error estimate of $e_{1,i-1}$ would be set to "$n" which is indeed the case. Consequently, when $e_{1,i+1}$ and
$e_{2,i}$ are in error at the $i$th time interval with "$m$" equals "$n$", no output error would result (see the upper portion of the third row of Table 3.5).

As a summary, the procedures in finding the number of remaining errors at the NEC output can be summarized as follows.

1. Determine the detection patterns by following the approach described in Chapter 2.
2. Test each error pattern which contains all possible combinations of the $2L$ error symbols having high occurrence probabilities.

Since different error patterns (even with the same number of errors) can give different number of remaining errors, it is necessary to consider patterns with different error combinations in order to determine the number of remaining output errors. Programs were written to count the number of remaining errors at the NEC receiver output for these error patterns. An example of the program listings used to count the number of remaining output errors for input error patterns containing three input errors in the double error correcting NEC receiver can be found in Appendix B.

Although an analytical proof of the relationship between Tables 3.2 - 3.4 is beyond the scope of this thesis, the following observations explain that there is no direct extension from the number of remaining output errors from one table to the other. First, every NEC receiver has a distinct set of detection patterns because of the different error symbols involved in the syndrome equations (see Tables 2.4, 2.5 and 2.6). Since one signal at one time interval takes part in the detection two symbols in each differential detector, there will be $2L$ input error symbols with high occurrence probability: four, six, and eight input error symbols with high occurrence probability for the single-, double- and the triple-error correcting NEC receivers, respectively. Furthermore, as can be noted from the previous illustrations of how the output remaining errors are obtained, the number of output errors is not only dependent on the combination of the values that the error symbols have, but also on the location of the error symbols in the input error patterns. As a result, when counting the number of remaining errors due to the different combinations of the
error patterns for each NEC receiver, a different set of detection patterns has to be employed. Thus, it can be deduced that there is no direct extension from Table 3.2 to Table 3.3 or to Table 3.4.

Table 3.2 Remaining output errors as a function of input errors with relatively high probability of single error correcting NEC receiver.

<table>
<thead>
<tr>
<th>Number of Errors (Input)</th>
<th>Number of Remaining Errors with NEC (Output)</th>
<th>Number of Patterns</th>
<th>Occurrence Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>8</td>
<td>$P'_1$</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>10</td>
<td>$P'_2$</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>8</td>
<td>$P'_3$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>8</td>
<td>$P'_4$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>16</td>
<td>$P'_5$</td>
</tr>
</tbody>
</table>

Table 3.3 Remaining output errors as a function of input errors with relatively high probability of double error correcting NEC receiver.

<table>
<thead>
<tr>
<th>Number of Errors (Input)</th>
<th>Number of Remaining Errors with NEC (Output)</th>
<th>Number of Patterns</th>
<th>Occurrence Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>12</td>
<td>$P'_1$</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>60</td>
<td>$P'_2$</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>96</td>
<td>$P'_3$</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>38</td>
<td>$P'_4$</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>66</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>136</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>52</td>
<td>$P'_5$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>140</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>64</td>
<td>$P'_6$</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 3.4 Remaining output errors as a function of input errors with relatively high probability of a triple error correcting NEC receiver.

<table>
<thead>
<tr>
<th>Number of Errors (Input)</th>
<th>Number of Remaining Errors with NEC (Output)</th>
<th>Number of Patterns</th>
<th>Occurrence Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>16</td>
<td>$P_1'$</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>112</td>
<td>$P_2'$</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>448</td>
<td>$P_3'$</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>738</td>
<td>$P_4'$</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>116</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>250</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>516</td>
<td>$P_5'$</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>442</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>834</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>242</td>
<td>$P_6'$</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>494</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1056</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>236</td>
<td>$P_7'$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>788</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>256</td>
<td>$P_8'$</td>
</tr>
</tbody>
</table>
Table 3.5 Output remaining errors as a function of input error patterns with relatively high probability of a single error correcting NEC receiver at the \( i \)th time interval. "x" indicates an error of values 1 or 3.

<table>
<thead>
<tr>
<th>Number of Errors (Input)</th>
<th>Input Error Patterns</th>
<th>Condition</th>
<th>Number of Patterns</th>
<th>Remaining Errors (Output)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( e_{1,i} ) ( e_{1,i+1} ) ( e_{2,i} ) ( e_{2,i+2} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0 0 x x</td>
<td>N/A</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>x 0 0 x</td>
<td>N/A</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0 x x* 0</td>
<td>( x = x^* ) ( x = -x^* ) 2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 x 0 x</td>
<td>N/A</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>x 0 x 0</td>
<td>N/A</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>x x 0 0</td>
<td>N/A</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>x x x 0</td>
<td>N/A</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>x x 0 x</td>
<td>N/A</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>0 x* x** x</td>
<td>( x^* = x^{<strong>} ) ( x^* = -x^{</strong>} ) 4</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 x* x** x</td>
<td>( x^* = x^{<strong>} ) ( x^* = -x^{</strong>} ) 4</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>x* 0 x x**</td>
<td>( x^* = x^{<strong>} ) ( x^* = -x^{</strong>} ) 4</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>x x x x</td>
<td>N/A</td>
<td>16</td>
<td>2</td>
</tr>
</tbody>
</table>

With the number of remaining output errors and the occurrence probabilities in Table 3.2, the output error probability of the NEC receiver with single error correction capability can be given by [33] as

\[
P_s = 20P_2' + 56P_3' + 32P_4'. \tag{3.25}
\]

Similarly, the output error probability of the NEC receiver with double error correction capability can be derived from Table 3.3 and is given by

\[
P_D = 104P_3' + 338P_4' + 332P_5' + 128P_6'. \tag{3.26}
\]
Finally, the output error probability of the NEC receiver with triple error correction capability can be derived from Table 3.4 and is given by

\[ P_T = 616P_4' + 2110P_5' + 2606P_6' + 1812P_7' + 512P_8'. \] (3.27)

Under the condition that the phase error of the reference signal is \( \alpha_2 \), the conditional symbol error probability \( P(E/\alpha_2) \) of the \( \pi/4 \)-shift DQPSK system for incorrect decision is given by

\[
P(E/\alpha_2) = 1 - P(C/\alpha_2) = 1 - \int_{\alpha_2 - \frac{\pi}{4}}^{\alpha_2 + \frac{\pi}{4}} f_A(\alpha_1) \, d\alpha_1 \] (3.28)

where \( P(C/\alpha_2) \) is the conditional symbol error probability for correct decision.

Furthermore, with the assumption that the signal is independently corrupted, the symbol error probability of two errors conditioned on the phase error of the reference signal is

\[
P(E_i, E_j/\alpha_2) = P(E_i/\alpha_2)P(E_j/\alpha_2, E_i) = P(E_i/\alpha_2)P(E_j/\alpha_2) = [P(E_i/\alpha_2)]^2. \] (3.29)

In general, the symbol error probability of \( t \) errors can be expressed as

\[
P(E_{i_1}, E_{i_2}, \ldots, E_{i_t}/\alpha_2) = [P(E_i/\alpha_2)]^t. \] (3.30)

Consequently, the occurrence probability of \( t \)-fold errors is

\[
P_t = \int_{-\pi}^{\pi} [P(E_{i_1}, E_{i_2}, \ldots, E_{i_t}/\alpha_2)] f_A(\alpha_2) \, d\alpha_2 = \int_{-\pi}^{\pi} [P(E_i/\alpha_2)]^t f_A(\alpha_2) \, d\alpha_2 \] (3.31)
Following the fact that the values of the error symbols are either 1 or 3 (see the DQPSK transformation concept in Chapter 2), there are \(2^{L+j}\) patterns for every set of error symbols. By defining the occurrence probability of having errors in \((L+j)\) specific received signals among the \(2L\) received signals as \(P'_{L+j}\), the occurrence probability of a pattern with \((L+j)\) specific errors \(P'_{L+j}\) is given by

\[
P'_{L+j} = \frac{1}{2^{L+j}} P''_{L+j}.
\] (3.32)

The occurrence probability of \((L+j)\)-fold errors \(P_{L+j}\) is the sum of the occurrence probability of having errors in a specific set of \((L+j)\) received signals among the \(2L\) received signals and the occurrence probabilities of having additional errors in the remaining \([2L-(L+j)] = (L-j)\) received signals. In general, the occurrence probability of \((L+j)\)-fold errors, \(P_{L+j}\), is

\[
P_{L+j} = P''_{L+j} + \binom{L-j}{1} P''_{L+j+1} + \cdots + \binom{L-j}{k-j} P''_{L+k} + \cdots + \binom{L-j}{L-j} P''_{2L}.
\] (3.33)

By solving these \((L+1)\) linear equations for \(P''_{L+j}\), \(j = 0, 1, \ldots, L\),

\[
P''_{L+j} = \sum_{k=j}^{L} (-1)^{k-j} \binom{L-j}{k-j} P_{L+j}.
\] (3.34)

By combining (3.32) and (3.34), the occurrence probability of a pattern with \((L+j)\) specific errors can be given by [33] as

\[
P'_{L+j} = \frac{1}{2^{L+j}} \sum_{k=j}^{L} (-1)^{k-j} \binom{L-j}{k-j} P_{L+j}.
\] (3.35)

where \(P_{L+j}\) is given by (3.31).

For the single error correcting NEC receiver in which there are two differential detectors, i.e., \(L=2\), the occurrence probabilities can be obtained by substituting \(L=2\) and \(j=0, 1,\) and 2
Performance of \(\pi/4\)-shift DQPSK systems with NEC in CCI-AWGN Environment

in (3.35),

\[
P'_2 = \frac{1}{4} P_2 - \frac{1}{2} P_3 + \frac{1}{4} P_4
\]
\[
P'_3 = \frac{1}{8} P_3 - \frac{1}{8} P_4
\]
\[
P'_4 = \frac{1}{16} P_4.
\]

Putting (3.36) in (3.25), the symbol error probability at the output of the NEC receiver with single error correction capability can be expressed as

\[
P_s = 20 P'_2 + 56 P'_3 + 32 P'_4
\]
\[
= 5 P_2 - 3 P_3.
\]

Similarly, since the NEC receiver with double error correction capability is made up of three differential detectors, the occurrence probabilities can be determined by putting \(L=3\) and \(j=0, 1, 2\) and 3 in (3.35),

\[
P'_3 = \frac{1}{8} P_3 - \frac{3}{8} P_4 + \frac{3}{8} P_5 - \frac{1}{8} P_6
\]
\[
P'_4 = \frac{1}{16} P_4 - \frac{1}{8} P_5 + \frac{1}{16} P_6
\]
\[
P'_5 = \frac{1}{32} P_5 - \frac{1}{32} P_6
\]
\[
P'_6 = \frac{1}{64} P_6
\]

The output symbol error probability of the double error correcting NEC receiver can be determined by substituting (3.38) in (3.26) and can be given as

\[
P_D = 104 P'_3 + 338 P'_4 + 332 P'_5 + 128 P'_6
\]
\[
= 13 P_3 - 17.875 P_4 + 7.125 P_5 - 0.25 P_6.
\]

Finally, for the triple error correcting NEC receivers in which there are four differential detectors, the occurrence probabilities can be determined by substituting \(L = 4\) and \(j = 0, 1,\)
Chapter 3 Performance of π/4-shift DQPSK systems with NEC in CCI-AWGN Environment

2, 3, and 4,

\[
P_4' = \frac{1}{16} P_4 - \frac{1}{4} P_5 + \frac{3}{8} P_6 - \frac{1}{4} P_7 + \frac{1}{16} P_8
\]

\[
P_5' = \frac{1}{32} P_5 - \frac{3}{32} P_6 + \frac{3}{32} P_7 - \frac{1}{32} P_8
\]

\[
P_6' = \frac{1}{64} P_6 - \frac{1}{32} P_7 + \frac{1}{64} P_8
\]

\[
P_7' = \frac{1}{128} P_7 - \frac{1}{128} P_8
\]

\[
P_8' = \frac{1}{256} P_8.
\]

The output symbol error probability at the output of the NEC receiver with triple error correction capability can be found by putting (3.40) in (3.27) is given as

\[
P_T = 616P_4' + 2110P_5' + 2606P_6' + 1812P_7' + 512P_8'
\]

\[
= 38.5P_4 - 88.0625P_5 + 73.90625P_6 - 23.46875P_7 + 1.125P_8.
\]

3.4 Computer Simulation Model Description

For the verification of some of the theoretical performance evaluation results which have been obtained from the material of the previous section, the computer simulation approach was employed. The simulation model used is an equivalent complex baseband representation of the digital communication system illustrated in Fig. 3.1. The actual simulated bit error rate results were obtained with Monte Carlo error counting technique. A total of 65536 bits at 16 samples per symbol had been employed throughout the simulation for each point. Following [54], the confidence bands of the simulation points are summarized in Table 3.6. It is noteworthy that SER \( \approx \frac{1}{2} \) BER due to the fact that each π/4-shift DQPSK signal is encoded by two bits per symbol.

Table 3.6 Confidence bands on BER simulation results for 65536 bits observed.

<table>
<thead>
<tr>
<th>BER</th>
<th>90% Percentage Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^{-1}</td>
<td>±0.1 \times 10^{-1}</td>
</tr>
<tr>
<td>10^{-2}</td>
<td>±0.2 \times 10^{-2}</td>
</tr>
<tr>
<td>10^{-3}</td>
<td>±1.8 \times 10^{-3}</td>
</tr>
</tbody>
</table>
The three parameters which are of interest are: C/N, C/I and \( M \). The C/N is defined as the ratio of received filtered signal power to the received filtered noise power. The C/I is defined as the ratio of the received filtered signal power to the total received filtered interference power. The power calculation for the C/N and C/I is defined as

\[
\text{Power} = \frac{I^2 + Q^2}{\text{no of samples}}
\]  

(3.42)

where I and Q are the in-phase and the quadrature-phase amplitude components of a complex signal.

Simulation of the transmitted \( \pi/4 \)-shift DQPSK signal encoded each pair of the incoming information bits as a differential phase shift according to (3.2). The encoded signal is a complex baseband signal represented by the in-phase (I-) and the quadrature phase (Q-) channel. The transmitter and the receiver filters which are ideal square-root-of-raised-cosine filters have a roll-off factor of 0.2, with an \( x/sin(x) \) amplitude equalizer added to the transmitter filter so that the overall filtering strategy satisfies the first Nyquist criterion [31]. The CCI corrupting the transmitted \( \pi/4 \)-shift DQPSK signal is assumed to consist of \( M \) equal power, statistically independent and randomized \( \pi/4 \)-shift DQPSK signals. The white Gaussian noise is generated by a random Gaussian noise generator subroutine given by [55].

A typical state space diagram of a computer simulated \( \pi/4 \)-shift DQPSK signal operating in the presence of four, i.e., \( M=4 \), independent, equal amplitude interferers with C/I=55 dB and AWGN with C/N=60 dB is illustrated in Fig. 3.4. The square root raised cosine filter with \( \eta=0.35 \) was employed.
Figure 3.4 State-space diagram of a computer-simulated $\pi/4$-shift DQPSK system employing raised cosine filters with an excess bandwidth of 35% and which is operated in the presence of CCI ($C/I = 55$ dB) and Gaussian noise ($C/N = 60$ dB). The number of interferers is equal to 4.
3.5 Error Rate Performance Evaluation Results

Analytical evaluation of the performance of the differentially detected \(\pi/4\)-shift DQPSK schemes which employ the previously proposed NEC receivers will use (3.12) – (3.41). Since the methodology of calculating the performance is the same for all three NEC receivers, the procedure for only one of them is described.

For example, for the NEC receiver with triple error correction capability, the symbol error probability can be obtained by substituting in (3.41) the occurrence probabilities \(P_4 - P_8\) which, in turn, can be obtained from (3.31). The obtained performance evaluation results which are given in terms of symbol error rate (SER) versus C/N having C/I and \(M\) as parameters are presented in Figs. 3.6 – 3.20. For comparison purposes, the coherently detected \(\pi/4\)-shift DQPSK operated in the same environment is always included in the figures. The following three typical C/I ratios are chosen to present the various performance evaluation results:

i. 10 dB (see Figs. 3.17 – 3.20)

ii. 14 dB (see Figs. 3.6 – 3.12)

iii. 18 dB (see Figs. 3.13 – 3.16).

For each of these C/I ratios, the number of cochannel interferers was assumed to vary from one to six. In addition, the case in which \(M\) approaches infinity was also considered.

Before presenting the performance evaluation results, it should be noted that since the Hermite polynomials inside the summation of (3.12) fluctuate between large positive numbers and large negative numbers. The number of terms “\(n\)” of (3.12) should be made large enough for proper convergence of the series. A series is said to be converged when the values of the remaining terms in the series approaches zero. Here, the series is considered as converged when the square bracket term \([\cdot]\) in (3.12) reached a value of \(10^{-16}\) which is almost equal to zero. In other words, the value of “\(n\)” was chosen so that the square bracket term \([\cdot]\) in (3.12) reached a value of \(10^{-16}\).
For example, Fig. 3.5 shows that the pdf of the phase error for different values of “n” at C/I = 10 dB, C/N = 18 dB, and M = 1. When “n” is equal to 40, the bracket term \([\cdot]\) of (3.12) reaches the value of \(10^{-16}\) for any \(\alpha\). A value less than 40 gives a pdf in which the series does not converge. For example, when “n” is equal to 10, the curve obtained fluctuates from large positive to large negative values possible due to the Hermite polynomials in the formula (see Fig. 3.5). On the other hand, using “n” equals to 80 in the series does not provide a noticeable difference compared with the case when “n” equals 40. As a result, for this particular example, “n” should be chosen as 40 in order to achieve accurate and efficient results for this pdf. In general, the same procedure has to be repeated for different values of C/I, C/N, and M so that the proper “n” which ensures convergence of the series can be found.

![Figure 3.5](image)

Figure 3.5 Probability density function (pdf) of the phase error given by (3.12) at C/I = 10 dB and C/N = 18 dB for various values of “n”.

\[ f_n(\alpha) \]
Numerical integration of (3.31) uses the Simpson's one-third integration rule [55]. The accuracy of this integration is normally dependent on the step size taken. Since (3.31) is a function of the integration of the pdf $f_A(\alpha)$, while it is well known that the area under a pdf is one, the stepsize should be chosen such that the sum of the areas under the pdf is approximately equal to one. Similar to looking for the proper "n" for convergence, the value of the proper stepsize has to be found on a trial-and-error basis. Here, the stepsize which gives rise to a result within $10^{-8}\%$ of accuracy is considered as appropriate to ensure the result, i.e., the area under the pdf, very close to one. For example, using the same parameters as before with "n" chosen as 40, the results obtained by the numerical integration of the pdf $f_A(\alpha)$ can be summarized in Table 3.7. It can be seen that the stepsize of $\pi/144$ gives 0.9999999999 for the area under the pdf and results in a percentage difference of $1 \times 10^{-8}$ and is therefore considered as adequate for accurate integration of the pdf $f_A(\alpha)$ for these set of parameters. Table 3.8 illustrates the error probability obtained by (3.31) as a function of various stepsizes. For comparison purposes, the same set of stepsizes used in Table 3.7 is employed. It is clear that the stepsize of $\pi/144$ is adequate for accurate computation of (3.31) and it reflects the fact that finding a proper stepsize for the integration of the pdf $f_A(\alpha)$ ensures accurate evaluation of (3.31).

<table>
<thead>
<tr>
<th>Stepsize [rad]</th>
<th>$-\pi \int f_A(\alpha) d\alpha$</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi/36$</td>
<td>1.0004295218</td>
<td>0.0423</td>
</tr>
<tr>
<td>$\pi/72$</td>
<td>0.9999999998</td>
<td>$2 \times 10^{-8}$</td>
</tr>
<tr>
<td>$\pi/144$</td>
<td>0.9999999999</td>
<td>$1 \times 10^{-8}$</td>
</tr>
<tr>
<td>$\pi/288$</td>
<td>0.9999999999</td>
<td>$1 \times 10^{-8}$</td>
</tr>
</tbody>
</table>
Table 3.8 The evaluation results of (3.31) as a function of the stepsizes in Table 3.7 for \( n = 40, \) \( C/I = 10 \) dB, and \( C/N = 18 \) dB.

<table>
<thead>
<tr>
<th>Stepsize [rad]</th>
<th>( P_1 \times 10^{-2} )</th>
<th>( P_2 \times 10^{-4} )</th>
<th>( P_3 \times 10^{-4} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi/36 )</td>
<td>1.027335753</td>
<td>8.343376904</td>
<td>1.191092748</td>
</tr>
<tr>
<td>( \pi/72 )</td>
<td>1.031858996</td>
<td>8.436324455</td>
<td>1.248774541</td>
</tr>
<tr>
<td>( \pi/144 )</td>
<td>1.031785337</td>
<td>8.438234975</td>
<td>1.248906865</td>
</tr>
<tr>
<td>( \pi/288 )</td>
<td>1.031781098</td>
<td>8.438351874</td>
<td>1.248915267</td>
</tr>
</tbody>
</table>
Figure 3.6 Symbol error rate performance of the NEC with 1 interferer at C/I = 14 dB.
Figure 3.7 Symbol error rate performance of the NEC with 2 interferers at C/I = 14 dB.
Figure 3.8 Symbol error rate performance of the NEC with 3 interferers at C/I=14 dB.
Figure 3.9 Symbol error rate performance of the NEC with 4 interferers at C/I=14 dB.
Figure 3.10 Symbol error rate performance of the NEC with 5 interferers at C/I=14 dB.
Figure 3.11 Symbol error rate performance of the NEC with 6 interferers at C/I=14 dB.
Figure 3.12 Symbol error rate performance of the NEC with infinitive interferers at C/I=14 dB.
Figure 3.13 Conventional differentially detected $\pi/4$-shift DQPSK in CCI at $C/I=18$ dB.
Chapter 3. Performance of π/4–shift DQPSK systems with NEC in CCI-AWGN Environment

Figure 3.14 π/4–shift DQPSK with single error NEC in CCI at C/I=18 dB.
Figure 3.15 \(\pi/4\)-shift DQPSK with double error NEC in CCI at C/I=18 dB.
Figure 3.16 $\pi/4$-shift DQPSK with triple error NEC in CCI at C/I=18 dB.
Figure 3.17 Conventional differentially detected $\pi/4$-shift DQPSK in CCI at $C/I=10$ dB.
Figure 3.18 π/4-shift DQPSK with single error NEC in CCI at C/I=10 dB.
Figure 3.19 $\pi/4$-shift DQPSK with double error NEC in CCI at C/I=10 dB.
Chapter 3. Performance of $\pi/4$-shift DQPSK systems with NEC in CCI-AWGN Environment

Figure 3.20 $\pi/4$-shift DQPSK with triple error NEC in CCI at C/I=10 dB.
3.6 Analysis and Discussion of Performance Evaluation Results

The results show that significant performance improvements over conventional differentially detected $\pi/4$-shift DQPSK systems. For example, Fig. 3.12 shows that, at SER = $10^{-3}$ and at C/I = 14 dB and as $M$ approaches infinity, a gain of more than 4 dB (for the triple-error NEC system) is obtained. This gain is even higher at lower values of SER. It is noteworthy that for this case the gap between coherent and noncoherent (with NEC) detection of $\pi/4$-shift DQPSK is reduced to about 1.2 dB.

The most important observations as well as some heuristic interpretations of the performance evaluation results illustrated in Figs. 3.6 – 3.20 are:

i. As expected, the gains of the single error NEC receiver are the highest, followed by the double error and triple error NEC receivers. The reason for this is that multiple errors do not appear as often as single errors. This trend will continue for higher order NEC receivers. These observations here are in agreement with those of Masamura [37] for the AWGN channel.

ii. The lower the C/I, the higher the gains obtained by the NEC systems will be. This fact makes the NEC systems useful and powerful in environments where there are a large number of users employing the same frequency, e.g., in cellular system with very high traffic.

iii. Under the condition that C/I is constant, increasing the number of interferes deteriorates the overall system performance because, as $M$ increases, the interference signal becomes more Gaussian. In the limiting case in which $M$ approaches infinity, the CCI becomes Gaussian so that the worst SER performance results. These observations are in agreement with similar results reported in [46, 48]. It should be noted, however, that the gains obtained by the NEC receivers are higher as $M$ increases.

iv. For relatively low C/I, the performance of conventional differentially detected $\pi/4$-shift DQPSK exhibits error floors. It can be observed from these results that the NEC system
reduces these error floors. For example in Fig. 3.12 for $C/I = 14$ dB and as $M$ approaches infinity, the error floor is reduced from $2 \times 10^{-4}$ to $10^{-5}$.

It is interesting that the performance improvements of the NEC system are less as $C/N$ decreases. This occurs because, when the channel is noisier, the number of error symbols exceeds the error correction capability of the NEC system. For the limiting case of very low $C/N$, it was found that the performance of a conventional $\pi/4$-shift DQPSK is better without the NEC because an error in the parity symbol will lead to a decoded error even though the decoding signal is actually correct. Nevertheless, it should be pointed out that this situation has no practical interest since the SER is already very high ($> 10^{-1}$).

A summary of the gains obtained by the NEC receiver is given in Table 3.9. The highest gain shown is 7.4 dB. The method employed to compute the SER was based on the series of (3.12). For combinations of low $C/I$ and high $C/N$ ratios, the series converge very slowly. This slow convergence is the reason why the SER results in Figs. 3.6 – 3.20 are plotted in a non-uniform fashion.

Finally, the computer simulation model described in Section 3.3 was employed to verify some of the analytical results. Fig. 3.21 shows some of these results which are in very good agreement with the theoretical results.

Finally, the computer simulation model described in Section 3.4 was employed to verify some of the analytical results. In Fig. 3.21 we show some of these results which indeed are in very good agreement with the equivalent theoretical performance evaluation results.
Table 3.9 Theoretical performance gains of a differentially detected $\pi/4$-shift DQPSK system employing various NEC receivers and operated in a CCI environment.

All gains reported are with reference to a conventional differentially detected $\pi/4$-shift DQPSK system. Although here "error floor" refers to the performance of a conventional differentially detected $\pi/4$-shift DQPSK system, it is clear from the obtained performance evaluation results that the NEC system reduces these error floors.

<table>
<thead>
<tr>
<th>C/I [dB]</th>
<th>$M$</th>
<th>C/N Gain at SER = $10^{-2}$ [dB]</th>
<th>C/N Gain at SER = $10^{-4}$ [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1</td>
<td>2.0</td>
<td>3.8</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>error floor</td>
<td>error floor</td>
</tr>
<tr>
<td></td>
<td>$\infty$</td>
<td>error floor</td>
<td>error floor</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>1.1</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>1.2</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>$\infty$</td>
<td>1.3</td>
<td>2.2</td>
</tr>
<tr>
<td>18</td>
<td>1</td>
<td>0.8</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.8</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>$\infty$</td>
<td>0.8</td>
<td>1.5</td>
</tr>
</tbody>
</table>
Figure 3.21 Comparison of theoretical and computer simulation performance evaluation results at C/I=14 dB with 4 interferers
3.7 Summary

The performance evaluation results of $\pi/4$-shift DQPSK systems employing NEC receivers operated in a combined AWGN and CCI environment show that significant performance improvements as compared to conventional differentially detected $\pi/4$-shift DQPSK systems have been obtained. Error floor reductions of more than one order of magnitude have been reported. Some of these theoretical performance evaluation results have been verified by computer simulation.

Based on the significant performance gains (more than 7 dB) especially for low C/I and increased number of interferers, it is concluded, since the NEC technique does not require any bandwidth expansion, it is a powerful technique for improved performance of digital communication systems operated in cochannel interference environment.
Chapter 4
Performance Analysis and Evaluation of $\pi/4$-shift DQPSK Systems
Employing NEC Technique Operated in a Multipath Fading Environment

4.1 Introduction

Although the effects of fading has been known for decades and are well documented in many publications, including [29, 30, 56–61], it still remains as one of the most challenging environments for reliable transmission/reception of information. In the past, various methods have been proposed and analyzed which reduce the effects of multipath fading on the performance of digital modulation schemes. These methods include the use of pilot tone calibration techniques [62–64], trellis-coded modulation (TCM) [65–67], prediction/cancellation techniques [68] and Orthogonal Frequency Division Multiplexing (OFDM) [69] and OFDM/FM [70]. All these methods have their own advantages and disadvantages and their usefulness depends on the specific application in which they are being employed.

This chapter analyses and evaluates the performance of the $\pi/4$-shift DQPSK systems employing NEC receivers operated in a fading environment. By virtue of the fact that in mobile communication systems channels with relative small RF bandwidth are being used (for example, 30 kHz for the North American cellular systems), it will be assumed that the fading model considered here is frequency nonselective (or equivalently flat). Section 4.2 describes the system model and discuss the modeling of the fading process. Section 4.3 analyzes the single error correcting NEC receiver and derives the theoretical error rate performance of the NEC receiver in a frequency non-selective fading environment. Section 4.4 describes the computer simulation model employed. Section 4.5 gives the performance evaluation of the NEC receivers in various fading environments. Interpretation of these results is presented in Section 4.6. Section 4.7 summarizes the findings of this chapter.
4.2 System Model Description and Analysis

The block diagram of the communication system model under consideration here is illustrated in Fig. 4.1, where for mathematical convenience complex baseband representation is been used.

\[ s_d(t) = \sum_{i=-\infty}^{\infty} e^{j\Phi_i} h_T(t - iT). \]  \hspace{1cm} (4.1)

where \( h_T(t) \) is the impulse response of the transmitter filter [31].

In a Rician fading environment, the received signal \( s(t) \) at the input of the receiver filter, \( H_R(f) \), can be expressed as

\[ s(t) = \sqrt{2S} s_d(t) + s_f(t) s_d(t) + n(t) \]  \hspace{1cm} (4.2)

where \( n(t) \) is the additive white Gaussian noise which had been previously described in Section 3.2, \( \sqrt{2S} s_d(t) \) is the direct path signal, and \( s_d(t) s_f(t) \) is the faded signal. The \( s_f(t) \) is modelled as a complex summation of two independent zero mean Gaussian random processes

\[ s_f(t) = X(t) + jY(t). \]  \hspace{1cm} (4.3)

* Since \( s_d(t) \) is a unity power signal, it is clear that \( S \) is the average power of the direct path signal.
Figure 4.2 The modelling of the fading signal.

As illustrated in Fig. 4.2, $X(t)$ and $Y(t)$ are generated by passing two independent white Gaussian noise processes $n_1(t)$ and $n_2(t)$ through two identical shaping filters which serve to specify the type of fading model under consideration, for example, land-mobile [29] and Mobile Satellite [57].

The spectra and the corresponding autocorrelation function for land–mobile fading channel are, respectively, given by [29] as

$$S_\xi(f) = D \left[ \pi \sqrt{f^2 - B_D^2} \right]$$

and

$$R_\xi(\tau) = DJ_0(2\pi B_D \tau) \triangleq D \rho(t)$$

where $B_D$ is the Doppler spread (or fading bandwidth) and is given by

$$B_D = uf_c/c$$

where $u$ is the velocity of the vehicle, $c$ is the velocity of light, and $f_c$ is the carrier wavelength.

The spectra and the corresponding autocorrelation function for the Gaussian channel are, respectively, given by

$$S_\xi(f) = D \exp \left[ -f^2/B_D^2 \right]/(\sqrt{\pi}B_D)$$
and

\[ R_\xi(\tau) = D \exp\left(-\pi^2 B_D^2 \tau^2\right) \triangleq D p(t). \quad (4.8) \]

With the assumption that the receiver filter \( H_R(f) \) has a square-root raised cosine shape, the direct signal will be undistorted and ISI free. It is also reasonable to assume that the filter bandwidth is larger than the fading bandwidth so that the fading signal passes through the filter without distortion. As a result, the filtered received signal in baseband can be given by

\[ v(t) = \sqrt{2} S s_d'(t) + [X(t) + j Y(t)] s_d'(t) + [n_I(t) + j n_Q(t)] \quad (4.9) \]

where \( s_d'(t) \) is the signal after the receiver filter, \( n_I(t) \) and \( n_Q(t) \) are the in-phase and the quadrature-phase low pass equivalent of \( n(t) \) and were defined in (3.5).

Finally, an important parameter in comparing system performance in the Rician fading environment is the ratio of the direct path signal power \( S \) to the multipath power \( D \). This ratio \( S/D \) is often referred to as the \( K \)-factor [56].

### 4.3 Theoretical Analysis of the Single Error Correcting NEC Receivers in a Frequency-Flat Fading Environment

The output error rate performance of the single error correcting NEC receiver can be analytically determined by following the approach described in in Section 3.3 in which input error patterns containing more errors than its error correction capability are considered. As one signal received at any time interval takes part in determining two symbols in each differential detector (see Table 3.1), this incorrect reference signal may give rise to two output errors in each differential detector. Unlike the CCI-AWGN environment in which the interference is independent from symbol and symbol, fading usually affects several symbols consecutively, thus creating a correlation between these symbols. This correlation can be seen from the autocorrelation functions in (4.5) and (4.8) and is the reason that why the occurrence probability of each input error pattern
is different. In fact, it turns out that this correlation makes the problem of theoretically evaluating the error rate performance of the NEC receiver much more complicated as compared to the CCI-AWGN case.

The output error rate performance of the NEC receivers can be approximated by using the number of remaining output errors and the occurrence probabilities of the corresponding input error pattern which produces output errors. Table 4.1 summarizes the various input error patterns which produce output errors, together with the corresponding remaining output errors and occurrence probabilities. Table 4.1 is in fact a summary of Table 3.5. The total number of output errors for each input error pattern in the table is simply a multiplication of the number of patterns with the number of remaining output errors for each specific input error pattern from Table 3.5. For example, when both \( e_{1,i} \) and \( e_{1,i+1} \) are in error, the number of remaining errors is equal to two and the corresponding number of input error patterns is equal to 4 (see the third row of Table 3.5). The total output errors is equal to \( 2 \times 4 = 8 \) as shown in the fourth row of Table 4.1. The output error rate at the decoder output of the NEC receiver is

\[
P_o = 4P'_a + 4P'_b + 4P'_c + 8P'_d + 16P'_e + 16P'_f + 12P'_g + 12P'_h + 32P'_m. \tag{4.10}
\]

Since the values of each error symbols are 1 or 3, there are \( 2^{2+j}, j = 0, 1, 2 \), combinations for each input error pattern with \( (2+j) \) errors. The occurrence probabilities of having \( (2+j) \) specific errors, \( P'' \) among the four error symbols in Table 4.1 are

\[
P'' = 2^{2+j} P'
\]

where \( P' \) is defined as any one of the set \( \{ P'_a, P'_b, \ldots, P'_m \} \) given in (4.10). By the substitution of (4.11) in (4.10), the output error rate equation can be rewritten as

\[
P_o = P''_a + P''_b + P''_c + 2P''_d + 2P''_e + 2P''_f + 1.5P''_g + 1.5P''_h + 2P''_m. \tag{4.12}
\]
Table 4.1 Occurrence probability as a function of various input error patterns for single error correcting NEC with relatively high probability at the ith time interval. "x" denotes an error with values 1 or 3, and "0" denotes no error.

<table>
<thead>
<tr>
<th>Number of Errors (Input)</th>
<th>Input Error Patterns</th>
<th>Total Output Errors</th>
<th>Occurrence Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>e1,i</td>
<td>e1,i+1</td>
<td>e2,i</td>
<td>e2,i+2</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>x</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>0</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>x</td>
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<td>3</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>x</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>0</td>
<td>x</td>
</tr>
<tr>
<td>4</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

The phase difference can be obtained by taking the argument of the product and the complex conjugate of the delayed signal [31] following which the output $\hat{\phi}_i$ of the kth order differential detector* at the ith sampling instant is given as

$$\hat{\phi}_i = \arg \{v^*[((i - k)T)v(iT)] \} \triangleq \phi_i + \theta_i$$  \hspace{1cm} (4.13)

where * denotes the complex conjugate, $\phi_i$ is the differential phase difference between the two received signals, and the $\theta_i$ is the corresponding differential phase error. The decision of a symbol which is based on the phase difference between two received signals will be in error, if the differential phase error forces the phase difference beyond the decision boundaries. It is thus vital to determine the pdf of this differential phase error so that the performance can be obtained.

The differential phase error $\theta_i$ of a kth order differential detector is

$$\theta_i = \arg (V_{i-k}^* \cdot V_i), \hspace{1cm} -\pi < \theta_i \leq \pi$$  \hspace{1cm} (4.14)

* A kth order differential detector is a differential detector with delay elements of k symbols duration.
where \( V_i \) is the \( i \)th sample of the received signal. This \( i \)th sample is given by

\[
V_i = \sqrt{2S} + X_i + jY_i + (n_{I_i} + jn_{Q_i})\exp(-j\Phi_i)
\]  

(4.15)

where \( X_i, Y_i, n_{I_i}, \) and \( n_{Q_i} \), are the sampled values of the processes \( X(t), Y(t), n_I(t) \) and \( n_Q(t) \) at the \( i \)th sampling instant, respectively.

The following analysis is based on the methodology described by [57]. Fig. 4.3 illustrates the phasor representations of the signals received at the \((i - 2)\)th, \((i - 1)\)th, \(i\)th, \((i + 1)\)th, and \((i + 2)\)th time intervals. The \( x_0, x_1, x_2, x_3, x_4 \) and the \( y_0, y_1, y_2, y_3, y_4 \) are the in-phase and the quadrature-phase components of the phasors \( V_0, V_1, V_2, V_3, V_4 \), respectively, and are given as

\[
x_i = \sqrt{2S} + X_i + n_{I_i}\cos\Phi_i + n_{Q_i}\sin\Phi_i
\]

\[
y_i = Y_i - n_{I_i}\sin\Phi_i + n_{Q_i}\cos\Phi_i.
\]

(4.16)

Table 3.1 illustrated that the signal received at the \( i \)th time interval is used as a reference for detecting the phase differences between itself and signals received at the \((i - 2)\)th, \((i - 1)\)th, \((i + 1)\)th, and \((i + 2)\)th time intervals. It follows that the phasor \( R_2 \) serves as a reference for the other four phasors (see Fig. 4.3). A decision error is made at the \( \pi/4 \)-shift DQPSK detector when the differential phase error is greater than \( \pi/4 \) or less than \( -\pi/4 \), i.e., \( |\theta_i| \geq \pi/4 \). The occurrence probabilities of having \( (L + j) \) specific errors among the \( 2L \) received signals can be determined by considering the corresponding \( (L + j) \) differential phase errors being greater than \( \pi/4 \) or less than \( -\pi/4 \), and the remaining \( [2L - (L + j)] = (L - j) \) differential phase errors being less than or equal to \( |\pi/4| \). For instance, the occurrence probability, \( P''_a \) in (4.12) is given by

\[
P''_a = P(|\epsilon_{1,i+1}, \epsilon_{2,i}| \geq \pi/4, |\epsilon_{1,i}, \epsilon_{2,i+2}| < \pi/4)
\]

\[
= \int \int \int \int p(\theta_1, \theta_2, \theta_3, \theta_4)d\theta_1 d\theta_2 d\theta_3 d\theta_4
\]

(4.17)

where \( p(\theta_1, \theta_2, \theta_3, \theta_4) \) is the joint probability density function (jpdf) of the differential phase errors \( \theta_1, \theta_2, \theta_3 \) and \( \theta_4 \) (see Fig. 4.3). The rest of the error probabilities in the set \( \{P''\} \) can
be found by a similar fashion. For the determination of the occurrence probabilities of having $(L + j)$ specific errors, the pdf of the set of the differential phase errors, i.e., $\theta_1, \theta_2, \theta_3,$ and $\theta_4$ is required and will be derived next.

Because of the independency of the in-phase $(x_0, x_1, x_2, x_3, x_4)$ and the quadrature-phase $(y_0, y_1, y_2, y_3, y_4)$ components, their jpdf of can be expressed as

$$f(x_0, x_1, x_2, x_3, x_4, y_0, y_1, y_2, y_3, y_4) = f(x_0, x_1, x_2, x_3, x_4) \cdot g(y_0, y_1, y_2, y_3, y_4). \quad (4.18)$$

Following [57], the pdf $f(x_0, x_1, x_2, x_3, x_4)$ and $g(y_0, y_1, y_2, y_3, y_4)$ which are jointly Gaussian can be expressed as

$$f(x_0, x_1, x_2, x_3, x_4) = \left[(2\pi)^5 \det(K)\right]^{-\frac{1}{2}} \cdot \exp \left[-\frac{1}{2}(x - s)K^{-1}(x - s)^T\right]$$

$$g(y_0, y_1, y_2, y_3, y_4) = \left[(2\pi)^5 \det(K)\right]^{-\frac{1}{2}} \cdot \exp \left[-\frac{1}{2}yK^{-1}y^T\right] \quad (4.19)$$
where \( \mathbf{x} = (x_0, x_1, x_2, x_3, x_4), \mathbf{y} = (y_0, y_1, y_2, y_3, y_4), \mathbf{s} = \left( \sqrt{2S}, \sqrt{2S}, \sqrt{2S}, \sqrt{2S}, \sqrt{2S} \right) \).

\( \det(\cdot) \) denotes the determinant and \((\cdot)^T\) denotes the transpose of a vector. \( \mathbf{K}^{-1} \) is the inverse of the covariance matrix \( \mathbf{K} \) which is given as

\[
\mathbf{K} = (D + N) \begin{bmatrix}
1 & k_1 & k_2 & k_3 & k_4 \\
k_1 & 1 & k_2 & k_3 & k_2 \\
k_2 & k_1 & 1 & k_2 & k_1 \\
k_3 & k_2 & k_1 & 1 & k_1 \\
k_4 & k_3 & k_2 & k_1 & 1
\end{bmatrix} \tag{4.20}
\]

with

\[
k_1 = \left[ D/(D + N) \right] \rho(T) \\
k_2 = \left[ D/(D + N) \right] \rho(2T) \\
k_3 = \left[ D/(D + N) \right] \rho(3T) \\
k_4 = \left[ D/(D + N) \right] \rho(4T) \tag{4.21}
\]

where \( \rho(\cdot) \) was previously defined in (4.8). The determinant of \( \mathbf{K} \) is given by

\[
(D + N)^5 c_0 \tag{4.22}
\]

where \( c_0 \) is given in the Appendix A.

Referring to Fig. 4.3, the in-phase, \( x \), and the quadrature-phase, \( y \), components can be expressed in polar form by using the following substitutions

\[
x_0 = R_0 \cos (\phi - \theta) \\
y_0 = R_0 \sin (\phi - \theta) \\
x_1 = R_1 \cos (\phi - \theta) \\
y_1 = R_1 \sin (\phi - \theta) \\
x_2 = R_2 \cos \phi \\
y_2 = R_2 \sin \phi
\]
\[ x_3 = R_3 \cos (\phi + \theta_3) \]
\[ y_3 = R_3 \sin (\phi + \theta_3) \]
\[ x_4 = R_4 \cos (\phi + \theta_4) \]
\[ y_4 = R_4 \sin (\phi + \theta_4). \]  

(4.23)

Substituting (4.19), (4.20) and (4.23) into (4.18) and by using the software Maple® for some algebraic simplification,

\[
\tilde{f}(R_0, R_1, R_2, R_3, R_4, \theta_1, \theta_2, \theta_3, \theta_4, \phi) = \frac{R_0 R_1 R_2 R_3 R_4}{(2\pi)^5(D + N)^5} \frac{S}{(D + N)c_0} \exp \left\{ -\frac{1}{(D + N)c_0} B(R_0, R_1, R_2, R_3, R_4, \theta_1, \theta_2, \theta_3, \theta_4) \right\} \exp \left\{ \frac{\sqrt{2S}}{(D + N)c_0} E(R_0, R_1, R_2, R_3, R_4, \theta_1, \theta_2, \theta_3, \theta_4, \phi) \right\}
\]

(4.24)

where the constant A is given in the Appendix A and the variables \( B(R_0, R_1, R_2, R_3, R_4, \theta_1, \theta_2, \theta_3, \theta_4) \), \( E(R_0, R_1, R_2, R_3, R_4, \theta_1, \theta_2, \theta_3, \theta_4, \phi) \) are defined as follows

\[
B(R_0, R_1, R_2, R_3, R_4, \theta_1, \theta_2, \theta_3, \theta_4) =
\]
\[
c_7(R_0^2 + R_4^2) + c_8(R_1^2 + R_3^2) + c_6R_2^2 + c_{17}R_0R_3 \cos (\theta_1 + \theta_3)
+ c_{10}R_0R_1 \cos (\theta_1 - \theta_2) + c_{11}R_0R_4 \cos (\theta_1 + \theta_4) + c_{12}R_1R_2 \cos \theta_2
+ c_{13}R_1R_4 \cos (\theta_2 + \theta_4) + c_{14}R_1R_3 \cos (\theta_2 + \theta_3) + c_{15}R_2R_3 \cos \theta_3
+ c_{16}R_3R_4 \cos (\theta_3 - \theta_4) + c_{17}R_2R_4 \cos \theta_4 + c_{18}R_0R_2 \cos \theta_1
\]

(4.25)

and

\[
E(R_0, R_1, R_2, R_3, R_4, \theta_1, \theta_2, \theta_3, \theta_4, \phi) =
\]
\[
c_1R_0 \cos (\phi - \theta_1) + c_2R_4 \cos (\phi + \theta_4) + c_3R_3 \cos (\phi + \theta_3)
+ c_4R_1 \cos (\phi - \theta_2) + c_5R_2 \cos \phi
\]

(4.26)
where the mathematical expressions for the constant set \{c_j| j = 1, 2, \ldots, 18\} are given in Appendix A. Expanding (4.26),

\[
E(R_0, R_1, R_2, R_3, R_4, \theta_1, \theta_2, \theta_3, \theta_4, \phi) = \\
(c_1 R_0 \cos \theta_1 + c_2 R_4 \cos \theta_4 + c_3 R_3 \cos \theta_3 + c_4 R_1 \cos \theta_2 + c_5 R_2) \cos \phi \\
+ (c_1 R_0 \sin \theta_1 + c_2 R_4 \sin \theta_4 + c_3 R_3 \sin \theta_3 + c_4 R_1 \sin \theta_2) \sin \phi.
\]

(4.27)

With the relationship [71]

\[
P_1 \cos \omega - P_2 \sin \omega = \sqrt{P_1^2 + P_2^2} \cos \left[ \omega + \tan^{-1} \left( \frac{P_1}{P_2} \right) \right]
\]

(4.28)

(4.27) can be rewritten as

\[
E(R_0, R_1, R_2, R_3, R_4, \theta_1, \theta_2, \theta_3, \theta_4, \phi) = \sqrt{Z(R_0, R_1, R_2, R_3, R_4, \theta_1, \theta_2, \theta_3, \theta_4, \phi)} \cos (\phi + \Delta)
\]

(4.29)

where \(\Delta\) is a dummy variable and \(Z(R_0, R_1, R_2, R_3, R_4, \theta_1, \theta_2, \theta_3, \theta_4, \phi)\) is given by

\[
Z(R_0, R_1, R_2, R_3, R_4, \theta_1, \theta_2, \theta_3, \theta_4, \phi) = \\
(c_1 R_0 \cos \theta_1 + c_2 R_4 \cos \theta_4 + c_3 R_3 \cos \theta_3 + c_4 R_1 \cos \theta_2 + c_5 R_2)^2 \\
+ (c_1 R_0 \sin \theta_1 + c_2 R_4 \sin \theta_4 + c_3 R_3 \sin \theta_3 + c_4 R_1 \sin \theta_2)^2.
\]

(4.30)

Finally, by using (4.25) and (4.29), (4.24) can be rewritten as

\[
f(R_0, R_1, R_2, R_3, R_4, \theta_1, \theta_2, \theta_3, \theta_4, \phi) = \\
\frac{R_0 R_1 R_2 R_3 R_4}{(2\pi)^5 (D + N)^5 c_0} \exp \left[ \frac{S}{(D + N)c_0} A \right] \exp \left\{ -\frac{1}{(D + N)c_0} B(R_0, R_1, R_2, R_3, R_4, \theta_1, \theta_2, \theta_3, \theta_4) \right\} \\
\cdot \exp \left\{ \frac{\sqrt{2S}}{(D + N)c_0} \sqrt{Z(R_0, R_1, R_2, R_3, R_4, \theta_1, \theta_2, \theta_3, \theta_4)} \cos (\phi + \Delta) \right\}.
\]

(4.31)

By integrating \(\phi\) from 0 to \(2\pi\), along with the use of the relationship that for any \(\Delta\) [72]

\[
I_0(x) = \frac{1}{2\pi} \int_{0}^{2\pi} \exp [x \cos (\phi + \Delta)] d\phi.
\]

(4.32)
where \( I_0 \) is the modified Bessel function of the first kind and zeroth order, (4.31) can be written as

\[
f(R_0, R_1, R_2, R_3, R_4, \theta_1, \theta_2, \theta_3, \theta_4) = \frac{R_0 R_1 R_2 R_3 R_4}{(2\pi)^4(D + N)^5 c_0} \exp \left[ -\frac{S}{(D + N)c_0} A \right] \exp \left\{ \frac{-1}{(D + N)c_0} B(R_0, R_1, R_2, R_3, R_4, \theta_1, \theta_2, \theta_3, \theta_4) \right\} \cdot I_0 \left\{ \frac{\sqrt{25}}{(D + N)c_0} \sqrt{Z(R_0, R_1, R_2, R_3, R_4, \theta_1, \theta_2, \theta_3, \theta_4) \right\}.
\]

(4.33)

Since it would be very tedious and time consuming to do the integration of the (4.33) from 0 to \( \infty \) through 5 dimensions, the \( R_0, R_1, R_2, R_3, R_4 \) would be replaced by using the following change of variables

\[
R_0 = R \cos (\mu/2)
\]
\[
R_1 = R \sin (\mu/2) \sin (\lambda/2) \sin (\iota/2) \cos (\eta/2)
\]
\[
R_2 = R \sin (\mu/2) \sin (\lambda/2) \cos (\iota/2)
\]
\[
R_3 = R \sin (\mu/2) \sin (\lambda/2) \sin (\iota/2) \sin (\eta/2)
\]
\[
R_4 = R \sin (\mu/2) \cos (\lambda/2).
\]

(4.34)

and the following transformation of variables

\[
dR_0 dR_1 dR_2 dR_3 dR_4 = |J| dR d\mu d\lambda d\eta
\]

(4.35)

where \( J \) is the Jacobian matrix and is given by

\[
|J| = \frac{R^4}{2^6} \sin (\mu/2) \sin (\iota/2)(1 - \cos \mu)(1 - \cos \lambda).
\]

(4.36)

By substituting (4.34) – (4.36) in (4.33), followed by some algebraic manipulations along with the use of the software Maple\textsuperscript{®}, (4.33) can be given as

\[
f(R, \mu, \lambda, \iota, \eta, \theta_1, \theta_2, \theta_3, \theta_4) = \frac{R^9 [\sin \mu \sin (\lambda/2)]^2 \sin \eta \sin \lambda \sin \iota (1 - \cos \iota)(1 - \cos \mu)^2}{21^1(D + N)^5 c_0 (2\pi)^4} \cdot \exp \left[ \frac{S}{(D + N)c_0} A \right] \cdot \exp \left\{ -a R^2 \right\} \cdot I_0[b R]
\]

(4.37)
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with

\[
a = \frac{F(\mu, \lambda, \iota, \eta, \theta_1, \theta_2, \theta_3, \theta_4)}{2(D + N)c_0}
\]
\[
b = \sqrt{\frac{2}{(D + N)c_0}} G(\mu, \lambda, \iota, \eta, \theta_1, \theta_2, \theta_3, \theta_4)
\]

(4.38)

where \(F(\mu, \lambda, \iota, \eta, \theta_1, \theta_2, \theta_3, \theta_4)\) and \(G(\mu, \lambda, \iota, \eta, \theta_1, \theta_2, \theta_3, \theta_4)\) are given as

\[
F(\mu, \lambda, \iota, \eta, \theta_1, \theta_2, \theta_3, \theta_4) = \]
\[
c_0\alpha + c_{10}\beta + c_{11}\gamma + c_{16}\xi + c_{13}\chi + c_{14}\psi + c_{16}\rho + c_{17}\sigma + (c_6 - c_8)\zeta
\]
\[
+ (c_8 - c_7)\varpi + 2c_7
\]

(4.39)

and

\[
G(\mu, \lambda, \iota, \eta, \theta_1, \theta_2, \theta_3, \theta_4) =
\frac{S}{c_0(D + N)} \cdot \left\{ 0.5(c_2^2 - c_3^2)\zeta + (c_2^2 - c_3^2)[1 - 0.5(1 - \cos \lambda)]0.5(1 - \cos \mu) + c_1^2 \right. \\
+ c_3^2 - 0.5c_1^2(1 - \cos \mu) + 0.25(c_4^2 - c_5^2)(1 + \cos \eta)(\varpi - \zeta) + c_1c_3\alpha + c_1c_4\beta \\
+ c_1c_2\gamma + c_1c_5\tau + c_3c_6\nu + c_4c_5\xi + c_2c_5\sigma + c_3c_4\psi + c_2c_3\rho + c_2c_4\chi \right\}. 
\]

(4.40)

The expressions for the constant set \(\{c_j | j = 0, 1, \ldots, 18\}\) in (4.39) and (4.40) is given in the Appendix whereas the rest of the variables are defined as

\[
\alpha = \sin \mu \sin (\lambda/2) \sin (\iota/2) \sin (\eta/2) \cos (\theta_1 + \theta_3)
\]
\[
\beta = \sin \mu \sin (\lambda/2) \sin (\iota/2) \cos (\eta/2) \cos (\theta_1 - \theta_2)
\]
\[
\gamma = \sin \mu \cos (\lambda/2) \cos (\theta_1 + \theta_4)
\]
\[
\tau = \sin \mu \sin (\lambda/2) \cos (\iota/2) \cos \theta_1
\]
\[
\xi = \sin \iota \cos (\eta/2) \cos \theta_2 \cdot 0.5(1 - \cos \mu) \cdot 0.5(1 - \cos \lambda)
\]
\[
\chi = \sin \lambda \sin (\iota/2) \cos (\eta/2) \cos (\theta_2 + \theta_4) \cdot 0.5(1 - \cos \mu)
\]
\[
\psi = \sin \eta \cdot 0.5(1 - \cos \iota) \cdot 0.5(1 - \cos \lambda) \cdot 0.5(1 - \cos \mu) \cos (\theta_2 + \theta_3)
\]
\[
\nu = \sin \iota \sin (\eta/2) \cos \theta_3 \cdot 0.5(1 - \cos \lambda) \cdot 0.5(1 - \cos \mu)
\]

\[
\rho = \sin \lambda \sin (\eta/2) \sin (\iota/2) \cos (\theta_3 - \theta_4) \cdot 0.5(1 - \cos \mu)
\]

\[
\sigma = \sin \lambda \cos (\iota/2) \cos \theta_4 \cdot 0.5(1 - \cos \mu)
\]

\[
\zeta = 2[1 - 0.5(1 - \cos \iota)] \cdot 0.5(1 - \cos \mu) \cdot 0.5(1 - \cos \lambda)
\]

\[
\omega = 20.5(1 - \cos \lambda) \cdot 0.5(1 - \cos \mu).
\]

Integrating \( R \) of (4.37) over \((0, \infty)\) and \(\mu, \lambda, \iota, \eta\) over \((0, \pi)\) together with the use of the following relationship

\[
\int_0^\infty R^b \exp \left( -aR^2 \right) I_0(bR) dR = \frac{12}{a^5} \exp \left( \frac{b^2}{4a} \right) \left[ 1 + 4 \left( \frac{b^2}{4a} \right) + 3 \left( \frac{b^2}{4a} \right)^2 + \frac{2}{3} \left( \frac{b^2}{4a} \right)^3 + \frac{1}{24} \left( \frac{b^2}{4a} \right)^4 \right]
\]

(4.42)

(4.37) becomes

\[
p(\theta_1, \theta_2, \theta_3, \theta_4)
\]

\[
= \int_0^\pi \int_0^\pi \int_0^\pi \int_0^\pi f(R, \mu, \lambda, i, \eta, \theta_1, \theta_2, \theta_3, \theta_4) dR d\mu d\lambda di d\eta
\]

\[
= \frac{3c_0^4}{2(2\pi)^5} \cdot \exp \left[ \frac{S}{(D + N)c_0} \right] \cdot \frac{T(\mu, \lambda, \iota, \eta)}{[F(\mu, \lambda, \iota, \eta, \theta_1, \theta_2, \theta_3, \theta_4)]^5}
\]

\[
\cdot \exp \left[ \frac{G(\mu, \lambda, \iota, \eta, \theta_1, \theta_2, \theta_3, \theta_4)}{F(\mu, \lambda, \iota, \eta, \theta_1, \theta_2, \theta_3, \theta_4)} \right] \cdot \left\{ 1 + 4 \left[ \frac{G(\mu, \lambda, \iota, \eta, \theta_1, \theta_2, \theta_3, \theta_4)}{F(\mu, \lambda, \iota, \eta, \theta_1, \theta_2, \theta_3, \theta_4)} \right] \right\}^2
\]

\[
+ 3 \left[ \frac{G(\mu, \lambda, \iota, \eta, \theta_1, \theta_2, \theta_3, \theta_4)}{F(\mu, \lambda, \iota, \eta, \theta_1, \theta_2, \theta_3, \theta_4)} \right]^2 + \frac{2}{3} \left[ \frac{G(\mu, \lambda, \iota, \eta, \theta_1, \theta_2, \theta_3, \theta_4)}{F(\mu, \lambda, \iota, \eta, \theta_1, \theta_2, \theta_3, \theta_4)} \right]^3
\]

\[
+ \frac{1}{24} \left[ \frac{G(\mu, \lambda, \iota, \eta, \theta_1, \theta_2, \theta_3, \theta_4)}{F(\mu, \lambda, \iota, \eta, \theta_1, \theta_2, \theta_3, \theta_4)} \right]^4 \right\} d\mu d\lambda di d\eta
\]

(4.43)

where \( T(\mu, \lambda, \iota, \eta) = [\sin \mu \sin (\lambda/2)]^2 \sin \iota \sin \lambda \sin \iota (1 - \cos \iota) [1 - \cos \mu] \). By substituting (4.43) in (4.17), the occurrence probability \( P''_a \) can be determined. The rest of the error probabilities in the set \( \{P''\} \) of (4.12) can be obtained in a similar manner. Finally, the
substitution of the values \( \{P_a', P_b', \ldots, P_m'\} \) in (4.12) yields the output error probability of the single-error correcting NEC receiver.

### 4.4 Computer Simulation Model Description

The computer simulation model used is an equivalent complex baseband representation of the digital communication system (see Fig. 4.1). The bit error rate (BER) results were obtained with Monte Carlo error counting techniques. A total of 131072 bits at 16 samples per symbol had been employed throughout the simulation for each point. Following [54], the confidence bands of the simulation points are summarized in Table 4.2.

<table>
<thead>
<tr>
<th>BER</th>
<th>90% Percentage Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10^{-1})</td>
<td>(\pm 0.1 \times 10^{-1})</td>
</tr>
<tr>
<td>(10^{-2})</td>
<td>(\pm 0.5 \times 10^{-2})</td>
</tr>
<tr>
<td>(10^{-3})</td>
<td>(\pm 1.0 \times 10^{-3})</td>
</tr>
<tr>
<td>(10^{-4})</td>
<td>(\pm 3.7 \times 10^{-4})</td>
</tr>
</tbody>
</table>

The three parameters which are of interest are: C/N, \(K\) and \(B_D T\) product. The carrier power \(C\) refers to the total received signal power, i.e., the summation of the direct path power \(S\) and the reflected path components power \(D\), following which the C/N is defined as:

\[
\frac{C}{N} = \frac{S + D}{N}.
\]  (4.44)

\(K\) is defined as the ratio of the direct path signal power divided by the ratio of the reflected path components power \(D\), i.e.,

\[
K = \frac{S}{D}.
\]  (4.45)

The power calculation for the C/N and \(K\) (= \(S/D\)) is defined in (3.42).
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The transmitter and the receiver filters which are ideal square-root-of-raised-cosine filters have a roll-off factor of 0.2, with an \( x/\sin(x) \) amplitude equalizer added to the transmitter filter so that the overall filtering strategy satisfies the first Nyquist criterion. The Rayleigh fading process was simulated by passing white Gaussian noise processes through two identical shaping filters with Gaussian spectrum (see Fig. 4.2 and equation 4.7 and equation 4.4).

4.5 Performance Evaluation Results

Although the output error rate equation for the single-error correcting NEC receiver has been derived, it is too time consuming and thus expensive to be evaluated with the accessible computing facilities, including the CRAY supercomputer of the Ontario Center for Large Scale Computation of the University of Toronto. By running smaller programs in this CRAY computer, it was estimated that more than 100 CPU hours (@ $100/hour) was needed in order to obtain only one BER point for the single-error NEC receiver. This occurs because, as can be seen from (4.12), the final output error probability is a sum of nine different error probabilities and each of these error probabilities is a function of the eight-fold integrals (see equation 4.17 and 4.43). Because the correlations among the signals diminish under the circumstances of fast fading, the number of the multiple integrals in these formulas may be reduced. Later computer simulation results show that under fast fading environment, the receiver filter plays a significant role in rejecting the fading components. For accurate evaluation of the output error probabilities, the effect of the receiver filter can no longer be ignored from the previous derivation of (4.43). The reduction of the number of multiple integrals under the assumption of independency in fast fading is therefore accompanied by an increase in the complexity of the formula due to consideration of the receiver filter.

As seen from (4.16), the in-phase components of the phasors in Fig. 4.3 will be approximately equal to \( \sqrt{2S} \) for high \( K \)-factor so that the differential phase error between phasor \( R_i \),
and \( R_j \) can be approximated by [57]

\[
\theta_i = \frac{\text{Im}(V_i) - \text{Im}(V_j)}{\sqrt{2S}} 
\]  

(4.46)

where \( \text{Im}(\cdot) \) refers to the imaginary part of its argument. The occurrence error probability will become a function of four integrals instead of the former eight integrals. Regardless of the approximation by the four integrals, the output error probability is a total of nine occurrence probabilities (see equation 4.12) and would lead to inaccurate results due to accumulation error and approximation inaccuracies. It was thus decided to employ the computer simulation approach for the evaluation of the performance of the \( \pi/4 \)-shift DQPSK signals employing the proposed NEC receivers.

In the rest of this section, various BER performance results of \( \pi/4 \)-shift DQPSK systems operated in a frequency nonselective fading environment and which employ single- and double-error correcting NEC receivers are presented. In the evaluation of the overall performance, the following three parameters are of great importance: the \( C/N \), the \( K \)-factor, and the \( B_D T \) product. The obtained performance evaluation results are given in terms of bit error rate (BER) as shown in Figs. 4.4 – 4.6. In these figures, the values of the \( K \)-factor considered are 1 dB, 10 dB, and 15 dB whereas the \( B_D T \) takes values of 0.01, 0.05, 0.13, and 1.26. Fig. 4.7 presents the performance of the proposed NEC receivers in an extremely fast fading environment, for example, an aeronautical channel. Finally, Fig. 4.8 illustrates a comparison of the performance of the conventional differentially detected \( \pi/4 \)-shift DQPSK systems operated in the presence of fading and CCI.
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Figure 4.4 BER performance of differential detected $\pi/4$-shift DQPSK systems employing NEC receivers operated in a Rician fading environment at $K = 1$ dB with various values of $B_D T$. 
Figure 4.5 BER performance of differential detected $\pi/4$-shift DQPSK systems employing NEC receivers operated in a Rician fading environment at $K = 10$ dB with various values of $B_D T$. 
Figure 4.6 BER performance of differential detected $\pi/4$-shift DQPSK systems employing NEC receivers operated in a Rician fading environment at $K = 15$ dB with various values of $B_D T$. 
Figure 4.7 BER performance of differential detected $\pi/4$-shift DQPSK systems employing NEC receivers in an aeronautical channel with $B_D T$ of 6.29.
Figure 4.8 Comparison of the error rate performance of the conventional differentially detected $\pi/4$-shift DQPSK systems in the presence of CCI and fading with equal power.
4.6 Analysis and Discussion of Performance Evaluation Results

As it can be seen from Figs. 4.4 - 4.7, in general the performance improvements obtained by the employment of the NEC receivers are not as high as the ones reported in Chapter 3 where CCI was the source of interference. In fact, for K-factor with relatively low values (for example, 1 dB) and not very fast fading (for example, $B_D T = 0.01, 0.05, \text{ and } 0.13$) the performance of the NEC systems is worse as compared to the conventional differential detected $\pi/4$-shift DQPSK systems, i.e., without NEC. For higher values of $B_D T$, significant improvements have been obtained. For example, for $B_D T = 1.26$, $K = 1 \text{ dB}$, and at $BER = 3 \times 10^{-3}$, a performance gain of approximately 4 dB is obtained.

Based upon the performance results presented in Figs. 4.4 - 4.8, the most important findings, including some heuristic interpretation and explanation of these results, will be summarized.

i. By considering first the BER performance of conventional differentially detected $\pi/4$-shift DQPSK systems operated in a fading environment, it is clear that they perform better than in an “equivalent” CCI environment. We refer to “equivalent” interference, if we have the same signal-to-interference ratio, i.e., $K = C/I$. The main reason for that is the fact that CCI corrupts the $\pi/4$-shift DQPSK signals independently from symbol to symbol and thus the performance of differential detection is not satisfactory. On the other hand, for the fading channel where, depending upon the value of $B_D T$, there is a strong correlation between adjacent symbols, differential detection will at least partially cancel its effects on the overall performance. In fact, the higher this correlation is (i.e., low $B_D T$), the better the differential detection performs. It should be mentioned that, as it was pointed out by Mason [56], for very high values of $B_D T$ (for example, $>1.0$), the receiver filter $H_R(f)$ attenuates the fading component, thus causing a reduction in the error probability. This phenomena can also be seen from our results for $B_D T = 1.26$. 
ii. For small values of $B_D T$ (for example, 0.01), the gains acquired by the NEC receivers increase with increasing $K$. However, when $K$ is small (for example, 1 dB) the improvements in the BER performance are small. Moreover, the improvement of the double-error correcting NEC receiver is negligibly small as compared to that of single-error correcting NEC receiver. In order to understand the above behavior, it should first be recalled that the error correction capability of the NEC receivers comes from the parity symbols, i.e., the outputs of the higher-order detectors. In other words, the correctness of the parity symbols determines the error correction performance of the NEC receivers. Thus, it would be interesting to examine the performance of each of the differential detectors, i.e., first-, second-, and third-order differential detectors. These error rate results have been obtained by means of computer simulation and are summarized in Figs. 4.9-4.12. For comparison purposes, the same values of $B_D T$ and the $K$-factor which have previously been used, have also been selected here. It can be seen from Fig. 4.9, that for $K=1$ dB the performance of the third-order differential detectors is the worst, whereas, the second-order differential is inferior to that of the conventional differential detector, i.e., first-order differential detector. As a result, the performance of the NEC is not satisfactory for this small $K$ and small $B_D T$, as illustrated in Fig. 4.4. On the other hand, as the values of the $K$-factor increases, the performance of all three differential detectors becomes better, therefore, the performance of the NEC becomes better for these higher $K$. The above observations clearly explain most of their performance results for the relatively small $B_D T$, such as 0.01, reported in Figs. 4.4-4.6.

iii. For very high $B_D T$ (1.26 as illustrated in Figs. 4.4-4.6 or 6.29 as illustrated in Fig. 4.7) the gains offered by the NEC receivers increase with decreasing $K$. Furthermore, it appears here that the double-error correcting NEC receiver provides a noticeable improvement as compared to the single-error correcting NEC receiver. Clearly these observations are opposite to the results and comments discussed in the previous paragraph. In order to explain this behavior,
the correlation characteristics in fading has to be taken into account. As pointed out earlier, for very high $B_D T$, the receiver filter rejects the fading components and thus decreases the correlation between signals. As a result, the error probability of the overall systems is relatively less and thus the higher order differential detectors perform more effectively. This can be seen from Fig. 4.12 that the error probabilities of the first-, second- and the third-order differential detectors are approximately the same. This explains the increased improvement acquired by the double-error correcting NEC systems. Although decreasing the values of the $K$-factor would increase the overall error probability, the fact the signals become somewhat uncorrelated, allows the NEC receivers to effectively improve the overall performance. Therefore, the effectiveness of the NEC receivers becomes more noticeable at smaller values of $K$, despite the overall higher probability of error. Finally, it is worth to mention that the highest gain obtained is approximately 6 dB for a mobile-satellite channel with $B_D T = 6.29$, as shown in Fig. 4.7. This suggests that the NEC technique will perform well in application with very fast fading, such as in the aeronautical channel.

iv. At moderate $B_D T$ values, for example, at $B_D T = 0.05$ or 0.13, the performance of the NEC receivers is similar to the situation when $B_D T$ is very small, i.e., increasing $K$ enhances the gain provided by the NEC receivers. However, it is worth notice that when $K = 1$ dB, the performance of the NEC receivers is worse than the situation when the $B_D T$ is very small. Moreover, increasing the $B_D T$ (from 0.05 to 0.13 in our case here) deteriorates the performance of the NEC receivers at this small $K$ value. This can be explained by the poor performance of the higher order detectors as illustrated in Figs. 4.10–4.11. In addition, increasing the $B_D T$ decreases the correlation among the signals, and lead to a higher error probability in conventional system. As a result, the overall system performance deteriorated.
Figure 4.9 Output symbol error rate for differential detectors of different orders at $B_D T = 0.01$. 
Figure 4.10 Output symbol error rate for differential detectors of different orders at $B_D T = 0.05$. 

Symbol error rate (SER) vs. C/N [dB]
Figure 4.11 Output symbol error rate for differential detectors of different orders at $B_D T = 0.13$. 
Figure 4.12 Output symbol error rate for differential detectors of different orders at $B_D T = 1.26$. 

Symbol error rate (SER) vs. $C/N [\text{dB}]$. 

- $K = 1 \text{ dB}$ 
- $K = 10 \text{ dB}$ 
- $K = 15 \text{ dB}$ 
- $3T$ 
- $2T$ 
- $1T$
4.7 Summary

In this chapter, we have applied the NEC technique to the π/4-shift DQPSK system operated in a frequency nonselective fading environment. First, the theoretical error rate equation for a single-error correcting NEC receiver has been derived. Afterwards, the performance of single- and double-error correcting NEC receivers have been obtained by means of computer simulation. It was found that for very fast fading the NEC receivers resulted in significant improved performance with a gain up to 6 dB, as compared to the conventional π/4-shift DQPSK systems. Furthermore, it was found that in general the performance of a conventional differential detected π/4-shift DQPSK systems operated in the CCI is inferior to that operated in fading.
Chapter 5.
Conclusions and Future Research Suggestions

5.1 Conclusions

This thesis has dealt with the application of the NEC technique to the $\pi/4$-shift DQPSK modulation format, which is the new transmission standard for the new North American and Japanese digital mobile cellular network. It was assumed that the environment in which such systems would operate is the CCI as well as the fading channel.

First, the NEC technique was analyzed from the convolutional coding point of view. It was shown that it could be viewed as a feedback decoding technique. Various issues inherent of the NEC technique, such as infinitive error propagation, decoding depth and constraint length were discussed in detail. Afterwards, based on a phase transformation concept, the NEC receiver, originally suggested by Samejima et al. [33] for DQPSK signals, is modified in order to accommodate for the $\pi/4$-shift DQPSK scheme. For the first time, we have theoretically analyzed the performance of the NEC receivers with up to three error correction capability. Furthermore, the performance of the $\pi/4$-shift DQPSK systems in the presence of a mixture of AWGN and CCI was theoretically evaluated. The obtained performance evaluation results have indicated significant gains in performance as well as reductions in error floors as compared to the conventional differential detected $\pi/4$-shift DQPSK. These gains increased with increasing number of interferers and decreasing C/I ratio.

Finally, in this thesis, the NEC technique was applied to the $\pi/4$-shift DQPSK systems operated in a frequency nonselective Rician fading channel. The error rate equation for the single-error correcting NEC receiver was for the first time derived. Performance evaluation results for both single- and double-error correcting NEC receivers obtained by means of computer simulation have shown that for very fast fading and decreasing $K$-factor, the obtained gains increase. In general, these gains were not as significant as those obtained for the CCI. It was found that
the reason behind this observation was the fact that the CCI corrupts the transmitted symbols independently and thus the performance of differential detection was poor.

Next, the most important findings would be summarized, so as to provide a guideline for the future researchers whom would be interested in working with the NEC technique.

i. Since the NEC technique makes use of the output symbols from the higher order detectors as parity symbols for the output symbols from the conventional detector (or the information symbols), the output error probability of the higher order detectors has to be at least close to that of the conventional detector in order for the NEC technique to operate effectively. Otherwise, increasing the number of higher order detectors may not increase the error correction capability as it should. These lead to two other important observations which would be described as follows.

- Optimum improvement from the NEC technique can be achieved when the signals are independently corrupted. It is because if signals are independently corrupted, the error probability of any two signals should give the same probability of error despite the time separation between these two signals.

- If there is a strong correlation between adjacent signals, increase the number of higher order detectors may not improve the performance significantly, unless the higher order detectors can yield at least approximately the same error probability as the conventional detectors.

ii. Under very low C/N condition, for example, 1 dB, virtually no improvement can be obtained from the NEC technique. It is because error may occur so frequent that the error correction capabilities of the NEC receiver have already exceeded. Furthermore, because of the feedback path in the NEC receiver, the frequent occurrence of errors does not give a chance to the system to recover. Consequently, the errors continue to propagate over time, thus resulting
in possible higher probability of error than conventional system without employing NEC. It should, however, be noted that this error propagation is different from the infinite error propagation where one error propagates throughout the network even though there is no more new incoming errors. However, these situations usually occur at error probability higher than $10^{-2}$ which is usually of no practical interest in actual environment.

iii. Since both the information and the parity symbols of the NEC receivers come from the detectors, well-designed detectors are important and will directly affect the performance of NEC receivers. Thus, rather than seeking ways to improve the decoding algorithm, it is more important to have a well-designed detectors. It is also worth notice that although more sophisticated decoding algorithm may possibly lead to a better improvement, the extra hardware circuitry involved may diminish the attractiveness of the NEC schemes in which simplicity is one of the prime concern.

### 5.2 Suggestions for Future Research

#### 5.2.1. Adjacent Channel Interference (ACI)

It seems promising to apply the NEC technique to systems operating in an ACI environment which is currently encountered in cellular system application.

#### 5.2.2. Combinations of CCI and fading

It could be interesting to investigate the performance of the NEC system in a combined CCI and fading environment, since in practice both interferences appear simultaneously.

#### 5.2.3. Application to GMSK

It would be interesting to apply the NEC technique to the Pan European digital cellular transmission standard, namely the GMSK.
References


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[74] J. P. McGehee and A. J. Bateman, "Theoretical and experimental investigation of feed-forward signal regeneration as a means of combating multipath propagation effects in pilot-
References


Appendices
Appendix A  Definition of Constants

\[ c_0 = 1 - 8k_2^2k_3k_1 + 4k_2k_3k_1^3 + 6k_2k_1^2 - 3k_2^2 - 4k_1^4 + 8k_1k_2k_3 - 6k_1^2k_2k_4 + 4k_1k_3k_4 + 3k_1^4 + 2k_3^2 - 2k_2^2 - 2k_3^2 + 2k_2k_4 + 2k_2^2k_3^2 + k_4^4 - 4k_1k_2k_3k_4 - 4k_2k_4k_1^4 + 2k_3^2k_1^2 - 2k_4^2k_1 + 4k_2k_4k_2k_4 + k_2^2k_3^2 + 2k_3^2k_4 - 2k_3^2k_2k_4 - 2k_1^2k_2k_4 \]

\[ A = 16k_2^2k_3k_1 + 16k_2k_4k_1 + 4k_1k_2k_3^3 + 8k_3k_1k_2^2 - 2k_2^2k_1^2 + 7k_2^2 - 12k_1k_3 \]

\[ c_1 = -k_3^2k_1^2 - k_3^2k_1^2 + 1 - 2k_3^2k_3k_1 + k_3^3 - k_3^2 - 2k_1^2 + k_2k_3^2 - k_1k_2k_3 + k_2^2 - k_3^2 \]

\[ \ldots \]

\[ A = 16k_2^2k_3k_1 + 16k_2k_4k_1 + 4k_1k_2k_3^3 + 8k_3k_1k_2^2 - 2k_2^2k_1^2 + 7k_2^2 - 12k_1k_3 \]

\[ c_2 = k_2k_4 + k_2k_3 - k_1 - k_3^2 + k_2^2k_3 + k_3k_4 - k_1k_3k_4 - k_3k_1^2 - k_2^3k_1^2 + k_2^2 - 2k_2^3k_1^2 \]

\[ \ldots \]

\[ c_4 = -k_3^2 + k_3^2k_1k_4 - k_4^2 - 2k_2k_3k_4 + k_3^2k_3k_4 + k_1k_4^2 + k_2k_4^2 + k_2^2k_1^2 - k_2^2k_2^2 \]

\[ \ldots \]
Appendix A Definition of Constants

\[c_5 = -2k_3^2k_4 + 2k_2k_3^2 - k_1^4 + 2k_2k_3^2k_1 + 2k_4k_3^2k_3 - k_2^4 + 4k_1k_2 + 2k_2k_3k_1^2 - 4k_3^2k_3 \]
-2k_3^2k_2 + 4k_3^2k_1^2 + k_3^4 + 2k_2k_3^2 - 2k_3^2k_1 - 2k_1k_2k_4 + 2k_2k_3^2 - 2k_3^2k_2k_4 + 2k_1k_3
-2k_1k_2k_3 + 2k_1k_3k_4 + 2k_2k_3^2 - 2k_2k_4^2 - 2k_2 - 2k_3^2k_1 - 2k_1k_2k_4 - 2k_3^2 + 2k_1k_2
+2k_4k_1k_3^2 + 2k_3^2 - 2k_1 - 2k_2k_3^2 - k_3^2 - 2k_2k_1k_4^2 + 2k_3^2k_1 + 2k_3^2k_4k_1 + 2k_4k_1^3
-2k_1k_2k_1^2 + k_2k_3^2k_4 + 2k_3k_1^3 + 2k_2k_3^2 - 4k_3^2k_3k_1 + 2k_1k_2k_3k_4 + 2k_3^2 + 1

\[c_6 = -(k_2^3 - 0.5k_1^4 - 0.5 + 0.5k_1^4 + 0.5k_3^4 + 0.5k_2^2 + k_3^2k_2k_4 - 0.5k_2^2k_4^2 - 2k_1k_3k_4
+k_2k_3^2k_4 + k_1^2 + k_1^2k_3^2 - 2k_1k_2k_3)\]

\[c_7 = -(0.5k_3^2 - 0.5k_1^4 + k_3^2 + k_2k_3k_1 - 0.5k_3^2k_1 - 2k_2k_1^2 - 2k_1k_2k_3 + k_3k_1^3 - 0.5
+k_3^2k_1 - 0.5k_1^2 + 1.5k_1^2)\]

\[c_8 = -(-0.5k_3^2k_1 + k_3^2 + k_1k_2k_3k_4 - k_1k_3k_4 - 0.5k_2^2k_3^2 + 0.5k_4^2 + k_2k_4k_1^2 - 0.5
-0.5k_3^2k_1 + k_2k_3k_1 - k_2k_3^2 + k_2^2 + 0.5k_3^2 - k_2k_4 - k_1k_3k_3)\]

\[c_9 = -(k_2k_3^2 + 3k_2k_3k_1 + k_1k_2k_3 - 2k_2k_3k_1^2 - k_2k_3^2 + k_2k_3k_4 - k_2^2k_3
-k_2k_3 + 2k_1k_2k_2^2 + k_1^2 - k_2k_4k_1 - k_1k_4 - 2k_1k_2 + k_2k_4 + k_2^2k_1
+k_3 - k_3^2)\]

\[c_{10} = -(k_3^2k_3 + k_3^2k_3 - 2k_2k_3k_2^2 + 3k_2k_3^3 - k_4k_1^2 - k_1k_2k_3^2 + k_2^2k_3 - k_2^2k_4k_1
+k_3k_4k_1^2 + 2k_1k_3k_4 + k_1 + k_1k_2k_4 - k_3k_4 - k_1k_2 - k_1k_3 - k_3k_3 - k_2^2k_4)\]

\[c_{11} = -(k_2^2 + k_3^2 - k_2^2 + 2k_3^2k_2k_4 - 2k_2k_3k_1 + 2k_1k_2k_3 - k_2k_3^2 + k_2^2k_4 - 2k_2k_1
-2k_2k_3 + 3k_2k_3^2 + 2k_3k_2k_4 - 2k_4k_3^2 - k_2^2k_4^2 - k_2^2)\]

\[c_{12} = -(k_1 + k_2k_3k_4 - 2k_2k_3^2k_1 - k_1k_2k_4 - k_4k_3^2 - k_2^2k_4 - 2k_2^2k_3^2k_1 + k_1k_2k_4 - k_3k_3 - k_2^2k_4k_1
+k_2k_4k_1 - k_3^2k_4 + k_1k_2k_4 - k_2^2k_4 + k_2^2k_4 + k_2k_4 + k_2k_4k_1
+k_3 - k_2k_3 + 2k_1k_2 - k_2^2k_3k_3 - k_3^2)\]

\[c_{13} = -(k_3^2k_4 + 2k_2k_3k_4^2 - k_2k_4 - 2k_2k_3^2k_1 + 3k_2k_3^2k_1 + k_1k_2k_4 - k_3^2 + k_2^2)\]

\[c_{14} = -(k_3^2k_4^2 + 2k_2k_3k_4^2 - 2k_1k_2k_3^2k_4 - 2k_1k_3^2 + k_3^2k_4^2 + 2k_1k_2k_3 - k_2^2k_3
+k_4k_3^2 - k_2^2k_4^2 - 2k_2k_3k_4^2 + 2k_2k_4k_4^2 + 2k_2k_4k_2 - 2k_2^2 - k_2^2)\]

\[c_{15} = -(k_3^2k_4 + 2k_2k_3k_4 - k_2k_4 - 2k_2k_4 - k_1k_2k_4 - k_2k_3^2k_4 + k_3^2k_4 + k_2k_4 + k_2k_3^2
+k_4^2k_2 - k_2k_3k_4 + k_2k_4k_3^2 - k_2^2k_3k_4 + k_4k_4^2 + k_4k_3^2 + k_1k_2k_4 - k_2^2k_3
-k_3k_3 + 2k_2k_3 - 2k_1k_2 + k_1 - k_1)\]

\[c_{16} = -(k_1 + k_2k_3 - k_4k_3^2 + k_3^2k_3 - k_4k_2 + 3k_4^2k_2 + 2k_2k_3 - k_1k_3 + k_2k_3 + 2k_1k_2k_4
+k_1k_2k_4 - k_4^2k_2 - 2k_2k_4k_1 - k_3k_4 - 2k_2k_3k_1 - k_3k_2k_3 - 2k_3k_4 - 2k_1k_2k_4
+k_1k_3k_4 - k_1k_3 - k_2k_4 - k_2^2k_3^2 - k_2^2k_2^2 + 2k_2k_1k_3 - k_1k_2k_3k_4 + k_3^2k_1 - k_2k_3^2
+k_4^2 - k_3^2)\]

(A.3)
This star models the SINGLE ERROR CORRECTION oct. It has two inputs and 1 output. Among the two inputs, one comes from the 1T period delay, while the other one comes from 2T period delay. The output is matched to the original values to maintain consistency with the transmitter end, followed by transforming these numbers to bit stream.

```c
if ((si = (a + b - c) % 4) < 0) si += 4;
if ((output = ( b - ei_l ) % 4) < 0) output += 4;
if ((si_l = (si - ei_l) % 4) < 0) si_l += 4;
/* convert the decoded output to original values to maintain consistency */
switch (output)
{
    case 0:
        output = 2;
        break;
    case 1:
        output = 0;
        break;
    case 2:
        output = 1;
        break;
    case 3:
        output = 3;
        break;
    default:
        output = 0;
        break;
} /* switch */
```

This code handles single error correction in a star model, taking into account the delays from different periods and converting the decoded output to maintain consistency with the transmitter end.
This star models the DOUBLE ERROR CORRECTION cct. It has three inputs and 1 output. Among the three inputs, one comes from the IT period delay, while the others come from 2T period delay and 3T period delay. The output is matched to the original values to maintain consistency with the transmitter end, followed by transforming these numbers to bit stream.

```c
/* Fix input and output cell sizes */
set_cellsize_in (0, sizeof(int));
set_cellsize_in (1, sizeof(int));
set_cellsize_in (2, sizeof(int));
set_cellsize_out(0, sizeof(int));

if (NARR.narrative > MEDIUM_NARRATION) {
    fprintf(NARR.file, "**************
    fprintf(NARR.file, "no parameters for star necl.s
    fprintf(NARR.file, "no of input buffers : 3 (int)"
    fprintf(NARR.file, "no of output buffers : 1 (int)"
    fprintf(NARR.file, "**************
}

/* Initialize the syndrome patterns. */
t = 0;
for (n=1; n<=3; n++)
{
    /* single error */
    a[t] = 0;
b[t] = c[t] = d[t] = e[t] = f[t] = n;
t++;
    /* double error */
    for (m=1; m<=3; m++)
    {
        a[t] = m;
b[t] = (m+n)%4;
c[t] = n;
d[t] = (m+n)%4;
e[t] = n;
f[t] = n;
t++;
    }
}
for (m=1; m<=3; m++)
{
    a[t] = m;
b[t] = (m+n)%4;
c[t] = n;
d[t] = (m+n)%4;
e[t] = (m+n)%4;
f[t] = n;
t++;
}
for (m=1; m<=3; m++)
{
    a[t] = m;
b[t] = n;
c[t] = n;
d[t] = n;
e[t] = n;
f[t] = n;
t++;
}
```
for (m=1; m<=3; m++)
{
    a[t] = 0;
    if (b[t] = (n - m) % 4 < 0) b[t] += 4;
    c[t] = n;
    d[t] = n;
    e[t] = n;
    f[t] = n;
    t++;
}

for (m=1; m<=3; m++)
{
    a[t] = 0;
    b[t] = n;
    c[t] = n;
    if (d[t] = (n - m) % 4 < 0) d[t] += 4;
    e[t] = n;
    f[t] = n;
    t++;
}

for (m=1; m<=3; m++)
{
    a[t] = 0;
    b[t] = n;
    c[t] = n;
    if (d[t] = (n - m) % 4 < 0) d[t] += 4;
    e[t] = n;
    f[t] = n;
    t++;
}

for (m=1; m<=3; m++)
{
    a[t] = 0;
    b[t] = n;
    c[t] = n;
    if (d[t] = (n - m) % 4 < 0) d[t] += 4;
    e[t] = n;
    f[t] = n;
    t++;
}

sli = sli_1 = sli_2 = s2l_1 = s2l_2 = ei_2 = output = x = found = 0
ss = bs = cs = ds = es = s2l = SW = 0;

return (0);
Mint  *)it_out(0) = output/2; /* MSB goes first. */
Mint  *)it_out(0) = output%2;
if (pass++ == 1) SW = 1;;
/* if */
return (0);
end /* main */

---

// d3.c

/*
/*****************************/
Programmer: Dominic Mong
Date: Sep 1990
This program counts the number of remaining errors at the decoded output for the double error correcting NEC when the number of input error is 3.
*******************************************************************/

#include <math.h>
#include <stdio.h>
int z[3],x[3],y[3],t[3],s[3];
int error,testing;
int pattern00=0,pattern01=0,pattern02=0;
void pattern1(), pattern2(), pattern3(), pattern4(), pattern5(), pattern6(), pattern7(), pattern8(), pattern9(), pattern10(), pattern11(), pattern12(), pattern13(), pattern14(), pattern15(), pattern16(), pattern17(), pattern18(), pattern19(), pattern20();
int check(int *);
int mod(int);

void checkPattern();
void checkPattern()
{
  switch (error-testing)
  { 
  case 0:
    pattern00++;
    break;
  case 1:
    pattern01++;
    break;
  case 2:
    pattern02++;
    break;
  default:
    printf("error\n");
  }
}
int check (int *corr)
{
if ((sli==1) && (sli_1==3) && (sli_2==3)
    (sli_3==0) && (sli_4==3) && (sli_5==3))
{
  *corr = 3;
  return 0;
}
if ((sli==1) && (sli_1==1) && (sli_2==1) &&
    (sli_3==0) && (sli_4==1) && (sli_5==1))
{
  *corr = 1;
  return 0;
}
if ((sli==1) && (sli_1==1) && (sli_2==1) &&
    (sli_3==2) && (sli_4==1) && (sli_5==1))
{
  *corr = 1;
  return 0;
}
if ((sli==1) && (sli_1==1) && (sli_2==1) &&
    (sli_3==0) && (sli_4==0) && (sli_5==0))
{
  *corr = 3;
  return 0;
}
if ((sli==1) && (sli_1==0) && (sli_2==3) &&
    (sli_3==0) && (sli_4==0) && (sli_5==0))
{
  *corr = 3;
  return 0;
}
if ((sli==1) && (sli_1==0) && (sli_2==1) &&
    (sli_3==0) && (sli_4==0) && (sli_5==0))
{
  *corr = 3;
  return 0;
}
if ((sli==1) && (sli_1==3) && (sli_2==1) &&
    (sli_3==0) && (sli_4==1) && (sli_5==1))
{
  *corr = 1;
  return 0;
}
if ((sli==3) && (sli_1==1) && (sli_2==1) &&
    (sli_3==2) && (sli_4==1) && (sli_5==1))
{
  *corr = 3;
  return 0;
}
if ((sli==3) && (sli_1==3) && (sli_2==3) &&
    (sli_3==3) && (sli_4==3) && (sli_5==3))
{
  *corr = 3;
  return 0;
}
if ((sli==1) && (sli_1==1) && (sli_2==1) &&
    (sli_3==1) && (sli_4==1) && (sli_5==1))
{
  *corr = 1;
  return 0;
}
```c
for (i=0;i<j+1;i++)
    for (j=0;j<k+1;j++)
        for (k=0;k<=l;k++)
            {
                testing-error;
                corr=0;
                sli=0;
                al=2+2*j*s2;
                s2=mod(s2i-y7[k]);
                sli=sli+1;
                sli1=sli1;j;
                sli2=sli2+s8[i];
                check(scorr);
                if (corr != s2) error++;    
                sli1 = mod(sli1 - corr);
                sli2 = mod(sli2 - corr);
                sli1 = sli;
                sli2 = sli2 - corr;
                sli1 = sli1;
                check(scorr);
                if (corr != s2) error++;    
            }
        }
    }
/* z8,y7,y9 are in error. */
void pattern2()
    {
        int i,j,k;
        int corr=0;
        for (i=0;i<=1;i++)
            for (j=0;j<=1;j++)
                for (k=0;k<=1;k++)
                    {
                        testing-error;
                        corr=0;
                        sli=0;
                        al=2+2*j*s2;
                        s2=mod(s2i-y7[k]);
                        sli=sli+1;
                        sli1=sli1;j;
                        sli2=sli2+s8[i];
                        check(scorr);
                        if (corr != s2) error++;    
                        sli1 = mod(sli1 - corr);
                        sli2 = mod(sli2 - corr);
                        sli1 = sli;
                        sli2 = sli2 - corr;
                        sli1 = sli1;
                        check(scorr);
                        if (corr != s2) error++;    
                    }
            }
/* z8,y9,y7 are in error. */
void pattern1()
    {
        int i,j,k;
        int corr=0;
        for (i=0;i<=1;i++)
            for (j=0;j<=1;j++)
                for (k=0;k<=1;k++)
                    {
                        testing-error;
                        corr=0;
                        sli=0;
                        al=2+2*j*s2;
                        s2=mod(s2i-y7[k]);
                        sli=sli+1;
                        sli1=sli1;j;
                        sli2=sli2+s8[i];
                        check(scorr);
                        if (corr != s2) error++;    
                        sli1 = mod(sli1 - corr);
                        sli2 = mod(sli2 - corr);
                        sli1 = sli;
                        sli2 = sli2 - corr;
                        sli1 = sli1;
                        check(scorr);
                        if (corr != s2) error++;    
                    }
            }
```

```c
/* module arithmetic */
int mod(int p)
    {
        if (p<0) return (p+4);
        else return (p+4);
    }
/* init the error symbols. */
void init()
    {
        int i,j=1;
        for (i=0;i<=1;i++)
            {
                s8[i]=s9[i]=y7[i]=y9[i]=t6[i]=t9[i]=j;
                j=3;
            }
/* z8,s9,y7 are in error. */
void pattern1()
    {
        int i,j,k;
        int corr=0;
        for (i=0;i<=1;i++)
            for (j=0;j<=1;j++)
                for (k=0;k<=1;k++)
                    {
                        testing-error;
                        corr=0;
                        sli=0;
                        al=2+2*j*s2;
                        s2=mod(s2i-y7[k]);
                        sli=sli+1;
                        sli1=sli1;j;
                        sli2=sli2+s8[i];
                        check(scorr);
                        if (corr != s2) error++;    
                        sli1 = mod(sli1 - corr);
                        sli2 = mod(sli2 - corr);
                        sli1 = sli;
                        sli2 = sli2 - corr;
                        sli1 = sli1;
                        check(scorr);
                        if (corr != s2) error++;    
                    }
            }
```
sli=0;
s2i=0;
check(&corr);
if (corr != 0) error++;
checkPattern();
}
}

/* z8,y9,t6 are in error. */
void pattern3()
{
  int i,j,k;
  int corr=0:
  for (i=0;i<=l;i++)
  {
    for (j=0;j<=l;j++)
    {
      for (k=0;k<=l;k++)
      {
        testing=error:
        corr=0;
        sli1=mod(-y9[i]);
        sli1_1=x8[i];
        sli1_2=x8[i];
        sli2=x8[i];
        sli2_1=x8[i];
        sli2_2=mod(x8[i]-t6[k]);
        check(&corr);
        if (corr != x8[i]) error++;
        sli2_1=mod(sli1_1 - corr);
        sli1_1 = ali;
        sli2_2 = mod(sli2_1 - corr);
        sli2_1 = mod(sli2 - corr);
        sli1_2=0;
        sli2=0;
        check(&corr);
        if (corr != 0) error++; 
        checkPattern();
      }
    }
  }
}

/* z9,y7,y9 are in error. */
void pattern5()
{
  int i,j,k;
  int corr=0:
  for (i=0;i<=l;i++)
  {
    for (j=0;j<=l;j++)
    {
      for (k=0;k<=l;k++)
      {
        for (k=0:k<=1:k++)
        {
          corr=0:
          testing=error:
          sli1=0;
          sli1_1=x8[i];
          sli2=x8[i];
          sli2_1=x8[i];
          sli2_2=mod(x8[i]-t6[j]);
          check(&corr);
          if (corr != x8[i]) error++;
          sli2_1=mod(sli1_1 - corr);
          sli1_1 = ali;
          sli2_2 = mod(sli2_1 - corr);
          sli2_1 = mod(sli2 - corr);
          sli1_2=0;
          sli2=0;
          check(&corr);
          if (corr != 0) error++; 
          checkPattern();
        }
      }
    }
  }
}
check(&corr);
if (corr != s9[i]) error++;
checkPattern();
}
}
}
*/
/* s9,y7,t9 are in error. */
void pattern7()
{
int i,j,k;
int corr=0;
for (i=0;i<1;i++)
{
for (j=0;j<1;j++)
{
for (k=0;k<1;k++)
{
corr=0;
}
}
}
}
*/
/* s9,y7,t9 are in error. */
void pattern8()
{
int i,j,k;
int corr=0;
for (i=0;i<1;i++)
{
for (j=0;j<1;j++)
{
for (k=0;k<1;k++)
{
corr=0;
}
}
}
}
}
/* y7,y9,t9 are in error. */

int i,j,k;
int corr=0;

for (i=0;i<l;i++)
{
    for (j=0;j<l;j++)
    {
        corr=0;
        testing=error;

        sli=mod(-y9[i]);
        sli_1=0;
        sli_2=0;
        sli_2=mod(-t6[j]);
        check(&corr);
        if (corr != 0) error++;

        sli_1 = mod(sli_1 - corr);
        sli_2 = mod(sli_2 - corr);
        sli_1 = y9[i];
        sli_2 = mod(-t6[j]);
        check(&corr);
        if (corr != 0) error++;

        sli_1 = 0;
        sli_2 = mod(-y9[i]);
        sli_1 = 0;
        sli_2 = mod(-t6[j]);
        check(&corr);
        if (corr != 0) error++;

    }
}

/* y7,y9,t9 are in error. */

int i,j,k;
int corr=0;

for (i=0;i<l;i++)
{
    for (j=0;j<l;j++)
    {
        corr=0;
        testing=error;

        sli=mod(-y9[i]);
        sli_1=0;
        sli_2=0;
        sli_2=mod(-t6[j]);
        check(&corr);
        if (corr != 0) error++;

        sli_1 = mod(sli_1 - corr);
        sli_1 = y9[i];
        sli_2 = mod(-t6[j]);
        check(&corr);
        if (corr != 0) error++;

        sli_1 = 0;
        sli_2 = mod(-y9[i]);
        sli_2 = mod(-t6[j]);
        check(&corr);
        if (corr != 0) error++;

    }
}

/* y9,t6,t9 are in error. */

int i,j,k;
int corr=0;

for (i=0;i<l;i++)
{
    for (j=0;j<l;j++)
    {
        corr=0;
        testing=error;

        sli=mod(-y9[i]);
        sli_1=0;
        sli_2=0;
        sli_2=mod(-t6[j]);
        check(&corr);
        if (corr != 0) error++;

        sli_1 = mod(sli_1 - corr);
        sli_1 = 0;
        sli_2 = mod(-t6[j]);
        check(&corr);
        if (corr != 0) error++;

        sli_1 = y9[i];
        sli_2 = mod(-t6[j]);
        check(&corr);
        if (corr != 0) error++;

    }
}

/* y9,t6,t9 are in error. */

int i,j,k;
int corr=0;

for (i=0;i<l;i++)
{
    for (j=0;j<l;j++)
    {
        corr=0;
        testing=error;

        sli=mod(-y9[i]);
        sli_1=0;
        sli_2=0;
        sli_2=mod(-t6[j]);
        check(&corr);
        if (corr != 0) error++;

        sli_1 = mod(sli_1 - corr);
        sli_1 = y9[i];
        sli_2 = mod(-t6[j]);
        check(&corr);
        if (corr != 0) error++;

        sli_1 = 0;
        sli_2 = mod(-y9[i]);
        sli_2 = mod(-t6[j]);
        check(&corr);
        if (corr != 0) error++;

    }
}

/* y9,t6,t9 are in error. */

int i,j,k;
int corr=0;

for (i=0;i<l;i++)
{
    for (j=0;j<l;j++)
    {
        corr=0;
        testing=error;

        sli=mod(-y9[i]);
        sli_1=0;
        sli_2=0;
        sli_2=mod(-t6[j]);
        check(&corr);
        if (corr != 0) error++;

        sli_1 = mod(sli_1 - corr);
        sli_1 = y9[i];
        sli_2 = mod(-t6[j]);
        check(&corr);
        if (corr != 0) error++;

        sli_1 = 0;
        sli_2 = mod(-y9[i]);
        sli_2 = mod(-t6[j]);
        check(&corr);
        if (corr != 0) error++;

    }
}

/* y9,t6,t9 are in error. */

int i,j,k;
int corr=0;

for (i=0;i<l;i++)
{
    for (j=0;j<l;j++)
    {
        corr=0;
        testing=error;

        sli=mod(-y9[i]);
        sli_1=0;
        sli_2=0;
        sli_2=mod(-t6[j]);
        check(&corr);
        if (corr != 0) error++;

        sli_1 = mod(sli_1 - corr);
        sli_1 = y9[i];
        sli_2 = mod(-t6[j]);
        check(&corr);
        if (corr != 0) error++;

        sli_1 = 0;
        sli_2 = mod(-y9[i]);
        sli_2 = mod(-t6[j]);
        check(&corr);
        if (corr != 0) error++;

    }
}
void pattern2() {
    int i, j, k;
    int corr = 0;
    for (i = 0; i < l; i++)
        for (j = 0; j < l; j++)
            for (k = 0; k < l; k++)
                if (corr != corr)
                    error++;
    corr = 0;
}

void pattern3() {
    int i, j, k;
    int corr = 0;
    for (i = 0; i < l; i++)
        for (j = 0; j < l; j++)
            for (k = 0; k < l; k++)
                if (corr != corr)
                    error++;
    corr = 0;
}

void pattern4() {
    int i, j, k;
    int corr = 0;
    for (i = 0; i < l; i++)
        for (j = 0; j < l; j++)
            for (k = 0; k < l; k++)
                if (corr != corr)
                    error++;
    corr = 0;
}
/* z9,t6,t9 are in error. */
void pattern15()
{
    int i,j,k;
    int corr=0;
    for (i=0;i<=l;i++)
    {
        for (j=0;j<=l;j++)
        {
            for (k=0;k<=l;k++)
            {
                corr=0;
                testing=error;
                sli=z9[i];
                sli_1=z9[i];
                sli_2=0;
                s2i=z9[i];
                s2i_1=mod(-t6[j]);
                check(&corr);
                if (corr != 0) error++;
                sli_2 = mod(sli_1 - corr);
                s1l_1 = sli;
                s2i_2 = mod(s2i_1 - corr);
                sli_1 = mod(sli_2 - corr);
                sli_1 = mod(sli[i]-t9[k]);
                check(&corr);
                if (corr != 0) error++;
                checkPattern();
                corr=0;
            }
        }
    }
}

/* y7,t6,t9 are in error. */
void pattern16()
{
    int i,j,k;
    int corr=0;
    for (i=0;i<=l;i++)
    {
        for (j=0;j<=l;j++)
        {
            for (k=0;k<=l;k++)
            {
                corr=0;
                testing=error;
                sli=0;
                sli_1=0;
                sli_2=mod(-y7[i]);
                s2i=0;
                s2i_1=0;
                s2i_2=mod(-t6[j]);
                check(&corr);
                if (corr != 0) error++;
                sli_2 = mod(sli_1 - corr);
                s1l_1 = sli;
                s2i_2 = mod(s2i_1 - corr);
                s2i_1 = mod(s2i_2 - corr);
                sli_1=0;
                s2i=mod(-t9[k]);
                check(&corr);
                if (corr != 0) error++;
                checkPattern();
            }
        }
    }
}

/* z8,z9,t6 are in error. */
void pattern17()
{
    int i,j,k;
    int corr=0;
    for (i=0;i<=l;i++)
    {
        for (j=0;j<=l;j++)
        {
            for (k=0;k<=l;k++)
            {
                corr=0;
                testing=error;
                sli=z9[i];
                sli_1=(z9[i]+z8[i])%4;
                sli_2=z8[i];
                s2i_2=mod(s2i_1 - corr);
                s1l_1 = sli;
                s2i_2 = mod(s2i_1 - corr);
                s2i_1 = mod(s2i_2 - corr);
                sli_1=0;
                s2i=mod(z8[i]-t6[k]);
                check(&corr);
                if (corr != z9[i]) error++;
                checkPattern();
                corr=0;
            }
        }
    }
}
if (corr != 0)
error++;

void pattern18()
{
    int i, j, k;
    int corr = 0;
    for (i=0; i<=l; i++)
    {
        for (j=0; j<=l; j++)
        {
            for (k=0; k<=l; k++)
            {
                corr = 0;
                testing = error;

                sli_2 = mod(sli_l - corr);
                sli_l = sli;
                s2i_2 = mod(s2i_l - corr);
                s2i_l = s2i;
                sli = 0;
                s2i = z9[i];
                testing = error;
            }
        }
    }
}

/* z8, z9, y9 are in error. */

void pattern20()
{
    int i, j, k;
    int corr = 0;
    for (i=0; i<=l; i++)
    {
        for (j=0; j<=l; j++)
        {
            for (k=0; k<=l; k++)
            {
                corr = 0;
                testing = error;

                sli_2 = mod(sli_l - corr);
                sli_l = sli;
                s2i_2 = mod(s2i_l - corr);
                s2i_l = s2i;
                sli = 0;
                s2i = z9[i];
                testing = error;
            }
        }
    }
}

/* z8, y9, t9 are in error. */
pattern1();
pattern2();
pattern3();
pattern4();
pattern5();
pattern6();
pattern7();
pattern8();
pattern9();
pattern10();
pattern11();
pattern12();
pattern13();
pattern14();
pattern15();
pattern16();
pattern17();
pattern18();
pattern19();
pattern20();
printf("Number of errors = %d\n",error);
printf("Number of pattern with 0 rem error=%d\n",pattern00);
printf("Number of pattern with 1 rem error=%d\n",pattern01);
printf("Number of pattern with 2 rem error=%d\n",pattern02);
return 0;
}