DIVERSITY PSK SIGNALS IN IMPULSIVE NOISE AND GENERALIZED FADING

by

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Abstract

Diversity combining techniques are well-known and useful to improve the performance of digital communication systems that experience fading. However, most research topics deal with the performance evaluation of modulation schemes in fading and AWGN channels, with diversity reception. Relatively few research topics cope with the performance evaluation of diversity techniques in non-Gaussian noise and fading. Therefore, to broaden research in the area of diversity combining techniques in a more realistic noise model, this thesis deals in particular with the performance evaluation of diversity combining techniques in the presence of impulsive noise and fading. Several contributions are made to the system model of modulation schemes in impulsive noise and fading, with diversity reception.

In the first part, the PDFs of the sum of impulsive noises are derived. Then, a system model without fading is considered and theoretical expressions of the error rate performance of modulation schemes in impulsive noise, with or without diversity reception, are derived. It is demonstrated that the diversity technique chosen affects the PDF of the sum of impulsive noises and makes the signal performance with diversity reception different from the signal performance without diversity reception. This is in contrast to the signal performance in the AWGN channel.

In the second part, for the digital communication system in impulsive noise and fading, analytical expressions of the signal performance in impulsive noise and fading are derived and validated through simulation.

The final part contains a system in impulsive noise and fading with diversity reception. Performance evaluations of the error rate performance of modulation schemes in impulsive
noise and fading, with diversity reception, are derived and compared with the previously
derived results. All analytical expressions are validated through numerous simulation
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<tr>
<td>8PSK</td>
<td>8-ary Phase Shift Keying</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>BER</td>
<td>Bit Error Rate</td>
</tr>
<tr>
<td>BPSK</td>
<td>Binary Phase Shift Keying</td>
</tr>
<tr>
<td>CBFSK</td>
<td>Coherent Binary Frequency Shift Keying</td>
</tr>
<tr>
<td>CDF</td>
<td>Cumulative Distribution Function</td>
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<tr>
<td>CHF</td>
<td>Characteristic Function</td>
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<tr>
<td>DPSK</td>
<td>Differential Phase Shift Keying</td>
</tr>
<tr>
<td>EGC</td>
<td>Equal-Gain Combining</td>
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<tr>
<td>EM</td>
<td>Electromagnetic</td>
</tr>
<tr>
<td>fad</td>
<td>fading</td>
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<tr>
<td>I</td>
<td>In-Phase</td>
</tr>
<tr>
<td>ImpA</td>
<td>Class A Impulsive Noise</td>
</tr>
<tr>
<td>ISM</td>
<td>Industrial, Scientific and Medical</td>
</tr>
<tr>
<td>MEDS</td>
<td>Method of Exact Doppler Spread</td>
</tr>
<tr>
<td>MPSK</td>
<td>M-ary Phase Shift Keying</td>
</tr>
<tr>
<td>MQAM</td>
<td>M-ary Quadrature Amplitude Modulation</td>
</tr>
<tr>
<td>Nak</td>
<td>Nakagami</td>
</tr>
<tr>
<td>NCBFSK</td>
<td>Non-Coherent Binary Frequency Shift Keying</td>
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<tr>
<td>PC</td>
<td>Personal Computer</td>
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<tr>
<td>PDF</td>
<td>Probability Density Function</td>
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<td>PSD</td>
<td>Power Spectral Density</td>
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<td>PSK</td>
<td>Phase Shift Keying</td>
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<tr>
<td>Q</td>
<td>Quadrature</td>
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<td>QAM</td>
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<td>Quadrature Phase Shift Keying</td>
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<td>Ray</td>
<td>Rayleigh</td>
</tr>
<tr>
<td>SC</td>
<td>Selection Combining</td>
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<tr>
<td>Sim</td>
<td>Simulation</td>
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<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
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CHAPTER 1
Introduction

1.1 PSK Signals

A commonly used technique for digital signal transmission is the Phase Shift Keying (PSK) technique [44, p. 349]. When using the PSK technique, signal amplitudes are the same, but signal phases are different, with equal spacing between phases. The general form of the PSK signals with \( M \) different phases is called \( M \)-ary Phase Shift Keying (MPSK). When \( M = 2, 4, 8 \), the PSK signals are called Binary Phase Shift Keying (BPSK), Quadrature Phase Shift Keying (QPSK) and 8-ary Phase Shift Keying (8PSK), respectively. Figure 1.1 shows the geometrical representation of these PSK signals.

![Figure 1.1: The signal space of (a) BPSK, (b) QPSK and (c) 8PSK.](image)

The signal information lies within these phases. Thus, the more phases the PSK signals have, the more binary information the PSK signals can represent. However, this causes a lower resistance of the PSK signals to noise.

1.2 Non-Gaussian Noise Models

The simplest and most frequently used additive noise model in digital transmission is
Gaussian noise [23, Chapter 5]. Other types of noise are usually referred to as non-Gaussian noise. There are many types of non-Gaussian noise models [17] such as the alpha-stable process [37],[42] and mixture process [21]. [34] shows that the electrical ignition circuits of vehicles have produced non-Gaussian noise that interferes with communication systems. This study also shows that the noise intensity is higher in urban areas that have more traffic, than in rural areas. Aside from vehicular interference, [35] shows that microwave ovens also generate non-Gaussian noise that interferes with signals at high frequency band, such as the 2.45 GHz used in the Industrial, Scientific and Medical (ISM) band. More studies about other non-Gaussian noise environments can be found in [8],[50]. Thus, non-Gaussian noise represents man-made atmospheric noise, for example microwave ovens and vehicular electrical ignition circuit interferences, and natural noise, such as thunder and ice-breaking interferences. Some of these non-Gaussian types of noise have burst characteristics, and are thus usually called impulsive noise [12].

One of the most acceptable non-Gaussian noise models thus chosen to be studied in this thesis is the so-called class $A$ impulsive noise model, introduced by Middleton [12]—[16]. The class A impulsive noise model is studied in many papers [29],[31],[46]—[49]. The analytical expressions of the performance of signals in class A impulsive noise models are derived in [10],[11],[27],[28],[30],[41],[43],[45]. In a recent paper by Middleton discussing his impulsive noise model [16], the class A impulsive noise model is compared with the alpha-stable noise model. The class A noise model is superior to the alpha-stable model, because it includes an additive Gaussian background component. Table I of [16] gives some examples of interferences that can be modelled as the class A impulsive noise model. Considering the acceptable and precise noise model for both man-made and natural inter-
ference, it is worth studying how the digital communication systems perform in class A impulsive noise environments.

1.3 Fading Models

It is well-known that when a signal propagates through a wireless channel, it is reflected and scattered, and arrives at the receiver with a slight time differential [23, p. 800]. The received signal is a combined version of these reflected and scattered signals. The received signal’s envelope and phase fluctuate over time. This phenomenon is called fading [51],[54]. In this thesis, fading is modelled by using various types of statistical envelope probabilities, while the fluctuation of the phase of the received signal is assumed to be compensated for by a perfectly coherent detection system.

The fading models used in this thesis are the Rayleigh, Rician and Nakagami fading. The Rayleigh fading is used to model multipath fading that doesn’t have a line-of-sight path. The probability density function (PDF) of the Rayleigh fading envelope $r$ is well-known to be given by [23, p. 44]

$$p_r(r) = \frac{r}{\sigma^2} \exp \left( -\frac{r^2}{2\sigma^2} \right)$$  \hspace{1cm} (1.1)$$

where $r \geq 0$ and $r^2 = 2\sigma^2$ where $\langle . \rangle$ denotes an average of $(.)$. The Rician fading is used to model the multipath fading that has a line-of-sight path and the PDF of the Rician fading envelope $r$ can be expressed as [23, p. 46]

$$p_r(r) = \frac{r}{\sigma^2} \exp \left( -\frac{r^2 + A_r^2}{2\sigma^2} \right) I_0 \left( \frac{rA_r}{\sigma^2} \right)$$  \hspace{1cm} (1.2)$$
where \( r \geq 0 \), \( A_r \) is the specular amplitude, \( \sigma^2 \) is the Gaussian’s variance and \( I_0(.) \) is the zero-order modified Bessel function of the first kind. For the special case of the Rician PDF with \( A_r = 0 \), Equation 1.2 simplifies to Equation 1.1, which is the Rayleigh PDF. The Nakagami-\( m \) fading, also used to model the multipath fading channel that has no line-of-sight path, is more general than the Rayleigh fading model, because it has the parameter \( m \) to control the severity of the fading. The smaller the value of \( m \) is, the more severe the Nakagami fading becomes. For the PDF of the Nakagami fading envelope \( r \), it is known that [23, p. 47]

\[
\begin{align*}
    p_r(r) &= \frac{2}{\Gamma(m)} \left( \frac{m}{\gamma} \right)^m r^{2m-1} \exp \left( -\frac{mr^2}{\gamma} \right) \\
    \end{align*}
\]

where \( r \geq 0 \), \( \gamma = r^2 \) and \( 0.5 \leq m < \infty \). When the parameter \( m = 1 \), Equation 1.3 reduces to Equation 1.1 that is the Rayleigh PDF.

These fading channels degrade the signal performance substantially [23, Chapter 14] depending on the type of fading. For example, the Rician fading channel with \( A_r > 0 \) does not degrade the signal performance as much as the Rayleigh fading channel, because of the line-of-sight path.

### 1.4 Diversity Combining Techniques

One of the most widely used techniques to combat fading is the diversity technique [22, Chapter 9]. By employing a combination of antennas (i.e. two or more), the received diversity signal usually gives a higher signal-to-noise ratio (SNR) than from one received signal alone.

The diversity combining techniques used in this thesis are the Selection Combining
(SC) [4],[7],[18],[28],[32],[41], and Equal-Gain Combining (EGC) [1]−[3],[5],[33],[38],[39] techniques. These two techniques give very good performance improvement and are easily implemented in the receiver hardware [51, Chapter 6]. In the SC technique, the receiver chooses the instantaneous signal from an antenna that has the highest fading envelope and discards the rest of the received signals from the other antennas. In the EGC technique, the receiver combines all received signals and makes a decision from this combined signal. In a case where there are two receiver antennas in the EGC diversity, the system is called the dual-branch ($L = 2$) EGC diversity.

1.5 Research Objective and Thesis Organization

In the past, there have been relatively few publications dealing with the subject of the performance evaluation of digital communication systems in the presence of class A impulsive noise (see Section 3.1) and fading (see Section 4.1) with diversity reception (see Section 5.1). Furthermore, to the best of our knowledge, the subject of diversity reception over impulsive channels has not been investigated in the open technical literature. Motivated by these observations, in this thesis we make the following contributions:

1) theoretical expressions of the error rate performance of PSK signals in class A impulsive noise, with or without diversity reception;

2) analytical expressions of the performance of BPSK in class A impulsive noise and different classes of fading;

3) performance evaluations of BPSK in class A impulsive noise and different kinds of fading, with the SC or EGC diversity reception.
Including this chapter, this thesis is composed of six chapters and an appendix. After this introductory chapter, the organization of this thesis is as follows.

Chapter 2 introduces the system model description and computer simulation methodology of the PSK signals in class A impulsive noise and fading, with diversity reception. Furthermore, the PDFs of class A impulsive noise in the SC and EGC diversity reception are investigated and derived.

Chapter 3 is based on the previously derived PDFs, and consists of derivations of analytical expressions of the error rate performance of BPSK and MPSK in impulsive noise with or without diversity reception.

Chapter 4 considers the effect of the fading channel, and comprises derivations of the performance of BPSK in the class A impulsive noise and the Rayleigh, Rician or Nakagami fading.

Chapter 5 is composed of derivations for the expressions of the performance evaluation of BPSK in the class A impulsive noise and different classes of fading channels with the SC or EGC diversity reception.

Chapter 6 concludes this thesis, and suggests future work.

Finally, in Appendix A, some useful theoretical derivations are given.
CHAPTER 2
System Impulsive Noise and Simulation Modelling

2.1 Introduction

This chapter presents the system design and simulation model of a PSK system assuming diversity combining reception with fading and impulsive noise. Furthermore, impulsive noise modelling in the receiver without ($L = 1$) or with dual-branch ($L = 2$) EGC diversity reception is investigated and novel expressions of the PDF of the sum of two impulsive noises are derived. The organization of this chapter is as follows. Section 2.2 presents the PSK system model in fading and impulsive noise with the dual-branch diversity reception. In Section 2.3, the impulsive noise in the receiver without and with dual-branch EGC diversity reception is modelled and the results are evaluated. Section 2.4 presents the employed computer simulation methodology. Conclusions can be found in Section 2.5.

2.2 System Model Description

The block diagram of the system model of PSK systems in dual-branch diversity combining techniques with fading and impulsive noise is illustrated in Figure 2.1. It consists of an MPSK encoder (a binary to phase converter), multiplicative fading and additive impulsive noise channels, a diversity combining device, a phase detector, a decision device and an MPSK decoder (a phase to binary converter). The MPSK encoder converts the
2.2 System Model Description

binary sequences $a_k$ into transmitted MPSK signal $S_k$ that can be expressed as

$$S_k = \sqrt{E_s} \exp(j\theta_s)$$  \hspace{1cm} (2.1)$$

where $E_s = \text{Energy/symbol}$, $\theta_s = 2(m_s - 1)\pi/M$ and $m_s = 1, 2, \ldots, M$. The signal $S_k$ is passed through two independent multiplicative fading and additive impulsive noise channels. Each channel multiplies the signal $S_k$ with the fading envelope $r_i$ ($i = 1, 2$)$^1$, and adds the signal $S_k$ with impulsive noise $\eta_i \exp(j\theta_{\eta_i})$. Then, each received signal $R_i \exp(j\theta_{R_i})$ enters the diversity combining device. In the SC system, the diversity combining device selects one of the received signals with the largest fading envelope $r_i$ to be the output. In the EGC system, the diversity combining device is merely a combiner, with its output being the sum of the two received signals. This output signal $R \exp(j\theta_R)$ is passed through the phase detector. The phase $\theta_R$ is passed through the decision device to determine and regenerate the signal $\hat{S}_k$. The regenerated signal $\hat{S}_k$ is passed through the MPSK decoder to convert the phase to the binary sequences $\hat{a}_k$.

![Figure 2.1: System model of PSK systems in fading and impulsive noise with dual-branch diversity reception.](image)

The noise in the SC diversity receiver is composed of one impulsive noise, no matter

$^1$From now on $i = 1, 2$, unless otherwise is stated.
how many branches of diversity reception are used. Therefore, the performance evaluation of SC diversity can be studied up to $L$ branches of diversity. The results are investigated only up to $L = 4$, because of the difficulty in obtaining the theoretical and simulation results for $L > 4$. On the other hand, the noise in the EGC diversity receiver comprises the sum of impulsive noises from all channels. Thus, the PDF of impulsive noise in the EGC diversity reception must be numerically calculated for each total number of diversity. In this thesis, only the performance evaluation of dual-branch EGC diversity is studied, because the highest diversity gain is obtained when the diversity branch increases from $L=1$ to 2, and only the PDF of the sum of two impulsive noises is derived. The PDF of one impulsive noise and the PDF of the sum of two impulsive noises are investigated in the next section.

### 2.3 Impulsive Noise Modelling

Middleton's class A impulsive noise represents electromagnetic (EM) interference that has bandwidth comparable to or narrower than the receiver. The PDF of the impulsive noise envelope $\eta_i$ given by Equation 84 of [16] is expressed as

$$p_{\eta_i}(\eta_i) = \exp(-A_i) \sum_{m_i=0}^{\infty} \frac{A_i^{m_i}}{m_i! \sigma_{m_i}^2} \exp\left(-\frac{\eta_i^2}{2\sigma_{m_i}^2}\right)$$

(2.2)

where $\eta_i \geq 0$, $\sigma_{m_i}^2 = (m_i/A_i+\Gamma'_i)/(1+\Gamma'_i)$, $A_i$ is impulsive index and $\Gamma'_i$ is the Gaussian factor. $A_i$ is the product of the received average number of impulses per unit time, multiplied by the mean duration of the impulses. The value of $A_i$ measures how non-Gaussian the noise is. On the other hand, when the value of $A_i$ is small (i.e. $A_i \ll 10$), the noise events
and durations are less, and the noise becomes more impulsive or highly non-Gaussian. On the other hand, when the value of $A_i$ is large (e.g. $A_i > 10$), the noise tends to be more Gaussian. For instance, $A_i = 10$ means that non-Gaussian noise acts similarly to Gaussian noise. In contrast, $A_i \ll 10$, such as 1, 0.35 and 0.01, means that the non-Gaussian noise is more impulsive. $\Gamma_i' = \sigma_G^2/\Omega_{2A}$ is the mean power ratio of the Gaussian noise component $\sigma_G^2$ to the non-Gaussian noise component $\Omega_{2A}$. The lower the value $\Gamma_i'$ is, the more non-Gaussian the noise becomes.

The PDF of the impulsive noise phase $\theta_{\eta_i}$ is uniformly distributed in $(0,2\pi]$ and given by

$$p_{\theta_{\eta_i}}(\theta_{\eta_i}) = \frac{1}{2\pi}.$$ (2.3)

Using Equations 2.2 and 2.3 leads to the joint PDF between the impulsive noise envelope $\eta_1$ and the impulsive noise phase $\theta_{\eta_1}$

$$p_{\eta_1, \theta_{\eta_1}}(\eta_1, \theta_{\eta_1}) = \exp(-A_1) \sum_{m_1=0}^{\infty} \frac{A_1^{m_1}}{m_1!} \frac{\eta_1}{2\pi \sigma_{m_1}^2} \exp \left( -\frac{\eta_1^2}{2\sigma_{m_1}^2} \right).$$ (2.4)

This joint PDF of the impulsive noise is used in Chapter 3 to derive the performance of MPSK in impulsive noise without EGC diversity reception.

The PDF of the in-phase ($I$) component of impulsive noise $\eta_{II} = \text{Re}\{\eta_1 \exp(j\theta_{\eta_1})\}$, given by Equation 7 of [10], can be expressed as

$$p_{\eta_{II}}(\eta_{II}) = \exp(-A_1) \sum_{m_1=0}^{\infty} \frac{A_1^{m_1}}{m_1!} \frac{1}{\sqrt{2\pi} \sigma_{m_1}^2} \exp \left( -\frac{\eta_{II}^2}{2\sigma_{m_1}^2} \right)$$

$$= \exp(-A_1) \sum_{m_1=0}^{\infty} \frac{A_1^{m_1} \sqrt{G_1}}{m_1! \sqrt{\pi}} \exp \left( -G_1 \eta_{II}^2 \right)$$ (2.5)
where $G_1 = 1/(2\sigma^2_{\eta_1})$ and $-\infty \leq \eta_1 \leq \infty$. This PDF of the $I$-component of impulsive noise is used in Chapter 3 to derive an analytical expression of the error rate performance of BPSK in impulsive noise without EGC diversity reception. It is interesting to note that this PDF of the $I$-component of impulsive noise is intentionally modelled on a one-dimensional scheme impulsive noise only. This PDF cannot be used to find the PDF of an envelope, which is a two-dimensional scheme noise, of impulsive noise, because the in-phase ($I$) and quadrature ($Q$) components of impulsive noise are dependent.

In a dual-branch EGC diversity system, the impulsive noise in the receiver is the sum of two impulsive noises and can be expressed as

$$\eta \exp(j\theta_{\eta}) = \eta_1 \exp(j\theta_{\eta_1}) + \eta_2 \exp(j\theta_{\eta_2}) \quad (2.6)$$

where $\eta$ is the envelope of the sum of two impulsive noises and $\theta_{\eta}$ is the phase of the sum of two impulsive noises. As shown in [6], the phase $\theta_{\eta}$ of the sum of random vectors is still uniformly distributed in $(0,2\pi]$ so that its PDF is given by

$$p_{\theta_{\eta}}(\theta_{\eta}) = \frac{1}{2\pi}. \quad (2.7)$$

Using Equations 9 and 10 of [6] leads to the PDF of the envelope of the sum of two impulsive noises

$$p_{\eta}(\eta) = \eta \int_0^\infty \rho J_0(\eta\rho) \Lambda(\rho) \, d\rho \quad (2.8)$$

where

$$\Lambda(\rho) = E_{\eta_1,\eta_2} \left[ \prod_{t=1}^2 J_0(\eta_t \rho) \right] = E_{\eta_1} \left[ J_0(\eta_1 \rho) \right] E_{\eta_2} \left[ J_0(\eta_2 \rho) \right] \quad (2.9)$$
and $\eta_1$, $\eta_2$ are independent. The term $E_{\eta_1} [J_0(\eta_1\rho)]$ can be rewritten as

$$E_{\eta_1} [J_0(\eta_1\rho)] = \int_{-\infty}^{\infty} J_0(\eta_1\rho) p_{\eta_1}(\eta_1) \, d\eta_1$$

$$= \exp(-A_i) \sum_{m_i=0}^{\infty} \frac{A_i^{m_i}}{m_i!} 2G_i \int_0^{\infty} \eta_i J_0(\eta_i\rho) \exp \left(-G_i \eta_i^2\right) \, d\eta_i$$  \hspace{1cm} (2.10)

where $G_i = 1/\sigma_{m_i}^2$. Applying Equation 6.631.1 of [19, p. 698] leads to the following equation

$$E_{\eta_1} [J_0(\eta_1\rho)] = \exp(-A_i) \sum_{m_i=0}^{\infty} \frac{A_i^{m_i}}{m_i!} \exp \left(-\frac{\rho^2}{4G_i}\right).$$  \hspace{1cm} (2.11)

Substituting the above equation into Equation 2.9 we can obtain

$$\Lambda(\rho) = \exp[-(A_1 + A_2)] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1! m_2!} \exp \left(-\frac{\rho^2}{4C_I}\right)$$  \hspace{1cm} (2.12)

where $C_I = G_1 G_2 / (G_1 + G_2) = 1/[2(\sigma_{m_1}^2 + \sigma_{m_2}^2)]$ and thus, Equation 2.8 can be rewritten as

$$p_{\eta}(\eta) = \exp[-(A_1 + A_2)] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1! m_2!} \int_0^{\infty} \rho J_0(\eta \rho) \exp \left(-\frac{\rho^2}{4C_I}\right) \, d\rho.$$  \hspace{1cm} (2.13)

By using Equation 6.631.1 of [19, p. 698], the above equation yields the PDF of the envelope of the sum of two impulsive noises

$$p_{\eta}(\eta) = \exp[-(A_1 + A_2)] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1! m_2!} 2\eta C_I \exp(-\eta^2 C_I).$$  \hspace{1cm} (2.14)

Using Equations 2.7 and 2.14 leads to the joint PDF between the envelope $\eta$ and phase $\theta_\eta$.
2.3 Impulsive Noise Modelling

of the sum of two impulsive noises

\[ p_{\eta,\theta_\eta}(\eta, \theta_\eta) = \exp(-(A_1 + A_2)) \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1! m_2!} \eta C_I \exp(-\eta^2 C_I). \quad (2.15) \]

This joint PDF is used in Chapter 3 to derive an analytical expression of the performance of MPSK in impulsive noise with dual-branch EGC diversity reception.

Furthermore, the PDF of the \( I \)-component of the sum of two impulsive noises \( \eta_I = \Re\{\eta \exp(j\theta_\eta)\} = \Re\{\eta_1 \exp(j\theta_{\eta_1})\} + \Re\{\eta_2 \exp(j\theta_{\eta_2})\} = \eta_{I1} + \eta_{I2} \) can be expressed as

\[ p_{\eta_I}(\eta_I) = \int_{-\infty}^{\infty} p_{\eta_{I1}, \eta_{I2}}(\eta_I - x, x) \, dx \]
\[ = \exp(-(A_1 + A_2)) \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2} \sqrt{G_I G_2}}{m_1! m_2!} \exp(-G_I \eta_I^2) \]
\[ \times \int_{-\infty}^{\infty} \exp\left[-(G_1 + G_2)x^2 + 2\eta_I x G_1\right] \, dx \]
\[ = \exp(-(A_1 + A_2)) \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2} \sqrt{C_I}}{m_1! m_2!} \exp(-C_I \eta_I^2). \quad (2.16) \]

The above PDF is used in Chapter 3 to derive an analytical expression of the error rate performance of BPSK in impulsive noise with dual-branch EGC diversity reception.

Figures 2.2 and 2.3 illustrate the PDF of the impulsive noise envelope (see Equation 2.2) for different values of \( \Gamma' \) and \( A \).² It can be seen that, when values of \( \Gamma' \) or \( A \) are high, i.e. \( \Gamma' \geq 100 \) or \( A \geq 10 \), the PDF of the impulsive noise envelope is closed to the PDF of Rayleigh. On the other hand, when values of \( \Gamma' \) or \( A \) are low, i.e. \( \Gamma' \ll 100 \) or \( A \ll 10 \), the impulsive noise becomes more impulsive and has a long PDF tail. The values of \( \Gamma' = 10^{-4} - 10^{-3} \) are frequently used to represent the highly non-Gaussian noise

²From now on \( \Gamma' = \Gamma'_1 = \Gamma'_2 \) and \( A = A_1 = A_2 \), unless \( \Gamma'_i \) and \( A_i \) are assigned specifically.
characteristic (e.g. [10]). Simulation results, which are obtained by means of computer simulation (see the next section), are seen to very well match the theoretical curves.

Figures 2.4 and 2.5 show the PDF of the $I$-component of impulsive noise (see Equation 2.5) for different values of $\Gamma'$ and $A$. As it can be seen from the plots, similar remarks to Figures 2.2 and 2.3 can be made so that when the values of $\Gamma'$ or $A$ are high, the PDF of the $I$-component of impulsive noise is very similar to the PDF of Gaussian. On the other hand, when the values of $\Gamma'$ or $A$ are low, the impulsive noise becomes more impulsive and has a long PDF tail. Simulation results obtained by computer simulation can be seen to match the theoretical curves very well.

In order to see how the dual-branch EGC reception influences the impulsive noise, the PDF of the envelope of the sum of two impulsive noises (see Equation 2.14) is compared with the PDF of one impulsive noise envelope (see Equation 2.2). However, the PDF of the sum of two impulsive noises cannot be directly compared with the PDF of one impulsive noise, because the power of the sum of two impulsive noises is twice as much as the power of one impulsive noise. Thus, the envelope of the sum of two impulsive noises $\eta$ is needed to be normalized and the PDF of the normalized envelope of the sum of two impulsive noises $\tilde{\eta} = \eta / \sqrt{2}$ is obtained by

$$p_{\tilde{\eta}}(\tilde{\eta}) = \sqrt{2} p_{\eta} \left( \sqrt{2} \tilde{\eta} \right) = \exp[-(A_1 + A_2)] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1! m_2!} 4\tilde{\eta} C_I \exp(-2\tilde{\eta}^2 C_I). \quad (2.17)$$

Figure 2.6 shows the PDF of the impulsive noise envelope (see Equation 2.2) versus the PDF of the normalized envelope of the sum of two impulsive noises (see Equation 2.17) for $\Gamma' = 10^{-4}$ and different values of $A$. It can be seen that, for the value of $A = 10$, $p_{\tilde{\eta}}(\tilde{\eta})$
is slightly lower than $p_{\eta_i}(\eta_i)$ at high values of the envelope. On the other hand, $p_{\eta_i}(\eta)$ is identical to $p_{\eta_i}(\eta_i)$ at low values of the envelope. For low values of $A$ (e.g. $A \ll 10$), $p_{\eta_i}(\eta)$ is largely lower than $p_{\eta_i}(\eta_i)$ at high values of the envelope. On the other hand, $p_{\eta_i}(\eta)$ is higher than $p_{\eta_i}(\eta_i)$ at low values of the envelope. Thus, it can be concluded that the summation of impulsive noises reduces the PDF of their sum at high values of the envelope and increases the PDF of their sum at low values of the envelope.

Similarly, in order to compare the PDF of the $I$-component of impulsive noise (see Equation 2.5) and the PDF of the $I$-component of the sum of two impulsive noises (see Equation 2.16), $\eta_i$ needs to be normalized. The PDF of the normalized $I$-component of the sum of two impulsive noises $\tilde{\eta}_i = \eta_i/\sqrt{2}$ can be expressed as

$$p_{\tilde{\eta}_i}(\tilde{\eta}_i) = \sqrt{2} p_{\eta_i}(\sqrt{2}\tilde{\eta}_i) = \exp[-(A_1 + A_2)] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2} \sqrt{2C_i}}{m_1! m_2! \sqrt{\pi}} \exp(-2\tilde{\eta}_i^2 C_i). \quad (2.18)$$

Figure 2.7 compares the PDF of the $I$-component of impulsive noise $p_{\eta_{1I}}(\eta_{1I})$ and the PDF of the normalized $I$-component of the sum of two impulsive noises $p_{\tilde{\eta}_i}(\tilde{\eta}_i)$ for $\Gamma' = 10^{-3}$ and different values of $A$. The plot illustrates that for the value of $A = 10$, $p_{\tilde{\eta}_i}(\tilde{\eta}_i)$ is slightly lower than $p_{\eta_{1I}}(\eta_{1I})$ at high values of the amplitude. On the other hand, $p_{\tilde{\eta}_i}(\tilde{\eta}_i)$ is identical to $p_{\eta_{1I}}(\eta_{1I})$ at low values of the amplitude. For low values of $A$ (i.e. $A \ll 10$), $p_{\tilde{\eta}_i}(\tilde{\eta}_i)$ is largely lower than $p_{\eta_{1I}}(\eta_{1I})$ at high values of the amplitude. On the other hand, $p_{\tilde{\eta}_i}(\tilde{\eta}_i)$ is higher than $p_{\eta_{1I}}(\eta_{1I})$ at low values of the amplitude. Similar comments to those in Figure 2.6 can also be made to illustrate that the summation of impulsive noise reduces the PDF of the in-phase component of the sum of two impulsive noises at high values of the amplitude and increases the PDF of the in-phase component of the sum of two impulsive
2.3 Impulsive Noise Modelling

Figure 2.2: PDF of the impulsive noise envelope $\eta_1$ for $\Gamma' = 10^{-4}$ and different values of $A$.

Figure 2.3: PDF of the impulsive noise envelope $\eta_1$ for $A = 0.01$ and different values of $\Gamma'$. 
2.3 Impulsive Noise Modelling

Figure 2.4: PDF of the in-phase amplitude component for $\Gamma' = 10^{-3}$ and different values of $A$.

Figure 2.5: PDF of the in-phase amplitude component for $A = 0.01$ and different values of $\Gamma'$. 
2.3 Impulsive Noise Modelling

Figure 2.6: Comparison between the PDF of the envelope of impulsive noise $p_{\bar{\eta}}(\eta)$ and the PDF of the normalized envelope of the sum of two impulsive noises $p_{\bar{\eta}}(\bar{\eta})$ for $\Gamma' = 10^{-4}$ and different values of $A$.

Figure 2.7: Comparison between the PDF of the in-phase amplitude component $p_{\eta_{1}}(\eta_{1})$ and the PDF of the normalized in-phase amplitude component of the sum of two impulsive noises $p_{\bar{\eta}}(\bar{\eta})$ for $\Gamma' = 10^{-4}$ and different values of $A$. 
noises at low values of the amplitude.

It should be noted that in the case of Gaussian noise, the PDF of the normalized sum of two (or more) Gaussian noises is identical to the PDF of one Gaussian noise.

### 2.4 Computer Simulation Methodology

This section presents the computer simulation methodology used to evaluate the performance of the PSK systems [24, Section 7.3.1] in impulsive noise and fading with diversity reception. The program codes for the simulation are written in C++ language, which can be run on both PC (i.e. Borland C++) and Unix. The simulation results are checked against the theoretical results on the same figure, drawn by using Matlab program. The basic idea behind the simulation of the performance evaluation is to generate the random variables, that have the required PDF, of impulsive noise and fading.

In order to generate the correct random variable, the percentile transformation method [9, p. 226], [24, Section 2.2] is used, so any random variable can be generated with its cumulative distribution function (CDF). To generate the impulsive noise, the CDF of the class A impulsive noise envelope given by Equation 85 of [16] is used. However, this method requires the inverse of the CDF, which is difficult to derive. Thus, the trial-and-error method is used to evaluate the random variable with the percentile transformation method.

For the fading simulator, the Rayleigh fading random variable is generated by using the method of an exact doppler spread (MEDS) of [36, Section 5.1.6]. The random variables generated using this method have the Rayleigh PDF and Jake's power spectral density (PSD) [54, p. 19]. Then, the Rician fading random variable is generated from the
2.5 Conclusions

Rayleigh fading random variable added by a constant. The Nakagami fading random variable is generated by using the method in [52]. Furthermore, a simpler method of generating the fading random variable is to use the same method as the impulsive noise generation. This necessitates the writing of fewer program codes and less simulation run time.

2.5 Conclusions

In this chapter, the PSK system model in fading and impulsive noise with diversity combining reception is presented in Section 2.2. In Section 2.3, the PDFs of impulsive noise with non-diversity reception are evaluated and verified with computer simulation results. Moreover, novel expressions of the PDF of the envelope and in-phase component of the sum of two impulsive noises are derived and evaluated. The results show that the sum of impulsive noises gives lower PDFs at high values of the envelope and in-phase amplitude component of impulsive noise, but it gives higher PDFs at low values of the envelope and in-phase amplitude component of impulsive noise. Finally, Section 2.4 presents the simulation methodology.
CHAPTER 3
Performance in Impulsive Noise with Diversity Reception

3.1 Introduction

For over two decades, since Middleton introduced the non-Gaussian noise model [12], relatively few researchers have published work on performance evaluations of various modulation schemes in impulsive noise. For instance, Spaulding et al. present the performance evaluations of BPSK, CBFSK [10] and NCBFSK [11] in impulsive noise. Then, Seo et al. [27] derive the analytical expression of the performance of MQAM in class A impulsive noise by considering the independence of the I and Q components. On the other hand, Miyamoto et al. [45] derive the theoretical expression of the performance of QAM in class A impulsive noise by considering the dependence of the I and Q components. In [43], Kosmopoulos et al. present the performance evaluation of MQAM in the presence of combined Gaussian and non-Gaussian noise. Prasad et al. [41] give the analytical expression of the performance of DPSK in class A impulsive noise. However, the performance evaluation of modulation scheme in impulsive noise with diversity reception has not been studied in any previous paper.

This chapter presents an analytical expression of the performance of BPSK in impulsive noise without diversity reception. Furthermore, it presents a novel theoretical expression of the performance of BPSK in impulsive noise with dual-branch EGC diversity reception, and the new integral representations of the performance of MPSK in impulsive noise without or with dual-branch EGC diversity reception. The organization of this
chapter is as follows. Section 3.2 presents the analytical expression of the performance of BPSK without diversity reception in impulsive noise. In Section 3.3, the derivation of the analytical expression of the performance of BPSK with dual-branch EGC diversity reception in impulsive noise is presented. Section 3.4 presents the integral representation of the performance of MPSK without diversity reception in impulsive noise. In Section 3.5, the integral representation of the performance of MPSK with dual-branch EGC diversity reception in impulsive noise is derived. Conclusions are presented in Section 3.6.

3.2 BPSK without Diversity

The performance of BPSK in impulsive noise without diversity reception is easily derived with the knowledge of the PDF of the in-phase component of impulsive noise (see Equation 2.5) and can be expressed as

$$P_{e,BPSK} = Pr(R_{11} \leq 0) = Pr(\eta_{11} \leq -\sqrt{E_b}) = \int_{-\infty}^{-\sqrt{E_b}} p_{\eta_{11}}(x) \, dx$$

where $R_{11} = Re\{R_1 e^{j\theta R_1}\}$, assuming that signal $S_k = \sqrt{E_b}$ where $E_b =$ Energy/bit. Changing variable $z = -x/\sqrt{2\sigma_{m_1}^2}$, the above equation becomes

$$P_{e,BPSK} = \frac{1}{2} \exp(-A_1) \sum_{m_1=0}^{\infty} \frac{A_{1m_1}^{m_1}}{m_1! \sqrt{2\pi\sigma_{m_1}^2}} \int_{-\infty}^{-\sqrt{E_b}} \exp\left( -\frac{x^2}{2\sigma_{m_1}^2} \right) \, dx$$

$$= \frac{1}{2} \exp(-A_1) \sum_{m_1=0}^{\infty} \frac{A_{1m_1}^{m_1}}{m_1!} \text{erfc} \left( \sqrt{\frac{E_b}{2\sigma_{m_1}^2}} \right)$$

(3.1)
The SNR/bit $\gamma_b$ and $E_b$ are related as follows:

$$\gamma_b = \frac{E_b}{2N_1} \quad (3.3)$$

where $N_1$ is the average power of the in-phase component of impulsive noise $\eta_{II}^2$ and can be calculated by

$$N_1 = \int_{-\infty}^{\infty} x^2 p_m(x) \, dx$$

$$= \exp(-A_1) \sum_{m_1=0}^{\infty} \frac{A_1^{m_1}}{m_1!} \frac{2}{\sqrt{2\pi\sigma_{m_1}^2}} \int_{0}^{\infty} x^2 \exp\left(-\frac{x^2}{2\sigma_{m_1}^2}\right) \, dx$$

$$= \exp(-A_1) \sum_{m_1=0}^{\infty} \frac{A_1^{m_1}\sigma_{m_1}^2}{m_1!} \quad (3.4)$$

Using Equations 3.2 and 3.3, the performance of BPSK in impulsive noise without diversity reception can be expressed mathematically as

$$P_{e,BPSK}^I = \frac{1}{2} \exp(-A_1) \sum_{m_1=0}^{\infty} \frac{A_1^{m_1}}{m_1!} \text{erfc}\left(\sqrt{\frac{N_1\gamma_b}{\sigma_{m_1}^2}}\right) \quad (3.5)$$

Figure 3.1 illustrates the bit error rate (BER) of BPSK in class A impulsive noise for $\Gamma' = 10^{-4}$ and different values of $A$ without diversity reception. As can be observed, on one hand, for $A = 10$ the performance of BPSK in impulsive noise is very close to the performance of BPSK in Gaussian noise. On the other, for low values of $A$ (e.g. $A \ll 10$), the performance of BPSK in impulsive noise is better than in Gaussian at low values of SNR, but poorer at high values of SNR. The BERs obtained by simulation are also included and are seen to match the theoretical curves very well. This plot is identical to
the plot in Figure 10 using Equation 30 of [10], although the two analytical expressions of
the performance of BPSK in impulsive noise are derived using different approaches.

3.3 BPSK with Dual-Branch EGC Diversity

As in the previous section, the performance of BPSK in impulsive noise with dual-
branch EGC diversity reception can easily be derived with the knowledge of the PDF of
the in-phase component of the sum of two impulsive noises (see Equation 2.16). It can be
expressed as

\[
P_{e,BPSK}^{H-EGC} = Pr(R_I \leq 0) = Pr \left( \eta_I \leq -2\sqrt{E_b} \right) = \int_{-\infty}^{-2\sqrt{E_b}} p_{\eta_I}(y) dy
\]

\[
= \exp[-(A_1 + A_2)] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2} \sqrt{C_I}}{m_1! m_2! \sqrt{\pi}} \int_{-\infty}^{-2\sqrt{E_b}} \exp(-C_I y^2) dy
\] (3.6)
3.3 BPSK with Dual-Branch EGC Diversity

where \( R_l = \text{Re}\{R e^{j\theta}\} \). Changing variable \( t = -\sqrt{C_i} y \), Equation 3.6 becomes

\[
P_{e,\text{BPSK}}^{\text{II-EGC}} = \exp\left[-(A_1 + A_2)\right] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1! m_2! \sqrt{\pi}} \int_{2\sqrt{C_i E_b}}^{\infty} \exp\left(-t^2\right) dt
\]

\[
= \frac{1}{2} \exp\left[-(A_1 + A_2)\right] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1! m_2!} \text{erfc}\left(\sqrt{4C_i E_b}\right).
\]

(3.7)

The \( E_b \) and \( \gamma_b \) are related as follows:

\[
\gamma_b = \gamma_1 + \gamma_2 = \frac{E_b}{2N_1} + \frac{E_b}{2N_2}
\]

\[
E_b = \frac{2N_1 N_2 \gamma_b}{N_1 + N_2}
\]

(3.8)

where \( \gamma_1 \) and \( \gamma_2 \) are the SNR/bit for channels 1 and 2, respectively, and \( N_1 \) and \( N_2 \) are the average powers of the I-component of impulsive noise for channels 1 and 2, respectively, (see Equation 3.4). Using Equations 3.7 and 3.8, the performance of BPSK in impulsive noise with dual-branch EGC diversity is obtained as

\[
P_{e,\text{BPSK}}^{\text{II-EGC}} = \frac{1}{2} \exp\left[-(A_1 + A_2)\right] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1! m_2!} \text{erfc}\left(\sqrt{\frac{8N_1 N_2 \gamma_b C_i}{N_1 + N_2}}\right).
\]

(3.9)

Figures 3.2 and 3.3 show the BER of BPSK with dual-branch EGC reception in impulsive noise for \( \Gamma' = 10^{-4} \) and different values of \( A \). In Figure 3.2, the impulsive noises in both channels are identical and have the same values of \( A \) (i.e. \( A = A_1 = A_2 \)). In Figure 3.3, the impulsive noises are different for both channels and have different values of \( A \) (e.g. \( A_1 = 10, A_2 = 0.01 \)). It can be seen from the plots that the simulations match the analytical curves very well.
Figure 3.4 compares the performance of BPSK without diversity reception with the performance of BPSK with dual-branch EGC reception in impulsive noise for $\Gamma' = 10^{-4}$ and different values of $A$. The plot shows that the dual-branch EGC diversity reception improves the performance of BPSK in impulsive noise at high values of SNR, but it degrades the performance at low values of SNR. This is because the dual-branch EGC diversity increases the PDF of the $I$ component of impulsive noise at low values of amplitude, but it decreases the PDF of the $I$ component of impulsive noise at high values of amplitude (see Figure 2.7).

In the Gaussian noise case, it should be noted that the performance of BPSK with...
3.3 BPSK with Dual-Branch EGC Diversity

Figure 3.3: Performance of BPSK with dual-branch EGC diversity in impulsive noise for $\Gamma' = 10^{-4}$ and different values of $A$ in each channel ($A_1 = 10$, $A_2 = 0.01$).

Figure 3.4: Comparison of the performance of BPSK with dual-branch EGC and without diversity reception in impulsive noise for $\Gamma' = 10^{-4}$ and different values of $A$ ($A = A_1 = A_2$).
\( (L > 1) \) and without diversity reception in Gaussian noise are identical and can be expressed as

\[
P_{e_{e_{\text{BPSK}}}}^{L-\text{EGC}} = P_{e_{\text{BPSK}}}^{I} = \frac{1}{2} \text{erfc} (\sqrt{\gamma_b}). \tag{3.10}
\]

Equation 3.10 is derived in Appendix A.1.

### 3.4 MPSK without Diversity

In [20], an integral representation of the performance of MPSK in Gaussian noise is derived with the knowledge of the joint PDF of the envelope and phase of Gaussian noise. Similarly, for the impulsive noise case, an integral representation of the performance of MPSK in impulsive noise can be derived with the knowledge of its joint PDF of the envelope and phase (see Equation 2.4).

In Figure 3.5, a geometrical representation of the received signal with the envelope \( R \) and phase \( \pi - \Psi \) is composed of one of the MPSK signals with the envelope \( \sqrt{E_s} \) and phase \( \pi \) and impulsive noise with the envelope \( \eta_{\min 1} \) and phase \( \theta_{\eta 1} \). Using the law of sines, \( \eta_{\min 1} \) can be expressed as

\[
\eta_{\min 1} = \frac{\sqrt{E_s} \sin \Psi}{\sin(\theta_{\eta 1} + \Psi)}. \tag{3.11}
\]

![Figure 3.5: A geometrical representation of one of MPSK signals at point S with the envelope \( \sqrt{E_s} \) and phase \( \pi \) added with impulsive noise at point Z with the envelope \( \eta_{\min 1} \) and phase \( \theta_{\eta 1} \) to form the received signal with the envelope \( R \) and phase \( \pi - \Psi \). The -- - line is the decision boundary of the signal with the phase \( \pi \).]
An error occurs when the envelope of impulsive noise is \( \eta_1 \geq \eta_{\text{min1}} \) and the phase of impulsive noise is \(-\pi + \Psi \leq \theta_{\eta 1} \leq \pi - \Psi \). An integral representation of the performance of MPSK in impulsive noise without diversity can be expressed as

\[
P_{e,\text{MPSK}}^I = 2 \int_{\eta_{\text{min1}}}^{\pi - \Psi} \int_{\eta_{\text{min1}}}^{\infty} p(\eta_1, \theta_{\eta 1}) \, d\eta_1 \, d\theta_{\eta 1}
= 2 \int_{0}^{\pi - \Psi} \int_{\eta_{\text{min1}}}^{\infty} \exp(-A_1) \sum_{m_1=0}^{\infty} \frac{A_{1}^{m_1}}{m_1!} \frac{\eta_1}{2\pi \sigma_{m_1}^2} \exp\left(-\frac{\eta_{1}^2}{2\sigma_{m_1}^2}\right) \, d\eta_1 \, d\theta_{\eta 1}
= \int_{0}^{\pi - \Psi} \frac{1}{\pi} \exp(-A_1) \sum_{m_1=0}^{\infty} \frac{A_{1}^{m_1}}{m_1!} \exp\left(-\frac{\eta_{\text{min1}}^2}{2\sigma_{m_1}^2}\right) \, d\theta_{\eta 1}.
\]

Using Equation 3.11, the above equation becomes

\[
P_{e,\text{MPSK}}^I = \int_{0}^{\pi - \Psi} \frac{1}{\pi} \exp(-A_1) \sum_{m_1=0}^{\infty} \frac{A_{1}^{m_1}}{m_1!} \exp\left(-\frac{E_s \sin^2 \Psi}{2\sigma_{m_1}^2 (\Psi + \theta_{\eta 1})}\right) \, d\theta_{\eta 1}.
\]

The SNR/symbol \( \gamma_s \) and signal energy/symbol \( E_s \) are related as follow

\[
\gamma_s = \frac{E_s}{N_{\eta 1}}
\]

where the average power of impulsive noise \( N_{\eta 1} \) is expressed as

\[
N_{\eta 1} = \overline{\eta_1^2} = \int_{-\infty}^{\infty} \eta_1^2 \, p_{\eta_1}(\eta_1) \, d\eta_1
= \exp(-A_1) \sum_{m_1=0}^{\infty} \frac{A_{1}^{m_1}}{m_1!} \int_{0}^{\infty} \eta_1^3 \frac{\eta_1^3}{\sigma_{m_1}^2} \exp\left(-\frac{\eta_{1}^2}{2\sigma_{m_1}^2}\right) \, d\eta_1
= \exp(-A_1) \sum_{m_1=0}^{\infty} \frac{A_{1}^{m_1}}{m_1!} 2\sigma_{m_1}^2.
\]

Substituting Equation 3.14 into Equation 3.13 leads to the following integral representation
of the performance of MPSK in impulsive noise without diversity reception

\[
P_{e,MPSK}^I = \frac{1}{\pi} \exp(-A) \sum_{m_1=0}^{\infty} \frac{A_{1m_1}}{m_1!} \int_0^{\pi-\Psi} \exp \left( -\frac{\gamma_s N_{q1} \sin^2 \Psi}{2\sigma^2_{m_1} \sin^2(\Psi + \theta_{q1})} \right) d\theta_{q1}. \tag{3.16}
\]

In order to evaluate Equation 3.16, the Romberg integration of [40, p. 207] is used and the results are plotted in Figures 3.6-3.9. For the special case of \( M = 2 \) (BPSK), the results of Equation 3.16 are identical to the results of Equation 3.5. Similar remarks to those in Figure 3.1 can also be made in Figures 3.6-3.9. For example, as it can be seen from the plot, the performance of MPSK in impulsive noise for the high value of \( A \) (i.e. \( A = 10 \)) is very similar to the performance of MPSK in Gaussian noise. However, for low values of \( A \) (e.g. \( A \ll 10 \)), the performance of MPSK is improved at low values of SNR, but it is degraded at high values of SNR. Simulation results are also included in the plots and are

Figure 3.6: Performance of MPSK without diversity in impulsive noise for \( \Gamma' = 10^{-4} \) and \( A = 10 \).
3.4 MPSK without Diversity

Figure 3.7: Same caption as in Figure 3.6 but with $A = 1$.

Figure 3.8: Same caption as in Figure 3.6 but with $A = 0.35$. 
3.5 MPSK with Dual-Branch EGC Diversity

Using the same approach as in the previous section, an integral representation of the performance of MPSK in impulsive noise with dual-branch EGC diversity can be derived with the knowledge of the joint PDF of the envelope and phase of the sum of two impulsive noises (see Equation 2.15).

Similarly to Figure 3.5, Figure 3.10 also shows a geometrical representation of an MPSK signal, impulsive noise and received signal. However, now the received signal with the envelope $R$ and phase $\pi - \Psi$ is composed of the sum of two MPSK signals with the total envelope $2\sqrt{E_s}$ and phase $\pi$ and the sum of two impulsive noises with the total envelope
$\eta_{min2}$ and phase $\theta_\eta$. Comparable to the previous section by using the law of sines, $\eta_{min2}$ can be expressed as

$$\eta_{min2} = \frac{2\sqrt{E_s} \sin \Psi}{\sin(\theta_\eta + \Psi)} \quad (3.17)$$

Figure 3.10: A geometrical representation of the sum of two MPSK signals at point $S$ with the total envelope $2\sqrt{E_s}$ and phase $\pi$ added with the sum of two impulsive noises at point $Z$ with the total envelope $\eta_{min2}$ and phase $\theta_\eta$ to form the received signal with the envelope $R$ and phase $\pi - \Psi$. The dotted line is the decision boundary of the signal with the phase $\pi$.

Similar remarks to those used in Figure 3.5 can be made in Figure 3.10 that an error occurs, when the envelope of the sum of impulsive noises is $\eta \geq \eta_{min2}$ and the phase of the sum of impulsive noises is $-\pi + \Psi \leq \theta_\eta \leq \pi - \Psi$. The integral representation of the performance of MPSK in impulsive noise with dual-branch EGC diversity can be expressed as

$$P_{e,\text{MPSK}}^{II-\text{EGC}} = \frac{1}{\pi} \exp\left[-(A_1 + A_2)\right] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1! m_2!} \int_0^{\pi - \Psi} \int_{\eta_{min2}}^\infty \exp\left(-\eta^2 C_1\right) d\eta d\theta_\eta \quad (3.18)$$

Using Equation 3.17, the above equation becomes

$$P_{e,\text{MPSK}}^{II-\text{EGC}} = \frac{1}{\pi} \exp\left[-(A_1 + A_2)\right] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1! m_2!} \int_0^{\pi - \Psi} \exp\left(-\frac{4E_s C_1 \sin^2 \Psi}{\sin^2(\theta_\eta + \Psi)}\right) d\theta_\eta. \quad (3.19)$$
The $\gamma_s$ and $E_s$ are related as follow

\[ \gamma_s = \frac{4E_s}{N_\eta} \]  

(3.20)

where the average power of impulsive noise $N_\eta$ can be calculated by

\[ N_\eta = \overline{\eta^2} = \int_{-\infty}^{\infty} \eta^2 p_\eta(\eta) \, d\eta \]

\[ = \exp[-(A_1 + A_2)] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1! m_2!} 2C_1 \int_0^{\infty} \eta^3 \exp(-\eta^2 C_1) \, d\eta \]

\[ = \exp[-(A_1 + A_2)] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1! m_2! C_1} \]  

(3.21)

By using Equations 3.19 and 3.20 leads to the integral representation of the performance of MPSK with dual-branch EGC diversity in impulsive noise

\[ P_{e,\text{MPSK}}^{\text{II-EGC}} = \frac{1}{\pi} \exp[-(A_1 + A_2)] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1! m_2!} \int_0^{\pi-\Psi} \exp \left( -\frac{\gamma_s N_\eta C_1 \sin^2 \Psi}{\sin^2(\theta_\eta + \Psi)} \right) \, d\theta_\eta. \]

(3.22)

Similarly to Equation 3.16, the above equation is also evaluated using the Romberg integration and the results are plotted in Figures 3.11-3.14. For the special case of $M = 2$ (BPSK with diversity), the results of Equation 3.22 are identical to the results of Equation 3.9. Similar comments to Figure 3.4 can also be made to Figures 3.11-3.14 in that the dual-branch EGC diversity improves the performance of MPSK in impulsive noise at high values of SNR. However, the diversity reception degrades the performance of MPSK in impulsive noise at low values of SNR. This can be explained by noting the PDF of the envelope of the sum of two impulsive noises (see Figure 2.6). The EGC diversity reception increases the PDF of the envelope of impulsive noise at low values of amplitude, but
3.5 MPSK with Dual-Branch EGC Diversity

Figure 3.11: Performance of MPSK with dual-branch EGC diversity in impulsive noise for $\Gamma' = 10^{-4}$ and $A = 10$.

Figure 3.12: Same caption as in Figure 3.11 but with $A = 1$. 
3.5 MPSK with Dual-Branch EGC Diversity

Figure 3.13: Same caption as in Figure 3.11 but with $A = 0.35$.

Figure 3.14: Same caption as in Figure 3.11 but with $A = 0.01$. 
decreases the PDF of the envelope of impulsive noise at high values of amplitude. The
SERs obtained by computer simulations are seen to match very well to the analytical
curves.

3.6 Conclusions

The goal of this chapter is to provide the performance evaluation of PSK signals in
impulsive noise with or without diversity reception, especially the performance of modu-
lation schemes in impulsive noise with diversity reception that had not previously been
studied and presented in any paper. The alternatively derived performance evaluation of
BPSK in impulsive noise is presented. Then, the novel analytical expression of the error
rate performance of BPSK in impulsive noise with dual-branch EGC diversity reception
is derived. Furthermore, the new integral representations of the performance of MPSK in
impulsive noise with or without dual-branch EGC diversity reception are presented. The
results show that the EGC diversity reception improves the performance of modulation
schemes at high values of SNR. However, the EGC diversity reception degrades the per-
formance of modulation schemes at low values of SNR. In all cases, the theoretical results
are thoroughly validated using computer simulations.
CHAPTER 4
Performance in Fading and Impulsive Noise

4.1 Introduction

Although, in the past, the subject of performance evaluation of various modulation schemes in fading channels has been extensively investigated [23, p. 818],[25],[26],[53], relatively few researchers have studied their performance in a combination of fading and impulsive noise. To the best of our knowledge, in [30], an analytical expression is derived for the performance of high level QAM with Nakagami fading and non-Gaussian noise by considering the independence of the I and Q components.

This chapter presents novel analytical expressions for the performance of BPSK in Rayleigh, Rician or Nakagami fading with class A impulsive noise. The organization of the chapter is as follows. After this introduction, in Section 4.2, the analytical expression for the performance of BPSK with Rayleigh fading and impulsive noise is derived and evaluated. In Section 4.3, the derivation of the performance of BPSK with Rician fading and impulsive noise is presented. Section 4.4 presents an analytical expression of the performance of BPSK with Nakagami fading and impulsive noise. The conclusions of this chapter can be found in Section 4.5.

4.2 Rayleigh Fading

It is well-known that the performance of signals in the presence of fading can be derived by averaging the error rate probability $P_e$ of signal over the PDF of fading envelope square
4.2 Rayleigh Fading

\( \gamma = r^2 \) as shown below

\[
\bar{P}_e = \int_0^\infty P_e p_\gamma(\gamma) \, d\gamma. \tag{4.1}
\]

Using Equation 1.1, the PDF of \( \gamma \) for the Rayleigh fading channel is given by

\[
p_\gamma(\gamma) = \frac{p_r(\sqrt{\gamma})}{2\sqrt{\gamma}} = \frac{1}{\bar{\gamma}_b} \exp \left(-\frac{\gamma}{\bar{\gamma}_b}\right) \tag{4.2}
\]

where \( \bar{\gamma}_b = 2\sigma^2 = \text{SNR}/\text{bit} \). By averaging Equation 3.5 with Equation 4.2, the performance of BPSK in Rayleigh fading and impulsive noise can be expressed as

\[
P_{e,\text{Ray}}^{I,\text{BPSK}} = \int_0^\infty P_{e,\text{BPSK}} p_\gamma(\gamma) \, d\gamma
\]

\[
= \int_0^\infty \frac{1}{2} \exp(-A) \sum_{m_1=0}^{\infty} \frac{A^{m_1}}{m_1!} \text{erfc} \left( \sqrt{\frac{N_1}{2}} \right) \frac{1}{\bar{\gamma}_b} \exp \left(-\frac{\gamma}{\bar{\gamma}_b}\right) \, d\gamma
\]

\[
= \frac{1}{2} \exp(-A) \sum_{m_1=0}^{\infty} \frac{A^{m_1}}{m_1!} \frac{1}{\bar{\gamma}_b} \int_0^\infty \frac{2}{\sqrt{\pi}} \sqrt{B} \gamma \exp \left(-t^2\right) dt \exp \left(-\frac{\gamma}{\bar{\gamma}_b}\right) \, d\gamma \quad (4.3)
\]

where \( B = N_1/\sigma^2_{m_1} \). Changing variable \( t = \sqrt{\gamma} z \), Equation 4.3 can be rewritten as

\[
P_{e,\text{Ray}}^{I,\text{BPSK}} = \frac{1}{2} \exp(-A) \sum_{m_1=0}^{\infty} \frac{A^{m_1}}{m_1!} \frac{1}{\bar{\gamma}_b} \sqrt{\pi} \int_0^\infty \frac{2}{\sqrt{B}} \gamma^{3/2-1} \exp \left[- \left( z^2 + \frac{1}{\bar{\gamma}_b}\right) \gamma \right] \, d\gamma \, dz. \quad (4.4)
\]

Using Equation 3.381.4 of [19, p. 342], the above equation becomes

\[
P_{e,\text{BPSK}}^{I,\text{Ray}} = \frac{1}{2} \exp(-A) \sum_{m_1=0}^{\infty} \frac{A^{m_1}}{m_1!} \frac{1}{\bar{\gamma}_b} \sqrt{B} \int_0^\infty \frac{dz}{(z^2 + 1/\bar{\gamma}_b)^{1+1/2}}. \quad (4.5)
\]
Appendix A.2 presents the derivation for the closed-form solution for the integral

$$
\int_{a}^{\infty} \frac{dx}{(x^2 + b)^{c+1/2}} = 2^{2c-1} \left[ \frac{1}{2b} \left( 1 - \sqrt{\frac{a^2}{a^2 + b}} \right) \right]^{c-1} \sum_{k=0}^{c-1} \frac{1}{k!} \left[ \frac{1}{2} \left( 1 + \sqrt{\frac{a^2}{a^2 + b}} \right) \right]^k
\times \frac{\Gamma(k + c)\Gamma(c)}{\Gamma(2c)}
$$

(4.6)

where $a$ and $b$ are real numbers, and $c$ is a positive integer. Using Equation 4.6 into Equation 4.5 leads to the following compact expression of the performance of BPSK in Rayleigh fading and impulsive noise

$$
P_{e,\text{BPSK}}^{\gamma_{\text{Ray}}} = \frac{1}{2} \exp(-A) \sum_{m_1=0}^{\infty} \frac{A^{m_1}}{m_1!} \left( 1 - \sqrt{\frac{B}{B + 1/\gamma_b}} \right).$$

(4.7)

Figure 4.1 shows the BER (as a function of the SNR/bit) of BPSK in Rayleigh fading and
impulsive noise for $\Gamma' = 10^{-4}$ and different values of $A$. Simulation results are also included and very well validate the theoretical curves.

4.3 Rician Fading

Similar to the previous section, the performance of BPSK in Rician fading and impulsive noise can be derived by using the Equation 4.1. Using Equation 1.2, the PDF of Rician envelope square $\gamma = r^2$ is given by

$$
p_\gamma(\gamma) = \frac{p_r(\sqrt{\gamma})}{2\sqrt{\gamma}} = \frac{1 + K}{\overline{\gamma}_b} \exp \left( -\frac{\gamma(1 + K) + K\overline{\gamma}_b}{\overline{\gamma}_b} \right) I_0 \left( \sqrt{\frac{4(1 + K)K\gamma}{\overline{\gamma}_b}} \right) \tag{4.8}$$

where $\overline{\gamma}_b = A_r^2 + 2\sigma^2$ and the specular-to-random ratio $K = A_r^2/(2\sigma^2) = \text{power in steady component}/\text{power in random component}$. By averaging Equation 3.5 with Equation 4.8, the performance of BPSK in Rician fading and impulsive noise can be expressed as

$$
P_{e,BPSK}^{R} = \int_0^\infty P_{e,BPSK}^{R} p_\gamma(\gamma) \, d\gamma$$

$$= \int_0^\infty \frac{1}{2} \exp(-A) \sum_{m_1=0}^{\infty} \frac{A^{m_1}}{m_1!} \text{erfc} \left( \frac{\sqrt{B\gamma}}{\overline{\gamma}_b} \right) \frac{1 + K}{\overline{\gamma}_b} \exp \left( -\frac{\gamma(1 + K) + K\overline{\gamma}_b}{\overline{\gamma}_b} \right)$$

$$\times I_0 \left( \sqrt{\frac{4(1 + K)K\gamma}{\overline{\gamma}_b}} \right) d\gamma$$

$$= \frac{1}{2} \exp[-(A + K)] \sum_{m_1=0}^{\infty} \frac{A^{m_1} 1 + K}{m_1! \overline{\gamma}_b} \int_0^\infty \int_0^\infty \exp \left( -t^2 \right) dt$$

$$\times \exp \left( -\frac{\gamma(1 + K)}{\overline{\gamma}_b} \right) I_0 \left( \sqrt{\frac{4(1 + K)K\gamma}{\overline{\gamma}_b}} \right) d\gamma \tag{4.9}$$
where \( B = N_1 / \sigma_{m1}^2 \). Changing variable \( t = \sqrt{\gamma} z \) to the previous equation results in

\[
P_{e, BPSK}^{f, \text{Rice}} = \frac{1}{2} \exp[-(A + K)] \sum_{m_1=0}^{\infty} \frac{A^{m_1}}{m_1!} \frac{1 + K}{\gamma_b} \frac{2}{\sqrt{\pi}} \int_{\sqrt{B}}^{\infty} \int_0^\infty \gamma^{1/2} \times \exp \left[ - \left( z^2 + \frac{1 + K}{\gamma_b} \right) \gamma \right] I_0 \left( \sqrt{\frac{4(1 + K)K\gamma}{\gamma_b}} \right) d\gamma dz.
\]

(4.10)

Using Equation 6.643.2 of [19, p. 701], the above equation can be rewritten as

\[
P_{e, BPSK}^{f, \text{Rice}} = \frac{1}{2} \exp[-(A + K)] \sum_{m_1=0}^{\infty} \frac{A^{m_1}}{m_1!} \sqrt{\frac{1 + K}{K}} \sqrt{\gamma_b} \int_{\sqrt{B}}^{\infty} \exp \left[ \frac{(1 + K)K}{2(z^2\gamma_b + 1 + K)} \right] \times \frac{1}{z^2\gamma_b + 1 + K} M_{-1,0} \left( \frac{(1 + K)K}{z^2\gamma_b + 1 + K} \right) dz
\]

(4.11)

where \( M_{\lambda, \mu}(z) \) is the Whittaker function. Applying Equation 9.220.2 of [19, p. 1014] leads to the following equation

\[
P_{e, BPSK}^{f, \text{Rice}} = \frac{1}{2} \exp[-(A + K)] \sum_{m_1=0}^{\infty} \frac{A^{m_1}}{m_1!} (1 + K)^{1/2} \int_{\sqrt{B}}^{\infty} \frac{1}{z^2\gamma_b + 1 + K}^{3/2} \times \Gamma \left( \frac{3}{2} ; \frac{(1 + K)K}{z^2\gamma_b + 1 + K} \right) dz
\]

\[
= \frac{1}{2} \exp[-(A + K)] \sum_{m_1=0}^{\infty} \sum_{n=0}^{\infty} \frac{A^{m_1}}{m_1!} (1 + K)^{n+1} K^n \left( \frac{3}{2} \right)_n \frac{1}{(1)_n n!} \times \int_{\sqrt{B}}^{\infty} \frac{dz}{z^2 + (1 + K)/\gamma_b}^{n+1+1/2}
\]

(4.12)

where \( \Gamma(.,.) \) is a confluent hypergeometric function given by Equation 9.210.1 of [19, p. 1013]. By using Equation 4.6 into Equation 4.12, we can obtain the following closed-form
solution for the performance of BPSK in Rician fading and impulsive noise

\[ P_{e,BPSK}^{Rice} = \exp\left[-(A + K)\right] \sum_{m_1=0}^{\infty} \sum_{n=0}^{\infty} \sum_{q=0}^{n} \frac{A^{m_1} K^n (n + q)!}{m_1! n! n! q!} \times \left[ \frac{1}{2} \left( 1 - \sqrt{\frac{B\gamma_b}{B\gamma_b + 1 + K}} \right) \right]^{n+1} \left[ \frac{1}{2} \left( 1 + \sqrt{\frac{B\gamma_b}{B\gamma_b + 1 + K}} \right) \right]^q. \] (4.13)

For the special case of \( K = 0 \) (i.e. Rayleigh fading), Equation 4.13 simplifies to Equation 4.7. Figures 4.2-4.5 illustrate the performance of BPSK in Rician fading and impulsive noise for \( \Gamma' = 10^{-4} \) and different values of \( K \) and \( A \). Simulation results are seen to very well match the theoretical curves.

Figure 4.2: Performance of BPSK in Rician fading for different values of \( K \) and impulsive noise for \( \Gamma' = 10^{-4} \) and \( A = 10 \).
4.3 Rician Fading

Figure 4.3: Similar caption to Figure 4.2 but with $A = 1$.

Figure 4.4: Similar caption to Figure 4.2 but with $A = 0.35$. 
4.4 Nakagami Fading

Using Equation 1.3, the PDF of Nakagami envelope square $\gamma = r^2$ is given by

$$p_\gamma(\gamma) = \frac{p_r(\sqrt{\gamma})}{2\sqrt{\gamma}} = \frac{1}{\Gamma(m)} \left( \frac{m}{\bar{\gamma}_b} \right)^m \gamma^{m-1} \exp \left( -\frac{m\gamma}{\bar{\gamma}_b} \right). \quad (4.14)$$

By substituting Equations 3.5 and 4.14 into Equation 4.1, the performance of BPSK in
Nakagami fading and impulsive noise can be obtained as follows

\[
P_{e,Nak}^{I,BPSK} = \int_0^\infty P_e^{I,BPSK} p_\gamma(\gamma) \, d\gamma
\]

\[
= \int_0^\infty \exp\left(-\frac{A}{2}\right) \sum_{m_1=0}^\infty \frac{A^{m_1}}{m_1!} \text{erfc}\left(\sqrt{B}\gamma\right) \frac{1}{\Gamma(m)} \left(\frac{m}{\gamma_b}\right)^m \gamma^{m-1} \exp\left(-\frac{m\gamma}{\gamma_b}\right) \, d\gamma
\]

\[
= \exp\left(-\frac{A}{2}\right) \sum_{m_1=0}^\infty \frac{A^{m_1}}{m_1!} \frac{1}{\Gamma(m)} \left(\frac{m}{\gamma_b}\right)^m \int_0^\infty \frac{2}{\sqrt{\pi}} \int_0^\infty \exp\left(-t^2\right) \, dt \, \gamma^{m-1} \times \exp\left(-\frac{m\gamma}{\gamma_b}\right) \, d\gamma
\]

where \(B = N_1/\sigma^2_{m_1}\). Changing variable \(t = \sqrt{\gamma}z\), the above equation can be rewritten as

\[
P_{e,Nak}^{I,BPSK} = \exp\left(-\frac{A}{2}\right) \sum_{m_1=0}^\infty \frac{A^{m_1}}{m_1!} \frac{1}{\Gamma(m)} \left(\frac{m}{\gamma_b}\right)^m \frac{2}{\sqrt{\pi}} \int_0^\infty \int_0^\infty \gamma^{m+1/2-1} \times \exp[-(z^2 + m/\gamma_b)\gamma] \, d\gamma \, dz. \quad (4.16)
\]

Using Equation 3.381.4 of [19, p. 342], the above equation becomes

\[
P_{e,Nak}^{I,BPSK} = \exp\left(-\frac{A}{2}\right) \sum_{m_1=0}^\infty \frac{A^{m_1}}{m_1!} \frac{1}{\Gamma(m)} \left(\frac{m}{\gamma_b}\right)^m \frac{2}{\sqrt{\pi}} \int_0^\infty \frac{\Gamma(m+1/2)}{\sqrt{B} (z^2 + m/\gamma_b)^{m+1/2}} \, dz. \quad (4.17)
\]

Considering the case where \(m \in I^+\), substituting \(\Gamma(m+1/2) = \Gamma(2m)\Gamma(1/2)/[\Gamma(m)2^{2m-1}]\)

and using Equation 4.6 into Equation 4.17, the performance of BPSK in Nakagami fading
and impulsive noise can be expressed as

\[
P_{e,BPSK}^{\text{Nak}} = \exp(-A) \sum_{m_1=0}^{\infty} \sum_{k=0}^{m_1} \frac{A^{m_1} (m+k-1)!}{m_1! (m-1)!k!} \left[ \frac{1}{2} \left( 1 - \sqrt{\frac{B}{B + m/\gamma_b}} \right) \right]^m \times \left[ \frac{1}{2} \left( 1 + \sqrt{\frac{B}{B + m/\gamma_b}} \right) \right]^k.
\]

(4.18)

For the special case of \( m = 1 \) (i.e. Rayleigh fading), Equation 4.18 becomes Equation 4.7. Figure 4.6 shows the BER of BPSK in Nakagami fading and impulsive noise for \( \Gamma' = 10^{-4} \) and different values of \( m \) and \( A \). It can be seen that the BER obtained from the simulator very well match the analytical curves.

Figure 4.6: Performance of BPSK in Nakagami fading for \( m = 1, 2 \) and impulsive noise for \( \Gamma' = 10^{-4} \) and different values of \( A \).
4.5 Conclusions

This chapter presents several new theoretical expressions for the performance of BPSK in Rayleigh, Rician or Nakagami fading and class A impulsive noise. Furthermore, in order to derive the performance in fading, the PDFs of Rayleigh, Rician or Nakagami envelope square $\gamma$ are also presented. It can be seen from the plots that the Rayleigh fading degrades the performance of BPSK in impulsive noise for all values of $A$. For Rician fading, the degree of degradation from the fading depends on the value of $K$. The lower the value of $K$ is, the higher the degree of degradation from the Rician fading causes to the performance of BPSK with impulsive noise. In the most general cases of fading, the Nakagami fading channel for the low value of $m$ impairs the BPSK performance more than the Nakagami fading for the high value of $m$. 
CHAPTER 5
Performance of Diversity Combining Techniques

5.1 Introduction

Numerous research topics presented in the past decade deal with the performance evaluations of various modulation schemes in Gaussian noise and fading channels with diversity combining reception [1]–[5],[7],[18],[32],[33],[38],[39]. However, to the best of our knowledge, very few research topics have tackled the performance evaluations of modulation schemes in impulsive noise and fading with diversity reception. More specifically in [41], the analytical expression of the performance of DPSK in Rician fading and impulsive noise with the SC system is derived, while in [28] the theoretical expressions of the performance of DPSK in Rician fading and impulsive noise with SC or EGC diversity reception are presented.

This chapter presents the novel theoretical expressions for the performance evaluation of BPSK in impulsive noise and different types of fading (e.g. Rayleigh, Rician and Nakagami) with SC or EGC diversity reception. The organization of this chapter is as follows. After this introduction, in Section 5.2 the analytical expressions for the performance of BPSK in impulsive noise and different classes of fading with $L$-branch SC diversity reception are derived. In order to derive the performance in SC diversity reception, the PDFs of SC technique with different kinds of fading are also presented. In Section 5.3, the theoretical expressions for the performance evaluation of BPSK in impulsive noise and different kinds of fading with dual-branch EGC diversity reception are derived. In order to derive
the performance of EGC diversity reception, the characteristic function (CHF) method is used and the EGC CHFs with different sorts of fading are presented. In Section 5.4, a comparison for the performance of BPSK in impulsive noise and fading with dual-branch SC and EGC diversity reception is made. Conclusions can be found in Section 5.5.

5.2 Selection Combining (SC)

To derive analytical expressions for the SC system, the same methodology as the previous chapter (See Equation 4.1) is used. First, the PDF of SC with fading needs first to be derived. Then, the error probability of BPSK in fading and impulsive noise with SC diversity is derived by averaging the error rate probability of BPSK in impulsive noise with the previously derived SC PDF with fading.

5.2.1 PDF with Rayleigh Fading

In a $L$-branch SC system, there is only one channel with the largest fading envelope $r$ or largest $\gamma = r^2$ and the remainder in $L - 1$ channels have lower $\gamma$. The SC PDF with Rayleigh fading can be expressed as

$$p_{SC, Ray}(\gamma) = L p(\gamma) P(\hat{\gamma} < \gamma)^{L-1}$$  \hspace{1cm} (5.1)

where $p(\gamma)$ is the Rayleigh PDF (see Equation 4.2) and $P(\hat{\gamma} < \gamma)$ is the Rayleigh CDF at $\gamma$ that can be calculated by

$$P(\hat{\gamma} < \gamma) = \int_0^\gamma \frac{1}{\hat{\gamma}} \exp \left( -\frac{\hat{\gamma}}{\hat{\gamma}_c} \right) d\hat{\gamma} = 1 - \exp \left( -\frac{\gamma}{\hat{\gamma}_c} \right)$$  \hspace{1cm} (5.2)
where $\tilde{\gamma}_c = \text{SNR}/\text{bit}/\text{channel}$. Substituting Equation 5.2 into Equation 5.1 results in the SC PDF with Rayleigh fading

$$p_{\text{SC, Ray}}(\gamma) = \frac{L}{\tilde{\gamma}_c} \exp \left( -\frac{\gamma}{\tilde{\gamma}_c} \right) \left[ 1 - \exp \left( -\frac{\gamma}{\tilde{\gamma}_c} \right) \right]^{L-1}$$

$$= \frac{L}{\tilde{\gamma}_c} \sum_{l=0}^{L-1} \left( \frac{L - 1}{l} \right) (-1)^l \exp \left( -\frac{(1 + l)\gamma}{\tilde{\gamma}_c} \right). \quad (5.3)$$

### 5.2.2 Performance in Rayleigh Fading

The performance of BPSK in impulsive noise and Rayleigh fading with SC diversity reception, derived by averaging Equation 3.5 with Equation 5.3, can be expressed as

$$P_{e, \text{BPSK}}^{L-\text{SC, Ray}} = \int_0^\infty P_{e, \text{BPSK}} p_{\text{SC, Ray}}(\gamma) \, d\gamma$$

$$= \int_0^\infty \frac{1}{2} \exp(-A) \sum_{m_1=0}^\infty \frac{A^{m_1}}{m_1!} \text{erfc} \left( \sqrt{\frac{N_1}{\sigma^2_{m_1}}} \right) \frac{L}{\tilde{\gamma}_c} \sum_{l=0}^{L-1} \left( \frac{L - 1}{l} \right) (-1)^l \exp \left( -\frac{(1 + l)\gamma}{\tilde{\gamma}_c} \right) \, d\gamma$$

$$= \frac{1}{2} \exp(-A) \frac{L}{\tilde{\gamma}_c} \sum_{m_1=0}^\infty \sum_{l=0}^{L-1} \frac{A^{m_1}}{m_1!} \left( \frac{L - 1}{l} \right) (-1)^l \int_0^\infty \frac{2}{\sqrt{\pi}} \int_{\sqrt{B}}^\infty \exp(-t^2) \, dt$$

$$\times \exp \left( -\frac{(1 + l)\gamma}{\tilde{\gamma}_c} \right) \, d\gamma \quad (5.4)$$

where $B = N_1/\sigma^2_{m_1}$. Changing variable $t = \sqrt{\gamma}z$, the above equation becomes

$$P_{e, \text{BPSK}}^{L-\text{SC, Ray}} = \frac{1}{2} \exp(-A) \frac{L}{\tilde{\gamma}_c} \sum_{m_1=0}^\infty \sum_{l=0}^{L-1} \frac{A^{m_1}}{m_1!} \left( \frac{L - 1}{l} \right) (-1)^l \frac{2}{\sqrt{\pi}} \int_{\sqrt{B}}^\infty \gamma^{3/2-1}$$

$$\times \exp \left( -z^2 + \frac{(1 + l)\gamma}{\tilde{\gamma}_c} \right) \, d\gamma dz. \quad (5.5)$$

Using Equation 3.381.4 of [19, p. 342], the previous equation can be written alternatively
5.2.2 Performance in Rayleigh Fading

as

\[ P_{e,BPSK}^{L-SC,Ray} = \frac{1}{2} \exp(-A) \frac{L}{\gamma_c} \sum_{m_1=0}^{\infty} \sum_{l=0}^{L-1} \frac{A^{m_1}}{m_1!} \left( L - 1 \right) (-1)^l \int_\gamma B \frac{dz}{\sqrt{(z^2 + (1 + l)/\gamma_c)^{1+1/2}}} \]  

(5.6)

Applying Equation 4.6 into Equation 5.6 yields the performance of BPSK in impulsive noise and Rayleigh fading with L-branch SC diversity reception and impulsive noise

\[ P_{e,BPSK}^{L-SC,Ray} = \frac{1}{2} L \exp(-A) \sum_{m_1=0}^{\infty} \sum_{l=0}^{L-1} \frac{A^{m_1}}{m_1!} \left( L - 1 \right) (-1)^l \left( 1 - \sqrt{B \gamma_c} \right) \]  

(5.7)

Figures 5.1-5.4 show the BER of BPSK in impulsive noise and Rayleigh fading with SC diversity reception for \( \Gamma' = 10^{-4} \) and different values of \( A \) and \( L \). It can be seen from these results that the L-branch SC reception improves the performance of BPSK in impulsive noise and Rayleigh fading at high values of SNR. The greatest performance improvement is obtained when the diversity branch increases from \( L = 1 \) to 2. On the other hand, the SC diversity degrades the performance at some values of SNR, as the value of \( L \) increases. It happens, because the power of a fading signal is not completely dominating the power of noise. When the SC diversity is employed to increase the chance of receiving the higher fading signal power, the chance of receiving the higher noise power increases as well. Thus, these degradations come from receiving the noise with higher power than the fading signal. The theoretical curves and computer simulation results are in very close agreement.
5.2.2 Performance in Rayleigh Fading

Figure 5.1: Performance of BPSK in Rayleigh fading and impulsive noise for $\Gamma' = 10^{-4}$ and $A = 10$ with SC diversity reception for different values of $L$.

Figure 5.2: Same caption as in Figure 5.1 but with $A = 1$. 
5.2.2 Performance in Rayleigh Fading

Figure 5.3: Same caption as in Figure 5.1 but with $A = 0.35$.

Figure 5.4: Same caption as in Figure 5.1 but with $A = 0.01$. 
5.2.3 PDF with Rician Fading

Similarly to Equation 5.1, the SC PDF with Rician fading is given by

\[ p_{SC, \text{Rice}}(\gamma) = L p(\gamma) P(\tilde{\gamma} < \gamma)^{L-1} \]  

(5.8)

where \( p(\gamma) \) is the Rician PDF (see Equation 4.8) and \( P(\tilde{\gamma} < \gamma) \) is the Rician CDF at \( \gamma \) that can be calculated by

\[
P(\tilde{\gamma} < \gamma) = \int_{0}^{\gamma} \frac{1 + K}{\tilde{\gamma}_c} \exp\left(-\frac{\tilde{\gamma}(1 + K) + K\tilde{\gamma}_c}{\tilde{\gamma}_c}\right) I_0\left(\sqrt{\frac{4(1 + K)K\tilde{\gamma}}{\tilde{\gamma}_c}}\right) d\tilde{\gamma} 
= \beta_c \exp(-K) \int_{0}^{\gamma} \exp(-\beta_c \tilde{\gamma}) I_0\left(2\sqrt{K\beta_c \tilde{\gamma}}\right) d\tilde{\gamma} \]  

(5.9)

where \( \beta_c = (K + 1)/\tilde{\gamma}_c \). Changing variable \( \tilde{\gamma} = z^2/(2\beta_c) \), the above equation becomes

\[
P(\tilde{\gamma} < \gamma) = \int_{0}^{\sqrt{2\beta_c \gamma}} z \exp\left(-\frac{z^2 + (\sqrt{2K})^2}{2}\right) I_0\left(\sqrt{2K}z\right) dz. \]  

(5.10)

Using the well-known Marcum’s Q function Equations 2.1-122 and 2.1-123 of [23, p. 43-44], the above equation can be written alternatively as

\[
P(\tilde{\gamma} < \gamma) = 1 - Q_1(\sqrt{2K}, \sqrt{2\beta_c \gamma}) 
= 1 - \exp[-(K + \beta_c \gamma)] \sum_{n=0}^{\infty} \left(\frac{K}{\beta_c \gamma}\right)^{n/2} I_n(\sqrt{4K\beta_c \gamma}). \]  

(5.11)
Applying Equation 5.11, the term \([P(\bar{\gamma} < \gamma)]^{L-1}\) in Equation 5.8 can be expressed as

\[
[P(\bar{\gamma} < \gamma)]^{L-1} = \left[ 1 - \exp[-(K + \beta_c \gamma)] \sum_{n=0}^{\infty} \left( \frac{K}{\beta_c \gamma} \right)^{n/2} I_n(\sqrt{4K\beta_c \gamma}) \right]^{L-1}
\]

\[
= \sum_{l=0}^{L-1} \binom{L-1}{l} (-1)^l \exp[-(K + \beta_c \gamma)] \left[ \sum_{n=0}^{\infty} \left( \frac{K}{\beta_c \gamma} \right)^{n/2} I_n(\sqrt{4K\beta_c \gamma}) \right]^l.
\]

(5.12)

Expanding the \(n\)th-order modified Bessel function of the first kind by using Equation 2.1-120 of [23, p. 43], the following equation is obtained

\[
[P(\bar{\gamma} < \gamma)]^{L-1} = \sum_{l=0}^{L-1} \binom{L-1}{l} (-1)^l \exp[-(K + \beta_c \gamma)] \left[ \sum_{n=0}^{\infty} \left( \frac{K}{\beta_c \gamma} \right)^{n/2} \sum_{q=0}^{\infty} \frac{K^n}{q!\Gamma(n+q+1)} \right]^l
\]

\[
\times \left( \frac{\sqrt{K\beta_c \gamma}^{n+2q}}{q!\Gamma(n+q+1)} \right)^l
\]

\[
= \sum_{l=0}^{L-1} \binom{L-1}{l} (-1)^l \exp[-(K + \beta_c \gamma)] \left[ \sum_{q=0}^{\infty} \left( \sum_{n=0}^{\infty} \frac{K^n}{q!\Gamma(n+q+1)} \right) \right]^l
\]

\[
\times (K\beta_c \gamma)^{q}.
\]

(5.13)

By using Equation 0.314 of [19, p. 17], the above equation reduces to

\[
[P(\bar{\gamma} < \gamma)]^{L-1} = \sum_{l=0}^{L-1} \sum_{h=0}^{\infty} \binom{L-1}{l} (-1)^l c_h \exp[-(K + \beta_c \gamma)] (K\beta_c \gamma)^h
\]

(5.14)

where \(c_0 = \exp(Kl)\) and

\[
c_h = \frac{1}{h \exp(K)} \sum_{s=1}^{h} \sum_{n=0}^{\infty} \frac{s(l+1)-h\Gamma^n}{s!(n+s)!} c_{h-s}
\]

(5.15)
with \( h = 1, 2, 3, \ldots \). The SC PDF with Rician fading can now be expressed as

\[
p_{SC, \text{Rice}}(\gamma) = L\beta_c \sum_{l=0}^{L-1} \sum_{h=0}^{\infty} \binom{L-1}{l} (-1)^l c_h (K\beta_c \gamma)^h \exp[-(K + \beta_c \gamma)(l + 1)] I_0(2\sqrt{K\beta_c \gamma}).
\]

(5.16)

5.2.4 Performance in Rician Fading

Similarly to Section 5.2.2, the performance of BPSK in Rician fading and impulsive noise with SC diversity reception derived by averaging Equation 3.5 with Equation 5.16 can be expressed as

\[
P_{e, \text{BPSK}}^{L-\text{SC}, \text{Rice}} = \int_0^\infty P_{e, \text{BPSK}}^{L-\text{SC}, \text{Rice}}(\gamma) \, d\gamma
\]

\[
= \int_0^\infty \frac{1}{2} \exp(-A) \sum_{m_1=0}^{\infty} \frac{A^{m_1}}{m_1!} \text{erfc} \left( \sqrt{\frac{N_1\gamma}{\sigma^2 m_1}} \right) L\beta_c \sum_{l=0}^{L-1} \sum_{h=0}^{\infty} \binom{L-1}{l} (-1)^l c_h (K\beta_c \gamma)^h \exp[-(K + \beta_c \gamma)(l + 1)] I_0(2\sqrt{K\beta_c \gamma}) \, d\gamma
\]

(5.17)

where \( B = N_1/\sigma^2 m_1 \). Changing variable \( t = \sqrt{\gamma}z \), the above equation becomes

\[
P_{e, \text{BPSK}}^{L-\text{SC}, \text{Rice}} = \frac{1}{2} L\beta_c \exp(-A) \sum_{m_1=0}^{\infty} \sum_{l=0}^{L-1} \sum_{h=0}^{\infty} \frac{A^{m_1}}{m_1!} \binom{L-1}{l} (-1)^l c_h (K\beta_c \gamma)^h \exp[-K(l + 1)] \frac{2}{\sqrt{\pi}} \int_0^\infty \int_0^{\sqrt{B}\gamma} \exp(-t^2) \, dt \, \gamma^h \exp[-\beta_c \gamma(l + 1)] I_0(2\sqrt{K\beta_c \gamma}) \, d\gamma \, dz.
\]

(5.18)
Using Equation 6.643.2 of [19, p. 701], the above equation can be written alternatively as

\[
P_{\text{e,BPSK}}^{L, \text{SC,Rice}} = \frac{1}{2}L\beta_c \exp(-A) \sum_{m_1=0}^{\infty} \sum_{l=0}^{L-1} \sum_{h=0}^{\infty} \frac{A^{m_1}}{m_1!} \left(\begin{array}{c} L-1 \\ l \\ \end{array}\right) (-1)^l c_h (K\beta_c)^h \exp[-K(l+1)] \frac{2}{\sqrt{\pi}} \int_{\sqrt{B}}^{\infty} \frac{\Gamma(h+3/2)}{\sqrt{K\beta_c}} \exp \left( \frac{K\beta_c}{2[\beta_c(l+1)+z^2]} \right) \left[ \beta_c(l+1)+z^2 \right]^{-(h+1)} x M_{-(h+1),0} \left( \frac{K\beta_c}{\beta_c(l+1)+z^2} \right) dz.
\]

(5.19)

Applying Equation 9.220.2 of [19, p. 1014] to the above equation yields

\[
P_{\text{e,BPSK}}^{L, \text{SC,Rice}} = \frac{1}{2}L\beta_c \exp(-A) \sum_{m_1=0}^{\infty} \sum_{l=0}^{L-1} \sum_{h=0}^{\infty} \frac{A^{m_1}}{m_1!} \left(\begin{array}{c} L-1 \\ l \\ \end{array}\right) (-1)^l c_h (K\beta_c)^h \exp[-K(l+1)] \frac{2}{\sqrt{\pi}} \int_{\sqrt{B}}^{\infty} \frac{\Gamma(h+3/2)}{\sqrt{K\beta_c}} \exp \left( \frac{K\beta_c}{2[\beta_c(l+1)+z^2]} \right) \left[ \beta_c(l+1)+z^2 \right]^{-(h+1/2)} x \left( \frac{K\beta_c}{\beta_c(l+1)+z^2} \right)^t dz.
\]

(5.20)

Substituting \( (1)_t = t! \), \( (h+3/2)_t = (2h+2t+1)!/[(2t+1)!/2^{2t}(2h+1)!(h+t)!] \) and \( \Gamma(h+3/2) = \Gamma(2h+2)\Gamma(1/2)/[\Gamma(h+1)2^{2h+1}] \), the following expression is obtained

\[
P_{\text{e,BPSK}}^{L, \text{SC,Rice}} = L\beta_c \exp(-A) \sum_{m_1=0}^{\infty} \sum_{l=0}^{L-1} \sum_{h=0}^{\infty} \frac{A^{m_1}}{m_1!} \left(\begin{array}{c} L-1 \\ l \\ \end{array}\right) (-1)^l c_h (K\beta_c)^h \exp[-K(l+1)] \frac{1}{2^{2h+1}} \sum_{t=0}^{\infty} \frac{(2h+2t+1)! (K\beta_c)^t}{2^t(h+t)! (t)!} \int_{\sqrt{B}}^{\infty} \frac{dz}{z^{(h+1/2)+t+1/2}}.
\]

(5.21)
5.2.4 Performance in Rician Fading

Using Equation 4.6, the performance of BPSK in Rician fading and impulsive noise with $L$-branch SC diversity reception is obtained as

$$P_{e,BPSK}^{L-SC,Rice} = L \exp(-A) \sum_{m_1=0}^{\infty} \sum_{l=0}^{L-1} \sum_{h=0}^{\infty} \sum_{t=0}^{\infty} \sum_{g=0}^{h+t} \frac{A^{m_1}}{m_1!} \binom{L-1}{l} \frac{(-1)^l c_h K^{h+t}}{(l!)^2 g!} \exp[-K(l+1)] \Gamma(h + t + g + 1) \left[ \frac{1}{2(l+1)} \left( 1 - \sqrt{\frac{B}{B + \beta_c(l+1)}} \right) \right]^{h+t+1} \times \left[ \frac{1}{2} \left( 1 + \sqrt{\frac{B}{B + \beta_c(l+1)}} \right) \right]^g.$$  \hspace{1cm} (5.22)

For the special case of $K = 0$ (i.e. Rayleigh fading), Equation 5.22 becomes Equation 5.7. The above equation has been evaluated for $\Gamma' = 10^{-4}$ and different values of $K$, $L$ and $A$ and the results are summarized in Figures 5.5-5.16. It can be observed that, the SC technique improves the performance for all values of $K$ only at the high values of SNR. However, at some values of SNR, SC diversity degrades the performance for the high value of $K$ more than the performance for the low value of $K$. Similar remarks to Figures 5.1-5.4 can be made in that using the SC diversity at the low values of SNR will degrade the performance, because the chance of receiving the higher power of noise increases, as the value of $L$ increases. Thus, it can be concluded that the SC diversity technique is not an efficient technique to combat fading that has a line-of-sight path such as Rician fading. The simulated BERs obtained from computer simulations are seen to match the analytical curves very well.
Figure 5.5: Performance of BPSK in Rician fading for different values of $K$ and impulsive noise for $\Gamma' = 10^{-4}$ and $A = 10$ with SC diversity reception for $L = 2$.

Figure 5.6: Similar caption to Figure 5.5 but with $L = 3$. 
5.2.4 Performance in Rician Fading

Figure 5.7: Similar caption to Figure 5.5 but with $L = 4$.

Figure 5.8: Performance of BPSK in Rician fading for different values of $K$ and impulsive noise for $\Gamma' = 10^{-4}$ and $A = 1$ with SC diversity reception for $L = 2$. 
5.2.4 Performance in Rician Fading

Figure 5.9: Same caption as in Figure 5.8 but with $L = 3$.

Figure 5.10: Same caption as in Figure 5.8 but with $L = 4$.
5.2.4 Performance in Rician Fading

Figure 5.11: Performance of BPSK in Rician fading for different values of $K$ and impulsive noise for $\Gamma = 10^{-4}$ and $A = 0.35$ with SC diversity reception for $L = 2$.

Figure 5.12: Similar caption to Figure 5.11 but with $L = 3$. 
5.2.4 Performance in Rician Fading

Figure 5.13: Similar caption to Figure 5.11 but with $L = 4$.

Figure 5.14: Performance of BPSK in Rician fading for different values of $K$ and impulsive noise for $\Gamma = 10^{-4}$ and $A = 0.01$ with SC diversity reception for $L = 2$. 
5.2.4 Performance in Rician Fading

Figure 5.15: Same caption as in Figure 5.14 but with $L = 3$.

Figure 5.16: Same caption as in Figure 5.14 but with $L = 4$.
5.2.5 PDF with Nakagami Fading

Similarly to Equations 5.1 and 5.8, the SC PDF with Nakagami fading is obtained by

\[ p_{SC,Nak}(\gamma) = L \cdot p(\gamma) \cdot P(\tilde{\gamma} < \gamma)^{L-1} \]  \hspace{1cm} (5.23)

where \( p(\gamma) \) is the Nakagami PDF (see Equation 4.14) and \( P(\tilde{\gamma} < \gamma) \) is the Nakagami CDF at \( \gamma \) that can be calculated by

\[ P(\tilde{\gamma} < \gamma) = \left( \frac{m}{\gamma_c} \right)^m \frac{1}{\Gamma(m)} \int_0^{\gamma} \tilde{\gamma}^{m-1} \exp \left( -\frac{m\tilde{\gamma}}{\gamma_c} \right) d\tilde{\gamma}. \]  \hspace{1cm} (5.24)

Using Equation 3.381.1 of [19, p. 342], the above equation becomes

\[ P(\tilde{\gamma} < \gamma) = \frac{\gamma_{fn}(m, m\gamma/\gamma_c)}{\Gamma(m)} \]  \hspace{1cm} (5.25)

where \( \gamma_{fn} \) is the incomplete gamma function given by Equation 8.350.1 of [19, p. 890].

Equation 5.23 can be written alternatively as

\[ p_{SC,Nak}(\gamma) = \left( \frac{m}{\gamma_c} \right)^m \frac{L^{m-1}}{\Gamma(m)} [\gamma_{fn}(m, m\gamma/\gamma_c)]^{L-1} \exp \left( -\frac{m\gamma}{\gamma_c} \right). \]  \hspace{1cm} (5.26)

Considering \( m \) is a positive integer and using Equation 8.352.1 of [19, p. 890] yields the
following equation

\[
P_{SC,Nak}(\gamma) = \left( \frac{m}{\gamma_c} \right)^m \frac{L^{m-1}}{\Gamma(m)} \exp \left( -\frac{m\gamma}{\gamma_c} \right) [(m-1)!]^{L-1} \left( 1 - \exp \left( -\frac{m\gamma}{\gamma_c} \right) \right)
\]

\[
\times \left[ \sum_{k=0}^{m-1} \frac{1}{k!} \left( \frac{m\gamma}{\gamma_c} \right)^k \right]^{L-1}
\]

\[
= \left( \frac{m}{\gamma_c} \right)^m \frac{L^{m-1}}{\Gamma(m)} \exp \left( -\frac{m\gamma}{\gamma_c} \right) [(m-1)!]^{L-1} \sum_{l=0}^{L-1} \left( \frac{L-1}{l} \right)(-1)^l
\]

\[
\times \exp \left( -\frac{m\gamma l}{\gamma_c} \right) \left[ \sum_{k=0}^{m-1} \frac{1}{k!} \left( \frac{m\gamma}{\gamma_c} \right)^k \right]^l
\]

(5.27)

Using Equation 3.14 of [19, p. 17] and Equation 16 of [32] leads to the SC PDF with Nakagami fading

\[
P_{SC,Nak}(\gamma) = \frac{L}{\Gamma(m)} \sum_{l=0}^{L-1} \sum_{q=0}^{l(m-1)} \left( \frac{m}{\gamma_c} \right)^{m+q} c_q \left( \frac{L-1}{l} \right)(-1)^l \exp \left[ -\frac{(l+1)m\gamma}{\gamma_c} \right] \gamma^{m+q-1}
\]

(5.28)

where the coefficient \( c_q \) can be computed recursively as \( c_0 = 1, c_1 = l, c_{l(m-1)} = [1/(m-1)!]^l \) and

\[
c_q = \frac{1}{q} \sum_{h=1}^{\min(m-1,q)} \frac{[h(l+1) - q]}{h!} c_{q-h}
\]

(5.29)

with \( 2 \leq q \leq l(m-1) - 1 \).

5.2.6 Performance in Nakagami Fading

Similarly to Sections 5.2.2 and 5.2.4, the performance of BPSK in Nakagami fading and impulsive noise with SC diversity reception derived by averaging Equation 3.5 with
Equation 5.28 can be expressed as

\[
P_{e,\text{BPSK}}^{L-\text{SC},\text{Nak}} = \int_0^\infty P_{e,\text{BPSK}} \ p_{\text{SC},\text{Nak}}(\gamma) \ d\gamma \\
= \int_0^\infty \frac{1}{2} \exp(-A) \sum_{m_1=0}^{\infty} \frac{A^{m_1}}{m_1!} \ erfc \left( \sqrt{\frac{N_1 \gamma}{\sigma^2_{m_1}}} \right) \frac{L}{\Gamma(m)} \sum_{l=0}^{L-1} \sum_{q=0}^{l(m-1)} \left( \frac{m}{\gamma_c} \right)^{m+q} c_q \\
\times \left( \frac{L-1}{l} \right)^l (-1)^l \exp \left[ -\frac{(l+1)m \gamma}{\gamma_c} \right] \gamma^{m+q-1} \ d\gamma \\
= \frac{1}{2} \exp(-A) \frac{L}{\Gamma(m)} \sum_{m_1=0}^{\infty} \sum_{l=0}^{L-1} \sum_{q=0}^{l(m-1)} \frac{A^{m_1}}{m_1!} \left( \frac{m}{\gamma_c} \right)^{m+q} c_q \left( \frac{L-1}{l} \right)^l (-1)^l \frac{2}{\sqrt{\pi}} \\
\times \int_0^\infty \int_0^\infty \exp(-t^2) \ dt \ exp \left[ -\frac{(l+1)m \gamma}{\gamma_c} \right] \gamma^{m+q-1} \ d\gamma \\n\tag{5.30}
\]

where \( B = N_1/\sigma^2_{m_1} \). Changing variable \( t = \sqrt{\gamma} z \), the above equation becomes

\[
P_{e,\text{BPSK}}^{L-\text{SC},\text{Nak}} = \frac{1}{2} \exp(-A) \frac{L}{\Gamma(m)} \sum_{m_1=0}^{\infty} \sum_{l=0}^{L-1} \sum_{q=0}^{l(m-1)} \frac{A^{m_1}}{m_1!} \left( \frac{m}{\gamma_c} \right)^{m+q} c_q \left( \frac{L-1}{l} \right)^l (-1)^l \frac{2}{\sqrt{\pi}} \\
\times \int_0^\infty \int_0^\infty \gamma^{m+q+1/2-1} \ exp \left( -\left[ z^2 + (l+1)\frac{m}{\gamma_c} \right] \gamma \right) \ d\gamma \ dz. \\n\tag{5.31}
\]

Using Equation 3.381.4 of [19, p. 342], the above equation can be written alternatively as

\[
P_{e,\text{BPSK}}^{L-\text{SC},\text{Nak}} = \frac{1}{2} \exp(-A) \frac{L}{\Gamma(m)} \sum_{m_1=0}^{\infty} \sum_{l=0}^{L-1} \sum_{q=0}^{l(m-1)} \frac{A^{m_1}}{m_1!} \left( \frac{m}{\gamma_c} \right)^{m+q} c_q \left( \frac{L-1}{l} \right)^l (-1)^l \frac{2}{\sqrt{\pi}} \\
\times \int_0^\infty \frac{\Gamma(m + q + 1/2)}{\sqrt{\pi} \left( z^2 + (l+1)m/\gamma_c \right)^{m+q+1/2}} \ dz. \\n\tag{5.32}
\]

Substituting \( \Gamma(m + q + 1/2) = \Gamma(2[m + q]) \Gamma(1/2)/[\Gamma(m + q) 2^{(m+q)-1}] \) and using Equation 4.6 leads to the performance of BPSK in Nakagami fading and impulsive noise with \( L-\)
branch SC diversity reception

\[
P_{e,\text{BPSK}}^{L-\text{SC},\text{Nak}} = \frac{1}{\Gamma(m)} L \exp(-A) \sum_{m_1=0}^{\infty} \sum_{l=0}^{L-1} \sum_{q=0}^{l(m-1)} \sum_{k=0}^{m+q-1} \frac{A^{m_1}}{m_1!} c_q \left( \frac{L - 1}{l} \right) \times \frac{(-1)^l \Gamma(m + q + k)}{k!(l + 1)^{m+q}} \left[ \frac{1}{2} \left( 1 - \sqrt{\frac{B}{B + (l + 1)m/\gamma_c}} \right) \right]^{m+q} \times \left[ \frac{1}{2} \left( 1 + \sqrt{\frac{B}{B + (l + 1)m/\gamma_c}} \right) \right]^k.
\]

Equation 5.33

For the special case of \( m = 1 \) (i.e. Rayleigh fading), Equation 5.33 simplifies to Equation 5.7. The above equation has been numerically evaluated for \( \Gamma' = 10^{-4} \), \( m = 2 \) and different values of \( L \) and \( A \) and the obtained results are presented in Figures 5.17-5.20. As it can be seen from the plots, SC diversity improves the performance in Nakagami fading at high values of SNR. The greatest diversity gain is received, when the diversity branch increases from \( L = 1 \) to 2. However, at some values of SNR, the SC diversity technique degrades the performance in Nakagami fading. Similar comments to Figures 5.1-5.4 can be made here that SC diversity increases chances of receiving the noise with higher power than the fading signal. Simulation results are also included and are seen to very well validate the theoretical curves.
5.2.6 Performance in Nakagami Fading

Figure 5.17. Performance of BPSK in Nakagami fading for $m = 2$ and impulsive noise for $I' = 10^{-4}$ and $A = 10$ with SC diversity reception for different values of $L$.

Figure 5.18. Same caption as in Figure 5.17 but with $A = 1$. 
5.2.6 Performance in Nakagami Fading

Figure 5.19: Same caption as in Figure 5.17 but with $A = 0.35$.

Figure 5.20: Same caption as in Figure 5.17 but with $A = 0.01$. 
5.3 Equal-Gain Combining (EGC)

To find analytical expressions of the performance of BPSK in EGC, the performance of BPSK in impulsive noise with diversity is averaged with the EGC PDF. However, as compared to the SC case, the derivation of the EGC PDF is much more difficult. Instead, a characteristic function (CHF) approach, introduced in [3],[39], is used to find the performance of EGC diversity. The EGC CHF is more easily derived than the EGC PDF. The performance of a modulation scheme in impulsive noise and fading with EGC diversity, is derived by averaging (see Equation 4.1) the performance of a modulation scheme in impulsive noise and diversity reception with the EGC PDF, needs to be rewritten into a function of the EGC CHF instead. Then, the earlier derived EGC CHF can be substituted and the performance of EGC diversity is obtained.

The EGC CHF is given by

\[ \phi_{\text{EGC}}(\omega) = \int_{-\infty}^{\infty} p_{\text{EGC}}(r) \exp(j\omega r) \, dr. \]  

(5.34)

In order to use the EGC CHF approach in the averaged integral of BPSK, the performance of BPSK in impulsive noise with dual-branch EGC reception (see Equation 3.9) is required to be alternatively written as

\[ P_{e,\text{BPSK}}^{\text{II-EGC}} = \frac{1}{2} \exp[-(A_1 + A_2)] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1!m_2!} \text{erfc} \left( \sqrt{B_2} \right) 
= \frac{1}{2} \exp[-(A_1 + A_2)] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1!m_2!} \left[ 1 - \text{erf} \left( r \sqrt{B_2} \right) \right] \]  

(5.35)

where \( r = \sqrt{\gamma} \) and \( B_2 = 8N_1N_2C_1/(N_1 + N_2) \). Using Equation 3.952.6 of [19, p. 497], the
above equation becomes

\[
P_{e,\text{BPSK}}^{\text{II-EGC}} = \frac{1}{2} \exp[-(A_1 + A_2)] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1!m_2!} \left[ 1 - \frac{2}{\pi} \int_0^\infty \exp(-t^2) \right. \\
\left. \times \sin \left( 2rt \sqrt{B_2} \right) \frac{dt}{t} \right].
\] (5.36)

Changing variable \( y = t^2 \), the performance of BPSK in impulsive noise with dual-branch EGC diversity reception can be expressed as

\[
P_{e,\text{BPSK}}^\text{II-EGC} = \frac{1}{2} \exp[-(A_1 + A_2)] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1!m_2!} \left[ 1 - \frac{1}{\pi} \int_0^\infty y^{-\frac{1}{2}} \exp(-y) \right. \\
\left. \times \sin \left( 2r \sqrt{B_2 y} \right) dy \right].
\] (5.37)

Applying Equation 4.1 leads to the performance of BPSK in impulsive noise and fading with dual-branch EGC diversity reception

\[
P_{e,\text{BPSK}}^\text{II-EGC,fad} = \int_0^\infty P_{e,\text{BPSK}}^\text{II-EGC}(r) p_{\text{EGC}}(r) dr \\
= \frac{1}{2} \exp[-(A_1 + A_2)] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1!m_2!} \left[ 1 - \frac{1}{\pi} \int_0^\infty \int_0^\infty y^{-\frac{1}{2}} \exp(-y) \right. \\
\left. \times \sin \left( 2r \sqrt{B_2 y} \right) p_{\text{EGC}}(r) dy dr \right].
\] (5.38)

Substituting the term \( \int_0^\infty \sin(2r \sqrt{B_2 y}) p_{\text{EGC}}(r) dr \), which is the imaginary part of the EGC CHF (see Equation 5.34), in the above equation leads to the performance of BPSK
in impulsive noise and fading with dual-branch EGC diversity reception using the CHF as

$$P_{e,BPSK}^{II-EGC,fad} = \frac{1}{2} \exp[-(A_1 + A_2)] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1!m_2!} \left[ 1 - \frac{1}{\pi} \int_0^\infty y^{-1} \exp(-y) \right] \times \text{Imag} \left\{ \phi_{EGC} \left( 2\sqrt{\frac{E_2}{y}} \right) \right\} dy. \quad (5.39)$$

The previous equation will be used to derive analytical expressions of the error rate performance of BPSK in impulsive noise and Rayleigh, Rician and Nakagami fading with dual-branch EGC diversity reception in Sections 5.3.2, 5.3.4 and 5.3.6, respectively.

### 5.3.1 CHF with Rayleigh Fading

By using Equation 5.34, the EGC CHF with Rayleigh fading can be calculated by

$$\phi_{EGC,Ray} (\omega) = E[\exp(j\omega r)] = E \left[ \exp \left( j\omega \sum_{l=1}^{L} \frac{r_l}{\sqrt{L}} \right) \right] = \prod_{l=1}^{L} E \left[ \exp \left( \frac{j\omega r_l}{\sqrt{L}} \right) \right]$$

$$= \prod_{l=1}^{L} \int_0^\infty p_{r_l}(r_l) \exp \left( \frac{j\omega r_l}{\sqrt{L}} \right) dr_l \quad (5.40)$$

where $r$ is a normalized combined envelope, $r_l$ is a Rayleigh envelope in the $l^{th}$ channel and the Rayleigh PDF of $r_l$ (see Equation 1.1) is given by

$$p_{r_l}(r_l) = \frac{2r_l}{\gamma_c} \exp \left( -\frac{r_l^2}{\gamma_c} \right). \quad (5.41)$$
Equation 5.40 can be written alternatively as

\[ \phi_{\text{EGC, Ray}}(\omega) = \prod_{l=1}^{L} \int_{0}^{\infty} p_{r_{l}}(r_{l}) \left[ \cos \left( \frac{\omega r_{l}}{\sqrt{L}} \right) + j \sin \left( \frac{\omega r_{l}}{\sqrt{L}} \right) \right] dr_{l} \]

\[ = \prod_{l=1}^{L} \left[ \phi_{\text{EGC, Re, Ray}}^{lth}(\omega) + j \phi_{\text{EGC, Im, Ray}}^{lth}(\omega) \right] \quad (5.42) \]

where \( \phi_{\text{EGC, Re, Ray}}^{lth}(\omega) \) and \( \phi_{\text{EGC, Im, Ray}}^{lth}(\omega) \) are the real and imaginary parts of the EGC-Rayleigh CHF of the \( l \)th channel, respectively. Using Equations 5.41 and 5.42, the \( \phi_{\text{EGC, Re, Ray}}^{lth}(\omega) \) can be expressed mathematically as

\[ \phi_{\text{EGC, Re, Ray}}^{lth}(\omega) = \int_{0}^{\infty} p_{r_{l}}(r_{l}) \cos \left( \frac{\omega r_{l}}{\sqrt{L}} \right) dr_{l} = \int_{0}^{\infty} \frac{2r_{l}}{\gamma_{c}} \exp \left( -\frac{r_{l}^{2}}{\gamma_{c}} \right) \cos \left( \frac{\omega r_{l}}{\sqrt{L}} \right) dr_{l}. \quad (5.43) \]

Using Equation 3.952.8 of [19, p. 497] reduces the above equation to

\[ \phi_{\text{EGC, Re, Ray}}^{lth}(\omega) = \text{e} F_{1} \left( 1; \frac{1}{2}; -\frac{\omega^{2} \gamma_{c}}{4L} \right). \quad (5.44) \]

Similarly, the \( \phi_{\text{EGC, Im, Ray}}^{lth}(\omega) \) can be expressed as

\[ \phi_{\text{EGC, Im, Ray}}^{lth}(\omega) = \int_{0}^{\infty} \frac{2r_{l}}{\gamma_{c}} \exp \left( -\frac{r_{l}^{2}}{\gamma_{c}} \right) \sin \left( \frac{\omega r_{l}}{\sqrt{L}} \right) dr_{l}. \quad (5.45) \]

Applying Equation 3.952.7 of [19, p. 497], the above equation becomes

\[ \phi_{\text{EGC, Im, Ray}}^{lth}(\omega) = \frac{\omega}{2} \sqrt{\frac{\pi \gamma_{c}}{L}} \exp \left( -\frac{\omega^{2} \gamma_{c}}{4L} \right). \quad (5.46) \]

By substituting Equations 5.44 and 5.46 into Equation 5.42, the following equation is
obtained

$$\phi_{\text{EGC,Ray}}(\omega) = \prod_{l=1}^{L} \left[ F_1 \left( 1; \frac{1}{2}; -\frac{\omega^2 \gamma_c}{4L} \right) + j \frac{\omega}{2} \sqrt{\frac{\pi \gamma_c}{L}} \exp \left( -\frac{\omega^2 \gamma_c}{4L} \right) \right]. \quad (5.47)$$

In the above equation, considering dual-branch diversity ($L = 2$) results in the imaginary part of $\phi_{\text{EGC,Ray}}(\omega)$

$$\text{Imag} \{ \phi_{\text{EGC,Ray}}(\omega) \} = \omega \sqrt{\frac{\pi \gamma_c}{2}} \exp \left( -\frac{\omega^2 \gamma_c}{8} \right) \text{F}_1 \left( 1; \frac{1}{2}; -\frac{\omega^2 \gamma_c}{8} \right). \quad (5.48)$$

5.3.2 Performance in Rayleigh Fading

Substituting Equation 5.48 into Equation 5.39 yields the performance of BPSK in Rayleigh fading and impulsive noise with dual-branch EGC reception

$$P_{e,\text{BPSK}}^{\text{EGC,Ray}} = \frac{1}{2} \exp \left[ -(A_1 + A_2) \right] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1! m_2!} \left[ 1 - \sqrt{\frac{2B_2 \gamma_c}{\pi}} \int_0^\infty y^{\frac{1}{2} - 1} \times \exp \left[ -\left( 1 + \frac{B_2 \gamma_c}{2} \right) y \right] \text{F}_1 \left( 1; \frac{1}{2}; -\frac{B_2 \gamma_c y}{2} \right) dy \right]. \quad (5.49)$$

Using Equation C.1 of [3], the following analytical expression of the performance of BPSK in Rayleigh fading and impulsive noise with dual-branch EGC diversity reception can be obtained

$$P_{e,\text{BPSK}}^{\text{II,EGC,Ray}} = \frac{1}{2} \exp \left[ -(A_1 + A_2) \right] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1! m_2!} \left[ 1 - \frac{\sqrt{B_2 \gamma_c (2 + B_2 \gamma_c)}}{(1 + B_2 \gamma_c)} \right]. \quad (5.50)$$

Figures 5.21-5.24 illustrate the theoretical BER of BPSK in Rayleigh fading and impulsive noise with dual-branch EGC diversity versus the SNR/bit. It can be seen from the plots
5.3.2 Performance in Rayleigh Fading

Figure 5.21: Performance of BPSK in Rayleigh fading and impulsive noise for $V = 10^{-4}$ and $A = 10$ with dual-branch EGC diversity reception.

Figure 5.22: Same caption as in Figure 5.21 but with $A = 1$. 
5.3.2 Performance in Rayleigh Fading

Figure 5.23: Same caption as in Figure 5.21 but with $A = 0.35$.

Figure 5.24: Same caption as in Figure 5.21 but with $A = 0.01$. 
that EGC diversity improves the performance at high values of SNR. However, when the
value of $A$ decreases (i.e. $A \ll 10$, see Figures 5.22-5.24), the EGC diversity degrades
the performance at low values of SNR as compared to the performance in fading without
diversity reception. This is because the EGC diversity increases the PDF of the sum of
two impulsive noises at the low values of amplitude (see Figure 2.7). Simulated BERs are
also included and are seen to match the theoretical curves very well.

5.3.3 CHF with Rician Fading

Using a similar methodology as the one presented in Section 5.3.1, Equation 5.42 can
be rewritten for the Rician fading as

$$
\phi_{EGRic} = \prod_{l=1}^{L} \int_{0}^{\infty} p_{r_l}(r_l) \left[ \cos \left( \frac{\omega r_l}{\sqrt{L}} \right) + j \sin \left( \frac{\omega r_l}{\sqrt{L}} \right) \right] dr_l
$$

\[
= \prod_{l=1}^{L} \left[ \phi_{EGRicRe} + j \phi_{EGRicIm} \right] \tag{5.51}
\]

where $\phi_{EGRicRe}$ and $\phi_{EGRicIm}$ are the real and imaginary parts of EGC-Rician
CHF of the $l^{th}$ channel, respectively, $r_l$ is a Rician envelope in the $l^{th}$ channel. The Rician
PDF of $r_l$ (see Equation 1.2) is given by

$$
p_{r_l}(r_l) = 2 \beta_c r_l \exp \left( -K - \beta_c r_l^2 \right) I_0 \left( 2r_l \sqrt{K \beta_c} \right) \tag{5.52}
$$

with $\beta_c = (1 + K)/\gamma_c$. Using Equations 5.51 and 5.52, $\phi_{EGRicRe}$ can be expressed
mathematically as

$$\phi_{\text{EGC,Re,Rice}}^{lth}(\omega) = \int_{0}^{\infty} p_{r_i}(r_i) \cos \left( \frac{\omega r_i}{\sqrt{L}} \right) dr_i$$

$$= \int_{0}^{\infty} 2\beta_c r_i \exp \left( -K - \beta_c r_i^2 \right) I_0 \left( 2r_i \sqrt{K \beta_c} \right) \cos \left( \frac{\omega r_i}{\sqrt{L}} \right) dr_i. \quad (5.53)$$

Expanding $I_0(.)$ in the above equation by using Equation 8.447.1 of [19, p. 909] leads to the following equation

$$\phi_{\text{EGC,Re,Rice}}^{lth}(\omega) = 2\beta_c \exp(-K) \sum_{n_l=0}^{\infty} \frac{(K \beta_c)^{n_l}}{(n_l)!^2} \int_{0}^{\infty} r_i^{2n_l+1} \exp \left( -\beta_c r_i^2 \right) \cos \left( \frac{\omega r_i}{\sqrt{L}} \right) dr_i. \quad (5.54)$$

Using Equation 3.952.8 of [19, p. 497] reduces to the following closed-form solution for the real part of EGC-Rician CHF of the $l^{th}$ channel

$$\phi_{\text{EGC,Re,Rice}}^{lth}(\omega) = \exp(-K) \sum_{n_l=0}^{\infty} \frac{K^{n_l}}{n_l!} {_1F_1} \left( n_l + 1; \frac{1}{2}; -\frac{\omega^2}{4\beta_c L} \right). \quad (5.55)$$

Similarly, $\phi_{\text{EGC,Im,Rice}}^{lth}(\omega)$ can be expressed as

$$\phi_{\text{EGC,Im,Rice}}^{lth}(\omega) = \int_{0}^{\infty} p_{r_i}(r_i) \sin \left( \frac{\omega r_i}{\sqrt{L}} \right) dr_i$$

$$= 2\beta_c \exp(-K) \int_{0}^{\infty} r_i \exp \left( -\beta_c r_i^2 \right) \sin \left( \frac{\omega r_i}{\sqrt{L}} \right) I_0 \left( 2r_i \sqrt{K \beta_c} \right) dr_i. \quad (5.56)$$

Expanding $I_0(.)$ by using Equation 8.447.1 of [19, p. 909], the above equation becomes

$$\phi_{\text{EGC,Im,Rice}}^{lth}(\omega) = 2\beta_c \exp(-K) \sum_{n_l=0}^{\infty} \frac{(K \beta_c)^{n_l}}{(n_l)!^2} \int_{0}^{\infty} r_i^{2n_l+1} \exp \left( -\beta_c r_i^2 \right) \sin \left( \frac{\omega r_i}{\sqrt{L}} \right) dr_i. \quad (5.57)$$
Applying Equation 3.952.7 of [19, p. 497] to the above equation, the imaginary part of EGC-Rician CHF of the \( l \)th channel simplifies to

\[
\phi_{\text{EGC,Im,Rice}}^{lth}(\omega) = \frac{\omega \exp(-K)}{\sqrt{\beta_c L}} \sum_{n_l=0}^{\infty} \frac{K^{n_l}}{(n_l!)^2} \Gamma \left( n_l + \frac{3}{2} \right) _1F_1 \left( n_l + \frac{3}{2}; \frac{3}{2}; -\frac{\omega^2}{4\beta_c L} \right). 
\] (5.58)

Substituting Equations 5.55 and 5.58 into Equation 5.51 yields the following expression

\[
\phi_{\text{EGC,Rice}}(\omega) = \prod_{i=1}^{L} \left[ \exp(-K) \sum_{n_l=0}^{\infty} \frac{K^{n_l}}{n_l!} _1F_1 \left( n_l + 1; \frac{1}{2}; -\frac{\omega^2}{4\beta_c L} \right) 
+ j \frac{\omega}{\sqrt{\beta_c L}} \sum_{n_l=0}^{\infty} \frac{K^{n_l}}{(n_l!)^2} \Gamma \left( n_l + \frac{3}{2} \right) _1F_1 \left( n_l + \frac{3}{2}; \frac{3}{2}; -\frac{\omega^2}{4\beta_c L} \right) \right]. 
\] (5.59)

In the previous equation, considering dual-branch diversity \((L = 2)\), the imaginary part of \( \phi_{\text{EGC,Rice}}(\omega) \) can be expressed as

\[
\text{Imag} \{ \phi_{\text{EGC,Rice}}(\omega) \} = \frac{2\omega \exp(-2K)}{\sqrt{2\beta_c}} \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \frac{K^{n_1+n_2}}{(n_1!)^2 n_2!} \Gamma \left( n_1 + \frac{3}{2} \right) 
\times _1F_1 \left( n_1 + \frac{3}{2}; \frac{3}{2}; -\frac{\omega^2}{8\beta_c} \right) _1F_1 \left( n_2 + 1; \frac{1}{2}; -\frac{\omega^2}{8\beta_c} \right). 
\] (5.60)

**5.3.4 Performance in Rician Fading**

Similarly to Section 5.3.2, by substituting Equation 5.60 into Equation 5.39, the performance of BPSK in Rician fading and impulsive noise with dual-branch EGC diversity
5.3.4 Performance in Rician Fading

reception can be expressed as

\[
P_{e,BPSK}^\text{II-EGC,Rice} = \frac{1}{2} \exp\left[-(A_1 + A_2)\right] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1! m_2!} \left[ 1 - \frac{2}{\pi} \sqrt{\frac{2B_2}{\beta_c}} \exp(-2K) \right] \\
\times \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \frac{K^{n_1+n_2}}{(n_1!)^2 n_2!} \Gamma\left(n_1 + \frac{3}{2}\right) \Gamma\left(n_2 + \frac{3}{2}\right) \int_{0}^{\infty} y^{\frac{1}{2}-1} \exp(-y) \\
\times {}_1F_1\left(n_2 + 1; \frac{1}{2}; -\frac{B_2y}{2\beta_c}\right) dy.
\]  \hspace{1cm} (5.61)

Using Equation C.1 of [3], the above equation becomes

\[
P_{e,BPSK}^\text{II-EGC,Rice} = \frac{1}{2} \exp\left[-(A_1 + A_2)\right] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1! m_2!} \left[ 1 - \frac{2}{\pi} \sqrt{\frac{2B_2}{\beta_c}} \exp(-2K) \right] \\
\times \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \frac{K^{n_1+n_2}}{(n_1!)^2 n_2!} \Gamma\left(n_1 + \frac{3}{2}\right) \Gamma\left(1/2\right) \frac{1}{F_2\left(\frac{1}{2}; n_1 + \frac{3}{2}, n_2 + 1; \frac{3}{2}, \frac{1}{2}; -\frac{B_2}{2\beta_c}\right)}
\]  \hspace{1cm} (5.62)

where \(F_2(.)\) is a hypergeometric function of two variables given by Equation 9.180.2 of [19,
5.3.4 Performance in Rician Fading

Applying Equation 9.183.2 of [19, p. 1010] to the above equation yields

\[
P_{e,BPSK}^{I-ECG,Rice} = \frac{1}{2} \exp[-(A_1 + A_2)] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1! m_2!} \left[ 1 - \frac{2}{\pi} \sqrt{\frac{2B_2}{\beta_c + B_2}} \exp(-2K) \right] \\
\times \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \frac{K^{n_1+n_2}}{(n_1!^2 n_2!)} \Gamma\left(n_1 + \frac{3}{2}\right) \Gamma(1/2) F_2 \left(\frac{1}{2}; -n_1, -n_2; \frac{1}{2}; \frac{3}{2}, \frac{1}{2}; \frac{B_2}{2(\beta_c + B_2)}, \frac{B_2}{2(\beta_c + B_2)}\right) \\
\times \frac{B_2}{2(\beta_c + B_2)} \left(\frac{B_2}{2(\beta_c + B_2)}\right)^q \\
2F_1 \left(\frac{1}{2} + q, -n_2 - \frac{1}{2}; \frac{1}{2}; 2(\beta_c + B_2)\right)
\]  \hspace{1cm} (5.63)

where \(2F_1(.)\) is a hypergeometric function given by Equation 9.100 of [19, p. 995]. Substituting \((1/2)_q = (2q)!/(2^q q!)\), \((3/2)_q = (2q + 1)!/(2^q q!)\) and \((-n_1)_q = (-1)^q n_1!/(n_1 - q)!\) to the above equation leads to the performance of BPSK in Rician fading and impulsive noise with dual-branch EGC diversity reception.

\[
P_{e,BPSK}^{I-ECG,Rice} = \frac{1}{2} \exp[-(A_1 + A_2)] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1! m_2!} \left[ 1 - \frac{4}{\pi} \exp(-2K) \right] \\
\times \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \sum_{q=0}^{n_1} \frac{K^{n_1+n_2}}{(n_1! n_2!(n_1 - q)!)q!} \Gamma\left(n_1 + \frac{3}{2}\right) \Gamma(1/2) \Gamma\left(\frac{1}{2}\right) (-1)^q \left[ \frac{B_2}{2(\beta_c + B_2)} \right]^{q+\frac{1}{2}} \\
\times 2F_1 \left(q + \frac{1}{2}, -n_2 - \frac{1}{2}; \frac{1}{2}; 2(\beta_c + B_2)\right)
\]  \hspace{1cm} (5.64)

For the special case of \(K = 0\) (e.g. Rayleigh fading), Equation 5.64 simplifies to Equation 5.50. The above equation has been numerically evaluated for \(\Gamma' = 10^{-4}\) and different values of \(K\) and \(A\). The obtained results are illustrated in Figures 5.25-5.28. As can be seen from
5.3.4 Performance in Rician Fading

Figure 5.25: Performance of BPSK in Rician fading for different values of $K$ and impulsive noise for $\Gamma' = 10^{-4}$ and $A = 10$ with dual-branch EGC diversity reception.

Figure 5.26: Similar caption to Figure 5.25 but with $A = 1$. 
5.3.4 Performance in Rician Fading

Figure 5.27: Similar caption to Figure 5.25 but with $A = 0.35$.

Figure 5.28: Similar caption to Figure 5.25 but with $A = 0.01$. 
the plots, the EGC diversity very well improves the performance in Rician fading and impulsive noise, especially at high values of SNR. Similar remarks to Figures 5.21-5.24 can also be made in that the EGC diversity degrades the performance in Rician fading and impulsive noise, particularly at low values of SNR and \( A \) (i.e. \( A \ll 10 \)), because the EGC diversity increases the PDF of the sum of two impulsive noises at low values of amplitude. Simulation results included in the plots are seen to be very close to the theoretical ones.

### 5.3.5 CHF with Nakagami Fading

Using a similar methodology as the one presented in Section 5.3.1, Equation 5.42 can be rewritten for the Nakagami fading as

\[
\phi_{\text{EGC,Nak}}(\omega) = \prod_{l=1}^{L} \int_{0}^{\infty} p_{r_l}(r_l) \left[ \cos \left( \frac{\omega r_l}{\sqrt{L}} \right) + j \sin \left( \frac{\omega r_l}{\sqrt{L}} \right) \right] dr_l \\
= \prod_{l=1}^{L} \left[ \phi_{\text{EGC,Re,Nak}}^{lth}(\omega) + j\phi_{\text{EGC,Im,Nak}}^{lth}(\omega) \right] \tag{5.65}
\]

where \( \phi_{\text{EGC,Re,Nak}}^{lth}(\omega) \) and \( \phi_{\text{EGC,Im,Nak}}^{lth}(\omega) \) are the real and imaginary parts of EGC-Nakagami CHF of the \( l^{th} \) channel, respectively, \( r_l \) is a Nakagami envelope in the \( l^{th} \) channel. The Nakagami PDF of \( r_l \) (see Equation 1.3) is given by

\[
p_{r_l}(r_l) = \frac{2}{\Gamma(m)} \left( \frac{m}{\gamma_c} \right)^{m} r_l^{2m-1} \exp \left( -\frac{mr_l^2}{\gamma_c} \right) . \tag{5.66}
\]
Using Equations 5.65 and 5.66, \( \phi_{\text{EGC,Re,Nak}}^{th}(\omega) \) can be expressed mathematically as

\[
\phi_{\text{EGC,Re,Nak}}^{th}(\omega) = \int_0^\infty p_r(r_l) \cos \left( \frac{\omega r_l}{\sqrt{L}} \right) dr_l \\
= \int_0^\infty \frac{2}{\Gamma(m)} \left( \frac{m}{\gamma_c} \right)^m r_l^{2m-1} \exp \left( -\frac{m r_l^2}{\gamma_c} \right) \cos \left( \frac{\omega r_l}{\sqrt{L}} \right) dr_l. \tag{5.67}
\]

Applying Equation 3.952.8 of [19, p. 497] to the above equation results in the real part of EGC-Nakagami CHF of the \( l^{th} \) channel

\[
\phi_{\text{EGC,Re,Nak}}^{th}(\omega) = \text{Re} \left( \frac{1}{1} \left( m; \frac{1}{2}; -\frac{\omega^2 \gamma_c}{4mL} \right) \right). \tag{5.68}
\]

Similarly, the \( \phi_{\text{EGC,Im,Nak}}^{th}(\omega) \) can be expressed as

\[
\phi_{\text{EGC,Im,Nak}}^{th}(\omega) = \int_0^\infty p_r(r_l) \sin \left( \frac{\omega r_l}{\sqrt{L}} \right) dr_l \\
= \int_0^\infty \frac{2}{\Gamma(m)} \left( \frac{m}{\gamma_c} \right)^m r_l^{2m-1} \exp \left( -\frac{m r_l^2}{\gamma_c} \right) \sin \left( \frac{\omega r_l}{\sqrt{L}} \right) dr_l. \tag{5.69}
\]

Using Equation 3.952.7 of [19, p. 497], the imaginary part of EGC-Nakagami CHF of the \( l^{th} \) channel is obtained as

\[
\phi_{\text{EGC,Im,Nak}}^{th}(\omega) = \frac{\omega}{\Gamma(m)} \left( \frac{\gamma_c}{mL} \right)^{1/2} \Gamma \left( m + \frac{1}{2} \right) \text{\,}_1F_1 \left( m + \frac{3}{2}; -\frac{\omega^2 \gamma_c}{4mL} \right). \tag{5.70}
\]
Substituting Equations 5.68 and 5.70 into Equation 5.65 yields the following equation

\[
\phi_{\text{EGC,Nak}}(\omega) = \prod_{l=1}^{L} \left[ \begin{array}{c}
\frac{1}{2} \left( \frac{1}{2}; \frac{\omega^2 \gamma_c}{4mL} \right) + j \frac{\omega}{\Gamma(m)} \left( \frac{\gamma_c}{mL} \right)^{1/2} \Gamma \left( m + \frac{1}{2} \right) \\
\end{array} \right] \times \Gamma(m) \left( \frac{1}{2}; \frac{\omega^2 \gamma_c}{4mL} \right) \right].
\] (5.71)

By considering dual-branch diversity \((L = 2)\) in the above equation, the imaginary part of \(\phi_{\text{EGC,Nak}}(\omega)\) can be expressed as

\[
\text{Imag} \{ \phi_{\text{EGC,Nak}}(\omega) \} = \frac{2\omega}{\Gamma(m)} \left( \frac{\gamma_c}{2m} \right)^{1/2} \Gamma \left( m + \frac{1}{2} \right) \Gamma(m) \left( \frac{1}{2}; \frac{\omega^2 \gamma_c}{8m} \right) \times \Gamma(m) \left( \frac{1}{2}; \frac{\omega^2 \gamma_c}{8m} \right) \right].
\] (5.72)

5.3.6 Performance in Nakagami Fading

Similarly to Sections 5.3.2 and 5.3.4, by substituting Equation 5.72 into Equation 5.39, the performance of BPSK in Nakagami fading and impulsive noise with dual-branch EGC diversity reception can be expressed as

\[
P_{\text{e,BPSK}}^{\text{II,EGC,Nak}} = \frac{1}{2} \exp[-(A_1 + A_2)] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1! m_2!} \left[ 1 - \frac{4}{\pi} \frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)} \sqrt{\frac{\gamma_c B_2}{2m}} \right]
\times \int_{0}^{\infty} y^\frac{1}{2} \exp(-y) \Gamma \left( m + \frac{1}{2}; \frac{\gamma_c B_2 y}{2m} \right) \frac{1}{2} \Gamma \left( m + \frac{1}{2}; \frac{\gamma_c B_2 y}{2m} \right) dy.
\] (5.73)
Using Equation C.1 of [3], the above equation becomes

\[
P_{e,\text{BPSK}}^{II-\text{EGC,Nak}} = \frac{1}{2} \exp[-(A_1 + A_2)] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1! m_2!} \left[ 1 - \frac{4}{\sqrt{\pi}} \frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)} \sqrt{\frac{\gamma_c B_2}{2m}} \right] 
\times F_2 \left( \frac{1}{2}; m, m + \frac{1}{2}; \frac{3}{2}; -\frac{\gamma_c B_2}{2m}, -\frac{\gamma_c B_2}{2m} \right). \tag{5.74}\]

Applying Equation 9.183.2 of [19, p. 1010] to the above equation results in the performance of BPSK in Nakagami fading and impulsive noise with dual-branch EGC diversity reception

\[
P_{e,\text{BPSK}}^{II-\text{EGC,Nak}} = \frac{1}{2} \exp[-(A_1 + A_2)] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1! m_2!} \left[ 1 - \frac{4}{\sqrt{\pi}} \frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)} \right] 
\times \sqrt{\frac{\gamma_c B_2}{2(\gamma_c B_2 + m)}} F_2 \left( \frac{1}{2}; 2 - m, 1 - m; \frac{3}{2}; \frac{\gamma_c B_2}{2(\gamma_c B_2 + m)}; \frac{\gamma_c B_2}{2(\gamma_c B_2 + m)} \right). \tag{5.75}\]

Considering \(m\) is a positive integer, the above equation can be expressed alternatively as

\[
P_{e,\text{BPSK}}^{II-\text{EGC,Nak}} = \frac{1}{2} \exp[-(A_1 + A_2)] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1! m_2!} \left[ 1 - \frac{4}{\sqrt{\pi}} \frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)} \sum_{q=0}^{m-1} \frac{(1-m)_q \left[ \frac{\gamma_c B_2}{2(\gamma_c B_2 + m)} \right]^{q-\frac{1}{2}}}{(2q+1) q!} {}_2F_1 \left( q + \frac{1}{2}; \frac{1}{2}; 2 - m; \frac{\gamma_c B_2}{2(\gamma_c B_2 + m)} \right) \right]. \tag{5.76}\]

For the special case of \(m = 1\) (i.e. Rayleigh fading), the above equation simplifies to Equation 5.50 which is indeed the equivalent results for the Rayleigh fading channel. For
5.3.6 Performance in Nakagami Fading

$m = 1/2, 3/2, 5/2, \cdots$, Equation 5.75 can be rewritten as

$$P_{e,\text{BPSK}}^{\text{II-EGC,Nak}} = \frac{1}{2} \exp[-(A_1 + A_2)] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1! m_2!} \left[ 1 - \frac{4}{\sqrt{\pi}} \frac{\Gamma\left(m + \frac{1}{2}\right)}{\Gamma(m)} \sum_{q=0}^{m-\frac{1}{2}} \frac{(-1)^q}{q!} \left( \frac{\overline{\gamma}_c B_2}{2(\overline{\gamma}_c B_2 + m)} \right)^{q+\frac{1}{2}} \right] \times \left( \frac{\frac{1}{2} - m}{q!} \right)^q 2F_1 \left( q + \frac{1}{2}, 1 - m, \frac{3}{2} - \frac{\overline{\gamma}_c B_2}{2(\overline{\gamma}_c B_2 + m)} \right).$$

(5.77)

From the above equation for the special case of Equation 5.77 at $m = 1/2$, it simplifies to

$$P_{e,\text{BPSK}}^{\text{II-EGC,Nak}} = \frac{1}{2} \exp[-(A_1 + A_2)] \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{A_1^{m_1} A_2^{m_2}}{m_1! m_2!} \left[ 1 - \frac{4}{\pi} \sin^{-1} \left( \sqrt{\frac{\overline{\gamma}_c B_2}{2(\overline{\gamma}_c B_2 + \frac{1}{2})}} \right) \right].$$

(5.78)

which can be compared with the Gaussian expression presented in Equation 34 of [3].

Figures 5.29-5.32 illustrate the performance of BPSK in Nakagami fading and impulsive noise with dual-branch EGC diversity reception for $\Gamma' = 10^{-4}$ and considering different values of $m$ and $A$. As expected and similar to the Rayleigh and Rician fading channels, EGC diversity significantly improves the performance in Nakagami fading and impulsive noise, especially at high values of SNR. However, as the value of $A$ decreases (i.e. $A \ll 10$, see Figures 5.30-5.32), EGC diversity degrades the performance in Nakagami fading and impulsive noise at low values of SNR, because EGC diversity increases the PDF of the sum of two impulsive noises at low values of amplitude. As shown in the figures, the validity of the theoretical results have been verified by means of computer simulation.
5.3.6 Performance in Nakagami Fading

Figure 5.29: Performance of BPSK in Nakagami fading for different values of $m$ and impulsive noise for $\Gamma' = 10^{-4}$ and $A = 10$ with dual-branch EGC diversity reception.

Figure 5.30: Same caption as in Figure 5.29 but with $A = 1$. 
5.3.6 Performance in Nakagami Fading

Figure 5.31: Same caption as in Figure 5.29 but with $A = 0.35$.

Figure 5.32: Same caption as in Figure 5.29 but with $A = 0.01$. 
5.4 SC and EGC Performance Comparisons

The purpose of this section is to compare the BER performance of SC and EGC diversity methods for the various fading channels, i.e. Rayleigh (see Figures 5.33-5.36), Rician (see Figures 5.37-5.40) and Nakagami (see Figures 5.41-5.44). It can be seen from the plots that EGC diversity has better performance than the SC diversity for all cases of fading at high values of SNR, because the EGC diversity raises the mean of the received signal and also reduces the PDF of the received impulsive noise at high values of amplitude (see Figure 2.7). However, as the values of $A$ decreases (e.g. $A \ll 10$), the EGC diversity degrades the BER at low values of SNR, because it increases the PDF of the received impulsive noise at low values of amplitude, while the SC diversity does nothing to the PDF of the received impulsive noise.

Figure 5.33: Comparison between the performance of BPSK with dual-branch SC and EGC diversity reception in Rayleigh fading and impulsive noise for $\Gamma^* = 10^{-4}$ and $A = 10$. 
5.4 SC and EGC Performance Comparisons

Figure 5.34: Similar caption to Figure 5.33 but with $A = 1$.

Figure 5.35: Similar caption to Figure 5.33 but with $A = 0.35$. 
5.4 SC and EGC Performance Comparisons

Figure 5.36: Similar caption to Figure 5.33 but with $A = 0.01$.

Figure 5.37: Comparison between the performance of BPSK with dual-branch SC and EGC diversity reception in Rician fading for different values of $K$ and impulsive noise for $\Gamma'' = 10^{-4}$ and $A = 10$. 
5.4 SC and EGC Performance Comparisons

Figure 5.38: Same caption as in Figure 5.37 but with $A = 1$.

Figure 5.39: Same caption as in Figure 5.37 but with $A = 0.35$. 
Figure 5.40: Same caption as in Figure 5.37 but with $A = 0.01$.

Figure 5.41: Comparison between the performance of BPSK with dual-branch SC and EGC diversity reception in Nakagami fading for $m = 2$ and impulsive noise for $T^* = 10^{-4}$ and $A = 10$. 
5.4 SC and EGC Performance Comparisons

Figure 5.42: Similar caption to Figure 5.41 but with $A = 1$.

Figure 5.43: Similar caption to Figure 5.41 but with $A = 0.35$. 
5.5 Conclusions

This chapter presents several new expressions for the performance of diversity combining techniques. After an introduction, Section 5.2 analytical expressions are derived for the performance evaluation of BPSK in impulsive noise and Rayleigh, Rician and Nakagami fading with $L$-branch SC diversity reception and the results are evaluated and compared with simulation results. Additionally, in this section the SC PDFs are presented with different kinds of fading. In Section 5.3, several theoretical expressions of the error rate performance of BPSK in impulsive noise and different types of fading with dual-branch EGC diversity reception are presented. Furthermore, the EGC CHFs with different classes of fading are introduced. The numerically evaluated theoretical and computer simulated results are presented together in the plots. In Section 5.4, the theoretical results from
Sections 5.2 and 5.3 are compared in the same figures by considering identical fading and impulsive noise channels.
CHAPTER 6
Conclusions and Suggestions for Future Research

6.1 Conclusions

The major contribution of this thesis is the evaluation of the performance of modulation schemes in the appearance of class A impulsive noise and fading with diversity reception through analysis and simulation. The contributions made throughout this thesis are summarized as follows.

- The PDFs of the envelope and the I-component of the sum of two impulsive noises are obtained. Then, performance evaluations of BPSK and MPSK in impulsive noise with dual-branch EGC or without diversity reception are presented. The theoretical results are thoroughly validated by computer simulation.

- Theoretical expressions of the error rate performance of BPSK in impulsive noise and different types of fading (i.e. Rayleigh, Rician and Nakagami) are proposed. The theoretical BERs very well match the simulated BERs.

- Analytical expressions of the performance of BPSK in impulsive noise and different classes of fading with $L$-branch SC or dual-branch EGC diversity reception are derived and the theoretical results are verified by means of simulation.

6.2 Suggestions for Future Research
The work done in this thesis can be complemented by several extensions suggested in the following sections.

6.2.1 MPSK or MQAM with Diversity Combining Techniques

The theoretical expression of the error rate performance of MQAM in class A impulsive noise with the dependence of the $I$ and $Q$ parts, with or without dual-branch EGC diversity reception, have yet to be derived. Then, the analytical expressions of the performance of MPSK or MQAM in class A impulsive noise and fading in the $L$-branch SC or dual-branch EGC diversity can be derived. Furthermore, deriving performance evaluations for MPSK or MQAM in impulsive noise and fading with the $L = 3$ EGC diversity reception is an interesting and challenging open research problem.

6.2.2 MPSK or MQAM in Correlated Fading with Diversity Combining Techniques

Assuming close spacing between the antennas and using diversity techniques, the received diversity signals are more likely subject to dependent fading than to independent fading. Thus, it would be interesting and useful to investigate the performance of MPSK and MQAM in SC or EGC techniques with correlated fading and impulsive noise and compare the difference between the performance in non-correlated and correlated fading.

6.2.3 Performance of Diversity Combining Techniques with Coding

It is well-known that coding techniques improve the BER and channel bandwidth efficiency. It will be interesting to find out how coding techniques improve the performance
of modulation schemes (i.e. MPSK or MQAM) in impulsive noise and fading (or correlated fading) with diversity reception.
References


APPENDIX A
Theoretical Derivations

A.1 Derivation of Equation 3.10

In L-branch EGC diversity with Gaussian noise, the sum $y$ of Gaussian noises from all channels can be expressed as

$$y = x_1 + x_2 + \cdots + x_L$$  \hspace{1cm} (A.1)

where $x_i$ is Gaussian noise in the $i^{th}$ channel and $i = 1, 2, \cdots , L$. The PDF of $x_i$ is given by

$$p_{x_i}(x_i) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{x_i^2}{2\sigma^2}\right).$$  \hspace{1cm} (A.2)

The characteristic function of $x_i$ can be calculated by

$$\phi_{x_i}(\omega) = \frac{1}{\sqrt{2\pi\sigma}} \int_{-\infty}^{\infty} \exp\left(-\frac{x_i^2}{2\sigma^2} + j\omega x_i\right) dx_i.$$  \hspace{1cm} (A.3)

Using Equation 3.323.2 of [19, p. 333] to the above equation leads to

$$\phi_{x_i}(\omega) = \exp\left(-\frac{\omega^2\sigma^2}{2}\right).$$  \hspace{1cm} (A.4)

Then, the characteristic function of $y$ is obtained by

$$\phi_y(\omega) = [\phi_{x_i}(\omega)]^L = \exp\left(-\frac{\omega^2\sigma^2 L}{2}\right).$$  \hspace{1cm} (A.5)
The PDF of $y$ can be expressed as

$$p_y(y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_y(\omega) \exp(-j\omega y) d\omega = \frac{1}{\sqrt{2\pi L \sigma^2}} \exp \left( -\frac{y^2}{2L\sigma^2} \right) . \hspace{1cm} (A.6)$$

The performance of BPSK in Gaussian noise with $L$-branch EGC diversity can be derived by

$$P_{e,BPSK}^{L-EGC} = Pr(R_I \leq 0) = Pr \left( y \leq -L\sqrt{E_b} \right) = \int_{-\infty}^{-L\sqrt{E_b}} p_y(y) dy$$

$$= \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E_bL}{2\sigma^2}} \right) . \hspace{1cm} (A.7)$$

The relation between signal energy per bit $E_b$ and SNR/bit $\gamma_b$ is

$$\gamma_b = \gamma_1 + \gamma_2 + \cdots + \gamma_L$$

$$= \frac{E_b}{2\sigma^2} + \frac{E_b}{2\sigma^2} + \cdots + \frac{E_b}{2\sigma^2}$$

$$= \frac{LE_b}{2\sigma^2} \hspace{1cm} (A.8)$$

where $\gamma_i$ is the signal-to-noise ratio per bit of the $i^{th}$ channel. Substituting Equation A.8 into Equation A.7, Equation 3.10 follows.

A.2 Derivation of Equation 4.6

To find the closed-form solution for the integral

$$\int_{a}^{\infty} \frac{dx}{(x^2 + b)^{c+1/2}} . \hspace{1cm} (A.9)$$
where \(a\) and \(b\) are real numbers, and \(c\) is a positive integer. The variable \(y = \left[1 - \sqrt{x^2/(x^2 + b)}\right]/2\) is introduced. Then, Equation A.9 can be rewritten as

\[
\int_a^\infty \frac{dx}{(x^2 + b)^{c+1/2}} = 2^{2c-1}b^{-c} \int_0^d y^{c-1}(1 - y)^{c-1} dy
\] (A.10)

where \(d = \left[1 - \sqrt{a^2/(a^2 + b)}\right]/2\). Using Equation 8.391 of [19, p. 900], the above equation becomes

\[
\int_a^\infty \frac{dx}{(x^2 + b)^{c+1/2}} = 2^{2c-1}b^{-c} \frac{d^c}{c} \frac{\Gamma(c, 1 - c; c + 1; d)}{\Gamma(c, 1 - c; c + 1)}
\]

\[
= 2^{2c-1}b^{-c} \sum_{n=0}^{c-1} \frac{(-1)^n\Gamma(c)\Gamma(c + n)}{n!\Gamma(c - n)\Gamma(c + n + 1)} [1 - (1 - d)]^n. \quad (A.11)
\]

Substituting \((c)_n = (c + n - 1)!/(c - 1)!, \ (1 - c)_n = (-1)^n(c - 1)!/(c - n - 1)!\) and \((c + 1)_n = (n + c)!/c!\) into Equation A.11 gives

\[
\int_a^\infty \frac{dx}{(x^2 + b)^{c+1/2}} = 2^{2c-1}b^{-c} d^c \sum_{n=0}^{c-1} \frac{(-1)^n\Gamma(c)\Gamma(c + n)}{n!\Gamma(c - n)\Gamma(c + n + 1)} [1 - (1 - d)]^n. \quad (A.12)
\]

The last term \([1 - (1 - d)]^n\) can be expanded by using power series Equation 1.111 of [19, p. 25] and then the following expression is obtained

\[
\int_a^\infty \frac{dx}{(x^2 + b)^{c+1/2}} = 2^{2c-1}b^{-c} d^c \sum_{n=0}^{c-1} \frac{(-1)^n\Gamma(c)\Gamma(n + c)}{\Gamma(c - n)\Gamma(n + c + 1)} \sum_{k=0}^n \frac{1}{k!(n-k)!} (-1)^k \times (1 - d)^k
\]

\[
= 2^{2c-1}b^{-c} d^c \sum_{k=0}^c \sum_{n=k}^{c-1} \frac{\Gamma(c)\Gamma(n + c)(-1)^{k+n}}{\Gamma(c - n)\Gamma(n + c + 1)k!(n-k)!} (1 - d)^k. \quad (A.13)
\]
Changing variable \( n = l + k \) in Equation A.13 results in

\[
\int_a^\infty \frac{dx}{(x^2 + b)^{c+1/2}} = 2^{2c-1} b^{-c} d^c \sum_{k=0}^{c-1} \frac{1}{k!} \frac{(1 - d)^k \Gamma(c) \Gamma(l + k + c)(-1)^l}{\Gamma(c - l - k) \Gamma(l + k + c + 1) l!} (1 - d)^k. \tag{A.14}
\]

Substituting the term \((-1)^l/\Gamma(c - l - k)\) with \(\Gamma(l + k - c + 1)/[\Gamma(c - k) \Gamma(k - c + 1)]\) into the above equation leads to

\[
\int_a^\infty \frac{dx}{(x^2 + b)^{c+1/2}} = 2^{2c-1} b^{-c} d^c \sum_{k=0}^{c-1} \frac{(1 - d)^k \Gamma(c) \Gamma(l + k + c)(l + k - c + 1)}{k! \Gamma(l + k + c + 1) \Gamma(c - k) l!} \times \frac{1}{\Gamma(k - c + 1)}. \tag{A.15}
\]

At \( l \geq c - k \), the term \(\Gamma(l + k - c + 1)\) is zero, and the above equation can be expressed alternatively as

\[
\int_a^\infty \frac{dx}{(x^2 + b)^{c+1/2}} = 2^{2c-1} b^{-c} d^c \sum_{k=0}^{c-1} \frac{(1 - d)^k \Gamma(c) \Gamma(l + k + c)}{k! \Gamma(c - k) \Gamma(k + c + 1)} \times \sum_{l=0}^{\infty} \frac{\Gamma(l + k + c) \Gamma(l + k - c + 1) \Gamma(k + c + 1)}{l! \Gamma(k + c) \Gamma(k - c + 1) \Gamma(l + k + c + 1)}. \tag{A.16}
\]

Applying Equation 9.122.1 of [19, p. 998] to the last summation term, the above equation is reduced to

\[
\int_a^\infty \frac{dx}{(x^2 + b)^{c+1/2}} = 2^{2c-1} b^{-c} d^c \sum_{k=0}^{c-1} \frac{(1 - d)^k \Gamma(k + c) \Gamma(c)}{k! \Gamma(2c)}. \tag{A.17}
\]

Rewriting Equation A.17 by substituting \( d \) yields the closed-form solution for Equation
A.9

\[ \int_{a}^{\infty} \frac{dx}{(x^2 + b)^{c+1/2}} = 2^{2c-1} \left[ \frac{1}{2b} \left( 1 - \sqrt{\frac{a^2}{a^2 + b}} \right) \right]^c \sum_{k=0}^{c-1} \frac{1}{k!} \left[ \frac{1}{2} \left( 1 + \sqrt{\frac{a^2}{a^2 + b}} \right) \right]^k \times \frac{\Gamma(k + c)\Gamma(c)}{\Gamma(2c)}. \] (A.18)