Application of Catastrophe Theory to Transient Stability Analysis of Multimachine Power Systems

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Abstract

Transient stability analysis is an important part of power planning and operation. For large power systems, such analysis is very time consuming and expensive. Therefore, an on-line transient stability assessment will be required as these large power systems are operated close to their maximum limits. In this thesis swallowtail catastrophe is used to determine the transient stability regions. The bifurcation set represents the transient stability region in terms of power system transient parameters bounded by the transient stability limits. The system modelling is generalized in such, that the analysis could handle either one or any number of critical machines. This generalized model is then tested on a three-machine as well as a seven-machine system. The results of the stability analysis done with the generalized method is compared with the time solution and the results were satisfactory. The transient stability regions determined are valid for any changes in loading conditions and fault location. This method is a good candidate for on-line assessment of transient stability of power systems.

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Chapter 1
Introduction

Instability in electric power systems, leading to loss of system synchronization, is a very sensitive problem for power utility engineers. In assessing power system stability there are two separate criteria to be considered, viz :

- Steady-state stability, for small perturbations, i.e., leading effectively to linear system analysis.
- Transient stability, for large system disturbances and involving non-linear system analysis.

The stability problem of power systems became very important following the famous power blackout in north eastern U.S.A. in 1965 . Planning, operations and control procedures of power systems had to be revised to ensure secure and reliable operation of power systems. Considerable research effort has gone into the stability investigation both for off-line and on-line purposes [1] .

A stable power system implies that all its interconnected generators are operating in synchronism with the network and with each other. These generators start to oscillate when a disturbance occurs due to a transmission fault or switching operation. Loss of synchronism must be prevented or controlled because it has a disturbing effect on voltages, frequency and power, and it may cause serious damage to generators, which are the most expensive components in a power system [2]. The generators which are losing synchronism due to the disturbance should be tripped, i.e. disconnected from the system before any serious damage occurs, and afterwards brought back to synchronism. Loss of synchronism may also cause some protective relays to operate falsely and trip the circuit breakers of unfaulted lines. In such cases the problem is very complicated and may result in more generators losing synchronism.

Therefore, an understanding of system stability requires a thorough knowledge of both the mathematical modelling of the system and effective numerical techniques. In most cases, the model

consists of a set of linear or non-linear algebraic and/or differential equations depending upon the type of study that is to be performed. It is important to select a numerical method which will provide accurate results, but the rapid growth of power systems makes it extremely difficult, expensive and time consuming to carry out these careful and detailed stability studies through solution of the system equations. Two possibilities for improving the speed of transient studies are : [3]

- Reduction of the total system to a smaller one, that could be solved faster but has the disadvantage of inaccuracy because of approximations.
- Improvement in the numerical solution techniques such as the *trapezoidal rule of integration method* which has already been used successfully for the solution of switching transients at the Bonneville Power Administration (BPA) [4] .

Much work has been done to find ways to reduce the amount of computation required for stability studies and to find direct methods to solve the transient stability problem that do not require the solution of the system equations . One direct method used is Lyapunov's direct method of determining stability [5]. However, this method is still not suitable for on-line applications [6] and has practical difficulties such as :

- The method is conservative and, although some stable points are readily identified, others are inconclusive.
- This method has difficulty deducing the Lyapunov's function which defines all possible stability regions, and it cannot predict instability, therefore, it may produce false alarms.

Other *Direct* methods of stability analysis are currently under consideration and investigation [7] .

Catastrophe theory is a new way of thinking about changes such as in a course of events, a systems behavior, or even change in ideas themselves. Its name suggests disaster, and indeed the theory can be applied to literal catastrophes. The mathematical principles we are used to are ideally suited to analyse smooth, continuous, qualitative change, [8] but there is another kind of change,

that is less suited to mathematical analysis : such as the discontinuous transition from ice at its melting point to water at its freezing point or the transition from stable to unstable state for a power system following a disturbance. The foundations of catastrophe theory were developed by the French mathematician *Rene'Thom* and became widely known through his book *Stabilite' Structurelle et Morphogenese* in which he proposed them as a foundation for biology.

A catastrophe, in the very broad sense Thom gives to the word, is any discontinuous transition that occurs when a system can have more than one stable state, or can follow more than one stable pathway of change. The catastrophe is the *jump* from one state or pathway to another [9] . The elementary catastrophes are the seven simplest ways such a transaction from one state to another state can occur. This is true for any system governed by a potential, and in which the behavior of the system is determined by no more than four different factors, then only seven qualitatively different types of discontinuity are possible [10] . The qualitative type of any stable discontinuity does not depend on the specific nature of the potential involved, merely on its existence, i.e. on the existence of cause-and-effect relationship between conditions. Now we can see how the elementary catastrophes are comparable to the regular forms of classical geometry. Just as we can say that any three dimensional object, if it is regular (i.e. all its faces are identical polygons), must be one of the five solids, so the catastrophe theory asserts that any discontinuous process whose behavior can be described by a graph in as many as six dimensions, if structurally stable, must correspond to one of the seven elementary catastrophes [11] . The seven elementary catastrophes is shown in Table 1.1.

Catastrophe theory is used for stability analysis of multimachine power systems. This theory has been applied previously to the steady-state stability of power systems, one-machine infinite bus system, as well as multimachine systems [12] with the worst case approach i.e., only one generator becoming critical for a three-phase fault. Catastrophe theory has also been used as a tool for determining synchronous power system dynamic stability [13]. The application of catastrophe theory to steady-state and transient stability of power systems is attractive because it provides comprehensive

stability regions with minimal computation. The transient stability regions have been shown to be applicable for changes in loading conditions and fault locations [12] .

This thesis generalizes the use of catastrophe theory to the case of multimachine power systems, with more than one machine being critical (likely to go unstable). Chapter 2 of this thesis derives the mathematical ideas involved in reducing the multimachine power system and the application of catastrophe theory by first briefly going through the single machine infinite bus system. In deriving the equations for the multimachine system, the general dynamic equivalent approach is used, grouping all the critical generators as one equivalent machine and grouping the rest of the system as another single equivalent machine. Then the swallowtail catastrophe is applied to this general system. Chapter 3 contains test results from two power systems, and the conclusion of the application of catastrophe theory to multimachine power systems is in Chapter 4. The program listing for the catastrophe application is given in Appendix.

Catastrophe	Control Dimensions	Catastrophe Manifold
Fold	1	$x^2 + u$
Cusp	2	$x^3 + ux + v$
Swallowtail	3	$x^4 + ux^2 + vx + w$
Butterfly	4	$6x^5 + ux^3 + vx^2 + wx + r$
Elliptic	3	$3x^2 - 3y^2 + 2ux + v$ $-6xy + 2uy + w$
Hyperbolic	3	$3x^2 + uy + v$ $3y^2 + ux + w$
Parabolic	4	$2xy + 2ux + w$ $4y^3 + x^2 + 2vy + r$

Table 1.1: The seven elementary catastrophes

Chapter 2
Transient Stability Analysis of Multimachine Power Systems Using Catastrophe Theory

2.1 Introduction

The transient stability analysis of multimachine power systems is more complicated than that of a single machine infinite bus system because the behavior of each machine is effected by and has an effect on the behavior of all the other machines coupled to it. During a large system disturbance, usually there are two switching done, one during the occurrence of the fault and the other at the time of clearance of the fault.

For transient stability analysis the following assumptions are made :

- Each generator, i is modelled by a constant voltage, $| E_i |$, behind its direct axis transient reactance, x'_{di}.
- Turbine dynamics are ignored so that the mechanical power input to each generator, P_{mi}, is assumed constant.
- Mechanical damping is ignored.
- The loads are modelled as constant impedances.

In this chapter we briefly review the application of catastrophe theory to a single machine infinite bus system [14] . Then we apply the General Dynamic Equivalent Method [15] [16]to multimachine power systems such that a suitable energy function can be defined for the application of catastrophe theory. This general method requires the identification of the critical machines involved for each fault being considered. The group of critical machines is then replaced by an equivalent machine and the rest of the system which is not significantly affected by the disturbance is also replaced by an equivalent machine.

The transient stability regions are found by use of catastrophe theory in terms of system parameters.

2.2 Catastrophe Theory Applied to Single Machine Infinite Bus System

Consider the one machine-infinite bus system [17]in Figure 2.1 which has two transmission lines.

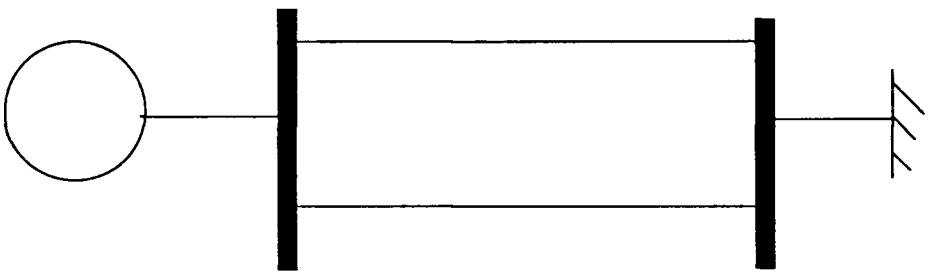


Figure 2.1: Single-machine Infinite-bus power system

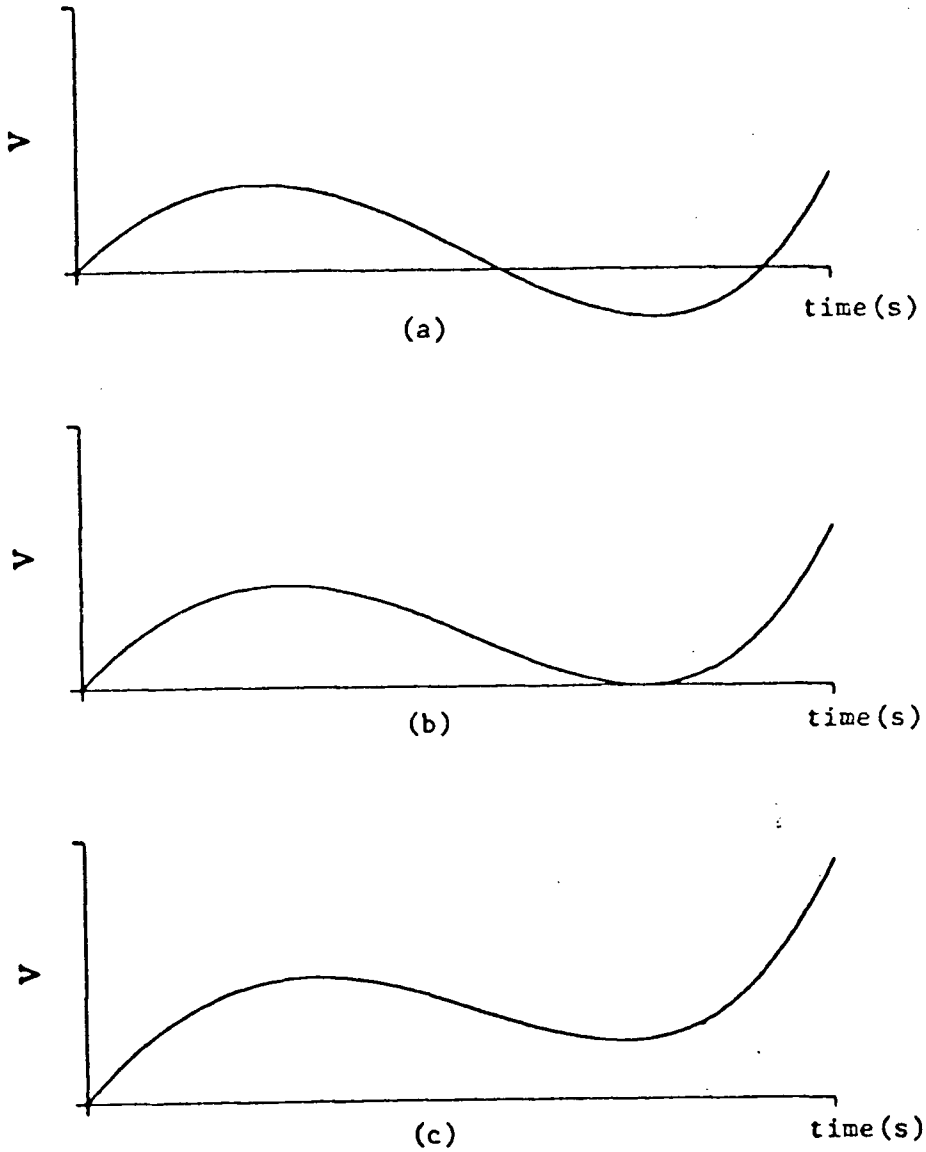
The swing equation is given by

$$\begin{aligned} M\frac{d^2\psi}{dt^2} &= P_i - P_e \\ &= P_i - P_{\max}\sin\psi \\ &= P_a \end{aligned} \tag{2.1}$$

where

$$\begin{aligned} M &= \text{inertia constant of machine} \\ P_e &= \text{electrical power output} \\ P_i &= \text{mechanical power input} \\ P_a &= \text{accelerating power} \\ \psi &= \text{rotor angle of the machine} \\ P_{max} &= \text{maximum power for post fault condition} \end{aligned} \tag{2.2}$$

If a three-phase fault occurs on one of the transmission lines near the generator bus, the rotor will start to accelerate and hence the machine would gain kinetic energy. If the fault is cleared at a clearing time such that the kinetic energy produced by the fault is absorbed by the potential energy produced after the clearance of the fault and the gained energy is less than zero then the system is stable and, if exactly zero the system is critically stable. This is shown in figure 2.2



a : Stable

b : Critically Stable

c : Unstable

Consider the critical clearing case for a three-phase fault initiated near the generator bus. Then

$$\text{kinetic energy} = \text{potential energy} \tag{2.3}$$

Equation 2.3 can be derived by multiplying equation 2.1 by $\dot{\psi}$ and integrating with respect to time once between ψ_0, ψ_c and next between ψ_c, ψ_m to obtain

$$\frac{1}{2}M\dot{\psi}_c^2 = P_m \cos\psi_c + P_i\psi_c - P_i\psi_m - P_m \cos\psi_m \tag{2.4}$$

where

$$\begin{aligned} \psi_c &= \text{critical clearing angle} \\ P_m &= \text{maximum power of post-fault network} \\ \psi_m &= \text{maximum angle} \end{aligned} \tag{2.5}$$

Using *Taylor series* expansion to approximate $\dot{\psi}_c$ and ψ_c as a function of time we get

$$\begin{aligned} \dot{\psi}_c &= \omega_c \\ &= \gamma t_c \end{aligned} \tag{2.6}$$

and

$$\psi_c = \psi_0 + \frac{1}{2}\gamma t_c^2 \tag{2.7}$$

where

$$\begin{aligned} \gamma &= \text{acceleration at instant of occurrence of fault} \\ &= \frac{1}{M}\left[P_i - P_e(t_{0+})\right] \end{aligned} \tag{2.8}$$

Replacing $\cos\psi_c$ in equation (2.4) by cosine series expansion and defining

$$x \triangleq \frac{1}{2}\gamma t_c^2 \tag{2.9}$$

$$k \triangleq P_i\psi_m + P_m \cos\psi_m \tag{2.10}$$

we obtain

$$\begin{aligned} -\frac{P_m}{24}x^4 - \frac{P_m}{6}\psi_0 x^3 + \left(\frac{2-\psi_0^2}{4}\right) P_m x^2 \\ + \left(M\gamma + P_m\psi_0 - \frac{P_m}{6}\psi_0^3 - P_i\right) x \\ + \left(\frac{P_m}{2}\psi_0^2 - \frac{P_m}{24}\psi_0^4 - P_m - P_i\psi_0 + k\right) = 0 \end{aligned} \tag{2.11}$$

For the above equation to be in the form of swallowtail catastrophe manifold divide the equation by $-\frac{24}{P_m}$ and eliminate the cubic term, by setting

$$x = y - \psi_0 \tag{2.12}$$

Therefore,

$$y^4 - 12y^2 + \frac{24}{P_m}\left[P_i - M\gamma\right] y + \left[\frac{24}{P_m}(M\gamma\psi_0 - k) + 24\right] = 0 \tag{2.13}$$

This is in the form of the standard swallowtail catastrophe manifold namely:

$$y^4 + uy^2 + vy + w = 0 \tag{2.14}$$

where

$$\begin{aligned} u &= -12 \\ v &= \frac{24}{P_m}\left[P_i - M\gamma\right] \\ w &= \frac{24}{P_m}\left[M\gamma\psi_0 - k\right] + 24 \end{aligned} \tag{2.15}$$

The bifurcation set can then be defined by

$$4y^3 + 2uy + v = 0 \tag{2.16}$$

and the transient stability region in terms of the power system parameters then takes the shape of the swallowtail bifurcation set. The region is defined by the above u, v and w parameters.

2.3 General Dynamic Equivalent Method for Multimachine Power Systems

The general dynamic equivalent method [15] [16]will be discussed in this section.

The equation of motion of machine i in a multimachine power system using classical model representation is given by

$$\dot{\delta}_i = \omega_i \qquad i = 1, \ldots, n$$
$$M_i \ddot{\delta}_i = P_{mi} - P_{ei} \tag{2.17}$$

$$\begin{aligned} P_{ei} &= \text{electrical power output of machines} \\ &= \sum_{j=1}^{n} \left[D_{ij} \cos \delta_{ij} + C_{ij} \sin \delta_{ij} \right] \end{aligned} \tag{2.18}$$

$$\begin{aligned} P_{mi} &= \text{mechanical power input} \\ M_i &= \text{inertia constant} \\ E_i &= \text{internal generator voltage} \\ \omega_i &= \text{rotor speed} \\ b_{ij} &= \text{transfer susceptance} \\ g_{ij} &= \text{transfer conductance} \end{aligned} \tag{2.19}$$

$$\begin{aligned} \delta_i &= \text{rotor angle} \\ \delta_{ij} &= \delta_i - \delta_j \\ D_{ij} &= E_i E_j g_{ij} \\ C_{ij} &= E_i E_j b_{ij} \end{aligned} \tag{2.20}$$

The critical machines are those machines that tend to respond actively to the occurrence of the fault and may lose synchronism. Therefore, in order to determine the transient stability of the power system it would be sufficient to group these critical machines as one equivalent machine and study the response of this equivalent machine with respect to the undisturbed equivalent machine representing the rest of the system.

Consider that A represents the critical machines for a specific three-phase fault. These machines are considered as one equivalent critical machine which oscillates against B, the rest of the power

system which is not significantly disturbed by the fault and also considered to be equivalent to one machine. Let

$$\begin{aligned} M_c &= \sum_{k\in A} M_k \\ M_0 &= \sum_{i\in B} M_i \\ \delta_c &= \frac{1}{M_c}\sum_{k\in A} M_k\delta_k \\ \delta_0 &= \frac{1}{M_0}\sum_{i\in B} M_i\delta_i \end{aligned} \tag{2.21}$$

where M_0 and δ_0 are , respectively , the inertia constant and the angle of the centre of angle of the power system with the critical machines excluded.

Let

$$\eta_k = \delta_k - \delta_c$$

$$\theta_i = \delta_i - \delta_0$$

then

$$\psi = \delta_c - \delta_0$$

$$\psi = \frac{1}{M_c}\sum_{k\in A} M_k\delta_k - \frac{1}{M_0}\sum_{i\in B} M_i\delta_i \tag{2.24}$$

$$\begin{aligned} \ddot{\psi} &= \frac{1}{M_c}\sum_{k\in A}\left(M_k\ddot{\delta}_k\right) - \frac{1}{M_0}\sum_{i\in B}\left(M_i\ddot{\delta}_i\right) \\ M_c\ddot{\psi} &= \sum_{k\in A}\left(M_k\ddot{\delta}_k\right) - \frac{M_c}{M_0}\sum_{i\in B}\left(M_i\ddot{\delta}_i\right) \end{aligned} \tag{2.25}$$

But

$$\begin{aligned} M_k\ddot{\delta}_k &= P_{mk} - P_{ek} \\ &= P_{mk} - \sum_{l=1}^{n}\Big[D_{lk}\cos(\eta_k - \eta_l) + C_{kl}\sin(\eta_k - \eta_l)\Big] \end{aligned} \tag{2.26}$$

and similarly

$$\begin{aligned} M_i\ddot{\delta}_i &= P_{mi} - P_{ei} \\ &= P_{mi} - \sum_{j=1}^{n}\Big[D_{ij}\cos(\theta_i - \theta_j) + C_{ij}\sin(\theta_i - \theta_j)\Big] \end{aligned} \tag{2.27}$$

Substituting equation (2.26) and (2.27) in (2.25) and following some mathematical manipulation we obtain the swing equation of the critical machines against the rest of the system.

This is explained in the following steps:

$$M_c\ddot{\psi} = \sum_{k\in A}\left\{P_{mk} - \sum_{l=1}^{n}[D_{kl}\cos(\eta_k-\eta_l) + C_{kl}\sin(\eta_k-\eta_l)]\right\} - \frac{M_c}{M_0}\sum_{i\in B}\left\{P_{mi} - \sum_{j=1}^{n}[D_{ij}\cos(\theta_i-\theta_j) + C_{ij}\sin(\theta_i-\theta_j)]\right\} \tag{2.28}$$

separating the D_{kk} term from the rest of the equation we obtain

$$\begin{aligned} M_c\ddot{\psi} = \sum_{k\in A}\Big\{ & P_{mk} - \sum_{l=k}^{n}\Big[D_{kl}\cos(\eta_k-\eta_l) + C_{kl}\sin(\eta_k-\eta_l)\Big] \\ & + \sum_{l\neq k}^{n}\Big[D_{kl}\cos(\eta_k-\eta_l) + C_{kl}\sin(\eta_k-\eta_l)\Big]\Big\} \\ -\frac{M_c}{M_0}\sum_{i\in B}\Big\{ & P_{mi} - \sum_{j\in A}\Big[D_{ij}\cos(\theta_i-\theta_j) + C_{ij}\sin(\theta_i-\theta_j)\Big] \\ & + \sum_{j\in B}\Big[D_{ij}\cos(\theta_i-\theta_j) + C_{ij}\sin(\theta_i-\theta_j)\Big]\Big\} \end{aligned} \tag{2.29}$$

grouping the appropriate terms together i.e.

$$\begin{aligned} M_c\ddot{\psi} = & \left\{\sum_{k\in A}P_{mk} - \frac{M_c}{M_0}\sum_{i\in B}P_{mi}\right\} \\ & - \Big\{\sum_{k\in A}\sum_{l=k}^{n}\Big[D_{kl}\cos(\eta_k-\eta_l) + C_{kl}\sin(\eta_k-\eta_l)\Big] \\ & - \frac{M_c}{M_0}\sum_{i\in B}\sum_{j\in B}[D_{ij}\cos(\theta_i-\theta_j) + C_{ij}\sin(\theta_i-\theta_j)]\Big\} \\ & - \Big\{\sum_{k\in A}\sum_{l\neq k}^{n}\Big[D_{kl}\cos(\eta_k-\eta_l) + C_{kl}\sin(\eta_k-\eta_l)\Big] \\ & - \frac{M_c}{M_0}\sum_{i\in B}\sum_{j\in A}\Big[D_{ij}\cos(\theta_i-\theta_j) + C_{ij}\sin(\theta_i-\theta_j)\Big]\Big\} \end{aligned} \tag{2.30}$$

Let

$$P_m = \left\{\sum_{k\in A}P_{mk} - \frac{M_c}{M_0}\sum_{i\in B}P_{mi}\right\} \tag{2.31}$$

$$\begin{aligned} P_c = & \Big\{\sum_{k\in A}\sum_{l=k}^{n}\Big[D_{kl}\cos(\eta_k-\eta_l) + C_{kl}\sin(\eta_k-\eta_l)\Big] \\ & - \frac{M_c}{M_0}\sum_{i\in B}\sum_{j\in B}\Big[D_{ij}\cos(\theta_i-\theta_j) + C_{ij}\sin(\theta_i-\theta_j)\Big]\Big\} \end{aligned} \tag{2.32}$$

Simplifying the P_c term

$$P_c = \left\{ \sum_{k \in A} D_{kk} - \frac{M_c}{M_0} \sum_{i \in B} \sum_{j \in B} \left[D_{ij} \cos(\theta_i - \theta_j) \right] \right\} \tag{2.33}$$

Therefore ,

$$\begin{aligned} M_c \ddot{\psi} = P_k - \Bigg\{ & \sum_{k \in A} \sum_{l \neq k}^{n} \left[D_{kl} \cos(\eta_k - \eta_l) + C_{kl} \sin(\eta_k - \eta_l) \right] \\ & - \frac{M_c}{M_0} \sum_{i \in B} \sum_{j \in A} \left[D_{ij} \cos(\theta_i - \theta_j) + C_{ij} \sin(\theta_i - \theta_j) \right] \Bigg\} \end{aligned} \tag{2.34}$$

where we have defined

$$P_k = P_m - P_c \tag{2.35}$$

Since

$$\begin{aligned} \psi &= \delta_c - \delta_0 \\ \eta_i &= \delta_i - \delta_c \\ \theta_k &= \delta_k - \delta_0 \end{aligned} \tag{2.36}$$

therefore we can write

$$\begin{aligned} \theta_k &= \eta_k + \psi \\ \eta_i &= \theta_i - \psi \end{aligned} \tag{2.37}$$

Substituting equation (2.37) in equation (2.34) and rearranging :

$$\begin{aligned} M_c \ddot{\psi} = P_k - \Bigg\{ & \sum_{k \in A} \sum_{i \neq k} \left[D_{ki} \cos(\eta_k - \theta_i + \psi) + C_{ki} \sin(\eta_k - \theta_i + \psi) \right] \\ & - \frac{M_c}{M_0} \sum_{k \in A} \sum_{i \in B} \left[D_{ki} \cos(\eta_k - \theta_i + \psi) - C_{ki} \sin(\eta_k - \theta_i + \psi) \right] \Bigg\} \end{aligned} \tag{2.38}$$

Expanding the *sine* and *cosine* terms of the above equation we obtain :

$$\begin{aligned} M_c\ddot{\psi} = P_k - \Bigg\{ & \sum_{k\in A}\sum_{i\neq k} D_{ki}\Big[\cos(\theta_i-\eta_k)\cos\psi + \sin(\theta_i-\eta_k)\sin\psi\Big] \\ & - \frac{M_c}{M_0}\sum_{k\in A}\sum_{i\in B} D_{ki}\Big[\cos(\theta_i-\eta_k)\cos\psi + \sin(\theta_i-\eta_k)\sin\psi\Big] \\ & - \sum_{k\in A}\sum_{i\neq k}^{n} C_{ki}\Big[\sin(\theta_i-\eta_k)\cos\psi - \cos(\theta_i-\eta_k)\sin\psi\Big] \\ & - \frac{M_c}{M_0}\sum_{k\in A}\sum_{i\in B} C_{ki}\Big[\sin(\theta_i-\eta_k)\cos\psi - \cos(\theta_i-\eta_k)\sin\psi\Big]\Bigg\} \end{aligned} \tag{2.39}$$

$$\begin{aligned} M_c\ddot{\psi} = P_k - \Bigg\{ & \Big\{\sum_{k\in A}\sum_{i\neq k}^{n}[D_{ki}\sin(\theta_i-\eta_k) + C_{ki}\cos(\theta_i-\eta_k)] \\ & - \frac{M_c}{M_0}\sum_{k\in A}\sum_{i\in B}[D_{ki}\sin(\theta_i-\eta_k) - C_{ki}\cos(\theta_i-\eta_k)]\Big\}\sin\psi \\ & - \Big\{\frac{M_c}{M_0}\sum_{k\in A}\sum_{i\in B}[D_{ki}\cos(\theta_i-\eta_k) + C_{ki}\sin(\theta_i-\eta_k)] \\ & + \sum_{k\in A}\sum_{i\neq k}^{n}[C_{ki}\sin(\theta_i-\eta_k) - D_{ki}\cos(\theta_i-\eta_k)]\Big\}\cos\psi\Bigg\} \end{aligned}$$

Therefore, the swing equation of the single machine representing the group of critical machines has the form :

$$M_c\ddot{\psi} = P_k - T_k\sin(\psi - \alpha_k) \tag{2.41}$$

where

$$\begin{aligned} P_k = & \Big\{\sum_{k\in A} P_{mk} - \frac{M_c}{M_0}\sum_{i\in B} P_{mi}\Big\} \\ & - \Big\{\sum_{k\in A} D_{kk} - \frac{M_c}{M_0}\sum_{i\in B}\sum_{j\in B}[D_{ij}\cos(\theta_i-\theta_j)]\Big\} \end{aligned} \tag{2.42}$$

$$T_k = \sqrt{a_k^2 + b_k^2} \tag{2.43}$$

$$\alpha_k = \tan^{-1} \frac{a_k}{b_k}$$

$$\begin{aligned} a_k = \frac{M_c}{M_0} \sum_{k \in A} \sum_{i \in B} \Big[D_{ki} \cos(\theta_i - \eta_k) + C_{ki} \sin(\theta_i - \eta_k) \Big] \\ + \sum_{k \in A} \sum_{i \neq k}^{n} \Big[C_{ki} \sin(\theta_i - \eta_k) - D_{ki} \cos(\theta_i - \eta_k) \Big] \end{aligned} \tag{2.45}$$

$$\begin{aligned} b_k = -\frac{M_c}{M0} \sum_{k \in A} \sum_{i \in B} \Big[D_{ki} \sin(\theta_i - \eta_k) - C_{ki} \cos(\theta_i - \eta_k) \Big] \\ + \sum_{k \in A} \sum_{i \neq k}^{n} \Big[D_{ki} \sin(\theta_i - \eta_k) + C_{ki} \cos(\theta_i - \eta_k) \Big] \end{aligned} \tag{2.46}$$

2.4 Application of Catastrophe Theory to Multimachine Power Systems

During the transient period an exchange of energy takes place between the rotor of the critical machines and the post-fault network [18] . The kinetic energy generated by the accelerating power during the fault-on period must be fully absorbed by the post-fault network in order to maintain stability.

Using equation (2.41) from the previous section, namely :

$$M_k \ddot{\psi}_k = P_k - T_k \sin(\psi_k - \alpha_k) \tag{2.47}$$

which represents the motion of the group of critical machines represented as a single machine with respect to the rest of the system, also represented as a single machine, for a certain three-phase fault. Since we have assumed that the rest of the system is not responding to the disturbance, it is reasonable to use the pre-disturbance angles θ_0 and η_0 to calculate the parameters P_k , T_k and α_k.

By solving equation (2.47) for ψ_k, the stable and unstable points are computed i.e.

$$P_k - T_k \sin(\psi_k^s - \alpha_k) = 0 \tag{2.48}$$

$$\psi_k^s = \text{stable equlibrium point} \tag{2.49}$$

and the unstable equilibrium point (UEP) is

$$\psi_k^u = \pi - \psi_k^s \tag{2.50}$$

Multiply equation (2.47) by $\dot{\psi}_k$ and integrate between ψ_k^0 and ψ_k^c with respect to time we, obtain the kinetic energy generated by the fault :

$$\begin{aligned} \frac{1}{2}M_k\dot{\psi}_k^{c^2} &= \text{K.E.} \\ &= P_k^f\left(\psi_k^c - \psi_k^0\right) - T_k^f\left[\cos\left(\psi_k^0 - \alpha_k^f\right) - \cos\left(\psi_k^c - \alpha_k^0\right)\right] \end{aligned} \tag{2.51}$$

where P_k^f, T_k^f and α_k^f are the fault-on parameters and ψ_k^c is the clearing angle.

The potential energy of the post-fault system is derived in the same fashion but is integrated between ψ_k^c and ψ_k^u using the post-fault parameters

$$\begin{aligned} -\frac{1}{2}M_k\dot{\psi}_k^{c^2} &= \text{P.E.} \\ &= P_k^p(\psi_k^u - \psi_k^c) - T_k^p\left[\cos\left(\psi_k^c - \alpha_k^p\right) - \cos\left(\psi_k^u - \alpha_k^p\right)\right] \end{aligned} \tag{2.52}$$

The L.H.S. of equation (2.52) represents the kinetic energy produced during the fault and the R.H.S. represents the potential energy of the post-fault network. In order for the system to be stable the kinetic energy should be equal to or less than the potential energy. Therefore,

$$\frac{1}{2}M_k\dot{\psi}_k^{c^2} - P_k^p\psi_k^c - T_k^p\cos\left(\psi_k^c - \alpha_k^p\right) + k^u = 0 \tag{2.53}$$

where we define

$$k^u \triangleq P_k^p\psi_k^u + T_k^p\cos\left(\psi_k^u - \alpha_k^p\right) \tag{2.54}$$

Expanding ψ_k^c by a Taylor series and using the first two terms only :

$$\psi_k^c = \psi_k^0 + \frac{1}{2}\gamma_k t_c^2 \tag{2.55}$$

and

$$\dot{\psi}_k^c = \gamma_k t_c \tag{2.56}$$

where

$$\gamma_k = \frac{1}{M_k}\Big[P_k - P_{ek}^t(t_{0+})\Big] \tag{2.57}$$

Replacing the *cosine* term by cosine series expansion up to the fourth order and defining

$$\begin{aligned} x &\triangleq \frac{1}{2}\gamma_k t_c^2 \\ \beta &\triangleq \psi_k^0 - \alpha_k^p \end{aligned} \tag{2.58}$$

then after some mathematical manipulation, we get the catastrophe manifold equation as shown in the following steps :

$$\begin{aligned} &\frac{1}{2}M_k(\gamma_k t_c)^2 - P_k^p\left(\psi_k^0 + x\right) - T_k^p \cos\left(\psi_k^0 + x - \alpha_k^p\right) + k^u = 0 \\ &M_k\gamma_k x - P_k^p\left(\psi_k^0 + x\right) - T_k^p \cos\Bigg[1 - \frac{\left(x + \psi_k^0 - \alpha_k^p\right)^2}{2!} \\ &\qquad\qquad \frac{\left(x + \psi_k^0 - \alpha_k^p\right)^4}{4!}\Bigg] + k^u = 0 \end{aligned} \tag{2.59}$$

$$M_k\gamma_k x - P_k^p\left(\psi_k^0 + x\right) - T_k^p\Big[1 - \frac{(x+\beta)^2}{2!} + \frac{(x+\beta)^4}{4!}\Big] + k^u = 0 \tag{2.60}$$

Therefore, expanding and simplifying equation (2.60) we obtain :

$$\begin{aligned} -\frac{T_k^p}{24}\,x^4 - \frac{T_k^p}{6}\beta\,x^3 + &\Big[\frac{T_k^p}{2} - \frac{T_k^p}{4}\beta^2\Big]\,x^2 \\ &+ \Big[T_k^p\beta - P_k^p - \frac{T_k^p}{6}\beta^3 + M_k\gamma_k\Big]\,x \\ &+ \Big[\frac{T_k^p}{2}\beta^2 - P_k^p\psi_k^0 - \frac{T_k^p}{24}\beta^4\Big] + k^u = 0 \end{aligned} \tag{2.61}$$

dividing equation (2.61) by $-\frac{24}{T_k^p}$ to give :

$$\begin{aligned} x^4 + (4\beta)\, x^3 - 12\left[1 - \frac{\beta^2}{2}\right] x^2 \\ - \frac{24}{T_k^p}\left[M_k\gamma_k + T_k^p\beta - P_k^p - \frac{T_k^p}{6}\beta^3\right] x \\ - \frac{24}{T_k^p}\left[\frac{T_k^p}{2}\beta^2 - P_k^p\psi_k^0 - \frac{T_k^p}{24}\beta^4 - T_k^p + k^u \right] = 0 \end{aligned} \tag{2.62}$$

In order to obtain the swallowtail catastrophe manifold , we have to eliminate the third order term in equation (2.62), by letting :

$$x = y - \beta \tag{2.63}$$

Therefore,

$$\begin{aligned} (y-\beta)^4 + 4\beta\,(y-\beta)^3 - 12\left[1 - \frac{\beta^2}{2}\right](y-\beta)^2 \\ - \frac{24}{T_k^p}\left[M_k\gamma_k + T_k^p\beta - P_k^p - \frac{T_k^p}{6}\beta^3\right](y-\beta) \\ - \frac{24}{T_k^p}\left[\frac{T_k^p}{2}\beta^2 - P_k^p\psi_k^0 - \frac{T_k^p}{24}\beta^4 - T_k^p + k^u \right] = 0 \end{aligned} \tag{2.64}$$

and expanding we get

$$\begin{aligned} (y^4 - 4y^3\beta + 6y^2\beta^2 - 4y\beta^3 + \beta^4) \\ + 4\beta\,(y^3 - 3y^2\beta + 3y\beta^2 - \beta^3) \\ + (6\beta^2 - 12)\,(y^2 - 2y\beta + \beta^2) \\ - \frac{24}{T_k^p}\left[M_k\gamma_k + T_k^p\beta - P_k^p - \frac{T_k^p}{6}\beta^3\right](y-\beta) \\ - \frac{24}{T_k^p}\left[\frac{T_k^p}{2}\beta^2 - P_k^p\psi_k^0 - \frac{T_k^p}{24}\beta^4 - T_k^p + k^u \right] = 0 \end{aligned} \tag{2.65}$$

rearranging the terms and simplifying the above equation

$$\begin{aligned} y^4 - 12y^2 + \frac{24}{T_k^p}\,(P_k^p - M_k\gamma_k\,)\,y \\ + \left[\frac{24}{T_k^p}(P_k^p\psi_k^0 - k^u - P_k^p\beta + M_k\gamma_k\beta) + 24\right] = 0 \end{aligned} \tag{2.66}$$

equation (2.66) is in the form of the swallowtail catastrophe manifold i.e :

$$y^4 + uy^3 + vy + w = 0 \tag{2.67}$$

where

$$
\begin{aligned}
u &= -12 \\
v &= \frac{24}{T_k^p}\left[P_k^p - M_k\gamma_k\right] \\
w &= \frac{24}{T_k^p}\left[P_k^p\psi_k^0 - k^u - P_k^p\beta + M_k\gamma_k\beta\right] + 24
\end{aligned}
\tag{2.68}
$$

The control variables u , v , w for the swallowtail catastrophe obtained for the multimachine power system can be compared to the control variables of the single machine infinite bus system namely equation (2.15) . It is seen that the control variables have the same form and the equations derived for the multimachine power system reduces to that of the single machine infinite bus system when the number of the critical machines is one and the rest of the system is also one.

The bifurcation manifold is reduced from three dimensions to only two dimensions in v and w as $u = -12$. The boundaries of the bifurcation set of Figure 2.3 represents the degenerate transient stability limits of the power system. It should be noted that for a generator the stable points are in the region of a positive v and w and for a motor the stable points lie between a negative v and a positive w.

Several comments are in order here :

- During a three-phase short-circuit of a generation bus the transfer admittances between machine k and other machines are zero i.e. $g_{kj} = b_{kj} = 0$ so $T_k = 0$ and the electric power output during fault-on period is found using equation 2.18
- When combining critical machines in an equivalent machine if the fault duration is short, the machine angle offsets will not change ; thus we may use the prefault steady-state values along with fault-on values for the b's and the g's to compute the fault-on parameters.

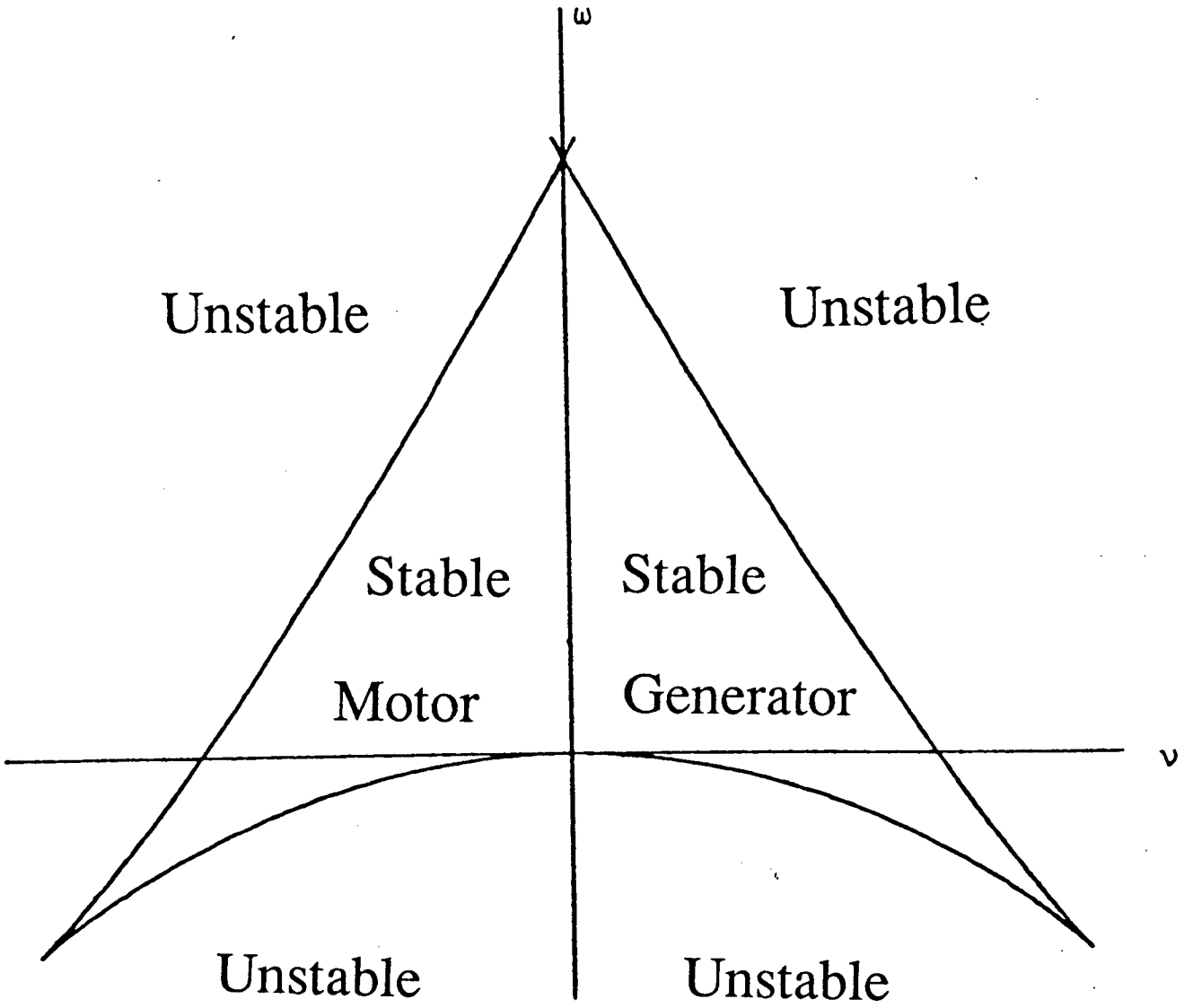


Figure 2.3: The transient stability limits given by the swallowtail catastrophe for a multimachine power system.

2.5 Identification of the Critical Machines

The method presented depends upon the accurate identification of the critical machines for a specified disturbance. Correct identification could be achieved by calculating the unstable equilibrium points for all machines in the power system; the machine having the highest unstable equilibrium point would be identified as the critical machine [19] , but its drawback is the calculation of the unstable equilibrium points, which is time consuming.

In this thesis for a certain three-phase fault sequence occurring at either generator or non-generator buses, the critical machine (s) are identified as follows :

- Calculate the initial acceleration for each machine using

$$\gamma_i = \frac{1}{M_i}\left[P_{mi} - P_{ei}\left(t_0^+\right)\right] \tag{2.69}$$

  where $P_{ei}\left(t_0^+\right)$ is the electrical power output during fault at the instant of fault occurrence.

- The machines which have high and positive initial accelerations are injecting kinetic energy to the system; therefore, they all contribute to the system instability and should be combined to form a single critical machine. In practice, only two, at times three, machines having the largest initial acceleration will be declared as candidates [15].

Chapter 3
Numerical Examples

In this chapter two test systems are presented to demonstrate the validity and advantages of the application of catastrophe theory to transient stability assessment of power systems. Three and seven machine power systems are used, where three-phase short circuits are considered at different locations. For each test system there will be a one line diagram, the steady state loadflow and the systems data. Transient stability regions in terms of systems catastrophe control parameters are given for each example used and they are compared with the time solution.

Each three-phase short-circuit case considered is evaluated by the following steps :

- Construct the systems reduced prefault, during-fault and the post-fault matrices.
- Identify the critical machine or machines for each case.
- Calculate the general dynamic equivalent parameters i.e. P_k, T_k, α_k, ψ_k and M_k.
- Calculate the bifurcation set parameters for the swallowtail catastrophe.
- Each case is then compared with the time solution.

3.1 The Three-Machine System

This system has nine buses, three machines and three loads [20] . It is widely referred to in the literature as the Western Systems Coordinating Council (WSCC) test system. A one-line diagram for the system is given in Figure 3.4 . The prefault normal load flow is given in Figure 3.5. Transmission line parameters and loads impedances are given in per unit on a 100 MVA base in Table 3.2. Generator data and initial operating conditions are given in Table 3.3.

Three-phase short circuits are considered at different locations. The transient stability of each fault location is evaluated by the use of the swallowtail catastrophe.

	Bus No.	Admittances (pu)	
		G	B
Generators			
1	1 - 4	0.0	-8.446
2	2 - 7	0.0	-5.485
3	3 - 9	0.0	-4.168
Transmission Lines			
	4 - 5	1.365	-11.604
	4 - 6	1.942	-10.511
	5 - 7	1.188	-5.975
	6 - 9	1.282	-5.588
	7 - 8	1.617	-13.698
	8 - 9	1.155	-9.784
Shunt Admittances			
Load A	5 - 0	1.261	-0.263
Load B	6 - 0	0.878	-0.035
Load C	8 - 0	0.969	-0.160
	4 - 0		0.167
	7 - 0		0.227
	9 - 0		0.283

Table 3.2: Network parameters of the three-machine system

Generator Data			Initial Operating Conditions		
Gen. No.	H (Mw/MVA)	x'_d	P_m	E (pu)	δ_0(deg)
1	23.64	0.0608	7.16	1.056	2.272
2	6.40	0.1198	1.63	1.050	19.732
3	3.01	0.1813	0.85	1.017	13.175

Table 3.3: Three-machine generator data and operating conditions

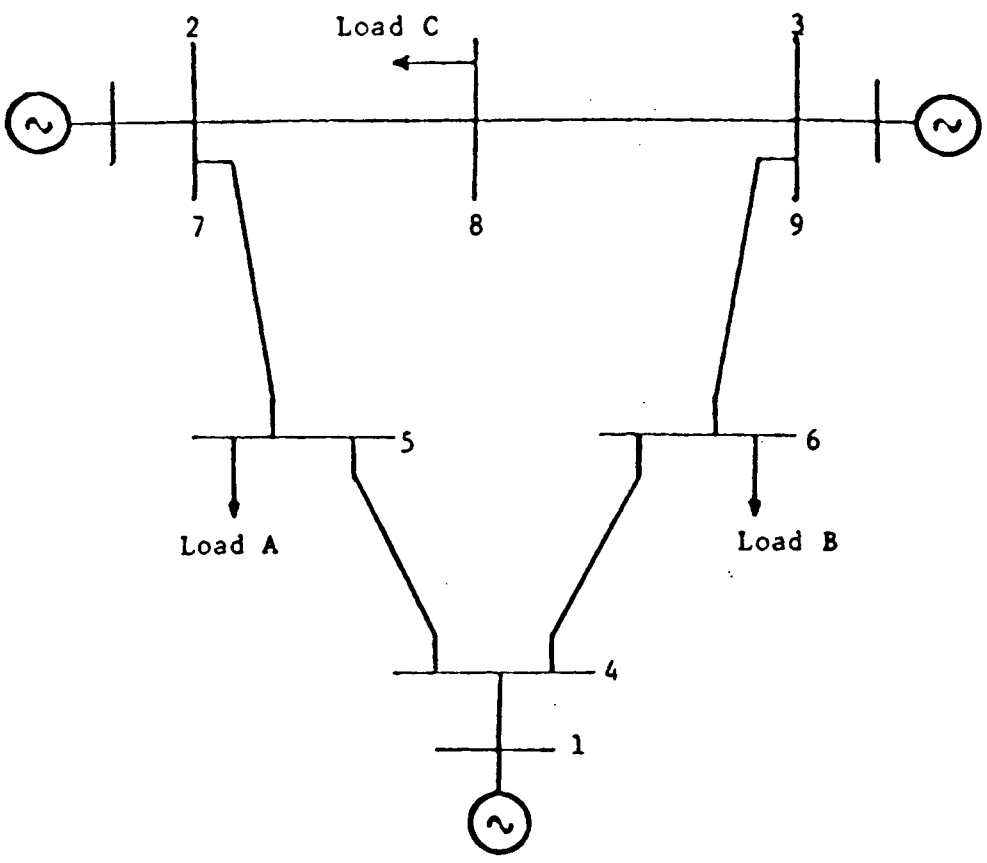


Figure 3.4: Nine-bus three-machine power system

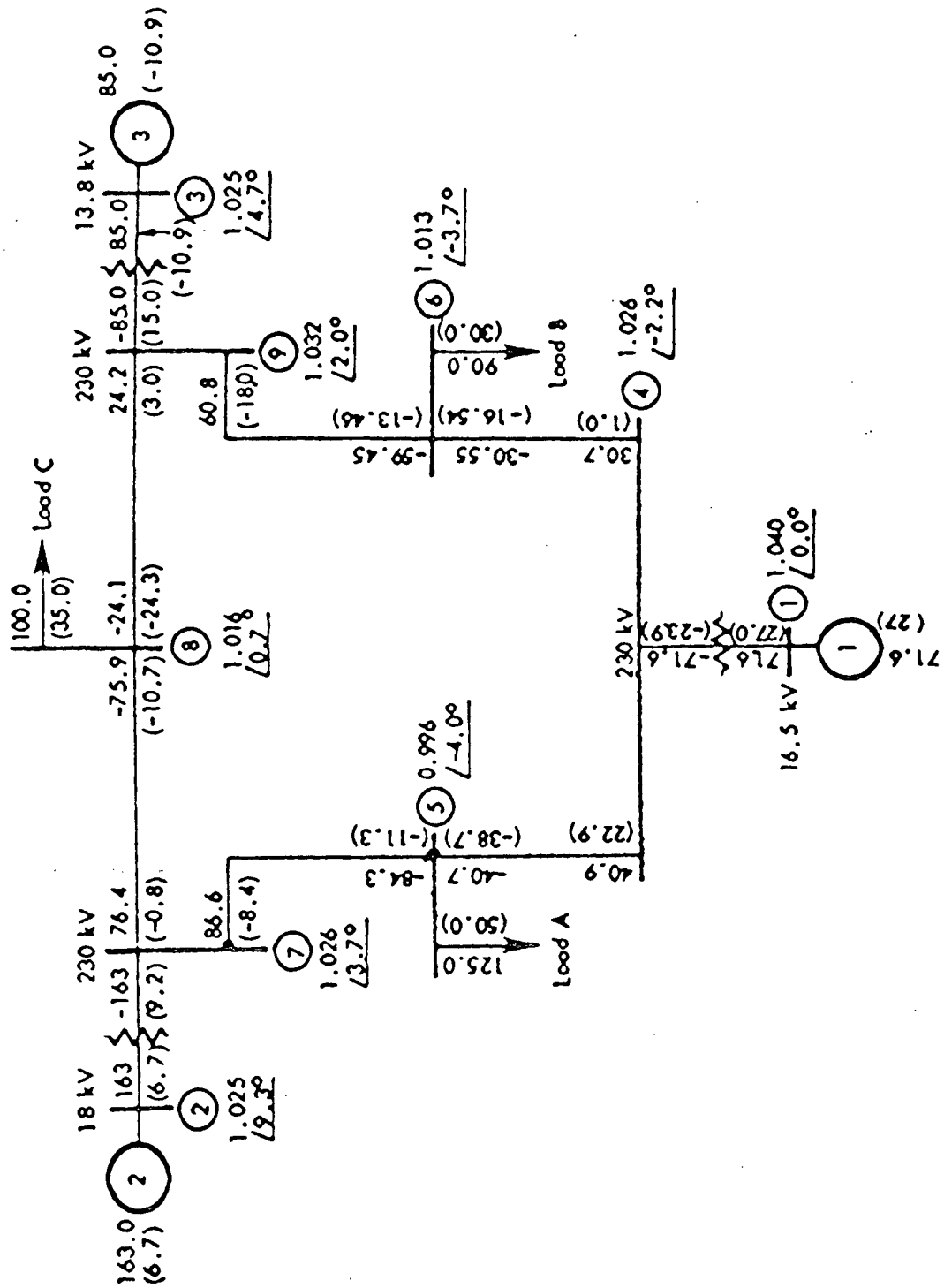


Figure 3.5: Three-machine system load flow showing prefault conditions ; all flows in MW and MVAR.

Three-phase faults applied at the generator buses, the lines that were opened, the number of critical generators that were involved, the electrical power produced during the fault as well as the catastrophe control parameters v and w are shown in Table 3.4. Those buses which are not generator buses are shown in Table 3.5.

The transient stability region using the general dynamic method is shown for generation buses in Figure 3.6 and for non-generating buses in Figure3.7 . All stable cases are shown inside the region in terms of the catastrophe control parameters.

Node Grounded	Line Opened	No. Critical Generators	Elec. Power DuringFault	V	W
7	7 - 8	1	0.0	1.9	13.5
7′	7 - 5	1	0.0	1.05	16.1
9′	9 - 6	1	0.0	0.7	28.5
9	9 - 8	1	0.0	1.02	27.1
4	4 - 5	3	0.304	1.02	31.7
4′	4 -6	3	0.304	1.16	31.9

Table 3.4: Cases of faulted generating buses

Node Grounded	Line Opened	No. Critical Generators	Elec. Power DuringFault	V	W
5′	4 - 5	1	0.652	2.0	27.0
5	5 - 7	1	0.652	7.2	14.3
6′	6 - 4	2	0.720	1.8	29.0
6	6 - 9	2	0.720	3.8	23.1
8′	8 - 9	2	0.487	1.0	27.2
8	8 - 7	2	0.487	1.5	26.0

Table 3.5: Cases of non-generating buses

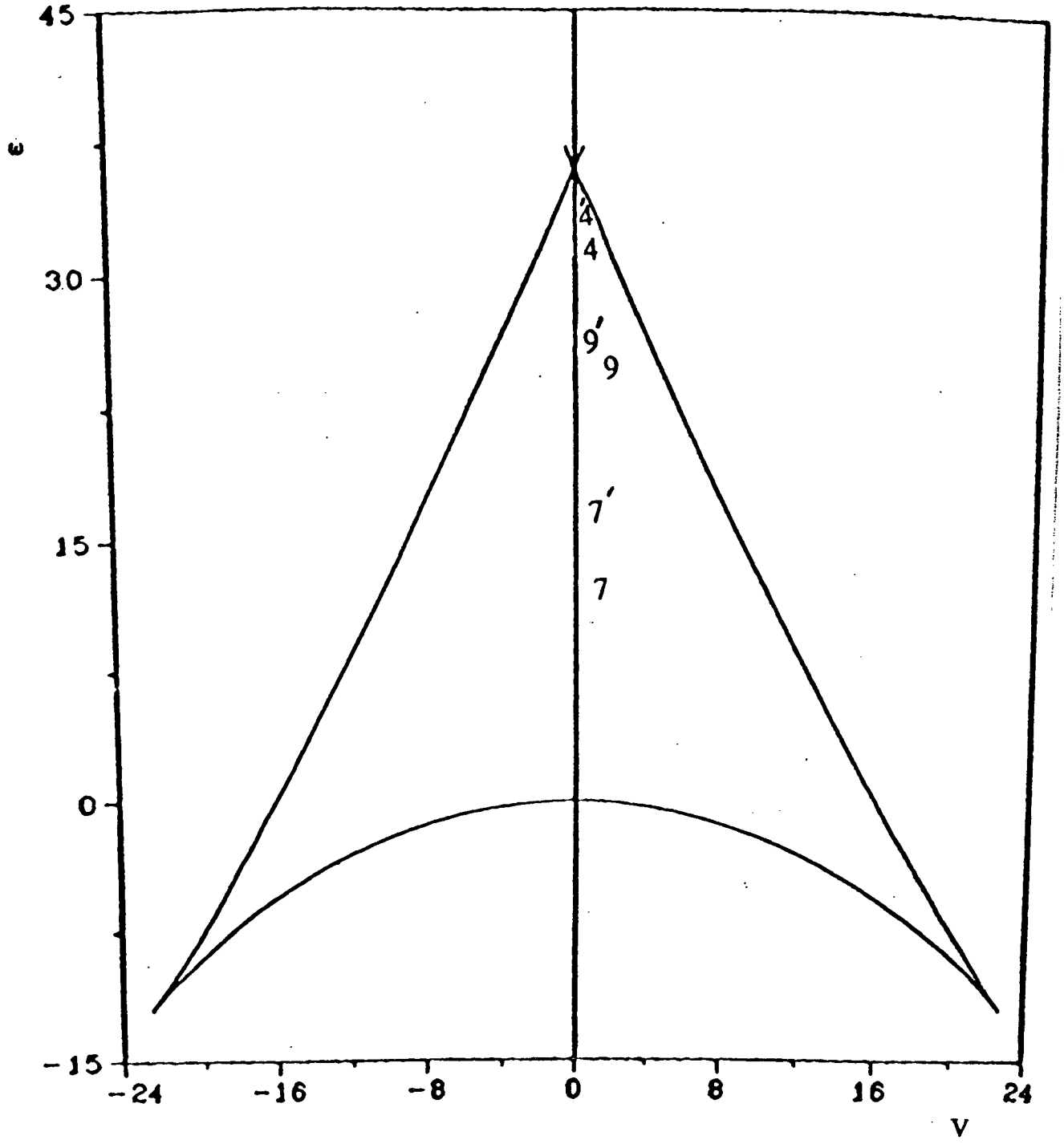


Figure 3.6: Results for the 3–machine power system with faulted generator buses . All cases are stable.

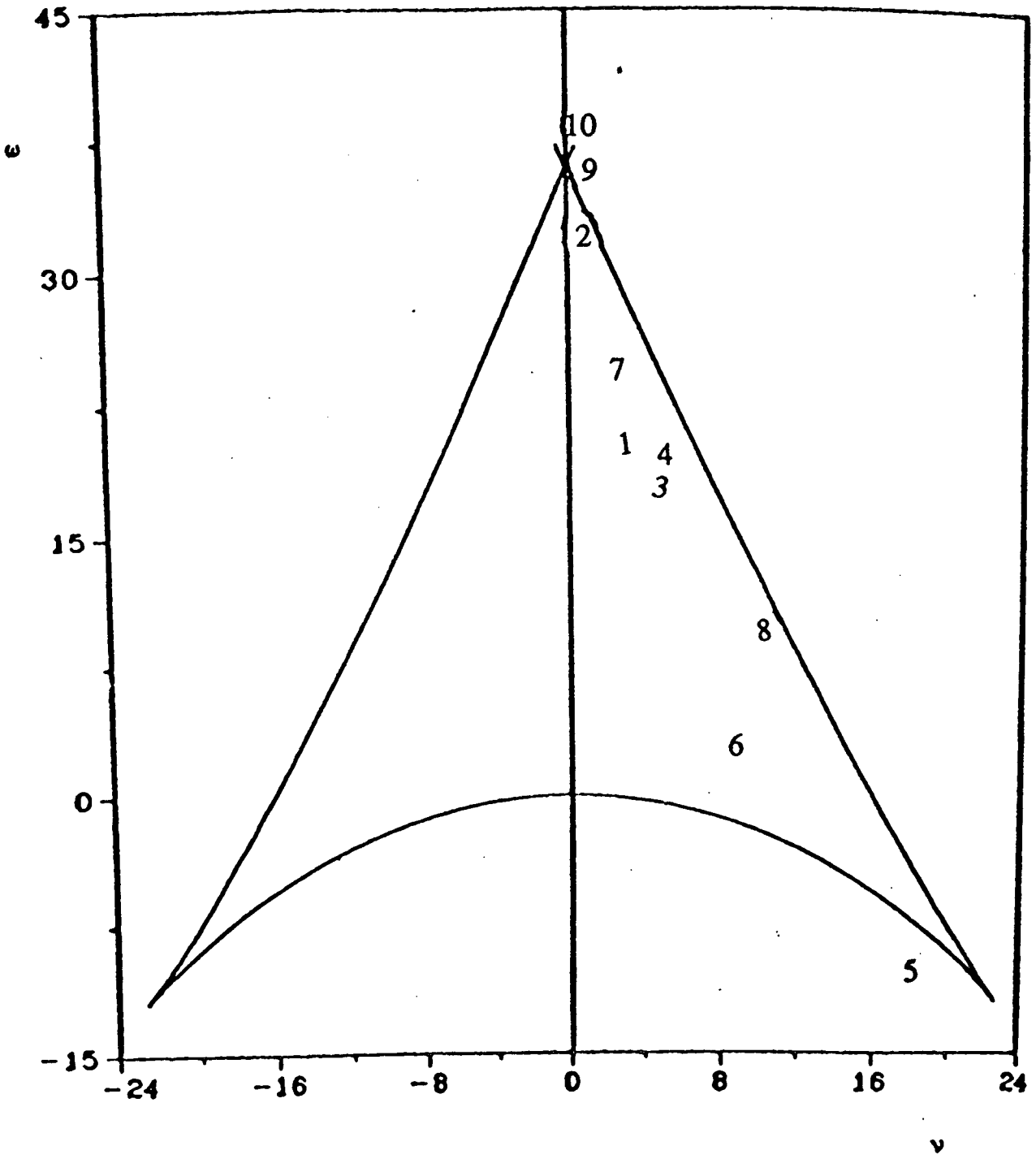


Figure 3.9: Results for the CIGRE test system. All cases except for faults at buses 5, 9 and 10 are stable.

3.2 The Seven-Machine System

The CIGRE 225 KV test system is shown in Figure 3.8. This system has 10 buses and 13 unique branches. Buses 1 through 7 are generating buses while loads are located at buses 2, 4, 6, 7, 8, 9, and 10. The base values used are 225 KV and 100 MVA [21] . The systems bus data, branch data and the systems loadflow summary is given in Table 3.6, Table 3.7and Table 3.8 respectively.

Bus #	P gen. (MW)	X (%)	P load (MW)	Q load (MVAR)
1	217.00	7.4	0.00	0.00
2	120.00	11.8	200.00	120.00
3	256.00	6.2	0.00	0.00
4	300.00	4.9	650.00	405.00
5	230.00	7.4	0.00	0.00
6	160.00	7.1	80.00	30.00
7	174.00	8.7	90.00	40.00
8	0.00	0.0	100.00	50.00
9	0.00	0.0	230.00	140.00
10	0.00	0.0	90.00	45.00

Table 3.6: CIGRE 7–machine bus data

Bus	Bus	R (pu)	X (pu)	CHARG (MVAR)
1	3	0.0099	0.0484	20.250
1	4	0.0099	0.0484	10.125
2	3	0.0450	0.1237	20.250
2	10	0.0164	0.0638	30.375
3	4	0.0119	0.0780	30.375
3	9	0.0114	0.0553	20.250
4	5	0.0040	0.0198	20.250
4	6	0.0075	0.0198	121.50
4	9	0.0488	0.1916	20.250
4	10	0.0164	0.0652	30.375
6	8	0.0188	0.0628	20.250
7	8	0.0119	0.0780	30.375
8	9	0.0488	0.1916	20.250

Table 3.7: CIGRE 7 - machine branch data

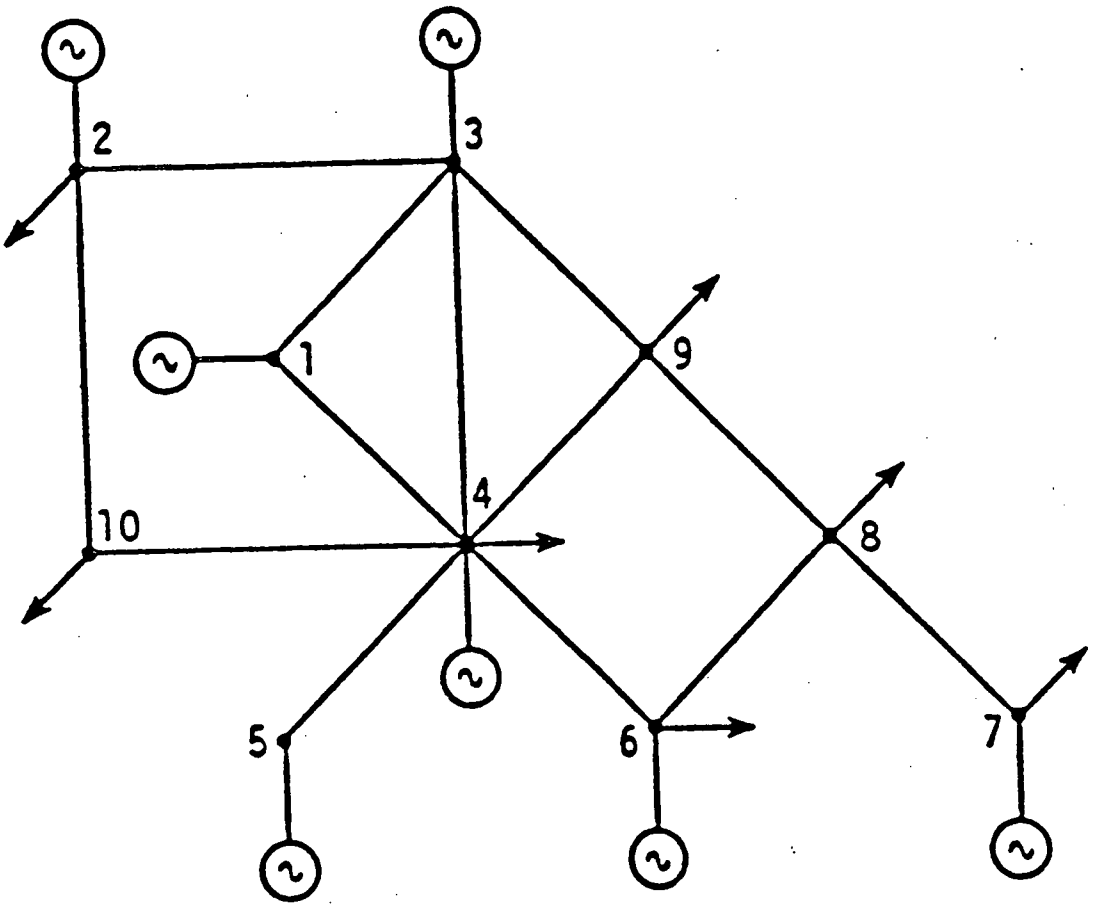


Figure 3.8: CIGRE 7 – machine test system

Three-phase faults are applied and the transient stability is evaluated for each fault. The bus which the fault is applied, the number of critical generators involved, the values of the catastrophe control parameters and weather the system is stable or not is shown in Table 3.9. The transient stability regions in terms of the swallowtail catastrophe control parameters are shown in Figure 3.9which show good agreement with the time solution.

Bus #	Vmag (pu)	Vang (deg)	Pgen (MW)	Qgen (MVAR)
1	1.106	7.9	227.83	-49.54
2	1.156	0.35	120.00	232.96
3	1.098	6.42	256.00	-59.687
4	1.110	4.07	300.00	746.462
5	1.118	6.17	230.00	-9.748
6	1.039	5.89	160.00	-434.255
7	1.054	7.84	174.00	39.866
8	1.034	4.50	0.00	0.00
9	1.032	1.95	0.00	0.00
10	1.124	0.88	0.00	0.00

Table 3.8: CIGRE 7 – machine voltage and power summary

Node Grounded	No. Critical Generators	Elec. Power DuringFault	V	W	Stable
1	2	1.0404	2.9	25.1	yes
2	2	1.057	0.9	33.5	yes
3	2	0.8045	3.9	19.3	yes
4	2	0.317	5.4	21.04	yes
5	1	0.00	12.76	-26.75	no
6	1	0.00	9.4	5.5	yes
7	1	0.0	5.24	21.4	yes
8	1	0.7152	12.53	11.27	yes
9	3	1.098	0.33	44.0	no
10	2	0.1507	0.245	40.0	no

Table 3.9: Cases of CIGRE 7–machine grounded buses

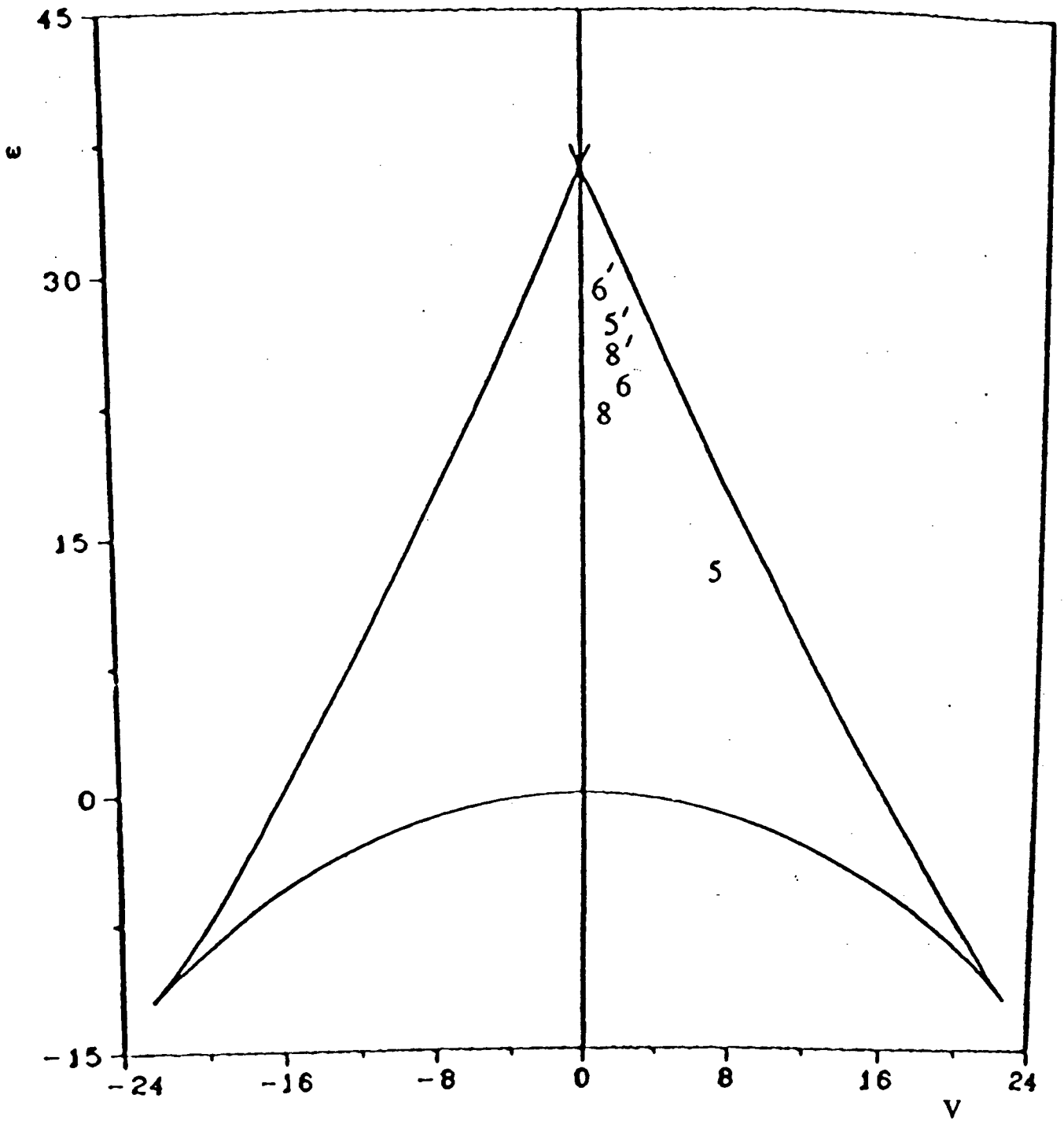


Figure 3.7: Results for the 3–machine power system with faulted load buses. All cases are stable.

Chapter 4
Conclusion

Catastrophe theory has been applied to the study of stability of various dynamic systems such as aircraft stability [22] , and in recent years to the steady state stability problem of power systems [23]. However, that application was limited only to salient-pole type synchronous generators. Then after swallow tail catastrophe was applied to transient stability of single-machine infinite bus system [17] and also to transient stability of multimachine power systems with the worst case approach i.e. only one generator being critical [18].

This thesis suggests a method to solve the transient stability problem of multimachine power systems with the system having more than one critical machine for a specified disturbance. Here the critical machines during a three-phase fault are identified, singled out and combined to be one equivalent machine and also the rest of the system as another single equivalent machine using the general dynamic equivalent approach. Then the energy balance equation is derived from the equation of motion of the equivalent critical machine against the rest of the system. The energy balance equation is then used to form the equilibrium surface of the swallowtail catastrophe manifold from which the transient stability region is derived by the bifurcation technique. The results obtained by this general swallowtail catastrophe approach is in good agreement with those obtained by the time solution method.

It should be noted that the application of swallowtail catastrophe to transient stability of multimachine power systems has the following advantages :

- The regions of stability are well defined in terms of the swallowtail catastrophe control parameters u, v and w.
- The computations required to define the stability regions are few and done in a very short time.
- The generator swing equations need *not* be solved.

New areas of research need to be explored in order to reach the goal of an efficient on-line direct method of transient stability analysis. The future research should include the following :

- Stability controls such as fast valving, braking resistors, single pole switching, series capacitors and generator trippings are usually applied in practice to restore transient stability of power systems. The inclusion of these controls in the transient stability using the swallowtail catastrophe approach would be of great interest to power utility companies.
- In this thesis we only considered three-phase faults. However single-phase faults as well as multiple disturbances also occur.

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Appendix
Fortran Program Listing of Swallowtail Catastrophe Applied to Multimachine Power Systems

```
      INTEGER NL,BUS(80),NGEN,LN(80),NLLL
      INTEGER FR(80),TO(80),NLL,NBB,NODE,CANCEL
      INTEGER N,R,NN,NB,NNF,RF,II,JJ,KK


C     NB: NUMBER OF BUSES
C      N: NUMBER OF GENERATORS IN THE SYSTEM
C     NN:MAXIMUM DIMENSION OF THE PREFAULT AND POST FAULT MATRIX
C     NNF:MAXIMUM DIMENSION OF THE DURING FAULT MATRIX
C       R: NN-N
C      RF: NNF-N
C     ..................................................


      PARAMETER(NB=9,N=3)
      PARAMETER(NN=9,R=6)
      PARAMETER(NNF=8,RF=5)


      INTEGER NPRSTA(N),BASE(N),KG(50)

      INTEGER CLIM(50),NCLIM,CRI(80),NCRI(80)

      REAL V(80),ANG(80),PGEN(80),QGEN(80),PLOAD(80)
      REAL QLOAD(80),MAGV(80),RV(80),IV(80),A
      REAL MAGE(80),G(N,N),MAGDF(N,N),TETAF(N,N),DELTA(80)
      REAL MAGI(NB),ALFA(NB),MAGDP(N,N),DELTA1(80)
      REAL TETAP(N,N)
      REAL PM(80),PM1(50),PEF(50),GAMMA(50),PEP(50)
      REAL RATING(N),H(N),M(50)
      REAL TOL,DELT,TC

      REAL MNOT,CM,NCM,CMECH,NCMECH,CD,CDF,CDP
      REAL CDEL,NCDEL,CANG,NCANG,RATIO,PMECH

      REAL ATEMP,A1TEMP,A2TEMP,A3TEMP,BTEMP,B1TEMP,B2TEMP
      REAL B3TEMP,CTEMP,C1TEMP,C2TEMP,C3TEMP

      REAL PRED(N,N),PREC(N,N),PREPK(80),PREAK,PREBK,PREALF
      REAL PRETK,PRECY1(50),PRECY2,PREPC

      REAL DURD(N,N),DURC(N,N),DURPK,DURAK,DURBK,DURALF
      REAL DURTK,DURPC,DURPE(50)

      REAL POSD(N,N),POSC(N,N),POSPK,POSAK,POSBK,POSALF
      REAL POSTK,POSPC,POSPE

      REAL ETAK(80),TETAI(80),BETA,SPOSCY,UPOSCY,MKGAMK(50)
      REAL UK,UCAT,VCAT,WCAT


      COMPLEX SGEN(80),V1(80),CUR(80),SLOAD(80)
      COMPLEX YLOAD(80),EPRIM(80)
      COMPLEX XPRIMD(80),TEMP,Y(80),YSH(80)
      COMPLEX YBUS(NB,NB),YBUS1(NB,NB),YBUS2(NB-1,NB-1)
      COMPLEX YBUS3(NB,NB)
      COMPLEX YNN(N,N),YNR(N,R),YRN(R,N)
```

```
      COMPLEX YRR(R,R),YRRR(R,R),AA(R,R),DET,COND
      COMPLEX B(N,R),C(N,N),D(N,N) ,TEM
      COMPLEX YNNF(N,N),YNRF(N,RF),YRNF(RF,N)
      COMPLEX YRRF(RF,RF),YRRRF(RF,RF),AAF(RF,RF)
      COMPLEX BF(N,RF),CF(N,N),DF(N,N)
      COMPLEX YNNP(N,N),YNRP(N,R),YRNP(R,N)
      COMPLEX YRRP(R,R),YRRRP(R,R),AAP(R,R)
      COMPLEX BP(N,R),CP(N,N),DP(N,N)

      OPEN(UNIT=5,FILE='FOUAD',STATUS='OLD')
C      OPEN(UNIT=6,FILE='OUT',STATUS='UNKNOWN')

C                 READ STATEMENTS
C                 ***************

          READ(5,*)NL,NGEN,NODE,CANCEL,KK

      DO 228 I=1,NL
          READ(5,*)BUS(I),V(I),ANG(I),PGEN(I),QGEN(I),
     &    PLOAD(I),QLOAD(I),XPRIMD(I)
 228  CONTINUE

          READ(5,*)NLL

      DO 111 I=1,NLL
          READ(5,*)LN(I),FR(I),TO(I),Y(I),YSH(I)
 111  CONTINUE

      DO 199 I=1,N
          READ(5,*)NPRSTA(I),RATING(I),H(I),BASE(I)
 199  CONTINUE


C     CALCULATE TRANSIENT VOLTAGE,INITIAL OPERATING
C         ANGLE,LOAD IMPEDANCE,CURRENT AT
C               GENERATING NODE.
C        ..............................


      A=3.14159/180.00
      DO 191 I=1,NL
         EPRIM(I)=(0.0,0.0)
         YLOAD(I)=(0.0,0.0)
         DELTA(I)=0.0
         DELTA1(I)=0.0
         CUR(I)=(0.0,0.0)
         ALFA(I)=0.0
         MAGI(I)=0.0
         PM(I)=0.0
 191  CONTINUE

      DO 100 I=1,NL
         RV(I)=V(I)*COS(ANG(I)*A)
         IV(I)=V(I)*SIN(ANG(I)*A)
```

```
          V1(I)=CMPLX(RV(I),IV(I))
          MAGV(I)=SQRT((RV(I)*RV(I))+(IV(I)*IV(I)))
         IF((PGEN(I).GT.0.0).OR.(QGEN(I).GT.0.0))THEN
           SGEN(I)=CMPLX(PGEN(I),QGEN(I))
           CUR(I)=(CONJG(SGEN(I)))/(CONJG(V1(I)))
           EPRIM(I)=V1(I)+(CUR(I)*XPRIMD(I))
         ELSE
           EPRIM(I)=(0.0,0.0)
         ENDIF
           SLOAD(I)=CMPLX(PLOAD(I),QLOAD(I))
           YLOAD(I)=(CONJG(SLOAD(I)))/(MAGV(I)*MAGV(I))
  100   CONTINUE


C        CALCULATE ABOVE VALUES IN POLAR FORM
C        ************************************


        DO 192 I=1,NL
           MAGE(I)=SQRT((REAL(EPRIM(I))*REAL(EPRIM(I)))
     &                +(AIMAG(EPRIM(I))*AIMAG(EPRIM(I))))
           MAGI(I)=SQRT((REAL(CUR(I))*REAL(CUR(I)))
     &                +(AIMAG(CUR(I))*AIMAG(CUR(I))))
      IF((AIMAG(EPRIM(I)).NE.0.0).OR.(REAL(EPRIM(I)).NE.0.0))THEN
           DELTA(I)=ATAN2(AIMAG(EPRIM(I)),REAL(EPRIM(I)))
      ELSE
           DELTA(I)=0.0
      ENDIF
      IF((AIMAG(CUR(I)).NE.0.0).OR.(REAL(CUR(I)).NE.0.0))THEN
           ALFA(I)=ATAN2(AIMAG(CUR(I)),REAL(CUR(I)))
      ELSE
           CUR(I)=0.0
      ENDIF
           DELTA(I)=DELTA(I)/A
           DELTA1(I)=DELTA(I)*A
           ALFA(I)=ALFA(I)/A
 192     CONTINUE


C       INITIALISE ALL THREE MATRICES
C       *****************************


        DO 112 I=1,NB
        DO 112 J=1,NB
           YBUS(I,J)=(0.0,0.0)
           YBUS1(I,J)=(0.0,0.0)
           YBUS3(I,J)=(0.0,0.0)
  112   CONTINUE

        DO 131 I=1,(NB-1)
        DO 131 J=1,(NB-1)
           YBUS2(I,J)=(0.0,0.0)
  131   CONTINUE


C       FORMULATE THE PREFAULT MATRIX
```

```
C       ****************************

        DO 113 I=1,NB
        DO 114 J=1,NLL
           IF((FR(J).EQ.LN(I)).OR.(TO(J).EQ.LN(I)))THEN
             YBUS(I,I)=YBUS(I,I)+(Y(J)+YSH(J))
           ENDIF
  114   CONTINUE
           YBUS(I,I)=YBUS(I,I)+YLOAD(I)
  113   CONTINUE

        DO 115 K=1,NLL
           IF((FR(K).NE.LN(K)).OR.(TO(K).NE.LN(K)))THEN
             I=FR(K)
             J=TO(K)
             YBUS(I,J)=YBUS(I,J)-Y(K)
             YBUS(J,I)=YBUS(I,J)
           ENDIF
 115    CONTINUE

C       REDUCE THE PREFAULT MATRIX
C       **************************

        DO 146 I=(N+1),NN
        DO 132 J=(N+1),NN
           TEMP=YBUS(I,J)
           II=I-N
           JJ=J-N
           YRR(II,JJ)=TEMP
           YRRR(II,JJ)=YRR(II,JJ)
  132   CONTINUE
  146   CONTINUE

        DO 134 I=1,N
        DO 134 J=1,N
           YNN(I,J)=YBUS(I,J)
  134   CONTINUE

        DO 136 I=1,N
        DO 136 J=(N+1),NN
           TEMP=YBUS(I,J)
           II=I
           JJ=J-N
           YNR(II,JJ)=TEMP
  136   CONTINUE

        DO 138 I=(N+1),NN
        DO 138 J=1,N
           TEMP=YBUS(I,J)
           II=I-N
           JJ=J
           YRN(II,JJ)=TEMP
  138   CONTINUE

        CALL CINVRT(YRR,R,R,DET,COND)
```

```
      CALL CMULT(YRRR,YRR,AA,R,R,R,R,R,R)
      CALL CMULT(YNR,YRR,B,N,R,R,N,R,N)
      CALL CMULT(B,YRN,C,N,R,N,N,R,N)
      CALL CSUB(YNN,C,D,N,N,N,N,N)

C     FORMULATE THE DURING FAULT MATRIX
C     *********************************

      DO 125 I=1,NB
      DO 125 J=1,NB
         YBUS1(I,J)=YBUS(I,J)
  125 CONTINUE

      DO 117 I=1,NB
      DO 117 J=1,NB
         IF((I.EQ.NODE).OR.(J.EQ.NODE))YBUS1(I,J)=(0.0,0.0)
  117 CONTINUE

      DO 119 I=1,NB
      DO 120 J=1,NB
         K=J+1
         IF((K.LE.NB).AND.(J.GE.NODE))THEN
           YBUS1(I,J)=YBUS1(I,K)
           YBUS1(I,K)=(0.0,0.0)
         ELSE
         ENDIF
  120 CONTINUE
  119 CONTINUE

      DO 121 I=1,NB
      DO 121 J=1,NB
         K=I+1
         IF((K.LE.NB).AND.(I.GE.NODE))THEN
           YBUS1(I,J)=YBUS1(K,J)
           YBUS1(K,J)=(0.0,0.0)
         ELSE
         ENDIF
  121 CONTINUE

      DO 123 I=1,NB-1
      DO 123 J=1,NB-1
         YBUS2(I,J)=YBUS1(I,J)
  123 CONTINUE

C     REDUCE THE FAULTED MATRIX
C     *************************

      DO 160 I=(N+1),NNF
      DO 161 J=(N+1),NNF
         TEMP=YBUS2(I,J)
         II=I-N
         JJ=J-N
         YRRF(II,JJ)=TEMP
         YRRRF(II,JJ)=YRRF(II,JJ)
```

```
 161    CONTINUE
 160    CONTINUE

        DO 163 I=1,N
        DO 163 J=1,N
           YNNF(I,J)=YBUS2(I,J)
 163    CONTINUE

        DO 165 I=1,N
        DO 165 J=(N+1),NNF
           TEMP=YBUS2(I,J)
           II=I
           JJ=J-N
           YNRF(II,JJ)=TEMP
 165    CONTINUE

        DO 167 I=(N+1),NNF
        DO 167 J=1,N
           TEMP=YBUS2(I,J)
           II=I-N
           JJ=J
           YRNF(II,JJ)=TEMP
 167    CONTINUE

        CALL CINVRT(YRRF,RF,RF,DET,COND)
        CALL CMULT(YRRRF,YRRF,AAF,RF,RF,RF,RF,RF,RF)
        CALL CMULT(YNRF,YRRF,BF,N,RF,RF,N,RF,N)
        CALL CMULT(BF,YRNF,CF,N,RF,N,N,RF,N)
        CALL CSUB(YNNF,CF,DF,N,N,N,N,N)

        DO 126 I=1,NB
        DO 126 J=1,NB
           YBUS3(I,J)=YBUS(I,J)
 126    CONTINUE

C       FORMULATE THE AFTER FAULT MATRIX
C       ********************************


        DO 127 I=1,NB
        DO 127 J=1,NB
           IF((I.EQ.FR(CANCEL)).AND.(J.EQ.TO(CANCEL)))THEN
             YBUS3(I,J)=(0.0,0.0)
             YBUS3(J,I)=(0.0,0.0)
           ENDIF
 127    CONTINUE

        DO 128 I=1,NB
        DO 128 J=1,NB
           IF((I.EQ.FR(CANCEL)).AND.(J.EQ.FR(CANCEL)))THEN
             YBUS3(I,I)=YBUS3(I,I)-(Y(CANCEL)+YSH(CANCEL))
C            YBUS3(I,I)=YBUS3(I,I)-Y(CANCEL)
           ENDIF
 128    CONTINUE

        DO 129 I=1,NB
        DO 129 J=1,NB
```

```
            IF((I.EQ.TO(CANCEL)).AND.(J.EQ.TO(CANCEL)))THEN
              YBUS3(I,I)=YBUS3(I,I)-(Y(CANCEL)+YSH(CANCEL))
C             YBUS3(I,I)=YBUS3(I,I)-Y(CANCEL)
            ENDIF
 129      CONTINUE


C         REDUCE THE AFTER FAULT MATRIX
C         *****************************


          DO 172 I=(N+1),NN
          DO 173 J=(N+1),NN
             TEMP=YBUS3(I,J)
             II=I-N
             JJ=J-N
             YRRP(II,JJ)=TEMP
             YRRRP(II,JJ)=YRRP(II,JJ)
  173     CONTINUE
  172     CONTINUE

          DO 175 I=1,N
          DO 175 J=1,N
             YNNP(I,J)=YBUS3(I,J)
  175     CONTINUE

          DO 177 I=1,N
          DO 177 J=(N+1),NN
             TEMP=YBUS3(I,J)
             II=I
             JJ=J-N
             YNRP(II,JJ)=TEMP
  177     CONTINUE

          DO 179 I=(N+1),NN
          DO 179 J=1,N
             TEMP=YBUS3(I,J)
             II=I-N
             JJ=J
             YRNP(II,JJ)=TEMP
  179     CONTINUE

          CALL CINVRT(YRRP,R,R,DET,COND)
          CALL CMULT(YRRRP,YRRP,AAP,R,R,R,R,R,R)
          CALL CMULT(YNRP,YRRP,BP,N,R,R,N,R,N)
          CALL CMULT(BP,YRNP,CP,N,R,N,N,R,N)
          CALL CSUB(YNNP,CP,DP,N,N,N,N,N)


C         FAULTED AND AFTER FAULT MATRIX IN POLAR FORM
C         ********************************************


          DO 193 I=1,N
          DO 194 J=1,N
             MAGDF(I,J)=SQRT((REAL(DF(I,J))*REAL(DF(I,J)))
     &                  +(AIMAG(DF(I,J))*AIMAG(DF(I,J))))
             MAGDP(I,J)=SQRT((REAL(DP(I,J))*REAL(DP(I,J)))
```

```
     &                  +(AIMAG(DP(I,J))*AIMAG(DP(I,J))))
        IF(REAL(DF(I,J)).NE.0.0.OR.AIMAG(DF(I,J)).NE.0.0)THEN
           TETAF(I,J)=ATAN2(AIMAG(DF(I,J)),REAL(DF(I,J)))
        ELSE
           TETAF(I,J)=0.0
        ENDIF
        IF(REAL(DP(I,J)).NE.0.0.OR.AIMAG(DP(I,J)).NE.0.0)THEN
           TETAP(I,J)=ATAN2(AIMAG(DP(I,J)),REAL(DP(I,J)))
        ELSE
           TETAP(I,J)=0.0
        ENDIF
        TETAF(I,J)=TETAF(I,J)/A
        TETAP(I,J)=TETAP(I,J)/A
 194    CONTINUE
 193    CONTINUE


C       Dij , Cij  USING PRE,DURING AND POST REDUCED MATRIX
C       ***************************************************


        DO 222 I=1,N
           DO 223 J=1,N
              PRED(I,J)=MAGE(I)*MAGE(J)*(REAL(D(I,J)))
              PREC(I,J)=MAGE(I)*MAGE(J)*(AIMAG(D(I,J)))
              DURD(I,J)=MAGE(I)*MAGE(J)*(REAL(DF(I,J)))
              DURC(I,J)=MAGE(I)*MAGE(J)*(AIMAG(DF(I,J)))
              POSD(I,J)=MAGE(I)*MAGE(J)*(REAL(DP(I,J)))
              POSC(I,J)=MAGE(I)*MAGE(J)*(AIMAG(DP(I,J)))
 223        CONTINUE
 222    CONTINUE


C       MECHANICAL INPUT=GENERATED POWER - LOACAL LOAD
C       **********************************************

        DO 198 I=1,NL
           IF(PGEN(I).GT.0.0)PM(I)=PGEN(I)-PLOAD(I)
 198    CONTINUE


        DO 221 I=1,N
           PEF(I)=0.0
           PM1(I)=0.0
            DO 220 J=1,N
               ATEMP=DELTA1(I)-DELTA1(J)
               PM1(I)=PM1(I)+((PRED(I,J)*COS(ATEMP))
     &                        +(PREC(I,J)*SIN(ATEMP)))

               PEF(I)=PEF(I)+((DURD(I,J)*COS(ATEMP))
     &                        +(DURC(I,J)*SIN(ATEMP)))
 220         CONTINUE
 221       CONTINUE
```

```
C       CALCULATE THE INERTIA CONSTANTS OF EACH MACHINE
C       ***********************************************

          DO 200 I=1,N
C            TEMP3=NPRSTA(I)*RATING(I)*H(I)
             TEMP3=NPRSTA(I)*H(I)
             M(I)=TEMP3/(60.0*3.14159*BASE(I))
             IF(M(I).NE.0)THEN
               GAMMA(I)=(PM1(I)-PEF(I))/M(I)
               KG(I)=I
            ENDIF
 200      CONTINUE


C            SORTING
C            *******


             DO 610 I=1,(N-1)
               DO 611 J=(I+1),N
                 IF(GAMMA(I).LT.GAMMA(J))THEN
                   ATEMP=GAMMA(I)
                   BTEMP=KG(I)
                   CTEMP=PEF(I)

                   GAMMA(I)=GAMMA(J)
                   KG(I)=KG(J)
                   PEF(I)=PEF(J)

                   GAMMA(J)=ATEMP
                   KG(J)=BTEMP
                   PEF(J)=CTEMP
                 ENDIF
 611           CONTINUE
 610         CONTINUE


C              WRITE(6,700)
C 700          FORMAT(////)
C              WRITE(6,701)
C 701          FORMAT(4X,'GAMMA',3X,'GEN',5X,'DUR FAULT PE')
C              DO 999 I=1,N
C              WRITE(6,702)GAMMA(I),KG(I),PEF(I)
C 702          FORMAT(/4X,F11.5,5X,I3,4X,F11.5)
C 999          CONTINUE

C            CM=CRITICAL MOMENT OF INERTIA
C            CANG=CRITICAL EQUIVALENT ANGLE
C            NCANG=NON CRITICAL EQUIVALENT ANGLE
C            CMECH=CRITICAL MECHANICAL POWER
C            NCMECH=NON CRITICAL MECHANICAL POWER
C            PMECH=EQUIVALENT MECHANICAL POWER
C            RATIO=CRITICAL MOMENT OF INERTIA/NON CRITICAL
C            ==============================================
```

```
      CLIM(KK)=0.0
      CLIM(KK)=KK

      DO 612 I=1,CLIM(KK)
        CRI(I)=KG(I)
612   CONTINUE

      NCLIM=NGEN-CLIM(KK)
      IF(NCLIM.EQ.0)THEN
        NCLIM=1
        DO 637 I=1,NCLIM
          NCRI(I)=0
637     CONTINUE

      ELSE
        DO 613 I=1,NCLIM
          J=I+CLIM(KK)
          NCRI(I)=KG(J)
613     CONTINUE
      ENDIF


     CMECH=0.0
     CDEL=0.0
     MNOT=0.0
     CM=0.0
     CD=0.0
     CDF=0.0
     CDP=0.0
     NCM=0.0
     NCDEL=0.0
     NCMECH=0.0


     DO 600 I=1,CLIM(KK)
       CM=CM+M(CRI(I))
       CDEL=CDEL+(M(CRI(I))*DELTA1(CRI(I)))
       CMECH=CMECH+PM1(CRI(I))
        CD=CD+PRED(CRI(I),CRI(I))
       CDF=CDF+DURD(CRI(I),CRI(I))
       CDP=CDP+POSD(CRI(I),CRI(I))
600  CONTINUE

     CANG=CDEL/CM

     DO 601 I=1,NCLIM
       IF(NCRI(I).NE.0)THEN

          NCM=NCM+M(NCRI(I))
          NCDEL=NCDEL+(M(NCRI(I))*DELTA1(NCRI(I)))
          NCMECH=NCMECH+PM1(NCRI(I))

          NCANG=NCDEL/NCM
          RATIO=CM/NCM

       ELSE

          NCANG=0.0
```

```
            RATIO=0.0

          ENDIF
601     CONTINUE

        PMECH=CMECH-(RATIO*NCMECH)


        DO 602 I=1,N
          ETAK(I)=DELTA1(I)-CANG
          TETAI(I)=DELTA1(I)-NCANG
602     CONTINUE


        ATEMP=0.0
        A1TEMP=0.0
        A2TEMP=0.0
        A3TEMP=0.0
        BTEMP=0.0
        B1TEMP=0.0
        B2TEMP=0.0
        B3TEMP=0.0
        CTEMP=0.0
        C1TEMP=0.0
        C2TEMP=0.0
        C3TEMP=0.0

         DO 622 I=1,NCLIM
           DO 623 J=1,NCLIM
             IF((NCRI(I).NE.0).OR.(NCRI(J).NE.0))THEN
                ATEMP=ATEMP+(PRED(NCRI(I),NCRI(J))*
     &                 COS(TETAI(NCRI(I))-TETAI(NCRI(J))))

               CTEMP=CTEMP+(POSD(NCRI(I),NCRI(J))*
     &                 COS(TETAI(NCRI(I))-TETAI(NCRI(J))))
             ENDIF
623        CONTINUE
622     CONTINUE

        PREPK(CLIM(KK))=PMECH-(CD-(RATIO*ATEMP))
        POSPK=PMECH-(CDP-(RATIO*CTEMP))

        PRECY1(CLIM(KK))=CANG-NCANG

        IF(CLIM(K).EQ.1)THEN
          DURPE(CLIM(KK))=PEF(CLIM(KK))

          DO 641 I=1,CLIM(KK)
            DO 638 J=1,NCLIM
              IF(NCRI(J).NE.0.0)THEN
                DTEMP=TETAI(NCRI(J))-ETAK(CRI(I))

                CTEMP=CTEMP+((POSD(CRI(I),NCRI(J))*COS(DTEMP))+
     &                 (POSC(CRI(I),NCRI(J))*SIN(DTEMP)))

                C1TEMP=C1TEMP+((POSD(CRI(I),NCRI(J))*SIN(DTEMP))-
     &                  (POSC(CRI(I),NCRI(J))*COS(DTEMP)))
```

```
            ENDIF
638       CONTINUE
641     CONTINUE


        DO 639 I=1,CLIM(KK)
          DO 640 J=1,N
            IF(J.NE.CRI(I))THEN

              DTEMP=TETAI(J)-ETAK(CRI(I))

              C2TEMP=C2TEMP+((POSD(CRI(I),J)*SIN(DTEMP))+
     &                (POSC(CRI(I),J)*COS(DTEMP)))

              C3TEMP=C3TEMP+((POSC(CRI(I),J)*SIN(DTEMP))-
     &                (POSD(CRI(I),J)*COS(DTEMP)))

            ENDIF
640       CONTINUE
639     CONTINUE

      ELSE
        DO 616 I=1,CLIM(KK)
          DO 617 J=1,NCLIM
            IF(NCRI(J).NE.0.0)THEN
              DTEMP=TETAI(NCRI(J))-ETAK(CRI(I))

              BTEMP=BTEMP+((DURD(CRI(I),NCRI(J))*COS(DTEMP))+
     &               (DURC(CRI(I),NCRI(J))*SIN(DTEMP)))
              CTEMP=CTEMP+((POSD(CRI(I),NCRI(J))*COS(DTEMP))+
     &               (POSC(CRI(I),NCRI(J))*SIN(DTEMP)))


              B1TEMP=B1TEMP+((DURD(CRI(I),NCRI(J))*SIN(DTEMP))-
     &                (DURC(CRI(I),NCRI(J))*COS(DTEMP)))

              C1TEMP=C1TEMP+((POSD(CRI(I),NCRI(J))*SIN(DTEMP))-
     &                (POSC(CRI(I),NCRI(J))*COS(DTEMP)))

            ENDIF
617       CONTINUE
616     CONTINUE


        DO 618 I=1,CLIM(KK)
          DO 619 J=1,N
            IF(J.NE.CRI(I))THEN

              DTEMP=TETAI(J)-ETAK(CRI(I))


              B2TEMP=B2TEMP+((DURD(CRI(I),J)*SIN(DTEMP))+
     &                (DURC(CRI(I),J)*COS(DTEMP)))

              C2TEMP=C2TEMP+((POSD(CRI(I),J)*SIN(DTEMP))+
     &                (POSC(CRI(I),J)*COS(DTEMP)))
```

```
                B3TEMP=B3TEMP+((DURC(CRI(I),J)*SIN(DTEMP))-
     &                 (DURD(CRI(I),J)*COS(DTEMP)))

                C3TEMP=C3TEMP+((POSC(CRI(I),J)*SIN(DTEMP))-
     &                 (POSD(CRI(I),J)*COS(DTEMP)))

              ENDIF
 619        CONTINUE
 618      CONTINUE


          DURAK=B3TEMP+(RATIO*BTEMP)
          DURBK=B2TEMP-(RATIO*B1TEMP)
          DURALF=ATAN2(DURAK,DURBK)
          DURTK=SQRT((DURAK*DURAK)+(DURBK*DURBK))
          DURPE(CLIM(K))=DURTK*SIN(PRECY1(CLIM(K))-DURALF)
        ENDIF

          POSAK=C3TEMP+(RATIO*CTEMP)
          POSBK=C2TEMP-(RATIO*C1TEMP)
          POSALF=ATAN2(POSAK,POSBK)
          POSTK=SQRT((POSAK*POSAK)+(POSBK*POSBK))
          SPOSCY=POSALF+ASIN(POSPK/POSTK)
          UPOSCY=3.14159-SPOSCY
          POSPE=POSTK*SIN(SPOSCY-POSALF)

       MKGAMK(CLIM(KK))=PREPK(CLIM(KK))-DURPE(CLIM(KK))


       BETA=PRECY1(CLIM(KK))-POSALF

       UK=(POSPK*UPOSCY)+(POSTK*COS(UPOSCY-POSALF))

       UCAT=-12.000

       VCAT=(24.0/POSTK)*(POSPK-MKGAMK(CLIM(KK)))

       WCAT=(24.0/POSTK)*((POSPK*PRECY1(CLIM(KK)))-UK-
     &      (POSPK*BETA)+(MKGAMK(KK)*BETA))+24.00

C        WRITE(6,700)
C        WRITE(6,703)
C 703    FORMAT(5X,'CRIT GENS',4X,'DURPE',5X,'MKGAMK')
C        WRITE(6,704)CLIM(KK),DURPE(CLIM(KK)),MKGAMK(CLIM(KK))
C 704    FORMAT(/6X,I3,5X,F11.5,5X,F11.5)
C        WRITE(6,700)
C        WRITE(6,705)
C 705    FORMAT(5X,'CRIT GENS',4X,'POST CYE',4X,'UK')
C        WRITE(6,706)CLIM(KK),SPOSCY,UK
C 706    FORMAT(/5X,I3,5X,F11.5,5X,F11.5)
C        WRITE(6,700)
       WRITE(6,707)
 707   FORMAT(5X,'UCAT',4X,'VCAT',6X,'WCAT')
       WRITE(6,708)UCAT,VCAT,WCAT
 708   FORMAT(/4X,F11.5,4X,F11.5,6X,F11.5)
         STOP
         END
```