THE DEVELOPMENT OF A REACTIVE POWER MANAGEMENT TECHNIQUE FOR A PLANNING ENVIRONMENT

by

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Abstract

A computer-aided algorithm is developed for the management of reactive power flow in an electric power system. The technique is designed to assist transmission planning engineers in establishing satisfactory base-case power flow solutions. The objective in the algorithm is to reduce real power loss in the system through control of reactive power flow, and so is different than conventional "VAr allocation" algorithms. The minimisation is performed by a specially adapted gradient search with a sub-optimal step-size, which can be simply incorporated into a standard Newton-Raphson power flow program.

A special feature of this thesis is the presentation of a set of contour plots of the objective function versus various pairs of control variables. An analysis of these plots is presented, and is used to demonstrate the validity of the steepest descent minimisation technique for this problem.

Comments are given on tests conducted with this technique on a typical British Columbia Hydro and Power Authority power flow simulation consisting of 245 busses and 327 branches, with 47 controllable generators and 44 controllable variable-tap transformers. The algorithm is claimed to be effective and efficient for studies of this size.

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NOTATION

Notes:

1) Scalar quantities are written as simple or subscripted variables (e.g. V). Vector quantities are written as simple variables with an overhead bar (e.g. $\vec{V}$). Matrix quantities are written as simple variables within square brackets (e.g. $[Y]$).

2) The null matrix is written $[0]$, while the identity matrix is written $[1]$. The zero vector is written $\bar{0}$.

3) A "Δ" preceding some variable (e.g. ΔV) indicates a variation in that variable. The real and reactive power mismatches are the exceptions to this rule. They are written ΔP and ΔQ. That these are mismatches will always be pointed out in the text.

4) A subscript (i, j, or k) applied to a scalar variable (e.g. V_i) indicates that the variable pertains to the bus indicated by the subscript. Similarly, a variable with two subscripts indicates that the variable pertains to the pair of busses indicated. A partial derivative of subscripted variables indicates that the derivative is an element of the derivative matrix (e.g. $\frac{\partial Q_i}{\partial V_j}$ is the (i,j) element of the matrix $\left[\frac{dQ}{dV}\right]$). A special subscript, s, is used to indicate the slack bus.

5) The unqualified term "power" refers to the complex power S.

6) Absolute value is indicated with two vertical bars (e.g. $|-1| = 1$), while the Euclidean norm is indicated with two pairs of vertical bars (e.g. $\|1 + j\,1\| = \sqrt{2}$).

7) Summations are given over elements of a set. For example, the sum of the squares of the reactive power mismatches for the power system is $\sum_{i \in NL} (\Delta Q_i)^2$.

8) The superscript "des" indicates the desired value of the variable. Similarly, "sched" indicates the scheduled value of the variable. E.g. V_i^{des} indicates the desired value of the voltage magnitude at bus i.

9) The superscript "max" indicates the maximum desired value of the variable. Similarly, "min" indicates the minimum desired value.

10) Where a vector or vector-valued expression is shown to be greater than $\overline{0}$, this is intended to mean that each element of the vector (or expression) is greater than 0.

11) An asterisk superscript indicates the complex conjugate, e.g. S^* is the complex conjugate of the power.

12) A "T" superscript indicates the transpose of the vector or matrix. E.g. $\overline{V}^T$ is the transpose of the voltage vector.

13) A subscript "o" indicates the initial value of the variable,

e.g. $\bar{x}_o$ is the initial value of vector $\bar{x}$.

14) A "∇" preceding a variable indicates the gradient of that variable, e.g. $\overline{\nabla F_u}$ indicates the gradient of F with respect to $\bar{u}$. A "∇^2" similarly indicates the Hessian of the variable.

15) A superscript such as "k" or "k+1" is used to indicate the value of the variable at step k or k+1 in the iterative process.

Symbols:

S	-	complex power. Sometimes used for the sensitivity matrix $\left[\frac{dV}{dQ}\right]$, but always indicated as such.
P	-	real (active) power = $\mathrm{Re}\{S\}$. Positive from bus into system (or ground).
Q	-	imaginary (reactive) power = $\mathrm{Im}\{S\}$. Positive from bus into system (or ground).
V	-	voltage magnitude.
θ	-	voltage angle relative to an arbitrary reference voltage (usually the slack bus voltage).
Z	-	impedance (complex).
R	-	resistance = $\mathrm{Re}\{Z\}$.
X	-	reactance = $\mathrm{Im}\{Z\}$.
Y	-	admittance (complex) = Z^{-1}.
G	-	conductance = $\mathrm{Re}\{Y\}$.
B	-	susceptance = $\mathrm{Im}\{Y\}$.
$\mathcal{L}$	-	transmission real power loss.
f	-	objective function to be minimised.
$\bar{g}$	-	the vector of equality constraints (equal to $\bar{0}$).
$\bar{h}$	-	the vector of inequality constraints (greater than $\bar{0}$).
$\bar{u}$	-	vector of control variables: generator voltages, transformer taps, and allocated reactive power.

$\overline{x}$	-	vector of independent variables: load bus voltages, voltage angles.
w	-	weighting factors for voltage penalty term.
z	-	weighting factors for reactive allocation term.
t	-	tap setting for variable tap transformers of the set NT.
N	-	set of all busses in the system.
NL	-	set of all load and other busses in the system for which the bus voltage is not fixed.
NG	-	set of all generators in the system eligible for voltage adjustment during optimisation.
NQ	-	set of all busses in the system eligible as locations for shunt reactive banks during optimisation.
NV	-	set of all busses at which the voltage is to be held to within some tolerance of nominal.
NT	-	set of all variable transformer taps eligible for adjustment during optimisation.
NCI	-	set of all busses with a connecting branch to bus i.
NVH	-	subset of NV for which voltages are higher than the desired maximum.
NVL	-	subset of NV for which voltages are lower than the desired minimum.

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INTRODUCTION

With power systems becoming larger and more tightly meshed, the development of good power flow simulations for each year and loading condition of a capricious ten-year system plan is becoming one of the most tedious procedures involved in planning transmission networks.

The procedure involved in producing a good power flow solution is sufficiently well defined that the majority of the adjustments can be done by an automated technique. Someone familiar with the power system (e.g. a good technologist) can set up the constraints for the process and evaluate the results.

If the results are not exactly right the first time, as is likely (depending on the ability of the user), the technique can be used iteratively. Provided the person "at the controls" understands what constitutes a good solution, it is possible to achieve a good simulation in much less time, and with much less frustration, than is possible with standard manual power flow simulation techniques.

There are basically only three steps in producing a power flow simulation. First, the transmission and generation must be modelled, and the system loads must be adjusted to the load forecast for the year and loading condition (e.g. 1984 heavy winter peak) under consideration. Second, a reasonable generation schedule, including import/export schedules, must be established to support the total load. This schedule must take into account the effect of upstream hydro plants on downstream plants (e.g. B.C. Hydro and Power Authority's Site One plant is dependent on the upstream G.M. Shrum plant), the availability of water for hydro plants, the desirability of operating thermal plants, the merit order of available hydro and thermal plants, etc. Third, the generator voltages, variable transformer taps, switchable capacitor/reactor banks and the like all must be adjusted so that the voltages around the system are within safe (and stable) operating limits. The voltages are then further adjusted using available and planned reactive power sources, so as to obtain a reasonable* voltage profile. This final trimming of voltages is carried out in increasing detail according to the nearness of the study date to the actual date, with the most attention being given to the first year of the plan.

This third and last step, the adjustment of generators,

* Usually this is a subjective evaluation, which involves many inter-relating factors, such as whether the simulation is for a normal or emergency (outage) condition, and the location, size, and nature of the affected load.

variable transformer taps, switched reactive banks, etc., and particularly the detailed investigation involved in early budget years, generally requires the most work in any power flow simulation, and it is thus for the resolution of this step that many computer-aided techniques have been proposed.

This thesis summarizes several of the various techniques proposed for reactive power management*, and then presents the development and analysis of an objective function different from that adopted by most other authors. Instead of minimising the deviation of system voltages from "standard" values as is usually done, the technique presented here reduces the transmission losses in the system, which, in the opinion of the author, provides the solution which good "voltage deviation" techniques only approximate. This amounts to solving the reactive half of the general optimal power flow problem addressed by Dommel [1]

* Reactive power management is defined here to be the control of generator voltages, variable transformer tap settings, and existing switchable shunt capacitor and reactor banks, and the allocation of new capacitor and reactor banks, in a manner which best achieves the desired goal of voltage control, loss reduction, or both. This is more general than reactive power allocation ("VAr allocation") which deals only with the allocation of new and existing blocks of shunt compensation.

and Sasson [2].

Finally, the analysis of the objective function is used to select and test a computationally efficient solution technique which will require a minimum of user interaction to be effective.

CHAPTER I

Principles and Techniques of Reactive Power Management

Although reactive power does not usefully contribute to the flow of energy in the power system, it has a significant effect on power system performance and efficiency. By reducing unnecessary reactive power flows, it is possible to increase thermally restricted active power capacities of lines, transformers, and generators. System stability is improved by reducing the wide variation in bus voltages which characteristically accompany high reactive power flows. The lower currents improve voltage regulation on distribution circuits, and reduce energy losses throughout the system.

There are several ways in which unnecessary reactive power flows can be reduced. The most common way is to supply the necessary reactive power generation at the load itself. This is usually done using shunt capacitors or reactors.

Although, ideally, shunt reactive devices should be provided at all busses where the power factor is less than unity, the cost of these devices prohibits this practice. Instead, it is usual to install shunt devices in such a way that the benefits are shared over several adjacent busses. A tradeoff is thus made between the effectiveness of the correction, and the cost of the equipment. The allocation of new and existing shunt devices in a manner which offers the greatest benefit for the lowest cost is the problem addressed by most "VAr allocation" techniques.

Some techniques perform true reactive power management, offering additional ways of controlling reactive power flow. Within stator current, field current, and stability restrictions, generator voltages may be controlled, altering both the reactive power production (or absorption) at the generator itself, and the reactive power flow through adjacent, strongly connected busses.

Another method of controlling the reactive power flow in a system is transformer tap adjustment. In overhead transmission networks, the line inductance is typically much greater than line resistance. Under these conditions, the active power flow along a transmission line is nearly independent of the difference between the voltage magnitudes at the ends of the line, while the reactive power flow is nearly proportional to this voltage difference. By using transformer taps to alter one or more bus voltages, it is possible to control the flow of reactive power through these busses independently of the flow of real power. (Note that only the reactive power flowing <u>through</u> a bus may be controlled. Any reactive power absorbed by a load must always be supplied, irrespective of any adjustments in transformer taps.)

Manual Techniques

Perhaps the most obvious method for allocating reactive power sources is by inspection of the power flows on various lines. Adjustments to generator voltages, existing (switchable) shunt reactive banks, and variable transformer tap settings can be determined from careful study of bus voltages and circuit reactive power flows. After these adjustments have been determined and checked with a new power flow simulation, remaining regions of high or low voltage, and transmission lines with high reactive power flows are identified from the newly calculated results. New shunt reactive devices can then be located at busses central to the problem areas. This procedure is repeated for each loading condition to determine the total system requirements for the year being studied.

This technique, which is often referred to as the trial and error method, can also be used to plan voltage support for outage cases. Lines can be removed from the study, as necessary, to investigate each system contingency. The power flows are calculated as for the normal condition base case, and reactive compensation located so as to correct for adverse voltages, and to minimise the power flows along heavily loaded transmission lines. The shunt compensation requirements for all cases would then be assimilated into a single system plan.

While this method of allocating reactive power sources gives exact power flow solutions, it does have several drawbacks. Firstly, at least two power flows--one initial to find the uncorrected voltages and power flows, and at least one other to test the allocation scheme--are required for each loading and contingency condition.

Secondly, it is difficult to determine the correct size for each reactive installation, particularly when it is necessary to keep voltages at several adjacent busses within some tolerance of nominal. This problem is further complicated by the need to combine reactive requirements for all contingencies into a single plan for the system, in that compensation added for one contingency will affect the amount of compensation needed for other contingencies.

In spite of the amount of work involved in this type of reactive power allocation study, the trial and error technique is the one used by most Canadian utilities [3] for studies of the first one or two years of the system plan.

Another manual technique which is sometimes used (and is presently in use at B.C. Hydro) is the <u>sensitivity</u> technique [4]. In this method, the partial derivatives of voltage change with respect to reactive power ($\frac{\partial \Delta V_i}{\partial Q_j}$) are calculated from the network data (these derivatives can easily be obtained from the Jacobian matrix of a Newton-Raphson power flow program). It is then possible to find the bus voltages which accompany a change in reactive power:

$$\overline{\Delta V} = [S_J]\ \overline{\Delta Q} \tag{1}$$

where

$\overline{\Delta V}$ = expected voltage changes

$\overline{\Delta Q}$ = changes in reactive power out of busses

$[S_J] = \left[\frac{dV}{dQ}\right]$ (sensitivity matrix derived from Jacobian matrix).

From this equation, it is possible to calculate the changes in bus reactive shunts which will adjust the voltage at bus i by ΔV_i. The most effective location for a shunt reactive bank is then at the bus with the highest sensitivity coefficient (element of $[S_J]$) for the voltage at bus i (i.e. bus k, where $|S_{ik}| = \max\{|S_{ij}|,\ \text{for all } j \in NQ\}$). Both the location and the sizing of the bank are thus obtained directly.

It is interesting to note that for differences $(\theta_i - \theta_j) \approx 0$, voltage magnitudes $V_i \approx 1$ p.u., and low resistance branches ($R_{ij} \ll X_{ij}$), then

$$\frac{\partial Q_i}{\partial V_j} \approx -V_i\, B_{ij} \cos(\theta_i - \theta_j) \approx -B_{ij} = \frac{X_{ij}}{\|Z_{ij}\|^2} \approx \frac{1}{X_{ij}}$$

which is just the short-circuit ratio of the branch between busses i and j. This, along with the approximation

$$\frac{\partial V_j}{\partial Q_i} \approx \left(\frac{\partial Q_i}{\partial V_j}\right)^{-1}$$

has been exploited by Maliszewski, Garver, and Wood [5].

Dommel [6] has shown that the sensitivity technique can be modified to determine the amount of reactive compensation Q_i required at a single bus i to minimise the objective function

$$f = \sum_{i \in NV} (V_i - V_i^{des})^2$$

where

V_i = voltage at bus i

V_i^{des} = desired voltage at bus i.

Inserting the first-order sensitivity relation of V_i about V_{o_i} gives

$$f = \sum_{i \in NV} (V_{oi} + S_{ij}\, \Delta Q_j - V_i^{des})^2$$

where

$S_{ij}\, \Delta Q_j$ is the expression for $\Delta V_i = \frac{\partial V_i}{\partial Q_j} \Delta Q_j$ from (1).

At the minimum

$$\frac{df}{d\Delta Q_j} = 0, \text{ or}$$

$$2 \sum_{i \in NV} (V_{oi} + S_{ij}\Delta Q_j - V_i^{des})\, S_{ij}$$

$$= 2 \left\{ \sum_{i \in NV} (V_{oi} - V_i^{des})\, S_{ij} + \Delta Q_j \sum_{i \in NV} S_{ij}^2 \right\} = 0$$

from which

$$\Delta Q_j = \frac{-\sum_{i \in NV} (V_{oi} - V_i^{des})\, S_{ij}}{\sum_{i \in NV} S_{ij}^2}$$

The sensitivity technique has the advantage of being a true reactive power management technique, and can be used to calculate required adjustments in transformer taps and generator voltages in the same way as was done above for shunt compensation.

The major disadvantage of the sensitivity technique is that it assumes that the relationship between $\overline{\Delta V}$ and $\overline{\Delta Q}$ is a linear one, while in practice it is very complex. The calculated values, therefore, are valid only for small changes in bus voltage.* After an allocation scheme is selected, it must be tested with a power flow to find the exact bus voltages. If the voltages are still unsatisfactory, the procedure can be repeated.

Note the similarity between this technique and the trial and error approach: they both begin with the solution of an initial power flow, and end with the solution of a confirming power flow. The difference is solely in the way the reactive installations are located and sized. In the trial and error technique, the planning engineer selects the location on the basis of the reactive power flows and voltage levels. He must then use judgement to size the installation--a task complicated by the difficulty of predicting the effect of the new installation on voltages at adjacent busses. The sensitivity technique provides this information directly (albeit approximately), and much more quickly.

The sensitivity technique is fast, fairly simple, handles all contingencies, all loading conditions, and all types of

* The accuracy of the predictions always improves as the magnitude of voltage correction decreases. For example, using this technique on a typical B.C. Hydro system, to change a bus voltage from 0.998 p.u. to 1.025 p.u., the predicted change in Q at that bus brings the voltage to 1.026--an error of 0.12%. For a starting voltage (at the same bus) of 1.007 p.u., the new predicted change in Q brings the voltage to 1.0253 p.u.--an error of only 0.03%.

reactive power management (shunt compensation, transformer taps, and generator voltages). Unfortunately, it is only approximate, and it shares with all manual techniques, and many of the "optimal" automatic techniques, the difficulty of combining allocations for all loading conditions and contingencies into a single, least-cost system plan.

Optimal (Automatic) Techniques

The major difference between manual and optimal reactive power allocation techniques is that optimal techniques both locate and size shunt reactive banks automatically. The term "optimal" implies--in some cases correctly--that the resultant allocation scheme is the best possible, subject to constraints such as bus voltage limits, maximum and minimum acceptable installation sizes, etc.

Optimal reactive power allocation techniques can be generally divided into two types. The simplest, and the one on which the majority of the literature has been written, is the group of linearized techniques: linear programming, integer programming, and 0-1 programming. These techniques usually deal only with the allocation of shunt reactive banks, and always assume a linear "objective function" (which in this case means that $\overline{\Delta V}$

and $\widetilde{\Delta Q}$ are assumed proportional), subject to linear constraints. (There is a variation of linear programming known as quadratic programming, which allows the use of a quadratic objective function subject to linear constraints.)

The second type of optimal technique, and the most complex, is the group of nonlinear techniques often termed nonlinear programming. These techniques deal with all types of reactive power management, can handle a wide range of objective functions, and can have both linear and nonlinear constraints. This great flexibility is not without penalty, however, as the computation time required for solution increases rapidly with problem complexity, and the solution technique used must often be tailored to the problem in order to find any solution at all. Nevertheless, if the extra flexibility is required, it is nearly always possible to develop a workable algorithm. Since the objective function is unrestricted, these techniques produce as exact an answer as the precision of the computer, the data, and the convergence behaviour of the algorithm will permit.

Linear Optimal Techniques

The basic linear optimal reactive power allocation technique is linear programming. In the linear programming approach, the voltage changes in the system are assumed to be proportional to the changes in reactive power;

$$\overline{\Delta V} = [S]\ \overline{\Delta Q} \tag{2}$$

where

$\overline{\Delta V}$ = expected changes in bus voltages

$\overline{\Delta Q}$ = changes in reactive power out of busses

$[S]$ = constant matrix, which may or may not be the $[S_J]$ of equation (1).

The constraints on $\overline{\Delta V}$,

$$\left.\begin{aligned} \overline{\Delta V}_i &\geq V_i^{min} - V_i \\ \overline{\Delta V}_i &\leq V_i^{max} - V_i \end{aligned}\right\} \quad \text{for all } i \in NV$$

are combined with (2) to obtain the set of inequalities:

$$\left.\begin{aligned} \sum_{j \in NQ} S_{ij}\, \Delta Q_j &\geq V_i^{min} - V_i \qquad (3) \\ \sum_{j \in NQ} S_{ij}\, \Delta Q_j &\leq V_i^{max} - V_i \qquad (4) \end{aligned}\right\} \quad \text{for all } i \in NV$$

The constraints on $\overline{\Delta Q}$ are:

$$\left.\begin{array}{l} \Delta Q_j \leq Q_j^{max} - Q_j \\ \Delta Q_j \geq 0, \text{ or} \\ -\Delta Q_j \geq 0 \end{array}\right\} \quad \text{for all } j \in NQ \qquad \begin{array}{r} (5) \\ (6a) \\ (6b) \end{array}$$

where

equation (6a) is used for inductive allocations, and

equation (6b) is used for capacitive allocations.

The objective function to be minimised is the sum of the absolute values of all the reactive additions:

$$f(\bar{Q}) = \sum_{j \in NQ} |\Delta Q_j| \; . \tag{7}$$

The series of equations (3) - (7) form a standard linear programming problem, and can be solved by any of the available linear programming algorithms. The result of the optimisation is a set of reactive allocations $\overline{\Delta Q}$ for the busses chosen by the planning engineer. In some methods, $\overline{\Delta Q}$ is permitted to assume any value, while in others it is constrained to be one of a set of standard sizes. In the case where any value is permissible, the sizes must later be rounded to the nearest standard size by the planning engineer. A power flow can then be run with the standardized allocations to obtain the exact bus voltages.

Although differing in some points--especially in the formulation of equation (2)--all linear programming allocation techniques have this general form [5,7]. They are usually solved with the Simplex linear programming algorithm, for which standard code is available.

These techniques can be made to handle multiple contingencies automatically. Maliszewski et al. [5] accomplish this by solving all contingencies together as one massive linear programming problem. If necessary, some of the capacity needed for one contingency can be removed during other contingencies to prevent the voltage from exceeding that experienced under normal conditions. The installed capacity is permitted to take any value, later being rounded to the nearest standard value by the user.

The linear programming techniques based on (2) through (7) use linearized sensitivity information. A different technique, by Kohli and Kohli [8], uses an <u>integer programming</u> technique to allocate capacitors in unit sizes. This technique builds a "tree" of all possible capacitor configurations, subject to the restrictions determined by the planning engineer. These configurations are then systematically tested to see if they satisfy bus voltage constraints. The search is ordered so that the first feasible solution will be optimal (least number of capacitors). The authors suggest that the extent of the (otherwise substantial) search can be reduced by assuming that the voltage on a bus will be affected the most by capacitors at that bus. Unfortunately this is not always true. (The voltage increase which accompanies capacitor additions is due to a reduction of current in the transmission feeding the bus in question. The current reduction--and hence voltage increase--will be the greatest when the capacity is added directly at the point of the reactive load, due to the saving in reactive power losses between the bus in question and the load point. The exception to this is when there is a constant voltage bus in the vicinity, in which case prediction is complex, and is best done directly from the sensitivities.)

As a result, the search remains substantial, and so is of little, if any, practical value for use with other than small subsystems of usual power networks.

Convergence of all of these techniques may be hampered by automatic transformer tap switching or generator Q limits, as these upset the assumed network linearity.

Nonlinear Optimal Techniques

Nonlinear optimal reactive power management techniques may treat the reactive power problem separately, or as a part of the complete optimal power flow problem, which would also optimise real power flows.

Kuppurajulu and Nayar [9] treat the reactive power problem separately. They minimise the total capacity allocated, subject to voltage and shunt capacity constraints.

This technique solves the nonlinear problem with a series of linear approximations designed to make each gradient step terminate on a constraint boundary. After each gradient step, a power flow is solved to find the exact bus voltages. By assuming that the optimal solution lies at the intersection of constraint boundaries, this technique applies <u>linear approximation programming</u> [10] to find the solution. This technique is very similar to the linear programming approach of [5].

Sachdeva and Billinton [11] solve the reactive power management problem as a portion of a complete optimal power flow. This technique performs true reactive power management including transformer taps and generator voltages. The shunt compensation is allocated in unit sizes.

Although the technique does handle multiple contingencies, this increases the data storage requirements considerably. A modified method which uses less storage is presented in [12]. This modified technique solves the optimal real power flow first, and then the optimal reactive power flow (with reactive shunt allocation). This cycle is repeated if the optimal reactive step alters the voltage angles by an amount sufficient to change the real power flow significantly from the optimum.

Both of these methods, the full and the decoupled, use a Fletcher-Reeves or Fletcher-Powell technique for the minimisation, with all constraints treated as penalty functions (similarly to [2]).

Two other optimal power flow techniques are also applicable to reactive power management. Sasson et al. [13] use a technique similar to that of [11], with all constraints handled as penalty functions. The coefficients of the penalty terms are altered during the solution to speed convergence to a feasible solution. The major difference between the techniques of [13] and [11] is in the minimisation, for which Sasson et al. use the Hessian, which is computed from the Newton-Raphson Jacobian matrix.

Dommel and Tinney [1] use a gradient search (and also a Hessian approximation) for minimisation, treating the power flow equations (equality constraints) with Lagrange multipliers. The inequality constraints are treated as absolute limits for independent variables (e.g. generator voltages), and as penalty functions for dependent variables (e.g. load bus voltages). The gradient technique, while sometimes slower to converge than Fletcher-Powell or Hessian techniques, requires considerably less storage and computation time per step.

All of the techniques {1,2,11-13} solve the optimal power flow (and the reactive power management extension) without assuming objective function linearity.

CHAPTER II

The Management of Reactive Power Using Loss Reduction

Most of the many papers written on reactive power allocation propose the use of linear programming and related techniques to determine the least amount of shunt capacitance which must be added at various busses to ensure that system voltages are above user-specified minimums. Most of these linear programming techniques are adaptable to shunt inductor switching also, allowing the scheduling of all shunt reactive devices in a roughly* least cost way. The computer program thus does in an orderly fashion exactly what a planning engineer would do in an "educated" manual way: add capacitors or inductors as necessary to adjust all voltages to within set limits. If this is all that is desired, these techniques work very well: they are fast, easy to use, and inexpensive to run.

The allocation of new shunt installations cannot reasonably begin, however, until all generator voltages, variable transformer taps, static and synchronous compensators, and existing switched shunt reactive banks have been adjusted to obtain the

* These techniques minimise the total additional capacity, neglecting the fact that the cost of a new installation is likely much more than the cost of enlarging an existing one.

best possible base case. Only then is it reasonable to attempt identifying sizes and locations for any new installations.

The task of obtaining the initial "optimum" base case is considerably more difficult--at least for major transmission networks--than locating new reactive banks. A transmission system is planned for continuous expansion. With a load growth of 7% or less per year, it should never be necessary to add extra reactive banks at more than a few busses at a time--even in large systems. This is especially true in view of the fact that it is cheaper to add one or two large banks (or expand existing banks) than to add several smaller ones, due mainly to the cost of the switchgear, buswork, concrete pads, etc. that are required to transform a symbol on a diagram into a reality on the system. It is even reasonable to do this manually, using V-Q sensitivities when necessary. The major difficulty, not only for reactive power allocation, but whenever power flow studies are made, is to obtain the best possible base case.

To many people, particularly practising power engineers, a "good" base case is one with a uniform, or nearly so, voltage profile. This explains the frequent appearance in the literature of "VAr allocation" techniques designed to correct bus voltages to within specified bounds. The need for specifying these bounds, however, and in particular the difficulty and importance of determining the "correct" bounds to use, limits the effectiveness of this approach. A poor choice of voltage limits for any of the linear programming techniques discussed earlier can prevent convergence by placing incompatible constraints on the solution. Some busses, particularly those near constant voltage busses, will be insensitive to changes in shunt reactive power at busses to which they are connected through relatively large branch impedances. If the voltage constraints on the bus and its neighbour do not allow for a realistic relationship between the two voltages, the problem will be either unsolvable, or the solution will require an unreasonably large amount of shunt compensation.

The avoidance of this type of difficulty is generally left as the responsibility of the user. Although it is not, perhaps, unreasonable to assume that an experienced engineer can set satisfactory voltage constraints, one of the most common arguments given in favour of reactive power allocation techniques in general is that they require less experience with the power system to use effectively than do manual techniques. The difficulty of setting proper voltage limits weakens this argument considerably for voltage-correction techniques.

Although there is little question that voltage-correction techniques can assist with the allocation process, and may even occasionally provide allocation schemes requiring little or no further modification, the requirement for user-selected voltage limits can lead to an unnecessary amount of preliminary work, and the resulting allocation scheme is always sensitive to the limits selected.

The more experience one gains with power flow studies, the more apparent it becomes that uniform (or other selected) voltages are not of primary importance at transmission, and to a lesser extent subtransmission system levels. Of more importance are the magnitudes, and the relative magnitudes, of the real and reactive power flows.

Whereas real power flows are usually determined by availability of (extremely expensive) generation sources, energy reserves, and the current system load, reactive power flows are more flexible. As there is no energy required for the generation of reactive power, it is much easier and less expensive to generate than is real power, and so can be produced nearer the load, reducing transmission losses, equipment loadings, and voltage drops. By reducing unnecessary reactive power flows, it is possible to approximate a uniform voltage condition. Indeed, it is by controlling reactive power flows that voltage-correction type "VAr allocation" techniques attempt to satisfy voltage constraints.

Reactive power flow is, in this way, more fundamental to a "good" base case than is a selected voltage profile. It is always possible to minimise reactive power flows; it is not always possible to achieve a desired voltage profile (given normal operating constraints). It is for these reasons that minimum reactive power flow, and not voltage profile, will be used in this thesis as the primary criterion for the selection of a "good" base case.

The Development of an Objective Function

From the foregoing discussion, it should be clear that a suitable objective function for a minimisation process will be related directly to the reactive power flow in the transmission system. There are several such functions which would be suitable:-

(a) the sum of the squares of the currents in each branch

(b) the sum of the squares of the reactive power flows in each branch

(c) the reactive power loss in the system (sum of $\|I\|^2|X|$ for all branches, which is effectively a weighted sum of squares of currents)

(d) the real power loss in the system (sum of $\|I\|^2$ R for all branches, similarly to (c))

(e) the slack bus real power (constant term + real power loss if real power injections are not altered)

or any similar function.

Function (a) is general, since it is the current which produces both the real and the reactive power losses. Unfortunately, it would tend to equalise branch currents irrespective of the equipment represented by the branch. Single-circuit, low capacity branches would thus be loaded at current levels comparable to multi-circuit, high capacity branches--obviously an untenable prospect.

Function (b) partially avoids the problem, since it places no restrictions on the real power flow. It does tend to balance reactive power flows between branches in the same way as (a) balances currents, however, which, although not resulting in quite such an unreasonable situation as (a), is still an undesirable characteristic for reasons analogous to those given above for (a).

Function (c) is a variation of (a) which tends to avoid the problem of equalising current flows by weighting the squares of the currents in each branch with a factor equal to the branch reactance. Effectively, this function tends to equalise the product $\|I\| |X|$*. A branch of twice the reactance of another branch will, therefore, be scheduled to carry about half of the current in the low reactance branch (assuming that the branches are effectively in parallel, and that the total current is constant). This is a near ideal sharing of current, and has the advantage of minimising the total reactive power generation which must be provided to meet a (fixed) reactive power load.

Function (d) is an interesting analog to (c). It produces a result similar to that of (c) in most cases, and identical if the ratio X/R is constant for every branch. The advantage of this function over (c) lies in the cost difference, economically, environmentally, and socially, between real and reactive power. Real power comes from large dams and reservoirs, conventional thermal stations burning irreplaceable and expensive supplies of coal or petroleum, or nuclear thermal stations which produce unmanageable, or nearly so, fission by-products. Reactive power, in contrast, is generated naturally by transmission lines and cables, and can be generated deliberately in shunt capacitor banks.

Function (d) shares with functions (a) to (c) a difficulty

* See appendix section A6 for a proof of this.

of calculation, in that currents must be calculated for each branch, and then summed according to the function employed. Function (e) is more elegant, as the slack bus power may be calculated with trivial effort from the solution voltages. For constant real power injections at busses other than the slack bus, as is the case here, this function produces results identical to those of (d). It is this objective function which will be used in this thesis.

In order to prevent the solution algorithm from driving the bus voltages excessively high, it is necessary either to augment the objective function with a term designed to increase or "penalise" the objective function as the bus voltages deviate from nominal, or to formally constrain the objective function with a voltage constraint during the minimisation. The former technique permits a voltage to deviate far from nominal if to do so significantly reduces either other voltage deviations, or the primary objective function (system real power loss). The latter technique will not permit the voltages to violate their respective constraints even if the constraints are preventing solution of the problem. The augmented objective function ("penalty factor") technique is therefore preferable for this use, as it at least ensures the existence of a solution.

If the algorithm is to be useful for the allocation of new shunt reactive power sources, the objective function will require an additional term accounting for the cost of the additional shunt capacity which is to be supplied. This may easily be accomplished by adding a term similar to the voltage (penalty) term, which will add to the objective function an amount equal to the weighted sum of squares of the compensation added.

The final form of the objective function is, therefore,

$$f(\bar{u},\bar{x}) = P_s(\bar{u},\bar{x}) + \sum_{i \in NVL} w_i (V_i - V_i^{min})^2 + \sum_{j \in NVH} w_j (V_j - V_j^{max})^2 + \sum_{k \in NQ} z_k B_k^2 \qquad (8)$$

where

term $w_i(V_i - V_i^{min})^2$ is low voltage penalty for bus i

term $w_j(V_j - V_j^{max})^2$ is high voltage penalty for bus j

term $z_k B_k^2$ is shunt capacity penalty for bus k

P_s = slack bus real power

w_i = voltage weighting factor for bus i

z_k = shunt capacity weighting factor for bus k

B_k = reactive shunt added at bus k

The constraints on $f(\bar{u},\bar{x})$ are:

$$\bar{g} = \begin{bmatrix} \overline{\Delta P} \\ \overline{\Delta Q} \end{bmatrix} = \bar{0} \qquad \text{for all busses} \qquad (9)$$

which are the power flow equations requiring the power mismatches to be zero at all busses ($\overline{\Delta P}$ and $\overline{\Delta Q}$ are the vectors of real and reactive power mismatches, respectively),

$$\bar{V} - \bar{V}^{min} > \bar{0} \qquad (10a)$$

$$\bar{V}^{max} - \bar{V} > \bar{0} \qquad (10b)$$

for all busses $\in$ NG

which are the minimum and maximum voltage limits for all controlled voltage (generator) busses,

$$\bar{T} - \bar{T}^{min} > \bar{0} \qquad (11a)$$

$$\bar{T}^{max} - \bar{T} > \bar{0} \qquad (11b)$$

for all busses $\in$ NT

which are the minimum and maximum tap limits for all controlled transformers,

$$\bar{B} - \bar{B}^{min} > \bar{0} \qquad (12a)$$

$$\bar{B}^{max} - \bar{B} > \bar{0} \qquad (12b)$$

for all busses $\in$ NQ

which are the minimum and maximum limits of reactive compensation to be added to eligible busses (of the set NQ), and finally

$$\bar{Q} - \bar{Q}^{min} > \bar{0} \qquad (13a)$$

for all generator busses

$$\bar{Q}^{max} - \bar{Q} > \bar{0} \qquad (13b)$$

which are the minimum and maximum reactive power limits for the generators. The inequalities (10) - (13) collectively constitute the inequality constraint set $\bar{h} > \bar{0}$.

Note that the limits on the control variables, equations (10) through (12) are linear, which will permit the use of a simpler constrained optimisation technique.

CHAPTER III

The Investigation of the Constrained Objective Function

The choice of a numerical minimisation technique requires knowledge of the nature of the constrained objective function. One of the simplest ways of getting this information is through the use of contour plots, in which contours of constant objective function value are plotted versus the various control variables. Such contour plots can be produced by using a power flow program to evaluate the objective function for various values of two control variables. The contour plots in this thesis were produced by a power flow program which automatically varied the control variables on the two axes through each of eight values, giving a total of 64 power flows (or function evaluations). The values of the objective function were then interpolated between these points to obtain the contours for plotting.

The objective function chosen (equation 8) is composed of three parts: the slack bus real power*, the voltage penalties,

* The real power loss is equal to the slack bus real power plus a constant. The contours of real power loss thus have identical form to the contours of slack bus real power. The contours of real power loss will be used herein.

and the allocated reactive power penalties. Although it is not difficult to plot the contours for this objective function, it will be difficult to analyse such a complex set of contours directly. It is simpler to analyse the loss contours first, and then analyse the effect that the penalties will have on these contours.

Contours of Constant Loss

The system for which contours have been obtained is shown in figure 1. This three-bus system is a simplified representation of the portion of B.C. Hydro's 230 kV system from Bridge River (bus 2) through Cheekeye (bus 3) to Vancouver (bus 1).

The contours of constant loss versus the voltages at busses 1 and 2 are shown in figure 2. These contours appear to be strongly parabolic, with the axis parallel to and slightly displaced from the line $V_1 = V_2$.

The reduction of loss with increasing voltage at busses 1 and 2 is due to the resultant increase in voltage at bus 3, which reduces the current necessary to provide the load P and Q. This reduced current flow then results in reduced loss in the two branches.

On either side of the axis the loss increases. This is <u>not</u>

due to a variation in the voltage at bus 3, which is essentially constant along a locus at right angles to the axis (parallel to the directrix). For example, along the line AB defined by A = (1.20,0.85) and B = (0.85,1.20), the per unit bus 3 voltages are: 1.00, 1.00, 1.00, 1.00, 1.00, 1.01, 1.01, 1.01 for increments of 0.05 pu in V_1 and V_2. (The fact that the voltages increase slightly as V_2 increases and V_1 decreases indicates that the line AB defined above actually intersects the axis at an angle slightly less than 90°, which means that the axis is not quite parallel to the line $V_1 = V_2$.)

The reason for the increase of loss to either side of the axis becomes apparent upon close examination of the power flows corresponding to each point on the plot: points off the axis correspond to a transfer of reactive power (and hence current) from one generator to the other. The axis of the contours is the locus of solution voltages for which this interchange of reactive power is zero. The following table shows this effect for three points on the plot of figure 2:-

V_1 (pu)	V_2 (pu)	Q from 2 to 3 (pu)	Q from 1 to 3 (pu)	system loss (pu)
0.95	1.10	2.364	-1.014	0.239
1.00	1.05	0.746	0.485	0.216
1.05	1.00	-0.718	2.128	0.250

The axis in this case runs approximately along the line CD defined by C = (0.85,0.93) and D = (1.15,1.20). The reactive power is seen to flow from generator 2 to generator 1 for voltage (0.95,1.10), and from 1 to 2 for voltage (1.05,1.00). The location of the axis is affected by such parameters as the relative impedances of the branches, the real power flows along each of the branches, and the R/X ratios of the branches. By way of example, for the case of figure 2, the real power flows from bus 2 to bus 1. For the case of figure 3, however, both generators supply power to bus 3 equally, resulting in a smaller displacement of the axis from the line $V_1 = V_2$ than is the case for figure 2.

Figure 3, which is for a case identical to that of figure 2 except for the generation, also shows more elongated contours due to the reduced power, and so current, flowing on the branch from bus 2 to bus 3. The loss ($\mathcal{L}$) thus decreases less rapidly with increasing voltage than for the more heavily loaded case (the derivative $\frac{d\mathcal{L}}{dI} = 2IR$ and so is proportional to the current flow), while the loss due to the interchange of reactive power between generators is affected relatively less by the reduction in load current. Hence the greater elongation of the contours.

The relationship between the shape of the contours and the line loading is further apparent from inspection of figure 5.

This is the contour plot for the situation of figure 4, with a (hypothetical) branch from bus 1 to bus 2 paralleling the previous path. Any reactive power flowing from generator 2 to generator 1 therefore has an alternate path around the load bus 3. This reduces significantly the current on the other lines. The contours have been elongated to the point where they are virtually parallel lines.

This extreme elongation of the contours is due, as with the case of figure 3, to the fact that the currents associated with the transfer of real power are now very small (due to equal load-sharing by the two generators), and the variation in loss with current is significantly less than would be the case for a higher current flow. The variation in loss due to the exchange of reactive power between the two generators, however, is much greater than for the case of figure 3. This is due to the fact that with a reduced impedance between the two generator busses, a given difference in voltage magnitude produces higher current than previously. Since the current increases more rapidly than before, the square of the current increases much more rapidly, with a correspondingly rapid increase in loss.

Figure 6 shows the contours for the same case as figure 2, but with a load at bus 3 of only 40% of the previous value.

Here again the dramatic elongation of the contours is very evident. This is again due to the substantial decrease in loading of the transmission line between bus 2 and the load at bus 3.

The contours of figure 7 were obtained by placing a transformer of zero impedance in the transmission line between busses 2 and 3 (with the tap at bus 3). In this way the tap of the transformer could control the voltage at load bus 3 without directly affecting the current flow along the line between busses 2 and 3. The contours of figure 7 therefore, are essentially the constant loss contours for the transmission line between bus 3 and bus 1. These contours are very similar to the light load contours for the whole system.

Figure 8 shows contours for a case identical to figure 4, but with a zero-impedance transformer as for figure 7. The contours are again elongated, but this time they appear to be elliptical, with a minimum within the plot range. The voltage at bus 2 for this case, as for the case of figure 7, is one per-unit. This explains the minimum located about a tap of one and a voltage on bus 1 of one per-unit. The closed contours in this example result from the fact that the tap now controls a circulating reactive power flow throughout the system. The minimum loss condition occurs for a circulating Q flow of zero, which occurs for a tap setting of 1 and for no transfer of reactive power between generators--i.e. when the voltage at bus 1 approximately equals the voltage of bus 2 at one per-unit.

One of the objectives of the algorithm being developed is to allocate reactive power in such a way as to minimise the transmission losses in the system, and so it is reasonable to investigate the effect of the power factor of the load at bus 3 on the transmission losses. For the cases of figures 9 and 10 the power factor of the load was reduced from the 95% of the previous cases to 80%. As can be seen from comparison with the previous cases (figures 2 and 6, respectively), altering the power factor has a negligible effect on the shape of the contours.

This is further demonstrated by the contours of figures 11 to 13. In each case, and particularly in the case of figure 13 (branch between bus 1 and bus 2), the contours are very elongated for the variation in shunt at bus 3. This is true because, in this particular case, the voltage magnitude at bus 3 is relatively insensitive to changes in the shunt at bus 3. Since the reactive power flow, and so the system loss, is determined by the voltage of bus 3 relative to the voltages of busses 1 and 2, the system loss is also relatively insensitive to changes in the shunt at bus 3. (For V_2 = 1.0 pu and V_1 = 1.0 pu,

V_3 varies from 0.9791 pu to 0.9893 pu over the range of shunts 0.0 to 0.7 pu for the case of figure 11.)

There are four conclusions, then, that can be drawn from these results. Firstly, in the absence of extreme conditions, the loss will depend more on generator voltages and on transformer taps than on load power factors. Secondly, the loss decreases with a simultaneous increase in generator voltages. Thirdly, the loss decreases as the generator voltage magnitudes approach a uniform value.* And lastly, the degree of elongation of the contours increases very rapidly as the loading decreases. (Conversely, the contours become more circular as the loading increases. This last point suggests that the optimisation process will be more difficult under light-load conditions because of the elongation of the contours, indicating poor scaling of the variables. As the magnitude of the loss is less under light-load, however, the optimisation can be terminated before completion with little penalty; this difficulty is not a serious one.)

* This point is important in that it spans the gap between the "equal voltage" criterion used by most utility engineers and the "minimum loss" criterion presented here. Generally the two criteria produce very similar results, as they must if the "minimum loss" criterion is to carry credibility.

The Effect of the Penalty Terms

The effect of the penalty terms depends to a large extent on the penalty factors used. By adjusting the penalty factors, the balance in the objective function between the loss and the two penalty terms can be shifted to obtain different performance characteristics. Because the dependence of the objective function on loss is linear, while the dependence on the penalties is quadratic, it is not possible to equate, for example, a loss of 1 MW with a voltage deviation of 0.01 pu or a reactive shunt allocation of 1 MVAr. Nevertheless, it is possible to determine the general effect of the penalty terms on the contours by using values of 7.5 and 1.0 for the voltage and reactive power penalties, respectively. (These values were found to give acceptable performance in the final program.)

Figure 14 illustrates the effect of the voltage penalty on the contours of figure 2. The contours have been closed at the high voltage end of the plot range, and have become generally rounded. The most extreme effect of the voltage penalty is observed in figure 15, which corresponds to the light-load case of figure 6. Here the elongated contours have been rounded to near-circular. In both cases, the minimum is clearly bounded, as opposed to the original cases.

The rounding of the contours occurs in all cases except those in which one parameter is the shunt on bus 3. Figure 16 shows the contours of figure 11 augmented with the voltage penalty term. This lack of rounding is due mainly to the relative insensitivity of the bus 3 voltage to the bus 3 shunt--a characteristic which was pointed out earlier. This voltage insensitivity results in only a very small change in voltage penalty with shunt, so that the moderating effect of the voltage penalty on the contour shape is minimal.

The effect of the reactive power penalty can be seen in figure 17, which corresponds to the same case as figure 16. The contours have now become nearly circular. This is again due to the relative insensitivity of the voltage of bus 3 to the shunt, which results in the reactive power penalty term dominating the objective function in the absence of reactive power transfer between generators. Both the reactive power penalty term and the loss due to an interchange of reactive power are quadratic terms, and so will produce circular contours with a suitable choice of reactive penalty factor.

It may appear from the foregoing discussion that there is little point in altering bus shunts. For the example used here, this is quite true. In cases where the sensitivity of a load bus voltage to a bus shunt is great, however, there will be a strong variation of loss with shunt, and the associated contours will be distinctly more rounded than for this example system. As it is only for cases exhibiting this strong sensitivity that shunt control would be used, there is no cause to doubt the effectiveness or usefulness of shunt control from the preceding results.

Reactive Power Limits on Generators

The set of equations (8) to (12) does not quite describe the reactive power management problem completely. All generators have limits on the reactive power they may absorb or produce, and so are constrained by equations (13), analogous to the equations (12) for allocation busses. Because of the analogous situation with the reactive allocation busses, it may seem desirable to treat generator reactive power limits in the same manner--i.e. as penalty terms.

This would, in fact, be a bad choice, as the contours of figure 18 indicate. Generator busses 1 and 2 in this case were penalised (quite lightly, in fact) to 0.95 power factor. The elongation of the contours--even with the voltage term included--is clearly the worst yet encountered. The explanation is that the real and reactive losses in a power system are very closely related--in fact, they are proportional for each branch in the system. Although the sum of the reactive losses in the system will not be quite proportional to the sum of the real losses (unless the X/R ratio is constant for every branch), it is nevertheless true that the minimum reactive power loss situation will correspond generally to the minimum real power loss situation. Since the minimum reactive power loss situation is the minimum reactive power generation condition, the generators will be called upon to generate the most reactive power when the real power loss is high, and vice versa. Penalising the generator Q violations as a square term is thus very much like using an objective function of

$$f = \mathcal{L} + k\mathcal{L}^2$$

where

k = constant

which exhibits very elongated contours.

It is better, then, to handle the generator Q limits in another way, thereby avoiding the minimisation problems which attend very poor scaling. An effective method of handling the generator limits will be presented in the following chapter.

CHAPTER IV

The Choice of a Suitable Minimisation Technique

The problem described by equations (8) through (13) requires the minimisation of a nonlinear objective function subject to linear and nonlinear constraints. There are a great many possible approaches to the solution of this type of problem. These many approaches differ mainly in the degree to which they utilise information about the objective function (such as first, second, and higher order derivatives), and the manner in which they treat the constraints. Constrained optimisation is generally much more complex than unconstrained optimisation, and it is therefore important that the constraints be treated in the way least likely to upset the minimisation procedure.

The Treatment of Constraints

Both equality and inequality constraints can be handled in either of two ways. Firstly, they can be handled directly, wherein the equality constraints are solved along with the other conditions for a minimum as a set of simultaneous equations, and the active inequality constraints are observed at each step as additional equations on the control variables. In each case, the constraints are met to within the precision of the calculations at every step, so that all intermediate solutions are feasible. This approach is essentially identical whether it is implemented as Lagrange multipliers, gradient projection, or gradient reduction.

The second approach is to use the penalty function technique, in which the steps are permitted to enter and leave the feasible region at will. For steps terminating outside the feasible region, a "penalty term" is added to the objective function. This penalty term usually increases quadratically as the step leaves the feasible region. The steps are thus "encouraged", and not, as with the previous approach, "forced" to remain within the boundaries of the feasible region. As the minimisation progresses, the penalty term is often multiplied by an ever-increasing factor, which tends to keep the intermediate solutions successively less infeasible until, at the solution point, the solution is feasible to within some tolerance.

Note that both techniques are essentially the same--as of course they must be--in that the second approach uses the minimisation process to solve the same equations as are solved algebraically by the first technique. Because the two techniques solve the constraint equations differently, it is reasonable to expect one technique to be superior to the other for certain forms of constraints. If the equations of constraint can be solved analytically without an extraordinary amount of computation, the first method is clearly superior, especially as the intermediate solutions for this method are all feasible, and so usable--although sub-optimal. An important disadvantage of the penalty function approach is that the penalty terms distort the contours, often making the resulting augmented objective function much more difficult to minimise (the generator Q limits of the last chapter are an example). For cases of constraint equations which may not be easily solved analytically, however, the penalty function approach is preferable.

The constraints on the control variables--transformer taps, generator voltages, and shunt reactive source allocations--can be easily treated as absolute limits on the allowable variation in the control variables, and so handled directly. The other constraints, however, must be treated by one of the two methods discussed above. While the equality constraints could likely be treated with either of the two techniques, the inequality constraints on the generator reactive power limits should not be handled as penalty terms. As was demonstrated in the last chapter, these penalties have a severe influence on the objective function contours, even with a small multiplying factor. Both equality and generator reactive power inequality constraints will thus be treated here using the analytic method of constraint handling.

It is not possible to solve explicitly the nonlinear equation (9) for the dependent variables as a function of the independent variables. If the function g (equation 9) is expanded about the current intermediate solution point in a first-order Taylor expansion, however, it is possible to solve the new linear relation explicitly for the independent variables. This relationship between the dependent and independent variables may then be substituted into the expression for the gradient of the objective function to obtain a reduced expression in which the gradient is a function of the independent variables only.* The equations are calculated in appendix section A2.

Because of the linearization of the constraint equation, each intermediate step will not necessarily end within the feasible region, and it is thus necessary to adjust the solution vector at each step to correct this. The most efficient way to do this for the equality constraints (9) is to solve the set of

* It is worth noting here that if the objective function were linearized along with the constraints, and if the minimum was assumed to lie along a constraint boundary, the problem could be solved using a standard linear programming program. This is the method used in [9].

simultaneous nonlinear equations using a conventional Newton-Raphson power flow program (the reader will recall that constraints (9) are just the set of power flow equations).

There are a number of advantages to this approach. First, the power flow program provides the slack bus power and load bus voltages which are necessary for the evaluation of the objective function at each step. Second, as was pointed out by Dommel and Tinney in [1], the gradient may be easily formulated at each step from terms of the Jacobian matrix (derived in appendix section A1) produced in the power flow program. As an additional advantage, this approach permits the bus-type switching portion of the power flow program to ensure the satisfaction of the generator reactive power constraints of equation (13).

Bus-type switching is the most common way of ensuring the operation of generator busses within their reactive power generation restrictions. It acts by switching generator (constant P, constant V) busses to load (constant P, constant Q) busses whenever they are no longer able to hold the scheduled voltage without exceeding a Q limit (i.e. when the constraints become active). When the scheduled voltage may again be held without exceeding a Q limit, the switched bus is permitted to revert back to constant voltage.

Switching busses from type P,V to P,Q amounts to changing the active inequality constraints into equality constraints, reducing the dimension of the solution space by one for each bus switched. This moves one bus voltage from the set of independent variables to the set of dependent variables, thereby reducing the dimensionality of the gradient by one (which is the same as projecting the full gradient onto the appropriate constraint boundary). The bus-type switching technique used in power flow programs is therefore identical in its effect to gradient reduction or gradient projection in a constrained optimisation.

Summarizing, it is not possible to use penalty function methods on the generator inequality constraints due to the adverse effects these penalties have on the objective function contours. It is possible, however, to use a gradient reduction technique, by using the linearized equations for the equality and active inequality constraints to reduce the dimensionality of the gradient, and then correcting for the effects of the linearization at the end of each step. If this correction is done using a conventional Newton-Raphson power flow program, the objective function and its gradient may be evaluated with little extra effort, and the generator reactive power (inequality) constraints are automatically satisfied by being converted to equality constraints, when they become active, by the bus-type switching algorithm.

The Method of Minimisation

It has been presumed above that the gradient would be an essential part of any selected minimisation technique. While the minimisation can, of course, be performed without knowledge of the gradient, better performance can usually be realized by taking advantage of this and any other information about the objective function. With the method of constraint handling described above, the calculation of the gradient requires a relatively minor amount of computation (most of which consists of one repeat solution with the factorized Jacobian matrix from the power flow step).

It is also possible, as has been pointed out by Sasson [13], to calculate the matrix of second partial derivatives--called the Hessian matrix--using the terms of the Jacobian matrix as for the gradient.* Using the Hessian matrix the minimisation problem may be solved using a generalized version of the Newton-Raphson method of solving nonlinear equations. For objective functions with elongated contours, the generalized Newton-Raphson method exhibits more rapid and reliable convergence than steepest descent or modified steepest descent methods,

* See appendix section A3.

which make use of the gradient only.

The disadvantage of Hessian-based techniques in this application is the large amount of storage and computation required to produce the Hessian matrix. Further, Himmelblau [14] points out that, while Hessian-based techniques exhibit quadratic convergence in the vicinity of the minimum, steepest descent methods may be superior far away from the minimum. For this application, it is not necessary to know the optimum exactly, but only to within, perhaps, a few percent, so that the major portion of the optimisation effort will occur away from the minimum. Bearing in mind the observations of the last chapter, where the objective function contours were found to be only moderately elliptical (for a reasonable selection of penalty factors), with no irregularities in shape to cause convergence failure, the steepest descent method appears to be a slightly better choice for this application than Hessian-based methods. It was the steepest descent method, coupled with a Lagrangian treatment of equality constraints, which was chosen by Dommel and Tinney in [1].

In order to gain the maximum improvement at each step, steepest descent searches generally use step lengths calculated to terminate each step at the function minimum in each successive search direction. These searches are termed "optimal step-size" searches.

The optimal step length can be approximated from the value of the objective function, and perhaps its derivatives, at one or more points in the current direction of search. The number of values needed is dependent upon the desired accuracy of the approximation, which determines the order of the polynomial used for interpolation (or extrapolation) in the current search direction. If only the function value is known at each point, various types of direct searches (see, for example, Himmelblau [15]) may be used.

The first derivative of the objective function in the direction of steepest descent is the negative of the gradient of the objective function (this is the directional derivative of the objective function in the direction of search). As the gradient is evaluated at each point to determine the next direction of search, both the value of the objective function and its first directional derivative are available immediately, without further work. To interpolate with a second-order polynomial (the lowest order polynomial which can reasonably describe the objective function in the direction of search), one other piece of information is required.

While this missing piece of information could be obtained from a further function evaluation in the direction of search, it is computationally more efficient to approximate the second directional derivative of the objective function. The reason the second directional derivative must be approximated is that its exact calculation would require the Hessian (second-order gradient) matrix. If this matrix were available, which would require considerable effort, it could be used directly for a Hessian-based minimisation.

Two observations permit the Hessian to be easily approximated. First, Smirnov [16] has pointed out that, as can be illustrated graphically, for each two-dimensional projection of the (elliptical) contour space, a steepest descent search converges to the optimum along the major axis of the ellipse. In the multi-dimensional case, the search will converge along the major axis of the hyper-ellipsoid, which is in the direction of the eigen-vector corresponding to the minimum eigenvalue of the Hessian matrix. The second derivative in subsequent directions of search may, therefore, be approximated as a constant equal to the minimum eigenvalue.

Second, as has already been pointed out, the contours discussed in the previous chapter are nearly circular. This indicates that the Hessian matrix has diagonal terms which are all of the same order of magnitude, and off-diagonal terms which are relatively small. This further improves the usefulness of of Smirnov's observation for this type of problem.

The second directional derivative may, therefore, be calculated on the basis of the previous step, and then used in a second-order Taylor expansion to predict the optimum step length for the current step (see appendix section A4). Although this method for calculating the step-size is approximate, the approximation improves during the minimisation, and substantially less computer time can be required than for the calculation of the Hessian matrix.

Summary

The best scheme for the solution of equations (8) through (13) is, therefore,

1) Newton-Raphson power flow solution, satisfying equality constraints, and evaluating the objective function and its gradient.

(2) steepest-descent search, using a sub-optimal step-size calculated from the preceding step assuming circular objective function contours (i.e. assuming that the Hessian is of the form $[H] = k[1]$, where k = constant).

(3) equality constraints (9) are handled using gradient reduction.

(4) inequality constraints (10) - (12) are handled as absolute limits on control variable variations (analogous to gradient reduction for active constraints).

(5) inequality constraints (13) are automatically handled using gradient reduction by a bus-type switching feature in the power flow program used at stage (1).

This approach is virtually identical to the general approach of Dommel and Tinney, and is in contrast to the more computationally complex scheme of Sasson et al. It is worth noting that, although Dommel and Tinney treated the equality constraints as Lagrange terms, the equations for the optimisation are mathematically identical to those developed for the gradient reduction approach used here. If the equality constraints are to be considered using Lagrange multiplier theory, then the bus-type switching scheme in the power flow routine causes the active inequality constraints of (13) to be treated as Kuhn-Tucker terms. The approach of [1] is thus identical to that outlined here.

CHAPTER V

The Performance of the Technique

Because the technique outlined in the last chapter is essentially an enhancement (to account for generator reactive power constraints and shunt capacitor and reactor allocations) of the reactive power optimisation technique described by Dommel and Tinney, it was implemented by modifying an available program based on the technique described in [1].

As there was an interpolation scheme in the original program, it was retained on the assumption that it would improve the estimate of optimal step-size near the minimum, thereby speeding convergence. As it is in the vicinity of the minimum that steepest descent exhibits the worst performance, the interpolation process was activated only near the end of the optimisation process.

This interpolation routine calculates the approximate second derivative assuming that the first directional derivative depends only on the control variables (u_i), using the equations derived in appendix section A5.

This was the only modification to the scheme developed in the last chapter.

The progress of the optimisation technique when performing an unconstrained minimisation on the three-bus system of figure 1 is plotted in figure 19 on the objective function contours of figure 14 (for the control of two generator voltages). In figure 20, the progress is plotted on the contours for the control of one transformer tap and one generator voltage (as for figure 7), and in figure 21, it is plotted on the contours of figure 17 for the control of one bus voltage and one reactive shunt (at bus 3). As can be seen from these plots, the optimisation progresses well in the first few steps, and reaches the minimum (to within practical tolerances) after 3 steps.

To test the procedure on a realistic, fully constrained problem, a 1976 winter heavy load representation of the B.C. Hydro system was used, consisting of 245 busses and 327 branches, with 47 controllable generators, and 44 controllable (on-load tap-changing) transformers.

After 16 iterations of the minimisation, which required a total of 72 power flow iterations, the solution was acceptably close to the optimum. Careful observation of the progress of convergence revealed that occasionally an iteration would apparently diverge, resulting in an increase in the value of the objective function and/or the derivative, both of which must reduce for the process to be convergent. This apparent divergence would occur for one or two steps, with the next several steps converging normally. This process may occur several times during a minimisation (see Table I).

The most probable cause for this peculiar behaviour is that the step-size chosen at each iteration is only an approximation to the optimal step. This approximation is based on the assumptions that:

a) the objective function is of order 2 or less in the direction of search.

b) the second derivative of the objective function is constant for all directions of search (i.e. the Hessian matrix is diagonal, with all diagonal terms equal).

c) the equality and active generator reactive power inequality constraints are approximately linear over the region of the step.

d) no inactive inequality constraints will become active during the step, and no active inequality constraints will become inactive.

If any of these assumptions are invalid for a given step--as, in general, at least one will be-- the calculated step-size will be sub-optimal. Depending on the degree to which the assumptions are invalid, the calculated step-size may become sub-

TABLE I

Progress of Convergence for 245 Bus Problem

Step No.	First Directional Derivative	Slack Bus Power plus penalties
1	7.37	15.88
2	2.68	15.58
3	2.23	15.52
4	3.27	15.50
5	2.61	15.48
6	1.03	15.46
7	0.853	15.45
8	0.674	15.44
9	0.797	15.42
10	9.37	15.59
11	9.05	15.64
12	1.14	15.44
13	0.911	15.43
14	1.56	15.42
15	1.73	15.43
16	0.382	15.41
17	0.271	15.41
18	0.238	15.41
19	0.364	15.41
20	0.175	15.41
21	0.122	15.41
22	0.098	15.41

optimal to the point of being divergent.

While--for a steepest descent search--a step-size of less than the optimal amount will generally only hold the convergence rate down somewhat, too large a step-size can cause the new value for the objective function and/or gradient to be greater than the previous value, giving a divergent step. For a step-size larger than the optimum, whether or not the step itself will be divergent depends on by how much the step length is too large, and on how rapidly the objective function and gradient change in the new direction of search.

Of the above four assumptions, there are two which are the most likely to disturb the approximation to the optimum step-size. For problems with a large number of inequality constraints, the fourth assumption will likely be violated at most steps. The switching of constraints from the inactive set to the active set produces a discontinuity in the optimisation process which could give rise to sporadic divergence. This constraint switching was observed to be occurring at most steps in the minimisation.

The second assumption is also known to be sometimes unreliable, as it implies that the objective function contours must be circular, whereas they are known to be always elliptical to some degree. From this observation, we can predict that the calculated step-size will be too small in some directions, and too large in others.

Since the estimate for the optimal step-size is based upon the rate of change of the gradient in the direction of search for the previous step (and evaluated over the span of that step), a greater-than-optimal step-size will most likely result when the new direction of search is more perpendicular to the major axis of the contours than was the last. The effect is more pronounced, and more readily leads to divergence, when the contours are strongly elliptical. It is mitigated somewhat by constraint activation, in that the successive search directions are thereby altered from the usual near-orthogonal search directions of an unconstrained (nearly-optimal step-size) search.

Of the two assumptions listed above as potential causes of the observed sporadic divergence, it is the second which is the probable major contributor. This would account for the fact that the divergent steps occur rarely; the steps would usually be convergent until the contours became too elliptical.

One way this could occur, for example, is when a generator on a long, lightly loaded feeder "came off" a minimum-Q limit (that is, the generator had been, but is no longer, limited to its minimum available reactive power output). In the plane of this and some other generator voltage, it is reasonable that the second generator voltage may affect the objective function considerably more than the first--leading to elliptical contours in the plane corresponding to these two generator voltages. When the first generator reaches a minimum-Q limit (which is more likely, under the circumstances, than a high limit), this plane vanishes because the first generator voltage is no longer a control variable. This would then (effectively) reduce the ellipticity of the multi-dimensional contours, stabilizing the adaptive step-size and thus the convergence.

It may appear from the above discussion that a better approximation to the optimal step-size, or perhaps even a more powerful unconstrained minimisation technique is needed. These are not necessarily solutions, however, in that the improvement gained will be much less than would be expected due to the effect of the inequality constraints, and may not be sufficient to warrant the extra calculation necessary.

The effect of the inequality constraints is rather hard to predict, other than that they are likely to disturb the steady convergence of the same unconstrained problem. This disturbance is so powerful that improvements in the unconstrained optimisation technique used at each step do not necessarily speed up the overall solution. In particular, knowing the optimal step-size is of little advantage if the search direction is deflected from the negative gradient direction by inequality constraints. The same is true for Hessian and related searches, in that the search direction calculated may bear little relation to the final deflected search direction.

Another problem which exhibited itself was the effect of the voltage penalty factors on the convergence rate. Values for the voltage penalty factors which are too large can lead to erratic convergence, probably due to the resulting sensitivity of the objective function to the voltage penalties. This is not unduly surprising, since if the "circularizing" effect of the voltage penalties demonstrated in chapter III is carried to extremes, the contours will become elliptical again, this time with the minor and major axes interchanged. Machine precision may also be a problem with large voltage penalty multipliers due to the larger second derivative terms and consequent shorter step lengths.

The use of moderate voltage penalty factors permitted the 500 kV busses at remote generator sites to rise well over the desired value of 1.05 per unit to values as high as 1.10 pu. Larger values of voltage penalty factors on these busses worsened convergence without decreasing the final voltages significantly, indicating that the high remote site voltages were important to the minimisation of the balance of the total objective function--i.e. the maintenance of reasonable voltages on other system busses and the reduction of system loss.

The best way to reduce these voltages, where necessary, is to control the maximum value of generator voltage on the associated generator busses. Since the generator voltage is treated as an absolute limit, this will prevent the high-voltage bus from exceeding safe voltage levels. This problem is most likely to arise when transformer taps, reactor banks, etc. which were not made controllable are poorly set. This was, in fact, the problem with the test case, as too many reactors had been used at stations between the remote generation and the load center.

It is important to realize that, while problems such as the high voltages noted above appear serious, they can in fact be important clues to deficiencies in the power system on which the minimisation was operating. The optimisation process acts on the control variables in any way necessary to achieve its objective. Provided always that the objective is a reasonable one, unconventional solutions may indicate poor adjustment of other parameters not controllable by the program, or--as is perhaps too often the case--merely inveterate thinking on the part of the person evaluating the solution.

Other than these isolated high voltages, the resulting system state was much better than the author had been able to achieve when using this case previously for system studies with a conventional manual power flow, one major improvement being the system voltage profile.

CHAPTER VI

Conclusions

The optimal reactive power flow problem can be solved using a steepest descent search with gradient reduction (or equivalently, Lagrange) constraint terms, using an objective function composed of the slack bus real power, and voltage and reactive power penalty terms for load bus voltages and allocation (load) busses, respectively. This objective function is generally well-scaled, and a sub-optimal step-size search performs effectively on the fully constrained problem, provided that the voltage penalty factors are not excessive.

The resulting set of generator and transformer settings, and sizings for shunt reactive compensation banks produce a generally good power flow case with less engineering effort than would be required using conventional manual adjustments.

CHAPTER VII

Directions for Further Work

The most important remaining work is the evaluation of the technique in a production environment. The two problems encountered--the sporadic divergence and the difficulty in holding down remote bus voltages with penalty terms alone--are not thought to be serious. The only way of confirming this belief however, is by obtaining production experience.

In addition to the aforementioned production testing, the technique may need to be extended to cover the many automatic features available in modern power flow programs (e.g. blocks of shunt reactive capacity activated by voltage magnitude, generators with reactive power adjusted to hold remote bus voltages within limits, etc.). In many cases, these features of power flow programs will be made unnecessary by an optimal reactive power management feature. Nevertheless, allowance must be made for possible conflicts between the optimisation process and the (generally rather crude) automatic control provided by such features, and the features should be removed, inhibited during optimisation, or formally incorporated into the optimisation process.

One further problem which should be investigated is multiple contingency optimisation. This is somewhat different than optimisation for a normal operating condition, in that the outage conditions must meet certain minimum operating limits (generally on load bus voltage) with only on-load transformer taps, generator voltages, and switchable banks of shunt compensation being adjusted from normal operating settings. This means that off-load transformer taps must be set so that it is possible to achieve the minimum operating limits using only the adjustable parameters. Although the optimisation may be performed separately on each contingency condition, a method is required for efficiently combining the separate contingency optima into a single result.

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APPENDIX

Derivation of Relevant Equations

A1 The Terms of the Jacobian Matrix

The power entering the system at bus i is given by

$$S_i^* = (P_i - j\, Q_i) = V_i^2\ (G_{ii} + j\, B_{ii}) + V_i\ (\cos\theta_i - j \sin\theta_i)\ * \sum_{j \in NCI} (G_{ij} + j\, B_{ij})\ V_j\ (\cos\theta_j + j \sin\theta_j)$$

Breaking this equation into real and imaginary components:

$$P_i = V_i \sum_{j \in NCI} V_j \left[\cos\theta_i\ (G_{ij} \cos\theta_j - B_{ij} \sin\theta_j) + \sin\theta_i\ (G_{ij} \sin\theta_j + B_{ij} \cos\theta_j)\right] + V_i^2 G_{ii}$$

$$Q_i = -V_i \sum_{j \in NCI} V_j \left[\cos\theta_i\ (G_{ij} \sin\theta_j + B_{ij} \cos\theta_j) - \sin\theta_i\ (G_{ij} \cos\theta_j - B_{ij} \sin\theta_j)\right] - V_i^2 B_{ii}$$

The terms of the Jacobian matrix are:

$$H_{ij} = \frac{\partial \Delta P_i}{\partial \theta_j} \qquad N_{ij} = V_j \frac{\partial \Delta P_i}{\partial V_j} \qquad J_{ij} = \frac{\partial \Delta Q_i}{\partial \theta_j} \qquad L_{ij} = V_j \frac{\partial \Delta Q_i}{\partial V_j}$$

$$H_{ii} = \frac{\partial \Delta P_i}{\partial \theta_i} \qquad N_{ii} = V_i \frac{\partial \Delta P_i}{\partial V_i} \qquad J_{ii} = \frac{\partial \Delta Q_i}{\partial \theta_i} \qquad L_{ii} = V_i \frac{\partial \Delta Q_i}{\partial V_i}$$

where

ΔP and ΔQ are the real and reactive power mismatches (positive into bus), respectively

Define

$$\alpha_{ij} = G_{ij} \cos \theta_j - B_{ij} \sin \theta_j$$

$$\beta_{ij} = G_{ij} \sin \theta_j + B_{ij} \cos \theta_j$$

so

$$\frac{\partial \alpha_{ij}}{\partial \theta_j} = -\beta_{ij} \qquad \frac{\partial \beta_{ij}}{\partial \theta_j} = \alpha_{ij}$$

Now, since

$$\Delta P_i = P_{gen_i} - P_{load_i} - P_i$$

$$\Delta Q_i = Q_{gen_i} - Q_{load_i} - Q_i$$

where

P_{gen_i}, P_{load_i}, Q_{gen_i}, and Q_{load_i} are constant,

then

$$H_{ij} = -\frac{\partial P_i}{\partial \theta_j} = -V_i \; V_j \; (\alpha_{ij} \sin \theta_i - \beta_{ij} \cos \theta_i)$$

$$H_{ii} = -\frac{\partial P_i}{\partial \theta_i} = -V_i \sum_{j \in NCI} V_j \; (\beta_{ij} \cos \theta_i - \alpha_{ij} \sin \theta_i) = Q_i + V_i^2 B_{ii}$$

$$N_{ij} = -V_j \frac{\partial P_i}{\partial V_j} = -V_i V_j (\alpha_{ij} \cos\theta_i + \beta_{ij} \sin\theta_i)$$

$$N_{ii} = -V_i \frac{\partial P_i}{\partial V_i} = -V_i \sum_{j \in NCI} V_j (\alpha_{ij} \cos\theta_i + \beta_{ij} \sin\theta_i) - 2V_i^2 G_{ii} = -P_i - V_i^2 G_{ii}$$

$$J_{ij} = -\frac{\partial Q_i}{\partial \theta_j} = V_i V_j (\alpha_{ij} \cos\theta_i + \beta_{ij} \sin\theta_i)$$

$$J_{ii} = -\frac{\partial Q_i}{\partial \theta_i} = -V_i \sum_{j \in NCI} V_j (\alpha_{ij} \cos\theta_i + \beta_{ij} \sin\theta_i) = -P_i + V_i^2 G_{ii}$$

$$L_{ij} = -V_j \frac{\partial Q_i}{\partial V_j} = V_i V_j (\beta_{ij} \cos\theta_i - \alpha_{ij} \sin\theta_i)$$

$$L_{ii} = -V_i \frac{\partial Q_i}{\partial V_i} = V_i \sum_{j \in NCI} V_j (\beta_{ij} \cos\theta_i - \alpha_{ij} \sin\theta_i) - 2V_i^2 B_{ii} = -Q_i + V_i^2 B_{ii}$$

A2 The Terms of the Gradient

The objective function is

$$f(\bar{x},\bar{u}) = P_s(\bar{x},\bar{u}) + \sum_{i \in NV} w_i (V_i - V_i^{sched})^2 + \sum_{j \in NQ} z_j (B_j - B_j^{sched})^2$$

where

$$\bar{x}^T = [\theta \;\; V]$$

$$\bar{u}^T = [t \;\; V_c \;\; B]$$

V_c = voltage on busses of the set NG

B = reactive shunt on busses of the set NQ

Let $F(\bar{u}) = f(\bar{x},\bar{u})$. The gradient $\overline{\nabla F}_u$ may be found by observing that the first-order variation of F is given by

$$\Delta F = \overline{\frac{\partial f}{\partial u}}^T \overline{\Delta u} + \overline{\frac{\partial f}{\partial x}}^T \overline{\Delta x} = \overline{\nabla F}_u^T \, \overline{\Delta u} \qquad \text{(A2.1)}$$

Now, expanding the equality constraint (9) in a first-order Taylor expansion:

$$\bar{g}(\bar{x},\bar{u}) = \bar{g}(\bar{x}_o,\bar{u}_o) + \left[\frac{\partial g}{\partial x}\right]\overline{\Delta x} + \left[\frac{\partial g}{\partial u}\right]\overline{\Delta u}$$

Since $\bar{g}(\bar{x},\bar{u}) = \bar{g}(\bar{x}_o,\bar{u}_o) = \bar{0}$,

$$\overline{\Delta x} = -\left[\frac{\partial g}{\partial x}\right]^{-1}\left[\frac{\partial g}{\partial u}\right]\overline{\Delta u} \qquad \text{(A2.2)}$$

Substituting (A2.2) into (A2.1):

$$\Delta F = \overline{\frac{\partial f}{\partial u}}^T \overline{\Delta u} - \overline{\frac{\partial f}{\partial x}}^T \left[\frac{\partial g}{\partial x}\right]^{-1}\left[\frac{\partial g}{\partial u}\right]\overline{\Delta u} = \overline{\nabla F}_u^T \, \overline{\Delta u}$$

The gradient $\overline{\nabla F}_u$ is thus given by

$$\begin{aligned}\overline{\nabla F}_u &= \left(\overline{\frac{\partial f}{\partial u}}^T - \overline{\frac{\partial f}{\partial x}}^T \left[\frac{\partial g}{\partial x}\right]^{-1}\left[\frac{\partial g}{\partial u}\right]\right)^T \\ &= \overline{\frac{\partial f}{\partial u}} - \left[\frac{\partial g}{\partial u}\right]^T\left[\frac{\partial g}{\partial x}\right]^{T-1}\overline{\frac{\partial f}{\partial x}} \end{aligned} \qquad \text{(A2.3)}$$

This is the same expression for the gradient as Dommel and Tinney obtain for the gradient in [1] using the Lagrange multiplier approach.

The terms of this expression are:

$$\frac{\partial g}{\partial x}^T = \begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta Q}{\partial \theta} \\ \frac{\partial \Delta P}{\partial V} & \frac{\partial \Delta Q}{\partial V} \end{bmatrix} \tag{A2.4}$$

$$\frac{\partial f}{\partial x}^T = \begin{bmatrix} \frac{\partial P_s}{\partial \theta} & \frac{\partial P_s}{\partial V} + 2w(V - V^{sched}) \end{bmatrix} \tag{A2.5}$$

$$\frac{\partial g}{\partial u}^T = \begin{bmatrix} \frac{\partial \Delta P}{\partial t} & \frac{\partial \Delta Q}{\partial t} \\ \frac{\partial \Delta P}{\partial V_c} & \frac{\partial \Delta Q}{\partial V_c} \\ 0 & \frac{\partial \Delta Q}{\partial B} \end{bmatrix} \tag{A2.6}$$

$$\frac{\partial f}{\partial u}^T = \begin{bmatrix} \frac{\partial P_s}{\partial t} & \frac{\partial P_s}{\partial V_c} & 2z(B - B^{sched}) \end{bmatrix} \tag{A2.7}$$

Equation (A2.3) can be calculated easily by taking advantage of the fact that the expression

$$\left[\frac{\partial g}{\partial x}\right]^{T^{-1}} \overline{\frac{\partial f}{\partial x}}$$

(which corresponds to the vector of Lagrange multipliers in the

Dommel and Tinney approach) can be obtained from one repeat solution using the transposed factorized Jacobian matrix from the Newton-Raphson power flow.

Expressed in terms of the Jacobian matrix, this expression becomes

$$\begin{bmatrix} -H & -J \\ -N & -L \end{bmatrix}^{-1} \begin{bmatrix} -H_S \\ -N_S + 2wV(V - V^{sched}) \end{bmatrix}$$

The terms for (A2.4) and (A2.5) can be obtained directly from the Jacobian matrix. The terms for (A2.6) and (A2.7) are

$$\frac{\partial \Delta P_i}{\partial t_{ij}} = \frac{V_i V_j}{t_{ij}} (\alpha_{ij} \cos \theta_i + \beta_{ij} \sin \theta_i) = -\frac{N_{ij}}{t_{ij}}$$

if i is non-tap side bus, or

$$= -\frac{N_{ij}}{t_{ij}} - \frac{2V_i^2 G_{ij}}{t_{ij}^2}$$

if i is tap side bus.

Similarly,

$$\frac{\partial \Delta Q_i}{\partial t_{ij}} = -\frac{L_{ij}}{t_{ij}} \qquad \text{if i is non-tap side bus, or}$$

$$= -\frac{L_{ij}}{t_{ij}} + \frac{2V_i^2 B_{ij}}{t_{ij}^2} \qquad \text{if i is tap side bus.}$$

$$\frac{\partial \Delta P_i}{\partial V_j} = N_{ij}$$

$$\frac{\partial \Delta P_i}{\partial V_i} = N_{ii}$$

$$\frac{\partial \Delta Q_i}{\partial V_j} = L_{ij}$$

$$\frac{\partial \Delta Q_i}{\partial V_i} = L_{ii}$$

$$\frac{\partial \Delta Q_i}{\partial B_j} = 0$$

$$\frac{\partial \Delta Q_i}{\partial B_i} = V_i^2$$

A3 The Terms of the Hessian Matrix

Following the same procedure as for the calculation of the terms of the gradient, let $\bar{F}(\bar{u}) = \bar{f}(\bar{x},\bar{u})$, so that

$$\begin{aligned}\overline{\Delta F} &= \overline{\nabla F}_u^T \overline{\Delta u} + \tfrac{1}{2}\, \overline{\Delta u}^T [\nabla F_{uu}]\, \overline{\Delta u} \\ &= \overline{\Delta f} = \overline{\frac{\partial f}{\partial u}}^T \overline{\Delta u} + \overline{\frac{\partial f}{\partial x}}^T \overline{\Delta x} + \tfrac{1}{2}\, \overline{\Delta u}^T \left[\frac{\partial^2 f}{\partial u^2}\right] \overline{\Delta u} + \tfrac{1}{2}\, \overline{\Delta x}^T \left[\frac{\partial^2 f}{\partial x^2}\right] \overline{\Delta x} \\ &\quad + \overline{\Delta u}^T \left[\frac{\partial^2 f}{\partial u\, \partial x}\right] \overline{\Delta x}\end{aligned}$$

Using the same expression for $\overline{\Delta x}$ in terms of $\overline{\Delta u}$ as in section A2,

$$\begin{aligned}\overline{\Delta F} &= \overline{\frac{\partial f}{\partial u}}^T \overline{\Delta u} - \overline{\frac{\partial f}{\partial x}}^T \left[\frac{\partial g}{\partial x}\right]^{-1} \left[\frac{\partial g}{\partial u}\right] \overline{\Delta u} + \tfrac{1}{2}\, \overline{\Delta u}^T \left[\frac{\partial^2 f}{\partial u^2}\right] \overline{\Delta u} \\ &\quad + \tfrac{1}{2} \left(\left[\frac{\partial g}{\partial x}\right]^{-1} \left[\frac{\partial g}{\partial u}\right] \overline{\Delta u}\right)^T \left[\frac{\partial^2 f}{\partial x^2}\right] \left[\frac{\partial g}{\partial x}\right]^{-1} \left[\frac{\partial g}{\partial u}\right] \overline{\Delta u} \\ &\quad - \overline{\Delta u}^T \left[\frac{\partial^2 f}{\partial u\, \partial x}\right] \left[\frac{\partial g}{\partial x}\right]^{-1} \left[\frac{\partial g}{\partial u}\right] \overline{\Delta u}\end{aligned}$$

The gradient term is as before, but the term $[\nabla F_{uu}]$ is

$$[\nabla F_{uu}] = \left[\frac{\partial^2 f}{\partial u^2}\right] + \left[\frac{\partial g}{\partial u}\right]^T \left[\frac{\partial g}{\partial x}\right]^{T^{-1}} \left[\frac{\partial^2 f}{\partial x^2}\right] \left[\frac{\partial g}{\partial x}\right]^{-1} \left[\frac{\partial g}{\partial u}\right]$$

$$- 2\left[\frac{\partial^2 f}{\partial u\, \partial x}\right] \left[\frac{\partial g}{\partial x}\right]^{-1} \left[\frac{\partial g}{\partial u}\right]$$

The evaluation of this matrix requires a considerably greater amount of computation than does the evaluation of the gradient.

The additional matrices needed are:

$$\nabla^2 f_{uu} = \begin{bmatrix} \dfrac{\partial^2 P_s}{\partial t\, \partial t} & \dfrac{\partial^2 P_s}{\partial t\, \partial V_c} & 0 \\ \dfrac{\partial^2 P_s}{\partial V_c\, \partial t} & \dfrac{\partial^2 P_s}{\partial V_c\, \partial V_c} & 0 \\ 0 & 0 & 2z \end{bmatrix}$$

$$\nabla^2 f_{xx} = \begin{bmatrix} \dfrac{\partial^2 P_s}{\partial \theta\, \partial \theta} & \dfrac{\partial^2 P_s}{\partial \theta\, \partial V} \\ \dfrac{\partial^2 P_s}{\partial V\, \partial \theta} & \dfrac{\partial^2 P_s}{\partial V\, \partial V} + 2w \end{bmatrix}$$

$$\nabla^2 f_{xu} = \begin{bmatrix} \dfrac{\partial^2 P_s}{\partial \theta\, \partial t} & \dfrac{\partial^2 P_s}{\partial \theta\, \partial V_c} & 0 \\ \dfrac{\partial^2 P_s}{\partial V\, \partial t} & \dfrac{\partial^2 P_s}{\partial V\, \partial V_c} & 0 \end{bmatrix}$$

for which the terms are:

$$\frac{\partial^2 P_s}{\partial t_{sj} \partial t_{sk}} = 0$$

$$\frac{\partial^2 P_s}{\partial t^2_{sj}} = -\frac{2N_{sj}}{t^2_{sj}} \qquad s = \text{non-tap side bus, or}$$

$$= -\frac{2N_{sj}}{t^2_{sj}} - \frac{6V_s^2 G_{sj}}{t^3_{sj}} \qquad s = \text{tap side bus.}$$

$$\frac{\partial^2 P_s}{\partial t_{sj} \partial V_k} = 0$$

$$\frac{\partial^2 P_s}{\partial t_{sj} \partial V_s} = \frac{N_{sj}}{V_s t_{sj}} \qquad s = \text{non-tap side bus, or}$$

$$= \frac{N_{sj}}{V_s t_{sj}} + \frac{4V_s G_{sj}}{t^2_{sj}} \qquad s = \text{tap side bus.}$$

$$\frac{\partial^2 P_s}{\partial t_{sj} \partial V_j} = \frac{N_{sj}}{V_j t_{sj}}$$

$$\frac{\partial^2 P_s}{\partial V_s \partial V_j} = -\frac{N_{sj}}{V_s V_j}$$

$$\frac{\partial^2 P_s}{\partial V_s^2} = 2\ G_{ss}$$

$$\frac{\partial^2 P_s}{\partial V_j^2} = 0$$

$$\frac{\partial^2 P_s}{\partial \theta_s \partial \theta_j} = -\ N_{sj}$$

$$\frac{\partial^2 P_s}{\partial \theta_s^2} = -\ P_s + V_s^2 G_{ss}$$

$$\frac{\partial^2 P_s}{\partial \theta_j^2} = N_{sj}$$

$$\frac{\partial^2 P_s}{\partial \theta_s \partial t_{sj}} = -\frac{L_{sj}}{t_{sj}}$$

$$\frac{\partial^2 P_s}{\partial \theta_j \partial t_{sk}} = 0$$

$$\frac{\partial^2 P_s}{\partial \theta_s \partial V_s} = -\frac{Q_s}{V_s} - V_s B_{ss}$$

$$\frac{\partial^2 P_s}{\partial \theta_s \partial V_j} = \frac{L_{sj}}{V_j}$$

$$\frac{\partial^2 P_s}{\partial V_s \partial \theta_j} = -\frac{L_{sj}}{V_s}$$

$$\frac{\partial^2 P_s}{\partial \theta_j \partial V_j} = -\frac{L_{sj}}{V_j}$$

$$\frac{\partial^2 P_s}{\partial \theta_s \partial t_{sj}} = \frac{L_{sj}}{t_{sj}}$$

A4 The Approximation to the Optimal Step-size

Let $F(\bar{u}) = f(\bar{x},\bar{u})$ as in section A2. Now expand $F(\bar{u})$ to second order:

$$F(\bar{u}) = F_o + \overline{\nabla^T F_u}\ \overline{\Delta u} + \tfrac{1}{2}\ \overline{\Delta u}^T \left[\nabla^2 F_{uu}\right] \overline{\Delta u}$$

For steepest descent

$$\overline{\Delta u} = -\frac{c\,[A]\,\overline{\nabla F_u}}{\|\,[A]\,\overline{\nabla F_u}\,\|}$$

where

$[A]$ represents the rotation and scaling of $\overline{\Delta u}$ as a result of constraint reflection (diagonal matrix).

Substituting,

$$F(\bar{u}) = F_o - \frac{\overline{\nabla^T F_u}\, c\,[A]\,\overline{\nabla F_u}}{\|\,[A]\,\overline{\nabla F_u}\,\|} + \frac{1}{2}\,\frac{c^2\,\overline{\nabla^T F_u}\,[A]^T[\nabla^2 F_{uu}][A]\,\overline{\nabla F_u}}{\|\,[A]\,\overline{\nabla F_u}\,\|^2}$$

so that

$$\frac{dF}{dc} = -\frac{\overline{\nabla^T F_u}\,[A]\,\overline{\nabla F_u}}{\|\,[A]\,\overline{\nabla F_u}\,\|} + \frac{c\,\overline{\nabla^T F_u}\,[A]^T[\nabla^2 F_{uu}][A]\,\overline{\nabla F_u}}{\|\,[A]\,\overline{\nabla F_u}\,\|^2}$$

$$\frac{d^2F}{dc^2} = \frac{\overline{\nabla^T F_u}\,[A]^T[\nabla^2 F_{uu}][A]\,\overline{\nabla F_u}}{\|[A]\,\overline{\nabla F_u}\|^2}$$

Assuming that the objective function is of order 2 or less in the direction of search, so that $\frac{d^2F}{dc^2} = G$ (constant), then

$$\frac{\frac{dF^{k+1}}{dc} - \frac{dF^k}{dc}}{c^k} = G$$

The derivative at step k + 1 in the direction of step k is

$$\frac{dF^{k+1}}{dc} = -\frac{\overline{\nabla^T F_u}^{k+1}\,[A]^k\,\overline{\nabla F_u}^k}{\|[A]^k\,\overline{\nabla F_u}^k\|}$$

so that

$$G = \frac{-\overline{\nabla^T F_u}^{k+1}[A]^k\overline{\nabla F_u}^k + \overline{\nabla^T F_u}^k[A]^k\overline{\nabla F_u}^k}{\|[A]^k\overline{\nabla F_u}^k\|\,c^k}$$

The optimal step-size will ensure that $\frac{dF^{k+2}}{dc} = 0$, and is given by

$$c^{k+1} = \frac{\overline{\nabla^T F_u}^{k+1}[A]^{k+1}\overline{\nabla F_u}^{k+1}}{G\,\|[A]^{k+1}\overline{\nabla F_u}^{k+1}\|}$$

$$= \frac{c^k \|[A]^k \overline{\nabla F_u}^k\| \, \overline{\nabla^T F_u}^{k+1} [A]^{k+1} \overline{\nabla F_u}^{k+1}}{\|[A]^{k+1} \overline{\nabla F_u}^{k+1}\| \left(-\overline{\nabla^T F_u}^{k+1} [A]^k \overline{\nabla F_u}^k + \overline{\nabla^T F_u}^k [A]^k \overline{\nabla F_u}^k\right)}$$

and

$$\overline{\Delta u}^{k+1} = \frac{-c^{k+1} [A]^{k+1} \overline{\nabla F_u}^{k+1}}{\|[A]^{k+1} \overline{\nabla F_u}^{k+1}\|}$$

$$= \frac{-c^k \|[A]^k \overline{\nabla F_u}^k\| \, \overline{\nabla^T F_u}^{k+1} [A]^{k+1} \overline{\nabla F_u}^{k+1} [A]^{k+1} \overline{\nabla F_u}^{k+1}}{\|[A]^{k+1} \overline{\nabla F_u}^{k+1}\|^2 \left(-\overline{\nabla^T F_u}^{k+1} [A]^k \overline{\nabla F_u}^k + \overline{\nabla^T F_u}^k [A]^k \overline{\nabla F_u}^k\right)}$$

Substituting $[A]^k \overline{\nabla F}_u^k = \dfrac{-\overline{\Delta u}^k \|[A]^k \overline{\nabla F_u}^k\|}{c^k}$ and $\|\overline{\Delta u}^k\|^2 = (c^k)^2$

$$\overline{\Delta u}^{k+1} = \frac{-\|\overline{\Delta u}^k\|^2 \, \overline{\nabla^T F_u}^{k+1} [A]^{k+1} \overline{\nabla F_u}^{k+1} [A]^{k+1} \overline{\nabla F_u}^{k+1}}{\|[A]^{k+1} \overline{\nabla F_u}^{k+1}\|^2 \left(\overline{\nabla^T F_u}^{k+1} \overline{\Delta u}^k - \overline{\nabla^T F}^k \overline{\Delta u}^k\right)}$$

At step k+1, $[A]^{k+1}$ is still unknown, and so we assume

$$[A]^{k+1} = [1]$$

giving

$$\overline{\Delta u}^{k+1} = -\frac{\|\overline{\Delta u}^k\|^2 \, \overline{\nabla F_u}^{k+1}}{\left(\overline{\nabla^T F_u}^{k+1} \overline{\Delta u}^k - \overline{\nabla^T F_u}^k \overline{\Delta u}^k\right)}$$

For the first step, there is no previous information, so there is no information for $\dfrac{d^2 F}{d c^2}$ in

$$F^2 = F^1 - \|\overline{\nabla F}_u^1\| \, c^1 + \tfrac{1}{2}(c^1)^2 \frac{d^2F}{dc^2} .$$

In order to solve this for c^1 without knowing $\frac{d^2F}{dc^2}$, we assume that an optimal step will reduce the objective function by an arbitrary amount. Experience indicates that 2% is reasonable, so

$$F^2 - F^1 = -0.02 \; F^1$$

and

$$-0.02 \; F^1 = -\|\overline{\nabla F}_u^1\| c^1 + \tfrac{1}{2}(c^1)^2 \frac{d^2F}{dc^2} .$$

Since the step is optimal,

$$\frac{dF^2}{dc} = -\|\overline{\nabla F}_u^1\| + c^1 \frac{d^2F}{dc^2} = 0$$

which implies that

$$\frac{d^2F}{dc^2} = \frac{\|\overline{\nabla F}_u^1\|}{c^1}$$

and

$$-0.02 \; F^1 = -\|\overline{\nabla F}_u^1\| c^1 + \tfrac{1}{2} \, c^1 \|\overline{\nabla F}_u^1\| = -\tfrac{1}{2} \, c^1 \|\overline{\nabla F}_u^1\|$$

Thus

$$c^1 = \frac{2 \cdot 0.02 F^1}{\|\overline{\nabla F_u^1}\|} = \frac{0.04 F^1}{\|\overline{\nabla F_u^1}\|}$$

and

$$\overline{\Delta u}^1 = \frac{-0.04 F^1}{\|\overline{\nabla F_u^1}\|^2} \overline{\nabla F_u^1}.$$

A5 : Interpolated Step-size

If the sign of any partial derivative changes, the corresponding variable can be interpolated, as the minimum in that direction has been passed.

Assuming

$$\frac{\partial^2 F}{\partial u_i^2} \approx \frac{\frac{\partial F^{k+1}}{\partial u_i} - \frac{\partial F^k}{\partial u_i}}{\Delta u_i^k} = R_i \text{ (constant)}$$

then

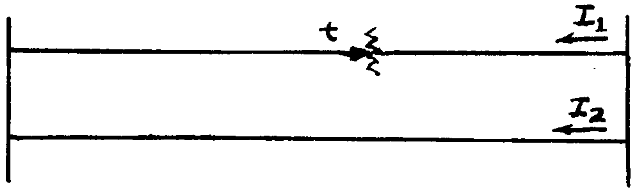
$$\Delta u_i^{k+1} = \frac{\frac{\partial F^{k+2}}{\partial u_i} - \frac{\partial F^{k+1}}{\partial u_i}}{R_i}$$

For an optimal step, $\frac{\partial F^{k+2}}{\partial u_i} = 0$ and

$$u_i^{k+1} = \frac{-\frac{\partial F^{k+1}}{\partial u_i}}{R_i} = \frac{-\frac{\partial F^{k+1}}{\partial u_i} \Delta u_i^k}{\frac{\partial F^{k+1}}{\partial u_i} - \frac{\partial F^k}{\partial u_i}}$$

A6 Proof that the Objective Function $\Sigma \|I\|^2 |X|$ for all System Branches Equalizes the Product $\|I\| |X|$

Consider the following section of a power system:



The total current $I_T = I_1 + I_2$ is assumed constant, while the two component currents I_1 and I_2 may be altered by adjustment of the transformer tap t.

The reactive power loss in these two branches is given by

$$\mathcal{L}_q = I_1^2 X_1 + I_2^2 X_2 = I_1^2 X_1 + (I_T - I_1)^2 X_2 .$$

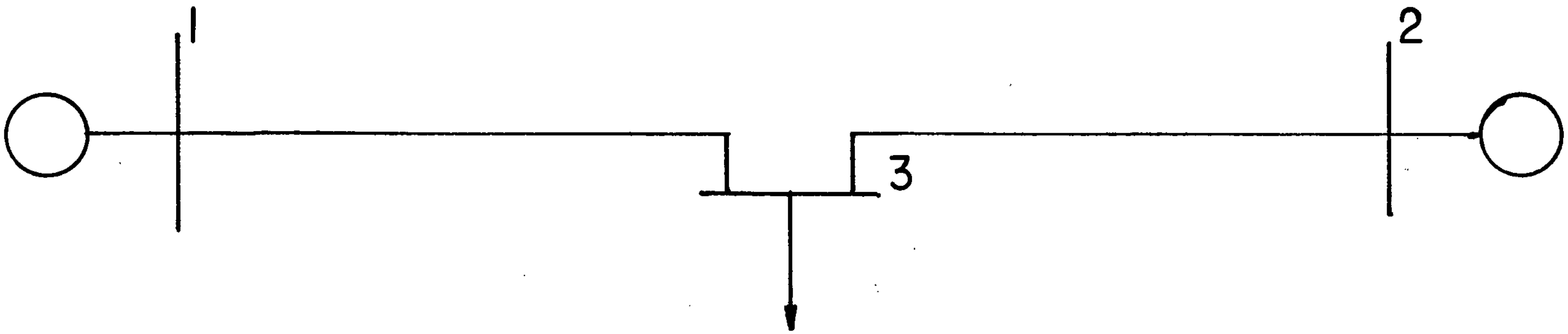
The value of I_1 for which $\mathcal{L}_q$ is a minimum can be determined by setting the first derivative of $\mathcal{L}_q$ with respect to I_1 equal to zero.

$$\frac{d\mathcal{L}_q}{dI_1} = 2I_1X_1 - 2(I_T - I_1)X_2 = 0$$

which implies that

$$I_1X_1 = I_2X_2.$$

Q.E.D.



$P_2 = 5.14$ pu
$P_3 = -2.076$ pu
$Q_3 = -0.535$ pu

$Y_{11} = 4.989 - j\,29.72$ pu
$Y_{13} = -4.989 + j\,29.84$ pu
$Y_{22} = 6.063 - j\,28.4$ pu
$Y_{23} = -6.063 + j\,28.67$ pu
$Y_{33} = 11.05 - j\,58.12$ pu

on 100 MVA & 230 kV

Figure 1. Three-bus example system.

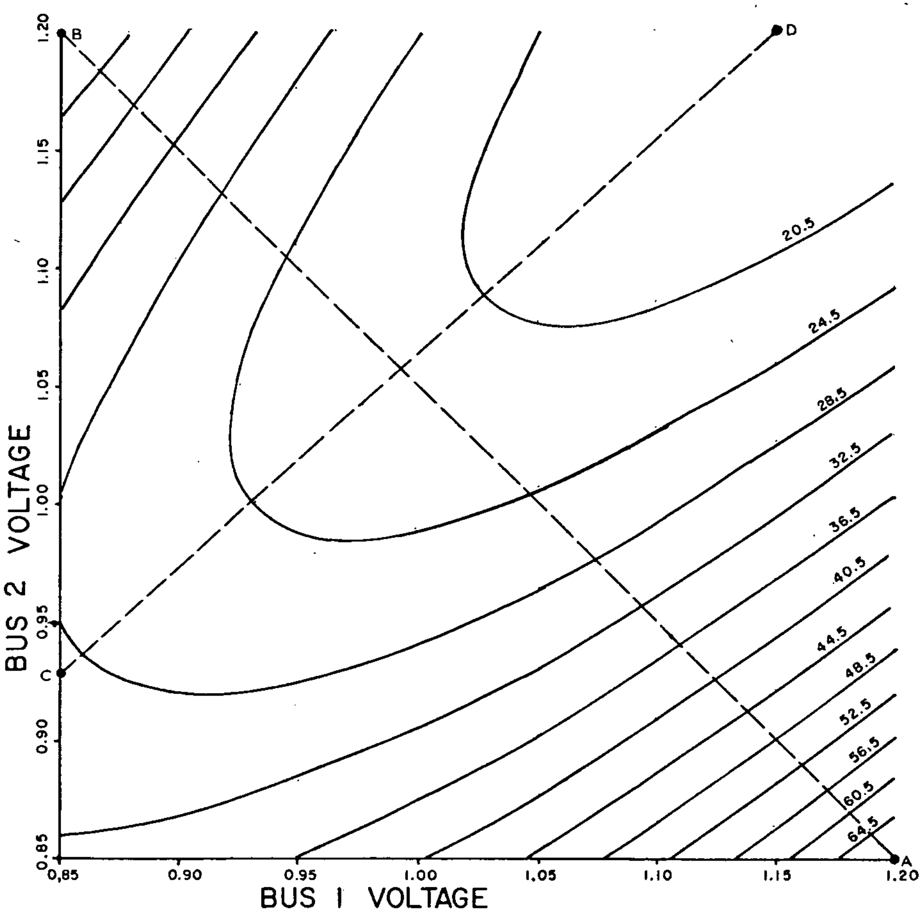


Figure 2. Contours of constant loss for system of figure 1. Voltages are per unit based on 230 kV, and contour values are MW.

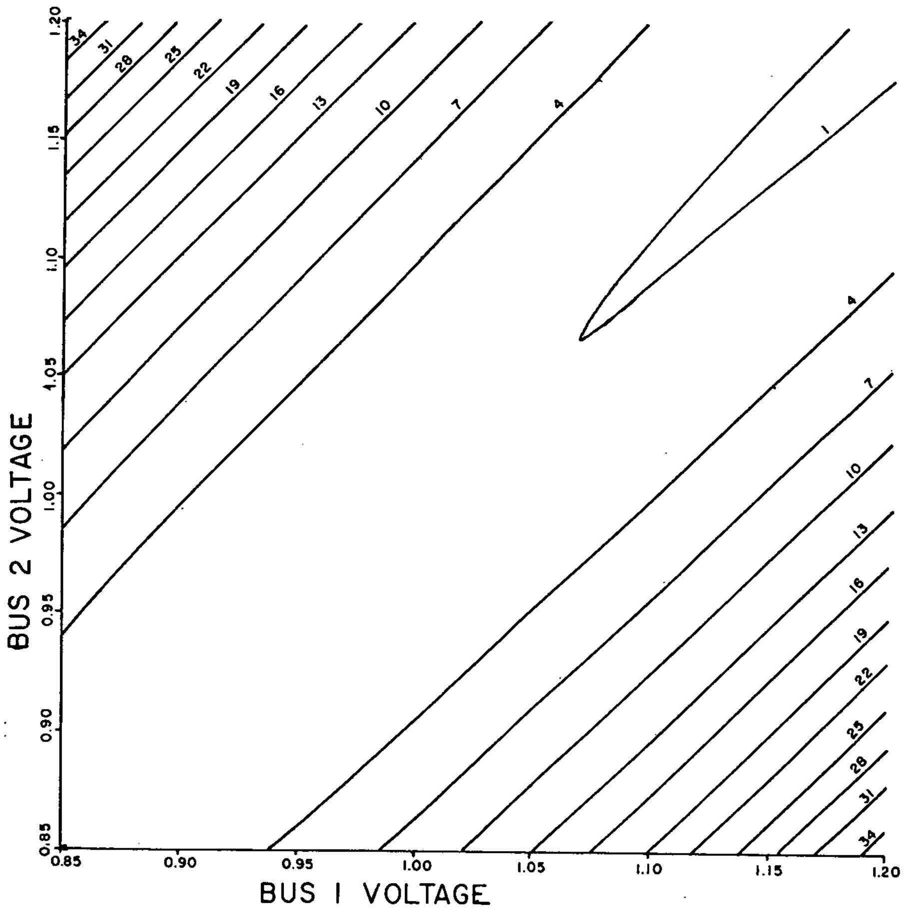
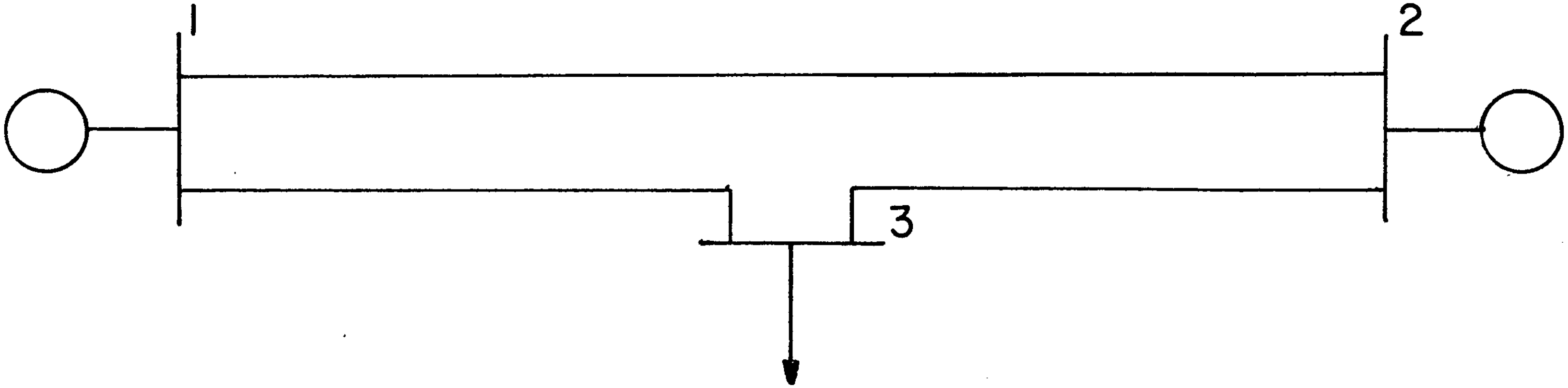


Figure 3. Contours of constant loss for system of figure 1, except that $P_2 = 1.0$ pu, so that both generators provide approximately half of the bus 3 real power each.



$P_2 = 1.0$ pu
$P_3 = -2.076$ pu
$Q_3 = -0.535$ pu

$Y_{11} = 6.985 - j\,41.37$ pu
$Y_{12} = -1.996 + j\,11.94$ pu
$Y_{13} = -4.989 + j\,29.84$ pu
$Y_{22} = 8.059 - j\,40.05$ pu
$Y_{23} = -6.063 + j\,28.67$ pu
$Y_{33} = 11.05 - j\,58.12$ pu

on 100 MVA & 230 kV

Figure 4. Three-bus example system of figure 1, with additional hypothetical branch between busses 1 and 2, and $P_2 = 1.0$ pu.

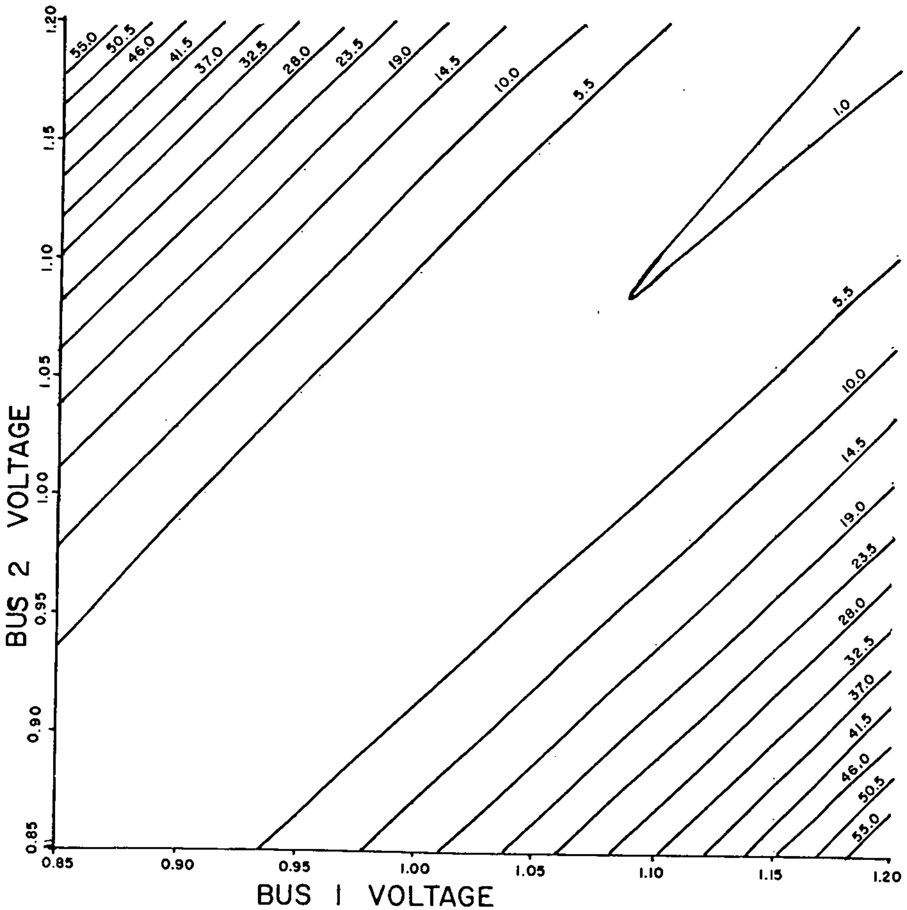


Figure 5. Contours of constant loss for system of figure 4.

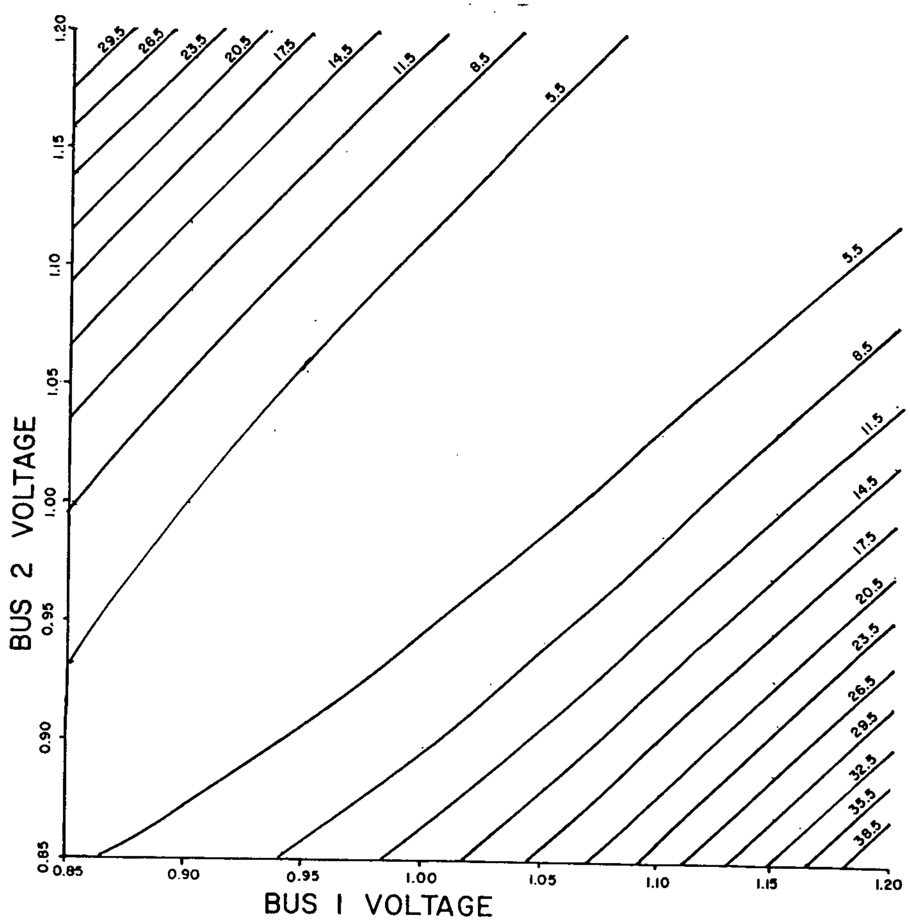


Figure 6. Contours of constant loss for system of figure 1, but with only 40% of the load at bus 3 (P_2 = 2.056 pu, S_3 = -0.83 - j 0.214 pu).

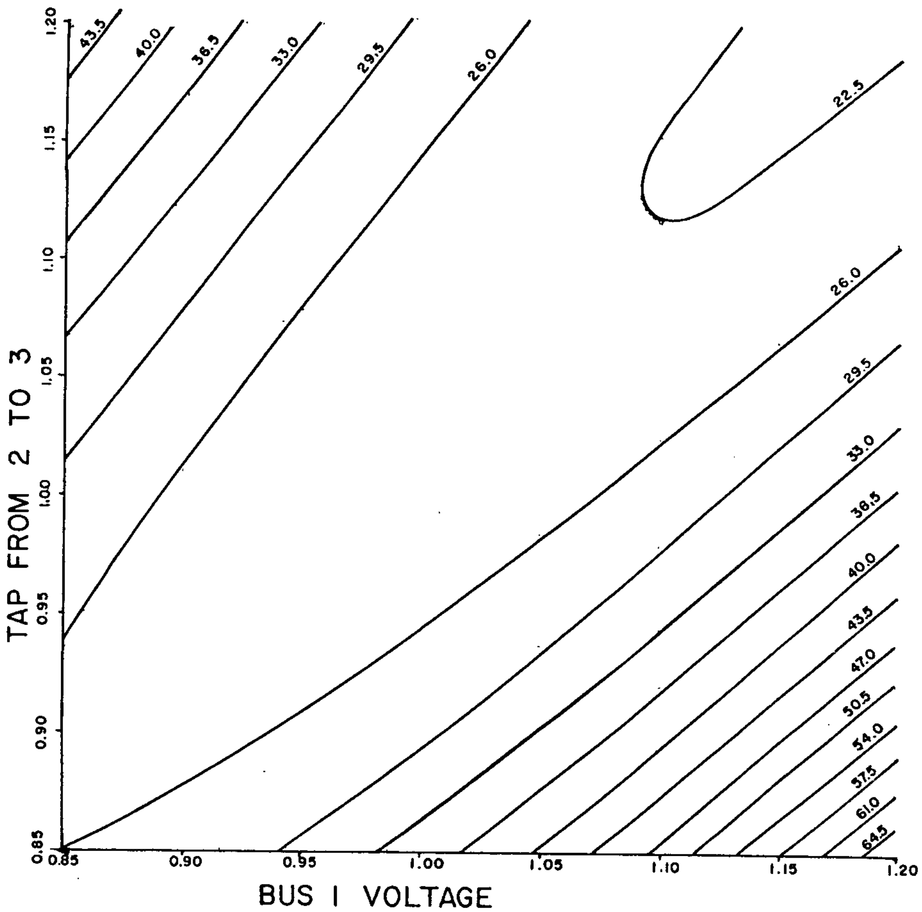


Figure 7. Contours of constant loss versus voltage at bus 1, and tap setting of hypothetical, zero impedance transformer inserted at the bus 3 end of the branch between busses 1 and 3. The tap is on the bus 3 side. Otherwise the system is identical to that of figure 1.

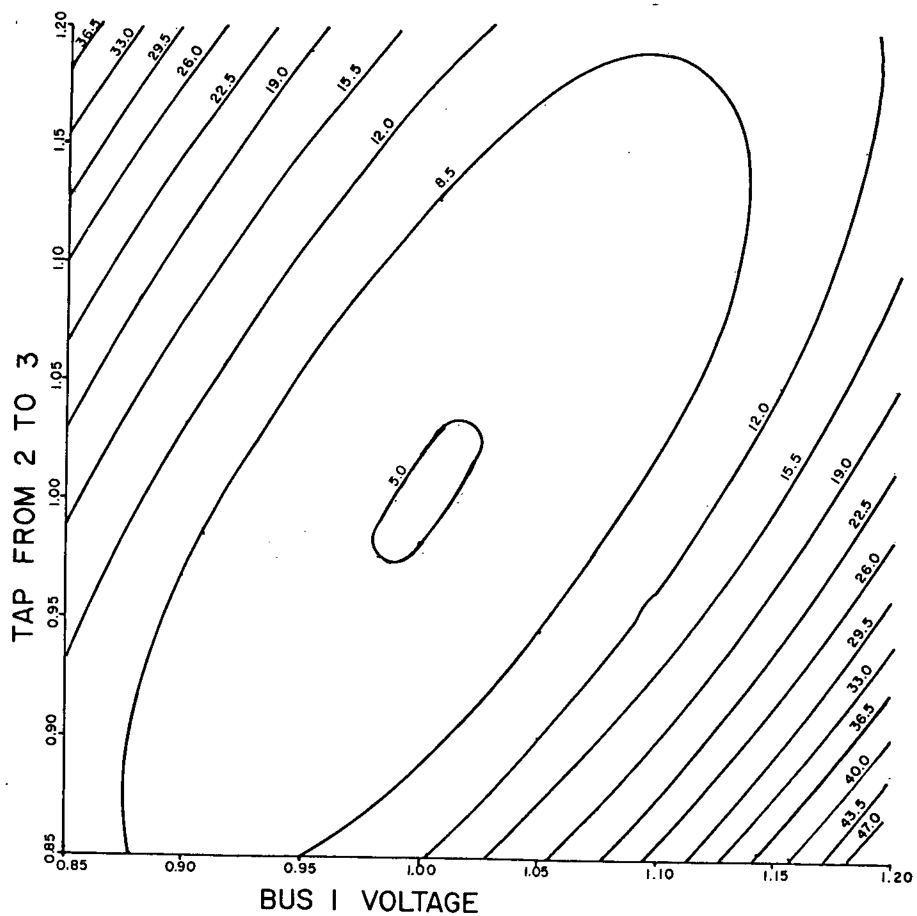


Figure 8. Contours of constant loss versus voltage at bus 1, and tap setting of hypothetical transformer identical to that for figure 7. Otherwise the system is identical to that of figure 4.

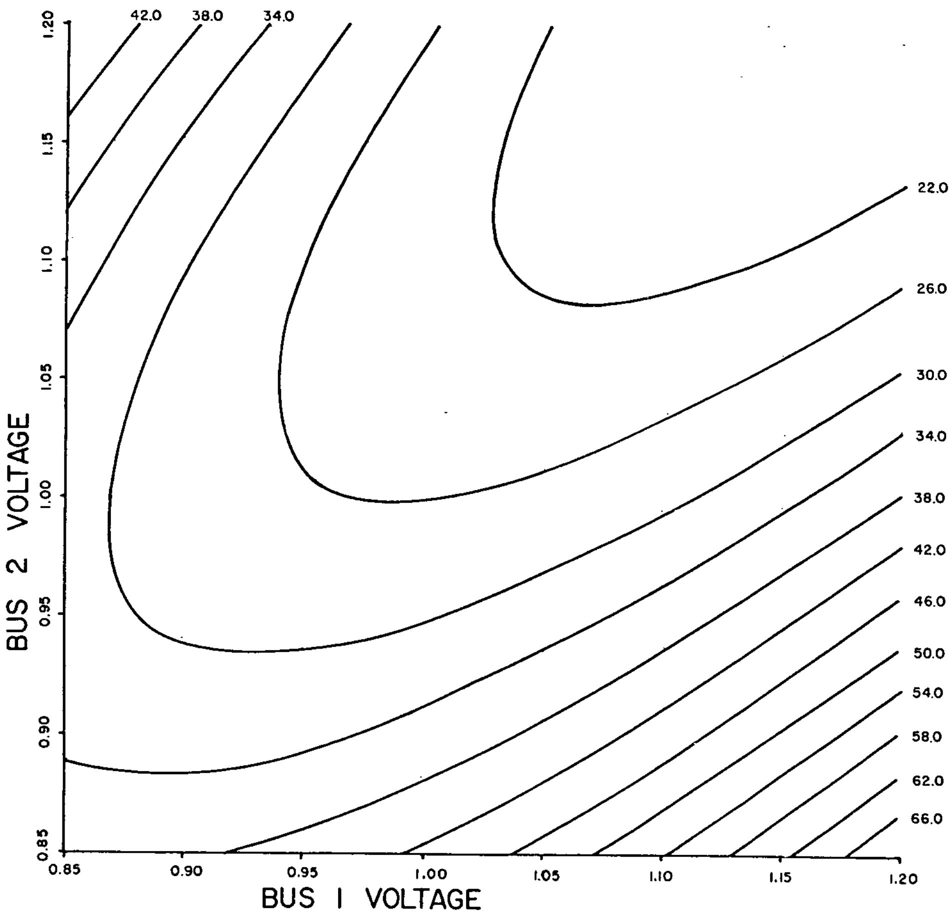


Figure 9. Contours of constant loss versus voltage. System is that of figure 1, except that the power factor of the load at bus 3 is only 80% (so that S_3 = -1.715 - j 1.286 pu).

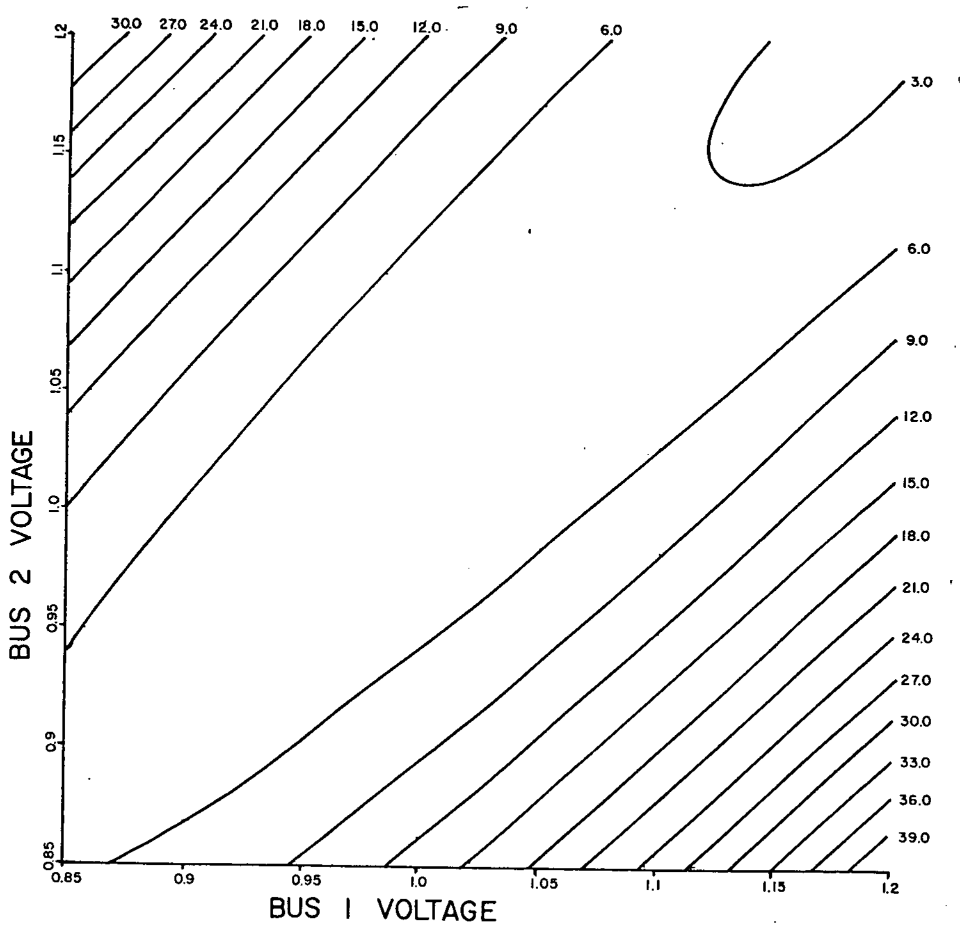


Figure 10. Contours of constant loss versus voltage. System is that of figure 1, except that the load at bus 3 has been reduced to only 40% (as for figure 6), and the power factor reduced to 80%. Thus S_3 = -0.686 - j 0.515 pu.

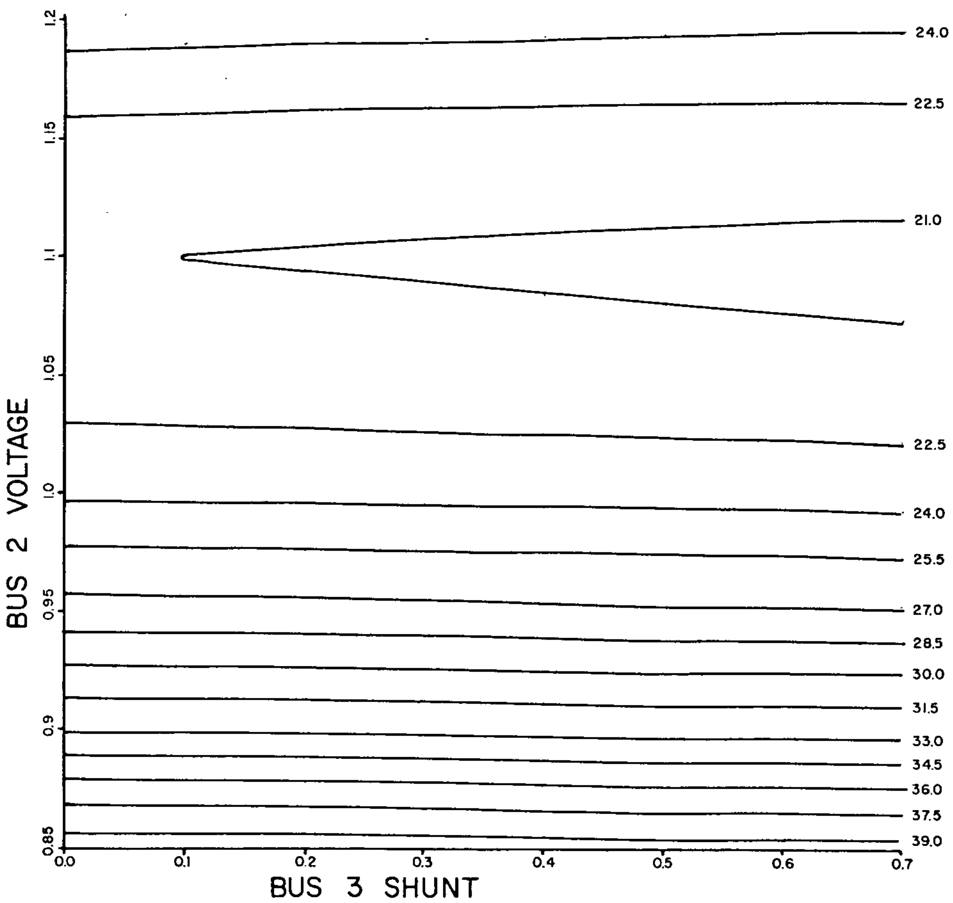


Figure 11. Contours of constant loss versus voltage at bus 2, and the value of reactive shunt at bus 3. Other than the bus 3 shunt, the system is that of figure 1.

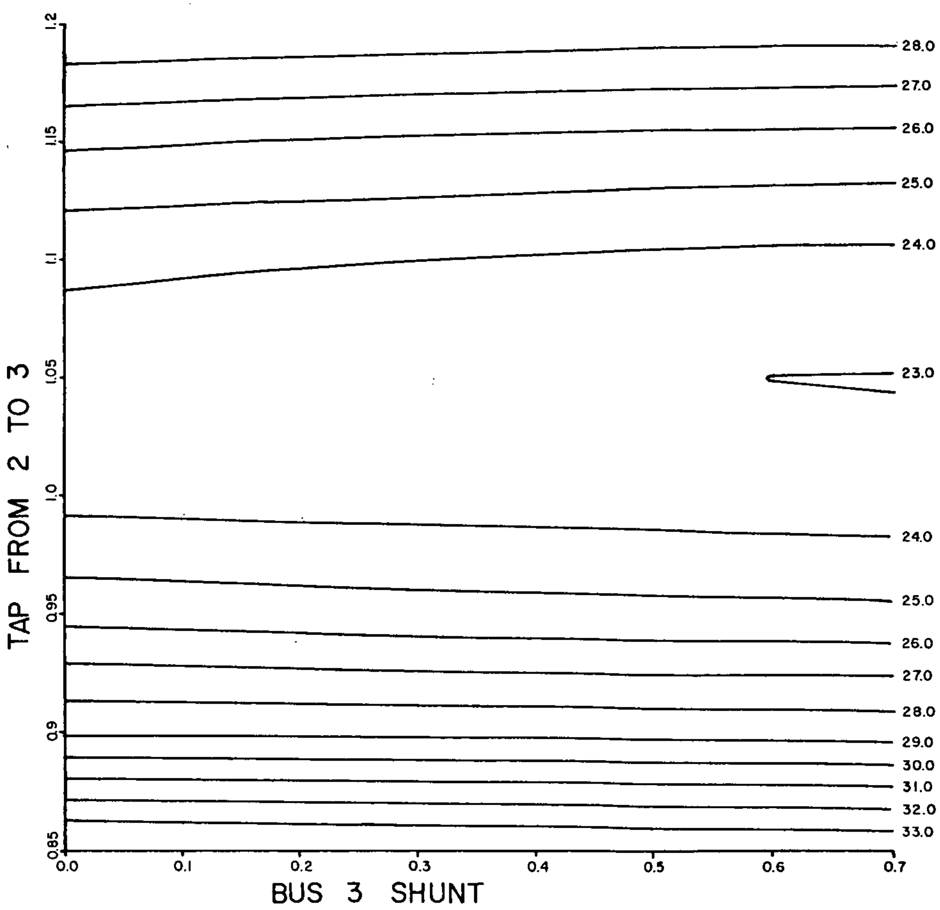


Figure 12. Contours of constant loss versus the tap on the hypothetical transformer (as for figure 7), and bus 3 reactive shunt (as for figure 11). Otherwise the system is that of figure 1.

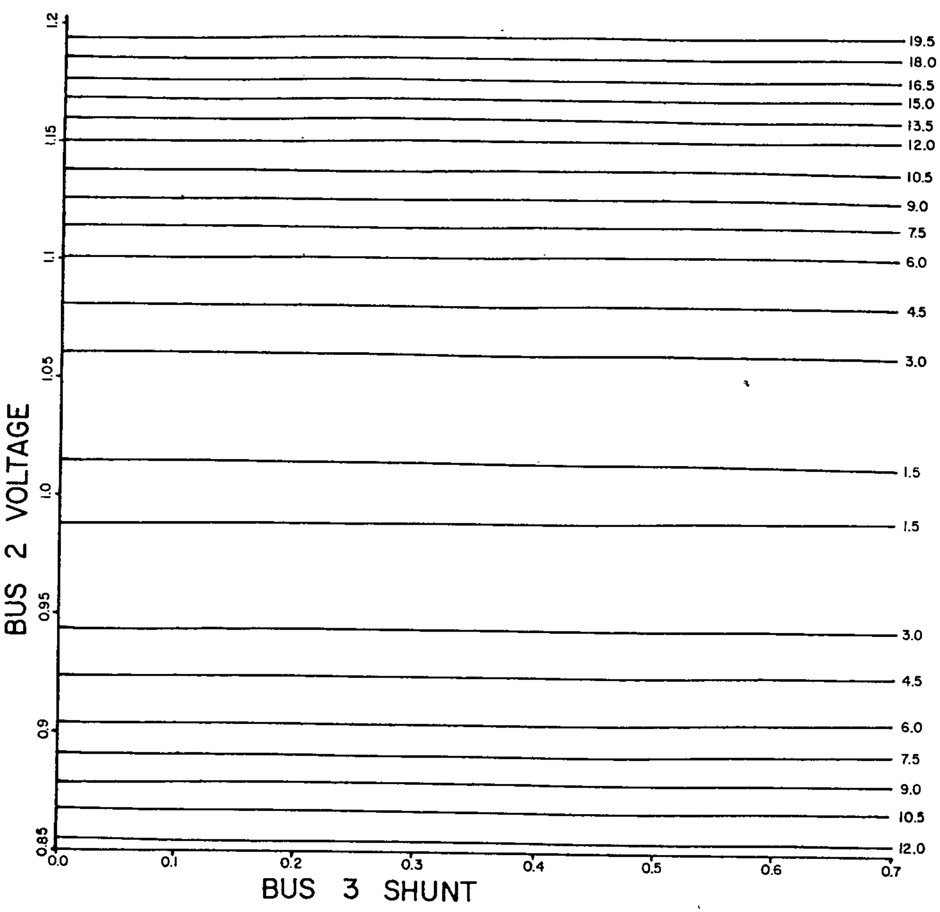


Figure 13. Contours of constant loss versus voltage at bus 2, and the value of reactive shunt at bus 3. Other than the bus 3 shunt, the system is that of figure 4.

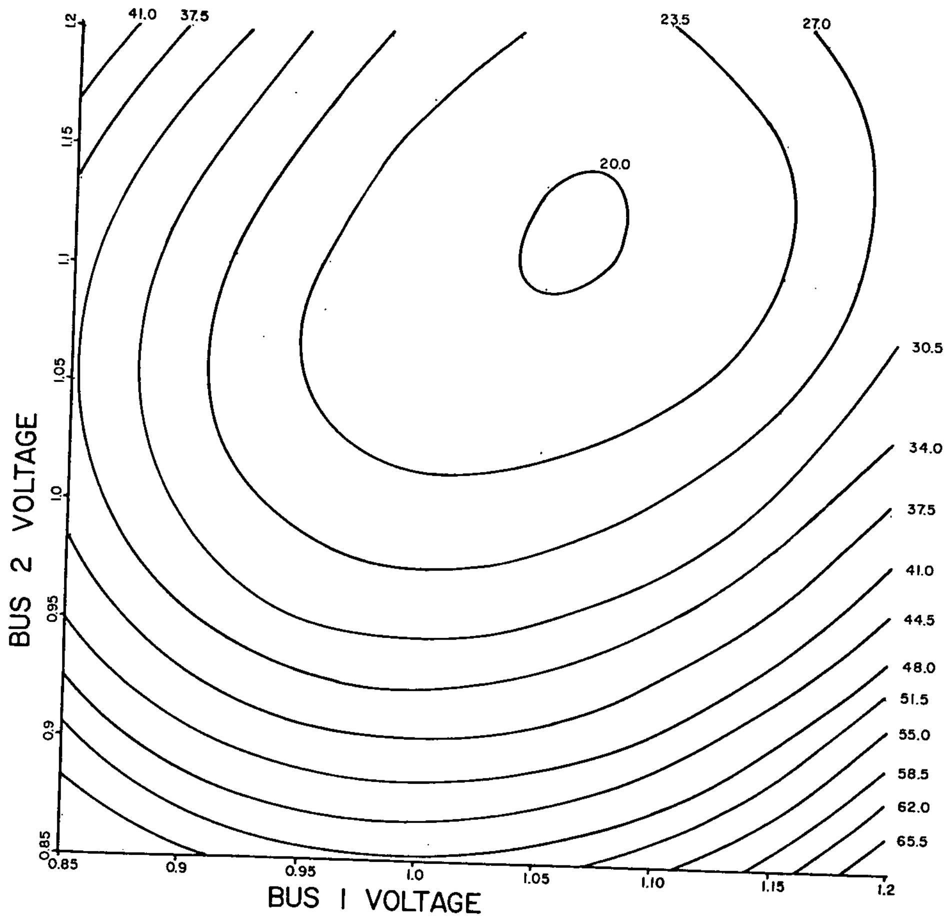


Figure 14. Contours of figure 2 augmented with a voltage penalty term for bus 3 voltage. The penalty factor is 7.5, and the maximum and minimum unpenalized voltages are 1.05 and 1.00 pu, respectively.

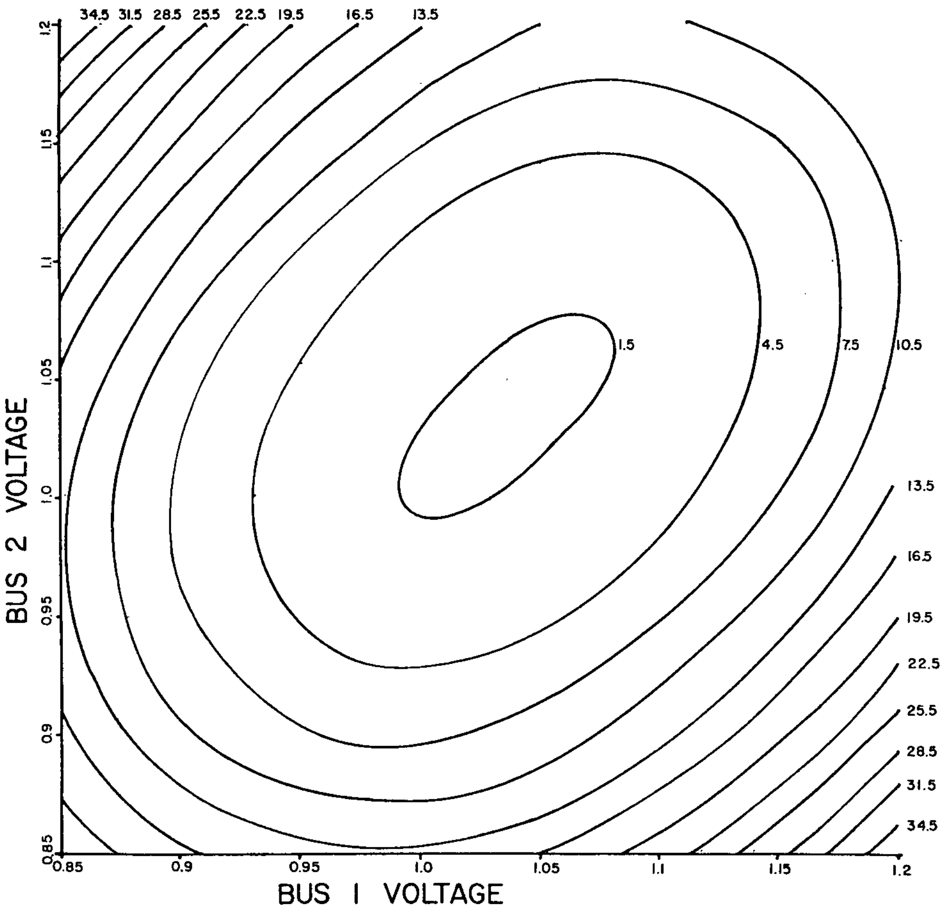


Figure 15. Contours of figure 3 augmented with a voltage penalty term as for figure 14.

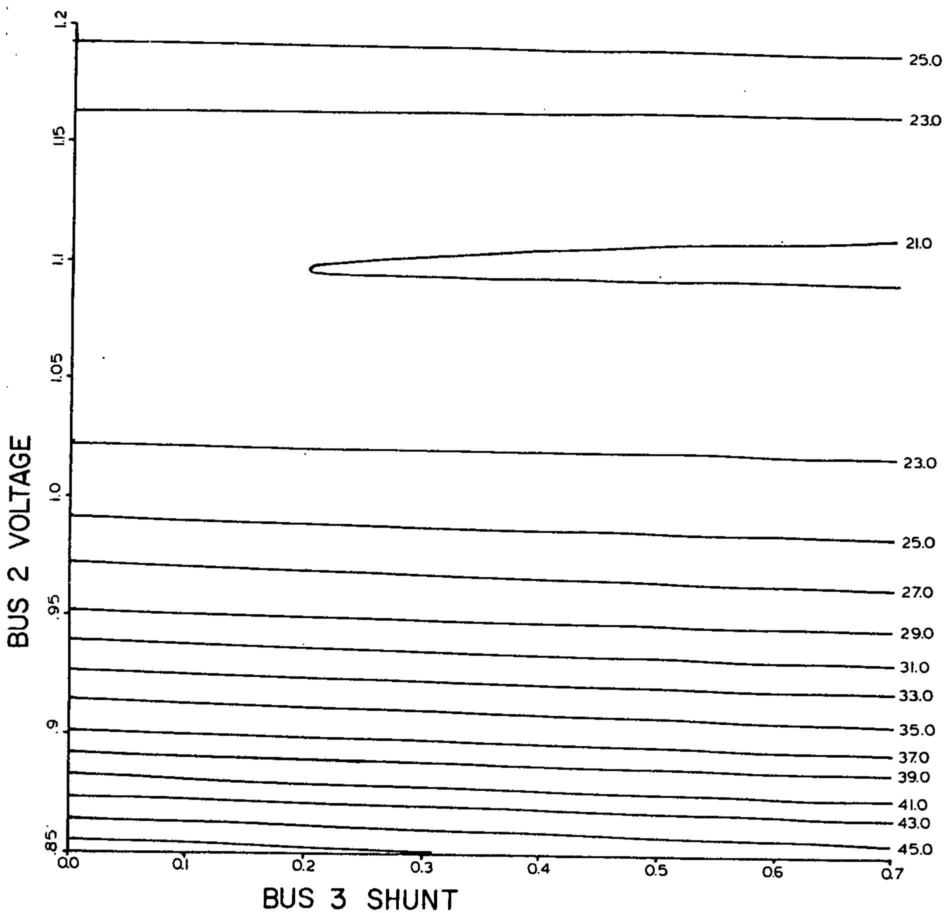


Figure 16. Contours of figure 11 augmented with a voltage penalty term as for figure 14.

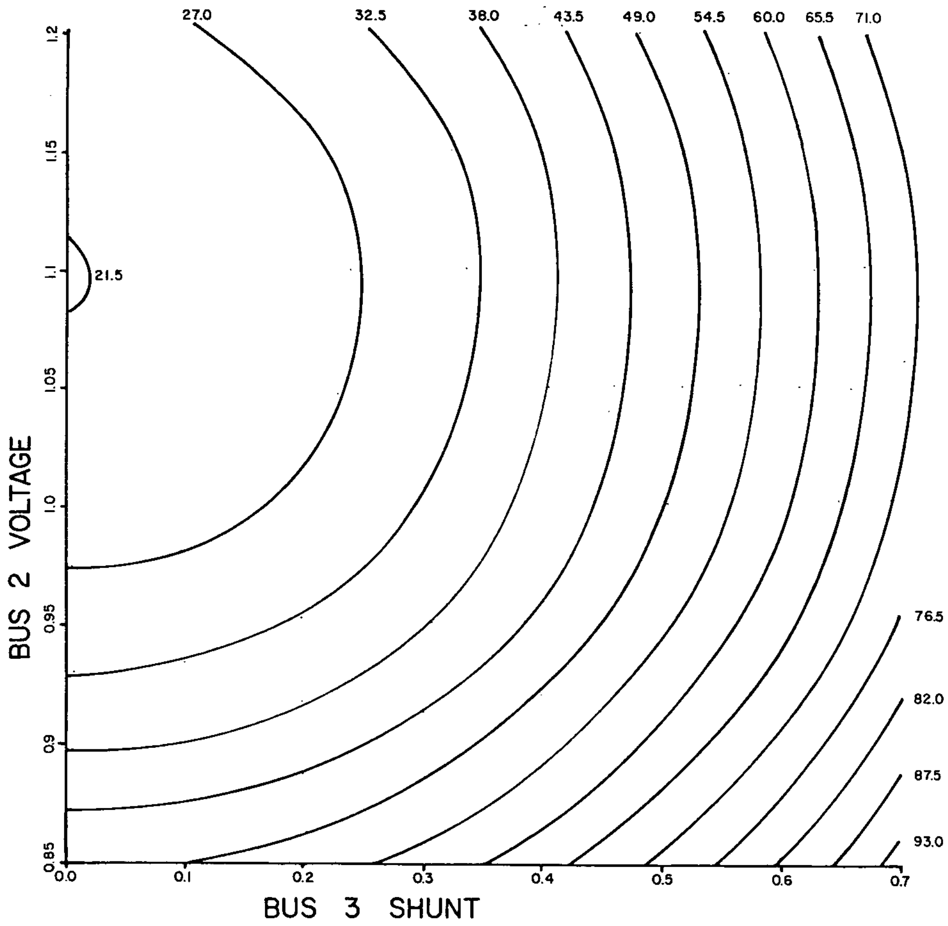


Figure 17. Contours of figure 16 augmented with a penalty term for the shunt reactive power injected at bus 3. Any amount of reactive injection is penalized, with the penalty factor being 1.0.

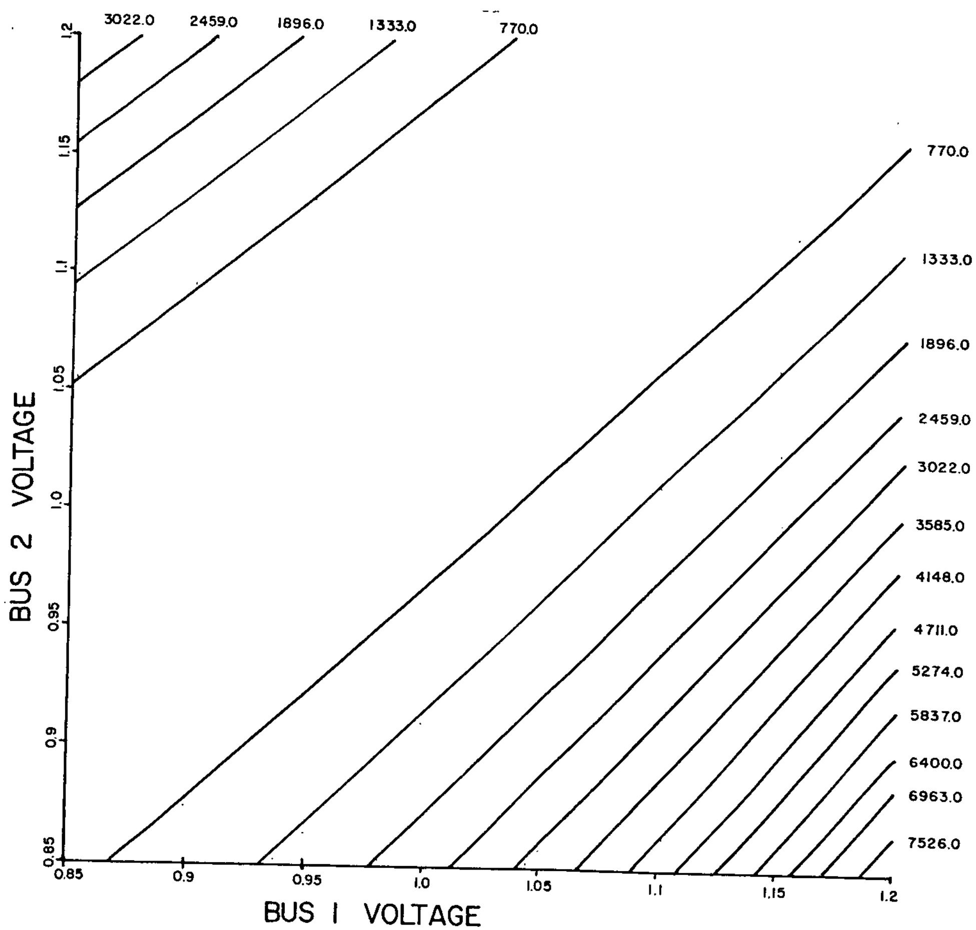


Figure 18. Contours of figure 14 augmented with a penalty term for the reactive power produced or absorbed by generators 1 and 2. The generators are allowed to produce (or absorb) reactive power to a power factor of 0.95, excess reactive power being penalized with a penalty factor of 1.0.

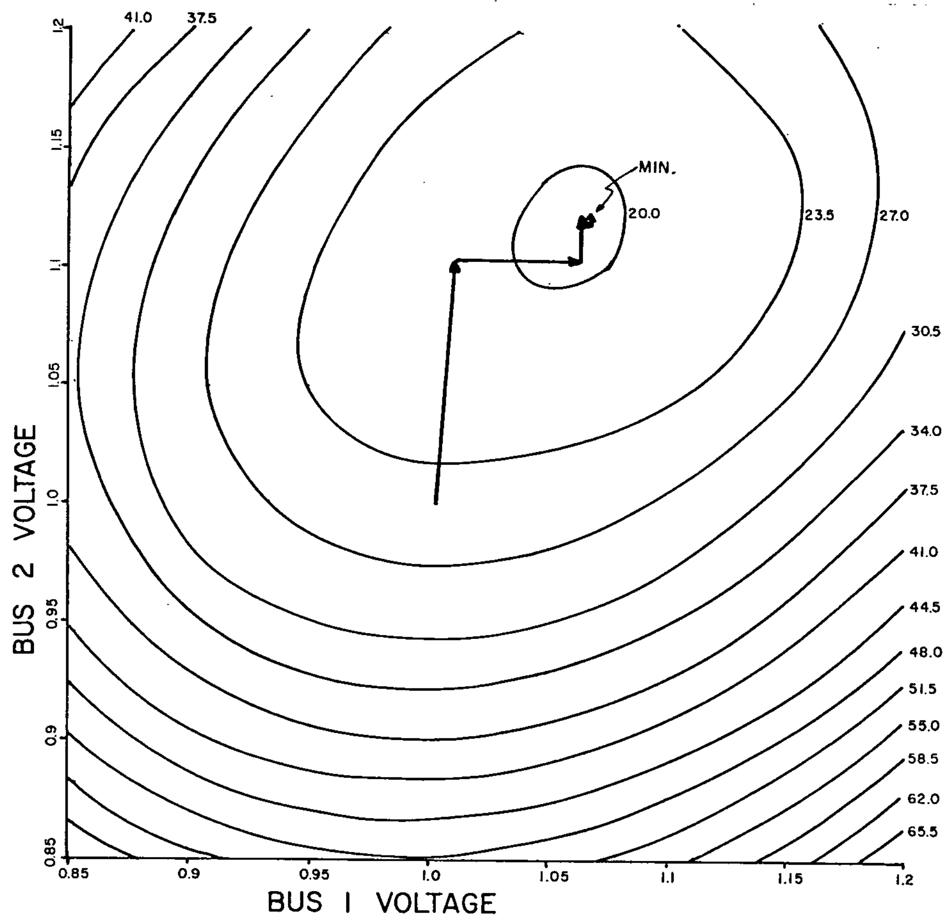


Figure 19. Contours of figure 14, on which has been plotted the progress of the programmed optimisation method.

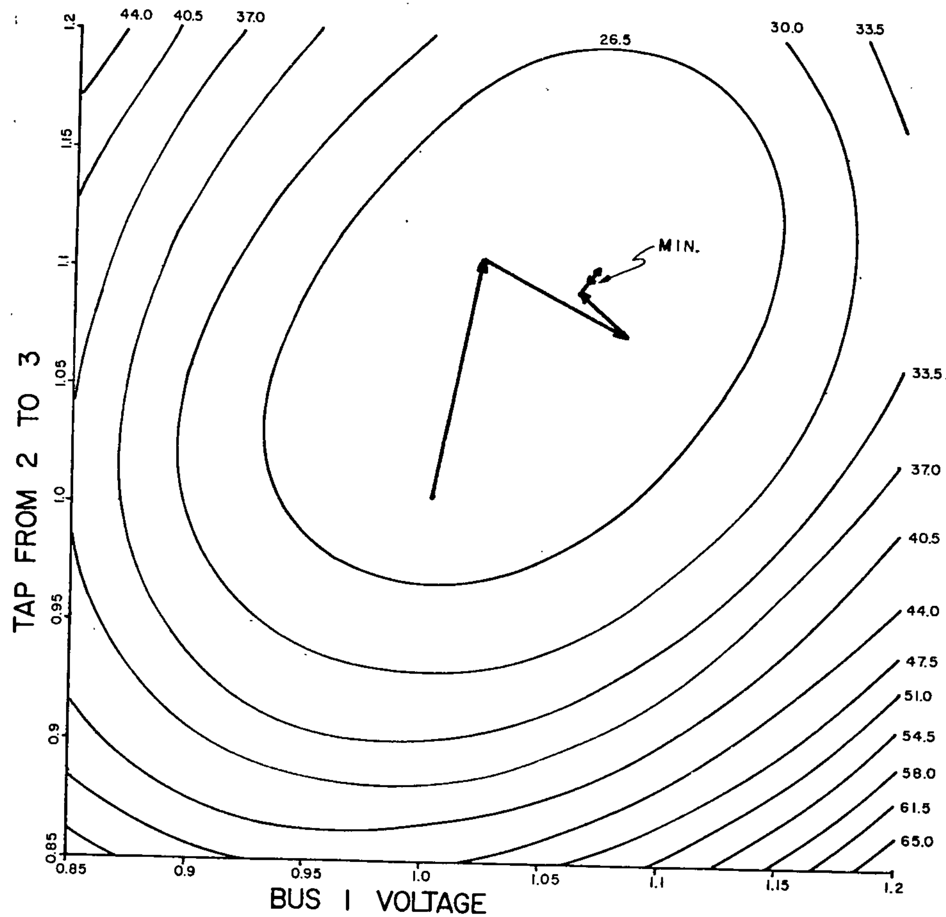


Figure 20. Contours of figure 7 augmented with a voltage penalty term as for figure 14, on which has been plotted the progress of the programmed optimisation method.

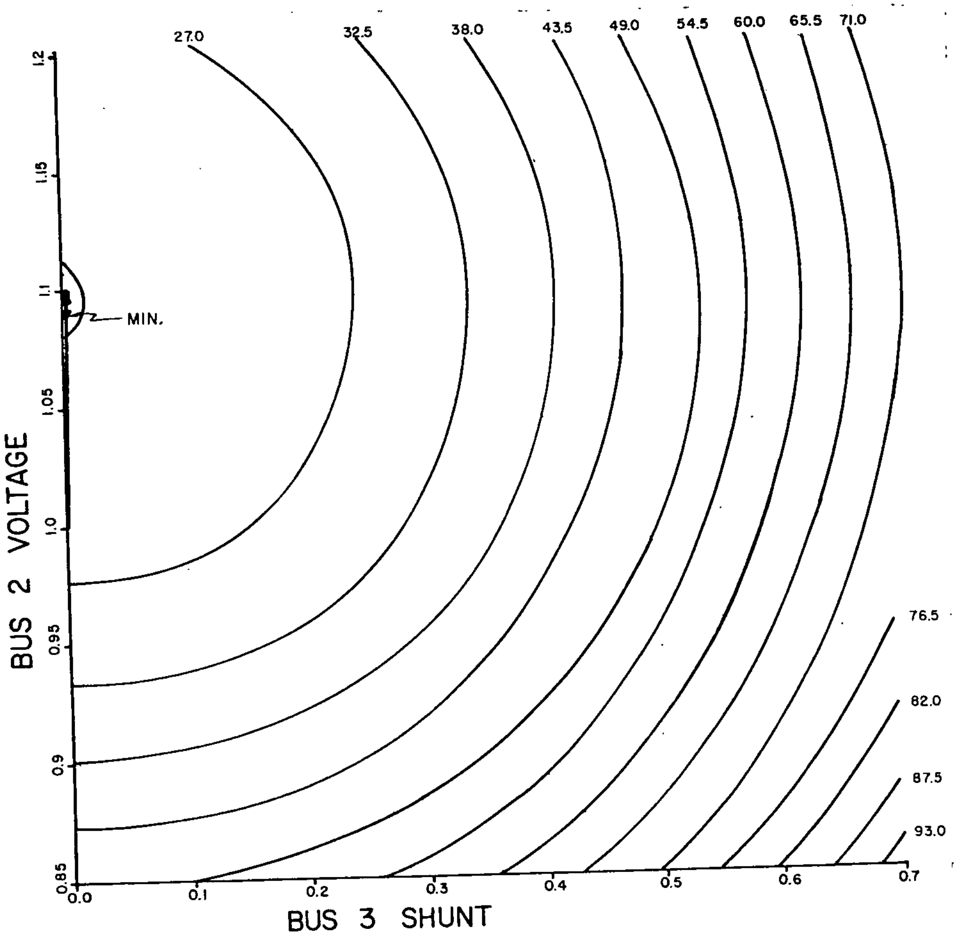


Figure 21. Contours of figure 17, on which has been plotted the progress of the programmed optimisation method.