Channel Aware Strategies in Wireless Communications

by

Maged Elkashlan

B.Sc., Arab Academy for Science and Technology, 1997
M.Sc., Arab Academy for Science and Technology, 2000

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

in

THE FACULTY OF GRADUATE STUDIES

(Electrical and Computer Engineering)

THE UNIVERSITY OF BRITISH COLUMBIA

August 2006

© Maged Elkashlan, 2006
Abstract

Channel-aware resource allocation is widely considered to be crucial for realizing high data rates in wireless networks. This thesis considers the resource allocation problem in a multiuser wireless network using link adaptation. Link Adaptation (LA), which loosely refers to changing transmission parameters over a link, such as modulation, coding rate, power, etc., in response to changing channel conditions is considered to be a powerful means of achieving higher efficiency or throughput in wireless networks. The adaptation of the transmission parameters is performed according to the predicted future quality of the channel, also referred to as the channel state (CS).

The objectives of this thesis are threefold: devise new methods for the improvement of multicarrier code division multiple access (MC-CDMA) as an effective signaling scheme in correlated fading channels, the development of new channel aware algorithms with adaptive subchannel allocation, and the investigation of the statistics of general order selection in correlated Nakagami fading channels.

A novel adaptive subcarrier allocation algorithm is developed for MC-CDMA to improve the overall bit error rate (BER) performance. The proposed method is suitable for use in correlated fading channels. This algorithm assigns users to subcarrier groups that provide favorable fading characteristics, while simultaneously reducing the amount
of interference caused to other users. This method examines the effect of equalizing the interference in each subcarrier group while maintaining a reduced correlation among the subcarrier fading processes for a group. Consequently, subcarriers are separated into non-contiguous groups to maximize frequency diversity and minimize multiple access interference (MAI).

Previously proposed adaptive subcarrier allocation algorithms, are considered to be greedy algorithms. They are greedy in the sense that they consider every reservation request individually, and make the choice that looks best at the moment. Adaptive allocation using the simple greedy method gives an optimal solution for a single user system. Adaptive bit allocation using the greedy method cannot give an optimal solution for multiuser cases. It is possible that the sub-channels with the largest channel gain for one user are also the largest for another user. We present in this thesis a new class of dynamic resource allocation schemes that are based on all the users’ subband gains. Due to the time-varying nature of the wireless channel, dynamic resource allocation makes full use of multiuser diversity to achieve higher performance. In this thesis, we formulate the multiuser subcarrier allocation problem and propose an iterative algorithm to perform the subcarrier allocation. This method involves the ordering of subband gains. Thus, the analytical foundation is the theory of order statistics. The users are recursively assigned to that subcarrier which provides the highest possible signal-to-noise ratio (SNR). While ensuring that no more than one user is assigned to the same subband. Our objective is to find low-complexity schemes which can improve system capacity and throughput, and simultaneously minimizing the MAI.
Finally, the *cumulative distribution function* (cdf) (and hence outage probability) of the $r$-th order branch SNR in correlated Nakagami–$m$ fading is studied. The accuracy of a simple exchangeable approximation to reduce the computational load is examined. The effects of correlation on the error performance of channel aware systems that involve the ordering of channel gains in a correlated Nakagami fading environment is investigated. A useful analytical formula for the BER of the $r$-th order statistic of a set of arbitrarily correlated and not necessarily exchangeable diversity branch gains is described and shown to be applicable in the performance analysis of various diversity systems.
Contents

Abstract .................................................. ii

Contents .................................................. v

List of Figures ............................................ ix

List of Abbreviations ....................................... xii

Acknowledgements .......................................... xvi

Dedication .................................................. xviii

1 Introduction .............................................. 1

1.1 Mobile Communications Systems: Past, Present, and Future 1

1.2 From Second- to Third-Generation Multiple Access Schemes 3

1.3 From Third- to Fourth-Generation Multiple Access Schemes 5

1.4 Motivation ............................................. 8

1.5 Link Adaptation Fundamentals .......................... 10

1.6 Subchannel and Power Allocation ........................ 12

1.7 This Thesis ........................................... 17
<table>
<thead>
<tr>
<th>Contents</th>
<th>vi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.7.1 Objectives</td>
<td>17</td>
</tr>
<tr>
<td>1.7.2 Contributions</td>
<td>17</td>
</tr>
<tr>
<td>1.7.3 Road Map</td>
<td>20</td>
</tr>
<tr>
<td>2 Matched MC-CDMA in Slowly Fading Dispersive Channels</td>
<td>22</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>22</td>
</tr>
<tr>
<td>2.2 System Model</td>
<td>23</td>
</tr>
<tr>
<td>2.3 Numerical Results</td>
<td>25</td>
</tr>
<tr>
<td>3 Frequency-Hopping Multicarrier CDMA in Rayleigh Fading</td>
<td>31</td>
</tr>
<tr>
<td>3.1 Introduction</td>
<td>31</td>
</tr>
<tr>
<td>3.2 FH-MC-CDMA System Model</td>
<td>33</td>
</tr>
<tr>
<td>3.3 Numerical Results and Discussions</td>
<td>38</td>
</tr>
<tr>
<td>4 Frequency-Hopping Multicarrier CDMA in Correlated Rayleigh Fading</td>
<td>47</td>
</tr>
<tr>
<td>4.1 Introduction</td>
<td>47</td>
</tr>
<tr>
<td>4.2 System Model</td>
<td>48</td>
</tr>
<tr>
<td>4.2.1 FH-MC-CDMA</td>
<td>48</td>
</tr>
<tr>
<td>4.2.2 Channel Model</td>
<td>50</td>
</tr>
<tr>
<td>4.3 BER Analysis of FH-MC-CDMA</td>
<td>51</td>
</tr>
<tr>
<td>4.3.1 Uniform FH-MC-CDMA</td>
<td>52</td>
</tr>
<tr>
<td>4.3.2 Random FH-MC-CDMA</td>
<td>53</td>
</tr>
<tr>
<td>4.3.3 Optimum FH-MC-CDMA</td>
<td>53</td>
</tr>
<tr>
<td>4.4 Numerical Results</td>
<td>54</td>
</tr>
</tbody>
</table>
## Contents

5 A Channel Aware Frequency Hopping Multiple Access Scheme

- 5.1 Introduction ........................................ 63
- 5.2 System Model ........................................ 64
- 5.3 CAFH Scheme ....................................... 65
- 5.4 BER analysis ........................................ 66
  - 5.4.1 BER analysis of CAFH for $r=1$ ............... 66
  - 5.4.2 Numerical Results of CAFH for $r=1$ .......... 70
  - 5.4.3 BER Analysis of CAFH for $r > 1$ ............ 72
  - 5.4.4 Numerical Results of CAFH for $r > 1$ ....... 74

6 Order Statistics for Correlated Fading Channels ............... 88

- 6.1 Introduction ........................................ 88
- 6.2 Preliminaries ....................................... 89
- 6.3 $\text{cdf of } \gamma_{r:n}$ ............................. 91
  - 6.3.1 Correlated Fading ............................. 91
  - 6.3.2 Independent Fading ............................ 93
- 6.4 Numerical results ................................... 93
  - 6.4.1 Constant Correlation Model .................... 94
  - 6.4.2 Linear Model .................................. 96

7 Error Analysis of Order Selection in Correlated Fading Channels ... 102

- 7.1 Introduction ........................................ 102
- 7.2 Channel and System Model .......................... 103
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.3</td>
<td>Error Performance for Order Selection BPSK</td>
<td>104</td>
</tr>
<tr>
<td>7.4</td>
<td>Numerical Results and Discussion</td>
<td>108</td>
</tr>
<tr>
<td>7.4.1</td>
<td>BER evaluation for Order Selection BPSK</td>
<td>109</td>
</tr>
<tr>
<td>7.4.2</td>
<td>Application: BER evaluation for CAFH</td>
<td>110</td>
</tr>
<tr>
<td>8</td>
<td>Conclusions and Future Research</td>
<td>118</td>
</tr>
<tr>
<td>8.1</td>
<td>Summary</td>
<td>118</td>
</tr>
<tr>
<td>8.2</td>
<td>Future Work</td>
<td>122</td>
</tr>
<tr>
<td></td>
<td>Bibliography</td>
<td>126</td>
</tr>
</tbody>
</table>
List of Figures

2.1 (a) Transmitter model (b) Receiver model .............................. 27
2.2 Matched MC-CDMA subcarrier pattern, channel transfer function TF, and integral $\Psi(\cdot)$ with $\tau_d = 2/B_T$. The frequency corresponding to the $l^{th}$ subcarrier is $f_c + F_l B_{ch}$ .................................................... 28
2.3 BER as a function of number of users for SNR = 9 dB. ................. 29
2.4 BER as a function of SNR for $M = 64$ users. ............................ 30
3.1 (a) Transmitter model (b) Receiver model .............................. 40
3.2 FH-MC-CDMA pattern .......................................................... 41
3.3 Theoretical and simulation BER comparison for $N = M$ ......... 42
3.4 Theoretical and simulation BER comparison for $N < M$. .......... 43
3.5 BER comparison between random and coordinated FH-MC-CDMA for $M = 32$ users. ......................................................... 44
3.6 BER comparison between random and coordinated FH-MC-CDMA for $M = 64$ users. ......................................................... 45
3.7 BER as a function of SNR for $M = 64$ users. ............................ 46
4.1 (a) Transmitter model; (b) Receiver model .............................. 57
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2</td>
<td>Frequency groups when $N_{G_j} = N \forall j$ with $N = L = 4$, and $F = g = s/N$.</td>
<td>58</td>
</tr>
<tr>
<td>4.3</td>
<td>BER versus SNR for $M = 32$ MS's, and $L = 32$.</td>
<td>59</td>
</tr>
<tr>
<td>4.4</td>
<td>BER versus SNR for $M = 32$ MS's, and $L = 8$.</td>
<td>60</td>
</tr>
<tr>
<td>4.5</td>
<td>BER versus $N$ for $L = 16$, and SNR = 10 dB.</td>
<td>61</td>
</tr>
<tr>
<td>4.6</td>
<td>BER versus $M$ for $L = 16$, and SNR = 10 dB.</td>
<td>62</td>
</tr>
<tr>
<td>5.1</td>
<td>A CAFH flow chart describing the proposed subband assignment scheme.</td>
<td>77</td>
</tr>
<tr>
<td>5.2</td>
<td>BER of conventional FH and CAFH with BPSK modulation, $L = 32$.</td>
<td>78</td>
</tr>
<tr>
<td>5.3</td>
<td>BER of conventional FH and CAFH with NCFSK modulation, $L = 32$.</td>
<td>79</td>
</tr>
<tr>
<td>5.4</td>
<td>BER of CAFH with BPSK for $r = 1$ and $2$, $L = 32$.</td>
<td>80</td>
</tr>
<tr>
<td>5.5</td>
<td>BER of CAFH with NCFSK for $r = 1$ and $2$, $L = 32$.</td>
<td>81</td>
</tr>
<tr>
<td>5.6</td>
<td>BER of conventional FH and CAFH with BPSK modulation. $\gamma_0 = 2$ dB and $L = 48$.</td>
<td>82</td>
</tr>
<tr>
<td>5.7</td>
<td>BER of conventional FH and CAFH with BPSK modulation. $\gamma_0 = 8$ dB and $L = 48$.</td>
<td>83</td>
</tr>
<tr>
<td>5.8</td>
<td>BER vs. average SNR, $\gamma_0$, of CAFH with BPSK for $r = 1$ and $2$, $L = 48$.</td>
<td>84</td>
</tr>
<tr>
<td>5.9</td>
<td>BER vs. average SNR, $\gamma_0$, of CAFH with NCFSK for $r = 1$ and $2$, $L = 48$.</td>
<td>85</td>
</tr>
<tr>
<td>5.10</td>
<td>BER vs. the number of rounds, $r$, of CAFH with BPSK. $\gamma_0 = 8$ dB and $L = 48$.</td>
<td>86</td>
</tr>
<tr>
<td>5.11</td>
<td>BER versus number of MS's of random allocation, CAFH with BPSK for $r = 1$ and $2$, and exhaustive search method. $\gamma_0 = 4$ dB and $L = 8$.</td>
<td>87</td>
</tr>
<tr>
<td>6.1</td>
<td>Plot of cdf of $\gamma_{1,3}$, $\gamma_{2,3}$ and $\gamma_{3,3}$ with $m = 1$ and $\overline{N} = 1$: theoretical (dotted) and simulation (solid).</td>
<td>98</td>
</tr>
</tbody>
</table>
List of Figures

6.2 Plot of cdf of $\gamma_{1:3}$, $\gamma_{2:3}$ and $\gamma_{3:3}$ with $m = 1$ and $\bar{\gamma} = 1$: independent (dotted) and correlated (solid) fading channels. ........................................... 99

6.3 Plot of cdf of $\gamma_{1:3}$ with $\bar{\gamma} = 1$ for different values of $m$. .................. 100

6.4 Plot of cdf of $\gamma_{r:5}$ with $m = 1$ and $\bar{\gamma} = 1$ for a linear model (solid) and its exchangeable approximation (dotted). .............................. 101

7.1 BER vs. average SNR for $B_n = 0.9$. .......................................................... 112

7.2 BER vs. average SNR for $B_n = 0.5$. .......................................................... 113

7.3 BER vs. average SNR for $B_n = 0.25$. ......................................................... 114

7.4 CAFH BER vs. number of mobile stations, for $\bar{\gamma} = 2$ and $8$ dB, and $B_n = 0.9$. .......................................................... 115

7.5 CAFH BER vs. number of MS's, for $\bar{\gamma} = 2$ and $8$ dB, and $B_n = 0.25$. .... 116

7.6 CAFH BER vs. number of MS's, for $\bar{\gamma} = 2$ and $10$ dB, and $B_n = 0.33$. .... 117
# List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2G</td>
<td>2nd Generation</td>
</tr>
<tr>
<td>3G</td>
<td>3rd Generation</td>
</tr>
<tr>
<td>4G</td>
<td>4th Generation</td>
</tr>
<tr>
<td>ACE</td>
<td>Active Constellation Extension</td>
</tr>
<tr>
<td>AMC</td>
<td>Adaptive Modulation and Coding</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>BER</td>
<td>Bit Error Rate</td>
</tr>
<tr>
<td>BPSK</td>
<td>Binary Phase-Shift Keying</td>
</tr>
<tr>
<td>BS</td>
<td>Base Station</td>
</tr>
<tr>
<td>CAFH</td>
<td>Channel Aware Frequency Hopping</td>
</tr>
<tr>
<td>cdf</td>
<td>Cumulative Distribution Function</td>
</tr>
<tr>
<td>CDMA</td>
<td>Code Division Multiple Access</td>
</tr>
<tr>
<td>CF</td>
<td>Characteristic Function</td>
</tr>
<tr>
<td>CP</td>
<td>Cycle Prefix</td>
</tr>
<tr>
<td>CS</td>
<td>Channel State</td>
</tr>
<tr>
<td>CSI</td>
<td>Channel State Information</td>
</tr>
<tr>
<td>CQI</td>
<td>Channel Quality Information</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>DAB</td>
<td>Digital Audio Broadcasting</td>
</tr>
<tr>
<td>D-AMPS</td>
<td>Digital-Advanced Mobile Phone Service</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
</tr>
<tr>
<td>DMT</td>
<td>Discrete Multitone</td>
</tr>
<tr>
<td>DS</td>
<td>Direct Sequence</td>
</tr>
<tr>
<td>DS-CDMA</td>
<td>Direct Sequence CDMA</td>
</tr>
<tr>
<td>DSL</td>
<td>Digital Subscriber Line</td>
</tr>
<tr>
<td>DVB</td>
<td>Digital Video Broadcasting</td>
</tr>
<tr>
<td>EDGE</td>
<td>Enhanced Data Rates for GSM Evolution</td>
</tr>
<tr>
<td>EGC</td>
<td>Equal Gain Combining</td>
</tr>
<tr>
<td>FDMA</td>
<td>Frequency Division Multiple Access</td>
</tr>
<tr>
<td>FH</td>
<td>Frequency Hopping</td>
</tr>
<tr>
<td>FH-MC-CDMA</td>
<td>Frequency Hopping MC-CDMA</td>
</tr>
<tr>
<td>GPRS</td>
<td>General Packet Radio System</td>
</tr>
<tr>
<td>GSM</td>
<td>Global System for Mobile Communications</td>
</tr>
<tr>
<td>HDR</td>
<td>High Data Rate</td>
</tr>
<tr>
<td>H-S/MRC</td>
<td>hybrid selection/maximal-ratio combining</td>
</tr>
<tr>
<td>ICI</td>
<td>Interchannel Interference</td>
</tr>
<tr>
<td>IP</td>
<td>Internet protocol</td>
</tr>
<tr>
<td>ISI</td>
<td>Intersymbol Interference</td>
</tr>
<tr>
<td>ITU</td>
<td>International Telecommunications Union</td>
</tr>
<tr>
<td>LA</td>
<td>Link Adaptation</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>LAN</td>
<td>Local Area Network</td>
</tr>
<tr>
<td>MA</td>
<td>Margin Adaptive</td>
</tr>
<tr>
<td>MAI</td>
<td>Multiple Access Interference</td>
</tr>
<tr>
<td>MC</td>
<td>Multicarrier</td>
</tr>
<tr>
<td>MC-CDMA</td>
<td>Multicarrier Code Division Multiple Access</td>
</tr>
<tr>
<td>MC-SS</td>
<td>Multicarrier Spread Spectrum</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multiple-Input Multiple-Output</td>
</tr>
<tr>
<td>MISO</td>
<td>Multi-input single-output</td>
</tr>
<tr>
<td>MMSE</td>
<td>Minimum Mean Square Error Estimation</td>
</tr>
<tr>
<td>MRC</td>
<td>Maximal Ratio Combining</td>
</tr>
<tr>
<td>MS</td>
<td>Mobile Station</td>
</tr>
<tr>
<td>MU-OFDM</td>
<td>Multiuser OFDM</td>
</tr>
<tr>
<td>NCFSK</td>
<td>Non-Coherent Frequency-Shift Keying</td>
</tr>
<tr>
<td>OFDM</td>
<td>Orthogonal Frequency Division Multiplexing</td>
</tr>
<tr>
<td>PAPR</td>
<td>Peak-to-Average Power Ratio</td>
</tr>
<tr>
<td>PDC</td>
<td>Personal Digital Cellular</td>
</tr>
<tr>
<td>pdf</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>PER</td>
<td>Packet Error Rate</td>
</tr>
<tr>
<td>PF</td>
<td>Proportional Fair</td>
</tr>
<tr>
<td>PG</td>
<td>Processing Gain</td>
</tr>
<tr>
<td>PSAM</td>
<td>Pilot Symbol Aided Modulation</td>
</tr>
<tr>
<td>PSD</td>
<td>Power Spectral Density</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>-------------</td>
<td>--------------------------------------------</td>
</tr>
<tr>
<td>PTS</td>
<td>Partial Transmit Sequence</td>
</tr>
<tr>
<td>QoS</td>
<td>Quality of Service</td>
</tr>
<tr>
<td>RA</td>
<td>Rate Adaptive</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>rv</td>
<td>Random Variable</td>
</tr>
<tr>
<td>SC</td>
<td>Selection Combining</td>
</tr>
<tr>
<td>SFH/CDMA</td>
<td>Slow FH/CDMA</td>
</tr>
<tr>
<td>SINR</td>
<td>Signal to Interference plus Noise Ratio</td>
</tr>
<tr>
<td>SISO</td>
<td>Single-Input Single-Output</td>
</tr>
<tr>
<td>SLM</td>
<td>Selected Mapping</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
</tr>
<tr>
<td>SNR&lt;sub&gt;r&lt;/sub&gt;</td>
<td>Received Signal-to-Noise Ratio</td>
</tr>
<tr>
<td>TDL</td>
<td>Tapped Delay Line</td>
</tr>
<tr>
<td>TDMA</td>
<td>Time Division Multiple Access</td>
</tr>
<tr>
<td>TI</td>
<td>Tone Injection</td>
</tr>
<tr>
<td>TR</td>
<td>Tone Reservation</td>
</tr>
<tr>
<td>UMTS</td>
<td>Universal Mobile Telecommunications System</td>
</tr>
<tr>
<td>WCDMA</td>
<td>Wideband Code-Division Multiple Access</td>
</tr>
<tr>
<td>WLAN</td>
<td>Wireless Local Area Network</td>
</tr>
<tr>
<td>WLL</td>
<td>Wireless Local Loop</td>
</tr>
</tbody>
</table>
Acknowledgements

First and foremost, I am obediently thankful to God, praise and glory be to him, for his countless blessings and mercy. Without his guidance and mercy I can do nothing.

I would like to express my heartfelt gratitude to my supervisor, Dr. Cyril Leung, for his guidance, technical advice, invaluable feedback, understanding, generous support, and friendship. I have passed through many personal and difficult circumstances during the period of my Ph.D. program. Without Dr. Leung's genuine kindness, I do not think you would be reading these words today.

I am very grateful to my co-supervisor, Dr. Robert Schober, for his valuable suggestions, and technical assistance. Although he is very busy, I always found him easy to approach and available for help.

I would like to express my appreciation to my thesis committee for their valuable comments which enhanced the presentation of this thesis.

I have been blessed to come in contact with many wonderful people throughout my study period at UBC. This page is definitely not enough to mention all of them, but, the least I can say to them is: thank you for making my stay at UBC such a pleasant experience.

Words cannot describe my feelings toward my parents. I can only pray to God to
reward them for their constant support, persistent encouragement, and most of all for their unwavering love.
Dedication

To my beloved parents.

البي أمي الحبيبة و البي أبي الحبيب
ٍربما أظرفُقا كما ربياني صغيرًا
Chapter 1

Introduction

1.1 Mobile Communications Systems: Past, Present, and Future

The common feature of next generation wireless technologies will be the convergence of multimedia services such as speech, audio, video, image, and data. This implies that a future wireless terminal, by guaranteeing high-speed data, will be able to connect to different networks in order to support various services: switched traffic, IP data packets and broadband streaming services such as video. The development of wireless terminals with generic protocols and multiple-physical layers or software-defined radio interfaces is expected to allow users to seamlessly switch between networks based on different standards.

The rapid increase in the number of wireless mobile terminal subscribers, which currently exceeds 1 billion users, highlights the importance of wireless communications in the new millennium. The increase has been sustained through the continuous evolution of standards and products. The adaptation of wireless technologies to the users' rapidly changing demands has been a hallmark of this rapid growth. It is expected that a variety
of standards and systems will continue to exist. This plethora of wireless communication systems is not limited to cellular mobile telecommunication systems such as Global System for Mobile communications (GSM), IS-95, Digital Advanced Mobile Phone Service (D-AMPS), Personal Digital Cellular (PDC), Universal Mobile Telecommunications System (UMTS) or cdma2000, but also includes wireless local area networks (WLANs), e.g., HIPERLAN/2, IEEE802.11 and Bluetooth, and wireless local loops (WLL), e.g., HIPERMAN, HIPERACCESS, and IEEE 802.16 as well as broadcast systems such as digital audio broadcasting (DAB) and digital video broadcasting (DVB).

Currently, the research community is focusing its activity on beyond 3G, i.e. 4th generation (4G) systems, with more ambitious technological challenges. The primary goal of next-generation wireless systems (4G) will not only be the introduction of new technologies to cover the need for higher data rates and new services, but also the integration of existing technologies in a common platform. Hence, the selection of a generic air-interface for future generation wireless systems will be of great importance. Although the exact requirements for 4G have not yet been defined, its new air interface should fulfill at least the following requirements [1]:

- generic architecture, enabling the integration of existing technologies,

- high spectral efficiency, offering higher data rates in a given scarce spectrum,

- high scalability with different cell configurations (hot spot, ad hoc), hence better coverage,

- high adaptability and reconfigurability, supporting different standards and tech-
Chapter 1. Introduction

- low cost, enabling a rapid market introduction, and

- future proof, opening the door for new technologies.

1.2 From Second- to Third-Generation Multiple Access Schemes

Second Generation (2G) wireless systems are mainly characterized by the transition from analog towards a fully digital technology and comprise the GSM, IS-95, PDC and D-AMPS standards. Work on the pan-European digital cellular standard GSM started in 1982 [2, 3], and now accounts for two-thirds of the world mobile market. In 1989, the technical specifications of GSM were approved by the European Telecommunication Standard Institute (ETSI), and commercialization began in 1993. Although GSM is optimized for circuit-switched services such as voice, it offers low-rate data services up to 14.4 kbit/s. High speed data services up to 115.2 kbit/s are possible with the enhancement of the GSM standard General Packet Radio Service (GPRS) by using a larger number of time slots. GPRS uses the same modulation, frequency band, and frame structure as GSM. However, the Enhanced Data Rates for Global Evolution (EDGE) [4] system which further improves the data rate up to 384 kbit/s introduces a new modulation scheme.

Parallel to GSM, the U.S. IS-95 standard [5] (recently renamed cdmaOne) was approved by the Telecommunication Industry Association (TIA) in 1993, and commercialization started in 1995. As in GSM, the first version of this standard (IS-95A) offered data services up to 14.4 kbit/s. In its second version, IS-95B, up to 64 kbit/s data ser-
vices are possible. During the same time period, two other 2G mobile radio systems were introduced: D-AMPS/IS-136, called TDMA in the USA and PDC in Japan [6].

Demands for more capacity for mobile receivers, new multimedia services, new frequencies, and new technologies are behind the introduction of 3G systems. A unique international standard was targeted: Universal/International Mobile Telecommunication System (UMTS-IMT-2000) which would enable a new generation of mobile personal communications services world-wide. The objectives of the 3G standards, namely UMTS [7] and cdma2000 [8] went far beyond those of 2G systems, especially with respect to [9]:

- the wide range of multimedia services (speech, audio, image, video, data) and bit rates (up to 2 Mbit/s for indoor and hot spot applications),

- the high quality of service requirements (better speech/image quality, lower bit error rate (BER), higher number of active users),

- operation in mixed cell scenarios (macro, micro, pico),

- operation in different environments (indoor/outdoor, business/domestic, cellular/cordless),

- and finally flexibility in frequency (variable bandwidth), data rate (variable), and radio resource management (variable power/channel allocation).

The commonly used multiple access schemes for 2G and 3G wireless mobile communication systems are based on either Time Division Multiple Access (TDMA), Code Division Multiple Access (CDMA) or a combinations of TDMA/CDMA in conjunction with an additional Frequency Division Multiple Access (FDMA) component:
Chapter 1. Introduction

• The GSM standard, employed in the 900 MHz and 1800 MHz bands, first divides the allocated bandwidth into 200 kHz FDMA sub-channels. Then, in each sub-channel, up to 8 users share the 8 time slots in a TDMA manner [3].

• In the IS-95 standard, up to 64 users share the 1.25 MHz channel by CDMA [5]. The system is used in the 850 MHz and 1900 MHz bands.

• The aim of D-AMPS (TDMA IS-136) is to coexist with analog AMPS, and the 30kHz channel of AMPS is divided into three channels, allowing three users to share a single radio channel by allocating unique time slots to each user [10].

• The recent International Telecommunications Union (ITU) adopted standards for 3G (UMTS and cdma2000) are both based on CDMA [7, 8]. For UMTS, the CDMA-FDD mode, which is known as wideband CDMA, employs separate 5 MHz channels for both the uplink and downlink directions. Within the 5 MHz bandwidth, each user is separated by a specific code, resulting in an end-user data rate of up to 2 Mbit/s.

1.3 From Third- to Fourth-Generation Multiple Access Schemes

Besides offering new services and applications, the success of 4G wireless systems will strongly depend on the choice of the concept and technology innovations in architecture, spectrum allocation, spectrum utilization, and exploitation [11, 12]. New high-performance physical layer and multiple access technologies are needed to provide high speed data rates with flexible bandwidth allocation. A low-cost generic radio interface,
operational in different environments with scalable bandwidth and data rates, is expected to have better acceptance.

The technique of *spread spectrum* may allow the above requirements to be at least partially fulfilled. A multiple access scheme based on *direct sequence code division multiple access* (DS-CDMA) relies on spreading the data stream using an assigned spreading code for each user in the time domain [13]-[16]. The capability of minimizing *multiple access interference* (MAI) is given by the cross-correlation properties of the spreading codes. In the case of severe multipath propagation in mobile communications, the capability of distinguishing one component from others in the composite received signal is offered by the autocorrelation properties of the spreading codes [14]. The so-called rake receiver should contain multiple correlators, each matched to a different resolvable path in the received composite signal [13]. Therefore, the performance of a DS-CDMA system will strongly depend on the number of active users, the channel characteristics, and the number of branches employed in the rake. System capacity is typically limited by self-interference and MAI, which results from the imperfect auto- and cross-correlation properties of spreading codes. It will be difficult for a DS-CDMA receiver to make full use of the received signal energy scattered in the time domain and hence to handle full load conditions [13].

The technique of *multicarrier transmission* has recently been receiving widespread interest, especially for high data-rate broadcast applications. The history of orthogonal multicarrier transmission dates back to the mid-1960s, when Chang published his paper on the synthesis of band-limited signals for multi-channel transmission [17]-[19]. The
basic principle is to transmit data simultaneously through a band-limited channel without interference between sub-channels (without interchannel interference, ICI) and without interference between consecutive transmitted symbols (without intersymbol interference, ISI) in the time domain. A major contribution to multicarrier transmission was made in [20] when Fourier transform for base-band processing was proposed in place of a bank of subcarrier oscillators. To combat ICI and ISI, a guard time was introduced between successive transmitted symbols.

The main advantages of multicarrier transmission are its robustness in frequency selective fading channels and, in particular, the reduced signal processing complexity by equalization in the frequency domain. The basic principle of multicarrier modulation is to divide a high-rate data stream into several low-rate sub-streams. These sub-streams are modulated on to different subcarriers [21]-[23]. By using a large number of subcarriers, a high immunity against multipath dispersion can be provided since the useful symbol duration $T_s$ on each sub-stream will typically be much larger than the channel dispersion time. Hence, the effects of ISI will be minimized. Since the number of filters and oscillators necessary is considerable for a large number of subcarriers, an efficient digital implementation of a special form of multicarrier modulation, called OFDM, with rectangular pulse-shaping and guard time was proposed in [21]. OFDM can be easily realized by using the fast Fourier transform (FFT). OFDM, having densely spaced subcarriers with overlapping spectra of the modulated signals, obviates the use of steep band-pass filters to detect each subcarrier as in FDMA schemes. Therefore, it offers high spectral efficiency.
Chapter 1. Introduction

OFDM came into prominence in the 1990’s as it was the modulation method chosen for ADSL in the USA [24], and for the European DAB standard [25]. This success continued with the choice of OFDM for the European DVB-T standard [26] and for the WLAN standards HIPERLAN/2 and IEEE802.11a [27, 28] and recently for the interactive terrestrial return channel (DVB-RCT) [29]. It is also a candidate for wireless MAN standards, HIPERMAN and IEEE802.16a [30, 31].

The advantages of multicarrier modulation on the one hand and the flexibility offered by spread spectrum techniques on the other hand have motivated many researchers to investigate the combination of both techniques, known as multicarrier spread spectrum (MC-SS). Such combinations [32]-[38], have resulted in new multiple access schemes called MC-CDMA and MC-DS-CDMA. They offer attractive features such as high flexibility, high spectral efficiency, simple and robust detection techniques, and narrow band interference rejection capacity. MC and MC-SS are today considered promising candidates to meet the requirements of next generation (4G) high-speed wireless multimedia communications systems, where spectral efficiency and flexibility will be the most important criteria for the choice of the air interface.

1.4 Motivation

The wireless communications industry is growing rapidly for both fixed and mobile applications. The increasing demand for all types of wireless services (voice, data, and multimedia) is fueling the need for higher capacity and data rates. Although improved compression technologies have reduced the bit rate for voice calls, data traffic will re-
quire much more bandwidth as new services come online. In this context, emerging technologies that improve wireless systems' spectrum efficiency are becoming a necessity, especially in broadband applications. Some examples include smart antennas, multiple-input multiple-output (MIMO) technology, coded multicarrier modulation, power control, channel allocation, link-level retransmission, and adaptive modulation and coding (AMC) techniques [39].

Popularized by cellular wireless standards such as EDGE, AMC techniques that adapt to the time-varying characteristics of wireless channels allow significantly higher data rates, reliability, and spectrum efficiency for future wireless data-centric networks. The set of algorithms and protocols governing AMC is often referred to as link adaptation (LA).

While substantial progress has been made in this area to understand the theoretical aspects of time adaptation in LA protocols, more challenges surface as dynamic transmission techniques must take into account the additional signaling dimensions explored in future broadband wireless networks. More specifically, the growing popularity of both MIMO and multicarrier (MC) systems that take channel aware algorithms into account create the need for LA solutions that integrate temporal, spatial, and spectral components together.

Currently a few existing and emerging wireless systems such as IS-95B, WCDMA, CDMA2000, HiperLAN/2 and (E)GPRS of GSM have already introduced some kind of LA [40]. While it is desirable to adapt the transmission parameters according to the channel state information (CSI) to capture even small-scale variations, there are
practical limitations to channel state prediction and link adaptation. Frequent adaptation increases the number of mode-change messages transmitted over the channel, consuming bandwidth, and time resources [41]. Moreover, predicting the future channel quality may also consume significant amount of system resources (e.g., time, bandwidth and power), since it may involve transmission of training-sequences, pilot tones, or feedback messages carrying the CSI. Naturally, the additional cost and complication of predicting CS and LA increase with the desired prediction accuracy.

Common channel estimation methods documented in the literature are pilot symbol aided modulation (PSAM) and minimum mean square error (MMSE) estimation. A classical paper on pilot symbol aided modulation for Rayleigh fading was presented in [42] where the optimum Wiener interpolation filter was derived to minimize the variance of the estimation error and system performance for BPSK, QPSK and 16-QAM was studied. Because the optimum Wiener filter requires a priori information of the channel and is computationally complex, several suboptimal interpolation filters have been proposed among which the sinc interpolator is most widely used due to its simple implementation and close-to-optimum performance [43]. The bit error rate (BER) of pilot symbol aided MQAM in Rayleigh fading using the sinc interpolator has been analyzed in [44].

1.5 Link Adaptation Fundamentals

The basic idea behind LA is to adapt the transmission parameters to take advantage of the prevailing channel conditions. The fundamental parameters to be adapted include modulation and coding levels, but other quantities can also be adjusted such as power
level (as in power control and power allocation), channel allocation, spreading factor, and signaling bandwidth. LA is now widely recognized as a key solution to increasing the spectral efficiency of wireless systems. LA techniques are incorporated in current proposals for third-generation CDMA wireless packet data services, such as cdma2000 and wideband CDMA (WCDMA), GPRS and GPRS-136 [39].

In practical LA implementations, the values for the transmission parameters are quantized and grouped into what we refer to as a set of modes. An example for a mode might be a pair of modulation level and coding rate. Since each mode has a different data rate (expressed in information bits per second) and robustness level (minimum SNR needed to activate the mode), they are optimal for use in different channel/link quality regions. When using packet-switching technology and multiple modulation and coding levels, the EDGE system employs a link-adaptation technique to adapt packet transmission to one of six coding levels [45] (a recent proposal has nine modulation and coding levels [46]), where the highest data rate can exceed 550 kb/s. When the channel condition is poor, a low modulation level (i.e., few information bits per symbol) and/or heavy coding should be used for packet transmission to enable correct signal detection. On the other hand, if the channel is in a good state, a high modulation level and/or light coding can be used to increase data rate.

Due to the unreliable nature of radio links, the provision of quality of service (QoS) such as packet error rate (PER) in wireless networks is challenging. For real-time services such as Internet protocol (IP) voice, music and video, stringent delay requirements severely limit or even preclude the retransmission of lost packets. Thus, tight delay
requirements often translate into stringent PER requirements.

The goal of an LA algorithm is to ensure that the most efficient mode is used for given channel conditions, based on a mode selection criterion (maximum data rate, minimum transmit power, minimum PER, etc). Making modes available that enable communication even in poor channel conditions increases system robustness; under good channel conditions, spectrally efficient modes can be used to increase throughput. In contrast, systems without LA are constrained to use a single mode that is often designed to maintain acceptable performance when the channel quality is poor to get maximum coverage. Such systems are designed for worst-case channel conditions and do not fully exploit the channel when good conditions prevail.

In practice, performance can be degraded by problems such as imperfect synchronization or quantization effects. However in such cases, these degradations can be reduced to manageable levels. To avoid unnecessary complication of the models, these effects are ignored in this thesis.

1.6 Subchannel and Power Allocation

With the introduction of new IP-based multi-rate, multi-QoS, future networks should be designed for economic packet data transfers [47] with decreasing cost per megabyte in order to make them more attractive to the users. These services are highly asymmetrical and require high transmission bandwidth. However, due to the limitations of the available frequency spectrum, its efficient use is crucial to the success of next generation wireless networks. Novel access methods coupled with adaptive resource management techniques,
for both uplink and downlink transmission, are required to improve spectrum efficiency. It is well established that dynamically allocating transmission rate and power are effective techniques to improve the performance of wireless networks. For example, [48]-[52] consider these problems in the context of the information theoretic capacity region of a multi-access fading channel under various assumptions.

Multicarrier transmission schemes have been introduced into *code-division multiple access* (CDMA) systems to enable high data rate transmission. One of the methods is to transmit identical narrowband *direct sequence* (DS) waveforms in parallel over a number of subchannels using frequency diversity. In [53], a multicarrier CDMA system with an adaptive subchannel allocation method for forward links was proposed. In this system, instead of identical DS waveforms being transmitted over a number of subchannels in parallel, each user's DS waveform is transmitted over the user's preferred subchannel which has the largest fading amplitude among all the subchannels. In [54] subchannel allocation in forward links for MC-CDMA with random signature sequences is studied. A near optimal subchannel allocation policy is formulated by maximizing the total average *signal-to-interference-and-noise ratio* (SINR). An iterative algorithm similar to the water filling algorithm was also proposed. In general, efficient subcarrier and power allocation have been shown to reduce performance degradation caused by channel dispersion [55]-[60].

Among the potential performance-enhancing technologies is *orthogonal frequency division multiplexing* (OFDM). It is suitable for high speed downlink transmission as it can yield high spectral efficiency due to its robust performance over heavily impaired
Chapter 1. Introduction

links. OFDM has been demonstrated as an efficient way to mitigate the adverse effects of frequency selective multi-path fading by transmitting signals over a number of flat-faded narrow-band channels. The inherent multi-carrier nature of OFDM also allows the use of adaptive modulation and power allocation. Furthermore, dynamic allocation techniques that efficiently use resources such as bandwidth, power and modulation, need to be devised to increase the system spectral efficiency.

OFDM is a promising technique for the next generation of wireless communication systems [61], [62]. OFDM divides the available bandwidth into $N$ orthogonal subchannels. By adding a cyclic prefix (CP) to each OFDM symbol, the channel appears to be circular if the CP length is longer than the channel length. Each subchannel can, thus, be modeled as a time-varying gain plus additive white Gaussian noise (AWGN). Besides the improved immunity to multipath fading [22] gained by the multicarrier property of OFDM systems, multiple access is also possible, because the subchannels are orthogonal to each other. In [59]-[67], adaptive bit and power allocation are studied for OFDM. In these studies, optimal allocation of resources requires a centralized controller with knowledge of every user's channel state.

Resource allocation for OFDM has been given a lot of attention. Different distributed resource allocation algorithms for circuit-switched traffic in an OFDM multi-cell environment are compared in [68]. For packet-switched traffic, a distributed resource allocation algorithm that employs channel segregation and interference sensing for efficient resource management is proposed in [69]. A non-frequency selective channel is considered in both papers. In [70], a distributed dynamic resource allocation scheme for a downlink OFDM
packet-based cellular system that adaptively uses channel information and individual rate requirements to determine the subcarrier, bit and power allocation was proposed and its performance was evaluated for real-time multimedia traffic via simulation whereas for non-real-time traffic, the performance was evaluated in [71]. In [72], an analytical model of this algorithm is developed to provide insight into the performance gain in terms of achievable system load for voice and data.

Two classes of resource allocation schemes exist, namely: 1) fixed resource allocation [65]; and 2) dynamic resource allocation [59][63][66][67]. Fixed resource allocation schemes, such as TDMA and FDMA, assign an independent dimension, e.g., time slot or subchannel, to each user. A fixed resource allocation scheme may not be efficient since the scheme is fixed regardless of the current channel condition. On the other hand, in dynamic resource allocation, resources are adaptively assigned to users based on their channel gains. Due to the time-varying nature of the wireless channel, dynamic resource allocation makes full use of multiuser diversity to achieve improved performance.

Two classes of optimization techniques have been proposed for multiuser OFDM, namely: 1) margin adaptive (MA) [63]; and 2) rate adaptive (RA) [66][67]. The MA objective is to achieve the minimum overall transmit power given the constraints on the users' bit error rate (BER). The RA objective is to maximize each user's error free capacity with a total transmit power constraint. These optimization problems are nonlinear and, hence, computationally intensive to solve. In [59], the nonlinear optimization problems were transformed into a linear optimization problem with integer variables. The optimal solution can be achieved by integer programming. However, even with integer
programming, the complexity increases exponentially with the number of constraints and variables.

In [66], it is shown that for RA the sum capacity is maximized when each subchannel is assigned to the user with the best subchannel gain and power is then distributed by the water-filling algorithm. However, fairness is not considered in [66]. When the path loss differences among users are large, it is possible that users with higher average channel gains will be allocated most of the resources, i.e., subchannels and power, for a significant portion of the time. The users with lower average channel gains may be starved. In [67], a max-min problem approach is used to ensure that all users achieve a similar data rate. However, the maxmin optimization problem can only provide maximum fairness among the users. In most wireless systems of interest, different users require different data rates, which may be accommodated by allowing users to subscribe to different levels of service.

In [73], an adaptive subchannel allocation and bit loading scheme for the multiuser MIMO/OFDMA system is proposed through some modification of the approach for OFDMA systems in [74]. A distributed power control scheme involving iterative water-filling is proposed [75] for the digital subscriber line (DSL) interference channel. An iterative power control algorithm is proposed [76] for an MIMO interference system with feedback. The capacity of MIMO interference systems without feedback is treated in [77], and it is shown that putting all power into one antenna is optimal when the interference is sufficiently strong. Optimum signaling for MIMO interference systems with feedback for flat Rayleigh fading channels is treated in [78]. In [79], long-term proportional fairness resource allocation with dumb antennas is studied. It was pointed out that in multiuser
systems, channel fading can be exploited as a source of randomness, i.e., multiuser diversity. However, in some scenarios, due to slow channel variation, the dynamic range of channel fluctuations in the time scale of interest may be small.

1.7 This Thesis

1.7.1 Objectives

This thesis has the following main objectives. To develop new methods for MC-CDMA that involve efficient subcarrier allocation to improve system capacity for cellular systems. Subcarrier correlation is also considered. To introduce and study an adaptive subchannel allocation algorithm based on order statistics suitable for channel-aware systems. To investigate the effects of correlation on the error performance of order selection, a multiuser diversity method that selects the r-th order channel gain for transmission.

1.7.2 Contributions

The main contributions of this thesis are:

- A channel-matched multicarrier code division multiple access (matched MC-CDMA) scheme is proposed for downlink multipath slowly fading channels. The design overcomes the performance degradation caused by the channel dispersion. In contrast to a conventional MC-CDMA scheme, subcarriers are chosen so as to be located in frequency subbands characterized by favorable channel transmission gains. Two diversity reception techniques, equal gain combining (EGC) and maximum ratio
Chapter 1. Introduction

Combining (MRC), are considered. The bit error rate performance improvement over conventional MC-CDMA is examined. The results have appeared in [80].

- A new transmission scheme, *frequency hopping multicarrier code division multiple access* (FH-MC-CDMA), is proposed and investigated. This scheme is compatible with existing narrowband second-generation frequency hopping and third-generation wideband CDMA systems. An analysis of the downlink bit error rate with EGC and MRC in slow frequency-selective Rayleigh fading is provided. Two *frequency hopping* (FH) schemes, namely random and coordinated FH, are used. The performance improvement of FH-MC-CDMA over conventional frequency hopping is demonstrated, with a capacity gain approaching that of MC-CDMA but using a much lower number of subcarriers. The results have appeared in [81].

- The FH-MC-CDMA scheme is modified for operation in correlated fading channels. The *bit error rate* (BER) performance of the modified FH-MC-CDMA scheme is compared to those of conventional FH and MC-CDMA, respectively, assuming a *tapped delay line* (TDL) channel model in which the subcarriers undergo correlated fading. Numerical results indicate that the proposed FH-MC-CDMA scheme can yield a much lower BER than conventional FH; with a proper choice of parameter values, the modified FH-MC-CDMA also outperforms MC-CDMA. This work has been reported in [82].

- A channel aware multiple access scheme based on frequency hopping (CAFH) is proposed. In contrast to conventional FH, which uses a channel state independent
Chapter 1. Introduction

hopping sequence, the transmitter in the channel aware scheme hops to the available frequency subband which currently has the largest transmission gain. It is shown that the proposed scheme can offer large performance gains over the conventional FH scheme [83][84]. An efficient method is presented to evaluate the performance of CAFH with \( r \) rounds. The resulting closed-form expressions are used to investigate the effect of the number, \( r \), of rounds on the error probability for a cellular communication system. Numerical examples are presented to illustrate the application of the new analysis. This work has been reported in [85].

- A procedure for computing the cumulative distribution function (cdf) of the \( r \)-th order statistic of a set of arbitrarily correlated fading channel gains is studied. The method should prove useful in the study of various communication problems. For example, it can be applied to the performance analyses of diversity systems operating over correlated Nakagami-\( m \) fading channels. Numerical results are presented to illustrate the effect of fading correlation and the fading severity parameter as well as the accuracy of an “exchangeable” approximation. The results have been reported in [86].

- A systematic characteristic function (CF)—based methodology has been developed for computing the BER of the \( r \)-th largest channel gain for correlated Nakagami fading channels for any given covariance matrix. The method should prove useful in the study of various communication problems. Numerical results are presented to point out the application of the new analysis. The expressions derived are then used to evaluate the error performance for CAFH. The effect of the fading correlation
on the BER performance is illustrated.

1.7.3 Road Map

In Chapter 2, matched MC-CDMA is proposed as an adaptive subchannel allocation scheme with non-uniformly spaced subcarriers matched to the channel transfer function to improve BER performance.

A new scheme, Frequency-Hopping Multicarrier CDMA (FH-MC-CDMA), is described in Chapter 3. The performance of FH-MC-CDMA used over a Rayleigh fading channel is analyzed. FH-MC-CDMA may be viewed as a combination of FH and MC-CDMA in which MC-CDMA is applied to portions or subbands of the available bandwidth.

A new analytical model for correlated FH-MC-CDMA is presented in Chapter 4. The effect of equalizing the interference in each subcarrier group while maintaining a reduced correlation among the subcarrier fading processes for a group is studied.

A novel channel aware frequency hopping (CAFH) multiple access scheme is described in Chapter 5. The BER of CAFH is analyzed assuming independently Rayleigh faded subbands. It is shown that CAFH can offer significant performance improvements over conventional FH. The resulting channel assignment algorithm is iterative in nature. The performance analysis of CAFH with \( r \) rounds is also considered.

Expressions for the cumulative distribution function (cdf) of the \( r \)-th order statistic, \( r = 1, 2, \ldots, n \), for correlated Nakagami fading branches are obtained in Chapter 6. The BER of a channel aware diversity system, hereafter referred to as order selection, is
evaluated in Chapter 7 for correlated Nakagami fading channels, when the $r$-th order branch is selected for transmission. Chapter 8 concludes the thesis and outlines future research problems.
Chapter 2

Matched MC-CDMA in Slowly Fading Dispersive Channels

2.1 Introduction

Frequency selective multipath fading is common in urban and indoor environments and can significantly degrade the performance of wideband, high rate mobile communication systems. MC-CDMA, a combination of OFDM and CDMA, has been proposed as a viable transmission technique [32, 33, 35, 38, 87]. With MC-CDMA, the conventional serial transmission of a data stream is converted into a parallel transmission of data symbols using a large number of narrowband orthogonal subcarriers.

In this chapter, a modified MC-CDMA technique, hereafter referred to as matched MC-CDMA, with non-uniformly spaced subcarriers matched to the channel transfer function is proposed to improve the BER performance. Two receiver diversity combining techniques, EGC and MRC, are considered for combining subcarrier signals.

\[\text{A version of this Chapter has been published [80].}\]
2.2 System Model

The matched MC-CDMA transmitter spreads the original data stream over different subcarriers in the frequency domain using a given spreading code. Subcarriers are concentrated in the regions of the channel with low attenuation, thereby reducing the performance degradation caused by the frequency-selective channel. An appropriate channel measurement technique is assumed. The channel is modeled using a two-path transfer function developed by Rummler of the form [88], [89]

\[ C(f) = \alpha \left[ 1 - \beta e^{-j2\pi (f-f_0)\tau_d} \right], \quad f \in (f_c, f_c + B_T) \quad (2.1) \]

where \( B_T \) is the total available bandwidth, \( \alpha \) and \( \beta \) are the overall attenuation parameter and shape parameter respectively, \( f_o \) is a fade minimum frequency, and \( \tau_d \) is the relative time delay between the two paths. The effect of the multipath component is to create deep fades at \( f = f_o + i/\tau_d, i = 0, \pm 1, \pm 2, \cdots \). Because of the reduced MC-CDMA symbol rate, a synchronous downlink channel is assumed [87]. The distribution of the matched frequency subcarrier locations is proportional to the square of the magnitude of the channel transfer function, \( |C(f)|^2 \), and is obtained from the cumulative probability distribution function (cdf) of \( |C(f)|^2 \) as in [55]

\[ \Psi(f_c + f) = \begin{cases} \frac{\int_{f_c}^{f_c+iB_T} |C(\gamma)|^2 d\gamma}{\int_{f_c}^{f_c+iB_T} |C(\gamma)|^2 d\gamma}, & f \in (0, B_T) \\ 0, & \text{elsewhere}. \end{cases} \quad (2.2) \]

A matched MC-CDMA signal is generated by replicating a single data bit into \( L \)
copies, each multiplied by a different element (chip), \( c_j[l] \in \{-1, +1\} \), \( l = 1, 2, \ldots, L \) of the spreading code assigned to user \( j \) and then binary phase-shift keying (BPSK) modulated onto a subcarrier. The transmitted matched MC-CDMA signal can be written as

\[
s_j(t) = \sqrt{\frac{2}{T_b}} \sum_{k=-\infty}^{+\infty} \sum_{l=1}^{L} c_j[l] a_j[k] \cdot \cos \left( 2\pi (f_c + \frac{F_i}{B}) t \right) \cdot p(t - kT_b), \quad F_i \in Z
\]

where \( a_j[k] \in \{-1, +1\} \) is the \( k^{th} \) data bit of user \( j \), \( T_b \) is the bit duration, \( p(t) = \text{rect}(t/T_b) \), and \( \{F_i\} \) are the \( L \) subcarrier indices selected for channel matching. The total number of available orthogonal subcarriers, uniformly located within \( B_T \), is denoted by \( N \). Each subcarrier has a bandwidth of \( B_{ch} \). Conventional MC-CDMA can be viewed as a special case of matched MC-CDMA with \( F_i = LF \), \( F \in Z \). The received signal to noise ratio (SNR) for the \( l^{th} \) subcarrier is

\[
X_l = \left| C(f_l) \right|^2 \frac{E_b}{N_o L} = \left| A_l \right|^2 \frac{E_b}{N_o L}, \quad l = 1, \ldots, L.
\]

where \( f_l = f_c + \frac{F_l}{B} \) and \( A_l = \left| A_l \right| e^{j\theta_l} = C(f_c + \frac{F_l}{B}) \) is the \( l^{th} \) subcarrier complex channel gain and \( E_b \) is the bit energy. Matched MC-CDMA is expected to provide an improved performance over conventional MC-CDMA since

\[
\sum_{l=1}^{L} \left| A_{l,\text{match}} \right|^2 > \sum_{l=1}^{L} \left| A_{l,\text{conv}} \right|^2, \quad l = 1, \ldots, L.
\]

where \( A_{l,\text{match}} \) and \( A_{l,\text{conv}} \) are the complex channel gains corresponding to the \( l^{th} \) subcarrier in the matched and conventional MC-CDMA schemes respectively. The received
signal with \( M \) active transmitters can then be written as

\[
\begin{align*}
    r(t) &= \sqrt{\frac{2}{T}} \sum_{k=-\infty}^{\infty} \sum_{m=1}^{M} \sum_{l=1}^{L} |A_l|c_{m[l]}a_m[k] \cdot \cos \left( 2\pi \left( f_c + \frac{f_t}{T} \right) t + \theta_{A_l} \right) \cdot p(t - kT_b) + n(t) \\
    &\quad (2.6)
\end{align*}
\]

where \( n(t) \) is a sample function of a white Gaussian noise process with zero mean and power spectral density \( N_0/2 \).

Assuming perfect phase recovery, the decision variable for the \( k^{th} \) symbol of user \( j \) is

\[
D_{MC}^j(kT_b) = \sum_{l=1}^{L} \left\{ G_{j,l} \left[ \sum_{m=1}^{M} |A_l|c_{m[l]}a_m[k] \right] + n_l(kT_b) \right\} \quad (2.7)
\]

where \( G_{j,l} \) is the gain factor for user \( j \) at subcarrier \( l \) and depends on the combining scheme used, and \( \{n_l(kT_b), \ l = 1,2,\cdots,L\} \) are sample values of independent Gaussian random variables with mean zero and variance \( \frac{N_0}{2} \).

### 2.3 Numerical Results

The Walsh-Hadamard sequences with a processing gain (PG) of 64 are assigned to users as follows: the reference user (user 1) is assigned each of the 64 sequences in turn. Assume that user 1 is assigned sequence number \( j \). If \( j = 1 \) or \( j > M \), user \( i \), \( i = 2, \cdots, M \) is assigned sequence number \( i \). If \( 2 \leq j \leq M \), then user \( i \) is assigned sequence number \( i \) if \( i < j \), and \( i + 1 \) if \( i \geq j \).

Numerical results show that the proposed matched MC-CDMA scheme can provide a much lower BER than conventional MC-CDMA. As shown in Fig. 2.2, the number,
$N$, of available subcarrier channels is 128 from which $L = 64$ matched subcarriers are selected. The channel parameters are $\alpha = -1$ dB, $\beta = 0.5$, $f_0 = 0.2B_T$ and $\tau_d = 2/B_T$. Fig. 2.3 shows the BER as a function of the number of users for matched MC-CDMA and conventional MC-CDMA with $F = 2$, and SNR $\triangleq E_b/N_0$ of 9 dB, where $E_b$ is the transmitted energy per bit. The figure shows that the proposed matched MC-CDMA scheme can yield up to a 10-fold reduction in BER compared to conventional MC-CDMA.

A BER comparison as a function of SNR for $M=64$ users is illustrated in Fig. 2.4. For the same combining scheme, the SNR difference between matched and conventional MC-CDMA required to achieve a given target BER increases as this target BER decreases. For 64 users and a BER of $10^{-2}$, matched MC-CDMA with EGC provides a gain of about 4 dB compared to conventional MC-CDMA with EGC. It should be noted that for a small number of users (i.e., in a noise limited environment), MRC outperforms EGC. However, for a large number of users (i.e., in a interference limited environment), EGC has a superior performance. This is likely due to the fact that the orthogonality of the codes is affected to a greater extent by MRC. MRC is only optimal in an additive white Gaussian noise (AWGN) environment, which is the case for an uplink scenario.
Figure 2.1: (a) Transmitter model (b) Receiver model.
Figure 2.2: Matched MC-CDMA subcarrier pattern, channel transfer function TF, and integral $\Psi(f)$ with $\tau_d = 2/B_T$. The frequency corresponding to the $l^{th}$ subcarrier is $f_c + f_l B_{ch}$. 
Chapter 2. Matched MC-CDMA in Slowly Fading Dispersive Channels

Matched MC-CDMA (EGC)
Conventional MC-CDMA (EGC)
Matched MC-CDMA (MRC)
Conventional MC-CDMA (MRC)

Figure 2.3: BER as a function of number of users for SNR = 9 dB.
Figure 2.4: BER as a function of SNR for $M = 64$ users.
Chapter 3

Frequency-Hopping Multicarrier

CDMA in Rayleigh Fading

3.1 Introduction

Multicarrier modulation is an attractive scheme for high data rate digital transmission because it allows a reduction of the symbol rate, at the cost of increasing the number of subcarriers used to transmit the information. In CDMA, multiple users can transmit simultaneously within the same frequency band by using a number of different pseudo-noise sequences. MC-CDMA is a combination of these two techniques. Unlike DS-CDMA, in MC-CDMA the input data is duplicated and transmitted through a number of narrow-band subcarriers. The signaling period can thus be much larger than the delay spread of the channel making the system less susceptible to ISI due to multipath propagation. On each subcarrier, the transmitted bit is binary antipodally modulated based on the spreading codes assigned to the user. At the receiver side, the received signals at the different subcarriers are weighted and summed to produce a decision variable. For the downlink system, orthogonal spreading codes can be used to reduce multiuser interference. How-

\footnote{A version of this Chapter has been published [81].}
ever, since the channel gains for different subcarriers are different, strict orthogonality among users is lost and multiuser interference is introduced.

The two principal types of CDMA are DS and FH. A major advantage of FH is that it can be implemented over a much larger frequency band than direct-sequence spreading, and the band can be noncontiguous [90]. A second advantage is that frequency hopping provides resistance to multiple-access interference while not requiring power control to deal with the near-far problem. In DS-CDMA systems, accurate power control is critical on the uplink [91]. A form of FH which allows simultaneous transmission in several frequency bins has also been investigated [92].

Because at each frequency hop, a frequency-hopping multicarrier CDMA (FH-MC-CDMA) user transmits in only a small portion of the band at a given time, multiple transmissions may occur simultaneously. In addition, users are separated through the use of different spreading codes in the frequency domain thereby reducing the effects of hits. In a synchronous down-link mobile radio communication channel, we may use orthogonal Walsh Hadamard codes. The proposed FH-MC-CDMA may be viewed as a combination of FH and MC-CDMA in which MC-CDMA is applied to portions or subbands of the available bandwidth, to which users may hop to. MC-CDMA has disadvantages [93] and problems associated with nonlinear amplification [94] and high peak-to-average power ratios (PAPRs), which result from the fact that the MC-CDMA signal is composed of many subcarriers, are reduced, as FH-MC-CDMA generally uses a smaller number of subcarriers than MC-CDMA. In an FH-MC-CDMA receiver, at the different hops, the received signal is combined in the frequency domain, therefore, the receiver can employ
all the received signal energy scattered in the frequency domain.

### 3.2 FH-MC-CDMA System Model

Consider a system where $M$ users are transmitting over the channel. Each user transmits one message bit per hop in one of $g$ available frequency groups. A synchronous frequency-hopping system is assumed, with the hop intervals (of period $T_b$), of all transmitters aligned at the receiver. The users' hop sequences and message bits are assumed to be independent random sequences, uniformly distributed over the $g$ frequency groups and the set {$-1, 1$}, respectively. To evaluate the performance of the system, it suffices to consider one of the users, hereafter referred to as the *reference user*. The proposed transmitter and receiver are illustrated in Fig. 3.1. The $l^{th}$ bit $d_m[l]$ of the $m^{th}$ user is first mixed with a frequency-hopping tone of frequency $f_{h}^{(l,m)}$. MC-CDMA processing with a spreading code of length $N$ is then performed. The transmitted signal for the $l^{th}$ bit of the $m^{th}$ user can be written as

$$s_m(t) = \sum_{i=1}^{N} d_m[l]c_{m}[i] \cdot \cos \left( 2\pi (f_{h}^{(l,m)} + f_i) t \right), \quad lT_b \leq t \leq (l + 1)T_b \quad (3.1)$$

with the subcarrier tone frequency $f_i = \frac{iF}{T_b}$, where $F$ and $i$ are integers.

In FH-MC-CDMA, the total *radio frequency* (RF) *bandwidth* ($BW_{RF}$) is divided into $s$ subcarriers which are organized into $g$ frequency groups. Each frequency group consists of $N$ subcarriers ($s = gN$). The RF signal from a given transmitter is hopped from group to group by changing the carrier frequency as illustrated in Fig. 3.2. An FH-MC-CDMA
system can be viewed at one extreme as a MC-CDMA system when \( g = 1 \), and at the other extreme as a conventional FH system when \( N = 1 \), for a fixed total bandwidth, i.e., a fixed value for \( s \). A pseudo-FH system is obtained by choosing a large number of frequency groups, each with a small number of subcarriers. For example, with \( s = 128 \), we might choose \( g = 64 \) and \( N = 2 \). In this way, some of the advantages of both FH and MC-CDMA systems may be obtained in a single hybrid scheme with those of the FH dominating. On the other hand, a pseudo-MC-CDMA is obtained by choosing a large number of subcarriers and a small number of frequency groups, e.g., \( N = 64 \) and \( g = 2 \).

Assuming that signals in different groups do not interfere with each other and that delay spread effects are negligible, the received signal corresponding to the transmission of the \( i^{th} \) message bit is given by

\[
r(t) = \sum_{m=1}^{M_h} d_m[l] \sum_{i=1}^{N} \rho_{m,i} c_{m}[l] \cdot \cos \left( 2\pi (f_{h}^{(l,m)} + f_{i}) t + \theta_{i}^{(l,m)} \right) + n(t),
\]

where \( M_h \) is the number of users in a given group, \( \rho_{m,i} \) is the channel gain at the \( i^{th} \) subcarrier of the \( m^{th} \) user, \( \theta_{i}^{(l,m)} \) is a random phase offset and \( n(t) \) is a sample function of a white Gaussian noise process with zero mean and power spectral density \( N_0/2 \). It is assumed that \( \{\rho_{m,i}\}_{m=1,...,M_h;i=1,...,N} \) are outcomes of statistically independent Rayleigh distributed random variables and \( \{\theta_{i}^{(l,m)}\}_{m=1,...,M_h;i=1,...,N} \) are statistically independent random variables, each of which is uniformly distributed in \([0, 2\pi)\).

The probability that \( M_h - 1 \) other transmitters (interfering users) share the same
frequency group as the reference user is given by

$$P(M_h - 1) = \binom{M - 1}{M_h - 1} P_h^{M_h - 1}(1 - P_h)^{M - M_h}, \quad (3.3)$$

where $P_h = \frac{1}{g}$ is the probability of another transmitter hopping to the reference frequency group. Two FH-MC-CDMA schemes are considered, namely random and coordinated FH-MC-CDMA. In random FH-MC-CDMA each user randomly selects a spreading code (out of the $N$ possible Walsh Hadamard codes) and a frequency group (out of the $g$ possible groups). In coordinated FH-MC-CDMA, the $M$ users are assigned to the $g$ frequency groups in such a way as to equalize as much as possible the number in each group. For each frequency group, each of the $M_h$ users is assigned a distinct code, so as to avoid multiple users sharing the same code and group simultaneously (i.e., hit). This is done by picking at random a set of $M_h$ distinct codes from the $N$ available codes. There are a total of $\binom{N}{M_h}$ such sets.

Assuming that the receiver is synchronized with the desired reference user ($m = 1$), we can express the decision variable for the $l^{th}$ bit as follows

$$Z = \frac{2}{T_b} \int_{lt_b}^{(l+1)t_b} r(t) \sum_{i=1}^{N} q_{1,i} \cdot \cos \left( 2\pi (f_h^{(l,1)} + f_i) t + \tilde{\theta}_i^{(l,1)} \right) dt, \quad (3.4)$$

where $q_{1,i}$ is the reference receiver gain factor (which depends on the diversity reception combining scheme) at subcarrier $i$, and $\tilde{\theta}_i^{(l,1)}$ is the estimated phase at the $i^{th}$ subcarrier of $f_h^{(l,1)}$. Assuming perfect phase correction ($\tilde{\theta}_i^{(l,1)} = \theta_i^{(l,1)}$), the decision variable reduces
Chapter 3. Frequency-Hopping Multicarrier CDMA in Rayleigh Fading

\[ Z = d_1[l] \sum_{i=1}^{N} \rho_{1,i} c_1[i] q_{1,i} + \sum_{m=2}^{M_h} d_m[l] \sum_{i=1}^{N} \rho_{m,i} c_m[i] q_{1,i} \cdot \cos(\bar{\theta}_i^{(l,m)}) + \mu_i \]  

(3.5)

where \( \bar{\theta}_i^{(l,m)} = \theta_i^{(l,1)} - \theta_i^{(l,m)} \) and \( \mu \) is a noise term. With EGC, the gain factor at the \( i^{th} \) subcarrier is given as \( q_{1,i} = c_1[i] \), and the decision variable becomes

\[ Z_{EGC} = d_1[l] \sum_{i=1}^{N} \rho_{1,i} + \sum_{m=2}^{M_h} d_m[l] \sum_{i=1}^{N} \rho_{m,i} c_m[i] c_1[i] \cdot \cos(\bar{\theta}_i^{(l,m)}) + \mu_{EGC}. \]  

(3.6)

The three terms in (3.6) may be expressed as

\[ s_1^{EGC} = d_1[l] \sum_{i=1}^{N} \rho_{1,i} \]

\[ I_{int}^{EGC} = \sum_{m=2}^{M_h} d_m[l] \sum_{i=1}^{N} \rho_{m,i} c_m[i] c_1[i] \cdot \cos(\bar{\theta}_i^{(l,m)}) \]

\[ \mu_{EGC} = \frac{2}{T_b} \int_{T_b}^{(l+1)T_b} n(t) \sum_{i=1}^{N} c_1[i] \cdot \cos \left( 2\pi (f_h^{(l,1)} + f_t) t + \bar{\theta}_i^{(l,1)} \right) dt. \]

The mean and variance of the random variable for the interference term are given by

\[ E[I_{int}^{EGC}] = 0 \]

\[ \sigma_{I_{int}^{EGC}}^2 = E[(I_{int}^{EGC})^2] \]

\[ = 2(M_h - 1) \bar{P}_1 (1 - \frac{\pi}{4}), \]  

(3.7)

where \( \bar{P}_m = N \frac{E[P_{m,i}]}{2} \) represents the mean power of the \( m^{th} \) user. The mean and variance
of the noise is given by

\[ E[\mu_{EGC}] = 0 \]

\[ \sigma_{\mu_{EGC}}^2 = E[\mu_{EGC}^2] = \frac{N}{I_N^2} \]

With MRC, the gain factor at the \( i \)th subcarrier is given as \( q_{1,i} = c_1[i] \rho_{1,i} \), and the decision variable becomes

\[ Z_{MRC} = d_1[l] \sum_{i=1}^{N} \rho_{1,i}^2 + \sum_{m=2}^{M_h} d_m[l] \sum_{i=1}^{N} \rho_{m,i} \rho_{1,i} c_m[i] c_1[i] \cdot \cos(\tilde{\theta}_i^{(l,m)}) + \mu_{MRC}. \]  

(3.8)

The three terms in (3.8) may be expressed as

\[ s_{MRC}^i = d_1[l] \sum_{i=1}^{N} \rho_{1,i}^2 \]

\[ I_{int}^{MRC} = \sum_{m=2}^{M_h} d_m[l] \sum_{i=1}^{N} \rho_{m,i} \rho_{1,i} c_m[i] c_1[i] \cdot \cos(\tilde{\theta}_i^{(l,m)}) \]

\[ \mu_{MRC} = \frac{2}{I_N^2} \int_{T_2}^{(l+1)T_h} n(t) \sum_{i=1}^{N} \rho_{1,i} c_1[i] \cdot \cos \left( 2\pi (f_{h}^{(1,1)} + f_t) t + \tilde{\theta}_i^{(l,1)} \right) dt. \]

The mean and variance of the random variable for the interference term are given by

\[ E[I_{int}^{MRC}] = 0 \]

\[ \sigma_{I_{int}^{MRC}}^2 = E[I_{int}^{MRC^2}] \]

(3.9)

\[ = (M_h - 1)N[E[\rho_{1,i}^4] - (E[\rho_{1,i}^2])^2]. \]
The mean and variance of the noise are given by

$$E[\mu_{MRC}] = 0$$

$$\sigma_{\mu_{MRC}}^2 = E[\mu_{MRC}^2] = 2\bar{P}_i \frac{N_0}{k_b}.$$ 

The BER for a FH-MC-CDMA system can be calculated as

$$\text{BER}_{FMC} = \sum_{M_h=1}^{N} P(M_h - 1) \text{BER}_{(MC-CDMA),EGC}^{M_h,N}$$

$$\text{BER}_{FMC}^{MRC} = \sum_{M_h=1}^{N} P(M_h - 1) \text{BER}_{(MC-CDMA),MRC}^{M_h,N}$$

where $\text{BER}_{(MC-CDMA),EGC}^{M_h,N}$ and $\text{BER}_{(MC-CDMA),MRC}^{M_h,N}$ are the downlink BER’s of an MC-CDMA system employing EGC and MRC respectively. Approximating the interference, $I_{int}$, from other users by a Gaussian random variable results in an MC-CDMA BER given by [38]

$$\text{BER}_{(MC-CDMA),EGC}^{M_h,N} \approx \frac{1}{2} \text{erfc} \left( \sqrt{\frac{\frac{P_0 T_b}{N_0} - \frac{1}{2} \frac{(M_h-1)}{N} P_0 T_b + N_0}} \right).$$

$$\text{BER}_{(MC-CDMA),MRC}^{M_h,N} \approx \frac{1}{2} \text{erfc} \left( \sqrt{\frac{\frac{P_0 T_b}{N_0} - \frac{1}{2} \frac{(M_h-1)}{N} P_0 T_b + N_0}} \right).$$

### 3.3 Numerical Results and Discussions

The BER values for coordinated FH-MC-CDMA calculated from (3.10) and obtained from computer simulations for $N = M$, are plotted as a function of the average SNR in Fig. 3.3. The SNR is defined as $\bar{P}_i T_b / N_0$. The simulation results are represented by the
symbols and the theoretical results by the dotted curves with symbols. Fig. 3.4 shows the theoretical and simulation results for coordinated FH-MC-CDMA with \( N < M \). We find that simulation results are well approximated by the analytical results when the number of users is large. For a small number of users the Gaussian approximation of (3.10) is not as accurate, e.g., 32 users using EGC.

The BER performance of random FH-MC-CDMA and coordinated FH-MC-CDMA using EGC for different number, \( N \), of subcarriers is compared in Fig. 3.5, for \( M = 32 \) users. A BER comparison of random and coordinated FH-MC-CDMA for \( M = 64 \) users for different subcarriers and employing EGC is illustrated in Fig. 3.6. It is clear that coordinated FH-MC-CDMA provides a lower BER than random FH-MC-CDMA.

The BER comparison between conventional FH employing 128 subbands, MC-CDMA having 128 subcarriers and coordinated FH-MC-CDMA employing different number, \( N \), of subcarriers for \( M = 64 \) using EGC and MRC are shown in Fig. 3.7. As expected, for a given average SNR, the BER decreases with \( N \), due to the bigger diversity gain. The figures show that the proposed FH-MC-CDMA scheme can yield up to a 10-fold reduction in BER compared to conventional FH when employing two subcarriers in the interference-limited region. In the noise-limited region FH-MC-CDMA becomes comparable with MC-CDMA. We find that the performance difference between a FH-MC-CDMA scheme employing 64, 32, 16, 8 and 4 subcarriers and a 128 subcarrier MC-CDMA is small. The performance difference is even smaller at low SNR values.
Figure 3.1: (a) Transmitter model (b) Receiver model
Figure 3.2: FH-MC-CDMA pattern.
Figure 3.3: Theoretical and simulation BER comparison for $N = M$
Figure 3.4: Theoretical and simulation BER comparison for $N < M$. 
Figure 3.5: BER comparison between random and coordinated FH-MC-CDMA for $M = 32$ users.
Figure 3.6: BER comparison between random and coordinated FH-MC-CDMA for $M = 64$ users.
Chapter 3. Frequency-Hopping Multicarrier CDMA in Rayleigh Fading

Figure 3.7: BER as a function of SNR for $M = 64$ users.
Chapter 4

Frequency-Hopping Multicarrier CDMA in Correlated Rayleigh Fading

4.1 Introduction

In MC-CDMA, the available system bandwidth is partitioned into $s$ subbands, each associated with a subcarrier frequency. Each data bit of a mobile station (MS) is multiplied by a chip sequence of length $s$; each of the resulting $s$ chips is then transmitted using a different subcarrier so as to exploit the diversity available in frequency-selective channels.

In [81], FH-MC-CDMA was proposed for use in a frequency-selective channel in which different subbands fade independently. In FH-MC-CDMA, in each hop, a MS transmits simultaneously over a predefined set of frequency subcarriers, referred to as a group.

In this chapter, we propose a modification of the FH-MC-CDMA scheme in [81] that is suitable for use in correlated fading channels. In particular, with the appropriate choice of the number of subcarriers in each group and subcarrier interleaving, MAI and

\footnote{A version of this Chapter has been published [82].}
correlation among the subcarrier fading processes for a group are largely reduced. We compare the BER performance of the modified FH-MC-CDMA scheme with those of conventional FH and MC-CDMA, respectively. The results show that FH-MC-CDMA with appropriate subcarrier interleaving and an appropriate number of subcarriers for each group can provide a significantly lower BER than MC-CDMA.

4.2 System Model

4.2.1 FH-MC-CDMA

Let the total number of subcarriers which can be accommodated in the available system bandwidth, $W$, be denoted by $s$. The subcarrier spacing can be chosen to be approximately $1/T_b$. Thus, for a system which fully utilizes the available bandwidth, $W \approx s/T_b$. In FH-MC-CDMA, the $s$ subcarriers are partitioned into $g$ frequency groups $G_1, G_2, \ldots, G_g$, each consisting of $N_{G_j}$ subcarriers so that

$$s = \sum_{j=1}^{g} N_{G_j}. \quad (4.1)$$

In each hop of bit duration $T_b$, one bit from each MS is sent using one of the $g$ groups. The $M_{G_j}$ MS's in a given group $G_j$ transmit data bits simultaneously to the base station (BS). Each of the $M_{G_j}$ MS's is assigned a distinct spreading code of length $N_{G_j}$.

The FH-MC-CDMA scheme in [81] was proposed for the case in which the fading gains in the $s$ subbands are uncorrelated. For the case when these gains are correlated we propose a modified FH-MC-CDMA scheme in which the frequency separation between
adjacent subcarriers in a given group $G_j$ is increased so as to exceed the coherence bandwidth, $\Delta f_c$, by interleaving the subcarriers of group $G_j$ with those of other groups. This allows subcarriers assigned to a given MS to experience largely uncorrelated fading. We note that an MC-CDMA system with a larger subcarrier spacing would require a larger system bandwidth when compared to that of the proposed FH-MC-CDMA scheme. Based on MS allocation and subcarrier assignment, three FH-MC-CDMA schemes are considered:

1) In *uniform* FH-MC-CDMA, the $M$ MS's are assigned to the $g$ groups in such a way as to equalize as far as possible the number of MS's in each group. Each group consists of $N_{G_j} = \frac{N}{g} \forall j$ subcarriers.

2) For comparison purposes, we also consider *random* FH-MC-CDMA, where each MS randomly selects one frequency group (out of the $g$ possible groups) for use. Each group $G_j$ has $N_{G_j} = \frac{N}{g} \forall j$ subcarriers, and $M_{G_j} \in \{1, \cdots, N\}$ MS's.

3) The numerical results in Section 4.4 suggest that optimum error performance is achieved when choosing $M_{G_j} = 1 \forall j$. In this case, hereafter referred to as *optimum* FH-MC-CDMA, we have $g = M$.

The proposed system is illustrated in Fig. 4.1. For a given MS $m$, each component of the spreading code $(c_m[0], c_m[1], \ldots, c_m[N_{G_j}-1])$, where $c_m[i] \in \{-1, +1\}$, is multiplied by the $k^{th}$ data bit, $d_m[k] \in \{-1, +1\}$. The $i^{th}$ product signal, where $i = \{0, 1, \ldots, N_{G_j}-1\}$, is used to modulate a baseband subcarrier of frequency $f_i = \frac{iF}{N_{G_j}}$, where $F$ is an integer that determines the subcarrier separation. The $N_{G_j}$ modulated baseband subcarriers are then summed to form the composite baseband signal. The RF signal is formed by frequency
shifting the composite baseband signal by an amount equal to the carrier frequency, \( f_h^{(k,m)} \). For FH-MC-CDMA, the transmitted signal of MS \( m \) for bit \( k \) can be written as

\[
s_m(t) = \sum_{i=0}^{N_{\sigma_j} - 1} d_m[k] c_m[i] \cdot \cos\left(2\pi f_h^{(k,m)} t + \phi_i\right), \quad kT_b \leq t \leq (k + 1)T_b. \tag{4.2}
\]

### 4.2.2 Channel Model

The channel is modeled as a tapped-delay line (TDL) with a baseband equivalent impulse response [95]

\[
h^{(k,m)}(t) = \sum_{l=0}^{L-1} h^{(k,m)}_l(t) \delta(t - \frac{l}{W}), \tag{4.3}
\]

where \( \delta(\cdot) \) is the unit impulse function, and the complex channel coefficients, \( h^{(k,m)}_l \), for path \( l \) are independent, identically distributed zero-mean complex Gaussian random variables, and \( L \) is the total number of resolvable paths. The complex subband coefficient of channel subcarrier \( i \), located at \( f = f_{\text{min}} + \frac{i}{T_b} \), \( i = 0, 1, \ldots, s - 1 \), with \( f_{\text{min}} \) representing the lowest subcarrier frequency in the available transmission bandwidth, \( W \), is

\[
\hat{h}^{(k,m)}_i = \sum_{l=0}^{L-1} e^{-j2\pi f_{\text{min}} T_b} h^{(k,m)}_l \cdot e^{-j2\pi i T_b}, \tag{4.4}
\]

where \( \hat{h}^{(k,m)}_i = \rho^{(k,m)}_i e^{j\theta^{(k,m)}_i} \), with \( \rho^{(k,m)}_i \) representing the Rayleigh distributed channel amplitude gain and \( \theta^{(k,m)}_i \) representing the random phase offset. For FH-MC-CDMA, the signal received by the BS from group \( G_j \), over a slowly time varying, frequency selective
Rayleigh fading channel can be expressed as

\[ r(t) = \sum_{m=1}^{M} d_m[k] \sum_{i=0}^{N-1} \rho_{I(i)}^{(k,m)} c_m[i] \cdot \cos \left( 2\pi \left( f_h^{(k,m)} + f_i \right) t + \theta_{I(i)}^{(k,m)} \right) + n(t), \]

\[ kT_b \leq t \leq (k + 1)T_b \]

where \( I(i) = (f_h^{(k,m)} - f_{\text{min}} + f_i) \cdot T_b \) and \( n(t) \) is a sample function of a white Gaussian noise process with power spectral density \( N_0/2 \). Assuming that \( L \) divides \( s \), subcarrier indices separated by \( n_f \), \( n \in \{1, 2, \ldots, L - 1\} \), experience uncorrelated fading since

\[ E[h_i^{(k,m)}] E[h_{i+n_f}^{(k,m)}] = E \left[ \sum_{l=0}^{L-1} e^{-j2\pi f_{\text{min}} \frac{t}{T_b}} h_i^{(k,m)} e^{-j2\pi \frac{t}{T_b}} \right] \left[ \sum_{l'=0}^{L-1} e^{j2\pi f_{\text{min}} \frac{t'}{T_b}} (h_{i+n_f}^{(k,m)})^* e^{j2\pi \frac{t'}{T_b}} \right] = A \sum_{l=0}^{L-1} e^{j2\pi \frac{n_f}{L}} = 0 \]

(4.5)

where \( A = E[|h_i|^2] \) and \( E[\cdot] \) denotes expectation.

### 4.3 BER Analysis of FH-MC-CDMA

Assuming that the BS is phase synchronized with the target MS, \( m \), we can express the decision variable for the \( k^{th} \) bit as

\[ Z = \frac{2}{T_b} \int_{kT_b}^{(k+1)T_b} r(t) \sum_{i=0}^{N-1} q_{m,i} \cdot \cos \left( 2\pi \left( f_h^{(k,m)} + f_i \right) t + \theta_{I(i)}^{(k,m)} \right) dt, \]

(4.6)

where \( q_{m,i} = \rho_{I(i)}^{(k,m)} c_m[i] \) is the receiver weighting factor for subcarrier \( I(i) \) when maximum ratio combining (MRC) is used by the BS.
4.3.1 Uniform FH-MC-CDMA

In this case, each group consists of \( N_{G_j} = N \) ∀ \( j \) subcarriers. Improved error performance is achieved by setting \( N \leq L \), \( F = g = \frac{N}{N} \), and \( f_{h}^{(k,m)} - f_{\min} \in \{0, \frac{1}{N}, \frac{2}{N}, \ldots, \frac{F-1}{N}\} \) as shown in Fig. 4.2. This may be explained by the fact that a group \( G_j \) will experience larger MAI for larger values of \( N \) (i.e., larger values of \( M_{G_j} \) in a given group \( G_j \)), and correlated fading among its subcarriers as \( N \) increases above \( L \). Here, we choose \( j \) groups to contain \( i + 1 \) MS's and the remaining \( g - j \) groups to contain \( i \) MS's, i.e.,

\[
M = j \cdot (i + 1) + (g - j) \cdot i
\]

\[
= i \cdot g + j,
\]

where \( i = \left\lfloor \frac{M}{g} \right\rfloor \) and \( j = M \mod g \); \([x]\) and "mod" denote the largest integer smaller than \( x \) and the modulo-operator, respectively. The average BER can then be calculated as

\[
P_{\text{uniform}} = \frac{M-(i+1)j}{M} P_{o}^{i,N} + \frac{(i+1)j}{M} P_{o}^{i+1,N},
\]

where \( P_{o}^{i,N} \) is the average BER on the uplink of an MC-CDMA system using MRC, when having \( i \) MS's and \( N \) subcarriers \([89], [38], [95]\)

\[
P_{o}^{i,N} = \begin{cases} 
\left[\frac{1}{2} (1 - \mu)\right]^N \sum_{n=0}^{N-1} \binom{N - 1 + n}{n} \left[\frac{1}{2} (1 + \mu)\right]^n, & N \leq L, \ i = 1 \\
Q\left(\sqrt{\frac{2N}{(N-1)^2+1}}\right), & N \leq L, \ i > 1 \\
\int_0^{\frac{L^2x^2 - Lx - 1}{(L-1)^3}} Q\left(\sqrt{\frac{2N}{(N-1)^2+1}}\right) dx, & N > L, \ i \geq 1
\end{cases}
\]
where \( Q(z) = \int_{z}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy \), \( \bar{\gamma} = N \frac{E[\Phi_{\text{tol}}^2]}{2N_0} \) is the average signal-to-noise ratio (SNR) of the \( m \)th MS, and \( \mu = \sqrt{\frac{\gamma_0}{1+\gamma_0}} \), where \( \gamma_0 = \frac{\bar{\gamma}}{N} \) is the average SNR per channel.

### 4.3.2 Random FH-MC-CDMA

As in uniform FH-MC-CDMA, each group consists of \( N_G = N \forall j \) subcarriers. In this case, each MS randomly selects one frequency group for use (i.e., \( M_{G_j} \in \{1, \cdots, N\} \)). The average BER for random FH-MC-CDMA can be calculated as

\[
P_{\text{random}} = \sum_{i=1}^{N} P(i-1)P_{\mu}^{i,N},
\]

where \( P(i-1) \) is the probability that \( i-1 \) interfering MS's are in the same group as the desired reference MS, and is given by

\[
P(i-1) = \binom{M-1}{i-1} \left( \frac{1}{\mu} \right)^{i-1} (1 - \frac{1}{\mu})^{M-i}.
\]

### 4.3.3 Optimum FH-MC-CDMA

The total number of MS's may be written as

\[
M = 2^i + j,
\]

where \( j = \{0, 1, \cdots, 2^i - 1\} \) and \( \bar{N} = \frac{2}{\bar{\gamma}} \). Our results in Section 4.4 suggest that optimum error performance is achieved by choosing \( (2^i - j) \) groups to have \( \bar{N} \) sub-
carriers and the remaining $M - (2^t - j)$ groups to have $\frac{N}{2}$ subcarriers. In this case, $f_h^{(k,m)} - f_{\text{min}} \in \{0, \frac{1}{T_h}, \frac{2}{T_h}, \ldots, \frac{F-1}{T_h}, \frac{s}{2} + \frac{1}{T_h}, \frac{s}{2} + \frac{2}{T_h}, \ldots, \frac{s}{2} + \frac{F-1}{T_h}\}$, $F = \frac{s}{N}$, $M G_j = 1 \forall j$, and $g = M$. Using this method, MAI is eliminated by having one MS in a group, while maximizing the diversity for a group. Reducing the number of subcarriers in a group below the proposed number will result in diversity reduction, while increasing the number of subcarriers in a group beyond the proposed number will result in more MS's in a group, thus, MAI is introduced. The average BER can then be calculated as

$$P_{\text{optimum}} = \frac{M - (2^t - j)}{M} P_e^{\frac{s}{N}} + \frac{(2^t - j)}{M} P_e^{\frac{s}{N}}.$$  (4.13)

### 4.4 Numerical Results

Numerical results are now presented to compare the performance of the proposed modified FH-MC-CDMA with that of MC-CDMA and conventional FH. Although not shown, simulation results were obtained and found to agree closely with the theoretical curves.

In Fig. 4.3, the number of resolvable paths, the total number of subcarriers, and the total number of MS's are chosen as $L = 32$, $s = 256$, and $M = 32$, respectively. When operating at $N = 8$ subcarriers, and $F = g = s/N$ set to 32, uniform FH-MC-CDMA reduces to optimum FH-MC-CDMA having a single MS ($M G_j = 1 \forall j$) in each group. In this case, FH-MC-CDMA may be viewed as an optimum access scheme with no MAI, thus, outperforming all other schemes. This result is expected due to the fact that a
group $G_j$ will experience larger MAI if $N > 8$, and a diversity gain reduction if $N < 8$. When operating at $N = 16$, uniform FH-MC-CDMA has $M_{G_j} = 2 \forall j$. As expected, the BER performance degrades with an increase in $N$. The error performance rapidly worsens when $N > L$ due to the correlated fading among the subcarriers of a group. MC-CDMA which is a special case of FH-MC-CDMA when $N = s$ and $g = 1$, has the highest MAI and the largest correlation among its $s = 256$ subcarriers. The figure clearly shows that FH-MC-CDMA outperforms MC-CDMA. In particular, smaller values of $M_{G_j}$ result in a lower BER.

To demonstrate the effect of lower diversity orders, $L = 8$ is chosen in Fig. 4.4. When operating at $N = 8$, uniform FH-MC-CDMA achieves independent fading among its $N$ subcarriers with no MAI (i.e., $M_{G_j} = 1 \forall j$). Comparing Figs. 3 and 4 for $N = 8$, the error performance is the same for both cases ($L = 8$ and $32$). For $N > 8$, it is clear that a smaller value of $L$ results in a significant performance deterioration due to the increasing correlation among the subcarrier fading processes.

Fig. 4.5 depicts the BER of uniform FH-MC-CDMA as a function of $N$ when $L = 16$ and SNR = 10 dB. For a given $M$, the figure clearly indicates that $N = 1$ (i.e., FH) and $N = s$ (i.e., MC-CDMA) are not favorable choices for $N$. The BER performance improves when $N$ is varied between 1 and $s$. In particular, smaller values of $M_{G_j}$ result in a lower BER. This is due to the fact that for a given $M$, a reduction in the number of MS's in each group results in smaller MAI. Optimum error performance is achieved when $M_{G_j} = 1 \forall j$ (i.e., optimum FH-MC-CDMA).

Finally, in Fig. 4.6, we show the BER as a function of $M$ when $L = 16$ and SNR = 10
Clearly, FH-MC-CDMA outperforms all other schemes over a wide range of active MS's. More specifically, optimum FH-MC-CDMA provides the lowest BER.
Chapter 4. Frequency-Hopping Multicarrier CDMA in Correlated Rayleigh Fading

\[ d_m[k] \]

\[
\begin{align*}
&c_m[0] \cos(2\pi \frac{0F}{T_b} t) \\
&c_m[1] \cos(2\pi \frac{1F}{T_b} t) \\
&\vdots \\
&c_m[1N_q-1] \cos(2\pi \frac{(N_q-1)F}{T_b} t)
\end{align*}
\]

\[ \sum \]

Band Pass Filter

\[ 2\cos(2\pi f_m t) \]

\[ \sum \]

Transmitted signal for group \( G_j \)

\[ s(t) \]

\[ m_G \]

\[ \text{Transmitter model} \]

\[ \text{Receiver model} \]

\[ \text{Integrator} \]

\[ \hat{d}_m[k] \]

\[ \text{Frequency Synthesizer} \]

\[ r(t) \]

\[ \frac{2}{T_b} \cos(2\pi f_m t) \]

\[ \text{Band Pass Filter} \]

\[ 2\cos(2\pi f_m t + \theta_m) \]

\[ q_m \]

\[ \text{Band Pass Filter} \]

\[ 2\cos(2\pi f_m t + \theta_m) \]

\[ q_m \]

\[ \text{Band Pass Filter} \]

\[ 2\cos(2\pi f_m t + \theta_m) \]

\[ q_m \]

\[ \text{Integrator} \]

\[ \hat{d}_m[k] \]

\[ \text{Frequency Synthesizer} \]

\[ r(t) \]

\[ \frac{2}{T_b} \cos(2\pi f_m t) \]

\[ \text{Band Pass Filter} \]

\[ 2\cos(2\pi f_m t + \theta_m) \]

\[ q_m \]

\[ \text{Band Pass Filter} \]

\[ 2\cos(2\pi f_m t + \theta_m) \]

\[ q_m \]

\[ \text{Band Pass Filter} \]

\[ 2\cos(2\pi f_m t + \theta_m) \]

\[ q_m \]

\[ \text{Integrator} \]

\[ \hat{d}_m[k] \]

\[ \text{Figure 4.1: (a) Transmitter model; (b) Receiver model} \]
Figure 4.2: Frequency groups when $N_{G_j} = N \forall j$ with $N = L = 4$, and $F = g = s/N$. 
Figure 4.3: BER versus SNR for $M = 32$ MS's, and $L = 32$. 
Figure 4.4: BER versus SNR for $M = 32$ MS's, and $L = 8$. 
Figure 4.5: BER versus $N$ for $L = 16$, and SNR = 10 dB.
Figure 4.6: BER versus $M$ for $L = 16$, and SNR = 10 dB.
Chapter 5

A Channel Aware Frequency Hopping Multiple Access Scheme

5.1 Introduction

The BER of FH systems has been studied in [96]. Although a great deal of emphasis has been placed on DS, recent developments in FH technology [97, 98, 99, 100] have resulted in commercial applications such as mobile cellular communications, personal communications and wireless local area networks (LANs). On the other hand, FH systems have been considered for a variety of military applications due to their frequency diversity and resistance to near-far problem [91, 101]. In an FH system, the total available bandwidth is divided into a number of subbands. FH allows many mobile stations (MS’s) to share a common channel by employing user-specific hopping patterns which specify the subband occupied by each MS at a given time and allow the data of different MS’s to be recovered at the base station (BS). Thus, the transmitted signal appears as a data-modulated carrier that is hopping from one frequency to the next. An advantage of FH over DS is that it can be implemented over a larger frequency band, and the band can be non-

\[^{0}\text{A version of this Chapter has been published [83, 84, 85].}\]
contiguous [90]. A second advantage is that FH provides resistance to multiple-access interference while not requiring power control to deal with the near-far problem. In [55], matched FH was described for a single-user system to provide performance improvement on a time-invariant frequency selective channel. The subbands were designated as either usable or unusable (i.e., very poor) and hops were only allowed to usable subbands.

In this chapter, a channel aware frequency hopping (CAFH) multiple access scheme is described. In this scheme, a suitable channel measurement technique is assumed. The utilization of one of the recently developed blind identification techniques [102] would be useful in this aspect. In CAFH, the BS monitors the channel to estimate the subband gains for each MS. The BS then uses this knowledge to assign to each MS the subband in which the MS has the highest gain, while ensuring that at most one MS is assigned to each subband. The BER of CAFH is analyzed assuming independently Rayleigh faded subbands. It is shown that CAFH can offer significant performance improvements over conventional FH.

5.2 System Model

We adopt a slowly time-varying, frequency-selective Rayleigh fading channel model which is commonly used for wideband systems [89, 103]. The instantaneous SNR, $\Gamma$, on each subband of the channel is distributed according to an exponential distribution given by

$$p_\Gamma(\gamma) = \frac{1}{\gamma_0} e^{-\frac{\gamma}{\gamma_0}}, \quad \gamma \geq 0 \tag{5.1}$$
where \( \gamma_0 \) is the average value of \( \Gamma \). The slowly varying nature of the channel implies that it can be treated as time-invariant over the duration of a few symbols.

Consider the uplink of a system in which \( K \) MS's transmit over \( L \) subbands to the BS. The signal of MS \( k \) in signaling interval \( i \) is

\[
s_{k,i}(t) = \sqrt{\frac{2E}{T}} d_{k,i} \cos(2\pi(f_{h_{k,i}} + f_c)t), \quad (i-1)T < t < iT
\]  

(5.2)

where \( T \) is the bit duration, \( f_c \) is the carrier frequency, \( f_{h_{k,i}} \) is the frequency offset, \( E \) is the bit energy and \( d_{k,i} \in \{\pm1\} \) depends on the information bit to be sent. The signal received during the \( i^{th} \) symbol interval at the BS is

\[
r(t) = \sum_{k=1}^{K} g_{k,i} s_{k,i}(t - \tau_k) + n(t)
\]  

(5.3)

where \( \tau_k \) is the \( k^{th} \) MS time delay, \( n(t) \) represents additive white Gaussian noise (AWGN) with mean zero and two-sided power spectral density (PSD) \( N_0/2 \) and \( g_{k,i} \) is the \( k^{th} \) MS subband gain during the \( i^{th} \) signaling interval. The subband gain, \( g_{k,i} \), depends on the scheme that is used to assign subbands to MS's.

### 5.3 CAFH Scheme

In the CAFH scheme with \( r \) rounds, the BS assigns subbands to active MS's as follows. The round number is initially set to 1. In round \( j, j = 1, 2, \cdots, r \), for each MS, \( k \), which has not yet received its subband assignment, the subband \( \ell_k^j \in \{1, \cdots, L\} \), with
the \( j^{th} \) best subband gain, \( g_{k,i}^{(j)} \), out of \( L \) subbands is determined. If a given subband has the \( j^{th} \) best gain for exactly one MS, the subband is assigned to that MS and the scheme enters round \( j + 1 \). If an available subband has the \( j^{th} \) best gain for several MS's, the subband is assigned to the MS with the largest gain and the scheme enters round \( j + 1 \). Once a subband is assigned to a MS, it is not available in subsequent rounds. The subband assignment procedure continues until all \( K \) MS's have been assigned subbands or round \( r + 1 \) is reached. In the latter event, i.e., there are still unassigned MS's after round \( r \), the BS chooses each such MS in turn in a random order and assigns to it the unoccupied subband for which its gain is highest. A flow chart describing the CAFH scheme is provided in Fig. 5.1. It might be noted that the number of rounds required is at most \( \min\{r, K\} \).

### 5.4 BER analysis

#### 5.4.1 BER analysis of CAFH for \( r = 1 \)

For simplicity of the analysis, we consider a synchronous system, i.e., \( \tau_1 = \tau_2 = \ldots \tau_K = 0 \) [104], with \( r = 1 \) round. The BER, \( P_{CAFH} \), for the CAFH scheme can be written as

\[
P_{CAFH} = A + B, \quad (5.4)
\]
where

\[ A = \sum_{k=1}^{K} \frac{1}{k} z(k) P_{e,1,kL} \tag{5.5} \]

represents the BER for the case when a subband is assigned to the target MS in round 1, and

\[ z(k) = \binom{K-1}{k-1} \left( \frac{1}{L} \right)^{k-1} \left( \frac{L-1}{L} \right)^{K-k} \tag{5.6} \]

represents the probability that \( k - 1 \) other MS's occupy the subband of the target MS in round 1. For the case in which a subband is not assigned to the target MS in round 1 (i.e., the highest gain among all the \( L \) subband gains of the target MS was smaller than that of some other MS that shared a common highest gain subband as the target MS), the BER can be approximated by

\[ B \approx (1 - Z) \frac{1}{K-1} \sum_{k=2}^{K} P_{e,k,L} \tag{5.7} \]

where

\[ Z = \sum_{k=1}^{K} \frac{1}{k} z(k) \tag{5.8} \]

is the probability that the target MS is selected in the first round. Let \( G^{(j)} \) be the random variable corresponding to the \( j^{th} \) largest gain out of \( h \) gain samples. The term \( P_{e,j,h} \) represents the BER when the transmitted signal from an MS experiences a subband gain \( G^{(j)} \) and is contaminated by AWGN with two-sided PSD \( N_0/2 \) and is given by

\[ P_{e,j,h}^{\text{BPSK}} = \int_0^\infty Q(\sqrt{2/\gamma}) p_{T^{(j)}}(\gamma) d\gamma \tag{5.9} \]
for BPSK modulation, where $Q(z) = \int_{z}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$. For non-coherent FSK (NCFSK) modulation

$$P_{e,j,h}^{NCFSK} = \int_{0}^{\infty} \frac{1}{2} e^{-\gamma/2} p_{\Gamma\cup\Omega}(\gamma) d\gamma. \tag{5.10}$$

where $\Gamma^{(j)} = [G^{(j)}] E/N_0$ is the $j$th largest SNR out of $h$ samples. For independent, identically distributed Rayleigh gain samples, the probability density function, $p_{\Gamma\cup\Omega}(\gamma)$ of $\Gamma^{(j)}$ can be obtained using a standard result in ordered statistics [105] as

$$p_{\Gamma\cup\Omega}(\gamma) = \frac{h!}{(h-j)!(j-1)!} \left(1 - e^{-\gamma/\gamma_0}\right)^{h-j} \cdot e^{-\gamma/\gamma_0(j-1)} \cdot \frac{1}{\gamma_0} e^{-\gamma/\gamma_0}, \tag{5.11}$$

From (5.9) and (5.11), the single-user BER when using BPSK may be written as

$$P_{e,j,h}^{BPSK} = \frac{h!}{(h-j)!(j-1)!} \int_{0}^{\infty} Q(\sqrt{2\gamma}) \cdot \left(1 - e^{-\gamma/\gamma_0}\right)^{h-j} \cdot e^{-\gamma/\gamma_0(j-1)} \cdot \frac{1}{\gamma_0} e^{-\gamma/\gamma_0} d\gamma, \tag{5.12}$$

Integration by parts as

$$P_{e,j,h}^{BPSK} = \frac{h!}{(h-j)!(j-1)!} \int_{0}^{\infty} Q(\sqrt{2\gamma}) \cdot \left\{ \sum_{i=0}^{h-j} \binom{h-j}{i} \left( -e^{-\gamma/\gamma_0} \right)^i \right\} \cdot \frac{1}{\gamma_0} e^{-\gamma/\gamma_0(j)} d\gamma, \tag{5.13}$$

From (5.10) and (5.11), the single-user BER when using NCFSK may be written as

$$P_{e,j,h}^{NCFSK} = \frac{h!}{(h-j)!(j-1)!} \int_{0}^{\infty} Q(\sqrt{2\gamma}) \cdot \left\{ \sum_{i=0}^{h-j} \binom{h-j}{i} \left( -e^{-\gamma/\gamma_0} \right)^i \right\} \cdot \frac{1}{\gamma_0} e^{-\gamma/\gamma_0(j)} d\gamma, \tag{5.14}$$
yields

\[
P_{e,j,h}^{BPSK} = \frac{h!}{(h-j)!(j-1)!} \sum_{i=0}^{h-j} \frac{(-1)^i}{i!} \binom{h-j}{i} \left[ \frac{1}{2} - \frac{1}{\sqrt{\pi}} \int_{0}^{\infty} e^{-x^2/2\sigma^2} \, dx \right].
\]  

(5.15)

Using

\[
\frac{1}{\sqrt{2\pi \sigma^2}} \int_{0}^{\infty} e^{-x^2/2\sigma^2} \, dx = \frac{1}{2},
\]

with \( \sigma^2 = \frac{\gamma_0}{2(\gamma_0 + i+1)} \) in (5.14), we have

\[
P_{e,j,h}^{BPSK} = \frac{h!}{2(h-j)!(j-1)!} \sum_{i=0}^{h-j} \frac{(-1)^i}{i!} \binom{h-j}{i} \left[ 1 - \sqrt{\frac{\gamma_0}{\gamma_0 + i+1}} \right].
\]  

(5.17)

For the best SNR case (i.e., \( j = 1 \)),

\[
P_{e,1,h}^{BPSK} = \frac{h}{2} \sum_{i=0}^{h-1} \frac{(-1)^i}{i!} \binom{h-1}{i} \left[ 1 - \sqrt{\frac{\gamma_0}{\gamma_0 + i+1}} \right].
\]  

(5.18)

From (5.10) and (5.11), the single-user BER when using NCFSK may be computed as follows

\[
P_{e,j,h}^{NCFSK} = \frac{h!}{(h-j)!(j-1)!} \sum_{i=0}^{h-j} \frac{(-1)^i}{i!} \binom{h-j}{i} \frac{1}{\gamma_0 + 2(i+1)}.\]

(5.19)

(5.19) can be simplified to

\[
P_{e,j,h}^{NCFSK} = \frac{h![h-j+1]!}{(h-j+1)!(h-j)!(j-1)!} \cdot \frac{1}{2 \prod_{i=0}^{h-j-1} (\frac{3\sigma + i+1}{2})}.\]

(5.20)
For $j = 1$, we have the result given in [106], i.e.,

$$P_{NCFSK}^{NCFSK} = \frac{h!}{2\prod_{i=1}^{h}(\gamma_{0i}^2 + i)}.$$  

(5.21)

For comparison purposes, we note that the BER, $P_{\text{conv}}$, for conventional FH in Rayleigh fading on the uplink is given by [107]

$$P_{\text{conv,BPSK}} = \frac{1}{2} \left( 1 - \sqrt{\frac{\gamma_{0}}{1 + \gamma_{0}}} \right) (1 - P_{h}) + \frac{1}{2} P_{h}.$$  

(5.22)

for BPSK modulation. For NCFSK modulation it is given by

$$P_{\text{conv,NCFSK}} = \frac{1}{2 + \gamma_{0}} (1 - P_{h}) + \frac{1}{2} P_{h}.$$  

(5.23)

where $P_{h} = 1 - (1 - \frac{1}{L})^{K-1}$ is the probability of a hit.

5.4.2 Numerical Results of CAFH for $r=1$

The BER curves on an uplink channel for CAFH with $r = 1$ are compared with conventional FH in Fig. 5.2 assuming BPSK modulation. The curves are plotted using (5.4) assuming that the $L = 32$ subbands undergo independent Rayleigh fading. The results show that CAFH/BPSK can provide a much lower BER than conventional FH/BPSK over a wide range of number of MS's. The BER of CAFH/BPSK decreases rapidly with SNR. At an average SNR, $\gamma_{0}$, value of 2 dB, CAFH/BPSK provides roughly a 100-fold
reduction in BER compared to conventional FH/BPSK. The BER difference between CAFH/BPSK and conventional FH/BPSK increases with $\gamma_0$.

Fig. 5.3 shows the BER curves of CAFH and conventional FH for NCFSK modulation. Comparing Figs. 5.2 and 5.3, it can be seen that CAFH/BPSK outperforms CAFH/NCFSK. The BER of CAFH/BPSK is one to three orders of magnitude smaller than that of CAFH/NCFSK for a given SNR and a small number of MS’s. With a large number of MS’s, CAFH/BPSK provides roughly an order of magnitude reduction in BER. CAFH/BPSK and CAFH/NCFSK performance differences tend to increase with SNR.

Computer simulation results were obtained in order to ascertain the accuracy of the analysis and are shown as dotted curves in Figs. 5.2 and 5.3. It can be seen that the simulation curves agree closely with the (approximate) analytic results. It might be noted that the analytic BER results are closer to the simulation results when the network is lightly loaded. The performance difference increases with the number of MS’s.

Fig. 5.4 shows the BER of CAFH/BPSK for $r = 1$ and 2. A larger value of $r$ results in a substantial decrease in BER as the number of MS’s increases. The performance difference between $r = 1$ and 2 increases with SNR. For example when $K = 8$ MS’s are transmitting, the performance difference is roughly 3-fold, 4-fold, 10-fold and 40-fold when operating at an average SNR of $\gamma_0 = 2, 4, 6$ and 8 dB, respectively. The BER curves of CAFH/NCFSK for $r = 1$ and 2 are shown in Fig. 5.5 and are qualitatively similar to the CAFH/BPSK curves.
5.4.3 BER Analysis of CAFH for $r > 1$

The CAFH BER with $r$ rounds can be obtained as

$$P_{CAFH} = P_1 + (1 - Z_1)\{P_2 + (1 - Z_2)\{P_3 + (1 - Z_3)\{ \cdots \{P_{r-1} + (1 - Z_{r-1})\{P_r + (1 - Z_r)P_{\text{random}}\}\} \cdots \}\}}}, \quad (5.24)$$

where $Z_j, P_j, 1 \leq j \leq r$, and $P_{\text{random}}$ denote the probability that the target MS is selected in round $j$, the BER when a subband is assigned to the target MS in round $j$, and the BER when a subband is assigned to the target MS after round $r$, respectively. The terms $Z_j, P_j$, and $P_{\text{random}}$ can be expressed as

$$Z_j = \begin{cases} \sum_{k=1}^{K} \frac{1}{k} \Phi(k, 0), & j = 1 \\ \sum_{k=1}^{K-j+1} \frac{1}{k} \sum_{n=j-1}^{K-k} \Phi(k, n)P^{(j)}(n)T^{(j)}(n), & j > 1 \end{cases} \quad (5.25)$$

$$P_j = \begin{cases} \sum_{k=1}^{K} \frac{1}{k} \Phi(k, 0)P_{e,1,kL}, & j = 1 \\ \sum_{k=1}^{K-j+1} \frac{1}{k} \sum_{n=j-1}^{K-k} \Phi(k, n)P^{(j)}(n)T^{(j)}(n)P_{e,1,k(L-n)}, & j > 1 \end{cases} \quad (5.26)$$

and

$$P_{\text{random}} \approx \frac{1}{K-r} \sum_{i=r+1}^{K} P_{e,i,L} \quad (5.27)$$

respectively. For the sake of mathematical tractability, we assume in (5.27) that when a subband is not assigned to the target MS in round $j \leq r$, one of the subbands with the $\{r+1, r+2, \cdots, K\}$ largest gains is assigned at random to the target MS. This procedure
is only an approximation of the CAFH assignment scheme described in Section 5.3, of course. However, as our results will show, (5.27) is a very accurate approximation of the true error rate. In (5.25) and (5.26), $T^{(j)}(n)$ is the probability that the subband with the $j^{th}$ largest gain is available when $n \geq j - 1$ subbands have been occupied by MS’s in the previous $j - 1$ rounds. $T^{(j)}(n)$ is given by

$$T^{(j)}(n) = \frac{L - n}{L - (j - 1)}.$$  \hspace{1cm} (5.28)

Furthermore, the probability, $\Phi(k, n)$, that $k - 1$ other MS’s occupy the subband of the target MS in round $j$ when $n$ subbands have been assigned in the previous $j - 1$ rounds is given by

$$\Phi(k, n) = \binom{K - n - 1}{k - 1} \left(\frac{1}{L - n}\right)^{k-1} \left(\frac{L - 1 - n}{L - n}\right)^{K - k - n}.$$  \hspace{1cm} (5.29)

In obtaining (5.29), it is assumed that the $j^{th}$ largest gain of each of the $K - n - 1$ other remaining MS’s is randomly located in one of the $L - n$ remaining subbands. In (6.25) and (5.26), $P^{(j)}(n)$ denotes the probability that $n$ subbands have been assigned in the previous $j - 1$ rounds. $P^{(j)}(n)$ may be obtained recursively as

$$P^{(j)}(n) = \sum_{i=j-2}^{n-1} P^{(j-1)}(i)q(L - i, n - i), \quad j > 2,$$  \hspace{1cm} (5.30)

with the initial condition

$$P^{(2)}(n) = q(L, n),$$  \hspace{1cm} (5.31)
where $q(l, m)$ is the probability that $m$ subbands are assigned in the previous round (i.e., round $j - 1$) when $l$ subbands were available. The term $q(l, m)$ can be expressed as follows

$$q(l, m) = \binom{l}{m} \left( \frac{1}{M} \right)^{M-1} \sum_{i_1=1}^{M-m} \sum_{i_2=1}^{M-m+1-i_1} \sum_{i_3=1}^{M-m+2-i_1-i_2} \cdots \sum_{i_{m-1}=1}^{M-2-\sum_{j=1}^{m-2} i_j} \binom{M-1}{i_1} \binom{M-1-i_1}{i_2} \binom{M-1-i_1-i_2}{i_3} \cdots \binom{M-1-\sum_{j=1}^{m-2} i_j}{i_{m-1}},$$

(5.32)

where $M = K - (L - l)$ is the number of unassigned MS's at the beginning of round $j - 1$. In (5.32), the summation indices $i_1, i_2, \ldots, i_{m-1}$ represent the number of MS's that could be assigned to each of the first $m - 1$ subbands and the combinatorial terms represent the number of ways of selecting the MS's in each subband from the available set of unassigned MS's.

Let $G^{(j)}$ be the random variable corresponding to the $j^{th}$ largest gain out of $h$ gain samples. In (5.26) and (5.27), $P_{e,j,h}$ denotes the BER of a MS that experiences a subband gain $G^{(j)}$ and is impaired by AWGN with two-sided PSD $N_0/2$.

### 5.4.4 Numerical Results of CAFH for $r > 1$

The BER curves for the uplink of a system using CAFH with $r = 1$ and 2 are compared to conventional FH in Fig. 5.6 assuming BPSK modulation and an average SNR of $\gamma_0 = 2$ dB. The results show that CAFH/BPSK can provide a much lower BER than conventional FH/BPSK over a wide range of number of MS's. At $\gamma_0 = 2$ dB, it is observed
that CAFH/BPSK can yield over a 100-fold reduction in BER compared to conventional FH/BPSK. The curves are plotted using (5.24) assuming that the $L = 48$ subbands undergo independent Rayleigh fading. Computer simulation results were obtained in order to ascertain the accuracy of the analysis and are shown as dotted curves. It can be seen that the simulation curves agree closely with the analytic results.

It is evident from Fig. 5.6 that for a fixed target BER, the number, $K$, of MS's that can be supported by a $r = 2$ CAFH can be significantly larger than in the $r = 1$ case. At $\gamma_0 = 2$ dB, and a fixed target BER of $10^{-3}$, it is found that the system capacity is 18 MS's for $r = 2$ compared to only 12 MS's for $r = 1$. Moreover, the BER ratio between $r = 1$ and $r = 2$ increases with $K$, for $K \leq 15$. Beyond $K = 15$, the BER ratio is fairly constant. The curves for $\gamma_0 = 8$ dB are plotted in Fig. 5.7 and are qualitatively similar to the $\gamma_0 = 2$ dB results. The BER ratio between conventional FH/BPSK and CAFH/BPSK increases with $\gamma_0$.

Fig. 5.8 illustrates the BER versus average SNR for CAFH/BPSK with $L = 48$ subbands. Clearly, the performance difference between the different numbers of MS's ($K = 1, 10, 20$) increases with $\gamma_0$. The curves for CAFH/NCFSK are plotted in Fig. 5.9. It is evident that as $\gamma_0$ increases, so does the improvement offered by $r = 2$ compared to $r = 1$.

Fig. 5.10 shows the BER versus the number, $r$, of rounds for CAFH/BPSK with $\gamma_0 = 8$ dB and $L = 48$ subbands. As expected, the performance difference between the different rounds gradually decreases with $r$ as the CAFH scheme reaches saturation at a BER of $10^{-8}$ and $2 \times 10^{-6}$ for 10 and 20 MS's, respectively.
To compare the performance of CAFH with an optimal allocation scheme, an exhaustive search method that searches over all the possible \( L(L-1)(L-2)\cdots(L-K+1) \) channel allocation permutations is used. The exhaustive search tries all the different channel allocation possibilities and selects the one that gives the lowest BER, on which the current information bit is transmitted. It can be seen from the results in Fig. 5.11 that the difference in performance between CAFH and the optimal exhaustive search method is very small. However, the channel utilization of CAFH is much higher than that of the exhaustive search method. This can be explained by noticing that CAFH requires the knowledge of the channel gains at each information bit transmission, which requires a single channel measurement session per information bit, regardless of the number of MS's and number of channels. On the other hand, the exhaustive search method requires the knowledge of the BER for each of the different channel allocation permutations. This requires \( L(L-1)(L-2)\cdots(L-K+1) \) channel measurement sessions per information bit. Since the exhaustive search calculates the BER, a sufficient number of information bits is required for transmission. Moreover, system delay should be considered. The time taken to measure the channel, and to transmit the information bits for BER calculation must be less than the channel coherence time, i.e., constant channel behavior. This becomes extremely difficult to achieve for the exhaustive search method. The exhaustive search is a hypothetical approach used only for comparison purposes.
Figure 5.1: A CAFH flow chart describing the proposed subband assignment scheme.
Figure 5.2: BER of conventional FH and CAFH with BPSK modulation, $L = 32$. 

*Legend:*
- $\text{FH} (\gamma_0 = 2 \text{ dB})$
- $\text{FH} (\gamma_0 = 4 \text{ dB})$
- $\text{FH} (\gamma_0 = 6 \text{ dB})$
- $\text{FH} (\gamma_0 = 8 \text{ dB})$
- $\text{CAFH} (\gamma_0 = 2 \text{ dB})$
- $\text{CAFH} (\gamma_0 = 4 \text{ dB})$
- $\text{CAFH} (\gamma_0 = 6 \text{ dB})$
- $\text{CAFH} (\gamma_0 = 8 \text{ dB})$
Figure 5.3: BER of conventional FH and CAFH with NCFSK modulation, $L = 32$. 
Figure 5.4: BER of CAFH with BPSK for \( r = 1 \) and \( 2, L = 32 \).
Figure 5.5: BER of CAFH with NCFSK for $r = 1$ and 2, $L = 32$. 
Figure 5.6: BER of conventional FH and CAFH with BPSK modulation. $\gamma_0 = 2$ dB and $L = 48$. 

<table>
<thead>
<tr>
<th>Number of MS's</th>
<th>BER</th>
<th>Conventional FH</th>
<th>CAFH ($r = 1$)</th>
<th>Simulation CAFH ($r = 1$)</th>
<th>CAFH ($r = 2$)</th>
<th>Simulation CAFH ($r = 2$)</th>
<th>CAFH ($r = 1$)</th>
<th>CAFH ($r = 2$)</th>
<th>Simulation CAFH ($r = 1$)</th>
<th>Simulation CAFH ($r = 2$)</th>
</tr>
</thead>
</table>
Figure 5.7: BER of conventional FH and CAFH with BPSK modulation. $\gamma_0 = 8 \text{ dB}$ and $L = 48$. 
Figure 5.8: BER vs. average SNR, $\gamma_0$, of CAFH with BPSK for $r = 1$ and 2, $L = 48$. 
Figure 5.9: BER vs. average SNR, $\gamma_0$, of CAFH with NCFSK for $r = 1$ and 2, $L = 48$. 
Figure 5.10: BER vs. the number of rounds, \( r \), of CAFH with BPSK. \( \gamma_0 = 8 \) dB and \( L = 48 \).
Chapter 5. A Channel Aware Frequency Hopping Multiple Access Scheme

Figure 5.11: BER versus number of MS's of random allocation, CAFH with BPSK for $r = 1$ and 2, and exhaustive search method. $\gamma_0 = 4$ dB and $L = 8$. 
Chapter 6

Order Statistics for Correlated Fading Channels

6.1 Introduction

In the design of wireless communication systems, much research effort has been devoted to the study of diversity schemes for mitigating the effects of channel fading and improving the received SNR. The theory of order statistics underpins the performance analysis of many diversity reception techniques. Selection combining (SC) [108] is a simple diversity scheme in which the signal from only one of the diversity branches (the branch with the largest SNR) is processed. In signal detection and estimation problems, the cdf of the $r$-th order statistic is often required. For example, it is useful in calculating the performance loss when the receiver makes an error in selecting the largest SNR branch. It is also applicable to the calculation of the average symbol error rates for various modulation schemes.

Most studies on the ordering of channel gains assume that the fading processes on the different branches are uncorrelated. This is partly due to the lack of a simple procedure.

\footnote{A version of this Chapter has been published [86].}
Chapter 6. Order Statistics for Correlated Fading Channels

for analyzing order statistics for fading envelopes with arbitrary cross-correlations. For correlated fading, expressions for the pdf of the highest SNR for two and three branch diversity systems are available [109, 110, 111, 112]. For a larger number of diversity branches, [113] and [108] present an expression for the correlated SC output pdf. In [114], the performance of a hybrid selection/maximal-ratio combining (H-S/MRC) diversity scheme over an exchangeable\(^1\) correlated Nakagami fading channel is analyzed.

Expressions for the cdf of the \(r\)-th order statistic, \(r = 1, 2, \ldots, n\), for correlated Nakagami fading branches are not available in the literature except for \(r = n\) (i.e. the largest order statistic). In this chapter, we apply a result from the theory of order statistics [115] for evaluating the cdf of the \(r\)-th order rv out of a set of \(n\) arbitrarily correlated rv's to the case of correlated Nakagami-\(m\) fading. Specifically, numerical results are obtained which illustrate the effects of the fading parameter \(m\) and the branch correlation matrix.

For a large number of branches, the numerical procedure can be quite computationally intensive. Thus, the accuracy of using a simplifying exchangeable approximation is also investigated.

6.2 Preliminaries

Let \(g_1, \ldots, g_n\) be \(n\) arbitrarily correlated rv's which represent the branch gains in a diversity communication system. If \(g_1, \ldots, g_n\) are arranged in increasing order of their magnitudes and written as

\[
g_{1:n} \leq \cdots \leq g_{n:n},
\]

(6.1)

\(^1\)A set of random variables \(A_1, \ldots, A_n\) are said to be exchangeable if the joint distribution of \(A_{\pi_1}, \ldots, A_{\pi_n}\) does not depend on the permutation \(\pi\).
we refer to $g_{r:n}$ as the $r$-th order branch gain. In this chapter, $g_1, \ldots, g_n$ are assumed to be statistically dependent and not necessarily exchangeable branch gains. The corresponding instantaneous branch SNR's are denoted $\gamma_1, \gamma_2, \ldots, \gamma_n$ with

$$\gamma_i \triangleq [g_i]^2 E/N_0, i = 1, 2, \ldots, n$$

(6.2)

where $E$ is the transmitted bit energy and $N_0$ is the one-sided noise PSD. The $r$-th order branch SNR is

$$\gamma_{r:n} = [g_{r:n}]^2 E/N_0.$$  

(6.3)

The joint cdf of $\gamma_1, \ldots, \gamma_n$ can be expressed as \cite[p. 140]{116}

$$F_{\gamma_1, \ldots, \gamma_n}(v_1, \ldots, v_n) = \frac{1}{(2\pi)^n} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \phi(t_1, \ldots, t_n) \prod_{k=1}^{n} \left( \frac{1-e^{-j_t/v_k}}{j_t} \right) dt_1 \cdots dt_n,$$

(6.4)

where $\phi(t_1, \ldots, t_n)$ is the joint characteristic function (CF) of $\gamma_1, \ldots, \gamma_n$. Hence, the cdf of the highest order statistic $\gamma_{n:n} \triangleq \max\{\gamma_1, \ldots, \gamma_n\}$ can be written as

$$F_{\gamma_{n:n}}(v) = \Pr\{\gamma_{n:n} \leq v\}$$

(6.5)

$$= \Pr\{\text{all } \gamma_i \leq v\}$$

(6.6)

$$= \frac{1}{(2\pi)^n} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \phi(t_1, \ldots, t_n) \prod_{k=1}^{n} \left( \frac{1-e^{-j_t/v_k}}{j_t} \right) dt_1 \cdots dt_n.$$  

(6.7)
6.3 cdf of $\gamma_{r:n}$

6.3.1 Correlated Fading

In this section, we derive an expression for the cdf, $F_{\gamma_{r:n}}$, of the $r$-th order branch SNR, $\gamma_{r:n}$, in terms of $\{F_{\gamma_{j:j}}(v), j = r, r+1, \ldots, n\}$. Recall that the probability, $p_{r:n}$, of the occurrence of at least $r$ out of $n$ events $\{A_1, \ldots, A_n\}$ is given by [117] [11, pp. 109]

$$p_{r:n} = \sum_{j=r}^{n} (-1)^{j-r} \binom{j-1}{r-1} S_j, r = 1, 2, \ldots, n \tag{6.8}$$

where

$$S_j = \sum_{1 \leq i_1 < \cdots < i_j \leq n} \Pr\{A_{i_1}, \ldots, A_{i_j}\}. \tag{6.9}$$

In (6.9), the summation is over all different ways of selecting $j$ out of $n$ rv's. Letting $A_i, i = 1, \ldots, n$ denote the event $\{\gamma_i \leq v\}$, (6.9) can be re-written as

$$S_j = \sum_{1 \leq i_1 < \cdots < i_j \leq n} \Pr\{\gamma_{i_1} \leq v, \ldots, \gamma_{i_j} \leq v\} \tag{6.10}$$

$$= \sum_{1 \leq i_1 < \cdots < i_j \leq n} \Pr\{\max(\gamma_{i_1}, \ldots, \gamma_{i_j}) \leq v\} \tag{6.11}$$

$$= \sum_{1 \leq i_1 < \cdots < i_j \leq n} \Pr\{\gamma_{i_1, \ldots, i_j}^{(i_1, \ldots, i_j)} \leq v\} \tag{6.12}$$

$$= \sum_{1 \leq i_1 < \cdots < i_j \leq n} F_{\gamma_{i_1, \ldots, i_j}}^{(i_1, \ldots, i_j)}(v), \tag{6.13}$$

In (6.13), the sum is over all possible ways of selecting $j$ out of $n$ rv's and the superscript notation in $F_{\gamma_{i_1, \ldots, i_j}}^{(i_1, \ldots, i_j)}$ indicates that only $\gamma_{i_1}, \ldots, \gamma_{i_j}$ are included in the selection. It is
assumed that the sum is equal to $F_{r:n}(v)$ for $j = n$. The term $F_{r:j}(v)$ is obtained using (6.7). From the definition of $A_i$, we can write

$$p_{r,n} = \Pr\{\gamma_1:n \leq v, \cdots, \gamma_r:n \leq v\}$$

(6.14)

$$= \Pr\{\gamma_r:n \leq v\}$$

(6.15)

$$= F_{r:n}(v).$$

(6.16)

Following the result in [115, p. 99], and substituting (6.13) and (6.16) into (6.8), we can obtain the cdf of the $r$-th order statistic

$$F_{r:n}(v) = \sum_{j=r}^{n} \left[ (-1)^{j-r} \binom{j-1}{r-1} \sum_{1 \leq i_1 < \cdots < i_j \leq n} F_{\gamma_{i_1}, \cdots, \gamma_{i_j}}(v) \right].$$

(6.17)

When $n$ is large, the evaluation of (6.17) may be time-consuming. However, if the $\gamma_i$'s are exchangeable, (6.17) can be easily evaluated as it reduces to

$$F_{r:n}(v) = \sum_{j=r}^{n} (-1)^{j-r} \binom{j-1}{r-1} \binom{n}{j} F_{\gamma_{i_j}}(v).$$

(6.18)
6.3.2 Independent Fading

Suppose that \( g_1, \ldots, g_n \) are \( n \) independent, but not necessarily identically distributed rv's, (6.7) simplifies to

\[
F_{\gamma_{n:n}}(v) = \Pr\{\gamma_1, \ldots, \gamma_n \leq v\}
\]

\[
= \Pr\{\gamma_1 \leq v\} \cdot \ldots \cdot \Pr\{\gamma_n \leq v\}
\]

\[
= \prod_{k=1}^{n} F_{\gamma_k}(v).
\]

By substituting (6.19) in (6.17) we get

\[
F_{\gamma_{r:n}}(v) = \sum_{j=r}^{n} (-1)^{j-r} \left( \begin{array}{c} j-1 \end{array} \right) \sum_{1 \leq i_{j+1} < \ldots < i_n \leq n} \prod_{k=1}^{j} F_{\gamma_{i_k}}(v).
\]

If the channel gains are now assumed to be independent, identically distributed (i.i.d) rv's, using the property of exchangeability, (6.20) reduces to

\[
F_{\gamma_{r:n}}(v) = \sum_{j=r}^{n} (-1)^{j-r} \left( \begin{array}{c} j-1 \end{array} \right) \left( \begin{array}{c} n \end{array} \right) [F_{\gamma}(v)]^j.
\]

6.4 Numerical results

To illustrate the application of the results, we evaluate the cdf of the \( r \)-th order statistic for correlated Nakagami-\( m \) fading branches. In this case, the joint CF is given by [118, p. 359]

\[
\phi(t_1, \ldots, t_n) = |I - jTS|^{-m}
\]

(6.22)
where \( I \) is the identity matrix of size \( n \times n \), \(|\cdot|\) denotes the determinant, \( T = \text{diag}\{t_1, t_2, \ldots, t_n\}, \) \( m \in [0.5, \infty) \), and \( S \) is a symmetric matrix with elements

\[
S_{k,\ell} = \sqrt{\frac{R_{k,\ell}}{m}}, \tag{6.23}
\]

In (6.23), \( R_{k,\ell} \triangleq \text{Cov}(\gamma_k, \gamma_\ell) \) is the element in row \( k \) and column \( \ell \) of the correlation matrix \( R \).

Of particular interest for wireless communication systems are the values \( m = \frac{1}{2} \), \( m = 1 \), and \( m > 1 \). The Nakagami distribution is a one-sided Gaussian distribution (severe fading) for \( m = \frac{1}{2} \), a Rayleigh distribution for \( m = 1 \), and a good approximation to a Rician distribution when \( m > 1 \).

### 6.4.1 Constant Correlation Model

The constant correlation model, discussed in [119, 120, 121], may be used either to approximate closely spaced antennas or to perform a first-order analysis using the average value of the correlation coefficients for all the off-diagonal entries of the correlation matrix [121]. Suppose we have a diversity receiver with an array of \( n = 3 \) antennas placed at the vertices of an equilateral triangle. Then the branch gains are exchangeable Nakagami-\( m \) rv's and the cdf can be obtained using (6.18). As an example, we use the correlation
matrix from [121]

\[
R = \bar{\gamma} \begin{bmatrix} 1.0 & 0.6 & 0.6 \\ 0.6 & 1.0 & 0.6 \\ 0.6 & 0.6 & 1.0 \end{bmatrix}
\] (6.24)

where \( \bar{\gamma} \) denotes the average SNR.

Fig. 6.1 shows the cdf of the \( r \)-th order statistic in a correlated Nakagami-\( m \) fading environment with the correlation matrix in (6.24), \( m = 1 \) and \( \bar{\gamma} = 1 \). Computer simulation results (shown as dotted curves) were used to verify the analysis results (solid curves). As to be expected, it can be seen that a lower order statistic, e.g. \( \gamma_{1:3} \), has a steeper curve than a higher order statistic, e.g. \( \gamma_{3:3} \). In a SC system, this provides the performance loss when the receiver incorrectly selects a branch with a lower gain instead of the highest gain branch.

Fig. 6.2 shows the cdf of the \( r \)-th order statistic in a correlated Nakagami-\( m \) fading environment (solid curves) with the correlation matrix in (6.24), \( m = 1 \) and \( \bar{\gamma} = 1 \). The cdf curves for independent fading (dotted curves) are obtained using (6.21) and are shown for comparison. When the highest SNR branch is selected (i.e. \( \gamma_{3:3} \)), the performance is better when the branches are independently fading; when the lowest SNR branch is selected (i.e. \( \gamma_{1:3} \)), correlated fading yields better performance.

The cdf of \( \gamma_{1:3} \) is plotted in Fig. 6.3 for different values of \( m \) in correlated fading using the correlation matrix in (6.24). It can be seen that the performance degrades as fading severity increases (i.e. \( m \) decreases).

Calculating the cdf of the \( r \)-th order statistic can be applied to evaluating the average
error performance of various modulation schemes in correlated fading channels. In diversity applications, this provides the performance loss when the receiver incorrectly selects a channel with a lower gain. In multiuser diversity applications, this is a good measure for the channel gain loss from one user to another, which implies a total throughput improvement when selecting only the user(s) with stronger instantaneous SNR.

### 6.4.2 Linear Model

This type of correlation model applies to linear antenna arrays. We consider the correlation matrix example for a five-element array in [122]

\[
R = \begin{bmatrix}
1.000 & 0.795 & 0.605 & 0.375 & 0.283 \\
0.795 & 1.000 & 0.795 & 0.605 & 0.375 \\
0.605 & 0.795 & 1.000 & 0.795 & 0.605 \\
0.375 & 0.605 & 0.795 & 1.000 & 0.795 \\
0.283 & 0.375 & 0.605 & 0.795 & 1.000
\end{bmatrix}
\] (6.25)

The correlation coefficient value decreases as the distance between array elements increases. In this case, the channel SNR rv's are not exchangeable and the cdf has to be obtained using (6.17). Using the correlation matrix in (6.25), Fig. 6.4 shows the cdf for \(\gamma_{1:5}, \gamma_{2:5}, \ldots, \gamma_{5:5}\) with \(m = 1\) and \(\overline{\gamma} = 1\) (solid curves). Approximate cdf curves (dotted curves), obtained using (6.18) and a 5 x 5 exchangeable matrix in which an average value of 0.514 is used for all off-diagonal entries, are also plotted. It can be seen that there is a significant difference between the exact and exchangeable approximation cdf curves for
\gamma_{7.5} \text{ and } \gamma_{5.5}.
Figure 6.1: Plot of cdf of $\gamma_{1:3}, \gamma_{2:3}$ and $\gamma_{3:3}$ with $m = 1$ and $\overline{\gamma} = 1$: theoretical (dotted) and simulation (solid).
Figure 6.2: Plot of cdf of $\gamma_{1:3}$, $\gamma_{2:3}$ and $\gamma_{3:3}$ with $m = 1$ and $\gamma = 1$: independent (dotted) and correlated (solid) fading channels.
Figure 6.3: Plot of cdf of $\gamma_{1:3}$ with $\overline{\gamma} = 1$ for different values of $m$. 
Figure 6.4: Plot of cdf of $\gamma_{r;5}$ with $m = 1$ and $\bar{\gamma} = 1$ for a linear model (solid) and its exchangeable approximation (dotted).
Chapter 7

Error Analysis of Order Selection in Correlated Fading Channels

7.1 Introduction

In order to increase system's capacity and to improve the offered QoS in wireless communication systems, several techniques have been proposed to mitigate the effects of channel fading and to improve the received SNR. Such techniques are diversity reception, dynamic channel allocation, and power control. The theory of order statistics underpins the performance analysis of many diversity reception techniques that involve efficient channel allocation and signal processing algorithms for signal detection and estimation.

Expressions for the cdf of the $r$-th order statistic, $r = 1, 2, \ldots, L$, for correlated Nakagami fading branches have appeared in [86]. In this chapter, we apply a general result from the theory of order statistics [86] and [115] to evaluate the BER of a channel aware diversity system, hereafter referred to as order selection, when the $r$-th order branch is selected for transmission when using BPSK modulation. The expressions derived are then used to evaluate the error performance for CAFH. The CAFH evaluation is carried out assuming correlated Nakagami-$m$ fading when the coherence bandwidth notably exceeds
the transmission bandwidth, which is the real scenario in practical CAFH systems.

### 7.2 Channel and System Model

Let $g_1, g_2, \ldots, g_L$ be $L$ arbitrarily correlated rv's which represent the branch gains in a diversity communication system. The corresponding branch SNR's are denoted by $\gamma_1, \gamma_2, \ldots, \gamma_L$, with

$$\gamma_i \triangleq [g_i]^2 E/\text{No}, i = 1, 2, \ldots, L \quad (7.1)$$

where $E$ is the transmitted bit energy and $N_0$ is the one-sided noise PSD. Consider an $L$-branch diversity receiver of a cellular system in which $K$ MS's transmit over $L$, $L \geq K$, branches to the BS. The received base-band signal on the $\pi_k$-th branch of the $k$-th MS at the BS is

$$r_{\pi_k}(t) = g_{\pi_k} s_k(t) e^{-j\psi_{\pi_k}} + n(t), \quad (7.2)$$

where $s_k(t)$ is the transmitted signal of the $k$-th MS and $n(t)$ is independently identically distributed AWGN with zero mean and one-sided spectral density $N_0$. The random phase $\psi_{\pi_k}$ is uniformly distributed over the range $[0, 2\pi)$ and $g_{\pi_k}$ is the random magnitude of the $\pi_k$-th diversity branch gain. Here we model the $g_{\pi_k}$'s as correlated rv's with a marginal Nakagami pdf given by [123]

$$f_{g_{\pi_k}}(v) = \frac{2}{\Gamma(m_{\pi_k})} \left( \frac{m_{\pi_k}}{\text{No}} \right)^{m_{\pi_k}} v^{2m_{\pi_k} - 1} e^{-(m_{\pi_k}/\text{No})v} v^2, \quad v \geq 0 \quad (7.3)$$

where the fading parameter $m_{\pi_k}$ is assumed to be a positive integer, $\Gamma(\cdot)$ is the Gamma
function defined by $\Gamma(p) = \int_0^\infty v^{p-1}e^{-v}dv$, and $\Omega_{\pi_k} = \mathbf{E}[(g_{\pi_k})^2]$, $\mathbf{E}[\cdot]$ being the expectation factor.

Let $\gamma_{\pi_k}$ denote the instantaneous signal-to-noise ratio (SNR) on the $\pi_k$-th branch, expressed as

$$\gamma_{\pi_k} = \frac{E}{N_0} (g_{\pi_k})^2,$$  \hspace{1cm} (7.4)

where $E/N_0$ is the ratio of the bit energy to the Gaussian noise spectral density. It is well known that the marginal pdf of $\gamma_{\pi_k}$ follows the Gamma distribution

$$f_{\gamma_{\pi_k}}(v) = \frac{m_{\pi_k}^m}{\Gamma(m_{\pi_k})} \frac{v^{m_{\pi_k}-1}}{(\gamma_{\pi_k})^m_{\pi_k}} \cdot e^{-m_{\pi_k}(v/\gamma_{\pi_k})}, \quad v \geq 0$$  \hspace{1cm} (7.5)

where the average received SNR on the $\pi_k$-th branch is given by $\overline{\gamma_{\pi_k}} = \mathbf{E}[\gamma_{\pi_k}] = \frac{E}{N_0} \Omega_{\pi_k}$.

### 7.3 Error Performance for Order Selection BPSK

If the rv's $g_1, \cdots, g_L$ are arranged in increasing order of their magnitudes and written as

$$g_{1:L} \leq \cdots \leq g_{L:L},$$  \hspace{1cm} (7.6)

we refer to the $r$-th order rv, $g_{r:L}$, as the $r$-th order statistic ($r = 1, \cdots, L$). In this section, $g_1, \cdots, g_L$ are assumed to be statistically dependent and not necessarily exchangeable branch gains. Accordingly, the SNR of the $r$-th order statistic is then

$$\gamma_{r:L} = [g_{r:L}]^2 E/N_0.$$  \hspace{1cm} (7.7)
Chapter 7. Error Analysis of Order Selection in Correlated Fading Channels

The average error probability when transmitting on the \( r \)-th order branch out of \( L \) branches for a BPSK modulation over an AWGN channel with two-sided PSD \( N_0/2 \) is obtained by averaging the conditional error probability over the pdf of \( \gamma_{r:L} \), i.e.,

\[
P_e^{r:L} = \int_0^\infty P(e|v) f_{\gamma_{r:L}}(v) dv, \tag{7.8}
\]

where \( P(e|v) \) is the conditional error probability. By differentiating (6.17), we can write the pdf of \( \gamma_{r:L} \) as

\[
f_{\gamma_{r:L}}(v) = \frac{dF_{\gamma_{r:L}}(v)}{dv} \]

\[
= \sum_{i=r}^{L} \left[ (-1)^{i-r} \binom{i-1}{r-1} \sum_{1 \leq \pi_1 < \cdots < \pi_i \leq L} \frac{dF_{\gamma_{\pi_1,\ldots,\pi_i}}(v)}{dv} \right] \tag{7.9}
\]

where the pdf of the largest order statistic, \( \gamma_{L:L} \), out of \( L \) rv's is given by [108]

\[
f_{\gamma_{L:L}}(v) = \frac{1}{(2\pi)^L} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \phi(t_1, \ldots, t_L) h(v, t) dt_1 \cdots dt_L, \tag{7.10}
\]

with

\[
h(v, t) = \frac{d}{dv} \left[ \prod_{\ell=1}^{L} \left( \frac{1-e^{-t\pi \nu}}{j\pi \nu} \right) \right] \]

\[
= \left[ \prod_{\ell=1}^{L} (j\ell \pi) \right]^{-1} \left[ \sum_{n=1}^{L} (-1)^{n+1} \times \sum_{b_1+\cdots+b_L=n} \exp(j(b_1t_1+\cdots+b_Lt_L))) \right]. \tag{7.11}
\]
Chapter 7. Error Analysis of Order Selection in Correlated Fading Channels

Inserting (7.9) into (7.8) yields

\[
P_{e}^{r-L} = \sum_{i=r}^{L} (-1)^{r-i} \binom{L-1}{r-1} \times \\
\sum_{1 \leq \pi_{1} < \cdots < \pi_{r} \leq L} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \phi^{(\pi_{1}, \cdots, \pi_{r})}(t_{\pi_{1}}, \cdots, t_{\pi_{r}}) w^{(\pi_{1}, \cdots, \pi_{r})}(t_{\pi_{1}}, \cdots, t_{\pi_{r}}) dt_{1} \cdots dt_{\pi_{r}}
\]

(7.12)

with

\[
w^{(\pi_{1}, \cdots, \pi_{r})}(t_{\pi_{1}}, \cdots, t_{\pi_{r}}) = \frac{1}{(2\pi)^{r}} \int_{0}^{\infty} P(e|\gamma) h^{(\pi_{1}, \cdots, \pi_{r})}(v, t) dv.
\]

(7.13)

In (7.12) the integrand is decomposed into two factors. The first factor \(\phi(t)\) is the CF, dependent solely on the channel characteristics. The second is the weighting function \(w(t)\); it depends solely on the modulation scheme. The conditional error probability for BPSK is given by [89]

\[
P(e|\gamma) = Q(\sqrt{2\gamma}).
\]

(7.14)

It follows from inserting (7.14) into (7.13) that

\[
w^{(\pi_{1}, \cdots, \pi_{r})}(t_{\pi_{1}}, \cdots, t_{\pi_{r}}) = \frac{1}{(2\pi)^{r}} \int_{0}^{\infty} Q(\sqrt{2v}) h^{(\pi_{1}, \cdots, \pi_{r})}(v, t) dv.
\]

(7.15)

Integration by parts as

\[
w^{(\pi_{1}, \cdots, \pi_{r})}(t_{\pi_{1}}, \cdots, t_{\pi_{r}}) = \frac{1}{(2\pi)^{r}} \left[ \prod_{\ell=1}^{r} (jt_{\pi_{\ell}}) \right]^{-1} \times \sum_{n=1}^{i} (-1)^{n+1} \\
\sum_{b_{\pi_{1}} + \cdots + b_{\pi_{r}} = \pi} \int_{0}^{\infty} Q(\sqrt{2v}) \cdot \frac{j(b_{\pi_{1}} t_{\pi_{1}} + \cdots + b_{\pi_{r}} t_{\pi_{r}})}{\exp(jv[b_{\pi_{1}} t_{\pi_{1}} + \cdots + b_{\pi_{r}} t_{\pi_{r}}])} dv.
\]

(7.16)
yields

\[ w(\pi_1, \cdots, \pi_i)(t_{\pi_1}, \cdots, t_{\pi_i}) = \frac{1}{(2\pi)^i} \left[ \prod_{\ell=1}^{i} (jt_{\pi_{\ell}}) \right]^{-1} \times \sum_{n=1}^{i} (-1)^{n+1} \sum_{b_{\pi_1} + \cdots + b_{\pi_i} = n}^{\frac{1}{2}} \left[ 1 - \frac{1}{\sqrt{\pi}} \int_{0}^{\infty} e^{-u(1+j)b_{\pi_1} t_{\pi_1} + \cdots + b_{\pi_i} t_{\pi_i}} du \right]. \]

\[ (7.17) \]

Letting \( x = \sqrt{v} \), we obtain

\[ w(\pi_1, \cdots, \pi_i)(t_{\pi_1}, \cdots, t_{\pi_i}) = \frac{1}{(2\pi)^i} \left[ \prod_{\ell=1}^{i} (jt_{\pi_{\ell}}) \right]^{-1} \times \sum_{n=1}^{i} (-1)^{n+1} \sum_{b_{\pi_1} + \cdots + b_{\pi_i} = n}^{\frac{1}{2}} \left[ 1 - \frac{1}{\sqrt{\pi}} \int_{0}^{\infty} e^{-x^2(1+j) b_{\pi_1} t_{\pi_1} + \cdots + b_{\pi_i} t_{\pi_i}} dx \right]. \]

\[ (7.18) \]

Using

\[ \frac{1}{\sqrt{2\pi\sigma^2}} \int_{0}^{\infty} e^{-x^2/2\sigma^2} dx = \frac{1}{2} \]

in (7.18) with \( \sigma^2 = \frac{1}{2(1+jb_{\pi_1} t_{\pi_1} + \cdots + b_{\pi_i} t_{\pi_i})} \), we have

\[ w(\pi_1, \cdots, \pi_i)(t_{\pi_1}, \cdots, t_{\pi_i}) = \frac{1}{(2\pi)^i} \left[ \prod_{\ell=1}^{i} (jt_{\pi_{\ell}}) \right]^{-1} \times \sum_{n=1}^{i} (-1)^{n+1} \sum_{b_{\pi_1} + \cdots + b_{\pi_i} = n}^{\frac{1}{2}} \left[ 1 - \frac{1}{\sqrt{1+j(b_{\pi_1} t_{\pi_1} + \cdots + b_{\pi_i} t_{\pi_i})}} \right]. \]

\[ (7.20) \]

The error performance can then be obtained by inserting \( w(\pi_1, \cdots, \pi_i)(t_{\pi_1}, \cdots, t_{\pi_i}) \) shown in (7.20) and \( \phi(\pi_1, \cdots, \pi_i)(t_{\pi_1}, \cdots, t_{\pi_i}) \) for a given operational environment into (7.12).
7.4 Numerical Results and Discussion

In this section, numerical examples are provided for several correlation models, well known from practical diversity systems, in order to provide the applicability and check the usefulness of the proposed analysis. We consider the uplink of a cellular system in which a MS transmits over $L = 8$ subbands to the BS. For a given MS, we assume that the various subbands are subject to correlated fading, where the amount of correlation depends on, among other things, the subband frequency separation. The correlation at any instant of time between the fade envelopes, $g_i$ and $g_j$ of the $i^{th}$ and $j^{th}$ subbands, respectively, is taken to be [124]

$$E\{g_i, g_j\} = \frac{1}{1 + \left(\frac{f_i - f_j}{B_c}\right)^2} \equiv \rho_{i,j} \quad (7.21)$$

where $B_c$ is the coherence bandwidth of the channel. Therefore, the $L \times L$ covariance matrix $\mathbf{R}$ is given by

$$\mathbf{R} = \begin{bmatrix}
1 & \rho_{1,2} & \cdots & \rho_{1,L} \\
\rho_{2,1} & 1 & \cdots & \rho_{2,L} \\
\vdots & \vdots & \ddots & \vdots \\
\rho_{L,1} & \rho_{L,2} & \cdots & 1
\end{bmatrix}.$$
where
\[ \rho_{i,i} = 1, \quad 1 \leq i \leq L, \]

and \( \mathbf{R} \) is symmetric and positive definite [106]. For correlated Nakagami-\( m \) fading channels, the joint CF is given by (6.22). We shall assume that each subband experiences the same extent of fading, i.e., \( m_\ell = m \) for \( \ell = 1, 2, \ldots, L \).

### 7.4.1 BER evaluation for Order Selection BPSK

Substituting the component correlation coefficients into the joint CF expression (6.22) and using BER expression (7.12), we will see how those parameters affect BER performance in a correlated Nakagami-\( m \) fading channel. To illustrate the application of the presented mathematical model, we evaluate the BER, \( P_{e,r}^L \), when transmitting on the \( r \)-th order subband out of \( L \) available subbands for a BPSK modulation in \( m = 1 \) correlated fading environment.

In Fig. 7.1 we select \( B_n = B_s/B_c = 0.9 \), where \( B_n \) is the subband bandwidth normalized by the coherence bandwidth of the channel, and the bandwidth of each subband is given by \( B_s \). Fig. 7.1 shows the BER, \( P_{e,r}^L \), of BPSK when \( r = 8, 7, 4, 1 \) for \( L = 8 \) subbands. In Fig. 7.1, \( P_{e,r}^L \) is plotted versus the SNR in correlated fading environment (solid line). In the same figure, \( P_{e,r}^{r:L} \) is depicted for an uncorrelated fading environment (dotted line). For comparison, the BER of conventional BPSK without ordering is plotted (dashed line).

When the highest SNR subband is selected (i.e. \( \gamma_{8:8} \)), the performance is better when the subbands are independently fading. On the other hand, if the lowest SNR subband
is selected (i.e., $\gamma_{1:8}$), correlated fading yields better performance. It can also be seen that the performance difference between correlated and uncorrelated fading subbands increases with $r$. At a SNR of 10 dB, operating in an uncorrelated fading environment provides roughly a 20-fold, 7-fold and 2-fold reduction in BER compared to a correlated case when operating at $r = 8, 7$ and 4, respectively. At $r = 1$ the performance difference is quite negligible.

To demonstrate the impact of the fading correlation to the error performance, $B_n = 0.5$ is chosen in Fig. 7.2. It is evident that the effect of the correlation to the BER performance is greater for larger correlation coefficients. As expected, the BER worsens for smaller $B_n$ values when the correlation coefficient increases. For example when transmitting at a SNR of 10 dB, the performance difference is roughly 100-fold, 30-fold and 4-fold between uncorrelated and correlated fading channels at $r = 8, 7$ and 4, respectively. At $r = 1$ the performance difference is more noticeable. Following the same trend, the BER for $B_n = 0.25$ with even a larger performance difference between the correlated and uncorrelated cases is depicted in Fig. 7.3.

### 7.4.2 Application: BER evaluation for CAFH

The presented CAFH analysis is carried out assuming correlated Nakagami-$m$ fading when the coherence bandwidth notably exceeds the transmission bandwidth, which is the real scenario in practical CAFH systems. The BER for CAFH with one round can
Chapter 7. Error Analysis of Order Selection in Correlated Fading Channels

be calculated as [83]

\[ P_{CAFH} = \sum_{k=1}^{K} \frac{1}{k} z(k) P_e^{kL,kL} + \left( 1 - \sum_{k=1}^{K} \frac{1}{k} z(k) \right) \frac{1}{K-1} \sum_{i=L-K}^{L-1} P_e^{iL}, \quad (7.22) \]

where the first term in (7.22) represents the BER for the case when a subband is assigned to the target MS in round 1. Accordingly, when the target MS was not considered in round 1 (i.e., a MS with a larger gain occupied the same subband as target MS), the target MS is chosen by the BS in a random order and is assigned to an unoccupied subband, this constitutes the second term in (7.22). And

\[ z(k) = \binom{K-1}{k-1} \binom{1}{1} \binom{L-1}{L} k^{-1} \]

represents the probability that \( k - 1 \) other MS's occupy the subband of the target MS in round 1.

To illustrate the sensitivity of CAFH to the fade envelopes correlation, Figs. 7.4 and 7.5 plot the BER performance for CAFH versus number of MS's for \( B_n = 0.9 \) and \( B_n = 0.25 \), respectively when \( L = 8 \). For smaller \( L \), Fig. 7.6 plots the BER performance for \( B_n = 0.33 \) and \( L = 3 \). It is observed that the performance difference between correlated and uncorrelated fading channels increases with \( \bar{\gamma} \). For a fixed \( \bar{\gamma} \), the performance difference decreases with the number of MS's.
Chapter 7. Error Analysis of Order Selection in Correlated Fading Channels

Figure 7.1: BER vs. average SNR for $B_n = 0.9$. 
Figure 7.2: BER vs. average SNR for $B_n = 0.5$. 
Figure 7.3: BER vs. average SNR for $B_n = 0.25$. 

- Conventional BPSK without ordering
- BPSK with ordering (correlated fading channels)
- BPSK with ordering (uncorrelated fading channels)
Figure 7.4: CAFH BER vs. number of mobile stations, for $\bar{\gamma} = 2$ and $8 \text{ dB}$, and $B_n = 0.9$. 
Chapter 7. Error Analysis of Order Selection in Correlated Fading Channels

Figure 7.5: CAFH BER vs. number of MS's, for $\bar{\gamma} = 2$ and $8$ dB, and $B_n = 0.25$. 
Figure 7.6: CAFH BER vs. number of MS's, for $\overline{\gamma} = 2$ and 10 dB, and $B_n = 0.33$. 
Chapter 8

Conclusions and Future Research

We conclude this thesis with a summary of our contributions and suggestions for future work.

8.1 Summary

The main objectives of this thesis were:

- The proposal of new methods for the improvement of MC-CDMA as an effective signaling scheme.
- The development of new channel aware systems with adaptive subchannel allocation.
- The investigation of statistics of general order selection in correlated Nakagami fading channels.

The following is a summary of Chapters 2-7. In Chapter 2, we have presented a new MC-CDMA scheme, *matched* MC-CDMA. In matched MC-CDMA, preference is given to the use of subcarriers with low channel attenuation. The BER as a function of the number of users and the SNR for the proposed scheme and conventional MC-CDMA
was investigated. It was found that matched MC-CDMA provides a lower BER. The performance difference between matched and conventional MC-CDMA increases as the target BER decreases. The results in this chapter also have appeared in [80].

The performance of a new FH-MC-CDMA scheme for transmission over a Rayleigh fading channel has been analyzed in Chapter 3. The proposed system can combine efficiently the techniques of slow FH, OFDM, and MC-CDMA. Both FH and MC-CDMA can be viewed as special cases of the proposed scheme. The performance of FH-MC-CDMA using EGC and MRC was evaluated and compared with those of corresponding FH and MC-CDMA systems. The results show that FH-MC-CDMA is an attractive wireless multiple access candidate, which can be integrated into existing 2G and 3G CDMA systems. FH-MC-CDMA provides a lower BER than conventional FH and a comparable BER to MC-CDMA while employing a lower number of subcarriers. This can avoid some of the disadvantages of MC-CDMA. Results of this work have been published in [81].

Chapter 4 has presented a detailed architecture for a modified FH-MC-CDMA system as a new transmission scheme for wireless communications. We propose a modification of the FH-MC-CDMA scheme in [81] that is suitable for use in correlated fading channels. Both FH and MC-CDMA can be viewed as special cases of the proposed generalized FH-MC-CDMA when the number of subcarrier in a group \( j \), \( N_{Gj} = 1 \), and \( N_{Gj} = s \), respectively. It is found that \( N_{Gj} = 1 \) and \( N_{Gj} = s \) are not favorable choices for \( N_{Gj} \). Optimum error performance is achieved by varying \( N_{Gj} \) between 1 and \( s \) while keeping the number of MS’s in a group \( j \), \( M_{Gj} = 1 \), \( \forall j \). The BER performance of FH-MC-
CDMA using MRC on an uplink correlated Rayleigh fading channel has been compared with that of FH and MC-CDMA, respectively. The results show that FH-MC-CDMA can achieve a much lower BER than FH and MC-CDMA whilst employing a smaller number of subcarriers in a group. This work has been published in [82].

In Chapter 5, we have presented an effective access scheme for slowly time-varying, frequency selective channels. A CAFH multiple access scheme was proposed for independently Rayleigh faded subbands. CAFH involves the ordering of subband gains of the MS's. In CAFH, the channel status is monitored at the base station (BS). The BS then determines the subband gains for each MS and uses this knowledge to assign to each MS the subband that enjoys the highest gain, while ensuring that no more than one MS is assigned to the same subband. It was shown that CAFH can provide a much lower BER than conventional FH over a wide range of SNR values and number of MS's. This is achieved by exploiting information about the MS's subband gains. The performance difference between conventional and CAFH increases with SNR. For a fixed target BER, the number of MS's which can be supported by CAFH is much larger than that with conventional FH. This work has also appeared in [83] and [84]. The performance of CAFH with \( r \) rounds over slowly time-varying, frequency selective channels is analyzed. In [83], the analysis was limited to just one round. In this chapter, closed-form expressions that are recursively-based are derived to evaluate the BER for CAFH with \( r \) rounds and its performance in a cellular communication system is studied. The BER improvement achieved when using \( r > 1 \) is depicted. Numerical results show that the improvement obtained using \( r > 1 \) is considerable compared to the \( r = 1 \) case. Moreover,
it is found that CAFH reaches saturation at larger \( r \) values; the dependence of the system performance on \( r \) is less significant for large \( r \). This work has been reported in [85].

In chapter 6, the cdf (and hence outage probability) of the \( r \)-th order branch SNR in correlated Nakagami–\( m \) fading has been studied. The accuracy of a simple exchangeable approximation to reduce the computational load has been examined. Results from this work has been reported in [86] and [125].

Chapter 7 investigates the effects of correlation on the error performance of channel aware systems that involve the ordering of channel gains in correlative Nakagami fading environments. A useful analytical formula for the BER of the \( r \)-th order statistic of a set of arbitrarily correlated and not necessarily exchangeable diversity branch gains is described and may be efficiently applied in the performance analysis of various diversity systems. In the case of coherent BPSK, a closed-form expression is extracted for the BER when transmitting on the \( r \)-th order branch. This result is then employed for the BER calculation of CAFH. Depending on the level of correlation, it can be observed from the results in Figs. 7.1, 7.2, and 7.3 that the performance over the \( r \)-th order branch is significantly degraded under correlated fading conditions when compared to the uncorrelated fading case, as a consequence, the average BER performance of CAFH is noticeably affected. This work has been submitted for publication [126].

In conclusion, among the main contributions of this thesis, we proposed:

- Matched CDMA and FH-MC-CDMA for wireless communications.

- Channel aware frequency hopping (CAFH) as an effective access scheme for correlated and independently faded subbands.
• BER of order selection when the r-th order branch is selected for transmission.

8.2 Future Work

We present below a list of research problems that can be investigated as possible extensions for research work reported in this thesis.

1. The use of multiple antennas to provide antenna diversity over a point-to-point wireless link has been studied extensively since the early 1960s. It is still a popular research topic for improving energy efficiency through the use of both transmit and receive diversity. Recently, with the increasing demand for high-speed wireless data services, antenna diversity in a multiuser data network has received particular attention [127][128], as it requires a broader multilink perspective for the spatial diversity gain in the system [79]. Recent progress in information theory [49][129] has shown that in a data network with multiple active users requesting services simultaneously, multiuser diversity can be obtained by a packet scheduler, which always allocates the common radio resource to the user having the best channel quality, on the condition that the instantaneous user channel quality information (CQI) in terms of SNR is available to the scheduler. Multiple transmit antennas can be used either to obtain transmit diversity or to form MIMO channels. We may refer to this multiuser scheduling scheme as greedy scheduling. Adaptive loading (or adaptive modulation) technique is an efficient technique to achieve power and rate optimization based on the sub-channel gains. Adaptive bit allocation using the simple Greedy method gives an optimal solution for a single user system. Adaptive
Chapter 8. Conclusions and Future Research

Bit allocation using the Greedy method cannot give an optimal solution for multiuser cases. It is probable that the sub-channels with the largest channel gain for one user also be the largest for another user. A practical channel-aware scheduling algorithm, proportional fair (PF) scheduling [79][130], has been used in the downlink of cellular packet data system IS-856 [131] [also known as 1xEV-DO or high data rate (HDR)], which trades some total system throughput for resource fairness among the active users. When users cannot share the same sub-channel at the same time, it is a difficult problem to find the best way to assign the sub-channels to the users. It would be interesting to study the improvement in average throughput due to multiuser diversity in MIMO cellular systems by assigning the transmit antennas to the users using the channel aware algorithm described in Chapter 5.

2. Multiuser diversity helps on two accounts. First, one can pick the user(s) that is (are) in a favorable channel condition, thereby increasing the network throughput for the forward link. Second, one can exploit the multiuser diversity to decrease feedback requirements in the reverse link. This novelty has been introduced in [79], where the transmitter pre-codes in a pseudo random fashion and that user is picked whose ratio of instantaneous data rate to its own average data rate is the largest. This idea can be extended to include users with different ratios. Hence, studying the effect of choosing users with different SNR levels on the network throughput. Using the concepts covered in Chaper 7, expressions for the BER and throughput may be obtained.

3. High PAPR of the transmit signal is a major drawback of multicarrier transmis-
sion such as OFDM, MC-CDMA and discrete multitone (DMT). It is possible to avoid high PAPR signals by employing a technique named clustered OFDM [132][133][134]. In this technique the subcarriers are clustered into several smaller blocks and transmitted over separate antennas. The PAPR is reduced since there are fewer subcarriers per transmitter. FH-MC-CDMA is another method that may be applied to PAPR reduction in multicarrier systems, as FH-MC-CDMA generally uses a smaller number of subcarriers. Since FH-MC-CDMA requires a single antenna for transmission, it may provide an effective method for reducing both cost and complexity when compared to clustered OFDM.

4. Multiple transmit and receive antennas can be used to improve the performance and increase the capacity of wireless communications systems. It is shown that when multiple transmit and receive antennas are used to form a MIMO system, the system capacity can be improved by a factor proportional to the minimum of the number of transmit and receive antennas compared to a single-input single-output (SISO) system over a flat narrowband Rayleigh fading channel [135][136]. However, for wideband channels OFDM has to be used with MIMO techniques for intersymbol interference mitigation and capacity improvement. This MIMO-OFDM system is investigated in [137][138][139]. An alternative to OFDM is FH-MC-CDMA that can be applied to MIMO technologies. In principle, FH-MC-CDMA and MIMO can be synergistically integrated to offer the benefits of both system simplicity and high performance. MIMO-FH-MC-CDMA could be effective in reducing ICI as well as in obtaining a diversity gain even for highly-correlated fast fading channels. Since
MIMO-OFDM systems are based on OFDM, they also suffer from the problem of inherent PAPR. MIMO-FH-CDMA can reduce such drawbacks.

To deal with the increasing number of system users and fading correlation, a subcarrier allocation scheme may be applied to MIMO systems with channel state information (CSI) at the transmitter or receiver. CAFH for MIMO channels (MIMO-CAFH) can be considered for wideband transmission to mitigate intersymbol interference and enhance system capacity. Studying the capacity of MIMO enabled CAFH system under a generic multiuser multicarrier framework may reduce ICI caused by high-speed mobiles in cellular environments. Multiple transmit and receive antennas can be used with CAFH to further improve system performance. Using MIMO-CAFH, PAPR related problems can be dramatically reduced.
Bibliography


