NONCOHERENT DETECTION FOR DIFFERENTIAL OFFSET QPSK

by

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Abstract

At the physical layer of wireless communication systems, differential offset quaternary phase-shift keying (DOQPSK) offers a great avenue for low-cost implementation. However, its unfavorable error performance with conventional differential detection (DD) has made DOQPSK unpopular in practical use. Recently, Dr. M. Simon proposed a multiple-bit differential detection (MBDD) scheme that is derived from optimal maximum-likelihood block detection (MLBD) [1]. Asymptotically, MBDD still suffers a 3 dB loss in power efficiency compared to that of coherent detection (CD). To close the performance gap between MBDD and CD, and to make the receiver a simple and versatile solution, five novel noncoherent receiver designs for DOQPSK are proposed in this thesis: full-size MBDD (F-MBDD), improved MBDD (I-MBDD), noncoherent linear equalization (NLE), noncoherent decision-feedback equalization (NDFE), and noncoherent sequence estimation (NSE). In F-MBDD, we use an appropriate model for the overall channel to overcome the performance gap of Simon’s receiver. I-MBDD improves the slow convergence of the error value in F-MBDD by truncating the detection window size from $N - 1$ to $N - 3$. For NLE and NDFE, computational complexity is reduced by employing suboptimal equalization techniques to suppress inter-symbol interference (ISI) introduced by the OQPSK modulation. Finally, NSE uses ML detection with a trellis decoder that is based on the Viterbi algorithm (VA). When designed carefully, all the schemes proposed in this work have shown performances close to that of CD. In addition, receiver parameters can be controlled to balance performance, complexity, and versatility.
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List of Abbreviations and Symbols

Acronyms

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<th>Description</th>
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<tr>
<td>ACF</td>
<td>Auto correlation function</td>
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<tr>
<td>AWGN</td>
<td>Additive white Gaussian noise</td>
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<td>BEP</td>
<td>Bit error probability</td>
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<td>BER</td>
<td>Bit error rate</td>
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<td>BPSK</td>
<td>Binary phase–shift keying</td>
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<td>BT</td>
<td>Time–bandwidth</td>
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<tr>
<td>CD</td>
<td>Coherent detection</td>
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<td>CEF</td>
<td>Complementary error function</td>
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<td>CIR</td>
<td>Channel impulse response</td>
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<tr>
<td>CSI</td>
<td>Channel state information</td>
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<td>DD</td>
<td>Differential decoding</td>
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<td>DE</td>
<td>Differential encoding</td>
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<td>DFE</td>
<td>Decision–feedback equalization</td>
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<tr>
<td>DQPSK</td>
<td>Differential quadrature phase–shift keying</td>
</tr>
<tr>
<td>DOQPSK</td>
<td>Differential offset quadrature phase–shift keying</td>
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<tr>
<td>FBF</td>
<td>Feedback filter</td>
</tr>
<tr>
<td>FFF</td>
<td>Feed–forward filter</td>
</tr>
<tr>
<td>FSM</td>
<td>Finite state machine</td>
</tr>
<tr>
<td>GFSK</td>
<td>Gaussian frequency–shift keying</td>
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<tr>
<td>Abbreviation</td>
<td>Description</td>
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<td>--------------</td>
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<tr>
<td>GMSK</td>
<td>Gaussian minimum–shift keying</td>
</tr>
<tr>
<td>i.i.d.</td>
<td>Independent and identically distributed</td>
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<tr>
<td>ISI</td>
<td>Inter–symbol interference</td>
</tr>
<tr>
<td>LE</td>
<td>Linear equalization</td>
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<tr>
<td>MBDD</td>
<td>Multiple–bit differential detection</td>
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<td>MLBD</td>
<td>Maximum–likelihood block detection</td>
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<td>MMSE</td>
<td>Minimum mean–square error</td>
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<tr>
<td>MSDD</td>
<td>Multiple–symbol differential detection</td>
</tr>
<tr>
<td>MSE</td>
<td>Mean–square error</td>
</tr>
<tr>
<td>MSK</td>
<td>Minimum–shift keying</td>
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<tr>
<td>NDFE</td>
<td>Noncoherent decision–feedback equalization</td>
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<tr>
<td>NLE</td>
<td>Noncoherent linear equalization</td>
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<tr>
<td>NLF</td>
<td>Noise–limiting filter</td>
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<tr>
<td>NSE</td>
<td>Noncoherent sequence estimation</td>
</tr>
<tr>
<td>OQPSK</td>
<td>Offset quadrature phase–shift keying</td>
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<tr>
<td>PAPR</td>
<td>Peak–to–average power ratio</td>
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<tr>
<td>pdf</td>
<td>probability density function</td>
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<tr>
<td>PEP</td>
<td>Pairwise error probability</td>
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<td>PLL</td>
<td>Phase–lock loop</td>
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<td>PSK</td>
<td>Phase–shift keying</td>
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<td>QPSK</td>
<td>Quadrature phase shift keying</td>
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<td>RTZ</td>
<td>Return–to–zero</td>
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<td>SNR</td>
<td>Signal–to–noise ratio</td>
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<tr>
<td>SRC</td>
<td>Square–root raised cosine</td>
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<tr>
<td>VA</td>
<td>Viterbi algorithm</td>
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<tr>
<td>WLAN</td>
<td>Wireless Local Area Network</td>
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<tr>
<td>WPAN</td>
<td>Wireless Personal Area Network</td>
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<tr>
<td>WMF</td>
<td>Whitened matched filter</td>
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<tr>
<td>ZF</td>
<td>Zero–forcing</td>
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Operators and Notation

$|\cdot|$  Absolute value of a complex number
$\Re\{\cdot\}$  Real value of a complex number
$\Im\{\cdot\}$  Imaginary value of a complex number
$\ast$  Convolution
$\delta(\cdot)$  Dirac delta function
$Q(\cdot, \cdot)$  Marcum Q–function
$I_0(\cdot)$  Zeroth-order modified Bessel function of the first kind
$\text{sign}(x)$  Signum function, equals to 1 if $x > 0$, 0 if $x = 0$,
$-1$ if $x < 0$
$u_r(x)$  returns 1 if $0 \leq x < r$, 0 otherwise
$u(x)$  returns 1 if $0 \leq x$, 0 otherwise
$x \mod y$  Modulo function, equal to the remainder of $x/y$
$\mathbb{E}\{\cdot\}$  Expectation
$[x]$  Smallest integer equal to or larger than $x$
$[\cdot]^*$  Complex conjugate
$[\cdot]^T$  Matrix or vector transposition
$[\cdot]^H$  Matrix or vector hermitian transposition
$0_m$  Zero vector with $m$ elements
$I_{m \times m}$  Identity matrix with dimension $m \times m$
$\hat{x}$  Decision vector
$\tilde{x}$  A hypothetical vector
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Undoubtedly, the completion of this thesis project came from the support of many individuals. First and foremost, I would like to thank my academic supervisor Professor Robert Schober for giving me an opportunity to work on this challenging project. He provided me his invaluable knowledge and experience on problem-solving, simulation techniques, and publications. I would also like to thank him for his understanding of my unexpected schedule this summer.

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Vancouver, British Columbia

Ivan C.H. Ho

September 20, 2004
In Memory of My Grandfather.
Chapter 1

Introduction

This chapter is divided into four parts: First, the background information and motivation for this work are described in detail. Second, we review the related work that has been proposed by other researchers in this field. Finally, a summary of our contributions to the field is presented before we briefly outline the chapters in this thesis.

1.1 Background and Motivation

For the past few years, the demand for communication devices to become more accessible, affordable and portable has grown tremendously. This has pushed the current standards for Wireless Personal Area Networks (WPANs) and Wireless Local Area Networks (WLANs)\(^1\) to facilitate lower costs, faster deployment, and higher market acceptance. Coupled with the on-going development towards higher data-rate transmission, physical-layer specifications are now even more important to wireless radio communications between notebook computers, personal digital assistants, cellular phones, other handheld devices, and connectivity to the Internet.

\(^1\)Please refer to IEEE Wireless Standards Zone for details of current specifications http://standards.ieee.org/wireless/.
For example, the current Bluetooth standard\textsuperscript{2} specifies the use of binary Gaussian frequency-shift keying (GFSK) as its modulation scheme to achieve a maximum data-rate of 1 Mb/s.

Higher-order signalling schemes can be a great alternative to their binary counterparts. In particular, quaternary phase-shift keying (QPSK) takes advantage of the in-phase and quadrature channels to achieve the same error performance as binary PSK when the signal is coherently detected\textsuperscript{3} at the receiver. Nonetheless, QPSK is often impractical because of its high peak-to-average power ratio (PAPR). The high PAPR makes the signal difficult to contain spectrally when low-cost, non-linear and band-limited power amplifiers are deployed at the transmitter. Another signalling scheme called $\pi/4$-QPSK can be considered as an interim solution to the problem stipulated by QPSK because of its lower PAPR, but its improvement is diminutive compared to the PAPR achieved by offset QPSK (OQPSK).

Even though OQPSK is a proficient modulation scheme for inexpensive amplifiers, it was previously thought that the bit-error performance of OQPSK reception is comparable to that of QPSK only when coherently detected. However, since random-phase distortions from the channel often occur during transmission, coherent detection (CD) becomes a costly technique as it must rely on carrier phase and frequency synchronization circuits such as the phase-lock loop (PLL) to track the absolute phase of the signal at real-time \cite{3}.

Another way to combat random phase shifts is to integrate differential encoding (DE) into the modulation. Assuming that the phase shift is relatively constant for a duration of two sample periods, differential detection (DD) may be used to estimate the data despite the distortion. The idea of DD can be extended to include all receivers that do not rely on absolute phase information. Such receivers are generally called noncoherent receivers. In conventional DD,\textsuperscript{2}Bluetooth v1.1\textsuperscript{TM}_2002 is the current release standard at the time of writing \cite{2}.
\textsuperscript{3}Coherent detection (CD) implies that the absolute phase of each received signal sample is known for the entire reception interval.
the receiver decodes the data by studying the phase difference between the successive samples, and therefore it does not require the knowledge of absolute phase. Unfortunately, conventional DD cannot overcome the inherent crosstalk between quadrature channels in differential OQPSK (DOQPSK), which causes a loss in error performance greater than that for regular DQPSK [4, 5].

1.2 Related Work

Before 2003, only a few noncoherent receivers for DOQPSK have shown improved error performances over conventional DD [4, 6, 7, 5, 8]. In 2003, Simon [1] proposed a multiple–bit differential detection (MBDD) scheme for DOQPSK, which is based on maximum–likelihood block detection (MLBD) [9]. MBDD increases the reliability of the data estimates by maximizing the joint likelihood of a block of received data. By interpreting DOQPSK as a continuous–phase modulated (CPM) signal, MBDD takes advantage of the block detection rule proposed in [10] to estimate the data. With increasing observation interval, MBDD asymptotically achieves a power efficiency that is approximately 3 dB worse than that of CD. Therefore, even though Simon’s MBDD shows a better error performance than the receivers in [4, 6, 7, 5, 8], its performance is still worse than that of similar noncoherent schemes for DQPSK.

Early in 2004, Phoel [11] argued that the less–than–expected performance in Simon’s strategy comes from the differential encoding scheme, which is based on a one–bit differential encoder. Phoel then proposed a more complex, two–bit differential encoder and proved that MBDD can indeed reach the performance of CD with increasing detection block size. Although Phoel’s result is encouraging, the receiver must pay a penalty as the two–bit decoding scheme is more intricate than the one–bit decoder. Also, MBDD requires a computational complexity that is exponential in the observation window size. Furthermore, MBDD is rigid against phase variations from the channel. The possibility of
decreasing the complexity while maintaining performance and versatility of the receiver motivated the start of this research.

1.3 Contributions

Upon investigation, we found that in contrast to previous results, not only can the common one-bit DE scheme attain the limits of CD through a receiver based on MLBD, but these limits can also be achieved with suboptimal, but more versatile and computationally efficient receiver structures. Here is a list of contributions accomplished in this work:

- We propose two MBDD receivers that can have error performances comparable to that of CD without compromising the one-bit DE scheme. For this proposal we first use a new approach to describe the DOQPSK signal as a binary PSK (BPSK) scheme transmitted over a time-variant inter-symbol interference (ISI) channel. Then a complementary detection strategy, which we call full-size MBDD (F-MBDD), succeeds in approaching the desired error performance in the limit of $N \to \infty$. Furthermore, with a shortened MBDD decision interval, improved MBDD (I-MBDD) makes the error performance converge faster than that of F-MBDD.

- For receivers with lower computational complexity we turn to suboptimal techniques. Among them three are newly derived for DOQPSK: noncoherent linear equalization (NLE) [12], noncoherent decision-feedback equalization (NDFE) [12], and noncoherent sequence estimation (NSE) [13]. Both NLE and NDFE schemes have a complexity that is linear in the observation window size while the NSE design has a complexity exponential in the channel length. In NLE and NDFE, equalization is used to compensate for ISI. On the other hand, NSE uses a trellis decoder based on the Viterbi algorithm (VA) to reduce the memory required in MLBD [14].
• The suboptimal receivers also provide flexibility to the system by allowing different pulse shapes (or waveforms) for transmission. In all of the MBDD schemes mentioned, the signal is transmitted using a rectangular pulse shape. For NLE, NDFE, and NSE schemes, the transmitted pulse shape can be arbitrarily chosen as long as the waveform is time–limited\(^4\). The time–limiting property of the signal is required for practical finite impulse response (FIR) equalization to be successful, though the resulting error performance varies depending on the adopted pulse–shaping filter.

• The three suboptimal schemes also provide versatility against phase variations. In the receivers, the phase reference can be estimated via nonrecursive or recursive summation based on previously detected data. Depending on the parameters in the summation, the receivers can maintain high error performance against a constant phase drift.

1.4 Outline

To explain the above findings in detail, this thesis starts with an overview on digital communication concepts, including an in–depth discussion on DOQPSK modulation and conventional demodulation in Chapter 2. Then in Chapter 3, we revisit MBDD and show that with one–bit encoding and a different interpretation of the channel, it can still approach the lower–bounds of CD. The subsequent Chapters (4 to 6) introduce three new receiver designs for DOQPSK that are based upon suboptimal NLE, NDFE, and NSE techniques, respectively. In Chapter 7, the error performances of previously proposed receivers are compared to those of the above schemes. Finally, evaluation, summary, and possible future work are discussed in Chapter 8 as a conclusion to this thesis.

\(^4\)Time–limited implies that the energy of the pulse is confined to a finite time slot, see [3].
Chapter 2

Basic Concepts

In this chapter, the basic concepts that underlie the analysis and design of digital communication systems are presented. First, the fundamental building blocks of a communication system are briefly described. We then focus on the modulation and demodulation blocks, as well as the characteristics of the channel, all in a continuous-time model. The DOQPSK modulation scheme and the signal model used in this research are then presented mathematically in detail. For simplicity, we further describe the overall continuous-time channel model in an equivalent discrete-time manner. Finally, common receiver designs for DOQPSK are reviewed for comparison with the designs proposed in later chapters.

2.1 Communication Model

Referring to Proakis\textsuperscript{1}, Fig. 2.1 illustrates the functional diagram and the basic elements of a digital communication system. Starting from the transmitter, the information source can be either analog or digital information, which is then converted to binary digits by the source encoder. The binary data is then passed to the channel encoder, where it introduces controlled redundancy to the information sequence.

\textsuperscript{1}Fig. 1.1-1 of [3]
Figure 2.1: Basic elements of a digital communication system.

The binary data\(^2\) at the output of the channel encoder is transferred to the digital modulator, where it is converted into a signal waveform. \(M\) distinct waveforms may be used to map more than one bit of information for transmission. Such schemes are commonly called \(M\)-ary modulation. For the purpose of this thesis we focus on offset quadrature phase-shift keying (OQPSK), which belongs to a family of phase modulation schemes, as discussed in Section 2.3.

The communication channel is the physical medium used to send the signal from the transmitter to the receiver. One common property among all types of channels is that they corrupt the transmitted signal in a random manner by mechanisms such as the additive thermal noise, atmospheric noise, and other distortions such as time dispersion and random phase. For this thesis the channel is described as a mathematical model in Section 2.2.1.

The first block at the receiver of the communication system is the digital demodulator, which processes the channel--corrupted, transmitted waveform and reduces it to a sequence of estimated numbers that represent the original data symbols. The output of the demodulator then connects to the channel decoder where it reconvertrs the redundant digits back to compressed information with error correction. Finally, the source decoder accepts the sequence of decoded data and reconfigures it to become the original information.

\(^2\)One common assumption is that the binary data contains approximately equal amounts of 1's and 0's. This is not necessarily true for all systems but it is true for the systems considered in this thesis.
2.2 Continuous–Time Transmission Model

As mentioned in the previous paragraphs, modulation and demodulation play a crucial role in the reliability of the system. Fig. 2.2 depicts a block diagram of the adopted continuous–time transmission model, starting from the output of the channel encoder to the input of the channel decoder. Note that the model uses the equivalent complex baseband representation rather than the real passband model [15, 3]. From the channel encoder a symbol $a[i] \in \{0,1\}$, $i \in \mathbb{Z}$, is emitted every $T_b$ seconds, where the digits are assumed to be independent and identically distributed (i.i.d.).

The mapper functions as an encoder and arranges $l = \log_2(M)$ binary symbols into one $M$–ary symbol $\Delta s[k]$, $k \in \mathbb{Z}$, according to some mapping rule. Differential QPSK signals, for example, rely on Gray mapping [3] to achieve optimum bit–error performance. Each resulting symbol at the output of the mapper has a duration of $T_s = l \cdot T_b$. The symbols are further encoded by the differential encoder, which is described in detail in Section 2.4.

The encoded symbols $s[k]$ then pass through a pulse–shaping filter (PSF) with impulse response $h_T(t)$, which transforms the discrete–time data into continuous–time waveforms and provides spectral shaping to the signal. The signal to be transmitted through the channel, $s_T(t)$, can be described by the following formula.

$$s_T(t) = \sum_{k=-\infty}^{\infty} s[k] h_T(t - kT)$$  \hspace{1cm} (2.1)
2.2.1 Channel Model

The random effects of the channel on the transmitted signal can be modeled mathematically with the support of statistical theories. The equivalent complex baseband continuous–time channel adopted for this research has the following characteristics, which are illustrated in Fig. 2.3.

- Channel impulse response \( h_c(t) \)
  
  For simplicity and since we are mainly interested in indoor channels, we assume \( h_c(t) = \delta(t) \), where \( \delta(\cdot) \) is the Dirac delta function.

- Random phase shift \( \Theta_0 \)
  
  The random phase shift is usually introduced by the channel or the local oscillator at the receiver during synchronization. The phase shift may be regarded as constant over the sequence of data samples at the receiver. It is further assumed to be uniformly distributed over the interval \( (-\pi, \pi] \).

- Frequency Offset \( \Delta f \)
  
  \( \Delta f \) is the carrier frequency offset between the transmitter and the receiver. For the derivations in this thesis, the offset is assumed to be zero. Nonetheless, the effects of a non-zero frequency offset on the signal are examined in Chapters 4 to 6 to show the robustness of the receivers described in the respective sections.

- Additive white Gaussian noise (AWGN) \( n(t) \)
  
  Physically, the AWGN process may arise from electronic components and amplifiers at the receiver of the communication system. AWGN is often regarded as the most common noise model in the evaluation of communication systems thanks to its application to a broad class of physical communication channels and its mathematical tractability. Statistically, the mean of the single–sided, real pass–band noise process is regarded as zero, with variance equal to \( N_0 \). In equivalent baseband
representation, the noise is complex, but maintains a zero mean and a variance of $N_0$. In addition, the in-phase and quadrature components of $n(t)$ are considered independent of each other.

![Diagram](image)

**Figure 2.3:** Continuous-time model for frequency-selective channels.

At the receiver, the signal is filtered by a noise-limiting filter (NLF) with impulse response $h_R(t)$ (cf. Fig. 2.2). The continuous-time output $r(t)$ can be expressed as

$$r(t) = e^{j\phi_0} \int_{-\infty}^{\infty} e^{j2\pi f \mu} s_T(\mu) h_R(t - \mu) d\mu$$

$$+ \int_{-\infty}^{\infty} n(\mu) h_R(t - \mu) d\mu$$

$$= e^{j\phi_0} \sum_{k=-\infty}^{\infty} s[k] \int_{-\infty}^{\infty} e^{j2\pi f \mu} h_T(\mu - kT) h_R(t - \mu) d\mu$$

$$+ \int_{-\infty}^{\infty} n(\mu) h_R(t - \mu) d\mu. \quad (2.2)$$

In Eq. (2.2), $j = \sqrt{-1}$ represents the imaginary unit. The continuous-time received signal $r(t)$ is then sampled at rate $kT_b + t_0$ to become $r[k]$, where the time shift factor $t_0$ is for receiver time synchronization purposes. The data samples are further propagated to the *noncoherent receiver*, where it provides estimates of the original mapped symbols, denoted $\hat{s}[k - k_0]$. $k_0$ is a decision delay, which can be predetermined to enhance performance especially in time-dispersive channels where equalization is employed [16]. Eventually, the estimated sequence is reverted by the *de-mapper* to become $\hat{a}[i - i_0]$.

The continuous-time model provides us with a framework for all receiver design and analysis in later chapters. We now describe OQPSK modulation and its
signal model in detail.

2.3 OQPSK Modulation

Phase-shift keying (PSK) is a modulation technique regularly used in digital communications. As the name suggests, PSK embeds the information in the phase of the signal. Quaternary, or quadrature, PSK (QPSK) is often preferred over binary PSK (BPSK) because it offers higher spectral efficiency with little or no penalty in performance when coherently detected.

As communication systems become more portable, however, the use of cost-effective, power efficient nonlinear components has made conventional QPSK an unattractive scheme. For example, rectangular pulse-shaped QPSK allows transitions between symbols to pass through the origin, which results in a large peak-to-average power ratio (PAPR). The above characteristic distorts the desired signal and produces unwanted side-lobe regrowth in nonlinear channels, causing a performance loss in practical use [17]. An intermediate scheme called $\pi/4$-QPSK [18] offers moderate improvement in PAPR and has therefore been considered for several standards$^3$. More convincing than $\pi/4$-QPSK, offset QPSK (OQPSK) counterbalances the transitions on the in-phase (I) and quadrature (Q) components to reduce the undesired artifacts from regular QPSK while maintaining the same spectral occupancy.

Fig. 2.4 shows the signal-space constellation of the three QPSK schemes if rectangular pulse-shaping is applied. In the diagrams the dots represent the signalling points of the modulation with magnitude and phase while the lines show the trajectories in which they travel during the signalling intervals. Notice that for all three schemes, the magnitude of the signalling points is constant. For QPSK, maximum phase transition occurs between points at the diagonal and is equal to 180 degrees. For $\pi/4$-QPSK, the constellation alter-

$^3$\(\pi/4\)-QPSK was chosen as the standard for the American Digital Cellular second-generation standard and the second-generation Japanese standard [18].
nates between two sets of signalling points (the second set indicated by X's) with maximum phase transition occurring at 135 degrees. OQPSK has the advantage of signalling at the bit rate, which is twice the baud rate of the former two schemes, and can have a maximum phase transition of only 90 degrees. The smaller the phase transition, the lower the PAPR, which translates to OQPSK being the most practical modulation scheme.

![Figure 2.4](image.png)

Figure 2.4: Signal constellation and traversal diagram for (a) QPSK, (b) π/4-QPSK, and (c) OQPSK.

### 2.4 Differential OQPSK

When the absolute phase knowledge of the received signal is known, OQPSK modulation can be coherently detected. The resulting error performance is equivalent to that for QPSK. However, having the phase knowledge is sometimes at a premium when severe fading occurs in the channel, or when synchronization is impossible between the transmitter and the receiver. Differential, or noncoherent, detection is preferred instead because it does not require the absolute phase knowledge during the estimation process. In PSK–type signals, differential encoding (DE) is a necessary feature for the functional success of noncoherent receivers. The term DE comes from the fact that each data symbol is embedded as the phase difference between the signal at the current instance and the one from the previous period. At the receiver, the information can then
be extracted from the phase difference between successive received samples.

To illustrate the DE process of OQPSK we refer to Fig. 2.5. We assume that the signal source emits $\Delta s[k] \in \{\pm 1\}$ from the mapper in Fig. 2.2, where $k$ represents each bit iteration. The signal $c[k]$ at the output of the differential encoder may be expressed as

$$c[k] = \Delta s[k] \cdot c[k - 2].$$

(2.3)

Again, we assume the binary source to contain i.i.d. symbol elements $\Delta s[k]$. Then the multiplier $j^{(k \mod 2)}$, where $x \mod y$ is equal to the modulo function, acts as a de-demultiplexer and alternately feeds $c[k]$ to the I and Q components of the signal $s[k]$. Finally, the stream of data goes to the pulse-shaping filter (PSF).

![Figure 2.5: Block diagram of Differential OQPSK.](image)

The signal $s[k]$ before entering the PSF is represented as

$$s[k] = j^{(k \mod 2)} c[k]$$

$$= \begin{cases} 
  c[k], & \text{for } k \text{ even;} \\
  j c[k], & \text{for } k \text{ odd.}
\end{cases}$$

(2.4)

Notice that $s[k]$ is deliberately arranged so that the data in the I and Q channels are not sent at the same time instance, which constitutes the term offset in OQPSK.

---

4 $x \mod y$ is equal to computing the remainder of $x/y$.

5 Offset modulation is sometimes referred to as staggered modulation [3, 19].
2.5 Complex Baseband Discrete-Time Model

From Fig. 2.2 we see that the continuous-time received signal \( r(t) \) is sampled at time instances \( kT_b + t_0 \), resulting in discrete-time data \( r[k] \). Here, we attempt to define \( r[k] \) based on Eq. (2.2) with the following approximation: the frequency offset \( e^{j2\pi \Delta f \mu} \) in Eq. (2.2) is assumed to be constant in the time interval where \( h_R(t - \mu) \) is large compared to zero. In this case, \( e^{j2\pi \Delta f \mu} \) can be moved out of the integration in Eq. (2.2) to become \( e^{j2\pi \Delta f t} \).

With the above assumption, Eq. (2.2) can be rewritten as

\[
\begin{align*}
    r(t) &= e^{j\theta_0} e^{j2\pi \Delta f t} \sum_{k=-\infty}^{\infty} s[k] \int_{-\infty}^{\infty} h_T(\mu - kT) h_R(t - \mu) d\mu \\
    &\quad+ \int_{-\infty}^{\infty} n(\mu) h_R(t - \mu) d\mu. \\
\end{align*}
\]

(2.5)

To further simplify the equation, we create the following definition:

\[
    h_G(t) \triangleq h_T(t) * h_R(t) = \int_{-\infty}^{\infty} h_T(\mu) h_R(t - \mu) d\mu,
\]

(2.6)

where \( h_G(t) \) represents the overall impulse response of the channel. Then Eq. (2.5) can be expressed as

\[
\begin{align*}
    r(t) &= e^{j\theta_0} e^{j2\pi \Delta f t} \sum_{k=-\infty}^{\infty} s[k] h_G(t - kT) \\
    &\quad+ \int_{-\infty}^{\infty} n(\mu) h_R(t - \mu) d\mu. \\
\end{align*}
\]

(2.7)

Finally, at the sampling process, \( r[k] \) can be defined as

\[
\begin{align*}
    r[k] &\triangleq r(kT_b + t_0) = e^{j\theta} e^{j2\pi \Delta f T_b k} \sum_{\nu=-\infty}^{\infty} h_G[\nu] s[k - \nu] + z[k], \\
\end{align*}
\]

(2.8)

where the discrete-time function \( h_G[\nu] \), \( \nu \in \mathbb{Z} \), represents the overall CIR and is defined as

\[
    h_G[\nu] \triangleq h_G(kT_b + t_0).
\]

(2.9)
In Eq. (2.8), $\Theta$ is equal to

$$\Theta \triangleq (\theta_0 + 2\pi \Delta f t_0) \mod 2\pi$$  \hspace{1cm} (2.10)

$\Theta$ is random and uniformly distributed over $(-\pi, \pi]$. It has a probability density function (pdf)

$$f_\Theta(\Theta) = \begin{cases} \frac{1}{2\pi}, & -\pi < \Theta \leq \pi; \\ 0, & \text{otherwise.} \end{cases}$$  \hspace{1cm} (2.11)

Furthermore, $z[k]$ is a complex discrete-time noise process which is equal to

$$z[k] = \int_{-\infty}^{\infty} n(\mu) h_R(kT_b + t_0 - \mu) d\mu,$$  \hspace{1cm} (2.12)

with mean equal to zero. The auto-correlation function (ACF) $\phi_{zz}[\lambda]$ is given by

$$\phi_{zz}[\lambda] = \mathcal{E}\{z^*[k]z[k + \lambda]\}$$  \hspace{1cm} (2.13)

$$= N_0 \phi_R[\lambda],$$

where $\phi_R[\lambda] \triangleq \phi_R(\lambda T_b)$, and $\phi_R(t)$ is equal to

$$\phi_R(t) = h_R(t) * h^*_R(-t)$$  \hspace{1cm} (2.14)

$$= \int_{-\infty}^{\infty} h^*_R(\tau) h_R(t + \tau) d\tau.$$  

In the event that $\phi_R[\lambda]$ has more than one non-zero components, the variance of $z[k]$ is considered to be temporally correlated. The correlation indicates that the noise process is colored. For a complete derivation of $\phi_{zz}[\lambda]$ please refer to Appendix A.

We now describe the effects of $h_G[\nu]$ on the overall sampled data $r[k]$ based on different impulse responses.

### 2.6 Overall Filter Impulse Response

Before entering the communication channel, the readily coded data sequence must go through a PSF in order to create a physical form. In addition, the
PSF shapes the power spectral density (PSD) of the signalling data, which allows band-limited amplifiers to better contain the signal at the output of the transmitter.

In the time domain, we are interested in calculating the energy of the transmitted signal, $s_T(t)$, since the amount of energy used to transmit the data is a necessary parameter in the evaluation of the system. For DOQPSK modulation, the bit energy $E_b$ of the transmitted signal is given by

$$E_b = \int_{-\infty}^{\infty} |h_T(t)|^2 dt. \quad (2.15)$$

At the receiver, one common noise filtering component is called the matched filter [3], which implies that the receiver filter’s impulse response $h_R(t)$ matches with that of the transmit filter, like so

$$h_R(t) = h_T^*(-t). \quad (2.16)$$

The matched filter is the optimal receiver filter for retaining sufficient statistics in the output samples $r[k]$ [3]. Different pulse shapes represent different advantages, as well as challenges, for the system. The following subsections describe three waveforms that are examined in this work.

### 2.6.1 Rectangular PSF

The rectangular pulse is one of the most commonly used waveforms for research purposes due to its simplicity. The waveform $h_T(t)$ for DOQPSK may be expressed as

$$h_{T-REC}(t) = \sqrt{\frac{E_b}{2T_b}} u_{2T_b}(t) \quad (2.17)$$

where $T_b$ is the bit period. $u_{\tau}(t)$ is equal to 1 for $0 \leq t < \tau$, and 0 elsewhere. The term $\sqrt{\frac{E_b}{2T_b}}$ is used to normalize the energy of the signal that passes through the filter. The following diagram from [20] illustrates the OQPSK waveform when it is transmitted through a rectangular PSF.

The advantage of this waveform is that the amplitude of the signal remains constant throughout the symbol period. However, the sharp transitions at
Figure 2.6: Output waveform of OQPSK in I and Q channels through a rectangular PSF.

the end of the symbol periods cause a spread in the spectral energy and are therefore not ideal for band-limited transmission in practice. But for the purpose of making suitable comparisons with other proposed structures and the ease of derivation, we have chosen this waveform as the common pulse shape for all of the receivers described in this thesis.

When coupled with a matched filter at the receiver, the overall discrete-time impulse response $h_G[k]$ is given by

$$h_{G-REC}[k] = E_b \left[ \frac{1}{2} \delta[k - 1] + \delta[k] + \frac{1}{2} \delta[k + 1] \right],$$

where $\delta[k]$ is the discrete-time Dirac delta function.$^6$

### 2.6.2 Half–Sine PSF

The half–sine filter takes the following form as opposed to its rectangular counterpart.

$$h_{T-SIN}(t) = \sqrt{\frac{2E_b}{T_b}} \sin \left( \frac{\pi t}{2T_b} \right) u_{2T_b}(t)$$

One advantage of the half–sine pulse is that it always starts and ends at zero amplitude, which means that the signal does not endure abrupt changes in the

---

$^6\delta[k]$ is equal to 1 at $k = 0$, and 0 elsewhere.
phase of the signal. The following diagram from [20] illustrates how OQPSK behaves with the half-sine PSF.

![Output waveform of OQPSK in I and Q channels through a half-sine PSF.](image)

Figure 2.7: Output waveform of OQPSK in I and Q channels through a half-sine PSF.

Coupled with a matched filter at the receiver, the combined discrete-time impulse response $h_{G-SIN}[k]$ is expressed as

$$h_{G-SIN}[k] = E_b \left[ \frac{1}{\pi} \delta[k - 1] + \delta[k] + \frac{1}{\pi} \delta[k + 1] \right].$$

(2.20)

This thesis uses the half-sine filter for evaluating the DOQPSK receivers described in Chapters 4 and 5.

Incidentally, the half-sine filter is the designated PSF for minimum shift keying (MSK) [20]. In fact, DOQPSK with a half-sine PSF and MSK are differed by their encoding scheme only. As discussed in Section 2.4, the input to the PSF $s[k]$ for DOQPSK is given by Eq. (2.30). On the other hand, the input to the half-sine filter $s_{MSK}[k]$ for MSK is related to the input signals $\Delta s[k]$ by [21]

$$s_{MSK}[k] = \gamma s_{MSK}[k - 1] \Delta s[k].$$

(2.21)

Numerous papers have been published on the performance of MSK decoders [21, 22, 23]. In Chapter 7, we compare the performance of the designs in this thesis using a half-sine filter with that of the proposed MSK receivers.
2.6.3 Return-to-Zero PSF

One of the disadvantages of both half-sine and rectangular PSFs is that the received signal, after match-filtering and being sampled, has residual effects called inter-symbol interference (ISI). ISI occurs when information from one period spans across adjacent bit periods, possibly altering the decisions made by the receiver if not designed properly.

There are a number of solutions to mitigate the effects of ISI. One simple way is to generate a pulse shape such that the information received at one instance would not involve data at other time instances after filtering and sampling. One such desired pulse, which we call return-to-zero (RTZ), can be represented mathematically as

\[ h_{RTZ}(t) = \sqrt{\frac{E_b}{2T_b}} \left[ -u_{3T_b/4}(t) + u_{T_b/2} \left( t - \frac{3T_b}{4} \right) - u_{3T_b/4} \left( t - \frac{5T_b}{4} \right) \right] \]  \hspace{1cm} (2.22)

Combined with a matched filter and sampled at the bit rate of the transmission, the overall discrete-time impulse response becomes a constant at \( k = 0 \), but zero at other bit periods, as expressed below:

\[ h_{G_{RTZ}}[k] = E_b \delta[k]. \]  \hspace{1cm} (2.23)

This means that ISI is removed from the channel. Unfortunately, the RTZ pulse is not spectrally efficient and therefore not suitable for band-limited transmission. Nonetheless, we use this filter to evaluate the performance of the receiver in Chapter 4.

Fig. 2.8 shows the continuous-time impulse response of the three waveforms described in this Section.

2.7 Continuous-Phase Modulation (CPM)

The instant transition in the phase of the signal in common PSK schemes causes the signal to have low spectral efficiency [3]. This deficiency can be avoided by the use of continuous-phase modulation (CPM). CPM allows the
information-bearing signal phase to carry over to the next instance. Simon in [1] shows that the DOQPSK signal can be modeled as a CPM signal. To express the signal mathematically, we first redefine the signal \( s_T(t) \) in terms of its magnitude and phase, namely

\[
s_T(t) = \sqrt{\frac{E_b}{2T_b}} e^{j\phi(s[k], t)},
\]

where \( \phi(s[k], t) \) is valid for \( nT_b \leq t \leq (n+1)T_b \) and is equal to

\[
\phi(s[k], t) = 2\pi \sum_{k=-\infty}^{n} s[k]u(t - kT_b),
\]

and \( u(t) \) is the unit step function\(^7\). The data sequence \( s[k] \) is differentially encoded according to the rule given in [1]. The resulting transmitted waveform, however, is identical to the one generated by the rectangular PSF.

At the receiver, the signal goes through an integrate-and-dump filter.

\(^7u(t) = 1 \text{ for } t \geq 0, 0 \text{ otherwise.}\)
The filtered samples can be expressed as

\[ r_{CPM}[k] = \int_{kT_b}^{(k+1)T_b} r(t) dt. \]  

(2.26)

Alternatively, the integrate-and-dump filter can be regarded as filtering with \( h_R(t) = u_{T_b}(t) \) plus sampling at \( kT_b \), as shown in [11].

The CPM interpretation of DOQPSK is dealt with in the analysis of the MBDD receivers in [1] and [11]. The derivation of CPM signals is shown here for comparison with the designs in Chapter 3.

2.8 Conventional Receivers

Several receiver designs have been presented in the past for DOQPSK, all of which fall into two categories: coherent and noncoherent. The following subsections provide a glimpse of the conventional receivers for comparisons with the receiver structures proposed in this work.

2.8.1 Coherent Detection (CD)

A prerequisite for CD at the receiver is the knowledge of absolute phase in the signal. Such information can be acquired through the use of a phase lock loop (PLL) or other synchronization circuits. A block diagram of a conventional coherent receiver is shown in Fig. 2.9. The received samples \( r[k] \) from Eq. (2.8) are fed to a decision block, where the estimates \( \hat{c}[k] \) can be obtained from the following manipulation.

\[ \hat{c}[k] = \text{sign} \{ \Re\{(-j)^{(k+2) \mod 2} r[k]\} \}, \]  

(2.27)

where \( \text{sign}(\cdot) \) denotes the Signum function\(^8\) and \( \Re\{\cdot\} \) is the real part of a complex number.

\(^8\)For \( x \in \mathbb{R} \), \( \text{sign}(x) \) returns 1 if the element is greater than zero, 0 if it equals zero, and -1 if it is smaller than zero.
Then in the next stage the estimates $\Delta \hat{s}[k]$ are obtained from

$$\Delta \hat{s}[k] = \hat{c}[k] \cdot \hat{c}[k - 2]. \quad (2.28)$$

While the separation of CD and the inversion of DE are suboptimum in terms of error probability [24], the two-stage approach is simple with negligible loss in performance.

![Block diagram of CD for DOQPSK.](image)

Common CD schemes require the use of extra hardware components, which are often costly. Components such as PLL may also introduce error propagations. Furthermore, in harsh cases such as fading channels, or when low-cost local oscillators are employed, phase tracking may be difficult, or even impossible. This is because the PLL circuitry cannot provide a stable phase reference. The next section describes another class of receivers called noncoherent receivers [3].

### 2.8.2 Noncoherent Detection (NCD)

When dealing with received signals that have unknown phase, a more robust reception technique than using PLL circuits is to apply signal processing methods to decipher them without the knowledge of the absolute phase. Such designs are called noncoherent, or differential, detection. A block diagram of conventional DD for DOQPSK is illustrated in Fig. 2.10. First, the differential receiver determines the decision variable $d[k]$ from $r[k]$ using the following function.

$$d[k] \triangleq r[k] \cdot r^*[k - 2] \quad (2.29)$$
Notice that the inversion of differential encoding takes place before the decision is made. The estimates $\Delta \hat{s}[k]$ are provided by the following decision rule.

$$\Delta \hat{s}[k] = \text{sign} \{ \Re \{ d[k] \} \}. \quad (2.30)$$

![Figure 2.10: Block diagram of conventional DD for DOQPSK.](image)

Papers on NCD for DOQPSK are rather scarce compared to the DQPSK case [6, 4, 23, 7, 5, 8]. In addition, the DE applied to OQPSK causes inherent crosstalk between quadrature channels, thereby penalizes its error performance when conventional DD (or another suboptimal receiver scheme) is applied.
Chapter 3

Multiple–Bit Differential Detection (MBDD)

DD is an attractive alternative to CD in applications where simplicity and robustness of implementation take precedence over achieving the best system performance. In particular, conventional DD of $M$–ary PSK ($M$–PSK) has been accomplished by comparing the phases between the received samples in consecutive intervals and making a decision based on the difference. The detector assumes that the information is carried in the phase difference through differential encoding at the transmitter, and that the received reference phase is constant over at least two symbol intervals.

Although DD eliminates the need for carrier acquisition and tracking in the receiver, it suffers from performance penalty when compared to CD. In 1990, Divsalar and Simon [9] proposed multiple–symbol DD (MSDD) for $M$–PSK. It compensates for the performance loss in conventional DD by first extending the observation interval to more than two symbols. MSDD then makes a joint decision on several symbols simultaneously rather than making one symbol decision per period of observation. At infinite observation window size, MSDD can achieve the same performance as CD.

The scheme in [9] is considered an optimal noncoherent solution for signals with random phase in AWGN channel because it uses the joint maximum–
likelihood (ML) approach to statistical detection, which minimizes the bit error rate (BER) [3]. In 2003, the same principle was used to derive multiple-bit DD (MBDD) for DOQPSK [1]. Simon overcame the crosstalk between quadrature channels in DOQPSK by first modeling the signal as a CPM, then devising a detector similar to the one proposed in [10]. An observation window size of $N$ bit intervals was used with a corresponding decision window size of $N - 1$ bit intervals. Since the data is differentially decoded, $N - 1$ is the maximum number of bits that can be detected from $N$ received signal samples. We refer to this type of MBDD scheme as full-size MBDD (F-MBDD). One would expect that such an approach would eliminate the effects of crosstalk and therefore achieve the same performance as CD. Contrastingly, Simon's F-MBDD for DOQPSK still requires 3 dB more power to accomplish the same BER as CD even when $N \rightarrow \infty$.

Early in 2004, Phoel conjectured in [11] that the performance loss suffered by Simon’s F-MBDD is due to the simple, independent, one-bit differential encoding (DE) of the in-phase (I) and the quadrature (Q) components in DOQPSK. He then proposed a new, more complex, two-bit DE that is based on Gray coding [3]. With a matching MBDD scheme, Phoel’s system approaches the performance of CD for $N \rightarrow \infty$.

Unsatisfied with the findings by both Simon and Phoel, we set out to find an MBDD solution that gives the performance of CD without compromising the simple encoding scheme. In doing so, we first interpret the DOQPSK signal as a binary PSK (BPSK) signal transmitted over a time-variant ISI channel. This alternative description leads us to a simple F-MBDD decision rule which is similar to Simon’s F-MBDD decision strategy. In contrast to Simon’s F-MBDD, the proposed F-MBDD scheme approaches the performance of CD as $N$ increases. However, the BER of F-MBDD converges very slowly to the BER of CD. A careful analysis of the pairwise error probabilities (PEP) of F-MBDD reveals that single bit errors at the beginning and the end of the detection interval are responsible for this slow convergence. Therefore,
for finite $N$, performance can be significantly enhanced by discarding the bit decision at the beginning and the end of the F-MBDD detection window. The resulting improved MBDD (I-MBDD) employs a reduced decision window size of $N - 3$ bit intervals, which is the same as the one used in Phoel’s receiver. The error performance of I-MBDD is shown to converge as quickly as Phoel’s scheme even at short observation window sizes$^1$.

### 3.1 Time–Variant ISI Channel

The channel for the DOQPSK system described in [1] is based on a CPM scheme that is reviewed in Section 2.7. Although the complex sampled data $r_{CPM}[k]$ from the integrate-and-dump filter (cf. Eq. (2.26)) is statistically sufficient for an optimal estimation of $\Delta s[k]$ [3], Simon’s MBDD fails to achieve a high performance. Here, we wish to show a different, but equivalent, channel interpretation that can be used for MBDD. The advantage of this new channel description lies in the impulse response, where it is represented by the I and Q components. The I and Q representation of the channel, rather than the magnitude and phase interpretation from Eq. (2.24), is crucial to the decision on the complex data $r_{CPM}[k]$.

The transmitted signal of the new channel description is essentially the same as $s_T(t)$ in Eq. (2.1), with $h_T(t)$ being the rectangular pulse-shaping filter (PSF). The resulting signal $s_T(t)$ can be rewritten as

$$s_T(t) = \sum_{k=-\infty}^{\infty} \left( s[2k]h_{T-REC}(t-2kT_b) + s[2k-1]h_{T-REC}(t-(2k-1)T_b) \right), \quad (3.1)$$

where $s[k]$ and $h_{T-REC}(t)$ are defined in Eqs. (2.4) and (2.17), respectively.

The transmitted signal is impaired by a constant unknown phase $\Theta$ and complex zero-mean AWGN, $n(t)$, as explained in Section 2.2.1. We further assume that the frequency drift $\Delta f$ is zero. The received signal is filtered with a rectangular filter $h_R(t)$ of duration $T_b$. Although $h_R(t)$ is not the matched filter of

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$^1$Two papers on F-MBDD and I-MBDD have been submitted for publication [25, 26].
$h_T(t)$, after $T_b$-spaced sampling the data $r[k]$ constitutes a sufficient statistic
for detection of $\Delta s[k]$ [11, 3]. Assuming an appropriate normalization, the
sampled received signal can be expressed as

$$r[k] = \frac{1}{\sqrt{2}} e^{i\Theta} \left( c[2\lfloor (k - 1)/2 \rfloor] + jc[2\lfloor k/2 \rfloor - 1] \right) + n[k], \quad (3.2)$$

where $\lfloor x \rfloor$ and $n[k]$ represent the ceiling function\(^2\) and the discrete-time complex zero-mean AWGN with variance $\sigma_n^2 = 2N_0/E_b$, respectively. $c[k]$ are the differentially encoded bits from Fig. 2.5 and defined in Eq. (2.3).

The above channel contains ISI as the overall CIR spans over two sampling intervals. We also note that $r[k]$ in Eq. (3.2) can be interpreted as the received signal of a BPSK transmission over a time-variant channel. From Fig. 2.6, we see that the time variance comes from the real and imaginary parts of $c[2\lfloor (k - 1)/2 \rfloor] + jc[2\lfloor k/2 \rfloor - 1]$ being changed in alternate intervals.

### 3.2 Full-Size MBDD (F-MBDD)

In this section, we derive and analyze the proposed F-MBDD decision rule. In order to simplify our notation, we assume in the following that the considered discrete-time index $k$ and $N - 3$ are even. However, the generalization to odd $k$ and even $N$ is straightforward.

#### 3.2.1 Derivation of Decision Rule

Assuming the signal model described in Section 3.1, the conditional probability of the received vector $r[k]$ given the transmitted vector $c[k]$ and $\Theta$ can be expressed as

$$p(r[k]|c[k], \Theta) = \frac{1}{(2\pi\sigma_n^2)^N} \exp \left\{ -\frac{1}{2\sigma_n^2} \sum_{\kappa=0}^{N-1} |r[k - \kappa] - y[k - \kappa]e^{i\Theta}|^2 \right\}, \quad (3.3)$$

according to [9, 10]. $y[k]$ is defined as

$$y[k] = \frac{1}{\sqrt{2}} \left( c[2\lfloor (k - 1)/2 \rfloor] + jc[2\lfloor k/2 \rfloor - 1] \right)$$

\(^2\lfloor x \rfloor\) is equal to the smallest integer greater than or equal to $x$.\(^2\)
while $r[k]$ and $c[k]$ are given by

$$r[k] = [r[k] \cdots r[k-N+1]]^T$$

(3.5)

$$c[k] = [c[k] \cdots c[k-N]]^T.$$  

(3.6)

Since $\Theta$ is assumed to be uniformly distributed, the conditional probability of $r[k]$ given $c[k]$ is simply

$$p(r[k] | c[k]) = \frac{1}{2\pi} \int_0^\pi p(r[k] | c[k], \theta) d\theta$$

$$= \frac{1}{2\pi (2\pi \sigma^2)^N} \exp \left\{ -\frac{1}{2\sigma^2} \sum_{\kappa=0}^{N-1} \left( |r[k-\kappa]|^2 + |y[k-\kappa]|^2 \right) \right\}$$

(3.7)

$$\times I_0 \left( \frac{1}{\sigma^2} \sum_{\kappa=0}^{N-1} r[k-\kappa] y^*[k-\kappa] \right),$$

where $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind [3].

Since the signal energy $|c[k]|^2$ is constant, and that $I_0(\cdot)$ is a monotonically increasing function of its argument, we can define the MBDD decision metric $\eta(\tilde{y}_0^{N-2})$ as

$$\eta(\tilde{y}_0^{N-2}) \triangleq \left| \sum_{\kappa=0}^{N-1} r[k-\kappa] \frac{1}{\sqrt{2}} (\bar{c}[k-2[\kappa/2]] + j\bar{c}[k - 2[(\kappa+1)/2] + 1])^* \right|$$

$$= \left| \sum_{\kappa=0}^{N-1} r[k-\kappa] \rho^*(k-\kappa, \tilde{y}_0^{N-2}) \right|,$$

(3.8)

where we used the definition

$$\rho(k-\kappa, \tilde{y}_0^{N-2}) \triangleq \frac{1}{\sqrt{2}} \left( \prod_{\nu=[(\kappa+1)/2]}^{(N-3)/2} \Delta \tilde{s}[k-2\nu] + j\bar{m}[k] \prod_{\nu=[(\kappa+1)/2]}^{(N-3)/2+1} \Delta \tilde{s}[k-2\nu+1] \right).$$

(3.9)

$\tilde{y}_\mu^\nu \triangleq [\Delta \tilde{s}_\mu^\nu, \bar{m}[k]]$ represents the hypothetical data sequence in the metric. $\Delta \tilde{s}_\mu^\nu$ and $\bar{m}[k]$ are correspondingly expressed as

$$\Delta \tilde{s}_\mu^\nu \triangleq \left[ \Delta \tilde{s}[k-\mu] \Delta \tilde{s}[k-\mu-1] \cdots \Delta \tilde{s}[k-\nu] \right]_{\nu \geq \mu}$$

(3.10)

$$\bar{m}[k] \triangleq \bar{c}[k-N] \bar{c}[k-N+1].$$
The joint multiplication of the hypothetical sequence $\Delta \bar{s}$ in Eq. (3.9) is the result of expanding the summation in Eq. (3.8) using Eq. (2.3). Also note that $\tilde{m}[k]$ is the product of the bits transmitted in I and Q, respectively. This means that $\tilde{m}[k] \in \{\pm 1\}$ does not correspond to a differential bit and basically reflects the uncertainty whether the relative phase shift of I and Q is $+\pi/2$ or $-\pi/2$. This uncertainty is a direct consequence of the one-bit DE of I and Q.

The F-MBDD decision rule is given by

$$\Delta \hat{s}_0^{N-2} = \arg \max_{\tilde{y}_0^{N-2}} \{ \eta (\tilde{y}_0^{N-2}) \}, \quad (3.12)$$

with $\Delta \hat{s}_0^{N-2}$ equal to the decision vector. This MBDD decision rule requires $2^N/(N-1)$ metric calculations per bit decision. Since each observation window of size $N$ corresponds to a detection window of size $N-1$, successive observation windows have to overlap by one bit interval.

The decision rule differs from the F-MBDD decision rule given by Simon because the signal models used for description of DOQPSK are not the same. Here, we directly decide on the differential bits $\Delta s_0^{N-2}$, whereas in [1, 11] first a decision is made on a phase vector $\alpha$ of CPM symbols, which are related to the differential bits by a certain precoding scheme. The estimate $\hat{\alpha}$ is subsequently used to obtain $\Delta s_0^{N-2}$. The main difference between the F-MBDD decision rule for DOQPSK and the F-MSDD decision rule [9] for DQPSK is due to the dependence of $\eta (\tilde{y}_0^{N-2})$ on $\tilde{m}[k]$. Although we are only interested in the differential bits $\Delta \tilde{s}_0^{N-2}$, we also have to optimize for $\tilde{m}[k]$ in Eq. (3.8), which is not necessary in case of F-MSDD for DQPSK.

### 3.2.2 Performance Analysis

The BER of F-MBDD can be mathematically upper bounded by the union bound [27, 1, 11]

$$\text{BER}_F \leq \frac{1}{N-1} \frac{1}{2^N} \sum_{\tilde{y}_0^{N-2}} \sum_{\tilde{y}_0^{N-2}} w (\Delta \tilde{s}_0^{N-2}, \tilde{y}_0^{N-2}) \Pr \{ \eta (\tilde{y}_0^{N-2}) > \eta (\tilde{y}_0^{N-2}) \},$$

$$\Delta \tilde{s}_0^{N-2} \neq \Delta \tilde{s}_0^{N-2} \quad (3.13)$$
where \( w(\Delta s_0^{N-2}, \Delta s_0^{N-2}) \) denotes the Hamming distance\(^3\) between \( \Delta s_0^{N-2} \) and \( \Delta s_0^{N-2} \). The pairwise error probability (PEP) \( \Pr\{\eta(\hat{g}_0^{N-2}) > \eta(y_0^{N-2})\} \) is the probability that \( \hat{g}_0^{N-2} \) is detected if \( y_0^{N-2} \) contains the correct set of \( \Delta s_0^{N-2} \) and \( m[k] \). Deriving from [1], we express the PEP as

\[
\Pr\{\eta(\hat{g}_0^{N-2}) > \eta(y_0^{N-2})\} = \frac{1}{2} \left( 1 - Q(\sqrt{a_+}, \sqrt{a_-}) + Q(\sqrt{a_-}, \sqrt{a_+}) \right),
\]

(3.14)

where \( Q(\cdot, \cdot) \) is the Marcum Q-function [28], and

\[
a_{\pm} \triangleq \frac{E_b}{N_0} \left( N \pm \sqrt{N^2 - |\gamma(y_0^{N-2}, \hat{g}_0^{N-2})|^2} \right)
\]

(3.15)

\[
\gamma(y_0^{N-2}, \hat{g}_0^{N-2}) \triangleq \sum_{\kappa=0}^{N-1} \rho(k - \kappa, y_0^{N-2}) \rho^*(k - \kappa, \hat{g}_0^{N-2}).
\]

(3.16)

More insight can be gained from the high signal-to-noise ratio (SNR) approximation for the PEP [1]

\[
\Pr\{\eta(\hat{g}_0^{N-2}) > \eta(y_0^{N-2})\} \approx \frac{N + |\gamma(y_0^{N-2}, \hat{g}_0^{N-2})|}{8|\gamma(y_0^{N-2}, \hat{g}_0^{N-2})|} \text{erfc} \left( \sqrt{\frac{E_b}{N_0} \left( N - |\gamma(y_0^{N-2}, \hat{g}_0^{N-2})| \right)} \right)
\]

(3.17)

where \( \text{erfc}(\cdot) \) denotes the complementary error function (CEF) [3]. Since the PEP depends on \( |\gamma(y_0^{N-2}, \hat{g}_0^{N-2})| \), we study this function in the following for the most important error events.

**Single bit errors in I or Q at bit position \( \nu \):**

Consider the case \( \Delta s[k-\nu] \neq \Delta s[k-\nu], \ 0 \leq \nu \leq N-2, \Delta s[k-\mu] = \Delta s[k-\mu], \mu \neq \nu \). Using Eqs. (3.9) and (3.15) it can be shown that

\[
|\gamma(y_0^{N-2}, \hat{g}_0^{N-2})| \triangleq |\gamma|_a = \begin{cases} 
\sqrt{(N - \nu - 1)^2 + \alpha_{a,1}^2(\nu + 1)^2}, & m[k] = \hat{m}[k] \\
\sqrt{\alpha_{a,2}^2(N - \nu - 1)^2 + (\nu + 1)^2}, & m[k] \neq \hat{m}[k]
\end{cases}
\]

(3.18)

where the magnitude of \( \alpha_{a,2} \) is limited to \( |\alpha_{a,2}| \leq 1, \ l \in \{1, 2\} \). Similarly, for the error events discussed later in Sections 3.2.2 and 3.2.2 we introduce \( \alpha_{b,2} \)

\(^3\)Hamming distance \( w(a, b) \) represents the number of bits in which vectors \( a \) and \( b \) differ.
and \( \alpha_{c,l} \), \( l \in \{1, 2\} \), respectively, whose magnitudes are also limited to unity. The exact value of the \( \alpha_{x,l} \), \( x \in \{a, b, c\} \), \( l \in \{1, 2\} \) depends on \( y_0^{N-1} \) and the error event. For example, for the above considered error event \( \alpha_{a,1} \) equals

\[
\alpha_{a,1} = \frac{1}{\nu + 1} \left( \sum_{\nu=0}^{(N-3)/2} \prod_{\mu=\left[(\nu+1)/2\right]}^{(N-3)/2+1} c[k-2\mu] \prod_{\mu=\left[(\nu+1)/2\right]} c[k-2\mu+1] \right). \tag{3.19}
\]

However, in most cases only the property \( |\alpha_{x,l}| \leq 1, x \in \{a, b, c\}, l \in \{1, 2\} \) is important while the exact definition and the exact value of \( \alpha_{x,l} \) are not relevant for our asymptotic analysis and will be omitted.

We observe from Eq. (3.18) that \( |\gamma|_a \) assumes its maximum value \( |\gamma|_{\text{max}} \triangleq \sqrt{(N-1)^2+1} \) for single errors in positions \( \nu = 0 \) and \( \nu = N-2 \), respectively. All other single error events result in a smaller \( |\gamma|_a \) and correspond therefore to a smaller PEP. Assuming \( m[k] = \bar{m}[k] \) there are \( 2^N \) and 4 combinations of \( y_0^{N-2} \) and \( \hat{y}_0^{N-2} \) that result in a single error event with \( |\gamma|_a = |\gamma|_{\text{max}} \) at position \( \nu = 0 \) and \( \nu = N-2 \), respectively. Note that for \( m[k] = \bar{m}[k] \) and \( \nu = N-2 \), \( |\gamma|_a = |\gamma|_{\text{max}} \) only results if \( |\alpha_{a,1}| = 1 \), which is the case only for \( c[k-\mu] = 1 \), \( 0 \leq \mu \leq N-3 \). Since similar statements are true for the case \( m[k] \neq \bar{m}[k] \), the total number of single error events with \( |\gamma|_a = |\gamma|_{\text{max}} \) is \( e_a = 2(2^N + 4) \). Using this and the approximation for the PEP in Eq. (3.14), it is easy to show that for large \( N \) and high SNRs the contribution of single errors in I and Q to the right hand side (RHS) of the union bound in Eq. (3.13) can be approximated by

\[
P_a \approx \frac{1}{N-1} \text{erfc} \left( \sqrt{\frac{E_b}{N_0}} (N - \sqrt{(N-1)^2+1}) \right). \tag{3.20}
\]

Note that we have used the assumption that \( N \) is large to simplify the factor in front of the CEF. \( P_a \) should be compared to the BER of coherently detected DOQPSK with one-bit DE which is identical to the BER of coherently detected differential BPSK (DBPSK) \( \text{BER}_{\text{CD}} = \text{erfc} \left( \sqrt{\frac{E_b}{N_0}} (1 - \frac{1}{2} \text{erfc}(\sqrt{\frac{E_b}{N_0}})) \right) \) [3]. For \( N \rightarrow \infty \) the argument of the CEF in Eq. (3.20) becomes \( \sqrt{\frac{E_b}{N_0}} \), which for high SNRs implies a loss of 3 dB compared to the BER of CD. However, the influence of single errors at the beginning and end of the detection interval on
the overall BER decreases slowly with increasing \( N \) due to the factor in front of the CEF in Eq. (3.20). Nevertheless, for finite \( N \) this type of error event dominates the performance loss of F-MBDD compared to CD as will be shown in Section 3.4.

**Double bit errors in I or Q:**

Assume that \( \Delta s[k-\nu] \neq \Delta \tilde{s}[k-\nu], \Delta s[k-\nu-2] \neq \Delta \tilde{s}[k-\nu-2], 0 \leq \nu \leq N-4, \Delta s[k-\mu] = \Delta \tilde{s}[k-\mu], \mu \neq \{\nu, \nu+1\} \). A careful inspection of Eqs. (3.9) and (3.15) shows that in this case

\[
|\gamma \left( y_0^{N-2}, \tilde{g}_0^{N-2} \right) | \triangleq |\gamma|_b = \begin{cases} 
\sqrt{(N-2)^2 + 4\alpha_{b,1}^2}, & m[k] = \hat{m}[k] \\
\sqrt{\alpha_{b,2}^2(N-2)^2 + 4}, & m[k] \neq \hat{m}[k]
\end{cases}.
\]

(3.21)

The maximum value of \( |\gamma|_b \) is \( \sqrt{(N-2)^2 + 4} \). For \( m[k] = \hat{m}[k] \) the total number of error events that cause double errors in I or Q is \( e_b = 2^N(N-3) \). For large \( N \) the contribution of the case \( m[k] \neq \hat{m}[k] \) becomes negligible since \( |\alpha_{b,2}| = 1 \) results only for certain \( y_0^{N-1} \). Therefore, using again Eq. (3.14), for large \( N \) the contribution of double errors in I or Q to the RHS of the union bound in Eq. (3.13) can be approximated by

\[
P_b \approx \text{erfc} \left( \frac{E_b}{N_0} (N - \sqrt{(N-2)^2 + 4}) \right). 
\]

(3.22)

Note that for \( N \to \infty \) \( P_b \) approaches the BER of coherently detected DOQPSK.

**Successive bit errors in I and Q:**

Finally, consider the case \( \Delta s[k-\nu] \neq \Delta \tilde{s}[k-\nu], \Delta s[k-\nu-1] \neq \Delta \tilde{s}[k-\nu-1], 0 \leq \nu \leq N-3, \Delta s[k-\mu] = \Delta \tilde{s}[k-\mu], \mu \neq \{\nu, \nu+1\} \). It can be shown that

\[
|\gamma \left( y_0^{N-2}, \tilde{g}_0^{N-2} \right) | \triangleq |\gamma|_c = \begin{cases} 
\sqrt{(N-2\nu-3)^2 + 1}, & m[k] = \hat{m}[k] \\
\sqrt{\alpha_{c,2}^2(N-1)^2 + 1}, & m[k] \neq \hat{m}[k]
\end{cases}.
\]

(3.23)
where $\alpha_{c,2}$ is given by

$$\alpha_{c,2} = \frac{1}{N-1} \left( \sum_{\kappa=0}^{N-1} \prod_{\mu=\lfloor \kappa/2 \rfloor}^{(N-3)/2} c[k-2\mu] \prod_{\mu=\lfloor (\kappa+1)/2 \rfloor}^{(N-3)/2+1} c[k-2\mu+1] \right)$$

$$- \sum_{\kappa=\nu+2}^{N-1} \prod_{\mu=\lfloor \kappa/2 \rfloor}^{(N-3)/2} c[k-2\mu] \prod_{\mu=\lfloor (\kappa+1)/2 \rfloor}^{(N-3)/2+1} c[k-2\mu+1] \right).$$

The combination $m[k] \neq \tilde{m}[k]$ and $|\alpha_{c,2}| = 1$ is critical as it results in $|\gamma| = |\gamma|_{\text{max}}$. A careful inspection of Eq. (3.24) reveals that $|\alpha_{c,2}| = 1$ only results if all non-erroneous bits in $\Delta \hat{s}_0^{N-1}$ are equal to one, i.e., $\Delta s[k - \mu] = 1$, $\mu \neq \{\nu, \nu+1\}$. Therefore, it is easy to show that for a given $N$ the total number of successive single errors in I and Q that result in $|\alpha_{c,2}| = 1$ is $e_c = 4(N-2)$. As a consequence, for large $N$ and high SNRs the total contribution of these error events to the union bound in Eq. (3.13) can be approximated as

$$P_c \approx \frac{4}{2^N} \text{erfc} \left( \sqrt{\frac{E_b}{N_0} \left( \frac{N-\sqrt{(N-1)^2+1}}{N} \right)} \right).$$

Thus, the influence of successive single errors in I and Q on the overall BER decreases approximately exponentially with increasing $N$.

We note that the above considered error events dominate the BER of DOQPSK with F–MBDD, and the influence of all other error events is negligible. Therefore, for large $N$ and high SNRs the BER of F–MBDD can be approximated as

$$\text{BER}_F \approx P_a + P_b + P_c.$$  \hfill (3.26)

As can be observed from Eqs. (3.20), (3.22), and (3.25), for $N \to \infty$ BER$_F$ converges to the BER of CD. Note that this result is in contrast to the findings in [1, 11], where it was shown that Simon's F–MBDD scheme cannot approach CD. This result also contradicts the statement in [11] that the performance of DOQPSK is limited by the one-bit DE. On the other hand, the numerical evaluation in Section 3.4 shows that F–MBDD approaches the performance of CD very slowly as $N$ increases. Responsible for this slow convergence are single errors at the beginning and the end of the detection interval since the corresponding error rate $P_a$ decreases only slowly with increasing $N$.  

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3.3 Improved MBDD (I–MBDD)

Our analysis in the previous section suggests that the performance of F–MBDD is mainly limited by single bit errors at the beginning and the end of the detection interval. Therefore, in this section, we propose a simple extension of F–MBDD that improves performance by simply discarding the decisions made on the first and the last bit, i.e., the detection interval size of the resulting I–MBDD scheme is reduced to $N - 3$.

3.3.1 Derivation of Decision Rule

Assuming $N \geq 5$ (and $N$ odd) the decision rule for I–MBDD is

$$\Delta \hat{s}_1^{N-3} = \arg \max_{\hat{y}_0^{N-2}} \{ \eta (\hat{y}_0^{N-2}) \},$$

(3.27)

where $\eta (\hat{y}_0^{N-2})$ is still given by Eq. (3.8). We note that now successive observation windows overlap by 3 bit intervals, and $2^N/(N - 3)$ metric calculations per bit decision are required. Therefore, the complexity of I–MBDD is slightly higher than that of F–MBDD. However, this difference becomes negligible for large $N$.

3.3.2 Performance Analysis

The union bound for the I–MBDD scheme is given by

$$\text{BER}_I \leq \frac{1}{N - 3} \frac{1}{2^N} \sum_{y_0^{N-2}} \sum_{\Delta \hat{s}_1^{N-3} \neq \Delta \hat{s}_1^{N-3}} w(\Delta s_1^{N-3}, \hat{s}_1^{N-3}) \Pr \{ \eta (\hat{y}_0^{N-2}) > \eta (y_0^{N-2}) \}. $$

(3.28)

Using a similar approach as for F–MBDD, for large $N$ and high SNRs we obtain for I–MBDD the approximate BER

$$\text{BER}_I \approx P_b + P_c,$$

(3.29)

where $P_b$ and $P_c$ are given by Eqs. (3.22) and (3.25), respectively. Note that the contributions of single bit errors in I and Q are neglected in Eq. (3.29).
Eq. (3.18) shows that for single bit errors in positions $\nu = 2$ and $\nu = N - 3$, $|\gamma|_a = \sqrt{(N-2)^2 + 4}$ results, which is identical to the maximum value of $|\gamma|_b$. However, since the total number of error events of this type is only approximately $2^{N+1}$, for moderately large $N$ its contribution to $BER_t$ is much smaller than $P_b$ and can be neglected.

We expect that $BER_t$ approaches the BER of CD much faster than $BER_F$, since $P_a$ is absent in Eq. (3.29) and the factor in front of the CEF in the analytical expression for $P_c$ decreases exponentially with increasing $N$.

As shown in Section 3.4, the speed of convergence does not increase when $\mu < N - 3$ and $\nu > 1$ in $\Delta\delta^\nu_\mu$. The reason is that the most probable erroneous bits have been neglected already in the decision of $\Delta\delta_1^{N-3}$.

Table 3.1 summarizes the parametrical differences between F-MBDD and I-MBDD.

<table>
<thead>
<tr>
<th>Detection scheme</th>
<th>F-MBDD</th>
<th>I-MBDD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observation window size in bits</td>
<td>$N$</td>
<td>$N$</td>
</tr>
<tr>
<td>Number of decision bits</td>
<td>$N - 1$</td>
<td>$N - 3$</td>
</tr>
<tr>
<td>Number of metric computations per bit</td>
<td>$2^N/(N - 1)$</td>
<td>$2^N/(N - 3)$</td>
</tr>
<tr>
<td>Approximated BER</td>
<td>$P_a + P_b + P_c$</td>
<td>$P_b + P_c$</td>
</tr>
</tbody>
</table>

3.4 Simulations and Results

In this section, the performances of F-MBDD and I-MBDD are evaluated by simulations and compared to the analytical results derived in Sections 3.2 and 3.3. If not stated otherwise, we assume DOQPSK transmission with one-bit DE as discussed in Section 3.1.

In Fig. 3.1, the BERs of DOQPSK with F-MBDD, I-MBDD, and CD are shown. As expected, I-MBDD performs significantly better than F-MBDD. For example, for $N = 9$ and $BER = 10^{-5}$ I-MBDD realizes a power efficiency
gain of 1.4 dB over F-MBDD. On the other hand, a comparison of Fig. 3.1 with the results given in [11] suggests that for \( N = 9 \) and \( \text{BER} = 10^{-5} \) the F-MBDD scheme proposed in this work performs about 1 dB better than Simon's F-MBDD scheme. We note that for high SNRs the bounds for F-MBDD and I-MBDD given in Eqs. (3.13) and (3.28), respectively, are in excellent agreement with our simulation results.

In Fig. 3.2, we compare the BERs of DOQPSK with F-MBDD and I-MBDD to the BER of DQPSK with F-MSDD [9]. The markers denote simulation points, whereas the solid and dashed lines refer to analytical results obtained from the BER approximations for F-MBDD [Eq. (3.26)] and I-MBDD [Eq. (3.29)], respectively. The dotted lines were generated by using the BER approximation for F-MSDD of DQPSK [9]

\[
\text{BER}_{\text{F-MSDD}} \approx \text{erfc} \left( \frac{E_b}{N_0} \sqrt{\frac{N}{N - \sqrt{(N - 1)^2 + 1}}} \right),
\]

where now \( N \) refers to the number of QPSK symbols within the observation window. We note that all approximations are well confirmed by the simulation results. As the observation window size \( N \) increases the BERs of all considered
schemes converge to the BER of coherently detected DOQPSK. However, the speed of convergence of F-MSDD is higher than that of I-MBDD, while I-MBDD has a higher speed of convergence than F-MBDD. For example, at BER = $10^{-6}$ F-MSDD of DQPSK and I-MBDD perform only 0.2 dB and 0.7 dB worse than CD, whereas F-MBDD entails a performance loss of 1.1 dB compared to CD even for $N = 101$.

![Figure 3.2: BERs of F-MBDD and I-MBDD for DOQPSK, and F-MSDD for DQPSK.](image)

The convergence behavior of the BERs of F-MBDD, I-MBDD, and F-MSDD is studied more in detail in Fig. 3.3. Here, we show BER vs. $N$ for $10 \log_{10}(E_b/N_0) = 10$ dB. Markers denote again simulation results whereas the solid lines refer to analytical results obtained from the BER approximations given in this work. The different speeds of convergence of the different detection schemes can be well explained by the obtained BER approximations. For $N \to \infty$ the argument of the CEF in Eq. (3.30) approaches $\sqrt{E_b/N_0}$, i.e., in contrast to F-MBDD and I-MBDD, a CEF term with asymptotic argument $\sqrt{E_b/N_0}$ (which negatively affects the BER performance) is absent. Therefore, the BER of F-MSDD of DQPSK has the highest speed of convergence. On the other hand,
the BER of I-MBDD converges faster to CD than that of F-MBDD since the term $P_a$ is absent in BER$_I$ (cf. Eq. (3.29)). It is also interesting to compare F-MSDD with $N = N_{MS}$ and I-MBDD with $N = N_{MB} = 2N_{MS}$. Since $N$ is defined with respect QPSK symbols and bits for F-MSDD and I-MBDD, respectively, $N_{MB} = 2N_{MS}$ guarantees that both observation windows contain approximately the same number of bits. Fig. 3.3 suggests that under these conditions F-MSDD and I-MBDD achieve approximately the same performance, i.e., DOQPSK does not incur any performance penalty compared to DQPSK. For example, I-MBDD with $N = 10$ achieves approximately the same BER as F-MSDD with $N = 5$.

![Figure 3.3: BER convergence of F-MBDD and I-MBDD for DOQPSK, and F-MSDD for DQPSK.](image)

Finally, we compare the proposed I-MBDD of DOQPSK with one-bit DE to the MBDD of DOQPSK with two-bit DE reported in [11]. Fig. 3.4 shows that MBDD of DOQPSK with two-bit DE achieves a small performance gain over I-MBDD of DOQPSK with one-bit DE. However, as $N$ increases this gain diminishes and for $N = 9$ both schemes achieve practically the same performance. The small performance difference is probably due to the uncertainty
of the relative phase of I and Q in case of one-bit DE. This uncertainty does not exist for the more complicated two-bit DE since in that case the bits transmitted over I and Q are coupled.

3.5 Summary and Extensions

In this chapter, we have proposed and analyzed two novel MBDD schemes for DOQPSK with one-bit DE. F-MBDD, which makes use of the full detection window size of $N - 1$ bit intervals, outperforms Simon’s F-MBDD in [1]. However, our BER analysis of I-MBDD has shown that decreasing the detection window size to $N - 3$ bits leads to significant performance improvements. For moderate to large $N$, I-MBDD achieves a similar performance as Phoele’s MBDD of DOQPSK with two-bit DE proposed in [11]. Our simulation results and analysis show in good agreement that for sufficiently large $N$ both F-MBDD and I-MBDD approach the performance of CD. Therefore, the inability of Simon’s F-MBDD to approach CD for $N \to \infty$ is due to the detection scheme and not a consequence of the one-bit DE as was conjectured in [11].
Although we have proven that MBDD is indeed the optimal noncoherent solution for DOQPSK, we have not addressed the issue that MBDD is too complex computationally to achieve significant improvements when $N$ is large. There exists fast decoding algorithms that can make MBDD for DOQPSK practically feasible and relevant, such as the fast algorithm (FA) by Mackethum [29] and decision-feedback MSDD (DF–MSDD) by Adachi and Sawahashi [30]. In fact, two papers have been submitted for publication with the proposal of FA and DF for the MBDD schemes described above [25, 26]. Both algorithms achieve the same error performances as their original version. In this thesis, however, we would like to explore suboptimum, but computationally more efficient than MBDD, receiver structures for DOQPSK. In particular, three schemes are presented and analyzed in Chapters 4 to 6.
Chapter 4

Noncoherent Linear
Equalization (NLE)

The channel described for DOQPSK in Section 2.5 has ISI, which is produced by the modulation and pulse-shaping. ISI causes high error rates at the receiver if it is left uncompensated. For reliable detection, the receiver needs to employ a means for mitigating the ISI in the received signal. Such a compensator is called an equalizer.

One common equalization technique is called linear equalization (LE). Essentially, the receiver has a linear filter to invert the effects of ISI. This filter structure has a computational complexity that is a linear function of the channel dispersion length \( L_H \). Additionally, the filter has two important characteristics: the number of taps in the filter (specifically, we pay attention to whether it has a finite impulse response (FIR) or an infinite impulse response (IIR)), and the optimizing criterion for its coefficients.

In the case of the PSFs described for DOQPSK in Section 2.6, the overall channel length \( L_H \) is finite and is equal to 3. We also let the equalizer’s length \( L_F \) be sufficiently larger than \( L_H \) to compensate for the finite filter length, since a complete elimination of ISI is in general not possible with FIR filters. For the optimizing criterion for the coefficients of the filter, it would be desirable to choose the coefficients to minimize the average probability of error.
However, the probability of error as a performance index for optimizing the tap weight coefficients is computationally complex since the function is highly nonlinear [3]. Other criteria were considered, such as peak distortion (PD), zero-forcing (ZF), and minimum mean-square-error (MMSE) [3, 31]. The PD criterion is based on minimizing the maximum possible distortion of the signal at the equalizer output due to ISI. In the ZF equalizer, the filter aims to force the residual ISI in the decision variable to zero while the MMSE filter minimizes the variance of the error in the decision variable.

In our work, we have decided to use MMSE as the criterion to derive the linear equalizer. This criterion ensures an optimum trade-off between residual ISI and noise enhancement, which allows the equalizer to achieve a significantly lower BER compared to ZF and PD equalizers at low-to-moderate SNRs.

For the DOQPSK channel considered in this thesis, we derive two noncoherent LE (NLE) schemes to combat ISI, AWGN, and random phase distortion. The first design estimates the phase reference via an averager in a non-recursive manner. The second one is based on a recursive estimation of the phase reference to decide on the sent data. Since the two schemes differ only in the estimation of the phase reference, we start this chapter by describing the overall CIR before deriving the MMSE–LE filter. We then show the recursive and non-recursive algorithms for NLE in detail. Furthermore, we analyze the performance of LE for DOQPSK with varying parameters. Finally, simulations and results are presented.

4.1 Overall Channel Impulse Response (CIR)

Recall that in Section 2.5, we have expressed the received samples $r[k]$ as a function of the general CIR $h_G[k]$ (cf. Eq. (2.8)), where $h_G[k]$ is specific to the channel and PSFs used in this work. We further assume that the phase drift $\Delta f$ is zero. The received sample data $r[k]$ from Fig. 2.2 and in Eq. (2.8) can
then be rewritten as

\[ r[k] = e^{j\theta} \sum_{\nu=0}^{L_H-1} h_G[\nu]s[k - \nu] + z[k], \]  

(4.1)

For the PSFs considered in Section 2.6, the CIR may be generalized as

\[ h_G[k] = E_b \left[ \beta \delta[k - 1] + \delta[k] + \beta \delta[k + 1] \right], \]  

(4.2)

where \( \delta[\cdot] \) is the discrete-time Dirac delta function \([3]\). The value of \( \beta \) varies with respect to the PSF used. For rectangular filter \( \beta = 1/2 \); for half-sine filter \( \beta \) is \( 1/\pi \); and for RTZ filter \( \beta = 0 \), according to Eqs. (2.18), (2.20), and (2.23), respectively. In all of the cases, we have assumed the use of a matched filter at the receiver.

Because the signalling data \( s[k] \) alternates between real and imaginary components, the resulting channel is time-variant and therefore the sampled data \( r[k] \) may be represented in separate sets for even and odd samples, respectively as

\[ r_{\text{even}}[k] = e^{j\theta} E_b \left[ j\beta c[k - 1] + c[k] + j\beta c[k + 1] \right] + z[k], \]  

(4.3)

\[ r_{\text{odd}}[k] = e^{j\theta} E_b \left[ \beta c[k - 1] + j c[k] + \beta c[k + 1] \right] + z[k], \]  

(4.4)

where substitutions have been made on transmitted data \( s[k] \) for \( c[k] \) according to Eq. (2.4). Furthermore, the noise component \( z[k] \) is colored by the matched filter at the receiver, and therefore has an auto-correlation function (ACF) of

\[ \phi_{zz}[\lambda] = N_0 \left( h_R^*[-\lambda] * h_R[\lambda] \right) \]

\[ = N_0 h_G[\lambda] \]

\[ = N_0 E_b \left[ \beta \delta[\lambda - 1] + \delta[\lambda] + \beta \delta[\lambda + 1] \right], \]  

(4.5)

where \( N_0 \) is equal to the variance of the complex AWGN noise process \( n(t) \) described in Section 2.2.1. Also, due to match filtering, we can substitute \( h_G[\lambda] \) for \( h_R^*[-\lambda] * h_R[\lambda] \) (cf. Appendix A).
4.2 Receiver Structure

The purpose of the MMSE filter $f[k]$ is to suppress the effects of channel ISI on the signal. However, even after filtering, the signal continues to sustain phase distortions and therefore must be noncoherently detected. Fig. 4.1 shows the complete receiver structure of NLE. We choose to use the same principle of conventional DD reviewed in Section 2.10, except that we add decision feedback to estimate the phase reference of the previous samples [32]. $q[k]$ is the output of the linear equalizer and $d[k]$ is the decision variable, which is defined as

$$ d[k] = q[k] \cdot p_{\text{ref}}^*[k - 2] $$

(4.6)

According to [33], the phase reference $p_{\text{ref}}[k - 2]$ can be obtained from one of two ways: nonrecursively and recursively.

In the case of nonrecursive phase reference calculation, an averaging device accumulates $N - 1$ previous decisions $\Delta s[k - k_0]$ with the filter output $q[k]$ to yield an estimate of phase reference $p_{\text{ref}}[k - 2]$ [34]

$$ p_{\text{ref}}[k - 2] = \frac{1}{N - 1} \sum_{n=1}^{N-1} q[k - 2n] \prod_{\mu=1}^{n-1} \Delta s[k - k_0 - 2\mu]. $$

(4.7)

Each of the terms in the summation in Eq. (4.7) is an estimate of the previous phase reference. The result is then normalized by the number of terms in the summation.
In the recursive calculation of the phase reference, preference may be given to the more recent decision $\Delta \hat{s}[k-k_0-2]$ and filter output $q[k-2]$ by introducing a forgetting factor $\zeta$ [32]. The resulting phase reference is calculated as
\[ p_{\text{ref}}[k-2] = (1 - \zeta)q[k-2] + \zeta \Delta \hat{s}[k-k_0-2]p_{\text{ref}}[k-4]. \] (4.8)
The value of $\zeta$, $0 \leq \zeta \leq 1$, controls the robustness of the receiver against possible variations in the phase. A small forgetting factor ensures that the most recent phase has the most contribution and vice versa.

The decision rule for $\Delta \hat{s}[k-k_0]$ is essentially the same as the one expressed in Eq. (2.30). Similar to conventional DD, the term $j(k-k_0)^{\text{mod}2}$ is not required in the decision because the differentially decoded estimate of $\Delta s[k-k_0]$ is always embedded in the real component of $d[k]$.

### 4.3 MMSE–LE Filter Optimization

Ideally, MMSE linear equalization reverses the effects of the channel on the signal at the receiver by minimizing the mean-square-error on the decision variable [19, 12, 16, 33, 31]. Referring to Fig. 4.1, we first define the channel as a transfer function
\[ H_G(z) = \sum_{k=0}^{L_H-1} h_G[k]z^{-k}, \] (4.9)
which is the $Z$-transform [35] of the CIR $h_G[k]$ defined in Eq. (4.2). Likewise, $F(z)$ denotes the $Z$–transform of the linear equalizer impulse response $f[k]$, and is equal to
\[ F(z) = \sum_{k=0}^{L_F-1} f[k]z^{-k}. \] (4.10)
$L_F$ is defined as the length of the filter and is treated as finite in this work.

In Fig. 4.1, the equalizer output signal is represented as $q[k]$ while $d[k]$ is the decision variable. $q[k]$ can be expressed as
\[ q[k] = \sum_{m=0}^{L_F-1} f[m]r[k-m] \] (4.11)
\[ = f^H r[k]. \]
where $\mathbf{f}$ and $\mathbf{r}$ are vector notations of $F(z)$ coefficients and received samples $r[k - m], 0 \leq m \leq L_F - 1$, respectively, as expressed below

$$
\mathbf{f} = \begin{bmatrix} f[0] & \cdots & f[q_F] \end{bmatrix}^H \tag{4.12}
$$

$$
\mathbf{r}[k] = \begin{bmatrix} r[k] & \cdots & r[k - q_F] \end{bmatrix} \tag{4.13}
$$

where we further define $q_F = L_F - 1$.

Combining Eqs. (4.11) and (2.29), $d[k]$ can be rewritten as

$$
d[k] = \mathbf{f}^H \mathbf{r}[k] \cdot p_{ref}^*[k - 2]. \tag{4.14}
$$

After the decisions $\Delta \hat{s}[k - k_0]$ are made, we can measure the error signal $e[k]$ as

$$
e[k] = d[k] - \Delta \hat{s}[k - k_0]. \tag{4.15}
$$

$d[k]$ and $\Delta \hat{s}[k - k_0]$ are the decision variable and the estimates of the original data, respectively. In order to cope with non-causal components of the channel, a decision delay $k_0 \geq 0$ is introduced. This means that at time $k$, we estimate $\Delta \hat{s}[k - k_0]$.

The error $e[k]$ is a random process that has zero mean and variance equal to the expected value of the squared error. For filter optimization we can assume that $\Delta \hat{s}[.] = \Delta \hat{s}[.]$. The cost function $J_{NLE}(\mathbf{f})$ on which we base our MMSE filter optimization is equal to

$$
J_{NLE}(\mathbf{f}) \triangleq \mathcal{E}\{ |e[k]|^2 \}. \tag{4.16}
$$

Unfortunately, for $N < \infty$ and $\zeta < 1$ a closed form solution of $\mathbf{f}$ is difficult to obtain as pointed out in [16]. The difficulty is due to the mutual correlation between $q[k]$ and $p_{ref}[k - 2]$. However, an approximate solution can be found under the condition that the optimum filter will suppress ISI, i.e., the overall impulse response $h_{ov}[k] = h_G[k] * f[k]$ is close to zero for $0 \leq k \leq L_F + L_H - 2$, $k \neq k_0$. From Eqs. (4.7), (4.8) and (4.11), it can be observed that $q[k]$ and $p_{ref}[k - 2]$ are uncorrelated.
With the above assumptions, we can now expand the cost function $J_{NLE}(f)$ according to \[36\]

$$
J(f) = \mathcal{E}\left\{ (f^H r \cdot p_{ref}^*[k-2] - \Delta s[k-k_0]) (f^H r \cdot p_{ref}^*[k-2] - \Delta s[k-k_0])^H \right\}
= \rho (f^H \Phi_{rr} f)^2 - f^H \varphi_{\Delta sr} \varphi_{\Delta sr}^H f (2 - (1 - \rho) f^H \Phi_{rr} f) + 1.
$$

(4.17)

$\Phi_{rr}$ denotes the autocorrelation matrix of the received vector sequence $r$, and $\varphi_{\Delta sr}$ is the cross correlation vector between $\Delta s[k-k_0]$ and $r$. They are correspondingly expressed as

$$
\Phi_{rr} = \begin{bmatrix}
\phi_{rr}[0] & \phi_{rr}[1] & \cdots & \phi_{rr}[q_r]
\\
\phi_{rr}[-1] & \phi_{rr}[0] & \cdots & \phi_{rr}[q_r - 1]
\\
\vdots & \vdots & \ddots & \vdots
\\
\phi_{rr}[-q_r] & \phi_{rr}[-q_r + 1] & \cdots & \phi_{rr}[0]
\end{bmatrix}
$$

(4.18)

$$
\varphi_{\Delta sr} = \begin{bmatrix}
\phi_{\Delta sr}[k_0] & \phi_{\Delta sr}[k_0 - 1] & \cdots & \phi_{\Delta sr}[k_0 - q_r]
\end{bmatrix}^T,
$$

(4.19)

where $\phi_{mn}[\lambda] = \mathcal{E}\{m^*[k]n[k+\lambda]\}$. Assuming that the data $\Delta s[k]$ are i.i.d., the correlation functions can be derived as

$$
\phi_{rr}[\lambda] = h_G[\lambda] * h_G[\lambda] + N_0 h_G[\lambda]
$$

(4.20)

$$
\phi_{\Delta sr}[\lambda] = h_G[k_0 + \lambda].
$$

(4.21)

Furthermore, $\rho \triangleq 1/(N-1)$ and $\rho \triangleq (1 - \zeta)/(1 + \zeta)$ are valid for nonrecursive and recursive phase reference, respectively.

For optimization of the cost function, we let the gradient of $J_{NLE}(f)$ with respect to $f^*$ equal to zero

$$
\frac{\partial J_{NLE}(f)}{\partial f^*} = 0.
$$

(4.22)

The solution to the above equation is often referred to as the Wiener solution [31]. The optimal filter coefficients $f_{opt}$ are given by

$$
f_{opt-NLE} = \chi \Phi_{rr}^{-1} \varphi_{\Delta sr},
$$

(4.23)

where $\chi \triangleq 1/\sqrt{\rho + (1 - \rho) \varphi_{\Delta sr}^H \Phi_{rr}^{-1} \varphi_{\Delta sr}}$ is defined. Eq. (4.23) shows that the obtained solution is a scaled version of the coherent LE (CLE) MMSE solution ($f_{opt-CLE} = \Phi_{rr}^{-1} \varphi_{\Delta sr}$) [3].

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4.4 Performance Analysis

The minimum error variance can be obtained by inserting $f_{\text{opt-NLE}}$ from Eq. (4.23) to Eq. (4.16). After some manipulations similar to that in [36], the following equation results:

$$\sigma_{\text{min-NLE}}^2 = 1 - \frac{(\varphi_{\Delta sr}^H \Phi_{rr}^{-1} \varphi_{\Delta sr})^2}{\rho + (1 - \rho)\varphi_{\Delta sr}^H \Phi_{rr}^{-1} \varphi_{\Delta sr}}. \quad (4.24)$$

In the limit of $\rho \to 0$, Eq. (4.24) simplifies to $\sigma_{\text{min-NLE}}^2 = 1 - \varphi_{\Delta sr}^H \Phi_{rr}^{-1} \varphi_{\Delta sr}$, which is equal to the coherent case. On the other hand, as $\rho \to 1$ the error variance becomes $\sigma_{\text{min-NLE}}^2 = 1 - (\varphi_{\Delta sr}^H \Phi_{rr}^{-1} \varphi_{\Delta sr})^2$. Since $\varphi_{\Delta sr}^H \Phi_{rr}^{-1} \varphi_{\Delta sr} < 1$ in general, the worst case error is larger than the minimum error variance in the coherent case.

To evaluate the MMSE-NLE performance we first consider the signal-to-distortion ratio (SDR). From [31] and Eq. (4.24), the SDR of the NLE system can be expressed as

$$\text{SDR}_{\text{NLE}} = \frac{1 - \sigma_{\text{min-NLE}}^2}{\sigma_{\text{min-NLE}}^2} = \frac{(\varphi_{\Delta sr}^H \Phi_{rr}^{-1} \varphi_{\Delta sr})^2}{\rho + (1 - \rho)\varphi_{\Delta sr}^H \Phi_{rr}^{-1} \varphi_{\Delta sr} - (\varphi_{\Delta sr}^H \Phi_{rr}^{-1} \varphi_{\Delta sr})^2}. \quad (4.25)$$

Again we consider the limit of $\rho \to 0$, the SDR becomes

$$\text{SDR}_{\text{LE}} = \frac{\varphi_{\Delta sr}^H \Phi_{rr}^{-1} \varphi_{\Delta sr}}{(1 - \varphi_{\Delta sr}^H \Phi_{rr}^{-1} \varphi_{\Delta sr})}, \quad (4.26)$$

which is the SDR for coherent LE. Thus Eq. (4.25) can be related to 4.26 as

$$\text{SDR}_{\text{NLE}} = \frac{(\text{SDR}_{\text{LE}})^2}{\rho + (1 - \rho)\text{SDR}_{\text{LE}}}. \quad (4.27)$$

For small coherent SDRs ($\text{SDR}_{\text{LE}} \ll \rho \leq 1$) $10 \log_{10}(\text{SDR}_{\text{LE}}/\text{SDR}_{\text{NLE}}) \approx 10 \log_{10}(\rho/\text{SDR}_{\text{NLE}})$ dB holds and the noncoherent receiver experiences a large loss in performance. On the other hand, for the practical interesting case $\text{SDR}_{\text{LE}} \gg 1$, the performance penalty for NLE is $10 \log_{10}(1 + \rho)$ dB and attains its maximum of 3 dB for $\rho = 1$ (corresponding to $N = 2$ or $\zeta = 0$). Therefore, if coherent LE can achieve a moderate or high SDR, the loss in
power efficiency of the proposed NLE is determined exclusively by the parameter $\rho$. Since the robustness of NLE against channel phase variations also depends on $\rho$, this parameter can be used by the receiver designer to optimize performance for a given channel.

Before showing the simulation and results, we also wish to show the effect of the pulse-shaping filter (PSF) coefficients on the performance of the system. From Eq. (4.2) we learn that the PSF tap weight $\beta$ changes the characteristics of the overall CIR $h_G[k]$, and ultimately the LE impulse response $f[k]$. For simplicity and without loss in generality, we express the error variance in terms of a coherent LE with infinite impulse response (IIR) rather than its FIR NLE counterpart. According to [3], the error variance of an IIR coherent LE for DOQPSK is

$$
\sigma^2_{e-IIR-LE} = T_b \int_{-1/2T_b}^{1/2T_b} \frac{N_0 G(e^{j2\pi f T_b})}{|H_G(e^{j2\pi f T_b})|^2 + N_0 H_G(e^{j2\pi f T_b})} df,
$$

(4.28)

where $H_G(e^{j2\pi f T_b})$ is the Z-transform of CIR with $z = e^{j2\pi f T_b}$. After some manipulation (see Appendix B), the error variance can be simplified to

$$
\sigma^2_{e-IIR-LE} = \frac{N_0}{\sqrt{(1 + N_0)^2 - 4\beta^2}}.
$$

(4.29)

We may now substitute different values of $\beta$ into Eq. (4.29) for evaluation. For example, when $\beta = 1/2$ (corresponding to a CIR with rectangular PSF and a matched filter) the error variance simplifies to $\sqrt{2N_0 + N_0^2}$. This means that for high SNRs (i.e., $N_0 \ll 1$) the performance is asymptotically worse than that of CD. For the case $\beta = 1/\pi$, which corresponds to a CIR with half-sine PSF and a matched filter, the error variance reduces to $N_0/\sqrt{(1 + N_0)^2 - 4/\pi^2}$. In the limit of high SNR ($N_0 \ll 1$), the error is approximately 1.1 dB away from that of CD. Finally, for $\beta = 0$, such as the CIR of the RTZ filter described in Eq. (2.22), technically since there is no ISI, the error variance is equal to

$$
\sigma^2_{e-IIR-LE} = \frac{N_0}{1 + N_0}.
$$

(4.30)
4.5 Simulation and Results

In the following subsections, we first show the bit-error performance of MMSE-NLE for DOQPSK, varying different parameters in the receiver. We then show the plots of SDR vs. SNR, as they represent how well the equalizer suppresses ISI. Finally, the versatility of MMSE-NLE is demonstrated by the BER performance against a constant phase drift $\Delta fT_s$.

4.5.1 Bit–Error Rate (BER) vs. Signal–to–Noise Ratio (SNR)

In all of the simulations, the length of the LE filter $L_F$ and the decision delay $k_0$ are optimized to 9 and 4, respectively, through simulations. Fig. 4.2 depicts the BER versus SNR of NLE with nonrecursive phase reference estimation. SNR is expressed as $10\log_{10}(E_b/N_0)$ in dB. The PSF used in this case is the rectangular filter. The BERs of DOQPSK–CD and conventional DD are also included as references. From the figure we see that MMSE–NLE provides significant improvement in error performance over conventional DD. This is the direct result of applying the linear equalizer to suppress ISI. The BER of MMSE–NLE also increases progressively with an increase in phase reference estimation size $N$, which is in agreement with our predictions. In the limit of $N \to \infty$, represented by the curve LE–RECT, the BER continues to worsen relative to that of CD at increasing SNR. From our analysis, the approximated error variance of MMSE–LE is $\sqrt{2N_0 + N_0^2}$, which contributes to the poor performance at high SNRs ($N_0 < 1$).

In Fig. 4.3, similar achievements are shown for the BER curves of NLE with the phase reference estimated recursively. When the recursive parameter $\zeta$ is equal to 1, the error performance is the same as $N \to \infty$ in the nonrecursive case. As much as MMSE–NLE has improved over conventional DD, its performance is significantly lower than that of CD due to residual ISI.

Fig. 4.4 illustrates the BER performance of NLE when different PSFs are
Figure 4.2: BERs of NLE with nonrecursive phase reference, and DOQPSK–CD.

Figure 4.3: BERs of NLE with recursive phase reference, and DOQPSK–CD.
used. In all cases, the nonrecursive scheme is employed for phase estimation. 
As expected, the BER curves improve when PSFs with a lower value of $\beta$
are used. When the half-sine filter is used ($\beta = 1/\pi$) with $N = 10$, the
power efficiency is within approximately 1.2 dB of CD, which matches with
our expectations. In the case of the RTZ filter ($\beta = 0$), the error performance
is virtually the same as that of CD for $N = 10$.

Figure 4.4: BERs of NLE with three different PSFs: RECT, SINE, and RTZ, 
varying $N$.

4.5.2 Signal–to–Distortion Ratio (SDR) vs. SNR

To evaluate how well the LE filter suppresses ISI, SDR versus SNR is plotted
in Figs. 4.5 (a) and (b) with nonrecursive and recursive phase estimations,
respectively. Rectangular PSF is used in both simulations. Also, both SDR
and SNR are expressed in dB. Ideally, if the effects of ISI were completely
eliminated, SDR would be equal to $(1 + N_0)\text{SNR}$ according to Eq. (4.30). 
But due to the coefficient $\beta$ resulting from MMSE–LE in Eq. (4.29), the slope of the
curves at high SNRs appears to be lower than that at low SNRs. This means
that MMSE–NLE performs well only at low SNRs. It is also worth noting that as $N$ increases from 2 to 10, the improvement in performance diminishes gradually. On the other hand, increasing $\zeta$ from 0.2 to 1 proportionally gives better error performance.

\[ \text{Figure 4.5: SDR vs. SNR of NLE with (a) nonrecursive phase reference } N, \text{ and (b) recursive phase reference } \zeta. \]

### 4.5.3 BEP vs. Frequency Offset ($\Delta fT_s$)

The plots that show the robustness of MMSE–NLE against frequency offset $\Delta fT_s$ with nonrecursive and recursive phase estimation are depicted in Figs. 4.6 (a) and (b), respectively. In all cases, rectangular PSF is used and $10 \log_{10} (E_b/N_0) = 10$ dB is valid. $\Delta fT_s$ indicates the speed at which the phase drifts per symbol period $T_s$. The plots show that although the BER performance is poor, receivers with low phase estimation parameters (e.g. $N = 2$ or $\zeta = 0.2$) are more tolerable towards phase drifts. This is because in those cases, only the most recent decisions are accounted for in the estimation of the phase reference. At the limit of $N \to \infty$ (or $\zeta \to 1$), the NLE receiver can-
not tolerate any phase drifts and immediately converges to high BERs. Also note that a recursive phase reference estimation shows higher stability than estimating the phase reference nonrecursively.

4.6 Summary

Although MBDD has proven to be an optimum noncoherent receiver for DOQPSK in terms of bit-error probability (BEP), the detection strategy is computationally complex. NLE addresses the issue of complexity by using a linear equalizer to suppress ISI from the channel, then using a phase reference estimator to differentially detect the signal $\Delta s[k]$. The linear equalizer derived from MMSE criterion balances residual ISI with noise enhancement and therefore performs particularly well at low to moderate SNRs. However, the filtering technique has made the performance sensitive to the overall CIR. We have considered three different pulse shapes that are variable by only one parameter, $\beta$. For rectangular PSF ($\beta = 1/2$), the error performance of NLE cannot
equal that of CD even at the limit of $N \to \infty$, or $\zeta \to 1$. Improvements are found from using half-sine and RTZ PSFs. Ideally, the performance of CD can only be achieved when $\beta$ is equal to zero, implying no ISI from the channel.
Chapter 5

Noncoherent Decision–Feedback Equalization (NDFE)

Although the MMSE–NLE design in Chapter 4 is simple to implement and effective for channels with low ISI, some undesirable properties hinder the receiver’s error performance. For example, the linear equalizer suffers from noise enhancement, especially in severely distorted channels with roots close to the unit circle [31]. Moreover, the noise after equalization is colored and the noise variance increased.

The above drawbacks of LE can be avoided by applying noise prediction [3, 31]. The noise predictor uses the previous estimates to reconstruct what may be the remaining ISI of the incoming data. If the predictor coefficients are suitably chosen in conjunction with the linear equalizer, we expect that the variance of the new error signal to be smaller than the variance without noise prediction. A smaller error variance implies a better error performance. The combination of the linear equalizer and the noise predictor results in a new design called decision–feedback equalization (DFE). In DFE, the linear filter before the decision stage is usually referred to as the feed–forward filter (FFF) while the noise prediction component is called the feedback filter (FFB) [33]. The addition of linear noise prediction to subtract residual ISI from the filtered data makes the DFE a superior technique over LE. We once again choose MMSE as
the criterion to optimize both filters in the DFE for better performance at low SNR and the ease of comparison with the LE receiver described in Chapter 4. To resolve random phase distortion in the DOQPSK signal, noncoherent detection is added to the DFE structure in this work. Similar to the NLE case, the phase reference in NDFE is calculated from previous estimates of the data. In addition, both nonrecursive and recursive phase estimation techniques are considered.

The channel for MMSE–NDFE is the same as the one described in Section 2.2.1 and further elaborated in Section 4.1. We start this chapter by describing the structure of the MMSE–NDFE receiver and the calculation of the phase reference in detail. We then derive the optimum FFF and FBF for the DFE design. Finally, the performance of the NDFE receiver is analyzed before simulations with varying parameters are presented.

5.1 Receiver Structure

A complete structure of the NDFE is illustrated in Fig. 5.1. In accordance with the NLE receiver, the received signal samples $r[k]$ consist of the transmit data $s[k]$ convolved with a channel impulse response (CIR) that has a transfer function $H_C(z)$, as expressed in Eq. (4.9). Additionally, the signal is impaired by both a constant phase shift $e^{j\Theta}$ and an additive noise component $z[k]$ that is colored by the matched filter $h_R(t)$. $z[k]$ has zero mean and an ACF equal to that described in Eq. (4.5) of Section 4.1.

The samples $r[k]$ are first passed through the FFF, with $f_F[k]$ as the coefficients of the filter's impulse response. The output of the filter $q[k]$ can be expressed as

$$q[k] = \sum_{m=0}^{q_F} f_F[m] r[k-m], \quad (5.1)$$

where $q_F = L_F - 1$, and $f_F[k]$ is the impulse response of the FFF. The decision variable $d[k]$ is then obtained from the following equation:

$$d[k] = e^{-j\theta_{ref}} q[k] - b[k-1]. \quad (5.2)$$
Figure 5.1: Complete receiver structure for NDFE.

Eq. (5.2) may be broken down into two parts: first the output of the FFF \( q[k] \) is multiplied by the conjugate of the random phase estimate \( e^{j\theta_{ref}[k-1]} \) based on previous samples. The result is then subtracted by the output of the FBF \( b[k-1] \). The phase estimate is defined as

\[
e^{j\theta_{ref}[k-1]} \triangleq \frac{p_{ref}[k-1]}{|p_{ref}[k-1]|},
\]

where \( p_{ref}[k-1] \) constitutes the phase reference and can be calculated nonrecursively as [37]

\[
p_{ref}[k-1] = \sum_{n=1}^{N-1} q[k-n]y^*[k-n],
\]

with \( N - 1 \) equal to the number of previous estimates used. \( y[k] \triangleq s[k-k_0] + b[k-1] \) may be considered as a reconstruction of the previous filter output \( q[k-1] \). Each of the terms \( q[k-n]y^*[k-n] \) in the summation in Eq. (5.4) is an estimate of the phase offset \( e^{j\theta} \) based on the previous decisions.

Alternatively, \( p_{ref}[k-1] \) can be determined recursively and is equal to

\[
p_{ref}[k-1] = (1 - \zeta)q[k-1]y^*[k-1] + \zeta p_{ref}[k-2].
\]

\( \zeta, 0 \leq \zeta \leq 1 \), is a forgetting factor similar to that used in recursive NLE. It is important to note that the use of the phase reference here in NDFE is slightly different from that in NLE. Specifically, the NDFE scheme uses \( p_{ref}[k-1] \) to produce an estimate of the random phase distortion \( e^{j\theta} \) while in NLE we factor \( p_{ref}[k-2] \) directly into the differential decoding process, i.e., the decision rule for conventional DD reviewed in Section 2.10 is not employed here in NDFE.
Instead, a decision of $c[k]$ is first made before inverse differential encoding takes place at the end of the receiver (cf. Fig. 5.1).

The output of the feedback filter $b[k]$ is denoted by

$$b[k] = \sum_{n=1}^{q_B} f_B[n]j^{[(k-k_0-n)\mod 2]}\hat{c}[k - k_0 - n],$$  \hspace{1cm} (5.6)

where $q_B = L_B - 1$ and $L_B$ is equal to the length of the FBF. The estimates of the differentially encoded data $\hat{c}[k]$ are obtained by the following decision rule:

$$\hat{c}[k - k_0] = \text{sign}[\Re \{(-j)^{[(k \mod 2)]}d[k]\}].$$  \hspace{1cm} (5.7)

The receiver then reverses the process of differential encoding to retrieve estimates of the original data, $\Delta\hat{s}[k - k_0]$ using the following function:

$$\Delta\hat{s}[k - k_0] = \hat{c}[k - k_0] \cdot \hat{c}[k - k_0 - 2].$$  \hspace{1cm} (5.8)

### 5.2 MMSE–DFE Filter Optimization

Since IIR filters cannot be realized, we derive the MMSE–DFE filters with FIR filters [31]. Referring to Fig. 5.1 and taking advantage of the derivations made in Section 4.1, we define the FFF's transfer function $F_F(z)$ as the $Z$–transform of the filter impulse response $f_F[k]$, and is equal to

$$F_F(z) = \sum_{k=0}^{L_F-1} f_F[k]z^{-k}.$$  \hspace{1cm} (5.9)

$L_F$ is redefined as the length of the FFF and is treated as finite in this work. In Fig. 5.1, the equalizer output signal is $q[k]$ and is defined in Eq. (5.1). Here, we wish to shorten the notation to

$$q[k] = f_F^H r[k],$$  \hspace{1cm} (5.10)

where $f_F$ and $r[k]$ are vector notations of $F_F(z)$ coefficients and received samples $r[k - m]$, $0 \leq m \leq L_F - 1$, respectively, as expressed below

$$f_F = \begin{bmatrix} f_F[0] & \cdots & f_F[q_F] \end{bmatrix}^H$$  \hspace{1cm} (5.11)

$$r[k] = \begin{bmatrix} r[k] & \cdots & r[k - q_F] \end{bmatrix}^T$$  \hspace{1cm} (5.12)
Combining Eqs. (5.10) and (5.2), $d[k]$ can be rewritten as
\[
d[k] = e^{-j\theta_{\text{rel}}[k-1]} f_F^H r[k] - f_B^H \hat{s}[k],
\]
with
\[
f_B = \left[ f_B[1] \cdots f_B[q_B] \right]^H
\]
\[
\hat{s}[k] = \left[ j^{(k-k_0-1)\mod 2} \hat{c}[k-k_0-1] \cdots j^{(k-k_0-q_B)\mod 2} \hat{c}[k-k_0-q_B] \right]^T
\]
(5.13)

We measure the error signal $e[k]$ as
\[
e[k] = d[k] - \hat{c}[k - k_0].
\]
(5.16)

$d[k]$ and $\hat{c}[k - k_0]$ are the decision variable and the estimates of the differentially encoded data, respectively. A decision delay $k_0 \geq 0$ is added to deal with non-causal components of the channel. The error $e[k]$ is a random process that has zero mean and variance equal to the expected value of the squared error. For filter optimization we can assume that $\hat{c}[:]=c[:]$. The cost function \( J_{\text{NDFE}}(f_F, f_B) \) on which we base our MMSE filter optimization is then equal to following according to [8]
\[
J_{\text{NDFE}}(f_F, f_B) \triangleq \mathcal{E}\{ |e[k]|^2 \}
= 1 + f_F^H \Phi_{rr} f_F + f_B^H f_B
- 2 \Re \{ \mathcal{E} \{ e^{-j\theta_{\text{rel}}[k-1]} f_F^H r[k] (\hat{c}[k - k_0] + f_B^H \hat{s}[k])^* \} \}
\]
(5.17)

with $\Phi_{rr}$ equal to
\[
\Phi_{rr} = \begin{bmatrix}
\phi_{rr}[0] & \phi_{rr}[1] & \cdots & \phi_{rr}[q_F] \\
\phi_{rr}[-1] & \phi_{rr}[0] & \cdots & \phi_{rr}[q_F-1] \\
\vdots & \vdots & \ddots & \vdots \\
\phi_{rr}[-q_F] & \phi_{rr}[-q_F+1] & \cdots & \phi_{rr}[0]
\end{bmatrix}
\]
(5.18)

where $\phi_{rr}[\lambda]$ is the correlation in Eq. (4.20). Unfortunately, for NDFE a closed form solution of $f_F$ and $f_B$ is difficult to obtain as pointed out in [33]. Nonetheless, it can be shown that the optimum solution of $f_F$ and $f_B$
for coherent MMSE-DFE is also valid for NDFE [16]. To determine the optimal filter settings for coherent DFE, we first define the coherent MMSE-DFE cost function $J_{DFE}$ [31], which is equal to

$$J_{DFE}(f_F, f_B) = \mathcal{E}\{ |e[k]|^2 \} = 1 + f_F^H \Phi_{rr} f_F + f_B^H f_B - f_F^H \Phi_{sr} f_B - f_B^H \Phi_{sr}^H f_F,$$

where we have assumed $s[r] = s[k]$ and

$$\Phi_{sr} = \mathcal{E}\{ s^H r[k] \} = h_G[k_0 + 1] h_G[k_0 + 2] \cdots h_G[k_0 + q_B]$$

$$= \left[ \begin{array}{cccc} h_G[k_0] & h_G[k_0 + 1] & \cdots & h_G[k_0 + q_B - 1] \\ \vdots & \vdots & \ddots & \vdots \\ h_G[k_0 + 1 - q_F] & h_G[k_0 + 2 - q_F] & \cdots & h_G[k_0 + q_B - q_F] \end{array} \right].$$

Then by differentiating $J_{DFE}$ with respect to $f_F^*$ and $f_B^*$, respectively, we get

$$\frac{\partial J_{DFE}(f_F, f_B)}{\partial f_F^*} = (\Phi_{rr} - \Phi_{sr} \Phi_{sr}^H) f_F - \Phi_{sr}$$

$$\frac{\partial J_{DFE}(f_F, f_B)}{\partial f_B^*} = f_B - \Phi_{sr}^H f_F,$$

By setting the gradients to zero, the optimum filter coefficients $f_{F-opt}$ and $f_{B-opt}$ are given by [31]

$$f_{F-opt-DFE} = (\Phi_{rr} - \Phi_{sr} \Phi_{sr}^H)^{-1} \Phi_{sr}$$

$$f_{B-opt-DFE} = \Phi_{sr}^H f_{F-opt-DFE}.$$

### 5.3 Performance Analysis

The minimum error variance can be obtained by inserting $f_{F-opt-DFE}$ and $f_{B-opt-DFE}$ from Eq. (5.26) into Eq. (5.17). After some manipulations similar
to that in the Appendix of [33], the following equation results:

\[
\sigma_{\text{min-NDFE}}^2 \approx \sigma_{\text{min-DFE}}^2 \left( 1 + \frac{1}{2} \rho \left\{ f_{F-\text{opt-DFE}}^H (\varphi_{sr} + \Phi_{sr} f_{B-\text{opt-DFE}}) \right\} \right). 
\]

(5.28)

Although the approximations necessary for derivation of this result are not strictly valid for finite \( N \) and \( \zeta < 1 \), comparisons with simulation results suggest that Eq. (5.28) is quite accurate for small \( N \) and \( \zeta \) (cf. Section 5.4). In the limit of \( \rho \rightarrow 0 \), Eq. (5.28) simplifies to \( \sigma_{\text{min-DFE}}^2 \), which represents the coherent case. On the other hand, as \( \rho \rightarrow 1 \) the performance loss of NDFE is maximized.

The highly nonlinear nature of the error variance makes performance analysis difficult. Nevertheless, using several approximations, which are very tight for \( N \gg 1 \) (\( \zeta \rightarrow 1 \)), high SNRs, and high SDRs for coherent DFE, it is possible to derive a closed-form expression for the SDR for NDFE [33]. From [31] and Eq. (5.28), the SDR for the NDFE system can be expressed as

\[
\text{SDR}_{\text{NDFE}} = \frac{\text{SDR}_{\text{DFE}} + 1}{1 + \frac{1}{2} \rho \left\{ f_{F-\text{opt-DFE}}^H (\varphi_{sr} + \Phi_{sr} f_{B-\text{opt-DFE}}) \right\}} - 1. 
\]

(5.29)

\( \text{SDR}_{\text{DFE}} \) denotes the SDR for coherent MMSE-DFE and \( \rho \) is defined in Section 4.3. Although the approximations necessary for derivation of this result are not strictly valid for finite \( N \) and \( \zeta < 1 \), comparisons with simulation results suggest that Eq. (5.29) is accurate for small \( N \) and \( \zeta \). Again, in the limit of \( \rho \rightarrow 0 \), the SDR becomes that of coherent DFE.

As well as the MMSE-NDFE can suppress ISI, the effect of the PSF coefficients on the error performance continues to exist. We can analyze the effect from derivations of the error variance with respect to the value of the PSF coefficient \( \beta \). For simplicity and without loss in generality, we express the error variance in terms of a coherent DFE with infinite impulse response (IIR) rather than that for FIR NDFE. Deriving from [3], the error variance of an IIR coherent
DFE for DOQPSK is
\[
\sigma_{e-\text{IIR-DFE}}^2 = \exp \left( -T_b \int_{-1/2T_b}^{1/2T_b} \ln \left[ \frac{|H_G(e^{j2\pi fT_b})|^2}{N_0 H_G(e^{j2\pi fT_b}) + 1} \right] df \right),
\]
where \( H_G(e^{j2\pi fT_b}) \) is the Z-transform of CIR with \( z = e^{j2\pi fT_b} \). After some manipulations (cf. Appendix C), the error variance can be simplified to
\[
\sigma_{e-\text{IIR-DFE}}^2 = \frac{2N_0}{1 + N_0 + \sqrt{(1 + N_0)^2 - 4\beta^2}}.
\]

We now try different values of \( \beta \) in Eq. (5.31) for evaluation. For example, when \( \beta = 1/2 \) (corresponding to a CIR with rectangular PSF and a matched filter) the error variance simplifies to \( 2N_0 \). For \( N_0 \ll 1 \), which is true at high SNRs, this result is comparatively smaller than the equivalent NLE error variance. Likewise for the case \( \beta = 1/\pi \), which corresponds to a CIR with half-sine PSF and a matched filter, the error variance reduces to \( 2N_0/(1 + N_0 + \sqrt{(1 + N_0)^2 - 4/\pi^2}) \). In the limit of high SNR, the error variance is approximately 0.5 dB away from that of CD. Finally, for \( \beta = 0 \), such as the CIR of the RTZ filter in Eq. (2.22), technically since there is no ISI, the error variance is \( \sigma_{e-\text{IIR-DFE}}^2 = N_0/(1 + N_0) \).

### 5.4 Simulation and Results

In the following subsections we first compare the bit-error performance of NDFE with that of NLE for DOQPSK, varying different parameters in the receiver. We then show plots of SDR vs. SNR, as they represent how well the equalizer suppresses ISI. Finally, the versatility of NDFE is demonstrated by the BER performance against a constant phase drift \( \Delta fT_s \).

#### 5.4.1 BER vs. SNR

For comparisons with the results from Section 4.5, the length of the FF filter \( L_F \), the length of the FB filter \( L_B \), and the decision delay \( k_0 \) are equal to 9, 2,
and 4, respectively. Fig. 5.2 shows the BER versus SNR of NDFE with nonrecursive phase reference estimation. SNR is expressed as $10 \log_{10}(E_b/N_0)$ in dB. The PSF used in this case is the rectangular filter. The BER of DOQPSK–CD and the performance of NLE are also included as references. From the figure we see substantial BER improvement by NDFE over NLE. At $\text{BER} = 10^{-4}$, and for the same phase reference parameter $N = 10$, NDFE power efficiency is more than 2 dB better than that of NLE. In the limit of $N \rightarrow \infty$, the BER still appears to worsen relative to that of CD at increasing SNR. At $\text{BER} = 10^{-5}$, the SNR of MMSE–NDFE is approximately 2 dB away from that of CD. As predicted, the MMSE–NDFE error variance is about 3 dB worst than that of CD at high SNRs.

![Figure 5.2: BERs of NDFE and NLE with nonrecursive phase reference, and DOQPSK–CD.](image)

Similarly, the BER curves of NDFE with recursive phase reference estimation are presented in Fig. 4.3. Recursive phase estimation in NDFE outperforms its NLE counterpart by a large margin. When the recursive parameter $\zeta$ is equal to 1, the error performance is the same as that for $N \rightarrow \infty$ in the nonrecursive case, which is still at least 2 dB away from the performance of CD.
Figure 5.3: BERs of NDFE and NLE with recursive phase reference, and DOQPSK–CD.

Compared to the results from NLE, we wish to see how the tap weight coefficient $\beta$ in the impulse response $h_G[k]$ from Eq. (4.2) play a role in the error performance of NDFE. Fig. 5.4 displays the BER performance of NLE when the rectangular and the half–sine PSFs are used. In all cases of simulation, the nonrecursive scheme is employed for phase estimation. As expected, the BER curves improve when a PSF with a lower value of $\beta$ is used. When the half–sine filter is used ($\beta = 1/\pi$) with $N = 10$, the power efficiency is within approximately 0.5 dB of CD, which agrees with our expectations.

5.4.2 SDR vs. SNR

Here we evaluate how well the DFE filters suppress ISI. The SDR versus SNR curves are plotted in Figs. 5.5 (a) and (b) with nonrecursive and recursive phase estimation, respectively. We match all the simulations with the same rectangular PSF. Also, both SDR and SNR are expressed in dB. The results show that MMSE–NDFE performs very well in terms of suppressing ISI, as
Figure 5.4: BERs of NDFE with rectangular and half-sine PSFs, varying N. exemplified by the relatively constant slope of the curves. The high SDR performance by NDFE is largely due to the FB filter, which cancels the residual ISI formed at the output of the FF filter.

5.4.3 BER vs. $\Delta fT_s$

The robustness of MMSE-NDFE against frequency offset $\Delta fT_s$ with non-recursive and recursive phase estimation is shown in Figs. 5.6 (a) and (b), respectively. In all of the simulations, a rectangular PSF is used and $10 \log_{10}(E_b/N_0) = 10$ dB is valid. $\Delta fT_s$ once again indicates the speed at which the phase drifts per symbol period $T_s$. The MMSE-NDFE plots show a more stable profile than that of MMSE-NLE.

When the receiver has low phase estimation parameters (e.g. $N = 2$ or $\zeta = 0.2$), the BER curves are most tolerable towards phase drifts. This is because in those cases, only the most recent decisions are accounted for in the estimation of the phase reference.
Figure 5.5: SDR vs. SNR of NDFE with (a) nonrecursive phase reference, and (b) recursive phase reference.

Figure 5.6: BERs of NDFE with constant phase drift $\Delta f T_s$. 
5.5 Summary

In this chapter, the NDFE scheme has been presented. Two filters have been used at the FF and FB positions to better cancel the effects of ISI. Based on a MMSE criterion similar to the one for NLE, the constraint on the FF filter has been reduced to suppressing only the non-causal components of the ISI. The FB filter then subtracts the residual components by noise prediction from previously detected bits. Again with a nonrecursive, or recursive, phase reference estimator similar to NLE, NDFE achieves even better performance than its NLE counterpart.

The error performance for NDFE may be improved even more by lengthening the equalizing filters, or by increasing the nonrecursive and recursive parameters, \( N \) and \( \zeta \), respectively, in the phase reference estimator. On the other hand, lowering \( N \) or \( \zeta \) strengthens the receiver’s robustness against phase drifts, compromising very little in error performance.

The sensitivity of error performance to the CIR is not as severe in NDFE as in NLE. Even with a half-sine PSF that has the ISI parameter \( \beta = 1/\pi \), the BER approaches closely to that of CD.
Chapter 6

Noncoherent Sequence Estimation (NSE)

The NLE and NDFE receivers described in Chapters 4 and 5, respectively, have offered considerable improvements in suppressing ISI from received DOQPSK signals over conventional DD. In addition to their low computational complexity, both equalization techniques have provided stability against constant phase drifts. However, in both cases, the error performance depends significantly on the characteristics of the channel, as shown in Eq. (4.29) for NLE, and Eq. (5.31) for NDFE. In some cases, the properties of the CIR can be controlled in part by the shape of the transmit and receive filters, as analyzed in Sections 4.4 and 5.3.

Regardless of channel characteristics, the F-MBDD and I-MBDD shown in Chapter 3 of this thesis remain to be the optimal solution in terms of minimizing the bit error probability (BEP) of the system. Nonetheless, the block-by-block approach to maximum ML detection increases the number of computations drastically when the observation window widens. ML block detection (MLBD) can be feasible using a trellis-based decoding algorithm like the Viterbi algorithm (VA) [14], which can reduce complexity to a moderate level. Trellis-based decoding has been considered in [38] and [39] for regular M-ary PSK, where the authors proposed approximations of the optimal sequence met-
ric. As expected, the performance of the above noncoherent schemes improves with increasing observation length and receiver complexity, and approaches that of CD. However, the performance degrades severely even with small phase variations with large observation length $N$ [33].

A suboptimal scheme based on the VA may be used to increase the versatility of the receiver against phase drifts. The noncoherent sequence estimation (NSE) scheme proposed for DOQPSK in this chapter is a suboptimal scheme that belongs to the class of trellis–based noncoherent receivers described in [13]. NSE overcomes some limitations of block–by–block MBDD by truncating the optimal sequence metric to a finite size $N$. With respect to other schemes, they are characterized by the fact that a performance gain may be achieved with acceptable levels of complexity. In the NSE case, the tradeoff between complexity and performance can be controlled by the metric window size $N$ and the number of trellis states $K$. The smaller the parameters, the lower the complexity, but at the expense of higher error probability.

The presented approach is applicable to the case of ISI–affected channels such as the one formulated in this work. From the discussions in [13], a whitened matched filter (WMF) front–end is found to be practically essential for NSE against ISI channels.

In the next section, we first derive the optimal decision rule for DOQPSK. The realization of this detection strategy entails a maximization of a suitable sequence metric by an exhaustive method. Second, we present the suboptimal NSE decision strategy in detail. Both a nonrecursive and a recursive scheme are presented. The algorithm for a four–state trellis decoder is then proposed for DOQPSK. We also describe the algorithm for a lower complexity decoder that uses only two states. In both decoders, the receiver front–end is installed with a linear filter equal to the feed–forward filter (FFF) in MMSE–NDFE. The linear filter not only whitens the noise by lowering the noise variance, as suggested in [13, 16, 33], it also shapes the overall impulse response favorably for state reduction in the trellis decoder. Section 6.3 follows with a performance
discussion of the NSE scheme before simulations and results are compared in Section 6.4.

### 6.1 Derivation of Decision Metric

Fig. 6.1 shows a block diagram of the NSE receiver for DOQPSK. As before, we assume that the signal model for NSE is the one described in Section 2.5. The DOQPSK signal \( s[k] \) is impaired by a constant phase shift \( e^{j\Theta} \), with random variable \( \Theta \) uniformly distributed in \((-\pi, \pi]\), and complex AWGN \( n(t) \) with zero mean and variance \( N_0 \). The overall impulse response \( h_G[k] \) is defined in Eq. (4.2). The received sample data \( r[k] \) is presented in Eq. (4.1) but shown here again for convenience.

\[
r[k] = e^{j\Theta} \sum_{\nu=-\infty}^{\infty} h_G[\nu] s[k - \nu] + z[k],
\]  

(6.1)

\( L_H = q_H + 1 \) is the length of the channel and \( z[k] \) is noise that is colored by the matched filter. It has a zero mean and ACF of \( \phi_{zz}[\lambda] \) as defined in Eq. (4.5).

![Figure 6.1: Block diagram of NSE receiver.](image)

In addition to the matched filter, a linear filter \( f_F[k] \) may be used to further reduce the noise. The output of the filter \( q[k] \) can be compactly described by

\[
q[k] = e^{j\Theta} y[k] + w[k],
\]  

(6.2)

where \( w[k] \) is the noise component of the signal, which consists of the colored
noise \( z[k] \) convolved with the linear filter \( f_F[k] \). \( y[k] \) can be defined as

\[
y[k] = \sum_{m=0}^{L} h_{ov}[m] s[k - m] = \sum_{m=0}^{L} h_{ov}[m] g^{((k-m) \text{mod } 2)} c[k - m],
\]

with \( L = L_H + L_F - 1 \) equal to the length of the overall discrete–time channel \( h_{ov}[k] \), which is equal to \( h_G[k] * f_F[k] \). \( L_F \) is the length of the linear filter \( f_F[k] \). The linear filter we have chosen to use is equal to the optimum FFF for MMSE–NDFE derived in Eq. (5.26). The overall CIR \( h_{ov}[k] \) is equal to

\[
h_{ov}[k] = \sum_{m=0}^{L_H-1} h_G[m] f_{F-opt}[k - m].
\]

Assuming perfect bit timing, the formation of the optimal decision strategy based on received samples \( q[k] \) according to [40, 13] is

\[
\hat{c} = \arg \max \hat{c} \left\{ -\frac{1}{N_0} \sum_{k=0}^{N_T-1} \left| \tilde{y}[k] \right|^2 + \log I_0 \left( \frac{1}{N_0} \sum_{k=0}^{N_T-1} q[k] \tilde{y}[k]^* \right) \right\}.
\]

\( \hat{c} \) is the detected sequence while \( \hat{c} \) is a hypothetical sequence of the differentially coded data \( c \). \( I_0(\cdot) \) is the zeroth–order modified Bessel function of the first kind [3]. \( N_T \) is the length of the entire data sequence transmitted. \( \tilde{y}[k] \) is defined in terms of the hypothetical code sequence \( \tilde{c}[k] \) as in Eq. (6.3). The optimal noncoherent strategy in Eq. (6.5) may be approximated by letting \( \log I_0(x) \approx x \). The longer the transmission length, the better the approximation quality. Finally, the decisions on the original data \( \Delta s[k] \) are determined by inverting the differential encoding process.

The terms defined in the argument of Eq. (6.5) can be considered as a decision metric for the entire signal sequence in the interval \( N_T \). We can further define a partial sequence metric \( \Lambda[k] \) in the \( k^{th} \) signalling interval as

\[
\Lambda[k] = \left| \sum_{n=0}^{k-1} q[n] \tilde{y}^*[n] \right| - \frac{1}{2} \sum_{n=0}^{k-1} |\tilde{y}[n]|^2,
\]

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and an incremental metric \( \Delta[k] \)
\[
\Delta[k] \equiv \Lambda[k + 1] - \Lambda[k] \\
= \left| \sum_{n=0}^{k} q[n] \hat{y}^*[n] \right| - \left| \sum_{n=0}^{k-1} q[n] \hat{y}^*[n] \right| - \frac{1}{2} |\hat{y}[n]|^2. \tag{6.7}
\]

Thus the entire sequence metric \( \Lambda[N_T] \) may be calculated recursively from Eqs. (6.6) and (6.7).

The difficulty inherent in the optimal decision metric in Eq. (6.7) is its unlimited memory. The evaluation of the optimal decision must depend on the entire previous code sequence. Unless a very short transmission length is assumed, the maximization of the metric may be realized by a search on a tree diagram, for example, which makes the search method impractical.

In order to limit the memory of the incremental metric in Eq. (6.7), the NSE receiver truncates the number of summation terms to a size of \( N \ll N_T \). The Viterbi algorithm can then be used to store the metric decisions previously determined based on a trellis decoder [41, 14]. The memory-truncated incremental metric \( \lambda[k] \) for the VA relative to the strategy in Eq. (6.7) is

\[
\lambda[k] \equiv \left| \sum_{n=0}^{N-1} q[k - n] \hat{y}^*[k - n] \right| - \left| \sum_{n=1}^{N-1} q[k - n] \hat{y}^*[k - n] \right| - \frac{1}{2} |\hat{y}[k]|^2. \tag{6.8}
\]

The NSE receiver with branch metrics from Eq. (6.8) succeeds by correlating the filter output \( q[k] \) with \( \hat{y}[k] \). The term \( \hat{y}[k] \) takes into account the overall CIR, which is significant in the decision. Another advantage of using the NSE detector is that the memory truncation increases the robustness of the system against constant phase drifts.

The incremental metric in Eq. (6.8) is a nonrecursive approach to NSE. Similarly, a recursive scheme can be considered by adding a factor \( \zeta \) into the equation [42]. The branch metric in Eq. (6.8) can be rewritten as

\[
\lambda[k] \equiv \left| q[k] \hat{y}^*[k] + \bar{p}_{\text{ref}}[k - 1] \right| - \left| \bar{p}_{\text{ref}}[k - 1] \right| - \frac{1}{2} |\hat{y}[k]|^2, \tag{6.9}
\]

where \( \bar{p}_{\text{ref}}[k - 1] \) is a hypothetical output from the preceding bits, and is defined as

\[
\bar{p}_{\text{ref}}[k - 1] \equiv \zeta \bar{p}_{\text{ref}}[k - 2] + (1 - \zeta) q[k - 1] \hat{y}^*[k - 1]. \tag{6.10}
\]
The amount of contribution \( q_{\text{ref}}[k - 1] \) makes to the branch metric \( \lambda[k] \) can then be adjusted by the variable \( \zeta \), \( 0 \leq \zeta \leq 1 \). The larger the value of \( \zeta \), the more reliable the decision.

Although the shortening of the memory in the decision metrics makes the NSE receiver suboptimal, vast error improvements can be achieved with small to moderate observation window size \( N \) (or \( \zeta \) ). The optimal solution is achieved with \( N \rightarrow N_T \) (or \( \zeta \rightarrow 1 \)).

### 6.2 Detection Algorithm

The idea behind the VA proposed by Forney in 1973 [14] is to interpret the incoming signal as a traversal of states in a finite-state machine (FSM) according to branch metrics \( \lambda \). The algorithm reduces memory by allowing the previous metric decisions to be stored in the states. Visually, a trellis diagram is often used to portray the transitions of the states over a number of intervals. As an example, Fig. 6.2 illustrates such a diagram with four states starting from a time interval indicated by \( \kappa \). As the time interval advances, the branch metrics for each of the possible state transitions are calculated. The main feature of the VA is displayed when more than one path \( P \) diverge from the same state at time \( \mu \), and merge in a state at another time \( \nu \). In this case, only one of the paths may survive as the decision path. The survivor path \( \hat{P} \) is selected usually by minimizing the cost function of all the possible paths, as stated mathematically below:

\[
\hat{P} = \arg \min_{\tilde{P}} \left\{ \sum_{k=\mu}^{\nu} \lambda_{\tilde{P}}[k] \right\}.
\]  

(6.11)

\( \hat{P} \) is one of the possible survivor paths and \( \lambda_{\hat{P}}[k] \) is the branch metric of a particular path at time \( k \). The branch metric \( \lambda[k] \) can be defined differently according to different modulations and decision strategies [12, 42, 16, 33]. For DOQPSK, we use the one written in Eq. (6.8) as the branch metric for selecting survivor paths.
The complexity of the VA can be adjusted by varying the number of states in the trellis diagram. As indicated in [13], the fewer the states, the lower the complexity and error performance. Here, we derive the algorithms for a four-state and a two-state decoder. Both decoders use the linear filter described in Section 6.1 to improve error performance.

In the four-state algorithm, the states $S$ are defined as the possible values of the current data bit $c[k] \in \{\pm 1\}$ and the previous one $c[k - 1] \in \{\pm 1\}$. Then for the subsequent interval $k + 1$, a new bit $c[k + 1]$ is received and the transitions advance. Each of the states at time $k$ results in a total of two possible transitions. Paths in contention are then resolved using the rule given in Eq. (6.11). Therefore, at each interval, only four survivor paths are retained. At the end of the transmission, the bits representing the path that has the lowest accumulated cost function $\sum_{k=\mu}^{\nu} \lambda_\rho[k]$ are selected as the decision sequence.

On the other hand, the two-state decoder marks the states $S$ by the possible values of the current bit $\tilde{c}[k] \in \{\pm 1\}$ only. For the next interval $k + 1$, each state also has two possible transitions and the survivor paths are chosen in the same manner as the four-state case. At the end of the transmission, only two paths remain for decision. Clearly, the two-state decoder has a simpler approach and requires fewer calculations per received bit than its four-state counterpart. However, the four-state algorithm makes more reliable decisions because more survivor paths are stored for evaluation.

Figure 6.2: Sample four-state trellis diagram.
6.3 Performance Analysis

Although performance analysis has been discussed for NSE in [40], the evaluation is based on $M$-ary PSK signals without ISI. For DOQPSK signals with ISI, the evaluation of NSE error performance is assessed by means of computer simulation in terms of bit-error rate (BER) versus signal-to-noise ratio (SNR). Despite the lack of theoretical derivation of the BER, we expect that the error performance improves as we increase the truncation size $N$ for the calculation of nonrecursive branch metrics, or the factor $\zeta$ for the calculation of the recursive metrics. In the limit of $N \rightarrow N_T$ ($\zeta \rightarrow 1$), the performance of NSE approaches that of CD. For optimal performance, NSE can be performed on a tree, i.e., a trellis where the number of states $K$ grows with time.

6.4 Simulation Results

In the following subsections we first show the BER performance of the NSE receiver with a four-state trellis decoder, with comparisons to NLE and NDFE schemes. We then contrast the bit-error performance of the four-state NSE decoder against the two-state decoder, varying different parameters in the receiver. Finally, the versatility of the NSE is revealed and discussed.

6.4.1 BER vs. SNR

For consistency with the simulations for NLE and NDFE, the length of the linear filter $L_F$, and the decision delay $k_0$ for NSE are equal to 9 and 4, respectively. Fig. 6.3 shows the BER versus SNR of NSE with a four-state trellis decoder and nonrecursive metric calculation. SNR is expressed as $10 \log_{10}(E_b/N_0)$ in dB. The PSF used in this case is the rectangular filter. The BER for DOQPSK–CD, NLE and NDFE are also included as references. From the figure we see that the four-state NSE decoder outperforms NLE and NDFE for $N \geq 4$. When the observation window size $N$ reaches the length of the
transmission sequence, \( N_T = 50000 \) in this case, the BER curve is virtually indistinguishable from that of CD. The power efficiency achieved by NSE with a rectangular PSF is simply not realized by either of the NLE and NDFE schemes. Even for low \( N \), the bit-error performance of NSE still follows closely to that of CD, which is not the case in NLE or NDFE.

![Figure 6.3: BERs of NSE four-state trellis decoder with nonrecursive phase reference, varying \( N \).](image)

Fig. 6.4 shows the performance of the same four-state NSE decoder but with recursive metric calculations. In this case, NSE performs well even for low values of the forgetting factor \( \zeta \). At the limit of \( \zeta = 1.0 \), the BER performance of NSE matches that of CD.

We then compare the performance of the four-state NSE decoder with a simplified one that uses only two states. Figs. 6.5 and 6.6 depict the error performance of a two-state NSE decoder against a four-state decoder, based on nonrecursive and recursive metric calculations, respectively. For low values of \( N \), and \( \zeta \), the power efficiency of the two-state decoder cannot compete with that of the four-state counterpart. However, for \( N = 10 \), and \( \zeta = 1.0 \) there is very little difference in performance between the two decoders. This implies
that asymptotically the number of states in the trellis decoder has no effect on the overall error performance. Hypothetically, a NSE trellis decoder that has only one state is in fact identical to MMSE-NDFE discussed in Chapter 5.

6.4.2 BER vs. $\Delta f T_s$

NSE's robustness against frequency offset $\Delta f T_s$ is represented in Figs. 6.7 (a) and (b). The plots show the BER vs. $\Delta f T_s$ performance of a two-state NSE decoder with nonrecursive and recursive metric calculations, respectively. Each of the graphs also contains the same plots from NDFE as references. For all of the simulations, rectangular PSF is used and $10 \log_{10}(E_b/N_0) = 10$ dB is valid. In both nonrecursive and recursive cases, the NSE decoder exhibits a steady decrease in performance over the speed of the phase drift. Also, increasing $N$, or $\zeta$, results in a lower tolerance against phase variations. Moreover, the NSE decoder shows comparatively lower tolerance than NDFE, particularly at high $\Delta f T_s$. This implies that NDFE is more robust than NSE in response to
Figure 6.5: BERs of NSE two-state trellis decoder with nonrecursive phase reference, varying $N$.

Figure 6.6: BERs of NSE two-state trellis decoder with recursive phase reference, varying $\zeta$. 
frequency offsets, due to the increased number of states (i.e. memory) in the NSE trellis decoder.

Figure 6.7: BERs of (a) nonrecursive and (b) recursive NSE and NDFE with constant frequency offset $\Delta fT_s$.

6.5 Summary

Among all of the receivers considered, NSE is the most well-rounded in terms of balancing error performance, computational complexity, and robustness against variations of the phase. NSE uses the same ML approach as MBDD, combined with a trellis decoder of $K$ states based on the VA. The NSE decoder is suboptimum in that the calculation of the branch metrics $\lambda$ is either based on a phase reference with observation size $N$, or from a recursive schedule with forgetting factor $\zeta$. The VA succeeds in reducing the memory required in storing the metrics via survivor path selection. Furthermore, the effects of ISI that is apparent in the equalization receivers, is now considered in the NSE decision metric. This implies that the NSE scheme is robust against ISI with minimal deviations in error performance. One other advantage is that
the complexity of the decoder can be lowered by having fewer states $K$ in the decoder. Ultimately, NSE with $K = 1$ is equivalent to MMSE–NDFE.
Chapter 7

Performance Comparisons

In this chapter, several receiver designs by other authors are compared to the ones presented in this thesis. First, the general BER results of all the receivers considered for DOQPSK are discussed. Second, the BER results of NSE are compared to that of the general likelihood ratio test (GLRT) proposed by Phoel [11]. Finally, minimum-shift keying (MSK) is a modulation scheme similar to OQPSK modulation and several receivers have been proposed for MSK in particular. We therefore compare the performance of those receivers against the ones in this work.

7.1 Comparison of Noncoherent Receivers for DOQPSK

Fig. 7.1 represents a consolidation of the benchmark BER results for various noncoherent receivers proposed or discussed in this work. Despite the different proposals for noncoherent detection (NCD), they all fall into three categories. The first one is so-called conventional differential detection (CDD), where decisions are made based on the phase difference of the received samples between consecutive intervals. CDD schemes have been successful for DBPSK and DQPSK signals [3]. But as exemplified by the BER curve in Fig. 7.1,
CDD does not perform well for DOQPSK due to cross correlation and ISI.

![Figure 7.1: BERs of different DOQPSK receivers.](image)

The receiver by Gunther, Hischke and Habermann [7, 5], is also based on the CDD technique, combined with a trellis decoder. The results by Gunther et al in Fig. 7.1 are directly obtained from [5]. When compared to the results for the NSE decoder with metric size $N = 2$ in Fig. 6.3, Gunther et al’s results show remarkable similarities. A closer look at both schemes indicates that they use a similar VA decoder, which calculates the branch metrics based on the last two received samples (i.e., $N = 2$). This means that the NSE scheme proposed in this work can be viewed as an extension to the scheme by Gunther et al. in that the size of the metric $N$, and the number of states $K$, can be variable. However, Gunther et al’s system assumes a square–root raised–cosine–filter\(^1\) whereas our NSE decoder assumes a rectangular PSF.

The second category of NCD receivers generally uses a form of feedback to estimate the phase reference of the received signal. As in both NLE and NDFE schemes, decisions from previously received samples are used to generate phase

\(^1\)When coupled with a matched–filter, a square–root raised–cosine filter is a PSF that satisfies Nyquist’s theorem for ISI–free transmission [3].
estimates for the decision on the next sample data. In addition, these receivers use equalization techniques to suppress the ISI inherent in the channel. These designs show improvements in error performance over the receivers from the first category.

The rest of the receivers mentioned in this work (F-MBDD, I-MBDD, and NSE) are categorized by their use of maximum likelihood (ML) approach to detecting the data. In F-MBDD and I-MBDD, ML block detection (MLBD) is used to estimate a sequence of data in each observation interval. NSE simplifies the complexity in MBDD by using a trellis decoder. All the receivers in this category are asymptotically optimal and their performance can therefore reach the BER of CD.

### 7.2 General Likelihood Ratio Test (GLRT)

Another scheme proposed by Phoel for DOQPSK is called general likelihood ratio test (GLRT) [11]. Based on [43], GLRT uses a set of predetermined phase estimates to test and find the most likely phase shift constant for a particular received sample. Fig. 7.2 shows a comparison in performance between GLRT and NSE receivers. In the GLRT simulations, the parameter $N$ is defined as the number of previous decisions used for the current decision. The number of predetermined phase estimates is 16. GLRT has linear computational complexity but the performance approaches the limits of CD only at high observation intervals ($N \geq 35$), which makes the receiver very sensitive to phase variations. On the other hand, NSE performs comparatively well even for $N = 10$.

### 7.3 Receivers for Minimum-Shift Keying

Finally, the receivers for minimum-shift keying (MSK) are compared to the NDFE receiver with half-sine PSF. In total, three schemes are considered: differential detection (DD) via VA by Kaleh [23], DD by Vitetta, Mengali and
Figure 7.2: BERs of GLRT and NSE varying observation window size $N$.

Morelli [21], and block demodulation by Mehlan and Meyr [22]. Fig. 7.3 shows the error performance of the four schemes including NDFE.

The receiver design by Kaleh is similar to the one by Gunther et al., which is just a special case of the NSE scheme. The performance of Kaleh’s receiver is poor in this case because the metric size $N$ is restricted to two. Also Kaleh’s scheme requires computational complexity that is exponential in the number of states. The BER plots by Vitetta et al. and Mehlan et al. are based on optimal MLBD with asymptotic behavior, which performs very well with respect to Kaleh’s scheme. In the case of NDFE, although it is suboptimal in terms of bit-error performance, the BER curve actually shows performance similar to that of the optimal schemes with only linear complexity.

Note that although the encoding scheme in MSK and the one in DOQPSK are slightly different (cf. Section 2.6.2), the results from the receivers in Chapters 4 to 6 can be compared to that from the above MSK receivers. This is because the overall CIR $h_G[k]$ is the same for both MSK and DOQPSK schemes given the half-sine filter is used. In NLE, NDFE, and NSE, the receivers succeed by suppressing the effects of ISI in $h_G[k]$ regardless of the encoding scheme.
Therefore, without any changes in the decision rule, similar receivers can be designed for MSK and we would expect them to perform just as well as the receivers for DOQPSK.
Chapter 8

Conclusions and Future Work

This chapter concludes the thesis with some general comments on the receivers proposed in this work, followed by a discussion on the possible future work for DOQPSK noncoherent detection.

8.1 General Comments

In this thesis, five novel noncoherent receiver designs for DOQPSK have been proposed: F-MBDD, I-MBDD, NLE, NDFE, and NSE. First in Chapter 3, the cause of the 3 dB gap in Simon’s MBDD performance against that of CD was found to be in the inadequate detection scheme. With a redefined system model and detection rule (F-MBDD), we proved that MBDD can indeed approach the error performance of CD in the limit of large observation window size $N$. I-MBDD further improves the slow convergence rate of F-MBDD by truncating the decision block size from $N-1$ to $N-3$. We also believe that this truncation is the same reason why Phoel’s two–bit DE scheme performs better than the one–bit scheme.

In Chapters 4 and 5, the high computational complexity in MBDD was simplified by two noncoherent receivers based on equalization techniques. The NLE scheme in Chapter 4 suppresses ISI via a linear equalizer based on the MMSE criterion. Together with a DD scheme that includes a phase reference
estimator, NLE achieves considerable error improvements over conventional DD. On the other hand, NDFE in Chapter 5 deploys two filters at the feed-forward (FF) and feedback (FB) positions to better cancel the effects of ISI. Based on the same MMSE criterion and phase reference estimator as the ones in NLE, NDFE achieves even better performance than its NLE counterpart in almost all aspects. However, the equalizers make the receivers' performance sensitive to the overall CIR. For the three different pulse shapes considered, the error performance of both NLE and NDFE schemes progressively approach that of CD with decreasing values of the channel coefficient parameter $\beta$. Another set of parameters, $N$ and $\zeta$, corresponding to nonrecursive and recursive phase reference calculations, can be controlled to balance error performance and robustness in phase variations.

The NSE technique proposed in Chapter 6 uses the same ML approach as MBDD, and further combines a trellis decoder of $K$ states based on the VA to provide a balanced performance in power efficiency, computational complexity, and robustness against phase variations. The NSE decoder is suboptimal in that the calculation of the branch metrics $\lambda[k]$ relies on phase reference parameters $N$ and $\zeta$. Also, the VA reduces the memory required to store the metrics via survivor path selection. Moreover, unlike in NLE and NDFE, the consideration of ISI is contained in the decision metric in NSE, which makes the scheme robust against ISI with minimal deviations in error performance.

### 8.2 Future Work

The DOQPSK receivers proposed in this work provided a variety of solutions for generating error performances that were previously unattainable. Nonetheless, many topics can still be considered as future work in the area of DOQPSK noncoherent detection. For example, a rectangular filter has been assumed at the transmitter in all of the aforementioned MBDD designs. As an extension to this work, the analysis of MBDD with other pulse shapes may be consid-
ered. However, the detection strategy may have to be adjusted depending on the overall impulse response of the PSFs. For example, the frequent decision errors in the beginning and end bits of the F-MBDD decision block are the result of the CIR characteristics. During block detection, the beginning and end bits contain only half as much energy as each of the bits in the rest of the data sequence, meaning that the average signal-to-noise ratio (SNR) of those bits is exactly 3 dB less than that of the remaining sequence. Asymptotically, these errors dominate the error probability, thereby slowing the convergence in error performance. Therefore, the size of the detection window will need to be optimized accordingly if other pulse-shaping techniques are to be used for MBDD.

Another consideration is to derive an optimal detection strategy for DOQPSK in the presence of fading, since the equalizers in the NLE and NDFE schemes will not perform well against time-variant channels. In addition, throughout this thesis we have assumed that the CIR is known at the receiver. In the absence of such information, adaptive algorithms will need to be included in the decoder to estimate the characteristics of the channel. Finally, if Nyquist-compatible PSFs\textsuperscript{1} are to be used for DOQPSK, techniques other than NLE and NDFE will be required since equalization is only applicable to signals with ISI.

\textsuperscript{1}PSFs that satisfy the Nyquist criterion for band-limited, ISI-free transmission [3].
Appendix A

Derivation of Output Noise Process $z[k]$

$z[k]$ is the sampled output of the AWGN process $n(t)$ through the linear time-invariant filter $h_R(t)$. Mathematically, $z[k]$ is expressed in Eq. (2.12) and conveniently redefined here as

$$z[k] = z(kT_b + t_0)$$

$$= \int_{-\infty}^{\infty} h_R(\mu)n(kT_b + t_0 - \mu)d\mu,$$  \hspace{1cm} (A.1)

where $n(t)$ is a complex AWGN process with zero mean and variance $N_0$ and $h_R(t)$ is the impulse response of the received filter. The mean of the output $z[k]$ is

$$\mathcal{E}\{z[k]\} = \mathcal{E}\left\{ \int_{-\infty}^{\infty} h_R(\mu)n(kT_b + t_0 - \mu)d\mu \right\}$$

$$= \int_{-\infty}^{\infty} h_R(\mu)\mathcal{E}\{n(kT_b + t_0 - \mu)\}d\mu$$  \hspace{1cm} (A.2)

$$= 0.$$

Since the mean of the AWGN process, $\mathcal{E}\{n(t)\}$, is zero for all $t$, Eq. (A.2) is valid.
Consider next the auto-correlation function (ACF) of the output random process $z[k]$, which is defined as

$$\phi_{zz}[\lambda] = \mathcal{E}\{z^*[k]z[k+\lambda]\}$$

$$= \mathcal{E}\left\{ \int_{-\infty}^{\infty} h^*_R(\tau_1)n^*(kT_b + t_0 - \tau_1)dt_1 \int_{-\infty}^{\infty} h_R(\tau_2)n((k + \lambda)T_b + t_0 - \tau_2)dt_2 \right\}$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h^*_R(\tau_1)h_R(\tau_2)\phi_{nn}(\lambda T_b + \tau_1 - \tau_2)d\tau_1 d\tau_2,$$

(A.3)

where $\lambda T_b$ is the time difference and $\phi_{nn}(\tau) = \mathcal{E}\{n^*(t)n(t + \tau)\} = N_0\delta(\tau)$ is the ACF of $n(t)$. One of the integrals can then be evaluated by integrating with respect to $\tau$ to become

$$\phi_{zz}[\lambda] = N_0 \int_{-\infty}^{\infty} h^*_R(\alpha)h_R(\lambda T_b + \alpha)d\alpha$$

(A.4)

$$= N_0\phi_{R}(\lambda T_b)$$

$$= N_0\phi_{R}[\lambda].$$

When $h_R(t)$ is the matched filter to the transmit filter $h_T(t)$, the integral in Eq. (A.4) is equivalent to the convolution $h_T(t) \ast h_R(t)$, which corresponds to the combined impulse response $h_G(t)$ in Eq. (2.6).

Consequently, in discrete–time, the sampled noise $z[k]$ has an ACF equal to

$$\phi_{zz}[\lambda] = N_0 h_G(\lambda T_b)$$

(A.5)

$$= N_0 h_G[\lambda].$$
Appendix B

Derivation of IIR MMSE–LE Error Variance

From [3], the error variance of IIR linear equalization with MMSE criterion, $\sigma_{e-IIR-LE}^2$, is defined as

$$
\sigma_{e-IIR-LE}^2 \triangleq \mathcal{E}\{|e[k]|^2\}
= T_b \int_{-1/2T_b}^{1/2T_b} \frac{\sigma_n^2 H_G(e^{j2\pi fT_b})}{|H_G(e^{j2\pi fT_b})|^2 + \sigma_n^2 H_G(e^{j2\pi fT_b})} df,
$$

(B.1)

with $e[k] = d[k] - \Delta \delta[k - k_0]$, as in Eq. (4.15). $T_b$ is the bit period and $\sigma_n^2 = N_0$ is the variance of the AWGN $n(t)$ from the channel. $H_G(e^{j2\pi fT_b})$ is the $Z$–transform of the combined impulse response $h_G(t)$ from Eq. (2.6), with $z = e^{j2\pi fT_b}$. For the PSFs considered in Section 4.1, the $Z$–transform can be substituted as

$$
H_G(e^{j2\pi fT_b}) = 1 + 2\beta \cos(2\pi x),
$$

(B.2)

where $\beta$ is the tap weight coefficient of $h_G[k]$ in Eq. (4.2) and $x = fT_b$. For
positive values of $H_G(e^{2\pi f T_b})$, i.e., $0 \leq \beta \leq 1/2$, Eq. (B.1) can be simplified to

$$
\sigma_{e-\text{IR-LE}}^2 = \sigma_n^2 \int_{-1/2}^{1/2} \frac{1}{H_G(e^{2\pi f T_b}) + \sigma_n^2} \, dx
$$

$$
= \frac{\sigma_n^2}{\sigma_n^2 + 1 - 2\beta} \int_{-1/2}^{1/2} \frac{1}{1 + \gamma \cos^2(\pi x)} \, dx.
$$

(B.3)

In Eq. (B.3), the identity $\cos(2\pi x) = 2\cos^2(\pi x) - 1$ is used and $\gamma$ is defined as

$$
\gamma \triangleq \frac{4\beta}{\sigma_n^2 + 1 - 2\beta}.
$$

(B.4)

The integral in Eq. (B.3) can be evaluated using trigonometric identities and partial fractions, and is reduced to

$$
\sigma_{e-\text{IR-LE}}^2 = \frac{\sigma_n^2}{\sigma_n^2 + 1 - 2\beta} \left[ \arctan \left( \frac{\tan(\pi x)}{\sqrt{1 + \gamma}} \right) \right]^{1/2}.
$$

(B.5)

We then substitute the limits $(1/2, -1/2)$ into Eq. (B.5) for $x$. Given small values of $\gamma$ (valid for $0 \leq \beta \leq 1/2$ and $0 \leq \sigma_n^2 \leq 1$), the term $1/\sqrt{1 + \gamma}$ in $\arctan(\cdot)$ can be neglected in Eq. (B.5) since, respectively, $\tan(\pm \pi/2) = \pm \infty$. $\arctan(\pm \infty)$ can then be evaluated to $\pm \pi/2$, respectively. Finally, the error variance can be expressed as

$$
\sigma_{e-\text{IR-LE}}^2 = \frac{\sigma_n^2}{\sigma_n^2 + 1 - 2\beta} \frac{1}{\sqrt{1 + \gamma}}
$$

$$
= \frac{\sigma_n^2}{\sqrt{(\sigma_n^2 + 1)^2 - 4\beta^2}}.
$$

(B.6)
Appendix C

Derivation of IIR MMSE–DFE Error Variance

From [3], the error variance of IIR decision-feedback equalization with MMSE criterion, $\sigma_{e-IIR-DFE}^2$, is defined as

$$
\sigma_{e-IIR-DFE}^2 \triangleq \mathcal{E}\{|e[k]|^2\}
= \exp\left(-T_b \int_{-1/2T_b}^{1/2T_b} \ln \left[ \frac{|H_G(e^{j2\pi fT_b})|^2}{\sigma_n^2 H_G(e^{j2\pi fT_b}) + 1} \right] df \right),
$$

with $e[k] = d[k] - \hat{c}[k - k_0]$, as in Eq. (5.16). $T_b$ is the bit period and $\sigma_n^2 = N_0$ is the variance of the AWGN $n(t)$ from the channel. $H_G(e^{j2\pi fT_b})$ is the $Z$-transform of the combined impulse response $h_G(t)$ from Eq. (2.6), with $z = e^{j2\pi fT_b}$. For the PSFs considered in Section 4.1, the $Z$-transform can be substituted as

$$
H_G(e^{j2\pi fT_b}) = 1 + 2\beta \cos(2\pi x),
$$

where $\beta$ is the tap weight coefficient of $h_G[k]$ in Eq. (4.2) and $x = fT_b$. For
positive values of $H_G(e^{2\pi f T_k})$, i.e., $0 \leq \beta \leq 1/2$, Eq. (C.1) can be simplified to

$$\sigma_{e-IIR-DFE}^2 = \sigma_n^2 \exp \left(- \int_{-1/2}^{1/2} \ln \left[ H_G(e^{2\pi f T_k}) + \sigma_n^2 \right] dx \right)$$

(C.3)

$$\sigma_{e-IIR-DFE}^2 = \sigma_n^2 \exp \left(- \int_{-1/2}^{1/2} \ln \left[ \sigma_n^2 + 1 + 2\beta \cos(2\pi x) \right] dx \right).$$

The integral in the exponent in Eq. (C.3) can be evaluated by substituting $u = 2\pi x$ [3]. The error variance can be simplified to

$$\sigma_{e-IIR-DFE}^2 = \sigma_n^2 \exp \left(- \int_0^\pi \ln \left[ \sigma_n^2 + 1 + 2\beta \cos(u) \right] du \right)$$

(C.4)

$$\sigma_{e-IIR-DFE}^2 = \sigma_n^2 \exp \left(- \ln \left[ \frac{\sigma_n^2 + 1 + \sqrt{(\sigma_n^2 + 1)^2 - 4\beta^2}}{2} \right] \right)$$

$$= \frac{2\sigma_n^2}{\sigma_n^2 + 1 + \sqrt{(\sigma_n^2 + 1)^2 - 4\beta^2}}.$$
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