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HIGH-VOLTAGE MEASUREMENT TECHNIQUES

by



Armand Gregoire Halim

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Department of Electrical Engineering

The University of British Columbia
2075 Wesbrook Place
Vancouver, Canada
V6T 1W5

Date June 25, 1980

ABSTRACT

The Department of Electrical Engineering at the University of British Columbia acquired a high-voltage test set in 1979 for teaching and research purposes. To make this test set useful for experiments which undergraduate students can do themselves, various additions and modifications had to be made.

This thesis describes these additions and modifications. First, a Faraday cage had to be constructed with interlocking safety circuits. Experiments were then developed to show basic high-voltage phenomena with AC voltage, with DC voltage, and with impulse voltages. Considerable modifications were required to eliminate noise in the impulse measuring system.

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1. INTRODUCTION

A withstand voltage test subjects an insulation for a restricted period of time to a voltage stress greater than that encountered under normal service conditions. The voltage specified for a withstand test may be of AC, DC (having a specified polarity with respect to ground), or a pulse having a specified polarity and wave-shape. For AC or DC test voltages, the duration of application is specified, typically 1 minute after the desired test value has been reached.¹ For an impulse the waveshape is determined by the virtual rise time and the virtual time to half value. A standard lightning pulse has 1.2 μ sec virtual rise time and 50 μ sec virtual time to half value.

1.1 Significance

The withstand voltage test is a demonstration that an insulation can withstand a specified over-voltage for a specified length of time. Successful completion of the test gives some assurance that no gross defect is present in the insulation structure. However, this does not mean that the insulation is absolutely free of defects. For this reason the test is often supplemented with measurements of insulation characteristics (Practical Dielectric Strength).

Practical Dielectric Strength tests are done with continued application of a voltage below the value required to cause immediate breakdown. This will cause the temperature to rise in local weak regions. The temperature rise further reduces the dielectric strength of these regions, and consequently the overall dielectric strength of the material, and may result in eventual failure without further increase in the applied voltage. Figure 1 illustrates the general behaviour of an insulator in terms of withstand voltage versus

time of voltage application.¹

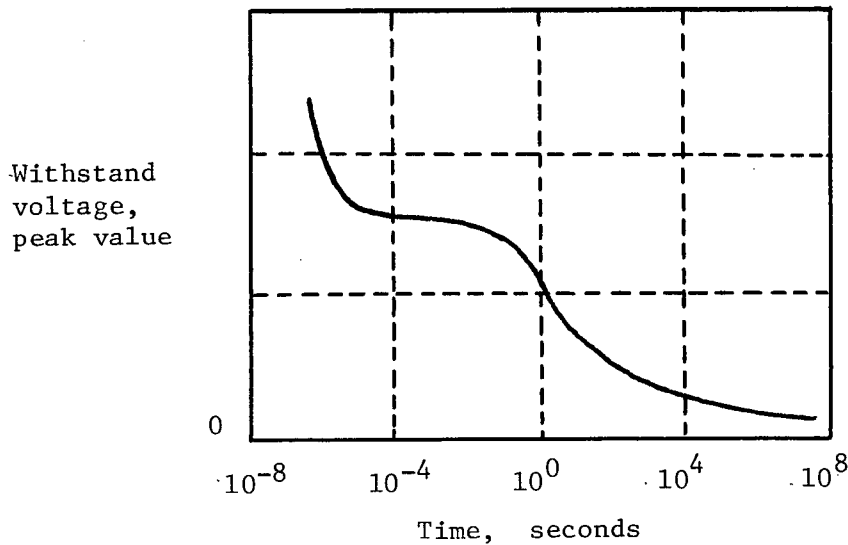


Figure 1: Effect of time on withstand voltage.

1.2 Test Equipment

Test equipment typically consists of a voltage source and a measuring system. The voltage source which is either of AC, DC, or Impulse type can be constructed with different methods. A review of these various voltage sources will be done in this thesis. Various measuring apparatus will also be discussed in this thesis.

1.3 Scope of this Thesis

First, the generation of high voltages and their measurements are discussed in general and in particular as they relate to the UBC high-voltage test set. An extensive investigation of the impulse measuring system of the UBC high-voltage test set is then described. Various techniques of shielding had to be used to obtain reasonably accurate oscillograms.

Finally, a series of exercises for undergraduate students is presented which was developed with the help of Dr. Van Dommelen.

2. GENERATION OF HIGH VOLTAGES

2.1. Introduction

High-voltage power equipment is tested with three types of voltages to verify its performance. These are:

- (a) AC voltage
- (b) DC voltage
- (c) Impulse voltage

Generation of high voltages in laboratories is usually done using transformers, therefore, the first type of high voltage produced is of AC type. With additional circuits, the AC voltage can then be transformed into DC and impulse voltages. DC voltage can also be produced directly by electrostatic generators.

The UBC-High Voltage Test Set can produce all three forms, namely 60 Hz AC voltage up to 75 KVRms, DC voltage up to 200 KV, and impulse voltage up to 200 KV.

2.2 Alternating Voltage

Transformers for generating high alternating voltages in laboratories generally have considerably lower power rating and frequently much larger turn ratios than power transformers. The primary current is usually supplied by a regulating transformer fed from mains supply. One end of the high-voltage winding is usually grounded, except for transformers to be connected in cascade where the windings must be completely isolated.

Figure 2 shows two basic circuits of test transformers. The length of voltage arrows indicates the magnitude of the stress on the insulation between the high voltage winding H and the excitation winding E or iron core

F. The fully isolated winding may be grounded if necessary at either of the two terminals or at the center tap, as shown.

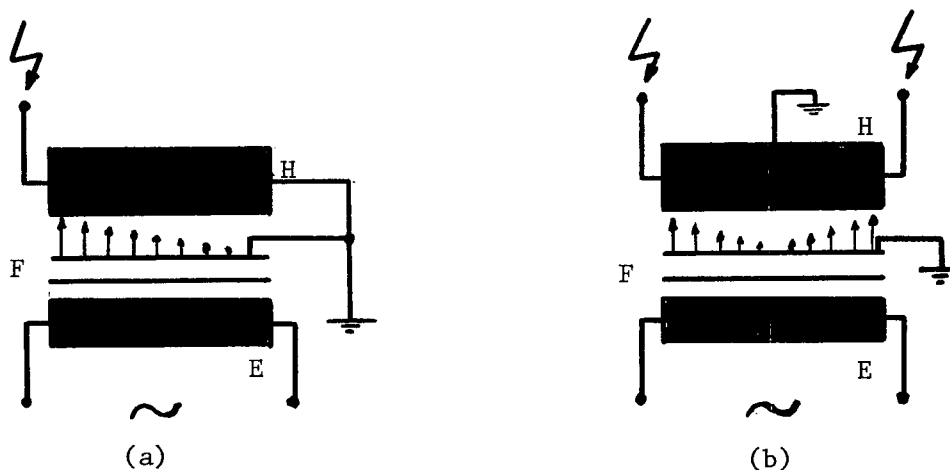


Figure 2: Single stage test transformer circuits

E - Excitation winding	(a) Single pole isolation
H - High-voltage winding	(b) Fully isolated
F - Iron core	

To generate voltages above a few hundred KV, single-stage transformers according to Figure 2 are now rarely used; for economical and technical reasons one employs instead a series connection of the high voltage windings of several transformers (introduced in 1915 by W. Petersen F. Dessauer and E. Welter).² The circuit configuration is shown in Figure 3. The excitation windings E of the upperstages are supplied from the coupling windings K of the stages immediately below. The individual stages, except for the uppermost, consist of three-winding transformers.

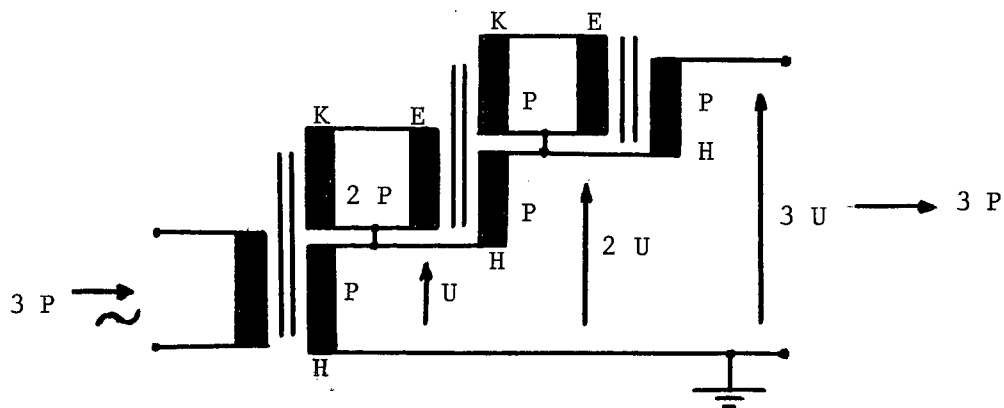


Figure 3: Three-stage test transformer cascade

E - Excitation winding
H - High-voltage winding
K - Coupling winding

For test objects with large capacitance, for example cables, a series resonant circuit is usually used to generate the high voltage.³ The basic circuit is shown in Figure 4. It comprises the load, which is almost purely capacitive, in series with a continuously variable inductance. The inductance is varied to produce series resonance with the capacitive load at the supply frequency. High voltages are then obtained by injecting current into the series circuit. Control of the high voltage is obtained by regulation of the supply current.

Some of the advantages of series resonant circuits are as follows:

1. Harmonics, which are caused by saturation in the transformers, are attenuated.

2. If failure of the test object occurs, a power arc does not develop.

Instead, the voltage collapses immediately after the load capacitance is short-circuited. This is of great importance to the cable industry where a power arc can sometimes lead to dangerous explosions.

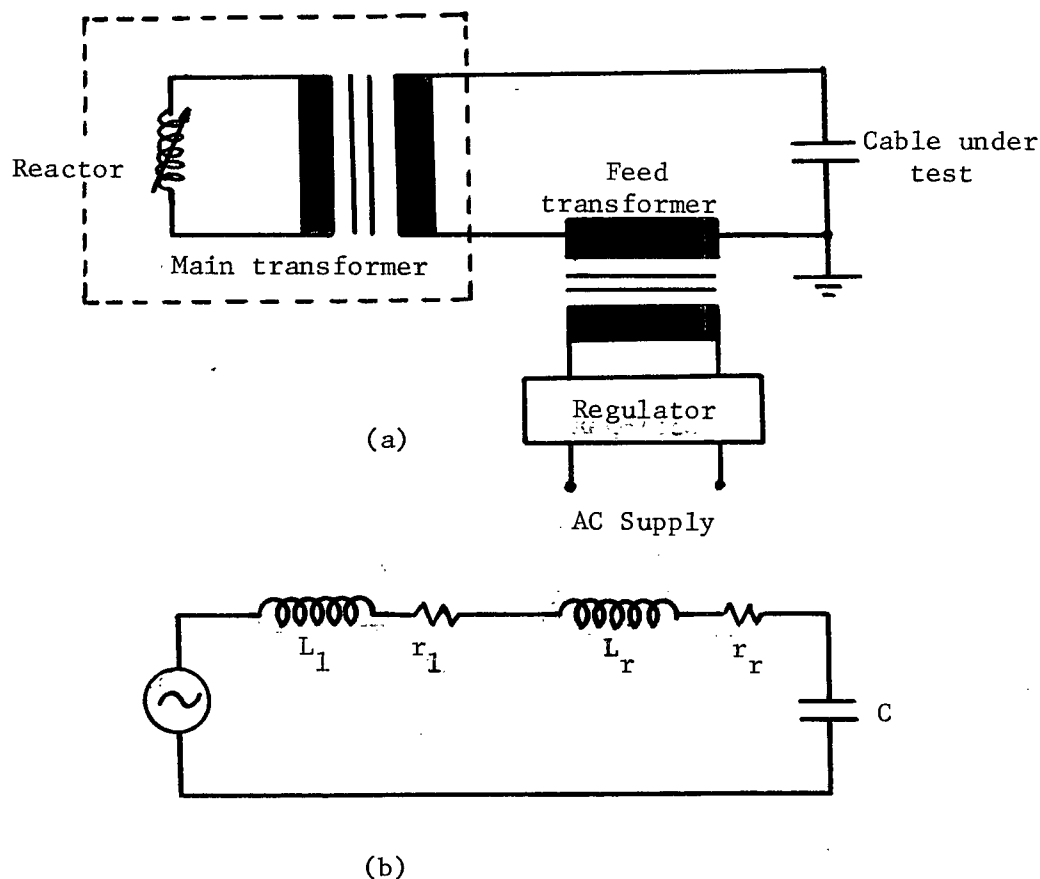


Figure 4: Series resonant circuit for single transformer/reactor unit.

(a) actual circuit (b) equivalent circuit

The series resonant circuit has now become accepted for cable testing. Many other laboratories have also used the circuit for general applications.

The UBC high-voltage test set uses a single transformer with one end grounded.

2.3 Direct Voltage

The simplest circuit for the generation of a high DC voltage is the half-period rectifier, shown in Figure 5. The circuit without the smoothing capacitor C will give a pulsating direct voltage, and with smoothing capacitor C a smoothed direct voltage with residual ripple is obtained.

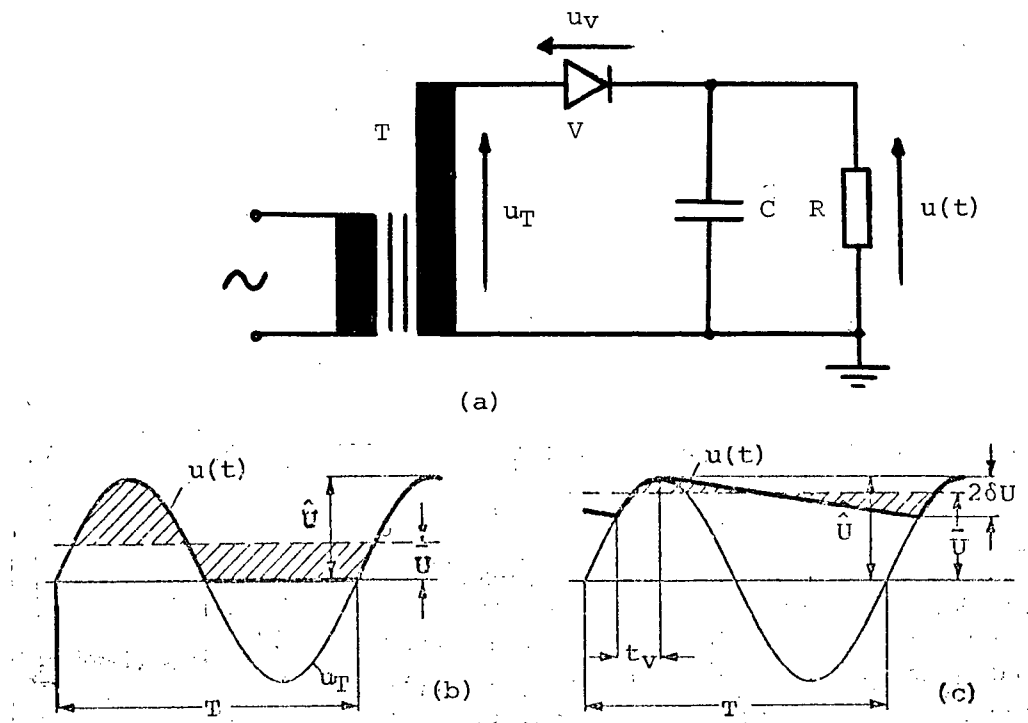


Figure 5: Half-period rectification with ideal circuit elements.

- a) Circuit
- b) Output voltage curve without smoothing capacitor C
- c) Output voltage curve with smoothing capacitor C

To obtain higher direct voltages, voltage multiplier circuits are used.

Some of these voltage multiplier circuits are:

1. Villard circuit.
2. Greinacher Doubler-circuit.
3. Zimmermann-Wittka circuit.
4. Greinacher cascade circuit.
5. Separate-rectifier cascade circuit.

Villard circuit: This circuit, shown in Figure 6, is the simplest doubling circuit. The blocking capacitor C is charged to the peak value \hat{U}_T and thus increases the potential of the high-voltage output terminal with respect to the transformer voltage by this amount. However, smoothing of the output voltage $u(t)$ is impossible.²

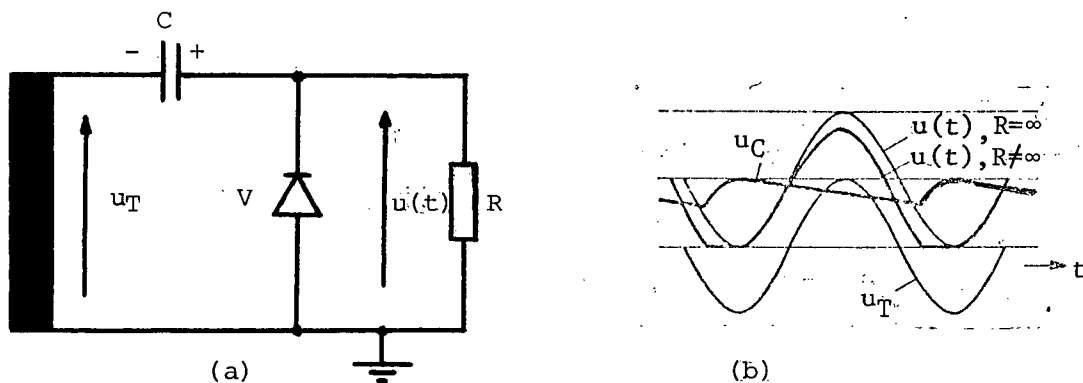


Figure 6: Villard circuit

- a) Circuit diagram
- b) Voltage curve

Greinacher Doubler-circuit: Extension of the Villard circuit by a rectifier V_2 and a smoothing capacitor C_2 enables the no-load output voltage of the Villard circuit to be smoothed. The complete circuit is shown in Figure 7.2,3

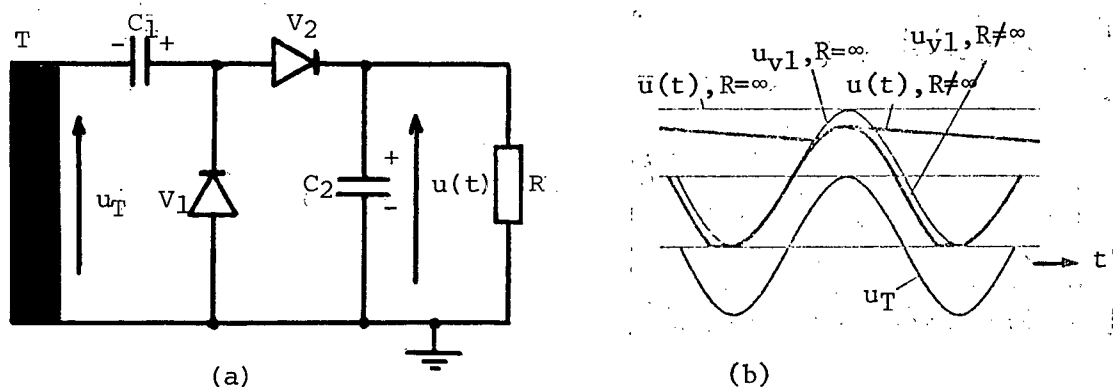


Figure 7: Greinacher doubler-circuit

- a) Circuit diagram
- b) Voltage curve

Zimmermann-Wittka circuit: If two Villard circuits are connected in opposition as in Figure 8, an unsmoothed direct voltage is produced between

the output terminals, with a peak value three times that of the transformer voltage and, under no load conditions, a mean output voltage $\bar{U} = 2 \hat{U}_T$.²

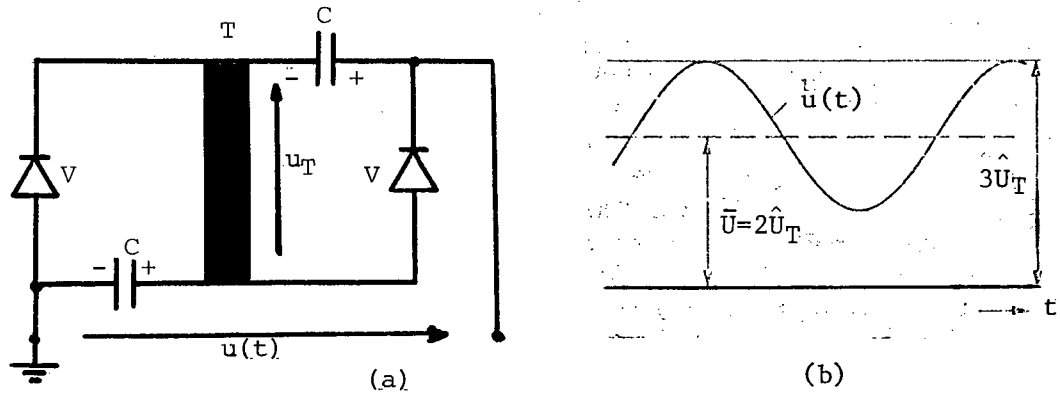


Figure 8: Zimmermann-Wittka circuit (no-load condition)

- a) Circuit diagram
- b) Voltage curve

Greinacher cascade circuit: This is an extension of Greinacher Doubler-circuit.² A three-stage circuit is shown in Figure 9 as an example; many practical circuits comprise only the parts shown in bold lines.

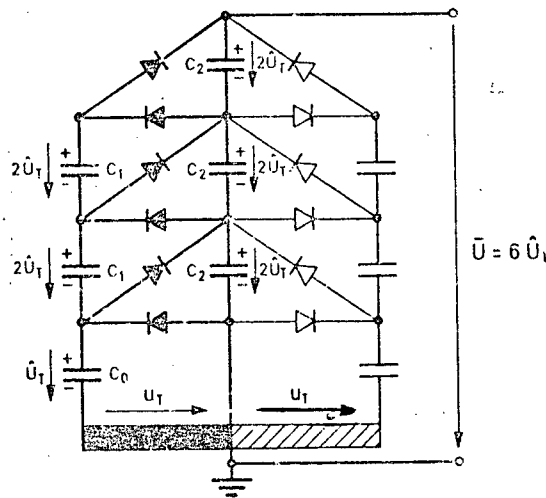


Figure 9: Greinacher cascade circuit (no-load condition).²

Separate-rectifier cascade circuit: This circuit, shown in Figure 10, gives low ripple and voltage drops even when output currents are high.²

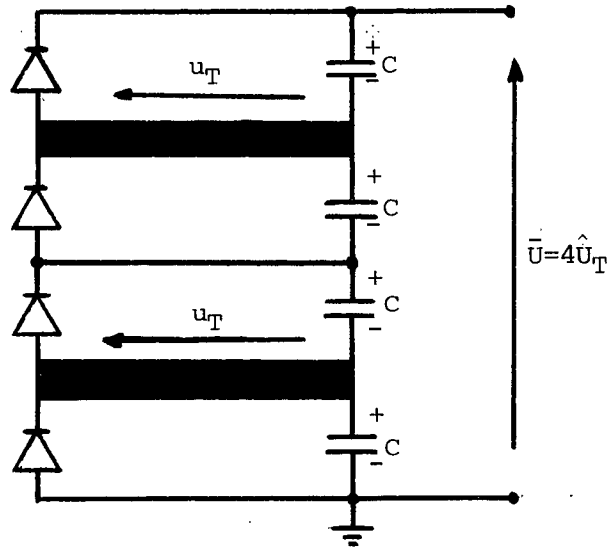


Figure 10: Example of cascade rectifier circuits (no-load condition)

DC voltage can also be generated electrostatically, namely, by using electrostatic generators. Because of its low power rating, this method is not used very often.

The generation of high DC voltages in the UBC laboratory is done with the half-period rectifier circuit for the 100 kV level and the Greinacher doubler-circuit for the 200 kV level.

2.4 Impulse Voltage

Figure 11 shows the two most important basic circuits used for the generation of impulse voltages. The impulse capacitor C_s is charged via a high-ohmic charging resistance to a DC voltage U_0 and then discharged by ignition of the switch gap F . The desired impulse voltage $u(t)$ appears across the load capacitor C_b .

The values of the circuit elements determine the wave shape of the impulse voltage. A short rise time requires rapid charging of C_b to the peak value \hat{U} , and long decay times require slow discharging. This is achieved by

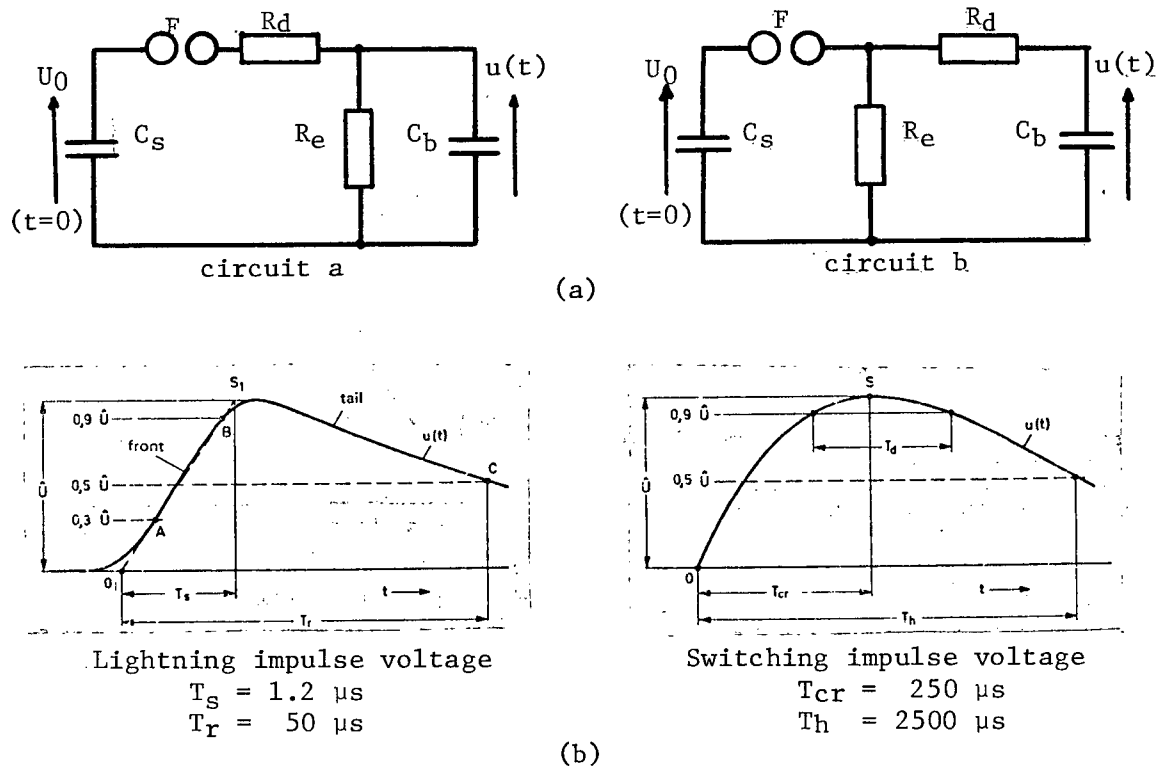


Figure 11: Basic impulse-voltage circuits.

a) Circuits b) Voltage curves

$R_e \gg R_d$. To obtain a peak value \hat{U} as high as possible one has to choose $C_s \gg C_b$. The exponential rise on the wave front has a time constant $R_d C_b$ whereas the decay on the tail has a time constant $C_s(R_d + R_e)$ for circuit a, $C_s R_e$ for circuit b. The output voltage curves of the lightning impulse and the switching impulse are shown in Figure 11b.

To generate impulse voltages with a peak value higher than the DC charging voltage, one commonly uses the multiplier circuit proposed by E. Marx in 1923.^{2,3} Several identical impulse capacitors are charged in parallel and then discharged in series, giving a multiplied total charging voltage which corresponds to the number of stages.

As an example, a three-stage multiplier circuit is shown in Figure 12. All impulse capacitors C_s' are charged to the stage charging-voltage U_0' ,

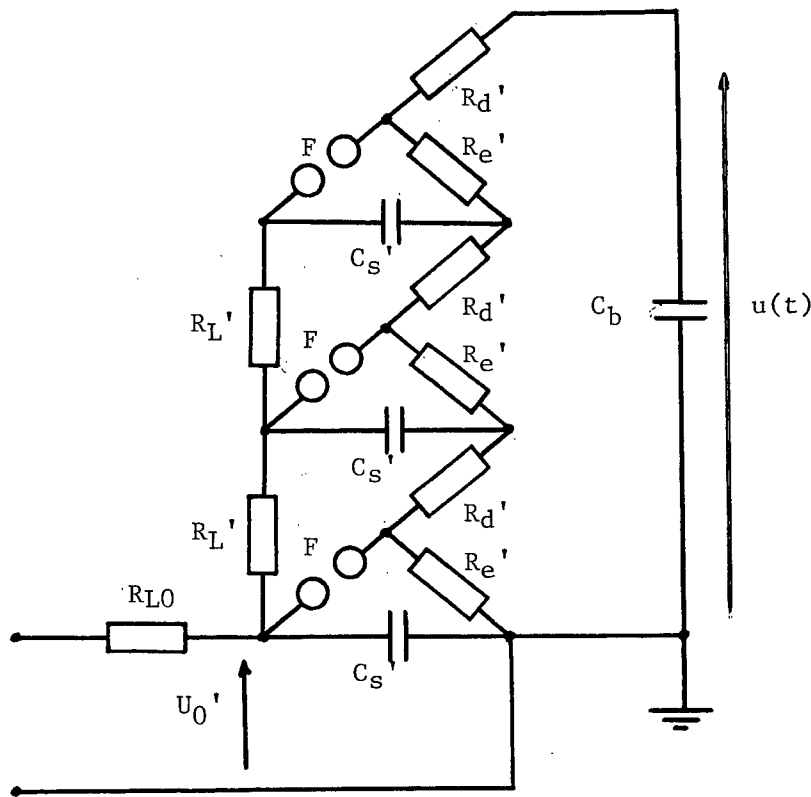


Figure 12: Multiplier circuit after Marx for 3 stages in circuit 11b connection.

via the high charging resistors R_L' . When all the switch gaps F break down, the capacitors C_S' will be connected in series so that C_b is charged via the series connection of all damping resistors R_d' ; finally, all C_S' and C_b will discharge again via the resistors R_e' and R_d' . The n -stage circuit can be reduced to a single stage equivalent circuit where the following relationships hold:

$$\begin{aligned} U_0 &= n U_0' & R_d &= n R_d' \\ C_s &= \frac{1}{n} C_s' & R_e &= n R_e' \end{aligned}$$

The UBC high-voltage test set uses the circuit shown in Figure 11b and can produce both the standard lightning impulse voltage and the switching impulse voltage.

3. MEASUREMENTS

3.1 Introduction

It is generally not practical to measure high voltages or currents directly. The usual procedure is to convert the quantity to be measured to a low voltage or current which can be measured with conventional instruments or oscilloscopes. A high-voltage or high-current measuring system generally comprises:

- (a) A converting device; for example, a voltage divider, a voltage transformer, a high-voltage measuring impedance, etc.
- (b) The leads required for connecting this device into the test circuit.
- (c) A measuring cable, together with any attenuating, terminating, and adapting impedances or networks.
- (d) The indicating or recording instrument.

In the UBC laboratory both resistive and capacitive dividers are used. The resistive divider is used for DC measurements, and the capacitive divider for AC and impulse measurements. For impulse measurements the capacitive divider is a part of the impulse generator circuit.

The measuring cable, which connects the low voltage side of the divider with the recording instrument, is of coaxial type. Coaxial cables have the advantage of small self-inductance and a shielding effect which minimizes the distortion of the signal.

AC/DC voltmeters and impulse peak-voltmeters are very common indicating instruments. They are usually provided with the HV-test set. To observe the waveform an oscilloscope is generally used. However, newer equipment often uses digital transient recorders, with which impulse waveforms can be analyzed more easily.

In regard to direct measurements of high voltages, voltage gaps such as sphere gaps are commonly used; peak voltages are then obtained. Direct measurement of DC voltages can also be done by an electrostatic voltmeter.

3.2 AC Voltage Measurements

3.2.1 Objective

The main objective of AC measurements is to measure the peak or rms value of the voltage, typically with an error of not more than 3%.⁴ This error requirement will be met if the voltage ratio of the voltage divider or voltage transformer is stable and known with an error of less than 1%.⁴ In the case of high-impedance systems, such as a voltage dividers, it may not be possible to comply with this error requirement. In such cases an overall error of slightly more than 3% may have to be accepted.

The secondary objective is to measure the amplitude of harmonics, typically with an error of not more than 10% of the harmonic amplitude or not more than 1% of the fundamental, whichever is larger.⁴ Harmonic measurements require a wave analyser in addition to the existing equipment. A measuring error of not more than 5% for harmonics up to the seventh and not more than 10% for those up to the twenty seventh, is required for the wave analyser.⁴

3.2.2 Measuring Devices

The four most common AC-voltage measuring devices are:

1. Sphere gaps
2. Measuring capacitors (Chubb & Fortescue)
3. Capacitive voltage dividers
4. Voltage transformers

Sphere gaps:

Sphere gaps are commonly used for the measurement of the peak value of high voltages, and as a result of extensive investigations, calibration tables giving breakdown voltages as a function of the gap length for different sizes of spheres have been obtained. The calibration data recommended in the "British Standard Rules for Measurement of Voltage with Sphere gaps"³ are included in Table 1.

Breakdown of a sphere gap occurs within a few μsec once the applied voltage exceeds the "static breakdown discharge voltage". Over such a short period the peak value of a power frequency voltage or of voltages with frequencies up to 500 KHz can be considered to be constant. Breakdown will always occur on the peak of low frequency AC voltages if the voltage amplitude is raised slowly.

Figure 13 shows the basic arrangement for voltage measurement with sphere gaps. The ratio S/D (Spacing/Diameter) must not be too large, because with increasing ratio S/D the field becomes increasingly inhomogeneous and at the same time the breakdown voltages become random.² This is demonstrated in Figure 14 where the curves begin to level off as S/D increases.

Humidity has no significant influence on the breakdown voltage of sphere gaps, however, the breakdown voltage \hat{U}_d is proportional to the relative air density d . The actual breakdown voltage \hat{U}_d at air density d may be found from the tabulated value \hat{U}_{d0} (standard value) by applying the following formula:^{2,3}

Sphere gap spacing, cm	Kilovolts peak at 20°C ; 1013 millibars											
	Sphere diameter, cm											
	2	5	6.25	10	12.5	15	25	50	75	100	150	200
0.05	2.8											
0.10	4.7											
0.15	6.4											
0.20	8.0	8.0										
0.25	9.6	9.6										
0.30	11.2	11.2										
0.40	14.4	14.3	14.2									
0.50	17.4	17.4	17.2	16.8	16.8	16.8						
0.60	20.4	20.4	20.2	19.9	19.9	19.9						
0.70	23.2	23.4	23.2	23.0	23.0	23.0						
0.80	25.8	26.3	26.2	26.0	26.0	26.0						
0.90	28.3	29.2	29.1	28.9	28.9	28.9						
1.0	30.7	32.0	31.9	31.7	31.7	31.7	31.7					
1.2	(35.1)	37.6	37.5	37.4	37.4	37.4	37.4					
1.4	(38.5)	42.9	42.9	42.9	42.9	42.9	42.9					
1.5	(40.0)	45.5	45.5	45.5	45.5	45.5	45.5					
1.6		48.1	48.1	48.1	48.1	48.1	48.1					
1.8		53.0	53.5	53.5	53.5	53.5	53.5					
2.0		57.5	58.5	59.0	59.0	59.0	59.0	59.0	59.0			
2.2		61.5	63.0	64.5	64.5	64.5	64.5	64.5	64.5			
2.4		65.5	67.5	69.5	70.0	70.0	70.0	70.0	70.0			
2.6		(69.0)	72.0	74.5	75.0	75.5	75.5	75.5	75.5			
2.8		(72.5)	76.0	79.5	79.5	80.0	81.0	81.0	81.0			
3.0		(75.5)	79.5	84.0	85.0	85.5	86.0	86.0	86.0	86.0		
3.5		(82.5)	(87.5)	95.0	97.0	98.0	99.0	99.0	99.0	99.0		
4.0		(88.5)	(95.0)	105	108	110	112	112	112			
4.5			(101)	115	119	122	125	125	125	125		
5.0			(107)	123	129	133	137	138	138	138	138	
5.5				(131)	138	143	149	151	151	151	151	
6.0				(138)	146	152	161	164	164	164	164	
6.5				(144)	(154)	161	173	177	177	177	177	
7.0				(150)	(161)	169	184	189	190	190	190	
7.5				(155)	(168)	177	195	202	203	203	203	
8.0					(174)	(185)	206	214	215	215	215	
9.0					(185)	(198)	226	239	240	241	241	
10					(195)	(209)	244	263	265	266	266	266
11						(219)	261	286	290	292	292	292
12						(229)	275	309	315	318	318	318
13							(289)	331	339	342	342	342
14							(302)	353	363	366	366	366
15							(314)	373	387	390	390	390
16							(326)	392	410	414	414	414
17							(337)	411	432	438	438	438
18							(347)	429	453	462	462	462
19							(357)	445	473	486	486	486
20							(366)	460	492	510	510	510
22								489	530	555	560	560
24								515	565	595	610	610
26								(540)	600	635	655	660
28								(565)	635	675	700	705

This table is not valid for the measurement of impulse voltages below 10 kV.
The figures in the brackets, which are for spacings of more than 0.5D are of doubtful accuracy.

Table 1: Flashover voltages for AC voltages, for DC voltages of either polarity, and for full negative standard impulses and impulses with longer tails: One sphere earthed.³

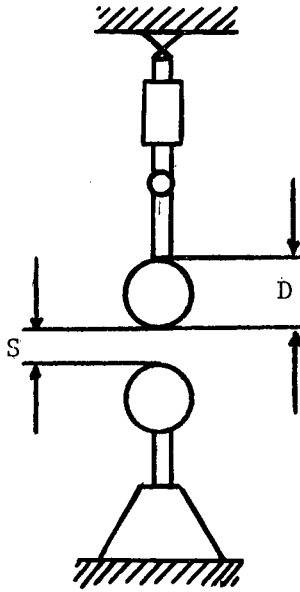


Figure 13: Sphere gaps for voltage measurement.

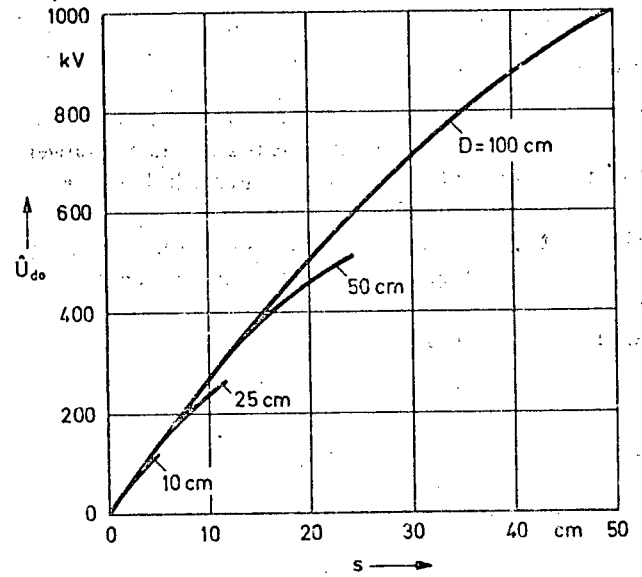


Figure 14: Breakdown voltage U_{d0} of sphere gaps as function of gap spacing s , for various sphere diameters D .

$$\hat{U}_d \approx d \hat{U}_{d0} = \frac{b}{1013} \frac{273 + 20}{273 + t} \hat{U}_{d0}$$

$$= 0.289 \frac{b}{273 + t} \hat{U}_{d0} \quad (1)$$

where: b : pressure in mbar

t : temperature in $^{\circ}\text{C}$

\hat{U}_{d0} : breakdown voltage at pressure 1013 mbar and temperature 20°C .

Even under apparently ideal conditions, having made allowances for such factors as air density, minimum clearances, smooth exactly spherical electrode surface and proper adjustment of the spacing, a measuring uncertainty of 3% remains. Sphere gaps are now rarely used for measuring voltages above 1 MV, because they require excessive space and are expensive. Continuous voltage measurements are obviously impossible with sphere gaps, since the voltage source is short-circuited at the instant of measurement.

Inspite of their disadvantages, sphere gaps can be useful and versatile devices in high-voltage laboratories. Apart from voltage measurements, they can also be used as voltage limiters, as voltage-dependent switches, as pulse sharpening gaps and as variable high-voltage capacitors, etc.² The UBC High-voltage test set uses sphere gaps for transformer protection against over-voltages. They have also been adapted for AC voltage measurements as part of this thesis project.

Measuring Capacitors:

As opposed to sphere gaps, the circuit suggested by Chubb and Fortescue in 1913² is capable of measuring the peak value of a high AC voltage continuously and accurately. Figure 15 shows the circuit with its current and voltage curves.

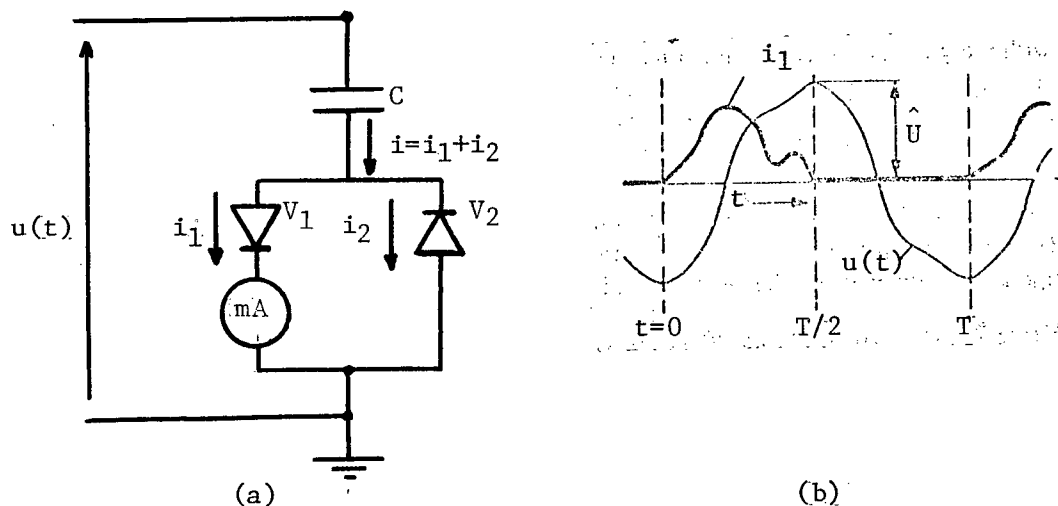


Figure 15: Peak voltage measurement according to Chubb and Fortescue.

- a) Circuit
- b) Current and voltage curves

A charging current i , given by the rate of change of the applied voltage $u(t)$, flows through the high-voltage capacitor C and is passed through two antiparallel rectifiers V_1 and V_2 to ground. The arithmetic mean value \bar{I}_1 of current i_1 in the left-hand branch is measured with a moving-coil instrument. As shown below, this current is proportional to the peak value \hat{U} of the high voltage provided that certain conditions are fulfilled.

If the behaviour of the rectifiers is assumed ideal, then for the conducting period of V_1 one has:

$$i_1 = i = C \frac{du}{dt} \quad \text{for} \quad t = 0 \text{ to } T/2$$

$$\bar{I}_1 = \frac{1}{T} \int_0^T i_1 dt = \frac{1}{T} \int_{u(0)}^{u(T/2)} C du = \frac{C}{T} [u(T/2) - u(0)] \quad (2)$$

If the voltage is symmetrical with reference to the zero line:

$$u(T/2) - u(0) = 2 \hat{U} \quad (3)$$

and with $T = \frac{1}{f}$, one obtains

$$\hat{U} = \bar{I}_1 \frac{1}{2fC} \quad (4)$$

In the derivation of this expression, it was not assumed that $u(t)$ is a sinusoid, but when semiconductor rectifiers are used, only one maximum per half period can occur. The use of synchronous mechanical rectifiers or controllable rectifiers (oscillating contacts, rotating rectifiers) allows correct measurement of AC voltages with more than one maximum per half-period. Oscillographic monitoring of high-voltage shape is necessary and is usually done by observing the current i_1 , which is allowed to have only one crossover in each half-period.

As the frequency f , the measuring capacitor C , and the current \bar{I}_1 can be determined precisely, measurement of symmetrical AC voltages using the technique of Chubb & Fortescue with the appropriate layout is very accurate, and is suitable for the calibration of other peak-voltage measuring devices. The disadvantages of this technique are the dependence of the reading upon the frequency and the need to monitor the wave.

Capacitive Voltage Dividers:

Several rectifier circuits have been developed which permit the measurement of peak values of high AC-voltages with the aid of capacitive dividers. Compared with the circuit of Chubb and Fortescue, most of these methods have the advantage that the reading is practically independent of frequency, and multiple extrema per half-period of the voltage to be measured can be permitted.²

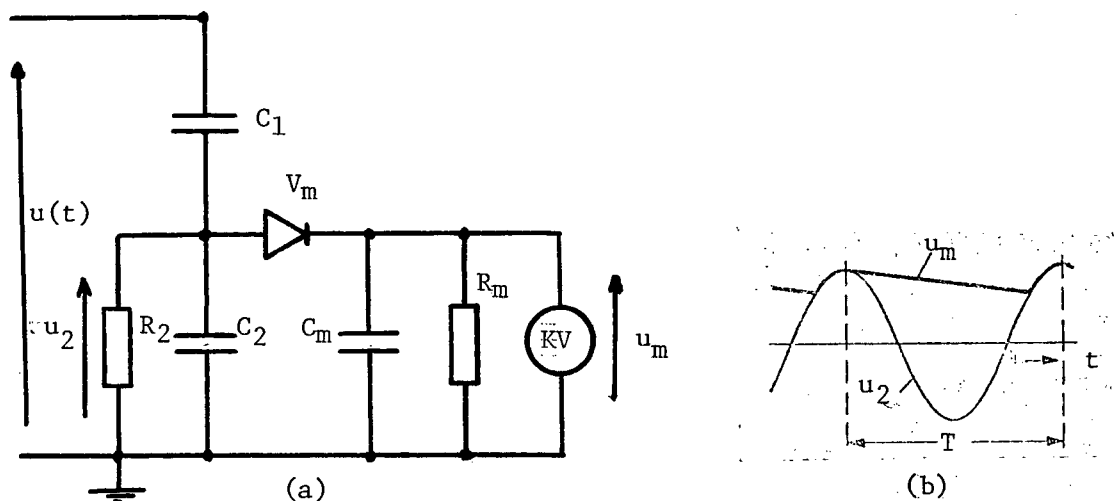


Figure 16: Peak voltage measurement with a capacitive divider

a) Circuit b) General form of the voltage

The half-period circuit is shown in figure 16. In this circuit the measuring capacitor C_m is charged to the peak value \hat{U}_2 of the lower arm voltage $u_2(t)$ of the capacitive divider. The resistor R_m which discharges the capacitor C_m is meant to follow reductions of the applied voltage. The choice of time constant for this discharge process is determined by the desired response of the measuring system. In general one chooses,

$$R_m C_m < 1 \text{ second, and} \quad (5)$$

$$R_m C_m \gg \frac{1}{f} \quad (6)$$

The resistor R_2 is necessary to minimize charging of C_2 by the current flowing through the rectifier V_m . The value of R_2 must be chosen in such a way that the DC voltage drop across R_2 which causes DC charging of C_2 remains as small as possible. In this case one must have $R_2 \ll R_m$. On the other hand, the capacitive divider ratio should not be affected much by R_2 , which requires $R_2 \gg \frac{1}{\omega C_2}$.

With all these conditions, the relation between the peak value of the high voltage and the indicated voltage \hat{U}_m is given by:

$$\hat{U} = \frac{C_1 + C_2}{C_1} \hat{U}_m \quad (7)$$

The working principle of the "Impulse Peak Voltmeter" of the UBC high-voltage test set is based on this technique, even though a more elaborate circuit is employed. In the control box, there is also an AC voltmeter which operates on the same principle but indicates peak value divided by $\sqrt{2}$ ($\frac{\hat{U}}{\sqrt{2}}$) instead of peak value.

Voltage Transformers:

High AC voltages can be measured very accurately with voltage trans-

formers.² The basic circuits of single pole isolated inductive and capacitive voltage transformers are shown in Figure 17.

Inductive voltage transformers for very high voltages are very expensive to build since they require very large number of turns of the high-voltage winding. The type of capacitive voltage transformer used extensively in supply networks is often considered unsuitable for normal testing work, mainly because it imposes a high capacitive load upon the voltage source.

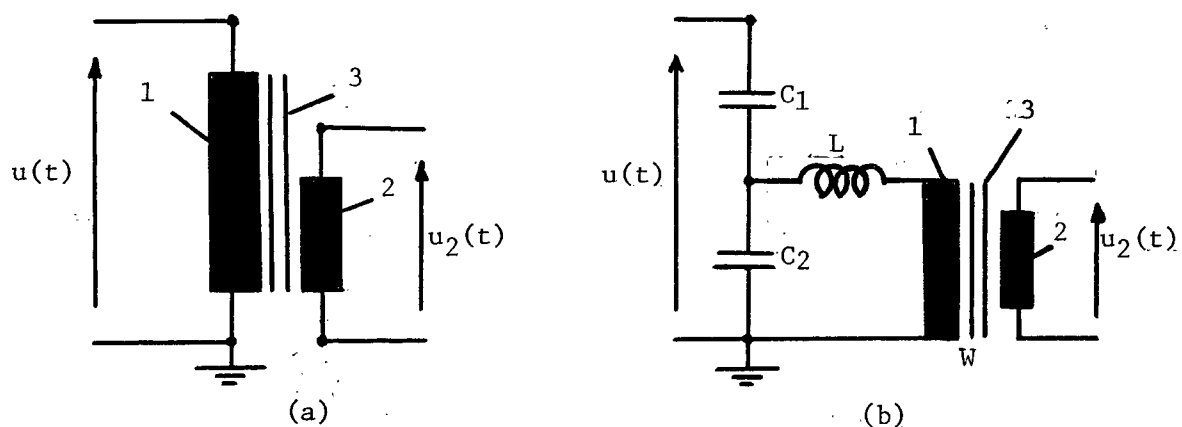


Figure 17: Basic circuits of voltage transformers

- | | |
|-----------------------------------|-----------------------------------|
| a) Inductive voltage transformers | b) Capacitive voltage transformer |
| 1 Primary winding | C_1, C_2 Divider capacitors |
| 2 Secondary winding | L Resonance inductor |
| 3 Iron core | W Matching transformer |
| | (marking as under a) |

Inductive and capacitive voltage transformers are used in laboratory measurements only when particularly precise measurements of moderate voltages are required. The secondary voltage of a voltage transformer will reproduce the shape of the primary voltage, if the load resistance is not low. Depending on the type of measuring device connected, it is possible to measure the peak value, the rms value or to display the wave shape.

3.3 DC Voltage Measurements

3.3.1 Objective

The general objective of DC voltage measurements are:⁴

- to measure the arithmetic mean value of the voltage, typically with an error of not more than 3%.
- to measure the ripple amplitude, typically with an error of not more than 10% of the actual ripple amplitude or not more than 1% of the arithmetic mean value of the DC voltage, whichever is larger.

To fulfill the above error requirements, the measuring system has to satisfy certain specifications:⁴

- (a) The voltage ratio of the voltage divider is stable and known with an error of not more than 1%. In the case of high-impedance systems where it may not be possible to comply with this specification, an overall error slightly exceeding 3% may have to be accepted.
- (b) The current drawn from the high-voltage source at full voltage is not less than 0.05 mA.
- (c) The frequency response of the system used for measuring ripple voltage is adequate and known within 10% for frequencies from the fundamental of the ripple frequency up to five times this frequency.

3.3.2 Measuring Devices

Measurement of the arithmetic mean value of a high DC voltage can be done with the following:

- (a) High-voltage resistors and voltage dividers.
- (b) Electrostatic voltmeters

(c) Field strength meter.

For the ripple voltage measurement, a voltage divider made up of a capacitor and a resistor is used.

There are other methods for DC voltage measurements. Sphere gaps are suitable for the determination of the peak value \hat{U} of high DC voltages. In physics laboratories the nuclear resonance method is often applied to measure DC voltages. Protons are accelerated in an electric field which is proportional to the voltage to be measured. At certain kinetic energies these protons collide with light atomic nuclei, producing resonant nuclear transformations which permit very accurate determination of the DC voltage.

High-voltage Resistors and Voltage dividers:

The current flowing through a resistor connected to a DC source can indicate the voltage to be measured. However, for high-voltage applications the current must be very small, of the order of 1 mA for example, because otherwise excessive loading of the voltage source and excessive heating of the measuring resistor will occur.² On the other hand, a small current is easily falsified by error currents; these occur in the form of leakage currents in insulating materials and on insulating surfaces, and also as a result of corona discharges. To avoid these problems, a special design for high-voltage resistors is needed.

The simple DC-voltage measuring circuit is shown in Figure 18.

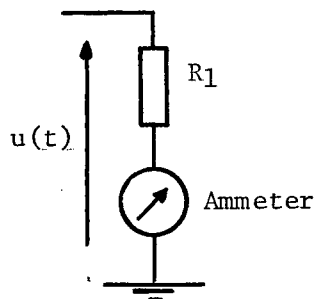


Figure 18: Measurement of DC voltage by means of a high-voltage resistor.

The ammeter is assumed at earth potential. A sensitive moving-coil instrument is usually chosen, the indication of which is the arithmetic mean value \bar{U} of the DC voltage.

Replacing the ammeter with a parallel connection of a voltmeter and a resistor R_2 , one obtains a voltage divider for measuring DC voltages which is shown in Figure 19.

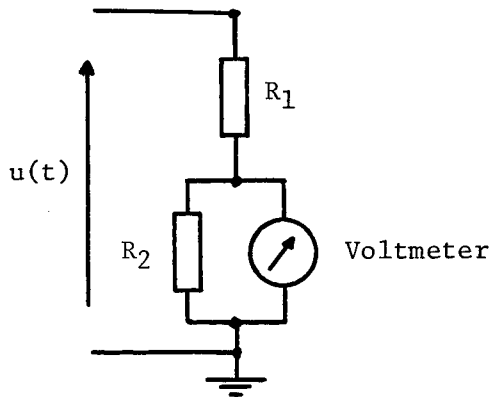


Figure 19: Measurement of a DC voltage by means of a resistive divider.

The UBC high-voltage test system uses this divider method to measure DC voltages. The Impulse Peak Voltmeter (IPV) can be used, instead of the "Control Box Voltmeter", to measure the peak value of a DC voltage.

Electrostatic Voltmeters:

Electrostatic voltmeters have the advantage of very high internal resistance and very small capacitance, which makes them useful for measurement of low energy high voltages.²

The schematic of the device is shown in Figure 20. When a voltage $u(t)$ is applied, the electric field produces a force $F(t)$ which tends to reduce the spacing s of the electrodes. This attractive force can be calculated from the change of energy of the electric field:

$$W(t) = \frac{1}{2} C u^2(t) \quad (8)$$

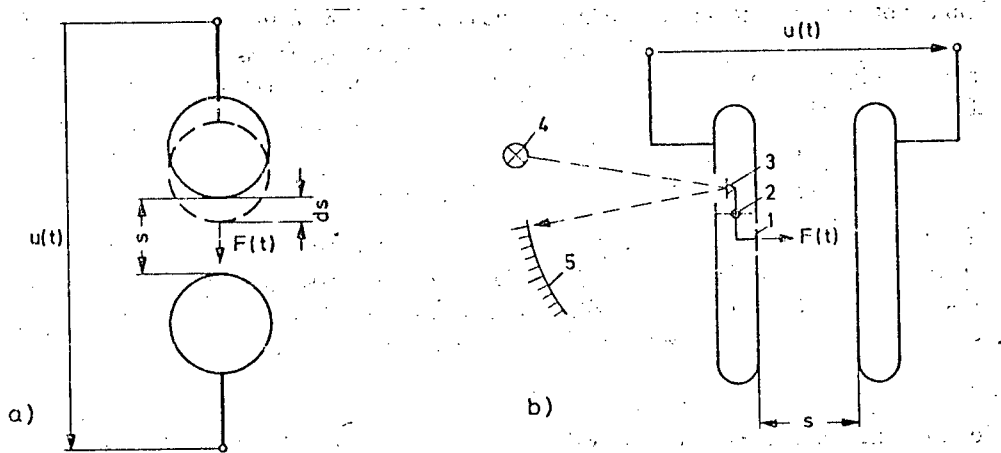


Figure 20: Electrostatic voltmeters for high voltages.²

- a) Using spherical electrodes (after Hueter)
- b) Using a movable electrode segment (after Starke and Schröder)

- 1. Movable electrode segment
- 2. Axis of rotation
- 3. Mirror
- 4. Light source
- 5. Scale

The capacitance C depends on the spacing s . Using the law of conservation of energy $dW + F ds = 0$ and assuming disconnection of the voltage source one obtains:

$$F(t) = - \frac{d W(t)}{ds} = \frac{1}{2} u^2(t) \frac{dC}{ds} \quad (9)$$

Taking the arithmetic mean value \bar{F} of the force,

$$\bar{F} = \frac{1}{2} \frac{dC}{ds} \frac{1}{T} \int_0^T u^2(t) dt = \frac{1}{2} \frac{dC}{ds} u_{\text{rms}}^2 \quad (10)$$

This force is counter-balanced by a spring. At equilibrium there is some defined value of spring extension which can be translated into a voltage reading.

Field-Strength Meters:

Variable capacitance is the basic principle of this device.²

The schematic of the device is shown in Figure 21. The two measuring electrodes 1 and 1' are alternately passed under the semi-circular opening 2 of the grounded plate 3; this produces a variable capacitance between each electrode and the high-voltage electrode 4.

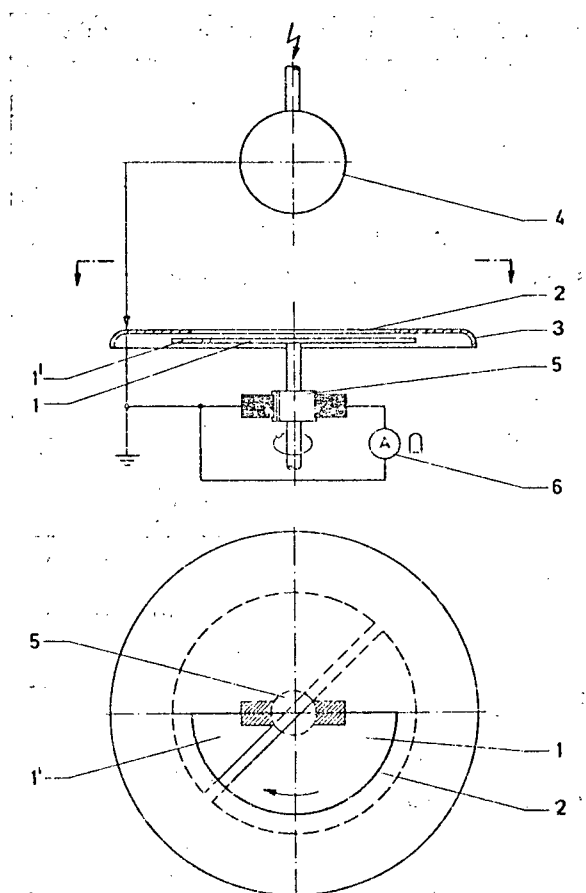


Figure 21: Voltmeter with the sphere-plate electrode configuration.²

- 1,1' Revolving semicircular discs
- 2 Semicircular opening
- 3 Earthed covering plate
- 4 High-voltage electrode
- 5 Commutator
- 6 Ammeter

At constant rate of revolution, a periodic alternating current $i(t)$ flows between the measuring electrodes, which is rectified by a commutator 5. The arithmetic mean value \bar{I} after rectification is read by a moving-coil ammeter b. Since the current \bar{I} is proportional to the measured voltage, the reading of the ammeter can be converted into a voltage reading.

Ripple-voltage measurement circuits:

Ripple voltages are AC components superimposed on the DC voltage. For smoothed DC voltages, the peak values ∂U of the ripple voltages are always much smaller than the mean value \hat{U} , which is why an oscilloscopic measurement, performed with a resistive divider, is too insensitive.

To separate the ripple from the DC voltage, a voltage divider made up of a resistor and a capacitor is used. The circuit is shown in Figure 22. The divider ratio of this

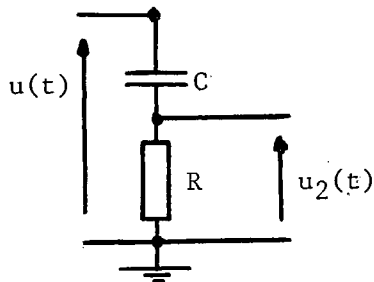


Figure 22: Circuit for measuring ripple voltages.

circuit is given by:

$$\frac{u_2}{u} = \frac{j\omega RC}{1 + j\omega RC} \quad (11)$$

If the full magnitude of the ripple is to appear on the lower arm of the divider, the divider ratio must be as close to one as possible for all frequencies in the ripple spectrum, which requires

$$\omega RC \gg 1 \quad (12)$$

In the UBC high-voltage laboratory, a resistive divider is installed, instead of a single resistor, as a practical development of this thesis project. The ripple voltage which appears on the lower arm of the divider is, therefore, reduced and can be displayed on an oscilloscope. A surge arrestor is also installed in parallel with the lower arm resistor to protect the oscilloscope in case the voltage $u(t)$ collapses to zero. Sudden voltage drop of $u(t)$ will cause the whole voltage, applied previously, to appear on the resistive part of the divider. This can destroy the oscilloscope.

3.4 Impulse Voltage Measurements

3.4.1 Objective

The general objectives of impulse-voltage measurements are:⁴

- To measure the peak value of full impulses and impulses chopped in the vicinity of the peak or on the tail, typically with an error not exceeding 3%.
- To measure the peak value of impulses chopped on the front, typically with an error Δ which is dependent on the time to chopping T_c as follows:

$$\text{if } T_c > 2 \mu\text{s}, \Delta \leq 3\%$$

$$\text{if } 0.5 \mu\text{s} \leq T_c \leq 2 \mu\text{s}, \Delta \leq 5\%$$

- To measure the time parameters which define the impulse shape, typically with an error not exceeding 10%.

The above requirements will be met if the system meets the following measurement qualifications:⁴

- (a) The voltage ratio of the divider should be stable and known with an error not exceeding 1%.
- (b) The scale factor of the oscilloscope or peak voltmeter (including attenuator or coupling devices) should be stable and known with an error not exceeding 2%.
- (c) The time scale of the oscilloscope should be stable and known with an error not exceeding 2%.
- (d) The response time requirements for measuring systems depend on the impulse shapes, such as the following:
 - Full 1.2 μ s lightning impulse and lightning impulses chopped on the peak or tail $|T| \leq 0.2 \mu$ s.
 - Switching impulses $|T| \leq 0.03 T_c$ and $|T| \leq 0.03 T_{cr}$
 where T_c is chopping time
 T_{cr} is time to crest
 $|T|$ is response time

3.4.2 Setup of Measuring System

A measuring system for impulse voltages generally consist of:

- (a) a Faraday Cage
- (b) a Wiring System
- (c) a Voltage Divider
- (d) a Measuring Instrument and its connection to the divider

Faraday Cage:

In practice, a high-voltage circuit behaves as an antenna which receives external electromagnetic waves. Electromagnetic waves are also produced during breakdown discharge processes in the high-voltage circuits themselves, and these can in turn disturb the surroundings. However, the

disturbing effect of the surroundings on the high-voltage circuit is generally worse than that exerted by the high-voltage circuit on the surroundings. To eliminate these interferences one uses a highly conductive metal shield in the form of a Faraday cage.^{5,6}

Strong electromagnetic fields, associated with high rapidly changing voltages and currents, also cause interference of a special type. Stray ground capacitances are charged and discharged by these strong fields which produce transient potential rises. Figure 23 shows schematically a high voltage circuit consisting of an impulse generator G and a test object P ; Z_g represents the unavoidable ground impedance.

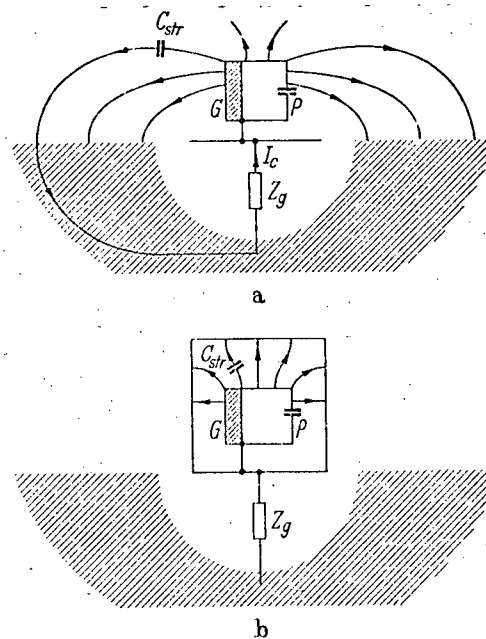


Figure 23: Jumping potential in impulse generator system.⁵ Figure 23a shows lines of stray flux in normal setup. Figure 23b shows lines of stray flux with test setup in Faraday cage.

G impulse generator, P test object,
 C_{str} stray ground capacitances, Z_g ground impedance,
 I_c charging current for stray ground capacitances.

Electric field lines exist between the high voltage electrodes and the grounded surroundings. These can be represented as stray ground capacitances C_g which are rapidly charged and discharged during transients. Because of the high rate of voltage change, the charging currents may have values as high as some kiloamperes, which when returning to the surge generator's ground through Z_g , will result in considerable transient ground potentials. If the entire high-voltage circuit is located inside a Faraday cage, Figure 23b, all stray field lines will terminate on the cage wall. The charging currents will then flow on the inner surface of the cage wall and can not raise the potential of the ground system of the high-voltage circuit.

As mentioned previously, Faraday cages are usually made of highly conductive metal in order to get rid of background interferences. However, this highly conductive metal enclosure acts as a huge cavity resonator when excited from inside by the fast discharge of an impulse generator.^{8,10} Several modes of oscillations occur with a very high Q-factor which are determined by the formula:⁸

$$f_{m,n,p} = \frac{C_0}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2} \quad (13)$$

(See appendix 2)

where:

m, n, p , integers identifying the resonance mode.

a, b, c , dimensions of the shield.

$C_0 = 300 \text{ m}/\mu\text{s}.$

Due to large dimensions of some high-voltage laboratories, the lower resonant frequencies are in the order of one MHz and are well within the pass bandwidth of the impulse-voltage measuring system. These lightly damped oscillations last for tens of microseconds and therefore cause

distortions.

A practical remedy for this interference is a reduction of the Q -factor of the electromagnetic shield. By covering the inner wall of the shield with a special resistive coating the cavity resonance can be damped out after a few oscillations. The resistive coating does not affect the laboratory shielding since the currents induced in the shield by external interference sources circulate in the outer layer of the metal wall.

In order to keep the currents induced by the cavity resonance within the resistive coating layer its thickness has to be not smaller than the current penetration depth. An analysis of the total resistance required for an effective damping, as well as of the available resistive coating indicate that a special material should be composed on a basis of magnetic powders and resistive paints having both high resistivity and high magnetic permeability.⁸

In the UBC High-voltage laboratory the Faraday cage is made of aluminum sheets which are joined together by folding them. Since aluminum is always oxidized in air, these folding connections have a high resistance at high frequency and therefore the shielding behaviour is lessened. However, with relatively low surrounding interference and low operating voltage (200 KV), this shielding is sufficient for our purposes.

Wiring System:

The presence of electromagnetic fields around the impulse generator and the measuring system tends to induce noise in the system, especially when loops are present in the system. Therefore, it is desirable to minimize the area of the loops by arranging the wiring system in such a way that all cables extend from a cable tree to provide branch wiring

rather than loops.⁵ Figure 24 shows the wrong and the correct arrangements of a wiring system. The schematic shown in Figure 24b is the arrangement used in the UBC high-voltage test system.

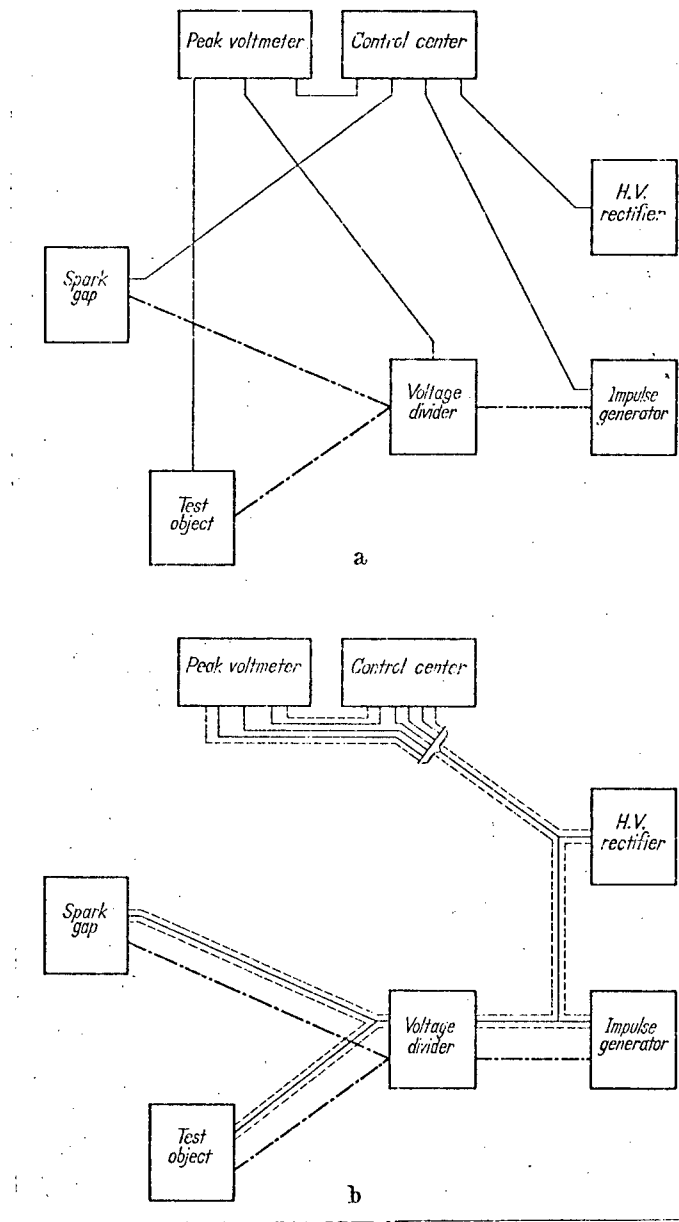


Figure 24: Block diagram of impulse test facility.⁵

- a) wrong arrangement of control and signal cables
(existence of loops);
- b) correct wiring of control and signal cables (branch wiring).

Voltage Divider:

A divider for recording high transient voltages, may consist of resistors or capacitors or combinations of both. Each type of divider should reproduce the wave shape of the voltage to be measured with a known reduction ratio.

The main sources of error common to all types of dividers are:^{3,4}

1. Residual inductance in any resistive or capacitive element.
2. Stray capacitance: (a) from any section of the divider to the high-voltage lead.
(b) from any section of the divider to ground.
(c) Between sections of the divider.
3. Impedance drop in the interconnecting leads.
4. Oscillations in the divider circuit caused by capacitance from divider high-voltage terminal to ground and lead inductance.

In the case of a resistive divider, the residual inductance of the divider generates an $L(\frac{dI}{dt})$ voltage which is superimposed upon the IR drop. This causes frequency-dependent behaviour of the divider. A resistive divider is normally acceptable for measuring the standard impulse of a 1.2/50 μ sec wave. However, when the duration of the surge is less than 1 μ sec a resistive divider may give large errors due to stray capacitance, which causes the response time to be large.³

The response time can be evaluated by comparing the "measured" voltage-time curve, for linearly rising impulse voltages of constant rate S , with the "true" voltage-time curve of the same wave.² Figure 25 shows the response of a system showing RC behaviour and RLC behaviour. If the steepness S is accurately known, the response time T can be determined from the voltage error ST . The response of the divider can be

improved by choosing a low value of resistance or compensating the earth capacitance by placing a special electrode at the high-voltage end of the divider to give a uniform stray-capacitance distribution along the surface of the divider.^{3,4,5}

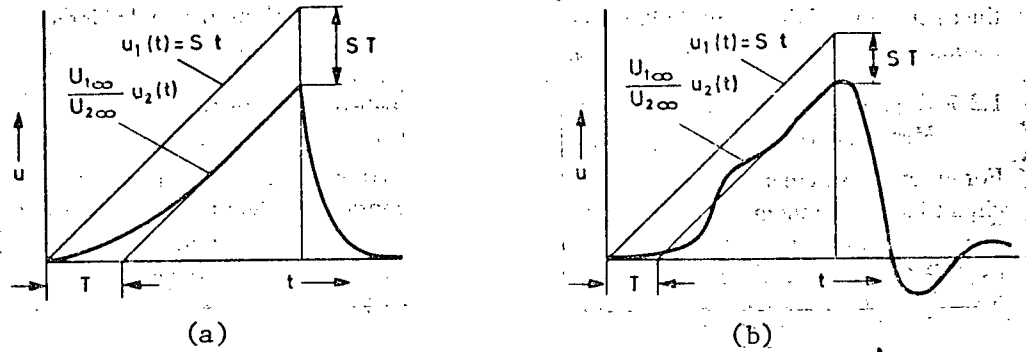


Figure 25: Display of a wedge-shaped impulse voltage.²

- a) System showing RC behaviour
- b) System showing RLC behaviour

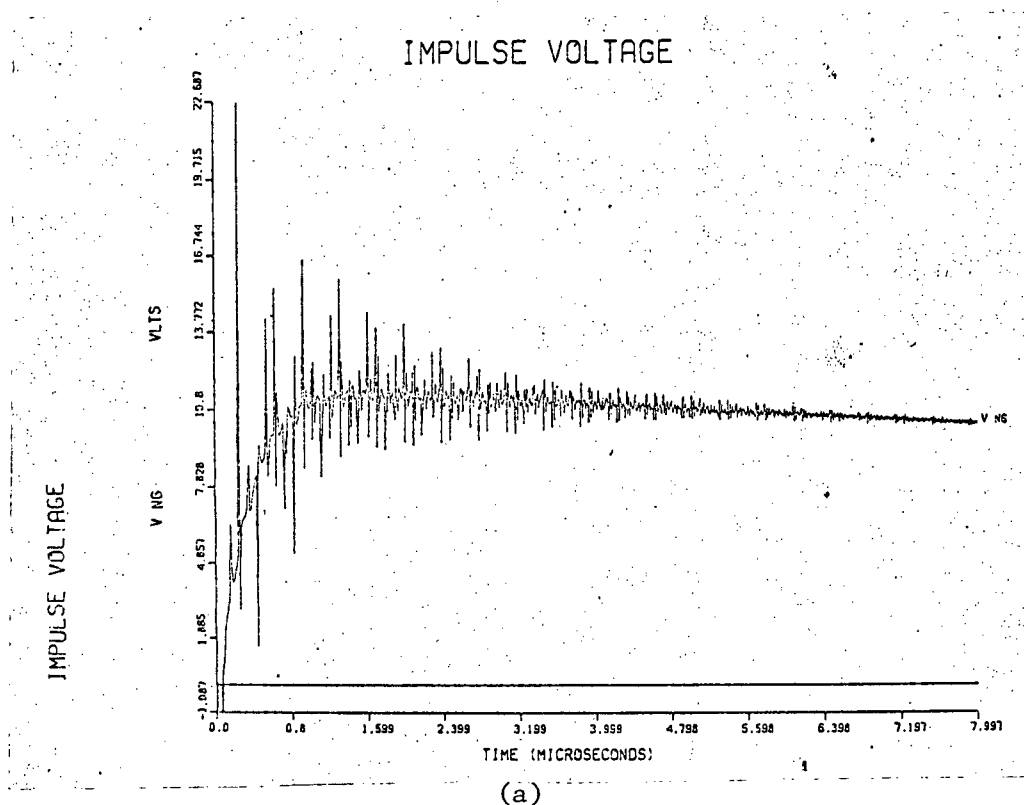
When the divider is constructed from pure capacitors, the response is theoretically perfect on fast as well as slow transients; however, with the existence of internal impedance of the measuring instrument and residual inductance of the divider and its leads, a certain limitation is imposed. From the divider transfer function, which is given below, it can be seen that the product $R(C_1 + C_2)$ has to be quite large to minimize the loading effect.

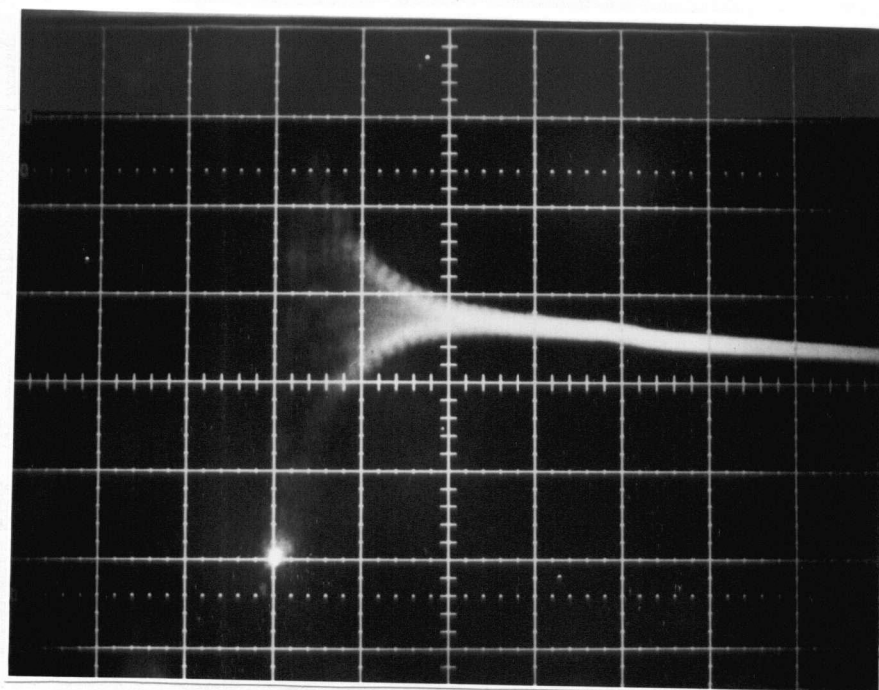
$$\frac{V_2(s)}{V_1(s)} = \frac{C_1}{C_1 + C_2} \cdot \frac{s}{s + \frac{1}{R(C_1 + C_2)}} \quad (14)$$

Otherwise, a large error will occur during slow transients. For fast transients the natural frequency of the divider is a major concern. The

capacitance and residual inductance of the divider determine this natural frequency which is usually around 200 MHz. However, if one uses large values of capacitances, the natural frequency will be lowered and may fall within the bandwidth of the measuring instrument and as a result, the recorded waveform would be distorted. This behaviour was observed experimentally in the UBC laboratory. With the capacitors supplied by the manufacturer, oscillations do not appear on the oscilloscope, whereas with a large capacitance value on the lower arm of the divider, oscillations do appear. The impulse waveshapes obtained with the capacitive divider of $4.14 \mu\text{F}$ lower capacitance value have been produced both numerically (using the UBC Electromagnetic Transients Program)¹⁴ and experimentally (shown in Figure 26).

Oscillations can also occur due to travelling wave reflections. The residual inductance and the stray capacitance to ground of the high-voltage arm and lead cause the divider to behave as a transmission line.¹³





Time scale:
2 $\mu\text{sec/div.}$
Voltage scale:
5 V/div.

(b)

Figure 26: Impulse waveshapes obtained with the capacitive divider of 4.14 μF lower capacitance value.

(a) numerically

(b) experimentally

Reflections occur because there is no impedance matching at both ends of the line. One end is short-circuited by the capacitor of the lower arm and the other end is usually badly matched. These oscillations are attenuated very little since the capacitors and the lead have low losses. To obtain high attenuation continuously-damped capacitive voltage dividers have been developed which are composed of series connections of resistors and capacitors.^{2,5}

Measuring Instrument and its connection to the divider:

Depending on the measurement quality, the required measuring instrument could be an impulse peak voltmeter, an oscilloscope or a digital transient recorder. Impulse peak voltmeters can only measure the peak value of the impulse wave and therefore do not supply enough information about the shape of the impulse. With an oscilloscope or digital transient recorder, full information of the impulse wave can be obtained.

The measuring instrument is usually connected to the voltage divider by a coaxial cable. Depending upon the amplitude level and the type of oscilloscope, the signal may either be fed directly to the deflection plates or it may be connected to the input terminals of the vertical amplifier of the oscilloscope. Additional attenuators are often needed to reduce the amplitude of the signal. In the UBC high-voltage laboratory an electronic oscilloscope and a hundred-to-one attenuator have been used.

Impedance matching is always required to avoid travelling-wave oscillations. For a resistive voltage divider, the signal cable is terminated at the measuring instrument end with its surge impedance. The circuit diagram and its equivalent circuit are shown in Figure 27.

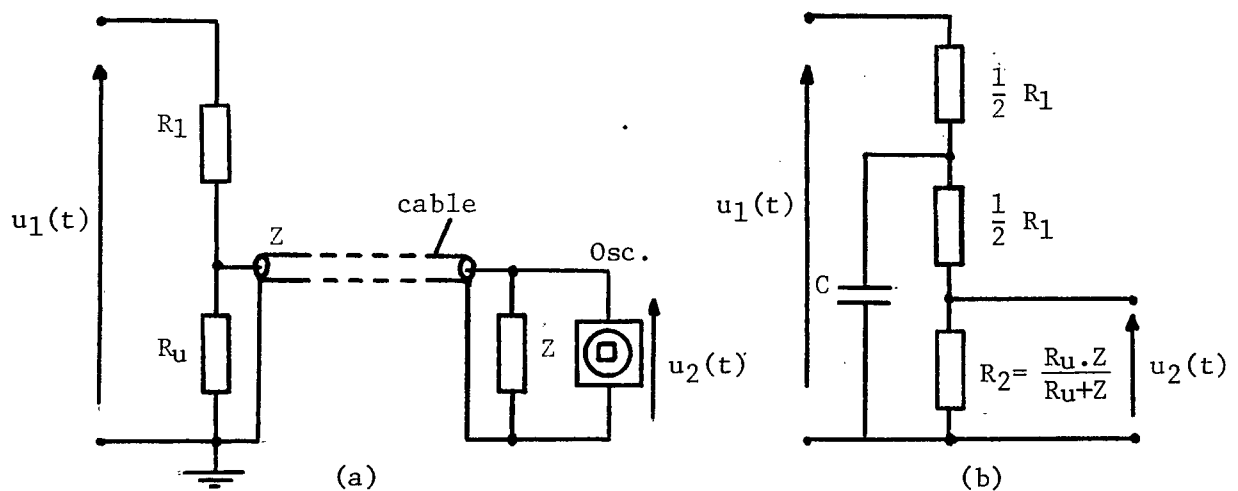


Figure 27: Impulse voltage measuring system with resistive divider

a) Circuit diagram b) Equivalent circuit with earth capacitance.

In measuring systems with capacitive dividers, as in Figure 28, termination is usually done with a series matching at the input end. The UBC test set uses this type of matching. This matching has the effect

that only half the voltage at the divider tap enters the cable, however, this is doubled again at the open end, so that full voltage will be measured at the measuring instrument again. For fast transients the

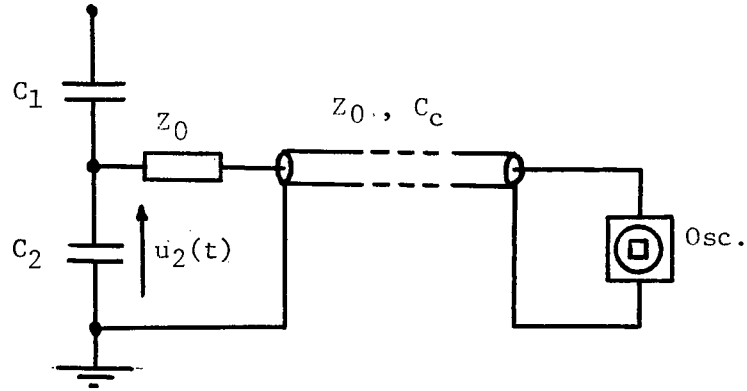


Figure 28: Connection of the capacitive voltage divider to a cathode-ray oscilloscope.

Z_0 = characteristic impedance of the signal cable, C_c = signal cable capacitance.

voltage ratio of this system is

$$a = \frac{V_1(t)}{V_2(t)} = \frac{C_1 + C_2}{C_1}, \quad (15)$$

but for slow transients the cable capacitance C_c increases the ratio as indicated below:

$$a = \frac{C_1 + C_2 + C_c}{C_1} \quad (16)$$

This error may be reduced by a complex termination proposed by Burch,⁵

Figure 29. By adjusting the additional capacitor C_3 , so that the equation $C_1 + C_2 = C_3 + C_c$ is satisfied, the ratio will be independent of fre-

quency as a first approximation.

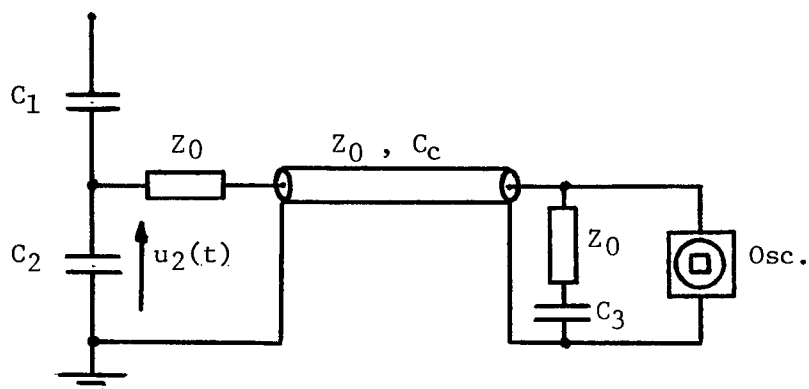


Figure 29: Compensation of signal cable capacitance by a complex cable termination. Z_0 = characteristic impedance of the signal cable, C_c = signal cable capacitance, C_3 = auxiliary capacitance.

In the case of a damped capacitive divider, series matching is also applied; however, the termination resistor at the cable input must be reduced by resistance R_2 contained in the low-voltage arm,⁵ Figure 30.

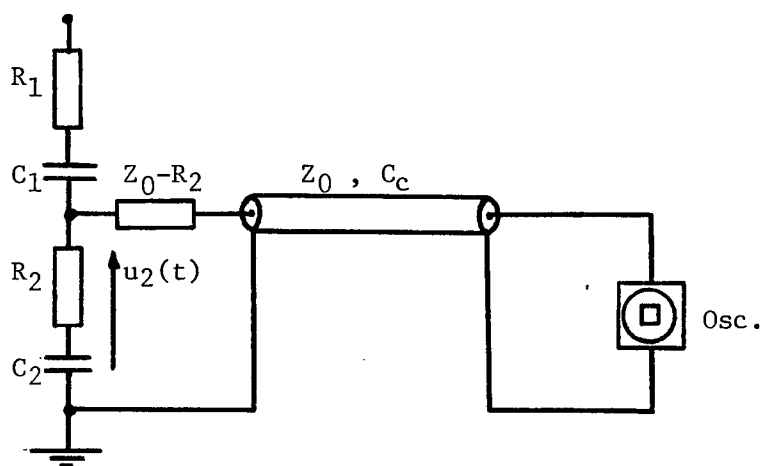


Figure 30: Impedance matching for damped capacitive voltage dividers.

3.4.3 Noise in Measuring Systems

Different sources of noise may be described with reference to the typical impulse voltage measuring system shown in Figure 31.

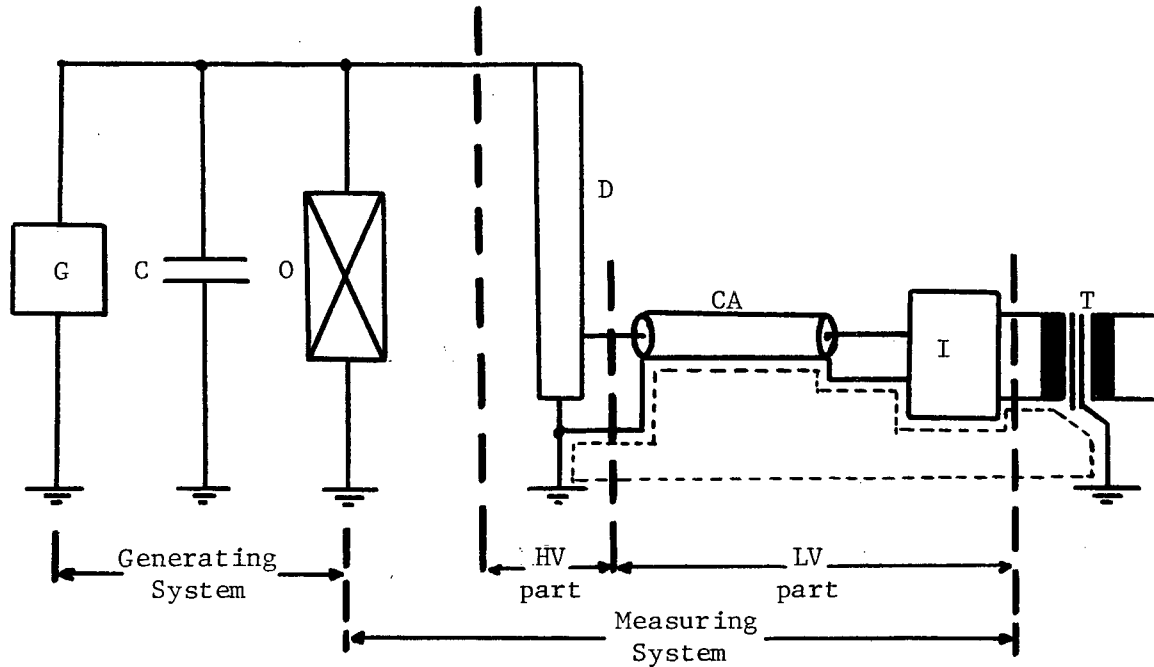


Figure 31: Impulse voltage generating and measuring systems.

G - impulse generator	O - test object
C - front capacitor	CA - measuring cable
D - voltage divider	T - isolating transformer
I - recording instrument	

The noise takes the form of current (or voltages) injected into various components of the system which give rise to measuring errors in the form of potential differences superimposed upon the actual signal. The following types of noise can be identified:^{5,6,7,9}

- a) Currents induced in the shield of the measuring cable due to ground potential differences between the divider's ground and the measuring instrument's ground during transients. To eliminate these currents, both the divider and the measuring instrument have to be grounded only at one point, which is

usually at the divider side. In the UBC laboratory, the oscilloscope is not grounded directly, but only indirectly through the shield of the measuring cable.

- b) Currents induced in the shield of the measuring cable if it forms part of a loop made up of the divider ground connection, the cable shield, the instrument case, and the ground return; such a loop is represented by a dotted line in Figure 31. The induced currents may be due to quasi-stationary (magnetic and electric) fields as well as to radiation fields. Radiation fields are generally built up by very high frequency phenomena such as triggering of sphere gaps of the impulse generator, or discharges in the test circuit. The quasi-stationary fields may be generated by current flowing in the high-voltage circuit. Currents may also be induced due to capacitive coupling between the cable shield and the high-voltage circuit.
- c) Signals penetrating directly into the active parts of the measuring instrument due to lack of screening. They are mainly due to radiation fields.
- d) Currents induced into the mains wire due to stationary fields as well as to radiation fields. These currents may or may not penetrate into the measuring instrument, depending on the effectiveness of the isolating transformer and the high frequency blocking devices (low-pass filters).

There are two techniques to suppress the high-frequency shield currents. One way is to increase the shield impedance, which can be achieved by winding the measuring cable (of coaxial type) on a ferrite

core or by sliding a number of ferrite toroids over the length of the cable. The disadvantage of this technique is that for long signal cables or very rapid pulses, the voltage and current distribution along the line is non-uniform (standing waves) and a lumped dissipative inductance no longer provides wide-band attenuation, because at a particular frequency the location of the core may coincide with a zero-current location.⁵

The other alternative is applying additional cable shields. This method permits the cable currents, originally flowing through the signal cable's braid and the oscilloscope cabinet, to bypass both and hence to eliminate the interfering voltage drops. The outermost shield is grounded at both ends or at many places throughout its length. Figure 32 compares the two cases of a simple coaxial cable and a cable with a double shield.

In Figure 32, i_1 is the inducing current flowing in the high-voltage circuit; i_2 (or i_3) is the induced current flowing in the secondary (or tertiary) loop. C_2 represents the capacitance between the instrument case and the ground, (in practice, the capacitance of winding to screen of the isolating transformer).

For the simple coaxial cable the induced current is given by the expression:⁷

$$I_2(s) = \frac{s^2 L_{12} C_2}{1 + s^2 L_2 C_2} I_1(s) \quad s = \text{Laplace operator} \quad (17)$$

where L_{12} is the mutual inductance, and L_1 and L_2 are the self inductances of the primary and secondary loops respectively.

If the instrument case is grounded, $C_2 = \infty$, the induced current is maximum. This condition leads to maximum noise, and should therefore be avoided.

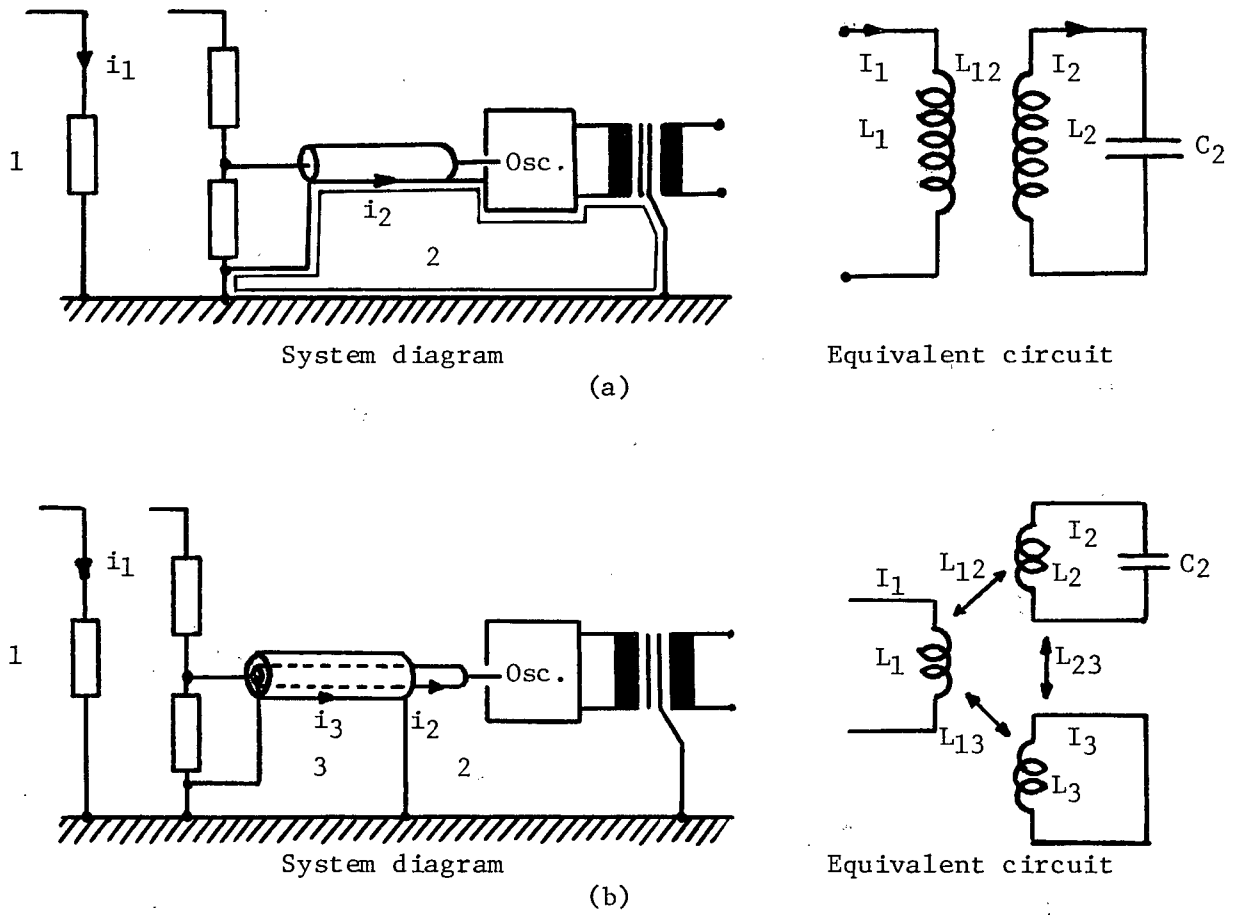


Figure 32: Currents induced in the cable shields by quasi-stationary magnetic fields.

a) Simple coaxial b) Double coaxial

For the double-shield coaxial cable, provided that the outer shield is grounded at both ends, the current in the inner shield is given by:⁷

$$I_2(s) = \frac{s^2(L_{12} - L_{13}) \cdot C_2}{1 + s^2(L_2 - L_{23}) \cdot C_2} \cdot I_1(s) \quad (18)$$

and the current in the outer shield is given by the equation:⁷

$$I_3(s) \cong \frac{L_{13}}{L_3} I_1(s) \quad (19)$$

Since $L_{12} - L_{13} \approx 0$, (loop 2 and loop 3 almost have the same area), the induced current in the inner shield is negligible, while the outer-shield takes most of the current, thus giving a good protective effect.

To have better shielding effect, an iron conduit is often used as the outermost shield. This will shield the measuring cable from electric as well as magnetic fields. In addition, the self inductance of the measuring cable shield is increased.

Direct penetration of signals into the active parts of the measuring instrument is avoided by putting the measuring instrument inside a metal enclosure.

The overall layout of the shielding of the measuring system is shown in Figure 33. This layout also describes the shielding system in the UBC high-voltage laboratory with the exception that ferrite cores are not used.

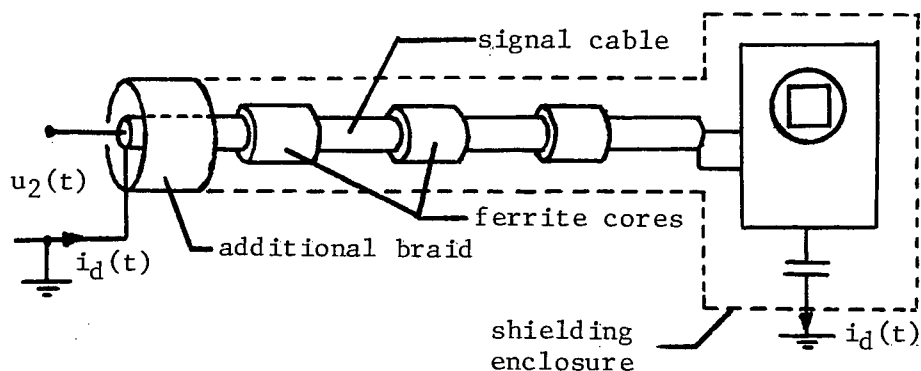


Figure 33: Correct measurement circuit layout, avoiding cable braid and cabinet current interference.

Finally, to block the currents induced into the mains wire, an isolating transformer and a low pass filter are installed before the power input of the measuring instrument. It is often sufficient to

wind the power cord on a ferrite core, which was also done in the UBC high voltage laboratory.

4. EXPERIMENT EXERCISES

4.1 Introduction

All tests are done on the "HAEFELY MULTI TEST SET" which is a versatile system capable of producing all major voltage types:

- AC voltages 50/60 Hz, up to 75 KV rms.
- DC voltages up to 200 KV.
- Impulse voltages up to 200 KV.

This flexibility allows the system to be used in many areas such as the following:

Industrial applications: Factory tests on insulators, bushings, capacitors, switchgear, instrument transformers, cables and distribution transformers.

Utility applications: Field testing of laid-in cables and completely assembled switchgear.

Teaching and research applications:

Demonstrates with AC, DC and impulse voltages.

Generation and measurement of high test voltages.

Laboratory training in high-voltage technology.

Experiments with insulator and electrode configurations.

Teaching and research application will be discussed in this chapter.

4.2 AC test

4.2.1 Breakdown voltage of sphere gaps

A minor modification to the system has to be made for this experiment. A resistor, which has enough rated value, has to be inserted in the primary circuit of the transformer to limit the primary current during breakdown. A five-ohm resistor was installed so that the maximum primary current during breakdown is less than the operating current of the protection unit (37A).

Before starting the experiment, the surfaces of the spheres should be polished and several breakdown tests made to remove any dust particles. Then five readings should be taken for each spacing, from which the arithmetic mean value can be determined.^{2,3}

The breakdown voltages of a sphere gap of 10 cm diameter, taken at 22.8°C and 990 mbar, for different gap spacings are shown in Table 2. Using the correction factor formula of equation (1), the corresponding standard breakdown voltages are calculated and compared with the accepted standard breakdown voltages in Table 2.

Table 2: Breakdown voltages of a sphere gap of 10 cm diameter for different gap spacings.

Gap (mm)	U_d (KV)	Corresponding U_{do} (KV)	Accepted U_{do} (KV)	% error
10	31.25	32.3	31.6	2.2
20	57.23	59.0	59.1	0.2
30	82.00	84.7	84.1	0.7

It can be seen that the error is still below the measuring uncertainty of 3%; therefore, the result is acceptable.

4.2.2 Corona voltage of a single conductor and a bundle conductor

Either an AC or DC source can be used for this experiment. Corona shields must be placed on both ends of the conductor to avoid sharp point effects. A simple corona shield can be built from aluminum foil.

Having connected the conductor with the voltage source, one can start the experiment. With no light in the laboratory, voltage is increased until corona begins to appear on the conductor. The voltage at this state is the corona onset voltage. This voltage can be compared with the calculated value which is obtained from the formula:^{11,12}

$$E_o = \frac{18 C_e V_{co}}{nr} \left\{ 1 + \frac{2r(n-1)}{s} \sin \left(\frac{\pi}{n} \right) \right\} \frac{KV}{cm} \quad (20)$$

where:

$$E_o = 30 m\delta \cdot \left(1 + \frac{0.426}{\sqrt{2\delta r}} \right) \frac{KV}{cm} \quad (21)$$

C_e : self capacitance ($\mu F/km$)

V_{co} : Corona onset voltage (KV)

n : number of subconductors

r : subconductor radius (cm)

s : spacing between adjacent subconductors (cm)

m : Surface or roughness factor ($0 < m \leq 1$)

δ : Relative air density

For a single conductor and a bundle conductor (2 conductors per bundle) of 39.5 cm height from the ground platform and 0.35 cm radius, the calculated and the experimental corona onset voltages are shown in Table 3.

Table 3: Corona onset voltages of a single conductor and a bundle conductor.

h = 39.5 cm r = .35 cm	Calculated Value (KV)	Experimental Value (KV)	% difference
Single Conductor	85.88	84.85	1.20
Bundled Conductor (2 Conductors) s = 2 cm	126.59	123.04	2.81

Since the differences between the two values are fairly small, one can conclude that the experiment is consistent with the theoretical formula.

4.3 DC test

4.3.1 Ripple measurement

The high DC voltages, whose ripple is to be measured, are produced from high AC voltages. The rectification can be either half wave or Greinacher doubler as explained in chapter 2. Figure 34 shows the complete circuit of a DC voltage generator and its measuring devices.

The DC voltage is measured with a resistive divider and a DC voltmeter, and the ripple by a capacitive resistive divider and either a peak voltmeter or an oscilloscope. The capacitor is meant to block the DC component such that only the ripple appears on the lower resistive divider. An arrester has to be put in parallel with the measuring device as a

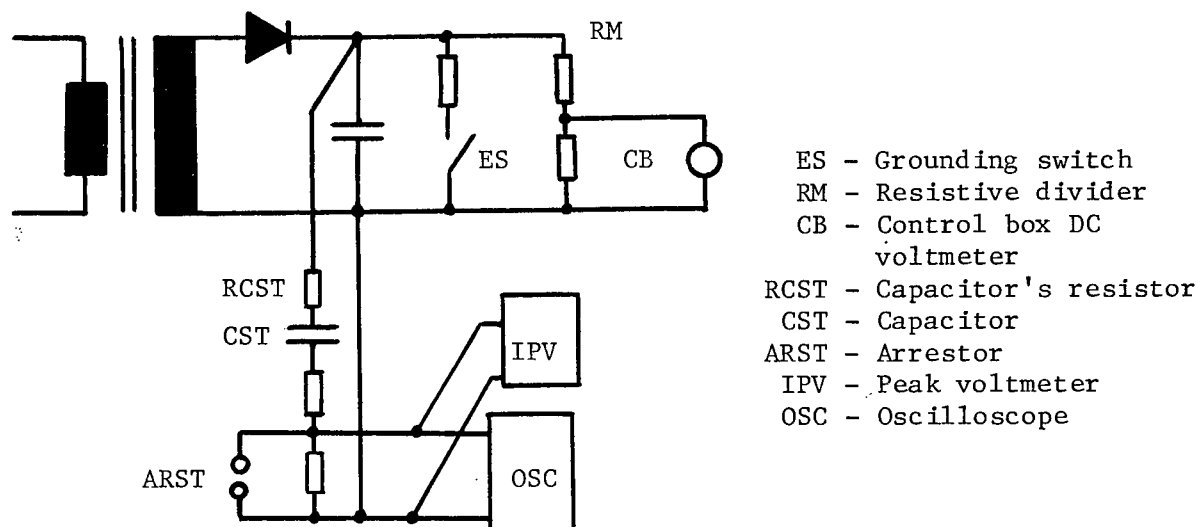


Figure 34: Overall circuit of the ripple measurement experiment.

protection against overvoltages, because a considerable amount of the blocked DC voltage will appear on the terminal of the measuring device if the voltage suddenly drops to zero. This fault can cause damage to the measuring device if no arrestor is used.

The percent ripples for different values of DC voltages are shown in table 4:

\bar{U}_{DC} (KV)	% ripple
12.25	0.12
24.90	0.11
37.50	0.12
50.00	0.13
62.50	0.14
74.80	0.14
87.50	0.15

Table 4: Percent ripples for different values of DC voltages.

4.3.2 Polarity effect in a Point-Plane gap

Positive and negative DC voltages are applied to observe the polarity effect of this gap. A protective resistor ($6000\ \Omega$) is used to protect the smoothing capacitor C_s against sudden short circuits during breakdown. In addition, the voltage may not be increased beyond 70 KV to avoid overloading of the rectifiers and capacitors.

The arrangement of the point-plane gap, which was used for the experiment, and the corresponding relationship between breakdown voltage and spacing are shown in Figure 35. One can see that for larger spacings, a positive point electrode has lower breakdown voltages than a negative point electrode. For, a positive point electrode the electrons move

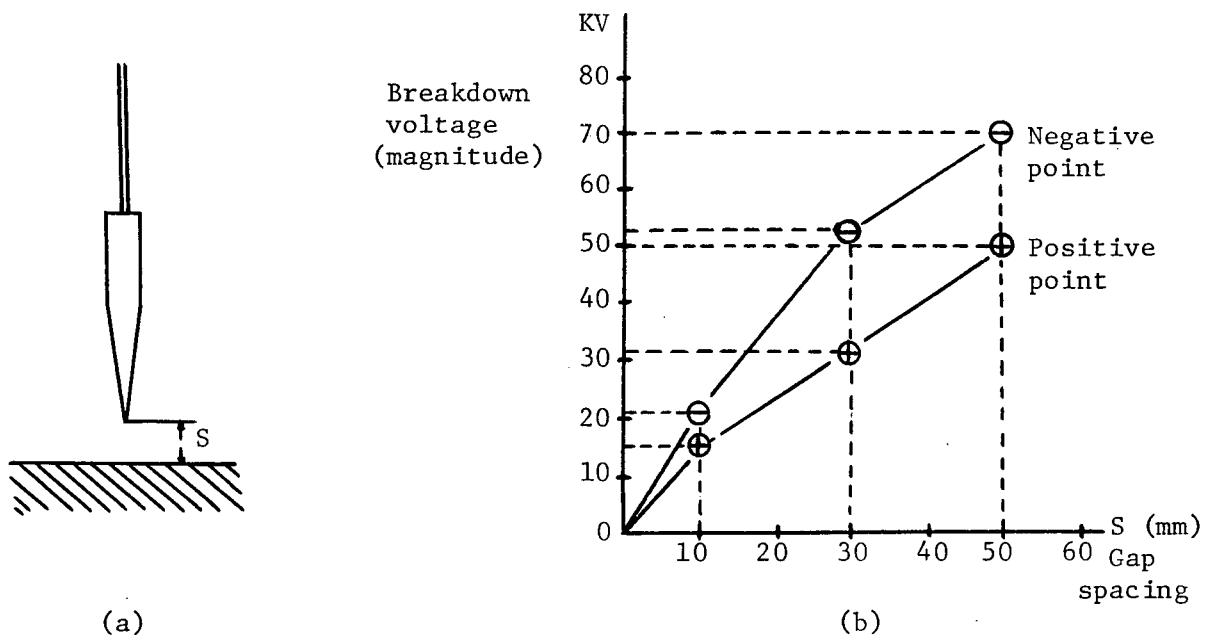


Figure 35: Polarity effect in a point-plane gap.

- (a) Electrodes configuration
- (b) Breakdown voltage versus spacing curve

towards it, producing excess positive charges in the direction of the plates, and therefore the growth of discharge channel is stimulated.

4.4 Impulse Test

4.4.1 Preliminary Preparations

A. Oscilloscope and its attenuator

To determine the required bandwidth of the oscilloscope the highest frequency, f_{\max} , has to be considered, which is a function of the size of the generating system and can be determined by the formula:⁷

$$f_{\max} \approx \frac{C}{4(H_g + H_c)} \text{ MHz} \quad (22)$$

where: C = velocity of light, $300 \text{ m}/\mu\text{s}$
 H_g = the height of generator in m
 H_c = the height of front capacitor in m

The approximate bandwidth of the oscilloscope is then given by the formula:⁷

$$f_{\lim} (-3\text{dB}) \approx \frac{1}{2\pi T(\text{osc})} \quad (23)$$

where: $T(\text{osc}) \approx \frac{1}{4\pi f_{\max}}$

For the "HAEFELY" test system, the generator and the front capacitor are both of 1 m height and consequently the upper limit of oscilloscope bandwidth is:

$$f_{\max} = \frac{300}{4(1 + 1)} = 37.5 \text{ MHz} , \quad (24)$$

$$f_{\lim} (-3\text{dB}) = \frac{1}{2\pi \frac{1}{4\pi f_{\max}}} = 75 \text{ MHz} \quad (25)$$

The TEKTRONIX 475 oscilloscope has a bandwidth of 200 MHz which clearly

satisfies the requirement.

The attenuator has to be adjusted correctly so that neither overcompensation nor undercompensation occur. The behaviour of a compensated attenuator excited by a square wave is shown in Figure 36.

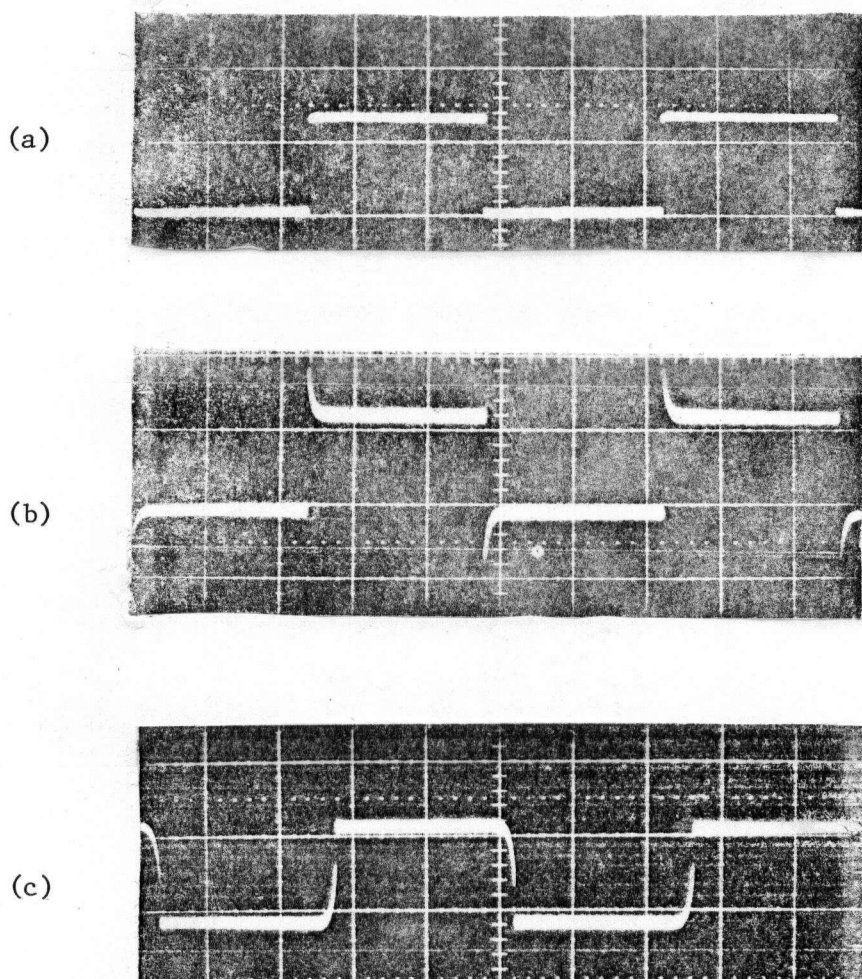


Figure 36: Output of a compensated attenuator for different degrees of compensation.

- (a) Correct compensation
- (b) Overcompensated
- (c) Undercompensated

B. Circuit Analysis

The impulse waveform can be predicted with the UBC Electromagnetic Transients Program (EMTP).¹⁴ The input of the program is the model of the circuit. Theoretically, the impulse circuit consists of capacitors and resistors only, but since the circuit interconnection is quite long unavoidable inherent inductance exists in the circuit. This inductance was measured with a simple resonant circuit and an approximate value of 1 μH was obtained. Including this inherent inductance in the model of the circuit, one finds that there is a curvature at the beginning of the waveform (shown in Figure 37).

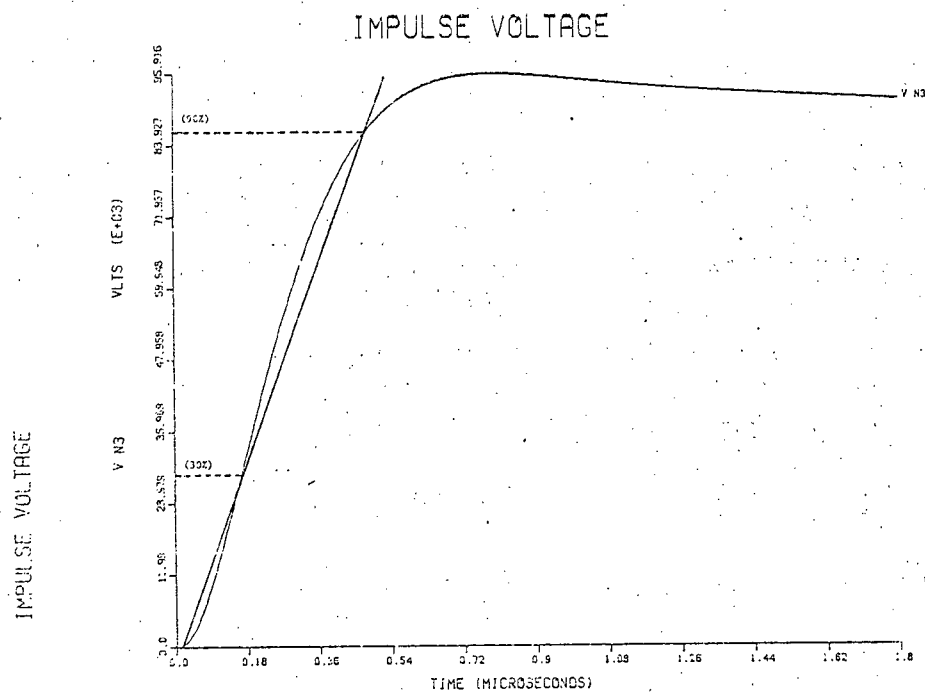


Figure 37: Impulse voltage waveform obtained by using EMTP.

Thus, to determine the origin of the waveform, the customary straight line through 30% and 90% points has to be drawn.

The inherent inductance can be reduced by putting all the components of the impulse circuit on the ground platform, thus, eliminating the

supplied metal base. This procedure was used in the laboratory not only to reduce the inductance but also to eliminate or minimize the ground loops.

4.4.2 Noise Reductions

A. Ground loop elimination

Due to the existence of radiation fields and quasi-stationary fields, ground loops have to be eliminated or minimized to reduce common mode interference. This can be achieved by putting the whole impulse circuit on the ground platform and laying the measuring cable as close as possible to the ground platform.

B. Further noise reduction

In addition to ground loop elimination, common mode interference can further be reduced by using a multishield cable instead of a simple coaxial cable. This will allow the ground current, which formerly flowed in the inner cable shield, to flow in the outer shields. In the laboratory, a triaxial cable laid in a grounded copper tube is used.

As a comparison, the impulse oscillogram of the circuit with ground loops and a simple coaxial cable and that of the circuit with reduced ground loops and a shielded cable are shown in Figure 38. It can be seen that the noise has been reduced considerably.

Some minor interferences can still be seen in Figure 38b. These interferences are due to radiation fields and quasi-stationary fields which penetrate into the oscilloscope directly and/or through the power line.

(a)

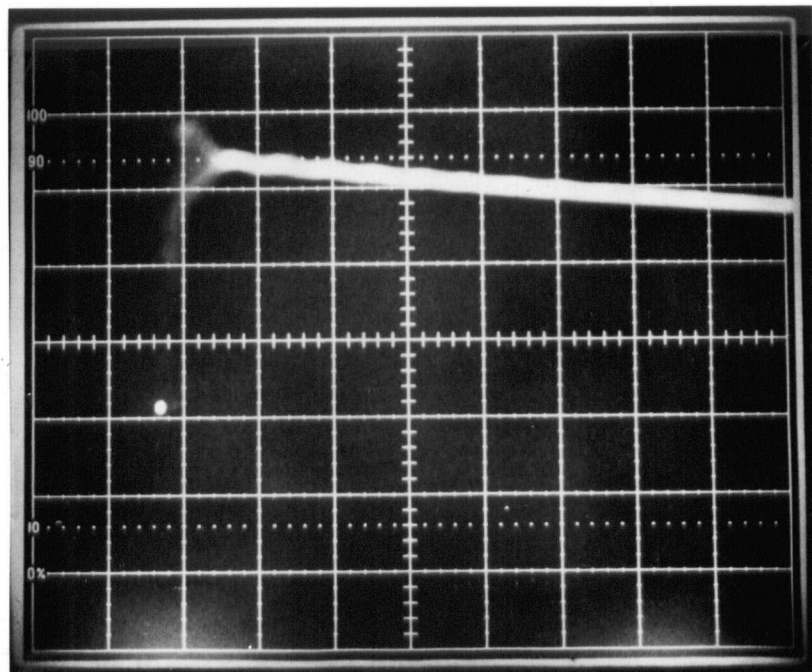
Time scale: 2 μ sec/div.

Figure 38a: Impulse oscillogram of the circuit with ground loops and a simple coaxial cable.

(b)

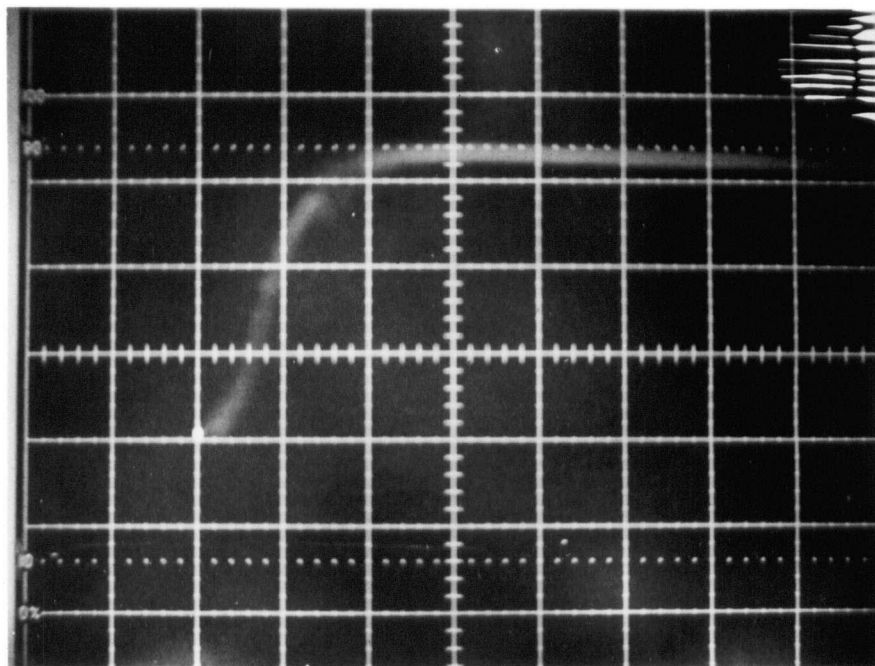
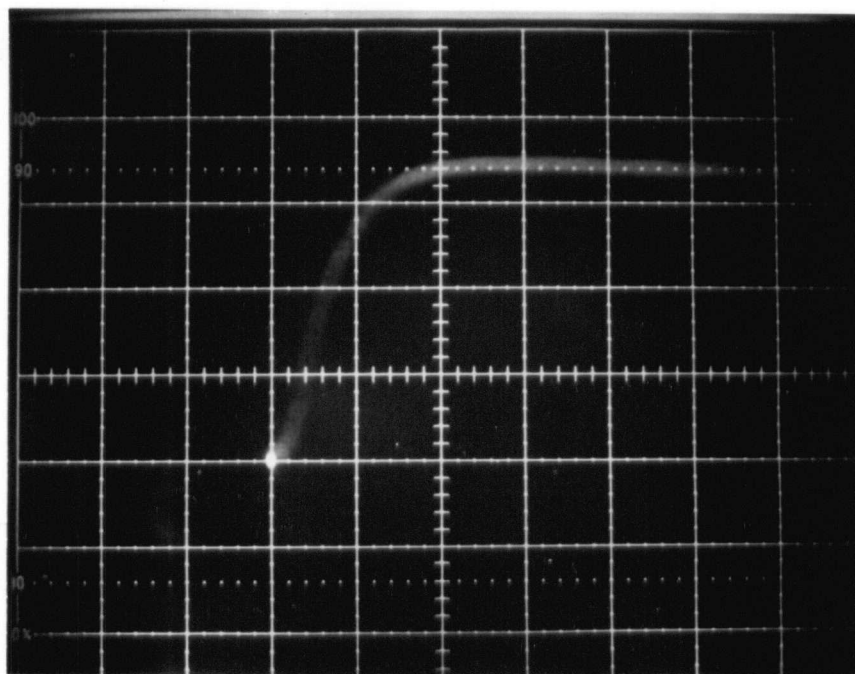
Time scale: 0.5 μ sec/div.

Figure 38b: Impulse oscillogram of the circuit with reduced ground loops and shielded cable.

To eliminate them, the oscilloscope is shielded with a metal box and a low-pass filter is inserted in the incoming power line. As a result, a continuous and clean oscillogram was obtained (shown in Figure 39).



Time scale: 0.5 μ sec/div.

Figure 39: Impulse oscillogram of the circuit with reduced ground loops and a shielded cable. Oscilloscope is inside a metal box.

5. CONCLUSIONS

The UBC high-voltage test set, which has been slightly modified and expanded, can now be used for various experiments for undergraduate students. AC and DC tests have been performed repeatedly with reproducible results.

For impulse tests various techniques of shielding have been applied in the UBC high-voltage laboratory to obtain reasonably accurate impulse oscillograms.

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APPENDIX I

SAFETY REGULATIONS FOR HIGH-VOLTAGE EXPERIMENTS

Experiments with high voltages could become particularly hazardous for the participants should the safety precautions be inadequate. To give an idea of the required safety measures, as an example the safety regulations of the High-Voltage Institute of The Technical University of Braunschweig shall be described below.² These supplement the appropriate safety regulations and as far as possible prevent risk to persons. Strict observance is therefore the duty of everyone working in the laboratory. Here any voltage greater than 250 V against earth is understood to be a high voltage.

Fundamental Rule: Before entering a high-voltage setup everyone must convince himself by personal observation that all the conductors which can assume high potential and lie in contact zone are earthed, and that all the main leads are interrupted.

Fencing

All high-voltage setups must be protected against unintentional entry of the danger zone. This is appropriately done with the aid of metallic fences. When setting up the fences for voltages up to 1 MV the following minimum clearances to the components at high-voltage should not be reduced:

for alternating and direct voltages	50 cm for every 100 KV
for impulse voltages	20 cm for every 100 KV

However, for voltages less than 100 KV a minimum clearance of 50 cm has to be maintained, independent of the type of voltage.

For voltages over 1 MV, in particular for switching impulse voltages, the values quoted could be inadequate; special protective measures must then be introduced.

The fences should be reliably connected with one another conductively, earthed and provided with warning boards inscribed: "High-voltage! Caution! Highly dangerous!" It is forbidden to introduce conductive objects through the fence whilst the setup is in use.

Safety-Locking

In high-voltage setups each door must be provided with safety switches; these allow the door to be opened only when all the main leads to test setup are interrupted.

Instead of direct interruption, the safety switches may also operate the no-voltage relay of a power circuit breaker, which, on opening the door, interrupts all the main leads to the setup. These power circuit breakers may only be switched on again when the door is closed. For direct supply from a high-voltage network (e.g. 10 KV city network), the main leads must be interrupted visibly before entry to the setup by an additional open isolating switch.

The switched condition of a setup must be indicated by a red lamp "Setup switched on" and by a green lamp "Setup switched off".

Earthing

A high-voltage setup may be entered only when all the parts which can assume high-voltage in the contact zone are earthed. Earthing may only be effected by a conductor earthed inside the fence. Fixing the

earthing leads onto the parts to be earthed should be done with the aid of insulating rods. Earthing switches with a clearly visible operating position, are also permissible. In high-power setups with direct supply from the high-voltage network, earthing is achieved by earthing isolator. Earthing may only follow after switching the current source off, and may be removed only when there is no longer anyone present within the fence or if the setup is vacated after removal of earth. All metallic parts of the setup which do not carry potential during normal service must be earthed reliably and with an adequate cross section of at least 1.5 mm^2 .

Circuit and Test Setup

Inasmuch as the setup is not supplied from ready wired desks, clearly marked isolating switches must be provided in all leads to the low-voltage circuits of high-voltage transformers and arranged at an easily identifiable position outside the fence. These must be opened before earthing and before entering the setup.

All leads must be laid so that there are no loosely hanging ends. Low voltage leads which can assume high potentials during breakdown or flashovers and lead out of the fenced area, e.g. measuring cable, control cable, supply cable, must be laid inside the setup in earthed sleeving. All components of the setup must be either rigidly fixed or suspended so that they cannot topple during operation or be pulled down by the leads.

For all setups intended for research purposes, a circuit diagram shall be fixed outside the fence in clearly visible position.

A test setup may be put into operation only after the circuit has been checked and permission to begin work given by an authorized

person.

Conducting the Experiments

Everyone carrying out experiments in the laboratory is personally responsible for the setup placed at his disposal and for the experiments performed with it. For experiments during working hours one should try, in the interest of personal safety, to make sure that a second person is present in the testing room. If this is not possible, then at least the times of the beginning and end of an experiment should be communicated to a second person.

When working with high-voltages outside working hours, a second person familiar with the experimental setups must be present in the same room.

Explosion and Fire Risk, Radiation Protection

In experiments with oil and other easily inflammable materials, special care is necessary owing to the danger of explosion and fire. In each room where work is carried out with these materials, suitable fire extinguishers must be to hand, ready for use. Easily inflammable waste products, e.g. paper or used cotton waste, should always be disposed of immediately in metal cans. Special regulations must be observed when radioactive sources are used.

APPENDIX II
FORMULA OF MODE OSCILLATIONS

The formula of mode oscillations in a rectangular resonator can be derived from MAXWELL's equations:

$$\nabla \times \mathbf{H} = \epsilon \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times \mathbf{E} = - \mu \frac{\partial \mathbf{H}}{\partial t}$$

After an extensive manipulation of these differential equations, the electric and the magnetic fields of TM modes and TE modes are obtained for the boundary conditions $x = 0$, $x = a$ and $y = 0$, $y = b$:

TM modes:

$$E_{ox1}(x,y) = \text{Re} \left\{ \frac{-j\beta m\pi}{h^2 a} C_{mn1} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{j(\omega t - \beta z)} \right\} \hat{a}_x$$

$$E_{oy1}(x,y) = \text{Re} \left\{ \frac{-j\beta n\pi}{h^2 b} C_{mn1} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{j(\omega t - \beta z)} \right\} \hat{a}_y$$

$$E_{oz1}(x,y) = \text{Re} \left\{ C_{mn1} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{j(\omega t - \beta z)} \right\} \hat{a}_z$$

$$H_{ox1}(x,y) = \text{Re} \left\{ \frac{j\omega\epsilon n\pi}{h^2 b} C_{mn1} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{j(\omega t - \beta z)} \right\} \hat{a}_x$$

$$H_{oy1}(x,y) = \text{Re} \left\{ \frac{-j\omega\epsilon m\pi}{h^2 a} C_{mn1} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{j(\omega t - \beta z)} \right\} \hat{a}_y$$

where: C_{mn1} corresponds to the particular mode defined by a given choice of m and n . (m, n are integers)

$$\beta^2 = \mu\epsilon\omega^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2$$

$$h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

The electric fields traveling in the opposite direction are:

$$E_{ox2}(x,y) = \text{Re} \left\{ \frac{-j\beta m\pi}{h^2 a} C_{mn2} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{j(\omega t + \beta z)} \right\} \hat{a}_x$$

$$E_{oy2}(x,y) = \text{Re} \left\{ \frac{-j\beta n\pi}{h^2 b} C_{mn2} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{j(\omega t + \beta z)} \right\} \hat{a}_y$$

Applying boundary condition $z = 0$ and $z = C$, the following is obtained:

$$\underline{z = 0}: \quad E_{ox1}(x,y) + E_{ox2}(x,y) = 0$$

$$(M_1 + M_2) \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \sin \omega t = 0$$

$$\text{where:} \quad M_1 = \frac{\beta m\pi}{h^2 a} C_{mn1}$$

$$M_2 = \frac{\beta m\pi}{h^2 a} C_{mn2}$$

$$\text{Therefore,} \quad M_1 = -M_2$$

$$\underline{z = C}: \quad E_{ox1}(x,y) + E_{ox2}(x,y) = 0$$

$$M_1 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \{ \sin(\omega t - \beta C) - \sin(\omega t + \beta C) \} = 0$$

$$-2 \cos \omega t \sin \beta C$$

$$\sin \beta C = 0 \quad \beta C = p\pi$$

where: p is an integer

$$\beta = \frac{p\pi}{C}$$

But
$$\beta^2 = \mu\epsilon\omega^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2$$

Therefore,

$$\omega = \frac{1}{\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2}$$

or

$$f = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2}$$

The same expression can be derived from $E_{oy_1}(x,y)$ and $E_{oy_2}(x,y)$ and from TE modes.