ESTIMATION OF THE LEVEL OF ANESTHESIA DURING SURGERY
BY AUTOMATIC EEG PATTERN RECOGNITION

by

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ABSTRACT

The feasibility of developing an automatic electroencephalographic (EEG) pattern recognition system for reliably estimating the level of consciousness of surgical patients during general anesthesia is investigated.

An effort was made to establish a valid methodology, by identifying and controlling as many extraneous variables as possible and by ensuring that the work would be relevant to current anesthetic practice. The data base that was established for use in all experimental investigations consists of 938 EEG pattern samples from 72 subjects and three types of anesthesia. Each EEG pattern sample corresponds to one of five possible clinical levels of anesthesia.

The use of automatic pattern recognition techniques, in conjunction with heuristic techniques of clinical EEG analysis, to develop spectral and time domain EEG pattern recognition systems is described. All of the initially developed systems extract a small number of heuristically derived features from unknown EEG pattern samples. The classifiers in these systems employ Bayes decision rule under the assumption that the extracted features are statistically independent. A rationale concerning the choice of this particular feature extraction scheme and pattern classification algorithm is presented and discussed.

Consideration is given to the general problem of how to use a relatively small set of available EEG pattern samples to effectively evaluate the performance of EEG pattern recognition systems. Two non-parametric techniques which provide particularly informative and efficient estimates of the performance of such systems are formulated. Results obtained by employing these techniques to estimate the performance of the initially developed spectral and time domain EEG pattern recognition systems are presented. The results clearly demonstrate the feasibility
of estimating the level of anesthesia by means of automatic EEG pattern recognition. However, the results also indicate that the initially developed systems are not sufficiently reliable for immediate and general clinical application.

Theoretical techniques are developed to model some relevant statistical properties of spontaneous EEG activity, with a view to improving the performance of the initially developed EEG pattern recognition systems. Results which were obtained by applying the modelling techniques to some specific ensembles of EEG pattern samples are presented. The comparative advantages of employing alternate methods of EEG analysis are then discussed in relation to the estimated statistical characteristics of the particular EEG ensembles under consideration.

Several factors which could adversely affect the reliable performance of EEG pattern recognition systems in general, and the initially developed systems in particular, are identified and discussed. Various schemes for improving the performance of the initially developed systems are suggested and an evaluation of the practicability of each is presented.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>ii</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF ILLUSTRATIONS</td>
<td>viii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>x</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENT</td>
<td>xi</td>
</tr>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Problem Area</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Evaluation of Previous Research</td>
<td>3</td>
</tr>
<tr>
<td>1.3 Scope of Thesis</td>
<td>5</td>
</tr>
<tr>
<td>II. EXPERIMENTAL CONTROLS AND DATA ACQUISITION</td>
<td>10</td>
</tr>
<tr>
<td>2.1 Objectives</td>
<td>10</td>
</tr>
<tr>
<td>2.2 Establishment of Anesthesia Levels</td>
<td>11</td>
</tr>
<tr>
<td>2.2.1 Introduction</td>
<td>11</td>
</tr>
<tr>
<td>2.2.2 Historical perspective</td>
<td>11</td>
</tr>
<tr>
<td>2.2.3 Definition of anesthesia levels</td>
<td>12</td>
</tr>
<tr>
<td>2.3 Acquisition of Experimental Data</td>
<td>14</td>
</tr>
<tr>
<td>2.3.1 Types of anesthesia considered</td>
<td>14</td>
</tr>
<tr>
<td>2.3.2 Standardized anesthetic technique</td>
<td>14</td>
</tr>
<tr>
<td>2.3.3 Data acquisition</td>
<td>16</td>
</tr>
<tr>
<td>2.3.4 Control of variables during data acquisition</td>
<td>19</td>
</tr>
<tr>
<td>2.4 Establishment of EEG Data Base</td>
<td>21</td>
</tr>
<tr>
<td>2.4.1 Description of analog EEG data collected</td>
<td>21</td>
</tr>
<tr>
<td>2.4.2 Digitization and preparation of digital EEG data base</td>
<td>23</td>
</tr>
<tr>
<td>III. DEVELOPMENT OF EEG PATTERN RECOGNITION SYSTEMS</td>
<td>28</td>
</tr>
<tr>
<td>3.1 EEG Pattern Recognition Systems</td>
<td>28</td>
</tr>
<tr>
<td>3.1.1 Basic description</td>
<td>28</td>
</tr>
<tr>
<td>3.1.2 Development and performance evaluation</td>
<td>29</td>
</tr>
<tr>
<td>3.2 Spectral Feature Extraction</td>
<td>30</td>
</tr>
<tr>
<td>3.2.1 EEG spectral analysis</td>
<td>30</td>
</tr>
</tbody>
</table>
3.2.2 Computation of EEG spectra ........................................ 31
3.2.3 Spectral feature vectors ......................................... 33

3.3 Time Domain EEG Feature Extraction .............................. 35
  3.3.1 Time domain EEG analysis ....................................... 35
  3.3.2 Time domain feature vectors .................................. 37

3.4 Classification Algorithm .......................................... 39

3.5 Evaluation of System Performance ............................... 43
  3.5.1 The performance estimation problem ......................... 43
  3.5.2 Performance estimation techniques ......................... 43
  3.5.3 The \( \Pi^* \) technique ....................................... 46
  3.5.4 The \( U^* \) technique ....................................... 47

3.6 Results ............................................................ 48
  3.6.1 EEG spectral pattern recognition systems .................. 48
  3.6.2 Time domain EEG pattern recognition systems .............. 53

3.7 Discussion
  3.7.1 Spectral and time domain EEG pattern recognition systems
       ................................................................. 57
  3.7.2 Evaluation of EEG pattern recognition approach .......... 60
  3.7.3 Further work .................................................. 63

IV. MODELLING THE STATIONARITY AND GAUSSIANITY OF EEG ACTIVITY .. 65

4.1 Introduction ........................................................ 65
  4.1.1 Motivation ..................................................... 65
  4.1.2 Evaluation of previous investigations ..................... 65
  4.1.3 Outline of chapter .......................................... 67

4.2 Random Process Characterization ................................ 67

4.3 Establishment of Empirical Testing Procedures ............... 70
  4.3.1 Testing for wide-sense stationarity ......................... 70
  4.3.2 Testing for Gaussianity ...................................... 71

4.4 Experiment ........................................................ 72
  4.4.1 Selection of sample EEG data ................................ 72
  4.4.2 Determination of optimum sampling rate ..................... 73
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.4.3 Application of tests for wide-sense stationarity and Gaussianity</td>
<td>75</td>
</tr>
<tr>
<td>4.5 Results</td>
<td>78</td>
</tr>
<tr>
<td>4.5.1 Interpretation of results</td>
<td>78</td>
</tr>
<tr>
<td>4.5.2 Effect of sampling rate on empirical tests</td>
<td>79</td>
</tr>
<tr>
<td>4.5.3 Estimated baseline EEG characteristics</td>
<td>83</td>
</tr>
<tr>
<td>4.5.4 Wide-sense stationarity</td>
<td>84</td>
</tr>
<tr>
<td>4.5.5 Gaussianity</td>
<td>84</td>
</tr>
<tr>
<td>4.5.6 Wide-sense stationarity and Gaussianity</td>
<td>85</td>
</tr>
<tr>
<td>4.6 Significance of Results</td>
<td>85</td>
</tr>
<tr>
<td>4.6.1 Development of EEG monitoring systems</td>
<td>85</td>
</tr>
<tr>
<td>4.6.2 Evaluation of alternate analytic techniques</td>
<td>87</td>
</tr>
<tr>
<td>4.6.3 Further work</td>
<td>88</td>
</tr>
<tr>
<td>V. PERFORMANCE IMPROVEMENT SCHEMES</td>
<td>90</td>
</tr>
<tr>
<td>5.1 Introduction</td>
<td>90</td>
</tr>
<tr>
<td>5.2 Extraction of Additional Features</td>
<td>93</td>
</tr>
<tr>
<td>5.2.1 Rationale</td>
<td>93</td>
</tr>
<tr>
<td>5.2.2 Definition of additional features</td>
<td>94</td>
</tr>
<tr>
<td>5.2.3 Feature selection</td>
<td>98</td>
</tr>
<tr>
<td>5.2.4 Resulting improvement in performance</td>
<td>101</td>
</tr>
<tr>
<td>5.3 Exploitation of Statistical Interdependencies Among Features</td>
<td>107</td>
</tr>
<tr>
<td>5.3.1 Method of investigation</td>
<td>107</td>
</tr>
<tr>
<td>5.3.2 Results and discussion</td>
<td>110</td>
</tr>
<tr>
<td>5.4 &quot;Nearest Subject&quot; Scheme</td>
<td>113</td>
</tr>
<tr>
<td>5.4.1 Rationale</td>
<td>113</td>
</tr>
<tr>
<td>5.4.2 Feasibility</td>
<td>114</td>
</tr>
<tr>
<td>5.4.3 Discussion</td>
<td>116</td>
</tr>
<tr>
<td>VI. CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH</td>
<td>118</td>
</tr>
<tr>
<td>6.1 Conclusions</td>
<td>118</td>
</tr>
<tr>
<td>6.1.1 Summary</td>
<td>118</td>
</tr>
<tr>
<td>6.1.2 Major Original Contributions</td>
<td>118</td>
</tr>
<tr>
<td>6.1.3 Establishment of a valid research methodology</td>
<td>119</td>
</tr>
</tbody>
</table>
6.1.4 Introduction of automatic pattern recognition techniques ........................................... 120
6.1.5 Formulation of performance estimation techniques ...................................................... 120
6.1.6 Demonstration of feasibility ......................................................................................... 121
6.1.7 Development of theoretical modelling techniques ......................................................... 121
6.1.8 Establishment of a statistical model of EEG activity ...................................................... 121
6.1.9 Evaluation of performance improvement schemes ......................................................... 122

6.2 Suggestions for Future Research ........................................................... 122

6.2.1 Performance improvement schemes ................................................................. 122
6.2.2 Experimental controls ......................................................................................... 123
6.2.3 Time domain EEG pattern recognition systems ......................................................... 123
6.2.4 The reliability of visual EEG assessment .................................................................. 124
6.2.5 Modelling ............................................................................................................... 125
6.2.6 Identification of artifact ........................................................................................... 126

APPENDIX A LEVEL OF ANESTHESIA EVALUATION FORM ........................................ 127
APPENDIX B DESCRIPTION OF EEG DATA BASE ......................................................... 128
APPENDIX C COMPUTATION OF EEG SPECTRA .............................................................. 131
APPENDIX D SPECTRAL FEATURE EXTRACTION PROGRAM ............................................. 134
APPENDIX E TIME DOMAIN ANALYSIS OF FEATURE EXTRACTION PROGRAM ................. 136
APPENDIX F PERFORMANCE ESTIMATION BY THE II* TECHNIQUE .................................. 140
APPENDIX G PERFORMANCE ESTIMATION BY THE U* TECHNIQUE ................................ 143
APPENDIX H EVALUATION OF K-S STATISTICS FOR EEG AMPLITUDE DISTRIBUTIONS ................................................................. 147
APPENDIX I EVALUATION OF K-S STATISTICS FOR EEG SPECTRAL DISTRIBUTIONS ................................................................. 150
APPENDIX J TESTS OF K-S STATISTICS .......................................................................... 154

REFERENCES ............................................................................................................... 156
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>(a) Data acquisition equipment</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>(b) Acquisition of data in the operating room</td>
<td>17</td>
</tr>
<tr>
<td>2-2</td>
<td>Sample segments of multichannel EEG activity</td>
<td>22</td>
</tr>
<tr>
<td>2-3</td>
<td>Configuration of system used for preparing and</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>screening EEG pattern samples</td>
<td></td>
</tr>
<tr>
<td>3-1</td>
<td>EEG pattern recognition system</td>
<td>29</td>
</tr>
<tr>
<td>3-2</td>
<td>Preparation of spectral and time domain feature vectors</td>
<td>39</td>
</tr>
<tr>
<td>3-3</td>
<td>Estimating classifier performance</td>
<td>44</td>
</tr>
<tr>
<td>4-1</td>
<td>Effect of increased sampling rates on K-S goodness of fit tests for</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>Gaussianity</td>
<td></td>
</tr>
<tr>
<td>4-2</td>
<td>Mean ensemble characteristics of the baseline EEG</td>
<td>79</td>
</tr>
<tr>
<td></td>
<td>activity of 30 subjects who were resting with eyes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>closed</td>
<td></td>
</tr>
<tr>
<td>4-3</td>
<td>Estimated percentage of EEG segments of various</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>durations from three different ensembles which can</td>
<td></td>
</tr>
<tr>
<td></td>
<td>be modelled as wide-sense stationary</td>
<td></td>
</tr>
<tr>
<td>4-4</td>
<td>Estimated percentage of EEG segments of various</td>
<td>81</td>
</tr>
<tr>
<td></td>
<td>durations from three different ensembles which can</td>
<td></td>
</tr>
<tr>
<td></td>
<td>be modelled as Gaussian</td>
<td></td>
</tr>
<tr>
<td>4-5</td>
<td>Estimated percentage of EEG segments of various</td>
<td>82</td>
</tr>
<tr>
<td></td>
<td>durations from three different ensembles which can</td>
<td></td>
</tr>
<tr>
<td></td>
<td>be modelled as both wide-sense stationary and</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gaussian</td>
<td></td>
</tr>
<tr>
<td>5-1</td>
<td>Confusion matrices for systems which extracted 13</td>
<td>91</td>
</tr>
<tr>
<td></td>
<td>spectral features</td>
<td></td>
</tr>
<tr>
<td>5-2</td>
<td>Improvement in the performance of an EEG spectral</td>
<td>102</td>
</tr>
<tr>
<td></td>
<td>pattern recognition system developed for halothane anestheisa</td>
<td></td>
</tr>
<tr>
<td>5-3</td>
<td>Improvement in the performance of an EEG spectral</td>
<td>103</td>
</tr>
<tr>
<td></td>
<td>pattern recognition system developed for narcotic anestheisa</td>
<td></td>
</tr>
<tr>
<td>5-4</td>
<td>Improvement in the performance of an EEG spectral</td>
<td>104</td>
</tr>
<tr>
<td></td>
<td>pattern recognition system developed for enflurane anestheisa</td>
<td></td>
</tr>
<tr>
<td>5-5</td>
<td>Confusion matrices for systems which extracted 26</td>
<td>106</td>
</tr>
<tr>
<td></td>
<td>spectral and coherence features</td>
<td></td>
</tr>
<tr>
<td>Figure</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>5-6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a) Spectral feature correlation matrix for halothane anesthesia data</td>
<td>111</td>
<td></td>
</tr>
<tr>
<td>(b) Spectral feature correlation matrix for narcotic anesthesia data</td>
<td>111</td>
<td></td>
</tr>
<tr>
<td>(c) Spectral feature correlation matrix for enflurane anesthesia data</td>
<td>112</td>
<td></td>
</tr>
</tbody>
</table>
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>Clinical criteria for estimating levels of anesthesia</td>
<td>13</td>
</tr>
<tr>
<td>2-2</td>
<td>Description of resulting EEG data base</td>
<td>27</td>
</tr>
<tr>
<td>3-1</td>
<td>Description of spectral feature set</td>
<td>34</td>
</tr>
<tr>
<td>3-2</td>
<td>Description of time domain EEG feature set</td>
<td>37</td>
</tr>
<tr>
<td>3-3</td>
<td>Performance of spectral pattern recognition systems on EEG data from halothane anesthesia</td>
<td>51</td>
</tr>
<tr>
<td>3-4</td>
<td>Performance of spectral pattern recognition systems on EEG data from narcotic anesthesia</td>
<td>51</td>
</tr>
<tr>
<td>3-5</td>
<td>Performance of spectral pattern recognition systems on EEG data from enflurane anesthesia</td>
<td>52</td>
</tr>
<tr>
<td>3-6</td>
<td>Performance of time domain pattern recognition systems on EEG data from halothane anesthesia</td>
<td>55</td>
</tr>
<tr>
<td>3-7</td>
<td>Performance of time domain pattern recognition systems on EEG data from narcotic anesthesia</td>
<td>56</td>
</tr>
<tr>
<td>3-8</td>
<td>Performance of time domain pattern recognition systems on EEG data from enflurane anesthesia</td>
<td>57</td>
</tr>
<tr>
<td>5-1</td>
<td>Spectral and coherence features chosen for extraction from each EEG channel</td>
<td>97</td>
</tr>
<tr>
<td>5-2</td>
<td>Summary of selected spectral and coherence features</td>
<td>100</td>
</tr>
<tr>
<td>5-3</td>
<td>Average correlation coefficient magnitudes</td>
<td>110</td>
</tr>
<tr>
<td>B-1</td>
<td>EEG data base</td>
<td>128</td>
</tr>
</tbody>
</table>
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CHAPTER I
INTRODUCTION

1.1 Problem Area

The need for a reliable method of assessing the level of consciousness of surgical patients during general anesthesia has existed since the introduction of the first general anesthetic agents more than a century ago. The clinical signs and stages of anesthetic depth that traditionally have been employed have never been entirely satisfactory. However, as a result of recent advances in anesthesiology, many of these traditional signs and stages have clearly become unreliable and inadequate in terms of modern anesthetic practice. It is significant that the electroencephalogram, an intuitively appealing indicator of the gross electrical activity of the brain, is not among the indicators which are routinely evaluated in attempting to assess the level of anesthesia. In fact, electroencephalographic activity is rarely even monitored during general anesthesia at present. The possibility of developing a computer-based system for reliably estimating the level of anesthesia by means of electroencephalographic pattern recognition is the subject of this thesis.

General anesthesia can be defined as a state of unconsciousness, produced by anesthetic agents, with an absence of pain sensation over the entire body and a greater or lesser degree of muscular relaxation. An electroencephalogram (EEG) is an electrical signal which is generated by the brain and recorded from electrodes attached to the scalp. Spontaneous electroencephalographic activity (or EEG activity) is characterized by voltages which are usually less than 100 µV, by frequencies which are essentially bandlimited to 30 Hz and by a wide range of patterns or waveforms, some of which are associated with different states of consciousness.
Because a number of general references are available in the areas of anesthesiology (e.g. [1-4]) and electroencephalography (e.g. [5-8]), further information of a fundamental nature in these particular areas will not be included in the thesis.

Intuitively, because general anesthesia is defined primarily in terms of brain function, it is reasonable to suspect that different levels of anesthesia, i.e. different levels of consciousness, could be manifested by different spontaneous EEG patterns. Considerable motivation exists for the development of an automatic system which could reliably estimate the level of anesthesia by means of spontaneous EEG pattern recognition. Some of the potential applications are immediately apparent.

1) Such a system could be employed to monitor the level of anesthesia throughout surgery, i.e. to provide a continually-updated estimate of the anesthesia level. This would permit an anesthesiologist to more accurately control the administration of anesthetic agents, thereby reducing the probability of subjecting the patient to unnecessarily deep, life-threatening levels of anesthesia or, alternatively, to very light levels of anesthesia which might result in periods of consciousness or awareness during surgery.

2) The system could provide a rapid and sensitive indication of the occurrence of anesthetic accidents.

3) It would be particularly valuable in certain kinds of operations where most clinical, non-EEG signs of anesthetic depth are unavailable, e.g. during the critical cardiopulmonary bypass phase of open-heart surgery.

4) It could be employed in the clinical evaluation of new anesthetic agents.

5) It could be of value in the instruction of anesthesiologists.
1.2 Evaluation of Previous Research

The prospect of using the EEG to estimate the level of anesthesia was first suggested in 1937 as a practical application of observed correlations between different EEG patterns and various levels of anesthesia induced by ether [9]. During the next two decades similar correlations between observed EEG patterns and anesthetic depth were described for other anesthetic agents including cyclopropane, nitrous oxide - ether and nitrous oxide - thiopental [10-12]. More quantitative correlations were also investigated by relating observed EEG patterns to the arterial blood concentrations of different anesthetic agents [13,14]. Over the years, subjective descriptions of recognizable time domain EEG patterns at various anesthetic levels have been reported for most of the commonly used anesthetic agents. An extensive review of the correlations between various general anesthetics and observed EEG patterns was recently published [15].

In a 1959 review paper, Martin et al. proposed that most of the general anesthetics that were then in common use had a similar, dose-dependent relationship to a recognizable sequence of EEG patterns [16]. This relationship seemed to suggest that a reliable method for estimating the level of anesthesia could eventually be developed by identifying and classifying the various EEG patterns produced by different patients and different anesthetics. This expectation was not realized, however, largely because of a variety of methodological problems relating to the validity and reliability of previous work. EEG validity in this instance may be defined as the extent to which the EEG contains information concerning the actual level of anesthesia, while reliability refers to the dependability of a particular method for extracting such information from the EEG in order to correctly estimate the level of anesthesia.
Four major unresolved problems relating to the validity and reliability of previous work can be identified. Martin et al. recognized the basic problem of level definition: a precise definition of the different possible levels of anesthesia is necessary before one can properly consider the question of whether or not the EEG constitutes a valid indicator of those levels. A second problem involving the reliability of EEG pattern definition has also been acknowledged: different investigators may vary considerably in their subjective definitions of what constitutes recognizably different EEG patterns [17]. In addition, the use of a variety of anesthetic agents results in pattern variability, thereby increasing the complexity of the pattern recognition task. Finally, the inter-rater reliability of visual EEG assessment among experienced clinical raters, even with an established set of objective criteria for pattern identification, may be surprisingly low. No study of inter-rater reliability has been conducted using EEG data from different levels of anesthesia. However, in a recent study based on clinical EEG data, the highest average intraclass correlation reported among seven experienced clinical EEG raters was 0.56 [18].

Largely because of such methodological problems, the results of many attempts to estimate anesthesia levels on the basis of visual EEG assessment have been confusing and contradictory. For example, one group which studied EEG activity at different levels of halothane anesthesia reported that seven distinct EEG patterns were observed [19], but a second group which studied the same type of anesthesia reported that only two distinct EEG patterns could be identified [20]. Furthermore, the second group stated that the classical sequence of EEG changes associated with progressively deeper levels of ether anesthesia could not be observed during halothane anesthesia.

It should be noted that the issue of whether or not the EEG constitutes a valid indicator of the level of anesthesia was not resolved
simply because the results of investigations based on visual EEG assessment were not reliable. Intuitively, the EEG still appears to be the single, most valid parameter to evaluate in attempting to estimate the level of anesthesia. From a practical viewpoint EEG monitoring is safe, non-invasive, and can usually be performed with relative ease in the operating room.

Recent advances in the fields of automatic EEG analysis [21-23] and pattern recognition [24-26] have provided a valuable new perspective for reconsidering the anesthetic level estimation problem. A few automatic techniques have already been used to analyse EEG activity during anesthesia, e.g. [27-32], but this work has been confined to the implementation of various methods of EEG data compression and parameter identification. Hence the pattern recognition task, i.e. the identification and interpretation of any changes in EEG characteristics during anesthesia, would still be performed subjectively, presumably by an experienced anesthesiologist. The work to be described in this thesis represents a significant departure from previous research: it constitutes the first comprehensive investigation into the possibility of developing a computer-based EEG pattern recognition system for reliably estimating the level of anesthesia during surgery.

1.3 Scope of Thesis

The overall objective of the research described in this thesis was to investigate the feasibility of reliably estimating the level of anesthesia during surgery by means of an EEG pattern recognition system. The specific objectives were:

1) to define a set of clinically valid levels of anesthesia in terms of objective, non-EEG criteria;

2) to establish a sample EEG data base, consisting of a set of EEG pattern samples corresponding to known clinical anesthesia levels,
for one or more commonly used types of anesthesia;
3) to develop systems for estimating anesthesia levels by the recognition of different spectral and time domain EEG patterns;
4) to establish a method for effectively evaluating the performance of EEG pattern recognition systems on the basis of a finite set of available EEG pattern samples;
5) to evaluate the performance of the initially developed spectral and time domain EEG pattern recognition systems;
6) to develop theoretical techniques which enable the degree of wide-sense stationarity and Gaussianity of spontaneous EEG activity to be modelled;
7) to model the degree of wide-sense stationarity and Gaussianity of some specific ensembles of EEG pattern samples, with a view to improving the performance of the initially developed pattern recognition systems;
8) to identify the major factors which affect the performance of EEG pattern recognition systems; and
9) to investigate any schemes which appear likely to improve the performance of the initially developed systems.

Chapter II describes the establishment of a sample EEG data base, consisting of a number of digitized, multichannel EEG segments which correspond to different levels of anesthesia. In the course of establishing the data base, a considerable effort was made to control a wide range of extraneous variables because it was recognized that the control of such variables was crucial to the success of subsequent work involving the data base. Accordingly, in addition to describing the preparation and organization of the sample EEG data base, Chapter II outlines the effort that
was made to identify and control as many extraneous variables as possible. For example, explicit definitions of the different possible levels of anesthesia were established to control the incidence of errors in clinical, non-EEG assessments of anesthetic depth. Chapter II also describes how a number of other potential sources of variability were controlled, e.g. by restricting the number of different types of anesthesia under consideration, by establishing a standardized anesthetic technique and by taking a variety of precautions during the preparation of digital EEG pattern samples.

Chapter III describes the initial development and performance evaluation of spectral and time domain EEG pattern recognition systems. All of the initially developed systems extract a small number of heuristically derived features from unknown EEG pattern samples. The classifiers in these systems employ Bayes decision rule under the assumption that the extracted features are statistically independent. A rationale concerning the choice of this particular feature extraction scheme and classification rule is presented and discussed in Chapter III. Then the general problem of how to use a relatively small set of available EEG pattern samples to effectively evaluate the performance of an EEG pattern recognition system is discussed. Two nonparametric techniques which provide particularly informative and efficient estimates of the performance of such systems are suggested. Results which were obtained by employing these techniques to estimate the performance of the initially developed spectral and time domain EEG pattern recognition systems are then presented. These results clearly demonstrate the feasibility of estimating the level of anesthesia by means of automatic EEG pattern recognition.

Chapter IV describes the development of a statistical model of spontaneous EEG activity. It was thought that such a model could be of value in improving the performance of the initially developed EEG pattern
recognition systems. Almost all methods of quantitative EEG analysis are based on certain implicit assumptions regarding the statistical characteristics of the underlying random process, particularly with respect to the extent of stationarity and Gaussianity of the process. The efficacy of alternate methods of analysis therefore depends upon the degree to which such assumptions are justified by the characteristics of the particular ensembles of EEG segments being analysed. In Chapter IV, theoretical techniques are developed which enable the degree of wide-sense stationarity and Gaussianity of spontaneous EEG activity to be modelled. Results which were obtained by applying these techniques to some specific ensembles of EEG pattern samples are presented. The comparative advantages of employing alternate methods of EEG analysis are then discussed in relation to the estimated degree of stationarity and Gaussianity of the particular EEG ensembles under consideration.

Chapter IV contains a discussion of possible methods for improving the performance of the initially developed pattern recognition systems by taking into account the actual statistical characteristics of the EEG data being analysed. Chapter V describes the investigation of other possible strategies for improving the performance of the initially developed systems. Most of these strategies involve changes in the initial feature extraction scheme and pattern classification algorithm. In the same chapter, it is argued that intersubject EEG variation is one of the major factors which adversely affect the performance of EEG pattern recognition systems. Accordingly, most of the work described in Chapter V was directed toward estimating and reducing the effect of intersubject EEG variation.

A few concluding remarks are presented in Chapter VI. In addition, the major original contributions of the research described in the thesis are briefly summarized and some suggestions are made regarding
possible areas for further research. The Appendices contain detailed information about the sample EEG data base that was established. This information should be sufficient to allow the data base to be readily used and expanded in future investigations. The Appendices also contain listings of the major programs that were written in the course of this investigation. The program listings serve a dual purpose: they provide detailed documentation concerning specific computational procedures and they facilitate the use of such procedures by others.

For reference purposes, it should be noted that some of the original results presented in subsequent chapters have already been published elsewhere [33-39,140].
CHAPTER II
EXPERIMENTAL CONTROLS AND DATA ACQUISITION

2.1 Objectives

This chapter describes the establishment of a data base, consisting of a relatively large number of sample EEG segments which correspond to different clinical anesthesia levels. During the establishment of this data base a substantial effort was made to identify and control as many extraneous variables as practicable, because it was recognized that the subsequent value of the acquired data would obviously be dependent on the extent to which such variables could be identified and controlled. To control the incidence of errors in clinical, non-EEG assessments of the level of anesthesia, it was necessary to establish explicit definitions of the different possible anesthesia levels in terms of reliable clinical criteria. Section 2.2.2 discusses the inadequacy of the traditional stages and signs of anesthesia for this purpose; section 2.2.3 describes how five clinically significant levels of anesthesia were defined, in terms of relatively objective non-EEG criteria, for this research. To eliminate some potential sources of variability, the number of different types of anesthesia under consideration was restricted and a standardized anesthetic technique was established, as described in section 2.3.1 and section 2.3.2. The data acquisition procedure which was followed is outlined in section 2.3.3 and the control of extraneous variables during data acquisition is discussed in section 2.3.4. Finally, sections 2.4.1 and 2.4.2 describe the preparation of a digital EEG data base from the experimental data collected.
2.2 Establishment of Anesthesia Levels

2.2.1 Introduction

General anesthesia may be defined as a state of unconsciousness produced by anesthetic agents, with absence of pain sensation over the entire body and a greater or lesser degree of muscular relaxation [40]. At present, different dosages of a wide variety of anesthetic agents and drugs, administered either by inhalation or intravenously, can be used to produce different levels of general anesthesia. For the purposes of this research a set of five possible levels of general anesthesia was explicitly defined in terms of clinical, non-EEG signs of anesthetic depth.

2.2.2 Historical Perspective

The first description of different stages of anesthesia was contained in a monograph published in 1847 [41]. The monograph described five recognizable stages of anesthesia with ether, based primarily on changes in the character of respiration and the degree of suppression of reflex activity. In subsequent years, various possible clinical signs of different anesthesia stages were investigated, including heart rate, blood pressure, respiration, pupil diameter and reactivity to light, tearing and eye movement. Several of these signs were eventually incorporated into a detailed description of four different stages of anesthesia which was published in a fairly complete form in 1937 [42]. For many years this description of clinical signs and stages served as the standard reference for inhalational anesthesia. It should be noted, however, that only a small number of inhalational anesthetic agents were then in common use and the primary goal of the anesthesiologist in this period was simply to administer one of the available agents in sufficient concentration to produce a stage of anesthesia associated with unconsciousness and an adequate degree of muscular relaxation, without seriously endangering the
patient's life. Unfortunately, this rather admirable goal was not always satisfactorily achieved.

Recent developments in anesthesiology have decreased the mortality rate associated with the administration of general anesthesia, but have eliminated or obscured many of the traditional clinical signs and stages of anesthesia [43]. For example, the clinical use of drugs which specifically produce good muscle relaxation and the emergence of controlled respiration to assure adequate patient ventilation have largely eliminated two formerly valuable clinical signs: the degree of muscle relaxation and the character of respiration [44]. Furthermore, factors such as the introduction of pre-anesthetic medication, the use of a combination of drugs during anesthesia and the increasing variety of anesthetic agents have contributed to the complexity of correctly interpreting changes in many of the remaining clinical signs [43-46]. In addition, modern anesthetic practise has reduced the significance of some of the traditional stages of anesthesia and has provided increased motivation for the definition of some new stages: for example, the current practise of rapid induction of anesthesia has essentially eliminated one of the traditional stages, while recent reports of consciousness occurring at apparent surgical levels of anesthesia [47-52] indicate the need for a new definition of anesthesia levels.

Thus, at least two important problems associated with the definition of anesthesia levels are apparent. First, many of the traditional clinical signs and stages of anesthesia are not relevant to modern anesthetic practise. Second, any available clinical signs may often be equivocal and require considerable subjective interpretation.

2.2.3 Definition of Anesthesia Levels

For this research, the set of possible levels of anesthesia was defined in a unique manner to clearly establish its validity in terms of
modern anesthetic practise. After considerable discussion, experienced
anesthesiologists\(^1\) defined five clinically significant levels of anesthesia
in terms of non-EEG criteria that they considered to be meaningful and ap­
propriate. Subsequently, minor revisions of the criteria were made to re­
solve possible ambiguities in the wording, to allow for a more objective
differentiation of levels, and to facilitate the use of the same set of
criteria with three common types of general anesthesia (to be described in
section 2.3.1). The resultant set of clinical criteria is given in Table
2-1. The criteria are based primarily upon a patient's responsiveness to
various stimuli and upon changes in his blood pressure and pulse rate. A
concerted effort was made to keep all criteria as objective and quantitative
as practicable.

Table 2-1 Clinical Criteria for Estimating Levels of Anesthesia

<table>
<thead>
<tr>
<th>Level</th>
<th>Clinical Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(Consciousness) Patient is alert with spontaneous speech.</td>
</tr>
<tr>
<td>2</td>
<td>(Light Surgical Anesthesia) Movement in response to surgical stimulation but not in response to the preparation or similar light stimulation. Tachycardia and hypertension during surgery.</td>
</tr>
<tr>
<td>3</td>
<td>(Surgical Anesthesia) No movement in response to the preparation or surgical stimulation. No tachycardia or bradycardia. Patient is either normotensive or mildly hypotensive, i.e. within 20 percent of normal.</td>
</tr>
<tr>
<td>4</td>
<td>(Deep Surgical Anesthesia) No movement in response to the preparation or surgical stimulation. Bradycardia and hypotension, i.e. greater than a 20 percent deviation from normal.</td>
</tr>
</tbody>
</table>

\(^1\) Dr. L.C. Jenkins, Professor and Head of the Department, and Dr. B.A. Saunders, Clinical Assistant Professor, Department of Anaesthesiology, Faculty of Medicine, University of B.C.
2.3 Acquisition of Experimental Data

2.3.1 Types of Anesthesia Considered

The different types of general anesthesia are commonly differentiated by referring to the combination of agents employed to maintain an adequate level of anesthesia. Hence the three types of anesthesia to be considered in this thesis are generally known to anesthesiologists as halothane-nitrous oxide-relaxant anesthesia, narcotic-nitrous oxide-relaxant anesthesia and enflurane-nitrous oxide-relaxant anesthesia. The first two types of anesthesia account for most of the general anesthetics administered in North America today. For example, of 28,988 inhalational anesthetics which were administered at the Vancouver General Hospital in 1974, approximately 33 percent employed some variation of the halothane-nitrous oxide-relaxant technique and 63 percent employed some variation of the narcotic-nitrous oxide-relaxant technique [53]. The third type of anesthesia, i.e. enflurane-nitrous oxide-relaxant anesthesia, is relatively new but is rapidly gaining in popularity and may be in common usage within a few years. For convenience, these three types of anesthesia will subsequently be referred to in the thesis as halothane anesthesia, narcotic anesthesia and enflurane anesthesia, respectively.

2.3.2 Standardized Anesthetic Technique

The following standardized technique was established for the administration of all three types of anesthesia. Approximately one hour before surgery, a premedication consisting of morphine (10-15 mg) or meperidine (50-100 mg) and atropine (0.6 mg) or scopolamine (0.4 mg) was administered. Induction of anesthesia was accomplished with sodium thiopentone (5 mg/kg body weight) and tracheal intubation was facilitated by the administration of succinylcholine (1 mg/kg). Halothane anesthesia
was maintained with halothane vapour (0.75 percent initially) as the primary anesthetic agent supplemented by a mixture of 60 percent nitrous oxide and 40 percent oxygen. Similarly, enflurane anesthesia was maintained with enflurane vapour (1.0 percent initially) as the primary anesthetic agent supplemented by a mixture of 60 percent nitrous oxide and 40 percent oxygen. The administered concentration of both primary anesthetic agents was changed occasionally during surgery to change the level of anesthesia of patients. The third type of anesthesia, narcotic anesthesia, was maintained with a mixture of 60 percent nitrous oxide and 40 percent oxygen in conjunction with small increments (5-15 mg) of alphaprodine, a narcotic analgesic, which were given intravenously as necessary during surgery. In all cases, adequate muscle relaxation was obtained with d-tubocurarine (0.3 mg/kg initially, with more as required during longer operations). A Bird Mark 8 Respirator was used to provide controlled respiration, with respiratory rates and tidal volumes initially determined by a Radford nomogram [54]. To ensure adequate ventilation during anesthesia, a Beckman LB-1 Medical Gas Analyzer was used to monitor each patient's end-tidal carbon dioxide concentration and the respirator was adjusted so that the end-tidal carbon dioxide concentration and the respirator was adjusted so that the end-tidal carbon dioxide concentration was always between 35 and 45 mm Hg. At the end of each operation the action of the muscle relaxant was reversed with atropine (1.2 mg) and neostigmine (2.5 mg).

Detailed information regarding the different anesthetic techniques and procedures and the properties of various anesthetic agents and drugs can be found in many general references, e.g. [7, 8], and will not be given here. Thorough reviews of possible EEG effects of a wide variety of general anesthetics are also available, e.g. [15-17], as are many
papers dealing specifically with relevant EEG and cardiovascular effects of the anesthetic agents and drugs used in the thesis research, e.g. nitrous oxide [55,56], sodium thiopentone [12], d-tubocurarine [57], halothane [19,20,58-60], narcotics [50,61] and enflurane [62-66]. At present, the most serious clinical problems associated with the three types of anesthesia considered in the thesis are: possible hepatitis resulting from halothane anesthesia [67-69], reported incidents of awareness during narcotic anesthesia [47-52] and occasional central nervous system irritability during enflurane anesthesia [62,64]. It should be noted in passing that the latter two problems are currently being investigated by means of EEG analysis.

2.3.3 Data Acquisition

Fig. 2-la shows most of the equipment employed to acquire experimental data, as well as some of the usual anesthetic equipment in the operating room. Fig. 2-lb shows the actual configuration of the equipment for data acquisition during an operation. The anesthetic equipment cart seen in Fig. 2-la contains a Bird Respirator, an anesthetic gas vaporizer and supplies of various anesthetic agents and drugs. The EEG electrodes seen in Fig. 2-lb are standard cup electrodes, filled with conductive paste, which have been attached to the patient at positions defined by the International 10-20 System [70] to establish four differential EEG channels: F3-C3, C3-O1, F4-C4 and C4-O2. The relative locations of these two bilaterally symmetric pairs of channels are indicated in Fig. 2-3. The EEG electrodes were connected to a termination box (Fig. 2-la) which can be used in one mode to measure the electrode contact resistance and in another mode as a preamplifier for the EEG machine. A Beckman 8-channel EEG machine, with its lowpass filters set at 50 Hz and its highpass
Fig. 2-la Data Acquisition Equipment

Fig. 2-lb Acquisition of Data in the Operating Room
filters set at 0.54 Hz to reduce artifact, was used to amplify the EEG and to plot the amplified EEG on chart paper for immediate visual inspection. A Hewlett-Packard Model 3960A instrumentation tape recorder was connected to the EEG machine; at a tape speed of 15/16 ips the recorder could store four channels of EEG activity for more than four hours on one reel of 3M Type 871 instrumentation tape. The pulse generator seen in Fig. 2-1b was connected to the recorder so that short pulses could be inserted into one channel of the recording to identify EEG segments of interest. As mentioned in section 2.3.2, an infrared CO₂ analyser was used to monitor the end-tidal carbon dioxide concentration throughout each operation. Not evident in either Fig. 2-1a or Fig. 2-1b is a Tektronix 410 Monitor which was used to monitor electrocardiographic activity.

The acquisition of experimental data proceeded in the following manner. After a suitable surgical patient had been identified by one of the anesthesiologists participating in this research, the patient was visited pre-operatively and informed consent was obtained. EEG electrodes were then attached and, before the standard premedication was administered, the patient's baseline EEG activity was recorded for several minutes while he or she was resting with eyes closed; the pulse generator was used to mark at least two 64s segments of baseline EEG activity for subsequent analysis. EEG recording was later resumed when the patient entered the operating room and was continued until the patient was moved to a post-operative recovery area. Estimations of the level of anesthesia, based on the clinical criteria given in Table 2-1, were made by an anesthesiologist at intervals of approximately five minutes during the operation. The pulse generator was used to mark 64s EEG segments which corresponded to the clinically estimated anesthesia levels. If the anesthesiologist was uncertain of the level of anesthesia as defined by the clinical
criteria, e.g. during a period of transition between levels, no further attempt was made to estimate the level at that time. Similarly, no attempt was made to estimate the level of anesthesia when the EEG contained obvious and excessive artifact, e.g. while the electrosurgical unit was in use. The Level of Anesthesia Evaluation Form shown in Appendix A was employed to record each estimated level of anesthesia and the number of the pulse which identified the corresponding EEG segment, as well as all other relevant information about the operation.

2.3.4 Control of Variables During Data Acquisition

An attempt was made to control several extraneous variables during the acquisition of experimental data. Many of these variables tended to increase the range of EEG pattern variability and the incidence of errors in clinically estimated anesthesia levels. Obviously the subsequent value of the acquired data is highly dependent on the extent to which such extraneous variables could be controlled.

To reduce the range of EEG variability resulting from the use of different anesthetic agents and drugs, only the three most common types of general anesthesia were considered and a standardized anesthetic technique was established. Furthermore, data was acquired only from healthy adult patients who underwent similar kinds of surgery, thus reducing the extent of EEG variability due to differences in age, general health status, intensity of surgical stimulation and duration of anesthesia. EEG variability associated with abnormal carbon dioxide levels in the blood [71] was controlled by monitoring the patient's carbon dioxide level and adjusting the respirator to keep it within normal limits, as described in section 2.3.2.

Additional precautions were taken to reduce the amount of
artifact present in recorded EEG activity. The EEG electrodes were firmly attached with gauze pads soaked in collodion, a special glue and sealant which prevented the electrode paste from drying out during the operation and thus reduced the possibility of artifact due to poor electrode contacts. Any artifact above 50 Hz, e.g. 60 Hz electrical interference, and below 0.54 Hz, e.g. some movement artifact, was eliminated by setting the lowpass and highpass filters on the EEG machine to 50 Hz and 0.54 Hz respectively. EEG activity was not recorded while the electrosurgical unit was being used because artifact from the unit saturated the EEG amplifiers.

Attempts were also made to reduce the incidence of incorrect estimations of anesthesia levels caused by errors in clinical judgement and by possible non-stationarities in the actual level of anesthesia over the 64s duration of the corresponding EEG segment. Errors in clinical judgement were reduced by developing an explicit set of objective clinical criteria (Table 2-1) and by minimizing the number of anesthesiologists who made clinical estimations of levels; these anesthesiologists became familiar with the standardized anesthetic technique and became quite proficient at estimating anesthesia levels on the basis of the clinical criteria. When they could not confidently estimate levels on the basis of the criteria, they were asked to refrain from guessing. The incidence of non-stationary anesthesia levels within the 64s intervals corresponding to identified EEG segments was reduced in two ways. First, whenever possible, a clinical level estimation was made at the beginning and end of a 64s interval and the corresponding EEG segment was only retained for analysis if both estimations were the same. Second, at least three minutes was allowed to elapse between a change in the administered concentration of the primary anesthetic agent and the time that the next clinical level estimation was made, so that the concentration of anesthetic agents in the blood could approach equilibrium;
it would have been preferable to determine a state of equilibrium by directly monitoring the arterial blood concentrations of the various anesthetic agents, but it was not possible to do so because the appropriate equipment was not available.

2.4 Establishment of EEG Data Base

2.4.1 Description of Analog EEG Data Collected

As stated previously, the operations from which data was collected consisted primarily of general surgical cases involving patients who were in the best surgical risk categories, i.e. who were in either Class I or Class II as defined by the American Society of Anesthesiologists ([8], pp. 401-402). Data which was collected from an operation was not retained for analysis when there was a significant deviation from the standardized anesthetic procedure outlined in section 2.3.2, or when it was apparent that the control of variables described in section 2.3.4 was inadequate. In total, EEG recordings and clinical data from 72 operations were retained for analysis. Of this total, halothane anesthesia was used in 21 cases, narcotic anesthesia was used in 26 cases and enflurane anesthesia accounted for the remaining 25 cases. Fig. 2-2 shows sample multichannel segments of baseline EEG activity (Level 0) and EEG activity at a surgical level of anesthesia (Level 3) for the three different types of anesthesia.

The halothane anesthesia data was obtained from 8 male and 13 female patients ranging in age from 17 to 65 years, with an average age of 46 years. The average duration of anesthesia was 70 min, although the duration of individual cases varied from 30 min to 135 min. The number of anesthesiologists who made clinical estimations of the level of anesthesia during halothane anesthesia was limited to three.

Of the 26 narcotic anesthesia cases, 9 involved male patients
Fig. 2-2 Sample Segments of Multichannel EEG Activity. Samples of EEG activity at Anesthesia Level 0 and Anesthesia Level 3 for three subjects having similar baseline EEG characteristics are shown in (a)-(b), (c)-(d), and (e)-(f). Segments (b), (d) and (f) were recorded during halothane anesthesia, narcotic anesthesia and enflurane anesthesia, respectively.
and 17 involved female patients. Their ages ranged from 20 to 64 years, with an average age of 44 years. The anesthetic was administered for between 30 min and 150 min; the average duration was 90 min. Thirteen anesthesiologists made clinical estimations of the level of anesthesia during narcotic anesthesia.

The enflurane anesthesia data was obtained from 9 male and 16 female patients. All were between 23 and 70 years of age, with an average age of 47 years. The anesthesia varied from 60 min to 150 min in duration, with an average duration of approximately 90 min. Three anesthesiologists were involved in making clinical estimations of anesthesia levels.

For reasons which will be given elsewhere in the thesis, it was considered desirable to collect some data from patients who were undergoing two successive operations within a short period of time. This was possible in a few instances, i.e. where female patients underwent tissue biopsies followed by mastectomies or hysterectomies. Consequently, the halothane anesthesia data included data from one pair of operations performed on the same patient and the narcotic anesthesia data included data from three such pairs of operations.

2.4.2 Digitization and Preparation of Digital EEG Data Base

Fig. 2-3 shows the general configuration of the system that was developed to prepare and screen digitized EEG pattern samples. As described in section 2.3.3, throughout each operation an instrumentation tape recorder was used to record four channels (F3-C3, C3-O1, F4-C4 and C4-O2) of spontaneous EEG activity. Short pulses which were inserted in one channel of the recording identified all EEG segments corresponding to known clinical anesthesia levels. The system shown in Fig. 2-3 was used to digitize these EEG recordings, to separate digitized EEG pattern samples
Fig. 2-3 Configuration of System Used for Preparing and Screening EEG Pattern Samples
corresponding to known anesthesia levels, and to plot these pattern samples for visual screening.

The Nova 840 Signal Processing Facility at the U.B.C. Electrical Engineering Department was used to convert all analog EEG records to digital records stored on digital data tapes. To accomplish this, as illustrated in Fig. 2-3, the recorded EEG activity was first reproduced on the instrumentation recorder, lowpass filtered at 30.0 Hz with Krohn-Hite 3342R filters and then the filtered data was digitized and stored on 9-track, IBM-compatible tapes using the Nova 840 Signal Processing Facility. The digitizer consisted of a multiplexer which sampled each EEG channel at 128 samples/s and a 10-bit analog/digital converter which converted each sample to binary form. For programming ease, each digital sample value was stored in two successive bytes on tape although the maximum resolution was limited to 10 bits.

The digital data tapes were transferred to the IBM 370/168 computer at the U.B.C. Computing Centre. A FORTRAN program was used to find the pulse locations on each tape. After the pulse locations were verified by checking the pulse information which had been recorded on the Level of Anesthesia Evaluation Forms (Appendix A), a second program was used to extract a 64s EEG segment from each location and then to copy each extracted segment into a separate file on a new digital tape. Thus, each file on the new tape contained a digitized 64s EEG pattern sample corresponding to a known level of anesthesia.

After all EEG pattern samples had been extracted, they were visually screened in order to reject samples containing obvious and excessive artifact. The visual screening procedure was facilitated by reproducing all EEG pattern samples in analog form on standard EEG chart paper. To do this, as indicated in Fig. 2-3, the digitized EEG pattern
samples were first converted to analog form using the Nova 840 Signal Processing Facility: each digitized EEG pattern sample was read from a tape, demultiplexed and transferred to digital/analog converters. The resultant analog EEG samples were lowpass filtered at 30.0 Hz and were then recorded on the instrumentation tape recorder. By later connecting the recorder to an 8-channel EEG machine at the Vancouver General Hospital, the EEG pattern samples could be reproduced on standard EEG chart paper in a format suitable for visual screening.

An EEG pattern sample was usually rejected if it contained more than 10s of visually apparent artifact in more than one channel. Major sources of visually recognizable artifact included interference from electrosurgical units in the operating rooms, poor electrode contacts, eyeblinks, electrocardiographic activity, movement and muscle activity. EEG pattern samples containing primarily low frequency artifact, e.g. movement artifact below 0.5 Hz, were not rejected because it was known that all data would again be highpass filtered (digitally) at 0.54 Hz before being analysed. EEG pattern samples containing small amounts of visually apparent artifact were not rejected in order to retain as large a data base as possible. Approximately 20 percent of the EEG pattern samples which were visually screened were rejected because of artifact.

Table 2-2 indicates the number of EEG pattern samples which were retained after visual screening. A total of 938 samples from 72 subjects and three types of anesthesia were retained for subsequent analysis. The screened EEG pattern samples associated with each type of anesthesia were transferred to the digital tapes listed in Table 2-2. In addition, the three disk files identified in Table 2-2 were used to store the following information about each EEG pattern sample: its location on the appropriate tape, its corresponding level of anesthesia and the identity of the patient.
from which it was obtained. The structure of data on the digital tapes and in the disk files is documented in Appendix B. In addition, Appendix B contains the listing for an input subroutine which can be used to transfer a specified EEG pattern sample from tape to a FORTRAN array.

Table 2-2 Description of Resulting EEG Data Base

<table>
<thead>
<tr>
<th>EEG Data Base Information</th>
<th>Type of Anesthesia</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Halothane</td>
</tr>
<tr>
<td>Number of EEG pattern samples:</td>
<td></td>
</tr>
<tr>
<td>Level 0</td>
<td>56</td>
</tr>
<tr>
<td>Level 1</td>
<td>37</td>
</tr>
<tr>
<td>Level 2</td>
<td>12</td>
</tr>
<tr>
<td>Level 3</td>
<td>125</td>
</tr>
<tr>
<td>Level 4</td>
<td>50</td>
</tr>
<tr>
<td>Total number of EEG pattern samples</td>
<td>280</td>
</tr>
<tr>
<td>Number of cases from which the samples were obtained</td>
<td>21</td>
</tr>
<tr>
<td>Rack number of the digital tape which contains the EEG pattern samples</td>
<td>RA0562</td>
</tr>
<tr>
<td>Name of disk file which contains labels for the EEG pattern samples</td>
<td>HS.I</td>
</tr>
</tbody>
</table>
CHAPTER III
DEVELOPMENT OF EEG PATTERN RECOGNITION SYSTEMS

3.1 EEG Pattern Recognition Systems

3.1.1 Basic Description

This chapter describes the initial development and performance evaluation of various systems for estimating the level of anesthesia by means of EEG pattern recognition. Fig. 3-1 contains a simple block diagram of an EEG pattern recognition system. The preprocessor transforms an EEG pattern sample into a form which allows meaningful features to be more easily extracted. The amplification, filtering and digitization of EEG pattern samples could all be considered to be examples of preprocessing. As indicated in Fig. 3-1, a feature extractor analyses each preprocessed sample and quantitatively evaluates it in terms of a specified set of features. For example, feature extraction might consist of the calculation of a power density spectrum for each preprocessed EEG pattern sample, followed by the evaluation of features such as the peak frequency and the relative energy in different frequency bands. Each set of extracted feature values is transferred to a classifier which employs some algorithm, in conjunction with stored data, to classify the corresponding EEG pattern sample into one of five possible classes, i.e. five possible levels of anesthesia.

There is no optimum procedure for selecting the best features to be used in discriminating among EEG pattern samples corresponding to different levels of anesthesia. However, in selecting features for specific pattern recognition problems, experience has shown that a few well chosen, heuristically derived features are usually better than a
larger number chosen more randomly. This is primarily because processing many features requires more computing time, more storage and more data for training a classifier [72,73]. Consequently, the EEG features considered in this research were restricted to a relatively small number of features which had an established clinical significance or which had previously been described as meaningful in the literature on automatic EEG analysis.

3.1.2 Development and Performance Evaluation

With the exception of highpass filtering, all preprocessing had been performed during the preparation of the sets of EEG pattern samples which are listed in Table 2-2. The calculation of EEG power spectra from the preprocessed pattern samples and the subsequent extraction of spectral features is described in section 3.2. A description of relevant time domain EEG measurements and the extraction of time domain features is given in section 3.3. Section 3.4 outlines the classification algorithm which was employed in all spectral and time domain EEG pattern recognition systems. The problem of estimating the performance of such systems is described in section 3.5.1; the development of two nonparametric techniques which provide particularly useful and efficient estimates of the performance of EEG pattern recognition systems is then described in sections 3.5.2 - 3.5.4. Results obtained
by using these techniques to estimate the performance of various spectral and time domain EEG pattern recognition systems are presented in section 3.6. Finally, in section 3.7, the significance of the results is discussed.

3.2 Spectral Feature Extraction

3.2.1 EEG Spectral Analysis

Spectral analysis of EEG activity only became a popular analytic technique after 1965, when the introduction of the Fast Fourier Transform algorithm made digital spectral analysis fast and economically feasible [21,74-76]. During the last decade EEG spectral analysis has been employed with mixed success in a wide variety of diagnostic investigations (e.g. [77-79]), monitoring studies (e.g. [31,80] and sleep research projects (e.g. [81-83]).

EEG spectral analysis treats the amplitude of spontaneous EEG activity as a random variable. If the EEG activity from one channel is denoted by \( x(t) \) then, if it is assumed that the underlying random process is ergodic ([84], pp. 343-344), the EEG power density spectrum (or more simply, the EEG spectrum) can be defined:

\[
S(f) = E\{|X(f)|^2\} \\
= E\{X(f)X^*(f)\} \\
= \lim_{T \to \infty} \frac{1}{T} [X(f)X^*(f)] \\
\tag{3.1}
\]

where \( X(f) \) denotes the Fourier transform of \( x(t) \) in the interval

\[-\frac{T}{2} \leq t \leq \frac{T}{2},\]

i.e.

\[
X(f) = \mathcal{F}[x(t)] \\
= \int_{-T/2}^{T/2} x(t)e^{-j2\pi ft} \, dt, \tag{3.2}
\]

and where \( X^*(f) \) denotes the complex conjugate of \( X(f) \) [76]. The
relevance to EEG spectral analysis of certain assumptions concerning the
stationarity and Gaussianity of the underlying random process will be
considered in Chapter IV.

At present, EEG spectra can be computed by three different
methods: digital bandpass filtering [86], Fourier transformation of
autocorrelation functions [87], or the Direct Method, i.e. direct Fourier
transformation with subsequent smoothing [21,88]. The Direct Method was
employed in the computation of all EEG spectra in this research because
it was found to be the fastest and most convenient of the three methods.

3.2.2 Computation of EEG Spectra

As described in section 2.4.2, all of the EEG pattern samples
listed in Table 2-2 had been lowpass filtered at 30.0 Hz and digitized at
128 samples/s. By considering every second sample value it was therefore
possible to analyse data with an effective sampling rate of 64 samples/s.
Assume that \( \{x_1, \ldots, x_N\} \) represents the set of samples obtained by sampling
one EEG channel at 64 samples/s for 64s, i.e. \( N = 4096 \). The discrete
Fourier transform of \( \{x_1, \ldots, x_N\} \) was computed as follows:

\[
T(f_k) = \sum_{\ell=1}^{N} x_\ell \exp\left(-\frac{2\pi ik(\ell-1)}{N}\right)
\]  

(3.3)

for \( k = 0,1,\ldots,(N/2) \), where \( T(f_k) \) is the kth complex coefficient of
the transform at the fundamental frequency

\[
f_k = \frac{k}{N\Delta t}
\]

\[
= \frac{k}{64} \text{ Hz}
\]  

(3.4)

since \( \Delta t = (1/64)\text{s} \), the sampling interval [89,90]. To remove any arti-
fact below 0.54 Hz, as mentioned in section 2.4.2, the data was highpass
filtered in the frequency domain:

\[
C(f_k) = H(f_k)T(f_k) \quad k = 0,1,\ldots,(N/2)
\]  

(3.5)
where
\[
H(f_k) = \begin{cases} 
0 & 0.0 \leq f_k < 0.50 \\
(f_k - 0.50)/0.8 & 0.50 \leq f_k < 0.58 \\
1 & 0.58 \leq f_k \leq 32.0 
\end{cases} \quad (3.6)
\]

From the filtered Fourier coefficients \( C(f_k) \) a periodogram was calculated:
\[
I(f_k) = \frac{\Delta t}{N} |C(f_k)|^2 \quad k = 0,1,\ldots,(N/2). \quad (3.7)
\]

To improve the statistical properties of the raw spectral estimates provided by (3.7), averaging was performed over adjacent frequencies by means of a spectral window \( G_i \) to yield the smoothed periodogram
\[
\bar{I}(f_k) = \sum_{i=-W}^{W} G_i I(f_{k-i}) \quad (3.8)
\]
where
\[
\sum_{i=-W}^{W} G_i = 1. \quad (3.9)
\]

\( G_i \) was chosen to be a rectangular window of width 15/64 Hz, i.e.
\[
G_i = \begin{cases} 
\frac{1}{2W+1} & i = -W,\ldots,W \\
0 & \text{otherwise} 
\end{cases} \quad (3.10)
\]

where \( W = 7 \). Finally, from (3.8) a smoothed EEG spectrum with spectral estimates at 0.125 Hz intervals from 0 - 32 Hz was computed:
\[
\hat{S}(f_m) = \frac{1}{8} \sum_{k=1}^{8} \bar{I}(f_{8m+k}) \quad (3.11)
\]
for
\[
\frac{(m-1)}{8} \leq f_m < \frac{m}{8}, \quad m = 1,\ldots,256. \quad (3.12)
\]

More detailed information concerning the computation of EEG spectra in this manner may be found elsewhere ([85], pp. 43-52). Appendix C contains a listing of the program which was used to compute EEG spectra by the method described above.
3.2.3 Spectral Feature Vectors

Table 3-1 contains a description of the set of features \( \{ \sigma_i \} \), \( 1 \leq i \leq 13 \), chosen for extraction from the spectra corresponding to each EEG pattern sample. It should be noted that, for the reasons given in section 3.1.1, only a relatively small number of features from two EEG channels were initially considered. These particular features were heuristically chosen after reviewing the literature on computer-based EEG spectral analysis (e.g. [77-83, 91]) and after considerable consultation with an academically well qualified and clinically experienced electroencephalographer\(^1\). From (3.11) - (3.12) and from the description of features given in Table 3-1, it is evident that

\[
\begin{align*}
\sigma_1 &= \frac{f_{b_1}}{\sum_{f_m = f_{a_1}}^{f_{b_1}} \hat{S}(f_m) \Delta f} \\
&= \frac{\Delta f}{256} \sum_{m=a_1}^{b_1} \hat{S}(f_m) \\
&= 0.125 \sum_{m=1}^{256} \hat{S}(f_m),
\end{align*}
\]

(3.13)

because \( \Delta f = 0.125 \), \( a_1 \) is the smallest integer greater than \( 8f_{a_1} \) and \( b_1 = 8f_{b_1} \). Knowing \( \sigma_1 \), the subset of features \( \{ \sigma_i \} \), \( 2 \leq i \leq 7 \), can be evaluated:

\[
\begin{align*}
\sigma_i &= \frac{100}{\sigma_1} \frac{f_{b_1}}{\sum_{f_m = f_{a_1}}^{f_{b_1}} \hat{S}(f_m) \Delta f} \\
&= \frac{12.5}{\sigma_1} \sum_{m=a_1}^{b_1} \hat{S}(f_m), \quad 2 \leq i \leq 7.
\end{align*}
\]

(3.14)

\(^1\)Dr. M.D. Low, Associate Professor of Neurology at the University of British Columbia and Director of the EEG Department at the Vancouver General Hospital.
Table 3-1 Description of Spectral Feature Set

<table>
<thead>
<tr>
<th>Spectral Feature $\sigma_i$</th>
<th>Frequency Range (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>i=</td>
<td>Channel</td>
</tr>
<tr>
<td>1</td>
<td>Total spectral energy</td>
</tr>
<tr>
<td>2</td>
<td>Relative energy: $\Delta$ band</td>
</tr>
<tr>
<td>3</td>
<td>Relative energy: $\theta$ band</td>
</tr>
<tr>
<td>4</td>
<td>Relative energy: $\alpha$ band</td>
</tr>
<tr>
<td>5</td>
<td>Relative energy: $\sigma$ band</td>
</tr>
<tr>
<td>6</td>
<td>Relative energy: $\beta_1$ band</td>
</tr>
<tr>
<td>7</td>
<td>Relative energy: $\beta_2$ band</td>
</tr>
<tr>
<td>8</td>
<td>Mean spectral frequency</td>
</tr>
<tr>
<td>9</td>
<td>Second moment</td>
</tr>
<tr>
<td>10</td>
<td>Peak intensity: $\alpha$ band</td>
</tr>
<tr>
<td>11</td>
<td>Peak frequency: $\alpha$ band</td>
</tr>
<tr>
<td>12</td>
<td>Peak intensity: $\alpha$ band</td>
</tr>
<tr>
<td>13</td>
<td>Peak frequency: $\alpha$ band</td>
</tr>
</tbody>
</table>

The features corresponding to the first and second moments of the spectrum, i.e.

$$\sigma_8 = \frac{1}{\sigma_1} \sum_{m=\sigma_1}^{b_1} \hat{S}(f_m)f_m\Delta f$$

and

$$\sigma_9 = \frac{1}{\sigma_1} \sum_{m=\sigma_1}^{b_1} \hat{S}(f_m)f_m^2\Delta f,$$

(3.15) (3.16)

can easily be computed. The value of $\sigma_8$ indicates the mean spectral frequency. The value of $\sigma_9$ is of interest because, assuming that the underlying random process is stationary and Gaussian with zero mean ([84], pp. 485-495), $\sigma_9$ is related to a popular time domain EEG feature: the mean EEG zero-crossing rate [92-94]. The remaining subset of features $\{\sigma_i\}$, $10 \leq i \leq 13$, can be quickly evaluated: for $i = 10$ (with spectral data from C4-02) and $i = 12$ (with spectral data from F4-C4), if
\[ S(f_m) > S(f_n) \quad \text{for} \quad \begin{cases} f_{a_1} \leq f_n \leq f_{b_1} \\ f_{a_1} \leq f_m \leq f_{b_1} \\ f_m \neq f_n \end{cases} \] (3.17)

then

\[ \sigma_i = \hat{S}(f_m) \] (3.18)

and

\[ \sigma_{i+1} = f_m. \] (3.19)

The extraction of spectral features proceeded in the following manner. First, EEG spectra were computed for all of the 938 pattern samples listed in Table 2-2. Then, for each pattern sample, the set of 13 features summarized in Table 3-1 was evaluated. Appendix D contains the listing of a program that was written to evaluate spectral features. The resultant 13-element feature vectors were stored for subsequent use in the development and evaluation of various pattern classifiers.

3.3 Time Domain EEG Feature Extraction

3.3.1 Time Domain EEG Analysis

It is known that EEG spectra will contain complete statistical information about the underlying random processes if the processes are stationary and Gaussian ([84], pp. 474-475). However it was initially suspected, and subsequently confirmed by the results in Chapter IV, that the assumptions of stationarity and Gaussianity are not generally valid. It was also known that visual EEG assessment is based primarily on the evaluation of time domain EEG features, not spectral features [22,95]. Therefore it was decided to develop EEG pattern recognition systems based on clinically relevant time domain features, so that their performance could be evaluated and compared to the performance of spectral pattern recognition systems.

After reviewing much of the literature on automatic time domain EEG analysis (e.g. [21,81, 92-94, 96]), and after discussions with Dr.
M.D. Low, it was decided that the clinically relevant features described in Table 3-2 would be extracted from EEG pattern samples. It should be noted that, as with spectral analysis, only two channels of EEG data (F4-C4 and C4-O2) were considered initially. Of the 10 features in the set \( \{ \tau_i \} \), \( 1 \leq i \leq 10 \), four are derived from a period analysis of EEG activity and six are derived from an amplitude analysis. If \( x(t) \) denotes the EEG activity from one channel then the mean zero-crossing rate is the average number of times per second that \( x(t) = 0 \). The mean zero-crossing rate of the time derivative corresponds to the average number of times per second that \( x(t) \) reaches an extremum, i.e. that

\[
\frac{d x(t)}{dt} = 0. \tag{3.20}
\]

All of the EEG amplitude features can be derived from \( p(x) \), the amplitude probability distribution of \( x(t) \): if

\[
m_1 = \int_{-\infty}^{\infty} x p(x) \, dx \tag{3.21}
\]

and

\[
m_n = \int_{-\infty}^{\infty} (x - m_1)^n p(x) \, dx, \quad n = 2, 3, 4 \tag{3.22}
\]

then the standard deviation of the amplitude

\[
\beta_0 = \left( m_2 \right)^{1/2}, \tag{3.23}
\]

the skewness

\[
\beta_1 = \frac{m_3}{(m_2)^{3/2}} \tag{3.24}
\]

and the excess of kurtosis

\[
\beta_2 = \frac{m_4}{(m_2)^2} - 3 \tag{3.25}
\]

are easily obtained [97]. The skewness feature indicates the relative asymmetry of \( p(x) \), i.e. in the case of a symmetrical distribution \( \beta_1 = 0 \); the excess of kurtosis indicates the relative flatness of \( p(x) \) in comparison to a Gaussian distribution, for which \( \beta_2 = 0 \) ([85], pp. 39-40).
Table 3-2 Description of Time Domain EEG Feature Set

<table>
<thead>
<tr>
<th>i=</th>
<th>Time Domain Feature $\tau_i$</th>
<th>Channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mean zero-crossing rate</td>
<td>F4-C4</td>
</tr>
<tr>
<td>2</td>
<td>Mean zero-crossing rate of first derivative</td>
<td>F4-C4</td>
</tr>
<tr>
<td>3</td>
<td>Standard deviation of amplitude</td>
<td>F4-C4</td>
</tr>
<tr>
<td>4</td>
<td>Skewness</td>
<td>F4-C4</td>
</tr>
<tr>
<td>5</td>
<td>Excess of kurtosis</td>
<td>F4-C4</td>
</tr>
<tr>
<td>6</td>
<td>Mean zero-crossing rate</td>
<td>C4-02</td>
</tr>
<tr>
<td>7</td>
<td>Mean zero-crossing rate of first derivative</td>
<td>C4-02</td>
</tr>
<tr>
<td>8</td>
<td>Standard deviation of amplitude</td>
<td>C4-02</td>
</tr>
<tr>
<td>9</td>
<td>Skewness</td>
<td>C4-02</td>
</tr>
<tr>
<td>10</td>
<td>Excess of kurtosis</td>
<td>C4-02</td>
</tr>
</tbody>
</table>

3.3.2 Time Domain Feature Vectors

This section outlines the procedure for evaluating individual pattern samples in terms of the feature set $\{\tau_i\}$, $1 \leq i \leq 10$, summarized in Table 3-2 and described in the previous section. Before any features were evaluated, EEG pattern samples were digitally filtered with

$$B(f) = \begin{cases} 
0 & 0 \leq f < 0.50 \\
(f-0.50)/0.8 & 0.50 \leq f < 0.58 \\
1 & 0.58 \leq f \leq f_{LP} \\
0 & f > f_{LP}
\end{cases}$$

(3.26)

to remove any artifact below 0.54 Hz and to remove high frequency EEG activity above $f_{LP}$ Hz which, in visual EEG assessment at least, often tended to obscure significant changes in time domain feature values. The different choices for $f_{LP}$ will be described in section 3.6.2. To illustrate the feature evaluation procedure let $\{x_1, \ldots, x_N\}$ denote the set of values obtained by digitizing the EEG activity from channel F4-C4 at 64 samples/s for $T = 64s$, i.e. $N = 4096$, and then bandpass filtering the digitized EEG
with $B(f)$ in (3.26). The mean zero-crossing rates of the EEG and its first derivative can be evaluated from the following equations:

$$\tau_1 = \frac{1}{2T} \sum_{k=1}^{N} \left[ 1 - \text{sgn}(x_{k+1} - x_k) \right]$$ \hspace{1cm} (3.27)

and

$$\tau_2 = \frac{1}{2T} \sum_{k=1}^{N} \left[ 1 - \text{sgn}(\Delta_{k+1} - \Delta_k) \right]$$ \hspace{1cm} (3.28)

where

$$\Delta_k = |x_{k+1} - x_k|.$$ \hspace{1cm} (3.29)

To evaluate the amplitude features defined in (3.23)-(3.25), the sample mean

$$\hat{m}_1 = \frac{1}{N} \sum_{k=1}^{N} x_k$$ \hspace{1cm} (3.30)

and higher order central moments

$$\hat{m}_n = \frac{1}{(N-1)} \sum_{k=1}^{N} (x_k - \hat{m}_1)^n \hspace{1cm} n = 2, 3, 4,$$ \hspace{1cm} (3.31)

are employed:

$$\tau_3 = \left( \frac{\hat{m}_2}{\hat{m}_3} \right)^{\frac{3}{2}}$$ \hspace{1cm} (3.32)

$$\tau_4 = \frac{\hat{m}_3}{(\hat{m}_2)^{3/2}}$$ \hspace{1cm} (3.33)

$$\tau_5 = \frac{\hat{m}_4}{(\hat{m}_2)^2} - 3.$$ \hspace{1cm} (3.34)

Similarly, features $\tau_6 - \tau_{10}$ can be evaluated using the sample EEG data from channel C4-02.

Appendix E contains the listing of a program that was written to evaluate EEG pattern samples in terms of the time domain features in Table 3-2. This program was used to prepare time domain feature vectors for all available pattern samples. The resultant feature vectors were stored for later use.
Fig. 3.2 summarizes the procedure for preparing spectral and time domain EEG feature vectors for subsequent use in classifier development and performance evaluation.

Fig. 3-2 Preparation of Spectral and Time Domain Feature Vectors

3.4 Classification Algorithm

A wide variety of algorithms have been developed to classify unknown pattern samples on the basis of a specified set of extracted feature values [24]. In EEG pattern recognition, many of the classifiers described in the literature have been heuristically derived and are based on ad hoc decision rules (e.g. [32, 79, 98]). Consequently the conditions under which such classifiers may be optimal are unknown, and meaningful comparisons of performance are often difficult or impossible.

Of the few EEG pattern classification algorithms which have a firm theoretical basis, the most popular is an algorithm based on linear discriminant analysis [99]. Under the assumptions that all sample feature values are from a multivariate normal population and that the feature
covariance matrices for the different classes are identical, this algorithm creates linear discriminant functions (in a stepwise manner) which can be used to classify unknown feature vectors \([77,100]\). However in many EEG applications the assumptions of normality and identical covariance matrices are obviously invalid (e.g. \([101]\)), thus affecting the optimality of the classifier and the accuracy of parametric performance estimates. Despite these and other problems, stepwise discriminant analysis is at present perhaps the most widely used EEG pattern classification algorithm (e.g. \([77,78,83,91,101]\)).

The classification algorithm chosen for this investigation makes only one assumption about the feature data: it is based on Bayes decision rule ([102], p. 13) under the assumption that all features are statistically independent. Although the algorithm has certain characteristics which indicate that it might be particularly appropriate for EEG pattern classification problems, apparently it has not been extensively studied in this context previous to this investigation. To explain the algorithm, let \((d_u, \theta_u)\) represent an observed EEG pattern sample from an unknown class: \(d_u\) is a row vector containing \(N\) feature values or measurements from the pattern sample and \(\theta_u\) is the label identifying the class to which the pattern sample belongs. The purpose of the classification algorithm is to decide on a value for \(\theta_u\). It is known that the observed feature vector \(d_u\) must belong to one of \(M\) possible classes \(C_0, \ldots, C_{M-1}\); in this problem \(M = 5\) and the five possible classes correspond to the five different levels of anesthesia.

The classification algorithm is based on the maximum likelihood principle, i.e. one asks which class (or level of anesthesia) was most likely to produce the observed sample vector \(d_u\) and decides \(\hat{\theta}_u = C_j\), 0 ≤ \(j\) ≤ \((M-1)\), if
By using Bayes Rule ([102], p.11) the a posteriori probabilities in (3.35) can be expressed in terms of conditional and a priori probabilities, e.g.

\[
P(C_m | d_u) = \frac{P(d_u | C_m)P(C_m)}{P(d_u)}.
\]

(3.36)

Therefore, using (3.36), the decision rule in (3.35) becomes: decide

\[\hat{u} = C_j \text{ if } \]

\[
P(d_u | C_j)P(C_j) > P(d_u | C_m)P(C_m)
\]

or

\[
P(d_u | C_j)P(C_j) > P(d_u | C_m)P(C_m)
\]

for \(m=0,...,M-1\) \(\{m\neq j\} \).

The amounts of storage, computation time and training data required to implement (3.38) are greatly reduced [24,72,73] if it is assumed that the vector components \(d_{un}\), \(1 \leq n \leq N\), are statistically independent of one another, i.e. if it is assumed that

\[
P(d_u | C_m) = \prod_{n=1}^{N} P(d_{un} | C_m).
\]

(3.39)

Under this assumption, and after taking logarithms of both sides, the decision rule in (3.38) becomes: decide \(\hat{u} = C_j \) if

\[
R_j > R_m \quad \{m = 0,...,M-1\}
\]

(3.40)

where

\[
R_m = \sum_{n=1}^{N} \ln[P(d_{un} | C_m)] + \ln[P(C_m)].
\]

(3.41)

This classification rule minimizes the probability of an error when the features are statistically independent and when \(P(d_{un} | C_m)\) and \(P(C_m)\) in (3.41) are either known exactly or estimated using Bayes estimation procedure [103]. Assume that a total of \(S\) pattern samples are available for estimating the
probability distributions and that the kth pattern sample is represented by 
\((d_k, \theta_k), 1 \leq k \leq S\), where \(d_k\) is the extracted feature vector and \(\theta_k\) is the 
label which identifies the corresponding level of anesthesia. If each of 
the N feature measurements is scaled and quantized to some value \(\ell\), 
\(1 \leq \ell \leq L\), then Bayes estimates of \(P(d_{\text{un}} = \ell | C_m)\) and \(P(C_m)\), denoted by 
\(\hat{P}(d_{\text{un}} = \ell | C_m)\) and \(\hat{P}(C_m)\) respectively, are given by

\[
\hat{P}(d_{\text{un}} = \ell | C_m) = \frac{q_{n/m}^\ell + 1}{s_m + L}
\]

and

\[
\hat{P}(C_m) = \frac{s_m + 1}{s + M}. \tag{3.43}
\]

In (3.42) \(q_{n/m}^\ell\) denotes the number of available pattern samples belonging 
to class \(C_m\) in which \(d_{kn} = \ell\), while \(s_m\) in (3.43) denotes the total number 
of available pattern samples belonging to \(C_m\), i.e.

\[
q_{n/m}^\ell = \sum_{k=1}^{S} g(d_{kn}, \theta_k, \ell, C_m) \tag{3.44}
\]

and

\[
s_m = \sum_{k=1}^{S} f(\theta_k, C_m) \tag{3.45}
\]

where

\[
g(d_{kn}, \theta_k, \ell, C_m) = \begin{cases} 
1 & \text{if } \theta_k = C_m \text{ and } d_{kn} = \ell \\
0 & \text{otherwise.}
\end{cases} \tag{3.46}
\]

and

\[
f(\theta_k, C_m) = \begin{cases} 
1 & \text{if } \theta_k = C_m \\
0 & \text{otherwise.}
\end{cases} \tag{3.47}
\]

In general, the assumption that the features are statistically 
independent may not be valid. However the performance of a classifier 
based on Bayes decision rule, under the assumption of statistically inde-
dependent features, does provide a bound on the performance that would 
be possible if any existing feature interdependence could be exploited.
This follows from the argument that an invalid assumption regarding the feature probability distributions cannot increase the probability that unknown pattern samples will be correctly classified.

3.5 Evaluation of System Performance

3.5.1 The Performance Estimation Problem

The criterion usually adopted for assessing the overall performance of a pattern recognition system is its probability of misclassification error, denoted here by $P_e$. If the system preprocessor and feature extractor have been specified, then evaluating the performance of the system is equivalent to evaluating the performance of the pattern classifier. However, $P_e$ for the classifier is not readily evaluated. Assume that the set of available pattern samples $\{d, \theta\}$ contains a total of $S$ pattern samples from $J$ subjects, where each pattern sample consists of an extracted feature vector and a label which identifies the corresponding anesthesia level. If the complete set $\{d, \theta\}$ is used to train the pattern classifier, i.e. to estimate $P(d_{\text{un}} | C_m)$ and $P(C_m)$ in (3.41) using (3.42)-(3.47), then $P_e$ is defined as the probability that future pattern samples will be incorrectly classified. Obviously $P_e$ cannot be evaluated because, by definition, all available pattern samples would be used for training the classifier and none would be left for testing its performance. Hence, as depicted in Fig. 3-3, some technique must be employed to estimate $P_e$ on the basis of the set of available pattern samples.

3.5.2 Performance Estimation Techniques

Several parametric and nonparametric methods have been developed to estimate $P_e$ for different types of classifiers on the basis of a finite set of pattern samples [25]. However, only a few of these methods are appropriate for estimating the performance of EEG pattern classifiers.
A method of performance estimation that is appropriate for EEG classifiers should possess some or all of the following characteristics. First, it should be a nonparametric method because, in general, little is known about the underlying nature of the feature distributions. Second, the method should make efficient use of the available pattern samples because in most EEG pattern recognition investigations the set of available pattern samples is relatively small. Third, the method should yield an estimate of $P_e$ that is as unbiased as possible, i.e. an estimate that is neither overly optimistic nor overly pessimistic [104]. Finally, it should provide
an indication of the variability of the estimate: it is important to have some indication of the extent to which the performance of the classifier will be affected by the normal range of variability among pattern samples. It appears that the major source of variability in small sets of EEG pattern samples is due to differences in EEG characteristics among different subjects, i.e., intersubject EEG variation. Some early investigations of EEG pattern recognition systems indicated that intersubject EEG variation apparently had a significant effect on the performance of such systems [77,105]. These findings were supported by some initial results which were obtained in the course of this research [37]. Therefore, it was concluded that a satisfactory method of performance estimation should also be capable of providing an indication of the expected effect of intersubject EEG variation on classifier performance.

No single, existing method of performance estimation was found to satisfy all of the above requirements. However, two nonparametric techniques were formulated which, together, satisfied many of the above requirements and provided particularly useful estimates of the performance of EEG pattern classifiers. Because these techniques were based on two popular nonparametric methods of performance estimation, known in the literature as the II method [106,107] and the U method [108,109], they will subsequently be referred to in this thesis as the \( \Pi^* \) technique and the \( U^* \) technique, respectively. The \( \Pi^* \) technique, to be described in section 3.5.3, produces an estimate of \( P_e \) which indicates the expected performance of the classifier on future EEG data from a population of subjects. The \( U^* \) technique, to be described in section 3.5.4, produces an estimate of \( P_e \) which indicates the expected performance of the classifier on future EEG data from only one subject, or the performance that would be possible across a
subject population if the effect of intersubject EEG variation could somehow be eliminated.

To permit concise descriptions of the $II^*$ and $U^*$ techniques in the following sections, let the set of $S$ available pattern samples from $J$ subjects be partitioned into $J$ mutually exclusive sets, denoted by

$$\{d_s, \theta\}^1, \{d_s, \theta\}^2, \ldots, \{d_s, \theta\}^J,$$

where each set corresponds to the pattern samples obtained from one subject. Then

$$\{d_s, \theta\}^j \triangleq \{d^j_{1}, \theta^j_{1}; \ldots; d^j_{p(j)}, \theta^j_{p(j)}\}$$

for $j = 1, \ldots, J$ where $d^j_k$ and $\theta^j_k$ denote, respectively, the feature vector and the label of the $k$th pattern sample from the $j$th subject, and

$$\sum_{j=1}^{J} p(j) = S, \quad (3.49)$$

i.e. $p(j)$ denotes the number of available pattern samples from the $j$th subject.

### 3.5.3 The $II^*$ Technique

Let the estimate of $P_e$ produced by the $II^*$ technique be denoted by $\hat{P}_e[II^*]$. Then the $II^*$ technique for estimating classifier performance can be conveniently described by the following algorithm.

1) Set aside $\{d_s, \theta\}^j$, the set of pattern samples from the $j$th subject, for testing the classifier.

2) Train the classifier on all pattern samples from the $J-1$ remaining sets, i.e. from

$$\{d_s, \theta\}^m \begin{cases} m = 1, \ldots, J \cr m \neq j. \end{cases}$$

3) Test the classifier on $\{d_s, \theta\}^j$ to obtain a proportion of
errors denoted by

\[ \hat{P}_{e \{\Pi^*\}} j = \frac{1}{P(j)} \sum_{k=1}^{j} e^j_k \]  

(3.50)

where

\[ e^j_k = \begin{cases} 0 & \text{if } d^j_k \text{ is correctly classified} \\ 1 & \text{otherwise,} \end{cases} \]  

(3.51)

i.e. \( e^j_k \) acts as an error indicator.

4) Repeat steps 1)-3) for \( j = 1, \ldots, J \) to obtain the proportions of errors \( \hat{P}_{e \{\Pi^*\}} j \) for \( j = 1, \ldots, J \).

5) The \( \Pi^* \) estimate of \( P_e \) can then be computed:

\[ \hat{P}_{e \{\Pi^*\}} = \sum_{j=1}^{J} P(j) \hat{P}_{e \{\Pi^*\}} j. \]  

(3.52)

Appendix F contains the listing of a program that was written to compute \( \hat{P}_{e \{\Pi^*\}} \) for the classifier described in section 3.4. The program can accommodate up to 500 pattern samples, i.e. up to 500 spectral or time domain feature vectors and their labels. The program allows the classifier's feature quantization scheme to be varied, and also permits the a priori class probabilities \( P(C_m) \) in equation (3.41) to be assumed equal or to be estimated by (3.43).

3.5.4 The \( U^* \) Technique

Let the estimate of \( P_e \) produced by the \( U^* \) technique be denoted by \( \hat{P}_{e \{U^*\}} \). Then \( \hat{P}_{e \{U^*\}} \) for a given classifier can be computed by means of the following algorithm:

1) Consider only the set of pattern samples from the \( j \)th subject, i.e. \( \{d, \theta\}^J_j \) for \( 1 \leq j \leq J \).

2) Take out the \( k \)th pattern sample \( (d^j_k, \theta^j_k) \) and then define

\[ \{d, \theta\}^j_k \triangleq \{d^j_1, \theta^j_1; \ldots; d^j_{k-1}, \theta^j_{k-1}; d^j_{k+1}, \theta^j_{k+1}; \ldots; d^j_J, \theta^j_J\}. \]  

(3.53)
3) Train the classifier on \( \{d, \theta_j^1\} \).

4) Test the classifier on \( \{d_k^j, \theta_k^j\} \) and use \( e_k^j \) for an error indicator, as in equation (3.51).

5) Repeat steps 2)-4) for \( k = 1, \ldots, p(j) \) to obtain \( e_k^j \) values for some fixed \( j \) and for \( k = 1, \ldots, p(j) \).

6) Repeat steps 1)-5) for \( j = 1, \ldots, J \). Thus \( e_k^j \) values are obtained for all \( j = 1, \ldots, J \) and \( k = 1, \ldots, p(j) \).

7) The \( \hat{U}^* \) estimate of \( P_e \) is then computed in the following manner:

\[
\hat{P}_e[\hat{U}^*] = \frac{1}{S} \sum_{j=1}^{J} \sum_{k=1}^{p(j)} e_k^j.
\] (3.54)

The program listed in Appendix G was written to compute \( \hat{P}_e[\hat{U}^*] \) for the classifier described in section 3.4. The only exception to the above algorithm was in the case of very small sets \( \{d, \theta\}^j \): the classifier was not tested on \( \{d_k^j, \theta_k^j\} \) if the training set \( \{d, \theta\}^j \) did not include at least one pattern sample from the level corresponding to \( \theta_k^j \). As with the \( \Pi^* \) performance estimation program, the \( \hat{U}^* \) performance estimation program listed in Appendix G permits changes in the classifier's feature quantization scheme and \textit{a priori} class probability assignments, and can accommodate up to 500 pattern samples.

3.6 Results

3.6.1 EEG Spectral Pattern Recognition Systems

The \( \Pi^* \) and \( \hat{U}^* \) techniques described in the previous sections were used to estimate the performance of various EEG spectral pattern recognition systems. To simplify the description of these different systems, it should be recalled that all EEG pattern recognition systems can be regarded as consisting of the three basic elements depicted in
Fig. 3-1: A preprocessor, a feature extractor and a classifier. In all of the spectral pattern recognition systems which were considered, the preprocessor and feature extractor remained unchanged. Therefore, the different systems varied only in the structure of their classifiers.

As stated in section 3.1.1, the basic function of a system preprocessor is to transform an EEG sample into a form which allows features to be more easily extracted. The preprocessor chosen for all spectral pattern recognition systems consisted of an amplifier to increase EEG amplitudes to convenient levels, a bandpass filter (0.54–30.0 Hz) to reduce artifact, and a digitizer to convert each amplified and filtered EEG sample to digital form. The spectral feature extractor had two functions: the computation of spectra corresponding to each preprocessed 64s EEG sample and the subsequent evaluation of the 13 spectral features listed in Table 3-1. In the feature extractor, the spectra were to be computed in the manner outlined in section 3.2.2 and the spectral features were to be evaluated as described in section 3.2.3.

Fig. 3-3 shows the basic configuration of the classifier employed in all spectral pattern recognition systems. It consists of a feature quantizer, a decision device and a memory for storing estimates of the class-conditional feature probabilities and the a priori class probabilities. The feature quantizer was linear in all systems, but the quantization range and the number of possible quantization levels were changed to study their possible effect on performance.

The different quantization ranges were defined in terms of a specified maximum number of standard deviations from the mean feature values, where the means and standard deviations were calculated from the available training data. The decision rule that was described in section 3.4 constituted the "decision device" shown in Fig. 3-3. In some
classifiers the *a priori* class probabilities $P(C_m)$ were estimated by the Bayes probability estimates defined in equation (3.43). For comparative purposes, similar classifiers were also considered in which the *a priori* class probabilities were assumed to be equal.

The performance of each different spectral pattern recognition system was estimated by the $\Pi^*$ and $U^*$ techniques. These techniques made use of the three sets of available spectral feature vectors, corresponding to the three types of anesthesia, which had been prepared as described in section 3.2.3. Estimating the overall performance of a system on the basis of a set of available pattern samples was therefore equivalent to estimating the performance of the system's classifier on the basis of the corresponding set of spectral feature vectors, because all preprocessing and feature extraction operations had already been performed on the EEG samples during the preparation of the feature vectors. The results obtained for many of the spectral pattern recognition systems which were developed for halothane anesthesia, narcotic anesthesia and enflurane anesthesia are summarized in Tables 3-3, 3-4 and 3-5, respectively. As stated in section 3.5.2, the estimate of misclassification error probability provided by the $\Pi^*$ technique, i.e. the value of $\hat{P}_e[\Pi^*]$, indicates the expected performance of the system on future EEG data from a population of subjects. Alternatively, the $U^*$ performance estimate ($P_e[U^*]$) for the same system indicates its expected performance on future EEG data from only one subject, or the performance that would be possible across a subject population if the effect of intersubject EEG variation could somehow be eliminated.

The best spectral pattern recognition system among those compared in Table 3-3, Table 3-4 or Table 3-5 was considered to be the one which minimized the mean of the two estimates of error probability, i.e.
the one which minimized

\[
\hat{P}_e[\text{Mean}] \triangleq \frac{1}{2} \left\{ \hat{P}_e[\Pi^*] + \hat{P}_e[U^*] \right\}.
\]  

(3.55)

Table 3-3 Performance of Spectral Pattern Recognition Systems on EEG Data from Halothane Anesthesia

<table>
<thead>
<tr>
<th>Feature Quantizer</th>
<th>Number of Levels</th>
<th>Type of (P(C_m)) Estimates Employed</th>
<th>Estimated Error Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pm 5.0 \text{ sd})</td>
<td>16</td>
<td>Equal</td>
<td>(P_e[\Pi^*] = 0.454)</td>
</tr>
<tr>
<td>(\pm 5.0 \text{ sd})</td>
<td>32</td>
<td>Equal</td>
<td>(P_e[\Pi^*] = 0.393)</td>
</tr>
<tr>
<td>(\pm 5.0 \text{ sd})</td>
<td>64</td>
<td>Equal</td>
<td>(P_e[\Pi^<em>] = 0.389^</em>)</td>
</tr>
<tr>
<td>(\pm 5.0 \text{ sd})</td>
<td>128</td>
<td>Equal</td>
<td>(P_e[\Pi^*] = 0.471)</td>
</tr>
<tr>
<td>(\pm 5.0 \text{ sd})</td>
<td>64</td>
<td>Bayes</td>
<td>(P_e[\Pi^*] = 0.404)</td>
</tr>
<tr>
<td>(\pm 1.0 \text{ sd})</td>
<td>64</td>
<td>Equal</td>
<td>(P_e[\Pi^*] = 0.475)</td>
</tr>
<tr>
<td>(\pm 50.0 \text{ sd})</td>
<td>64</td>
<td>Equal</td>
<td>(P_e[\Pi^*] = 0.396)</td>
</tr>
</tbody>
</table>

Table 3-4 Performance of Spectral Pattern Recognition Systems on EEG Data from Narcotic Anesthesia

<table>
<thead>
<tr>
<th>Feature Quantizer</th>
<th>Number of Levels</th>
<th>Type of (P(C_m)) Estimates Employed</th>
<th>Estimated Error Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pm 5.0 \text{ sd})</td>
<td>16</td>
<td>Equal</td>
<td>(\hat{P}_e[\Pi^*] = 0.463)</td>
</tr>
<tr>
<td>(\pm 5.0 \text{ sd})</td>
<td>32</td>
<td>Equal</td>
<td>(\hat{P}_e[\Pi^*] = 0.460)</td>
</tr>
<tr>
<td>(\pm 5.0 \text{ sd})</td>
<td>64</td>
<td>Equal</td>
<td>(\hat{P}_e[\Pi^<em>] = 0.449^</em>)</td>
</tr>
<tr>
<td>(\pm 5.0 \text{ sd})</td>
<td>128</td>
<td>Equal</td>
<td>(\hat{P}_e[\Pi^*] = 0.519)</td>
</tr>
<tr>
<td>(\pm 5.0 \text{ sd})</td>
<td>64</td>
<td>Bayes</td>
<td>(\hat{P}_e[\Pi^*] = 0.478)</td>
</tr>
<tr>
<td>(\pm 1.0 \text{ sd})</td>
<td>64</td>
<td>Equal</td>
<td>(\hat{P}_e[\Pi^*] = 0.490)</td>
</tr>
<tr>
<td>(\pm 50.0 \text{ sd})</td>
<td>64</td>
<td>Equal</td>
<td>(\hat{P}_e[\Pi^*] = 0.481)</td>
</tr>
</tbody>
</table>
Table 3-5 Performance of Spectral Pattern Recognition Systems on EEG Data from Enflurane Anesthesia

<table>
<thead>
<tr>
<th>Feature Quantizer</th>
<th>Type of Estimates Employed</th>
<th>Estimated Error Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>Number of Levels</td>
<td>( P_e [\Pi^*] )</td>
</tr>
<tr>
<td>± 5.0 sd</td>
<td>16</td>
<td>Equal</td>
</tr>
<tr>
<td>± 5.0 sd</td>
<td>32</td>
<td>Equal</td>
</tr>
<tr>
<td>± 5.0 sd</td>
<td>64</td>
<td>Equal</td>
</tr>
<tr>
<td>± 5.0 sd</td>
<td>128</td>
<td>Equal</td>
</tr>
<tr>
<td>± 5.0 sd</td>
<td>64</td>
<td>Bayes</td>
</tr>
<tr>
<td>± 1.0 sd</td>
<td>64</td>
<td>Equal</td>
</tr>
<tr>
<td>±50.0 sd</td>
<td>64</td>
<td>Equal</td>
</tr>
<tr>
<td>± 5.0 sd</td>
<td>16</td>
<td>Bayes</td>
</tr>
<tr>
<td>± 1.0 sd</td>
<td>16</td>
<td>Equal</td>
</tr>
<tr>
<td>±50.0 sd</td>
<td>16</td>
<td>Equal</td>
</tr>
</tbody>
</table>

In Table 3-3, Table 3-4 and Table 3-5 the best system is identified with asterisks. Accordingly, from the results in Table 3-3, the best spectral pattern recognition system developed for halothane anesthesia can be expected to classify between 61.1 percent and 89.2 percent of future EEG samples correctly. This system has a linear feature quantizer with 64 possible quantization levels over a range of ±5.0 sd (standard deviations) and employs equal a priori class probability estimates. From the results in Table 3-4 it is evident that the best spectral pattern recognition system for narcotic anesthesia has the same feature quantization scheme and uses the same probability estimates. However, its performance is slightly inferior: it can only be expected to correctly classify between 55.1
percent and 78.9 percent of future EEG samples. Finally, the results in Table 3-5 indicate that between 58.7 percent and 87.8 percent of future EEG samples from enflurane anesthesia will be correctly classified by the best spectral pattern recognition system. This system employs equal class probability estimates, as do the best systems for halothane and narcotic anesthesia, but has a feature quantizer with only 16 possible quantization levels over a range of ±50.0 sd.

3.6.2 Time Domain EEG Pattern Recognition Systems

In addition to spectral pattern recognition systems, various systems based on the recognition of time domain EEG patterns were developed. As stated in section 3.3.1, these systems were investigated because it was suspected that the conditions under which some form of spectral pattern recognition system would be optimal were not satisfied, and because it was known that visual EEG assessment is based primarily on the evaluation of time domain EEG features, not spectral features. Time domain EEG pattern recognition systems were therefore developed so that their performance could be estimated and compared to the estimated performance of spectral pattern recognition systems.

The structure of all time domain EEG pattern recognition systems which were considered was similar to the structure of the spectral pattern recognition systems described in section 3.6.1. Both consisted of the three basic elements shown in Fig. 3-1: a preprocessor, a feature extractor and a classifier. The preprocessors in all time domain systems were identical to the preprocessors in spectral pattern recognition systems, with the following exception: instead of a band-pass filter from 0.54–30.0 Hz, the filter defined in (3.26) was employed and the lowpass filter frequency \( f_{LP} \) was set at 8.0, 16.0, 24.0 and 30.0
Hz in different systems to study the effect of prefiltering on system performance. As mentioned in section 3.3.2, this additional prefiltering was performed to eliminate high frequency EEG activity above $f_{LP}$ Hz which, at least in visual EEG assessment, seemed to obscure significant changes in time domain feature values.

The function of the feature extractor employed in all time domain EEG pattern recognition systems was to evaluate each preprocessed 64s EEG sample in terms of the set of 10 time domain features listed in Table 3-2, so that the EEG sample could subsequently be classified on the basis of the set of extracted feature values. The classifiers in all time domain systems had the same basic structure as the classifiers in spectral systems, consisting of a feature quantizer, a decision device and a memory for storing estimates of the relevant probability distributions, as indicated in Fig. 3-3. The quantization range and the number of possible quantization levels were changed in different systems, in the manner described in section 3.6.1, in an attempt to establish the best linear feature quantization scheme. An implementation of the decision rule described in section 3.4 constituted the "decision device" in all time domain system classifiers. For comparative purposes the a priori class probabilities were assumed to be equal in some systems, while the Bayes probability estimates defined in (3.43) were employed in other systems.

Using the sets of available pattern samples from halothane anesthesia, narcotic anesthesia and enflurane anesthesia, estimates of the misclassification error probability for various time domain EEG pattern recognition systems were obtained by the $\Pi^*$ and $\Sigma^*$ techniques described in section 3.5.3 and section 3.5.4, respectively. The resulting values of $\hat{P}_e[\Pi^*]$ and $\hat{P}_e[\Sigma^*]$ are presented in Table 3-6, Table 3-7 and
Table 3-6 Performance of Time Domain Pattern Recognition Systems on EEG Data from Halothane Anesthesia

<table>
<thead>
<tr>
<th>Lowpass Prefilter Frequency $f_{LP}$ (Hz)</th>
<th>Feature Quantizer</th>
<th>Estimated Error Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Range</td>
<td>Number of Levels</td>
</tr>
<tr>
<td>8.0</td>
<td>± 5.0 sd</td>
<td>8</td>
</tr>
<tr>
<td>8.0</td>
<td>± 5.0 sd</td>
<td>16</td>
</tr>
<tr>
<td>8.0</td>
<td>± 5.0 sd</td>
<td>32</td>
</tr>
<tr>
<td>16.0</td>
<td>± 5.0 sd</td>
<td>8</td>
</tr>
<tr>
<td>16.0</td>
<td>± 5.0 sd</td>
<td>16</td>
</tr>
<tr>
<td>16.0</td>
<td>± 5.0 sd</td>
<td>32</td>
</tr>
<tr>
<td>24.0</td>
<td>± 5.0 sd</td>
<td>8</td>
</tr>
<tr>
<td>24.0</td>
<td>± 5.0 sd</td>
<td>16</td>
</tr>
<tr>
<td>24.0</td>
<td>± 5.0 sd</td>
<td>32</td>
</tr>
<tr>
<td>16.0</td>
<td>± 1.0 sd</td>
<td>8</td>
</tr>
<tr>
<td>16.0</td>
<td>± 50.0 sd</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 3-8. Because their performance was consistently better, only systems which employ equal, rather than Bayes, a priori class probability estimates are described in these three tables. The best system among those presented in each table was considered to be the one which minimized the mean of the two error estimates, i.e. the one which minimized (3.55). The best system in each of the three tables is identified with asterisks. From the results presented in Table 3-6, the best time domain EEG pattern recognition system developed for halothane anesthesia can be expected to classify between 51.4 percent and 87.3 percent of future EEG samples correctly. In this system the lowpass filter frequency $f_{LP}$ is 16.0 Hz and the feature quantizer has 8 possible quantization levels extending over a range of 50.0 sd. The results in Table 3-7 indicate that between
### Table 3-7 Performance of Time Domain Pattern Recognition Systems on EEG Data from Narcotic Anesthesia

<table>
<thead>
<tr>
<th>Lowpass Prefilter Frequency $f_{LP}$ (Hz)</th>
<th>Feature Quantizer</th>
<th>Estimated Error Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Range</td>
<td>Number of Levels</td>
</tr>
<tr>
<td>8.0</td>
<td>± 5.0 sd</td>
<td>8</td>
</tr>
<tr>
<td>8.0</td>
<td>± 5.0 sd</td>
<td>16</td>
</tr>
<tr>
<td>8.0</td>
<td>± 5.0 sd</td>
<td>32</td>
</tr>
<tr>
<td>16.0</td>
<td>± 5.0 sd</td>
<td>8</td>
</tr>
<tr>
<td>16.0</td>
<td>± 5.0 sd</td>
<td>16</td>
</tr>
<tr>
<td>16.0</td>
<td>± 5.0 sd</td>
<td>32</td>
</tr>
<tr>
<td>24.0</td>
<td>± 5.0 sd</td>
<td>8</td>
</tr>
<tr>
<td>24.0</td>
<td>± 5.0 sd</td>
<td>16</td>
</tr>
<tr>
<td>24.0</td>
<td>± 5.0 sd</td>
<td>32</td>
</tr>
<tr>
<td>8.0</td>
<td>± 1.0 sd</td>
<td>8</td>
</tr>
<tr>
<td>8.0</td>
<td>± 50.0 sd</td>
<td>8</td>
</tr>
</tbody>
</table>

49.6 percent and 68.0 percent of future EEG samples from narcotic anesthesia could be correctly classified by the best time domain EEG pattern recognition system. The lowpass filter frequency $f_{LP}$ is 8.0 Hz in the preprocessor of this system and the feature quantizer has 8 possible quantization levels over a range of ±5.0 sd. Finally, from the results in Table 3-8, the best time domain EEG pattern recognition system developed for enflurane anesthesia can be expected to correctly classify between 62.8 percent and 89.8 percent of future EEG samples. In this system the feature quantizer has 8 possible quantization levels extending over a range of ±5.0 sd and, in contrast to the best systems in Table 3-6 and Table 3-7, the lowpass filter frequency $f_{LP}$ is 30.0 Hz, i.e. the
Table 3-8 Performance of Time Domain Pattern Recognition Systems on EEG Data from Enflurane Anesthesia

<table>
<thead>
<tr>
<th>Lowpass Prefilter Frequency f_LP (Hz)</th>
<th>Feature Quantizer</th>
<th>Estimated Error Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Range</td>
<td>Number of Levels</td>
</tr>
<tr>
<td>8.0</td>
<td>± 5.0 sd</td>
<td>8</td>
</tr>
<tr>
<td>8.0</td>
<td>± 5.0 sd</td>
<td>16</td>
</tr>
<tr>
<td>8.0</td>
<td>± 5.0 sd</td>
<td>32</td>
</tr>
<tr>
<td>16.0</td>
<td>± 5.0 sd</td>
<td>8</td>
</tr>
<tr>
<td>16.0</td>
<td>± 5.0 sd</td>
<td>16</td>
</tr>
<tr>
<td>16.0</td>
<td>± 5.0 sd</td>
<td>32</td>
</tr>
<tr>
<td>24.0</td>
<td>± 5.0 sd</td>
<td>8</td>
</tr>
<tr>
<td>24.0</td>
<td>± 5.0 sd</td>
<td>16</td>
</tr>
<tr>
<td>24.0</td>
<td>± 5.0 sd</td>
<td>32</td>
</tr>
<tr>
<td>30.0</td>
<td>± 5.0 sd</td>
<td>8</td>
</tr>
<tr>
<td>30.0</td>
<td>± 5.0 sd</td>
<td>16</td>
</tr>
<tr>
<td>30.0</td>
<td>± 5.0 sd</td>
<td>32</td>
</tr>
<tr>
<td>30.0</td>
<td>± 1.0 sd</td>
<td>8</td>
</tr>
<tr>
<td>30.0</td>
<td>± 50.0 sd</td>
<td>8</td>
</tr>
</tbody>
</table>

elimination of high frequency EEG activity did not result in an improvement in system performance.

3.7 Discussion

3.7.1 Spectral and Time Domain EEG Pattern Recognition Systems

The primary objective of the work described in this chapter was to study the feasibility of estimating anesthesia levels by means of EEG
pattern recognition. It was assumed that the clinical, non-EEG criteria listed in Table 2-1 defined five clinically valid levels of anesthesia for patients during halothane, narcotic or enflurane anesthesia. The various spectral and time domain EEG pattern recognition systems described in sections 3.6.1 - 3.6.2 were developed in an attempt to reliably estimate the different levels of anesthesia, i.e. to agree with assessments made by anesthesiologists on the basis of the non-EEG criteria. Specifically, the function of each EEG pattern recognition system was to estimate the level of anesthesia by classifying an unknown EEG sample on the basis of a set of extracted feature values, corresponding to the spectral features listed in Table 3-1 or the time domain features listed in Table 3-2.

The performance of each EEG pattern recognition system on future data was evaluated, in terms of the estimated probability of misclassification error, by means of the \( \Pi^* \) technique and the \( U^* \) technique as described in sections 3.5.2 - 3.5.4. The resulting values of \( \hat{P}_e[\Pi^*] \) and \( \hat{P}_e[U^*] \) for various spectral and time domain EEG pattern recognition systems are summarized in Tables 3-3 to 3-8. In section 3.5.2 it was pointed out that the \( \Pi^* \) and \( U^* \) techniques provide particularly informative and efficient estimates of the performance of EEG pattern recognition systems. It is suspected that the values of \( \hat{P}_e[\Pi^*] \) and \( \hat{P}_e[U^*] \) obtained for a specific system could be decreased, i.e. performance could be improved, by increasing the number of EEG pattern samples available for training the system. This follows from the argument that the error between the actual feature probability distributions needed to evaluate the decision rule in (3.41) and the Bayes estimates of those distributions, defined in (3.42), can be expected to decrease with an increase in sample size.
The best systems among those compared in Tables 3-3 to 3-8 were considered to be the ones which minimized the mean error estimate defined in (3.55). It should be noted that all of the best systems in terms of this criterion employed equal, rather than Bayes, estimates of the \textit{a priori} class probabilities. Assuming that future EEG samples will be from \( M = 5 \) equiprobable classes, the performance of these systems can reasonably be compared to the expected performance of a completely random pattern classification system, for which

\[
\hat{p}_e[\Pi^*] = \hat{p}_e[U^*] = 1 - M^{-1} = 0.8, \quad (3.56)
\]

i.e. only 20 percent of future EEG samples would be correctly classified. In contrast, the results obtained for the best spectral pattern recognition systems (in Tables 3-3 to 3-5) indicate that between 61.1 - 89.2 percent, 55.1 - 78.9 percent and 58.7 - 87.8 percent of future EEG samples from, respectively, halothane anesthesia, narcotic anesthesia and enflurane anesthesia will be correctly classified. The best time domain EEG pattern recognition systems (in Tables 3-6 to 3-8) have slightly inferior performance compared to the best spectral systems for halothane and narcotic anesthesia, but slightly superior performance for enflurane anesthesia: it is expected that between 51.4 - 87.3 percent, 49.6 - 68.0 percent and 62.8 - 89.8 percent of future EEG samples from halothane, narcotic and enflurane anesthesia, respectively, could be correctly classified by these systems.

It would obviously be desirable to compare the results obtained for the best spectral and time domain EEG pattern recognition systems to the expected reliability of visual EEG assessment. Although an exact
comparison is not possible, the results of a recent investigation con­
cerning the reliability of visual EEG assessment, discussed previously
in Chapter I, are relevant. The results of this investigation indi­
cate that, even with an established set of objective criteria for pattern
identification, the reliability of visual EEG assessment may be surprisingly
low: the highest average intraclass correlation coefficient among seven
experienced clinical EEG raters was reported to be 0.56 [18].

One additional point concerning the results is worthy of con­
sideration. An IBM 370/168 computer was used to develop all EEG pattern
recognition systems and to estimate their performance. Consequently,
the differing amounts of memory and processing time required by the var­
ious systems were not apparent. However, these factors are of practical
significance since one would obviously prefer to use the smallest, least
expensive computer when actually implementing such a system for use on a
routine basis in a hospital environment. It was calculated that the im­
plementation of the best spectral pattern recognition systems would require
a computer with at least 8000 bytes of memory and an efficient version of
the Fast Fourier Transform (FFT) algorithm. In contrast, the best time
domain EEG pattern recognition systems would require approximately 50 per­
cent less memory and would be computationally faster and simpler, primarily
because the FFT would not be required.

3.7.2 Evaluation of EEG Pattern Recognition Approach

The results presented in this chapter have clearly demonstrated
the feasibility of obtaining reliable estimations of the level of anes­
thesia during surgical operations by means of computer-based EEG pattern
recognition. Perhaps the value of this approach, in relation to earlier
attempts to visually evaluate EEG activity during anesthesia, can best be
assessed in terms of the four methodological problems associated with earlier work which were discussed previously (section 1.2.4): the definition of anesthesia levels, the definition of EEG patterns, EEG pattern variability and the extent of inter-rater reliability.

In this research, the different levels of anesthesia were defined by a set of clinically valid, non-EEG criteria. Estimations of the level of anesthesia which were made on the basis of these criteria were assumed to be correct and EEG pattern recognition systems were then designed to agree with the estimations. Of course, any error introduced by the inability of anesthesiologists to consistently identify levels of anesthesia on the basis of the clinical criteria would obviously be incorporated into such systems. Attempts to control this possible source of error were described in sections 2.3.3 - 2.3.4.

The earlier difficulty associated with the reliability of EEG pattern definition was resolved in this research by explicitly defining sets of heuristically derived features so that EEG samples could be quantitatively evaluated in terms of these features. Although it was assumed that all features were statistically independent for computational simplicity, this assumption is not necessarily justified. As stated in section 3.4, it is therefore theoretically possible to develop a more reliable EEG pattern recognition system by exploiting any statistical dependence that may exist among features. However, taking statistical dependencies into account can easily prove to be a formidable task because of an exponential increase in the measurement complexity, where the measurement complexity refers to the total number of discrete probability values to be estimated [73]. For example, the measurement complexity $C$ of a set of $N$ statistically independent features, each of which can assume $L$ possible quantization values, is given by

$$C = L \cdot N; \quad (3.57)$$
however, if the features are interdependent the measurement complexity is given by
\[ C = L^N, \]
(3.58)
an enormous increase for reasonable values of L and N. This is significant because as a rule of thumb the amount of data required to adequately train a classifier, as well as the memory and computation time required in its subsequent utilization, is proportional to the measurement complexity of the feature data. In the event that many of the features are thought to be strongly interdependent the use of a different pattern recognition technique such as stepwise discriminant analysis, which does not assume statistically independent features, might prove to be more tractable. However, in the theoretical development of stepwise discriminant analysis other simplifying assumptions concerning the statistical properties of the feature set are made which are also not necessarily valid.

The previously encountered methodological problems associated with the variability of EEG patterns among different anesthetic agents and different patients for the same level of anesthesia were reduced in three ways. First, only three specified combinations of anesthetic agents were considered in this initial investigation. In addition, only healthy adult patients in the best surgical risk categories were selected as subjects. Finally, because the EEG pattern recognition systems were developed by processing all available training data and storing the extracted feature values, no simplifying assumptions concerning the underlying feature distributions were necessary. However the fact that the available data base is relatively small, corresponding to a limited number of patients, means that EEG pattern variability must still be regarded as a major potential source of variability in the performance of EEG pattern recognition systems which were trained on the available set of pattern samples.
Obviously, the inter-rater reliability problems which were evident in earlier studies based on visual EEG assessment are effectively eliminated by the computer-based EEG pattern recognition approach. Once one reliable EEG pattern recognition system has been developed, other replicas can easily be produced to provide consistent and continuous estimations of the level of anesthesia during surgery.

3.7.3 Further Work

Aside from the factors already mentioned, the performance of the EEG pattern recognition systems described in this chapter could have been affected by invalid assumptions concerning the underlying statistical characteristics of the EEG data, by the limited number and type of features extracted, by the presence of undetected artifact in EEG samples and by a marked degree of EEG pattern variability and intersubject EEG variation. Each of these factors should be investigated further with a view to improving system performance.

In Chapter IV, for example, some relevant statistical characteristics of spontaneous EEG activity will be investigated. Specifically, it would be useful to know over what time interval (if any) the EEG can be considered to be a sample function from a stationary, or at least a wide-sense stationary, random process. In addition, it would be potentially useful to have an indication of the extent to which a sample EEG amplitude distribution deviates from a Gaussian distribution. With such information an appropriate EEG source model could be generated and subsequently employed in developing improved computer-based systems for monitoring the level of anesthesia.

All systems considered in this chapter were based on the extraction of relatively small sets of spectral or time domain features. These
features were chosen either because they had an established clinical significance or because they had previously been described as meaningful in the literature on automatic EEG analysis. The extraction of additional, heuristically derived features to improve the performance of specific EEG pattern recognition systems will be considered in Chapter V. Alternatively, although beyond the scope of this thesis, the use of statistical feature selection techniques (e.g. [110]) to choose a small set of good features from a large number of more randomly chosen ones might also be explored.

As stated in section 3.4, it is theoretically possible to develop more reliable EEG pattern recognition systems by exploiting any statistical interdependencies that may exist among spectral or time domain EEG features. In Chapter V the magnitude of any interdependencies, or at least the magnitude of any intercorrelations, that may exist among spectral features will be investigated. Also in Chapter V, methods for reducing the effect of intersubject EEG variation on the performance of EEG pattern recognition systems will be explored.

Finally, instead of considering only spontaneous EEG pattern recognition systems, the possibility of developing systems which are based on the recognition of different sensory evoked responses during anesthesia (e.g., see [15]) might also be considered.
CHAPTER IV

MODELLING THE STATIONARITY AND GAUSSIANITY OF EEG ACTIVITY

4.1 Introduction

4.1.1 Motivation

Considerable motivation exists for the development of an adequate statistical model for spontaneous EEG activity. For example, it was mentioned in section 3.7.3 that such a model might be of value in the development of EEG pattern recognition systems for monitoring anesthesia levels. More generally, almost all methods of quantitative EEG analysis are based on certain implicit assumptions regarding the statistical characteristics of the underlying random process, particularly with respect to the extent of stationarity and Gaussianity of the process. The efficacy of alternate analytic techniques depends upon the degree to which such assumptions are justified by the characteristics of the particular ensemble of EEG segments being analysed. In addition, a better understanding of some of the statistical properties of different EEG ensembles might eventually result in a better understanding of the neurophysiological mechanism of spontaneous EEG generation, a mechanism which is still not well understood. Despite such motivation, relatively few investigations of the statistical properties of specific EEG ensembles have been described in the literature.

4.1.2 Evaluation of Previous Investigations

The first studies of the EEG amplitude probability distribution suggested a striking similarity to the normal or Gaussian distribution [111,112]. A later analysis of one 8.33s EEG segment from each of four subjects also showed that in two cases the amplitude distributions closely
fitted a Gaussian distribution [113]. However, subsequent reports by others contained rather contradictory results. For example, tests of thirty 52.8s EEG segments for Gaussianity resulted in 29 rejections; the investigators concluded that the spontaneous EEG could not be modelled as a normal random process because not even its amplitude distribution was Gaussian [114]. Elul suggested that this study illustrated an extreme case where non-stationarity of the EEG was erroneously construed as indicative of a non-Gaussian distribution [115]. He tested successive 2s EEG segments from one subject and reported that the EEG was Gaussian 66 percent of the time in the resting state, shifting to 32 percent during a mental arithmetic task. Although the results of some later studies appear to agree with those of Elul (e.g. [116]), others do not. For example, Dumermuth et al. commented that most of the 40s EEG segments which they had analysed deviated from Gaussianity [21]. Following the suggestion of Elul they also analysed 4s EEG segments in an attempt to reduce effects due to non-stationarity but reported even stronger deviations from a Gaussian model [117,118].

Several factors can be identified which have contributed to the previously described inconsistencies in the literature. Many early investigations involved relatively small ensembles of EEG segments from very few subjects. Frequently, EEG data from only one non-standardized channel was considered. The reliability and comparability of the results obtained in such studies were therefore affected by topological differences, by statistical variability due to small sample sizes and by inter-subject EEG variation. Another factor contributing to discrepancies among published findings concerns the different EEG digitization rates which were used: it will be shown in this chapter that different sampling
rates change the efficacy of statistical hypothesis tests. Finally, the problem of estimating the degree of stationarity of a particular ensemble of EEG segments has seldom been considered directly in such investigations. Attempts were instead made to circumvent the problem of stationarity when investigating Gaussianity by subdividing the EEG into very short segments in the expectation that any non-stationary effects would be reduced.

4.1.3 Outline of Chapter

In this chapter, a technique is proposed for estimating the degree of wide-sense stationarity and the degree of Gaussianity of an ensemble of EEG records. Results which have been obtained by applying this technique to three relatively large ensembles of multichannel EEG data are also described. In addition, the comparative advantages of employing alternate methods of EEG analysis are discussed in relation to the estimated degree of stationarity and Gaussianity of the particular EEG ensembles under consideration. Finally, the specific relevance of the results presented in this chapter to the development of EEG pattern recognition systems for monitoring anesthesia levels is discussed.

4.2 Random Process Characterization

The ensemble of all possible time functions which can be generated by a particular source together with their respective probabilities of occurrence defines a random process. Spontaneous EEG activity may therefore be modelled as a random process. Any such process, denoted by X(t), is said to be completely characterized or modelled if its nth order
distribution function

\[ F[x_1, \ldots, x_n; \ t_1, \ldots, t_n] = P \left[ X(t_1) \leq x_1, \ldots, X(t_n) \leq x_n \right] \quad (4.1) \]

is known for any \( n \) and any set of sampling times \( t_1, \ldots, t_n \) ([84], pp. 296-297). For most random processes it is difficult to obtain empirical estimates of (4.1). However, if a particular random process is both Gaussian and stationary then the problem of modelling it by estimating (4.1) is greatly simplified.

Briefly, a random process \( X(t) \) is said to be Gaussian or normal if its \( n \)th order probability density function

\[ f[x_1, \ldots, x_n; \ t_1, \ldots, t_n], \]

obtained by differentiating (4.1) with respect to all variables \( x_i \), takes the form of a jointly Gaussian distribution, i.e.

\[ f(x_1, \ldots, x_n; \ t_1, \ldots, t_n) = \frac{\exp\left(-\frac{1}{2}(x - u)[K]^{-1}(x - u)^T\right)}{(2\pi)^{n/2}(|K|)^{1/2}}, \quad (4.2) \]

where

\[ x = [x_1, \ldots, x_n], \quad (4.3) \]

\[ u = [E(X(t_1)), \ldots, E(X(t_n))] \]

\[ = [u_1, \ldots, u_n], \quad (4.4) \]

\[ [K] = \begin{bmatrix} k_{11} & \cdots & k_{1n} \\
     \vdots & \ddots & \vdots \\
     k_{n1} & \cdots & k_{nn} \end{bmatrix}, \quad (4.5) \]

\[ k_{ij} = E((x_i - u_i)(x_j - u_j)), \quad (4.6) \]

and \( |K| \) is the determinant of \( [K] \), the covariance matrix ([119], pp. 111-112).

A random process \( X(t) \) is said to be strictly stationary if none of its statistics are affected by a shift in time origin, i.e. if the two
processes \( X(t) \) and \( X(t+\xi) \) have the same statistics for any \( \xi \). A much weaker condition is that of wide-sense stationarity in a finite time interval: if

\[
E[X(t)] = \mu = \text{constant} \quad (4.7)
\]

and if the autocorrelation function is given by

\[
R(t_i, t_j) = R(\tau), \quad (4.8)
\]

where

\[
\tau = |t_j - t_i|, \quad (4.9)
\]

for all \( t, t_i \) and \( t_j \in [0,T] \) then \( X(t) \) is said to be wide-sense stationary in the interval \([0,T]\) ([84], pp. 300-304). Under this condition, \((4.6)\) becomes

\[
k_{ij} = E \{(x_i - u_i)(x_j - u_j)\}
\]

\[
= E\{x_ix_j\} - u_iu_j
\]

\[
= R(t_i, t_j) - \mu^2
\]

\[
= R(\tau) - \mu^2 \quad (4.10)
\]

for all \( t_i, t_j \in [0,T] \). From \((4.2)\) and \((4.10)\) it is therefore evident that, under the condition of wide-sense stationarity in the interval \([0,T]\), a Gaussian random process \( X(t) \) is completely specified by its mean and autocorrelation function in the interval.

If a random process \( X(t) \) is ergodic [120] then such statistics as the mean and autocorrelation function can be calculated from a single sample function, denoted by \( x(t) \), i.e.

\[
E[X(t)] = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) \, dt = \mu \quad (4.11)
\]

and

\[
R(\tau) = E[X(t)X(t+\tau)]
\]

\[
= \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x(t+\tau) \, dt
\]

\[
= \mathcal{R}(\tau) \quad , \quad (4.12)
\]
where \( R(t) \) represents the time autocorrelation function. However, an empirical test for ergodicity would require extensive ensemble calculations and would certainly not be feasible when only a limited number of sample functions of relatively short duration are available. Under these conditions ergodicity is usually assumed and any desired ensemble statistics are estimated from the individual characteristics of all available sample functions. For example, if all sample functions can be modelled as the output of a wide-sense stationary Gaussian process in an interval \([0,T]\) then the mean and autocorrelation function are sufficient statistical descriptors of the process in the interval. These ensemble descriptors can be estimated in practice by averaging the means and the time autocorrelation functions (or equivalently the power spectra) of the available sample functions.

A necessary requirement before any such modelling of observed EEG activity can be attempted is that some empirical procedures be established for testing individual EEG segments, at a specified significance level, for wide-sense stationarity and Gaussianity.

4.3 Establishment of Empirical Testing Procedures

4.3.1 Testing for Wide-Sense Stationarity

Assume that \([x_1, \ldots, x_{2n}]\) has been obtained by sampling a band-limited EEG signal \(x(t)\) at or above the Nyquist rate during the time interval \([0,2T]\). Although an exact determination of the degree of wide-sense stationarity and Gaussianity of \(x(t)\) in the given interval is not possible, useful estimates of these statistical properties can be obtained by the application of certain hypothesis testing procedures.

A procedure for determining whether or not \([x_1, \ldots, x_{2n}]\) can be considered to be a set of samples from a wide-sense stationary function
can be based on the requirement that the amplitude distributions and the power spectra calculated for the sample subsets \([x_1, \ldots, x_n]\) and \([x_{n+1}, \ldots, x_{2n}]\) must not be significantly different. Specifically, a test for the wide-sense stationarity of a given sample set can be constructed by first dividing the set into two equal subsets and calculating an amplitude histogram and power spectrum for each. Then the two-sample Kolmogorov-Smirnov (K-S) test [121, 122] can be employed to compare the sample amplitude and spectral distribution functions of each. The two-sample K-S test is based on the statistic

\[ D_2 = \sup_{\text{all } s} |F_n(s) - G_n(s)| \]

(4.13)

where \(F_n(s)\) and \(G_n(s)\) are distribution functions calculated from a set of samples of size \(n\) from populations \(F\) and \(G\) respectively. A large value of \(D_2\) resulting from application of the two-sample K-S test would indicate rejection, at some significance level, of the null hypothesis that \(F\) and \(G\) are identical. When \([x_1, \ldots, x_n]\) and \([x_{n+1}, \ldots, x_{2n}]\) are tested in this manner, rejection of either the hypothesis of identical amplitude distributions or the hypothesis of identical spectral distributions indicates that the original EEG signal cannot be modelled with confidence as a sample function of a random process that is wide-sense stationary over the interval \([0, 2T]\). Thus, rejection of either hypothesis for a given set of samples constitutes an empirical upper bound on the interval of wide-sense stationarity, i.e. in this instance the interval of wide-sense stationarity for the random process of which \(x(t)\) is a sample function is assumed to be less than \(2T\).

4.3.2 Testing for Gaussianity

Testing the amplitude distribution of a set of EEG samples
for Gaussianity or normality is accomplished by means of a goodness of fit test. The K-S goodness of fit test is employed because it has been shown that, with the population mean and variance estimated by the sample mean and variance, it yields a test for normality which is more powerful than the more popular chi-square test [121-123]. The K-S statistic $D_1$ represents the least upper bound of the differences between the empirical and assumed distribution functions:

$$D_1 = \sup_{s} \left| F_{2n}(s) - F(s) \right|,$$

where $F_{2n}(s)$ is the distribution function calculated from the set of $2n$ samples and $F(s)$ is the assumed distribution function. If $D_1$ is too large, the null hypothesis that $F(s)$ represents the population distribution function is rejected.

4.4 Experiment

4.4.1 Selection of Sample EEG Data

In order to apply the previously described tests for Gaussianity and wide-sense stationarity to some actual EEG ensembles, three sets of sample EEG segments were selected from the available EEG data base (described in Table 2-2). Because of the extensive computation involved in testing for Gaussianity and wide-sense stationarity, it was necessary from a practical standpoint to limit the amount of sample EEG data under consideration. Consequently, only sample EEG segments from the two most common types of general anesthesia, previously referred to in this thesis as halothane and narcotic anesthesia, were considered. It was also necessary to restrict the number of sample EEG segments from each type of anesthesia because of computational time and cost considerations. Accordingly, it was decided that four multichannel EEG segments without
visually apparent artifact from each of 30 subjects would be considered: 15 of the subjects who were chosen had received halothane anesthesia and the other 15 subjects had received narcotic anesthesia.

Detailed descriptions of the EEG data acquisition procedure and the preparation of sample EEG segments corresponding to clinical anesthesia levels were given in Chapter II. Briefly, EEG activity was recorded from two pairs of bilaterally symmetric, differential channels: F3-C3, C3-O1, F4-C4 and C4-O2, according to the International 10-20 System of electrode placement [70]. The recorded data was later lowpass filtered at 30.0 Hz and then the 4-channel, 64s sample EEG segments were prepared. As stated above, four filtered multichannel EEG segments were selected from each of 30 different subjects for the modelling investigation. Two of the 64s segments from each subject were baseline EEG segments corresponding to Anesthesia Level 0, i.e. they were recorded while the subject was awake and resting with eyes closed, approximately one hour before surgery. The two additional EEG segments from the same subject corresponded to Anesthesia Level 3, i.e. they were recorded at a surgical level of anesthesia.

Three different sets of multichannel EEG segments were thus selected for consideration: one set of 60 baseline segments from 30 awake and resting subjects, a second set of 30 segments from 15 of these subjects during halothane anesthesia, and a third set of 30 segments from the other 15 subjects during narcotic anesthesia. Some samples of multichannel EEG activity from each of these three sets of data can be seen in Fig. 2-2(a)-(d).

4.4.2 Determination of Optimum Sampling Rate

After the three sets of sample EEG data had been selected for the modelling investigation, it was desired to determine the best rate at
which to sample and digitize the data. Because the EEG segments had already been lowpass filtered at 30.0 Hz, the theoretical minimum sampling rate, as given by the Sampling Theorem ([119], pp. 400-405), was 60.0 Hz, i.e. the Nyquist rate. The filter roll-off characteristics and the computational desirability of setting the sampling rate to a power of two indicated that the most practical minimum sampling rate, denoted by $F_s$, would be 64 Hz. Most of the previous investigations of Gaussianity or stationarity have considered EEG data sampled at rates of from $2F_s$ to $4F_s$ and even higher. However, statistical hypothesis tests such as the K-S and chi-square tests assume that the set of samples to be tested corresponds to a set of statistically independent random variables or observations. Therefore, when this assumption of statistical independence is violated because of an unnecessarily high sampling rate, one can expect the efficacy of such tests to decrease accordingly.

To examine and illustrate the effect of different sampling rates on statistical hypothesis tests, 30 of the recorded 64s baseline EEG segments from channel C4-02 were reproduced, bandpass filtered from 0.54 Hz to 30.0 Hz, and digitized at a rate of 512 Hz or $8F_s$. By considering every second, fourth or eighth sample it was also possible to study EEG data with an effective sampling rate of $4F_s$, $2F_s$, or $F_s$, respectively. At each of these sampling rates a K-S goodness of fit test for Gaussianity, at the 0.05 significance level, was performed on each of the $M$ available EEG segments of $T$ sec duration, where

$$ T = 2^i , i = 0,1,...,6 , \quad (4.15) $$

and

$$ M = \frac{30 \cdot 64}{T} \quad (4.16) $$

The results of these tests are summarized in Fig. 4-1 and clearly indicate the desirability of using a sampling rate as little above the Nyquist rate as practicable.
Fig. 4-1 Effect of Increased Sampling Rates on K-S Goodness of Fit Tests for Gaussianity. $F_s$ is equal to 64 Hz, slightly above the Nyquist rate. The percentage of EEG segments of a specified duration which could be modelled as Gaussian is plotted for 4 different sampling rates.

4.4.3 Application of Tests for Wide-Sense Stationarity and Gaussianity

To reduce error in the computation of power spectra, a sampling rate of 128 Hz was used to digitize all 120 EEG segments from the three ensembles under consideration. However, in view of the results in Fig. 4-1, EEG data with an effective sampling rate of 64 Hz was prepared by considering every second sample value and was used to compute all sample amplitude distribution functions needed for the previously described tests.
for wide-sense stationarity and Gaussianity.

Recall from section 4.3.1 that, for an EEG segment \( x(t) \) to be modelled as a sample function of a process that is wide-sense stationary in the interval \([0,2T]\), a necessary condition is that the amplitude distribution functions and the power spectral distribution functions of \( x(t) \) in the intervals \([0,T]\) and \([T,2T]\) must not be significantly different. The distribution functions can be compared by means of the two-sample K-S test. It should also be recalled from section 4.3.2 that \( x(t) \) in the interval \([0,2T]\) can be tested for Gaussianity by means of the K-S goodness of fit test, with the mean and variance of the Gaussian population estimated by the sample mean and variance. Values for the two-sample K-S test ([121], p.487) and for the K-S goodness of fit test with unknown mean and variance [124], at the 0.05 level of significance, were used. After testing all 120 EEG segments of 64s duration for wide-sense stationarity and Gaussianity, each segment was subdivided into two segments of 32s duration which were also tested in the same manner. This procedure of successively subdividing and testing was repeated until all available EEG segments of 1s duration were tested. In total, 4M EEG segments of \( T \) seconds duration were tested, 2M segments from the baseline ensemble and M segments from each of the anesthesia ensembles, where \( T \) and \( M \) are given by (4.15) and (4.16) respectively. For each of the three ensembles, the percentage of EEG segments of a specified duration which could be modelled as being wide-sense stationary, Gaussian, or both wide-sense stationary and Gaussian was calculated. All results were then corrected for type I errors arising from false rejection of the hypotheses being tested.

The computation of power spectra required as part of the previously described test for wide-sense stationarity was performed by the
Direct Method, i.e. direct Fourier transformation of the data with consecutive averaging over frequency, as described in section 3.2.1. Before Fourier transformation, each digitized EEG segment of T seconds duration, consisting of a set of 128T sample values, was first tapered with a time window \( W(t) \) of the form

\[
W(t) = \begin{cases} 
\frac{1}{2}(1 - \cos(\frac{\pi t}{0.1T})) & 0 \leq t < 0.1T \\
1 & 0.1T \leq t < 0.9T \\
\frac{1}{2}(1 - \cos(\pi \frac{T-t}{0.1T})) & 0.9T \leq t \leq T.
\end{cases}
\]

Each tapered EEG segment was then transformed via the Fast Fourier Transform algorithm. A periodogram was calculated from the complex Fourier coefficients for each fundamental frequency \( k/T \) Hz, where \( k=0,1,\ldots,64T \). Smoothing of the periodogram was performed using a rectangular window with 7 non-zero coefficients. In this manner a set of \((64T + 1)\) smoothed spectral estimates from 0-64 Hz was calculated for each EEG segment of T seconds duration. The distribution function of the subset of spectral estimates between 1 Hz and 30 Hz was then used in the previously described test for wide-sense stationarity.

Appendix H contains a listing of the program that was used to compute EEG amplitude distribution functions and to evaluate the appropriate one-sample and two-sample K-S statistics. A companion program that was used to compute EEG power spectra and to evaluate the two-sample K-S statistics for the appropriate spectral distribution functions is listed in Appendix I. Finally a third program, listed in Appendix J, performed K-S tests on the sample statistics evaluated by the first two programs, and calculated the corrected percentages of EEG segments of different durations which could be modelled as Gaussian, or wide-sense stationary, or both.
4.5 Results

4.5.1 Interpretation of Results

The results of the modelling investigation are summarized graphically in Fig. 4-1 to Fig. 4-5. In Fig. 4-2 to Fig. 4-5, the results for each EEG channel are presented topologically, i.e. the results are located on a stylized representation of the head in a position corresponding to the location of the electrodes from which the EEG activity was recorded. Although all results have already been corrected for type I errors due to false rejections of the hypothesis being tested, type II errors due to false acceptances of the hypothesis may still exist. Also, these results are based on empirical tests for necessary, but not sufficient, properties that sample EEG segments must possess in order to be modelled as the output of a particular type of random process. For these reasons, the estimated percentages given in Fig. 4-1 to Fig. 4-5 therefore represent useful empirical upper bounds on the corresponding "true" percentages.

4.5.2 Effect of Sampling Rate of Empirical Tests

The effect of different sampling rates upon the outcome of statistical hypothesis tests is illustrated in Fig. 4-1. This marked and previously unexplored relationship may account for some discrepancies apparent in the literature. The problem arises from the assumption, made in the formulation of both the chi-square and the K-S tests, that the set of samples to be tested represents a set of independent random observations. In practice, as the rate of sampling a bandlimited EEG segment increases above the Nyquist rate, successive samples become more interdependent and the efficacy of statistical hypotheses tests is consequently affected [125,126]. It is therefore not surprising that one study of 2s EEG segments which were sampled at 200 Hz concluded that resting EEG activity is
Fig. 4-2 Mean Ensemble Characteristics of the Baseline EEG Activity of 30 Subjects Who Were Resting With Eyes Closed. Results are based on a total of 3840s of EEG activity from each of 4 channels, collected in the form of two multichannel EEG samples of 64s duration per subject.
Fig. 4-3 Estimated Percentage of EEG Segments of Various Durations From Three Different Ensembles Which Can Be Modelled as Wide-Sense Stationary.
Fig. 4-4 Estimated Percentage of EEG Segments of Various Durations from Three Different Ensembles Which Can Be Modelled as Gaussian.
Fig. 4-5 Estimated Percentage of EEG Segments of Various Durations from Three Different Ensembles Which Can Be Modelled as Both Wide-Sense Stationary and Gaussian.
Gaussian 66 percent of the time [115], while other studies of EEG segments of similar duration which were sampled at 5000 Hz concluded that resting EEG activity is strongly non-Gaussian [21,117,118]. Fig. 4-1 indicates that, if it is desired to investigate the characteristics of EEG segments by means of statistical hypothesis tests, the best tradeoff between the requirement to adequately sample a bandlimited signal and the desirability of satisfying the assumption of a statistically independent sample set is reached if the sampling rate is set as little above the Nyquist rate as is practicable.

4.5.3 Estimated Baseline EEG Characteristics

The estimated statistical characteristics of the ensemble of baseline EEG activity are presented in Fig. 4-2. The percentage of EEG segments which can be modelled as being Gaussian, wide-sense stationary, or both is given for each of the 4 differential channels under consideration. In Fig. 4-2 the strong dependence of the results on the duration of the EEG segments being tested is apparent. This dependence accounts for many of the discrepancies in the literature, e.g. the results presented here are consistent with one previous finding [113] that two of four baseline EEG segments (of 8.33s duration) tested were Gaussian and they are also consistent with another report that only 3.3 percent of 30 baseline EEG segments (of 52.8s duration) were found to be Gaussian [114]. The results in Fig. 4-2 also clearly differentiate between the properties of Gaussianity and stationarity: for example, in channel C4–02 over 57 percent of EEG segments of 64s duration were modelled as wide-sense stationary but only 5.3 percent were found to be Gaussian and less than 2.0 percent could be considered both Gaussian and wide-sense stationary. Fig. 4-2 also reveals striking similarities among corresponding results for
all 4 channels, and even stronger similarities between results for pairs of bilaterally symmetric channels. Thus, while no obvious inter-hemispheric EEG differences were found, occipital EEG activity appears to be consistently more Gaussian and more stationary than frontal EEG activity.

4.5.4 Wide-Sense Stationarity

In Fig. 4-3 to Fig. 4-5 the estimated statistical characteristics of baseline EEG activity are compared to the corresponding characteristics during narcotic anesthesia and during halothane anesthesia. The data base for each type of anesthesia consisted of 1920s of EEG activity from 15 subjects, i.e. two 64s segments per subject, and the baseline data consisted of a total of 3840s of EEG activity from all 30 subjects.

Fig. 4-3 shows the estimated percentage of sample EEG segments of various durations from each of the three different ensembles which can be modelled as wide-sense stationary. If the stationarity of EEG segments of the same duration is considered, it appears that EEG activity during halothane anesthesia is marginally more stationary than baseline activity while EEG activity during narcotic anesthesia is slightly less stationary than baseline activity. The results in Fig. 4-3 indicate that, for sample EEG segments less than 32s in duration from any channel and from any of the three ensembles, the assumption of wide-sense stationarity may be valid more than 50 percent of the time.

4.5.5 Gaussianity

Fig. 4-4 gives the estimated percentage of sample EEG segments from each ensemble which can be modelled as Gaussian. EEG segments from halothane anesthesia are generally less Gaussian than the baseline activity,
particularly in channels C3-01 and C4-02, while EEG segments from narcotic anesthesia are marginally more Gaussian than the baseline activity. In all channels, EEG activity during halothane anesthesia is consistently less Gaussian than EEG activity during narcotic anesthesia.

4.5.6 Wide-Sense Stationarity and Gaussianity

In Fig. 4-5, the percentage of sample EEG segments from each of the three ensembles which can be modelled as both Gaussian and wide-sense stationary is presented. A bilateral symmetry is immediately apparent in these results. In all channels, the percentage of EEG segments from halothane anesthesia which are wide-sense stationary and Gaussian is markedly smaller than the corresponding percentage from narcotic anesthesia. Also, from Fig. 4-5 it is evident that less than 10 percent of the 64s EEG segments from any ensemble can be modelled as wide-sense stationary and Gaussian.

4.6 Significance of Results

4.6.1 Development of EEG Monitoring Systems

The estimated degree to which ensembles of EEG activity may be modelled as stationary and Gaussian, e.g. the results presented in Fig. 4-3 and Fig. 4-5, should be an important consideration in the choice of an appropriate technique for analysing sample EEG segments from those ensembles. For example, the primary motivation for investigating the statistical characteristics of the three specific ensembles of EEG activity described in this chapter was the expectation that the results would assist in the development of a computer-based system for monitoring the level of anesthesia during surgery by means of an automatic analysis of spontaneous EEG activity. In the development of such a system, decisions must be made with respect to the duration of the EEG segments to
be analysed, the rate at which the estimated level should be updated, the choice of an analytic technique, and the significance which may be attached to the results of the analysis. It should be noted that the feasibility of employing EEG monitoring systems to continuously assess a patient's status during sleep, serious illness, coma, and possible cerebral death is also currently being investigated by others, e.g. [83,127,128]. The statistical characteristics of the particular ensembles of EEG activity being analysed in each instance should be an important consideration in the development of the appropriate monitoring system.

To illustrate how knowledge concerning the degree of stationarity of the three ensembles described in this chapter might influence the development of a system for monitoring and analysing EEG activity during anesthesia, the problem of selecting the most appropriate duration for sample EEG segments on the basis of the results in Fig. 4-3 will be briefly considered. It would obviously be desirable to analyse EEG segments of long duration because the significance of any transient noise and artifact is thereby reduced, because a high resolution in the estimation of power spectra is possible, and because a potentially large data reduction can be realized if such segments can be adequately characterized. However, in the theoretical development of most analytic techniques the assumption is made that the signal under consideration represents a sample function from a random process that is at least stationary to some extent over the interval of interest. Fig. 4-3 indicates that the assumption of wide-sense stationarity for the three ensembles under consideration is only partially justified, even for EEG segments of relatively short duration. The a priori selection of the most suitable analytic technique therefore cannot be made on a firm theoretical basis. Under
such conditions, the results in Fig. 4-3 indicate that the choice of 32s duration for sample EEG segments might, in this instance, represent a reasonable compromise. For all three ensembles at least one half of the EEG segments of this duration could be modelled as wide-sense stationary. An analytic technique which assumes wide-sense stationarity could then reasonably be applied to the 32s segments and any inherent non-stationarity could be taken into account by some ancillary technique. For example, the previously described K-S D₂ statistics could be included in the analysis as parameters indicating the degree of non-stationarity of the segment being analysed and hence could be used in interpreting the significance of the results. Alternatively, individual EEG segments could be tested for wide-sense stationarity as described previously and only those segments found to be stationary would be analysed. If non-stationarities are to be considered for some particular EEG ensembles, and they cannot adequately be taken into account by such ancillary techniques, then a non-stationary analysis of the EEG could be attempted [129-131].

4.6.2 Evaluation of Alternate Analytic Techniques

This section will consider some implications of the results in Fig. 4-5 with respect to the choice of the most appropriate technique for analysing EEG segments of a specified duration from any of the three ensembles. For the reasons stated previously in section 4.2, power spectrum analysis of the EEG segments would be preferable if the segments could be modelled as both wide-sense stationary and Gaussian. However, Fig. 4-5 shows that only a certain proportion of sample EEG segments may be so modelled, e.g. for all ensembles less than 50 percent of the 8s segments from any channel could be considered wide-sense stationary and Gaussian. It cannot therefore be assumed that spectral analysis will provide a
sufficient characterization of such sample EEG segments. When it is known that a certain proportion of the EEG segments to be analysed cannot be modelled as the output of a stationary Gaussian random process, alternate analytic strategies might be considered. Of course, any analytic technique could arbitrarily be applied to the data in the hope that the results might somehow provide an ad hoc justification for its usage. However, if it can be assumed that most of the segments under consideration are wide-sense stationary, or that any inherent non-stationarity has been taken into account by one of the techniques described previously, then certain analytic strategies might be more profitably investigated. For example, if the EEG segments are stationary and only slightly non-Gaussian, ancillary parameters which indicate the degree of non-Gaussianity (e.g. skewness and kurtosis [97] or the previously described K-S D₁ statistic) might be employed in addition to spectral analysis. Alternatively, if the EEG segments to be analysed are stationary but very non-Gaussian, then the information provided by EEG spectral analysis could be supplemented by the use of other analytic techniques, e.g., bispectral analysis [117].

4.6.3 Further Work

The modelling investigation described in this chapter also indicates some areas for further work that are beyond the scope of this thesis. It has been suggested that, on the basis of the Central Limit Theorem, increased Gaussianity in observed EEG activity may reflect an increased degree of independence among individual cortical neural generators [115]. If one accepts this premise, then Fig. 4-4 and Fig. 4-5 indicate that the cortical generators are considerably more interdependent during halothane anesthesia than during narcotic anesthesia. The possible neurophysiological significance of this result could be investigated,
perhaps by studies of EEG coherence in individual subjects and by consider­
ering more sample data from more channels. In addition, the technique
described in this chapter for estimating the degree of wide-sense station­
arity and Gaussianity of an ensemble of EEG segments could obviously be
applied to many other ensembles of EEG activity corresponding to other
states of consciousness.
CHAPTER V  PERFORMANCE IMPROVEMENT SCHEMES

5.1  Introduction  

The initial results presented in Chapter III demonstrated the feasibility of using EEG pattern recognition systems to estimate the level of anesthesia. To a large extent, the modelling results presented in Chapter IV vindicated the initial EEG pattern recognition approach. In addition, Chapter IV contained a discussion of possible methods for improving the performance of the initially developed systems by giving greater consideration to the actual statistical characteristics of the EEG data. In this chapter, other possible methods for improving performance will be investigated; for illustrative purposes each of these methods will be investigated with a view to improving the performance of three specific EEG spectral pattern recognition systems. It should be recalled that all such systems classified an unknown EEG pattern sample on the basis of a set of thirteen extracted spectral feature values. The Bayes classifier that was employed in all systems was optimal only if all features were statistically independent and if the required class-conditional feature probabilities either were known exactly or were given by the corresponding Bayes estimates. Most of the performance improvement schemes considered in this chapter involve changes in the initial feature extraction procedure and pattern classification algorithm.

The three initially developed EEG spectral pattern recognition systems which were employed in the work described in this chapter had the same structure, i.e. all contained a linear feature quantizer with 64 possible levels and a classifier which assumed equal a priori class probabilities. However, each was trained on the set of available EEG pattern samples from a different type of anesthesia. Fig. 5-1 depicts the "confusion" matrices which were calculated for these three systems. The i-jth element in each confusion matrix contains the number of pattern samples from class
Halothane Anesthesia

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Fig. 5-1 Confusion Matrices for Systems Which Extracted 13 Spectral Features. The matrices on the left resulted from performance estimation by the H* technique and those on the right resulted from performance estimation by the U* technique.
1, 0 ≤ i ≤ 4, which the system identified as belonging to class j, 0 ≤ j ≤ 4. The actual numbers are given, rather than the corresponding probabilities, to indicate the unequal number of available pattern samples per class for the different types of anesthesia. The matrices on the left in Fig. 5-1 resulted from performance estimation by the n* technique and those on the right resulted from performance estimation by the U* technique. Thus the performance estimates for the three systems, which were given previously in Tables 3-3 to 3-5, can be derived from the appropriate confusion matrices in Fig. 5-1: \( \hat{P}_e[n^*] \) and \( \hat{P}_e[U^*] \) were 0.389 and 0.108 for the halothane anesthesia system, 0.449 and 0.211 for the narcotic anesthesia system, and 0.420 and 0.132 for the enflurane anesthesia system.

From the definitions of the n* and U* techniques (section 3.5), it is evident that the difference between \( \hat{P}_e[n^*] \) and \( \hat{P}_e[U^*] \) for a specific system provides an indication of the effect of intersubject EEG variation on system performance [39]. The relatively large magnitude of this effect is apparent when the difference between \( \hat{P}_e[n^*] \) and \( \hat{P}_e[U^*] \) is evaluated for each of the systems considered in Chapter III. Similar results have been reported in the literature for other types of EEG pattern recognition systems (e.g. [77,132]). Accordingly, intersubject EEG variation must be regarded as a major obstacle preventing the development of more reliable systems. Much of the work described in this chapter was directed toward reducing the effect of intersubject EEG variation.

In section 5.2 the possibility of improving performance by increasing the number of extracted features is considered. The feasibility of exploiting statistical interdependencies among features is discussed in section 5.3. In section 5.4 a "nearest subject" scheme for reducing the effect of intersubject EEG variation on classifier performance is explored.
5.2 Extraction of Additional Features

5.2.1 Rationale

The EEG spectral pattern recognition systems that were initially developed were based on the extraction of a total of 13 spectral features from two EEG channels. All of these features were heuristically derived, i.e. they either had an established clinical relevance or they had previously been described as meaningful in the literature on EEG pattern recognition. Each EEG pattern sample was evaluated in terms of these features and was subsequently classified on the basis of the extracted set of feature values. Because of computational time and cost considerations in the initial phase of the research it was necessary to limit the number of extracted features, i.e. to limit the extent to which EEG pattern samples could be characterized. In spite of this limitation, the results of the initial phase of the research (as described in Chapter III) clearly established the feasibility of estimating the level of anesthesia by means of EEG pattern recognition systems. Consequently, after the feasibility had been established it seemed worthwhile to investigate the possibility that the performance of the initially developed EEG spectral pattern recognition systems could be improved by the inclusion of additional features in the extracted feature set.

To investigate this possibility, it was decided that the selection of an appropriate set of additional features would proceed in the following manner. First, a large set of additional, heuristically derived features would be defined. It was recognized that adding each of these features to the extracted feature set would not necessarily result in an improvement in performance. It was also recognized that there was a practical constraint on the large number of additional features that should be selected from the large set, because of the limited computational time.
that would be available for extracting features from successive EEG pattern samples in an on-line monitoring system. For the purposes of this investigation, therefore, the maximum number of additional features to be selected was arbitrarily set at 13, i.e. it was decided that the total number of extracted features would be increased by a factor of two. However, in general there is no optimal procedure for selecting the best subset of features from a large set, except by the exhaustive evaluation of all possible subsets [110]. Since that would be computationally impractical here, it was decided that various suboptimal feature selection criteria would be used to choose alternate sets of 13 additional features. EEG spectral pattern recognition systems which included these additional features in their extracted feature sets would then be developed and their performance would be estimated.

5.2.2 Definition of Additional Features

To define the relatively large number of additional, heuristically derived features from which various sets of 13 features would later be selected, the notation that was introduced in section 3.2.1 will be extended: let \( x(t) \) and \( y(t) \) denote the sample EEG activity from two specified channels, let \( X(f) \) and \( Y(f) \) represent their Fourier transforms, as defined in (3.2), and let \( X^*(f) \) and \( Y^*(f) \) denote the complex conjugates of \( X(f) \) and \( Y(f) \), respectively. From (3.1) it follows that the EEG spectra, or more specifically the EEG autospectra, corresponding to \( x(t) \) and \( y(t) \) are given by

\[
S_{xx}(f) = E\{X(f) X^*(f)\} \tag{5.1}
\]

and

\[
S_{yy}(f) = E\{Y(f) Y^*(f)\} \tag{5.2}
\]

The 13 features which were initially chosen for extraction from the EEG autospectra corresponding to two of the four available channels were described previously in section 3.2.3.
Many of the additional features which were chosen for extraction are derived from the EEG autospectra corresponding to all four available channels. Other features were defined in terms of the EEG coherence spectrum: if

$$S_{xy}(f) = E\{X(f)Y^*(f)\},$$

(5.3)

i.e. $S_{xy}(f)$ denotes the cross-spectrum, then the coherence spectrum $C_{xy}(f)$ is defined as

$$C_{xy}(f) \triangleq \frac{|S_{xy}(f)|}{|S_{xx}(f)S_{yy}(f)|^{1/2}},$$

(5.4)

where $S_{xx}(f)$ and $S_{yy}(f)$ are given by (5.1) and (5.2), respectively [21,133]. It should be pointed out that the quantity in (5.4) is the square root of the quantity defined as coherence in some references (e.g. [91,134]). From the definition in (5.4) it is evident that the coherence spectrum $C_{xy}(f)$ is a real-valued function of frequency for which

$$C_{xy}(f) = C_{yx}(f)$$

(5.5)

and for which

$$0 \leq C_{xy}(f) \leq 1.$$  

(5.6)

It should also be noted from (5.4) that, if $x(t)$ and $y(t)$ are linearly related, i.e. if

$$Y(f) = H(f)X(f)$$

(5.7)

for some $H(f)$, then $C_{xy}(f) = 1$. Accordingly, the coherence spectrum can be regarded as a measure of the degree of linear relationship between the EEG activity from two specified channels as a function of frequency [134, 135]. This has motivated the investigation of various "coherence features", i.e. features derived from the coherence spectrum, as potentially significant descriptors of multichannel EEG activity (e.g. [78,81,91]).

In this research, additional features were derived from "bilateral" coherence spectra and from "unilateral" coherence spectra. For
convenience in defining these features, let channels F3-C3, C3-O1, F4-C4 and C4-O2 (in Fig. 2-3) be denoted as channels 1, 2, 3 and 4, respectively. Then bilateral coherence features refer to features derived from a coherence spectrum corresponding to a symmetrically located pair of channels, i.e. channels 1 and 3 or channels 2 and 4. Unilateral coherence features refer to those features derived from the coherence spectrum corresponding to an anterior-posterior channel pair, i.e. channels 1 and 2 or channels 3 and 4.

Coherence spectra were computed, smoothed and averaged in a manner analogous to the procedure outlined previously in section 3.2.2 for autospectra. Appendix C contains a listing of the program that was used to compute the autospectra and the coherence spectrum for sample EEG data from any two specified channels. The results of all spectral and coherence calculations that were performed on each EEG pattern sample consisted of four smoothed autospectra

$$\hat{S}_{jj}(f_m), \quad \text{for } j = 1, 2, 3, 4$$

and four smoothed coherence spectra

$$\hat{C}_{jk}(f_m) \quad \text{for }\begin{cases} j = 1, k = 2 \\ j = 1, k = 3 \\ j = 2, k = 4 \\ j = 3, k = 4 \end{cases}$$

where j and k correspond to the appropriate channel numbers and where

$$f_m = \frac{m-1}{8} \text{ Hz, for } m=1,\ldots,256. \quad (5.8)$$

Table 5-1 describes all of the spectral and coherence features chosen for extraction from each EEG channel. In Table 5-1, channel j refers to the channel under consideration and channels k and l refer, respectively, to the corresponding unilateral and bilateral channels. Three autospectral features and two coherence features were chosen for extraction.
### Table 5-1  Spectral and Coherence Features Chosen for Extraction From Each EEG Channel

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<th>Description</th>
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<tr>
<td></td>
<td>Mean coherence (unilateral)</td>
<td>u_Δjk</td>
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<td>E_j</td>
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<tr>
<td>0.00 - 32.00 Hz</td>
<td>Mean spectral frequency</td>
<td>f_j</td>
<td></td>
</tr>
<tr>
<td>(Total)</td>
<td>Second moment</td>
<td>f_2</td>
<td></td>
</tr>
</tbody>
</table>
from each of the five traditional EEG frequency bands. The last three features listed in Table 5-1, i.e. the total spectral energy, the mean spectral frequency and the mean second moment, were defined previously in equations (3.13), (3.15) and (3.16), respectively. All other autospectral features describing the relative energy, the peak frequency and the peak intensity in the traditional frequency bands were evaluated as indicated in (3.14) and (3.17) - (3.19) for the corresponding α-band features. The coherence features described in Table 5-1 were evaluated in a similar manner.

In total, Table 5-1 describes 76 spectral features and 20 coherence features corresponding to four EEG channels. However, because 13 of these features constituted the initially chosen spectral feature set, only 83 additional spectral and coherence features are described by Table 5-1. To facilitate the subsequent selection of various sets of 13 additional features, all available EEG pattern samples were evaluated in terms of the additional features in Table 5-1 and the resultant 83-element feature vectors were stored for later use.

5.2.3 Feature Selection

The purpose of selecting additional features was to explore the possibility of improving the performance of the initially developed EEG spectral pattern recognition systems by expanding their extracted feature sets. As stated in section 5.2.1, it was decided to increase the size of the extracted feature set by a factor of two, i.e. to select 13 additional features. Alternate sets of 13 additional features were therefore chosen from the 83 spectral and coherence features described in section 5.2.2 by means of various feature selection criteria. EEG spectral pattern recognition systems which extracted the additional features thus selected
were then developed and their performance was estimated. The systems employed 64 feature quantization levels over a range of ±5.0 sd and assumed that the \textit{a priori} class probabilities were equal. A summary of their estimated performance, based on the extraction of 13 spectral features, was given in section 5.1.

Several alternate feature selection criteria were considered. In each instance, a set of the 13 "best" features was selected after all 83 available features had been ranked on the basis of some criterion such as the magnitude of their interclass/intraclass $F$ ratios [136,137], their relative lack of correlation with other features, and their estimated error probabilities when used separately [110]. The performance of each EEG spectral pattern recognition system which employed a set of additional features selected in this manner was estimated by the $\Pi^*$ and $U^*$ techniques. Results indicated that only marginal improvements in system performance could be achieved with most of the feature selection criteria that were initially considered.

However, the use of one particular criterion in conjunction with a stepwise feature selection algorithm did result in significant improvements in system performance. To describe the criterion and the algorithm, let \{s$_i$\}, $1 \leq i \leq n$, denote the complete set of features chosen for extraction from each EEG pattern sample ($n = 13$ initially) and let \{a$_j$\}, $1 \leq j \leq N$, denote the set of additional features described in section 5.2.2 which have not yet been included in the extracted feature set ($N = 83$ initially). Furthermore, let

\[
(\hat{P}_e[\Pi^*])_{n+1} = a_j
\]

indicate the misclassification error probability, as estimated by the $\Pi^*$ technique, for an EEG spectral pattern recognition system in which a$_j$ was selected to be the additional extracted feature s$_{n+1}$. The feature selection
criterion can then be described as follows: at each step choose \( \sigma_{n+1} = a_j \) if
\[
(\hat{P}_e[\pi^k]) \sigma_{n+1} = a_j < (\hat{P}_e[\pi^k]) \sigma_{n+1} = a_k
\]
(5.9)

for \( k = 1, \ldots, N \) and \( k \neq j \). An algorithm was implemented to select 13 additional features, in terms of the above criterion, in a stepwise manner. Table 5–2 lists the additional features which were selected in this way for each of the three different types of anesthesia under consideration. The symbols used in Table 5–2 correspond to those defined previously in Table 5–1.

Table 5–2 Summary of Selected Spectral and Coherence Features

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<td>26</td>
<td>( f_{\sigma 4} )</td>
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</table>

EEG spectral pattern recognition systems which extracted the additional features listed in Table 5–2, as well as the 13 initially chosen features, were developed and their performance was estimated. The results
are summarized in Figs. 5-2 to 5-4. In each figure the estimated probability of correct classification for a given system is plotted as a function of the number of features included in the extracted feature set. It should be noted that, to facilitate the subjective interpretation of results, estimated probabilities of correct classification are given in Figs. 5-2 to 5-4, i.e.

\[ \hat{p}_c[U^*] = 1 - \hat{p}_e[U^*], \] (5.10)

\[ \hat{p}_c[\Pi^*] = 1 - \hat{p}_e[\Pi^*] \] (5.11)

and

\[ \hat{p}_c[\text{Mean}] = (\hat{p}_c[U^*] - \hat{p}_c[\Pi^*])/2 = 1 - \hat{p}_e[\text{Mean}], \] (5.12)

where \( \hat{p}_e[\Pi^*] \) and \( \hat{p}_e[U^*] \) were described in sections 3.5.3 - 3.5.4 and \( \hat{p}_e[\text{Mean}] \) was defined in (3.55).

5.2.4 Resulting Improvement in Performance

The results presented in Figs. 5-2 to 5-4 indicate that significant improvements in performance have been achieved by the selection of additional, heuristically derived features for inclusion in the extracted feature set. The improvement in performance is reflected by increased values of \( \hat{p}_c[U^*], \hat{p}_c[\Pi^*] \), and hence \( \hat{p}_c[\text{Mean}] \) for the systems under consideration. Improved performance is also indicated by a decrease in the value of

\[ \Delta = |\hat{p}_c[U^*] - \hat{p}_c[\Pi^*]| \] (5.13)

for systems which extracted the additional features, as shown in Figs. 5-2 to 5-4. From the definitions of the \( \Pi^* \) and \( U^* \) techniques (section 3.5), it is evident that the value of \( \Delta \), i.e. the magnitude of the difference between the two estimates of performance, can be regarded as an estimate of the effect of intersubject EEG variation on system performance [39].

In considering the results presented in Fig. 5-2 for halothane anesthesia, it is evident that the values of \( \hat{p}_c[\Pi^*] \) and \( \hat{p}_c[U^*] \) changed
Fig. 5-2 Improvement in the Performance of an EEG Spectral Pattern Recognition System Developed for Halothane Anesthesia
Fig. 5-3 Improvement in the Performance of an EEG Spectral Pattern Recognition System Developed for Narcotic Anesthesia
ENFLURANE ANESTHESIA

\[ \hat{P}_c [U^*] \]
\[ \hat{P}_c [\text{MEAN}] \]
\[ \hat{P}_c [TT^*] \]

\[ \Delta = 0.128 \]
\[ \Delta = 0.288 \]

Fig. 5-4 Improvement in the Performance of an EEG Spectral Pattern Recognition System Developed for Enflurane Anesthesia
from 0.611 and 0.892, respectively, for 13 extracted features to 0.700 and 0.866 for 26 extracted features. There was a corresponding increase in the value of $\hat{P}_c[\text{Mean}]$ from 0.751 to 0.783. It can also be seen in Fig. 5-2 that $\Delta$ decreased from 0.281 initially to 0.166 finally, a relative decrease of more than 40 percent in the value of $\Delta$.

For narcotic anesthesia, the results in Fig. 5-3 show that the extraction of the 13 additional features listed in Table 5-2 resulted in a change of $\hat{P}_c[\text{H*}]$ and $\hat{P}_c[\text{U*}]$ from 0.551 and 0.788 initially to 0.613 and 0.724. This did not represent an improvement in the value of $\hat{P}_c[\text{Mean}]$, which changed from 0.670 to 0.669. However, Fig. 5-3 shows that for narcotic anesthesia the value of $\Delta$ decreased from 0.237 for 13 extracted features to 0.111 for 26 features, a decrease of more than 53 percent.

The results presented in Fig. 5-4 for enflurane anesthesia indicate the greatest improvement in performance. In Fig. 5-4 it can be seen that the values of $\hat{P}_c[\text{H*}]$ and $\hat{P}_c[\text{U*}]$ were 0.580 and 0.868 initially, but increased to 0.751 and 0.878 with the inclusion of the 13 additional features in the extracted feature set. The value of $\hat{P}_c[\text{Mean}]$ showed a significant increase, from 0.724 for 13 extracted features to 0.815 for 26 extracted features. There was also a marked decrease of more than 55 percent in the value of $\Delta$, from 0.288 to 0.128.

The confusion matrices for the systems which extracted 26 features are presented in Fig. 5-5. The improvement in the performance of these systems is evident when the matrices in Fig. 5-5 are compared with those in Fig. 5-1. To summarize, the results indicate that the initially developed EEG spectral pattern recognition systems were significantly improved by expanding the extracted feature set to include an appropriate set of additional features listed in Table 5-2. The manner in which these additional features were selected suggests some promising areas for further work. For example, a larger number and a wider variety of possible
### Halothane Anesthesia

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### Enflurane Anesthesia

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### Confusion Matrices

Fig. 5-5 Confusion Matrices for Systems Which Extracted 26 Spectral and Coherence Features. The matrices on the left resulted from performance estimation by the \( \Pi^* \) technique and those on the right resulted from performance estimation by the \( U^* \) technique.
additional features could be defined. Other feature selection criteria, such as those suggested in [110], might also be explored. In fact, the efficacy of choosing the complete extracted feature set on the basis of some feature selection criterion could be investigated. Alternatively, the effect of including more than 13 additional features in the extracted feature set might be considered. It should be recalled, however, that the maximum number of features that could actually be extracted from successive EEG pattern samples in an on-line monitoring environment must ultimately be determined by the nature of the pattern recognition system implementation.

5.3 Exploitation of Statistical Interdependencies Among Features

5.3.1 Method of Investigation

In the initial development of EEG pattern classifiers it was assumed that all of the features chosen for extraction were statistically independent. This assumption allowed the decision rule in (3.38) to be simplified and thereby reduced the amount of storage, computation time and training data required to implement various classifiers based on that decision rule. Such classifiers are optimal only if the assumption of statistically independent features is valid. Otherwise the decision rule in (3.38) will not be evaluated correctly because the estimates of \( P(d_u|C_j) \), i.e. the estimates of the class-conditional feature probabilities formed by these classifiers, will not be accurate. Therefore, if the features are not in fact statistically independent, the initially developed classifiers are suboptimal and classifiers with improved performance could theoretically be developed by exploiting statistical interdependencies among features.

To obtain some indication of the feasibility of improving
performance in this manner, it was decided to investigate the validity of
the assumption of statistically independent features. Because no practi-
cal method of directly determining the degree of statistical interdependence
among the features was available, the following property was employed to
obtain an indirect indication: if two features (or two random variables)
are statistically independent then they are uncorrelated, i.e. the lack
of correlation is a necessary condition for statistical independence
([84], pp. 211-212). It should be noted that this is a necessary but
not sufficient condition: two random variables can be uncorrelated but
not statistically independent (for an example, see [138], P. 135). How­
ever, a non-zero correlation coefficient does indicate that the features
in question are not statistically independent.

To be more specific, let \( \{\sigma_i\} \), \( 1 \leq i \leq N \), represent the set
of spectral features chosen for extraction; descriptions of the \( N=13 \)
initially chosen spectral features can be found in Table 3-1. The cor­
relation coefficient for any two features \( \sigma_i \) and \( \sigma_j \) is given by

\[
\rho_{ij} = \frac{\mathbb{E}((\sigma_i - \bar{\sigma}_i)(\sigma_j - \bar{\sigma}_j))}{\sqrt{\mathbb{E}((\sigma_i - \bar{\sigma}_i)^2) \cdot \mathbb{E}((\sigma_j - \bar{\sigma}_j)^2)}}
\]  

(5.14)

for \( 1 \leq i \leq N \) and \( 1 \leq j \leq N \). If the features are statistically inde­
dependent then, by the definition of statistical independence, (5.14)
becomes

\[
\rho_{ij} = \frac{\mathbb{E}((\sigma_i - \bar{\sigma}_i)) \cdot \mathbb{E}((\sigma_j - \bar{\sigma}_j))}{\sqrt{\mathbb{E}((\sigma_i - \bar{\sigma}_i)^2) \cdot \mathbb{E}((\sigma_j - \bar{\sigma}_j)^2)}}
\]  

(5.15)

\[ = 0, \]

i.e. the features are also uncorrelated.

To estimate the magnitudes of any intercorrelations among the
13 initially chosen spectral features, a sample correlation matrix
\[ R = (r_{ij}) \quad \text{for} \quad \begin{cases} i = 1, \ldots, N \\ j = 1, \ldots, N \end{cases} \quad (5.16) \]

was calculated for each of the three available sets of spectral feature vectors (described in section 3.2.3), which correspond to the three types of anesthesia under consideration. Let each spectral feature \( \sigma_i \) be regarded as a random variable which assumes the value \( d_{ki} \) in the \( k \)th feature vector, where \( 1 \leq k \leq S \). To calculate (5.16), the sample means

\[ \bar{d}_i = \frac{1}{S} \sum_{k=1}^{S} d_{ki}, \quad \text{for} \quad i = 1, \ldots, N, \quad (5.17) \]

were first obtained. The sample covariance matrix

\[ T = (t_{ij}) \quad (5.18) \]

was then formed by evaluating

\[ t_{ij} = \frac{1}{S-1} \sum_{k=1}^{S} (d_{ki} - \bar{d}_i)(d_{kj} - \bar{d}_j) \quad (5.19) \]

for \( \begin{cases} i = 1, \ldots, N \\ j = 1, \ldots, N \end{cases} \).

After (5.18) had been formed, the sample correlation matrix in (5.16) was computed:

\[ r_{ij} = \frac{t_{ij}}{\sqrt{t_{ii}t_{jj}}} \quad \text{for} \quad \begin{cases} i = 1, \ldots, N \\ j = 1, \ldots, N \end{cases}. \quad (5.20) \]

Only half of the elements in each sample correlation matrix were computed because \( r_{ij} = r_{ji} \), i.e. \( R \) is symmetric. Finally, the average correlation coefficient magnitude for each feature was evaluated:

\[ a_i = \frac{1}{N-1} \sum_{j \neq i}^{N} |r_{ij}|, \quad \text{for} \quad i = 1, \ldots, N. \quad (5.21) \]

The quantity defined in (5.21) indicates the average correlation of a specific feature with all other features in the set. Table 5-3 lists the values of \( a_i \), for \( i = 1, \ldots, 13 \), which were obtained for each of the three types of anesthesia under consideration.
Table 5-3 Average Correlation Coefficient Magnitudes

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5.3.2 Results and Discussion

It is evident from the results in Table 5-3 that many spectral features are strongly correlated with other features in the set. Individual correlation coefficients for specific pairs of features can be seen in Fig. 5-6. The sample correlation matrices in Figs. 5-6(a) to 5-6(c) were obtained by evaluating (5.20) with the available sets of feature vectors from halothane anesthesia, narcotic anesthesia and enflurane anesthesia, respectively. Strong correlations between several pairs of features are evident in Fig. 5-6. For example, at least eight pairs of features in each sample correlation matrix have correlation coefficient magnitudes which are greater than 0.80. In view of such strong correlations, it
Fig. 5-6a Spectral Feature Correlation Matrix for Halothane Anesthesia Data

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Fig. 5-6b Spectral Feature Correlation Matrix for Narcotic Anesthesia Data

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is apparent that the assumption of statistically independent features is generally invalid. Therefore, the initially developed classifiers are suboptimal and the development of better classifiers is theoretically feasible.

However, as indicated previously in section 3.7.2, the exploitation of all possible statistical interdependencies would increase the amount of required memory, computation time and training data by a factor of

\[ F = \frac{L^N}{L \cdot N} \]

\[ = \frac{64^{13}}{64 \cdot 13} \]

(5.22)

for a classifier with \( N=13 \) features and \( L=64 \) possible feature quantization levels. Even with a substantial reduction in both the number of statistical interdependencies taken into consideration and the number of possible feature quantization levels, the complexity of the classifiers
would be greatly increased. There would be a corresponding increase in the size of the EEG data base required to adequately train such classifiers. However, the relatively small EEG data base that was acquired in the course of this research was less than adequate for training classifiers which assumed statistically independent features. Clearly, a much larger EEG data base should be acquired before any attempts are made to exploit even the strongest of the observed feature intercorrelations.

5.4 "Nearest Subject" Scheme

5.4.1 Rationale

As stated in section 5.1, the magnitude of the difference between \( \hat{P}_e[U^*] \) and \( \hat{P}_e[I^*] \) for a specific EEG pattern recognition system provides an indication of the effect of intersubject EEG variation on system performance. Based on this measure, it is evident from the results summarized in section 5.1 that the initially developed systems were significantly affected by intersubject EEG variation. Accordingly, considerable attention was directed toward the development of schemes for adapting the classifiers in these systems to the particular EEG characteristics of the subject to be monitored. However, the small size of the available EEG data base greatly limited the types of adaptive schemes which could be studied experimentally. One intuitively appealing adaptive scheme that was investigated was based on the following notion: a classifier trained only on data from a subject with EEG characteristics which are very similar to those of the test subject should perform more reliably than a classifier trained on all available data from the subject population. This scheme is analogous to schemes which have been considered previously in the context of multifont print recognition and multiauthor character recognition problems (e.g., see [139]). Among the
subjects represented in the set of available EEG training data, the one with EEG characteristics which are most similar to those of the test subject will be referred to as the "nearest subject". Assuming the availability of training data from a sufficiently large number of subjects, it was anticipated that the performance of a classifier trained only on data from the "nearest subject" could approach the U* estimate of classifier performance. This was anticipated because the U* technique (section 3.5.4) provides an estimate of the expected system performance when both training and testing data are from the same subject.

The feasibility of employing the "nearest subject" scheme for improving classifier performance was studied in two phases. The objective of the first phase was to determine the feasibility of training a classifier on data from a subject other than the one on which the classifier would be tested. Because this was established as feasible, the second phase of the study was undertaken. The objective was to determine whether the "nearest subject" could be identified on the basis of EEG pattern samples from class \( C_0 \) alone. To gain some insight into why the second phase of the study was undertaken, it should first be recalled from the description of the data acquisition procedure given in section 2.3.3 that EEG pattern samples from class \( C_0 \) could be obtained from a particular test subject before the induction of anesthesia. If the "nearest subject" could be identified on the basis of these pre-anesthesia EEG pattern samples, then the classifier could be trained with the appropriate subset of pattern samples at that time. Accordingly, the "nearest subject" scheme would have been shown to be a practicable means of improving classifier performance.

5.4.2 Feasibility

The following training/testing paradigm was used in the initial
phase of the feasibility study: an EEG classifier was first trained on the subset of available data from subject \( j \), \( 1 \leq j \leq J \), and was then tested on the subset of available data from subject \( i \), \( 1 \leq i \leq J \). Using this paradigm a \( J \times J \) matrix was calculated for each of the three types of anesthesia; each element contained the percentage of EEG pattern samples from subject \( i \) which had been correctly classified by a classifier trained only on data from subject \( j \). Some difficulties in the computation of these matrices arose because the subsets of available EEG pattern samples from individual subjects were frequently too small and because all of the classes which were represented in the test data were not necessarily represented in the training data. The latter problem was resolved as described in section 3.5.4.

The results of the first phase of the feasibility study were encouraging. For most test subjects, one or more appropriate training subjects were identified; when the classifier had been trained on the available data from any one of these subjects, its performance on data from the test subject was superior to the \( \Pi^* \) estimate (section 3.5.3) of the expected classifier performance on data from a subject population. Consequently, the second phase of the feasibility study was undertaken to ascertain whether certain techniques could be employed to identify the most appropriate training subject, i.e. to identify the "nearest subject" to the test subject, on the basis of the available \( C_0 \) pattern samples. Consideration was given to the possibility of matching subjects by evaluating the relative dominance of alpha-band activity [82] or by using the mean \( C_0 \) spectra as templates in a clustering algorithm (e.g. [132]). Euclidean distances, likelihoods and correlations (e.g. [24]) between \( C_0 \) spectral feature vectors were also considered. However the initial results were inconclusive: the "nearest subjects" which were identified by these techniques did not consistently match the best
training subjects which had been identified in the first phase of the study.

Thus the objective of the second phase might be infeasible, i.e. it might not be possible to identify the "nearest subject" on the basis of pre-anesthesia data from C_0 only. Alternatively, the initial attempts to do so may have been hampered by the inadequacies of the available EEG data base. It has already been noted that many of the subsets of pattern samples corresponding to individual subjects were very small and/or did not contain samples from all possible classes. In addition, some subsets did not contain any artifact-free, pre-anesthesia EEG pattern samples. Finally, the relatively small number of subjects represented in the available data base may have prevented the accurate identification of the "nearest subject".

5.4.3 Discussion

The results of the initial phase of the feasibility study indicated that it was possible to train a classifier on data from one subject so that it would perform reliably on test data from another subject. However the results of the second phase of the study were equivocal: the practicability of using a small number of EEG pattern samples from class C_0 to identify the most appropriate training subject, i.e. the "nearest subject", was not established. The resolution of this issue by the techniques mentioned in section 5.4.2 would be greatly facilitated if the available EEG data base could be expanded to include a larger number of subjects. The subset of data corresponding to each subject should also be expanded to include a larger number of artifact-free, pre-anesthesia EEG pattern samples, as well as an adequate number of pattern samples from all possible classes or levels of anesthesia.

It should perhaps be recalled that the "nearest subject" scheme
for improving performance was investigated because it was thought that such a scheme could be readily employed in some practical monitoring situations. Pre-anesthesia samples of baseline EEG activity could be obtained from a particular test subject and used to identify the "nearest subject" represented in the available data base. An EEG pattern recognition system could then be trained with the available subset of pattern samples corresponding to this subject. In some anesthesia monitoring situations the identification of the "nearest subject" might not be necessary, i.e. it might be possible to train the system with EEG pattern samples from the same subject. For example, sample EEG data which had been collected from a subject during one operation might be used to train an EEG pattern recognition system for monitoring the same subject during subsequent operations. Another example involves the development of a reliable system for estimating the level of anesthesia during open-heart surgery. In this situation, sample EEG data could be collected during the initial phase of the operation and could be used to train an EEG pattern recognition system; the trained system could then be employed during the critical cardiopulmonary bypass phase of the operation, when most clinical non-EEG signs of anesthetic depth are unavailable.
CHAPTER VI

CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH

6.1 Conclusions

6.1.1 Summary

The work described in this thesis constitutes the first comprehensive investigation into the question of whether or not the level of anesthesia can be reliably estimated during surgery by means of an automatic EEG pattern recognition system. A valid methodology for conducting the research was first established and a digital EEG data base was prepared. Automatic pattern recognition techniques, in conjunction with heuristic techniques of clinical EEG analysis, were employed in the development of spectral and time domain EEG pattern recognition systems for three different types of general anesthesia. An evaluation of the performance of the initially developed systems clearly demonstrated the validity of the EEG pattern recognition approach, but also indicated that such systems are not sufficiently reliable for immediate and general clinical application. Accordingly, theoretical techniques were developed to model some relevant statistical properties of spontaneous EEG activity, with a view to improving the performance of the initially developed systems. Several factors which could adversely affect the reliable performance of EEG pattern recognition systems in general, and the initially developed systems in particular, were identified and discussed. Various schemes for improving the performance of the initially developed systems were suggested and an evaluation of the practicability of each was presented.

6.1.2 Major Original Contributions

The following items constitute the major original contributions of this work:

1) the establishment of a valid methodology for conducting research
into the question of whether or not the level of anesthesia can be estimated by EEG pattern recognition;

2) the first comprehensive application of automatic pattern recognition techniques to this problem area;

3) the formulation of nonparametric techniques for effectively estimating the performance of EEG pattern recognition systems on future EEG data;

4) the demonstration that, with specified experimental controls, it is feasible to estimate the level of anesthesia by means of automatic EEG pattern recognition;

5) the development of theoretical techniques for modelling the degree of wide-sense stationarity and Gaussianity of spontaneous EEG activity;

6) the establishment of the first model of the degree of wide-sense stationarity and Gaussianity of spontaneous EEG activity; and

7) the suggestion and evaluation of a number of promising schemes for improving the performance of the initially developed EEG pattern recognition systems.

These points are discussed in more detail in the following sections.

6.1.3 Establishment of a Valid Research Methodology

It was largely because of methodological problems that the results of many previous attempts to estimate anesthesia levels by means of visual EEG assessment were confusing and contradictory. Therefore, a considerable effort was made throughout the present research to establish a valid methodology, by identifying and controlling as many extraneous variables as possible and by ensuring that the work would be relevant to current anesthetic practice. The methodology that was established was crucial to
the success of the research reported here and should also facilitate future research in the same area.

6.1.4 Introduction of Automatic Pattern Recognition Techniques

The present work does not constitute the first attempt to employ automatic techniques in the analysis of EEG activity during surgical anesthesia. However, previous work in this area has primarily been limited to considering various schemes for EEG data compression and parameter identification (e.g., see [27-32]). The work reported here is apparently the first attempt to develop an automatic EEG pattern recognition system capable of reliably estimating clinically relevant anesthesia levels. As such, it constitutes the first comprehensive application of automatic pattern recognition techniques, including preprocessing, feature extraction, feature selection, pattern classification and performance evaluation techniques, to this problem area.

6.1.5 Formulation of Performance Estimation Techniques

The two nonparametric performance estimation techniques formulated in this work are particularly suitable for estimating the performance of EEG pattern recognition systems. In most potential applications, such as the one under consideration, the set of available EEG pattern samples is relatively small. By making efficient use of the pattern samples which are available, the two techniques estimate the performance of a given system on future EEG data from only one subject, as well as its performance on future EEG data from a subject population. More generally, because the two performance estimation techniques are nonparametric, they can be applied to a wide variety of EEG pattern recognition systems to produce meaningful and comparable performance estimates. This is a potentially significant advance because, as noted elsewhere [22,39], meaningful
performance evaluations are conspicuously absent from much of the current literature on automatic EEG pattern recognition.

6.1.6 Demonstration of Feasibility

The demonstration that it is feasible to estimate the level of anesthesia by means of automatic EEG pattern recognition is the most important single contribution of this work. It should be emphasized that feasibility in this instance does not imply immediate practicability, i.e. the initially developed EEG pattern recognition systems are not sufficiently reliable for immediate and general clinical application. It should also be noted that the demonstration of feasibility was accomplished by the implementation of a wide range of experimental controls; the effect of modifying or relaxing these controls was not investigated.

6.1.7 Development of Theoretical Modelling Techniques

Theoretical techniques have been developed for modelling the degree of wide-sense stationarity and Gaussianity of spontaneous EEG activity. This is significant because almost all methods of quantitative EEG analysis are based on certain implicit assumptions regarding the statistical characteristics of the underlying random process, particularly with respect to the extent of stationarity and Gaussianity of the process. Therefore the efficacy of alternate methods of analysis depends upon the degree to which such assumptions are justified by the characteristics of the particular ensembles of EEG segments being analysed.

6.1.8 Establishment of a Statistical Model of EEG Activity

Relatively few investigations of the statistical properties of specific EEG ensembles have been reported in the literature. In this work, a model of the degree of wide-sense stationarity and Gaussianity of spontaneous EEG activity is established. The model resolves most of the major
inconsistencies in the literature with regard to the estimated degree of Gaussianity of spontaneous EEG activity. More significantly, the model provides the first comprehensive estimates of the extent to which ensembles of spontaneous EEG segments exhibit the properties of wide-sense stationarity and Gaussianity.

6.1.9 Evaluation of Performance Improvement Schemes

An evaluation of several promising schemes for improving the performance of the initially developed EEG pattern recognition systems is presented. For example, it is shown that the performance of the initially developed spectral pattern recognition systems can be significantly improved by doubling the number of extracted features. Some improvements in the initial pattern classification algorithm are also suggested, but only preliminary feasibility evaluations are possible because of the relatively small size of the available EEG data base. Finally, it appears that some schemes which were suggested for improving performance by reducing the effect of intersubject EEG variation could be of immediate practical significance.

6.2 Suggestions for Future Research

6.2.1 Performance Improvement Schemes

Many of the suggested schemes for improving the performance of the initially developed EEG pattern recognition systems should be explored further. A few of these schemes can be readily investigated but the exploration of others, particularly some of the most promising performance improvement schemes considered in Chapter V, cannot be undertaken at present because of the inadequacy of the available EEG data base.

The inadequacy of the available EEG data base also prevented the consideration of some appealing schemes for adapting the pattern classifiers to the particular EEG characteristics of individual test subjects.
Thus, a future expansion of the EEG data base is necessary if the efficacy of some promising performance improvement schemes is to be investigated. In any future expansion of the data base for this purpose, an effort should be made to collect as many EEG pattern samples as possible, corresponding to all levels of anesthesia, from each additional subject. Also, to facilitate future investigations into the feasibility of classifier adaptation and "nearest subject" identification (see Chapter V), the subset of data corresponding to each subject should contain a large number of artifact-free, pre-anesthesia EEG pattern samples.

Before the acquisition of more sample EEG data, an inter-rater reliability study might be conducted to estimate the rate of error in clinical assessments of the level of anesthesia on the basis of the criteria employed in this work. If warranted, anesthesiologists might then be asked to suggest refinements in the criteria and improvements in the clinical assessment procedure.

6.2.2 Experimental Controls

The effect of modifying or relaxing the experimental controls which were implemented in the work reported here should be explored. Hopefully, such research would identify the major clinical sources of variability which could adversely affect the reliable performance of the EEG pattern recognition systems developed in this work. This would provide a clear indication of the variables that must be adequately controlled if such systems are to be employed in a clinical environment. In addition, research in this area might eventually result in the development of adaptive systems which could take such variables into consideration, thereby improving their reliability and extending their range of applicability.

6.2.3 Time Domain EEG Pattern Recognition Systems

The feasibility of developing more reliable time domain EEG
pattern recognition systems should be studied. On the basis of the initial results reported in this work, the best time domain systems developed for halothane anesthesia and narcotic anesthesia were slightly less reliable than the corresponding spectral systems, but for enflurane anesthesia the best time domain system was more reliable than the best spectral system. From a practical viewpoint, implementation of the time domain systems considered here would be simpler and less expensive than implementation of the corresponding spectral systems. This is primarily because many of the time domain features could be more easily extracted, e.g. an implementation of the FFT algorithm would not be necessary. Thus, both experimental results and practical considerations provide motivation for attempting to increase the reliability of the initially developed time domain systems to a clinically acceptable level. In this regard, most of the performance improvement schemes which were suggested in this work and applied to spectral systems could also be applied to time domain systems.

6.2.4 The Reliability of Visual EEG Assessment

In attempting to view the performance of automatic EEG pattern recognition systems in perspective, it would be desirable to be able to compare their reliability to the expected reliability of experienced clinical EEG raters performing the same task. Unfortunately, almost no data is available concerning the expected reliability of visual EEG assessment. The few papers which have been published in this area indicate that the reliability of visual EEG assessment, even among experienced clinical EEG raters, may be surprisingly low (e.g. see [18]). Accordingly, a future interdisciplinary study, perhaps employing the EEG data base prepared in this work, seems to be warranted in order to obtain a quantitative estimate of the expected inter-rater reliability of visual EEG
6.2.5 Modelling

The estimated statistical characteristics of spontaneous EEG activity should be exploited in a future attempt to develop more reliable EEG pattern recognition systems for monitoring the level of anesthesia. The model of spontaneous EEG activity established in this work should be of considerable value in a future reconsideration of the often arbitrary decisions which were made in the initial development of the EEG pattern recognition systems, e.g. decisions regarding the choice of analytic techniques, the duration of EEG segments to be analysed and the rate at which the estimated level of anesthesia should be updated.

The modelling techniques developed in this work could also be applied to many other ensembles of EEG activity corresponding to other states of consciousness. For example, it was noted previously that the feasibility of employing EEG pattern recognition systems to monitor the status of subjects during sleep, intensive care, coma and possible cerebral death is currently being investigated by others. In each instance, the statistical characteristics of the particular ensembles of EEG activity being analysed should be an important consideration in the development of the most appropriate monitoring system.

6.2.6 Identification of Artifact

Another area deserving further exploration, but beyond the scope of the present investigation, concerns the identification of EEG artifact. It should be recalled that artifact was defined as that component of the EEG which does not originate in the brain. Most of the visually recognizable artifact encountered in the work reported here may be attributed to interference from electrosurgical units in the operating rooms, poor
electrode contacts, eyeblinks, electrocardiographic activity, movement and muscle activity. In this work, digitized EEG segments were visually screened to eliminate those segments which contained excessive artifact. However, an EEG pattern recognition system suitable for monitoring the level of anesthesia should be capable of automatically identifying EEG pattern samples which contain excessive artifact. Therefore, the development of algorithms for the automatic identification of EEG artifact should be undertaken.
**APPENDIX A**

**LEVEL OF ANESTHESIA EVALUATION FORM**

Date: ____________________________  Page __________ of ______

Patient: __________________________ Age: ______ Weight: ______ Sex: ______

Surgical Procedure: __________________________ Anesthetist: __________________________

Premedication: __________________________ Anesthetic Agents: __________________________

Analog Tape Number: __________________________ Footage: Start ______ End ______

EEG Machine Gain: ______ LP Filter Frequency: ______ Time Constant: ______

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APPENDIX B

DESCRIPTION OF EEG DATA BASE

Information concerning the sample EEG data base for each of the three types of anesthesia is given in Table B-1. All of the digital tapes listed in Table B-1 are 9-track, IBM-compatible tapes with a density of 1600 BPI and a block size of 4096 bytes. The tapes are unlabelled. Documentation which describes how to mount and use such tapes under the Michigan Terminal System (MTS) can be obtained from the U.B.C. Computing Centre.

Table B-1 EEG Data Base

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</tbody>
</table>
Each EEG pattern sample, i.e. the digital representation of each four-channel EEG segment of 64s duration, is stored in a separate file on the appropriate tape. Each file on the tape therefore contains a total of 32768 sample values, the result of sampling four EEG channels (F3-C3, C3-O1, F4-C4 and C4-O2) at 128 Hz/channel for 64s. For programming ease, each sample value is stored in two bytes although the maximum resolution is limited to 10 bits. Each successive set of 8 bytes in a file therefore contains one sample value from each channel: F3-C3, C3-O1, F4-C4 and C4-O2, in that order. The 32768 samples in each file on the tape are grouped into 16 blocks, with 2048 samples (4096 bytes) per block.

The disk files listed in Table B-1 contain the following information about each EEG pattern sample: the sample identification number, the level of anesthesia and the subject identification number. This information is stored in integer form, with one disk file line per EEG sample, in the following FORTRAN format: (I5, 5X, 2I5). The "sample identification number" represents the number of the file on the appropriate tape which contains the sample EEG. The "level of anesthesia" represents the clinically estimated anesthesia level associated with the sample EEG. The "subject identification number" refers to the individual patient from which the sample EEG was obtained.

The following FORTRAN subroutine can be used to (i) read a sample identification number, (ii) locate the appropriate tape file, (iii) read the 16 blocks of sample data from the file, (iv) sort the
data by channels, and (v) store the sorted data in an array:

SUBROUTINE INPUT(NFLAG)

C
C NFLAG=0 INITIALLY; NFLAG=1 AT TAPE END
C INDEXF CONTAINS CURRENT FILE NO
C LUNIT INDICATES TAPE LOGICAL UNIT NO
C
REAL DATIN(4,8192)
COMMON /DATIN/ DATIN
INTEGER*2 BLOCK(2048), LEN1
INTEGER INDEXF /0/
LUNIT=1
NSKIP=0
C
C READ THE FILE NO
READ(5,20,END=10)NFILE
20 FORMAT(15)
WRITE(6,12)NFILE
12 FORMAT('****',15)
C
C PREPARE TO SKIP TO THE APPROPRIATE FILE
ITEMP=NFILE-INDEXF-1
CALL SKIP(ITEMP,NSKIP,LUNIT)
INDEXF=NFILE-1
C
C READ FILE DATA AND STORE IN ARRAY
DO 14 IBLK=1,16
INDX=(IBLK-1)*512
CALL READ(BLOCK,LEN1,LUNIT,&10)
DO 14 ICH=1,4
DO 14 ISAM=1,2048,4
IR=INDX+1+(ISAM-1)/4
IRR=(ICH-1)+ISAM
14 DATIN(ICH,IR)=BLOCK(IRR)
C
RETURN
10 NFLAG=1
RETURN
END
APPENDIX C

COMPUTATION OF EEG SPECTRA

This program computes the power spectra and coherence spectrum for two selected channels of EEG data.

Input:
- **LUNIT 1**: Input data tape (see Appendix B)
- **LUNIT 5**: File containing data labels

Output:
- **LUNIT 8**: Power spectrum for channel "A"
- **LUNIT 9**: Power spectrum for channel "B"
- **LUNIT 10**: Coherence spectrum

Last update: January 6, 1975

```
0001 INTEGER NCHANE/1/,NCHANE/2/
0002 COMPLEX TRAN (2049),TR (2049)
0003 REAL DATA (4096), LATE (4096)
0004 COMMON /TRAN/ TRAN,TR
0005 EQUIVALENCE (TRAN,DATA)
0006 EQUIVALENCE (TR,LATE)
0007 REAL LATN(4,8192)
0008 COMMON /DATIN/ DATIN,INDEXF,NFILE
0009 NFLAG=0
0010 INDEXF=0
0011 NSAMP=8192
0012 N=4096
0013 SRATE=64.

GET ALL FOUR EEG CHANS FROM A 64 SEC SA AND PUT IN DATIN

0014 CALL INPUT(NFLAG)
0015 IF (NFLAG.EQ.1) GO TO 4

COPY TWO CHANS AT 64 SA/SEC (NOT THE ORIG 128/SEC)

0016 DO 1 J=1,NSAMP,2
0017 DATA ((J-1)/2,J)=DATIN(NCHANE,J)
0018 DATE((J-1)/2,J+1)=LATIN(NCHANE,J)
0019 1 CONTINUE

CALCULATE POWER SPECTRA AND COHERENCE (AND OUTPUT SAME)

0020 CALL COHEF(N,SRATE,NFILE)
0021 GO TO 3

STOP

0022 END

SUBROUTINE INPUT(NFLAG)

0001 READ (5,20,END=10) NFILE

0002 DATA (J,0,192)=REAL DATA(J)

0003 COMMON /DATIN/ DATIN,INDEXF,NFILE

0004 INTEGER *2 BLOCK(2048),LEN1

0005 LUNIT=1

0006 NSKIP=0

READ THE FILE NO

0007 READ (5,20,END=10) NFILE

0008 FORMAT(15)

0009 WRITE (6,12) NFILE

0010 FORMAT(* *****,15)

0011 CALL SKIP (ITEMP,NFILE,INDEXF)

0012 CALL SKIP (ITEMP,NFILE,LUNIT)

0013 INDEXF=NFILE-1

0014 DO 14 NSKIP=1,16
```
INDX= (IBLK-1) * 512
CALL READ (BLOCK, IEN1,0, IEN1,LUNIT,F10)
DO 14 ISC=1,4
IQ=INDX+1*(ISA=1)/4
IB=INDX+1*(ISA=0)/4
14 DATIN (ISC,IP)=BLOCK (IRJ)
RETURN
C
IFLAG=1
RETURN
END

SUBROUTINE COHER (N,SB ATI,NIILI)
COMPUTES THE SPECTRA AND COHERENCE VIA METHOD OF EUMERHURTH ET AL.
IEEE TRANS AUDIO EEC+70.
C
COMMON TRANS(2049),TRANS(2049)
REAL DATA (4096), DATAT (4096)
REAL SPECT (4096), SMOOTH (4096)
COMPLEX XSPEC(2049),SB3 (2049)
INTEGER NN (1)

GET FOURIER TRANSFORM VIA FFT
KN(1) = N
ISIGN=-1
CALL FOUR2(DATB.NN,1,ISIGN,0)

HP FILTERING
LI=32
LB=5
DO 3 J=1,LI
TR (J) = (0.,0.)
TB (J) = (0.,0.)
DO 4 J=1,LB
TR (LI*J) = TR (LI*J) * (FLOAT (J)/FLOAT (LB) )
TB (LI*J) = TB (LI*J) * (FLOAT (J)/FLOAT (LB) )
3 4 CONTINUE

OBTAIN RAW SPECTRAL ESTIMATES (INCL CROSS-SPECTRA)
FACT0R= 1./ (SRATE*F1CAT (N) )
BID=N/2*1
DO 99 1=1,BID
T=CABS (THAN (I))
IT=CABS (TB (I) )
XSPEC (I)= (CONJG (TH (I)) *TRAN (I))
DATA (I) = T*T*FACT0R
XSPEC (I) = XSPEC (I) *FACT0R
CONTINUE
SMOOTHED SPECTRAL ESTIMATES OBTAINED VIA SQUARE HINDCH
THE FIRST AND LAST 8 ECINTS ARE NOT SMOOTHED
DO 52 I=1,8
SM2 (I) = XSPEC (I)
SM3 (I) = SMOOTH (I)
SM2 (I) =SM2 (I) * (I-MID-7) /(MID-7)
SM3 (I) =SM3 (I) * (I-MID-7) /(MID-7)
52 CONTINUE
SM2 (I) =SM2 (I) * (I-MID-7) /(MID-7)
SM3 (I) =SM3 (I) * (I-MID-7) /(MID-7)
11 CONTINUE

COHERENCE CALCULATION
DO 1012 I=1,32
SV2=0.
SV3=0.
DO 51 J=1,16
SV2=SV2+DATA (I-1)+J-7
SV3=SV1+XSPEC (I-1)+J-7
51 CONTINUE

CONTINUE

DO 1012 I=1,32
SV2=0.
SV3=0.
DO 51 J=1,16
SV2=SV2+DATA (I-1)+J-7
SV3=SV1+XSPEC (I-1)+J-7
51 CONTINUE

CONTINUE

DO 1012 I=1,32
SV2=0.
SV3=0.
DO 51 J=1,16
SV2=SV2+DATA (I-1)+J-7
SV3=SV1+XSPEC (I-1)+J-7
51 CONTINUE

CONTINUE
IF (SMOOTH(I).EQ.0.) SMOOTHE(I)=.000001

DO 101 I=1,256
    SM2 =0.
    SUM4=0.
    SUM=0.

DO 101 J=1,8
    SUM2=SUM2+SM2((I-1)*8+J)

SUM=SUM+SUM4((I-1)*8+J)

SUB=SUM+SMOOTH((I-1)*8+J)

SF(I)=SUB/SM2/DENOM

WRITE (8,58) NFILE

WRITE (10,58) NFILE

WRITE (9,59) NFILE

RETURN

C

OUTPUT PHASE BEGINS

WRITE (8,58) NFILE

WRITE (9,59) NFILE

RETURN

END
APPENDIX D
SPECTRAL FEATURE EXTRACTION PROGRAM

This program calculates one 44-element autospectral feature vector corresponding to each 64 sec EEG pattern sample.

**INPUT:**

- LIMIT 1: Spectral data from Chan 1 (E3:C3)
- LIMIT 2: Spectral data from Chan 2 (C3:C3)
- LIMIT 3: Spectral data from Chan 3 (E4:C4)
- LIMIT 4: Spectral data from Chan 4 (C4:C4)
- LIMIT 5: Indexing data for each EEG segment (F/LEV/PTNT)

**OUTPUT:**

- LIMIT 11: Feature vectors
- LIMIT 6: Error msgs
- LIMIT 7: Spectral feature elements calculated for each EEG channel

**Limitations:**

- No more than 500 pattern samples
- Only data from 64 sec spectral calculations

**Notes:**

1. Data was prepared in fmt specified in "APPENDIX C"
2. Output file (LIMIT 11) must be sequential
3. Spectrum is assumed to exist from 0-32 Hz, with 8 samples/Hz

**Last Update:**

NCV 23 1974

---

### Subroutine SPECTF (LUNIT)

```fortran
REAL DATA (256), FEATUR (44)
COMMON /COM/ DATA, FEATUR

N = (IUNIT-1) * 11

ENERGY:

DO 1 I=2,256

ENERGY(I)=ENERGY(I)+DATA(I)

STOP
```

---

This subroutine calculates the energy of the EEG data. It sums the square of the data points for each channel, multiplying the sum by a scaling factor to get the energy value.

**Energy Calculation:**

The energy of each EEG channel is calculated as follows:

```fortran
ENERGY(I)=ENERGY(I)+DATA(I)^2
```

**Stop Statement:**

The subroutine ends with the statement `STOP` to indicate the completion of the energy calculation for all channels.
DO 5 K=2, 32
5 FEATUB ((M+2)) = FEATUB ((M+3)) • DATA(K)
C
DO 6 K=33,64
6 FEATUB ((M+3)) = FEATUB ((M+4)) • DATA(K)
C
DO 7 K=65,104
7 FEATUB ((M+4)) = FEATUB ((M+5)) • DATA(K)
C
DO 8 K=105, 120
8 FEATUB ((M+5)) = FEATUB ((M+6)) • DATA(K)
C
DO 9 K=121, 256
9 FEATUB ((M+6)) = FEATUB ((M+7)) • DATA(K)
C
DO 10 K = 2, 10
10 FEATUB ((M+7)) = FEATUB ((M+8)) • DATA(K)
C
END
APPENDIX E

TIME DOMAIN ANALYSIS AND FEATURE EXTRACTION PROGRAM

C

APPENDIX E TIME DOMAIN ANALYSIS AND FEATURE EXTRACTION PROGRAM

C

THIS PROGRAM CALCULATES ONE 10-ELEMENT TIME DOMAIN FEATURE VECTOR

C FOR EACH 64 SEC EEG PATTERN SAMPLE.

C INPUT:

*UNIT 1: INPUT DATA TAPE (SEE APPENDIX B)

*UNIT 4: FILE CONTAINING DATA LABELS

*UNIT 5: FILE CONTAINING DATA LABELS

C OUTPUT:

*UNIT 7: OUTPUT FILE FOR 10-ELEMENT FEATURE VECTORS

C LAST UPDATE:

JANUARY 20 1975

C

0001 INTEGER NCHAN/3/,NCHANB/4/

0002 COMPLEX TRAN (2049)

0003 REAL DATA (U096),SRATE/64./

0004 COMMON TRAN

0005 EQUIVALENCE (TRAN,DATA)

0006 REAL LATIN (N,8192)

0007 COMMON / LATIN / LATIN,INITIX,NOFILE

0008 INTEGER NLAG/0/,NSEC/64/,NSAMP/8192/,N/4096/

0009 INDEXF=0

0010 PRN=N

0011 PREQLP=16

C GET ALL FOUR EEG CHANS PER A 64 SEC SA AND PUT IN LAT

0012 CALL INPUT (NFLAG)

0013 IF (NFLAG.EQ.1) GO TO 4

0014 DO 1 J=1,NSAMP,2

0015 DATA ( (J-U2)/2 )= LATIN (NCHANJA,J)

0016 CALL FILTLP (N,SRATE,FREQLP)

0017 CALL NORM (DATA,N,SM2,SM3,SM4)

0018 CALL PKEW (TM,SM2,SKREW,VARSK,SDSK,SN)

0019 CALL RKURT (FM4,SM2,FRK,FRKRT,VARFRK)

0020 XXXM2=SQRT (SM2)

0021 CALL ZCROSS (DATA,N,ZRATE,NSEC)

0022 CALL DCROSS (DATA,N,DRATE,NSEC)

0023 ZRATE=ZRATE/2.

0024 ZRATE=ZRATE/2.

C COPY THE OTHER CHAN AND COMPUTE TIME DOMAIN FEATURE EL'S

0025 DO 11 J=1,NSAMP,2

0026 DATA ( (J-U2+1))/2=LATIN (NCHANJB,J)

0027 CALL FILTLP (N,SRATE,FREQLP)

0028 CALL NORM (DATA,N,SM2,SM3,SM4)

0029 CALL PKEW (TM,SM2,SKREW,VARSK,SDSK,SN)

0030 CALL RKURT (FM4,SM2,FRK,FRKRT,VARFRK)

0031 XXXM2=SQRT (SM2)

0032 CALL ZCROSS (DATA,N,ZRATE2,NSEC)

0033 CALL DCROSS (DATA,N,DRATE2,NSEC)

0034 ZRATE2=ZRATE2/2.

0035 ZRATE2=ZRATE2/2.

C COMPLETE AND WRITE ONE FEATURE VECTOR

0036 READ (4,12)NF,LEV,NPTNT

0037 IF (NFLAG.EQ.1) GO TO 4

0038 WRITE (7,13)XXXM2,ZRATE,ZRATE2,DRATE,SKRE,

0039 DCROSS,1DKURT2,N,LEV,NPTNT

0040 GO TO 3

C

END
SUBROUTINE INPUT(#FLAG)

THIS SubR was adapted from COST 4.5 Jan 10 1978

REAL DATIN(0,0192)

COMMON /DATIN/ DATIN,INDEXF,NFILE

INTEGER*2 BLOCK(2048),LTM1

LUNIT=1

NSKIP=0

READ THE FILE NO.

READ(5,20,END=10) NFILE

20 FORMAT (15)

WRITE(6,12) NFILE

12 FORMAT (" ")

PREPARE TO SKIP TO THE APPROPRIATE FILE.

ITEMP=FILE-INDEX-1

CALL SKIP(ITEMP,NSKIP,LUNIT)

INDEXF=FILE-1

DO 10 IBLK=1,16

INDX=(IBLK-1)*512

CALL READ(BLOCK,LEN1,0,LTNE1,LUNIT,CIO)

DO 10 ICIL=1,10

IR=INDX*I+1

IRR=ICIL-1*ICIL

DATIN(ICH,IR) = BLOCK(IBM)

RETURN

NSKIP=1

RETURN

END

SUBROUTINE ZCROSS(X,N,AVCZC,NSIC)

THIS SubR counts the no. of zero crossings in an array of nob.

SIZE and returns the average crossing rate

REAL X(N)

VC=0.

LIN=N-1

DO 1 T=1,LIN

IF (X(I).GE.0.) GCTE 2
SUBROUTINE DCROSS (DEL, N, AVERDC, NSEC)

THIS SUBROUTINE COMPUTES THE AVER ZEBO CROSSING RATE OF THE DERIVATIVE OF THE SIGNAL STORED IN ARRAY X.

NOTE: CONTENTS OF ARG ARRAY ARE CHANGED.

REAL DEL (N)

N=N-1

DO J=1,N

DEL (J) = DEL (J-1) - DEL (J-2)

CONTINUE

AV2DC=DEL/NSEC

RETURN

END

SUBROUTINE NORM (ARRAY, N, AVER, VAR, TM, IN)

C

C THIS PROGRAM ACCEPTS AN ARRAY OF SIZE TC N=61*128=8192 AND

C COMPUTES THE MEAN, VARIANCE OF THE SAMPLE AND

C COMPUTES THE THIRD AND FOURTH MOMENTS,

DIMENSION ARRAY (N)

SUM=0.

SVAR=0.

SS3=0.

SSU=0.

BN=N

C

C MEAN:

DO J=1,N

SUN=SUM+ARRAY (J)

AVER=SUN/BN

CONTINUE

C

C SAMPLE VARIANCE:

DO J=1,N

Z=ARRAY (J)-AVER

SVAR=SVAR+Z*Z

SUM=SUM

SVAR=SVAR/(BN-1)

SDEV=SQRT (SVAR)

CONTINUE

C

C COMPUTE THIRD AND FOURTH MOMENTS...

DO J=1,N

SM=SM+Z**3

SS3=SS3+ARRAY (J)**3

SS4=SS4+ARRAY (J)**4

CONTINUE

C

RETURN

END

SUBROUTINE PSKEW (TM3, SM2, SKEW, VARSK, SDSK, RN)

C

C THIS SUBROUTINE CALCULATES THE MOMENT OF SKEWNESS AND ITS

C VARIANCE AND STANDARD CIVILIZATION

C TM3 IS THE 3RD MOMENT

C SM2 IS THE SECOND MOMENT (VARIANCE)

C SKEW WILL CONTAIN THE MEASURE OF SKEWNESS
VARSK WILL CONTAIN ITS VARIANCE
SDSK WILL CONTAIN ITS STANDARD DEVIATION
SK IS THE NUMBER OF OBSERVATIONS
TK IS K3
SK IS K2

TK = (RN*RN/((RN-1.)*(RN-2.)))*SK3
SDSK = TK/SCRT(SK*3)

C CALCULATE VARIANCE OF SKEWNESS
VARSK = 6. * RN * (RN-1.)
VARSK = VARSK/((RN-2.)*(RN+1.)*(RN+3.))

C GET S.D. OF SKEWNESS
SDSK = SQRT(VARSK)
END

SUBROUTINE RKURT(FH4,SH2,FN,tKURT,VAKRT,SDKRT)

THIS SUBROUTINE CALCULATES A MEASURE OF KURTOSIS
FM4 IS THE 4TH MOMENT ABOUT THE MEAN
SM2 IS THE SECOND MOMENT ABOUT THE MEAN (VARIANCE)
RN IS THE NUMBER OF OBSERVATIONS
DKURT WILL CONTAIN THE MEASURE OF KURTOSIS
VAKRT WILL CONTAIN ITS VARIANCE
SDKRT WILL CONTAIN ITS STANDARD DEVIATION
FK IS K4
TK IS K2

FK = RN*RN/((RN-1.)*(RN-2.)*(RN-3.))
FK = FK/((RN+1.)*SN4 - 3. *(RN-1.)*SM2*SM2)
TK = (RN/((RN-1.))*SK2
DKURT = FK/(TK*TK)

C CALCULATE THE VARIANCE OF THE KURTOSIS
VAKRT=24. * RN * (RN-1.) * (RN-1.)
VAKRT=VAKRT/((RN-3.)*(RN-2.)*(RN+1.)*(RN+3.))

C GET STANDARD DEVIATION OF THE KURTOSIS
SDKRT=SCRT(VAKRT)
END
APPENDIX F

PERFORMANCE ESTIMATION BY THE II* TECHNIQUE.

THIS PROGRAM CAN BE USED TO ESTIMATE THE PERFORMANCE OF SPECTRAL AND TIME DOMAIN PATTERN RECOGNITION SYSTEMS BY THE II* TECHNIQUE.

INPUT:
- LIMIT 4: FEATURE VECTORS AND LABELS
- LIMIT 5: QUANTIZER PARAMETERS

OUTPUT:
- LIMIT 6: ALL CUTOFF PARAMETERS

PARAMETERS:
- NEL = NUMBER OF ELEMENTS IN FEATURE VECTOR
- NQUANT = NUMBER OF POSSIBLE QUANTIZATION LEVELS
- SD = NUMBER OF ST. DEV. ALLOWED FOR FEATURE VARIATION
- IPROB = 0 IF EQUAL A PRIORI CLASS PROB'S ARE TO BE USED
- IPRINT = 0 TO PRINT TEST RESULTS AT EACH STEP

LAST UPDATE: OCTOBER 19 1974

REAL DATA (500, 80)
INTEGER IX(500,80), LEV(500), N:MF(500)
COMMON /CHAIN/ IX, LEV, NUMP
COMMON /PROP/ DATA
COMMON /CHAIN/

REAL DATA (500, 80), LEV(500), N:MF(500)
COMMON /CHAIN/ IX, LEV, NUMP
COMMON /PROP/ DATA
COMMON /CHAIN/

INITIALIZATION:
- NEL=80
- DO 1 J=1,501
- 1 READ (U, 2, E=ID = 3) (DATA (I,J) ,J=1 .NEL) , LEV (I) , NUMP (I)

FORMAT(80F1U.6,0X,2I0)

NSAMP = I-1

INITIALIZATION OF PARAMETERS:
- IPROC=1
- IPRINT=1
- 56 READ(5,55,TEND=50) NCUANT,SE
- 55 FORMAT(I3,F5.0)

*PI METHOD* OF PERFORMANCE ESTIMATION:
- LOW=1
- HIGH=HIGH
- IF (HIGH,ILEC, NSAMP) GC TO 7
- IF NUMP ((HIGH+1)),N, NUMP (LOW)) GC TO 7
- HIGH=HIGH
- GO TO 8
- CALL QUANT (LOW, HIGH, NSAMP, NEL, NCUANT, SD)
- CALL TEST (LOW, HIGH, NEL, IPRINT, IPROB)
- CALL PRINT (NSAMP, NEL, NCUANT, IFRCE)
- CALL PRINT (NSAMP, NEL, NCUANT, IFRCE)
- CALL PRINT (NSAMP, NEL, NCUANT, IFRCE)

END

SUBROUTINE QUANT (IMAT, IEND, NSAMP, NEL, NCUANT, SE)
CONSIDERS ALL FEATURE VALUES FROM THE TRAINING DATA (IE, NOT
SAMPLES FROM IMAT,...,IEND) FOR EACH FEATURE. THE MIN, MAX, MEAN
AND ST. DEV. ARE CALCULATED. ALL FEATURE VALUES ARE THEN QUANTIZED

INTEGER IX(500,80)
REAL DATA (500, 80), LEV(80), N:MF(80), ESUM(80), IFRCE(80)
COMMON /PROP/ DATA
COMMON /CHAIN/ IX

INITIALIZATION:
- NSAMP=NSAMP-1-TEND+IMID
- DO 1 J=1,80
- 1 DMIN(J)=999999.
- DMAX(J)=999999.
FIND MIN, MAX, MEAN AND SD. DEV. FOR EACH FEATURE:

DO 1 I=1,NSAMP

IF (DATA(I,J).GT.UMAX(J))DMAX(J)=DATA(I,J)

DO 2 J=1,NEI

IF (INHIG.EQ.NSAMP), CR. (IEND.EQ. NSAMP) GO TO 8

LOW=IEND*1

IF (DATA(I,J).LT. CMN(J)) CMN(J)=DATA(I,J)

LHOUR=IEND*1

ONE=NSAMP

GO TO 5

FIND CLASS WIDTH FOR LINEAR QUANTIZATION:

DO 6 J=1,NEI

DAVER (J) = DAVER (J)/RNSAMP

DSDEV (J) = 50RT ((DSCEV (J) - (RNSAMP* CAVER (J) * CAVER (J) )) / (RNSAMP- 1 .) )

IF (DMIN (J) .LT. (DAVER (J) -SE*DSDEV  (J)))DKIN(J)  = EAVER(J)-SE*DSDEVIJ)

IF (DMAX (J) .GT. (CAVER (J) » CS

QUANTIZE ALL SAMPLE DATA (TRAINING AND TESTING DATA):

DO 10 1=1,NSAMP

DO 10 J=1,NEI.

IX (I ,J) = (CAT A (I,J) - CM N (J) )/F (J)

IF (IX (I,J).GT.NQUANT)IX (I,J)  = NQUANT

RETURN

SUBROUTINE TRAIN(LCW,IESC,NSAMP,NEL,NQUANT)

INTEGER IX(500,80)  .LEV (500)

REAL PRCOND (5, 80, 128) .PRCLAS

COMMON /CMAIN/ IX,LEV

COMMON /TST/  PRCOND,PRCLAS

INITIALIZATION:

DO 1

DO 1 K=1,5

PRCLAS (K)=0.

DO 1 J=1,NEI

DO 1 K=1,NQUANT

PRCCND (I, J, K) =0.

PRCCND (I,J,K) = PRCCND (I,J,K) +1.

PRCLAS (I) = PRCLAS (I) +1.

RETURN

SUBROUTINE TEST ( LC W, IFC N, NEI, IP R INT , IPSO E)

INTEGER IX  (500 ,80) ,1 EV (500) , HUMP (500)

SEAL CLASijM (5.5),PRCOND (5,MO, 12H) ,ri)CL(,S (5) , PTEST

CHECK TRAINING DATA FOR UNREPRESENTED CLASSES:

DO 9

IF (PRCLAS (K).EC.0.)WRITE

APPLY BAYES ESTIMATION  PROCEDURE TC PROB MATRICES:

S=PBCLAS(1)»PRCLAS|2)*FFCLAS

DO 7 1=1,5

DO 8 J=1,NEI

DO 8 K= 1, NQUANT

PRCCND (I, J, K) = (PRCCND (I,J,K) *1.  ) / (PRCLAS ( 3) « FLO AT (NQUANT) )

PRCLAS (I) = (PRCLAS (I) *1.) / (S*5.)

RETURN

SUBROUTINE TEST (LC W, IFC N, NEI, IP R INT , IPSO E)

SUBROUTINE TRAIN(LCW,IESC,NSAMP,NEL,NQUANT)
0004 COMMON /CPAIN/, IX, IY, NUM
0005 COMMON /TST/, PROCM, PCLAS
0006 COMMON /PRNT/, CLAS

C INITIALIZATION ON FIRST CALL:

0007 IF (LOW. NE. 1) GO TO 1
0008 DO 1 I=1,5
0009 DO 2 J=1,5
0010 2 CLAS (I,J)=0.

C GET CLASS CONDITIONAL PROB ESTIMATES FOR TESTING SAMPLE(S):

0011 DO 10 I=LOW, IEND
0012 DO 3 K=1,5
0013 3 PTEST (K)=0.
0014 DO 4 ICLASS=1,5
0015 4 DO 5 IEL=1, NEL
0016 5 PTEST (ICLASS)=PTEST (ICLASS) + PORG (ICLASS, IEL, IX (I, IEL))

C INCLUDE A PRIORI CLASS PROB'S AND ESTIMATE ANESTHESIA LEVEL:

0017 IF (IPRINT.EQ.0) GO TO 5
0018 DO 6 TCLASS=1,5
0019 6 DO 7 ICLASS=1,5
0020 7 IF (PTEST (ICLASS), GT, 5) PTEST (ICLASS)=PTEST (ICLASS) * ALG (FBCONC (ICLASS, IEL, IX (I, IEL)))

C UPDATE CLASSIFICATION MATRIX AND PRINT RESULTS IF DESIRED:

0023 II=LEV (I)+1
0024 CLASS (II, ICLASS)=CLASS (II, ICLASS)+1
0025 IF (IPRINT.NE.0) GO TO 10
0026 IR=1
0027 FORMAT (6, 10X, ' SUMMARY '////)
0028 WRITE (6, 16)

C SUBROUTINE PRINT (NSAMP, NEL, NQUANT, IPORB)

0032 RETURN

C END

0034 WRITE (6, 11)

C REAL CLAS (5, 5), TOTAL (5)

0035 COMMON /FRMT/, CLAS
0036 COMMON /PRNT/, NSAMP

0038 WRITE (6, 17)

C END

0040 RETURN

C WRITE (6, 19)

C FORMAT (6, 'CLASSIFICATION MATRIX: *//)

0041 WRITE (6, 20)

C DO 19 IX=1,5
0042 DO 20 JX=1,5
0043 20 TOTAL (I) = TOTAL (I) + CLAS (J, I)
0044 WRITE (6, 21) (CLAS (J, I), J=1,5), TOTAL (I)

C WRITE (6, 22)

C FORMAT (6, 'CLASS (I,J)/TOTAL (I)*100.

C WRITE (6, 23)

C FORMAT (6, 'CLASS (I,J)/TOTAL (I)*100.

C WRITE (6, 24)

C WRITE (6, 25)

C WRITE (6, 26)

C WRITE (6, 27)

C WRITE (6, 28)

C WRITE (6, 29)

C WRITE (6, 30)

C WRITE (6, 31)

C WRITE (6, 32)

C WRITE (6, 33)

C WRITE (6, 34)

C WRITE (6, 35)

C WRITE (6, 36)

C WRITE (6, 37)

C WRITE (6, 38)

C WRITE (6, 39)

C WRITE (6, 40)

C WRITE (6, 41)
APPENDIX G

PERFORMANCE ESTIMATION BY THE U* TECHNIQUE

THIS PROGRAM CAN BE USED TO ESTIMATE THE PERFORMANCE OF SPECTRAL AND
TIME DOMAIN EEG PATTERN RECOGNITION SYSTEMS BY THE U* TECHNIQUE.

INPUT:
* LIMIT 4: NUMBER OF THE SUBJECTS TO BE CONSIDERED
* LIMIT 5: FEATURE VECTORS AND LABELS

OUTPUT:
* LIMIT 6: ALL OUTPUT

PARAMETERS:
NEL = NUMBER OF ELEMENTS IN FEATURE VECTOR
NQUANT = NUMBER OF POSSIBLE QUANTIZATION LEVELS
SE = NUMBER OF ST. LIV. THAT ALLOWED FOR FEATURE VARIATION
IPBMIN = 0 IF EQUAL PRIOR CLASS PROBS ARE TO BE USED
IPRINT = 0 TO PRINT TEST RESULTS AT EACH STEP

LAST UPDATE:
JANUARY 20 1975

REAL DATA (SOO, 80)
INTEGER IX (SOO, 80), LEV (SOO), NUHP (SOO)
COMMON /CIF/ IX, LEV, SUMP

INITIALIZATION OF PARAMETERS:
NEL=13
NQUANT=64
IPBMIN=0
IPRINT=0

READ IN ALL AVAILABLE FEATURE DATA:
1 READ(4,10,END=12) NTNT
10 FORMAT (F5.0)

DO
5 NSTP=NSTP+1
2 IF (NUHP(I) .NE. NTNT) GO TO 1
1=1
GO TO 1
3 NSTP=NSTP-1

*U* METHOD* CF PERFORMANCE ESTIMATION:
DO 5 I=1, NSTP

CALL QUANT(I, I, NSTP, NEL, NQUANT)

CALL TRAIN(I, I, NSTP, NEL, NQUANT)

CALL TEST(I, I, NSTP, NEL, NQUANT, IPBMIN)

CALL PRINT(NSTP, NEL, NQUANT, IPBMIN)

RE KIND 5
GO TO 11

STOP
END

SUBROUTINE QUANT(INO, IX, NSTP, NEL, NQUANT, SD)

CONSIDERS ALL FEATURE VALUES FROM THE TRAINING DATA FOR EACH FEATURE. THE MIN, MAX, MEAN
AND ST. DEV. ARE CALCULATED. ALL FEATURE VALUES ARE THEN QUANTIZED

INTEGER IX (500, 80)
REAL DATA (SOO, 80), DMIN (SOO), DMAX (SOO), DAVER (SOO), DSDEV (SOO), F (SOO)

COMMON /PERCG/ DATA

INITIALIZATION:
END

COMMON /CMVN/ IX

REAL DATA (500, 80), LEV (500), NUHP (500)
COMMON /PERCG/ DATA

INPUT:
* LIMIT 4:

REAL DATA (500, 80), LEV (500), NUHP (500)

COMMON /CMVN/ IX, LEV, SUMP

COMMON /PERCG/ DATA

INPUT:
* LIMIT 4:

REAL DATA (500, 80), LEV (500), NUHP (500)

COMMON /CMVN/ IX, LEV, SUMP

COMMON /PERCG/ DATA

INPUT:
* LIMIT 4:

REAL DATA (500, 80), LEV (500), NUHP (500)

COMMON /CMVN/ IX, LEV, SUMP

COMMON /PERCG/ DATA

INPUT:
* LIMIT 4:

REAL DATA (500, 80), LEV (500), NUHP (500)

COMMON /CMVN/ IX, LEV, SUMP

COMMON /PERCG/ DATA

INPUT:
* LIMIT 4:

REAL DATA (500, 80), LEV (500), NUHP (500)

COMMON /CMVN/ IX, LEV, SUMP

COMMON /PERCG/ DATA

INPUT:
* LIMIT 4:

REAL DATA (500, 80), LEV (500), NUHP (500)

COMMON /CMVN/ IX, LEV, SUMP

COMMON /PERCG/ DATA

INPUT:
* LIMIT 4:

REAL DATA (500, 80), LEV (500), NUHP (500)

COMMON /CMVN/ IX, LEV, SUMP

COMMON /PERCG/ DATA

INPUT:
* LIMIT 4:

REAL DATA (500, 80), LEV (500), NUHP (500)

COMMON /CMVN/ IX, LEV, SUMP

COMMON /PERCG/ DATA

INPUT:
* LIMIT 4:

REAL DATA (500, 80), LEV (500), NUHP (500)

COMMON /CMVN/ IX, LEV, SUMP

COMMON /PERCG/ DATA

INPUT:
* LIMIT 4:

REAL DATA (500, 80), LEV (500), NUHP (500)

COMMON /CMVN/ IX, LEV, SUMP

COMMON /PERCG/ DATA

INPUT:
* LIMIT 4:

REAL DATA (500, 80), LEV (500), NUHP (500)

COMMON /CMVN/ IX, LEV, SUMP

COMMON /PERCG/ DATA

INPUT:
* LIMIT 4:

REAL DATA (500, 80), LEV (500), NUHP (500)

COMMON /CMVN/ IX, LEV, SUMP

COMMON /PERCG/ DATA

INPUT:
* LIMIT 4:

REAL DATA (500, 80), LEV (500), NUHP (500)

COMMON /CMVN/ IX, LEV, SUMP

COMMON /PERCG/ DATA

INPUT:
* LIMIT 4:

REAL DATA (500, 80), LEV (500), NUHP (500)
C
FIND MIN, MAX, MEAN AND STD. DEV. FOR EACH FEATURE:
0015
5 DO 3 I=LOW, HIGH
0016
DO 3 J=1, NEL
0017
IF (DATA(I,J) .LT. DMIN(J)) DMIN(J) = DATA(I,J)
0018
IF (DATA(I,J) .GT. DATA(J)) DMA(J) = DATA(I,J)
0019
DAVE(J) = DAVE(J) + DATA(I,J)
0020
DSDE(J) = DSDE(J) * (DATA(I,J) - DAVE(J)) / (N - 1)
0021
IF (J .GE. Nsamp) GO TO 4
0022
LOW = I + 1
0023
HIGH = NSAMP
0024
GO TO 5

C
FIND CLASS WIDTH FOR LINEAR QUANTIZATION:
0025
4 DO 6 J=1, NEL
0026
DATA(J) = DAVE(J) / N
0027
ESDEV(J) = DSDE(J) / ((N - 1) * Nsamp)
0028
IF (DATA(J) .LT. DMIN(J)) GO TO 9
0029
IF (DATA(J) .GT. DMA(J)) GO TO 9
0030
F(J) = (DATA(J) - DMIN(J)) / DSDE(J)
0031
QUANTIZE ALL SAMPLE DATA (TRAINING AND TESTING DATA):
0032
DO 10 I=1, Nsamp
0033
DO 10 J=1, NEL
0034
IX(I,J) = (DATA(I,J) - DMIN(J)) / F(J)
0035
IX(I,J) = IX(I,J) * N
0036
IF (IX(I,J) .LT. 1) IX(I,J) = 1
0037
IF (IX(I,J) .GT. NQUANT) IX(I,J) = NQUANT
0038
RETURN
0039
SUBROUTINE TRAIN (LOW, IEND, NSAMP, NEL, NQUANT)

C
INITIALIZE:
0040
DO 1 I=1, 5
0041
IFLAG(I) = 0
0042
DO 1 I=1, NEL
0043
IFLAG(I) = 0
0044
DO 1 I=1, NSAMP
0045
I = I + 1
0046
RETURN

C
USZ TRAINING DATA TO ESTIMATE PROB DISTRIBUTIONS:
0047
IF (LOW .LE. 1) GO TO 2
0048
IFLAG(1) = 1
0049
DO 3 I = 1, NSAMP
0050
DO 3 J = 1, 5
0051
IFLAG(2) = 0
0052
IFLAG(3) = 0
0053
IFLAG(4) = 0
0054
IFLAG(5) = 0
0055
DO 3 J = 1, NEL
0056
RETURN

C
CHECK TRAINING DATA FOR UNREPRESENTED CLASSES:
0057
AND FOR CLASSES WITH O1 SAMPLE ONLY:
0058
DO 9 K = 1, 5
0059
DO 9 J = 1, NSAMP
0060
RETURN

C
APPLY BAYES ESTIMATION PROCEDURE TO PROB MATRICES:
0061
DO 9 J = 1, 5
0062
RETURN

END
DO 8 J=1,5
8 PROCM(I,J,K) = PROCM(I,J,K)+1.)/(PRCLAS(I)*FLOAT(NQUANT))
DO 7 PRCLAS(I)=(PRCLAS(I)+1.)/(5.*1.)
RETURN
C SUBROUTINE TEST(LCW,IL,EL,IPRM,IPFOE).
C
INTEGER IFLAG(S),IX(500,80),LEV(500),NUMP(500)
REAL CLASCM(5,5),PCONCD(5,80,12),PRCLAS(5),PTEST(5)
COMMON /CIA/ IX,LEV,NUMP
COMMON /TEST/ PCONCD,PRCLAS,IFLAG
C
C INITIALIZATION ON FIRST SUBS CALL:
C
IF (LOW.EQ.1) GO TO 1
DO 2 I=1,5
2 CLASSM(I,J)=0.
C GET CLASS CONDITIONAL PDF ESTIMATES FOR TESTING SAMPLE(S):
DO 10 I=1
10 IF (IFLAG(I).GT.1)) .EQ. 1) GO TO 11
DO 13 IK=1,5
13 IF(IX(IK).EQ.1) WRITE (6,14) IX(IK)
14 FORMAT (5X,'LEVEL',15)
CONTINUE
C
IF (IPRINT. NE.0) GO TO 10
C
DO 20 J = 1,5
WRITE(6,15)
20 FORMAT (//10X,21) (CLASSM(I,J), J = 1,5)
RETURN
C INCLUDE A PRIORI CLASS PROBS AND ESTIMATE ANESTHESIA LEVEL:
C
IF (IPRINT. NE.0) GO TO 10
C
DO 19 J=1,5
19 FORMAT (5X,'METHOD OF EFEECT ESTIMATION: "U" METHOD')
C
DO 21 J=1,5
21 FORMAT (5X,'TOTAL NUMBER OF PATTERN SAMPLES=',I3)
C
DO 23 J=1,5
23 FORMAT (5X,'ELEMENTS IN FEATURE VECTOR WITH',I4,' QUANTIZATION LEVELS FOR FEATUE')
C
DO 30 I=1,5
30 FORMAT (19,3X,'SUBJECT NUMBER: ',13)
C
WRITE (6,16) NSAMP
16 FORMAT (5X,'SUBROUTINE FIRST(NASEF,REI,NQUANT,IPRCE,NPNT)
C
REAL CLASCM(5,5),TCTL(5),TOK/0./,TCP/0./
COMMON /PNT/ CLASCM
WRITE(6,17) NPTMT
17 FORMAT (5X,'TOTAL NUMBER OF PATTERN SAMPLES=',I3)
WRITE(6,18) TCTL(5)
18 FORMAT (5X,'SUBJECT NUMBER: ',13)
WRITE(6,19) NSAFE
19 FORMAT (5X,'TOTAL NUMBER OF PATTERN SAMPLES=',I3)
WRITE(6,20) TCTL(5)
20 FORMAT (5X,'METHOD OF PERFORMANCE ESTIMATION: "U" METHOD')
WRITE(6,21) TCTL(5)
21 FORMAT (5X,'METHOD OF PERFORMANCE ESTIMATION: "P" METHOD')
WRITE(6,22) TCTL(5)
22 FORMAT (5X,'EQUAL A PRIORI CLASS PROBABILITIES WERE USED')
WRITE(6,23) TCTL(5)
23 FORMAT (5X,'UNEQUAL A PRIORI CLASS PROBABILITIES WERE USED')
WRITE(6,24) TCTL(5)
24 FORMAT (5X,'TOTAL(I)=TOTAL(1)+CLASSM(I,J)
WRITE(6,25) OK=CK(5,1)
25 FORMAT (5X,'TOTAL(I)=TOTAL(1)+CLASSM(I,J)
WRITE(6,26) TCTL(5)
26 FORMAT (5X,'TOTAL(I)=TOTAL(1)+CLASSM(I,J)
WRITE(6,27) TCTL(5)
27 FORMAT (5X,'TOTAL(I)=TOTAL(2)+TOTAL(5)')
C
DO 50 I=1,5
50 TOTAL(I)=0.
DO 60 K=1,5
60 TOTAL(5)=TOTAL(1)+CLASSM(I,J)
DO 70 K=1,5
70 TOTAL(5)=TOTAL(2)+TOTAL(3)+TOTAL(4)+TOTAL(5)
APPENDIX H

EVALUATION OF K-S STATISTICS FOR EEG AMPLITUDE DISTRIBUTIONS

INPUT:
* LUNIT 3: INPUT DATA TAPE
* LUNIT 5: NUMBERS OF THE TAPE FILES TO BE ANALYSED
* LUNIT 1: K-S C1 VALUES
* LUNIT 2: K-S D2 VALUES

LAST UPDATE:
JUNE 25 1974

C INTEGER I1(1),I2(2),KREC(0) / 
REAL XA(4096),XB(4096),D(17,2,64,64) /
C INTEGER NSAMP/8192/,NFLAG/0/,NCHAN/4/ 
REAL DATA(6192) / 
COMMON /CAT A/ CATA,INDEXF 
INDEXF=0 
SIGLEV = 0.05 
SRATE=64. 

INPUT ONE CHANNEL OF DATA AND CHANGE SA RATE
3 CALL INPUT(NFLAG,NCHAN) 
IF (NFLAG. EQ. 1) GO TO 44, 
HREC=NREC+1 
DO 10 K = 2, H192, 2 
KK=K/2 
10 DATA (KK) =DATA (K) 

TEST SEGMENTS OF **N SEC DURATION, N = 0, 1, 2
DO 1000 K=1,7 
NS2C=(K-1) 
N=(NS2C*SRATE)*0.1 
MQ=N-1 
MO=(K/2)-1 
CALL KS(MQ,SIGLEV,I1,DTHE01) 
CALL KS(MO,SIGLEV,12,DTHE02) 
WRITE (6,111) NSEC, NQ,MQ,0THSO1.DTHE02 
111 FORMAT (' NSEC= ',I3,' NQ= ',I5,' MQ= ',I5,' OTS1= ',F9.6,' DTHE02= ',F9.6) 
DO 4 JJ=NSEC,64,NSEC 
LLIB=(JJ-NSEC)*SRATE*0.1 
INDFX=JJ/NSEC 
DO 5 JJJ=1,N 
XA (JJJ) =DATA (L11*JJJ) 
XB (JJJ) = DATA (L11+JJJ) 
C CALCULATE THE D VALUES AND STORE THEM 
M=N/2: 
CALL CDFDEV(XE(1),XE(N/2+1),D2) 
CALL DBCMC(XA,XE,N,L) 
D(K,11,INDEX,NREC)=D2 
D(K,11,INDEX,NREC)=E1 
1000 CONTINUE 
GO TO 3 

C OUTPUT ALL D VALUES 
DO 45 K=1,7 
LIM=64/(2**(K-1)) 
DO 45 IREC=1,NREC 
WRITE(11,46) (E(K,I1,LL,REC),LL=1,LIM) 

C--APPENDIX H--EVALUATION OF K-S STATISTICS FOR EEG AMPLITUDE DISTRIBUTIONS
C THIS PROGRAM CALCULATES D1 (FOR GAUSSIANS) AND D2 (FOR FIRST-COMES 
C STATIONARITY) FOR EEG DATA SAMPLE AT A RATE OF 64 Hz. THE DATA HAS 
C PREVIOUSLY BEEN (DIGITALLY) HP FILTERED AT 0.54 Hz AND LP FILTERED AT 
C 30.0 Hz. 
C INPUT: 
* LUNIT 3: INPUT DATA TAPE 
* LUNIT 5: NUMBERS OF THE TAPE FILES TO BE ANALYSED 
C OUTPUT: 
* LUNIT 1: K-S C1 VALUES 
* LUNIT 2: K-S D2 VALUES 
C LAST UPDATE: JUNE 25 1974
SUBROUTINE INPUT (NPLAG,ICH)

BEADS IN ON £ CHANNEL OF DATA SAMPLED AT 128 HZ.
REAL DATA (8192)
COMMON /DATA/ DATA,INE
INTEGER*2 BLOCK (2048),LEN1
LUNIT=3
NSKIP=0

READ THE FILE NO
READ (5 ,20 ,END=10)NPIII
FORMAT (15)
WRITE (6,12) NFILE
FORMAT (' »••»•, 15)
PREPARE TO SKIP TO THE APPROPRIATE FILE
ITEMP=NPIIE-INDEXF-1
CALL SKIP(ITEMP,NSKIP,LUNIT)
INDEXF=NFILE-1
DO 14 IELK=1,16
INCX=(IBLK-1)*512
CALL READ (BLOCK, LEN1,0,LINE1,IUNIT,$10)
DO 14 ISAH=1,2048,4
IR=ISDX*1*(ISAM-1)/4
IRR= (ICH-1)*ISAH
DATA (IR) = bLCCK (IBB)
RETURN
NFLAG=1
RETURN
END

SUBROUTINE DNCBME(X1,X2,N,E)

THIS SUBREHFCRMS THE FCLLCWIS ECNS:
1. COMPUTES THE CDF FOR DATA IN XI
2. GETS S A M F MEAN
3. CALCULATES  A CDF FOR THE COR 3ESPONDING 'NORMAL DISTN
4. FINDS THE MAX DEV BETUEZN THE TWO COP'S
REAL XI(N),X2(N)
ID = 0
BN = N
FIRST,COMPUTE THE DIST FCN BY SORTING ARRAY VAL'S
CALL SSC3I(X1,N,3,C10,G10)
GO TO 2
WRITE (6,1)
FORMAT (' ••** SORTING ERROR •••••)
RETURN
CALC A CDF FOR A NORMAL CIST  WITH SAMPLE MEAN
! CAIL STAT
DO 100 JJ = 1,N
ZUL=(XI(JJ)-AVERJ/SCEV
X2 (JJ)=0.5* ERF (ZUL/ 1.41421) *0.5
NEXT, FIND MAX DEV BETWEEN ARRAY INDICES  FOR EACH SUCCESSIVE  VAL
OF X,USING X1 AS THE STANCAFC. NCT E: ARRAY INCIX 1->N IS IQUIV TO
0->N-1 OS 0->1
D=0.
DO 3 J=1,N
DEV=ABS ((FLOAT (J) /FLCAT (N) ) -X2(JJ))
IF (DEV. GT.D) D=DEV
RETURN
END

SUBROUTINE STAT (ARRAY,N,AVER,SDEV)

THIS SUB COMPUTES THE MEAN AN C STANEARE EIVIATICK  CF THE
SAMPLES STORED IN ARFAY(N)
INITIALIZATION
DIMENSION ARRAY (N)
SUH=0.
SVAH=0.
RN = N
MEAN:
DO 1 J=1,N
SUM=SUM+ARRAY (J)
RETURN
END
SUBROUTINE KS(NSAMP,SIGLEV,NSIDES,ICRIT)

THIS SUBR/finds the crit value of I for the one-sample OR 2-SAMPLE K-S test at these levels of significance: 0.01, 0.05, 0.10, 0.15, 0.20.


Restrictions: Samples must be greater than 100 and in the 2-sample test, sizes must be equal.

REAL DNSKS(5)/1.031,0.086,0.085,0.0768,0.0736/

REAL TWOKS(5)/1.63,1.36,1.22,1.14,1.07/

I=0

IF(SIGLEV.EQ.0.01)I=1

IF (SIGLEV.EQ.0.05) I=2

IF (SIGLEV.EQ.0.10)  I=3

IF •(SIGLEV.EQ.0.15)  I=4-

IF (SIGLEV.EQ.0.20)  I=5

IF (I.EQ.0) WRITE (6,1)

C

GOODNESS OF FIT TEST (WITH MEAN AND VARIANCE UNKNOWN)

IF (NSIDES.EQ.2) GOTO 2

IF (NSIDES.EQ.1) WRITE (6,1)

DCRIT = DNSKS(I)/RCCT

RETURN

C

TWO SAMPLE TEST (EQUAL SAMPLE SIZES)

RETURN

C

SUBROUTINE CDFDEV (X1,X2,N,C)

THIS SUBR takes two arrays of equal size, computes the list FCFs for each, and then calculates the maximum deviation between the two list FCFNs....JAN 23, 1971.

BEAL XI (N),X2 (N)

10 = 0

FIRST, COMPUTE THE 2 DIST FCFNS BY SORTING ARRAY VALS

CALL SSCRT (XI,N,3,610,C10)

CALL SSORT (X2,N,3,610,610)

GO TO 2

DO 3 J=1,N

XTEMP=X1 (J)

DO 4 K=J,N

IF(X2(K).GE.XTEMP) GO TO 7

4 CONTINUE

IZ=IZZ-1

IF (X2(K-IZ).LE.XTEMP) GO TO 5

5 CONTINUE

IDTEMP=IATIS (J-K*IZ)

IF (IDTEMP.GT.ID) ID = IDTEMP

RETURN

C

NEXT, FIND MAX DEVIATION BETWEEN ARRAY INDICES FOR EACH SUCCESSIVE VAL OF X USING XI AS THE STANDARD. NOTE: ARRAY INDEX 1->N IS EQUIVALENT TO 0->N-1 OR 0->N.

C

NEXT, FIND MAX DEVIATION BETWEEN ARRAY INDICES FOR EACH SUCCESSIVE VAL OF X USING XI AS THE STANDARD. NOTE: ARRAY INDEX 1->N IS EQUIVALENT TO 0->N-1 OR 0->N.

C

DO 3 J=1,N

XTEMP=X1 (J)

DO 4 K=J,N

IF(X2(K).GE.XTEMP) GO TO 7

4 CONTINUE

IZ=IZZ-1

IF (X2(K-IZ).LE.XTEMP) GO TO 5

5 CONTINUE

IDTEMP=IATIS (J-K*IZ)

IF (IDTEMP.GT.ID) ID = IDTEMP

RETURN

C

NOW, COMPUTE THE TRUE VAL OF DEVIATION I FOR USE IN 2 SAMPLE K-S TEST.

C

RETURN
APPENDIX I

EVALUATION OF K-S STATISTICS FOR EEG SPECTRAL DISTRIBUTIONS

C-APPENDIX I  EVALUATION OF K-S STATISTICS FOR EEG SPECTRAL DISTRIBUTIONS

C THIS PROGRAM CALCULATES D2 (FOR SPECTRAL DISTRIBUTION FUNCTIONS) FOR
C EEG DATA SAMPLED AT A RATE OF 128 HZ. THE DATA HAS PREVIOUSLY BEEN
C (DIGITALLY) HP FILTERED AT 0.54 HZ AND LP FILTERED AT 30.6 HZ.
C INPUT:
C  *UNIT 3: INPUT DATA TAPE
C  *UNIT 5: NUMBERS OF THE TAPE FILES TO BE ANALYSED
C OUTPUT:
C  *UNIT 2: K-S D2 (SPECTRAL) VALUES
C LAST UPDATE:
C  JUNE 25 1974

INTEGER 1,100/1/,12/2/,NREC/0/
REAL SMCOOH (2049) ,S2 (2049) ,S (7,64,60)
COMMON /S1100TH/SMOOTH INTEGRAL NSAMP/3192/,NFLAC/0/,NCFAN/4/
REAL DATA (8192),DATB (4098)
COMMON /THAN/ DATE
COMMON /DATA/ DATA,INDEXF
INDEXF=0
SIGLEV=0.05
SRATE=128.

INPUT THE CHANNEL OF DATA.
3 CALL INPUT (NFLAG,NCHAN)
IF (NFLAG.EQ.1) GC 44
NRBC=NRBC
1 DO 7 K=1,7
NSEC=2** (K-1)
N=SPEC/(NSEC»64)/2+1
IX=ISPEC- (2+hSEC)
ISTHEO=IX-NX
CALL KS (ISTHEO,SIGLEV,I2,ETHEOB)
WRITE (6,111) NSEC,ISTHSO,DTHEOR
111 FORMAT ('NSEC=',13,'ISTHESO=',15,'CF.OR=',19.6)

DO 4 JJ=NSEC,64,NS5C
LLIH= (JJ-NSEC) *SKATE
INDEX=JJ/NSEC
4 CALL SPECT (M,SRATE)
DO 33 J=1,ISPEC
33 S2(J)=SMOOTH (J)
DO 332 J=1,M
332 DATB (J)=DATA (LLIM+J*M*J)
CALL SPECT (M,SRATE)
CALL CDFDEV (SMOOTH (NX.),S2 (NX) ,IX
4 D(K,INDEX,NPEC)=C3
1000 CONTINUE
GO TO 3

OUTPUT ALL D VALUES
44 DO 45 K=1,7
45 LLIM=64/(2***(K-1))
51.000
52.000
53.000
54.000
55.000
56.000
57.000
58.000
59.000
60.000
61.000
62.000
63.000
64.000
65.000
66.000
67.000
68.000

SUBROUTINE SPECT(N,SRATI)

C COMPUTES THE POWER SPECTRUM VIA METHOD OF LUMMERTH ET AL.
C IEEE TRANS AUDIO DEC '70.

REAL DATA (8192)

REAL SMCCTH (2049)
COMM / SMOOTH / SMOOTH

INTEGER NN(1)
REAL PI/3.141592/

ences WINDOW DATA BEFORE FFT...SEE HANDBOOK, V5-A, PSO

ILIF=(N/10)+1
LIMUP=M-LLIM
XINT=PLCAT (LLIH)

DO 1 IQ=1,LLIM
1 DATA (IQ) = DATA (IQ) *0.5* (1. - COS (PI*FLCAT (IQ)/XINT)

DO 2 IQ=L1MUP,N
2 DATA (IQ) = DATA (IQ) *0.5* (1. -COS (PI*FLCAT (N-IQ)/XINT)

GET RAW SPECTRAL ESTIMATES VIA FFT

NN(1)=N
ISIGN=-1
CALL FOUR2 (DATA,NN, 1,ISIGN)

DELT=1./S RATE
FACTCR= (DELT/FLCAT (N) ) * 1.14625
MID=N/2*1

DO 99 1 =1,MID
T= CABS (TRAN (I) )
99 CONTINUE

SMOOTHED SPECTRAL ESTIMATES OBTAINED VIA SQUARE WINDOW (2W+1)=7

THE FIRST AND LAST 3 POINTS ARE NOT SMOOTHED

DO 52 1 =1,3
52 SMCCTH (I) = DATA (I)

DO 11 I=1,3
11 SMCCTH (I-3*1) = DATA (I-3*1)

RETURN
END

SUBROUTINE INPUT (N FLAG, ICH)

REAL DATA (8192)
COMMON /DATA/ DATA,INDEXF
INTEGER*2 BLOCK (2048) ,LEN1
LUNIT=3
NSKIP=0

READ (5,20,END=10) NFILE

WRITE (6, 12) NFILE

CALL SKIP (ITEMP, INDEXF, LUNIT)
INDEXF=NFILE-1

DO 14 ISAM=1,2048,4
IR = INDEXF «1 • (ISAM-1 )/4
IRR= (ICH- 1) »ISAH
14 DATA (IR) = bLCCK (IRR)

RETURN
END

SUBROUTINE INPUT (N FLAG, ICH)

REAL DATA (8192)
COMMON /DATA/ DATA,INDEXF
INTEGER*2 BLOCK (2048) ,LEN1
LUNIT=3
NSKIP=0

READ (5,20,END=10) NFILE

WRITE (6, 12) NFILE

CALL SKIP (ITEMP, INDEXF, LUNIT)
INDEXF=NFILE-1

DO 14 ISAM=1,2048,4
IR = INDEXF «1 • (ISAM-1 )/4
IRR= (ICH- 1) »ISAH
14 DATA (IR) = bLCCK (IRR)

RETURN
END
SUBROUTINE STAT(ABBAY, N, AVFB, SEIV)
C THIS SUBR COMPUTES THE MEAN AND STANDARD DEVIATION OF THE SAMPLES STORED IN ARRAY(N)
C INITIALIZATION
DIMENSION ARRAY(N)
NFLAG=0
RETURN
END

SUBROUTINE KS(NSAMP, SIGLEV, NSIDES, DCRIT)
C C C C C
C THIS SUBR FINDS THE CRIT VALUE OF D FOR THE CHI-SAMPLE OR 2-SAMPLE K-S TEST AT THE
C LEVELS OF SIGNIFICANCE: 0.01, 0.05, 0.10, 0.15, 0.20
C RESTRICTIONS: SAMPLES MUST BE GREATER THAN 100 AND IN THE 2-SAMPLE TEST, SIZES
C MUST BE EQUAL.
REAL DNKS(5), TWOKS(5)
B=N=NSAMP
BOOT = SQRT(N)
IF(SIGLEV.EQ.0.01)I=1
IF(SIGLEV.EQ.0.05)I=2
IF(SIGLEV.EQ.0.10)I=3
IF(SIGLEV.EQ.0.15)I=4
IF(SIGLEV.EQ.0.20)I=5
IF(I.EQ.0)WRITE(6,1)
RETURN
1 FORMAT(' ERROR IN KS *** ')
IF(NSIDES.EQ.2) GO TO 2
IF(NSIDES.EQ.1) WRITE(6,1)
DCRIT=DNKS(I)/ROOT
RETURN
2 FACTOR=SQRT(2./N)
DCRIT=FACTOR*TWOKS(I)
RETURN
END

SUBROUTINE CDFDEV(X1, X2, N, D)
C THIS SUBR TAKES TWO ARRAYS OF EQUAL SIZE, COMPUTES THE DIST FCN FCB AND THEN
REAL X1(N), X2(N)
I=0
RETURN
END
C 0->N-1 OR 0->1

C

0010 2 DO 3 J=1,N
0011 XTEMP=X(1,J)

C

0012 4 DO 4 K=J,N
0013 IF (X2(K).GE.XTEMP) GC TC 7
0014 7 CONTINUE
0015 DO 6 IZZ=1,K
0016 IZ=IZZ-1
0017 IF (X2(K-IZ).LE.XTEMP) GC TC 5
0018 6 CONTINUE

C

0019 5 IDTEMP=IABS(J-K+IZ)
0020 IF (IDTEMP.GT.ID) IC=IDTEMP
0021 3 CONTINUE

C

NOW, COMPUTE THE TRUE VAL OF DEVIATION D FOR USE IN 2 SAMPLE
K-S TEST.

0022 C=FLOAT(ID)/FLOAT((N-1))
0023 RETURN
0024 END
APPENDIX J

TESTS OF K-S STATISTICS

THIS PROGRAM INTERPRETS THE SIGNIFICANCE OF THE K-S VALUES PRODUCED BY MEANS OF THE PROGRAMS LISTED IN "APPENDIX F" AND "APPENDIX I".

INPUT:

- LUNIT 1: K-S C1 (AMPLITUDE) VALUES
- LUNIT 2: K-S C2 (AMPLITUDE) VALUES
- LUNIT 3: K-S C2 (SPECTRAL) VALUES

OUTPUT:

- LUNIT 7: GAUSSIAN PERCENTAGES
- LUNIT 8: FIRST-ORDER STATIONARY PERCENTAGES
- LUNIT 9: W-S STATIONARY AND GAUSSIAN PERCENTAGES
- LUNIT 10: W-S STATIONARY AND GAUSSIAN PERCENTAGES

LAST UPDATE: JUNE 28 1974

CALCULATE VALUES FOR K-S TESTS AT SOME SIGNIFICANCE LEVEL:

DO 11 = 1, 7
CALL KS(IG(I),SIGLEV,11,GKS(I))
CALL KS(IF(I),SIGLEV,12,FSKS(I))
CALL KS(IS(I),SIGLEV,13,WSKS(I))
WRITE(6,1U2)IG(I),GKS(I),IFS(I),FSKS(I),ISP(I),WSKS(I)
1 CONTINUE

INPUT D VALUES AND TEST TEES:

CALL SKIP(0,ISKIP,1)
CALL SKIP(0,ISKIP,2)
CALL SKIP(0,ISKIP,3)
DO 17 J = 1, 7
LIM=4/(J)**2
DO 16 N=1,30
READ(2,100) (X(L),L=1,LIM)
READ(2,100) (Y(L),L=1,LIM)
READ(2,100) (Z(L),L=1,LIM)
100 FORMAT(64F8.6)
1 CONTINUE

CALCULATE PERCENTAGES AND CORRECT FOR TYPE I ERRORS:

CORREC=1.-SIGLEV
DO 14 J=1,7
BRCH=(64**30.)/(J**2)**2*CORREC
14 CONTINUE

DO 24 L=1,LIM
IF(X(L)+.LE.GKS(J))G(J)=G(J)+1.
DO 26 J=1,7
IF(Y(L)+.LE.FSKS(J))FS(J)=FS(J)+1.
DO 28 J=1,7
IF(Z(L)+.LE.WSKS(J))WS(J)=WS(J)+1.
1WSKS(J)=WS(J)+1.
24 CONTINUE

CALL SKIP(0,ISKIP,1)
CALL SKIP(0,ISKIP,2)
CALL SKIP(0,ISKIP,3)
DO 33 J=1,7
C CALCULATE PERCENTAGES AND CORRECT FOR TYPE I ERRORS:
034 CORREC=1.-SIGLEV
035 DO 35 J=1,7
036 BRCH=(64**30.)/(J**2)**2*CORREC
037 IX=J-1
038 DO 36 J=1,7
039 IF(Y(L)+.LE.FSKS(J))FS(J)=FS(J)+1.
040 IF(Z(L)+.LE.WSKS(J))WS(J)=WS(J)+1.
041 IF(WSS(J)=WSS(J)+1.
042 WSS(J)=WSS(J)+1.
154
C prepare a plotfile:

DO 14 J=7,10
  WRITE (7,13) XX,G(J),L1
13 FORMAT(2F10.2,2X,I2)
14 WRITE (8,13) XX,FS(J),L3
15 WRITE (9,13) XX,WSS(J),L2
16 WRITE (10,13) XX,WSSG(J),L1
STOP
END

SUBROUTINE KS (NSAMP,SIGLEV,NSIDES,DCRIT)
THIS SUBR FINDS THE CRITICAL VALUE OF D FOR THE ONE-SAMPLE OR 2-SAMPLE K-S TEST AT THESE LEVELS OF SIGNIFICANCE:
/.01,.05,.10,.15,.20 /
VALUES FOR THE ONE-SAMPLE TEST ARE FROM JASA,P399,1967.
VALUES FOR 2-SAMPLE TEST FROM AN.M.STAT.,P279,1948.
RESTRICTIONS: SAMPLES MUST BE GREATER THAN 100 AND IN
THE 2-SAMPLE TEST, SIZES MUST BE EQUAL.
REAL DNKS(5)/1.031,0.886,0.805,0.768,0.736/
REAL TWKS(5)/1.63,1.36,1.22,1.14,1.07/
RN=NSAMP
ROOT=SQRT(RN)
10 DO I=1,5
  IF(SIGLEV.EQ.0.01)I=1
  IF(SIGLEV.EQ.0.05)I=2
  IF(SIGLEV.EQ.0.10)I=3
  IF(SIGLEV.EQ.0.15)I=4
  IF(SIGLEV.EQ.0.20)I=5
  IF(I.EQ.0)WRITE(6,4)
4 FORMAT(' 

GOODNESS OF FIT TEST (WITH MEAN AND VARIANCE UNKNOWN)

IF(NSIDES.EQ.2)GO TO 2
IF(NSIDES.EQ.1)WRITE(6,1)
RETURN
RETURN
2 DCRIT=DNKS(I)/ROOT
RETURN
END
REFERENCES


PUBLICATIONS:


3. J.A. McEwen, "Modelling spontaneous electroencephalographic activity," in Fifth Canadian Medical and Biological Engineering Conf. Dig. (Montreal, Canada), pp. 10.4a - 10.4b, 1974.


