VARIABLE BIT RATE VIDEO TRANSMISSION FOR CODE-DIVISION MULTIPLE-ACCESS SYSTEMS IN WIDEBAND FAADING CHANNELS

by

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ABSTRACT

Efficient real-time transmission of video data over bandwidth-constrained wireless channels is challenging in several ways: in particular, due to the underlying compression algorithms, the source rate can vary in bursts, which complicates the resource allocation problem, isolated channel errors can totally corrupt a video frame if sensitive information is affected, and errors in earlier frames can cause damage to later frames due to error propagation. This thesis will deal in particular with the effect of source rate variability on current and future cellular systems which employ code-division as the multiple-access strategy, such as IS-95B and IS-2000 systems. The problem will be approached from a physical-layer perspective: hence issues relating to the channel- and cellular-level performances will be addressed in detail, and then integrated into the system-level performance. This nonconventional cross-layer approach allows us to obtain additional insights over studies which tackle the issue mainly or exclusively at the higher system layers.

In the first part of this thesis, several contributions are made to the theory of wideband fading channels, which will be considered as the physical channel model throughout the thesis. We derive the analytical level-crossing rates, average fade durations, envelope autocorrelations and baseband spectra of several channel models for some common diversity techniques. Based on some of the previously derived properties we design a fast wideband Nakagami channel simulator. We then derive the exact analytical error probabilities of several linear modulation schemes with diversity in correlated Nakagami channels, and validate them through simulation.

In a second part, we derive accurate analytical or semi-analytical error probability expressions for the multicode and multirate configurations used in the physical layers of both the uplink and downlink of IS-95B and IS-2000 systems, in the presence of wideband fading. It is demonstrated that the effect of the multicode interference must be precisely taken into account to obtain reliable error statistics in wideband channels, especially for cellular systems with a low number of users. To this end, the fading dependence across
multiple codes of a given user must be taken into account in the analysis, whereas for single-code systems this situation didn’t occur. We consider systems which employ either maximal-ratio or equal-gain combining. The proposed methodology places no restrictions on the type of fading distribution, and examples are given for the cases of Rayleigh, Rice, Nakagami and lognormal fading, for both independent and correlated diversity branches. For the IS-95B uplink, the analysis is extended to deal with closed-loop power control using the inverse update algorithm, successive interference cancellation, and multicell systems. All analytical results are thoroughly validated through numerous entire system simulations, for different values of several transceiver and channel parameters.

In the final part of this thesis, we demonstrate the benefits of employing rate smoothing for variable bit rate video applications in DS/CDMA cellular systems, and present and evaluate practical algorithms to achieve these gains. To support our exposition, a generic rate smoothing algorithm is developed, whose goal is to minimize the degradation caused by source bursts in such systems. Its performance in terms of decoded video quality is compared to that of a popular algorithm which was developed in the context of wireline communications, and which serves as a benchmark. It is shown that for systems subject to certain practical constraints, in particular concerning the granularity of the transmission rates, the proposed algorithm can offer an improved decoded video quality with respect to the benchmark algorithm. The influence of smoothing-related parameters such as the startup buffering delay and sliding window length are quantified. In addition, the effects of some transceiver and channel parameters on the decoded video quality are presented. To carry out these performance evaluations, a flexible software platform has been developed which emulates the transmission of video data at the physical/link layers in IS-95B and IS-2000 cellular systems with wideband fading, and allows the user to objectively measure the decoded video quality directly at the application layer.
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<td>Activity Factor</td>
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<td>AL</td>
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<td>BS</td>
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<td>Bit Error Rate</td>
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<td>BH</td>
<td>Basic Header</td>
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<td>Bits per second</td>
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<td>BPSK</td>
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<td>cdf</td>
<td>Cumulative Distribution Function</td>
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<td>EAF</td>
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<td>QPSK</td>
<td>Quadrature Phase-Shift Keying</td>
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<td>RAN</td>
<td>Radio Access Network</td>
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<tr>
<td>RC</td>
<td>Radio Configuration</td>
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<td>RCPC</td>
<td>Rate Compatible Punctured Convolutional</td>
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<td>RTP</td>
<td>Real-Time Protocol</td>
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<tr>
<td>SC</td>
<td>Selection Combining</td>
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<td>SCH</td>
<td>Supplemental Channel</td>
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<td>SDU</td>
<td>Service Data Unit</td>
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<td>SER</td>
<td>Symbol Error Rate</td>
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<td>SF</td>
<td>Synchronization Flag</td>
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<td>SIC</td>
<td>Successive Interference Cancellation</td>
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<td>SLWIN</td>
<td>Sliding Window Algorithm</td>
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<td>SLWIN2</td>
<td>Sliding Window Algorithm 2</td>
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<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
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<tr>
<td>SR</td>
<td>Source Rate / Spreading Rate</td>
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<tr>
<td>SRC</td>
<td>Source Rate Control</td>
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<tr>
<td>S-TRC</td>
<td>Source and Transmission Rate Control</td>
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<td>TB</td>
<td>Transmission Buffer / Tail Bits</td>
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<td>TR</td>
<td>Transmission Rate</td>
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<td>TRC</td>
<td>Transmission Rate Control</td>
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<tr>
<td>TDMA</td>
<td>Time-Division Multiple-Access</td>
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<td>TMN</td>
<td>Test Model Near-term</td>
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<td>VBR</td>
<td>Variable Bit Rate</td>
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<tr>
<td>Acronym</td>
<td>Definition</td>
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<td>VCEG</td>
<td>Video Coding Experts Group</td>
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<td>VLBR</td>
<td>Very Low Bit Rate</td>
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<td>VLC</td>
<td>Variable Length Coding</td>
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<td>Video Packet</td>
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<td>VSG</td>
<td>Variable Spreading Gain</td>
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<td>W-CDMA</td>
<td>Wideband CDMA</td>
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CHAPTER 1

INTRODUCTION

1.1 Introduction to Wireless Video Communications

Wireless mobile communications can truly be described as multimedia when they include a video component in addition to voice and data. The rapid advances in digital video compression [1], [2], matched with techniques for increasing the capacity of wireless systems [3], are likely within the next decade to permit the widespread delivery of real-time video over mobile cellular networks, which has in fact recently begun in certain countries. While digital video codecs were originally designed for high-bandwidth, high-quality wirelines, the latest set of codecs were conceived with low bit-rate and error-prone channels in mind. The H.263 [4] and MPEG-4 [5] standards build on techniques which produce very low bit-rates (in the order of tens of kbps), and have a number of modes allowing them to cope with errors. These characteristics make them more suitable for next-generation wireless networks. However, as time will pass by, it is expected that higher-quality video, necessitating higher data rates and fewer errors, will become in demand. Thus it will not suffice to simply transmit the video stream the same way voice or data are currently delivered. Techniques which make use of the capabilities of both the video codecs and the transceivers are to be developed to maximize the quality of the received video.

Early papers on wireless video can be traced back to a decade ago [6], [7], however the bulk of the research was conducted in the last six years or so. Contributions are made not only by academic institutions, but also by research and development firms. For example, Texas Instruments is developing digital signal processors (DSP's) customized for wireless MPEG-4 applications [8], and a number of large or small firms have patented technologies relating to wireless video [9].
In this thesis we are considering the transmission of video over current and next-generation direct-sequence code-division multiple-access (DS/CDMA) cellular networks [10], [11], [12], [13], based on standards such as IS-95B and IS-2000, which are part of the cdma2000 family [14]. IS-95B systems have already been deployed in North America, and currently support voice and data, while IS-2000 systems are expected to be deployed on a large scale in the near future. Such systems experience wideband fading (i.e. multipath fading where several components are resolved at the receiver) and multiple-access interference from other users: these are the two main factors which make the study of video transmission over wireless CDMA networks much different than that of wireline video systems. The wireless link is indeed the bottleneck in future global communication networks, since the fading and interference limit the transmission rates over large ranges. Our goal is to develop rate control methods to allow the reliable transmission of a variable bit rate video stream by a CDMA system with rate and power constraints, for delay-sensitive applications. We will approach the problem from a physical/link layer perspective. Hence, before tackling the rate control problem, a detailed understanding of the impairments caused by wideband fading and multiple-access interference is necessary: we will thus consider the analysis and simulation of variable rate IS-95B/IS-2000 transmission over wideband channels.

In this introduction, we begin by giving a brief general review of wideband fading channels and DS/CDMA systems in Sections 1.2 and 1.3, respectively. Section 1.4 presents some important characteristics of video applications and low bit-rate video coding, together with rate control methods needed in bandwidth-constrained environments. Then, in Section 1.5 we summarize our contributions in the areas of fading channel theory, DS/CDMA systems, and wireless video communications, and outline the organization of the thesis. Finally, in Section 1.6 we present for reference purposes a list of the publications which have resulted from the research reported in this thesis.

1.2 Wideband Fading Channels

In wireless mobile communications, the transmitted signal is subject to path loss, large-scale fading (shadowing) and small-scale fading ([15], Chaps. 1-2, [16], Chaps. 3-
While accurate power control can mitigate the effects of the first two phenomena, which usually vary slowly in time, the small-scale fading can vary too rapidly in time to be accurately tracked by a power control mechanism, particularly for mobiles moving at high speed. Hence other mechanisms are necessary to counter the effect of the fading: diversity techniques ([15], Chaps. 5-6, [16], Chap. 6), which strive to capture the maximum energy from a transmitted signal, are by far the most popular due to their relative simplicity. The type of diversity circuit which should be used for maximum efficiency depends on the particular nature of the fading which is expected, as discussed below.

Small-scale fading is due to spreading (distortion) of the transmitted signal in both the time and frequency domains, or either one ([16], Chap. 4). Temporal distortion is due to the signal traveling on multiple propagation paths (multipath fading), due to reflections from surrounding terrain features or objects, which causes scaled replicas of the signal to arrive at the receiver at different time instants. The difference between the arrival times of the first and last components is called the delay spread. The inverse of the delay spread is proportional to the coherence bandwidth, which is the range of frequencies over which the channel passes all spectral components with approximately equal gain and linear phase. Frequency distortion is attributable to the random frequency modulation phenomenon caused by the motion of a mobile user, or by the motion of objects moving near a fixed user: the energy of the received signal is contained in a continuum of frequencies alongside the carrier frequency. The difference between the maximum and minimum frequencies (on one side of the carrier frequency) where the signal energy is non-zero is called the Doppler spread, or alternatively the Doppler shift. The inverse of the Doppler shift is proportional to the coherence time, which is the time duration over which the channel impulse response is mostly invariant. Among other classifications, the small-scale fading that affects a transmitted signal can be roughly categorized as either frequency-nonselective (or flat) or frequency-selective, although there is no clear-cut demarcation between both attributes, and rules of thumb are used instead [16]. Flat fading, also termed narrowband fading, occurs when the bandwidth of the signal is smaller than the coherence bandwidth of the channel, and the delay spread of the channel is smaller than the symbol period of the signal [17], [16]. The multipath components combine constructively and destructively at the receiver, which cannot separate them in the time domain. Frequency-selective fading,
also termed wideband fading, occurs when the bandwidth of the signal is larger than the coherence bandwidth of the channel, and the delay spread of the channel is larger than the symbol period of the signal [17], [16]. In this case, the receiver can resolve some of the multipath components. The small-scale fading can also be categorized as either fast or slow. Fast fading corresponds to a coherence time smaller than the symbol period (hence a high Doppler spread) and channel variations faster than baseband signal variations. Slow fading corresponds to a coherence time larger than the symbol period (hence a low Doppler spread) and channel variations slower than baseband signal variations.

First and second-generation (1G and 2G) cellular systems with low data rates typically had to deal with narrowband fading (except for spread-spectrum IS-95 systems). In this case, antenna spatial diversity techniques were useful in collecting energy from multipath components received at different spatial locations, at about the same times. Third-generation (3G) cellular systems based on spread-spectrum technology (so as their ancestors such as IS-95, and other high data-rate systems) typically experience wideband fading instead. Spatial antenna diversity techniques can still be used, but in addition these systems can now make use of temporal diversity: Rake receivers [18] with resolutions as low as one chip interval can be used to capture multipath components which are closely-spaced in the time domain. High-rate systems will typically experience slow fading, especially if the mobile speed is low: this thesis will hence essentially be concerned with frequency-selective slow fading channels.

The use of diversity is crucial in 3G systems in order to obtain a higher signal-to-noise ratio at the receiver end, and satisfy the higher quality-of-service required by certain new applications such as video telephony. In fact, the IS-2000 standard specifies the support of transmit diversity techniques [19], such as orthogonal transmit diversity (OTD). Hence our simulation models for video communications over 2.5G (IS-95B) and 3G (IS-2000) DS/CDMA systems will include diversity techniques. While extensive research has been carried out for decades in the field of diversity techniques [20], [21], [22], there are still many open issues, and there will continue to be as long as new methods are proposed and more accurate characterizations in different fading environments are required. Part of this thesis will tackle several issues related to the statistics, simulation and effect on communication performance of wideband Nakagami fading channels [23]. The Nakagami
distribution has been chosen due to its generality (it reverts to the Rayleigh distribution as a special case), and its reported better fit to urban and suburban fading conditions [24]. In particular, the effect of correlation between diversity branches will be examined. Some of the results obtained for single-user diversity systems will prove useful in our work on multiuser systems.

1.3 DS/CDMA Cellular Networks

IS-95 and IS-2000 DS/CDMA cellular systems use spread-spectrum modulation, which uses a bandwidth much wider than that of the information signal, due to spreading by pseudorandom sequences with high chip rates ([25], [26], Chap. 13). Spread-spectrum modulation has several advantages over narrowband modulation, in particular enhanced interference rejection capability, resistance to multipath fading, and increased communication privacy ([16], Chap. 5). In a DS/CDMA system multiple users transmit simultaneously in the same bandwidth and time slots: the user separation is realized in the code domain, i.e. each user is assigned a particular code ([26], Chap. 15). However, in most conditions this code domain separation is not perfect, and hence users will experience multiple-access interference from users in the same cell and other cells. Since the seminal work of [27] on how to approximately determine the error rate of a user in a DS/CDMA system, a large amount of studies have been carried out to either refine or improve the results of [27] (e.g. [28], [29], [30], [31], [32]), or to extend them to deal with fading channels (e.g. [33], [34], [35], [36], [37], [38], [39], [40]) and a plethora of other conditions. In order to support high-data rate transmissions, e.g. for video telephony, multicode and multirate transmission have been recently introduced. These will be described in detail in Section 3.1. Part of this thesis (Chapter 3) will tackle the problem of accurately determining the error rate of multicode/multirate systems in wideband fading channels, for the main system configurations encountered in IS-95 and IS-2000 systems.
1.4 Video Communications: Characteristics and Methods

1.4.1 Requirements of Video Transmission

From a communication viewpoint, video signals have characteristics which make their treatment different from that of voice and data signals. These characteristics are presented below.

1.4.1.1 Bandwidth Requirements

H.263 video (and its later versions such as H.263+ [41], H.263++) necessitates higher rates than voice (typically around 64 kbps for low-motion video, compared to 9.6 kbps for compressed speech in IS-95). However, the rates specified are usually average rates, and much higher peak rates need to be dealt with. Indeed, due to the coding mechanisms involved (c.f. Section 4.2.2), H.263 produces a variable bit rate (VBR) video stream. This variability can be short-term (at the individual frame level), middle-term (at the scene level) or long-term (between scenes). Short-term variations are due mainly to variable-length coding (VLC) and the possible different modes used to encode blocks of video data. The type of frames used (I versus P or B, c.f. Section 4.2.2) is essentially responsible for middle-term variations. Long-term variations are caused by changes in scene contents: motion (inside the scene, or by the camera) will significantly increase the required bit rate compared to a quasi-still scene. While variations on a short time scale could be handled by smoothing while keeping a low delay, middle and long-term variations will in most cases require changes in the bandwidth allocated (unless an arbitrary long delay is permitted).

1.4.1.2 Delay Requirements

Different applications can tolerate different amounts of end-to-end delay. Real-time interactive video telephony cannot withstand more than a few hundred msec of delay (typically 150 to 400 msec one-way [42]), in order to avoid annoying periods of stalling in the conversation. Other real-time but non-conversational applications, such as streaming video, can tolerate a higher initial delay: an end-user logging on to a live one-way video-conference would be willing to wait a few (tens of) seconds of startup time in order to
have a higher-quality stream. The downloading of pre-compressed stored video can afford a much longer delay, in order to make sure that the connection isn't broken up due to insufficient resources. Hence, the amount of delay that can be supported will dictate in part the bandwidth requirements discussed above: bursty live interactive video will require higher peak rates, and a continuous allocation of bandwidth, while stored video (whose treatment is closer to that of data) can be handled with discontinuous transmission and lower, smoother rates.

1.4.1.3 Error Requirements

Like voice, video can tolerate a certain level of degradation (such that the user hardly notices it), however too many errors can have a catastrophic impact ([43], Chap. 8). Moreover, the video stream is unevenly sensitive to errors: transmission errors in segment headers can cause the loss of a whole picture, while those affecting texture coefficients and motion vectors will usually have a limited and containable impact ([43], Chap. 6, [44]). This has lead to research on robust video coding, which resulted for example in the error-resilient modes used in H.263+ [45], [46], [47].

1.4.2 Impairments to Video Transmission

The VBR nature of compressed video can result in frame losses due to transmission buffer overflow or receiver buffer underflow ([43], Chap. 9). Transmission buffer overflow occurs when the rate at which the transmitter processes frames is consistently lower than the rate at which these frames are delivered to it from the source. This same situation can lead to receiver buffer underflow, that is when the video decoder at the receiving end processes frames at a rate higher than the rate at which it receives them.

The techniques which allow increased video compression also lead to high vulnerability of the resulting stream to random, burst and erasure errors. The following impairments can be caused by such transmission errors:

- Loss of video frame or segment. Errors in segment headers can cause the whole segment to be discarded, depending on the bits which are affected.
• *Loss of synchronization.* Because of variable length coding, when the decoder encounters an error in a codeword, it doesn't know where to restart the decoding, and can possibly lose synchronization.

• *Temporal and spatial error propagation.* Due to motion-compensated predictive coding, errors occurring in the current frames can carry over to the following frames, until the sequence is refreshed by an I-frame (or intra-coded macroblocks), or the effect of the errors is neutralized by a leaky prediction mechanism. The distortion can also propagate spatially in future frames, especially when high motion is present.

1.4.3 Rate Control for VBR Video

In a VBR video communication system, the source encoder outputs the generated bitstream to a transmission buffer (TB) ([43], Chap. 9). The latter delivers its contents to the transmitter, according to a certain algorithm. The transmitter then performs on the bitstream the necessary operations specified by the corresponding standard (such as framing, error-control coding, interleaving and demultiplexing), and modulates it onto a signal suitable for transmission.

The source rate (SR) is constrained by the maximal transmission rate (TR) that can be supported. The maximal TR is in turn determined by the transmitter specifications, i.e. the maximum rate supported by the transceiver, and the maximal bandwidth allocated at a given time to the transmitter in a wireless network, which is influenced by the network load. The channel conditions can further impose time-varying limitations on the TR. Hence a certain control must be exercised on the SR seen by the transmitter or the TR made available to the source, in order to allow the VBR video to be transported adequately. Three scenarios can be envisioned for the rate-controlled transport of VBR video for such a communication system.

In the first scenario, there is no feedback from the transmitter to the source. The source has no information about the maximum TR supported at any time. Hence it cannot adapt its rate according to constraints posed by the transmitter and network. The transmitter must thus handle by itself the variable bit rate produced by the source:
this can usually be done either by transcoding [48], smoothing [49], rate adaptation, or by a combination of all of these. This situation arises, for example, in the case of pre-recorded pre-compressed video. It is also common to certain systems supporting live or streaming video in which no feedback path is present from the transmitter to the source, for complexity or system design reasons. We will call this scenario the transmission rate control (TRC) case, to denote the fact that the rate control is carried out by the transmitter. Note that we are dealing here with the control of the actual transmission rate.

In the second scenario, there is an interaction between the source and the transmitter. The source receives feedback from the transmitter concerning the maximal TR that can be supported (explicit rate feedback [50]), or elements that determine this rate (non-explicit rate feedback). These feedback messages can be periodic or occur upon certain events, e.g. when there is a change in the maximum TR. If the source is performing live encoding, it can therefore adjust its output rate in order to match the TR. The transmitter does not need to adapt its TR to the VBR source which is fed to it, even though it can still perform rate adaptation related to varying channel or network conditions. We will call this scenario the source rate control (SRC) case, to emphasize the fact that the rate control is performed by the source encoder. We are thus concerned here with the control of the source output rate.

The third possibility is to combine the feedback capability of the second scenario with the transmitter rate control of the first scenario. In this case both source and transmitter strive to optimize the video quality under given rate constraints. For example, if after being rate-controlled by the source encoder, the video stream has a rate which is still too high, the transmitter can perform TRC in order to attempt the adequate delivery of the stream. Since source rate control and transmission rate control can be jointly performed, we will call this scenario the source and transmission rate control (S-TRC) case. It generalizes the previous two cases.

In all of the previous cases, the receiver can play a role in the video delivery process. A certain number of video frames are stored in the receiver buffer before video decoding starts. This delay between the reception of frames and their playout is called the lookahead interval [51]: it alleviates the delay constraints placed on the transmission of
video, and hence gives the source and/or transmitter more flexibility in performing rate adaptation.

In the following we present an overview of the most common rate control approaches used in each of these scenarios.

1.4.3.1 Source Rate Control (SRC)

Indicators for Rate Adaptation

There are three ways that the source can be informed that a change in its output rate is desirable: by direct feedback from the receiver (through a return path), by examination of the buffer(s) present at the transmitter, or by a combination of the latter two approaches. Different indicators can be used for each approach.

In the first case, indications about the current network or channel conditions are transmitted back to the source encoder via a control channel (or piggybacked on a traffic channel). Some examples of indicators used in previous studies are:

- The channel state information which is determined at the receiver. In [52], the channel is determined to be in one of a fixed number of states. The index of the state serves as an indicator of the channel quality. Estimates of the signal-to-noise-ratio (SNR) at the receiver or of the fading coefficient(s) can also be used.

- The number of negative acknowledgements (NAK’s), in a system using automatic-repeat request (ARQ), as in [53].

Some systems cannot support a return channel needed for receiver feedback: this could be the case for example for a large multicast configuration. Moreover, certain systems would prefer an approach which is not feedback-based for one or more of the following reasons:

- The need for feedback requires additional signalling on the reverse channel: this increases bandwidth requirements and complexity.

- The feedback information is returned to the transmitter with a certain delay. If this delay is too large compared to the rapidity of changes in network conditions, the information can be outdated and lead to sub-optimal or incorrect source adaptation.
This brings us to the second case, where the source adapts its rate only from information available at the transmitter. The following indicators were used previously:

- The transmitter buffer fullness. In [54], if the number of video bits waiting to be transmitted gets too large, e.g. because of a mismatch between SR and TR, or bad channel conditions, the source decreases its output rate.

- The ARQ buffer fullness. In [55], a separate buffer is maintained to store the frames which haven't been positively acknowledged yet. The status of this buffer determines the source's rate adaptation.

The transmitter and ARQ buffers could also be combined into a single larger buffer, and therefore the two previous indicators would merge.

Some proposals use a combination of feedback and transmitter-based adaptations, which constitutes the third case of source rate adaptation. More reliable results are expected, since indicators from independent sources are used in deciding the type of rate adaptation to adopt. This translates of course in increased complexity, due to the additional resources required and the need for a more advanced decision process which encompasses all the information sources. In [56], the proposed algorithm takes into account both the channel state information and the ARQ buffer fullness.

Note that rate adaptation based on transmitter information only can also be subject to the problem of outdated information, as in the feedback-based case. Indeed, by the time buffer congestion builds up due to bad network conditions or SR/TR mismatch, the latter can improve: thus the decrease in source rate which is requested will not be useful anymore.

An alternative to the observation of current (or recent) conditions would be to try to predict the future network conditions or source rate. Numerous papers have been published on the topic of source rate prediction, for various video coders [57], [58], [59]. They most often base their prediction scheme on a statistical or dynamic model developed for the source of concern. However, the goal of such predictions was in most cases to obtain traffic management algorithms needed for middle/long-term traffic variations. A new topic of research would be on how to use source prediction algorithms to regulate the source or transmission rates on a short-time basis. Fading channel prediction is a
fairly recent research area [60], which could also be exploited for SR/TR adaptation.

Source Rate Adaptation Techniques
The following mechanisms are available to a video encoder in order to vary its output rate:

- Adjustment of quantization step size. By choosing a larger quantization step size, the video sequence will be more coarsely encoded and thus use up less bits, at the expense of a lower visual quality. The step size can be selected from a pre-determined set, as in H.263. Some authors opt for a simple “embedded quantization” scheme [55], where the least significant bit is abandoned in the event of a request for rate decrease.

- Selection of coding modes. Since there are various coding options available for encoders such as H.263+ and MPEG-4, these options can be varied during the encoding process to maintain the target source rate. For example, the interval between intra-coded frames is varied adaptively in [61]. The number of intra-coded macroblocks (MB’s) is adapted to the maximal TR in [54].

- Frame skipping. When the buffer occupancy reaches a certain level, certain frames belonging to the original raw (uncoded) video sequence are skipped in the encoding process, in order to decrease the total bitrate. While the number of transmitted frames is indeed lower, the number of bits carried by the latter is increased: the differentially-encoded motion vectors and/or texture coefficients will be larger because of an increase in the prediction range. The video sequence will also appear more jerky if too much frame skipping is used. The rate control algorithm used in the Test Model Near-term version 8 (TMN8) of H.263 uses frame skipping to complement quantization step size control. A modification of this algorithm presented in [56], and the framework of [53] also use frame skipping for wireless channels.

- Layer dropping. In scalable video coders, enhancement layers can be dropped during periods of buffer congestion. This can be seen as a form of frame skipping, since all the frames belonging to a certain enhancement layer are dropped. Such a mechanism was exploited in [55].
The rate control algorithm used in TMN8 of H.263 [62], [63] performs in a first step frame-layer rate control (the total number of bits per frame is determined), then follows on with macro-block-layer rate control (the bit budget is split between the MB's). Several other authors have opted for such an approach [54], [56].

1.4.3.2 Transmission Rate Control (TRC)

Very different approaches can be used and combined to assure the timely transmission of the video stream delivered by the source encoder. They are individually reviewed below.

*Video Transcoding*

Video transcoding consists in recoding the video stream produced by the original encoder, in order to make it suitable for transmission or adapt it to the capabilities of the user. A clear drawback is the extra complexity required and the associated delay. It was nonetheless privileged in the following works:

- In [48], the authors use transcoding in order to adapt the transmitted video sequence to the capabilities of the end-user and the network conditions. The transcoding is carried out at a proxy, which serves as an interface between the video server and the air interface of the network.

- In [64], transcoding is also performed by a proxy to match the bit rate to the wireless link capacity.

- In [65], transcoding for error-resilient purposes was reported.

*Smoothing*

To avoid dealing with the source coding mechanisms as in source rate control or transcoding, one can perform smoothing of the input source traffic, which consists in scheduling the future transmission rates in a manner that the rate variations are minimized, so that the source appears close to a constant bit rate one. To this end, a certain amount of buffering must be introduced either at the transmitter or the receiver. This will however inevitably result in a certain delay in the decoding. Methods to achieve smoothing
depend on whether the video frames are available progressively to the transmitter, as in real-time interactive or streaming video, or they are all available beforehand, as in stored video. In the first case (the *online* case), the transmitter can introduce a delay of a few frames in order to obtain a certain margin for smoothing. It has to determine the best sequence of transmission rates (or *schedule*) in a progressive way, either on a frame-by-frame basis, or based on a sliding window of frames. In the second case (the *offline* case), since the transmitter knows all of the frame lengths in advance, it can determine the optimal schedule before it initiates transmission, and then keeps it frozen.

Several studies have been carried out for both offline and online smoothing algorithms, but mainly with wireline networks in mind [66], [49], [67], [68], [69], [51]. In Section 4.3.1, we give more details about some of the algorithms, and explain what are the new constraints and their implications when dealing with power and bandwidth-limited cellular CDMA systems.

*Physical and Link-Layer Techniques for Rate Adaptation*

For time-division multiple-access (TDMA) systems, many studies have been carried out on variable modulation for video [70], [71], [72]. Comprehensive reviews are available in [73] and [74]. Essentially, the transmitter chooses from a set of predefined Quadrature Amplitude Modulation (QAM) constellations the modulation format which best matches the channel conditions. However, most studies seem to overlook the impact of the VBR source on the adaptation schemes, or consider transmission rates which are well above those required for low-bit rate video.

For CDMA systems, as will be detailed in Section 3.1, multicode and multirate transmission are available to the transmitter in order to adapt its output rate. Some studies have considered multicode transmission for video [75], [76], [77], however the results are usually preliminary. We will show how to combine smoothing and link-layer rate adaptation techniques in Section 4.3.
1.5 Thesis Contributions and Organization

1.5.1 Statistics and Simulation of Wideband Fading Channels

Since our work targets transmission over wireless wideband channels, it is necessary to be able to understand the properties of such channels, so as to efficiently simulate them, and their effect on the performance of communication systems. In Chapter 2, we make several novel contributions to the theory of wideband fading channels. The Nakagami distribution, which is a generalization of the Rayleigh distribution (c.f. Section 2.2), will be used to model the envelope of the fading process.

A first objective consists in designing an efficient Nakagami fading simulator. To do so, we start by deriving several analytical expressions for the most common second-order statistics of Nakagami fading channels. In particular, in Section 2.3 we obtain the level crossing rates, average fade durations, envelope correlation and baseband spectrum of channels with diversity combining, which provide us with some insights into the behavior of such channels. The obtained theoretical level crossing rates are then used to construct in Section 2.4.2 a finite-state Markov channel simulator for slow fading, whose advantages and limitations are evaluated quantitatively. In Section 2.4.1 we also present a review of previous methods used to generate spatially or temporally correlated Nakagami random variables. We give some tricks on how to improve some of the spatially correlated simulators, and discuss the limitations of current temporally correlated simulators.

A second objective is to obtain generic error probability formulas for different modulation schemes in correlated Nakagami fading channels, for both maximal-ratio or equal-gain combining. In Section 2.5, we consider the special but widespread case of two diversity branches, and derive in a unified manner several novel expressions in single-integral form. These expressions simplify or complement previous work on diversity combining in correlated channels. In some special cases, closed-form solutions (as infinite series summations) can be found.

The results of Chapter 2 are of very general scope, and as such this chapter could stand on its own. In fact several of the analytical results and simulation methods presented here will be useful in Chapter 3, when we consider the performance of multiple-access systems in wideband fading channels.
1.5.2 Analysis of Multicode and Multirate DS/CDMA Systems in Wideband Fading Channels

Since we are interested in VBR video transmission over multicode/multirate CDMA systems, it is necessary to be able to simulate such systems, by deriving the decision metrics which are used at the receiver, and to predict their performance in fading channels. Chapter 3 presents a detailed and accurate analysis of the multicode/multirate configurations used in the physical layers of IS-95B and IS-2000 systems, when diversity receivers are used to improve performance in fading channels. The methodology takes into account in a precise manner the effect of the fading (which was overlooked or misunderstood in previous studies), and leads to theoretical bit error rates very close to the actual (simulated) ones. The analysis is carried out separately for:

- Reverse link multicode DS/CDMA with $M$-ary orthogonal noncoherent modulation and equal-gain combining (as used in IS-95B), in Section 3.2;

- Reverse link multicode DS/CDMA with binary coherent modulation and maximal-ratio/equal-gain combining, and complex spreading sequences (as used in IS-2000), in Section 3.3;

- Forward link multicode DS/CDMA with quaternary coherent modulation and maximal-ratio/equal-gain combining, and real spreading sequences (as used in IS-95B), in Section 3.4;

- Forward link multicode DS/CDMA with quaternary coherent modulation and maximal-ratio/equal-gain combining, and complex spreading sequences (as used in IS-2000), in Section 3.5.

The proposed methodology allows us to use any fading distribution: examples will be given for Rayleigh, Nakagami, Rician and lognormal fading. Moreover, correlated diversity branches (which represent a more realistic fading scenario) are easily integrated in the analysis. Some extensions to systems using advanced techniques are also presented in Section 3.2.
1.5.3 VBR Video Transmission for Multicode and Multirate DS/CDMA Systems in Wideband Fading Channels

As stated in Section 1.1, we are interested in providing algorithms to support the efficient and delay-conscious transmission of variable bit rate sources over multicode and multirate DS/CDMA systems. Chapter 4 takes the IS-95B and IS-2000 cellular systems as case examples to support our research: it is stressed that our work is intended to be more general in scope, and not specific to particular standards, however to obtain more realistic results we have abided by many of the parameters specified in current and future DS/CDMA physical layer standards.

In this chapter, we start by describing in Section 4.2 a software implementation of an end-to-end video communication system, which comprises the video coder, the packetizer/multiplexer, and the physical layer components of the reverse link (mobile user to base station, or uplink) and forward link (base station to mobile user, or downlink) of IS-95B and IS-2000 systems (which include error-control encoders, interleaver, spreading sequences and modulator). The system is subject to multipath fading and multiple-access interference, the latter being obtained by simulating all of the interfering users or codes. This platform will be used in the performance evaluations of the rest of the chapter.

We then propose in Section 4.3 a smoothing algorithm optimized for multicode and multirate DS/CDMA systems, and compare it to a previous benchmark algorithm which we adapted to deal with the system at hand. End-to-end simulations are carried out using the previously described software platform, for both the uplink and downlink of IS-95B and IS-2000 systems, and the performance of both smoothing algorithms is assessed in terms of the decoded video quality. The effects of the startup buffering delay and of the length of the sliding window are investigated in detail.
1.6 Publications

Based on the research reported in this thesis, the following publications have resulted:

Refereed Papers in Journals


Refereed Papers in Conference Proceedings


CHAPTER 2

STATISTICS AND SIMULATION OF WIDEBAND FADING CHANNELS

2.1 Introduction

This chapter makes several novel contributions to the theory of wideband fading channels. Section 2.2 briefly reviews three types of fading channels commonly used in the study of mobile radio, namely Rayleigh, Rician and Nakagami channels. Section 2.3 analytically derives some important second order statistics of such channels, when diversity is used. Making use of some of the presented first- and second-order statistics of Nakagami channels, Section 2.4 proposes a new wideband Nakagami fading channel simulator, and also reviews and discusses existing ones. Then Section 2.5 derives new analytical expressions for the error rates of several common modulation techniques, in a wideband Nakagami fading environment with correlation between the diversity-received signals; these theoretical results are checked against computer simulation results obtained by implementing some of the previously described channel simulators.

2.2 A Review of Common Fading Models and Distributions

2.2.1 Rayleigh Fading

The most common model used in the radio communications literature to describe flat fading in urban/suburban environments is Clarke's model [78]. It assumes a fixed transmitter with a vertically polarized antenna, and a mobile terminal; however it has also been used (with possible modifications) in several other scenarios which do not necessarily conform to these assumptions in a strict sense.
Clarke’s original model considers an electric field incident on a mobile antenna. This field consists of $N$ azimuthal plane waves (also often called scatterers), with:

- Arbitrary carrier phases $\phi_n$;
- Arbitrary azimuthal angles of arrivals $\alpha_n$;
- Equal average amplitudes.

Mathematically, this field can be expressed as:

$$E_z(t) = \sum_{n=1}^{N} E_n(t) = \sum_{n=1}^{N} \hat{E}_n C_n \cos(2\pi f_c t + \theta_n)$$

$$= \hat{E} \sum_{n=1}^{N} C_n \cos(2\pi f_c t + \theta_n)$$

$$= T_c(t) \cos(2\pi f_c t) - T_s(t) \sin(2\pi f_c t) \quad (2.1)$$

where:

$$T_c(t) = \hat{E} \sum_{n=1}^{N} C_n \cos(\theta_n) \quad (2.2)$$

$$T_s(t) = \hat{E} \sum_{n=1}^{N} C_n \sin(\theta_n) \quad (2.3)$$

are the uncorrelated in-phase and quadrature components of the electric field, respectively, $\hat{E}$ is a constant, $C_n$ are mutually independent normalized random variables representing the fading on each path, such that $E[\sum_{n=1}^{N} C_n^2] = 1$, where $E[\cdot]$ denotes statistical expectation, and:

$$\theta_n = 2\pi f_n t + \phi_n \quad (2.4)$$

are the phases, with $f_n = (v/\lambda) \cos \alpha_n$ the Doppler shift of the $n^{th}$ wave, given a mobile speed $v$ (m/s), a carrier frequency $f_c$ (Hz) and a carrier wavelength $\lambda = c/f_c$ ($c \approx 3.10^8$ m/s is the speed of light in free space). $f_m = v/\lambda$ is called the maximum Doppler shift. For a large number of waves $N$, as per the central limit theorem ([79], p. 266), samples
$T_c$ and $T_s$ of the in-phase and quadrature processes $T_c(t)$ and $T_s(t)$ become normally distributed with zero means and variances $E[T_c^2] = E[T_s^2] = \bar{E}^2/2 = \Omega/2 = \sigma^2$. Hence the envelope of a sample $E_z$ of the electric field $E_z(t)$:

$$r = |E_z| = \sqrt{T_c^2 + T_s^2}$$  \(2.5\)

where $| \cdot |$ denotes the absolute value of $\cdot$, can be shown ([79], p. 195) to be Rayleigh-distributed, i.e. with probability density function (pdf):

$$p_R(r) = \frac{2r}{\Omega} e^{-\frac{r^2}{\Omega}}, \quad r \geq 0$$  \(2.6\)

and cumulative distribution function (cdf):

$$F_R(r) = 1 - e^{-\frac{r^2}{\Omega}}$$  \(2.7\)

where $\Omega = 2\sigma^2 = E[r^2]$. The moments of the Rayleigh distribution are given by [26]:

$$E[r^k] = \Omega^\frac{k}{2} \Gamma\left(\frac{k}{2} + 1\right)$$  \(2.8\)

where $\Gamma(x) = \int_0^\infty e^{-t}t^{x-1}dt$ is the gamma function (Eq. 8.310.1 of [80]). The mean of $r$ is hence given by $\mu_r = E[r] = \sigma \sqrt{\pi}/2$ and its variance by $\sigma_r^2 = E[r^2] - E^2[r] = \sigma^2(2 - \pi/2)$. The quantity $\tan^{-1}(T_s/T_c)$ can be shown ([79], pp. 200-201) to be independent of $r$ and uniformly distributed over $[-\pi/2, \pi/2]$. By periodicity, the phase of $E_z$, $\theta$, is then independent of $r$ and uniformly distributed over $[0, 2\pi]$:

$$p_\theta(\theta) = \frac{1}{2\pi}, \quad 0 \leq \theta \leq 2\pi.$$  \(2.9\)

Based on Clarke's model, Gans [81] developed a power spectral theory for the mobile radio channel. Assuming that $N$ is very large, that the $\alpha_n$'s are uniformly distributed over $[0, 2\pi]$, and that the signal is transmitted by a omnidirectional vertical $\lambda/4$ antenna with gain $G(\alpha) = 1.5$, the power spectrum of the electric field Eq. (2.12) was shown to
be (c.f. [15], Eq. 1.2-11):

\[
S(f) = \frac{1.5\Omega}{2\pi f_m \sqrt{1 - \left(\frac{f - f_m}{f_m}\right)^2}}. \tag{2.10}
\]

The power spectrum of the baseband envelope Eq. (2.5) was further shown to be (c.f. [15], Eq. (1.3-27) p. 29, or [16], Eq. (4.79)):

\[
S_b(f) = \frac{\Omega}{16\pi f_m} K \left(\sqrt{1 - \left(\frac{f}{2f_m}\right)^2}\right) \tag{2.11}
\]

where \(K(\cdot)\) is the complete elliptic integral of the first kind [80].

### 2.2.2 Rician Fading

If a dominant stationary wave \(E_0(t) = \bar{E}_0 C_0 \cos(2\pi f_c t + \theta_0)\) of constant amplitude \(A = \bar{E}_0 C_0\) is included in the received signal (e.g. a nonfading line-of-sight component), the expression for the electric field becomes:

\[
E_z(t) = E_0(t) + \sum_{n=1}^{N} \tilde{E}_n C_n \cos(2\pi f_c t + \theta_n)
\]

\[
= \tilde{\bar{E}} \sum_{n=0}^{N} C_n \cos(2\pi f_c t + \theta_n)
\]

\[
= T_c(t) \cos(2\pi f_c t) - T_s(t) \sin(2\pi f_c t) \tag{2.12}
\]

where now \(T_c\) and \(T_s\) are Gaussian with means \(A \cos(\phi_0)\), \(A \sin(\phi_0)\) (taking \(f_0 = 0\)), and powers \(E[T_c^2] = A^2 \cos^2(\phi_0) + \sigma^2\), \(E[T_s^2] = A^2 \sin^2(\phi_0) + \sigma^2\). The envelope \(r\) can then be shown ([79], pp. 196-197) to be Rician-distributed, i.e. with pdf:

\[
p_R(r) = \frac{2r}{\Omega} e^{-\frac{Ar^2}{\Omega}} I_0 \left(\frac{2Ar}{\Omega}\right), \quad r \geq 0 \tag{2.13}
\]

and cdf:

\[
F_R(r) = 1 - Q_1 \left(\frac{A}{\sqrt{\Omega/2}}, \frac{r}{\sqrt{\Omega/2}}\right) \tag{2.14}
\]
where $I_0(\cdot)$ is the modified Bessel function of order 0, and $Q_1(\cdot)$ is the Marcum-Q function of order 1. $K = 10 \log_{10}(A^2/\Omega)$ (dB) is termed the Rician factor. When $A = 0$ (i.e. $K \to -\infty$), the Rice pdf reverts to the Rayleigh pdf. Note that the Rice pdf was also independently discovered by Nakagami [23] in his studies of mobile radio propagation, which he termed the $n$-distribution. Hence the Rice pdf is also called the Nakagami-$n$ or Nakagami-Rice pdf. The moments of the Rice distribution are given by [26]:

$$E[r^k] = \Omega^{\frac{k}{2}} \Gamma \left( \frac{k}{2} + 1 \right) \Phi \left( -\frac{k}{2}; 1; -\frac{A^2}{\Omega} \right)$$

(2.15)

where $\Phi(a; b; x)$ is the confluent hypergeometric function ([80], Eq. 9.210.1). The mean of $r$ is hence given by $\mu_r = E[r] = \frac{1}{2} \sqrt{\Omega \pi} \Phi \left( -\frac{1}{2}; 1; -\frac{A^2}{\Omega} \right)$ and its variance by $\sigma_r^2 = A^2 + \Omega - \mu_r^2$. The phase of $E_z$ can be shown ([79], pp. 499-501) to be given by ($\phi_0 = 0$):

$$p_\theta(\theta) = \frac{e^{-\frac{A^2}{2\sigma^2}}}{2\pi} + \frac{A \cos(\theta)e^{-\frac{A^2 \sin^2(\theta)}{2\sigma^2}}}{2\sigma \sqrt{2\pi}} \left[ 1 + 2\text{erf} \left( \frac{A \cos(\theta)}{\sigma} \right) \right], \quad 0 \leq \theta \leq 2\pi$$

(2.16)

where $\text{erf}(x) = 2/\sqrt{\pi} \int_0^x e^{-z^2} dz$ is the error function.

The power spectrum of the electric field is similar to that of the Rayleigh case, but with an impulse at the frequency $f_c$.

### 2.2.3 Nakagami Fading

While the Rayleigh and Rice distributions can indeed be used to model the envelope of fading channels in many cases of interest, it has been found experimentally [23] that the Nakagami distribution offers a better fit for a wider range of fading conditions. The Nakagami distribution was proposed in the early forties for characterizing urban and suburban fading channels, and was originally deduced from a series of experiments. It was later shown that it constitutes an approximation to the pdf of the amplitude of a sum of phasors with random moduli and phases [23], [82]. Contrarily to the Rice pdf, it doesn’t assume a line-of-sight (LOS) condition. Hence, while the Rice distribution can only describe better-than-Rayleigh fading conditions, the Nakagami pdf with parameter $m < 1$ models worse-than-Rayleigh conditions. Moreover, for $m = 1$, the Nakagami pdf reduces to the Rayleigh pdf, and can thus be seen as a generalization of the latter. It
was verified in several other independent experimental researches that the Nakagami-\(m\) pdf could indeed accurately represent the wide range of commonly encountered fading conditions [24], [83], [84]. As a result, it has been adopted in some software and hardware fading channel simulators for 3rd generation (3G) cellular networks [85], [86]. It is also increasingly used in the analysis and modeling of wideband channels, in particular for CDMA systems in frequency-selective fading [39]; this is also encouraged by the fact that its analytical form is more amenable to manipulations, compared to the Rice pdf, which contains a modified Bessel function of the first kind.

We briefly review the theoretical development leading to the Nakagami pdf, along with the approximations made [23]. Using a complex notation for the plane waves, the sample envelope of the received signal can be written as:

\[
\begin{align*}
\sum_{n=1}^{N} |E_n e^{j\theta_n}| & = \xi(E_1 e^{j\theta_1}, E_2 e^{j\theta_2}, \ldots, E_N e^{j\theta_N}) \quad (2.17)
\end{align*}
\]

where no specific assumptions are made about the pdfs of the random variables \(E_n\) and \(\theta_n\), \(n = 1, 2, \ldots, N\), and \(\xi = \xi(E_1 e^{j\theta_1}, E_2 e^{j\theta_2}, \ldots, E_N e^{j\theta_N})\) is a positive definite function of \(N\) complex random vectors. The pdf of \(r\) can be written as:

\[
\begin{align*}
\rho_N(r) & = E[\delta(r - \xi)] \quad (2.18) \\
& = E\left[ r^{\nu+1} \int_0^\infty \lambda J_\nu(\lambda r) \frac{J_\nu(\lambda \xi)}{\xi^\nu} d\lambda \right] \\
& = r^{\nu+1} \int_0^\infty \lambda J_\nu(\lambda r) \frac{\lambda^v}{2^v \Gamma(v + 1)} \left( \frac{2}{\lambda} \right)^v \Gamma(v + 1) E \left[ \frac{J_\nu(\lambda \xi)}{\xi^v} \right] d\lambda \\
& = \frac{r^{\nu+1}}{2^v \Gamma(v + 1)} \int_0^\infty \lambda^{v+1} J_\nu(\lambda r) F_v(\lambda) d\lambda \quad (2.19)
\end{align*}
\]

where

\[
F_v(\lambda) = \left( \frac{2}{\lambda} \right)^v \Gamma(v + 1) E \left[ \frac{J_\nu(\lambda \xi)}{\xi^v} \right] . \quad (2.20)
\]

In Eqs. (2.18) and (2.20), the expectations are taken with respect to the \(N\) complex variables \(\{E_n e^{j\theta_n}\}\), \(\delta(\cdot)\) denotes the Dirac functional, \(v = N/2 - 1\), and \(J_\nu(\cdot)\) is the Bessel
function of order $v \geq \frac{1}{2}$. Using the infinite series expansion (A.5) for $J_v(\cdot)$:

$$F_v(\lambda) = \Gamma(v + 1) \sum_{k=0}^{\infty} \frac{(-1)^k}{\Gamma(v + k + 1)k!} \left(\frac{\lambda^2}{4}\right)^k E[(\xi^2)^k].$$

(2.21)

By replacing the Lyapunov inequality $E[\xi^{2k+2}] \geq E[\xi^{2k}]E[\xi^2]$ by an equality, Nakagami obtains the approximation:

$$F_v(\lambda) \simeq e^{-\frac{\Omega \lambda^2}{4m}}$$

(2.22)

where $\Omega = E[r^2]$ and

$$m = \frac{(E[r^2])^2}{E[(r^2 - E[r^2])^2]} = \frac{\Omega^2}{E[r^4] - \Omega^2} = v + 1 \geq \frac{1}{2}$$

(2.23)

is called the Nakagami fading figure. Using Eq. (2.22) in Eq. (2.19), and making use of Eq. (8.6.10) of [87], one obtains:

$$p_R(r) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m r^{2m-1} e^{-\frac{m}{\Omega}r^2}, \quad r \geq 0$$

(2.24)

which is the Nakagami pdf. Note that while the above derivation was carried out with the assumption that $m$ was an integer or half-integer, the pdf Eq. (2.24) holds for any real $m \geq 0.5$, as was also verified experimentally [23]. For $m = 1$, Eq. (2.24) reverts to Eq. (2.6). The moments of the Nakagami distribution are given by [26]:

$$E[r^k] = \left(\frac{\Omega}{m}\right)^{\frac{k}{2}} \frac{\Gamma(\frac{k}{2} + m)}{\Gamma(m)}.$$

(2.25)

The mean of $r$ is hence given by $\mu_r = \left(\frac{\Omega}{m}\right)^{\frac{1}{2}} \frac{\Gamma(\frac{1}{2} + m)}{\Gamma(m)}$ and its variance by:

$$\sigma_r^2 = \Omega \left(1 - \frac{1}{m} \frac{\Gamma(\frac{1}{2} + m)}{\Gamma(m)} \right).$$

(2.26)

There was no phase pdf associated with the Nakagami distribution in the original paper [23]. While it has been argued on several occasions [88] that a uniform phase can be
assumed (as for the Rayleigh case), there is still no formal theoretical (or experimental) justification for this assumption. Determining the pdf (or proving the uniformity) of the phase is thus still an open problem.

2.3 Analytical Second-Order Statistics of Fading Channels with Diversity

2.3.1 Analytical Level Crossing Rates and Average Fade Durations of Diversity-Combined Nakagami Fading Signals

2.3.1.1 Introduction

The level crossing rates and the average fade durations are two quantities which statistically characterize a fading communication channel. The level crossing rate (LCR) is defined as the number of times per unit duration that the envelope of a fading channel crosses a given value in the negative direction [15]. The average duration of fades (AFD) corresponds to the average length of time the envelope remains under this value once it crosses it in the negative direction. These quantities reflect the correlation properties, and thus the second-order statistics, of a fading channel. They provide a dynamic representation of the channel. They complement the pdf and cdf, which are first-order statistics, and can only be used to obtain static metrics associated with the channel, such as the bit error rate (BER). The LCR and the AFD have found a variety of applications in the modeling and design of wireless communication systems, such as the finite-state Markov modeling of fading channels [89], the analysis of handoff algorithms [90] and the estimation of packet error rates [91]. Pioneering work on the subject was done by Rice [92], which examined LOS (Rician) channels. Much later, some expressions for the LCR and AFD of the combined envelope of diversity Rayleigh channels were published. For example, Lee [93] derived these quantities for equal-gain combining (EGC). Adachi et al. [94] provided general expressions in the case of dual correlated channels with selection combining (SC), maximal-ratio combining (MRC) and EGC diversity; these expressions could be put in closed-form for independent channels. Recently, Yacoub et al. [95]
also presented expressions in the case of EGC and MRC with an arbitrary number of independent channels.

In [96], using results from [97], the authors derived the LCR and AFD for Nakagami channels without diversity, and for a special case of MRC diversity. They also presented an approximate result for EGC, relying on Eqs. (81)-(83) of [23]. The LCR and AFD of non-diversity Nakagami channels with isotropic scattering were also later obtained in [98] using a different approach, which relied on the decomposition of the distribution. Note that the LCR had been previously obtained in an intuitive manner in the original paper of Nakagami ([23], Eqs. (23)-(25)), where they were denoted by "fineness". Field trials were carried out in [99] and [100] for the non-diversity case, and good agreements were reported between the experimental LCR and the analytical expressions.

In this section, we present a very general approach which can be used to analytically evaluate the LCR and AFD for Nakagami channels with diversity reception. Our methodology is the following: we straightforwardly rearrange the expression for the LCR, such that it can be expressed as the product of the probability density function of the received signal and an integral involving the conditional pdf of the derivative of this signal. Depending on the cases, the first term either can be found in the literature or has to be derived. The conditional pdf in the second term is found by examining the expression for the derivative of the received signal. It should be noted that the proposed methodology does not have limitations nor makes any simplifying assumptions. However, we shall only present the cases where a simple analytical closed-form solution for the LCR and AFD is possible, which generally requires the diversity channels to fade independently. It will also be shown that our general analytical expressions for Nakagami fading reduce to previously known results. In contrast to this work, earlier derivations of the LCR and AFD for channels with diversity reception were usually specific to a particular channel/diversity pair, and can’t always be used in obtaining the same quantities for different situations. For example, in [96] the LCR were obtained by finding a closed-form expression for the joint pdf of the received signal and its derivative for the case of Nakagami fading, which is not always possible; whereas in [94] and [95], the analytical derivations were conducted for specific situations only (dual-diversity for the former) or under certain assumptions (identical channels for both).
The organization of this section is as follows. After this introduction, our analytical approach and the steps needed to apply it to Nakagami channels (using the physical insights of [98]) are presented in Section 2.3.1.2. These results are used to obtain the LCR and AFD for Nakagami channels with SC, MRC and EGC diversity reception in the following subsections. The analytical expressions obtained are evaluated numerically and discussed in Section 2.3.1.6. The last section summarizes our contributions and cites some applications.

2.3.1.2 General Expressions

Let \( r \) be the sampled value of the diversity combined envelope \( R(t) \) of a fading channel. The LCR \( N_R(r) \) and AFD \( \tau_R(r) \) are defined as a function of \( r \) by:

\[
N_R(r) = \int_0^\infty r p_{R_R}(r, \dot{r}) d\dot{r}, \tag{2.27}
\]

\[
\tau_R(r) = \frac{F_R(r)}{N_R(r)} \tag{2.28}
\]

where (') denotes the derivation operator with respect to time, \( F_R(r) = \int_0^r p_R(\alpha) d\alpha \) is the cdf of the fading channel, and \( p_R(r) \) is the corresponding pdf. The LCR can be rewritten in terms of \( p_R(r) \) and the conditional distribution \( p_R(\dot{r}|r) \) as

\[
N_R(r) = \int_0^\infty \dot{r} p_{R_R}(\dot{r}|r)p_R(r) d\dot{r} \]

\[
= p_R(r) \int_0^\infty \dot{r} p_R(\dot{r}|r) d\dot{r}. \tag{2.29}
\]

This generic expression for \( N_R(r) \) will be the basis for all later derivations. It is indeed applicable to all forms of diversity, and can be used in conjunction with any fading distribution. It reduces directly, for example, to Eq. (6) of [93], (15) of [96], and (16) of [98] for the special cases treated in these papers.

The output sampled envelope of an \( L \)-branch diversity combiner can be expressed in the generic form of

\[
r = f(r_1, r_2, \ldots, r_L) \tag{2.30}
\]
where $r_l$, with $l = 1, 2, \ldots, L$ is the envelope of the $l^{th}$ diversity channel seen by the receiver, and $f(\cdot)$ is a function which depends on the diversity technique used. In the case of Nakagami fading, the pdf of $r_l$ is mathematically expressed as

$$p_{R_l}(r_l) = 2 \left( \frac{m_l}{\Omega_l} \right)^{m_l} \frac{r_l^{2m_l-1}}{\Gamma(m_l)} e^{-\frac{m_l}{\Omega_l} r_l^2}, \quad r_l \geq 0$$

(2.31)

where $\Omega_l = E[r_l^2]$ and $m_l$ are the average power and the fading figure of the $l^{th}$ channel, respectively. The cdf of $r_l$ is given by

$$F_{R_l}(r_l) = \frac{\gamma(m_l, \frac{m_l r_l^2}{\Omega_l})}{\Gamma(m_l)}$$

(2.32)

where $\gamma(x, \alpha) = \int_0^x t^{\alpha-1} e^{-t} dt$ is the incomplete gamma function of the first kind (Eq. 8.350.1 of [80]).

By analogy with [98], when $2m_l$ is an integer, the envelope of the $l^{th}$ diversity channel can be written as

$$r_l^2 = \begin{cases} r_{l0}^2 + \sum_{i=1}^{m_l} r_{li}^2 & \text{with } 2m_l \text{ odd} \\ \sum_{i=1}^{m_l} r_{li}^2 & \text{with } 2m_l \text{ even} \end{cases}$$

(2.33)

where $r_{l0}^2 = x_{l0}^2$, $r_{li}^2 = x_{li}^2 + y_{li}^2$, and the $x_{li}$'s, $y_{li}$'s are Gaussian random variables with zero mean and variance $\sigma_l^2 = \Omega_l/(2m_l)$. The derivatives of the $r_l$'s can then be calculated using

$$\dot{r}_l = \begin{cases} \frac{(r_{l0} \dot{r}_{l0} + \sum_{i=1}^{m_l} r_{li} \dot{r}_{li})}{r_l} & \text{with } 2m_l \text{ odd} \\ \frac{(\sum_{i=1}^{m_l} r_{li} \dot{r}_{li})}{r_l} & \text{with } 2m_l \text{ even}. \end{cases}$$

(2.34)

From [15], for isotropic scattering, the $\dot{r}_{li}$'s are Gaussian-distributed with zero mean and variance $\tilde{\sigma}_{r_{li}}^2 = \text{Var}[\dot{r}_{li}] = \sigma_l^2 2\pi^2 f_m^2$. We let $\hat{\sigma}_l^2 = \sigma_l^2 2\pi^2 f_m^2$ to alleviate the notation. Since $\dot{r}_l$ is a sum of zero-mean Gaussian variables, it is also zero-mean Gaussian, conditioned
on \( r_l \). Using Eqs. (2.34) and (2.33), its variance is found to be \( \sigma_r^2 = \text{Var}[\hat{r}_l] = \hat{\sigma}_l^2 \), which is independent of \( r_l \). As asserted in [98], \( \hat{r}_l \) and \( r_l \) are thus independent, so that \( p(r_l, \hat{r}_l) = p(r_l)p(\hat{r}_l) \).

Using the above, the analytical LCR and AFD for the diversity methods of concern will be derived in the next sections.

### 2.3.1.3 Selection Combining (SC)

In [94], the authors present an expression for the LCR for dual SC in Rayleigh fading. It is generalized in [101] for an arbitrary number of independent and identically distributed (i.i.d.) channels. Below, using Eq. (2.29), we derive an expression for the LCR of SC for \( L \) independent but not necessarily identical channels. We then apply it to the case of Nakagami fading. The channel envelope at the output of a SC diversity system is well-known to be given by:

\[
\hat{r} = \max \{ r_l, l = 1, 2, \ldots , L \}. 
\]  

(2.35)

Its derivative is:

\[
\hat{r}' = \hat{r}_j, \quad r_j = \max \{ r_l, l = 1, 2, \ldots , L \}. 
\]  

(2.36)

\( \hat{r} \) is thus a Gaussian random variable when conditioned on the \( r_l \)'s, with zero mean and variance:

\[
\hat{\sigma}_r^2 = \hat{\sigma}_j^2 \text{ if } (r_j = \max_{l=1,2,\ldots,L} r_l | r_j = r). 
\]  

(2.37)

Consequently, \( \hat{\sigma}_r \) is a discrete random variable with pdf:

\[
p_{\hat{\sigma}_r}(\hat{\sigma}_r) = \sum_{j=1}^{L} P(\hat{\sigma}_r = \hat{\sigma}_j) \delta(\hat{\sigma}_r - \hat{\sigma}_j) \\
= \sum_{j=1}^{L} P(r_j = \max_{l=1,2,\ldots,L} r_l | r_j = r) \delta(\hat{\sigma}_r - \hat{\sigma}_j). 
\]  

(2.38)
From Eq. (2.29), the LCR, conditional on \( \dot{\sigma}_r \), are given by:

\[
N_R(r|\dot{\sigma}_r) = p_R(r) \int_0^\infty \frac{1}{\sqrt{2\pi} \dot{\sigma}_r} e^{-\frac{-\dot{\sigma}_r^2}{2\dot{\sigma}_r^2}} \, d\dot{\sigma}_r
\]

\[= p_R(r) \frac{\dot{\sigma}_r}{\sqrt{2\pi}}. \tag{2.39}\]

Eq. (2.39) is averaged over the pdf for \( \dot{\sigma}_r \), i.e. Eq. (2.38), to obtain:

\[
N_R(r) = \int_0^\infty N_R(r|\dot{\sigma}_r) p_{\dot{\sigma}_r}(\dot{\sigma}_r) \, d\dot{\sigma}_r
\]

\[= \sum_{j=1}^L p_{R_j}(r) \frac{\dot{\sigma}_j}{\sqrt{2\pi}} P(r_j = \max_{l=1,2,...,L} r_l | r_j = r). \tag{2.40}\]

By taking into account the independence assumption, the term \( P(r_j = \max_{l=1,2,...,L} r_l | r_j = r) \) can be evaluated as:

\[
P(r_j = \max_{l=1,2,...,L} r_l | r_j = r) = P(r_l < r, l = 1, 2, \ldots, L, l \neq j | r_j = r)
= P(r_l < r, l = 1, 2, \ldots, L, l \neq j)
= \prod_{l=1}^L P(r_l < r) = \prod_{l \neq j} F_{R_l}(r). \tag{2.41}\]

From Eqs. (2.40) and (2.41) the LCR can be expressed as:

\[
N_R(r) = \sum_{j=1}^L p_{R_j}(r) \frac{\dot{\sigma}_j}{\sqrt{2\pi}} \prod_{l \neq j} F_{R_l}(r). \tag{2.42}\]

Substituting Eqs. (2.31) and (2.32) into (2.42), and using \( \dot{\sigma}_j = \sigma_j \sqrt{2\pi} f_m \) leads to the LCR for a Nakagami fading channel with arbitrary parameters and SC:

\[
N_R(r) = \sqrt{2\pi f_m} \sum_{j=1}^L \frac{1}{\Gamma(m_j)} \left( \frac{m_j r^2}{\Omega_j} \right)^{m_j - \frac{1}{2}} e^{-\frac{m_j r^2}{\Omega_j}} \prod_{l \neq j} \frac{\gamma(m_l, \frac{m_l r^2}{\Omega_l})}{\Gamma(m_l)}. \tag{2.43}\]

For identical channel parameters, \( m_l = m, \Omega_l = \Omega, l = 1, 2, \ldots, L \), Eq. (2.43) reverts to the expression given in [102], and when \( m = 1 \), to the one given in [101], Eq. (45) for
Rayleigh fading:

\[ N_R(r) = L \frac{\sqrt{2\pi} f_m}{\sqrt{\Omega}} \frac{r}{e^{-\frac{r^2}{\Omega}}} \left( 1 - e^{-\frac{r^2}{\Omega}} \right)^{L-1}. \]  

(2.44)

It can be verified (c.f. Appendix B) that the approach taken in [101] for obtaining the LCR, when extended to include arbitrary parameters, also leads to Eq. (2.43) for Nakagami fading.

Eq. (2.43) gives the average number of times per second that the output \( R(t) \) of a selection diversity combiner falls below a specified value \( r \), given \( L, f_m, \) and the channel parameters \( m_i, \Omega_i, i = 1, 2, \ldots, L \). Hence, if the channel conditions can be estimated and a maximum speed for the mobile is assumed, based on Eq. (2.43), one can determine the number of diversity branches needed \( L \) so that the combined signal \( R(t) \) doesn't fall below a threshold \( r_T \) more than a specified maximum number of times \( N_T \). This can be done by evaluating \( N_R(r_T) \) for increasing values of \( L \), until \( N_R(r_T) < N_T \).

The cdf for SC is given by:

\[ F_R(r) = P(r_1 < r, r_2 < r, \ldots, r_L < r) \]  

which reduces for independent Nakagami channels to:

\[ F_R(r) = \prod_{i=1}^{L} F_{R_i}(r) = \prod_{i=1}^{L} \frac{\gamma(m_i, \frac{m_i}{\Omega_i} r^2)}{\Gamma(m_i)}. \]  

(2.46)

The AFD for arbitrary Nakagami channels with SC can then be obtained straightforwardly by substituting Eqs. (2.43) and (2.46) in (2.28):

\[ \tau_R(r) = \frac{\prod_{i=1}^{L} \frac{\gamma(m_i, \frac{m_i}{\Omega_i} r^2)}{\Gamma(m_i)}}{\sqrt{2\pi f_m} \sum_{j=1}^{L} \left( \frac{m_j}{\Omega_j} r^2 \right)^{m_j-\frac{3}{2}} e^{-\frac{m_j}{\Omega_j} r^2} \prod_{i=1}^{L} \frac{\gamma(m_i, \frac{m_i}{\Omega_i} r^2)}{\Gamma(m_i)}}. \]  

(2.47)
Eq. (2.47) gives the average time (in seconds) that the combined signal $R(t)$ stays below a specified level $r$, once it has crossed it in the downward direction, again given $L$, $f_m$ and the channel parameters. Hence, one can again evaluate the required $L$ so that, on average, the combined signal $R(t)$ doesn’t stay below a threshold $r_T$ more than a specified maximum period of time $\tau_{\text{max}}$. The quantity $\tau_{\text{max}}$ can correspond, for example, to the average period of time a receiver can demodulate a signal of amplitude $r_T$, without going into outage or losing synchronization. Similar insights can be obtained for the cases of maximal-ratio and equal-gain diversity, thanks to the expressions for the LCR and AFD derived in the following sections.

2.3.1.4 Maximal-Ratio Combining (MRC)

The output of a MRC diversity system is given by [15]:

$$ r = \left[ \sum_{l=1}^{L} r_{l}^2 \right]^{\frac{1}{2}} \quad (2.48) $$

and its derivative by:

$$ \dot{r} = \frac{\sum_{l=1}^{L} r_{l} \dot{r}_{l}}{r} \quad (2.49) $$

As in the SC case, $\dot{r}$ is a Gaussian random variable when conditioned on the $r_{l}$'s, with zero mean and variance:

$$ \hat{\sigma}_{r}^2 = E \left[ \left( \sum_{l=1}^{L} r_{l} \dot{r}_{l} \right)^2 / r^2 \right] $$

$$ = \sum_{l=1}^{L} r_{l}^2 E[\dot{r}_{l}^2] / \sum_{l=1}^{L} r_{l}^2 \quad (2.50) $$

where the last equation was obtained using the independence assumption between the branches. If the diversity channels are identically distributed, $E[\dot{r}_{l}^2] = \hat{\sigma}_{r_{l}}^2 = \sigma^2 2 \pi^2 f_m^2$, 

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and Eq. (2.50) reduces to:

\[ \sigma_r^2 = \sigma^2 2\pi^2 f_m^2. \]  

(2.51)

In that case, Eq. (2.29) can be solved to give:

\[ N_R(r) = p_R(r) \frac{\sigma_r}{\sqrt{2\pi}}. \]  

(2.52)

The pdf of \( r \) (again for the special case of i.i.d. channels) is known to be given by [23]:

\[ p_R(r) = 2 \left( \frac{m_T}{\Omega_T} \right)^{m_T} \frac{r^{2m_T-1}}{\Gamma(m_T)} e^{-\frac{m_T r^2}{\Omega_T}}, \quad r > 0 \]  

(2.53)

where \( m_T = mL, \Omega_T = \Omega L \), and the cdf is given by:

\[ F_R(r) = \frac{\gamma(m_T, \frac{m_T}{\Omega_T} r^2)}{\Gamma(m_T)}. \]  

(2.54)

Using Eqs. (2.53) and (2.51) in Eq. (2.52) leads to the following result for the LCR, which was also derived in [96] using the approach mentioned in the introduction:

\[ N_R(r) = \frac{\sqrt{2\pi} f_m}{\Gamma(m_T)} \left( \frac{m_T}{\Omega_T} r^2 \right)^{m_T-\frac{1}{2}} e^{-\frac{m_T r^2}{\Omega_T}}. \]  

(2.55)

Substituting Eqs. (2.54) and (2.55) in Eq. (2.28) yields the AFD:

\[ \tau_R(r) = \frac{\gamma(m_T, \frac{m_T}{\Omega_T} r^2)e^{\frac{m_T r^2}{\Omega_T}}}{\sqrt{2\pi} f_m \left( \frac{m_T}{\Omega_T} r^2 \right)^{m_T-\frac{1}{2}}}. \]  

(2.56)

For Rayleigh fading \( m = 1 \), Eq. (2.55) reduces to the following expression \(^1\):

\[ N_R(r) = \frac{\sqrt{2\pi} f_m}{\Gamma(L)} \left( \frac{r^2}{\Omega} \right)^{L-\frac{1}{2}} e^{-\frac{r^2}{\Omega}}. \]  

(2.57)

\(^1\)It should be noted that it is similar to Eq. (13) of [95], but the latter has possibly a misprint in the exponential term.
2.3.1.5 Equal-Gain Combining (EGC)

The output of a EGC diversity system is given by

\[ r = \frac{1}{\sqrt{L}} \sum_{l=1}^{L} r_l \]  \hspace{1cm} (2.58)

whereas its derivative by [93]

\[ \dot{r} = \frac{1}{\sqrt{L}} \sum_{l=1}^{L} \dot{r}_l. \]  \hspace{1cm} (2.59)

As opposed to the previous cases, \( \dot{r} \) is now a Gaussian random variable independently of the \( r_l \)'s, with zero mean and variance

\[ \sigma_r^2 = \frac{1}{L} \sum_{l=1}^{L} E[\dot{r}_l^2] = 2\pi^2 f_m^2 \frac{1}{L} \sum_{l=1}^{L} \sigma_i^2 \]  \hspace{1cm} (2.60)

where the last equation was obtained using the independence assumption between the branches. Solving (2.29) leads to

\[ N_R(r) = p_R(r) \frac{\sigma_r}{\sqrt{2\pi}}. \]  \hspace{1cm} (2.61)

It is similar to Eq. (2.52) in the previous section, however for the MRC case this equation required the i.i.d. assumption in order to be valid, while this is not the case for EGC. For i.i.d. channels, the cdf and pdf of \( r \) were presented in [20] and [95] respectively, in integral form, for an arbitrary \( L \). For independent channels with arbitrary parameters, the pdf can be written as

\[ p_R(r) = \sqrt{L} \int_0^{\sqrt{Lr}} \int_0^{\sqrt{Lr} - r_L} \cdots \int_0^{\sqrt{Lr} - \sum_{l=2}^{L} r_l} \left[ p_{R_l}(r_L - \sum_{l=2}^{L} r_l) \prod_{l=2}^{L} p_{R_l}(r_l) \right] dr_2 \cdots dr_L. \]  \hspace{1cm} (2.62)

For Rayleigh fading, a simple closed-form solution is available only for \( L \leq 2 \), and is implicitly presented in [94]. For Nakagami fading with identical parameters and \( L = 2 \),
(2.62) reduces to

\[ p_R(r) = \sqrt{2} \int_{0}^{\sqrt{2}r} p_R(r' - r_2)p_R(r_2)dr_2. \]  

(2.63)

Substituting Eq. (2.31) in Eq. (2.63):

\[ p_R(r) = \frac{4\sqrt{2}}{[\Gamma(m)]^2} (\frac{m}{\Omega})^{2m} e^{-\frac{m}{\Omega}r^2} \int_{0}^{\sqrt{2}r} \left[ r_1(\sqrt{2}r - r_1) \right]^{2m-1} e^{-\frac{m}{\Omega}(\sqrt{2}r_1 - r)^2} dr_1. \]  

(2.64)

The variable transformation \( y = \frac{m}{\Omega}(\sqrt{2}r_1 - r) \) in Eq. (2.64) gives

\[ p_R(r) = \frac{2(\frac{m}{\Omega})^{\frac{1}{2}}}{[\Gamma(m)]^2 2^{2m-2}} \int_{-\frac{m}{\Omega}r^2}^{\frac{m}{\Omega}r^2} y^{-\frac{1}{2}} \left[ \frac{m}{\Omega} r^2 - y \right]^{2m-1} e^{-y} dy. \]  

(2.65)

Using Eq. 3.383.1 of [80] then leads to the following closed-form solution:

\[ p_R(r) = (\frac{m}{\Omega})^{2m} \frac{2B(2m, \frac{1}{2})}{[\Gamma(m)]^2 2^{2m-2}} r^{4m-1} e^{-2\frac{m}{\Omega}r^2} \Phi \left( 2m, 2m + \frac{1}{2}, \frac{m}{\Omega}r^2 \right) \]  

(2.66)

where \( B(x, y) \) is the beta function and \( \Phi(a, c, x) \) the confluent hypergeometric function, given by Eqs. 8.380.1 and 9.210.1 of [80], respectively. Substituting Eqs. (2.63) and (2.60) in Eq. (2.61) yields the following expression for the LCR of dual-branch EGC:

\[ N_R(r) = \frac{\sqrt{2\pi} f m B(2m, \frac{1}{2})}{[\Gamma(m)]^2 2^{2m-2}} \left( \frac{m}{\Omega} r^2 \right)^{2m-\frac{1}{2}} \frac{m}{\Omega} e^{-2\frac{m}{\Omega}r^2} \Phi \left( 2m, 2m + \frac{1}{2}, \frac{m}{\Omega}r^2 \right). \]  

(2.67)

The cdf is given by:

\[ F_R(r) = (\frac{m}{\Omega})^{2m} \frac{2B(2m, \frac{1}{2})}{[\Gamma(m)]^2 2^{2m-2}} \int_{0}^{r} \alpha^{4m-1} e^{-2\frac{m}{\Omega}a^2} \Phi \left( 2m, 2m + \frac{1}{2}, \frac{m}{\Omega}a^2 \right) da. \]  

(2.68)

Making the change of variable \( x = 2(m/\Omega)a^2 \), using the infinite series expansion

\[ \Phi(a, c, x) = \frac{\Gamma(c)}{\Gamma(a)} \sum_{k=0}^{\infty} \frac{\Gamma(a + k) x^k}{\Gamma(c + k) k!} \]  

(2.69)
and the relation \( B(x,y) = \Gamma(x)\Gamma(y)/\Gamma(x+y) \) (Eq. 8.384.1 of [80]) in Eq. (2.68) results in

\[
F_R(r) = \frac{1}{[\Gamma(m)]^2 2^{4m-2}} \sum_{n=0}^{\infty} \frac{\Gamma(n + 2m)}{\Gamma(n + 2m + 1/2) 2^n n!} \int_0^{2\Omega r^2} e^{-x} x^{2m+n-1} dx. \quad (2.70)
\]

Applying Eq. 8.350.1 of [80] to the integral above leads to the desired representation:

\[
F_R(r) = \frac{\sqrt{\pi}}{[\Gamma(m)]^2 2^{4m-2}} \sum_{n=0}^{\infty} \frac{\Gamma(2m+n)}{\Gamma(2m+n + 1/2) 2^n n!} \frac{1}{\gamma} \left( 2m+n, \frac{2m r^2}{\Omega} \right). \quad (2.71)
\]

By substituting Eqs. (2.71) and (2.67) in Eq. (2.28) we obtain the AFD

\[
\tau_R(r) = \frac{e^{2\Omega r^2} \sum_{n=0}^{\infty} \frac{\Gamma(2m+n)}{\Gamma(2m+n + 1/2) 2^n n!} \frac{1}{\gamma} \left( 2m+n, \frac{2m r^2}{\Omega} \right)}{2f_m B(2m, 1/2) (2m r^2)^{2m-1/2} \Phi \left( 2m, 2m + 1/2, \frac{m r^2}{\Omega} \right)}. \quad (2.72)
\]

For \( m = 1 \), Eq. (2.67) can be simplified using Eqs. 9.212.2, 9.212.4, 9.212.1 and 9.236.1 of [80] for \( \Phi(a,c,x) \), in that order, and Eq. 8.384.1 of [80] for \( B(x,y) \). This reduces to the following expression for the LCR of dual-branch EGC and i.i.d. Rayleigh fading channels, which was also presented in [94]:

\[
N_R(r) = \sqrt{2\pi} f_m e^{-r^2} \left[ \frac{r}{\sqrt{\Omega}} e^{-\frac{r^2}{\Omega}} + \left( \frac{r^2}{\Omega} - \frac{1}{2} \right) \sqrt{\pi} \text{erf} \left( \frac{r}{\sqrt{\Omega}} \right) \right]. \quad (2.73)
\]

For independent but non-identical Rayleigh fading channels, we make use of Eqs. (2.62), (2.60) and (2.61) to obtain a closed-form expression for the LCR of dual-diversity EGC:

\[
N_R(r) = \sqrt{2\pi} f_m \sqrt{\frac{\Omega_p}{\Omega_m}} e^{-\frac{r^2}{\Omega_m}} \times \left[ \frac{r}{\sqrt{\Omega_m}} \left( \sqrt{\Omega_{12}} e^{-\frac{r^2}{\Omega_{m1}}} + \sqrt{\Omega_{21}} e^{-\frac{r^2}{\Omega_{m2}}} \right) + \left( \frac{r^2}{\Omega_m} - \frac{1}{2} \right) \sqrt{\pi} \left( \text{erf} \left( \frac{\Omega_{21} r}{\sqrt{\Omega_m}} \right) + \text{erf} \left( \frac{\Omega_{12} r}{\sqrt{\Omega_m}} \right) \right) \right] \quad (2.74)
\]

with \( \Omega_m = (\Omega_1 + \Omega_2)/2 \), \( \Omega_{21} = \Omega_2/\Omega_1 \), \( \Omega_{12} = \Omega_1/\Omega_2 \) and \( \Omega_p = \Omega_1 \Omega_2/4 \).
2.3.1.6 Numerical Results and Discussion

The LCR and AFD expressions presented above are plotted in logarithmic scale against the normalized value of the combined received envelope, $r_n = r/\sqrt{\Omega}$ in dB. Figs. 2.1-2.3 compare the LCR (normalized by $f_m$) for the diversity techniques presented above with $L = 2$ and the no-diversity (ND) case ($L = 1$), for three values of the $m$-parameter: $m = 0.6$ corresponds to severe fading (worse than Rayleigh), $m = 1.3$ to fading conditions slightly better than Rayleigh, and $m = 3.0$ to line-of-sight conditions. For all the curves, it is observed that the LCR for MRC are the lowest for low values of $r_n$'s, and the highest for high values of $r_n$'s, while the opposite is true for the ND case. Indeed, in the ND case, fades occur more frequently due to the absence of diversity. As a consequence, the signal crosses lower values of $r_n$ more often than when diversity is used (with the lowest number of crossings occurring for the optimal diversity scheme, i.e. MRC), whereas it crosses high values of $r_n$ less often. Also, from these curves, the output of an EGC receiver fades less frequently than that of an SC receiver. However, the differences in LCR between MRC, EGC and SC depend on the Nakagami-$m$ parameter, and thus on the severity of the channel in terms of fading, and are commented below.

For $m = 0.6$, the LCR curves for SC and EGC are nearly identical, but differ from those of MRC, for which the combined envelope exhibits less severe fading. As $m$ is increased from 0.6 to 3.0, the LCR curve for EGC gets closer to that for MRC, and further away from that for SC. For $m = 3.0$ the LCR curves for MRC and EGC nearly overlap. This reflects the fact that, as the fading severity decreases (i.e. for higher values of $m$), the performance of EGC tends to approach that of MRC, while the performance margin between the latter two and SC increases. This could also be observed by comparing plots of the error probabilities for these diversity techniques and different $m$'s.

The results plotted in Figs. 2.4-2.6 present the AFD, normalized by $1/f_m$, for the same cases as before. From these curves it is seen that the AFD for all three diversity techniques remain very close for values of $r_n$ less than about -5 dB. This means that once the combined signal has faded below this value, it remains below for nearly the same amount of time for all of MRC, EGC or SC. However, from our previous examination of the LCR, since a MRC signal crosses low values of $r_n$ less often than EGC and SC signals, on average it will spend less time into deep fades than the latter two. For higher
values of $r_n$, it is observed that the AFD are lower for MRC than they are for EGC and SC: for each $r_n$, the combined signals obtained with EGC and SC spend more time below this value than that obtained with MRC, which reflects the fact that on average a stronger signal results from the use of MRC. This agrees with the previous discussion on LCR, in which it was pointed out that a MRC signal is more often in the high end of the signal strength $r_n$ than EGC and SC. As before, we observe that for severe fading ($m = 0.6$), the behavior of EGC follows closely that of SC, while for milder fading ($m = 3.0$), it compares to that of MRC. Thus, for a fixed set of parameters, there isn’t a one-to-one correspondence between the behavior of the LCR and that of the AFD (similar observations were reported in [95], for the case of Rayleigh fading): the LCR of MRC differed from that of SC (and EGC for low $m$’s) over the whole range of $r_n$’s, while the AFD are nearly identical for low $r_n$’s. This is due to the term $F_R(r)$ which intervenes in the relation (2.28) between the latter quantities.

Figs. 2.7 and 2.8 illustrate the LCR for SC and MRC, respectively, for a variable number of diversity branches $L$. In the case of SC, as $L$ increases, the frequency at which the received signal crosses high values (e.g. at approximately $r_n > 0$ dB) stays almost the same. Whereas in the case of MRC, $N_R(r_n)$ increases with $L$ for high values of $r_n$. Moreover, for low values of $r_n$, the LCR decrease faster for MRC than for SC as more diversity branches are added: e.g., for $r_n = -20$ dB, the decrease in the dB value of $N_R(r_n)$ is more than six-fold for MRC as $L$ goes from 2 to 4, while it is less than five-fold in the case of SC. This parallels the observations made in [20], according to which the advantage of MRC and EGC over SC (i.e. the strength of the signal) gets more pronounced as the number of diversity branches increases (with EGC following the behavior of MRC).

In summary, the numerical results support the assertion that the gain in performance made possible using MRC and EGC, as compared to using SC, gets more important as the fading gets less severe and the diversity order increases.
Figure 2.1 LCR’s with SC, MRC, and EGC dual-diversity ($L = 2$) and without diversity (ND, $L = 1$); $m = 0.6$.

Figure 2.2 LCR’s with SC, MRC, and EGC dual-diversity ($L = 2$) and without diversity (ND, $L = 1$); $m = 1.3$. 
Figure 2.3 LCR's with SC, MRC, and EGC dual-diversity ($L = 2$) and without diversity (ND, $L = 1$); $m = 3.0$.

Figure 2.4 AFD’s with SC, MRC, and EGC dual-diversity ($L = 2$) and without diversity (ND, $L = 1$); $m = 0.6$.  

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Figure 2.5 AFD's with SC, MRC, and EGC dual-diversity ($L = 2$) and without diversity (ND, $L = 1$); $m = 1.3$.

Figure 2.6 AFD's with SC, MRC, and EGC dual-diversity ($L = 2$) and without diversity (ND, $L = 1$); $m = 3.0$. 

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Figure 2.7 LCR for SC with different diversity orders and $m = 1.3$. 

Figure 2.8 LCR for MRC with different diversity orders and $m = 1.3$. 

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2.3.1.7 LCR for Diversity Techniques with Correlated Branches

In the previous sections, the assumption was made that the diversity branches were independent. In the cases presented, the combined signal $r$ was independent of its derivative $\dot{r}$, which allowed the derivation of closed-form solutions for the LCR. If the branches are correlated, this no longer holds. For the case $L = 2$, as shown in [94], $\dot{r}$ is now conditional on $r$ (or alternatively, on $r_1$ and $r_2$) and the phase difference $\theta_{12} = \theta_2 - \theta_1$, where $\theta_i, i = 1, 2$ are the phases associated with diversity channels 1 and 2. The conditional pdf of $\dot{r}$ can be written as:

$$p_{\dot{r}}(\dot{r} | r_1, r_2, \theta_{12}) = \frac{1}{\sqrt{2\pi \hat{\sigma}_r}} \exp \left[ - \frac{(\dot{r} - \hat{m}_r)^2}{2 \hat{\sigma}_r^2} \right]$$

(2.75)

where $\hat{m}_r = E[\dot{r}]$ and $\hat{\sigma}_r^2$ are now dependent on $r_1$, $r_2$ and $\theta_{12}$. As a result, the expression for the LCR in Eq. (2.29) will be written as function of $r_1$, $r_2$ and $\theta_{12}$:

$$N_R(r_1, r_2, \theta_{12}) = p_{\hat{r}_1, \hat{r}_2, \theta_{12}}(r_1, r_2, \theta_{12}) \int_0^\infty \hat{r} p_{\dot{r}}(\dot{r} | r_1, r_2, \theta_{12}) d\dot{r}.$$  (2.76)

From [94], the expressions for the LCR for SC, MRC and EGC as a function of $r$ can be obtained as:

$$N_R(r) = \int_{-\pi}^{\pi} \int_0^r \hat{n}r p_{\hat{r}_1, \hat{r}_2, \theta_{12}}(r_1 = r, r_2, \theta_{12}) dr_2 d\theta_{12}$$

$$+ \int_{-\pi}^{\pi} \int_0^r \hat{n}r p_{\hat{r}_1, \hat{r}_2, \theta_{12}}(r_1, r_2 = r, \theta_{12}) dr_1 d\theta_{12} \quad \text{for SC},$$

(2.77)

$$N_R(r) = \int_{-\pi}^{\pi} \int_0^{\pi/2} \hat{n}r p_{\hat{r}_1, \hat{r}_2, \theta_{12}}(r_1 = r \cos \phi, r_2 = r \sin \phi, \theta_{12}) d\phi d\theta_{12} \quad \text{for MRC},$$

(2.78)

$$N_R(r) = \int_{-\pi}^{\pi} \int_0^{r_{\sqrt{2}}} \hat{n}r p_{\hat{r}_1, \hat{r}_2, \theta_{12}}(r_1, r_2 = r \sqrt{2} - r_1, \theta_{12}) dr_1 d\theta_{12} \quad \text{for EGC},$$

(2.79)
In the case of Rayleigh fading, the following joint pdf is known ([103], p. 63):

\[
p_{R_1, R_2, \theta_{12}}(r_1, r_2, \theta_{12}) = \frac{r_1 r_2}{2\pi \sigma^4 (1 - \rho)} \exp \left[ -\frac{r_1^2 + r_2^2 - 2r_1 r_2 \sqrt{\rho} \cos(\theta_{12})}{2\sigma^2 (1 - \rho)} \right] (2.81)
\]

where \(\rho = \frac{\text{Cov}(r_1^2, r_2^2)}{\sqrt{\text{Var}(r_1^2)} \sqrt{\text{Var}(r_2^2)}}\) is the correlation coefficient between \(r_1\) and \(r_2\) in power terms, \(\text{Cov}(\cdot, \cdot)\) and \(\text{Var}(\cdot)\) denote the covariance and variance, respectively. Moreover, expressions for \(\hat{m}_r\) and \(\hat{\sigma}_r^2\) are obtained in [94]. The latter and Eqs. (2.80) and (2.81) were then substituted in Eqs. (2.77)-(2.79), and the LCR were evaluated by solving numerically the double integrals. Unlike for the independent fading case, the LCR were a function of the angle \(\alpha\) between the antenna axis and the direction of vehicle motion. The LCR were minimum for \(\alpha = 0\), and maximum for \(\alpha = \pi/2\).

In the case of Nakagami fading, the following joint pdf was proposed in [104], even though it was stated that it is not unique:

\[
p_{R_1, R_2, \theta_{12}}(r_1, r_2, \theta_{12}) = \frac{4(m - 1)(r_1 r_2)^{m-1}}{2\pi \Gamma(m)(1 - |\rho|) \rho^{1-m} \Omega^{m+1}} \int_0^1 z^{-1} I_{m-1} \left( \frac{2\sqrt{\rho} r_1 r_2 z}{\Omega(1 - \rho)} \right) \exp \left[ -\frac{r_1^2 + r_2^2 - 2r_1 r_2 (1 - z) \sqrt{\rho} \cos(\theta_{12})}{\Omega(1 - \rho)} \right] \, dz (2.82)
\]

where \(I_m(\cdot)\) is the modified Bessel function of order \(m\). However, expressions for \(\hat{m}_r\) and \(\hat{\sigma}_r^2\) are not easily obtainable, because the development used in [94] can’t be applied straightforwardly to the Nakagami case. Indeed, in the Rayleigh case, in order to derive such expressions, the authors of [94] made use of the fact that the complex signals \(z_i = r_i e^{j\theta_i}, i = 1, 2\), received on each branch are complex Gaussian variables. This is not the case for Nakagami fading, thus more work is needed on this issue.

Nonetheless, we examined the LCR for correlated diversity with \(L = 2\), when it is assumed that \(r\) and \(\hat{r}\) are independent. Following the development obtained in the
previous sections, for the identically distributed case, the LCR can be obtained as 
\( N_R(r) = p_R(r) \hat{\sigma}_r / \sqrt{2\pi} \), with \( \hat{\sigma}_r \) given by Eqs. (2.37), (2.50) or (2.60) and:

\[
p_R(r) = \frac{4(m)^{m+2m-1}}{\Gamma(m)} e^{-\frac{m^2}{2}} \left[ 1 - Q_m\left( \sqrt{\frac{2m\rho}{\Omega(1-\rho)^r}}, \sqrt{\frac{2m}{\Omega(1-\rho)^r}} \right) \right]
\]
for SC, (2.83)

\[
p_R(r) = \frac{2\sqrt{\pi}(m/\Omega)^{m+1/2}2^m e^{-\frac{m^2}{2}}}{\Gamma(m)(1-\rho)^{1/2} \rho^{2m-1}} I_{m-1} \left( \frac{\sqrt{\rho mr^2}}{\Omega(1-\rho)} \right)
\]
for MRC, (2.84)

\[
p_R(r) = \frac{2}{\Gamma(m)} e^{-\frac{m^2}{2(1-\rho)}} \sum_{k=0}^{\infty} \frac{\rho^k(1-\rho)^m B(2(m+k), \frac{1}{2})}{k! \Gamma(m+k)2^{2(m+k)-2}} \left( \frac{m}{\Omega(1-\rho)} \right)^{2(m+k)}
\]
\times x^{2(m+k)-1} \Phi \left( 2(m+k), 2(m+k) + \frac{1}{2}, \frac{mr^2}{\Omega(1-\rho)} \right)
\]
for EGC, (2.85)

where \( Q_m(\alpha, \beta) \) is the generalized Marcum Q-function (Eq. 2-1-22, p. 44, of [26]). Eqs. (2.83) and (2.84) are deduced from [105] and [23], respectively, while Eq. (2.85) is a novel contribution (c.f. Section 2.5.3.2). Figs. 2.9 and 2.10 plot Eqs. (2.77) and (2.78), respectively, for \( \alpha = 0 \) and \( \alpha = \pi/2 \), along with the LCR obtained by using Eqs. (2.83) and (2.84) with \( m = 1 \), for \( \rho = 0.6425 \). It can be seen that the latter equations give an upper bound on the LCR for low values of \( r \). Similar results were obtained with different \( \rho \)'s. Since it is the low range of \( r \)'s which is mostly of interest from a LCR standpoint (since one wants to determine the rate at which the signal goes into fade), the approximate expressions given above can be used to conveniently bound the maximum LCR obtained regardless of the angle \( \alpha \) (which varies with the movements of the mobile).
Figure 2.9 LCR for SC with $L = 2$ correlated branches, $m = 1.0$, $\rho = 0.6425$. — : upper bound; - - - : exact, $\alpha = 0$; - - - : exact, $\alpha = \pi/2$.

Figure 2.10 LCR for MRC with $L = 2$ correlated branches, $m = 1.0$, $\rho = 0.6425$. — : upper bound; - - - : exact, $\alpha = 0$; - - - : exact, $\alpha = \pi/2$. 49
2.3.1.8 Conclusions

Starting from a common representation for the LCR, we derived generalizations of expressions for the LCR of a diversity received signal in Rayleigh fading, in order to handle the more general Nakagami fading distribution. Closed-form solutions were presented for arbitrary $L$ in the case of SC and MRC, and for $L = 2$ in the case of EGC. The assumption of i.i.d. channels was made throughout Sections 2.3.1.3-2.3.1.6 (except for SC, where nonidentical parameters were allowed) in order to obtain these results in closed-form, however the methodology used is not limited by this assumption: the correlated case can be dealt with in the same manner (c.f. Section 2.3.1.7), but will require numerical evaluations of the LCR and AFD for most cases of interest. The material we presented can be used in designing finite-state channel simulators [102], analyzing error-correcting schemes for burst error channels [91], determining the minimum duration outages in fading channels [106], or determining the delay spread of frequency-selective channels [107].

2.3.2 Analytical Envelope Correlation and Spectrum of Maximal-Ratio Combined Nakagami and Rician Fading Signals

2.3.2.1 Introduction

The temporal variations of a mobile fading channel can be described by its envelope autocorrelation (or autocovariance) function. The latter can be used to determine the minimum average time interval that is needed for two samples of the channel envelope to be uncorrelated, or to have a correlation coefficient below a certain threshold. This can prove useful in determining the size of an interleaver, choosing an appropriate error-correcting code, or selecting a robust packet length. The autocorrelation function of a complex Gaussian random process is given for example in [111], p. 62 and [112]. The envelope autocorrelation function (EAF) of a flat-fading Rayleigh channel can be deduced

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2Since the time [108] was submitted on May 2nd, 2001 (and several months after many of the results of [108] initially appeared in [102]), other independent contributions dealing with the LCR and AFD of diversity Nakagami channels have been published [109], [110]. The material presented here differs from that of [109] and [110] in the methodology used, and/or the generality (non-identically versus identically distributed) or the representation (closed-form versus integral-form) of the original analytical results.
from it, by substituting an appropriate time-variant correlation coefficient which describes the variations of a mobile radio channel [113]. For systems with diversity combining, the variations of the *combined* received signal is now of interest, since it is this signal which is available to the rest of the transceiver. In [114], Lee derived an expression for the EAF of the combined signal for a receiver with EGC, for the general case where the diversity branches can be mutually correlated. However, to our knowledge no expression has been published in the open technical literature for the EAF of a combined signal for MRC. Drawing upon the theory of multidimensional Gaussian distributions, this section provides such expressions in closed-form or as an infinite series, for Rayleigh, Nakagami and Rician fading channels. For the special case of no diversity, these expressions revert to those presented for example in [113], p. 50, [23], and [115], respectively. The power spectrum of the combined envelope is also derived for the Rayleigh and Nakagami cases.

### 2.3.2.2 Rayleigh Fading

Let $r_1(t), r_2(t), \ldots, r_L(t)$ be the received signals on the $L$ uncorrelated diversity branches. They are assumed to be Rayleigh-distributed and stationary. The maximal-ratio combined signal is given by (c.f. also Eq. (2.48), where sampled values were used):

$$ r(t) = \left[ \sum_{i=1}^{L} r_i^2(t) \right]^{\frac{1}{2}}. \tag{2.86} $$

Let $r_{11}$ and $r_{12}$ be the values of $r_i(t)$ ($l = 1, 2, \ldots, L$) sampled at $t = T$ and $t = T + \tau$, respectively. $r_{11}$ and $r_{12}$ are then Rayleigh-distributed random variables, which can be expressed as:

$$ r_{11} = \sqrt{x_{i,1}^2 + x_{i,2}^2}, \tag{2.87} $$

$$ r_{12} = \sqrt{y_{i,1}^2 + y_{i,2}^2} \tag{2.88} $$
where $x_{i,1}, x_{i,2}, y_{i,1}$ and $y_{i,2}$ are zero-mean Gaussian random variables with common variance $\sigma^2$. The values of $r(t)$ sampled at $t = T$ and $t = T + \tau$ are then given by:

$$U = r(T) = \left[ r_{11}^2 + r_{21}^2 + \ldots + r_{L1}^2 \right]^{\frac{1}{2}}$$
$$V = r(T + \tau) = \left[ (y_{1,1}^2 + y_{1,2}^2) + (y_{2,1}^2 + y_{2,2}^2) + \ldots + (y_{L,1}^2 + y_{L,2}^2) \right]^{\frac{1}{2}}.$$  \hspace{1cm} (2.89)

$U$ and $V$ can thus be interpreted as the magnitudes of vectors $Z^{(1)}$ and $Z^{(2)}$, respectively, given by:

$$Z^{(1)} = \left[ x_{1,1} \ x_{1,2} \ldots \ x_{L,1} \ x_{L,2} \right]^T$$
$$Z^{(2)} = \left[ z_1^{(1)} \ z_2^{(1)} \ldots \ z_{2L}^{(1)} \right]^T.$$  \hspace{1cm} (2.91)

where $(\cdot)^T$ denotes the transpose of a vector, and $z_1^{(1)} = x_{1,1}$, $z_2^{(1)} = x_{1,2}$, $\ldots$, $z_{2L}^{(1)} = x_{L,2}$. $U$ and $V$ are generalized Rayleigh random variables ([116], Chap. 2), and $Z^{(1)}$ and $Z^{(2)}$ are $2L$-dimensional Gaussian random vectors. Let $P^{(i)} = [z_{i}^{(1)} z_{i}^{(2)}]^T$, i.e. $P^{(i)}$ is a random vector made up of the $i^{th}$ ($i = 1, \ldots, 2L$) components of $Z^{(j)}$, $j = 1, 2$. Its covariance is given by ([26], p. 50):

$$M^{(i)} = \begin{bmatrix} \sigma^2 & \sqrt{\rho}\sigma^2 \\ \sqrt{\rho}\sigma^2 & \sigma^2 \end{bmatrix} = M.$$  \hspace{1cm} (2.93)

The correlation coefficient $\rho$ depends on the spectrum of the fading signal. For example, for land-mobile radio channels, the correlation coefficient $\rho(\tau) = J_0^2(2\pi f_m\tau)$ is often used [113], where $\tau$ is the time separation between two envelope samples, and $J_0(\cdot)$ is the Bessel function of zero-order. For clarity of presentation, from now on in this section, the dependence of $\rho$ on $\tau$ is dropped. The inverse of the covariance matrix can be calculated as

$$W = (M)^{-1} = \begin{bmatrix} \frac{1}{\sigma^2(1-\rho)} & -\sqrt{\rho} \sigma^2 \\ -\sqrt{\rho} \sigma^2 & \frac{1}{\sigma^2(1-\rho)} \end{bmatrix}.$$  \hspace{1cm} (2.94)
and its determinant as \( \det(M) = \sigma^4(1 - \rho) \). Making use of the previous expressions for \( W \) and \( \det(M) \), the joint pdf of \( U \) and \( V \) can be obtained from the corollary to Theorem 3 (Chap. 2, Section 2) on pp. 32-34 of [116]:

\[
p_{U,V}(u,v) = \frac{4(uv)^{L-1} e^{-\frac{u^2 + v^2}{2(1-\rho)}} I_{L-1} \left( \frac{2uv\sqrt{\rho}}{\Omega(1-\rho)} \right)}{\Gamma(L)(\Omega)^{L+1}(\rho)^{\frac{L-1}{2}}(1-\rho)}
\]

(2.95)

where \( \Omega = 2\sigma^2 \) is the average power received on each of the diversity branches. With this pdf given, and taking into account the assumed stationarity of \( r(t) \), the determination of the EAF reduces to the evaluation of a two-dimensional integral (instead of a 2L dimensional integral, if the averaging was performed over the individual \( r_{ij} \)'s). The EAF is given by:

\[
R(\tau) = \mathbb{E}[r(t)r(t + \tau)] = \mathbb{E}[UV] = \int_0^\infty \int_0^\infty uvp_{U,V}(u,v) \, du \, dv.
\]

(2.96)

Substituting Eq. (2.95) into Eq. (2.96), using Eq. 2.5.6 on p. 115 of [116] and Eq. 7.621.4 of [80] to perform the integration with respect to the \( u \) and \( v \) variables, respectively, the following expression is obtained:

\[
R(\tau) = \left[ \frac{\Gamma(L + \frac{1}{2})}{\Gamma(L)} \right]^2 \Omega I_2F_1 \left( -\frac{1}{2}, -\frac{1}{2}; L; \rho \right)
\]

(2.97)

where \( 2F_1(a, b; c; x) \) is the Gaussian hypergeometric function ([80], Eq. 9.100). Eq. (2.97) is in fact a special case of Eq. (4), p. 73 of [116] (c.f. also Eq. 1.9 of [117]), which has thus been applied in this section to obtain the envelope correlation of maximal-ratio combined Rayleigh fading signals. It is pointed out that Eq. (2.97) can also be written as (c.f. Eq. 4.3 of [117])

\[
R(\tau) = \frac{\Omega(1 - \rho)^{L+1}}{\Gamma(L)} \frac{d^{L-1}}{d(\rho)^{L-1}} \left[ \frac{E(\sqrt{\rho})}{(1 - \rho)^2} - \frac{K(\sqrt{\rho})}{2(1 - \rho)} \right]
\]

(2.98)

where \( K(\cdot) \) and \( E(\cdot) \) are the complete elliptic integrals of the first and second kinds, respectively, and \( d^l(\cdot)/d(x)^l \) is the \( l \)th order derivative with respect to \( x \). For a system
with no diversity \((L = 1)\), Eq. (2.97) simplifies to the following well-known expression ([113], p. 50):

\[
R(\tau) = \frac{\pi}{4} \Omega \, _2F_1 \left( \frac{1}{2}, -\frac{1}{2}; 1; \rho \right).
\]

The mean of the combined envelope \(r(t)\) is given by:

\[
\mu_r = E[r(t)] = \left[ \frac{\Gamma(L + \frac{1}{2})}{\Gamma(L)} \right] \sqrt{\Omega}.
\]

The autocovariance of \(r(t)\) is then given by:

\[
C(\tau) = R(\tau) - \mu_r^2 = \left[ \frac{\Gamma(L + \frac{1}{2})}{\Gamma(L)} \right]^2 \Omega \left[ _2F_1 \left( -\frac{1}{2}, -\frac{1}{2}; L; \rho \right) - 1 \right].
\]

Proceeding in similar lines as in Section 1.3.2 of [15], if we make the expansion

\[
_2F_1 \left( -\frac{1}{2}, -\frac{1}{2}; L; \rho \right) = 1 + \frac{\rho}{4L} + \frac{\rho^2}{32L(L+1)} + \frac{3\rho^3}{128L(L+1)(L+2)} + \ldots,
\]

retain only the first two terms and substitute them in (2.101), the following approximate expression for the autocovariance is obtained:

\[
C(\tau) \approx \left[ \frac{\Gamma(L + \frac{1}{2})}{\Gamma(L)} \right]^2 \frac{\Omega}{4L} \rho(\tau).
\]

For \(L = 1\), the above reduces to \(C(\tau) \simeq \pi\Omega/16\rho(\tau)\) (c.f. [113], Eq. (2.55)).

Considering Clarke’s model with isotropic scattering [78], where the transmitting antenna is vertically polarized and the receiving antenna is a vertical monopole, the power spectrum of the passband signal is known to be given by \(S(f) = \frac{\Omega}{2\pi f_m} \left[ 1 - \left( \frac{f}{f_m} \right)^2 \right]^{-1/2}\) (where here, unlike in Eq. (2.10), the gain of the receiving antenna, e.g. 1.5 for a vertical whip antenna, is included in \(\Omega\)). The approximate spectrum of the baseband signal can be obtained using Eq. (2.103) and the development of Section 1.3.2 of [15]:

\[
S_b(f) \simeq \left[ \frac{\Gamma(L + \frac{1}{2})}{\Gamma(L)} \right]^2 \frac{\Omega}{4\pi^2 f_m} K \left( \sqrt{1 - \left( \frac{f}{2f_m} \right)^2} \right).
\]
Eq. (2.104) is a generalization of the case $L = 1$, for which $S_b(f)$ is given by Eq. (2.11).

### 2.3.2.3 Nakagami fading

To carry out the derivation, we assume the Nakagami-$m$ parameter to be an integer. The Nakagami pdf with $m$ real is equivalent to the pdf of the square-root of a gamma random variable $G(\alpha, \beta)$, with parameters $\alpha = m$ and $\beta = m/\Omega_{nak}$, where $\Omega_{nak} = m2\sigma^2 = m\Omega$. As a special case, the Nakagami distribution with $m$ integer is equivalent to the distribution of the square-root of a chi-square random variable $\chi^2(m, m/\Omega_{nak})$, with $2m$ degrees of freedom [98], which is also called a chi random variable [118]. Hence, the Nakagami-fading signal on the $l^{th}$ branch, sampled at $t = T$ can be written as

$$r_{l1} = \left[ \sum_{i=1}^{2m} x_{i,l}^2 \right]^{\frac{1}{2}}.$$  

(2.105)

The combined envelope at the output of a MRC receiver, sampled at $t = T$ can thus be expressed as:

$$U = r(T) = \left[ \sum_{l=1}^{L} r_{l1}^2 \right]^{\frac{1}{2}} = \left[ \sum_{l=1}^{L} \sum_{i=1}^{2m} x_{i,l}^2 \right]^{\frac{1}{2}}.$$  

(2.106)

As before, $U$ and $V = r(T + \tau)$ correspond to the magnitudes of vectors $Z^{(1)}$ and $Z^{(2)}$, respectively, given by:

$$Z^{(1)} = [x_{1,1} \ldots x_{1,2m} \ldots x_{L,1} \ldots x_{L,2m}]^T$$

$$= [z_1^{(1)} z_2^{(1)} \ldots z_{2mL}^{(1)}]^T,$$  

(2.107)

$$Z^{(2)} = [z_1^{(2)} z_2^{(2)} \ldots z_{2mL}^{(2)}]^T.$$  

(2.108)

$Z^{(1)}$ and $Z^{(2)}$ are also Gaussian random vectors, but of dimension $2mL$ instead of $2L$. In order to carry on the derivation, we make the assumption that the covariance matrix of $P^{(i)} = [x_i^{(1)} z_i^{(2)}]^T$ takes the same form as in the Rayleigh case, i.e. Eq. (2.93). While there are typical correlation coefficients available for the Rayleigh case, to the best of our knowledge there haven’t been any yet proposed on a physical basis for the Nakagami
channel. However, with the assumption that the covariance matrix can be put in the previously mentioned form (which necessitates \( m \) to be integer, in order for (2.105) to be valid), the relations developed below will hold for arbitrary correlation coefficients. Following the same steps as in Section 2.3.2.2 leads to:

\[
R(r) = \left[ \frac{\Gamma(mL + \frac{1}{2})}{\Gamma(mL)} \right]^{2} \left( \frac{\Omega_{nak}}{m} \right) {}_{2}F_{1} \left( \frac{-1}{2}, \frac{-1}{2}; mL; \rho \right). \tag{2.109}
\]

It can be observed that Eq. (2.109) is a generalization of Eq. (2.97), in which \( m = 1 \). For \( L = 1 \), Eq. (2.109) reduces to a special case of Eq. (137) of [23] (in which \( \Omega_{1} = \Omega_{2} = \Omega \) and \( n = l = 1 \)), i.e.:

\[
R(r) = \left[ \frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)} \right]^{2} \left( \frac{\Omega_{nak}}{m} \right) {}_{2}F_{1} \left( \frac{-1}{2}, \frac{-1}{2}; m; \rho \right). \tag{2.110}
\]

However, while Eq. (2.110) holds for arbitrary \( m \geq 1/2 \) and \( \rho^{(i)} \), certain restrictions on the latter quantities were needed to obtain our derivation of (2.109), as detailed above, even though we do not claim that these restrictions can't be relaxed. It is interesting to note that by replacing \( m \) by \( L \) in Eq. (2.110), Eq. (2.97) can be obtained. The mean of the combined envelope \( r(t) \) is given by:

\[
\mu_{r} = \left[ \frac{\Gamma(mL + \frac{1}{2})}{\Gamma(mL)} \right] \sqrt{\frac{\Omega_{nak}}{m}}. \tag{2.111}
\]

The autocovariance of \( r(t) \) is then given by:

\[
C(\tau) = \left[ \frac{\Gamma(mL + \frac{1}{2})}{\Gamma(mL)} \right]^{2} \left( \frac{\Omega_{nak}}{m} \right) {}_{2}F_{1} \left( \frac{-1}{2}, \frac{-1}{2}; mL; \rho \right) - 1. \tag{2.112}
\]

As in the previous section, considering Clarke's isotropic scattering model, the approximate spectrum of the baseband signal can be obtained by making the approximation

\[
C(\tau) \approx \left[ \frac{\Gamma(mL + \frac{1}{2})}{\Gamma(mL)} \right]^{2} \frac{\Omega_{nak}/m}{4\pi^{2}f_{m}mL} \rho(\tau):
\]

\[
S_{b}(f) \approx \left[ \frac{\Gamma(mL + \frac{1}{2})}{\Gamma(mL)} \right]^{2} \frac{\Omega_{nak}/m}{4\pi^{2}f_{m}mL} K \left( \sqrt{1 - \left( \frac{f}{2f_{m}} \right)^{2}} \right). \tag{2.113}
\]

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2.3.2.4 Rician Fading

In the Rician case, the development follows the Rayleigh case through Eqs. (2.86)-(2.94), except that \( x_{i,1}, x_{i,2} \) (and \( y_{i,1}, y_{i,2} \)) are now Gaussian variables with means \( a_{i,1}, a_{i,2} \), such that \( A_t = \sqrt{a_{i,1}^2 + a_{i,2}^2} \). The joint pdf of \( U \) and \( V \) is obtained from Theorem 3 (Chapter 2, Section 2) on pp. 32-34 of [116]:

\[
p_{U,V}(u,v) = \frac{4\Gamma(L-1)|\Omega(1-\rho)|^{L-3}(1-\rho)^{L-1}uv}{(-A_t^2 \sqrt{\rho})^{L-1}(1-\sqrt{\rho})^{2(L-1)}} e^{-\frac{u^2 + v^2 + 2(1-\sqrt{\rho})A_t^2}{\Omega(1-\rho)}}
\]

\[
\times \sum_{k=0}^{\infty} (-1)^k (L+k-1) \binom{2L+k-3}{2L-3} I_{L+k-1} \left( \frac{2uv(-\sqrt{\rho})}{\Omega(1-\rho)} \right)
\]

\[
\times I_{L+k-1} \left( \frac{2uA_t(1-\sqrt{\rho})}{\Omega(1-\rho)} \right) I_{L+k-1} \left( \frac{2vA_t(1-\sqrt{\rho})}{\Omega(1-\rho)} \right)
\]

(2.114)

where \( A_t^2 = \sum_{i=1}^L A_i^2 \), and \( \binom{n}{k} \) is the binomial coefficient. The EAF is obtained by substituting Eq. (2.114) in Eq. (2.96). One way to solve the double integral is to make the expansion \( I_{\nu}(z) = \sum_{j=0}^{\infty} x^{\nu+j} / (j!\Gamma(\nu + j + 1)) \) for the term \( I_{L+k-1} \left( \frac{2uv(-\sqrt{\rho})}{\Omega(1-\rho)} \right) \), and then use twice using Eq. 2.5.6 on p. 115 of [116], to perform the integrations with respect to the \( u \) and \( v \) variables, which gives the following infinite series expression (c.f. [119], Sec. 6, for the case \( L = 1 \); c.f. [120], Sec. 3, for arbitrary \( L \)):

\[
R(\tau) = \Gamma(L-1)\Omega(1-\rho)^{L+1} e^{-\frac{2(1-\sqrt{\rho})A_t^2}{\Omega(1-\rho)}} \sum_{k=0}^{\infty} (L+k-1) \binom{2L+k-3}{2L-3} \times \frac{\sqrt{\rho}^k}{[\Gamma(L+k)]^2} \left[ \frac{A_t^2(1-\sqrt{\rho})^2}{\Omega(1-\rho)} \right]^k \sum_{j=0}^{\infty} \frac{(\sqrt{\rho})^{2j}}{j!} \left[ \frac{\Gamma(L+k+j+\frac{1}{2})}{\Gamma(L+k+j)} \right]^2 \times \Phi \left( L + k + j + \frac{1}{2}; L + k; \frac{A_t^2(1-\sqrt{\rho})^2}{\Omega(1-\rho)} \right)^2
\]

(2.115)

For \( A_t = 0 \), Eq. (2.115) simplifies to Eq. (2.97): it can verified by nulling all terms for which \( k \geq 1 \) in the outer summation, and using the relations ([80], Eqs. 9.131.1 and
For $L = 1$, using the relation $\Phi(a; b; x) = e^{x} \Phi(b - a; b; -x)$ ([80], Eq. 9.212.1), Eq. (2.115) reduces to:

$$R(\tau) = \Omega(1 - \rho)^2 e^{-\frac{2\sqrt{A_t^2}}{\Omega(1 + \sqrt{\rho})}} \sum_{k=0}^{\infty} \epsilon_k \frac{(\sqrt{\rho})^k}{(k!)^2} \left[\frac{A_t^2(1 - \sqrt{\rho})}{\Omega(1 + \sqrt{\rho})}\right]^k$$

$$\times \sum_{j=0}^{\infty} \frac{(\sqrt{\rho})^{2j}}{j!} \left[\frac{\Gamma(k + j + \frac{3}{2})}{\Gamma(k + j + 1)}\right]^2 \left[\Phi(-j - \frac{1}{2}; k + 1; -\frac{A_t^2(1 - \sqrt{\rho})}{\Omega(1 + \sqrt{\rho})})\right]^2$$

where $\epsilon_k = 1$ for $k = 0$, $\epsilon_k = 2$ for $k \geq 1$, which is similar to [119], Eq. (6.13), along with the corrections made in [115]. The mean of the combined envelope $r(t)$ is given by [26]:

$$\mu_r = \sqrt{\Omega} e^{-\frac{A_t^2}{4\Omega}} \frac{\Gamma(L + \frac{1}{2})}{\Gamma(L)} \Phi(L + \frac{1}{2}; L; \frac{A_t^2}{\Omega}).$$

The autocovariance of $r(t)$ is obtained as per Eq. (2.101).

The calculation of the baseband power spectrum for the diversity case is more involved than for the Rayleigh case and is deferred for future research. For the case of no diversity, expressions can be found for example in [113] and [121].

### 2.3.2.5 Numerical Results and Discussion

We check the validity of Eq. (2.101) by comparing it against the simulation results obtained with a Rayleigh fading simulator. The latter uses the sum-of-sinusoids Monte Carlo technique described in [122] with $m = 1$, and intends to approximate the spectrum of a land-mobile radio channel with correlation coefficient $\beta_0(2\pi f_m \tau)$. The number of sinusoids was taken to be 250. The average power is normalized to $\Omega = 1$. The carrier frequency and mobile speed are taken to be $f_c = 2$ GHz and $v_c = 100$ km/h, respectively. This corresponds to a Doppler frequency of $f_m = 185.18$ Hz. Fig. 2.11 plots Eq. (2.101)
versus $\tau \times R_b$, for $L = 3$ and a bit rate $R_b = 64$ kbps, along with simulation results. As it can be observed, the theoretical and computer simulation curves are in very close agreement. The slight mismatches arise from the fact that the channel simulator is non-ideal. The same order of precision was observed for different diversity orders and bit rates. Fig. 2.12 compares Eq. (2.101) for $L = 1 - 3$, and the same parameters as before. We observe that the autocovariance between the outputs of the diversity combiner is higher for larger diversity orders, especially for time delays corresponding to the local peaks of the envelope autocovariance function. However, as $L$ increases, the differences between the EAF curves are gradually reduced. Fig. 2.13 plots $S_b(f)$ for $L = 1 - 3$. Fig. 2.14 plots the Rician envelope autocovariance versus $\tau \times R_b$, for $L = 3$ and a bit rate $R_b = 9.6$ kbps, for different values of $A_l = A$, $l = 1, 2, \ldots, L$. The zeros are at the same locations as for the Rayleigh case, but the amplitudes of the local peaks increase with $A$.

2.3.2.6 Conclusions

Based upon multidimensional Gaussian calculus, we have provided a simple closed-form expression for the envelope autocorrelation function of the output of a maximal-ratio diversity combiner, in the case of Rayleigh fading. Using the same method we provided such an expression for Rician fading and for Nakagami fading when the $m$-parameter is integer. It has been shown that for the non-diversity case, these expressions simplify to results obtained previously by Uhlenbeck [111], Middleton [119] and Nakagami [23]. It has been observed that the use of diversity slightly increases the autocovariance between receiver inputs, with the largest difference evidenced when the number of branches goes from one to two; in particular, for the land-mobile radio channel, the increase is most important around the peaks of the envelope autocovariance function.
**Figure 2.11** Envelope autocovariance function versus time-delay (normalized by symbol period) for the Rayleigh channel with $L = 3$.

**Figure 2.12** Envelope autocovariance function versus time-delay (normalized by symbol period) for the Rayleigh channel.
Figure 2.13 Spectrum of the baseband envelope versus frequency for the Rayleigh channel.

Figure 2.14 Envelope autocovariance function versus time-delay (normalized by symbol period) for the Rice channel with $L = 3$. 
2.4 Simulation of Wideband Correlated Nakagami Fading Channels

The simulation of Rayleigh and Rice fading channels is based on their underlying well-defined and widely accepted physical interpretations, which can be directly used to generate random variables with the desired statistics. In the case of Nakagami fading, the PDF represents only an approximation to the actual physical process, and hence, no simulation models have been found yet which rely on the actual physical mechanism, while providing the desired statistics. Simulation methods for Nakagami fading have essentially borrowed from well-known techniques for random variable generation [123].

We consider two family of approaches for Nakagami fading channel generation. The first one generates continuous random variables (r.v.'s), i.e. which can theoretically take any positive real value (the actual number of values is limited only by machine precision). The second one generates discrete r.v.'s, i.e. which can only be drawn from a limited (but possibly very large) set, with each value corresponding to a certain state. We will denote these approaches by continuous simulation models and discrete simulation models, respectively.

2.4.1 Continuous Channel Simulation

2.4.1.1 Introduction

We further classify continuous simulation models into those which can be used in generating multiple sequences of mutually-correlated r.v.'s, and those used in generating a sequence of time-correlated r.v.’s. The first class is useful in applications where space diversity is used (i.e. an antenna array with mutual correlation between each or some of its branches). The second class is useful in generating consecutive samples of a single fading channel, in which there is a positive correlation from one sample to the next one.
2.4.1.2 Mutually (Spatially) Correlated Nakagami Random Variables

We review simulation methods according to the number of mutually correlated sequences they can generate, which we denote by $L$ (i.e. the number of diversity branches).

$L = 1$

The simplest case consists in generating one sequence of independent Nakagami variables. As mentioned in Section 2.3.2.3, the Nakagami pdf with $m$ real is equivalent to the pdf of the square-root of a gamma random variable $G(\alpha, \beta)$, with parameters $\alpha = m$ and $\beta = m/\Omega$, where $\Omega = m2\sigma^2$. Indeed, the gamma pdf is given by:

$$p_G(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1}e^{-\beta x}, \quad x \geq 0$$

Hence the pdf of the square root of a gamma variable is obtained by simple transformation [26] as:

$$p_{\sqrt{G}(x)} = 2\frac{\beta^\alpha}{\Gamma(\alpha)} x^{2\alpha-1}e^{-\beta x}, \quad x \geq 0$$

from which the equivalence with the Nakagami pdf can be immediately made.

Hence, to generate a Nakagami variable $r$ with parameters $\{m, \Omega\}$, one can generate a gamma variable $g$ with parameters $\alpha = m$ and $\beta = m/\Omega$ and take its square root:

$$r = \sqrt{g}.$$  

Fortunately, several techniques are available to efficiently generate gamma variables. When $m \geq 1.0$, the following methods can be used [123]:

- Envelope rejection methods:
  - Cauchy method ([124], [123] pp. 108-110)
  - Log-logistic method ([125], [123] pp. 110-111)
  - t-distribution method ([126], [123] pp. 111-113)

- Ratio of uniforms method: ratio method ([123], p. 118-119)

When $m \leq 1.0$, the following methods can be used [123]:
• Envelope rejection method: switching algorithm ([123] pp. 116-118, [124])

• Ratio of uniforms method: power transformation method ([123] pp. 115-116)

• Beta method ([123] pp. 108-110)

We've implemented and tested each of the above methods. Table 2.1 shows the bit error rates (BER) for Binary Phase Shift Keying (BPSK) modulation obtained using the first group of methods, i.e. the Cauchy (C), log-logistic (LL), t-distribution (tD) and ratio (R) methods, for \( m = 2.0 \), with a fixed energy per symbol \( E_b = 1000 \) W, and a noise spectral density \( N_0 \) varying from 1 to 951 W in steps of 50. In each case 10 million samples were generated. It can be seen that the numerical results obtained with the different methods agree by usually at least two representative digits. Table 2.2 shows similar results obtained using the second group of methods, i.e. the switching (S), power transformation (PT) and beta (B) methods, for \( m = 0.6 \).

[123] finds that the log-logistic and t-distribution methods are the fastest in the first group if all the variables need to be reset for each iteration (otherwise the ratio method can be faster), while the switching method is the fastest in the second group. However, as we have verified and as detailed in [123], the execution times are all within the same order of magnitude within each group (and depend on the implementations), and hence there is no single method that greatly outperforms the others for the whole range of \( m \).

\[ L = 2 \]

[127] proposes an algorithm to generate 2 correlated Nakagami r.v.'s with identical \( m \)-parameters but unequal powers \( \{\Omega_1, \Omega_2\} \). Their method is based on a modification of the inverse transform method [123]. The algorithm is summarized as follows:

1) Generate a Nakagami r.v. as

\[
r_1 = \sqrt{-\frac{\Omega_1}{m} \ln \left( \prod_{j=1}^{m} u_j \right)}.
\]  

(2.123)
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<th>BER: C</th>
<th>BER: LL</th>
<th>BER: tD</th>
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</tbody>
</table>

Table 2.1 Comparison in terms of BER of methods for generating Nakagami random variables with $m \geq 1.0$: $m = 2.0$.

where $u_j$, $j = 1, 2, \ldots, m$, are uniformly distributed r.v.'s over $[0, 1]$.

2) Solve for $r_2$ in

$$F(r_2|r_1) = u \quad (2.124)$$
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<th>BER: PT</th>
<th>BER: B</th>
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Table 2.2 Comparison in terms of BER of methods for generating Nakagami random variables with $m \leq 1.0$: $m = 0.6$.

where $u$ is a uniformly distributed r.v. over $[0, 1]$, and

$$F(r_2|r_1) = \int_0^{r_2} p(r_2|r_1) \, dr_2 = \int_0^{r_2} \frac{p(r_1, r_2)}{p(r_1)} \, dr_2$$

$$= \int_0^{r_2} \frac{4m^{m+1}(r_1 r_2)^m}{\Gamma(m) \Omega_1 \Omega_2} \left(1 - \rho \right)^{-\frac{m}{2}} e^{-\frac{m}{2} \frac{r_1^2 + r_2^2}{\Omega_1^2 + \Omega_2^2}} I_{m-1} \left( \frac{2m \sqrt{\rho r_1 r_2}}{\sqrt{\Omega_1 \Omega_2 (1 - \rho)}} \right)$$

$$\times \frac{1}{\Gamma(m) r_2^{2m-1} e^{-\frac{m}{2} r_1^2}} \, dr_2$$

$$= 1 - Q_m \left( r_1 \sqrt{\frac{2 \rho}{(1 - \rho) \Omega_1^2}}, r_2 \sqrt{\frac{2}{(1 - \rho) \Omega_2^2}} \right)$$  \hspace{1cm} (2.125)$$

Due to Eq. (2.123) the algorithm as presented in [127] is restricted to $m$ integer. To extend it to $m$ real, we can substitute Eq. (2.123) with Eq. (2.122). An algorithm for the
Efficient calculation of the generalized Marcum $Q$-function $Q_m(\alpha, \beta)$ is given in Appendix C.

$L$ arbitrary

[128] proposes an algorithm to generate $L$ correlated Nakagami r.v.'s with identical $m$-parameters but unequal powers. The method is based on a decomposition of the fading process, by generating correlated Gaussian r.v.'s with a correlation matrix specified such that the Nakagami r.v.'s obtained by a transformation from these Gaussian r.v.'s have the desired correlation matrix. The method works for $m$ integer, however only an approximation is proposed for $m$ arbitrary real. The algorithm is summarized below.

$z = [z_1 z_2 \ldots z_L]$ is the vector of $L$ mutually correlated Nakagami envelopes, with $L \times L$ covariance matrix $C_z = \{\rho_{ij}\}$.

1) Determine $\rho_{ij}$, $i, j = 1, 2, \ldots, L$, $i \neq j$ ($v_{ii} = 1$) by solving:

$$\rho_{ij} = \frac{\Gamma^2(m + \frac{1}{2})}{\Gamma(m)\Gamma(m + 1) - \Gamma^2(m + \frac{1}{2})} \left[ {}^2F_1 \left( -\frac{1}{2}, -\frac{1}{2}; m; v_{ij} \right) - 1 \right]$$

(2.126)

2) Determine the $L \times L$ covariance matrix $C_x$ as:

$$C_x(i, j) = \begin{cases} \zeta \text{var}[z_i] & i = j \\ \zeta (\text{var}[z_i] \text{var}[z_j] v_{ij})^{\frac{1}{2}} & i \neq j \end{cases}$$

(2.127)

where:

$$\text{var}[z_i] = \left[ 1 - \frac{1}{m} \frac{\Gamma^2(m + \frac{1}{2})}{\Gamma^2(m)} \right] \Omega_i,$$

(2.128)

$$\zeta = \frac{1}{2m} \left[ 1 - \frac{1}{m} \frac{\Gamma^2(m + \frac{1}{2})}{\Gamma^2(m)} \right]^{-1}.$$
3) Determine $\mathbf{x}_k = [x_{k1} x_{k2} \ldots x_{kL}]$, $k = 1, 2, \ldots, N$, with $N = 2m$ if $2m$ is integer and $2m + 1$ otherwise:

$$
\mathbf{x}_k = \mathbf{L}_x \mathbf{e}_k
$$

(2.130)

where $\mathbf{L}_x$ is obtained by the Cholesky decomposition of $\mathbf{C}_x = \mathbf{L}_x \mathbf{L}_x^H$ ($\cdot^H$ denotes the Hermitian transpose) and $\mathbf{e}_k = [e_{k1} e_{k2} \ldots e_{kL}]$, $k = 1, 2, \ldots, N$, are Gaussian vectors with zero mean and covariance the $N \times N$ identity matrix $\mathbf{I}$.

4) Determine $\mathbf{y} = [y_1 y_2 \ldots y_L]$ as:

$$
\mathbf{y} = \begin{cases} 
\sum_{k=1}^{2m} x_k^2 & \text{if $2m$ integer} \\
\alpha \sum_{k=p}^{p+1} x_k^2 + \beta x_{p+1}^2 & \text{otherwise}
\end{cases}
$$

(2.131)

where the squaring operation applies to each element of the vectors $\mathbf{x}_k$ and:

$$
\alpha = \frac{2pm + \sqrt{2pm(p + 1 - 2m)}}{p(p + 1)},
$$

(2.132)

$$
\beta = 2m - p\alpha
$$

(2.133)

and $p = \lfloor 2m \rfloor$.

5) Determine $\mathbf{z}$ as (the square-root operation applies to each element of the vector):

$$
\mathbf{z} = \mathbf{y}^{\frac{1}{2}}.
$$

(2.134)

Note that a decomposition method relying on the same principles was also presented in [129], for $m$ integer.

2.4.1.3 Time-Correlated Nakagami Random Variables

Rayleigh Channel Simulators

Several methods are available to simulate temporally correlated Rayleigh fading channels, either in the time or frequency domain. Below, we give a brief overview of the main categories of simulators:
- Sum-of-sinusoids simulators: the fading distribution is obtained by taking the envelope of a sum of cosines (or sines) with given individual amplitudes and phases. This approach is used for example in the Jakes simulator for Rayleigh channels [15], [130], and was inspired by the work in [92]. Modifications to the Jakes simulator are proposed in [131], [132]. Other sum-of-sinusoids simulators are given in [133], [134]. By adding a constant term to this sum, a Rice simulator can be obtained, as in [122].

- Frequency-domain filtering simulators: a computationally efficient approach consists of generating in-phase and quadrature Gaussian samples in the frequency domain, shaping them using the desired spectrum, and then obtaining the time-domain envelope using the Inverse Discrete Frequency Transform (IDFT). Simulators built according to this principle are described in [135], [16], [136], [137].

- Time-domain filtering simulators: a sequence of Gaussian variables is processed by a time-domain filter in order to obtain the desired spectrum [138].

- Autoregressive (AR) simulators: the fading process is fitted to a (high-order) AR process [139].

Nakagami Channel Simulators
Several approaches have been used in the literature for simulating a temporally correlated Nakagami fading channel. However, due to the lack of a solid physical basis for the Nakagami fading channel, all of the proposed models need to make assumptions or approximations regarding the fading process. Below we briefly review some of the main approaches along with their limitations.

- Approximating the Nakagami channel by a Rice channel:
  - One of the first proposals for a Nakagami simulator appeared in [122]. The authors approximated the Nakagami channel as a Rice channel, by matching selected moments. This approximation was initially pointed out in [23]. However, as noted in [140], [141], the approximation doesn’t hold for the tails of the pdf, which are essential in the determination of the BER. We verified that a simulator based on such an approach approximates well the BER for low signal-to-noise ratios (SNR’s),
but for high SNR's a large discrepancy is observed.
- In [134], the authors considered a sum-of-sinusoids simulator, and tried to determine the optimal coefficients (amplitudes and phases of the cosines) to approximate a Nakagami distribution. Unfortunately, the resulting approximation also suffers from the same problem as the previous approach: there are large mismatches in the tails' areas.

- Modeling the fading as an AR process:
  - [142] uses an AR approximation for the fading process, coupled with the inverse transform method, in order to simulate an arbitrary number of Nakagami fading channels correlated both temporally and spatially. The resulting simulator is quite complex and only approximates the fading process.
  - [128] also uses an AR approximation for the fading process, and uses the decomposition method described above to produce the correlation. The resulting simulator also gave unprecise results.

- Modeling the fading as a Rayleigh process and mapping the Rayleigh amplitude to a Nakagami amplitude using the inversion method: [88] assumes that the phase of the Nakagami fading process has a uniform pdf (like the underlying Rayleigh process), however there is no strict theoretical or experimental justification for this assumption. Also, the envelope correlation is determined from the simulation model, and not the opposite.

- Modeling the fading envelope as the product of a square-root beta process and a complex Gaussian process: the method of [143] is restricted to $0.5 < m < 1$, and the implementation is quite complex.

Hence it is not yet clear whether some of the assumptions or approximations used above can be considered valid or not, and more theoretical and/or experimental work is needed in this direction.
2.4.2 Discrete Channel Simulation

2.4.2.1 Introduction

The assumption that a fading channel can be modeled as a Markov chain can be used to design various low-complexity simulators. Instead of generating the correlated envelope of the channel, which then needs to be processed to determine if a decision error is made, one can directly produce the error process seen at the output of the demodulator. For over half a century, researchers have tried to fit such discrete-time models to realistic radio channels [144]. These models range from the early Gilbert-Elliott two-state channels [145] to more complex models such as those based on hidden Markov chains [146]. The finite state Markov channel (FSMC) proposed in [89] has attracted quite some attention due to its good balance between accuracy and complexity. It is based on the partitioning of the received SNR in a finite number of states, an approach which was also presented independently in [147]. The use of a first-order Markov process to model the envelope of a Rayleigh channel has been shown to be a good approximation in [148], using an information theoretic criterion, and was discussed recently in [149]. A Rician channel was also simulated using the same approach in [150]. A higher-order model was proposed to represent Nakagami channels in [151], however the complexity of the calculations (requiring the numerical evaluation of double integrals) make its use less attractive. Several variations of the FSMC were examined for example in [152], [153], [154]. In previous papers FSMC's were mostly designed to model flat fading channels. If one needs to simulate the multiple paths of a channel in order to take into account diversity, the execution time of a waveform simulator increases with the number of diversity branches. Instead, one can generate directly the error process seen at the output of the diversity receiver. This technique was used in [101] for SC and Rayleigh fading, and was slightly discussed in [151] for an approximation to EGC in Nakagami fading.

This section tackles the design of FSMC's for SC, MRC and EGC, in a generalized fading (Nakagami) environment. Most of the Nakagami fading simulators proposed up to date are complex and time-consuming [142], [143], [128], and many are restricted to particular values of the m-parameter (i.e. m integer). Our goal is to design a Nakagami simulator which generates samples at a very high speed, while giving a satisfactory accu-
racy: this simulator will be useful in simulation studies requiring a very large number of samples, such as the performance evaluation of digital video signals over fading channels. Indeed, the use of FSMC's has proven popular in previous papers dealing with wireless video transmission [155], [101], however only Rayleigh statistics were usually used. This section extends the use of FSMC's to Nakagami fading channels with diversity.

In the next sections we derive the parameters used by the FSMC, for the three diversity combining methods, in addition to the non-diversity case: the analytical steady state, transition and error probabilities. These parameters are integrated in a low-complexity simulator, whose first and second order statistics are compared with theoretical expressions in Section 2.4.2.6.

2.4.2.2 Review of FSMC Model

We use the approach first proposed in [89] to construct a FSMC. Let \( r \) be the combined envelope of the channel at the output of the diversity receiver, and \( \gamma = r^2 E_s / N_0 \) the postdetection SNR per symbol of the received signal, where \( E_s = E_b \log_2 M \) is the average energy per symbol (with \( E_b \) the average energy per bit and \( M \) the constellation size) and \( N_0 \) is the single-sided power spectral density. Let \( p_T(\gamma) \) and \( F_T(\gamma) = \int_0^\gamma p_T(\alpha) d\alpha \) be the pdf and cdf of \( \gamma \). We define \( K \) partitions for \( \gamma \) such that if \( \Gamma_k < \gamma < \Gamma_{k+1}, k = 0,1,\ldots,K-1 \), then the FSMC is said to be in the state \( s_k \). The \( \Gamma_k \)'s are the thresholds of the partition, with \( \Gamma_0 = 0 \) and \( \Gamma_K \to \infty \). A simple way of choosing these thresholds consists in specifying that the steady-state probabilities \( \pi_k \) of each state be all equal, i.e.:

\[
\pi_k = \int_{\Gamma_k}^{\Gamma_{k+1}} p_T(\alpha) d\alpha = F_T(\Gamma_{k+1}) - F_T(\Gamma_k) = \frac{1}{K} \quad (2.135)
\]

for \( k = 0,1,\ldots,K-1 \). The set of equations (2.135) must be solved numerically (or analytically if a closed-form solution exists) for the thresholds \( \Gamma_k, k = 1,2,\ldots,K-1 \). This equal probability method (EPM) was proposed in [89]. Optimization of the thresholds using least squares quantization and the Lloyd-Max algorithm was later suggested in [155]. However, the latter requires much more computations, and as the number of
states increases the advantage in accuracy with respect to the EPM diminishes. We thus rely on the EPM throughout this work.

Let \( \bar{\gamma} = E[\gamma] = \Omega E_s / N_0 \) be the average SNR per symbol of the received signal, with \( \Omega = E[r^2] \). The average SNR corresponding to the state \( k \) is then:

\[
\bar{\gamma}_k = \frac{1}{\pi_k} \int_{\Gamma_k}^\Gamma \alpha p_\Gamma(\alpha) d\alpha.
\]

(2.136)

In a first-order Markov model, transitions are possible only between adjacent states. In a slow fading environment, the variations in the received SNR during a symbol period are slow enough that we can consider only adjacent state transitions without incurring a significant penalty. Let \( t_{i,j} \) denote the transition probability between states \( s_i \) and \( s_j \). Following [89], these can be approximated as:

\[
t_{k,k+1} \simeq \frac{N_{k+1}}{N_k}, \quad k = 0, 1, 2, \ldots, K - 2
\]

(2.137)

\[
t_{k,k-1} \simeq \frac{N_k}{R_s}, \quad k = 1, 2, \ldots, K - 1
\]

(2.138)

where \( N_k \) is the theoretical LCR evaluated at \( \Gamma_k \), and \( R_s = \pi_k R_s \) is the average number of symbols transmitted per second during which the SNR is in state \( s_k \), for a symbol rate \( R_s \). The remaining probabilities are deduced using:

\[
t_{0,0} = 1 - t_{0,1}, \quad t_{K-1,K-1} = 1 - t_{K-1,K-2},
\]

(2.139)

\[
t_{k,k} = 1 - t_{k,k-1} - t_{k,k+1}, \quad k = 1, 2, \ldots, K - 2.
\]

(2.140)

The symbol error probability for each state is calculated as:

\[
P_k = \frac{1}{\pi_k} \int_{\Gamma_k}^\Gamma P_s(\alpha)p_\Gamma(\alpha) d\alpha
\]

(2.141)

where \( P_s(\gamma) \) is the average symbol error probability for a nonquantized model, conditioned on the SNR.

For coherent detection and BPSK:

\[
P_s(\gamma) = Q(\sqrt{2\gamma})
\]

(2.142)
where \( Q(x) = 1/\sqrt{2\pi} \int_{-\infty}^{\infty} \exp(-\alpha^2/2) d\alpha \). Following the procedure given in the Appendix of [89], \( P_k \) can be written as:

\[
P_k = \frac{1}{\pi_k} \int_{\Gamma_k}^{\Gamma_{k+1}} \left[ \int_{\sqrt{2\Gamma_k}}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx \right] p_{\Gamma}(\alpha) d\alpha
\]

\[
= \frac{1}{\pi_k} \left( \int_{\sqrt{2\Gamma_k}}^{\sqrt{2\Gamma_{k+1}}} \left[ \int_{\Gamma_k}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx + \int_{\sqrt{2\Gamma_k}}^{\infty} \left[ \int_{\Gamma_k}^{\Gamma_{k+1}} p_{\Gamma}(\alpha) d\alpha \right] \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx \right) \right.
\]

\[
+ \int_{\sqrt{2\Gamma_k}}^{\sqrt{2\Gamma_{k+1}}} \left[ F_{\Gamma}(\Gamma_k) - F_{\Gamma}(\Gamma_{k+1}) \right] e^{-x^2/2} dx
\]

\[
= \frac{1}{\pi_k} \left( -F_{\Gamma}(\Gamma_k) \left[ Q(\sqrt{2\Gamma_k}) - Q(\sqrt{2\Gamma_{k+1}}) \right] + I + \left[ F_{\Gamma}(\Gamma_{k+1}) - F_{\Gamma}(\Gamma_k) \right] Q(\sqrt{2\Gamma_k}) \right)
\]

\[
= \frac{1}{\pi_k} \left( F_{\Gamma}(\Gamma_{k+1}) Q(\sqrt{2\Gamma_{k+1}}) - F_{\Gamma}(\Gamma_k) Q(\sqrt{2\Gamma_k}) + I \right) \quad (2.143)
\]

where

\[
I = I_{k+1} - I_k = \int_{\sqrt{2\Gamma_k}}^{\sqrt{2\Gamma_{k+1}}} F_{\Gamma} \left( \frac{x^2}{2} \right) e^{-x^2/2} dx. \quad (2.144)
\]

Eq. (2.143) can be rewritten as

\[
P_k = \frac{1}{\pi_k} (\xi_{k+1} - \xi_k) \quad (2.145)
\]

where:

\[
\xi_k = F_{\Gamma}(\Gamma_k) Q(\sqrt{2\Gamma_k}) + I_k, \quad (2.146)
\]

\[
I_k = I(\Gamma_k) = \int_{0}^{\sqrt{2\Gamma_k}} F_{\Gamma} \left( \frac{\alpha^2}{2} \right) e^{-\alpha^2/2} d\alpha. \quad (2.147)
\]
For noncoherent detection and \( M \)-ary orthogonal frequency-shift keying modulation (NC-\( M \)-FSK) [26]:

\[
P_s(\gamma) = \sum_{i=1}^{M-1} (-1)^{i+1} \binom{M-1}{i} \frac{1}{i+1} e^{-\frac{i}{M+1} \gamma}.
\] (2.148)

Below, analytical expressions are provided for the parameters \( \{N_k\} \) and \( \{I_k\} \), used in Eqs. (2.137)-(2.138) and Eqs. (2.145)-(2.146) respectively, for different diversity techniques in a Nakagami fading channel. Once the cdf of the SNR is known, the \( \{\Gamma_k\} \) are solved for numerically using Eq. (2.135). The obtained simulator needs only to compute these parameters at the beginning of a set of (multiple) simulations. After that, in the simulator’s software implementation, only two (uniform) random numbers need to be generated for each iteration: one to determine if a state transition has taken place, and one to determine if an error has occurred. This drastically reduces the computational complexity compared to waveform simulators, which need for example to compute a sum of sinusoids for each iteration (Jakes’ method, [15]), or to perform the Inverse Fast Fourier Transform (IFFT) of a block of samples the size of the total number of iterations (Smith’s method, [135]).

### 2.4.2.3 No Diversity

The pdf and cdf of the received signal SNR for a Nakagami fading channel and no diversity are given by:

\[
p_r(\gamma) = \left( \frac{m}{\gamma} \right)^m \frac{\gamma^{m-1}}{\Gamma(m)} e^{-\frac{m}{\gamma} \gamma},
\] (2.149)

\[
F_r(\gamma) = \frac{\gamma(m, \frac{m}{\gamma} \gamma)}{\Gamma(m)}.
\] (2.150)
From Eqs. (2.135) and (2.150), the thresholds are obtained by solving numerically (e.g. using the bisection method [156]) the following equation for \( \Gamma_k, k = 1, 2, \ldots, K - 1 \):

\[
\gamma \left( m, \frac{m}{\tilde{\gamma}} \Gamma_{k+1} \right) - \gamma \left( m, \frac{m}{\tilde{\gamma}} \Gamma_k \right) = \frac{\Gamma(m)}{K}
\]  

(2.151)

knowing that \( \Gamma_0 = 0 \). The average \( \text{SNR} \) corresponding to state \( k \) is

\[
\tilde{\gamma}_k = \frac{\tilde{\gamma}/m}{\pi_k \Gamma(m)} \left[ \gamma \left( m + 1, \frac{m}{\tilde{\gamma}} \Gamma_{k+1} \right) - \gamma \left( m + 1, \frac{m}{\tilde{\gamma}} \Gamma_k \right) \right].
\]  

(2.152)

From [96] or [98] (c.f. Eq. (2.43) for \( L = 1 \)), the LCR's can be obtained as:

\[
N_k = \frac{\sqrt{2\pi f_m}}{\Gamma(m)} \left( \frac{m}{\tilde{\gamma}} \Gamma_k \right)^{-\frac{1}{2}} e^{-\frac{m}{\tilde{\gamma}} \Gamma_k}.
\]  

(2.153)

**BPSK**

Using the following infinite series expansion for \( \gamma(\alpha, x) \) in Eq. (2.150):

\[
\gamma(\alpha, x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{\alpha+n}}{n! \alpha + n}
\]  

(2.154)

and substituting Eqs. (2.150) in (2.147) results in:

\[
I_k = \frac{1}{\Gamma(m)} \sum_{n=0}^{\infty} \frac{(-1)^n \left( \frac{m}{\tilde{\gamma}} \right)^{m+n}}{n! (m + n)} \int_0^{\sqrt{2m+1}k} \left( \frac{x}{\sqrt{2}} \right)^{2(m+n)} e^{-\frac{x^2}{2}} \, dx.
\]  

(2.155)

Making the change of variable \( y = x^2/2 \) and using Eq. 8.350 of [80] leads to :

\[
I_k = \frac{1}{2\sqrt{\pi} \Gamma(m)} \sum_{n=0}^{\infty} (-1)^n \left( \frac{m}{\tilde{\gamma}} \right)^{m+n} \gamma \left( m + n + \frac{1}{2}, \Gamma_k \right) \frac{1}{n! (m + n)}.
\]  

(2.156)
Substituting Eqs. (2.149) and (2.148) in Eq. (2.141) leads to

\[
P_k = \frac{1}{\pi_k \Gamma(m)} \left( \frac{m}{\gamma} \right)^m \sum_{i=1}^{M-1} (-1)^{i+1} \binom{M-1}{i} \frac{1}{i+1} \left( \frac{m}{\gamma} \right)^m \times \left[ \gamma \left( m, \left( \frac{i}{i+1} + \frac{m}{\gamma} \right) \Gamma_k+1 \right) - \gamma \left( m, \left( \frac{i}{i+1} + \frac{m}{\gamma} \right) \Gamma_k \right) \right].
\]  

(2.157)

### 2.4.2.4 Selection Combining (SC) Diversity

In the rest of this section we consider only the case of independent diversity channels and identical fading parameters for every channel, for which closed-form expressions for the LCR have been found in Section 2.3.1. The pdf and cdf of the output SNR of a \( L \)-branch diversity combiner are:

\[
p_{\Gamma}(\gamma) = L \left[ \frac{\gamma(m, \frac{m}{\gamma} \Gamma)}{\Gamma(m)} \right]^{L-1} \left( \frac{m}{\gamma} \right)^m \frac{\gamma^{m-1}}{\Gamma(m)} e^{-\frac{m}{\gamma} \gamma},
\]  

(2.158)

\[
F_{\Gamma}(\gamma) = \left[ \frac{\Gamma(m, \frac{m}{\gamma} \Gamma)}{\Gamma(m)} \right]^L.
\]  

(2.159)

From Eqs. (2.135) and (2.159), the thresholds are obtained by solving numerically the following equation for \( \Gamma_k, k = 1, 2, \ldots, K - 1 \):

\[
\left[ \gamma \left( m, \frac{m}{\gamma} \Gamma_k+1 \right) \right]^L - \left[ \gamma \left( m, \frac{m}{\gamma} \Gamma_k \right) \right]^L = \frac{[\Gamma(m)]^L}{K}.
\]  

(2.160)

For \( m \) integer, the incomplete gamma function can be written as (Eq. 8.352.1 of [80]):

\[
\gamma(m, x) = (m - 1)! \left[ 1 - e^{-x} \left( \sum_{n=0}^{m-1} \frac{x^n}{n!} \right) \right].
\]  

(2.161)
Making use of Eq. (2.161) in Eq. (2.158), of the binomial expansion (Eq. 1.111 of [80]), and of the following multinomial expansion [157]:

\[
\left( \sum_{k=0}^{L-1} \frac{x^k}{k!} \right)^l = \sum_{k=0}^{l(L-1)} \beta_{kl} x^k
\]  

(2.162)

the pdf for \( m \) integer can be written as:

\[
p_{\gamma}(\gamma) = \frac{L}{\Gamma(m)} \sum_{l=0}^{L-1} (-1)^l \binom{L-1}{l} e^{-\left(\frac{m}{\gamma}\right)^{m+k}} \sum_{k=0}^{l(m-1)} \beta_{kl} \left( \frac{m}{\gamma} \right)^{m+k} \gamma^{m+k-1}.
\]

(2.163)

In the above equations, \( \beta_{kl} \) are the coefficients of the multinomial expansion, evaluated using [157]:

\[
\beta_{kl} = \sum_{i=k-(L-1)}^{k} \frac{\beta_{l(l-1)}}{(k-i)!} I_{([0,(l-1)(L-1)])(i)}
\]

(2.164)

with \( \beta_{00} = \beta_{0l} = 1, \beta_{kl} = 1/k! \), \( \beta_{1l} = l \) and \( I_{[a,b]}(i) = 1 \) if \( a \leq i \leq b \) and 0 otherwise. Similarly, the cdf for \( m \) integer can be written as:

\[
F_{\gamma}(\gamma) = \sum_{l=0}^{L} (-1)^l \binom{L}{l} e^{-\left(\frac{m}{\gamma}\right)^{m+k}} \sum_{k=0}^{l(m-1)} \beta_{kl} \left( \frac{m}{\gamma} \right)^k.
\]

(2.165)

The average SNR corresponding to state \( k \) is then

\[
\bar{\gamma}_k = \frac{L \bar{\gamma}/m}{\pi_k \Gamma(m)} \sum_{l=0}^{L-1} (-1)^l \binom{L-1}{l} \sum_{n=0}^{l(m-1)} \frac{\beta_{nl}}{(l+1)^{m+n+1}} \times \left[ \gamma \left( m + n + 1, (l+1)^m \frac{m}{\gamma} \Gamma_{k+1} \right) - \gamma \left( m + n + 1, (l+1)^m \frac{m}{\gamma} \Gamma_k \right) \right].
\]

(2.166)

The LCR’s are obtained as (c.f. Eq. (2.43)):

\[
N_k = \frac{L \sqrt{2\pi f_m}}{\left[ \Gamma(m) \right]^L} \left( \frac{m}{\gamma} \Gamma_k \right)^{-\frac{1}{2}} e^{-\frac{m}{\gamma} \Gamma_k} \left[ \gamma \left( m, \frac{m}{\gamma} \Gamma_k \right) \right]^{L-1}.
\]

(2.167)
BPSK

Using Eq. (2.165) in Eq. (2.147), with the aid of Eq. 8.350 of [80] we obtain:

\[
I_k = \frac{1}{2\sqrt{\pi}} \sum_{l=0}^{L} (-1)^l \binom{L}{l} \sum_{k=0}^{l(m-1)} \beta_{kl} \left( \frac{m}{\gamma} \right)^k \left( \frac{\gamma}{lm + \gamma} \right)^{k+\frac{1}{2}} \times \gamma \left( k + \frac{1}{2}, \frac{lm + \gamma}{\gamma} \Gamma_k \right). \tag{2.168}
\]

Recall that this expression is valid only for \( m \) integer. For \( m \) arbitrary, there is no simple closed-form solution obtainable for \( I_k \), and the \( \epsilon_k \)'s must be calculated numerically via direct integration.

NC-M-FSK

Substituting Eqs. (2.163) and (2.148) in Eq. (2.141) leads to:

\[
P_k = \frac{L}{\pi_k \Gamma(m)} \sum_{i=1}^{M-1} (-1)^{i+1} \binom{M-1}{i} \frac{1}{i+1} \sum_{l=0}^{L-1} (-1)^l \binom{L-1}{l} \sum_{k=0}^{l(m-1)} \beta_{kl} \left( \frac{m}{\gamma} \right)^{m+k} \times \left[ \gamma \left( m + k, \left( \frac{i}{i+1} + (l + 1) \frac{m}{\gamma} \right) \Gamma_{k+1} \right) \right. \\
\left. - \gamma \left( m + k, \left( \frac{i}{i+1} + (l + 1) \frac{m}{\gamma} \right) \Gamma_k \right) \right]. \tag{2.169}
\]

2.4.2.5 Maximal Ratio Combining (MRC) Diversity

With the previous assumption of identical fading parameters, the pdf and cdf of the output SNR are [23]:

\[
p_T(\gamma) = \left( \frac{m_T}{\gamma_T} \right)^{m_T} \frac{\gamma_T^{m_T-1}}{\Gamma(m_T)} e^{-\frac{m_T}{\gamma_T} \gamma}, \tag{2.170}
\]

\[
F_T(\gamma) = \frac{\gamma(m_T, \frac{m_T}{\gamma_T} \gamma)}{\Gamma(m_T)} \tag{2.171}
\]

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with \( m_T = Lm \) and \( \gamma_T = L\gamma \). The desired quantities can be deduced from Eqs. (2.149)-(2.157) by substituting \( m \) with \( m_T \) and \( \gamma \) with \( \gamma_T \), i.e.:

\[
N_k = \frac{\sqrt{2\pi f_m}}{\Gamma(m_T)} \left( \frac{m_T}{\gamma_T \Gamma_k} \right)^{\frac{m_T}{2}} e^{-\frac{m_T}{2} \Gamma_k}
\]  

(2.172)

and:

\[
I_k = \frac{1}{2\sqrt{\pi} \Gamma(m_T)} \sum_{n=0}^{\infty} \frac{(-1)^n \left( \frac{m_T}{\gamma_T} \right)^{m_T+n}}{n! (m_T+n)} \gamma \left( m_T + n + \frac{1}{2} \right) \Gamma_k.
\]  

(2.173)

### 2.4.2.6 Equal Gain Combining (EGC) Diversity

We consider only the case of dual-branch diversity \((L = 2)\), for which we found in Section 2.3.1.5 the following closed-form and infinite series representations for the pdf and cdf, respectively:

\[
p_T(\gamma) = \frac{B(2m, \frac{1}{2})}{2^{2(m-1)}[\Gamma(m)]^2} \left( \frac{m}{\gamma} \right)^{2m-1} \gamma^{2m-1} e^{-\frac{2m}{\gamma}} \Phi \left( 2m, 2m + \frac{1}{2}, \frac{m}{\gamma} \right).
\]  

(2.174)

\[
F_T(\gamma) = \frac{\sqrt{\pi}}{[\Gamma(m)]^2 2^{2m-2}} \sum_{n=0}^{\infty} \frac{\Gamma(2m+n)}{\Gamma(2m+n+\frac{1}{2}) 2^n n!} \gamma \left( 2m + n, 2 \frac{m}{\gamma} \right).
\]  

(2.175)

From Eqs. (2.135) and (2.175), the thresholds are obtained by solving numerically the following equation for \( \Gamma_k \), \( k = 1, 2, \ldots, K - 1 \):

\[
\sum_{n=0}^{\infty} \frac{\Gamma(2m+n)}{\Gamma(2m+n+\frac{1}{2}) 2^n n!} \left[ \gamma \left( 2m + n, 2 \frac{m}{\gamma} \Gamma_{k+1} \right) - \gamma \left( 2m + n, 2 \frac{m}{\gamma} \Gamma_k \right) \right] = \frac{[\Gamma(m)]^2 \gamma^{4m-2}}{\sqrt{\pi} K}.
\]  

(2.176)
The average SNR corresponding to state \( k \) is
\[
\bar{\gamma}_k = \frac{\sqrt{\pi \gamma/m}}{\pi_k \Gamma(m)^2 2^{4m-2}} \sum_{n=0}^{\infty} \frac{\Gamma(n+2m)}{\Gamma(n+2m+1/2)} \frac{1}{2^n n!} \times \left[ \gamma \left(2m+n+1, \frac{2m}{\gamma} \Gamma_{k+1} \right) - \gamma \left(2m+n+1, \frac{2m}{\gamma} \Gamma_k \right) \right]. \quad (2.177)
\]

We previously obtained the LCR's as (c.f. Eq. (2.67)):
\[
N_k = \frac{\sqrt{2\pi} f_m B(2m+1/2)}{2^{2(m-1)} \Gamma(m)^2} \left( \frac{m}{\gamma} \right)^{2m-\frac{k}{2}} e^{-\frac{m}{2} \Gamma_k} \Phi \left(2m, 2m+1/2, \frac{m}{\gamma} \Gamma_k \right). \quad (2.178)
\]

Substituting Eqs. (2.175) in (2.147), and making use of Eq. (2.154) and Eq. 8.350 of [80] results in:
\[
I_k = \frac{1}{{\Gamma(m)^2 2^{4m-1}}} \sum_{n=0}^{\infty} \frac{\Gamma(2m+n)}{\Gamma(2m+n+1/2) 2^n n!} \times \sum_{j=0}^{\infty} \frac{(-1)^j}{j!} \frac{(2m)^j}{2m+n+j} \gamma \left(2m+n+j+1, \frac{2m}{\gamma} \Gamma_k \right). \quad (2.179)
\]

The previous doubly infinite series can become unstable when calculated using limited precision software, thus one might use direct numerical integration to evaluate \( I_k \).

2.4.2.7 Numerical Results and Discussion

**Uncoded BER**

The theoretical uncoded BER's for BPSK for the three types of combining schemes, in i.i.d. Nakagami channels, are obtained as
\[
P_s = \int_0^{\infty} Q(\sqrt{2\gamma}) p_{1\gamma}(\gamma) d\gamma \quad (2.180)
\]
where the $p_r(\gamma)$ were given previously. Closed-form expressions for these BER's have been obtained in previous literature and are given below.

**Selection Combining**

$$P_s = \frac{L}{(m-1)!} \sum_{l=0}^{L-1} (-1)^l \binom{L-1}{l} \frac{\beta_{kl}(m+k-1)!}{(l+1)^{m+k}} \left(\frac{1-\mu}{2}\right)^{m+k}$$

$$\times \sum_{h=0}^{m+k-1} \binom{m+k-1}{h} \left(1+\frac{\mu}{2}\right)^h$$

(2.181)

where $m$ is integer, and $\mu = \sqrt{\gamma/(\gamma + (l+1)m)}$. Eq. (2.181) is similar to Eq. (18) of [158], however some small typos in the latter have been corrected.

**Maximal Ratio Combining**

$$P_s = \frac{1}{2\sqrt{\pi} \left[(m + \gamma)/m\right]^{m+\frac{1}{2}} \Gamma(m + \frac{1}{2})} \left(\frac{1}{2}\right)^{m+\gamma} \frac{\Gamma(m+\frac{1}{2})}{\Gamma(m+1)}_2F_1 \left(1, m + \frac{1}{2}; m + 1; \frac{m}{m + \gamma}\right)$$

(2.182)

Eq. (2.182) is given for example in [39]. For $m$ integer, it reduces to [39]:

$$P_s = 2 \left[1 - \mu \sum_{k=0}^{m-1} \binom{2k}{k} \left(\frac{1-\mu^2}{4}\right)^k\right]$$

(2.183)

where $\mu = \sqrt{\gamma/(m + \gamma)}$.

**Equal Gain Combining, $L = 2$**

$$P_s = \frac{1}{2} - 2 \sqrt{\frac{1}{\pi} \frac{\eta^{2m}}{(1+2\eta)^{2m+\frac{1}{2}}} \frac{\Gamma(m+\frac{1}{2})}{\Gamma(m)}}$$

$$\times F_2 \left(2m + \frac{1}{2}; 1, 2m; \frac{3}{2}, 2m + \frac{1}{2}; \frac{1}{1+2\eta}, \frac{1}{1+2\eta}\right)$$

(2.184)

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where \( \eta = m/\bar{\gamma} \), and \( F_2(a; b_1, b_2; c_1, c_2; x, y) \) is the Appell hypergeometric function of two arguments ([80], Eq. (9.19)). Eq. (2.184) is derived in Section 2.5.3.2.

For each diversity type, the BER's obtained via the proposed simulator are compared to the theoretical values for different values of the \( m \) parameter. \( L = 2 \) branches are used in the results, however any number of branches can be accommodated (except for the EGC case). The fading power in each branch is normalized to 1.0. The FSMC has \( K = 16 \) states. The estimated BER was averaged over 100 simulation runs, each one producing \( 10^6 \) samples. Figs. 2.15-2.17 illustrate the BER versus the SNR per bit per diversity branch: it can be seen that the BER's obtained from the simulator match very well the analytical curves.

\[ \frac{E_b}{N_0} \text{ per diversity branch (dB)} \]

**Figure 2.15** BER for SC: (a) \( m = 1 \), (b) \( m = 2 \), (c) \( m = 3 \). FSMC simulation: ++ ; theory: ——.
Figure 2.16 BER for MRC: (a) $m = 1$, (b) $m = 2$, (c) $m = 3$. FSMC simulation: ++ ; theory: ——.

Figure 2.17 BER for EGC: (a) $m = 1$, (b) $m = 2$. FSMC simulation: ++ ; theory: ——.

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Figs. 2.18-2.20 compare the normalized LCR's of the FSMC ($N_n$) with the corresponding normalized theoretical expressions, where in both cases the normalization factor is $1/f_m$. The LCR's are plotted as a function of the normalized received envelope $r_n = r/\sqrt{\Omega}$ (in dB). For the FSMC, the value of the channel envelope when the model is in state $s_k$ is computed as $r_k = \sqrt{\tilde{b}_k}$. $K = 64$ states were used in order to get sufficient data points, and $L = 2$ diversity branches. As can be seen from the plots, the model generates level-crossing statistics very close to the theoretical ones.

**Figure 2.18** LCR for SC: (a) $m = 1$, (b) $m = 2$, (c) $m = 3$. FSMC simulation: ++ ; theory: —.
Figure 2.19 LCR for MRC: (a) $m = 1$, (b) $m = 2$, (c) $m = 3$. FSMC simulation: ++ ;
theory: ---.

Figure 2.20 LCR for EGC: (a) $m = 1$, (b) $m = 2$, (c) $m = 3$. FSMC simulation: ++ ;
threeory: ---.
Coded BER Performance

The previous two sections showed that the FSMC produces average LCR's (and consequently AFD's) and uncoded BER's very close to the theoretical ones: this is to be expected, since the FSMC dynamics are based on the theoretical LCR's, and the error rates in each state are derived from the theoretical BER's (one could alternatively add white Gaussian noise to the generated envelope \( r \) in order to calculate the decision metrics for any modulation scheme: this would add one more processing step, but wouldn't require the knowledge of the theoretical BER's). However, the generated envelope does not necessarily possess the other higher order statistics of the theoretical envelope: for example, [149] shows that depending on the fading rate (i.e. the maximum Doppler shift \( f_m \) times the symbol period \( T_s \)), the generated envelope does not necessarily match the autocorrelation function and the probability distribution of the fade durations (contrary to the average fade durations). Nevertheless, it is shown below that for small fading rates, the coded BER obtained by applying error correction to the output of the FSMC is reasonably close to the coded BER obtained using a traditional waveform simulator. Indeed, the design of the FSMC was based on the slow fading assumption [89], and if we depart significantly from this assumption the behavior of the FSMC will also diverge from the theoretical one.

In the following simulations, the bit rate is taken as 76.8 kbps, and groups of 1536 bits (including 8 added tail bits) form frames of 20ms. These frames are encoded with a rate 1/4 convolutional code, with generator functions (765), (671), (513) and (473) (in octal form). These parameters are taken from the cdma2000 standard [14], for medium-rate data or video transmission. The receiver performs hard-decision decoding using the Viterbi algorithm. Figs. 2.21-2.23 show the coded BER's (as a function of the SNR per branch \( E_b/N_0 \)) obtained with both the FSMC and a waveform simulator (WS), for a mobile speed of \( v = 6 \) km/h, a fading parameter \( m = 1 \), and maximal-ratio combining with \( L = 1, 2 \) and 3 diversity branches. The coded BER's are calculated using 160 frames, and are averaged over 100 and 20 simulation runs for the FSMC and WS, respectively. Results for the FSMC are shown for two numbers of states: \( K = 1024 \) and \( K = 2048 \).
It can be seen that the coded BER's for the FSMC and the WS are reasonably close together, especially for lower diversity orders. The number of states needs to be chosen with care. For systems with lower bit error rates, a higher number of states needs to be chosen: indeed, the quantization of the SNR needs to be small enough to allow the existence of several states with very small error probabilities. Otherwise, if the quantization of the SNR is too coarse, the FSMC will not be able to produce very small BER's. For example, in Fig. 2.23, for $E_b/N_0 = 7$ dB the FSMC with 2048 states gives a BER in the order of $10^{-6}$ - $10^{-5}$, while a BER of 0 is obtained with $K = 1024$. However, there is a limit on the number of states: with $K$ too large, the FSMC is not able to move quickly enough from one state to another (recall that in the current model, one state transition is allowed per symbol period), which leads to erroneous results. The higher the fading rate, the lower the maximum $K$: for the current simulations, it was found that for $v = 100$ km/h, $K$ must be smaller than about 200, while $K < 400$ for $v = 50$ km/h (i.e. it appears that the maximum $K$ is directly proportional to the fading rate).

Figs. 2.24-2.26 show the coded BER's for the same parameters but with $v = 50$ km/h, i.e. a higher fading rate. The accuracy of the FSMC is not as good as in the case $v = 6$ km/h (for the reasons given above), but overall the coded BER's are roughly in the same range.

2.4.2.8 Conclusions

This section presented the fast simulation of Nakagami fading channels using FSMC's. The simulators include the effect of diversity reception in order to directly simulate the envelope of the combined signal, thus avoiding the generation of multiple separate channels. It was verified that for slowly fading channels, the simulators can accurately reproduce the theoretical bit error rates and level crossing rates. The coded BER's can be reasonably approximated by a careful choice of the number of states. The simulators developed are thus useful in evaluating the performance of broadband systems, such as wireless video distribution, where a large number of samples need to be generated for performance evaluation.
**Figure 2.21** Coded BER with $v = 6$ km/h, $m = 1$, and $L = 1$.

**Figure 2.22** Coded BER with MRC and $v = 6$ km/h, $m = 1$, and $L = 2$. 

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Figure 2.23 Coded BER with MRC and \( v = 6 \, \text{km/h}, \, m = 1, \) and \( L = 3. \)

Figure 2.24 Coded BER with \( v = 50 \, \text{km/h}, \, m = 1, \) and \( L = 1. \)
Figure 2.25 Coded BER with MRC and $v = 50$ km/h, $m = 1$, and $L = 2$.

Figure 2.26 Coded BER with MRC and $v = 50$ km/h, $m = 1$, and $L = 3$. 

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2.5 Analytical Symbol Error Rates for Diversity Techniques in Correlated Nakagami Fading Channels

2.5.1 Introduction

MRC is known to be the optimal combining technique in white Gaussian noise for mobile radio systems employing diversity, however its use requires the careful estimation of the channel-induced fading and phase-shift at the receiver. In contrast, EGC only requires the estimation of the phase-shift (for coherent modulation schemes), which leads to a simpler implementation. Several analyses of MRC and EGC in Nakagami fading have been obtained for systems where it is assumed that the signals received at each diversity branch fade independently [159], [160]. However, in practical mobile radio systems the antennas are close together due to space constraints, which typically leads to non-zero correlation coefficients between the signals, and thus voids the independence assumption.

In the case of MRC, many analyses have considered correlated diversity branches with Nakagami fading. Without being exhaustive, [161] obtains the Symbol Error Rate (SER) in double integral form of M-ary Phase-Shift Keying (M-PSK) for two diversity branches ($L = 2$). For two special correlation models, Aalo [162] obtains closed-form expressions for Binary Phase-Shift Keying (BPSK) and Frequency Phase-Shift Keying (BFSK), for a general number of branches $L$. For more general correlation models, one-dimensional integral expressions with general $L$ are obtained in [163], [164], for BPSK/BFSK, and in [165], [166], [167] for M-ary Quadrature Amplitude Modulation (M-QAM) and M-PSK.

In the case of EGC, much fewer analytical results are available in the literature. It is well accepted that the accurate analysis of EGC over fading channels is a more challenging task, due to the difficulty in finding a simple exact expression for the pdf of the combined signal: indeed, there is still no general exact closed-form expression for the pdf of the sum of $L$ Rayleigh or Nakagami variables, for an arbitrary $L$. In the case of uncorrelated fading, in [168] the authors used the characteristic function method in order to obtain generic expressions for the SER of EGC, for a general number of branches $L$. In the special case of BPSK, their approach leads to closed-form solutions for the SER. However, in the case of higher-order modulations such as M-PSK and M-QAM, their approach leads to expressions containing double and single integrals, respectively. The
case of correlated fading has only been tackled for $L = 2$. The case of dual-diversity is an important one, since the greatest diversity gain is obtained when the number of antennas is increased from one to two: hence several commercial wireless systems (such as outdoor cellular and indoor cordless base stations) employ only two antennas. Previous work on dual-branch EGC in correlated fading includes [161], which obtains the SER of $M$-PSK in the form of a double integral, and [169] and [170], which obtain closed-form solutions for the SER of BPSK in Rayleigh and Nakagami fading, respectively.

In this section, we derive the exact theoretical SER of the coherent detection of several $M$-ary modulation schemes such as $M$-PSK, Differentially Encoded $M$-ary PSK (DE-$M$-PSK$^3$) and $M$-QAM, with MRC and EGC, for two correlated diversity branches in a Nakagami-$m$ fading environment. Several novel expressions are obtained, either in the form of a single-dimensional integral, or in closed-form as infinite series summations. In the case of MRC, closed-form solutions for the SER can be derived by using an alternative expression for the pdf of the sum of the squares of two correlated Nakagami-$m$ variables. In the case of EGC, the SER is obtained by first deriving the pdf of the sum of two correlated Nakagami-$m$ variables, for which, to our knowledge, no closed-form expression was previously available in the open technical literature on communications. We also derive the exact pdf of the sum of two independent Nakagami variables with unequal SNR's $\gamma_1, \gamma_2$ and fading parameters $m_1, m_2$ on each branch, when the $m_i$'s are arbitrary multiples of half integers. Using this pdf, expressions for the SER are obtained containing only a single integral (as opposed to a double integral in [168] for certain modulations such as $M$-PSK).

2.5.2 General Approach

Let $r = f(r_1, r_2)$ be the normalized output of the diversity combiner, where $f(\cdot)$ depends on the diversity technique used. We use the classical approach to the derivation of error probabilities for fading channels [26], by first deriving an expression for the pdf $p_R(r)$ of the combined signal $r$, and then directly averaging the conditional SER $P_s(\gamma)$ over the pdf of the signal-to-noise ratio after combining, which leads to the unconditional

---

$^3$Not to be confused with noncoherently-detected $M$-ary Differential PSK ($M$-DPSK) [22].
SER:

\[ P_s = \int_0^\infty P_s(\gamma) p_\gamma(\gamma) d\gamma, \]  
(2.185)

where \( \gamma = r^2 E_s/N_0 \) is the instantaneous SNR per symbol, with pdf

\[ p_\gamma(\gamma) = \frac{p_\gamma(\sqrt{\gamma/(E_s/N_0)})}{2\sqrt{\gamma E_s/N_0}} \]  
(2.186)

and average value \( \bar{\gamma} = E[r^2] E_s/N_0 = \Omega E_s/N_0 \). \( E_s = E_b \log_2 M \) is the energy per symbol (with \( E_b \) the energy per bit and \( M \) the constellation size) and \( N_0 \) is the single-sided power spectral density.

The conditional SER for several different \( M \)-ary modulations, such as \( M \)-PSK, \( M \)-QAM and \( M \)-PAM, can be expressed in a unified manner as [167]:

\[ P_s(\gamma) = \sum_{i=1}^{K} \int_0^{\Theta_i} a_i(\theta) e^{-\phi_i(\theta)\gamma} d\theta \]  
(2.187)

where the parameters \( K, a_i(\theta), \phi_i(\theta), \Theta_i \) are given in Table 2.3, in which \( f(\theta) = \frac{3 \cos(2\theta) - 1}{2 \cos^2(2\theta)} - 1 \) (c.f. [167], [171]).

<table>
<thead>
<tr>
<th>Modulation</th>
<th>( K )</th>
<th>( a_i(\theta) \cdot \pi )</th>
<th>( \phi_i(\theta) \cdot \sin^2 \theta )</th>
<th>( \Theta_i / \pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M )-PSK</td>
<td>1</td>
<td>1</td>
<td>( \sin^2 \left( \frac{\pi}{M} \right) )</td>
<td>( 1 - \frac{1}{M} )</td>
</tr>
<tr>
<td>( M )-QAM</td>
<td>2</td>
<td>( 4(1 - \frac{1}{\sqrt{M}}) )</td>
<td>( \frac{3}{2(M-1)} )</td>
<td>( 1/2 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( -4(1 - \frac{1}{\sqrt{M}})^2 )</td>
<td>( \frac{2}{2(M-1)} )</td>
<td>( 3/4 )</td>
</tr>
<tr>
<td>( M )-PAM</td>
<td>1</td>
<td>( 2(1 - \frac{1}{M}) )</td>
<td>( \frac{3}{M^2-1} )</td>
<td>( 1/2 )</td>
</tr>
<tr>
<td>( D)E-BPSK</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-2</td>
<td>1</td>
<td>1/4</td>
</tr>
<tr>
<td>( D)E-QPSK</td>
<td>4</td>
<td>4</td>
<td>1/2</td>
<td>1/2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-8</td>
<td>1/2</td>
<td>1/4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \frac{4}{\pi} \cos^{-1} f(\theta) )</td>
<td>( 1/2 )</td>
<td>( 1/6 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \frac{4}{\pi} (\pi - \cos^{-1} f(\theta)) )</td>
<td>( 1/2 )</td>
<td>( \frac{1}{\pi} \sin^{-1} \frac{1}{\sqrt{3}} )</td>
</tr>
</tbody>
</table>

Table 2.3 Parameters associated with the SER’s of \( M \)-ary modulations [167], [171].
Below, we also recall expressions for the individual SER's for M-PSK, BPSK, QPSK, M-QAM and DE-QPSK, conditioned on $\gamma$ [22]. We also give alternative forms, obtained using Eq. (A.31), which will be helpful in obtaining expressions in fading channels.

\[
P_{s}^{M-\text{PSK}}(\gamma) = \frac{1}{\pi} \int_{0}^{\pi} \exp \left( -\gamma \frac{\sin^2(\pi/M)}{\sin^2 \theta} \right) d\theta \tag{2.188}
\]

\[
P_{s}^{\text{BPSK}}(\gamma) = \frac{1}{2} \text{erfc}(\sqrt{\gamma}) \tag{2.189}
\]

\[
P_{s}^{\text{QPSK}}(\gamma) = \text{erfc}(\sqrt{\gamma/2}) - \frac{1}{4} \text{erfc}^2(\sqrt{\gamma/2}) \tag{2.191}
\]

\[
P_{s}^{M-\text{QAM}}(\gamma) = 2q \text{erfc}(\sqrt{p\gamma}) - q^2 \text{erfc}^2(\sqrt{p\gamma}) \tag{2.193}
\]

\[
P_{s}^{\text{DE-QPSK}}(\gamma) = 2\text{erfc} \left( \sqrt{\frac{\gamma}{2}} \right) - 2 \left[ \text{erfc} \left( \sqrt{\frac{\gamma}{2}} \right) \right]^2 + \left[ \text{erfc} \left( \sqrt{\frac{\gamma}{2}} \right) \right]^3 - \frac{1}{4} \left[ \text{erfc} \left( \sqrt{\frac{\gamma}{2}} \right) \right]^4 \tag{2.195}
\]

\[
P_{s}^{\text{DE-QPSK}}(\gamma) = \frac{3}{4} - \frac{\gamma}{\pi} e^{-\gamma} \left[ \Phi \left( 1, \frac{3}{2}, \gamma \right) \right]^2 - \frac{\gamma^2}{\pi^2} e^{-2\gamma} \left[ \Phi \left( 1, \frac{3}{2}, \gamma \right) \right]^4 \tag{2.196}
\]

Eq. (2.196) is obtained by using Eq. (A.31) for each term of (2.195), expanding the powers and simplifying.
2.5.3 Symbol Error Rate for Equal-Gain Combining

2.5.3.1 Unequal Branch SNR’s, Identical \( m \)-Parameters (2\( m \) Integer)

Let \( r = (r_1 + r_2)/\sqrt{2} \) be the normalized amplitude of the signal at the output of the equal-gain combiner, \( r_i \) being the envelope of the fading affecting the signal on the \( i \)th antenna. Then, the exact pdf of \( r \) is obtained as:

\[
p_R(r) = \sqrt{2} \int_0^{r\sqrt{2}} p_{R_1,R_2}(r_1,r_2 = r\sqrt{2} - r_1)dr_1 \tag{2.197}
\]

where \( p_{R_1,R_2}(r_1,r_2) \) is the joint pdf of \( r_1 \) and \( r_2 \). For Nakagami correlated r.v.'s with different powers and identical \( m \)-parameters:

\[
p_{R_1,R_2}(r_1,r_2) = \frac{4m^{m+1}(r_1r_2)^m}{\Gamma(m)(\Omega_1\Omega_2)^{m+1/2}} \frac{e^{-m(r_1^2 + r_2^2)/(\Omega_1\Omega_2)^{1/2}}}{(1 - \rho)\rho} I_{m-1} \left( \frac{2m\sqrt{\rho r_1r_2}}{\sqrt{\Omega_1\Omega_2}(1 - \rho)} \right) \tag{2.198}
\]

where \( m \geq 0.5 \) is the Nakagami fading parameter, \( \Omega_i = E[r_i^2] \) is the average fading power of each channel, \( \rho \) is the power correlation coefficient between \( r_1 \) and \( r_2 \). Hereafter we briefly summarize the main steps followed in solving (2.197).

We first use the series expansion ([80], Eq. 8.445)

\[
I_\nu(z) = \sum_{k=0}^{\infty} \frac{1}{k!\Gamma(\nu + k + 1)} \left( \frac{z}{2} \right)^{\nu + 2k} \tag{2.199}
\]

in Eq. (2.198). Upon substituting in Eq. (2.197), we complete the square in the exponential, make the change of variable \( y = \sqrt{m/(1 - \rho)}(r_1 \sqrt{(\Omega_1 + \Omega_2)/(\Omega_1\Omega_2)} - r \sqrt{2(\Omega_1/\Omega_2)/(\Omega_1 + \Omega_2)}) \), use twice the series expansion \( (y + a)^n = \sum_{i=0}^{n} \binom{n}{i} y^i a^{n-i} \) ([80], Eq. 1.111) (which necessitates \( 2m \) to be an integer), and solve for the integral using the incomplete gamma function \( \gamma(\alpha, x) = \int_0^x e^{-t} t^{\alpha-1} dt \). This results in the following novel expression for the pdf of the sum of two correlated Nakagami-\( m \) variables with arbitrary
powers and identical m-parameters (2m an integer):

\[ p_R(r) = \frac{2\sqrt{2}}{\Gamma(m) \bar{\Omega}_s^m} \left[ \frac{\Omega_p(1 - \rho)}{\Omega_s} \right]^{m - \frac{1}{2}} e^{-\frac{\Omega_p}{\bar{\Omega}_s}} \sum_{k=0}^{\infty} \frac{[\sqrt{\Omega_p\bar{\Omega}_s}]^{2k}}{k! \Gamma(m + k)} \times \sum_{i=0}^{2m-1+2k} \left( 2m - 1 + 2k \right) \left[ \frac{2m\bar{\Omega}_1}{1 - \rho} \right]^{m - \frac{1}{2} + k - \frac{1}{2} + \frac{2m-1+2k}{2} \left( 2m - 1 + 2k \right) \left[ 2m\bar{\Omega}_2 \right]^{m - \frac{1}{2} + k - \frac{1}{2} \left( 2m - 1 + 2k \right) \left[ 1 - \rho \right]^{m - \frac{1}{2} + k - \frac{1}{2}} \left( \frac{1}{2}, 2m\bar{\Omega}_2 \right) \right] 
\]

where we have defined \( \bar{\Omega}_s = \Omega_1 + \Omega_2, \bar{\Omega}_p = \Omega_1\Omega_2, \bar{\Omega}_1 = (\Omega_1/\Omega_2)/\bar{\Omega}_s, \bar{\Omega}_2 = (\Omega_2/\Omega_1)/\bar{\Omega}_s. \)

The pdf of the combined SNR is then:

\[ p_r(\gamma) = \frac{\sqrt{2}}{\Gamma(m) \bar{\Omega}_s^m} \left[ \frac{\Omega_p(1 - \rho)}{\Omega_s} \right]^{m - \frac{1}{2}} e^{-\frac{\Omega_p}{\bar{\Omega}_s}} \sum_{k=0}^{\infty} \frac{[\sqrt{\Omega_p\bar{\Omega}_s}]^{2k}}{k! \Gamma(m + k)} \times \sum_{i=0}^{2m-1+2k} \left( 2m - 1 + 2k \right) \left[ \frac{2m\bar{\Omega}_1}{1 - \rho} \right]^{m - \frac{1}{2} + k - \frac{1}{2} + \frac{2m-1+2k}{2} \left( 2m - 1 + 2k \right) \left[ 2m\bar{\Omega}_2 \right]^{m - \frac{1}{2} + k - \frac{1}{2} \left( 2m - 1 + 2k \right) \left[ 1 - \rho \right]^{m - \frac{1}{2} + k - \frac{1}{2}} \left( \frac{1}{2}, 2m\bar{\Omega}_2 \right) \right] 
\]

Substituting Eqs. (2.187) and (2.201) in Eq. (2.185) and using Eq. 6.455.2 of [80], the following expression is obtained:

\[ P_s = \frac{2\sqrt{2}}{\Gamma(m) \bar{\Omega}_s^m} \left[ \frac{\gamma_p(1 - \rho)}{\bar{\gamma}_s} \right]^{m - \frac{1}{2}} \sum_{k=0}^{\infty} \frac{\Gamma(2(m + k))}{k! \Gamma(m + k)} \left[ \frac{\sqrt{\gamma_p\bar{\gamma}_s}}{\bar{\gamma}_s} \right]^{2k} \times \sum_{i=0}^{2m-1+2k} \left( 2m - 1 + 2k \right) \left[ \frac{2m\bar{\gamma}_1}{1 - \rho} \right]^{m - \frac{1}{2} + k - \frac{1}{2} + \frac{2m-1+2k}{2} \left( 2m - 1 + 2k \right) \left[ 2m\bar{\gamma}_2 \right]^{m - \frac{1}{2} + k - \frac{1}{2} \left( 2m - 1 + 2k \right) \left[ 1 - \rho \right]^{m - \frac{1}{2} + k - \frac{1}{2}} \left( \frac{1}{2}, 2m\bar{\gamma}_2 \right) \right] 
\]

where we have defined \( \gamma_i = 2m\bar{\gamma}_i/(1 - \rho), i = 1,2, \beta(\theta) = \phi_i(\theta) + 2m/(\bar{\gamma}_s(1 - \rho)), \gamma_i = \bar{\gamma}_i + \bar{\gamma}_2, \bar{\gamma}_p = \bar{\gamma}_1\bar{\gamma}_2, \bar{\gamma}_1 = (\bar{\gamma}_1/\bar{\gamma}_2)/\bar{\gamma}_s, \bar{\gamma}_2 = (\bar{\gamma}_2/\bar{\gamma}_1)/\bar{\gamma}_s, \gamma_i = \Omega_i E_s/N_0, i = 1,2. \)
2.5.3.2 Identical Branch SNR's and m-Parameters

For Nakagami correlated random variables with identical parameters \((\Omega_1 = \Omega_2 = \Omega)\) [23]:

\[
p_{R_1, R_2}(r_1, r_2) = \frac{4(r_1 r_2)^m (m/\Omega)^{m+1}}{\Gamma(m)(1-\rho)^{m+1}} \exp \left( -\frac{m(r_1^2 + r_2^2)}{\Omega(1-\rho)} \right) I_{m-1} \left( \frac{2m\sqrt{\rho}r_1 r_2}{\Omega(1-\rho)} \right). \tag{2.203}
\]

Substituting Eq. (2.203) into Eq. (2.197) results in

\[
p_R(r) = \frac{\sqrt{24}(m/\Omega)^{m+1}}{\Gamma(m)(1-\rho)^{m+1}} \int_0^{\sqrt{2}} \left[ r_1(r\sqrt{2} - r_1) \right]^m \exp \left( -\frac{m[r_1^2 + (r\sqrt{2} - r_1)^2]}{\Omega(1-\rho)} \right)
\times I_{m-1} \left( \frac{2m\sqrt{\rho}r_1(r\sqrt{2} - r_1)}{\Omega(1-\rho)} \right) dr_1. \tag{2.204}
\]

In order to solve the integral in Eq. (2.204), we use the infinite series expansion Eq. (2.199) for \(I_{m-1}(\cdot)\), make the variable transformation \(y = m/(2\Omega(1-\rho))(\sqrt{2}r_1 - r)^2\), and use Eq. 3.383.1 of [80] to solve the integral, leading to the following expression for the exact pdf of the normalized sum of two correlated Nakagami variables with identical parameters:

\[
p_R(r) = \frac{2}{\Gamma(m)} \exp \left( -\frac{2mr_1^2}{\Omega(1-\rho)} \right) \sum_{k=0}^{\infty} \frac{\rho^k (1-\rho)^m B(2(m+k),\frac{1}{2}) (m/\Omega(1-\rho))^{2(m+k)}}{k! \Gamma(m+k) 2^{2(m+k-1)}} \times r^{4(m+k)-1} \Phi \left( 2(m+k), 2(m+k) + \frac{1}{2}, \frac{mr_1^2}{\Omega(1-\rho)} \right) \tag{2.205}
\]

Note that an expression for the pdf of the sum of two correlated chi variables was given in [118] (recalling that the chi pdf is equivalent to the Nakagami pdf when the m-parameter is an integer), however it contained a one-dimensional integral. The cdf is given by

\[
F_R(r) = \int_0^r p_R(\alpha) d\alpha,
\]

where \(p_R(r)\) is expressed in Eq. (2.205). By making the change of variable \(x = m/(\Omega(1-\rho))\alpha^2\), using the infinite series expansion

\[
\Phi(a, c, x) = \frac{\Gamma(c)}{\Gamma(a)} \sum_{k=0}^{\infty} \frac{\Gamma(a+k)x^k}{\Gamma(c+k)k!}, \tag{2.206}
\]

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and using Eqs. 8.384.1 and 8.350.1 of [80] to solve the integral, the following exact expression is obtained:

\[
F_R(r) = \frac{\sqrt{\pi}}{\Gamma(m)} \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \frac{\rho^k(1-\rho)^n}{k!n!2^{(m+k)+n-2} \Gamma(m+k)\Gamma(2(m+k)+n+\frac{1}{2})} \times \gamma \left(2(m+k)+n, \frac{2mr^2}{\Omega(1-\rho)}\right),
\]

The truncation of the infinite series in Eqs. (2.205) and (2.207) to 50 terms gave an excellent accuracy in all the cases we tried. In the case of uncorrelated fading, i.e. \( \rho = 0 \), it can be easily verified that Eqs. (2.205) and (2.207) reduce to Eqs. (38) and (40), respectively, of [108].

The pdf of \( r' = \sqrt{2}r = r_1 + r_2 \), obtained through Eq. (2.205), is plotted in Fig. 2.27 against the approximation of [172], for the two correlated cases presented in [172], i.e. \( m = 1.8, \rho = 0.7, \) and \( m = 3, \rho = 0.3 \). In [172], the pdf of the (un-normalized) sum of two correlated Nakagami variables with identical parameters \( \{m, \Omega\} \) is approximated by the Nakagami pdf with equivalent parameters:

\[
\Omega_e = 2\Omega + 2 \left( \frac{m}{\Omega} \right) \frac{\Gamma^2(m+\frac{1}{2})}{\Gamma^2(m)}
\]

\[
m_e = \frac{m}{\Omega_e} \left[ \frac{\Omega_e^2}{m} + 4\Omega_e^2 + 6\Omega_e^2 \rho - 4 \left( \frac{\Omega_e}{m} \right)^2 \frac{\Gamma^4(m+\frac{1}{2})}{\Gamma^4(m)} \left( \text{\( _2F_1 \)} \left( \frac{1}{2}, -\frac{1}{2}; m; \rho \right) \right)^2 \\
+ 8 \left( \frac{\Omega_e}{m} \right)^2 \frac{\Gamma(m+\frac{3}{2})\Gamma(m+\frac{3}{2})}{\Gamma^2(m)} \text{\( _2F_1 \)} \left( -\frac{1}{2}, -\frac{3}{2}; m; \rho \right) \\
- 8 \frac{\Omega_e^2 \Gamma^2(m+\frac{1}{2})}{m \Gamma^2(m)} \text{\( _2F_1 \)} \left( -\frac{1}{2}, -\frac{1}{2}; m; \rho \right) \right]^{-1}
\]

While overall the exact and approximate curves appear to be close, the discrepancy for low values or \( r' \) can have a significant impact on the evaluation of the SER, since the latter depends strongly on the lower tail of the pdf. The same can be observed from Fig. 2.28, which plots Eq. (2.207) and the cdf of a Nakagami variable [23] with the parameters specified in [172].
Figure 2.27 Probability density function of the sum of two correlated Nakagami-$m$ variables: —— exact (through Eq. (2.205)); - - - - approximation (using Eqs. (2.208)-(2.209)).

Figure 2.28 Cumulative distribution function of the sum of two correlated Nakagami-$m$ variables: —— exact (Eq. (2.207)); - - - - approximation (using Eqs. (2.208)-(2.209)).
From Eq. (2.205) the pdf of the combined SNR is obtained as:

\[ p_\gamma(\gamma) = \frac{1}{\Gamma(m)} \exp \left( -\frac{2m\gamma}{\Omega(1-\rho)} \right) \sum_{k=0}^{\infty} \frac{\rho^k(1-\rho)^m B(2(m+k),\frac{1}{2})}{k!\Gamma(m+k)2^{2(m+k-1)}} \left( \frac{m}{\Omega(1-\rho)} \right)^{2(m+k)} \times \gamma^{2(m+k)-1} \Phi \left( 2(m+k), 2(m+k) + \frac{1}{2}, \frac{m\gamma}{\Omega(1-\rho)} \right). \] (2.210)

To obtain the SER, one can substitute Eqs. (2.210) and (2.187) in Eq. (2.185), leading to the following expression valid for an arbitrary real \( m > 0.5 \):

\[ P_s = \frac{1}{\Gamma(m)} \sum_{k=0}^{\infty} \frac{\rho^k(1-\rho)^m B(2(m+k),\frac{1}{2})\Gamma(2(m+k))}{k!\Gamma(m+k)2^{2(m+k-1)}} \eta^{2(m+k)} \times \sum_{l=1}^{K} \int_{\vartheta_l}^{\vartheta_{l+1}} \frac{a_l(\theta)}{|\beta(\theta)|^{2(m+k)}} d\theta \] (2.211)

where we have defined \( \eta = m/(\gamma(1-\rho)) \), and now \( \beta(\theta) = \phi_l(\theta) + 2m/(\gamma(1-\rho)) \).

Using the above derived pdf (Eq. (2.210)) we can also obtain several expressions specific to each modulation scheme, some of them which can be put in closed-form.

**M-PSK**

Substituting Eqs. (2.188) and (2.205) in Eq. (2.185), and using Eq. 7.621.4 of [80], the following expression is obtained:

\[ P_s = \frac{1}{\pi \Gamma(m)} \sum_{k=0}^{\infty} \frac{\rho^k(1-\rho)^m B(2(m+k),\frac{1}{2})\Gamma(2(m+k))}{k!\Gamma(m+k)2^{2(m+k-1)}} \left( \frac{m}{\gamma(1-\rho)} \right)^{2(m+k)} \times \int_{0}^{\pi} \frac{1}{|\mathbb{N}(\theta)|^{2(m+k)}2F_1 \left( 2(m+k), 2(m+k), 2(m+k) + \frac{1}{2}, \frac{m}{\gamma(1-\rho)\mathbb{N}(\theta)} \right)} d\theta \] (2.212)

where \( \mathbb{N}(\theta) = \frac{\sin^2(\pi/M)}{\sin^2(\theta)} + \frac{2m}{\gamma(1-\rho)} \). Eq. (2.212) gives the exact SER of M-PSK with dual-branch EGC in correlated Nakagami fading. In the case of uncorrelated fading \( (\rho = 0) \), Eq. (2.212) reduces to

\[ P_s = \frac{1}{\pi} \left( \frac{m}{\gamma} \right)^{2m} B(2m,\frac{1}{2}) \Gamma(2m) \int_{0}^{\pi} \frac{1}{|\mathbb{N}(\theta)|^{2m}2F_1 \left( 2m, 2m, 2m + \frac{1}{2}, \frac{m}{\gamma\mathbb{N}(\theta)} \right)} d\theta \] (2.213)
where $N'(\theta) = \frac{\sin^2(\pi/M)}{\sin^2\theta} + \frac{2m}{\bar{\gamma}}$.

**Special case: BPSK ($M = 2$)**

Substituting Eq. (2.190) in Eq. (2.185) gives

$$P_s = \frac{1}{2} - \int_0^\infty \sqrt{\frac{\gamma}{\pi}} e^{-\gamma} \Phi\left(1, \frac{3}{2}, \gamma\right) p_\theta(\gamma) d\gamma. \quad (2.214)$$

The second term of Eq. (2.214) is

$$\int_0^\infty \gamma^{2(m+k)-\frac{1}{2}} e^{-(1+\frac{m}{\bar{\gamma}(1-\rho)})\gamma} \Phi\left(1, \frac{3}{2}, \gamma\right) \Phi\left(2(m+k), 2(m+k) + \frac{1}{2}, \frac{m}{\bar{\gamma}(1-\rho)}\right) \quad (2.215)$$

The integral in Eq. (2.215) can be solved using Eq. (A.38) (c.f. Eq. 7.622.3 of [80] and [162], Eq. (A-12)). Further straightforward simplifications then lead to:

$$P_s = \frac{1}{2} - \frac{2}{\Gamma(m)} \sqrt{\frac{1}{\pi}} \sum_{k=0}^{\infty} \frac{\rho^k(1-\rho)^m B(2(m+k), \frac{1}{2})}{k! \Gamma(m+k) 2^{2(m+k)-2}} \left[ \frac{m}{\bar{\gamma}(1-\rho)} \right]^{2(m+k)} \times \int_0^\infty \gamma^{2(m+k)-\frac{1}{2}} e^{-(1+\frac{m}{\bar{\gamma}(1-\rho)})\gamma} \Phi\left(1, \frac{3}{2}, \gamma\right) \Phi\left(2(m+k), 2(m+k) + \frac{1}{2}, \frac{1}{1+2\eta}, \frac{\eta}{1+2\eta}\right) \quad (2.216)$$

where we have defined $\eta = m/(\bar{\gamma}(1-\rho))$, and recall that $F_2(a; b_1, b_2; c_1, c_2; x, y)$ is the Appell hypergeometric function of two arguments. In the case of uncorrelated fading, Eq. (2.216) reduces to

$$P_s = \frac{1}{2} - 2 \sqrt{\frac{1}{\pi}} \frac{\eta^{2m}}{(1+2\eta)^{2m+\frac{1}{2}}} \frac{\Gamma(m+\frac{1}{2})}{\Gamma(m)} \times F_2\left(2m+\frac{1}{2}, 1, 2m; \frac{3}{2}, 2m+\frac{1}{2}; \frac{1}{1+2\eta}, \frac{\eta}{1+2\eta}\right) \quad (2.217)$$

where now $\eta = m/\bar{\gamma}$. Note that the SER for BPSK with coherent EGC in uncorrelated and correlated Nakagami-$m$ fading was also previously obtained in [168] and [170], respectively, for different parameters on each branch, using a different method based on the characteristic function. The results were also expressed in terms of the Appell hy-
pergeometric function.

Special case: QPSK \((M = 4)\)

Substituting Eq. (2.191) in Eq. (2.185) gives

\[
P_s = \int_0^\infty \text{erfc} \left( \sqrt{\gamma/2} \right) p_T(\gamma) d\gamma - \frac{1}{4} \int_0^\infty \text{erfc}^2 \left( \sqrt{\gamma/2} \right) p_T(\gamma) d\gamma. \tag{2.218}
\]

The first term in Eq. (2.218) is seen to be equal to \(A = P_{s,BPSK}(\gamma/2)\), where \(P_{s,BPSK}(\gamma)\) is given by Eq. (2.216). The second term in Eq. (2.218) can be expressed as:

\[
B = \frac{1}{4} \int_0^\infty \left[ 1 - 2\sqrt{\frac{\gamma}{2\pi}} e^{-\frac{\gamma}{2}} \Phi \left( 1, \frac{3}{2}, \frac{\gamma}{2} \right) \right]^2 p_T(\gamma) d\gamma
\]

\[
= \frac{1}{4} \int_0^\infty p_T(\gamma) d\gamma - \sqrt{\frac{1}{2\pi}} \int_0^\infty \gamma^{1/2} e^{-\gamma/2} \Phi \left( 1, \frac{3}{2}, \frac{\gamma}{2} \right) p_T(\gamma) d\gamma
\]

\[
+ \frac{1}{2\pi} \int_0^\infty \gamma e^{-\gamma} \left[ \Phi \left( 1, \frac{3}{2}, \frac{\gamma}{2} \right) \right]^2 p_T(\gamma) d\gamma. \tag{2.219}
\]

The first term of Eq. (2.219) readily evaluates to \(1/4\). The integrals of the second and third terms of Eq. (2.219) are solved using Eq. (A.38). Simplifications lead to:

\[
P_s = \frac{3}{4} - \sqrt{\frac{2}{\pi}} \frac{1}{\Gamma(m)} \sum_{k=0}^\infty \frac{\rho^k (1 - \rho)^m}{k!} \eta^{2(m+k)} \Gamma \left( m + k + \frac{1}{2} \right) \times \left[ \frac{1}{\left[ \frac{1}{2} + 2\eta \right]^{2(m+k)+\frac{1}{2}}} \right]
\]

\[
\times F_2 \left( 2(m + k) + \frac{1}{2}, 1, 2(m + k); \frac{3}{2}, 2(m + k) + \frac{1}{2}, \frac{1}{2} + 2\eta, \frac{1}{2} + 2\eta \right) +
\]

\[
\frac{\sqrt{\frac{2}{\pi}}}{\left[ 1 + 2\eta \right]^{2(m+k)+1}} \Gamma(2(m + k) + 1) \Gamma(2(m + k) + \frac{1}{2}) F_3 \left( 2(m + k) + 1; 1, 1, 2(m + k) \right.
\]

\[
\left. \frac{3}{2}, \frac{3}{2}, 2(m + k) + \frac{1}{2}, \frac{1}{2} + 2\eta, \frac{1}{2} + 2\eta, \frac{1}{2} + 2\eta \right) \tag{2.220}
\]

where \(F_3(a; b_1, b_2, b_3; c_1, c_2, c_3; x, y, z)\) is the Appell hypergeometric function of three arguments ([80], Eq. 9.19). The SER for the uncorrelated case can be obtained by setting \(\rho = 0\) in Eq. (2.220).
DE-QPSK (M = 4)

Substituting Eqs. (2.195) and (2.205) in Eq. (2.185), the following expression is obtained:

\[
P_s = \frac{3}{4} - \frac{1}{\pi \Gamma(m)} \sum_{k=0}^{\infty} \frac{\rho^k (1 - \rho)^m B(2(m + k), \frac{1}{2}) \eta^{2(m+k)}}{k! \Gamma(m + k) 2^{2(m+k-1)}} \times \left[ \frac{\Gamma(2(m + k) + 1)}{(1 + 2\eta)^{2(m+k)+1}} \right]
\]

\[
F_3 \left( 2(m + k) + 1; 1, 1, 2(m + k); \frac{3}{2}, \frac{3}{2}, 2(m + k) + 1; \frac{1}{2}, \frac{1}{2}, \eta \right)
\]

\[
\left( 2(m + k) + 1; 1, 1, 2(m + k); \frac{3}{2}, \frac{3}{2}, 2(m + k) + 1; \frac{1}{2}, \frac{1}{2}, \eta \right)
\]

\[
\frac{1}{1 + \eta}, \frac{1}{1 + \eta}, \frac{1}{1 + \eta}, \frac{1}{1 + \eta}
\]

where \( F_n(a; b_1, b_2, \ldots, b_n; c_1, c_2, \ldots, c_n; x_1, x_2, \ldots, x_n) \) is the Appell hypergeometric function of \( n \) arguments ([80], Eq. (9.19)). The SER's for DE-M-PSK with \( M > 4 \) can be similarly obtained, but the resulting expressions are increasingly complicated due to the larger number of terms.

M-QAM

The unconditional SER can be obtained following the same development as for the QPSK case, leading to:

\[
P_s = q(2 - q) - 8q \sqrt{\frac{p}{\pi \Gamma(m)}} \sum_{k=0}^{\infty} \frac{\rho^k (1 - \rho)^m \eta^{2(m+k)}}{k!} \Gamma \left( m + k + \frac{1}{2} \right) \times
\]

\[
\left[ \frac{1 - q}{[p + 2\eta]^{2(m+k)+\frac{1}{2}}} F_2 \left( 2(m + k) + \frac{1}{2}; 1, 2(m + k); \frac{3}{2}, 2(m + k) + 1; \frac{p}{p + 2\eta}, \eta \right) \right.
\]

\[
+ \frac{q \sqrt{\frac{p}{\pi}}}{[2p + 2\eta]^{2(m+k)+1}} \frac{\Gamma(2(m + k) + 1)}{\Gamma(2(m + k) + \frac{1}{2})} F_3 \left( 2(m + k) + 1; 1, 1, 2(m + k); \frac{3}{2}, \frac{3}{2}, 2(m + k) + 1; \frac{p}{p + 2\eta}, \frac{p}{p + 2\eta}, \eta \right)
\]

\[
\left. \frac{p}{2p + 2\eta}, \frac{p}{2p + 2\eta}, \frac{p}{2p + 2\eta} \right] .
\]

(2.222)
For $M = 4$, Eq. (2.222) reduces to Eq. (2.220). In the case of uncorrected fading, i.e. $ho = 0$, Eq. (2.222) reduces to

$$P_s = q(2 - q) - 8q \sqrt{\frac{p}{\pi}}^{2m} \frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)} \times \left[ \frac{1 - q}{(p + 2\eta)^{2m + \frac{1}{2}}} \right] F_1 \left(2m + 1, 1, 2m; \frac{3}{2}, 2m + 1; \frac{p}{2p + 2\eta}, \frac{\eta}{p + 2\eta}\right) + \frac{q \sqrt{\frac{p}{\pi}}}{\Gamma(2m + 1)} \times \frac{\Gamma(2m + 1)}{\Gamma(2m + \frac{1}{2})} F_1 \left(2m + 1, 1, 2m; \frac{3}{2}, 2m + 1; 2 \frac{p}{2p + 2\eta}, \frac{p}{2p + 2\eta}\right).$$

(2.223)

### 2.5.3.3 Uncorrelated Fading, Unequal Branch SNR’s and $m$-Parameters ($2m_i$ Integer)

For Nakagami uncorrelated ($\rho = 0$) random variables with arbitrary parameters:

$$p_R_1, R_2(r_1, r_2) = p_R_1(r_1)p_R_2(r_2) = \frac{4 \left( \frac{m_1}{m_1} \right)^{m_1} \left( \frac{m_2}{m_2} \right)^{m_2}}{\Gamma(m_1)\Gamma(m_2)} e^{- \frac{m_1 r_1^2 + m_2 r_2^2}{m}}. (2.224)$$

Substituting Eq. (2.224) in Eq. (2.197) and following a development parallel to that presented in Section 2.5.3.1, the following pdf is obtained for the sum of two uncorrelated Nakagami random variables with arbitrary parameters, but $2m_i$ integer:

$$p_R(r) = \frac{4 \left( \frac{m_1}{m_1} \right)^{m_1} \left( \frac{m_2}{m_2} \right)^{m_2}}{\Gamma(m_1)\Gamma(m_2)} \left[ \frac{\Omega_p}{2m} \right]^{m_1 + m_2 - \frac{1}{2}} e^{- \frac{2m_1 m_2 r^2}{m}} \times \sum_{i=0}^{2m_1 - 1} \binom{2m_1 - 1}{i} \left( \frac{4 \frac{m_1 \Omega_1}{r^2}}{m} \right)^{m_1 - \frac{1}{2} - \frac{2m_2 - 1}{2}} \sum_{j=0}^{2m_2 - 1} \binom{2m_2 - 1}{j} \left( \frac{4 \frac{m_2 \Omega_2}{r^2}}{m} \right)^{m_2 - \frac{1}{2} - \frac{j}{2}} 2^{i+j} \times \left[ (-1)^i \gamma \left( \frac{i + j + 1}{2}, 2 \frac{m_2 \Omega_1}{m} \frac{r^2}{r^2} \right) + (-1)^j \gamma \left( \frac{i + j + 1}{2}, 2 \frac{m_1 \Omega_2}{m} \frac{r^2}{r^2} \right) \right].$$

(2.225)
where $\Omega_p = \Omega_1 \Omega_2$, $\bar{m} = m_1 \Omega_2 + m_2 \Omega_1$. The cdf can also be obtained as:

$$F_R(r) = \frac{2 \left( \frac{m_1}{\bar{m}} \right)^{m_1} \left( \frac{m_2}{\bar{m}} \right)^{m_2} \Omega_p^{m_1 + m_2 - \frac{1}{2}}}{\Gamma(m_1) \Gamma(m_2) \left( \frac{2m_1}{\bar{m}} \right)^{m_1 + m_2 - \frac{1}{2}}} \times \sum_{i=0}^{2m_1-1} \left( \begin{array}{c} 2m_1 - 1 \\ i \end{array} \right) \left[ \frac{4m_2 \Omega_2}{\bar{m}} \right]^{m_1 - \frac{1}{2} - \frac{1}{2} 2m_2 - 1} \sum_{j=0}^{2m_2 - 1} \left( \begin{array}{c} 2m_2 - 1 \\ j \end{array} \right) \left[ \frac{4m_1 \Omega_1}{\bar{m}} \right]^{m_2 - \frac{1}{2} - \frac{1}{2}} \times 2^{i+j} \sum_{k=0}^{\infty} \frac{(-1)^k \gamma \left( m_1 + m_2 + \frac{k}{2}, \frac{2m_1 m_2 r^2}{\bar{m}} \right)}{k! \bar{m}^{m_1 + m_2 + \frac{k}{2} \left( i+j+1 \right)} + k} \times \left[ (-1)^i \left( \frac{2m_2 \Omega_2}{\bar{m}} \right)^{i+j+1+k} + (-1)^j \left( \frac{2m_1 \Omega_1}{\bar{m}} \right)^{i+j+1+k} \right].$$

The pdf of the combined SNR is then:

$$p_R(\gamma) = \frac{2 \left( \frac{m_1}{\bar{m}} \right)^{m_1} \left( \frac{m_2}{\bar{m}} \right)^{m_2} \Omega_p^{m_1 + m_2 - \frac{1}{2}}}{\Gamma(m_1) \Gamma(m_2) \left( \frac{2m_1}{\bar{m}} \right)} \times \sum_{i=0}^{2m_1-1} \left( \begin{array}{c} 2m_1 - 1 \\ i \end{array} \right) \left[ \frac{4m_2 \gamma_1 / \bar{m} \gamma_2}{m} \right]^{m_1 - \frac{1}{2} - \frac{1}{2} 2m_2 - 1} \sum_{j=0}^{2m_2 - 1} \left( \begin{array}{c} 2m_2 - 1 \\ j \end{array} \right) \left[ \frac{4m_1 \gamma_1 / \bar{m} \gamma_2}{m} \right]^{m_2 - \frac{1}{2} - \frac{1}{2}} \times 2^{i+j} \gamma^{\frac{1}{2}} \left[ (-1)^i \gamma \left( \frac{i+j+1}{2}, \frac{2m_2 \gamma_1 / \bar{m} \gamma_2}{m} \right) + (-1)^j \gamma \left( \frac{i+j+1}{2}, \frac{2m_1 \gamma_2 / \bar{m} \gamma_1}{m} \right) \right].$$

where $\bar{m} = m_1 \gamma_2 + m_2 \gamma_1$.

Substituting Eqs. (2.187) and (2.227) in Eq. (2.185) and using Eq. 6.455.2 of [80], the following expression is obtained:

$$P_s = \frac{4 \Gamma(m_1 + m_2) \left( \frac{m_1}{\bar{m}} \right)^{m_1} \left( \frac{m_2}{\bar{m}} \right)^{m_2} \gamma_p \gamma_1^{m_1 + m_2 - \frac{1}{2}}}{\Gamma(m_1) \Gamma(m_2) \left( \gamma_1 \right)^{m_1 - \frac{1}{2} - \frac{1}{2} 2m_2 - 1} \sum_{j=0}^{2m_2 - 1} \left( \begin{array}{c} 2m_2 - 1 \\ j \end{array} \right) \left[ \frac{2 \alpha_1 \gamma_1 / \bar{m} \gamma_2}{m} \right]^{m_1 - \frac{1}{2} - \frac{1}{2} 2m_2 - 1} \times K \int_{0}^{\Theta_1} a_1(\theta) \left[ \left( \frac{1 + j + 1}{2} \right) \frac{\alpha_1}{\alpha_1 + \beta(\theta)} m_1 + m_2 F_1 \left( 1, m_1 + m_2; \frac{i + j + 3}{2}; \frac{\alpha_1}{\alpha_1 + \beta(\theta)} \right) \right. \right. \left. \left. \right. \left( \frac{1 + j + 1}{2} \right) \frac{\alpha_2}{\alpha_2 + \beta(\theta)} m_1 + m_2 F_1 \left( 1, m_1 + m_2; \frac{i + j + 3}{2}; \frac{\alpha_2}{\alpha_2 + \beta(\theta)} \right) \right] d\theta.$$
where we have now defined $\alpha_1 = \frac{2m_2^2 \gamma_1 / \gamma_2}{m}$, $\alpha_2 = \frac{2m_2^2 \gamma_2 / \gamma_1}{m}$, $\beta(\theta) = \phi_1(\theta) + \frac{2m_1 m_2}{m}$.

### 2.5.4 Symbol Error Rate for Maximal-Ratio Combining

Let $r = \sqrt{r_1^2 + r_2^2}$ be the normalized amplitude of the signal at the output of the maximal-ratio combiner. Then for identical $m$-parameters on each branch it is known that ([23], Eq. (142)):

$$p_R(r) = \frac{2r\sqrt{\pi}}{\Gamma(m)} \left[ \frac{m^2}{\Omega_1 \Omega_2 (1 - \rho)} \right]^m e^{-\alpha r^2} \left( \frac{r^2}{2\beta} \right)^{m-\frac{1}{2}} I_{m-\frac{1}{2}} (\beta r^2) \quad (2.229)$$

where $\alpha = (m(\Omega_1 + \Omega_2))/(2\Omega_1 \Omega_2 (1 - \rho))$ and $\beta^2 = m^2(\Omega_1 - \Omega_2)^2 + 4\Omega_1 \Omega_2 \rho)/(4(\Omega_1 \Omega_2)^2 (1 - \rho)^2)$. Using Eq. (A.33), Eq. (2.229) can be alternatively written as:

$$p_R(r) = \frac{2}{\Gamma(2m)} \left[ \frac{m^2}{\Omega_1 \Omega_2 (1 - \rho)} \right]^m r^{4m-1} e^{-(\alpha + \beta) r^2} \Phi(m, 2m, 2\beta r^2) \quad (2.230)$$

or, in terms of SNR:

$$p_T(\gamma) = \frac{1}{\Gamma(2m)} \left[ \frac{m^2}{\gamma_1 \gamma_2 (1 - \rho)} \right]^m \gamma^{2m-1} e^{-(\alpha + \beta) \gamma} \Phi(m, 2m, 2\beta \gamma) \quad (2.231)$$

This form will facilitate the error probability derivations, which are carried out in the same way as for EGC.

Substituting Eqs. (2.187) and (2.231) in (2.185), and using Eq. (7.621.4) of [80], the following general expression is obtained:

$$P_s = \left( \frac{m^2}{\gamma_1 \gamma_2 (1 - \rho)} \right)^m \sum_{i=1}^{K} \int_{0}^{\Theta_i} a_i(\theta) [\mathcal{N}(\theta)]^{-2m} _2F_1 \left( 2m, m, 2m, \frac{2\beta}{\mathcal{N}(\theta)} \right) d\theta \quad (2.232)$$

where $\mathcal{N}(\theta) = \phi_i(\theta) + \alpha + \beta$. Solutions specific to different modulation schemes are derived separately below.
**M-PSK**

Using the parameters of Table 2.3, Eq. 2.232 reduces to:

\[
P_s^* = \frac{1}{\pi} \left( \frac{m^2}{\gamma_1 \gamma_2 (1 - \rho)} \right)^m \int_0^{\pi - \frac{\pi}{M}} [\mathcal{R}(\theta)]^{-2m} e_1 \left( 2m, m, 2m, \frac{2\beta}{\mathcal{N}(\theta)} \right) d\theta \tag{2.233}
\]

where now \( \mathcal{R}(\theta) = \frac{\sin^2(\pi/\theta)}{\sin^2 \theta} + \alpha + \beta \).

**Special case: BPSK \((M = 2)\)**

Substituting Eqs. (2.190) and (2.231) in Eq. (2.185), the following expression is obtained (c.f. development for the EGC case):

\[
P_s = \frac{1}{2} - \sqrt{\frac{1}{\pi} \frac{\Gamma(2m + \frac{1}{2}) \left( \frac{m^2}{\gamma_1 \gamma_2 (1 - \rho)} \right)^m}{\Gamma(2m) [1 + \alpha + \beta]^{2m + \frac{1}{2}}} \times F_2 \left( \begin{array}{c} 2m + \frac{1}{2}; 1, m; \frac{3}{2}, 2m; \frac{1}{1 + \alpha + \beta}; \frac{2\beta}{1 + \alpha + \beta} \end{array} \right). \tag{2.234}
\]

**DE-QPSK \((M = 4)\)**

Substituting Eqs. (2.196) and (2.231) in Eq. (2.185), the following expression is obtained (c.f. development for the EGC case):

\[
P_s = \frac{3}{4} - \frac{1}{\pi} \frac{1}{\Gamma(2m)} \left( \frac{m^2}{\gamma_1 \gamma_2 (1 - \rho)} \right)^m \times F_3 \left( \begin{array}{c} 2m + 1; 1, 1, m; \frac{3}{2}, 2m; \frac{1}{2}, \frac{1}{1 + \alpha + \beta}; \frac{1}{1 + \alpha + \beta}; \frac{2\beta}{1 + \alpha + \beta} \end{array} \right) + \frac{\Gamma(2m + 2)}{\pi (2 + \alpha + \beta)^{2m + 2}} F_5 \left( \begin{array}{c} 2m + 2; 1, 1, 1, 1, m; \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, 2m; \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{2\beta}{2 + \alpha + \beta}; \frac{2\beta}{2 + \alpha + \beta}; \frac{2\beta}{2 + \alpha + \beta}; \frac{2\beta}{2 + \alpha + \beta} \end{array} \right). \tag{2.235}
\]

**M-QAM**

The unconditional SER can be obtained following the same development as for the EGC

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case, leading to:

\[ P_s = 2q - q^2 - (1 - q)^4q \sqrt{\frac{p}{\pi}} \frac{\Gamma(2m + \frac{1}{2})}{\Gamma(2m)} \left( \frac{m^2}{\gamma_n \gamma_2 (1 - \rho)} \right)^m \times \]

\[ F_2 \left( 2m + \frac{1}{2}; 1, m; \frac{3}{2}; 2m; \frac{p}{p + \alpha + \beta}, \frac{2\beta}{p + \alpha + \beta} \right) - q^2 \frac{p}{\pi} \frac{\Gamma(2m + 1)}{\Gamma(2m)} \left( \frac{m^2}{\gamma_n \gamma_2 (1 - \rho)} \right)^m \]

\[ F_3 \left( 2m + 1; 1, 1, m; \frac{3}{2}, \frac{3}{2}, 2m; \frac{p}{2p + \alpha + \beta}, \frac{p}{2p + \alpha + \beta}, \frac{2\beta}{2p + \alpha + \beta} \right). \] (2.236)

### 2.5.5 Performance Evaluation Results

Figs. 2.29 and 2.30 show the SER of coherent 2-, 4- and 8-PSK with dual-branch EGC and MRC, respectively, for \( m = 2 \) and different correlation coefficients. Figs. 2.31-2.33 plot the SER for 2-, 4- and 8-PSK, respectively, with EGC and MRC, for \( \rho = 0.5 \) and different Nakagami-\( m \) fading parameters. Simulation results, obtained using the method of [128] (c.f. Section 2.4.1.2), are seen to match very well the theoretical curves.

Fig. 2.34 shows the SER of coherent 4-PSK and 8-PSK with dual-branch EGC, for \( m = 2, \rho = 0.5 \), and different sets of powers \( \{\Omega_1, \Omega_2\} \). As expected, the SER performance degrades with a decrease in the total power captured by the diversity combiner. Fig. 2.35 illustrates the SER of coherent 8-PSK and 16-QAM with dual-branch EGC, for \( m = 2, \{\Omega_1 = 1.0, \Omega_2 = 0.5\} \), and different \( \rho \)'s. Simulation results are included and are seen to validate very well the theoretical curves, which are obtained by truncating the infinite sum in (2.228) to only its first 11 terms.

Fig. 2.36 shows the SER for 2-PSK and 8-PSK with \( m_1 = 2.0, m_2 = 1.0 \), and different sets of branch powers \( \{\Omega_1, \Omega_2\} \). Similar results are shown in Fig. 2.37 in the cases of 4-QAM and 16-QAM, and in Fig. 2.38 in the cases of DE-2-PSK and DE-4-PSK.
Figure 2.29 SER of M-PSK with dual-branch EGC and $m = 2$, for different $\rho$'s.

Figure 2.30 SER of M-PSK with dual-branch MRC and $m = 2$, for different $\rho$'s.
Figure 2.31 SER of 2-PSK with dual-branch EGC and MRC, for $\rho = 0.5$ and different $m$'s.

Figure 2.32 SER of 4-PSK with dual-branch EGC and MRC, for $\rho = 0.5$ and different $m$'s.
Figure 2.33 SER of 8-PSK with dual-branch EGC and MRC, for $\rho = 0.5$ and different $m$'s.

Figure 2.34 SER of 4-PSK and 8-PSK with dual-branch EGC and $m = 2$, $\rho = 0.5$, for different $\{\Omega_1, \Omega_2\}$. 
Figure 2.35 SER of 8-PSK and 16-QAM with dual-branch EGC and $m = 2$, $\{\Omega_1 = 1.0, \Omega_2 = 0.5\}$, for different $\rho$'s.

Figure 2.36 SER of 2-PSK and 8-PSK with dual-branch EGC and $m_1 = 2.0, m_2 = 1.0$, for different $\{\Omega_1, \Omega_2\}$, and $\rho = 0$. 

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Figure 2.37 SER of 4-QAM and 16-QAM with dual-branch EGC and $m_1 = 2.0, m_2 = 1.0$, for different $\{\Omega_1, \Omega_2\}$, and $\rho = 0$.

Figure 2.38 SER of DE-2-PSK and DE-4-PSK with dual-branch EGC and $m_1 = 2.0, m_2 = 1.0$, for different $\{\Omega_1, \Omega_2\}$, and $\rho = 0$. 

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2.5.6 Conclusions

We derived novel expressions for the pdf of the sum of two correlated Nakagami variables with arbitrary powers and identical \( m \)-parameters, and for the pdf of the sum of two uncorrelated Nakagami variables with arbitrary powers and \( m \)-parameters, when the \( m_i \)'s are multiples of half-integers. In both cases of correlated and uncorrelated Nakagami fading, these pdf's were used to obtain unified SER expressions for several \( M \)-ary modulations with dual-branch unbalanced equal-gain combining. If the branch powers and \( m \)-parameters are equal, an alternative expression was provided which is valid for any real \( m \geq 0.5 \). Closed-form expressions were also obtained for several \( M \)-ary modulations with dual-branch unbalanced maximal-ratio combining in correlated and uncorrelated Nakagami fading. The SER's for Rayleigh fading are special cases which can be easily deduced from the previous general SER expressions by setting \( m = 1 \) in the latter.

2.6 Conclusions

This chapter presented several new results in the area of wideband fading channels, for the general Nakagami distribution (which includes Rayleigh as a special case).

After an introduction, in a first part (Sections 2.3 and 2.4) we considered the characterization and simulation of wideband Nakagami fading channels. In particular, we derived in a unified manner the analytical level crossing rates and average fade durations of systems with diversity combining, and the envelope correlation and baseband spectrum of channels with maximal-ratio combining. The expressions for the level crossing rates were used to design a discrete Nakagami fading simulator based on a finite-state Markov chain. We further compared techniques to generate samples of independent Nakagami samples, and reviewed methods to generate spatially and temporally correlated Nakagami channels. We pointed out some of the restrictive/unjustified assumptions which were previously made in designing continuous temporally correlated Nakagami channels. Since we consider the design of the latter to be still an open issue, in the rest of the thesis we will only use Nakagami simulators which produce independent or spatially correlated samples. The simulation of temporally correlated samples will be carried out
only for $m = 1$, which corresponds to Rayleigh fading, for which well-accepted simulation methods are available.

In a second part (Section 2.5), we considered the performance evaluation of some diversity techniques in Nakagami channels. In particular, we carried out an analysis of commonly used dual-branch diversity receivers, which employ either MRC or EGC, for the general case of correlated and/or unbalanced channels. The analytical results were presented in a unified manner for various $M$-ary modulation schemes as single-integral expressions, which generalized or simplified previous works. In some cases closed-form solutions were also obtained. The analytical expressions were thoroughly validated by comparing them against simulation results, obtained using the methods for generating spatially correlated Nakagami samples which were previously reviewed. Several of the results obtained in this chapter will be useful in the next chapter, where the analysis of multicode/multirate CDMA systems in general wideband fading channels will be tackled.
CHAPTER 3

ANALYSIS OF MULTICODE AND MULTIRATE DS/CDMA SYSTEMS IN WIDEBAND FADING CHANNELS

3.1 Introduction

As discussed in Chapter 1, the VBR nature of compressed video will require the transmitter to vary its rate in order to reduce bandwidth requirements, unless perfect smoothing is achieved. In CDMA systems, two main methods have been standardized for variable-rate transmission: multicode transmission and multirate (or variable spreading gain) transmission. The organization of this chapter is as follows. After this introduction, we first briefly introduce in Sections 3.1.1 and 3.1.2 these two approaches. Then, in the following sections, we present detailed theoretical analyses of the multicode/multirate configurations which are used by the forward and reverse links of IS-95B and IS-2000 systems. The analysis is intended to be more general than that specific to the IS-95B/IS-2000 standards.

3.1.1 Multicode Transmission

In the multicode approach, one or several channels are assigned to each user. Each channel is spread by a code specific to it, whose purpose is to differentiate it from every other channel. Variations on this scheme were proposed in [173], [174], [175]. Ideally, the channels should be made orthogonal to each other by the spreading mechanism, in order to totally eliminate the interference between users. This process of assigning orthogonal codes to the set of channels is called orthogonal covering [176]. In order to
remain orthogonal, the signals on each channel must be transmitted in a synchronous (time-aligned) fashion. For example, in the forward link of an IS-95 system, during each time (symbol) slot, the base station (BS) sends the information destined to each mobile station (MS), which renders the signals synchronous. This differs from the reverse link of an IS-95 system, where the MS's transmit asynchronously. While perfect orthogonal covering is theoretically possible in a flat-fading propagation channel with synchronous channels, this orthogonality is destroyed in wideband channels: indeed, the multipaths are asynchronous with the desired path, thus making a receiver vulnerable to multipath interference contributed by its own or other users' signals. In the forward link case, in order reduce the interference, the channels are further spread by an additional pseudonoise (PN) spreading sequence [177], with a period much longer than the symbol duration. This second stage of spreading reduces the correlation between asynchronous channel codes, and results in decreased multipath interference. The technique of spreading each channel by both a code member of an orthogonal set and a PN sequence is called concatenated spreading [178].

The multicode capability of a system can be exploited in different ways by each user. In one scenario, a user can assign a different service to each of its channels; the information rate of each of these services must be below or equal the maximum bandwidth supported by the corresponding channel. This is the case, for example, in the reverse link of an IS-2000 system, where up to 3 channels (the Fundamental Channel and two Supplemental Channels) can be used to transmit 3 types of services [14]. In a second scenario, only one type of service at a time can be supported. However, if the rate needed for a specific service exceeds the maximum bandwidth offered by the highest-rate channel, then two or more channels can be assigned to that service, in order to realize a rate aggregation. In that case, the information stream is split into several parallel sub-streams at the transmitter end, and each sub-stream is assigned to a separate channel. At the receiver, these sub-streams are multiplexed together in order to form the original information stream [174]. This is the method specified for the transport of high-rate data in the IS-95B standard, where all the channels (at most eight) can offer the same maximum information rate. A system could also operate according to a mixture of the first and second scenarios: certain low-rate services would be carried individually on a
set of channels (one service per channel), while a high-rate service would be split onto several of the remaining channels.

3.1.2 Multirate Transmission

To increase the transmission rate, the data rate of a channel can be increased, while the number of codes assigned to this channel stays the same. With the chip rate held constant, this results in a lower processing gain, hence the appellation variable spreading gain (VSG) [179]. If the power remains the same, a higher BER will result. Hence, in order to maintain the same error rate on a high-rate channel, the power assigned to it must be increased. However, this will inevitably result in a higher level of interference seen by the other users in the cellular system, unless the high-rate user is isolated from the others by using, for example, adaptive antennas. Hence, a tradeoff must be made between the BER sustained by the high-rate user and the requirements of the other users in the system.

3.2 Reverse Link Performance with Noncoherent M-ary Orthogonal Modulation and Real Spreading Sequences

3.2.1 Introduction

M-ary orthogonal modulation with noncoherent reception has been used successfully in the reverse link of IS-95 cellular systems, and is also specified in Radio Configurations 1 and 2 of the reverse link in the cdma2000 standard. Such an approach does not require the estimation of the phase of the received signal, which leads to reduced complexity implementations but also to a higher theoretical BER with respect to coherent demodulation. Two early analyses of the performance of this scheme in a CDMA environment were given in [180] and [181] for an additive white Gaussian noise channel. Extensions to the case of a multipath fading channel were presented in [182] for the Rayleigh distribution and in [183] for a general fading distribution. Some closed-form solutions for the BER in the case of Nakagami fading were presented in [184], [185] and [186] for $M = 2$, and in [187] for $M$ arbitrary, where the interference analysis followed the same method-
ology. An upper bound on the coded error probability of a system with soft-decision Viterbi decoding and Rice-lognormal fading was presented in [188]. In these studies, the users transmit only one code each. However, for high-rate applications such as data or low-quality video [189], [190], the IS-95B standard allows users to transmit up to eight codes in parallel [191]. Such a scheme is termed multicode CDMA, and variations of it were proposed in [173], [174], [175]. An analysis of multicode CDMA was given in [192] for coherent detection, in a reverse link scenario. Further results are available in [193], [194], also for the case of coherent detection. In [195] and [196], analytical and semi-analytical studies, respectively, of the capacity of a multicode CDMA system were also provided, but the effects of the modulation format and multipath fading were averaged out. [197] evaluates the performance of the IS-95B reverse link in terms of required SNR for a given Frame Error Rate (FER), using computer simulations. Despite the many recent contributions, to our knowledge no detailed mathematical analysis (which considers in a precise manner the effect of multipath fading) of multicode CDMA with noncoherent $M$-ary orthogonal modulation has been published. The goal of this section is to tackle this issue.

In the reverse link of IS-95B systems, i.e. the MS to BS communication link, the spreading sequences assigned to each code are not orthogonal, since they correspond to different offsets of a long PN sequence [177]. Hence, a correlation receiver is subject to multi-access interference from all codes different from the desired code, and to multipath interference from all users (including the desired user) if the channel is frequency-selective. In a first step, we express the interference terms as a function of aperiodic crosscorrelation functions [27], which was omitted in previous papers (except partly in [198]), and detail the statistics of these terms for multicode transmission. In [183] and [187] the BER was calculated using the standard Gaussian approximation (GA), by first replacing the values of all the fading coefficients in the interference terms by their expectations, and then either using Stirling's formula or averaging over a known fading distribution in order to reflect the effect of the fading. While this leads to a good approximation in the single-code case, our research will show that the same approach cannot be applied for multicode transmission. Indeed, if the desired user transmits many codes simultaneously, they all fade in unison. Therefore the interference resulting from each of the non-desired codes

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is not independent from the desired signal. To obtain a better approximation, we derive
the BER conditioned on the fading coefficients affecting the desired user, then integrate
it numerically over the pdf of the fading. The resulting BER curves match the ones
obtained by simulation of an IS-95B system better than if we use the previous methods:
the latter become less and less accurate as the diversity order increases, or as the number
of users decrease when no diversity is used, while our approach gives a good degree of
accuracy for most situations.

The organisation of this section is as follows. After this introduction, Section 3.2.2 de­
tails the signal, channel and receiver models for a single-cell system. It gives the decision
metrics of the receiver, which are used in deriving the probability of error for Nakagami,
Rician and lognormal fading channels. Extensions to the closed-loop power control, suc­
cessive interference cancellation, and multiple cells cases are detailed in Sections 3.2.2.6,
3.2.2.7 and 3.2.2.8, respectively. They are followed by illustrative numerical results in
Section 3.2.3 and a concise conclusion in Section 3.2.4.

3.2.2 Error Probability Analysis

3.2.2.1 Signal Model

The transmitter is a multicode Offset-QPSK (O-QPSK) M-ary orthogonal modulator,
whose diagram is shown in Fig. 3.1. User \( k \) transmits \( N^{(k)} \) streams in parallel, each one
formed by the concatenation of a data stream with a spreading code. The stream of data
symbols for code \( c \) \((c = 0, 1, \ldots, N^{(k)} - 1)\) of user \( k \) is given in the time-domain by:

\[
W^{(kc)}(t) = \sum_{j=-\infty}^{\infty} W_{i(kc,j)}(t - jT_w)
\]  

where \( W_{i(kc,j)}(t) \) is a Hadamard-Walsh function of dimension \( M \) and duration \( T_w \), whose
index \( i(kc,j) \in [1, 2, \ldots, M] \) depends on the indices of the user \( (k) \), code \( (c) \) and symbol
sequence number \( (j) \). In order to alleviate the notation, unless otherwise noted, we will
drop the dependence of \( i \) on these indices. The set of \( M \) Hadamard-Walsh functions are
orthogonal to each other, i.e. \( \int_0^{T_w} W_m(t)W_n(t)dt = T_w \delta_{mn} \), where \( \delta_{ij} = 1 \) if \( i = j \) and 0
otherwise. We can further expand Eq. (3.1) as:

\[
W^{(kc)}(t) = \sum_{j=-\infty}^{\infty} \sum_{r=0}^{M-1} w_{i,r} p_{Tw}(t - jTw - rTw)
\]  \hspace{1cm} (3.2)

where \(w_{i,r}, r = 0, 1, \ldots, M - 1\), form a sequence of \(M\) Walsh bits, which correspond to a particular \(M\)-ary symbol \(W_i = [w_{i,0}, w_{i,1}, \ldots, w_{i,M-1}]\) [177], and \(p_{Tw}(t)\) is a rectangular pulse of unit amplitude and duration \(T_w = T_W / M\) seconds. This symbol stream is spread by a long PN sequence \(a^{(kc)}_L(t) = \sum_{j=-\infty}^{\infty} a^{(kc)}_{r,j} p_{r}(t - jT_c)\) with chip period \(T_c\), which corresponds to a shifted version of a reference long sequence \(a_{r,L}(t)\). This assignment of different offsets to every code of each user is responsible for the multicode multi-access capability of the system \(^1\). The same stream is then mapped onto both the in-phase (I) and quadrature (Q) branches of the transmitter, with a delay of \(T_0\) (of half a chip duration in IS-95) introduced in the Q branch in order to achieve O-QPSK modulation. In the IS-95 and cdma2000 standards, two short spreading sequences \(a_{s,I}(t)\) and \(a_{s,Q}(t)\) with the same chip period \(T_c\) further spread the signal on the I and Q branches, respectively. These sequences are identical for all the users of the system. To alleviate the notation, we can combine the long and short PN sequences into two I and Q long PN sequences \(a^{(kc)}_I(t)\) and \(a^{(kc)}_Q(t)\). The transmitted signal for the \(k\)th user can then be written as:

\[
s^{(k)}(t) = \sqrt{P^{(k)}} \sum_{c=0}^{N^{(k)}-1} \left[ W^{(kc)}(t) a^{(kc)}_I(t) \cos(w_c t + \phi^{(kc)}) + W^{(kc)}(t - T_0) a^{(kc)}_Q(t - T_0) \sin(w_c t + \phi^{(kc)}) \right] \]  \hspace{1cm} (3.3)

where \(P^{(k)}\) is the average power of user \(k\) which, without loss of generality, can be assumed identical for all users, i.e. \(P^{(k)} = P \forall k\). \(w_c = 2\pi f_c\) is the carrier frequency in radians per second.

\(^1\)This corresponds to the code aggregation scheme of [189], which is used here. The subcode concatenation scheme of [189] further spreads each code stream of a same user by a different code taken from an orthogonal set: this cancels the multicode self-interference from the same path (but not the one from different paths, for which the orthogonality property is destroyed), and hence leads to a lower BER than that obtained using the code aggregation scheme.
3.2.2.2 Channel Model

The frequency-selective channel is modeled as a tapped delay line filter \[26\] with impulse response 
\[
h(t, \tau) = \sum_{i=0}^{L_c-1} a_i(t)e^{j\theta_i(t)}\delta(\tau - \tau_i),
\]
where \(L_c\) is the number of multipath components, \(a_i(t)\) and \(\theta_i(t)\) are the time-variant amplitude and phase, respectively, of the complex short-term fading coefficient of the \(i^{th}\) path, and \(\tau_i\) is the delay of the latter. It is assumed that either the delay spread of the channel \(\tau_{\text{max}}\) can be upper-bounded by the symbol period \(T_W\), in order to avoid intersymbol interference (ISI), or that the latter is removed by an equalization mechanism. Also, \(a_i(t)\) and \(\theta_i(t)\) are assumed constant during a symbol interval. Samples of \(a_i(t)\) are not restricted to any particular distribution. For illustration purposes, we will consider in particular the Nakagami distribution, given by \[23\]:

\[
p(\alpha_t) = 2 \left( \frac{m_t}{\Omega_t} \right)^{m_t} \frac{\alpha_t^{2m_t-1}}{\Gamma(m_t)} e^{-\frac{m_t}{\Omega_t} \alpha_t^2}, \quad \alpha_t > 0
\]  

(3.4)

where \(m_t > 0.5\) and \(\Omega_t = E[\alpha_t^2]\). If \(m_t = 1\), this density function reverts to the Rayleigh distribution. We will also consider the Rician distribution, given by:

\[
p(\alpha_t) = \frac{a_t}{\sigma_{R,I}^2}e^{-\frac{A_t^2 + \sigma_{R,I}^2}{2\sigma_{R,I}^2}} I_0 \left( \frac{A_t \alpha_t}{\sigma_{R,I}^2} \right), \quad \alpha_t > 0
\]  

(3.5)

where \(2\sigma_{R,I}^2\) is the power in the random component and \(A_t\) is the amplitude of the non-random specular component \[26\] of the \(i^{th}\) multipath, and the lognormal distribution,
given by:

\[ p(\alpha_l) = \frac{1}{\sqrt{2\pi\sigma_{LN,l}\alpha_l}} \exp \left( -\frac{[\ln(\alpha_l) - m_{LN,l}]^2}{2\sigma_{LN,l}^2} \right), \quad \alpha_l > 0 \]  

(3.6)

where \( m_{LN,l} \) and \( \sigma_{LN,l} \) are the mean and standard deviation of \( \ln(\alpha_l) \), with \( \ln(\cdot) \) denoting the natural logarithm.

The stationary complex additive Gaussian noise at the output of the received band-pass filter can be expressed mathematically as \( n(t) = n_c(t) \cos(\omega_c t) + n_s(t) \sin(\omega_c t) \), where \( n_c(t) \) and \( n_s(t) \) are uncorrelated white low-pass Gaussian processes with two-sided power spectral density (PSD) \( N_0 \).

3.2.2.3 Receiver Model and Decision Metrics

Using the above mentioned system and considering that there are \( K \) users, the composite signal received at the output of the channel is:

\[
\begin{align*}
\mathbf{r}(t) &= \sum_{k=1}^{K} \sum_{c=0}^{N^{(k)}-1} \sum_{l=0}^{L_c-1} \sqrt{P_{\alpha_l}^{(k)}(t)} [W^{(kc)}(t - \tau_l^{(k)})a_l^{(kc)}(t - \tau_l^{(k)}) \cos(\omega_c t + \varphi_l^{(kc)}) \\
&+ W^{(kc)}(t - T_0 - \tau_l^{(k)})a_l^{(kc)}(t - T_0 - \tau_l^{(k)}) \sin(\omega_c t + \varphi_l^{(kc)})] + n(t)
\end{align*}
\]

(3.7)

with \( \varphi_l^{(kc)} = \varphi_l^{(kc)} + \theta_l^{(k)} - w_c \tau_l^{(k)} \). Note that all the received codes pertaining to a same user are affected by the same fading coefficients and delay, since they are transmitted via the same channel. This fact will be crucial in determining the error probability for a low number of interfering users. The phases, however (as specified in the IS-95B standard), can be different if a specific phase shift is assigned to each channel by the transmitter.

The receiver consists of \( L \) Rake fingers and performs EGC. The schematic structure for the \( n^{th} \) finger is illustrated in Fig. 3.2, and its operation is detailed in [177]. In the following, the metrics calculated refer to code 0 of user 1, which is assumed to be the desired stream. Let \( Z_{IJ}^{(m,n)} \) be the output of the \( m^{th} \) correlator of the \( n^{th} \) Rake finger, in response to the received signal on the \( I^{th} \)-branch despread by \( a_l^{(10)}(t) \). It can be expressed
Figure 3.2 Demodulator for $n^{th}$ Rake finger of user 1 and code $c$.

by:

$$Z_{II}^{(m,n)} = \frac{1}{\sqrt{T_W}} \int_{\tau_n^{(1)}}^{\tau_n^{(1)} + T_W} (r(t) \cos(w_c t))_{LP} \alpha_l^{(10)}(t - \tau_n^{(1)}) W_m(t - \tau_n^{(1)}) dt \quad (3.8)$$

where $(\cdot)_{LP}$ denotes low-pass filtering, and the $\tau_n^{(1)}$ are perfectly estimated. Upon expanding terms in (3.8) and integrating, one obtains:

$$Z_{II}^{(m,n)} = S_{II}^{(m,n)} + I_{1II}^{(m,n)} + I_{2II}^{(m,n)} + I_{3II}^{(m,n)} + I_{II}^{(m,n)} + N_{II}^{(m,n)} \quad (3.9)$$

where $S_{II}^{(m,n)}$ is the signal term, $I_{1II}^{(m,n)}$ is the $I/Q$ crosstalk interference, $I_{2II}^{(m,n)}$ is the multipath self-interference, $I_{3II}^{(m,n)}$ is the multicode self-interference, $I_{II}^{(m,n)}$ is the other-user multi-access interference, and $N_{II}^{(m,n)}$ is an integrated noise term. Let $C = \frac{1}{2}\sqrt{P/T_W}$. 

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Then these terms are identified as [183]:

\[ S_{II}^{(m,n)} = \int_{\tau_n^{(1)}}^{\tau_n^{(1)} + T_W} \alpha_n^{(1)}(t) \cos(\varphi_n^{(10)}) W^{(10)}(t - \tau_n^{(1)}) W_m(t - \tau_n^{(1)}) dt, \]  
\[ (3.10) \]

\[ I_{II}^{(m,n)} = C \int_{\tau_n^{(1)}}^{\tau_n^{(1)} + T_W} \alpha_n^{(1)}(t) \sin(\varphi_n^{(10)}) W^{(10)}(t - \tau_n^{(1)}) a_Q^{(10)}(t - \tau_n^{(1)}) - T_0) \times W_m(t - \tau_n^{(1)}) a_I^{(10)}(t - \tau_n^{(1)}) dt, \]  
\[ (3.11) \]

\[ I_{II}^{(m,n)} = C \int_{\tau_n^{(1)}}^{\tau_n^{(1)} + T_W} \sum_{i=0}^{L_c-1} \sum_{l \neq n} \alpha_i^{(1)}(t) W^{(i1c)}(t - \tau_i^{(1)}) a_I^{(1c)}(t - \tau_i^{(1)}) \cos(\varphi_i^{(1c)}) \]  
\[ + W^{(1c)}(t - \tau_i^{(1)} - T_0) a_Q^{(1c)}(t - \tau_i^{(1)} - T_0) \sin(\varphi_i^{(1c)}) W_m(t - \tau_n^{(1)}) a_I^{(10)}(t - \tau_n^{(1)}) dt, \]  
\[ (3.12) \]

\[ I_{II}^{(m,n)} = C \int_{\tau_n^{(1)}}^{\tau_n^{(1)} + T_W} \sum_{i=1}^{N_i} \sum_{l=0}^{L_c-1} \alpha_i^{(1)}(t) W^{(i0)}(t - \tau_i^{(1)}) a_I^{(0)}(t - \tau_i^{(1)}) \cos(\varphi_i^{(0)}) \]  
\[ + W^{(0)}(t - \tau_i^{(1)} - T_0) a_Q^{(0)}(t - \tau_i^{(1)} - T_0) \sin(\varphi_i^{(0)}) W_m(t - \tau_n^{(1)}) a_I^{(10)}(t - \tau_n^{(1)}) dt, \]  
\[ (3.13) \]

\[ I_{II}^{(m,n)} = C \int_{\tau_n^{(1)}}^{\tau_n^{(1)} + T_W} \sum_{k=2}^{K} \sum_{c=0}^{N_i^{(k)}} \sum_{l=0}^{L_c-1} \alpha_i^{(k)}(t) W^{(k0)}(t - \tau_i^{(1)}) a_I^{(k0)}(t - \tau_i^{(1)}) \cos(\varphi_i^{(k0)}) \]  
\[ + W^{(k0)}(t - \tau_i^{(1)} - T_0) a_Q^{(k0)}(t - \tau_i^{(1)} - T_0) \sin(\varphi_i^{(k0)}) W_m(t - \tau_n^{(1)}) a_I^{(10)}(t - \tau_n^{(1)}) dt, \]  
\[ (3.14) \]

\[ N_{II}^{(m,n)} = \frac{C}{\sqrt{P}} \int_{\tau_n^{(1)}}^{\tau_n^{(1)} + T_W} n_i(t) W_m(t - \tau_n^{(1)}) a_I^{(10)}(t - \tau_n^{(1)}) dt. \]  
\[ (3.15) \]

The first term readily simplifies to:

\[ S_{II}^{(m,n)} = \alpha_n^{(1)} \frac{\sqrt{E_W}}{2} \cos(\varphi_n^{(10)}) \delta_{mi}. \]  
\[ (3.16) \]
where we recall that $i$ is the index of the desired Walsh data symbol in the current integration interval, and $E_W = PT_W$. The interference terms can be expressed as a function of aperiodic crosscorrelation functions. Similarly to [199], let:

$$C_{xy,muv}^{(kc)}(\tilde{l}) = \begin{cases} 
\sum_{j=0}^{N-1-l} w_{v,\lfloor \frac{l}{N} \rfloor}^{(kc)} a_{\tilde{x},j}^{(kc)} w_{m,\lfloor \frac{l+N-1}{N} \rfloor}^{(10)} a_{\tilde{y},j}^{(10)} & 0 \leq \tilde{l} \leq N-1 \\
\sum_{j=0}^{N-1-l} w_{u,\lfloor \frac{l-N-1}{N} \rfloor}^{(kc)} a_{\tilde{x},j-\tilde{l}}^{(kc)} w_{m,\lfloor \frac{l}{N} \rfloor}^{(10)} a_{\tilde{y},j}^{(10)} & 1 - N \leq \tilde{l} < 0 \\
0 & |\tilde{l}| \geq N
\end{cases}$$

(3.17)

where $N = T_W/T_c$ is the processing gain, $h = N/M$ is the number of PN chips per Walsh bit, $\tilde{l} = [(\tau^{(k)}_l - lT_c)/T_c]$ (with $\lfloor x \rfloor$ denoting the largest integer smaller than or equal to $x$), and $u$ and $v$ are the indices of two consecutive Walsh symbols belonging to code 0 of user 1 and contained in the integration interval $[\tau^{(1)}_n, \tau^{(1)}_n + T_W]$. The indices $\{xy\}$ are chosen from $\{II\}, \{IQ\}, \{QI\}$ or $\{QQ\}$. The aperiodic crosscorrelation functions can be written in terms of the previous quantity as [27]:

$$R_{xy,muv}^{(kc)}(\tau) = [C_{xy,muv}^{(kc)}(\tilde{l} + 1 - N) - C_{xy,muv}^{(kc)}(\tilde{l} - N)](\tau - \tilde{l}T_c)$$

$$+ [C_{xy,muv}^{(kc)}(\tilde{l} - N)]T_c,$$

(3.18)

$$R_{xy,muv}^{(kc)}(\tau) = [C_{xy,muv}^{(kc)}(\tilde{l} + 1) - C_{xy,muv}^{(kc)}(\tilde{l})](\tau - \tilde{l}T_c)$$

$$+ [C_{xy,muv}^{(kc)}(\tilde{l})]T_c.$$  

(3.19)

The interference terms in Eqs. (3.11)-(3.15) can then be expressed as a function of these functions (where, for convenience of presentation, the dependence on $m$, $u$ and $v$ has
been dropped), which was not done explicitly in [183]:

\[
I_{1lf}^{(m,n)} = C \alpha_n^{(1)} \sin(\varphi_n^{(10)}) [R_{Ql}^{(10)}(T_0) + \hat{R}_{Ql}^{(10)}(T_0)],
\]

\[
I_{2lf}^{(m,n)} = C \sum_{\substack{l=0 \\
l \neq n}}^{L_c-1} \alpha_l^{(1)} \bar{I}_{lf}^{(m,n)}(10),
\]

\[
I_{3lf}^{(m,n)} = C \sum_{c=1}^{N^{(1)}-1} \sum_{l=0}^{L_c-1} \alpha_l^{(1)} \bar{I}_{lf}^{(m,n)}(1c),
\]

\[
I_{lf}^{(m,n)} = C \sum_{k=2}^{K} \sum_{c=0}^{N^{(k)-1}} \sum_{l=0}^{L_c-1} \alpha_l^{(k)} I_{lf}^{(m,n)}(kc)
\]

where

\[
\bar{I}_{lf}^{(m,n)}(kc) = C \left[ (R_{lf}^{(kc)}(\tau_{nl}^{(k)}) + \hat{R}_{lf}^{(kc)}(\tau_{nl}^{(k)})) \cos(\varphi_i^{(kc)}) \right.
\]

\[+ (R_{Ql}^{(kc)}(\tau_{nl}^{(k)} + T_0) + \hat{R}_{Ql}^{(kc)}(\tau_{nl}^{(k)} + T_0)) \sin(\varphi_i^{(kc)}) \]

(3.24)

and \(\tau_{nl}^{(k)} = \tau_{n}^{(k)} - \tau_{n}^{(1)}\). Similarly, signal and interference terms can be derived for \(Z_{lf}^{(m,n)}\), \(Z_{Ql}^{(m,n)}\) and \(Z_{QQ}^{(m,n)}\), and these quantities are listed in Appendix D for completeness.

The final decision metrics are given by:

\[
U_m = \sum_{n=0}^{L_1-1} (|Z_{lf}^{(m,n)} + Z_{Ql}^{(m,n)}|^2 + |Z_{lf}^{(m,n)} - Z_{Ql}^{(m,n)}|^2)
\]

(3.25)

where \(m = 1, 2, \ldots, M\). The receiver selects the symbol \(W_j\) whose index \(j\) corresponds to the maximum \(U_j\) of all \(U_m\)‘s.

### 3.2.2.4 Statistics of Decision Metrics

As stated in [183], the noise terms \(N_{xy}^{(m,n)}\) can be easily shown to be mutually uncorrelated zero-mean Gaussian random variables with variance:

\[
\sigma_N^2 = \text{Var}[N_{xy}^{(m,n)}] = N_0/4.
\]

(3.26)
Using the standard GA as in [183], the interference terms of Eqs. (3.20)-(3.23) can be modeled as mutually uncorrelated zero-mean Gaussian random variables with given conditional variances. These variances are derived below, and are conditional on the random variable \( s = \sum_{l=0}^{L-1} (\alpha_l^{(1)})^2 \). If \( T_0 = 0 \), because of the orthogonality between the Walsh functions, the interference terms \( I_{1xy}^{(m,n)} \) vanish. Otherwise, \( \text{Var}[R_{xy}^{(10)}(T_0) + \hat{R}_{xy}^{(10)}(T_0)] = 2T_w^2/(3N) \); which results in a variance:

\[
\sigma_{I1}^2 = \text{Var}[I_{1xy}^{(m,n)}] = \frac{E_w}{12N} (\alpha_n^{(1)})^2, \quad T_0 \neq 0.
\] (3.27)

Again if \( T_0 = 0 \), the interference terms \( I_{2xy}^{(m,n)} \) and \( I_{3xy}^{(m,n)} \) are chip synchronous with the desired user signal, resulting in \( \text{Var}[(R_{xy}^{(1c)}(\tau_n^{(1)}) + \hat{R}_{xy}^{(1c)}(\tau_n^{(1)}))] = T_w^2/N \). If \( T_0 \neq 0 \), \( \text{Var}[R_{xy}^{(1c)}(\tau_n^{(1)} + T_0) + \hat{R}_{xy}^{(1c)}(\tau_n^{(1)} + T_0)] = 2T_w^2/(3N) \). This leads to the following variances:

\[
\sigma_{I2}^2 = \text{Var}[I_{2xy}^{(m,n)}] = \eta \frac{E_w}{N} \sum_{l=0}^{L-1} (\alpha_l^{(1)})^2,
\] (3.28)

\[
\sigma_{I3}^2 = \text{Var}[I_{3xy}^{(m,n)}] = \eta \frac{E_w}{N} (N^{(1)} - 1) \sum_{l=0}^{L-1} (\alpha_l^{(1)})^2,
\] (3.29)

where \( \eta = 1/4 \) if \( T_0 = 0 \) and \( \eta = 5/24 \) if \( T_0 \neq 0 \). Replacing \( (\alpha_n^{(1)})^2 \) with its expected value, we could approximate Eq. (3.28) by

\[
\sigma_{I2}^2 = \text{Var}[I_{2xy}^{(m,n)}] \approx \eta \frac{E_w}{N} \left[ \sum_{l=0}^{L-1} (\alpha_l^{(1)})^2 - \Omega_n^{(1)} \right]
\] (3.30)

in order to render the expression fully conditional on \( s \). Alternatively, as in [39], we could also approximate it by

\[
\sigma_{I2}^2 = \text{Var}[I_{2xy}^{(m,n)}] \approx \eta \frac{E_w}{N} \sum_{l=1}^{L-1} \Omega_l^{(1)}
\] (3.31)
in order to remove the conditioning on the \((\alpha_1^{(1)})^2\)'s. A comparison with numerical results shows that for similar \(\Omega_1^{(1)}\)'s, the use of Eq. (3.31) leads to a better estimate of the BER, especially for low values of \(L\). Indeed, when performing the numerical integration of Section 3.2.2.5, for certain low values of \(\alpha_1^{(1)}\), the variance \(\sigma_{f_2}^2\) given in Eq. (3.30) becomes negative (because of the term \(-\Omega_n^{(1)}\)), which doesn’t make sense. Hence we will favour Eq. (3.31) in our calculations.

Unlike with the previous variance \(\sigma_{f_2}^2\), the \((\alpha_1^{(1)})^2\)'s in the expression for \(\sigma_{f_3}^2\) cannot be approximated by their expected values. Indeed, we must take into account the fact that the instantaneous fading coefficients affecting the multicore self-interference of user 1 are all equal, and equal to the fading coefficient which affects the desired signal. This results from all the codes of user 1 being transmitted via the same channel.

Irrespective of \(T_0\), the interference terms \(I_{xy}^{(m,n)}\) are chip asynchronous with respect to the desired user signal, leading to \(\text{Var}[(R_{xy}^{(k)} (r_{n_i}^{(k)}) + \hat{R}_{xy}^{(k)} (r_{n_i}^{(k)}))] = 2T_W^2/(3N), k \neq 1\), and a variance:

\[
\sigma_i^2 = \text{Var}[I_{xy}^{(m,n)}] = \frac{E_W}{6N} \sum_{k=2}^K N^{(k)} \sum_{l=0}^{L_c-1} \Omega_l^{(k)}. \tag{3.32}
\]

We notice that this variance does not depend on \(s\). In the following, we will adopt \(T_0 = 0\) (as in [183] and [187]), in order to ease the development. Thus, we can neglect \(I_{1xy}^{(m,n)}\) (which we can in fact do even in the case \(T_0 \neq 0\). As sums of mutually uncorrelated Gaussian random variables, the terms \(Z_{11}^{(m,n)} + Z_{QQ}^{(m,n)}\) and \(Z_{1Q}^{(m,n)} - Z_{Q1}^{(m,n)}\) are therefore Gaussian random variables with means \(\alpha_n^{(1)} \sqrt{E_W \cos(\varphi_n^{(1)})} \delta_{mi}\) and \(\alpha_n^{(1)} \sqrt{E_W \sin(\varphi_n^{(1)})} \delta_{mi}\), respectively, and common variance \(\sigma^2\) conditioned on \(s\):

\[
\sigma^2 = 2(\sigma_{f_2}^2 + \sigma_{f_3}^2 + \sigma_i^2 + \sigma_N^2) = \frac{N_0}{2} + \frac{E_W}{3N} \left[ \frac{3}{2} \left( (N^{(1)} - 1)s + \sum_{i=1}^{L_c-1} \Omega_i^{(1)} \right) + \sum_{k=2}^K N^{(k)} \sum_{l=0}^{L_c-1} \Omega_l^{(k)} \right]. \tag{3.33}
\]

In the following, we take the number of fingers of the Rake receiver, \(L\), to be equal to the number of multipaths \(L_c\) (i.e. all the energy of the channel is collected), as was done in [183] and [187]. Let \(\nu = E_W s\). The quantity \(U_m\) is thus a chi-square random variable.
with \( 2L \) degrees of freedom, conditioned on \( \nu \), whose pdf is given by:

\[
p_{U_m}(u|\nu) = \begin{cases} 
\frac{1}{2\sigma^2} \left( \frac{\nu}{\sqrt{\pi}} \right)^{\frac{L-1}{2}} e^{-\frac{\nu + \frac{2m}{\sigma^2}}{2\sigma^2}} I_{L-1} \left( \frac{\sqrt{\nu} u}{\sigma^2} \right) & \text{if } m = i, \\
\frac{1}{(2\sigma^2)^L \Gamma(L)} u^{L-1} e^{-\frac{u}{2\sigma^2}} & \text{if } m \neq i.
\end{cases}
\]

### 3.2.2.5 Probability of Error

In this section, the method used for computing the probability of error of a multicode system in multipath fading is presented. Assume that the Walsh function \( W_1(t) \) is transmitted, and again that code 0 of user 1 is the desired data stream. The probability of a correct symbol decision, conditioned on \( \nu \), is given by:

\[
P_c(\nu) = P(U_2 < U_1, U_3 < U_1, \ldots, U_M < U_1|\nu)
\]

\[
= \int_{0}^{\infty} \left[ P(U_2 < u|U_1 = u) \right]^{M-1} p_{U_m}(u|\nu) du
\]

\[
= \int_{0}^{\infty} \left[ 1 - e^{-\frac{u}{2\sigma^2}} \sum_{n=0}^{L-1} \left( \frac{\nu}{2\sigma^2} \right)^n \frac{n!}{n!} \right]^{M-1} p_{U_m}(u|\nu) du
\]

\[
= \sum_{r=0}^{M-1} (-1)^r \binom{M-1}{r} \sum_{n=0}^{\infty} \beta_{nr} \frac{e^{-\frac{u}{2\sigma^2}}}{[2\sigma^2]^{n+1} \nu^{\frac{L-1}{2}}} \times \int_{0}^{\infty} u^{\frac{L-1}{2} + n} e^{-\frac{u(r+1)}{2\sigma^2}} I_{L-1} \left( \frac{\sqrt{\nu} u}{\sigma^2} \right) du
\]

where the coefficients \( \beta_{nr} \) are given by [157]:

\[
\beta_{nr} = \sum_{i=n-(L-1)}^{n} \frac{\beta_{i(r-1)}}{(n-i)!} I_{[0,(r-1)(L-1)]}(i)
\]

with \( \beta_{00} = \beta_{0r} = 1 \), \( \beta_{n1} = 1/n! \), \( \beta_{1r} = r \) and \( I_{[a,b]}(i) = 1 \) if \( a \leq i \leq b \) and 0 otherwise.

The integral in Eq. (3.35) can be evaluated with the help of Eq. 6.643.2 of [80], yielding:

\[
P_c(\nu) = \frac{1}{\Gamma(L)} \sum_{r=0}^{M-1} (-1)^r \binom{M-1}{r} \sum_{n=0}^{\infty} \beta_{nr} \times \frac{\Gamma(L+n)}{(r+1)^{L+n}} e^{-\frac{\nu}{2\sigma^2}} \Phi \left( L + n, L, \frac{\nu}{2\sigma^2(r+1)} \right).
\]

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The probability of symbol error, conditional on $s$ (recalling that $\nu = E_W s$), is thus:

\[
P_s(s) = 1 - P_c(s) \tag{3.38}
\]

\[
= \frac{1}{\Gamma(L)} \sum_{r=1}^{M-1} (-1)^{r+1} \binom{M-1}{r} \sum_{n=0}^{r(L-1)} \beta_{nr} \times \frac{\Gamma(L+n)}{(r+1)^{L+n}} e^{-\frac{E_W s}{2\sigma^2}} \Phi \left( L + n, L, \frac{E_W s}{2\sigma^2(r+1)} \right) \tag{3.39}
\]

In the above it is recalled that the variance $\sigma^2$ is a function of $s$, as made explicit in Eq. (3.33). The probability of symbol error is then obtained by averaging Eq. (3.39) over the pdf of $s$:

\[
P_s = \int_0^\infty P_s(s)p_s(s)ds. \tag{3.40}
\]

The probability of bit error is approximated as $P_b = \frac{M/2}{M-1} P_s$ [26].

In the case of multicode transmission, for most practical fading distributions, the dependence of $\sigma^2$ on $s$ in Eq. (3.39), and therefore in Eq. (3.40), forbids a simple closed-form solution for $P_s$. Numerical integration is thus used to evaluate Eq. (3.40). In the case of single-code transmission, $\sigma^2$ does not depend on $s$, and closed-form expressions for $P_s$ can be found in some special cases. In the following, the cases of Rayleigh, Nakagami, Rician and lognormal fading are treated individually. As previously mentioned, the Rayleigh case is a special case of both Nakagami and Rician, but a separate treatment is included for it due to its widespread use. The details of the analytical derivations are given in Appendix E.

**Rayleigh Fading**

In the case of Rayleigh fading with identical powers $\Omega_i^{(1)} = \Omega$ on each branch, the pdf of $s$ is given by:

\[
p_s(s) = \frac{s^{L-1}}{\Omega^L \Gamma(L)} e^{-\frac{s}{\Omega}}. \tag{3.41}
\]

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No simple closed-form expression for $P_s$ seems available for the general multicode case. In the case of single-code transmission, Eq. (3.40) can be put in closed-form by using Eqs. 7.621.4 and 9.121.1 of [80]:

$$
P_s = \frac{1}{\Gamma(L)} \sum_{r=1}^{M-1} \frac{(-1)^{r+1}}{(r + 1 + \frac{\Omega E_W r}{2a^2})^L} \binom{M - 1}{r} \sum_{n=0}^{r(L-1)} \beta_{nr} \Gamma(L + n) \left[ 1 + \frac{\Omega E_W r}{2a^2} \right]^n. \quad (3.42)
$$

This expression is identical to Eq. 14.4.46 of [26].

**Nakagami Fading**

In the case of Nakagami fading with identical fading parameters and powers on each branch, the pdf of $s$ is given by:

$$
p_s(s) = \left( \frac{m}{\Omega} \right)^{mL} s^{mL-1} \frac{\Gamma(mL)}{\Gamma(mL)} e^{-\frac{m}{\Omega} s^m}. \quad (3.43)
$$

It appears that there is no closed-form solution for $P_s$ for multicode transmission, for the same reasons as the Rayleigh fading case. In the case of single-code transmission, the following expression can be obtained, again through the use of Eq. 7.621.4 of [80]:

$$
P_s = \frac{1}{\Gamma(L)} \sum_{r=1}^{M-1} \frac{(-1)^{r+1}}{(1 + \frac{\Omega E_W r}{m2a^2})^{mL}} \binom{M - 1}{r} \times \sum_{n=0}^{r(L-1)} \beta_{nr} \Gamma(L + n) \frac{1}{(r + 1)(1 + \frac{\Omega E_W r}{2a^2})} F_1 \left( mL, L + n; L; \frac{1}{(r + 1)(1 + \frac{\Omega E_W r}{2a^2})} \right). \quad (3.44)
$$

This expression is identical to Eq. (36) of [187], and reduces to Eq. (3.42) for $m = 1$.

The analysis can also be applied to the more general case of correlated diversity branches, if an expression for the pdf of $s$ is known. For example, in the case of equally correlated diversity branches, it is known that [162], [187]:

$$
p_s(s) = \left[ \frac{\Omega}{m} \left( 1 - \sqrt{\rho} \right)^{m-1} \left( 1 - \sqrt{\rho} + L\sqrt{\rho} \right)^m \Gamma(mL) \right]^{-1} \left( \frac{m}{\Omega} \right)^{mL-1} \exp \left( -\frac{m}{\Omega(1 - \sqrt{\rho})^s} \right) \Phi \left( mL; mL; \frac{Lm}{\Omega(1 - \sqrt{\rho})(1 - \sqrt{\rho} + L\sqrt{\rho})^s} \right). \quad (3.45)
$$
For single-code transmission, a closed-form solution for the SER of equi-correlated diversity in Nakagami fading has been found as (Eq. (46) of [187]):

\[
P_s = \frac{1}{\Gamma(mL)} \sum_{r=1}^{M-1} (-1)^{r+1} \binom{M-1}{r} \left( \frac{m}{r+1} \right)^{\frac{m}{\Omega E_y}} \frac{\sum_{i=0}^{n} \sum_{j=0}^{m} \beta_{ni} \Gamma(L+n)}{(r+1)^{L+n}} \\
\times \binom{n}{i} \binom{i}{j} \frac{\Gamma(mL+i)L(m+j)}{\Gamma(mL+j)\Gamma(mL+i)} \left( \frac{\Omega E_y}{2\sigma^2} (1 - \sqrt{\rho}) \right)^i \\
\times \left( \frac{Lm\sqrt{\rho}}{(1 - \sqrt{\rho})(r+1)^2(1 - \sqrt{\rho}) + mL} \right)^j.
\]

(3.46)

An approximate expression for exponentially correlated diversity branches has also been proposed in [162], [187], but it was shown in [200], [163] to be inaccurate.

**Rician Fading**

In the case of Rician fading with identical fading parameters and powers on each branch, the pdf of \(s\) is given by [26]:

\[
p_s(s) = \frac{1}{2\sigma^2} \left( \frac{s}{\lambda} \right)^{\frac{L-1}{2}} e^{-\frac{\lambda s^2}{2\sigma^2}} I_{L-1} \left( \frac{\sqrt{s^2}}{\sigma} \right)
\]

(3.47)

where \(\lambda = LA^2\). In the case of single-code transmission, a closed-form expression can be obtained by first calculating the unconditional pdf, using Eq. (2.4.13), p. 115 of [116] (c.f. also [201], [202])

\[
p_{U_m}(u) = \int_0^\infty p_{U_m}(u|s)p_s(s)ds
\]

(3.48)

\[
= \left\{ \begin{array}{ll}
\frac{(u/(2\sigma^2))^2}{2\sigma^2(1+\beta)} e^{-\frac{u^2+2\sigma^2\beta}{2\sigma^2(1+\beta)}} I_{L-1} \left( \frac{\sqrt{2\sigma^2\beta}u}{\sigma^2(1+\beta)} \right) & \text{if } m = i \\
\frac{1}{(2\sigma^2)^2 \Gamma(L)} u^{L-1} e^{-\frac{u^2}{2\sigma^2}} & \text{if } m \neq i
\end{array} \right.
\]

(3.49)
where $\beta = 2\sigma_R^2 E_W$ and $\rho_T = L A^2 E_W$, and then substituting it into the third line of (3.35) and (3.38), which results in (after making use of Eq. 6.643.2 of [80]):

$$
\begin{align*}
P_s &= \sum_{r=1}^{M-1} \frac{(-1)^{r+1} e^{-\frac{\rho_T}{L+\beta}}}{[r(1+\beta)+1]^{L}} \binom{M-1}{r} \sum_{n=0}^{r(L-1)} \frac{\beta_{nr}}{\Gamma(L+n)} \frac{\Gamma(L+n)}{\Gamma(L)} \\
& \times \left[ \frac{1 + \beta}{r(1+\beta)+1} \right]^n \Phi \left( L + n, L, \frac{\rho_T/(1+\beta)}{[r(1+\beta)+1]} \right). \\
\end{align*}
$$

This expression is identical to Eq. (41) of [201], and reduces to Eq. (3.42) for $A = 0$.

**Lognormal Fading**

In the case of lognormal fading with $L = 1$, the pdf of $s$ is known to be given by:

$$
p_s(s) = \frac{1}{2\sqrt{2\pi\sigma_{LN}s}} \exp \left( \frac{1}{2} \ln(s)^2 - \frac{(\ln s - m_{LN})^2}{2\sigma_{LN}^2} \right). 
$$

However, for $L > 1$ an exact closed-form expression is difficult to obtain [203], although approximations can be made [204]. We will therefore consider only the flat-fading case, for which no exact closed-form solution is known either for Eq. (3.40) [22], for both the single- and multi-code cases.

### 3.2.2.6 Extension 1: Closed-Loop Power Control with the Inverse Update Algorithm

Closed-loop power control (CLPC) is used in the reverse link of IS-95 systems, and in both the forward and reverse links of IS-2000 systems [14]. Its goal is to compensate for the small-scale fading which affects the received signal, and it works on top of the open-loop power control mechanism which compensates for large-scale fading (shadowing) and path loss. In the reverse link of IS-95B, the CLPC algorithm sets a target received power at the BS, which should allow a certain FER to be attained while keeping to a minimum the interference to other users. The BS then instructs the MS to raise or lower its transmitted power in order to achieve this target, by sending it power control update commands at a certain rate (800 Hz for IS-95B and IS-2000). The MS adjusts its transmitted power in certain fixed size increments: for example, 1 dB, 0.5 dB and 0.25 dB.
increments are allowed in IS-95B. There is a necessary delay between the instant the BS makes a decision on the power control update command and the instant this command is executed by the MS. Fig. 3.3 gives a schematic block diagram of a CLPC algorithm, using the loglinear model of [205] (in which all the given powers are expressed in dB).

![Figure 3.3 Simplified loglinear closed-loop power control model [205].](image)

A purely analytical determination of the BER for a system using the previous CLPC algorithm is made difficult by the presence of nonlinearities (fixed size increments of the transmitted power) and delays. Hence, BER and FER studies of such a system have been based mostly on computer simulations [206], [207], [205], [208], [209].

Nevertheless, a simplified linear version of this algorithm, often denoted as the *inverse update algorithm* (IUA), has been used in the analysis of systems with both coherent [210] and noncoherent detection [211]. In the algorithm studied in the previous references, the power control increments have an infinite quantization precision, and in the case of [211], power control commands are issued by the BS at every symbol interval. Indeed, the MS multiplies its transmitted signal by the inverse of the square-root of the BS-estimated total power contained in the frequency-selective channel envelope. However this algorithm still considers the delay between the BS decision on the power control update command and the execution of this command. The analysis of such a theoretical system gives a lower bound on the BER which is attainable with a practical CLPC algorithm. If there was no power control updating delay in the system and the estimation process was perfect, the channel would revert to an AWGN channel.
Below, we extend the analysis given in [211] for the IUA to a multicode system. The BS issues power control commands updates at every symbol instant \((k = 1\) in Fig. 3.3), and these command bits are received error-free at the MS. There is a total loop delay of \(\tau_d = k_D T_w\) seconds. It is assumed that the BS estimates perfectly the power of the frequency-selective channel envelope at time \(t - \tau_d\), given for user 1 by \(\sum_{l=0}^{L_c-1} (\alpha_l^{(1)}(t - \tau_d))^2\) \((P_N(t) = 0\) in Fig. 3.3). At time \(t\), the MS (user 1) multiplies its transmitted signal by \(1/\sqrt{\sum_{l=0}^{L_c-1} (\alpha_l^{(1)}(t - \tau_d))^2}\) \((k_F = k_D\) in Fig. 3.3). Referring to the analysis given in previous sections, the \(\{II\}\) component of the desired received signal is now given by:

\[
S_{II}^{(m,n)} = \frac{\alpha_n^{(1)}}{\sqrt{\sum_{l=0}^{L_c-1} (\alpha_{l,F}^{(1)})^2}} \frac{\sqrt{E_W}}{2} \cos(\varphi_n^{(10)}) \delta_{mi}
\]

where \(\alpha_l^{(1)}\) is the sample of \(\alpha_l^{(1)}(t)\) taken \(\tau_d\) seconds before \(\alpha_l^{(1)}\). The variances of the multipath, multicode, and other-user multi-access interference terms are given respectively by:

\[
\sigma_I^2 = \frac{E_W}{N} \sum_{l=1}^{L_c-1} E \left[ \frac{(\alpha_l^{(1)})^2}{\sum_{l'=0}^{L_c-1} (\alpha_{l',F}^{(1)})^2} \right],
\]

\[
\sigma_I^3 = \frac{E_W}{N} (N^{(1)} - 1) \sum_{l=0}^{L_c-1} \frac{(\alpha_l^{(1)})^2}{\sum_{l'=0}^{L_c-1} (\alpha_{l',F}^{(1)})^2},
\]

\[
\sigma_I^2 = \frac{E_W}{6N} \sum_{k=2}^{K} N^{(k)} \sum_{l=0}^{L_c-1} E \left[ \frac{(\alpha_l^{(k)})^2}{\sum_{l'=0}^{L_c-1} (\alpha_{l',F}^{(k)})^2} \right].
\]

For i.i.d. Rayleigh fading multipaths, using Eq. 6, p. 73 of [116] (with \(m = 2\) along with Eqs. 9.131.1 and 9.100 of [80], it can be easily shown that:

\[
E \left[ \frac{(\alpha_l^{(k)})^2}{\sum_{l'=0}^{L_c-1} (\alpha_{l',F}^{(k)})^2} \right] = \frac{L - \rho^2(\tau_d)}{L(L - 1)}
\]
for $L = L_c > 1$ (which is given also in [211]), where $\rho(\tau)$ is the correlation coefficient of the channel. For example, for a land-mobile radio channel with isotropic scattering, it is recalled that $\rho(\tau) = J_0(2\pi f_m \tau)$ [113], where $f_m$ is the maximum Doppler frequency.

Again for the Rayleigh fading case, the pdf of $s' = \sum_{\tau=0}^{L-1}(\alpha_{\tau}^{(1)})^2/\sum_{\tau=0}^{L-1}(\alpha_{\tau}^{(1)})^2$, $p_{s'}(s')$, is obtained by a simple variable transformation of Eq. 1, p. 50 of [116] for i.i.d. channels (c.f. Eq. (8) of [211]), while its cdf is given by Eq. 44 of [210] for the general case of non-i.i.d. channels. The symbol error probability for this CLPC system can be obtained as before through Eq. (3.40), substituting $s$ with $s'$, and making use of Eqs. (3.53)-(3.56). It was shown in [211] that the pdf of $s'$ can be approximated by a lognormal pdf (Eq. 3.6) with parameters $m_{LN} = 0$ and $\sigma_{LN}^2 = \frac{1}{2}\ln[(L - \rho^2(\tau_p))/(L - 1)]$ (obtained by equating the first moments of the exact pdf and the lognormal pdf). It was also shown that this approximation was very satisfactory for an IS-95 system with more than $K = 8$ users. Section 3.2.3 presents the numerical results obtained using this approximation (which facilitates the numerical integration) for a multicode system.

3.2.2.7 Extension 2: Successive Interference Cancellation

Successive interference cancellation (SIC) is a multiuser detection technique [212] which consists in successively detecting each user, and substracting estimates of the previously detected users from the total signal-plus-interference signal received by the next user to be detected. Hence, in the ideal case, if all $K - 1$ users of a system with $K$ users have been correctly detected and substracted, then from the standpoint of the $K^{th}$ user the system will appear as an AWGN channel.

An early discussion of SIC applied to DS/CDMA, in terms of capacity achievable, was given by Viterbi in [213]. Other early papers are [214], in which SIC is used to cancel cochannel interference, and [215], in which SIC is performed in the frequency domain on a DS/CDMA system. In [216], the authors apply SIC to a system using noncoherent $M$-ary orthogonal modulation, similar to that used on the single-code IS-95 uplink, and perform a detailed BER analysis. Implementation aspects are discussed in [217]. In this section we extend (and somewhat modify, as discussed below) the analysis of [216] to deal with a multicode system.

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As in [216], at each iteration the correlation metrics (i.e. the signal and interference terms, instead of just the signal part, which would be difficult to extract from the total received composite signal-plus-interference) are used to remodulate the detected symbol which has been chosen in the previous iteration, and the resulting remodulated signal is subtracted from the remaining composite signal-plus-interference. Hence, taking into account the possibility of incorrect decisions on previously detected symbols, the low-pass filtered received signal on the $I$-branch at the $(h+1)^{th}$ iteration can be mathematically expressed as (taking, for clarity, $L = L_{c}$):

$$\tau_{I,LP}^{h+1}(t) = \tau_{I,LP}^{h}(t) - \sum_{l=0}^{L-1} Z_{II}^{(j(h),l),h-1} W_{j(h)}(t - \tau_{i}^{(h)}) a_{I}^{(h)}(t - \tau_{i}^{(h)})$$

$$- \sum_{l=0}^{L-1} Z_{IQ}^{(j(h),l),h-1} W_{j(h)}(t - T_{0} - \tau_{i}^{(h)}) a_{Q}^{(h)}(t - T_{0} - \tau_{i}^{(h)})$$

$$= \frac{1}{2} \sum_{k=1}^{K} \sum_{c=0}^{N(h)-1} \sum_{l=0}^{L-1} \sqrt{2} \alpha_{l}^{(k)}(t) \left[ W^{(kc)}(t - \tau_{i}^{(k)}) a_{I}^{(kc)}(t - \tau_{i}^{(k)}) \cos(\varphi_{I}^{(kc)})

+ W^{(kc)}(t - T_{0} - \tau_{i}^{(k)}) a_{Q}^{(kc)}(t - T_{0} - \tau_{i}^{(k)}) \sin(\varphi_{I}^{(kc)}) \right]

- \sum_{l=0}^{L-1} \sum_{i=0}^{h} \left[ c_{II}^{(j(h),l),i} W_{j(h)}(t - \tau_{i}^{(i)}) a_{I}^{(i)}(t - \tau_{i}^{(i)})

+ c_{IQ}^{(j(h),l),i} W_{j(h)}(t - T_{0} - \tau_{i}^{(i)}) a_{Q}^{(i)}(t - T_{0} - \tau_{i}^{(i)}) \right]$$

(3.57)

where $\tau_{I,LP}^{h+1}(t) = \tau_{I,LP}^{h}(t)$ is the low-pass filtered version of the $I$-component of Eq. (3.7). $h+1$ ($h = 0, 1, \ldots, K-1$) is the index of the user which is being detected, which we also choose, without loss of generality, to be equal to the iteration number (i.e. the users are successively detected according to their index). $K_{r}$ is the set of users which have not yet been decoded, or which have been incorrectly decoded in previous iterations. The index $j(h)$ corresponds to the index of the symbol which has been chosen in iteration $h$ (which can be either correct or incorrect). The $c_{II}^{(j(h),l),i}$ and $c_{IQ}^{(j(h),l),i}$ are interference terms due to imperfect cancellation of the correctly or incorrectly decoded users (since the correlation metrics $Z_{II}^{(j(h),l),h-1}, Z_{IQ}^{(j(h),l),h-1}$, instead of only the desired signals $S_{II}^{(j(h),l),h-1}, S_{IQ}^{(j(h),l),h-1}$, are used for cancellation), and will be detailed below.
Still at the $(h+1)^{th}$ iteration, the output of the $m^{th}$ correlator of the $n^{th}$ Rake finger, in response to the low-pass filtered received signal on the $I$-branch despread by $a_f^{(h+1,0)}(t)$, can be expressed as (c.f. Eq. (3.8)):

$$Z_{I,I}^{(m,n),h+1} = \frac{1}{\sqrt{T_W}} \int_{r_n^{(h+1)}}^{r_n^{(h+1)}+T_W} r_{I,I,P}(t) a_I^{(h+1,0)}(t - r_n^{(h+1)}) W_m(t - r_n^{(h+1)}) dt$$

$$= S_{I,I}^{(m,n),h+1} + I_{II}^{(m,n),h+1} + I_{II}^{(m,n),h+1} + I_{III}^{(m,n),h+1} + N_{II}^{(m,n),h+1}$$

$$- \frac{1}{\sqrt{T_W}} \int_{r_n^{(h+1)}}^{r_n^{(h+1)}+T_W} \sum_{i=0}^{L-1} \sum_{i=1}^{h} \left[ c_{II}^{(j(i),j,i)} W_j(i) (t - r_n^{(i)}) x a_I^{(h+1,0)}(t - r_n^{(h+1)}) W_m(t - r_n^{(h+1)}) dt$$

where:

$$S_{I,I}^{(m,n),h+1} = \alpha_n^{(h+1)} \sqrt{E_W} \cos(\varphi_n^{(h+1,0)}) \delta_{m,j(h+1)},$$

$$c_{I,I}^{(m,n),h+1} = I_{III}^{(m,n),h+1} + I_{II}^{(m,n),h+1} + I_{III}^{(m,n),h+1} + N_{II}^{(m,n),h+1}$$

$$c_{I,I}^{(m,n),1} = I_{III}^{(m,n),1} + I_{II}^{(m,n),1} + I_{III}^{(m,n),1} + N_{II}^{(m,n),1},$$

$$c_{I,Q}^{(m,n),h+1} = I_{IQ}^{(m,n),h+1} + I_{II}^{(m,n),h+1} + I_{III}^{(m,n),h+1} + N_{II}^{(m,n),h+1}$$

$$- \frac{1}{\sqrt{T_W}} \int_{r_n^{(h+1)}}^{r_n^{(h+1)}+T_W} \sum_{i=0}^{L-1} \sum_{i=1}^{h} \left[ c_{II}^{(j(i),j,i)} W_j(i) (r_n^{(i)} - T_0) + \hat{R}_{II}^{(i)} (r_n^{(i)} - T_0)$$

$$+ c_{I,Q}^{(j(i),j,i)} (R_{QQ}^{(i)} (r_n^{(i)} + T_0) + \hat{R}_{II}^{(i)} (r_n^{(i)} + T_0)),$$

$$c_{I,Q}^{(m,n),1} = I_{IQ}^{(m,n),1} + I_{II}^{(m,n),1} + I_{III}^{(m,n),1} + N_{II}^{(m,n),1},$$

$$(3.58)$$

$$(3.59)$$

$$(3.60)$$

$$(3.61)$$

$$(3.62)$$

$$(3.63)$$

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\[
I_{1II}^{(m,n),h+1} = \frac{\sqrt{E_W}}{2} \alpha_n^{(h+1)} \sin(\varphi_n^{(h+1,0)}) [R_{QII}^{(h+1,0)}(T_0) + \hat{R}_{QII}^{(h+1,0)}(T_0)],
\]
\[
I_{2II}^{(m,n),h+1} = \frac{\sqrt{E_W}}{2} \sum_{l=0}^{L-1} \alpha_l^{(h+1)} I_{II}^{(m,n),h+1}(h+1,0),
\]
\[
I_{3II}^{(m,n),h+1} = \frac{\sqrt{E_W}}{2} \sum_{c=1}^{N^{(h+1)-1}} \sum_{l=0}^{L-1} \alpha_l^{(h+1)} I_{II}^{(m,n),h+1}(h+1,c),
\]
\[
I_{II}^{(m,n),h+1} = \frac{\sqrt{E_W}}{2} \sum_{k=1}^{K} \sum_{c=0}^{N^{(k)}-1} \sum_{l=0}^{L-1} \alpha_l^{(k)} I_{II}^{(m,n),h+1}(kc),
\]
\[
N_{II}^{(m,n),h+1} = \frac{1}{2\sqrt{T_W}} \int_{\tau_{n}^{(h+1)} + T_W}^{\tau_{n}^{(h+1)}} n_c(t) a_{l}^{(h+1,0)}(t - \tau_{n}^{(h+1)}) W_{m}(t - \tau_{n}^{(h+1)}) dt,
\]

where

\[
I_{II}^{(m,n),h+1}(kc) = \frac{\sqrt{E_W}}{2} \left[ (R_{II}^{(kc)}(\tau_{nl}^{(k)}) + \hat{R}_{II}^{(kc)}(\tau_{nl}^{(k)})) \cos(\varphi_l^{(kc)}) \right. \\
\left. + (R_{QII}^{(kc)}(\tau_{nl}^{(k)} + T_0) + \hat{R}_{QII}^{(kc)}(\tau_{nl}^{(k)} + T_0)) \sin(\varphi_l^{(kc)}) \right]
\]

and \( \tau_{nl}^{(k)} = \tau_l^{(k)} - \tau_{n}^{(h+1)} \). The terms \( \{R_{xy}^{(k)}(\tau), \hat{R}_{xy}^{(k)}(\tau)\} \) are given by Eqs. (3.18)-(3.19), but now with:

\[
C_{x,y,mut}(\bar{l}) = \begin{cases} 
\sum_{j=0}^{N-1-l} w_{\frac{j}{p}, l, j+1}^{(kc)} a_{x,j}^{(h+1,0)} & 0 \leq \bar{l} \leq N - 1 \\
\sum_{j=0}^{N-1-l} w_{\frac{j}{p}, l - N}^{(kc)} a_{x,j}^{(h+1,0)} & 1 - N \leq \bar{l} < 0 \\
0 & |\bar{l}| \geq N
\end{cases}
\]

with \( p = N/M \). The term \( I_{II}^{(m,n),h+1} \) can be further broken down into:

\[
I_{II,und}^{(m,n),h+1} = \frac{\sqrt{E_W}}{2} \sum_{k=h+2}^{K} \sum_{c=0}^{N^{(k)}-1} \sum_{l=0}^{L-1} \alpha_l^{(k)} I_{II}^{(m,n),h+1}(kc),
\]
\[
I_{II,inc}^{(m,n),h+1} = \frac{\sqrt{E_W}}{2} \sum_{k=1}^{h} \sum_{c=0}^{N^{(k)}-1} \sum_{l=0}^{L-1} \alpha_l^{(k)} I_{II}^{(m,n),h+1}(kc)
\]
where \( I_{\text{fit}, \text{und}}^{(m,n),h+1} \) is the interference corresponding to the users which haven’t been decoded yet, and \( I_{\text{fit}, \text{inc}}^{(m,n),h+1} \) is the interference due to the users which have been incorrectly decoded and hence haven’t been cancelled (belonging to the set \( K_{\text{inc}} \)).

Similarly, signal and interference terms can be derived for \( Z_{iQ}^{(m,n),h+1} \), \( Z_{QI}^{(m,n),h+1} \) and \( Z_{QQ}^{(m,n),h+1} \), as was done in Appendix D, with:

\[
c_{QQ}^{(m,n),h+1} = I_{iQQ}^{(m,n),h+1} + I_{2QQ}^{(m,n),h+1} + I_{3QQ}^{(m,n),h+1} + I_{QQ}^{(m,n),h+1} + N_{QQ}^{(m,n),h+1} \]
\[
- \frac{1}{\sqrt{T_W}} \sum_{i=0}^{L-1} \sum_{i=1}^{h} \left[ c_{QQ}^{(j(i),l),i} \left( R_{QQ}^{(i)} (\gamma_{nl}) + \hat{R}_{QQ}^{(i)} (\tau_{nl}) \right) \right] + c_{QQ}^{(j(i),l),i} \left( R_{QI}^{(i)} (\gamma_{nl} - T_0) + \hat{R}_{QI}^{(i)} (\tau_{nl} - T_0) \right),
\]

(3.73)

\[
c_{QQ}^{(m,n),1} = I_{iQQ}^{(m,n),1} + I_{2QQ}^{(m,n),1} + I_{3QQ}^{(m,n),1} + I_{QQ}^{(m,n),1} + N_{QQ}^{(m,n),1},
\]

(3.74)

\[
c_{QI}^{(m,n),h+1} = I_{iQI}^{(m,n),h+1} + I_{2QI}^{(m,n),h+1} + I_{3QI}^{(m,n),h+1} + I_{QI}^{(m,n),h+1} + N_{QI}^{(m,n),h+1} \]
\[
- \frac{1}{\sqrt{T_W}} \sum_{i=0}^{L-1} \sum_{i=1}^{h} \left[ c_{QI}^{(j(i),l),i} \left( R_{QI}^{(i)} (\gamma_{nl} + T_0) + \hat{R}_{QI}^{(i)} (\tau_{nl} + T_0) \right) \right] - c_{QI}^{(j(i),l),i} \left( R_{QI}^{(i)} (\gamma_{nl} + T_0) + \hat{R}_{QI}^{(i)} (\tau_{nl} + T_0) \right),
\]

(3.75)

\[
c_{QI}^{(m,n),1} = I_{iQI}^{(m,n),1} + I_{2QI}^{(m,n),1} + I_{3QI}^{(m,n),1} + I_{QI}^{(m,n),1} + N_{QI}^{(m,n),1},
\]

(3.76)

The variances of the different interference terms can be obtained as (following Section 3.2.2.4):

\[
\sigma_N^2 = \text{Var}[N_{xy}^{(m,n),h+1}] = \frac{N_0}{4},
\]

(3.77)

\[
\sigma_{I_1}^2 = \text{Var}[I_{2xy}^{(m,n),h+1}] \approx \eta \frac{E_W}{N} \sum_{i=1}^{L-1} \Omega_i^{(h+1)},
\]

(3.78)

\[
\sigma_{I_3}^2 = \text{Var}[I_{3xy}^{(m,n),h+1}] = \eta \frac{E_W}{N} (N^{(h+1)} - 1) \sum_{i=0}^{L-1} (\alpha_i^{(h+1)})^2,
\]

(3.79)

\[
\sigma_{I_\text{und}}^2 = \text{Var}[I_{xy,\text{und}}^{(m,n),h+1}] = \frac{E_W}{6N} \sum_{k=h+2}^{K} N^{(k)} \sum_{l=0}^{L-1} \Omega_i^{(k)} P_s^k,
\]

(3.80)

\[
\sigma_{I_\text{inc}}^2 = \text{Var}[I_{xy,\text{inc}}^{(m,n),h+1}] = \frac{E_W}{6N} \sum_{k=1}^{h} N^{(k)} \sum_{l=0}^{L-1} \Omega_i^{(k)} P_s^k,
\]

(3.81)
where \( \eta = 1/4 \) if \( T_0 = 0 \) and \( \eta = 5/24 \) if \( T_0 \neq 0 \). In Eq. (3.81), \( P_s^k \) denotes the probability of symbol error at the \( k^{th} \) iteration. It is included in order to account for the effect of error propagation due to incorrect decisions during previous iterations. This serves only as an approximation, and its validity/limitations will be examined in Section 3.2.3. As mentioned in Section 3.2.2.4, \( \text{Var}[(R_{xy}^{(k)}(\tau_{n}^{(k)}) + \hat{R}_{xy}^{(k)}(\tau_{n}^{(k)}))] = 2T_w^2/(3N), k \neq h + 1. \)

Using Eqs. (3.77)-(3.81) and Eq. (3.60), the total variance of \( Z_{II}^{(m,n),h+1} + Z_{QQ}^{(m,n),h+1} \) is:

\[
\sigma_{h+1}^2 = \text{Var}[c_{II}^{(m,n),h+1}] + \text{Var}[c_{QQ}^{(m,n),h+1}]
\]

\[
= \frac{N_0}{2} + \frac{E_W}{2N}(N^{(h+1)} - 1)s + \frac{E_W}{2N} \sum_{i=1}^{L-1} \Omega_i^{(h+1)}
\]

\[
+ \frac{E_W}{3N} \sum_{k=h+2}^{K} N^{(k)} \sum_{i=0}^{L-1} \Omega_i^{(k)} + \frac{E_W}{3N} \sum_{k=1}^{h} (N^{(k)} \sum_{i=0}^{L-1} \Omega_i^{(k)}) P_s^k
\]

\[
+ \frac{2T_w}{3N} \sum_{i=0}^{L-1} \sum_{j=1}^{h} \left[ \text{Var}[c_{II}^{(j(i),i)}] + \text{Var}[c_{IQ}^{(j(i),i)}] + \text{Var}[c_{QQ}^{(j(i),i)}] + \text{Var}[c_{QI}^{(j(i),i)}] \right]
\] (3.82)

where \( s = \sum_{i=1}^{L-1} (\alpha_i^{(h+1)})^2 \).

The error probability at iteration \( h+1 \) is thus obtained by first successively computing the error probabilities \( P_s^k, k = 1, \ldots, h \), substituting these values in Eq. (3.82), and then making use of Eq. (3.40).

**3.2.2.8 Extension 3: Multiple-Cells System with Hard Handoff**

The analysis in the previous sections only took into account the effect of interference from users of the same cell as that of the desired one. In this section we extend this analysis to the multiple-cells case, in order to reflect a more realistic cellular environment. The cellular environment considered is sketched in Fig. 3.4. We assume a cluster of seven hexagonal equal-size cells, corresponding to the home cell (0) and the first tier of interfering cells (1-6). More than one tier can be considered, but the insights gained are small compared to the increase in simulation time. We employ the commonly used model where the mobile stations are uniformly distributed over each cell. The emitted signals are subject to path loss, with a commonly chosen path loss exponent \( \gamma = 4 \) [218], and lognormal shadowing with standard deviation \( \sigma_L = 8 \) dB. We make the assumption
that each MS is power-controlled by the BS of the cell in which it is physically located: we leave the effects of soft handoff and BS macrodiversity for future consideration. As a result of the reverse-link open-loop power control mechanism, the path loss and lognormal fading are compensated for in the case of the MS's of cell 0, as seen by the home cell BS, while they must be taken into account when characterizing the interference from the first-tier cells. Closed-loop power control is not considered, but can be straightforwardly integrated as per the previous section.

Let user \{q, k\} denote the kth user of cell q, for \( k = 1,2,\ldots,K, \) \( q = 0,1,\ldots,Q' - 1 \) \((Q' = 7)\). The composite signal received at BS 0 can be mathematically expressed as:

\[
r(t) = n(t) + \sum_{q=0}^{Q'-1} \sum_{k=1}^{K} \sum_{c=0}^{N(qk)-1} \sum_{l=0}^{L_c-1} \sqrt{P_{qk} g_{qk} \alpha_{qk}^{(qk)}(t)} \times \left[ W_{qkc}^{(qkc)}(t - \tau_{t}^{(qk)}) a_{qk}^{(qkc)}(t - \tau_{t}^{(qk)}) \cos(\omega_c t + \phi_{qkc}^{(qkc)}) \right. \\
\left. + W_{qkc}^{(qkc)}(t - T_0 - \tau_{t}^{(qk)}) a_{qk}^{(qkc)}(t - T_0 - \tau_{t}^{(qk)}) \sin(\omega_c t + \phi_{qkc}^{(qkc)}) \right] \tag{3.83}
\]
where \( g^{(q,k)} = \left( r_{0}^{(q,k)} \right)^{-\gamma} 10^{\xi_{0}^{(q,k)}/10} \) is the combined path loss and lognormal attenuation function for the path between user \( \{q, k\} \) and BS 0, and \( \zeta^{(q,k)} = \left( r_{q}^{(q,k)} \right)^{-\gamma} 10^{-\xi_{q}^{(q,k)}/10} \) is the open-loop power control function applied to user \( \{q, k\} \) by its home BS \( q \). \( r_{0}^{(q,k)} \) and \( r_{q}^{(q,k)} \) are the distances between user \( \{q, k\} \) and BS’s 0 and \( q \), respectively. \( \xi_{q}^{(q,k)} \) and \( \xi_{0}^{(q,k)} \) are Gaussian random variables with standard deviation \( \sigma_{L} = 8 \text{ dB} \), and correlation coefficient \( E[\xi_{q}^{(q,k)}\xi_{0}^{(q,k)}]/\sigma_{L}^{2} = 1/2 \) [219]. The presence of the subscript \( q \) on the remaining terms and quantities means that the latter are related to cell \( q \).

An outer-cell interference term \( I_{II,o}^{(m,n)} \) is now added to the decision metric of Eq. (3.9).

This term is defined as:

\[
I_{II,o}^{(m,n)} = C \int_{\tau_{n}^{(0,1)}}^{\tau_{n}^{(0,1)}+T_{W}} \sum_{q=1}^{Q-1} \sum_{k=1}^{K} \sum_{c=0}^{N^{(q,k)}-1} \sum_{l=0}^{L_{c}-1} \sqrt{\rho^{(q,k)}\xi^{(q,k)}}(t)W_{m}(t - \tau_{l}^{(0,1)})\alpha_{l}^{(0,1)}(t - \tau_{n}^{(0,1)})
\]

\[
\times [W^{(q,k)c}(t - \tau_{l}^{(q,k)})\alpha_{l}^{(q,k)}(t - \tau_{l}^{(q,k)}) \cos(\phi^{(q,k)c})
\]

\[
+ W^{(q,k)c}(t - \tau_{l}^{(q,k)} - T_{0})\alpha_{Q}^{(q,k)}(t - \tau_{l}^{(q,k)} - T_{0}) \sin(\phi^{(q,k)c})]dt
\]

(3.84)

where:

\[
\rho^{(q,k)} = g^{(q,k)}\zeta^{(q,k)} = \left( \frac{r_{q}^{(q,k)}}{r_{0}^{(q,k)}} \right)^{\gamma} 10^{-\xi_{0}^{(q,k)}/10}. \tag{3.85}
\]

Its variance is given by:

\[
\sigma_{I_{II,o}}^{2} = \text{Var}[I_{II,o}^{(m,n)}] = \frac{E_{W}}{6N^{2}} \sum_{q=1}^{Q} \sum_{k=1}^{K} \sum_{c=0}^{N^{(q,k)}-1} \sum_{l=0}^{L_{c}-1} \alpha_{l}^{(q,k)}. \tag{3.86}
\]

with

\[
\bar{\rho} = E[\rho^{(q,k)}] = E \left[ \left( \frac{r_{q}^{(q,k)}}{r_{0}^{(q,k)}} \right)^{\gamma} \right] E \left[ 10^{\frac{\xi_{0}^{(q,k)}-\xi_{q}^{(q,k)}}{10}} \right], \tag{3.87}
\]

by assuming that the quantities \( \xi_{q}^{(q,k)} \) are independent of \( r_{q}^{(q,k)} \). The first expectation on the right-hand side of Eq. (3.87) can be computed as in Chap. 10 of [177] or [220], which
assume circular cells, and results in:
\begin{equation}
E \left[ \left( \frac{\tau_{q}^{(qk)}}{\tau_{q}^{(qk)}} \right)^{2} \right] = 12 \ln 3 - \frac{19}{4} \approx 0.11558.
\end{equation}

The second expectation can be evaluated as in [218], which leads to:
\begin{equation}
E \left[ \frac{\sigma_{l}^{(qk)} - \xi_{l}^{(q)}}{10} \right] = e^{\frac{1}{2}(\sigma_{l} \ln 10)^{2}}.
\end{equation}

Hence \( \bar{\rho} \approx 0.6304 \). The new variance of the total interference is then:
\begin{equation}
\sigma_{mc}^{2} = 2(\sigma_{q2}^{2} + \sigma_{l3}^{2} + \sigma_{l1}^{2} + \sigma_{l0}^{2} + \sigma_{h}^{2}).
\end{equation}

Thus in the multiple-cells environment with hard handoff, the BER is obtained by using the same equations as before, but with \( \sigma^{2} \) replaced by \( \sigma_{mc}^{2} \).

### 3.2.3 Performance Evaluation Results and Discussion

We implemented a multi-cell IS-95B software simulator in order to check the validity of our equations. The BER is evaluated through Monte Carlo error counting. The PN sequences are as specified in [14]. Without loss of generality, we have used \( M = 8 \) instead of \( M = 64 \) (as used in the standard) in order to speed up the evaluation of Eq. (3.40). With a bit rate \( R_b = 28.8 \text{ kbps} \) and a chip rate \( R_c = 1.228800 \text{ Mbps} \), this results in a processing gain of \( N = (\log_2 M) R_c / R_b = 128 \).

Unless stated otherwise, the channel is Rayleigh fading with \( \Omega_{l}^{(k)} = 1 \) for all \( l, k \), and \( N^{(k)} = 1, k = 2, 3, \ldots, K \), while \( N^{(1)} \) can take different values. At first we consider a system with \( N^{(1)} = 8 \) codes. We compare the results obtained in two cases:

i) Through the use of Eq. (3.40), which is the proposed approach;

ii) Through the use of Eq. (3.42), where

\begin{equation}
\sigma^{2} = \frac{N_{0}}{2} + E_{\text{w}} \left( \frac{3}{2} \left[ (N^{(1)} - 1) \sum_{l=0}^{L_{c}-1} \Omega_{l}^{(1)} + \sum_{l=1}^{L_{c}-1} \Omega_{l}^{(1)} \right] + \sum_{k=2}^{K} N^{(k)} \sum_{l=0}^{L_{c}-1} \Omega_{l}^{(k)} \right)
\end{equation}
The second approach has been used in the past for example in [75] and [193], but for the case of coherent reception. The BER curves obtained using both methods are plotted for \( L = 1, 2, 3, 4, 5 \), along with the simulation results, in Figs. 3.5-3.9.

From these it is apparent that the use of Eq. (3.42) is not appropriate for the evaluation of the BER in a multicode CDMA system. In the case \( L = 1 \), i.e. when no multipath interference is present, the approximation provided by Eq. (3.42) gets worse as the number of users in the system decreases. Indeed, this equation assumes that the fading coefficients affecting the multicode self-interference of the desired user \( (I_{31f}) \) are independent of the fading experienced by the latter, which is obviously incorrect since all the codes of this user fade in unison. As a consequence, it does not take into account the fact that when the desired user's received signal is low due to deep fading, the multicode self-interference will also be decreased proportionally, leading to a better BER than that predicted by Eq. (3.42). When multipath interference is present, we observe that Eq. (3.42) differs from the simulated results for most values of \( K \). For higher values of \( L \), we notice that for values of \( K \) greater than a certain breakpoint, \( K_b \), Eq. (3.42) is optimistic compared to the simulation results. As \( L \) increases, \( K_b \) becomes smaller and the gap between the two curves gets more important. For \( L = 5 \), Eq. (3.42) underestimates the BER for all values of \( K \). In contrast to the previous observations, Eq. (3.40) leads to values of the BER very close to the simulated ones over the whole range of \( K \). Indeed it captures the dependence between the fading coefficients of all of the desired user's codes, which is necessary for an accurate BER evaluation.

Next, Fig. 3.10 illustrates the use of Eq. (3.40) in evaluating the BER of a system with \( N^{(1)} = 4 \), for \( L = 1, 2, 3 \). For low values of interference (smaller \( K \)'s), the use of diversity produces a small improvement, with the largest performance gain obtained by going from one diversity branch to two, which is a well-known fact [20]. However, after a certain value of \( K \), the use of diversity slightly increases the BER: indeed, the level of interference becomes such that the use of EGC worsens the situation, which is due to the noncoherent combining loss described for example in [26], Section 12.1.1, or [177], Section 9.3.2.1. The same type of results are plotted for \( N^{(1)} = 8 \) in Fig. 3.11. As before, for a high level of interference the use of diversity actually increases the BER. However, in this case the use of \( L = 3 \) does not bring any improvement over \( L = 2 \), and
even slightly worsens the performance for most of the $K$'s: because of the high number of parallel codes transmitted by the desired user, the degradation due to the increase in multicode self-interference offsets the gain realized by diversity.

Figs. 3.12-3.14 illustrate the influence of the number of codes $N^{(1)}$ on the system BER, for $L = 1, 2, 3$. When $L = 1$, the difference in BER for the cases $N^{(1)} = 1, 4$ and $8$ is relatively small. As the number of branches grows ($L = 3$), the gap in BER increases. As before, this can be explained by the larger amount of multicode self-interference due to the multipaths, since the multiple-access interference from other users remains the same.

If $N^{(k)} > 1$ for any user $k > 1$, the extra interference produced by the additional codes of this user is accounted for through Eq. (3.32). With respect to the desired user, the interference produced by each supplemental code of another same-cell multi-code user is equivalent to the interference of a separate same-cell single-code user. Fig. 3.15 illustrates the BER of the desired user (with $N^{(1)} = 1$), when user 2 transmits $N^{(2)} = 4$ codes (for $L = 1, 2, 3$). Fig. 3.16 illustrates similar results when $N^{(2)} = 8$.

![Figure 3.5 BER vs K for m = 1, L = 1, N^{(1)} = 8. - : Eq. (3.40); - - : Eq. (3.42); + : simulation.](image-url)
Figure 3.6 BER vs $K$ for $m = 1$, $L = 2$, $N^{(1)} = 8$. - : Eq. (3.40); - - : Eq. (3.42); + : simulation.

Figure 3.7 BER vs $K$ for $m = 1$, $L = 3$, $N^{(1)} = 8$. - : Eq. (3.40); - - : Eq. (3.42); + : simulation.
Figure 3.8 BER vs $K$ for $m = 1$, $L = 4$, $N^{(1)} = 8$. - : Eq. (3.40); - - : Eq. (3.42); + : simulation.

Figure 3.9 BER vs $K$ for $m = 1$, $L = 5$, $N^{(1)} = 8$. - : Eq. (3.40); - - : Eq. (3.42); + : simulation.
Figure 3.10 BER vs $K$ for $m = 1$, $N^{(1)} = 4$. — (†): $L = 1$; - . - (∗): $L = 2$; - - (○): $L = 3$.

Figure 3.11 BER vs $K$ for $m = 1$, $N^{(1)} = 8$. — (†): $L = 1$; - . - (∗): $L = 2$; - - (○): $L = 3$. 
Figure 3.12 BER vs $K$ for $m = 1$, $L = 1$. — (+): $N^{(1)} = 1$; – - (·): $N^{(1)} = 4$; – – (○): $N^{(1)} = 8$.

Figure 3.13 BER vs $K$ for $m = 1$, $L = 2$. — (+): $N^{(1)} = 1$; – - (·): $N^{(1)} = 4$; – – (○): $N^{(1)} = 8$. 

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Figure 3.14 BER vs $K$ for $m = 1$, $L = 3$. — (+): $N^{(1)} = 1$; - - (•): $N^{(1)} = 4$; - - (o): $N^{(1)} = 8$.

Figure 3.15 BER vs $K$ for $m = 1$, $N^{(1)} = 1$, $N^{(2)} = 4$. — (+): $L = 1$; - - (•): $L = 2$; - - (o): $L = 3$. 

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Figure 3.16 BER vs K for \( m = 1, N^{(1)} = 1, N^{(2)} = 8. \) --- (+): \( L = 1; \) -- (*) : \( L = 2; \) --- (o): \( L = 3. \)

Figs. 3.17-3.19 show the effect of the Nakagami-m parameter on the system, for \( L = 1, 2, 3 \) and \( N^{(1)} = 8. \) As expected, it is noticed that as the \( m \)-parameter increases, the performance improves. However, this gain in performance becomes smaller with the number of diversity branches: for example, with \( L = 3, \) there is little difference between systems with \( m = 2 \) and \( m = 3. \) Indeed, while a high value for the \( m \)-parameter increases the desired signal, it also results in larger multicode self-interference, which grows with the number of multipaths.

Figs. 3.20 and 3.21 show the effect of the number of diversity branches on systems with \( m = 2 \) and 3, respectively.

Fig. 3.22 plots the BER of a system with 2 correlated diversity branches (\( \rho = 0.5 \)) in Nakagami fading with \( m = 2, \) for different numbers of codes. Fig. 3.23 shows the effect of the correlation coefficient \( \rho \) on a similar system with \( N^{(1)} = 8: \) for \( \rho \) smaller than 0.7, the correlation between branches doesn't significantly affect the BER. Simulation results were obtained using the method of 2.4.1.2 (\( L \) arbitrary) and are seen to match very well the theoretical curves.
Figure 3.17 BER vs K for \( L = 1, N^{(1)} = 8 \). \(- (+): m = 1; - . - (*): m = 2; - - (o): m = 3.\)

Figure 3.18 BER vs K for \( L = 2, N^{(1)} = 8 \). \(- (-): m = 1; - . - (*): m = 2; - - (o): m = 3.\)
Figure 3.19 BER vs $K$ for $L = 3$, $N^{(1)} = 8$. — (+): $m = 1$; - - (*): $m = 2$; - - (o): $m = 3$.

Figure 3.20 BER vs $K$ for $m = 2$, $N^{(1)} = 8$. — (+): $L = 1$; - - (*): $L = 2$; - - (o): $L = 3$. 
Figure 3.21 BER vs $K$ for $m = 3$, $N^{(1)} = 8$. — (+): $L = 1$; . . . (*) $L = 2$; -- (o): $L = 3$.

Figure 3.22 BER vs $K$ for $m = 2$, $L = 2$, $\rho = 0.5$. — (+): $N^{(1)} = 1$; . . . (*) $N^{(1)} = 4$; -- (o): $N^{(1)} = 8$. 
Figs. 3.24 and 3.25 plot the BER for a system with Rician fading and $A = 1.0, 2.0$, respectively, for $L = 1, 2, 3$ and $N^{(1)} = 8$. It is seen that the BER increases with the number of diversity branches for large $K$'s, which is explained by a larger noncoherent combining loss due to the non-fading component.

![Figure 3.23 BER vs K for m = 2, L = 2, N^{(1)} = 8. — (+): \rho = 0.0; -- (--): \rho = 0.3; -- (o): \rho = 0.5; -- (o): \rho = 0.7.](image)

Figure 3.23 BER vs $K$ for $m = 2, L = 2, N^{(1)} = 8$. — (+): $\rho = 0.0$; -- (--): $\rho = 0.3$; -- (o): $\rho = 0.5$; -- (o): $\rho = 0.7$.

Fig. 3.26 shows results for the lognormal fading case with $L = 1, N^{(1)} = 8, m_{LN} = 0$ and $\sigma_{LN} = 0.2, 0.5, 1.0$. Slight discrepancies (less than a factor of 2) are noticeable for $\sigma_{LN} = 1$ in the range $2 < K < 15$. The value of $\sigma_{LN}$ has an important impact on the BER, especially for low $K$'s.

Figs. 3.27 and 3.28 plot the BER for a system using the CLPC inverse update algorithm of Section 3.2.2.6, for $L = 2$ and $3$, respectively, with a loop delay $k_D = 4$ and different number of codes $N^{(1)}$. The channel fading is Rayleigh-distributed. It can be seen that the lognormal approximation which is used in the analysis fares better for larger $N^{(1)}$'s, i.e. for a larger value of the total interference, which is consistent with the observations of [211]. Hence, while this approximation is accurate for a single-code when $K$ is at least 8 or so, for a multicode system it is accurate for a lower number of
Figure 3.24 BER vs $K$ for a Rician channel with $A = 1.0$, $N^{(1)} = 8$. --- (+): $L = 1$; --: $L = 2$; -- (*) $L = 3$.

interfering users, depending on the value of $N^{(1)}$ (e.g. for $N^{(1)} = 8$, it is accurate for all $K$).

Figs. 3.29 and 3.30 illustrate the effect of the loop delay $k_D$ on the BER, for $L = 2$, and $N^{(1)} = 4$ and 8, respectively. A performance degradation is noticed for higher values of $k_D$, however it is less important for higher values of $L$.

Figs. 3.31 and 3.32 show results for a system of $K$ users using SIC, where the BER is that of the last user to be decoded (user $K$), given that all previous $K - 1$ users have been tentatively cancelled. All cancelled users transmit only one code ($N^{(k)} = 1$, $k = 1, \ldots, K - 1$), while the last one transmits $N^{(K)}$ codes. From both the theoretical and simulation results it can be seen that as more codes are used (a higher $N^{(K)}$), more incorrect decisions are made in the earlier cancellation iterations, leading to error propagation and a higher BER. The theoretical results in general overestimate the BER, especially in the low-user region for $L > 1$: the discrepancies are imputable to the approximation which had to be made in the analysis in order to include detection errors and error propagation, the effect of which is very difficult to account for in an exact manner.
Figure 3.25 BER vs $K$ for a Rician channel with $A = 2.0$, $N^{(1)} = 8$. — (+): $L = 1$; — — (*): $L = 2$; — — (o): $L = 3$.

Figure 3.26 BER vs $K$ for lognormal fading, $L = 1$, $N^{(1)} = 8$, $m_{LN} = 0$. — (+): $\sigma_{LN} = 1.0$; — — (*): $\sigma_{LN} = 0.5$; — — (o): $\sigma_{LN} = 0.2$. 

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Figure 3.27 BER vs $K$ for $m = 1$, $L = 2$, and CLPC with $k_D = 4$. — (+): $N^{(1)} = 1$; — (-): $N^{(1)} = 4$; — (o): $N^{(1)} = 8$.

Figure 3.28 BER vs $K$ for $m = 1$, $L = 3$, and CLPC with $k_D = 4$. — (+): $N^{(1)} = 1$; — (-): $N^{(1)} = 4$; — (o): $N^{(1)} = 8$. 
Figure 3.29 BER vs $K$ for $m = 1$, $L = 2$, $N^{(1)} = 4$ and CLPC for different $k_D$. — (+): $k_D = 1$; — — (*): $k_D = 2$; — . — (o): $k_D = 4$.

Figure 3.30 BER vs $K$ for $m = 1$, $L = 2$, $N^{(1)} = 8$ and CLPC for different $k_D$. — (+): $k_D = 1$; — — (*): $k_D = 2$; — . — (o): $k_D = 4$. 
Figure 3.31 BER vs $K$ for $m = 1$, $L = 1$, and SIC for different $N^{(1)}$. — (+): $N^{(1)} = 1$; — (*): $N^{(1)} = 4$; — (o): $N^{(1)} = 8$.

Figure 3.32 BER vs $K$ for $m = 1$, $L = 2$, and SIC for different $N^{(1)}$. — (+): $N^{(1)} = 1$; — (*): $N^{(1)} = 4$; — (o): $N^{(1)} = 8$. 

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Results for a multi-cell system are illustrated in Figs. 3.33 and 3.34, with \( N^{(1)} = 4 \) and \( N^{(1)} = 8 \), respectively. Similar remarks as for Figs. 3.10 and 3.11 can be made: in particular, the use of more than two diversity branches does not improve the BER for all values of \( K \), especially that the multiple-access interference is now higher because of the interference contributions of other cells. Figs. 3.35-3.37 further illustrate the effect of the \( m \)-parameter for Nakagami fading, for \( L = 1, 2, 3 \) and \( N^{(1)} = 8 \). Figs. 3.38-3.40 plot the BER as a function of the number of codes \( N^{(1)} \) for a flat-fading \( (L = 1) \) lognormal channel and different variances \( \sigma_{LN} \).

### 3.2.4 Conclusions

The goal of this section was to provide an accurate analysis of reverse link multicode CDMA systems with noncoherent \( M \)-ary modulation and equal-gain combining in multipath fading channels. We provided convenient expressions for the interference terms as a function of the aperiodic crosscorrelation functions. The derivation of the error probability relied on the fact that the multicode self-interference is dependent on the desired received signal, because both are subject to the same level of fading. This led to a considerable improvement in accuracy as compared to a simpler method. Using this expression for the BER, we verified that the use of EGC allows improvement only for a certain range of values of the total interference seen at the receiver. In particular, when the number of users is too large, the BER is slightly increased by the use of EGC. Also, for users with a large number of codes, the use of diversity increases considerably the multicode self-interference, which can lead to reduced performance compared to a system with little or no diversity. The analysis applies to any type of fading, and results were given for the common cases of Rayleigh, Nakagami, Rician and lognormal fading. The case of correlated diversity branches was also treated within the same framework. The analysis was extended to deal with closed-loop power control using the inverse update algorithm, successive interference cancellation, and multi-cells systems.
Figure 3.33 Multicell system: BER vs $K$ for $m = 1$, $N^{(1)} = 4$. — (+): $L = 1$; -- (*) $L = 2$; -- (o): $L = 3$.

Figure 3.34 Multicell system: BER vs $K$ for $m = 1$, $N^{(1)} = 8$. — (+): $L = 1$; -- (*) $L = 2$; -- (o): $L = 3$. 
Figure 3.35 Multicell system: BER vs $K$ for $L = 1$, $N^{(1)} = 8$. — (+): $m = 1$; -- (-): $m = 2$; -- (o): $m = 3$.

Figure 3.36 Multicell system: BER vs $K$ for $L = 2$, $N^{(1)} = 8$. — (+): $m = 1$; -- (-): $m = 2$; -- (o): $m = 3$. 
Figure 3.37 Multicell system: BER vs \( K \) for \( L = 3, N^{(1)} = 8 \). — (+): \( m = 1 \); — (*): \( m = 2 \); — (o): \( m = 3 \).

Figure 3.38 Multicell system: BER vs \( K \) for lognormal fading, \( L = 1, \sigma_{LN} = 0.2, m_{LN} = 0 \). — (+): \( N^{(1)} = 1 \); — (*): \( N^{(1)} = 4 \); — (o): \( N^{(1)} = 8 \).
Figure 3.39 Multicell system: BER vs $K$ for lognormal fading, $L = 1$, $\sigma_{LN} = 0.5$, $m_{LN} = 0$. 
— (+): $N^{(1)} = 1$; - - (*): $N^{(1)} = 4$; . - (o): $N^{(1)} = 8$.

Figure 3.40 Multicell system: BER vs $K$ for lognormal fading, $L = 1$, $\sigma_{LN} = 1.0$, $m_{LN} = 0$. 
— (+): $N^{(1)} = 1$; - - (*): $N^{(1)} = 4$; . - (o): $N^{(1)} = 8$. 

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3.3 Reverse Link Performance with Coherent BPSK Modulation and Complex Spreading Sequences

3.3.1 Introduction

The reverse link of an IS-2000 system supports simultaneously a number of control or data channels, each one spread by a certain Walsh code in order to maintain orthogonality. As such, we can view its operation as a form of multicode transmission. In addition, some of the channels (such as the Supplemental Channels) support multiple spreading rates, leading to multirate transmission. Unlike the IS-95 systems from which it partially evolved, the IS-2000 system makes use of complex spreading sequences in order to limit the power imbalance between its in-phase \((I)\) and quadrature \((Q)\) branches. Also, coherent modulation is used on both forward and reverse links. An analysis of a IS-2000-type uplink with coherent modulation was provided in [221]. However, it assumed only one channel per \(I/Q\) branch (the pilot channel on the \(I\), and the data channel on the \(Q\)). Other earlier works which derived the SER of CDMA systems with complex sequences for various modulation schemes include [222], [223], [224], [225]. All considered single-code transmission.

In this section, we consider the general situation where multiple channels are transmitted on each branch, and derive the BER performance of such a multicode CDMA system with complex spreading sequences, in a multipath fading environment. In such a system, the fading which affects the multicode self-interference and the multipath self-interference is not independent of the desired code: indeed, since all codes of a same user are transmitted through the same physical channel, they fade in unison. Therefore, unlike most recent analyses of coherent multicode CDMA which assumed independent fading, we take into account the dependence between the fading of self-interference and that of the desired code. We will show that this is necessary to obtain reliable results, especially for the case of few interfering users. We derive an expression for the BER where the self-interference is conditional on the fading coefficients affecting the desired code. We then integrate numerically this expression over the pdf of the fading. This method was also used in the previous section for the case of noncoherent \(M\)-ary modulation with equal-gain combining.
The organization of this section is as follows. Section 3.3.2.1 describes the signal model, and Section 3.3.2.2 details the receiver model and the decision metrics, whose statistics are derived in Section 3.3.2.3. Section 3.3.2.4 presents the error probability analysis, which is validated against simulation results in Section 3.3.3.

3.3.2 Error Probability Analysis

3.3.2.1 Signal and Channel Models

The transmitter uses BPSK modulation with quadrature branches, in which different channels are mapped to each of the $I$ and $Q$ branches. For example, in the reverse link of a cdma2000 system, the Pilot Channel (PCH) and the Supplemental Channel 2 (SCH2) are mapped onto the $I$-branch, while the Fundamental Channel (FCH), the Dedicated Control Channel (DCCH) and the Supplemental Channel 1 (SCH1) are mapped onto the $Q$-branch. Let $N_I^{(k)}$ and $N_Q^{(k)}$ be the number of channels (and thus codes) assigned to branches $I$ and $Q$ of user $k$, where $k = 1, 2, \ldots, K$. We denote by $\{kc\}$ code $c$ ($c = 0, 1, \ldots, N_I^{(k)} - 1$ or $N_Q^{(k)} - 1$) of user $k$, for either the $I$ or $Q$-branch. The streams of binary data symbols for code $\{kc\}$ of branches $I$ and $Q$ are given in the time-domain by:

$$b_I^{(kc)}(t) = \sum_{j=-\infty}^{\infty} b_{I,j}^{(kc)}(t - jT_b), \quad b_{I,j}^{(kc)} \in [-1, 1]$$

$$b_Q^{(kc)}(t) = \sum_{j=-\infty}^{\infty} b_{Q,j}^{(kc)}(t - jT_b), \quad b_{Q,j}^{(kc)} \in [-1, 1]$$

where $T_b$ is the symbol (bit) period. The transmitter uses concatenated spreading, as defined in the introduction. Walsh sequences are used for orthogonal covering, as in the cdma2000 standard. Each Walsh sequence corresponds to a row of a Hadamard matrix of size $M$ ([177], Ch.5), and thus consists of $M$ elements taking values of 1 or -1 (mapped from symbols 0 and 1, respectively, of the Hadamard matrix). The Walsh sequence
assigned to code \{kc\} of the I-branch, \( W_{I}^{(kc)} = [w_{I,0}^{(kc)} w_{I,1}^{(kc)} \ldots w_{I,M-1}^{(kc)}] \), satisfies:

\[
(W_{I}^{(kc)} \cdot W_{I}^{(jd)}) = \begin{cases} 
M & \text{if } (kc) = (jd) \\
0 & \text{if } (kc) \neq (jd)
\end{cases}
\]

where \((x \cdot y)\) is the scalar product operation between vectors \(x\) and \(y\). The periodic orthogonal covering sequence can be expressed in the time domain as:

\[
W_{I,p}^{(kc)}(t) = \sum_{h=-\infty}^{\infty} W_{I}^{(kc)}(t - hT_W) = \sum_{h=-\infty}^{\infty} \sum_{i=0}^{M-1} w_{I,i}^{(kc)} p_{Wc}(t - hT_W - iT_{wc}) \tag{3.92}
\]

where \(T_W\) is the duration of \(W_{I}^{(kc)}(t)\) and period of \(W_{I,p}^{(kc)}(t)\). \(p_{Wc}(t)\) is a rectangular pulse of unit amplitude and duration \(T_{wc} = T_W / M\) seconds. The Walsh sequence assigned to code \{kc\} of the Q-branch, \(W_{Q}^{(kc)}\), is defined in a similar manner.

After the orthogonal covering phase, the data on the I and Q-branches are spread by the same complex spreading sequence \(a^{(k)}(t) = a_{I}^{(k)}(t) + ja_{Q}^{(k)}(t)\), where \(j = \sqrt{-1}\) and:

\[
a_{I}^{(k)}(t) = \sum_{j=-\infty}^{\infty} a_{I,j}^{(k)} p_{Tc}(t - jT_c), \quad a_{I,j}^{(k)} \in [-1,1] \tag{3.93}
\]

\[
a_{Q}^{(k)}(t) = \sum_{j=-\infty}^{\infty} a_{Q,j}^{(k)} p_{Tc}(t - jT_c), \quad a_{Q,j}^{(k)} \in [-1,1] \tag{3.94}
\]

where \(T_c\) is the PN chip duration. The chip durations of the Walsh and PN sequences are not necessarily equal. For example, in cdma2000 systems \(T_{wc}\) can be a multiple of \(T_c\). After complex spreading, the real and imaginary parts of the output are separated and modulated onto orthogonal carriers, as illustrated in Fig. 3.41.
The transmitted signal of the \( k \)th user can be expressed as:

\[
\begin{align*}
    s^{(k)}(t) &= \sum_{c=0}^{N_I^{(k)}-1} s_{II}^{(k)}(t) \cos(w_c t + \phi_I^{(kc)}) - \sum_{c=0}^{N_Q^{(k)}-1} s_{IQ}^{(k)}(t) \cos(w_c t + \phi_Q^{(kc)}) \\
    &+ \sum_{c=0}^{N_I^{(k)}-1} s_{IQ}^{(k)}(t) \sin(w_c t + \phi_I^{(kc)}) + \sum_{c=0}^{N_Q^{(k)}-1} s_{QQ}^{(k)}(t) \sin(w_c t + \phi_Q^{(kc)})
\end{align*}
\] (3.95)

where

\[
\begin{align*}
    s_{II}^{(k)}(t) &= \sqrt{P_I^{(kc)} b_I^{(kc)}(t)} W_I^{(kc)}(t) a_I^{(k)}(t), \\
    s_{IQ}^{(k)}(t) &= \sqrt{P_Q^{(kc)} b_Q^{(kc)}(t)} W_Q^{(kc)}(t) a_Q^{(k)}(t), \\
    s_{IQ}^{(k)}(t) &= \sqrt{P_I^{(kc)} b_I^{(kc)}(t)} W_I^{(kc)}(t) a_Q^{(k)}(t), \\
    s_{QQ}^{(k)}(t) &= \sqrt{P_Q^{(kc)} b_Q^{(kc)}(t)} W_Q^{(kc)}(t) a_I^{(k)}(t)
\end{align*}
\]

and \( P_x^{(kc)} = E_{b_x}^{(kc)} P_x^{(kc)} \) is the average power of code \( \{kc\} \) of branch \( x \) (\( x = I \) or \( Q \)). The channel model is the same as that used in the previous section.

### 3.3.2.2 Receiver Model and Decision Metrics

The composite signal received at the output of the channel is:

\[
r(t) = \sum_{k=1}^{K} \sum_{l=0}^{L_c-1} \alpha_l^{(k)}(t) s^{(k)}(t - \tau_l^{(k)}) + n(t)
\] (3.96)
$n(t)$ is AWGN with double-sided power spectral density $N_0/2$. Note that all the received codes pertaining to a same user are affected by the same fading coefficients and delay, since they are transmitted via the same channel. The phases, however, can be different if a specific phase shift is assigned to each channel by the transmitter (as in IS-95B).

The receiver consists of $L$ Rake fingers and performs either MRC or EGC. One finger of the receiver is illustrated in Fig. 3.42. In the following, the metrics calculated refer to code 0 of user 1 of branch $I$, which is assumed to be the desired stream. They further correspond to finger $n$ of the Rake receiver. Without any loss of generality, we assume that $N_{I}^{(k)} = N_{Q}^{(k)} = N^{(k)} \forall k$ in order to ease the notation.

The quantities $u_I^{(n)}(t)$ and $u_Q^{(n)}(t)$ at the output of the lowpass filters are given by:

$$u_I^{(n)}(t) = (r(t) \cos(w_c t + \varphi_{I,n}^{(10)}))_{LP}$$

$$= \frac{1}{2} \sum_{k=1}^{K} \sum_{c=0}^{N^{(k)}-1} \sum_{l=0}^{L_c-1} \alpha_I^{(k)}(t) (s_{I}^{(k)}(t - \tau_{l}^{(k)}) \cos(\varphi_{I,n,l}^{(10)}) - s_{Q}^{(k)}(t - \tau_{l}^{(k)}) \cos(\varphi_{Q,n,l}^{(10)}))$$

$$+ s_{I}^{(k)}(t - \tau_{l}^{(k)}) \sin(\varphi_{I,n,l}^{(10)}) + s_{Q}^{(k)}(t - \tau_{l}^{(k)}) \sin(\varphi_{Q,n,l}^{(10)}))$$

$$+ (n(t) \cos(w_c t + \varphi_{I,n}^{(10)}))_{LP}$$

(3.97)

$$u_Q^{(n)}(t) = (r(t) \sin(w_c t + \varphi_{I,n}^{(10)}))_{LP}$$

$$= \frac{1}{2} \sum_{k=1}^{K} \sum_{c=0}^{N^{(k)}-1} \sum_{l=0}^{L_c-1} \alpha_I^{(k)}(t) (s_{I}^{(k)}(t - \tau_{l}^{(k)}) (- \sin(\varphi_{I,n,l}^{(10)})) - s_{Q}^{(k)}(t - \tau_{l}^{(k)}) (- \sin(\varphi_{Q,n,l}^{(10)}))$$

$$+ s_{I}^{(k)}(t - \tau_{l}^{(k)}) \cos(\varphi_{I,n,l}^{(10)}) + s_{Q}^{(k)}(t - \tau_{l}^{(k)}) \cos(\varphi_{Q,n,l}^{(10)}))$$

$$+ (n(t) \sin(w_c t + \varphi_{I,n}^{(10)}))_{LP}$$

(3.98)
where (·)_{LP} denotes low-pass filtering, \( \varphi^{(kc)}_{x,i} = \phi^{(k)} - \theta^{(k)} - w^{(k)} \), and \( \varphi^{(kc)}_{x,i} = \varphi^{(kc)}_{x,i} - \varphi^{(10)}_{I,n} \).

The output of the complex spreading operation is given by:

\[
U^{(n)}(t) = (u^{(n)}_i(t) + ju^{(n)}_Q(t))a^{(1)}(t - \tau^{(1)}_{I,n})^*
\]

\[
= [u^{(n)}_i(t)a^{(1)}(t - \tau^{(1)}_{I,n}) + u^{(n)}_Q(t)a^{(1)}(t - \tau^{(1)}_{I,n})] + j[-u^{(n)}_i(t)a^{(1)}(t - \tau^{(1)}_{I,n}) + u^{(n)}_Q(t)a^{(1)}(t - \tau^{(1)}_{I,n})]
\]

(3.99)

where (·)* stands for complex conjugate. From Fig. 3.42, the decision metric for Rake finger \( n \) is thus given by:

\[
U^{(n)}_I = \int_{\tau^{(1)}_{I,n}}^{\tau^{(10)}_{I,n}} \text{Re}[U^{(n)}(t)]W^{(10)}_I(t - \tau^{(1)}_{I,n})dt
\]

\[
= \frac{1}{2} \sum_{k=1}^{K} \sum_{c=0}^{N^{(k)}-1} \sum_{l=0}^{L^{(k)}-1} \int_{\tau^{(1)}_{I,n}}^{\tau^{(10)}_{I,n}} a^{(k)}_l(t) \times
\]

\[
[\sqrt{P^{(k)}_I}b^{(k)}_I(t - \tau^{(k)}_I)W^{(k)}_I(t - \tau^{(k)}_I)a^{(k)}_I(t)W^{(10)}_I(t - \tau^{(1)}_{I,n})a^{(1)}_I(t - \tau^{(1)}_{I,n})cos(\varphi^{(10)}_{I,n})
\]

\[-\sqrt{P^{(k)}_Q}b^{(k)}_Q(t - \tau^{(k)}_I)W^{(k)}_Q(t - \tau^{(k)}_I)a^{(k)}_Q(t - \tau^{(k)}_I)W^{(10)}_I(t - \tau^{(1)}_{I,n})a^{(1)}_I(t - \tau^{(1)}_{I,n})cos(\varphi^{(10)}_{I,n})
\]

\[+\sqrt{P^{(k)}_I}b^{(k)}_I(t - \tau^{(k)}_I)W^{(k)}_I(t - \tau^{(k)}_I)a^{(k)}_I(t - \tau^{(k)}_I)W^{(10)}_I(t - \tau^{(1)}_{I,n})a^{(1)}_I(t - \tau^{(1)}_{I,n})sin(\varphi^{(10)}_{I,n})
\]

\[+\sqrt{P^{(k)}_Q}b^{(k)}_Q(t - \tau^{(k)}_I)W^{(k)}_Q(t - \tau^{(k)}_I)a^{(k)}_Q(t - \tau^{(k)}_I)W^{(10)}_I(t - \tau^{(1)}_{I,n})a^{(1)}_I(t - \tau^{(1)}_{I,n})sin(\varphi^{(10)}_{I,n})
\]

\[-\sqrt{P^{(k)}_I}b^{(k)}_I(t - \tau^{(k)}_I)W^{(k)}_I(t - \tau^{(k)}_I)a^{(k)}_I(t - \tau^{(k)}_I)W^{(10)}_I(t - \tau^{(1)}_{I,n})a^{(1)}_Q(t - \tau^{(1)}_{I,n})sin(\varphi^{(10)}_{I,n})
\]

\[+\sqrt{P^{(k)}_Q}b^{(k)}_Q(t - \tau^{(k)}_I)W^{(k)}_Q(t - \tau^{(k)}_I)a^{(k)}_Q(t - \tau^{(k)}_I)W^{(10)}_I(t - \tau^{(1)}_{I,n})a^{(1)}_Q(t - \tau^{(1)}_{I,n})sin(\varphi^{(10)}_{I,n})
\]

\[+\sqrt{P^{(k)}_I}b^{(k)}_I(t - \tau^{(k)}_I)W^{(k)}_I(t - \tau^{(k)}_I)a^{(k)}_I(t - \tau^{(k)}_I)W^{(10)}_I(t - \tau^{(1)}_{I,n})a^{(1)}_Q(t - \tau^{(1)}_{I,n})cos(\varphi^{(10)}_{I,n})
\]

\[+\sqrt{P^{(k)}_Q}b^{(k)}_Q(t - \tau^{(k)}_I)W^{(k)}_Q(t - \tau^{(k)}_I)a^{(k)}_Q(t - \tau^{(k)}_I)W^{(10)}_I(t - \tau^{(1)}_{I,n})a^{(1)}_Q(t - \tau^{(1)}_{I,n})cos(\varphi^{(10)}_{I,n})
\]

\[+I^{(n)}_{N,I}
\]

\[= S^{(n)}_I + I^{(n)}_{MP,I} + I^{(n)}_{CS,I} + I^{(n)}_{MC,I} + I^{(n)}_{MA,I} + I^{(n)}_{N,I}
\]

(3.100)

where Re[·] is the real part operator. \( S^{(n)}_I \) is the signal term, \( I^{(n)}_{MP,I} \) is the multipath self-interference, \( I^{(n)}_{CS,I} \) is the complex spreading self-interference, \( I^{(n)}_{MC,I} \) is the multicode self-interference, \( I^{(n)}_{MA,I} \) is the other-user multi-access interference, and \( I^{(n)}_{N,I} \) is an integrated noise term, all referring to the \( n \)th finger. In order to obtain closed-form expressions for
the previous signal and interference terms, we use the following aperiodic crosscorrelation functions:

\[ R_{xy,wz}^{(kc)}(\tau) = [C_{xy,wz}^{(kc)}(\bar{i} + 1 - N) - C_{xy,wz}^{(kc)}(\bar{i} - N)](\tau - \bar{T}_c) + [C_{xy,wz}^{(kc)}(\bar{i} - N)]T_c. \tag{3.101} \]

\[ \hat{R}_{xy,wz}^{(kc)}(\tau) = [C_{xy,wz}^{(kc)}(\bar{i} + 1) - C_{xy,wz}^{(kc)}(\bar{i})](\tau - \bar{T}_c) + [C_{xy,wz}^{(kc)}(\bar{i})]T_c. \tag{3.102} \]

with:

\[
C_{xy,wz}^{(kc)}(\bar{i}) = \begin{cases} 
\sum_{j=0}^{N-1-\bar{i}} w_{w,j} a_{x,j}^{(kc)} w_{z,\lfloor \frac{\bar{i}+T}{N} \rfloor} a_{y,j+\bar{i}}^{(10)} & 0 \leq \bar{i} \leq N - 1 \\
\sum_{j=0}^{N-1+\bar{i}} w_{w,j} a_{x,j-\bar{i}-N} w_{z,\lfloor \frac{\bar{i}}{N} \rfloor} a_{y,j}^{(10)} & 1 - N \leq \bar{i} < 0 \\
0 & |\bar{i}| \geq N 
\end{cases} \tag{3.103}
\]

and where \( N = T_b/T_c \) is the processing gain, \( h = T_{wc}/T_c \) is the number of PN chips per Walsh chip, \( \bar{i} = [(T^k - \bar{T}_c)/T_c] \). The indices \( \{xy\} \) and \( \{wz\} \) are chosen from \( \{II\}, \{IQ\}, \{QI\} \) or \( \{QQ\} \). After expanding the expression for \( U^{(n)}_I \), the terms in Eq. (3.100) can be written as follows:

\[
S^{(n)}_I = \alpha^{(1)}_n \sqrt{P^{(10)}_I b^{(10)}_{I,0} T^{(10)}_c}, \tag{3.104}
\]

\[
I^{(n)}_{N,I} = \int_{\tau^{(1)}_n}^{\tau^{(1)}_n + \tau^{(1)}_{b,I}} W^{(10)}_I(t - \tau^{(1)}_n) \times \\
[(n(t) \cos(w_c t + \varphi^{(10)}_{I,n}))_{LP} a^{(1)}_{I}(t - \tau^{(1)}_n) + (n(t) \sin(w_c t + \varphi^{(10)}_{I,n}))_{LP} a^{(1)}_{Q}(t - \tau^{(1)}_n)] dt, \tag{3.105}
\]

\[
I^{(n)}_{MP,I} = \frac{1}{2} \sum_{l=0}^{L-1} \sum_{l \neq n} \alpha^{(1)}_l \sqrt{P^{(10)}_I} \cos(\varphi^{(10)}_{I,ml}) \times \\
[b^{(10)}_{I,m} (R^{(10)}_{II,II}(\tau^{(1)}) + R^{(10)}_{QQ,II}(\tau^{(1)})) + b^{(10)}_{I,w} (\hat{R}^{(10)}_{II,II}(\tau^{(1)}) + \hat{R}^{(10)}_{QQ,II}(\tau^{(1)}))] \tag{3.106}
\]

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\[ I_{CS,I}^{(n)} = \frac{1}{2} \sum_{l=0}^{L_c-1} \alpha_{l}^{(1)} \left[ -\sqrt{P_{Q}^{(10)}} (b_{Q,u}^{(10)} R_{Q,I}^{(10)} (\tau^{(1)}) + b_{Q,v}^{(10)} \tilde{R}_{Q,I}^{(10)} (\tau^{(1)})) \cos(\varphi_{Q,nl}) \\
+ \sqrt{P_{Q}^{(10)}} (b_{Q,u}^{(10)} R_{Q,I}^{(10)} (\tau^{(1)}) + b_{Q,v}^{(10)} \tilde{R}_{Q,I}^{(10)} (\tau^{(1)})) \sin(\varphi_{Q,nl}) \\
+ \sqrt{P_{Q}^{(10)}} (b_{Q,u}^{(10)} R_{Q,I}^{(10)} (\tau^{(1)}) + b_{Q,v}^{(10)} \tilde{R}_{Q,I}^{(10)} (\tau^{(1)})) \sin(\varphi_{Q,nl}) \\
- \sqrt{P_{I}^{(10)}} (b_{I,u}^{(10)} R_{I,I}^{(10)} (\tau^{(1)}) + b_{I,v}^{(10)} \tilde{R}_{I,I}^{(10)} (\tau^{(1)})) \sin(\varphi_{I,nl}) \right] \] (3.107)

\[ I_{MC,I}^{(n)} = \sum_{c=1}^{N^{(1)}-1} \sum_{l=0}^{L_c-1} \alpha_{l}^{(1)} I_{I}^{(n)} (1, c), \] (3.108)

\[ I_{MA,I}^{(n)} = \sum_{k=2}^{K} \sum_{c=0}^{N^{(k)}-1} \sum_{l=0}^{L_c-1} \alpha_{l}^{(k)} I_{I}^{(n)} (k, c), \] (3.109)

where:

\[ I_{I}^{(n)} (k, c) = \frac{1}{2} \left[ -\sqrt{P_{I}^{(k)}} (b_{I,I}^{(k)} R_{I,I}^{(k)} (\tau^{(k)}) + b_{I,I}^{(k)} \tilde{R}_{I,I}^{(k)} (\tau^{(k)})) \cos(\varphi_{I,nl}) \\
- \sqrt{P_{Q}^{(k)}} (b_{Q,u}^{(k)} R_{Q,I}^{(k)} (\tau^{(k)}) + b_{Q,v}^{(k)} \tilde{R}_{Q,I}^{(k)} (\tau^{(k)})) \cos(\varphi_{Q,nl}) \\
+ \sqrt{P_{Q}^{(k)}} (b_{Q,u}^{(k)} R_{Q,I}^{(k)} (\tau^{(k)}) + b_{Q,v}^{(k)} \tilde{R}_{Q,I}^{(k)} (\tau^{(k)})) \sin(\varphi_{Q,nl}) \\
- \sqrt{P_{I}^{(k)}} (b_{I,u}^{(k)} R_{I,I}^{(k)} (\tau^{(k)}) + b_{I,v}^{(k)} \tilde{R}_{I,I}^{(k)} (\tau^{(k)})) \sin(\varphi_{I,nl}) \right] \] (3.110)
and where \( \{u,v\} = \{-1,0\} \) and \( \tau^{(k)} = \tau_{nl}^{(k)} \) if \( \tau_{nl}^{(k)} > 0 \), and \( \{u,v\} = \{0,1\} \) and \( \tau^{(k)} = \tau_{nl}^{(k)} + T_b \) if \( \tau_{nl}^{(k)} < 0 \). Notice that for \( I_{MC,I}^{(n)} \) the component for which indice \( l = n \) has been removed. Indeed, when the collected multipath components are synchronous with the desired signal (which happens when \( l = n \)), the interference produced by these components is null due to the orthogonal covering of the spreading sequences.

A similar development can be carried out for the decision metrics on the \( Q \) branch, leading to the following:

\[
U_Q^{(n)} = \int_{\tau_n^{(1)}}^{\tau_n^{(1)} + \tau_b^{(10)}} \text{Im}[U_Q^{(n)}(t)]W_Q^{(10)}(t - \tau^{(1)}) dt
\]

\[
= S_Q^{(n)} + I_{MP,Q}^{(n)} + I_{CS,Q}^{(n)} + I_{MC,Q}^{(n)} + I_{MA,Q}^{(n)} + I_{N,Q}^{(n)} \quad (3.111)
\]

where \( \text{Im}[\cdot] \) is the imaginary part operator, and:

\[
S_Q^{(n)} = a_n^{(1)} \sqrt{P_Q^{(10)} b_Q^{(10)} T_b^{(10)}} , \quad (3.112)
\]

\[
I_{N,Q}^{(n)} = \int_{\tau_n^{(1)}}^{\tau_n^{(1)} + \tau_b^{(10)}} W_f^{(10)}(t - \tau_n^{(1)}) \times
\]

\[
[-(n(t) \cos(w_c t + \varphi_{I,n}^{(10)}))_{LP} a_Q^{(1)}(t - \tau_n^{(1)})
\]

\[
+ (n(t) \sin(w_c t + \varphi_{I,n}^{(10)}))_{LP} a_f^{(1)}(t - \tau_n^{(1)})] dt \quad (3.113)
\]

\[
I_{MP,Q}^{(n)} = \frac{1}{2} \sum_{l=0}^{L_v-1} \alpha_l^{(1)} \sqrt{P_Q^{(10)} \cos(\varphi_{Q,nl}^{(10)})} \times
\]

\[
[b_{Q,u}^{(10)} (R_Q^{(10)} Q^{(1)} + R_Q^{(10)} Q^{(1)}) + b_{Q,v}^{(10)} (R_Q^{(10)} Q^{(1)} + R_Q^{(10)} Q^{(1)})] \quad (3.114)
\]

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\[ I_{CS,Q}^{(n)} = \frac{1}{2} \sum_{l=0 \atop l \neq n}^{L_e-1} \alpha_l^{(1)} \left[ -\sqrt{P_l^{(10)}} b_{I,I,Q}^{(10)}(\tau) + b_{I,v}^{(10)} \hat{R}_{I,I,Q}^{(10)}(\tau) \right] \cos(\varphi_{I,I}) \]
\[ + \sqrt{P_l^{(10)}} b_{I,I,I,I,Q}^{(10)}(\tau) + b_{I,v}^{(10)} \hat{R}_{I,I,I,I,Q}^{(10)}(\tau) \sin(\varphi_{I,I}) \]
\[ + \sqrt{P_Q^{(10)}} b_{Q,I,I,I,Q}^{(10)}(\tau) + b_{Q,v}^{(10)} \hat{R}_{Q,I,I,I,Q}^{(10)}(\tau) \cos(\varphi_{Q,I}) \]
\[ - \sqrt{P_Q^{(10)}} b_{Q,I,I,I,I,Q}^{(10)}(\tau) + b_{Q,v}^{(10)} \hat{R}_{Q,I,I,I,I,Q}^{(10)}(\tau) \sin(\varphi_{Q,I}) \]
\[ + \sqrt{P_I^{(10)}} b_{I,I,I,I,I,I,Q}^{(10)}(\tau) + b_{I,v}^{(10)} \hat{R}_{I,I,I,I,I,I,Q}^{(10)}(\tau) \cos(\varphi_{I,I}) \]
\[ + \sqrt{P_I^{(10)}} b_{I,I,I,I,I,I,I,I,Q}^{(10)}(\tau) + b_{I,v}^{(10)} \hat{R}_{I,I,I,I,I,I,I,I,Q}^{(10)}(\tau) \cos(\varphi_{I,I,I}) \]
\[ + \sqrt{P_Q^{(10)}} b_{Q,I,I,I,I,I,I,Q}^{(10)}(\tau) + b_{Q,v}^{(10)} \hat{R}_{Q,I,I,I,I,I,I,Q}^{(10)}(\tau) \cos(\varphi_{Q,I,I}) \]
\[ + \sqrt{P_Q^{(10)}} b_{Q,I,I,I,I,I,I,I,I,Q}^{(10)}(\tau) + b_{Q,v}^{(10)} \hat{R}_{Q,I,I,I,I,I,I,I,I,Q}^{(10)}(\tau) \cos(\varphi_{Q,I,I,I}) \]  
\[ + \sqrt{P_I^{(10)}} b_{I,I,I,I,I,I,I,I,I,I,Q}^{(10)}(\tau) + b_{I,v}^{(10)} \hat{R}_{I,I,I,I,I,I,I,I,I,I,Q}^{(10)}(\tau) \cos(\varphi_{I,I,I,I}) \]
\[ + \sqrt{P_Q^{(10)}} b_{Q,I,I,I,I,I,I,I,I,I,I,Q}^{(10)}(\tau) + b_{Q,v}^{(10)} \hat{R}_{Q,I,I,I,I,I,I,I,I,I,I,Q}^{(10)}(\tau) \cos(\varphi_{Q,I,I,I,I}) \]  
\[ + \sqrt{P_I^{(10)}} b_{I,I,I,I,I,I,I,I,I,I,I,I,I,I,Q}^{(10)}(\tau) + b_{I,v}^{(10)} \hat{R}_{I,I,I,I,I,I,I,I,I,I,I,I,I,I,Q}^{(10)}(\tau) \cos(\varphi_{I,I,I,I,I}) \]
### 3.3.2.3 Statistics of Decision Metrics

**Maximal-Ratio Combining**

Let $E_b = E_{b,l}^{(10)}$ and $T_b = T_{b,l}^{(10)}$. With MRC, the expected value of the desired received signal $S_I = \sum_{n=0}^{L-1} \alpha_n^{(1)} S^{(n)}_I$ is obtained from Eq. (3.104):

$$
\bar{S}_I = \sqrt{E_b T_b} \sum_{n=0}^{L-1} |\alpha_n^{(1)}|^2.
$$

(3.119)

The noise term $I_{N,I} = \sum_{n=0}^{L-1} \alpha_n^{(1)} I_{N,I}^{(n)}$ can be easily shown to be a zero-mean Gaussian random variables with variance:

$$
\sigma_{N,I}^2 = \text{Var}[I_{N,I}] = \frac{T_b N_0}{2} \sum_{n=0}^{L-1} |\alpha_n^{(1)}|^2.
$$

(3.120)

Using the standard Gaussian approximation, we model the interference terms Eqs. (3.106)-(3.109) as mutually uncorrelated zero-mean Gaussian random variables with given conditional variances. These variances are derived below, and are conditional on the random variables $\alpha_l^{(1)}$, $l = 1, 2, \ldots, L$. The interference terms $I_{MP,I} = \sum_{n=0}^{L-1} \alpha_n^{(1)} I_{MP,I}^{(n)}$, $I_{CS,I} = \sum_{n=0}^{L-1} \alpha_n^{(1)} I_{CS,I}^{(n)}$, and $I_{MC,I} = \sum_{n=0}^{L-1} \alpha_n^{(1)} I_{MC,I}^{(n)}$ are chip synchronous with the desired user signal, since they originate from signals which are transmitted synchronously on the reverse link. Therefore $E_{nl}^{(1c)} = E_{nl}^{(1)}$ for all $k, n$ and $l$. This results in $\text{Var}[R_{xy,uz}(\tau_{nl}^{(1c)}) + \hat{R}_{xy,uz}^{(1c)}(\tau_{nl}^{(1c)})] = T_b^2 / N$ (with $l \neq n$). However, the interference term $I_{MA,I} = \sum_{n=0}^{L-1} \alpha_n^{(1)} I_{MA,I}^{(n)}$ is chip asynchronous with the desired user signal, from which $\text{Var}[R_{xy,uz}^{(1c)}(\tau_{nl}^{(k)}) + \hat{R}_{xy,uz}^{(1c)}(\tau_{nl}^{(k)})] = 2T_b^2 / (3N)$, $k = 2, 3, \ldots, K$. From these observations, the following variances are obtained:

$$
\sigma_{IMP,I}^2 = \text{Var}[I_{MP,I}] = \frac{P_I^{(10)} T_b^2}{4N} \sum_{n=0}^{L-1} \alpha_n^{(1)} \sum_{l=0}^{L_e-1} \alpha_l^{(1)}|^2,
$$

(3.121)

$$
\sigma_{ICS,I}^2 = \text{Var}[I_{CS,I}] = \frac{T_b^2}{4N} \sum_{n=0}^{L-1} \alpha_n^{(1)} \sum_{l=0}^{L_e-1} \alpha_l^{(1)}|^2 + 2P_Q \sum_{l=0}^{L_e-1} \alpha_l^{(1)}|^2.
$$

(3.122)
\[
\sigma_{MC,I}^2 = \text{Var}[I_{MC,I}] = \sum_{n=0}^{L-1} [\alpha_{n}^{(1)}]^2 \sum_{c=1}^{N(k) - 1} \frac{(P_c^{(1c)} + P_Q^{(1c)}) T_b^2 L_c - 1}{2N} \sum_{l=0}^{2L_c - 1} [\alpha_{l}^{(1)}]^2, \quad (3.123)
\]

\[
\sigma_{MA,I}^2 = \text{Var}[I_{MA,I}] = \sum_{n=0}^{L-1} [\alpha_{n}^{(1)}]^2 \sum_{k=2}^{K} \sum_{c=0}^{N(k) - 1} \frac{(P_c^{(kc)} + P_Q^{(kc)}) T_k^2 L_c - 1}{3N} \sum_{l=0}^{\Omega_{l}^{(k)}}. \quad (3.124)
\]

In Eqs. (3.121), (3.122) and (3.123), the variances are seen to be conditional on the term \(\sum_{l=0}^{L_c - 1} [\alpha_{l}^{(1)}]^2\). Indeed, the self-interference terms \(I_{MP,I}, I_{CS,I}\) and \(I_{MC,I}\) are affected by the same coefficients \(\alpha_{l}^{(1)}, l = 0, 1, \ldots, L - 1\) as the desired signal \(S_I\), since they all fade in unison. Thus these interference terms are correlated with the signal term, and this fact must be taken into account when determining the error probability. In contrast, the other-user multiple-access interference term \(I_{MA,I}\) is affected by the fading coefficients \(\alpha_{l}^{(k)}, l = 0, 1, \ldots, L - 1, k = 2, 3, \ldots, K\), which are independent from \(\alpha_{l}^{(1)}, l = 0, 1, \ldots, L - 1\): indeed, the signals from each user are assumed to travel along different paths to the base station, and hence experience independent fading. As a consequence, the squared fading coefficients \((\alpha_{l}^{(k)})^2, l = 0, 1, \ldots, L - 1, k = 2, 3, \ldots, K\) can be replaced by their expected values \(\Omega_{l}^{(k)} = E[(\alpha_{l}^{(k)})^2]\), as was done in most analyses using the Gaussian approximation [39].

The total variance from the interference and thermal noise sums up to:

\[
\sigma_I^2 = \sigma_{I_{MP,I}}^2 + \sigma_{I_{MC,I}}^2 + \sigma_{I_{CS,I}}^2 + \sigma_{I_{MA,I}}^2 + \sigma_N^2. \quad (3.125)
\]

**Equal-Gain Combining**

With EGC, the signal and interference terms are now \(S_I = \sum_{n=0}^{L-1} S_I^{(n)}, I_{N,I} = \sum_{n=0}^{L-1} I_{N,I}^{(n)}, I_{MP,I} = \sum_{n=0}^{L-1} I_{MP,I}^{(n)}, I_{CS,I} = \sum_{n=0}^{L-1} I_{CS,I}^{(n)}, I_{MC,I} = \sum_{n=0}^{L-1} I_{MC,I}^{(n)},\) and \(I_{MA,I} = \sum_{n=0}^{L-1} I_{MA,I}^{(n)}\).

Proceeding as in the previous section, the signal term is now:

\[
\bar{S}_I = \sqrt{E_b T_b} \sum_{n=0}^{L-1} \alpha_{n}^{(1)} \quad (3.126)
\]
and the variances of the (Gaussian-modeled) interference terms:

\[
\sigma_{IMP,I}^2 = \text{Var}[I_{MP,I}] = \frac{P_I^{(10)}T_b^2}{4N} L \sum_{l=0}^{L_c-1} [\alpha_l^{(1)}]^2,
\]

\[(3.127)\]

\[
\sigma_{ICS,I}^2 = \text{Var}[I_{CS,I}] = \frac{T_b^2}{4N} L (P_I^{(10)} \sum_{l=0}^{L_c-1} [\alpha_l^{(1)}]^2 + 2P_Q^{(10)} \sum_{l=0}^{L_c-1} [\alpha_l^{(1)}]^2),
\]

\[(3.128)\]

\[
\sigma_{IMC,I}^2 = \text{Var}[I_{MC,I}] = L \sum_{c=1}^{N^{(k)}-1} \frac{(P_I^{(1c)} + P_Q^{(1c)})T_b^2}{2N} \sum_{l=0}^{L_c-1} [\alpha_l^{(1)}]^2,
\]

\[(3.129)\]

\[
\sigma_{IMA,I}^2 = \text{Var}[I_{MA,I}] = L \sum_{k=2}^{K} \sum_{c=0}^{N^{(k)}-1} \frac{(P_I^{(kc)} + P_Q^{(kc)})T_b^2}{3N} \sum_{l=0}^{L_c-1} \Omega_l^{(k)}.
\]

\[(3.130)\]

The total variance from the interference and thermal noise is obtained as per Eq. (3.125).

### 3.3.2.4 Probability of Error

In order to clarify the presentation of the BER derivation, but without loss of generality, we take the number of fingers of the Rake receiver, \(L\), to be equal to the number of multipaths \(L_c\) (i.e. all the energy of the channel is collected). If some paths are not collected (i.e. \(L_c > L\)), then the interference resulting from these paths can be considered as additional independent Gaussian noise, and incorporated in the term \(I_{MA,I}\), without having an impact on the validity of the BER derivation: indeed, the fading coefficients \(\alpha_l^{(1)}, l = L, \ldots, L_c - 1\) are independent of the desired signal, because in this case they don't appear in the expression for \(\bar{S}_I\), i.e. Eq. (3.119). It is considered unlikely to have \(L_c < L\), because in this case the Rake receiver would simply switch off its fingers which aren't collecting any signal, instead of adding background noise to the received signal. Moreover, if the powers in each multipath are similar, we can replace the term
\[ \sum_{i=0}^{L-1} |\alpha_i^{(1)}|^2 \text{ by } \sum_{i=1}^{L-1} |\alpha_i^{(1)}|^2 \text{ in order to ease the analysis, as in [39].} \]

**Maximal-Ratio Combining**

From Eqs. (3.119), (3.125) and (3.121)-(3.124), and using the previous assumptions, the conditional output SNR at the receiver, \( SNR_0 = \frac{S_f^2}{\sigma_f^2} \), takes the following form:

\[
SNR_0 = \frac{1}{A \left( \sum_{i=0}^{L-1} |\alpha_i^{(1)}|^2 \right) + B \left( \sum_{i=1}^{L-1} |\alpha_i^{(1)}|^2 \right) + C}
\]

where the terms \( A \), \( B \) and \( C \) are constants given by:

\[
A = \frac{1}{4N}, \quad (3.132)
\]

\[
B = \frac{1}{4N} \left( 1 + 2 \frac{E_{b,Q}^{(1)}}{E_b} \frac{T_{b,Q}^{(10)}}{T_{b,Q}^{(10)}} \right) + \frac{1}{2N} \sum_{c=1}^{N(1)-1} \left( \frac{E_{b,I}^{(1)} T_b}{E_b T_{b,I}^{(1c)}} + \frac{E_{b,Q}^{(1)} T_b}{E_b T_{b,Q}^{(1c)}} \right), \quad (3.133)
\]

\[
C = \frac{1}{3N} \sum_{k=2}^{K} \sum_{c=0}^{N(k)-1} \left( \frac{E_{b,I}^{(k,c)} T_b}{E_b T_{b,I}^{(k,c)}} + \frac{E_{b,Q}^{(k,c)} T_b}{E_b T_{b,Q}^{(k,c)}} \right) \sum_{i=0}^{L-1} \omega_i^{(k)} + \frac{N_0}{2E_b}. \quad (3.134)
\]

If all the energies and bit periods are the same, the expressions for \( B \) and \( C \) reduce to:

\[
B = \frac{3}{4N} + \frac{1}{N} (N^{(1)} - 1), \quad (3.135)
\]

\[
C = \frac{2}{3N} \sum_{k=2}^{K} \sum_{c=0}^{N(k)-1} \sum_{i=0}^{L-1} \omega_i^{(k)} + \frac{N_0}{2E_b}. \quad (3.136)
\]

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Moreover, if no data is transmitted on the Q-branch, i.e. BPSK modulation is used, these expressions become:

\[ B = \frac{1}{4N} + \frac{1}{2N}(N^{(1)} - 1), \]  
\[ (3.137) \]

\[ C = \frac{1}{3N} \sum_{k=2}^{K} \sum_{c=0}^{N^{(k)}-1} \sum_{t=0}^{L-1} \Omega^{(k)} + \frac{N_0}{2E_0}. \]  
\[ (3.138) \]

It can be seen that \( SNR_0 \), as given by Eq. (3.131), is a ratio of sums of weighted gamma variables, with some of these variables present in both the numerator and the denominator. A general expression for the pdf of such a ratio is given on pp. 155-156 of [226]. It is everything but simple. Instead, we decided to try out an approximation. If we replace the second term in the denominator of the right-hand side of (3.131) by \( B \sum_{i=0}^{L-1} |\alpha_i^{(1)}|^2 \) (i.e. the indice \( l = 0 \) is included in the summation), and use the previous assumption that \( L = L_c \), the approximate output SNR becomes:

\[ SNR_0 = \frac{s}{(A + B)s + C} \]  
\[ (3.139) \]

where \( s = \sum_{n=0}^{L-1} |\alpha_n^{(1)}|^2 \). The pdf of \( s \) was given for example in Eqs. (3.41) and (3.43) for independent Rayleigh and Nakagami fading, respectively, and in Eq. (2.231) for Nakagami fading with 2 correlated branches. The BER can then be written as:

\[ P_s = \int_0^\infty Q\left(\sqrt{\frac{s}{(A + B)s + C}}\right) p_s(s)ds. \]  
\[ (3.140) \]

Since we haven't found any closed-form solution for (3.140), we evaluated it numerically.
Equal-Gain Combining

From Eqs. (3.126)-(3.130), the conditional output SNR at the receiver is now:

\[
SNR_0 = \frac{\left(\sum_{n=0}^{L-1} \alpha_{n}^{(1)}\right)^2 / L}{A \left(\sum_{l=0}^{L-1} [\alpha_l^{(1)}]^2\right) + B \left(\sum_{l=1}^{L-1} [\alpha_l^{(1)}]^2\right) + C}
\]  

(3.141)

where the terms \(A\), \(B\) and \(C\) are as given previously. In the above, the numerator term \(s = [\sum_{n=0}^{L-1} \alpha_n^{(1)}]/L\) is the weighted square of the sum of the fading coefficients, while the terms \(\sum_{l=0}^{L-1} [\alpha_l^{(1)}]^2\) and \(\sum_{l=1}^{L-1} [\alpha_l^{(1)}]^2\) in the denominator are sums of the squares of the fading coefficients. Hence, contrary to the MRC case where by replacing \(B \sum_{l=0}^{L-1} [\alpha_l^{(1)}]^2\) with \(B \sum_{l=0}^{L-1} [\alpha_l^{(1)}]^2\) it was possible to obtain an expression for \(SNR_0\) conditional solely on \(\sum_{n=0}^{L-1} [\alpha_n^{(1)}]^2\), here no such simplification is possible. Nevertheless, considering the case \(L = 2\) for which a closed-form expression for the pdf of \(s\) is known (c.f. Eq. (2.201), with \(\rho = 0\) for uncorrelated channels), we attempted to replace the terms \(\sum_{l=0}^{L-1} [\alpha_l^{(1)}]^2\) and \(\sum_{l=1}^{L-1} [\alpha_l^{(1)}]^2\) with \(s\). As shown in the next section, satisfactory results were obtained for \(L = 2\) using this approximation.

3.3.3 Performance Evaluation Results and Discussion

We implemented an IS-2000 software simulator (c.f. Section 4.2.7) in order to check the validity of our equations, with the parameters specified in [14]. Throughout our simulations, we assume that \(N^{(k)} = 1, k = 2, 3, \ldots, K\), while \(N^{(1)}\) can take different values. Also, with no loss in generality, we run simulations only for the case where no data is transmitted on the Q-branch, and the power and symbol periods are equal for each channel (although our simulation model is general enough to include data on both \(I\) and Q-branches, and different powers and symbol periods).

Simulation and analytical results are plotted in Fig. 3.43 and 3.44, for \(N^{(1)} = 8\) and \(N^{(1)} = 4\), respectively, and \(L = 1, 2, 3\), for a Rayleigh fading channel with \(\Omega_l^{(k)} = 1\) for all \(l, k\). In the case of 8 codes, Eq. (3.140) gives a very good approximation to the actual BER. However, for a low number of codes (e.g. for \(N^{(1)} = 4\), there are slight
discrepancies when \( L > 1 \), especially when the number of users is small. This is due to the fact that the Gaussian approximation is used, in conjunction with the approximation made in deriving Eq. (3.131). Figs. 3.45 and 3.46 compare the BER obtained by using our method with that resulting from overlooking the dependence between the fading affecting the codes of a same user, for \( N^{(1)} = 8 \) and \( L = 2, 3 \), respectively: a significant improvement in accuracy can be noted, especially for a low number of users.

Figs. 3.47-3.49 illustrate results for \( N^{(1)} = 8, N^{(1)} = 4 \) and \( N^{(1)} = 1 \), respectively, and \( L = 1, 2, 3 \), for a Nakagami fading channel with \( m = 2.0 \) and \( \Omega_l^{(k)} = 1 \) for all \( l, k \). As for the case \( m = 1.0 \), it is seen that the accuracy of the approximation increases with the number of multicodes. In all cases the results are more accurate (or equal for \( N^{(1)} = 1 \)) than if we made the independence assumption between the codes of a same user.

The effect of the \( m \)-parameter is shown in Figs. 3.50-3.52 for diversity orders \( L = 1, 2, 3 \), respectively, with \( N^{(1)} = 8 \). For increasing \( L \), it is seen that the effect of the \( m \)-parameter on the BER is reduced: indeed, with a higher diversity order the combined signal is less likely to be in a deep fade, and is thus less sensitive to the severity of the fading determined by \( m \).

Figs. 3.53 and 3.54 present results for a Rician fading channel with \( L = 1 \) and \( L = 2 \), respectively, for different values of the Rician parameter \( A \). For \( L = 1 \) there is no multicode interference, and the match between theory and simulation is very good for all \( K \). For \( L = 2 \) the theoretical curves become more accurate as \( K \) increases, especially for higher \( A \)'s: for \( K > 10 \) the match is very good in all cases. As in the Nakagami case, when a higher diversity order is used, the effect of the parameter \( A \) on the BER is reduced, since the combined signal becomes less sensitive to the severity of the fading.

Next, results are presented for a system with two correlated diversity branches. In Fig. 3.55 a correlation coefficient \( \rho = 0.5 \) is used, and the channel is Rayleigh fading. The theoretical BER curves are close together for different numbers of codes \( N^{(1)} = 1, 4, 8 \), and agree reasonably well with the simulation results, especially for a higher \( N^{(1)} \). Figs. 3.56 and 3.57 illustrate the effect of the correlation coefficient \( \rho \) for a Nakagami fading channel with \( m = 2.0 \), for \( N^{(1)} = 8 \) and \( N^{(1)} = 4 \), respectively. As before, the accuracy of the analysis is slightly better for a higher \( N^{(1)} \).
Fig. 3.58 compares the approximate analytical error probability of EGC of Section 3.3.2.4 with simulation results, for a Rayleigh fading channel, $L = 2$, and different $N^{(1)}$'s. There is good agreement between both, especially for $N^{(1)} = 8$, despite the rough approximations which needed to be made in the analysis. Figs. 3.59 and 3.60 show results obtained with EGC and two correlated diversity branches, in Nakagami fading with $m = 2$, for different correlation coefficients. Again due to the approximation in the analysis for EGC, the theoretical curves underestimate the simulated SER's, with a better match seen for the case $N^{(1)} = 8$.

### 3.3.4 Conclusions

This section provided an analysis of reverse link multicode CDMA systems with coherent BPSK modulation and maximal-ratio or equal-gain combining in multipath fading channels. As in the previous section, the analysis took into account the fact that both the multicode interference and the desired signal were affected by the identical fading process. Using some approximations to facilitate the analysis, good matches were obtained between the theoretical curves produced by the method and the simulation results. The analytical results were seen to be especially accurate for a higher number of codes (e.g. 8 codes). The analysis applies to any type of fading, and results were illustrated for the cases of Rayleigh and Nakagami fading, for both independent and correlated diversity branches.
Figure 3.43 BER vs $K$ for $N^{(1)} = 8$. $++ + : L = 1$; $* * * : L = 2$; $o o o : L = 3$.

Figure 3.44 BER vs $K$ for $N^{(1)} = 4$. $++ + : L = 1$; $* * * : L = 2$; $o o o : L = 3$. 
Figure 3.45 BER vs $K$ for $N^{(1)} = 8$, $L = 2$. — — : Eq. (3.140); -- : approximation.

Figure 3.46 BER vs $K$ for $N^{(1)} = 8$, $L = 3$. — — : Eq. (3.140); -- : approximation.
Figure 3.47 BER vs $K$ for $N^{(1)} = 8$, and $m = 2.0$. -- (•): $L = 1$; - (- •): $L = 2$; - (•): $L = 3$.

Figure 3.48 BER vs $K$ for $N^{(1)} = 4$, and $m = 2.0$. -- (•): $L = 1$; - (- •): $L = 2$; - (•): $L = 3$. 

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Figure 3.49 BER vs $K$ for $N^{(1)} = 1$, and $m = 2.0$. — (+): $L = 1$; – .. (*) : $L = 2$; – – (o) : $L = 3$.

Figure 3.50 BER vs $K$ for $N^{(1)} = 8$, and $L = 1$. — (+): $m = 1.0$; – .. (*) : $m = 2.0$; – – (o) : $m = 3.0$. 

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Figure 3.51 BER vs $K$ for $N^{(1)} = 8$, and $L = 2$. — (+): $m = 1.0$; — (*): $m = 2.0$; — (o): $m = 3.0$.

Figure 3.52 BER vs $K$ for $N^{(1)} = 8$, and $L = 3$. — (+): $m = 1.0$; — (*): $m = 2.0$; — (o): $m = 3.0$. 
Figure 3.53 BER vs $K$ for $N^{(1)} = 8$, and $L = 1$.  — ( + ) : $A = 1.0$; — ( * ) : $A = 2.0$; — ( o ) : $A = 3.0$.

Figure 3.54 BER vs $K$ for $N^{(1)} = 8$, and $L = 2$.  — ( + ) : $A = 1.0$; — ( * ) : $A = 2.0$; — ( o ) : $A = 3.0$. 

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Figure 3.55 BER vs $K$ for $L = 2$ correlated branches with $\rho = 0.5$ and $m = 1.0$. — (+): $N^{(1)} = 1$; - - (*): $N^{(1)} = 4$; - - (o): $N^{(1)} = 8$.

Figure 3.56 BER vs $K$ for $L = 2$ correlated branches and $m = 2.0$, with $N^{(1)} = 8$. — (+): $\rho = 0.0$; - - (*): $\rho = 0.3$; - - (o): $\rho = 0.5$; · · · (o): $\rho = 0.7$. 

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Figure 3.57 BER vs $K$ for $L = 2$ correlated branches and $m = 2.0$, with $N^{(1)} = 4$. — (+): $\rho = 0.0$; -- (*) : $\rho = 0.3$; -- (o) : $\rho = 0.5$; • • • (©) : $\rho = 0.7$.

Figure 3.58 BER vs $K$ for EGC with $L = 2$ and $m = 1.0$. — (•): $N^{(1)} = 1$; -- (*) : $N^{(1)} = 4$; -- (o) : $N^{(1)} = 8$. 

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Figure 3.59 BER vs $K$ for EGC with $L = 2$, $m = 2.0$ and $N^{(1)} = 8$. --- (+): $\rho = 0.0$; -- (o): $\rho = 0.3$; --- (o): $\rho = 0.7$.

Figure 3.60 BER vs $K$ for EGC with $L = 2$, $m = 2.0$ and $N^{(1)} = 4$. --- (+): $\rho = 0.0$; -- (o): $\rho = 0.3$; --- (o): $\rho = 0.7$. 

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3.4 Forward Link Performance with Coherent QPSK Modulation and Real Spreading Sequences

3.4.1 Introduction

As mentioned in Section 3.1.1, the forward link of IS-95B uses concatenated Walsh/PN spreading. Since the BS transmissions to each mobile user are chip synchronous, in the absence of multipath there won't be any intra-cell interference (due to the orthogonality of the Walsh sequences), but in a multipath environment a mobile user will suffer from the interference due to the multipaths of the signals intended for other users (which are no longer orthogonal). In a forward link configuration the interference affecting a mobile user travels on the same propagation path as the desired signal, hence the signal and interference from all users fade in unison, unlike in a reverse link configuration where only the multicode interference fades in unison.

Most analyses of the CDMA forward link BER considered asynchronous transmission and non-concatenated spreading [227], [228], [229], [230], [220], [231], [232], [233]. A few studies have considered synchronous transmission and concatenated Walsh/PN spreading [178], [234], but either the theory and simulation weren't compared ([178], possibly due to approximations used in the analysis, which can cause a significant departure from the exact BER values), or they didn't match ([234], possibly due to a mistake in the analysis). In this section, we provide a detailed analysis of the IS-95B forward link, using the actual modulation scheme (while other studies often used a simplified version), and derive the BER in a semi-analytical manner. We show that the Gaussian approximation works quite well for a wide range of values, which is seen by a good match between theory and simulation. Moreover, we obtain results for the general case of correlated diversity branches, for Nakagami fading with MRC or EGC, thus extending the scope of previous work which only considered independent Rayleigh fading with MRC [178], [234].
3.4.2 Error Probability Analysis

3.4.2.1 Signal Model

The transmitter uses the type of QPSK modulation specified in IS-95, i.e. the same information sequence is mapped onto both its I and Q branches, as opposed to the traditional QPSK scheme where consecutive symbols are mapped alternatively onto each branch. This leads to quadrature spreading, i.e. the information sequence is spread by two sequences. Let \( N^{(k)} \) be the number of channels (and thus codes) assigned to user \( k \), where \( k = 1, 2, \ldots, K \). We denote by \( \{kc\} \) code \( c \) \((c = 0, 1, \ldots, N^{(k)} \) of user \( k \). The stream of binary data symbols for code \( \{kc\} \) is given in the time-domain by:

\[
 b^{(kc)}(t) = \sum_{j=-\infty}^{\infty} b^{(kc)}_j (t - jT_b), \quad b^{(kc)}_j \in [-1, 1] \tag{3.142}
\]

where \( T_b \) (\( = T_s \)) is the bit (symbol) period. The transmitter uses concatenated spreading, with Walsh sequences (c.f. previous section) used for orthogonal covering. The Walsh sequence assigned to code \( \{kc\} \), \( W^{(kc)} = [w^{(kc)}_0 w^{(kc)}_1 \ldots w^{(kc)}_{M-1}] \), satisfies:

\[
 (W^{(kc)} . W^{(jd)}) = \begin{cases} 
 M & \text{if } (kc) = (jd), \\
 0 & \text{if } (kc) \neq (jd).
\end{cases}
\]

The periodic orthogonal covering sequence can be expressed in the time domain as:

\[
 W^{(kc)}_P(t) = \sum_{h=-\infty}^{\infty} W^{(kc)}(t - hT_W) \\
 = \sum_{h=-\infty}^{\infty} \sum_{i=0}^{M-1} w^{(kc)}_i p_{T_{wc}}(t - hT_W - iT_{wc}) \tag{3.143}
\]

where \( T_W \) is the duration of \( W^{(kc)}(t) \) and period of \( W^{(kc)}_P(t) \), \( p_{T_{wc}}(t) \) is a rectangular pulse of unit amplitude and duration \( T_{wc} = T_W/M \) seconds.

Two different PN sequences, \( a_I(t) \) and \( a_Q(t) \), are used on the I and Q branches of the transmitter, respectively. These same sequences are used to spread all of the information channels associated with the users of one cell, in a forward-link configuration. They may
be expressed as:

\[ a_I(t) = \sum_{j=-\infty}^{\infty} a_{I,j}p_{T_c}(t-jT_c), \quad a_{I,j} \in [-1,1], \quad (3.144) \]

\[ a_Q(t) = \sum_{j=-\infty}^{\infty} a_{Q,j}p_{T_c}(t-jT_c), \quad a_{Q,j} \in [-1,1] \quad (3.145) \]

where \( T_c \) is the PN chip duration. In the following, for simplicity, we take the chip durations of the Walsh and PN sequences to be equal, i.e. \( T_{wc} = T_c \). This is the case for IS-95, but not necessarily for cdma2000, where \( T_{wc} \) can be a multiple of \( T_c \). In the latter case, one simply needs to replicate \( N_u = T_{wc}/T_c \) times each chip of the Walsh sequence \( W_{wc}^{(kc)} \), in order to obtain a new upsampled sequence \( W_{u}^{(kc)} \) of length \( N_uM \): the new associated Walsh function \( W_{u}^{(kc)}(t) \) now has chip duration \( T_c \). The analysis, however, will remain the same. The concatenated Walsh/PN sequences are given by:

\[ a_{I,c}^{(kc)}(t) = a_I(t)W_{P_c}^{(kc)}(t) \]

\[ = \sum_{j=-\infty}^{\infty} a_{I,c,j}^{(kc)}p_{T_c}(t-jT_c), \quad a_{I,c,j}^{(kc)} \in [-1,1], \quad (3.146) \]

\[ a_{Q,c}^{(kc)}(t) = a_Q(t)W_{P_c}^{(kc)}(t) \]

\[ = \sum_{j=-\infty}^{\infty} a_{Q,c,j}^{(kc)}p_{T_c}(t-jT_c), \quad a_{Q,c,j}^{(kc)} \in [-1,1]. \quad (3.147) \]

The transmitted signal of the \( k^{th} \) user can be expressed as:

\[ s^{(k)}(t) = \sum_{c=0}^{N^{(k)}-1} \sqrt{P^{(kc)}}[b^{(kc)}(t)a_{I,c}^{(k)}(t)\cos(w_c t + \phi^{(kc)}) + b^{(kc)}(t)a_{Q,c}^{(k)}(t)\sin(w_c t + \phi^{(kc)})] \quad (3.148) \]

where \( P^{(kc)} = E_b^{(kc)}/T_b \) is the average power of code \( c \) of user \( k \), which we assume identical for all users without loss of generality \( (P^{(kc)} = P \text{ for all } \{k, c\}) \). The IS-95B forward link transmitter is illustrated in Fig. 3.61.
3.4.2.2 Receiver Model and Decision Metrics

The composite signal received at the output of the channel is:

\[
\tau(t) = \sum_{k=1}^{K} \sum_{c=0}^{N(k)-1} \sum_{l=0}^{L_c-1} \sqrt{P^{(kc)}} \alpha_i^{(k)}(t) [b^{(kc)}(t - \tau_i^{(k)}) a_{I,c}^{(kc)}(t - \tau_i^{(k)}) \cos(w_c t + \varphi_i^{(kc)}) \\
+ b^{(kc)}(t - \tau_i^{(k)}) a_{Q,c}^{(kc)}(t - \tau_i^{(k)}) \sin(w_c t + \varphi_i^{(kc)})] + n(t) \tag{3.149}
\]

with \(\varphi_i^{(kc)} = \varphi_i^{(kc)} + \phi_i^{(k)} - \omega_c \tau_i^{(k)}\). \(n(t)\) is AWGN with double-sided power spectral density \(N_0/2\). As in the reverse link case, note that all the received codes pertaining to a same user are affected by the same fading coefficients and delay, since they are transmitted via the same channel, while the phases can be different if a specific phase shift is assigned to each channel by the transmitter.

The receiver consists of \(L\) Rake fingers and performs either MRC or EGC. In the following, the metrics calculated refer to code 0 of user 1, which is assumed to be the desired stream. The decision metric at the output of the receiver, \(U\) is the sum of the decision variables at the output of the \(I\) branch, \(U_I\) and the \(Q\) branch, \(U_Q\). The former is given for MRC by \(U_I = \sum_{n=0}^{L-1} \alpha_n^{(1)} U_I^{(n)}\) and for EGC by \(U_I = \sum_{n=0}^{L-1} U_I^{(n)}\), with:

\[
U_I^{(n)} = \int_{\tau_n^{(1)} + T_b}^{\tau_n^{(1)}} (r(t) \cos(w_c t + \varphi_n^{(10)})) L P a_{I,c}^{(10)}(t - \tau_n^{(10)}) dt. \tag{3.150}
\]
Upon expanding terms in Eq. (3.150) and integrating, one obtains:

\[ U_l^{(n)} = S_l^{(n)} + I_{MP,l}^{(n)} + I_{MC,l}^{(n)} + I_{MA,l}^{(n)} + I_{N,l}^{(n)} \]  

(3.151)

where the terms in Eq. (3.151) are defined as in the previous section. Let \( C = \frac{1}{2}\sqrt{P} \).

The signal component (corresponding to the terms in Eq. (3.150) for which \( k = 1, c = 0 \) and \( l = n \)) is given by:

\[ S_l^{(n)} = C b_0^{(10)} T_b \alpha_n^{(1)} . \]  

(3.152)

The interference terms are given as follows:

\[ I_{MP,l}^{(n)} = C \int_{\tau_n^{(1)}}^{\tau_n^{(1)} + T_b} \sum_{l=0}^{L_c-1} \alpha_l^{(1)} [b^{(10)}(t - \tau_l^{(1)}) a_{l,c}^{(10)}(t - \tau_l^{(1)}) \cos(\varphi_{nl}^{(10)})]
\]

\[ + b^{(10)}(t - \tau_l^{(1)}) a_{Q,c}^{(10)}(t - \tau_l^{(1)}) \sin(\varphi_{nl}^{(10)})] a_{l,c}^{(10)}(t - \tau_n^{(1)}) dt, \]  

(3.153)

\[ I_{MC,l}^{(n)} = C \int_{\tau_n^{(1)}}^{\tau_n^{(1)} + T_b} \sum_{c=1}^{N_c^{(1)}} \sum_{l=0}^{L_c-1} \alpha_l^{(1)} [b^{(10)}(t - \tau_l^{(1)}) a_{l,c}^{(10)}(t - \tau_l^{(1)}) \cos(\varphi_{nl}^{(10)})]
\]

\[ + b^{(10)}(t - \tau_l^{(1)}) a_{Q,c}^{(10)}(t - \tau_l^{(1)}) \sin(\varphi_{nl}^{(10)})] a_{l,c}^{(10)}(t - \tau_n^{(1)}) dt, \]  

(3.154)

\[ I_{MA,l}^{(n)} = C \int_{\tau_n^{(1)}}^{\tau_n^{(1)} + T_b} \sum_{k=2}^{K} \sum_{c=0}^{N_c^{(k)}} \sum_{l=0}^{L_c-1} \alpha_l^{(k)} [b^{(k0)}(t - \tau_l^{(k)}) a_{l,c}^{(k0)}(t - \tau_l^{(k)}) \cos(\varphi_{nl}^{(k0)})]
\]

\[ + b^{(k0)}(t - \tau_l^{(k)}) a_{Q,c}^{(k0)}(t - \tau_l^{(k)}) \sin(\varphi_{nl}^{(k0)})] a_{l,c}^{(k0)}(t - \tau_n^{(1)}) dt, \]  

(3.155)

\[ I_{N,l}^{(n)} = \frac{C}{\sqrt{P}} \int_{\tau_n^{(1)}}^{\tau_n^{(1)} + T_b} n(t) a_{l,c}^{(10)}(t - \tau_n^{(1)}) dt. \]  

(3.156)

They can be expressed as a function of the following aperiodic crosscorrelation functions:

\[ R_{xy}^{(k)}(\tau) = [C_{xy}^{(k)}(\bar{l} + 1 - N) - C_{xy}^{(k)}(\bar{l} - N)](\tau - \bar{l}T_c) + [C_{xy}^{(k)}(\bar{l} - N)]T_c, \]  

(3.157)

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\[ \hat{R}^{(k)}_{xy}(\tau) = [C^{(k)}_{xy}(\bar{l} + 1) - C^{(k)}_{xy}(\bar{l})](\tau - \bar{l}T_c) + [C^{(k)}_{xy}(\bar{l})]T_c \quad (3.158) \]

where

\[ C^{(k)}_{xy}(\bar{l}) = \left\{ \begin{array}{ll}
\sum_{j=0}^{N-1-\bar{l}} a^{(k)}_{x,j} a^{(10)}_{y,j+\bar{l}} & 0 \leq \bar{l} \leq N - 1 \\
\sum_{j=0}^{N-1+\bar{l}} a^{(k)}_{x,j-N} a^{(10)}_{y,j} & 1 - N \leq \bar{l} < 0 \\
0 & |\bar{l}| \geq N
\end{array} \right. \quad (3.159) \]

and \( N = T_b/T_c \) is the processing gain, \( \bar{l} = \lceil (\tau^{(k)} - lT_c)/T_c \rceil \), and the indices \( \{xy\} \) are chosen from \( \{II\}, \{IQ\}, \{QI\} \) or \( \{QQ\} \). The interference terms in (3.153)-(3.155) can then be expressed through these functions:

\[ I^{(n)}_{M_{P,I}} = C \sum_{l=0}^{L_{c}-1} \sum_{l \neq n} \alpha^{(1)}_{l} [(b^{(10)}_u R^{(10)}_{II} (\tau^{(1)})) + b^{(10)}_v R^{(10)}_{II} (\tau^{(1)})] \cos(\varphi^{(10)}_{nl}) \\
+ (b^{(10)}_u R^{(10)}_{II} (\tau^{(1)}) + b^{(10)}_v R^{(10)}_{II} (\tau^{(1)}) \sin(\varphi^{(10)}_{nl})), \quad (3.160) \]

\[ I^{(n)}_{M_{C,I}} = C \sum_{c=1}^{N^{(1)}-1} \sum_{l \neq n} \alpha^{(1)}_{l} [(b^{(10)}_u R^{(10)}_{II} (\tau^{(1)})) + b^{(10)}_v R^{(10)}_{II} (\tau^{(1)})] \cos(\varphi^{(10)}_{nl}) \\
+ (b^{(10)}_u R^{(10)}_{II} (\tau^{(1)}) + b^{(10)}_v R^{(10)}_{II} (\tau^{(1)})) \sin(\varphi^{(10)}_{nl})), \quad (3.161) \]

\[ I^{(n)}_{M_{A,I}} = C \sum_{k=2}^{K} \sum_{c=0}^{N^{(k)}-1} \sum_{l \neq n} \alpha^{(1)}_{l} [(b^{(k)}_u R^{(k)}_{II} (\tau^{(k)})) + b^{(k)}_v R^{(k)}_{II} (\tau^{(k)})] \cos(\varphi^{(k)}_{nl}) \\
+ (b^{(k)}_u R^{(k)}_{II} (\tau^{(k)}) + b^{(k)}_v R^{(k)}_{II} (\tau^{(k)})) \sin(\varphi^{(k)}_{nl})), \quad (3.162) \]

where \( \{u,v\} = \{-1,0\} \) and \( \tau^{(k)} = \tau^{(k)}_{nl} \) if \( \tau^{(k)}_{nl} > 0 \), and \( \{u,v\} = \{0,1\} \) and \( \tau^{(k)} = \tau^{(k)}_{nl} + T_b \) if \( \tau^{(k)}_{nl} < 0 \). Notice that for \( I_{M_{C,I}} \) and \( I_{M_{A,I}} \), the components for which indice \( l = n \) have been removed. Indeed, when the collected multipath components are synchronous with the desired signal (which happens when \( l = n \)), the interference produced by these
components is nil due to the orthogonal covering of the spreading sequences. Signal and interference terms can be derived in a similar fashion for $U_Q$, and are given as follows:

\[ S_Q^{(n)} = C b_0^{(10)} T_b \alpha_n^{(1)}, \]  
(3.163)

\[ I_{M,F,Q}^{(n)} = C \sum_{i=0}^{L_o-1} \sum_{\substack{i=0 \\ i \neq n}}^{\alpha_i^{(1)} \left( b_u^{(10)} R_{IQ}^{(10)} (\tau_i^{(1)}) + b_v^{(10)} \tilde{R}_{IQ}^{(10)} (\tau_i^{(1)}) \right) \cos(\varphi_i^{(10)}) \right) \] 
\[ + \left( b_u^{(10)} R_{QQ}^{(10)} (\tau_i^{(1)}) + b_v^{(10)} \tilde{R}_{QQ}^{(10)} (\tau_i^{(1)}) \right) \sin(\varphi_i^{(10)}), \]  
(3.164)

\[ I_{M,C,Q}^{(n)} = C \sum_{c=1}^{N(k)-1} \sum_{\substack{i=0 \\ i \neq n}}^{\alpha_i^{(1)} \left( b_u^{(1c)} R_{IQ}^{(1c)} (\tau_i^{(1)}) + b_v^{(1c)} \tilde{R}_{IQ}^{(1c)} (\tau_i^{(1)}) \right) \cos(\varphi_i^{(1c)}) \right) \] 
\[ + \left( b_u^{(1c)} R_{QQ}^{(1c)} (\tau_i^{(1)}) + b_v^{(1c)} \tilde{R}_{QQ}^{(1c)} (\tau_i^{(1)}) \right) \sin(\varphi_i^{(1c)}), \]  
(3.165)

\[ I_{M,A,Q}^{(n)} = C \sum_{k=2}^{K} \sum_{c=0}^{N(k)-1} \sum_{\substack{i=0 \\ i \neq n}}^{\alpha_i^{(k)} \left( b_u^{(k)} R_{IQ}^{(k)} (\tau_i^{(k)}) + b_v^{(k)} \tilde{R}_{IQ}^{(k)} (\tau_i^{(k)}) \right) \cos(\varphi_i^{(k)}) \right) \] 
\[ + \left( b_u^{(k)} R_{QQ}^{(k)} (\tau_i^{(k)}) + b_v^{(k)} \tilde{R}_{QQ}^{(k)} (\tau_i^{(k)}) \right) \sin(\varphi_i^{(k)}). \]  
(3.166)

### 3.4.2.3 Statistics of Decision Metrics

**Maximal-Ratio Combining**

The conditional expected value of the desired received signal $S = S_I + S_Q = \sum_{n=0}^{L-1} \alpha_n^{(1)} (S_Q^{(n)} + S_Q^{(n)})$ is obtained from Eqs. (3.152) and (3.163):

\[ S = \sqrt{\frac{E_b T_b}{2}} \sum_{n=0}^{L-1} [\alpha_n^{(1)}]^2. \]  
(3.167)
The noise term $I_N = I_{N,I} + I_{N,Q} = \sum_{n=0}^{L-1} \alpha_n^{(1)} (I_{N,I}^{(n)} + I_{N,Q}^{(n)})$ can be easily shown to be a zero-mean Gaussian random variable with conditional variance:

$$\sigma_N^2 = \text{Var}[I_N] = \frac{T_b N_0}{4} \sum_{n=0}^{L-1} [\alpha_n^{(1)}]^2. \quad (3.168)$$

Using the standard Gaussian approximation, we model the interference terms Eqs. (3.160)-(3.162) as mutually uncorrelated zero-mean Gaussian random variables with given conditional variances. These variances are derived next. The interference terms $I_{MP} = I_{MP,I} + I_{MP,Q} = \sum_{n=0}^{L-1} \alpha_n^{(1)} (I_{MP,I}^{(n)} + I_{MP,Q}^{(n)})$, $I_{MC} = I_{MC,I} + I_{MC,Q} = \sum_{n=0}^{L-1} \alpha_n^{(1)} (I_{MC,I}^{(n)} + I_{MC,Q}^{(n)})$ and $I_{MA} = I_{MA,I} + I_{MA,Q} = \sum_{n=0}^{L-1} \alpha_n^{(1)} (I_{MA,I}^{(n)} + I_{MA,Q}^{(n)})$ are all chip synchronous with the desired user signal, since they originate from signals which are transmitted synchronously on the forward link. Therefore $\tau_{nl}^{(k)} = \tau_{nl}^{(1)}$ for all $k$, $n$ and $l$. This results in $\text{Var}[R_{cy}^{(k c)} (\tau_{nl}^{(k)})] = T_b^2 / N$ (with $l \neq n$ if $\{kc\} = \{10\}$), and in the following variances:

$$\sigma_{I_{MP}}^2 = \text{Var}[I_{MP}] = \frac{E_b T_b}{4N} \sum_{n=0}^{L-1} \sum_{l=0 \atop l \neq n}^{L_e-1} [\alpha_l^{(1)}]^2, \quad (3.169)$$

$$\sigma_{I_{MC}}^2 = \text{Var}[I_{MC}] = \frac{E_b T_b}{4N} (N^{(1)}-1) \sum_{n=0}^{L-1} \sum_{l=0 \atop l \neq n}^{L_e-1} [\alpha_l^{(1)}]^2, \quad (3.170)$$

$$\sigma_{I_{MA}}^2 = \text{Var}[I_{MA}] = \frac{E_b T_b}{4N} \sum_{n=0}^{L-1} \sum_{k=2}^{K} N^{(k)} \sum_{l=0 \atop l \neq n}^{L_e-1} [\alpha_l^{(k)}]^2. \quad (3.171)$$

Since all channels are transmitted via the same physical propagation paths, we have that $\alpha_l^{(k)} = \alpha_l^{(1)}$ for all $k$ and $l$. The total variance from the interference and thermal noise
sums up to:

\[
\sigma^2 = \sigma_{I_{MP}}^2 + \sigma_{I_{MC}}^2 + \sigma_{I_{MA}}^2 + \sigma_N^2
\]

\[
= \frac{E_bT_b}{4} \sum_{n=0}^{L-1} (\alpha_n^{(1)})^2 \left( \frac{N_0}{E_b} + \frac{1}{N} \sum_{l=0}^{L-1} \sum_{k=1}^{K} N^{(k)} \right)
\]

(3.172)

**Equal-Gain Combining**

With EGC, the signal and interference terms are now

\[
S_I = \sum_{n=0}^{L-1} (S_{I}^{(n)} + S_Q^{(n)})
\]

\[
I_{N,I} = \sum_{n=0}^{L-1} (I_{N,I}^{(n)} + I_{N,Q}^{(n)})
\]

\[
I_{MP,I} = \sum_{n=0}^{L-1} (I_{MP,I}^{(n)} + I_{MP,Q}^{(n)})
\]

\[
I_{MC,I} = \sum_{n=0}^{L-1} (I_{MC,I}^{(n)} + I_{MC,Q}^{(n)})
\]

\[
I_{MA,I} = \sum_{n=0}^{L-1} (I_{MA,I}^{(n)} + I_{MA,Q}^{(n)})
\]

Proceeding as in the previous section, the signal term is now:

\[
\bar{S} = \sqrt{\frac{E_bT_b}{2}} \sum_{n=0}^{L-1} \alpha_n^{(1)}.
\]

(3.173)

and the variances of the (Gaussian-modeled) interference terms:

\[
\sigma_N^2 = \text{Var}[I_N] = \frac{T_b N_0 L}{4}
\]

(3.174)

\[
\sigma_{I_{MP}}^2 = \text{Var}[I_{MP}] = \frac{E_bT_b}{4N} \sum_{n=0}^{L-1} \sum_{l \neq n} (\alpha_l^{(1)})^2
\]

(3.175)

\[
\sigma_{I_{MC}}^2 = \text{Var}[I_{MC}] = \frac{E_bT_b}{4N} (N^{(1)}-1) \sum_{n=0}^{L-1} \sum_{l \neq n} (\alpha_l^{(1)})^2
\]

(3.176)

\[
\sigma_{I_{MA}}^2 = \text{Var}[I_{MA}] = \frac{E_bT_b}{4N} \sum_{n=0}^{L-1} \sum_{k=2}^{K} N^{(k)} \sum_{l \neq n} (\alpha_l^{(1)})^2
\]

(3.177)

The total variance from the interference and thermal noise sums up to:

\[
\sigma^2 = \frac{E_bT_b}{4} \left( \frac{N_0}{E_b} L + \frac{1}{N} \sum_{k=1}^{K} N^{(k)} \sum_{n=0}^{L-1} \sum_{l \neq n} (\alpha_l^{(1)})^2 \right)
\]

(3.178)
3.4.2.4 Probability of Error

Maximal-Ratio Combining

From Eqs. (3.167) and (3.172), and using (without loss of generality) the assumption that \( L_c = L \), the conditional output SNR at the receiver then takes the form\(^2\):

\[
SNR_O = \frac{\left( \sum_{n=0}^{L-1} |\alpha^{(1)}_n|^2 \right)^2}{\sum_{n=0}^{L-1} |\alpha^{(1)}_n|^2 \left( \frac{N_0}{2E_b} + \frac{1}{2N} \sum_{k=1}^{K} N^{(k)} \sum_{l=0 \atop l \neq n}^{L-1} |\alpha^{(1)}_l|^2 \right)}.
\]

(3.179)

It is seen that the fading coefficients \([\alpha^{(1)}_l]^2, l = 1, \ldots, L-1\) appear both in the numerator and denominator of the SNR. Indeed, the interference terms are subject to the same fading as is the received desired signal. Therefore the terms \([\alpha^{(1)}_l]^2\) in the denominator cannot be replaced by their expected values, like it can be done in the single-code reverse-link scenario. The expression above is thus the ratio of a combination of products and weighted sums of correlated random variables, and a closed-form solution for the pdf of the SNR is thus difficult to find, even for low diversity orders involving only a few terms.

To circumvent this difficulty, we have used a semi-analytical approach to obtain the theoretical error probability. The conditional BER (conditioned on the set \(\{\alpha^{(1)}_l, l = 0, \ldots, L - 1\}\)), is given for BPSK as:

\[
P_s(\{\alpha^{(1)}_l\}) = Q\left(\sqrt{SNR_O}\right)
\]

(3.180)

with \(SNR_O\) as given above. The unconditional BER is found by Monte Carlo integration [156], i.e. by averaging the above conditional BER over a set of randomly generated

\(^2\)This expression is similar to [178], Eqs. (44) and (47), and [234], Eqs. (36) and (38), for a single-cell environment with \( L_c = L \), with the exception that [178] and [234] use a factor \(1/(3N)\) for the multipath interference, which corresponds to multipath which is not chip synchronous with the desired signal, while Eq. (3.179) uses a factor \(1/(2N)\), which corresponds to chip synchronous multipath. However, [178] further makes an approximation in its Eq. (44) to derive the BER, but doesn’t check its accuracy by simulation, while the theoretical and simulation results of [234] are curiously very far apart.
\{\alpha_i^{(1)}, l = 0, \ldots, L - 1\}:

\[ P_s = \frac{1}{N_{\text{iter}}} \sum_{i=1}^{N_{\text{iter}}} P_s(\{\alpha_i^{(1)}\}) \]  \hspace{1cm} (3.181)

where \(N_{\text{iter}}\) is the number of generated sets of \(\{\alpha_i^{(1)}\}\). Methods for generating independent or correlated sets of \(\{\alpha_i^{(1)}\}\) for Rayleigh and Nakagami pdfs have been reviewed in Section 2.4.1.2.

**Equal-Gain Combining**

The conditional output SNR at the receiver now takes the form:

\[
SNR_O = \frac{\left( \sum_{n=0}^{L-1} \alpha_n^{(1)} \right)^2}{\frac{N_0}{2E_b} L + \frac{1}{2N} \sum_{k=1}^{K} N^{(k)} \sum_{n=0}^{L-1} \sum_{l=0}^{L-1} \alpha_l^{(1)} \alpha_l^{(1)}}. \]  \hspace{1cm} (3.182)

The BER can be obtained following the same semi-analytical Monte Carlo integration approach outlined in the previous section.

### 3.4.3 Performance Evaluation Results and Discussion

We simulated the forward link of a cellular system employing coherent QPSK Modulation and real spreading sequences. The orthogonal Walsh sequences and the short code spreading sequences are as specified in the IS-95 standard [177]. The theoretical BER is obtained by numerical Monte Carlo integration as explained above: 10 million samples were used in each case. The BER’s obtained via simulation are averaged over 1 million samples in order to maintain a reasonable simulation time (especially when the number of users is large): hence the accuracy is lower than that obtained theoretically, and simulation results will typically show a good fit for above 20 users, but looser fits (or even outliers) for a lower number of interfering users, where the BER becomes very low.

Fig. 3.62 plots the BER for MRC in Rayleigh fading, for different diversity orders. In the case \(L = 1\), since all the users are synchronous and there is no multipath, there
is no multiuser interference due to the orthogonality of the Walsh spreading sequences. Hence only AWGN is present, and the BER is constant across the range of numbers of users. In the cases $L = 2$ and $L = 3$, there is now multipath interference from all the users (including the desired one). It can be seen that for certain values of the number of users (e.g. for $K > 25$ in the case $L = 2$), the use of diversity actually increases the BER: indeed the gain made possible by diversity is now offset by the higher level of interference present (due to the nonorthogonality between multipaths). In fact, for $L = 3$ the performance is seen to be worse than for $L = 2$: this is imputable to the larger number of multipaths, each contributing additional interference. There is a very good match between the simulated results and the theoretical curves obtained through Monte Carlo integration, especially for $K > 20$. For lower values of $K$ the large number of samples needed for averaging (given the very low BER's) make it more difficult to obtain reliable values for the simulation results, although the general tendency still remains.

Figs. 3.63 and 3.64 plot the BER for MRC and EGC, respectively, with $L = 2$ in Nakagami fading, for different values of the $m$-parameter. In the case of MRC, a larger $m$ (less fading) improves the BER for a low number of users $K < 15$, but leads to a higher BER for larger $K$'s. It can be explained by the fact that for a larger $m$, the power in the interfering multipaths also becomes stronger, and hence for a large number of users (and hence multipaths) the degradation caused by the total interference offsets the benefit of having less fading of the desired signal. However, in the case of EGC, a larger $m$ actually improves the BER across all $K$: a possible explanation is that EGC doesn't amplify the interference as much as MRC, which assigns higher weights to strong multipaths, and hence to strong interference.

The same phenomenon can be observed in the case of Rician fading. Figs. 3.65 and 3.66 plot the BER for MRC and EGC, respectively, with $L = 2$, for different values of the Rician parameter $A$. As in the Nakagami case, it can be seen that as the fading gets less severe (higher $A$), the BER increases for MRC, but decreases for EGC.

The effect of correlation between $L = 2$ diversity branches is shown in Figs. 3.67 and 3.68 for MRC and EGC, respectively, where the channel is Nakagami fading with $m = 2.0$. In the case of MRC, the presence of correlation slightly increases the BER, while for EGC, the opposite is true. The outliers which can be seen for lower numbers
of users are imputable to the finite number of samples used (given the very high number of samples which would be needed to obtain accurate results for very low BER's).

3.4.4 Conclusions

This section analyzed a forward link multicode CDMA system with coherent BPSK modulation and maximal-ratio or equal-gain combining in multipath fading channels. All users fade in unison since they are transmitted along the same propagation path. Taking into account this fact was crucial in obtaining accurate analytical results. Due to the difficulty of finding general closed-form solutions (if they do exist), Monte Carlo integration was used to obtain the semi-analytical BER's, which matched closely the simulation results. The method of analysis was applied to Rayleigh, Rice and Nakagami channels (although any fading distribution can be used), for both independent and correlated diversity branches.

Figure 3.62 BER vs $K$ for MRC and $m = 1.0$. — (+): $L = 1$; — — (*): $L = 2$; — — (o): $L = 3$. 

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Figure 3.63 BER vs $K$ for MRC and $L = 2$. — (+): $m = 1.0$; – – (*): $m = 2.0$; – – (o): $m = 3.0$.

Figure 3.64 BER vs $K$ for EGC and $L = 2$. — (+): $m = 1.0$; – – (*): $m = 2.0$; – – (o): $m = 3.0$. 
Figure 3.65 BER vs $K$ for MRC and $L = 2$. -- (†): $A = 1.0$; - . - (*): $A = 2.0$; -- (o): $A = 3.0$.

Figure 3.66 BER vs $K$ for EGC and $L = 2$. -- (†): $A = 1.0$; - . - (*): $A = 2.0$; -- (o): $A = 3.0$. 

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Figure 3.67 BER vs $K$ for $L = 2$ correlated branches and $m = 2.0$, with MRC. — (+): $\rho = 0.0$; -- (*) $\rho = 0.3$; -- (o): $\rho = 0.5$; · · · (o): $\rho = 0.7$.

Figure 3.68 BER vs $K$ for $L = 2$ correlated branches and $m = 2.0$, with EGC. — (+): $\rho = 0.0$; -- (*) $\rho = 0.3$; -- (o): $\rho = 0.5$; · · · (o): $\rho = 0.7$.  

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3.5 Forward Link Performance with Coherent QPSK Modulation and Complex Spreading Sequences

3.5.1 Introduction

The physical layers of the IS-2000 and IS-95B forward links share essential features: both use concatenated Walsh/PN spreading and coherent demodulation. There are however two major differences between the systems (at the spreading/modulation levels):

- IS-95B uses a form of QPSK where the same bit is mapped to both I and Q branches, while IS-2000 uses "true" QPSK, as detailed below.
- IS-95B uses real short-code spreading sequences, while IS-2000 employs complex short-code spreading sequences in order to reduce power imbalances on the I and Q branches.

In this section we present a detailed analysis of the IS-2000 downlink, and show that under the same conditions the BER is theoretically similar (although not exactly identical) to that of the IS-95B downlink. In practice however IS-2000 systems are expected to perform better than their IS-95B counterparts, as they support advanced communication and signal processing techniques (e.g. transmit diversity, fast closed-loop power control).

3.5.2 Error Probability Analysis

3.5.2.1 Signal and Channel Models

The transmitter uses QPSK modulation in the conventional sense, i.e. in which alternating bits from a same channel are mapped to the I and Q branches. Let $N^{(k)}$ be the number of channels (and thus codes) assigned to both branches I and Q of user $k$, where $k = 1, 2, \ldots, K$. We denote by $\{k_c\}$ code $c$ ($c = 0, 1, \ldots, N^{(k)} - 1$) of user $k$. The streams of complex data symbols for code $\{k_c\}$ are given in the time-domain by:

$$b^{(k_c)}(t) = b_i^{(k_c)}(t) + jb_Q^{(k_c)}(t)$$  \hspace{1cm} (3.183)
where

\[ b^{(kc)}_I(t) = \sum_{j=-\infty}^{\infty} b^{(kc)}_{I,j}(t-jT_b), \quad b^{(kc)}_{I,j} \in [-1,1] \]

\[ b^{(kc)}_Q(t) = \sum_{j=-\infty}^{\infty} b^{(kc)}_{Q,j}(t-jT_b), \quad b^{(kc)}_{Q,j} \in [-1,1] \]

and \( T_b \) is the bit period, and \( T_s = 2T_b \) is the symbol period. The transmitter uses concatenated spreading, and the Walsh sequences used for orthogonal covering were described in the previous section.

After the orthogonal covering phase, the data symbols from each user are spread by the same complex spreading sequence \( a(t) = a_I(t) + ja_Q(t) \), where \( a_I(t) \) and \( a_Q(t) \) are given by Eqs. (3.144) and (3.145), respectively. The real and imaginary parts of the output are then separated and modulated onto orthogonal carriers, as illustrated in Fig. 3.69.

\[ s^{(k)}(t) = \sum_{c=0}^{N^{(k)}-1} \left[ s^{(k)}_{I,I}(t) \cos(w_ct + \phi^{(kc)}_I) - s^{(k)}_{Q,I}(t) \cos(w_ct + \phi^{(kc)}_Q) \\
+ s^{(k)}_{I,Q}(t) \sin(w_ct + \phi^{(kc)}_I) + s^{(k)}_{Q,Q}(t) \sin(w_ct + \phi^{(kc)}_Q) \right] \quad (3.184) \]

Figure 3.69 Channelization/Spreading/Modulation subsystem.
where

\[ s_{II}^{(k)}(t) = [\sqrt{P^{(kc)}} b_I^{(k)}(t) W^{(kc)}(t)] a_I(t), \]
\[ s_{IQ}^{(k)}(t) = [\sqrt{P^{(kc)}} b_Q^{(k)}(t) W^{(kc)}(t)] a_Q(t), \]
\[ s_{IQ}^{(k)}(t) = [\sqrt{P^{(kc)}} b_I^{(k)}(t) W^{(kc)}(t)] a_Q(t), \]
\[ s_{QQ}^{(k)}(t) = [\sqrt{P^{(kc)}} b_Q^{(k)}(t) W^{(kc)}(t)] a_I(t) \]

and \( P^{(kc)} = E_b^{(kc)} / T_s^{(kc)} \) is the average power of code \{kc\} on each branch \( I \) and \( Q \).

3.5.2.2 Receiver Model and Decision Metrics

If there are \( K \) users in the system, the composite signal received at the output of the channel is:

\[(t) = \sum_{k=1}^{K} \sum_{l=0}^{L_c-1} \alpha_l^{(k)}(t) s_l^{(k)}(t - \tau_l^{(k)}) + n(t) \tag{3.185}
\]

where \( n(t) \) is AWGN with double-sided power spectral density \( N_0/2 \).

The receiver consists of \( L \) Rake fingers and performs either MRC or EGC. One finger of the receiver is illustrated in Fig. 3.70. In the following, the metrics calculated refer to code 0 of user 1, which is assumed to be the desired stream. They further correspond to finger \( n \) of the Rake receiver.

![Figure 3.70 Demodulator for the \( n^{th} \) Rake finger.](image)

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The quantities \( u_l^{(n)}(t) \) and \( u_Q^{(n)}(t) \) at the output of the lowpass filters are given by:

\[
\begin{align*}
u_l^{(n)}(t) &= (r(t) \cos(w_c t + \varphi_{l,n}) )_{LP} \\
&= \frac{1}{2} \sum_{k=1}^{K} \sum_{c=0}^{N(k)-1} \sum_{l=0}^{L_c-1} a_{l}^{(k)}(t) \left( s_{l}^{(k)}(t - \tau_l^{(k)}) \cos(\varphi_{l,n}) - s_{Q}^{(k)}(t - \tau_l^{(k)}) \cos(\varphi_{Q,n}) \right) \\
&\quad + s_{l}^{(k)}(t - \tau_l^{(k)}) \sin(\varphi_{l,n}) + s_{Q}^{(k)}(t - \tau_l^{(k)}) \sin(\varphi_{Q,n}) \\
&\quad + (n(t) \cos(w_c t + \varphi_{l,n}^{(10)}))_{LP} \\
&= \frac{1}{2} \sum_{k=1}^{K} \sum_{c=0}^{N(k)-1} \sum_{l=0}^{L_c-1} a_{l}^{(k)}(t) \left( s_{l}^{(k)}(t - \tau_l^{(k)}) \cos(\varphi_{l,n}) - s_{Q}^{(k)}(t - \tau_l^{(k)}) \cos(\varphi_{Q,n}) \right) \\
&\quad + s_{l}^{(k)}(t - \tau_l^{(k)}) \sin(\varphi_{l,n}) + s_{Q}^{(k)}(t - \tau_l^{(k)}) \sin(\varphi_{Q,n}) \\
&\quad + (n(t) \cos(w_c t + \varphi_{l,n}^{(10)}))_{LP} \tag{3.186}
\end{align*}
\]

\[
\begin{align*}
u_Q^{(n)}(t) &= (r(t) \sin(w_c t + \varphi_{Q,n}^{(10)}))_{LP} \\
&= \frac{1}{2} \sum_{k=1}^{K} \sum_{c=0}^{N(k)-1} \sum_{l=0}^{L_c-1} a_{l}^{(k)}(t) \left( s_{l}^{(k)}(t - \tau_l^{(k)}) \sin(\varphi_{l,n}) - s_{Q}^{(k)}(t - \tau_l^{(k)}) \sin(\varphi_{Q,n}) \right) \\
&\quad + s_{l}^{(k)}(t - \tau_l^{(k)}) \cos(\varphi_{l,n}) + s_{Q}^{(k)}(t - \tau_l^{(k)}) \cos(\varphi_{Q,n}) \\
&\quad + (n(t) \sin(w_c t + \varphi_{Q,n}^{(10)}))_{LP} \tag{3.187}
\end{align*}
\]

where \( \varphi_{x,n}^{(k)} = \varphi_{x}^{(k)} + \theta_l^{(k)} - w_c \tau_l^{(k)} \), and \( \varphi_{x,n}^{(k)} = \varphi_{x,n}^{(k)} - \varphi_{l,n}^{(10)} \). The output of the complex spreading operation is given by:

\[
\begin{align*}
U^{(n)}(t) &= (u_l^{(n)}(t) + j u_Q^{(n)}(t)) a(t - \tau_n^{(1)})^* \\
&= [u_l^{(n)}(t)a_l(t - \tau_n^{(1)}) + u_Q^{(n)}(t)a_Q(t - \tau_n^{(1)})] \\
&\quad + j[-u_l^{(n)}(t)a_Q(t - \tau_n^{(1)}) + u_Q^{(n)}(t)a_l(t - \tau_n^{(1)})]. \tag{3.188}
\end{align*}
\]
From Fig. 3.42, the decision metric on branch $I$ for Rake finger $n$ is thus given by:

$$U_I^{(n)} = \int_{\tau_n^{(1)}}^{r_n^{(1)} + T_s^{(10)}} \Re[U_I^{(n)}(t)]W^{(10)}(t - \tau_n^{(1)})dt$$

$$= \frac{1}{2} \sum_{k=1}^{K} \sum_{c=0}^{N^{(k)} - 1} \sum_{l=0}^{L_c - 1} \sqrt{P^{(kc)}} \int_{\tau_n^{(1)}}^{r_n^{(1)} + T_s^{(10)}} a_{l}^{(k)}(t) \times$$

$$[b_{l}^{(kc)}(t - \tau_{l}^{(k)})W^{(kc)}(t - \tau_{l}^{(k)})a_I(t - \tau_{l}^{(k)})W^{(10)}(t - \tau_n^{(1)})a_I(t - \tau_n^{(1)})\cos(\varphi_{I,nI}^{(10)})$$

$$- b_{l}^{(kc)}(t - \tau_{l}^{(k)})W^{(kc)}(t - \tau_{l}^{(k)})a_Q(t - \tau_{l}^{(k)})W^{(10)}(t - \tau_n^{(1)})a_I(t - \tau_n^{(1)})\cos(\varphi_{Q,ni}^{(10)})$$

$$+ b_{l}^{(kc)}(t - \tau_{l}^{(k)})W^{(kc)}(t - \tau_{l}^{(k)})a_I(t - \tau_{l}^{(k)})W^{(10)}(t - \tau_n^{(1)})a_I(t - \tau_n^{(1)})\sin(\varphi_{I,nI}^{(10)})$$

$$+ b_{l}^{(kc)}(t - \tau_{l}^{(k)})W^{(kc)}(t - \tau_{l}^{(k)})a_Q(t - \tau_{l}^{(k)})W^{(10)}(t - \tau_n^{(1)})a_I(t - \tau_n^{(1)})\sin(\varphi_{Q,ni}^{(10)})$$

$$- b_{l}^{(kc)}(t - \tau_{l}^{(k)})W^{(kc)}(t - \tau_{l}^{(k)})a_I(t - \tau_{l}^{(k)})W^{(10)}(t - \tau_n^{(1)})a_Q(t - \tau_n^{(1)})\sin(\varphi_{Q,ni}^{(10)})$$

$$+ b_{l}^{(kc)}(t - \tau_{l}^{(k)})W^{(kc)}(t - \tau_{l}^{(k)})a_Q(t - \tau_{l}^{(k)})W^{(10)}(t - \tau_n^{(1)})a_I(t - \tau_n^{(1)})\cos(\varphi_{I,nI}^{(10)})$$

$$+ b_{l}^{(kc)}(t - \tau_{l}^{(k)})W^{(kc)}(t - \tau_{l}^{(k)})a_Q(t - \tau_{l}^{(k)})W^{(10)}(t - \tau_n^{(1)})a_Q(t - \tau_n^{(1)})\cos(\varphi_{Q,ni}^{(10)})$$

$$+ b_{l}^{(kc)}(t - \tau_{l}^{(k)})W^{(kc)}(t - \tau_{l}^{(k)})a_I(t - \tau_{l}^{(k)})W^{(10)}(t - \tau_n^{(1)})a_Q(t - \tau_n^{(1)})\cos(\varphi_{Q,ni}^{(10)})]$$

$$+ I_{N/I}^{(n)}$$

$$= S_I^{(n)} + I_{MP,I}^{(n)} + I_{CM,I}^{(n)} + I_{MMC,I}^{(n)} + I_{MPA,I}^{(n)} + I_{N/I}^{(n)} \tag{3.189}$$

where the terms in Eq. (3.189) are as defined in Section 3.3. After expanding the expression for $U_I^{(n)}$, the terms in Eq. (3.189) can be written as follows:

$$S_I^{(n)} = a_{n}^{(1)}\sqrt{P^{(10)}}b_{I,0}^{(10)}T_s^{(10)}, \tag{3.190}$$

$$I_{N/I}^{(n)} = \int_{\tau_n^{(1)}}^{r_n^{(1)} + T_s^{(10)}} W^{(10)}(t - \tau_n^{(1)}) \times$$

$$[(n(t)\cos(w_c t + \varphi_{I,n}^{(10)}))_L a_I(t - \tau_n^{(1)})$$

$$+ (n(t)\sin(w_c t + \varphi_{I,n}^{(10)}))_L a_Q(t - \tau_n^{(1)})]dt, \tag{3.191}$$

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\[ I_{MP,I}^{(n)} \quad = \quad \frac{1}{2} \sum_{l=0}^{L_{a}-1} \alpha_{l}^{(1)} \sqrt{P^{(10)}} \cos(\varphi_{\alpha,ni}) \times \\
[ b_{I,u}^{(10)} (R_{II}^{(10)} (\tau^{(1)}) + R_{QQ}^{(10)} (\tau^{(1)})) + b_{I,v}^{(10)} (\hat{R}_{II}^{(10)} (\tau^{(1)}) + \hat{R}_{QQ}^{(10)} (\tau^{(1)}))], \quad (3.192) \]

\[ I_{CS,I}^{(n)} \quad = \quad \frac{1}{2} \sum_{l=0}^{L_{a}-1} \alpha_{l}^{(1)} \sqrt{P^{(10)}} \left[ - (b_{Q,u}^{(10)} R_{QI}^{(10)} (\tau^{(1)}) + b_{Q,v}^{(10)} \hat{R}_{QI}^{(10)} (\tau^{(1)})) \cos(\varphi_{Q,ni}) \\
+ (b_{Q,v}^{(10)} R_{IQ}^{(10)} (\tau^{(1)})) \cos(\varphi_{Q,ni}) \right] \\
+ \frac{1}{2} \sum_{l=0}^{L_{a}-1} \alpha_{l}^{(1)} \sqrt{P^{(10)}} \left[ (b_{I,u}^{(10)} R_{QI}^{(10)} (\tau^{(1)}) + b_{I,v}^{(10)} \hat{R}_{QI}^{(10)} (\tau^{(1)})) \sin(\varphi_{Q,ni}) \\
+ (b_{Q,v}^{(10)} R_{II}^{(10)} (\tau^{(1)})) \sin(\varphi_{Q,ni}) \right] \\
+ (b_{I,u}^{(10)} R_{IQ}^{(10)} (\tau^{(1)})) \sin(\varphi_{Q,ni}) \\
- (b_{I,v}^{(10)} R_{IQ}^{(10)} (\tau^{(1)})) \sin(\varphi_{Q,ni}) \\
+ (b_{Q,u}^{(10)} R_{QQ}^{(10)} (\tau^{(1)})) \sin(\varphi_{Q,ni}) \right], \quad (3.193) \]

\[ I_{MC,I}^{(n)} \quad = \quad \sum_{c=1}^{N^{(1)}-1} \sum_{l=0}^{L_{a}-1} \alpha_{l}^{(1)} I_{I}^{(n)} (1, c), \quad (3.194) \]

\[ I_{MA,I}^{(n)} \quad = \quad \sum_{k=2}^{K} \sum_{c=0}^{N^{(k)}-1} \sum_{l=0}^{L_{a}-1} \alpha_{l}^{(k)} I_{I}^{(n)} (k, c), \quad (3.195) \]
where:

\[ I_I^{(n)}(k, c) = \frac{\sqrt{P(kc)}}{2} \left[ (b_{I,u}^{(kc)} R_{II}^{(kc)} (\tau^{(k)}) + b_{I,v}^{(kc)} R_{II}^{(kc)} (\tau^{(k)})) \cos(\varphi_{I,nc}) \right. \]
\[ + (b_{Q,u}^{(kc)} R_{II}^{(kc)} (\tau^{(k)}) + b_{Q,v}^{(kc)} R_{II}^{(kc)} (\tau^{(k)})) \sin(\varphi_{I,nc}) \]
\[ - (b_{Q,u}^{(kc)} R_{II}^{(kc)} (\tau^{(k)}) + b_{Q,v}^{(kc)} R_{II}^{(kc)} (\tau^{(k)})) \sin(\varphi_{Q,nc}) \]
\[ + (b_{Q,u}^{(kc)} R_{II}^{(kc)} (\tau^{(k)}) + b_{Q,v}^{(kc)} R_{II}^{(kc)} (\tau^{(k)})) \sin(\varphi_{Q,nc}) \]
\[ + (b_{Q,u}^{(kc)} R_{II}^{(kc)} (\tau^{(k)}) + b_{Q,v}^{(kc)} R_{II}^{(kc)} (\tau^{(k)})) \sin(\varphi_{Q,nc}) \]
\[ + (b_{Q,u}^{(kc)} R_{II}^{(kc)} (\tau^{(k)}) + b_{Q,v}^{(kc)} R_{II}^{(kc)} (\tau^{(k)})) \sin(\varphi_{Q,nc}) \]
\[ \left. + (b_{Q,u}^{(kc)} R_{II}^{(kc)} (\tau^{(k)}) + b_{Q,v}^{(kc)} R_{II}^{(kc)} (\tau^{(k)})) \sin(\varphi_{Q,nc}) \right] \right) \] 

(3.196)

and where \( \{u, v\} = \{-1, 0\} \) and \( \tau^{(k)} = \tau^{(k)}_{nl} \) if \( \tau^{(k)}_{nl} > 0 \), and \( \{u, v\} = \{0, 1\} \) and \( \tau^{(k)} = \tau^{(k)}_{nl} + T_b \) if \( \tau^{(k)}_{nl} < 0 \). The terms \( R_{xy}^{(kc)}(\tau) \) are as defined in Section 3.4, but with \( N = T_s / T_c = 2T_b / T_c \). Notice that for \( I_{MC,l}^{(n)} \) and \( I_{MA,l}^{(n)} \) the component for which indice \( l = n \) has been removed. Indeed, when the collected multipath components are synchronous with the desired signal (which happens when \( l = n \)), the interference produced by these components is null due to the orthogonal covering of the spreading sequences.

A similar development can be carried out for the decision metrics on the \( Q \) branch, leading to the following:

\[ U_Q^{(n)} = \int_{\tau^{(1)}_n}^{\tau^{(1)}_n + T_s^{(10)}} \text{Im}[U^{(n)}(t)] W^{(10)}(t - \tau^{(1)}_n) dt \]
\[ = S_Q^{(n)} + I_{MP,Q}^{(n)} + I_{CS,Q}^{(n)} + I_{MC,Q}^{(n)} + I_{MA,Q}^{(n)} + I_{N,Q}^{(n)} \] 

(3.197)

where:

\[ S_Q^{(n)} = \alpha_n^{(1)} \sqrt{P^{(10)}} b_{Q,0}^{(10)} T_s^{(10)} \] 

(3.198)
\[ I_{N,Q}^{(n)} = \int_{\tau_0^{(1)}}^{\tau_0^{(1)} + \tau_0^{(10)}} W^{(10)}(t - \tau_n^{(1)}) \times \\
[(n(t) \cos(w_c t + \varphi_{1,n}^{(10)})LPA I(t - \tau_n^{(1)}) \\
+ (n(t) \sin(w_c t + \varphi_{1,n}^{(10)})LPA Q(t - \tau_n^{(1)}))]dt, \quad (3.199) \]

\[ I_{M,P,Q}^{(n)} = \frac{1}{2} \sum_{l=0}^{L_n - 1} \alpha_l^{(1)} \sqrt{P^{(10)}} \cos(\varphi_{1,ni}) \times \\
[b_{Q,n}^{(10)}(R_{II}^{(10)}(\tau^{(1)}) + R_{QQ}^{(10)}(\tau^{(1)})) + b_{Q,v}^{(10)}(\dot{R}_{II}^{(10)}(\tau^{(1)}) + \dot{R}_{QQ}^{(10)}(\tau^{(1)}))], \quad (3.200) \]

\[ I_{C,S,Q}^{(n)} = \frac{1}{2} \sum_{l=0}^{L_n - 1} \alpha_l^{(1)} \sqrt{P^{(10)}}[-(b_{Q,n}^{(10)}R_{II}^{(10)}(\tau^{(1)}) + b_{Q,v}^{(10)}\dot{R}_{II}^{(10)}(\tau^{(1)})) \cos(\varphi_{1,ni}) \\
+ (b_{Q,n}^{(10)}R_{QQ}^{(10)}(\tau^{(1)}) + b_{Q,v}^{(10)}\dot{R}_{QQ}^{(10)}(\tau^{(1)})) \cos(\varphi_{1,ni})] \\
+ \frac{1}{2} \sum_{l=0}^{L_n - 1} \alpha_l^{(1)} \sqrt{P^{(10)}}[-(b_{Q,n}^{(10)}R_{II}^{(10)}(\tau^{(1)}) + b_{Q,v}^{(10)}\dot{R}_{II}^{(10)}(\tau^{(1)})) \sin(\varphi_{1,ni}) \\
-(b_{Q,n}^{(10)}R_{QQ}^{(10)}(\tau^{(1)}) + b_{Q,v}^{(10)}\dot{R}_{QQ}^{(10)}(\tau^{(1)})) \sin(\varphi_{1,ni})] \\
+ (b_{Q,n}^{(10)}R_{II}^{(10)}(\tau^{(1)}) + b_{Q,v}^{(10)}\dot{R}_{II}^{(10)}(\tau^{(1)})) \sin(\varphi_{1,ni})], \quad (3.201) \]

\[ I_{M,C,Q}^{(n)} = \sum_{c=1}^{N^{(1)} - 1} \sum_{l=0}^{L_n - 1} \alpha_l^{(1)} I_I^{(n)}(1, c), \quad (3.202) \]

\[ I_{M,A,Q}^{(n)} = \sum_{k=2}^{K} \sum_{c=0}^{N^{(k)} - 1} \sum_{l=0}^{L_n - 1} \alpha_l^{(k)} I_I^{(n)}(k, c), \quad (3.203) \]
where:

\[
I_Q^{(n)}(k, c) = \frac{\sqrt{P^{(c)}}}{2} \left[ \left( -b_{I,w}^{(k)} \hat{R}_{II}^{(k)}(\tau^{(k)}) + b_{I,w}^{(k)} \hat{R}_{II}^{(k)}(\tau^{(k)}) \right) \cos(\varphi_{I,ni}) \right. \\
+ (b_{Q,w}^{(k)} \hat{R}_{QQ}^{(k)}(\tau^{(k)}) + b_{Q,w}^{(k)} \hat{R}_{QQ}^{(k)}(\tau^{(k)}) \cos(\varphi_{Q,ni}) \\
- (b_{I,w}^{(k)} \hat{R}_{QQ}^{(k)}(\tau^{(k)}) + b_{I,w}^{(k)} \hat{R}_{QQ}^{(k)}(\tau^{(k)}) \sin(\varphi_{Q,ni}) \\
- (b_{Q,w}^{(k)} \hat{R}_{QQ}^{(k)}(\tau^{(k)}) + b_{Q,w}^{(k)} \hat{R}_{QQ}^{(k)}(\tau^{(k)}) \sin(\varphi_{Q,ni}) \\
- (b_{I,w}^{(k)} \hat{R}_{II}^{(k)}(\tau^{(k)}) + b_{I,w}^{(k)} \hat{R}_{II}^{(k)}(\tau^{(k)}) \sin(\varphi_{I,ni}) \\
+ (b_{Q,w}^{(k)} \hat{R}_{II}^{(k)}(\tau^{(k)}) + b_{Q,w}^{(k)} \hat{R}_{II}^{(k)}(\tau^{(k)}) \sin(\varphi_{Q,ni}) \\
+ (b_{Q,w}^{(k)} \hat{R}_{II}^{(k)}(\tau^{(k)}) + b_{Q,w}^{(k)} \hat{R}_{II}^{(k)}(\tau^{(k)}) \cos(\varphi_{Q,ni}) \right] . \tag{3.204}
\]

### 3.5.2.3 Statistics of Decision Metrics

Let \( E_b = E_{b_{1,0}}^{(10)} \) and \( T_b = T_{b_{1,0}}^{(10)} \). Considering MRC, the conditional expected value of the desired received signal \( S_I = \sum_{n=0}^{L-1} \alpha_n^{(1)} S_I^{(n)} \) is obtained from (3.190):

\[
\bar{S}_I = \sqrt{E_b T_s} \sum_{n=0}^{L-1} [\alpha_n^{(1)}]^2. \tag{3.205}
\]

The noise term \( I_{N,I} = \sum_{n=0}^{L-1} \alpha_n^{(1)} I_{N,I}^{(n)} \) can be easily shown to be a zero-mean Gaussian random variables with variance:

\[
\sigma_{I_{N,I}}^2 = \text{Var}[I_{N,I}] = \frac{T_s N_0}{2} \sum_{n=0}^{L-1} [\alpha_n^{(1)}]^2. \tag{3.206}
\]

Using the standard Gaussian approximation, we model the interference terms (3.192)-(3.195) as mutually uncorrelated zero-mean Gaussian random variables with given conditional variances. These variances are derived below, and are conditional on the random variables \( \alpha_i^{(1)}, l = 1, 2, \ldots, L \). The interference terms \( I_{MP,I} = \sum_{n=0}^{L-1} \alpha_n^{(1)} I_{MP,I}^{(n)} \), \( I_{CS,I} = \sum_{n=0}^{L-1} \alpha_n^{(1)} I_{CS,I}^{(n)} \), \( I_{MC,I} = \sum_{n=0}^{L-1} \alpha_n^{(1)} I_{MC,I}^{(n)} \), and \( I_{MA,I} = \sum_{n=0}^{L-1} \alpha_n^{(1)} I_{MA,I}^{(n)} \) are all chip synchronous with the desired user signal, since they originate from signals which are transmitted synchronously on the forward link. Therefore \( \tau_{nl}^{(k)} = \tau_{nl}^{(1)} \) for all \( k, n \) and
This results in $\text{Var}\left[\hat{R}^{(1c)}_{xy}(r_{nl}^{(1)}) + \hat{R}^{(1c)}_{xy}(r_{nl}^{(1)})\right] = T_s^2/N$ (with $l \neq n$). The following variances are then obtained:

$$\sigma_{IM_P,I}^2 = \text{Var}[I_{M_P,I}] = \sum_{n=0}^{L-1} [\alpha_n^{(1)}]^2 \frac{P^{(10)}T_s^2}{4N} \sum_{l=0}^{L_c-1} \sum_{l \neq n} [\alpha_l^{(1)}]^2,$$

$$\sigma_{ICS,I}^2 = \text{Var}[I_{CS,I}] = \sum_{n=0}^{L-1} [\alpha_n^{(1)}]^2 \frac{P^{(10)}T_s^2}{4N} \left( \sum_{l=0}^{L_c-1} [\alpha_l^{(1)}]^2 + 2 \sum_{l \neq n} [\alpha_l^{(1)}]^2 \right),$$

$$\sigma_{IM_C,I}^2 = \text{Var}[I_{MC,I}] = \sum_{n=0}^{L-1} [\alpha_n^{(1)}]^2 \sum_{c=1}^{N^{(1)}} \frac{P^{(1c)}T_s^2}{N} \sum_{l=0}^{L_c-1} \sum_{l \neq n} [\alpha_l^{(1)}]^2,$$

$$\sigma_{IMA,I}^2 = \text{Var}[I_{MA,I}] = \sum_{n=0}^{L-1} [\alpha_n^{(1)}]^2 \sum_{k=2}^{K} \sum_{c=0}^{N^{(k)-1}} \frac{P^{(kc)}T_s^2}{N} \sum_{l=0}^{L_c-1} \sum_{l \neq n} [\alpha_l^{(1)}]^2.$$

In Eqs. (3.207)-(3.210), the variances are seen to be conditional on the term $\sum_{l=0}^{L_c-1} [\alpha_l^{(1)}]^2$. Indeed, the interference terms are affected by the same coefficients $\alpha_l^{(1)}$, $l = 0, \ldots, L-1$ as the desired signal $S_l$, since they all fade in unison. Thus these interference terms are correlated with the signal term, and this fact must be taken into account when determining the error probability.

Assuming equal powers $P^{(kc)} = P = E_b/T_s$ for all $k$ and $c$, the total variance from the interference and thermal noise sums up to:

$$\sigma_I^2 = \sigma_{IM_P,I}^2 + \sigma_{IM_C,I}^2 + \sigma_{ICS,I}^2 + \sigma_{IMA,I}^2 + \sigma_{N,I}^2$$

$$= \sum_{n=0}^{L-1} [\alpha_n^{(1)}]^2 \left( \frac{N_0 T_s}{2} + \sum_{l=0}^{L_c-1} \sum_{l \neq n} [\alpha_l^{(1)}]^2 \sum_{k=1}^{K} N^{(k)} \frac{E_b T_s}{4N} [\alpha_n^{(1)}]^2 \right).$$
3.5.2.4 Probability of Error

From Eqs. (3.205) and (3.211), and using our assumption that \( L_c = L \), the conditional output SNR at the \( I \)-branch of the receiver for MRC then takes the form:

\[
SNR_{O,I} = \frac{\left( \sum_{n=0}^{L-1} [\alpha_n^{(1)}]^2 \right)^2}{\sum_{n=0}^{L-1} [\alpha_n^{(1)}]^2 \left( \frac{N_0}{2E_b} + \frac{1}{N} \sum_{k=1}^{K} N(k) \sum_{l=0}^{L-1} [\alpha_l^{(1)}]^2 + \frac{1}{4N} [\alpha_0^{(1)}]^2 \right)}.
\] (3.212)

As in Section 3.4.2.4, it is seen that the fading coefficients \([\alpha_l^{(1)}]^2, l = 1, \ldots, L - 1\) appear both in the numerator and denominator of the SNR, since the interference terms are subject to the same fading as is the received desired signal. Neglecting the term \(\frac{1}{4N} [\alpha_n^{(1)}]^2\) due to the complex spreading self-interference, Eq. (3.212) is seen to be similar to Eq. (3.179) (recalling that the processing gain \(N = T_s/T_c = 2T_b/T_c\) in (3.212) is twice the processing gain used in Eq. (3.179)), which is to be expected. Letting \(P_{b,I} = Q(\sqrt{SNR_{O,I}})\) and \(P_{b,Q} = Q(\sqrt{SNR_{O,Q}})\) be the conditional probabilities of bit error on the \( I \)- and \( Q \)-branches, respectively, the conditional probability of symbol error is obtained as:

\[
P_s(\{\alpha_l^{(1)}\}) = 1 - (1 - P_{b,I})(1 - P_{b,Q}) = 2Q(\sqrt{SNR_O}) + [Q(\sqrt{SNR_O})]^2
\] (3.213)

where \(SNR_O = SNR_{O,I} = SNR_{O,Q}\) due to symmetry. The conditional BER can be approximated as \([26] P_b(\{\alpha_l^{(1)}\}) \approx \frac{1}{2} P_s(\{\alpha_l^{(1)}\}) \approx Q(\sqrt{SNR_O})\), which is similar to Eq. (3.181). The unconditional BER is then obtained in a semi-analytical fashion as in Section 3.4.2.4, through Monte Carlo integration. The numerical results will be similar to those presented in Section 3.4.3, and hence are not presented again to avoid repetition.
3.6 Conclusions

This chapter presented the analysis of multicode DS/CDMA systems in wideband fading channels, which hadn't been tackled yet in a precise manner in previous works. By taking into account the effect of dependent fading between the multicode or multiuser interference and the desired signal, accurate analytical or semi-analytical theoretical results were obtained for the BER, and interesting insights into the behavior of multicode systems were given. Analyses were provided for the four main types of configurations encountered in IS-95B and IS-2000 systems: reverse link with noncoherent \( M \)-ary modulation and real spreading, reverse link with BPSK modulation and complex spreading, forward link with QPSK modulation and real spreading, and forward link with QPSK modulation and complex spreading. In the first case, the system employed non-orthogonal long spreading sequences to differentiate the codes of a same user, while in the other cases the systems used orthogonal Walsh covering. The analysis was general enough to be applicable to different fading distributions, to systems with correlated diversity branches, to multi-cell systems, and was extended to deal with advanced techniques such as closed-loop power control and multiuser detection. In all cases, the theoretical results were thoroughly validated using entire system simulations.

The analysis presented in the present chapter was essential in obtaining the results reported in the next chapter, which considers VBR video transmission over multicode/multirate IS-95B and IS-2000 DS/CDMA systems, given that:

- It provided the decision metrics used in the simulation implementations.
- It derived BER expressions which were used to validate the simulations.

While this analysis considered several different fading environments, the next chapter will mainly make the assumption of Rayleigh fading and uncorrelated diversity branches, which is commonly used in order to simplify simulations. The Rayleigh distribution represents the worst case of the Rice and Nakagami (for \( m \geq 1 \); the case \( m < 1 \) is more rare) distributions, which will hence allow us to observe to the full extent the degradations caused by the channel fading. The Rayleigh fading model (along with its methods of simulation) is also more widely accepted than the Nakagami model.
CHAPTER 4
VBR VIDEO TRANSMISSION FOR MULTICODE
AND MULTIRATE DS/CDMA SYSTEMS IN
WIDEBAND FADING CHANNELS

4.1 Introduction

In the previous chapters, we provided detailed analyses of the performance of multi-code and multirate DS/CDMA systems used to support high-rate services (in our case streaming or interactive video), in the presence of wideband fading and multiple-access interference. The systems were evaluated in terms of BER at the modulation/demodulation level, i.e. at the lowest level of the physical layer. This was helpful in understanding the effects of the channel parameters (e.g. severity of fading, correlation between branches) and of the transmitter/receiver parameters (e.g. number of assigned parallel codes, data rate and processing gain, number of diversity branches). In this chapter, we move further up the transmission chain to include the error-control coding/decoding, interleaving/deinterleaving, framing, packetizing and video coding/decoding operations.

While both analysis and simulation were consistently used in Chapters 2 and 3, in this chapter we will essentially rely on simulation results, due to the high complexity of the full transmission system, and the resulting difficulty in finding exact or even accurate analytical solutions. Indeed, our main performance criterion will be the peak signal-to-noise ratio (PSNR), which is an objective measure of the decoded video quality, and is hence more representative of the actual user-perceived performance than measures such as the bit and frame error rates. The PSNR is a highly nonlinear function of the BER and the distribution of the errors, which themselves depend in a nonlinear fashion on the peak, mean and variance of the rate (as will be discussed in Section 4.3). Most previously
published papers which try to obtain relations between the PSNR and the channel characteristics usually assume a simplified time-invariant channel model (such as a Markov chain), and yet the analysis remains approximate [235], [236]. In our case, we are further dealing with a time-variant channel (due to the changing rates and thus varying processing gain or number of multicodes), which includes sophisticated elements such as a channel coder/decoder, an interleaver/deinterleaver, and which is additionally subject to not-necessarily-Gaussian noise (the multiple-access interference from other users). We thus believe that a formal analysis in terms of PSNR of such an end-to-end system remains beyond the current state-of-the-art of the analysis of video communication systems (which is in its early stages). For our purposes simulations are much more useful. As a result we implemented a sophisticated customizable software platform which includes the video coder/decoder, the packetizer/d depacketizer, and the physical layer components of the IS-95B and IS-2000 systems, for both uplink and downlink.

The organization of this chapter is as follows. In Section 4.2, we describe in detail the characteristics of all of the elements of the software platform, according to the corresponding industry standards. This is necessary to appreciate under which conditions the simulation results were obtained, and in order to be able to reproduce these conditions if the need arises. In Section 4.3, we tackle the issue of transmission rate control for VBR video over multicode/multirate DS/CDMA. In particular, we compare the performances of a proposed smoothing algorithm and of a benchmark one, which are both adapted to deal with transmission formats compatible with IS-95B/IS-2000 systems. To evaluate the PSNR in the presence of wideband fading and multiple-access interference, the smoothing algorithms are used in conjunction with the previously described simulation platform.

4.2 System Description

4.2.1 Overview of a Video Communication System for IS-95B/IS-2000 Networks

Fig. 4.1 gives a high-level representation of a video communication system for 2.5G/3G cellular networks. The representation is based on the elements which compose the sys-
tem: a video source, a transport coder, a network and a radio link. Each of these entities will be briefly reviewed separately.

![Diagram of video communication system for cellular network.](image)

**Figure 4.1** Video communication system for cellular network.

### 4.2.1.1 Video Services

**Video Applications**

A wide range of video applications can be conceived for wireless networks. From a delay viewpoint, they can be roughly categorized in the following three groups:

- Low-latency video conversational applications, e.g. video telephony, video teleconferencing. The video encoding is done in real-time, and the end-to-end allowable delay must be very limited (at most within a few hundred milliseconds);

- Low-to-medium-latency video streaming applications, e.g. video-on-demand, video broadcast, video surveillance. The allowable end-to-end delay is variable, but in the order of seconds or tens of seconds;

- High-latency video messaging applications, e.g. video e-mail. There is no hard constraint on the delay, which must however be reasonably bounded.

From an encoding viewpoint, these applications can also be categorized in the following two groups:

- Live video applications, e.g. video telephony, video teleconferencing, video surveillance. The video encoding is done in real-time;
• Prerecorded video applications, e.g. video-on-demand, video e-mail. The video encoding has already been done before the beginning of transmission.

This chapter will mainly deal with conversational and streaming video, for both live and prerecorded applications. Indeed, messaging applications can use protocols which are used for conventional data applications.

**Video Encoding Architecture**

In Fig. 4.1, at the sender end, the video bitstream can be produced in real-time by compressing a captured video source, such as for live conversational or broadcast video. It can also have been produced earlier and stored in digital format, and be retrieved from its storage location and streamed to a mobile user, such as for video-on-demand. If the video application is duplex, such as conversational video, the sender also needs to be able to decode an incoming video bitstream and display it in real-time.

Fig. 4.2 illustrates the main functions performed by a real-time video encoder: image capture, compression, bitstream generation and rate control. The image capture (or frame grabbing) step consists in converting a sequence of analog pictures into a sequence of digital images which can be processed by the video compressor. The goal of the compression step is to reduce a very large quantity of image data to a much smaller number of bits, while minimizing the effect that the loss of information due to lossy coding will have on the reconstructed video quality. The ratio between the uncompressed and compressed rates (i.e. the compression efficiency) depends on the attributes of the video sequence (such as the amount of motion, the number of scene changes, the scene complexity, the level of details, etc.); for H.263 and H.264 it can be higher than 100 for some video streams. The goal of the bitstream generation step is to organize the information produced by the compression step in a bit sequence which can be handled efficiently by the transport coder/decoder and the video decoder, while lending itself to error resilient techniques.

All of the previous operations can be viewed as belonging to the application layer of the ISO (International Standards Organization) layering model [237].

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Standards for (Very) Low Bit Rate (VLBR) Video Coding

Currently, H.263/H.264 and MPEG-4 are the most popular video codecs for VLBR mobile applications: they have been recommended as the video components for some mobile multimedia terminals [238], [239].

H.263 is the ITU-T (International Telecommunications Union - Telecom) standard for VLBR coding. It evolved from another ITU-T standard, H.261 [240], originally proposed for videoconferencing over the public switched telephone network (PSTN). The first H.263 standard was released in 1996 [241]. Revision 2 (also termed H.263+) was issued in 1998 [41], and revision 3 (or H.263++) in 2000. A detailed description of H.263 can be found in book Chapters 11 of [242] (version 1) and 1 of [43] (version 2), for example, and a short description will be given in Section 4.2.2. By H.263, we will denote the latest revision. The H.263 standard describes the syntax and the functional operation of the decoder, and does not mandate any particular algorithm to be used at the encoder side, as long as the bitstream produced by the encoder complies with the syntax expected at the decoder for proper operation. The image capture procedure and the rate control strategy are outside the scope of the H.263 standard. The standard consists of the main body for baseline syntax and decoder specification and of a set of Annexes (A through X in H.263++) which describe optional modes and some additional specifications. A set of levels and profiles (Annex X) recommend combinations of these optional modes as a function of the application and/or type of network connection. While allowing different levels of operation for different systems, the large number of optional modes in H.263 also has its share of disadvantages, such as implementation complexity and overhead. In 1999, opting for a “back-to-basics” approach, the working group in charge of H.263 standardization, ITU-T Study Group (SG) 16/ Video Coding Experts Group (VCEG), Question 6 (Q.6, formerly Q.15), started working on a new VLBR coding standard,
H.264 (also previously denoted as H.26L or JVT), whose main objectives were better compression efficiency and network friendliness [243]. Since the operations of H.263 and H.264 codecs have many fundamental similarities, the description in Section 4.2.2 will apply to both codecs, denoted by H.26x.

### 4.2.1.2 Transport and Network Services

**Transport and Network Layer Functions**

The video data generated at the application layer is passed on to the transport and network layers. The specifications of the transport and network layers describe how the packetized video stream is to be transported over a circuit-switched or packet-switched network connection, in order to be delivered to/from the cellular BS serving the MS.

The main tasks supported by the transport layer are the following:

- Segmentation and packetization of the video stream in a format suitable for transmission over the network layer;

- Multiplexing of the packets of the video connection with packets from other connections/applications;

- Error control at the transport layer, either by error detection, Forward Error Correction (FEC), retransmission ARQ), or combinations of these;

- Flow control (if there is a need for it at this stage).

The main tasks supported by the network layer are the following:

- Routing of the packets delivered by the transport layer across the core wireline network and the radio access network (RAN), to/from the BS;

- Congestion (or flow) control.

Fig. 4.3 gives a simplified view of the architecture of a cdma2000 radio access network, which supports both circuit-switched and packet-switched traffic, according to [244]. Considering the MS-to-network link, the circuit-switched data is sent from the BS to a
mobile switching center (MSC), which then directs it to the PSTN, or to a packet data
network (PDN) such as the Internet. The packet-switched data is first processed by a
packet control function (PCF), which relays it to a packet data serving node (PDSN),
whose functions are to establish, maintain and terminate link layer sessions to mobile
stations. The data is then routed to a PDN.

![Diagram of cdma2000 wireless network model](image)

**Figure 4.3** cdma2000 wireless network model.

**Standards for Transport and Network Protocols**

The importance and relevance of the tasks described above differ according to whether the
network is circuit-switched or packet-switched: therefore, different protocols are defined
for each case. In particular, the functions performed at the network layer are more
relevant to packet-switched networks than they are to circuit-switched networks, since
the latter benefit from a dedicated connection and are therefore less concerned about
packet routing.

For circuit-switched networks, H.324/M [245] is the leading standard defining the
operation of mobile multimedia terminals. It mandates in particular the use of the
following set of standards:

- H.261 [240] and H.263 [246] for video coding, G.723.1 for audio coding (optionally
  G.728 and G.729);
- H.223 [247] for multimedia multiplex and synchronization;
- H.245 [248] for system control.
Other standards are also recommended (T.120 for data, H.226 for multilink operation). The main features of H.223, which will be used in our circuit-switched simulation model, are summarized in Section 4.2.3.

4.2.1.3 Radio Link Services

The radio link is responsible for the communication between the BS and MS. It covers both the link layer and the physical layer. The tasks of the link layer are to:

- Provide link access control (LAC), which includes management of signaling control information;
- Provide medium access control (MAC), which includes optional best effort delivery (retransmission, using the Radio Link Protocol, RLP), optional Quality-of-Service (QoS) control, resource allocation between services and multiplexing of the traffic channels onto the physical layer channels.

The point-to-point protocol (PPP) [249] is used to interface between the LAC layer and the IP layer.

The physical layer specifies the wireless transmission technology. In our simulation model we will be using the IS-95B and IS-2000 standards, whose main features of interest to us are described in Sections 4.2.4-4.2.7.

4.2.2 H.26x Video Coding and Bitstream Generation

This section first describes the picture formats used by H.26x coders and the organization of pictures into smaller structures. It then reviews the general coding principles used by these coders. The principles behind the bitstream generation process are also summarized.

4.2.2.1 Picture Format and Organization

Different picture formats corresponding to different resolutions can be used by the codec. Five of them are explicitly specified in H.263: CIF (common intermediate format), QCIF (quarter-CIF), sub-QCIF, 4CIF and 16CIF. Custom picture formats can also be
negotiated by the encoder. However only the QCIF and sub-QCIF are mandatory for an H.263 decoder, and the encoder only needs to support one of them. These formats are obtained by subsampling PAL/SECAM or NTSC video signals, the conversion process being outside the scope of the H.263 standard.

A QCIF picture consists of 144 lines and 176 pels (or pixels) per line. Such a picture is divided into a number of groups of blocks (GOB's). Each GOB spans one or more rows of the picture, with each row consisting of 11 macroblocks (MB's). In Fig. 4.4, the picture is divided in 9 GOB's, with each GOB containing one row of the picture. MB's are composed of 6 blocks, 4 of which are luminance (Y) blocks and 2 chrominance (Cb and Cr) blocks. Blocks are square and consist of 8 by 8 pixels. While blocks are the basic structure used for many encoding/decoding operations in H.263, smaller structures (e.g. groups of 4×4 pixels) are preferred in H.264 for certain operations. A slice is a grouping of an arbitrary number of MB's (within a picture): the use of slices is optional in H.263 (Annex K: Slice Structured mode), but is part of the baseline coder in H.264.

4.2.2.2 Overview of the Core Video Coding Process

Fig. 4.5 depicts the schematic of an H.26x encoder, whose operation is based on hybrid differential/transform encoding, and is a combination of lossy and lossless coding. There are two fundamental modes which are jointly used for maximum compression efficiency: the intra and inter modes. Different types of frames correspond to these modes.

In the intra mode, the contents of a video frame are first processed by a transform for energy compaction and thus bit-rate reduction. The resulting coefficients are quantized with a chosen quantizer step size, thus leading to a loss of information. The quantized coefficients are encoded using a certain entropy coding strategy, scanned across the picture (often using a zig-zag strategy), and delivered to an encoder buffer. An optional rate control algorithm, which probes the buffer fullness, is used to adapt the quantizer step size or decide the number of frames to be skipped.

In the inter mode, the same operations are applied to the motion-predicted difference between the current frame and the previous (or earlier) frame, instead of to the frame itself. To this end a motion estimation algorithm is applied to the input frame, and the extracted motion information (in the form of motion vectors, MV's) is used in predicting
the following frame(s), through a motion-compensation circuit. In order to avoid a drift between the encoder and decoder due to motion prediction, the motion compensation circuit needs to use a locally reconstructed version of the compressed frame being sent: this explains the presence of an inverse quantizer and an inverse transform in the feedback loop. The MV's are differentially encoded in order to realize bit rate savings. A deblocking filter can also be included in the motion-compensation loop in order to reduce visual artefacts.

The intra mode produces an intra frame or I-frame. This type of frame is needed for the decoder to have a reference for prediction. I-frames should also be transmitted at a certain frequency (the H.263 standard specifies at least one I-frame for every 132 frames) in order to refresh the prediction, or when considerable motion is detected in the sequence, which can forbid any accurate prediction. However, I-frames use a large number
of bits, so that they should be used sparingly in low bit-rate applications. The inter mode produces prediction frames or P-frames, which can be predicted from I-frames or other P-frames. These in general use considerably less bits than I-frames, and are responsible for the large compression gain. The inter mode can also produce bidirectionally predicted frames, or B-frames: these are predicted from two pictures (which are allowed to serve as reference pictures), one being temporally precedent and the other temporally subsequent. B-frames are not used as reference pictures for other frames. In H.263, B-frames are either coded jointly with a P-frame as a PB-frame (Annex G: PB-frames mode) or Improved PB-frame (Annex M: Improved PB-frames mode), or coded separately as a standalone ("true") B-frame if the temporal scalability mode of Annex O (temporal, SNR and spatial scalability mode) is used. In H.264, B-frames are coded separately.

Figure 4.5 H.26x video encoder.
Other frame types are used for scalability or error resilience purposes. Annex O of H.263+ allows the encoding of enhancement frames, such as EI (Enhanced Intra) and EP (Enhanced Predictive) frames, in addition to true B frames. H.264 allows for interstream transitional SI and SP-frames to be encoded.

4.2.2.3 Bitstream Syntax and Generation

An H.26x-compliant bitstream can be characterized in terms of a number of syntax layers, which are organized in the following top-to-bottom hierarchy:

- Picture layer;
- Slice or GOB layer;
- MB layer;
- Block layer;

An element from each layer contains a group of elements from the previous layer, along with a layer-specific header. For example, a picture consists of a picture header followed by a group of slices or GOB's. The picture header contains information on how to decode elements from the slice/GOB layer, and possibly from the lower layers. The headers belonging to the higher layer elements are more important than those of the lower layer elements: for example a lost or corrupted picture header will likely render unusable the data received for all of the lower layer elements, since the decoder relies on this picture header to process the next syntax elements.

4.2.3 H.324/H.223 Packetization and Multiplexing

This section describes how units from the video layer are packetized and multiplexed in order to be transported over the underlying network.

The first step, which is the task of the application layer, is to map the bitstream into a sequence of service data units (SDU). Each SDU is defined as the logical unit of information used in the transfer from one protocol layer entity to the peer protocol layer entity [247]. For video applications a SDU consists of a video packet. In H.263 and
H.264, a video packet contains either a picture, a GOB (for H.263), a slice or a slice data partition.

The resulting SDU's are then handed over from the application layer to the lower layers, for transport-layer and network-layer multiplexing and packetization. At these layers, the protocols used for packetization and multiplexing are ITU-T H.223 (Multiplexing protocol for low bit rate multimedia communication) for H.324-based circuit-switched networks, and RTP/UDP/IP for packet-switched networks. The packetization and multiplexing aspects for H.324/H.223 are described below.

4.2.3.1 H.324/H.223 Packetization and Multiplexing for Circuit-Switched Networks

H.223 is a packet-oriented multiplexing protocol which can be used between either two low bit rate multimedia terminals, or a low bit rate multimedia terminal and an interworking adapter (IWA) or multipoint control unit (MCU). Its role is to multiplex audio, video and data information streams over a single communication link using packets. It can optionally add sequence numbering information and perform error-control procedures. The control of H.223 is handled by the ITU-T H.245 recommendation [248].

Each information stream handled by H.223 can correspond to one or many logical channels. Each logical channel is assigned a logical channel number (LCN), an integer between 0 and 65535, with LCNO being the H.245 control channel. In order to keep the delay and packetization overhead low, H.223 uses segmentation and reassembly, and can combine information from different logical channels into a single packet [247].

The baseline H.223 protocol specification is complemented by a set of Annexes which provide optional features in the case of communication over error-prone environments:

- H.223 Annex A [250] is a multiplexing protocol for low error-prone channels;
- H.223 Annex B [251] is a multiplexing protocol for moderate error-prone channels;
- H.223 Annex C [252] is a multiplexing protocol for highly error-prone channels;
- H.223 Annex D [253] is an optional multiplexing protocol for highly error-prone channels;
The protocol stack and data structures of H.223 are illustrated in Fig. 4.6. H.223 consists of two layers: the adaptation layer (AL) and the multiplex (MUX) layer. The functions performed by each layer are reviewed below.

**Adaptation Layer (AL)**
The AL receives a SDU from the application layer, which is then called an AL-SDU and consists of an integer number of bytes. If the AL-SDU is not segmentable, it is conveyed as a single AL-PDU (protocol data unit) to the MUX layer. If the AL-SDU
is segmentable, then it can be broken down and conveyed as multiple AL-PDU’s to the MUX layer. Depending on the type of AL used, bytes can be added to each AL-PDU for error detection (for audio and video), and optionally for sequence numbering (for audio and video) and retransmission (for video). There are three types of AL’s in baseline H.223:

- AL1 is geared towards the transfer of data and control information. It doesn’t support error control, and works in either framed (the application layer sequence is framed) or unframed (the application layer sequence is unframed) transfer mode;

- AL2 is geared towards the transfer of digital audio. It provides error detection (1-byte cyclic redundancy check, or CRC) and optional sequence numbering (1 byte sequence number);

- AL3 is geared towards the transfer of digital video. It provides error detection (2-byte CRC), optional sequence numbering and retransmission.

The format of an AL-PDU for AL3 is given in Fig. 4.7, along with the syntax for the optional control field.

![Diagram of AL-PDU for AL3](image)

**Figure 4.7** AL-PDU for AL3 with optional control field (CF).

The same AL’s are defined for H.223 Annexes A and B. However, distinct AL’s are defined for Annexes C and D: in both Annexes they are denoted by AL1M (data, control),
AL2M (audio) and AL3M (video). Fig. 4.8 illustrates the format of an AL-PDU for the AL3M of both Annexes C and D, along with the syntax for the optional control field.

<table>
<thead>
<tr>
<th>Control field (2 or 3 bytes)</th>
<th>AL-PDU payload (&gt;= 0 byte)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRC field (Annex C: 4, 12, 20 or 28 bits, Annex D: 8, 16 or 32 bits)</td>
<td>TB: Tail Bits (4) (Annex C only)</td>
</tr>
<tr>
<td>RCPC or RS parity bytes (variable length)</td>
<td>P: Control Error Code (CEC) Parity for SN, RN and X fields:</td>
</tr>
<tr>
<td>SN: Sequence Number (5 or 10 bits)</td>
<td></td>
</tr>
<tr>
<td>RN: Receive Number (1 bit)</td>
<td></td>
</tr>
<tr>
<td>X: Odd (X=1)/Even(X=0) parity of length (in bytes) of non-empty AL-PDU (1 bit)</td>
<td></td>
</tr>
<tr>
<td>P: 9 bits: Systematic Extended Bose–Chaudhuri–Hocquenghem (SEBCH) (16,7,6) code</td>
<td></td>
</tr>
<tr>
<td>P: 12 bits: Extended (E) Golay (24,12,8) code</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.8 AL-PDU for AL3M (Annexes C and D) with control field (CF).

Fig. 4.9 illustrates the structure of the AL3M of Annex C. It uses rate compatible punctured convolutional (RCPC) coding [254] and interleaving to protect the contents of the AL-SDU, and allows optional retransmission procedures (hybrid ARQ types I and II [255]). The following steps are used for encoding a (possibly partial) AL-SDU to an AL-PDU:

- Calculate the length of the AL-PDU;
- Add a CRC of length 4, 12, 20 or 28 bits;
- Add 4 tail bits (TB);
- Perform convolutional encoding using a systematic recursive convolutional code;
- Puncture the output of the encoder, while maintaining rate compatibility, and fill it in a linear buffer according to a given mapping procedure;
• Output the contents of the linear buffer to the AL-PDU payload field;

• Add a CF if the retransmission mode is used;

• Perform block interleaving of the whole AL-PDU.

The details of the above steps can be found in [252]. At the receiver side, the decoding of the AL-PDU in order to reconstruct an AL-SDU follows the inverse steps. If the received AL-PDU is invalid (the number of bytes of the AL-PDU is outside the range of authorized values or is not an integer, or the number of bytes of the reconstructed AL-SDU is not an integer), it is discarded. If it is valid and there is no CRC error, the AL-PDU is delivered to the AL1M along with an error indicator (EI) set to 0. If it is valid but there is a CRC error, the presence of a CF field is checked. If there is an error-free CF, then one of two retransmission procedures, ARQ I and ARQII, can be used. If there is no CF or it is errored, then the AL-PDU is delivered to the AL1M with an EI set to 1.

The encoding/procedures for Annex D parallel those of Annex C, except that Reed-Solomon encoding is now used instead of RCPC coding. Only ARQ I is supported.

**Multiplex (MUX) Layer**

The AL-PDU's are transferred to the MUX layer as MUX-SDU's, where each MUX-SDU contains data from a single logical channel and consists of an integer number of octets. The MUX layer allows to multiplex several segmentable logical channels contained in MUX-SDU's into a packet called MUX-PDU, whose size is typically smaller than that of a MUX-SDU (e.g. 254 bytes). Each logical channel is assigned a certain pattern of slots. This assignment is made through a 4-bit multiplex code (MC), which references an entry from a multiplex table containing 16 possible patterns. The entries of this multiplex table are controlled by the transmitter. In baseline H.223, the MC is included in a 1-byte header prepended to the MUX-PDU payload, along with a 1-bit packet marker (PM) field, which marks the end of MUX-SDU's of segmentable logical channels, and a 3-bit header error control (HEC) field, used for error detection over the MC. An 8-bit synchronization flag is prepended to the concatenation of the header and the MUX-PDU payload, leading to the MUX-PDU format given in Fig. 4.10. Bit stuffing is performed in
the MUX-PDU payload in order to prevent the emulation of the synch flag, by inserting a 0 bit after a sequence of five consecutive 1 bits.

The MUX-PDU formats for H.223 Annexes A and B are given in Figs. 4.11 and 4.12, respectively. Annex B also contains an optional header mode, in which 5 header bits (PM, MC) from the previous MUX-PDU are prepended to the current header, and protected with a 3-bit HEC. The MUX-PDU formats for H.223 Annexes C and D are similar to that of Annex B, except for its stuffing mode (used when no information is available).

### 4.2.4 IS-95B Forward Link

**Channels**
The IS-95B forward link consists of up to 64 code-division multiplexed channels, each
HEC: Header Error Control (3 bits)
MC: Multiplex Code (4 bits)
PM: Packet Marker (1 bit)

**Figure 4.10** Format of H.223 MUX-PDU.

<table>
<thead>
<tr>
<th>Synch Flag</th>
<th>HEC</th>
<th>MC</th>
<th>PM</th>
<th>Bit-stuffed payload</th>
</tr>
</thead>
<tbody>
<tr>
<td>01111110</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

HEC: Header Error Control (3 bits)
MC: Multiplex Code (4 bits)
PM: Packet Marker (1 bit)

**Figure 4.11** Format of H.223 Annex A MUX-PDU.

<table>
<thead>
<tr>
<th>Synch Flag</th>
<th>HEC</th>
<th>MC</th>
<th>PM</th>
<th>Non bit-stuffed payload</th>
</tr>
</thead>
<tbody>
<tr>
<td>01001111100001</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

HEC: Header Error Control (3 bits)
MC: Multiplex Code (4 bits)
PM: Packet Marker (1 bit)

channel being assigned a different orthogonal Hadamard-Walsh sequence taken from the set \( \{ W_i^{64} \}, i = 0, \ldots, 63 \). These channels are divided among the following:
- A pilot channel \( (W_0^{64}) \): serves as a coherent phase reference for demodulating the other channels, and doesn't carry any data modulation;
- A synchronization (synch) channel \( (W_{32}^{64}) \): used to continuously broadcast to all mobile users the *synch channel message*, which contains information allowing the mobiles to synchronize to the system clock;
- Up to 7 paging channels \( (W_1^{64}, W_7^{64}) \): used to alert mobiles of incoming calls, convey channel assignments, and transmit system overhead information. They can also be assigned as traffic channels in the case of a heavily loaded system;
- Up to 55 traffic channels \( (W_8^{64}, W_{31}^{64}, W_{32}^{64}, W_{63}^{64}) \): carry the digital information to the

<table>
<thead>
<tr>
<th>Synch Flag</th>
<th>P</th>
<th>MPL</th>
<th>MC</th>
<th>Non bit-stuffed payload</th>
</tr>
</thead>
<tbody>
<tr>
<td>010011011100001</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

MC: Multiplex Code (4 bits)
MPL: Multiplex Payload Length (8 bits)
P: Parity bits (12 bits), Extended (E) Golay (24,12,8) code

**Figure 4.12** Format of H.223 Annex B MUX-PDU.
Several data rates are supported on a traffic channel: 0.8, 2.0, 4.0 and 8.6 kbps. For IS-95B systems, up to 8 traffic channels can be assigned in parallel to a single user, resulting in a maximum data rate of $R_b = 8.6 \times 8 = 68.8$ kbps per user. In the following, specifications are given only for the traffic channels. A block diagram of the air interface for the IS-95B forward link traffic channels is given in Fig. 4.13. The air interface for other channels is detailed in Chapter 4 of [177].

**Figure 4.13** Air interface for the IS-95B forward link traffic channels [177].

Framing

An IS-95 forward link traffic frame spans 20ms. It is formed by:
- Grouping $n_{data}$ data bits together;
- Appending a CRC of $n_{CRC}$ bits, which serves as a frame quality indicator (FQI), for
  the two highest rates (4.0 and 8.6 kbps);
- Appending an 8-bit convolutional encoder tail;
- Convolutionally encoding the concatenation of the previous fields, using a rate $R = 1/2$
  encoder;
- Repeating $n_{rep}$ times the encoded symbols.

Table 4.1 details the constitution of the traffic frames for the different data rates, in
which $R_b$ is the data rate, $n_{data}$ is the number of data bits, $n_{CRC}$ is the number of CRC
bits, $n_{tail}$ is the number of tail bits, $n_{rep}$ is the repetition factor, and $n_{sym}$ is the total
number of symbols in a frame.

<table>
<thead>
<tr>
<th>$R_b$ (kbps)</th>
<th>$n_{data}$</th>
<th>$n_{CRC}$</th>
<th>$n_{tail}$</th>
<th>$R$</th>
<th>$n_{rep}$</th>
<th>$n_{sym}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>16</td>
<td>0</td>
<td>8</td>
<td>1/2</td>
<td>8</td>
<td>384</td>
</tr>
<tr>
<td>2.0</td>
<td>40</td>
<td>0</td>
<td>8</td>
<td>1/2</td>
<td>4</td>
<td>384</td>
</tr>
<tr>
<td>4.0</td>
<td>80</td>
<td>8</td>
<td>8</td>
<td>1/2</td>
<td>2</td>
<td>384</td>
</tr>
<tr>
<td>8.6</td>
<td>172</td>
<td>12</td>
<td>8</td>
<td>1/2</td>
<td>1</td>
<td>384</td>
</tr>
</tbody>
</table>

Table 4.1 Frame structure of the IS-95B forward link [14].

Coding

The 8.6 and 4.0 data rates use the following generator polynomials to compute the 12
and 8-bit CRC’s, respectively:

\[
g(x) = 1 + x + x^3 + x^4 + x^7 + x^8, \quad 8.6 \text{ kbps} \quad (4.1)
\]

\[
g(x) = 1 + x + x^4 + x^8 + x^9 + x^{10} + x^{11} + x^{12}, \quad 4.0 \text{ kbps} \quad (4.2)
\]

The convolutional encoder has rate 1/2 and constraint length $K = 9$. Its generation
functions are, in octal form, $g_0 = (753)$ and $g_1 = (561)$. The resulting transmission rate
for the 8.6 kbps data rate is 19.2 kbps.
Block Interleaving
The block interleaver corresponds to a matrix consisting of 64 rows and 6 columns. The coded bits are serially input into the matrix by column, starting with the upper element of the leftmost column. Once the matrix has been filled with \( N = 64 \times 6 = 384 \) bits, its rows are permuted by bit reversing the indice of each row (also called the bit reversal technique). The bits are then read out by rows, starting with the leftmost element of the first row. This set of operations can be mathematically expressed as follows:

\[
A_i = 2^m(i \mod J) + \text{BRO}_m\left(\frac{i}{J}\right)
\]  

(4.3)

where \( A_i \) is the address from which input symbol \( i \) (\( i = 0,1,\ldots,N - 1 \)) is read out, \( \text{BRO}_m(x) \) denotes the bit-reversed \( m \)-bit value of \( x \), and \( m \) and \( J \) are 2 integer parameters which are determined by the size of the interleaver array (total number of elements \( N \)) as given by Table 4.2. The block deinterleaver simply reverses the previous operation. Interleaving and deinterleaving cause a total delay of \( 2 \times 20 \text{ ms} = 40 \text{ ms} \).

Scrambling and Power Control
The interleaved sequence is scrambled by the decimated version (1 out of every 64 symbols) of a user-specific phase offset of a long-code PN sequence running at 1.2288 Mcps. This PN sequence, \( PN_L \), is generated by a 42-stage shift register of period \( 2^{42} - 1 \), with characteristic polynomial [177]:

\[
f(x) = x^7 + x^9 + x^{11} + x^{15} + x^{16} + x^{17} + x^{20} + x^{21} + x^{23}
+ x^{24} + x^{25} + x^{26} + x^{32} + x^{35} + x^{36} + x^{37} + x^{39}
+ x^{40} + x^{41} + x^{42}
\]

(4.4)

or, equivalently, reciprocal characteristic polynomial (= \( x^n f(x^{-1}) \)) [14]:

\[
P(x) = 1 + x + x^2 + x^3 + x^5 + x^6 + x^7 + x^{10} + x^{16} + x^{17}
+ x^{18} + x^{19} + x^{21} + x^{22} + x^{25} + x^{26} + x^{27} + x^{31}
+ x^{33} + x^{35} + x^{42}
\]

(4.5)
<table>
<thead>
<tr>
<th>Block Size $N$</th>
<th>$m$</th>
<th>$J$</th>
</tr>
</thead>
<tbody>
<tr>
<td>48</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>96</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>192</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>384</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>768</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>1536</td>
<td>6</td>
<td>24</td>
</tr>
<tr>
<td>3072</td>
<td>6</td>
<td>48</td>
</tr>
<tr>
<td>6144</td>
<td>7</td>
<td>48</td>
</tr>
<tr>
<td>12288</td>
<td>7</td>
<td>96</td>
</tr>
<tr>
<td>144</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>288</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>576</td>
<td>5</td>
<td>18</td>
</tr>
<tr>
<td>1152</td>
<td>6</td>
<td>18</td>
</tr>
<tr>
<td>2304</td>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>4608</td>
<td>7</td>
<td>36</td>
</tr>
<tr>
<td>9216</td>
<td>7</td>
<td>72</td>
</tr>
<tr>
<td>18432</td>
<td>8</td>
<td>72</td>
</tr>
<tr>
<td>36864</td>
<td>8</td>
<td>144</td>
</tr>
<tr>
<td>128</td>
<td>7</td>
<td>1</td>
</tr>
</tbody>
</table>

**Table 4.2** Parameters of the IS-95B forward link interleaver [14].

The scrambled sequence is punctured at a rate of 800 bps in order to insert power control bits.

**Orthogonal (Walsh) Covering**

Each channel is spread by a distinct sequence consisting of the repetition of a 64-chips Walsh sequence running at a rate of 1.2288 Mcps. There are hence $1228.8/19.2 = 64$ Walsh chips per coded bit. Each Walsh sequence $W_i^{64}$, $i = 0, 1, \ldots, 63$, of length 64, corresponds to row $i$ of a Hadamard matrix $H_{64}$ of size $64 \times 64$ (with the mapping $0 \rightarrow +1, 1 \rightarrow -1$), which is generated according to the following:

$$ H_{2n} = \begin{bmatrix} H_n & H_n \\ H_n & H_n \end{bmatrix} $$

(4.6)
where \( n = 1, 2, 4, 8, 32, H_0 = 0, \) and \( \bar{T} \) denotes the complement (such that \( \bar{H}_0 = 1 \)). The rows of the Hadamard matrix are orthogonal to each other, which allows the channels to be differentiated at the receiver. The assignment of the \( W_i^{64} \)'s to the different channels was described previously.

**Quadrature Spreading**

The same Walsh-spread sequence is passed to both the \( I \) and \( Q \) branches of the QPSK transmitter, where it is further spread by different short-code PN sequences \( PN_I \) and \( PN_Q \) on branches \( I \) and \( Q \), respectively. \( PN_I \) and \( PN_Q \) are maximal-length sequences generated by 15-stage shift registers, and lengthened by 1 chip period: their period is thus \( (2^{15} - 1) + 1 = 32768 \). Each base station assigns a different offset (a multiple of 64 chips) to \( PN_I \) and \( PN_Q \); the latter are the same for each user corresponding to a same base station. \( PN_I \) and \( PN_Q \) are generated using the following characteristic polynomials, respectively [177]:

\[
\begin{align*}
    f_I(x) &= 1 + x^2 + x^6 + x^7 + x^8 + x^{10} + x^{15} \\
    f_Q(x) &= 1 + x^3 + x^4 + x^5 + x^9 + x^{10} + x^{11} + x^{12} + x^{15}
\end{align*}
\]  

(4.7)  

(4.8)

The previous can be expressed as \( f_y(x) = x^{15}P_y(x^{-1}), y \in \{I, Q\}, \) where \( P_I, P_Q \) are the reciprocal polynomials of the PN sequences, as given in the IS-95 standard. Modular shift registers used to generate \( PN_I \) and \( PN_Q \) along with their offsets are illustrated in Figs. 4.14 and 4.15.

\[\text{Figure 4.14 Generation of } PN_I \text{ and its offsets [177].}\]

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Pulse Shaping and Modulation
The PN-spread sequences on each branch are shaped using a baseband finite impulse response (FIR) filter, in order to constrain the spectral bandwidth of the transmitted signal, and to minimize the ISI. The resulting shaped sequences are modulated by $I$ and $Q$ carriers ($\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$, respectively), combined and transmitted.

### 4.2.5 IS-95B Reverse Link

#### Channels
The IS-95B reverse link consists of up to 64 code-division multiplexed channels, each channel being assigned a different offset of the long-code PN sequence of (4.4):
- Up to 32 access channels per forward link paging channel: used to initiate communication with the BS, or to respond to messages received on the paging channel;
- Up to 62 traffic channels (equal to the number of forward traffic channels).

Similarly to the forward link, data rates of rates 0.8, 2.0, 4.0 and 8.6 kbps are supported on each traffic channel, and up to 8 traffic channels can be assigned in parallel to a single user. In the following, specifications are again given only for the traffic channels. A block diagram of the air interface for the IS-95B reverse link traffic channels is given in Fig. 4.16.

#### Framing and Coding
Traffic frames are generated similarly as on the forward link, with the difference that the convolutional coder has a rate of 1/3. The coder has constraint length $K = 9$ and generation functions (in octal form) $g_0 = (557)$, $g_1 = (663)$, and $g_2 = (711)$. For the 8.6 kbps data rate, the resulting transmission rate is 28.8 kbps, and frames consist of 576
Information bits

- 8.6 kbps
- 4.0 kbps
- 2.0 kbps
- 0.8 kbps

Add frame quality bit indicators (CRC) for 9.6 and 4.8 kbps

Add 8-bit encoder tail

Convolutional encoder R=1/3, K=9

Code symbols

Symbol repetition
- 28.8: x1
- 14.4: x2
- 7.2: x4
- 3.6: x8

Encoder tail

9.6 kbps

Add 8-bit encoder tail

9.2 kbps

9.6 kbps

2.0 kbps

4.4 kbps

4.8 kbps

1.2 kbps

28.8 kbps

14.4 kbps

7.2 kbps

3.6 kbps

28.8 ksps

14.4 ksps

7.2 ksps

3.6 ksps

28.8 kbps

28.8 kbps

4.8 kbps

4.8 kbps

7.2 kbps

7.2 kbps

3.6 kbps

3.6 kbps

28.8: x1

14.4: x2

7.2: x4

3.6: x8

Block interleaver 32 x 18 = 576-symbol array

W(64,6) Walsh orthogonal modulator

Modulation symbols

Data burst randomizer

Frame data rate

Control bits

42-bit long code mask

42-stage long-code PN generator

1.2288 Mcps

15-stage short code PN

15-stage short code PN

1/2 chip delay

Baseband filter

Baseband filter

\cos(wt)

\sin(wt)

\cos(wt)

\sin(wt)

Figure 4.16 Air interface for the IS-95B reverse link traffic channels [177].

bits, divided into 172 data bits, 12 CRC bits, 8 encoder tail bits, and 384 redundancy bits.

Block Interleaving

The block interleaver corresponds to a matrix consisting of 32 rows and 18 columns. The coded bits are serially input into the matrix by column, starting with the upper element of the leftmost column. Once the matrix has been filled with \( N = 32 \times 18 = 576 \) bits, the bits are then read out by rows, starting with the leftmost element of the first row. The block deinterleaver simply reverses the previous operation. Interleaving and deinterleaving cause a total delay of \( 2 \times 20\text{ms} = 40 \text{ms} \).
Orthogonal (Walsh) Modulation

Every group of 6 binary code symbols $\{b_0, b_1, \ldots, b_5\}$ is mapped into one of $2^6 = 64$ Walsh sequences $W_i^{64}$, by the relation:

$$i = b_0 + 2b_1 + 4b_2 + 8c_3 + 16c_4 + 32c_5$$

(4.9)

This leads to a Walsh symbol rate of $28.8/6 = 4.8$ ksps, or alternatively a Walsh chip rate of $4.8 \times 64 = 307.2$ kcps. Note that Walsh sequences are used here for modulation, and not for orthogonal covering (as on the forward link).

Long-code PN Spreading

The Walsh modulation symbols are spread by a phase offset of the long PN sequence $PN_L$ of (4.4). Each code channel of each user employs a different phase offset: this is what permits the BS to differentiate between channels. Fig. 4.17 illustrates the modular shift register used to generate $PN_L$ along with its offsets.

Quadrature Spreading

The quadrature spreading operation is the same as that used on the forward link.

Pulse Shaping and Modulation

The PN-spread sequence on the Q-branch is delayed by half a chip: this allows to implement a form of O-QPSK, in order to have a more constant envelope. The PN-spread
sequences on each branch are then shaped and modulated as on the forward link.

4.2.6 IS-2000 Forward Link

Multiple radio configurations (RC) are specified in the IS-2000 forward link, each one supporting a certain set of data rates. Each RC is assigned a spreading rate: either spreading rate 1 (SR1, 1.2288 Mcps) or spreading rate 3 (SR3, $3 \times 1.2288 = 3.6864$ Mcps). The modes of operation, types of channels, coding methods and rates, spreading sequences, orthogonal covering sequences and modulation schemes depend on the spreading rates and radio configurations, and are presented separately below.

Radio Configurations and Channels

Table 4.3 presents the different radio configurations (RC1-9), along with the sets of supported data rates $R_b$ and coding rates $R$. RC1 and RC2 are similar to the IS-95B forward link, and are included in IS-2000 for backwards compatibility. Hence in the following we will only describe the operations associated with RC3-9.

<table>
<thead>
<tr>
<th>RC</th>
<th>SR</th>
<th>$R_b$ (kbps)</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1.2 2.4 4.8 9.6</td>
<td>1/2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1.8 3.6 7.2 14.4</td>
<td>1/2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1.5 2.7 4.8 9.6 19.2 38.4 76.8 153.6</td>
<td>1/4</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1.5 2.7 4.8 9.6 19.2 38.4 76.8 153.6 307.2</td>
<td>1/2</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1.8 3.6 7.2 14.4 28.8 57.6 115.2 230.4</td>
<td>1/4</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>1.5 2.7 4.8 9.6 19.2 38.4 76.8 153.6 307.2</td>
<td>1/6</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>1.5 2.7 4.8 9.6 19.2 38.4 76.8 153.6 307.2 614.4</td>
<td>1/3</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>1.8 3.6 7.2 14.4 28.8 57.6 115.2 230.4 460.8</td>
<td>1/4</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>1.8 3.6 7.2 14.4 28.8 57.6 115.2 230.4 460.8 1036.8</td>
<td>1/2</td>
</tr>
</tbody>
</table>

Table 4.3 Radio configurations (RC) of the IS-2000 forward link [14].

The channels supported by the IS-2000 forward link are listed below, along with their maximum numbers (NS means no maximum number is specified, while NA means not available). Their functions can be found in the previous sections or in [14]. Unless noted otherwise, they can be used for both SR1 and SR3:
- Common Assignment Channel (CACH) (NS)
- Common Power Control Subchannels (CPCSCH) (NS)
- Pilot Channels:
  - Forward Pilot Channel (PCH) (1)
  - Transmit Diversity Channel (TDCH) (1)
  - Auxiliary Pilot Channel (APCH) (NS)
  - Auxiliary Transmit Diversity Pilot Channel (ATDPCH) (NS)
- Common Control Channels (CCCH) (NS)
- Synch Channel (SCH) (1)
- Traffic Channels:
  - Dedicated Control Channel (DCCH) (1)
  - Fundamental Channel (FCH) (1)
  - Power Control Subchannels (PCSCH) (1)
  - Supplemental Channels (SCH) (2) (RC 3-9)
  - Supplemental Code Channels (SCCH) (7) (RC 1-2)
- Broadcast Channels (BCH) (NS)
- Paging Channels (PGCH) (7 for SR1, NA for SR3)
- Quick Paging Channels (QPGCH) (3 for SR1, NS for SR3)

In the following we will only describe in detail the air interfaces of the Fundamental and Supplemental Channels (which are similar). A block diagram of the air interface for the IS-2000 forward link traffic channels is given in Fig. 4.18. The air interface of the Supplemental Code Channels is similar to that of the IS-95B forward link.

**Framing**

Tables 4.4 and 4.5 illustrate the frame structures associated with RC3 and RC4, respectively, which both use SR1. \( n_{\text{punc}} \) denotes the number of punctured symbols. Tables 4.6 and 4.7 illustrate the frame structures associated with RC6 and RC7, respectively, which both use SR3. These are the RC's which will be used in our simulation models. Further details about the other RC's (i.e. RC5, RC8-9) can be obtained from [14].
Coding
The Quick Paging and Common Power Control Channels are not coded. All other channels use convolutional codes, with rates depending on the SR and the RC. The Forward Supplemental Channels can use turbo codes for high data rates ($R_s \geq 19.2$ kbps).

Convolutional Coding
The convolutional encoders have the following generation functions (in octal form):
- $R = 1/6$: $g_0 = (457), \ g_1 = (755), \ g_2 = (551), \ g_3 = (637), \ g_4 = (625), \ g_5 = (727)$.
- $R = 1/4$: $g_0 = (765), \ g_1 = (671), \ g_2 = (513), \ g_3 = (473)$.
- $R = 1/3$: $g_0 = (557), \ g_1 = (663), \ g_2 = (711)$.
- $R = 1/2$: $g_0 = (753), \ g_1 = (561)$.
Table 4.4 Frame structure of RC3 of the IS-2000 forward link [14].

<table>
<thead>
<tr>
<th>$R_b$ (kbps)</th>
<th>$n_{data}$</th>
<th>$n_{CRC}$</th>
<th>$n_{tail}$</th>
<th>$R$</th>
<th>$n_{rep}$</th>
<th>$n_{punc}$</th>
<th>$n_{sym}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>16</td>
<td>6</td>
<td>8</td>
<td>1/4</td>
<td>8</td>
<td>1 of 5</td>
<td>768</td>
</tr>
<tr>
<td>2.0</td>
<td>40</td>
<td>6</td>
<td>8</td>
<td>1/4</td>
<td>4</td>
<td>1 of 9</td>
<td>768</td>
</tr>
<tr>
<td>4.0</td>
<td>80</td>
<td>8</td>
<td>8</td>
<td>1/4</td>
<td>2</td>
<td>0</td>
<td>768</td>
</tr>
<tr>
<td>8.6</td>
<td>172</td>
<td>12</td>
<td>8</td>
<td>1/4</td>
<td>1</td>
<td>0</td>
<td>768</td>
</tr>
<tr>
<td>18.0</td>
<td>360</td>
<td>16</td>
<td>8</td>
<td>1/4</td>
<td>1</td>
<td>0</td>
<td>1536</td>
</tr>
<tr>
<td>37.0</td>
<td>744</td>
<td>16</td>
<td>8</td>
<td>1/4</td>
<td>1</td>
<td>0</td>
<td>3072</td>
</tr>
<tr>
<td>75.6</td>
<td>1512</td>
<td>16</td>
<td>8</td>
<td>1/4</td>
<td>1</td>
<td>0</td>
<td>6144</td>
</tr>
<tr>
<td>152.4</td>
<td>3048</td>
<td>16</td>
<td>8</td>
<td>1/4</td>
<td>1</td>
<td>0</td>
<td>12288</td>
</tr>
</tbody>
</table>

Table 4.5 Frame structure of RC4 of the IS-2000 forward link [14].

Turbo Coding

The turbo encoder is illustrated in Fig. 4.19. The transfer function of the constituent code is the same for both upper and lower recursive systematic convolutional encoders, and is given by:

$$G(D) = \begin{bmatrix} n_0(D) & n_1(D) \end{bmatrix} \frac{d(D)}{d(D)}$$  \hspace{1cm} (4.10)

where $d(D) = 1 + D^2 + D^3$, $n_0(D) = 1 + D + D^3$ and $n_1(D) = 1 + D + D^2 + D^3$.

The algorithm for the turbo interleaver is given as follows:

1) Write consecutive values of the input stream into an array of $2^5$ rows by $2^n$ columns,
<table>
<thead>
<tr>
<th>$R_b$ (kbps)</th>
<th>$n_{data}$</th>
<th>$n_{CRC}$</th>
<th>$n_{tail}$</th>
<th>$R$</th>
<th>$n_{rep}$</th>
<th>$n_{punc}$</th>
<th>$n_{sym}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>16</td>
<td>6</td>
<td>8</td>
<td>1/6</td>
<td>8</td>
<td>1 of 5</td>
<td>1152</td>
</tr>
<tr>
<td>2.0</td>
<td>40</td>
<td>6</td>
<td>8</td>
<td>1/6</td>
<td>4</td>
<td>1 of 9</td>
<td>1152</td>
</tr>
<tr>
<td>4.0</td>
<td>80</td>
<td>8</td>
<td>8</td>
<td>1/6</td>
<td>2</td>
<td>0</td>
<td>1152</td>
</tr>
<tr>
<td>8.6</td>
<td>172</td>
<td>12</td>
<td>8</td>
<td>1/6</td>
<td>1</td>
<td>0</td>
<td>1152</td>
</tr>
<tr>
<td>18.0</td>
<td>360</td>
<td>16</td>
<td>8</td>
<td>1/6</td>
<td>1</td>
<td>0</td>
<td>2304</td>
</tr>
<tr>
<td>37.0</td>
<td>744</td>
<td>16</td>
<td>8</td>
<td>1/6</td>
<td>1</td>
<td>0</td>
<td>4608</td>
</tr>
<tr>
<td>75.6</td>
<td>1512</td>
<td>16</td>
<td>8</td>
<td>1/6</td>
<td>1</td>
<td>0</td>
<td>9216</td>
</tr>
<tr>
<td>152.4</td>
<td>3048</td>
<td>16</td>
<td>8</td>
<td>1/6</td>
<td>1</td>
<td>0</td>
<td>18432</td>
</tr>
<tr>
<td>306.0</td>
<td>6120</td>
<td>16</td>
<td>8</td>
<td>1/6</td>
<td>1</td>
<td>0</td>
<td>36864</td>
</tr>
</tbody>
</table>

Table 4.6 Frame structure of RC6 of the IS-2000 forward link [14].

<table>
<thead>
<tr>
<th>$R_b$ (kbps)</th>
<th>$n_{data}$</th>
<th>$n_{CRC}$</th>
<th>$n_{tail}$</th>
<th>$R$</th>
<th>$n_{rep}$</th>
<th>$n_{punc}$</th>
<th>$n_{sym}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>16</td>
<td>6</td>
<td>8</td>
<td>1/3</td>
<td>8</td>
<td>1 of 5</td>
<td>576</td>
</tr>
<tr>
<td>2.0</td>
<td>40</td>
<td>6</td>
<td>8</td>
<td>1/3</td>
<td>4</td>
<td>1 of 9</td>
<td>576</td>
</tr>
<tr>
<td>4.0</td>
<td>80</td>
<td>8</td>
<td>8</td>
<td>1/3</td>
<td>2</td>
<td>0</td>
<td>576</td>
</tr>
<tr>
<td>8.6</td>
<td>172</td>
<td>12</td>
<td>8</td>
<td>1/3</td>
<td>1</td>
<td>0</td>
<td>576</td>
</tr>
<tr>
<td>18.0</td>
<td>360</td>
<td>16</td>
<td>8</td>
<td>1/3</td>
<td>1</td>
<td>0</td>
<td>1152</td>
</tr>
<tr>
<td>37.0</td>
<td>744</td>
<td>16</td>
<td>8</td>
<td>1/3</td>
<td>1</td>
<td>0</td>
<td>2304</td>
</tr>
<tr>
<td>75.6</td>
<td>1512</td>
<td>16</td>
<td>8</td>
<td>1/3</td>
<td>1</td>
<td>0</td>
<td>4608</td>
</tr>
<tr>
<td>152.4</td>
<td>3048</td>
<td>16</td>
<td>8</td>
<td>1/3</td>
<td>1</td>
<td>0</td>
<td>9216</td>
</tr>
<tr>
<td>306.0</td>
<td>6120</td>
<td>16</td>
<td>8</td>
<td>1/3</td>
<td>1</td>
<td>0</td>
<td>18432</td>
</tr>
<tr>
<td>613.2</td>
<td>12264</td>
<td>16</td>
<td>8</td>
<td>1/3</td>
<td>1</td>
<td>0</td>
<td>36864</td>
</tr>
</tbody>
</table>

Table 4.7 Frame structure of RC7 of the IS-2000 forward link [14].

in a row-by-row fashion. The parameter $n$ is given in Table 4.8.

2) Permute the elements within each row according to the following row-specific linear congruential sequence rule: $x[i + 1] = (x[i] + c) \mod 2^n$, where $x[0] = c$ and $c$ is determined from $n$ and the row number according to the table lookup of Table 4.9.

3) Permute the rows by bit-reversing the row numbers.

4) Output the elements of the array in a column-by-column fashion.

An alternative formulation of the above algorithm can be found in [14].
Figure 4.19 Turbo encoder for the IS-2000 forward link [14].

<table>
<thead>
<tr>
<th>Block Size</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>378</td>
<td>4</td>
</tr>
<tr>
<td>570</td>
<td>5</td>
</tr>
<tr>
<td>762</td>
<td>5</td>
</tr>
<tr>
<td>1146</td>
<td>6</td>
</tr>
<tr>
<td>1530</td>
<td>6</td>
</tr>
<tr>
<td>2298</td>
<td>7</td>
</tr>
<tr>
<td>3066</td>
<td>7</td>
</tr>
<tr>
<td>4602</td>
<td>8</td>
</tr>
<tr>
<td>6138</td>
<td>8</td>
</tr>
<tr>
<td>9210</td>
<td>9</td>
</tr>
<tr>
<td>12282</td>
<td>9</td>
</tr>
<tr>
<td>20730</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 4.8 Parameter $n$ of the IS-2000 turbo interleaver [14].

256
<table>
<thead>
<tr>
<th>Index</th>
<th>$n = 4$</th>
<th>$n = 5$</th>
<th>$n = 6$</th>
<th>$n = 7$</th>
<th>$n = 8$</th>
<th>$n = 9$</th>
<th>$n = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
<td>27</td>
<td>3</td>
<td>15</td>
<td>3</td>
<td>13</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>15</td>
<td>3</td>
<td>27</td>
<td>127</td>
<td>1</td>
<td>335</td>
<td>349</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>1</td>
<td>15</td>
<td>89</td>
<td>5</td>
<td>87</td>
<td>303</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>15</td>
<td>13</td>
<td>1</td>
<td>83</td>
<td>15</td>
<td>721</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>13</td>
<td>29</td>
<td>31</td>
<td>19</td>
<td>15</td>
<td>973</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>17</td>
<td>5</td>
<td>15</td>
<td>179</td>
<td>1</td>
<td>703</td>
</tr>
<tr>
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<td>33</td>
<td>69</td>
<td>67</td>
<td>391</td>
<td>163</td>
</tr>
</tbody>
</table>

**Table 4.9** Table lookup for the IS-2000 turbo interleaver [14].
In order to support different code rates, symbols can be punctured prior to transmission. The puncturing patterns for the different code rates are given in Table 4.10.

<table>
<thead>
<tr>
<th>Output</th>
<th>$R = 1/2$</th>
<th>$R = 1/3$</th>
<th>$R = 1/4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{k}^{1,1}$</td>
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<td>11</td>
<td>11</td>
</tr>
<tr>
<td>$x_{k}^{p,11}$</td>
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<td>11</td>
<td>11</td>
</tr>
<tr>
<td>$x_{k}^{p,12}$</td>
<td>00</td>
<td>00</td>
<td>10</td>
</tr>
<tr>
<td>$x_{k}^{p,2}$</td>
<td>00</td>
<td>00</td>
<td>00</td>
</tr>
<tr>
<td>$x_{k}^{p,21}$</td>
<td>01</td>
<td>11</td>
<td>01</td>
</tr>
<tr>
<td>$x_{k}^{p,21}$</td>
<td>00</td>
<td>00</td>
<td>11</td>
</tr>
</tbody>
</table>

Table 4.10 Puncturing patterns for the IS-2000 turbo encoder [14].

Block Interleaving

The forward link block interleaving can be mathematically expressed as follows, with a block size of $N$:

$$A_i = \begin{cases} 2^m \left[ \frac{i}{2} \mod J \right] + \text{BRO}_m \left( \left\lfloor \frac{N}{2} \right\rfloor \right) & i = 0, 2, \ldots, N - 2 \\ 2^m \left[ \left( N - \frac{i+1}{2} \right) \mod J \right] + \text{BRO}_m \left( \left\lfloor \frac{N - \frac{i+1}{2}}{J} \right\rfloor \right) & i = 1, 3, \ldots, N - 1 \end{cases}$$

where the symbols in the above equation are as defined previously, and $m$ and $J$ are determined as in Table 4.2.

Scrambling and Power Control

The interleaved sequence is scrambled by the decimated version of a user-specific phase offset of a long-code PN sequence running at 1.2288 Mcps (c.f. Eq. (4.4)). The scrambled sequence is punctured at a rate of 800 bps in order to insert power control bits.

Orthogonal (Walsh) Covering

Each traffic channel is spread by a different Walsh sequence $W_{i}^{N}$, taken from a $N$-ary orthogonal set, where $N$ depends on the RC and the data rate. For RC3, $N = 4, 8, 16, 32, 64$ are used for $R_b = 152.4, 75.6, 37.0, 18.0, 8.6$ kbps, respectively. For RC4 and RC6, $N = 4, 8, 16, 32, 64, 128$ are used for $R_b = 306.0, 152.4, 75.6, 37.0, 18.0, 8.6$ kbps, respectively.
For RC7, \( N = 4, 8, 16, 32, 64, 128, 256 \) are used for \( R_b = 613.2, 306.0, 152.4, 75.6, 37.0, 18.0, 8.6 \) kbps, respectively. The Walsh sequences run at rates of 1.2288 Mcps for SR1 and 3.6864 Mcps for SR3, and with periods of \( N/1.2288 \mu\text{s} \) (SR1) and \( N/3.6864 \mu\text{s} \) (SR3). In order to be mutually orthogonal, the Walsh sequences of different lengths, denoted as orthogonal variable spreading factor (OVSF) sequences, can be generated according to the method described in [256].

Modulation, Complex Spreading and Pulse Shaping

After the orthogonal covering, each channel is split into \( I \) and \( Q \) branches, with consecutive bits being mapped to different branches in alternation, leading to QPSK-modulated symbols. The resulting \( I/Q \) channel is spread by a complex spreading sequence. For SR1, the real and imaginary parts of the complex spreading sequence have characteristic polynomials given by Eqs. (4.7) and (4.8), respectively. For SR3, the real and imaginary parts of the complex spreading sequence are the first \( 3 \times 2^{15} \) chips of the offset versions (by 19 and \( 19 + 2^{19} \) chips, respectively) of the PN sequence with characteristic polynomial:

\[
f(x) = 1 + x^{11} + x^{15} + x^{17} + x^{20}.
\] (4.12)

The PN-spread sequences on each branch are then shaped using a baseband FIR filter, modulated by \( I \) and \( Q \) carriers, combined and transmitted.

4.2.7 IS-2000 Reverse Link

Radio Configurations and Channels

Table 4.11 presents the different radio configurations (RC1-6), along with the sets of supported data and coding rates. RC1 and RC2 are similar to the IS-95B reverse link, and are included in IS2000 for backwards compatibility. Hence in the following we will only describe the operations associated with RC3-6.

The channels supported by the IS-2000 reverse link are listed below, along with their maximum numbers. Unless noted otherwise, they can be used for both SR1 and SR3:

- Access Channel (ACH) (1 for SR1, NA for SR3)
- Enhanced Access Channel (EACH) (1)
Table 4.11 Radio Configurations of the IS-2000 reverse link [14].

- Reverse Common Control Channel (RCCCH) (1)
- Traffic Channels:
  - Pilot Channel (PCH) (1)
  - Dedicated Control Channel (DCCH) (1)
  - Fundamental Channel (FCH) (1)
  - Power Control Subchannel (PCCH) (1)
  - Supplemental Channels (SCH) (2) (RC 3-6)
  - Supplemental Code Channels (SCCH) (7) (RC 1-2)

In the following we will only describe in detail the air interfaces of the Fundamental and Supplemental Channels (which are similar). A block diagram of the air interface for the IS-2000 reverse link Fundamental and Supplemental Channels is given in Fig. 4.20. The air interface of the Supplemental Code Channels is similar to that of the IS-95B reverse link.

Framing
Table 4.12 illustrates the frame structures associated with RC3, which uses SR1. Table 4.13 illustrates the frame structures associated with RC5, which uses SR3. These are the RC's which will be used in our simulation models. Further details about the other RC's (i.e. RC4, RC6) can be obtained from [14].
Coding
The convolutional and turbo encoders are the same as those on the forward link.

Block Interleaving
The reverse link block interleaving can be mathematically expressed as follows, with a block size of $N$:

$$A_i = 2^m [i \mod J] + \text{BRO}_m \left( \left\lfloor \frac{i}{J} \right\rfloor \right) \quad i = 0, 1, \ldots, N - 1$$  \hspace{1cm} (4.13)

where the symbols in the above equation are as defined previously, and $m$ and $J$ are determined as in Table 4.14.

Orthogonal (Walsh) Covering
Each channel is spread by a distinct Walsh sequence $W_i^N$, which corresponds to the $i^{th}$ row of a Hadamard matrix of size $N \times N$. All Walsh sequences used are mutually or-
<table>
<thead>
<tr>
<th>$R_b$ (kbps)</th>
<th>$n_{data}$</th>
<th>$n_{CRC}$</th>
<th>$n_{tail}$</th>
<th>$R$</th>
<th>$n_{rep}$</th>
<th>$n_{punc}$</th>
<th>$n_{sym}$</th>
</tr>
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<tbody>
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<td>6</td>
<td>8</td>
<td>1/4</td>
<td>16</td>
<td>1 of 5</td>
<td>1536</td>
</tr>
<tr>
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<td>8</td>
<td>1/4</td>
<td>8</td>
<td>1 of 9</td>
<td>1536</td>
</tr>
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<td>8</td>
<td>1/4</td>
<td>4</td>
<td>0</td>
<td>1536</td>
</tr>
<tr>
<td>8.6</td>
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<td>12</td>
<td>8</td>
<td>1/4</td>
<td>2</td>
<td>0</td>
<td>1536</td>
</tr>
<tr>
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<td>16</td>
<td>8</td>
<td>1/4</td>
<td>1</td>
<td>0</td>
<td>1536</td>
</tr>
<tr>
<td>37.0</td>
<td>744</td>
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<td>8</td>
<td>1/4</td>
<td>1</td>
<td>0</td>
<td>3072</td>
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<td>8</td>
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<tr>
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<td>8</td>
<td>1/4</td>
<td>1</td>
<td>0</td>
<td>12288</td>
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<tr>
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<td>8</td>
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<td>0</td>
<td>12288</td>
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<tr>
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<td>8</td>
<td>1/3</td>
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<td>0</td>
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Table 4.12 Frame structure of RC3 of the IS-2000 reverse link [14].

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<th>$n_{data}$</th>
<th>$n_{CRC}$</th>
<th>$n_{tail}$</th>
<th>$R$</th>
<th>$n_{rep}$</th>
<th>$n_{punc}$</th>
<th>$n_{sym}$</th>
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<tbody>
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<td>0.8</td>
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<td>6</td>
<td>8</td>
<td>1/4</td>
<td>16</td>
<td>1 of 5</td>
<td>1536</td>
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<tr>
<td>2.0</td>
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<td>1/4</td>
<td>8</td>
<td>1 of 9</td>
<td>1536</td>
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<tr>
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<td>1/4</td>
<td>4</td>
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<td>8.6</td>
<td>172</td>
<td>12</td>
<td>8</td>
<td>1/4</td>
<td>2</td>
<td>0</td>
<td>1536</td>
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<td>1/4</td>
<td>1</td>
<td>0</td>
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<td>1</td>
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<td>8</td>
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<td>1</td>
<td>0</td>
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<td>0</td>
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<td>8</td>
<td>1/3</td>
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<td>12288</td>
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<td>8</td>
<td>1/3</td>
<td>1</td>
<td>0</td>
<td>36864</td>
</tr>
</tbody>
</table>

Table 4.13 Frame structure of RC5 of the IS-2000 reverse link [14].

orthogonal, and are assigned as follows (+1 is abbreviated by + and -1 by -):
- PCH: $W_6^{32} = (+ + + + + + + + + + + + + + + + + + + + + +)$
- EACH: $W_8^8 = (+ + - - + + + +)$
- RCCCH: $W_8^8 = (+ + - - + + + +)$
- DCCH: $W_8^{16} = (+ + + + + + - - - - - - - -)$
- FCH: $W_4^{16} = (+ + + - - - - + + + + - - -)$
- SCH1: $W_1^2 = (+ - )$ or $W_2^4 = (+ + - )$
- SCH2: $W_2^4 = (+ + - - )$ or $W_6^8 = (+ + - - + +)$

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Table 4.14 Parameters of the IS-2000 reverse link interleaver [14].

<table>
<thead>
<tr>
<th>Block Size $N$</th>
<th>$m$</th>
<th>$J$</th>
</tr>
</thead>
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<td>384</td>
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<td>6</td>
</tr>
<tr>
<td>768</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>1536</td>
<td>6</td>
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<td>12288</td>
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<td>96</td>
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<td>72</td>
</tr>
<tr>
<td>36864</td>
<td>8</td>
<td>144</td>
</tr>
</tbody>
</table>

Modulation, Complex Spreading and Pulse Shaping

The data channels are mapped onto either the $I$ or the $Q$ branches, but not both: this corresponds to BPSK modulation, as opposed to QPSK modulation in the case of the forward link. The PCH, DCCH and SCH2 are mapped onto the $I$-branch, while the FCH, SCH1, EACH and RCCCH are mapped onto the $Q$-branch. The resulting $I/Q$ channel is spread by a complex spreading sequence. The real part of the latter is the product of a short PN sequence $PN_i$ with a long PN sequence $PN_i$. The imaginary part is obtained by multiplying the 1-chip delayed version of $PN_i$ with a short PN sequence $PN_q$, decimating by a factor 2 the resulting sequence, and multiplying the decimated sequence by $PN_i$ and $W_1^2 = (+-)$. For SR1, $PN_i$, $PN_q$ and $PN_i$ are the same as those specified for the reverse link of IS-95B: $PN_i = PN_I$, $PN_q = PN_Q$, $PN_i = PN_L$. For SR3, $PN_i$ and $PN_q$ are the first $3 \times 2^{15}$ chips of the offset versions (by 19 and $19 + 2^{19}$ chips, respectively) of the PN sequence with generator polynomial given by (4.12). $PN_i$ is obtained by multiplexing the following 3 sequences: $PN_I$, the product of $PN_I$ and its 1-chip delayed version, and the product of $PN_I$ and its 2-chips delayed version.

The PN-spread sequences on each branch are then shaped using a baseband FIR filter, modulated by $I$ and $Q$ carriers, combined and transmitted.
4.3 Smoothing for VBR Video over Multicode and Multirate DS/CDMA

4.3.1 Introduction

As discussed in Chapter 3 and in the previous section, 2.5G and 3G DS/CDMA mobile networks have the ability to change the transmission rate either on a connection or frame-by-frame basis [191]. The adaptation method used depends on the type of system. IS-95B systems use multicode transmission in order to provide higher transmission rates [191]: several codes are assigned to a high-rate service requested by a user, and the data stream is demultiplexed among these codes. IS-2000 systems use multirate transmission instead [14]: the data rate of a traffic channel can be increased (with the chip rate remaining the same) to support a higher source rate. Wideband CDMA (W-CDMA, [257], [258]) uses a combination of multirate and multicode transmission. Various methods have been proposed to support VBR video over DS/CDMA networks. If the source rate is assumed constant (e.g. due to perfect source rate control), or if the peak source rate is within the maximum transmission rate, then a simple approach consists in transmitting at a constant rate. This will however result in a reduced video quality (in the source rate control case), or a high bandwidth requirement (in the peak rate transmission case). However, for simplicity and benchmarking purposes, such an approach was used in [259] for the forward link of an IS-95 system and in [190] for the reverse link of a multicode IS-95B system. Alternatively, the transmission rate can be varied adaptively in order to follow the variations of the video source. This was done for example in [75] for the multicode case. If each code retains the same power as in the single-code case, this approach will lead to higher interference to other users, and thus a reduction in the cell capacity. The multirate (variable spreading gain) case was treated for example in [260] for IS-2000 systems, where a decoding delay was introduced in order to be able to better support the rate variations. However, an increase in the transmission data rate will lower the processing gain, which will result in a higher BER if the transmission power is kept constant.

One way to minimize the drawbacks of the previous approaches is to perform real-time smoothing of the video stream. The ideal goal of smoothing is to permit the transmission
at a nearly constant rate of a VBR source. To allow this, a certain amount of buffering must be introduced either at the transmitter or the receiver, resulting in a certain delay in the decoding. If the video frames are all available beforehand to the transmitter (as in stored video), the latter can determine the optimal sequence of transmission rates (or schedule [49]) before it initiates transmission, and then keep it frozen. For this offline case, in [49], the authors present an algorithm which achieves the "greatest possible reduction in rate variability" for stored video. They derive a formal proof of the optimality of the algorithm, and propose a practical $O(n)$ implementation for it based on a shortest-path approach. A review and comparison of various offline smoothing algorithms (including the previous) are offered in [68]. If the video frames are available progressively to the transmitter (as in real-time interactive or streaming video), the latter has to determine the best schedule in a progressive way, either on a frame-by-frame basis, or based on a sliding window of frames. An online version of the optimal algorithm of [49] is proposed in [51]. Further work on online smoothing is presented for example in [66], where the proposed algorithm tries to send as many frames as possible at the same rate with a given look-ahead interval, or in [69], which also proposes online versions of the algorithm of [49]. An optimal algorithm for both online and offline video was proposed in [67] using dynamic programming.

While there have been many studies on source rate control for wireless video (e.g. [55], [54], [261], [56]), most previous published work on smoothing was carried out in the framework of wireline communications, such as the Internet or asynchronous transfer mode (ATM) networks, which are relatively free of errors due to noise or interference. In these systems, a rate increase doesn't necessarily translate into a BER increase or a capacity reduction, as is the case for current DS/CDMA cellular systems. Hence most previous work on smoothing was thus concerned mainly with optimizing certain characteristics of the transmission rate (such as peak and variance), but without looking at the effect on the PSNR of the decoded video sequence (which is influenced by the BER of the communication link). In DS/CDMA cellular systems, the choice of the transmission rates will have an effect on the processing gain and/or transmitted power, and hence on the resulting BER (and hence on the PSNR) or cell capacity. Hence the
goal of this section is to illustrate such an interplay between rate, delay, and PSNR, through the application of smoothing concepts to DS/CDMA cellular systems.

To this end, we describe a real-time smoothing scheme whose goal is to maximize the SNR over the highest number of frames, and hence obtain a minimum BER for a minimum transmitted power. The presented algorithm relies on some principles introduced in [51] for the design of the Sliding Window algorithm (SLWIN), but is designed for best operation in a DS/CDMA cellular system. It will be denoted as SLWIN2. It attempts to minimize the peak rate and the rate variability of the transmitted signal (as does SLWIN), but subject to the additional constraint of maintaining the minimum average transmission bandwidth at all times. This last constraint is introduced in order to maximize the processing gain (for multirate systems) or minimize the number of codes transmitted in parallel (for multicode systems), which will lead to a lower BER, if the total power assigned to the video user has to remain constant, or a higher cell capacity, if the total power assigned to the video user can be varied. No source rate control is applied (i.e. varying the quantizer step size or skipping frames) in order to maintain a constant encoded video quality. The algorithm must also take into account the fact that in standardized communication systems, the set of allowable transmission rates is limited, and the transmission frames' periods usually differ from the video frames' periods; in previous descriptions of smoothing algorithms for wireline systems, with few exceptions, no constraints were imposed on the allowable set of transmission rates, and the transmission and video frame sizes were often assumed equal. While the difference in frame sizes only affects the practical implementation, it will be shown through examples that the limited set of transmission rates can lead to different rate schedules and PSNR performances for SLWIN and SLWIN2.

We perform two case studies using both SLWIN and SLWIN2: we apply them to real-time H.263 video communication over the reverse and forward links of both multicode IS-95B and multirate IS-2000 systems, using detailed simulation models, in the case of a circuit-switched H.324 connection [245]. We compare the results obtained with both schemes in terms of peak, mean and variance of rate, and PSNR. The smoothing algorithms can be applied equally well to the reverse or forward links of such systems, and to packet-switched connections (c.f. [49], Section III-C). Indeed, our work considers
"non-collaborative" smoothing schemes [262], which can be used on both forward and reverse links. In contrast, a recent paper [262] (which also considered smoothing over wireless DS/CDMA systems) focused on "collaborative" schemes, which are useful on the forward link of cellular systems; however it presented numerical results for stored video only, using a simplified channel model.

The plan of this section is outlined as follows. After describing the motivations behind using smoothing in Section 4.3.2, Section 4.3.3 presents a smoothing framework for the transmission of VBR video, which is used to derive the SLWIN and SLWIN2 algorithms described in Section 4.3.4. Section 4.3.5 presents and discusses representative numerical results obtained with the algorithms and simulation model of the previous sections.

4.3.2 Motivation for Smoothing

The traffic generated by video sequences can be very bursty, especially when the interval between I-frames is small. Let $N_I$ be the interval (in number of video frames) between two intra-coded (I) frames. Examples of a video trace, corresponding to the H.263-encoded Miss America video sequence, are given in Figs. 4.21 and 4.22 for $N_I = 132$ and $N_I = 20$, respectively. The spikes correspond to I-frames, while the rest of low-rate traffic is made out of P-frames. The Miss America video sequence features the head and shoulders of a woman talking to the camera and making occasional head movements, and is of low motion; it is typical of a videoconferencing application. Other examples of a video trace, corresponding to the H.263-encoded Foreman video sequence, are given in Figs. 4.23 and 4.24 for $N_I = 132$ and $N_I = 20$, respectively. This sequence features a foreman talking in an expressive way to the camera, and includes more motion than the Miss America sequence, as can be evidenced by comparing the rate variability of the traces; it is typical of a videophone application.

In order to support the frequent source rate variations, IS-95B and IS-2000 systems could be made to adapt their transmission rate, preferably on a frame-by-frame basis.

---

1In practical very-low bit-rate live video communication systems, only segments of frames (e.g. macroblocks) instead of whole frames are intra-coded, in order to minimize the bit rate. This does not require any changes to the smoothing algorithms discussed in this chapter.
In the case of multicode IS-95B systems, this could be done by varying the number of codes assigned to the video service. However, if the power of each code is equal to that of a single-code user, the total transmitted power of the video user will vary frequently and exhibit a high peak-to-average ratio (PAR). Conversely, if the total power assigned to a multicode user needs to remain constant regardless of the number of codes, the SNR per code will decrease resulting in a higher BER.

In the case of multirate IS-2000 systems, if the transmission rate of the video user is increased by a certain factor $F$ to accommodate a higher source rate, the processing gain $PG$ of the receiver decreases by the same factor $F$. This results in a reduction of robustness to multiple-access and multipath interference, and thus a higher BER for the high-rate user. To compensate for the lower $PG$ in order to maintain a constant BER, the transmitted power of the video user needs to be increased to $F$ times the power needed when the processing gain is $F$ times higher. If the transmitted power (in addition to the transmission rate) is adapted to the source rate, this will also lead to frequent power variations and high PAR’s for the transmitted power.

These rapid power variations and high PAR’s are not desirable due to the following:

- The increased power of the video user will create extra multiple-access interference to the other users of the system, thus reducing the cell capacity. The high-power video user can further swarm a voice user which operates close to him;

- High PAR’s reduce the power amplifier efficiency;

- Frequent rate and power changes increase the signaling overhead.

Moreover, the burstiness of the video stream may lead to peak source rates which are higher than the maximum rate for transmission.

It would thus be favorable to minimize the peak, average and variance of the transmission rate, which will lead to lower and more constant power requirements for a variable-power system, or a lower BER for a fixed-power system. In the next section we examine the constraints which govern the selection of the transmission rates; these constraints are then taken into account in the design of a smoothing algorithm, whose goal is to minimize the peak and variance of the transmission rate, while maintaining the minimum average bandwidth at all times.
Figure 4.21 Video trace of Miss America, with 1 out of 132 I frames.

Figure 4.22 Video trace of Miss America, with 1 out of 20 I frames.
**Figure 4.23** Video trace of Foreman, with 1 out of 132 I frames.

**Figure 4.24** Video trace of Foreman, with 1 out of 20 I frames.
4.3.3 VBR Video Constraints

We first present the system parameters and notations used to describe the rate adaptation framework, then lay down the rate constraints which must be observed by the system.

4.3.3.1 Video Source

The source outputs video frames of equal duration $T_F$ seconds. Let $s(i)$ denote the source rate of video frame $i$, in bits/second. Let $S(i) = s(i)T_F$ be the number of bits carried by video frame $i$, and $S_T(i) = \sum_{j=1}^{i} S(j)$ be the total number of bits in video frames 1 through $i$.

4.3.3.2 Transmitter

Let $T_c$ be the duration (in seconds) of an IS-95B or IS-2000 frame, which is not necessarily equal to $T_F$. Similarly to before, $r(i)$ denotes the data rate for CDMA frame $i$, $R(i) = r(i)T_c$ is the number of data bits in CDMA frame $i$, and $R_T(i) = \sum_{j=1}^{i} R(j)$ is the total number of bits in CDMA frames 1 through $i$.

4.3.3.3 Video Decoder

Let $T_{max}$ be the maximum end-to-end delay that can be tolerated between video users. The actual delay is given by $T_d = T_p + T_b$, where:

- $T_p = N_{pd}T_F$, where $N_{pd}$ is a real number, is the sum of the propagation delay and processing delays (due to video and channel coding/decoding, interleaving, etc.).

- $T_b = N_{bd}T_F$, where $N_{bd}$ is a real number (which is chosen to be an integer to clarify expositions), is a buffering delay introduced in order to smooth the video stream.

We can then write $T_d = N_dT_F$, where $N_d = N_{pd} + N_{bd}$ is the total number of video frames by which the decoding is delayed, from the start of the transmission. Hence, the video decoder starts processing the received bitstream $T_d$ seconds after the transmission began. In the following, without loss of generality, we will take $T_p = 0$ and thus $N_d = N_{bd}$ in order to simplify the exposition.
The video encoder and decoder are synchronized, thus the rate of decoded video frame \( i \), \( d(i) \) is equal to the rate of encoded frame \( i \), \( s(i) \). However, the decoded video frames lag their corresponding encoded video frames by \( T_d \), as illustrated in Fig. 4.25. Hence, \( d(i) = s(i - N_d) \) if \( i > N_d \), and 0 if \( i \leq N_d \). Also, define \( D(i) = d(i)T_C \), and \( D_T(i) = \sum_{j=1}^{i} D(j) \). If the decoder buffer has a finite buffer capacity of \( b \) bits, then the maximum number of bits that can be received by the decoder by frame \( i \) is \( B_T(i) = D_T(i) + b \).

\[

t_s(i) \quad t_s(i+1) \quad t_s(i+2) \quad t_s(i+N_d) \quad t_s(i+N_d+1) \\
\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
\text{Video frames (encoder end)} \\
\text{CDMA frames} \quad j \quad j+1 \quad j+2 \quad j+3 \quad j+4 \quad j+5 \quad j+6 \quad j+7 \quad \cdots \\
\text{Video frames (decoder end)} \quad \cdots \quad \cdots \\
\downarrow \quad \downarrow \quad \downarrow \\
T_C \quad T_d \quad T_F \\
\text{Figure 4.25 Rate adaptation framework.}
\]

### 4.3.3.4 Rate Constraints

Both video encoder and decoder are equipped with a buffer used to smooth out rate variations. Let \( B^e(i) \) be the number of bits which need to be buffered at the video encoder at the end of encoded video frame \( i \). Similarly, let \( B^d(i) \) be the number of bits which need to be buffered at the video decoder at the end of encoded video frame \( i \). Equivalently, \( B^d(i + N_d) \) is the number of bits which need to be buffered at the video decoder at the end of decoded video frame \( i \) (c.f. Fig. 4.25). Then the constraints which must be observed by the video communication system, for each video frame \( i \), are given by [263]:

\[
0 \leq B^e(i) \leq B^e_{\text{max}}, \quad (4.14)
\]

\[
0 \leq B^d(i + N_d) \leq B^d_{\text{max}}, \quad (4.15)
\]
where $B_{e\max}^e$ and $B_{d\max}^d$ are the encoder and decoder buffer sizes, respectively (in bits).

In this work, we make the assumption that the video encoder buffer is of sufficient size so as to avoid any overflow, i.e. $B^e(i) \leq B_{e\max}^e$ for all $i$. Moreover, encoder underflow, which occurs when the encoder is not delivering enough bits to the transmitter, can be prevented by reducing the transmission rate or suspending transmission of video data altogether (it is unlikely for the case at study, due to the high source bit rate), hence $0 \leq B^e(i)$ for all $i$. Then, the constraints of Eq. (4.14) need not be dealt with. Our main concerns are thus the possibilities of decoder buffer underflow and overflow [263], i.e. the violation of the constraints of Eq. (4.15). Decoder buffer underflow, i.e. when $B^d(i + N_d) < 0$, occurs when the receiver does not deliver enough bits on time to the video decoder, which leads to a loss of one or several video frames by the decoder. Decoder buffer overflow, i.e. when $B^d(i + N_d) > B_{d\max}^d$, occurs when too many bits need to be stored in the decoder buffer before the beginning of video decoding, which leads to the discarding by the buffer of one or several received video frames.

Let $R_T'(i + N_d)$ (different from $R_T(i + N_d)$) be the total number of bits transmitted by the end of video frame $i + N_d$, hence $B^d(i + N_d) = R_T'(i + N_d) - S_T(i)$. Then Eq. (4.15) can be written as:

$$0 \leq R_T'(i + N_d) - S_T(i) \leq B_{d\max}^d. \tag{4.16}$$

In the case of our system with unequal video and CDMA frame sizes, Eq. (4.16) becomes:

$$0 \leq \sum_{j=1}^{l} r(j)T_C + r(l + 1)(hT_F - lT_C) - \sum_{j=1}^{i} s(j)T_F \leq B_{d\max}^d \tag{4.17}$$

where $h = i + N_d - 1$, and $l = \lceil hT_F/T_C \rceil$, which means that in the time spanned by $h$ video frames there are $l$ full CDMA frames (the first term on the left-hand side of Eq. (4.17)), and possibly an additional fraction of a CDMA frame (the second term on the left-hand side of (4.17)) if $hT_F/T_C$ is not an integer. Note that for Eq. (4.17) to be equivalent to the decoder buffer underflow constraint given in [263], we should have $h = i + N_d$. However, this will allow the constraint Eq. (4.17) to hold throughout decoded video frame $i$ only if there were no buffer underflow at the beginning of this
frame, and if the transmission rate was constant during the whole duration of this video frame (as in [263]). This doesn't apply to the general case of unequal video and CDMA frame sizes, since a transmission rate change during decoded video frame $i$ can cause the decoder buffer to underflow even if the constraint Eq. (4.17) is satisfied at the beginning of the video frame. However, by choosing $h = i + N_d - 1$, all of decoded video frame $i$ will be received on time if there is no buffer underflow at the beginning of it. Hence constraint Eq. (4.17) is more conservative than the one given in [263]. In order for the decoder buffer overflow constraint to still be respected, as indicated in Eq. (4.17), $B_{max}^{d}$ has been replaced with $B_{max}^{d} = B_{max}^{d} - r_{max}T_f$, where $r_{max}T_f$ is the maximum number of bits received over the interval of one video frame.

In the special case where $T_F = T_C$, one can choose $h = i + N_d$, and Eq. (4.17) reduces to $0 \leq R_T(l) - S_T(i) \leq B_{max}^{d}$, i.e. the term $r(l + 1)(hT_F - lT_C)$ disappears from the inequalities. If $T_F \neq T_C$, the term $r(l + 1)(hT_F - lT_C)$ can lead to a non-integer number of bits in the evaluation of (4.17), and in all cases makes the latter less easy. We've therefore used the following simplified form in our algorithms:

$$0 \leq \sum_{j=1}^{l'} r(j)T_C - \sum_{j=1}^{i} s(j)T_F \leq B_{max}^{d}$$ (4.18)

where $l' = \lfloor hT_F/T_C \rfloor$, and $\lfloor x \rfloor$ denotes the smallest integer larger than $x$. In order to make Eq. (4.18) as conservative as (or more than) Eq. (4.17), we can simply replace $N_d$ with $N_d - 1$ in the algorithms.

### 4.3.4 Smoothing Algorithms for VBR Video over DS/CDMA

Two smoothing algorithms are described in the next sections. The first one, termed SLWIN2, is a more general version of that initially presented in [264], [265] (where it was denoted as Method 2), by allowing a sliding window length $N_w \geq 1$ ($N_w = 1$ in [264], [265]), and imposing constraints on the decoder buffer capacity ($B_{max}^{d} \to \infty$ in [264], [265]). The second algorithm is equivalent to that presented earlier in [51] and denoted as SLWIN, and will serve as a benchmark. It has been modified in order to be able to deal
with unequal source and transmission frame lengths. It should be noted that the case of different transmission granularities has also been treated in [266], for an IP network.

Both algorithms are based on a sliding window mechanism in which the transmission schedule is updated for every new video frame made available (larger updating intervals can be considered [51], in order to decrease computational complexity). For a sliding window of length \( N_w \), in order to incorporate video frame \( i \) into the schedule computation, the algorithms need to have knowledge of the following \( N_w - 1 \) video frames. With this mechanism, the total end-to-end delay will now be \( T_d = T_b + T_w = N_d T_f \), where \( T_w = (N_w - 1)T_F \) and thus \( N_d = N_{bd} + N_w - 1 \). An alternative in order to avoid this extra delay would be to try to predict the following \( N_w - 1 \) frames (c.f. [66], [51]).

At the start of video frame \( i \), thus at time \( t_s[i] = (i - 1)T_F \), the algorithms can modify the rates of the CDMA frames whose transmission begins before time \( t_d[i] = t_s[i] + T_d \). There are \( N_f = N_d T_F / T_C \) or \( N_d T_F / T_C + 1 \) such frames, the first case occurring if there are no incomplete frames within the \( T_d \) duration. At the end of video frame \( i \), thus at time \( t_e[i] = t_s[i] + T_F = iT_F \), the CDMA frames whose transmission started before \( t_c[i] \) now have their rate determined; thus if a CDMA frame is shared by video frames \( i \) and \( i + 1 \) (such as CDMA frame \( j + 1 \) in Fig. 4.25), its rate will be determined by the former only.

### 4.3.4.1 SLWIN2

The goal of SLWIN2 is to minimize the peak and variability of the transmission schedule, subject to the constraint of maintaining at all times the minimum necessary transmission bandwidth required to respect constraints Eq. (4.18). In other words, for each new available video frame, SLWIN2 chooses among all schedules with the minimum necessary bandwidth the one which provides the smallest peak rate and variance. In order to verify the previous constraint, it is allowed to change the transmission rates on a frame-by-frame basis. The way in which the transmission rates are chosen is described below, in response to both buffer underflow and overflow conditions.

When a higher bandwidth is necessary in order to avoid buffer underflow, the algorithm progressively increases the rates of the CDMA frames with the lowest rates, starting with the earliest frames, until Eq. (4.18) is verified. More specifically, when a
rate increase is needed, the algorithm identifies the earliest frame with the lowest rate, and determines whether it can assign to it a higher rate without overflowing the decoder buffer. If it's possible, it increases its rate to the next higher one, and starts over the same procedure. If it's not possible, it identifies the next earliest frame with the same rate, or if there isn't any, the earliest frame with the next higher rate.

When the transmission bandwidth must be decreased in order to avoid buffer overflow, the algorithm progressively decreases the rates of the CDMA frames with the highest rates, starting with the latest frames, until Eq. (4.18) is verified. This procedure is simply the reverse of that detailed above.

The pseudocode for an implementation of this algorithm is given below, along with explanatory comments. The following indices and quantities are used in the pseudocode:
- \( i_f \) (\( i_c \)) is the indice of the video (CDMA) frame currently under consideration, and \( t_v \) (\( t_c \)) is its ending time;
- \( i_w \) (\( i_{w,c} \)) is the indice of the earliest video (CDMA) frame of the sliding window, and \( t_w \) (\( t_{w,c} \)) is its starting time;
- \( i_{sp} \) (\( i_{sp,c} \)) is the indice of the earliest video (CDMA) frame to which the algorithm can backtrack (i.e. all previous frames have their rates fixed), given that \( i_{sp} \geq i_w \) (\( i_{sp,c} > i_{w,c} \)), and \( t_{sp} \) (\( t_{sp,c} \)) is its starting time;
- \( i_d \) (\( i_{d,c} \)) is the indice of the last video (CDMA) frame at which a buffer underflow condition was detected, and \( i_b \) (\( i_{b,c} \)) is the indice of the last video (CDMA) frame at which a buffer overflow condition was detected;
- Rates[] is an array containing all the permissible transmission rates, and \( i_r[i] \) is the indice of the rate assigned to CDMA frame \( i \), with minimum (maximum) value \( i_{r,min} \) (\( i_{r,max} \)).

**Initialization**

\[
\begin{align*}
  i_f &= 0, \ i_c = 0, \ t_v = 0, \ t_c = 0 \\
  i_w &= 1, \ i_{w,c} = 1, \ t_w = 0, \ t_{w,c} = 0 \\
  i_{sp} &= 1, \ i_{sp,c} = 1, \ t_{sp} = 0, \ t_{sp,c} = 0 \\
  i_d &= 1, \ i_{d,c} = 1, \ i_b = 1, \ i_{b,c} = 1 \\
  // \text{ Assign the minimum rate to all CDMA frames} 
\end{align*}
\]
\[ i_r[i] = 1, r[i] = \text{Rates}[i_r[i]] \text{ for all } i \]

**Loop**

\[ i_f = i_f + 1, t_v = t_v + T_f, i_c = i_c + 1, t_c = t_c + T_c, R_T[0] = 0, \text{ continue } = 1 \]

While (continue == 1) {

// Update \( R_T \)

\[ R_T[i_c] = R_T[i_c - 1] + r[i_c]T_c \]

While (\( t_c < t_v \)) {

\[ i_c = i_c + 1, t_c = t_c + T_c, R_T[i_c] = R_T[i_c - 1] + r[i_c]T_c \]

}// Buffer underflow is detected

If (\( R_T[i_c] < D_T[i_f] \)) {

// Find the earliest CDMA frame with the min. rate

\[ i_{\text{min}} = \min \arg \min \{r[i] : i \in [\max(i_{sp,c}, i_{w,c}), \ldots, i_c]\} \]

// Assign to it the next higher rate, if possible

If (\( i_r[i_{\text{min}}] + 1 > i_r[\text{max}] \)) {

If (\( i_{\text{min}} == i_c \)) { Signal buffer underflow }

Else { \( i_r[i_{\text{min}}] = i_r[i_{\text{min}}] + 1 \) }

\[ r[i_{\text{min}}] = \text{Rates}[i_r[i_{\text{min}}]] \]

}// The underflow position is stored, the algorithm backtracks to the earliest possible frame

\[ i_d = i_f, i_f = \max(i_{sp}, i_{w}), t_v = \max(t_{sp}, t_{w}) + T_f \]

\[ i_{d,c} = i_c, i_c = \max(i_{sp,c}, i_{w,c}), t_c = \max(t_{sp,c}, t_{w,c}) + T_c \]

}// Buffer overflow is detected

Else if (\( R_T[i_c] > B_T[i_f] \)) {

// Find the latest CDMA frame with the max. rate

\[ i_{\text{max}} = \max \arg \max \{r[i] : i \in [\max(i_{sp,c}, i_{w,c}), \ldots, i_c]\} \]

// Assign to it the next lower rate, if possible

If (\( i_r[i_{\text{max}}] - 1 < i_r[\text{min}] \)) {

If (\( i_{\text{max}} == \max(i_{sp,c}, i_{w,c}) \)) { Signal buffer overflow }

Else { \( i_r[i_{\text{max}}] = i_r[i_{\text{max}}] - 1 \) }

\[ r[i_{\text{max}}] = \text{Rates}[i_r[i_{\text{max}}]] \]

}// The overflow position is stored, the algorithm backtracks to the earliest possible frame,

Updates \( i_{sp}/i_{sp,c} \)

\[ i_b = i_f, i_f = \max(i_{sp}, i_{w}), t_v = \max(t_{sp}, t_{w}) + T_f, i_{sp} = i_b + 1, t_{sp} = (i_{sp} - 1)T_f \]

\[ i_{b,c} = i_c, i_c = \max(i_{sp,c}, i_{w,c}), t_c = \max(t_{sp,c}, t_{w,c}) + T_c, i_{sp,c} = i_{b,c} + 1, t_{sp,c} = (i_{sp,c} - 1)T_c \]}

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// The constraints are respected
else {
    // Process the next video frame, unless the last one has already been reached
    if (if == Nd + Nf) { continue = 0 }
    else { if = if + 1, tv = tv + Tf, ic = ic + 1, tc = tc + Tc }
}

// Update the starting position of the sliding window
if (if >iw + Nd + Nw) { iw = iw + 1, tw = tw + Tf }

while (tw,c < tw) { iw,c = iw,c + 1, tw,c = tw,c + Tc }

In [260], we used another algorithm to calculate the schedule. When a higher bandwidth is necessary in order to avoid buffer underflow, the algorithm progressively increases the rates of the earliest CDMA frames, until Eq. (4.18) is verified (instead of the CDMA frames with the lowest rates). When the transmission bandwidth must be decreased in order to avoid buffer overflow, the algorithm progressively decreases the rates of the latest CDMA frames (instead of the CDMA frames with the highest rates). This algorithm will be denoted as SLWIN2' (Method 1 of [264]). Unlike SLWIN2, it does not attempt to make the transmission schedule as smooth as possible: it simply chooses the rates so as to avoid buffer underflow and overflow. The advantage of using smoothing algorithms over such a scheme will be quantified in Section 4.3.5.

4.3.4.2 SLWIN

The goal of SLWIN is to make the transmission schedule as smooth as possible, i.e. with minimum peak rate, rate variance and effective bandwidth [51]. To this end, it creates a series of constant-rate runs, by attempting to minimize the rate and maximize the length of each run. Hence, unlike SLWIN2, the transmission rate is not changed on a frame-by-frame basis, but on a run-by-run basis, as further detailed below.

When a rate change is needed, a new run at a higher/lower rate is started at the earliest (leftmost) point in the schedule. Hence, when a higher bandwidth is necessary in order to avoid buffer underflow, the algorithm increases the rates of all the CDMA frames comprised between the start of the run (max(i_{sp,c}, i_{w,c})) and the occurrence of
the underflow \((i_{d,c})\), and repeats it until Eq. \((4.18)\) is verified. When the transmission bandwidth must be decreased in order to avoid buffer overflow, the algorithm decreases the rates of all the CDMA frames comprised between the start of the run \((\max(i_{sp,c}, i_{w,c}))\) and the occurrence of the overflow \((i_{b,c})\), and repeats it until Eq. \((4.18)\) is verified.

The pseudocode for SLWIN, tailored to deal with unequal video and transmission frame sizes, is obtained by making the following additions and modifications to the pseudocode for SLWIN2:

**Initialization**

... 

**Loop**

... 

while \((continue == 1)\) {
  // Update \(R_T\)
  ...
  // Buffer underflow is detected
  if \((R_T[i_{c}] < D_T[i_{f}])\) {
    ...
    // Assign the same rate to the following CDMA frames which begin within the same video frame
    \(i = i_{c}, t = t_{c}\)
    while \((t < T_v)\) {
      \(i = i + 1, t = t + T_c\)
      if \((i > i_{min})\) {
        \(r[i] = r[i - 1], Rates[i] = Rates[r[i]]\)
      }
    }
  }
  // Buffer overflow is detected
  else if \((R_T[i_{c}] > B_T[i_{f}])\) {
    // Find the earliest CDMA frame with the max. rate
    \(i_{max} = \min \arg \max\{r[i] : i \in [\max(i_{sp,c}, i_{w,c}), \ldots, i_{c}]\}\)
    ...
    // The overflow position is stored, the algorithm backtracks to the earliest possible frame,
    updates \(i_{sp}/i_{sp,c}\)
    if \((i_d > i_f)\) {
      \(i_b = i_f, i_f = \max(i_{sp}, i_w), t_v = \max(t_{sp}, t_w) + T_f, i_{sp} = i_b + 1, t_{sp} = (i_{sp} - 1)T_f\)
  }
\[ i_{b,c} = i_c, \quad i_c = \max(i_{sp,c}, i_{w,c}), \quad t_c = \max(t_{sp,c}, t_{w,c}), \quad i_{sp,c} = i_{b,c} + 1, \quad t_{sp,c} = (i_{sp,c} - 1)T_c \]

else {

\[ i_f = i_f, \quad i_{sp} = \max(i_d + 1, i_w), \quad t_{sp} = (i_{sp} - 1)T_f, \quad i_f = i_{sp}, \quad t_v = t_{sp} + T_f \]

\[ i_{b,c} = i_c, \quad i_{sp,c} = \max(i_{d,c} + 1, i_{w,c}), \quad t_{sp,c} = (i_{sp,c} - 1)T_c, \quad i_c = i_{sp,c}, \quad t_c = t_{sp,c} \]

// Assign the same rate to the following CDMA frames which begin within the same video frame
\[ i = i_c, \quad t = t_c \]

while \( (t < T_v) \) \{ \n\[ i = i + 1, \quad t = t + T_c \]

if \( (i > i_{\text{min}}) \) \{ \n\[ i_r[i] = i_r[i - 1], \quad r[i] = Rates[i_r[i]] \] \}

\}

// The constraints are respected
else {

if \( (i_f == N_d + N_f) \) \{

if \( (i_d == N_d + N_f) \) \{ continue = 0 \} else {

// Tries to reduce the rate of the last run, if possible
\[ i_{sp} = \max(i_d + 1, i_w), \quad t_{sp} = (i_{sp} - 1)T_f, \quad i_f = i_{sp}, \quad t_v = t_{sp} + T_f \]

\[ i_{sp,c} = \max(i_{d,c} + 1, i_{w,c}), \quad t_{sp,c} = (i_{sp,c} - 1)T_c, \quad i_c = i_{sp,c}, \quad t_c = t_{sp,c} + T_c \]

if \( (i_r[i_c] - 1 < i_r_{\text{min}}) \) \{ continue = 0 \} else \{ \n\[ i_r[i_c] = i_r[i_c] - 1, \quad r[i_c] = Rates[i_r[i_c]] \] \}

// Assign the same rate to the following CDMA frames which begin within the same video frame
\[ i = i_c, \quad t = t_c \]

while \( (t < T_v) \) \{ \n\[ i = i + 1, \quad t = t + T_c \]

if \( (i > i_{\text{min}}) \) \{ \n\[ i_r[i] = i_r[i - 1], \quad r[i] = Rates[i_r[i]] \] \}

\}

else {

\[ i_f = i_f + 1, \quad t_v = t_v + T_f, \quad i_c = i_c + 1, \quad t_c = t_c + T_c \]

// Holds the same rate throughout the current run
if \( (i_f == i_d + 1 \text{ OR } i_f = i_b + 1 \text{ OR } i_c \leq i_{\text{min}} \text{ OR } i_c \leq i_{\text{max}}) \) \{ \n\[ r[i_c] = Rates[i_r[i_c]] \] \}

else \{ \n\[ i_r[i_c] = i_r[i_c] - 1, \quad r[i_c] = Rates[i_r[i_c]] \] \}

\[ i = i_c, \quad t = t_c \]

while \( (t < T_v) \) \{ \n\[ i = i + 1, \quad t = t + T_c \]

if \( (i_f == i_d + 1 \text{ OR } i_f = i_b + 1 \text{ OR } i \leq i_{\text{min}} \text{ OR } i \leq i_{\text{max}}) \) \{ \n\[ r[i] = Rates[i_r[i]] \] \}

\}

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else { \( i_r[i] = i_r[i-1] \), \( r[i] = Rates[i_r[i]] \) } }

// Update the starting position of the sliding window

4.3.4.3 Comparison between SLWIN and SLWIN2

The main difference between SLWIN and SLWIN2 lies in their adaptation of the transmission rate on a run-by-run and frame-by-frame basis, respectively.

Given that SLWIN tries to maintain long constant-rate runs between “critical points”, at the end of the run it may have consumed more bandwidth than actually needed by the data. This can lead to a higher mean rate than the SLWIN2 algorithm, which is more flexible in its assignment of rates in order to consume the minimum necessary bandwidth (c.f. Fig. 4.31 and associated discussion in Section 4.3.5.2). One possibility could be to use the “gating” capability of cdma2000 systems, i.e. the transmitter would stop transmitting (i.e. the rate is 0 kbps) when there is no more useful data to send. However, this would still result in a higher mean rate for SLWIN than for SLWIN2, since the same amount of useful data would be transmitted in a shorter time interval.

Nevertheless, changing the transmission rate on a frame-by-frame basis presents some drawbacks. The number of rate changes can be larger for SLWIN2 than for SLWIN: frequent switching between rates can be undesirable (c.f. [267]). SLWIN2 has a higher computational complexity than SLWIN, with the gap increasing with the size of the smoothing window.

If there is no restriction on the values of the rates which can be used, i.e. in the case of a fluid model (used for example in the theoretical discussions of [49], [267]), then SLWIN and SLWIN2 will give identical performances.

Section 4.3.5 presents sample numerical results illustrating differences between SLWIN and SLWIN2 in terms of peak, mean and variance of rate.
4.3.5 Performance Evaluation Results and Discussion

4.3.5.1 Simulation Setup

Fig. 4.26 is a general diagram of the simulated IS-95B or IS-2000-based video communication system. Below we describe the various components which make up the video transmission link: the source, the transceiver and the multiple-access channel. In this thesis we do not tackle issues related to the network layer, and assume throughout that the circuit-switched video user is granted the required (maximum) transmission bandwidth at the start of the connection: this allows us to focus on the mechanisms affecting the lower layers, as stated in the introduction.

![Diagram of IS-95B or IS-2000-based video communication system](image)

**Video Source**
The source coder is a H.263-compliant video compressor, which encodes QCIF frames at a rate of 30 frames/second. The optional modes are disabled. The video bitstream is then packetized according to the level 2 protocol of H.223 Annex B [251]. Each picture corresponds to one video packet (VP), which is then broken down into one or more multiplex packets (MP), such that the payload of each MP does not exceed 254 bytes. A synchronization flag (SF) of 2 bytes and a basic header (BH) of 3 bytes are prepended to
each MP, according to the syntax of [251]. Moreover, an optional header (OH) of 1 byte follows the BH. Its purpose is to allow the recovery of the previous MP, in the case the latter is lost due to an error in its SF or BH. A CRC of 2 bytes is appended to each VP.

To evaluate the average PSNR, we use $N_F = 120$ frames of the Miss America or Foreman H.263 video sequences, playing at 30 frames/s. Results are averaged over 20 independent simulation runs, each run using the same $N_F$ video frames. The average PSNR for each simulation run, $PSNR_r$, is calculated as the average of the PSNR’s $PSNR_f(i)$ of each decoded frame $i = 1, 2, \ldots, N_F$, i.e.:

$$PSNR_r = \frac{1}{N_F} \sum_{i=1}^{N_F} PSNR_f(i).$$

(4.19)

The PSNR of each frame is calculated as in [259] as the weighted sum of the PSNR’s for the weighted luminance and chrominance components of that frame:

$$PSNR_f(i) = PSNR_{f,Y}(i) + 0.3PSNR_{f,Cb}(i) + 0.3PSNR_{f,Cr}(i)$$

(4.20)

where:

$$PSNR_{f,x}(i) = 10 \log \left( \frac{255^2}{N_tN_p \sum_{k=1}^{N_t} \sum_{l=1}^{N_p} (o_x(k,l) - r_x(k,l))^2} \right).$$

(4.21)

In Eq. (4.21), $N_t$ denotes the number of lines in the picture (=144 for QCIF), $N_p$ denotes the number of pixels per line (=176 for QCIF), $o_x(k,l)$ and $r_x(k,l)$ are the values of the original and reconstructed 8-bit pixel values, respectively, for component $x \in \{Y, Cb, Cr\}$.

No error concealment is used in the simulations. If the decoder crashes during a run, the PSNR’s for the individual unprocessed frames are taken to be zero. If the decoder freezes during a run, the PSNR for this run is taken to be equal to the average of the PSNR’s of the other runs.

**Reverse Link IS-2000 Transceiver**

The IS-2000 uplink transceiver is as described in Section 4.2.7. In our system, the video data is transported by the SCH1, while the FCH is used to carry voice and/or
control information. The DCCH\textsuperscript{2} and SCH2 channels are omitted for our purposes. The spreading rate is $R_c = 1.2288 \times N \text{ Mcps}$, where $N = 3$ (Spreading rate 3), and the chip period $T_c = 1/R_c$. Data is processed by a rate 1/4 convolutional encoder, and interleaved\textsuperscript{3} according to the pattern specified in Section 4.2.7.

At the BS receiver, the channel multipaths are collected by a Rake receiver with $L_r$ fingers and a resolution of $T_c$, and are maximal-ratio combined. The reverse despreading and coherent demodulation operations are then performed, followed by deinterleaving and hard-decision Viterbi decoding. The analytical uncoded BER performance of such a system was derived in Chapter 3.

In our simulations, the FCH has a fixed data rate of $R_F = 9600$ bps. The SCH1 can take one of the following rates: 9600, 19200, 38400, 76800 or 153600 bps. Note that lower and higher rates are possible, but will not be considered here due to the high source rate and the high mobility scenario, respectively. We assume that the powers of the PCH and the FCH are equal, which is a worst-case assumption since the power of the PCH is likely to be chosen a few factors smaller than that of the FCH. The power of the SCH1 is also taken to be equal to that of the FCH, resulting in lower SNR's and higher BER's for higher data rates. This allows us to observe the effect of smoothing on the video user, in terms of PSNR. If the power was increased proportionally to the rate on a frame-by-frame basis (as we did in [260]), the PSNR of the video user would remain constant, but the capacity of the cell would be decreased due to the extra interference contributed by the video user. While numerically obtaining the cell capacity as a function of the smoothing algorithms or parameters would be interesting, it would prove to be a formidable task in terms of simulation time, given the fact that each user's physical layer is simulated. Moreover, as mentioned in Section 4.3.2, varying the power on a frame-by-frame basis would increase the signaling complexity. These practical facts motivate our decision to keep the power of the video user constant, despite our knowledge that the degradations

\textsuperscript{2}For all the systems under consideration in this section, the signaling information associated with the rate changes is assumed to be carried by the control channels (e.g. DCCH for the IS-2000 uplink) and handled by higher-layer protocols. Since the multiple-access interference contributed by this extra signaling isn't large enough to have a noticeable effect on the simulation results, the control channels haven't been included in the simulation models in order to limit their complexity.

\textsuperscript{3}In [264], [265], the interleaver was switched off, leading to higher BER's.
incurred by a variable BER might not be acceptable to the end user.

**Forward Link IS-2000 Transceiver**

The IS-2000 downlink transceiver is as described in Section 4.2.6. As in the reverse link, the video data is transported by the SCH1, while the FCH is used to carry voice and control information. The other channels are omitted. We use the parameters of Radio Configuration 3 [14]: the spreading rate is $R_c = 1.2288$ Mcps, the code rate is 1/4. The transmitter uses turbo encoding; the turbo encoder and interleaver are as specified in Section 4.2.6.

At the MS receiver, the channel multipaths are collected by a two-finger Rake receiver with a resolution of $T_c$ and are maximal-ratio combined (alternatively, transmitter diversity could be used). The reverse despreading and coherent demodulation operations are then performed, followed by channel deinterleaving and turbo decoding. The turbo decoder uses the MAP (i.e. BCJR) algorithm (c.f. Appendix F). The analytical uncoded BER performance of such a system was derived in Chapter 3.

In our simulations, the SCH's can take one of the following rates: 9600, 19200, 38400, 76800 or 153600 bps. The power of the SCH is made to remain constant for all rates, resulting in lower SNR's for higher data rates, but thus avoiding a reduction in the cell capacity, as discussed in the previous section on the IS-2000 reverse link.

**Reverse Link IS-95B Transceiver**

The IS-95B uplink transceiver is as described in Section 4.2.5. The coded and interleaved symbol stream carrying the video information is demultiplexed into a number of sub-streams. Each symbol sub-stream $c$ of user $k$ is assigned to a different traffic channel.

At the BS receiver, the channel multipaths collected by the Rake receiver are equal-gain combined with a resolution of $T_c$. The reverse despreading and noncoherent demodulation operations are then performed, followed by deinterleaving and hard-decision Viterbi decoding. The analytical uncoded BER performance of such a system was derived in Chapter 3.

Each channel has a maximum data bit rate of 9.6 kbps. A user can then transmit at a total aggregated data bit rate of $9.6 \times N_{codes}$ kbps, where the number of codes $N_{codes}$
varies from 1 to 8. With a coded bit rate \( R_b = 3 \times 9.6 = 28.8 \) kbps and a chip rate \( R_c = 1.228800 \) Mbps, this results in a processing gain of \( N = (\log_2 M)R_c/R_b = 256 \) for each channel. In our simulations, a system with \( N_{\text{codes}} \) codes will be limited to transmit the same total power as a system with only one code: the power of each traffic channel of the multicode video user will be equal to \( 1/N_{\text{codes}} \) times the power of the traffic channel of a single-code user. This preserves system capacity at the expense of a higher BER for the multicode video user, and hence a degradation in the received video quality. We will show how the smoothing algorithms manage to limit this degradation.

**Forward Link IS-95B Transceiver**
The IS-95B downlink transceiver is as described in Section 4.2.4. The management of the multiple codes, the set of allowed rates and the peak power specification are similar to those described in the previous section on the IS-95B uplink. The MS uses a Rake receiver with a resolution of \( T_c \), which performs MRC on the collected multipaths. The analytical uncoded BER performance of such a system was derived in Chapter 3.

**Cellular Environment**
As in the previous chapter, the frequency-selective channel is modeled as a tapped delay line filter with an impulse response given by \( h(t, \tau) = \sum_{i=0}^{L_c-1} a_i(t)e^{j\theta_i(t)}\delta(\tau - \tau_i) \), where \( L_c \) is the number of multipath components, \( a_i(t) \) and \( \theta_i(t) \) are the time-variant amplitude and phase, respectively, of the complex short-term fading coefficient of the \( l^{th} \) path, and \( \tau_i \) is the delay of the latter. It is assumed that there is no ISI. Samples of each \( a_i(t) \) are Rayleigh-distributed. A single-cell system is considered in order to simplify the simulations, however numerical results could be obtained the same way for a multiple-cell environment as we have done in in [190], [260].

We assume that all the users in the system, except for the desired video user, transmit voice only (using a single code for IS-95B, or the FCH at 9.6 kbps for IS-2000). Moreover they are uniformly distributed over the cell, which is a common assumption. The activity factor (AF) of the voice is 0.375, while that of the video source is 1. The mobile speed is \( v = 100 \) km/h, and there are \( L_r = 2 \) Rake fingers. Perfect power control, synchronization and channel estimation are also assumed for simplicity.
4.3.5.2 Comparison of Transmission Rates for SLWIN and SLWIN2

This section compares the transmission schedules produced by SLWIN and SLWIN2, in terms of peak, variance and mean of the rates, for different systems and parameters. The effects of these quantities on the decoded PSNR will be assessed in Section 4.3.5.3.

IS-2000 Reverse Link

Fig. 4.27 plots the peak and mean, and Fig. 4.28 the variance of the transmission rates for the SLWIN and SLWIN2 algorithms, as the delay $N_d$ is varied. The sliding window length is $N_w = 10$ and the decoder buffer size $B_{max}^d = 64k$. For this set of parameters, it can be seen that while the peak rate is similar for both algorithms, the mean and variance of SLWIN2 is lower than that of SLWIN. It can be explained by the better ability of SLWIN2 to adjust to source rate variations: indeed, SLWIN smooths the video stream by producing piecewise constant-rate transmission segments, while SLWIN2 changes the transmission rate on a frame-by-frame basis. However, as mentioned previously, this can result in many more rate changes for SLWIN2, and the computational complexity of SLWIN2 is higher than SLWIN. The peak rate decreases with $N_d$ for both SLWIN and SLWIN2. The variance and mean of the transmission rates decrease in quasi-monotonous and monotonous fashions, respectively, for SLWIN2, while they vary in an irregular (but overall decreasing) fashion for SLWIN.

Fig. 4.29 plots the peak and mean, and Fig. 4.30 the variance of the transmission rates for the SLWIN and SLWIN2 algorithms, as the sliding window length $N_w$ is varied. The delay is $N_d = 9$ and the decoder buffer size $B_{max}^d = 64k$. The peak and variance decrease with $N_w$ for both SLWIN and SLWIN2, but for $N_w$ larger than 10 there is not much improvement. The average rate is constant for SLWIN2 but fluctuates lightly for SLWIN.

Figs. 4.31 and 4.32 plot the cumulative schedule $R_T$ with $N_w = 1$ and $N_w = 10$, respectively, for both SLWIN and SLWIN2, with $N_d = 9$ and $B_{max}^d = 64k$, along with the cumulative decoded data $D_T$. It is noticed that the schedule of SLWIN2 follows $D_T$ more closely than SLWIN does, i.e. it exhibits a lower variance and consumes a lower average bandwith, as evidenced previously in Figs. 4.27-4.30. In the case of SLWIN, the high variability of the rate is due to the fact that the schedule is computed on a
run-by-run basis. For example, in Fig. 4.31 a new constant-rate run is started at video frame 21, and extends until video frame 30 (an underflow critical point): since the rate stays constant between these two points, at the end of the run the cumulative rate \( R_T \) is much bigger than the minimum needed \( D_T \). In contrast, in the case of SLWIN2, the rate is changed on a frame-by-frame basis between these two locations, and thus at the end of the variable-rate run the cumulative rate \( R_T \) is very close to \( D_T \). For larger \( N_w \)'s, e.g. in Fig 4.32, the schedules become smoother for both SLWIN and SLWIN2, due to the latters' knowledge of future frames. Fig. 4.33 plots the same quantities but for \( N_d = 120, N_w = 1 \) and \( B_{max}^d = 8k \), and also along the cumulative buffer capacity \( B_T \). In this case the schedules are very close alike.

Fig. 4.34 plots the transmissions rates of the \( N_C = [(N_F + N_d - 1)T_F] = 214 \) CDMA frames used to transport the \( N_F = 120 \) frames of the Miss America video sequence, with \( N_d = 9 \). It can be seen that, for this particular case, SLWIN2' has a peak transmission rate of 153600 bps, versus 76800 bps for SLWIN2. Moreover, it can be seen that more rate variations are observed with SLWIN2'.

Fig. 4.35 plots the transmissions rates of the \( N_C = [(N_F + N_d - 1)T_F] = 399 \) CDMA frames used to transport the same \( N_F = 120 \) frames, but now with \( N_d = 120 \). In this case SLWIN2' still has a peak transmission rate of 153600 bps, while that of SLWIN2 is merely 19200 bps. The transmission schedule for SLWIN2 is much smoother than in the previous case. The transmission schedule for SLWIN2' has significant variations over a certain number of frames, and then exhibits the minimum rate for the rest of the sequence: indeed, the algorithm of SLWIN2' tries to meet the rate constraint as early as possible.

**IS-2000 Forward Link**

Figs. 4.36, 4.37 and 4.38 plot the peak, variance and mean of the transmission rates, respectively, for the SLWIN and SLWIN2 algorithms, as the delay \( N_d \) is varied. The sliding window length is \( N_w = 10 \) and the decoder buffer size \( B_{max}^d = 64k \). For this set of parameters, it can be seen that the peak rate is similar for both algorithms, and decreases with \( N_d \) for both SLWIN and SLWIN2. However, the mean and variance of
SLWIN2 are lower than that of SLWIN. As mentioned before, it can be explained by the better ability of SLWIN2 to adjust to source rate variations.

Figs. 4.39 and 4.40 plot the variance and mean, respectively, of the transmission rates for the SLWIN and SLWIN2 algorithms, as the sliding window length $N_w$ is varied. The delay is $N_d = 12$ and the decoder buffer size $B_{max}^d = 64k$. The variance decreases with $N_w$ for both SLWIN and SLWIN2, but for $N_w$ larger than 10 there is not much improvement. The average rate is constant for SLWIN2 but fluctuates lightly for SLWIN.

Fig. 4.41 plots the cumulative schedule $R_T$ for both SLWIN and SLWIN2, with $N_d = 12$, $N_w = 10$ and $B_{max}^d = 64k$, along with the cumulative decoded data $D_T$. It is noticed that the schedule of SLWIN2 follows $D_T$ more closely than SLWIN does, i.e. it consumes a lower average bandwidth, as evidenced previously in Fig. 4.38.

![Graph showing IS-2000 peak and mean transmission rates against $N_d/N_F$ for SLWIN and SLWIN2 with $N_T = 20$, $N_w = 10$, $B_{max}^d = 64k$, for the Miss America bitstream.]

**Figure 4.27** IS-2000 peak and mean transmission rates against $N_d/N_F$ for SLWIN and SLWIN2 with $N_T = 20$, $N_w = 10$, $B_{max}^d = 64k$, for the Miss America bitstream.
Figure 4.28 IS-2000 variance of the transmission rates against $N_d/N_F$ for SLWIN and SLWIN2 with $N_I = 20$, $N_w = 10$, $B_{max}^d = 64k$, for the Miss America bitstream.

Figure 4.29 IS-2000 peak and mean transmission rates against $N_w$ for SLWIN and SLWIN2 with $N_I = 20$, $N_d = 9$, $B_{max}^d = 64k$, for the Miss America bitstream.
Figure 4.30 IS-2000 variance of the transmission rates against $N_w$ for SLWIN and SLWIN2 with $N_I = 20$, $N_d = 9$, $B^d_{max} = 64k$, for the Miss America bitstream.

Figure 4.31 IS-2000 transmission rate schedule for SLWIN and SLWIN2 with $N_I = 20$, $N_d = 9$, $N_w = 1$, $B^d_{max} = 64k$, for the Miss America bitstream.
Figure 4.32 IS-2000 transmission rate schedules for SLWIN and SLWIN2 with $N_I = 20$, $N_d = 9$, $N_w = 10$, $B_{max}^d = 64k$, for the Miss America bitstream.

Figure 4.33 IS-2000 transmission rate schedules for SLWIN and SLWIN2 with $N_I = 20$, $N_d = 120$, $N_w = 1$, $B_{max}^d = 8k$, for the Miss America bitstream.
Figure 4.34 IS-2000 transmission rates for SLWIN2 (i.e. Method 2) and SLWIN2′ (i.e. Method 1) with $N_f = 20$, $N_d = 9$, $N_w = 1$, $B_{\text{max}}^d = 64k$, for the Miss America bitstream.

Figure 4.35 IS-2000 transmission rates for SLWIN2 (i.e. Method 2) and SLWIN2′ (i.e. Method 1) with $N_f = 20$, $N_d = 120$, $N_w = 1$, $B_{\text{max}}^d = 64k$, for the Miss America bitstream.
Figure 4.36 IS-2000 peak transmission rates against $N_d$ for SLWIN and SLWIN2 with $N_I = 20$, $N_w = 10$, $B_{max}^d = 64k$, for the Foreman bitstream.

Figure 4.37 IS-2000 variance of the transmission rates against $N_d$ for SLWIN and SLWIN2 with $N_I = 20$, $N_w = 10$, $N_F = 120$ and $B_{max}^d = 64k$, for the Foreman bitstream.
Figure 4.38 IS-2000 mean transmission rates against $N_d$ for SLWIN and SLWIN2 with $N_t = 20$, $N_w = 10$, $N_F = 120$ and $B_{max}^d = 64k$, for the Foreman bitstream.

Figure 4.39 IS-2000 variance of the transmission rates against $N_w$ for SLWIN and SLWIN2 with $N_t = 20$, $N_d = 12$, $B_{max}^d = 64k$, for the Foreman bitstream.
Figure 4.40 IS-2000 mean transmission rates against $N_w$ for SLWIN and SLWIN2 with $N_f = 20$, $N_d = 12$, $B_{\text{max}}^d = 64k$, for the Foreman bitstream.

Figure 4.41 IS-2000 transmission rate schedules for SLWIN and SLWIN2 with $N_f = 20$, $N_d = 12$, $N_w = 10$, $B_{\text{max}}^d = 64k$, for the Foreman bitstream.
**IS-95B Reverse Link**

Fig. 4.42 plots the peak and mean, and Fig. 4.43 the variance of the transmission rates for the SLWIN and SLWIN2 algorithms, as the delay $N_d$ is varied. The sliding window length is $N_w = 10$ and the decoder buffer size $B_{max}^d = 64k$. In comparison to the results of the previous section, these quantities are closer together for SLWIN and SLWIN2. Indeed, in this case there is a finer granularity of the rates as compared with the IS-2000 case: SLWIN then offers a performance closer to its ideal one, which occurs when a fluid model for the rates is assumed. The same can be said from an examination of plots of these quantities versus $N_w$.

Figs. 4.44 and 4.45 plot the transmission rates of the CDMA frames used to transport the $N_F = 120$ frames of the Miss America video sequence, for $N_d = 9$ and $N_d = 120$ respectively. In both cases, SLWIN2' has a peak transmission rate of 76.8 kbps, as compared to 67.2 kbps (for $N_d = 9$) and 28.8 kbps (for $N_d = 120$) with SLWIN2. Also, more rate variations are observed with SLWIN2'. In the case $N_d = 9$, the small buffering delay limits the smoothing ability of SLWIN2, while in the case $N_d = 120$, the transmission schedule for SLWIN2 is much smoother. In contrast, for $N_d = 120$, the transmission schedule for SLWIN2' behaves as described for the IS-2000 system: it has significant variations over a certain number of frames, and then exhibits the minimum rate for the rest of the sequence.

**IS-95B Forward Link**

Fig. 4.46 plots the cumulative schedule $R_T$ for both SLWIN and SLWIN2, with $N_d = 60$, $N_w = 10$ and $B_{max}^d = 64k$, along with the cumulative decoded data $D_T$. The schedules are seen to be very close together, and in fact mostly overlap: as explained in the previous section on the IS-95B uplink, the finer granularity of the transmission rates makes in sort that SLWIN and SLWIN2 produce similar outputs.

The effect of $N_w$ on the schedule produced by SLWIN2 is illustrated in Fig. 4.47, with the same parameters as before: larger $N_w$'s are seen to produce smoother schedules.
Figure 4.42 IS-95B peak transmission rates against $N_d/N_F$ for SLWIN and SLWIN2 with $N_I = 20$, $N_w = 10$, $B_{\text{max}} = 64k$, for the Miss America bitstream.

Figure 4.43 IS-95B variance of the transmission rates against $N_d/N_F$ for SLWIN and SLWIN2 with $N_I = 20$, $N_w = 10$, $B_{\text{max}} = 64k$, for the Miss America bitstream.
Figure 4.44 IS-95B transmission rates for SLWIN2 (i.e. Method 2) and SLWIN2' (i.e. Method 1), with $N_I = 20$, $N_d = 9$, $N_w = 1$, $B_{max}^d = 64k$, for the Miss America bitstream.

Figure 4.45 IS-95B transmission rates for SLWIN2 (i.e. Method 2) and SLWIN2' (i.e. Method 1), with $N_I = 20$, $N_d = 120$, $N_w = 1$, $B_{max}^d = 64k$, for the Miss America bitstream.
Figure 4.46 IS-95B transmission rate schedules for SLWIN and SLWIN2 with $N_f = 132$, $N_d = 60$, $N_w = 10$, $B_{max}^d = 64k$, for the Foreman bitstream.

Figure 4.47 IS-95B transmission rate schedules for SLWIN2 with $N_f = 132$, $N_d = 60$, $B_{max}^d = 64k$, and different $N_w$'s, for the Foreman bitstream.
4.3.5.3 Simulation Results: PSNR

This section presents the performance evaluation and comparison of the SLWIN and SLWIN2 algorithms in terms of PSNR. The results are obtained through computer simulations, and illustrate the effects of smoothing and transceiver parameters.

**IS-2000 Reverse Link**

Fig. 4.48 plots the average PSNR as a function of the number of users $K$, for $N_f = 20$, $N_d = 9$, for the SLWIN, SLWIN2 and SLWIN2' algorithms with $N_w = 1$ and $N_w = 10$. In the case $N_w = 1$, SLWIN2 performs better than SLWIN, especially for a larger number of users. When $N_w = 10$, the two algorithms show a similar performance. This stems from the observations made in Section 4.3.5.2: when a small sliding window length is used, e.g. $N_w = 1$, with SLWIN the variance and mean of the rate are higher than with SLWIN2. Hence more CDMA frames are transmitted at high rates, which leads to a lower average processing gain and thus a higher BER (with the assumption made of no power adaptation). Furthermore, many of these high-rate high-BER frames are used to transport the I-frames from the video sequence (since these I-frames are responsible for the high-rate), which will have a greater impact on the decoded video quality since they are used for predicting the following P-frames. While a higher $N_w$ improves the performance for SLWIN, it also introduces a larger end-to-end delay, which may not be tolerable. Fig. 4.49 plots the results for $N_f = 132$. Similar observations can be made, however for a large number of users SLWIN2 performs better than SLWIN even for a higher $N_w$. In both figures, it can be seen that SLWIN and SLWIN2 allow a dramatic gain in the PSNR over SLWIN2': this shows the benefit of using smoothing algorithms.

Fig. 4.50 illustrates the effect of the buffering delay $N_d$ on the average PSNR, using SLWIN2. With a delay $N_d = 3$, the video quality degrades rapidly for $K > 15$. However, if all of the video frames are buffered prior to decoding ($N_d = 120$), the PSNR remains almost constant, at the expense of a longer delay. The effect of $N_f$ on the PSNR is shown in Fig. 4.51, with $N_d = 9$. A higher $N_f$ means a higher-rate for the video sequence, and thus a higher BER. It also means that the video sequence is refreshed more often, and thus error propagation can be stopped earlier. However, in this case it is seen that the higher BER offsets the gain in PSNR made possible by more frequent refreshing.
Figure 4.48 Average PSNR for $N_f = 20$, $N_d = 9$, $B_{max}^d = 64k$, for SLWIN, SLWIN2, and SLWIN2', in the IS-2000 uplink case, for the Miss America bitstream.

Figure 4.49 Average PSNR for $N_f = 132$, $N_d = 9$, $B_{max}^d = 64k$, for SLWIN, SLWIN2, and SLWIN2', in the IS-2000 uplink case, for the Miss America bitstream.
Figure 4.50 Average PSNR for $N_I = 20$, $N_w = 1$, $B_{max}^d = 64k$ and various $N_d$’s, for SLWIN2 in the IS-2000 uplink case, for the Miss America bitstream.

Figure 4.51 Average PSNR for $N_d = 9$, $N_w = 1$, $B_{max}^d = 64k$ and various $N_I$’s, for SLWIN2 in the IS-2000 uplink case, for the Miss America bitstream.
**IS-2000 Forward Link**

Let $N_{\text{iter}}$ denote the number of turbo decoding iterations. $T_{\text{max}} = N_{\text{max}}T_F$ denotes the maximum total end-to-end delay allowed. Then, assuming a pipeline architecture for the turbo decoder (which leads to a worst-case turbo decoding delay), $N_{\text{iter}}$ and $N_{\text{bd}}$ must satisfy $N_{\text{iter}}T_C + N_{\text{bd}}T_F < T_{\text{max}}$, where $T_p = N_{\text{iter}}T_C$ and $T_b = N_{\text{bd}}T_F$. Fig. 4.52 plots the average PSNR as a function of the number of users $K$, with $N_f = 20$, $N_{\text{max}} = 13$, $N_w = 1$, for the SLWIN and SLWIN2 algorithms and different sets of $\{N_{\text{iter}}, N_{\text{bd}}\}$ which satisfy the previous constraint. It can also be seen that increasing the number of iterations at the expense of $N_{\text{bd}}$ is beneficial for up to about 6-8 iterations: after this the returns of the turbo decoder diminish, and thus it is preferable to rely on a larger $N_{\text{bd}}$ to improve performance. Surely if the turbo decoding delay is made much smaller than $N_{\text{iter}}T_C$, the number of iterations will have a negligible effect on the delay budget, and hence a larger $N_{\text{bd}}$ will be affordable.

Fig. 4.53 shows the effect of $N_{\text{bd}}$ for $N_{\text{iter}}$ fixed and equal to 1, with $N_f = 20$, $N_w = 1$, for the SLWIN2 algorithm. It can be seen that in this case the PSNR is not greatly influenced by varying $N_{\text{bd}}$ over the previous range of 6-9. Hence this confirms that the number of turbo decoding iterations was the determining factor in improving the PSNR in Fig. 4.52.

The effect of the number of diversity branches $L_r$ on the PSNR is examined in Fig. 4.54, with $N_f = 20$, $N_{\text{bd}} = 9$, $N_w = 1$, $B_{\text{max}} = 64k$, $N_{\text{iter}} = 6$, and SLWIN2. A higher $L_r$ results in a degradation of the PSNR, given a certain number of users. This corroborates the observations made in Section 3.4.3, where it was explained that the use of diversity could actually increase the BER due to increased multicode interference. Note however that these results are for a single-cell system with low background noise: for a multi-cell system, diversity will be useful in combating interference (which isn’t synchronous anymore) contributed from other cells.

Fig. 4.55 indicates that as the speed of the mobile increases, so does the PSNR. Indeed, the errors induced by the fading process are less correlated because of the higher Doppler shift, enabling the turbo decoder to correct more errors. However, if CLPC and the effect of estimation errors are taken into account, the reverse is most likely to be
true: the gain due to a more accurate tracking of the channel variations can offset the BER penalty due to correlated errors.

Fig. 4.56 plots the PSNR for code rates $R = 1/3$ and $R = 1/4$, and the same parameters as before. It is seen that the PSNR is considerably improved for $K > 15$ when $R = 1/4$ is used instead of $R = 1/3$.

![Graph showing PSNR vs Number of users K](image)

**Figure 4.52** Average PSNR for $N_f = 20$, $N_w = 1$, $B_{max}^d = 64k$, for SLWIN and SLWIN2, in the IS-2000 downlink case, for the Foreman bitstream.

**IS-95B Reverse Link**

Fig. 4.57 plots the average PSNR as a function of the number of users $K$, for $N_f = 132$, $N_w = 1$ and $B_{max}^d = 64k$ and different $N_d$’s, for the two algorithms. The difference in performance between SLWIN and SLWIN2 is now smaller than for the previous case, as was explained in Section 4.3.5.2, and also due to the fact that less smoothing needs to be done since the source rate is less bursty (only one I-frame is present). It is seen again that higher delays cause the PSNR to degrade more gracefully, but at the expense of a latency penalty. As illustrated in Figs. 4.58 and 4.59, for SLWIN and SLWIN2 respectively, higher $N_w$’s weren’t found to improve much the PSNR.
Figure 4.53 Average PSNR for $N_I = 20$, $N_w = 1$, $B_{max}^d = 64k$, 1 turbo iteration, and various $N_{bd}$'s, for SLWIN2 in the IS-2000 downlink case, for the Foreman bitstream.

Figure 4.54 Average PSNR for $N_I = 20$, $N_{bd} = 9$, $N_w = 1$, $B_{max}^d = 64k$, 6 turbo iterations, and various numbers of diversity branches $L_r$, for SLWIN2 in the IS-2000 downlink case, for the Foreman bitstream.
Figure 4.55 Average PSNR for $N_f = 20$, $N_{bd} = 9$, $N_w = 1$, $B_{\text{max}}^d = 64k$, 6 turbo iterations, and various values of the mobile speed $v$, for SLWIN2 in the IS-2000 downlink case, for the Foreman bitstream.

Figure 4.56 Average PSNR for $N_f = 20$, $N_{bd} = 9$, $N_w = 1$, $B_{\text{max}}^d = 64k$, 6 turbo iterations, and different code rates $R$, for SLWIN2 in the IS-2000 downlink case, for the Foreman bitstream.
Fig. 4.60 plots the average PSNR for a higher number of I-frames, i.e. $N_I = 20$, and $N_w = 10$. A much higher $N_d$ is seen to be needed to maintain an acceptable decoded video quality at high interference levels.

The effect of the number of diversity branches $L_r$ on the PSNR is illustrated in Fig. 4.61, with $N_I = 132$, $N_{bd} = 9$, $N_w = 1$, $B_{max}^d = 64k$, and SLWIN2. As $L_r$ is increased from 1 to 2, a clear improvement of the PSNR is observed. However, the improvements are much smaller for higher $L_r$'s. This is in line with the constatations made in Section 3.2.3 (c.f. in particular Figs. 3.10, 3.11), i.e. for multicode DS/CDMA systems using noncoherent $M$-ary modulation and EGC, having more than two diversity branches doesn't bring a much bigger improvement due to the combining loss.

Fig. 4.62 shows the effect of the mobile speed on the PSNR: as in Fig. 4.55, for all other parameters being constant, a lower Doppler shift due to a smaller $v$ leads to burstier error patterns and a lower average PSNR.

![Figure 4.57](image-url)  

**Figure 4.57** Average PSNR for $N_I = 132$, $N_w = 1$, $B_{max}^d = 64k$ and various $N_d$'s, for SLWIN and SLWIN2 in the IS-95B uplink case, for the Miss America bitstream.
Figure 4.58 Average PSNR for $N_I = 132$, $N_d = 9$, $B_{max}^d = 64k$ and various $N_w$'s, for SLWIN in the IS-95B uplink case, for the Miss America bitstream.

Figure 4.59 Average PSNR for $N_I = 132$, $N_d = 9$, $B_{max}^d = 64k$ and various $N_w$'s, for SLWIN2 in the IS-95B uplink case, for the Miss America bitstream.
Figure 4.60 Average PSNR for $N_I = 20$, $N_w = 10$, $B_{\text{max}}^d = 64k$ and different $N_d$'s, for SLWIN2 in the IS-95B uplink case, for the Miss America bitstream.

Figure 4.61 Average PSNR for $N_I = 132$, $N_d = 9$, $N_w = 1$, $B_{\text{max}}^d = 64k$ and various numbers of diversity branches $L_r$, for SLWIN2 in the IS-95B uplink case, for the Miss America bitstream.
**IS-95B Forward Link**

Figs. 4.63 and 4.64 plot the average PSNR for SLWIN and SLWIN2 with $N_w = 1$ and $B_{max}^d = 64k$, for the cases $N_I = 132$, $N_d = 60$, and $N_I = 20$, $N_d = 120$, respectively. Large buffering delays $N_d$ were necessary to support the high-rate Foreman sequence. As for the IS-95B reverse link, the difference in performance between SLWIN and SLWIN2 is small, due to the finer granularity of the transmission rates.

Fig. 4.65 shows the effect of the sliding window length $N_w$ on the performance of SLWIN2. Increasing $N_w$ from 1 to 20 results in a slight but noticeable improvement in the PSNR. This gain is even more visible for SLWIN, as evidenced by Fig. 4.66.
Figure 4.63 Average PSNR for $N_I = 132$, $N_d = 60$, $N_w = 1$, $D_{\text{max}}^d = 64k$, for SLWIN and SLWIN2 in the IS-95B downlink case, for the Foreman bitstream.

Figure 4.64 Average PSNR for $N_I = 20$, $N_d = 120$, $N_w = 1$, $D_{\text{max}}^d = 64k$, for SLWIN and SLWIN2 in the IS-95B downlink case, for the Foreman bitstream.
Figure 4.65 Average PSNR for $N_l = 132$, $N_d = 60$, $B_{max}^d = 64k$ and different $N_w$'s, for SLWIN2 in the IS-95B downlink case, for the Foreman bitstream.

Figure 4.66 Average PSNR for $N_l = 132$, $N_d = 60$, $B_{max}^d = 64k$ and different $N_w$'s, for SLWIN in the IS-95B downlink case, for the Foreman bitstream.
4.3.6 Conclusions

This section showed the benefit or even necessity of using rate smoothing for H.263 video transmission over bandwidth-limited mobile CDMA networks. It presented a simple algorithm (SLWIN2) for smoothing applicable to H.263 (and similar codecs) for circuit-switched IS-95B and IS-2000 links, and compared it to a previous online smoothing algorithm (SLWIN). When the set of transmission rates is constrained, SLWIN2 has more flexibility than SLWIN, and can lead to a lower bandwidth use and less variability. This section also illustrated the interplay between source rate, decoding delay, SNR and video quality, using end-to-end system simulations of standardized systems.
CHAPTER 5

CONCLUSIONS AND TOPICS FOR FUTURE RESEARCH

5.1 Contributions

The goal of this thesis was to evaluate the performance of variable bit rate video transmission over multicode/multirate DS/CDMA systems such as IS-95B and IS-2000, through analysis (where possible) and simulation. The work was carried out at different levels of the system: at the channel level (Chapter 2), at the cellular level (Chapter 3), and at the video application level (Chapter 4). Indeed, in designing efficient multimedia systems over wireless it is desirable to be able to precisely take into account the effects of the lower layers of the system (physical and link layers) on the higher ones (transport and application layers). Issues relating to the network layer weren’t considered in this thesis, given that we were mainly considering a fixed circuit-switched connection. However, for packet-switched networks especially, an ambitious research project would consist in taking into account the network layer in the simulation and even analysis. The contributions made at the different levels are summarized below.

At the channel level:

- We derived in a unified manner the analytical level crossing rates and average fade durations of Nakagami fading channels for different diversity techniques (SC, MRC, EGC).

- We provided closed-form solutions for the analytical envelope correlation and baseband spectrum of MRC signals for Rayleigh, Nakagami and Rician channels.

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• We assessed the performance of several methods used to generate spatially correlated Nakagami random variables.

• Making use of the previously derived LCR's, we proposed a finite-state Markov channel simulator to speed up the simulation of temporally correlated Nakagami random variables, for different diversity techniques (SC, MRC, EGC), and evaluated its accuracy in terms of LCR, and both uncoded and (convolutionally) coded BER. We pointed out design guidelines and limitations.

• We derived analytical expressions for the SER's of several \( M \)-ary modulation schemes, for dual-branch EGC and MRC in correlated Nakagami fading, using a different approach than those previously used to obtain performance expressions for binary signals. The theoretical expressions were verified by means of simulation, using the previously described generators of spatially correlated Nakagami random variables.

At the cellular level:

• We obtained in single-integral form accurate BER expressions for reverse link multicode DS/CDMA with noncoherent \( M \)-ary modulation and EGC (as used in the IS-95B uplink). The methodology was general enough to be applied to different fading environments (e.g. Rayleigh, Nakagami, Rician, lognormal), multicell systems, systems with closed-loop power control (with the inverse update algorithm) and with successive interference cancellation. The theoretical results were thoroughly validated by entire system simulation, which was often skipped in previous studies.

• We obtained in single-integral form approximate but sufficiently accurate BER expressions for reverse link multicode DS/CDMA with coherent BPSK and complex spreading sequences (as used in the IS-2000 uplink), for MRC and EGC. The methodology allows to accommodate different fading environments and correlated diversity branches, and could be extended to deal with advanced techniques. Results were also thoroughly validate through entire system simulation.

• We analyzed the performance of forward link multicode DS/CDMA with coherent QPSK, for both real (as in IS-95B) and complex (as in IS-2000) spreading sequences,
and MRC or EGC. The BER was obtained in a semi-analytical manner, for different channel environments and correlated diversity branches, and a very good fit with simulated results was observed.

At the video application level:

- We implemented a software platform to test the end-to-end performance of circuit-switched packetized compressed video across different radio links (the IS-95B and IS-2000 forward and reverse links). The platform simulates at the link and physical layers the transmissions of multiple CDMA users from multiple cells (including framing, convolutional and turbo coding/decoding, spreading and modulation), and evaluates the performance degradation incurred by a given video user due to fading and multiple-access interference.

- We extended a benchmark smoothing algorithm to be able to deal with constraints due to the IS-95B/IS-2000 specifications. We proposed another smoothing algorithm, which relies on the same basic principles as the benchmark algorithm, but is optimized to deal with power and bandwidth-constrained cellular CDMA systems. The algorithm seeks to avoid both buffer underflow and overflow, while maximizing the decoded video quality. We compared the performance of both algorithms for different radio links using the developed software platform, in terms of the peak, mean and variance of the transmission rates, and of the decoded video quality. The effects of the startup buffering delay and of the sliding window length were evaluated.

5.2 Topics for Future Research

The work carried out in this thesis could be complemented by several extensions, or be a starting point for other interesting research initiatives. We describe a few of these below:

- In Chapter 3, we mainly considered systems where the parameters associated with transceiver/receiver are fixed (i.e. the size of the modulation constellation, the processing gain). Future work could consist in providing new analytical models for
systems employing dynamic adaptation of parameters, such as channel prediction-aided adaptive modulation.

- Another analytical challenge consists in obtaining models in terms of other performance measures, such as the packet/frame error rate and throughput. Indeed, these quantities are more useful when dealing with system design at higher layers: for example, when dealing with packet-switched video communications, the probability that a RTP packet is corrupted is more relevant to the designer. However, obtaining accurate (or exact) expressions such as those derived for the BER is likely to be very difficult, and bounds could be more feasible.

- There are several open areas in the design of rate adaptation algorithms for wireless video. In particular, the following topics are of interest, and research on these can build on the material presented in Chapter 4: 1) Joint source coding and rate smoothing for time-varying channels; 2) Rate smoothing for mobile-to-mobile tandem links (uplink/downlink/uplink); 3) Video transport over heterogeneous links, such as wireline-to-wireless.

- In order to better exploit the time-varying conditions experienced by mobile wireless systems, adaptive techniques are being introduced in standards such as cdma2000 1xEV. These techniques rely on estimating/predicting the interference seen by a user, or the strength of the propagation channel, which is best done by employing adaptive signal processing algorithms with the ability to track variations. Most studies have considered environments with high SNR’s, in which certain classical prediction algorithms can be used with relative success. However, in practice the received signal can be buried in noise due to multiple-access interference: it is thus important to devise robust algorithms which can handle a wide range of conditions. The use of adaptive techniques in conjunction with VBR video sources is another area of future research opportunities.

- End-to-end simulation studies of radio protocols or video communications require a very large number of samples in order to obtain reliable results. For multiple-access communication systems employing advanced techniques, execution times on
software platforms become a bottleneck, even for highly optimized code. There is thus a motivation for moving some (or all) of the elements of the system on hardware platforms, enabling near real-time simulations. Rapid prototyping consists in developing complex simulation systems with high-level software tools, and then running these systems on target hardware platforms, using specifically designed interfaces. The usefulness of rapid prototyping is a two-way street: simulation models conveniently developed using high-level tools with slow execution times are run on fast hardware, while the tedious coding required on hardware platforms is bypassed thanks to the high-level tools. Thus a challenging but rewarding research and development effort would consist in implementing fast simulation platforms for computation-intensive simulation models. An objective would be to develop a wireless video transmission testbed.
APPENDIX A: Special Functions, Series Expansions and Definite Integrals

This section summarizes the special functions used in the thesis, and presents some useful relations.

Error Function and Related Functions

- Error function:
  \[ \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \, dt \quad ([80], 8.250.1) \quad (A.1) \]

- Complementary error function:
  \[ \text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} \, dt = 1 - \text{erf}(x) \quad ([80], 8.250.4) \quad (A.2) \]

- Q-function:
  \[ Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} \, dt = \frac{1}{2} \text{erfc} \left( \frac{x}{\sqrt{2}} \right) \quad (A.3) \]

Bessel Functions

- Bessel function of the first kind of order \( \nu \):
  \[ J_\nu(x) = \frac{(x^2/2)^\nu}{\Gamma(\nu + 1/2)\Gamma(1/2)} \int_0^\pi e^{i x \cos(\theta)} \sin^{2\nu} \theta \, d\theta \quad \text{Re}(\nu) > -\frac{1}{2} \quad (A.4) \]
  \[ = \sum_{k=0}^{\infty} \frac{(-1)^k \left( x^2/2 \right)^{2k+\nu}}{k!\Gamma(\nu + k + 1)} \quad \text{arg}(x) < \pi \quad ([80], 8.411.7, 8.440) \quad (A.5) \]

- Modified Bessel function of the first kind of order \( \nu \):
  \[ I_\nu(x) = \frac{(x^2/2)^\nu}{\Gamma(\nu + 1/2)\Gamma(1/2)} \int_0^\pi e^{x \cos(\theta)} \sin^{2\nu} \theta \, d\theta \quad \text{Re}(\nu) > -\frac{1}{2} \quad (A.6) \]
  \[ = \sum_{k=0}^{\infty} \frac{(x^2/2)^{\nu+2k}}{k!\Gamma(\nu + k + 1)} \quad ([80], 8.431.3, 8.445) \quad (A.7) \]
Elliptic Integrals

- Complete elliptic integral of the first kind:

\[
K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}} \quad 0 \leq k \leq 1
\]  
\[
= \int_0^1 \frac{dt}{\sqrt{(1-t^2)(1-k^2 t^2)}} \quad ([80], 8.112.1, 8.111.2) \tag{A.8}
\]

- Complete elliptic integral of the second kind:

\[
E(k) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \theta} \, d\theta \quad 0 \leq k \leq 1
\]  
\[
= \int_0^1 \frac{\sqrt{1-k^2 t^2}}{\sqrt{1-t^2}} \, dt \quad ([80], 8.112.2, 8.111.3) \tag{A.10}
\]

Generalized Marcum-Q Function

\[
Q_m(\alpha, \beta) = \int_0^\infty u \left( \frac{u}{\alpha} \right)^{m-1} e^{-\alpha^2 + \beta^2} I_{m-1}(\alpha u) \, du \tag{A.12}
\]

Gamma and Beta Functions

- Gamma function:

\[
\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} \, dt \quad [\text{Re}(x) > 0] \quad ([80], 8.310.1) \tag{A.13}
\]

- Incomplete gamma function:

\[
\gamma(x, \alpha) = \int_0^\alpha e^{-t} t^{x-1} \, dt \quad [\text{Re}(x) > 0] \quad ([80], 8.350.1) \tag{A.14}
\]
- Beta function:

\[ B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt \quad [\text{Re}(x) > 0, \ \text{Re}(y) > 0] \]  

(A.15)

\[ B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)} \]  

([80], 8.380.1, 8.384.1) (A.16)

Hypergeometric Functions

- Gaussian hypergeometric function:

\[ _2F_1(a, b; c; x) = \frac{1}{B(b, c-b)} \int_0^1 t^{b-1}(1-t)^{c-b-1}(1-tx)^{-a}dt \quad [\text{Re}(c) > \text{Re}(b) > 0] \]  

(A.17)

where the Pochhammer symbol \((\lambda)_k\) is defined as:

\[
(\lambda)_0 = 1 \\
(\lambda)_k = \lambda(\lambda+1)(\lambda+2)\cdots(\lambda+k-1) = \frac{\Gamma(\lambda+k)}{\Gamma(\lambda)}
\]  

(A.18)

Useful relations:

\[ _2F_1(a, b; c; x) = (1-x)^{-a-b} _2F_1(c-a, c-b; c; x) \]  

(A.19)

\[ = (1-x)^{-a} _2F_1\left(a, c-b; c; \frac{x}{x-1}\right) \]  

(A.20)

\[ = (1-x)^{-b} _2F_1\left(b, c-a; c; \frac{x}{x-1}\right) \]  

([80], 9.131.1) (A.21)

- Generalized hypergeometric function:

\[ _pF_q(a_1, a_2, \ldots, a_p; b_1, b_2, \ldots, b_q; x) = \sum_{k=0}^\infty \frac{(a_1)_k \cdots (a_p)_k x^k}{(b_1)_k \cdots (b_q)_k k!} \]  

([80], 9.14.1) (A.22)
Appell hypergeometric functions of several variables:

\[ F_n(a; b_1, b_2, \ldots, b_n; c_1, c_2, \ldots, c_n; x_1, \ldots, x_n) \]

\[ = \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \cdots \sum_{k_n=0}^{\infty} \frac{(a)_{k_1+\ldots+k_n}(b)_{k_1} \ldots (b)_{k_n}}{(c)_{k_1} \ldots (c)_{k_n}} \frac{x_1^{k_1} \ldots x_n^{k_n}}{k_1! \ldots k_n!} \]  \[ ([80], 9.19) \quad (A.23) \]

Confluent Hypergeometric Functions

- Confluent hypergeometric function:

\[ \Phi(a, c; x) = \frac{1}{B(a, c-a)} x^{1-c} \int_0^x e^{t(a-1)}(x-t)^{c-a-1} \quad [\text{Re}(c) > \text{Re}(a) > 0] (A.24) \]

\[ = \sum_{k=0}^{\infty} \frac{(a)_k x^k}{(c)_k k!} \quad ([80], 9.210.1, 9.211.2) \quad (A.25) \]

\[ = _1F_1(a, c; x) \quad \text{(alternate notation)} \quad (A.26) \]

Useful relations:

\[ \Phi(a, c; x) = e^x \Phi(c-a, c; -x) \quad ([80], 9.212.1) \quad (A.27) \]

\[ \frac{x}{c} \Phi(a + 1, c + 1; x) = \Phi(a + 1, c; x) - \Phi(a, c; x) \quad ([80], 9.212.2) \quad (A.28) \]

\[ a \Phi(a + 1, c + 1; x) = (a-c) \Phi(a, c + 1; x) + c \Phi(a, c; x) \quad ([80], 9.212.3) \quad (A.29) \]

\[ a \Phi(a + 1, c; x) = (x + 2a - c) \Phi(a, c; x) + (c - a) \Phi(a - 1, c; x) \quad ([80], 9.212.4) \quad (A.30) \]

\[ \text{erf}(x) = \frac{2x}{\sqrt{\pi}} \Phi \left( \frac{1}{2}, \frac{3}{2}; -x^2 \right) \quad ([80], 9.236.1) \quad (A.31) \]

\[ \gamma(x, \alpha) = \frac{x^\alpha}{\alpha} \Phi(\alpha, \alpha + 1; -x) \quad ([80], 9.236.4) \quad (A.32) \]

\[ I_\nu(x) = \frac{2^{-\nu}}{\Gamma(\nu + 1)} x^\nu e^{-x} \Phi \left( \frac{1}{2} + \nu, 1 + 2\nu; 2x \right) \quad ([80], 9.238.2) \quad (A.33) \]
Binomial Series Expansion

\[(y + a)^n = \sum_{i=0}^{n} \binom{n}{i} y^i a^{n-i} \quad ([80], 1.111) \quad (A.34)\]

Some Integrals Involving or Solved Using Special Functions

\[\int_0^u x^{\nu-1}(u - x)^{\mu-1} e^{\beta x} \, dx = B(\mu, \nu) u^{\mu+\nu-1} F_1(\mu, \mu + \nu; \beta u) \quad \text{Re}(\mu) > 0, \; \text{Re}(\nu) > 0 \quad ([80], 3.383.1) \quad (A.35)\]

\[\int_0^u x^{\nu-1}(u - x)^{\mu-1} e^{\beta x^n} \, dx = \]

\[B(\mu, \nu) u^{\mu+\nu-1} F_n \left( \frac{\mu + 1}{n}, \ldots, \frac{\mu + n - 1}{n}, \frac{\mu + \nu}{n}, \frac{\mu + \nu + 1}{n}, \ldots, \frac{\mu + \nu + n - 1}{n}; \beta u^n \right) \quad \text{Re}(\mu) > 0, \; \text{Re}(\nu) > 0, \; n = 2, 3, \ldots \quad ([80], 3.478.3) \quad (A.36)\]

\[\int_0^\infty x^{\nu-1} e^{-bx} F_1(a, b; cx) \, dx = b^{-\nu} \Gamma(\nu) F_1 \left( \nu, a; b; \frac{c}{b} \right) \quad ([80], 7.621.4) \quad (A.37)\]

\[\int_0^\infty x^{\nu-1} e^{-bx} \prod_{k=1}^n F_1(a_k, b_k; c_k x) \, dx = b^{-\nu} \Gamma(\nu) F_n \left( \nu, a_1, \ldots, a_n; b_1, \ldots, b_n; \frac{c_1}{b}, \ldots, \frac{c_n}{b} \right) \quad \left[ b_k > 0, \; \nu > 0, \; \sum c_k < b \right] \quad ([80], 7.622.3) \quad (A.38)\]

\[\int_0^\infty x^{\mu-1} e^{-\beta x} \Gamma(\nu, \alpha x) \, dx = \frac{\Gamma(\mu + \nu)}{\nu} \frac{\alpha^n}{(\alpha + \beta)^{\mu+\nu}} F_1 \left( 1, \mu + \nu; \nu + 1; \frac{\alpha}{\alpha + \beta} \right) \quad ([80], 6.455.2) \quad (A.39)\]
\[ \int_0^\infty x^\beta e^{-\alpha x^2} I_\mu(\gamma x) \, dx = \frac{1}{2a^{\frac{1}{2}(\beta+\mu+1)}} \left( \frac{1}{2\gamma} \right)^{\mu} \frac{\Gamma\left(\frac{1}{2}(\beta + \mu + 1)\right)}{\Gamma(\mu + 1)} \times {}_1F_1\left(\frac{1}{2}(\beta + \mu + 1); \mu + 1; \frac{\gamma^2}{4a}\right) \] (A.40)

\[ \int_0^\infty x e^{-\gamma^2} I_\mu(\alpha x) I_\mu(\beta x) \, dx = \frac{1}{2\gamma} e^{-\alpha^2 + \beta^2} I_\mu\left(\frac{\alpha \beta}{2\gamma}\right) \] (A.41)
APPENDIX B: Alternative Derivation of the LCR for SC

The purpose of this appendix is to show that the analytical approach given in [101], with some adjustments, also leads to Eq. (2.43). For channels with nonidentical parameters, Eq. (44) of [101] can be modified as

\[
N_R(r) = \sum_{l=1}^{L} \left( \prod_{j=1}^{L} \int_{0}^{2\pi} p(\theta_{ij})d\theta_{ij} \right) p_{R_l}(r) \left( \prod_{j=1}^{L} \int_{0}^{r} p_{R_l}(r_l)dr_l \right) \int_{0}^{\infty} \hat{r}_l p_{R_l}(\hat{r}_l|r_l = r)d\hat{r}_l
\]

(A.42)

where

\[
N_R(r) = \sum_{l=1}^{L} p_{R_l}(r) \left( \prod_{j=1}^{L} F_{R_l}(r) \right) n_l
\]

(A.43)

The usual assumption is made that the phases associated with the Nakagami signals are uniformly distributed over \([0, 2\pi]\). Given that the phases, envelopes, and \(\hat{r}_l\) are mutually independent, (A.42) can be rewritten as

\[
N_R(r) = \sum_{l=1}^{L} \left( \prod_{j=1}^{L} \int_{0}^{2\pi} p(\theta_{ij})d\theta_{ij} \right) p_{R_l}(r) \left( \prod_{j=1}^{L} \int_{0}^{r} p_{R_l}(r_l)dr_l \right) \int_{0}^{\infty} \hat{r}_l p_{R_l}(\hat{r}_l|r_l = r)d\hat{r}_l
\]

(A.43)

For a Nakagami channel with arbitrary parameters and SC, (A.43) reverts to Eq. (2.43).
APPENDIX C: Calculation of the Generalized Marcum-Q Function

Several methods are available [268] to calculate the Generalized Marcum-Q function, given by Eq. (A.12). In this thesis, we make use of the generalization of the algorithm of [269] for the Marcum Q-function, which is summarized as follows [270] ($\epsilon$ is a small constant):

\begin{align*}
    aa &= \alpha \times \alpha / 2 \\
    bb &= \beta \times \beta / 2 \\
    d &= e^{-aa} \\
    h &= d \\
    f &= bb^m \times e^{-bb} / m! \\
    f_{err} &= e^{-bb} \\
    errbnd &= 1 - f_{err} \\
    k &= 1 \\
    \delta &= f \times h \\
    sum &= \delta \\
    \text{if } ((errbnd > 4 \times eps) \text{ AND } ((1 - sum) > 8 \times \epsilon)) \ j = 1 \ \text{else } j = 0 \\
    \text{while } ((j > 0) \text{ OR } (k \leq m)) \\
    \quad d &= aa \times d / k \\
    \quad h &= h + d \\
    \quad f &= bb \times f / (k + m) \\
    \quad \delta &= f \times h \\
    \quad sum &= sum + \delta \\
    \quad f_{err} &= f_{err} * bb / k \\
    \quad errbnd &= errbnd - f_{err} \\
    \text{if } ((errbnd > 4 \times \epsilon) \text{ AND } ((1 - sum) > 8 \times \epsilon)) \ j = 1 \ \text{else } j = 0 \\
    \quad k &= k + 1 \\
    \quad \text{if } (k > 100000) : \text{ if } ((j == 1) \text{ AND } (\delta > \epsilon * (1 - sum))) \ j = 1 \ \text{else } j = 0 \\
    \quad \text{if } (k > 100000) : \text{ print(The algorithm failed to converge. Results may be incorrect)} \\
    q &= 1 - sum
\end{align*}
APPENDIX D: Derivation of Signal and Interference Terms for Multicode DS/CDMA with Noncoherent M-ary Orthogonal Modulation

The low-pass filtered received signal on the $I$-branch is given by:

\[
\begin{aligned}
    r_{I,LP}(t) &= (r(t) \cos(\omega_c t))_{LP} \\
    &= \frac{n_c(t)}{2} + \frac{1}{2} \sum_{k=1}^{K} \sum_{c=0}^{N^{(k)}-1} \sum_{l=0}^{L_c-1} \sqrt{\alpha_l^{(k)}}(t) \times \\
    &\quad W^{(kc)}(t - \tau_i^{(k)})a_i^{(kc)}(t - \tau_i^{(k)}) \cos(\varphi_i^{(kc)}) \\
    &\quad + W^{(kc)}(t - T_0 - \tau_i^{(k)})a_i^{(kc)}(t - T_0 - \tau_i^{(k)}) \sin(\varphi_i^{(kc)}). \\
\end{aligned}
\]  

(A.45)

The correlator output $Z_{II}^{(m,n)}$ is then:

\[
\begin{aligned}
    Z_{II}^{(m,n)} &= \frac{1}{\sqrt{T_W}} \int_{\tau_n^{(1)}}^{\tau_n^{(1)}+T_W} r_{I,LP}(t)a_i^{(10)}(t - \tau_i^{(1)})W_m(t - \tau_n^{(1)})dt \\
    &= S_{II}^{(m,n)} + I_{I_{II}}^{(m,n)} + I_{II}^{(m,n)} + I_{II_{II}}^{(m,n)} + N_{II}^{(m,n)}. \\
\end{aligned}
\]  

(A.46)

The interference terms can be written as follows:

\[
\begin{aligned}
    I_{I_{II}}^{(m,n)} &= C \int_{\tau_n^{(1)}}^{\tau_n^{(1)}+T_W} \alpha_n^{(1)}(t) \sin(\varphi_n^{(10)})W^{(10)}(t - \tau_n^{(1)} - T_0)a_Q^{(10)}(t - \tau_n^{(1)} - T_0) \\
    &\quad \times W_m(t - \tau_n^{(1)})a_i^{(10)}(t - \tau_i^{(1)})dt \\
    &= C \int_{0}^{T_0} \alpha_n^{(1)}(t) \sin(\varphi_n^{(10)})W^{(10)}(t - T_0)a_Q^{(10)}(t - T_0)W_m(t)a_i^{(10)}(t)dt \\
    &= C \sin(\varphi_n^{(10)})\alpha_n^{(1)}[ \int_{0}^{T_0} W_i(\tau_n^{(1)} - 1)(t - T_0)a_Q^{(10)}(t - T_0)W_m(t)a_i^{(10)}(t)dt \\
    &\quad + \int_{T_0}^{T_W} W_i(\tau_n^{(1)} - 0)(t - T_0)a_Q^{(10)}(t - T_0)W_m(t)a_i^{(10)}(t)dt] \\
    &= C\alpha_n^{(1)}\sin(\varphi_n^{(10)})[R_{QI}^{(10)}(T_0) + R_{QI}^{(10)}(T_0)]
\end{aligned}
\]  

(A.47)
\[ I^{(m,n)}_{2II} = C \int_{\tau_1^{(1)}}^{\tau_H^{(1)}} \sum_{l=0}^{L_e-1} \alpha_l^{(1)}(t) [ W^{(10)}(t - \tau_1^{(1)}) a_I^{(10)}(t - \tau_1^{(1)}) \cos(\varphi_l^{(10)}) \\
+ W^{(10)}(t - \tau_1^{(1)} - T_0) a_Q^{(10)}(t - \tau_1^{(1)} - T_0) \sin(\varphi_l^{(10)})] W_m(t - \tau_n^{(1)}) a_I^{(10)}(t - \tau_n^{(1)}) dt \\
= C \sum_{l=0}^{L_e-1} \int_0^{T_w} \alpha_l^{(1)}(t) [ W^{(10)}(t - \tau_1^{(1)}) a_I^{(10)}(t - \tau_1^{(1)}) \cos(\varphi_l^{(10)}) \\
+ W^{(10)}(t - \tau_1^{(1)} - T_0) a_Q^{(10)}(t - \tau_1^{(1)} - T_0) \sin(\varphi_l^{(10)})] W_m(t) a_I^{(10)}(t) dt \\
= C \sum_{l=0}^{L_e-1} \alpha_l^{(1)} \left[ \int_{\tau_1^{(1)}}^{\tau_n^{(1)}} W_{i10,-1}(t - \tau_n^{(1)}) a_I^{(10)}(t - \tau_n^{(1)}) W_m(t) a_I^{(10)}(t) \cos(\varphi_l^{(10)}) dt \\
+ \int_{\tau_1^{(1)}}^{T_w} W_{i10,0}(t - \tau_n^{(1)}) a_I^{(10)}(t - \tau_n^{(1)}) W_m(t) a_I^{(10)}(t) \cos(\varphi_l^{(10)}) dt \\
+ \int_0^{T_0} + \int_{T_0}^{T_1} W_{i10,1}(t - \tau_n^{(1)} - T_0) a_Q^{(10)}(t - \tau_n^{(1)} - T_0) W_m(t) a_I^{(10)}(t) \sin(\varphi_l^{(10)}) dt \\
+ \int_{T_0}^{T_1} W_{i10,1}(t - \tau_n^{(1)} - T_0) a_Q^{(10)}(t - \tau_n^{(1)} - T_0) W_m(t) a_I^{(10)}(t) \sin(\varphi_l^{(10)}) dt \right] \\
= C \sum_{l=0}^{L_e-1} \alpha_l^{(1)} \left[ (R_{II}^{10}(\tau_n^{(1)}) + \hat{R}_{II}^{10}(\tau_n^{(1)})) \cos(\varphi_l^{(10)}) \\
+ (R_{II}^{10}(\tau_n^{(1)} + T_0) + \hat{R}_{II}^{10}(\tau_n^{(1)} + T_0)) \sin(\varphi_l^{(10)}) \right] \tag{A.48} \]

\[ I^{(m,n)}_{3II} = C \int_{\tau_1^{(1)}}^{\tau_H^{(1)}} \sum_{c=1}^{N^{(1)-1}} \sum_{l=0}^{L_e-1} \alpha_l^{(1)}(t) [ W^{(1c)}(t - \tau_1^{(1)}) a_I^{(1c)}(t - \tau_1^{(1)}) \cos(\varphi_l^{(1c)}) \\
+ W^{(1c)}(t - \tau_1^{(1)} - T_0) a_Q^{(1c)}(t - \tau_1^{(1)} - T_0) \sin(\varphi_l^{(1c)})] W_m(t - \tau_n^{(1)}) a_I^{(1c)}(t - \tau_n^{(1)}) dt \\
= C \sum_{c=1}^{N^{(1)-1}} \sum_{l=0}^{L_e-1} \int_0^{T_w} \alpha_l^{(1)}(t) [ W^{(1c)}(t - \tau_n^{(1)}) a_I^{(1c)}(t - \tau_n^{(1)}) \cos(\varphi_l^{(1c)}) \\
+ W^{(1c)}(t - \tau_n^{(1)} - T_0) a_Q^{(1c)}(t - \tau_n^{(1)} - T_0) \sin(\varphi_l^{(1c)})] W_m(t) a_I^{(1c)}(t) dt \\
= C \sum_{c=1}^{N^{(1)-1}} \sum_{l=0}^{L_e-1} \alpha_l^{(1)} \left[ (R_{II}^{1c}(\tau_n^{(1)}) + \hat{R}_{II}^{1c}(\tau_n^{(1)})) \cos(\varphi_l^{(1c)}) \\
+ (R_{II}^{1c}(\tau_n^{(1)} + T_0) + \hat{R}_{II}^{1c}(\tau_n^{(1)} + T_0)) \sin(\varphi_l^{(1c)}) \right] \tag{A.49} \]

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\[ I_{II}^{(m,n)} = C \int_{r_{n}^{(1)}+T_{W}}^{r_{n}^{(1)}+T_{W}+T_{0}} \sum_{k=2}^{K} \sum_{c=0}^{N^{(k)}-1} \sum_{l=0}^{L_{c}-1} \alpha_i^{(k)}(t) [W^{(kc)}(t - \tau_{l}^{(k)})a_{I}^{(kc)}(t - \tau_{l}^{(k)}) \cos(\varphi_{l}^{(kc)}) + W^{(kc)}(t - \tau_{l}^{(k)} - T_{0})a_{Q}^{(kc)}(t - \tau_{l}^{(k)} - T_{0}) \sin(\varphi_{l}^{(kc)})]W_{m}(t - \tau_{n}^{(1)})a_{I}^{(10)}(t - \tau_{n}^{(1)})dt \]
\[ = C \sum_{k=2}^{K} \sum_{c=0}^{N^{(k)}-1} \sum_{l=0}^{L_{c}-1} \int_{0}^{T_{W}} \alpha_i^{(k)}(t) [W^{(kc)}(t - \tau_{nl}^{(k)})a_{I}^{(kc)}(t - \tau_{nl}^{(k)}) \cos(\varphi_{l}^{(kc)}) + W^{(kc)}(t - \tau_{nl}^{(k)} - T_{0})a_{Q}^{(kc)}(t - \tau_{nl}^{(k)} - T_{0}) \sin(\varphi_{l}^{(kc)})] \times W_{m}(t - \tau_{n}^{(1)})a_{I}^{(10)}(t - \tau_{n}^{(1)})dt \]
\[ = C \sum_{k=2}^{K} \sum_{c=0}^{N^{(k)}-1} \sum_{l=0}^{L_{c}-1} \alpha_i^{(k)} [(R_{II}^{(kc)}(\tau_{nl}^{(k)}) + \hat{R}_{II}^{(kc)}(\tau_{nl}^{(k)})) \cos(\varphi_{l}^{(kc)})] \times [R_{QI}^{(k)}(\tau_{nl}^{(k)} + T_{0}) + \hat{R}_{QI}^{(k)}(\tau_{nl}^{(k)} + T_{0}) \sin(\varphi_{l}^{(kc)})]. \] (A.50)

The correlator output \( Z_{IQ}^{(m,n)} \) is:
\[ Z_{IQ}^{(m,n)} = \frac{1}{T_{W}} \int_{r_{n}^{(1)}+T_{W}}^{T_{W}+T_{0}} r_{I,L,P}(t) a_{Q}^{(10)}(t - \tau_{n}^{(1)} - T_{0})W_{m}(t - \tau_{n}^{(1)} - T_{0})dt \]
\[ = S_{IQ}^{(m,n)} + I_{1IQ}^{(m,n)} + I_{2IQ}^{(m,n)} + I_{3IQ}^{(m,n)} + N_{IQ}^{(m,n)}. \] (A.51)

The above signal and interference terms can be written as follows:
\[ S_{IQ}^{(m,n)} = \alpha_n^{(1)} \frac{\sqrt{E_{w}}}{2} \sin(\varphi_{n}^{(10)}) \delta_{mi}, \] (A.52)

\[ I_{1IQ}^{(m,n)} = C \int_{r_{n}^{(1)}+T_{W}+T_{0}}^{r_{n}^{(1)}+T_{W}+T_{0}} \alpha_n^{(1)}(t) \cos(\varphi_{n}^{(10)})W^{(10)}(t - \tau_{n}^{(1)})a_{I}^{(10)}(t - \tau_{n}^{(1)}) \times W_{m}(t - \tau_{n}^{(1)} - T_{0})a_{Q}^{(10)}(t - \tau_{n}^{(1)} - T_{0})dt \]
\[ = C \int_{T_{W}}^{T_{W}+T_{0}} \alpha_n^{(1)}(t) \cos(\varphi_{n}^{(10)})W^{(10)}(t - (-T_{0}))a_{I}^{(10)}(t - (-T_{0}))W_{m}(t)a_{Q}^{(10)}(t)dt \]
\[ = C \alpha_n^{(1)} [R_{IQ}^{(10)}(-T_{0}) + \hat{R}_{IQ}^{(10)}(-T_{0})] \cos(\varphi_{n}^{(10)}) \] (A.53)
\[
I_{2IQ}^{(m,n)} = C \int_{\tau_{n1}^{(1)} + T_0}^{\tau_{n1}^{(1)} + T_0 + T_W} \sum_{\substack{l=0 \\ l \neq n}}^{L_e-1} \alpha_i^{(1)}(t)[W^{(10)}(t - \tau_i^{(1)} - T_0) a_i^{(10)}(t - \tau_i^{(1)}) \cos(\varphi_i^{(10)})
\]
\[
+ W^{(10)}(t - \tau_i^{(1)} - T_0) a_i^{(10)}(t - \tau_i^{(1)}) \sin(\varphi_i^{(10)})]
\]
\[
\times W_m(t - \tau_i^{(1)} - T_0) a_Q^{(10)}(t - \tau_i^{(1)} - T_0) dt
\]
\[
= C \sum_{\substack{l=0 \\ l \neq n}}^{L_e-1} \int_0^{T_W} \alpha_i^{(1)}(t)[W^{(10)}(t - \tau_{nl}^{(1)} - T_0) a_i^{(10)}(t - \tau_{nl}^{(1)}) \cos(\varphi_i^{(10)})
\]
\[
+ W^{(10)}(t - \tau_{nl}^{(1)}) a_i^{(10)}(t - \tau_{nl}^{(1)}) \sin(\varphi_i^{(10)})]W_m(t) a_Q^{(10)}(t) dt
\]
\[
= C \sum_{\substack{l=0 \\ l \neq n}}^{L_e-1} \alpha_i^{(1)}[\left(R_{IQ}^{(10)}(\tau_{nl}^{(1)} - T_0) + \hat{R}_{IQ}^{(10)}(\tau_{nl}^{(1)} - T_0)\right) \cos(\varphi_i^{(10)})
\]
\[
+ (\hat{R}_{IQ}^{(10)}(\tau_{nl}^{(1)}) + \hat{R}_{IQ}^{(10)}(\tau_{nl}^{(1)})) \sin(\varphi_i^{(10)})]
\] (A.54)

\[
I_{3IQ}^{(m,n)} = C \int_{\tau_{n1}^{(1)} + T_0}^{\tau_{n1}^{(1)} + T_0 + T_W} \sum_{c=1}^{N^{(1)-1}} \sum_{l=0}^{L_e-1} \alpha_i^{(1)}(t)[W^{(1c)}(t - \tau_i^{(1)} - T_0) a_i^{(1c)}(t - \tau_i^{(1)}) \cos(\varphi_i^{(1c)})
\]
\[
+ W^{(1c)}(t - \tau_i^{(1)} - T_0) a_i^{(1c)}(t - \tau_i^{(1)} - T_0) \sin(\varphi_i^{(1c)})]
\]
\[
\times W_m(t - \tau_i^{(1)} - T_0) a_Q^{(1c)}(t - \tau_i^{(1)} - T_0) dt
\]
\[
= C \sum_{c=1}^{N^{(1)-1}} \sum_{l=0}^{L_e-1} \int_0^{T_W} \alpha_i^{(1)}(t)[W^{(1c)}(t - \tau_{nl}^{(1)} - T_0) a_i^{(1c)}(t - \tau_{nl}^{(1)}) \cos(\varphi_i^{(1c)})
\]
\[
+ W^{(1c)}(t - \tau_{nl}^{(1)}) a_i^{(1c)}(t - \tau_{nl}^{(1)}) \sin(\varphi_i^{(1c)})]W_m(t) a_Q^{(1c)}(t) dt
\]
\[
= C \sum_{c=1}^{N^{(1)-1}} \sum_{l=0}^{L_e-1} \alpha_i^{(1)} \left[(\hat{R}_{IQ}^{(1c)}(\tau_{nl}^{(1)} - T_0) + \hat{R}_{IQ}^{(1c)}(\tau_{nl}^{(1)} - T_0)) \cos(\varphi_i^{(1c)})
\]
\[
+ \hat{R}_{IQ}^{(1c)}(\tau_{nl}^{(1)}) + \hat{R}_{IQ}^{(1c)}(\tau_{nl}^{(1)})) \sin(\varphi_i^{(1c)})\right]
\] (A.55)
\[ I_{IQ}^{(m,n)} = C \sum_{k=1}^{n_m+T_w+T_0} \prod_{j=0}^{N(k)-1} L - 1 \sum_{c=0}^{K} \sum_{l=0}^{K} \sum_{c=0}^{K} \sum_{l=0}^{K} \alpha_i^{(k)}(t)[W^{(k)}(t - \tau_i^{(k)})a_I^{(k)}(t - \tau_i^{(k)}) \cos(\varphi_i^{(k)})
\]
\[ + W^{(k)}(t - \tau_i^{(k)})a_Q^{(k)}(t - \tau_i^{(k)}) - T_0) \sin(\varphi_i^{(k)})][W_m(t) + W_{0}(t) - T_0) \sin(\varphi_i^{(k)})])\]
\[ \times W_m(t - \tau_i^{(1)} - T_0) a_Q^{(10)}(t - \tau_i^{(1)} - T_0) dt \]
\[ = C \sum_{c=1}^{N(k)-1} L - 1 \sum_{l=0}^{K} \int_{0}^{T_w} \alpha_i^{(k)}(t)[W^{(k)}(t - (\tau_i^{(k)} - T_0))a_I^{(k)}(t - (\tau_i^{(k)} - T_0)) \cos(\varphi_i^{(k)})
\]
\[ + W^{(k)}(t - \tau_i^{(k)})a_Q^{(k)}(t - \tau_i^{(k)}) \sin(\varphi_i^{(k)})]W_m(t) + W_{0}(t) - T_0) \sin(\varphi_i^{(k)})])\]
\[ = C \sum_{c=1}^{N(k)-1} L - 1 \sum_{l=0}^{K} \sum_{c=0}^{K} \sum_{l=0}^{K} \alpha_i^{(k)}[(R_{IQ}^{(k)}(\tau_i^{(k)} - T_0) + \hat{R}_{IQ}^{(k)}(\tau_i^{(k)} - T_0)) \cos(\varphi_i^{(k)})
\]
\[ + (R_{QQ}^{(k)}(\tau_i^{(k)} - T_0) + \hat{R}_{QQ}^{(k)}(\tau_i^{(k)} - T_0)) \sin(\varphi_i^{(k)})]] \] (A.56)

The correlator output \( Z_{QQ}^{(m,n)} \) is then:
\[ r_{Q,LP}(t) = (r(t) \sin(w_c t))_{LP} \]
\[ = \frac{r_m(t)}{2} + \sum_{k=1}^{K} \sum_{c=0}^{K} \sum_{l=0}^{K} \sqrt{P} \alpha_i^{(k)}(t) \times \]
\[ [W^{(k)}(t - \tau_i^{(k)})a_I^{(k)}(t - \tau_i^{(k)}) \sin(\varphi_i^{(k)})]) \]
\[ + W^{(k)}(t - T_0 - \tau_i^{(k)})a_Q^{(k)}(t - T_0 - \tau_i^{(k)}) \cos(\varphi_i^{(k)})]. \] (A.57)

The above signal and interference terms can be written as follows:
\[ Z_{QQ}^{(m,n)} = 1 \sqrt{T_w} \int_{r_m^{(1)}}^{T_w} r_{Q,LP}(t) a_Q^{(10)}(t - \tau_n^{(1)} - T_0) W_m(t - \tau_n^{(1)} - T_0) dt \]
\[ = S_{QQ}^{(m,n)} + I_{1QQ}^{(m,n)} + I_{2QQ}^{(m,n)} + I_{3QQ}^{(m,n)} + N_{QQ}^{(m,n)}. \] (A.58)

The correlator output \( Z_{QQ}^{(m,n)} \) is then:
\[ Z_{QQ}^{(m,n)} = 1 \sqrt{E_w} \cos(\varphi_n^{(10)}) \delta_{n,m} \] (A.59)
\[ I_{12Q}^{(m,n)} = C \int_{\tau_n^{(1)}}^{\tau_n^{(1)}+T_W+T_0} \alpha_n^{(1)}(t)(-\sin(\varphi_n^{(10)}))W_n^{(10)}(t - \tau_n^{(1)})a_f^{(10)}(t - \tau_n^{(1)}) \times W_m(t - \tau_n^{(1)} - T_0)a_Q^{(10)}(t - \tau_n^{(1)} - T_0)dt \]
\[ = C \int_{\tau_n^{(1)}}^{T_W} \alpha_n^{(1)}(t)(-\sin(\varphi_n^{(10)}))W_n^{(10)}(t - (-T_0))a_f^{(10)}(t - (-T_0))W_m(t)a_Q^{(10)}(t)dt \]
\[ = C \alpha_n^{(1)}[R_Q^{(10)}(-T_0) + \tilde{R}_Q^{(10)}(-T_0)](-\sin(\varphi_n^{(10)})) \]  \hspace{1cm} (A.60)

\[ I_{2QQ}^{(m,n)} = C \int_{\tau_n^{(1)}}^{\tau_n^{(1)}+T_W+T_0} L_{c-1}^{l=0} \sum_{l\neq n} \alpha_l^{(1)}(t)[W_n^{(10)}(t - \tau_l^{(1)})a_f^{(10)}(t - \tau_l^{(1)})(-\sin(\varphi_l^{(10)})) \]
\[ + W_n^{(10)}(t - \tau_l^{(1)} - T_0)a_Q^{(10)}(t - \tau_l^{(1)} - T_0)\cos(\varphi_l^{(10)})] \times W_m(t - \tau_l^{(1)} - T_0)a_Q^{(10)}(t - \tau_l^{(1)} - T_0)dt \]
\[ = C \sum_{l\neq n} L_{c-1}^{l=0} \int_{0}^{T_W} \alpha_l^{(1)}(t)[W_n^{(10)}(t - \tau_l^{(1)} - T_0)a_f^{(10)}(t - \tau_l^{(1)} - T_0)(-\sin(\varphi_l^{(10)})) \]
\[ + W_n^{(10)}(t - \tau_l^{(1)} - T_0)a_Q^{(10)}(t - \tau_l^{(1)} - T_0)\cos(\varphi_l^{(10)})]W_m(t)a_Q^{(10)}(t)dt \]
\[ = C \sum_{l\neq n} \alpha_l^{(1)}[(R_Q^{(10)}(\tau_l^{(1)} - T_0) + \tilde{R}_Q^{(10)}(\tau_l^{(1)} - T_0))(\tau_l^{(1)})\cos(\varphi_l^{(10)})] \]
\[ + (R_Q^{(10)}(\tau_l^{(1)}) + \tilde{R}_Q^{(10)}(\tau_l^{(1)})\cos(\varphi_l^{(10)})) \]  \hspace{1cm} (A.61)

\[ I_{3Q}^{(m,n)} = C \int_{\tau_n^{(1)}}^{\tau_n^{(1)}+T_W+T_0} L_{c-1}^{N^{(1)}-1} L_{c-1}^{l=0} \sum_{c=1}^{N^{(1)}-1} \sum_{l=0}^{L_{c-1}} \alpha_l^{(1)}(t)[W_n^{(1c)}(t - \tau_l^{(1)})a_f^{(1c)}(t - \tau_l^{(1)})(-\sin(\varphi_l^{(1c)})) \]
\[ + W_n^{(1c)}(t - \tau_l^{(1)} - T_0)a_Q^{(1c)}(t - \tau_l^{(1)} - T_0)\cos(\varphi_l^{(1c)})] \times W_m(t - \tau_l^{(1)} - T_0)a_Q^{(1c)}(t - \tau_l^{(1)} - T_0)dt \]
\[ = C \sum_{c=1}^{N^{(1)}-1} \sum_{l=0}^{L_{c-1}} \int_{0}^{T_W} \alpha_l^{(1)}(t)[W_n^{(1c)}(t - \tau_l^{(1)} - T_0)a_f^{(1c)}(t - \tau_l^{(1)} - T_0)(-\sin(\varphi_l^{(1c)})) \]
\[ + W_n^{(1c)}(t - \tau_l^{(1)} - T_0)a_Q^{(1c)}(t - \tau_l^{(1)} - T_0)\cos(\varphi_l^{(1c)})]W_m(t)a_Q^{(1c)}(t)dt \]
\[ = C \sum_{c=1}^{N^{(1)}-1} \sum_{l=0}^{L_{c-1}} \alpha_l^{(1)}[(R_Q^{(1c)}(\tau_l^{(1)} - T_0) + \tilde{R}_Q^{(1c)}(\tau_l^{(1)} - T_0))(\tau_l^{(1)})\cos(\varphi_l^{(1c)})] \]
\[ + (R_Q^{(1c)}(\tau_l^{(1)}) + \tilde{R}_Q^{(1c)}(\tau_l^{(1)})\cos(\varphi_l^{(1c)})) \]  \hspace{1cm} (A.62)
\[ I_{QQ}^{(m,n)} = C \sum_{k=2}^{K} \sum_{c=0}^{N(k)-1} \sum_{l=0}^{L_{c}-1} \alpha_i^{(k)}(t)[W^{(kc)}(t - \tau_l^{(k)})a_l^{(kc)}(t - \tau_l^{(k)})(-\sin(\varphi_i^{(kc)})) + W^{(kc)}(t - \tau_l^{(k)}) - T_0)a_Q^{(10)}(t - \tau_l^{(k)} - T_0) \cos(\varphi_i^{(kc)})] \times W_m(t - \tau_l^{(k)} - T_0) \alpha_Q^{(10)}(t - \tau_l^{(k)} - T_0)dt \]

\[ = C \sum_{k=2}^{K} \sum_{c=0}^{N(k)-1} \sum_{l=0}^{L_{c}-1} \int_0^{T_W} \alpha_i^{(k)}(t)[W^{(kc)}(t - (\tau_{nl}^{(k)} - T_0))a_l^{(kc)}(t - (\tau_{nl}^{(k)} - T_0))(-\sin(\varphi_i^{(kc)})) + W^{(kc)}(t - \tau_{nl}^{(k)})a_Q^{(10)}(t - \tau_{nl}^{(k)}) \cos(\varphi_i^{(kc)})]W_m(t) \alpha_Q^{(10)}(t)dt \]

\[ = C \sum_{k=2}^{K} \sum_{c=0}^{N(k)-1} \sum_{l=0}^{L_{c}-1} \alpha_i^{(k)}[(R_{I'Q}^{(kc)}(\tau_{nl}^{(k)} - T_0) + \hat{R}_{I'Q}^{(kc)}(\tau_{nl}^{(k)} - T_0))(-\sin(\varphi_i^{(kc)}))] \]

\[ + (R_{I'Q}^{(kc)}(\tau_{nl}^{(k)})) + \hat{R}_{I'Q}^{(kc)}(\tau_{nl}^{(k)}) \cos(\varphi_i^{(kc)})]. \quad (A.63) \]

The correlator output \( Z_{QI}^{(m,n)} \) is:

\[ Z_{QI}^{(m,n)} = \frac{1}{\sqrt{T_W}} \int_{\tau_1^{(1)}}^{T_W} \tau_{QIP}(t)a_Q^{(10)}(t - \tau_1^{(1)})W_m(t - \tau_1^{(1)})dt \]

\[ = S_{QI}^{(m,n)} + I_{1QI}^{(m,n)} + I_{2QI}^{(m,n)} + I_{3QI}^{(m,n)} + I_{QI}^{(m,n)} + N_{QI}^{(m,n)} \quad (A.64) \]

The above signal and interference terms can be written as follows:

\[ S_{QI}^{(m,n)} = -\alpha_n^{(1)} \frac{\sqrt{E_w}}{2} \sin(\varphi_n^{(10)}) \delta_{mi} \quad (A.65) \]

\[ I_{1QI}^{(m,n)} = C \int_{\tau_1^{(1)}}^{T_W} \alpha_n^{(1)}(t) \cos(\varphi_n^{(10)})W^{(10)}(t - \tau_1^{(1)} - T_0)a_Q^{(10)}(t - \tau_1^{(1)} - T_0) \]

\[ \times W_m(t - \tau_1^{(1)})a_Q^{(10)}(t - \tau_1^{(1)})dt \]

\[ = C \int_0^{T_W} \alpha_n^{(1)}(t) \cos(\varphi_n^{(10)})W^{(10)}(t - T_0)a_Q^{(10)}(t - T_0)W_m(t)a_Q^{(10)}(t)dt \]

\[ = C\alpha_n^{(1)}[R_{Q1}^{(10)}(T_0) + \hat{R}_{Q1}^{(10)}(T_0)] \cos(\varphi_n^{(10)}) \quad (A.66) \]
$$I_{2QI}^{(m,n)} = C \int_{\tau_n^{(1)}}^{\tau_n^{(1)}+T_w} \sum_{l=0}^{L_a-1} \sum_{l \neq n} \alpha_l^{(1)}(t)[W^{(10)}(t - \tau_l^{(1)})a_I^{(10)}(t - \tau_l^{(1)})(-\sin(\varphi_l^{(10)}))$$

$$+W^{(10)}(t - \tau_l^{(1)} - T_0)a_Q^{(10)}(t - \tau_l^{(1)} - T_0)\cos(\varphi_l^{(10)})]W_m(t - \tau_l^{(1)})a_I^{(10)}(t - \tau_l^{(1)})dt$$

$$= C \sum_{l=0}^{L_a-1} \int_0^{T_w} \alpha_l^{(1)}(t)[W^{(10)}(t - \tau_l^{(1)})a_I^{(10)}(t - \tau_l^{(1)})(-\sin(\varphi_l^{(10)}))$$

$$+W^{(10)}(t - \tau_l^{(1)} + T_0)a_Q^{(10)}(t - (\tau_l^{(1)} + T_0))\cos(\varphi_l^{(10)})]W_m(t)a_I^{(10)}(t)dt$$

$$= C \sum_{l=0}^{L_a-1} \alpha_l^{(1)}[(R_{II}^{(10)}(\tau_l^{(1)}) + \hat{R}_{II}^{(10)}(\tau_l^{(1)}))(-\sin(\varphi_l^{(10)}))$$

$$+(R_{QI}^{(10)}(\tau_l^{(1)} + T_0) + \hat{R}_{QI}^{(10)}(\tau_l^{(1)} + T_0))\cos(\varphi_l^{(10)}))$$

(A.67)

$$I_{3QI}^{(m,n)} = C \int_{\tau_n^{(1)}}^{\tau_n^{(1)}+T_w} \sum_{c=1}^{N^{(1)}-1} \sum_{l=0}^{L_a-1} \alpha_l^{(1)}(t)[W^{(1c)}(t - \tau_l^{(1)})a_I^{(1c)}(t - \tau_l^{(1)})(-\sin(\varphi_l^{(1c)}))$$

$$+W^{(1c)}(t - \tau_l^{(1)} - T_0)a_Q^{(1c)}(t - \tau_l^{(1)} - T_0)\cos(\varphi_l^{(1c)})]W_m(t - \tau_l^{(1)})a_I^{(10)}(t - \tau_l^{(1)})dt$$

$$= C \sum_{c=1}^{N^{(1)}-1} \sum_{l=0}^{L_a-1} \alpha_l^{(1)}(t) \int_0^{T_w} \alpha_l^{(1)}(t)[W^{(1c)}(t - \tau_l^{(1)})a_I^{(1c)}(t - \tau_l^{(1)})(-\sin(\varphi_l^{(1c)}))$$

$$+W^{(1c)}(t - (\tau_l^{(1)} + T_0))a_Q^{(1c)}(t - (\tau_l^{(1)} + T_0))\cos(\varphi_l^{(1c)})]W_m(t)a_I^{(10)}(t)dt$$

$$= C \sum_{c=1}^{N^{(1)}-1} \sum_{l=0}^{L_a-1} \alpha_l^{(1)}[(R_{II}^{(1c)}(\tau_l^{(1)}) + \hat{R}_{II}^{(1c)}(\tau_l^{(1)}))(-\sin(\varphi_l^{(1c)}))$$

$$+(R_{QI}^{(1c)}(\tau_l^{(1)} + T_0) + \hat{R}_{QI}^{(1c)}(\tau_l^{(1)} + T_0))\cos(\varphi_l^{(1c)}))$$

(A.68)
\[ I_{QI}^{(m,n)} = C \int_{\tau_{n}^{(1)}}^{\tau_{n}^{(1)}+T_W} \sum_{k=2}^{K} \sum_{c=0}^{N_{n}^{(1)}-1} \sum_{l=0}^{L_{n}-1} \alpha_{l}^{(k)}(t)[W^{(k)}(t-\tau_{l}^{(k)})a_{l}^{(k)}(t-\tau_{l}^{(k)})(-\sin(\varphi_{l}^{(k)}))] \\
+ \sum_{k=2}^{K} \sum_{c=0}^{N_{n}^{(1)}-1} \sum_{l=0}^{L_{n}-1} \int_{0}^{T_W} \alpha_{l}^{(k)}(t)[W^{(k)}(t-\tau_{nl}^{(k)})a_{l}^{(k)}(t-\tau_{nl}^{(k)})(-\sin(\varphi_{l}^{(k)}))] \\
+ W^{(k)}(t-(\tau_{nl}^{(k)}+T_0))a_{Q}^{(k)}(t-(\tau_{nl}^{(k)}+T_0)) \cos(\varphi_{l}^{(k)}))]W_{m}(t)a_{I}^{(10)}(t)dt \]

\[ = C \sum_{k=2}^{K} \sum_{c=0}^{N_{n}^{(1)}-1} \sum_{l=0}^{L_{n}-1} \alpha_{l}^{(k)}[(R_{II}^{(k)}(\tau_{nl}^{(k)}) + \dot{R}_{II}^{(k)}(\tau_{nl}^{(k)}))(\sin(\varphi_{l}^{(k)}))] \\
+(R_{QI}^{(k)}(\tau_{nl}^{(k)}+T_0) + \dot{R}_{QI}^{(k)}(\tau_{nl}^{(k)}+T_0)) \cos(\varphi_{l}^{(k)})] \]  \hspace{1cm} (A.69)
APPENDIX E: Derivation of Symbol Error Probabilities of Noncoherent
M-ary Orthogonal Modulation with EGC in Fading Channels

Rayleigh Fading

The probability of symbol error is given by:

\[
P_s = \frac{1}{\Gamma(L)} \sum_{r=1}^{M-1} (-1)^{r+1} \binom{M-1}{r} \sum_{n=0}^{r(L-1)} \beta_{nr} \Gamma(L+n) \frac{1}{(r+1)^{L+n} \Omega L \Gamma(L)} \times \int_0^\infty e^{-\frac{E_W s}{2\sigma^2}} \Phi \left( L + n, L, \frac{E_W s}{2\sigma^2(r+1)} \right) s^{L-1} e^{-\frac{s}{\Omega}} ds. \tag{A.70}\]

The integral above, denoted by \( I \), can be solved by making the change of variable \( y = s \left( \frac{E_W}{2\sigma^2} + \frac{1}{\Omega} \right) \), i.e.:

\[
I = \left[ \frac{2\sigma^2 \Omega}{2\sigma^2 + \Omega E_W} \right]^L \int_0^\infty y^{L-1} e^{-y} \Phi \left( L + n, L, \frac{\Omega E_W}{(2\sigma^2 + \Omega E_W)(r+1)} \right) dy

= \left[ \frac{2\sigma^2 \Omega}{2\sigma^2 + \Omega E_W} \right]^L \Gamma(L) \Phi \left( L, L + n, L, \frac{\Omega E_W}{(2\sigma^2 + \Omega E_W)(r+1)} \right)

= \left[ \frac{2\sigma^2 \Omega}{2\sigma^2 + \Omega E_W} \right]^L \Gamma(L) \left[ 1 - \frac{\Omega E_W}{(2\sigma^2 + \Omega E_W)(r+1)} \right]^{-(L+n)} \tag{A.71}.
\]

where Eq. (A.37) was used in the 2nd line, and Eq. 9.121.1 of [80] was used in the 3rd line. Simplifying then leads to Eq. (3.42).

Nakagami Fading

The probability of symbol error is given by:

\[
P_s = \frac{1}{\Gamma(L)} \sum_{r=1}^{M-1} (-1)^{r+1} \binom{M-1}{r} \sum_{n=0}^{r(L-1)} \beta_{nr} \Gamma(L+n) \frac{1}{(r+1)^{L+n} \Gamma(L) \Gamma(mL)} (\frac{m}{\Omega})^{mL} \times \int_0^\infty e^{-\frac{E_W s}{2\sigma^2}} \Phi \left( L + n, L, \frac{E_W s}{2\sigma^2(r+1)} \right) s^{mL-1} e^{-\frac{s}{\Omega}} ds. \tag{A.72}\]
The integral above, denoted by $I$, can be solved by making the change of variable $y = s^2 \left( \frac{E_W}{2 \sigma^2} + \frac{m}{n} \right)$, i.e.:

$$I = \left[ \frac{2 \sigma^2 \Omega}{m^2 \sigma^2 + \Omega E_W} \right]^{mL} \int_0^\infty y^{mL-1} \exp(-y) \left( L + n, L, \frac{\Omega E_W}{(m^2 \sigma^2 + \Omega E_W)(r + 1)} y \right) dy$$

$$= \left[ \frac{2 \sigma^2 \Omega}{m^2 \sigma^2 + \Omega E_W} \right]^{mL} \Gamma(mL) \Gamma \left( mL + n + L, \frac{\Omega E_W}{(m^2 \sigma^2 + \Omega E_W)(r + 1)} \right) \quad \text{(A.73)}$$

where Eq. (A.37) was used in the 2nd line. Simplifying then leads to Eq. (3.44).

### Rician Fading

The probability of symbol error is given by:

$$P_s = \frac{1}{\Gamma(L)} \sum_{r=1}^{M-1} (-1)^{r+1} \left( M - 1 \right) \sum_{n=0}^{r(L-1)} n \frac{\Gamma(L + n)}{(r + 1)L + n + 1} \frac{1}{2 \sigma_R^2} \frac{1}{\lambda} \left( \frac{\lambda}{2 \sigma_R^2} \right)^{1/2}$$

$$\times \int_0^\infty e^{-\frac{E_W}{2 \sigma^2}} \left( L + n, L, \frac{E_W s}{2 \sigma^2} \right) s^{L-1} \exp(-s/2 \sigma^2) I_{L-1} \left( \frac{\sqrt{\lambda s}}{2 \sigma_R^2} \right) ds \quad \text{(A.74)}$$

The above integral is difficult to solve directly, as we have not found any closed-form solution for it. Instead, we proceed as in [201], by first finding the unconditional pdf $p_{U_m}(u)$. Given that:

$$p_{U_m}(u) = \begin{cases} \frac{1}{2 \sigma^2} \left( \frac{u}{E_W} \right)^{k-1} \exp \left( -\frac{E_W u}{2 \sigma^2} \right) I_{L-1} \left( \frac{\sqrt{E_W u}}{\sigma^2} \right) & \text{if } m = i \\ \left( \frac{1}{2 \sigma^2} \right)^{L(L)} u^{L-1} \exp \left( -\frac{u}{2 \sigma^2} \right) & \text{if } m \neq i \end{cases} \quad \text{(A.75)}$$

we can then determine $p_{U_m}(u)$ using Eq. (3.47):

$$p_{U_m}(u) = \int_0^\infty p_{U_m}(u|s)p_s(s)ds$$

$$= \frac{1}{2 \sigma^2} \left( \frac{u}{E_W} \right)^{k-1} \exp \left( -\frac{E_W u}{2 \sigma^2} \right) \frac{1}{2 \sigma_R^2} \frac{1}{\lambda} \left( \frac{1}{\lambda} \right)^{k-1} \exp \left( -\frac{\lambda u}{2 \sigma_R^2} \right)$$

$$\times \int_0^\infty \left( \frac{1}{s} \right)^{k-1} s^{L-1} \exp \left( -s \left( \frac{E_W u + \frac{1}{2 \sigma^2} \lambda \sigma_R^2} {2 \sigma^2} \right) I_{L-1} \left( \frac{\sqrt{E_W u}}{\sigma^2} \right) \right) I_{L-1} \left( \frac{\sqrt{\lambda s}}{2 \sigma_R^2} \right) ds \quad \text{(A.76)}$$
Making the change of variable \( x = \sqrt{\beta} \):

\[
p_{U_m}(u) = \frac{1}{(2\sigma^2)(2\sigma_R^2)} \left( \frac{u}{\lambda E_W} \right)^{\frac{L-1}{2}} e^{-\left(\frac{u^2}{2\sigma^2} + \frac{\lambda u}{2\sigma_R^2}\right)}
\]

\[
\times 2\int_0^\infty xe^{-x^2 \left( \frac{E_W}{2\sigma^2} + \frac{1}{2\sigma_R^2} \right)} I_{L-1} \left( \frac{\sqrt{E_W u x}}{\sigma^2} \right) I_{L-1} \left( \frac{\sqrt{\lambda x}}{\sigma_R^2} \right) dx. \quad (A.77)
\]

The integral above, denoted by \( I \), can be solved using Eq. 2.4.13 of \([116]\), giving:

\[
I = \frac{1}{2 \left( \frac{E_W}{2\sigma^2} + \frac{1}{2\sigma_R^2} \right)} \exp \left[ \frac{E_W u}{(\sigma^2)^2} + \frac{\lambda}{(\sigma_R^2)^2} \right] I_{L-1} \left( \frac{\sqrt{E_W U x}}{\sigma^2 (1 + \beta)} \right) \left( \frac{\lambda}{\sigma_R^2 (1 + \beta)} \right). \quad (A.78)
\]

Simplifying then leads to Eq. (3.49). The probability of correct decision is obtained as:

\[
P_c = P(U_2 < U_1, U_3 < U_1, \ldots, U_M < U_1) = \int_0^\infty \left[ P(U_2 < u | U_1 = u) \right]^{M-1} p_{U_1}(u) du
\]

\[
= \sum_{r=0}^{M-1} (-1)^r \left( \begin{array}{c} M - 1 \\ r \end{array} \right) \frac{\beta_n^r}{2(\sigma^2)^n} \frac{1}{2\sigma^2(1 + \beta)} \left( \frac{1}{2\sigma^2 \rho_T} \right)^{\frac{L-1}{2}} e^{-\rho_T 2\sigma^2} 
\]

\[
\times \int_0^\infty x^n e^{-\frac{E_W}{2\sigma^2} x} I_{L-1} \left( \frac{\sqrt{E_W \sigma^2 x}}{\sigma^2 (1 + \beta)} \right) dx. \quad (A.79)
\]

The integral above, denoted by \( I \), can be solved by making the change of variable \( z = \sqrt{x} \):

\[
I = \int_0^\infty x^{n+\frac{L-1}{2}} e^{-\frac{E_W}{2\sigma^2} (1+\beta) x} I_{L-1} \left( \frac{\sqrt{E_W \sigma^2 x}}{\sigma^2 (1 + \beta)} \right) dx
\]

\[
= 2\int_0^\infty z^{2n+L} e^{-\frac{E_W}{2\sigma^2 (1+\beta)} z^2} I_{L-1} \left( \frac{\sqrt{E_W \sigma^2 z}}{\sigma^2 (1 + \beta)} \right) dz
\]

\[
= 2 \left[ \frac{\sqrt{E_W \sigma^2}}{2\sigma^2 (1+\beta)} \right]^{L-1} \frac{\Gamma(L+n)}{\Gamma(L)} \Phi \left( L+n, \frac{\rho_T}{(1+\beta)[r(1+\beta)+1]} \right) \quad (A.80)
\]

where Eq. 2.4.3 of \([116]\) was used in the 3rd line. Simplifying, and making use of \( P_s = 1 - P_c \) then leads to Eq. (3.50).
APPENDIX F: MAP Decoding Algorithm

Notation
- \( R \): code rate
- \( N \): length of input word
- \( x^s = (x^s_1, x^s_2, ..., x^s_N) = u = (u_1, u_2, ..., u_N) \): encoder input word
- \( P[\cdot] \): permutation array such that \( u'_k = u_{P[k]}, k = 1 : N \), is the interleaved version of \( u \)
- \( Pinv[\cdot] \): de-permutation array such that \( u_k = u'_{Pinv[k]}, k = 1 : N \), is the de-interleaved version of \( u' \)
- \( s_k \): state of encoder at time \( k \)
- \( S^+ \): set of ordered pairs \((s' = s_{k-1}, s = s_k)\) corresponding to all state transitions \( s' \to s \) caused by input \( u_k = +1 \)
- \( S^- \): set of ordered pairs \((s' = s_{k-1}, s = s_k)\) corresponding to all state transitions \( s' \to s \) caused by input \( u_k = -1 \)
- \( n + 1 \): number of outputs of encoder (including systematic output) (here \( n = 2 \))
- \( x^{p,1} = (x^{p,11}_1, x^{p,11}_2, x^{p,12}_2, x^{p,12}_3, ..., x^{p,11}_N, x^{p,12}_N) \): parity word generated by encoder 1
- \( x^{p,2} = (x^{p,21}_1, x^{p,21}_2, x^{p,22}_2, x^{p,22}_3, ..., x^{p,21}_N, x^{p,22}_N) \): parity word generated by encoder 2
- \( y^{p,1}_k = (y^{p,11}_k, y^{p,11}_k, y^{p,12}_k) \): noisy (faded) version of \( x^{p,1}_k = (x^{p,11}_k, x^{p,11}_k, x^{p,12}_k) \)
- \( y^{p,2}_k = (y^{p,21}_k, y^{p,21}_k, y^{p,22}_k) \): noisy (faded) version of \( x^{p,2}_k = (x^{p,21}_k, x^{p,21}_k, x^{p,22}_k) \)
- \( a_k \): value of (diversity-combined) fading amplitude at time \( k \)

MAP decoding algorithm (c.f. [271])

Initialization

Decoder 1:

\[
\alpha^{(1)}_0(s) = \begin{cases} 
1 & s = 0 \\
0 & s \neq 0
\end{cases}
\]

\[
\beta^{(1)}_N(s) = \begin{cases} 
1 & s = 0 \\
0 & s \neq 0
\end{cases}
\]

\( L_{21}(u_k) = 0, \ k = 1, 2, ..., N \)
Decoder 2:
\[ \alpha_0^{(2)}(s) = \begin{cases} 
1 & s = 0 \\
0 & s \neq 0 
\end{cases} \]
\[ \beta_N^{(2)}(s) = \alpha_N^{(2)}(s) \forall s \]

\textit{j}^{th} \text{ iteration}

Decoder 1:
For \( k = 1 : N \)
\[ \gamma_k(s', s) = \exp \left[ \frac{1}{2} u_k (L_{21}^e(u_{Pinv[k]}) + L_{c,k} y_k^{s,1}) \right] \gamma_k^e(s', s), \forall (s', s) \]
with \( L_{c,k} = 4 E_{a_k} \), \( E_c = RE_b \), \( \gamma_k^e(s', s) = \exp \left[ \frac{1}{2} \sum_{v=1}^{n} L_{c,k} y_k^{p,1v} x_k^{p,1v} \right] \)
\[ \alpha_k^{(1)}(s) = \frac{\sum_{s'} \alpha_{k-1}^{(1)}(s') \gamma_k(s', s)}{\sum_s \sum_{s'} \alpha_{k-1}^{(1)}(s') \gamma_k(s', s)}, \forall s \]
end

For \( k = N : -1 : 2 \)
\[ \beta_{k-1}^{(1)}(s') = \frac{\sum_s \beta_k^{(1)}(s) \gamma_k(s', s)}{\sum_s \sum_{s'} \alpha_{k-1}^{(1)}(s') \gamma_k(s', s)}, \forall s' \]
end

For \( k = 1 : N \)
\[ L_{12}^e(u_k) = \log \left( \frac{\sum_{s'} \alpha_{k-1}^{(1)}(s') \gamma_k(s', s) \beta_k^{(1)}(s')}{\sum_s \sum_{s'} \alpha_{k-1}^{(1)}(s') \gamma_k(s', s) \beta_k^{(1)}(s)} \right) \]
end

Decoder 2:
For \( k = 1 : N \)
\[ \gamma_k(s', s) = \exp \left[ \frac{1}{2} u_k (L_{12}^e(u_{P|k}) + L_{c,k} y_k^{s,2}) \right] \gamma_k^e(s', s), \forall (s', s) \]
with \( \gamma_k^e(s', s) = \exp \left[ \frac{1}{2} \sum_{v=1}^{n} L_{c,k} y_k^{p,2v} x_k^{p,2v} \right] \)
\[ \alpha_k^{(2)}(s) = \frac{\sum_{s'} \alpha_{k-1}^{(2)}(s') \gamma_k(s', s)}{\sum_s \sum_{s'} \alpha_{k-1}^{(2)}(s') \gamma_k(s', s)}, \forall s \]
end

\[
\beta^{(2)}_{k-1}(s') = \frac{\sum_{s'} \beta^{(2)}_k(s) \gamma_k(s', s)}{\sum_{s} \sum_{s'} \alpha^{(2)}_{k-1}(s') \gamma_k(s', s)}, \quad \forall s'
\]
end

\[
\text{For } k = 1 : N
\]
\[
L^{(s)}_{21}(u_k) = \log \left( \frac{\sum_{s} \alpha^{(2)}_{k-1}(s') \gamma_k(s', s) \beta^{(2)}_k(s)}{\sum_{s} \alpha^{(2)}_{k-1}(s') \gamma_k(s', s) \beta^{(2)}_k(s)} \right)
\]
end

After last iteration

\[
\text{For } k = 1 : N
\]
\[
L_1(u_k) = L_{c,k} u_k^{s,1} + L^{(s)}_{21}(u_{Pinv[k]}) + L^{(s)}_{12}(u_k)
\]
if \( L_1(u_k) > 0 \)
Decide \( \hat{u}_k = +1 \)
else
Decide \( \hat{u}_k = -1 \)
end

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REFERENCES


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