### An Indirect Method for Non-Contact Sensing of Robot Joint Angles Using Accelerometers with Automatic In-Situ Calibration

by

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### Abstract

An indirect, self-calibrating, easy to install, and robust joint angle sensing method is presented in this thesis. The approach is based on the use of a pair of accelerometers placed on each link near the joint axis. Two different methods are described for automatic, in-situ, integrated calibration of the accelerometers, which significantly improve joint angle estimation accuracy. The angle sensing method is suitable for harsh environments and applications where traditional contact-type angle sensors cannot be deployed, or problems are associated with their use. It is believed joint angle sensing in heavy-duty hydraulic manipulators is one of the best applications for this method. A Takeuchi TB035 mini-excavator in the Robotics and Control Laboratory of the University of British Columbia is used in this thesis to evaluate the performance of the developed system. This machine is equipped with digital resolvers at each joint. The outputs of the resolvers are compared to the estimated joint angles in various conditions. According to the experimental results presented in this thesis, the achieved accuracy with the accelerometer-based system is  $\pm 1.33\%$ of the full-scale angle ( $\pm 1.6^{\circ}$  in 120°). The performance of the proposed method is also evaluated in position control of the machine and dynamic measuring of its payload. It is shown that the performance of this method is comparable to the performance of the digital resolvers in both tasks.

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### Chapter 1

## Introduction

#### 1.1 Review and Motivation

The design and implementation of a novel accelerometer-based angle sensing system is presented in this thesis. The technique is quite general and can be used for estimating manipulator joint angles. However, it is particularly suited for heavyduty manipulators. For conventional one-axis joints, two biaxial accelerometers are installed on the adjacent links of the joint, near its center of rotation (Figure 1.1). A computer-based system estimates the angles from the accelerometer outputs. To ensure accurate measurement, two calibration procedures have been devised which can easily be conducted on-site.

In this method, joint angle values are obtained without integrating the accelerometer outputs. Instead, the outputs from the two accelerometers are compared in order to resolve the relative angle between their frames. Naturally, similar to a tilt sensor (or inclinometer), gravity plays an important role as it is the main component of the measured acceleration. However, while a tilt sensor functions properly in static conditions only, the proposed method is still applicable in dynamic conditions. This property is achieved by placing the two accelerometers very close to the joint in the range of several centimeters. The constant gravitational acceleration is



Figure 1.1: Configuration of accelerometers for joint angle sensing.

also exploited for calibration of the accelerometers in the proposed method. Hence, it is crucial that the accelerometers used in this method be capable of measuring gravity. Some types of accelerometers are not responsive to the Earth's gravitational field [8][14], and therefore, can not be used here.

The problems associated with the use of traditional contact-type angle sensors on heavy-duty single or multi-link hydraulic manipulators were the original motivation for designing this new angle sensing system. Examples of these machines include loaders, mining shovels, and excavators. The joint angles are needed to implement many computer-assisted control and monitoring features on these machines which facilitate their operation. Traditional contact-type angle sensors such as potentiometers, encoders, or resolvers [6] have a housing which mounts on one link and a shaft that couples to the other link. Therefore, they require some retrofitting to joints before installation (Figure 1.2). Most often, this is costly and burdensome. These sensors also wear out fast due to their moving parts, and break easily by hitting objects that are very common in the working environment of these machines. In



Figure 1.2: Digital resolver installed on a mini-excavator joint.

the proposed method, installation of accelerometers does not require any mechanical or structural modifications to machines. It is as easy as finding two suitable locations on the adjacent links and near enough to the joints to mount the accelerometers. Moreover, by using a new generation of accelerometers, called *microaccelerometers*, superior durability can also be achieved for this system. Microaccelerometers are tiny silicon-based accelerometers with a high shock survivability. They are packaged in rugged enclosures that allows them to be used in hostile environments.

In particular, this work was initiated due to an urgent need to develop an alternative angle sensing method for dynamic payload (weight) monitoring of multilink hydraulic manipulators. The referenced payload monitoring system is proprietary to Motion Metrics Inc. It has been implemented on a mini-excavator (see Section 1.2 for description of this machine) and requires real-time measurement of the boom and stick joint angles. Consequently, the main focus of this work is on angle estimation for the boom and stick joints only. For a similar reason, only angle estimation for one-axis joints is discussed here. However, the methodology presented here can be easily extended to be used for universal joints by replacing the biaxial accelerometers with triaxial ones.

The idea of using multiple accelerometers to estimate joint angles, without integrating their outputs, was originally proposed in [33] to measure the lower and upper extremity angles of the human body. These angles are required in feedback control systems used in electrostimulation of paralyzed muscles for regaining functional movements. In this application, microaccelerometers were investigated for their potential use as non-contact, implantable angle sensors. This solution, however, was found to be highly problematic due to the flexibility of the human body, its fast dynamics, and the fact that its joints are not perfectly two-dimensional. The standard deviation of the measurement error reported in this case was 5.15° [33]. Evidently, angle sensing for hydraulic manipulators is a better application for the proposed method, as none of the foregoing issues is a concern anymore.

#### **1.2** Test Platform and System Setup

The experimental setup used in this thesis is an instrumented Takeuchi mini-excavator model TAB035 [30]. This industrial human-operated mobile hydraulic machine has 4 degrees of freedom. The machine is available in the Robotics and Control laboratory of the Department of Electrical and Computer Engineering at the University of British Columbia.

The structure of the mini-excavator is shown in Figure 1.3. The *bucket* is the movable end-effector of the machine. The upper structure of the machine, the *cab*, can rotate on the carriage using a hydraulic reversible swing motor, operated by a reduction gear. The main links, the *boom* and the *stick*, together with the swing motion, serve to control the position of the bucket. Thus, the major degrees of freedom are as follows: bucket in and out, stick in and out, boom up and down, and cab swing motion.

The actuators used for the backhoe links (boom, stick and bucket) are all



Figure 1.3: The instrumented mini-excavator (from the Motion Metrics website with permission).

single-rod (asymmetric) cylinders with limited linear motions. Since the joints are revolute, use of linear actuators results in joint angle limitations. This limitation is further accentuated for the boom joint angle if the machine is used on a flat surface, as the bucket touches the ground before the boom cylinder is fully retracted. The full ranges of motion for the boom, stick and bucket joints are 131°, 119°, and 183° respectively. The boom, however, can only be moved 79° on a flat surface. The mini-excavator is instrumented with digital resolvers on each joint, fluid pressure sensors and load pins. The pilot stage of the main valves has also been modified to enable control of the machine by computer. A pair of fast on/off solenoid valves are installed in the pilot stage of each actuator [29]. Accelerometers (uniaxial, biaxial, and triaxial) were also added to the sensors in the course of this project.

The outputs of the resolvers were used to evaluate the results of the new angle sensing system. Digital resolvers have a resolution of 0.02°. The Denavit-Hartenberg (DH) joint angles [26] are extracted from their readings. The reference position of the mini-excavator arm used to calibrate the resolvers is achieved by fully extending all the cylinders. In this position, the boom, stick, and bucket joint angles must read 69.18°,  $-154.26^{\circ}$ , and  $-153.80^{\circ}$ , respectively. The DH joint angles extracted from the resolvers are considered as the *actual* angles (benchmark) in this thesis as opposed to the *estimated* angles obtained from the new angle sensing system.

The sensors and the pilot values on the machine are all connected to a VMEbus based computer system which consists of data acquisition boards and a Sun SPARC 1E CPU board running VxWorks<sup>®</sup> real-time operating system. The computer system is networked to the local Ethernet and programs (written in C) are cross-developed in the UNIX environment. This computer setup was used to develop the accelerometer-based angle sensing system and evaluate its performance.

As the first step towards the commercialization of the developed angle sensing technology, it was also implemented on a PC-104 system running MS-DOS. The PC-104 platform is the *de facto* standard for many embedded applications which offers full architecture, hardware, and software compatibility with the PC bus, but in compact  $(3.6" \times 3.8")$  stackable modules suitable for industry [16].

#### **1.3** Thesis Organization

This thesis consists of 6 chapters:

Chapter 1 is the introduction.

In Chapter 2, some background information related to the accelerometerbased angle sensing is presented. The structure of accelerometers is studied in great detail and a primitive method of accelerometer-based angle sensing is discussed to familiarize the reader with the challenges involved.

In Chapter 3, calibration of accelerometers is extensively discussed. Like many other sensors, the parameters of accelerometers, especially silicon-based ones, are affected by environmental factors, and therefore, these devices need regular calibration. In this chapter, the calibration procedures are explored that require no extra hardware and can be carried out without major interruption in the operation of the machine. In Chapter 4, the theory of the angle sensing system is described and supporting experimental results are presented. The system is studied under various conditions and the findings of Chapter 3 are applied to improve the results of the angle estimator.

In Chapter 5, the performance of the proposed angle sensing system is tested in two applications and compared to the performance of the resolvers in the same applications. The first application is payload monitoring, in which the weight of the load in the bucket of the mini-excavator is estimated from the outputs of the angle and pressure sensors. The second application involves the position control of the mini-excavator in Cartesian space. This application uses the joint angle as the feedback variables for closed-loop position control of the corresponding cylinders.

In Chapter 6, the conclusions are outlined with some suggestions for further research.

Specifications for the accelerometers used in this work can be found in the Appendix.

### Chapter 2

## Background

Accelerometers are the main devices used in the proposed angle sensing system. Therefore, it is important to be familiar with their principles of operation and their measurement errors. These issues are discussed in Section 2.1.

As Table 2.1 suggests, a wide range of technologies is used to make accelerometers. From this list, microaccelerometers (also known as *micromachined accelerometers*) have been used in the proposed angle sensing system in this thesis, and are studied in Section 2.2. Microaccelerometers are particularly suited for acceleration sensing with modest accuracy and are widely used in industry for their overall benefits.

In Section 2.3, a primitive method of measuring the boom joint angle of the mini-excavator is discussed, which uses a uniaxial accelerometer as the primary sensor. This method cannot be used for angle sensing due to its poor performance, however, the discussion presented in this section illustrates the basic concept of angle sensing using accelerometers and reveals associated challenges.

8

| Accelerometers      |                   |  |  |
|---------------------|-------------------|--|--|
| Micromachined       | Others            |  |  |
| Capacitive(Surface) | Fiber Optic       |  |  |
| Capacitive(Bulk)    | Piezoelectric     |  |  |
| Electromagnetic     | Servo(Capacitive) |  |  |
| Optical             | Servo(Inductive)  |  |  |
| Piezofilm           | Seismic           |  |  |
| Piezoresistive      | Strain Gage       |  |  |
| Resonant            |                   |  |  |
| Thermal             |                   |  |  |
| Tunneling           |                   |  |  |

Table 2.1: Accelerometer types.

#### 2.1 Acceleration Sensing

Acceleration sensing devices are called accelerometers. Acceleration, the rate at which the velocity of an object is changing, is an abstract concept and cannot be measured directly in the physical world. However, Newton's second law expressed by the equation:

$$\mathbf{f} = M\mathbf{a} \tag{2.1}$$

establishes a direct relation between force,  $\mathbf{f}$ , applied to a mass, M, and its resulting acceleration,  $\mathbf{a}$ . In many accelerometers, this relation is employed directly or indirectly to compute acceleration from force. Force, on the other hand, is usually measured either through relative displacement sensing or through stress sensing of the components of the structure on which it is applied. This structure is usually modeled with a mass-damper-spring system (Figure 2.1). It consists of a *proof* mass (also known as a reference mass or seismic mass) suspended by a spring, K, anchored on a fixed frame, and a damper, B, for effects involving the dynamic movement of the mass. Once external acceleration is applied, it displaces the support frame relative to the proof mass, which in turn changes the internal stress in



Figure 2.1: Electromechanical structure of an accelerometer [13].

the suspension spring. A transduction and interface unit, T, senses the relative displacement or the stress and converts it into a desired signal, which can be a varying electrical voltage, displacement of a moving pointer over a fixed scale, and so forth. This output can even be scaled to display other quantities such as slope or pressure.

In general, there are six degrees of freedom for proof mass motion (3 for translation and 3 for orientation). Typically, however, this is not desirable, since neither knowledge of nor control over the orientation of measured acceleration can be obtained this way. Usually the geometrical structure of the suspension is made arbitrarily dominant in one direction, such that the proof mass can only be responsive to accelerations in that direction. This direction is called the *sensitive axis* of the accelerometer. Along this direction, the dynamic equations governing the mass-damper-spring system can be written as follows:

$$M\ddot{x}_m + B(\dot{x}_m - \dot{x}_a) + K(x_m - x_a) = 0$$
(2.2)

which can be rewritten as:

$$M\ddot{x}_m = Kx + B\dot{x} \tag{2.3}$$

$$x \stackrel{\Delta}{=} x_a - x_m \tag{2.4}$$

where  $x_m$  is the position of the proof mass,  $x_a$  is the position of the accelerometer frame relative to a reference frame, and x is the position of the proof mass relative to the accelerometer frame. Combining Equations 2.3 and 2.4, the following transfer function is obtained:

$$H_a(s) = \frac{x(s)}{a(s)} = \frac{1}{s^2 + \frac{B}{M}s + \frac{K}{M}}$$
(2.5)

where s is the Laplace transform variable and  $a = \ddot{x}_a$  is the external acceleration. After combining with the transfer function of the transduction and interface unit, T(s), the overall transfer function of the sensor can be written as follows:

$$H_e(s) = \frac{e(s)}{x(s)} \frac{x(s)}{a(s)} = T(s) \frac{1}{s^2 + \frac{B}{M}s + \frac{K}{M}}$$
(2.6)

As is evident, the frequency characteristics of the accelerometer and its bandwidth of operation can be changed by modifying the values of M, K and B, as well as the characteristics of T(s). Since the mass-damper-spring system behaves as a lowpass filter, accelerometers made with this structure should be capable of detecting constant accelerations, *e.g.* gravity. In such circumstances, the static acceleration,  $a_{static}$ , results in a fixed displacement,  $x_{static}$ , such that:

$$a_{static} = \frac{Kx_{static}}{M} \tag{2.7}$$

However, in some types of accelerometers such as *piezoelectric accelerometers* [8][14], the characteristics of T(s) totally dampens low-frequency measurement, so these accelerometers cannot measure constant accelerations. As a result, accelerometers are divided into two categories, depending on their ability to measure low-frequency accelerations. In practice, there is a good correlation between the magnitude of an acceleration and its frequency. Constant and low-frequency accelerations can rarely exceed several g's (the Earth's gravitational acceleration =  $9.8m/s^2$ ), while high-frequency shock and vibration type accelerations can reach up to thousands of g's of magnitude. Hence, the terms low-g and high-g which are commonly used to refer to the measurement range of accelerometers, are also an indication of the operational bandwidth of devices.

Very often, the static relation in Equation 2.7 satisfactorily describes the behavior of an accelerometer in the low bandwidth operation range and there is no need to use the dynamic relation of Equation 2.6. When the output signal of the accelerometer is in the form of an electrical voltage, it is even more convenient to modify the static equation one step further to establish a direct relation from the output voltage,  $V_x$ , to the acceleration,  $a_x$ . Index x indicates that acceleration is sensed along the direction of the sensitive axis of the device. This equation can be rewritten in the following form:

$$a_x = \frac{V_x - O_x}{S_x} \tag{2.8}$$

where  $O_x$  is the offset voltage (*i.e.* the output voltage when no acceleration is applied to the device along its sensitive axis), and  $S_x$  is sensitivity. Typically, this relationship is normalized to g, in which case, the units of  $a_x$  and  $S_x$  are g and V/g, respectively. This convention has been adopted throughout this thesis.

Obviously acquiring the exact values of offset and sensitivity parameters is of great importance for accurate acceleration sensing. This is usually done in the factory through complicated procedures using complex apparatus [9][19]. Nevertheless, aging, temperature variations and other environmental factors affect accelerometers and change their parameters. For instance, the offset parameter of an accelerometer used in the extended temperature range (from -40°C to 85°C) can drift up to 10% which results in a 0.2g error in an acceleration reading <sup>1</sup>. Therefore, regular

 $<sup>^{1}</sup>$ All the figures given in this chapter are based on the specifications of the accelerometers used in this work.

calibration is usually needed for the sensors, especially if they are used in harsh environments. The factory-calibrated values of the parameters will be referred to as the *pre-calibrated* values in this work as opposed to *calibrated* values which are obtained from the on-site calibration procedures.

Several other sources, in addition to parameter drift, contribute to measurement error in accelerometers. Nonlinearity is one of them. The spring and damper in the accelerometer model do not necessarily follow linear patterns, and consequently, the actual input-output mapping of the accelerometer is nonlinear. A typical accelerometer has 0.2% nonlinearity at full scale. A higher order model might be sought if it is found that the linear model of Equation 2.8 is inadequate for describing the behavior of the device.

Cross-axis excitation is another source of error. A uniaxial accelerometer presumably responds to acceleration along its sensitive axis only. However, the unidirectional dominance of a suspension cannot be perfectly achieved and accelerometers are slightly excited by accelerations along other directions. *Cross-axis sensitivity* (or in short *cross-sensitivity*) parameters of an accelerometer indicate how much an accelerometer can be excited by accelerations perpendicular to its sensitive axis. Cross-sensitivity parameters of a good accelerometer should not be more than 3% of the main axis sensitivity.

To extend acceleration measurement to multiple axes, the sense elements are duplicated along the desired directions, usually normal to each other. For a biaxial accelerometer, the linear input-output equations can be found by extending Equation 2.8 as follows:

$$V_x - O_x = S_{xx}a_x + S_{xy}a_y (2.9)$$

$$V_y - O_y = S_{yx}a_x + S_{yy}a_y (2.10)$$

where  $S_{ij}$  is (cross-)sensitivity along axis *i* for an acceleration along the *j* direction (see Figure 2.2). The acceleration components on the X and Y axes can be



Figure 2.2: Cross-axis excitation of an accelerometer.

calculated accordingly:

$$a_x = \frac{S_{yy}(V_x - O_x) - S_{xy}(V_y - O_y)}{S_{xx}S_{yy} - S_{xy}S_{yx}}$$
(2.11)

$$a_y = \frac{S_{xx}(V_y - O_y) - S_{yx}(V_x - O_x)}{S_{xx}S_{yy} - S_{xy}S_{yx}}$$
(2.12)

Another measurement error occurs in this case when the sense elements cannot be exactly aligned along the desired directions. Figure 2.3 depicts a case in which there is a deviation of  $\alpha$  in the Y axis of a biaxial accelerometer from its proper direction. This deviation generates errors in readings along the Y axis, as well as in computation of the magnitude and orientation of the total acceleration vector. Since the magnitude of the acceleration vector is important for the accelerometer calibration procedures described in Chapter 3, the error caused due to misalignment is briefly investigated here.

Let us assume the actual acceleration has a magnitude of a and makes an angle  $\theta$  with the X-axis of the accelerometer. In this non-ideal case, the output of each axis would be:

$$a_x = a\cos\theta \tag{2.13}$$

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$$a_{y'} = a\sin(\theta - \alpha) \tag{2.14}$$



Figure 2.3: Analysis of misalignment axes in an accelerometer.

If the magnitude of the total acceleration is computed using  $a_x$  and  $a_{y'}$  without taking misalignment into consideration, the result would be this:

$$a^{\prime 2} = a_x^2 + a_{y^{\prime}}^2 \tag{2.15}$$

However, by eliminating  $\theta$  between Equations 2.13 and 2.14, the true value of  $a^2$  can be obtained from the following relation:

$$a^{2} = \frac{1}{\cos^{2} \alpha} (a_{x}^{2} + a_{y'}^{2} + 2a_{x}a_{y'}\sin\alpha)$$
(2.16)

It can be shown that for a given acceleration, assuming a small misalignment  $\alpha$ , the relative error in the magnitude measurement would approximately be:

$$e \triangleq \frac{|a'-a|}{a} \approx \alpha/2 \tag{2.17}$$

A typical biaxial accelerometer has about  $2^{\circ}$  misalignment in its axes, therefore, there is a relative error of 1.74% in the acceleration magnitude measured by this device associated with the axis misalignment.

One approach to improve the performance of accelerometers is to operate them closed-loop. These types of accelerometers are known as *force-balance*, *servo*, or *null-balance* accelerometers. In force-balance accelerometers, the interface circuitry reads the sensor signal and uses feedback to generate a force to maintain the proof mass in the equilibrium position in a stable manner. The system stability can be ensured by overdamping the accelerometer with a dominant pole. As expected, closed-loop operation improves the overall sensor linearity, eliminates hysteresis effects as compared to mechanical springs, and can be used to control the bandwidth of operation [7]. In some cases, electrical damping can also be provided, which is much less sensitive to temperature variations. Another very important feature of a force-balance accelerometer is the possibility of testing the device performance by introducing electrically excited test forces into the system. This self-checking feature can be quite convenient for diagnostic purposes [8].

#### 2.2 Microaccelerometers

In the past, accelerometers were made in larger sizes as a combination of mechanical and electrical components. These sensors were very accurate and reliable and are still used for very demanding applications. However, they tend to be fragile and expensive, and because of their larger size, they cannot be easily integrated with other mechanical or electrical parts in a system. Because of these issues, applications of accelerometers remained in very specific areas until the late 1980's, when new knowledge and technology for fabricating electromechanical structures (*micromachining*) made the first commercial microaccelerometers available [1][5]. With the introduction of microaccelerometers, acceleration sensing devices found many new applications. Microaccelerometers are now used in biomedical applications for activity monitoring; in the automotive industry to activate safety systems, including airbags and to implement vehicle stability systems, and electronic suspension; in consumer applications, such as active stabilization of the picture in camcorders, head-mounted displays, and virtual reality, three-dimensional mice, and sporting equipment; in military applications for impact and void detection; in inertial navigation and autopilot systems; in industrial vibration test equipment; in detection of earthquakes (*seismography*) and oil exploration; and in microgravity measurements and platform stabilization in space [34].

Operationally, microaccelerometers are not very different from their larger counterparts, and they both operate based on the same concept. For instance, a capacitive accelerometer (see Section 2.2.2) can be fabricated on a chip or is manufactured with electromechanical parts. However, micromachined devices are less expensive and more durable. They are available in small rugged packages which protect them in harsh environments. In most devices, onboard signal conditioning circuitry and self-testing features are integrated on the same chip with the sensor. They are less likely to modify the dynamic response of the mechanical structure to which they are attached. Multiaxial acceleration sensing is much easier to implement with them, since alignment of sense elements is accomplished at the fabrication level, whereas in conventional accelerometers, this involves the challenging task of aligning mechanical parts.

On the negative side, the silicon-based structure of microaccelerometers is more sensitive to temperature variation and has a higher nonlinearity error. Mechanical noise is also an issue in microaccelerometers because of their small proof mass (typically about 5 mg). The primary mechanical noise source in these devices is the Brownian motion of the gas molecules surrounding the proof mass and the Brownian motion of the proof mass suspension or anchors.

There are two major techniques to build micromachined devices: bulk micromachining and surface micromachining. In bulk micromachining, all components of the accelerometer are made from bulk silicon by etching machining material out of the wafer itself. In surface micromachining, layers of material are built on top of a silicon wafer and then selectively etched away to make the sensor structure. High resolution and good signal to noise ratio can be obtained using bulk micromachined devices, since a large proof mass can be implemented on the wafer using this technique, however, they are usually more sensitive to temperature variations. The surface approach, on the other hand, offers lower temperature sensitivity and better versatility in design. This technique allows one to develop accelerometers with the integrated interface circuitry and force-balance feedback loop to further improve the performance of the device. The disadvantage with surface micromachining is a lower resolution limit and higher mechanical noise, because the same size proof mass cannot be achieved through this process compared to bulk micromachining. This was not of major concern for low accuracy airbag crash sensors, one of the first applications of microaccelerometers, in which lower cost and higher reliability were more important than performance [11].

Three types of the microaccelerometers listed in Table 2.1 have proven more successful commercially. These are piezoelectric, piezoresistive, and capacitive devices. Piezoelectric devices cannot measure constant acceleration (*e.g.* gravity) for more than a few seconds due to their leakage problem, and therefore, they cannot be used for low bandwidth applications. The two others, however, are capable of measuring constant accelerations and are discussed in more detail below.

#### 2.2.1 **Piezoresistive Accelerometers**

In these accelerometers, piezoresistors (*i.e.* strain gages) are placed on the suspension beams (Figure 2.4). As the support frame moves relative to the proof mass, the suspension beams will be stretched or compressed, which changes their stress profile and hence the resistance of their embedded piezoresistor [34]. A Wheatstone bridge is used to measure the change in resistance.

The first micromachined accelerometer [21], and one of the first commercialized microaccelerometers were piezoresistive [5]. Most of the piezoresistive accelerometers are manufactured using bulk micromachining techniques. The main



Figure 2.4: Structure of a piezoresistive accelerometer.

advantage of piezoresistive accelerometers is the simplicity of their readout circuitry, since the resistive bridge generates a voltage with low output-impedance. However, because of the development of surface micromachined devices, this feature is not as appealing as it used to be. Piezoresistive accelerometers also have higher temperature sensitivity, and lower overall sensitivity, as compared to capacitive devices [34].

#### 2.2.2 Capacitive Accelerometers

Some of the most commercially successful micromachined accelerometers, including the ones used in the proposed angle sensing system in this thesis, are surface micromachined, closed-loop capacitive devices [2][27][28]. Capacitive accelerometers operate based on the variation of a capacitance (Figure 2.5). In the presence of external acceleration, the support frame of the accelerometer moves from its rest position, thus changing the capacitance between the proof mass and fixed conductive electrode separated from it by a narrow gap. This capacitance can be measured using electronic circuitry.

Early capacitive micromachined accelerometers [24] utilized bulk micromachining, but a new generation of surface micromachined capacitive accelerometers outperformed the early devices and soon took over. Silicon capacitive accelerometers have several advantages that make them very attractive for numerous applications, ranging from low-cost, large-volume automotive safety systems to high-precision navigational ones. They have high sensitivity, good D.C. response and noise perfor-



Figure 2.5: Structure of a capacitive accelerometer.

mance, low drift, low temperature sensitivity, and low power dissipation [34].

#### 2.3 Joint Angle Sensing Using Uniaxial Accelerometers

A primitive method for joint angle sensing can be devised by using uniaxial accelerometers. In this method, one uniaxial accelerometer is placed on each adjacent link of the joint. In static conditions, the output voltage of each device is a function of the absolute angle between its sensitive axis and gravity. By determining this angle for the two adjacent links, the corresponding joint angle can be computed.

This method was experimentally tested on the mini-excavator to measure the boom joint angle. The boom and the cab are the adjacent links for this joint. If it is assumed that the machine is on a flat surface, the angle that the cab makes with gravity is known. Therefore, one accelerometer only is enough to estimate the boom joint angle. This configuration is shown schematically in Figure 2.6. The boom angle,  $\theta$ , can be estimated from the measured acceleration using the following relation:

$$\theta = \sin^{-1}(-a_x) + \theta_0 \tag{2.18}$$

where  $\theta_0$  is an angle offset to adjust the estimated angle to any desired reference angle. The acceleration  $a_x$  is obtained from Equation 2.8.

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5 N. BOARD



Figure 2.6: Configuration of the uniaxial accelerometer for boom joint angle sensing.

The experimental results of angle estimation are shown in Figure 2.7. For the experiment shown in this figure, a typical profile of the link motion was applied to the machine arm, including one touch-down to the ground at t = 4s. The accelerometer output voltage is plotted in Figure 2.7A, in which the effect of shock at t = 4s is visible. The estimated and actual boom angles are plotted together in Figure 2.7B and the corresponding error is shown in Figure 2.7C. According to the last plot, the range of estimation error is approximately 10°, which is fairly large and unacceptable for most applications. Two major sources contribute to the observed error. They are discussed next.

#### 2.3.1 Modeling Errors

The effect of the accelerometer modeling errors in angle estimation can be better understood through an example. Assume that the output voltage of the accelerometer in one position is 2.488V. By using the pre-calibrated values of the offset and sensitivity parameters of the sensor ( $O_x = 2.488V$  and  $S_x = 0.51V/g$ ), and presum-



Figure 2.7: Boom joint angle estimation using a uniaxial accelerometer.

ing  $\theta_0 = 0^\circ$ , the boom joint angle is estimated according to Equations 2.8 and 2.18 to be 0°. However, the actual offset parameter value can be different from the precalibrated value at the time of this observation. If the offset parameter value is actually 2.600V, then the correct estimation would be 12.69°. This error becomes even more serious if the nonlinear characteristics of the device are taken into account. To improve the angle estimation results, the modeling parameters must be adjusted by calibration. Unfortunately, there is no easy and feasible way available to calibrate uniaxial accelerometers at the location of use.



Figure 2.8: Error in tilt sensing due to motion acceleration.

#### 2.3.2 Dynamic Effects

Dynamic effects observed in the angle estimation results can be divided into two types: (i) link motion-related (or inertia) acceleration and cab shaking, and (ii) external shock and vibration. Both types contribute to estimation error, but in a different frequency spectrum. Shock and vibration effects appear in a higher frequency range while the motion-related effects appear mainly in the low frequency range.

With a uniaxial accelerometer, it is almost impossible to distinguish between the motion-related and gravitational accelerations since they both have similar frequency contents. As soon as the links start moving or when the cab is shaking, an acceleration component due to the motion is generated, which in turn causes the accelerometer to sense an apparent tilt (Figure 2.8). Therefore, there is no way to eliminate the effect of link motion and cab shaking in angle sensing using a uniaxial accelerometer. These effects can only be partially reduced on the boom by placing the accelerometer as close as possible to the joint axis, because the effects of the angular velocity and angular acceleration sensed by the accelerometer are proportional to its distance from the joint axis. Therefore, by minimizing this distance, the dynamic effects are reduced. For the stick joint, even this solution would not be very helpful. In this case, the accelerometer on the stick would always be affected by the movements of the stick as well as those of the boom. While the dynamic effects of stick movements can be partially reduced by placing the sensor close to



Figure 2.9: Effects of starting the machine engine on the accelerometer output ( the engine has been started at t = 6.7s).

this joint, there is no way to eliminate the effects of the boom movements.

Shock impulses appear on the accelerometer output when the bucket touches the ground. Vibration is induced from the engine movements (Figure 2.9). By means of a low-pass filter, these effects can be drastically reduced. In fact, for the plots of Figure 2.7, such a filter has already been applied to the accelerometer output voltage before its incorporation into Equation 2.8; otherwise the results of the angle estimation would be even worse. Using this filter, however, introduces an undesirable delay in angle estimation, which is visible in Figure 2.7B<sup>2</sup>.

Overall, it can be seen that uniaxial accelerometers cannot be efficiently used for angle estimation, as there exist two challenges in accelerometer-based angle sensing: calibration problems and inertia effects. The rest of this thesis attempts to find alternative solutions.

<sup>&</sup>lt;sup>2</sup>To plot the estimation error, the estimated angle was actually first shifted in time properly.

### Chapter 3

## **Calibration of Accelerometers**

In this chapter, we study in-situ calibration procedures for accelerometers that require no extra hardware. Two major procedures are discussed here: *static calibration*, and dynamic *cross-calibration*. Static calibration is a general self-calibrating procedure for low-g accelerometers in which each sensor is individually calibrated. This method of calibration has been devised based on the work in [17], [19], and [31], with some modifications, and it is studied in detail in Section 3.1. Cross-calibration, on the other hand, is a novel approach for simultaneous calibration of a pair of accelerometers, which is discussed in Section 3.2. It benefits from the configuration of the accelerometers used for angle sensing to calibrate them (Figure 1.1). As this chapter is concerned with modeling of accelerometers, the possibility of using a second order model for accelerometers is also investigated in Section 3.3. If successfully calibrated, this model can characterize the nonlinear behavior of accelerometers, and therefore, generate more accurate readings.

It should be stressed that the contents of this chapter are very much linked to Chapter 4 where the results of the calibration procedures are actually applied for angle estimation. The final judgment on the performance of a calibration procedure can only be made when it is actually tested for angle estimation.

For most of the discussion and experimental results in this chapter, a pair of

Summit Instruments 23203A low-g biaxial micromachined capacitive accelerometers are used. As shown in Figure 1.1, these accelerometers are mounted on the boom and stick links near the stick joint for angle sensing. The input-output equations of these accelerometers were given in Equations 2.9 and 2.10 in the previous chapter. The accelerometers will be referred to according to their link names in this chapter.

#### **3.1** Static Calibration

Traditionally, low-g accelerometers are calibrated by placing them in different orientations in the gravitational field and solving the input-output equations for the unknown parameters of offset and sensitivity [10]. This method requires explicit knowledge of the orientation of the sensitive axes, meaning another angle sensing system, therefore, it cannot be used here.

Nevertheless, gravity can still be employed as a measure to calibrate accelerometers. [17] and [23] describe such a procedure for triaxial accelerometers. If calibrated, a triaxial accelerometer must fully capture gravity in static conditions. Otherwise, the device can be calibrated by collecting samples from its output voltages in various static poses, and adjusting its model parameters to minimize a cost function that compares gravity with the magnitudes of acceleration samples in those static poses. The described method can also be extended to biaxial accelerometers under one condition. It must be ensured that the X-Y plane of the biaxial sensor is normal to the ground. Only in such a case, can biaxial accelerometers be fully exposed to gravity. By considering the structure of the mini-excavator, this requirement can almost be achieved if one, the accelerometers are mounted on the vertical sides of links such that their X-Y planes are parallel with the side, and two, the machine is standing on a flat surface during calibration. For now, it is assumed that these conditions are fully satisfied. In Section 3.1.5, it is explored what happens if these conditions are not met.

In the implementation of this procedure on the mini-excavator, the user


Figure 3.1: Block diagram of the static calibration [17].

must first issue the calibration command. Calibration might be required if the sensors have just been installed or if the user personally judges that the devices need calibration. After the calibration command is issued, the user is asked to position the arm in a number of different and arbitrary poses and keep the links in those poses for a few seconds. A *static moment detector* automatically detects each new static pose and stores a few samples from the output voltages of the accelerometers in that pose. Once a sufficient number of samples is collected, another module, the *parameter estimator*, is invoked, which identifies the new values of parameters by processing the collected samples. The block diagram of the static calibration procedure is shown schematically in Figure 3.1.

To collect a rich set of samples, it would be ideal if the accelerometers could be placed in a set of poses that cover a full 360° range of rotation. However, the sensors are attached to the body of the machine and have only a limited range of motion depending on their location. For instance, the accelerometer mounted on the cab for boom joint angle estimation cannot be calibrated using this method due to lack of sufficient motion relative to the gravitational field. This accelerometer can only be used with its pre-calibrated values. Similarly (as will be seen later), the calibration procedure fails relatively more often for those accelerometers mounted on the boom, than for those mounted on the stick, as the former link has a smaller range of motion.

# 3.1.1 Static Moment Detector

Almost any movement of the links causes acceleration and affects the outputs of the accelerometers. These variations of the output voltages of the accelerometers can be used to detect static moments. In [31], a (quasi)static state detecting algorithm has been proposed, which magnifies these variations using a high-pass filter, and then detects the static moments by comparing the filtered signal with a threshold. This algorithm can be summarized in the following steps:

- 1. Collect the output voltage of each axis,  $V_x$  and  $V_y$ .
- 2. Define  $V_d = \sqrt{V_{x.}^2 + V_y^2}$ .
- 3. Pass  $V_d$  through a High-Pass Filter (HPF) to amplify the changes in the signal.
- 4. Apply a rectifier (absolute value function) to the high-pass filtered signal,  $V_h$ , to create an effective value,  $V_{rh}$ .
- 5. Pass  $V_{rh}$  through a Low-Pass Filter(LPF) to smooth the signal.
- 6. Compare the low-pass filtered signal,  $V_{lrh}$ , with a threshold voltage,  $V_t$ , to decide whether a static moment has occured or not.

Figure 3.2 shows the flow chart of the above algorithm with some additional features. To design the high-pass and low-pass filters needed in steps 3 and 5, the approach presented in [29] was adopted. For high-pass filtering, a first order low-pass filtered differentiator with the following transfer function in the *s*-domain was used:

$$H_h(s) = \frac{s}{1 + T_h s} \tag{3.1}$$



Figure 3.2: Flow chart of the static moment detector.

where  $T_h > 0$  is the time constant of the filter. The following bilinear transformation was used to discretize this filter where  $T_s$  is the sampling time:

$$s = \frac{2}{T_s} \frac{1 - z^{-1}}{1 + z^{-1}} \tag{3.2}$$

Hence, the discrete approximation is:

$$H_h(z) = \frac{\alpha(1 - z^{-1})}{1 - \beta z^{-1}}$$
(3.3)

with the coefficients  $\alpha$  and  $\beta$  defined below:

$$\begin{cases} \alpha = 2/(2T_h + T_s) \\ \beta = (2T_h - T_s)/(2T_h + T_s) \end{cases}$$
(3.4)

For low-pass filtering, a first order filter with the following transfer function was used:

$$H_l(s) = \frac{1}{1 + T_l s}$$
(3.5)

where  $T_l > 0$  is the time constant of the filter. By using the bilinear transformation of Equation 3.2, the discretized form of this filter can be expressed in this way:

$$H_l(z) = \frac{\gamma(1+z^{-1})}{1-\lambda z^{-1}}$$
(3.6)

with the coefficients  $\gamma$  and  $\lambda$  defined as:

$$\begin{cases} \gamma = T_s / (2T_l + T_s) \\ \lambda = (2T_l - T_s) / (2T_l + T_s) \end{cases}$$
(3.7)

The time constants of both filters were experimentally set to  $20T_s$  while the threshold voltage,  $V_t$ , was set to 150mV. The threshold voltage can actually vary from one sensor to another as it depends on the sensor parameters as well as the parameters of the filters. Here, the same  $V_t$  was found to be usable for all devices, since the same type of accelerometers with similar sensitivity and offset parameters was used. For other cases, if the sensitivity parameters of each accelerometer are of the same order, an acceleration threshold,  $a_t$ , can be found using the following relation:

$$a_t = \frac{V_t}{\sqrt{S_{xx}^2 + S_{yy}^2}}$$
(3.8)

which is virtually independent of sensitivity parameters [23]. The value of  $V_t$  for a new sensor can then be computed from Equation 3.8 by solving it for  $V_t$  using the new sensor sensitivity parameters.

To make sure that transient behavior is not declared as a static pose, the algorithm requires  $V_{trh}$  to remain below  $V_t$  for a sufficient period of time. Considering the mini-excavator dynamics, it was experimentally found that a time period of 2 to 3 seconds is enough for this purpose. After this time elapses, the accelerometer output voltage samples are stored in a buffer for parameter estimation. To avoid collecting redundant data from the same pose, the mean value of each set of samples in each collected pose is also stored. Before storing the new set of samples, the mean of the new set is compared with those of already stored sets. If the mean of the new set differs from the means of others above a certain threshold, the new samples are stored in the buffer, otherwise they are discarded. The threshold for this comparison again depends on the sensitivity parameters of the accelerometer. For the accelerometers used in this work, this threshold was set to 10mV. In each pose, 5 samples from 10 different poses, the parameter estimator routine is called.

The static moment detection can be carried out more efficiently by adding a few features to the algorithm: one can realize that not all the accelerometers require monitoring for static moment detection. If there are two accelerometers on the same link, monitoring one is enough to determine whether the link is moving or not. Moreover, if it is found that a link closer to the base of the machine is moving, it can be concluded that the distal links are also moving. This way, the accelerometers closer to the base of the machine have higher precedence in declaring a static moment. A static moment detector can also be used to automatically trigger the calibration procedure by examining the acceleration magnitudes at static moments and issuing the calibration command whenever the acceleration magnitudes are significantly different from 1g at those moments.

Figure 3.3 shows the outputs of the various blocks in the static moment detector. Samples for this experiment were collected from the boom accelerometer during a typical calibration procedure in which the links were moved gradually to various static poses. It can be seen from the figure that once the boom is stationary, the output of the low-pass filter,  $V_{lrh}$ , goes below the threshold voltage and, therefore, the static moments are safely declared.

#### **3.1.2** Parameter Estimator

The parameters of the accelerometers are estimated based on an iterative least squares scheme. For this purpose, the squared magnitude of the acceleration vector,  $H(\mathbf{v}, \mathbf{p})$ , sensed at collected static moments, is computed. For a biaxial accelerometer, the following equation must be satisfied for the collected samples after calibration:

$$H(\mathbf{v}, \mathbf{p}) = 1 \ [g^2] = a_x^2 + a_y^2 \tag{3.9}$$

where  $\mathbf{v} = (V_x, V_y)^T$  is the output voltage vector, and  $\mathbf{p}$  is an arbitrary parameter vector to be estimated.  $a_x$  and  $a_y$  are obtained from Equations 2.9 and 2.10. Ac-



Figure 3.3: Signals in various parts of the static moment detector during calibration of the boom accelerometer.

cording to these equations, each biaxial accelerometer has 6 parameters,  $O_x$ ,  $O_y$ ,  $S_{xx}$ ,  $S_{xy}$ ,  $S_{yx}$ ,  $S_{yy}$ . As indicated in Chapter 1, the cross-sensitivity parameters,  $S_{xy}$  and  $S_{yx}$ , are too small to have a significant effect on computations. Among the other four parameters, the offset parameters,  $O_x$ ,  $O_y$ , are more sensitive to temperature and consequently are more likely to vary in the working environment. Therefore, either all four parameters (estimation in 4-parameter mode) or only the offset parameters (estimation in 2-parameter mode) need to be considered for calibration. In the first case,  $\mathbf{p} = (O_x, O_y, S_{xx}, S_{yy})^T$  and in the latter case,  $\mathbf{p} = (O_x, O_y)$ .

Since  $H(\mathbf{v}, \mathbf{p})$  is not linearly expressed in terms of the sensor parameters, linear least squares cannot be utilized directly to estimate the parameter vector  $\mathbf{p}$ . Instead  $H(\mathbf{v}, \mathbf{p})$  can be first linearized around the pre-calibrated values of the parameters. Then least squares is applied to identify the *deviation* of parameters from their true values. This procedure is iteratively run by using the new values of the parameters for linearization in the succeeding steps until the parameters converge to their final values within a certain threshold.

The parameter identification equations for the 4-parameter mode can be obtained by deriving the Taylor expansion of  $H(\mathbf{v}, \mathbf{p})$  around  $\mathbf{p_0}$ , the initial parameter vector (pre-calibrated values at the first step)<sup>1</sup>:

$$H(\mathbf{v}, \mathbf{p}) = 1 \left[g^2\right] = H(\mathbf{v}, \mathbf{p_0}) + \frac{\partial H(\mathbf{v}, \mathbf{p})}{\partial \mathbf{p}} \bigg|_{\mathbf{p}=\mathbf{p_0}} (\mathbf{p} - \mathbf{p_0}) + h.o.t.$$
(3.10)

where *h.o.t.* stands for higher order terms. Assuming,

$$y \triangleq H(\mathbf{v}, \mathbf{p}) - H(\mathbf{v}, \mathbf{p_0}) = 1 [g^2] - H(\mathbf{v}, \mathbf{p_0})$$
(3.11)

$$\varphi \triangleq \frac{\partial H(\mathbf{v}, \mathbf{p})}{\partial \mathbf{p}} \bigg|_{\mathbf{p} = \mathbf{p}_0} = \begin{bmatrix} \varphi_1 & \varphi_2 & \varphi_3 & \varphi_4 \end{bmatrix}$$
(3.12)

$$\boldsymbol{\xi} \triangleq \mathbf{p} - \mathbf{p}_{\mathbf{0}} = \begin{bmatrix} \Delta O_x & \Delta O_y & \Delta S_{xx} & \Delta S_{yy} \end{bmatrix}^T$$
(3.13)

where,

$$\varphi_1 = \frac{\partial H}{\partial O_x} = \frac{2(S_{yx}a_y - S_{yy}a_x)}{S_{xx}S_{yy} - S_{xy}S_{yx}}$$
(3.14)

$$\varphi_2 = \frac{\partial H}{\partial O_y} = \frac{2(S_{xy}a_x - S_{xx}a_y)}{S_{xx}S_{yy} - S_{xy}S_{yx}}$$
(3.15)

<sup>&</sup>lt;sup>1</sup>Since the 2-parameter mode is a subset of the 4-parameter mode, the equations are derived for the 4-parameter mode only.

$$\varphi_3 = \frac{\partial H}{\partial S_{xx}} = \frac{2(S_{yx}a_xa_y - S_{yy}a_x^2)}{S_{xx}S_{yy} - S_{xy}S_{yx}}$$
(3.16)

$$\varphi_4 = \frac{\partial H}{\partial S_{yy}} = \frac{2(S_{xy}a_xa_y - S_{xx}a_y^2)}{S_{xx}S_{yy} - S_{xy}S_{yx}}$$
(3.17)

and ignoring the higher order terms, Equation 3.10 can be expressed in the following regression model [4]:

$$y = \varphi \boldsymbol{\xi} \tag{3.18}$$

By extending this scalar equation to N > 4 measurements, the following vectorial equation is obtained:

$$\mathbf{y} = \boldsymbol{\Phi}\boldsymbol{\xi} \tag{3.19}$$

where

$$\mathbf{y} = \left[ \begin{array}{ccc} y(t_1) & y(t_2) & \dots & y(t_N) \end{array} \right]^T$$

$$\boldsymbol{\Phi} = \left[ \begin{array}{ccc} \boldsymbol{\varphi}(t_1) & \boldsymbol{\varphi}(t_2) & \dots & \boldsymbol{\varphi}(t_N) \end{array} \right]^T$$

Finally, using the least squares estimator, the optimal  $\boldsymbol{\xi}$  is obtained from the following equation:

$$\widehat{\boldsymbol{\xi}} = (\boldsymbol{\Phi}^T \boldsymbol{\Phi})^{-1} \boldsymbol{\Phi}^T \mathbf{y}$$
(3.20)

and the parameter vector is updated as follows:



Figure 3.4: Geometrical interpretation of the parameter estimation task.

$$\widehat{\mathbf{p}} = \mathbf{p}_0 + \widehat{\boldsymbol{\xi}} \tag{3.21}$$

The new parameter vector replaces  $\mathbf{p}_0$  in the next iteration.

The task of the parameter estimator may be better understood by studying the geometrical interpretation given in Figure 3.4 for a set of simulated data. Before calibration, the location of the static acceleration points in the  $(a_x, a_y)$  coordinates, marked with 'x' in the figure, does not lie exactly on the unit circle. After calibration, the estimator adjusts the sensor parameters such that the new acceleration points are placed much closer to the unit circle.

# 3.1.3 Analysis of the Parameter Estimator Using Simulated Data

To ensure that the estimated parameters satisfactorily converge to their true values, the behavior of the parameter estimator is analyzed by feeding it with simulated voltages. This analysis is particularly important since in each iteration, the estimator is subject to truncation and linearization. In addition, the behavior of the estimator in the presence of noise can be studied this way. Towards this end, a voltage simulator was implemented in Matlab<sup>®</sup>. The inputs to this simulator were the orientation angle of the sensor with respect to gravity and the presumed nominal sensor parameters. Using the orientation angle  $(\theta, \text{ the angle that the } X \text{ axis of the device makes with gravity})$ , the simulator first determines the accelerations that should be sensed on each axis according to the following:

$$a_x = \cos\theta \tag{3.22}$$

$$a_y = \sin\theta \tag{3.23}$$

Then the simulator generates the voltages that should be observed at the output of the accelerometers for the given orientation angle and parameters:

$$V_x = S'_{xx}a_x + S'_{xy}a_y + O'_x \tag{3.24}$$

$$V_y = S'_{yx}a_x + S'_{yy}a_y + O'_y (3.25)$$

where  $S'_{ij}$  and  $O'_i$  are the nominal values of the parameters that should be later determined by the estimator. After generating several simulated voltages for different orientation angles, the parameter estimator is applied to the simulated voltages to find the nominal parameters.

In the first simulation, ten sets of voltages were generated by changing the orientation angle from  $\theta = 5^{\circ}$  to  $55^{\circ}$  in the 5° steps. To generate these sets of voltages, the pre-calibrated values of the offset parameters of one of the accelerometers were perturbed by 10% from their original values, and used as the nominal values of  $O'_x$  and  $O'_y$  in Equations 3.24 and 3.25. The sensitivity parameters were left intact. The range of  $\theta$  for simulation is arbitrarily chosen to be less than the minimum of the full range of motion for the boom and stick joints, in order to study the performance of the parameter estimator in one of the worst case scenarios. The parameter

| Parameter | Pre-Calibrated<br>Value | Nominal<br>Value | 2-Parameter Mode<br>Estimate | 4-Parameter Mode<br>Estimate |
|-----------|-------------------------|------------------|------------------------------|------------------------------|
| $O_x$     | 2.2012                  | 2.4113           | 2.4113                       | 2.4113                       |
| $O_y$     | 2.5462                  | 2.2916           | 2.2916                       | 2.2916                       |
| $S_{xx}$  | -1.28464                | -1.28464         | Not Applicable               | -1.28464                     |
| $S_{yy}$  | -1.25536                | -1.25536         | Not Applicable               | -1.25536                     |

Table 3.1: Analysis of the parameter estimator using simulation and in the absence of noise.

| Parameter | Pre-Calibrated<br>Value | Nominal<br>Value | 2-Parameter Mode<br>Estimate | 4-Parameter Mode<br>Estimate |
|-----------|-------------------------|------------------|------------------------------|------------------------------|
| $O_x$     | 2.2012                  | 2.4113           | 2.41121                      | 2.49746                      |
| $O_y$     | 2.5462                  | 2.2916           | 2.29155                      | 2.30359                      |
| $S_{xx}$  | -1.28464                | -1.28464         | Not Applicable               | -1.36954                     |
| $S_{yy}$  | -1.25536                | -1.25536         | Not Applicable               | -1.27364                     |

Table 3.2: Analysis of the performance of the parameter estimator using simulation and in the presence of noise.

estimator was able to restore exactly the nominal values in both the 2-parameter and the 4-parameter modes after 4 iterations (Table 3.1).

In the second simulation, the same set of nominal parameters were used but in the presence of white noise with standard deviation of 3mV (similar to the actual standard deviation of the sensor output voltages) added to the generated voltages. In the 2-parameter mode, the estimator successfully restored the nominal values of the parameters with an accuracy of  $\pm 0.01\%$  while in the 4-parameter mode, the accuracy was reduced to  $\pm 10\%$ , a significant error for the estimator(Table 3.2).

In another observation, the behavior of the parameter estimator was examined for various different initial parameter vectors,  $P_0$ 's. It was found that if the initial values were selected with 50% perturbation from the nominal values, the estimator diverged in the 4-parameter mode, whereas in the 2-parameter mode, the estimator converged even with the null initial values ( $O_x = O_y = 0V$ ). As a result, it was concluded that for robust, accurate, and unbiased estimation, only the offset parameters should be adjusted. The poor performance of the 4-parameter mode can be attributed to the lack of enough excitation on the mini-excavator arm and the larger number of possible solutions in this mode. In fact, the estimates obtained in the 4-parameter mode in all above simulations could always perfectly map the acceleration points to the unit circle. However, since the solutions for these mappings are not unique, chances are that the parameter estimator converges to another set of values rather than to the nominal values. If samples from the full 360° range of motion were available, the likelihood of this error would be substantially less. However, since the range of motion is limited, divergence from the nominal values is expected, especially in the 4-parameter mode, as the possible number of solutions is overwhelmingly larger in this case. After an incorrect mapping, the  $a_x$  and  $a_y$  represent incorrect values, and thereby any angle estimation based on these values would be inaccurate too.

# 3.1.4 On-Site Calibration

To calibrate the stick accelerometer, the stick cylinder was fully retracted and extended in 10 steps. The outputs of the stick DH angle at those steps were -154.65, -149.04, -141.70, -130.45, -117.05, -100.46, -82.00, -66.97, -54.14, -35.27°, respectively. The estimator was run in the 2-parameter mode for reasons explained in Section 3.1.3, and converged within 5 significant digits after 3 iterations. Table 3.3 shows the adjusted (estimated) values of the parameters.

For cross-validation of parameter estimation, the links were positioned in four other static poses and the sensed accelerations were derived from the output voltages employing the pre-calibrated and calibrated values of the parameters. Table 3.4 shows the results and indicates an improvement in acceleration sensing after calibration as the magnitudes are pushed closer to 1g with the adjusted parameters.

Although the estimator enhances the accelerometer accuracy in static conditions, improvement in angle estimation has yet to be explored. This comparison is made in Chapter 4.

| Parameter | Pre-Calibrated Value | Adjusted Value | % of Change |
|-----------|----------------------|----------------|-------------|
| $O_x$     | 2.2012               | 2.3140         | 5.12        |
| $O_y$     | 2.5462               | 2.4775         | -2.70       |

Table 3.3: Values of the offset parameters of the stick accelerometer before and after on-site static calibration.

| Exp. | Acceleration<br>(before calibration) | Acceleration<br>(after calibration) | Boom<br>Joint Angle | Stick<br>Joint Angle |
|------|--------------------------------------|-------------------------------------|---------------------|----------------------|
| 1    | 0.8981                               | 0.9998                              | 19.22°              | -114.92°             |
| 2    | 1.0084                               | 0.9958                              | 19.07°              | - 34.89°             |
| 3    | 0.9495                               | 1.0017                              | -10.58°             | -154.65°             |
| 4    | 0.9032                               | 1.0023                              | 30.14°              | -154.65°             |

Table 3.4: Comparison of acceleration magnitudes sensed by the stick accelerometer in various static poses using the pre-calibrated and calibrated values.

To calibrate the boom accelerometer, the boom cylinder was fully retracted and extended in 10 steps (the boom DH angle = 66.85, 50.84, 42.11, 33.92, 19.82, 12.65, 6.21, 2.95, -0.48,  $-9.64^{\circ}$ ). Similar tables for the boom accelerometer adjusted parameters and the acceleration values before and after calibration were generated (Tables 3.5 and 3.6). It was again observed that calibration improves the accuracy of acceleration sensing.

### 3.1.5 Non-ideal Cases

For the simulated and experimental results in the last two sections, it was always assumed that the biaxial accelerometers are actually sensing gravity. In the real world, however, it might be difficult to find a flat surface for calibration. With this in mind, one might think that static calibration is not implementable in practice. Our tests on the mini-excavator, however, have shown that a certain amount of unevenness can be easily tolerated during calibration. One should note that only the angles that the cab makes with its sides are of concern (*i.e.* roll angles in Figure 3.5). Yaw and pitch angles in the cab basically do not change the angle

| Parameter | Pre-Calibrated Value | Adjusted value | % of Change |
|-----------|----------------------|----------------|-------------|
| $O_x$     | 2.3954               | 2.5487         | 6.40        |
| $O_y$     | 2.4242               | 2.4635         | 1.62        |

 

 Table 3.5: Values of the offset parameters of the boom accelerometer before and after on-site static calibration.

| Exp. | Acceleration<br>(before calibration) | Acceleration<br>(after calibration) | Boom<br>Joint Angle | Stick<br>Joint Angle |
|------|--------------------------------------|-------------------------------------|---------------------|----------------------|
| 1    | 0.8959                               | 1.0010                              | 19.22°              | -114.92°             |
| 2    | 0.8958                               | 1.0005                              | 19.07°              | - 34.89°             |
| 3    | 0.9386                               | 0.9977                              | -10.58°             | -154.65°             |
| 4    | 0.8834                               | 0.9991                              | 30.14°              | -154.65°             |

Table 3.6: Comparison of acceleration magnitudes sensed by the boom accelerometer in various static poses using the pre-calibrated and calibrated values.

that the X-Y plane of the accelerometers makes with gravity, and therefore, have no impact on calibration. For testing the effects of roll angles, the machine was tilted to one side by 8.3°, as shown in Figure 3.5. This angle is believed to be fairly large and the operator should easily be able to find a better surface for calibration. After tilting the machine, the accelerometers were calibrated. Table 3.7 gives the calibrated values of the parameters and compares them with the values obtained by calibration on the flat surface. Clearly, only a slight change has occurred in the values of the parameters. This is further confirmed by comparing the change in the magnitudes of acceleration vectors once the calibrated values obtained in the horizontal position are replaced with the ones obtained in the tilted position. According to Table 3.8, this change is not insignificant either. In the next chapter, when the results of angle estimation based on these two calibration procedures are compared, it is shown that using the calibration data obtained in the tilted position

The small amount of change in the estimated values can be explained by the fact that even in this tilted position, the accelerometers are still sensing a =



Figure 3.5: The tilted mini-excavator.

 $(1g).(\cos 8.3^{\circ}) = 0.9895g$ , which is very close to the range of the accuracy of the estimator.

Nevertheless, if this error cannot be tolerated, several remedies can be recommended to eliminate or at least reduce it. In the simplest configuration, a uniaxial accelerometer, mounted on the cab, can be used as a tilt sensor to determine the roll angle of the cab. The cab roll angle information can then be used either to warn the operator of the possible inappropriate position for calibration, or can be incorporated into the parameter estimator. In the latter case, the sensor parameters are adjusted according to the new static acceleration  $a = 1.\cos\theta_r[g]$ , where  $\theta_r$  is the cab roll angle. In a more costly approach, all the biaxial accelerometers can be replaced with triaxial ones where the Z-axis outputs are only used for calibration. This guarantees that gravity is always fully sensed by the sensors. In this case, the equations of the parameter estimator should be expanded accordingly to include the Z-axis output. The steps involved in this task are explained below. The

CHAPTER 3. CALIBRATION OF ACCELEROMETERS

| Parameter           | Pre-Calibrated<br>Value | Adjusted Value<br>in Horizontal<br>Position | Adjusted Value<br>in Tilted<br>Position | % of Change<br>Between<br>Two Adjusted Values |
|---------------------|-------------------------|---|---|---|
| Boom acc            | elerometer              |   |   |   |
| $O_x$               | 2.3954                  | 2.5487                                      | 2.5634                                  | 0.58  |
| $O_y$               | 2.4242                  | 2.4635                                      | 2.4770                                  | 0.55  |
| Stick accelerometer |                         |   |   |   |
| $O_x$               | 2.2012                  | 2.3140                                      | 2.3323                                  | 0.79  |
| $O_y$               | 2.5462                  | 2.4775                                      | 2.4903                                  | 0.52  |

Table 3.7: Comparison of static calibration results in the horizontal and tilted positions.

input-output equations for a triaxial accelerometer can be written as:

| Exp. | Acceleration<br>(before calibration) | Acceleration<br>(calibration in<br>horizontal position) | Acceleration<br>(calibration in<br>tilted position) | Boom<br>Joint Angle | Stick<br>Joint Angle |
|------|--------------------------------------|---|---|---------------------|----------------------|
| 1    | 0.8981                               | 0.9998  | 1.0099  | 19.22°              | -114.92°             |
| 2    | 1.0084                               | 0.9958  | 1.0114  | 19.07°              | - 34.89°             |
| 3    | 0.9495                               | 1.0017  | 1.0030  | -10.58°             | -154.65°             |
| 4    | 0.9032                               | 1.0023  | 1.0040  | 30.14°              | -154.65°             |

Table 3.8: Comparison of acceleration magnitudes sensed by the stick accelerometer in various static poses using the pre-calibrated and calibrated values obtained in the horizontal and titled positions.

$$V_x - O_x = S_{xx}a_x + S_{xy}a_y + S_{xz}a_z (3.26)$$

$$V_y - O_y = S_{yx}a_x + S_{yy}a_y + S_{yz}a_z (3.27)$$

$$V_z - O_z = S_{zx}a_x + S_{zy}a_y + S_{zz}a_z (3.28)$$

These equations can be solved for  $a_x$ ,  $a_y$ , and  $a_z$ . Then, similar to the biaxial case (Equation 3.9),  $a_x$ ,  $a_y$ , and  $a_z$  can be incorporated to obtain the squared magnitude of the acceleration vector:

$$H(\mathbf{v}, \mathbf{p}) = 1 \ [g^2] = a_x^2 + a_y^2 + a_z^2 \tag{3.29}$$

By linearizing  $H(\mathbf{v}, \mathbf{p})$  through truncating the higher order terms in its Taylor expansion, the regressor model can be realized. In this case,  $\varphi_1$  and  $\varphi_2$ , the elements of the regression matrix are derived as follows:

$$\varphi_{1} = \frac{\partial H}{\partial O_{x}} = \frac{2}{p} [(S_{yz}S_{zy} - S_{zz}S_{yy})a_{x} + (S_{zz}S_{yx} - S_{yz}S_{zx})a_{y} + (3.30)$$

$$(S_{yy}S_{zx} - S_{zy}S_{yx})a_{z}]$$

$$\varphi_{2} = \frac{\partial H}{\partial O_{y}} = \frac{2}{p} [(S_{zz}S_{xy} - S_{xz}S_{zy})a_{x} + (S_{xz}S_{zx} - S_{zz}S_{xx})a_{y} + (3.31)$$

$$(S_{xx}S_{zy} - S_{zx}S_{xy})a_{z}]$$

where,

$$p = S_{xx}S_{yy}S_{zz} - S_{zz}S_{xy}S_{yx} - S_{xx}S_{yz}S_{zy} - S_{yy}S_{xz}S_{zx} + S_{xy}S_{yz}S_{zx} + S_{xz}S_{zy}S_{yx}$$

In general,  $O_z$  can also be calibrated. However, for the sensors installed on the mini-excavator,  $a_z$  remains close to zero most of the time. Hence, due to the lack of sufficient excitation,  $O_z$  cannot be estimated.

In another approach to calibrate accelerometers on uneven surfaces, the squared magnitude of acceleration in static conditions can be added to the list of the parameters for estimation. Originally, this magnitude was assumed to be one. In the new form, Equation 3.10 is modified to:

$$H(\mathbf{v}, \mathbf{p}) = M + \Delta M = H(\mathbf{v}, \mathbf{p_0}) + \frac{\partial H(\mathbf{v}, \mathbf{p})}{\partial \mathbf{p}} \bigg|_{\mathbf{p}=\mathbf{p_0}} (\mathbf{p} - \mathbf{p_0}) + h.o.t.$$
(3.32)

where M is the presumed value of the squared magnitude of acceleration, and  $\Delta M$  is the deviation of the squared magnitude from its true value to be identified by

the estimator. The equation for the parameter estimator can then be rewritten according to:

$$y = M - H(\mathbf{v}, \mathbf{p_0}) = \begin{bmatrix} \varphi & -1 \end{bmatrix} \begin{bmatrix} \boldsymbol{\xi} \\ \Delta M \end{bmatrix}$$
(3.33)

where  $\varphi$  and  $\xi$  are defined previously in Equations 3.12 and 3.13. Evidently, the best initial value for M is one. In each step of estimation, M is updated by adding  $\Delta M$  to the previous value. Note that in this approach, it is automatically assumed that the roll angle is not changing during calibration.

# 3.2 Cross-Calibration

The configuration in which accelerometers are used for angle estimation permits a unique approach to calibrating them. According to this configuration, the pair of accelerometers are placed very close to each other near the joint, such that the magnitude of acceleration sensed by them should be nearly the same. If this is not the case, their parameters are adjusted to force the difference to approach zero.

Cross-calibration has several advantages over static calibration. Unlike static calibration, the restriction on the position of accelerometers relative to gravity is no longer a concern. In this case, even if the machine is tilted or the accelerometers are not fully exposed to gravity for any reason, they should still sense the same acceleration; therefore, calibration can still be carried out. For a similar reason, the restriction on the link motion is also relaxed to some extent and the links can be moved during calibration. However, one should note that due to the small displacement between the two devices, they do not sense exactly the same acceleration in dynamic conditions. Therefore, harsh movements should preferably be avoided, or alternatively, low-pass filters could be applied to the accelerometer output voltages before calibration. The benefit of using low-pass filters is that they remove the high-frequency components from the signals, which are more likely to cause the difference.

To obtain the equations of the parameter estimator for the cross-calibration procedure, the difference between the squared magnitudes of acceleration vectors measured by each sensor of a pair should be determined:

$$H(\mathbf{v_1}, \mathbf{p_1}) - H(\mathbf{v_2}, \mathbf{p_2}) = 0$$
(3.34)

Then  $H(\mathbf{v_1}, \mathbf{p_1})$  and  $H(\mathbf{v_2}, \mathbf{p_2})$  are expanded around the current values of the sensor parameters,  $\mathbf{p_{01}}$  and  $\mathbf{p_{02}}$ , as follows:

$$H(\mathbf{v_{1}}, \mathbf{p_{01}}) + \frac{\partial H(\mathbf{v_{1}}, \mathbf{p_{1}})}{\partial \mathbf{p_{1}}} \bigg|_{\mathbf{p_{1}}=\mathbf{p_{01}}} (\mathbf{p_{1}} - \mathbf{p_{01}}) - H(\mathbf{v_{2}}, \mathbf{p_{02}}) - \frac{\partial H_{2}(\mathbf{v}, \mathbf{p_{2}})}{\partial \mathbf{p_{2}}} \bigg|_{\mathbf{p_{2}}=\mathbf{p_{02}}} (\mathbf{p_{2}} - \mathbf{p_{02}}) = 0$$
(3.35)

By ignoring the second and higher order terms and assuming the following:

$$y \triangleq H(\mathbf{v_1}, \mathbf{p_{01}}) - H(\mathbf{v_2}, \mathbf{p_{02}})$$
(3.36)

$$\varphi = \left[ \begin{array}{cc} -\varphi_1 & \varphi_2 \end{array} \right] \tag{3.37}$$

$$\boldsymbol{\xi} = \begin{bmatrix} \boldsymbol{\xi}_1 \\ \boldsymbol{\xi}_2 \end{bmatrix} \tag{3.38}$$

Equation 3.35 is written in the regression form as below:

$$y = \varphi \boldsymbol{\xi} \tag{3.39}$$

where  $\varphi_1$  and  $\varphi_2$  are defined according to Equation 3.12 and  $\xi_1$  and  $\xi_2$  are defined according to Equation 3.13. Similar to the static calibration case, Equation 3.39 is

| Parameter  | Pre-Calibrated<br>Value                | Adjusted Value<br>in Horizontal<br>Position | Adjusted Value<br>in Tilted<br>Position | % of Change<br>Between<br>Two Adjusted Values |  |  |
|------------|--|---|---|---|--|--|
| The biaxia | The biaxial accelerometer on the boom  |   |   |   |  |  |
| $O_x$      | 2.3954                                 | 2.5585                                      | 2.5532                                  | -0.21   |  |  |
| $O_y$      | 2.4242                                 | 2.4739                                      | 2.4717                                  | -0.08   |  |  |
| The biaxia | The biaxial accelerometer on the stick |   |   |   |  |  |
| $O_x$      | 2.2012                                 | 2.3256                                      | 2.3211                                  | -0.19   |  |  |
| $O_y$      | 2.5462                                 | 2.4904                                      | 2.4888                                  | -0.06   |  |  |

Table 3.9: Values of the offset parameters of the stick accelerometer before and after on-site cross-calibration.

then extended to include more measurements, and the same iterative least squares scheme described in Section 3.1.2 is run to obtain the new values of the parameters.

For the experimental results, Table 3.9 shows the values obtained from crosscalibration of the stick pair conducted in both the horizontal and tilted positions. During both procedures, the links were moved arbitrarily for 60 seconds and any contact of the bucket with the ground was avoided. In these experiments, only the offset parameters were adjusted and similar filters used for angle estimation (see Section 4.2.2) were applied to the accelerometer output voltages before using them for calibration. Attempts to calibrate the sensitivity parameters failed as the estimator diverged. According to Table 3.9, calibration in the tilted position has only caused a minor change in the estimation results, as expected.

# **3.3** Second Order Model for Accelerometers

The input-output mapping of an accelerometer is not perfectly linear. Obviously if the nonlinear behavior of the accelerometer can be included in its model, the sensor readings become more accurate. Here, the possibility of using a second order model for the accelerometers is examined. Similar to the linear model, the parameters of this model must be easily adjustable through a simple procedure. A second order model can be proposed for the accelerometers as follows:

$$a_x = \frac{S_{yy}(\alpha_x V_x^2 + V_x - O_x) - S_{xy}(\alpha_y V_y^2 + V_y - Oy)}{S_{xx} S_{yy} - S_{xy} S_{yx}}$$
(3.40)

$$a_y = \frac{S_{xx}(\alpha_x V_x^2 + V_x - O_x) - S_{yx}(\alpha_y V_y^2 + V_y - Oy)}{S_{xx} S_{yy} - S_{xy} S_{yx}}$$
(3.41)

where  $\alpha_i$  is the coefficient for the second order effects of  $V_i$ . Note that if  $\alpha_x = \alpha_y = 0$ , then the original first order model is obtained (Equations 2.11 and 2.12). The same static calibration procedure, used for the first order model, can be applied to calibrate the parameters of the second order model. In the simplest form, the offset parameters and second order coefficients can be calibrated. In this case, most of the equations of the parameter estimator (Equations 3.10 to 3.21) remain intact. Only derivatives of  $H(\mathbf{v}, \mathbf{p})$  with respect to  $\alpha_x$  and  $\alpha_y$  must be used, instead of the ones used for the sensitivity parameters in  $\varphi$ . These derivatives are the following:

$$\varphi_3 = \frac{\partial H}{\partial \alpha_x} = -V_x^2 \varphi_1 \tag{3.42}$$

$$\varphi_4 = \frac{\partial H}{\partial \alpha_y} = -V_y^2 \varphi_2 \tag{3.43}$$

where  $\varphi_1$  and  $\varphi_1$  are obtained from Equations 3.14 and 3.15. A good choice for the initial values of the parameters is the pre-calibrated values for the offset parameters and zero for  $\alpha_x$  and  $\alpha_y$  as they are expected to be very close to zero anyway.

In practice, however, it was found that the second order model cannot be used on the mini-excavator arm, as the parameter estimator did not converge for the boom accelerometer. This can be related to insufficient excitation on this link. Once the same accelerometer was moved and placed on the stick, the estimator normally converged. This model might be useful for other applications.

# Chapter 4

# Angle Estimation

In this chapter, accelerometer-based angle estimation is studied in detail. The theory behind this methodology is explained in Section 4.1. The experimental results with error analysis are presented in Section 4.2. Several sources contribute to the errors in angle sensing with accelerometers, and solutions for reducing these errors are discussed.

# 4.1 Accelerometer-Based Angle Estimation

Assume a pair of biaxial accelerometers are placed extremely close to the axis of a joint, on the two adjacent links that form it (Figure 4.1). In this configuration, the two sensors are exposed to almost the same acceleration, regardless of the position of the links or how they move. When the links are stationary, this acceleration is gravity. In dynamic conditions, it becomes a combination of gravitational and inertia accelerations. The projection of this acceleration vector on the axes of these accelerometers can be used to determine the rotation angle between their frames, which corresponds to the joint angle with an offset.

The relation between the acceleration vector measured by the first accelerometer, that is  $\mathbf{a_1} = (a_{x1}, a_{y1})^T$ , and the acceleration vector measured by the second



Figure 4.1: Joint angle sensing using biaxial accelerometers.

accelerometer, that is  $\mathbf{a_2} = (a_{x2}, a_{y2})^T$ , can be written as follows:

$$\begin{bmatrix} a_{x1} \\ a_{y1} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} a_{x2} \\ a_{y2} \end{bmatrix}$$
(4.1)

where  $\theta$  is the rotation angle. Equivalently  $\theta$  can be found from:

$$\tan \theta = \frac{a_{x2} \cdot a_{y1} - a_{y2} \cdot a_{x1}}{a_{x1} \cdot a_{x2} + a_{y1} \cdot a_{y2}} \tag{4.2}$$

In reality, however, it is impossible to mount two sensors with a zero proximity to the joint axis, therefore,  $\mathbf{a_1}$  and  $\mathbf{a_2}$  represent slightly different accelerations and Equations 4.1 and 4.2 are not exactly held anymore. Two approaches can be taken to deal with this error:

- 1. The error is ignored [12]. This is possible only in the systems with slow dynamics as the difference between the sensed accelerations can be negligible.
- 2. The accelerations at the end-points are somehow estimated [32] and then the estimated end-point accelerations are used in Equation 4.2 for angle estimation. Acceleration at the end-points or any other point on a rigid body can be



Figure 4.2: Estimation of the acceleration at  $P_0$  using the accelerations at  $P_1$  and  $P_2$ .

estimated if accelerations at two other points on the body are known, and the body has planar motion. Accordingly, as Figure 4.2 illustrates, to estimate the acceleration at point  $P_0$ , two accelerometers must be mounted at two arbitrary points,  $P_1$  and  $P_2$ , on the body. The acceleration at those points can be expressed as:

$$\mathbf{a}_{\mathbf{P1}} = \mathbf{a}_{\mathbf{P0}} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_1) + \boldsymbol{\dot{\omega}} \times \mathbf{r}_1 \tag{4.3}$$

$$\mathbf{a}_{\mathbf{P2}} = \mathbf{a}_{\mathbf{P0}} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r_2}) + \boldsymbol{\dot{\omega}} \times \mathbf{r_2}$$
(4.4)

where,

 $\mathbf{a_{Pi}} = (a_{Pix}, a_{Piy})^T$  = the acceleration vector at  $P_i$ , i = 0, 1, 2  $\mathbf{r_i} = (r_{xi}, r_{yi})^T$  = the position vector of  $P_i$  relative to  $P_0$ , i = 1, 2  $\boldsymbol{\omega} = \boldsymbol{\omega}(0, 0, 1)^T$  = the angular velocity of the rigid body relative to the arbitrary inertia frame  $X_I$ - $Y_I$ 

 $\dot{\omega} = \dot{\omega}(0,0,1)^T$  = the angular acceleration of the rigid body relative to the

arbitrary inertia frame  $X_I$ - $Y_I$ 

By further manipulation, Equations 4.3 and 4.4 can be written as:

$$\begin{bmatrix} a_{P1x} \\ a_{P1y} \end{bmatrix} = \begin{bmatrix} a_{P0x} \\ a_{P0y} \end{bmatrix} - \omega^2 \begin{bmatrix} r_{x1} \\ r_{y1} \end{bmatrix} + \dot{\omega} \begin{bmatrix} -r_{y1} \\ r_{x1} \end{bmatrix}$$
(4.5)
$$\begin{bmatrix} a_{P2x} \\ a_{P2y} \end{bmatrix} = \begin{bmatrix} a_{P0x} \\ a_{P0y} \end{bmatrix} - \omega^2 \begin{bmatrix} r_{x2} \\ r_{y2} \end{bmatrix} + \dot{\omega} \begin{bmatrix} -r_{y2} \\ r_{x2} \end{bmatrix}$$
(4.6)

and solved in the  $X_0 - Y_0$  frame for the unknown parameters of  $a_{P0x}$ ,  $a_{P0y}$ ,  $\omega^2$  and  $\dot{\omega}$  using Cramer's rule [3]. For the special case, when  $\mathbf{r_1} = r_1 \mathbf{e}$  and  $\mathbf{r_2} = r_2 \mathbf{e}$  (meaning that  $P_0$ ,  $P_1$  and  $P_2$  are on the same line), the solutions for  $a_{P0x}$  and  $a_{P0x}$ , can be written in the following simple form:

$$a_{P0x} = \frac{r_2 a_{P1x} - r_1 a_{P2x}}{r_2 - r_1} \tag{4.7}$$

$$a_{P0y} = \frac{r_2 a_{P1y} - r_1 a_{P2y}}{r_2 - r_1} \tag{4.8}$$

Going back to the joint angle estimation problem, by placing one pair of accelerometers on one link and choosing  $P_0$  at the center of the rotation, the estimated values of  $\mathbf{a_1}$  in Equation 4.2 can be obtained according to this method. Similarly, by placing another pair of accelerometers on the other link and choosing the same  $P_0$ , the estimated values of  $\mathbf{a_2}$  can also be obtained. The estimated acceleration values then used to compute the joint angle.

Of the two approaches, the first one appears to be better for angle estimation on the mini-excavator. The dynamics of the mini-excavator are slow, which qualifies it for this approach. Moreover, only two accelerometers are required in this approach to estimate one joint angle, as opposed to four sensors in the second approach. The installation of the sensors is also easier in the first approach, as the distance between sensors and the joint axis should be precisely known. The second approach is only recommended for systems like the human body, whose dynamics are so fast that a small displacement in the positions of the sensors can generate a significant difference in the sensed accelerations on each sensor. Since the dynamics of the mini-excavator and heavy-duty manipulators are generally not that fast, the second approach is not appropriate for them.

### 4.1.1 Angle Offset

Equation 4.2 only gives the rotation angle between the frames of two accelerometers. Depending on how the sensors are installed on the links, the estimation would be different in the same pose. Therefore, an angle offset, determined with reference to a fixed position, should be added to the estimated angle. The procedure to find the angle offset is similar to the one described in Section 1.2 to calibrate the digital resolvers in which the arm cylinders are fully extended and the difference between the estimated angle and the desired angle is considered as the angle offset.

### 4.1.2 Limitations of the Angle Estimator

There is only one case in which the angle estimator is unable to determine the joint angle from the acceleration values. In this case, the rotation axis between the frames of the two accelerometers coincides with the acceleration vector, that is, when the overall acceleration on the field becomes exactly perpendicular to the frame of the accelerometers, *i.e.*  $a_{x1} = a_{y1} = a_{x2} = a_{y2} = 0$ . In this circumstance, an ambiguity occurs in Equation 4.2, and  $\theta$  cannot be found.

This case, however, never occurs on the mini-excavator arm during normal operation of the machine. The major field acceleration is gravity. In order that gravity and the joint axes of the mini-excavator lie in the same plane, the machine must be fully tilted onto its side.

# 4.2 Practical Aspects and Experimental Results

If Equation 4.2 were simply applied to the acceleration values obtained from the raw output voltages of the accelerometers, the results of angle estimation would be inaccurate. Figure 4.3 shows the results of this approach for stick joint angle estimation. As can be seen, the estimation error is in the range of 25°. Similar to estimation with a uniaxial accelerometer, there are two main sources for this error: modeling errors and dynamic effects.

Modeling errors are also observed in static conditions. In static conditions, both sensors are exposed to the same acceleration, which is gravity. However, because of the modeling errors, the measured values of this acceleration are different on each device, causing the rotation angle to be falsely determined. Modeling errors can be reduced by proper calibration procedures. Dynamic effects are related to the small displacement between the two sensors, which leads to their sensing different accelerations in non-stationary conditions. These kinds of errors can be reduced by using proper filtering methods. In the following section, as angle sensing is studied under static and dynamic conditions, it is shown how each type of error can be reduced with the proper approach.

### 4.2.1 Angle Sensing in Static Conditions

Chapter 3 extensively discussed how the model parameters of accelerometers can be experimentally estimated at the location of use. Now the results of those calibration techniques are applied to the angle estimation system to reduce modeling errors. To study the impact of accelerometer calibration on angle estimation, the stick joint was positioned in several arbitrary static poses, and the joint angles in these poses were estimated by using both the pre-calibrated and the calibrated values of the parameters given in Tables 3.3 and 3.9 (obtained from the static and crosscalibration procedures). During all these calibration procedures and measurement tests, the machine was in the horizontal position. Table 4.1 lists the estimation



Figure 4.3: Stick joint angle estimation with no calibration and filtering.

error in all cases, and Figure 4.4 visualizes them. The table shows that the error in estimation is about 12° before calibration, whereas after employing the calibrated parameters, this error is reduced to 1.73° and 2.19° in the cases of static calibration and cross-calibration, respectively. This is a significant improvement in accuracy. But even after calibration, there can still be some error in angle estimation. Possible reasons for this are:

• Due to noise, limited motion of the links, truncation of high order terms and so on, the calibration procedures are never capable of finding the exact values

| Exp. | Actual Stick<br>Joint Angle | Error in Estimation (before calibration) | Error in Estimation (static calibration) | Error in Estimation<br>(cross-calibration) |
|------|-----------------------------|--|--|--|
| • 1  | -154.65°                    | · 0°                                     | 0°                                       | 0°   |
| 2    | -145.07°                    | -2.22°                                   | -1.10°                                   | -1.14°                                     |
| 3    | -134.87°                    | -3.34°                                   | -1.05°                                   | -1.12°                                     |
| 4    | -125.14°                    | -4.51°                                   | -1.02°                                   | -1.14°                                     |
| 5    | -114.92°                    | -5.84°                                   | -1.07°                                   | -1.24°                                     |
| 6    | -105.01°                    | -7.23°                                   | -1.23°                                   | -1.45°                                     |
| 7    | -94.95°                     | -8.52°                                   | -1.35°                                   | -1.63°                                     |
| 8    | -85.38°                     | -9.73°                                   | -1.57°                                   | -1.89°                                     |
| 9    | -75.11°                     | -10.70°                                  | -1.65°                                   | -2.02°                                     |
| 10   | -64.95°                     | -11.43°                                  | -1.70°                                   | -2.12°                                     |
| 11   | -55.07°                     | -11.93°                                  | -1.73°                                   | -2.19°                                     |
| 12   | -44.91°                     | -12.14°                                  | -1.67°                                   | -2.16°                                     |
| 13   | -34.89°                     | -12.00°                                  | -1.46°                                   | -1.94°                                     |

Table 4.1: Stick joint angle estimation error in a full range set of static poses (boom joint angle =  $19.27^{\circ}$  in all poses).

of the parameters.

- As long as a linear model is used for the accelerometers, the inherent nonlinear characteristics of the sensors are ignored. Naturally, this can influence the estimation results. As discussed in Chapter 3, it is not easy to find the parameters of a higher order model on the mini-excavator arm.
- The X and Y axes of the accelerometers may be misaligned. This defect has impacts both on the angle estimation and calibration results by changing the magnitude of the acceleration vectors, as previously discussed in Chapter 2.
- The X-Y plane of the two accelerometers may not be coplanar. This can affect both the angle estimator and calibration results. The basic assumption that the accelerometers are actually exposed to the same acceleration is not valid anymore. The definition of the angle between the two frames is also vague in this case, as the two frames are not coplanar.



Figure 4.4: Comparison of angle estimation results in static conditions for different calibration methods.

It is also important to explore how calibration with the mini-excavator in a tilted position affects angle estimation. For this purpose, the estimated parameters of Tables 3.7, obtained from the static and cross-calibration procedures carried out in a tilted position (see Figure 3.5), were applied to the static poses of the early experiments of this section. Table 4.2 presents the results. Estimation with static calibrated values is slightly less accurate (about  $1^{\circ}$ ), as expected. However, this change is small. As a result it can be claimed that static calibration can still be safely run even with roll angles of up to 8.3°. The change in angle estimation results is even less significant (about  $0.5^{\circ}$ ) with cross-calibration. This was also expected as this method of calibration is robust to tilt angles.

# 4.2.2 Angle Sensing in Dynamic Conditions

As discussed in Chapter 2, low-pass filtering can be used to remove high-frequency components generated by shock and vibration from the accelerometer output voltages. It should be also noted that the calibration procedures discussed in Chapter 3

| Exp. | Actual Stick<br>Joint Angle | Error in Estimation<br>(static calibration) | Error in Estimation<br>(cross-calibration) |
|------|-----------------------------|---|--|
| 1    | -154.65°                    | 0°  | 0°   |
| 2    | -145.07°                    | -1.10°                                      | -1.15°                                     |
| 3    | -134.87°                    | -1.08°                                      | -1.15°                                     |
| 4    | -125.14°                    | -1.11°                                      | -1.18°                                     |
| 5    | -114.92°                    | -1.25°                                      | -1.31°                                     |
| 6    | -105.01°                    | -1.53°                                      | -1.55°                                     |
| 7    | -94.95°                     | -1.79°                                      | -1.76°                                     |
| 8    | -85.38°                     | -2.15°                                      | -2.07°                                     |
| 9    | -75.11°                     | -2.40°                                      | -2.23°                                     |
| 10   | -64.95°                     | -2.62°                                      | -2.37°                                     |
| 11   | -55.07°                     | -2.82°                                      | -2.47°                                     |
| 12   | -44.91°                     | -2.94°                                      | -2.48°                                     |
| 13   | -34.89°                     | -2.88°                                      | -2.33°                                     |

Table 4.2: Stick joint angle estimation error in a full range set of static poses with calibration procedures performed in a tilted position (8.3° of tilt).

were all carried out in static conditions, or at the very low bandwidth of operation. As a result, the accelerometers are better calibrated at low frequencies. Employing low-pass filters has the advantage of using the same bandwidth for angle estimation. Based on this argument, it is recommended that the filters used for cross-calibration and angle estimation be identical to improve accuracy.

Since employing low-pass filters introduces a delay in estimation, a trade-off exists between the delay and the accuracy in angle estimation. Depending on the application, the order and other specifications of the filters may change. In the rest of this section, possible designs for low-pass filters for achieving greater accuracy are discussed.

For design of digital low-pass filters, there are two options: Infinite-duration Impulse Response (IIR), and Finite-duration Impulse Response (FIR) filters [20]. The sampling frequency of the angle estimator was set to 100Hz, in accordance with most applications that run on the machine and need joint angle positions. It was also experimentally decided to set the bandwidth of the low-pass filters to less than 5Hz, in order to achieve good accuracy in angle estimation. Compared to the sampling rate, this is a very narrow bandwidth. Design of such narrow bandwidth filters is possible with the lower order IIR filters. However, these filters have nonlinear phase characteristics (varying group delay) and distort the signal in the time domain. The FIR filters, on the other hand, require a much higher order to achieve the same magnitude response, but can be designed with linear phase characteristic (constant known delay). Since the outputs of the filters in the time domain are what we are concerned with, the FIR design was adopted.

The window method is commonly used to design low-pass FIR filters [20]. In this method, the impulse response of an ideal low-pass filter is shifted and truncated to construct a linear phase FIR filter. Several truncation methods are available for this purpose. They differ mainly in the compromises they make between the attenuation in the stop-band and the sharpness of transition from pass-band. Rectangular windowing is one of these truncation methods, and was found to yield the best results for angle estimation. The impulse response of the filter obtained by this truncation is the product of the impulse response of the ideal filter and the rectangular window. This method generates the sharpest roll-off compared to the other methods, but it also results in a low stop-band attenuation. A filter of this type of order 30 and with the cut-off frequency of 3 Hz was used for the accelerometer voltages.

When the filtered signals are incorporated in Equation 4.2 to compute the joint angle, the nonlinear nature of this equation results in higher frequency components in the measured angle. Hence, another low-pass filter can be applied to the estimated angles to minimize this effect. For the experimental results of this chapter, an FIR filter of order 10 with the cut-off frequency of 4 Hz was applied to the estimated angles. The role of this filter, however, is trivial and can be removed for time-critical applications. The block diagram of the complete angle estimator is



Figure 4.5: Block diagram of the angle estimator.

shown in Figure 4.5.

since each linear phase Nth-order FIR filter delays the input signal by N/2 samples. Therefore, the overall delay of the estimated angle is:

$$d = \frac{N_1}{2} + \frac{N_2}{2} = 20 \ samples \tag{4.9}$$

where  $N_1$  and  $N_2$  are orders of the voltage and angle filters respectively. For instance, with the sampling rate of 100Hz and the filter orders of 30 and 10, this delay is 0.2s.

As the order of the FIR filters increases, they become computationally demanding. This is not a serious problem at the sampling rate of 100Hz with the current state-of-the-art microprocessors. Nevertheless if the computational load is a concern, a technique suggested in [18] can be exploited to reduce the load. This technique can only be applied to a limited class of FIR filters called *moving average filters*. In an N + 1-point moving average filter, the current output is the average of the current and the last N samples of the input. The coefficients of the filter are all 1/(N + 1). In this technique, the moving average filter can be rewritten in the following recursive form:

$$y(n) = y(n-1) + \frac{1}{N+1}(x(n) - x(n-N-1))$$
(4.10)

where x(n) and y(n) are the current input and output, respectively. By using the recursive form of the moving average filter, the output can be obtained with 2



Figure 4.6: Magnitude responses of the modified moving average filter and the FIR filter designed by windowing.

additions and 1 multiplication, while it requires N addition and 1 multiplication in the conventional form.

Experimentally, it was found that by replacing the FIR filter designed by the rectangular window method with a modified moving average filter of the same order, the range of the stick joint angle error was only increased by 0.5°. This can be further confirmed by looking at the magnitude responses of both filters in Figure 4.6. The difference between the magnitude responses is insignificant.

Figure 4.7 represents the results of the angle estimation for the same experiment used to draw the plots of Figure 4.3. However, this time, the calibrated values obtained from the cross-calibration procedure on the flat surface (Table 3.9) and the filters discussed in this section were employed to compensate for the modeling errors and dynamic effects. As can be seen, the range of the estimation error has been substantially reduced to  $3.2^{\circ}$ . The link movement profile in this experiment, in fact, contains one ground touch-down at t = 29.1s and two time intervals between t = 5 - 10s and t = 21 - 24s, in which the cab is severely shaking. This profile can be considered as one of the worst-case scenarios for the angle estimation system. To calculate the delay in Figure 4.7B, the estimated angle signal was shifted by 20



Figure 4.7: Stick joint angle estimation in dynamic conditions.

samples to match it with the actual angle signal. The standard deviation of the estimation error is  $0.53^{\circ}$  in this experiment. It was also found that cross-calibration surpasses static calibration in performance and generates better results. Once the statically calibrated values are used, the error in estimation is increased by  $1^{\circ}$ .

It should be noted that the estimation errors both in static and dynamic conditions tend to be negative numbers. This is because angle offsetting is performed at one end of the joint motion (Section 4.1.1). If the angle offset is obtained by using the information from both extremes of the motion, a bias-free angle estimation can be achieved. *Thus it can be claimed that the proposed method successfully*
#### estimates the stick joint angle with an accuracy of $\pm 1.6^{\circ}$ .

The boom joint angle estimation is not as challenging as that of the stick joint angle. From Tables 4.1, one can see that as the stick joint is moving away from its reference angle, the static estimation error increases proportionally. The same pattern is observed for the boom joint, however, since this joint has a smaller range of motion compared to the stick joint, the static error never becomes as evident as it is on the stick. In addition, the boom joint is closer to the body of the machine, therefore, the dynamic effects of the cab shaking are less observed by the accelerometers mounted near it. The only disadvantage for the boom joint is the fact that the accelerometer mounted on the cab cannot be calibrated. Nevertheless, even when using the pre-calibrated values for this accelerometer, an accuracy greater than  $\pm 0.75^{\circ}$  is achievable for this joint.

### Chapter 5

# Applications

The proposed angle sensing method in the previous chapter is useful only if similar results are obtained with it in applications where other angle sensing techniques were originally utilized. In this chapter, this performance validation is investigated for the accelerometer-based angle sensing system in two applications: dynamic payload monitoring (Section 5.1), and closed-loop position control of the joints (Section 5.2) where digital resolvers originally measured the angles.

#### 5.1 Dynamic Payload Monitoring

A payload monitoring system has been previously designed and implemented on the mini-excavator. This system dynamically calculates the mass being handled by the manipulator using hydraulic pressures and angles of the boom and stick joints. The performance of the payload monitoring system was studied by using the two alternative position sensing systems available on the machine: digital resolvers and the accelerometer-based system. A pair of triaxial accelerometers was used to estimate the boom joint angle, and a pair of biaxial accelerometers was used to estimate the stick joint angle. Only the X and Y-axis outputs of the triaxial accelerometers were used for the boom joint angle estimation. The third axis output



Figure 5.1: Results of the dynamic payload monitoring using two different methods of angle sensing (load = 0Kg).

of the cab accelerometer was used to determine the roll and pitch angles of the machine. This information is useful for payload monitoring on an uneven surface, if needed. Since the accelerometer-based angles are generated within a fraction of a second, the pressure signals were also delayed for synchronization of all the inputs.

Two sets of experiments, one with and one without payload, were carried out in order to compare the performance of the payload monitoring algorithm in the presence of the two different angle sensing systems. The experiment with the loaded bucket included both free-space and contact tasks. It involved three touch-



Figure 5.2: Results of the dynamic payload monitoring using two different methods of angle sensing (load = 130Kg).

downs with the ground (about 15 to 20 seconds apart from each other). The payload in the loaded experiment was 130Kg. The results are shown in Figures 5.1 and 5.2 for the no-load and loaded experiments, respectively. The performance of the accelerometer-based system can be validated by looking at these figures. Both figures indicate that the new angle sensing system has successfully generated a pattern for the estimated payload similar to what has been achieved with the resolvers. The statistical analysis of the results is summarized in Table 5.1. In this table,  $M_{mean}$ ,  $M_{std}$ ,  $M_{min}$ , and  $M_{max}$  refer to the mean, standard deviation, minimum,

| Angle<br>Sensing System | $\mathrm{M}_{\mathrm{mean}}$ | $M_{\text{std}}$ | $M_{min}$ | $M_{max}$ |
|-------------------------|------------------------------|------------------|-----------|-----------|
| Load = 0Kg              |                              |                  |           |           |
| Resolver                | 1.75                         | 4.81             | -9.88     | 14.48     |
| Accelerometer-Based     | 1.27                         | 5.85             | -13.86    | 19.71     |
| Load = 130Kg            |                              |                  |           |           |
| Resolver                | 128.53                       | 6.75             | 107.64    | 141.42    |
| Accelerometer-Based     | 128.47                       | 6.69             | 105.69    | 143.55    |

Table 5.1: Statistics of the dynamic payload monitoring results reported in Figures 5.1 and 5.2.

and maximum of the estimates. The statistics for the loaded experiments were obtained during the time interval 34s < t < 46s after leaving contact and arriving at the steady-state regime.

#### 5.2 Computer-Assisted Closed-Loop Position Control

A coordinating computer-assisted control system has been designed for hydraulic excavators to allow users to remotely operate the machine in Cartesian space [15][25]. Traditionally, the main four links of the hydraulic excavators are controlled in the joint space by means of two two degree of freedom hand levers provided in the cab. For the machine in our laboratory, the two hand levers have been replaced by a four degree of freedom joystick, and the user can directly control the motion of the implement in Cartesian space. In the control loop, the joystick inputs are sampled, properly scaled and adjusted to represent the desired position of the endeffector (bucket). Alternatively, the desired trajectory can also be provided from previously computer-generated data. By solving the inverse kinematics, the desired end-effector positions are transformed to the desired joint angles. The desired joint angles and the actual angles obtained from the installed angle position sensors are then transformed into the cylinder space. A PD controller[7] uses the difference between two transformed values to drive the electrically-actuated pilot valves. In this control, the boom, stick, and bucket joint angles are used to obtain the current position of the end-effector. Since angle estimation with accelerometers is not conducted on the bucket joint, the output of the bucket resolver was also used for control using the accelerometer-based system. The design and implementation of the position controller for the mini-excavator has been discussed in detail in [22].

Obviously, the same performance cannot be expected from the control system if the digital resolvers are replaced with the accelerometer-based system which uses high order FIR filters, such as those described in Section 4.2.2 for angle estimation. The delays that these filters introduce to the control loop deteriorate stability [7]. Poor performance was experimentally observed on the machine when a computer generated trajectory along a circular path (with sine and cosine waveforms for the X and Y directions) was given for control. The accelerometer-based system was used with the 30th and 10th-order FIR filters for the accelerometer voltages and estimated angles, respectively. Harsh oscillatory behavior was evident at the endeffector during the experiment. As mentioned in the previous chapter, employing the filters is a trade-off between accuracy and speed of angle estimation. Therefore, to reduce the delay, the high order filters were replaced with 6-point moving average filters. This time, the filters were only applied to the accelerometer voltages and the angle filter was removed. Based on the sampling rate of 100 Hz, the new arrangement only introduces a 0.025s delay, while the previous arrangement had a delay of 0.2s. When the same experiment was conducted with the new filters, the control performance was significantly improved and oscillation was considerably reduced. Figure 5.3 illustrates the results of trajectory tracking. In this figure, a comparison has also been made between the control performance using the accelerometer-based system, and the control performance using the digital resolvers. While small oscillatory behavior is still visible on the accelerometer-based trajectory tracking, it has achieved its goal of generating a similar path. Most of the high-frequency variations in the accelerometer-based systems have been filtered out by the slow dynamics of the manipulator actuators in this case.



Figure 5.3: Comparison of closed-loop angle control using two different methods of angle sensing.

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### Chapter 6

## **Conclusions and Future Work**

#### 6.1 Contributions and Recommendation

In this thesis, for the first time, an angle sensing method based on accelerometers was designed and implemented on a mini-excavator arm to estimate the boom and stick joint angles. The calibration procedure described in [31] was revised and examined with the angle sensing system (static calibration). A novel calibration procedure was also developed and fully integrated with the angle sensing system (cross-calibration). Various tests were performed on the machine to emulate the performance of the accelerometer-based angle sensing system in a real world application. The statistical results are summarized in Table 6.1.

The following observations were also made regarding the use of this method on the mini-excavator. These observations can also be taken into account for implementation on other hydraulic and non-hydraulic manipulators:

1. Both calibration procedures considerably improved the estimation results. In particular, the cross-calibration procedure, which can be carried out virtually under any condition, resulted in 6-fold error reduction. Therefore, the use of these procedures with the angle sensing system is highly recommended.

| Joint | Range of Motion | Absolute Error     | Relative Error | Standard Deviation of<br>Error |
|-------|-----------------|--------------------|----------------|--------------------------------|
| Boom  | 79°             | ±0.75°             | $\pm 0.94\%$   | 0.30°                          |
| Stick | 119°            | $\pm 1.60^{\circ}$ | $\pm 1.33\%$   | 0.53°                          |

Table 6.1: Statistics of angle estimation for the boom and stick joint using accelerometers.

- 2. Dynamic effects due to link motion and cab shaking deteriorate angle estimation results. Low-pass filters can be applied to reduce these effects. In the experiments, surges in angle estimation results due to dynamic effects with magnitudes of up to 10° were eliminated in this manner.
- 3. There is a trade-off between accuracy and delay in estimation caused by lowpass filters. Decisions on the order and specifications of the filters must be made according to each specific application.
- 4. The performance of this system was found comparable to the performance of the resolvers in dynamic payload monitoring and arm position control. This was specifically true for dynamic payload monitoring where the difference in payload estimation was less than 0.5Kg with two different angle sensing methods. In dynamic payload monitoring, a certain amount of delay in angle estimation can be tolerated, therefore, greater accuracy in angle estimation results can be achieved.

#### 6.2 Suggestions for Further Work

Several suggestions can be made to further improve or study this angle sensing system:

1. The possibility of using adaptive filters can be examined, or any other filtering technique that may generate better results with less or the same amount of delay.

- 2. A function can be added to the system according to the instructions in Section 3.1.1 to automatically determine when a calibration procedure must be carried out.
- 3. Angle estimation on the bucket can be implemented and tested individually or with other estimated joint angles in various applications.
- 4. The system can be further scrutinized by studying it on a more flexible platform which would allow the performance of the experiments that cannot be run on the mini-excavator, such as calibration in the full 360° range of motion.
- 5. The angle sensing algorithm and its hardware can be further optimized towards the production of a new stand-alone angle sensor. This task was partially completed during this work by implementing this algorithm on a PC-104 system.
- 6. To fully exploit the signal processing power available onboard, the angle sensor can be upgraded to provide the user with additional measurements, such as angular velocity of the joint (obtained by numerical differentiation of the estimated angle), linear acceleration of the distal and proximal links (the accelerometer outputs), and link temperatures (if accelerometers are equipped with temperature transducers).
- 7. A survey can also be conducted to find other applications for this method.

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## Appendix

## **Specifications of Accelerometers**

All the accelerometers used in this work have capacitive micromachined sense elements inside their rugged packages. The benefits of capacitive micromachined accelerometers were explained in Chapter 2. The uniaxial accelerometer that was used in this work was an ADXL05EM-1 from Analog Devices [2]. Two 23203A biaxial accelerometers and two 34103A triaxial accelerometers from Summit Instruments were used for other experiments [27][28]. The triaxial pair was mostly used for the boom joint angle measurement, while the biaxial ones were used for the stick joint angle measurement. The pre-calibrated values of the offset and (cross-)sensitivity parameters of the accelerometers are given in Table A.1. Other specifications for these devices are as follows:

#### Uniaxial Accelerometers (ADXL05EM-1):

Range: $\pm 4g$ Bandwidth:0-400 HzNoise: $0.5mg/\sqrt{\text{Hz}}$ Offset Drift ( $-40^{\circ}C$  to  $85^{\circ}C$ ): $\pm 0.2g$ Nonlinearity: $\pm 0.2\%\text{FS}$ Shock Survivability: $\pm 500g$ Power Supply: $+5 \pm 0.25V$ 



FigureA.1: TheADXL05EM-1uniaxialaccelerometer.

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| Accelerometer                       | Uniaxial<br>(on the boom) | Triaxial<br>(on the cab) | Triaxial<br>(on the boom) | Biaxial<br>(on the boom) | Biaxial<br>(on the stick) |
|-------------------------------------|---------------------------|--------------------------|---------------------------|--------------------------|---------------------------|
| O <sub>x</sub>                      | 2.488                     | 2.6182                   | 2.8233                    | 2.3954                   | 2.2012                    |
| Oy                                  | -                         | 2.5985                   | 2.5923                    | 2.4242                   | 2.5462                    |
| Oz                                  | -                         | 2.4827                   | 2.5841                    | -                        | -                         |
| S <sub>xx</sub>                     | 0.501                     | -1.28850                 | -1.29808                  | -1.28629                 | -1.28464                  |
| $S_{xy}$                            | -                         | 0.00337                  | -0.00240                  | -0.00822                 | 0.00094                   |
| $\mathbf{S_{xz}}$                   | -                         | 0.00029                  | -0.00134                  | -0.02456                 | -0.00897                  |
| $S_{yx}$                            | -                         | 0.00265                  | -0.00000                  | 0.00675                  | -0.02982                  |
| $\mathbf{S}_{\mathbf{y}\mathbf{y}}$ | -                         | -1.29647                 | -1.30034                  | -1.30951                 | -1.25536                  |
| $S_{yz}$                            | -                         | -0.00120                 | 0.00134                   | -0.00334                 | 0.02164                   |
| $S_{zx}$                            | -                         | 0.00245                  | -0.00066                  | -                        | -                         |
| Szy                                 | -                         | -0.00119                 | -0.00059                  | -                        | -                         |
| $S_{zz}$                            | -                         | -1.17172                 | -1.29822                  | -                        | -                         |

Table A.1: Pre-calibrated values of the offset and sensitivity parameters of the accelerometers.

| Biaxial | Accelerometers | (23203A): |
|---------|----------------|-----------|
|---------|----------------|-----------|

| Range:  | $\pm 1.5g$                  |
|---|-----------------------------|
| Bandwidth:  | 0-15Hz                      |
| Noise:  | $0.5 mg/\sqrt{\mathrm{Hz}}$ |
| Offset Drift $(-40^{\circ}C \text{ to } 85^{\circ}C)$ : | $\pm 0.2g$                  |
| Nonlinearity:   | $\pm 0.2\% \mathrm{FS}$     |
| Misalignment:   | $\pm 2^{\circ}$             |
| Shock Survivability:                                    | $\pm 500g$                  |
| Power Supply:   | +5 to $30V$                 |

#### Triaxial Accelerometers (34103A):

| Range:  | $\pm 1.5g$                  |
|---|-----------------------------|
| Bandwidth:  | $0-6.9 \mathrm{Hz}$         |
| Noise:  | $0.5 mg/\sqrt{\mathrm{Hz}}$ |
| Offset Drift $(-40^{\circ}C \text{ to } 85^{\circ}C)$ : | $\pm 0.2g$                  |
| Nonlinearity:   | $\pm 0.2\% \mathrm{FS}$     |
| Misalignment:   | $\pm 2^{\circ}$             |
| Shock Survivability:                                    | $\pm 500g$                  |
| Power Supply:   | $+5\pm0.25V$                |



Figure A.2: The 23203A biaxial accelerometer.



FigureA.3: The34103Atriaxialaccelerometer.