Investigation of Biases in Doppler Centroid Estimation Algorithms

by

Tonghua Zhang

M.Sc., Peking University, China, 1996
B.S., University of Science and Technology of China, 1993

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Department of Electrical & Computer Engineering
The University of British Columbia
Vancouver, Canada

Date Aug 12, 1999
Abstract

Synthetic Aperture Radar (SAR) is a microwave imaging system capable of producing high-resolution imagery from data collected by a relatively small antenna. The Doppler centroid is an important parameter in the SAR signal processing. In principle, it is possible to calculate the Doppler centroid from orbit and attitude data. But the measurement uncertainties on these parameters will limit the accuracy of the estimation. Alternatively, the Doppler centroid can be estimated from the received data.

In the past a few years, a number of Doppler centroid estimation algorithms have been developed. These algorithms can be categorized as one of two kinds. The first kind of algorithm utilizes the signal amplitude. The second kind of algorithm utilizes the phase of the received signal, such as the DLR algorithm, the MLCC and the MLBF algorithms. It is assumed that the estimation algorithms based on the signal phase can obtain more accurate estimates.

The objective of this research is to examine and test the performance of the phased-based Doppler estimation algorithms with different scene contrasts, SNR levels and different squint angles, and examine the sensitivity of some phase-based Doppler estimation algorithms to radiometric discontinuities to find out how the radiometric discontinuities affect these estimation algorithms.

First, the signal model is carefully examined. The effect of range sampling is
discussed. The three candidate algorithms, the DLR, MLCC and MLBF, are introduced. Mathematical analysis of the ACCC angle and the contrast model are performed to obtain an insight of the operation of these algorithms.

Experiments on simulated data with different scene contrast and SNR level are performed to compare the performance of these candidate algorithms.

The MLCC algorithm works well with the ERS and J-ERS data, which normally have a low squint. However, it does not work reliably with the RADARSAT data. Since the RADARSAT data has a higher squint, simulations are performed to examine the effect of squint on the DLR, the MLCC and the MLBF algorithms.

Radiometric discontinuity has significant effect on the estimate of the phased-based algorithms. This thesis proposed a theory on the mechanism of how the radiometric discontinuity affects these algorithms in different ways. This thesis also proposed that, the MLBF is not affected by the radiometric discontinuity, the DLR algorithm is more sensitive to the azimuth discontinuity than the MLCC algorithm, whereas the MLCC algorithm is more sensitive to the range discontinuity than the DLR algorithm. These theories are proven by simulations and real data experiments.
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TONGHUA ZHANG
Chapter 1

Introduction

1.1 Background

Synthetic Aperture Radar (SAR) is a microwave imaging system capable of producing high-resolution imagery from data collected by a relatively small antenna [1, 2, 3, 4]. It is a technique for creating high resolution images of the earth's surface. It can be used to make remote observations of the earth during day or night, through cloud cover and even through light rain [5]. SAR images are widely applied in many areas, such as oceanography [6], geology, forestry [7] and national defense.

A SAR system consists of a microwave transmitter/receiver and an antenna mounted on a moving platform such as an orbiting satellite. The antenna transmits a beam of chirp signals to the ground and receives the reflected signal from the ground. The received signal consists of a superposition of a large amount of reflections from scatterers.
As in conventional radars, the high resolution in range of a SAR is attained using long transmitted pulses that are compressed to a short duration by range compression. The main advantage of SAR compared to conventional radar systems is that SAR achieves higher resolution in azimuth by coherently processing the phase history of targets in the azimuth direction [8].

SAR began with an observation by Carl Wiley in 1951 that a radar beam oriented obliquely to the radar platform velocity will receive signals having frequencies offset from the radar carrier frequency due to the Doppler effect [9]. Activities continued on analyses and signal processing techniques and many signal processing techniques are successfully developed, such as the Range/Doppler (R/D) algorithm [10, 11], the SPECAN algorithm [12] and the Chirp Scaling algorithm [13].

Basically the SAR signal processing consists of range processing and azimuth processing. An important parameter in relation to the SAR signal processing is the Doppler centroid. The Doppler centroid is the Doppler frequency received from a given point target on the ground when the target is centered in the azimuth antenna beam pattern. Because the transmitted signal of a SAR is pulsed with the Pulse Repetition Frequency (PRF), and the PRF is designed according to the azimuth bandwidth instead of the absolute azimuth frequency, the spectrum of the azimuth signal can be aliased by several PRF ambiguities. Thus the absolute Doppler centroid can be considered to be made up of an "integer PRF part" and a "fractional PRF part". The fractional PRF part is the centroid wrapped around to the fundamental frequency range of the PRF, and the integer PRF part is referred to as the Doppler ambiguity [14].
Since the Doppler centroid is an important parameter in the SAR signal processing, a lot of effort has been devoted to obtaining an accurate estimate of it. In principle, it is possible to calculate the Doppler centroid from orbit and attitude data. But the measurement uncertainties on these parameters will limit the accuracy of the estimation [14]. Alternatively, the Doppler centroid can be estimated from the received data [15].

In the past a few years, a number of Doppler centroid estimation algorithms have been developed. These algorithms can be categorized as one of two kinds. The first kind of algorithm utilizes the signal amplitude, such as the look range cross-correlation technique [16, 17], and the multiple PRF technique [18, 19]. The second kind of algorithm utilizes the phase of the received signal, such as the “DLR” algorithm [20], and the MLCC and the MLBF algorithms [14]. It is assumed that the estimation algorithms based on the signal phase can obtain more accurate estimates [14]. Thus, a in depth understanding of these algorithms is necessary.

The squint angle is another important parameter in the SAR geometry model. The MLCC algorithm works well with ERS and J-ERS data, which have a low squint. However, it does not work reliably with the RADARSAT data, which has a higher squint. The effect of the squint on the phase-based algorithms should be well examined.

Radiometric discontinuity in a SAR image is due to the difference of the properties of scatters on the ground surface. Some echoes of the chirp from a certain part of the ground surface are strong, whereas some echoes from another part of the ground surface are weak. The boundary of these two parts forms a discontinuity. This structure is very common in a SAR image, such as the boundary of ocean and land. Radiometric
discontinuity is an important factor that affects the performance of DOPCEN estimators, especially of the phase-based estimators.

1.2 Research Objectives

In this research, three phased-based Doppler centroid estimation algorithms (the DLR, the MLCC and the MLBF algorithms) will be examined. The objectives of the research project are to:

- examine and test the performance of the three candidate algorithms with different scene contrasts and SNR levels,
- examine the performance of these algorithms with higher squint data,
- examine the sensitivity of the candidate algorithms to radiometric discontinuities,

1.3 Outline

To obtain an insight of the performance of different DOPpler CENtroid (DOPCEN) estimators, a mathematical model of the received data should be established and well studied. In Chapter 2, the form of the received signal after demodulation is derived. Then the concept of DOPCEN is introduced. The effect of range sampling is discussed, illustrating that the Range Cell Migration (RCM) in the raw data causes a shift in the azimuth spectra, forming the dependence of DOPCEN on range time. Finally, the Principle of Stationary Phase is applied to approximate the spectrum of range-compressed
signal, showing that the DOPCEN is a linear function of range frequency under this approximation, which is the basis of the DOPCEN estimators discussed in the following chapter.

In Chapter 3, the concept of Average Cross Correlation Coefficient (ACCC) angle is introduced, which is used to estimate the fractional PRF part of the DOPCEN. Then the “DLR algorithm”, “Multi-Look Cross Correlation (MLCC) algorithm” and the “Multi-Look Beat Frequency (MLBF) algorithm” are introduced. Single target simulations are performed to illustrate the operation of these algorithms. The DLR and the MLCC algorithm use the ACCC angle to obtain the fractional PRF part of the DOPCEN and ambiguity number, whereas the MLBF algorithm uses the beat frequency to obtain the ambiguity number. The sensitivity of the ACCC angle and the beat frequency to the azimuth partial exposure is discussed.

In Chapter 4, experiments with simulated data and real SAR data are performed to compare the performance of the DLR, MLCC and MLBF algorithms. The method of generating the simulated data with multiple point targets in the low squint case is introduced. Simulation results are then presented. The effect of scene contrast and the effect of noise are then discussed. The raw data of ERS-1 in the yaw-steering mode are used to compare the performance of these algorithms on real data in the low squint case.

The effect of a squint mode imaging geometry on SAR signal properties is quite complicated. When the squint increases, the properties of the signal structure become more complicated as the cross-coupling between the range and azimuth signals increases. In Chapter 5, the effect of the squint on the ACCC angle is studied by simulations. Then simulations of the DLR, the MLCC and the MLBF algorithms on single point target and
multiple point target are performed to examine the performance these algorithms with higher squint. It is found that these three algorithms still work accurately to obtain the correct estimates. This proves that the approximation to the spectrum of a compressed target using the Principle of Stationary Phase is still accurate enough with high squint.

Radiometric discontinuity is an important factor that affects the performance of DOPCEN estimators, including the phase-based estimators. In Chapter 6, we will discuss the effect of the radiometric discontinuity on the DLR, MLCC and MLBF algorithms. Simulations are first performed. Then the performance of the DLR algorithm and the MLCC algorithm are compared on the discontinuity in the range and azimuth directions respectively. We explain that how the radiometric discontinuity affect the DLR and the MLCC algorithm, why the DLR algorithm is more sensitive to the azimuth discontinuity than the MLCC algorithm and why the MLCC algorithm is more sensitive to the range discontinuity than the DLR algorithm. We also explain why the MLBF algorithm is not affected by the radiometric discontinuity. These three algorithms are also performed on ERS-1 raw data to illustrate our conclusions.
Chapter 2

Model of the Received Signal

To obtain an insight into the performance of different DOPpler CENtroid (DOPCEN) estimators, the mathematical model of the received data should be established and well understood. In this chapter, the form of the received signal after demodulation is derived. Then the concept of DOPCEN is introduced.

The effect of range sampling is discussed, illustrating that the Range Cell Migration (RCM) in the raw data causes a shift in the azimuth spectra, forming the dependence of DOPCEN on range time for each target. Finally, the Principle of Stationary Phase is applied to approximate the spectrum of the range-compressed signal, showing that the DOPCEN is a linear function of range frequency under this approximation. This linear relationship is the basis of the DOPCEN estimators discussed in the following chapters.
2.1 Form of the Received Signal

The geometry model of SAR imaging of a single point target is shown in Figure 2.1.

The satellite carrying the SAR antenna moves with an effective velocity $V_r$ relative to the point target. The slant range $R$ is the distance from the antenna to the point target. Pulses are transmitted from the antenna to the ground with a constant frequency, the Pulse Repetition Frequency (PRF).

In the range direction, the received signal is an analog signal as a function of range time $\tau$. The "timing" of the received signal is reset at the beginning of each transmitted pulse, to constitute the beginning of a new range line. In this way the received signals
form a discrete signal along the azimuth direction with each new range line forming a new “sample” in azimuth time \( \eta \).

The nominal range to the point target \( R_0 \) is usually defined as the range when the point target is closest to the antenna, as the radar system passes by the point target. The time at which the slant range equals \( R_0 \) is defined as \( \eta_0 \). The exposure time is defined as the duration during which the point target is fully exposed in the radar beam, \( i.e. \) the time required to generate the Doppler bandwidth. The squint is defined as the angle between the antenna pointing direction and the direction perpendicular to the velocity, as \( \theta \) shown in Figure 2.1.

After transmitting each pulse, the SAR antenna receives the reflected signals. The properties of the received signals are determined by parameters of the SAR system and the characteristics of the reflectors.

The transmitted signal is a linear FM signal or a chirp, which is given by:

\[
S_t(\eta, \tau) = P(\tau) W(\eta - \eta_c) \cos[2\pi f_0 \tau + \pi K_r (\tau - \tau_l/2)^2], \quad \tau = [0, \tau_l] \quad (2.1)
\]

where \( f_0 \) is the center frequency of the transmitted chirp

\( \tau \) range time

\( \tau_l \) the exposure time

\( \eta \) azimuth time

\( K_r \) FM rate of the transmitted chirp

\( \eta_c \) azimuth time offset of beam center from zero Doppler

\( P(\tau) \) envelope of chirp, and

\( W(\eta) \) azimuth beam pattern magnitude
The received signal from the ideal point target is assumed to be the same as the transmitted signal except for a time delay $\tau_d$, and is given by:

$$S_r(\eta, \tau) = P(\tau-\tau_d)W(\eta-\eta_c)\cos(2\pi f_0(\tau-\tau_d)+\pi K(\tau-\tau_d-\eta/2)^2), \quad \tau = [\tau_d, \tau_d+\eta]$$

(2.2)

The time delay due to the varying slant range from the SAR antenna to the point target at different azimuth positions is given by:

$$\tau_d(\eta) = \frac{2R(\eta)}{c}, \quad \eta = [\eta_c - \eta_l/2, \eta_c + \eta_l/2]$$

(2.3)

where $\eta_l$ is the azimuth exposure time, and $R(\eta)$ is the instantaneous slant range from the antenna to the point target given by:

$$R(\eta) = \sqrt{R_0^2 + V_r^2 (\eta - \eta_0)^2}$$

(2.4)

By expanding (2.4) in a Taylor series around azimuth time $\eta_0$ and selecting $\eta_0$ as the azimuth time origin, $R(\eta)$ can be approximated by:

$$R(\eta) \approx R_0 + \frac{V_r^2}{2R_0} \eta^2$$

(2.5)

where $R_0$ is the closest slant range from the antenna to the point target, and $V_r$ is the effective velocity of the antenna with respect to the point target.

Consider the case of demodulation to baseband, which converts the signal to complex form. This is done by multiplying the received signal by the coherent oscillator signal given by:

$$h(\tau) = \exp\{-j2\pi f_0 \tau\}$$

(2.6)
The demodulated signal is given by:

\[ S_d(\eta, \tau) = h(\tau) S_r(\eta, \tau) \]

\[ = \frac{1}{2} P(\tau - \tau_d) W(\eta - \eta_c) \left( \exp\{j 2\pi f_0 (\tau - \tau_d) + j \pi K_r (\tau - \tau_d - \tau_1/2)^2\} + \exp\{-j 2\pi f_0 (\tau - \tau_d) - j \pi K_r (\tau - \tau_d - \tau_1/2)^2\} \right) \exp\{-j 2\pi f_0 \tau\} \]

\[ = \frac{1}{2} P(\tau - \tau_d) W(\eta - \eta_c) \left( \exp\{-j 2\pi f_0 (2\tau - \tau_d) - j \pi K_r (\tau - \tau_d - \tau_1/2)^2\} + \exp\{-j 2\pi f_0 (2\tau - \tau_d) - j \pi K_r (\tau - \tau_d - \tau_1/2)^2\} \right) \quad (2.7) \]

After a low pass filter is used to remove the \( 2f_0 \) component, the demodulated signal is given by:

\[ S_d(\eta, \tau) = \frac{1}{2} P(\tau - \tau_d) W(\eta - \eta_c) \exp\{-j 2\pi f_0 \tau_d + j \pi K_r (\tau - \tau_d - \tau_1/2)^2\} \quad (2.8) \]

### 2.2 The Doppler Centroid

From Equation (2.8), the received signal can be expressed as the product of an azimuth signal \( S_\alpha(\eta) \) and a range signal \( S_r(\tau) \) given by:

\[ S_\alpha(\eta) = W(\eta - \eta_c) \exp\{-j2\pi f_0 \tau_d\} \quad (2.9) \]

\[ S_r(\tau) = P(\tau - \tau_d) \exp\{j\pi K_r (\tau - \tau_d - \tau_1/2)^2\} \quad (2.10) \]

Note that because of the range-azimuth coupling, the range signal actually is still a function of azimuth time as well as of range time. However, after Range Cell Migration
Correction (RCMC), the locus of energy in the 2-D memory is aligned with the azimuth time axis.

The fine range resolution is achieved by compressing the range signal, as conventional radar systems do. The high azimuth resolution of the SAR system is achieved by compressing the azimuth signal. The azimuth signal can be approximated by a chirp because of the $\eta^2$ relation in (2.5). From Equation (2.5) and Equation (2.9), the azimuth signal can be given by:

$$S_a(\eta) = W(\eta - \eta_c) \exp \left\{ -j \frac{4\pi R_0}{c} f_0 \right\} \exp \left\{ -j 2\pi \frac{V_r^2}{c R_0} f_0 \eta^2 \right\} \quad (2.11)$$

Ignoring the constant part, the phase of the azimuth signal is

$$\phi(\eta) = -2\pi \frac{V_r^2}{c R_0} f_0 \eta^2 \quad (2.12)$$

Thus, the azimuth frequency or Doppler is given by:

$$f_a(\eta) = \frac{1}{2\pi} \frac{\partial \phi(\eta)}{\partial \eta} = -\frac{2 V_r^2}{c R_0} f_0 \eta \quad (2.13)$$

Equation (2.13) shows that the azimuth signal is also a chirp signal with the FM rate given by:

$$K_a = \frac{2 V_r^2}{c R_0} f_0 \quad (2.14)$$

The DOPCEN is defined as the Doppler or azimuth frequency of a given target, at the pulse or azimuth time when the target lies in the center of the beam. Thus the DOPCEN is given by:

$$F_{dc,0} = f_a(\eta_c) = -\frac{2 V_r^2}{c R_0} f_0 \eta_c \quad (2.15)$$
Since the transmitted signal is repeated at the PRF, and the PRF is chosen to properly sample the azimuth bandwidth instead of the absolute azimuth frequency, the spectrum of the azimuth signal is aliased by the PRF. Expressing the DOPCEN in units of one PRF, and considering the integer and fractional parts separately in PRF units, the DOPCEN can be considered to be made up of an "integer PRF part" and a "fractional PRF part". The fractional PRF part is the centroid wrapped around to the fundamental frequency range of the PRF and is the part estimated by finding the peak of the Doppler spectrum. The integer PRF part is referred to as Doppler Ambiguity, and must be estimated by other means.

2.3 The Effect of Range Sampling

After demodulation, the baseband range signal is sampled with the frequency $F_s$ and stored in a 2-D memory. Each echo of a transmitted chirp forms a range line and each range sample at the same sampling time forms an azimuth line. Figure 2.2 illustrates the locus of energy of an uncompressed point target in SAR signal memory.

The instantaneous range sampling time is shown by the vertical dotted lines. The interval between each horizontal dashed line is the time interval between each pulse, i.e. $1/PRF$. The echo of each transmitted chirp from the point target is shown by the solid line along the range time direction. The samples containing non-zero point target energy are shown by circles.

Assume that the first echo arrives exactly at a sampling time, then the interval of the range time is given by:
Figure 2.2: Illustrating the Sampling of Point Target Energy in SAR Signal Space

\[ \tau = [\tau_d(\eta_c - \eta_l/2), \tau_d(\eta_c - \eta_l/2) + \tau_i] \]  

(2.16)

where \( \tau_d(\eta_c - \eta_l/2) \) is given by:

\[ \tau_d = \frac{2R_0}{c} + \frac{V_c^2}{c R_0} (\eta_c - \eta_l)^2 \]  

(2.17)

Let:

\[ \tau = \mu + \tau_d(\eta_c - \eta_l/2) \]  

(2.18)

\[ \xi = \mu - \tau_l/2 \]  

(2.19)

\[ \alpha = \frac{V_c^2}{c R_0} \]  

(2.20)
and \( \beta = (\eta_c - \eta_l/2) \) \hspace{1cm} (2.21)

then from Equation (2.8), the 2-D phase of the demodulated signal is given by:

\[
\phi(\eta, \tau) = -2\pi f_0 \tau_d + \pi K_r \left[ \xi + \alpha(\beta^2 - \eta_l^2) \right]^2
\] \hspace{1cm} (2.22)

Thus the Doppler or the azimuth frequency is given by:

\[
f_a = \frac{1}{2\pi} \frac{\partial \phi(\eta, \tau)}{\partial \eta} = -2\alpha \left[ f_0 + K_r \xi + \alpha \beta^2 K_r \right] \eta + 2\alpha^2 K_r \eta^3
\] \hspace{1cm} (2.23)

where \( \xi = [-\pi/2, \pi/2] \). Thus the term \( f_0 + K_r \xi \) in Equation (2.23) is actually the instantaneous range frequency. For the low squint case, such as ERS-1 in yaw-steering mode, \( \alpha \approx 2.0 \times 10^{-7}, \beta \approx -0.15 \text{ s}, \) and \( K_r = 0.5 \text{ GHz/s} \). The term \( \alpha \beta^2 K_r \) in Equation (2.23) is about 2 KHz. Compared to the range bandwidth, which is about 15 MHz, it is negligible. The cubic item \( 2\alpha^2 K_r \eta^3 \) in Equation (2.23) is less than 0.02 Hz, which is also negligible. Thus the azimuth frequency can be approximated in low squint case by:

\[
f_a = -2\alpha f \eta = -2 \frac{V_r^2}{c R_0} f \eta
\] \hspace{1cm} (2.24)

where \( f \) is the instantaneous transmitted range frequency.

The DOPCEN is then given by:

\[
F_{dc} = -2 \frac{V_r^2}{c R_0} \eta_c f
\] \hspace{1cm} (2.25)

From Equation (2.25), it can be seen that the DOPCEN is a linear function of the transmitted range frequency. Compared to Equation (2.15), where there is no Range Cell Migration (RCM), it can be concluded that, if there is no RCM, the DOPCEN is
a constant for each azimuth line. If RCM does exist, as happens in the raw data, the DOPCEN is a linear function of range frequency in low squint case.

2.4 The Spectrum of the Range Compressed Signal

Range compression is the first main step in many SAR processing algorithms. Most DOPCEN estimators work on range compressed data. Range compression is performed by convolving a matched filter with each range line. Fast convolution is usually used to obtain computing efficiency. Each range line and the matched filter are transferred from the range time domain to the range frequency domain using range FFTs and are multiplied together.

The demodulated signal in the azimuth time, range frequency domain is given by:

\[
S_d(n, f_r) = \int_{-\infty}^{+\infty} S_d(\eta, \tau) \exp\{-j 2\pi f_r \tau\} d\tau \\
= \int_{-\infty}^{+\infty} S_a(\eta) S_r(\eta, \tau) \exp\{-j 2\pi f_r \tau\} d\tau \\
= S_a(\eta) \int_{-\infty}^{+\infty} P(\tau - \tau_d) \exp\{j \pi K_r(\tau - \tau_d - \tau_f/2)\} \exp\{-j 2\pi f_r \tau\} d\tau \\
= S_a(\eta) S_r(\eta, f_r) \tag{2.26}
\]

where \( S_r(\eta, f_r) \) is the Fourier transform of \( S_r(\eta, \tau) \), \( f_r \) is the fundamental range frequency and is given by:

\[
f_r = f - f_0, \quad -BW_r/2 \leq f_r \leq BW_r/2 \tag{2.27}
\]
where $BW_r$ is the range bandwidth given by:

$$BW_r = K_r \xi \quad -\tau_l/2 \leq \xi \leq \tau_l/2 \quad (2.28)$$

It has been proven that $S_r(\eta, f)$ has no analytical expression since it involves a Fresnel integral. The Principle of Stationary Phase is applied to approximate the expression of the range spectrum. The Principle of Stationary Phase states that, when the phase $\pi[K_r(\tau - \tau_d - \tau_l/2)^2 - 2f\tau]$ changes rapidly, the positive and negative values of the function will be approximately canceled by each other in the integration and have negligible contribution to the integral. Only those parts where the phase changes slowly have the most significant contribution to the integral. By using the Principle of Stationary Phase, the range spectrum can be approximated by [21]:

$$S_r(\eta, f_r) = |K_r|^{-1/2} \exp\left(j \frac{\pi}{4} \text{sgn}(K_r)\right) \exp\left(-j \pi \left[\frac{f_r^2}{K_r} + 2f_r \tau_d + f_r \tau_l\right]\right) \quad (2.29)$$

The range matched filter of $S_r(\eta, \tau)$ is generated according to the complex envelope of the transmitted chirp and is given by:

$$M_r(\mu) = S_r^*(\eta, -\mu) = \exp\left(-j \pi K_r (-\mu - \tau_l/2)^2\right)$$

(2.30)

Again applying the Principle of Stationary Phase, the spectrum of the range matched filter is given by:

$$M_r(f_r) = \int_{-\infty}^{+\infty} M_r(\mu) \exp\{-j 2\pi f_r \tau\} d\tau$$

$$= |K_r|^{-1/2} \exp\left\{j \frac{\pi}{4} \text{sgn}(K_r)\right\} \exp\left(-j \pi \left[-\frac{f_r^2}{K_r} - \pi f_r \tau_l\right]\right) \quad (2.31)$$
Thus the range compressed signal is given by:

\[ S_{rc}(\eta, f_r) = S_a(\eta) S_r(\eta, f_r) M_r(f_r) \]

\[ = W(\eta - \eta_c) \exp\{-j 2\pi f_0 \tau_d\} \frac{1}{|K_r|} \exp\left\{j \frac{\pi}{2} \text{sgn}(K_r)\right\} \exp\{-j 2\pi f_r \tau_d\} \]

\[ = \frac{W(\eta - \eta_c)}{|K_r|} \exp\left\{j \frac{\pi}{2} \text{sgn}(K_r)\right\} \exp\{-j 2\pi (f_0 + f_r) \tau_d\} \]

\[ = \frac{W(\eta - \eta_c)}{|K_r|} \exp\left\{j \frac{\pi}{2} \text{sgn}(K_r)\right\} \exp\{-j 2\pi f \tau_d\} \quad (2.32) \]

We can see that the azimuth frequency can be given by:

\[ f_a = -\frac{\partial}{\partial\eta} (f \tau_d) = -\frac{2V_r^2}{c R_0} f \eta \quad (2.33) \]

Thus the DOPCEN is given by:

\[ F_{dc} = -\frac{2V_r^2}{c R_0} \eta_c f \quad (2.34) \]

Thus we see that from Equation (2.34), after range compression, the DOPCEN of the range compressed signal is a linear function of the range frequency.
Chapter 3

Phase-based DOPCEN Estimation Algorithms

In Chapter 2, the model of a point target and the concept of the Doppler centroid (DOPCEN) were introduced. There we conclude that, after range compression, the DOPCEN is approximately a linear function of range frequency. This is the basic principle of two of the “phase-based” DOPCEN algorithms we will review in this chapter.

In this chapter, the concept of Average Cross Correlation Coefficient (ACCC) angle is introduced, which is used in two of the phase-based DOPCEN estimators. Then the main phase-based algorithms are introduced:

1. the “DLR” algorithm,

2. the “Multi-Look Cross Correlation” (MLCC) algorithm, and

3. the “Multi-Look Beat Frequency” (MLBF) algorithm.
The first two algorithms, the DLR and the MLCC algorithms, use the ACCC angle to obtain the fractional PRF part of the DOPCEN and the ambiguity. The MLBF algorithm uses beat frequency to obtain the ambiguity.

Single point target simulations are performed to illustrate the operation and properties of these algorithms.

Finally, we discuss the distinction of the ACCC-angle-based algorithms and the beat-frequency-based algorithm.
3.1 The Average Cross Correlation Coeff. Angle

Most of the early DOPCEN estimators used the distribution of spectral energy to form the estimate [10, 15]. Recently, several estimators have been developed which make use of the phase of the received signal to get more accurate DOPCEN estimates. In 1989, Madsen [22] used an algorithm based on using azimuth phase increments in raw signal data to estimate the fractional PRF part of DOPCEN. The phase increments are estimated by taking the average cross correlation coefficient (ACCC) between adjacent azimuth samples\(^1\).

From Equation (2.32), after demodulation and range compression, the azimuth signal in the azimuth-time, range-frequency domain can be written as:

\[
S_a(\eta) = W(\eta - \eta_c) \exp\{-j 2\pi f \tau_d\}
\]

\[
= W(\eta - \eta_c) \exp\left\{-j \frac{4\pi f}{c} R(\eta)\right\}
\]

(3.1)

Ignoring the large constant \(R_0\) term in the range equation and using the quadratic approximation (2.5), the range compressed signal can be expressed as:

\[
S_a(\eta) = W(\eta - \eta_c) \exp\{-j \pi K_a \eta^2\}
\]

(3.2)

where \(K_a\) is the azimuth FM rate:

\[
K_a = \frac{2 V_r^2 f}{c R_0}
\]

(3.3)

The average cross correlation coefficient ACCC is defined as the average correla-

\(^1\)Madsen actually used an approximation to the ACCC by taking the sign of the data samples. This was done for computing efficiency.
tion between one azimuth sample and the next, and can be computed by the sum over azimuth time:

\[ C = \sum_{\eta} S_a(\eta) S^*_a(\eta + \Delta\eta) \]  

(3.4)

where \( \Delta\eta = 1/\text{PRF} = 1/F_a \) is the time between consecutive azimuth samples.

From Equation (3.2), the ACCC of the range compressed azimuth signal is given by:

\[ C = \sum_{\eta} W(\eta - \eta_c) W(\eta + \Delta\eta - \eta_c) \exp\{-j\pi K_a \eta^2\} \exp\{j\pi K_a(\eta + \Delta\eta)^2\} \]
\[ \approx \sum_{\eta} W^2(\eta - \eta_c) \exp\{j2\pi K_a \eta \Delta\eta\} \]
\[ = \sum_{\eta} M(\eta) \cos(2\pi K_a \eta \Delta\eta) + j \sum_{\eta} M(\eta) \sin(2\pi K_a \eta \Delta\eta) \]  

(3.5)

where \( M(\eta) = W^2(\eta - \eta_c) \) and is symmetric with respect to \( \eta_c \). Thus the ACCC angle is given by:

\[ \phi = \arg\{C\} \]
\[ = \tan^{-1}\left[ \frac{\sum_{\eta} M(\eta) \sin(2\pi K_a \eta \Delta\eta)}{\sum_{\eta} M(\eta) \cos(2\pi K_a \eta \Delta\eta)} \right] \]
\[ = \tan^{-1}\left[ \frac{M(\eta - \delta_1) \sin(2\pi K_a(\eta_c - \delta_1)\Delta\eta) + M(\eta_c + \delta_1) \sin(2\pi K_a(\eta_c + \delta_1)\Delta\eta) + \ldots}{M(\eta - \delta_1) \cos(2\pi K_a(\eta_c - \delta_1)\Delta\eta) + M(\eta_c + \delta_1) \cos(2\pi K_a(\eta_c + \delta_1)\Delta\eta) + \ldots} \right] \]
\[ = \tan^{-1}\left[ \frac{\sin(2\pi K_a \eta_c \Delta\eta)}{\cos(2\pi K_a \eta_c \Delta\eta)} \right] \quad \text{by symmetry} \]
\[ = \frac{2\pi K_a \eta_c}{F_a} \]  

(3.6)

where \( \delta_1, \delta_2 \ldots \) are the azimuth time intervals \( n\Delta\eta \), symmetric with respect to \( \eta_c \).
The relationship between the DOPCEN and the ACCC angle would be given by:

\[ F_{dc} = K_a \eta_c = \frac{F_a}{2\pi} \phi \]  

(3.7)

if \( \phi \) could be computed without the wraparound of the arc tan function. But this estimate is affected by the wraparound, and so Equation (3.7) can only estimate the component of the DOPCEN lying in the frequency range \((0, F_a)\) or \((-F_a/2, F_a/2)\), i.e. the fractional PRF part. We denote the fractional PRF part as \( F_{dca} \) which is given by:

\[ F_{dca} = \frac{F_a}{2\pi} \lfloor \phi \rfloor \mod 2\pi \]  

(3.8)

### 3.1.1 Illustrating ACCC using a Point Target

Consider a single point target that exhibits a quadratic azimuth phase history, as shown in the solid line of Figure 3.1(a). The target duration is chosen so that the target sweeps through Doppler frequency \([-0.5F_a/O_s, 0.5F_a/O_s]\) plus an offset given by the selected Doppler centroid. In Figure 3.1, \( F_a = 1600 \) Hz, the DOPCEN is -200 Hz and the oversampling ratio \( O_s = 1.1 \).

The phase increments per azimuth sample are examined. These phase increments are drawn in the dashed line in Figure 3.1(a). When the current azimuth sample is multiplied by the complex conjugate of the previous sample, a complex number is obtained whose amplitude represents the square of the signal amplitude, and whose phase represents the phase increment \( \phi \) discussed above.

These complex numbers can be drawn as vectors in the complex plane, as shown in Figure 3.1(b). The line marked Z represents the product at the zero Doppler point,
where the phase increment is zero. Note how the length of the vectors is small at the end of the target exposure, at points A and B, and is larger near the middle of the exposure. This effect is due to the azimuth antenna pattern $W(\eta - \eta_c)$.

When the vectors of Figure 3.1(b) are added together coherently, i.e. they are averaged, the long vector shown in Figure 3.1(b) is obtained, indicating the preferred direction of the phase increments. Thus the angle of the long vector represents the fractional part of the DOPCEN. In this case, it is estimated as -200 Hz.
(a) Target phase (*1) and phase increment per sample (*10)

(b) Individual and average phase increment

Estimated Dopcen = -200 Hz

Figure 3.1: Illustration of an ACCC Calculation
3.2 The DLR Algorithm

In 1991, the German Aerospace Establishment (DLR) developed a DOPCEN estimator [20] based upon the property that the absolute or unaliased DOPCEN is a function of the frequency of the transmitted signal, as we discussed in Chapter 2.

From Equation (2.34), the unaliased DOPCEN is given by:

\[ F_{dc} = -\frac{2V_r^2}{c R_0} \eta_c f = kf \]  \hspace{1cm} (3.9)

where \( k = -2 V_r^2 \eta_c / c R_0 \) is the slope of this linear function of range frequency.

From the Chapter 2, the DOPCEN can be considered to be made up of two parts, the fractional PRF part and the Doppler ambiguity, as given by:

\[ F_{dc} = F_{dca} + m F_a \]  \hspace{1cm} (3.10)

where \( m \) is the Doppler ambiguity number.

After demodulation by \( f_0 \), the complex signal has the range frequency:

\[ f_r = f - f_0, \quad \frac{-BW_r}{2} \leq f_r \leq \frac{BW_r}{2} \]  \hspace{1cm} (3.11)

Thus the absolute Doppler centroid is:

\[ F_{dc} = kf = k (f_0 + f_r) = kf_0 + kf_r \]  \hspace{1cm} (3.12)

One of the main purposes of DOPCEN estimators is to estimate the average value of \( F_{dc} \), i.e. to estimate the DOPCEN at the centre range frequency \( f_0 \) given by\(^2\):

\(^2\)The DOPCEN is also a function of range time, and is usually estimated at several ranges. However, we will not address that aspect of the estimation in this report.
\[ F_{dc,0} = k f_0 = -\frac{2 V_r^2}{c R_0} \eta_c f_0 \] (3.13)

From the SAR system specifications, the centre range frequency \( f_0 \) can be obtained accurately. Thus we only need to estimate the slope \( k \). From Equation (3.10):

\[ F_{dca} = F_{dc} - m F_a = k f_0 + k f_r - m F_a \] (3.14)

Thus the slope \( k \) can be estimated using:

\[ \frac{\partial F_{dca}}{\partial f_r} = \frac{\partial}{\partial f_r} (k f_0 + k f_r - m F_a) = k \] (3.15)

The DLR algorithm works in the range frequency domain, in which the various range frequencies in the demodulated signal can be accessed by FFTing the compressed range signal\(^3\). As we discussed in the last section, the algorithm can use the ACCC angle to estimate the fractional PRF part of DOPCEN. The ACCC between adjacent azimuth samples at each range frequency cell is computed, then the slope \( k \) of the ACCC angles as a function of range frequency is estimated. Then the Doppler ambiguity can be obtained from:

\[ m = \text{round} \left( k f_0 - F_{dca} \right) \] (3.16)

and the unaliased DOPCEN is obtained from the ambiguity number \( m \) and the estimation of the fractional PRF part \( F_{dca} \) using (3.10).

In the real case, due to the variation of the DOPCEN in the along and across track directions, the data must be segmented in blocks at the beginning of this algorithm.

\(^3\)Or not performing the range IFFT just yet in range compression.
The required segmentation, on the other hand, raises the problem of partially covered chirps, both in range and azimuth. Therefore, the data must be range compressed before segmentation, whenever the range segments are smaller or in the order of a range chirp [20].

The DLR algorithm is a Doppler Ambiguity Resolver (DAR) applied to different range frequencies. Hence, its accuracy can be predicted using the well-established theories for DOPCEN estimators [22, 23].

3.2.1 Illustration of DLR with a Single Point Target

To illustrate the principle of the DLR algorithm, a single point target simulation is performed. In the point target simulation, parameters close to those of the RADARSAT and ERS-1 SAR systems are used. However, to keep the simulation to reasonable array sizes, the parameters are modified slightly, and are given in Table 3.1.

Compared to the real SAR parameters, we use somewhat lower range and azimuth bandwidths. The Time Bandwidth Products (TBPs) we use in the simulations are 340 and 311 in the range and azimuth directions respectively. Although the TBPs are reduced in both the range and azimuth directions, they are still large enough to ensure the accuracy of the simulation by using the Principle of Stationary Phase in the low squint case.

Range compression is performed on each range line. Next the ACCC of each azimuth line is computed according to Equation (3.4) in the range frequency, azimuth time domain. ACCC angles are then computed as a function of range frequency, as
<table>
<thead>
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<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
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<td>km</td>
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<tr>
<td>effective radar velocity</td>
<td>( V_r )</td>
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<td>m/s</td>
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<td>MHz</td>
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<td>( \tau_l )</td>
<td>20</td>
<td>( \mu \text{s} )</td>
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<td>Doppler bandwidth</td>
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<td>( F_a )</td>
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<tr>
<td>DOPCEN</td>
<td>( F_{dc,0} )</td>
<td>-400</td>
<td>Hz</td>
</tr>
</tbody>
</table>

Table 3.1: Parameters of the Single Point Target Simulation

A straight line is then fitted to the high-energy regions of Figure 3.2, and is shown after conversion to Hz units in Figure 3.3.\(^4\) From the straight line fit, the mean value is found to be \(-400.1\) Hz and the slope is estimated as \(-82\) mrad/MHz. Thus the fractional PRF part of the DOPCEN is estimated as \(-400.1\) Hz, and the integer PRF part of the DOPCEN is estimated as \(-431\) Hz, which are both very accurate for their respective purposes.

In Figure 3.3, the absolute centroid is estimated by projecting the sloped line back to the radar centre frequency, at which point it should intercept the vertical axis at \( F_{dc} \).\(^5\)

\(^4\)Note only those values within the dashed line in Figure 3.2 are used to estimate the slope since this part is the region within which the main energy of the signal is located.

\(^5\)Note that in the present simulation, we have not included any variation of beam pointing angle with range frequency, so the bias \( F_{0i} \), often found in the DLR and MLCC estimators is zero.
The ambiguity number is then obtained from (3.16), and the correct value of $m = 0$ is found.
Fit the ACCC Angle in a 1-Order Polynomial, DOPCEN = -400 Hz

Figure 3.2: ACCC angles as a linear function of range frequency

Estimation of DOPCEN, DOPCEN = -400 Hz

Figure 3.3: Estimation of the Absolute DOPCEN in the DLR Algorithm
3.3 The MLCC Algorithm

In 1996, a combined SAR DOPCEN estimation scheme based on signal phase was published [14]. This scheme uses two complementary Doppler estimation algorithms, both utilizing the phase information embedded in the radar signal. In each algorithm, upper and lower parts of the available range bandwidth of the received signal are extracted to form two range looks.

The first algorithm, called Multilook Cross Correlation (MLCC), computes the average cross correlation coefficient between adjacent azimuth samples for each of the two looks and then takes the difference between the ACCC angles of the two range looks. The Doppler ambiguity is determined from the angle difference, again projecting the difference back to zero range frequency. The fractional PRF part is also determined from the cross correlation coefficients. In this section, the MLCC algorithm will be introduced. The second algorithm, called Multi-Look Beat Frequency (MLBF) will be reviewed in the next section.

The principle of these algorithms is to generate two independent range looks to emulate two SAR systems imaging the same region of the earth's surface. Each system works at a slightly different frequency given by:

\[ f_1 = f_0 - \frac{\Delta f}{2} \]  

(3.17)

\[ f_2 = f_0 + \frac{\Delta f}{2} \]  

(3.18)

where \( f_0 \) is the centre frequency of the real SAR system and \( \Delta f \) is the separation of the centre frequencies of the two emulated SAR systems.
The operation of the MLCC algorithm is illustrated by considering two range compressed looks of a single point target. From Equation (2.32), ignoring the target's complex amplitude and range envelope, the range compressed signals of the two looks are given by:

\[
S_1(\eta) = W(\eta - \eta_c) \exp\{-j2\pi f_1 \tau_d\} \\
= W(\eta - \eta_c) \exp\{-j\frac{4\pi f_1 R(\eta)}{c}\} \quad (3.19)
\]

\[
S_2(\eta) = W(\eta - \eta_c) \exp\{-j2\pi f_2 \tau_d\} \\
= W(\eta - \eta_c) \exp\{-j\frac{4\pi f_2 R(\eta)}{c}\} \quad (3.20)
\]

The phase arguments in Equations (3.19) and (3.20) give the azimuth phase history of the target, which are different between looks 1 and 2 because of the frequency separation \( \Delta f \).

By expanding the instantaneous slant range \( R(\eta) \) in Taylor series at \( \eta = \eta_0 \), as shown in Equation (2.4), and ignoring the constant phase terms, the two range look signals can be expressed as:

\[
S_1(\eta) = W(\eta - \eta_c) \exp\{-j\pi K_{a1} \eta^2\} \quad (3.21)
\]

\[
S_2(\eta) = W(\eta - \eta_c) \exp\{-j\pi K_{a2} \eta^2\} \quad (3.22)
\]

where \( K_{a1} \) and \( K_{a2} \) are the azimuth FM rates of the two looks given respectively by:

\[
K_{a1} = \frac{2 V_r^2 f_1}{c R_0} = \frac{2 V_r^2 (f_0 - \Delta f/2)}{c R_0} \quad (3.23)
\]
The difference between the azimuth FM rates is given by:

\[ K_{a2} - K_{a1} = \frac{2 V_r^2 \Delta f}{c R_0} = K_{a0} \frac{\Delta f}{f_0} \]

where \( K_{a0} \) is the azimuth FM rate for the azimuth signal centered at \( f_0 \):

\[ K_{a0} = \frac{2 V_r^2 f_0}{c R_0} = \frac{1}{2} (K_{a1} + K_{a2}) \]

The ACCCs of look 1 and look 2 are given by:

\[ C_1 = \sum_{\eta} S_1(\eta) S_1^*(\eta + \Delta \eta) \]

\[ C_2 = \sum_{\eta} S_2(\eta) S_2^*(\eta + \Delta \eta) \]

From Equation (3.6), the ACCC angles of look 1 and look 2 are given by:

\[ \phi_{L1} = \text{arg}[C_1] = \frac{2 \pi K_{a1} \eta_c}{F_a} \]

\[ \phi_{L2} = \text{arg}[C_2] = \frac{2 \pi K_{a2} \eta_c}{F_a} \]

From Equation (3.13) and (3.25), the difference of the ACCC angles of the two range looks is given by:

\[ \Delta \phi = \phi_{L2} - \phi_{L1} = \frac{2 \pi (K_{a2} - K_{a1}) \eta_c}{F_a} \]
\[
\Delta \phi = 2\pi \Delta f \frac{K_{a0} \eta_c}{F_a} = -2\pi \frac{\Delta f}{f_0} \frac{F_{dc,0}}{F_a} \quad (3.31)
\]

Since \(\Delta \phi\) is much less than a radian, angle wrap around is not a problem and thus the absolute DOPCEN at centre frequency \(f_0\) is given by:

\[
F_{dc,0} = -\frac{f_0 F_a \Delta \phi}{2\pi \Delta f} \quad (3.32)
\]

In practice, the value of \(F_{dc,0}\) determined by Equation (3.32) may not be accurate enough. To improve its accuracy, the fractional PRF part \(F_{dca}\) is determined directly from the ACCC angle [14], and the estimation of Equation (3.32) is used only to obtain the Doppler ambiguity \(m\).

From Equation (3.29), (3.30) and (3.26), the fractional PRF part is obtained from the average (aliased) phase increments by:

\[
F_{dca} = -\frac{F_a}{2\pi} \frac{\phi_{L1} + \phi_{L2}}{2} = -K_{a0} \eta_c \quad (3.33)
\]

This method is similar to that proposed by Madsen [22], except that the sign approximation is not made here. The error tolerance in the two ACCC angles is relatively robust, as an error as high as 5° in the ACCC angles causes an error of only 0.0014 \(F_a\) in \(F_{dca}\) [14].

The Doppler ambiguity is then estimated by:

\[
m = \text{round} \left( \frac{F_{dc,0} - F_{dca}}{F_a} \right) \quad (3.34)
\]
Finally, the absolute DOPCEN is then obtained by:

$$F_{dc} = F_{dcn} + m F_a$$  \hspace{1cm} (3.35)

The accuracy of the MLCC method depends on the range look bandwidth and the look separation. The optimal separation of the looks is found to be $\Delta f = 2W_s/3$ and the optimal look bandwidth is $W_s/3$, where $W_s$ be the range bandwidth of the signal [14].

3.3.1 Illustration of MLCC with a Single Point Target

To illustrate the operation of MLCC algorithm, the same point target which is used in the DLR algorithm simulation is used. The range bandwidth is 17 MHz, thus the separation of the two range looks is set to 11 MHz and the bandwidth of each look is set to 5.6 MHz. Figure 3.4 shows the range spectrum of a compressed range line. After range look extraction, IFFTs are performed on each look to transfer the signal back to the range time domain.

Figure 3.5 shows the compressed pulses of the two range looks. ACCC angles of each look are computed and the DOPCEN is obtained. In this simulation, the estimation of the fractional PRF part of the DOPCEN is -401 Hz and the integer part is -425 Hz. Since the principle of the MLCC and DLR algorithms is same, it is easy to predict that their estimation properties will be almost the same.
The Spectrum of a Range Line, DOPCEN = -400 Hz

Figure 3.4: The Spectrum of a Range Line

(a) Compressed Pulse of Look 1, DOPCEN = -400 Hz

(b) Compressed Pulse of Look 2, DOPCEN = -400 Hz

Figure 3.5: Compressed Pulses of the Two Range Looks
3.4 The MLBF Algorithm

In the second algorithm, called Multilook Beat Frequency (MLBF), the two range looks are multiplied together to generate a beat signal. The beat frequency is then estimated and the Doppler ambiguity is determined from the beat frequency.

The operation of MLBF algorithm can also be understood by examining what happens to a single point target. From Equation (3.21) and (3.22), the resultant beat signal $S_b(\eta)$ for a point target is given by:

$$S_b(\eta) = S_1(\eta) S_2^*(\eta) = |W(\eta - \eta_c)|^2 \exp\{j\pi(K_{a2} - K_{a1})\eta^2\}$$  \hspace{1cm} (3.36)

Because $K_{a1}$ and $K_{a2}$ are quite close to one another and the Doppler bandwidth is limited, the frequencies of the beat signal are confined to a narrow bandwidth. Thus, a distinct beat frequency is discernible and the average beat frequency $f_b$ is given by:

$$f_b = (K_{a2} - K_{a1})\eta_c$$
$$= K_{a0} \frac{\Delta f}{f_0} \eta_c$$
$$= -\frac{\Delta f}{f_0} F_{dc,0}$$  \hspace{1cm} (3.37)

The beat frequency is estimated by taking the FFT of the beat signal, often finding the peak only to the nearest cell. The absolute DOPCEN is then estimated by:

$$F_{dc,0} = -\frac{f_0}{\Delta f} f_b$$  \hspace{1cm} (3.38)
3.4.1 Illustration of MLBF with a Single Point Target

The operation of MLBF algorithm also can be illustrated by the single point target simulation. The two range looks generated in the MLCC algorithm can be used to generate the beat signal according to Equation (3.36). Then FFTs are performed on the beat signal to estimate the average beat frequency.

Figure 3.6 shows the spectrum of the beat signal. Since the envelope of the beat signal is approximately symmetric with respect to $\eta = \eta_c$, the peak of the spectrum can be identified. In the plot, the peak is located at the 3rd range frequency sample, corresponding to a beat frequency of 0.9 Hz. From Equation (3.38), the integer part of the DOPCEN is estimated to be $-457$ Hz.
Figure 3.6: Spectrum of the MLBF beat signal with a single point target
3.5 Discussion

The operation of the DLR, MLCC and MLBF algorithms have been introduced in this chapter. All of these algorithms are based on the principle that the DOPCEN is approximately a linear function of the range frequency. Single point target simulations are performed to illustrate the operations of these algorithms. From the simulations, the estimates are almost the same since they are based on the same principle.

However, the DLR and the MLCC algorithm use the ACCC angle to obtain the fractional part of the DOPCEN and the ambiguity, whereas the MLBF algorithm uses the beat frequency. This makes the DLR and the MLCC algorithm more sensitive than the MLBF algorithm to azimuth partial exposures.

When a target is not fully exposed in the azimuth beam or it is partially covered by the azimuth beam, we say it is partially exposed. Figure 3.7 illustrates the effect of azimuth partial exposure on the ACCC angles. The DOPCEN we used here is 0 Hz.

In Figure 3.7, the azimuth partial exposure is emulated by taking away some ACCC vectors at the end of the exposure. The ACCC angle is biased significantly. Figure 3.8 shows the error in the ACCC angle introduced by the azimuth partial coverage. When 3/4 of the ACCC vectors are taken away, the error is about \(-496\) Hz, which is about 30% of the PRF.

The MLBF algorithm uses the beat frequency. The change of the beat frequency along the azimuth exposure is very small. For example, using the same parameters in the above experiment, the average beat frequency at the first 1/4 of the azimuth exposure is about \(-0.7\) Hz, leading to an estimate error of about 324 Hz, which is about 20% of
Figure 3.7: Illustration of an ACCC Calculation with azimuth partial exposure when $PRF = 1600 \ Hz$. 

42
Effect of partial azimuth exposures on ACCC angles

Figure 3.8: Error in ACCC Calculation with azimuth partial exposure
the PRF. The average beat frequency at the last 1/4 of the azimuth exposure is about 0.7 Hz, leading to an estimate error of about $-324$ Hz, which is also about 20\% of the PRF. The change of the beat frequency is 1.4 Hz. Both estimates can give the correct ambiguity number. Note that the beat frequency tolerance for $\pm 0.5$ PRF is about $\pm 0.9$ Hz. Thus, the MLBF algorithm is less sensitive to partial exposure than the DLR and the MLCC algorithm, which are based on the ACCC angle. This conclusion will be used to explain the effect of the radiometric discontinuity on these algorithms in Chapter 6.

In the next chapter, we will see that the ACCC angle gives the most accurate estimate of the fractional PRF part of the DOPCEN. The DLR and the MLCC algorithms use the ACCC angle, whereas the MLBF algorithm does not. For this reason, the MLBF algorithm should not be used to estimate the fractional PRF part.
Chapter 4

Experiments in the Low-Squint Case

In this chapter, experiments with simulated data and real SAR data are performed to compare the performance of the DLR, MLCC and MLBF algorithms when the antenna squint angle is relatively low.

First, the method of generating the simulated data with multiple point targets is outlined. Simulation results are presented, and the effects of scene contrast and noise on the algorithm performance are then discussed.

Finally, experiments are done with low-squint SAR data to illustrate and compare the performance of the algorithms. ERS data is used, as it has low squints when operated in yaw-steering mode. In most yaw-steering scenes, the ERS DOPCEN is close to -400 Hz.
4.1 Generation of Multiple-Target Simulated Data

4.1.1 Methodology

To compare the performance of the three algorithms, simulated data are generated. Considering the computer memory and computing efficiency, we generate a $1K \times 1K$ data block, with the same length of 1024 samples in the range and azimuth directions.

To obtain the most realistic simulation, each image sample will have an independent target, whose amplitude and phase we can specify. Then a 2D convolution will be used to generate the received signal from these targets, with particular attention paid to obtaining a symmetrical set of partially exposed targets in the final simulation array. Figure 4.1 illustrates the simulation methodology.

![Diagram](image.png)

Figure 4.1: Convolution to Generate the Simulated Data of Multiple Point Targets
First, the same single point target is generated as used in the last chapter. Therefore, the specified DOPCEN is -400 Hz, corresponding to a squint angle of about 0.1°, which is a low squint case. The length of a single point target in the range direction is denoted as \( nr \) samples and in the azimuth direction is denoted as \( na \). The received data used to form a SAR image is the superposition of signals from a distribution of a single point target [24]. A matrix of reflectivity coefficients is generated to describe the distribution of scatterers on the ground surface and to emulate the ground reflection of the microwave energy. The amplitude of the reflectivity coefficients can be controlled to emulate the ground with selectable levels of contrast or radiometric discontinuities. The phase of the reflectivity coefficients is generated randomly with a normal distribution. The size of the reflectivity matrix is \( 1024 + nr - 1 \) samples in the range direction and \( 1024 + na - 1 \) samples in the azimuth direction, as shown in Figure 4.1.

The simulated data is generated by convolving the expanded single point target with the reflectivity matrix. For computational efficiency, a 2D fast convolution is used. The single point target and the reflectivity matrix are zero-padded to the size of \( 1024 + 2nr - 2 \) by \( 1024 + 2na - 2 \) in the range direction and the azimuth direction respectively. 2D FFTs are performed on the zero-padded point target and the reflectivity matrix. The product of the single point target and the matrix is then transformed back to the range and azimuth time domain with a 2D IFFT.

This method has an obvious physical meaning. It assumes that in each range and azimuth cell, there is a single effective point target. The properties of the ground surface can be described by the elements of the reflectivity matrix. The amplitude of each coefficient describes the amplitude of the ground reflectivity and the phase of each coefficient is \((4\pi \frac{R}{\lambda})_{\text{mod} 2\pi}\), where \( R \) is the range to the effective scattering center of
each pixel and $\lambda$ is the radar wavelength. By controlling the amplitude of the reflectivity coefficients, we can generate a simulated data block with different scene contrast and radiometric discontinuities.

To simulate the partial coverage of targets at the edge of the radar beam, only a square at the center of the product matrix with a size of $1024 \times 1024$ is selected as the simulated data. Since the zero-padding is enough, no wrap-around errors occur in the fast convolution.

The partial coverage of the antenna beam at each edge is simulated by throwing away both sides of the convolution result in the azimuth direction with a size of $na - 1$ azimuth samples. This ensures that all possible partial exposures are included, from 1 azimuth sample to $na - 1$ azimuth samples, as shown in Figure 4.2. In Figure 4.2, Target 1 only has its last sample covered, Target 4 only has its first sample covered, Target 2 only has its first sample excluded and Target 3 only has its last sample excluded.

In addition, partial target coverage is also included in the range direction, although DOPCEN estimators usually work on data after range compression, in which case partial range targets are only caused by uncorrected range cell migration.

### 4.1.2 Simulated Data with Noise

Simulated data with different Signal to Noise Ratios (SNR) are required to compare the performance of the algorithms under different noise levels.

To generate simulated data with a specified SNR, a noise matrix is generated. We assume the noise is complex, white, Gaussian noise. The power of the noise can be
controlled according to the specified SNR. Then the noise is added to the signal to form a simulated data block with specified SNR. This process is shown in Figure 4.3.

4.1.3 Experimental Database

A database for multiple target simulations is generated. First, 10 low contrast scenes are generated using a constant amplitude but different random phase for each element (target) in 10 reflectivity matrices.

Then we create 10 higher-contrast scenes by setting the amplitude of every 50th data point to a higher value (constant within each scene) from 10 to 100 with a step of 10. The same phase noise is used in each of the 10 scenes, so the effect of contrast can be seen. While these big targets have constant amplitude and spacing, the key to a
correct simulation is to use random phases for each target. The contrast of each scene is measured by the commonly-used formula [22]:

\[
\text{Contrast} = \frac{< I^2 >}{< I >^2}
\]  

(4.1)

where \( I \) is the magnitude of each pixel and \(< . >\) is the expected value.

We also specified 10 different noise levels and generated 10 noise matrices. By adding them to the same low contrast scene, we have 10 low contrast scenes with different SNRs.

Figure 4.3: Generate and Add Noise to the Simulated Data
4.2 Simulations with Low Contrast Data

DLR Low-Squint, Low-Contrast Simulations

First, the DLR algorithm was run on the 10 simulated data blocks with low contrast. Table 4.1 gives the simulation results.

As an example, the estimation on data block 2 is shown in Figure 4.4 where the ACCC angles as a function of the range frequency is given, as well as a straight line giving the best linear fit to the ACCC angle. Note that only the ACCC angles within the dotted box are used in the estimation of the straight line, since the ACCC angles are noisy outside this high-energy portion of the range spectrum.

The fractional part of the DOPCEN is estimated by the average height of the fitted line (the value at zero range frequency). The Doppler ambiguity (in Hz) is estimated by projecting the slope of the fitted line to the radar center frequency. The ambiguity number is then found by dividing by the PRF and rounding to the nearest integer.

We see that the DLR algorithm works well on all 10 of these multi-target, low-contrast scenes. It gives an estimate of the fractional part which is almost perfect. It gives ambiguity DOPCEN estimates which are within a few 10’s of Hz of the correct value of -400 Hz, well within the limits needed to estimate the correct ambiguity number on all of the 10 data blocks. The average estimate of the ambiguity DOPCEN is -413.9 Hz, and the standard deviation is 24.3 Hz.
<table>
<thead>
<tr>
<th>Data Block</th>
<th>Frac. part Hz</th>
<th>Ambiguity Hz</th>
<th>Error Hz</th>
<th>Ambiguity Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-400</td>
<td>-401</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>-400</td>
<td>-438</td>
<td>-38</td>
<td>0</td>
</tr>
<tr>
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<td>-390</td>
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<td>0</td>
</tr>
<tr>
<td>4</td>
<td>-400</td>
<td>-444</td>
<td>-44</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>-399</td>
<td>-458</td>
<td>-58</td>
<td>0</td>
</tr>
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<td>-399</td>
<td>+1</td>
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<td>-424</td>
<td>-24</td>
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</tr>
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<td>-385</td>
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<td>9</td>
<td>-400</td>
<td>-390</td>
<td>+10</td>
<td>0</td>
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<tr>
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<td>Mean</td>
<td>-400</td>
<td>-414</td>
<td>-14</td>
<td>0</td>
</tr>
<tr>
<td>St.Dev.</td>
<td>0</td>
<td>24</td>
<td>24</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.1: Ambiguity estimation results of DLR algorithm with low scene contrast (correct answer = -400 Hz)
Figure 4.4: ACCC Angles as a Function of Range Frequency (DLR Algorithm)

Slope = \(-8.330 \times 10^{-8}\) rad/Hz
Standard Dev. = 0.052 rad
MLCC Low-Squint, Low-Contrast Simulations

The MLCC algorithm was then performed on the same data set. Table 4.2 gives the simulation results. As an example, the ACCC angles found on data block 2 are shown in the following two figures.

Figure 4.5 shows the measured ACCC angles of range look 1 and range look 2 for data block 2. Figure 4.6(a) shows the sum of the ACCC angles of the two range looks, from which the fractional part of the DOPCEN is estimated by finding the mean ACCC angle. Figure 4.6(b) shows the difference of the ACCC angles of the two range looks, from which the Doppler ambiguity is found by estimating the slope of the data.

<table>
<thead>
<tr>
<th>Data Block</th>
<th>Frac. part Hz</th>
<th>Ambiguity Hz</th>
<th>Error Hz</th>
<th>Ambiguity Number</th>
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<tr>
<td>1</td>
<td>-401</td>
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<td>+4</td>
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<td>-2</td>
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<td>Mean</td>
<td>-400</td>
<td>-412</td>
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<tr>
<td>St.Dev.</td>
<td>0</td>
<td>21</td>
<td>21</td>
<td>0</td>
</tr>
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</table>

Table 4.2: Ambiguity estimation results of MLCC algorithm with low scene contrast (correct answer = -400 Hz)

We see from Table 4.2 that the MLCC algorithm also works well on all of the
10 scenes. It gives near-perfect fractional DOPCEN estimates, and gives the correct Doppler ambiguity on all of the 10 data blocks. The average estimate of the ambiguity DOPCEN is -412.0 Hz (a bias of -12.0 Hz), and the standard deviation is 21.3 Hz. The bias and standard deviation are a little lower than in the DLR algorithm.
Figure 4.5: ACCC angles of the two range looks in experiment #2

Figure 4.6: Sum and difference of ACCC angles of the two range looks in experiment #2
MLBF Low-Squint, Low-Contrast Simulations

The simulation results of the MLBF algorithm on the same data set are given in Table 4.3.

As an example, the estimation on a data block 1 is shown in Figure 4.7. From this figure, it is seen that there is no clear single beat frequency. In this case, we fit a quadratic curve to the data in Figure 4.7 using the MATLAB polyfit function, and find the peak of the parabola. The quadratic curve fitting is used whenever the peak beat spectrum energy is less than 1.4 of the average energy. But in general, the MLBF algorithm will be less reliable when a clear beat frequency is not observable.

<table>
<thead>
<tr>
<th>Data Block</th>
<th>Beat Freq. Hz</th>
<th>DOPCEN KHz</th>
<th>Ambiguity No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-38</td>
<td>+18.7</td>
<td>19</td>
</tr>
<tr>
<td>2</td>
<td>-32</td>
<td>+15.5</td>
<td>16</td>
</tr>
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<td>3</td>
<td>-67</td>
<td>+32.4</td>
<td>34</td>
</tr>
<tr>
<td>4</td>
<td>+43</td>
<td>+21.0</td>
<td>22</td>
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<td>+62</td>
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</tr>
<tr>
<td>10</td>
<td>-38</td>
<td>+18.7</td>
<td>19</td>
</tr>
</tbody>
</table>

| Mean       | -11           | +9.4        | 10            |
| St.Dev.    | 47            | 21          | 22            |

Table 4.3: Ambiguity estimation results of MLBF algorithm with low scene contrast (the answers should be zero)
Spectrum of the Azimuth Beat Signal, DOPCEN = -400 Hz

Peak location = -41
Beat freq. = -38.4 Hz
Fdc = 18718.8 Hz
Ambiguity = 19

Figure 4.7: Estimation of DOPCEN Ambiguity by MLBF algorithm with low-contrast scene (note the exaggerated vertical scale)
4.3 Simulations with Higher Contrast Data

A set of data with different scene contrasts is used to compare the performance of the three candidate algorithms, with only the magnitude of the large targets varied to change the contrast. The DLR, MLCC and MLBF algorithms are run on the same data sets. The results are summarized in Tables 4.4 and 4.5.

From the tables, we can see that the DLR and MLCC algorithms work well for low contrast scenes, but begin to take on substantial biases when the scene contrast increases. On the other hand, the MLBF algorithm works well with high contrast scenes, but breaks down when the scene contrast is low. These properties have been observed previously [14], but have not been quantified in this manner before\(^1\).

From Tables 4.4 and 4.5 we note that the MLCC algorithm gives better fractional DOPCEN estimates than the DLR algorithm, but the reverse is true for the ambiguity estimates.

\(^1\)Note that the results of these simulations are somewhat pessimistic, because 4K×4K data arrays are usually used in the production processors.
<table>
<thead>
<tr>
<th>Scene Contr</th>
<th>DLR</th>
<th>MLCC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frac. Part</td>
<td>Error</td>
</tr>
<tr>
<td></td>
<td>Hz</td>
<td>Hz</td>
</tr>
<tr>
<td>1.1</td>
<td>-398.9</td>
<td>1.1</td>
</tr>
<tr>
<td>1.2</td>
<td>-396.2</td>
<td>3.8</td>
</tr>
<tr>
<td>1.3</td>
<td>-392.5</td>
<td>7.5</td>
</tr>
<tr>
<td>1.6</td>
<td>-388.5</td>
<td>11.5</td>
</tr>
<tr>
<td>2.0</td>
<td>-384.9</td>
<td>15.1</td>
</tr>
<tr>
<td>2.4</td>
<td>-381.8</td>
<td>18.2</td>
</tr>
<tr>
<td>2.9</td>
<td>-379.1</td>
<td>20.9</td>
</tr>
<tr>
<td>3.5</td>
<td>-377.0</td>
<td>23.0</td>
</tr>
<tr>
<td>4.1</td>
<td>-375.2</td>
<td>24.8</td>
</tr>
<tr>
<td>4.8</td>
<td>-373.8</td>
<td>26.2</td>
</tr>
<tr>
<td>Mean</td>
<td>-384.8</td>
<td>15.2</td>
</tr>
<tr>
<td>St.Dev.</td>
<td>8.9</td>
<td>8.9</td>
</tr>
</tbody>
</table>

Table 4.4: Fractional PRF estimation results with low squint and varying scene contrast (the answer should be -400)
Table 4.5: Ambiguity estimation results with low squint and varying scene contrast (the DOPCEN should be -400 and the ambiguity numbers should be zero)

<table>
<thead>
<tr>
<th>Scene Contr</th>
<th>DLR DCen Hz</th>
<th>DLR Error Hz</th>
<th>Amb No.</th>
<th>MLCC DCen Hz</th>
<th>MLCC Error Hz</th>
<th>Amb No.</th>
<th>MLBF BF Hz</th>
<th>MLBF DCen Hz</th>
<th>MLBF Amb No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>-433</td>
<td>-33</td>
<td>0</td>
<td>-454</td>
<td>-54</td>
<td>0</td>
<td>-12.2</td>
<td>5941</td>
<td>6</td>
</tr>
<tr>
<td>1.2</td>
<td>-340</td>
<td>60</td>
<td>0</td>
<td>-439</td>
<td>-39</td>
<td>0</td>
<td>-24.5</td>
<td>11931</td>
<td>12</td>
</tr>
<tr>
<td>1.3</td>
<td>-251</td>
<td>149</td>
<td>0</td>
<td>-59</td>
<td>341</td>
<td>0</td>
<td>4.7</td>
<td>-2283</td>
<td>-2</td>
</tr>
<tr>
<td>1.6</td>
<td>-41</td>
<td>359</td>
<td>0</td>
<td>173</td>
<td>573</td>
<td>1</td>
<td>0.0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2.0</td>
<td>-25</td>
<td>375</td>
<td>0</td>
<td>160</td>
<td>560</td>
<td>1</td>
<td>0.9</td>
<td>-456</td>
<td>0</td>
</tr>
<tr>
<td>2.4</td>
<td>83</td>
<td>483</td>
<td>1</td>
<td>533</td>
<td>933</td>
<td>1</td>
<td>0.9</td>
<td>-456</td>
<td>0</td>
</tr>
<tr>
<td>2.9</td>
<td>218</td>
<td>618</td>
<td>1</td>
<td>583</td>
<td>983</td>
<td>1</td>
<td>0.9</td>
<td>-456</td>
<td>0</td>
</tr>
<tr>
<td>3.5</td>
<td>336</td>
<td>736</td>
<td>1</td>
<td>626</td>
<td>1026</td>
<td>1</td>
<td>0.9</td>
<td>-456</td>
<td>0</td>
</tr>
<tr>
<td>4.1</td>
<td>439</td>
<td>839</td>
<td>1</td>
<td>666</td>
<td>1066</td>
<td>1</td>
<td>0.9</td>
<td>-456</td>
<td>0</td>
</tr>
<tr>
<td>4.8</td>
<td>527</td>
<td>927</td>
<td>1</td>
<td>703</td>
<td>1103</td>
<td>1</td>
<td>0.9</td>
<td>-456</td>
<td>0</td>
</tr>
<tr>
<td>Mean</td>
<td>51</td>
<td>451</td>
<td>0.5</td>
<td>249</td>
<td>649</td>
<td>0.7</td>
<td>-2.7</td>
<td>1285</td>
<td>1.6</td>
</tr>
<tr>
<td>St.Dev.</td>
<td>330</td>
<td>330</td>
<td>0.5</td>
<td>446</td>
<td>446</td>
<td>0.5</td>
<td>8.9</td>
<td>4315</td>
<td>4.2</td>
</tr>
</tbody>
</table>

Figure 4.8 shows the spectrum of the beat signal when the scene contrast is 4.8. The beat frequency is easily identified in this case and the MLBF algorithm gives a correct estimate of the Doppler ambiguity, which is 0 in this case.

More interesting, however, is how the MLBF algorithm behaves with the two scenes of contrast 1.3 and 1.6, which are the contrast values at which the MLBF algorithm changes from not working to working. In Figure 4.9, the contrast 1.3 case is shown, where it can be seen that the expected beat frequency is hidden by the multiple beating of many
targets of almost equal size. But when the contrast is raised to 1.6, the extra contrast between strong and weaker targets allows the expected beat frequency to be observed, and the correct ambiguity number is obtained.
Figure 4.9: Estimation of DOPCEN by MLBF Algorithm with Scene Contrast of 1.3

Figure 4.10: Estimation of DOPCEN by MLBF Algorithm with Scene Contrast of 1.6
4.3.1 Discussion on the Effect of Scene Contrast

From the experiments, we can see the scene contrast has an important effect on the candidate algorithms. Figure 4.11 shows the estimation errors of the DLR and the MLCC algorithms on low contrast data blocks. The x-axis is the number of the data block, each with the same low contrast. The y-axis is the estimation error. Figure 4.12 gives the error of the MLBF algorithm on the same data set.

Tables 4.1, 4.2 and 4.3 give the average error and standard deviation of the DLR, MLCC and the MLBF algorithms on the 10 low contrast data blocks processed in Section 4.2. We see that the DLR and MLCC performance is similar, but the MLBF algorithm performs very badly.

When the scene contrast becomes high, the performance of the three algorithms changes. Figure 4.13 shows the estimation errors of the DLR and MLCC algorithms as the scene contrast changes from low to high. The x-axis is the scene contrast. The y-axis is the estimation error. Figure 4.14 gives the error of the MLBF algorithm on the same data set.

Note that only when the estimation error is within the two dashed lines in Figures 4.11 - 4.14 is the estimation of the Doppler ambiguity correct. The limits of the allowed error are \([-F_a/2, F_a/2]\). In the present simulation, the limits are \([-480, 480]\) Hz.

The scene contrast of the low contrast data set is equal to 1. The DLR algorithm and the MLCC algorithm work well on this set of data. The MLBF algorithm does not work on the low contrast data set. When the scene contrast increases, the MLBF algorithm begins to work well while the error of the DLR algorithm and the MLCC
algorithm becomes large, leading to an incorrect estimation of the Doppler ambiguity.

This can be explained by considering the ACCC when multiple point targets are involved. In Chapter 3, we proved that the average increment of the azimuth frequency could be estimated by the ACCC angle in the case that only single point target is involved. When multiple targets are involved, the correlation between these targets will affect the ACCC angle, thus affect the estimation of the increment of the azimuth frequency.
Figure 4.11: Ambiguity estimation error of DLR and MLCC on low contrast data

Figure 4.12: Ambiguity estimation error of the MLBF algorithm on low contrast data (note the vastly different vertical scale compared with Figure 4.11 — the estimate must be within the dotted lines of ±PRF to get the correct ambiguity number)
Figure 4.13: Ambiguity estimation error of DLR and MLCC algorithms on scenes of increasing contrast.

Figure 4.14: Ambiguity estimation error of MLBF algorithm on scenes of increasing contrast with quadratic curve fitting (note the different vertical scale compared with Figure 4.13 — the estimate must be within the dotted lines of ±PRF to get the correct ambiguity number in each case)
Consider two point targets P and Q which overlap in the SAR signal domain in the azimuth direction, as shown in Figure 4.15 [14]. \( \eta_P \) is the azimuth time for closest approach for target P. \( \eta_Q \) is the azimuth time for closest approach for target Q. \( \varepsilon \) is the separation between target P and target Q. We assume target P and target Q in the same range cell and they have the same power, thus their envelopes have the same magnitude.

The azimuth signal is given by:

\[
S(\eta) = S_P(\eta) + S_Q(\eta)
\]  

(4.2)

where \( S_P(\eta) \) and \( S_Q(\eta) \) are given by:

\[
S_P(\eta) = P W(\eta - \eta_c) \exp\{-j\pi K_a(\eta - \eta_P)^2\}
\]  

(4.3)

\[
S_Q(\eta) = Q W(\eta - \eta_c) \exp\{-j\pi K_a(\eta - \eta_Q)^2\}
\]  

(4.4)

where \( P \) is the complex envelope of target P, \( Q \) is the complex envelope of target Q and \( W(\eta - \eta_c) \) is the azimuth antenna profile. From Equation (3.4), the ACCC is given by:
\[ C = \sum_{\eta} S(\eta) S^*(\eta + \Delta\eta) \]
\[ = \sum_{\eta} [S_P(\eta) + S_Q(\eta)] [S'^*_P(\eta + \Delta\eta) + S'^*_Q(\eta + \Delta\eta)] \]
\[ = C_{PP} + C_{QQ} + C_{PQ} + C_{QP} \quad (4.5) \]

The first two items, \( C_{PP} \) and \( C_{QQ} \), are the ACCCs of target P and target Q respectively:

\[ \text{arg}\{C_{PP}\} = \text{arg}\{C_{QQ}\} = \frac{2\pi K_a \eta_c}{F_a} \quad (4.6) \]

They have the same phase angle, which is the desired one. The other two terms, \( C_{PQ} \) and \( C_{QP} \), are due to the overlap of the exposures of target P and target Q. If there is no overlap between target P and target Q, these two items will be zero and the phase angle of the ACCC of \( S(\eta) \) will be the desired one. However, overlap does happen in the real case.

Consider the third term, \( C_{PQ} \). To generalize the analysis, suppose target Q is fixed and target P moves from left to right. One important parameter is the separation \( \varepsilon \) between target P and target Q. If the origin of the azimuth time axis is set to \( \eta_Q \), the third term in Equation (4.5) is given by:

\[ C_{PQ} = PQ^* \sum_{\eta} W(\eta - \eta_c - \varepsilon) W(\eta - \eta_c) \exp\{j\pi K_a (2\eta + \Delta\eta - \varepsilon) (\Delta\eta + \varepsilon)\} \]
\[ = PQ^* \sum_{\eta} M(\eta, \varepsilon) \exp\{j\phi(\eta, \varepsilon)\} \quad (4.7) \]

where \[ M(\eta, \varepsilon) = W(\eta - \eta_c - \varepsilon) W(\eta - \eta_c) \quad (4.8) \]

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and \[ \phi(\eta, \varepsilon) = \pi K_s (2\eta + \Delta \eta - \varepsilon) (\Delta \eta + \varepsilon) \] (4.9)

Note that \( M(\eta, \varepsilon) \) and \( \phi(\eta, \varepsilon) \) are both symmetrical about \( \eta_s = \eta_c + \varepsilon/2 \).

Assume \( \eta_l \) is the length of the azimuth aperture, we have:

\[ -\eta_l \leq \varepsilon \leq \eta_l \] (4.10)

and

\[ \varepsilon < 0 : \quad \eta_c - \eta_l/2 \leq \eta \leq \eta_c + \varepsilon + \eta_l/2 \]

\[ \varepsilon > 0 : \quad \eta_c + \varepsilon - \eta_l/2 \leq \eta \leq \eta_c + \eta_l/2 \]

After a mathematical derivation, the phase angle of \( C_{PQ*} \) is given by:

\[ \text{arg}\{C_{PQ*}\} = \phi(\eta_s, \varepsilon) = \pi K_s (2\eta_c + \Delta \eta - \varepsilon) (\Delta \eta + \varepsilon) \] (4.11)

The amplitude of \( C_{PQ*} \) is shown in Figure 4.16 as a function of \( \varepsilon \). The phase angle of \( C_{PQ*} \) is shown in Figure 4.17 also as a function of \( \varepsilon \).
Figure 4.16: Amplitude of \( C_{PQ} \) as a Function of \( \varepsilon \)

Figure 4.17: Phase of \( C_{PQ} \) as a Function of \( \varepsilon \)
From Figure 4.16 and Figure 4.17, we can see that the amplitude of the ACCCs between target P and target Q is symmetric about $\varepsilon = -\Delta \eta$. The phase of the ACCCs is an odd function of $\varepsilon$. The ACCC is complex conjugate about $\varepsilon$. Thus the sum of all ACCCs between target Q and all its neighbor targets in the same range cell is approximately a real number. In an ideal case, the reflection of all point target has the same power and random phase. In this case, for each point target, we have a sum of ACCCs between it and all its neighbor targets in the same range cell and all the sums have the same amplitude and random phase. The mean of all ACCCs due to overlap is zero. Thus the ACCC angle should be unbiased in the ideal case, which has the lowest scene contrast of 1. This is why the DLR algorithm and MLCC algorithm work well on low contrast scenes.

When the scene contrast is higher than 1, in which case the reflectivity of targets in the same range cell are likely to have different powers, the mean of all ACCCs due to overlap may not be zero, leading to a random bias in the estimation of the phase increment at each range cell. This random bias will affect the estimation of the slope, which is an important parameter in the DLR and MLCC algorithms, leading to an estimation error in the DOPCEN. This is in fact because of insufficient averaging of the ACCCs due to overlap since the strong targets have dominant effect over weak targets. The higher the scene contrast becomes, the worse the insufficient average is. This is why the estimation error of the DLR and the MLCC algorithms becomes large when the scene contrast becomes high.

Reference [14] explains why the MLBF does not work on low contrast scenes and why it works well on high contrast scenes in details. In summary, when two targets are present, there are three peak frequency components in the FFT of the beat signal, one
corresponding to the Doppler centroid, and two due to the cross beating between the
two targets. When more than two significant targets are present in the same range cell,
the distortion of the peak beat frequency gets worse. As the number of dominant targets
increase, the power due to the cross beating can eventually mask out the required beat
frequency, as we observed in Figure 4.7.

Thus, it can be concluded that, the DLR and the MLCC algorithm work well on
low contrast scenes, while the MLBF algorithm works well on high contrast scene. Since
the DLR and the MLCC algorithms are based on similar principles, the performance of
these two algorithms on low contrast scenes is almost the same. From Figure 4.13, the
MLCC algorithm is more sensitive to the scene contrast than the DLR algorithm. This
effect is explained in Chapter 6.
4.4 Simulations with Noise

As the DLR and MLCC algorithms behave almost the same in terms of scene contrast, it is interesting to also compare them in terms of their response to scene noise. To perform this comparison, we generate a set of simulated data with different noise levels. The signal to noise ratio is defined as:

\[ \text{SNR} = 10 \log \frac{P}{N} \]  

(4.12)

where \( P \) is the power of the signal and \( N \) is the power of the noise.

The simulation results are shown in Table 4.6. The first row indicates the different noise levels in units of dB. The second row is the estimation results of the DLR algorithm and the third row is the estimation results of the MLCC algorithm. Figure 4.18 shows the trend of the estimation error of the DLR and MLCC algorithms as the noise level decreases.

<table>
<thead>
<tr>
<th>SNR (dB)</th>
<th>30</th>
<th>20</th>
<th>10</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>DLR (Hz)</td>
<td>-401.2</td>
<td>-397.2</td>
<td>-384.0</td>
<td>-383.1</td>
<td>-369.9</td>
<td>-369.5</td>
<td>-369.1</td>
<td>-363.7</td>
</tr>
<tr>
<td>MLCC (Hz)</td>
<td>-396.5</td>
<td>-393.9</td>
<td>-379.6</td>
<td>-377.1</td>
<td>-375.4</td>
<td>-386.3</td>
<td>-381.3</td>
<td>-358.0</td>
</tr>
</tbody>
</table>

Table 4.6: Simulation Results of DLR and MLCC with Different SNRs
Figure 4.18: Comparison of ambiguity estimation errors of the DLR and MLCC algorithms with increasing scene SNR (low contrast scenes)
4.4.1 Discussion of the Effect of Noise

We see from Figure 4.18 that the estimation error decreases when the SNR increases. Note that even when the SNR is low, the DLR and the MLCC algorithms still work reliably, as the ambiguity estimation errors are well below PRF/2.

Thus we can conclude that the DLR algorithm and the MLCC algorithm are robust to white noise on low contrast scenes. The estimation errors of the DLR and the MLCC algorithms on same noise level are almost the same.

Consider the spectrum of a range chirp, as sketched in Figure 4.19. The dashed line is the power level of the white noise. From the figure, it can be seen that the ratio of the signal power to the noise power either in the whole range bandwidth or in the bandwidths of look 1 and look 2 are the same under the assumption that the noise is white. Thus the performance of the DLR algorithm and the MLCC algorithm in white noise is essentially the same.

Figure 4.19: Illustrating the distribution of noise and signal as seen by the DLR and MLCC algorithms

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4.5 Experiments on SAR Data

4.5.1 Experiments on a Low Contrast Scene

Real SAR data is used to test the performance of the three candidate algorithms. An ERS-1 Bathurst Island scene in yaw-steering mode is used to test the algorithms under low-squint conditions. The acquisition data is Oct. 25, 1995 and the orbit/track is 22365/1539. Figure 4.20 shows the detected image of this scene. The left part is snow and rock and the right part is sea ice. It is noticed that the scene contrast is fairly low.

The absolute DOPCEN in the middle of the scene is \(-159\) Hz as obtained by MDA's dtSAR processor. This reference DOPCEN is used in the processing of the raw data. It is a good estimate since the detected image has a high quality. The horizontal direction is the range direction with near range on the left. The azimuth direction is vertical, with azimuth time increasing in the upward direction.

We extracted 6 consecutive data blocks along the azimuth direction. Each data block is 4096 \times 4096 samples in the range and azimuth directions. The overlap of each block is 1696 azimuth samples. Table 4.7 gives the fractional PRF estimates of the DLR and MLCC algorithms. Table 4.8 summarizes the ambiguity estimation results of the DLR, MLCC and MLBF algorithms.

Note, in the real case, there is a frequency offset (FOS) in the estimation of the DLR algorithm and the MLCC algorithm. The FOS does not exist in the estimation of the MLBF algorithm [14]. This effect is probably caused by a frequency dependency of the beam squint angle [20]. The frequency offset in the estimation on ERS-1 yaw-steering
mode data is about 1400 Hz [25]. In the table, the FOS value has been subtracted from the estimation of the DLR and the MLCC algorithms.

<table>
<thead>
<tr>
<th>Block No.</th>
<th>Scene Contr.</th>
<th>DLR Fract.</th>
<th>DLR Error</th>
<th>MLCC Fract.</th>
<th>MLCC Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.144</td>
<td>-162</td>
<td>-3</td>
<td>-158</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1.145</td>
<td>-151</td>
<td>8</td>
<td>-150</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>1.145</td>
<td>-147</td>
<td>12</td>
<td>-152</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>1.146</td>
<td>-147</td>
<td>12</td>
<td>-148</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>1.153</td>
<td>-158</td>
<td>1</td>
<td>-155</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>1.164</td>
<td>-149</td>
<td>10</td>
<td>-148</td>
<td>11</td>
</tr>
<tr>
<td>Mean</td>
<td>1.150</td>
<td>-152</td>
<td>7</td>
<td>-152</td>
<td>7</td>
</tr>
<tr>
<td>St.Dev.</td>
<td>0.008</td>
<td>6</td>
<td>6</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 4.7: Fractional PRF estimation results on the low-squint low-contrast Bathurst Island ERS scene (the DOPCEN should be -159 Hz)
Figure 4.20: The detected image of the ERS-1 Bathurst Island scene
Table 4.8: Ambiguity estimation results on the low-squint low-contrast Bathurst Island ERS scene with $F_{os} = 1400 \text{ Hz}$ (the DOPCEN should be $-159 \text{ Hz}$ and the ambiguity numbers should be zero)

<table>
<thead>
<tr>
<th>Block No.</th>
<th>Scene Contr</th>
<th>DLR</th>
<th>MLCC</th>
<th>MLBF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>DCEn</td>
<td>Err</td>
<td>Am No.</td>
</tr>
<tr>
<td>5</td>
<td>1.144</td>
<td>-232</td>
<td>-73</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>1.145</td>
<td>280</td>
<td>439</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1.145</td>
<td>-210</td>
<td>-51</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1.146</td>
<td>2</td>
<td>161</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1.153</td>
<td>85</td>
<td>244</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>1.164</td>
<td>202</td>
<td>361</td>
<td>0</td>
</tr>
<tr>
<td>Mean</td>
<td>1.150</td>
<td>21</td>
<td>180</td>
<td>0</td>
</tr>
<tr>
<td>St Dev</td>
<td>0.007</td>
<td>192</td>
<td>192</td>
<td>0</td>
</tr>
</tbody>
</table>

All three algorithms give the correct estimate of the Doppler ambiguity on the 6 data blocks. Note that the MLCC algorithm has a lower mean error, and a lower standard deviation than the DLR algorithm.

Though the scene contrast is fairly low, the MLBF algorithm still works reliably. Thus, the MLBF algorithm is reasonably robust to scene contrast in real SAR data.

Figure 4.21 shows the estimation error of the three algorithms in different scene contrasts. Note that the scene contrast is computed on the detected image and is different form the contrast metric used in the simulation, which is computed from the reflectivity coefficients. The estimation error increases when the scene contrast increases, as we have seen in the simulations, but the range of contrast is too narrow to draw definite
conclusions.

Figure 4.21: Estimation Error on the Bathurst Island Scene
4.5.2 Experiments on a High Contrast Scene

An ERS-1 North Cascade Glacier scene in yaw-steering mode, which has a high scene contrast, is used to test the algorithms. The acquisition data is Sep. 23, 1995 and the orbit/track is 21916/385. Figure 4.22 shows the detected image of this scene. This scene mainly consists of mountains, including forest, snow and glaciers.

The absolute DOPCEN in the middle of the scene is 485 Hz as obtained by MDA’s dtSAR processor. This value is the best independent indication we have to check the accuracy of our estimates. However, as we do not know how accurate the dtSAR estimate really is, the best indication of the accuracy of our algorithm is the standard deviation of the estimates from block to block. This is because the true DOPCEN changes very slowly with azimuth. The horizontal direction is the range direction with near range on the left. The azimuth direction is vertical, with azimuth time increasing in the upward direction.

We also extract 6 consecutive data blocks along the azimuth direction. Each data block is 4096 x 4096 samples in the range and azimuth directions. The overlap of each block is 1696 azimuth samples.

Table 4.9 gives the fractional PRF estimates of the DLR and MLCC algorithms. Table 4.10 summarizes the ambiguity estimation results of the DLR, MLCC and MLBF algorithms. In the table, the FOS value, which is about 1400 Hz, has been subtracted from the estimation of the DLR and the MLCC algorithms. The results of Table 4.10 suggests that the $F_0$ is wrong at least for this scene. If we set $F_0 = 2477$ Hz, the mean error in the estimate of the ambiguity is close to zero, and the DLR and the MLCC algorithm will give the correct estimate for every block.
<table>
<thead>
<tr>
<th>Block No.</th>
<th>Scene Contr</th>
<th>Fract. Hz</th>
<th>Error Hz</th>
<th>Fract. Hz</th>
<th>Error Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1.333</td>
<td>467</td>
<td>-18</td>
<td>465</td>
<td>-20</td>
</tr>
<tr>
<td>5</td>
<td>1.332</td>
<td>462</td>
<td>-23</td>
<td>461</td>
<td>-24</td>
</tr>
<tr>
<td>1</td>
<td>1.328</td>
<td>470</td>
<td>-15</td>
<td>467</td>
<td>-18</td>
</tr>
<tr>
<td>3</td>
<td>1.309</td>
<td>469</td>
<td>-16</td>
<td>466</td>
<td>-19</td>
</tr>
<tr>
<td>6</td>
<td>1.284</td>
<td>463</td>
<td>-22</td>
<td>462</td>
<td>-23</td>
</tr>
<tr>
<td>2</td>
<td>1.256</td>
<td>466</td>
<td>-19</td>
<td>467</td>
<td>-18</td>
</tr>
<tr>
<td>Mean</td>
<td>1.307</td>
<td>466</td>
<td>-19</td>
<td>465</td>
<td>-20</td>
</tr>
<tr>
<td>St.Dev.</td>
<td>0.029</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 4.9: Fractional PRF estimation results on the high-contrast North Cascade Glacier ERS scene (the DOPCEN should be 485 Hz according to dtSAR, but the true fractional DOPCEN is likely 466 Hz according to these estimates)
Figure 4.22: The detected image of the ERS-1 North Cascade Glacier scene
Table 4.10: Ambiguity estimation results on the low-squint high-contrast North Cascade Glacier ERS scene (the DOPCEN should be 485 Hz and the ambiguity numbers should be zero)

<table>
<thead>
<tr>
<th>Block No.</th>
<th>Scene Contr</th>
<th>DLR DCen Hz</th>
<th>DLR Err Hz</th>
<th>DLR Am No.</th>
<th>MLCC DCen Hz</th>
<th>MLCC Err Hz</th>
<th>MLCC Am No.</th>
<th>MLBF DCen Hz</th>
<th>MLBF Err Hz</th>
<th>MLBF Am No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1.333</td>
<td>1837</td>
<td>1352</td>
<td>1</td>
<td>1295</td>
<td>810</td>
<td>0</td>
<td>201</td>
<td>-284</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>1.332</td>
<td>1601</td>
<td>1116</td>
<td>1</td>
<td>1845</td>
<td>1360</td>
<td>1</td>
<td>-201</td>
<td>-686</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1.328</td>
<td>1897</td>
<td>1412</td>
<td>1</td>
<td>1363</td>
<td>878</td>
<td>1</td>
<td>-201</td>
<td>-686</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1.309</td>
<td>1505</td>
<td>1020</td>
<td>1</td>
<td>1333</td>
<td>848</td>
<td>1</td>
<td>201</td>
<td>-284</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>1.284</td>
<td>1149</td>
<td>664</td>
<td>0</td>
<td>1890</td>
<td>1405</td>
<td>1</td>
<td>-201</td>
<td>-686</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1.256</td>
<td>1269</td>
<td>381</td>
<td>0</td>
<td>1871</td>
<td>1386</td>
<td>1</td>
<td>-201</td>
<td>-686</td>
<td>0</td>
</tr>
<tr>
<td>Mean</td>
<td>1.307</td>
<td>1543</td>
<td>1058</td>
<td>1</td>
<td>1599</td>
<td>1145</td>
<td>1</td>
<td>-67</td>
<td>-552</td>
<td>0</td>
</tr>
<tr>
<td>St Dev</td>
<td>0.029</td>
<td>273</td>
<td>273</td>
<td>0.5</td>
<td>270</td>
<td>270</td>
<td>0.4</td>
<td>189</td>
<td>190</td>
<td>0</td>
</tr>
</tbody>
</table>

Only the MLBF algorithm gives the correct estimate of the Doppler ambiguity on the 6 data blocks since for each data block, a clear dominant frequency was always observed in the beat spectrum. As an example, Figure 4.23 shows the beat spectrum of data block 1. From Figure 4.23, we found the peak or dominant beat frequency, which in this case is 0.4 Hz. This beat frequency estimate must be accurate to ±1.6 Hz in order for the correct ambiguity to be found.

The DLR and MLCC algorithm do not work reliably on this high-contrast scene. Thus, the MLBF algorithm is reasonably robust to scene contrast in real SAR data. Figure 4.24 shows the estimation error of the three algorithms in different scene contrasts.
Figure 4.23: The beat spectrum of data block 1 of the North Cascade Glacier ERS scene
Figure 4.24: Estimation Error on the North Cascade Glacier Scene
4.6 Summary

Since the MLCC algorithm works best with scenes of low, uniform contrast, while the MLBF algorithm works best with scenes of high contrast, and since they all need the processing of range look extraction, these two algorithms can be efficiently combined together to form a reliable DOPCEN estimator over large ranges of scene contrast. The criteria which have been found useful for selecting the best of the MLBF and the MLCC results are “Normalized Correlation”, “Consistency of estimate” and the “Scene Contrast” [14].
Chapter 5

Higher Squint Considerations

The effect of a squint mode imaging geometry on SAR signal properties is quite complicated [26]. However, as far as the DOPCEN estimation is concerned, when the squint is low, the approximation using the Principle of Stationary Phase is accurate enough.

When the squint increases, the properties of the signal structure become more complicated as the cross-coupling between the range and azimuth signals increases [27]. This may effect the performance of the DLR and the MLCC algorithms, which use the ACCC angle, and the performance of the MLBF algorithm, which uses the beat frequency estimation. Since the computation of the spectrum of a chirp involves the calculation of Fresnel integrals, for which no analytical expressions can be obtained, we cannot obtain an analytical expression for the spectrum of a compressed chirp. Therefore, the properties of the ACCC angles and the beat frequency should be studied in detail in the higher squint case using non-analytical means.
In this chapter, the effect of the squint on the ACCC angle and the effect of the squint on the beat spectrum is studied by simulations. Then simulations of the DLR, the MLCC and the MLBF algorithms on single point target and multiple point target are performed to examine the performance of these algorithms with higher squint. It is found that these three algorithms still work accurately to obtain correct estimates. This proves that the approximation to the spectrum of a compressed target using the Principle of Stationary Phase is still accurate enough with high squint (see Chapter 3).

Note that the simulations are ideal in the sense that we assume that the beam pointing angle does not vary with range frequency, i.e., $F_{os} = 0$. 
5.1 Effect of Squint on the ACCC Angle

The DLR, the MLCC and the MLBF algorithms were found to work reliably and accurately with ERS and J-ERS data, which normally have a low squint. When the squint increases, the properties of the signal structure become more complicated as the cross-coupling between the range and azimuth signals increases [27]. MDA found that MLCC algorithm did not work reliably with RADARSAT data, which generally has a higher squint.

Since the standard beam of RADARSAT has a higher squint than that of ERS, we will examine the effect of squint on the ACCC angle, which is the most important parameter that the MLCC and the DLR algorithms use.

To examine the effect of squint on the ACCC angles, computer simulations are performed. The point target we use in the simulations is the same as used in the last chapter. We can change the squint angle by specifying the DOPCEN when the target is generated.

Table 5.1 gives the corresponding squint angle of different DOPCEN values used in the experiments of this chapter. The radar parameters used are those adapted from the ERS and RADARSAT systems, and are given in Table 3.1. Figures 5.1 - 5.6 show the ACCC angles as a function of the range frequency with the 6 different squint values given in Table 5.1.

<table>
<thead>
<tr>
<th>DOPCEN (KHz)</th>
<th>-2</th>
<th>-5</th>
<th>-10</th>
<th>-15</th>
<th>-20</th>
<th>-50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Squint (degree)</td>
<td>0.46</td>
<td>1.56</td>
<td>2.31</td>
<td>3.50</td>
<td>4.62</td>
<td>11.42</td>
</tr>
</tbody>
</table>

Table 5.1: The squint angles corresponding to different DOPCENs
Figure 5.1: ACCC Angles with DOPCEN = -2KHz

Figure 5.2: ACCC Angles with DOPCEN = -5KHz
ACCC Angles vs. Range Frequency with DOPCEN = -10 KHz

Figure 5.3: ACCC Angles with DOPCEN = -10 KHz

ACCC Angles vs. Range Frequency with DOPCEN = -15 KHz

Figure 5.4: ACCC Angles with DOPCEN = -15 KHz
ACCC Angles vs. Range Frequency with DOPCEN = -20 KHz

Figure 5.5: ACCC Angles with DOPCEN = -20KHz

ACCC Angles vs. Range Frequency with DOPCEN = -50 KHz

Figure 5.6: ACCC Angles with DOPCEN = -50KHz
From the simulations, we can see that, with the increase of the squint, the linearity of the ACCC angles as a function of the range frequency remains unchanged. Thus, we can conclude that the approximation of the chirp spectrum using the Principle of Stationary Phase holds in the sense that the ACCC angle is a linear function of the range frequencies when the squint increases.

5.2 Effect of Squint on the Beat Spectrum

To examine the effect of squint on the beat spectrum, computer simulations are performed. The point target we use in the simulations is the same as used in the last chapter.

The same DOPCENs described in Table 5.1 are used in the simulations. The radar parameters used are those adapted from the ERS and RADARSAT systems, and are given in Table 3.1. Figures 5.7 - 5.12 show the beat spectra with the 6 different squint values given in Table 5.1.

From the simulations, we can see that there is always a dominant frequency in the beat spectrum even with a very high squint ($DOPCEN = -50 \text{ KHz}$). We also found that the beat frequency bandwidth increases with the squint. This is because of the short duration of the azimuth signal (or the beat signal) within any one range cell. The beat frequency bandwidth is inversely proportional to the duration of the signal in one range cell.
Spectrum of Azimuth Beat Signal with true DOPCEN = -2 KHz

Figure 5.7: Beat spectrum with DOPCEN = -2 KHz
Figure 5.8: Beat spectrum with DOPCEN = -5KHz
Figure 5.9: Beat spectrum with DOPCEN = -10KHz
Figure 5.10: Beat spectrum with DOPCEN = -15 KHz
Figure 5.11: Beat spectrum with DOPCEN = -20 KHz
Figure 5.12: Beat spectrum with DOPCEN = -50KHz
In the next two sections, we will examine the accuracy of the slope of the ACCC angle vs. range frequency by performing the DLR and the MLCC algorithms on single point target and multiple point targets.

The MLBF algorithm is also performed on single point targets and high contrast, multiple point targets data to examine the accuracy of the peak of the beat frequency under high squint.

5.3 Single Point Target Simulations

In this section, the DLR and the MLCC algorithms are performed on single point targets with the DOPCEN increasing from -2 \( KHz \) to -50 \( KHz \) to examine the accuracy of the slope that the DLR and the MLCC algorithm use. The MLBF algorithm is also performed to examine the accuracy of the peak of the beat frequency.

The method to generate the single point is the same as described in Chapter 4 except that the DOPCEN is specified corresponding to different squint shown in Table 5.1. The PRF we use is still 960 \( Hz \). The specified DOPCEN, the fractional PRF part of the DOPCEN and the ambiguity number are given in Table 5.2. Table 5.3 gives the estimates of the fractional PRF part of the DOPCEN from the DLR and the MLCC algorithms. Table 5.4 gives the estimates of the ambiguity by the DLR, the MLCC and the MLBF algorithms.
Using the correct values given in Table 5.2, the estimates of the fractional PRF part of the DOPCEN given by the DLR and the MLCC algorithms are very accurate (See Table 5.3). Thus, we can conclude that the average of the ACCC angles is still accurate when the squint is high.

From Table 5.4, the estimates of the ambiguity given by the DLR and the MLCC algorithms are still accurate even when the squint is high. From the single point target simulations, we can see that, when the squint is high, the slope of the ACCC angle is accurate enough for ambiguity estimation. Also the estimates of the ambiguity given by the MLBF algorithm is accurate even when the squint is high. Thus, we can conclude that the location of the peak frequency in the beat spectrum is still accurate when the squint is high.

In the next section, we use multiple point targets to demonstrate that the DLR, the MLCC and the MLBF algorithms work reliably when the squint is high.

### Table 5.2: The specified DOPCEN in simulations

<table>
<thead>
<tr>
<th>DOPCEN (KHz)</th>
<th>Frac. part (Hz)</th>
<th>Ambiguity</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-80</td>
<td>-2</td>
</tr>
<tr>
<td>-5</td>
<td>-200</td>
<td>-5</td>
</tr>
<tr>
<td>-10</td>
<td>-400</td>
<td>-10</td>
</tr>
<tr>
<td>-15</td>
<td>360</td>
<td>-16</td>
</tr>
<tr>
<td>-20</td>
<td>160</td>
<td>-21</td>
</tr>
<tr>
<td>-50</td>
<td>-80</td>
<td>-52</td>
</tr>
</tbody>
</table>
Table 5.3: Fractional PRF estimation results with higher squint on single point target

<table>
<thead>
<tr>
<th>DOPCEN</th>
<th>DLR</th>
<th>MLCC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frac. Part</td>
<td>Error</td>
</tr>
<tr>
<td>KHz</td>
<td>Hz</td>
<td>Hz</td>
</tr>
<tr>
<td>-2</td>
<td>-80</td>
<td>0</td>
</tr>
<tr>
<td>-5</td>
<td>-200</td>
<td>0</td>
</tr>
<tr>
<td>-10</td>
<td>-400</td>
<td>0</td>
</tr>
<tr>
<td>-15</td>
<td>362</td>
<td>2</td>
</tr>
<tr>
<td>-20</td>
<td>160</td>
<td>0</td>
</tr>
<tr>
<td>-50</td>
<td>-79</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5.4: Ambiguity estimation results with higher squint on single point target

<table>
<thead>
<tr>
<th>DOPCEN</th>
<th>DLR</th>
<th>MLCC</th>
<th>MLBF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DCen</td>
<td>Error</td>
<td>Amb</td>
</tr>
<tr>
<td>KHz</td>
<td>Hz</td>
<td>Hz</td>
<td>No.</td>
</tr>
<tr>
<td></td>
<td>DCen</td>
<td>Error</td>
<td>Amb</td>
</tr>
<tr>
<td></td>
<td>Hz</td>
<td>Hz</td>
<td>No.</td>
</tr>
<tr>
<td></td>
<td>DCen</td>
<td>Error</td>
<td>Amb</td>
</tr>
<tr>
<td></td>
<td>Hz</td>
<td>Hz</td>
<td>No.</td>
</tr>
<tr>
<td>-2</td>
<td>-1999</td>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>-5</td>
<td>-5067</td>
<td>-67</td>
<td>-5</td>
</tr>
<tr>
<td>-10</td>
<td>-10051</td>
<td>-51</td>
<td>-10</td>
</tr>
<tr>
<td>-15</td>
<td>-15007</td>
<td>-7</td>
<td>-16</td>
</tr>
<tr>
<td>-20</td>
<td>-19982</td>
<td>18</td>
<td>-21</td>
</tr>
<tr>
<td>-50</td>
<td>-49954</td>
<td>46</td>
<td>-52</td>
</tr>
</tbody>
</table>

Table 5.4: Ambiguity estimation results with higher squint on single point target
5.4 Multiple Point Targets Simulation

To demonstrate that there is no bias in the estimation of the DLR, the MLCC and the MLBF algorithm when the squint is high, multi-target simulations are performed. The DLR and the MLCC algorithm are performed on 6 low-contrast data blocks with the scene contrast of 1. The MLBF is performed on 6 high-contrast data blocks with the scene contrast of 4.8. The DOPCENs we use are shown in Table 5.1. These data blocks are generated using the same method described in Chapter 4.

Table 5.5 gives the estimates of the fractional PRF part of the DOPCEN from the DLR and the MLCC algorithms performed on the 6 low-contrast data blocks. Table 5.6 gives the estimates of the ambiguity by the DLR, the MLCC and the MLBF algorithms. The DLR and the MLCC algorithms are performed on the 6 low-contrast data blocks, and the MLBF algorithm is performed on the 6 high-contrast data blocks.

In the simulations, the estimates of the fractional PRF part given by the DLR and the MLCC algorithms are very accurate. All the DLR, the MLCC and the MLBF algorithms gave the correct estimates of the ambiguity. Looking at the mean absolute error for each estimator, we note that the MLCC algorithm has a slightly better accuracy than the DLR algorithm on the low contrast scenes. Even with the high contrast scenes, the MLBF algorithm has a higher mean absolute error than the DLR and the MLCC algorithms have on low contrast scenes.

Till now we can conclude that, the linearity of the ACCC angle as a function of the range frequency holds when the squint is high. The slope of this line is accurate enough for the DLR and the MLCC algorithms to obtain the correct ambiguity number.
We can also conclude that, the peak of the beat frequency will not be masked when the squint is high. The location of the peak frequency is accurate for the MLBF algorithm to get a correct estimate of the ambiguity number.

<table>
<thead>
<tr>
<th>DOPCEN KHz</th>
<th>DLR</th>
<th></th>
<th></th>
<th>MLCC</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frac. Part</td>
<td>Error</td>
<td>Frac. part</td>
<td>Error</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td>-80</td>
<td>0</td>
<td>-80</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-5</td>
<td>-200</td>
<td>0</td>
<td>-200</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-10</td>
<td>-400</td>
<td>0</td>
<td>-400</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-15</td>
<td>360</td>
<td>0</td>
<td>361</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-20</td>
<td>159</td>
<td>1</td>
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</tr>
<tr>
<td>-50</td>
<td>-81</td>
<td>-1</td>
<td>-79</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.5: Fractional PRF estimation results with higher squint on multiple point targets
<table>
<thead>
<tr>
<th>DOPCEN KHz</th>
<th>DLR</th>
<th>MLCC</th>
<th>MLBF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DCent</td>
<td>Error</td>
<td>Amb</td>
</tr>
<tr>
<td>-5</td>
<td>-5030</td>
<td>-30</td>
<td>-5</td>
</tr>
<tr>
<td>-10</td>
<td>-9987</td>
<td>13</td>
<td>-10</td>
</tr>
<tr>
<td>-15</td>
<td>-15028</td>
<td>-28</td>
<td>-16</td>
</tr>
<tr>
<td>-20</td>
<td>-20019</td>
<td>-19</td>
<td>-21</td>
</tr>
<tr>
<td>MAE</td>
<td>24</td>
<td>18</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.6: Ambiguity estimation results with higher squint on multiple point targets ("MAE" means "Mean Absolute Error")
5.5 Summary

In this chapter, we examined the linearity of the ACCC angle as a function of the range frequency when the squint increases. When the squint becomes high, this linearity still holds. Single point target simulations and multiple point target simulations are performed. From the performance of the DLR and the MLCC algorithms under high squint, we conclude that the linearity and the slope is accurate enough for these algorithms to obtain correct estimates of the ambiguity number.

We also examined the spectrum of the azimuth beat signal when the squint increases. When the squint becomes high, the bandwidth of the beat frequency increases. However, the peak frequency in the beat spectrum is not masked and the location of the peak frequency is still accurate enough for the MLBF algorithm to obtain correct estimates of the ambiguity number.

The MLCC algorithm works well on ERS data, but may not work on RADARSAT data, which has a higher squint. From the simulations in this chapter, we can conclude that the origin of any bias in the MLCC algorithm on RADARSAT data is not due to the high squint.
Chapter 6

Radiometric Sensitivities

Radiometric discontinuities in a SAR image are due to the difference of the reflectivity of scatterers on the ground surface. Echoes of the chirp from a certain part of the ground surface are strong, whereas echoes from another part are weak. The boundary of these two parts forms a discontinuity in the SAR data after compression. These discontinuities are very common in SAR images, such as at the boundaries of water and land.

Radiometric discontinuities have a significant effect on the performance of DOP-CEN estimators, including the phase-based estimators. In this chapter, we will discuss the effect of radiometric discontinuities on the DLR, MLCC and MLBF algorithms. Simulations are first performed to investigate the effect of the discontinuities. We use single-direction discontinuities, first in the azimuth direction, then in the range direction, and compare the performance of the three algorithms with each discontinuity. For the DLR and MLCC algorithms, we use low-squint, low-contrast data sets, except for the contrast change introduced by the radiometric boundary. For the MLBF algorithm, we
use low-squint, high contrast data sets.

We explain how each radiometric discontinuity affects each algorithm, showing why the DLR algorithm is more sensitive to the azimuth discontinuity and why the MLCC algorithm is more sensitive to the range discontinuity. We also explain why the MLBF algorithm is not affected by the radiometric discontinuity.

The three DOPCEN estimation algorithms are also run on ERS-1 data to illustrate our conclusions.
6.1 Experiments with Simulated Data

6.1.1 Simulation Methodology

In this part, the methodology of simulations is introduced. To examine the effect of radiometric discontinuity on the DLR, the MLCC and the MLBF algorithms, we examine the discontinuity in each of the azimuth and range directions separately.

The method of generating low-contrast simulated data for the DLR and the MLCC algorithm is the same as that described in Chapter 4. The discontinuity is generated in the reflectivity coefficient matrix. Figure 6.1 illustrates the pattern of the azimuth-direction discontinuity.

After an uniform reflectivity matrix is generated, in which the magnitude of each coefficient is equal to 1 and the phase of each coefficient is random, we create a discontinuity at the central azimuth cell. We keep the magnitude of all coefficients in Part A the same and amplify the magnitude of all coefficients in Part B by a factor $M$. We define the Magnitude Ratio $M$ as:

$$M = \frac{I_B}{I_A}$$  
(6.1)

where $I_A$ is the magnitude of the coefficients in Part A and $I_B$ is the magnitude of the coefficients in Part B. Figure 6.2 shows an azimuth-direction discontinuity in the central azimuth cell when $M = 5$. Note that the plot is subsampled for plotting efficiency.

After the reflectivity matrix is generated, it is convolved with the expanded point target array by fast convolution. Since the magnitude of the coefficients in the matrix represents the strength of the scatterers, a radiometric discontinuity is generated in the
simulated data. Figure 6.3 shows the discontinuity in the simulated data, illustrating that the azimuth data encoding has smoothed the edge, as azimuth compression has not taken place yet.

To generate the radiometric discontinuity in the range direction, the same method is used except that the boundary between Part A and Part B is at the central range cell, as shown in Figure 6.4. Figure 6.5 shows the magnitude of the reflectivity matrix which contains a radiometric discontinuity in the range direction. Note that the plot is also subsampled.

Figure 6.6 shows the range discontinuity in the simulated data after the convolution with the point target. Note that this data will be range compressed before the DOPCEN algorithms are run, so that the discontinuity becomes sharp again, as shown in Figure 6.7.
Figure 6.1: Illustrating the Generation of an Azimuth Radiometric Discontinuity

Figure 6.2: Magnitude of the Reflectivity Matrix with an azimuth discontinuity when $M = 5$
Figure 6.3: Azimuth Discontinuity in the Simulated Data when $M = 5$
Figure 6.4: Illustrating the Generation of a Range Radiometric Discontinuity

Figure 6.5: Magnitude of the Reflectivity Matrix with a range discontinuity when $M = 5$
Figure 6.6: Range Discontinuity in the Simulated Data when $M = 5$

Figure 6.7: Range Discontinuity in the Simulated Data After Range Compression
To generate high-contrast simulated data for the MLBF algorithm, the same method is used except that, before generating the radiometric discontinuity, high contrast is generated in the reflectivity matrix, as described in 4. In the simulations, the scene contrast before generating radiometric discontinuities is 4.8. This guarantees that the MLBF algorithm works well without radiometric discontinuities so that our research can focus on the effect of the radiometric discontinuity.
6.1.2 Simulation Results with an Azimuth Discontinuity

Nineteen low-contrast simulated data blocks containing a radiometric discontinuity in the azimuth direction are generated with the Magnitude Ratio changing from small to large. The DLR and the MLCC algorithms are run on these data blocks. The estimation results of the fractional PRF part of DOPCEN are shown in Table 6.1. The estimation results of the Doppler ambiguity are shown in Table 6.2. In the ambiguity case, the results are given in Hz units, which is the intercept when the ACCC slope is projected to the radar frequency.

In a similar fashion, another set of simulated data consists of nineteen high-contrast simulated data blocks is generated with the Magnitude Ratio changing from small to large to test the MLBF algorithm. The results are also shown in Table 6.2.

Note that the parameters we used in the simulations are given in Table 3.1, with a true DOPCEN of -400 Hz. They represent low-squint data, except for the contrast change at the radiometric discontinuity.
Table 6.1: **Fractional** PRF estimation results with an **azimuth** discontinuity
Table 6.2: Ambiguity estimation results with an azimuth discontinuity
6.1.3 Simulation Results with a Range Discontinuity

Nineteen low-contrast simulated data blocks containing a radiometric discontinuity in the range direction are generated with the Magnitude Ratio changing from small to large. The DLR and the MLCC algorithms are run on these data blocks. The estimation results of the fractional PRF part of DOPCEN are shown in Table 6.3. The estimation results of the Doppler ambiguity are shown in Table 6.4. In the ambiguity case, the results are given in Hz units, which is the intercept when the ACCC slope is projected to the radar frequency.

In a similar fashion, another set of simulated data consists of nineteen high-contrast simulated data blocks is generated with the Magnitude Ratio changing from small to large to test the MLBF algorithm. The results are also shown in Table 6.4.
<table>
<thead>
<tr>
<th>M</th>
<th>DLR Frac. Part Hz</th>
<th>DLR Error Hz</th>
<th>MLCC Frac. part Hz</th>
<th>MLCC Error Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>-400</td>
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<td>-400</td>
<td>0</td>
</tr>
<tr>
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<td>0</td>
<td>-401</td>
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<td>-401</td>
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<td>-401</td>
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<td>1</td>
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</tr>
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</tr>
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<td>1</td>
<td>-402</td>
<td>2</td>
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<td>120.0</td>
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<td>1</td>
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Table 6.3: Fractional PRF estimation results with a range discontinuity
<table>
<thead>
<tr>
<th>M</th>
<th>DLR DCen Hz</th>
<th>Error Hz</th>
<th>Amb No.</th>
<th>MLCC DCen Hz</th>
<th>Error Hz</th>
<th>Amb No.</th>
<th>MLBF DCen Hz</th>
<th>Amb No.</th>
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<td>1.0</td>
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<td>-4</td>
<td>0</td>
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<tr>
<td>1.1</td>
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<td>-3</td>
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<td>-394</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
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Table 6.4: Ambiguity estimation results with a range discontinuity
6.2 Discussion of Simulation Results

From the simulation results, we can see that the radiometric discontinuity has different effects on the DLR and the MLCC algorithms in the range and in the azimuth direction. However, both the azimuth and the range discontinuities have little effect on the MLBF algorithm. In this section, we discuss the following four aspects of the results:

- How the azimuth discontinuity affects the DLR, the MLCC and the MLBF algorithms,
- Which ACCC-based algorithm performs better with the azimuth discontinuity,
- How the range discontinuity affects the DLR, the MLCC and the MLBF algorithms, and
- Which ACCC-based algorithm performs better with the range discontinuity.

6.2.1 Effects of the Azimuth-Direction Radiometric Discontinuity

From Table 6.1 and Table 6.2, we can see that the discontinuity in the azimuth direction has a large effect on both the DLR and the MLCC algorithms. In the case of the fractional part, both estimators suffer a bias of about 55 Hz fairly independent of the Magnitude Ratio. In other words, the average ACCC angle is biased.

In the case of the Doppler ambiguity, the results are much more random, indicating that the slope of the ACCC vs. range frequency has a random error. In some cases, the
error is large enough to create an ambiguity error of one PRF.\(^1\)

The reason for this behavior of the estimators is easy to understand. In Chapter 4, we concluded that, when only one target is considered, the ACCC angle is given by Equation (3.6). This is the desired value which can be used to get the correct estimation of the Doppler ambiguity and the fractional PRF part of the DOPCEN.

When multiple point targets are involved, the ACCC angle will have an error component due to the cross correlations between two overlapped targets, as shown in Equation (4.5). In Chapter 4, we concluded that, for a fixed target, the sum of the cross correlation coefficients between the fixed target and its neighboring targets due to overlap is a random complex number. Only when the power of all targets is the same, does the mean of all the sums for each target due to overlap become zero, and the error in the ACCC angle tends to average out. When the strength of targets are not the same, the cross correlation coefficients due to overlap cannot be totally averaged out, leading to an error in the ACCC angle estimates.

In this simulation, targets in Part A are weak while targets in Part B are strong. Then near the boundary of Part A and Part B, the sum of all ACCCs between a target and all its neighboring targets in the same range cell is a random complex number. All the sums for each target cannot be averaged out due to the difference of the target strength near the boundary, and will cause a random error in the estimation of the phase increment. This random variation in the estimation of the ACCC angles at each range cell introduces a random error into the estimation of the slope, thus leading to a random

\(^1\)Note that the production estimators are usually applied to 4K×4K data blocks, which will reduce the random component of the error considerably. Here we use a 1K×1K data block. The larger data block may not improve the bias, unless the larger data block contains many discontinuities of different sizes and directions, so that their effect cancels out.
error in the estimation of the DOPCEN ambiguity.

In addition, an azimuth radiometric discontinuity can be modeled as an azimuth partial exposure of the weak target since a strong target will hide or upset the angle of a small target. Since the ACCC angle is very sensitive to the azimuth partial exposure, as explained in Chapter 3, the partial exposure of the weak target introduces a bias in the estimate.

From the estimation of the fractional PRF part of the DOPCEN shown in Table 6.1, we can see that the mean of the random variation in the ACCC angles at each range cell is not zero, thus the estimation of the fractional PRF part of the DOPCEN is significantly biased.

Figure 6.8(a) shows the estimation error in the estimates for Doppler ambiguity of the DLR and MLCC algorithms as a function of magnitude ratio. Figure 6.8(b) shows a zoomed view on the Magnitude Ratio from 1 to 5. Note only when the error falls in between $-PRF/2$ and $PRF/2$, which is represented by the two dotted lines in the figure, may the estimator gives a correct estimate of the DOPCEN ambiguity. We can see that the error is random, although it seems to increase with very large Magnitude Ratios.

The MLBF algorithm is not affected significantly by the azimuth discontinuity. This is easy to understand. In Chapter 4, we conclude that, when more than one significant target is present in the same range cell, the peak beat frequency becomes masked by cross-beating. As the number of dominant targets increase, the power due to the cross beating can eventually mask out the required beat frequency, as we observed in Figure 4.7. Since the azimuth discontinuity does not increase the number of the
dominant targets, it does not affect the performance of the MLBF algorithm, as we have seen in the simulations.

In addition, the azimuth discontinuity behaves like a partial azimuth exposure. We have explained in Chapter 3 that the beat frequency is relatively independent of partial azimuth exposure. This can also explain why the MLBF algorithm is not significantly affected by the azimuth discontinuity.
Figure 6.8: Ambiguity estimation error of the DLR and the MLCC algorithms caused by the azimuth discontinuity
6.2.2 Comparison of the DLR and the MLCC algorithms with an AZIMUTH discontinuity

In addition to the effect of partial exposure on the ACCC-based vs. the MLBF algorithms, there is also a distinction between the DLR and the MLCC algorithms due to the different operation of these two algorithms.

From Figure 6.8, we can see that, when estimate the Doppler ambiguity, the DLR algorithm is more sensitive to the azimuth discontinuity than the MLCC algorithm. Assuming that the bias in the ACCC angle due to the discontinuity is a random variable $\gamma$, the measured ACCC angle can be written as:

$$
\phi = \frac{2\pi K_a \eta_c}{F_a} + \gamma
$$

(6.2)

Since $\gamma$ is a random variable, it introduces a random error at each range cell or each range frequency. This leads to an error in the estimation of the slope, which can lead to an error in the estimated ambiguity number with either algorithm.

For the MLCC algorithm, the biased ACCC angles of range look 1 and range look 2 is given by:

$$
\phi_{L1} = \frac{2\pi K_{a1} \eta_c}{F_a} + \gamma_1
$$

(6.3)

$$
\phi_{L2} = \frac{2\pi K_{a2} \eta_c}{F_a} + \gamma_2
$$

(6.4)

where $\gamma_1$ and $\gamma_2$ are the errors in the ACCC angles of look 1 and look 2 due to the azimuth discontinuity. The difference of $\phi_{L1}$ and $\phi_{L2}$ is given by:

$$
\Delta \phi = \phi_{L2} - \phi_{L1}
$$
\[ \frac{2\pi (K_{a2} - K_{a1}) \eta_c}{F_a} + (\gamma_2 - \gamma_1) \]
\[ \frac{2\pi (K_{a2} - K_{a1}) \eta_c}{F_a} + \Delta \gamma \]

(6.5)

where \( \Delta \gamma \) is equal to \( \gamma_2 - \gamma_1 \).

Since the two range looks image the same scatterers on the ground, we can assume that the random variables, \( \gamma_1 \) and \( \gamma_1 \), are correlated with each other. Then the random variable, \( \Delta \gamma \), has a smaller variation than \( \gamma \) in Equation (6.2).

If the correlation is significant, the error in the ambiguity estimate of the MLCC algorithm should be smaller than that in the DLR algorithm. In the simulations, the average estimation of the DLR algorithm is -140 Hz and the standard deviation is 771 Hz. The average estimation of the MLCC algorithm is -238 Hz and the standard deviation is 550 Hz, which supports the above theory.

Thus we see that the process of finding the difference of the ACCC angles of look 1 and look 2 in the MLCC algorithm reduces the variation of the error due to the azimuth discontinuity. The lack of this step in the DLR algorithm makes the DLR algorithm more sensitive to the azimuth discontinuity than the MLCC algorithm.
6.2.3 Effects of the Range-Direction Radiometric Discontinuity

From Table 6.3 and Table 6.4, we can see that, when we estimate the Doppler ambiguity, the discontinuity in the range direction has effects on the DLR algorithm and the MLCC algorithm, especially on the MLCC algorithm. However, when we estimate the fractional PRF part of the DOPCEN, the range discontinuity has negligible effect on either the DLR or the MLCC algorithm.

In Chapter 3, when we discuss the azimuth signal after range compression, as shown in Equation (3.2), we ignored the effect of range cell migration (RCM). In fact, the azimuth signal after range compression is a curve instead of a line confined to one range cell. The higher the squint is, the more obvious the curvature is. In a low contrast scene, simulations show that this effect can be ignored.

However, when there is a large discontinuity in the range direction, a part of the azimuth signal of a strong target extends into the weak region, and affects the phase of the weak targets. Figure 6.9 illustrates this situation. In Figure 6.9, the thick line represents a strong target near the boundary and the thin line represents a weak target near the boundary. Since both the DLR and the MLCC algorithms work in the azimuth direction, the strong target will affects the estimate of the phase of the weak target. This seems to affect the distribution of the errors in the ACCC angle, in such a way that it biases the slope estimate without affecting the average ACCC angle over the whole range time or frequency domain.

The nature of the slope bias is shown in Figure 6.10(a), which shows the error of the ambiguity estimate of the DLR and MLCC algorithms as a function of the magnitude ratio. Figure 6.8(b) shows a zoomed view on the Magnitude Ratio from 1 to 5.
The MLBF algorithm is not affected by the range discontinuity. This is because that, the estimation of the MLBF algorithm uses the average of the spectrum of the beat signal at each range cell. The effect of the range discontinuity on the the beat signal near the range discontinuity is averaged out, as we have seen in the simulation results.

In addition, the partial exposure, or weak targets masked by strong targets, have a large effect on the ACCC angle but little effect on the beat frequency. This can also explain why the DLR and the MLCC algorithm are affected by the range discontinuity, whereas the MLBF algorithm is not.

Figure 6.9: Illustration of the effect of a range discontinuity
Figure 6.10: Ambiguity estimation error of the DLR and the MLCC algorithms caused by the range discontinuity.
6.2.4 Comparison of the DLR and the MLCC algorithms with the RANGE discontinuity

Even though the main difference in reaction to range discontinuities is between the ACCC-based algorithms and the MLBF algorithm, there are differences between the two ACCC-based algorithms.

From Figure 6.10, we can see that the MLCC algorithm is more sensitive than the DLR algorithm to the range discontinuity. Both algorithms are biased by the range discontinuity, as discussed above. The range discontinuity also causes another bias only in the MLCC algorithm, which makes the MLCC algorithm more sensitive to the range discontinuity than the DLR algorithm.

To illustrate this effect, we first consider a single point target. Figure 6.11(a) shows the range compressed pulse of a range look. In this case, the location of the peak is at the 82nd range cell. Figure 6.11(b) shows the difference of ACCC angles of range look 1 and range look 2. Note that the MLCC algorithm works in the range time domain.

Table 6.5 gives the value of the difference of ACCC angles around the peak and the corresponding estimation of the DOPCEN according to Equation (3.32).

From Table 6.5, we can see that only the $\Delta \phi$ value at the peak of the target gives the correct estimate of the DOPCEN, in this case, $-400 \text{ Hz}$ . Others around the peak are too noisy to be useful. This is because, after range compression, almost all the signal energy is compressed to the peak index. Other parts of the compressed pulse have a very low SNR. Thus we can conclude that, in the MLCC algorithm, only the range
sample at the peak location is "useful" for the estimation process. Other parts of the range compressed signal are noisy and the difference of the ACCC angles of these parts lead to a high phase noise.

Now consider two targets near one another. After range compression, two peaks appear and these two peaks are next to each other in the range time domain, as shown in Figure 6.12. From the conclusion above, only the values of $\Delta \phi$ of the two peaks have high SNR and their average can give the correct estimation of the DOPCEN. The phase noise of the strong pulse will affect the useful phase value at the peak of the weak pulse, depending upon the relative strengths of each target.

When the MLCC algorithm works on a low contrast scene, the phase noise is relatively low compared to the peak it affects. However, when there is a discontinuity in the range direction, the "noise" from the strong target will effectively distort the weak
Table 6.5: Difference of ACCC Angles and DOPCENs (MLCC effect only)

<table>
<thead>
<tr>
<th>Range Cell No.</th>
<th>$\Delta\phi$ (mrad)</th>
<th>DOPCEN (Hz)</th>
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</thead>
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<tr>
<td>79</td>
<td>-8.2</td>
<td>614</td>
</tr>
<tr>
<td>80</td>
<td>0.0</td>
<td>0</td>
</tr>
<tr>
<td>81</td>
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<td><strong>-425</strong></td>
</tr>
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</tr>
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<td>85</td>
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<td>745</td>
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</table>

peak, leading to a random bias in the average of all the $\Delta\phi$ values at each peak near the range discontinuity. This results in an additional bias in the estimates of the MLCC algorithm.

Since the DLR algorithm works in the range frequency domain, after range FFTs, the spectra of strong targets and weak targets are all aligned. Thus the strong targets and the weak targets near the discontinuity do not interfere with each other in the way that they do in the MLCC algorithm. This may explain why the MLCC algorithm is more sensitive to range discontinuities than the DLR algorithm.
Figure 6.12: Illustration of the effect of a strong target on a weak target
6.3 Experiments with ERS-1 Data

To demonstrate the effect of the radiometric discontinuity in real data and its effects on the DLR algorithm, the MLCC algorithm and the MLBF algorithm, ERS-1 raw data are used. Note in this case, the estimation of the DOPCEN given by the DLR algorithm and the MLCC algorithm contains a frequency offset, which is attributed to the variation of the antenna pointing angle with transmitted frequency.

6.3.1 Azimuth Discontinuity in ERS-1 Data

An ERS-1 scene in yaw-steering mode is used. This image, Kamloops, is a medium contrast image, thus, all the three algorithms are expected work well on this scene. The acquisition data of this scene is Sep. 23, 1995 and the orbit/track is 21916/385. This scene contains a lake, which forms a discontinuity in the azimuth direction. We took a $1024 \times 1024$ data block containing this discontinuity selected from the right hand side of Figure 6.13.

Figure 6.13: The Detected Image of the Kamloops ERS Scene

The absolute DOPCEN in the middle of the scene is $527 \text{ Hz}$, obtained by MDA's
dtSAR processor. This value is used in the processing of the raw data to obtain the detected image shown in Figure 6.13. This estimate is likely reliable since the detected image has a high quality. The horizontal direction is the range direction with near range on the left. The azimuth direction is vertical, with azimuth time increasing in the upward direction.

The estimate of fractional PRF part of the DLR algorithm on this data block is 435 Hz. The estimate of the MLCC algorithm is 467 Hz. The estimate error of these two algorithms are -92 Hz and -60 Hz respectively. Both of these estimates exhibit a small bias, likely due to the azimuth discontinuity, as discussed above.

The ambiguity estimate of the DLR algorithm on this data block is 2447 Hz. The estimate of the MLCC algorithm is 944 Hz. Since the PRF is about 1680 Hz, the MLCC algorithm can gives the correct estimate of the Doppler ambiguity and the DLR algorithm does not. This observation agrees with the simulation results of Section 6.1.2.

The ambiguity estimate of the MLBF algorithm is 201 Hz, which is accurate enough to get the correct ambiguity, zero.

This experiment illustrates that the DLR algorithm is more sensitive to the azimuth discontinuity than the MLCC algorithm. It also demonstrates that the azimuth radiometric discontinuity has little effect on the MLBF algorithm.

6.3.2 Range Discontinuity in ERS-1 Raw Data

Another ERS-1 scene in yaw-steering mode is used which contains a radiometric discontinuity in the range direction. This image, Port Alice, is obtained on Sep. 23, 1995 and
the orbit/track is 361/999. The left part of this scene is ocean and the right part is land. The boundary between the ocean and the land forms a discontinuity in the range direction, as the ocean is smoother and has low reflectivity near the shoreline. We select a data block containing this discontinuity and the size of the data block is $1024 \times 1024$ samples. Figure 6.14 shows the detected image from which this block was taken near the ocean/land boundary.

![Figure 6.14: The Detected Image of the Port Alice ERS Scene](image)

The absolute DOPCEN in the middle of the scene is $-385 \text{ Hz}$, obtained by MDA's dtSAR processor. The horizontal direction is the range direction with near range on the left. The azimuth direction is vertical, with azimuth time increasing in the upward direction.

The fractional PRF part estimate of the DLR algorithm is $-358 \text{ Hz}$. The estimate of the MLCC algorithm is $-367 \text{ Hz}$. The estimate errors are 27 Hz and 18 Hz respectively. This bias is relatively small and may be caused by the high scene contrast. Thus, this experiment demonstrates that the range discontinuity has a small effect on the estimation of the fractional PRF part of the DOPCEN.

The ambiguity estimate of the DLR algorithm on this data block is $-926 \text{ Hz}$.
The ambiguity estimate of the MLCC algorithm on this data block is -1116 Hz. Since the PRF is about 1680 Hz, both algorithms give the correct estimate of the Doppler ambiguity number, but the better estimate of the DLR algorithm leaves more room for error. This observation agrees with the simulation results.

The ambiguity estimate of the MLBF algorithm is -805 Hz. It is also accurate enough to get the correct estimate of the ambiguity, zero.

Thus this experiment illustrates that the MLCC algorithm is more sensitive to the range discontinuity than the DLR algorithm. It also demonstrates that the range radiometric discontinuity has little effect on the MLBF algorithm.
6.4 Summary

In this chapter, we discussed the effects of radiometric discontinuities on the DLR, MLCC and MLBF DOPCEN estimation algorithms.

The azimuth discontinuity has no effect on the MLBF algorithm since it does not increase the number of the dominant targets and the MLBF algorithm is not sensitive to the resulting partial exposure. However, the azimuth discontinuity has a significant effect on the DLR and the MLCC algorithms since both of these two algorithms average the ACCC angle over the azimuth direction (See Chapter 3). The discontinuity in the azimuth direction introduces a bias in the ACCC angles at each range cell, leading to an error in the estimation of the slope. Thus the estimation of the projected DOPCEN, which is used to obtain the Doppler ambiguity, can have a substantial error since small errors in the slope causes a large error in the estimate of the DOPCEN. The fractional PRF estimates are also significantly biased.

The process of finding the difference of the ACCC angles of look 1 and look 2 in the MLCC algorithm reduces the variation of the bias due to the azimuth discontinuity. The lack of this step in the DLR algorithm makes the DLR algorithm more sensitive to the azimuth discontinuity than the MLCC algorithm.

The range discontinuity has no effect on the MLBF algorithm since the beat frequency is relatively independent of the position in the azimuth exposure and the MLBF algorithm works on the average of the beat spectrum over range. Also, the range discontinuity has a small effect on the DLR and the MLCC algorithm compared to the azimuth discontinuity. This is because only the ACCC angles near the discontinuity is affected. A range-compressed signal is actually a curve instead of a straight line confined
in one range cell. In a low contrast scene, simulations show that this effect can be ignored. However, when there is a large discontinuity in the range direction, a part of a azimuth signal of a weak target extends into the strong region, thus is largely reduced due to the strong targets. This may introduce a bias into the ACCC angles in the vicinity of the discontinuity, thus a bias exists in the estimation of the DOPCEN.

Since the MLCC algorithm works in the range time domain, the phase noise of a strong target will distort the desired phase value at the peak of a weak target. This may lead to a bias in the average of the difference of the ACCC angles at each peak near the range discontinuity. However, the DLR algorithm works in the range frequency domain. The spectra of strong targets and weak targets are all aligned. Thus the strong targets and the weak targets near the discontinuity do not interfere with each other in the way that they do in the MLCC algorithm. This makes the DLR algorithm more robust to the range discontinuity than the MLCC algorithm.

The results also point out that the two algorithms use the same method of estimating the fractional PRF part of the DOPCEN, as they both simply use the average ACCC angle over the range swath. Thus the fractional PRF estimates are substantially the same in each experiment. However, it is clear that the performance in estimating the Doppler ambiguity is quite different, which we attribute to the different way in which the data is aligned when estimating the change in ACCC with range frequency.
Chapter 7

Conclusions

7.1 Summary

The objective of this research is to examine and test the performance of the phased-based Doppler estimation algorithms with different scene contrasts and SNR levels, examine the performance of these algorithms with higher squint data, and examine the sensitivity of some phase-based Doppler estimation algorithms to radiometric discontinuities to find out how the radiometric discontinuities affect these estimation algorithms.

The form of the received signal after demodulation is derived. The effect of range sampling is discussed, illustrating that the RCM in the raw data causes a shift in the azimuth spectra, forming the dependence of DOPCEN on range time. The Principle of Stationary Phase is applied to approximate the spectrum of range-compressed signal, showing that the DOPCEN is a linear function of range frequency under this approxi-
mation, which is the basic principle of the phase-based DOPCEN estimators.

The operation of the DLR, MLCC and MLBF algorithms have been introduced in this chapter. All of these algorithms are based on the principle that the DOPCEN is approximately a linear function of the range frequency. Single point target simulations are performed to illustrate the operations of these algorithms. From the simulations, the estimates are almost the same since they are based on the same principle. It also shown that the linear approximation in the low squint case is accurate enough to obtain an accurate estimate of the DOPCEN.

Performance of the DLR, MLCC and MLBF algorithms are examined. The DLR algorithm and the MLCC algorithm work well on this set of data. The MLBF algorithm does not work on the low contrast data set. When the scene contrast increases, the MLBF algorithm begins to work well while the error of the DLR algorithm and the MLCC algorithm becomes large, leading to an incorrect estimation of the Doppler ambiguity.

As the DLR and MLCC algorithms behave almost the same in terms of scene contrast, it is interesting to also compare them in terms of their response to scene noise. We conclude that the DLR algorithm and the MLCC algorithm are robust to white noise on low contrast scenes. The estimation errors of the DLR and the MLCC algorithms on same noise level are almost the same.

The linearity of the ACCC angle as a function of the range frequency is examined when the squint increases. When the squint becomes high, this linearity still holds. From the performance of the DLR, the MLCC and the MLBF algorithms under high squint, we conclude that the linearity and the slope is good enough for these algorithms to obtain correct estimates of the ambiguity number. The MLCC algorithm works well
on ERS data, whereas may not work on RADARSAT data, which has a higher squint. From the simulations, we can conclude that the origin of the bias in the MLCC algorithm on RADARSAT data is not due to the higher squint.

Radiometric discontinuities have a significant effect on the performance of DOPCEN estimators, including the phase-based estimators. From simulations, we can see that the discontinuity in the azimuth direction has a large effect on both the DLR and the MLCC algorithms. In the case of the Doppler ambiguity, the results are much more random, indicating that the slope of the ACCC vs. range frequency has a random error. In some cases, the error is large enough to create an ambiguity error of one PRF. When multiple point targets are involved, the ACCC angle will have an error component due to the cross correlations between two overlapped targets. For a fixed target, the sum of the cross correlation coefficients between the fixed target and its neighboring targets due to overlap is a random complex number. Only when the power of all targets is the same, does the mean of all the sums for each target due to overlap become zero, and the error in the ACCC angle tends to average out. When the strength of targets are not the same, the cross correlation coefficients due to overlap cannot be totally averaged out, leading to an error in the ACCC angle estimates.

From simulations, we can see that, when estimate the Doppler ambiguity, the DLR algorithm is more sensitive to the azimuth discontinuity than the MLCC algorithm. The process of finding the difference of the ACCC angles of look 1 and look 2 in the MLCC algorithm reduces the variation of the error due to the azimuth discontinuity. The lack of this step in the DLR algorithm makes the DLR algorithm more sensitive to the azimuth discontinuity than the MLCC algorithm.
The MLBF algorithm is not affected by the azimuth discontinuity. When more than two significant targets are present in the same range cell, the distortion of the peak beat frequency gets worse. As the number of dominant targets increase, the power due to the cross beating can eventually mask out the required beat frequency. Since the azimuth discontinuity does not increase the number of the dominant targets, it does not affect the performance of the MLBF algorithm.

When we estimate the Doppler ambiguity, the discontinuity in the range direction has effects on the DLR algorithm and the MLCC algorithm, especially on the MLCC algorithm. However, when we estimate the fractional PRF part of the DOPCEN, the range discontinuity has negligible effect on either the DLR or the MLCC algorithm.

Since the DLR algorithm works in the range frequency domain, after range FFTs, the spectra of strong targets and weak targets are all aligned. Thus the strong targets and the weak targets near the discontinuity do not interfere with each other in the way that they do in the MLCC algorithm. This may explain why the MLCC algorithm is more sensitive to range discontinuities than the DLR algorithm.

The MLBF algorithm is not affected by the range discontinuity. This is because that, the estimation of the MLBF algorithm uses the average of the spectrum of the beat signal at each range cell. The distortion of the the beat signal near the range discontinuity is averaged out.

Since the MLCC algorithm works best with scenes of low, uniform contrast, while the MLBF algorithm works best with scenes of high contrast, and since they all need the processing of range look extraction, these two algorithms can be efficiently combined together to form a reliable DOPCEN estimator over large ranges of scene contrast.
7.2 Contributions

The contributions of this research work are:

- Examined the effect of range sampling, and concluded that, if there is not RCM, the DOPCEN is a constant for all azimuth lines. If RCM does exist, the DOPCEN is a linear function of the range frequency.

- Examined the cross correlation item in the ACCC angle due to overlap when multiple targets are involved, and concluded that, only when the strength of all targets are same can these items be averaged out.

- Understood the difference between the ACCC-based algorithms and the MLBF approach.

- Implemented quadratic curve fitting to the MLBF estimator when the scene contrast is low.

- Proposed the theory on how the radiometric discontinuity affects the performance of the DLR, MLCC and MLBF algorithms, and proved it by simulations and real data experiments.

- Proposed the theory on why the DLR algorithm is more sensitive to the azimuth discontinuity than the MLCC algorithm, whereas the MLCC algorithm is more sensitive to the range discontinuity than the DLR algorithm, and proved it by simulations and real data experiments.
7.3 Future Work

Our results suggest the following topics for future research:

- More experiments are needed to quantify the effect of radiometric discontinuities and to improve the performance of the MLCC and the DLR algorithms.

- All these algorithms should be examined with real data, which have a higher squint, for example, RADARSAT data.
Bibliography


