Design and Development of a Magnetostrictive Actuator

by

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Abstract

In this joint project between NRC and UBC, the author has developed an actuator using Terfenol-D, an optical precision displacement sensor, and a few control strategies. These control strategies include proportional-plus-integral-plus-derivative (PID), cascaded PID, and sliding mode control strategies. Each of these controllers was designed and tested experimentally on the actuator. The controllers were very difficult to design because the dynamics of Terfenol-D has hysteresis. The system with the designed PID controller showed sign of instability under load. However, the designed cascaded PID and the designed discrete-time sliding mode controllers are capable of controlling the actuator under loaded condition and fulfilling the requirements of NRC. Real-time experimental results are attached in chapter 5.
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1 Introduction

1.1 Motivation

1.1.1 Background

Fretting is a very common kind of wear in machine components. It is caused by an oscillatory slip between two contacting surfaces. Since fretting affects our daily life quite significantly, it is necessary to study it. Nowadays, one of the major concerns is the fretting wear in pressure tubes of a nuclear reactor. This will have major consequences from both financial and safety point of view. A group of researchers from National Research Council's (NRC) Tribology Laboratory are interested in solving this problem. Hence, it is necessary to study the wear characteristics of various materials, especially under high pressure, high temperature water environment.

The test system will consist of an autoclave, a powerful yet compact actuator, a specimen assembly, a normal force applicator, and a computer based dynamic control system. The autoclave will allow the researchers to simulate different test environments for studying wear. The actuator was developed from a magnetostrictive material, Terfenol-D, which is an actuation material with high enough strain and stiffness to yield a compact size. The NRC required the actuator to be able to push a load of $500N$ over a displacement of $130\mu m$ at a maximum frequency of $70\text{Hz}$. The developed actuator will fit into the autoclave and be insulated from the test compartment's temperature. This minimizes the problem of sealing the autoclave. Compactness is very important because the construction of the autoclave will be too expensive and too large if the internal volume is not restricted. An optical sensor will be used to sense the actual displacement of a test specimen. Also, a computer subsystem will be employed to control the vibration of the actuator.
1.1.2 Objective of the Research

The main objective of this project is to design an effective controller for the Terfenol-D based actuator. In addition, the author will have to design the mechanisms of the actuator and an optical position sensor. The presence of hysteresis characteristics makes the design of controllers a challenge and there are not many published results of controlled Terfenol-D actuators.

1.2 Development and Implementation of Terfenol-D Based Actuator

Terfenol-D is a recently developed material composed of highly magnetostrictive rare-earths, Tb and Dy, combined with the magnetic transition metals, Ni, Co, and Fe. This material will change its shape and size in the presence of changing magnetization state. Terfenol-D has a strain of $2000 \text{ ppm}$. Since Terfenol-D became commercially available in the mid 1980s, there have been lots of interest and designs using Terfenol-D.

In a conference held in March 1986 in Spain, several application areas were proposed and discussed. The applications considered are listed below [14]:

1. Sonar Transmitter: This application required the conversion of an electrical signal into a controlled frequency sonar signal generator. Terfenol-D was used in mechanically amplifying the signal.
2. High Force Robot Gripper for Armaments: Terfenol-D was proposed to be used as a force sensor in this application.
3. Electrical Circuit Breaker
4. Internal Combustion Engine Valve Lifter
5. Diesel Fuel Injection Valve
6. Vibration Isolator

No control strategy was considered in this conference, however.

In a paper published by Olof Vingsbo and Joakim Schon [28], a Terfenol-D based actuator was used in a fretting test system as well. The required displacement is very small, no more
than 5\textmu m, with an actuator capable of providing 150\textmu m. The paper mentioned nothing about the implemented controller. In fact, operating in a very small minor loop within a relatively large hysteresis loop, the hysteresis characteristic is insignificant. It behaves as if it is linear; thus, the process of designing a controller is simplified. A proportional-plus-integral-plus-derivative (PID) controller is capable of fulfilling the requirement. In another work by Michael Bryant et al. [3], they used three Terfenol-D actuators to actively suppress the vibration of a table. The controller implemented was a combination of a neural network and a PID controller. The article demonstrated the ability of the system to eliminate disturbances up to 100Hz. Terfenol-D also has been proposed to be used in a multiple degree-of-freedom Stewart Platform-type mechanism by Geng and Haynes for active vibration control [10]. Robust adaptive control algorithms for active vibration control were formulated by them as well.

The actuators mentioned are very different in structure comparing to our design. The existing designs used a single rod in each of the actuators. In our design, we employed two rods in a push-pull design. This design has several advantages over the existing designs. Details will be discussed in the following chapter.

1.3 Controller Design

Since the dynamic properties of a Terfenol-D rod consist of a hysteresis loop, the description of the plant is not easy. In the control processes mentioned in the previous section, some ingenious idea was incorporated into the design of the controllers. This allows those controllers to adapt themselves on-line under the change of plant parameters.

In this thesis, three different control strategies were tested. The strategies considered are PID, cascaded PID, and sliding mode controllers. The latter two control strategies are capable of dealing with changes in system parameters better than PID controller. PID controller was considered simply because it is the most commonly used controller in process control. There are several advantages in implementing PID control algorithm. Firstly, it is relatively easy to implement analogically or digitally. Secondly, no knowledge of the plant is required. There
are three adjustable gains in a PID controller, to achieve a desired output. Cascaded PID controller is an extension of PID controller; it is robust against some change in disturbance and also system parameters. These two control strategies need no knowledge of system's model. Details on PID and cascade PID controllers, and the tuning strategies for their parameters are discussed in chapter 3.

Sliding mode control was introduced in the Soviet Union more than 30 years ago which relies on high frequency switching between control values [25]. The author considered the implementation of the sliding mode control in this project largely because of its robustness against model uncertainties. The uncertainties of a model include unstructured uncertainties and structured uncertainties. The former consist of the inaccuracies on the system order while the latter consist of the inaccuracies in the terms of a model [24]. By treating the plant as a linear system, least squares estimation technique was employed in the identification of system parameters. Based on the limited knowledge of the plant thus derived, a sliding mode controller was then designed. Since the actual plant's dynamics consists of hysteresis, the robustness of sliding mode control against model uncertainties is essential for controlling the plant.

The sliding mode control was developed based upon the continuous time system and its robustness was well known [9]. In normal practice, one would design the control law based on the continuous time dynamics of the plant and then implement the law in discrete time [11]. In this thesis, a sliding mode control law was synthesized in discrete-time and implemented in discrete-time. In [6], [9], [19], [20], [22], and [26], the control laws were derived to minimize disturbance. Servo problem was not solved directly. Combining the ideas of PID control and the discrete-time sliding mode control mentioned in those papers, the author derived a more direct approach in tracking time varying signal. Details are shown in chapter 4.

1.4 Contributions

The objectives of this project are to develop a computer controlled Terfenol-D actuator,
and to experimentally test different controllers on the actuator. The contributions of this project are summarized as follows:

- Development of a Terfenol-D actuator using a unique, push-pull, design. This design has some advantages over the existing designs.
- Derivation of a discrete-time sliding mode control law for a highly nonlinear system, based on an estimated linear model, to track time varying signal.
- Verification of the derived discrete-time sliding mode control law by testing it on the actuator experimentally.
- Development of a computer controlled Terfenol-D actuator capable of fulfilling the requirements of the NRC.
2 Mechanical Design and System Identification

2.1 Introduction

This chapter describes the design of the actual hardware for both experiment and application. Some comparisons are shown between the existing designs and our design at UBC. To fully utilize the advantages of our design, frictionless joints are needed. The mechanism of the optical position sensor is introduced. After the actual hardware has been designed, a model of the actuator is required for the purpose of controller design and simulation. Some modeling methods are also discussed later.

2.2 Design of Actuator

2.2.1 Existing Designs

To increase the strain of a Terfenol-D rod, mechanical pre-stress is needed to change the properties of the material. In almost all existing designs that were published or commercially sold, a simple spring type mechanism was employed to provide this pre-stress. This type of pre-stress mechanism is very simple, but the stress applied on a rod may not be constant. Since pre-stress will change the properties of a Terfenol-D rod, changes in the pre-stress will result in an actuator that is more difficult to control as the model will vary with time. This is not a problem in our design. Details will be discussed later in this chapter.

In most of the existing designs, a single Terfenol-D rod is responsible for the pushing action while the pulling action is done by the pre-stress spring mechanism. This is because Terfenol-D is good for pushing only. To eliminate the reliance on a spring, a push-pull configuration was developed for the present design. This arrangement allows for full range operation to be carried out symmetrically.
Since the direction or polarity of external magnetic field has no effect on the direction of a Terfenol-D rod's expansion, a magnetic biasing is necessary to have a bi-directional movement. Figure 2.1 demonstrates why a magnetic bias is necessary. In some of the existing designs, the biasing was achieved using DC current instead of a permanent magnet. There are some drawbacks if DC current is employed for biasing. The obvious effect of using DC current for biasing is the overheating problem due to heat loss. Since Terfenol-D is temperature sensitive, the operating range decreases as the temperature around it increases. The curie point of this material is around 380°C; that means a Terfenol-D rod will not expand or contract at this temperature. Thus, in most cases, a permanent magnet is used for biasing instead. In our design, we used permanent magnets to create the bias, then compensate the remaining offset by a small DC current. The undesired effect of losing operating range due to heating is minimized.

2.2.2 In-House Design

After addressing some weaknesses of existing designs, we decided on a new design, which minimized the undesired effects mentioned. The main idea behind the design is two Terfenol-D rods in a push-pull mechanism. We rely on the pushing of the rods only; thus, we eliminate the reliance of a spring mechanism to pull the actuator back in. In this design, there must be a stiff pivoting beam. This pivoting beam maintains constant mechanical pre-stress applied on the rods independent of the motion of the actuator. Constant pre-stress
also means that the properties of the rods remain the same in both directions of actuation. Also, we chose to use permanent magnets for the biasing so that the undesirable effect of losing range due to over heating is minimized.

In figure 2.2, there are two rods on each side of the pivoting beam. The stiffness of this overall structure is equivalent to the stiffness of a single rod and the range of operation is increased.

In the following example, we simplify the model of the rods as spring (see figure 2.3). A force $F$ is forcing down on one side of the pivoting beam. By summing the moment around
the fulcrum of the pivoting beam in clockwise direction, we have:

\[ Fl - F_1 l - F_2 l = 0 \]

\[ \therefore F_1 = -kx, F_2 = -kx \]

\[ F = 2F_1 = -2kx \]  \hspace{1cm} (2.1)

where \( k \) and \( l \) are the spring constant and half the length of the pivoting beam, respectively. From equation (2.1), the stiffness of the structure shown in figure 2.3 is doubled. However, if one stacks two rods in series alone by themselves, stiffness will actually be decreased by one half. Therefore, in our design as shown in figure 2.2, the overall stiffness is equivalent to the stiffness of a single rod.

From the above, one might argue that we could use a longer Terfenol-D rod with larger diameter in order to have higher displacement without sacrificing the stiffness. However, because of the skin effect of Terfenol-D, this will result in limiting the frequency response of the actuator. From a financial point of view, the push-pull design should not increase the cost very much, because with a push-pull configuration, the rods can be thinner; that means, instead of using a larger rod, we can use two thinner rods of the same length. In addition, we can gain all the advantages mentioned by employing the push-pull configuration. However, to employ such a configuration, we need a special technique that minimizes wearing between joints, to support the structure. More details about these frictionless joints will be discussed in the following section.

### 2.3 Frictionless Flexure Pivots

In the last section, we discussed the advantages of the push-pull configuration. To fully utilize the advantages of this configuration, we require frictionless joints between the supporting casing and the Terfenol-D rods. Some studies done previously at UBC concluded that flexure pivots are the best joints for connecting the Terfenol-D rods together [8]. In our design, there are three flexure pivots used; they are simply made of metal plates. Figure 2.4
shows the use of the flexure pivots in our design. One of the flexure pivots is used as the
fulcrum of the pivoting beam. The other two are used as mechanical couplings between the
pivoting beam and the Terfenol-D rods. These flexure pivots are made out of stainless steel
because the actuator will be placed into an environment which is likely to corrode carbon
steel. The flexure pivots used in this project have the dimension of 0.8mm in thickness,
11mm and 50mm in height and length, respectively. These plates are inserted about 3mm
on each end into a flexure clamp. Each flexure pivot has a free-standing height of 5mm.

After deciding on the dimension of these flexure pivots, we have to determine if these
plates can withstand the stress applied. This is to prevent any potential material failure.
The maximum force experienced by these flexure pivots is the sum of the mechanical pre-
stress plus the maximum expected load. The cross-section area of a Terfenol-D rod used
is $\pi \left(\frac{13mm}{2}\right)^2 = 132.7mm^2 = 0.133 \times 10^{-3}m^2$. Commonly applied mechanical pre-stress
and the maximum expected external load for actuators of similar dimensions used in the
present study, are $10MPa$ and $500N$, respectively [14]. The pre-stress results in a force of
$(10 \times 10^6Pa) (0.133 \times 10^{-3}m^2) = 1.33kN$ applying on each flexure pivot. Since the external
load is shared by two rods, the actual external force experienced by each flexure pivot is
only 250N. A safety margin of 100N is also included into the design of the flexure pivots. Hence, the maximum expected force applying on each plate, due to pre-stress and external load, is \(1.33kN + 250N + 100N = 1.68kN\) and this force results in a compressive stress of 
\[
\left( \frac{1.68kN}{0.8 \text{mm} \times 50 \text{mm}} \right) = 42.0 \text{MPa}
\]
in each flexure pivot.

Beside the compressive stress caused by the mechanical pre-stress and the external load, there is an additional stress in each flexure pivot induced by bending. To calculate the stress induced by bending, we simplified the problem by assuming that each flexure pivot forms a perfect circular arc of \(l = R\theta = 5\text{mm}\) (see figure 2.5). Also, the length of each plate is assumed to be the same along the centre when it is bent. The outside of each flexure pivot has been stretched and has a length of:

\[
(l + \Delta l) = \left( R + \frac{1}{2}d \right) \theta
\]

where \(d\) is the thickness of the plate. The induced strain can then be computed as:

\[
\varepsilon = \frac{\Delta l}{l} = \frac{(R + \frac{1}{2}d)\theta - l}{l} = \frac{\frac{1}{2}d\theta}{l} = \frac{\frac{1}{2}(0.8\text{mm})}{5\text{mm}}\theta = 0.08\theta.
\]

The angle of rotation can be estimated by dividing the output movement over half the length of the pivoting beam. To include a safety margin, we assumed the required maximum displacement to be 200\(\mu\text{m}\). The angle of rotation is thus \(\theta = \frac{0.2\text{mm}}{60\text{mm}} = 3.33 \times 10^{-3}\text{rad}\).
because the designed pivoting beam has a length of 120\text{mm}. The maximum strain in each flexure pivot due to bending is $\varepsilon = 0.08 \times 3.33 \times 10^{-3} = 0.2664 \times 10^{-3}$. We can convert the bending strain into a bending stress of $\varepsilon E = (0.2664 \times 10^{-3})(186.2GPa) = 49.6MPa$ by multiplying an elasticity modulus of $E = 186.2GPa$. The maximum expected stress applied on each flexure pivot is, therefore, around $49.6MPa + 42.0MPa = 91.6MPa$, which is much smaller than the yield strength (tension) of $\sigma_y = 1034.3MPa$ for stainless steel. Also, from the equation for Euler load, we can calculate the critical height of each plate.

$$P = \frac{4\pi^2EI}{L_c^2} \Rightarrow L_c = 2\pi \sqrt{\frac{EI}{P}}$$  \hspace{1cm} (2.4)

where $E$, $I$, $P$, and $L_c$ are the elasticity modulus, moment of inertia, load, and the critical length, respectively.

Previously, we calculated the maximum expected stress applied on each plate was around $91.6MPa$, which implied that the load on each flexure plate was $3.664\text{kN}$. From the dimension of the flexure pivots, the moment of inertia, $I$, is:

$$I = \frac{1}{12}(50\text{mm})(5\text{mm})^3 = 5.208 \times 10^2\text{mm}^4 = 5.208 \times 10^{-10}\text{m}^4.$$  \hspace{1cm} (2.5)

Thus, the critical height of each plate is:

$$L_c = 2\pi \sqrt{\frac{(186.2GPa)(5.208 \times 10^{-10}\text{m}^4)}{3.664\text{kN}}} = 1.022\text{m} = 1.022 \times 10^3\text{mm}.$$  \hspace{1cm} (2.6)

We have decided on plates with a free-standing height of $5\text{mm}$, which is much shorter than the critical length. As a result, we can conclude that the designed flexure plates are capable of withstanding the force applied in this project.

### 2.4 Mechanical Pre-stress Adjustment

Among the advantages mentioned about the push-pull configuration, one of them is about maintaining constant mechanical pre-stress. In our design, the spring mechanism, which is

\footnote{data obtained from [16]}
commonly used, was abandoned. We implemented a wedge to alter mechanical pre-stress applied on the Terfenol-D rods. In figure 2.4, the wedge is located underneath the rods on the left. The pivoting beam maintains constant mechanical pre-stress on the rods. Figure 2.6 shows the pre-stress mechanism employed in our design. The design is such that at least half of a rod is standing on the wedge when maximum mechanical pre-stress is applied. This is to prevent the structure from collapsing during operation. The wedge is moved to alter mechanical pre-stress applied on the rods using a long lead screw. Pin stoppers are used to hold the lead screw in place, but the lead screw remains free to turn. To increase the mechanical pre-stress, we move the wedge inward by turning a thumb screw (see figure 2.6). To allow the wedge to move more freely, a brass shim was wrapped around the wedge to reduce friction. This mechanism applied constant pre-stress on the rods so that the properties of the Terfenol-D rods remain the same during operation. Thus, a more reliable model can be identified for the design of controllers.

Figure 2.6 Pre-stress Mechanism using Wedge
2.5 Position Sensing Device

The sensing device used in this project is an optical device. The mechanism of the sensor is very simple and it is shown in figure 2.7. The actuator is connected to a specimen holder, which is attached to the lever of the sensor (see figure 2.8). As the specimen holder vibrates so does the sensor’s lever. Since the light source can only pass through the magnifying lens and shine onto the position sensing device (PSD) (see figure 2.7), the spot, where the light source shines on the PSD, changes as the specimen holder is vibrated. When light shines on the PSD, an electric charge proportional to the light energy is generated at the incident position. This electric charge is driven through the P-layer and collected by electrodes of the PSD (see figure 2.7). Since the resistivity of the P-layer is uniform, current collected by an electrode is inversely proportional to the distance between the incident position and the electrode [12]. An innovative circuit was developed at UBC to amplify these currents, $I_1$ and $I_2$, and to convert them to voltages. Difference in these voltages can be measured at one of the circuit’s output channels. This voltage difference corresponds to actual displacement from the centre position of the actuator. At the beginning of operation, the incident position does not have to be at the centre of the PSD because the circuit has a built-in potentiometer.
to set the sensor's output to zero. This initialization can also be done through software and was programmed in the real-time controllers developed.

### 2.6 Sensor Calibration

Since the sensor was designed and developed in house, we needed to figure out how the voltage difference corresponded to the actual displacement. A micrometer was used to measure the displacement of the actuator. Then, the corresponding sensor output was recorded. Large numbers of samples were collected to minimize errors introduced. The average gain of the sensor was calculated to be $8.9909 \mu m/V$.

### 2.7 Overall Design of the Fretting Test System

At the time this thesis was written, the overall design of the fretting test system was not finalized yet because of some delays at the NRC. A sketch of the tentative idea of the design is shown in figure 2.8. Since Terfenol-D is very sensitive to temperature, the chambers with Terfenol-D actuators need to be cooled. This is to ensure that the performance of the actuators will not be sacrificed. The chambers with actuators will also be insulated from the hot-water chamber. Since we are using an optical positioning sensor, it is important to prevent moisture from entering the chambers because condensation on the sensor's lens can impair the performance of the sensor.

After designing the actuator, the next thing to do is to have a model of the actuator in order to design controllers for it. In the following section, a couple system modelling methods are discussed.
Figure 2.8 Sketch of the Overall Fretting Test System
2.8 System Identification

The dynamics of Terfenol-D is highly nonlinear and consists of hysteresis loop. Preisach model is the most widely used method in describing hysteresis. This method was not implemented in this project because of the complexity of mathematics involved. However, the implementation of Preisach model is outlined later in this section. The Preisach model is very useful in simulation processes but its usefulness in designing controllers still needs to be studied.

2.8.1 Least Squares Estimation

First, we will discuss the identification method used in this project. We simplified the model of the designed actuator into a linear model. This model has a general structure of an ARX model (equation (2.7)) which is a special case of an ARMAX model [13].

\[ Ay(t) = Bu(t) + Ce(t) \]  \hspace{1cm} (2.7)

where

\[ A = 1 + a_1 z^{-1} + \cdots + a_n z^{-n} \]

\[ B = b_1 z^{-1} + b_2 z^{-2} + \cdots + b_m z^{-m} \]  \hspace{1cm} (2.8)

\[ C = 1. \]

We can substitute equation (2.8) into equation (2.7) and have [30][31]:

\[ y(t) = -a_1 y(t - 1) - \cdots - a_n y(t - n) + b_1 u(t - 1) + b_2 u(t - 2) + \cdots + b_m u(t - m) + e(t) \]

\[ = [-y(t - 1) \cdots - y(t - n) \ u(t - 1) \cdots u(t - m)] \theta + e(t) \]  \hspace{1cm} (2.9)

\[ \therefore y(t) = x \hat{\theta} + e(t). \]

In equation (2.9), true system parameter vector, \( \theta \), and noise error, \( e \), can be replaced by an adjustable model parameter vector, \( \hat{\theta} \), and the corresponding estimation error at time \( t \), \( \hat{e}(t) \), respectively. Using least squares algorithm after a set of input-output data has been collected, we can select \( \hat{\theta} \) so that the overall estimation error, \( \hat{e} \), is minimized. Therefore,
we can write:

\[
\begin{bmatrix}
  y(1) \\
  \vdots \\
  y(N)
\end{bmatrix} = \begin{bmatrix}
  x_1 \\
  \vdots \\
  x_N
\end{bmatrix} \hat{\theta} + \begin{bmatrix}
  \hat{e}(1) \\
  \vdots \\
  \hat{e}(N)
\end{bmatrix}
\]

\[Y_N = X_N \hat{\theta} + \hat{e}_N\]  \hfill (2.10)

Rearranging equation (2.10) in terms of the error vector, we have:

\[\hat{e}_N = Y_N - X_N \hat{\theta}\]  \hfill (2.11)

Let the sum of squares of errors be,

\[J = \hat{e}_N^T \hat{e}_N = \sum_t \hat{e}(t)\]  \hfill (2.12)

The sum can also be rewritten as:

\[J = (Y_N - X_N \hat{\theta})^T (Y_N - X_N \hat{\theta})\]  \hfill (2.13)

\[= Y_N^T Y_N - \hat{\theta}^T X_N^T Y_N - Y_N^T X_N \hat{\theta} + \hat{\theta}^T X_N^T X_N \hat{\theta}\]

To minimize the sum of squares of errors, we have:

\[
\frac{dJ}{d\hat{\theta}} = -2X_N^T Y_N + 2X_N^T X_N \hat{\theta} = 0
\]

\[\Rightarrow \hat{\theta} = \left(X_N^T X_N\right)^{-1} X_N^T Y_N\]  \hfill (2.14)

Later in Chapter 5, we can see how the sum of squares of errors, \(J\), is used as a performance index to select an order and a parameter vector, \(\hat{\theta}\), for the model.

### 2.8.2 Preisach Model

Preisach Model is a mathematical model proposed by F. Preisach in 1935 [18]. This method makes no reference to the physical properties or underlying mechanism that produces hysteresis. It supposes that hysteresis consists of an infinite number of hysteresis operators, \(\hat{\gamma}_{\alpha\beta}\) (refer to figure 2.9). Because the model contains no physics, it is applicable to many hysteresis phenomena. It has been shown in a paper by J.B. Restorff et al. that this method
is effective in describing hysteresis in Terfenol-D's dynamics [21]. In figure 2.9, $\alpha$ and $\beta$ are

$\gamma_{\alpha\beta}$

+1

-1

Figure 2.9 Hysteresis Operator, $\gamma_{\alpha\beta}$

“up” and “down” switching values of input, respectively. It is natural from a physical point of view that $\alpha \geq \beta$. Output of the operators is assumed to have two values only: either $+1$ or $-1$.

The Preisach model is defined as [15]:

$$f(t) = \int_{\alpha \geq \beta} \int \mu(\alpha, \beta) \gamma_{\alpha\beta} u(t) d\alpha d\beta$$  \hspace{1cm} (2.15)

The output and input of a hysteresis nonlinearity are $f(t)$ and $u(t)$, respectively. Weighting function, $\mu(\alpha, \beta)$, is determined by matching some experimental data. The output depends mainly on the difference between the number of “up” and “down” operators.

Let us consider the geometric interpretation of the Preisach model. Figure 2.10 shows a right-angle triangle called the limiting triangle. Weighting function, $\mu(\alpha, \beta)$, is zero outside of this area. As the input monotonically increases until $u = \alpha'$, a horizontal link moves upward along the $\alpha$—axis. The limiting triangle will look as in figure 2.11 (a). When the input is then monotonically decreased to $u = \beta'$, a new vertical link moves from right to left along the $\beta$—axis. Refer to figure 2.11 (b), a new interface $I(t)$ has formed which divides the limiting
triangle into two sets: $S^+(t)$ consists of points $(\alpha, \beta)$ for which the operators are in "up" positions, and $S^-(t)$ consists of points $(\alpha, \beta)$ for which the operators are in "down" positions.

Equation (2.15) can then be rewritten as:

$$f(t) = \int_{S^+} \int \mu(\alpha, \beta) \dot{\gamma}_{\alpha\beta} u(t) d\alpha d\beta + \int_{S^-} \int \mu(\alpha, \beta) \dot{\gamma}_{\alpha\beta} u(t) d\alpha d\beta.$$  \hspace{1cm} (2.16)

Since $\dot{\gamma}_{\alpha\beta} u(t) = +1$, if $(\alpha, \beta) \in S^+(t)$, and $\dot{\gamma}_{\alpha\beta} u(t) = -1$, if $(\alpha, \beta) \in S^-(t)$, thus

$$f(t) = \int_{S^+} \int \mu(\alpha, \beta) d\alpha d\beta - \int_{S^-} \int \mu(\alpha, \beta) d\alpha d\beta.$$  \hspace{1cm} (2.17)

An instantaneous output depends on a particular subdivision of the limiting triangle by an interface $L(t)$, which depends on the past extremum of inputs. This is also how the model could memorize the input history.

Figure 2.10 The Limiting Triangle $L(t)$

Figure 2.11 Forming of an Interface
To implement the Preisach model numerically, we first define the function

\[ F(\alpha', \beta') = \frac{1}{2}(f_{\alpha'} - f_{\alpha', \beta'}) \]  
(2.18)

(see figure 2.12). From equation (2.17) and figure 2.11 (b), the above equation can be rewritten as:

\[ f_{\alpha', \beta'} - f_{\alpha'} = -2 \int \int_{T(\alpha', \beta')} \mu(\alpha, \beta) d\alpha d\beta. \]  
(2.19)

Assuming that a hysteresis loop is symmetric, we have:

\[ F(\alpha_0, \beta_0) = \int \int_{T} \mu(\alpha, \beta) d\alpha d\beta \]  
(2.20)

\[ f(t) = - \int_{T} \int_{S^+(t)} \mu(\alpha, \beta) d\alpha d\beta + 2 \int_{S^+(t)} \mu(\alpha, \beta) d\alpha d\beta \]

\[ \Rightarrow \int_{T} \int \mu(\alpha, \beta) d\alpha d\beta = \int_{S^+(t)} \mu(\alpha, \beta) d\alpha d\beta - \left( - \int_{S^-(t)} \mu(\alpha, \beta) d\alpha d\beta \right) \]  
(2.21)

\[ - \int_{S^-} \int \mu(\alpha, \beta) d\alpha d\beta = - \int_{T} \int_{S^+(t)} \mu(\alpha, \beta) d\alpha d\beta + \int_{S^+(t)} \int \mu(\alpha, \beta) d\alpha d\beta \]
The positive set $S^+(t)$ can be subdivided into $n$ trapezoids $Q_k$ (see figure 2.13). As a result, we have:

$$
\int_{S^+(t)} \mu(\alpha, \beta) d\alpha d\beta = \sum_{k=1}^{n} \int_{Q_k(t)} \mu(\alpha, \beta) d\alpha d\beta \quad (2.22)
$$

Each trapezoid $Q_k$ can be represented by the difference of two triangles, $T(M_k, m_{k-1})$ and $T(M_k, m_k)$. Thus,

$$
\int_{Q_k(t)} \mu(\alpha, \beta) d\alpha d\beta = F(M_k, m_{k-1}) - F(M_k, m_k). \quad (2.23)
$$

From equations (2.20), (2.21), (2.22), and (2.23), we obtain:

$$
f(t) = -F(\alpha_0, \beta_0) + 2 \sum_{k=1}^{n} [F(M_k, m_{k-1}) - F(M_k, m_k)] \quad (2.24)
$$

In figure 2.13 (a), one can see that $m_n = u(t)$. Hence, the above expression can be rewritten as:

$$
f(t) = -F(\alpha_0, \beta_0) + 2 \sum_{k=1}^{n-1} [F(M_k, m_{k-1}) - F(M_k, m_k)] + 2[F(M_n, m_{n-1}) - F(M_n, u(t))] \quad (2.25)
$$

In the case of a monotonically increasing input (see figure 2.13 (b)), $m_n = M_n(t) = u(t)$. Since $F(M_n, m_n) = F(u(t), u(t)) = 0$, equation (2.24) becomes:

$$
f(t) = -F(\alpha_0, \beta_0) + 2 \sum_{k=1}^{n-1} [F(M_k, m_{k-1}) - F(M_k, m_k)] + 2F(u(t), m_{n-1}) \quad (2.26)
$$

In terms of experimental data, equations (2.25) and (2.26) can be rewritten as:

$$
f(t) = -f^+ + \sum_{k=1}^{n-1} (f_{M_k m_k} - f_{M_k m_{k-1}}) + f_{M_n u(t)} - f_{M_n m_{n-1}} \quad (2.27)
$$
and

\[ f(t) = -f^+ + \sum_{k=1}^{n-1} (f_{M_k m_k} - f_{M_k m_k-1}) + f_u(t) - f_u(t)_{m_{n-1}}, \] (2.28)

respectively. The \( f^+ \) is the positive saturation value of output.

Assuming that a large number of experimental input-output data in the limiting triangle was collected, every \( f_{\alpha \beta} \) can be calculated by interpolation, if necessary. Thus, an instantaneous output can then be computed from equations (2.27) and (2.28). For detailed information, one can refer to a book written by I.D. Mayergoyz [15].
3 Design of PID Controller

3.1 Introduction

After designing the actuator, the sensor, and obtaining a model of the actuator, it is time to consider some control strategies that will regulate the actuator to achieve the specifications of the NRC. Since PID controller has been used to solve many process control problems, the author decided to start a series of experiments with this control algorithm. In this chapter, PID and cascaded PID controllers are presented. Cascaded PID controller is a strategy that connects multiple PID controllers in a series. This control strategy has several advantages over a single PID controller. The advantages are discussed later in this chapter.

3.2 PID Control Algorithm

PID control algorithm has a general formula of:

\[ u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt} \]  \hspace{1cm} (3.1)

where \( e(t) \) is the difference between the reference signal, \( r \), and the process output, \( y \). Laplace transform of equation (3.1) is:

\[ U(s) = \left( K_p + \frac{K_i}{s} + sK_d \right) E(s) \]  \hspace{1cm} (3.2)

The differential term shown in equation (3.2) should not be implemented directly because noise can be amplified and this effect is undesirable. To reduce amplification of noise, gain of the differential term needs to be limited. We can approximate the differential term as if there is a low-pass filter connected in series with it. Thus, the differential term has a new formula of:

\[ sK_d E(s) \approx \frac{sK_d}{1 + s/N} E(s) \]  \hspace{1cm} (3.3)
From equation (3.3), it is clear that the approximation at low frequency is good. However, at high frequency, the gain is limited to $N$ [2]. Also, to avoid a large control signal whenever there is a sudden change in the reference signal, we only differentiate the measured output signal [1]. Therefore, equation (3.2) is modified to be:

$$U(s) = \left[ K_p + \frac{K_i}{s} \right] E(s) - \frac{sK_d}{1 + s/N} Y(s)$$  \hspace{1cm} (3.4)

where $Y(s)$ is the output signal. Figure 3.1 is a block diagram of a system described by equation (3.4). To implement equation (3.4) digitally, we need to discretize it. The most common way of discretizing it is to make an Euler approximation of the integral term and a backward-difference approximation of the differential term. The following include discretized terms of a PID controller and some discussion for each term. The $r(t)$ and $y(t)$ are the reference signal and the output signal, respectively.

- **Proportional term:**

$$P(t) = K_p(r(t) - y(t))$$  \hspace{1cm} (3.5)

An increase in $K_p$ gain can speed up the response of a system because it moves the system roots increasingly either to the left of the real axis or moves them up the imaginary
axis. However, damping in a system is decreased with the increase of $K_p$. A decrease in damping can lead to an unstable system. Typically, there is an upper limit on this gain in order to have a well-damped and stable system response.

- **Integral term:**

\[
I(t + 1) = I(t) + K_i \cdot \Delta t \cdot (r(t) - y(t))
\]

\[
= I(t) + K_i \cdot \Delta t \cdot e(t)
\]

The primary reason for having an integral controller is to reduce or eliminate steady-state errors because this term can provide a finite value to the control signal, $u$, with no error-signal, $e$. Signal from an integrator is a function of past errors, $e$, rather than of current error. The past errors “charge up” the integrator to some value that will remain even when the present error becomes zero [7]. However, depending on the integral gain, addition of this term can lead to a less stable or less damped system.

- **Differential term:**

Let $D = \frac{sK_d}{1+s/N}Y(s)$

\[
D(t) + \frac{1}{N} \frac{dD}{dt} = K_d \frac{dy}{dt}
\]

\[
D(t) + \frac{1}{N} \left[ \frac{D(t) - D(t - 1)}{\Delta t} \right] = K_d \left[ \frac{y(t) - y(t - 1)}{\Delta t} \right]
\]

\[
\therefore D(t) = \frac{D(t - 1)}{1 + N \cdot \Delta t} + \frac{N \cdot K_d}{1 + N \cdot \Delta t} [y(t) - y(t - 1)]
\]

This controller itself cannot reduce tracking error unless the plant consists of a natural spring. Differentiator is commonly implemented in conjunction with proportional and integral controllers to increase damping in a system. PI controller could make a system response faster but this controller could bring some negative effects, such as oscillation and large overshoot, to the output. Addition of a differential term can reduce the overshoot and the oscillation at the output while maintaining a relatively fast system response.

Thus, the discretized control signal is given by:

\[
u(t) = P(t) + I(t) - D(t)
\]
where $P(t)$, $I(t)$, and $D(t)$ are derived from equations (3.5), (3.6), and (3.7), respectively. A properly tuned PID controller allows advantages of one term to compensate disadvantages of other terms. There are several methods we can use to tune PID controller parameters. Three commonly used tuning strategies are introduced here.

Trial-and-error method is the first one to be introduced. In practice, all PID controller parameters are set to zero at the beginning. The proportional gain, $K_p$, is the first one to be tuned until a desired rise-time of the output is obtained. $K_i$ is then tuned to reduce any steady-state error while maintaining a fast system response. However, the combination of proportional and integral terms can cause the system response to oscillate and have a large overshoot while trying to maintain a fast response. Increases in $K_d$ can increase the damping in a system. With the increase of damping, the overshoot and oscillation at the output are reduced. Thus, a good system response is acquired.

Trial-and-error method could be difficult to implement without adequate experience in tuning PID controller parameters because change in one parameter tends to affect the performance of other terms as well. Ziegler and Nichols proposed two systematic methods for tuning: transient-response method and ultimate-sensitivity method [1][2].

- Transient-response method: This method is based on a decay ratio of approximately 0.25, which means a dominant transient decays to a quarter of its magnitude after one cycle. This corresponds to $\zeta = 0.21$, which is a good compromise between response and stability [7]. From a step response of an open-loop system, the steepest slope, $R$, and delay time, $L$, are measured (see figure 3.2). Parameters for a PID controller can then be obtained from table 3.1. Thus, we have:

\[
\begin{align*}
K_p &= K \\
K_i &= \frac{K_p}{T_i} \\
K_d &= K_p T_d.
\end{align*}
\]

(3.9)

To implement the above parameters in a discretized PID controller, the effect of sampling
time on the parameters should be taken into account.

![Figure 3.2 Variables $R$ and $L$ in an open loop system step response](image)

Table 3.1 Controller Parameters using Transient-Response Method [2][7]

<table>
<thead>
<tr>
<th></th>
<th>$K$</th>
<th>$T_i$</th>
<th>$T_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>$\frac{1}{RL}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI</td>
<td>$\frac{0.9}{RL}$</td>
<td>$3L$</td>
<td></td>
</tr>
<tr>
<td>PID</td>
<td>$\frac{1.2}{RL}$</td>
<td>$2L$</td>
<td>$0.5L$</td>
</tr>
</tbody>
</table>

- Ultimative-sensitivity method: In this method, we use only the proportional term to control a system at the beginning. The proportional gain is increased until the system response is continuously oscillating. This system is now on the stability boundary. This gain is called the ultimate gain, $K_u$, and the period of oscillation, $T_p$ (called ultimate period), is measured (see figure 3.3). Using equation (3.9) and table 3.2, parameters of a PID controller are obtained.

![Figure 3.3 Determination of the Ultimate Gain and Period](image)
3. Design of PID Controller

From experiments, variables $R$ and $L$ in the transient-response method were changing because the step response of the actuator was different depending on whether the actuator was going up or down. Thus, in this project, the author decided to use the ultimate-sensitivity method, instead. An initial set of PID controller parameters was acquired using the ultimate-sensitivity method. Controller parameters were further tuned by trial-and-error to enhance the system performance.

In general, PID controllers are effective in reducing steady-state error and improving the transient response of a system, which is either a first or second-order plant (or may be approximated by a second-order plant) [5]. If a system can only be described by a model higher than second-order, a more complex controller should be considered as a replacement.

### 3.3 Cascaded PID Controller

Cascaded PID controller is effective in regulating a system with additional measurable variables other than the output signal. In general, a block diagram for a cascaded control system looks like the following:

![Figure 3.4 Block Diagram for Cascaded PID Control System [32]](image-url)
PID 1 and PID 2 in figure 3.4 are named the master and the slave controllers, respectively. A cascaded PID control system has the following advantages over a system with a single controller [23):

1. Cascaded PID control system is more robust against load disturbance. From figure 3.4, the transfer function of the inner loop is given by:

\[ F_2 = \frac{Y_2}{R_2} = \frac{G_2 C_2}{1 + G_2 C_2} \]  

(3.10)

thus the overall closed loop transfer function will be:

\[ F_1 = \frac{Y_1}{R_1} = \frac{F_2 C_1 G_1}{1 + F_2 C_1 G_1} = \frac{C_1 C_2 G_1 G_2}{1 + C_2 G_2 + C_1 C_2 G_1 G_2}. \]  

(3.11)

For a load disturbance in the inner loop, i.e. \( d_1 = 0 \), the transfer function to the output is:

\[ \frac{Y_1}{D_2} = \frac{G_1}{1 + C_2 G_2 + C_1 C_2 G_1 G_2}. \]  

(3.12)

Without the inner loop and the inner controller, the transfer function for the same variables is:

\[ \frac{Y_1}{D_2} = \frac{G_1}{1 + C_1 G_1 G_2}. \]  

(3.13)

If the master and slave controllers are chosen correctly, disturbance, \( d_2 \), has less effect in a cascaded PID control system.

2. The inner loop allows more manipulation of the flow of energy from the outer loop. Thus, a cascaded PID controller can improve system response more effectively than a single PID controller.

3. Cascaded PID control system is more robust against system parameter variation because parameter variation in Plant 2 can be corrected within its own loop.

In this project, only the output signal is measurable. However, cascaded PID control strategy was considered because, intuitively, cascading PID controllers in series can increase the order of the overall regulator. A high order regulator allows more freedom to obtain a desired
dynamic response for a complex system. Hence, it can be considered as an alternative controller for a complex system that cannot be controlled satisfactorily by a single controller.

Tuning of cascaded PID controller parameters is done sequentially starting from the inner loop because the inner loop exists as an element in the outer loop [29]. Taking the transfer function of the inner loop into account, the master controller parameters are tuned. In our case, the slave controller was tuned using both ultimative-sensitivity and trial-and-error methods. Taking the inner loop into consideration, the master controller was tuned by trial-and-error method. Later in chapter 5, results using PID and cascaded PID controllers are shown and discussed.
4 Sliding Mode Control

4.1 Introduction

In this chapter, the author presents a discrete-time sliding mode control law derived from a discrete-time differential Lyapunov function and a sufficient condition for the existence of a sliding surface. This control law was tested experimentally on the actuator and results were attached in chapter 5. In the next few sections, the derivation of the control law is discussed.

4.2 Determination of Sliding Surface

To derive a discrete-time sliding mode control law, we first assumed we have a plant that can be described by the following equation.

\[ X(k + 1) = AX(k) + BU(k) \]  

(4.1)

\( X \in \mathbb{R}^n \) is the state variable at current sampling instant of \( kT \), where \( T \) is the sampling period. \( A \in \mathbb{R}^{n \times n} \) is the system matrix and \( B \in \mathbb{R}^{n \times m} \) is the input matrix.

In publications such as [9] and [26]\(^2\), they have defined the sliding surface, \( S \), as:

\[ S(k) = GX(k) \]  

(4.2)

\( G \in \mathbb{R}^{1 \times n} \) is a vector of constants. In the sliding mode control, a problem of tracking a \( n \)-dimensional vector is transformed to a problem of driving a scalar \( S(k) \) approaches and then stays on the sliding surface, \( S(k) = 0 \) [24]. Equation (4.2) is useful in solving the disturbance and noise reduction problems. To allow a system to track a time-varying signal, the definition of the sliding surface needs to be modified.

\(^2\) [19] and [20]
The selection of vector $G$ in equation (4.2) depends on the desired pole locations. The sliding mode and the pole placement control algorithms are similar because both of them have a state feedback gain that depends on the desired pole locations. In linear feedback control, an addition of an integral term in the feedback loop reduces the steady-state error; this allows the system to track a reference signal. A feedforward term can be added to reduce disturbances generated by changes in the reference signal. Based on this knowledge, the author decided to define a sliding surface differently from equation (4.2) in order to obtain a system that is capable of tracking a time varying signal well. The newly defined sliding surface is:

$$ S(k) = K_w R(k) + K_i I(k) - GX(k) $$  \hspace{1cm} (4.3)

where $R(k)$ is the reference signal, and $I(k)$ is the integration of the error between the reference signal and the actual output. $K_w$ and $K_i$ are their corresponding scalar gains.

![Figure 4.1 Sliding Mode Closed Loop System](image)

A similar definition of sliding surface, $S$, as shown in equation (4.3) was proposed for a continuous-time system in [33]. Figure 4.1 is a block diagram of the newly defined sliding mode control system.

After defining a sliding surface, the next step in the sliding mode controller design is to solve for vector $G$. In the next section, the procedure of computing vector $G$ is presented.
4.3 Determination of States Feedback Gain 'G'

To compute vector $G$, let us define a new variable $\Delta-$ as:

$$\Delta- = S(k + 1) - S(k)$$

(4.4)

When a system is in sliding mode, the variable $S(k)$ in equation (4.3) equals to zero for all $k$ and equation (4.4) becomes:

$$\Delta- = S(k + 1) - S(k) = S(k + 1) = 0$$

(4.5)

From equation (4.5), we have:

$$S(k + 1) = -GX(k + 1) + K_w R(k + 1) + K_i I(k + 1)$$

$$= -GAX(k) - GBU(k) + K_w R(k + 1) + K_i I(k + 1) = 0$$

(4.6)

Thus, we can obtain an equivalent control law:

$$U_{eq}(k) = (GB)^{-1}[−GAX(k) + K_w R(k + 1) + K_i I(k + 1)]$$

(4.7)

Replacing the $U(k)$ in equation (4.1) with that in equation (4.7), we have:

$$X(k + 1) = \left(A - B(GB)^{-1}GA\right)X(k) + B(GB)^{-1}(K_w R(k + 1) + K_i I(k + 1)).$$

(4.8)

Since it is difficult to solve for all three gains: state feedback gain, $G$, feedforward gain, $K_w$, and integral gain, $K_i$, simultaneously, it was decided to solve for $G$ first mathematically, then select $K_w$ and $K_i$ by tuning. Vector $G$ can be solved by placing the inner loop poles to some desired locations. To do this, $K_w$ and $K_i$ can be assumed zero, which yields:

$$X(k + 1) = \left(A - B(GB)^{-1}GA\right)X(k)$$

$$= (A - BK)X(k).$$

(4.9)

where $K = (G \cdot B)^{-1}G \cdot A$. The approach shown in the following for solving vector $G$ was proposed in [6] and described in [19] and [20]. When $S(k) = 0$, the overall system is a
reduced order system; in other words, only \((n - m)\) eigenvalues need to be assigned. So, to compute vector \(G\), one specifies desired poles in a diagonal (for distinct real poles), block diagonal (for complex conjugate poles) or Jordan block diagonal (for multiple real poles) matrix \(J \in \mathbb{R}^{(n-m) \times (n-m)}\). Let \(W \in \mathbb{R}^{n \times (n-m)}\) be a matrix containing the corresponding eigenvectors. From the relationship between eigenvalues and eigenvectors, we have an equation:

\[
(A - B \cdot K) \cdot W = W \cdot J
\]  
(4.10)

The above equation can be rewritten as:

\[
A \cdot W - W \cdot J = B \cdot K \cdot W
\]  
(4.11)

This means the columns of \((A \cdot W - W \cdot J)\) belongs to the range of \(B\). That is,

\[
\text{col}(A \cdot W - W \cdot J) \in \text{R}(B)
\]  
(4.12)

Thus, the matrix \(W\) can be solved, using the following equation:

\[
A \cdot W - W \cdot J = B \cdot M
\]  
(4.13)

where \(M\) is an arbitrary \(m \times (n - m)\) non-zero matrix [19]. Equation (4.13) is a Lyapunov equation and it can be solved for \(W\), using a Matlab function, \text{lyap}, since matrices \(A, B\) and \(J\) are known. After solving for \(W\), we can obtain vector \(G\) with the following:

\[
G = \text{first column of } [B;W]^{-1}
\]  
(4.14)

The next step is to derive a control law that is capable of maintaining the stability of a system.

### 4.4 Discrete Differential Lyapunov Function and the Control Law

In this section, a control law based upon satisfying the condition of a differential Lyapunov function candidate is derived. To ensure stability, a Lyapunov function candidate for our system is selected to be:
After differentiating the above equation, we have:

\[ \dot{V} = S \cdot \dot{S} \]  \hspace{1cm} (4.16)

Ensuring that equation (4.16) is always non-positive will be sufficient to conclude that the system is stable \([27]\). For a discrete-time system, a similar condition for maintaining stability in a system is:

\[ S(k) \cdot (S(k + 1) - \dot{S}(k)) \leq 0, \quad \forall k \]

or

\[ \dot{S}(k) \cdot \Delta_- \leq 0, \quad \forall k \]  \hspace{1cm} (4.17)

The equation (4.17) is a sufficient condition for ensuring the stability in a discrete-time system, but it does not guarantee the existence of a sliding surface. An oscillatory sliding surface, \(S\), can satisfy the condition given by equation (4.17). This condition is used to derive a control law because it can lead to a stable sliding mode controller, depending on the selection of gains.

A control law satisfying the condition given by equation (4.17) has a formula as follows:

\[ u(k) = -\sum_{i=1}^{n-1} \psi_i x_i(k) + K_i I_i(k) + K_w R(k) + \delta_- \]  \hspace{1cm} (4.18)

The summation term in the equation (4.18) does not include the \(n\)th term because this is a reduced order system. The \(\delta_-\) term is included to drive the system back to a sliding surface once it slides off the surface.

From equation (4.4), we have:

\[ \Delta_- = S(k + 1) - S(k) \]

\[ = [K_w R(k + 1) + K_i I(k + 1) - GX(k + 1)] - [K_w R(k) + K_i I(k) - GX(k)] \]

\[ = K_w (R(k + 1) - R(k)) + K_i (I(k + 1) - I(k)) - G(X(k + 1) - X(k)) \]  \hspace{1cm} (4.19)

\[ = K_w (R(k + 1) - R(k)) + K_i (I(k + 1) - I(k)) - G(AX(k) + BU(k) - X(k)) \]
Let
\[ A_\perp = A - I \] (4.20)
\[ \Delta R = R(k + 1) - R(k) \]

The Euler approximation of an integral term is:
\[ I(k + 1) = I(k) + e(k) \cdot \delta t \] (4.21)

Thus, equation (4.19) becomes:
\[ \Delta_\perp = K_w \Delta R + K_i e(k) \delta t - G(A_\perp X(k) + BU(k)) \] (4.22)

Substitute the control law given by equation (4.18) into the above equation, and we have:
\[ \Delta_\perp = K_w \Delta R + K_i e(k) \delta t - GA_\perp X(k) - GB \left[ \sum_{i=1}^{n-1} \psi_{-i} x_i(k) + K_i I(k) + K_w R(k) + \delta_- \right] \] (4.23)

Let
\[ M1 = K_w \Delta R - GB K_w R(k) + K_i e(k) \delta t - GB K_i I(k) \] (4.24)

then,
\[ \Delta_\perp = M1 - GA_\perp X(k) + GB \sum_{i=1}^{n-1} \psi_{-i} x_i(k) - GB \delta_- \]
\[ = M1 + \sum_{i=1}^{n-1} (-GA_{-i} + GB \psi_{-i}) x_i(k) - GA_{-n} x_n(k) - GB \delta_- \] (4.25)

where \( A_{-i} \) is the \( i \)th column of the \( A_- \) matrix.

From the definition of a sliding surface (equation (4.3)), we have:
\[ S(k) = K_w R(k) + K_i I(k) - G X(k) \]
\[ = K_w R(k) + K_i I(k) - \left[ g_1 x_1(k) + g_2 x_2(k) + \cdots + g_n x_n(k) \right] \]
\[ \therefore x_n(k) = \frac{1}{g_n} \left[ K_w R(k) + K_i I(k) - g_1 x_1(k) - g_2 x_2(k) - \cdots - g_{n-1} x_{n-1}(k) - S(k) \right] \cdot \]
\[ \therefore \Delta_\perp = M1 + \sum_{i=1}^{n-1} (-GA_{-i} + GB \psi_{-i} + \frac{g_i}{g_n} GA_{-n}) x_i(k) - GA_{-n} \left( \frac{K_w R(k) + K_i I(k) - S(k)}{g_n} \right) - GB \delta_- \] (4.26)
To fulfill the condition given by the equation (4.17), \( \psi_{-i} \) and \( \delta_- \) have to satisfy the following conditions:

\[
\begin{align*}
\psi_{-i} &\leq (GB)^{-1} \left( GA_{-i} - \frac{g_i}{g_n} GA_n \right) \text{ if } S(k)x_i(k) \geq 0 \\
\psi_{-i} &\geq (GB)^{-1} \left( GA_{-i} - \frac{g_i}{g_n} GA_n \right) \text{ if } S(k)x_i(k) \leq 0 \\
\delta_- &\geq (GB)^{-1} \left[ M_1 - GA_n \left( \frac{K_w R(k) + K_i I(k) - S(k)}{g_n} \right) \right] \text{ if } S(k) \geq 0 \\
\delta_- &\leq (GB)^{-1} \left[ M_1 - GA_n \left( \frac{K_w R(k) + K_i I(k) - S(k)}{g_n} \right) \right] \text{ if } S(k) \leq 0
\end{align*}
\]

(4.27)

Let \( A_i \) and \( A_n \) be the \( i \)th column and the \( n \)th column of the \( A \) matrix, respectively.

\[
GA_{-i} = G(A_i - I_i) = GA_i - g_i
\]

(4.28)

and,

\[
GA_{-n} = G(A_n - I_n) = GA_n - g_n.
\]

(4.29)

The \( I_i \) and \( I_n \) are the \( i \)th column and the \( n \)th column of the \( I \) matrix, respectively.

\[
GA_{-i} - \frac{g_i}{g_n} GA_{-n} = GA_i - \frac{g_i}{g_n} GA_n,
\]

(4.30)

and

\[
GA_{-n} \left( \frac{K_w R(k) + K_i I(k) - S(k)}{g_n} \right) = \left( GA_n - g_n \right) \left( \frac{K_w R(k) + K_i I(k) - S(k)}{g_n} \right).
\]

(4.31)

Equation (4.27) becomes:

\[
\begin{align*}
\psi_{-i} &\leq (GB)^{-1} \left( GA_i - \frac{g_i}{g_n} GA_n \right) \text{ if } S(k)x_i(k) \geq 0 \\
\psi_{-i} &\geq (GB)^{-1} \left( GA_i - \frac{g_i}{g_n} GA_n \right) \text{ if } S(k)x_i(k) \leq 0 \\
\delta_- &\geq (GB)^{-1} \left[ M_1 - (GA_n - g_n) \left( \frac{K_w R(k) + K_i I(k) - S(k)}{g_n} \right) \right] \text{ if } S(k) \geq 0 \\
\delta_- &\leq (GB)^{-1} \left[ M_1 - (GA_n - g_n) \left( \frac{K_w R(k) + K_i I(k) - S(k)}{g_n} \right) \right] \text{ if } S(k) \leq 0
\end{align*}
\]

(4.32)

As mentioned, this control law is derived based on a sufficient condition for maintaining the stability in a system. However, the existence of a sliding surface is not guaranteed.
4. Sliding Mode Control

4.5 Sliding Surface Existence Condition and the Control Law

In the previous section, the existence of a sliding surface is not guaranteed because an oscillatory sliding surface, $S$, can satisfy the condition defined by equation (4.17). Sarpturk et al. proposed a necessary and sufficient condition for the existence of an asymptotic sliding surface [22]. The condition is stated as follows:

$$|S(k + 1)| \leq |S(k)|$$  \hspace{1cm} (4.33)

A control law satisfying the above condition guarantees a decreasing sliding trajectory. The equation (4.33) also expresses the following:

$$\begin{cases} -S(k) < S(k + 1) < S(k) & \text{if } S(k) > 0 \\ -S(k) > S(k + 1) > S(k) & \text{if } S(k) < 0 \\ S(k + 1) = 0 & \text{if } S(k) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} S(k + 1) - S(k) < 0 \text{ and } S(k + 1) + S(k) > 0 & \text{if } S(k) > 0 \\ S(k + 1) - S(k) > 0 \text{ and } S(k + 1) + S(k) < 0 & \text{if } S(k) < 0 \\ S(k + 1) = 0 & \text{if } S(k) = 0 \end{cases}$$ \hspace{1cm} (4.34)

Let

$$\Delta_+ = S(k + 1) + S(k),$$ \hspace{1cm} (4.35)

The equation (4.34) in terms of $\Delta_-$ and $\Delta_+$ can be expressed:

$$\begin{cases} S(k)\Delta_- < 0 \text{ and } S(k)\Delta_+ > 0 & \text{if } S(k) \neq 0 \\ S(k + 1) = 0 & \text{if } S(k) = 0 \end{cases}$$ \hspace{1cm} (4.36)

From the previous section, a control law for $S(k)\Delta_- < 0$ was derived. In this section, the derivation of a control law for $S(k)\Delta_+ > 0$ is shown. The control law has a similar form of:

$$u(k) = -\sum_{i=1}^{n-1} \psi_{+i} x_i(k) + K_I I(k) + K_w R(k) + \delta_+$$ \hspace{1cm} (4.37)

where

$$\begin{cases} \psi_{+i} > (GB)^{-1} \left( GA_i - \frac{\dot{x}_i}{g_n} GA_n \right) & \text{if } S(k)x_i(k) > 0 \\ \psi_{+i} < (GB)^{-1} \left( GA_i - \frac{\dot{x}_i}{g_n} GA_n \right) & \text{if } S(k)x_i(k) < 0 \end{cases}$$ \hspace{1cm} (4.38)
and
\[
\begin{cases}
\delta_+ < (GB)^{-1} \left( M_2 - GA_n \frac{K_w R(k) + K_i I(k) - S(k)}{g_n} \right) & \text{if } S(k) > 0 \\
\delta_+ > (GB)^{-1} \left( M_2 - GA_n \frac{K_w R(k) + K_i I(k) - S(k)}{g_n} \right) & \text{if } S(k) < 0
\end{cases}
\]
\hspace{1cm} (4.39)

The \( M_2 \) in equation (4.39) is:
\[
M_2 = K_w (R(k + 1) + R(k)) + 2 K_i I(k) + K_i e(k) \delta t - G B K_i I(k) - G B K_w R(k)
\]
\[
= K_w \Sigma R + 2 K_i I(k) + K_i e(k) \delta t - G B K_i I(k) - G B K_w R(k)
\]
\hspace{1cm} (4.40)

The derivation of the control law for \( S(k) A_+ > 0 \) is omitted because the procedure is similar to the one shown in the previous section.

Let \( U_{c1} \) be the control law satisfying \( S(k) A_- < 0 \). \( U_{c2} \) is the control law satisfying \( S(k) A_+ > 0 \). Combining \( U_{c1} \) and \( U_{c2} \), one can obtain a control law satisfying all the conditions mentioned: the discrete-time differential Lyapunov function and the sufficient condition for the existence of a sliding surface. One can deduce that the control effort \( U_{c2} \) is larger than \( U_{c1} \) because the latter control effort tries to keep sliding trajectory \( S(k + 1) \) at the same level as \( S(k) \), while the former one drives \( S(k + 1) \) to \( -S(k) \). So, the ideal control effort should lie between these two values. A control law suggested in [19] and [20] is:
\[
u(k) = U_{c1}(k) + (U_{c2}(k) - U_{c1}(k)) \cdot \left| \frac{S(k)}{M_s} \right|
\]
\hspace{1cm} (4.41)

where \( M_s \geq \max \{|S(k)|\} \) is a 'tuning knob' which adjusts the tightness of the control signal. \( M_s \) is initially set to be a small value which must be greater than zero. The 'tuning knob' also has a smoothing effect on the control signal.

To obtain \( U_{c1} \) and \( U_{c2} \), let us look at \( U_{c1} \); it is defined as:
\[
U_{c1} = - \sum_{i=1}^{n-1} \tilde{\psi}_i x_i(k) + K_i I(k) + K_w R(k) + \delta_-
\]
\hspace{1cm} (4.42)

where
\[
\tilde{\psi}_i = (GB)^{-1} \left( G A_i - \frac{g_i}{g_n} G A_n \right)
\]
\hspace{1cm} (4.43)
4. Sliding Mode Control

and

\[ \delta_- = (GB)^{-1} \left( M1 - GA_n \left( \frac{K_w R(k) + K_i I(k) - S(k)}{g_n} \right) + (K_w R(k) + K_i I(k) - S(k)) \right). \] (4.44)

And \( U_{c2} \) is defined as:

\[ U_{c2} = - \sum_{i=1}^{n-1} \tilde{\psi}_{+i} x_i(k) + K_i I(k) + K_w R(k) + \delta_+ \] (4.45)

where

\[ \tilde{\psi}_{+i} = (GB)^{-1} \left( GA_i - \frac{g_i}{g_n} GA_n \right) \] (4.46)

and

\[ \delta_+ = (GB)^{-1} \left( M2 - GA_n \left( \frac{K_w R(k) + K_i I(k) - S(k)}{g_n} \right) - (K_w R(k) + K_i I(k) - S(k)) \right). \] (4.47)

Therefore,

\[ U_{c2} - U_{c1} = 2(GB)^{-1} S(k). \] (4.48)

4.6 Further Modification of Sliding Surface and the Control Law

Referring to figure 4.1, we can include a proportional and a differential term into the block diagram. A new sliding surface is defined as:

\[ S(k) = K_w R(k) + K_p e(k) + K_i I(k) - K_d D(k) - GX(k) \] (4.49)

where \( e(k) = R(k) - Y(k) \), and \( D(k) \) is the differentiation of the output signal (see figure 4.2).
The overall control law still has a form as in equation (4.41).

\[ u(k) = U_{c1} + (U_{c2} - U_{c1}) \cdot \frac{|S(k)|}{M_s} \]  (4.50)

where \( M_s = \max\{|S(k)|\} \). However, in the presence of two new terms in the definition of the sliding surface, \( U_{c1} \) and \( U_{c2} \) have changed. New \( U_{c1} \) and \( U_{c2} \) are shown as follows:

- \( U_{c1} = -\sum_{i=1}^{n-1} \tilde{\psi}_{-i} x_i(k) + K_w R(k) + K_p e(k) + K_i I(k) - K_d D(k) + \delta_- \)
  where
  \[ \tilde{\psi}_{-i} = (GB)^{-1} \left( GA_i - \frac{g_i}{g_n} GA_n \right), \]  (4.51)
  \[ \delta_- = (GB)^{-1} \left( M1 + (1 - \frac{GA_n}{g_n}) \cdot (K_w R(k) + K_p e(k) + K_i I(k) - K_d D(k) - S(k)) \right), \]  (4.52)
  and
  \[ M1 = K_I e(k) \delta t + GB(K_D D(k) - K_w R(k) - K_I I(k) - K_p e(k)). \]  (4.53)

- \( U_{c2} = -\sum_{i=1}^{n-1} \tilde{\psi}_{+i} x_i(k) + K_w R(k) + K_p e(k) + K_i I(k) - K_d D(k) + \delta_+ \)
  where
  \[ \tilde{\psi}_{+i} = (GB)^{-1} \left( GA_i - \frac{g_i}{g_n} GA_n \right), \]  (4.54)
  \[ \delta_+ = (GB)^{-1} \left( M2 - \left( 1 + \frac{GA_n}{g_n} \right) (K_w R(k) + K_i I(k) + K_p e(k) - K_d D(k) - S(k)) \right), \]  (4.55)
and

\[
M2 = 2K_w R(k) + 2K_i I(k) + K_i e(k) \delta t + 2K_p e(k) - 2K_d D(k) + GB(K_d D(k) - K_w R(k) - K_i I(k) - K_p e(k))
\]  

(4.56)

Thus,

\[
U_{c2} - U_{c1} = 2(GB)^{-1} S(k)
\]  

(4.57)

In the process of deriving the control laws, the author replaced all \( R(k+1) \) and \( Y(k+1) \) with \( R(k) \) and \( Y(k) \), respectively, because it is discovered that the sampling period chosen in this project is much shorter than the period of the fastest change in the reference signal. This simplified the controller design process. We can still introduce a delay into a system, which increases the order of the model, to handle the \((k+1)\)th term. However, this technique did not work in our simulations.

In this chapter, the derivation of a discrete-time sliding mode control law is presented. This control law was derived based upon two sufficient conditions: the existence of a sliding surface and the maintenance of the stability in a system. In the following chapter, some simulation and experimental results, using the control law derived in section 4.6, are shown and discussed. Also, the selection of the parameters are discussed in more detail.
5 Experimental Setup and Results

5.1 Introduction

After completing the mechanical design of the actuator and deciding on the control strategies, experiments to test the actuator and the designed controllers were carried out. In this chapter, the experimental setup, the selection of the controller parameters, simulation and experimental results are shown and discussed.

5.2 Experimental Setup

A series of experiments was performed on the actuator, using different controllers. Figure 5.1 shows a basic structure of this setup. A Sun Workstation is used as a terminal for the Sparc CPU in the VME cage through a local area network (LAN). A controller, which processes all the signals handled by the A/D and the D/A channels of the VME 612 board, was programmed and uploaded onto the Sparc CPU. The displacement of the actuator was measured by the designed optical sensor. Before being read by an A/D channel, the signal was amplified; this signal was used by the controller to compute a control signal. The control signal was then sent to the actuator from a D/A channel through a PWM servo amplifier.

Figure 5.1 Basic Structure of the Experimental Setup
The PWM servo amplifier was set to be in torque mode, in which the output current was proportional to the input voltage. The proportional gain between the input and the output signal was set to $1A/V$. Figure 5.2 shows a block diagram of the overall system.

![Block Diagram of the Overall Control System](image)

**Figure 5.2** Block Diagram of the Overall Control System

### 5.3 Response of the Actuator

The dynamics of the Terfenol-D based actuator possesses hysteresis. By applying a slowly-varying excitation, one can observe an open-loop response of the actuator (see figure 5.3). The overall operating range is about $130\mu m$, which is required by the NRC. The excitation range was determined experimentally. With an excitation lower than $-1.5V$, the output of the actuator shows a wrapping effect. To ensure the actuator will operate over the full range, the excitation, $u$, is limited to $-1.5V \leq u \leq 4.0V$. Before each experiment, the actuator was initialized with two complete cycles of a low frequency sine wave, which has an amplitude and an offset of $2.75V$ and $1.25V$, respectively. This is necessary to ensure that the actuator is always starting at the same position before each experiment. After the initialization, the sensor's output was then read and stored for the purpose of initializing the sensor's reading.
5. Experimental Setup and Results

5.4 System Modeling

Even though the dynamics of the actuator consists of hysteresis, it was modeled as a linear plant to simplify the process of designing a controller. The method used here to identify the model parameters was the least squares estimation, which was introduced in Chapter 2.

To identify the system model, white noise, with a variance of 1 and a mean of 1.25, was used to excite the actuator. The sensor's outputs were recorded in a data file; the collected data were then analyzed, using Matlab. A nonlinear system response depends on both the frequency and the amplitude of the excitation. It is important to ensure that the selected white noise covers the whole excitation range.

Figure 5.3 DC Response of the Actuator
5. Experimental Setup and Results

Applying no excitation into the actuator, one could measure the sensor noise. The power spectrum density (see figure 5.4) indicates that the noise is quite white; thus, one could assume the estimated linear model of the actuator has a general formula as follows:

\[ y(t) = -a_1 y(t-1) - a_2 y(t-2) - \cdots - a_n y(t-n) + b_1 u(t-1) + b_2 u(t-2) + \cdots + b_m u(t-m) \]  

(5.1)

where \( n \geq m \). The performance indices, \( J = \sum_i e^2(t) \), for systems with a different order, \( n \), were compared in table 5.1. This table shows that the estimated linear model of the actuator is at least a third-order model, i.e. \( n = 3 \). To simplify the process of designing a discrete-time sliding mode controller, the author chose \( n = 3 \) as the order of the estimated model. The \( m \) was chosen to be 3 as well in order to have a better estimated model of the actuator. Using Matlab’s Identification Toolbox to analyze the collected data, one could obtain an estimated

<table>
<thead>
<tr>
<th>System Order (( n ))</th>
<th>Performance Index (( J ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>22.83</td>
</tr>
<tr>
<td>3</td>
<td>16.71</td>
</tr>
<tr>
<td>4</td>
<td>15.42</td>
</tr>
<tr>
<td>5</td>
<td>15.38</td>
</tr>
<tr>
<td>6</td>
<td>15.31</td>
</tr>
</tbody>
</table>

Table 5.1 System Orders vs. Estimating Performance Indices
5. Experimental Setup and Results

linear model of the actuator as follows:

\[
y(t) = 1.2225y(t - 1) - 0.0557y(t - 2) - 0.2480y(t - 3) - 0.0052u(t - 1) + 0.0134u(t - 2) + 0.0577u(t - 3)
\]  \hspace{1cm} (5.2)

Thus, a state space description of the model is:

\[
X(k + 1) = AX(k) + Bu(k)
\]

\[
Y(k) = CX(k) + Du(k),
\]  \hspace{1cm} (5.3)

where

\[
A = \begin{bmatrix} 0 & 0 & -0.2480 \\ 1 & 0 & -0.0557 \\ 0 & 1 & 1.2225 \end{bmatrix}, B = \begin{bmatrix} 0.0577 \\ 0.0134 \\ -0.0052 \end{bmatrix}, C = [0 \ 0 \ 1], D = 0.
\]  \hspace{1cm} (5.4)

From the pole-zero map shown in figure 5.5, the estimated model has a right-hand zero; in other words, the system is nonminimum-phase. According to the papers [3] and [4] by M.D. Bryant et al., they also observed the existence of one (or more) right-hand zero in the displacement to the coil voltage transfer function of their Terfenol-D based actuator. The existence of right-hand zeros limits the bandwidth of a feedback control system because it can contribute to the instability of the overall system. This makes active vibration control difficult over an extended bandwidth [4]. However, from the position of the right-hand zero (see figure 5.5), it may not be significant in our application because the frequency of the fastest reference signal is relatively low at 70Hz or 439.8rad/s, compared to the position of the right-hand zero.

Figure 5.6 are the Bode Plots of the model in equation (5.2). After obtaining an estimated linear model for the actuator, we can design a discrete-time sliding mode controller based on the algorithm mentioned in the previous chapter. The following section includes all the information of designing a discrete-time sliding mode controller.
5. Experimental Setup and Results

Figure 5.5 Pole-Zero Map for the Estimated Linear Plant

Figure 5.6 Bode Plots for the Estimated Plant
5.5 Designs of Discrete-time Sliding Mode Controller and State Observer

Beside the requirement of 130μm for the displacement, the NRC also requires the actuator to be able of tracking a 70Hz (439.82 rad/s) signal. The cut-off frequency of the estimated linear model is around 400 rad/s (see figure 5.6). The actuator is able to track a 70Hz signal with the assistance of a properly-designed feedback controller.

In order to obtain an overall system that is capable of satisfying all the requirements, the author decided to place the desired poles at \( s = 1000 \) and \( s = 1500 \). There are only two poles specified here because the order of the overall system will be reduced as discussed in the previous chapter. Using equations (4.12) or (5.5), we can compute the state feedback gain, \( G \), as the first column of \( B'W^{-1} \), where \( W \) is a solution of the following equation.

\[
A \cdot W - W \cdot J = B \cdot M
\]  
(5.5)

Since the selected poles are real and distinct, the \( J \) matrix in equation (5.5) is a diagonal matrix. With a sampling time of \( \frac{1}{2000 \text{sec}} \), the transformation of the desired poles from the s-domain to the z-domain is shown as follows:

\[
\begin{align*}
\{ s_1 &= 1000 \} \Rightarrow \{ z_1 &= 0.6065 \} \\
\{ s_2 &= 1500 \} \Rightarrow \{ z_2 &= 0.4724 \}
\end{align*}
\]  
(5.6)

Thus, the \( J \) matrix is:

\[
J = \begin{bmatrix} 0.6065 & 0 \\ 0 & 0.4724 \end{bmatrix}
\]

The \( M \) matrix is arbitrarily set to \( M = [1 \ 1] \). The matrix \( W \) in equation (5.5) can thus be solved, knowing the matrices \( A \) and \( B \) from the estimated linear model. Using a Matlab function, \texttt{lyap}, the matrix \( W \) was solved to be:

\[
W = \begin{bmatrix} 0.3784 & 0.1866 \\ 0.7082 & 0.4359 \\ -1.1581 & -0.5880 \end{bmatrix}
\]
From equation (4.13), we have:

\[ G = \text{first column of } \begin{bmatrix} B & W \end{bmatrix}^{-1} \]  

(5.9)

Thus, the state feedback gain, \( G \), (see figure 4.1) is:

\[ G = \begin{bmatrix} 17.6216 & 1.2894 & 6.5463 \end{bmatrix}. \]

The displacement of the actuator is the only physically accessible signal. Since the system states were needed in a sliding mode control algorithm, a discrete-time state observer was designed. A block diagram of the observer is shown in figure 5.7. In order to have a deadbeat response, the desired characteristic equation of the observer was chosen to be:

\[ z^3 = 0. \]  

(5.11)

The observer feedback gain, \( K_e \), can thus be given by Ackermann's equation [17]:

\[ K_e = \phi(A) \begin{bmatrix} C \\ CA \\ \vdots \\ CA \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \]  

(5.12)
where \( \phi(A) = A^n + \alpha_1 A^{n-1} + \cdots + \alpha_{n-1} A + \alpha_n I \). In our case, \( n = 3 \) and \( \phi(A) = A^3 \). The matrix \( K_e \) was calculated to be:

\[
K_e = \begin{bmatrix}
-0.2480 \\
-0.0557 \\
1.2225
\end{bmatrix}.
\] (5.13)

Since the system states were available through the state observer, one might argue that a pole placement control method, instead of a more complicated sliding mode control method, could be used. However, the state observer was designed, based upon the estimated linear model; the actual plant is highly nonlinear. Pole placement control method is not good at maintaining the stability of a system in the presence of system uncertainties; thus, the sliding mode control method was implemented. In fact, the author tested the pole placement control method on the actuator, but it failed to maintain the actuator stability.

After designing a discrete-time sliding mode controller and an observer, simulations of the overall system were carried out.

### 5.6 Simulation Results and Discussion

A series of simulations was conducted before the derived discrete-time sliding mode control algorithm was actually implemented on the actuator. In the first simulation, we wanted the system to track a zero reference signal with a non-zero initial value at the system output. The controller which was used, was the state feedback gain, \( G \), only. The result is shown in figure 5.8. The output signal went up first before coming down to the reference. This is due to the existence of a right-hand zero in the estimated linear model. The rise time of the response is around 0.007s. This shows that the system is fast enough to fulfill the requirements with properly-tuned feedforward and PID gains. More simulations were carried out to determine how well the system output track a reference signal by adding a feedforward term and PID terms to the state feedback gain, \( G \), (see figure 4.2). The reference signals used in the simulations were a step input and different frequency sine waves.

Figures 5.9, 5.10, and 5.11 show the performance of the system tracking different reference signals. The tuning of the PID gains and the feedforward gain were done by
trial-and-error. Observing the unit step response while tuning the controller gains, the author finally decided on the PID gains as $K_p = 0.1$, $K_i = 100.0$, $K_d = 0.2$, and the feedforward gain as $K_w = 3.0$. With these gains, the system step response has a rise time and a settling time of 0.005s and 0.05s, respectively. The overshoot is about 10%. Sine waves were used to test how well the system was capable of tracking time varying signals. Figures 5.10 and 5.11 show the results of the system tracking a $30Hz$ and a $70Hz$ sine wave, respectively. There are phase shifts in the outputs. In the case of a $30Hz$ sine wave, the phase shift is $27^\circ$. A phase shift of $50^\circ$ was observed when the system was tracking a $70Hz$ sine wave. These simulation results show that the designed discrete-time sliding mode controller is capable of tracking time varying signals.

The addition of a feedforward term helps to reject possible disturbance generated by changes in the reference signal, and it minimizes a phase shift error between the actual output and the reference signal; however, the system response will have a large overshoot if too much emphasis is put on the feedforward gain.

Figure 5.8 Simulation Result with Zero Input Signal
5. Experimental Setup and Results

Figure 5.9 Simulation Results with a Step Input

Figure 5.10 Simulation Results with a Sine Wave of 30Hz
5. Experimental Setup and Results

5.7 Real-Time Experimental Results

A series of real-time experiments was performed, using different control strategies. In the previous section, only simulation results of the discrete-time sliding mode controller were shown because the sliding mode controller was the only control strategy that required some knowledge of the model among all the control strategies chosen. In this section, the experimental results of systems with a PID, a cascaded PID, or a sliding mode controller are presented.

5.7.1 Experimental Results with PID Controller

The PID controller was the first one tested because the tuning of the parameters needed no knowledge of the plant; furthermore, it was the most commonly used controller. The methods used for tuning the parameters were the ultimate-sensitivity and the trial-and-error methods (refer to chapter 3).
At first, all the controller parameters were set to zero. The proportional gain, $K_p$, was then slowly increased until the overall system was marginally unstable. The system was marginally unstable when $K_p = 1$. Thus, the ultimate gain is $K_u = 1$. Under a marginally unstable condition, the period of the output response was measured as $T_p = 0.0115s$. According to table 3.2, we have:

$$K = 0.6 \times K_u = 0.6$$
$$T_i = \frac{T_p}{2} = 5.75 \times 10^{-3}$$
$$T_d = \frac{T_p}{8} = 1.438 \times 10^{-3}$$

Hence, the parameters for the designed PID controller based upon the ultimate-sensitivity method proposed by Ziegler and Nichols are:

$$K_p = K = 0.6$$
$$K_i = \frac{K}{T_i} = 104.3$$
$$K_d = KT_d = 8.625 \times 10^{-4}.$$  \hspace{1cm} (5.15)

These parameters were tested on the actuator, but the system response was too slow for our purpose. The controller parameters were further tuned by trial-and-error to improve the performance of the system. The final parameters the author settled on are:

$$K_p = K = 0.6$$
$$K_i = 120.0$$
$$K_d = 0.0012$$  \hspace{1cm} (5.16)

The system has a step response shown in figure 5.12. The rise time of the system is around 0.01s. An increase in $K_i$ also resulted in an increase in $K_d$, in order to reduce large overshoot and/or oscillation in the system output.
The parameters were tuned when the system was operating over its full range because the actuator’s dynamics is highly nonlinear, the response being operating range dependent. The system with parameters tuned under a non-full range operating condition may fail to perform well when the system is tracking a smaller or a larger signal. This depends on how well the controller is capable of handling system parameter variations.

The following are the results extracted in real-time to show how well the system performed. The tracking of the system was performed with little or no error, when it was operating at a low frequency, e.g. 1 Hz. The system was also tested to track a 30 Hz and a 70 Hz sine wave.
5. Experimental Setup and Results

The results are as follows:

- At 30Hz:
  
  Phase shift: 48.6°
  
  System gain: 0.925

- At 70Hz:
  
  Phase shift: 151.2°
  
  System gain: 0.8
When the actuator was operating in a much smaller range at a relatively high frequency, we observed some distortion in the output signal caused by the nonlinearity in the system (see figure 5.14). Higher frequency decreases the 'fatness' of a hysteresis loop; furthermore, a small operating range also makes the hysteresis characteristics less noticeable; in other words, a plant with hysteresis characteristics behaves like a linear system under a high frequency, small operating range condition. One might think that a near-linear plant should be easily controlled by a PID controller. The PID was tuned under a very different circumstance; however, for a plant with hysteresis characteristics, changes in operating range and frequency also mean changes in system parameters. The distorted output (see figure 5.14) shows the incapability of the PID controller to handle the variations of the system parameters. To further improve the performance of the system, we considered another PID controller cascading with the present PID controller. This arrangement, rather than the use of one PID controller, provides greater stability in the presence of system parameter variations.
5.7.2 Experimental Results with Cascaded PID Controller

In this experiment, an additional PID controller was cascaded with the previously-designed PID controller. Generally, a cascaded PID controller is employed when there is an additional measurable signal; however, in our case, displacement of the actuator is the only measurable signal. The reason of employing this control algorithm was mentioned earlier. Figure 5.15 shows the block diagram of the system.

The parameters for the slave controller (PID 2) came from the previously-designed PID controller. The parameters for the master controller (PID 1) has yet to be determined. Based on the trial-and-error method, the parameters for the master controller were determined as:

\[
K_p = 0.6 \\
K_i = 150.0 \\
K_d = 0.001.
\]  

Figure 5.16 shows the step response of the system; the output response has an overshoot.
of 2.5%. The rise time of the system response is around 0.01s, which is very much the same as the single PID control system's response.

Figure 5.17 shows the tracking performance of the system with a cascaded PID controller. Experimental results are summarized as follows:

- At $30\,Hz$:
  
  Phase shift: $43.2^\circ$
  
  System gain: 0.925

- At $70\,Hz$:
  
  Phase shift: $151.2^\circ$
  
  System gain: 0.8
They are very close to those obtained from the system with a single PID controller. In order
to demonstrate the superiority of a cascaded PID control system to a PID control system,
the system was tested under the same conditions in which the PID control system failed to
perform well. With a cascaded PID controller, the output had been smoothed out (see figure
5.18). This demonstrates that the capability of a cascaded PID controller was better than the
use of a single PID controller in handling parameter variations.
In the next experiment, a sliding mode controller was tested on the actuator. This allows one to observe how well the derived sliding mode control algorithm can stack up with more conventional control algorithms like the PID and the cascaded PID control algorithms.

![Experimental Output with 30Hz Sine Wave](image)

**Figure 5.18** System Output in Small Operating Range Using Cascaded PID Controller

### 5.7.3 Experimental Results with Discrete-time Sliding Mode Controller

A discrete-time sliding mode control algorithm was also considered in these experiments. The control law implemented was derived in section 4.6 of chapter 4.

States of the actuator are required in this control algorithm, the displacement of the actuator is the only measurable signal. A state observer, which was designed in section 5.5, was implemented to obtain all the system states. Figure 5.19 shows a block diagram
of the overall control system. The procedure of selecting suitable $G$, $K_p$, $K_i$, $K_d$, and $K_w$ was described in sections 5.5 and 5.6. With the selection of $G = \begin{bmatrix} 17.6216 & 1.2894 & 6.5463 \end{bmatrix}$, $K_p = 0.1$, $K_i = 100.0$, $K_d = 0.2$, and $K_w = 3.0$, the real-time experimental result was relatively
slow. The rise time was slow at around 0.08s. Thus, the controller parameters were further tuned by trial-and-error to improve the overall system performance. The $K_p$, $K_i$, $K_d$, and $K_w$ terms were changed on line. Tuning the controller parameters while observing the step response of the system, the author decided on the parameters as $K_p = 0.1$, $K_i = 300.0$, $K_d = 0.15$, and $K_w = 0.04$. Figure 5.20 shows the overall step response, which has an overshoot of 7.5% and a rise time of 0.008s. Some major changes were made in the $K_i$ and the $K_w$ terms. The $K_i$ term was increased to reduce the rise time of the system. The value decided on gave a much-needed faster response with a large overshoot; however, the $K_w$ term was then greatly reduced to eliminate the large overshoot. The final system was then tested, oscillating at a different frequency. The sliding mode controller, which was based on an estimated linear model of the actuator, was capable of maintaining the system stability in the presence of system uncertainties. Figure 5.21 shows the experimental results obtained; they are summarized as follows:

- **At 30Hz:**
  
  Phase shift: 64.8°  
  System gain: 0.8

- **At 70Hz:**
  
  Phase shift: 126°  
  System gain: 0.73

The system was also tested operating over small range. The result shows no further improvement over that obtained using a cascaded PID controller (see figure 5.22).
5. Experimental Setup and Results

Figure 5.21 System Outputs with Different Reference Signals Using Sliding Mode Controller
5. Experimental Setup and Results

5.8 More Experiments and Discussions

The results obtained using a PID, a cascaded PID, and a sliding mode controller showed good tracking performance under no load condition. Under a high operating frequency and small operating range condition, the performance of the system with either a cascaded PID controller or a sliding mode controller is significantly better than the performance of the system with a single PID controller. To further distinguish the superiority in performance between these controllers under more realistic conditions, the system was tested with an external load.

The system was tested with an external load of 15 kg due to the difficulty in mounting more external load onto the actuator at that point; it was then controlled using different controllers. Figure 5.23 shows the step responses of systems with different controllers. It is clear that the system controlled by either a sliding mode controller or a cascaded controller gives a better result than the one controlled by a PID controller. Their output signals look as if the load did not exist. The output signal of the PID control system showed some signal of instability whenever there was a change in the reference signal. The rise time of the system with either
5. Experimental Setup and Results

A cascaded PID or a sliding mode controller is around 0.01s; the variance in the rise time, under both no load and load conditions, is negligible. The overshoot in the system with a cascaded PID controller has increased to 6.25%. In the case of using a PID controller, the system response was oscillating at about 100Hz for 0.05s before the oscillation attenuated completely. The overshoot was also very large at 36.5%.

All these experiments showed that the cascaded PID controller is much more robust against changes in system parameters and external load disturbance than its counterpart, the single PID controller. This is because the existence of the inner loop made the system more inert to parameter variations and to load disturbance. The sliding mode control algorithm, derived from the discrete Lyapunov function and a sufficient condition for the existence of a sliding surface, could perform as well as the cascaded PID controller. The controller extracted the states needed for the controlling purpose from a state observer, which was based on an estimated linear model of the actuator; it managed to stabilize the system in the presence of parameter variations and load disturbance. It can be claimed that the robustness of the derived sliding mode control law has been proven experimentally in this project.

In this project, the design of the discrete-time sliding mode controller relied on an estimated linear model of a highly nonlinear plant. For a complex-highly-nonlinear plant, it can be described better by a number of linear models, depending on the operating region, than by a single linear model. Thus, it is proposed that multiple linear models could be tried in designing a discrete-time sliding mode controller in future studies. As a result of using multiple linear models in designing a controller, a number of state feedback gains, \( G \), can thus be obtained. A vector \( G \), depending on the operating region, is selected as the state feedback gain, during a control process. Thus, the state feedback gain in a sliding mode controller is considered as self-adaptive. One could introduce a weighting function in smoothing out a 'crisp' boundary between operating regions. This method of using multiple linear models in designing a controller for a highly nonlinear plant, has great potential in exploring the plant's capacity; however, it has yet been conclusively proven.
5. Experimental Setup and Results

Figure 5.23 Load Test
6 Conclusion

6.1 Summary

This thesis is summarized as follows:

• A Terfenol-D based actuator with a push-pull configuration was designed and fabricated, and was found to have several advantages over existing designs. The advantages are:
  a. It allows active actuation in both directions.
  b. It maintains constant pre-stress on the Terfenol-D at all times.
  c. It allows an external load to be shared by two rods; thus, more external load can be handled by the actuator.
  d. It decreases the 'fatness' of hysteresis loop which makes controller design simpler.

• A discrete-time sliding mode control algorithm based on a differential Lyapunov function and a condition for the existence of a sliding surface was derived. Based on an estimated linear model of the highly nonlinear plant, the controller parameters were designed and were tested on the actual plant. The derived sliding mode control algorithm was experimentally proven to be capable of maintaining the stability of the system in the presence of system uncertainties.

• The actuator was also test-controlled by a PID controller and a cascaded PID controller. The system with a PID controller performed well under no load conditions; however, external load disturbance caused instability in the system. Comparing the experimental results, one could observe that the system with a cascaded PID controller was capable of performing better under both conditions.
• The system with a cascaded controller showed similar performance as the system with a sliding mode controller. Since the cascaded PID controller is easier to implement by software, the author recommends its use in the fretting test system.
• A user manual for the designed actuator was written to assist the next researcher to continue with the development of this actuator.

6.2 Future Work

• Effect of high temperature and high pressure on the actuator have yet to be tested. The controlled actuator showed good performance under room temperature condition.
• The potential of using the Preisach model directly for designing a controller should be studied because this model has a flavour of a state space description.
• A more theoretical approach for solving $K_w$, $K_p$, $K_i$, and $K_d$ needs to be investigated. These gains and the state feedback gain, $G$, determine the desired system pole locations.
• Multiple $G$ vectors in a discrete-time sliding mode control algorithm should be studied because it has great potential in solving tracking problems involving a complex and highly nonlinear system.
References


