PERFORMANCE OF TRANSMISSION STRATEGIES
IN MULTIHOP PACKET RADIO NETWORKS

by

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Abstract

The problem of data communication in a multihop packet radio network (PRN) with a number of mobiles or randomly distributed stationary nodes using a slotted ALOHA channel access scheme is addressed. A multihop PRN is an extension of a single-hop PRN; in the latter system, a node can communicate directly with any other nodes. A multihop PRN is necessary if a node cannot reach another node due to transmit power constraints. In such a case, the node requires repeaters to forward packets on to the destination. If a network has a large number of nodes, employing spatial frequency reuse to achieve better spectrum is one of the advantages of a multihop PRN over a single-hop PRN.

In multihop PRNs, a transmission strategy involves designing a routing algorithm and determining the transmission probability for each node in the network. Routing is one of the major problems in a multihop PRN. In this thesis, three new routing schemes, MAD, ARR, and MTP, are proposed. The performances of five transmission strategies based on the MFR, NFP, MAD, ARR, and MTP routing schemes with capture and fading are studied and compared. Results show that the transmission strategies based on the ARR and MTP routing schemes perform better than the other three transmission strategies. All five transmission strategies perform worse in the presence of Rayleigh fading; this is in contrast to contention-limited single-hop systems in which Rayleigh fading generally does not result in a poorer performance.

A multiple receiver antenna system for a multihop PRN is proposed in which the nodes make use of multiple receiver antennas so as to provide diversity reception. The performances of the five transmission strategies based on the MFR, NFP, MAD, ARR, and MTP routing schemes in a Rayleigh fading environment are investigated. Results show
that the performances of all five transmission strategies can be substantially improved by employing multiple antennas.

The performance improvements of the transmission strategies arising from the use of directional transmitter antennas are also examined. It is found that the performances of all five transmission strategies can be improved by reducing the beamwidth of the antennas.
Table of Contents

Abstract ii

List of Figures vii

Glossary xi

Acknowledgement xiii

1 Introduction 1

2 Preliminaries 5

  2.1 Slotted ALOHA 5

  2.2 Channel Models 6

  2.3 Capture Models 6

  2.4 Interference Models 7

  2.5 Antenna Patterns 9

  2.6 Network Model 10

  2.7 Performance Measure 11

  2.8 Review of Related Works 13

3 Routing Schemes 17

  3.1 Most Forward with Fixed Range (MFR) Scheme 19

  3.2 Nearest with Forward Progress (NFP) Scheme 21

  3.3 Minimal Angular Deviation (MAD) Scheme 22

  3.4 Angular Deviation to Transmission Range Ratio (ARR) Scheme 24

  3.5 Maximal Transmission Potential (MTP) Scheme 28
4 Performance Evaluation of Transmission Strategies

4.1 Interference Model 1.2 ........................................... 31

4.1.1 Probability of Interference ..................................... 31

4.1.1.1 Calculation of Pr(E_{int}|E_+) ................................ 32

4.1.1.2 Calculation of Pr(E_{int}|E_-) .............................. 37

4.1.2 Throughput and Normalized Average Progress ............... 40

4.2 Interference Model 2.1 ........................................... 42

4.3 Interference Model 2.2 ........................................... 44

4.3.1 Capture Probability ............................................. 44

4.3.2 Throughput and Normalized Average Progress ............... 49

4.4 Analytic and Simulation Results .................................. 51

4.4.1 Interference Model 1.2 ........................................... 52

4.4.2 Interference Model 2.1 ........................................... 53

4.4.3 Interference Model 2.2 ........................................... 54

4.4.4 Regular Structure Networks ..................................... 56

5 Multiple Receiver Antenna Selection ................................. 65

5.1 Multiple Receiver Antenna System ............................... 65

5.2 Capture Probability ............................................... 66

5.3 Throughput and Normalized Average Progress .................... 68

5.4 Analytic and Simulation Results .................................. 69
List of Figures

2.1 Antenna radiation patterns, rect function and sinc function, with half-power beamwidth $B_W = \frac{\pi}{2}$. ................................................................. 9

2.2 Illustration of the progress towards the destination with a successful transmission. 12

3.1 Receiving node selection and excluded region for MFR. ............................... 20

3.2 Receiving node selection and excluded region for NFP. ............................... 22

3.3 Receiving node selection and excluded region for MAD. ............................ 23

3.4 Receiving node selection and excluded region for ARR. ............................. 25

3.5 Illustration for the derivation of $f_T(t)$ for ARR. ..................................... 26

3.6 Illustration for the derivation of $f_\theta T (\theta | t)$ for ARR. .......................... 27

4.1 Illustration of the calculation of $Pr(E_{\text{int}} | E_+)$ for MFR. .................... 33

4.2 Illustration of the calculation of $Pr(E_{\text{int}} | E_+)$ for NFP. .................... 35

4.3 Illustration of the calculation of $Pr(E_{\text{int}} | E_+)$ for MAD. ................... 36

4.4 Illustration of the calculation of $Pr(E_{\text{int}} | E_+)$ for ARR. ................... 37

4.5 Illustration of the interference region for interference model 2.1. ............... 43

4.6 The uniform spatial and the bell-shaped spatial probability density functions $f_{R_m}(r_m)$ of distance between Y and M with $r_e = 0$ and $r_d = 10r_{\text{max}}$. 46

4.7 Comparison of normalized average progress for MFR, NFP, MAD, and ARR with interference model 1.2. ......................................................... 58

4.8 Comparison of throughput for MFR, NFP, MAD, and ARR with interference model 1.2. Legend is the same as in Figure 4.7. ................................. 58

4.9 Comparison of normalized average progress for MFR, NFP, MAD, ARR, and MTP with interference model 2.1. ..................................................... 59
4.10 Comparison of throughput for MFR, NFP, MAD, ARR, and MTP with interference model 2.1. Legend is the same as in Figure 4.9.

4.11 Comparison of normalized average progress for MFR, NFP, MAD, ARR, and MTP with interference model 2.2 in a non-fading channel.

4.12 Comparison of throughput for MFR, NFP, MAD, ARR, and MTP with interference model 2.2 in a non-fading channel. Legend is the same as in Figure 4.11.

4.13 Comparison of normalized average progress for MFR, NFP, MAD, ARR, and MTP with interference model 2.2 in a Rayleigh fading channel. Legend is the same as in Figure 4.11.

4.14 Comparison of throughput for MFR, NFP, MAD, ARR, and MTP with interference model 2.2 in a Rayleigh fading channel. Legend is the same as in Figure 4.11.

4.15 Distribution of nodes in a square grid network.

4.16 Distribution of nodes in a hexagonal grid network.

4.17 Comparison of normalized average progress for MFR, NFP, MAD, ARR, and MTP with interference model 2.2 in a square grid network.

4.18 Comparison of throughput for MFR, NFP, MAD, ARR, and MTP with interference model 2.2 in a square grid network. Legend is the same as in Figure 4.17.

4.19 Comparison of normalized average progress for MFR, NFP, MAD, ARR, and MTP with interference model 2.2 in a hexagonal grid network. Legend is the same as in Figure 4.17.

4.20 Comparison of throughput for MFR, NFP, MAD, ARR, and MTP with interference model 2.2 in a hexagonal grid network. Legend is the same as in Figure 4.17.

5.1 Normalized average progress for MFR with various $N_r$ receiver antennas. Legend is the same as in Figure 5.2.

5.2 Throughput for MFR with various $N_r$ receiver antennas.

5.3 Normalized average progress for NFP with various $N_r$ receiver antennas. Legend is the same as in Figure 5.2.

5.4 Throughput for NFP with various $N_r$ receiver antennas. Legend is the same as in Figure 5.2.
5.5 Normalized average progress for MAD with various $N_r$ receiver antennas. Legend is the same as in Figure 5.2. .................................................. 74

5.6 Throughput for MAD with various $N_r$ receiver antennas. Legend is the same as in Figure 5.2. .................................................. 74

5.7 Normalized average progress for ARR with various $N_r$ receiver antennas. Legend is the same as in Figure 5.2. .................................................. 75

5.8 Throughput for ARR with various $N_r$ receiver antennas. Legend is the same as in Figure 5.2. .................................................. 75

5.9 Normalized average progress for MTP with various $N_r$ receiver antennas. .................................................. 76

5.10 Throughput for MTP with various $N_r$ receiver antennas. Legend is the same as in Figure 5.9. .................................................. 76

6.1 Illustration of $Y$, a transmitting node $M$, $M$'s intended receiving node $W$, and $M$’s destination $U$. .................................................. 79

6.2 Normalized average progress for MFR in a non-fading channel with rect pattern and different values of $B_W$. Legend is the same as in Figure 6.5. ............. 86

6.3 Normalized average progress for MFR in a non-fading channel with sinc pattern and different values of $B_W$. Legend is the same as in Figure 6.5. ............. 86

6.4 Normalized average progress for NFP in a non-fading channel with rect pattern and different values of $B_W$. Legend is the same as in Figure 6.5. ............. 87

6.5 Normalized average progress for NFP in a non-fading channel with sinc pattern and different values of $B_W$. .................................................. 87

6.6 Normalized average progress for MAD in a non-fading channel with rect pattern and different values of $B_W$. Legend is the same as in Figure 6.5. ............. 88

6.7 Normalized average progress for MAD in a non-fading channel with sinc pattern and different values of $B_W$. Legend is the same as in Figure 6.5. ............. 88

6.8 Normalized average progress for ARR in a non-fading channel with rect pattern and different values of $B_W$. Legend is the same as in Figure 6.5. ............. 89

6.9 Normalized average progress for ARR in a non-fading channel with sinc pattern and different values of $B_W$. Legend is the same as in Figure 6.5. ............. 89
6.10 Normalized average progress for MTP in a non-fading channel with \textit{rect} pattern and different values of $B_W$. Legend is the same as in Figure 6.5. \\
6.11 Normalized average progress for MTP in a non-fading channel with \textit{sinc} pattern and different values of $B_W$. Legend is the same as in Figure 6.5. \\
6.12 Normalized average progress for MFR in a Rayleigh fading channel with \textit{rect} pattern and different values of $B_W$. Legend is the same as in Figure 6.15. \\
6.13 Normalized average progress for MFR in a Rayleigh fading channel with \textit{sinc} pattern and different values of $B_W$. Legend is the same as in Figure 6.15. \\
6.14 Normalized average progress for NFP in a Rayleigh fading channel with \textit{rect} pattern and different values of $B_W$. Legend is the same as in Figure 6.15. \\
6.15 Normalized average progress for NFP in a Rayleigh fading channel with \textit{sinc} pattern and different values of $B_W$. \\
6.16 Normalized average progress for MAD in a Rayleigh fading channel with \textit{rect} pattern and different values of $B_W$. Legend is the same as in Figure 6.15. \\
6.17 Normalized average progress for MAD in a Rayleigh fading channel with \textit{sinc} pattern and different values of $B_W$. Legend is the same as in Figure 6.15. \\
6.18 Normalized average progress for ARR in a Rayleigh fading channel with \textit{rect} pattern and different values of $B_W$. Legend is the same as in Figure 6.15. \\
6.19 Normalized average progress for ARR in a Rayleigh fading channel with \textit{sinc} pattern and different values of $B_W$. Legend is the same as in Figure 6.15. \\
6.20 Normalized average progress for MTP in a Rayleigh fading channel with \textit{rect} pattern and different values of $B_W$. Legend is the same as in Figure 6.5. \\
6.21 Normalized average progress for MTP in a Rayleigh fading channel with \textit{sinc} pattern and different values of $B_W$. Legend is the same as in Figure 6.5.

A.1 Illustration of the partial excluded region for MFR. \\
A.2 Illustration of the partial excluded region for NFP. \\
A.3 Illustration of the partial excluded region for MAD. \\
A.4 Illustration of the partial excluded region for ARR.
Glossary

Acronyms

ARR  
Angular Deviation to Transmission Range Ratio

cdf  
Cumulative distribution function

GPS  
Global Positioning System

MAD  
Minimal Angular Deviation

MFR  
Most Forward with Fixed Range

MVR  
Most Forward with Variable Radius

NFP  
Nearest with Forward Progress

MTP  
Maximal Transmission Potential

pdf  
Probability density function

PRN  
Packet radio network

rv  
Random variable

Notations

$\beta$  
Power loss factor

$\Gamma, \gamma$  
received power

$\Theta, \theta$  
Angle between $XD$ and $XY$

$\lambda$  
Average number of nodes per unit area

$A_e$  
Area of an excluded region

$A_e(r_c)$  
Area of a partial excluded region within distance $r_c$ of node $Y$
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_W$</td>
<td>Beamwidth of a directional antenna</td>
</tr>
<tr>
<td>$c$</td>
<td>Capture ratio</td>
</tr>
<tr>
<td>$D$</td>
<td>Node $X$'s destination node</td>
</tr>
<tr>
<td>$E_f$</td>
<td>Event that a node finds a neighboring node in its forward direction</td>
</tr>
<tr>
<td>$F_{\Gamma}(\gamma)$</td>
<td>Cumulative distribution function of $\Gamma$</td>
</tr>
<tr>
<td>$f_{\Gamma}(\gamma)$</td>
<td>Probability density function of $\Gamma$</td>
</tr>
<tr>
<td>$f_{R,\Theta}(r, \Theta)$</td>
<td>Joint probability density function of $R$ and $\Theta$</td>
</tr>
<tr>
<td>$K$</td>
<td>A packet transmitted by node $X$ to node $Y$ onto node $D$</td>
</tr>
<tr>
<td>$N$</td>
<td>Connectivity, average number of nodes in a circular area of radius $r_{max}$</td>
</tr>
<tr>
<td>$N_r$</td>
<td>Number of receiver antennas at a node</td>
</tr>
<tr>
<td>$Pr(E_f)$</td>
<td>Probability of the event $E_f$</td>
</tr>
<tr>
<td>$p_t$</td>
<td>Probability of a node being in transmission mode</td>
</tr>
<tr>
<td>$q_i(r, \Theta)$</td>
<td>Capture probability of packet $K$ by node $Y$ given $i$ interfering nodes</td>
</tr>
<tr>
<td>$R, r$</td>
<td>Distance between node $X$ and node $Y$</td>
</tr>
<tr>
<td>$r_{max}$</td>
<td>Maximum transmission range</td>
</tr>
<tr>
<td>$S$</td>
<td>Throughput</td>
</tr>
<tr>
<td>$X$</td>
<td>A transmitting node</td>
</tr>
<tr>
<td>$Y$</td>
<td>$X$'s intended receiving node</td>
</tr>
<tr>
<td>$Z, z$</td>
<td>Progress of packet $K$ towards node $D$</td>
</tr>
<tr>
<td>$\bar{Z}$</td>
<td>Average progress</td>
</tr>
<tr>
<td>$Z_n$</td>
<td>Normalized average progress</td>
</tr>
</tbody>
</table>
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Chapter 1

Introduction

Packet radio is a technology that extends the application of packet switching. Packet radio networks (PRNs) have evolved from the traditional networks of point-to-point communication lines to the networks of broadcasting on VHF/UHF radio channels [1]. PRNs permit mobile digital communication applications over a wide geographic area and provide a degree of flexibility in rapid deployment and reconfiguration which currently is not possible with fixed plant installations [2]. When the channel is occupied by bursty traffic from a large number of nodes, PRNs with random access schemes, such as ALOHA [3], slotted ALOHA [4], Carrier Sense Multiple Access (CSMA) [5], etc. offer efficient ways of using the multiple access channel.

Although random access schemes can provide highly efficient use of a radio channel, they give rise to the possibility of a collision when several packets are transmitted at about the same time. If the packets are received with more or less the same powers, it is reasonable to assume that they will all be destroyed [3], [5], [6]. However, this assumption is somewhat pessimistic in a radio transmission environment. The performance improvement brought about by the fact that the transmitting nodes may be at different distances from the receiver (and hence their respective packets received with different power levels due to attenuation) has been discussed [4], [7]–[10]. This phenomenon has been referred to as the near-far or capture effect.

Another phenomenon in the radio environment is fading [11], [12]. Propagation of VHF/UHF radio waves in urban areas takes place mostly by diffraction or reflection by
obstacles in the vicinity of the node. Under certain conditions, it can be shown that the short-term amplitude of the received signal is well approximated by a Rayleigh distribution. The effects of Rayleigh fading in ALOHA systems have been studied in the literature [13]–[18].

So far, most of the published papers relating to PRNs address single-hop systems in which a node can communicate directly with all other nodes. Hence, no packet routing problem exists in single-hop PRNs. As a network becomes larger, a node may not be able to reach another node in one hop due to transmit power constraints. As the number of nodes increases, it may also be desirable to go to a multihop system which employs spatial frequency reuse to achieve better spectrum efficiency. In this event, a node will require repeaters to forward packets on to the destination. Such networks have been referred to as multihop PRNs [19], [20]. The key point in a multihop PRN is that different nodes can transmit with the same carrier frequency in different parts of the network, and more than one of these transmissions could be received successfully by their corresponding intended receiving nodes in the same time slot. In multihop PRNs, a transmission strategy involves designing a routing algorithm, determining the range $r_{max}$ and the transmission probability for each node in the network. Routing is one of the major problems in multihop PRNs [2], [21]. Another important problem is the determination of $r_{max}$ for each node in the network. A transmitting node $X$ chooses one of the nodes, within distance $r_{max}$ of $X$, as its intended receiving node. A multihop system starts to resemble a single-hop system as $r_{max}$ increases. A larger $r_{max}$ will increase the probability of finding a receiving node in the desired direction and contribute a larger progress towards the destination if the transmission is successful. However, it will also result in a higher probability of interfering with other transmissions. A smaller $r_{max}$ can reduce the interference but will contribute a smaller
progress towards the destination if the transmission is successful. This is the trade-off in choosing the optimal $r_{\text{max}}$ in multihop PRNs.

In this thesis, we study the performances of five transmission strategies in a multihop PRN with various interference models. The nodes are assumed to be randomly distributed in an infinitely large plane according to a two-dimensional Poisson distribution. Hence, the results obtained in this thesis can apply to a mobile network or a stationary network in which the nodes are randomly located.

This thesis is organized as follows. In Chapter 2, a number of models and assumptions are discussed. The previous and related works in multihop PRNs are also summarized in this chapter. In Chapter 3, two previously proposed routing schemes, Most Forward with Fixed Range (MFR) [19], [20] and Nearest with Forward Progress (NFP) [22], are discussed. Three new routing schemes, Minimal Angular Deviation (MAD) [23], [24], Angular Deviation to Transmission Range Ratio (ARR) [23], [24], and Maximal Transmission Potential (MTP) are also proposed.

In Chapter 4, the performances of the five transmission strategies based on the MFR, NFP, MAD, ARR, and MTP routing schemes are examined with a number of different interference models. The transmission strategies based on ARR and MTP are shown to generally have better performances than the other three transmission strategies.

In Chapter 5, a multiple antenna system is proposed in which the nodes in the network make use of multiple receiver antennas so as to provide diversity reception. The performances of the five transmission strategies in a Rayleigh fading environment are examined. Results show that the performances of all five transmission strategies can be substantially improved by employing multiple antennas.

In Chapter 6, we study the performance improvement of the transmission strategies
brought about by using directional transmitter antennas with a more realistic model than those in [25] and [26]. Two antenna patterns, \textit{rect} function and \textit{sinc} function, are considered. Results show that the performances of all five transmission strategies can be improved by reducing the beamwidth of the antennas.

The main contributions of the thesis are summarized in Chapter 7. A number of suggestions for further research are also outlined.
Chapter 2
Preliminaries

In this chapter, a number of models and assumptions which will be used in subsequent chapters are discussed. In Section 2.1, we described a slotted ALOHA system which is used in evaluating the performances of the transmission strategies throughout this thesis. Two channel models, a non-fading channel and a Rayleigh fading channel, are discussed in Section 2.2. Two capture models and various interference models are discussed in Sections 2.3 and 2.4, respectively. In Section 2.5, two antenna patterns, rect function and sinc function, for the directional antennas considered in Chapter 6 are described. The network model for the multihop PRN and the performance measure used for the transmission strategies are described in Sections 2.6 and 2.7, respectively. The previous and related works are summarized in Section 2.8.

2.1 Slotted ALOHA

In a pure ALOHA system [3], each node transmits as soon as a packet arrives. The node then waits for an acknowledgement from the receiver. If no acknowledgement arrives after some timeout interval, the node retransmits its packet after some random time interval. Since each node transmits its packet independently, there is a possibility that two or more packets will collide at the receiver. Packet synchronization is employed in a slotted ALOHA system in order to reduce the likelihood of a collision [4]. Any node which has a packet ready to send will start transmitting only at the beginning of a slot. In this case, no partial collision can occur.
2.2 Channel Models

Two types of channels are considered: non-fading and Rayleigh Fading. The non-fading channel is based on a flat terrestrial propagation model [27] in which the normalized received signal power $\Gamma$, hereafter referred to simply as the received power, at the receiver varies with the transmitter-to-receiver distance $r$ as

$$\Gamma(r) = \frac{1}{r^\beta}$$  \hspace{1cm} (2.1)

where $\beta$ is the power loss factor. For land mobile radio systems, a typical value of $\beta$ is 4 [8], [9], [14] and we will use this value throughout this thesis.

The fading channel model assumes that the received signal amplitude is Rayleigh. This results in the received signal power having an exponential distribution with a mean $\mu$ given by (2.1), i.e.,

$$f_\Gamma(\gamma) = \frac{1}{\mu} e^{-\frac{\gamma}{\mu}}.$$  \hspace{1cm} (2.2)

2.3 Capture Models

Various conditions under which a data packet is assumed to be successfully received ("captured") have appeared in the literature [4], [8]–[10], [13]–[16], [28]–[36]. The following two capture models [4], [8]–[10], [13], [14], [16], [28] are commonly used. Suppose that node $X$ transmits packet $K$ to a receiving node $Y$ in a given time slot and the received power at $Y$ is $\gamma_t$. Let the probability of successful reception of $K$ by $Y$ given a set of $i$ interfering signals (excluding any transmission by $Y$) with received powers at $Y \{ \gamma_1, \gamma_2, \ldots, \gamma_i \}$ be denoted by $p_c(\gamma_t, \gamma_1, \gamma_2, \ldots, \gamma_i)$. It is assumed that the received signal powers are more or less constant over a packet duration and that only the packet with the largest received power can be captured [4], [8]–[10], [13]–[16], [28]–[36].
Then the capture model in [4], [9], [16], and [28] corresponds to

\[ p_c (\gamma_t, \gamma_1, \gamma_2, \cdots, \gamma_i) = \begin{cases} 1, & \text{if } \gamma_t > c \max \{ \gamma_1, \gamma_2, \cdots, \gamma_i \} \\ 0, & \text{otherwise} \end{cases} \tag{2.3} \]

and the capture model in [8], [10], [13], and [14] corresponds to

\[ p_c (\gamma_t, \gamma_1, \gamma_2, \cdots, \gamma_i) = \begin{cases} 1, & \text{if } \gamma_t > c \left( \sum_{j=1}^{i} \gamma_j \right) \\ 0, & \text{otherwise} \end{cases} \tag{2.4} \]

In (2.3) and (2.4), the parameter \( c \) is referred to as the capture ratio, and is greater than 1 for non-spread-spectrum systems. The actual value for \( c \) depends on the modulation and coding schemes used. For convenience, we will refer to conditions (2.3) and (2.4) as capture models 1 and 2, respectively. Even though capture model 1 is somewhat unrealistic and leads to optimistic results, its use often allows for simpler analytic derivations.

### 2.4 Interference Models

Suppose node \( X \) transmits packet \( K \) to node \( Y \). The reception of \( K \) by \( Y \) could be unsuccessful due to interference from other transmitting nodes. Let \( M \) be an interfering node which transmits in the same time slot as \( X \) transmits to \( Y \). Four different models for the interference to \( Y \) by \( M \) are considered:

**Interference model 1.1:**

In this model, every transmitting node in the network has a fixed transmission range [19], [20]. Only the nodes within the transmission range can hear the transmission; other nodes cannot hear the transmission at all. Since the transmission range is fixed, \( Y \) will receive \( K \) successfully if and only if there is no interfering node within the fixed transmission range. The fixed transmission range assumption implies that every node transmits with the same constant power. However, this model suggests that the probability of successful reception
of $K$ by $Y$ is independent of the distance between $X$ and $Y$. Since this is unrealistic, we will not use this interference model.

**Interference model 1.2:**

Every transmitting node in the network adjusts its transmission power to be strong enough to reach its intended receiving node [22]. In this case, the distance between node $X$ and its intended receiving node $Y$ is defined as the transmission range of $X$. In this model, every transmitting node transmits with variable range and hence, the interference to other nodes can be reduced. Note that $Y$ will receive $K$ successfully if and only if $Y$ is not within the transmission range of any interfering node. This interference model always yields a better performance than interference model 1.1 for a given set of nodes because the interference at $Y$ is decreased by reducing the transmission ranges of the interfering nodes.

**Interference model 2.1:**

Every transmitting node transmits with constant power, and the non-fading or the Rayleigh fading channel model and capture model 1 are assumed [26]. In this model, $Y$ will receive $K$ successfully if the received power of $K$ at $Y$ exceeds the strongest interfering power at $Y$ by a capture ratio $c$.

**Interference model 2.2:**

Every transmitting node transmits with constant power, and the non-fading or the Rayleigh fading channel model and capture model 2 are assumed [37]. In this model, $Y$ will receive $K$ successfully if the received power of $K$ at $Y$ exceeds the total interfering power at $Y$ by $c$. The only difference between interference models 2.1 and 2.2 is the capture model. Since capture model 2 is more realistic than capture model 1, the use of interference model 2.2 should lead to more realistic results.
Chapter 2. Preliminaries

2.5 Antenna Patterns

Two different antenna radiation patterns are considered. The first antenna pattern is a \textit{rect} function:

\[
g(\phi) = \begin{cases} 
1, & -\frac{B_w}{2} \leq \phi \leq \frac{B_w}{2} \\
0, & \text{otherwise}
\end{cases}
\]  \hfill (2.5)

where \( g(\cdot) \) is the normalized power gain, \( \phi \) is the orientation angle and \( B_w \) is the half-power beamwidth. This is the model of an ideal antenna. Although no real antenna pattern can be represented by a \textit{rect} function, the antenna pattern of (2.5) is considered for comparative purposes.

The second antenna pattern is a \textit{sinc} function:

\[
g(\phi) = \left( \frac{\sin \left( \frac{2.78 \phi}{B_w} \right)}{\frac{2.78 \phi}{B_w}} \right)^2 \]  \hfill (2.6)

Figure 2.1 Antenna radiation patterns, \textit{rect} function and \textit{sinc} function, with half-power beamwidth \( B_w = \frac{\pi}{2} \).
where \( g(\cdot) \), \( \phi \), and \( B_W \) are as defined in (2.5). Some antenna patterns can be modelled approximately by a \( \text{sinc} \) function [38].

Figure 2.1 shows the two antenna patterns with \( B_W = \frac{\pi}{2} \). It is preferable that the gain fall off rapidly at the two half-power angles \( \pm \frac{B_W}{2} \) and remain as low as possible outside the main lobe. If a node transmits with such a directional antenna, the interference to nodes beyond the half-power beamwidth region will be substantially reduced.

2.6 Network Model

The network model and assumptions are basically the same as those used in [19], [20], [22], and [24].

- Transmission protocol: slotted ALOHA. The slot duration is equal to the transmission time of a packet and the acknowledgement channel is noiseless.
- Transmission probability: all nodes are assumed to have packets ready for transmission at all times (heavy traffic assumption). For every slot, each node is in transmission mode with probability \( p_t \) and not in transmission mode with probability \( 1 - p_t \), where \( 0 \leq p_t \leq 1 \). A node can transmit only if it is in transmission mode.
- Spatial distribution of nodes: two-dimensional Poisson point process with average number of nodes per unit area equal to \( \lambda \). For a given region with area \( A \), the probability \( p_k \) that there are \( k \) nodes in the region is
  \[
  p_k = \frac{(\lambda A)^k \exp(-\lambda A)}{k!},
  \]
  (2.7)
  and these nodes are uniformly distributed in the region.
- Knowledge of node locations: each node is assumed to know the positions of those nodes within distance \( r_{\text{max}} \) of its location; they are said to be its neighbors (or neighboring
nodes). The connectivity (average number of nodes within \( r_{\text{max}} \)) is denoted by \( N \) and is equal to \( \lambda \pi r_{\text{max}}^2 \).

- Packet addressing: each transmitted packet contains the address of the receiving node. A node will discard a packet which is not intended for it.

- Direction of the destination: each node is assumed to know the direction of the destination. For every slot, the direction of the destination of a packet for each node is assumed to be uniformly distributed in \([-\pi, \pi]\).

- Transmitting-receiving capability: each node can either transmit or receive a packet, but cannot do both, in the same time slot.

With the assumption of the spatial distribution of nodes, we can interpret the results in Sections 4.4.1 to 4.4.3, 5.4, and 6.4 as the performance for a mobile network or the average performance for a network in which the nodes are stationary, but whose locations are chosen at random. For a mobile network, the assumption that a node would know the locations of the other nodes may seem somewhat unrealistic. One possible approach is for each node to broadcast its location at regular time intervals. A node can determine its own location using a system such as Global Positioning System (GPS) [39] which can provide an accuracy of a location within 5 meters. The broadcast could be done using a pure ALOHA random access scheme over a land radio channel or a satellite channel. A node would send a location update whenever it moved beyond a certain distance from its last reported location. It should also periodically send a location update so as to inform the network that it is still active. In the land radio channel case, the location information is transmitted at high power.

### 2.7 Performance Measure

For a single-hop PRN, a commonly used system performance measure is the throughput,
defined as the average number of successful packet transmissions per time slot. In a multihop PRN, throughput is defined as the number of successful (one-hop) transmissions for a node per time slot. Since every node in a single-hop PRN can communicate directly with all other nodes in one hop, the throughput of the network also gives the end-to-end throughput. However, this is not the case in a multihop PRN and hence, the use of throughput as a system performance measure may not be appropriate. The end-to-end throughput of a transmission strategy in a multihop PRN appears to be difficult to obtain. A common performance measure for a multihop PRN transmission strategy is the normalized average progress, measured on a per slot basis, of a packet towards its destination [19], [20], [22].

In Figure 2.2, X transmits K to Y as the first step in forwarding K to the destination D. If the distance between X and D is much larger than $r_{\text{max}}$, the progress $z$ can be approximated by $r \cos \theta$. Let the average progress be denoted by $\bar{Z}$. The normalized average progress $Z_n$ is defined as $\bar{Z} \sqrt{\lambda}$ where $\lambda$ is the average number of nodes per unit area. Since the

![Figure 2.2 Illustration of the progress towards the destination with a successful transmission.](image)
average distance between a node and the node nearest to it is \( \frac{1}{2\sqrt{\lambda}} \), the term \( \sqrt{\lambda} \) in \( Z_n \) is a normalization factor. The normalized average progress is used as a dimensionless measure. For a given network model, interference model, and routing scheme, \( Z_n \) depends on \( \lambda \) and \( r_{max} \) only through the connectivity \( N = \lambda \pi r_{max}^2 \). This is because for a fixed value of \( N \), a change in \( \lambda \) amounts merely to a rescaling of \( r_{max} \). If |\( \theta \)| < \( \frac{\pi}{2} \), we say that there is forward progress and that \( Y \) is in the forward direction of \( X \). Otherwise, we say that \( Y \) is in the backward direction of \( X \).

2.8 Review of Related Works

Relatively few papers have dealt with multihop PRNs compared to the literature on single-hop PRNs. The previous works and results related to multihop PRNs with randomly distributed nodes are summarized in this chapter. The network models assumed in these papers are mostly the same as the model described in Section 2.6, and interference model 1.1, 1.2, or 2.1 is assumed.

Routing and the determination of the transmission range for each node in the network are the two problems which have been studied in multihop PRNs. A number of papers have dealt with the selection of transmission ranges to optimize the system performance using slotted ALOHA. Kleinrock and Silvester [19] first proposed a routing algorithm Most Forward with Fixed Range (MFR) in which interference model 1.1 is assumed. In [19], a node transmits with a fixed range to another node which results in the most forward progress. If no forward progress is made, then the least backward progress is preferred. The goal of MFR is to minimize the number of hops needed for a packet to reach its destination. It is shown in [19] that a connectivity of 5.89 optimizes the normalized average progress \( Z_n \). Given the average number of nodes per unit area \( \lambda \), this allows the optimal transmission range to be determined. Later Takagi and Kleinrock [20] improved on this model and showed that the maximum
value of $Z_n$ is 0.043 at a connectivity $N = 7.7$. This result is obtained by optimizing the probability $p_t$ of a node in a transmission mode. Hou and Li [22] provided a more precise analysis and concluded that 6.0 is a better estimate of the connectivity to optimize $Z_n$. They also found that the maximum value of $Z_n$ is 0.047. In [22], transmissions resulting in backward progress are not allowed, and a more detailed investigation of the progress is done taking account of the fact that an interfering node cannot be located in a certain region. Although the disallowance of backward progress in MFR can improve $Z_n$, the improvement is not large and becomes negligible as the connectivity increases.

Hou and Li [22] modified MFR to *Most Forward with Variable Radius* (MVR), and introduced another routing scheme, *Nearest with Forward Progress* (NFP). In MVR and NFP, interference model 1.2 is assumed. MVR is similar to Hou and Li's MFR except that interference model 1.2 is used instead of interference model 1.1. In MVR, a transmitting node adjusts its transmission power to be just strong enough to reach the receiving node in forward direction. Thus, it tries to reduce interference to other packets while maintaining the goal of obtaining the largest progress possible if the transmission is successful. In NFP, a node transmits to the nearest neighbor which results in forward progress. The transmission range is also adjusted to be equal to the distance between the transmitter and the receiver. The goal here is to reduce conflict as much as possible. For MVR, the maximum value of $Z_n$ shown in [22] is 0.056 which occurs at $N = 6.0$, and the maximum value of $Z_n$ for NFP is 0.055 at $N = 7.7$. For MFR and MVR, $Z_n$ decreases as $N$ increases whereas for NFP $Z_n$ tends to remain constant. Therefore, NFP has the best performance for $N > 7$.

Yee and Shiao [40], [41] optimized the end-to-end throughput for a network of arbitrary topology with fixed node locations and interference model 1.1. In [40] and [41], the optimization problem is formulated as a non-linear programming problem and nodes are
allowed to have different transmission probabilities. The end-to-end throughput of such an optimal routing scheme can be improved substantially over that of MFR for a network with a small number of nodes. The improvement is reduced as the number of nodes increases. Yee and Shiao [42] simplified the optimization problem to a linear programming problem for a subclass of PRNs considered in [40] and [41]. In [42], the nodes are restricted to transmit with the same probability.

Hajek [43] used a different measure, efficiency, to study the network performance. The efficiency is the average progress divided by the area covered by the transmission. In [43], Hajek determined the optimal efficiency under the same assumptions made in [19]. He also showed that if a transmitting node can adjust its transmission power to be just strong enough to reach the receiving node, the optimal connectivity is about 3, and the optimal efficiency in this case results in an increase of 85% compared to the case of [19] in which the optimal connectivity is about 6.

Chang and Chang [25] have studied the use of directional transmitter antennas with interference model 1.1 and showed that performance is substantially increased since interference is largely eliminated by reducing the beamwidth of a directional transmitter antenna. Zander [26] also studied the use of directional transmitter antennas but his work focuses on networks with high connectivity with interference model 2.1. Shacham and King [44] have proposed a scheme which uses several subchannels. They showed that there is little or no performance gain achieved by splitting a channel into multiple subchannels.

Nelson and Kleinrock [45] studied the effect of capture under a different routing algorithm. In their model, a node forwards a packet to one of its neighboring nodes, in forward direction, with equal probability. Takagi and Kleinrock [20] investigated the capture effect on MFR. They showed that the normalized average progress in a system with
perfect capture is about 36% better than that in the system without capture. Hou and Li [46] also investigated the capture effect on MFR. The capture model used in [20], [45], and [46] is somewhat similar to capture model 1 described in Section 2.3. Zander [26] studied the performance improvement using directional transmitter antennas with capture model 1.
Chapter 3
Routing Schemes

In this chapter, we describe two previously proposed routing schemes, Most Forward with Fixed Range (MFR) and Nearest with Forward Progress (NFP), and three new routing schemes, Minimal Angular Deviation (MAD), Angular Deviation to Transmission Range Ratio (ARR), and Maximal Transmission Potential (MTP), for a multihop PRN. Suppose node X wants to send a packet $K$ to the destination node $D$. If $D$ is out of range of $X$, $X$ may need to send $K$ to a neighboring node $Y$ as a first step towards $D$. The choice of the receiving node $Y$ in a routing scheme involves a trade-off: a larger distance between $X$ and $Y$ will tend to contribute a larger progress of $K$ towards the destination if the transmission is successful. For interference model 1.2, a larger distance is equivalent to a larger transmission range and hence, more nodes will experience interference by the transmitting nodes. For interference models 2.1 and 2.2, a larger distance translates to a smaller average received signal power (at $Y$) for $K$. Therefore, a larger distance generally results in $K$ becoming more vulnerable due to the interference by other transmitting nodes and hence, reducing the chance of a successful reception of $K$ by $Y$. For all five routing schemes, if $X$ cannot find a neighboring node in the forward direction, it refrains from transmitting.

The joint probability density functions (pdf’s) $f_{R,\Theta}(r, \theta)$ of the distance $R$ between $X$ and $Y$ and the angle $\Theta$ between the destination direction of $X$ and $XY$ for the MFR, NFP, MAD, and ARR routing schemes are derived in Sections 3.1 to 3.4. These pdf’s will be used in Chapters 4 to 6. Throughout this thesis, it is understood that $R$ is in the range $[0, r_{max}]$ and $\Theta$ is in the range $[-\frac{\pi}{2}, \frac{\pi}{2}]$. 

If $X$ transmits to $Y$, there is a region in which no node can be located. We will refer to this as an *excluded region*. The area of the excluded region is required in the derivation of $f_{R, \Theta}(r, \theta)$, and the evaluation of the throughput and normalized average progress in Chapters 4 to 6. The excluded regions as defined here are slightly different from those in [22]. In [22], the portion of an excluded region beyond distance $r_{\text{max}}$ from $Y$ is not considered. For interference models 1.2 and 2.1, any transmitting node, excluding $X$ and $Y$, beyond distance $r_c$ from $Y$ does not interfere with $Y$. The values of $r_c$ for interference models 1.2 and 2.1 are $r_{\text{max}}$ and $c \frac{1}{3} r$, respectively. We refer to the portion of an excluded region within distance $r_c$ of $Y$ as a *partial excluded region*. The area of a partial excluded region $A_e(r_c)$ depends not only on $r_c$ but also on $r$, $\theta$, and the routing scheme. As $r_c$ increases, the area $A_e(r_c)$ becomes larger and finally equals to the area of the excluded region $A_e$ as defined in this thesis. In any case, $A_e(r_c) = A_e$ for $r_c \geq 2r_{\text{max}}$. The areas $A_e(r_c)$ for the MFR, NFP, MAD, and ARR routing schemes are given in Appendix A.

Let $E_f$ be the event that $X$ can find a neighboring node in the forward direction and $\Pr(E_f)$ be the probability that the event $E_f$ occurs. We have

$$
\Pr(E_f) = 1 - \Pr(\text{No neighboring node in the forward direction})
= 1 - \exp \left( -\frac{1}{2} \lambda \pi r_{\text{max}}^2 \right)
= 1 - \exp \left( -\frac{N}{2} \right).
$$

(3.1)

Unless otherwise stated, all pdf's and cumulative distribution functions (cdf's) discussed in this chapter are conditioned on $E_f$. The conditioning is not shown explicitly in order to simplify the notation.
3.1 Most Forward with Fixed Range (MFR) Scheme

In MFR [19], $X$ transmits to the receiving node $Y$ which results in the most forward progress (the largest value of $r \cos \theta$). If no node with forward progress is found, the node with the smallest backward progress is chosen. Hou and Li [22] modified MFR so as to allow only forward progress. In this routing scheme, a successful transmission results in the largest progress possible. However, a larger progress translates to a larger distance between $X$ and $Y$; hence the chance of a successful reception by $Y$ also decreases. Hou and Li referred to this routing scheme as MVR when it is associated with interference model 1.2. Hereafter, we will refer to this routing scheme as MFR regardless of the interference models assumed.

The receiving node selection for MFR is shown in Figure 3.1.

The area $A_e$ of the excluded region, shown as shaded in Figure 3.1, is derived in [20], [22] as

$$A_e = A_z = r_{\text{max}}^2 \cos^{-1} \left( \frac{z}{r_{\text{max}}} \right) - z \sqrt{r_{\text{max}}^2 - z^2}$$

$$= r_{\text{max}}^2 \left[ \cos^{-1} \left( \frac{z}{r_{\text{max}}} \right) - \frac{z}{r_{\text{max}}} \sqrt{1 - \frac{z^2}{r_{\text{max}}^2}} \right], \quad (3.2)$$

where $z = r \cos \theta$. Let $Z$ be the random variable (rv) representing the progress, i.e., the projection of $XY$ onto $XD$. The cdf $F_Z(z)$ of $Z$, for $0 \leq z \leq r_{\text{max}}$, is

$$F_Z(z) = 1 - \Pr \left( Z \geq z \mid E_f \right)$$

$$= 1 - \Pr \left( \text{at least one node in } A_z \mid E_f \right)$$

$$= 1 - \frac{1 - \exp \left( -\lambda A_z \right)}{1 - \exp \left( -\frac{N}{2} \right)}$$

$$= \frac{\exp \left( -\lambda A_z \right) - \exp \left( -\frac{N}{2} \right)}{1 - \exp \left( -\frac{N}{2} \right)}. \quad (3.3)$$
The pdf \( f_Z(z) \) of \( Z \), for \( 0 \leq z \leq r_{\text{max}} \), can then be written as

\[
f_Z(z) = \frac{2 \lambda \sqrt{r_{\text{max}}^2 - z^2} \exp(-\lambda A_z)}{1 - \exp(-N/2)}.
\] (3.4)

Let \( L \) be the rv denoting the vertical distance from \( Y \) to \( XD \). Since the nodes are uniformly distributed outside the excluded region, the conditional pdf of \( L \), given \( Z \), is

\[
f_{L|Z}(l|z) = \frac{1}{2 \sqrt{r_{\text{max}}^2 - z^2}}
\] (3.5)

for \( -\sqrt{r_{\text{max}}^2 - z^2} \leq l \leq \sqrt{r_{\text{max}}^2 - z^2} \). We can then obtain the joint pdf of \( Z \) and \( L \) as

\[
f_{Z,L}(z, l) = \frac{\lambda \exp(-\lambda A_z)}{1 - \exp(-N/2)}.
\] (3.6)

Since \( Z = z = r \cos \theta \) and \( L = l = r \sin \theta \), we can obtain the joint pdf of \( R \) and \( \Theta \) by a change from rectangular to polar co-ordinates [22] as

\[
f_{R,\Theta}(r, \theta) = \frac{\lambda r \exp(-\lambda A_z)}{1 - \exp(-N/2)}.
\] (3.7)
3.2 Nearest with Forward Progress (NFP) Scheme

In NFP [22], $X$ transmits to its nearest neighbor $Y$ (with the smallest value of $r$) in the forward direction as shown in Figure 3.2. For interference models 2.1 and 2.2, the nearer $Y$ is to $X$, the larger is the received signal power at $Y$, and the less vulnerable $Y$ is to interference from other transmitting nodes; hence the probability of successful reception is improved. For interference model 1.2, the nearer $Y$ is to $X$, the less interference (smaller transmission range) is caused to other nodes; hence the overall probability of successful reception is improved. However, the resulting progress is small even if the transmission is successful.

For NFP, the area $A_e$ of the excluded region, shown as shaded in Figure 3.2, is [22]

$$A_e = A_r = \frac{1}{2} r^2 \pi. \quad (3.8)$$

The cdf of $R$ is

$$F_R(r) = \Pr (R \leq r \mid E_f)$$

$$= \Pr (\text{at least one node in } A_r \mid E_f)$$

$$= \frac{1 - \exp(-\lambda A_r)}{1 - \exp\left(-\frac{N}{2}\right)}. \quad (3.9)$$

The pdf of $R$ can then be written as

$$f_R(r) = \frac{\lambda \pi r \exp(-\lambda A_r)}{1 - \exp\left(-\frac{N}{2}\right)}. \quad (3.10)$$

Since the nodes are uniformly distributed, the conditional pdf of $\Theta$, given $R$, is

$$f_{\Theta \mid R}(\theta \mid r) = \frac{1}{\pi}. \quad (3.11)$$

Hence, the joint pdf of $R$ and $\Theta$ is [22]

$$f_{R, \Theta}(r, \theta) = \frac{\lambda r \exp(-\lambda A_r)}{1 - \exp\left(-\frac{N}{2}\right)}. \quad (3.12)$$

Note that this pdf is independent of $\Theta$. 
3.3 Minimal Angular Deviation (MAD) Scheme

In MAD, $X$ transmits to the receiving node with the smallest absolute value of $\theta$, the angle between the destination direction of $X$ and the line joining $X$ and its receiving node. For a given distance, $r$, between $X$ and $Y$, minimizing $\theta$ corresponds to maximizing progress.

The selection of the receiving node for MAD is shown in Figure 3.3.

For MAD, the area $A_e$ of the excluded region, shown as shaded in Figure 3.3, is

$$A_e = A_\theta = r_{max}^2 |\theta|.$$

(3.13)

From Figure 3.3, it follows that the cdf of $|\Theta|$ is given by

$$F_{|\Theta|}(\theta) = \Pr (|\Theta| \leq \theta \mid E_f)$$

$$= \frac{1 - \exp (-\lambda A_\theta)}{1 - \exp (-\frac{N}{2})}.$$  

(3.14)
By taking the derivative of (3.14), the pdf of $|\Theta|$ is obtained as

$$f_{|\theta|}(\theta) = \lambda r_{\text{max}}^2 \frac{\exp(-\lambda A_{\theta})}{1 - \exp(-\frac{\lambda N}{2})}.$$ \hspace{1cm} (3.15)

Because of circular symmetry, we can write

$$f_{\theta}(\theta) = \frac{1}{2} \lambda r_{\text{max}}^2 \frac{\exp(-\lambda A_{\theta})}{1 - \exp(-\frac{\lambda N}{2})}.$$ \hspace{1cm} (3.16)

Since the nodes are uniformly distributed, the conditional pdf of $R$, given $\Theta$, is proportional to $r$ and is given by

$$f_{R|\Theta}(r \mid \theta) = \frac{2r}{r_{\text{max}}^2}.$$ \hspace{1cm} (3.17)

which is independent of $\Theta$. The joint pdf of $R$ and $\Theta$ for MAD can therefore be written as [23], [24]

$$f_{R,\Theta}(r, \theta) = \frac{\lambda r \exp(-\lambda A_{\theta})}{1 - \exp(-\frac{\lambda N}{2})}.$$ \hspace{1cm} (3.18)
Chapter 3. Routing Schemes

The pdf’s \( f_{r, \theta}(r, \theta) \) for MFR, NFP, and MAD, given by (3.7), (3.12), and (3.18), respectively, appear similar; however, it is important to note that the areas \( A_z, A_r, \) and \( A_\theta \) of the excluded regions for the three routing are not the same.

3.4 Angular Deviation to Transmission Range Ratio (ARR) Scheme

In ARR, \( X \) transmits to the receiving node with the largest value of \( \nu_r \), and \( \nu_r \) is defined as

\[
\nu_r = \frac{\cos \theta}{r}.
\]

(3.19)

ARR is a combination of NFP and MAD in that it tries to minimize \( r \) and \( \theta \). As shown in Figure 3.4, the motivation in this scheme is for \( X \) to choose the receiving node \( Y \) whose pair \((r, \theta)\) results in the largest value of \( \nu_r \).

For ARR, we introduce a rv \( T \) which is the reciprocal of \( \nu_r \) defined in (3.19). Since \( T = \frac{r}{\cos \theta} \) represents a circle in polar co-ordinates, it can easily be shown from Figures 3.4 and 3.5 that the locus of points corresponding to a fixed value, \( t \), of \( T \) is a circle of radius \( \frac{t}{2} \). If \( X \) is at the origin, the center of the circle is at \( (\frac{t_0}{2}, 0) \). For \( T = t \leq r_{\text{max}} \), the area \( A_e \) of the excluded region, shown as shaded in Figure 3.4, is

\[
A_e = A_t = \frac{\pi}{4} t^2.
\]

(3.20)

For \( t > r_{\text{max}} \) (see Figure 3.5),

\[
A_e = A_t = r_{\text{max}}^2 \alpha + \int_0^{\frac{\pi}{2}} (t \cos \phi)^2 d\phi
\]

\[
= r_{\text{max}}^2 \alpha + \frac{t^2}{2} \left[ \frac{\pi}{2} - \alpha - \frac{1}{2} \sin(2\alpha) \right]
\]

\[
= \frac{\pi}{4} t^2 + \left( r_{\text{max}}^2 - \frac{t^2}{2} \right) \alpha - \frac{r_{\text{max}}^2}{2} \sqrt{t^2 - r_{\text{max}}^2},
\]

(3.21)
where \( \alpha = \cos^{-1}(\frac{r_{\text{max}}}{t}) \). The cdf of \( T \) is

\[
F_T(t) = \Pr(T \leq t \mid E_f) = \frac{1 - \exp(-\lambda A_t)}{1 - \exp(-\frac{N}{t})},
\]

and the pdf of \( T \) is

\[
f_T(t) = \frac{\lambda A'_t \exp(-\lambda A_t)}{1 - \exp(-\frac{N}{t})}
\]

where \( A'_t \) is the derivative of \( A_t \) with respect to \( t \).

Using the uniform distribution property of the user spatial distribution, we can obtain the conditional pdf of \( \Theta \), given \( T = t \). Let the area of the shaded region in Figure 3.6 be denoted by \( A_{t,\theta} \). Then we have,

\[
A_{t,\theta} = \left(\frac{t}{2}\right)^2 |\theta| + \left(\frac{t}{2}\right)^2 \sin |\theta| \cos \theta
\]
and the partial derivative of $A_{t,\theta}$ can be obtained as

$$\frac{\partial}{\partial t} \frac{\partial}{\partial \theta} A_{t,\theta} = \pm \frac{t}{2} (1 + \cos 2\theta).$$  

(3.25)

For $t \leq r_{\text{max}}$, $\theta$ is in the range $[-\frac{\pi}{2}, \frac{\pi}{2}]$ and the integration of the partial derivative over such a range is

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left( \frac{\partial}{\partial t} \frac{\partial}{\partial \theta} A_{t,\theta} \right) d\theta = \pm \frac{t\pi}{2}.$$  

(3.26)

We can obtain the conditional pdf of $\Theta$, given $T = t \leq r_{\text{max}}$, from (3.25) and then normalized by (3.26) as

$$f_{\Theta|T}(\theta | t) = \frac{1 + \cos 2\theta}{\pi}.$$  

(3.27)
For $t > r_{\text{max}}$, $\theta$ is in the range \([-\frac{\pi}{2}, -\alpha]\) or \([\alpha, \frac{\pi}{2}]\) where $\alpha = \cos^{-1} \left( \frac{r_{\text{max}}}{t} \right)$. We have

$$2 \int_{\alpha}^{\frac{\pi}{2}} \left( \frac{\partial}{\partial t} \frac{\partial}{\partial \theta} A_{t, \theta} \right) d\theta = \pm \frac{t \pi}{2} \mp t \alpha = \frac{r_{\text{max}}}{t} \sqrt{t^2 - r_{\text{max}}^2}. \quad (3.28)$$

Similarly, the conditional pdf of $\Theta$, given $T = t > r_{\text{max}}$, can be obtained as

$$f_{\Theta|T}(\theta | t) = \frac{t^2 (1 + \cos 2\theta)}{\pi t^2 - 2t^2 \alpha - 2r_{\text{max}} \sqrt{t^2 - r_{\text{max}}^2}}. \quad (3.29)$$

The joint pdf of $T$ and $\Theta$ can be obtained from (3.23), and (3.27) or (3.29) as

$$f_{T, \Theta}(t, \theta) = f_{\Theta|T}(\theta | t) f_T(t). \quad (3.30)$$

Finally the joint pdf of $R$ and $\Theta$ for ARR can be derived from (3.30) by changing variable $T$ to variable $R$ using $R = T \cos \Theta$, and is given by [23], [24]

$$f_{R, \Theta}(r, \theta) = \frac{1}{\cos \theta} f_{T, \Theta}(t, \theta). \quad (3.31)$$
3.5 Maximal Transmission Potential (MTP) Scheme

Ideally, $X$ should choose the receiving node which yields the largest average progress in each time slot. Let $p_s$ be the probability of successful reception of $K$ by $Y$ given that $X$ transmits to $Y$. The average progress $\overline{Z}_K$ of $K$ given that $X$ transmits to $Y$ is

$$\overline{Z}_K = z p_s$$

where $z = r \cos \theta$ is the resulting progress if $K$ is successfully received by $Y$. We can express $\overline{Z}_K$ as

$$\overline{Z}_K = z p_{sc} \Pr(E_s,Y),$$

where $p_{sc}$ is the probability of successful reception of $K$ by $Y$ given that $X$ transmits to $Y$ and $Y$ does not transmit, and $\Pr(E_s,Y)$ is the probability that $Y$ does not transmit given that $X$ transmits to $Y$. Simulation results show that the effect of the excluded region can be neglected in calculating $\Pr(E_s,Y)$. Results also show that the probability that any neighboring node of $X$ does not transmit is typically close to $\Pr(E_s,Y)$. Hence, maximizing $\overline{Z}_K$ can be approximated by maximizing $zp_{sc}$. The expressions of $p_{sc}$ are implicitly shown in (4.26), (4.33), and (4.50), for interference models 1.2, 2.1, and 2.2, respectively. In MTP, $X$ transmits to the receiving node $Y$ with the largest transmission potential. The transmission potential $\nu_p$ of $Y$ located at $(r, \theta)$ is defined as

$$\nu_p = z \tilde{p}_{sc}$$

where $\tilde{p}_{sc}$ is an approximation of $p_{sc}$ in which the effect of the excluded region is ignored. We choose the receiving node with the largest $\nu_p$ rather than the largest $\overline{Z}_K$ because it is difficult to work with $\overline{Z}_K$. 
Chapter 3. Routing Schemes

For interference model 1.2, it is difficult to obtain a closed form expression for $\tilde{p}_{sc}$. For interference model 2.1 with the non-fading channel, it can be derived from (4.33) that

$$\tilde{p}_{sc} = \exp \left( -\tilde{N}_t \right),$$  \hspace{1cm} (3.35)

where $\tilde{N}_t$ is obtained from (4.31) by setting $r_e$ to 0. If $c^\frac{1}{2} r > r_{max}$, where $c$ is the capture ratio,

$$\tilde{N}_t = \lambda \pi p_t \left[ r_{max}^2 \left( 1 - \frac{1}{2} p_n \right) + \left( c^\frac{1}{2} r^2 - r_{max}^2 \right) (1 - p_n) \right],$$  \hspace{1cm} (3.36)

where

$$p_n = \exp \left( -\frac{N}{2} \right).$$  \hspace{1cm} (3.37)

If $c^\frac{1}{2} r \leq r_{max}$,

$$\tilde{N}_t = \lambda \pi p_t c^\frac{1}{2} r^2 \left( 1 - \frac{1}{2} p_n \right).$$  \hspace{1cm} (3.38)

For interference model 2.2, we can obtain $\tilde{p}_{sc}$ from (4.53) by setting $r_e$ to 0. For the non-fading case,

$$\tilde{p}_{sc} = \text{erfc} \left( \frac{1}{2} \sqrt{c \pi} \kappa r^2 \right),$$  \hspace{1cm} (3.39)

where $\kappa = \lambda \pi p_t \Pr (E_f)$ and $\text{erfc}(u) \triangleq \frac{2}{\sqrt{\pi}} \int_0^\infty \exp \left( -t^2 \right) dt$ is the complementary error function.

For the Rayleigh fading case, from (4.57) we can obtain

$$\tilde{p}_{sc} = \exp \left\{ -\frac{\pi}{2} \sqrt{c \kappa r^2} \right\}.$$

Note that (3.39) and (3.40) are obtained assuming $\beta = 4$. They show that the receiving node chosen by $X$ depends on $\beta$ and $c$. This is in contrast to the other four routing schemes discussed earlier in which the receiving node chosen does not depend on $\beta$ or $c$. 
Chapter 4

Performance Evaluation of Transmission Strategies

In this chapter, we analyze the throughput $S$ and the normalized average progress $Z_n$ for the four transmission strategies based on the MFR, NFP, MAD, and ARR routing schemes in a multihop PRN as described in Section 2.6. Three interference models 1.2, 2.1, and 2.2 are considered. Simulation results are provided to validate the simplifying assumptions made in the analysis. Simulation results for the transmission strategy based on the MTP routing scheme with interference models 2.1 and 2.2 are also provided for comparison.

Suppose node $X$ transmits a packet $K$ to a neighboring node $Y$. Let $E_{s,Y}$ be the event that $Y$ does not interfere with the transmission from $X$ to $Y$. This event will occur if $Y$ is not in transmission mode or if it is in transmission mode but cannot find a node, including $X$, in its forward direction. The probability that $X$ is not in the forward direction of $Y$ is $\frac{1}{2}$. We can thus write

$$\Pr(E_{s,Y}) = (1 - p_t) + \frac{p_t}{2} \left[ 1 - \Pr(E_f) \right]$$

$$= 1 - p_t + \frac{p_t}{2} \exp \left( -\frac{N}{2} \right) \quad (4.1)$$

where $p_t$ is the probability that a node is in transmission mode. In (4.1), we have ignored the effect of the excluded region since it was found from simulation results that taking the excluded region into account yields only a slight change in $\Pr(E_{s,Y})$, especially for large values of connectivity $N$. In [22], $\Pr(E_{s,Y})$ is approximated by $(1 - p_t)$ and the difference from (4.1) can be substantial for small values of $N$. 

30
Chapter 4. Performance Evaluation of Transmission Strategies

Let $E_s$ be the event that $K$ is successfully received by $Y$, given that $X$ transmits to $Y$. The throughput $S$, defined as the average number of successful transmissions per slot per node, can be expressed as

$$S = p_t \Pr(E_f) \int_0^{r_{\text{max}}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \Pr(E_s) f_{R,\Theta}(r, \theta) \, d\theta \, dr,$$

(4.2)

where $\Pr(E_f)$ of $X$ is given by (3.1), and $f_{R,\Theta}(r, \theta)$ is defined in Chapter 3 for the MFR, NFP, MAD, and ARR routing schemes. The probability $\Pr(E_s)$ is a function of $R$ and $\Theta$, and it also depends on the interference model and routing scheme. The normalized average progress $Z_n$ can be obtained from (4.2) by multiplying the integrand by $\sqrt{\lambda} r \cos \theta$, resulting in

$$Z_n = p_t \sqrt{\lambda} \Pr(E_f) \int_0^{r_{\text{max}}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \Pr(E_s) r \cos \theta f_{R,\Theta}(r, \theta) \, d\theta \, dr.$$

(4.3)

4.1 Interference Model 1.2

In interference model 1.2, a receiving node $Y$ is interfered with if it is within the transmission range of an interfering node. In such a case, $Y$ cannot receive any packet successfully. Since the maximum transmission range is $r_{\text{max}}$, only neighbors of $Y$ can interfere with $Y$. In Section 4.1.1, we analyze the probability of interference to $Y$ by its neighboring nodes. The probability of interference is then used to obtain $S$ and $Z_n$ in Section 4.1.2. The performance analysis of the transmission strategies based on the MFR and NFP routing schemes provided here is more accurate than that given in [22].

4.1.1 Probability of Interference

If the transmission from $X$ to $Y$ is to be successful, every neighbor of $Y$, excluding $X$, must not interfere with $Y$. In this section, we will examine the probability of interference
with $Y$ by a neighbor of $Y$. Let node $M$ be a neighbor of $Y$ and $E_{\text{int}}$ be the event that $M$ will interfere with $Y$ given that $M$ is in transmission mode. Let $p_{\text{int}}$ be the probability of this event which will occur if the transmission range of $M$ is larger than the distance between $M$ and $Y$. Let $E_+$ and $E_-$ denote the events that $Y$ is in $M$'s forward and backward direction, respectively. Since $Y$ is either in $M$'s forward or backward direction with probability $\frac{1}{2}$, i.e., $\Pr(E_+) = \Pr(E_-) = \frac{1}{2}$, we have

$$
p_{\text{int}} = \Pr(E_{\text{int}} | E_+) \Pr(E_+) + \Pr(E_{\text{int}} | E_-) \Pr(E_-)
= \frac{1}{2} \Pr(E_{\text{int}} | E_+) + \frac{1}{2} \Pr(E_{\text{int}} | E_-).
$$

(4.4)

In the following, we assume that $M$ is in transmission mode. Let $Y$ be located at $(r_0, \theta_0)$ in a polar coordinate plan centered at $M$. Let the transmission range of $M$ be $r_t$ so that $M$ will interfere with $Y$ if $r_t \geq r_0$.

### 4.1.1.1 Calculation of $\Pr(E_{\text{int}}|E_+)$

In calculating $\Pr(E_{\text{int}}|E_+)$ for MFR, NFP, MAD, and ARR, we ignore the effect of the excluded region and hence, $Y$ is uniformly located in $M$'s forward direction. The effect of the excluded region complicates the analysis, and simulation results show that it affects $\Pr(E_{\text{int}}|E_+)$ by less than 5%, except for $N \leq 2$.

In MFR, if $Y$ is in $M$'s forward direction, the calculation of the probability of interference is complicated. Consider nodes which are located in the shaded region with area $A_s$ as shown in Figure 4.1(a), and let $Q$ be the node in this region which produces the largest progress if the transmission from $M$ to $Q$ is successful. Denote the progress resulting from a successful transmission of $M$ to $Q$ by $z_Q$. Let $A_i$ be the area of the shaded region shown in Figure 4.1(b). Note that all points in this shaded region have progress larger than $z_Q$. Then $M$ will
not interfere with \( Y \) if and only if there exists a node \( Q \) and no node exists in the region with area \( A_i \). As shown in Figure 4.1, \( A_i \) depends on the location of \( Q \), but in any case it is larger than \( A_n \) and less than \((A_n + 2A_b)\). For simplicity, we assume that

\[
A_i = A_n + A_b
\]  

(4.5)

where

\[
A_n = r_{\text{max}}^2 \left[ \cos^{-1} \left( \frac{r_o}{r_{\text{max}}} \right) - \frac{r_o}{r_{\text{max}}} \sqrt{1 - \left( \frac{r_o}{r_{\text{max}}} \right)^2} \right],
\]

(4.6)

\[
A_b = \frac{1}{2} \left\{ r_{\text{max}}^2 \left[ \cos^{-1} \left( \frac{z_o}{r_{\text{max}}} \right) - \frac{z_o}{r_{\text{max}}} \sqrt{1 - \left( \frac{z_o}{r_{\text{max}}} \right)^2} \right] - A_s - A_n \right\},
\]

(4.7)

where \( z_o = r_o \cos \theta_o \) and

\[
A_s = r_o^2 \left( \theta_o - \frac{1}{2} \sin 2\theta_o \right).
\]

(4.8)

Assuming that \( Y \) is uniformly located in \( M \)'s forward direction, the probability that \( Q \) is in the region with area \( A_s \) is \([1 - \exp(-\lambda A_s)]\) and the probability that no node is in the region

Figure 4.1 Illustration of the calculation of \( \Pr(E_{\text{int}} \mid E_+) \) for MFR.
with area $A_i$ is $\exp(-\lambda A_i)$. We have for MFR [23]

$$
\Pr(E_{\text{int}}|E_+) = \frac{1}{\pi} \frac{1}{r_{\text{max}}} \int_0^{r_{\text{max}}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left\{ 1 - [1 - \exp(-\lambda A_i)] \exp(-\lambda A_i) \right\} \frac{2r_o}{r_{\text{max}}} \, dr_o \, d\theta_o. \quad (4.9)
$$

It might be mentioned that in [22], $\Pr(E_{\text{int}}|E_+)$ is assumed to be 1; this differs from (4.9) by more than 10% for most values of $N$. Simulation results show that the expression in (4.9) gives a fairly good approximation (less than 2% difference, except for $N \leq 4$) to the actual value of $\Pr(E_{\text{int}}|E_+)$.

In NFP, if $Y$ is the nearest node to $M$ in $M$'s forward direction, $M$ will transmit to $Y$ and hence interfere with $Y$. In this case, there cannot be any node in the shaded region as shown in Figure 4.2, and the area of the shaded region is $\frac{1}{2}\pi r_o^2$. If $Y$ is not the nearest node to $M$ in $M$'s forward direction, $M$ will transmit to the nearest node and not interfere with $Y$. The probability of no node, not counting $X$, being located in the shaded region is $\exp\left(-\frac{\lambda \pi r_o^2}{2}\right)$. If there is a node other than $X$ in the shaded region, then $M$ will not interfere with $Y$; otherwise, $M$ will not interfere with $Y$ if and only if $X$ is in the shaded region and hence, the presence of $X$ will result in a decrease in $\Pr(E_{\text{int}}|E_+)$. Since $Y$ is the nearest neighboring node in the forward direction of $X$, the probability that $X$ is located in the shaded region is less than $\frac{1}{2}$. From the simulation results, it was found that the presence of $X$ decreases the value of $\Pr(E_{\text{int}}|E_+)$ by about 25%. Using this value and assuming that $Y$ is uniformly located in $M$'s forward direction, we have for NFP [23],

$$
\Pr(E_{\text{int}}|E_+) = \frac{3}{4} \frac{1}{r_{\text{max}}} \int_0^{r_{\text{max}}} \frac{2r_o}{r_{\text{max}}} \exp\left(-\frac{\lambda \pi r_o^2}{2}\right) \, dr_o
$$

$$
= \frac{3}{2N} \left[ 1 - \exp\left(-\frac{N}{2}\right) \right]. \quad (4.10)
$$
In calculating $\Pr (E_{\text{int}} | E_+)$ in [22], the pdf of the transmission range of $M$ is assumed to be the same as that of $X$. This assumption is valid only if the probability that no node is located in the shaded region, given by $\exp \left( -\frac{\lambda r^2}{2} \right)$, is negligible.

In MAD, if $Y$ is in $M$'s forward direction and there is no node is in the shaded region as shown in Figure 4.3, $M$ will transmit to $Y$ and hence interfere with $Y$. The area of the shaded region is $r_{\text{max}}^2 \theta_0$. If there are nodes in shaded region, $M$ will choose the node with the smallest angle deviation from its destination direction. In this case, the simulation results indicate that the effect of the excluded region is negligible. Therefore, we can assume that the probability of interference averaged over all values of $r_o$ is $\frac{1}{2}$ for any given value of $\theta_0$. The probability of at least one node, not counting $X$, being located in the shaded region is $\left[ 1 - \exp \left( -\lambda r_{\text{max}}^2 |\theta_0| \right) \right]$. We assume that the probability that $X$ is located in the shaded region is $\frac{|\theta_0|}{\pi}$ and the probability of interference, given that only $X$ is located in the shaded region.
region, is $\frac{1}{2}$. Therefore, the probability of interference given that no node, except possibly for $X$, is located in the shaded region is $\left(1 - \frac{|\theta_o|}{2\pi}\right)$. Hence, we have for MAD [23],

$$ \Pr(E_{\text{int}} | E_\downarrow) = \frac{2}{\pi} \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \left\{ \frac{1}{2} \left[ 1 - \exp(-\lambda r_{\text{max}}^2 \theta_o) \right] + \left(1 - \frac{\theta_o}{2\pi} \right) \exp(-\lambda r_{\text{max}}^2 \theta_o) \right\} d\theta_o $$

$$ = \frac{1}{\pi} \int_{0}^{\frac{\pi}{2}} \left[ 1 + \exp(-\lambda r_{\text{max}}^2 \theta_o) - \frac{\theta_o}{\pi} \exp(-\lambda r_{\text{max}}^2 \theta_o) \right] d\theta_o $$

$$ = \frac{1}{2} + \frac{1 - \exp(-\frac{N}{2})}{N} + \frac{1 - \exp(-\frac{N}{2})}{N^2} + \frac{\exp(-\frac{N}{2})}{2N} $$

$$ = \frac{1}{2} + \frac{1}{N} - \frac{\exp(-\frac{N}{2})}{2N} + \frac{1 - \exp(-\frac{N}{2})}{N^2} . \quad (4.11) $$

In ARR, if $Y$ is in $M$'s forward direction, the calculation of the probability of interference is also complicated. Since $M$ chooses the receiving node with the largest value of $u_r$, defined
in (3.19), M may interfere with Y even though there are nodes in the shaded region as shown in Figure 4.4. Simulation results show that if there are nodes in the shaded region, the probability that M will interfere with Y is small. To simplify the analysis, we assume that M will interfere with Y if and only if there is no node in the shaded region. The area, \( A_r \), of the shaded region is given by

\[
A_r = r_o^2 |\theta_o| + \frac{t^2}{2} \left( \sin^{-1} \frac{r_o}{t} - \frac{r_o}{t} \sqrt{1 - \frac{r_o^2}{t^2}} \right). \tag{4.12}
\]

The probability that no node is located in the shaded region is \( \exp(-\lambda A_r) \). Assuming that Y is uniformly located in M’s forward direction, we have for ARR [23],

\[
Pr(E_{\text{int}} | E+) = \int_0^{\frac{r_{\text{max}}}{2}} \int_0^{\frac{\pi}{2}} \exp(-\lambda A_r) \frac{2}{\pi} d\theta_o \frac{2r_o}{r_{\text{max}}^2} dr_o. \tag{4.13}
\]

**4.1.1.2 Calculation of Pr\((E_{\text{int}} | E_-)\)**

As noted in Chapter 3, if node X transmits to its receiving node Y, there is an excluded region in which no node is located. For analyzing the performance of the transmission
strategies, we will make use of the area of the excluded region within the circle of radius $r_{max}$ centered at $Y$. We refer to the excluded region within the circle as a partial excluded region and denote its area by $A_e(r_{max})$. The derivation of $A_e(r_{max})$ for both MAD and ARR is given in Appendix A and for MFR and NFP appears in [22].

If there were no excluded region, the pdf of the distance between $M$ and $Y$ would be $\frac{2r_o}{r_{max}}$ for $0 \leq r_o \leq r_{max}$. Since the excluded region exists near $Y$, the pdf is smaller than $\frac{2r_o}{r_{max}}$ for some small values of $r_o$. In determining $\Pr(E_{int} | E_+)$, we did not consider the excluded region for simplicity. In calculating $\Pr(E_{int}|E_-)$, we model the distribution of the distance between $M$ and $Y$ as

$$f_{R_o}(r_o) = \begin{cases} \frac{3}{r_{max}^2 - \frac{1}{4}r_e^2}, & 0 \leq r_o \leq r_e \\ \frac{2}{r_{max}^2 - \frac{1}{4}r_e^2}, & r_e \leq r_o \leq r_{max} \end{cases}$$

where $r_e = \sqrt{\frac{4}{\pi}A_e}$. This model assumes that the excluded region is represented by one quarter of the circle with radius $r_e$ and centered at $M$. In the analysis in [22], it is assumed that $M$ is uniformly distributed within the range of $Y$; this results in a pdf for $R_o$ given by

$$f_{R_o}(r_o) = \frac{2r_o}{r_{max}^2}, \quad 0 \leq r_o \leq r_{max}.$$  

(4.15)

Simulation results suggest that use of (4.14) yields more accurate results than (4.15) when used in calculating $\Pr(E_{int}|E_-)$. This is because (4.15) ignores the excluded region. In this case, the probability of $M$ being located near $Y$ becomes larger and hence, $\Pr(E_{int}|E_-)$ is also larger. If $Y$ is in the backward direction of $M$, $Y$ will not be an intended receiving node of $M$. In most cases, $X$ is also not an intended receiving node of $M$. The simulation results suggest that the pdf of the transmission range of $M$, $f_{R_t}(r_t)$, is the same as that of $X$, and we make such an assumption in calculating $\Pr(E_{int}|E_-)$.

For MFR, NFP, MAD or ARR we can write

$$\Pr(E_{int} | E_-) = \Pr(E_f) \int_0^{r_{max}} \int_0^{r_t} f_{R_o}(r_o) dr_o f_{R_t}(r_t) dr_t$$

(4.16)
For both MFR and ARR, using (4.14) and (4.16) yields [23]

\[
\Pr (E_{\text{int}} | E_-) = \frac{1 - \exp \left( -\frac{N}{2} \right)}{r_{\text{max}}^2 - \frac{1}{4} r_e^2} \left[ \int_0^{r_e} \int_0^{r_t} \frac{3}{2} r_o \, dr_o \, f_{R_t}(r_t) \, dr_t 
\right.
\]

\[
+ \int_{r_e}^{r_{\text{max}}} \left( \int_{r_e}^{r_t} 2 r_o \, dr_o + \int_0^{r_e} \frac{3}{2} r_o \, dr_o \right) f_{R_t}(r_t) \, dr_t 
\]

\[
= \frac{1 - \exp \left( -\frac{N}{2} \right)}{r_{\text{max}}^2 - \frac{1}{4} r_e^2} \left[ \int_0^{r_e} \int_0^{r_t} \frac{3}{4} r_t^2 f_{R_t}(r_t) \, dr_t 
\right.
\]

\[
+ \int_{r_e}^{r_{\text{max}}} \left( r_t^2 - \frac{1}{4} r_e^2 \right) f_{R_t}(r_t) \, dr_t 
\]

(4.17)

where \( f_{R_t}(r_t) \) can be derived from (3.31) and (3.7) for MFR and ARR, respectively.

For NFP, using (3.12), (4.14) and (4.16), we have [23]

\[
\Pr (E_{\text{int}} | E_-) = \frac{1 - \exp \left( -\frac{N}{2} \right)}{r_{\text{max}}^2 - \frac{1}{4} r_e^2} \left[ \int_0^{r_e} \int_0^{r_t} \frac{3}{2} r_o \, dr_o \, \lambda \pi r_t \exp \left( -\frac{\lambda \pi r_t^2}{2} \right) \frac{1 - \exp \left( -\frac{N}{2} \right)}{1 - \exp \left( -\frac{N}{2} \right)} \, dr_t 
\right.
\]

\[
+ \int_{r_e}^{r_{\text{max}}} \left( \int_{r_e}^{r_t} 2 r_o dr_o + \int_0^{r_e} \frac{3}{2} r_o \, dr_o \right) \lambda \pi r_t \exp \left( -\frac{\lambda \pi r_t^2}{2} \right) \frac{1 - \exp \left( -\frac{N}{2} \right)}{1 - \exp \left( -\frac{N}{2} \right)} \, dr_t 
\]

\[
= \frac{r_{\text{max}}^2}{r_{\text{max}}^2 - \frac{1}{4} r_e^2} \left[ \frac{3}{2 N} - \frac{\exp \left( -\frac{\lambda \pi r_t^2}{2} \right)}{2 N} 
\right.
\]

\[
- \left( 1 + \frac{2}{N} - \frac{r_e^2}{4 r_{\text{max}}^2} \right) \exp \left( -\frac{N}{2} \right) \right] . \tag{4.18}
\]

For MAD, using (3.17), (4.14) and (4.16), we have [23]

\[
\Pr (E_{\text{int}} | E_-) = \frac{1 - \exp \left( -\frac{N}{2} \right)}{r_{\text{max}}^2 \left( r_{\text{max}}^2 - \frac{1}{4} r_e^2 \right)} \left[ \int_0^{r_e} \int_0^{r_t} \frac{3}{2} r_o \, dr_o \, 2 r_t \, dr_t 
\right.
\]

\[
+ \int_{r_e}^{r_{\text{max}}} \left( \int_{r_e}^{r_t} 2 r_o \, dr_o + \int_0^{r_e} \frac{3}{2} r_o \, dr_o \right) 2 r_t \, dr_t 
\]

\]
Chapter 4. Performance Evaluation of Transmission Strategies

\[ = \frac{1 - \exp \left(-\frac{N}{2}\right)}{r_{\text{max}}^{2}} \left( \frac{1}{2} r_{\text{max}}^4 - \frac{1}{4} r_{\text{e}}^2 r_{\text{max}}^2 + \frac{1}{8} r_{\text{e}}^4 \right) \] \quad (4.19)

### 4.1.2 Throughput and Normalized Average Progress

The approach used to obtain the throughput \( S \) and the normalized average progress \( Z_n \) is somewhat similar to that in [22]. Let \( E_{s,M} \) be the event that neighbor \( M \) does not interfere with \( Y \). This event will occur if \( M \) is not in transmission mode or it is in transmission mode but its transmission range is smaller than its distance from \( Y \). Therefore, we have

\[
\Pr (E_{s,M}) = (1 - p_t) + p_t (1 - p_{\text{int}}) \\
= 1 - p_t p_{\text{int}}
\] \quad (4.20)

where \( p_{\text{int}} \) is given by (4.4).

Let \( E_s \) be the event that the transmission from \( X \) to \( Y \) is successful, given that \( X \) transmits to \( Y \), and \( E_i \) be the event that \( Y \) has \( i \) neighbors, not counting \( X \). Then, we have

\[
\Pr (E_s | E_i) = \Pr (E_{s,Y}) \left[ \Pr (E_{s,M}) \right]^i
\] \quad (4.21)

for \( i = 0, 1, 2, \ldots \). The probability \( \Pr (E_{s,Y}) \) of \( Y \) not transmitting is given by (4.1). In (4.21), we have assumed that the events that nodes do not interfere with \( Y \) are independent. Such an assumption greatly simplifies the analysis. It was also verified from simulation results that the assumption only affects the performance slightly.

Let \( A_I(r_{\text{max}}) \) be the area of the non-excluded region within distance \( r_{\text{max}} \) of \( Y \), i.e.,

\[
A_I(r_{\text{max}}) = \pi r_{\text{max}}^2 - A_e(r_{\text{max}})
\] \quad (4.22)
where $A_e(r_{\max})$ is the area of the partial excluded region. The areas $A_e(r_{\max})$ for the MFR, NFP, MAD, and ARR routing schemes are given in Appendix A. We can then write

$$\Pr(E_i) = \int_0^{r_{\max}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \Pr(E_i \mid R, \Theta) f_{R,\Theta}(r, \theta) \, d\theta \, dr, \quad (4.23)$$

where

$$\Pr(E_i \mid R, \Theta) = \left[ \frac{\lambda A_i(r_{\max})}{i!} \right]^i \exp \left[ -\lambda A_i(r_{\max}) \right]$$

and $f_{R,\Theta}(r, \theta)$ for the four routing schemes are given in Chapter 3. We then have

$$\Pr(E_s) = \sum_{i=0}^{\infty} \Pr(E_s \mid E_i) \Pr(E_i)$$

$$= \Pr(E_{s,Y}) \sum_{i=0}^{\infty} \int_0^{r_{\max}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 - p_t p_{int})^i$$

$$\times \left[ \frac{-\lambda A_i(r_{\max})}{i!} \right]^i \exp \left[ -\lambda A_i(r_{\max}) \right] f_{R,\Theta}(r, \theta) \, d\theta \, dr$$

$$= \Pr(E_{s,Y}) \int_0^{r_{\max}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \exp \left[ -p_t p_{int} \lambda A_i(r_{\max}) \right] f_{R,\Theta}(r, \theta) \, d\theta \, dr. \quad (4.25)$$

Using (4.2) and (4.25), we can express the throughput as

$$S = p_t \Pr(E_f) \Pr(E_s)$$

$$= p_t \Pr(E_f) \Pr(E_{s,Y})$$

$$\times \int_0^{r_{\max}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \exp \left[ -p_t p_{int} \lambda A_i(r_{\max}) \right] f_{R,\Theta}(r, \theta) \, d\theta \, dr. \quad (4.26)$$

The normalized average progress can be obtained from (4.26) by multiplying the integrand by $\sqrt{\lambda} r \cos \theta$, resulting in

$$Z_n = p_t \sqrt{\lambda} \Pr(E_f) \Pr(E_{s,Y})$$
Chapter 4. Performance Evaluation of Transmission Strategies

\[ \times \int_0^{\frac{r_{max}}{2}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \exp \left[ -p_t p_{int} \lambda A_l(r_{max}) \right] r \cos \theta f_R, \theta(r, \theta) \, d\theta \, dr. \]  

(4.27)

4.2 Interference Model 2.1

If \( X \) transmits packet \( K \) to \( Y \) which is at a distance \( r \) from \( X \), the transmission from any other node may result in unsuccessful reception of \( K \) by \( Y \). Let the received power of \( K \) at \( Y \) be \( \gamma_t \). If the interference power at \( Y \) by a transmitting node \( M \) exceeds \( \frac{\gamma_t}{c} \), then \( Y \) will not receive \( K \) successfully. Assuming that the channel is non-fading, the received power of \( K \) at \( Y \) is given by (2.1) as \( \frac{1}{r^2} \). Let the distance between \( Y \) and \( M \) be \( r_m \); then \( M \) will interfere with \( Y \) if \( \frac{1}{r_m^2} > \frac{1}{cr^2} \). If \( X \) transmits to \( Y \) and \( Y \) does not transmit, \( Y \) will receive \( K \) successfully if and only if there is no other node transmitting within distance \( \alpha r \) of \( Y \) where \( \alpha = c^\frac{1}{2} \). Such a region is referred to as an interference region; a node in this region, excluding \( X \) and \( Y \), which transmits is referred to as an interfering node.

Let \( A_l(r_c) \) and \( A_e(r_c) \) be the areas of the non-excluded and excluded parts of the interference region within the distance \( r_c \) of \( Y \). No nodes are located in the excluded region. The areas \( A_e(r_c) \) for the MFR, NFP, MAD, and ARR routing schemes are given in Appendix A. Let the areas of the non-excluded part of the interference region within and beyond the distance \( r_{max} \) from \( Y \) be \( A_{l_1} \) and \( A_{l_2} \), as shown in Figure 4.5. If \( \alpha r > r_{max} \), we can show that

\[ A_{l_1} = A_l(r_{max}) \]
\[ = \frac{r_{max}^2 \pi}{4} - A_e(r_{max}) \]

(4.28)

and

\[ A_{l_2} = A_l(\alpha r) - A_l(r_{max}) \]
Figure 4.5 Illustration of the interference region for interference model 2.1.

\[
A_h = \pi (\alpha^2 r^2 - r_{\text{max}}^2) - A_e(\alpha r) + A_e(r_{\text{max}}). \quad (4.29)
\]

If \( \alpha r \leq r_{\text{max}} \), \( A_{h2} = 0 \) and we can write

\[
A_{h1} = A_i(\alpha r) = \alpha^2 r^2 \pi - A_e(\alpha r). \quad (4.30)
\]

The probability that \( Y \) is in the forward direction of its neighboring node is assumed to be \( \frac{1}{2} \) as simulation results show that the effect of the excluded region is negligible in this case. If \( X \) is ignored and \( M \) is a neighbor of \( Y \), the probability that \( M \) has at least one node in its forward direction is \( [1 - \frac{1}{2} \exp \left(-\frac{N}{2}\right)] \). If \( M \) is not a neighbor of \( Y \), such a probability reduces to \( [1 - \exp \left(-\frac{N}{2}\right)] \). Let \( N_t \) be the average number of interfering nodes, i.e.,

\[
N_t = \lambda p_t A_{h1} \left[ 1 - \frac{1}{2} \exp \left(-\frac{N}{2}\right) \right] + \lambda p_t A_{h2} \left[ 1 - \exp \left(-\frac{N}{2}\right) \right]. \quad (4.31)
\]
The probability that there is no interfering node is \( \exp (-N_t) \). Hence, the probability that the transmission from \( X \) to \( Y \) is successful, given that \( X \) transmits to \( Y \), is

\[
Pr(E_s) = Pr(E_{s,Y}) \exp (-N_t),
\]

(4.32)

where \( Pr(E_{s,Y}) \) is given by (4.1).

From (4.2) and (4.32), we can obtain the throughput as

\[
S = p_t \ Pr(E_f) \int_{0}^{r_{max}} \int_{-\pi/2}^{\pi/2} Pr(E_s) f_{R, \Theta}(r, \theta) d\theta dr
\]

\[
= p_t \ Pr(E_f) Pr(E_{s,Y}) \int_{0}^{r_{max}} \int_{-\pi/2}^{\pi/2} f_{R, \Omega}(r, \theta) \exp (-N_t) d\theta dr.
\]

(4.33)

The normalized average progress can be obtained from (4.33) by multiplying the integrand by \( \sqrt{\lambda} r \cos \theta \), resulting in

\[
Z_n = p_t \sqrt{\lambda} Pr(E_f) Pr(E_{s,Y}) \int_{0}^{r_{max}} \int_{-\pi/2}^{\pi/2} f_{R, \Omega}(r, \theta) r \cos \theta \exp (-N_t) d\theta dr.
\]

(4.34)

### 4.3 Interference Model 2.2

In this section, we derive the capture probability for both non-fading and Rayleigh fading channels with interference model 2.2 [47]. The capture probability is then used for evaluating the throughput and normalized average progress.

#### 4.3.1 Capture Probability

If \( X \) transmits \( K \) to \( Y \) which is at a distance \( r \) from \( X \), the transmission from any other node will result in some interference to the reception of \( K \) by \( Y \). Let \( q_i(r, \theta) \) be the capture probability of \( K \) by \( Y \) given that \( R = r \) and \( \Theta = \theta \), and that there are \( i \) transmitting nodes,
excluding \( X \) and \( Y \). These transmitting nodes are referred to as the interfering nodes. Let \( M \) be an interfering node and \( R_m \) denote the distance between \( Y \) and \( M \). We assume that the excluded region can be represented by a circle of radius \( r_e \), centered at \( Y \). Since the interfering nodes are uniformly distributed outside the excluded region, we assume the pdf of \( R_m \) to be

\[
f_{R_m}(r_m) = \frac{2r_m}{r_d^2 - r_e^2}, \quad r_e \leq r_m \leq r_d, \tag{4.35}
\]

where \( r_e = \sqrt{\frac{A_e}{\pi}} \). The areas of the excluded region \( A_e \) for the MFR, NFP, MAD, and ARR routing schemes are given in Chapter 3. The pdf in (4.35), referred to as the uniform spatial pdf, is valid if \( r_d >> r \) (i.e., \( r_d = 10r \)) because the interference to \( Y \) from nodes beyond distance \( r_d \) from \( Y \) will be negligible. Let \( \Gamma_m \) denote the received power at \( Y \) from a transmission by \( M \). The pdf \( f_{\Gamma m}(\gamma_m) \) of \( \Gamma_m \) can be obtained from (2.1) and (4.35). Let \( \Gamma_n \) be the sum of the interference powers from the \( i \) interfering nodes at \( Y \). The pdf \( f_{\Gamma n}^{(i)}(\gamma_n) \) of \( \Gamma_n \) is given by the \( i \)-fold convolution of \( f_{\Gamma m}(\gamma_m) \). Let \( \Gamma_t \) be the received power of \( K \) at \( Y \) and \( f_{\Gamma t}(\gamma_t) \) be the corresponding pdf. The capture probability can be written as

\[
q_i(r, \theta) = \int_0^\infty \int_0^\infty f_{\Gamma t}(\gamma_t) f_{\Gamma n}^{(i)}(\gamma_n) d\gamma_n d\gamma_t
= \int_0^\infty \int_0^{c\gamma_n} f_{\Gamma n}^{(i)}(\gamma_n) f_{\Gamma t}(\gamma_t) d\gamma_t d\gamma_n. \tag{4.36}
\]

We next derive \( q_i \) for both non-fading and Rayleigh fading cases.

In order to get a closed form expression of \( f_{\Gamma n}^{(i)}(\gamma_n) \) for the non-fading channel, we approximate the uniform spatial pdf by the following pdf:

\[
f_{R_m}(r_m) = \frac{2r_m \exp\left(-\frac{r_m^2}{4r_e^2}\right)}{r_d^2 \text{erfc}\left(\frac{\sqrt{\pi}r_m}{2r_e}\right)}, \quad r_e \leq r_m < \infty, \tag{4.37}
\]
where \( \text{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_{u}^{\infty} \exp(-t^2) dt \) is the complementary error function. We refer to the pdf in (4.37) as the bell-shaped spatial pdf; it results from assuming a bell-shaped spatial traffic density [14]. As shown in Figure 4.6, such an approximation is quite accurate for \( r_m << r_d \).

Since the most significant interference powers to \( Y \) are generated from the interfering nodes with the small values of \( r_m \), the bell-shaped spatial pdf is a good approximation to the uniform spatial pdf for obtaining the sum of the interference powers at \( Y \). The pdf of \( \Gamma_m \) can be obtained from (2.1) and (4.37) as

\[
\begin{align*}
    f_{\Gamma_m}(\gamma_m) &= \frac{\gamma_d^{\frac{1}{2}} \exp\left(\frac{\pi \gamma_d}{4 \gamma_m}\right)}{2 \gamma_m^{\frac{3}{2}} \text{erfc}\left(\frac{\sqrt{\pi} \gamma_d^{\frac{1}{2}}}{2 \gamma_m^{\frac{1}{2}}}\right)}, \\
    0 &\leq \gamma_m \leq \gamma_e,
\end{align*}
\]

(4.38)

where \( \gamma_e = \frac{1}{r_e^2} \) and \( \gamma_d = \frac{1}{r_d^2} \). The pdf \( f_{\Gamma_n}^{(i)}(\gamma_n) \) can be obtained using [48, equation (29.3.82)]
and written as

$$f^{(i)}_{\Gamma_n}(\gamma_n) = \begin{cases} \frac{i \sqrt{\gamma_d} \exp \left(-\frac{\pi^2 \gamma_d}{4 \gamma_n} \right)}{2 \gamma_n^{\frac{3}{2}} \left[ \text{erfc} \left( \frac{\sqrt{\gamma_d}}{2 \sqrt{\gamma_n}} \right) \right]^i}, & 0 \leq \gamma_n \leq \gamma_e, \\
\text{no closed form expression}, & \gamma_e < \gamma_n \leq i \gamma_e, \\
0, & i \gamma_e < \gamma_n. \end{cases} \quad (4.39)$$

Since we do not have a closed form expression for $f^{(i)}_{\Gamma_n}(\gamma_n)$ when $\gamma_e < \gamma_n \leq i \gamma_e$, the capture probability is also undetermined when the distance $r$ between $X$ and $Y$ is smaller than $c^{-\frac{1}{4}} r_e$ (or the corresponding received power at $Y$ is larger than $c \gamma_e$). In such a case, we increase the value of $\gamma_e$ in (4.39) to $\frac{1}{c^4}$ which would result in a lower capture probability. However, the probability that $R < c^{-\frac{1}{4}} r_e$ is quite small so that this assumption leads to a good approximation for $q_i(r, \theta)$. The received power of $K$ at $Y$ is $\Gamma_t = \frac{1}{r^2}$ for a given distance $r$ between $X$ and $Y$. Since the received power is a constant for a given value $r$, the pdf of $\Gamma_t$ is a Dirac-delta function. Using (4.36) and (4.39), we can obtain the capture probability for the non-fading channel as

$$q_i(r, \theta) = \int_0^{\frac{1}{c^4 r^2}} f^{(i)}_{\Gamma_n}(\gamma_n) \, d\gamma_n$$

$$= \int_0^{\frac{1}{c^4 r^2}} \frac{i \sqrt{\gamma_d} \exp \left(-\frac{\pi^2 \gamma_d}{4 \gamma_n} \right)}{2 \gamma_n^{\frac{3}{2}} \left[ \text{erfc} \left( \frac{\sqrt{\gamma_d}}{2 \sqrt{\gamma_n}} \right) \right]^i} \, d\gamma_n. \quad (4.40)$$

Letting $\zeta = \gamma_n^{-\frac{1}{2}}$ in (4.40) yields [24], [47]

$$q_i(r, \theta) = \int_{\sqrt{c} r^2}^{\infty} \frac{i \sqrt{\gamma_d} \exp \left(-\frac{\pi^2 \gamma_d \zeta^2}{4} \right)}{\left[ \text{erfc} \left( \frac{\sqrt{\gamma_d}}{2 \sqrt{\gamma_n}} \right) \right]^i} \, d\zeta$$

$$= \frac{\text{erfc} \left( \frac{i \pi^2 \sqrt{c} \gamma_d}{2} \right)}{\left[ \text{erfc} \left( \frac{i \pi \gamma_d}{2 \sqrt{\gamma_n}} \right) \right]^i}$$

$$= \frac{\text{erfc} \left( \frac{i \pi^2 \sqrt{c} \gamma_d}{2 r^2} \right)}{\left[ \text{erfc} \left( \frac{i \pi \gamma_d}{2 r^2} \right) \right]^i}. \quad (4.41)$$
For the Rayleigh fading channel, the average received power of $\Gamma_t$ is $\frac{1}{r^\beta}$ for a given value $r$. From (2.2), the pdf of $\Gamma_t$ is

$$f_{\Gamma_t}(\gamma_t) = r^\beta \exp \left(-r^\beta \gamma_t\right). \tag{4.42}$$

From (4.36) and (4.42), we can write the capture probability as [49]

$$q_t(r, \theta) = \int_0^\infty \int_0^\infty f_{\Gamma_n}^{(1)}(\gamma_n) r^\beta \exp \left(-r^\beta \gamma_t\right) d\gamma_t d\gamma_n$$

$$= \int_0^\infty \exp \left(-c r^\beta \gamma_n\right) f_{\Gamma_n}^{(1)}(\gamma_n) d\gamma_n$$

$$= \mathcal{L} \left\{ f_{\Gamma_n}^{(1)}(\gamma_n) \right\}$$

$$= \left[ \mathcal{L} \left\{ f_{\Gamma_n}^{(1)}(\gamma_n) \right\} \right]^i$$

$$= \left[ \int_0^\infty \exp \left(-c r^\beta \gamma_m\right) f_{\Gamma_m}(\gamma_m) d\gamma_m \right]^i, \tag{4.43}$$

where $\mathcal{L}\{\cdot\}$ denotes the Laplace transform and $f_{\Gamma_m}(\gamma_m) = f_{\Gamma_n}^{(1)}(\gamma_n)$. The pdf of $\Gamma_m$ can be obtained from the pdf of $R_m$ and (2.2) as

$$f_{\Gamma_m}(\gamma_m) = \int_0^\infty r_m^\beta \exp \left(-r_m^\beta \gamma_m\right) f_{R_m}(r_m) dr_m, \tag{4.44}$$

where $f_{R_m}(r_m)$ is as defined in (4.35). Using (4.43) and (4.44), the capture probability can be obtained as [24], [47]

$$q_t(r, \theta) = \left\{ \int_0^\infty \int_0^\infty r_m^\beta \exp \left[-(cr^\beta + r_m^\beta) \gamma_m\right] f_{R_m}(r_m) d\gamma_m dr_m \right\}^i$$

$$= \left[ \int_0^\infty \frac{f_{R_m}(r_m)}{1 + c \left(\frac{r}{r_m}\right)^\beta} dr_m \right]^i.$$
Chapter 4. Performance Evaluation of Transmission Strategies

\[
= \left[ \int_{r_e}^{r_d} \frac{2r_m}{r_d^2 - r_e^2} \frac{1}{1 + c \left( \frac{r}{r_m} \right)^4} dr_m \right]^i
\]
\[
= \left\{ 1 - \frac{\sqrt{c} r^2}{r_d^2 - r_e^2} \left[ \tan^{-1} \left( \frac{r_d^2}{\sqrt{c} r^2} \right) - \tan^{-1} \left( \frac{r_e^2}{\sqrt{c} r^2} \right) \right] \right\}^i. \tag{4.45}
\]

Note that \( q_i(r, \theta) \) depends on \( \theta \) since \( r_e = \sqrt{\frac{A_e}{\pi}} \) and \( A_e \) depends on \( \theta \).

4.3.2 Throughput and Normalized Average Progress

In the case where the uniform spatial pdf is used, let \( A_l \) be the area of the non-excluded region, where the nodes can be located, i.e.,

\[
A_l = r_e^2 \pi - A_e. \tag{4.46}
\]

Let \( N_t \) be the average number of interfering nodes in the non-excluded region with area \( A_l \), i.e.,

\[
N_t = \lambda A_l p_i \Pr (E_f). \tag{4.47}
\]

In (4.47), we have assumed that the probability that any node, excluding \( Y \), has at least one node in its forward direction is \( \Pr (E_f) \). This assumption is valid if the number of neighbors of \( X \) or \( Y \) is much smaller than the total number of nodes. For the case where the bell-shaped spatial pdf is used, we also use the same value of \( N_t \) for comparison. Let \( p_i \) be the probability that there are \( i \) interfering nodes in a given time slot. Then

\[
p_i = \frac{N_t^i \exp (-N_t)}{i!}. \tag{4.48}
\]

The probability that the transmission from \( X \) to \( Y \) is successful, given that \( X \) transmits to \( Y \), is

\[
\Pr (E_s) = \Pr (E_{s,Y}) \sum_{i=0}^{\infty} p_i q_i(r, \theta), \tag{4.49}
\]
where $\Pr(E_{s,Y})$ is given by (4.1).

We can obtain the throughput from (4.2) and (4.49) as

$$S = p_t \Pr(E_f) \int_0^{r_{\text{max}}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \Pr(E_s) f_{R,\Theta}(r, \theta) \, d\theta \, dr$$

$$= p_t \Pr(E_f) \Pr(E_{s,Y}) \int_0^{r_{\text{max}}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f_{R,\Theta}(r, \theta) \sum_{i=0}^{\infty} p_i q_i(r, \theta) \, d\theta \, dr. \quad (4.50)$$

The normalized average progress can be obtained from (4.50) by multiplying the integrand by $\sqrt{\lambda}r \cos \theta$, resulting in

$$Z_n = p_t \sqrt{\lambda} \Pr(E_f) \Pr(E_{s,Y}) \int_0^{r_{\text{max}}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f_{R,\Theta}(r, \theta) r \cos \theta \sum_{i=0}^{\infty} p_i q_i(r, \theta) \, d\theta \, dr. \quad (4.51)$$

The summation in both (4.50) and (4.51) for the non-fading case can be obtained from (4.41) and (4.48) as

$$\sum_{i=0}^{\infty} p_i q_i(r, \theta) = \sum_{i=0}^{\infty} \frac{N_i^*}{i!} \frac{\exp(-N_i^*)}{\sqrt{\pi}} \left[ \text{erfc}\left(\frac{\sqrt{c}r^2}{2r_d^2}\right) \right]^i. \quad (4.52)$$

It is shown in Appendix B that if $r_d \to \infty$, the summation can be reduced to

$$\sum_{i=0}^{\infty} p_i q_i(r, \theta) = \exp\left(\kappa r_d^2\right) \text{erfc}\left(\frac{1}{2}\sqrt{c\pi \kappa r^2}\right) \quad (4.53)$$

where $\kappa = \lambda \pi p_i \Pr(E_f)$. For the Rayleigh fading case, the summation can be obtained from (4.45) and (4.48) as

$$\sum_{i=0}^{\infty} p_i q_i(r, \theta) = \sum_{i=0}^{\infty} \frac{N_i^*}{i!} \frac{\exp(-N_i^*)}{\sqrt{\pi}} \left\{ 1 - \frac{\sqrt{c}r^2}{r_d^2 - r_c^2} \right\}^i$$

$$\times \left[ \tan^{-1}\left(\frac{r_d^2}{\sqrt{c}r_c^2}\right) - \tan^{-1}\left(\frac{r_c^2}{\sqrt{c}r_c^2}\right) \right] \right\}^i$$

$$= \exp\left\{ -\frac{\sqrt{c}N_i^* r^2}{r_d^2 - r_c^2} \left[ \tan^{-1}\left(\frac{r_d^2}{\sqrt{c}r_c^2}\right) - \tan^{-1}\left(\frac{r_c^2}{\sqrt{c}r_c^2}\right) \right] \right\}. \quad (4.54)$$
From (4.47), we have

\[ N_t = \lambda \left( r_d^2 \pi - A_e \right) p_t \Pr \left( E_f \right). \]  

(4.55)

If \( r_d \to \infty \), we have

\[ \lim_{r_d \to \infty} \frac{N_t}{r_d^2} = \kappa. \]  

(4.56)

Hence, (4.54) can be reduced to

\[ \sum_{i=0}^{\infty} p_i q_i(r, \theta) = \exp \left\{ -\sqrt{c} \kappa r^2 \left[ \frac{\pi}{2} - \tan^{-1} \left( \frac{r_e^2}{\sqrt{r^2 - r_e^2}} \right) \right] \right\} \]  

(4.57)

for \( r_d \to \infty \).

### 4.4 Analytic and Simulation Results

In this section, the terms MFR, NFP, MAD, ARR, and MTP are used to refer to the transmission strategies based on the corresponding routing schemes. A number of simplifying assumptions are used in the analysis of the throughput \( S \) and normalized average progress \( Z_n \) for MFR, NFP, MAD, and ARR with different interference models discussed in the previous sections of this chapter. In this section, the validity of these assumptions is verified by comparing the analytic results with those obtained from a Monte-Carlo simulation of the network model with interference models 1.2, 2.1, and 2.2. An optimal value of \( p_t \) (obtained by searching in \( p_t \) steps of 0.01) is chosen which maximizes \( Z_n \) for a given routing scheme and a given connectivity \( N \). It might be noted that the optimal \( p_t \) value generally decreases with increasing \( N \). Throughout this thesis, the 95% confidence intervals of the simulation results are within ±2% of the average values shown. In the figures to be presented, the analytic and simulation results are represented by curves and symbols, respectively. The simulation program is outlined in Appendix C.
In Sections 4.4.1 to 4.4.3, the nodes are assumed to be randomly distributed. Hence, the results in these sections can be interpreted as the performance for a mobile network or the average performance for a network in which the nodes are stationary, but their locations are chosen at random. In Section 4.4.4, we examine the performances of the five transmission strategies in two networks in which nodes are regularly located.

4.4.1 Interference Model 1.2

In this section, analytic and simulation results for MFR, NFP, MAD, and ARR with interference model 1.2 are given. Figure 4.7 shows $Z_n$ as a function of connectivity, $N$, for the four transmission strategies; the $Z_n$ values are obtained using either (4.27) or simulation. The corresponding throughput curves are shown in Figure 4.8. For all four transmission strategies, the analytic and simulation results agree quite closely over the range of connectivity values considered.

The greatest difference between the analytic and simulation results for $Z_n$ occurs at small values of $N$. For $N \geq 3$, the difference for MAD is less than 2%, and for MFR, NFP, and ARR is less than 5%. For comparison, the analytic results provided in [22] for MFR and NFP are significantly smaller than the simulation results. For MFR, the difference is in the range of 10% to 30% depending on the value of $N$. The difference is larger for a smaller value of $N$. For NFP, the difference is about 10% for $N \geq 3$. From Figure 4.7, it can be seen that MAD always performs better than MFR, and that $Z_n$ of both strategies increases first and then decreases as $N$ increases. The reason is that in MFR and MAD, the transmission range is not reduced with increasing traffic so that these two strategies do not cope well with the increasing interference which results from a larger value of $N$. For NFP and ARR, $Z_n$ increases and then stays fairly constant as $N$ increases. This is because the transmission range in NFP and ARR decreases as $N$ increases. The maximum value of $Z_n$ for MFR is
0.062 and occurs at $N = 6$. The maximum value of $Z_n$ for MAD occurs at $N = 7$ and is 0.069, which is about 15% better than that for NFP. For MAD, $Z_n$ degrades for $N \geq 7$, and becomes worse than NFP for $N \geq 16$. For NFP, $Z_n$ stays at the maximum value of 0.060 for $N \geq 9$. It can also be seen that ARR performs better than NFP by over 20% for $N \geq 8$. The improvement comes about mostly from those cases in which the node transmits to the second nearest neighbor (with a smaller value of $\theta$). For ARR, $Z_n$ stays at about 0.073 for $N \geq 11$.

Figure 4.8 shows the throughput curves for the four transmission strategies. Figure 4.8 is somewhat similar to Figure 4.7, and NFP has the largest throughput. The reason is that the transmission range in NFP is always the smallest among the four transmission strategies and hence, the interference is also the least for NFP.

4.4.2 Interference Model 2.1

In this section, the analytic and simulation results for MFR, NFP, MAD, and ARR with interference model 2.1 are given. The simulation results for MTP are also provided for comparison. The value of $c = 4$ is used [4], [14] for both analytic and simulation results. Figure 4.9 shows $Z_n$ as a function of $N$ for the five transmission strategies as obtained using either (4.34) or simulation. The corresponding throughput curves are shown in Figure 4.10. For MFR, NFP, MAD, and ARR, the analytic and simulation results agree quite closely over the range of connectivity values considered. The greatest difference between the analytic and simulation results of $Z_n$ occurs at small values of $N$.

For $N \geq 5$, the difference between the analytic and simulation results for MFR, NFP, MAD, and ARR is less than 2%. Similar to the case of interference model 1.2, Figure 4.9 shows that MAD always performs better than MFR. The maximum value of $Z_n$ for MFR is 0.40 and occurs at $N = 4$. The maximum value of $Z_n$ for MAD also occurs at $N = 4$ and is 0.042. This is over 10% better than that for NFP, but the performance of MAD degrades
for $N \geq 5$. For $N \geq 12$, MAD becomes worse than NFP. The reason is that in MAD the received power of $K$ at $Y$ does not increase with the increasing interference which results from a larger value of $N$. It can also be seen that ARR performs better than NFP by over 13%, and MTP performs better than ARR by about 10% for $N \geq 6$. For $N \geq 10$, $Z_n$ stays at about 0.036, 0.041, and 0.045 for NFP, ARR, and MTP, respectively.

Figure 4.10 shows the throughput curves for the five transmission strategies. Figure 4.10 is similar to Figure 4.8 in which NFP has the largest throughput. The reason is that the received power of $K$ at $Y$ in NFP is always the largest among the five transmission strategies and hence, the probability of capture is also the largest for NFP.

### 4.4.3 Interference Model 2.2

In this section, the analytic and simulation results for MFR, NFP, MAD, and ARR with interference model 2.2 are given. The simulation results for MTP are also provided for comparison. The value of $r_d$ is assumed to be $10r_{\text{max}}$ in the simulation. The simulation indicates that there is little difference in the results by increasing the value of $r_d$ beyond $10r_{\text{max}}$. As in the previous section, the value of $c=4$ is used.

Figure 4.11 shows $Z_n$ as a function of $N$ for the five transmission strategies in a non-fading channel as obtained using either (4.51) or simulation. The corresponding throughput curves are shown in Figure 4.12. For MFR, NFP, MAD, and ARR, the analytic and simulation results agree quite closely over the range of connectivity values considered. The greatest difference between the analytic and simulation results for $Z_n$ occurs at small values of $N$. For $N \geq 4$, the difference is less than 2%. From Figure 4.11, it can be seen that $Z_n$ of both MFR and MAD decreases as $N$ increases while the performance of NFP, ARR, and MTP does not change much. The reason is that in NFP, ARR, and MTP, the average received signal power increases (or the average distance between $X$ and $Y$ decreases) with
the increasing interference which results from a larger value of $N$. It can also be seen that
ARR performs better than NFP by about 10%, and MTP performs better than ARR by about
13% for $N \geq 3$. MFR, MAD, and ARR have a maximum value of $Z_n$ at $N = 3$ and is
about 0.032; for NFP, the maximum value also occurs at $N = 3$ and is about 0.029. The
maximum value of $Z_n$ for MTP is about 0.034 and occurs at $N = 4$. Figure 4.11 shows
that ARR is better than MFR, NFP, and MAD for $N \geq 6$, and MTP is the best among the
cfive transmission strategies for all values of $N$. As noted in Section 3.5, the receiving node
$Y$ chosen by $X$ in MTP depends on $\beta$ and $c$. The values of $\beta$ and $c$ in the expression for
$\tilde{p}_{sc}$ in (3.39) were set to 4. In the actual system, $\beta$ and $c$ may be different from 4. The
effect of this difference on the performance of MTP was studied by using $\beta = 3, 4, \text{ and } 5$
and $c = 1, 4, \text{ and } 10$. In all cases, the relative performance improvement of MTP over the
other four transmission strategies was about the same. This conclusion was also found to
hold for interference model 2.1.

Figure 4.13 shows $Z_n$ for the five transmission strategies in a Rayleigh fading channel.
The corresponding throughput curves are shown in Figure 4.14. Compared to the case of no
fading, $Z_n$ of all five transmission strategies is lowered by about 5 to 11%, depending on
the value of $N$. The throughput curves of these transmission strategies also decrease in the
presence of Rayleigh fading. This is in contrast to a contention limited single-hop system
in which fading does not generally result in a poorer performance [14], [15]. Simulation
results also indicate that the use of (3.39) instead of (3.40) for $\tilde{p}_{sc}$ in choosing the receiving
node in MTP has little effect on the performance.

The shapes of the $Z_n$ curves for a given transmission strategy are similar for interference
models 1.2, 2.1, and 2.2. The relative performances of the transmission strategies are also
about the same for the three interference models. Note that for a given transmission strategy
with a given value of $N$, $Z_n$ is largest for interference model 1.2 and smallest for interference model 2.2. The value of $N$ which maximizes $Z_n$ decreases as we change from interference model 1.2 to 2.1 to 2.2.

### 4.4.4 Regular Structure Networks

In the previous sections, we have examined the performance for MFR, NFP, MAD, ARR, and MTP in a randomly distributed network with different interference models. In this section, we examine the performance of the five transmission strategies in two regular structure networks with interference model 2.2 and no fading. Figures 4.15 and 4.16 show the distributions of nodes in a square grid network and a hexagonal grid network, respectively. The network assumptions are the same as those in Section 2.6, except that the nodes are now located according to some deterministic pattern.

The results in this section are obtained by simulation. If $X$ transmits packet $K$ to $Y$, only a certain number of combinations of interfering nodes can cause unsuccessful packet reception by $Y$. The probability of successful reception by $Y$ can be determined in a brute-force way by finding all such combinations. If the distance between $X$ and $Y$ is large, the number of such combinations becomes very large. Except for MFR and MAD with larger values of $N$, the simulation results were verified by this brute-force method. Figure 4.17 shows $Z_n$ of the five transmission strategies in a square grid network. The corresponding throughput curves are shown in Figure 4.18. The maximum value of $Z_n$ for MFR and MAD is about 0.038 at the lowest connectivity value of 5 corresponding to a node and its four nearest neighboring nodes. The value of $Z_n$ decreases as $N$ increases. For NFP and ARR, $Z_n$ is independent of $N$. The value of $Z_n$ remains at about 0.027 and 0.038 for NFP and ARR, respectively, and MTP performs the same as ARR. For NFP, $X$ chooses one of the nearest neighboring nodes in the forward direction regardless of the resulting progress. For ARR and MTP, $X$
chooses the neighboring node in the forward direction with the smallest angle deviation from the destination direction. Since \( X \) always transmits to its nearest neighboring node in NFP, ARR, and MTP, the received power at the receiving node is the same and hence, the capture probability as well as the throughput is the same. The substantial improvement in \( Z_n \) for ARR and MTP over NFP is due to the larger resulting progress.

Figure 4.19 shows \( Z_n \) for the five transmission strategies in a hexagonal grid network. The corresponding throughput curves are shown in Figure 4.20. The transmission strategies behave relatively the same in both regular structure networks. However, they perform better in the hexagonal grid network. This is because a transmitting node in the hexagonal grid network has more choices in selecting a receiving node at a given distance. The maximum value of \( Z_n \) for MFR, MAD, ARR, and MTP is about 0.045, and for NFP is about 0.039.
Figure 4.7 Comparison of normalized average progress for MFR, NFP, MAD, and ARR with interference model 1.2.

Figure 4.8 Comparison of throughput for MFR, NFP, MAD, and ARR with interference model 1.2. Legend is the same as in Figure 4.7.
Chapter 4. Performance Evaluation of Transmission Strategies

Figure 4.9 Comparison of normalized average progress for MFR, NFP, MAD, ARR, and MTP with interference model 2.1.

Figure 4.10 Comparison of throughput for MFR, NFP, MAD, ARR, and MTP with interference model 2.1. Legend is the same as in Figure 4.9.
Figure 4.11 Comparison of normalized average progress for MFR, NFP, MAD, ARR, and MTP with interference model 2.2 in a non-fading channel.

Figure 4.12 Comparison of throughput for MFR, NFP, MAD, ARR, and MTP with interference model 2.2 in a non-fading channel. Legend is the same as in Figure 4.11.
Figure 4.13 Comparison of normalized average progress for MFR, NFP, MAD, ARR, and MTP with interference model 2.2 in a Rayleigh fading channel. Legend is the same as in Figure 4.11.

Figure 4.14 Comparison of throughput for MFR, NFP, MAD, ARR, and MTP with interference model 2.2 in a Rayleigh fading channel. Legend is the same as in Figure 4.11.
Chapter 4. Performance Evaluation of Transmission Strategies

Figure 4.15 Distribution of nodes in a square grid network.

Figure 4.16 Distribution of nodes in a hexagonal grid network.
Figure 4.17 Comparison of normalized average progress for MFR, NFP, MAD, ARR, and MTP with interference model 2.2 in a square grid network.

Figure 4.18 Comparison of throughput for MFR, NFP, MAD, ARR, and MTP with interference model 2.2 in a square grid network. Legend is the same as in Figure 4.17.
Chapter 4. Performance Evaluation of Transmission Strategies

Figure 4.19 Comparison of normalized average progress for MFR, NFP, MAD, ARR, and MTP with interference model 2.2 in a hexagonal grid network. Legend is the same as in Figure 4.17.

Figure 4.20 Comparison of throughput for MFR, NFP, MAD, ARR, and MTP with interference model 2.2 in a hexagonal grid network. Legend is the same as in Figure 4.17.
Chapter 5

Multiple Receiver Antenna Selection

In this chapter, we consider a multihop PRN in which each node has $N_r$ receiving antennas. A Rayleigh fading environment with interference model 2.2 is used in this study. In Section 5.1, a multiple receiver antenna system is described. The capture probability for the multiple receiver antenna system is derived in Section 5.2, and its performance is analyzed in Section 5.3. In Section 5.4, the analytic and simulation results of the four transmission strategies based on MFR, NFP, MAD, and ARR routing schemes are provided. Simulation results of the transmission strategy based on the MTP routing scheme are also provided. Results show that the performance of the five transmission strategies can be improved substantially if each node has $N_r > 1$ antennas.

5.1 Multiple Receiver Antenna System

We consider a multihop PRN in which nodes make use of $N_r$ receiver antennas so as to provide diversity reception [50]. It is assumed that each node has only one radio receiver and hence, a node can only decode the signal at one of the $N_r$ antennas in a given time slot. If the $N_r$ antennas are separated from one another by more than half a wave length, the received signal powers at the antennas are more or less uncorrelated [12] in a Rayleigh fading environment. The receiving node chooses which antenna signal to decode based on its knowledge of the signal power received at each of the $N_r$ antennas. It acquires this knowledge by briefly monitoring the signal strengths at each antenna in turn at the beginning of each time slot. Ideally, the receiving node would want to choose the antenna which yields the largest normalized average progress $Z_n$ of the packets intended for it. Unfortunately such
an optimal choice appears difficult to implement. Simulation results show that $Z_n$ increases and then decreases gradually (for MFR) or stays constant (for NFP) with the received power $\Gamma$ at the receiving node. They also indicate that $P_r (\Gamma \leq \gamma^*)$, where $\gamma^*$ is the received power which yields the maximum $Z_n$, is larger than 0.8 for all five transmission strategies. Hence, a scheme in which the receiving node decodes the signal from the antenna with the largest power is close to optimal in most cases. We will examine the performance gain from using such an antenna selection algorithm in Section 5.4.

5.2 Capture Probability

Suppose $X$ transmits packet $K$ to $Y$ with $R = r$ and $\Theta = \theta$. As in Section 4.3.1, let $q_i(r, \theta)$ be the capture probability of $K$ by $Y$ given that there are $i$ interfering nodes. The capture probability will be used to analyze the performance of the transmission strategies in Section 5.3. The exact analysis of the capture probability of the transmission strategies in the multiple receiver antenna system described in the previous section is difficult. In this section, we derive the capture probability of the multiple receiver antenna system assuming that $Y$ decodes the signal from the antenna with the largest received power of $K$ from $X$. This is in contrast to the proposed system in which $Y$ decodes the antenna with the largest received power (not necessary the one with the largest received power of $K$). It is expected that such an assumption leads to optimistic results.

The pdf of the received power, $\Gamma_i$, of $K$ at each of the $N_r$ antennas is given by (4.42). Let $\Gamma_s$ be the largest received power among the received powers of $K$ at the $N_r$ antennas. The pdf of $\Gamma_s$ is given by [51, Theorem 3.6]

$$f_{\Gamma_s}(\gamma_s) = N_r \left[ F_{\Gamma_s}(\gamma_s) \right]^{N_r - 1} f_{\Gamma_s}(\gamma_s)$$ (5.1)
where

\[ F_{\Gamma_t}(\gamma_t) = 1 - \exp \left(-\frac{\gamma_t}{\mu} \right) \]  

(5.2)

is the cdf of \( \Gamma_t \) and \( \mu = \frac{1}{r^s} \). From (4.42), (5.1), and (5.2), the pdf of \( \Gamma_s \) can be written as

\[
 f_{\Gamma_s}(\gamma_s) = \frac{N_r}{\mu} \exp \left(-\frac{\gamma_s}{\mu} \right) \left[ 1 - \exp \left(-\frac{\gamma_s}{\mu} \right) \right]^{N_r-1} 
 = \frac{N_r}{\mu} \sum_{j=1}^{N_r} (-1)^{j+1} \binom{N_r-1}{j-1} \exp \left(-\frac{j\gamma_s}{\mu} \right).
\]

(5.3)

Let the distance between \( Y \) and an interfering node \( M \) be denoted by \( R_m \). The excluded region with area \( A_e \), modelled in the same way as in Section 4.3.1, is represented by a circle of radius \( r_e \), where \( r_e = \sqrt{\frac{A_e}{\pi}} \), centered at \( Y \). Node \( M \) is uniformly distributed in the non-excluded region, and the pdf \( f_{R_m}(r_m) \) of \( R_m \) is as defined in (4.35). Only the interfering nodes within distance \( r_d \), where \( r_d >> r \), of \( Y \) are considered because the interference to \( Y \) from nodes beyond distance \( r_d \) from \( Y \) is negligible. Let \( \Gamma_m \) denote the received power at \( Y \) from a transmission by \( M \), and \( \Gamma_n \) be the sum of the interference powers from the \( i \) interfering nodes at \( Y \). The pdf \( f_{\Gamma_m}^{(i)}(\gamma_m) \) of \( \Gamma_n \) is given by the \( i \)-fold convolution of \( f_{\Gamma_m}(\gamma_m) \). Similar to the case of \( N_r = 1 \) in (4.43), the capture probability for \( N_r \geq 1 \) can be written as

\[
 q_i(r, \theta) = \int_0^{\infty} \int_{c_{\gamma_n}}^{\infty} f_{\Gamma_m}^{(i)}(\gamma_m) f_{\Gamma_s}(\gamma_s) \, d\gamma_s \, d\gamma_n 
 = \frac{N_r}{\mu} \sum_{j=1}^{N_r} (-1)^{j+1} \binom{N_r-1}{j-1} \int_0^{\infty} \int_{c_{\gamma_n}}^{\infty} f_{\Gamma_m}^{(i)}(\gamma_m) \exp \left(-\frac{j\gamma_s}{\mu} \right) \, d\gamma_s \, d\gamma_n 
 = \sum_{j=1}^{N_r} (-1)^{j+1} \frac{N_r}{j} \binom{N_r-1}{j-1} \int_0^{\infty} \exp \left(-j c r^\beta \gamma_m \right) f_{\Gamma_m}^{(i)}(\gamma_m) \, d\gamma_m 
 = \sum_{j=1}^{N_r} (-1)^{j+1} \frac{N_r}{j} \binom{N_r-1}{j-1} \left[ \int_0^{\infty} \exp \left(-j c r^\beta \gamma_m \right) f_{\Gamma_m}(\gamma_m) \, d\gamma_m \right]^i, (5.4)
\]
Chapter 5. Multiple Receiver Antenna Selection

where \( f_{R_m}(\gamma_m) = f_{T_n}^{(1)}(\gamma_n) \) is given by (4.44). Similar to (4.43), the last step in (5.4) is obtained by applying the Laplace transform. Using (4.44) and (5.4), the capture probability can be obtained as [50]

\[
q_i(r, \theta) = \sum_{j=1}^{N_r} (-1)^{j+1} \frac{N_r}{j} \left( \frac{N_r - 1}{j - 1} \right) \\
\times \left\{ \int_0^\infty \int_0^\infty r_m^\beta \exp \left[ - \left( j c r^\beta + r_m^\beta \right) \gamma_m \right] f_{R_m}(r_m) \, d\gamma_m \, dr_m \right\}^i \\
= \sum_{j=1}^{N_r} (-1)^{j+1} \frac{N_r}{j} \left( \frac{N_r - 1}{j - 1} \right) \left[ \int_0^\infty \frac{f_{R_m}(r_m)}{1 + j c \left( \frac{r_m}{r_e} \right)^\beta} \, dr_m \right]^i \\
= \sum_{j=1}^{N_r} (-1)^{j+1} \frac{N_r}{j} \left( \frac{N_r - 1}{j - 1} \right) \left\{ 1 - \frac{2 r_m}{r_d^2 - r_e^2} \right\} \\
\times \left[ \tan^{-1} \left( \frac{r_d^2}{\sqrt{j c r^2}} \right) - \tan^{-1} \left( \frac{r_e^2}{\sqrt{j c r^2}} \right) \right]^i. \tag{5.5} \\
\]

Note that \( q_i(r, \theta) \) depends on \( \theta \) since \( r_e = \sqrt{\frac{A_e}{\pi}} \) and \( A_e \) depends on \( \theta \).

5.3 Throughput and Normalized Average Progress

Similar to the case of \( N_r = 1 \) in Section 4.3.2, the throughput for \( N_r \geq 1 \) can be expressed as

\[
S = p_t \Pr(E_f) \Pr(E_{s,Y}) \int_0^{r_{\text{max}}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f_{R,\Theta}(r, \theta) \sum_{i=0}^{\infty} p_i q_i(r, \theta) \, d\theta \, dr, \tag{5.6} \\
\]

where \( \Pr(E_f) \) is the probability of \( X \) finding a neighboring node in its forward direction as given by (3.1), \( \Pr(E_{s,Y}) \) is the probability of \( Y \) not transmitting as given by (4.1), and
$p_i$ is the probability of $i$ interfering nodes in the non-excluded region as given by (4.48). The normalized average progress can be obtained from (5.6) by multiplying the integrand by $\sqrt{\lambda} r \cos \theta$, resulting in

$$Z_n = p_t \sqrt{\lambda} \Pr (E_f) \Pr (E_{s,Y}) \int_0^{r_{\max}} \int_{-\frac{\pi}{2}}^\frac{\pi}{2} f_{R_r}(r, \theta) r \cos \theta \sum_{i=0}^{\infty} p_i q_i(r, \theta) d\theta dr. \quad (5.7)$$

Similar to (4.54) for the case of $N_r = 1$, the summation in (5.6) and (5.7) can be simplified as

$$\sum_{i=0}^{\infty} p_i q_i(r, \theta) = \sum_{j=1}^{N_r} (-1)^{j+1} \frac{N_r}{j} \left( \frac{N_r - 1}{j - 1} \right) \exp \left\{ -\frac{\sqrt{\lambda} c N_t r^2}{r_d^2 - r_e^2} \right\}$$

$$\times \left[ \tan^{-1} \left( \frac{r_d^2}{\sqrt{\lambda} c r^2} \right) - \tan^{-1} \left( \frac{r_e^2}{\sqrt{\lambda} c r^2} \right) \right], \quad (5.8)$$

where $N_t$, as given by (4.47), is the average number of interfering nodes in the non-excluded region. If $r_d \to \infty$, (5.8) reduces to

$$\sum_{i=0}^{\infty} p_i q_i(r, \theta) = \sum_{j=1}^{N_r} (-1)^{j+1} \frac{N_r}{j} \left( \frac{N_r - 1}{j - 1} \right) \exp \left\{ -\sqrt{\lambda} c \frac{\pi}{2} p_t \Pr (E_f) r^2 \right\}$$

$$\times \left[ \frac{\pi}{2} - \tan^{-1} \left( \frac{r_e^2}{\sqrt{\lambda} c r^2} \right) \right]. \quad (5.9)$$

### 5.4 Analytic and Simulation Results

In this section, the terms MFR, NFP, MAD, ARR, and MTP are used to refer to the transmission strategies based on the corresponding routing schemes. Similar to the case of $N_r = 1$ in Section 4.3.2, a number of simplifying assumptions are used in the analysis of the throughput $S$ and normalized average progress $Z_n$ for MFR, NFP, MAD, and ARR. The validity of these assumptions is verified by comparing the analytic results with those obtained from a Monte-Carlo simulation of the model described in Section 2.6. Simulation
results for MTP are also provided for comparison. The value of \( r_d \) is assumed to be 10 \( r_{max} \) in the simulation. Based on the simulation results, it appears that there is little difference in the results by increasing the value of \( r_d \) beyond 10 \( r_{max} \). As in Sections 4.4.2 and 4.4.3, the value of \( c=4 \) is used. An optimal value of \( p_t \) (obtained by searching in \( p_t \) steps of 0.01) is chosen which maximizes \( Z_n \) for a given routing scheme and a given connectivity \( N \). The optimal \( p_t \) value generally decreases with increasing \( N \).

Figure 5.1 shows \( Z_n \) as a function of \( N \) for MFR with different values of \( N_r \) as obtained using either (5.7) or simulation. The \( Z_n \) values for NFP, MAD, ARR, and MTP are shown in Figures 5.3, 5.5, 5.7, and 5.9, respectively. The corresponding throughput curves are shown in Figures 5.2, 5.4, 5.6, 5.8, and 5.10. For MFR, NFP, MAD, and ARR, the analytic and simulation results for \( N_r \leq 2 \) agree quite closely over the range of connectivity values considered. The simulation results are always smaller than the corresponding analytic results because we assume that \( Y \) decodes the signal with the strongest received power of \( K \) among the \( N_r \) antennas in obtaining the analytic results. However, \( Y \) actually decodes the signal from the antenna with the largest received power (of \( K \) and interference) in the simulation. Simulation results ran assuming that \( Y \) decodes the signal from the antenna with the largest received power of \( K \) agree well with the analytic results. The antenna with the largest received power is not necessarily the one with the largest received power of \( K \). The chance that the former antenna coincides with the latter antenna becomes smaller as \( N_r \) increases. Hence, the difference between analytic and simulation results becomes larger as \( N_r \) increases. The greatest difference for \( Z_n \) occurs at small values of \( N \). For the four transmission strategies with \( N \geq 4 \), the differences are about 1\%, 3\%, and 7\% for \( N_r = 1, 2, \) and 4, respectively.

For all five transmission strategies, the simulation results show that \( S \) and \( Z_n \) can be
improved by about 25%, and 40% if $N_r$ is increased from 1 to 2, and 4, respectively. The results show that $Z_n$ for both MFR and MAD decreases as $N$ increases while the performance of NFP, ARR, and MTP does not change much. The reason is that in NFP, ARR, and MTP the average received signal power increases (because the average distance between $X$ and $Y$ decreases) with the increasing interference which results from a larger value of $N$. It can also be seen that ARR has a better performance than MFR, NFP, and MAD for almost all values of $N$ and $N_r$, and MTP performs better than ARR by about 10%.
Chapter 5. Multiple Receiver Antenna Selection

Figure 5.1 Normalized average progress for MFR with various $N_r$ receiver antennas. Legend is the same as in Figure 5.2.

Figure 5.2 Throughput for MFR with various $N_r$ receiver antennas.
Chapter 5. Multiple Receiver Antenna Selection

Figure 5.3 Normalized average progress for NFP with various $N_r$ receiver antennas. Legend is the same as in Figure 5.2.

Figure 5.4 Throughput for NFP with various $N_r$ receiver antennas. Legend is the same as in Figure 5.2.
Chapter 5. Multiple Receiver Antenna Selection

Figure 5.5 Normalized average progress for MAD with various $N_r$ receiver antennas. Legend is the same as in Figure 5.2.

Figure 5.6 Throughput for MAD with various $N_r$ receiver antennas. Legend is the same as in Figure 5.2.
Chapter 5. Multiple Receiver Antenna Selection

Figure 5.7 Normalized average progress for ARR with various $N_r$ receiver antennas. Legend is the same as in Figure 5.2.

Figure 5.8 Throughput for ARR with various $N_r$ receiver antennas. Legend is the same as in Figure 5.2.
Chapter 5. Multiple Receiver Antenna Selection

Figure 5.9 Normalized average progress for MTP with various $N_r$ receiver antennas.

Figure 5.10 Throughput for MTP with various $N_r$ receiver antennas. Legend is the same as in Figure 5.9.
Chapter 6

Directional Transmitter Antennas

In this chapter, we study the performance improvement resulting from the use of directional transmitter antennas by the nodes in a multihop PRN. Interference model 2.2 is used in this study. We also examine the effect of a non-ideal antenna pattern with and without Rayleigh fading. In Section 6.1, a directional transmitter antenna system is described. The capture probability for the directional transmitter antenna system is derived in Section 6.2, and its performance is analyzed in Section 6.3. In Section 6.4, the analytic and simulation results for the four transmission strategies based on the MFR, NFP, MAD, and ARR routing schemes are provided. Simulation results for the transmission strategy based on the MTP routing scheme are also provided. Results show that the performances of all five transmission strategies can be improved substantially if directional transmitter antennas are employed.

6.1 Directional Transmitter Antenna System

Omnidirectional transmitter antennas are employed in conventional multihop PRNs [19], [20], [22]. As a consequence, a transmission causes the same amount of interference in all directions. Employing directional antennas in transmission can limit the interference to other nodes and hence improve system performance [25], [26]. In this chapter, we consider a multihop PRN in which nodes employ directional antennas in transmission and omnidirectional antennas in reception. In such a system, a transmitter always aims its directional antenna at the intended receiving node. The performance improvement brought about by the use of directional transmitter antennas is examined by using a more realistic
interference model than those in [25] and [26]. Interference model 1.1 is used in [25], and interference model 2.1 is assumed in [26]. In both papers, the antenna pattern is assumed to be a rect function, and the transmission strategy based on the MFR routing scheme in a non-fading environment is examined. In this chapter, we examine the five transmission strategies based on the MFR, NFP, MAD, ARR, and MTP routing schemes with interference model 2.2. We also consider both rect and sinc function antenna patterns with and without Rayleigh fading.

6.2 Capture Probability

In this section, we derive the capture probability for the Rayleigh fading case; the capture probability for the non-fading case appears to be difficult to obtain because there is no closed form expression for the pdf of the interference power at a receiving node. Suppose $X$ transmits packet $K$ to $Y$ with $R = r$ and $\Theta = \theta$. Let $q_i(r, \theta)$ be the probability of $K$ being captured by $Y$ given that there are $i$ interfering nodes. The capture probability derived in this section will be used to obtain the throughput $S$ and normalized average progress $Z_n$ in Section 6.3. Let $M$ be an interfering node and $R_m$ denote the distance between $Y$ and $M$. There is an excluded region with area $A_e$, described in Chapter 3, given that $X$ transmits to $Y$. The excluded region, modelled in the same way as in Section 4.3.1, is represented by a circle of radius $r_e$, where $r_e = \sqrt{\frac{A_e}{\pi}}$, centered at $Y$. Since the interference power to $Y$ by $M$ is negligible compared to the received power of $K$ at $Y$ if $r_m >> r$, we only consider the nodes within distance $r_d$, where $r_d >> r$, of $Y$. Node $M$ is uniformly distributed in the non-excluded region, and the pdf of $R_m$ is given by (4.35). Let $U$ be the destination of the packet transmitted by $M$ and $\Theta_m$ be the angle between $MU$ and $MY$. Since the destination
direction of $M$ is uniformly distributed in $[-\pi, \pi)$, the pdf of $\Theta_m$ is

$$f_{\Theta_m}(\theta_m) = \frac{1}{2\pi}. \quad (6.1)$$

Based on the model of the non-excluded region assumed above, $R_m$ and $\Theta_m$ are independent.

Let $\Phi$ be the angle between $MY$ and $MW$ where $W$ is the intended receiving node of $M$. Figure (6.1) illustrates the angles $\Theta_m$ and $\Phi$, and the nodes $Y, M, W, and U$. In the following derivation of the pdf of $\Phi$, we ignore the effects of the presence of $X$ and the excluded region. For $R_m > r_{max}$, $\Phi$ is uniformly distributed in $[-\pi, \pi)$ because $Y$ cannot be the intended receiving node of $M$ in this case. If $Y$ is a neighbor of $M$ and also in the forward direction of $M$, i.e., $R_m \leq r_{max}$ and $|\Theta_m| \leq \frac{\pi}{2}$, the distribution of $\Phi$ is no longer uniform because of the presence of $Y$. Let the probability that $M$ transmits to $Y$ be denoted by $p_{Y}$. In such a case, the intended receiving node $W$ is $Y$, and the angle $\Phi = 0$. Then we have

$$p_Y = \exp(-\lambda A_{e,m}), \quad (6.2)$$
where $A_{e,m}$ in (6.2) is the area of the excluded region due to the transmission from $M$ to $Y$. The area $A_{e,m}$ can be obtained from $A_e$ by replacing $r$ and $\theta$ by $r_m$ and $\theta_m$, respectively. The area $A_e$ is defined in Chapter 3 and Appendix A. The probability $p_Y$ can be interpreted as the probability that no node is located in the region with area $A_e(r_m, \theta_m)$. For $R_m \leq r_{\text{max}}$ and $|\theta_m| \leq \frac{\pi}{2}$, we model the pdf of $\Phi$ as

$$f_{\Phi}(\phi) = \begin{cases} \frac{p_Y}{1 - 2p_Y}, & \phi = 0 \\ \frac{1}{2\pi}, & 0 < |\phi| \leq \frac{\pi}{2} \\ \frac{1}{2\pi}, & \frac{\pi}{2} < |\phi| \leq \pi. \end{cases} \quad (6.3)$$

Otherwise, the pdf of $\Phi$ is assumed to be uniformly distributed in $[-\pi, \pi]$, i.e.,

$$f_{\Phi}(\phi) = \frac{1}{2\pi}. \quad (6.4)$$

In [25] and [26], the pdf of $\Phi$ is assumed to be uniformly distributed in $[-\pi, \pi]$ regardless of the values of $R_m$. Such an assumption decreases the probability of interference to $Y$ and hence leads to optimistic results.

Let $\Gamma_m$ denote the received power at $Y$ from a transmission by $M$. The pdf $f_{\Gamma_m}(\gamma_m)$ of $\Gamma_m$ can be obtained from (2.1), (4.35), and the pdf of $\Phi$. We can then express $f_{\Gamma_m}(\gamma_m)$ as

$$f_{\Gamma_m}(\gamma_m) = \int_{r_m} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{r_m^{2}\beta}{g(\phi)} \exp \left[ -\frac{r_m^{2}\beta}{g(\phi)} \gamma_m \right] \\
\times f_{\Phi}(\phi) f_{\Theta_m}(\theta_m) f_{R_m}(r_m) d\phi d\theta_m dr_m. \quad (6.5)$$

where $g(\phi)$, defined in Section 2.5, is the antenna power gain at the orientation angle $\phi$. Let $\Gamma_i$ be the sum of the interference powers from the $i$ interfering nodes at $Y$. The pdf $f_{\Gamma_i}(\gamma_i)$ of $\Gamma_i$ is given by the $i$-fold convolution of $f_{\Gamma_m}(\gamma_m)$. The pdf $f_{\Gamma_i}(\gamma_i)$ of the received power, $\Gamma_i$, of $K$ at $Y$ is given by (4.42). From (4.36), the capture probability can be written as

$$q_i(r, \theta) = \int_{\gamma_i} \int_{\gamma_i} f_{\Gamma_i}(\gamma_i) f_{\Gamma_i}(\gamma_i) d\gamma_i d\gamma_i.$$
\[
q_i(r, \theta) = \left\{ \int_{r_e}^{r_m} \int_{\pi}^{\pi} \int_{\pi}^{\pi} \int_{0}^{\infty} \frac{r^\beta}{g(\phi)} \exp \left[ -\left( c r^\beta + \frac{r_m^\beta}{g(\phi)} \right) \right] \gamma_m \right\} \\
\times f_\phi(\phi) f_\Theta_m(\theta_m) f_R_m(r_m) d\gamma_m d\phi d\theta_m dr_m \right\}^i \\
= \left[ \int_{r_e}^{r_m} \int_{\pi}^{\pi} \int_{\pi}^{\pi} \int_{0}^{\infty} \frac{f_\phi(\phi) f_\Theta_m(\theta_m) f_R_m(r_m)}{c g(\phi) r_m^\beta + 1} \right] \gamma_m \right\}^i \\
= \left[ \int_{r_e}^{r_m} \int_{\pi}^{\pi} \int_{\pi}^{\pi} \int_{0}^{\infty} \frac{f_\phi(\phi) f_\Theta_m(\theta_m) f_R_m(r_m)}{c g(\phi) r_m^\beta + 1} \right] \gamma_m \right\}^i \\
+ \frac{1}{2\pi} \int_{r_m}^{r_d} \int_{\pi}^{\pi} \int_{\pi}^{\pi} \frac{f_R_m(r_m)}{c g(\phi) r_m^\beta + 1} d\phi dr_m \right\}^i \\
= \left\{ \int_{r_e}^{r_m} \int_{\pi}^{\pi} \int_{\pi}^{\pi} \int_{0}^{\infty} \frac{f_\phi(\phi) f_\Theta_m(\theta_m) f_R_m(r_m)}{c g(\phi) r_m^\beta + 1} \right\} \gamma_m \right\}^i \\
+ \frac{r_d^2 - r_m^2}{r_d^2 - r_e^2} - \frac{1}{2\pi} \int_{\pi}^{\pi} \frac{\sqrt{c g(\phi) r_m^2}}{r_d^2 - r_m^2} \right\}^i \\
\times \left[ \tan^{-1} \left( \frac{r_d^2}{\sqrt{c g(\phi) r_m^2}} \right) - \tan^{-1} \left( \frac{r_m^2}{\sqrt{c g(\phi) r_m^2}} \right) \right] d\phi \right\}^i \quad (6.7)
\]
Note that $q_i(r, \theta)$ depends on $\theta$ since $r_e = \sqrt{\frac{A_e}{2}}$ and $A_e$ depends on $\theta$. If omnidirectional antennas are employed, i.e., $g(\phi) = 1$, (6.7) is reduced to (4.45).

### 6.3 Throughput and Normalized Average Progress

Similar to the case of omnidirectional antennas in Section 4.3.2, we can express the throughput as

$$ S = p_t \Pr (E_f) \Pr (E_{s,Y}) \int_0^{\frac{\pi}{2}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f_{R,\Theta}(r, \theta) \sum_{i=0}^{\infty} p_i q_i(r, \theta) \, d\theta \, dr, \quad (6.8) $$

where $\Pr (E_f)$ is the probability of $X$ finding a neighboring node in its forward direction as given by (3.1), $\Pr (E_{s,Y})$ is the probability of $Y$ not transmitting as given by (4.1), and $p_i$ is the probability of $i$ interfering nodes in the non-excluded region as given by (4.48). The normalized average progress can be obtained from (6.8) by multiplying the integrand by $\sqrt{\lambda} r \cos \theta$, resulting in

$$ Z_n = p_t \sqrt{\lambda} \Pr (E_f) \Pr (E_{s,Y}) \int_0^{\frac{\pi}{2}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f_{R,\Theta}(r, \theta) r \cos \theta \sum_{i=0}^{\infty} p_i q_i(r, \theta) \, d\theta \, dr. \quad (6.9) $$

Since $q_i(r, \theta) = [q_i(r, \theta)]^i$ in (6.7), the summation in both (6.8) and (6.9) can be simplified as

$$ \sum_{i=0}^{\infty} p_i q_i(r, \theta) = [N_t q_1(r, \theta)]^i \exp \left( -N_t \right) $$

$$ = \exp \left\{ -[1 - q_1(r, \theta)] N_t \right\} \quad (6.10) $$

where $N_t$, given by (4.47), is the average number of interfering nodes in the non-excluded region.

In a system using directional antennas, the transmission potential $\nu_p$ for MTP is not the same as for the omnidirectional antennas case. In order to simplify $\nu_p$ for the
case of directional antennas, we assume that the amount of interference at $Y$ is reduced by a factor of $\frac{B_W}{2\pi}$, where $B_W$ is the half-power beamwidth, compared to the case of omnidirectional antennas. Hence, we can obtain $\nu_p$ by multiplying $\kappa$ in (3.39) and (3.40) for the omnidirectional case by $\frac{B_W}{2\pi}$.

6.4 Analytic and Simulation Results

In this section, the terms MFR, NFP, MAD, ARR, and MTP are used to refer to the transmission strategies based on the corresponding routing schemes. For the non-fading case, only simulation results are provided. For the Rayleigh fading case, simulation results as well as analytic results (simulation only for MTP) are provided for comparison. The simulation results obtained are based on the model described in Section 2.6. The value of $r_d$ is assumed to be $10r_{max}$ in the simulation because there is little difference in the results by increasing the value of $r_d$ beyond $10r_{max}$. As in Sections 4.4.2, 4.4.3, and 5.4, the value of $c = 4$ is used [4], [14]. An optimal value of $p_t$ (obtained by searching in $p_t$ steps of 0.01) is chosen which maximizes the normalized average progress $Z_n$ for a given routing scheme and a given connectivity $N$. It might be noted that the optimal $p_t$ value generally decreases with increasing $N$.

Figure 6.2 shows the simulation results of $Z_n$ as a function of $N$ for MFR in a non-fading channel. The results are for the $\text{rect}$ function antenna pattern with different values of beamwidth $B_W$. In Figure 6.2, $Z_n$ for the omnidirectional antennas case, labelled as "Omni", is also provided for comparison. Figure 6.3 shows the corresponding results for MFR with the $\text{sinc}$ function antenna pattern and different values of $B_W$. The $Z_n$ values for NFP, MAD, ARR, and MTP with the $\text{rect}$ function antenna pattern are shown in Figures 6.4, 6.6, 6.8, and 6.10, respectively. The corresponding results with the $\text{sinc}$ function antenna pattern are shown in Figures 6.5, 6.7, 6.9, and 6.11. For the Rayleigh fading case, the
simulation results of $Z_n$ for the five transmission strategies with the two antenna patterns are shown in Figures 6.12 to 6.21. The analytic results for MFR, NFP, MAD, and ARR are also provided for comparison.

Similar to the case of omnidirectional antenna, the performances of all five transmission strategies for the directional antenna cases decrease by about 10% in the presence of Rayleigh fading. For all five transmission strategies, $Z_n$ can be improved substantially by employing directional antennas with the $\text{rect}$ function antenna pattern. The ratio of the maximum value of $Z_n$ for the $B_W = \frac{\pi}{8}$ case to that of the omnidirectional antenna case is about 5.8, 2.7, 5.6, 3.7, and 5.4 for MFR, NFP, MAD, ARR, and MTP, respectively. MFR, MAD, and MTP show greater improvement on $Z_n$ than NFP and ARR. If the interference is small, a strong received signal power is not needed for capturing the packet. In this case, a receiving node with a larger progress (corresponding to a larger transmitter-to-receiver distance) is preferred. Thus MFR and MAD would be preferred over NFP and ARR. If the interference is large, NFP and ARR perform better than MFR and MAD. MTP has the best performance among the five transmission strategies because it can adapt to different interference levels. For the case of the $\text{sinc}$ function antenna pattern, The ratio of the maximum value of $Z_n$ for the $B_W = \frac{\pi}{8}$ case to that of the omnidirectional antenna case is about 3.8, 2.5, 3.9, 3.3, and 3.8 for MFR, NFP, MAD, ARR, and MTP, respectively. The performance improvement for the five transmission strategies, especially for MFR, MAD, and MTP, is reduced. The effect of the side lobes of the $\text{sinc}$ function antenna pattern has a larger impact on MFR and MAD because the received signal power is smaller for MFR and MAD compared to that for NFP and ARR.

In [25] and [26], the performance of MFR with the $\text{rect}$ function antenna pattern was studied. In this section, we had provided the results for all five transmission strategies with
the *rect* function and *sinc* function antenna patterns. The performances of the transmission strategies were also examined with and without fading, and the interference model used is more realistic than those used in [25] and [26]. Our results as well as those in [25] and [26] show that a smaller value of $B_W$ generally leads to a better performance. However, the *sinc* function antenna pattern results in a substantial performance degradation relative to the *rect* function antenna pattern for MFR, MAD, and MTP.
Figure 6.2 Normalized average progress for MFR in a non-fading channel with \textit{rect} pattern and different values of $B_W$. Legend is the same as in Figure 6.5.

Figure 6.3 Normalized average progress for MFR in a non-fading channel with \textit{sinc} pattern and different values of $B_W$. Legend is the same as in Figure 6.5.
Chapter 6. Directional Transmitter Antennas

Figure 6.4 Normalized average progress for NFP in a non-fading channel with rect pattern and different values of $B_W$. Legend is the same as in Figure 6.5.

Figure 6.5 Normalized average progress for NFP in a non-fading channel with sinc pattern and different values of $B_W$. 
Figure 6.6 Normalized average progress for MAD in a non-fading channel with rect pattern and different values of $B_W$. Legend is the same as in Figure 6.5.

Figure 6.7 Normalized average progress for MAD in a non-fading channel with sinc pattern and different values of $B_W$. Legend is the same as in Figure 6.5.
Figure 6.8 Normalized average progress for ARR in a non-fading channel with rect pattern and different values of $B_W$. Legend is the same as in Figure 6.5.

Figure 6.9 Normalized average progress for ARR in a non-fading channel with sinc pattern and different values of $B_W$. Legend is the same as in Figure 6.5.
Figure 6.10 Normalized average progress for MTP in a non-fading channel with \textit{rect} pattern and different values of $B_W$. Legend is the same as in Figure 6.5.

Figure 6.11 Normalized average progress for MTP in a non-fading channel with \textit{sinc} pattern and different values of $B_W$. Legend is the same as in Figure 6.5.
Chapter 6. Directional Transmitter Antennas

Figure 6.12 Normalized average progress for MFR in a Rayleigh fading channel with *rect* pattern and different values of $B_w$. Legend is the same as in Figure 6.15.

Figure 6.13 Normalized average progress for MFR in a Rayleigh fading channel with *sinc* pattern and different values of $B_w$. Legend is the same as in Figure 6.15.
Figure 6.14 Normalized average progress for NFP in a Rayleigh fading channel with *rect* pattern and different values of $B_w$. Legend is the same as in Figure 6.15.

Figure 6.15 Normalized average progress for NFP in a Rayleigh fading channel with *sinc* pattern and different values of $B_w$. 
Figure 6.16 Normalized average progress for MAD in a Rayleigh fading channel with \textit{rect} pattern and different values of $B_W$. Legend is the same as in Figure 6.15.

Figure 6.17 Normalized average progress for MAD in a Rayleigh fading channel with \textit{sinc} pattern and different values of $B_W$. Legend is the same as in Figure 6.15.
Figure 6.18 Normalized average progress for ARR in a Rayleigh fading channel with rect pattern and different values of $B_W$. Legend is the same as in Figure 6.15.

Figure 6.19 Normalized average progress for ARR in a Rayleigh fading channel with sinc pattern and different values of $B_W$. Legend is the same as in Figure 6.15.
Figure 6.20 Normalized average progress for MTP in a Rayleigh fading channel with \textit{rect} pattern and different values of $B_W$. Legend is the same as in Figure 6.5.

Figure 6.21 Normalized average progress for MTP in a Rayleigh fading channel with \textit{sinc} pattern and different values of $B_W$. Legend is the same as in Figure 6.5.
Chapter 7

Conclusions

The one-hop throughput $S$ and the average normalized progress $Z_n$ of the five transmission strategies (based on two previously proposed routing schemes, MFR and NFP, and three new routing schemes, MAD, ARR, and MTP) in a multihop PRN, as described in Section 2.6, with different interference models were studied. The study in this thesis can apply to a multihop PRN with randomly distributed nodes which are mobile or stationary. For the three interference models considered in this thesis, the $Z_n$ of the five transmission strategies initially increases rapidly with the connectivity $N$. As $N$ increases further, the performance of MFR and MAD degrades due to the increased interference from other transmitting nodes. However, the performance of NFP, ARR, and MTP does not degrade for larger values of $N$. The maximum value of $Z_n$ for the five transmission strategies (MTP with interference model 1.2 was not studied) occurs at the connectivity of 6 to 11, 4 to 5, and 3 to 4 for interference models 1.2, 2.1, and 2.2, respectively.

In this thesis, a refinement of the analysis of MFR and NFP in [22] (carried out assuming interference model 1.2) was presented and shown to yield more accurate results. The difference between the analytic and simulation results for MFR, NFP, MAD, and ARR with the three interference models is less than 2% in most cases. Simulation results for MTP are also provided for comparison. Results show that ARR has a better performance than MFR, NFP, and MAD for almost all values of $N$. They also show that MTP performs better than ARR by about 10%. It was found, using interference model 2.2, that all five transmission strategies perform worse in the presence of Rayleigh fading. This is in contrast
Chapter 7. Conclusions

to contention-limited single-hop systems in which Rayleigh fading generally does not result in poorer performance.

The performance improvement of the five transmission strategies in a multiple receiver antenna (diversity) system in a Rayleigh fading environment was studied. In the proposed system, every node in the network has $N_r$ receiver antennas. It was determined from simulation that a scheme in which nodes decode the antenna with the largest received power yields good performance. Results show that $Z_n$ of the five transmission strategies can be improved by about 25% if $N_r$ is increased from 1 to 2. An approximate analysis was provided, and the results were found to agree well with simulation results for $N_r \leq 2$. The difference becomes larger as $N_r$ increases because a simplification used in the analysis becomes less valid. For any given value of $N_r$, ARR has a larger $Z_n$ than MFR, NFP, and MAD for almost all values of $N$, and MTP again performs better than ARR by about 10%.

The performance improvement brought about by using directional transmitter antennas for the five transmission strategies was also studied. In all five cases, $Z_n$ can be improved substantially by employing directional antennas with a $\text{rect}$ function antenna pattern. The ratio of the maximum value of $Z_n$ for a beamwidth $B_w = \frac{\pi}{2}$ case to that of the omnidirectional antenna case is about 5.8, 2.7, 5.6, 3.7, and 5.4 for MFR, NFP, MAD, ARR, and MTP, respectively. MFR, MAD, and MTP show greater improvement than NFP and ARR. This is because interference is the primary factor limiting the performance of MFR and MAD, and the interference is reduced as the beamwidth decreases. The performance improvement for the five transmission strategies, especially for MFR, MAD, and MTP, is reduced in the case of a $\text{sinc}$ function antenna pattern. The side lobes of the $\text{sinc}$ function antenna pattern have a larger impact on MFR, MAD, and MTP because the received signal power at the receiving node is smaller for them compared to that for NFP and ARR.
Among the topics which could be further investigated are:

- To study multihop PRNs using other random access techniques such as CSMA.
- To find a routing scheme which yields the optimal normalized average progress under various interference models.
- To analyze the end-to-end throughput of the transmission strategies.
- To find a transmission strategy which yields the optimal end-to-end throughput under the more realistic interference models such as 2.1 and 2.2.
Bibliography


Bibliography


Appendix A

Partial Excluded Region

If node X transmits to its receiving node Y, there is a region, referred to as an excluded region, in which no node can be located. The area of the excluded region depends on the distance r between X and Y, and the angle θ between XY and the destination direction of X. We will refer to the part of the excluded region which is within the circle of radius \( r_c (\geq r) \) centered at Y as a partial excluded region. The values of \( r_c \) for interference models 1.2, 2.1, and 2.2 are \( r_{max}, \ c^2 r, \) and \( \infty \), respectively. In Chapters 4, 5, and 6, the area \( A_e(r_c) \) of the partial excluded region is needed in the performance analysis. In this appendix, we determine \( A_e(r_c) \) for the MFR, NFP, MAD, and ARR routing schemes. Since the area of the partial excluded region is the same for ±θ, we only consider θ in the range of \([0, \frac{\pi}{2}]\).

A.1 MFR

Figures A.1(a) and (b) illustrate the partial excluded region for MFR. Let the distance between \( W_1 \) and Y and the distance between \( W_2 \) and Y be denoted by \( d_1 \) and \( d_2 \), respectively. We have

\[
\begin{align*}
    d_1 &= r_{max} \sin \alpha - r \sin \theta, \\
    d_2 &= r_{max} \sin \alpha + r \sin \theta,
\end{align*}
\]

where \( \alpha = \cos^{-1} \left( \frac{r \cos \theta}{r_{max}} \right) \).

Case 1: \( d_2 \leq r_c \).

In this case, the entire excluded region is within the distance \( r_c \) from Y and

\[
A_e(r_c) = A_e = \frac{1}{2} r_{max} \alpha - \frac{1}{2} r_{max}^2 \sin 2\alpha.
\]
Case 2: $d_2 > r_c$ and $d_1 \leq r_c$.

This case is illustrated in Figure A.1(a). Denote the distance between $W_1$ and $W_3$ by $h$ where

$$h = 2 r_{\text{max}} \sin \left( \frac{\alpha + \theta}{2} \right). \quad \text{(A.3)}$$

We have

$$\delta = \cos^{-1} \left( \frac{r^2 + r_c^2 - r_{\text{max}}^2}{2 r r_c} \right),$$

$$\xi = \delta + \theta - \frac{\pi}{2}, \quad \text{(A.4)}$$

and we can express $A_e(r_c)$ as

$$A_e(r_c) = A_1 + A_2 + A_3, \quad \text{(A.5)}$$

where

$$A_1 = \frac{1}{2} r_c^2 \xi,$$

$$A_2 = \frac{1}{2} d_1 r_c \sin \xi,$$

$$A_3 = r_{\text{max}}^2 \left[ \sin^{-1} \left( \frac{h}{2 r_{\text{max}}} \right) - \frac{h}{2 r_{\text{max}}} \sqrt{1 - \frac{h^2}{4 r_{\text{max}}^2}} \right]. \quad \text{(A.6)}$$

Case 3: $d_1 > r_c$.

If $r + r_c > r_{\text{max}}$, we have the case illustrated in Figure A.1(b). Let the distance between $W_3$ and $W_4$ be denoted by $l$ where

$$l = 2 r_c \sin \delta. \quad \text{(A.7)}$$
The areas $A_1$, $A_2$, and $A_3$ are

$$A_1 = \frac{1}{2} r_c^2 \xi,$$

$$A_2 = \frac{1}{2} r_c^2 (2 \delta - \pi - \xi),$$

$$A_3 = r_{max}^2 \left[ \sin^{-1} \left( \frac{l}{2r_{max}} \right) - \frac{l}{2r_{max}} \sqrt{1 - \frac{l^2}{4r_{max}^2}} \right] + \frac{1}{2} l r_c \cos (\delta - \pi),$$  \hspace{1cm} (A.8)

where $\delta$ and $\xi$ are given in (A.4), and

$$A_e(r_c) = A_1 + A_2 + A_3.$$ \hspace{1cm} (A.9)

If $r + r_c \leq r_{max}$, we have

$$A_e(r_c) = \frac{r_c^2 \pi}{2}.$$ \hspace{1cm} (A.10)

### A.2 NFP

Figures A.2(a) and (b) illustrate the partial excluded region for NFP. Let the distance between $W_1$ and $Y$ and the distance between $W_2$ and $Y$ be denoted by $d_1$ and $d_2$, respectively. We have

$$d_1 = 2 r \sin \left( \frac{\pi}{4} - \frac{\theta}{2} \right),$$

$$d_2 = 2 r \sin \left( \frac{\pi}{4} + \frac{\theta}{2} \right).$$ \hspace{1cm} (A.11)

**Case 1: $d_2 \leq r_c$.**

In this case, the excluded region is a semi circle with radius $r$ and the area of the region is

$$A_e(r_c) = A_e = \frac{1}{2} \pi r^2.$$ \hspace{1cm} (A.12)
Case 2: $d_1 \leq r_c$ and $d_2 > r_c$.

This case is illustrated in Figure A.2(a) and we have

$$\alpha = 2 \sin^{-1} \left( \frac{r_c}{2r} \right),$$  
$$\delta = \frac{\pi - \alpha}{2},$$  
$$\xi = \frac{\pi}{2} - \theta - \sin^{-1} \left( \frac{r}{r_c} \cos \theta \right).$$  \hspace{1cm} (A.13)

$$A_1 = \frac{1}{2} r^2 \left( \frac{\pi}{2} - \theta \right),$$  
$$A_2 = \frac{1}{2} r r_c \sin \xi,$$  
$$A_3 = \frac{1}{2} r_c^2 (\delta - \xi),$$  
$$A_4 = \frac{1}{2} r^2 \alpha - \frac{1}{2} r r_c \cos \frac{\alpha}{2},$$  \hspace{1cm} (A.14)

and

$$A_e(r_c) = A_1 + A_2 + A_3 + A_4.$$  \hspace{1cm} (A.15)

Case 3: $d_1 > r_c$.

This case is illustrated in Figure A.2(b) and we have

$$A_1 = r_c^2 \delta - \frac{1}{2} r_c^2 \left[ \xi + \psi - \sin (\xi + \psi) \right],$$  
$$A_2 = r^2 \alpha - r r_c \cos \frac{\alpha}{2},$$  \hspace{1cm} (A.16)

where $\alpha$, $\delta$, and $\xi$ are given in (A.13), and

$$\psi = \frac{\pi}{2} + \theta - \sin^{-1} \left( \frac{r}{r_c} \cos \theta \right).$$  \hspace{1cm} (A.17)

The area of the partial excluded region is

$$A_e(r_c) = A_1 + A_2.$$  \hspace{1cm} (A.18)
A.3 MAD

Figures A.3(a) and (b) illustrate the partial excluded region for MAD. Let the distance between \( Y \) and \( W \) be denoted by \( d \) where

\[
d = \sqrt{r^2 + r_{\text{max}}^2 - 2rr_{\text{max}} \cos 2\theta}.
\]  

(A.19)

Case 1: \( d \leq r_c \).

In this case, the entire excluded region is within the distance \( r_c \) from \( Y \) and

\[
A_e(r_c) = A_e = r_{\text{max}}^2 \theta.
\]  

(A.20)

Case 2: \( d > r_c \) and \( r + r_c \leq r_{\text{max}} \).

This case is illustrated in Figure A.3(a). We have

\[
\delta = \pi - 2\theta - \sin \left( \frac{r}{r_c} \sin 2\theta \right)
\]  

(A.21)

and the area of the partial excluded region is

\[
A_e(r_c) = \frac{1}{2} r_c^2 (\pi - \delta) + \frac{1}{2} rr_c \sin \delta.
\]  

(A.22)

Case 3: \( d > r_c \) and \( r + r_c > r_{\text{max}} \).

This case is illustrated in Figure A.3(b). We have

\[
\alpha = \sin^{-1} \left( \frac{r}{r_{\text{max}}} \sin 2\theta \right),
\]

\[
\xi = \cos^{-1} \left( \frac{r^2 + r_c^2 - r_{\text{max}}^2}{2rr_c} \right),
\]  

(A.23)

and

\[
A_1 = \frac{1}{2} r r_{\text{max}} \sin \delta,
\]

\[
A_2 = \frac{1}{2} r_c^2 (\xi - \delta),
\]

\[
A_3 = \frac{1}{2} r_{\text{max}}^2 \alpha - \frac{1}{2} rr_{\text{max}} \sin \alpha.
\]  

(A.24)
Appendix A. Partial Excluded Region

The area of the partial excluded region is given by

\[ A_e(r_c) = A_1 + A_2 + A_3. \]  \hfill (A.25)

A.4 ARR

Figure A.4(a) and (b) illustrate the partial excluded region for ARR. Circle \( C_1 \) has radius \( r_{\text{max}} \) and is centered at \( X \). Circle \( C_2 \) has radius \( r_c \) and is centered at \( Y \). Circle \( C_3 \) has radius \( \frac{r}{2} \) and is centered at \( (\frac{r}{2}, 0) \) where \( t = \frac{r}{\cos \theta} \). The distance between \( Y \) and \( W \) is

\[ d = \sqrt{r^2 + r_{\text{max}}^2 - 2rr_{\text{max}} \cos(\theta + \alpha)} \]  \hfill (A.26)

where

\[ \alpha = \cos^{-1}\left(\frac{r_{\text{max}}}{t}\right). \]  \hfill (A.27)

Case 1: \( t \leq r_{\text{max}} \).

If \( t \leq r_c \),

\[ A_e(r_c) = A_e = \frac{\pi}{4} t^2. \]  \hfill (A.28)

If \( t > r_c \),

\[ A_e(r_c) = r_c^2 \cos^{-1}\left(\frac{r_c}{t}\right) + \frac{t^2}{2} \left[ \sin^{-1}\left(\frac{r_c}{t}\right) - \frac{r_c}{t} \sqrt{1 - \left(\frac{r_c}{t}\right)^2} \right]. \]  \hfill (A.29)

Case 2: \( t > r_{\text{max}} \) and \( d \leq r_c \).

If \( \theta + \alpha \leq \frac{\pi}{2} \) and \( r_c \leq t \), as illustrated in Figure A.4(a), the area of the partial excluded region is

\[ A_e(r_c) = \frac{\pi}{4} t^2 + \left( r_{\text{max}}^2 - \frac{t^2}{2} \right) \alpha - \frac{r_{\text{max}}^2}{2} \sqrt{t^2 - r_{\text{max}}^2} \]
\[ + \left( r_c^2 - \frac{t^2}{2} \right) \frac{\delta}{2} - \frac{r_c^2}{2} \sqrt{t^2 - r_c^2}, \]  \hfill (A.30)
where \( \delta = 2 \cos^{-1} \left( \frac{\pi}{2} \right) \). If \( \theta + \alpha > \frac{\pi}{2} \) or \( r_c > t \), we have from (3.21) that

\[
A_e(r_c) = \frac{\pi}{4} t^2 + \left( \frac{r_{\text{max}}^2 - t^2}{2} \right) \alpha - \frac{r_{\text{max}}}{2} \sqrt{t^2 - r_{\text{max}}^2}.
\] (A.31)

**Case 3: \( t > r_{\text{max}} \) and \( d > r_c \).**

Let the three intersection points of the circles \( C_1, C_2 \) and \( C_3 \) be denoted by \( J_1, J_2, \) and \( J_3 \) as illustrated in Figure A.4(b). Let the distances between these intersection points be \( s_1, s_2, \) and \( s_3 \). We can write

\[
A_e(r_c) = A_1 + A_2 + A_3 + A_4
\] (A.32)

where

\[
A_1 = r_{\text{max}}^2 \left[ \sin^{-1} \left( \frac{s_1}{2r_{\text{max}}} \right) - \frac{s_1}{2r_{\text{max}}} \sqrt{1 - \frac{s_1^2}{4r_{\text{max}}^2}} \right],
\]

\[
A_2 = r_c^2 \left[ \sin^{-1} \left( \frac{s_2}{2r_c} \right) - \frac{s_2}{2r_c} \sqrt{1 - \frac{s_2^2}{4r_c^2}} \right],
\]

\[
A_3 = \frac{t^2}{4} \left[ \sin^{-1} \left( \frac{s_3}{t} \right) - \frac{s_3}{t} \sqrt{1 - \frac{s_3^2}{t^2}} \right],
\]

\[
A_4 = \sqrt{s (s - s_1)(s - s_2)(s - s_3)}.
\] (A.33)

In (A.33), \( s = \frac{1}{2} (s_1 + s_2 + s_3) \). Let \((x_i, y_i)\) be the rectangular co-ordinates for the intersection point \( J_i \) where \( i = 1, 2, 3 \). The terms \( s_1, s_2 \) and \( s_3 \) can be calculated as follows.

\[
s_1 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2},
\]

\[
s_2 = \sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2},
\]

\[
s_3 = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}.
\] (A.34)
The equations for $C_1$, $C_2$ and $C_3$ are

$$x^2 + y^2 = r_{\text{max}}^2,$$

$$\left( x - r \cos \theta \right)^2 + \left( y - r \sin \theta \right)^2 = r_c^2,$$

$$\left( x - \frac{t}{2} \right)^2 + y^2 = \frac{t^2}{4}. \quad (A.35)$$

The co-ordinates $(x_i, y_i)$ for $i = 1, 2, 3$ are determined as follows. For $J_1$, we have

$$x_1 = \frac{r_{\text{max}}^2}{t},$$

$$y_1 = r_{\text{max}} \sqrt{1 - \frac{1}{t^2}}. \quad (A.36)$$

For $J_2$, we have

$$x_2 = \frac{r_s - 2 y_2 \sin \theta}{2 \cos \theta},$$

$$y_2 = \frac{1}{2} \left( r_s \sin \theta - \cos \theta \sqrt{4 r_{\text{max}}^2 - r_s^2} \right), \quad (A.37)$$

where $r_s = \frac{1}{t} (r_{\text{max}}^2 - r_c^2 + r^2)$. For $J_3$, we have

$$x_3 = \frac{-a_2 - \sqrt{a_2^2 - 4 a_1 a_3}}{2 a_1},$$

$$y_3 = \frac{(t - 2 r \cos \theta) x_3 - r_c^2 + r^2}{2 r \cos \theta}, \quad (A.38)$$

where

$$a_1 = t^2 - 4 r t \cos \theta + 4 r^2,$$

$$a_2 = 2 \left( r^2 - r_c^2 \right) (t - 2 r \cos \theta) - 4 r^2 t \sin^2 \theta,$$

$$a_3 = (r^2 - r_c^2)^2. \quad (A.39)$$
Figure A.1 Illustration of the partial excluded region for MFR.
Figure A.2 Illustration of the partial excluded region for NFP.
Figure A.3 Illustration of the partial excluded region for MAD.
Figure A.4 Illustration of the partial excluded region for ARR.
Appendix B

Asymptotic Value of $\sum p_i q_i(r, \theta)$

In this appendix, we derive the asymptotic values of $\sum_{i=0}^{\infty} p_i q_i(r, \theta)$ as $r_d \to \infty$ for the case of non-fading in Section 4.3.2. For a random variable $W$ with mean $\mu$ and variance $\sigma^2$ and for any $\varepsilon > 0$, Chebyshev's inequality [66] states that

$$\Pr(|W - \mu| > \varepsilon) \leq \frac{\sigma^2}{\varepsilon^2}$$

which can be rewritten as

$$\Pr(|W - \mu| < \varepsilon) \geq 1 - \frac{\sigma^2}{\varepsilon^2}.$$  \hspace{1cm} (B.2)

Let $I$ be the number of interfering nodes. From (4.48), we have

$$\Pr(I = i) = p_i = \frac{N_t^i \exp(-N_t)}{i!}.$$  \hspace{1cm} (B.3)

The mean and variance of $I$ are equal to $N_t$. Applying Chebyshev's inequality, we have

$$\Pr(|I - N_t| < \alpha N_t) \geq 1 - \frac{N_t}{\alpha^2 N_t^2}$$

for any $\alpha > 0$. From (4.47), we have

$$N_t = \lambda \left( r_d^2 \pi - A_e \right) p_t \Pr(E_f).$$  \hspace{1cm} (B.5)

If $r_d \to \infty$,

$$\lim_{r_d \to \infty} \frac{N_t}{r_d^2} = \kappa$$

where $\kappa = \lambda \pi p_t \Pr(E_f)$. 

117
Appendix B. Asymptotic Value of $\sum p_i q_i(r, \theta)$

Let the value of $a$ be set arbitrarily to $N_t^{-\frac{1}{3}}$. From (B.4), we can obtain that

$$\lim_{r_d \to \infty} \Pr( |I - N_t| < a N_t ) = \lim_{N_t \to \infty} \Pr( |I - N_t| < a N_t ) \geq \lim_{N_t \to \infty} 1 - \frac{N_t}{a^2 N_t^2} = 1.$$  \hspace{1cm} (B.7)

Equation (B.7) states that as $N_t \to \infty$, the probability that $I$ is within $aN_t$ from its mean $N_t$ approaches 1.

The capture probability for the non-fading case is shown in (4.41) as

$$q_i(r, \theta) = \frac{\text{erfc} \left( \frac{i r^2}{2 r_d^2 \sqrt{c \pi}} \right)}{\left[ \text{erfc} \left( \frac{\sqrt{c r^2}}{2 r_d^2} \right) \right]^i},$$  \hspace{1cm} (B.8)

where $\text{erfc}(u) \triangleq \frac{2}{\sqrt{\pi}} \int_u^\infty \exp(-t^2) dt$ is the complementary error function. As $r_d$ and $N_t \to \infty$, $I$ will be in the range $[(1-a)N_t, (1+a)N_t]$ with probability close to 1.

For $i \in [(1-a)N_t, (1+a)N_t]$, the numerator in (B.8) can be simplified using (B.7) as

$$\lim_{r_d \to \infty} \text{erfc} \left( \frac{i r^2}{2 r_d^2 \sqrt{c \pi}} \right) = \lim_{r_d \to \infty} \text{erfc} \left( \frac{N_t r^2}{2 r_d^2 \sqrt{c \pi}} \right) = \text{erfc} \left( \frac{1}{2 \sqrt{c \pi} \kappa r^2} \right).$$  \hspace{1cm} (B.9)

If $r_d \to \infty$, the denominator in (B.8) can be written as

$$\lim_{r_d \to \infty} \left[ \text{erfc} \left( \frac{\sqrt{c r^2}}{2 r_d^2} \right) \right]^i = \lim_{r_d \to \infty} \left[ \text{erfc} \left( \frac{\sqrt{c \kappa r^2}}{2 N_t} \right) \right]^i = \lim_{r_d \to \infty} \left[ 1 - \text{erf} \left( \frac{\sqrt{c \kappa r^2}}{2 N_t} \right) \right]^i = \lim_{r_d \to \infty} \left[ 1 - \frac{\kappa r^2}{N_t} \right]^i,$$  \hspace{1cm} (B.10)
Appendix B. Asymptotic Value of \( \sum p_i q_i(r, \theta) \)

where \( \text{erf}(u) \triangleq \frac{2}{\sqrt{\pi}} \int_0^u \exp(-t^2) \, dt \) is the error function. The last step of (B.10) is obtained by using the fact that \( \lim_{u \to 0} \text{erf}(u) = \frac{2u}{\sqrt{\pi}} \). Note that for \( I \) in the range of \([ (1 - a)N_t, (1 + a)N_t ]\), we have

\[
\lim_{r_d \to \infty} \left[ 1 - \frac{\kappa r_e^2}{N_t} \right]^i = \lim_{r_d \to \infty} \left[ 1 - \frac{\kappa r_e^2}{N_t} \right]^N_t \left[ 1 - \frac{\kappa r_e^2}{N_t} \right]^{-bN_t} = \lim_{r_d \to \infty} \exp\left(-\kappa r_e^2\right) \exp\left(b \kappa r_e^2\right) = \exp\left(-\kappa r_e^2\right), \tag{B.11}
\]

where \( b \in [-a, a] \). From (B.9) and (B.11), we obtain the capture probability for \( r_d \to \infty \) as

\[
\lim_{r_d \to \infty} q_i(r, \theta) = \exp\left(\kappa r_e^2\right) \text{erfc}\left(\frac{1}{2} \sqrt{c\pi \kappa r^2}\right). \tag{B.12}
\]

Since \( q_i(r, \theta) \) is independent of \( i \) for \((1 - a)N_t \leq i \leq (1 + a)N_t\), we have

\[
\lim_{r_d \to \infty} \sum_{i=0}^{\infty} p_i q_i(r, \theta) = \exp\left(\kappa r_e^2\right) \text{erfc}\left(\frac{1}{2} \sqrt{c\pi \kappa r^2}\right). \tag{B.13}
\]
Appendix C

Outline of Simulation Program

In this appendix, we provide an outline of the program which was used to obtain the simulation results for a randomly distributed network. These results are described in Sections 4.4, 5.4, and 6.4. Suppose node $X$ transmits packet $K$ to node $Y$, and that only the nodes within distance $r_d$ of $X$ are considered in the simulation. The values of $r_d$ are set to $3r_{\text{max}}$, $(2 + e^3)r_{\text{max}}$, and $10r_{\text{max}}$ for interference models 1.2, 2.1, and 2.2, respectively. For interference models 1.2 and 2.1, any transmitting node beyond distance $(r_d - r_{\text{max}})$ of $X$ cannot interfere with $Y$. A node within such a distance will transmit if it is in transmission mode and it has a neighboring node in its forward direction. For interference model 2.2, there is little difference in the results when the value of $r_d$ is increased beyond $10r_{\text{max}}$.

For a given routing scheme and a given value of connectivity $N = \lambda\pi r_{\text{max}}^2$, the steps of the simulation are described as follows.

Step 1. Set the maximum progress accumulator $a_{cm}$ and the probability, $p_t$, of a node in transmission mode to zero. Set the total number of trials $N_{tr}$ to a desired value.

Step 2. Set the progress accumulator $a_c$, the number, $n_t$, of trials, and the number, $n_s$, of successful receptions of $K$ by $Y$ to zero.

Step 3. Generate the number of nodes $N_n$ according to a Poisson point process with average number of nodes per unit area equal to $\lambda$ for a given area of $\pi r_t^2$.

Step 4. Locate the $N_n$ nodes uniformly in a circular area of radius $r_t$ with center $C$ at the origin. This can be done by choosing the polar co-ordinates $(r_n, \theta_n)$ according to
Appendix C. Outline of Simulation Program

the following pdf’s.

\[ f_{R_n}(r_n) = \frac{2r_n}{r_t^2}, \quad (C.1) \]

and

\[ f_{\Theta_n}(\theta_n) = \frac{1}{2\pi}, \quad (C.2) \]

where \(0 \leq r_n \leq r_t\) and \(0 \leq \theta_n < 2\pi\).

Step 5. Choose a node \(X\) at random. If \(X\) is within distance \(r_x, r_x = r_t - r_d\), of \(C\), then go to Step 6; otherwise, go to Step 3. This is because an edge effect will occur if \(X\) is too far away from \(C\).

Step 6. Increment \(n_t\). If \(X\) is in transmission mode, it sends \(K\) to \(Y\) according to the routing scheme.

Step 7. The event that \(Y\) receives \(K\) successfully depends on the interference model used. If \(Y\) receives \(K\) successfully, increment \(n_s\) and accumulate the resulting progress \(z\) to \(a_c\).

Step 8. If \(n_t = N_{tr}\), go to Step 9; otherwise, go to Step 3.

Step 9. If \(a_c > a_{cm}\), replace \(a_{cm}\) by \(a_c\). Increment \(p_t\) by a step of 0.01 and go to Step 2 until \(p_t = 1\).

The \(p_t\) value yields \(z_{max}\) is referred to as the optimal \(p_t\). The throughput \(S\) and the normalized average progress \(Z_n\) can be determined by \(n_t, n_s,\) and \(a_{cm}\) as follows.

\[ S = \frac{n_s}{N_{tr}} \quad (C.3) \]

and

\[ Z_n = \frac{a_{cm} \sqrt{\lambda}}{N_{tr}}. \quad (C.4) \]