PERFORMANCE ANALYSIS OF SOME NEW AND EXISTING TRANSMIT AND RECEIVE ANTENNA DIVERSITY SCHEMES

by

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A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF APPLIED SCIENCE

in

THE FACULTY OF GRADUATE STUDIES

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

We accept this thesis as conforming

to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

October 2000

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Date Oct. 11, 2000

DE-6 (2/88)

Abstract

A well-known method for symbol detection involving signals received over a number of independent diversity branches perturbed by additive white Gaussian noise is Maximum Ratio Combining (MRC). Two other diversity schemes, Simple Transmit Diversity (STD) and Maximum Ratio Transmission (MRT), have recently been proposed. The performances of these three schemes are compared. The STD scheme, which is applicable only to two transmit antennas, is shown to have the same error performance as MRC with the same diversity order when perfect channel estimates are available. The MRT scheme with N transmit and one receive antennas is shown to have the same performance as MRC with N receive antennas, but MRT with one transmit and $N > 1$ receive antennas or MRT with $N > 1$ transmit and $M > 1$ receive antennas do not perform as well as MRC.

The performance degradations of MRC, STD and MRT due to errors in estimating the channel parameters are analyzed and compared. It is found that STD is significantly more susceptible than MRC and MRT to errors in the channel estimates.

An improved scheme for MRT (IMRT) and a new optimal maximum ratio transmission and combining scheme (MRTC) are proposed. In the IMRT scheme, the same transmit weighting functions are used as in MRT. At the receiver, MRC combining rules are used to choose the weights for the received signals. The MRTC scheme maximizes the SNR using optimal transmit and receive weighting factors. Simulation results indicate that both IMRT and MRTC provide significant performance improvements over MRT.

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Acknowledgment

I would like to express my sincere thanks and deep gratitude to my thesis advisor, Dr. Cyril Leung, for his guidance and encouragement. His critical reviews and many constructive suggestions were very essential to the completion of this work. This work was partially supported by NSERC Grant OGP0001731.

Friends and fellow students have certainly made my studies here a memorable and interesting one. I would like to thank Mr. Kelvin Ho, Mr. Cyril Iskander, Mr. Lawrence Chen, Mr. Peter Chong, Mr. Shailesh Sheoran, Mr. Dave Martin and other people in the Communication Group for their support and help.

Finally I would like to thank my parents, Mr. Xixian Feng and Mrs. Shuhua Liu, my sister, Ms. Xiaoli Feng and her family, for their constant love and encouragement. They, too, have played a significant role in the completion of this work.

Chapter 1 Introduction

In recent years there has been an explosive growth in the mobile communication market worldwide. This demand is expected to grow unabated over the next decade as new services are offered and new markets developed. Multipath fading is a major obstacle to the efficient and reliable transmission of data over many radio channels. Possible solutions to this problem are to increase the transmission power, antenna size, or antenna height [1]. These solutions may not be compatible with the need for portability and reduced energy consumption. Another standard technique which can be used to mitigate the effects of fading is diversity. The motivation behind diversity techniques is that we provide several independent paths, hence there are hopefully always some paths that may have strong signals so as to reduce the probability that all the signal components will fade simultaneously.

Depending on the propagation mechanism, there are several effective diversity techniques. Independent transmission paths suitable for the diversity method could be obtained by using different frequencies, different transmission times or spatially separated antennas. In most scattering environments, space diversity (antenna diversity) is a practical, effective and widely applied technique for reducing the effects of multipath fading [2]. All the discussion for diversity has been premised on the assumption that the fading processes among the diversity branches are mutually statistically independent. The absence of correlation between the branches is an important desired feature for diversity techniques, since it would not help the receiver to have additional copies of the signal if the copies are all equally poor [3]. In practice, there will be some cases in which this cannot be achieved, for example, insufficient antenna spacing (due to siting limitation in spaced antenna diversity). A number of papers have appeared in this subject area [4], [5], [6], [7]. Bit error rate (BER) performance of space diversity systems with channel correlation was studied in [4]. In [5], numerical results demonstrate the impact of the different correlation coefficients in combating fading and reducing co-channel interference (CCI). In [6], a maximum likelihood sequence estimation receiver structure was derived for the case of correlated diversity sources.

A brief introduction to alternative diversity techniques is given in the latter part of this chapter.

In Chapter 2, related studies found in the literature are reviewed. A classical approach, maximal ratio combining (MRC), involves the use of multiple antennas at the receiver. The signals received at the various antennas are weighted such that the signal-to-noise ratio (SNR) of their sum is maximized. The major problem with using MRC is the cost, size, and power of the remote units. Simple transmit diversity (STD) is a simple but effective scheme proposed by Alamouti [8]. In STD, a pair of symbols is transmitted using two antennas during the first time unit, and a transformed version of the pair is transmitted during the second time unit to obtain a MRC-like diversity. Two transmit and M receive antennas are used for the generalization of the STD scheme so as to provide a diversity of the order of $2M$. A scheme called maximum ratio transmission (MRT) is suggested by Lo [9]. MRT can be generalized to any number of antennas for both transmission and reception.

In Chapter 3, bit error rate (BER) performances for MRC, STD and MRT are compared. The BER degradations due to imperfect channel estimates are analyzed. Both BPSK and QPSK modulation methods are considered. The analysis starts with the simplest case, two branch diversity, and then is generalized to multiple branches.

In Chapter 4, an improved MRT (IMRT) scheme and a new optimal maximum ratio transmission and combining scheme (MRTC) are introduced. Symbol error rate (SER) performances of these two new schemes are presented and compared with those of MRC and MRT. A summary of the main results of this work and some suggestions for possible future work appear in Chapter 5.

1.1 Frequency Diversity

In frequency diversity, the information bearing signals is transmitted on more than one carrier frequency. If the frequency separation is larger than the coherence bandwidth, independent fading variations can be assumed [10]. In [11], coherence bandwidth is defined as a statistical measure of the range of frequencies over which the channel can be considered "flat". Those spectral components passed through the "flat" channel are subject to an approximately equal gain and a linear phase shift. For the mobile radio case with a coherence bandwidth on the order of $500kHz$, it has been measured that the separation between the branches has to be at least $1 \sim 2MHz$ [2]. The advantages of the frequency diversity over the space diversity is the reduction of the number of antennas. However, to achieve M branch diversity, the bandwidth and transmitting power required will be M times larger. Therefore, this technique is not commonly used for land mobile communication systems in which spectrum efficiency and power savings are important issues.

1.2 Time Diversity

In time diversity, the information bearing signal is repeatedly transmitted so that the multiple repetitions of the signal undergo nearly independent fading, thereby providing diversity.

It has been shown in [12], [13] that the required time slot interval is at least as great as the reciprocal of the fading bandwidth, or $0.5/f_d$, in the mobile radio case to obtain diversity branch signals, where f_d is the maximum Doppler frequency. Time diversity is effective for CDMA systems, where the multipath channel provides redundancy in the transmitted message. However, it is less effective when the channel is slowly varying because a very long time slot interval is necessary to obtain sufficient diversity gain. Moreover, when the mobile is stationary, we may obtain no diversity gain at all.

1.3 Space Diversity

Space Diversity is another effective approach to combat multipath fading. It has historically been the most commonly used form of diversity in mobile radio link base station [14]. Sufficiently spaced antennas are an attractive means of obtaining this diversity advantage since they do not incur bandwidth expansion. The method is based upon the principles of using two or more attennas in order to receive uncorrelated signals. Conventional cellular radio systems consist of elevated base station antennas and mobile antennas close to the ground. The existence of a direct path between the transmitter and the receiver is not guaranteed and the possibility of a number of scatterers in the vicinity of the mobile suggests a Rayleigh fading signal. At the mobiles, an antenna spacing greater than $\lambda/2$ is sufficient to achieve very low fading correlation between branches [10], [14], [15], whereas 50λ and 100λ are necessary at the base station [16]. Space diversity can be implemented at either the mobile terminal or base station, or both, depending on the particular combining technique used and degree of the signal enhancement required. In [17], the research results show that theoretically, space diversity (with optimum combining) can substantially increase the capacity of most interference limited wireless communication systems.

The following combining techniques have been considered in the literature.

- Selection Diversity
- Feedback Diversity
- Equal Gain Diversity
- Maximal Ratio Combining

1.3.1 Selection Diversity

Referring to Figure 1.1, M receivers are used to achieve M branch diversity. The diversity branch having the highest instantaneous signal to noise ratio (SNR) is connected to the output [2]. In practice, a selection diversity system cannot function on a truly instantaneous basis. It must be designed so that the internal time constants of the selection circuitry are shorter than the reciprocal of the signal fading rate [11]. The drawback of this scheme is its sub-optimal performance since it does not use all of the branch channel information.

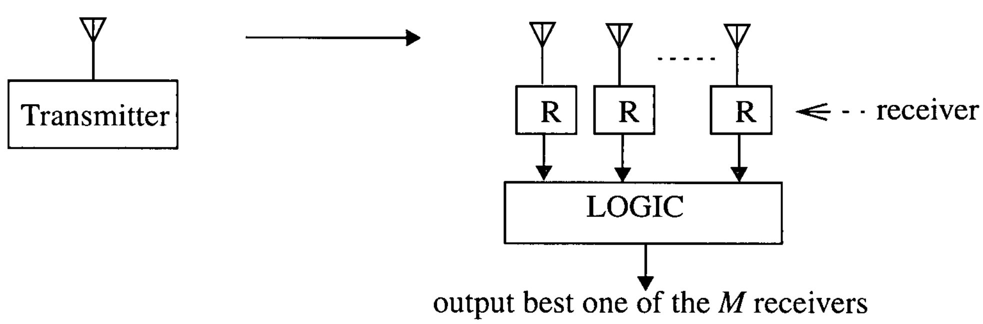


Figure 1.1 Principles of selection diversity from [2].

1.3.2 Feedback Diversity

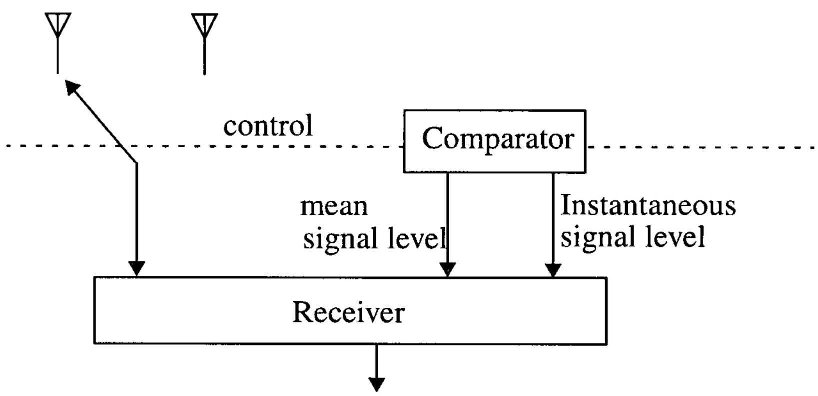


Figure 1.2 Basic form of feedback diversity.

As shown in Figure 1.2, the feedback diversity scheme is similar to selection diversity. A feedback link is provided to switch between transmitting antennas at the remote station within a limited amount of time delay. Instead of always using the best of M signals, the M signals are scanned in a fixed sequence until one is found which is above a predetermined threshold. This signal is then demodulated until it falls below a certain threshold and the scanning process is initiated again. This scheme avoids excessive switching when both antennas are in simultaneous fades [11]. The performance of this scheme is not as good as those obtained by other methods but it requires only one receiver.

1.3.3 Equal Gain Diversity

In equal gain combining, signals from all the branches are coherently combined using the same weighting factor. The signals received over the diversity channels are co-phased and added up. The performance of this method is marginally inferior to that of maximal ratio combining and superior to that of selection diversity.

1.3.4 Maximal Ratio Combining

It was stated that in selection diversity only one of the diversity branch signals is used for demodulation. In contrast, in MRC, all the branch information is used to improve the overall performance. The signals from the received antenna elements are co-phased and weighted so as to maximize the overall SNR [1]. Owing to the recent development of the pilot signal-aided scheme as well as DSP technologies, the maximum combining schemes can be implemented with a simple hardware configuration [18], which is especially effective for base station to implement three branch diversity or more.

Chapter 2 Performance Comparison with Perfect Channel Estimation

The maximal ratio combining (MRC) approach uses a maximal-ratio combining receiver to process the signals received at multiple antennas. The signals from the received antenna elements are cophased and weighted according to their individual signal voltage to noise power ratios. The realization of this combiner is based on the assumption that the channel state is known perfectly [19].

In some applications, e.g. the third generation cellular communication systems currently under development, such a receive diversity scheme may not be desirable for the mobile handsets because of cost, size and power considerations. A simple transmit diversity (STD) scheme [8], which uses two transmit antennas and M receive antennas, was proposed. There is no feedback required from the receiver to the transmitter. It was shown that, for a fixed level of radiated power per transmit antenna, this STD scheme has the same BER as MRC with the same diversity order. The STD scheme can be generalized so as to include any number of receive antennas, but cannot be easily extended to more than two transmit antennas.

The maximum ratio transmission (MRT) scheme [9] can be applied to any number, N, of transmit antennas and any number, M, of receive antennas, even though feedback is required from the receiver so that the transmitter can estimate the channel. For convenience, we refer to this as a ($N \times M$) MRT scheme.

Those aspects of the MRC, STD and MRT schemes which are necessary in this study are briefly reviewed in this section. Following [8], a complex baseband representation of the systems is used. The channel diversity branch from antenna n at the transmitter to antenna m at the receiver is denoted by $h_{nm} = \alpha_{nm} e^{j\theta_{nm}}$, $(n = 1 \ldots N, m = 1 \ldots M)$, where α_{nm} is the amplitude gain of the diversity branch and θ_{nm} is the phase distortion introduced by the channel.

Systems that coherently combine independent signals from spatially separated antennas have better carrier statistics and less random FM than selection diversity systems [2]. Coherent combining systems do not suffer degradation from phase transients that are inherent in antenna-switching systems. It has been shown in [12] that absolute phase coherent detection gives the best theoretical BER performance for BPSK and QPSK with a given number of diversity branches under flat Rayleigh fading conditions. Therefore, Coherent detection is considered to be more desirable when a large number of diversity branches are employed, and is used for those schemes reviewed in this section. Noncoherent detection is also used in practice. In this study, we use coherent detection.

2.1 Review of Maximal Ratio Combining (MRC)

Figure 2.1 shows the baseband representation of the MRC scheme with a diversity order of M. The signal received on branch i corresponding to the transmission of a signal s_0 is

$$r_{1i, MRC} = h_{1i} s_0 + n_i, \; i = 1 \ldots M, \tag{2.1}$$

where $\{n_i, i = 1 \ldots M\}$ are outcomes of independent complex Gaussian random variables (r.v.) representing noise and interference.

Through the thesis, we use uppercase letters to denote r.v.'s and the corresponding lowercase letters to denote their samples. The combined signal $\tilde{s}_{0,\,MRC}$ is then defined as

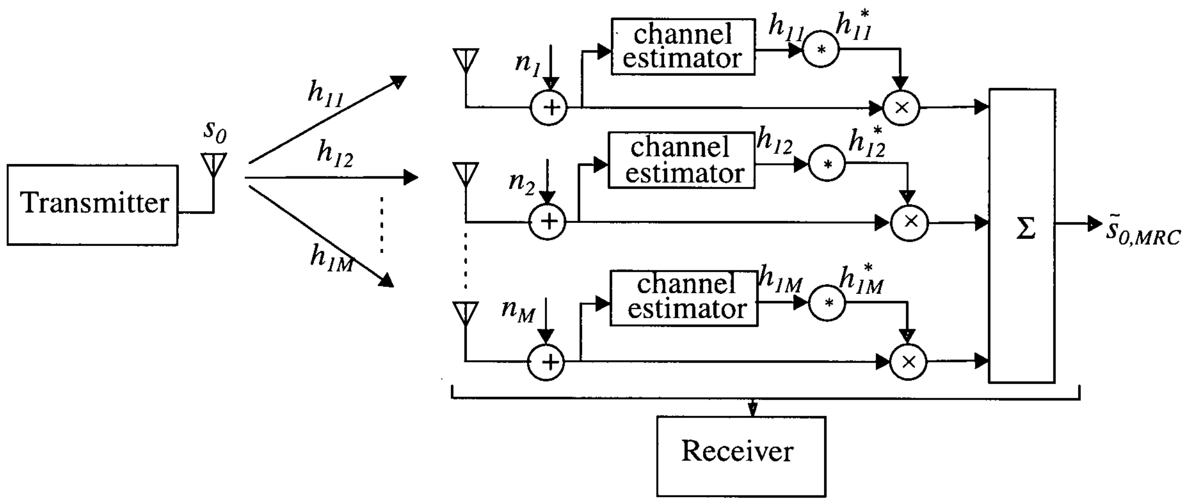


Figure 2.1 MRC with M receive antennas.

$$
\begin{aligned}
\tilde{s}_{0,\,MRC} &= \sum_{i=1}^{M} h_{1i}^{*} r_{1i,\,MRC} \\
&= \sum_{i=1}^{M} h_{1i}^{*}(h_{1i}s_0 + n_i) \\
&= \left(\sum_{i=1}^{M} \alpha_{1i}^{2}\right)s_0 + \sum_{i=1}^{M} h_{1i}^{*}n_i \ ,
\end{aligned} \tag{2.2}
$$

where h_{1i}^{*} denotes the complex conjugate of h_{1i}. The theoretical analysis of the error performance for a binary digital communications system with diversity has been discussed in [1][19] assuming Rayleigh faded diversity channels. The instantaneous output signal to noise ratio (SNR) from the combiner was shown to be the sum of the instantaneous SNR's on the individual branches,

$$\gamma = \sum_{i=1}^{M} \gamma_{1i}. \tag{2.3}$$

With coherent binary PSK, the output of the maximal ratio combiner can be expressed as a single decision variable in the form

$$U_{MRC} = Re(\tilde{S}_{0,MRC}), \tag{2.4}$$

where $Re(\tilde{S}_{0,MRC})$ denotes the real part of $\tilde{S}_{0,MRC}$. The data bits to be transmitted are assumed to be independent and equally likely to be 0 or 1.

The BER is the probability of that U_{MRC} is less than zero, i.e.

$$P(\gamma) = Q(\sqrt{2\gamma}), \tag{2.5}$$

where

$$\gamma = \frac{\zeta}{\sigma_N^2}\left(\sum_{i=1}^{M} \alpha_{1i}^2\right), \tag{2.6}$$

where ζ is the signal energy, and σ_N^2 is the variance of the real or imaginary components of N_i. The noise variance σ_N^2 has been assumed to be identical for all branches. It is assumed that the average energy gain of each diversity channel is the same, i.e. $E(\alpha_{1i}^2) = x$. The average SNR per branch is then written as

$$\gamma_0 = \frac{\zeta}{\sigma_N^2} E(\alpha_{1i}^2). \tag{2.7}$$

The p.d.f of Γ, $f_\Gamma(\gamma)$, is derived in [1][19] as

$$f_\Gamma(\gamma) = \frac{1}{(M-1)!} \frac{\gamma^{M-1}}{\gamma_0{}^M} e^{-\gamma/\gamma_0}. \tag{2.8}$$

The BER can be evaluated by averaging the conditional error probability given in (2.5) over the fading channel statistics in (2.8), i.e.

$$P_e = \int_0^\infty P(\gamma) f_\Gamma(\gamma) d\gamma. \tag{2.9}$$

The integral of (2.9) can be simplified as indicated in [20]

$$p_e = \left[\frac{1}{2}(1-u)\right]^M \sum_{i=0}^{M-1} \left\{ \binom{M-1+i}{i} \left[\frac{1}{2}(1+u)\right]^i \right\}, \tag{2.10}$$

where

$$u = \sqrt{\frac{\gamma_0}{1+\gamma_0}}. \tag{2.11}$$

2.2 Review of Simple Transmit Diversity (STD)

The system with two transmit and one receive antennas is introduced in this section. The generalization to a system with diversity order of $2M$ can be achieved by using the combiner for one receive antenna and adding the combined signals from M receive antennas.

2.2.1 Two-Branch STD

In the STD scheme, two bits represented by s_0 and s_1 are sent simultaneously during two consecutive bit periods. In the first bit period, s_0 is sent from antenna A and s_1 is sent from antenna B. In the second bit period, the signals sent from antennas A and B are $-s_1^*$ and s_0^* respectively.

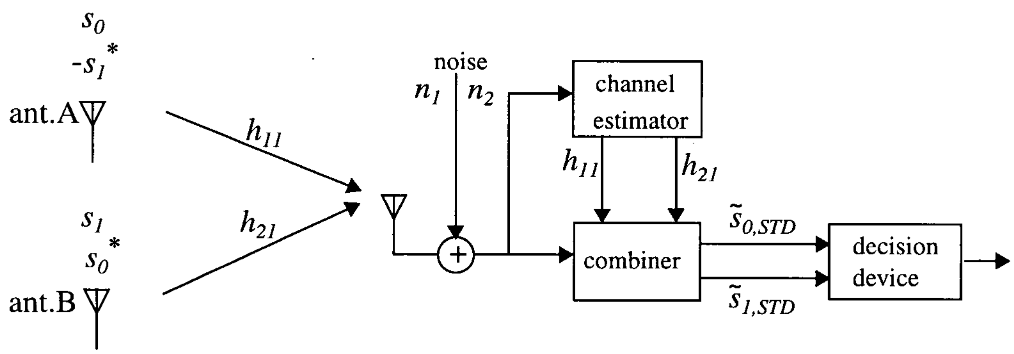


Figure 2.2 The two-branch STD scheme with one receiver.

Figure 2.2 shows the baseband representation of the STD scheme with a diversity order of two. Assuming that the channel fading does not change significantly during two consecutive bit periods, the received signals are

$$\begin{aligned} r_{0,STD} &= h_{11}s_0 + h_{21}s_1 + n_1 \\ r_{1,STD} &= -h_{11}s_1^* + h_{21}s_0^* + n_2 \end{aligned} \tag{2.12}$$

where n_1 and n_2 are complex random variables representing receiver thermal noise and interference.

It is proposed in [8] that the decoding of s_0 and s_1 be based on $\tilde{s}_{0,\,STD}$ and $\tilde{s}_{1,\,STD}$ respectively, where

$$\begin{aligned}
\tilde{s}_{0,\,STD} &= h_{11}^{*} r_{0,\,STD} + h_{21} r_{1,\,STD}^{*} \\
&= (\alpha_{11}^2 + \alpha_{21}^2)\; s_0 + h_{11}^{*} n_1 \; + h_{21} n_2^{*} \\
\tilde{s}_{1,\,STD} &= h_{21}^{*} r_{0,\,STD} - h_{11} r_{1,\,STD}^{*} \\
&= (\alpha_{11}^2 + \alpha_{21}^2)\; s_1 - h_{11} n_1^{*} + h_{21}^{*} n_2 \,.
\end{aligned} \tag{2.13}$$

Those combined signals are then sent to the decision device. The STD scheme yields the same BER as MRC for a fixed value of the power radiated per transmit antenna *assuming that the channel gains h_{11} and h_{21} can be perfectly estimated by the receiver.*

2.2.2 STD With Two Transmit and M Receive Antennas

Two transmit and M receive antennas can be used to yield a diversity order of $2M$. The case of two transmit and two receive antennas is shown in Figure 2.3.

The encoding and transmission sequence of the information symbols are identical to the case of one receiver. The received signals at the two receive antennas are:

$$\begin{aligned}
r_{1,\,STD} &= h_{11} s_0 + h_{21} s_1 + n_1 \\
r_{2,\,STD} &= -h_{11} s_1^{*} + h_{21} s_0^{*} + n_2 \\
r_{3,\,STD} &= h_{12} s_0 + h_{22} s_1 + n_3 \\
r_{4,\,STD} &= -h_{12} s_1^{*} + h_{22} s_0^{*} + n_4,
\end{aligned} \tag{2.14}$$

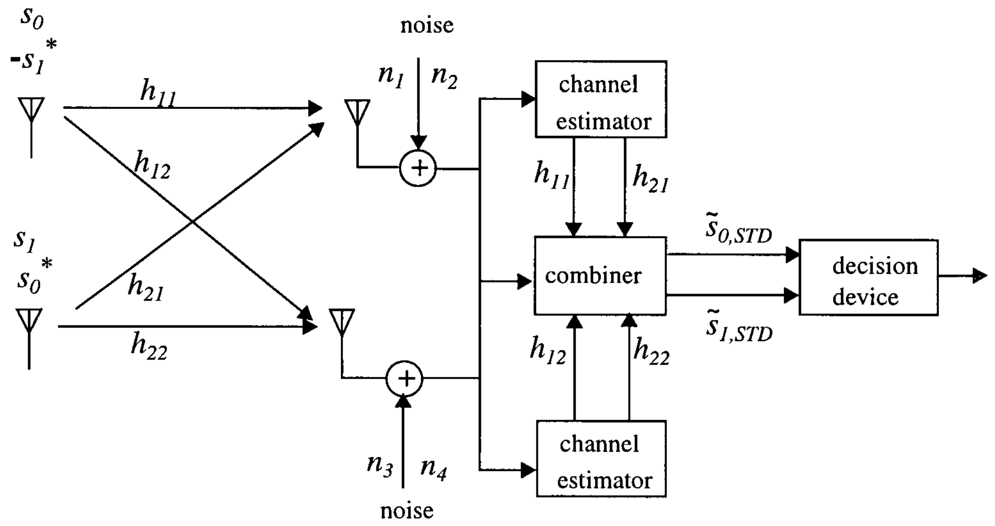


Figure 2.3 STD with two receive antennas.

where $r_{1,\,STD}$ and $r_{3,\,STD}$ are the signals received in the first symbol period, $r_{2,\,STD}$ and $r_{4,\,STD}$ are the received signals corresponding to the second symbol period. The output signals from the combiner are achieved as:

$$\begin{aligned}
\tilde{s}_{0,\,STD} &= h_{11}^* r_{1,\,STD} + h_{21} r_{2,\,STD}^* + h_{12}^* r_{3,\,STD} + h_{22} r_{4,\,STD}^* \\
&= (\alpha_{11}^2 + \alpha_{12}^2 + \alpha_{21}^2 + \alpha_{22}^2)\, s_0 + h_{11}^* n_1 + h_{21} n_2^* + h_{12}^* n_3 + h_{22} n_4^* \\
\tilde{s}_{1,\,STD} &= h_{21}^* r_{1,\,STD} - h_{11} r_{2,\,STD}^* + h_{22}^* r_{3,\,STD} - h_{12} r_{4,\,STD}^* \\
&= (\alpha_{11}^2 + \alpha_{12}^2 + \alpha_{21}^2 + \alpha_{22}^2)\, s_1 - h_{11} n_1^* + h_{21}^* n_2 - h_{12} n_4^* + h_{22}^* n_3\,.
\end{aligned} \tag{2.15}$$

Equation (2.15) shows that those two combined signals are the summation of the combined signals from each receive antenna. The combiner with two transmit and M receive antennas can be built by using the combiner for each receive antenna, and then adding the combined signals from all the receive antennas to obtain a diversity order of $2M$.

2.3 Review of Maximum Ratio Transmission (MRT)

The MRT scheme [9] can be applied to a system with N transmit and M receive antennas. The channel can be represented by a channel coefficient matrix,

$$\boldsymbol{H} = \begin{bmatrix} h_{11} & \dots & h_{1M} \\ \vdots & \ddots & \vdots \\ h_{N1} & \dots & h_{NM} \end{bmatrix} \tag{2.16}$$

where h_{nm}, $n = 1, 2, \dots, N$, $m = 1, 2, \dots, M$ represents the channel from antenna n at the transmitter to the antenna m at the receiver.

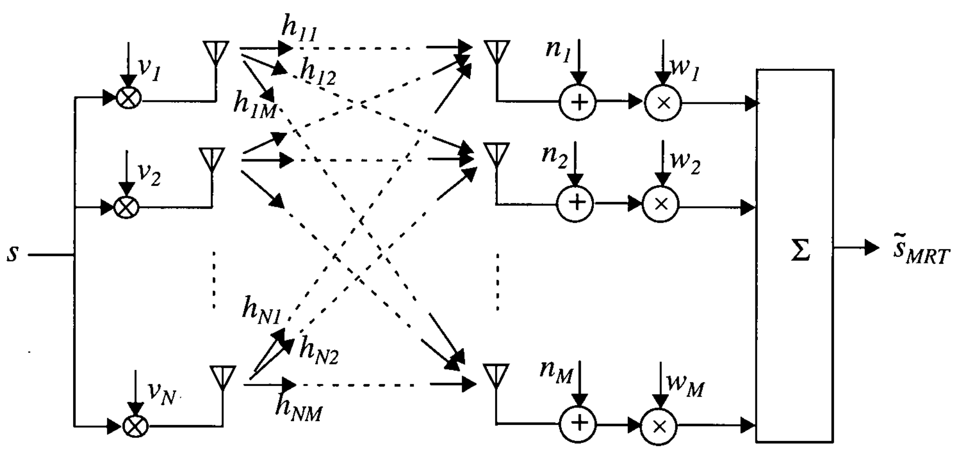


Figure 2.4 MRT with N transmit and M receive antennas.

As shown in Figure 2.4, the source signal s is weighted by a $(N \times 1)$ transmit weighting vector $\underline{V}$ before transmission, where $\underline{V}$ is defined as

$$\underline{V} = \frac{1}{a}(\boldsymbol{H}\underline{W})^H. \tag{2.17}$$

where $a = |\boldsymbol{H}\underline{W}|$, which is the length of the vector $\boldsymbol{H}\underline{W}$, is a normalization factor to ensure that the transmission power is normalized to an average value, and $\underline{W}$ is a $(M \times 1)$ receive weighting vector. The superscript ^{H} in (2.17) denotes the Hermitian operation, i.e. complex conjugate transposed. The transmitted signal vector can be expressed as

$$\underline{s}_t = \frac{s}{a}(\boldsymbol{H}\underline{W})^H, \tag{2.18}$$

and the received signal vector as

$$\underline{r} = \frac{s}{a}(\boldsymbol{H}\underline{W})^H\boldsymbol{H} + \underline{n}, \tag{2.19}$$

where $\underline{n} = [n_1, \ldots, n_M]$ is the noise vector. The received vector components are then weighted and sent to the combiner. The estimate of the signal is given by

$$\begin{aligned} \tilde{s}_{MRT} &= \frac{s}{a}(\boldsymbol{H}\underline{W})^H\boldsymbol{H}\underline{W} + \underline{n}\underline{W} \\ &= as + \underline{n}\underline{W} \end{aligned} \tag{2.20}$$

The overall output SNR can be written as

$$\gamma = \frac{a^2}{\underline{W}\,\underline{W}^H}\gamma_0 = \frac{a^2\gamma_{0,\,MRT}}{\sum_{i=1}^{M}|w_i|}, \tag{2.21}$$

where $\gamma_{0,\,MRT} = \zeta/\sigma_N^2$. To maximize the γ in (2.21), It is concluded in [9] that $\underline{W}$ should satisfy the condition $|w_1| = |w_2| = \ldots = |w_M|$, and

$$(w_p w_q^*)^* = \frac{\sum_{i=1}^{N} h_{pi} h_{qi}^*}{\left|\sum_{i=1}^{N} h_{pi} h_{qi}^*\right|}, \tag{2.22}$$

where p and q take on values in the set $\{1, 2, \ldots, N\}$.

2.4 Performance Comparison with Perfect Channel Estimates

The BER's of MRC, STD and MRT with diversity orders of two and four are compared in this section. It is assumed that each channel undergoes independent slow fading, whose amplitude is Rayleigh distributed. The interferences and noises are modeled as additive complex white Gaussian random variables. The complex channel gains h_{nm}, $n = 1, 2, \ldots, N$, $m = 1, 2, \ldots, M$ can be perfectly estimated by the receiver and/or the transmitter.

The theoretical BER performance of MRC is given by (2.10), and plotted in Figure 2.5. The simulation results for two-branch MRC and STD are also shown and agree very closely with the theoretical MRC curve.

Figure 2.6 shows BER's for two-branch MRC and MRT with uncoded coherent BPSK. It can be seen that the diversity schemes provide substantial improvement over the no diversity case. The improvement increases with SNR. For BER= 10^{-2}, there is an improvement of about 8 dB. MRC and STD are about 0.5 dB more efficient than MRT($N \times M = 1 \times 2$) at BER=10^{-2}. From Figure 2.5 and Figure 2.6, we can see that STD and ($N \times M = 2 \times 1$) MRT have the same perfor-

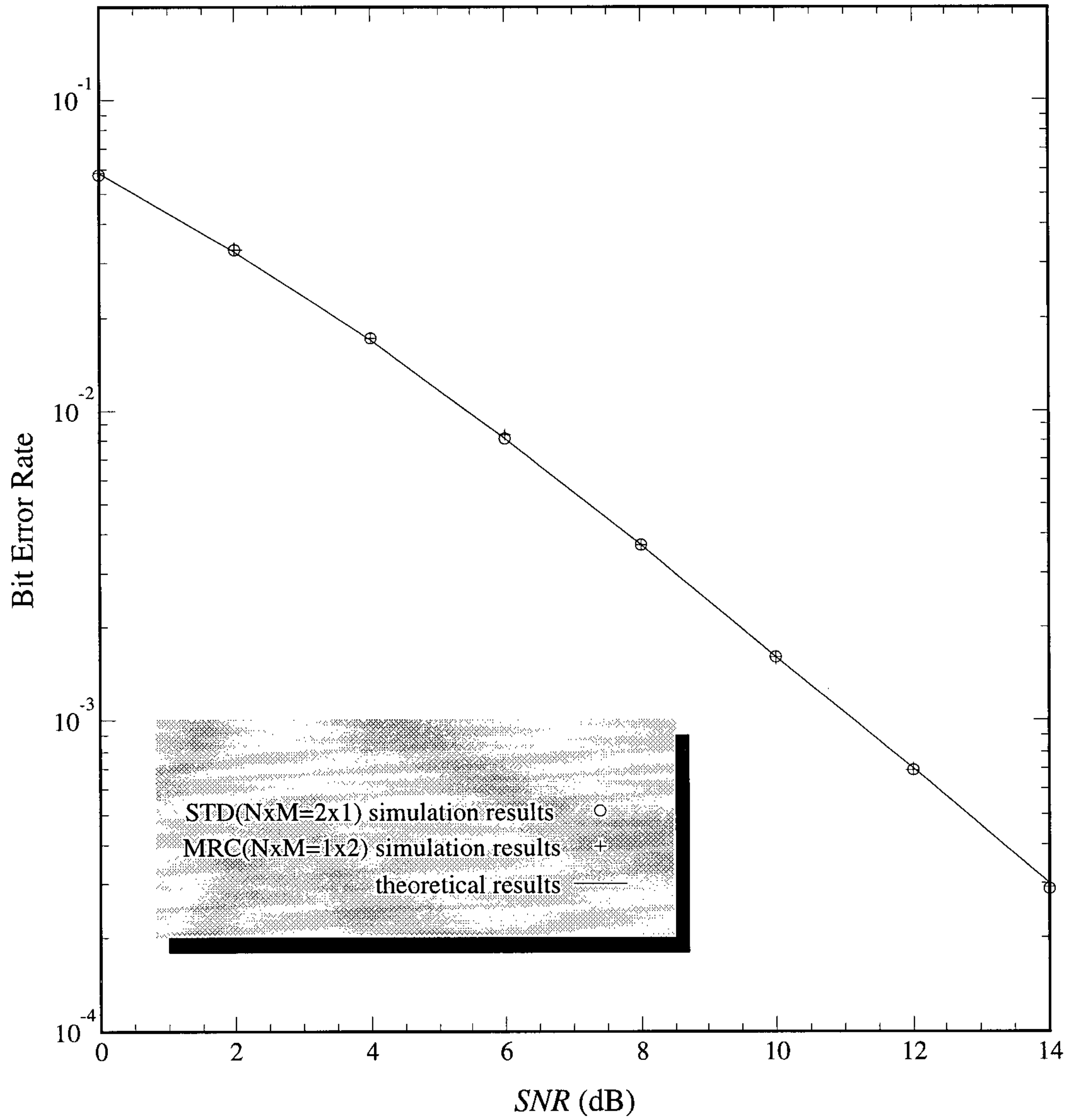
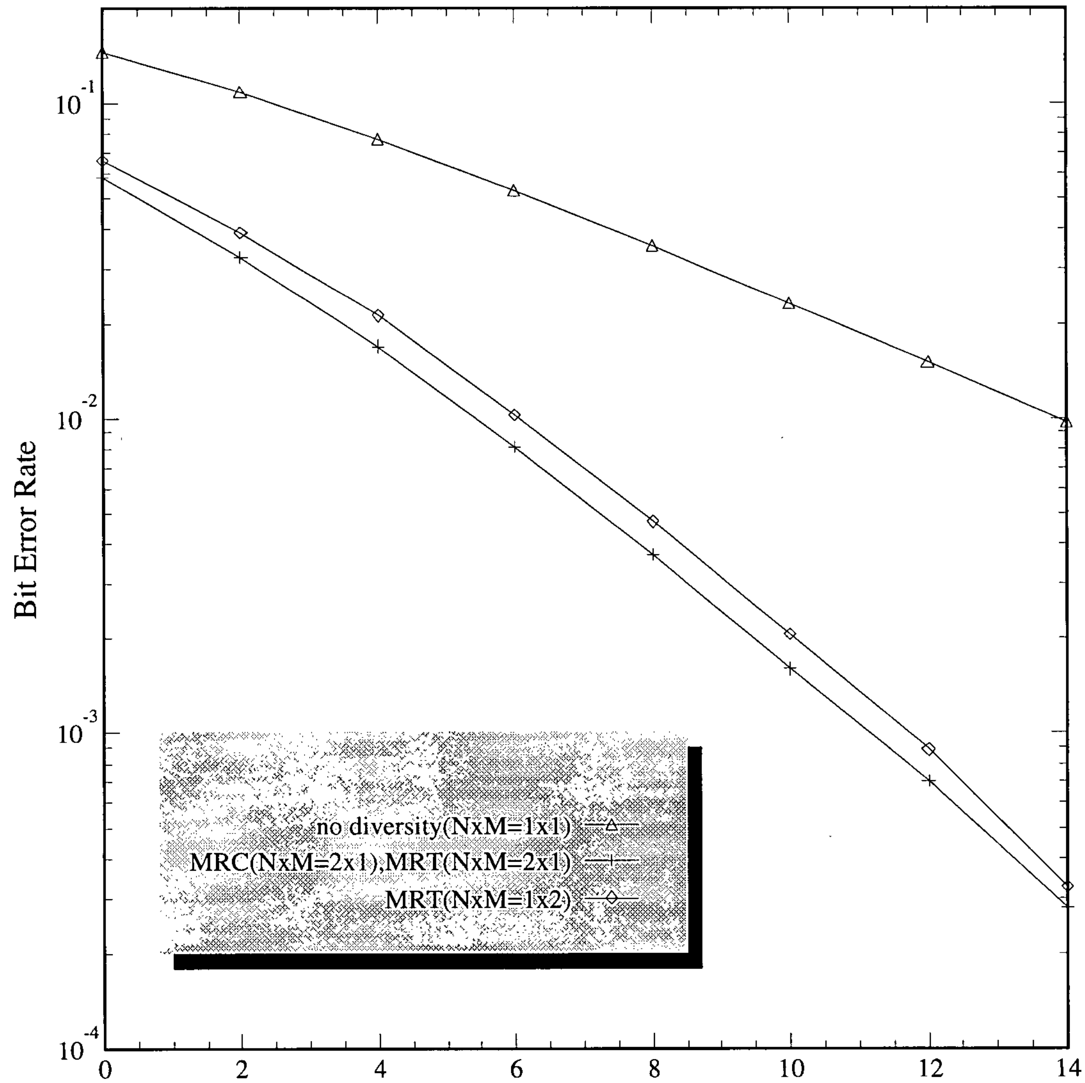


Figure 2.5 Comparison of theoretical and simulation results.

mances as $(N \times M = 1 \times 2)$ MRC. This is proved at the end of this section. It is interesting to note that the BER performance of $(N \times M = 1 \times 2)$ MRT is worse than that of $(N \times M = 2 \times 1)$ MRT.

The symbol error rate (SER) curves for diversity order four MRC, STD and MRT with QPSK modulation are plotted in Figure 2.7. It can be seen that MRC$(N \times M = 1 \times 4)$,



SNR (dB)

Figure 2.6 BER's of MRC and MRT schemes with diversity order of two.

MRT$(N \times M = 4 \times 1)$, and STD$(N \times M = 2 \times 2)$ have the same performances for a given value of the average power radiated per transmit antenna as shown below. For MRT with diversity order of four, the SER performances are given for three different cases (i.e. $(N \times M = 1 \times 4)$, $(N \times M = 2 \times 2)$ and $(N \times M = 4 \times 1)$). Comparing these cases, it can be seen that MRT with only transmit diversity $(N \times M = 4 \times 1)$ gives the same SER performance as MRC, and that

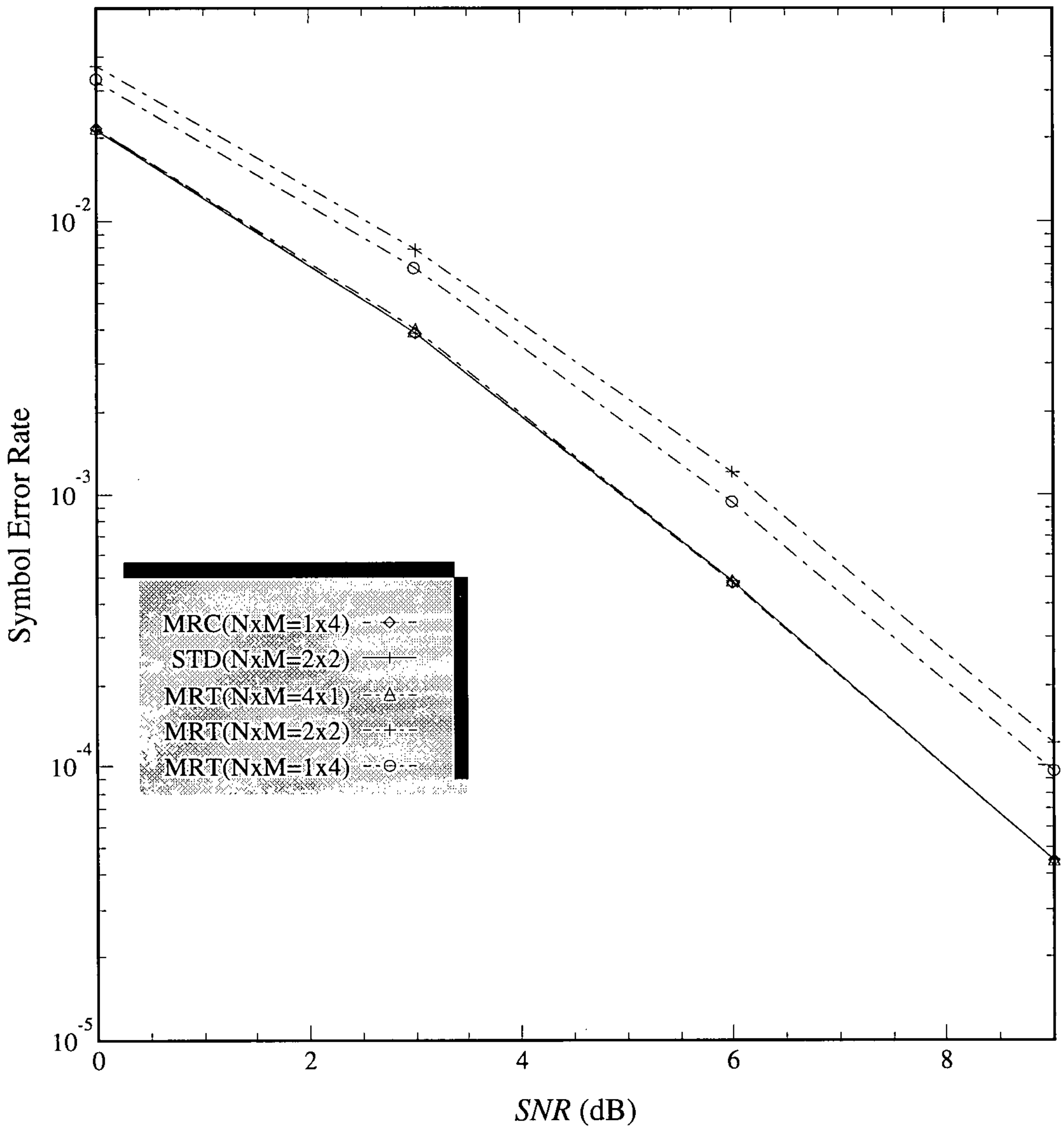


Figure 2.7 SER's of MRC, STD and MRT with diversity order of four.

MRT with both the transmit and receive diversity ($N \times M = 2 \times 2$) gives the worst SER performance. The performance of MRT with ($N \times M = 1 \times 4$) is inferior to that of ($N \times M = 4 \times 1$) MRT and superior to that of ($N \times M = 2 \times 2$). For SER=10^{-3}, there is a 0.9 dB degradation for MRT($N \times M = 1 \times 4$) and 1.3 dB for MRT($N \times M = 2 \times 2$) compared to

$\mathrm{MRT}(N \times M = 4 \times 1)$.

From Figure 2.6 and Figure 2.7, we notice that for $(N \times 1)$ MRT, the error probability is the same as that of MRC with the same diversity order. For $(N \times 1)$ MRT, the weighting function, w_1, at the receiver is set to 1 for convenience. Then, we have from (2.21)

$$\gamma = \frac{\zeta}{\sigma_N^2} a^2, \tag{2.23}$$

where $a^2 = |\boldsymbol{H}|^2 = \sum_{i=1}^{M} \alpha_{i1}^2$. The overall SNR γ can then be written as

$$\gamma = \frac{\zeta}{\sigma_N^2}\left(\sum_{i=1}^{M} \alpha_{i1}^2\right), \tag{2.24}$$

which is the same as (2.6), the output SNR from MRC combiner. From (2.9), we can see that P_e is a function of $f_\Gamma(\gamma)$. Since the output SNR p.d.f. for both $(1 \times N)$ MRC and $(N \times 1)$ MRT are the same, they have the same error performance.

Chapter 3 Performance with Imperfect Channel Estimation

It is of practical importance to study the performance degradations which result from the use of imperfect channel estimates. The BER's of MRC, STD and MRT with channel estimation errors are now analyzed. Two modulation methods BPSK and QPSK are considered in the analysis. We begin with the performance analysis for those three schemes. The channel model to be used in the analysis is then described. Performance results are compared in the last section. A two-branch diversity model is discussed in this section.

The estimated complex channel gains are expressed as

$$\hat{h}_{nm} = h_{nm} + z_{nm}, n, m = 1, 2 \ , \tag{3.1}$$

where $z_{nm} = \beta_{nm} e^{j\phi_{nm}}$ represents the estimation error for the diversity branch from the nth transmit antenna to the mth receive antenna.

3.1 BPSK Modulation

We first derive the BER for given values of $h_{nm}, n, m= 1, 2$ and $z_{nm}, n, m= 1, 2$. The results can then be used to obtain the BER when h_{nm} and z_{nm} are drawn according to any arbitrary probability distribution. It can be shown in [21] that the maximum likelihood decision rule, which minimizes the BER, is equivalent to choosing $s_0 \doteq 1$ if $Re(\tilde{s}_0) > 0$ and choosing $s_0 = -1$ otherwise, where $\tilde{s}_0$ denotes the combined signal.

3.1.1 MRC

The decision r.v. for MRC is denoted by $U_{MRC} = Re(\tilde{S}_{0,MRC})$. For given values of s_0, h_{11}, h_{12}, z_{11} and z_{12}, we have from (2.2) and (3.1) that

$$\begin{aligned}\tilde{S}_{0,MRC} &= (h_{11}+z_{11})^* r_{0,MRC} + (h_{12}+z_{12})^* r_{1,MRC} \\ &= (\alpha_{11}^2+\alpha_{12}^2)s_0 + z_{11}^* h_{11} s_0 + z_{12}^* h_{12} s_0 \\ &\quad + (h_{11}+z_{11})^* N_1 + (h_{12}+z_{12})^* N_2 .\end{aligned} \tag{3.2}$$

After simplification, the mean and variance of the decision r.v. can be written as

$$\begin{aligned}E(U_{MRC}) &= [(\alpha_{11}^2+\alpha_{12}^2) + \alpha_{11}\beta_{11}\cos(\theta_{11}-\phi_{11}) \\ &\quad + \alpha_{12}\beta_{12}\cos(\theta_{12}-\phi_{12})]s_0 ,\end{aligned} \tag{3.3}$$

and

$$\begin{aligned}\sigma_{U_{MRC}}^2 &= [\alpha_{11}^2+\beta_{11}^2+2\alpha_{11}\beta_{11}\cos(\theta_{11}-\phi_{11}) \\ &\quad + \alpha_{12}^2+\beta_{12}^2+2\alpha_{12}\beta_{12}\cos(\theta_{12}-\phi_{12})]\sigma_N^2 .\end{aligned} \tag{3.4}$$

Since U_{MRC} has a Gaussian distribution, the BER is given by

$$P_{e,MRC} = Q\left(\frac{a}{\sigma_{U_{MRC}}}\right), \tag{3.5}$$

where

$$a = (\alpha_{11}^2+\alpha_{12}^2) + \alpha_{11}\beta_{11}\cos(\theta_{11}-\phi_{11}) + \alpha_{12}\beta_{12}\cos(\theta_{12}-\phi_{12}), \tag{3.6}$$

and $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-y^2/2}dy$. A detailed derivation of (3.3) and (3.4) is given in Appendix A.

3.1.2 STD

Recall that in the STD scheme, two bits s_0 and s_1 are simultaneously transmitted. By symmetry, the BER's for both bits are equal so that we only need to consider the BER for one of the bits, say bit s_0. Denote the corresponding decision r.v. by $U_{STD} = Re(\tilde{S}_{0,STD})$. For given values of s_0, s_1, h_{11}, h_{21}, z_{11} and z_{21}, we have from (2.13) and (3.1) that

$$\begin{aligned}\tilde{S}_{0,STD} &= (h_{11}+z_{11})^* r_{0,STD} + (h_{21}+z_{21}) r_{1,STD}^* \\ &= (\alpha_{11}^2+\alpha_{21}^2)s_0 + z_{11}^* h_{11} s_0 + z_{21} h_{21}^* s_0 + z_{11}^* h_{21} s_1 \\ &\quad - z_{21} h_{11}^* s_1 + (h_{11}+z_{11})^* N_1 + (h_{21}+z_{21}) N_2^* .\end{aligned} \tag{3.7}$$

After simplification (see details in Appendix A.), it can be shown that the mean of the decision r.v. can be written as

$$\begin{aligned}E(U_{STD}) = {} & [(\alpha_{11}^2+\alpha_{21}^2) + \alpha_{11}\beta_{11}\cos(\theta_{11}-\phi_{11}) + \alpha_{21}\beta_{21}\cos(\theta_{21}-\phi_{21})]s_0 \\ & +[\alpha_{21}\beta_{11}\cos(\theta_{21}-\phi_{11}) - \alpha_{11}\beta_{21}\cos(\theta_{11}-\phi_{21})]s_1 .\end{aligned} \tag{3.8}$$

The variance of U_{STD} can be shown to be the same as the variance, $\sigma_{U_{MRC}}^2$, of U_{MRC} as given in (3.4). Since U_{STD} has a Gaussian distribution, the BER for STD is given by

$$P_{e,STD} = \frac{1}{2}\left[Q\left(\frac{a+b}{\sigma_{U_{MRC}}}\right) + Q\left(\frac{a-b}{\sigma_{U_{MRC}}}\right)\right], \tag{3.9}$$

where a is given by (3.6) and

$$b = \alpha_{21}\beta_{11}\cos(\theta_{21}-\phi_{11}) - \alpha_{11}\beta_{21}\cos(\theta_{11}-\phi_{21}). \tag{3.10}$$

3.1.3 MRT

It was shown in Section 2.4 that the BER performance for the $(N \times M = 1 \times 2)$ case is worse than that for the $(N \times M = 2 \times 1)$ case. Furthermore, the $(N \times M = 2 \times 1)$ case has the same performance as two-branch MRC and STD, when perfect channel estimates are available. The $(N \times M = 2 \times 1)$ case is considered in the following theoretical analysis to compare the system sensitivity performance with MRC and STD.

From the description of MRT$(N \times M = 2 \times 1)$ in Section 2.3, the weighting functions at the transmitter can be written as

$$\begin{aligned} v_1 &= \frac{h_{11}^*}{\sqrt{|h_{11}|^2 + |h_{21}|^2}} \\ v_2 &= \frac{h_{21}^*}{\sqrt{|h_{11}|^2 + |h_{21}|^2}} \end{aligned} \tag{3.11}$$

The weighting function at the receiver can be normalized to 1 in this case. When the channel estimates are erroneous, the estimated $\hat{v}_i$, $i = 1, 2$ can be expressed as

$$\begin{aligned} \hat{v}_1 &= \frac{(h_{11}+z_{11})^*}{\sqrt{|h_{11}+z_{11}|^2 + |h_{21}+z_{21}|^2}} \\ \hat{v}_2 &= \frac{(h_{21}+z_{21})^*}{\sqrt{|h_{11}+z_{11}|^2 + |h_{21}+z_{21}|^2}} \end{aligned} \tag{3.12}$$

From Section 2.3, the combined signal r.v. can then be written as

$$\begin{aligned} \tilde{S}_{0,MRT} &= \hat{v}_1 s_0 h_{11} + \hat{v}_2 s_0 h_{21} + N_1 \\ &= \frac{|h_{11}|^2 + h_{11} z_{11}^* + |h_{21}|^2 + h_{21} z_{21}^*}{\sqrt{|h_{11} + z_{11}|^2 + |h_{21} + z_{21}|^2}} s_0 + N_1 . \end{aligned} \tag{3.13}$$

The corresponding decision r.v. is denoted as $U_{MRT} = Re(\tilde{S}_{0,MRT})$. For given values of s_0, h_{11}, h_{21}, z_{11} and z_{21}, we have from (3.13) that

$$\begin{aligned} E(U_{MRT}) &= \frac{|h_{11}|^2 + |h_{21}|^2 + Re(h_{11} z_{11}^* + h_{21} z_{21}^*)}{\sqrt{|h_{11} + z_{11}|^2 + |h_{21} + z_{21}|^2}} s_0 \\ &= \frac{[(\alpha_{11}^2 + \alpha_{21}^2) + \alpha_{11}\beta_{11}\cos(\theta_{11} - \phi_{11}) + \alpha_{21}\beta_{21}\cos(\theta_{21} - \phi_{21})] s_0}{\sqrt{\alpha_{11}^2 + \beta_{11}^2 + 2\alpha_{11}\beta_{11}\cos(\theta_{11} - \phi_{11}) + \alpha_{21}^2 + \beta_{21}^2 + 2\alpha_{21}\beta_{21}\cos(\theta_{21} - \phi_{21})}} \end{aligned} \tag{3.14}$$

and $\sigma_{U_{MRT}}^2 = \sigma_N^2$. With (3.4) and (3.6), the mean of U_{MRT} can be expressed as

$$E(U_{MRT}) = \frac{a}{\sigma_{U_{MRC}} / \sigma_N} . \tag{3.15}$$

The BER for MRT is then given by

$$P_{e,MRT} = Q\left(\frac{a}{\sigma_{U_{MRC}}}\right). \tag{3.16}$$

3.1.4 Channel Model

As described in Section 2.4, a Rayleigh faded channel model is assumed. The channel gains h_{nm}, $n = 1 \ldots N$, $m = 1 \ldots M$ are then modeled as complex Gaussian r.v.'s. The fading processes among the $(N \times M)$ diversity channels are assumed to be independent. At the receiver, an ideal coherent detector is assumed. The signal in each branch is corrupted by an additive zero-mean white Gaussian noise process.

3.1.5 Numerical Results with BPSK modulation

From (3.5) and (3.16), it can be seen that $(N \times M = 1 \times 2)$ MRC and $(N \times M = 2 \times 1)$ MRT have identical channel error sensitivity performances. For two-branch STD, using the symmetry of the $Q(.)$ function, it can be shown from (3.5) and (3.9) that $P_{e,\,STD} \geq P_{e,\,MRC}$, i.e. STD has a higher BER than MRC and MRT$(N \times M = 2 \times 1)$, if a is positive. Details are given in Appendix B. From (3.6), a is positive for $\beta_{ik} < \alpha_{ik}$, $i = 1, 2$, $k = 1, 2$, since $\cos(.) \geq -1$. Typically, the magnitude of the estimation error will be small compared to that of the channel gain and the STD will have a higher BER.

For presentation of the numerical results, we consider two cases. For case 1, the channel gains and estimation errors are fixed with values chosen at random other than to ensure $\beta_{ik} < \alpha_{ik}$, $i = 1, 2$, $k = 1, 2$. The BER's for MRC and STD can be calculated using (3.5) and (3.9) respectively.

For the second case, a Rayleigh faded channel is used and the channel estimation errors z_{ik}, $i = 1, 2$, $k = 1, 2$ are modeled as samples of independent complex Gaussian r.v.'s. The estimate $\hat{h}_{nm}$, $n = 1, 2$, $m = 1, 2$, gain amplitude and phase shift of the $n \to m$th channel can be derived either from the transmission of a pilot signal or from demodulation of the information bearing signals received in previous signaling intervals [19]. The BER's for MRC and STD can then be obtained by averaging (3.5) and (3.9) respectively over the probability distributions for α_{ik}, θ_{ik}, β_{ik}, and ϕ_{ik}, $i = 1, 2, k = 1, 2$. These samples are all independent. The samples α_{ik}, β_{ik}, $i = 1, 2, k = 1, 2$ are Rayleigh distributed and θ_{ik}, ϕ_{ik}, $i = 1, 2, k = 1, 2$ are uniformly distributed in $[0, 2\pi]$.

The variances of the real and imaginary components of H_{ik} and Z_{ik} are denoted by σ_H^2 and σ_Z^2. The signal-to-noise ratio (SNR) and the estimation error-to-signal ratio (ESR) are defined as $(E_b\sigma_H^2)/\sigma_N^2$ and σ_Z^2/σ_H^2 respectively, where E_b is the bit energy. Since $|H_{ik}|$ has zero mean, here we have $\sigma_H^2 = E[|H_{ik}|^2]$.

Figure 3.1 shows the BER as a function of SNR for Case 1. For the parameter values shown in the figure caption, it can be seen that at a BER of 10^{-3}, STD is worse than MRC by about 1.5 dB.

The results for Case 2 are shown in Figure 3.2 and Figure 3.3. When perfect channel estimates are available, MRC and STD have the same BER, given by (2.10). For two branch diversity

$$P_{e,\,perfect} = \frac{1}{2} - \frac{3}{4}\sqrt{\frac{SNR}{1+SNR}} + \frac{1}{4}\left(\frac{SNR}{1+SNR}\right)^{3/2}. \tag{3.17}$$

In Figure 3.2, the BER's of the MRC and STD schemes are plotted as a function of ESR for $SNR = 4$ and 10 dB. From (3.17), the BER values of $SNR = 4$ and 10 dB are 1.69×10^{-2}

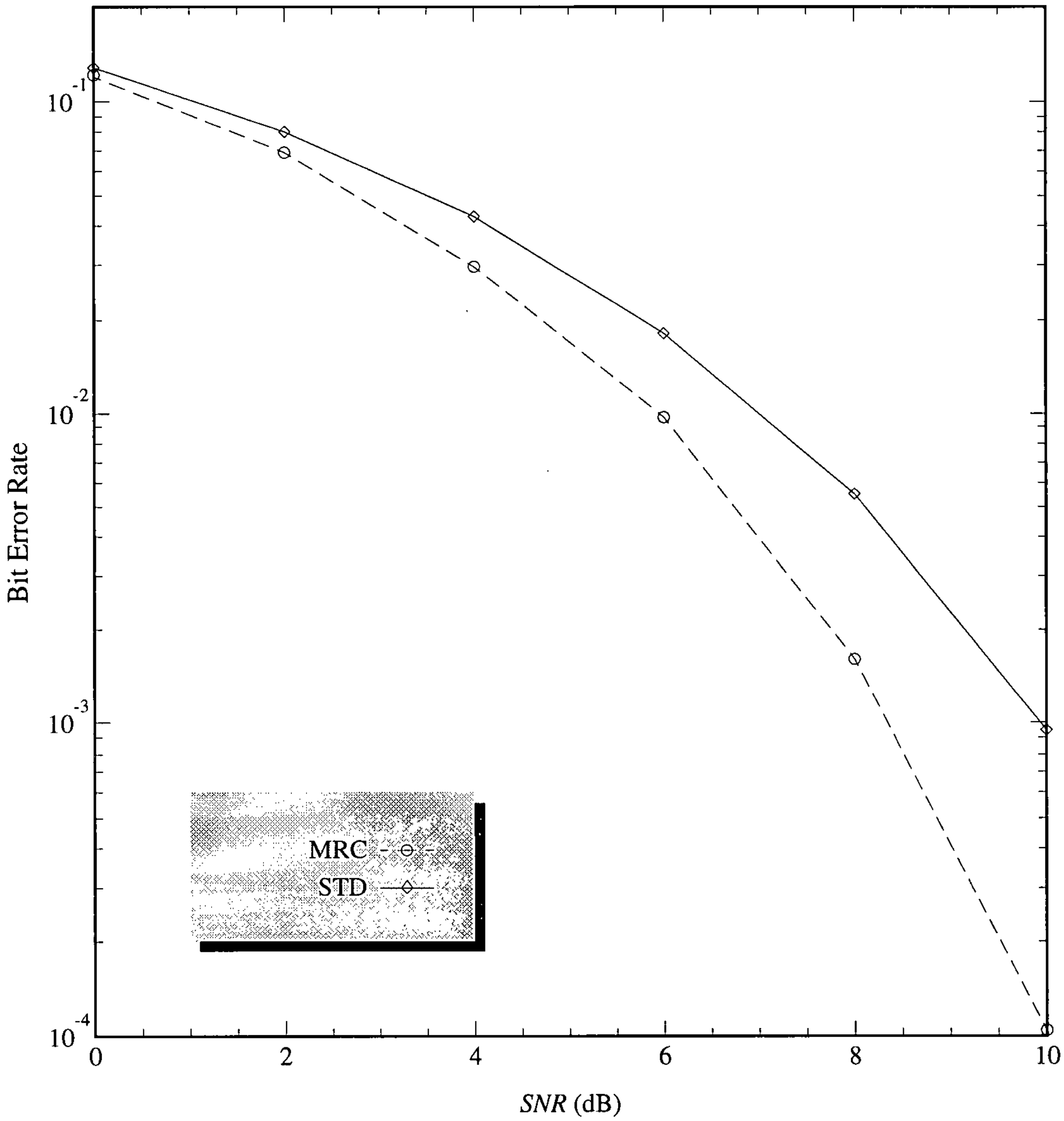


Figure 3.1 BER's of MRC and STD schemes as a function of SNR for a randomly chosen set of parameter values:

$$h_0 = -0.43 - 1.66j,\ h_1 = 0.13 + 0.29j,\ z_0 = -0.811 + 0.842j,\ z_1 = 0.841 - 0.027j.$$

and 1.6×10^{-3} respectively. It can be seen that the BER increases quite rapidly with ESR for $ESR > -10$ dB. For $SNR = 10$ dB and $BER = 10^{-2}$, there is a 2 dB degradation for STD compared to MRC.

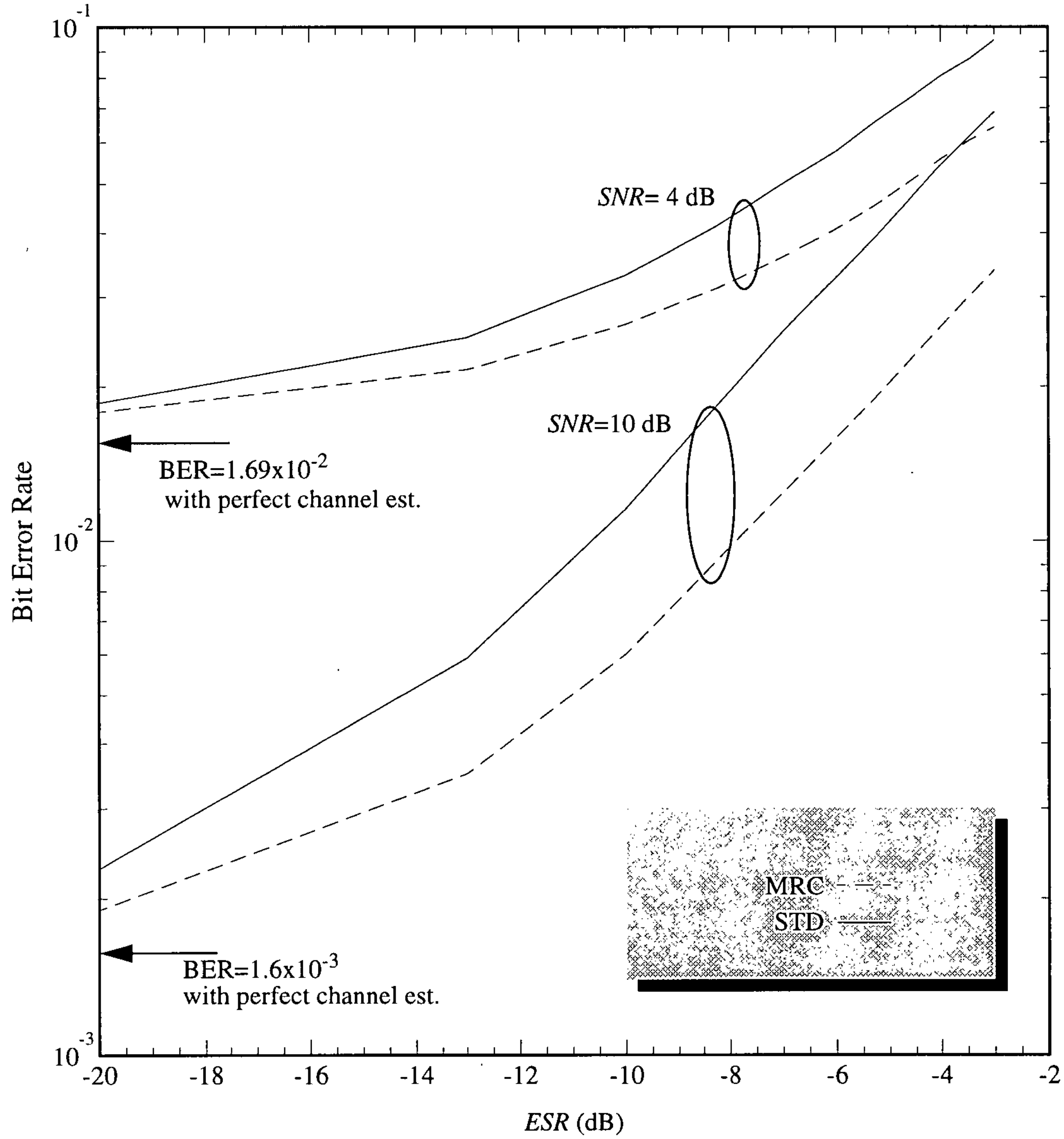


Figure 3.2 BER's of MRC and STD schemes as a function of ESR for $SNR = 4$ and 10 dB .

Figure 3.3 shows the BER's of the MRC, STD and MRT$(N \times M = 1 \times 2)$ schemes as a function of SNR for four different values of ESR. The curve for MRT$(N \times M = 2 \times 1)$ with perfect estimate appears in Figure 2.6 and has been omitted from Figure 3.3 to reduce cluttering. At low ESR values, estimation error tend to be small and there is little difference in the BER's between MRC and STD. At $ESR = -10$ dB and BER= 10^{-2}, there is a 3.5 dB degradation for STD and 0.6 dB for MRT($N \times M = 1 \times 2$) compared to MRC. It can be seen that the difference increases fairly rapidly with SNR.

From Figure 2.6, we can see that with perfect channel estimation, STD has better BER performance than MRT($N \times M = 1 \times 2$); however, with imperfect channel estimation, MRT may have a lower BER than STD. For $ESR = -20$ dB, the BER performance for STD is still better than that for MRT, but the difference decreases with SNR. It can be seen that for $SNR > 10$ dB, the BER for STD is almost the same as that for MRT. For $ESR = -10$ dB, STD has a better BER performance than MRT only for $SNR < 2$ dB. We concluded that STD is more sensitive to channel estimation errors than MRC and MRT. It is interesting to note that MRC and MRT show similar sensitivity to channel estimation errors.

3.2 QPSK Modulation

The performance sensitivity to channel error estimates for MRC, STD and MRT with QPSK modulation is now considered. We first derive the symbol error rate (SER) for given values of z_{nm}, $n= 1\ldots N$, $m= 1\ldots M$. The results are then used to obtain SER when z_{nm} is drawn according to any arbitrary probability distribution. Gray coding is used in this study. For a given symbol error, the most probable number of bit errors is one, subject to the mapping constraint [22]. In the analysis below, we make use of the fact that coherent demodulation ideally results in the two messages being separated at the outputs of the quadrature mixers [23]. Thus, the transmitted signal for a QPSK system can be viewed as two binary PSK signals.

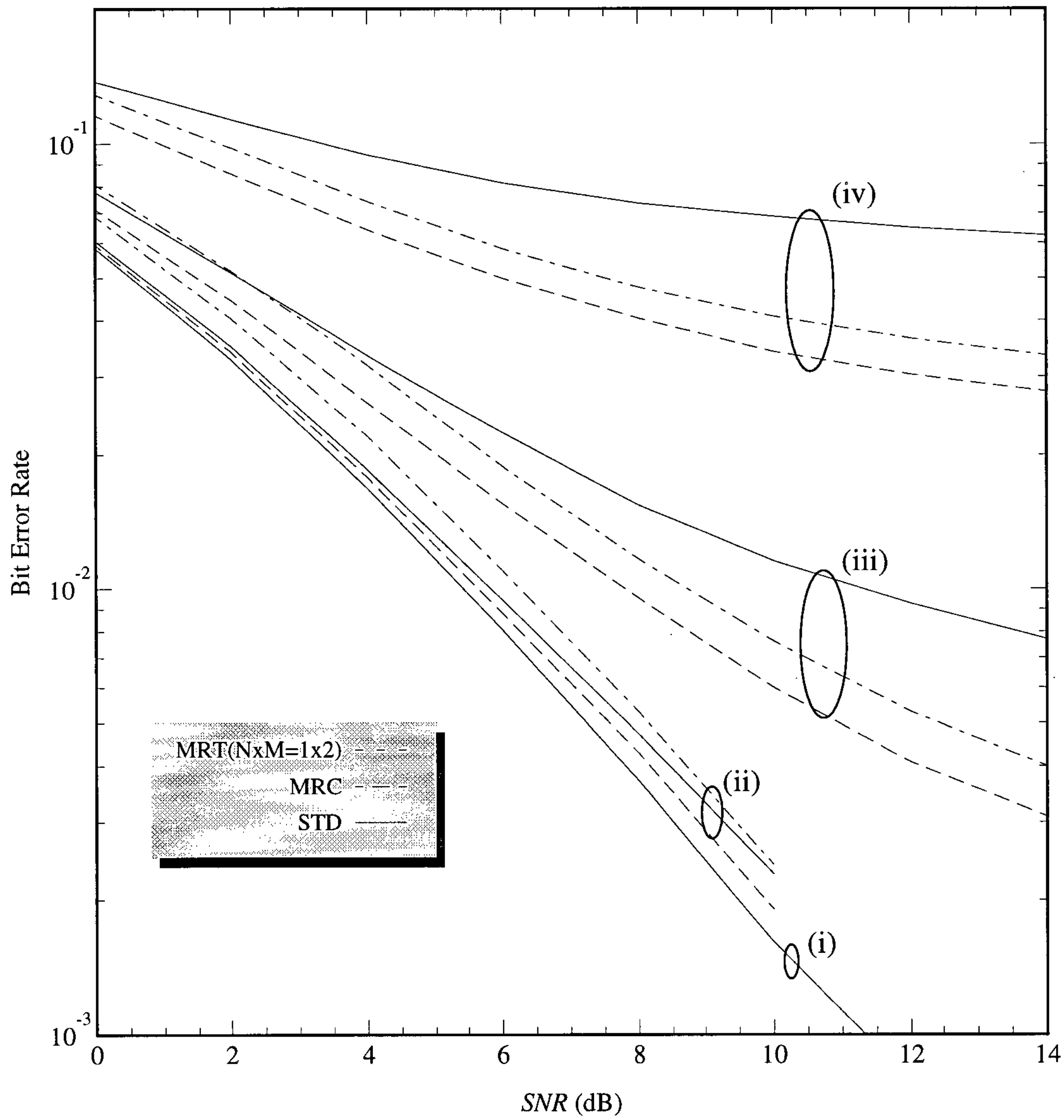


Figure 3.3 BER's of MRC and STD schemes as a function of SNR for four different ESR values:

(i) $ESR = -\infty$ dB, i.e. perfect estimate, MRC and STD

(ii) $ESR = -20$ dB

(iii) $ESR = -10$ dB

(iv) $ESR = -3$ dB

3.2.1 MRC

For given values of s_0, h_{11}, h_{12} z_{11} and z_{12}, the combined signal can be written as in

(3.2). After simplification, the means and variances of the decision r.v.'s can be written as

$$\begin{aligned} E[Re(\tilde{S}_{0,MRC})] &= [(\alpha_{11}^2+\alpha_{12}^2)+\alpha_{11}\beta_{11}\cos(\theta_{11}-\phi_{11})+\alpha_{12}\beta_{12}\cos(\theta_{12}-\phi_{12})]s_0 \\ E[Im(\tilde{S}_{0,MRC})] &= [(\alpha_{11}^2+\alpha_{12}^2)+\alpha_{11}\beta_{11}\sin(\theta_{11}-\phi_{11})+\alpha_{12}\beta_{12}\sin(\theta_{12}-\phi_{12})]s_0, \end{aligned} \tag{3.18}$$

where $Re(\tilde{S}_{0,MRC})$ denotes the real part of $\tilde{S}_{0,MRC}$ and $Im(\tilde{S}_{0,MRC})$ denotes the imaginary part of $\tilde{S}_{0,MRC}$, and

$$\begin{aligned} \sigma_{MRC}^2 = [&\alpha_{11}^2+\beta_{11}^2+2\alpha_{11}\beta_{11}\cos(\theta_{11}-\phi_{11}) \\ &+\alpha_{12}^2+\beta_{12}^2+2\alpha_{12}\beta_{12}\cos(\theta_{12}-\phi_{12})]\sigma_N^2, \end{aligned} \tag{3.19}$$

which is the same as $\sigma_{U_{MRC}}^2$ in (3.4).

Since both $Re(\tilde{s}_{MRC})$ and $Im(\tilde{s}_{MRC})$ are Gaussian distributed, the BER's of the two symbol bits are given by

$$\begin{aligned} P_{MRC,bit1} &= Q\left(\frac{c}{\sigma_{MRC}}\right) \\ P_{MRC,bit2} &= Q\left(\frac{d}{\sigma_{MRC}}\right), \end{aligned} \tag{3.20}$$

where

$$\begin{aligned} c &= \alpha_{11}^2+\alpha_{12}^2+\sqrt{2}\alpha_{11}\beta_{11}\cos(\theta_{11}-\phi_{11}+\pi/4)+\sqrt{2}\alpha_{12}\beta_{12}\cos(\theta_{12}-\phi_{12}+\pi/4) \\ d &= \alpha_{11}^2+\alpha_{12}^2+\sqrt{2}\alpha_{11}\beta_{11}\sin(\theta_{11}-\phi_{11}+\pi/4)+\sqrt{2}\alpha_{12}\beta_{12}\sin(\theta_{12}-\phi_{12}+\pi/4)\ . \end{aligned} \tag{3.21}$$

The SER can then be derived as

$$P_{MRC,\,QPSK} = 1-(1-P_{MRC,\,bit1})(1-P_{MRC,\,bit2})\,. \tag{3.22}$$

3.2.2 STD

For given values of s_0, s_1, h_{11}, h_{21}, z_{11} and z_{21}, we have the combined signal r.v. $\tilde{S}_{0,\,STD}$ as in (3.7). After simplification, the means for the decision symbol bits can be expressed as

$$\begin{aligned}
E[Re(\tilde{S}_{0,\,STD})] =& \ [(\alpha_{11}^2+\alpha_{21}^2)+\alpha_{11}\beta_{11}\cos(\theta_{11}-\phi_{11})+\alpha_{21}\beta_{21}\cos(\theta_{21}-\phi_{21})]s_0 \\
&+[\alpha_{21}\beta_{11}\cos(\theta_{21}-\phi_{11})-\ \alpha_{11}\beta_{21}\cos(\theta_{11}-\phi_{21})]s_1 \\
E[Im(\tilde{S}_{0,\,STD})] =& \ [(\alpha_{11}^2+\alpha_{21}^2)+\alpha_{11}\beta_{11}\sin(\theta_{11}-\phi_{11})+\alpha_{21}\beta_{21}\sin(\theta_{21}-\phi_{21})]s_0 \\
&+[\alpha_{21}\beta_{11}\sin(\theta_{21}-\phi_{11})-\ \alpha_{11}\beta_{21}\sin(\theta_{11}-\phi_{21})]s_1\,.
\end{aligned} \tag{3.23}$$

The variance σ_{STD}^2 is the same as σ_{MRC}^2 in (3.19). The BER's of the two symbol bits are given as

$$\begin{aligned}
P_{STD,\,bit1} &= \frac{1}{4}Q\left(\frac{c+e_1}{\sigma_{MRC}}\right)+\frac{1}{4}Q\left(\frac{c-e_1}{\sigma_{MRC}}\right)+\frac{1}{4}Q\left(\frac{c+e_2}{\sigma_{MRC}}\right)+\frac{1}{4}Q\left(\frac{c-e_2}{\sigma_{MRC}}\right) \\
P_{STD,\,bit2} &= \frac{1}{4}Q\left(\frac{d+f_1}{\sigma_{MRC}}\right)+\frac{1}{4}Q\left(\frac{d-f_1}{\sigma_{MRC}}\right)+\frac{1}{4}Q\left(\frac{d+f_2}{\sigma_{MRC}}\right)+\frac{1}{4}Q\left(\frac{d-f_2}{\sigma_{MRC}}\right)
\end{aligned} \tag{3.24}$$

where

$$\begin{aligned}
e_1 &= \sqrt{2}\alpha_{21}\beta_{11}\cos(\theta_{21}-\phi_{11}+\pi/4)-\sqrt{2}\alpha_{11}\beta_{21}\cos(\phi_{21}-\theta_{11}+\pi/4) \\
e_2 &= \sqrt{2}\alpha_{21}\beta_{11}\cos(\theta_{21}-\phi_{11}-\pi/4)-\sqrt{2}\alpha_{11}\beta_{21}\cos(\phi_{21}-\theta_{11}-\pi/4) \\
f_1 &= \sqrt{2}\alpha_{21}\beta_{11}\sin(\theta_{21}-\phi_{11}+\pi/4)-\sqrt{2}\alpha_{11}\beta_{21}\sin(\phi_{21}-\theta_{11}+\pi/4) \\
f_2 &= \sqrt{2}\alpha_{21}\beta_{11}\sin(\theta_{21}-\phi_{11}-\pi/4)-\sqrt{2}\alpha_{11}\beta_{21}\sin(\phi_{21}-\theta_{11}-\pi/4)
\end{aligned} \tag{3.25}$$

c and d are given in (3.21). The SER can then be written as

$$P_{STD,\,QPSK} = 1-(1-P_{STD,\,bit1})(1-P_{STD,\,bit2})\,. \tag{3.26}$$

3.2.3 MRT

The outcome of the combiner has been given in (3.13). The means of $Re(\tilde{S}_{0,\,MRT})$ and $Im(\tilde{S}_{0,\,MRT})$ can be written as

$$E[Re(\tilde{S}_{0,\,MRT})] = \frac{[(\alpha_{11}^2+\alpha_{21}^2)+\alpha_{11}\beta_{11}\cos(\theta_{11}-\phi_{11})+\alpha_{21}\beta_{21}\cos(\theta_{21}-\phi_{21})]s_0}{\sqrt{\alpha_{11}^2+\beta_{11}^2+2\alpha_{11}\beta_{11}\cos(\theta_{11}-\phi_{11})+\alpha_{21}^2+\beta_{21}^2+2\alpha_{21}\beta_{21}\cos(\theta_{21}-\phi_{21})}}$$
$$E[Im(\tilde{S}_{0,\,MRT})] = \frac{[(\alpha_{11}^2+\alpha_{21}^2)+\alpha_{11}\beta_{11}\sin(\theta_{11}-\phi_{11})+\alpha_{21}\beta_{21}\sin(\theta_{21}-\phi_{21})]s_0}{\sqrt{\alpha_{11}^2+\beta_{11}^2+2\alpha_{11}\beta_{11}\cos(\theta_{11}-\phi_{11})+\alpha_{21}^2+\beta_{21}^2+2\alpha_{21}\beta_{21}\cos(\theta_{21}-\phi_{21})}}$$

(3.27)

The variance for those two decision bits are identical to σ_N^2. Thus, the BER's for those two symbol bits can be derived as

$$\begin{aligned} P_{MRT,\,bit1} &= Q\left(\frac{c}{\sigma_{MRC}}\right) \\ P_{MRT,\,bit2} &= Q\left(\frac{d}{\sigma_{MRC}}\right), \end{aligned} \tag{3.28}$$

where c and d are as given in (3.22). The SER can then be obtained as

$$P_{MRT,\,QPSK} = 1-(1-P_{MRT,\,bit1})(1-P_{MRT,\,bit2}) \tag{3.29}$$

3.2.4 Numerical Results with QPSK modulation

We can see from (3.20) and (3.28) that MRT $(N \times M = 2 \times 1)$ has the same BER and

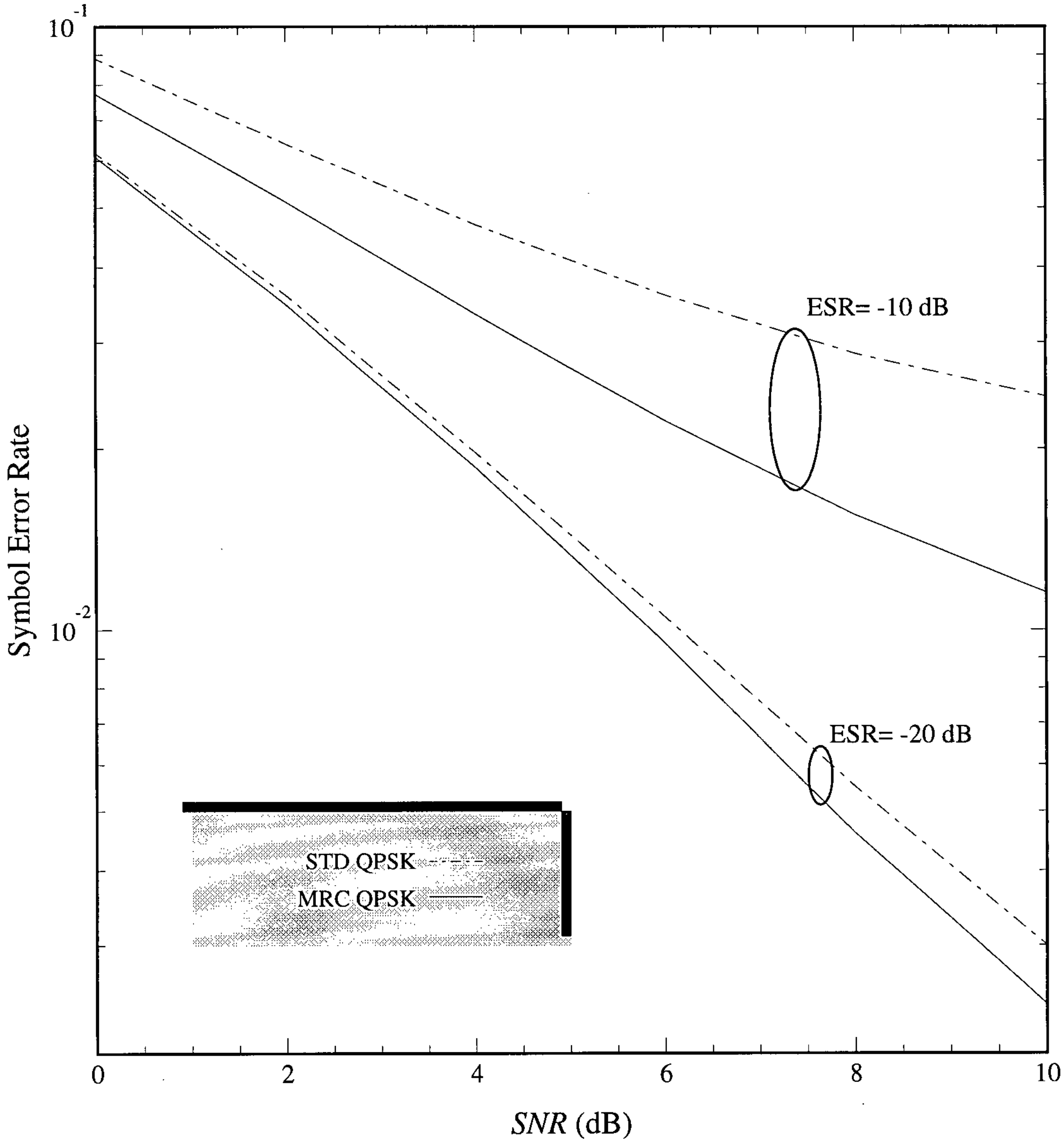


Figure 3.4 SER's of MRC and STD schemes as a function of SNR for $ESR = -20$ and -10 dB.

SER performance as MRC ($N \times M = 1 \times 2$). From (3.20) and (3.24), using the same procedure derived in Appendix B., it can be shown that

$$\begin{aligned} P_{STD,\,bit1} &\geq P_{MRC,\,bit1} \\ P_{STD,\,bit2} &\geq P_{MRC,\,bit2}\,, \end{aligned} \tag{3.30}$$

if c and d are positive. The SER's for MRC and STD with imperfect channel estimates are shown in Figure 3.4, using the same channel model as in Section 3.1.4. The SER performance for STD is worse than that for MRC, as in the case of BPSK. Figure 3.4 shows that the SER difference between the two schemes increases with SNR and ESR.

Chapter 4 Two New Diversity Schemes

A number of diversity combining schemes have been devised to exploit the uncorrelated fading exhibited by separate antennas in a space diversity array. The Maximal Ratio Combining (MRC) scheme is known to be optimal in the sense that it yields the best statistical reduction of fading of any linear diversity combiner [14]; however, the MRC technique has so far been exclusively for receiving applications. The Maximum Ratio Transmission (MRT) scheme is proposed for a system using both transmit and receive diversity [9]. In the MRT scheme, the weighting factors w_i, $i = 1...M$, in Figure 2.4, are set to have equal absolute values, i.e. $|w_1| = |w_2| = ...= |w_M|$; the absolute values are set to 1 for simplicity. However, it should be noted that this constraint affects the maximum SNR. An improved MRT scheme (IMRT) is proposed in Section 4.1. In the IMRT scheme, the receiver weights the signals on different branches according to MRC, but uses the same transmit weighting factors as in Figure 2.4. A new optimal maximum ratio transmission and combining (MRTC) scheme is then proposed in Section 4.2, which maximizes the SNR using optimal transmit weighting function and a MRC-like receive weighting function. Here we use a $(N \times M = 2 \times 2)$ model to illustrate its operation. The SER performance of the new scheme together with that of the IMRT is discussed in Section 4.3.

4.1 Improved MRT (IMRT) Scheme

In MRT, the received signals on all diversity branches are equally weighted. In this section, an improved MRT (IMRT) scheme is proposed in which the signals on each branch are weighted by a factor proportional to the received signal amplitude.

In the IMRT scheme, the two received signals in a $(N \times M = 2 \times 2)$ MRT scheme can be viewed as two inputs for another MRC combiner. The system model is shown in Figure 4.1.

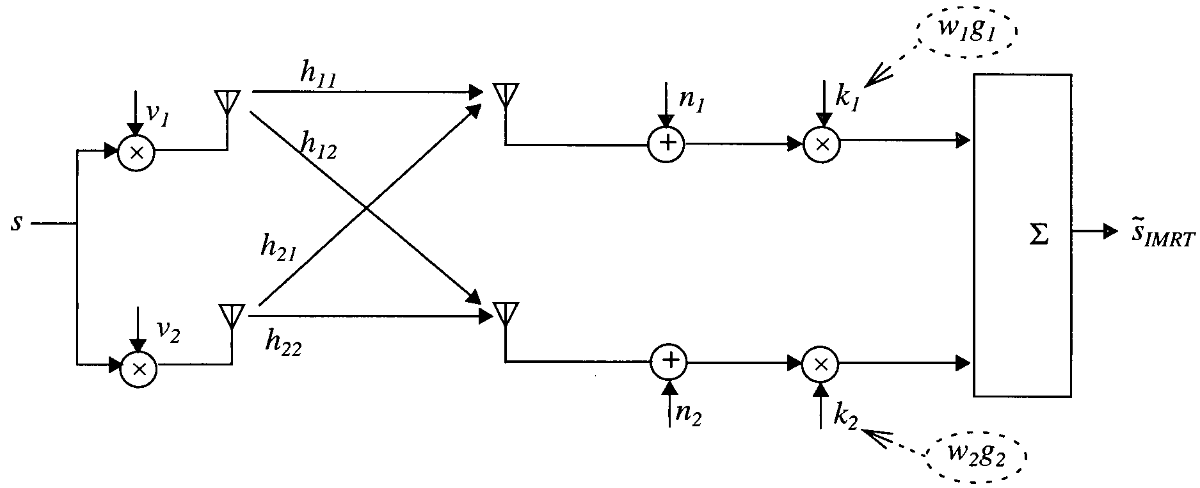


Figure 4.1 IMRT with two transmit and two receive antennas

The weighting factors $k_i, i = 1, 2$ are the products of the weighting factors $w_i, i = 1, 2$ from MRT and $g_i, i = 1, 2$. The values of $g_i, i = 1, 2$ are set according to the MRC rules. From (2.17), the transmit weighting factors are

$$\begin{aligned} v_1 &= \frac{1}{a}(w_1 h_{11} + w_2 h_{12})^* \\ v_2 &= \frac{1}{a}(w_1 h_{21} + w_2 h_{22})^*, \end{aligned} \tag{4.1}$$

where $a^2 = |h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2 + 2|h_{11}h_{12}^* + h_{21}h_{22}^*|$, and $|w_1| = |w_2| = 1$.

The combined signal is then given as

$$\begin{aligned}\tilde{s}_{IMRT} &= (sv_1h_{11} + sv_2h_{21} + n_1)k_1 + (sv_1h_{12} + sv_2h_{22} + n_2)k_2 \\ &= (sv_1h_{11} + sv_2h_{21} + n_1)w_1g_1 + (sv_1h_{12} + sv_2h_{22} + n_2)w_2, \\ &= \frac{s}{a}(|h_{11}|^2 + |h_{21}|^2 + |h_{11}^*h_{12} + h_{21}^*h_{22}|)g_1 + w_1g_1n_1 \\ &+ \frac{s}{a}(|h_{12}|^2 + |h_{22}|^2 + |h_{11}h_{12}^* + h_{21}h_{22}^*|)g_2 + w_2g_2n_2.\end{aligned} \tag{4.2}$$

According to the combining rule in MRC, the optimum weight for each branch has a magnitude proportional to the signal magnitude and inverse to the branch noise power level [1], here we choose

$$\begin{aligned}g_1 &= (|h_{11}|^2 + |h_{21}|^2 + |h_{11}^*h_{12} + h_{21}^*h_{22}|)/a \\ g_2 &= (|h_{12}|^2 + |h_{22}|^2 + |h_{11}h_{12}^* + h_{21}h_{22}^*|)/a.\end{aligned} \tag{4.3}$$

The combined signal can then be written as

$$\begin{aligned}\tilde{s}_{IMRT} &= \frac{s}{a^2}(|h_{11}|^2 + |h_{21}|^2 + |h_{11}^*h_{12} + h_{21}^*h_{22}|)^2 + k_1n_1 \\ &+ \frac{s}{a^2}(|h_{12}|^2 + |h_{22}|^2 + |h_{11}h_{12}^* + h_{21}h_{22}^*|)^2 + k_2n_2,\end{aligned} \tag{4.4}$$

where $k_1 = w_1g_1$, $k_2 = w_2g_2$. The IMRT is actually a combination of a MRT and a MRC-like receiver.

4.2 New Maximum Ratio Transmission and Combining (MRTC) Scheme

It has been shown in Section 4.1 that the IMRT scheme can improve the output SNR by using a MRC-like weighting function at the MRT receiver. However, the improvement is not optimal for maximizing SNR when multiple transmit and multiple receive antennas are used. In this section, we introduce a new maximum ratio transmission and combining scheme (MRTC) with both optimal transmit and receive weighting functions.

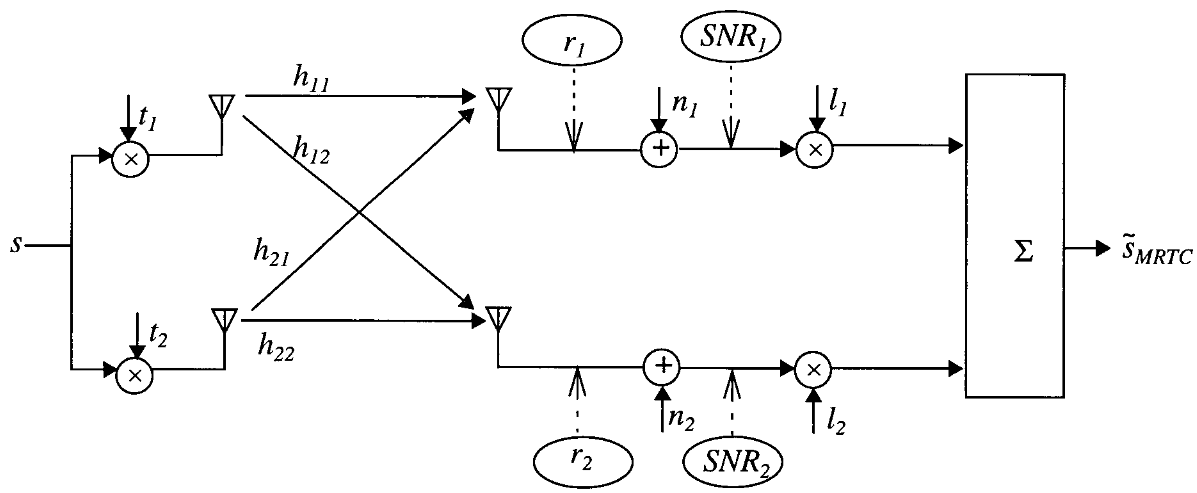


Figure 4.2 MRTC with two transmit and two receive antennas

As shown in Figure 4.2, $t_i, i = 1, 2$ are the transmit weighting factors, and $l_i, i = 1, 2$ are the receive weighting factors. The two received signal components can be written as

$$\begin{aligned} r_1 &= (t_1 h_{11} + t_2 h_{21}) s \\ r_2 &= (t_1 h_{12} + t_2 h_{22}) s. \end{aligned} \tag{4.5}$$

The weighting functions, $l_i, i = 1, 2$ at the receiver, are chosen according to the MRC scheme, i.e.

$$\begin{aligned} l_1 &= (t_1 h_{11} + t_2 h_{21})^* \\ l_2 &= (t_1 h_{12} + t_2 h_{22})^*. \end{aligned} \tag{4.6}$$

The combined signal can then be written as

$$\tilde{s}_{MRTC} = l_1(r_1 + n_1) + l_2(r_2 + n_2). \tag{4.7}$$

Assuming that each receiving branch has the same average noise power σ_{RN}^2, which is the sum of the powers, σ_N^2, in real and imaginary components, the total noise power σ_{NT}^2 is simply the weighted sum of the noise in each branch [11]. Thus,

$$\sigma_{NT}^2 = \sigma_{RN}^2 \sum_{i=1}^{2} |l_i|^2. \tag{4.8}$$

The SNR, γ, at the output of the combiner can be written as

$$\gamma = r^2/\sigma_{NT}^2, \tag{4.9}$$

where $r = l_1(t_1h_{11} + t_2h_{21})s + l_2(t_1h_{12} + t_2h_{22})s$. Using Schwarz Inequality [24], γ is maximized when $l_i = r_i/\sigma_{RN}^2$, which leads to

$$\gamma = \frac{\left(\sum_{i=1}^{2} r_i^2/\sigma_{RN}^2\right)^2}{\sum_{i=1}^{2} (r_i/\sigma_{RN}^2)^2} = \sum_{i=1}^{2} \frac{r_i^2}{\sigma_{RN}^2} = \sum_{i=1}^{2} \gamma_i^2. \tag{4.10}$$

The SNR at the output of the diversity combiner is simply the sum of the SNR's for each receiving branch, i.e. $SNR = SNR_1 + SNR_2$. From equation (4.5), we have

$$\begin{aligned} SNR_1 &= |t_1h_{11} + t_2h_{21}|^2\zeta/\sigma_{RN}^2 \\ SNR_2 &= |t_1h_{12} + t_2h_{22}|^2\zeta/\sigma_{RN}^2, \end{aligned} \tag{4.11}$$

where ζ is the signal energy. Maximizing SNR is then equivalent to maximizing

$$K = \left|t_1 h_{11} + t_2 h_{21}\right|^2 + \left|t_1 h_{12} + t_2 h_{22}\right|^2 \tag{4.12}$$

with respect to t_1 and t_2 subject to the transmit power constraint, i.e. $\left|t_1\right|^2 + \left|t_2\right|^2 \le 1$.

We use the same complex representation for $h_{nm} = \alpha_{nm} e^{j\theta_{nm}}, n, m=1, 2$ as in previous chapters. The transmit weights $t_i, i = 1, 2$ can be represented by $t_i = \psi_i e^{j\phi_i}, i = 1, 2$. With the constraint $\left|t_1\right|^2 + \left|t_2\right|^2 = 1$, i.e. $\psi_1^2 + \psi_2^2 = 1$, K can be expressed as

$$\begin{aligned} K &= \left|\psi_1\alpha_{11}e^{j(\theta_{11}+\phi_1)} + \psi_2\alpha_{21}e^{j(\theta_{21}+\phi_2)}\right|^2 + \left|\psi_1\alpha_{12}e^{j(\theta_{12}+\phi_1)} + \psi_2\alpha_{22}e^{j(\theta_{22}+\phi_2)}\right|^2 \\ &= \psi_1^2\alpha_{11}^2 + \psi_2^2\alpha_{21}^2 + 2\psi_1\psi_2\alpha_{11}\alpha_{21}\cos(\theta_{11}+\phi_1-\theta_{21}-\phi_2) \\ &\quad + \psi_1^2\alpha_{12}^2 + \psi_2^2\alpha_{22}^2 + 2\psi_1\psi_2\alpha_{12}\alpha_{22}\cos(\theta_{12}+\phi_1-\theta_{22}-\phi_2) \\ &= \psi_1^2(\alpha_{11}^2+\alpha_{12}^2) + \psi_2^2(\alpha_{21}^2+\alpha_{22}^2) + \psi_1\psi_2 Q, \end{aligned} \tag{4.13}$$

where $Q = 2\alpha_{11}\alpha_{21}\cos(\theta_{11}+\phi_1-\theta_{21}-\phi_2) + 2\alpha_{12}\alpha_{22}\cos(\theta_{12}+\phi_1-\theta_{22}-\phi_2)$. Since $\psi_2 = \sqrt{1-\psi_1^2}$, equation (4.13) can be re-written as

$$K = \psi_1^2(\alpha_{11}^2+\alpha_{12}^2) + (1-\psi_1^2)(\alpha_{21}^2+\alpha_{22}^2) + Q\psi_1\sqrt{1-\psi_1^2}. \tag{4.14}$$

For given values of $h_{nm} = \alpha_{nm}e^{j\theta_{nm}}, n, m=1, 2$, we wish to maximize K w.r.t. ψ_1, ϕ_1 and ϕ_2. To find the stationary points of ψ_1, ϕ_1 and ϕ_2, we set the partial derivation $\frac{\partial K}{\partial \psi_1}$, $\frac{\partial K}{\partial \phi_1}$ and $\frac{\partial K}{\partial \phi_2}$ to zero [25]. From $\frac{\partial K}{\partial \phi_1} = 0$ and $\frac{\partial K}{\partial \phi_2} = 0$, we have the same equation

$$\tan(\phi_1 - \phi_2) = \frac{\alpha_{11}\alpha_{21}\sin(\theta_{11} - \theta_{21}) + \alpha_{12}\alpha_{22}\sin(\theta_{12} - \theta_{22})}{-\alpha_{11}\alpha_{21}\sin(\theta_{11} - \theta_{21}) - \alpha_{12}\alpha_{22}\sin(\theta_{12} - \theta_{22})} = D. \tag{4.15}$$

Thus, the optimal solution for K depends only on the difference Δ between ϕ_1 and ϕ_2. From (4.15), in the interval $(-\pi, \pi)$, there are two roots for Δ, which is $(\operatorname{atan} D)$ and either $(\operatorname{atan} D + \pi)$ or $(\operatorname{atan} D - \pi)$. From $\frac{\partial K}{\partial \psi_1} = 0$, we have

$$\psi_1^2 = \frac{1 \pm \sqrt{1 - 4/(4 + C^2)}}{2}, \tag{4.16}$$

where $C^2 = 2(\alpha_{11}^2 + \alpha_{12}^2 - \alpha_{21}^2 - \alpha_{22}^2)/Q$. Since $0 \le \psi_1 \ge 1$, K will obtain its maximum value at one of the four possible points $\left\{0, (1 \pm \sqrt{1 - 4/(4 + C^2)})/2, 1\right\}$. Since Q depends on Δ, for each Δ, there are four possible $\psi's$. In our simulation, the eight possible points are compared to select the proper weighting functions which maximize the function K.

4.3 Numerical Results

The channel, interference and noise models are the same as those described in Section 3.1.4. QPSK is used in the simulation. It is assumed that perfect knowledge of channel fading coefficients are available to both transmitting and receiving stations. A $(N \times M = 2 \times 2)$ diversity model is used in this section.

The symbol error rate (SER) curves are shown in Figure 4.3. It can be seen that IMRT and

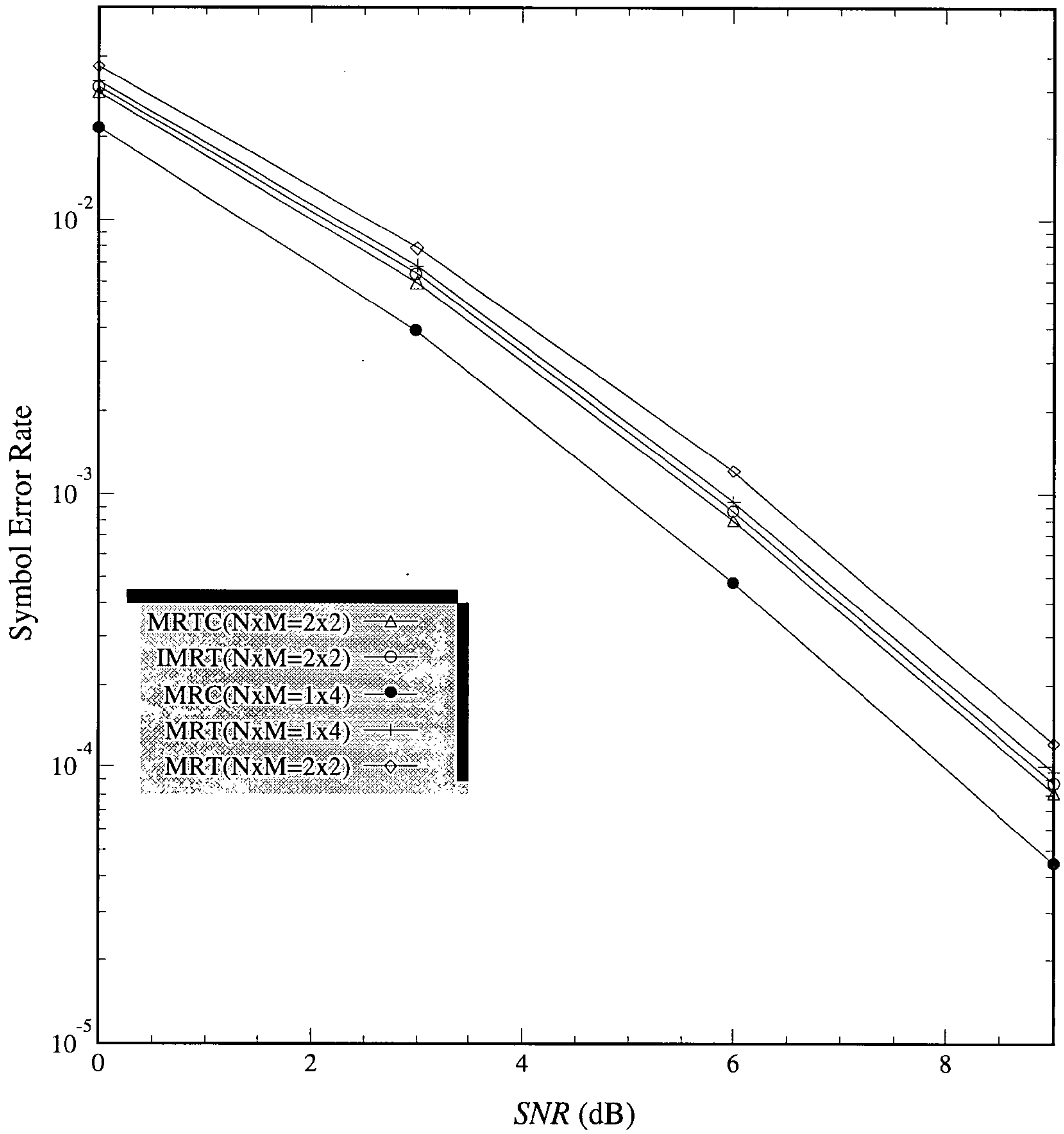


Figure 4.3 SER performances of 4-branch MRC, MRT, IMRT and MRTC

MRTC offer significant improvements over MRT. At a SER of 10^{-3}, IMRT and MRTC are about 0.5 dB and 0.6 dB more efficient than MRT$(N \times M = 2 \times 2)$. With optimal transmit weighting function, MRTC exhibits a slightly better SER performance than IMRT. Among the schemes shown in Figure 4.3, MRC ($N \times M = 1 \times 4$) has the best performance.

Chapter 5 Conclusion

5.1 Main Thesis Contributions

In this thesis, the effects of channel estimation errors on error probability performance of MRC, STD and MRT were studied. An improved scheme for MRT and a new optimal maximum ratio transmission and combining scheme were proposed.

- The MRC scheme yields the best statistical reductions of fading for any linear diversity combiner. However, a major problem with using the receive diversity approach in a cellular communication system is the cost, size and power of the mobile units. The STD and MRT schemes allow implementations of diversity without requiring multiple antennas at the receiver. This is attractive for small mobile handsets. The error probability performances of the three diversity schemes were compared in this thesis. The STD scheme has previously been shown to have the same BER as the MRC scheme with the same diversity order when perfect channel estimates are available. For MRT, when multiple antennas are used at the transmitter and only one antenna at the receiver, i.e. $(N > 1, M = 1)$, the BER performance is shown to be the same as that of MRC with the same diversity order. When multiple antennas are used at the receiver, the BER performance is much worse than that for the $(N > 1, M = 1)$ case. For a 4th order diversity scheme, at BER=10^{-3}, there is a 0.9 dB degradation for $\mathrm{MRT}(N \times M = 1 \times 4)$ and 1.3 dB for $\mathrm{MRT}(N \times M = 2 \times 2)$ compared to $\mathrm{MRT}(N \times M = 4 \times 1)$.

- The performance sensitivity of MRC, STD and MRT to channel estimation errors was studied. It was shown that $(N > 1, M = 1)$ MRT has the same BER as MRC, whereas STD can have a significantly worse performance than MRC when the channel cannot be estimated accurately. With BPSK, at a BER of 10^{-3}, STD is worse than MRC by about 1.5 dB. The BER and SER degradations for STD increase rapidly with ESR and SNR relative to MRC.

- Two new schemes, IMRT and MRTC, which yield improved error performances compared to the MRT scheme were proposed. In IMRT, the same transmit weighting function as in MRT is used. Unlike MRT, MRC combining is applied at the receiver. This results in a substantial error performance improvement over MRT. The MRTC scheme is an optimal maximum ratio transmission and combining scheme. It maximizes the SNR using both optimal transmit and receive weighting factors. For a $(N \times M = 2 \times 2)$ system, at BER=10^{-3}, IMRT and MRTC are about 0.5 dB and 0.6 dB more efficient than MRT$(N \times M = 2 \times 2)$. With an optimal transmit weighting function, MRTC was shown to have a slightly better SER performance than IMRT.

5.2 Topics for Further Study

- For $(N > 1, M > 1)$ MRT, further theoretical analysis of the error performances due to channel estimation errors would be useful.

- The sensitivity to channel estimation errors for the IMRT and MRTC schemes needs to be investigated.

- The results of this thesis is based on the assumption that channel fadings are independent. It would be interesting to study the effects of correlated fades on performance with and without channel estimation errors.

Glossary

Acronyms

BER - Bit Error Rate

BPSK - Binary Phase Shift Keying

CCI - Co - Channel Interference

CDMA - Code Division Multiple Access

dB - decibel

ESR - Estimation Error to Signal Ratio

IMRT - Improved Maximum Ratio Transmission

MRC - Maximal Ratio Combining

MRT - Maximum Ratio Transmission

MRTC - Maximal Ratio Transmission and Combining

QPSK - Quadrature Phase Shift Keying

SER - Symbol Error Rate

SNR - Signal to Noise Ratio

STD - Simple Transmit Diversity

Notations

f_d - Maximum Doppler frequency

λ - Wave length

N - Number of transmit antennas

M - Number of receive antennas

h_{nm} - Diversity branch gain from transmit antenna n to receive antenna m

α_{nm} - Amplitude of h_{nm}

θ_{nm} - Phase of h_{nm}

$r_{1i, MRC}$ - Received signal on branch $1 \rightarrow i$ in MRC scheme

s_0, s_1 - Transmitted signals

n_i - noise and interference on branch i

$\tilde{s}_{0, MRC}$ - Signal output from MRC combiner

γ - output SNR from combiner

γ_{1i} - SNR on $1 \rightarrow i$ branch

U_{MRC} - Decision r.v. for MRC

$\sigma^2_{U_{MRC}}$ - Variance of U_{MRC}

ζ - Signal energy

σ^2_N - Variance of the real (or imaginary) components of N_i

γ_0 - Average SNR per branch

$f_\Gamma(\gamma)$ - p.d.f of Γ

$P_{MRC, QPSK}$ - SER for MRC

P_e - BER

$r_{i, STD}$ - Received signal on branch i in STD scheme

$\tilde{s}_{0, STD}$, $\tilde{s}_{1, STD}$ - Signal outputs from STD combiner

$\boldsymbol{H}$ - Channel coefficient matrix

$\underline{V}$ - Transmit weighting vector

$\underline{W}$ - Receive weighting vector

^{H} - Hermitian operation

$\underline{s}_t$ - Transmitted signal vector

$\underline{n}$ - Noise vector

$\tilde{s}_{MRT}$ - Signal output from MRT combiner

a - A normalization factor

$\hat{h}_{nm}$ - Estimated complex branch gain

z_{nm} - Estimation error for the $n \times m$ branch

β_{nm} - Amplitude of z_{nm}

ϕ_{nm} - Phase of z_{nm}

U_{STD} - Decision r.v. in STD scheme

U_{MRT} - Decision r.v. in MRT scheme

$Re(\tilde{S}_{0,\,MRC})$ - Real part of $\tilde{S}_{0,\,MRC}$

$Im(\tilde{S}_{0,\,MRC})$ - Imaginary part of $\tilde{S}_{0,\,MRC}$

g_i - Receive weighting function in the IMRT scheme

$\tilde{s}_{IMRT}$ - Signal output form IMRT combiner

$\tilde{s}_{MRTC}$ - Signal output from MRTC combiner

t_i - Transmit weighting factor in the MRTC scheme

l_i - Receive weighting factor in the MRTC scheme

σ^2_{RN} - Average noise power in each receive branch in the MRTC scheme

σ^2_{NT} - Total noise power in the MRTC scheme

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Appendix A. Means and Variances of MRC and STD Decision Variables

Here we derive the means of the decision r.v.'s for MRC and STD, as given in (3.3) and (3.8) respectively, and the variance which is given in (3.4). For MRC, using $h_{nm} = \alpha_{nm}e^{j\theta_{nm}}$ and $z_{nm} = \beta_{nm}e^{j\phi_{nm}}$, $n, m = 1, 2$, (3.3) can be written as

$$\begin{aligned}\tilde{S}_{0,MRC} &= (\alpha_{11}^2 + \alpha_{12}^2)s_0 + \alpha_{11}e^{j\theta_{11}}\beta_{11}e^{-j\phi_{11}}s_0 + \alpha_{12}e^{j\theta_{12}}\beta_{12}e^{-j\phi_{12}}s_0 \\ &+(h_{11} + z_{11})^* N_1 + (h_{12} + z_{12})^* N_2 \ .\end{aligned} \quad \textbf{(A.1)}$$

The decision r.v. U_{MRC} can then be expressed as

$$\begin{aligned}U_{MRC} &= Re(\tilde{S}_{0,MRC}) \\ &= (\alpha_{11}^2 + \alpha_{12}^2)s_0 + \alpha_{11}\beta_{11}\cos(\theta_{11} - \phi_{11})s_0 + \alpha_{12}\beta_{12}\cos(\theta_{12} - \phi_{12})s_0 \\ &+ \mathrm{Re}[(h_{11} + z_{11})^* N_1] + Re[(h_{12} + z_{12})^* N_2].\end{aligned} \quad \textbf{(A.2)}$$

Since the channel noise is zero mean Gaussian, the mean of U_{MRC} can be derived as

$$E(U_{MRC}) = [(\alpha_{11}^2 + \alpha_{12}^2) + \alpha_{11}\beta_{11}\cos(\theta_{11} - \phi_{11}) + \alpha_{12}\beta_{12}\cos(\theta_{12} - \phi_{12})]s_0, \quad \textbf{(A.3)}$$

which is the same as (3.3).

The variance of U_{MRC} can be expressed as

$$\begin{aligned}\sigma^2_{U_{MRC}} &= E\left\{[Re(C_1^*N_1) + Re(C_2N_2^*)]^2\right\} \\ &= E[Re^2(C_1^*N_1) + Re^2(C_2N_2^*)] \ ,\end{aligned} \tag{A.4}$$

where we assume $C_1 = (h_{11} + z_{11})$, $C_2 = (h_{21} + z_{21})$, and the noise processes in the diversity branches are assumed to be statistically independent with equal powers. Let

$$\begin{aligned}N_1 &= X_1 + jY_1 \ ; C_1 = U_1 + jV_1 \\ N_2 &= X_2 + jY_2 \ ; C_2 = U_2 + jV_2,\end{aligned} \tag{A.5}$$

where

$$\begin{aligned}U_1 &= \alpha_{11}\cos\theta_{11} + \beta_{11}\cos\phi_{11} \\ U_2 &= \alpha_{21}\cos\theta_{21} + \beta_{21}\cos\phi_{21} \\ V_1 &= \alpha_{11}\sin\theta_{11} + \beta_{11}\sin\phi_{11} \\ V_2 &= \alpha_{21}\sin\theta_{21} + \beta_{21}\sin\phi_{21},\end{aligned} \tag{A.6}$$

we can then write

$$\begin{aligned}\sigma^2_{U_{MRC}} &= E[(X_1U_1 + Y_1V_1)^2 + (X_2U_2 + Y_2V_2)^2] \\ &= U_1^2E(X_1^2) + V_1^2E(Y_1^2) + U_2^2E(X_2^2) + V_2^2E(Y_2^2) \\ &= \sigma_N^2(U_1^2 + V_1^2 + U_2^2 + V_2^2) \\ &= \sigma_N^2[\alpha_{11}^2 + \beta_{11}^2 + 2\alpha_{11}\beta_{11}\cos(\theta_{11} - \phi_{11}) \\ &\quad + \alpha_{12}^2 + \beta_{12}^2 + 2\alpha_{12}\beta_{12}\cos(\theta_{12} - \phi_{12})]\end{aligned} \tag{A.7}$$

where $E(X_1^2) = E(Y_1^2) = E(X_2^2) = E(Y_2^2) = \sigma_N^2$.

For STD, the combined signal given in (3.7) can be written as

$$\begin{aligned}\tilde{S}_{0,STD} &= (\alpha_{11}^2+\alpha_{21}^2)s_0+\alpha_{11}e^{j\theta_{11}}\beta_{11}e^{-j\phi_{11}}s_0+\alpha_{21}e^{-j\theta_{21}}\beta_{21}e^{j\phi_{21}}s_0 \\ &+\alpha_{21}e^{j\theta_{21}}\beta_{11}e^{-j\phi_{11}}s_1-\alpha_{11}e^{-j\theta_{11}}\beta_{21}e^{j\phi_{21}}s_1 \\ &+(h_{11}+z_{11})^*N_1+(h_{21}+z_{21})\,N_2^*\ .\end{aligned} \tag{A.8}$$

The decision r.v., U_{STD}, is

$$\begin{aligned}U_{STD} &= Re(\tilde{S}_{0,STD}) \\ &= (\alpha_{11}^2+\alpha_{21}^2)s_0+\alpha_{11}\beta_{11}\cos(\theta_{11}-\phi_{11})s_0+\alpha_{21}\beta_{21}\cos(\theta_{21}-\phi_{21})s_0 \\ &+\alpha_{21}\beta_{11}\cos(\theta_{21}-\phi_{11})s_1-\alpha_{11}\beta_{21}\cos(\theta_{11}-\phi_{21})s_1 \\ &+\mathrm{Re}[(h_{11}+z_{11})^*N_1]+Re[(h_{21}+z_{21})\,N_2^*]\ .\end{aligned} \tag{A.9}$$

The mean of U_{STD} can then be derived as

$$\begin{aligned}E(U_{STD})= &[(\alpha_{11}^2+\alpha_{21}^2)+\alpha_{11}\beta_{11}\cos(\theta_{11}-\phi_{11})+\alpha_{21}\beta_{21}\cos(\theta_{21}-\phi_{21})]s_0 \\ &+[\alpha_{21}\beta_{11}\cos(\theta_{21}-\phi_{11})-\ \alpha_{11}\beta_{21}\cos(\theta_{11}-\phi_{21})]s_1,\end{aligned} \tag{A.10}$$

which is the same as (3.8).

The derivation of the variance of U_{STD} is the same as that for U_{MRC}.

Appendix B. Comparison of (3.5) and (3.9)

Here we compare the BER's of MRC and STD, as given by (3.5) and (3.9) respectively, using the symmetry property of the $Q(.)$ function, which is defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-y^2/2} dy. \tag{B.1}$$

From (3.5) and (3.9), to prove that $P_{e,\,STD} \geq P_{e,\,MRC}$ when a is positive is equivalent to proving that

$$Q\left(\frac{a}{\sigma_{U_{MRC}}}\right) \leq \frac{1}{2}\left[Q\left(\frac{a+b}{\sigma_{U_{MRC}}}\right) + Q\left(\frac{a-b}{\sigma_{U_{MRC}}}\right)\right],\ \mathrm{a} \geq 0. \tag{B.2}$$

The inequality (B.2) can be further written as

$$Q(c) \leq \frac{1}{2}[Q(c+d) + Q(c-d)],\ c \geq 0, \tag{B.3}$$

where $c = \dfrac{a}{\sigma_{U_{MRC}}}$, $d = \dfrac{b}{\sigma_{U_{MRC}}}$. Assuming that $c, d > 0$, i.e. $a, b > 0$, we have

$$\begin{aligned} Q(c) - Q(c+d) &= \frac{1}{\sqrt{2\pi}}\int_c^{c+d} e^{-y^2/2} dy \\ Q(c-d) - Q(c) &= \frac{1}{\sqrt{2\pi}}\int_{c-d}^{c} e^{-y^2/2} dy. \end{aligned} \tag{B.4}$$

Due to the symmetry property of the $Q(.)$ function, we can see from Figure B.1 that

$$Q(c-d) - Q(c) \geq Q(c) - Q(c+d) \text{ i.e. } Q(c) \leq \frac{1}{2}(Q(c-d) + Q(c+d)), \qquad \textbf{(B.5)}$$

with the equality holds if and only if $c = 0$.

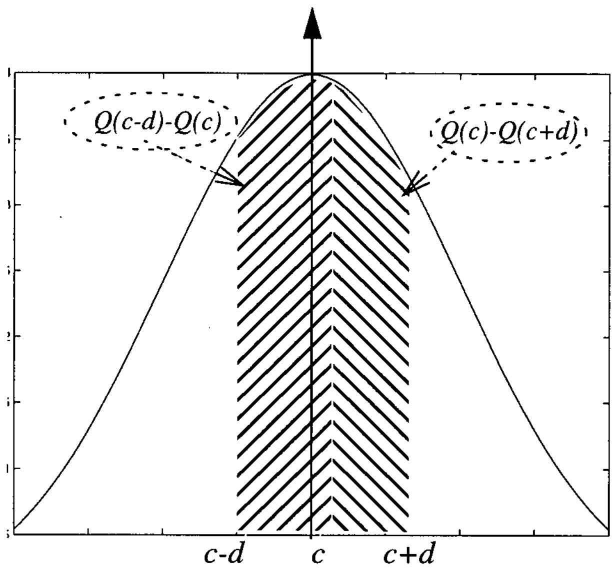

Figure B.1 Illustrating (B.5)