ADVANCED NONCOHERENT RECEIVERS FOR MOBILE FADING CHANNELS

By

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A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

in
THE FACULTY OF GRADUATE STUDIES
ELECTRICAL ENGINEERING

We accept this thesis as conforming
to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA
April 1995
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June 16, 1995
ABSTRACT

The purpose of this thesis, is to derive and evaluate the performance of noncoherent, maximum likelihood receivers with improved performance, for trellis coded PSK and QAM type signals, transmitted over Rician, correlated, fast, frequency non-selective and frequency selective fading channels, with and without diversity.

First we derive the optimal, in the maximum likelihood detection sense, receiver structure for frequency non-selective Rician fading channels, employing diversity reception. In order to reduce the complexity of the optimal receiver, we propose and evaluate the performance of suboptimal receiver structures, which show significant performance improvements as compared to conventional techniques. Investigation of the effects on performance of the proposed algorithms, due to imperfect statistical knowledge of the fading channel typical for a real life environment, demonstrates very small sensitivity even to large errors in estimates of channel parameters.

Complementing our work in frequency non-selective fading, we derive the optimal, in the maximum likelihood detection sense, receiver, for the correlated, fast, frequency selective Rician fading channel. In the interest of system simplicity, we propose and evaluate reduced complexity versions of the decoding algorithms. The impact of simplifying assumptions in the theoretical derivation, as well as the receiver sensitivity to non ideal channel knowledge, is investigated. The results show significant performance improvements over the fastest known channel equalization technique, accompanied by small sensitivity to imperfections.

Last, we derive analytical performance bounds for simplified versions of the optimal diversity receiver, for frequency non-selective, Rician fading channels. The tightness and accuracy
of the bounds is verified, through the excellent agreement between computer simulation results, and bound calculation. Performance evaluation demonstrates significant improvements, approaching the effectiveness of coherent detection in AWGN, even with a relatively small diversity order, for Rician, as well as shadowed EHF fading channels.
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## GLOSSARY

<table>
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<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>AVG</td>
<td>Average</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive white Gaussian noise</td>
</tr>
<tr>
<td>BER</td>
<td>Bit error rate</td>
</tr>
<tr>
<td>CCI</td>
<td>Co-channel interference</td>
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<tr>
<td>CDMA</td>
<td>Code division multiple access</td>
</tr>
<tr>
<td>CE</td>
<td>Convolutional encoder</td>
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<tr>
<td>DPSK</td>
<td>Differential phase shift keying</td>
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<tr>
<td>DQAM</td>
<td>Differential quadrature amplitude modulation</td>
</tr>
<tr>
<td>DQPSK</td>
<td>Differential quadrature phase shift keying</td>
</tr>
<tr>
<td>DRR</td>
<td>Direct to reflected power ratio</td>
</tr>
<tr>
<td>EGC</td>
<td>Equal gain combining diversity</td>
</tr>
<tr>
<td>EHF</td>
<td>Extremely high frequency</td>
</tr>
<tr>
<td>ETSI</td>
<td>European telecommunications standards institute</td>
</tr>
<tr>
<td>FDMA</td>
<td>Frequency division multiple access</td>
</tr>
<tr>
<td>FSK</td>
<td>Frequency shift keying</td>
</tr>
<tr>
<td>HF</td>
<td>High frequency</td>
</tr>
<tr>
<td>ISI</td>
<td>Intersymbol interference</td>
</tr>
<tr>
<td>MDD</td>
<td>Multiple differential detector</td>
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<td>MDTS</td>
<td>Mobile digital telecommunication systems</td>
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<td>MLSE</td>
<td>Maximum likelihood sequence estimation</td>
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<td>MMSPE</td>
<td>Minimum mean square prediction error</td>
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<td>MRC</td>
<td>Maximal ratio combining diversity</td>
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<td>MSAT</td>
<td>Mobile satellite</td>
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<td>MSK</td>
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<td>MSRK</td>
<td>Modified square root Kalman equalization</td>
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<tr>
<td>NEC</td>
<td>Nonredundant error correction</td>
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<tr>
<td>NMT</td>
<td>Nordic mobile telephone</td>
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<tr>
<td>NRZ</td>
<td>Non return to zero pulse</td>
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<tr>
<td>PCS</td>
<td>Personal communication systems</td>
</tr>
<tr>
<td>PD</td>
<td>Phase detector</td>
</tr>
<tr>
<td>pdf</td>
<td>Probability density function</td>
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<td>PSK</td>
<td>Phase shift keying</td>
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<td>QAM</td>
<td>Quadrature amplitude modulation</td>
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<tr>
<td>QPSK</td>
<td>Quadrature phase shift keying</td>
</tr>
<tr>
<td>RTMS</td>
<td>Radio telephone mobile system</td>
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<tr>
<td>SM</td>
<td>Signal mapper</td>
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<tr>
<td>SNR</td>
<td>Signal-to-noise ratio</td>
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<tr>
<td>SWC</td>
<td>Switched combining diversity</td>
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<tr>
<td>TACS</td>
<td>Total access communication system</td>
</tr>
<tr>
<td>TCM</td>
<td>Trellis coded modulation</td>
</tr>
<tr>
<td>TDMA</td>
<td>Time division multiple access</td>
</tr>
<tr>
<td>TIA</td>
<td>Telecommunications industry association</td>
</tr>
<tr>
<td>WPCS</td>
<td>Wireless personal communication systems</td>
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ACKNOWLEDGEMENTS

My deepest thanks go to my parents, Τούλα and Παναγιώτη, to which this Ph.D. thesis is dedicated, for their continuous support and encouragement throughout the years it took to complete.

I would like to express my most sincere thanks and appreciation to my thesis supervisor, Dr. P. Takis Mathiopoulos, for offering his valuable experience throughout the research effort, his moral support in trying times, as well as his encouragement and stimulation in critical points during the course of the work presented in the pages to follow.

Many thanks are also due to Dr. Dimitrios Makrakis, currently with the University of Ottawa, for his valuable feedback on many topics covered in this thesis, as well as his help with some of the complicated mathematical analysis of Chapter 4.

Taking this opportunity, I would also like to thank Dr. Vassilios Makios of the University of Patras, Greece, for positively influencing my decision on coming to Canada for post-graduate studies, and for all his help in providing an opportunity for my entrance to the M.A.Sc. programme at UBC. His later encouragement during the course of this Ph.D. thesis is also deeply appreciated.

I would like to thank all members of my Ph.D. committee, Dr. C. Leung (committee member), Dr. V. Leung (committee member), Dr. R. W. Donaldson (Head’s nominee) and Dr. R. K. Ward (chair, departmental examination), as well as Dr. Vasant K. Prabhu (external examiner) of the Department of Electrical Engineering, from the University of Texas at Arlington, Dr. M. J. Yedlin (university examiner), Dr. H. Chen (university examiner), and Dr. F. L. Curzon (chair, university final oral examination). Special thanks go to Dr. V. Leung who agreed to be a committee member despite the fact that he was on sabbatical leave.

I would also like to thank Mr. Ian Marsland for proof-reading portions of the draft of this thesis, and intercepting typographical mistakes in the complicated mathematical formulas of
Chapter 2.

Last, I would also like to acknowledge the financial support provided by the Natural Sciences and Engineering Research Council of Canada (NSERC) under Grant OGP-44312, the Centre for Integrated Computer Systems Research (CICSR), a University of British Columbia Graduate Fellowship (UGF), and a B.C. Advanced Systems Institute (ASI) Fellowship.
CHAPTER 1

INTRODUCTION

The second half of the twentieth century has been undoubtedly marked by the rapid growth of public wireline networks, providing reliable and affordable voice, and relatively low-rate data communications. During the last ten years there has also been a near exponential growth of specialized wire networks, focused primarily on providing high rate data communications, both on a local and a global scale [1]. These developments have changed our perception of possibilities in both business and scientific information processing, and in turn, have created a need of access to these resources, unconstrained by time, place or mobility [2]. Towards this need for unrestricted access, the idea of using radio for providing personal communications has taken on a renewed meaning during this last decade, giving birth to what we broadly refer to today as wireless personal communication systems (WPCS) [3]. This deep change in our perception of communication is also reflected in the suggestion of integrating WPCS with future high-end networks, rather than having two separate systems [4]. Furthermore, although wireline phone service penetration in developed countries has reached almost 100 percent, two thirds of the world’s population still does not even have access to a public telephone [5]. WPCS are also becoming a very attractive alternative to tethered solutions for communities trying to rapidly upgrade their telecommunication infrastructure, due to decreasing manufacturing costs as well as rapid and flexible deployment in both urban and rural areas [5].

The current analog North American cellular phone system, known as the Advanced Mobile Phone Service (AMPS) pioneered during the ‘70s by Bell Laboratories in the United States...
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[6], employs analog frequency modulation (FM) for speech transmission and frequency shift keying (FSK) for signaling. The same analog transmission techniques have also been used in similar cellular networks worldwide, including Total Access Communication System (TACS) in the United Kingdom, Italy, Spain, Austria and Ireland; Nordic Mobile Telephone (NMT) in many European, African and Southeast Asian countries; C-450 in Germany and Portugal; Radiocom 2000 in France; and Radio Telephone Mobile System (RTMS) in Italy [2]. In the interest of higher system capacity and ease of integration for information services other than voice, WPCS employ an all digital transmission format. Examples of possible applications for such WPCS include cellular phone service [7], land mobile radio phone and data networks [8], mobile satellite terminals [9] and indoor low-range personal communication systems (PCS) [10]. This thesis focuses on WPCS operated in a non frequency reuse environment, suffering signal distortion primarily caused by fading [11]. We will be referring to such WPCS as mobile digital telecommunication systems (MDTS).

Following the well known International Standards Organization (ISO) Open Systems Interconnection (OSI) reference model of a network architecture [12], the three lowest layers, namely the physical layer, the data-link layer and the network layer, are of particular interest to the design of a communication system, since they comprise each network node. The physical layer, the lowest in the hierarchy defined in the ISO model, takes care of the raw transmission and reception of the digital information. The data-link layer assures error-free transmission through the error prone physical layer, and the network layer implements the necessary intelligence for sending information from one node to an other, via a physical route through the mesh of nodes [12]. Among the most important issues in the design of a MDTS are a) the access technique by which users allocate network resources to themselves, b) the capacity of the communication system in terms of simultaneous number of users and c) the communication quality. Multiple user access techniques fall into three categories [13]: a) Frequency division multiple access (FDMA) where a transmit receive pair is allocated to communication between
two network nodes. b) time division multiple access (TDMA) where each user is allocated a
time slot within a time duration of fixed length called a time slot, and c) code division multiple
access (CDMA) where each node makes use of the same spectrum allocation as all other nodes,
by using spread spectrum signaling. The type of communication service provided by the net-
work, i.e., voice and/or low- and high-rate data, together with the type of access technique used
has a great influence on the capacity of a MDTS. Lastly, the quality of service reflected at a
high network layer, as for example in the voice quality of phone communications, or the speed
of data communications, depends on the percentage of information received erroneously at the
physical layer. Since in digital communication the information is conveyed in bits, taking either
one of two discrete values, 0 or 1, a commonly employed measure of quality is the average
number of bits in error, over a large sequence of bits received, called the bit-error-rate (BER).

This thesis is concentrated on the design issues particular to the physical layer of MDTS,
that is, the lowest network layer in such communication systems. Our effort is particularly
focused on proposing and evaluating new receiver structures for improving the physical layer
BER, which can also serve to increase the system capacity by accommodating systems of higher
complexity, with little or no penalty to communication quality. The proposed receiver structures
are shown to outperform other known receivers in applications where the rate of change of the
communication channel is relatively high, i.e., when fading is relatively fast\(^1\). Such fast fading
appears in numerous real life applications, including MDTS with relatively low transmission
rates, mobile stations traveling at high velocities, employing extremely high frequency carriers.
Specific examples of such systems are given in Section 1.2. The organization of this introdutory
chapter is as follows. In the next two sections, we summarize methods used for mobile digital
communication, and the impact of signal propagation on the signal quality at the receiving
end. In particular, Section 1.1 describes common ways of using physical qualities of the
transmitter signal for communicating digital states, i.e., modulation schemes, while Section

\(^1\) A precise definition of fast fading is given in Section 1.2.
1.2 introduces the types of distortion imposed on a radio signal in propagation environments typically encountered in MDTS. Section 1.3 presents an overview of previously investigated techniques, employed by such systems for communication over mobile fading channels. Section 1.4 discusses analytical methods used in the past for predicting the performance of MDTS, while Section 1.5 summarizes the research contributions of this thesis. Finally, Section 1.6 presents the thesis organization.

1.1 MODULATION SCHEMES

Among the various ways of categorizing digital modulation schemes, a widely accepted one is that in constant and non-constant envelope schemes [14]. Constant envelope schemes are those modulation schemes which produce signals with continuous phase [15], and as a result, have a constant envelope. Perhaps one of the most extensively studied is the Gaussian-filtered minimum shift keying (GMSK) [16]. Because of its excellent spectral properties, especially in a non-linear channel, GMSK has been a very popular modulation scheme for mobile communication systems, and it is the format adopted by the Groupe Spécial Mobile (GSM) in 1982\(^2\) for the pan-European digital cellular [17].

Non-constant envelope schemes, as the name implies, produce signals with variable, i.e., non constant envelope. Among the various such modulation formats (with multi-amplitude/-phase signals), \(M\)-ary phase shift keying (\(M\)-PSK) and \(M\)-ary quadrature amplitude modulation (\(M\)-QAM) type of modulation schemes appear to be the most popular. Typically, \(M = 2^k\), where \(k\) is the number of bits required to represent all possible transmitted symbols. PSK signals are multi-phase, whereas QAM signals are multi-amplitude/-phase. In \(M\)-ary PSK, the transmitted digital symbols are mapped to \(M\) distinct transmitter carrier phases, uniformly distributed in the interval \([0, 2\pi]\) [14]. An example of an 8-PSK scheme is shown in Fig.

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2Note that currently, GSM is also referred to as the Global System for Mobile communications, of the European Telecommunications Standards Institute (ETSI) [7].
1.1a; the arrangement of signal points on such a diagram of phase and amplitudes is called a signal constellation. In $M$-ary QAM, the $M$ digital symbols each correspond to one of $M$ possible combinations of carrier signal phase and amplitude, arranged in such a way as to produce a signal constellation of rectangular form [14]. An example of a 16-QAM signal constellation is shown in Fig. 1.1b. The number of possible signals on the constellation of

![Diagram of 8-PSK and 16-QAM constellations](a)(b)

**Figure 1.1:** Examples of PSK and QAM modulation formats; (a) 8-PSK (b) 16-QAM.

a modulation scheme directly translates to the maximum attainable spectral efficiency when using that scheme. It is well known, the maximum theoretical spectral efficiency for an $M$-ary modulation scheme, is $k$ bits/s/Hz [18]. For example, the 8-PSK scheme having 8 constellation points, can communicate with each signal 3 bits, and hence has a maximum theoretical spectral efficiency of 3 bits/s/Hz; for 16-QAM it is 4 bits/s/Hz. It is important to note that, QAM modulation has been traditionally associated with coherent communication systems, whereas PSK with both coherent and noncoherent systems [19, 20, 18, 14].

It is certain that the one modulation format common to QAM and PSK, that is, quadrature phase shift keying (QPSK), has been the most extensively investigated over the past two decades. QPSK has four constellation points, associated to the signal carrier phases of $\pi/4, 3\pi/4, 5\pi/4$ and $7\pi/4$, and thus a maximum theoretical spectral efficiency of 2 bits/s/Hz.
In the 1980’s it was extensively employed in conjunction with fixed point, satellite and, to a lesser extent, terrestrial microwave links [20]. In the 1990’s, a modified version of QPSK, namely, $\pi/4$-shift differential QPSK (DQPSK) was adopted as the modulation standard for the emerging North American [21] and the Japanese [22] digital cellular systems. $\pi/4$-shift DQPSK uses the four differential phase shifts of $\pi/4, 3\pi/4, 5\pi/4$ and $7\pi/4$, rather than the absolute carrier phase values of QPSK, and in so doing has reduced envelope fluctuation [23].

Note that QPSK can also be transmitted in another differential fashion, often referred to as differential QPSK (DQPSK), using differential phases of $0, \pi/2, \pi$ and $3\pi/2$ with considerably more envelope fluctuation than $\pi/4$-shift DQPSK. When using differential phase encoding at the transmitter, the receiver does not need to produce a coherent estimate of the transmitter carrier phase in order to detect the transmitted signal phases. Rather, each received signal phase can be used as a reference value, from which that of the next received signal can be differentially detected [23], yielding the information symbol transmitted. The envelope fluctuation of $\pi/4$-shift DQPSK, although much improved with respect to that of DQPSK, will nevertheless result in spectral spreading in a nonlinear channel. To reduce this spreading, amplifier linearization techniques have been proposed [23, 24], with excellent results. It should be noted, however, that both DQPSK and $\pi/4$-shift DQPSK have identical performance in linear channels.

There is no doubt, that $\pi/4$-shift DQPSK is an important modulation scheme for achieving spectral efficiencies of not more than 2 bits/s/Hz. For higher efficiencies, however, $M$-ary PSK ($M > 4$) and most importantly $M$-ary QAM ($M > 4$), have to be considered. In addition to providing higher spectral efficiency, increasing the number of signals in the constellation can also be used in conjunction with convolutional, or otherwise called, trellis coding [14], to improve the error rate of a communication system. The technique of combining modulation and trellis coding, for improving the performance of digital transmission over band-limited channels, is called trellis coded modulation (TCM) [25]. Its main advantage is that it offers significant gains over conventional uncoded modulation, without compromising bandwidth efficiency by
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requiring an increase in transmission rate, for accommodating the redundancy introduced by coding [25]. Instead of increasing the rate, a higher order modulation format is used, with twice as many signals in the constellation as compared to the uncoded case. The success of TCM lies in the special way of assigning signals to trellis code transitions, called set partitioning [25]. This thesis concentrates on trellis coded $M$-ary PSK and QAM schemes.

1.2 THE MOBILE FADEING CHANNEL

The most prominent from the variety of interference types imposed on a signal transmitted in a mobile-radio environment, is that often referred to as fading. Fading is the time varying signal strength, phase and spectral-shape distortion [26], its rate of change depending on the symbol rate of the digital transmission, appearing "faster" or "slower" according to the relative speed between transmitter and receiver, and following the Doppler\(^3\) frequency [14]. Denoting the maximum Doppler frequency by $B_F$, and the inverse of the digital transmission symbol rate as $T$\(^4\), the product $B_FT$ gives us a measure of the average rate of change of the fading distortion normalized to the transmission symbol rate. As there is no universally accepted value of $B_FT$ below which fading is characterized as slow, and above which as fast, for the purposes of work reported in this thesis, we have adopted the value of $B_FT = 0.001$ for the point of this transition. Values of $B_FT > 0.001$ will be henceforth regarded as fast fading, while values of $B_FT \leq 0.001$ as slow. In the emerging North American digital mobile cellular system, using carrier frequencies in the 800/900 MHz frequency range (ultra high frequency (UHF) band), and a symbol rate of about 25,000 Baud\(^5\), a vehicle moving at little over 50 km/h would cause a

\(^3\)The Doppler frequency is defined as the ratio of the relative speed $v$ between transmitter and receiver, over the carrier wavelength $\lambda$. For a more detailed discussion of Doppler frequency and its implications on the distortion caused to the transmitted signal, see the sections describing the channel models in Chapter 2 and subsequent chapters.

\(^4\)Clearly, $T$ is the symbol duration in seconds.

\(^5\)One Baud is one symbol per second; in the case of $\pi/4$-shift DQPSK, where 2 bits are transmitted in every symbol, one Baud corresponds to 2 bits per second.
B_F T product of 0.0015. On the other hand, in the mobile satellite system (MSAT) developed jointly by the United States and Canada [27], with carrier frequencies of about 1600 MHz (L-band) and a transmission rate of about 3200 Baud, assuming a 45° elevation angle pointing to the satellite, traveling at a little over than 70 km/h produces a B_F T of about 0.023. Assuming the same rate, in the case of an aeronautical channel [28] where the vehicle speed is an order of magnitude greater, or in NASA’s advanced communications technology satellite (ACTS) system [29] with carrier frequencies in the 20/30 GHz range (extremely high frequency (EHF) band), the B_F T values will be well within the range of 0.1 to 1.

The cause for the random signal amplitude and phase variations characteristic of fading, is the simultaneous arrival of signal reflections, produced in an environment possessing numerous objects capable of reflecting a considerable portion of the energy of incident radio signals, rather than absorbing them [26, 11]. As previously mentioned, well known examples of such an environment are the UHF terrestrial radio channel [26], and the EHF mobile satellite channel [29]. Depending on the time delay spread \( \tau \) by which such reflections arrive at the receiver, the random amplification/attenuation and phase shift imposed on the signal by the mobile fading channel is, either of essentially constant amplitude over the received signal frequency spectrum, or considerably varying, presenting peaks and nulls [26]. We define the normalized to the symbol duration delay spread as \( \hat{\tau} = \tau / T \). The value of \( \hat{\tau} \) can be used to characterize the type of fading, distinguishing frequency non-selective and frequency selective cases. Similarly to the distinction between slow and fast fading, there exists no universally accepted point in the range of values for \( \hat{\tau} \), where such a transition occurs. However, it is reasonable to state that \( \hat{\tau} \leq 0.1 \) characterizes the former case of frequency non-selective fading [26], whereas for \( \hat{\tau} > 0.1 \) we have frequency selective fading [14].

In the following two sub-sections, the mathematical modeling and the different cases for both the frequency non-selective and the frequency selective mobile fading channel will be presented.
1.2.1 Frequency Non-Selective Mobile Fading

Depending on whether or not there exists a line-of-sight (LOS) signal path between transmitter and receiver, flat mobile fading channels can have Rayleigh or Rician characteristics\(^6\) [30]. For the Rayleigh fading channel the probability density function of the faded signal amplitude \(r\) follows the Rayleigh distribution, i.e.,

\[
f(r) = \begin{cases} 
  \left(\frac{r}{\sigma^2}\right) \exp\left(-\frac{r^2}{2\sigma^2}\right), & r > 0 \\
  0 & \text{elsewhere}
\end{cases}
\]  

(1.1)

and its phase \(\theta\) is uniformly distributed in \([0, 2\pi)\). For the Rician fading channel case, in addition to the diffused signal component (Rayleigh fading) there is also a LOS path. The composite signal follows the Rice distribution given as [31]

\[
f(r) = 2r \sqrt{\frac{1+K}{S}} \exp\left[-K - (1 + K)r^2\right] I_0\left[2r \sqrt{K(1+K)}\right]
\]  

(1.2)

with the \(K\)-factor defined as

\[
K = \frac{D}{S}, \quad \text{and in dB} \quad 10 \log_{10} K
\]  

(1.3)

\(D\) denoting the average power of the LOS signal component, \(S\) the average power of the diffused component, and \(I_0(\cdot)\) the modified Bessel function of order 0 [32]. Note that \(K \to 0\), or \(K \to -\infty\) dB, results in Rayleigh fading. The phase statistics of the Rician channel are described by [31]

\[
f(\theta) = \frac{e^{-K}}{2\pi} + \frac{\sqrt{K} \cos \theta \exp\left(-K \sin^2 \theta\right)}{2\sqrt{\pi}} \cdot \left[2 - \text{erfc}\left(\sqrt{K} \cos \theta\right)\right]
\]  

(1.4)

with \(\text{erfc}(\cdot)\) the well known complementary error function [32]. Depending on the particular mobile fading channel environment, there exist various mathematical models adopted in the

\(^6\)As it will become apparent, and as a matter of fact, is well known, Rayleigh is a special case of Rician. However, for convenience in presentation, we will be using both terms.
literature for the autocorrelation function $R_F(\tau)$ of the diffused signal component, with a corresponding power spectral density $S_F(f)$ [30]. The most simple one is the rectangular model, with $S_F(f) = S/(2B_F)$ at frequencies $|f| < B_F$ and 0 elsewhere. For the aeronautical channel, the model adopted is Gaussian, with $S_F(f) = S \exp \left[-f^2/B_F^2\right]/(\sqrt{\pi}B_F)$. For the case of land-mobile fading, $S_F(f) = S J_0(2\pi B_F r)$, $J_0(\cdot)$ being the zero-order Bessel function of the first kind [33].

In the EHF bands, the advantage of being able to accommodate much higher bandwidths per channel does not come without a price. Since the wavelength at such frequencies is between 1 and 1.5 cm, i.e., roughly 20 to 30 times smaller than at UHF frequencies, it is very often that the signal path is blocked even by relatively small objects, such as tree leaves and small branches. Contrary to UHF where the signal would have traveled through such obstacles without any significant attenuation, in EHF this signal blockage brings the receiver temporarily within an electromagnetic "shadow" with respect to the transmitted signal. For this reason, fading in the EHF band is commonly referred to as shadowed fading, or simply shadowing. In a shadowed mobile fading channel, the direct path signal level can no longer be considered constant. Mathematical modeling based upon experimental data has shown that in such case, the channel can be assumed Rician with its local mean, i.e., the LOS signal component, following a lognormal statistical distribution [34, 35]. The probability density function (pdf) of the signal envelope $r$ can be expressed as

$$f(r) = \frac{r}{b_0 \sqrt{2\pi d_0}} \int_0^\infty \frac{1}{z} \exp \left[-\frac{(\ln z - \mu)^2}{2d_0} - \frac{r^2 + z^2}{2b_0}\right] I_0 \left(\frac{rz}{b_0}\right) dz$$

(1.5)

where $b_0$ is the power of the multipath signal, $\mu$ and $\sqrt{d_0}$ are the mean and standard deviation of the shadowing process. The pdf of the lognormally distributed LOS signal component can be written as

$$f(z) = \frac{1}{\sqrt{2\pi d_0}z} \exp \left[-\frac{(\ln z - \mu)^2}{2d_0}\right].$$

(1.6)

Specific values for $b_0$, $\mu$ and $\sqrt{d_0}$ have been determined experimentally for enabling this
mathematical model to accommodate three types of shadowed fading conditions, namely, light average and heavy [34].

1.2.2 Frequency Selective Mobile Fading

The particular situation giving rise to frequency selective mobile fading can perhaps be explained easier using the illustration of Fig. 1.2. In the simplified propagation instance shown,

the transmitted signal reaches the mobile receiver via three discrete paths. The delay $\tau_1$ of the direct signal component on path-1 is the reference and hence assumed equal to 0. The delay $\tau_2$ of the second path is still small with respect to the symbol period $T$ and its effect alone would still be regarded as non-selective fading. The third signal ray, however, arrives at the receiver with considerable delay. The cumulative effect of the three signals clearly causes intersymbol interference (ISI) [14]. It is well known, that in a mobile fading environment signal reflections arriving with large delay spreads at the receiver, e.g., with $\hat{\tau} > 0.1$ cause frequency selective fading [14]. With reference to the maximum value of $\hat{\tau}$, for the purpose of this thesis, we considered $\hat{\tau} \leq 1$. The main reason for this choice is that this range is recommended in the
model assumed by the EIA for the North American digital cellular standard [21], and is also commonly adopted by other researchers investigating the time dispersive frequency selective mobile fading channel [36, and references within].

In a more practical mobile radio environment than the one illustrated in Fig. 1.2, with reflectors of arbitrary size and orientation, we don't have discrete signal rays, but rather, concentrations of signal energy arriving with different delays at the receiver. Each one of these concentrations can be assumed to be a signal following Rayleigh fading statistics. If a LOS signal path exists, the concentration exhibiting the smallest time delay is by definition assumed to be the "direct" signal, for which the fading channel characteristic is Rician. In this respect, the "direct" signal will have a finite $K$, while all other signals, being Rayleigh, have $K \to -\infty$ dB.

1.3 MITIGATION TECHNIQUES FOR MOBILE FADING CHANNELS

The effect of fading distortion to digital communication systems is indeed quite devastating [26]. The random signal phase fluctuations of fading give rise to irreducible error rates, called error floors [11]. Such error floors, as the term implies, are independent of received signal strength, and in fast fading conditions can render a communication system completely unusable. Over the years, a very large number of fading mitigation techniques have appeared in the open technical literature for a wide variety of fading channel conditions. Since it is virtually impossible to present them all within the limited space of this section, we briefly summarize important relevant techniques which are related to the research effort reported in this thesis. The four following subsections each describe the most important qualities, advantages and disadvantages of fading combating signal reception methods, classified by the general directions of research carried out in this area.
1.3.1 Coherent Detection Techniques

In coherent transmission systems, an estimate of the carrier signal employed at the transmitter is constructed at the receiver, with the purpose of recovering the information conveyed by the received signal [14]. The vast majority of fading mitigation techniques for coherent systems involve the transmission of pilot tones [37, 38, 39] or pilot sequences [40, 41, 42, 43, 44]. The main idea behind such fading combating methods is the following. The signal impairment is extracted by processing the received tones or sequences, and a coherent reference signal is generated. Assuming that the fading effects are the same on both pilot and data, this coherent reference can be used to cancel, to great extent, the effects of fading on the information bearing signal. However, such techniques have several disadvantages. More specifically, for relatively fast fading applications, the overhead required to adequately offset the fading distortion is impractically high. As it has been pointed out in [45], in order to achieve acceptable performance results, the intervals between pilot symbol insertions must be less than $1/(2B_FT)$. In other words, the rate of pilot symbol insertion must be proportional to more than twice the average rate of the fading distortion, rendering the redundancy for a pilot symbol assisted scheme unacceptably high for a fast fading environment. For example, fading with $B_FT = 0.125$ would require a redundancy of at least 25%. Furthermore, such pilot calibration techniques tend to yield relatively complicated receiver structures, relying heavily on memory to store signal samples, and introducing delay in decoding decisions as a side-effect of estimating the distortion imposed by the channel.

1.3.2 Noncoherent Detection Techniques

Conventional Techniques

In contrast with coherent systems, noncoherent detection does not require an estimate of the transmitter carrier signal at the receiver. Instead of using absolute carrier phase values, the
transmitter translates information symbols to carrier phase differences [14]. Hence, information can be recovered at the receiver by comparing the received signal with its value at some time instant in the past, usually at the symbol rate. Noncoherent detection can be subdivided into limiter/discriminator detection [23, 46], and differential detection [47, 48, 49, 50, 51, 52, 53]. The limiter/discriminator detection, although simple and low-cost in terms of implementation, suffers from the disadvantages of the frequency discriminator it employs. A serious problem is the FM "click" effect [54], whereby an FM receiver will lock to a signal at an adjacent frequency, if its level is considerably higher than that of the desired signal. Differential detection, on the other hand, maintains the simplicity in implementation but does not exhibit the drawbacks associated to the frequency discriminator. It is, in this sense, more robust, providing extremely fast signal acquisition as compared to coherent detection techniques outlined in the previous subsection, and a relatively good performance in additive white Gaussian noise (AWGN) and slow fading channels, equivalent to that of limiter/discriminator detection. Note, however, that as well known, its performance in AWGN is worse than that offered by coherent detection [14]. The improvement in performance provided by this classical 1-symbol differential detector in a fading channel, can be understood by regarding its operation of subtracting adjacent symbol phases in order to recover the information bearing carrier phase change. In a slow fading channel, for most of the time, the phase distortion between adjacent symbols is sufficiently small, so that it is cancelled out to a large degree, by this phase subtraction. Performance comparisons for these noncoherent techniques can be found in [55], and particularly for limiter/discriminator detection versus differential detection in [56]. As awareness of the benefits inherent to differential detection in fading channels was established, the idea of expanding the signal observation from a adjacent symbols to a multi-symbol "window" was quick to follow, as it was expected to improve the receiver performance in fading.

\[7\] A 1-symbol differential detector gets its name by the fact that it operates on adjacent symbols, i.e., signal symbols with time distance of one symbol (1\ $T$).
Multiple Differential Detection Techniques

The hardware structure implementing an expanded observation interval spanning more than two symbols, is called a multiple differential detector (MDD). A block diagram illustrating this hardware structure is shown in Fig. 1.3. At a time instance $kT$, the received signal sample is denoted as $y_k$, and the output of a conventional differential detector as $d_1(k)$. Assuming that the MDD has access to $Z$ received signal inputs, the MDD hardware structure provides additional outputs $d_2(k)$ up to $d_{Z-1}$, by employing delay elements of progressively increasing multiples of $T$, covering all symbols within the observation interval. Clearly, the number of differential detectors used is $z = Z - 1$.

![Block diagram illustrating the multiple differential detector hardware structure. PD: phase detector.](image)

The MDD hardware structure was used in the past first by Chow and Ko [57] in the form of a hard decision error correction scheme referred to as nonredundant error correction (NEC). The work in [57] has shown that a receiver employing the MDD structure and a signal observation window of $Z$ symbols, employing $z - 1$ differential detectors, has the capability to correct up
to $z - 2$ errors. This improvement in receiver performance, as the name of this error correction scheme implies, is achieved without introducing any redundancy in the transmission. Note that the analysis and results reported in [57] assume only binary phase shift keying (BPSK)\(^8\) in AWGN, yielding relatively small gains in overall system BER performance. In [58], Masamura \textit{et al.} have applied the NEC technique to the minimum shift keying (MSK) constant envelope scheme. Samejima \textit{et al.} in [53], have extended the work of [57] to include differential PSK (DPSK) signals, including DQPSK. The performance of $\pi/4$-shift DQPSK signals employing the NEC scheme at the receiver, has been analyzed and evaluated by Wong and Mathiopoulos, for the static\(^9\) co-channel interference (CCI) channel in [59]. Due to the additive nature of the static CCI, significant BER performance improvements were reported in [59]. However, as shown in [60], in general, the NEC technique does yield any significant performance gains for mobile fading channel's. Only small error floor reductions for extremely fast fading conditions have been reported in [60].

Motivated by the implementation simplicity and robustness of differential detection, Makrakis, Mathiopoulos and Bouras proposed for the first time, the use of MDD receiver structures for the mobile fading channel [61, 62, 63]\(^10\). The general problem investigated dealt with optimal detection of PSK and QAM signals transmitted over frequency non-selective correlated fast fading channels [63]. By using maximum likelihood detection arguments, the optimal sequence estimator was derived, for coded digital multi-phase/multi-amplitude signals, corrupted by AWGN and multiplicative Rayleigh or Rician frequency non-selective fading. The derivation presented in [63] does not make any assumptions on the type of fading autocorrelation function, or on how slow or fast the fading might be, although both are assumed to be known to the

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\(^8\)In BPSK, as the name implies, there are only two points on the signal constellation, at phases of $0^\circ$ and $180^\circ$, each one conveying one of two possible states, 0 or 1.

\(^9\)Static, in this case, implies that transmitter, receiver and interfering transmitters are all stationary, i.e., there is no random phase/amplitude signal distortion caused by any signal reflections and movement, as in fading.

\(^10\)Earlier work related to MDD techniques in AWGN, published by Makrakis and Mathiopoulos, can be found in [64] for QAM and in [65] for PSK signals.
receiver. The authors do not adopt the simplified case where the fading is assumed to be constant over several symbol intervals ([66, 52, 67]), or even during one symbol interval. Rather, both the fading amplitude and phase distortion are assumed to change significantly, according to the fading's statistical characteristics. Furthermore, contrary to common practice ([66, 51, 52]), the fading correlation is not removed by using interleaving at the transmitter and de-interleaving at the receiver. Although the advantage of interleaving is that transmission schemes derived for AWGN can be readily applied to fading, the disadvantage is the increased implementation complexity and processing requirement introduced, and their resultant impact on the practical implementation of small, lightweight portable units as such desirable for MDTS. Instead of removing the fading correlation by interleaving, the optimal sequence estimator described in [63] uses it to the receiver's advantage. Its structure can be viewed as a combination of a squared envelope detector, a MDD hardware structure, and a coherent detector. The squared envelope detector provides information on the signal amplitude, and is only needed when schemes such as QAM, which have more than one signal levels, are employed. The coherent detector is used by the receiver only for providing an estimate of the transmitter modulator initial phase, and is only required for the case of Rician fading; it is not needed in Rayleigh fading conditions. In order to reduce the algorithmic complexity of the optimal sequence estimator derived, the authors of [63] also presented suboptimal, reduced complexity versions of these algorithms. Using computer simulation they also demonstrated that the large portion of performance gains available from these techniques, is obtained with relatively low complexity implementations of the suboptimal algorithms. The performance improvements presented in [63] are rather significant, as error floors are shown to decrease by orders of magnitude, compared to performance achieved by conventional detection methods.

Interleaving at the transmitter is accomplished, for example, by entering symbols to be transmitted in a matrix row by row, and then transmitting the matrix contents column by column. At the receiver, the inverse operation restores the symbol sequence to its original order. If the row/column length of the interleaving matrix is sufficiently large, the errors afflicted upon the received symbols due to the fading distortion are, for all practical purposes, uncorrelated.
We conclude this subsection by mentioning that since 1990, i.e., after the publication of [61] and [64], MDD based receivers have been investigated by various other researchers, and have resulted in several journal and conference publications. Concurrent with [61], in [68] Wilson et al. investigated the use of MDD for improving the performance of differential detection of QPSK and 8-PSK in AWGN, and presented some suboptimal detection procedures. Divsalar and Simon in [69] also proposed the use of MDD techniques\textsuperscript{12} for PSK signals in AWGN. This work is very similar to that of [65]. In [70], Ho and Fung have obtained analytical performance bounds for uncoded PSK signals in Rayleigh fading and AWGN. The work in [70], employs interleaving for reducing the correlation between faded signal samples, and only considers Rayleigh frequency non-selective fading and uncoded single amplitude schemes such as PSK.

1.3.3 Diversity Techniques

Diversity techniques for improving the signal reception in fading channels are classified into three general categories, namely, selection diversity combining (SC), equal-gain combining (EGC) and maximal ratio combining (MRC) [26, 11]. In SC, the least complicated of the three, the strongest signal is selected for processing at the receiver end. This method is impractical for use in mobile radio communication, as it requires maintaining a floating threshold level [11]. A more practical version of SC is called switched combining (SWC) and it amounts to having the receiver use the selected signal until it drops below a predetermined switching threshold. Following that, it switches to the strongest signal at that moment. In MRC, before combining, each signal is scaled according to an estimate of the signal strength on that channel. SC and its derivative SWC, both require switching signals at the receiver, while MRC requires more complicated and hence, more costly receiver design. EGC, on the other hand, is implemented very simply by incoherently summing up the signals available on all

\textsuperscript{12}They refer to MDD as multi-symbol differential detection.
Introduction

diversity channels. Analysis found in [11] shows that the performance of EGC is only slightly worse than that of MRC, while providing a considerable simplification in receiver design. As intuitively expected, the performance improvement available from all three methods decreases as the correlation between the available diversity signals increases [11].

More recent work, investigating the performance of DQPSK and $\pi/4$-shift DQPSK signals, employing all three types of signal combining in conjunction with differential detection, is presented by Adachi et al. in [71]. The signal is assumed to be corrupted by CCI and frequency selective fading. An expression for the BER is derived, for the MRC and the SC case, and numerical results show the MRC technique to be slightly superior to the SC technique, with virtually identical BER performance for $\pi/4$-shift DQPSK and DQPSK. The work reported in [71] does not address the more general case of Rician fading, and also uses simplifications on the nature of the fading distortion, based on the assumption of a slow fading rate, whereby the channel transfer function is assumed approximately constant during the symbol duration $T$. In [72, 73], Kam and Ching investigate sequence estimation of PSK signals with diversity reception over slow, Rayleigh fading channels. In this work, the correlation present in faded signal samples is removed by the use of interleaving at the transmitter and de-interleaving at the receiver, the drawbacks of which have already been mentioned in the previous section. In addition, reliable sequential estimation of the transmitted symbols depends heavily on faded signal amplitude estimation, which is the first of two stages incorporated in the proposed decoding algorithm. More recent work involving MRC used in conjunction with a QAM modulation scheme can be found in [74]. In that paper, experimental results from a hardware prototype and computer simulation are provided, for frequency non-selective Rayleigh fading and co-channel interference. Fading estimation is provided by inserting pilot symbols in the information sequence, and expectedly, the technique breaks down in fast fading environments. In [75], a new diversity method is introduced, called code combining (CC), for improving the performance of $\pi/4$-shift DQPSK, in both frequency selective and non-selective Rayleigh
fading, CCI and AWGN. The analytical performance bounds presented in this work assume that the interleaving degree is large enough to effectively eliminate all correlation in adjacent signal samples due to fading; also the fading $B_FT$ product is assumed approximately equal to 0, i.e., the fading is assumed to be very slow. Experimental results for SC and MRC are also presented in [76], where the performance of $\pi/4$-shift DQPSK employing block coding and diversity is investigated in CCI and frequency selective Rayleigh fading. This work also assumes interleaving sufficient to randomize the burst errors caused by fading, and does not consider the more general case of Rician fading.

### 1.3.4 Equalization Techniques

For more than two decades, several nonlinear equalization techniques, including decision feedback equalization and maximum likelihood sequence estimation (MLSE)\footnote{For a brief introduction to MLSE see Section 1.3.5.} - efficiently implemented by a Viterbi algorithm - have been developed for improving the performance of digital transmission systems, in order to provide reliable communication over time invariant or slowly varying channels, with severe inter-symbol interference (ISI) [14]. A considerable volume of techniques and results reported on this particular subject appear in the literature; as examples we mention equalization in the time invariant 300 Hz - 3 kHz telephone channel as found in [77], application to slowly varying high frequency (HF) radio channels in [78, 79], and for microwave LOS communications in [80]. In contrast to the wealth of techniques for equalization in static and slowly varying channel conditions, relatively little has been published in the open technical literature on combating both ISI and the rapidly varying characteristic of the frequency selective mobile fading channel [81]. Several techniques to this end have recently appeared, dealing with analysis and evaluation of various equalization structures [82, 83], including diversity [84, 85].
1.3.5 Maximum Likelihood Sequence Estimation (MLSE) Techniques

Interestingly enough, the vast majority of research work on combating frequency selective fading has concentrated on equalization rather than MLSE as a means of combating the combined effect of ISI and fading. The frequency selective multipath Rician fading channel presents two major obstacles when attempting to analyze MLSE detection; the first one being the time varying characteristic in the amplitude and phase of the fading distortion, and the second one being the non uniform attenuation over the signal spectrum (frequency selectivity). It is only relatively recently that research work employing MLSE has been reported, using various maximum likelihood detectors for digital signals transmitted over frequency selective Rayleigh fading channels [36, 86]. In [36], Alles and Pasupathy describe the derivation of optimal symbol-by-symbol detectors for a two-ray Rayleigh fading channel. The second publication [86], by Dai and Shwedyk, presents a more general approach of sequence estimation for a generalized frequency selective Rayleigh fading channel. More specifically, in [86] two sequence estimators, which are referred to as i) MLSE with Viterbi Algorithm (MLSE-VA) and ii) Sequential Sequence Estimator (SSE), have been proposed. It has been shown that for the same ISI channels, the SSE has much lower computational complexity while achieving a BER performance which is almost identical to that of the MLSE-VA. However, in terms of analysis and performance evaluation, both of these publications deal exclusively with Rayleigh fading. In addition, they only consider binary modulation schemes without the use of any coding, presenting a relatively limited set of results.

1.4 ANALYTICAL BER PERFORMANCE BOUNDS

Although deriving analytical bounds for the BER performance of digital communication systems is more often than not a formidable task, it is indeed a preferred method for evaluating such systems. There are two main reasons for this, both related to shortcomings of the other
method commonly employed to obtain such performance evaluation results, namely, computer simulation. The first one is related to the fact that simulation results are practical only down to error rate levels of about $10^{-4}$, and in some rare cases, at most down to $10^{-5}$ [87]. This is due mainly to the large time interval required to process long signal sequences, and the number of error events needed to acquire a good estimate of the error rate. The second reason why analytical bounds are attractive is related to rarely occurring phenomena which also require extremely long signal sequences if the estimate of their effect in a computer simulation is to be statistically significant. An interesting example of one such instance arises in the case of slow fading, where the occurrence of deep fades\textsuperscript{14} is much less frequent than in the fast fading case, in terms of signal samples processed in a digital simulation. This can even lead to the total absence of such a deep fade if the number of samples is not large enough, yielding, in turn, an erroneous estimate of the system error rate.

BER performance bounds for trellis coded coherent 4-PSK and 8-PSK are presented by Biglieri and McLane in [88]. In this publication, perfect channel state information is assumed, rendering the results practical only for channels exhibiting very slow fading conditions. In [89], McKay et al. present evaluation results using analytical bounds, for the performance of trellis coded 8-PSK in Rayleigh, Rician and EHF shadowing channels, with and without ideal channel state information. The authors of [89] assume interleaving/de-interleaving which increases the receiver implementation complexity, and do not examine higher spectral efficiency, multi-level modulation formats. Analytical results for the performance of receivers employing the MDD hardware structure have been presented by Ho and Fung in [70], where they have derived an expression of the pairwise error event probability of MDD for uncoded PSK signals transmitted over a Rayleigh fading channel. Their analysis is based upon a residue theorem technique reported in [90], using interleaving/de-interleaving and pilot sequences for channel

\textsuperscript{14}By the term "deep fade" we describe the severe attenuation randomly imposed on the received signal due to fading [26].
state estimation. In [91], Divsalar and Simon also provide analytical results for PSK systems using the MDD hardware structure, in AWGN and slow fading channels. They also provide some computer simulated BER results for a differentially detected 16-QAM scheme in an AWGN channel, but not for a fading channel. In conclusion, it should be pointed out that none of the aforementioned publications include the effects of diversity reception in their analytical derivations.

1.5 THESIS RESEARCH CONTRIBUTION

The foregoing discussion on noncoherently detected, PSK and QAM type of signals, transmitted over mobile fading channels with diversity, has established the context in which the three research contributions of this thesis are presented as follows.

1. We derive the optimal receiver, in the maximum likelihood detection sense, for fast frequency non-selective Rician fading channels, employing diversity reception. The receivers employ the MDD hardware structure, in conjunction with novel optimal detection algorithms. The proposed analysis is general enough to accommodate any type of modulation signal. However, for performance evaluations we considered the $\pi/4$-shift DQPSK, and the more spectrally efficient trellis coded 8-DPSK and $\pi/4$-shift 8-DQAM schemes, employing trellis coding in all cases. In order to reduce the overall implementation complexity, we propose suboptimal, reduced complexity versions of these algorithms, and evaluate their performance by using digital computer simulation. The results obtained demonstrate significant improvement over conventional schemes. Furthermore, taking into consideration the inaccuracies inherent to practical system implementations, we investigate cases where the receiver statistical knowledge of the fading channel is in considerable error with respect to the actual channel state. Results show relatively small sensitivity, even to large inaccuracies in estimates of channel parameters.
2. We present the theoretical derivation of the optimal receiver, in the maximum likelihood detection sense, for correlated fast Rician frequency selective fading channels. The impact of simplifying assumptions made to facilitate the theoretical derivation are investigated via digital simulation. Furthermore, we estimate the sensitivity of the receiver performance to inaccurate knowledge of channel parameters. The results show significant performance improvements over conventional signal detection methods, accompanied by small sensitivity to imperfections. The performance of the derived receiver structure is also compared to the fastest known equalization technique for frequency selective fading, which employs the modified square root Kalman equalization algorithm derived in [78] for use in HF dispersive channels. Results show the optimal receiver to outperform equalization in fast fading channel conditions.

3. We derive novel analytical BER performance bounds for simplified versions of the optimal diversity receiver, for frequency non-selective, Rician fading, in the most general case when no channel state information is available. Simplified versions of the diversity receiver structure are derived for varying degrees of channel state information, and are shown to be optimal in the case of slow fading channel conditions. Using digital simulation and computer aided calculation of the derived bounds, we evaluate the performance of these receivers in slow as well as fast Rician fading, employing different fading rates, and various orders of reception diversity and receiver implementation complexity. The excellent agreement between calculation and computer simulation results, demonstrates the tightness and accuracy of the derived bounds. Motivated by the performance improvement gained for increasing diversity order, and given the fact that antenna diversity is more practical in higher frequencies with smaller wavelengths, we present BER evaluation results of these simplified receivers in an EHF channel, experiencing light, average and heavy shadowing.
1.6 THESIS ORGANIZATION

After this introductory chapter, the three research contributions summarized in Section 1.5, are presented in Chapters 2, 3 and 4.

In Chapter 2, first, the model for the communication system employed throughout this thesis is described in Section 2.1. The optimal receiver structures for Rician frequency non-selective fading are derived in Section 2.2, where details regarding the decoding algorithms are presented. Section 2.3 includes the BER performance evaluation results for the optimal receivers derived, and results of the investigation of receiver sensitivity to errors in knowledge of channel statistics. Section 2.4 offers the conclusions of this chapter.

In Chapter 3, Section 3.1 presents the model employed for the frequency selective mobile fading channel, and Section 3.2 the implications of the channel imposed distortion to the received signal. In Section 3.3 we present the derivation of the optimal receiver structure and details on the associated decoding algorithms. BER performance evaluation and sensitivity investigation results are included in Section 3.4, with concluding remarks in Section 3.5.

In Chapter 4, Section 4.1 includes the derivation of the simplified receiver structures for the case of slow fading conditions. Three versions are presented, for varying degrees of channel state information. In Section 4.2, we present the derivation of the analytical BER performance bounds for the case of no channel state information, in Rician frequency non-selective fading. Section 4.3 includes results from computer simulation and computer aided calculation of the performance bounds derived, for Rician, as well as EHF shadowed fading channels. Section 4.4 offers the conclusions of this chapter.

The thesis concludes with Chapter 5, which is divided in two parts. Section 5.1 presents the concluding remarks of this thesis, and Section 5.2 offers suggestions on future work, as inspired during the course of this research effort.
CHAPTER 2

OPTIMAL SEQUENCE ESTIMATION FOR FAST
CORRELATED FREQUENCY NON-SELECTIVE RICIAN
FADING CHANNELS WITH DIVERSITY

In this chapter, we derive and evaluate the optimal sequence estimator for digital signals received over \( \Lambda \) different channels. Each of these channels corrupts the transmitted signal by a mixture of AWGN and frequency non-selective, correlated, fast Rician fading. The diversity assumed is of the equal gain combining type, and is implemented within the decoding metric.

The organization of the chapter is as follows. In Section 2.1, we describe the overall system model, including the transmitter and channel model under study. In Section 2.2, we present the theoretical derivation of the optimal equal combining diversity receiver. Section 2.3 summarizes various BER performance evaluation results. Finally, Section 2.4 contains the conclusions of this chapter.

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\(^{1}\)The research reported in this chapter has been presented in part at the 1991 IEEE Pacific Rim Conference, Victoria, Canada [92], and has been published as a full paper in the *IEEE Transactions on Vehicular Technology* [93].

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2.1 TRANSMITTER AND FREQUENCY-FLAT FADING DIVERSITY CHANNEL MODEL

The block diagram of the model assumed for the communication system transmitter\(^2\) appears in Fig. 2.1. The transmitter consists of a Convolutional Encoder (CE), a Signal Mapper (SM), a Differential Encoder (DE), a transmit pulse-shaping filter with transfer function \(H_T(f)\) and a modulator (complex multiplier). The \(p\) -bit information words \(\bar{a}_k^p = [a_k^1, a_k^2, \ldots, a_k^p]\) of the input data sequence \(\bar{A}\), consist of independent and equiprobable bits taking values from the alphabet \(\{0, 1\}\). This input sequence is transformed by the CE to \(q\) -bit words \(\bar{b}_k^q = [b_k^1, b_k^2, \ldots, b_k^q]\), by using a \(p/q\) rate convolutional code. The signal mapper then converts these words to symbols \(\delta_k = \gamma_k \exp(j\Omega_k)\), where \(\gamma_k\) represents the amplitude and \(\Omega_k\) the phase of \(\delta_k\), respectively. The sequence of \(\delta_k\)'s is then differentially encoded, resulting in the sequence of transmitted symbols

\[
c_k = \delta_k \frac{\gamma_{k-1}}{\gamma_{k-1}} = \gamma_k \exp \left[j(\Phi_{k-1} \oplus \Omega_k)\right] \tag{2.7}
\]

with \(\Phi_k\) denoting the phase of \(c_k\) and \(\oplus\) modulo \(2\pi\) addition. The non-return-to-zero (NRZ) sequence of \(c_k\)'s is shaped by the transmitting premodulation filter \(H_T(f)\), resulting to the baseband signal

\[
x_B[\bar{C}(\bar{A}), t] = \sum_{k=0}^{Z-1} c_k h_T(t - kT) \tag{2.8}
\]

\(^2\)The transmitter model illustrated in the block diagram of Fig. 2.1 is the one assumed not only for this chapter, but for all work reported throughout this thesis.
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where $T$ is the symbol duration, $Z$ is the number of symbols transmitted, $h_T(t)$ is the impulse response corresponding to $H_T(f)$, and $x_B \left[ \bar{C}(\bar{A}), t \right]$ the transmitted sequence generated by the information sequence $\bar{A}$. The frequency response shapes of the filters utilized throughout the work reported in this thesis, are the $\sqrt{\alpha}$ raised cosine (Nyquist filtering), and the 4th order Butterworth. These are given as the transfer function [14]

$$H(f) = \begin{cases} 
T & 0 \leq |f| \leq (1 - \alpha)/2T \\
\frac{T}{2} \left[ 1 - \sin \pi T \left( f - \frac{1}{2T} \right) / \alpha \right] & (1 - \alpha)/2T \leq |f| \leq (1 + \alpha)/2T 
\end{cases} \quad (2.9)$$

for Nyquist filtering, and as the transfer function [94]

$$H(f) = \frac{f_B^3}{\left[ f_B^4 + \frac{1}{2\pi} \cos \frac{\pi}{8} - f \right] \left[ f_B^5 + \frac{1}{2\pi} \cos \frac{3\pi}{8} - f \right]} \quad (2.10)$$

for Butterworth filtering, with $f$ denoting frequency and $f_B$ the 3 dB Butterworth filter cut-off, both in Hz. Assuming for example a rate-1/2 trellis coded $\pi/4$-shift DQPSK signal, we will have $p = 1, q = 2, \overline{a_k} = [\overline{a_1_k}, \overline{b_k}] = [\overline{b_1_k}, \overline{b_2_k}], c_k = \delta_k c_{k-1}/\gamma_k-1$ and $\bar{C}(\bar{A}) = [c_1, c_2, ..., c_Z]$, with $\gamma_k = 1$ and $\Omega_k \in \{\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\}$, resulting in the signal-space diagram of Fig. 2.2. For a rate 2/3 trellis coded $\pi/4$-shift 8-DQAM signal, we will have $p = 2, q = 3, \overline{a_k} = [\overline{a_1_k}, \overline{a_2_k}], \overline{b_k} = [\overline{b_1_k}, \overline{b_2_k}, \overline{b_3_k}], c_k = \delta_k c_{k-1}/\gamma_k-1$ and $\bar{C}(\bar{A}) = [c_1, c_2, ..., c_Z]$. Such a signal will have $\gamma_k \in \{1/3, 1\}$ and $\Omega_k \in \{\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\}$, resulting in the signal-space diagram shown in Fig. 2.3. The rate 1/2 and rate 2/3 trellis codes used for $\pi/4$-shift DQPSK and $\pi/4$-shift 8-DQAM respectively, are the best codes for AWGN channels found in [25]. The convolutional coders used as well as the signal phase and amplitude assignment to signal numbers are depicted in Fig. 2.4.

The modulator up-converts this signal to its carrier frequency $f_c$, resulting to the transmitted signal

$$x(t) = \text{Re} \left\{ x_B \left[ \bar{C}(\bar{A}), t \right] \exp[j(2\pi f_c t + \eta)] \right\} \quad (2.11)$$

with $\text{Re}\{\cdot\}$ denoting real part of $\{\cdot\}$ and $\eta$ the modulator initial phase.
Figure 2.2: The signal space of the π/4-shift DQPSK transmitter; a) before spectral shaping, b) at the output of the receiver filter in the absence of interference; raised cosine filter α = 0.7.

Figure 2.3: The signal space of the π/4-shift 8-DQAM transmitter; a) before spectral shaping, b) at the output of the receiver filter in the absence of interference; raised cosine filter α = 0.7.
Figure 2.4: Convolutional coders used for \(\pi/4\)-shift DQPSK, \(\pi/4\)-shift 8-DQAM and 8-DPSK, showing assignment of signal phases and amplitudes to signal numbers.
We investigate the case where \( x(t) \) is transmitted over \( 1 \leq l \leq \Lambda \) in general correlated channels which corrupt \( x(t) \) with a mixture of multiplicative nonselective fading \( f^l(t) \) and AWGN \( n^l(t) \) with a double-side power spectral density \( N_0/2 \). \( f^l(t) \) is modeled as a complex summation of two white and independent Gaussian noise processes, \( n'_f^l(t) \) and \( n''_f^l(t) \), filtered by two identical filters \( H^l_F(f) \) [26], as illustrated in Fig. 2.5. Hence \( f^l(t) = n'_f^l(t) + jn''_f^l(t) \),

\[
\begin{align*}
\text{Complex signal:} & \quad \rightarrow \\
\text{Real signal:} & \quad \rightarrow
\end{align*}
\]

Figure 2.5: Block diagram of the low-pass equivalent of the frequency selective fading model.

with \( f'^l(t) \) and \( f''^l(t) \) having the same autocorrelation function, \( R^l_F(\tau) \), which is given by

\[
R^l_F(\tau) = E \left\{ \left[ f'^l(t) - f'^l(t - \tau) \right] \left[ f'^l(t) - f'^l(t - \tau) \right] \right\} \\
= E \left\{ \left[ f''^l(t) - f''^l(t - \tau) \right] \left[ f''^l(t) - f''^l(t - \tau) \right] \right\} \\
= N_f^l \int_{-\infty}^{\infty} \left| H^l_F(f) \right|^2 e^{j2\pi f \tau} df. \quad (2.12)
\]

In the above equation, \( N_f^l \) is the power spectral density of the white Gaussian noise process generating \( f^l(t) \), \( f'^l(t) = E \{ f'^l(t) \} \) and \( f''^l(t) = E \{ f''^l(t) \} \), where \( E\{\cdot\} \) denotes expected value. As \( f'^l(t) \) and \( f''^l(t) \) are independent, their cross-correlation is zero. In the case where a LOS signal component exists, the fading follows Rician statistics. Furthermore, since the phase of Rayleigh faded component is uniformly distributed within \([0, 2\pi)\), the constant phase of the
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LOS signal component can be assumed to take any value within \([0, 2\pi)\). Without any loss of generality it will be assumed henceforth that \(\overline{f^Q(t)} = 0\), i.e., that phase of the LOS signal component is equal to 0. The transfer function \(H_p^f(f)\) of the fading filter employed is related to the spectral characteristics of the fading model considered [30]. As this thesis deals with land-mobile applications, the fading model adopted throughout is that of land-mobile fading, having the autocorrelation function [26, 30]

\[
R_p^l(\tau) = J_0(2\pi B_F \tau) \quad \forall \ 1 \leq l \leq \Lambda. \tag{2.13}
\]

For the Rician fading channel, comparisons under different fading conditions employ the \(K\)-factor, which in this particular case is given as

\[
K = 10\log_{10} \left[ \frac{\langle f^I(t) \rangle^2}{\sigma^2_{f^I} + \sigma^2_{f^Q}} \right] \text{dB} \tag{2.14}
\]

where \(\sigma^2_{f^I}\) and \(\sigma^2_{f^Q}\) are the variances of \(f^I(t)\) and \(f^Q(t)\), respectively.

In general, the diversity receiver consists of \(\Lambda\) different branches. Diversity can be implemented either in space, frequency or time, with advantages and disadvantages in each case [11]. For relatively low carrier frequencies (i.e., with large wavelengths), for example, it might not be practical to employ space diversity in the form of using \(\Lambda\) different receiver antennas, especially when considering a portable unit of small physical size. In such case time or frequency diversity would be preferable, requiring, nevertheless, additional bandwidth. On EHF frequencies, however, where the wavelength is within the 1-2 cm range, space diversity is very easily realizable, and since it does not require additional bandwidth, it is much more attractive solution than either one of the other two types. In terms of combining, we assume that the average power is equal for signals on all diversity channels, which leads to EGC, as will become apparent in the next section. The received signal at the \(l^{th}\) branch, after corruption by multiplicative fading and AWGN, can be mathematically expressed as

\[
x_p^l(t) = f^l(t)x(t) + n^l(t) \quad 1 \leq l \leq \Lambda. \tag{2.15}
\]
For the computer simulation performance results which will be reported in Section 2.3, the
genral case was considered where the fading interference $f'(t)$ introduced on paths $l' \in \{2, 3, ..., \Lambda \}$ can be correlated with that of path $l = 1$. A correlation coefficient $\rho_{1,l'}$ is used as a
measure of correlation between the amplitudes of the fading processes on the $\Lambda$ diversity paths.
During evaluation of the proposed diversity receiver structures through computer simulation, experimental values of $\rho_{1,l'}$ found in [95] were used.

![Block diagram of the diversity receiver](image)

Figure 2.6: Block diagram of the diversity receiver.

The general block diagram of the diversity receiver is illustrated in Fig. 2.6. Each block
contains a wideband (roofing) Band Pass Filter (BPF) which limits the Gaussian noise without
distorting the information bearing signal $f'(t)x(t)$, a coherent demodulator$^3$, a predetection filter

$^3$For purely mathematical convenience, $f_x$ is assumed to be known to the receiver, so that the derivation
of the optimal diversity detector can be carried out in the complex baseband domain. Notice that through this
complex demodulation, the rapid phase and amplitude changes caused by the fast fading, appear at the output of
the demodulator totally uncompensated. Note that, if desirable, this can be accomplished in practice by employing
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$H^I_R(f)$ matched to the pulse shaping filter $H_T(f)$ employed at the transmitter, and a receiver to be derived in Section 2.2. Observing Fig. 2.6, the signal at the output of $H^I_R(f)$ can be expressed as

$$y_l(t) = f^I_l(t) \sum_{k=0}^{Z} c_k e^{j\pi} h^I_l(t - kT) + n^I_s(t) - j n^I_c(t)$$  \hspace{1cm} (2.16)

where $h^I_l(t)$ represents the inverse Fourier transform of $H_T(f) \cdot H^I_R(f)$, and $n^I_s(t)$ and $n^I_c(t)$ represent the in-phase and quadrature baseband components of the narrowband Gaussian noise respectively. Under the assumptions that $H_T(f) \cdot H^I_R(f)$ satisfies the Nyquist I criterion and that the BPF does not alter the fading characteristics\(^4\), the sampled signal at the output of the predetection filter $H^I_R(f)$ of the $l$th branch can be expressed as

$$y^I_k = (f^{I;l}_k + j f^{Q;l}_k) c_k e^{j\pi} + n^{I;l}_k - j n^{Q;l}_k$$  \hspace{1cm} (2.17)

where $f^{I;l}_k = f^{I;l}(kT)$, $f^{Q;l}_k = f^{Q;l}(kT)$, $n^{I;l}_k = n^I_s(kT)$ and $n^{Q;l}_k = n^I_c(kT)$. The samples $y^I_k$ ($1 \leq k \leq Z$) are fed into the receiver, the optimal structure of which is derived in the following section.

### 2.2 DERIVATION OF THE STRUCTURE OF THE OPTIMAL DIVERSITY RECEIVER

The derivation of the diversity receiver structure is presented in two parts. In the first one, we derive the structure of the optimal diversity receiver and present the associated optimal algorithms. For mathematical convenience in the derivation we assume that the fading processes on the $\Lambda$ diversity channels are uncorrelated. In the second part, we present suboptimal but reduced complexity versions of the optimal structure and associated suboptimal algorithms.

\(^4\)This is a reasonable assumption, since the transmission rate in almost all cases will be at least an order of magnitude greater than the maximum Doppler frequency $B_F$. 

subsystem locked in frequency, but not in phase, with the transmitter local oscillator.
computer simulation. Because of the high complexity presented by the optimal algorithms and the limitations inherent to the computer simulation, only reduced complexity suboptimal algorithms were evaluated. The obtained BER performance evaluation results involving various modulation schemes can be found in Section 2.3.

### 2.2.1 Optimal Algorithms

The optimal receiver will choose the data sequence $\mathbf{A} = [a_1^p, a_2^p, \ldots, a_L^p]$ which maximizes the following probability density function (pdf)

$$
\zeta \left[ y_1^l, y_2^l, \ldots, y_{\Lambda}^l \right] = \prod_{l=1}^{\Lambda} \prod_{k=1}^{Z} \zeta \left[ y_k^l | y_{k-1}^l, y_{k-2}^l, \ldots, y_1^l, \mathcal{C}(\mathbf{A}), \eta \right] (2.18)
$$

where $y_l^l = [y_1^l, y_2^l, \ldots, y_{\Lambda}^l]$ (1 ≤ l ≤ Λ).

Let us define the following variables

$$
e_k^{l;1} = f_k^{l;1} + n_e^{l;1} = \frac{1}{|c_k|^2} \text{Re} \left( y_k^l c_k^* e^{-j\eta} \right)
$$

$$
e_k^{l;2} = f_k^{l;2} + n_e^{l;2} = \frac{1}{|c_k|^2} \text{Im} \left( y_k^l c_k^* e^{-j\eta} \right)
$$

(2.19)

where * denotes complex conjugate and

$$
n_e^{l;1} = \frac{1}{|c_k|^2} \left[ n_k^{l;1} \text{Re} \left( c_k^* e^{-j\eta} \right) + n_k^{l;2} \text{Im} \left( c_k^* e^{-j\eta} \right) \right]
$$

$$
n_e^{l;2} = \frac{1}{|c_k|^2} \left[ n_k^{l;1} \text{Im} \left( c_k^* e^{-j\eta} \right) - n_k^{l;2} \text{Re} \left( c_k^* e^{-j\eta} \right) \right]
$$

(2.20)

Clearly, as $e_k^{l;1}$ and $e_k^{l;2}$ are the sum of Gaussian random variables, they are also Gaussian random variables with autocorrelation function

$$
R_k^l \left[ \mathcal{C}(\mathbf{A}), (k - i)T \right] = R_k^l \delta(k - i) + \frac{(\sigma_G^l)^2}{|c_k|^2} \delta(k - i)
$$

(2.21)

where $(\sigma_G^l)^2 = N_0^l/(2T)$ and $\delta(\cdot)$ is the Kronecker $\delta$-function [96]. From Eq. (2.21) it is evident that for multilevel signals, e.g., QAM, the correlation properties of $e_k^{l;1}$ and $e_k^{l;2}$ depend upon the
transmitted sequence. For PSK signals however, these correlation properties are independent of the sequence.

Maximizing the pdf $\zeta$ is shown in Appendix A.1 to be equivalent to maximizing the following pdf

$$\prod_{l=1}^{L} \prod_{k=1}^{Z} \frac{1}{2\pi (\sigma_k^l [\overline{C(A)}]^2)} \times \exp \left\{ -\frac{1}{2(\sigma_k^l [\overline{C(A)}]^2)^2} \left[ \frac{1}{|c_k|^2} \Re \left( y_k^l c_k^* e^{-jn} \right) - \frac{1}{|c_{k-m}|^2} \Re \left( y_{k-m}^l c_{k-m}^* e^{-jn} - \frac{1}{|c_k|^2} \right) \right] \right\}$$

$$\times \exp \left\{ -\frac{1}{2(\sigma_k^l [\overline{C(A)}]^2)^2} \left[ \frac{1}{|c_k|^2} \Im \left( y_k^l c_k^* e^{-jn} \right) - \frac{1}{|c_{k-m}|^2} \Im \left( y_{k-m}^l c_{k-m}^* e^{-jn} \right) \right] \right\}$$

(2.22)

In the above equation, $\left( \sigma_k^l [\overline{C(A)}]^2 \right)^2$ represents the $k^{th}$ order minimum mean square prediction error (MMSPE) of the $l^{th}$ branch and $p_{k,m}^l [\overline{C(A)}]$ ($1 \leq k \leq Z; 0 \leq m \leq k$) are the $k^{th}$ order prediction coefficients, calculated according to the statistics of the $l^{th}$ fading channel. To simplify the representation, from now on the dependence of $\sigma_k^l$ and $p_{k,m}^l$ on $\overline{C(A)}$ will be dropped. These prediction coefficients depend upon the fading model employed, the signal-to-noise ratio (SNR), the ratio between the power of the fading signal and the Gaussian noise as well as on the transmitted sequence for multilevel signals. Their values can be obtained by solving the following set of Yule-Walker equations

$$\overline{P}_k^l = \left[ \overline{R}_k^l \right]^{-1} \overline{D}_k^l$$

(2.23)
where $\bar{R}_k^l$ is a $[k \times k]$ matrix, $\bar{D}_k^l$ is a $[k \times 1]$ matrix and $\bar{P}_k^l$ is a $[k \times 1]$ matrix with

\[
\bar{R}_k^l = \begin{bmatrix}
R_F^l(0) + \frac{(\sigma^2_k)^2}{|c_{k-1}|^2} & R_F^l(1) & \cdots & R_F^l(k-1) \\
R_F^l(1) & R_F^l(0) + \frac{(\sigma^2_k)^2}{|c_{k-1}|^2} & \cdots & R_F^l(k-2) \\
\vdots & \vdots & \ddots & \vdots \\
R_F^l(k-1) & \cdots & R_F^l(k-2) & R_F^l(0) + \frac{(\sigma^2_k)^2}{|c_{k-1}|^2}
\end{bmatrix}
\]

\[
\bar{D}_k^l = \begin{bmatrix}
R_F^l(1), R_F^l(2), \cdots, R_F^l(k)
\end{bmatrix}^T \quad \text{and} \quad \bar{P}_k^l = \begin{bmatrix}
p_{k,1}^l, p_{k,2}^l, \cdots, p_{k,k}^l
\end{bmatrix}^T
\]

where $T$ denotes transpose. From Eq. (2.24), it is straightforward to verify that when the fading rate is extremely slow, i.e., when $R_F^l(k) \approx 1$ for the range of values used for $k$, then the matrix $\bar{R}_k^l$ is singular, as the equations in the system of Eq. (2.23) become identical. In such a case, the maximum likelihood receiver structure is considerably simplified, if we choose the simplest solution of setting all prediction coefficients $p_{k,k}$ equal to 1. This subclass of receivers has been shown to be optimal for noncoherent sequence estimation in the AWGN channel [63, 64]. However, the interesting implications of using this simplified structure for receivers operating in fading channels, are further investigated in Chapter 4.

In Appendix A.2 it is shown that maximization of the pdf in Eq. (2.22) is equivalent to maximizing the following function

\[
2 \sum_{k=1}^{Z} \sum_{m=1}^{k} \sum_{l=1}^{\Lambda} \sum_{i=1}^{2} \text{Re} \left[ y_k^l (y_{k-m})^* \right] \text{Re} \left( c_k c_{k-m}^* \right) + \text{Im} \left[ y_k^l (y_{k-m})^* \right] \text{Im} \left( c_k c_{k-m}^* \right) \left[ \sum_{k=0}^{Z-k} \frac{p_{k+j,i}^l p_{k+j,m+j}^l}{(\sigma^2_{k+j})^2} \right]
\]

\[
+ \sum_{k=1}^{Z} \sum_{l=1}^{\Lambda} \left[ \sum_{m=0}^{Z-k} \frac{(p_{k+m,m})^2}{(\sigma^2_{k+m})^2} \right]
\]

\[
+ \sum_{l=1}^{Z} \sum_{k=1}^{f_{l,l}} \left[ \sum_{m=0}^{Z-k} \frac{p_{k+m,m}^l (p_{k+m,m})^2}{(\sigma^2_{k+m})^2} \sum_{j=0}^{k+m} p_{k+m,j}^l \right].
\]

(2.25)

From Eq. (2.25) it is clear that the basic hardware structure of the diversity receiver derived is the same as that in [63]. However, the difference lies in the decoding algorithm, and the way
samples from all $\Lambda$ diversity branches are combined to yield the overall metric. In the next two paragraphs, we discuss the physical meaning and implications stemming from each term in Eq. (2.25), translating it to the structure of the optimal sequence estimator.

The first term in the equation is that corresponding to the *multiple differential detectors*, the second to the *envelope detector* and the third to the *coherent detector*. As introduced in Chapter 1 (see also [63]), by the term "*multiple differential detectors*" we refer to differential detectors which decode the receiver signal over a multi-symbol interval. Such a detector employing a delay element of $mT$ seconds will provide $\text{Re} \left[ y_k^l \left( y_{k-m}^l \right)^* \right]$ and $\text{Im} \left[ y_k^l \left( y_{k-m}^l \right)^* \right]$ for the calculation of the first term of Eq. (2.25). Because of the summation $\sum_{m=1}^{k}$ which appears in this first term, it is clear that a bank of $k$ distinct multiple differential detectors are necessary, each of them employing a progressively increasing by $T$ time-delay element. The structure of such a detector bank is illustrated in Fig. 2.7, where for ease of notation we have defined $d_m^l(k) = y_k^l \left( y_{k-m}^l \right)^*$. The term "envelope detector" refers to a subsystem providing the instantaneous signal envelope $|y_k|$, which is then squared, to be used as $|y_k|^2$ in the metric expression of Eq. (2.25).

Finally, the term "*coherent detector*" refers to the estimation of $\eta$, the modulator initial phase shift$^5$. From the Eq. (2.25), it is evident that $\eta$ is required only when the received signal includes a direct component, i.e., when we have Rician fading. If no direct component is present, i.e., when we have Rayleigh fading, then $\sum_{j=1}^{J} f_j = 0$ and the third term disappears from Eq. (2.25). This also makes sense intuitively, as we do not expect the modulator initial phase to affect the decoding when the phase of the received signal is uniformly distributed in $[0, 2\pi)$, as is the case in Rayleigh fading. It is also important to point out that extensive BER performance evaluation via computer simulation for the Rician channel demonstrated quite small performance degradation, if the term including $\eta$ was omitted from the function to be

$^5$It should be noted that this "coherent detector" refers only to the estimation of the initial carrier phase $\eta$ and not the random FM which is introduced by fading. The rapid phase changes caused by the fading interference will be compensated for by the receiver which is derived in this section.
Figure 2.7: Block diagram of the bank of multiple differential detectors; PD: Phase Detector.

maximized. This seems to indicate that this term is of relatively less importance as compared with the other two terms, and especially with respect to the multiple differential detector term. This qualitative statement is also quantitatively expressed in the results presented in Section 2.3, where the sensitivity of the proposed receiver performance to inaccurate estimates of \( \overline{fT_1} \), \( R_F^i(\tau) \) and \( B_F T \) is shown to be very small. Furthermore, BER computer evaluation results for a Rician fading channel with \( K \) taking values between 5 and 10 dB have also indicated that the performance improvement gained when having a LOS signal is orders of magnitude greater than any degradation introduced by omitting the coherent detection term involving \( \eta \). In any case, when it is necessary, the estimation of \( \eta \) need only be performed once at the beginning of the receiver operation. We conclude this interpretive discussion of the receiver derived, by providing the complete block diagram illustrating the structure of the optimal diversity receiver in Fig. 2.8.

For an equal combining system, the following conditions apply: \( \overline{fT_1} = \overline{fT_2} = \overline{fT}, R_F^i = \overline{fT_1} = \overline{fT_2} = \overline{fT} \).
\[ R_F^2 = R_F \text{ and } (\sigma^2_G)^2 = (\sigma^2_G)^2 \quad \forall 1 \leq l_1, l_2 \leq \Lambda. \] In this case, \( p_{k,m}^1 = p_{k,m}^2 = p_{k,m} \) and \( (\sigma_k^1)^2 = (\sigma_k^2)^2 = \sigma_k^2 \quad \forall 1 \leq l_1, l_2 \leq \Lambda. \) For such a system the function to maximize simplifies to

\[
\begin{align*}
2 \sum_{k=1}^{\Lambda} & \sum_{m=1}^{\Lambda} \frac{1}{|c_k|^2 |c_{k-m}|^2} \sum_{j=0}^{\Lambda} \frac{p_{k+j,j} p_{k+j,m+j}}{(\sigma_{k+j})^2} \left[ \sum_{l=1}^{\Lambda} \text{Re} \left[ y_k^l (y_{k-m}^l)^* \right] \text{Re} \left( c_k c_{k-m}^* \right) \right] + \\
& + \sum_{k=1}^{\Lambda} \frac{1}{|c_k|^2} \sum_{m=0}^{\Lambda} \frac{(p_{k+m,m})^2 (\sigma_{k+m})^2}{(\sigma_k^2)^2} \left[ \sum_{l=1}^{\Lambda} |y_k^l|^2 \right] \\
& + 2 \sum_{k=1}^{Z} \frac{Z-k}{|c_k|^2} \sum_{m=0}^{\Lambda} \frac{p_{k+m,m}}{(\sigma_{k+m})^2} \sum_{j=0}^{k+m} p_{k+m,j} \sum_{l=1}^{\Lambda} \text{Re} \left( y_k^l c_k^* e^{-jn} \right). \end{align*}
\] (2.26)
2.2.2 Suboptimal Algorithms

According to Eq. (2.26), the receiver requires knowledge of the prediction coefficients with orders 1 to \( Z \). It also requires the use of \( Z - 1 \) order\(^6\) MDD. Even for relatively short sequences, as for example commonly encountered block sizes of \( Z = 256 \) or 512, the receiver would require impractically high levels of computational and implementation complexity. To resolve this problem, the following truncation approach is adopted. The number of differential detectors \( z \) used by the receiver, which for the optimal receiver is equal to \( Z - 1 \), is reduced to a value much smaller than \( Z - 1 \) (\( z \ll Z - 1 \)). By truncating the sums in Eq. (2.26) to the maximum prediction order \( z \), we end up with the following metric expression to be maximized

\[
\Theta_{y_1', y_2', \ldots, y_A}; \bar{c}(A), \eta = 2 \sum_{k=1}^{Z} \sum_{m=1}^{k} \frac{1}{|c_k|^2 |c_{k-m}|^2} \left\{ \Re \left( c_k c_{k-m}^* \right) \left[ \sum_{l=1}^{\Lambda} \Re \left[ d_{m}^l (k) \right] B_{z,m}^l \left[ \bar{c} (A) \right] \right] + \right. \\
\left. \Im \left( c_k c_{k-m}^* \right) \left[ \sum_{l=1}^{\Lambda} \Im \left[ d_{m}^l (k) \right] B_{z,m}^l \left[ \bar{c} (A) \right] \right] \right\} \\
+ \sum_{k=1}^{Z} \frac{1}{|c_k|^2} \sum_{l=1}^{\Lambda} \left| y_{k}^l \right|^2 \Gamma_{z,k}^l \left[ \bar{c} (A) \right] \\
+ \sum_{k=1}^{Z} \frac{1}{|c_k|^2} \sum_{l=1}^{\Lambda} \Re \left( y_{k}^l c_{k}^* e^{-jn} \right) \Delta_{z,k}^l \left[ \bar{c} (A) \right] \right] \right) (2.27)
\]

where

\[
d_{m}^l (k) = y_{k}^l \left( y_{k-m}^l \right)^*, \quad B_{z,m}^l \left[ \bar{c} (A) \right] = \sum_{j=0}^{z-m} \frac{p_{z,j}^l p_{z,m+j}^l}{\sigma_z^2}, \\
\Gamma_{z,k}^l = \frac{1}{\sigma_z^2} \sum_{j=0}^{q} \left( p_{z,j}^l \right)^2, \quad \Delta_{z,k}^l = \frac{2}{\sigma_z^2} \sum_{i=0}^{q} \sum_{j=0}^{z} p_{z,i}^l p_{z,j}^l \quad (2.28)
\]

with \( q = z \) for \( m \leq Z - z \) and \( q = Z - k \) for \( m > Z - z \). Using Eqs. (2.25) and (2.28), for PSK signals the metric expression to be maximized is simplified as

\[
\Psi_{y_1', y_2', \ldots, y_A}; \bar{c}(A), \eta = \sum_{k=1}^{Z} \sum_{l=1}^{\Lambda} \left( y_{k}^l c_{k}^* e^{-jn} \right) \Delta_{z,k}^l \left[ \bar{c} (A) \right]
\]

\(^6\)By \( Z - 1 \) order MDD we imply a MDD hardware structure with \( Z - 1 \) differential detectors, employing elements of time delay up to \( ZT \) seconds.
Optimal Sequence Estimation for Fast Flat Fading with Diversity

\[ +2 \sum_{k=1}^{Z} \sum_{m=1}^{Z} \sum_{i=1}^{\Lambda} \left\{ \begin{array}{c} \text{Re} \left[ d_m^i (k) \right] \text{Re} \left( c_k c_{k-m}^* \right) + \\ \text{Im} \left[ d_m^i (k) \right] \text{Im} \left( c_k c_{k-m}^* \right) \end{array} \right\} B_{z,k}^{i} \left[ C \left( \overline{A} \right) \right] \] (2.29)

Finally, assuming that the fading as well as the AWGN processes corrupting each one of the \( \Lambda \) diversity channels have identical statistics, the above equation is further simplified to

\[ \psi_{y^1, y^2, \ldots, y^n, C(\overline{A})} = \sum_{k=1}^{Z} \Delta_{z,k} \left[ C \left( \overline{A} \right) \right] \sum_{i=1}^{\Lambda} \text{Re} \left( y_k^i c_k^* e^{-2\eta} \right) + 
+ 2 \sum_{k=1}^{Z} \sum_{m=1}^{Z} B_{z,k} \left[ C \left( \overline{A} \right) \right] \left[ \sum_{i=1}^{\Lambda} \text{Re} \left[ d_m^i (k) \right] \text{Re} \left( c_k c_{k-m}^* \right) + \\
\sum_{i=1}^{\Lambda} \text{Im} \left[ d_m^i (k) \right] \text{Im} \left( c_k c_{k-m}^* \right) \right] \] (2.30)

where

\[ B_{z,k} \left[ C \left( \overline{A} \right) \right] = \sum_{j=0}^{Z-m} \frac{p_{z,j} p_{z,m+j}}{\sigma_z^2}, \]
\[ \Gamma_{z,k} \left[ C \left( \overline{A} \right) \right] = \frac{1}{\sigma_z^2} \sum_{j=0}^{Z} (p_{z,j})^2, \] (2.31)
\[ \Delta_{z,k} \left[ C \left( \overline{A} \right) \right] = \frac{2}{\sigma_z} \sum_{i=0}^{Z} p_{z,i} \sum_{j=0}^{Z} p_{z,j} \]

are now independent of the diversity channel \( l \).

In the next section we present BER evaluation results obtained via Monte-Carlo simulation, for \( \pi/4 \)-shift DQPSK, 8-ary DPSK (8-DPSK) and \( \pi/4 \)-shift 8-DQAM modulation formats, in Rayleigh and Rician frequency non-selective fading conditions, employing the proposed receivers with various degrees of diversity.
2.3 BER PERFORMANCE EVALUATION OF THE SUBOPTIMAL ALGORITHMS

The proposed diversity receivers employing suboptimal algorithms were evaluated by means of computer simulation. As previously discussed, the fading interference assumed follows the land-mobile correlated, fast Rayleigh and Rician channel model, with $B_f T = 0.125$. The BER results reported indicate average number of bits in error over long signal sequences and were obtained via computer simulation employing Monte-Carlo error counting techniques. The number of error bits counted for all simulation runs was much greater than 100, for all cases above the error rate level of $5 \times 10^{-4}$. For levels between $10^{-4}$ and $5 \times 10^{-4}$ where the number of errors were allowed to drop to around 50, the greater uncertainty introduced was resolved by averaging results from more than one statistically independent simulation runs for each point. All signals in the computer simulation were represented by their baseband equivalents [14].

The filtering process was also carried out in baseband [14], and more specifically in the discrete frequency domain, by employing the fast Fourier Transform (FFT) transform [94]. The SNR indicated on all figures in the following sections is equal to $E_s/N_0$, where $E_s$ is the signal energy in each symbol transmitted, and $N_0$ the AWGN power spectral density. This is because Nyquist filtering is employed. In later chapters where Butterworth filtering is used, this does not hold; in such case the SNR at the receiver is lower than the channel $E_s/N_0$. For more details on the digital simulation environment the reader is referred to [62].

In investigating the BER performance of the receivers employing the suboptimal algorithms derived in the previous section, we distinguish two cases: a) that where perfect knowledge of the statistical and deterministic channel parameters is available to the receiver, and b) when the receiver knowledge of those same parameters is inaccurate.
2.3.1 Perfect Knowledge of Channel Parameters

![Graph showing BER performance evaluation](image)

Figure 2.9: BER performance evaluation of raised-cosine filtered ($\alpha = 0.35$) rate 2/3, 8-state $\pi/4$-shift 8-DQAM in a Rayleigh land-mobile fading environment with $B_F T = 0.125$.

The BER performance of an 8-state trellis-coded $\pi/4$-shift 8-DQAM scheme is depicted in Fig. 2.9. The code used for the convolutional encoder is the best 8-state code found in [25], having parity check coefficients $h^2 = 04_8$, $h^1 = 02_8$, $h^0 = 11_8$ (see also Fig. 2.4b). Employing a second diversity path ($\Lambda = 2$) results in a dramatic increase in gain - approximately 22 dB - at a BER level of $10^{-3}$. The little over 3 dB of gain at the same BER level obtained by adding an extra diversity path, does not present a substantial performance improvement, given the implementation complexity of introducing a third receiver stage and associated hardware. Error floors for both the $\Lambda = 2$ and $\Lambda = 3$ cases are expected to exist below the $10^{-4}$ BER level. However, statistically significant results below $10^{-4}$ could not be obtained due to computer simulation limitations.
Figure 2.10: BER performance comparison of raised-cosine filtered ($\alpha = 0.35$) rate 1/2, 4-state $\pi$/4-shift DQPSK in a Rayleigh land-mobile fading environment with $B_p T = 0.125$ using the improved diversity receiver structure derived, to the performance of a 1-symbol differential detector (DD) and Viterbi decoder.

The performance of the proposed MDD scheme is compared against the Viterbi decoding algorithm in Figs. 2.10 and 2.11. The Viterbi decoder processes the output from a conventional 1-symbol differential detector and employs equal combining of the $\Lambda$ Euclidean distances in the metric calculation, i.e., the metric used during decoding is

$$\sum_{k=1}^{Z} \sum_{l=1}^{\Lambda} \left| d_l^i(k) - c_{k} c_{k-1}^* \right|^2$$

or equivalently, the incremental metric used by the Viterbi decoder is

$$\sum_{l=1}^{\Lambda} \left| d_l^i(k) - c_{k} c_{k-1}^* \right|^2.$$  \hfill (2.33)

In Fig. 2.10 the modulation scheme used is a 4-state trellis-coded $\pi$/4-shift DQPSK while
Figure 2.11: BER performance comparison of raised-cosine filtered ($\alpha = 0.35$) rate 2/3, 8-state 8-DPSK in a Rayleigh land-mobile fading environment with $B_F T = 0.125$ using the improved diversity receiver structure derived, to the performance of a 1-symbol differential detector (DD) and Viterbi decoder.

Fig. 2.11 illustrates the results for an 8-state 8-DPSK. The 4-state code used for the $\pi/4$-shift DQPSK scheme is the best code found in [25], having parity check coefficients $k^1 = 2$, $k^0 = 5$ (see also Fig. 2.4a). The 8-state code used for the 8-DPSK is the same as the one for the $\pi/4$-shift 8-DQAM (see Fig. 2.4c). The only thing that changes is the signal assignment, which is provided in Fig. 2.4, of Section 2.1. Both the diversity algorithms proposed and the Viterbi decoder, use the same trellis codes, with the same number of states and length of path memory. It is evident by comparing the two sets of results that the differential detector coupled with the Viterbi decoder performs reasonably well when the number of signals in the constellation is small, or otherwise when the distances between points in the constellation are substantial. As shown in Fig. 2.11, this is not the case for schemes of higher spectral
efficiency with "crowded" signal spaces. The proposed diversity scheme for trellis coded 8-DPSK outperforms the Euclidean-metric based Viterbi decoding by dramatically reducing the error floor. Using a prediction order of \( z = 2 \) the irreducible error rate is below \( 10^{-4} \), indicating improvement of more than two orders of magnitude. For the same 8-DPSK system, by introducing a block interleaver before the differential encoder at the transmitter, and a de-interleaver after the differential detector at the receiver as described in [70], we verified the optimal interleaving span and depth experimentally determined in [70] for land mobile fading. In cases where \( B_fT \leq 0.03 \), we observed that use of this block interleaver indeed has a beneficial effect on the BER performance, reducing the number of bits in error at least by a factor of 2. However, for the fast fading conditions considered in Fig. 2.11, use of interleaving did not yield any BER improvement for the conventional differential detector and Viterbi decoder, both for the cases of no diversity, and for diversity \( \Lambda = 2 \).

Fig. 2.12 demonstrates the effect of correlation between the faded signals received on two antennas, i.e., diversity \( \Lambda = 2 \) is assumed. The modulation format tested is the 4-state trellis-coded \( \pi/4 \)-shift DQPSK. The simulation results indicate that even severe correlation \( (\rho_{12} \approx 0.8) \) does not seriously affect the gain obtained by using diversity. At the \( 10^{-3} \) BER level, the loss is approximately 3.7 dB as compared to the completely uncorrelated case, the performance still around 5.5 dB better than the no-diversity case. A moderate correlation coefficient of \( \rho_{12} \approx 0.5 \) will produce a loss of no more than 1.6 dB at BER = \( 10^{-3} \). The \( \pi/4 \)-shift DQPSK scheme was also tested in the Rician fading channel environment. BER results for two different values of the \( K \)-factor, 5 dB and 10 dB, are illustrated in Fig. 2.13. As \( K \) is increased, aside from the expected improvement in BER performance, it is interesting to note that the degradation observed for a given fading correlation remains essentially constant; the \( \rho_{12} = 0.49 \) case, for both values of the \( K \)-factor shown (0 and 5 dB), is approximately 3 dB worse with respect to \( \rho_{12} = 0 \), at BER = \( 10^{-3} \).
Figure 2.12: Effects of correlation between signals received on two diversity paths on a raised-cosine filtered ($\alpha = 0.35$) rate 1/2, 4-state $\pi/4$-shift DQPSK scheme, in a Rayleigh land-mobile fading environment with $B_F T = 0.125$, using the optimal diversity receiver with prediction order $z = 2$.

2.3.2 Imperfect Knowledge of Channel Parameters

In Section 2.2, the derivation of the optimal sequence estimator assumed knowledge of the channel parameters characterizing the fading distortion. Such are the type of fading environment reflected in the choice of fading filter $H_f^1(f)$, the $B_F T$ product value, and the Rician fading $K$-factor. Using $\pi/4$-shift DQPSK and $\Lambda = 1$, we investigated via computer simulation, the sensitivity of the proposed detection algorithms, to imperfect knowledge of all the aforementioned channel parameters.

The receiver sensitivity was found to be very small with respect to the fading model employed. For example, for receivers with $z = 2$, $\Lambda = 1$ and at BER=$10^{-2}$ observed at SNR
Figure 2.13: Effects of fading correlation and direct/diffused power ratio $K$ on a raised-cosine filtered ($\alpha = 0.35$) rate 1/2, 4-state $\pi/4$-shift DQPSK scheme, in a Rician land-mobile fading environment with $B_F T = 0.125$, using the optimal diversity receiver with prediction order $z = 2$ and two ($\Lambda = 2$) diversity paths.

$\simeq 14.5$ dB, employing a rectangular fading filter for the channel, instead of the land-mobile characteristic assumed by the receiver, resulted in a performance of $1.5 \times 10^{-2}$ (degradation equivalent to less than 0.7 dB). Using a Gaussian fading filter for the channel, the obtained BER was approximately $2 \times 10^{-2}$ (degradation equivalent to less than 2 dB). The difference in degradation is explained by the fact that the impulse response of the Gaussian filter is significantly different from that of the land-mobile filter. On the other hand, the impulse responses of the rectangular and land-mobile filters are more similar. Nevertheless, in both cases the degradation is insignificant as compared to the overall gains obtained by the proposed receivers. Similarly, the performance was found to be very insensitive to variations of the
channel \( B_F T \) product, with the receiver assuming a fixed value. Again we assumed the land-mobile fading filter, at \( \text{SNR} \approx 14.5 \text{ dB}, \; z = 2, \; \Lambda = 1 \). In Table 2.1 the “IDEAL RX” is designed for the actual \( B_F T \) present in the channel, whereas “FIXED RX” is designed for \( B_F T = 0.125 \). By observing the simulation results summarized in the table, it is clear that even extreme variations of \( \pm 50\% \) on the \( B_F T \) product value used by the receiver result in relatively insignificant performance degradation, equivalent to not more than 2 dB of power loss.

Table 2.1: Performance degradation due to inaccurate estimate of channel \( B_F T \) at the receiver.

<table>
<thead>
<tr>
<th>( B_F T )</th>
<th>BER &quot;FIXED RX&quot; ( (B_F T = 0.125) )</th>
<th>BER &quot;IDEAL RX&quot; ( (B_F T) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.125</td>
<td>1.07E-2</td>
<td>1.07E-2</td>
</tr>
<tr>
<td>0.0625 (-50%)</td>
<td>1.564E-2</td>
<td>1.020E-2</td>
</tr>
<tr>
<td>0.100 (-20%)</td>
<td>1.271E-2</td>
<td>1.136E-2</td>
</tr>
<tr>
<td>0.150 (+20%)</td>
<td>1.326E-2</td>
<td>1.283E-2</td>
</tr>
<tr>
<td>0.1875 (+50%)</td>
<td>2.322E-2</td>
<td>1.680E-2</td>
</tr>
</tbody>
</table>

The performance of the proposed receivers was also tested against the third fading parameter of interest, namely the value of the \( K \)-factor, as it is reflected in the value of \( f_{\bar{T}}^{\bar{T}} \). For the purpose of this test only one channel was employed so \( f_{\bar{T}}^{\bar{T}} = \bar{T} \). A value of 0 indicates Rayleigh fading while anything above that implies a direct signal path and consequently a Rician fading channel. Two series of tests were performed in order to assess the impact of an inaccurate estimation of \( f_{\bar{T}}^{\bar{T}} \) in the receiver. In both cases, the fading channel \( B_F T \) product was assumed equal to 0.125, \( z = 2, \; \Lambda = 1 \). The results of the first series of tests are shown in Table 2.2, where the BER performance is given for three values of the channel SNR, around a BER \(^{7}\) It is evident from Eq. (2.14) that the \( K \)-factor is linearly related to the value of \( (f_{\bar{T}}^{\bar{T}})^2 \).
level of $10^{-3}$ (7.5, 10 and 12.5 dB) and various degrees of error in the estimation of $\overline{fI}$. The BER performance with accurate channel information is given in the column under "IDEAL RX". The channel is assumed to be Rician having a $K = 5$ dB. The receiver, on the other hand, uses values of $\overline{fI}$ which deviate from the correct value by some percentage. An extreme case occurs when the estimate of $\overline{fI}$ is equal to 0 (i.e., -100% error or assumed $K \rightarrow -\infty$ dB), in other words, the receiver assumes that the channel is Rayleigh while in fact it is Rician with $K = 5$ dB. The table summarizes the results for other percentage errors and gives performance degradation in dB for the three SNR values. The error percentages have also been translated in terms of $K$ and are given in dB. For example, a -90% error in $\overline{fI}$ (i.e., the receiver estimate is equal to 0.1 of the actual value) corresponds to -20 dB of error in $K$, which in this case means that the receiver has assumed $K = -15$ dB while the channel presents $K = 5$ dB. From the results summarized in Table 2.2 it is clear that a maximum degradation of about 1 dB will occur, around the $10^{-3}$ BER level, if the receiver estimate of the $K$-factor for the Rician fading channel is within ±15 dB off the actual value. It should also be noted that for errors in $\overline{fI}$ above -70% (error in $K$ above -10.5 dB) and below +180% (error in $K$ below 9 dB) the performance degradation is negligible.

The results from the second series of tests regarding the receiver performance sensitivity with respect to $\overline{fI}$ are shown in Table 2.3. In this case the receiver has assumed a Rician fading characteristic for the channel while in fact the channel is Rayleigh. BER performance and corresponding degradation in dB is given for three values of $K$ (-5, 0 and 5 dB) and five values of the channel SNR (10, 12.5, 15, 17.5 and 20 dB). The BER performance of the ideal receiver, i.e., one which has knowledge of the fading in the channel being Rayleigh, is also given in the first row and is marked as "Ideal". As expected, the performance degrades rapidly as the assumed $K$-factor is increased, since the channel in this case exhibits a $K \rightarrow -\infty$ dB.

---

8 By degradation of x dB here, we imply that the IDEAL RX will require x dB less signal power in order to provide the same BER performance.
Optimal Sequence Estimation for Fast Flat Fading with Diversity

Table 2.2: Performance degradation due to inaccurate estimate of channel \( \tilde{f}^T \) at the receiver. For the results summarized in this table, the Rician channel has been assumed to have a \( K = 5 \) dB and \( B_f T = 0.125 \). The estimate error for \( \tilde{f}^T \) has also been translated to an error in the value of \( K \) which is given in dB. The extreme case is when the receiver assumes the channel is Rayleigh (-100% error in \( \tilde{f}^T \) or \( K \to -\infty \) dB) while the channel has been assumed Rician with \( K = 5 \) dB. SNR degradation is given in parenthesis under each BER performance value.

<table>
<thead>
<tr>
<th>SNR [dB]</th>
<th>IDEAL RX</th>
<th>-100% error (Rayleigh)</th>
<th>-90% error (-20 dB error)</th>
<th>+200% error (+9.5 dB error)</th>
<th>+400% error (+14 dB error)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.5</td>
<td>7.00E-3</td>
<td>1.84E-2 (1.7 dB)</td>
<td>1.16E-2 (0.7 dB)</td>
<td>7.76E-3 (0.2 dB)</td>
<td>1.03E-2 (0.7 dB)</td>
</tr>
<tr>
<td>10</td>
<td>1.56E-3</td>
<td>7.09E-3 (2.5 dB)</td>
<td>3.06E-3 (1.0 dB)</td>
<td>1.58E-3 (0.2 dB)</td>
<td>2.57E-3 (0.8 dB)</td>
</tr>
<tr>
<td>12.5</td>
<td>3.30E-4</td>
<td>2.69E-3 (3.3 dB)</td>
<td>8.00E-4 (1.3 dB)</td>
<td>4.00E-4 (0.3 dB)</td>
<td>6.00E-4 (0.9 dB)</td>
</tr>
</tbody>
</table>

Note, however, that the performance degradation would be quite small (approximately 1 dB) at a BER level around \( 10^{-3} \), if the receiver had assumed a \( K \)-factor of \(-10 \) dB. The test results summarized in Tables 2.2 and 2.3 suggest that a practical approach towards system design would be: first to establish a most probable 30 dB range for the \( K \)-factor, i.e., within \( \pm 15 \) dB of a mean value and, second, to use this mean value to calculate \( \tilde{f}^T \) in the receiver. This guarantees that as long as the \( K \)-factor lies within the 30 dB range selected, the receiver SNR performance will be at most 1 dB worse as compared to the theoretically attainable. Degradation results presented in Table 2.3 suggest that the term of the metric to be maximized involving \( \tilde{f}^T \) (or in more general terms \( \tilde{f}^{T,i} \)) should not be dropped (effectively leading to a Rayleigh fading receiver design), unless the \( K \)-factor is guaranteed to be below \(-10 \) dB most of the time.
Table 2.3: Performance degradation due to inaccurate estimate of channel $\tilde{f}$ at the receiver. In this case, the receiver assumes that the channel is Rician whereas in fact it has a Rayleigh characteristic with $B_fT = 0.125$. The value of $K$ given is that assumed by the receiver. SNR degradation is given in parenthesis under each BER performance value.

<table>
<thead>
<tr>
<th>$K$ [dB]</th>
<th>10</th>
<th>12.5</th>
<th>15</th>
<th>17.5</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>6.26E-2</td>
<td>2.57E-2</td>
<td>9.23E-3</td>
<td>3.67E-3</td>
<td>1.77E-3</td>
</tr>
<tr>
<td>(Ideal Receiver)</td>
<td>(Ideal Receiver)</td>
<td>(Ideal Receiver)</td>
<td>(Ideal Receiver)</td>
<td>(Ideal Receiver)</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>7.28E-2</td>
<td>3.21E-2</td>
<td>1.30E-2</td>
<td>5.23E-3</td>
<td>3.12E-2</td>
</tr>
<tr>
<td>(0.5 dB)</td>
<td>(0.5 dB)</td>
<td>(0.7 dB)</td>
<td>(1.1 dB)</td>
<td>(1.8 dB)</td>
<td>(1.8 dB)</td>
</tr>
<tr>
<td>5</td>
<td>7.99E-2</td>
<td>3.92E-2</td>
<td>1.74E-2</td>
<td>8.86E-3</td>
<td>6.23E-3</td>
</tr>
<tr>
<td>(1.1 dB)</td>
<td>(1.1 dB)</td>
<td>(1.4 dB)</td>
<td>(2.2 dB)</td>
<td>(3.7 dB)</td>
<td>(3.7 dB)</td>
</tr>
<tr>
<td>9</td>
<td>9.97E-2</td>
<td>5.11E-2</td>
<td>2.78E-2</td>
<td>1.61E-2</td>
<td>1.22E-2</td>
</tr>
<tr>
<td>(1.8 dB)</td>
<td>(1.8 dB)</td>
<td>(2.5 dB)</td>
<td>(3.4 dB)</td>
<td>(5.4 dB)</td>
<td>(5.4 dB)</td>
</tr>
</tbody>
</table>

2.4 CONCLUSION

We have derived the optimal sequence estimator using diversity reception for digital signals transmitted over a frequency flat, correlated fast Rician fading channels. The analysis presented has considered the more general case where both phase and amplitude distortion due to fading vary significantly over the symbol duration. The decoding algorithm does not require the use of interleaving for removing the correlation between adjacent signal samples, a quality which greatly simplifies receiver design. Various reduced complexity versions of the optimal decoder structure, which consists of a combination of envelope, multiple differential and coherent detectors, have been presented for differentially encoded PSK and QAM signal constellations. Performance evaluation results obtained by means of computer simulation, have shown them to outperform conventional Viterbi decoding using differential detection, especially for more bandwidth efficient modulation schemes, i.e., more dense signal constellations. For example,
for a trellis coded 8-DPSK scheme using two diversity channels ($\Lambda = 2$) and two differential detectors in a fast Rayleigh fading channel ($B_R T = 0.125$) there was no error floor observed at $BER = 10^{-4}$. Under the same channel conditions, the conventional Viterbi decoder using a 1-symbol differential detector and two diversity channels, exhibits an irreducible error rate of about $4 \times 10^{-2}$. BER performance tests also demonstrated that the proposed sequence estimation receivers are quite insensitive to significantly varying channel conditions including the fading model (e.g., fading autocorrelation function, $B_R T$ and the $K$-factor), thus making them particularly well suited for practical system implementation.
CHAPTER 3

OPTIMAL SEQUENCE ESTIMATION FOR FAST CORRELATED FREQUENCY SELECTIVE RICIAN FADING CHANNELS

In this chapter, following our previous work on frequency flat (i.e., frequency non-selective) fading channels [63], we derive the optimal, in the maximum likelihood detection sense, receiver for coded digital signals transmitted over channels corrupted by correlated fast Rician frequency selective fading and AWGN.

The organization of the chapter is as follows. Section 3.1 describes the channel model assumed while Section 3.3 presents the derivation of the optimal receiver for the frequency selective fading channel. The performance evaluation results obtained by means of computer simulation for \(\pi/4\)-shift DQPSK signals are presented in Section 3.4, while in Section 3.5 we offer some concluding remarks.

3.1 FREQUENCY SELECTIVE FADING CHANNEL MODEL

For the frequency selective fading channel model we assume the existence of two independent propagation paths, the “direct” and the “reflected”, both corrupted by frequency non-selective

\[\text{The research reported in this chapter has been presented in part at the 1993 IEEE Pacific Rim Conference, Victoria, Canada [97], and is to be published as a paper in the IEEE Transactions on Vehicular Technology.}\]
Rician fading. As previously mentioned, this two-ray model has been recommended by the Telecommunications Industry Association (TIA) Standards Committee to evaluate the tolerance of delay spread in the new North-American digital cellular system [21]. The delay by which the reflected signal is assumed to arrive at the receiver, normalized to the symbol duration $T$, is denoted by $\hat{\tau}$. Upon arrival, it is superimposed on the direct signal. The multiplicative fading processes on the direct path (denoted as path 0) and the reflected path (denoted as path 1), $f^0(t)$ and $f^1(t)$ respectively, are modeled as a complex summation of two independent Gaussian noise processes. In other words, $f^0(t) = f^{0,0}(t) + j f^{0,Q}(t)$ and $f^1(t) = f^{1,1}(t) + j f^{1,Q}(t)$. Considering the same transmitter as that of Section 2.1 (see Eq. (2.8)), and that the faded signal is corrupted by AWGN $n(t)$, the signal arriving at the receiver can then be expressed as

$$r(t) = f^0(t)x(t) + f^1(t)x(t-\hat{\tau}T) + n(t). \quad (3.34)$$

Adopting the land-mobile fading model for both $f^0(t)$ and $f^1(t)$ fading processes, their autocorrelation function can be expressed identically to Eq. (2.13) as

$$R_F^m(\beta) = J_0(2\pi B_F \beta) \quad m \in \{0, 1\}. \quad (3.35)$$

Furthermore, and similarly to Eq. (2.12), the real and imaginary parts of both $f^m(t)$, $m \in \{0, 1\}$, have the same autocorrelation function, given by

$$R_F^m(\beta) = E \left\{ \left[ f^{1,m}(t) - \bar{f}^{1,m} \right] \left[ f^{1,m}(t-\beta) - \bar{f}^{1,m} \right] \right\} =$$

$$= E \left\{ \left[ f^{Q,m}(t) - \bar{f}^{Q,m} \right] \left[ f^{Q,m}(t-\beta) - \bar{f}^{Q,m} \right] \right\} =$$

$$= N_F^m \int_{-\infty}^{\infty} |H_F^m(f)|^2 e^{2\pi j \beta f} df. \quad (3.36)$$

In Eq. (3.36), $\bar{f}^{1,m} = E \left\{ f^{1,m}(t) \right\}$, $\bar{f}^{Q,m} = E \left\{ f^{Q,m}(t) \right\}$ is, without any loss of generality, assumed to be equal to zero, $m \in \{0, 1\}$ and $N_F^m$ is the power spectral density of the white Gaussian noise processes $n^m(t) = n^{1,m}(t) + j n^{Q,m}(t)$ generating the fading processes $f^m(t) = f^{1,m}(t) + j f^{Q,m}(t)$. As $n^{1,m}(t)$ and $n^{Q,m}(t)$ are assumed independent, the cross-correlation
between $f^{I,m}(t)$ and $f^{Q,m}(t)$ is zero. Considering the same fading filters and power spectral density for both paths, we have $H_P^m(f) = H_P(f)$, and $N_P^m = N_P$ for $m \in \{0, 1\}$. A block diagram of the low-pass equivalent of the channel model is depicted in Fig. 3.1. Similar to Eq. (2.14), the $K$-factor for the $m$-th path, $K_m$, is given in dB as

$$K_m = 10 \log_{10} \left[ \frac{(f^{I,m})^2}{\sigma_{f^{I,m}}^2 + \sigma_{f^{Q,m}}^2} \right] [\text{dB}]$$  

(3.37)

where $\sigma_{f^{I,m}}^2$ and $\sigma_{f^{Q,m}}^2$ are the variances of $f^{I,m}(t)$ and $f^{Q,m}(t)$, respectively.
3.2 RECEIVER MODEL

For simplicity in the theoretical derivation of the optimal sequential decoder we assume that the impulse response \( h_T(t) \) of the premodulation shaping filter is very closely approximated by a Dirac \( \delta \)-function. In other words, the signal up-converted by the modulator very closely resembles a non-return-to-zero (NRZ) [14] pulse-sequence, i.e., we consider an unfiltered transmitted sequence. It should be pointed out that including the effects of signal filtering in the analysis presented in the next section is straightforward but mathematically very tedious task. Because of this, the filtering effects, i.e., assessing the impact of both small and large deviations on the pulse shape from that of an NRZ signal, were included only in the performance evaluation results obtained via computer simulation (see Section 3.4).

For mathematical convenience we assume that the receiver front-end down-converts the incoming signal by making use of a local oscillator in frequency lock with the signal carrier\(^2\). By using an integrate-and-dump predetection filter and sampling its output every \( t = kT \), the signal provided as input to the detector can be expressed as

\[
y(kT) = c_k f^A_0(kT) + c_{k-1} f^{B_1}(kT) + c_k f^{C_1}(kT) + n(kT)
\]

(3.38)

with

\[
\begin{align*}
f^{A_0}(t) &= \frac{1}{T} \int_{t-T}^{t} f^0(a) da \\
f^{B_1}(t) &= \frac{1}{T} \int_{t-T}^{t+T} f^1(a) da \\
f^{C_1}(t) &= \frac{1}{T} \int_{t-T+T}^{t+T} f^1(a) da.
\end{align*}
\]

(3.39)

Since \( f^0(t) \) and \( f^1(t) \) are complex Gaussian random processes, \( f^{A_0}(t), f^{B_1}(t) \) and \( f^{C_1}(t) \) are also complex Gaussian random processes. Using well known results for the statistics of linear

\(^2\)Similarly to Section 2.1, this complex demodulation only serves as a mathematical tool for downconverting the signals to the complex baseband domain. Rapid phase and amplitude fluctuations caused by fading appear at the output of the demodulator totally uncompensated.
system response to random process input, it is shown in Appendix B.1 that their autocorrelation
functions are

\[
R_A(kT) = \int_{-1}^{1} (1 - |a|) J_0 \left[ 2\pi B_F T(k - a) \right] da \\
R_B(kT) = \tilde{\tau} \int_{-\tilde{\tau}}^{\tilde{\tau}} \left( 1 - \frac{|a|}{\tilde{\tau}} \right) J_0 \left[ 2\pi B_F T(k - a) \right] da \\
R_C(kT) = (1 - \tilde{\tau}) \int_{-(1-\tilde{\tau})}^{(1-\tilde{\tau})} \left( 1 - \frac{|a|}{1 - \tilde{\tau}} \right) J_0 \left[ 2\pi B_F T(k - a) \right] da
\]

(3.40)

while the cross-correlations between \( f^{B,1}(t) \) and \( f^{C,1}(t) \) are given by

\[
R_{BC}(kT) = \tilde{\tau} \left\{ \int_{-1}^{-\tilde{\tau}} \left( \frac{a + 1}{\tilde{\tau}} \right) J_0 \left[ 2\pi B_F T(k - a) \right] da \\
+ \int_{-\tilde{\tau}}^{0} J_0 \left[ 2\pi B_F T(k - a) \right] da \\
+ \int_{0}^{\tilde{\tau}} \left( \frac{-a}{\tilde{\tau}} \right) J_0 \left[ 2\pi B_F T(k - a) \right] da \right\}
\]

\[
R_{CB}(kT) = -R_{BC}(kT)
\]

(3.41)

for \( 0 \leq \tilde{\tau} < 0.5 \), and

\[
R_{BC}(kT) = (1 - \tilde{\tau}) \left\{ \int_{-1}^{-\tilde{\tau}} \left( \frac{a + 1}{\tilde{\tau}} \right) J_0 \left[ 2\pi B_F T(k - a) \right] da \\
+ \int_{-\tilde{\tau}}^{0} J_0 \left[ 2\pi B_F T(k - a) \right] da \\
+ \int_{0}^{\tilde{\tau}} \left( \frac{-a}{\tilde{\tau}} \right) J_0 \left[ 2\pi B_F T(k - a) \right] da \right\}
\]

\[
R_{CB}(kT) = -R_{BC}(kT)
\]

(3.42)

for \( 0.5 \leq \tilde{\tau} < 1 \). Using the notation \( y_k = y(kT) \), \( f^{(A,B,C),m}(kT) = f_k^{(A,B,C),m} \) and \( n(kT) = n_k \),
it is convenient to define the following random variable (r.v.)

\[
e(k) = \frac{y_k c_k e^{-j\eta}}{|c_k|^2} = f_k^{A,0} + f_k^{C,1} + f_k^{B,1} \frac{c_k c_{k-1}}{|c_k|^2} + \frac{n_k c_k e^{-j\eta}}{|c_k|^2}.
\]

(3.43)
The real and imaginary parts of \( e(k) \), \( e'(k) \) and \( e^Q(k) \) respectively, are given by

\[
e'(k) = f_k^{I;A,0} + f_k^{I;C,1} + \frac{\text{Re}\{c_k^*c_{k-1}\}}{|c_k|^2} f_k^{I;B,1} + \frac{\text{Im}\{c_k^*c_{k-1}\}}{|c_k|^2} f_k^{Q;B,1} + n_e'(k)
\]

\[
e^Q(k) = f_k^{Q;A,0} + f_k^{Q;C,1} + \frac{\text{Im}\{c_k^*c_{k-1}\}}{|c_k|^2} f_k^{I;B,1} + \frac{\text{Re}\{c_k^*c_{k-1}\}}{|c_k|^2} f_k^{Q;B,1} + n_e^Q(k)
\]

(3.44)

with the noise terms \( n_e'(k) \) and \( n_e^Q(k) \) given similarly to Eq. (2.20) as

\[
n_e'(k) = \frac{n_k^I \text{Re}\{c_k^*e^{-jn}\} - n_k^Q \text{Im}\{c_k^*e^{-jn}\}}{|c_k|^2}
\]

\[
n_e^Q(k) = \frac{n_k^I \text{Im}\{c_k^*e^{-jn}\} + n_k^Q \text{Re}\{c_k^*e^{-jn}\}}{|c_k|^2}
\]

(3.45)

with \( n_k^I \) and \( n_k^Q \) denoting the real and imaginary parts of \( n(kT) \), respectively. From Eqs. (3.44) and (3.45) it is not difficult to see that both \( e'(k) \) and \( e^Q(k) \) are Gaussian r.v.'s. In Appendix B.2 their autocorrelation and cross-correlation functions are derived, and are given by the following functions

\[
R_{II}(k, l) = E\left[e'(k)e'(k-l)\right] = R_A(l) + R_C(l) + \frac{\text{Re}\{c_k^*c_{k-1}c_{k-l}c_{k-l-1}\}}{|c_k|^2|c_{k-l}|^2} R_B(l)
\]

\[
+ \frac{\text{Re}\{c_k^*c_{k-1}\}}{|c_k|^2} R_{CB}(l) + \frac{\text{Re}\{c_{k-1}^*c_{k-l-1}\}}{|c_{k-l}|^2} R_{BC}(l)
\]

\[
+ \frac{\sigma_g^2}{|c_k|^2} \delta_K(l)
\]

\[
R_{QQ}(k, l) = E\left[e^Q(k)e^Q(k-l)\right] = E\left[e'(k)e'(k-l)\right] = R_{II}(k, l)
\]

(3.46)

with \( \delta_K(l) \) denoting the Kronecker \( \delta \)-function, and

\[
R_{IQ}(k, l) = E\left[e'(k)e^Q(k-l)\right] = \frac{\text{Im}\{c_k^*c_{k-1}c_{k-l}c_{k-l-1}\}}{|c_k|^2|c_{k-l}|^2} R_B(l) + \frac{\text{Im}\{c_{k-1}^*c_{k-l-1}\}}{|c_{k-l}|^2} R_{BC}(l)
\]
\[ - \frac{\text{Im} \{ c_k^* c_{k-1} \}}{|c_k|^2} R_{CB}(l) \]
\[ R_{QL}(k, l) = E \left[ e^Q(k) e^I(k - l) \right] \]
\[ = - R_{IQ}(k, l). \]  

Note that since \( E \left[ e^I(k) e^Q(k) \right] = 0 \), and since \( e^I(k), e^Q(k) \) are Gaussian, they are also independent.

### 3.3 DERIVATION OF THE MAXIMUM LIKELIHOOD SEQUENTIAL DECODER

The maximum likelihood sequential detector must choose that sequence which maximizes the conditional pdf \( \zeta[y_1, y_2, \ldots, y_Z | \overline{C(A), \eta}] \). Since \( e(k) \) is a scaled version of \( y_k \), maximization can be carried out on the joint pdf resulting from the sequence of \( e(k) \)'s rather than from the received signal samples. It is mathematically convenient to further rearrange the \( Z \) complex valued \( e(k) \)'s into a vector of real numbers, associating the real parts with the even vector indexes and the imaginary with the odd, i.e.,

\[ \{ e_0, e_1, e_2, \ldots, e_{2Z-2}, e_{2Z-1} \} \]
\[ e_{2k} = e^I(k), \quad e_{2k+1} = e^Q(k) \quad \forall \quad 0 \leq k \leq Z - 1. \]  

From Eq. (3.44), the expected values for \( e_{2k} \) and \( e_{2k+1} \), denoted \( \overline{e_{2k}} \) and \( \overline{e_{2k+1}} \) respectively, are given by

\[ \overline{e_{2k}} = \overline{f^I} + (1 - \overline{\tau}) \overline{f^I} + \overline{\tau} \text{Re} \left\{ c_k^* c_{k-1} \right\} \overline{f^I} \]
\[ \overline{e_{2k+1}} = \overline{\tau} \text{Im} \left\{ c_k^* c_{k-1} \right\} \overline{f^I}. \]  

Using the expression for the conditional probability density function of a Gaussian r.v. given a sample vector of Gaussian r.v.'s, [98], and a mathematical approach equivalent to the one in
Chapter 2 (for details see Appendix A.1), it is obtained that the maximum likelihood detector must choose the sequence \( \overline{C}(A) \) which maximizes the following pdf

\[
\begin{align*}
\mathbf{f}[z | \overline{C}(A)] &= \prod_{k=0}^{Z-1} \frac{1}{\sqrt{2\pi \sigma_{E}^{2}[\overline{C}(A), k]}} \\
&\times \exp \left\{ -\frac{1}{2 \sigma_{E}^{2}[\overline{C}(A), k]} \left[ (\varepsilon_{2k} - \overline{\varepsilon}_{2k}) - \sum_{i=1}^{2k} p_{E,i} [\overline{C}(A), k] \cdot (\varepsilon_{2k-1-i} - \overline{\varepsilon}_{2k-1-i}) \right] \right\} \\
&\times \frac{1}{\sqrt{2\pi \sigma_{O}^{2}[\overline{C}(A), k]}} \\
&\times \exp \left\{ -\frac{1}{2 \sigma_{O}^{2}[\overline{C}(A), k]} \left[ (\varepsilon_{2k+1} - \overline{\varepsilon}_{2k+1}) - \sum_{i=1}^{2k} p_{O,i} [\overline{C}(A), k] \cdot (\varepsilon_{2k+i} - \overline{\varepsilon}_{2k+i}) \right] \right\}.
\end{align*}
\]

(3.50)

In the above equation, similarly to Eq. (2.22), \( \sigma_{E}^{2}[\overline{C}(A), k] \) and \( \sigma_{O}^{2}[\overline{C}(A), k] \) represent the \( k \)th order MMSPE's and \( p_{E,i}(k) [\overline{C}(A)] \), \( p_{O,i}(k) [\overline{C}(A)] \) (1 \( \leq k \leq Z \)) are the \( k \)th order prediction coefficients [63]. For representation simplicity, in the equations henceforth, the dependence of \( \sigma_{2k}, \sigma_{2k+1} \) and \( p_{E,i}, p_{O,i} \) on \( \overline{C}(A) \) will be omitted. Similar to Section 2.2.1, the prediction coefficient values are found by solving the two sets of Yule-Walker equations

\[
\begin{align*}
\overline{P}_{E,k} &= [\overline{R}_{k}]^{-1} \overline{D}_{E,k} \\
\overline{P}_{O,k} &= [\overline{R}_{k}]^{-1} \overline{D}_{O,k}
\end{align*}
\]

(3.51)

and they depend upon the type of fading model employed, the ratio of powers between the fading signal and the Gaussian noise, as well as, in general, on the transmitted sequence. The difference in the two linear systems of Eq. (3.51) as compared the one in Eq. (2.23), stems from the fact that in the case of frequency non-selective fading in Chapter 2, samples from the I and Q channels are uncorrelated. In frequency selective fading, however, the ISI caused by
the delay spread combined with the signal distortion caused by fading, introduces correlation between the samples from the I and Q channels, as evident in Eq. (3.47). Due to this, prediction for each I and Q channels, is based not only on the $k$ signal samples from that channel, but on the $2k$ samples from both channels combined. In Eq. (3.51), $\tilde{R}_k$ is a $[2k \times 2k]$ matrix (or a $[k \times k]$ matrix of $[2 \times 2]$ sub-matrixes $\tilde{M}(m)$ - see Eq. (3.53) below), $\tilde{D}_{E,k}$ and $\tilde{D}_{O,k}$ are $[2k \times 1]$ matrixes and $\tilde{P}_{E,k}$ and $\tilde{P}_{O,k}$ are $[2k \times 1]$ matrixes with

$$
\tilde{R}_k = \begin{bmatrix}
\tilde{M}(k-1,0) & \tilde{M}(k-2,-1) & \ldots & \tilde{M}(1,2-k) & \tilde{M}(0,1-k) \\
\tilde{M}(k-1,1) & \tilde{M}(k-2,0) & \ldots & \tilde{M}(1,3-k) & \tilde{M}(0,2-k) \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\tilde{M}(k-1,k-2) & \tilde{M}(k-2,k-3) & \ldots & \tilde{M}(1,0) & \tilde{M}(0,1) \\
\tilde{M}(k-1,k-1) & \tilde{M}(k-2,k-2) & \ldots & \tilde{M}(1,1) & \tilde{M}(0,0)
\end{bmatrix}
$$

$$
\tilde{M}(k,m) = \begin{bmatrix}
R_{11}(k,m) & R_{Q1}(k,m) \\
R_{1Q}(k,m) & R_{QQ}(k,m)
\end{bmatrix}
$$

$$
(3.52)
$$

$$
\tilde{D}_{E,k} = [R_{11}(k,1), R_{1Q}(k,1), \ldots, R_{11}(k,k), R_{1Q}(k,k)]^T
$$

$$
\tilde{D}_{O,k} = [R_{Q1}(k,1), R_{QQ}(k,1), \ldots, R_{Q1}(k,k), R_{QQ}(k,k)]^T
$$

$$
\tilde{P}_{E,k} = [p_{E,1}, p_{E,2}, \ldots, p_{E,2k-1}, p_{E,2k}]
$$

$$
\tilde{P}_{O,k} = [p_{O,1}, p_{O,2}, \ldots, p_{O,2k-1}, p_{O,2k}]
$$

where $^T$ denotes transpose. The $k^{th}$ order MMSPE's are given by

$$
\sigma_E^2(k) = R_{11}(k,0) - \sum_{j=1}^{k} p_{E,2j-1}(k) R_{11}(k - j, -j) - \sum_{j=1}^{k} p_{E,2j}(k) R_{Q1}(k - j, -j)
$$

$$
\sigma_O^2(k) = R_{QQ}(k,0) - \sum_{j=1}^{k} p_{O,2j-1}(k) R_{1Q}(k - j, -j) - \sum_{j=1}^{k} p_{O,2j}(k) R_{QQ}(k - j, -j).
$$

(3.53)
Maximizing the product in the pdf of Eq. (3.50) is equivalent to minimizing the sum of logarithms in the following metric function

\[
\sum_{k=0}^{Z-1} \left\{ \frac{1}{\sigma_E^2(k)} \left[ \sum_{i=1}^{2k} p_{E,i}(k) \cdot (\varepsilon_{2k-i} - \bar{\varepsilon}_{2k-i}) \right]^2 + \ln \left[ 2\pi \sigma_E^2(k) \right] \right\} + \sum_{k=0}^{Z-1} \left\{ \frac{1}{\sigma_O^2(k)} \left[ \sum_{i=1}^{2k} p_{O,i}(k) \cdot (\varepsilon_{2k+i} - \bar{\varepsilon}_{2k+i}) \right]^2 + \ln \left[ 2\pi \sigma_O^2(k) \right] \right\}.
\] (3.54)

According to Eq. (3.54), in order to implement a sequential detection algorithm using this metric expression, we need to compute the \(2 \times 2Z\) prediction coefficients and also need storage for all \(Z\) complex received signal samples. The formidable computational load of solving the two linear systems to yield the prediction coefficients and then computing the metric expression, even for moderately large values of \(Z\), can be a serious limiting factor as far as practical system implementation is concerned. However, similarly to the method followed in Chapter 2, taking advantage of the statistical properties of the fading channel, i.e., knowing that the autocorrelation function \(R_F(\beta)\) values become quite small as we move away from \(\beta = 0\), we can truncate both sums in Eq. (3.54) up to a maximum prediction order \(z\) \([63]\). This decreases dramatically the computational load imposed by the algorithm without significantly compromising the attainable gain. A practical system would typically employ values of \(z\) ranging between 2 and 4. Substituting the expressions for \(\varepsilon_{2k}\) and \(\varepsilon_{2k+1}\) in Eq. (3.54) and truncating the sums beyond the maximum prediction order \(z\) yields the function to be maximized given in the following equation

\[
\sum_{k=0}^{Z-1} \left\{ \frac{1}{\sigma_E^2(k)} \left[ \sum_{i=1}^{z} p_{E,2i}(k) \left\{ \frac{y_k c_k e^{-jn}}{|c_k|^2} - \bar{\varepsilon}_{2k} \right\} - \bar{\varepsilon}_{2k-2i+1} \right] - \frac{y_k-1 c_k e^{-jn}}{|c_k-1|^2} - \bar{\varepsilon}_{2k-2i} \right\} + \ln \left[ 2\pi \sigma_E^2(k) \right] + \sum_{k=0}^{Z-1} \left\{ \frac{1}{\sigma_O^2(k)} \left[ \sum_{i=1}^{z} p_{E,2i-1}(k) \left\{ \frac{y_k c_k e^{-jn}}{|c_k|^2} - \bar{\varepsilon}_{2k-2i} \right\} - \bar{\varepsilon}_{2k-2i} \right] - \frac{y_k-1 c_k e^{-jn}}{|c_k-1|^2} - \bar{\varepsilon}_{2k-2i} \right\} + \ln \left[ 2\pi \sigma_O^2(k) \right] \right\}.
\]
3.4 BER PERFORMANCE EVALUATION RESULTS AND DISCUSSION

The BER performance of the maximum likelihood decoder derived, was evaluated via computer simulation, using Monte-Carlo error counting techniques. The modulation format used was 4-state, rate-1/2 trellis-coded π/4-shift DQPSK. For comparison purposes, as in Chapter 2, the code used is the best 4-state code listed in [25]. Fading channel frequency selectivity was simulated by one strong signal reflection arriving at the receiver at τ = 0.25 and τ = 0.75, or by a triangular delay spread profile. Both Rayleigh and Rician channel characteristics were investigated. The π/4-shift DQPSK NRZ sequences were filtered with square-root, 4-pole Butterworth or Nyquist filters. The Butterworth filters using a 3 dB cut-off frequency $f_B = 1/T$, and the Nyquist filters an excess bandwidth of $\alpha = 1.0$. The sensitivity of the receiving algorithms was also tested, by recording the BER degradation as a function of the error in the receiver's estimate of $\hat{\tau}$ and $B_T T$ product. For comparison purposes, using computer simulation, we also evaluated the BER performance of a receiver employing the modified square-root Kalman (MSRK) equalizer for fading dispersive channels found in [78]. We chose this equalization method to compare against, as it is the most well suited to the rapid signal phase and amplitude fluctuation caused by fading [14], exhibiting orders of magnitude faster convergence [78] as compared to conventional decision feedback equalization methods found in [14]. The performance of this receiver is compared to that of the maximum likelihood

\[
+ \sum_{k=0}^{Z-1} \frac{1}{\sigma_0^2(k)} \left[ \sum_{i=1}^{z} p_{0,2i}(k) \left( \left| \frac{y_{k-1} c_{k-1}}{c_k} e^{-j\eta} \right|^2 \right) - \frac{\varepsilon_{2k+1}}{2} \right]^{2} + \ln \left[ 2\pi \sigma_0^2(k) \right].
\]

\[\text{(3.55)}\]
decoder derived, in identical channel conditions, for increasing values of the \( B_F T \) product. In the following paragraphs, we will be presenting the detailed BER performance evaluation results.

Fig. 3.2 illustrates a comparison between the performance of the optimal sequence estimation receiver for frequency selective fading, versus the equivalent receiver for flat fading found in [63]. Results for complexities of \( z = 2 \) and 3 are shown, as well as two levels of the power ratio between direct-diffused and reflected-diffused paths, referred to as direct-to-reflected ratio (DRR). Both paths are Rayleigh faded, the reflected one arriving at \( \hat{\tau} = 0.75 \) of the symbol duration \( T \), while the \( B_F T \) product is 0.0625, corresponding for example, to a transmission rate of 1200 Baud on 850 MHz while the vehicle is moving at a speed of 95.3 km/h. In this figure, it is interesting to note the very small effect the DRR has on the frequency-selective receiver, and also the gain in performance when increasing the receiver complexity from \( z = 2 \) to 3. As expected, the frequency-flat receiver performance worsens when its receiver complexity is increased. This is because an increase in \( z \) implies an increase in the number of terms contributing to the decoding metric used by the frequency-flat receiver, thus increasing its inaccuracy as compared to the metric used by the frequency-selective receiver. The improvement in performance for both the frequency-flat and frequency-selective receivers when the reflection delay is decreased to \( \hat{\tau} = 0.25 \), is shown in Fig. 3.3. The complexity for both receivers is set to \( z = 3 \), and DRR takes values of 3 and 6 dB. Comparing the results illustrated in Figs. 3.2 and 3.3, it can be seen that, as intuitively expected, the frequency-flat receiver performance approaches that achieved by the frequency-selective receiver, as \( \hat{\tau} \to 0 \). Note also the contrast in performance variation between the two types of receivers with the change in DRR. As is intuitively expected, the frequency-flat receiver performance is more sensitive to the value of DRR, as compared to the the frequency-selective receiver, where the reflected ray signal is accounted for by the decoding algorithm. The slight performance improvement observed for the frequency-flat receiver in medium values of SNR, is attributed to the randomizing effect of
Figure 3.2: BER performance of rate-1/2 trellis coded Butterworth filtered \( f_B = 1/T \) \( \pi/4 \)-shift DQPSK scheme, using the frequency-selective maximum likelihood receiver, versus the equivalent frequency-flat maximum likelihood receiver, in the presence of a 2-ray Rayleigh fading channel with \( B_F T = 0.0625 \) and \( \tilde{\tau} = 0.75 \).

Figure 3.3: BER performance of rate-1/2 trellis coded Butterworth filtered \( \pi/4 \)-shift DQPSK scheme using the frequency-selective maximum likelihood receiver, versus the equivalent frequency-flat maximum likelihood receiver, both with \( z = 3 \), in the presence of a 2-ray Rayleigh fading channel with \( B_F T = 0.0625 \) and \( \tilde{\tau} = 0.25 \).
the AWGN, which cancels a portion of the error inherent to the frequency-flat receiver metric when used in such frequency-selective channel conditions.

Figs. 3.4 and 3.5 illustrate results for conditions same to those of Figs. 3.2 and 3.3, with the exception of a higher $B_F T$ product, set to 0.125. As expected, the performance for both receiver types is worse as compared to that for $B_F T = 0.0625$. Results in both figures are presented for receiver complexity $z = 3$. Indeed the receiver BER shown in Fig. 3.4 is quite high. While the illustration is provided for the purpose of comparison to the frequency-flat receiver case, it should be noted that the performance, even at such a high fading rate, could be enhanced, if the receiver complexity was increased to $z = 4$ or $5$. However, due to the fact that the time required for computer simulation at such receiver complexity was impractically high, we will not be presenting evaluation results for $z > 3$.

The sensitivity of the optimal receiver was investigated by obtaining the BER performance for various degrees of inaccuracy in the receiver’s estimate of the channel delay $\hat{\tau}$ and the fading $B_F T$ product. The results are depicted in Figs. 3.6 and 3.7. Since we are interested in both positive and negative errors in the receiver’s estimate of the delay spread, reflected ray delay is set to $\hat{\tau} = 0.5$; the receiver’s estimate of $B_F T$ is set to 0.0625. The value of $\hat{\tau}$ assumed by the receiver is allowed to take values between 0 and 1, while the channel’s $B_F T$ product varies within $\pm 50\%$. Note that this variation in $B_F T$ product could directly correspond to a $\pm 50\%$ change in vehicle speed, i.e., in our case, $B_F T = 0.03125 \rightarrow 0.09375$. The BER performance with both receiver estimates perfect, using a complexity of $z = 2$, is slightly over $10^{-3}$, while using $z = 3$, below $10^{-4}$, corresponding to the indentation on the two three-dimensional surfaces, at $B_F T$ 0% error and $\hat{\tau} = 0.5$. It is interesting to note that a moderate error in $B_F T$ of $\pm 20\%$ causes relatively little degradation. A decrease in the fading rate, although it translates to inaccuracy in the detection algorithms employed, yields better performance due to the fact that, the slower signal phase changes are cancelled out to a greater degree by the operation of the differential detectors. On the other hand, when the receiver’s
Figure 3.4: BER performance of rate-1/2 trellis coded Butterworth filtered $\pi/4$-shift DQPSK scheme using the frequency-selective maximum likelihood receiver, versus the equivalent frequency-flat maximum likelihood receiver, both with $z = 3$, in the presence of a 2-ray Rayleigh fading channel with $B_F T = 0.125$ and $\hat{\tau} = 0.75$.

Figure 3.5: BER performance of rate-1/2 trellis coded Butterworth filtered $\pi/4$-shift DQPSK scheme using the frequency-selective maximum likelihood receiver, versus the equivalent frequency-flat maximum likelihood receiver, both with $z = 3$, in the presence of a 2-ray Rayleigh fading channel with $B_F T = 0.0625$ and $\hat{\tau} = 0.25$. 
Figure 3.6: Frequency-selective maximum likelihood receiver ($z = 2$) performance degradation due to inaccurate estimate of channel delay ($\hat{\tau}$) and $B_FT$, in the presence of a 2-ray Rayleigh fading channel.
Figure 3.7: Frequency-selective maximum likelihood receiver \((z = 3)\) performance degradation due to inaccurate estimate of channel delay \((\hat{\tau})\) and \(B_T T\), in the presence of a 2-ray Rayleigh fading channel.
estimate of the reflected ray delay is decreased beyond the actual channel value, the performance deteriorates quite quickly. Interestingly enough, this is not the case when the receiver estimate of \( \hat{\tau} \) is greater than that corresponding to the actual channel delay. The trend implies that a practical system should implement a channel delay estimate which would be more likely to have a positive rather than a negative error. As intuitively expected, the higher complexity scheme of \( z = 3 \) is more sensitive to high inaccuracies in the channel delay estimate, than when employing \( z = 2 \).

![BER Performance Graph](image)

Figure 3.8: BER performance of rate-1/2 trellis coded Butterworth filtered (\( f_B = 1/T \)) \( \pi/4 \)-shift DQPSK scheme, using the frequency-selective maximum likelihood receiver, in a channel with triangular delay spread profile for the reflected signal, centered at \( 0.75T \), with \( B_{FT} = 0.0625 \) and DRR = 3 and 6 dB.

The effect of having a delay spread profile rather than a single reflected signal path, is illustrated in the BER results presented in Fig. 3.8. The channel conditions assumed in the simulation are the same as in Fig. 3.2; the results depicted in Fig. 3.2 are duplicated here for ease of comparison. The triangular delay profile used, simulated in discrete time as 5 signal rays centered around \( 0.75T \) and spaced \( T/16 \) apart. Total signal power for the single ray is the same as that for the delay profile, while powers of reflections before and after the central
reflection roll-off at $3 \times 16/T$ dB/s. In other words, assuming 0 dB for the central reflection, the ones adjacent to it are 3 dB down, and the two outer ones 6 dB down. Such an exaggerated reflected energy profile was chosen in order to investigate the sensitivity of the receiver to the dispersion of the delay profile; in a practical multipath environment, and for the transmission rates of interest, the signal power around a peak caused by a strong reflection will present a much steeper roll-off, resulting in much less degradation [99]. As seen in Fig. 3.8, the actual performance loss for $z = 3$ and DRR = 6 dB, is around 5 dB at a BER level of $2 \times 10^{-4}$, bringing the error floor previously below $10^{-4}$, at about $2 \times 10^{-4}$. For DRR = 3 dB, the error floor is slightly higher, as intuitively expected, and around $3 \times 10^{-4}$. For the $z = 2$ case, the irreducible error floor is higher and the observable loss in performance less. It is interesting to note that even in such an extreme case of high signal energy around the central reflection, the performance loss is within reasonable limits, given the fact that the receiver is designed for a single reflection.

Although the derivation of the investigated maximum likelihood receivers has assumed NRZ pulse signaling, the results presented thus far are for 4-pole Butterworth filtered NRZ pulses. Comparison to results generated for unfiltered NRZ signals has revealed that the receiver structures derived suffer no measurable performance degradation when 4-pole Butterworth filtering is employed. The results depicted in Figs. 3.9 and 3.10 demonstrate the relatively small sensitivity of the algorithm to the pulse shape used. Both figures present BER evaluation results for Nyquist filtered signals, using excess bandwidth $\alpha = 1.0$, under channel conditions identical to those in Figs. 3.2 and 3.5. It is interesting to note the great similarity between the two sets of results, keeping in mind the vast differences between the Butterworth and Nyquist pulse shapes. The slight degradation – rather than the expected enhancement – of performance when decreasing DRR in the Nyquist filter case, is attributed to the degradation introduced by the metric inaccuracies, due to pulses interfering from both sides of the current symbol rather than just the past symbol as in the Butterworth pulse case. The fact that the performance degradation
Figure 3.9: BER performance of rate-1/2 trellis coded Nyquist filtered ($\alpha = 1.0$), $\pi/4$-shift DQPSK scheme using the frequency-selective maximum likelihood receiver, versus the equivalent frequency-flat maximum likelihood receiver, in the presence of a 2-ray Rayleigh fading channel with $B_f T = 0.0625$ and $\hat{\tau} = 0.75$ (see also Fig. 3.2 for comparison).

Figure 3.10: BER performance of rate-1/2 trellis coded Nyquist filtered ($\alpha = 1.0$), $\pi/4$-shift DQPSK scheme using the frequency-selective maximum likelihood receiver, versus the equivalent frequency-flat maximum likelihood receiver, in the presence of a 2-ray Rayleigh fading channel with $B_f T = 0.0625$, $\hat{\tau} = 0.25$ (see also Fig. 3.3 for comparison).
is small, is a strong indication that this error could be rectified by modifying the derivation to include symbols on both sides of a sampling instant, something that would only moderately increase the metric and hence the receiver complexity. However, decreasing values of the roll-off factor $\alpha$ has the well known effect of increasing the amplitude of pulse side-lobes [14]. This, in turn, would increase the inaccuracies in the metric computation and have a detrimental effect on performance. If low values of $\alpha$ are to be accommodated successfully, the signal expression of Eq. (3.38) has to be modified to include more terms from previous and future symbols.

The BER performance of the optimal sequence estimation receiver in a Rician frequency selective fading environment is depicted in Figs. 3.11 and 3.12. In both cases the delay for the Rayleigh ($K_1 \rightarrow -\infty dB$) faded reflection is set to $\hat{\tau} = 0.75$. The first figure shows a comparison between receivers with complexities of $z = 2$ and 3, both for the frequency-flat and frequency-selective cases, at a Rician channel $K_0 = 5$ dB. As expected for this type of channel, the performance improvement gained by increasing the receiver complexity from $z = 2$ to 3 is quite smaller than for the Rayleigh fading case. Decreasing the $K$-factor from 5 dB down to 0 dB, as illustrated in Fig. 3.12, has quite an impact on the frequency-flat receiver by raising its BER performance error floor a little over an order of magnitude. The optimal receiver, on the other hand, suffers a loss of slightly less than 6 dB at a BER level of $10^{-3}$, but still presents no observable error floor above $10^{-4}$.

The performance of the derived optimal sequence estimation receiver is also compared to a receiver employing an decision feedback equalization method using the MSRK algorithm reported in [78]. As mentioned previously, this algorithm was chosen for its ultra fast convergence properties, as compared to other conventional weight-updating algorithms - a vital trait in the time-varying fading channel [14]. Since the communication format employs differentially encoded signals, we have made the simplifying assumption that the equalizer operates on the $\pi/4$-shift DQPSK signal space, rather than the space of differential phases (which is the QPSK
Figure 3.11: BER performance of rate-1/2 trellis coded Butterworth filtered ($f_B = 1/T$), \(\pi/4\)-shift DQPSK scheme using the frequency-selective maximum likelihood receiver, versus the equivalent frequency-flat maximum likelihood receiver, in the presence of a 2-ray Rician fading channel with $B_FT = 0.0625$, $K_0 = 5 \text{ dB}$, $K_1 \to -\infty \text{ dB}$ and $\hat{\tau} = 0.75$.

Figure 3.12: BER performance of rate-1/2 trellis coded Butterworth filtered, \(\pi/4\)-shift DQPSK scheme using the frequency-selective maximum likelihood receiver, versus the equivalent frequency-flat maximum likelihood receiver, in the presence of a 2-ray Rician fading channel with $B_FT = 0.0625$, $\hat{\tau} = 0.75$, $K = K_0$ and $K_1 \to -\infty \text{ dB}$.
Figure 3.13: BER performance comparison of the optimal receiver derived versus a receiver employing MSRK equalization, both employing a rate-1/2 trellis coded Butterworth filtered ($f_B = 1/T$) π/4-shift DQPSK scheme, in the presence of a 2-ray Rician fading channel with $B_F T = 0.001$ and 0.005, $K_0 = 0$ dB, $K_1 \to -\infty$ dB and $\hat{\tau} = 1.0$.

This assumption, implying knowledge of the transmitter initial phase at the receiver, shall under no circumstance make the MSRK equalizer-based receiver performance worse than it would be, if a suitable equalization method was found to operate on the output of a single-symbol differential detector. Such a modified equalizer is beyond the scope of the performance comparison sought here, since employing a differential detector before the equalizer input introduces nonlinearity in the channel [100], invalidating the assumptions on which linear-channel equalization is based. The MSRK equalizer output is processed by a differential detector, and the information-bearing phase shifts are then fed to a Viterbi decoder. Experimentation with this type of receiver, in slow and fast frequency selective Rician fading channels, while optimizing the equalizer parameters [78] by trial and error, demonstrated that reasonable operation is possible in fading channels with $B_F T$ products < 0.001. As the fading speed approaches $B_F T = 0.001$, and beyond that value, the irreducible error floor observed is
unacceptably high. Fig. 3.13 illustrates BER performance results comparing the optimal receiver derived to the MSRK equalizer-based receiver. Clearly the optimal sequence estimation receiver has superior performance for $B_F T$ values greater than 0.001, even though large values of training overhead (50% for $B_F T = 0.001$, and 100% for $B_F T = 0.005$) are required for securing the MSRK performance illustrated in Fig. 3.13.

3.5 CONCLUSION

We have presented the derivation of an optimal, in the maximum likelihood detection sense, sequence estimation receiver for coded digital signals, transmitted over frequency selective Rician fading and additive white Gaussian noise (AWGN) channels. Computer simulation results of several reduced complexity versions of the optimal detection algorithm, for $\pi/4$-shift DQPSK signals under varying channel conditions, have demonstrated the merit of using the derived receiver structures. Sensitivity evaluation employing various degrees of implementation complexity has also shown the optimal receiver derived to suffer relatively small performance degradation, while enduring appreciable errors in channel condition estimates, an important quality when considering practical system implementation. Comparison of performance with linear-channel equalization-based receiver structures specifically tailored to the fading dispersive channel, has demonstrated the merit of using the MLSE receiver derived in cases where the fading $B_F T$ product is greater than or equal to 0.001.
CHAPTER 4

PERFORMANCE ANALYSIS OF OPTIMAL NONCOHERENT DETECTION FOR SLOW CORRELATED FREQUENCY NON-SELECTIVE RICIAN FADING WITH DIVERSITY

In Chapter 2 we presented and evaluated the optimal multiple differential detector using diversity reception for fast Rician frequency non-selective fading. When fading is slow, i.e., assuming a transmission rate of two to three orders of magnitude higher than that assumed in Chapter 2, with vehicle speed and operating frequency remaining the same, considerable simplification to the receiver structure is possible. In such case it is straight-forward to verify that the values of the fading autocorrelation function $R_F(kT)$ for symbols close to the sampling instant, i.e., for small values of $k$, are approximately equal to 1. Then, the prediction coefficients $p_{k,m}[ar{C}(\bar{a})]$ can be all set equal to 1. The resulting simplified receiver structure also arises from the derivation of the maximum likelihood receiver under slow fading conditions presented in this chapter. Both the optimal receiver decoding metric for varying degrees of channel state information, and a performance bound for the case of no channel state information are presented. Although designed with slow fading in mind, computer simulation and analytical performance bound evaluation demonstrate the merits of using diversity reception.

\footnote{The research methodology for obtaining the analytical bounds reported in this chapter, has been published in part, as a full paper in the IEEE Journal on Selected Areas in Communications \cite{101}.}
and especially how its use offsets the inaccuracies inherent to using such receivers in moderate and even fast fading conditions. The BER performance is also investigated in shadowing EHF channel environments, where the small wavelength makes a higher order of diversity much more practical than in UHF frequencies. The rest of this chapter is organized as follows. Section 4.1 presents the decoder metric derivation, and Section 4.2 the analytical performance bound for the case of no channel state information. Monte-Carlo simulation results together with computer-aided evaluation of the performance bound are presented in Section 4.3, with the conclusions appearing in Section 4.4.

4.1 DERIVATION OF THE MAXIMUM LIKELIHOOD SEQUENTIAL DECODER FOR SLOW FADING

In Chapter 2 we pointed out that the modulator initial phase \( \eta \) has been proven of relatively small importance as far as the performance of the derived MDD receiver is concerned. At least theoretically, however, such a receiver must estimate \( \eta \). In this chapter, in order to further simplify the receiver structure, we assume that the receiver has no knowledge of \( \eta \). This is equivalent to assuming that \( \eta \) follows a uniform distribution in the interval \([0, 2\pi)\). For the purpose of the analysis presented here, we consider the signal received on the \( l \)th diversity channel to be as in Eq. (2.15), which is repeated here for the reader’s convenience.

\[
x_l^i(t) = f^i(t)x(t) + n_l^i(t) \quad 1 \leq i \leq \Lambda
\]  

(4.56)

where \( f^i(t) = f^i(t) + jf^Q(t) \) is the frequency non-selective Rician fading distortion and \( n_l^i(t) \) the AWGN with two-sided power spectral density \( N_0/2 \). Using Eq. (2.8), the above equation can be rewritten as

\[
x_l^i(t) = \xi_l^i(t)e^{j\psi_l^i(t)}x(t) + n_l^i(t) \\
= \xi_l^i(t)\exp[j\psi_l^i(t)]\exp[j(2\pi f_c t)]\sum_{k=0}^{Z-1}c_k h_T(t - kT) + n_l^i(t)
\]  

(4.57)
where

\[ \xi_P^i(t) = |f^i(t)|, \quad \psi_P^i(t) = \tan^{-1}\left( \frac{f_{Q,i}(t)}{f_{I,i}(t)} \right), \quad \psi^i_P(t) = \psi_P^i(t) + \eta \]  

(4.58)

with \( \eta \) denoting the modulator initial phase, and \( Z \) the number of symbols in the transmitted sequence of \( c_k \)’s. Clearly, \( \xi_P^i(t) \) represents the interference in the envelope, and \( \psi_P^i(t) \) the interference in the phase of the received signal. \( \psi^i_P(t) \) denotes the combined effect of the two phases, \( \psi_P^i(t) \) and \( \eta \). Since we consider a "totally" noncoherent receiver, \( \psi^i_P(t) \) is unknown to the receiver, and hence assumed to be uniformly distributed in \([0, 2\pi)\). We also make the usual assumption of uncorrelated diversity, i.e., \( \psi^i_P(t), \psi^l_P(t) \) are uncorrelated, \( 1 \leq i, l \leq \Lambda, i \neq l \).

We shall consider the following three cases as representative of the receiver’s knowledge of the channel state:

1. The receiver has knowledge of both the envelope \( \xi_P^i(t) \) and phase \( \psi_P^i(t) \) of the fading process \( \forall \ i, 1 \leq i \leq \Lambda \).

2. The receiver has knowledge only of the envelope \( \xi_P^i(t) \).

3. The receiver has no knowledge of either the envelope \( \xi_P^i(t) \) or the phase \( \psi_P^i(t) \).

For the first and second case, it is assumed that the fading is slow enough, i.e., not changing considerably over the transmission period, so that a suitable subsystem can be used to track the phase and/or amplitude variation with a reasonable degree of success.

For the first case, it is shown in Appendix C.1 that the optimal maximum likelihood sequence estimator must maximize the following metric expression

\[ \prod_{i=1}^{\Lambda} I_0 \left[ \frac{\sum_{k=0}^{Z-1} \hat{R}_P^i(kT)e^{-j\psi_P^i(kT)}y_k^i c_k^*}{N_0} \right] \]  

(4.59)

where \( \hat{R}_P^i \) is the estimate of the signal strength on the \( i \)th diversity channel, and \( y_k^i \) is the output of a \( \sqrt{\alpha} \) Nyquist filter with input \( x^i_r(t) \) after it has been downconverted by \( \exp[-j(2\pi f_c t)] \).
For the second and third case, the optimal decoding metric derivation which is almost identical to that used for the first case, is outlined in Appendix C.2. The resulting decoding metric for the second case is

\[ \prod_{i=1}^{\Lambda} I_0 \left[ \frac{\sum_{k=0}^{Z-1} \hat{R}_F(kT)y_k^i c_k^*}{N_0} \right] \]  
(4.60)

and for the third case it is given by

\[ \prod_{i=1}^{\Lambda} I_0 \left[ \frac{\sum_{k=0}^{Z-1} y_k^i c_k^*}{N_0} \right]. \]  
(4.61)

There are implications for the implementation of the receiver structure as expressed by Eqs. (4.59), (4.60) and (4.61), which bring about the necessity of performing simplifications. The problem is the complexity involved in implementing the Bessel function \( I_0(\cdot) \). However, by using an approximation for \( I_0(\cdot) \), and also distinguishing low SNR and high SNR channel operating conditions, we are able to arrive to the following two, reduced complexity asymptotically optimal\(^2\) forms of the decoding metrics.

### 4.1.1 Low SNR channel

For low SNR (e.g., \( \leq 5 \) dB), the Bessel function \( I_0(x) \) can be approximated as follows [33]

\[ I_0(x) = 1 + \frac{x^2}{2^2} + \frac{x^4}{2^44^2} + \frac{x^6}{2^64^26^2} + \ldots = \sum_{i=0}^{\infty} \frac{x^{2i}}{2^{2i}(i!)^2}. \]  
(4.62)

Use of Eq. (4.62) with Eq. (4.60), elimination of all terms which are independent from the considered sequence \( \overline{C}(\overline{A}) \) and truncation of all the terms where \( 1/N_0 \) appears with power higher than 2, leads to the following metric expression

\[ R_{NB}^Z(y^1, y^2, \ldots, y^{\Lambda}, \overline{C}(\overline{A})) = \sum_{i=1}^{\Lambda} \left[ \hat{R}_F^i(kT) \right]^2 \sum_{k=0}^{Z-1} y_k^i c_k^* \]  
(4.63)

\(^2\)For the high SNR case, "asymptotically optimal" implies that as the noise power after the receiver filter approaches zero, the performance of the simplified metric form becomes identical to that of the original metric before simplification. For low SNR, this convergence is achieved by using higher accuracy approximations of \( I_0(\cdot) \) for increasing noise power.
where \( \vec{y} = [y^i_0, y^i_1, \ldots, y^i_{Z-1}] \). Following a procedure very similar to that used in Appendix A.2, eliminating terms which are independent from the considered sequence \( \vec{C}(\vec{A}) \), leads to the following metric expression

\[
\sum_{k=1}^{Z-1} \sum_{i=1}^{k} \left[ \sum_{l=1}^{\Lambda} \left( \Re_F^i(kT) \right)^2 d_{i,i}^1(k) \cos [\Delta \Theta_i(k)] + \sum_{l=1}^{\Lambda} \left( \Im_F^i(kT) \right)^2 d_{i,i}^2(k) \sin [\Delta \Theta_i(k)] \right]
\] (4.64)

where

\[
d_{i,i}(k) = y^i_k(y^i_{k-1})^*, \quad d_{i,i}^1(k) = \Re \{d_{i,i}(k)\}, \quad d_{i,i}^2(k) = \Im \{d_{i,i}(k)\}, \quad \Delta \Theta_i(k) = \arg \left[ c_i^i(c_i^{i-1})^* \right].
\] (4.65)

The term \( d_{i,i}(k) \) is the product between the signal sample \( y_k \) and the complex conjugate of the \( y_{k-1} \) sample, that is, a signal sample received \( tT \) seconds before. This is the output of a differential detector with delay element equal to \( tT \). From Eq. (4.64) it is apparent that the maximum likelihood sequence estimation receiver processes the output of a multiple differential detector structure with a total of \( Z - 1 \) differential detectors. In this respect, much like in the case for fast fading in Chapter 2, this receiver employs a MDD hardware structure for processing the signal samples within an observation window of \( Z \) symbols in length. However, in contrast to the algorithm for fast fading, we do not have the added complexity of calculating prediction coefficients, or estimating the initial phase \( \eta \) of the transmitter modulator. This great reduction in algorithmic complexity does not come without a price, as will become evident in the performance evaluation results presented in Section 4.3.

### 4.1.2 High SNR channel

For high SNR, the following approximation for the \( I(x) \) will be used [33]

\[
I_0(|x|) \simeq \frac{1}{\sqrt{2\pi|x|}} \exp(|x|) \quad |x| > 3.
\] (4.66)
In Appendix C.2 it is shown that use of the above approximation in the metric expression of Eq. (4.60) leads to the following asymptotically optimal decoding metric

$$R_{NB}^Z(y^1, y^2, \ldots, y^\Lambda, \overline{C}(A)) = \sum_{\lambda=1}^{\Lambda} \left| \sum_{k=0}^{Z-1} \xi_{F}(kT)y_k^i z_k^i \right|.$$  \hspace{1cm} (4.67)

Eqs. (4.63) and (4.67) indicate that a square envelope structure is asymptotically optimal for low SNR, while an envelope structure for high SNR.

### 4.2 ANALYTICAL PERFORMANCE BOUND

From the cases of channel state information examined in the previous section, undoubtedly the most simple and, at the same time, the most desirable in a practical system implementation is that whereby no knowledge of channel state information is available at the receiver. Because of this, our effort was concentrated on deriving an analytical BER performance bound for this particular case. For mathematical convenience, the type of modulation format assumed in the derivation is single level, accommodating, for example, π/4-shift DQPSK, or 8-DPSK.

The bound calculation used in Section 4.3 employs averaging of the union bound [14] for all possible signal sequences of length Z. In calculating this union bound for every possible sequence, we need an expression of the pairwise error event probability, for any two sequences from the total number of sequences having length Z symbols. The derivation of this pairwise error event probability $P_{\Lambda}(\overline{C}(A^\lambda) \leftarrow \overline{C}(A^{\mu}))$ for a single level modulation format in a Rician fading channel with no channel state information available at the receiver is presented in Appendix C.3, the final result of which is the following.

For $\alpha_{[\nu,\xi]} > 0$,

$$P_{\Lambda}(\overline{C}(A^\lambda) \leftarrow \overline{C}(A^{\mu})) = Q_f \left( \frac{1}{\sigma_n} \alpha_{[\nu,\xi]}, \frac{1}{\sigma_n} \beta_{[\nu,\xi]} \right)$$

$$-I_o \left[ \frac{1}{\sigma_n} \alpha_{[\nu,\xi]} \frac{1}{\sigma_n} \beta_{[\nu,\xi]} \right] \exp \left[ -\frac{\alpha_{[\nu,\xi]} + \beta_{[\nu,\xi]}^2}{2\sigma_n^2} \right]$$  \hspace{1cm} (4.68)
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\[ \times \frac{(1 + D)\Lambda(1 - D)^{\Lambda - 1}}{2^{2\Lambda - 1}} \sum_{l=0}^{\Lambda - 1} \left( \frac{\Lambda - 1}{\Lambda + l} \right) (1 + D)^l(1 - D)^{-l} \]

\[ + \exp \left[ -\frac{\alpha_{[V,\zeta]}^2 + \beta_{[V,\zeta]}^2}{2\sigma_n^2} \right] \times \sum_{m=0}^{\Lambda - 1} I_m \left( \frac{\alpha_{[V,\zeta]} - \beta_{[V,\zeta]}}{\sigma_n} \right) \]

\[ \times \left\{ \gamma^m \left[ 1 - \frac{(1 + D)^{\Lambda-m}(1 - D)^{\Lambda+m-1}}{2^{2\Lambda - 1}} \sum_{l=0}^{\Lambda+m-1} \left( \frac{\Lambda - 1}{\Lambda + l - m} \right) (1 + D)^l(1 - D)^{-l} \right] \right. \]

\[ \left. - \frac{1}{\gamma^m} \left[ 1 - \frac{(1 + D)^{\Lambda+m-1}(1 - D)^{\Lambda-m}}{2^{2\Lambda - 1}} \sum_{l=0}^{\Lambda+m-1} \left( \frac{\Lambda - 1}{\Lambda + l - m} \right) (1 + D)^{-l}(1 - D)^l \right] \right\} \times \left[ 1 - \delta(\Lambda - 1) \right]. \]

In the above equation, \( \delta(\cdot) \) is the Kronecker \( \delta \)-function, \( Q_f(x, y) \) is Marcum's Q function [32, p. 585] and

\[ \begin{align*}
\left\{ \alpha_{[V,\zeta]} \right\} &= \left[ \frac{\Lambda}{\nu_{\nu^*} + \nu_{\zeta}} \left( \frac{E_{\nu}^\nu + E_{\zeta}^\nu}{1 - |\gamma_{\nu\zeta}|^2} - 2 \text{Re} \left\{ \left( E_{\nu}^\nu \right)^* E_{\zeta}^\nu \gamma_{\nu\zeta} \right\} \right) \\
&\quad \pm \frac{E_{\nu}^\nu - E_{\zeta}^\nu}{\sqrt{1 - |\gamma_{\nu\zeta}|^2}} \right]^{1/2}
\end{align*} \]

with

\[ \gamma_{\nu\zeta} = \frac{2\nu_{\zeta}}{\nu_{\nu^*} + \nu_{\nu^*}}, \]

\[ D = \frac{\nu_{\nu^*} - \nu_{\zeta}}{\sqrt{(\nu_{\nu^*} + \nu_{\zeta})^2 - 4 |\nu_{\zeta}|^2}}, \]

(4.69)
and

\[ y = \frac{\left| E_{\nu}^\nu \right|^2 + \left| E_{\zeta}^\nu \right|^2 - 2\text{Re} \left\{ \left( E_{\nu}^\nu \right)^* E_{\zeta}^\nu \gamma_{\nu\zeta} \right\} \sqrt{1 - \left| \gamma_{\nu\zeta} \right|^2}}{\left| E_{\nu}^\nu \right|^2 + \left| E_{\zeta}^\nu \right|^2 - 2\text{Re} \left\{ \left( E_{\nu}^\nu \right)^* E_{\zeta}^\nu \gamma_{\nu\zeta} \right\} \sqrt{1 - \left| \gamma_{\nu\zeta} \right|^2}}. \] (4.71)

For \( \alpha_{[\nu,\zeta]} = 0 \),

\[ P_A \left( C(\overline{A}^\nu) \leftarrow C(\overline{A}^\nu) \right) = \exp \left[ -\frac{\beta_{[\nu,\zeta]}^2}{2\sigma_n^2} \right] \left[ 1 - \frac{1}{2^{2\Lambda - 1}} \sum_{i=0}^{\Lambda - 1} 2^{2\Lambda - 1} \left( 2\Lambda - 1 \right) \right]. \] (4.72)

Denoting by

\[ \left| F_{Z,i}^\nu \left( \gamma^i, C(\overline{A}^\nu) \right) \right| = \left| \sum_{k=0}^{Z-1} y_k(c_k^\nu)^* \right| \] (4.73)

the argument of \( I_0(\cdot) \) in Eq. (4.61), with \( c_k \)'s taking values according to the information sequence \( \overline{A}^\lambda \), then \( E_{\nu}^\nu \), is the conditional expected value of \( F_{Z,i}^\nu \left( \gamma^i, C(\overline{A}^\nu) \right) \) given that \( \overline{A}^\nu \) was the transmitted sequence, and \( E_{\zeta}^\nu \), is the conditional expected value of \( F_{Z,i}^\nu \left( \gamma^i, C(\overline{A}^\nu) \right) \) given that \( \overline{A}^\nu \) was the transmitted sequence. \( v_{j[\nu,\zeta]} \) and \( v_{j[\nu,\nu]} \) are the conditional variances for \( F_{Z,i}^\nu \left( \gamma^i, C(\overline{A}^\nu) \right) \) and \( F_{Z,i}^\nu \left( \gamma^i, C(\overline{A}^\nu) \right) \) respectively, while \( v_{j[\nu,\zeta]} \) is their conditional cross covariance. Detailed expressions for all above moments are given in Appendix C.3.

In the derivation of the pairwise error event probability in Appendix C.3, in order to provide simplified expressions of Eqs. (4.71) and (4.72), we have distinguished two cases of commonly encountered fading channel conditions. These are

- Rician fading with strong direct signal component at high SNR, and
- Rayleigh fading, i.e., no LOS signal component.

In the following two paragraphs, we present the final results for the pairwise error event probability for these two special cases.
In a Rician fading channel with strong direct signal component, i.e., when $K \geq 5$ dB, the expression of Eq. (4.68) can be tightly upper bounded by the following expression

$$ P_1 \left( \{ \overline{C}(A^c) \leftarrow \overline{C}(A^u) \} \right) \simeq \mathcal{B} \mathcal{F}^{Z,A}_{[\nu,\zeta]} = \exp \left( -\frac{\alpha_{[\nu,\zeta]}^2 + \beta_{[\nu,\zeta]}^2}{2\sigma_n^2} \right) \Gamma^{Z,A}_{[\nu,\zeta]} . \quad (4.74) $$

For $\alpha_{[\nu,\zeta]} > 0$,

$$ \Gamma^{Z,A}_{[\nu,\zeta]} = \frac{1}{\sqrt{2\pi \sigma_n^2}} \frac{1}{\alpha_{[\nu,\zeta]} \beta_{[\nu,\zeta]}} \left\{ 1 - \exp \left[ \frac{\alpha_{[\nu,\zeta]} \beta_{[\nu,\zeta]}}{\sigma_n^2} \right] \left( 1 + D \right)^{\Lambda} \frac{(1 - D)^{\Lambda - 1}}{2^{2\Lambda - 1}} \sum_{l=0}^{\Lambda-1} \left( \begin{array}{c} 2\Lambda - 1 \\ \Lambda + l \end{array} \right) (1 + D)^l (1 - D)^{-l} \right\} 

+ \exp \left[ \frac{\alpha_{[\nu,\zeta]} \beta_{[\nu,\zeta]}}{\sigma_n^2} \right] \sum_{m=0}^{\Lambda-1} G_m \left( \frac{\alpha_{[\nu,\zeta]} \beta_{[\nu,\zeta]}}{\sigma_n^2} \right) 

\times \left\{ J_m \left[ 1 - \frac{(1 + D)^{\Lambda - m} (1 - D)^{\Lambda + m - 1}}{2^{2\Lambda - 1}} \sum_{l=0}^{\Lambda - m - 1} \left( \begin{array}{c} 2\Lambda - 1 \\ \Lambda + l - m \end{array} \right) (1 + D)^l (1 - D)^{-l} \right] 

- \frac{1}{J_m} \left[ 1 - \frac{(1 + D)^{\Lambda + m - 1} (1 - D)^{\Lambda - m}}{2^{2\Lambda - 1}} \sum_{l=0}^{\Lambda + m - 1} \left( \begin{array}{c} 2\Lambda - 1 \\ \Lambda + l - m \end{array} \right) (1 + D)^l (1 - D)^{-l} \right] \right\} 

\times \left[ 1 - \delta(\Lambda - 1) \right] \right\} . \quad (4.75) $$

For $\alpha_{[\nu,\zeta]} = 0$, the expression is identical to the one in Eq. (4.72).

For a Rayleigh fading channel, i.e., $K \to -\infty$, the pairwise error probability is reduced to

$$ P_1 \left( \{ \overline{C}(A^c) \leftarrow \overline{C}(A^u) \} \right) = \frac{(1 - D)^{\Lambda} (1 - D)^{\Lambda - 1}}{2^{2\Lambda - 1}} \sum_{l=0}^{\Lambda - 1} \left( \begin{array}{c} 2\Lambda - 1 \\ \Lambda + l \end{array} \right) (1 + D)^l (1 - D)^{-l} \right\} . \quad (4.76) $$
4.3 BER PERFORMANCE EVALUATION RESULTS AND DISCUSSION

This section offers BER evaluation results for the multiple differential decoder structure developed in Section 4.1, in the case when no channel state information is available to the receiver. There is no doubt that this is the most attractive case in terms of practical implementation, since no special subsystem is needed for providing an estimate of the amplitude and/or the phase of the faded signal. Furthermore, since it is our intention to investigate the receiver behaviour in fast fading channels, although the metric design is based on slow fading, assuming availability of an accurate signal amplitude and/or phase estimate is not an option. In Section 4.3.1 we present results for Rayleigh and Rician fading channels, at various fading rates, degrees of receiver structure complexity and diversity order. BER performance obtained via Monte-Carlo digital simulation is compared to the analytical results using the bound described in Section 4.2. In Section 4.3.2, the multiple-differential detector structure is also evaluated in an EHF Ka-band channel, exhibiting a LOS signal component assumed to be a random variable following a log-normal distribution. Motivation for providing these results was provided by the fact that the small wavelength for carrier frequencies around 20 GHz ($\lambda \simeq 1.5$ cm), makes implementation of antenna diversity much easier than at UHF frequencies.

4.3.1 UHF Land Mobile Fading Channel

Computer simulation BER evaluation

Using the decoder metric derived for the case where no channel state information is available at the receiver (see Eq. (4.61)), Monte-Carlo error counting techniques were used to evaluate the BER performance in a Rayleigh fading channel, using sequence lengths $Z$ of 2, 3 and 4, without diversity ($\Lambda = 1$), and with diversity $\Lambda = 2$ and 3. Note that $Z = 2$ corresponds to
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Figure 4.1: BER performance of rate-1/2 trellis coded Nyquist filtered ($\alpha = 0.35$) $\pi/4$-shift DQPSK scheme, in Rayleigh fading with $B_FT = 0.005$, for diversity $\Lambda = 1$, 2 and 3 and sequence length $Z = 2$, 3 and 4.

conventional differential detection. The fading $B_FT$ product was varied from a relatively slow fading rate of 0.005, to the very fast fading value of 0.1. Results for $B_FT = 0.005$ are depicted in Fig. 4.1. Evident in this figure is the merit of employing diversity, as opposed to using large values of $Z$ for reducing the BER at a specific SNR. The gain when going from no diversity to diversity $\Lambda = 2$, employing a sequence length $Z = 4$ at a BER level of $10^{-3}$, is almost 13 dB. Increasing the order of diversity by one to $\Lambda = 3$ yields an additional 5 dB. Increasing the fading rate by a factor of two, to $B_FT = 0.01$ has negligible effect on the trends observed in Fig. 4.1. The results presented in Fig. 4.2 reveal approximately the same amount of gain when introducing diversity of $\Lambda = 2$ as compared to the no diversity case, at the same BER level of $10^{-3}$. The gain available from increasing $\Lambda$ to 3 is again approximately equal to 5 dB. In both the $B_FT = 0.005$ case of Fig. 4.1 and the $B_FT = 0.01$ case of Fig. 4.2, when no diversity is employed, it is interesting to note that the incremental gain obtained when increasing $Z$ from 2 (simple differential detector) to 3, is considerably more than that obtained by increasing the
sequence length further, to \( Z = 4 \).

![Figure 4.2: BER performance of rate-1/2 trellis coded Nyquist filtered (\( \alpha = 0.35 \)) \( \pi/4 \)-shift DQPSK scheme, in Rayleigh fading with \( B_F T = 0.01 \), for diversity \( \Lambda = 1, 2 \) and 3 and sequence length \( Z = 2, 3 \) and 4.](image)

The problems caused by inaccuracies inherent to employing this type of decoder metric in channels with high fading rate are depicted in Figs. 4.3 and 4.4. In Fig. 4.3, in the case of no diversity, i.e., \( \Lambda = 1 \), and for a fading \( B_F T = 0.05 \) we observe an intuitively expected performance penalty when the sequence length used is increased to \( Z = 4 \). The very interesting point to note in this figure is that, employing diversity not only continues to secure the performance gain observed in the previous two cases, but also seems to offset the effect of the inaccuracies inherent to using the metric of Eq. (4.61) in such a high fading rate. Quantitatively, the inaccuracy introduced can be appreciated by comparing the solution of the linear system of Eq. (2.23), to setting all prediction coefficients equal to 1, as implied by Eq. (4.61) – the error increasing proportionally to an increase in the fading \( B_F T \) product. Diversity \( \Lambda = 2 \) seems to cure part of the problem, with a BER performance curve cross-over for \( Z = 3 \) and 4 still taking place around an SNR of 12.5 dB, while employing \( \Lambda = 3 \) seems to offset the losses due to
Figure 4.3: BER performance of rate-1/2 trellis coded Nyquist filtered ($\alpha = 0.35$) $\pi/4$-shift DQPSK scheme, in Rayleigh fading with $B_F T = 0.05$, for diversity $\Lambda = 1, 2$ and 3 and sequence length $Z = 2, 3$ and 4.

Figure 4.4: BER performance of rate-1/2 trellis coded Nyquist filtered ($\alpha = 0.35$) $\pi/4$-shift DQPSK scheme, in Rayleigh fading with $B_F T = 0.1$, for diversity $\Lambda = 1, 2$ and 3 and sequence length $Z = 2, 3$ and 4.
inaccuracy, at least down to the minimum observable BER level of $10^{-4}$, with the Monte-Carlo error counting techniques used. Fig. 4.4, however, clearly illustrates how the decoder breaks down, as the fading rate is increased further to the very fast fading value of $B_F T = 0.1$. As expected, the greatest performance penalty occurs with $Z = 4$. Note, that as in the case of $B_F T = 0.05$, using diversity of $\Lambda = 3$ appears to offset the loss caused by the mismatch between the fading statistics assumed by the receiver and those actually present in the channel, at least for $Z = 3$. The trend observed in Figs. 4.3 and 4.4 leads us to the conclusion that, if a high order of diversity is practically feasible, there is much more merit in using large values of $\Lambda$ and a simple differential detector for fast fading environments, than using no diversity or a diversity of 2, and a MDD based maximum likelihood receiver designed for slow fading, with $Z > 3$. If, on the other hand, diversity higher that $\Lambda = 2$ is not practical, the alternative for improving the performance would be to employ the receiver structures derived for fast fading, as presented in Chapter 2. Comparison of the performance in Fig. 2.10 with $z = 2$ and $\Lambda = 2$, to that of Fig. 4.4 with $Z = 3$ and $\Lambda = 2$ clearly illustrates this point. Both receivers use two differential detectors, and the one in Fig. 4.4 has the advantage of operating at 20% lower fading rate than that in Fig. 2.10. Nevertheless, the receiver designed for fast fading, performs about 3 dB better. This difference in performance will increase even further if $z$ is increased to 3, and the $B_F T$ product for Fig. 4.4 is increased to 0.125.

**Analytical BER evaluation**

The performance of the MDD receivers using the metric of Eq. (4.61) when no channel state information is available was also investigated using the performance bound developed in Section 4.2. The computer aided calculation of this bound greatly reduces the time required to evaluate the receiver structures, as it is between 3 and 4 orders of magnitude faster than that required for the Monte-Carlo digital simulation. Furthermore, it enables us to predict the
receiver performance at BER levels well below the practical limit of $10^{-4}$ in a typical computer simulation. Figs. 4.5 and 4.6 illustrate results obtained by averaging the union bound over all possible sequences generated by the rate-1/2 trellis-code used and associated error paths within $Z$ symbols. The probability for each error event is calculated by evaluating the expression of Eq. (4.68) and (4.72). Results illustrated for Rayleigh fading with $B_FT = 0.005$ in Fig. 4.5 and $B_FT = 0.01$, in Fig. 4.6, together with Monte-Carlo simulation results, demonstrate the tightness of the analytical bounds derived. As intuitively expected, the irreducible error rate (error-floor) due to the random FM is reduced by increasing the diversity order, while it increases in a faster fading environment. Results for a Rician fading channel with $B_FT = 0.05$ are presented in Fig. 4.7, for a $K$-factor of 5 and 10 dB, using diversity $\Lambda = 2$ and 3, and a sequence length $Z = 3$. The Monte-Carlo simulation results for each case illustrate once more the tightness of the analytical bound. At a BER level of $10^{-4}$ and $K = 5$ dB there is gain of approximately 4.5 dB when using diversity $\Lambda = 3$ as compared to $\Lambda = 2$. Increasing the value
Figure 4.6: BER performance bound evaluation versus Monte-Carlo simulation results for rate-1/2 trellis coded Nyquist filtered ($\alpha = 0.35$) $\pi/4$-shift DQPSK scheme, in Rayleigh fading with $B_FT = 0.01$, for diversity $\Lambda = 1, 2$ and 3 and sequence length $Z = 4$.

Figure 4.7: BER performance bound evaluation and Monte-Carlo simulation results for rate-1/2 trellis coded Nyquist filtered ($\alpha = 0.35$) $\pi/4$-shift DQPSK scheme, in Rician fading with $B_FT = 0.05$, for diversity $\Lambda = 2$ and 3, sequence length $Z = 4$, and $K = 5$ and 10 dB.
of $K$ to 10 dB improves the BER performance for both diversity cases, moving the calculated error-floor below $10^{-10}$ and, as expected, decreases the incremental gain between $\Lambda = 2$ and $3$. It is interesting to note that the BER receiver performance for the case of $\Lambda = 3$ and $K = 10$ dB is only slightly worse that of the rate-1/2 4-state trellis coded QPSK in AWGN.

4.3.2 EHF Mobile Satellite Channel

Propagation measurement campaigns have shown [34, 102] that the UHF (800-1000 MHz) and L-band (1.5 GHz) mobile-satellite radio channel can be modeled as Rician fading with its local mean, the LOS component, following a lognormal statistical distribution. The fading appearing in the $i^{th}$ diversity channel can be expressed as

$$f_i^p(t) = \rho_i^p(t) \exp \left[ j \psi_i^p(t) \right] = f_i^{FS}(t) \exp \left[ j \phi_i^{FS}(t) \right] + f_i^{FD}(t) \exp \left[ j \phi_i^{FD}(t) \right]$$

where $\rho_i^p(t)$, $\psi_i^p(t)$ are the amplitude and phase of the fading process, respectively. $\phi_i^{FS}(t)$ and $\phi_i^{FD}(t)$ are uniformly distributed over $[0, 2\pi)$. $f_i^{FS}(t)$ is a lognormally distributed stochastic process which represents the amplitude of the LOS signal, while $f_i^{FD}(t)$ is a Rayleigh distributed stochastic process which represents the envelope of the multipath component. The pdf of the received fading envelope $\rho_i^p(t)$ can be mathematically represented as [34]

$$p(\rho_i^p(t)) = \frac{r}{b_0 \sqrt{2\pi d_0}} \int_0^\infty \frac{1}{z} \exp \left[ -\frac{(\ln z - \mu)^2}{2d_0} - \frac{r^2 + z^2}{2b_0} \right] I_0 \left( \frac{rz}{b_0} \right) dz$$

where $b_0$ is the power of the multipath signal, and $\mu$ and $\sqrt{d_0}$ are the mean and standard deviation of the shadowing process. The pdf of the lognormally distributed LOS component is

$$p(z) = \frac{1}{\sqrt{2\pi d_0 z}} \exp \left( -\frac{\mu}{2d_0} \right)$$

The parameters for the L-band mobile satellite channel [34, 35] are shown in Table 4.4. The wavelength at the EHF Ka-band (20 GHz) is approximately 1.5 cm. As this is comparable
Table 4.4: L-band mobile-satellite radio channel model parameters.

<table>
<thead>
<tr>
<th>L-band channel</th>
<th>$b_0$</th>
<th>$\mu$</th>
<th>$\sqrt{d_0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>light shadowing</td>
<td>0.158</td>
<td>0.115</td>
<td>0.115</td>
</tr>
<tr>
<td>average shadowing</td>
<td>0.126</td>
<td>-0.115</td>
<td>0.161</td>
</tr>
<tr>
<td>heavy shadowing</td>
<td>0.0631</td>
<td>-3.91</td>
<td>0.806</td>
</tr>
</tbody>
</table>

Table 4.4: L-band mobile-satellite radio channel model parameters.

To even small obstructions in the signal path, such as tree leaves and small branches, we would expect higher losses from shadowing in such frequencies. Since, to the best of our knowledge, experimental data for the Ka-band is not presently available, we attempt to estimate an appropriate scaling factor for the model parameters, in order to account for this higher signal degradation. Using a scaling technique described in detail in Appendix C.4, we modify $\mu$ and $d_0$ according to the following equations

$$\sqrt{d_0}(F) \simeq \sqrt{d_0} \times 0.45 F^{0.284}$$

$$\mu(F) \simeq \mu \times 0.45 F^{0.284}$$

(4.80)

with $F$ in GHz, using $b_0$ and $\mu$ from Table 4.4. The model of the fading channel used in the Monte-Carlo simulation is depicted in Fig. 4.8. The independent white noise processes $n^I(t), n^Q(t)$ and $n(t)$ are filtered by identical filters having the land-mobile fading characteristic. The bandwidth $B_\text{S}$ for the real and imaginary part of the complex process for the diffused component is equal to the maximum Doppler frequency $B_F$, while for the lognormally distributed LOS component, the bandwidth $B_D$ is about 1/20 of $B_F$ in UHF frequencies [26, 34]. For comparison with the Rician fading case, the ratio of the LOS signal average power to the multipath power is derived in Appendix C.5 as

$$K_{\text{AVG}} = 10 \log_{10} \frac{\exp(2\mu + 2d_0)}{2b_0} \text{ dB.}$$

(4.81)

BER simulation results for an EHF channel with carrier frequency around 21 GHz, vehicle
speed of about 65 km/h and antenna pointing elevation of 20° are depicted in Figs. 4.9, 4.10 and 4.11. The parameter scaling factor is approximately equal to 2.1, the resulting maximum Doppler frequency is about 1200 Hz, and the bandwidth of the lognormally distributed LOS component approximately equal to 43 Hz. The $B_F T$ product for the diffused signal component is then 0.05, given a data rate of 24 kBaud. The results for light shadowing conditions with $K_{AVG} = 7.34$ dB, sequence lengths of $Z = 2, 3$ and 4, and diversity $\Lambda = 1, 2$ and 3, are illustrated in Fig. 4.9. Similarly to our previous observation for the UHF fading channel, it is interesting to note the large gain in performance by using diversity $\Lambda = 2$ as compared to the no diversity case ($\Lambda = 1$). At a BER level of $10^{-3}$ and using sequence length $Z = 4$ this gain

3 An elevation angle of 20° makes the effective speed with respect to the satellite equal to $65 \text{km/h} \times \cos(21°)$ which is approximately 61 km/h.

4 The reason why $B_S$ is more than $20 \times B_D$ in this EHF band example is that, the two phenomena of LOS shadowing and Doppler, are independent. The LOS shadowing rate is obtained by scaling the UHF band values according to the increase in frequency, while the Doppler depends on the carrier frequency as well as the vehicle speed.
is approximately 10 dB, while at a BER level of $10^{-4}$ it increases to over 13 dB. When using diversity order $\Lambda = 3$ besides the additional gain available, we also note a greater incremental gain when going from using $Z = 2$ to $Z = 3$ and 4, as compared to the case with $\Lambda = 2$. This is attributed to the fact that most of the distortion imposed by fading has been offset by the high diversity order, which in turn decreases the penalty in performance inherent to employing this type of receivers in fast fading channels. It is interesting to note that the receiver performance with $Z = 4$ and $\Lambda = 3$ is only approximately 1 dB worse than the rate-1/2 trellis-coded QPSK performance in AWGN.

![Figure 4.9: BER performance of rate-1/2 trellis coded Nyquist filtered ($\alpha = 0.35$) $\pi/4$-shift DQPSK, in EHF light shadowing conditions with $BF = 0.05$, for diversity $\Lambda = 1, 2$ and 3 and sequence length $Z = 2, 3$ and 4.](image)

Figs. 4.10 and 4.11 illustrate the results for average and heavy fading conditions, having $K_{AVG} = 4.36$ dB, and -50.48 dB, respectively. It is interesting to note that even for heavy shadowing, the receivers perform reasonably well, if sufficient order of diversity is employed. The inaccuracies due to fast fading are all the more visible as we go from light, to average and then to heavy shadowing. They are manifested as performance degradation for increasing
Figure 4.10: BER performance of rate-1/2 trellis coded Nyquist filtered ($\alpha = 0.35$) $\pi/4$-shift DQPSK, in EHF average shadowing conditions with $B_F T = 0.05$, for diversity $\Lambda = 1, 2$ and 3 and sequence length $Z = 2, 3$ and 4.

Figure 4.11: BER performance of rate-1/2 trellis coded Nyquist filtered ($\alpha = 0.35$) $\pi/4$-shift DQPSK, in EHF heavy shadowing conditions with $B_F T = 0.05$, for diversity $\Lambda = 1, 2$ and 3 and sequence length $Z = 2, 3$ and 4.
values of $Z$, and are more pronounced in higher SNR values, where the randomization effect of AWGN is negligible.

4.4 CONCLUSION

We have derived the optimal sequence estimator for slow frequency non-selective fading channels, and varying degrees of channel state information. The algorithmic structure of the estimator is a simplified version of that derived for frequency non-selective fading in Chapter 2. For the case when no such channel state information is available, we have derived analytical BER performance bounds for Rician and Rayleigh frequency non-selective fading channels. The very close agreement of Monte-Carlo computer simulation and computer aided bound calculation have demonstrated the accuracy and tightness of these analytical bounds. The results obtained for $\pi/4$-shift DQPSK demonstrate great merit in using diversity in conjunction with these simplified receiver structures, even in fast fading environments where considerable inaccuracies would normally prohibit their use. Furthermore, with practical systems in mind, we have used computer simulation to evaluate the performance offered by these receivers in the EHF shadowing channel. In this we were motivated by the fact that due to the very small wavelength in the 20 GHz and 30 GHz EHF bands, large orders of diversity are considerably easier to implement than in the UHF band. The results obtained demonstrate a great improvement in performance, as we increase the diversity order from 2 to 3. In the case of light shadowing, the attainable BER approaches that of trellis coded coherent QPSK, employing the same code.
5.1 CONCLUSIONS

In this thesis we have addressed the general problem of noncoherent detection in fast, correlated, Rician fading channels. In our derivation of improved receivers, for this most prominent of distortions among those present in mobile communication environments, we have distinguished cases of frequency non-selective and frequency selective fading. For all cases we have considered trellis coded PSK and QAM type signals, with particular emphasis on the emerging North American digital cellular standard $\pi/4$-shift DQPSK, and the more spectrally efficient 8-DPSK and $\pi/4$-shift 8-DQAM modulation formats.

For the frequency non-selective fading channel we have derived the optimal receiver, in the maximum likelihood detection sense, based on the MDD hardware structure, employing novel maximum likelihood sequence estimation algorithms, using diversity reception. In the interest of system simplicity, we have proposed and evaluated reduced complexity versions of the optimal receiver structures. Computer simulation results have demonstrated significant gains in performance, as compared to conventional signal detection schemes. Taking into consideration the inaccuracies inherent to practical system implementations, we have investigated cases where the receiver statistical knowledge of the fading channel is in considerable error with respect to the actual channel state. Results show relatively small sensitivity, even to large inaccuracies in
Conclusions and Suggestions for Future Work

estimates of channel parameters.

For the frequency selective fading channel, we have derived the optimal receiver, in the maximum likelihood detection sense. Although the effect of signal filtering was omitted for the purpose of reducing the mathematical complexity in the derivation, using digital simulation we have investigated the effects of such filtering on the receiver performance. Furthermore, we have also presented computer simulation results, illustrating the relatively small sensitivity of the proposed algorithms to inaccuracies in the receiver estimates of channel parameters. The performance of the derived receiver structure has been compared to the fastest known equalization technique for frequency selective fading, which employs the modified square root Kalman equalization algorithm, developed for use in HF dispersive channels. Results have shown the proposed receiver to significantly outperform equalization in fast fading channel conditions.

Finally, we have derived novel analytical BER performance bounds for simplified versions of the optimal diversity receiver, for frequency non-selective, Rician fading. These simplified receiver structures were derived for varying degrees of channel state information, and were shown to be optimal in the case of slow fading channel conditions. The bound derivation, however, assumed the most general case, where no channel state information is available at the receiver. Using digital simulation and computer aided calculation of the derived bounds, we have evaluated the performance of these receivers in slow as well as fast Rician fading, employing different fading rates, and various orders of reception diversity and receiver implementation complexity. The tightness and accuracy of the derived bounds was demonstrated by the excellent agreement between calculated and computer simulation results. BER evaluation has also indicated that, with increased diversity order, the performance gains are so great, that in most cases they offset completely the inaccuracies inherent in using this type of receivers in fast fading conditions. Motivated by this result, and with practical implementation in mind, we have also evaluated these receiver structures in the Ka-band EHF shadowing channel.
Computer simulation results, for both Rician and EHF shadowing conditions, have shown the receiver performance approach the effectiveness of coherent detection in AWGN, even when employing a relatively small diversity order.

5.2 SUGGESTIONS FOR FUTURE WORK

In the course of the work reported throughout this thesis, there have been instances in which interesting questions were raised; questions, which in our opinion, warrant further research. In the following subsections, we briefly present the most interesting of these topics.

5.2.1 Suboptimal Receiver Algorithms of Reduced Complexity

Both in Chapters 2 and 3, we pointed out the complexity involved in calculating the metric of the optimal receiver structures derived. This complexity is a limiting factor, both when attempting to evaluate such structures using computer simulation, and, perhaps more importantly, when implementing them in practical communication systems. Although the truncation method used in both aforementioned chapters resulted in considerable receiver simplification, the remaining complexity is still a problem, especially when trying to accommodate signal constellations with large numbers of signals, and large orders of diversity. The problem is definitely even more pronounced in the case of frequency selective fading, than for frequency flat fading, due to the correlation between I and Q channels introduced by the delay spread. In light of all this, it is worthwhile investigating the possibility of further simplifying the receiver structure, hopefully yielding forms of much reduced complexity, with only small sacrifice in receiver performance.
5.2.2 Receiver Structures for Nyquist Pulses in Frequency Selective Fading

In Chapter 3 we pointed out that, although straightforward, it is nevertheless tedious to include the effects of signal filtering in the derivation of the optimal receiver structure. For convenience in the mathematical derivation, we adopted the use of unfiltered NRZ signals. Although this approach provided considerable simplification, as mentioned in Section 3.4, the receiver performance is expected to deteriorate as the pulse shape employed departs considerably from that of the assumed NRZ pulse. This is especially the case for Nyquist filtering employing small values of the roll-off factor $\alpha$. Although modifying the derivation to include such effects is not expected to change the general structure of the receiver, it will certainly be of great importance, considering the fact that it will eliminate the performance penalty otherwise observed in systems using this type of filtering.

5.2.3 Frequency selective fading with $\tilde{\tau} > 1$

The normalized delay spread, in both the derivation and the evaluation in Chapter 3, was assumed to be at most equal to 1. This assumption was based on the fact that such range of values is typical for the digital cellular applications considered. However, since values of $\tilde{\tau}$ greater than 1 are possible in other digital communication applications, there certainly is merit in extending the derivation to include such cases.

5.2.4 Extension of Derivation of Bounds for Multi-level Schemes

As mentioned in Chapter 4, the analytical bound derivation for the performance of the MDD receivers in slow fading, assumed single level modulation formats such as $\pi/4$-shift DQPSK and 8-DPSK. Since higher order modulation schemes employing more than single level signals are desirable for their increased spectral efficiency, it would be very useful to extend the derivation in Chapter 4 to include multi-level signals. The results of such an undertaking would be useful
in predicting the performance of the receivers presented in Chapter 4, when employing such signals in Rician frequency non-selective fading channels, without the limitations inherent to Monte-Carlo computer simulations.

5.2.5 Performance Bounds for Fast Fading

The BER performance bounds derived in Chapter 4 are based on the simplified MDD receivers in the case of slow frequency non-selective fading. The derivation of such analytical bounds for the case of fast frequency non-selective and also frequency selective fading is not at all obvious. However, derivation of equations bounding the performance of the derived receiver structures in such channel conditions is quite desirable. Using the results of such a derivation, it would be possible to predict their performance at BER levels well below the $10^{-4}$ level, where Monte-Carlo computer simulation becomes impractical.

5.2.6 Investigation of MDD Techniques in Spread Spectrum Systems

A preliminary investigation in using the MDD hardware structure with systems employing spread-spectrum transmission, has yielded receiver structures similar to those derived for diversity reception. Nevertheless, considerably more research is warranted in this area, towards evaluating such receiver structures in fading conditions typical to spread-spectrum systems, and also deriving analytical bounds for their performance.

5.2.7 VLSI Implementation of Detection Algorithms Derived

In order to apply the findings of this thesis to receivers in practical systems, the algorithms derived for optimal detection in frequency flat as well as frequency selective fading, with or without diversity, must be implemented in hardware. The type of hardware implementation will certainly be affected by the processing speed requirement of the particular application.
Conclusions and Suggestions for Future Work

However, based on techno-economical criteria, the modern integrated circuit implementation in the form of a very large scale integration (VLSI) "chip" is perhaps the most preferred method of hardware realization. There are certainly many interesting issues in designing such a VLSI chip, ranging from techniques for parallelizing portions of the receiving algorithm to achieve high processing speed, to assessing the impact of signal value quantization, and implementing trade-offs between speed and chip physical area, to name just a few. It is without any doubt, though, that application of the receiving techniques presented throughout this thesis to a wide range of digital transmission systems, depend to a very large extent on successful hardware implementation of the associated algorithms.

5.2.8 Neural Net Based Adaptive Receivers

Although the receiver structures derived and presented in this thesis were shown to suffer relatively small performance degradation due to inaccuracies in estimates of channel parameters, there exist cases where it is desirable to obtain the best possible performance at any given time. To this end, it should be possible to derive neural net based receiver structures, employing the detection algorithms derived, with the additional advantage of being able to track varying channel conditions. Encouraging results from preliminary work, demonstrating the beneficial effect of using neural net assisted receivers in CCI and ACI channels, can be found in [103].

5.2.9 Digital Image Transmission in Fading Channels

In the recent past, we have investigated the effects on picture quality, of random errors caused in digital image transmission over noisy channels [104]. It certainly would be of great practical importance to assess the impact of correlated errors afflicted on image transmission in a mobile fading environment, and to investigate the benefits of using the receiver structures derived, in such systems.
REFERENCES


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A.1 Derivation of Eq. (2.22)

The derivation of the joint pdf of Eq. (2.18) is based on the following probabilistic theorem [98].

**Theorem:** For the zero mean Gaussian random variables $x_m, x_{m-1}, x_{m-2}, \ldots, x_0$ with covariance $R_{i-j} = \mathbb{E}\{x_i x_j\}$, the conditional pdf of $x_m$ given $x_{m-1}, x_{m-2}, \ldots, x_0$ is given by:

$$
\zeta(x_m|x_{m-1}, x_{m-2}, \ldots, x_0) = \frac{1}{\sqrt{2\pi \sigma_{p,m}^2}} \exp \left[ -\frac{(x_m - \bar{x}_m)^2}{2\sigma_{p,m}^2} \right]
$$

(A.1.1)

where $(\cdot)$ represents the prediction of $(\cdot)$. $\sigma_{p,m}^2$ is the $m$th order minimum mean square prediction error (MMSPE), with $\bar{x}_m$ based on $x_{m-1}, x_{m-2}, \ldots, x_0$ and given by:

$$
\bar{x}_m = \sum_{j=1}^{m} p_{m,j} x_{m-j}
$$

(A.1.2)

with $p_{m,j}$ being the linear prediction coefficients and $m$ the order of prediction. The MMSPE can be calculated from:

$$
\sigma_{p,m}^2 = R_0 - \sum_{j=1}^{m} p_{m,j} R_j.
$$

(A.1.3)

Since in Eq. (2.18) $y_k$ is a complex random variable with independent Re{$\cdot$} and Im{$\cdot$} parts, this equation can be rewritten as

$$
\zeta \left[ \bar{y}^1, \bar{y}^2, \ldots, \bar{y}^\Lambda, \bar{C} (\bar{A}) , \eta \right] = \prod_{\lambda=1}^{\Lambda} \prod_{k=1}^{Z} \left[ \zeta_{\text{Re}} \left[ \text{Re} \left( y_k^\lambda \right) \left| y_{k-1}^\lambda, y_{k-2}^\lambda, \ldots, y_1^\lambda, \bar{C} (\bar{A}) , \eta \right] \right], \right. \\
\zeta_{\text{Im}} \left[ \text{Re} \left( y_k^\lambda \right) \left| y_{k-1}^\lambda, y_{k-2}^\lambda, \ldots, y_1^\lambda, \bar{C} (\bar{A}) , \eta \right] \right]
$$

(A.1.4)
where \( \zeta_{Re} \) and \( \zeta_{Im} \) denote the pdf of \( \left[ \text{Re} \left( y_k \right) \left| y_{k-1}, y_{k-2}, \ldots, y_1, C(A), \eta \right. \right] \) and \( \left[ \text{Im} \left( y_k \right) \left| y_{k-1}, y_{k-2}, \ldots, y_1, C(A), \eta \right. \right] \), respectively. But from Eq. (2.19), \( \zeta_{Re} \) is a scaled version of the pdf of \( e^{TL} \) and \( \zeta_{Im} \) a scaled version of the pdf of \( e^{QI} \). Hence, instead of the pdf.

of \( \left[ y_k \left| y_{k-1}, y_{k-2}, \ldots, y_1, C(A), \eta \right. \right] \) we can use the pdf of \( \left[ e_k \left| e_{k-1}, e_{k-2}, \ldots, e_1, C(A), \eta \right. \right] \).

Note here, that in the general case \( e_k^{TI} \) is not a zero mean Gaussian random variable since \( \bar{f}^{TI} \neq 0 \) in which case we use the random variable \( e_k^{TI} - \bar{f}^{TI} \) instead. Using the above theorem and the fact that \( e_k^{TI} \) and \( e_k^{QI} \) are Gaussian random variables, maximization of the pdf in Eq. (2.18) is equivalent to maximizing the following pdf

\[
\prod_{l=1}^{L} \prod_{k=1}^{Z} \frac{1}{\sqrt{2\pi \left( \sigma_k^{IC} \left[ C(A) \right] \right)^2}} \exp \left[ -\frac{\left( e_k^{TI} - e_k^{TI} \right)^2}{2 \left( \sigma_k^{IC} \left[ C(A) \right] \right)^2} \right] \frac{1}{\sqrt{2\pi \left( \sigma_k^{IC} \left[ C(A) \right] \right)^2}} \exp \left[ -\frac{\left( e_k^{QI} - e_k^{QI} \right)^2}{2 \left( \sigma_k^{IC} \left[ C(A) \right] \right)^2} \right]
\]

which by substituting the expressions for \( e_k^{TI}, e_k^{QI} \) of Eq. (2.19), and

\[
\hat{e}_k^{TI} = \sum_{m=1}^{k} \frac{p_{km}}{\left| c_{k-m} \right|^2} \text{Re} \left( y_{k-m} c_k e^{-j\eta} - \bar{f}^{TI} \right)
\]

\[
\hat{e}_k^{QI} = \sum_{m=1}^{k} \frac{p_{km}}{\left| c_{k-m} \right|^2} \text{Im} \left( y_{k-m} c_k e^{-j\eta} \right)
\]

yields the pdf in Eq. (2.22).
A.2 Derivation of Eq. (2.25)

Because of the exponential nature of Eq. (2.22), maximization of this pdf is equivalent to maximizing the following function:

\[
\frac{1}{2} \sum_{k=1}^{Z} \sum_{l=1}^{\Lambda} \left( \sigma_k^l \right)^2 \left\{ \left[ - \sum_{m=0}^{k} \text{Re} \left\{ y_{k-m}^{l} c_{k-m} e^{-j\eta} \right\} \frac{p_{k,m}^{l}}{|c_{k-m}|^2} - \frac{1}{2} \sum_{m=0}^{k} p_{k,m}^{l} \right]^2 + \right. \\
\left. \left[ - \sum_{m=0}^{k} \text{Im} \left\{ y_{k-m}^{l} c_{k-m}^{*} e^{-j\eta} \right\} \frac{p_{k,m}^{l}}{|c_{k-m}|^2} \right]^2 \right\}
\]

(A.2.1)

where we have defined \( p_{m,0}^l = -1 \) \( \forall \ m, \ l, \ 0 \leq m \leq Z, \ 0 \leq l \leq \Lambda \). The dependence of \( \left( \sigma_k^l \right)^2 \) and \( p_{k,m}^l \) on the transmitted sequence \( C(\bar{A}) \) will be dropped for the rest of the derivation for clarity purposes. By expanding the squared terms inside the brackets of the above equation and dropping the 1/2 scaling factor, Eq. (A.2.1) becomes

\[
\sum_{k=1}^{Z} \sum_{l=1}^{\Lambda} \left( \sigma_k^l \right)^2 \left\{ \left( \frac{1}{2} \sum_{m=0}^{k} p_{k,m}^{l} \right)^2 + 2 \frac{1}{2} \sum_{m=0}^{k} \sum_{n=0}^{k} p_{k,n}^{l} p_{k,m}^{l} \text{Re} \left\{ y_{k-m}^{l} c_{k-m} e^{-j\eta} \right\} \right. \\
\left. + \sum_{m=0}^{k} \left( p_{k,m}^{l} \right)^2 \frac{|y_{k-m}^{l}|^2}{|c_{k-m}|^2} \right\} + \left. \right\{ \text{Re} \left\{ y_{k-n}^{l} \left( y_{k-m}^{l} \right)^{*} \right\} \text{Re} \left\{ c_{k-n} c_{k-m}^{*} \right\} + \right. \\
\left. \left[ \text{Re} \left\{ y_{k-n}^{l} \left( y_{k-m}^{l} \right)^{*} \right\} \text{Im} \left\{ c_{k-n} c_{k-m}^{*} \right\} \right] \right\} \\
\left. \left. + \sum_{n=0}^{k} \sum_{m=0}^{k} \sum_{m \neq n} p_{k,n}^{l} p_{k,m}^{l} \left( \frac{1}{2} \sum_{m=0}^{k} p_{k,m}^{l} \right)^2 \frac{|y_{k-m}^{l}|^2}{|c_{k-m}|^2} \left( p_{k,m}^{l} \right)^2 \right. \\
\left. \left. + \sum_{m=0}^{k} \left( p_{k,m}^{l} \right)^2 \frac{|y_{k-m}^{l}|^2}{|c_{k-m}|^2} \right\} \left. \right\} \right. \\
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Further simplification is possible if Eq. (A.2.2) is split into the following four distinct terms (\( T_1, T_2, T_3 \) and \( T_4 \)), the sum of which must be maximized

\[
T_1 = \sum_{k=1}^{Z} \sum_{l=1}^{\Lambda} \frac{1}{\sigma_k^2} \left( \sum_{m=0}^{k} \sum_{n=0}^{k} p_{k,m} p_{k,n} \right) \left[ \text{Re} \left\{ y_{k-m}^l (y_{k-n}^l)^* \right\} \text{Re} \left\{ c_{k-m} c_{k-n}^* \right\} \right. \\

\text{Im} \left\{ y_{k-m}^l (y_{k-n}^l)^* \right\} \text{Im} \left\{ c_{k-m} c_{k-n}^* \right\} \\
\left. \right|_{c_{k-m}^2 |c_{k-n}|^2} \\
T_2 = \sum_{k=1}^{Z} \sum_{l=1}^{\Lambda} \frac{1}{\sigma_k} \left( \sum_{m=0}^{k} p_{k,m}^l \right)^2 \frac{|y_{k-m}^l|^2}{|c_{k-m}|^2} \\
T_3 = \sum_{k=1}^{Z} \sum_{l=1}^{\Lambda} \frac{1}{\sigma_k^2} \left( f_{l}^{*} \right) \sum_{m=0}^{k} p_{k,m}^l \sum_{n=0}^{k} p_{k,n}^l \frac{\text{Re} \left\{ y_{k-m}^l c_{k-n} c_{k-n}^* e^{-jn} \right\}}{|c_{k-n}|^2} \\
T_4 = \sum_{k=1}^{Z} \sum_{l=1}^{\Lambda} \frac{1}{\sigma_k^2} \left( f_{l}^{*} \right) \sum_{m=0}^{k} p_{k,m}^l \frac{2}{|c_k|^2} 
\]

(A.2.3)

The term \( T_4 \) is independent of the transmitted sequence and hence can be removed from the maximization process. By writing out in more detail the sum in term \( T_1 \), it is not difficult to see that the individual terms can be grouped together in a different way, namely, with respect to combinations of the form \( [y_k^l (y_{k-m}^l)^*] \). In the same fashion, we group the terms in \( T_2 \) with respect to combinations of the form \( |y_k^l|^2 / |c_k|^2 \) and in \( T_3 \) with respect to combinations of the form \( \text{Re} \left\{ y_k^l c_k^* e^{-jn} \right\} / |c_k|^2 \). The resulting three terms, are the three terms in the sum of Eq. (2.25).
APPENDIX B

B.1 Derivation of Eqs. (3.40), (3.41) and (3.42)

The three fading functions in Eq. (3.39) can be regarded as the outputs from three corresponding linear filters with the following impulse responses

\[
\begin{align*}
    h_A(t) &= \begin{cases} 1/T & \text{if } 0 \leq t < T \\ 0 & \text{otherwise} \end{cases} \\
    h_B(t) &= \begin{cases} 1/T & \text{if } (1 - \hat{\tau})T \leq t < T \\ 0 & \text{otherwise} \end{cases} \\
    h_C(t) &= \begin{cases} 1/T & \text{if } 0 \leq t < (1 - \hat{\tau})T \\ 0 & \text{otherwise} \end{cases}
\end{align*}
\]  

(B.1.1)

The input to \( h_A(t) \) is \( f^0(t) \), and the input to \( h_B(t) \) and \( h_C(t) \) is \( f^1(t) \). The autocorrelation functions of the output random processes can then be calculated by

\[
\begin{align*}
    R_A(\lambda) &= E[f_A(t)f_a(t + \lambda)] = R^0(\lambda) \otimes h_A(\lambda) \otimes h_A(-\lambda) \\
    R_B(\lambda) &= E[f_B(t)f_B(t + \lambda)] = R^1(\lambda) \otimes h_B(\lambda) \otimes h_B(-\lambda) \\
    R_C(\lambda) &= E[f_C(t)f_C(t + \lambda)] = R^1(\lambda) \otimes h_C(\lambda) \otimes h_C(-\lambda)
\end{align*}
\]  

(B.1.2)

where \( \otimes \) denotes convolution. By substituting the expressions for \( h_A(t), h_B(t) \) and \( h_C(t) \) in the above equations and using the change of variable \( a = b/T \) where \( b \) is the integration variable for the convolution integrals, Eqs. (3.40) are easily obtained.
For the cross-correlations between \( f_B(t) \) and \( f_C(t) \) we use

\[
R_{BC}(\lambda) = E[f_B(t)f_C(t + \lambda)] = R^1(\lambda) \otimes h_B(-\lambda) \otimes f_C(\lambda)
\]
\[
R_{CB}(\lambda) = E[f_C(t)f_B(t + \lambda)] = R^1(\lambda) \otimes h_C(-\lambda) \otimes f_B(\lambda).
\]  

(B.1.3)

Substituting the impulse responses, distinguishing cases for \( 0 \leq \hat{\tau} < 0.5 \) and \( 0.5 \leq \hat{\tau} < 1.0 \), and performing the same change of variable as in the case of the calculation for the autocorrelations, one can arrive easily at Eq. (3.41) and (3.42).
B.2 Derivation of Eqs. (3.46) and (3.47)

Using the functions for the real and imaginary parts of \( e(k) \) as they are given in Eqs. (3.44) and (3.45), making use of their auto- and cross-correlation functions in Eqs. (3.40),(3.41) and (3.42), and eliminating non-zero terms, yields

\[
E \left[ e^I(k)e^I(k-l) \right] = E \left[ f_k^{I,A,0} f_{k-l}^{I,A,0} + f_k^{I,C,1} f_{k-l}^{I,C,1} + \frac{\Re \{ c_{k-1}^* c_{k-l-1} \} f_k^{I,C,1} f_{k-l}^{I,B,1}}{|c_{k-l}|^2} + \frac{\Re \{ c_{k}^* c_{k-l} \} f_k^{I,I,1} f_{k-l}^{I,B,1}}{|c_k|^2 |c_{k-l}|^2} + \frac{\Im \{ c_{k}^* c_{k-l} \} \Im \{ c_{k-1}^* c_{k-l-1} \} f_k^{Q,B,1} f_{k-l}^{Q,B,1}}{|c_k|^2 |c_{k-l}|^2} \right] + \nonumber
\]

\[
E \left[ n_{e}^I(k) n_{e}^I(k-l) \right] = R_A(l) + R_C(l) + \frac{\Re \{ c_{k-1}^* c_{k-l-1} \} R_B(l)}{|c_k|^2 |c_{k-l}|^2} + \frac{\Re \{ c_{k}^* c_{k-l} \} R_{CB}(l)}{|c_k|^2 |c_{k-l}|^2} + \frac{\Re \{ c_{k}^* c_{k-l} \} R_{BC}(l)}{|c_k|^2 |c_{k-l}|^2} + E \left[ n_{e}^I(k) n_{e}^I(k-l) \right] \tag{B.2.1}
\]

with \( E \left[ n_{e}^I(k) n_{e}^I(k-l) \right] \) calculated as

\[
E \left[ n_{e}^I(k) n_{e}^I(k-l) \right] = E \left[ n_{k}^I n_{k-l}^I \Re \{ c_{k}^* e^{-j\eta} \} \Re \{ c_{k-l}^* e^{-j\eta} \} + n_{k}^Q n_{k-l}^Q \Im \{ c_{k}^* e^{-j\eta} \} \Im \{ c_{k-l}^* e^{-j\eta} \} \right]. \tag{B.2.2}
\]

Since \( E \left[ n_{k}^I n_{k-l}^I \right] = E \left[ n_{k}^Q n_{k-l}^Q \right] = \sigma_G^2 \delta_K(l) \) with \( \delta_K(\cdot) \) denoting the Kronecker \( \delta \)-function, with the aid of the mathematical identity \( \Re(A)\Re(B) + \Im(A)\Im(B) = \Re(A^*B) \) the above equation becomes

\[
E \left[ n_{e}^I(k) n_{e}^I(k-l) \right] = \sigma_G^2 \delta_K(l) E \left[ \frac{\Re \{ c_{k} c_{k-l}^* \}^2}{|c_k|^2 |c_{k-l}|^2} \right] = \frac{\sigma_G^2}{|c_k|^2} \delta_K(l). \tag{B.2.3}
\]
Inserting Eq. (B.2.3) in Eq. (B.2.1), and following an identical procedure for deriving $E\left[e^{\phi(k)}e^{\theta(k-l)}\right]$, we obtain the expressions for $R_{11}(k,l)$ and $R_{QQ}(k,l)$ given in Eq. (3.46).

The non-zero terms of the cross-correlation function $E\left[e^{\phi(k)}e^{\theta(k-l)}\right]$ yield

\[
E\left[e^{\phi(k)}e^{\theta(k-l)}\right] = E\left[\frac{\text{Im}\{c_{k-l}\} f_{k,B} f_{k-l,B} +}{|c_{k-l}|^2} \frac{\text{Re}\{c_{k-l}^{*}\} \text{Im}\{c_{k-l}\} f_{k,B} f_{k-l,B} +}{|c_k|^2} \frac{\text{Im}\{c_{k-l}\}^{*} f_{k,B} f_{k-l,B} +}{|c_{k-l}|^2} \frac{\text{Re}\{c_{k-l}^{*}\} \text{Im}\{c_{k-l}\}^{*} f_{k,B} f_{k-l,B} +}{|c_k|^2}\right] +
\]

\[
E\left[n_{e}^{\phi}(k)n_{e}^{\theta}(k-l)\right]
\]

\[
= \frac{\text{Im}\{c_{k-l}\}^{*} f_{k,B} f_{k-l,B} +}{|c_k|^2} \frac{\text{Re}\{c_{k-l}^{*}\} \text{Im}\{c_{k-l}\} f_{k,B} f_{k-l,B} +}{|c_{k-l}|^2} \frac{\text{Im}\{c_{k-l}\}^{*} f_{k,B} f_{k-l,B} +}{|c_k|^2} \frac{\text{Re}\{c_{k-l}^{*}\} \text{Im}\{c_{k-l}\}^{*} f_{k,B} f_{k-l,B} +}{|c_{k-l}|^2}
\]

with $E\left[n_{e}^{l}(k)n_{e}^{Q}(k-l)\right]$ calculated as

\[
E\left[n_{e}^{l}(k)n_{e}^{Q}(k-l)\right] = E\left[n_{e}^{l}(k)\text{Re}\{c_{k-l}e^{-j\eta}\}\text{Im}\{c_{k-l}e^{-j\eta}\} - n_{e}^{Q}n_{e}^{Q}\text{Im}\{c_{k-l}e^{-j\eta}\}\text{Re}\{c_{k-l}e^{-j\eta}\}\right].
\]

Further, by using the mathematical identity $\text{Re}(A)\text{Im}(B) - \text{Im}(A)\text{Re}(B) = \text{Im}(A^*B)$ the equation becomes

\[
E\left[n_{e}^{l}(k)n_{e}^{Q}(k-l)\right] = \sigma_{G}^{2} \delta_{K}(l) E\left[\frac{\text{Im}\{c_{k-l}\}^{*}}{|c_k|^2} \frac{\text{Im}\{c_{k-l}\}^{*}}{|c_{k-l}|^2}\right] = 0.
\]

Inserting Eq. (B.2.6) in Eq. (B.2.4), and following an identical procedure for $E\left[e^{\phi(k)}e^{\phi(k-l)}\right]$, we obtain the expressions for $R_{IQ}(k,l)$ and $R_{QI}(k,l)$ given in Eq.
C.1 Derivation of Eq. (4.59)

Assuming the receiver has knowledge of $\xi_\nu(t)$ and $\psi_\nu(t), \forall \ i, 1 \leq i \leq \Lambda$, the maximum likelihood sequence estimation receiver must maximize the pdf of $x_1^i(t), \ldots, x^\Lambda(t)$ conditioned on $x_B[\mathcal{C}(\bar{A}), t], \eta, \hat{R}_F^i, \psi_\nu, \ldots, \psi^\Lambda_F$, i.e., the function

$$f (x_1^i(t), \ldots, x^\Lambda(t) | x_B[\mathcal{C}(\bar{A}), t], \hat{R}_F, \ldots, \hat{R}_F, \psi, \ldots, \psi^\Lambda_F, \eta) = K_c \prod_{i=1}^\Lambda \exp \left\{ -\frac{1}{2N_0} \int_{t_L}^{t_U} \left| x_1^i(t) - \hat{R}_F^i x_B[\mathcal{C}(\bar{A}), t] \exp \left\{ j(2\pi f_c t + \eta + \psi_\nu(t) + \eta) \right\} \right|^2 dt \right\}. \tag{C.1.1}$$

In the above equation, $K_c$ is a normalizing constant and $t_L, t_U$ represent the limits of the integration. Their values depend on the spreading of the signal $x_B[\mathcal{C}(\bar{A}), t]$ in the time domain. Considering a bandlimited channel, in which case $h_T(t)$ extends from $-\infty$ to $\infty$, we have $t_L \to -\infty$ and $t_U \to \infty$. Furthermore, since $\eta$ has been assumed uniformly distributed over $[0, 2\pi)$, the above equation can be rewritten as

$$f (x_1^i(t), \ldots, x^\Lambda(t) | x_B[\mathcal{C}(\bar{A}), t], \hat{R}_F, \ldots, \hat{R}_F, \psi, \ldots, \psi^\Lambda_F) = \int_0^{2\pi} f (x_1^i(t), \ldots, x^\Lambda(t) | x_B[\mathcal{C}(\bar{A}), t], \hat{R}_F(t), \ldots, \hat{R}_F, \psi, \ldots, \psi^\Lambda_F) d\eta$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \left\{ \prod_{i=1}^\Lambda K_c \exp \left[ -\frac{1}{2N_0} \int_{-\infty}^\infty |x_1^i(t)|^2 dt \right] \right\}$$

$$\times \exp \left[ -\frac{1}{2N_0} \int_{-\infty}^\infty (\hat{R}_F(t))^2 |x_B[\mathcal{C}(\bar{A}), t]|^2 dt \right]$$

$$\times \exp \left[ \frac{1}{2N_0} \int_{-\infty}^\infty [x_1^i(t) \hat{R}_F^i(t) x_B^*[\mathcal{C}(\bar{A}), t] \exp (-j2\pi f_c t - j\psi_\nu(t) - j\eta) + \right.$$
\[ [x_r^i(t)]^* \hat{R}_F(t) x_B [\overline{C}(\Lambda), t] \exp \left( j2\pi f_c t + j\psi_F^i(t) + j\eta \right) \] \] \] \[ d\eta. \]

Note that, as the first two exponentials in the integral are independent of \( \eta \) and the third one can be transformed by using the identity [32]

\[ I_0(2|\epsilon|) = \frac{1}{2\pi} \int_0^{2\pi} \exp \left[ \epsilon \exp(-j\psi) + \epsilon^* \exp(j\psi) \right] d\psi \] \] \[ (C.1.3) \]

where \( \epsilon \) is an arbitrary complex number, Eq. (C.1.2) is simplified to

\[ f \left( x_r^1(t), \ldots, x_r^\Lambda(t) \mid x_B [\overline{C}(\Lambda), t], \hat{R}_F^1, \ldots, \hat{R}_F^\Lambda, \psi_F^1, \ldots, \psi_F^\Lambda \right) = K_c \exp \left[ -\frac{1}{2N_0} \sum_{i=1}^{\Lambda} \int_{-\infty}^{\infty} |x_r^i(t)|^2 dt \right] \times \exp \left[ -\frac{1}{2N_0} \sum_{i=1}^{\Lambda} \int_{-\infty}^{\infty} |x_B [\overline{C}(\Lambda), t]|^2 dt \right] \times I_0 \left[ \frac{1}{N_0} \int_{-\infty}^{\infty} \hat{R}_F(t) x_r^i(t) x_B^* [\overline{C}(\Lambda), t] \exp \left( -j2\pi f_c t - j\psi_F^i(t) \right) dt \right]. \] \[ (C.1.4) \]

In the above equation, the first exponential term is common for all \( \overline{C}(\Lambda) \) and hence can be removed from the maximization process. Therefore, the expression to be maximized further simplifies to

\[ \exp \left[ \sum_{i=1}^{\Lambda} -\frac{(\hat{R}_F(t))^2}{2N_0} \int_{-\infty}^{\infty} |x_B [\overline{C}(\Lambda), t]|^2 dt \right] \times I_0 \left[ \frac{1}{N_0} \int_{-\infty}^{\infty} \hat{R}_F(t) x_r^i(t) x_B^* [\overline{C}(\Lambda), t] \exp \left( -j2\pi f_c t - j\psi_F^i(t) \right) dt \right]. \] \[ (C.1.6) \]

The integral inside the above exponential term can be expanded by using Eq. (2.8) as

\[ \int_{-\infty}^{\infty} |x_B [\overline{C}(\Lambda), t]|^2 dt = \int_{-\infty}^{\infty} \left[ \sum_{k=0}^{Z-1} c_k h_T(t - kT) \right] \left[ \sum_{l=0}^{Z-1} c_l^* h_T(t - lT) \right] dt \]

\[ = \sum_{k=0}^{Z-1} \sum_{l=0}^{Z-1} c_k c_l^* \int_{-\infty}^{\infty} h_T(t - kT) h_T^*(t - lT) dt \]
\[ z_1^{k} = \sum_{k=0}^{Z-1} c_k c_{i-k} h_{i-k} \]
\[ = \sum_{k=1}^{Z-1} Z-1 \sum_{l=0}^{l=k} c_l c_{i-l-k} h_{-k} + c_{i-k} c_{i-l-k} (h_{-k})^* + h_0 \sum_{k=0}^{Z-1} |c_k|^2 \quad \text{(C.1.8)} \]

where
\[ h_i = h(lT) = h_T(lT) \otimes h_T^*(-lT) = \int_{-\infty}^{\infty} h_T(t) h_T^*(t-lT) dt. \quad \text{(C.1.9)} \]

Note that the function \( h(t) \) represents the total impulse response of the concatenation of \( h_T(t) \) with its matched version \( h_T^*(-t) \), which is the receiver filter assumed for every diversity channel. When \( H_T(f) \) is a square root Nyquist I filter, i.e., \( H_T(f) = \sqrt{H(f)} \) where \( H(f) \) is a Nyquist filter with impulse response \( h(t) \), then
\[ h(t) = h_T(t) \otimes h_T^*(t) = h(t) \quad \text{(C.1.10)} \]
and therefore
\[ h_i = h(lT) = \begin{cases} h_0 > 0 & \text{for } l = 0 \\ 0 & \text{elsewhere.} \end{cases} \quad \text{(C.1.11)} \]

For this Nyquist channel, since there is no ISI, the first sum of Eq. (C.1.8) is 0. For single level schemes (e.g., PSK type signals), the second sum of Eq. (C.1.8) is independent of the transmitted sequence \( \overline{C(A)} \), and hence can be removed from the metric maximization process.

Now returning back to Eq. (C.1.7), the integral inside the \( I_0 \{ \cdot \} \) term can be rewritten as
\[ \int_{-\infty}^{\infty} x_i^t(t) \mathcal{R}_f^i(t) \mathcal{C}_B^* [\overline{C(A)}, t] \exp \left( -j2\pi f_c t - j\psi_i^t(t) \right) dt \quad \text{(C.1.12)} \]
\[ = \int_{-\infty}^{\infty} x_i^t(t) \mathcal{R}_f^i(t) \exp \left( -j\psi_i^t(t) \right) \sum_{k=0}^{Z-1} c_k^* h_T^*(t-kT) \exp \left( -j2\pi f_c t \right) dt \]
\[ = \sum_{k=0}^{Z-1} c_k^* \int_{-\infty}^{\infty} x_i^t(t) \mathcal{R}_f^i(t) \exp \left( -j\psi_i^t(t) \right) x_i^t(t) \exp \left( -j2\pi f_c t \right) h_T^*(t-kT) dt. \]

Given the fact the the fading process \( f_i^t(t) \) occupies a considerably smaller bandwidth as
compared to the receiver matched filter \( h_T^*(-t) \), the above equation is transformed to
\[
\sum_{k=0}^{Z-1} c_k^* \hat{\xi}_j^i(kT) \exp \left( -j \psi_F^j(kT) \right) \int_{-\infty}^{\infty} x_i^j(t) \exp \left( -j 2\pi f_c t \right) h_T^*(t - kT) dt = \sum_{k=0}^{Z-1} c_k^* \hat{\xi}_j^i(kT) \exp \left( -j \psi_F^j(kT) \right) y_k^i \tag{C.1.13}
\]
with
\[
y_k^i = y^i(kT) \\
y^i(t) = \int_{-\infty}^{\infty} x_i^j(\tau) \exp \left( -j 2\pi f_c \tau \right) h_T^*(\tau - kT) d\tau = \sum_{l=0}^{Z-1} c_l \int_{-\infty}^{\infty} \xi_j^l(t) \exp \left( \psi_F^j(t) + j \eta \right) h_T(\tau - lT) h_T^*(\tau - t) d\tau + n^i(t)
\]
\[
e^{-j} \sum_{l=0}^{Z-1} c_l \xi_j^l(lT) \exp \left( j \psi_F^j(lT) \right) h(t - lT) + n^i(t)
\]
and
\[
n^i(t) = \int_{-\infty}^{\infty} n_{i\omega}^l(\tau) \exp \left( -j 2\pi f_c \tau \right) h_T^*(\tau - kT) d\tau \\
n_k^i = n^i(kT). \tag{C.1.14}
\]

Note that the samples \( y^i(t) \) can be derived, for example, by downconverting \( x_i^j(t) \) using a local oscillator locked in frequency with the incoming signal\(^1\), and then passing it through a receiver filter \( H_R(f) \), which is matched to \( H_T(f) \). Finally, by straightforward substitution of Eqs. (C.1.12), (C.1.13) and (C.1.14) into Eq. (C.1.7), the metric to be maximized, as given in Eq. (4.59), is easily obtained.

\(^1\)By "locked in frequency" we imply a local oscillator generating the waveform \( \exp(-j2\pi f_c t) \). During this downconversion, the rapid phase and amplitude changes caused by fading, appear at the output of the demodulator totally uncompensated.
APPENDIX C.

C.2 Derivation of Eqs. (4.60) and (4.61)

When estimates $\hat{R}_P(t)$ for the $c_i(t)$ of the signal strength are available, the maximum likelihood sequence estimation receiver must maximize the pdf $f \left( x_1^*(t), \ldots, x_n^*(t) \mid x_B \left[ \bar{C}(A), t \right], \hat{R}_P(t) \right)$. This function can be calculated from Eq. (C.1.1), by integrating with respect to all $c_i^*(t) = \eta + \psi_i^*(t)$. This integration is over the interval $[0, 2\pi)$ where $c_i^*(t)$ is uniformly distributed. Also, since $c_i^*(t)$, $c_j^*(t)$ ($i \neq j$) have also been assumed to be independent random processes, then $c_i^*(t)$, $c_j^*(t)$ will also be independent. This yields

$$f \left( x_1^*(t), \ldots, x_n^*(t) \mid x_B \left[ \bar{C}(A), t \right], \hat{R}_P(t) \right) = \prod_{i=1}^{\Lambda} \frac{1}{2\pi} \int_0^{2\pi} f \left( x_1^*(t), \ldots, x_n^*(t) \mid x_B \left[ \bar{C}(A), t \right], \hat{R}_P(t), \psi_i^*(t) \right) d\psi_i^*(t)$$

Using a procedure identical to the one followed in Appendix C.1, eliminating all the terms in Eq. (C.2.1) which are independent from the transmitted sequence $\bar{C}(A)$, results to Eq. (4.60).

In the case where no estimates for the fading interference envelope $c_i^*(t)$ or the total random phase shift $c_i^*(t)$ is available at the receiver, the maximum likelihood sequence estimation receiver must maximize the pdf $f \left( x_1^*(t), \ldots, x_n^*(t) \mid x_B \left[ \bar{C}(A), t \right] \right)$. Since we have assumed $\eta$ to be uniformly distributed within $[0, 2\pi)$, the pdf to be maximized can be rewritten as

$$f \left( x_1^*(t), \ldots, x_n^*(t) \mid x_B \left[ \bar{C}(A), t \right] \right) = \prod_{i=1}^{\Lambda} \frac{1}{2\pi} \int_0^{2\pi} f \left( x_1^*(t), \ldots, x_n^*(t) \mid x_B \left[ \bar{C}(A), t \right], \eta \right) d\eta$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \left\{ \prod_{i=1}^{\Lambda} K_c \exp \left[ -\frac{1}{2N_0} \int_{-\infty}^{\infty} \left| x_i^*(t) \right|^2 dt \right] \right. \times \exp \left[ -\frac{1}{2N_0} \int_{-\infty}^{\infty} \left| x_B \left[ \bar{C}(A), t \right] \right|^2 dt \right] \times \exp \left[ \frac{1}{2N_0} \int_0^{\infty} \left. x_i^*(t) x_B \left[ \bar{C}(A), t \right] \exp (-j2\pi f_c t - j\eta) + \right. \right. \left. [x_i^*(t)]^* x_B \left[ \bar{C}(A), t \right] \exp (j2\pi f_c t + j\eta) \right] dt \} \right. d\eta.$$
Using a procedure identical to the one followed in Appendix C.1, eliminating all the terms in Eq. (C.2.2) which are independent from the transmitted sequence \( \overline{C(A)} \), results to Eq. (4.61).
C.3 Derivation of Eq. (4.68)

Let us assume that $\overline{A}^\nu$ is the transmitted information sequence, and that the signal sequence $\overline{C}(\overline{A}^\nu)$ corresponding to $\overline{A}^\nu$, is compared to the signal sequence $\overline{C}(\overline{A}^\zeta)$, corresponding to $\overline{A}^\zeta$, $\overline{A}^\zeta$ being one of the remaining possible information sequences of length $Z$. Since we have assumed no knowledge of channel state information, the maximum likelihood sequence estimation receiver will use the metric function of Eq. (4.61), and will base its decision on which one of the two sequences $\overline{C}(\overline{A}^\nu)$ and $\overline{C}(\overline{A}^\zeta)$ is more likely to have been transmitted on the following comparison

$$
\sum_{i=1}^{\Lambda} |F_{Z,i}^\nu (\overline{y}, \overline{C}(\overline{A}^\nu))|^2 \geq \sum_{i=1}^{\Lambda} |F_{Z,i}^\zeta (\overline{y}, \overline{C}(\overline{A}^\zeta))|^2
$$

(C.3.1)

with

$$
F_{Z,i}^\nu (\overline{y}, \overline{C}(\overline{A}^\nu)) = \sum_{k=0}^{Z-1} y_k^i (c_k^\nu)^* = e^{i\eta f^i (kT)} \sum_{k=0}^{Z-1} c_k^\nu (c_k^\nu)^* + \sum_{k=0}^{Z-1} n_k (c_k^\nu)^*. \quad (C.3.2)
$$

In the above equation, $1 \leq i \leq \Lambda$, and $c_k^\nu$ represents the value the symbol $c_k$ would have, if the sequence $\overline{A}^\nu$ was the one transmitted. $F_{Z,i}^\nu (\overline{y}, \overline{C}(\overline{A}^\nu))$, $F_{Z,i}^\zeta (\overline{y}, \overline{C}(\overline{A}^\zeta))$ can be expressed as

$$
F_{Z,i}^\nu (\overline{y}, \overline{C}(\overline{A}^\lambda)) = E_{\lambda}^{\nu,i} + N_{[\nu, \lambda]}^{F,i} + N_{[\nu, \lambda]}^{G,i} \quad (C.3.3)
$$

where $\lambda$ can be either $\nu$, i.e., $\zeta (\lambda \in \{\nu, \zeta\})$ and

$$
E_{\lambda}^{\nu,i} = e^{i\eta \sqrt{\frac{K}{K+1}}} \sum_{k=0}^{Z-1} c_k^\nu (c_k^\nu)^*, \quad (C.3.4)
$$

$$
N_{[\nu, \lambda]}^{F,i} = e^{i\eta \sum_{k=0}^{Z-1} (f_k^i)^* c_k^\nu (c_k^\nu)^*}, \quad (C.3.5)
$$

$$
N_{[\nu, \lambda]}^{G,i} = e^{i\eta \sum_{i=0}^{Z-1} n_i (c_i^\nu)^*} \quad (C.3.6)
$$

with $(f_k^i)^* = f_k^i - \overline{f}_{T,i}$. Assuming that the total signal power on each diversity channel is normalized to 1, i.e.,

$$
(\overline{f}_{T,i})^2 + (\sigma_{u,i})^2 = 1 \Rightarrow \overline{f}_{T,i} = \sqrt{\frac{K}{K+1}}, \quad (C.3.7)
$$
and that the Rician $K$-factor is the same in all $\Lambda$ diversity channels. Furthermore, assuming that the operating SNR is the same for all diversity paths, we have consequently that

$$
\overline{E}^{\nu,i}_{\lambda} = E^{\nu}_{\lambda}, \quad N^{F,i}_{[\nu,\Lambda]} = N^{F}_{[\nu,\Lambda]}, \quad \text{and} \quad N^{G,i}_{[\nu,\Lambda]} = N^{G}_{[\nu,\Lambda]} \quad \forall \quad i.
$$

(C.3.8)

Because of the zero mean complex Gaussian nature of $f_k^i$ and $n_k^i$, $F_{Z}^\nu \left( \overline{y}, \overline{C}(\Lambda)^{\lambda} \right)$ is a complex Gaussian random variable with average $\overline{E}^{\nu}_{\lambda}$ and variance

$$
\mu_{\nu,\lambda} = E \left\{ \left| F_{Z}^{\nu} \left( \overline{y}, \overline{C}(\Lambda)^{\lambda} \right) - \overline{E}^{\nu}_{\lambda} \right|^2 \right\} = E \left\{ \left| N^{F,i}_{[\nu,\Lambda]} + N^{G,i}_{[\nu,\Lambda]} \right|^2 \right\}
$$

$$
= R_{0}^{F} \sum_{k=0}^{Z-1} \left| c_{k}^{\nu} \right|^2 \left| c_{k}^{\lambda} \right|^2 + 2 \sum_{k=1}^{Z-1} R_{k}^{F} \sum_{l=k}^{Z-1} \text{Re} \left[ c_{k}^{\nu} \left( c_{l-k}^{\nu} \right)^* \left( c_{k}^{\lambda} \right)^* c_{l-k}^{\lambda} \right] + \sigma_{n}^{2} \sum_{k=0}^{Z-1} \left| c_{k}^{\nu} \right|^2
$$

$$
= R_{0}^{F} \left[ Z + \sum_{k=1}^{Z-1} 2 \rho_{k}^{F} \sum_{l=k}^{Z-1} \cos \left( \Delta \Theta_{k}^{\nu}(l) - \Delta \Theta_{k}^{\lambda}(l) \right) \right] + \sigma_{n}^{2} Z
$$

(C.3.9)

where $R_{k}^{F} = R_{F}(kT)$, and $\rho_{k}^{F} = R_{k}^{F} / R_{0}^{F}$. The cross covariance between $F_{Z}^{\nu} \left( \overline{y}, \overline{C}(\Lambda)^{\nu} \right)$ and $F_{Z}^{\nu} \left( \overline{y}, \overline{C}(\Lambda)^{\nu} \right)$ equals

$$
\mu_{\nu,\lambda} = R_{0}^{F} \sum_{k=0}^{Z-1} \left| c_{k}^{\nu} \right|^2 \left( c_{k}^{\nu} \right)^* c_{k}^{\lambda}
$$

$$
+ \sum_{k=1}^{Z-1} R_{k}^{F} \sum_{l=k}^{Z-1} \left\{ \left| c_{l-k}^{\nu} \right|^2 \left( c_{l-k}^{\nu} \right)^* c_{l-k}^{\lambda} + \left| c_{l-k}^{\nu} \right|^2 \left( c_{l}^{\nu} \right)^* c_{l}^{\lambda} \right\}
$$

$$
+ \sigma_{n}^{2} \sum_{k=0}^{Z-1} \left( c_{k}^{\nu} \right)^* c_{k}^{\nu}
$$

$$
= R_{0}^{F} \left[ \sum_{k=0}^{Z-1} \left( c_{k}^{\nu} \right)^* c_{k}^{\lambda} + \sum_{k=1}^{Z-1} \rho_{k}^{F} \sum_{l=k}^{Z-1} \left\{ \left( c_{l-k}^{\nu} \right)^* c_{l-k}^{\lambda} + \left( c_{l}^{\nu} \right)^* c_{l}^{\lambda} \right\} \right] + \sigma_{n}^{2} \sum_{k=0}^{Z-1} \left( c_{k}^{\nu} \right)^* c_{k}^{\nu}.
$$

(C.3.10)
We define the normalized covariance and crosscovariance as

\[ u_{\nu, \lambda} \triangleq \frac{\mu_{\nu, \lambda}}{\sigma_n^2} \]

\[ = \frac{\text{SNR}}{K + 1} \left\{ Z + \sum_{k=1}^{Z-1} 2 \rho_k^F \sum_{l=k}^{Z-1} \cos \left[ \Delta \Theta_k^\nu(l) - \Delta \Theta_k^\lambda(0) \right]\right\} + Z \]  

(C.3.11)

\[ u_{\nu, \zeta}^c \triangleq \frac{\mu_{\nu, \zeta}}{\sigma_n^2} \]

\[ = \frac{\text{SNR}}{K + 1} \left\{ \sum_{k=0}^{Z-1} (c_k^\nu)^* c_k^\zeta + \sum_{k=1}^{Z-1} \rho_k^F \sum_{l=k}^{Z-1} \left[ (c_l^\nu - k)^* c_l^\nu - k + (c_l^\nu)^* c_l^\nu \right]\right\} + \sum_{k=0}^{Z-1} (c_k^\nu)^* c_k^\zeta \]  

(C.3.12)

where we have used

\[ \frac{R_0^F}{\sigma_n^2} = \frac{1}{\sigma_n^2} \frac{1}{(K + 1)} = \frac{\text{SNR}}{K + 1}. \]  

(C.3.13)

Using the material in Appendix 4.B of [14, pp.223-228], substituting \( A = 1 \), \( B = -1 \) and \( C = 0 \) in Eq. (4B.1) of [14], we find the following expression for the \( P_A(\{\overline{C}(A^\nu) \leftarrow \overline{C}(A^\nu)\}) \).

For \( \alpha_{[\nu, \zeta]} > 0 \),

\[ P_A(\overline{C}(A^\zeta) \leftarrow \overline{C}(A^\nu)) = Q_f \left( \frac{1}{\sigma_n} \alpha_{[\nu, \zeta]}, \frac{1}{\sigma_n} \beta_{[\nu, \zeta]} \right) \]

\[ - \int_0^1 \left[ \frac{1}{\sigma_n} \alpha_{[\nu, \zeta]} \frac{1}{\sigma_n} \beta_{[\nu, \zeta]} \right] \exp \left[ - \frac{\alpha_{[\nu, \zeta]}^2 + \beta_{[\nu, \zeta]}^2}{2 \sigma_n^2} \right] \]

\[ \times \frac{(1 + D)^\Lambda (1 - D)^{\Lambda - 1}}{2^{2\Lambda - 1} \sum_{l=0}^{\Lambda - 1} \left( 2\Lambda - 1 \right)} \left( 1 + D \right)^{2\Lambda - 1} \left( 1 - D \right)^{-t} \]

\[ + \exp \left[ - \frac{\alpha_{[\nu, \zeta]}^2 + \beta_{[\nu, \zeta]}^2}{2 \sigma_n^2} \right] \]

\[ \times \sum_{m=1}^{\Lambda - 1} I_m \left( \frac{1}{\sigma_n} \alpha_{[\nu, \zeta]} \frac{1}{\sigma_n} \beta_{[\nu, \zeta]} \right) \]

\[ \times \left\{ \gamma_m \left[ 1 - \frac{(1 + D)^{\Lambda - m} (1 - D)^{\Lambda + m - 1}}{2^{2\Lambda - 1} \sum_{l=0}^{\Lambda - m} \left( 2\Lambda - 1 \right)} \left( 1 + D \right)^{2\Lambda - 1} \left( 1 - D \right)^{-t} \right] \right\} \]  

(C.3.14)
\[
- \frac{1}{2^m} \left[ 1 - \frac{(1 + D)^{\Lambda + m - 1}(1 - D)^{\Lambda - m}}{2^{2\Lambda - 1}} \sum_{l=0}^{\Lambda - m - 1} \left( \frac{2\Lambda - 1}{\Lambda + l - m} \right) (1 + D)^{-l}(1 - D)^l \right] \times \left[ 1 - \delta(\Lambda - 1) \right].
\]

In the above equation, \( Q_f(x, y) \) is Marcum's Q function [32, p. 585] and
\[
\left\{ \begin{array}{c}
\alpha_{\nu, \zeta} \\
\beta_{\nu, \zeta}
\end{array} \right\} = \left[ \frac{\Lambda}{\nu_\nu + \nu_\zeta} \left( \left| E_\nu^\nu \right|^2 + \left| E_\zeta^\nu \right|^2 - 2\text{Re} \left\{ \left( E_\nu^\nu \right)^* E_\zeta^\nu \gamma_{\nu, \zeta} \right\} \right) \right]^{1/2}
\]

where
\[
\gamma_{\nu, \zeta} = \frac{2\nu_\zeta}{(\nu_\nu + \nu_\zeta)},
\]
\[
D = \frac{\nu_\nu - \nu_\zeta}{\sqrt{(\nu_\nu + \nu_\zeta)^2 - 4 \left| \nu_\zeta \right|^2}}.
\]

and
\[
\mathcal{Y} = \frac{\left| E_\nu^\nu \right|^2 + \left| E_\zeta^\nu \right|^2 - 2\text{Re} \left\{ \left( E_\nu^\nu \right)^* E_\zeta^\nu \gamma_{\nu, \zeta} \right\} + \left| E_\nu^\nu \right|^2 - \left| E_\zeta^\nu \right|^2}{1 - \left| \gamma_{\nu, \zeta} \right|^2} \frac{\sqrt{1 - \left| \gamma_{\nu, \zeta} \right|^2}}{\sqrt{\left| E_\nu^\nu \right|^2 + \left| E_\zeta^\nu \right|^2 - 2\text{Re} \left\{ \left( E_\nu^\nu \right)^* E_\zeta^\nu \gamma_{\nu, \zeta} \right\} - \left| E_\nu^\nu \right|^2 - \left| E_\zeta^\nu \right|^2}}.
\]

For \( \alpha_{\nu, \zeta} = 0 \), Eq. (C.3.14) simplifies to
\[
P_\Lambda \left( \overline{C}(\overline{\Lambda}^\nu) \leftarrow \overline{C}(\overline{\Lambda}^\nu) \right) = \exp \left[ -\frac{\beta_{\nu, \zeta}}{2\sigma_n^2} \right] \left[ 1 - \frac{1}{2^{2\Lambda - 1}} \sum_{l=0}^{\Lambda - 1} \left( \frac{2\Lambda - 1}{\Lambda + l} \right) \right].
\]

For the case where diversity is not used, i.e., for \( \Lambda = 1 \), the pairwise error probability can be expressed as
\[
P_1 \left( \{ \overline{C}(\overline{\Lambda}^\nu) \leftarrow \overline{C}(\overline{\Lambda}^\nu) \} \right) = Q_f \left( \frac{1}{\sigma_n} \alpha_{\nu, \zeta}, \frac{1}{\sigma_n} \beta_{\nu, \zeta} \right) - \frac{1}{2} \left[ 1 + \frac{\nu_{\nu, \nu} - \nu_{\nu, \zeta}}{(\nu_{\nu, \nu} + \nu_{\nu, \zeta})^2 - 4 \left| \nu_{\nu, \zeta} \right|^2} \right]^{1/2} \times
\]
\[ \times I_0 \left( \frac{1}{\sigma_n} \alpha_{[\nu,\zeta]} \frac{1}{\sigma_n} \beta_{[\nu,\zeta]} \right) \exp \left( -\frac{1}{\sigma_n^2} \frac{\alpha_{[\nu,\zeta]}^2 + \beta_{[\nu,\zeta]}^2}{2} \right) \]  

(C.3.19)

It is clear that the expression provided by Eq. (C.3.14) is quite general, providing us with the means of evaluating the performance of maximum likelihood sequence estimation receivers using the MDD hardware structure, and operating under any SNR value and Rician fading \( K \)-factor. However, for some cases commonly encountered in practical communication systems, the expression provided by Eq. (C.3.14) can be simplified further. More specifically, we shall examine the cases of

- Rician fading with strong direct signal component at high SNR, and
- Rayleigh fading, i.e., no LOS signal component.

### C.3.1 Rician Fading, Strong LOS Component and High SNR

Under these fading channel conditions, the value of \( K/\left[ 1 + \frac{\Lambda(K+1)}{\text{SNR}} \right] \) is quite large. This makes the terms \( \left( \frac{1}{\sigma_n} \alpha_{[\nu,\zeta]} \right), \left( \frac{1}{\sigma_n} \beta_{[\nu,\zeta]} \right) \) quite large, and permits the use of the following approximations for the \( Q_f(x, y) \) and \( I_m(z) \) functions [33]

\[
Q_f(x, y) \approx \begin{cases} 
\exp \left( -\frac{y^2}{2} \right) & \text{for } x = 0 \\
(1/\sqrt{2\pi xy}) \exp \left( -\frac{\|y-x\|^2}{2} \right) & \text{for } x > 0,
\end{cases}
\]

\[
I_m(z) = \frac{\exp(|z|)}{\sqrt{2\pi |z|}} G_m(z) \]  

(C.3.21)

where

\[
G_m(z) = \begin{cases} 
1 & \text{for } m = 0 \\
\prod_{i=1}^{l-1} \left( 4m^2 - (2i - 1)^2 \right) / \left( (l)! (8|z|)^l \right) & \text{for } m \neq 0.
\end{cases}
\]  

(C.3.22)
Substituting these approximations in Eqs. (C.3.14), (C.3.18), yields the following expression for $P_D(\{\overline{C}(\overline{A}) \leftarrow \overline{C}(\overline{A}')\})$

$$P_D(\{\overline{C}(\overline{A}) \leftarrow \overline{C}(\overline{A}')\}) = \exp \left( - \frac{\alpha_{\nu,\zeta}^2 + \beta_{\nu,\zeta}^2}{2\sigma_n^2} \right) \Gamma_{\nu,\zeta}^{Z,A}. \quad (C.3.23)$$

In the above equation, for $\alpha_{\nu,\zeta} > 0$,

$$\Gamma_{\nu,\zeta}^{Z,A} = \frac{1}{\sqrt{2\pi \frac{1}{\sigma_n} \alpha_{\nu,\zeta} \beta_{\nu,\zeta}}} \times \begin{cases} 1 - \exp \left[ \frac{\alpha_{\nu,\zeta} \beta_{\nu,\zeta}}{\sigma_n^2} \right] \frac{(1 + D)^A (1 - A + 1)}{2^{2A-1}} \left( \sum_{l=0}^{A-1} \left( \frac{2A - 1}{\Lambda + l} \right) (1 + D)^l (1 - D)^{-l} \right) \\ + \exp \left[ \frac{\alpha_{\nu,\zeta} \beta_{\nu,\zeta}}{\sigma_n^2} \right] \sum_{m=1}^{A-1} G_m \left( \frac{\alpha_{\nu,\zeta} \beta_{\nu,\zeta}}{\sigma_n^2} \right) \times \begin{cases} 1 - \frac{(1 + D)^{A-m} (1 - D)^{A+m-1}}{2^{2A-1}} \sum_{l=0}^{A+m-1} \left( \frac{2A - 1}{\Lambda + l - m} \right) (1 + D)^l (1 - D)^{-l} \\ - \frac{1}{\nu_m} \left[ 1 - \frac{(1 + D)^{A+m-1} (1 - D)^{A-m}}{2^{2A-1}} \sum_{l=0}^{A+m-1} \left( \frac{2A - 1}{\Lambda + l - m} \right) (1 + D)^l (1 - D)^{-l} \right] \right) \end{cases} \right) \times \left[ 1 - \delta(\Lambda - 1) \right]. \quad (C.3.24)$$

For $\alpha_{\nu,\zeta} = 0$, the expression for $P_D(\{\overline{C}(\overline{A}) \leftarrow \overline{C}(\overline{A}')\})$ is identical to the one in Eq. (C.3.18).

**C.3.2 Rayleigh Fading Channels**

For a Rayleigh fading channel, $K \rightarrow -\infty$ dB, i.e., $K \rightarrow 0$, which leads to $K/\left(1 + \frac{K+1}{\text{SNR}}\right) \rightarrow 0$.

Under such channel conditions, from Eq. (C.3.4) we have that $E_{\nu,i} = 0$, and from Eq. (C.3.15)
it is easy to verify that $\alpha_{\nu,\xi} = \beta_{\nu,\xi} = 0$. Using the identities $Q_f(0,0) = 1$, $I_0(0) = 1$ and $I_m(0) = 0$ for $m > 0$ [33], in Eq.(C.3.14) yields

$$P_A \left( \{ \overline{C}(A^i) \leftarrow \overline{C}(A^\nu) \} \right) = 1 - \frac{(1 + D)^A (1 - D)^{A-1}}{2^{2A-1}} \sum_{i=0}^{A-1} \left( \begin{array}{c} 2A - 1 \\ A + l \end{array} \right) (1 + D)^i (1 - D)^{l-i}.$$  

(C.3.25)

For very slow Rayleigh fading, no diversity ($A = 1$), a BPSK signal and for $Z = 2$, which corresponds to the conventional differential detector, the pairwise error event probability simplifies to

$$P_1 \left( \{ \overline{C}(A^i) \leftarrow \overline{C}(A^\nu) \} \right) = \frac{1}{2} \frac{1}{\text{SNR} + 1}$$  

(C.3.26)

which is identical to the result given in [14, Eq. 7.3.10].
C.4 Derivation of Eq. (4.80)

The channel model parameters for a mobile-satellite radio channel as found in [34, 35] are based on data collected by propagation experiments in the UHF and L (1.5 GHz) frequency bands. Since experimental data for the Ka-band is not presently available, we use a parameter scaling technique based on the modified exponential decay (MED) model developed from static propagation measurements through deciduous trees [105]. The attenuation coefficient in dB/m can then be computed by the following equation [106]

\[
\alpha_F \simeq 0.45 F^{0.284} \quad \text{for } 0 \leq d_T \leq 14
\]
\[
\alpha_F \simeq 1.33 F^{0.284} (d_T)^{-0.412} \quad \text{for } 14 \leq d_T \leq 400
\]  

(C.4.1)

where \( F \) is the operating frequency in GHz and \( d_T \) the depth of trees in meters (m) intercepted by the LOS signal. Since in the case of satellite communications having a depth of trees of more than 14 m would be a rather rare occurrence, we use the first of the equations above for attenuation scaling. According to the log-normal model [34], the attenuation in signal amplitude at UHF and L-band frequencies is

\[
A = \exp(x)
\]  

(C.4.2)

with \( x \) a Gaussian random variable with mean \( \mu \) and variance \( d_0 \) given by Table 4.4. The attenuation scaling factor of Eq. (C.4.1) can be used to compute the new values of \( \mu \) and \( d_0 \) as follows. The signal power at a frequency \( F \) can be written as

\[
A_F^2 [\text{dB}] = \alpha_F \times A^2 [\text{dB}]
\]  

(C.4.3)

and taking the exponent of base 10 for both sides in the above equation,

\[
A_F^2 = (A^2)^{\alpha_F} \Rightarrow \\
A_F = A^{\alpha_F} = (\exp(x))^{\alpha_F} = \exp(\alpha_F x).
\]  

(C.4.4)
Consequently, calculation of the new mean $\mu$ and standard deviation $\sqrt{d_0}$ involves multiplying with the scaling factor $\alpha_F$. For the frequency of approximately 21 GHz used for the simulation results presented in Chapter 4, the scaling factor is approximately equal to 2.1, and the resulting parameters are shown in Table C.5. Using Eq. (4.81), the ratio of LOS signal average power to multipath signal power can be calculated for both the L-band and the EHF Ka-band. Results of this calculation are presented for comparison in Table C.6. As intuitively expected, in the case of heavy shadowing, the results in the above table show a considerable decrease of about 30 dB in $K_{AVG}$ at the 20 GHz Ka-band, as compared to that for the 1.5 GHz L-band.

### Table C.5: Ka-band satellite channel model parameters.

<table>
<thead>
<tr>
<th>L-band channel</th>
<th>$b_0$</th>
<th>$\mu$</th>
<th>$\sqrt{d_0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>light shadowing</td>
<td>0.158</td>
<td>0.2415</td>
<td>0.2415</td>
</tr>
<tr>
<td>average shadowing</td>
<td>0.126</td>
<td>-0.2415</td>
<td>0.3381</td>
</tr>
<tr>
<td>heavy shadowing</td>
<td>0.0631</td>
<td>-8.211</td>
<td>1.6926</td>
</tr>
</tbody>
</table>

### Table C.6: Ratio of LOS signal average power to multipath signal power ($K_{AVG}$); comparison between L and Ka bands.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Band</th>
<th>dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>light shadowing</td>
<td>L</td>
<td>6.12</td>
</tr>
<tr>
<td></td>
<td>Ka</td>
<td>7.34</td>
</tr>
<tr>
<td>average shadowing</td>
<td>L</td>
<td>5.21</td>
</tr>
<tr>
<td></td>
<td>Ka</td>
<td>4.36</td>
</tr>
<tr>
<td>heavy shadowing</td>
<td>L</td>
<td>-19.33</td>
</tr>
<tr>
<td></td>
<td>Ka</td>
<td>-50.48</td>
</tr>
</tbody>
</table>
C.5 Derivation of Eq. (4.81)

Calculation of the total average power of the LOS signal component amounts to calculating $E\{y\}$, where, $y = \exp(2x)$, and $x$ is a Gaussian random variable with variance $d_0$ and mean $\mu$. Using the well known theorem for calculating the expected value of a function of a random variable with known pdf [98], the average $E\{y\}$ can be written as

$$E\{y\} = \int_{-\infty}^{\infty} \exp(2x) \frac{1}{\sqrt{2\pi d_0}} \exp \left( -\frac{(x - \mu)^2}{2d_0} \right) \, dx. \quad (C.5.1)$$

Rearranging the argument of $\exp(\cdot)$ inside the above integral, $E\{y\}$ becomes

$$E\{y\} = \frac{1}{\sqrt{2\pi d_0}} \int_{-\infty}^{\infty} \exp \left\{ - \left[ \frac{1}{2d_0} x^2 - \frac{\mu + 2d_0}{d_0} + \frac{\mu^2}{2d_0} \right] \right\} \, dx. \quad (C.5.2)$$

Finally, making use of the following integral expression [33]

$$\int_{-\infty}^{\infty} \exp \left[ -(ax^2 + bx + c) \right] \, dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} \exp \left[ \frac{b^2 - 4ac}{4a} \right] \quad (C.5.3)$$

and by substituting $a = 1/(2d_0)$, $b = - (\mu + 2d_0)/d_0$ and $c = \mu^2/(2d_0)$, Eq. (4.81) results.