A LOW COST THREE-DIMENSIONAL VISION SYSTEM
USING SPACE-ENCODED SPOT PROJECTION
BY
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to the required standard

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Vision support plays an important role in intelligent robotic systems. It is often necessary to first recognize the location and the shape of the work piece before the robot arm can be directed to perform various tasks on it. A low cost three-dimensional (3-D) vision system based upon the space-encoded spot projection technique has been developed to measure the shape of a smooth, featureless curved surface.

The vision system projects a 32x32 array of spots onto the measurement surface and uses a binary space-encoding scheme to incorporate the column address of these projection spots in a series of five projection patterns. Binary thresholding is implemented to extract the spot features in the images of the projection patterns. The image centroid positions and the decoded column addresses of the spot features, as well as the transformation matrices of the camera and slide projector are used to compute the spatial coordinates of the projection spots.

This thesis establishes a calibration procedure to derive the transformation matrix of the camera using the measured spatial and image coordinates of a set of calibration image feature points. A similar calibration procedure is also established for the derivation of the
transformation matrix of the slide projector. The accuracy of these matrices are substantially improved due to the utilization of the random sample consensus (RANSAC) algorithm to eliminate the gross error data points in the sample populations and allow only the good samples to be used in the derivation of the final transformation matrices.

A surface reflectance model and a image centroid separation models are developed to describe the image intensity and spacings of the projection spots as a function of the measurement surface orientation. Using these models, the limiting orientations of a measurement surface with respect to the optical configuration of the vision system were found.

Three reconstruction algorithms for fitting a surface to the scattered amplitude samples obtained by the spot projection system are described in the thesis. The suitability of these algorithms for the measurement of aircraft-wing surfaces is also discussed.

The shape measurement results on a cylindrical surface showed that the average measurement error of points on a smooth featureless surface is less than a tenth of an inch, and the maximum error is less than a quarter of an inch in a total field of 20x20 inches.
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CHAPTER 1

INTRODUCTION

In recent years, intelligent robotic systems have been widely used in industrial applications, such as pick-and-place operations in the assembly lines [Kelly et al. 1981] and seam welding [Schmidt 1984], etc. Vision support plays an important role in these systems. It is often necessary to first recognize the location and the shape of the work piece before the robot arm can be directed to perform various tasks on it. Such systems are ideal to replace human operation in hazardous working environments and in tedious and repetitious functions. With the vision-sensing capability, the robotic systems can be programmed to recognize certain types of pre-defined adverse conditions, like the intrusion of a foreign object into the work space, so the appropriate responses can be taken.

The nondestructive evaluation (NDE) group at Defence Research Establishment Pacific (DREP) in Victoria, B.C. is actively involved in a development program to automate the ultrasonic inspections on the CF-18 aircraft. As shown in Figure 1.1, 40% of the external skin of the CF-18 is made of graphite-epoxy material. After the aircraft has been in service for a number of years, flaws, such as disbonds, porosity, delamination, etc., may develop on these materials and the structural integrity of the aircraft would be affected by these problems [Sturrock 1983]. These flaws can
Figure 1.1 CF-18 materials distribution.
(Courtesy of McDonnell Douglas, St. Louis)
be detected by the changes in the signal reflectance characteristics of the materials in an ultrasonic inspection process. Hence, a routine ultrasonic inspection program is needed for detecting these problems at the early stage before any further damage develops. Furthermore, the graphite-epoxy material can also be damaged due to impacts from sources such as ground servicing and maintenance equipment or runway debris. The ultrasonic inspection procedure is also used to assess the extent of the damage and to locate flaws which are not recognizable by visual inspections.

Presently, the inspection process is done manually by an inspector who holds an ultrasonic transducer against the materials and performs a raster scan operation over the problematic areas. Simultaneously, he must monitor the characteristics of the signal displayed on an oscilloscope for signal anomalies. The inspection task is very tedious and requires a high degree of hand/eye coordination. It prompted a requirement for an automated inspection system to be developed for this application. The first version of the robot inspection system is mainly for the inspection of the materials on the wing and stabilator surfaces. In order for the robot manipulator to guide the ultrasonic transducer along a surface, the three-dimensional (3-D) shape of the surface must be acquired prior to the inspection process. The overall objective of the thesis is to develop a vision system for measuring the 3-D shape of object surfaces in a laboratory environment, and to apply the experience and
insights gained from this prototyping vision system to the upcoming aircraft wing measurement application.

With the exception of a few fastening devices and identification markings on the wings and stabilators, these surfaces are basically considered to be smooth and featureless. Having defined the underlying properties of these specific measurement surfaces, the next step is to find the most appropriate technique for measuring the 3-D profile of aircraft-wing shaped surfaces.

Range finding techniques used in 3-D vision systems are divided into two broad categories, direct and image based methods. The direct method uses the time of flight [Polaroid] or the phase-related effects [Nitzan et al. 1977] to measure distances to the surface. The ultrasonic transducer available from Polaroid for the time of flight measurement system, exhibit a beam pattern with a major forward lobe of about 30° solid angle which translates into a 2.7 feet square cross sectional area at a working distance of 10 feet. Nitzan et al. [1977] used the phase shift of the laser pulses to obtain the range information. These methods are both quite time consuming and the equipment cost tends to be expensive. Because of the coaxial source/detector arrangements, these methods are not subject to the problem of "missing parts" as in the image-based triangulation methods. However, they are not commonly used because of these other disadvantages.

The image-based range finding techniques are further subdivided into two groups, passive and active methods.
Passive techniques consists of photometric [Woodham 1981] and stereopsis methods [Baker 1982, Grimson 1981 1985, Clark 1985]. The photometric method works best in finding the surface gradient and it does not measure the actual distances to the measurement surface. Stereopsis method involves a stereo pair of images and matching algorithms are used to identify the corresponding surface points in the stereo images. Geometric triangulation is then used to determine the range information of these surface points from their stereo disparities. Most of the processing time is used in the stereo matching process. Baker [1982] dealt with the matching problem by using line by line edge correlation procedures on the edge data of the stereo image pair to find the best correspondences in the left and right images of the scene. Grimson [1981] implemented the multi-resolution matching algorithm proposed by Marr and Poggio [1979], which involves the matching of the zero crossings of the $\nabla^2 G$ filtered stereo images at four different spatial resolution. The zero-crossing matches in the low resolution images are used to guide the matchings in the finer resolutions. The zero-crossing matching operation was only performed along the epipolar lines. In this case, the epipolar lines coincide with the horizontal scan lines of the stereo images. In a more recent paper, Grimson [1985] suggested the matching should be perform at the finest resolution and use the disparity scan at the lower resolutions for disambiguation of the possible match points. Clark [1985] proposed another matching scheme on the multi-resolution $\nabla^2 G$. 
filtered images. This scheme relies on the disparity function of one resolution level to provide a disparity estimate to help the matching at the next finer resolution level. All these stereopsis methods have had good success for scenes with complicated features, however, due to the lack of edge image features, they will have poor performance for scenes of smooth featureless curved surfaces.

The other group of image-based range finding technique is known as active stereo or structured light methods. This approach is the basis for the majority of the 3-D machine vision systems in use today [Corby 1983]. A survey article by Jarvis [1983] provides a good summary of the papers published on the various aspects on this topic. In this approach, structured light patterns, such as stripe lighting [Agin and Binford 1973, Sato et al. 1982], grid lighting [Hall et al. 1981, Le Moigne and Waxman 1984] and spot pattern [Altschuler et al. 1979, Posdamer and Altschuler 1982] are projected onto the scene. The images of the light patterns can be processed to yield range information. Agin and Binford [1973] used a laser beam light source and a cylindrical lens to project a sheet of light onto the scene. The light stripe was then scanned across the scene in a step-wise manner by a rotating mirror. Instead of having a scanning light source, Sato [1982] placed the object on a turntable and rotated the object with respect to two stationary slit-ray laser projection systems located on either sides of the camera. At each incremental position, the projection system will take turns to illuminate the
object. If there is more than one light stripe in the scene, it would be difficult for the processing algorithm to identify the originating light source of the image features. Hall et al. [1981] projected a 5x6 grid onto the surface of an object and then took the images of the grid pattern from two different positions. Using an interactive image display program, the image coordinates of each grid vertex were located and recorded manually. From the image coordinates of the matching vertices, the spatial coordinates of the surface points were triangulated. This method is very tedious and time consuming, and is only suitable for small grid lattices.

It is necessary to match the originating light sources to the image positions of the feature points in order to triangulate the spatial coordinates of the points. Posdamer et al. [1982] proposed a vision system which projects a 128x128 array of laser beams onto a surface and used a binary space-encoding projection scheme to incorporate the address of the projection spots into a series of spot projection patterns. It allows 16,384 surface samples to be acquired by using only eight projection patterns. In addition to the speed improvement, there are no moving parts required in this method, thus, the control circuitry and inaccuracies associated with the mechanical scanning devices can also be eliminated. Inokuchi [1984] improved the space-encoding scheme by using Gray-coded spot projection patterns. Although implementation methods were discussed in the cited references, measurement results of
the methods have yet emerged from the literature.

**Spot projection method**

It was Posdamer's proposal that inspired the idea of using a space-encoded spot projection scheme for the 3-D vision system developed in this thesis. In order to keep the equipment cost of the vision system low, a standard slide projector is used to produce the spot projection patterns instead of the custom-made laser projection system as suggested by Posdamer. Since the spot projection rays created by the slide projector are more dispersive than the fine collimated laser beams, a smaller size array of 32x32 spots is used in the slide projector implementation. By using the binary spacing-encoding scheme, the 3-D spatial coordinates of all the 1024 projection spots on the measurement surface can be obtained in six projections. In a field of view of 20x20 inches, the spatial resolution of a 32x32 spot pattern is five-eighths of an inch. The spot projection structured light technique was chosen as the most suitable range finding method for smooth, featureless curved surfaces.

**Objectives**

The primary objective of this thesis is to develop a low cost 3-D vision system based upon the spot projection technique to measure the shape of smooth, featureless curved
surfaces. Because of the cost factor, only standard commercially available equipment is used in the implementation of the system. A cylindrical surface in the shape of an aircraft wing was specially manufactured by the machine shop in the Electrical Engineering Department as the test measurement surface of the vision system. The developmental process of the system was divided into a number of intermediate stages. The objectives of these intermediate stages were listed as follows:

1) to assemble the optical equipment required for the spot projection system;
2) to derive camera and slide projector models;
3) to determine the model parameters experimentally;
4) to develop an image processing algorithm for determining the spatial coordinates of the projection spot on the measurement surface;
5) to investigate the limiting measurement configuration of the spot projection system;
6) to study the surface reconstruction algorithms for fitting a surface to the scattered samples obtained from the spot projection patterns.

The description of these intermediate stages are presented in Chapters 2-7 of this thesis in the order as given in the list above. Chapter 8 summarizes the results of the study and offers some recommendations for future improvements.
CHAPTER 2

IMAGING SYSTEM

2.1 GENERAL DESCRIPTION OF THE IMAGING SYSTEM

Active stereo imaging, or structured light projection, is a widely adopted technique used in industrial vision systems for three-dimensional shape measurements. Highly illuminated light patterns are projected onto the object of interest. The light rays or planes from the projected patterns form bright spots or curves on the surface of the object. The three-dimensional shape of the object can then be triangulated from the image positions of these illuminated features in the scene. This is basically the imaging acquisition scheme utilized in the space-encoded spot imaging system.

Figure 2.1 shows the hardware configuration of the space-encoded spot projection system. The imaging system is comprised of an industrial grade Leitz slide projector and a MTI 512x512 pixel resolution video camera. The video camera is connected to an Imaging Technology Inc. image digitizer and frame buffer unit attached to a LSI-11/23 microcomputer using the RT-11 operating system. The digitized data are then transferred to the VAX-11/750 computer facility on floppy diskettes for further image analysis. A Ramtek 512x512 pixel resolution colour display monitor interfaced to the VAX computer is available for image viewing. Hard
Fig. 2.1 Basic block diagram of the space-encoded imaging system.
copy output of binary image data and 3-D wire-frame plots are available on the Printronix printer/plotter.

2.2 SPACE-ENCODED SPOT PROJECTION SCHEME

A reference pattern of 32x32 regularly spaced light spots as shown in Figure 2.2, is projected from the slide projector onto an object located at the center of the scene. Bright illuminated spots are created at the intersections of the projected beams and the object surface.

In order to triangulate the 3-D spatial coordinates of the surface points, the system must be able to identify both the position of the originating transparent windows on the slide and the camera image location of the illuminated spots. A
A binary space-encoding scheme is implemented to identify the position of the transparent windows on the slide.

A single projection of the reference mask does not contain sufficient information to identify the column addresses of the light beams. The binary address encoding scheme using a sequence of space-encoded spot projection patterns, as shown in Figure 2.3, has been implemented for this application. The projection pattern on the reference slide consists of 32 rows and 32 columns of regularly spaced transparent windows in an opaque background. As shown in Figure 2.4, the projector lens center and each column of transparent windows on the slide defines a light plane which produces a trail of spots on the object surface. The optical path of each received image spot on the camera defines a ray-line which connects the illuminated spot on the object surface to the camera center. The spatial coordinates of the surface point can be uniquely located by the triangulation method. Detailed derivation of the spatial coordinates from the image coordinates will be given in the subsequent chapters.

For an \(NxN\) spot projection array, \(\log_2(N)\) space-encoded projections together with the reference pattern are needed to perform the necessary column address decoding function. For instance, a \(32\times32\) spot projection array requires five space-encoded patterns in addition to the reference pattern to implement the binary coding scheme, whereas, a \(128\times128\) projection array requires only eight projections for the decoding process. Thus, a significant
Fig. 2.3  (a) Reference spot projection pattern.
(b) – (f) Binary space-encoded spot patterns.
increase in the surface point resolution can be achieved with only a small increase in the number of projections. However, increasing the projection spot density will lead to a smaller spot size and closer spacing between the spots. Consequently, the probability of an address decoding error in the image analysis process will be higher. The maximum spot density is determined by two factors: (1) the capability of the imaging system to resolve adjacent projection spots in the camera image; and (2) the sophistication of the error correction scheme in the address decoding process. The resolution and light sensitivity of the camera also play an important role in the overall performance of the measurement system.
2.3 BINARY SPACE-ENCODING SCHEME FOR THE SPOT COLUMN ADDRESSES

The purpose of space-encoding projection is to determine the column address of each illuminated spot generated on the reference projection. The address decoding process can be accomplished by monitoring the presence or absence of the spot on each of the space-encoded projections. Using a 32x32 spot matrix pattern as an example, the address of each column can be uniquely represented by a 5-bit binary number, \(a_4a_3a_2a_1a_0\), where \(a_4\) is the most significant bit and \(a_0\) is the least significant bit. Thus, the range of the address would be from \((00000)_2\) to \((11111)_2\).

The first space-encoded spot pattern is formed by turning on only those columns which have a "one" in the least significant bit of their address. The second pattern is formed by turning on the columns having a "one" in address bit \(a_1\). This arrangement continues up to the fifth pattern, which is formed by only those columns with a "one" in the most significant bit of their address. A picture of the reference projection pattern and the space-encoded patterns are depicted in Figure 2.3. By examining the projection patterns on the object in the order specified above, the presence or absence of an illuminated spot determines the value of the associated address bit for that projection pattern. A "one" is assigned to the address bit
if the spot is present and a "zero" is assigned if the spot is absent. Hence, the presence/absence condition of the illuminated spot can be translated directly into the binary code of the spot column address.

**Slide Production**

The slides that generate the encoded projection patterns were generated using the International Imaging System Model 75 image processing facilities at Defence Research Establishment Pacific. The image data of these patterns are downloaded into the Quick Colour Recorder (QCR) made by IMAPRO Inc. The QCR is a high resolution (2048x2048 pixels) colour display unit equipped with a 35mm camera to capture the image on the display unit. Standard Kodachrome KR-135 colour films were used to produce the projection slides for the space-encoded imaging system.

**Slide Alignment**

A major limitation of the slide projection method is the requirement of precise registration among all the slides. Pattern misalignment will shift the positions of the projected spots of the space-encoded projections on the object surface relative to the spots generated by the reference projection. As a result, the wrong address may be decoded due to incorrect identification of the presence/absence condition of the space-encoded projection.
at the spot locations obtained from the reference projection. Slide films are usually not accurately aligned when they are mounted onto cardboard holders during the photo finishing process. Furthermore, the problem is compounded by the sloppiness of the slide position inside the slot of the projector. Thus, manual alignment procedures are usually required before each projection to align the top row of the projected spot with a reference marker in the scene.

The feasibility of implementing an automatic alignment correction algorithm was investigated. It was thought that the image offset of the projection spots due to the misalignment could be compensated by knowing the image offset of a marker in each of the space-encoded projection patterns relative to a marker in the reference projection. Subsequently, the offset of the rest of the projection spots could be computed from the initial offset. This idea was found not to be feasible because the amount of image offset experienced at the projection spots is generally not a simple function of the initial offset. In fact, the amount of offset in the image position is a function of both the slide misalignment and the height of the projected spot. Since the height of the projected spots varies with the shape of the object, it would be difficult to estimate the offset of the image position without knowing the z-coordinate of the object point. Manual alignment procedures which only take a few minutes to complete appear to be the
present solution of this problem. This set-up procedure can be eliminated by using an electro-optical light shutter device made of nematic liquid crystal. With a LCD light shutter installed in the projector, the brightness of each column of the spot pattern can be controlled electronically. The LCD shutter implementation is left as a future improvement of the spot projection system.

2.4 3-D MEASUREMENT OF THE CYLINDRICAL TEST SURFACE

2.4.1 OPTICAL CONFIGURATION OF THE PROJECTION SYSTEM

Optical Equipment

In order to keep the equipment cost of the imaging system low, the system comprises only of existing optical equipment which was readily available at the Electrical Engineering Department. The Dage-MTI model 65 camera, equipped with a vidicon sensor and a 17.5-120 mm zoom lens, was used to capture the images of the spot projection patterns. The video signal from the camera was digitized by the Imaging Technology Corporation IP-512 image processing unit to form a digital image with 512x480 pixel resolution. The digitized grey-scale intensities of the pixels have 8-bit resolution, or an intensity range between 0 and 255. The Leitz slide projector available for the spot projection is equipped with a 90 mm lens and a 250 watt projection lamp. The power of the projection lamp defines the brightness of
the projection spots, and the focal length of the lens defines the size of the spot pattern at a given working distance.

**Lighting condition**

The spot projection imaging system is operated in a dark, controlled lighting environment where the ambient light intensity is reduced to a minimum by eliminating all the surrounding light sources. The ambient light intensity is at a level such that the outlines of the cylindrical test surface were barely visible with the naked eye. The special lighting condition described above is referred to in the thesis as the low controlled lighting condition of the spot projection system.

In the absence of the spot projection pattern, the range of the ambient reflection intensity values obtained by the Dage-MTI camera equipped with a vidicon sensor was between 6 and 8. With the spot pattern projection on the test surface, the image intensity range of the projection spots was between 55 and 95. Because of the diffusive reflectance characteristics of the surface, the average intensity value of the non-illuminated background area in between the projection spots was 22. Full details of the reflectance characteristics of the cylindrical surface will be given in Chapter 6.
Optical Geometry

A cylindrically shaped test surface was manufactured by the machine shop according to the physical dimension as specified in Figure 2.5. In the shape measurement process, the cylindrical surface was placed in front of a background plane located on the x-y plane of the object-centered reference coordinate system. In order to produce a spot projection pattern about two inches larger than the cylindrical surface, the working distance of the Leitz slide projector was about 21 feet from the surface. The area on the background plane covered by the spot projection pattern is about 18x18 inches. Hence, the 32x32 spot pattern
provides a spatial resolution of five-eighths of an inch.

Sato et al. [1982] found that the measurement error of a slit-ray projection system due to the quantization resolution of the camera image plane is inversely proportional to \( \sin\beta \), where \( \beta \) is the angle between the incident light and the optical axis of the camera. Since the optical geometry of the spot projection system is identical to the slit-ray projection system developed by Sato, the error due to the quantization resolution of the camera in the spot projection system can be written as [Sato et al. 1982]:

\[
\Delta r = \frac{L}{f \sin\beta} \Delta \epsilon
\]

where \( L \) is the working distance of the camera, \( f \) is the focal length of the camera lens, and \( \Delta \epsilon \) is the pixel spacing of the camera. Obviously, the quantization error decreases as \( \beta \) approaches 90°, however, it also increases the area of the spot projection pattern to be occluded from the visibility of the camera. An angle of approximately 40° was found to be the maximum angle between the optical axes of the camera and slide projector, such that the entire spot projection pattern was still visible by the camera.

The working distance and the focal length of the camera were adjusted to have the whole cylindrical surface just included in the field of view of the camera and it was measured to be 20x20 inches. The working distance of the
camera and the focal length of the lens were measured to be 22 feet and 110 mm, respectively. The pixel spacing of the vidicon sensor of the camera was found to be about 0.001 inch. By substituting these optical parameters into Equation (2.1), the quantization error of the camera system was calculated to be 0.09 inches.

Without special measurement devices, the orientation angles of the camera and slide projector were not available. However, the coarse position measurements of the camera and projector with respect to the reference coordinate system were obtained and are given as (0.1ft,9ft,20ft) and (0.5ft,-5ft,21ft), respectively.

2.4.2 EXPERIMENTAL PROCEDURES

The cylindrical test surface was placed in front of a white background plane located on the x-y plane of the reference coordinate system, as shown earlier in Figure 2.4. The bottom right corner of the test surface was located at the point (0.0,0.87,0.0) with respect to the reference coordinate system. Grey-scale images of the cylindrical surface with the reference spot projection pattern and the five space-encoded projection patterns were acquired under the optical configuration as specified in Section 2.4.1. This set of grey-scale images was used in the development of the image processing algorithm and the reflectance model of the cylindrical test surface to be discussed later in this thesis. Due to the misalignment problem of the projection
slides, manual adjustment was performed before each space-encoded spot projection to align the top row of the projected spot with the reference marker on the background plane.

The sequence of grey-scale images will be analyzed by the algorithms to be presented in this thesis for determining the 3-D shape of the cylindrical test surface. In order to relate the object points in the 3-D space to their corresponding 2-D image positions, the camera and slide projector models are required to establish the connections between the corresponding object and image points. Full details of the models and their calibration procedures will be presented in Chapters 3 and 4.
CHAPTER 3

CAMERA AND PROJECTOR MODELS

3.1 INTRODUCTION

Homogeneous coordinates and transformation are widely used in robotics and computer graphics applications to describe position coordinates of object points with respect to various coordinate reference systems [Paul 1981, Foley and Van Dam 1983]. The camera and slide projector transformation matrices describe the spatial relationship of the surface points in an object-centered coordinate system with respect to the corresponding points in the image plane of the camera, and also the corresponding transparent windows on the projection slide [Hall et al. 1981, Duda and Hart 1973].

Camera and slide projector transformation matrices have been used in numerous computer vision applications, such as stereo reconstruction [Hwang 1980], 3-D shape measurements [Hall and McPherson 1983], and photogrammetry [Ghosh 1979]. If the locations and orientations of the camera and slide projector with respect to an object centered coordinate system are known, the transformation matrices can then be computed analytically [Ballard and Brown 1982]. However, special calibration devices are required to obtain accurate results for these measurements.
Without such devices, the camera model can also be derived from the spatial coordinates of a set of known object points and their corresponding image coordinates [Hwang 1980]. Similarly, the slide projector model can be derived from the spatial coordinates of a set of known projection spots and their corresponding spot addresses in the projection pattern [Posdamer and Altschuler 1982].

3.2 REVIEW OF HOMOGENEOUS TRANSFORMATIONS

The camera model defines the spatial transformation of an object point in the 3-D space onto a 2-D point on the camera image. The projector model defines the light projections through the transparent windows on the slides onto illuminated spots on the object surface. These object-image spatial relationships can be defined by a homogeneous transformation known as the perspective projection transformation. Familiarity with homogeneous coordinates is necessary to understand the derivation of the transformation matrices for the camera and projector models as explained in subsequent sections of this chapter.

For an imaging process as shown in Figure 3.1, a point \( p \) in the 3-D Cartesian space with coordinates \( (x_o, y_o, z_o) \) can be represented in the 4-dimensional homogeneous coordinate system by a line with coordinates \( (x_o, y_o, z_o, 1) \). Similarly, an image point \( q \) located at \( (x_c, y_c, z_c) \) in the camera image coordinate system can be defined in homogeneous coordinates as \( (x_c, y_c, z_c, s) \), where
s is also a non-zero arbitrary constant. For a given 4x4 transformation matrix $A$ of the imaging process, the position vector $Q$ of the image point is the result of applying the matrix $A$ to the position vector $P$. The image vector $Q$ is given by

$$ Q = A P $$

(3.1)

or

$$
\begin{bmatrix}
  x_s \\
  y_s \\
  z_s \\
  s
\end{bmatrix} =
\begin{bmatrix}
  x_0 \\
  y_0 \\
  z_0 \\
  1
\end{bmatrix}
$$

(3.2)

The homogeneous transformation matrix is a useful tool in computer vision and robotics applications. The 4x4
homogeneous matrix can represent any translation, rotation or perspective transformations individually or in any combination. For example, a new coordinate system is formed by translation of a displacement vector \((x_t, y_t, z_t)\) in the Cartesian system, followed by rotation of \(\theta\) degrees about the x-axis. The translation transformation matrix \(A_T\) and the rotation matrix \(A_R\) are given as

\[
A_T = \begin{bmatrix}
1 & 0 & 0 & x_t \\
0 & 1 & 0 & y_t \\
0 & 0 & 1 & z_t \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
A_R = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & c\theta & -s\theta & 0 \\
0 & s\theta & c\theta & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

where

\[c\theta = \cos\theta\]
\[s\theta = \sin\theta\]

The resultant transformation \(A_{RT}\) of the translation and rotation can be combined into a single matrix operation.

\[
A_{RT} = A_R \cdot A_T
\]

The matrix of the combined operation is simply the product of the two matrices. This is one of the useful properties of homogeneous transformations and is used repeatedly in the derivation of the camera and projector models.
3.3 DESCRIPTION OF THE CAMERA MODEL

It is often inconvenient to identify an object point by its coordinates with respect to the camera reference system. Thus, one would like to describe the position of the object point with respect to an object-centered reference frame $F_o$ adjacent to the work piece, as shown in Figure 3.2.

![Diagram showing the relationship between the camera and the object-centered reference frame $F_o$.](image)

**Figure 3.2** Spatial relationship between the camera and the object-centered reference frame, $F_o$.

The new global reference frame is defined by the following series of transformations: 1) translation of the origin from $O_c$ to location $O_o$; 2) rotation of the x-y plane about the z-axis by an angle $\theta$, the pan angle; 3) rotation of the new x-z plane about the new x axis by an angle $\phi$, the tilt.
angle; and 4) rotation of the new x-z plane around the new y axis by an angle \( \psi \), the swing angle. The sign of the rotation angles follows the right-handed rotation convention. Figure 3.3 shows a step by step transformation of the coordinate frames from the camera to the global reference system.

**Fig. 3.3** Step by step transformation from the object-centered reference frame \( F_o \) to the camera reference frame \( F_c \).

The translation transformation matrix \( A_T \) is given as

\[
A_T = \begin{bmatrix}
1 & 0 & 0 & x_T \\
0 & 1 & 0 & y_T \\
0 & 0 & 1 & z_T \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  

where \((x_T, y_T, z_T)\) is the location of the origin of the
The combined rotation matrix $A_R$ is determined by the pan, tilt and swing orientation of the camera with respect to the global reference frame. The combined rotation transformation matrix $A_R$ is

$$A_R = \text{Rot}(y, \psi) \text{Rot}(x, \phi) \text{Rot}(z, \theta) \quad (3.7)$$

where

$$\text{Rot}(z, \theta) = \begin{bmatrix}
\cos \theta & \sin \theta & 0 & 0 \\
-sin \theta & \cos \theta & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix} \quad (3.8)$$

$$\text{Rot}(x, \phi) = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \phi & \sin \phi & 0 \\
0 & -\sin \phi & \cos \phi & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix} \quad (3.9)$$

$$\text{Rot}(y, \psi) = \begin{bmatrix}
\cos \psi & 0 & -\sin \psi & 0 \\
0 & 1 & 0 & 0 \\
\sin \psi & 0 & \cos \psi & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix} \quad (3.10)$$

and

$\theta, \phi,$ and $\psi$ are the pan, tilt and swing angles of the camera, respectively.

As shown in Figure 3.4, the origin $O_c$ of the camera reference system $F_c$ is located at the center of the camera image plane, and the $y$-axis is parallel to the optical axis of the camera. The lens center of the camera system is positioned at the point $(0, -f, 0)$, where $f$ is the focal length of the camera lens. For an object point located at $(x_4', y_4', z_4')$ with respect to the rotated camera coordinate system $F_c$, the coordinates of its image point $(x_i, y_i, z_i)$ can be computed from the optical geometry of the system. The
image coordinates are given as:

\[ x_i = \frac{x_4}{y_4} f \]  
\[ y_i = 0 \]  
\[ z_i = \frac{z_4}{y_4} f \]  

![Diagram of camera lens center, camera image plane, and object point relationship](image)

Fig. 3.4 Spatial relationship between the image plane of the camera and the camera reference frame \( F_c \).

The image coordinates, \( x_i \) and \( z_i \), in the above discussion are the physical distances of the image point from the center of the camera image plane in the vertical and horizontal directions. Usually, the position of image points are specified in pixel units, measured in the horizontal and vertical directions from a corner of a
picture, such as the upper left corner. The image coordinate system $F_i$ with the origin located at the upper left corner of the image plane is shown in Figure 3.5. The spatial coordinates $(u,v)$ in pixel units, with respect to the new system $F_i$ can then be expressed in terms of the coordinates $(x_i, 0, z_i)$ of the original camera reference frame $F_c$. The new image coordinates, $u$ and $v$ are given by:

$$u = u_0 - \frac{z_i}{k_u}$$

$$= u_0 - \frac{z_4}{k_u y_4}$$

(3.14)
where $k_u$ and $k_v$ are the horizontal and vertical spacings between the image pixels, and $u_0$ and $v_0$ are the horizontal and vertical displacements from the top left hand corner of the image center, in pixel units. Let $K_1 = -f/k_u$, and $K_2 = -f/k_v$, then the transformation from the camera coordinate system $F_4$ to the new image coordinate system can be represented by the perspective transformation matrix $A_C$:

\[
\begin{bmatrix}
  u_t \\
  v_t \\
  t
\end{bmatrix} = A_C \begin{bmatrix}
  x_4 \\
  y_4 \\
  z_4 \\
  1
\end{bmatrix},
\]

where

\[
A_C = \begin{bmatrix}
  0 & u_0 & K_1 & 0 \\
  K_2 & v_0 & 0 & 0 \\
  0 & 1 & 0 & 0
\end{bmatrix}
\]

For cameras with unequal horizontal and vertical pixel spacing or non-unity aspect ratio, the matrix $A_C$ has conveniently taken this factor into account by permitting unequal values for $k_u$ and $k_v$. Consequently, no special calibration procedures are necessary to compensate for this property in such cameras.

Finally, all the transformation matrices can be
combined together to describe the perspective projection of the position vector $P$ in the 3-D space onto an image vector $Q$ on the 2-D image plane. The combined transformation matrix $T_{\text{cam}}$ is given as the product of all the individual transformation matrices:

$$T_{\text{cam}} = A_C A_R A_T$$

$$= \begin{bmatrix}
a_{11} & a_{12} & a_{13} & a_{14} \\
a_{21} & a_{22} & a_{23} & a_{24} \\
a_{31} & a_{32} & a_{33} & a_{34}
\end{bmatrix}$$

where $A_C$, $A_R$, and $A_T$ are the perspective, rotation and translation transformation matrices, respectively. Since $A_R A_T$ is a $4 \times 4$ matrix and $A_C$ is a $3 \times 4$ matrix, hence the product of $A_C A_R A_T$ becomes a $3 \times 4$ matrix as given in Equation (3.18). Hence, the relationship between an object point $P$ with respect to the global reference system and its corresponding image point with respect to the image coordinate system is given by:

$$Q = T_{\text{cam}} P$$

where

$$Q = \begin{bmatrix}
tu \\
tv \\
t \end{bmatrix}$$

(3.20)
\[ t = \text{an arbitrary scaling factor, and} \]

\[ P = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix} \quad (3.21) \]

3.4 DESCRIPTION OF THE PROJECTOR MODEL

A camera samples the light energy reflected from a surface to produce an image. Each pixel of the image is generated by a ray extending from a point originating on the object surface to the camera lens center. Conversely, the projector illuminates spots on the object surface by projecting rays from the lens center through the transparent windows on the slide. In the case of the camera, the light propagation direction is from an object point to the lens center. Although the propagation direction is reversed for the slide projector, the geometry of both devices is identical. Hence, the derivation of the transformation matrices for both devices is the same.

Similar to the case of the camera, the transformation matrix of the slide projector is determined by the focal length of the projector lens, as well as the displacement, pan, tilt, and swing angles of the slide projector with respect to the global reference frame. However, the transformation from the reference frame \( F_p \) of the rotated slide projector to the slide coordinate reference system \( F_s \), as shown in Figure 3.6, is different from the camera model.
Fig. 3.6 Spatial relationship between the slide plane coordinate system \( F_s \) and the projector reference system \( F_p \).

The optical axis of the projector is still parallel to the y-axis of \( F_p \), but the origin of the slide coordinate reference system is now at the bottoms left corner of the slide plane. The \( u \) and \( v \) axes of the slide plane are parallel to the \( x \) and \( z \) axes of \( F_p \), respectively. The homogeneous matrix \( A_p \) of the transformation from \( F_p \) to \( F_s \) is given as:

\[
A_p = \begin{bmatrix}
K_1 & u_0 & 0 & 0 \\
0 & v_0 & K_2 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (3.22)
where \( K_1 = -\frac{f}{k_u} \), \( K_2 = -\frac{f}{k_v} \), \( f \) is the focal length of the slide projector lens, and \( k_u \) and \( k_v \) are the horizontal and vertical spacings of the transparent windows of the reference projection pattern. From Equation (3.6), (3.7) and (3.22), the combined transformation matrix of the slide projector can be written as:

\[
T_{proj} = A_p A_R A_T
\]  

(3.23)

Given the centroid coordinates of the projected spot \((x_o, y_o, z_o)\) on the surface of an object, the column and row address \((u, v)\) of the transparent window in the projection pattern on the slide can be obtained from the model of the slide projector as

\[
\begin{bmatrix}
  u_s \\
  v_s \\
  s
\end{bmatrix} = \begin{bmatrix}
P_{11} & P_{12} & P_{13} & P_{14} \\
P_{21} & P_{22} & P_{23} & P_{24} \\
P_{31} & P_{32} & P_{33} & P_{34}
\end{bmatrix} \begin{bmatrix}
x_o \\
y_o \\
z_o \\
1
\end{bmatrix}
\]  

(3.24)

where

\[
T_{proj} = \begin{bmatrix}
P_{11} & P_{12} & P_{13} & P_{14} \\
P_{21} & P_{22} & P_{23} & P_{24} \\
P_{31} & P_{32} & P_{33} & P_{34}
\end{bmatrix}
\]

and \( s \) is the arbitrary scaling factor of the spot address vector.
3.5 MODEL CALIBRATION PROCEDURES

The purpose of the calibration process is to determine the coefficients of the perspective projection transformation matrix which describes the spatial relationship of an image point in the camera or projector coordinate system to its object point in the global reference frame. Several calibration methods are available. One method is to carefully measure all the optical parameters, such as, the focal length of the lens, the displacement between the global reference frame and the imaging system, and the pan, tilt and swing angles of the camera and projector. These values are then substituted into Equation (3.18) to compute the transformation matrix directly. In the absence of special hardware and calibration devices to provide accurate angular measurements, an alternative calibration process using the measured location of some known object points in space and their corresponding images is more desirable.

If the perspective transformation matrix of the imaging system under calibration is

\[
A = \begin{bmatrix}
a_{11} & a_{12} & a_{13} & a_{14} \\
a_{21} & a_{22} & a_{23} & a_{24} \\
a_{31} & a_{32} & a_{33} & a_{34}
\end{bmatrix},
\]

then using Equation (3.19), the image coordinates \((u,v)\) of a
feature point and its corresponding calibration point 
\((x_0, y_0, z_0)\) can be written as

\[
\begin{bmatrix}
    ut \\
    vt \\
    t
\end{bmatrix} = \begin{bmatrix}
a_{11} & a_{12} & a_{13} & a_{14} \\
a_{21} & a_{22} & a_{23} & a_{24} \\
a_{31} & a_{32} & a_{33} & a_{34}
\end{bmatrix} \begin{bmatrix}
x_0 \\
y_0 \\
z_0 \\
1
\end{bmatrix}
\] (3.26)

where \(t\) is the arbitrary scaling factor. Substituting for 
the variable \(t\) in Equation (3.26) yields,

\[
a_{11}x_o - a_{31}ux_o + a_{12}y_o - a_{32}uy_o + a_{13}z_o - a_{33}uz_o + a_{14} - a_{34}u = 0
\] (3.27)

\[
a_{21}x_o - a_{31}vx_o + a_{22}y_o - a_{32}vy_o + a_{23}z_o - a_{33}vz_o + a_{24} - a_{34}v = 0
\] (3.28)

Two linear equations are formed from each calibration point. 
Since there are twelve unknown coefficients, it requires at 
least six linearly independent calibration points to solve 
for these unknowns. If the minimum number of six feature 
points are utilized in this calibration process, a system of 
twelve linear equations with twelve unknown variables is 
formed. The system of linear equations in matrix 
representation is given as

\[
Ca = 0
\] (3.29)

where
\[ a = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{21} & a_{22} \\ a_{23} & a_{24} & a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}^T \]  \hspace{1cm} (3.30)

and the \( i^{th} \) and \( i-1^{st} \) row of the matrix \( C \) are

\[ c_i = \begin{bmatrix} 0 & 0 & 0 & 0 & x_k & y_k & z_k & 1 & -v_kx_k & -v_ky_k & -v_kz_k & -v_k \end{bmatrix} \]  \hspace{1cm} (3.31)

\[ c_{i-1} = \begin{bmatrix} x_k & y_k & z_k & 0 & 0 & 0 & -u_ky_k & -v_ky_k & -u_kz_k & -u_k \end{bmatrix} \]  \hspace{1cm} (3.32)

for \( i = 2,4,6,8,10,12 \)

\( k = 1,2,3,4,5,6 \).

Since the equation \( C \mathbf{a} = 0 \) is homogeneous, it implies that the solution for the unknown variables \( a_{ij} \)'s will all have a common scaling factor. If one of the unknown, say \( a_{34} \), is set to one, the solutions of the eleven remaining unknown variables in Equation (3.29) are normalized. A selection of six calibration points with no more than four coplanar points among them, satisfies the linear independency requirement. These calibration points form an over-determined linear system of 12 equations with eleven unknown variables. The solutions for this system can be solved by using generalized least squares regression method [Pratt 1978]. By rearranging Equations (3.29) to (3.32), the over-determined system of linear equations becomes
The least squares pseudoinverse estimator $T^+$ of $T$ in Equation (3.34) can be determined by, [Pratt 1978]

$$T^+ = (T^T T)^{-1} T^T$$  \(3.35\)

Therefore, the least square estimation of $A$ is

$$A = T^+ R$$  \(3.36\)

or,

$$A = (T^T T)^{-1} T^T R$$  \(3.37\)
3.6 IMPLEMENTATION AND EXPERIMENTAL RESULTS

3.6.1 CAMERA MODEL CALIBRATION

Implementation

In the camera model calibration process, a rectangular block is placed in front of a background plane located on the x-y plane of the reference coordinate system. The rectangular block is oriented as shown in Figure 3.7, so that the two front faces of the block are visible by the camera. A total of fifty evenly distributed calibration points were marked on the two faces of the block and on the background plane. The spatial coordinates of these calibration points with respect to the object-centered reference coordinate system were measured manually. The image coordinates of these calibration points were recorded manually by displaying the image on the RAMTEK image display system and positioning the image cursor sequentially through the image positions of these calibration points. By substituting the image and spatial coordinates of the calibration points into Equations (3.29) to (3.32), an overdetermined system of linear equations is formed. The least squares estimation of the twelve unknown coefficients of the camera transformation matrix can be calculated by using the pseudoinverse operator as described in the previous section.
Fig. 3.7 Reference object used in the camera model calibration process.

Using the calibration matrix, any two of the three spatial coordinates, say $x$ and $y$, can be computed from the third spatial coordinate, $z$, and the image coordinates of the calibration samples using Equations (3.27) and (3.28). The accuracy of the camera model can then be determined by comparing the calculated spatial coordinates of the sample with the measured coordinate values. The error terms, $\Delta R$, of the calibration points with respect to the camera model are defined as:

$$ \Delta R = \left[ (x_{\text{cal}} - x_m)^2 + (y_{\text{cal}} - y_m)^2 \right]^{1/2} \tag{3.38} $$

where $x_{\text{cal}}$ and $y_{\text{cal}}$ are the calculated spatial coordinates of the calibration points, and $x_m$ and $y_m$ are their measured
Experimental Results

The transformation matrix of the camera model obtained by using all 50 data points in the sample population is given in Table 3.1. The error terms \( \Delta R \), and the spatial and image coordinates of the calibration points are tabulated in Table 3.2. A calibration point is considered to be within the error tolerance of the model if its error term is less than the lateral spatial resolution of the spot projection pattern which is five-eighths of an inch or 3.1% of the camera's field of view.

As indicated in Table 3.2, 28 out of 50 calibration points in the sample population have exceeded the error tolerance of the model. Since the number of bad samples have exceeded 25% of the sample population, the model obtained by this process is not a good representation of the sample population. The results also implied that some calibration points might have large measurement errors associated with them. Because of the bad sample points, the least squares averaging process on the entire sample population might not be the most appropriate technique to generate the transformation matrix of the camera model. Hence, it is necessary to preprocess the data points in the sample population to determine their compatibility with the rest of the population. The incompatible samples will be discarded and will not be used for the derivation of the camera model.
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</table>

Table 3.1 The camera transformation matrix

Table 3.2 Spatial and image coordinates of the calibration points
This idea leads to the concept of random sample consensus which will be presented in the next chapter.

As shown in Figure 3.7, the markers of some calibration points are quite fuzzy in the picture. It is difficult to accurately locate the image coordinates of these points and consequently gross error points occur in the sample population. This situation can be alleviated by increasing the ambient light intensity in the scene and to increase the visibility of the markings for the calibration points.

3.6.2 SLIDE PROJECTOR MODEL CALIBRATION

The same rectangular block is used as the reference object for the slide projector calibration process. The reference spot projection pattern is projected onto this rectangular block. The block is oriented as shown in Figure 3.8, such that the two front faces of the block will show the illuminated spots from the slide projector. Sixty-nine evenly distributed projection spots from the reference spot pattern were selected in a pseudo-random manner as sample points for the projector model calibration process. The row and column addresses of these selected spots and the spatial coordinates at the center of these spots with respect to the reference coordinate system were measured manually. Using the column and row addresses and the spatial coordinates of these calibration points, an overdetermined system of linear equations is formed. The coefficients of the projector
Fig. 3.8 The reference object used in the slide projector model calibration process.

Transformation matrix are determined by using the pseudoinverse operator as in the case of the camera calibration process.

Similar to the camera calibration process, the accuracy of the slide projector model can be determined by comparing the calculated spatial coordinates with the calibration points with the measured coordinate values. The error terms, $\Delta R$, of the calibration points with respect to the slide projector model are also defined as:

$$\Delta R = \left[ (x_{\text{cal}} - x_m)^2 + (y_{\text{cal}} - y_m)^2 \right]^{1/2}$$  \hspace{1cm} (3.39)

where $x_{\text{cal}}$ and $y_{\text{cal}}$ are the calculated $x$ and $y$ coordinates.
of the calibration points, and \( x_m \) and \( y_m \) are the measured coordinate values.

**Experimental Results**

The transformation matrix of the slide projector obtained by using all 69 data points in the sample population is given in Table 3.3. The error terms \( \Delta R \) and the spatial and address coordinates of the calibration points are listed in Table 3.4. There are only eight samples with an error term greater than the error tolerance of the model. This implied that an acceptable model have been found. The model can further be improved by discarding the eight bad sample points and be rederived based on the remaining good data points. This idea leads to the concept of random sample consensus to be discussed in the next chapter for improving the accuracy of the model calibration process.

```
******** SLIDE PROJECTOR TRANSFORMATION MATRIX ********

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-1.7407   -0.0105   -0.1370   29.2784
0.0000    0.0009   -0.0067   1.0000
```

Table 3.3 Slide projector transformation matrix
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Table 3.4 The spatial and image coordinates of the calibration points in the slide projector sample population
3.7 DISCUSSION

A large number of spatial and image coordinate measurements are involved in these model calibration processes. Hence, the measurement results are liable to have errors occasionally. A few gross errors among these data points can have devastating effects on the accuracy of the least squares estimation of the model. For a given setup, the camera and projector need to be calibrated only once. Therefore, it would be worthwhile to pre-process the coordinate measurements and discard the samples which are grossly in error. Finally, only the good data points are used to derive the camera and slide projector model. This leads to the application of the random sample consensus (RANSAC) algorithm to the sampling population. Complete details of this algorithm will be presented in the next chapter.
CHAPTER 4

EXPERIMENTAL DETERMINATION OF MODEL PARAMETERS

4.1 PROBLEM DEFINITION

Given a camera image of a set of calibration points, or the scene of a spot pattern projected from a slide projector, it is necessary to instantiate a mathematical model to represent the perspective transformation of these imaging devices. The derivation of these models was given previously in Chapter 3. Theoretically, either the camera or projector model can be derived from a set of six known linearly independent calibration points. If the minimum number of calibration points is used, any measurement errors of the calibration points will be reflected in the estimated model. Traditionally, a set of sampling points much larger than the minimum required is collected. Approximation methods such as the least squares technique, are then used to smooth out any gross deviations in the measurements.

There are two major types of errors associated with the location measurement of each calibration point: 1) errors in the actual distance measurements of the feature locations; and 2) errors in the identification of the image coordinates of the feature points. Measurement errors
generally follow the normal distribution with a zero mean, whereas, the identification errors are often scene dependent and do not obey any statistical rules. The identification errors may cause gross errors in the model estimation process.

Generally, the traditional approximation methods can provide satisfactory results. Occasionally, these methods may fail due to insufficient size of the sample population or gross errors associated with some sample points. The Random Sample Consensus (RANSAC) algorithm, designed by Fischler and Bolles [1981], can be used to identify the sample points that are grossly in error within a sample population. After eliminating these error samples, the desired model can then be instantiated with the remaining good sample points using least squares approximation. The RANSAC algorithm has been incorporated into the camera and projector calibration procedures for eliminating the gross errors among the sample points.

4.2 RANDOM SAMPLE CONSENSUS ALGORITHM

In a sample population P of N data points, a minimum subset S of n random data points (n<N) are first selected to produce an instantiated model. The accuracy of this model is determined by comparing the measured spatial coordinates at each sample point in P with the computed sample coordinates obtained from the model. A sample point is considered to be consistent and will be included in the
consensus set associated with the model, if the computed coordinates at the sample point are within a preassigned error tolerance limit from the measured value. The instantiated model \( M_1 \) is accepted as a valid approximation of the sample population only if the number of elements in the consensus set is greater than some threshold, \( t \). This predetermined threshold value is usually equal to a percentage of the total sample population. Only the elements in its associated consensus set are then used to compute the final model of the sample population. On the other hand, if the size of the consensus set associated with the model is less than \( t \), this model will be rejected and the entire procedure is repeated with another minimum subset until an acceptable consensus set is found. If an acceptable consensus set is not found after a number of trials, the process can either terminate in failure or proceed to compute the final model of the population using the largest consensus set available.

4.3 IMPLEMENTATION OF THE RANDOM SAMPLE CONSENSUS ALGORITHM

In the camera model calibration scenario, each data point in the sample population consists of the spatial coordinates of the object point and its corresponding image coordinates. For the slide projector calibration process, each data point is defined by the centroid coordinates of the projected spot, as well as the row and column number of the corresponding transparent window on the slide. The
Algorithm begins with the selection of a list of m 6-tuples, each composed of six evenly distributed and linearly independent feature points on the scene. The restriction of allowing no more than four coplanar points of the six selected points is imposed to fulfill the linear independency requirement. All the points in the sample populations are located on three planar surfaces: the two faces of the rectangular block and the background surface on the x-y plane of the reference coordinate system. Each 6-tuple is composed of two randomly selected samples from each of the planar surfaces. The Venn diagram representation of the sample set and the 6-tuples is shown in Figure 4.1.

Fig. 4.1 Venn diagram representation of the sample population P and the subset of 6-TUPLES.
The mathematical model of the device to be calibrated is instantiated for the first 6-tuples using Equations (3.33). From this model, any two of the three spatial coordinates of an object point can be computed from the third spatial coordinate and the two image coordinates using Equations (3.27) and (3.28). For example, if the u, v and z coordinates of a point in the sample set are known, the x and y coordinates of the object point can then be calculated by using the instantiated transformation matrix. The deviation of the calculated values, \(x_{\text{cal}}\) and \(y_{\text{cal}}\), from the measured coordinates, \(x_m\) and \(y_m\), defines the degree of accuracy of the estimated model at that sample point. The error term \(\Delta R\) is given as

\[
\Delta R = \left[ (x_{\text{cal}} - x_m)^2 + (y_{\text{cal}} - y_m)^2 \right]^{1/2}
\]  

Any sample point with an error term greater than 0.625 inch, the lateral spatial resolution of the projection system, is considered to be an inconsistent point and is excluded from the consensus set associated with the 6-tuple. Experimental results show that if there are gross error points in the 6-tuple, then the consensus set of the instantiated model usually consists of less than 50 percent of the sample population. Thus, in the camera and projector calibration process, a 6-tuple is rejected if the size of the consensus set is less than 75 percent of the sample population, including a 25 percent safety margin. The instantiation process is repeated with the next 6-tuple on
the list until an acceptable model is found. After a 6-tuple is accepted, the inconsistent points are discarded from the sample set. Least squares approximation is applied only to the sample points in the consensus set for generation of the final model of the calibrating device. If the list of 6-tuples is exhausted before an acceptable consensus set is found, the program terminates with a failure message. This situation indicates that there are many gross error points in the sample set and the calibration process must be repeated with improved measurement accuracy.

The random sample consensus algorithm is summarized in the following pseudo high-level language procedure:

BEGIN:

Collect a sample set of N calibration points
Compile a list of m 6-tuples

DO each selected 6-tuple, WHILE finish = FALSE

Instantiate the model with the 6-tuple

DO all the points in the sample set P

Compute the x, y coordinates of the sample point with the instantiated model

Compute the error term ΔR

IF ΔR < 0.625 inch THEN

Add the calibration point into the consensus set

ELSE

Discard the calibration point
ENDIF
END DO
IF the size of the consensus set < .75*N THEN
Compute the final model using the entire consensus set
Set finish = TRUE
ELSE
Try the next 6-tuple on the list
Set finish = FALSE
ENDIF
END DO
IF finish = TRUE THEN
Calibration process complete
ELSE
Calibration process failed
ENDIF
END

4.4 DECOMPOSITION OF THE TRANSFORMATION MATRIX

Numerous papers have been published on the calibration process of camera devices, but methods in recovering the parameters of the optical configuration of the device from a given transformation matrix can only be found in a recent paper by Ganapathy [1984]. He suggested an algebraic method which utilizes the values of the twelve known coefficients of the transformation matrix to determine
the 10 unknown physical parameters of the optical device, $K_1$, $K_2$, $u_0$, $v_0$, $X_T$, $Y_T$, $Z_T$, $\Theta$, $\phi$, and $\psi$, as defined in Chapter 3. The calculated parameters can then be used to compare with the coarse physical measurements of a few of these parameters. This procedure can be used to check the validity of the transformation matrices obtained from the model calibration processes.

As defined in Equations (3.17) and (3.23), the transformation matrices of the camera and slide projector are given as:

$$ T_{\text{cam}} = A_C A_{RC} A_{TC} $$ (4.2)

and

$$ T_{\text{proj}} = A_P A_{RP} A_{TP} $$ (4.3)

where $A_C$, $A_{RC}$ and $A_{TC}$ are the perspective, rotation and translation transformation matrices of the camera, and $A_P$, $A_{RP}$, and $A_{TP}$ are the perspective, rotation and translation transformation matrices of the slide projector. In the decomposition procedure of the camera model, we can represent the rotation matrix $A_{RC}$ as:

$$ A_{RC} = \begin{bmatrix} a & b & c & 0 \\ d & e & f & 0 \\ g & h & i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$ (4.4)

and the $A_{RC} A_{TC}$ matrix is represented by:
\[ A_{RC} A_{TC} = \begin{bmatrix} a & b & c & p \\ d & e & f & q \\ g & h & i & r \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.5} \]

where \( a = C\psi S\theta - S\theta S\psi S\phi \), \( b = C\psi S\theta + C\theta S\phi S\psi \),
\( c = -C\phi S\psi \), \( d = -S\theta C\phi \),
\( e = C\theta C\phi \), \( f = S\phi \),
\( g = C\theta S\psi + S\theta S\phi C\psi \), \( h = S\theta S\psi - C\theta S\phi C\psi \),
\( i = C\phi C\psi \),
\( p = aX_T + bY_T + cZ_T \),
\( q = dX_T + eY_T + fZ_T \),
\( r = gX_T + hY_T + iZ_T \),
\( C = \text{cosine} \), \( S = \text{sine} \).

Let us represent the coefficients of the camera matrix by \( c_{11}, c_{12}, \ldots, c_{33} \) and \( c_{34} \). From Equations (4.5) and (3.17), the unknown parameters \( a, b, \ldots, i, p, q \) and \( r \) can be expressed in terms of the transformation matrix coefficients and given as:

\[
\begin{align*}
c_{11} &= K_1 g + u_0 d, \\
c_{12} &= K_1 h + u_0 e, \\
c_{13} &= K_1 i + u_0 f, \\
c_{14} &= K_1 r + u_0 q, \\
c_{21} &= K_2 a + v_0 d, \\
c_{22} &= K_2 b + v_0 e, \\
c_{23} &= K_2 i + v_0 f, \\
c_{24} &= K_2 p + v_0 q, \\
c_{31} &= d, \\
c_{32} &= e, \\
c_{33} &= f, \\
c_{34} &= q, \\
\end{align*} \tag{4.6} \]
Since $A_{RC}$ is an orthornormal rotation matrix [Goldstein 1950], the transpose of the matrix is the inverse of the matrix and the determinant of the matrix is unity. Hence, the following constraints can be imposed on the elements of $A_{RC}$:

\[ a^2 + b^2 + c^2 = d^2 + e^2 + f^2 = g^2 + h^2 + i^2 = 1, \]
\[ i = ae - bd, \quad g = bf - ce, \quad h = cd - af, \]
\[ a = ei - hf, \quad b = fg - di, \quad c = dh - eg, \]
\[ d = hc - bi, \quad e = ai - gc, \quad f = bg - ah, \]
\[ ad + be + cf = dg + eg + fi = ag + bh + ci = 0. \]  

(4.7)

Using the constraint equations specified in Equation (4.7) and the definitions of the matrix coefficients given in Equation (4.6), the ten unknown optical parameters of the camera configuration can be solved by following the algebraic procedures as suggested by Ganapathy. Since square root operations were involved in the calculations of the unknown variables $q, K_1, K_2, u_0$ and $v_0$, it is necessary to determine the signs of these variables as well as their magnitudes. It can be proven that the signs of any two among the three parameters $K_1, K_2$ and $q$ are positive. In our case, $K_1$ and $q$ are defined to be positive. The redundant constraint conditions given in Equation (4.6) and (4.7) can then be used to determine the signs of these unknown parameters $K_2, u_0$ and $v_0$.

In our situation, this matrix decomposition process
is used as an error checking tool on the validity of the transformation matrices produced by the model calibration procedures. The solutions of the ten optical parameters are calculated for each possible set of values in $K_2$, $u_o$, and $v_o$. The results of these optical parameters were checked against the coarse measurements of the camera and slide projector positions. Only one set of $K_2$, $u_o$, and $v_o$ values could generate the displacement vector for the camera that resembled the values given in the coarse measurements of the optical parameters.

4.5 RESULTS OF THE MODEL CALIBRATION PROCESS

The image and spatial coordinates of the samples used in the camera model calibration process are listed in Table 4.1. The row and column address and the spatial coordinates of the samples used in the slide projector model calibration process are given in Table 4.2. The lists of the eight 6-tuples chosen for the RANSAC algorithm used in these two processes are given in Table 4.3. The numbers associated with each 6-tuple are the index number of the sample as given in Tables 4.1 and 4.2. The sample population sizes of the camera and slide projector cases are 50 and 69, and their minimum consensus set sizes used in the RANSAC algorithm are 37 and 51, respectively.

In the camera calibration case, only the 6-tuples numbered 4 and 5 on the list given in Table 4.3 cannot form
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Table 4.1 Spatial and image coordinates of the sample points used in camera model calibration.

an acceptable consensus set. All the other 6-tuples have collected the same 41 points from the sample population to form their consensus set. In other words, nine points in the sample population are grossly in error. The average of the error terms ΔR, defined in Equation (4.20), associated with the initial models derived from all the 6-tuples, and the final models derived from the common consensus set of the acceptable 6-tuples, are listed in Table 4.4. The averages of the error terms have decreased from over half an inch to
Number of calibration points = 69

<table>
<thead>
<tr>
<th>Index</th>
<th>Spatial Coordinates</th>
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Table 4.2 Spot addresses and spatial coordinates of the sample points used in slide projector model calibration.

about 0.06 inches when the final model was used. The final model of the camera is given in Table 4.5.

Similar encouraging results were also experienced in the slide projector calibration process. Again, only two out of eight 6-tuples failed to accumulate an acceptable
6-TUPLES used in the Camera Model Calibration Process

1  8  12  28  23  42  37
2  1  12  28  23  42  37
3  3  12  28  23  42  37
4  1  12  28  23  44  37
5  5  15  20  34  36  45
6  8  13  30  20  40  41
7  16  7  33  29  38  35
8  9  11  34  23  35  39

6-TUPLES used in the Slide Projector Model Calibration Process

1  32  28  4  17  57  45
2  32  31  9  17  57  45
3  38  31  12  8  59  68
4  43  36  12  8  59  68
5  42  50  31  32  17  22
6  17  9  28  32  50  61
7  17  9  35  28  50  61
8  7  19  24  33  68  57

Table 4.3 Lists of the 6-tuples

<table>
<thead>
<tr>
<th>6-tuple index number</th>
<th>Average of the error terms $\Delta R$ (inches)</th>
<th>Initial Model</th>
<th>Final Model</th>
</tr>
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<tr>
<td>1</td>
<td>$.544</td>
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<td>.060</td>
</tr>
<tr>
<td>2</td>
<td>$.527</td>
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<td>.060</td>
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<td>4</td>
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<td></td>
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</tr>
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<td>8</td>
<td>$.635</td>
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<td>.060</td>
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</table>

Table 4.4 The average error terms using the initial and the final camera model.

The final camera model

<p>| | | | |</p>
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<td>-28.6809</td>
<td>-0.2483</td>
<td>-1.4001</td>
<td>422.1229</td>
</tr>
<tr>
<td>0.0009</td>
<td>-0.0001</td>
<td>-0.0043</td>
<td>1.0000</td>
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</tbody>
</table>

Table 4.5 The final camera transformation matrix
consensus set. Five out of the six accepted 6-tuples formed the same consensus set with the identical 61 data points and the sixth one formed a smaller consensus set with 57 data points. The larger consensus set was used to derive the final model of the slide projector. The average of the error terms $\Delta R$ associated with the initial model derived from the 6-tuples and the final model derived from the consensus set, are listed in Table 4.6. It was found that the average error decreased from .21 inches to .07 inches when the final model of the slide projector was used. The final model of the slide projector is given in Table 4.7.

<table>
<thead>
<tr>
<th>6-tuple index number</th>
<th>Average of the error terms $\Delta R$ (inches)</th>
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<tr>
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<td>.288</td>
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</table>

Table 4.6 The average error terms using the initial and the final slide projector model.

The final slide projector model

```
0.0233  -1.6979  -0.3640  30.0430
-1.7322  0.0043  -0.0807  29.0658
0.0007  0.0010  -0.0037  1.0000
```

Table 4.7 Final slide projector transformation matrix
If the probability in selecting a good data point from the sample population is \( w \), then the expected number of tries to pick \( N \) good data points from the sample population is \( w^{-N} \) [Fischler and Bolles 1981]. In the camera and projector model calibration processes, \( w \) is found to be equal to \( 41/50 \) and \( 61/69 \), respectively. Therefore, the expected number of tries to pick 6 good data points from the camera and projector sample populations are 3.29 and 2.65, respectively.

4.6 RESULTS OF MATRIX DECOMPOSITION

The values of the optical parameters obtained by using this set of values are given in Table 4.8. The identical procedure has been repeated for the decomposition of the slide projector transformation matrix to calculate the optical parameters in the slide projector configuration. The results are listed in Table 4.9.

\[
\begin{align*}
\phi &= -78.1^\circ \\
\theta &= 96.3^\circ \\
\psi &= -6.9^\circ \\
X_T &= 2.41\text{ inches} \\
Y_T &= 106.9\text{ inches} \\
Z_T &= 230.5\text{ inches}
\end{align*}
\]

Table 4.8 Optical parameters of the camera

\[
\begin{align*}
\phi &= -71.74^\circ \\
\theta &= 35.0^\circ \\
\psi &= 55.82 \\
X_T &= 4.77\text{ inches} \\
Y_T &= -62.72\text{ inches} \\
Z_T &= 254.22\text{ inches}
\end{align*}
\]

Table 4.9 Optical parameters of the slide projector
From the parameters given in Tables 4.8 and 4.9, the positions of the camera and slide projector with respect to the reference coordinate system can be obtained from $X_T$, $Y_T$ and $Z_T$, and the rotation transformation matrices $A_{RC}$ and $A_{RP}$ can be determined from $\Theta$, $\phi$ and $\psi$. Using these rotation transformation matrices, the orientation of the camera and projector can then be obtained. Since the camera optical axis is along the $y$-axis of the camera coordinate system, the unit vector of the optical axis is $(0,1,0)$. The projection of this unit vector onto the reference coordinate system can be represented by the following homogeneous transformation:

$$\begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}^T = A_{RC} \cdot \begin{bmatrix} x_{co} & y_{co} & z_{co} & 1 \end{bmatrix}^T \quad (4.8)$$

where $\begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}^T$ and $\begin{bmatrix} x_{co} & y_{co} & z_{co} & 1 \end{bmatrix}^T$ are the unit vector of the camera optical axis with respect to the camera and the reference coordinate systems, respectively. $\begin{bmatrix} x_{co} & y_{co} & z_{co} & 1 \end{bmatrix}^T$ is defined as the pointing vector of the camera. Similarly, the pointing vector of the slide projector can be defined by the following expression:

$$\begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}^T = A_{RP} \cdot \begin{bmatrix} x_{po} & y_{po} & z_{po} & 1 \end{bmatrix}^T \quad (4.9)$$

The pointing vectors and the physical representation of the optical parameters for the camera and slide projector are given in Table 4.10 and Figure 4.2, respectively.

If no prior knowledge of the optical configuration is
Fig. 4.2 Physical representation of the camera and slide projector optical configuration.
The pointing vector of the camera in homogeneous representation

\[ [0.02, -0.40, -0.98, 1] \]

The pointing vector of the slide projector in homogeneous representation

\[ [0.18, 0.26, -0.95, 1] \]

Table 4.10 The pointing vectors of the camera and slide projector

available, one can use the redundant constraint conditions in Equation (4.6) and (4.7) to determine the proper sign values for \( K_\alpha \), \( u_0 \) and \( v_0 \). The matrix decomposition method is successful in solving the optical parameters of the camera and slide projection configuration for the spot projection imaging system. However, it is an algebraic method that provides no insight into the underlying geometry of the optical equipment. It is more suited for checking the validity of a given transformation matrix.

4.7 EFFECTIVENESS OF THE RANSAC ALGORITHM

As a control experiment, the transformation matrices of the camera and slide projector were derived from the entire sample population of each calibration process without using the RANSAC algorithm to eliminate the gross error entries. They relied solely on the least squares approximation to reduce the measurement errors. When the RANSAC algorithm was incorporated, the average of the error
terms associated with the camera model was reduced from over half of an inch to .06 inch. Similarly, the average of the error terms for the projector model was reduced from 0.21 inch to 0.07 inch.

When matrix decomposition was performed on the models obtained without using RANSAC, the optical parameters recovered from these transformation matrices did not resemble any of the measured parameter values. More importantly, as described later in Chapter 5, the spatial coordinates of the projection spots calculated by using these model did not look like the original surface. On the contrary, favourable results were obtained from the transformation matrices generated by using the RANSAC algorithm.
CHAPTER 5

IMAGE PROCESSING ALGORITHM

5.1 OVERVIEW

A detailed description of the image processing algorithms used in determining the 3-D spatial coordinates of the illuminated spots on the cylindrical test surface, with respect to the reference coordinate system, is presented in this chapter. The image analysis process, depicted in Figure 5.1, consists of the following tasks:

1) location of the image outlines of the projection spots;
2) estimation of the image centroids of the projection spots;
3) determination of the column address of the projection spots;
4) determination of the 3-D spatial coordinates of the projection spots.

The description of these tasks is presented in section 2 to section 5 of this chapter in the order prescribed in the list above.

The six grey-scale images of the space-encoded spot projection patterns on a cylindrical surface were acquired and used as the input information to the image processing
algorithm; pictures of these images are shown in Figure 5.2. These grey-scale images of the projection patterns were acquired under the optical configuration as specified in section 2.4.

![Block diagram of the image processing algorithm.](image)

When operating the spot projection imaging system in the 3-D measurement of the cylindrical surface, the ambient light reflection from the object surface was substantially reduced to enhance the image intensity contrast between the spot projection pattern and the non-illuminated areas on the cylindrical surface. The high intensity contrast of the spot pattern from the background enables the spot features to be detected by simple image analysis techniques, such as binary thresholding [Bolles and Cain 1983, Horaud 1981, Chan 1985].
Figure 5.2 (a)-(f) Grey-scale images of the projection patterns.
5.2 EDGE DETECTION

Edges are commonly defined as abrupt intensity changes in an image. Intensity changes in an image of a scene can be caused by: 1) illumination changes, such as shadows or light projection patterns; 2) changes in surface orientation of the object in the scene with respect to the viewer; 3) changes in the distance between the visible surfaces and the viewer; and 4) changes in reflection characteristics of the surface. The edge features in the images of the space-encoded spot patterns projected onto a smooth and featureless curve surface are mainly due to the illumination changes between the spot patterns and the background.

If the size of the projection spot on the cylindrical surface is about half an inch square and the surface curvature within the spot is almost flat, then the image intensity of the pixels inside the spot is considered to be constant and much higher than the intensity levels in the surrounding background. However, the image irradiance of the projection spot on different regions of the object surface may vary substantially depending on the orientation of the surface with respect to the optical geometry of the imaging system.

The principle function of the edge detection process is to locate the outline of the illuminated spots in the images of the projection patterns.
5.2.1 BINARY THRESHOLDING

In the binary thresholding process, the image pixels with an intensity value higher than some predetermined threshold value, \( T \), are identified as feature elements of the spot projection pattern, and their intensity values are set to one. The pixels with an intensity level below the threshold value are known as background elements, and the image intensities of these pixels are set to zero.

Binary thresholding operation on grey-scale images can easily be implemented in software or in the image digitizer hardware. The hardware implementation takes advantage of the look-up-table feature in the image digitizer to perform the thresholding function. The resulting binary images of the spot patterns are displayed on the output monitor of the image acquisition system. In the hardware implementation, only the binary images of the spot patterns will be recorded. In the software implementation, the grey-scale images of the patterns are acquired from the image digitizer unit and the binary versions of the projection patterns are then generated. Therefore, the response time of the software implementation is longer. If the binary thresholding technique fails to provide satisfactory results, then other more sophisticated grey-scale edge operators will be required to perform the edge detection function. Therefore, it is advantageous to record the grey-scale images of the projection patterns.
during the development stage of the imaging system.

Since the irradiance intensities of the projection spots from the cylindrical surface vary as a function of the surface orientation, several threshold levels may need to be selected before a satisfactory binary image of the spot pattern is obtained. As given in Section 2.4.1, the image intensity range of the projection spots was between 55 and 95, the average intensity value of the non-illuminated background area in between the projection spots was 22. If the threshold level is set above 55, the some projection spots will be deleted by the process. On the other hand, if the threshold level is set below 22, some pixels in the background non-illuminated areas may be misclassified as spot feature elements. A binary image of the spot pattern is considered to be acceptable if all the visible spot features in the grey-scale images are identified and no pixel from the non-illuminated surface area is classified as feature element by the binary thresholding algorithm.

From the binary images of the projection patterns, the edge pixels of the spots can be identified by a transition of image intensity with any one of their immediate neighbours in the north, south, east or west directions. An edge segment can be formed by an edge pixel any one of its eight neighbour pixels. The 8-connectivity of an edge pixel is demonstrated in Figure 5.3. This property of the edge pixels is utilized by the edge-tracing algorithm in the image centroid estimation process to identify the border outlines of the projection spots.
5.3 IMAGE CENTROID ESTIMATION

The task of the image centroid estimation process is to locate the image centroids of the illuminated spots produced by the projection of the space-encoded spot patterns on the cylindrical surface. The image centroid positions can be determined from the edge outlines of the spots.

Fig. 5.3 8 - Connectivity of an edge pixel.
5.3.1 PROJECTION SPOT EDGE-TRACING

The edge-tracing process is an operation which locates the edge pixels of the projection spots given in the binary edge map of a spot projection pattern. The operation begins by raster scanning the binary edge map for an edge pixel. After an edge pixel is found, a neighbourhood edge pixel tracing procedure is initiated to find all the successively connected edge pixels which form the outline of the projection spot. As the outline of the spot is being traced, the contribution from each edge pixel to the coordinate values of the centroid is calculated. When the neighbourhood edge tracing procedure is complete, the centroid coordinates of the projection spot will be available. The image values of these edge pixels in the binary edge map can then be reset to zero before the search of a new edge curve begins. This will prevent the same edge curve from being traced again. The raster scan operation continues until all the spot outlines in the binary edge map have been traced. At such time, the image value of all the elements in the binary edge map should be zero.

Since the outlines of the illuminated spots on the object surface are closed, the image outlines of the projection spots must also be closed. Therefore, any edge outline in a binary edge map which does not form a closed path is considered to be invalid. Thus, the centroid position of the image feature inside an open border curve
5.3.2 CALCULATION OF PROJECTION SPOT CENTROID COORDINATES

The Freeman chain coding method [Freeman 1961] was used to estimate the centroid positions of the projection spots. This method allows the centroid calculation to be performed as each edge point is traced. Hence, it eliminates the extra procedure of storing the image coordinates in a memory buffer as in the method of centroid calculation based upon the Green's Integral Theorem [Elgazzar et al. 1984, Horaud et al. 1981]. Using the chain code representation, an edge curve in the binary edge map can be uniquely defined by the position of an edge pixel on the curve, followed by an ordered sequence of numbers describing the direction vectors which point to the next connected edge point along the curve. The ordered sequence will terminate when the initial edge point is reached again. The direction vectors of the eight neighbours with respect to an edge pixel, \( P \), is shown in Figure 5.4. The geometric parameters of a projection spot image, such as the area of the spot and the moments about the vertical and horizontal axes of the image plane, can be calculated from the sum of all the incremental contributions from each successive edge segment on the spot outline. The contributions from the edge segments oriented in the various directions are given in Table 5.1 [Freeman 1961].

If an edge outline of a projection spot consists of \( N \)
Fig. 5.4 Direction vectors of the edge segments.

<table>
<thead>
<tr>
<th>Vector Directions</th>
<th>ΔA_i</th>
<th>ΔM_{xi}</th>
<th>ΔM_{yi}</th>
</tr>
</thead>
<tbody>
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<td>8</td>
<td>Y</td>
<td>1/2Y^2</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>Y-1/2</td>
<td>Y^2/2-Y-Y/2+1/6</td>
<td>X^2/2+X/2+1/6</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>X^2/2</td>
</tr>
<tr>
<td>5</td>
<td>-Y+1/2</td>
<td>-Y^2/2+Y/2-1/6</td>
<td>X^2/2-X/2+1/6</td>
</tr>
<tr>
<td>4</td>
<td>-Y</td>
<td>-Y^2/2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>-Y-1/2</td>
<td>-Y^2/2+Y/2+1/6</td>
<td>-X^2/2+X/2-1/6</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>-X^2/2</td>
</tr>
<tr>
<td>1</td>
<td>Y+1/2</td>
<td>Y^2/2+Y/2+1/6</td>
<td>-X^2/2-X/2-1/6</td>
</tr>
</tbody>
</table>

Table 5.1 Area and moments contribution of the edge segments.
pixels, the total area, and the x and y moments of the enclosed image are given by

\[
\text{Area} = \sum_{i=1}^{N} \Delta A_i
\]

(5.1)

\[
M_x = \sum_{i=1}^{N} \Delta M_{xi}
\]

(5.2)

\[
M_y = \sum_{i=1}^{N} \Delta M_{yi}
\]

(5.3)

where \( \Delta A_i \), \( \Delta M_{xi} \), and \( \Delta M_{yi} \) are the incremental contributions to the area, x-moment and y-moment of the projection spot from the \( i^{th} \) edge segment of the boundary curve, as given in Table 5.1.

The image centroid coordinates \((X_c, Y_c)\) of the projection spot are given by

\[
X_c = \frac{M_x}{\text{Area}}, \quad \text{and}
\]

(5.4)

\[
Y_c = \frac{M_y}{\text{Area}}.
\]

(5.5)

5.4 PROJECTION SPOT COLUMN ADDRESS DECODING

The procedures used in decoding the column address of the illuminated spots on the object surface are presented in this section. Due to the misalignment of the projection slides, the image centroids of the projection spots produced by the space-encoded projection patterns do not usually line
up exactly with the corresponding spots generated by the reference projection pattern. Although the misalignment has been minimized by manual adjustments, the displacements among the corresponding spot centroids from the various projection patterns are still noticeable. However, the spatial variations among these spot centroids have been restricted to a four pixel radius neighbourhood from the spot centroids of the reference pattern. Since this is an inherent problem in the projection system, a scheme must be incorporated to compensate for the effects caused by the spatial variations of these projection spots.

To determine the presence/absence of the projection spots in the image of a space-encoded spot pattern, a nearest neighbour search operation must be performed about each of the centroids found in the image of the reference projection pattern. If a centroid is found within the search area about a reference centroid, the column address bit of the spot, which is associated with the space-encoded projection pattern, is set to one. If a centroid is not found, the associated column address bit of the spot is set to zero. The address words of the reference centroids are decoded by repeated applications of the above procedure on the images of all the space-encoded projection patterns.

5.4.1 EFFECTS OF SURFACE DISCONTINUITY ON ADDRESS DECODING

If there are depth discontinuities on the measurement surface, such as the edges of the cylindrical surface, the
projection spots onto these areas may produce illuminated spots on both sides of the discontinuities. Since the image position of the projection spots is a function of the surface depth, the spots on both sides of a depth discontinuity may appear as two separated spots on the camera image. Besides causing split images, depth discontinuities can sometimes join the images of different projection spots together. The column address decoding algorithm will usually fail to give the right column address for these projection spots.

Furthermore, because of the misalignment problem on the projection slides, the projection spots from one projection pattern may be on one side of the discontinuity while the corresponding spots from another projection pattern may be on the opposite side. The image positions of these corresponding spots may vary quite substantially depending on the magnitude of the depth discontinuity. These situations will also result in wrong address bit assignment for these projection spots. The binary encoding scheme currently used in the space-encoded projection pattern lacks error detection capability. However, other encoding schemes, such as the Gray code or the Hamming code, can be used to detect incorrectly decoded address values. In these cases, the projection spots with incorrectly decoded column address will be discarded from the surface sample population. The implementation of such error detection schemes is left as a future improvement to the imaging system.
5.4.2 NEAREST NEIGHBOUR SEARCH ALGORITHM

The nearest neighbour search algorithm implemented was developed by Hall [1982], and later modified by Clark [1985] for performing a diamond search pattern over a rectangular grid. The goal of the search operation is to find a centroid marker within a radius of four pixel units from the center of the search area. The search path generated by the Clark algorithm is an outward expanding diamond shaped spiral pattern, as shown in Figure 5.5. The search procedure consists of two stages: 1) four rounds of diamond shape spiral search operation; and 2) individual testing of the eight outside diagonal grid point. After the

![Fig. 5.5 Search sequence of the nearest neighbour search algorithm.](image-url)
Completion of four rounds of spiral operations, the search will stop at the grid point marked number 41 in the diagram. Individual tests are then used to examine the eight diagonal grid points, numbered 42 through 49, which are also within the four pixel radius search area. The complexity of the search algorithm increases when the search point reaches the boundary of a search region. A complicated scheme was developed to determine the location of the subsequent search points at the boundary. The detailed description of the implementation of this search algorithm can be found in Clark [1985].

5.5 DETERMINATION OF THE 3-D SPATIAL COORDINATES OF THE PROJECTION SPOTS

The 3-D spatial coordinates of a projection spot on the object surface, with respect to the global reference coordinate system, can be calculated from the image centroid coordinates and the column address of the spot. Let the spatial coordinates at the center of a projection spot on the cylindrical surface be \((x,y,z)\), and the column and row addresses of this projection spot in the projection pattern be \((u,v)\). The homogeneous relationship of the position vector of this surface point and the address vector of the spot can be given by:
where \( \{p_{ij}\} \) is the perspective transformation matrix of the slide projector, and \( t \) is an arbitrary scaling constant. Similarly, the homogeneous relationship between the point \((x,y,z)\) in the reference coordinate system, and the image centroid of the spot, \((x_c,y_c)\) in the image plane, can be represented by:

\[
\begin{bmatrix}
    x_c \\
    y_c \\
    s \\
\end{bmatrix} = \begin{bmatrix}
    c_{11} & c_{12} & c_{13} & c_{14} \\
    c_{21} & c_{22} & c_{23} & c_{24} \\
    c_{31} & c_{32} & c_{33} & c_{34} \\
\end{bmatrix} \begin{bmatrix}
    x \\
    y \\
    z \\
    1 \\
\end{bmatrix}
\]  

(5.7)

where \( \{c_{ij}\} \) is the perspective transformation matrix of the camera, and \( s \) is an arbitrary scaling constant. Substituting for \( t \) and \( s \) in Equations (5.6) and (5.7) yields,

\[
\begin{align*}
(c_{11} - c_{31} x_c)x + (c_{12} - c_{32} y_c)y + (c_{13} - c_{33} z_c)z &= c_{34} x_c - c_{14} \\
(c_{21} - c_{31} y_c)x + (c_{22} - c_{32} y_c)y + (c_{23} - c_{33} z_c)z &= c_{34} y_c - c_{24} \\
(p_{11} - p_{31})x + (p_{12} - p_{32})y + (p_{13} - p_{33})z &= p_{34} u - p_{14} \\
(p_{21} - p_{31})x + (p_{22} - p_{32})y + (p_{23} - p_{33})z &= p_{34} v - p_{24}
\end{align*}
\]  

(5.8)
It is sufficient to solve for the unknown variable \( x, y \) and \( z \) by using the first three equations in (5.8). A 3x3 system of linear equations can then be formed and given in matrix representation as:

\[
\begin{bmatrix}
c_{11} - c_{31} x & c_{12} - c_{32} x & c_{13} - c_{33} x \\
c_{21} - c_{31} y & c_{22} - c_{32} y & c_{23} - c_{33} y \\
p_{11} - p_{31} u & p_{12} - p_{32} u & p_{13} - p_{33} u
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
= \begin{bmatrix}
c_{34} x - c_{14} \\
c_{34} y - c_{24} \\
p_{34} u - p_{14}
\end{bmatrix}
\]  
(5.9)

or,

\[
A X = P
\]  
(5.10)

By computing the inverse of matrix \( A \), the spatial coordinates at the center of a projection spot on the cylindrical surface can be expressed as:

\[
X = A^{-1} P
\]  
(5.11)

The cylindrical surface was manufactured according to the following amplitude function of \( z \) (inches):

\[
z = \left[ 12^2 - (y - 8.625)^2 \right]^{1/2} - 9.20 , \quad 0.0 \leq x \leq 12.0 \quad \text{and} \quad 0.875 \leq y \leq 16.375
\]

\[
= 0 , \quad \text{otherwise}
\]  
(5.12)

Using this analytical function, the measurement error of the surface amplitude value, \( z \), of each projection spot on the
cylindrical surface can then be calculated.

5.6 SOFTWARE IMPLEMENTATION OF THE IMAGE PROCESSING ALGORITHM

All the image processing concepts described in this chapter are implemented in a Fortran software package called POS. The software package was developed on the VAX 11/750 computer at Electrical Engineering, UBC and Defence Research Establishment Pacific (DREP) in Victoria, B.C. The VMS operating system is used in the VAX computer in both locations. The RAMTEK 9300 and the International Imaging system model 75 image processor are available at the UBC and DREP computer facilities, respectively, for image viewing. Since two different computer facilities were involved in the program development process, it is essential that program and data file compatibility between the two systems must be maintained. A general purpose graphics package called MAGIC is available at DREP to generate 2-D or 3-D terminal graphics displays. Hard copy versions of the graphics are available on the Printronix Model 300 printer/plotter or on the QMS laser printer.

5.6.1 SOFTWARE DESIGN APPROACHES

The software package POS was developed using structural programming methodology and top-down programming techniques. The image processing software package is
designed to be a portable standalone package to be run on different computer systems. In order to obtain compatibility among computer systems, only standard Fortran 77 instructions were used in the program. Obviously, system library functions of the VMS operating system and externally referenced subroutines could not be utilized.

The modularity concept was also incorporated into the software package in anticipation of future expansion. An operational version of the spot projection imaging system is scheduled to be installed at DREP next year. In modular-structured programs, future tasks that are designed specially for the applications at DREP can be readily incorporated into the system. In this software package, each image processing task is implemented as a function module. Future tasks can be added to the system in the form of new function modules. An executive driver routine is responsible for invoking the required functional modules in the prescribed sequence to compute the spatial coordinates of the projection spots on the measurement surface.

5.6.2 DESCRIPTION OF SOFTWARE

The executive driver program, POS, is the nucleus of the software package. It fetches the image data from the data files and activates the functional modules to perform image processing tasks, such as binary thresholding, spot outline tracing, spot centroid estimation, spot column address decoding and spatial coordinate calculations. The
input information for the image processing program are the six grey-scale images of the space-encoded spot patterns, and the transformation matrices of the camera and slide projector. The program computes the spatial coordinates of the projection spots on the measurement surface from these input information. A listing of the executive drive program in pseudo high-level language is given as follows:

BEGIN: {PROGRAM 'POS' }

Get the image data of the reference projection pattern
CALL edge_filter
CALL trace
I = 2
DO 1 UNTIL I > 6

Get the image data of the \textit{I}th space-encoded pattern
CALL edge_filter
CALL trace
CALL mark_centroid
DO 2 J = 1,N \{N is the number of spot features found in the reference pattern\}

CALL search
IF (a centroid is found) THEN
address bit "I-2" is set to one
ELSE
address bit "I-2" is set to zero
ENDIF
1 = I + 1

2 CONTINUE

1 CONTINUE

Get the camera and projector transformation matrices

DO 3 J = 1,N

CALL position_3D

3 CONTINUE

END PROGRAM:

A number of subroutines are called from this program.

The description of these subroutines are given as follows:

1) edge_filter

This subroutine performs binary thresholding operation on the input grey scale image and generates a binary output image.

2) trace

This subroutine traces the edge outlines of the spot features found in the binary image. The centroid positions are calculated from the image coordinates of the edge points. It also returns the total number of spot features found in the image.

3) mark_centroid

This subroutine marks the centroids of the spot features on a 512x512 image matrix with the intensity value of 255 (full scale).
4) search

This subroutine performs a spiral search operation over a four pixel radius neighbourhood about a specified image position to search for a centroid marker, i.e. a pixel with a value of 255. The logical variable FOUND is set to TRUE if a centroid marker is found, otherwise, it is set to FALSE.

5) position_3D

This subroutine computes the 3-D spatial coordinates of the projection spots from the given transformation matrices for the camera and slide projector, and the spot column addresses and centroid coordinates.

5.7 EXPERIMENTAL RESULTS

A binary image of the spot pattern is considered to be acceptable if all the visible spot features in the grey-scale image are identified and no pixel of the non-illuminated areas is classified as a feature element. After analyzing the grey-scale images of the spot projection patterns acquired in the shape measurement of the cylindrical surface, a threshold level of 45 was empirically determined to produce a binary image of the spot patterns that satisfied the criteria suggested above. It must be emphasized that this threshold level is system dependent. The value of the threshold level should vary according to
the ambient light condition, the sensitivity of the camera and the brightness of the spot projection patterns. In this application, the projection spots were the only visible features in the images and an acceptable threshold level was obtained quite readily.

Two other edge detection algorithms, the Sobel gradient operator [Rosenfeld and Kak, 1982], and the $V^2G$ operator [Marr and Hildreth 1980], were also used for detecting the spot features in the images of the spot patterns. Satisfactory results were also reported for these two algorithms. By taking advantages of the controlled lighting operation condition of the projection system, comparable results were obtained using the binary thresholding method with substantially less computation time. Thus, binary thresholding was implemented into the image processing algorithm.

The binary images of the space-encoded projection patterns formed by using a threshold level of 45 are shown in Figure 5.6. From these binary images of the spot pattern, the edge pixels of the spots were identified. The binary edge maps of the projection spots are shown in Figure 5.7. The image centroid coordinates of the projection spots were estimated using the Freeman chain coding method and the spot centroid locations are marked as "+" in the centroid distribution maps as given in Figure 5.8. The list of the centroid coordinates and the decoded column address of the projection spots can be found in Appendix 1.

The edges of the cylindrical surface have created
Figure 5.7 (a)-(f) Binary edge maps of the projection patterns.
Figure 5.8 (a)-(f) Spot centroids distribution maps
depth discontinuities in the scene. It is evident from row number five of the projection pattern shown in Figures 5.6 (a) and 5.7 (a) that the depth discontinuities have caused some projection spots to appear as split images and some other spots images to be joined together. Furthermore, because of the misalignment problem of the projection slides, some spots from one projection pattern are on one side of the discontinuity while the corresponding spots from another projection pattern are on the opposite side. Hence, the image centroid positions of these spots from the different projection patterns vary more than 4 pixels with respect to their reference centroid positions. These situations are well illustrated by the spots in row number five of the projection patterns in Figures 5.6 (a)-f) and 5.7 (a)-f). The first error condition due to depth discontinuity is an inherent problem with spot projection scheme, and is unavoidable unless the system is only used with smooth, curved surfaces. However, the second error condition due to depth discontinuity and slide misalignment can be resolved by using an electro-optical LCD light shutter installed in the slide projector to generate the different space-encoded patterns under electronic control.

By examining the decoded column address of the projection spots listed in Appendix 1, it is found that the only projection spots which have incorrectly decoded column addresses are the ones in the immediate vicinity of the depth discontinuity. It is also evident that the address decoding errors are caused by the two situations discussed
above. From the results of the other projection spots, we can conclude that the manual alignment procedure performed before the projection of each spot pattern has managed to reduce the alignment errors among the projection slides to a maximum of four pixel units. Moreover, they have also indicated that the nearest neighbour search algorithm has accurately detected the appropriate centroid markers in the centroid distribution maps of the space-encoded projection patterns during the spot address decoding process.

The spatial coordinates of the projection spots obtained by the imaging processing algorithm and the measurement errors of the surface amplitude values are also listed in Appendix 1. From the shape measurement results of the cylindrical test surface, it is evident that the overall accuracy of the surface amplitude measurement results is better than a quarter of an inch, except for the samples that are in the immediate vicinity of the edges on the cylindrical surface. It is because the address decoding process failed to provide the correct column address of these sample points. Large surface amplitude errors at these samples are caused by the incorrect column address values being used in the spatial coordinates calculation process. If these special sample points are ignored, and only the interior projection spots on the cylindrical surface are considered, the average of the measurement error magnitudes of the interior projection spots is less than one-tenth of an inch. This result can then be used to describe the accuracy of the spot projection imaging system.
As described earlier in Section 2.4, a 32x32 spot pattern was used in the shape measurement of the cylindrical surface. The coverage area of the reference spot pattern on the background plane located on the x-y plane of the reference coordinate system was about 20 inches square. Thus, the lateral spatial resolution of the 32x32 spot projection pattern is only 0.65 inch. However, the spatial resolution will increase with the number of the spots in the projection pattern.

The immediate ramifications of the increase in the spot array size are the extra space-encoded patterns required for the spot column definition, and the reduction in the size of the projection spots, as well as the spacings in between them. The first consequence can easily be handled by using an additional space-encoded projection for the case of the 64x64 pattern, or two additional projections for the case of the 128x128 pattern, etc. However, the second ramification may impose several serious problems in the image analysis process. Firstly, due to the reduction in the size of the projection spots and their separation, the image intensity contrast between the illuminated areas and the background area on the test surface will decrease. Secondly, due to the reduction in the spot separation spacings, the four pixel radius search areas of some adjacent spot centroids used in the address decoding process may overlap. Thirdly, misalignment problem of the projection slides will be more critical on the spot column address decoding process.
A series of 64x64 space-encoded spot projections was experimented with on the cylindrical test surface using the same optical configuration as specified in Section 2.4. It was found that the binary thresholding process could not individually identify 30% of the projection spots on the cylindrical surface, especially for the ones on the side orientated away from the camera. Furthermore, because of the smaller spot spacings and the misalignment problem of the projection slides, incorrectly decoded column addresses were reported in over 50% of the projection spots. These results have indicated that the implementation of a 64x64 spot pattern is not feasible under the current system configuration. If the spatial resolution of a 64x64 spot pattern is required, the LCD light shutter should be used to generate the space-encoded projection patterns, and a more light sensitive camera might be more preferable.

5.8 SUMMARY

In this chapter, we have presented an image processing algorithm for determining the spatial coordinates of the projection spots on the cylindrical test surface. The algorithm uses the binary thresholding technique to detect the spot features in the grey-scale images of the space-encoded projection patterns. The centroid coordinates of the spots are then computed by using the image outlines of the spot features. By examining the presence/absence conditions of the spots in these images, the column addresses of the
spots are decoded and the spatial coordinates of the spots are then obtained.

It is clear from the results of the spot address decoding process that the surface discontinuities at the edges of the cylindrical surface have caused some problems in decoding the addresses of the projection spots near the discontinuity. Two possible causes for these address decoding errors were suggested. The first is due to an inherent problem with the spot projection method, while the second is related to the misalignment problem of the projection slides. The slide misalignment problem could be eliminated by using an electro-optical LCD light shutter to produce the spot projection patterns. The reliability of the decoded spot addresses would be improved by introducing an error detection space-encoding scheme to detect the incorrectly decoded addresses. Both suggestions are left to future work.

A 64x64 spot pattern was experimented with on the cylindrical test surface. Due to the low contrast between some of the projection spots and the background, and the misalignment problem of the projection slides, the error rate of the address decoding process in this case is above 50%. This makes the 64x64 spot pattern inoperative in the present system configuration. However, encouraging results were obtained by using the 32x32 spot pattern. The average and maximum surface amplitude measurement errors on the smooth interior region of the cylindrical surface are found to be less than .1 and .25 inch, respectively. A few
limitations of the $32 \times 32$ spot projection scheme will be discussed in the next chapter.
CHAPTER 6

LIMITATIONS OF THE MEASUREMENT SURFACE ORIENTATION

6.1 OVERVIEW

The performance of the spot projection imaging system depends on the light sensitivity of the camera, the brightness of the spot patterns produced by the slide projector, the reflectance characteristics of the measurement surface, and the optical geometry of the imaging devices with respect to the surface orientation of the measurement surface. Unfortunately, many of these design factors of the imaging system are restricted by technical, environmental and equipment limitations. For example, the working distance between the slide projector or camera to the measurement surface is limited to the physical size of the room available to a horizontal spot projection system. In an overhead projection configuration, the working distance is restricted by the height of the ceiling. In order to keep the equipment cost of the imaging system low, the system comprises only of existing equipment which is readily available at the Electrical Engineering Department. A vidicon sensor equipped DAGE-MTI Model 65 camera is available for the spot projection experiment. This camera was originally purchased for an outdoor application and intended to be used in daylight intensity conditions. For a spot projection system operating in low lighting condition,
a more sensitive image sensor such as the Newicon sensor, would be more suitable for this application. The Leitz slide projector available for the projection of spot patterns is equipped with a 90 mm lens and a 250 watt projection lamp. The power of the projection lamp determines the brightness of the illuminated spots, and the focal length of the projector lens defines the size of the projection pattern at a given working distance. The optical configuration of the spot projection imaging system was specified in Section 2.4.

For a measurement surface with uniform reflectance characteristics, as in the case of an aircraft wing surface, it is important to know the relationship of the image intensity with respect to the measurement surface orientation. Subsequently, this will affect the accuracy of the spatial coordinates measurements provided by the imaging system. It has been shown from the shape measurements using the cylindrical surface, that the spatial coordinate errors of the projection spots which have correctly decoded column addresses are less than one quarter of an inch. Large measurement errors are caused by using incorrect decoded column addresses in the spatial coordinate calculations. Thus, we would like to determine the effects of the measurement surface orientation on the spot column address decoding process.

It is critical for the image processing algorithm to identify all the projections spot in the images of the projection pattern. Misidentification of spot images will result in erroneous column addresses being assigned to these
projection spots. Spot identification error is caused by the follow factors:

1) the image irradiance intensity of the projection spot being below the threshold level used in the binary thresholding process;
2) the projection spots being too close together for the detection algorithm to separate the individual edge outlines of their images;
3) the image centroids of adjacent projection spots being less than nine pixels apart.

The causes of error in feature identification due to the first two cases are obvious. In the third case, because of the slide misalignment problem, the image centroids of the projection spots from the different space-encoded patterns can be located anywhere within a four pixel radius from the corresponding centroids in the image of the reference projection pattern. Nearest neighbour search operations are implemented to find the image centroids of the space-encoded spot patterns. As illustrated in Figure 6.1, the search areas about the two adjacent reference centroids in the image of a space-encoded pattern may overlap if they are less than nine pixels apart. If the corresponding centroid of C1, for example, is present in the overlapping search area, as shown in Figure 6.2, then the results of both search operations will be positive. Therefore, the column address bits associated with the
Fig. 6.1  Search areas of two adjacent reference centroids.

Fig. 6.2  Overlapping search areas of two adjacent centroids.
space-encoded pattern for these two spots, C1 and C2, will both be set to one. Consequently, the address bit for the spot C2 is incorrectly decoded to be a one. Hence, in order to prevent this situation from happening, the image spacing between two reference centroids must be greater than nine pixel units. If the size of the spot images are less than 8 pixels square and the centroids of two adjacent spots are less than nine pixels apart, then the image processing algorithm can locate the separate outlines of adjacent spots.

A reflectance model which describes the image irradiance measurement surface is presented in the next section. This model is useful in determining the limiting orientations of the measurement surface which cause the image irradiance intensity of the spot to drop below the binary thresholding level. Planar surfaces in various orientations with respect to the reference coordinate system can be represented by analytical functions. The analytical functions are used to develop the relationship of the image centroid spacings with respect to the measurement surface orientation. With this relationship, the limiting orientations of the measurement surface which cause the image centroid separations to be closer than nine pixels can be determined. The procedures for calculating the centroid spacings of adjacent projection spots on the planar surfaces, and their relationship with the surface orientation are presented in Section 6.3.
6.2 REFLECTANCE MODEL OF THE MEASUREMENT SURFACE

The image irradiance of a projection spot from a measurement surface is a function of the light reflectance properties of the surface material, the brightness of the spot projection pattern, and the orientation of the surface elements with respect to the imaging system. If the reflectance characteristics of the surface material is homogeneous, and the brightness of the projection pattern is uniform, then the changes in image irradiance from the projection spots are only due to the variation of the surface orientation. The image irradiance from any reflective surface can be decomposed into three components: the diffusive, specular, and ambient terms. Since the spot projection system is operating in a controlled lighting environment with very low ambient light intensity, the ambient reflective term from the surface is negligible compared to the two other terms. A reflectance model is suggested by Hall and McPherson [1983] to describe the diffusive and specular terms of a reflective surface.

6.2.1 DIFFUSIVE REFLECTANCE MODEL

As shown in Figure 6.3, an incident light vector \( \mathbf{i} \) is projected onto a surface element of the reflective surface \( S \), and the surface normal vector \( \mathbf{n} \) is at an angle \( \Theta_\mathbf{n} \) with respect to the y-axis of the reference coordinate system. If
Fig. 6.3 Optical geometry of a projection spot on a reflective surface.

The surface is an ideal diffuse reflector, the incident light is diffused equally in all directions and the image irradiance of the incident projection is a function only of the incidence angle, \( \theta \), between the \( \vec{i} \) and \( \vec{n} \) vectors. Surfaces with the ideal diffuse reflectance property are known as Lambertian surfaces. The reflectance model of Lambertian surfaces is given by [Hall and McPherson 1983]:

\[
I_{\text{diff}} = k_1 \cos \theta , \quad -90^0 \leq \theta \leq 90^0 \\
= 0 \quad , \quad \text{otherwise} \quad \quad (6.1)
\]

where \( k_1 \) is a constant determined by the camera light sensitivity and \( \theta \) is the angle of incidence.
6.2.2 SPECULAR REFLECTANCE MODEL

If the surface reflectance characteristic is specular, then the maximum irradiance of the light source can be observed in the direction of the reflection vector \( r \), which is located at an angle equal to the angle of incidence but on the opposite side of the surface normal vector. The irradiance intensity decreases as the viewing vector \( v \) is moved away from \( r \). The specular reflectance model can be expressed as [Hall and McPherson 1983]:

\[
I_{\text{spec}} = k_2 \cos^m \phi, \quad -90^\circ \leq \phi \leq 90^\circ, \\
= 0, \quad \text{otherwise}
\]  

(6.2)

where \( m \) represents the shininess of the surface, and \( \phi \) is the angle between \( v \) and \( r \). For an ideal mirror-like surface, \( m \) is infinitely large. For the surface of a nickel (5 cent piece), \( m \) can be between 10 and 100.

6.2.3 COMBINED REFLECTANCE MODEL

The reflectance model of most surfaces consists of the diffusive, specular and the ambient reflection terms. The specular component represents the reflected light intensity from the cylindrical surface. The diffusive component represents the light scattering intensity on the surface of the material [Cook and Torrance 1981]. Since the ambient term is small compared to the other two, it is
combined with the diffusive term as a common term, \( K \), in the reflectance model. The combined model of an reflective surface can then be expressed as:

\[
I = K + k_2 \cos^m \phi, \quad -90^\circ \leq \phi \leq 90^\circ,
= K, \quad \text{otherwise}
\] (6.3)

Now we would like to determine the combined reflectance model of the cylindrical surface using the image of the reference projection pattern acquired in the 3-D measurement process described in Section 2.4. The calibration procedures of the reflectance model will be presented in Section 6.2.5.

6.2.4 OPTICAL GEOMETRY OF THE PROJECTION SYSTEM

The optical geometries of the camera and slide projector used in the spot projection system were obtained by decomposing their transformation matrices as explained in Section 4.5. From these results, the angular offset of the camera and slide projector optical axes from the y-z plane of the reference coordinate system are only 1.33 and 10.6 degrees, respectively. Since these offsets are quite small, they are ignored in order to reduce the complexity of the optical configuration of the imaging system. Therefore, from this point onwards, we will assume the optical axes of the camera and slide projector to be parallel to the y-z plane of the coordinate system. The simplified optical geometry of
the imaging system is given in Figure 6.4. The inclination angle $\alpha$, projection angle $\beta$, camera angle $\gamma$ and the incidence angle $\theta$ are defined as: the angle between the projector axis and the y-axis of the reference system; the angle between the camera and projector axes; the angle between the camera and the y-axis; and the angle between the incident light vector and the surface normal, respectively.

The camera angle $\gamma$ can be obtained by:

$$\gamma = 180^\circ - \alpha - \beta$$

(6.4)

From the results given in Section 4.6, the values of $\alpha$ and $\beta$ are $74^\circ$ and $39^\circ$, respectively. From Equation (6.4), $\gamma$ is equal to $67^\circ$. 

Fig. 6.4 Optical configuration of the camera and slide projector with respect to the reference coordinate system $\text{Fo}$. 

---

From the results given in Section 4.6, the values of $\alpha$ and $\beta$ are $74^\circ$ and $39^\circ$, respectively. From Equation (6.4), $\gamma$ is equal to $67^\circ$. 

---
The size of the reference projection pattern on the slide is 8.89 mm square, as shown in Figure 6.5, and the focal length of the projector lens is 90 mm. Therefore, the maximum deviation of the projection light beams from the optical axis of the slide projector is 2.8 degrees. Due to this low maximum deviation, the incidence light vectors from the projector lens center to the illuminated spots on the cylindrical surface are approximated by the vector along the optical axis of the projector. Similarly, the viewing vectors from the projection spots to the camera lens center are approximated by the vector along the optical axis of the camera.

![Diagram](image)

**Fig. 6.5** The dimensions of the reference spot projection pattern.
The image irradiance intensity as a function of surface orientations in directions along the y-z plane and the x-z plane of the reference coordinate system are considered individually.

**y-z Plane Variations**

Consider the normal vectors to the cylindrical surface along the y-z plane of the reference coordinate system. The cylindrical surface is oriented such that the rectangular base of the surface is located on the x-y plane and the longitudinal axis of the cylinder is parallel to the x-axis of the reference system, as shown in Figure 6.6. The

![Diagram of cylindrical surface with normal vectors and projection pattern](image)

**Fig. 6.6** The cylindrical surface with longitudinal axis parallel to the x-axis and is placed in front of a background plane, located on the x-y plane of Fo.
cylindrical surface function is given by (all dimensions are given in inches):

\[
z(x,y) = \left[ 12^2 - (y-8.625)^2 \right]^{1/2} - 9.20 , \quad 0.0 \leq x \leq 12.0 \quad \text{and} \quad 0.875 \leq y \leq 16.375
\]

\[
= 0 , \quad \text{otherwise} \quad (6.5)
\]

The normal vector to the surface element located at the point \((x,y,z)\) is given by:

\[
n = 0.0i + \frac{(y-8.625)}{\sqrt{12^2-(y-8.625)^2}} j + 1.0k , \quad 0.0 \leq x \leq 12.0 \quad \text{and} \quad 0.875 \leq y \leq 16.375
\]

\[
= 1.0k , \quad \text{otherwise} \quad (6.6)
\]

where \(i, j, \) and \(k\) are the unit vectors of the reference coordinate system. The angle \(\theta_n\) between the surface normal vector and the \(y\)-axis of the reference system is given by:

\[
\theta_n = \tan^{-1} \frac{\sqrt{12^2 - (y-8.625)^2}}{(y-8.625)} , \quad 0.0 \leq x \leq 12.0 \quad \text{and} \quad 0.875 \leq y \leq 16.375
\]

\[
= 90^\circ , \quad \text{otherwise} \quad (6.7)
\]

Consider the optical geometry of the cylindrical surface, as shown in Figure 6.7. The light incident vector \(i\) and the viewing vector \(v\) are along the optical axes of the projector and camera, respectively. The angle \(\phi\) between the reflection vector \(r\) and the viewing vector \(v\) can be
Fig. 6.7  Optical geometry of a projection spot on the cylindrical surface, with normal vectors in the direction along the y-z plane of $F_0$. 

TV CAMERA

PROJECTOR
expressed in terms of $\theta_n$, $\alpha$, and $\beta$ as:

$$
\phi = 2\theta - \beta \\
= 2\left((180^\circ - \theta_n) - \alpha\right) - \beta
$$

(6.8)

where $\theta = 180^\circ - \theta_n - \alpha$.

Substituting for $\phi$ in Equation (6.8), the surface reflectance model becomes:

$$
I = K + k_2 \cos^m\left(2\left(180^\circ - \theta_n - \alpha\right) - \beta\right)
$$

(6.9)

where $K$, $k_2$, and $m$ are the coefficient of the reflectance model.

From the reflectance model of the cylindrical surface, the relationship between the spot image intensity and $y$-$z$ plane variations of the surface orientation can be established by using Equation (6.9). This relationship can then be used to determine the limiting orientations of the cylindrical surface subjected to the optical configuration of the spot projection system as specified in Section 2.4.

**x-z Plane Variations**

In this case, the axis of the cylindrical surface is parallel to the $y$-axis of the reference coordinate system as shown in Figure 6.8. The optical geometry of the camera and projector remains unchanged, but the surface normal vectors
Fig. 6.8  The cylindrical surface with longitudinal axis parallel to the y-axis and is placed in front of a background plane, located on the x–y plane of \( \vec{F}_0 \).

are now along the x–z plane. The surface normal vector, the reflection vector and the viewing vector are no longer coplanar as in the previous case. Hence, the optical geometry is now more complicated. As shown in Figure 6.9, the incidence light vector \( \vec{i} \) and the viewing vector \( \vec{v} \) are still on the y–z plane, but the surface normal is now along the x–z plane and is at an angle \( \theta_m \) from the z-axis. Thus, the incidence angle, \( \theta \), between the incident light vector and the surface normal is given as:

\[
\cos \theta = \cos \theta_m \sin \alpha \quad (6.10)
\]

where \( \alpha \) is the angle between the incident vector and the y-axis of the coordinate system. The cosine value of the angle
Fig. 6.9 Optical geometry of a projection spot on the cylindrical surface with normal vectors in the directions along the x-z plane of F₀.

The angle \( \phi \) between the reflection vector and the viewing vector is given as:

\[
\cos \phi = \frac{\cos 2\theta}{\cos(180-2\alpha)} \cos(\gamma - \alpha)
\]

Substituting for \( \cos \theta \) in Equation (6.11) yields,

\[
\cos \phi = \frac{(2\cos^2 \Theta - 1)}{\cos(180-2\alpha)} \cos(\gamma - \alpha)
\]

Substituting the expression of \( \cos \phi \) from Equation (6.10) into the reflectance model, as given in Equation
(6.3), the image irradiance from the projection spots on the measurement surface can be expressed as:

$$I = K + k_2 \left[ \frac{2\cos^2 \theta_m \sin^2 \alpha - 1}{\cos(180 - 2\alpha)} \cos(\gamma - \alpha) \right]^m$$  (6.13)

where $K$, $k_2$ and $m$ are the coefficients of the reflectance model.

Using this reflectance model given in Equation (6.13), the relationship between the spot image intensity and the $x$-$z$ plane variations of the cylindrical surface orientation can be established. The relationship can then be used to determine the limiting orientations of the cylindrical surface subjected to the optical configuration of the spot projection system as specified in Section 2.4.

6.2.5 REFLECTANCE MODEL CALIBRATION RESULTS

Now we would like to determine the reflectance model using the image of the reference projection pattern acquired in the 3-D measurement process of the cylindrical surface described in Section 2.4. The grey-scale image of the reference projection pattern is shown in Figure 6.10. A plot of the irradiance intensities of the pixels along row address 106 versus the column address of the pixel is shown in Figure 6.11. The peaks shown in the plot correspond to the image positions of the projection spots. These projection spots are used to derive the reflectance model of
Figure 6.10 Grey-scale image of the reference projection pattern

the cylindrical test surface.

The intensity levels and the image coordinates of the spots were obtained directly from the plot given in Figure 6.11. The corresponding 3-D spatial coordinates of these projection spots were obtained from Appendix 1. The image intensity, image coordinates and the spatial coordinates of the projection spots used in the model calibration process are listed in Table 6.1. Knowing the spatial coordinates of the projection spots, the surface normal vectors at the spots can then be defined by Equation (6.6). Finally, from Equations (6.7) and (6.8), the relative viewing angle $\phi$ between the reflection vector $r$ and the viewing vector $v$ at each of these projection spots were obtained and are listed.
Figure 6.11 Image intensity profile of the projection spots across the cylindrical surface
Table 6.1 Image and spatial coordinates and the intensity of projection spots used to determine the reflectance model of the cylindrical surface. (Pixel row coordinate of the spots is 106)

<table>
<thead>
<tr>
<th>Pixel column address</th>
<th>X (inches)</th>
<th>Y (inches)</th>
<th>Z (inches)</th>
<th>Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>133</td>
<td>10.83658</td>
<td>15.92423</td>
<td>0.53047</td>
<td>68.</td>
</tr>
<tr>
<td>152</td>
<td>10.79246</td>
<td>15.22983</td>
<td>0.98682</td>
<td>72.</td>
</tr>
<tr>
<td>171</td>
<td>10.78326</td>
<td>14.53871</td>
<td>1.44296</td>
<td>77.</td>
</tr>
<tr>
<td>189</td>
<td>10.77641</td>
<td>13.86571</td>
<td>1.82286</td>
<td>79.</td>
</tr>
<tr>
<td>205</td>
<td>10.77421</td>
<td>13.22508</td>
<td>2.05208</td>
<td>82.</td>
</tr>
<tr>
<td>221</td>
<td>10.77198</td>
<td>12.58607</td>
<td>2.28195</td>
<td>86.</td>
</tr>
<tr>
<td>237</td>
<td>10.73507</td>
<td>11.94848</td>
<td>2.51172</td>
<td>89.</td>
</tr>
<tr>
<td>252</td>
<td>10.73743</td>
<td>11.34142</td>
<td>2.59329</td>
<td>90.</td>
</tr>
<tr>
<td>266</td>
<td>10.73743</td>
<td>10.72105</td>
<td>2.75070</td>
<td>93.</td>
</tr>
<tr>
<td>280</td>
<td>10.74201</td>
<td>10.12984</td>
<td>2.75980</td>
<td>95.</td>
</tr>
<tr>
<td>293</td>
<td>10.74424</td>
<td>9.52535</td>
<td>2.84473</td>
<td>94.</td>
</tr>
<tr>
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<td>8.93532</td>
<td>2.85626</td>
<td>93.</td>
</tr>
<tr>
<td>319</td>
<td>10.75318</td>
<td>8.34582</td>
<td>2.86902</td>
<td>94.</td>
</tr>
<tr>
<td>331</td>
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<td>7.77009</td>
<td>2.80849</td>
<td>92.</td>
</tr>
<tr>
<td>343</td>
<td>10.76658</td>
<td>7.19456</td>
<td>2.74921</td>
<td>91.</td>
</tr>
<tr>
<td>355</td>
<td>10.77553</td>
<td>6.63216</td>
<td>2.61660</td>
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</tr>
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<td>84.</td>
</tr>
<tr>
<td>387</td>
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</tr>
<tr>
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<td>4.39370</td>
<td>2.02300</td>
<td>79.</td>
</tr>
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<td>10.82441</td>
<td>3.84275</td>
<td>1.82123</td>
<td>77.</td>
</tr>
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<td>1.54459</td>
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</tr>
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<td>1.26950</td>
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<td>10.86105</td>
<td>1.16216</td>
<td>0.29729</td>
<td>61.</td>
</tr>
</tbody>
</table>

table with their image intensity levels in Table 6.2.

It is evident from Figure 6.11 that there are two distinct bias levels in the image irradiance plot. The first one represents the ambient light reflection and has an intensity level of eight. The second one is the combined ambient and diffusive term K and has an intensity level of about 20. Geometric regression [Dorn and McCracken 1972] was used to determine the coefficients, m and k₂, of the
specular reflectance component from the image irradiances and the relative viewing angles of these spots as given in Table 6.2. The values of $m$ and $k_2$ were determined to be .34 and 70.2 respectively. Hence, the reflectance model of the cylindrical surface is given by:

$$I = 20 + 70.2 \cos^{0.34} \phi$$

(6.14)

where $I$ is the image irradiance of the spot feature in grey scale.

<table>
<thead>
<tr>
<th>$\theta_n$ (DEG)</th>
<th>$\phi$ (DEG)</th>
<th>$\cos \phi$</th>
<th>Meas. Intensity</th>
<th>Cal. Intensity</th>
<th>Error</th>
<th>Percentage Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>52.5</td>
<td>68.9</td>
<td>0.36</td>
<td>68.0</td>
<td>69.7</td>
<td>1.7</td>
<td>2.5</td>
</tr>
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<td>56.6</td>
<td>60.8</td>
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<td>72.0</td>
<td>75.1</td>
<td>3.1</td>
<td>4.3</td>
</tr>
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<td>77.0</td>
<td>79.1</td>
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<td>2.8</td>
</tr>
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<td>64.1</td>
<td>45.8</td>
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<td>79.0</td>
<td>82.2</td>
<td>3.2</td>
<td>4.0</td>
</tr>
<tr>
<td>67.5</td>
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<td>82.0</td>
<td>84.5</td>
<td>2.5</td>
<td>3.0</td>
</tr>
<tr>
<td>70.7</td>
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<td>86.0</td>
<td>86.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>73.9</td>
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<td>89.0</td>
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<td>-1.5</td>
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<td>90.0</td>
<td>88.7</td>
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<td>-1.4</td>
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<td>0.97</td>
<td>93.0</td>
<td>89.5</td>
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<td>-3.8</td>
</tr>
<tr>
<td>82.8</td>
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<td>0.99</td>
<td>95.0</td>
<td>89.9</td>
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<td>-5.3</td>
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<tr>
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<td>92.0</td>
<td>89.5</td>
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<td>-2.7</td>
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<td>88.8</td>
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<td>-2.4</td>
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</tr>
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<td>1.3</td>
<td>1.6</td>
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<td>0.7</td>
</tr>
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<td>3.3</td>
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<td>79.2</td>
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<td>73.0</td>
<td>76.3</td>
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<td>69.0</td>
<td>72.9</td>
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</tr>
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</tr>
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<td>61.0</td>
<td>54.7</td>
<td>-6.3</td>
<td>-10.4</td>
</tr>
</tbody>
</table>

Table 6.2 Image intensities and relative viewing angles $\phi$ of the projection spots on the cylindrical surface
scale levels and $\phi$ is the angle between the $r$ and $n$ vectors. The reflectance model of the cylindrical surface is shown in Figure 6.12 with the measured image intensity of the spots used in the reflectance model calibration process marked as "+". The deviations of the measured intensity values of these projection spots from the values predicted by the reflectance model and the percentage errors are given in Table 6.2. These results have demonstrated that the reflectance model given in Equation (6.14) gives a good representation of the reflectance characteristics of the cylindrical test surface.

6.2.6 LIMITING SURFACE ORIENTATION RESULTS

Based upon the reflectance model of the cylindrical surface and the optical geometry of the spot projection system, we can determine the range of the surface normal vector orientations so that the image intensities of the projection spots are above a given threshold level. In the binary thresholding process presented in Section 5.2, a binary threshold level of 45 was empirically selected to produce an acceptable binary image of the spot pattern. In order for a projection spot to be identified by the binary thresholding process, the intensity level of the spot must exceed 45.

For the normal vectors to the cylindrical test surface oriented along the y-z plane of the reference coordinate system, the relationship of the spot intensities
Figure 6.12 Reflectance model of the cylindrical surface
and the surface normal angles \( \theta_n \) was determined by using Equation (6.9). A plot of the intensity function versus the surface normal angle \( \theta_n \) is given in Figure 6.13. Using 45 as the minimum acceptable intensity level, the limiting angles of the surface normal vectors along the y-z plane are found to be 49° and 136°. Similarly, for the normal vectors oriented along the x-z plane, the relationship of the spot intensities and the surface normal angles \( \theta_m \) was determined by using Equation (6.13). A plot of the intensity level versus the surface normal angle \( \theta_m \) is given in Figure 6.14. The limiting angles of the surface normal vector along the x-z plane are found to be 48° and 131°.

6.3 IMAGE CENTROID SPACING MODEL

We would like to consider the separation between the image centroids of two adjacent projection spots with respect to the orientation of the measurement surface. The spot pattern is projected onto a series of planar surfaces with the normal vectors varying only along the y-z plane of the reference projection system, as shown in Figure 6.15. By varying the direction of the normal vectors of these planar surface, the relationship between the centroid spacings with respect to the surface orientation along the y-z plane can be obtained. The identical procedure can be repeated for another series of planar surfaces whose normal vectors vary only along the x-z plane, as shown in Figure 6.16. From these results, we can deduce the limiting orientation of the
Figure 6.13 The image intensities versus the surface normal angles with respect to the y-axis of $F_0$. 
Figure 6.14 The image intensities versus the surface normal angles with respect to the x-axis of $F_0$. 
Fig. 6.15  A planer surface with normal vectors in the directions along the y-z plane.

Fig. 6.16  A planar surface with normal vectors in the directions along the x-z plane of Fo.
measurement surface such that the minimum centroid spacing is nine pixel units.

6.3.1 ANALYTICAL REPRESENTATION OF PLANAR SURFACES

y-z Plane Variations

We would first consider the analytical surface function of a planar surface with the surface normal oriented along the y-z plane of the reference coordinate system, as shown in Figure 6.15. The surface function can be written as:

\[ f_{yz}(x,y,z) = z + y \tan \psi \]

\[ = 0 \]  

where \( \psi \) is the inclination angle between the planar surface and the y-axis of the coordinate system. The surface normal vector can be defined as:

\[ \mathbf{n} = \nabla f_{yz}(x,y,z) \]

\[ = 0.0 \mathbf{i} + \tan \psi \mathbf{j} + 1.0 \mathbf{k} \]  

where \( \mathbf{i}, \mathbf{j} \) and \( \mathbf{k} \) are the basis vectors of the reference coordinate system. The angle between the surface normal vector \( \mathbf{n} \) and the y-axis is \( \psi - 90^\circ \). The plan view of the surface geometry is shown in Figure 6.17. In order to have a meaningful physical representation of the projection pattern
onto these analytical surfaces, the range of the surface inclination angle $\psi$ must be between the camera angle, $-\gamma$, to the projector angle $\alpha$. If $\psi$ is less than $-\gamma$, the projection spots on the surface will be occluded from the view of the camera. On the other hand, the projection of the spot pattern will not be able to reach the surface if $\psi$ is greater than $\alpha$. Thus the analytical amplitude function $f_{yz}$ of a flat surface can be expressed as:

$$f_{yz}(x, y, z) = z + y \tan \psi, \text{ for } -\gamma \leq \psi \leq \alpha \quad (6.17)$$

As given in Section 6.2, $\alpha$ and $\gamma$ are 74° and 67°, respectively. Therefore, $\psi$ must be between $-74^\circ$ and $67^\circ$ with respect to the y-axis of the reference coordinate system.
x-z Plane Variations

Now, let us consider the planar surfaces whose normal vectors are in the direction along the x-z plane of the coordinate system, as shown in Figure 6.16. Similar to the previous case, the analytical functions of these surfaces are given as:

\[ f_{xz}(x,y,z) = z + x \tan \psi \]
\[ = 0 \]  

where \( \psi \) is the angle between the planar surface and the x-axis. The normal vector of a surface is at an angle \( \psi - 90^\circ \) with respect to the x-axis, where \( \psi \) is the angle between the plane and the x-axis of the coordinate system. The side view of the surface geometry is shown in Figure 6.18. Since the optical axes of the slide projector and camera are in the y-z plane, the range of the surface inclination angle for producing valid spot projection geometries is from \(-90^\circ\) to \(90^\circ\). The surface normal vector can be defined as:

\[ \mathbf{n} = \nabla f_{xz}(x,y,z) \]
\[ = \tan \psi \mathbf{i} + 0.0 \mathbf{j} + 1.0 \mathbf{k} \]  

(6.19)
6.3.2 SPOT SPACING CALCULATION PROCEDURE AND RESULTS

Let the spatial coordinates of a projection spot on the planar surface be \((x,y,z)\) and the address of the spot be \((u,v)\). The homogeneous relationship between the spatial coordinates \((x,y,z)\) and the spot pattern coordinates \((u,v)\) can be written as:

\[
\begin{bmatrix}
vt \\
vt \\
vt \\
\end{bmatrix} = \begin{bmatrix}
P_{11} & P_{12} & P_{13} & P_{14} \\
P_{21} & P_{22} & P_{23} & P_{24} \\
P_{31} & P_{32} & P_{33} & P_{34} \\
\end{bmatrix} \begin{bmatrix}
x \\
y \\
z \\
1 \\
\end{bmatrix}
\] (6.20)
where \( t \) is an arbitrary scaling factor, and \( \{p_{ij}\} \) is the transformation of the slide projector.

**y-z Plane Variations**

For the planar surfaces with normal vectors oriented along the y-z plane, the \( x \), \( y \), and \( z \) spatial coordinates of a projection spot on the cylindrical surface can be solved by Equations \((6.17)\) and \((6.20)\). The image centroid coordinates \((I,J)\) of the corresponding spot feature can then be obtained by applying the camera transformation matrix on the spatial coordinates of the projection spot. Hence, the image centroid coordinates are given as:

\[
\begin{bmatrix}
I_s \\
J_s \\
s
\end{bmatrix}
= 
\begin{bmatrix}
c_{11} & c_{12} & c_{13} & c_{14} \\
c_{21} & c_{22} & c_{23} & c_{24} \\
c_{31} & c_{32} & c_{33} & c_{34}
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
\tag{6.21}
\]

where \( s \) is an arbitrary scaling factor and \( \{c_{ij}\} \) is the transformation matrix of the camera. The same procedure is repeated for the adjacent spot on the same row as the previous one. If the image coordinates of the second spot feature is denoted by \((M,N)\), then the image spacing between the centroids of these two projection spots is defined by:
\[ \Delta R = \left[ (M-1)^2 + (N-J)^2 \right]^{1/2} \] (6.22)

The image spacing \( \Delta R \) for the planar surfaces with inclination angles, \( \psi \), varying from \( 74^\circ \) to \( -67^\circ \) in increments of five degrees, have been calculated and are plotted in Figure 6.19.

The limiting inclination angle \( \psi \) of the surface which causes the image spacing of two adjacent spots to be less than 9 pixels apart, is found to \( +28^\circ \) with respect to the y-axis of the reference coordinate system. The minimum included angle between the surface normal vectors and the y-axis is \( 62^\circ \).

**x-z Plane Variations**

For the planar surfaces with normal vectors oriented along the x-z plane, the x, y, and z spatial coordinates of a projection spot on the cylindrical surface can be solved by Equations (6.18) and (6.20). Using the identical procedures as described above, the image separations between two adjacent spots on the same column of the projection pattern were calculated. A plot of the image separations versus the surface inclination angle \( \psi \) is shown in Figure 6.20.

From Figure 6.20, we can show that any of the planar surfaces with the normal vector parallel to the x-z plane will have produced a minimum image spacing of 15 pixels
Figure 6.19  Centroid spacings versus surface inclination angles for surface normal vectors along the y-z plane.
Figure 6.20 Centroid spacings versus surface inclination angles for surface normal vectors along the x-z plane
between any two adjacent projection spots. Therefore, these surfaces will not cause any problems in providing the minimum image spacing between two adjacent spots required to give a reliable address decoding result.

6.4 Combined Results of the Reflectance and Centroid Spacing Models

In order to satisfy the intensity level and centroid spacing requirements as specified in Section 6.1, the operating range of the surface normal angle along the y-z plane of the reference coordinate system \( F_0 \) is between 49° and 136° for the intensity requirement, and a minimum of 62° for the centroid spacing requirement. Hence, the resulting range of the combined specification is between 62° and 136°. In other words, the included angle between the y-axis of \( F_0 \) and the normal vectors can vary from 62° to 136°.

For the surface normal vectors along the x-z plane of \( F_0 \), the centroid spacing produced by the spot projection system is always greater than the required width of 9 pixel units. However, the intensity requirement can only be fulfilled if the included angle between the surface normal and the x-axis of \( F_0 \) is between 49° and 131°.
Chapter 7

Surface Reconstruction

7.1 Overview

Range or depth values measured by active or passive stereopsis techniques are non-uniformly spaced samples. It is difficult to give an explicit representation of the surface properties, such as amplitude and orientation, using this scattered information. Furthermore, in a multi-sensor environment, range information available from various processes needs to be assimilated to give a complete surface description of the object. Therefore, a reconstruction algorithm is required to fill in the gaps of the unsampled area for the generation of the full surface representation over a regular grid pattern from all the scattered measurements. Reconstruction is defined as a process of fitting a surface to some scattered measurement points with additional constraints related to some known fundamental characteristics of the surface, such as surface smoothness, or the bandlimited property in the spectrum of the surface function.

This chapter deals with the basic problems of reconstructing a visible-surface representation to fit the amplitude measurements acquired by the spot projection imaging system. The three-dimensional spatial coordinates \((x_i, y_i, z_i)\) of each scattered sample point are obtained from
the spot projection system. Since some of the projection spots may be occluded from the view of the camera, the number of measurement points generated by the 32x32 spot projection pattern are limited to 1024 points or less. The irradiance pattern of the illuminated spots on the object surface is often irregular and varies according to the shape of the measurement surface. We would like to reconstruct a bivariate surface function over a specified planar region to fit these irregularly distributed samples.

The spot projection system is designed to cover an area of 20x20 inches on the x-y plane of the reference coordinate system. When an 80x80 line square sampling lattice is used over the work area of the projection system, each element in the tesselation corresponds to a square of .25 by .25 inches. In other words, the grid points in the sampling lattice have a planar spatial resolution of one quarter of an inch.

Three reconstruction algorithms, nearest neighbour, thin plate modelling and warping transformation, were implemented to reconstruct the measurement surface using the scattered samples obtained from the projection system [Terzopoulos 1984, Clark 1985]. The characteristics and constraints of these three reconstruction methods are summarized in the next three sections of this chapter. Detailed theoretical derivations of the reconstruction algorithms is beyond the scope of this thesis, and can be found in the cited references. A discussion of the results generated by these surface reconstruction methods, and the
adaptability of these methods for shape-measurement of aircraft wing surfaces, are given in the last section.

7.1.1 SURFACE CONSTRAINTS

There are some common constraints amongst the three reconstruction algorithms. First, a scattered sampling set is formed from the spot projection on the object surface. The scattered sampling set is a set of grid points on the $80 \times 80$ sampling lattice, where the illuminated spots on the object surface are projected onto the sampling grid and assigned to the closest grid point as illustrated in Figure 7.1. Thus, the surface amplitude values of these sampled grid points are equal to the z-coordinates of the

\[ x = \text{sampled grid points} \]
\[ o = \text{unsampled grid points} \]

Fig. 7.1 Sampled grid points assignments.
corresponding spot on the object surface. Grid points in the lattice with measurement samples affixed to them are defined as the sampled grid points. The others are known as unsampled grid points. The surface values at these sampled grid points become the amplitude constraints of the reconstructed surface. Second, it is reasonable to assume that the measurement surface is placed inside the designated work space of the spot projection system. Hence, the surface amplitude values of the grid points on the boundary of the work space can be set to zero. These boundary grid points are also added into the set of scattered sampling points. The third constraint is only applicable to the thin plate reconstruction algorithm. In this method, the first partial derivatives of the reconstructed surface is assumed to be continuous.

A cylindrical surface is placed on the x-y plane of the reference coordinate system, such that the rectangular base of the surface is bounded by the lines y = .875 inch, y = 16.375 inches, x = 0.0 and x = 12.00 inches. The 3-D wire-frame plot of the cylindrical surface generated by the analytical function is shown in Figure 7.2. A diagram of the scattered sample points is shown in Figure 7.3. The values of the unsampled grid points are represented by a negative surface value of -0.5 inch.

7.2 NEAREST NEIGHBOUR RECONSTRUCTION ALGORITHM

Nearest neighbour interpolation is the simplest
reconstruction algorithm. Under this algorithm, the reconstructed surface values of the sampled grid points are unchanged. The values of the unsampled grid points are obtained from the surface value of their nearest sampled grid points. No averaging or smoothing is attempted on the reconstructed surface values. As a result, the reconstructed surface obtained using this algorithm might not be smooth. If any surface discontinuities exist on the original surface, the same discontinuities will also appear on the reconstructed surface, although the edge position of the discontinuities might be altered by the process. If the
Figure 7.3 The scattered sampling points

Scattered sample points are too sparsely distributed, the unsampled grid point may be quite distant from its nearest sample point. Consequently, the surface value of this sample point may be inappropriate to be used in the unsampled grid point and may cause large reconstruction error. The nearest neighbour method works best if the sample points are dense and isotropically distributed.
7.2.1 IMPLEMENTATION

Nearest neighbour reconstruction is an iterative expansion algorithm which replaces the surface values of the unsampled grid points by the values of their nearest neighbouring sample points. Initially, the surface values of the sampled grid points are defined by the z-coordinates of the corresponding projection spots on the object surface. A large invalid surface value, $-10^{35}$, is assigned to the remaining unsampled grid points. The grid points with surface values equal to $-10^{35}$ are known as "undefined points". During each iteration, a raster scan operation is performed on each point of the grid lattice to check for invalid surface values. When an undefined point is encountered, the algorithm will search its four immediate neighbours for a valid surface value. The order of the search sequence around the grid points is shown in Figure 7.4. If a valid surface value is found, this surface value will be used to replace the value of the undefined point. If not, the surface value of this point remains invalid. The same raster scan operation is repeated in each iteration. The total number of iterations to be performed in the expansion algorithm determines the size of the region where the surface value of a sampled grid point will be expanded. In the spot projection application, the number of iterations to be performed in the reconstruction algorithm is five. The unsampled grid points which failed to have a defined surface
1-COORDINATES OF THE SAMPLED LATTICE

J-COORDINATES OF THE SAMPLED LATTICE

Fig. 7.4 The order of search sequence in the nearest neighbor algorithm.

value assigned, continue to have a large negative value.

7.3 THIN PLATE MODEL RECONSTRUCTION

It may not be possible to describe fully the geometric properties of the measured surface with only the 3-D spatial measurements of the scattered samples. Implicit assumptions or a priori knowledge of the surface characteristics, such as the smoothness and boundary conditions, can provide additional information for the interpolation process. These extra insights about the surface would improve the accuracy of the reconstructed surface. A thin plate model, a surface with continuous first order partial derivatives over the region of interest, was
suggested by Terzopoulos [1984] to fit the best surface through a set of scattered sample points.

7.3.1 PHYSICAL MODEL

The input to the thin plate reconstruction algorithm is the scattered sampling set defined over the 80x80 grid lattice in the work space of the spot projection system. The input constraints can be visualized as vertical pins attached to the sampled grid points within the region. The height of these pins are equal to the amplitude values of the associated projection spot on the object surface. A thin plate is then used to fit onto the tips of these vertical pins. Since measurement error exists for each surface sample, the thin plate cannot be fitted directly onto the tip of the pins. Instead, it rests on some ideal springs attached to the end of the vertical pins. If these ideal springs all have zero natural length and controllable stiffness, then the displacement of the springs from their natural positions can be used to simulate the measurement error of the surface amplitudes of the sample points. The best thin plate surface to fit this set of scattered sample points is the one which experiences the minimum amount of deformation potential energy on the plate surface and the attached springs. The graphical representation of the surface and the position constraints over the sampling lattice is shown in Figure 7.5.
7.3.2 SURFACE CONSTRAINTS

In addition to the position and potential energy constraints, a surface smoothness constraint is also imposed on the reconstructed surface. Ikeuchi and Horn [1981] have shown in their parallel cylinder experiment that discontinuities in surface depth and first partial derivatives are readily detected by human perception, but changes in the second partial derivatives of a visible surface are mostly unnoticeable. These results imply that surface models with continuous second or higher order partial derivatives are excessively smooth. A thin plate surface which has continuous first partial derivatives,
would provide the appropriate smoothness characteristic for our application.

As mentioned earlier, the measurement surface is always placed inside the designated workspace of the spot projection system. It is reasonable to assign zero amplitude values to the grid point along the boundary of the region. Furthermore, the reconstructed surface will also have zero partial derivative with respect to the outward normal vector along the surface boundary. This model is then known as clamped plate model.

7.3.3 IMPLEMENTATION

It has been shown that, the discrete potential energy functional of a reconstructed surface \( \mathbf{v} \) obtained by the thin plate model can be written as [Terzopoulos 1984, p86]:

\[
E(\mathbf{v}) = \frac{1}{2h^2} \left\{ \sum_{i=2}^{79} \sum_{j=1}^{79} \left[ v_{i+1,j} - 2v_{i,j} + v_{i-1,j} \right]^2 
+ \sum_{i=1}^{79} \sum_{j=1}^{79} \left[ v_{i+1,j+1} - v_{i,j+1} + v_{i,j-1} \right]^2 
+ \sum_{i=1}^{80} \sum_{j=1}^{79} \left[ v_{i,j+1} - 2v_{i,j} + v_{i,j-1} \right]^2 \right\} 
+ \frac{1}{2} \sum_{\{\mathbf{x}_s\}} \beta_{\mathbf{x}_s} (\mathbf{v}_{\mathbf{x}_s} - \mathbf{d}_{\mathbf{x}_s})^2
\]  

(7.1)

where \( v_{i,j} \) is the reconstructed surface value of any grid
point \((i,j)\), \(v_{xs}\) is the reconstructed surface value at a sampled grid point, \(d_{xs}\) is the measured surface value at a sampled grid point, \(h\) is the size of a grid lattice cell, and \(\beta_{xs}\) is the surface smoothness parameters at the sampled grid points. If the smoothness of the surface is even, the values of \(\beta_{xs}\) of all the grid points are equal to a constant, \(\beta\). Since the 20x20 working area of the spot projection system is partitioned into an 80x80 grid lattice, the size of the lattice cells is approximately equal to a quarter of an inch.

By first taking the partial derivatives of the potential energy functional with respect to the reconstructed surface value of each grid point and then equating the results to zero, the minimized surface amplitude value of a grid point \((i,j)\) is given by:

\[
(20+h^2\beta_{i,j})v_{i,j} = 8(v_{i-1,j}+v_{i+1,j}+v_{i,j-1}+v_{i,j+1}) - 2(v_{i-1,j}+v_{i+1,j}+v_{i,j-1}+v_{i,j+1}) - \left(v_{i-2,j}+v_{i+2,j}+v_{i,j-2}+v_{i,j+2}\right) - h^2\beta d_{i,j}
\]

(7.2)

where

\[
\beta_{i,j} = \beta, \quad (i,j) \text{ is a sampled grid point}
\]

\[
= 0, \quad \text{otherwise.}
\]

From the surface amplitude constraints of the grid points as specified in Section 7.1.1, the measured surface values \(d_{i,j}\) at the grid points are given as:
\[ d_{i,j} = z_{i,j}, \quad (i,j) \text{ is a sampled grid point} \]
\[ = 0, \quad \text{otherwise.} \quad (7.3) \]

where \( z_{i,j} \) is the surface amplitude of the projection spot associated with \((i,j)\).

The minimization process of the potential energy functional is carried out on every grid point of the sampling lattice over the work space of the projection system. Thus, a system of 6400 linear equations is formed from this process. The system of equations can be expressed in matrix form as:

\[ A \mathbf{v} = \beta \mathbf{d} \quad (7.4) \]

where \( A \) is the Hessian matrix of the potential energy function, and the entries of \( A \) are given by

\[ A = \begin{bmatrix} \frac{\partial^2 E(\mathbf{v})}{\partial v_i \partial v_j} & & & \text{1 \( \leq i, j \leq 80 \)} \\ & \frac{\partial^2 E(\mathbf{v})}{\partial v_i \partial v_k} & & \text{1 \( \leq i, k \leq 80 \)} \\ & & \text{\( \text{and} \)} & \end{bmatrix} \quad (7.5) \]

\( d \) is a 6400x1 column vector consisting of the measured surface amplitudes of the sample points, and \( \mathbf{v} \) is the solution vector of the minimized surface. The solution vector \( \mathbf{v} \) can be solved by using the Gauss-Seidel relaxation method [Dorn and McCracken 1972].

Due to the size of matrix \( A \), it is difficult to write out the entire matrix explicitly. It can be realized from
Equations (7.2) or (7.3) that the surface value of a grid point is a linear combination of the reconstructed surface values of its 12 neighbouring grid points. This linear combination relationship of the surface values can be better demonstrated by the computational molecules, as shown in Figure 7.6 [Terzopoulos, 1984]. The number given in each molecule represents the scalar multiplication factor associated with the grid point. For the grid points on or near the boundary of the sampling lattice, some of the 12 neighbouring grid values may not exist. Therefore, special molecules are arranged to handle these situations. The computational molecules for the grid points on or near the left corner and the lower edge of the sampling lattice is depicted in Figure 7.7. Rotated versions of the given

![Computational molecules](image)

Fig. 7.6 Computational molecules for the interior grid points (Terzopoulos, 1984.)
Figure 7.7 Computation molecules at boundary nodes. (Terzopoulos, 1984)
molecules can be used for the other corners and boundary edges of the lattice.

A set of initial values is required by the Gauss-Seidel method as the first approximation to the solution of the system of linear equations. The surface values obtained from the nearest neighbour method are used as the initial approximated solution of the relaxation process. This set of initial surface values is substituted into Equation (7.3) to form a new set of surface values for the next iteration. The calculations involved for the grid point in each iteration are represented by the computation molecules given in Figure 7.6 or Figure 7.7. The set of surface values obtained by the iterative relaxation process converges to form the solution vector $v$ of the minimum potential energy functional.

Automatic stopping criteria for the Gauss-Seidel relaxation process which check the differential changes between successive iterations can be implemented to test for the convergence condition of the solution. However, the solution of the reconstructed surface generated by the relaxation method always appears to be stable after a hundred iterations. Thus, the number of iterations performed by the Gauss-Seidel method to compute the solution of the reconstructed surface is empirically selected to be 100 iterations.

Terzopoulos [1984] suggested that the surface smoothness parameter is given by $\beta = \gamma h^{-k}$, where $\gamma$ is a constant and $k$ is the degree of the surface analytical
function. In the cylindrical surface measurement, \( k = 2 \) and \( h = .25 \) inches. The values of \( \gamma \) given in Terzopoulos' reconstruction examples were between 0.2 to 2.0. From Equation (7.1), \( \beta \) is found to be associated only with the measurement error term \((v_{xs} - d_{xs})\). Since the average measurement error of the surface amplitude was less than one tenth of an inch in the cylindrical surface measurement, the effects of \( \beta \) on the reconstructed surface values would be minimal. The surface reconstruction results obtained by using \( \gamma = 0.8 \) and 2.5 were found to be virtually identical.

7.4 WARPING TRANSFORMATION RECONSTRUCTION

The warping transformation algorithm is a surface reconstruction technique based on series expansions of the surface function rather than on the minimization of a variational principle as in the case of the thin plate spline method. Similar to the thin plate method, the starting point of this algorithm is the scattered sampling set collected over an 80x80 sampling lattice located in the work space of the spot projection imaging system. The sample points are the 32x32 projection spots on the object surface. The projection spots are usually non-uniformly spaced but homogeneously distributed over the object surface. This homogeneity property is used later to form the warping transformation in the surface reconstruction process.

The set of grid points on the 80x80 sampling lattice is formally defined by:
where \( \mathbf{v}_1 \) and \( \mathbf{v}_2 \) are the basis vectors of the square sampling lattice. The scattered sampling set \( Z_s \) obtained by the spot projection system is defined by:

\[
Z_s = \{ z_s | z_s = x_i \mathbf{v}_1 + y_i \mathbf{v}_2, \ i=1,...,N, \text{ and } 0 \leq x_i, y_i \leq 80 \} \tag{7.7}
\]

where \( N \) is the number of scattered sample points. It is obvious that \( Z_s \) is a subset of \( X \). The objective of the surface reconstruction process is to evaluate the values of the bivariate surface function \( f(x) \), for every \( x \in X \), from the given scattered sampling set \( Z_s \). The graphical representation of the points in \( X \) and \( Z_s \) is illustrated in Figure 7.8.
7.4.1 SAMPLING THEOREMS

For the sake of completeness, we will begin with a brief review of the fundamental 1-D and 2-D uniform sampling theorems, followed by a discussion of the extensions of these theorems for surface reconstruction from non-uniformly spaced samples. Reconstruction of a bandlimited 1-D function from a set of uniformly spaced samples has been a well studied subject since the days of E.T. Whittaker [1915] and C.E. Shannon [1949]. The uniform 1-D sampling theorem is stated as follows:

**Theorem 7.1 [Papoulis 1966]**

If a bandlimited signal $f(t)$ with Fourier transform

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} \, dt, \quad \text{for} \ |\omega| < \omega_1 = \pi/T \quad (7.8)$$

is sampled at the points \( t_n = nT, \ n=0, \pm 1, \pm 2, \ldots \), then, $f(t)$ can be reconstructed exactly from its samples $f(nT)$ as follows:

$$f(t) = \sum_{n=-\infty}^{\infty} f(nT) \frac{\sin[\omega_1(t-nT)]}{[\omega_1(t-nT)]} \quad (7.9)$$

As an extension of the 1-D sampling theorem, a multi-dimensional uniform sampling theorem was later developed by Petersen and Middleton [1966] for signals that
have been sampled on a multi-dimensional sampling lattice.

In the 2-D case, we consider a bivariate function \( h(\mathbf{\xi}) \) defined over a 2-D Euclidean space \( \Xi \), sampled at a set of regular sampling lattice points \( \Xi_s \), such that,

\[
\Xi_s = \{ \mathbf{\xi}_s \mid \mathbf{\xi}_s = l_1 \mathbf{v}_1 + l_2 \mathbf{v}_2, \ l_1, l_2 = 0, \pm 1, \pm 2, \ldots, \ \text{and} \ \mathbf{\xi}_s \in \Xi \} \quad (7.10)
\]

where \( \mathbf{v}_1 \) and \( \mathbf{v}_2 \) are the basis vectors of the sampling lattice. The Fourier transform of the sampled function, \( H_s(\omega) \), can then be expressed as:

\[
H_s(\omega) = \Sigma_{\Omega_s} H(\omega + \omega_s), \ \text{for any } \omega_s \in \Omega_s \quad (7.11)
\]

where \( H_s(\omega) \) is the spectrum of the original function \( h(\mathbf{\xi}) \) and \( \Omega_s \) is the set of periodic lattice points in the 2-D \( \Omega \)-domain. The elements in this periodic lattice are defined by:

\[
\Omega_s = \{ \omega_s \mid \omega_s = l_3 \mathbf{u}_1 + l_4 \mathbf{u}_2, \ l_3, l_4 = 0, \pm 1, \pm 2, \ldots \} \quad (7.12)
\]

where \( \mathbf{u}_1, \mathbf{u}_2 \) are the basis vector of the periodic lattice in the frequency domain. The basis vectors in the spatial and frequency domains are related by:

\[
\mathbf{u}_1 \cdot \mathbf{v}_1 = \mathbf{u}_2 \cdot \mathbf{v}_2 = 2\pi, \ \text{and} \ \mathbf{u}_1 \cdot \mathbf{v}_2 = \mathbf{u}_2 \cdot \mathbf{v}_1 = 0 \quad (7.13)
\]

If \( \Xi_s \) is on a hexagonal sampling lattice where \( \mathbf{v}_1 = (2/\sqrt{3}, 0) \) and \( \mathbf{v}_2 = (1/\sqrt{3}, 1) \), then the basis vectors of the frequency lattice become \( \mathbf{u}_1 = (\pi/\sqrt{3}, \pi/3) \) and \( \mathbf{u}_2 = (0, 2\pi/3) \). The hexagonal
sampling lattices in the spatial and frequency domain are depicted in Figure 7.9. The uniform 2-D sampling theorem can then be stated as follows:

**Theorem 7.2 [Petersen and Middleton, 1966]**

If there exists a 2-D function \( h(\xi) \) and its Fourier transform \( H(\omega) \), such that,

1) \( H(\omega) = 0 \) for any \( \omega \in \Omega_0 \), for some region \( \Omega_0 \) in the frequency domain;

2) the spectrum repetitions in the function \( H_s(\omega) \) do not overlap;

then, for any arbitrary \( \xi \in \Xi \), the bivariate function \( h(\xi) \) can be exactly reconstructed from its uniform samples as follows:

\[
h(\xi) = \sum_{\xi_s \in \Xi_s} h(\xi_s) g(\xi - \xi_s), \quad \text{for any } \xi_s \in \Xi_s
\]  

(7.14)

where \( g(\xi) \) is the inverse Fourier transform of a lowpass filter function \( G(\omega) \) defined by:

\[
G(\omega) = Q \quad \text{for any } \omega \in \Omega_0
\]

\[
= \text{arbitrary} \quad \text{for any } \omega \in \Omega_0, \omega - \omega_s \in \Omega_0
\]

\[
= 0 \quad \text{for any } \omega - \omega_s \in \Omega_0;
\]

where \( Q \) is a constant equal to the area of the sampling lattice cell in the \( \Xi_s \)-domain.

The uniform sampling theorems stated above are valid.
Fig. 7.9 Horizontal sampling lattice's in spatial and frequency domain's
only when the samples of the functions are taken from a regular sample lattice. They do not have any provisions to handle scattered samples from an irregularly spaced sampling lattice, such as the samples obtained by the spot projection system. Papoulis [1966] investigated the error of a 1-D function, \( f(t) \), reconstructed from samples taken at variable sampling intervals. He treated each sampling period as a regular interval with an additional random time delay term \( \theta \). The sampling time \( t_n \) of the \( n^{th} \) sample in the 1-D time series is

\[
t_n = nT - \theta(nT) \tag{7.15}
\]

where \( T \) is the regular sampling period. Substituting for \( \tau_n = nT \) yields, \( t_n = \tau_n - \theta(\tau_n) \). Papoulis stated that if there exists:

1) a non-linear transformation \( \gamma: t \rightarrow \tau \), such that, \( \tau = \gamma(t) \), for any arbitrary \( t \) \hspace{1cm} (7.16)

2) a new bandlimited function \( h(\tau) \) defined over the uniformly spaced interval, such that, \( h(\tau) \equiv f(\tau - \theta(\tau)) \), for any arbitrary \( t \) \hspace{1cm} (7.17)

and

\[
h(\tau_n) = f[nT - \theta(nT)] = f(t_n) \tag{7.18}
\]

then, the original function \( f(t) \) can be exactly reconstructed from the regularly spaced samples of \( h(\tau_n) \).

Based on Papoulis' results for the 1-D signal, Clark [1985]
utilized a non-linear or warping, transformation to formulate the non-uniform 2-D sampling theorem. The theorem is stated as follows:

**Theorem 7.3 [Clark, 1985]**

Consider a bivariate function $f(x)$ over the 2-D Euclidean space $X$, if this function is sampled at a set of scattered points $X_s$, and there exists:

1) a one-to-one continuous and invertible transformation,

$$\gamma: X \rightarrow \Xi$$

such that,

$$\xi = \gamma(x), \quad \text{for any arbitrary } x \in X, \text{ and } \xi \in \Xi \quad \text{(7.19)}$$

and

$$\xi_s = \gamma(x_s), \quad \text{for any } x_s \in X_s \in X, \text{ and } \xi_s \in \Xi_s \in \Xi; \quad \text{(7.20)}$$

2) a new function $h(\xi)$ is defined over the $\Xi$-space, such that,

a) $h(\xi) = f(\gamma^{-1}(\xi)) = f(x)$,

for any arbitrary $x \in X$; \quad \text{(7.21)}

b) the Fourier transform $H(\omega)$ of $h(\xi)$ satisfies conditions 1) and 2) of Theorem 7.2;

then, for any arbitrary $\xi \in \Xi$, the function $h(\xi)$ can be reconstructed exactly from the regularly spaced samples of $h(\xi_s)$ as follows:

$$h(\xi) = \sum_{s} h(\xi)g(\xi - \xi_s), \quad \text{for any arbitrary } \xi \in \Xi, \xi_s \in \Xi_s \quad \text{(7.22)}$$

where $g(\xi)$ is the reconstruction function whose frequency spectrum is given in Equation (7.14). Finally, for any $x \in X$,
the reconstructed bivariate function \( f(x) \), can be derived from Equation (7.22).

Hence,

\[
f(x) = h(\xi)
\]

\[
= \sum_{x_s \in X_s} f(x_s) g[\gamma(x) - \gamma(x_s)] , \text{ for } x_s \in X_s
\]  

(7.23)

7.4.2 IMPLEMENTATION

Now, with the sampling theorems all in place, we will reconstruct the surface function to fit a set of scattered samples obtained from the spot projection imaging system. More specifically, we would like to reconstruct the value of the surface function at any given grid point \( x \) on the 80x80 lines sampling lattice, from the scattered sampling set \( Z_s \) defined in Equation (7.7). For exact surface reconstruction, an infinite number of sample points are required. However, in practical applications, only samples taken from a finite region about the given grid point will be used. Consequently, the reconstructed surface is subject to truncation and aliasing errors. If only \( N_s \) samples are selected from the scattered sampling set to generate the reconstructed surface value, it has been shown [Clark 1985] that, the reconstruction errors can be minimized by choosing the set of \( N_s \) closest sampling points \( \{ z_s \} \), about \( x \). In our application, \( N_s \) is equal to seven.

It was mentioned earlier that the distribution of the
scattered sample points are non-uniform but homogeneous. By taking advantage of this property, Clark derived a heuristic algorithm to determine a one-to-one warping function $\gamma$, such that the points in $\{z_s\}_0$ are mapped onto the vertices and the center of a hexagon in the uniform hexagonal lattice in the $\xi$-space, as illustrated in Figure 7.10. Under this warping function, the image point $\xi=\gamma(x)$ of any arbitrary $x$ inside the triangular region shown in this diagram can be estimated using the trilinear interpolation function, from the vertices of the enclosing triangles both in the $x$-space and the $\xi$-space [Clark 1985]. If $\{x_1, x_2, x_3\}$ and $\{\xi_1, \xi_2, \xi_3\}$ are the vertex sets of the enclosing triangles in the $x$-space and $\xi$-space, respectively, then the image point $\xi$ is

\[\xi_1 = \gamma(z_1)\]
\[\xi_2 = \gamma(z_2)\]
\[\xi_3 = \gamma(z_3)\]
\[\xi_4 = \gamma(z_4)\]
\[\xi_5 = \gamma(z_5)\]
\[\xi_6 = \gamma(z_6)\]

\[\xi = \gamma(x)\]

**Fig. 7.10** Warping transformation from $X$-space to $\xi$-space
given by:

$$
\xi = \gamma(x)
= \xi_1 I(x,x_1,x_2,x_3) + \xi_2 (x,x_2,x_3,x_1) + \xi_3 I(x,x_3,x_1,x_2)
$$

(7.24)

where \( I(x,x_1,x_2,x_3) \) is the trilinear interpolation function defined by:

$$
I(x,x_1,x_2,x_3) = \frac{x^1(x^2_2-x^2_3)+x^2(x^1_3-x^1_2)+(x^2_3x^1_2-x^2_2x^1_3)}{\Delta}, \quad (7.25)
$$

and \( x_1 = (x^1_1,x^2_1), \quad x_2 = (x^1_2,x^2_2), \quad x_3 = (x^1_3,x^2_3) \) and \( x = (x^1,x^2) \)

A summary of the procedures used to calculate the reconstructed surface value of a grid point \( x \) is given as follows:

1) Find the closest scattered sample point, \( z_0 \), about \( x \);
2) Find the set of the closest sample points \( \{z_1, \ldots, z_6\} \) of \( z_0 \) in each of the six 60° sector regions, as shown in Figure 7.10;
3) Determine the image set \( \{\xi_s\}_0 \) of the elements in \( \{z_1, \ldots, z_6\} \);
4) Determine the vertex set of the triangle containing \( x \);
5) Determine the corresponding vertex set of the equilateral triangle containing \( \xi \), the image of \( x \);
6) Use trilinear interpolation to determine the position of \( \xi \) in the \( \xi \)-space;
7) Compute the surface value \( f(x) \) from \( \{z_s\}_0 \) and \( \{\xi_s\}_0 \) using the reconstruction equation [Clark 1985, p163]:
\[ f(x) = h(\gamma(x)) = \sum_{\xi_s} h(\xi_s) g(\xi - \xi_i), \quad \text{for } \xi_i \in \{\xi_s\}_0 \quad (7.25) \]

where

\[
g(\xi - \xi_i) = \frac{e^{-2.16|\xi - \xi_i|^2}}{\sum_{\xi_s} e^{-2.16|\xi - \xi_i|^2}}, \quad \text{for } \xi_i \in \{\xi_s\}_0 \quad (7.26)\]

### 7.5 SURFACE RECONSTRUCTION RESULTS

The reconstruction of the cylindrical surface from the set of known scattered samples using the nearest neighbour algorithm is shown in Figure 7.11 and the error map of the reconstructed surface is given in Figure 7.12. The 3-D wire-frame plot of the thin plate model reconstructed surface and its error map generated after 100 iterations are given in Figure 7.13 and 7.14, respectively. The reconstruction of the cylindrical surface by means of the warping transformation algorithm is shown in Figure 7.15 and the error map of this reconstructed surface is given in Figure 7.16.

Three quantitative error terms, maximum absolute error, mean error and root-mean-square error, are used to describe the deviation of the reconstructed surface from the original cylindrical surface shown in Figure 7.2. The error terms of the surfaces generated by the three reconstruction algorithms are listed in Table 7.1. As shown in the error
Figure 7.11 Nearest neighbour reconstruction surface maps of the reconstructed surfaces, it is evident that the major contributions to these error terms are from the grid points in the area around the edges of the cylindrical surface.

If surface discontinuities are ignored on the reconstructed surface and attention is focused on its smooth interior region, the error terms of the reconstructed surfaces inside the smooth region, as listed in Table 7.2,
Figure 7.12 Error map of the nearest neighbour reconstruction surface.

<table>
<thead>
<tr>
<th>Reconstruction Methods</th>
<th>Max. Absolute Error (inches)</th>
<th>Mean Errors (inches)</th>
<th>RMS Errors (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nearest Neighbour</td>
<td>2.87</td>
<td>.07</td>
<td>.31</td>
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<tr>
<td>Thin Plate</td>
<td>2.34</td>
<td>.09</td>
<td>.29</td>
</tr>
<tr>
<td>Warping Transformation</td>
<td>2.40</td>
<td>.08</td>
<td>.34</td>
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Table 7.1 The error terms of the reconstruction surfaces
Reconstruction Methods

<table>
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<tr>
<th>Method</th>
<th>Max. Absolute Error (inches)</th>
<th>Mean Errors (inches)</th>
<th>RMS Errors (inches)</th>
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</thead>
<tbody>
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<td>.04</td>
<td>.12</td>
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<tr>
<td>Thin Plate</td>
<td>.25</td>
<td>.05</td>
<td>.09</td>
</tr>
<tr>
<td>Warping Transformation</td>
<td>.35</td>
<td>.05</td>
<td>.10</td>
</tr>
</tbody>
</table>

Table 7.2 The error terms in the interior region of the reconstructed cylindrical surfaces.
Figure 7.14 Error map of thin plate reconstruction surface.

are significantly lower than in the previous case when the entire surface was considered. These results give a more typical representation of the accuracy of the measurement system for objects with continuous surface amplitude, such as a hemispherical surface.

The computation times on a VAX-11/750 required for the three surface reconstruction algorithms for an 80x80 grid surface are listed in Table 7.3. Because of the
Figure 7.15 Warping transformation reconstruction surface.

<table>
<thead>
<tr>
<th>Reconstruction Algorithm</th>
<th>CPU Time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nearest Neighbour</td>
<td>1.42</td>
</tr>
<tr>
<td>Thin Plate Model</td>
<td>101.0</td>
</tr>
<tr>
<td>Warping Transformation</td>
<td>142.0</td>
</tr>
</tbody>
</table>

Table 7.3 Computation times of the reconstruction algorithms over an 80x80 sampling lattice. (VAX-11/750)
simplicity of the nearest neighbour algorithm, its computation time is much less than the other two methods.

Figure 7.16 Error map of the warping transformation reconstruction surface
7.6 DISCUSSION

It has been demonstrated from the error maps that the reconstruction errors occur mainly near the edge of the cylindrical surface. This is caused by the aliasing error associated with 32x32 discrete sampling pattern utilized by the vision system. In its current configuration, the samples obtained from the spot projection system are not capable to describe the shape of the surface near the discontinuities. As with any surface reconstruction algorithm, they will not be possible to reconstruct new surface features that are not described by the sampled data. If a supplementary imaging system is available to locate the amplitude discontinuities, then a piece-wise surface reconstruction algorithm can be applied separately to each group of sample points obtained from the different continuous local regions.

A backlighting imaging system as illustrated in Figure 7.17, can be used to determine the border of the cylindrical surface on the x-y plane of the reference coordinate system. From the image outline and the perspective transformation matrix of the camera, the spatial coordinates of the boundary points on the x-y plane can be calculated. This boundary information can then be used to define the boundary grid points to be used in the piece-wise reconstruction algorithm. The backlighting system is not implemented and it is left as another future improvement for the spot projection imaging system.
Fig. 7.17. A suggested backlighting projection system.

The surface reconstruction results from the nearest neighbour method appears to be able to retain the shape of some simple discontinuity features, such as the straight edges of the cylindrical surface. Because of the surface smoothness constraint imposed in the thin plate model, and the low pass filter characteristics of the reconstruction equation in the warping transformation algorithm, the discontinuity features on the original surface cannot be reproduced. However, if the boundary of the surface discontinuity is located by the supplementary measurement system, a piece-wise method can be implemented to avoid the assignment of surface values from a different region across the boundary of the discontinuity.

Other than the rough textural appearance of the
reconstruction surface generated by the warping transformation method, the mean and rms errors of the surface, as given in Table 7.2, are almost identical to that of the smooth thin plate reconstruction surface. Currently, only seven scattered sample points in the neighbourhood of an unsampled grid point are used in the reconstruction process. If additional sample points are used, the truncation error of the reconstructed surface can then be reduced. Therefore, the warping transformation has the potential of providing a more accurate reconstructed surface for the expense of increased computation time.

It must be emphasized that the warping transformation algorithm does not impose any surface smoothness constraints on the reconstruction process. Therefore, measurement surfaces with shape characteristics which do not satisfy the continuous first partial derivatives condition can be better reconstructed by the warping transformation technique. This was Clark's primary intention to develop the warp reconstruction technique.

7.7 RECONSTRUCTION ALGORITHM SELECTION FOR AIRCRAFT-WING DATA

The shape of the aircraft wing surface is very similar to that of the cylindrical surface used in the space-encoded spot projection experiment. They both have smooth interior curve surfaces and are bounded by edges with sharp curvatures. Therefore, the concept of the spot
projection imaging system used in the shape measurement of the cylindrical surface should be directly applicable for aircraft-wing surfaces measurement.

Based on the results of the cylindrical surface, one would select the most appropriate reconstruction method in fitting a surface to the set of scattered sample points obtained by the spot projection system. Four criteria are considered in the selection process:

1) Accuracy of the reconstruction method.
2) Smoothness of the reconstructed surface.
3) Adaptability to piece-wise reconstruction.
4) Computation time requirement.

7.7.1 ACCURACY AND SMOOTHNESS

The three reconstruction methods tested with the cylindrical surface measurements, all started with a common set of scattered surface sample points. The three quantitative errors of the reconstructed surface generated by the thin plate model are consistently lower than the errors of the surfaces produced by the other two methods. Furthermore, the thin plate model reconstructed surface is smooth and has continuous first partial derivatives, whereas the other types of reconstructed surfaces are not. The aircraft-wing surface is considered to be smooth with the exception of a few fastening devices on the wing. In this respect, the thin plate reconstruction surface should provide a closer resemblance to the physical shape of the
wing surface.

7.7.2 PIECE-WISE RECONSTRUCTION

If the edges of the wing are located by a supplementary imaging system, the location and surface values at the discontinuities can then be defined prior to the reconstruction process. Separates region over the sampling lattice can be established from the discontinuity boundaries. In the warping transformation reconstruction, the spiral search operation is used to locate the seven nearest sample grid points about the reconstruction point. The discontinuity boundary must be used to prevent the search operations from selecting sample points from a different reconstruction region. Complex algorithms are required to determine the location of the search points at these irregularly shaped boundaries. In nearest neighbour reconstruction, the piece-wise reconstruction algorithm simply involves a test of the four immediate neighbouring grid points for discontinuity conditions. The reconstructed surface value is obtained only from the neighbouring sampled grid point that is not on the discontinuity boundaries. As recalled from the thin plate reconstruction, the surface value of a grid point is a linear combination of the surface values of its 12 neighbouring grid points as defined in the computational molecules depicted in Figure 7.8. To incorporate piece-wise reconstruction in the thin plate model, each of these 12 grid points must be tested for
discontinuity condition. If any discontinuity points are found among them, the surface values of these points cannot be used in the reconstruction equation. The appropriate computational molecules must be set-up to take care of these discontinuity points. An example of the computational molecules for a couple of surface discontinuity configurations are given in Figure 7.17. The piece-wise reconstruction algorithm has not been incorporated into the above surface reconstruction methods. It is left as a future improvement for the imaging system.

7.7.3 COMPUTATION TIME

Because of the simplistic approach taken by the nearest neighbour algorithm, the computation time required to perform an 80x80 grid surface is considerably faster than the other two methods. In real-time applications with stringent timing requirements, the nearest neighbour algorithm offers an advantage in computation time. Although the times for the other two methods are considerably greater than the nearest neighbour method, the computation times are still less than three minutes. Since the computation time of the reconstruction is proportional to the square of the grid size, the time required by the thin plate model and the warping transformation methods can increase quite rapidly as the grid size of the reconstructed surface increases.
7.7.4 DISCUSSIONS

In the final analysis, the choice of the appropriate reconstruction algorithm for aircraft wing surface shape measurement is a trade-off between the accuracy and smoothness of the reconstructed surface versus the computation time requirement. In an application where a more accurate shape of the surface is required, the thin plate reconstruction technique is the best method. For other applications where response time is critical, the nearest neighbour algorithm might be a more suitable selection.
CHAPTER 8

CONCLUSIONS AND FUTURE RECOMMENDATIONS

8.1 SUMMARY AND CONCLUSIONS

In this thesis, the algorithm of a 3-D vision system based upon the space-encoding spot projection technique was developed. The vision system was implemented using only optical and image processing equipment which was readily available in the Electrical Engineering Department. A cylindrical surface was made to the given shape specifications and was used as a calibration surface for the vision system. Besides this cylindrical surface, several other test objects including a landing gear cover plate for the CF-18 aircraft were used for testing the vision system. The measurement errors of the sample points on the cylindrical surface were obtained from the difference between the measured amplitudes and the calculated values from the shape specifications. The maximum and average measurement errors of the sample points in the smooth interior region of the cylindrical surface was found to be less than a quarter of an inch and a tenth of an inch, respectively. Similar encouraging measurement results on these other test objects were also obtained. Since the performance of the vision system had been well illustrated by the measurement results of the cylindrical surface, the
results of the other surfaces were not included in the thesis.

Procedures for calibrating the transformation matrices of the camera and slide projector used in the vision system were developed. The RANSAC algorithm was utilized to eliminate the gross error points in the calibration sample populations of these devices, so that only good data points were used for the derivation of the transformation matrices. The average error between the calculated spatial coordinates of the calibration points by using these matrices and their measured coordinate values were only 0.06 inch in the case of the camera, and 0.07 inch in the case of the slide projector. The optical parameters of the camera and slide projector were recovered from these matrices using the decomposition process suggested by Ganapathy [1984]. The measured displacement vectors of the camera and slide projector with respect to the reference coordinate system were found to be consistent with the calculated values. This confirmed the validity of the transformation matrices obtained by the calibration process.

An image processing algorithm was developed to recognize the spot features in the images of the space-encoded projection patterns. Binary thresholding technique was used to detect the spot features in these images. The edge feature detection results obtained by using the binary thresholding method in a controlled lighting environment were comparable to the results obtained by other more sophisticated methods. The column addresses of the projection
spots were decoded by the presence/absence conditions of the spots in the images of the space-encoded patterns. The spatial coordinates of the spots were computed using the decoded column addresses and the transformation matrices of the camera and slide projector. We have illustrated that surface depth discontinuity would affect the reliability of the address decoding process for the spots located in the immediate vicinity to the discontinuity. This was found to be an inherent problem of the spot projection structured-light method. The only solution to this problem was to implement an error detection space-encoding scheme for the projection patterns so the spots with an incorrectly decoded address would be deleted from the sample population.

It was found from the image intensities of the projection spots on the cylindrical surface that the irradiance intensities and the separations of the projection spots were a function of the orientation of the measurement surface. The detectability of the spot features and the reliability of the spot address decoding process were determined by these two variables. A reflectance model for the cylindrical surface was derived from the image intensities and the surface normal vectors at the projection spots on the surface. The relationship between the separation of the image centroids of the projection spots and the orientation of the surface was also established. From this reflectance and centroid separation models, the limiting orientation of a measurement surface with a similar reflectance characteristics as the cylindrical test surface
could be found.

Three reconstruction algorithms, nearest neighbour, thin plate model, and warping transformation, were investigated for fitting a surface to the scattered measurement samples obtained from the spot projection system. The nearest neighbour algorithm required the least amount of computation time, but the error of the reconstructed surface was the highest. The average and rms reconstruction errors of the other two methods were quite comparable. However, the thin plate reconstruction surface offered a smooth visible-surface representation and a lower maximum error. All three reconstruction surfaces experienced large errors in the areas near the edges of the cylindrical surface. These results indicated that the three implemented algorithms had all failed to provide a good representation of the surface discontinuities. By ignoring the reconstruction errors near the edges of the cylindrical surface, the errors of the points inside the smooth interior region were substantially lower. These results were a more typical representation of the accuracy of the system for smooth curved surfaces.

The main contribution of the thesis is the development of a low cost 3-D vision system for measuring the shape of smooth featureless curved surfaces. Three surface reconstruction algorithms are implemented in the system for generating a visible-surface representation from the scattered samples provided by the spot projection system. The thesis also provides a solid basis for the 3-D
vision system to be used for the future aircraft wing measurement application.

8.2 RECOMMENDATIONS FOR FUTURE WORK

A number of problems associated with the vision system were discussed in this thesis. Solutions to some of these problems were investigated and suggested as future improvements for the system.

The misalignment problem of the projection spot patterns can be eliminated by using a liquid crystal display (LCD) light shutter device to produce the spot patterns. This electro-optical device made of nematic liquid crystal display operating in the transmissive viewing mode is an ideal solution for this problem. The liquid crystal display can be configured to operate with light segments on a black background. It can be packaged into a thin slab, the size of a standard two inch by two inch cardboard holder for the 35 mm slides.

Similar to liquid crystal displays in watches and instrument panels, the nematic liquid crystal in the light shutter is sandwiched between two planes of thin glass. The nominal spacing between the front and back glass plates is 10 to 20 \( \mu \text{m} \). A sealing material is used to hermetically isolate the liquid crystal from moisture and oxygen in the atmosphere. Strips of transparent conductive coating material consisting of a mixture of indium and tin oxide are etched onto the inner sides of the glass planes in a criss-
cross fashion. The outer sides of the glass planes are laminated with polarizers oriented in perpendicular directions. When the electric field is applied to the appropriate electrodes, the liquid crystal molecules in the area under the influence of the field, reverse orientation and allow light energy to pass through. The unenergized portion of the liquid crystal remains unchanged and blocks all the incident light from passing through. Hence, a spot pattern is produced by projecting light through the liquid crystal shutter. One of the liquid crystal shutter manufacturers claims the intensity contrast ratio between the transparent and opaque condition to be 20:1 [Excel Industries 1985].

The reliability of the spot address decoding process can be improved by encoding the spot patterns with an error detection scheme, such as the Gray-code or the Hamming code, in place of the binary scheme which is currently used. As a result, the projection spots with incorrectly decoded column addresses can be recognized and removed from the surface sample population.

If surface depth discontinuities exist on the measurement surface, the location of the discontinuities should be detected before surface reconstruction is performed. With this information, piece-wise reconstruction algorithms can be implemented separately for the sample points obtained from each continuous local region. For example, a backlighting vision system may be used to determine the border outlines of the cylindrical surface and
the outlines can then be used to define the boundary of the local piece-wise reconstruction regions.
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# APPENDIX I

## SPATIAL COORDINATES OF THE PROJECTION SPOTS ON THE CYLINDRICAL SURFACE

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<tr>
<th>IMAGE COORD (PIXELS)</th>
<th>COL</th>
<th>SPATIAL COORDINATES ADD OF THE PROJECTION SPOTS</th>
<th>SURFACE AMP.</th>
<th>ERROR</th>
<th>ERROR</th>
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**References**

1. [Source Reference 1](#)
2. [Source Reference 2](#)
3. [Source Reference 3](#)