INTEGRATED OPTICAL DEVICES IN LITHIUM NIOBATE

by

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A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

in

THE FACULTY OF GRADUATE STUDIES
Department of Electrical Engineering

We accept this thesis as conforming
to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

December 1981

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ABSTRACT

Integrated optical devices use light to perform circuit functions. One class of devices using Ti indiffused waveguides in LiNbO$_3$ are investigated in this thesis. The applications studied include series and multibranch interferometers, a comparatorless A/D converter, and a high voltage sensor.

An impedance transformation equation for an exponential line is derived and used to obtain relations for the propagation constant of an optical waveguide with linearly graded index. The results are used to study the effect of a Ta$_2$O$_5$ film on the propagation constant of a Ti diffused waveguide. Heating in O$_2$ of the Ta$_2$O$_5$ film, loading one arm of the Mach-Zehnder modulator, is shown to tune the modulator.

The fabrication of devices in Y-cut LiNbO$_3$ by Ti diffusion is described. Microscope objectives are used to couple 0.6328 µm light into 4 µm wide waveguides through polished edges.

An application of the Mach-Zehnder modulator and a two mode BOA (Bifurcation optique active) modulator to high voltage measurement is discussed. The output intensity of the series interferometric filter is calculated. Measured results on a two section filter are presented. Also, the output intensity of a multibranch interferometric filter is determined. Experimental results on a three branch filter are included.
An elimination of comparators from the electrooptic A/D converters (ADCs) utilizing Mach-Zehnder modulators is proposed. Design parameters of a 4-bit precision comparatorless ADC are obtained. The measured results on such an ADC to produce a single bit are encouraging.
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Acknowledgement

I thank my supervisor Dr. L. Young, for support, guidance and encouragement during the course of this work. I am obliged to Dr. D.J. Smith for ellipsometry, and for many interesting and productive discussions. I thank Drs. T.R. Ranganath, R.V. Schmidt, G. Raimier, S. Jensen and G.L. Tangonan for a tour of Hughes Research Laboratories, Malibu, and pointers on making optical waveguides. I am grateful to Mr. B. Cranston for suggestions on polishing, and to Mr. A. Lacis for doing the electron microprobe analysis. The experimental set-up was made by Mr. J. Stuber. Mr. K.S. Lowe prepared the rubylith and Mr. L. Jones the devices for the HV Sensor. I thank Dr. D. Pulfrey for the platinum foil, and Mr. T. Lester for loan of the wafer fixture. I wish to express my appreciation to Mrs. Kathy Brindamour and Mrs. Gail Schmidt for typing the thesis. Moral support by my friends, mother, father and wife made the work easier. Financial support by NSERC and the B.C. Science Council is gratefully acknowledged.
CHAPTER I
INTRODUCTION

The term integrated optics [Miller 1969] refers to devices in which circuit functions are achieved using light instead of electrons and holes of conventional integrated circuits. Various realizations of this concept have been proposed. One class of circuits is based on using semiconductors such as GaAs. In the present work Ti indiffused waveguides in LiNbO$_3$ are used to form the devices.

Semiconductors GaAs and InP are used to make lasers and optical waveguides, but LiNbO$_3$ and LiTaO$_3$ to make more efficient modulators and switches. One method of making waveguides is by diffusing Ti into LiNbO$_3$. A typical Ti:LiNbO$_3$ single mode waveguide is 3 μm wide and 2 μm deep. Light can be coupled into and out of the waveguide by the use of prisms, gratings, lenses and fibres. In the future, the emphasis will be on coupling light by the butt joining of laser diodes or single mode fibres to integrated optical devices [Hsu et al. 1977; Boyd and Sriram 1978; Campbell 1979; Saruwatari and Nawata 1979]. With the development of low loss (0.5 dB at 1300 nm) optical fibres and solid state lasers, the need for integrated optical devices to perform modulation, switching and filtering etc. is increasing. The main advantages integrated optical devices would offer are very high speed (GHz), miniaturization, low power consumption, and less weight. Losses and optical damage in integrated optical devices are two problem areas. The fibre to
waveguide coupling loss is $> 1$ dB, and the waveguide propagation loss is 1 dB/cm. Bends, directional changes, and furcations in the waveguide network produce scattering losses. Branching angles of 1-2° that are required to keep the loss below 3 dB, make the devices long and limit the number of components that can be accommodated on a substrate. The waveguides are susceptible to optical damage even at a power level of 0.1 mW due to large power density, $> 1$ kW/cm$^2$.

Although integration of the light source and several integrated optical devices on a common substrate has not been realized so far, in the integrated optical spectrum analyzer [Anderson 1978], and the multichannel data processor [Vahey et al. 1978], several components have been successfully integrated. A large number of integrated optical devices like couplers [Marcatili 1969; Somekh et al. 1973], switches [Schmidt and Kogelnik 1976; Kogelnik and Schmidt 1976], modulators [Kaminow 1975; Martin 1975; Kubota et al. 1980], deflectors [Tsai and Saunier 1975; Bulmer et al. 1979], and filters [Flanders et al. 1974] have been developed.

In this thesis integrated optical devices investigated are; a high voltage sensor, interferometric filters/modulators, and a comparatorless A/D converter. In addition, adjustment of the propagation constant of a waveguide by film loading is achieved. A review of optical waveguides and integrated optical devices is given in Chapter II. Progress in switches, couplers, modulators and filters is covered.

An input impedance equation, for a complex impedance transformed through an exponentially tapered transmission line is derived in Chapter III. It is used to obtain a characteristic equation for the propagation constant of a
linearly graded optical waveguide. The "transverse resonance" method is used.

Some integrated optical modulators and couplers depend on the path length to perform properly, controlling it to fraction of guide wavelength is impractical. So there is a need for a means to adjust the propagation constant or the effective length of the waveguide, [Mikami et al. 1977; Mikami and Zembutsu 1979]. For example, when the two arm lengths in the integrated optical Mach-Zehnder modulator (Chapter VI) are not equal, then in the absence of applied voltage the output intensity is not maximum and the intrinsic phase difference $\psi \neq 0$. Phase tuning, that is, adjustment of the propagation constant is needed to shift $\psi$ or the operating point.

In order to establish the propagation constant requirements to solve the phase tuning problem, results of Chapter III are used to model a diffused waveguide loaded with a film. In Chapter IV, numerical results on a Ta$_2$O$_5$ film loading a Ti:LiNbO$_3$ diffused waveguide at 0.6328 $\mu$m are given. Heating in O$_2$ of a Ta$_2$O$_5$ film, which loads one arm of the Mach-Zehnder modulator is used to modify the effective arm length and hence the intrinsic phase of the modulator.

The fabrication procedure for Ti diffused waveguides in LiNbO$_3$ is described in Chapter V. A polishing technique to obtain defect free edge to facilitate end fire coupling is presented. An Al lift-off procedure, to form a sputtered Ta loading film on a diffused guide, is described. A description of the measurement set-up, and the alignment procedure for the laser, lens, crystal and the detector is included. Design and performance of the integrated optical Mach-Zehnder modulator is reviewed in Chapter VI.
An application of the integrated optical devices to high voltage (HV) measurement is discussed in Chapter VII. In particular, a use of the Mach-Zehnder modulator with 88-108 μm arm separation is considered. Also, the performance of a two-mode (BOA) modulator, is described. An interferometer composed of two parallel, contiguous, but uncoupled waveguides, only one of which is electrooptic, is proposed for the HV sensing.

A compression of the passband of an interferometric modulator, is proposed by use of the series and multibranch interferometric filters/modulators; Chapter VIII. An expression for the output intensity of $N$ series interferometers, with $2^N L$ length for the $N$th electrode is derived. The measured full width half maximum (FWHM) bandwidth of a two section interferometer agrees well with the theory. Similarly, an equation for the output intensity of a multibranch filter/modulator, with $NL$ length for the $N$th electrode is obtained. The measured performance compares favorably with the computed ones. Such filters are voltage tunable.

An elimination of external comparators, from the electrooptic A/D converter (ADC) utilizing the Mach-Zehnder modulators is proposed in Chapter IX. The comparators (maximum bandwidth 300 MHz) limit the potential speed, 1-2 GHz, of the above ADCs. Each comparator is replaced by three modulators. A design of a 4-bit comparatorless ADC is given. The measured performance of a circuit to produce 1 bit is encouraging. The design and performance of a three branch network used in conjunction with the three modulators is discussed.
CHAPTER II
A REVIEW OF INTEGRATED OPTICS

2.1 Introduction

The previous work on optical waveguides, integrated electrooptic modulators, switches, and filters is reviewed. These components are likely to be used in optical communication systems to perform modulation, multiplexing, demultiplexing and filtering of signals.

Various factors affecting the fabrication of Ti:LiNbO$_3$ waveguides are discussed. Single mode waveguides are needed for efficient modulation. The effective index method to estimate the parameters of a single mode waveguide is reviewed.

External modulators with up to 10 GHz bandwidth have been demonstrated [Izutsu et al. 1978]. Such modulators are likely to replace direct modulation of laser diodes, since the direct modulation rate is limited to < 1 GHz. The modulation rate could be extended to 2 GHz, if the self-pulsations and the relaxation oscillations in the laser diode could be suppressed [Arnold et al. 1980].

The subject of integrated optics has been reviewed frequently; [Tien 1971; Marcuse 1973a; Taylor and Yariv 1977; Tamir 1977; Yariv 1979; Botez and Herskowitz 1980; Alferness 1981; Noda 1981].

2.2 Optical waveguides

Schmidt and Kaminow (1974) reported metal (Ti, Ni and V) indiffused optical waveguides in LiNbO$_3$. With this technique, the waveguide depth and width can be controlled to form single mode waveguides. Planar and three
Dimensional waveguides have been demonstrated using films on substrates [Goell 1969; Tien et al. 1969; Tien 1971; Ulrich and Weber 1972]. Planar waveguides, formed by outdiffusion of Li$_2$O from LiNbO$_3$ produce only a small change in index ($\Delta n = 0.003$) between the guide and the substrate index. Due to the large diffusion depth (500 $\mu$m), as many as 198 modes can be supported [Kaminow and Carruthers 1973]. The Ti diffusion into LiNbO$_3$ produces a $\Delta n$ as much as 0.04, and it can be controlled by the metal thickness. The depth of the waveguide can be controlled by the diffusion time and temperature. It has been found that [Noda 1980] $2^+$ valence state ions increase the extraordinary index $n_e$; NiO and ZnO increase the ordinary index $n_o$, but MgO [Noda et al. 1978a] decreases it. However, $> 2^+$ valence state ions increase $n_e$ and $n_o$. Fe$^{3+}$ and Cr$^{3+}$ replace Li, but Ti$^{4+}$ replaces Nb$^{5+}$ or Ta$^{5+}$ [Noda 1980].

Typical propagation loss in a Ti diffused waveguide is 1 dB/cm and 0.5 dB/cm at 0.6328 $\mu$m and 1.153 $\mu$m respectively. There is no increase in absorption, 0.1 dB/cm at 1.15 $\mu$m, of bulk LiNbO$_3$ due to the presence of Ti. So, a low propagation loss should also be possible for a Ti diffused waveguide [Alferness 1980]. Use of thin (<300 A) Ti layers, and removal of the undiffused TiO$_2$ from the crystal surface have been found to reduce scattering [Vahey 1980]. A single mode waveguide can be formed by the diffusion of 300 A thick and 3 $\mu$m wide Ti strip, into LiNbO$_3$ for 6 hours, at 1020°C, in O$_2$. Some problems in forming Ti:LiNbO$_3$ waveguides, are:

1. Li$_2$O outdiffusion [Miyazawa 1977; Burns et al. 1978; Ranganath and Wang 1979]
2. Phase transformation [Vahey 1980]
3. Domain inversion [Miyazawa 1979]
4. Microdefects [Ramaswamy and Standley 1975]

During metal diffusion into LiNbO$_3$ around 1000°C, Li escapes as Li$_2$O. The loss of Li increases $n_e$. As a result, a guiding layer capable of supporting a large number of unwanted modes is formed. This problem can be solved by diffusing Ti in the presence of LiNbO$_3$ powder [Burns et al. 1978a; Ranganath and Wang 1979]. Although LiCO$_3$ has been used successfully, it requires longer compensation time [Miyazawa et al. 1977]. Recently, in an attempt to heal the optical damage, it was discovered that by exposing a hot crystal to moisture laden O$_2$, outdiffusion was suppressed [Jackel et al. 1981a; 1981b]. In the devices made for this work, Li$_2$O deficiency was compensated by carrying out diffusion for 4.5 hours without the LiNbO$_3$ powder, and 1.5 hour with the powder.

Phase transformation to LiNb$_3$O$_8$ can occur near 850°C following diffusion at ~1000°C [Vahey 1980]. This can produce surface craters and scratches and contribute to scattering losses. The damage can be minimized by a rapid passage through 850°C during heating and cooling.

Domain inversion can occur in Ti diffused in the +c plane, at temperatures exceeding 1020°C. This, and the degradation of the electrooptic coefficient is ascribed to a lowering of the Curie temperature [Miyazawa 1979]. Thus, Ti-diffused waveguides in Z-cut LiNbO$_3$ should be formed in the -c plane. The problem of misfit-dislocations, and microcracks running parallel and perpendicular to the c-axis are due to the stress created by the Ti diffusion [Ramaswamy and Standley 1975].
2.2.1 Optical damage

When visible light at power densities of 1-10 kW/cm$^2$ is fed into a waveguide, the near field light pattern changes [Tangonan et al. 1977]. This phenomenon is known as optical damage or the photorefractive effect. While it has been utilized for writing holograms [Young et al. 1974; Vahey et al. 1978], it has deleterious effect on other integrated optical devices with narrow waveguides. For example, it degrades the cross-talk of the coupler-switch [Schmidt et al. 1980], and it produces drift in the output of Mach-Zehnder modulator [Sasaki 1977].

LiNbO$_3$ has the same value for the $r_{33}$ electrooptic coefficient as LiTaO$_3$, but its optical threshold, $\sim$ 100 W/cm$^2$, is only 0.01 of LiTaO$_3$. The Curie temperature of LiNbO$_3$ is 1125°C, but that of LiTaO$_3$ is only 600°C, so that, repoling of LiTaO$_3$ is necessary after the diffusion at $\sim$ 1000°C. Recent work on the formation of 0.5 dB/cm loss optical waveguides by Ag and Li exchange at 250°C would eliminate Li$_2$O outdiffusion and repoling problems [Jackel 1980]. One limitation of this method is that waveguides can be formed only in X-cut LiNbO$_3$ and LiTaO$_3$ crystals. This would still permit utilization of $r_{33}$.

2.2.2 Drift phenomenon

The output of an integrated optical device, such as a Mach-Zehnder modulator, can drift at constant dc bias [Sasaki 1977]. One possible source of the short term drift is an incomplete oxidation of the SiO$_2$ or Al$_2$O$_3$ buffer layer between the crystal and the electrodes [Tangonan et al. 1978]. The buffer layer is required to reduce loss, particularly to the TM mode. The drift can be halted by an anneal in O$_2$, or by removal of the buffer layer from
the area between the electrode edges and on top of the waveguide. SiO₂ layer formed by chemical vapour deposition (CVD) works better than a sputtered layer. The long term drift is due to the optical damage, and it is virtually absent at wavelengths exceeding 1 μm.

2.3 Waveguide analysis

An analysis of waveguides is necessary to estimate the parameters of a single mode waveguide, and to compute modulation efficiency. The modulation efficiency is determined by overlap of the optical and the applied electric fields.

The number of modes a waveguide can support and the cut-off conditions, are determined by the waveguide depth and width, and refractive indices of the substrate (nₛ) and the waveguide (n₇). In general, the refractive index of the guide varies in the transverse direction. Waveguides with various profiles have been analyzed; linear [Chen et al. 1974], parabolic [Tamir 1979], exponential [Conwell 1973], etc. Exact solution for the propagation constant, is possible only for two profiles, namely, the exponential, and the Morse. For other index profiles, approximate methods like the WKB [Marcuse 1973b; Hocker and Burns 1977], transverse F-matrix method [Suematsu and Furuya 1972], and the variational method [Geshiro et al. 1978] must be used. The WKB approximation yields results with a 10⁻⁵ accuracy relative to the exact method for the exponential profile.
2.3.1 WKB and the effective index method [Noda 1980]

The WKB approximation in conjunction with the effective index method [Hocker and Burns 1977] is applied to a waveguide loaded with a film of refractive index $n_f$ and thickness $t$. The refractive index profile for a waveguide confined in both of the transverse dimensions is,

$$n(x, y) = n_s + \Delta n \frac{f(x/d_x)}{f(x/d_x)} \frac{g(y/d_y)}{g(y/d_y)}$$  \hspace{1cm} (2.1)

where, $n_s$ is the isotropic substrate refractive index, $\Delta n (=n_o - n_s)$ is the maximum change in refractive index at the surface, $f(x/d_x)$ is the refractive index profile function perpendicular to the surface, and $g(y/d_y)$ parallel to it. Terms $d_x$ and $d_y$ are the diffusion lengths. The index profile during diffusion is initially erfc, and Gaussian later on. Representative index profiles are shown in Fig. 2.1 and 2.2. With the WKB approximation and the effective index method, the x-axial mode dispersion equation is,

$$k \int_0^x \left[ n^2(x) - n_x^2 \right]^{1/2} dx = (p + 1/4) \pi + \phi$$  \hspace{1cm} (2.2)

$$n(x) = n_s + \Delta n \frac{f(x/d_x)}{f(x/d_x)} = n(x,o)$$  \hspace{1cm} (2.3)

$$\phi = \tan^{-1} \left[ \frac{n(P_a/P_f)R}{1 + (P_a/P_f)\tan^2(P_f t)} \right]$$  \hspace{1cm} (2.4)

$$R = \frac{\xi P_a/P_f - \tan^2(P_f t)}{1 + (P_a/P_f)\tan^2(P_f t)}$$  \hspace{1cm} (2.5)

$$P_i = k\left| n_i^2 - n_x^2 \right|^{1/2}, \hspace{1cm} i = a, f$$  \hspace{1cm} (2.6)

For TE modes; \hspace{1cm} $\eta = \xi = 1$  \hspace{1cm} (2.7)
Fig. 2.1. Index profile perpendicular to the surface (Noda 1980).

Fig. 2.2. Index profile parallel to the surface (Noda 1980).

Fig. 2.3. Linear segment approximation of the index profile (Noda 1980).
For TM modes; \[ n = \left(\frac{n_a}{n_f}\right)^2, \quad \xi = n_a^2 \] (2.8)

Here, \( k = 2\pi/\lambda_o \) is the free space wavenumber, \( x_t \) is the turning point, \( n_a \) and \( n_f \) are refractive indices of air and the loading film, and \( p \) is the mode number. And \( \tan^* = \tan \) for \( n_f > n(o) \), but \( \tan^* = \tanh \) for \( n_f < n(o) \).

In the absence of a loading film, \( t = 0 \), \( \Delta n/n_s \) is small, and \( n(o) \gg n_a \), so that \( \phi = \pi/2 \). Then (2.2) with \( k[n^2(x) - n_x^2]^{1/2} = b_1(x) \) becomes,

\[
\int_{x_0}^{x_t} b_1(x) \, dx = (p + 0.75)\pi, \quad p = 0, 1, 2, \ldots 
\] (2.9)

\[
b_1^2(x) + \beta^2 = k^2 n^2(x) 
\] (2.10)

and \( b_1(x_t) = 0 \) (2.11)

For a given index profile \( n(x) \), \( b_1(x) \) can be computed from (2.10) and \( x_t \) from (2.11). Then the integral in (2.9) can be calculated for any value of \( \beta \) [Tien et al. 1971]. Values of \( \beta \) are distinct for different modes. For an exponential profile [Conwell 1973], \( \Delta n = 0.043 \), \( n_s = 2.177 \), and \( d = 0.931 \) \( \mu m \), for the two possible modes, \( \beta = 2.1899 \) and 2.1790.

For a waveguide with uniform refractive index

\[
b_1 d = (q + \frac{1}{4})\pi + \phi \] (2.12)

When the waveguide is not planar but has a width \( 2w \), the \( y \)-axial mode dispersion equation, assuming step refractive index is, [Noda 1980],

\[
k \int_{-y_t}^{y_t} \left[ n^2(y) - n^2_y \right]^{1/2} dy = (q + \pi/2) - wS 
\] (2.13)
\[
S = k[n^2(o) - n_y^2]^{1/2} \quad (2.14)
\]

\[
n_y = n(x_t, y) \quad (2.15)
\]

Where, \(x_t\) is the turning point, and \(q\) the mode number. At the waveguide surface \(n(o)\) is replaced by \(n_x\).

2.3.2 Linear segment approximation analysis [Noda 1980]

The wave equation for the electric or magnetic field \(G\) of a planar waveguide along \(x\) is given by,

\[
\frac{d^2 G}{dx^2} + k^2[n^2(x) - n_x^2] G = 0 \quad (2.16)
\]

The derivative of the dielectric constant has been neglected, as the refractive index variation is very gradual with wavelength. By considering the index profile to be made up of several linear segments, as shown in Fig. 2.3, the solutions of (2.16) are [Noda and Fukuma 1980];

\[
G = C \exp[P_f(x+t)] \quad -\infty < x < -t
\]

\[
= A_f \cos(P_f x) + B_f \sin(P_f x), \quad n_f > n(o), -t < x < 0
\]

\[
= A_f \cosh(P_f x) + B_f \sinh(P_f x), \quad n_f < n(o), -t < x < 0
\]

\[
= \sqrt{w_i}[A_1 J_{1/3}(u_1) + B_1 J_{-1/3}(u_1)], \quad x < x_t, x_t < x < x_{t+1}
\]

\[
w_i = n_1^2 - a_i(x - x_t) - n_x^2
\]
\[ w_j = n_x^2 - n_j^2 + a_j(x - x_j) \]

\[ a_{i,j} = 2n_{1,j}(n_{1,j} - n_{i+1,j+1})/\Delta x \]

\[ u_{i,j} = (2k/3a_{i,j}) w_{i,j}^{3/2} \]  \hspace{1cm} (2.17)

Where, \( \Delta x = (x_{i+1} - x_i) \) is the layer thickness, and \( J_n \) are the Bessel functions. \( A, B \) and \( C \) are the amplitude components established by matching electric or magnetic fields, and their derivatives at layer boundaries. The above relations are useful in estimating the overlap of the optical and the applied electrical fields, and the overlap integral of fibre and guide fields for coupling estimation.

A planar waveguide is shown in Fig. 2.4. Various regimes of the propagation constant \( \beta \), and the corresponding field distributions are shown in Fig. 2.5 and 2.6. When \( \beta < kn_a \), only the air radiation modes are excited. With \( kn_a < \beta < kn_s \), the substrate radiation modes are supported. For the radiation modes, \( \beta \) is continuous. However, for discrete \( \beta \) in the range \( kn_s < \beta < k(n_s + \Delta n) \), guided modes propagate. In the case of guided modes, the field is nearly sinusoidal in the \( n \) region. It decays exponentially in the air and substrate regions. The transition from sinusoidal to exponential occurs at the turning point [Conwell 1973]. The smaller the value of \( \beta \) the higher the mode, and the larger is the distance at which oscillatory behaviour exists. The number of nodes correspond to the mode order, see Fig. 2.7. Also, the wave amplitude increases as the turning point is approached, as shown in Fig. 2.7. The propagation loss is greater for the higher order
Fig. 2.4. Planar optical waveguide.

Fig. 2.5. Propagation constants and electric field distributions of TE modes (Taylor and Yariv 1974).

Fig. 2.6. Typical $\omega-\beta$ diagram of a dielectric waveguide (Tamir 1979).
Fig. 2.7. TE mode electric field distribution (Conwell 1973).

Fig. 2.8. Optical intensity distributions in a 3-D waveguide (Noda and Fukuma 1980).
modes, as illustrated by the ray model [Tien 1971] in Fig. 2.6. The higher order modes undergo a large number of reflections accompanied by scattering, at the guide boundaries.

2.4 Modulators

Modulators are needed in optical communication systems. The information is impressed on the optical wave as phase modulation using the linear electrooptic effect. The phase modulation can be converted to intensity modulation through interference, mode conversion or diffraction. Two other types of modulators are the waveguide cut-off and the directional coupler modulators. In this section, the strip-waveguide modulator, the coupled guide modulator and the Y-branch intensity modulator are reviewed.

2.4.1 Parameters of a phase modulator

Change in the refractive index is given by

$$\Delta n = \frac{1}{2} \frac{n^3 r V}{d}$$

(2.18)

where $n$ is the refractive index, $r$ the appropriate electrooptic coefficient, $V$ the applied voltage and $d$ the electrode gap. The change in phase $\phi$ due to $\Delta n$, with electrode length $L$, at free space wavelength $\lambda_o$ is,

$$\phi = \eta \left( \frac{\pi}{\lambda_o} \right) n^3 \frac{r V}{d} L$$

(2.19)

where $\eta$ is determined by the optical and the electric field overlap. The power to drive the capacitance $C$ over bandwidth $\Delta f \approx (\pi RC)^{-1}$ is,
\[ P = \frac{V^2}{2R} = \frac{\pi}{2} \frac{\Delta f}{V} CV^2 \]  

Then the power required per bandwidth for phase shift \( \phi \) is,

\[ \frac{P}{\Delta f \phi^2} = \frac{\pi}{2} \frac{CV^2}{\eta^2} \left( \frac{\lambda_0}{\pi} \frac{d}{n^2 r} \frac{1}{V} \right)^2 = \frac{\lambda_0^2}{2\pi^2} \left( \frac{1}{n^2 r^2} \right) \left( \frac{Cd^2}{L} \right) \]

This is a measure of the performance of the modulator, and it is usually quoted in mW/MHz rad^2. The term \( 1/(n^6 r^2) \) is determined by the material properties, but the term \( Cd^2/L^2 \) depends on the electrode geometry.

2.4.2 Strip-waveguide modulator

The modulator electrode structure can be a lumped element network or a travelling wave structure. In the case of the lumped element version, the bandwidth is limited by the RC product, where R is the impedance of the system, and C the electrode capacitance. For the phase modulator shown in Fig. 2.9, the electrode capacitance is given by [Kaminow et al. 1975],

\[ C = \frac{\varepsilon_0}{\pi} L \left( 1 + K_m \right) \ln \left( \frac{W}{d} \right) F \]

where \( L, d \) and \( W \) are the electrode length, gap, and outer edge distance respectively, and \( K_m \) is related to the principal dielectric constants. The bandwidth can be increased by using shorter electrodes, but at the expense of higher modulating voltage to achieve the same phase change. The phase modulator shown in Fig. 2.9 requires only 0.3 V/rad and 1.7 \( \mu \)W/MHz rad^2 at 0.6328 \( \mu \)m.

The bandwidth of a lumped element modulator can be tripled by using a travelling wave modulator [Izutsu et al. 1977;1978]. The bandwidth is limited
Fig. 2.9. Schematic diagram of a strip-waveguide phase modulator (Kaminow et al. 1975).

Fig. 2.10. Travelling wave LiNbO$_3$ modulator using asymmetric electrodes (Izutsu et al. 1978).
by the optical and the electric field transit time difference [Izutsu et al. 1977];

\[ \Delta f = \frac{1.4c}{(\pi L |n_{rf} - n|)} \]  

(2.23)

For LiNbO$_3$ $n_{rf} = \sqrt{n_s n_{air}} = 4.3$ and $n = 2.2$, but for GaAs $n_{rf} = n$ so that electrode length can be increased without sacrificing the bandwidth [Alferness 1981]. A travelling wave phase modulator with a single guide is shown in Fig. 2.10. The electrodes form a microwave coplanar transmission line terminated in 50Ω. For a Ti diffused modulator in Y-cut LiNbO$_3$ at 0.6328 μm, a bandwidth of 10 GHz was achieved [Izutsu et al. 1977; 1978]. The electrode parameters were: length = 9.1 mm, width = 35 μm, gap = 55 μm, and thickness = 2 μm. The electrode loss was 3 dB at 10 GHz. The on/off ratio was 4.5 dB at 11V.

2.4.3 Directional coupler modulator

A directional coupler modulator with travelling wave electrodes is shown in Fig. 2.11, [Kubota et al. 1980]. In order to reduce the electrode losses, asymmetric 3-μm thick Al electrodes were used. The optical insertion loss was 5.4 dB, the 3 dB bandwidth was 3.6 GHz, the extinction ratio was 17 dB at 1.317 μm, and 100% modulation at 4V was obtained. A directional coupler modulator with alternating $\Delta \beta$ with a bandwidth of 1 GHz and rise time of 590 ps has been reported [Cross and Schmidt 1979].
Fig. 2.11. A directional coupler modulator with travelling wave electrodes (Kubota et al. 1980).

Fig. 2.12. Integrated optical Mach-Zehnder modulator.
2.4.4 Integrated optical Mach-Zehnder modulator

A Y-branch intensity modulator is an integrated optic version of a Mach-Zehnder modulator [Martin 1975]. The two Y's serve as a 3 dB splitter and a combiner as shown in Fig. 2.12. If the phase of the light in one arm of the modulator is retarded by 180°, then the output intensity is minimum upon recombination. The output is maximum with no phase retardation. For an electrode gap and length of 4 μm and 3 mm respectively, a $V_\pi$ of 4.5 was required at 0.6328 μm, in Y-cut LiNbO₂. The extinction ratio was 98% [Leonberger et al. 1979]. Push-pull configuration of electrodes is usually employed. A polarization insensitive modulator has been demonstrated [Burns et al. 1978b]. Use of the modulators as A/D converters [Taylor 1975] and as logic elements [Taylor 1977] has been proposed. The Y-branch modulator is covered in greater detail in Chapter VI.

2.5 Switches [Alferness 1981]

Switches change the spatial location of light in response to an applied voltage. Applications of switches include time division multiplexing and demultiplexing, in order to better utilize the bandwidth capability of optical systems by combining several low bit rate signals. Switches have also been used as ADCs and modulators. A 2 x 2 switch has two output and two input ports. The performance criteria of a good switch are; low insertion loss, small switching voltage, isolation or low crosstalk, and high speed. Two types of switches are, the directional coupler switch and the balanced modulator switch.
2.5.1 **Directional coupler switch**

A coupler switch consists of two parallel waveguides, separated by a gap g, over a length L, along which two codirectional coupled waves propagate. A schematic diagram of a typical device is shown in Fig. 2.13, it has been analyzed extensively [Marcatili 1969; Somekh et al. 1973; Taylor 1973; Tamir 1979].

In a coupler, the two guides are such that their propagation constants $\beta_1$ and $\beta_2$ are equal, the gap g is small so that the evanescent fields overlap, and L is long. The light $R_0$ that is initially in guide 1 gets transferred to guide 2 after travelling a distance L, and back to guide 1 after additional distance L, and so on. Here, L the critical length is given by

$$L = \frac{\pi}{2\kappa}$$  \hspace{1cm} (2.24)

Where $\kappa$ is the coupling/unit length, determined by the waveguide parameters, gap g, and the wavelength. When the two guides are not phase-matched,

$$\Delta \beta = \beta_2 - \beta_1 = k(N_2 - N_1) = 2\delta$$

where $N_1$, $N_2$ are the effective guide indices. The coupling is obtained by solving the coupled wave equations,

$$R' - j \delta R = -j \kappa S$$
$$S' - j \delta S = -j \kappa R$$  \hspace{1cm} (2.25)

The crossover intensity $\eta = I_1/I_2$ is given by,

$$\eta = \frac{2}{\kappa^2 + \delta^2} \sin^2 L \sqrt{\frac{\kappa^2}{\kappa^2 + \delta^2}}$$  \hspace{1cm} (2.26)

For identical waveguides, $N_1 = N_2$ and $\delta = 0$, so
Fig. 2.13. Schematic of an electrooptic directional coupler switch.

Fig. 2.14. Variation of power in the initially excited guide with the propagation length (Papuchon et al. 1975).
\[ n = \sin^2 \kappa L \]  

(2.27)

and complete crossover occurs at

\[ \kappa L = \frac{n\pi}{2}, \quad n = 1,3,5,... \]  

(2.28)

When \( \delta \neq 0 \), complete transfer is impossible. This is illustrated in Fig. 2.14. Phase mismatch, \( \Delta \beta \), can be introduced by applying voltage to the electrodes. For example, with electrodes on top of the waveguides in Z-cut LiNbO\(_3\) to utilize \( r_{33} \),

\[ \Delta \beta = \frac{2\pi}{\lambda} n e r_{33} \frac{V}{d} \Gamma \]  

(2.29)

where \( \Gamma \) is the overlap between the optical and electric fields, \( V \) is the applied voltage and \( d \) the electrode gap. The coupling length is 200 \( \mu \text{m} \cdot \text{cm} \).

The two coupling states are the cross state, and the through or bar state. The bar state is realized by applying voltage to produce \( \Delta \beta \), so that \( n = 0 \) for \( \kappa L = \frac{\pi}{2} \); then \( \Delta \beta \kappa = \sqrt{3} \pi \). The voltage required to switch from one state to the other, can be reduced by increasing \( \kappa \) or reducing \( d \). The crossover \( n \) can be modified through \( \Delta \beta \) or \( n \).

In order to have a cross state with low crosstalk, the value of \( \kappa L \) has to be controlled accurately. The difficulty of controlling \( L \) to within \( \pm 3.5\% \) for -25 dB crosstalk [Papuchon 1975], can be overcome by using a stepped \( \Delta \beta \) reversal in the coupler [Schmidt and Kogelnik 1976].

A switched directional coupler with alternating \( \Delta \beta \) over two sections is shown in Fig. 2.15. A phase mismatch \( \Delta \beta = \beta_1 - \beta_2 \), between the propagation constants of the two coupled waveguides is induced by the applied voltage.
Fig. 2.15. A reversed $\Delta \beta$ directional coupler switch (Schmidt and Kogelnik 1976).

Fig. 2.16. (a) Corrugated waveguide filter, and its response (Flanders et al. 1974).
When the phase mismatch is equal but opposite in the two sections, incident energy $R_o$ from the top guide is transferred to the bottom guide, i.e. crossover state exists. On the other hand, when there is uniform mismatch, any energy that is transferred from the top guide to the bottom guide in the first half, is returned to the top guide in the second half; then bar state exists. For sufficiently large $L/\lambda$, a voltage always exists to achieve complete cross state. To switch, from one state to the other, a change in $\Delta\beta L$ of less than $2\pi$ is required.

A two section reversed $\Delta\beta$ coupler [Schmidt and Kogelnik 1976], had a crosstalk of -27 dB, and a switching voltage of 27 V. Recently, a six section alternating $\Delta\beta$ coupler, with an interaction length of \~ 1 cm, required a drive voltage of \~ 3 V, and has a 10 dB extinction ratio. It had bandwidth of 1 GHz, and a rise time of 590 ps, [Cross and Schmidt 1979]. The device was in Z-cut LiNbO$_3$. A polarization insensitive stepped $\Delta\beta$ switch has been formed with a variable gap coupler [Alferness 1979]. The polarization independent cross state is achieved with reversed $\Delta\beta$, and at an operating point insensitive to $\Delta\beta$ variations. Shaped transfer characteristic with weighted coupling is used to obtain polarization independent bar state. Although $\Delta n_{TE} \neq \Delta n_{TM}$, the coupling strength is equalized by suitable choice of the waveguide and coupler parameters, that is, $\kappa$ and $L$ etc. are chosen so that $S_{TE} = S_{TM}$ at $\Delta\beta_{TE}/\Delta\beta_{TM} \approx 1/3$. 

$13/33 = 1/3$. 

s
2.6 **Filters** [Alferness 1981]

The information-carrying capacity of an optical fibre communication system can be better utilized by combining several channels. Wavelength selective filters are required to multiplex and demultiplex the channels. The filters are characterized by wavelength, bandwidth, insertion loss, and electrical tunability.

Narrow-band (5-15 Å) filters are needed to accommodate several narrow channels with small interchannel spacing. But broad-band (200-300 Å) filters are required because of source drift with temperature, and fabrication uncertainties. Thus, filters with a wide range of bandwidths are needed.

Three types of filters that have been demonstrated are; the corrugated or grating filter, TE→TM mode converter filter, and the directional coupler filter.

2.6.1 **Corrugated/grating filter** [Tamir 1979]

The corrugated filter has a periodic perturbation, which produces a periodic index change for phase matching. For a perturbation period \( \Lambda \), at an effective index \( N \), a backward reflection at wavelength \( \lambda_o \) meets the Bragg condition, \( \lambda_o = 2N\Lambda \). Typically \( \Lambda < 0.5 \) µm for \( \lambda_o = 0.5-1.5 \) µm. The fractional bandwidth is \( \frac{\Delta \lambda}{\lambda} = \frac{2\Lambda}{L} \). The coupling constant \( \kappa \) is a function of the corrugation depth and the waveguide parameters. Typical filter length is 1 cm. A filter and its response is shown in Fig. 2.16. A filter with as little bandwidth as 0.1 Å has been reported [Schmidt et al. 1974].
2.6.2 Mode converter filter [Alferness 1981]

In LiNbO$_3$ because of large birefringence, $(n_o-n_e=0.09$ at $\lambda=0.6328$ $\mu$m), the effective indices $N_{TE}$ and $N_{TM}$ are quite different. Since the birefringence changes with wavelength, the TE$\leftrightarrow$TM mode conversion varies with wavelength. However, if phase matching condition,

$$\frac{2\pi}{\lambda_o} |N_{TE}-N_{TM}| = \frac{2\pi}{\Lambda}$$ (2.30)

at $\lambda_o$ is met, then there is strong coupling between the asynchronous modes TE and TM. Here, $\Lambda$ is the electrode period. The mismatch at $\lambda = \lambda_o + \Delta\lambda$ is $\Delta\beta=-2\pi\Delta\lambda/\Lambda\lambda$. The conversion efficiency $\eta$ is given by (2.26), and the coupling constant $\kappa$ by (2.27). The full width half maximum (FWHM) bandwidth is $\Delta\lambda/\lambda=A/L$. In the case of LiNbO$_3$, $n_o-n_e = 0.086$ at $\lambda = 0.6328$ $\mu$m, and the electrode period $A = 7$ $\mu$m. For LiTaO$_3$, the birefringence is smaller and the bandwidth wider for the same $A$.

A filter with directed field utilizes the $r_{51} = 28 \times 10^{-12}$ m/V coefficient, and a bandwidth 50 $\Lambda$ - 5 $\Lambda$ can be obtained at $\lambda = 0.6328$ $\mu$m for $\Lambda = 0.5 - 6$ mm. The bandwidth is four times as much at 1.3 $\mu$m, all else being the same. Performance of a three channel converter is shown in Fig. 2.17 [Alferness 1981]. The electrodes were tilted to obtain small period differences. The bandwidth at 50% conversion efficiency is 15 $\Lambda$, and the channel separation 80 $\Lambda$, at a crosstalk of 20 dB.
Fig. 2.17. (a) Phase-matched electrooptic TE\leftrightarrow TM converter/filter; and (b) measured conversion efficiency (Alferness 1981).
2.6.3 Directional coupler filter [Alferness 1981]

This type of filter is made up of two coupled waveguides of unequal dimensions and refractive indices, Fig. 2.18. As a result, the modal characteristics of the two guides are distinct. If width $W_1 > W_2$ then index $N_1 < N_2$. At a wavelength $\lambda_o$, where phase matching exists, there is complete transfer of power from one guide to the other. At a different wavelength $\lambda = \lambda_o + \Delta \lambda$, the mismatch is $\frac{2\pi}{\lambda} \frac{\Delta \lambda}{\lambda}$, and the fractional bandwidth is;

$$\frac{\lambda}{L} \frac{1}{L} \left| \frac{d}{d\lambda} (N_2 - N_1) \right| = \frac{\lambda_o}{L} \geq 100 \text{ A} \tag{2.31}$$

The filter is tuned to a different wavelength by shifting the dispersion characteristics with an applied voltage. Complete crossover can be affected by reversed $\Delta \phi$ coupler [Alferness and Schmidt 1978]. For a Ti:LiNbO$_3$ directional coupler filter, with $L = 1.5 \text{ cm}$, the 3 dB bandwidth is 200 A at $\lambda = 6.33 \text{ \mu m}$, at a peak crossover efficiency $\eta = 100\%$. The tuning range is 600 A, with a tunability of 100 A/V.

The -3 dB bandwidth is,

$$\frac{\Delta \lambda_v}{\Delta \lambda_{BW}} = \frac{\Delta N_v L}{\lambda} \tag{2.32}$$

where a change in wavelength $\Delta \lambda_v$, corresponds to a change in electrically induced $\Delta N_v$ which is equal to $(N_{TE} - N_{TM})$ or $(N_2 - N_1)$. The electrical tunability of a filter is given by

$$\Delta \lambda_v = \Lambda \Delta N_v \tag{2.33}$$
Fig. 2.18. The directional coupler filter: (a) schematic representation; waveguide dispersion; and filter response (Alferness 1981).
CHAPTER III
TAPERED LINE AND LINEARLY GRADED WAVEGUIDES

3.1 Introduction

A new and simple set of characteristic equations for the propagation constant of an optical waveguide with linearly graded refractive index are obtained. To this end, tapered transmission line theory, and the method of "transverse resonance" are used.

A planar optical waveguide is made up of three dielectric layers; the substrate, film and cover, Fig. 2.4. In general, the refractive index \( n_f \) of the guiding layer varies in the transverse direction \( x \). The characteristic equations for \( k_z \) are obtained by solving Maxwell's equations. The solutions involve Bessel functions, Hermite polynomials, etc. Computations using such complicated functions are expensive. So, an alternative approach using the method of "transverse resonance" [Tamir 1973] is followed here for the case when \( n_f(x) \) is a linear function. First the substrate, film and cover that make up the optical waveguide are represented by an equivalent transmission network in the transverse \( x \)-direction. Then the propagation constants are determined by solving the eigenvalue equation. The refractive index of the film is variable in the transverse direction, consequently the characteristic impedance by which it is represented is variable too. After laying the ground-work in Section 3.2, an impedance transformation equation for a non-uniform transmission line is derived in Section 3.3. Finally, the
characteristic equations for the propagation constant corresponding to the TE and the TM modes are obtained in Sections 3, 4 and 5 respectively.

In Chapter IV this work is applied to the study of non-uniform films deposited on a diffused waveguide structure. Such film and waveguide combinations have the potential for the phase tuning of the integrated optical devices.

3.2 Differential equation for input impedance

In this section, a second order nonlinear differential equation for the input impedance of a tapered transmission line is derived. This equation is then solved for exponentially tapered lines [Ahmed 1981].

For a uniform-transmission line of characteristic impedance $Z_0$, a load impedance $Z_a$ when transformed a distance $\ell$ towards the generator is $Z_{in}$ [Ragan 1948],

$$\frac{Z_{in}}{Z_0} = \frac{Z_a + jZ_0 \tan \beta \ell}{Z_0 + jZ_a \tan \beta \ell}$$  \hspace{1cm} (3.1)

The characteristic impedance of a non-uniform transmission line is variable; it is a function of distance $\ell$, thus $Z_0$ becomes $Z(\ell) = Z$ (for brevity). To determine an expression for the input impedance consider Fig. 3.1, and let $Z_{in}$ be input impedance at $\ell$, and $(Z_{in} + dZ_{in})$ at $(\ell + d\ell)$. If it is assumed that the line is of uniform characteristic impedance $Z(\ell) = Z$, over the incremental distance $d\ell$, then using (3.1),

$$\frac{Z_{in} + dZ_{in}}{Z} = \frac{Z_{in} + jZ \tan (\beta d\ell)}{Z + jZ_{in} \tan (\beta d\ell)}$$  \hspace{1cm} (3.2)
Fig. 3.1. Schematic representation of a generalised tapered transmission line.

Fig. 3.2. Schematic representation of an exponentially tapered line.
For small $\beta d\xi$, $\tan(\beta d\xi) = \beta d\xi$, using this in (3.2)

\[
\frac{Z_{in} + dZ_{in}}{Z} = \frac{Z_{in} + jZ\beta d\xi}{Z[1 + \frac{jZ\beta d\xi}{Z}]} \tag{3.3}
\]

The denominator of (3.3) can be expressed as,

\[
[1 + \frac{jZ_{in}\beta d\xi}{Z}] = [1 - \frac{jZ_{in}\beta d\xi}{Z}] \tag{3.4}
\]

Using this in (3.3)

\[
Z_{in} + dZ_{in} = (Z_{in} + jZ\beta d\xi)(1 - jZ_{in}\beta d\xi) \tag{3.5}
\]

Ignoring the products of differential terms [Collin 1966];

\[
\frac{dZ_{in}}{d\xi} = \frac{-j\beta Z_{in}^2}{Z} + j\beta Z \tag{3.5}
\]

Letting

\[
Z_{in} = y \tag{3.6}
\]

\[
R = j\beta/Z \tag{3.7}
\]

Substituting (3.6) and (3.7) in (3.5)

\[
\frac{dy}{d\xi} = -Ry^2 - \frac{\beta^2}{R} \tag{3.8}
\]

Equation (3.8) is the well known Riccati equation, which can be transformed into a homogeneous linear differential equation of second order and then solved [Davis 1962].

Making the transformation

\[
y = \frac{1}{Ru} \left(\frac{du}{d\xi}\right) \tag{3.9}
\]

or

\[
y = \frac{Z}{j\beta} \left(\frac{du}{ud\xi}\right) \tag{3.10}
\]
Differentiating (3.10) with respect to \( \ell \),

\[ \frac{dy}{d\ell} = \frac{1}{j\beta} \frac{dZ}{d\ell} \frac{1}{d\ell} - \frac{Z}{j\beta u} \left[ \frac{du}{d\ell} \right]^2 + \frac{Z}{j\beta u} \frac{d^2u}{d\ell^2} \]  \hspace{2cm} (3.11)

Substituting (3.9) into (3.8),

\[ \frac{dy}{d\ell} = -uR^2 \left\{ \frac{1}{R_u} \right\}^2 \left[ \frac{du}{d\ell} \right]^2 - \frac{\beta^2}{R} \]  \hspace{2cm} (3.12)

Multiplying (3.11) and (3.12) by \(-uR^2\) and then equating the right hand sides and using (3.7),

\[ \frac{-uR^2}{J} \frac{dZ}{d\ell} \frac{du}{d\ell} + \frac{uR^2Z}{J} \left[ \frac{du}{d\ell} \right]^2 - \frac{uR^2Z}{J} \frac{d^2u}{d\ell^2} \]

\[ = uR^3 \left\{ \frac{1}{R_u} \right\}^2 \left[ \frac{du}{d\ell} \right]^2 + uR\beta^2 \]

Substituting value of \( R \) from (3.7),

\[ \frac{-uJ\beta \frac{dZ}{d\ell} \frac{1}{d\ell} - \frac{uj\beta \frac{d^2u}{d\ell^2}}{Z^2}}{Z u \frac{d\ell}{d\ell}} = \frac{uj\beta^2}{Z} \]

or

\[ \frac{d^2u}{d\ell^2} + \frac{1}{Z} \frac{dZ}{d\ell} \frac{du}{d\ell} + \beta^2u = 0 \]  \hspace{2cm} (3.13)

Equation (3.13) will serve as the springboard for deriving expressions for the input impedances of the exponentially tapered transmission line.

3.3 Exponentially tapered line

Along an exponential taper, Fig. 3.2, \( \text{ln} Z(\ell) \) varies linearly with \( \ell \) [Burrow 1938; Jasik 1961; Womack 1962; Berquist 1972];
\ln Z = \ln Z_1 + \frac{\xi}{L} \ln \left(\frac{Z_2}{Z_1}\right) \tag{3.14}

Differentiating with respect to \(\xi\),

\[ \frac{1}{Z} \frac{dZ}{d\xi} = \frac{1}{L} \ln \left(\frac{Z_2}{Z_1}\right) = 2k \text{ (say)} \tag{3.15} \]

Substituting (3.15) in (3.13),

\[ \frac{d^2u}{d\xi^2} + 2k \frac{du}{d\xi} + \beta^2 u = 0 \tag{3.16} \]

Solution of (3.16) is given in (A14), which after appropriate substitution yields,

\[ Z_{\text{in}} = \frac{Z_1^a \left\{ (\beta^2-k^2)^{1/2} - k \tan(\beta^2-k^2)^{1/2} \} + j\beta \tan(\beta^2-k^2)^{1/2} \right\}}{Z \left\{ (\beta^2-k^2)^{1/2} + k \tan(\beta^2-k^2)^{1/2} + \frac{j\beta Z_1}{Z_1^a} \tan(\beta^2-k^2)^{1/2} \right\}} \tag{3.17} \]

where

\[ k = \frac{1}{2} \left( \frac{1}{Z} \frac{dZ}{d\xi} \right) \]

\[ Z(\xi) = Z_1 \left\{ \exp \frac{\xi}{L} \ln \left(\frac{Z_2}{Z_1}\right) \right\} \tag{3.18} \]

Although exponential transmission lines have been studied [Ghose 1957; Das and Rustogi 1968; Arnold and Bailey 1974; Arnold et al. 1974], an input impedance expression in a convenient form like (3.17) is not available.
3.4 Dispersion equation for a linearly graded index guide

The method of transverse resonance [Tamir 1973] is used to obtain the dispersion equation for a planar optical waveguide with linearly graded refractive index. The three media structure is represented by an equivalent transmission network corresponding to the transverse x direction, as indicated in Fig. 3.3. The propagation constants in the longitudinal direction for the guided modes are determined by solving the eigenvalue equation,

\[ \hat{Z} + \hat{Z} = 0 \] (3.19)

or

\[ Z_{in} + Z_{r} = 0 \] (3.20)

For the waveguide structure of Fig. 3.3 where \( n_a, n(x), \) and \( n_r \) are the refractive indices of the substrate, film and cover respectively, and \( Z_a, Z(x) \) and \( Z_r \) are the corresponding impedances.

3.4.1 TE mode

The impedance equation for the TE mode is [Tamir 1979: 109],

\[ Z_1 = \omega \mu_0 / k_{x1} \] (3.21)

where \( Z_1 \) is the characteristic impedance and \( k_{x1} \) is the propagation factor in the transverse x direction. The dispersion relation is

\[ k_{x1}^2 + k_z^2 = k_n^2 n_1^2 \]

Dropping the suffix 1, and using \( k_z = k n_1 \), where \( n_1 \) is the effective index in the longitudinal propagation z direction. Refractive index \( n \) is a function of \( x \).
\[ k_x = k(n^2 - n_1^2)^{1/2} \quad (3.22) \]

\[ Z = \frac{\omega \mu_0}{k} \cdot \frac{1}{\sqrt{n^2 - n_1^2}} \]

\[ \frac{dZ}{dx} = \frac{-\omega \mu_0}{k} \cdot \frac{n}{(n^2 - n_1^2)^{3/2}} \frac{dn}{dx} \]

\[ \frac{1}{Z} \frac{dZ}{dx} = -\frac{n}{(n^2 - n_1^2)} \frac{dn}{dx} \]

If the variation in \( n \) is small \( n \approx n_c \) [Heibei and Voges 1978], that is, \( k_x \approx k_c \) then

\[ \frac{1}{Z} \frac{dZ}{dx} = -\frac{n_c}{(n_c^2 - n_1^2)} \frac{dn}{dx} \quad (3.23) \]

When the refractive index changes linearly with distance as shown in Fig. 3.3,

\[ n = (n_c + \delta n) + \delta n \frac{x}{t} \]

and

\[ \frac{dn}{dx} = \frac{\delta n}{t} \]

\[ \therefore \frac{1}{Z} \frac{dZ}{dx} = -\frac{n_c}{(n_c^2 - n_1^2)} \cdot \frac{\delta n}{t} = 2A \text{ (say)} \quad (3.24) \]

Comparing (3.24) with (3.13)
Fig. 3.3: Cross-section of a planar waveguide and its equivalent transmission line representation.
\[
\frac{d^2 u}{dx^2} + 2A \frac{du}{dx} + k_c^2 u = 0 \tag{3.25}
\]

Comparing (3.25), with (3.16) and its solution (3.17), for the configuration of Fig. 3.3,

\[
Z_{in} = Z_{f2} \frac{Z_a}{Z_{f1}} \left[ \left( k_c^2 - A^2 \right)^{1/2} + A \tan \left( k_c^2 - A^2 \right)^{1/2} \right] - jk_c \tan \left( k_c^2 - A^2 \right)^{1/2}
\]

\[
\frac{(k_c^2 - A^2)^{1/2}}{Z_{f1}} \frac{(k_c^2 - A^2)^{1/2}}{Z_{f2}} \frac{k_c Z_a}{Z_1} \tan \left( k_c^2 - A^2 \right)^{1/2} + jk_c \tan \left( k_c^2 - A^2 \right)^{1/2}
\]

Since \( Z_{in} = -Z_r \) from (3.20),

\[
\tan m t = \frac{(Z_r Z_{f1} + Z_a Z_{f2})}{A(Z_r Z_{f1} - Z_a Z_{f2}) + jk_c (Z_a Z_r + Z_{f1} Z_{f2})} \tag{3.27}
\]

where

\[
m = (k_c^2 - A^2)^{1/2} \tag{3.28}
\]

\[
k_c = k(n_c^2 - n_1^2)^{1/2}
\]

Here \( k = 2\pi/\lambda_o \), and \( \lambda_o \) is the free space wavelength.

Since, for the TE modes, \( Z_1 = \omega \mu_o / k_1 \),

\[
\tan m t = \frac{(k_{af2} + k_{rf1})}{A(k_{af2} - k_{rf1}) + jk_c (k_{af2} k_{f1} k_{f2} + k_{r1} k_{r2})}
\]

Finally, the dispersion relation for the TE modes in a film with linearly graded index is given by,

\[
\tan m t = \frac{(k_{af2} + k_{rf1})}{A(k_{af2} - k_{rf1}) + jk_c (k_{af2} k_{f1} k_{f2} - k_{r1} k_{r2} k_{f1} k_{f2})} \tag{3.29}
\]
Where the various decay and propagation constant are,

\[ k_a = k(n_a^2 - n_{1}^2)^{1/2} = jk'(n_a^2 - n_{1}^2)^{1/2} \]  \hspace{1cm} (3.30)

\[ k_r = k(n_r^2 - n_{1}^2)^{1/2} = jk'(n_r^2 - n_{1}^2)^{1/2} \]  \hspace{1cm} (3.31)

\[ k_{f1} = k(n_c^2 - n_{1}^2)^{1/2} \approx k_c \]  \hspace{1cm} (3.32)

\[ k_{f2} = k\{(n_c + \delta n)^2 - n_{1}^2\}^{1/2} \]  \hspace{1cm} (3.33)

\[ m = (k_c^2 - A^2)^{1/2} \]  \hspace{1cm} (3.28)

\[ A = \frac{-n_c \delta n}{2(n_c^2 - n_{1}^2)t} \]  \hspace{1cm} (3.24)

3.4.2 TM mode

The propagation constant \( k_{x1} \) and the characteristic impedance for the \( i \)-th medium for the TM modes are related by [Tamir 1979: 109],

\[ Z_i = \frac{k_{x1}}{\omega \varepsilon_o n_{1}^2} \]  \hspace{1cm} (3.34)

where \( k_{x1} + k_{2}^2 = k_{n1}^2 \).

Dropping the suffix \( i \), an expression for \( B \) analogous to \( A \), (3.24), for the TE modes is,

\[ B = \frac{-\delta n(n_c^2 - 2n_{1}^2)}{n_c(n_c^2 - n_{1}^2)t} \]  \hspace{1cm} (3.35)
The dispersion equation from (3.27), (3.34) and (3.35) is,

\[
\tan \frac{m_1 L}{m_1} = \frac{\left( \frac{k_r k_{f1}}{n_r^2 n_{f1}^2} + \frac{k_a k_{f2}}{n_a^2 n_{f2}^2} \right)}{B \left( \frac{k_r^2}{n_r^2} - \frac{k_{a f1}^2}{n_{f1}^2} \right) + j k_c \left( \frac{k_a k_r}{n_a n_r} + \frac{k_{f1} k_{f2}}{n_{f1} n_{f2}} \right)}
\]

(3.36)

\[
\tan \frac{m_1 L}{m_1} = \frac{(k'' k_{r f1} + k'' k_{a f2})}{B(k'' k_r f1 - k'' k_{a f2}) + k (k_{f1} k_{f2} - k'' k_{r f1})}
\]

(3.37)

Where,

\[
B = -\delta n (n_c^2 - 2n_1^2)/[(n_c^3 - n_c n_1^2) t]
\]

(3.38)

\[
m_1 = (k_c^2 - B^2)^{1/2}
\]

(3.39)

\[
k''_a = (n_1^2 - n_a^2)^{1/2}/n_a^2
\]

(3.40)

\[
k''_r = (n_1^2 - n_r^2)^{1/2}/n_r^2
\]

(3.41)

\[
k_{f1} = (n_{f1}^2 - n_1^2)^{1/2}/n_{f1}^2
\]

(3.42)

\[
k_{f2} = (n_{f2}^2 - n_1^2)^{1/2}/n_{f2}^2
\]

The above dispersion relations are applied to a study of a film loaded diffused waveguide in Chapter IV.
CHAPTER IV

PHASE COMPENSATION BY FILM LOADING

4.1 Introduction

The intrinsic phase difference of an integrated optical Mach-Zehnder modulator can be adjusted by means of a loading film as shown in Fig. 4.1. First, a review of various materials which may be suitable for this purpose is carried out. Next, a transmission line model and the theory for a film loading a Ti indiffused optical waveguide are described. Then, a description of the computer program and the computational results on the effect of a high index film loading a diffused waveguide are given. Finally, the experimental results are presented.

4.2 Phase compensation

Several devices depend on the path length to perform as intended. Examples of these are filters, directional couplers, and interferometers. The parallel guide directional coupler performance is critically dependent upon the coupling length. In case of the Mach-Zehnder interferometer, the intrinsic phase difference is determined by difference in the length of the two arms. In order to either operate the interferometer in the linear portion (e.g. for HV measurement) or to combine the output of several interferometers, it is necessary to have precise design lengths. But, control of the path length to a fraction of the guide wavelength (= \(\lambda/n\)) is impractical. So, an alternative is to tune the devices by adjusting the propagation constant. This had been done [Mikami et al. 1977; 1979] by loading a high refractive
Fig. 4.1. Location of the loading film on the Mach-Zehnder modulator.

Fig. 4.2. (a) Equivalent transverse network and (b) configuration of an optical waveguide loaded by a film.
index Se-S based chalcogenide glass on top of one of the directional coupler arms, and then irradiating the glass film with 0.5 μm halogen light. This increased the refractive index $n_f$ by as much as 0.03, and the change in index was reversed by heating at 190°C. By altering the refractive index and hence the propagation constant, the transfer of power was adjusted from minimum to maximum. Unfortunately, the chalcogenide glass film has high absorption in the visible spectrum, consequently it is not suitable for use at the He-Ne wavelength. Many different materials have been used to fabricate planar optical waveguides in the form of multilayer dielectric films. Some of these films are amenable to a change in the refractive index when subjected to heat, light or oxidation. Properties of several such materials are compared, with an application to the phase compensation of the interferometer, and ease of fabrication in mind.

Additional phase retardation introduced by a film of length $L_1$ on an arm of the interferometer in Fig. 4.1, is given by

$$\phi = (\beta_F - \beta_A)L_1$$

and

$$\phi = \frac{2\pi}{\lambda} (n_z - n_A)L_1$$

(4.1)

where $\beta_F$ and $\beta_A$ are the effective propagation constants for the waveguide with and without the loading film respectively. The change induced in the propagation constant with a change in film index can be considerable. For example, by changing the film refractive index from 2.2134 to 2.2084, $n_z$ changes: 2.225230 to 2.225156. For a film length $L_1 = 1$ mm, the phase retardation induced is 42°.
4.3 **Criteria for the loading film**

Characteristics of a film suitable for the phase compensation are:

1. The film be transparent at 0.6328 μm with low optical loss (< 2 dB/cm)
2. The film index $n_f$ be greater than the extraordinary index $n_e = 2.203$ of a Ti diffused waveguide
3. The film index $n_f$ be amenable to change by heat treatment, oxidation, etching etc.
4. Preferably, the film index $n_f$ be reversible by a simple chemical or physical action
5. The change in film index be sufficiently large to induce a phase change of up to 360°
6. The facilities for processing the film be readily available.
7. It would be very desirable if the film index could be modified while the device is operating

4.3.1 **A review of the materials**

Some of the candidate materials and their properties for the loading film are compared in Table 4.1. It appears that Ta$_2$O$_5$ may be appropriate. However, Ta is difficult to etch. The refractive index of Ta$_2$O$_5$ film depends on the duration for which it is kept in O$_2$ at 500°C.

4.4 **Modelling of the loading film** [Uchida et al. 1976; Uchida 1976; Noda 1978b]

An optical waveguide formed by diffusion and loaded by a film is shown in Fig. 4.2(a). Also, a representation of the film loaded guide by an equivalent
<table>
<thead>
<tr>
<th>Materials</th>
<th>Reference</th>
<th>$n$</th>
<th>Loss dB/cm</th>
<th>Change in $n$ after deposition</th>
<th>Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corning 7059</td>
<td>[Goell et al. 1969]</td>
<td>1.62</td>
<td>1</td>
<td>-</td>
<td>Sputtered in $O_2$</td>
</tr>
<tr>
<td>Vinyltrimethylsilane</td>
<td>[Tien et al. 1972]</td>
<td>1.531</td>
<td>0.04</td>
<td>Yes, in $O_2$ at 140°C</td>
<td>Rf polymerization</td>
</tr>
<tr>
<td>Polyurethane Polyester epoxy Organic polymer</td>
<td>[Ulrich and Weber 1972]</td>
<td>0.3</td>
<td></td>
<td>Yes, with UV if dye in film</td>
<td>Dipping</td>
</tr>
<tr>
<td>$Nb_2O_5$</td>
<td>[McGraw and Zernike 1974]</td>
<td>2.276</td>
<td></td>
<td>-</td>
<td>Sputtered $Nb$ in $Ar$, $O_2$</td>
</tr>
<tr>
<td>ZnO</td>
<td>[Tien et al. 1969]</td>
<td>1.973</td>
<td>20</td>
<td>-</td>
<td>Sputtered</td>
</tr>
<tr>
<td>ZnS</td>
<td>[Tien 1971]</td>
<td>2.342</td>
<td>5</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Ion exchange with X-cut $LiNbO_3$</td>
<td>[Jackel 1980]</td>
<td>2.23-2.26</td>
<td>1.5</td>
<td>Probably</td>
<td>Ag$NO_3$+$LiNbO_3$ at ~300°C</td>
</tr>
<tr>
<td>$Ta_2O_5$</td>
<td>[Hensler et al. 1971 and Fujumori et al. 1972]</td>
<td>2.2136</td>
<td>0.9</td>
<td>Yes, in $O_2$ at 500°C</td>
<td>Sputtered Ta followed by oxidation</td>
</tr>
</tbody>
</table>
transmission line network, in the transverse \( x \) direction, is shown in Fig. 4.2(b). The refractive index variations in the \( x \) direction are,

\[
\begin{align*}
n(x) &= n_a & -\infty < x < -t \\
      &= n_f(x) - n_c - \Delta n x/t & -t < x < 0 \\
      &= n_s + \Delta n \exp(-x/d) & 0 < x < \infty \\
\end{align*}
\]

(4.2)

Where \( n_a, n_f \) and \( n_s \) are the refractive indices of the air, film and substrate respectively, and \( \Delta n \) is maximum change in index due to diffusion. The film thickness is \( t \), and \( d \) is the diffusion depth at which \( \Delta n \) decreases to \( \Delta n/e \), i.e. an exponential diffusion profile is assumed. The characteristic equation for the TE mode propagation in a planar waveguide with an exponential diffusion profile is [Conwell 1973];

\[
\frac{J_{V-1}(g) - J_{V+1}(g)}{J_v(g)} = \frac{-p_1 \lambda}{\pi (2n_s \Delta n)^{1/2}}
\]

(4.3)

where

\[
g = 2 \frac{dk}{2n_s \Delta n}^{1/2} \exp(-x/d)
\]

(4.4)

\[
v = 2 \frac{dk}{(n_1^2 - n_s^2)^{1/2}}
\]

(4.5)

\[
p_1 = (k^2 n_1^2 - \epsilon_r \omega_c^2)^{1/2}
\]

(4.6)

and \( n_1 \) is the effective mode index in the propagation direction \( z \). In order to apply (4.3) to a waveguide loaded with a film, an appropriate value of \( p_1 \) has to be used. So \( p_1 \) corresponding to a film of index \( n_f(x) \) is used.
Equation similar to (4.3) corresponding to a film with linearly graded index was derived in the previous chapter. Special cases, when the film index is constant are determined and compared with [Noda et al. 1978b]. The eigenvalues using (4.3) are determined at the $x = 0$ plane, so $g(x) = g(0) = g$ for brevity.

4.4.1 TE modes, $n_f(x) > n_s$

The input impedance $Z_{in}$ at $x = 0$, for a film with linearly graded index of thickness $t$, covering a diffused waveguide, of configuration in Fig. 4.2 for the TE modes (3.26) is,

$$Z_{in} = \frac{\frac{k_{f1}}{k_a}}{\frac{k_{f2}}{m - A \tan mt - jk_c \tan mt}} \frac{(m + A \tan mt) - jk_c \tan mt}{m - A \tan mt - jk_c \tan mt}$$

(4.7)

where,

$$k_{f1} = k(n_c^2 - n_1^2)^{1/2} = k_c$$

$$k_{f2} = k\{(n_c^2 + \Delta n)^2 - n_1^2\}^{1/2}$$

$$k_a = k(n_a^2 - n_1^2)^{1/2}$$

$$m = (k_c^2 - A^2)^{1/2}$$
A = \frac{-n_c \delta n}{2(n_c^2 - n_1^2) t}

Recalling (4.6)

p_1 = k(n_1^2 - n_f^2)^{1/2} = \pm jk(n_1^2 - n_f^2)^{1/2}

p_1 = \pm j \omega_o / Z_i = \pm j \omega_o / Z_{in}

Using the value of $Z_{in}$ in $p_1$ the characteristic equation (4.3) becomes,

\[
\frac{J_{V-1}(g) - J_{V+1}(g)}{J_V(g)} = \frac{-\frac{j \omega_o}{(2n_s \Delta n_s)^{1/2}}}{Z_{in} k}
\]

(4.8)

For the case, when $n_f(x) = n_f$ (constant), $k_c = k_{f1} = k_{f2} = k_f$, $\delta n = 0$ and hence $A = 0$ and $m = k_c$, so that $Z_{in}$ reduces to

\[
Z_{in} = \frac{\omega_o (k_f / k_a) - j \tan tk_f}{k_f 1 - j(k_f / k_a) \tan tk_f}
\]

(4.9)

Thus from (4.8) and (4.9),

\[
\frac{J_{V-1}(g) - J_{V+1}(g)}{J_V(g)} = \frac{-2(n_f^2 - n_1^2)^{1/2}}{(2n_s \Delta n_s)^{1/2}} \frac{S - \tan t (n_f^2 - n_1^2)^{1/2}}{1 + S \tan t (n_f^2 - n_1^2)^{1/2} k}
\]

(4.10)

where,

\[
S = \frac{(n_1^2 - n_a^2)^{1/2}}{(n_f^2 - n_1^2)^{1/2}}
\]
Which agrees with (7) of [Noda et al. 1978b].

4.4.2 TE modes, \(n_f(x) < n_s\)

For this case which is of practical importance also, (4.7) for \(Z_{in}\) becomes,

\[
Z_{in} = \frac{k_f^1}{k_a} \frac{\frac{k_f^1}{k_a} (jm' + jA \tanh m' t) + j_k c \tanh m' t}{jm' - jA \tanh m' t + j \frac{k_c k_f^1}{k_a} \tanh m' t}
\]  

(4-11)

where

\[
j k_i' = k_i \quad \text{for } i = a, c, f_1, f_2
\]

\[
jk_i = k_i \quad \text{for } i = a, c, f_1, f_2
\]

Thus for the TE modes, when \(n_f < n_s\), the characteristic equation is given by (4.11) and (4.8)

\[
\frac{J_{V-1}(g) - J_{V+1}(g)}{J_V(g)} = \frac{-2 - jw_o}{(2n_s \Delta n)^{1/2} Z_{in}}
\]  

(4.12)

When the film index \(n_f(x)\) is a constant, \(\Delta n = 0\) and hence \(A=0\), \(m' = k_c\), with \(S' = k_a' / k_f'\) (4.12) reduces to,

\[
\frac{J_{V-1}(g) - J_{V+1}(g)}{J_V(g)} = \frac{-2(n_1^2 - n_f^2)^{1/2}}{(2n_s \Delta n)^{1/2}} \frac{S' + \tanh t(n_1^2 - n_f^2)^{1/2} k}{1 + S' \tanh t(n_1^2 - n_f^2)^{1/2} k}
\]  

(4.13)

which is analogous to (7) of [Noda et al. 1978b].
4.4.3 TM modes, $n_f(x) > n_s$

In case of the TM mode propagation, the characteristic equation differs from the one for the TE modes, it is [Conwell 1973];

$$\frac{J_{v-1}(g) - J_{v+1}(g)}{J_v(g)} + \frac{(2n_s \Delta n)^{1/2}}{d k n_s^2} = -\frac{n_s^2}{n_1} \frac{2p_1}{k(2n_s \Delta n)^{1/2}} \left( 1 + \frac{2\Delta n}{n_s} \right) \quad (4.14)$$

When the film index is linearly graded, then $Z_{in}$ is given by (3.26) with $B$ instead of $A$, for the TM modes it is,

$$Z_{in} = \frac{k f_2}{\omega c n_f^2} \frac{n_f^2}{n_a} \frac{k a}{k f_1} \left( m_1 + B \tan m_1 t \right) - jk_c \tan m_1 t \quad (4.15)$$

Where,

$$B = -\delta n \left( n_c^2 - 2n_f^2 \right) / \left[ \left( n_c^2 - n_f^2 \right) t \right] \quad (4.16)$$

$$m_1 = \left( k_c^2 - B^2 \right)^{1/2}$$

The term $p_1$ in (4.6) in this case is,

$$p_1 = k \left( n_f^2 - n_i^2 \right)^{1/2} = \pm jk \left( n_f^2 - n_i^2 \right)^{1/2}$$

$$p_1 = \pm j \omega c n_i Z_i = \pm j \omega c n_f^2 Z_{in} \quad (4.17)$$

Thus the characteristic equation for mode propagation using (4.15) and (4.17) in (4.14) is,
\[
\frac{J_{V-1}(g) - J_{V+1}(g)}{J_{V}(g)} + \frac{(2n_s \Delta n)^{1/2}}{dnk_s^2} = -\frac{n_s}{n_f^2} \cdot \frac{2 \sqrt{\Delta n} Z_{in}}{k(2n_s \Delta n)^{1/2}} \left(1 + \frac{2 \Delta n}{n_s}\right) \tag{4.18}
\]

For the case when the film index \(n_f\) is constant, \(n_f = n_{f1} = n_{f2} = n_c\), \(\Delta n = 0\) and hence \(B = 0\), \(Z_{in}\) simplifies, and (4.18) reduces to,

\[
\frac{J_{V-1}(g) - J_{V+1}(g)}{J_{V}(g)} + \frac{(2n_s \Delta n)^{1/2}}{dnk_s^2} = -\frac{n_s}{n_f^2} \cdot \frac{2 (n_f^2 - n_c^2)^{1/2}}{\left(2n_s \Delta n\right)^{1/2}} \cdot \xi s - \tan \theta \tan \theta_f \tag{4.19}
\]

where,

\[
\xi = \left(\frac{n_f}{n_a}\right)^2
\]

\[k_f = k(n_f^2 - n_a^2)^{1/2}\]

### 4.4.4 TM modes, \(n_f(x) \leq n_s\)

When the refractive index of the film \(n_f(x)\) is less than that of the substrate \(n_s\), then \(Z_{in}\) given by (4-15) modifies to,

\[
Z_{in} = \frac{jk'_{f2}}{\omega e n_f^2} \cdot \frac{n_{f2}^2 k'_{f2}}{n_{a}^2 k'_{f1}} \cdot \left(\frac{jm' + jB \tanh m't}{1} + jk' \tanh m't\right)
\]

\[
\tag{4.20}
\]

Where

\[
jk'_{i} = k(n_i^2 - n_i^2)^{1/2}, \text{ for } i = a, c, f1, f2
\]

\[
jm'_{i} = m_{i}
\]
The characteristic equation for the mode propagation constants is given by (4.18) and (4.21).

If the film refractive index is uniform so that \( n_f(x) = n_f = n_{f1} = n_{f2} = n_c \), \( \delta n = 0 \) and hence \( B = 0 \). The expression for \( Z_{in} \) simplifies and the characteristic equation reduces to

\[
\frac{J_{V-1}(g) - J_{V+1}(g)}{J_V(g)} + \frac{(2n_s \Delta n)^{1/2}}{dk n_s^2} = \frac{-n_s^2}{n_f^2} \frac{2(n_f^2 - n_s^2)^{1/2}}{(2n_s \Delta n)^{1/2}} \frac{\xi S' + \tanh \frac{tk_f'}{2}}{1 + \xi S' \tanh \frac{tk_f'}{2}}
\]

where

\[
S' = \frac{(n_f^2 - n_a^2)^{1/2}}{(n_f^2 - n_s^2)^{1/2}}
\]

(4.23)

### 4.5 Propagation constant of film loaded guide of width \( w \)

The effective index method [Hocker and Burns 1977; Ramaswamy 1973; Furuta et al. 1974] is used to determine the propagation constant \( \beta = kn_z \) of a film loaded guide of finite width. The effective index \( n_z \) is determined in two steps. First, the width of the film is considered to be infinite, Fig. 4.3(b) or (c), and the effective index \( n_1 \) is obtained by solving one of the applicable relations; (4.7), (4.10), (4.11), (4.14), etc. Second, the effective index \( n_z \) of the structure formed by a channel of width \( w \) sandwiched by the substrate, Fig. 4.3(d) or (e), is computed from;

\[
b_2w = 2 \tan^{-1} \left( \frac{\xi V_s}{b_2} \right) + q\pi
\]

(4.24)

where,
Fig. 4.3. Application of the effective index method;
(a) representation of the waveguide under consideration.
(b), (c) Planar waveguide and effective indices: with film \( n_{II} \) and without film \( n_{III} \).
(d), (e) Inclusion of the width constraint; effective indices: with film \( n_{zf} \) and without film \( n_{za} \).
\[ b_2 = k(n_f^2 - n_2^2)^{1/2} \]  
\[ V_s = k(n_s^2 - n_2^2)^{1/2} \]

and

\[ \zeta = \begin{cases} 
\left( \frac{n_1}{n_s} \right)^2, & \text{for TE}_q \text{ modes} \\
1, & \text{for TM}_q \text{ modes} 
\end{cases} \]

Therefore, \( \beta = k\sqrt{n_2} = k\sqrt{n_z} \) is the desired propagation constant of the film loaded guide of width \( w \), in the longitudinal direction \( z \).

### 4.6 Computations

Computations were done for a uniform \( Ta_2O_5 \) film on a Ti diffused waveguide in Y-cut \( LiNbO_3 \). Exponential refractive index profile was assumed. The calculations were carried out for the TE and TM modes using (4.10) and (4.19) respectively. The computer program was checked by confirming the results published in [Noda et al. 1978b] and [Mikami et al. 1979].

For the TE modes, the characteristic equation (4.10) was used. The roots of this equation lie close to where \( J_y(g) = 0 \). In order to avoid a division by zero, (4.10) was rearranged as,

\[ J_{v-1}(g) - J_{v+1}(g) = -J_v(g) \frac{2(n_f^2 - n_1^2)^{1/2}}{(2n \Delta n)_s^{1/2}} \frac{S - \tan t(n_f^2 - n_1^2)^{1/2}k}{1 + S \tan t(n_f^2 - n_1^2)^{1/2}k} \]

In the computer program FT is difference between the LHS and RHS (LHS = left hand side) of (4.27). The propagation constant with the film thickness \( t \) and
zero, are \( n_1 = n_{1I} \) and \( n_1 = n_{1III} \) respectively. The data used were; Y-cut LiNbO\(_3\); \( \lambda = 0.6328 \) \( \mu\)m; \( n_s = n_e = 2.203 \) for TE modes, \( n_s = n_o = 2.2868 \) for the TM modes [Nelson and Mikulyak 1974]; \( n_f = 2.2134 \) for the TE modes, \( n_f = 2.2038 \) for the TM modes [Hensler et al. 1971]; \( 0 < t < 0.6 \) \( \mu\)m; \( 0.5 < d < 6 \) \( \mu\)m; and \( 0.01 < \Delta n < 0.04 \).

Plots of \( F_I \) and \( F_{II} \), which correspond to (4.10) with and without the film respectively are shown in Fig. 4.4. Values of \( n_1 \) range from \( (n_s + \Delta n) \) to \( n_s \). Roots for the lowest order mode correspond to the largest value of the propagation constant \( n_1 \). The propagation constants with and without the film are \( n_{1I} \) and \( n_{1III} \) respectively.

To study the effect of the film with various parameters, the roots \( n_1 \) were computed and tabulated. The roots were found by using the half interval search method [Ley 1970]. The program given in [Ley 1970] has flaws, so it was modified. A flow chart and listing for the computer program are included in Appendix B. The search interval was kept small, so as not to include more than one root; it started at \( (n_s + \Delta n) \) and extended downwards. If no root was discovered in the interval, it was shifted towards \( n_s \); and the process repeated until a root was found. First, \( n_{1III} \) corresponding to zero thickness, and then \( n_{1I} \) corresponding to finite film thickness were located. When \( n_f > n_s \), \( n_{1I} \) was searched with the condition \( n_{1I} > n_{1III} \).

4.6.1 Description of computed results

The effective index, \( n_z \), as a function of the diffusion depth \( d \), is shown in Fig. 4.5. The plots compare \( n_z \) for planar and finite width \((w)\) waveguides, with \( (n_{fz}) \) and without \( (n_{za}) \) film. For fixed film thickness and \( \Delta n \), the
Fig. 4.4. Roots of (4.27), which correspond to effective indices of planar waveguide modes; with film $F_I$ and without film $F_{II}$.
Fig. 4.5. Effective index $n_z$ vs. $d$, for planar ($n_{1\ldots}$) and finite width waveguide ($n_{z\ldots}$); with and without film.
effective index increases with the diffusion depth. Everything else being the same, the effective index of a finite width guide is less than that of a planar guide, and the effective index is increased by the loading film. For the TE_{00} and TE_{11} modes the cut-off curves are shown in Fig. 4.6, and n_{z} vs. d with w = 2.5, 3 and 4 \mu m in Fig. 4.7.

Relative change produced in the propagation constant of 4 \mu m wide guide increases with the loading film thickness, as shown in Fig. 4.8. For fixed d, the change produced is greater for larger \Delta n. In the case of TM, the change levels off after a sharp increase at t < 0.1 \mu m.

The effect of a small change in the film index (\Delta n_{f} = 0.005), on n_{z} is depicted in Fig. 4.9. Here, a change in the effective index \delta n_{z} (= n_{zf} - n_{za}) for n_{f} = 2.2134 and 2.2084, is shown as a function of t. The effect of loading film is largest, at largest t and \Delta n.

Change in the effective index with \Delta n as the variable is plotted in Fig. 4.10, with film thickness t as the parameter. Finally, \delta n_{z} vs. d, with \Delta n as a parameter is shown in Fig. 4.11. From these results it is concluded, that for the TE mode, the change \delta n_{z} increases with t and \Delta n, but decreases with d.

An application of the above results is illustrated by an example. In Y-cut LiNbO_{3}, for TE_{00} mode, at \lambda = 0.633 \mu m, n_{e} = 2.203, \Delta n = 0.04, t = 0.2 \mu m, and d = 2 \mu m; it is computed that n_{zf} = 2.225230 and 2.225156 corresponding to n_{f} = 2.2134 and 2.2084 respectively. Thus, change in phase produced by a loading film of length L,

\[ \delta \psi = \delta \beta L = \left( \frac{2\pi}{\lambda} \delta n_{zf} \right) L = 42^\circ/\text{mm} \]

But, for t = 0.5 \mu m; \delta \psi = 160^\circ/\text{mm}. 
Y-CUT LITHIUM NIOBATE; 
$\text{TE}_\infty$ AND $\text{TE}_{11}$ CUT-OFF CURVES; 
$\lambda=0.6328\,\mu\text{m};\ n_e=2.203;\ t=0;\ w=4\,\mu\text{m}$

Fig. 4.6. Mode cut-off curves for waveguides 3 and 4 $\mu$m wide.
Fig. 4.7. Effective index $n_z$ vs. $d$, with $w$ as parameter; for $TE_{oo}$ and $TE_{11}$ modes.
Fig. 4.8. Change in the propagation constant as a function of loading film thickness, with $\Delta n$ as parameter.
Fig. 4.9. Change in effective index $dn$ due to loading film thickness $t$, with the film index $n_f$ as parameter.
Fig. 4.10. Change in effective index due to loading film; $\delta n_z$ vs. $\Delta n$, with film thickness as parameter.
Fig. 4.11. Change in the effective index due to a loading film; $\delta n_z$ vs. $d$, with $\Delta n$ as parameter.
4.7 Experimental Results

Intrinsic phase of a Mach-Zehnder modulator was changed with a Ta$_2$O$_5$ film. To this end, a Ta film was formed on one arm of a Y-branch interferometer. An Al lift-off procedure described in Chapter V was followed. The Ta film was oxidized for 75 minutes at 500°C, before the electrode formation. The waveguide and the Ta$_2$O$_5$ film turned out to be wider than intended; 6 and 8 μm respectively. Length of film loaded waveguide was ~ 3.2mm, and its thickness was estimated to be 0.1 - 0.2 μm. Output light intensity of the modulator was measured with an applied voltage. The results with no oxidation after the electrode formation are shown in Fig. 4.12. The top trace, a positive and negative triangular waveform corresponds to the applied voltage, and the bottom trace is the output light intensity. The extinction ratio is poor because of surface guiding. The intrinsic phase difference $\psi_0 = 56°$. The Ta$_2$O$_5$ was treated with O$_2$ at 500°C for 11 min. The measured results after the O$_2$ treatment are given in Fig. 4.13; and $\psi_2 = 111°$. Thus, 11 min. of oxidation produced a change in phase of 55°. The Ta$_2$O$_5$ film was treated in O$_2$ for an additional 41 min., after which $\psi_3 = 400°$, Fig. 4.14. The second O$_2$ treatment (41 min.) produced a change in phase of 289°. The average change in phase was ~ 6.6°/min. of oxidation. This technique of phase tuning may be used to adjust the intrinsic phase of the BOA modulators (Chapter VII), and the coupling length of directional couplers.
Fig. 4.12. Applied triangular voltage and the modulator response before treatment in oxygen.

Fig. 4.13. Applied triangular voltage and the modulator response after 11 minutes of treatment in oxygen; $\psi=111^\circ$. 
Fig. 4.14. Applied triangular voltage and the modulator response after 52 minutes of treatment in oxygen; $\psi=400^\circ$. 
CHAPTER V

FABRICATION AND TESTING OF DEVICES

5.1 Introduction

The fabrication procedure that was used to make Ti diffused waveguides in Y-cut LiNbO$_3$ and the technique to polish the crystal edges is described below. A description of the measurement apparatus and steps to line it up are given. A flow chart for the fabrication process is shown in Fig. 5.1

5.2 Mask

Line drawings of the devices were made, and the data for the coordinates were entered on the AMDAHL 470 UBC computer. The data were then transferred, via a data link to the PDP8/E computer in the Electrical Engineering Department, and stored on a disk. A 16"x38" rubylith mask was cut on a computer controlled coordinatograph using a knife blade. The knife blade is primarily meant for straight cuts, however, it was tried in this application where non-orthogonal cuts are required because of the minute fork angle of 1°. An omnidirectional point cutter was tried and was abandoned, since it tended to tear the plastic and produced ragged edges. The software that controlled the cutter motor drive was modified to incorporate gradual acceleration in the cutter speed. A drawing of a Mach-Zehnder interferometer with typical dimensions is shown in Fig. 5.2. The cutter followed the path ABCDEF, GHJKLM, POR, TSR, PN, and TN. When cutting sections similar to PNR, the cutter direction was P to N, and T to N as indicated in the diagram. This strategy
Fig. 5.1. A flow chart for the fabrication process.
Fig. 5.2. Line drawing of (a) straight waveguide and the Mach-Zehnder modulator.
of cutting avoided lifting of the rubylith adjacent to the coordinate N. Photograph masks were prepared from the rubylith masters by Precision Photomask Ltd., Montreal. The rubylith pattern was reduced 50 times, consequently the 4 micron wide waveguides were 0.008" wide on the rubylith. The pattern was stepped and repeated on the mask; 2 times in the long direction and 4 times in the narrow direction, with 1 mm inter-pattern spacing. Two negative glass masks were obtained, one for the waveguide circuitry and another for the electrode network. By negatives it is meant that areas that were transparent on the rubylith were dark on the mask.

5.3 Crystal

Optical waveguide devices were fabricated on LiNbO₃ crystals obtained from Crystal Technology Inc., Palo Alto. These were 2" diameter, 0.020" thick YZ wafers with the Y-axis (X-ray oriented) perpendicular to the major face within ±30 arc minutes, and the Z-axis (X-ray oriented) perpendicular to a 1" wide reference flat within ±6 arc minutes. The front surface had "SAW polish", i.e., a polish suitable for photofabrication of interdigital transducers, and the back surface had been fine ground to 9 microns lapped finish. The crystal surface on which waveguides are to be formed has to be scratch-free; at the same time, the substrate has to be defect-free since evanescent modes propagate through it. This particular cut of crystal was selected to utilize the highest electrooptic coefficient \( r_{33} \) \( (30.8 \times 10^{-8} \text{ m/V}) \) to modulate the TE mode propagating in the X-direction with planar electrodes.
5.4 Cleaning

Care is required in cleaning and handling of the substrates because of the small dimensions involved. Freedom from dust during photolithography is crucial, as a single dust particle would interrupt the wave path and render the device useless. The LiNbO₃ crystals require special care because they are brittle, very sensitive to mechanical and thermal shock, and attract dust. The waveguides are narrow (4 microns) and very long (18 mm). Because of the large aspect ratio (4500), unusual for an integrated circuit, the mask and the crystal have to be scratch and dust free. Also, as only four to six devices can be accommodated per wafer, the yield has to be high.

The glassware, fluoroware, baskets and tweezers were cleaned for three minutes with 30% hydrogen peroxide and 98% sulphuric acid (1:1). Polyethylene disposable gloves and latex finger cots were used to handle the glassware, masks, etc. Beakers were dedicated to various chemicals and not interchanged. Most of the work was carried out in a clean area with positive air pressure.

The fabrication steps are illustrated in Figs. 5.3. Prior to the deposition of Ti, the crystals were cleaned as follows:

1. Boiled in acetone for 10 minutes
2. Put in ultrasonic bath of acetone for 10 minutes
3. Rinsed in deionized (DI) water cascade for 10 minutes; 2 and 8 minutes in the downstream and upstream baths respectively
4. Etched in 5% hydrofluoric acid for 2 minutes
5. Rinsed in DI water cascade for 10 minutes, as above
Fig. 5.3. Fabrication steps; developing, etching, diffusing, sputtering, and metallizing.
6. Boiled in methanol for 5 minutes

As LiNbO$_3$ tends to attract dust, the crystal was transported in warm methanol. The crystal was then allowed to dry near its destination, such as vacuum evaporation chamber, furnace, etc. Alternative procedures are the so called RCA Clean [Kern and Puotinen 1970], and trichloroethylene, hydrogen peroxide, and ammonium hydroxide treatment [Stulz 1979].

5.5 Evaporation of Ti

About 450 Å thick layer of 99.99% Ti was evaporated onto the clean crystals, using the Veeco (VE400) system. The pressure prior to evaporation was < 5x10$^{-6}$ Torr as measured at the base plate with an Ionization Vacuum Gauge. A 0.045" diameter Ti wire wrapped around a tungsten filament was used as a source. A filament current of 80 A was required, and the evaporation rate was about 10 Å/s.

The Ti wrapped tungsten filament was obtained from Vacuum Atmospherics, Hawthorne, California. The main contaminants in the Ti, as quoted by the supplier were: carbon (0.009 ppm); iron (0.034 ppm); oxygen (0.126 ppm); hydrogen (0.004 ppm); and nitrogen (0.007 ppm).

The quartz crystal film thickness monitor (Inficon; Model 321) used to determine the thickness of the evaporated metal had to be calibrated. The calibration was done by two methods. In the first method, a clean dry crystal was weighed before and after evaporation of the Ti. The film thickness was calculated to be 761 Å, with density = 4.54 gm/cc, diameter = 2", and weight = 0.7 gm. Thus, the actual thickness is about 57% of the monitor reading. In the second method, Ti was evaporated over part of a clean silicon wafer to
form a step. Then a 10 kÅ thick layer of aluminium was evaporated over the Ti step. The Sloan Angstrometer (M100) was used to determine thickness of the Ti, and it was estimated to be 618 Å. Thus, the thickness is 47% of the monitor reading.

5.6 Exposing, developing, and etching

The crystal with the evaporated metal was taken from the vacuum system directly to the photoresist spinner, in order to eliminate baking to drive out moisture. The LiNbO₃ crystal was placed in a teflon fixture and spun at 2500 rpm for 15 seconds to dislodge any dust particles. The use of the fixture, Fig. 5.4, was necessary to hold the crystal, otherwise it tended to warp, fly off, and break. Positive photoresist (Waycoat Positive LSI 295 Resist) was spun on the metallized crystal at 2500 rpm for 15 seconds to obtain a resist thickness of about 2 microns.

Pre-bake of the photoresist was carried out at 90°C for 30 minutes. The crystal temperature was raised and lowered at a rate not exceeding 20°C/minute. The photoresist was exposed to ultraviolet light for 18 seconds through the waveguide mask. The mask aligner (Kasper Instruments), which is normally used for integrated circuits, was employed. The exposed crystal was loaded into a single substrate fluoroware clamped holder. The pattern was then developed until the photoresist came off, i.e., when the clouds and rings formed on the surface disappeared completely. The development was halted by soaking in deionized water for 10 minutes. The crystal was blown dry and then examined under the microscope. If the contrast in the resist pattern was not sufficient, further development was done. If, for some reason the resist
Fig. 5.4. Jig to hold the wafer; (a) stationary and (b) spinning.

Fig. 5.5. Arrangement for carrying out the diffusion.
pattern was unacceptable, it was necessary to dissolve the resist in boiling acetone, and bake the crystal at 200°C for 30 minutes to drive out the moisture before repeating the whole procedure. If the resist pattern was satisfactory, post-bake (pre-etch) was done at 120°C for 30 minutes, again with gradual heating and cooling of the crystal. The exposed Ti was etched in 5% HF for 6 seconds, and control of this duration is critical to avoid undercutting. Etching was halted with a 10 minute soak in fresh deionized water. The crystal was dried and examined under the microscope. If necessary, the etching processing was repeated only one second at a time until satisfactory results were obtained. If the etching was acceptable, the resist was stripped in boiling acetone. The pattern was then examined under the microscope, device by device. Of course, if the pattern was unacceptable, it was necessary to start all over again, i.e., repeat the cleaning, metallization and etching.

5.7 Diffusion

The crystal with Ti pattern was cleaned (acetone; DI water; methanol) and loaded into the centre of a 2.5" diameter Mini Brute furnace. The furnace had been turned off 12 hours earlier for it to cool to room temperature. The flow of oxygen (medical grade) through the furnace was adjusted to be 1 litre/minute, Fig. 5.5.

The complete diffusion cycle is shown in Fig. 5.6. When the furnace was turned on, the rate of increase of temperature was 20°C/minute. After the temperature reached 400°C, it was further increased to 500°C at 20°C/minute. The temperature was maintained at 500°C for two hours to oxidize the Ti, after
Fig. 5.6. The diffusion cycle.
which it was raised to 780°C at 20°C/minute. When the temperature reached 780°C the controls were adjusted to 950°C, to force as quick a passage through 850°C as possible. This is necessary to avoid the formation of surface craters due to the phase transformation of LiNbO$_3$ to LiNbO$_3$O. This occurs if the temperature is maintained near 850°C for more than 10 minutes [Vahey 1980].

Once the temperature reached 950°C, it was increased to 1020°C at 20°C/minute. The temperature of 1020°C and the oxygen flow of 1 litre/minute were held constant for 4.5 hours. During this period the oxidized Ti diffused into LiNbO$_3$. After 4.5 hours, the furnace was turned off and allowed to cool to room temperature. Initially, the temperature decreased at 20°C/minute, but the rate diminished with time.

During the diffusion cycle Li$_2$O escapes from the LiNbO$_3$ crystal, forming a layer of outdiffused waveguide all over the surface [Miyazawa et al. 1977; Burns et al. 1978; Esdaile 1978; Ranganath and Wang 1979]. The Li$_2$O deficiency increases the extraordinary refractive index, $n_e$. This affects propagation only of the TE but not the TM modes in Y-cut crystals. Outdiffusion was compensated by continuing the diffusion cycle in the presence of LiNbO$_3$ powder. The crystal was removed from the cooled furnace, and placed face down on top of a platinum box (2"x2"x0.25") containing 1 gm of fresh LiNbO$_3$ powder, Fig. 5.7. The crystal and the platinum box were repositioned in the centre of the furnace. The above diffusion cycle was repeated, except that the temperature was maintained at 1020°C for 90 minutes before the furnace was turned off. Once the furnace cooled to room temperature, the oxygen flow was terminated, the crystal withdrawn from the furnace, and examined under the microscope.
Fig. 5.7. Arrangement for Li$_2$O compensation during diffusion.
The diffusion of Ti atoms into LiNbO$_3$ produced bulging easily seen with a microscope. In addition to the transverse diffusion there was some lateral diffusion which widened the lines slightly. It also straightened the waveguide edges, and thus eliminated some of the irregularities of the mask and etching. A scrutiny of the crystal surface after diffusion revealed some worm-like striations about 0.2" long. These were probably induced by the heating and cooling of the crystal.

5.8 Sputtering of SiO$_2$

Metal electrodes in direct contact with diffused waveguides cause loading and produce losses, since decaying mode fields extend beyond the waveguide. A low-index dielectric buffer layer interposed between the waveguides and the electrodes reduces the loss due to metal loading. Subsequent to the Ti diffusion and the Li$_2$O compensation, the crystal surface was scrubbed clean with deionized water to remove any traces of LiNbO$_3$ powder. Then the substrate was cleaned, and a 2000 Å layer of SiO$_2$ was sputtered on it using the Perkin-Elmer Vacuum System, which had a 6" diameter and 1/8" thick water cooled SiO$_2$ target of 99.95% purity. The sputtering was done in an argon atmosphere at a 35 mTorr pressure, at a forward power of 150 W, and a reflected power of < 5 W. The separation between the target and the crystal was about 2". Adjustments were made to the gas pressure and the rf power every 10 minutes or so, to keep the sputtering conditions constant and to maintain a uniform and stable plasma. The sputtering rate was approximately 36 Å/minute. The deposition rate can be increased by boosting the gas
pressure [Maissel and Glang 1970], but at pressures exceeding 45 mTorr secondary plasmas were excited.

The sputtering rate was determined prior to depositing the SiO\(_2\) layer on the LiNbO\(_3\) crystal. Three pieces of Si (n-type, 1-2 ohm-cm, [100] orientation) were cleaned and SiO\(_2\) was sputtered on these for 15, 32 and 37 minutes. Using ellipsometry, the corresponding oxide thickness was determined to be 600, 1200 and 1350 Å. Measured data are indicated on the film thickness and refractive index curves in Fig. 5.8. The average sputtering rate was calculated to be 36 Å/minute. The sputtered layer growth is assumed to be linear with time, this is usually the case if the gas pressure and rf power conditions are stable. An estimation of the oxide thickness was made by consulting the IBM colour chart. When the sputtered film was viewed in daylight, the oxide layer on LiNbO\(_3\) crystal was not as perceptible as it was on the silicon crystal. Nevertheless, it could be detected by observing the reflection of fluorescent lights from the crystal surface. The 2000 Å layer of SiO\(_2\) was of yellow-green hue.

5.9 Forming Ta\(_2\)O\(_5\) Pattern

A Ta\(_2\)O\(_5\) film was formed on an arm of the Y-branch modulator to be able to change its intrinsic phase later on. The film was formed after the Ti diffusion, and without an SiO\(_2\) layer. An Al lift-off technique was used to form a Ta pattern; Fig. 5.9. First, a ~ 3 kÅ Al film was deposited on the wafer by evaporation. Then, a ~ 4 μm window was etched in the Al film on one arm of the modulator. After stripping the negative photoresist, a ~ 400 Å Ta film was sputtered in Ar at 25 mTorr. The sputtering rate was 62 Å/min at a
Fig. 5.8. Computed curves of SiO$_2$ on Si.

Fig.5.9. Sputtered Ta over the Ti indiffused waveguide and between Al, prior to Al lift-off.
forward power of 100 W. Unwanted Ta was lifted off by dissolving Al in phosphoric acid. The Ta pattern thus obtained was oxidized (O\textsubscript{2}:1 l/m) at 500°C, for 90 minutes. The resulting oxide film was about 1100 Å thick. Electrodes were then formed, and the crystal cut and polished. The intrinsic phase of the modulator was measured. The Ta\textsubscript{2}O\textsubscript{5} was oxidized at 500°C for 10 minutes, and the intrinsic phase remeasured. The results are discussed in Chapter VII.

5.10 Forming the Electrode Pattern

After sputtering the SiO\textsubscript{2} buffer layer, the crystal was cleaned in acetone, deionized water, and methanol, and then a 10 kÅ layer of aluminium was deposited by electron beam evaporation, using the Veeco. The evaporation rate was about 30 Å/second at a current of 280 mA. Next, the electrode pattern was etched using the procedure outlined earlier. A split field microscope on the mask aligner was used to align the electrodes with the waveguides. A solution of phosphoric acid (85%) and deionized water (1:1) at 50°C was used to etch the aluminium pattern; the etch time was about 7 minutes. If the electrode pattern was not satisfactory, it was necessary to strip the resist, remove the aluminium, and start all over again. A photograph of a typical diffused waveguide and the electrode pattern is in Fig. 5.10.
Fig. 5.10. Photograph of the diffused waveguides and the electrode pattern; magnification: (a) 14X; and (b) 280X.
5.11 Cutting, grinding, and polishing

To launch light into the waveguide by "end-firing", a polished crystal edge is required. For efficient coupling the edge has to be square, since the waveguide is only a couple of microns deep. Furthermore, the edge has to be defect-free and polished to a surface finish approaching a fraction of the guide wavelength. Edge polishing offers several advantages over cleaving [Stulz 1979]. The cleavage is accompanied by chipping, also it tends to wander for substrates wider than 2 mm [Stulz 1979]. An irregular edge causes light to reflect and scatter at the entrance and the exit to the guide. The ultimate operation of the device and its efficiency, depend on how well the light is coupled into and out of the waveguide. So, the polishing procedure established after much trial and error is outlined in some detail. This procedure, however, is suited to the polishing of only one crystal at a time.

5.11.1 Preparation of the crystal for cutting

After the waveguide pattern had been diffused, a SiO₂ layer sputtered, and the aluminium contact electrodes etched, the crystal was ready for cutting and polishing. The crystal was temporarily cemented to a steel plate using canada balsam. It was then cut with a diamond saw. The steel plate had surfaces flat to within ± 0.001", and the dimensions are shown in Fig. 5.11. The steel plate was placed on a hot plate and heated to about 110°C, just warm enough to form a molten layer of the canada balsam. Just enough balsam to form a strong bond was applied. Any balsam that flowed to the top surface of the wafer interfered with the cover pieces and smeared the electrodes, and it was very difficult to remove. During this period the crystal was placed on an
Fig. 5.11. Mounting of the crystal in preparation for cutting.
adjacent steel plate to warm up gradually. The warm LiNbO$_3$ crystal was very carefully placed on the molten balsam, and its flat 1'' reference edge was aligned accurately with the steel plate edge. The assembly was then allowed to cool.

As mentioned earlier, the waveguide edges were protected from chipping and rounding by permanently bonding LiNbO$_3$ cover pieces, about 1/10'' wide and 2'' long, on the crystal where it had to be cut transversely. It was necessary to use LiNbO$_3$ cover pieces, since any other material would have worn off at a different rate during polishing, and produced scratches. The cover pieces were bonded with a two part, 5 minute epoxy (Devcon, Danvers, Mass.). To ensure a strong bond, 24 hours were allowed for the epoxy to set. Once the epoxy set, it had the hardness to withstand cutting and polishing. A small bead of epoxy was formed where the cover piece was to be bonded. Then a cover piece with the rough side towards the crystal was pressed into place, to form as thin an epoxy layer as possible to facilitate polishing. The epoxy was used sparingly to avoid smearing the aluminium electrodes. After the epoxy was definitely set, the assembly was placed on a hot plate and heated to 70°C, the melting point of Pyseal wax (Fisher Scientific, Fairlawn, N.J.). The wax was applied to form a puddle over the exposed areas of the crystal. It was applied to protect the waveguides and the electrode pattern during cutting and polishing. After the wax hardened, the crystal was ready for cutting and dividing.

Cutting was done with a diamond saw which has a vertical blade 8'' in diameter, 0.02'' thick and 180 grit. The cutting rate was 2''/hour. Of course, the slower the cut, the better the finish. After all the cuts were made, the
assembly was removed, cleaned in deionized water, and dried. The assembly was warmed on a hot plate to melt the Canada balsam, and the cut pieces were removed and stored.

5.11.2 Grinding and polishing of the crystal

In order to obtain a defect-free polished edge, grinding and polishing was done. The grinding was done on an emery paper. After which the polishing was carried out using 9, 6, 1 and 1/4 micron diamond paste compound (Metadi II and I; Buehler Ltd., Evanston, Ill.).

In preparation for grinding and polishing, a piece of crystal was centred and bonded to a strip of glass or metal, Fig. 5.12. The strip was cut wide enough to provide adequate support to the crystal, yet narrow enough to give sufficient overhang so as not to interfere with the polishing. If necessary, another coat of the Pyseal wax was applied to protect the waveguides and the electrode pattern. The edge of the crystal was examined microscopically. To view the edge, one end of the glass strip was held in a lump of modelling clay seated on a glass slide. The crystal edge was levelled and viewed, Fig. 5.13. If the crystal edge was not smooth, or if it had epoxy on it, the edge was ground on an emery polishing paper (4/0 Z948, Closekote, Norton). A figure of eight or a circular motion was executed to obtain uniform grinding. Usually 15 to 30 minutes of grinding per edge was required to smooth the irregularities and scratches resulting from cutting. Grinding was done one minute at a time followed by a pause, to avoid heating the crystal and weakening the epoxy bond. A view of the ground edge is shown in Fig. 5.14. The grinding was terminated when there was no further improvement.
Fig. 5.12. Mounting of a crystal piece before polishing.

Fig. 5.13. Arrangement to view the crystal edge.
Fig. 5.14. The crystal after grinding; magnification: 140X.

Fig. 5.15. The crystal after polishing with 9 μm diamond paste for 90 minutes; magnification: 140X.
The crystal was cleaned in deionized water and then put in ultrasonic bath for 10 minutes; this prevented the transmission of ground particles from one stage to the next. Cotton swabs (0-tips), soaked in DI water were used to wipe the edge. The wiping was carried out in a single stroke, and no portion of the swab was reused.

The next step was polishing with a 9 micron diamond paste. A polishing cloth with adhesive backing (Buehler Ltd., Ill.) was applied to a glass plate 9"x9". The diamond paste, which comes in a syringe, was dispensed to form 3 or 4 dabs, 1/4" long, spaced at equal distances over 2 square inches. The diamond compound was spread lightly with a clean finger tip. Three or four drops of extender oil were applied with a dropper, over the area charged with paste. An atomizer should be used for a controlled and uniform application of the oil. The oil was used parsimoniously, since too much oil produced scratches. The crystal edge was then rubbed lightly on the oiled diamond compound with straight strokes, to wear down large particles. The charge was "broken-in", and the crystal edge was polished tracing a figure of eight to prevent rolling. The polishing with the paste was continued, until the gouges and scratches disappeared. This can be seen by comparing Figs. 5.15 and 5.16. After about 90 minutes of polishing, the edges on either side of the epoxy line became visible. These were usually jagged and chipped. With subsequent polishing small chips of LiNbO₃ broke off the crystal. These can be seen embedded in the epoxy. The polishing was continued until the edges became chip-free, and formed two smooth parallel lines on either side of the epoxy line. This required about 60 to 90 minutes of polishing.
Fig. 5.16. The crystal edge after polishing with 9 μm diamond paste for 3½ hours; magnification: (a) 14X; and (b) 140X.
Once the surface was smooth, it was a matter of continuing with finer grades of diamond paste until the desired finish was obtained. The crystal edge was cleaned with cotton swab soaked in DI water before proceeding with fine polishing. The use of solvents, trichloroethylene, xylene, etc., was avoided, to prevent weakening the epoxy bond. The unused area on the polishing cloth was charged with a 6 microns diamond paste. The diamonds were broken-in, and the polishing resumed with straight and figure of eight motions. The diamond compound was used sparingly, as excessive diamonds cannot find niches and skate on the surface, causing edge damage by scratching. After the edge acquired an appearance similar to Fig. 5.17, the crystal was cleaned, and the whole process of charging, breaking-in, and polishing repeated with 1 micron and 1/4 micron diamond paste for 30 minutes in each case. Ultimately, the surface was highly polished and the edge roughness less than one micron; it had the appearance of Fig. 5.18. The epoxy line was about 6 microns wide at the end of polishing. If the epoxy joint is wider, more polishing is required resulting in rounding of the edges.

An alternative method of grinding and polishing is to use an electrically driven polishing wheel. The polishing time is reduced considerably, but this method is risky without the use of appropriate fixtures: on two different occasions the crystal caught in the rotating wheel and broke.

After both crystal edges had been polished and cleaned, the Pyseal wax was removed with boiling trichloroethylene. The edges were wiped clean with acetone and methanol, the crystal was ready for testing by end-fired coupling. In order to remove unwanted canada balsam, warm xylene was used with partial
Fig. 5.17. The crystal edge after polishing with 6 μm diamond paste magnification: (a) 140X; and (b) 280X.
Fig. 5.18. The crystal edge after polishing with (a) 1 μm paste; 560X; 
and (b) 1/4 μm paste; 560X.
success. To dissolve epoxy (Devcon), a 24-hour soak in trichloroethylene was effective. Alternatively, the epoxy was burnt off at 500°C in 0.

5.12 Fabrication tolerances

During the fabrication of the integrated optical circuits, several factors affect the device dimensions. A typical device is shown in Fig. 5.2. A small branching angle of 0.5° at the waveguide junctions is needed to facilitate equal division of the light. Narrow gaps (6 microns) between the electrodes, and long waveguides (18000 microns), are required for efficient modulation. Tolerances in the processing steps that contribute to deviations from the above objectives are:

1. Cutting and peeling
2. Mask
3. Exposing, developing, and etching
4. Diffusion

Irregularities in the rubylith affect definition of the pattern on the mask and quality of the devices. A rubylith pattern only 50 times the actual device size (18 mm long) could be cut, as the coordinatograph bed is only 40" long. Therefore, any errors in cutting were reduced by a factor of 50 only. A knife blade designed for making straight cuts, was used for the skew lines, which as a result had ragged edges. Skew lines with opposite slopes had unequal widths due to the asymmetric knife blade. Also, the width of the skew lines was narrower than the design, due to triangular motion of the cutter. As the cutter could move only in 0°, 90° or 45° directions, a skew line could be approximated either by a staircase, or by a concatenation of 45° line
segments. The latter strategy minimizes deviations from the design, so it had been incorporated in the cutter drive program. The cutter was driven by stepping motors with a minimum step of 0.001", thus the error in each cut was ±0.001". A desired waveguide width of 4 microns which corresponds to 0.008" on the rubylith, was cut as 0.008 ± 0.002". Consequently, the waveguide width on the mask was 4±1 micron.

Masks from the rubyliths were prepared externally: some had aberrations. On one mask, an L-shaped line with a uniform design width of 20 microns, measured 24 and 28 microns on the two arms. The line widths were difficult to measure, because at very high magnification (560X) the pattern on the mask appeared granular with diffused edges. Also, repeated use and handling sullied the mask.

The exposure, development, and etch times were established by trial and error. By juggling these times, the width of the lines could be adjusted by a micron or two. Thickness of the photoresist was affected by the spin speed and time. Exposure and development times depend on the resist thickness and affect the contrast. Etching is usually accompanied by undercutting. In the case of the positive resist photolithography, the errors due to overexposure and overetch are additive. As a result, the etched pattern is narrower than the design. In contrast, with the negative resist photolithography, overexposure and overetch have opposite effects and tend to cancel each other. In addition to the transverse diffusion, there is lateral diffusion which increases the line width, but reduces the irregularities due to the mask and the etch.
5.13 End-fire coupling procedure

The following procedure was used to align the laser, the input and output microscope objectives, and the crystal. The measurement set-up of Fig. 5.19 was used. Light was focussed with an input objective at the polished end of the guide, and collected at the output by another objective. This method of coupling light is called "end-fire" coupling, it is suitable for use with optical fibres. Other methods of coupling light require the use of prisms, gratings, etc.

1. The laser L and the input objective J1 (100X), Fig. 5.19, were aligned to obtain maximum output. This was done by placing a paper beyond the objective J1, and obtaining a circular patch of uniform bright light.

2. The second objective J2 (40X), placed at a distance of 2 cm from J1. Then J2 was adjusted to obtain a uniform circular pattern of diffused light.

3. The test crystal was mounted on a platform; adjustable in X, Y and Z directions, and rotatable around the Y-axis. A photograph of the set-up is shown in Fig. 5.20. A piece of masking tape was used to secure the crystal-carrying glass slide to the platform.

4. The two objectives were lined-up using the electrode pattern as a guide. Next, the objectives were adjusted to be 1 mm from the crystal edges.

5. The crystal was then moved downwards to intercept the beam of light. Next, the objectives and the crystal were adjusted to
Fig. 5.19. Measurement set-up.
Fig. 5.20. Photograph of the experimental set-up.
obtain a focussed set of parallel lines, see Fig. 5.21(a). Then the crystal was moved downwards until the lines moved further apart, as shown in Fig. 5.21(b).

6. The crystal was moved back and forth (Y-direction) until a bushy interference pattern of Fig. 5.21(c) was seen. Such a bushy pattern indicated the presence of an optical waveguide. The pattern was bigger and more complex when there was more than one waveguide.

7. Once the bushy pattern was obtained, it was simply a matter of adjusting the platform height and the input objective Jl, to obtain a bright focused spot of light; see Fig. 5.21(d).

8. The TE polarization of the light was selected by placing the polarizer at the laser; moving the polarizer adjacent to the detector made negligible difference.

9. The screen was removed, and the output light allowed to impinge the detector. To eliminate extraneous light from the substrate, room lights, etc., a paper with 0.5 mm diameter pinhole was placed at the detector input.

10. The output intensity after detection and amplification was displayed on an oscilloscope.

11. The modulating voltage was applied through steel probes. While viewing the probe and its reflection through a microscope, the probe was lowered to the contact pad. The position of the probe was adjusted, if necessary, before the contact was made. Once the probe tip and its reflection coalesced, the probe movement
Fig. 5.21. Output light pattern as the microscope objective and the crystal are adjusted to display the guided light.
was halted. When the probe tip made contact with the pad, the speckle pattern on the crystal surface shifted.

12. Pressing the probe on the pad invariably decreased the output light level, but the level was regained by lowering the input objective. Excessive probe pressure was avoided to prevent scratching of the electrode pattern. Of course, once all the probes were in place, the platform was not moved.

13. The modulating signal was then turned on and displayed on the oscilloscope. The system was tweaked; that is, the input coupling and the detector positions were adjusted to obtain a symmetrical output waveform with as small a dc component as possible.
CHAPTER VI
INTEGRATED OPTICAL MACH-ZEHNDER MODULATOR

The design, measured performance and applications of the integrated electrooptic Mach-Zehnder modulator are reviewed below. The modulator is a component of the HV sensor, series and multibranch interferometer, and the A/D converter investigated in this work.

6.1 Y-branch modulator

The electrooptic Y-branch modulator shown in Fig. 5.2 is an integrated optical version of the bulk Mach-Zehnder type interferometer [Martin 1975; Ohmachi and Noda 1975; Ranganath and Wang 1979]. The output ($I_0$) and input ($I_i$) light intensities are related by,

$$I_0 = \frac{I_i}{2} [1 + \cos(KLV + \psi)] + I_{dc} \tag{6.1}$$

where $I_i$ and $I_{dc}$ are the input and unmodulated light intensities respectively, $L$ is the electrode length, $V$ is the applied voltage, and $\psi$ is the intrinsic phase difference due to unequal path lengths. Here, $\phi = KLV$ and $K$ is given by,

$$K = \frac{2\pi}{\lambda} \left( \frac{1}{2} n^3 r \right) \frac{\eta}{d} \tag{6.2}$$

Where, $\lambda$ is the wavelength, $n$ and $r$ are the refractive index and electrooptic coefficient of the waveguide respectively, $d$ is the gap between electrodes, and $\eta$ is overlap between optical and electrical fields. For TE modulation electrodes are on either side of the waveguide, $\eta \approx 0.6$. For Z-cut LiNbO$_3$, TM
polarization utilizes, $r_{33}$, and one electrode resides on top of the waveguide, and in this case $\eta \leq 0.4$ is smaller.

Various versions of this type of modulator have been reported [Martin 1975; Minakata 1979]. In one version [Ramaswamy et al. 1978], the Y-sections were replaced by two tunable 3 dB couplers. In another case [Ranganath and Wang 1977], the splitting of light was adjusted by placing electrodes at the input Y. In order to eliminate losses due to branching, two parallel line 3 dB couplers were used in lieu of the Y's, and an ion etched slot was used to decouple the parallel arms over the modulating part, [Minakata 1979].

The integrated Mach-Zehnder modulator, has found applications as an A/D converter [Taylor 1975], a D/A converter [Papuchon et al. 1980], astable multivibrator [Ito et al. 1980], bistable device [Ito et al. 1979], optical logic [Taylor 1977] and an electromagnetic field sensor [Bulmer et al. 1980]. A polarization independent modulator to modulate the TE and TM mode has been demonstrated also [Burns et al. 1978].

In order to utilize the highest electrooptic coefficient $r_{33}$ in LiNbO$_3$, it is necessary to use Y-cut for the TE mode. The problems of the branching angle, suppression of outdiffused waveguide due to Li$_2$O escape, and radiation modes were addressed in [Miyazawa et al. 1977; Burns et al. 1978; Ranganath and Wang 1979]. Considerable progress has been made towards understanding the problems caused by Li$_2$O escape and the solutions. Loss at the Y-sections remains a problem. An extinction ratio as high as 97% has been reported [Leonberger et al. 1979].
6.2 Design and fabrication

Several Y-branch modulator designs were tried. The first one had 6 μm wide guides, and a total branching angle $2\theta = 2^\circ$. There was very little output light compared to a straight waveguide. In order to diagnose the problem, the device was dissected, and light coming out of the two arms measured. The light from the two arms was very weak as compared to that from a straight waveguide. The poor performance was attributed to the branching angle ($2\theta = 2^\circ$). In the next design, the waveguide width was reduced to 4 μm, and the branching angle decreased to $2\theta = 1^\circ$. Because of the small angle, the inclined waveguide sections span 2520 μm for a guide separation of 22 μm, as shown in Fig. 5.2.

Two important parameters are the guide width $W$, and the branching angle $2\theta$, in the design of Y-branch modulators. A $W = 2.7$ μm is required for single mode propagation at 0.6328 μm [Ranganath and Wang 1977]. Since, the rubylith could be cut only a maximum of 50X the actual size, the pattern width for $W = 2.7$ μm would have been $0.005 \pm 0.002"$, taking into account the cutter tolerances. A minimum width $0.003"$ ($\equiv W = 1.6$ μm) would have been difficult to maintain over 18000 μm length. So, $W = 4$ μm was selected as a compromise.

A large branching angle conserves the device length, but at the expense of higher scattering loss. The angle also affects mode separating and recombining. The branching loss for $2\theta = 1^\circ$ and $2^\circ$ is 1 dB and 2 dB respectively [Ranganath and Wang 1977].
6.3 Measurement results

The Y-branch modulator of Fig. 5.2 was fabricated in Y-cut LiNbO₃. Evaluation was done at 0.6328 μm, where nₑ = 2.203. For r₃₃ = 30.8 x 10⁻¹² m/V [Turner 1966], d = 6 μm, and L = 7200 μm, Vᵢ is given by,

\[ Vᵢ = \frac{1}{L} \left( \frac{\lambda}{nₑ} \right) \left( \frac{2}{r₃₃} \right) \frac{d}{n} \]

(6.3)

A positive and negative going triangular modulating voltage was applied to the electrodes. This, along with the modulated output light intensity is shown in Fig. 6.1. An applied voltage, \( Vᵢ = 4 \) volts produces a phase change of 180°, thus \( \eta = \frac{1.6}{4} = 0.4 \). When the applied voltage is zero, the output has a minimum, so \( \psi = 180° \). The extinction ratio is 3.8/4.4 ≈ 86%. Measured results for larger magnitudes of applied voltage are shown in Fig. 6.2-6.4.

The extinction ratio is degraded by:

1. Unequal splitting of light
2. Unequal recombination of light
3. Unsymmetrical Y-junctions
4. Scattering of rough edges of Y's
5. Presence of more than one mode
6. Radiation of air and substrate modes
7. Unsymmetrical placement of the electrodes
8. Surface guiding due to Li₂O deficiency

When more than one mode is present, the phase shift \( \pi \) for all the modes is not achieved simultaneously, and there is only partial extinction. The substrate and air radiation modes scatter to combine with the guided wave to contribute
Fig. 6.1. Applied triangular voltage (8 V<sub>pp</sub>) and the output light intensity of the Mach-Zehnder modulator; $V = 4 \text{ V}, \psi = 180^\circ$.

Fig. 6.2. Applied triangular voltage (20 V<sub>pp</sub>) and the output light intensity of the Mach-Zehnder modulator.
Fig. 6.3. Applied triangular voltage (24 V<sub>pp</sub>) and the output light intensity of the Mach-Zehnder modulator.

Fig. 6.4. Applied sinusoidal voltage (20 V<sub>pp</sub>) and the output light intensity of the Mach-Zehnder modulator.
a dc component and mar the extinction. Effect of unequal splitting or recombinatio
of light is considered below. The output light intensity \( I_o = E_o^2 \) is,

\[
E_o^2 = (xE_i)^2 + (1-x)^2E_i^2 + 2x(1-x)E_i^2 \cos \theta \tag{6.4}
\]

\[
I_o/I_i = 2x^2 - 2x + 1 + 2x(1-x) \cos \theta \tag{6.5}
\]

where, \( I_i = E_i^2 \) is the input light intensity, and \( xE_i \) and \( (1-x)E_i \) are the signals in the two modulator arms. Extinction ratio \( R \) is given by (6.6), and it is plotted in Fig. 6.5

\[
R = \frac{4x(1-x)}{2x^2 - 2x + 1 + 2x(1-x)} = 4x(1-x) \tag{6.6}
\]
Fig. 6.5. Variation of the extinction ratio $R$, with increasing signal $xE_1$ in one arm of the modulator.
CHAPTER VII
HIGH VOLTAGE SENSOR

7.1 Introduction

The conventional methods to measure the current and voltage in high voltage lines (> 400 kV) use transformers. The transformers lack the transient accuracy to take advantage of the high speed solid state protective relays that have been developed [Mouton et al. 1978]. The cost of insulation at > 400 kV is high. The metallic components are susceptible to electromagnetic and radio frequency interference. Passive optical systems to measure and control HV generation and transmission may be feasible. The possible advantages would be;

1. Electrical isolation
2. Immunity from electrical noise
3. No power required - passive device
4. Absence of saturation effects
5. High sensitivity
6. Low cost - only slightly affected by voltage level
7. Easy interfacing with optical communication link
8. Wide bandwidth

The optical systems would be particularly suitable for; (1) sensing over-voltage and overcurrent transients in protective relaying, where a bandwidth > 10 kHz but an accuracy of only 10% is required; and (2) fault sensing, where a bandwidth > 1 MHz is needed but an accuracy of 50% is adequate. For
metering purposes, only 10 Hz bandwidth is needed, but an accuracy of 0.1% is necessary [Rogers 1977; Erickson 1979].

Bulk optical measuring systems utilizing magneto-optic, electro-optic, and gyro-optic effect have been demonstrated. A typical arrangement, shown in Fig. 7.1, consists of a light source, polarizer, glass/quartz rod, mirror, analyzer and a detector. Such systems are sensitive to mechanical vibrations, misalignment, and interruption of the optical beams. These problems, could be alleviated by use of an integrated optical device with optical fibres at the input and output, Fig. 7.2. The advantages of such a device would be compact size, weight and cost.

The basic idea, as illustrated in Fig. 7.2, is to immerse the electro-optic modulator in the HV field to be measured. The field modulates the phase of the light through the electro-optic effect. Input and output light propagates through fibres. The light source and the detection system can be remote, to ensure electrical-optical isolation. Three types of modulators are considered for HV sensing: (1) The Y-branch modulator; (2) The BOA modulator; and (3) A parallel line modulator.

7.2 The Y-branch modulator

This type of modulator, which was discussed in Chapter VI is well proven; and it is suitable for HV sensing. It is necessary to shield one arm of the modulator, but expose the other to the HV field, Fig. 7.3. To shield the arm the thick metal is required. Relative output intensity of the modulator is given by,
Fig. 7.1. Schematic of an optical E-field detector (Erickson 1979).
Fig. 7.2. Scheme for optical measurement of high voltage.

Fig. 7.3. Y-branch modulator as a high voltage sensor.
\[ I_o/I_1 = (1 + \cos \theta)/2 \] 

(7.1)

where \( \theta = (\phi + \psi) \). When the intrinsic phase difference \( \psi = \pi/2 \); \( I_o/I_1 = (1 - \sin \phi)/2 \). In addition, when \( \phi \), the electrooptically induced change in phase is small, viz. \( \phi \ll \pi/2 \), the output changes linearly with the field;

\[ I_o/I_1 = 0.5 (1 - \phi) = 0.5 \left[ 1 - \frac{2\pi}{\lambda} \left( \frac{1}{2} n^2 r \right) L \left( \frac{E_o}{\varepsilon_r} \right) \eta \right] \] 

(7.2)

Here, \( E_o \) is the HV field strength, and \( \varepsilon_r \) is the relative dielectric constant of the crystal.

For Y-cut LiNbO\(_3\), at \( \lambda = 0.6328 \ \mu m \), \( n_e = 2.203 \), \( r_{33} = 30.8 \times 10^{-12} \ m/V \), \( \varepsilon_r = 28 \), assuming \( \eta = 0.4 \), \( L = 1 \ mm \), at \( E_o = 10^6 \ V/m \), for the TE polarization, \( \phi = 3.785^\circ \). Thus, in order to obtain \( \Delta \phi = \pm 10^\circ \), an arm length of \( L = 5.3 \ mm \) is needed.

In order to shield one arm of the modulator from the HV field, wide arm separation is required because of mechanical tolerances. Extent of the arm separation is limited, however, due to the small branching angle (2\( \theta = 1^\circ \)) and the need to conserve size. Modulators shown in Fig. 7.4 with arm separations of 88, 98 and 108 \( \mu m \) were designed and fabricated. The measured results are shown in Fig. 7.5 and 7.6. The measured half-wave voltage \( V_w = 5.5 \) volts, as compared with a theoretical value of 3.6 volts. Thus the optical and electrical overlap factor \( \eta = 0.65 \). Also, \( \psi = 99^\circ \), and the extinction ratio is \( \sim 90\% \). The intrinsic phase \( \psi \) or operating point, was adjusted by heating.
Fig. 7.4. Y-branch modulators with various arm separations: (a) 88 μm; (b) 98 μm; and (c) 108 μm.
10 ms/DIV.

5 V/DIV.

0.5 V/DIV.

10 ms/DIV.

Fig. 7.5. Performance of the Y-branch modulator with 88 μm separation; with applied triangular voltage (5 V pp).

10 V/DIV.

-  

0.5 V/DIV.

10 ms/DIV.

Fig. 7.6. Applied triangular voltage (20 V pp) and the output light intensity of the Y-branch modulator with 88 μm arm separation.
in O$_2$ a Ta$_2$O$_5$ loading film on one arm of the modulator, as discussed in Chapter IV.

7.3 The BOA modulator

It may be possible to use the BOA (Bifurcate Optique Active) switch [Papuchon et al. 1977] for HV measurement. It has only a single straight waveguide, where the modulating field is applied. If such a switch were to be placed in HV electric field, the output light intensity would be amplitude modulated.

The BOA modulator, shown in Figs. 7.7 and 7.8, may be considered as zero gap parallel line coupler. The narrow (4 μm) waveguides are monomode. But, the wide (8 μm) waveguide can support two modes; a symmetric and an antisymmetric mode. The input signal upon entering the two mode guide, divides into the symmetric and the antisymmetric modes. As the signal travels in the two mode guide, the energy shifts from the "top" half to the "bottom" half, back to the "top" half, and so on. The distance over which complete energy shift occurs, is [Papuchon and Roy 1977],

$$L_o = \frac{m\pi}{(\beta_s - \beta_{as})} = \frac{m\pi}{\Delta\beta}$$

(7.3)

where $\beta_s$ and $\beta_{as}$ are the propagation constant of the symmetric and the antisymmetric modes respectively, and $m$ is a positive odd integer. Thus, the energy can be shifted from one branch to the other by changing $\Delta\beta$. It has been shown [Papuchon et al. 1977], that the voltage required to switch energy from one coupler arm to the other, is reduced by an order of magnitude as the gap between coupler arms approaches zero. As the applied voltage changes, it
Fig. 7.7. BOA modulator with electrodes.

Fig. 7.8. BOA modulator as a high voltage sensor.
alters the interaction length. That is, the voltage changes the distance required to go from a node at the bottom section at the output. Of course, as the applied voltage varies, the output intensity is amplitude modulated at any one output branch.

A BOA modulator, with the dimensions indicated in Fig. 7.7, was fabricated in Y-cut X-propagating LiNbO$_3$. For TE modes, with $L = 9004 \ \mu m$ an electrode separation $d = 12 \ \mu m$, and waveguide widths of 4 and 8 $\mu m$, a $V_\pi = 9V$ was measured, see Fig. 7.9. The extinction ratio was $\sim 74\%$, and the intrinsic phase $\psi = 260^\circ$. The original BOA modulator [Papuchon et al. 1977], had 2 and 4 $\mu m$ guides, an electrode separation of 5 $\mu m$, and electrode length of 5 mm, and $V_\pi = 28V$. By comparison, the $V_\pi = 9V$ measured (Fig. 7.9) was only 1/3, considering that wider electrode separation (2.5X) was approximately offset by longer electrodes (1.8X). The measured results indicate, that a HV field of $10^6 \ \text{V/m}$ would produce a change in the phase of the output of $\sim 2.7^\circ/\text{mm}$ modulator length.

The BOA modulator in z-cut LiNbO$_3$ was placed between HV electrodes 16 cm apart. The output light from one branch of the modulator is shown in Fig. 7.10. The output amplitude changed linearly with HV. The laser and the detection system were placed away from the HV electrodes to prevent unwanted modulation. The measurement result indicate that the BOA device can be used as a HV sensor.
Fig. 7.9. Applied triangular voltage (15 V pp) and the output light intensity of the BOA modulator out of one arm only.

Fig. 7.10. Output light intensity of the BOA modulator out of one arm only in (a) 10 kV field, and (b) no field.
7.4 Parallel line modulator

This proposed device consists of two contiguous, straight, parallel, uncoupled waveguides. One of the waveguides is electrooptic and the other one is not. If such a device were to be placed in an external electric field, only the phase of the signal propagating through the electrooptic guide would be altered. At the output, where the phase modulated and the unmodulated signals combine, a sinusoidal variation in intensity would result. A configuration for the above device is shown in Fig. 7.11. One branch of the modulator consists of a Ti diffused waveguide, and the other of a superstrate $\text{Ta}_{2}\text{O}_5$ waveguide. The effect of $10^6$ V/m HV field on the light propagation through the $\text{Ta}_{2}\text{O}_5$ waveguide is negligible, because the linear electrooptic coefficient of $\text{Ta}_{2}\text{O}_5$ is small [Yee and Young 1975]. The coupling between the guides is assumed to be negligible, as the contiguous area between the two waveguides is small. The output intensity $I_o$ of this modulator is,

$$ I_o/I_1 = \frac{1}{2} \left\{ 1 + \cos \left[ \left( \frac{n_3}{\lambda} k E \right) + (\beta_1 - \beta_2) \right] L \right\} \quad (7.4) $$

where, $E$ is the field intensity, $L$ the device length, and $\beta_1$ and $\beta_2$ the propagation constants of the diffused and superstrate guides respectively.

This proposed parallel line modulator offers the advantages of complete utilization of the substrate length, an absence of bends or furcations in the network, and amenability to TE mode ($Y$-cut $\text{LiNbO}_3$) or to TM mode ($Z$-cut $\text{LiNbO}_3$) modulation. Furthermore, the refractive index of the superstrate $\text{Ta}_{2}\text{O}_5$ waveguide may be changed gradually until a $\pi/2$ intrinsic phase difference is obtained.
Fig. 7.11. Parallel line modulator: (a) isometric view; and (b) top view.
CHAPTER VIII
SERIES AND MULTIBRANCH INTERFEROMETERS/FILTERS

8.1 Introduction

Series and multibranch interferometers/filters are proposed, analyzed and demonstrated. Initially, the bandwidth of series interferometers for various electrode sequences is estimated, and then that for the multibranch. Measured results are compared with the theoretical ones.

8.2 Series interferometer/filter [Ahmed and Young 1980a]

The output intensity \( I_0 \) of a single interferometric modulator (Mach-Zehnder) varies sinusoidally (8.1);

\[
I_0 = \frac{1}{2} (1 + \cos \theta) = I_1 \sin^2 \left( \frac{\theta}{2} \right) \tag{8.1}
\]

where \( \theta = (KLV + \psi) \) from (6.1). If a sharper response is desired, several modulators may be connected in series. One arrangement of electrooptic modulators to realize a narrow passband is shown in Fig. 8.1. The output intensity will be maximum at a voltage equal to \( 2V_\pi \), and it will fall off rapidly as voltage is varied away from \( 2V_\pi \). Alternatively, an interferometric filter can be obtained by incorporating path differences of \( (2 \frac{1\pi}{2}), (2 \frac{\pi}{2}) \), \( (2 \frac{3\pi}{2}) \), ..., \( (2N \frac{\pi}{2}) \) between the two branches of the \( 1^{st}, 2^{nd}, 3^{rd}, ..., N^{th} \) interferometer. Such a device could be realized on a single substrate capable of supporting optical waveguides. In conventional optics, an analogue of such a device is the Lyot polarization interference filter [Tolansky 1973], it is
Fig. 8.1(a). Interferometer composed of \( N \) electrooptic modulators in series.

Fig. 8.1(b). Interferometric filter composed of \( N \) Mach-Zehnder interferometers in series.
capable of passing a band only 4 Å wide. The Lyot filter is made up of alternating sheets of polarizing material $P_1, P_2 \ldots$ and parallel quartz plates $O_1, O_2, \ldots$; the latter thicknesses $2^0t, 2^1t, 2^2t \ldots$. However, there are difficulties in realizing a Lyot filter: (1) the thickness of the quartz plates has to be very accurate; (2) the thick good quality quartz is difficult to find; (3) the temperature has to be held constant; (4) the system has to be immersed in oil to discourage multiple reflections from various surfaces; and (5) the transmission loss is high, even with all the precautions 90% of the incident light is lost. Billing used electrooptic ADP (ammonium dihydrogen phosphate) crystals with temperature compensation to obtain a voltage tunable passband of 1.25 Å [Tolansky 1973]. An interferometer/filter composed of several Mach-Zehnder interferometers in series on a single substrate (e.g. LiNbO$_3$) offers the advantages of compactness, temperature stability, low loss, low cost, and voltage tunability.

8.2.1 $N$ interferometers in series

The output intensity after the first interferometer shown in Fig. 8.1 is given by (8.1). But, the resultant intensity $I_o$ after passage of incident light intensity $I_1$ through $N$ interferometers of Fig. 8.1 is;

$$I_o^{1/2} = I_1^{1/2} (\cos \frac{\theta}{2} \cdot \cos 2^1 \frac{\theta}{2} \cdot \cos 2^2 \frac{\theta}{2} \cdot \ldots \cdot \cos 2^N \frac{\theta}{2}) \quad (8.2)$$

Let $x = \exp(j \theta/2)$; then,
\[(\frac{I_o}{I_1})^{1/2} = \frac{1}{2^N} (x+x^{-1})(x^2+x^{-2})(x^4+x^{-4})\ldots(x^{2N-1}+x^{-2N-1})\]

\[= \frac{1}{2^N} (x+x^3+\ldots+x^{-1+2^N}+x^{-1}+x^{-3}+\ldots+x^{-1-2^N})\]

Summing the geometric series:

\[(\frac{I_o}{I_1})^{1/2} = \frac{1}{2^N} \left\{ \frac{x[1-(x^2)^{2N-1}]}{1-x^2} + \frac{x^{-1}[1-(x^{-2})^{2N-1}]}{1-x^{-2}} \right\}\]

\[= \frac{1}{2^N} \left\{ \frac{-1+x^2N+1-x^{-2N}}{(x-x^{-1})} \right\}\]

Substituting the value of \(x = \exp(j\theta/2)\),

\[(\frac{I_o}{I_1})^{1/2} = \frac{1}{2^N} \left\{ \frac{\exp(j\theta/2)^{2N}-(\exp-j\theta/2)^{-2N}}{\exp j\theta/2 - \exp -j\theta/2} \right\}\]

\[
\frac{I_o}{I_1} = \left( \frac{\sin(\frac{\theta}{2} \cdot 2^N)}{2^N \sin(\frac{\theta}{2})} \right)^2 \tag{8.3}
\]

As a check, for \(N = 1,2\) and 3 the above expression reduces to \((\cos \frac{\theta}{2})^2\), \((\cos \frac{\theta}{2} \cdot \cos\theta)^2\) and \((\cos \frac{\theta}{2} \cdot \cos\theta \cdot \cos4\theta)^2\) respectively.

The numerator \(\sin(\frac{\theta}{2} \cdot 2^N)\) undergoes rapid fluctuations as a function of \(\theta\), Fig. 8.2(a) whereas the modulating function \((2^N \sin \frac{\theta}{2})^{-2}\) changes relatively slowly, Fig. 8.2(b) [Hecht and Zajac 1979]. A plot of intensity for \(N=1,2,3\) and 4 with \(\psi=0\), are shown in Fig. 8.3. Applying L'Hospital's rule at \(2^N\theta_m=2m\pi\), to (8.3),
Fig. 8.2. Plot of series filter response with $N=1$ and $N=3$. 
\[
\frac{d(1 - \cos \theta^N)}{2^N d(1 - \cos \theta)} + \frac{d(2^N \sin \theta^N)}{2^N d(\sin \theta)} + \frac{2^N \cos \theta^N}{2^N \cos \theta} = 1
\]

The principal maxima \( I_o/I_1 = 1 \), occur at values of \( \theta_m \) when;

\[2^N \theta_m = 2m\pi, \quad m = 0, \pm 1, \pm 2, \ldots \]

This is to be expected since a lossless system is assumed, and for \( 2^N \theta = 2m\pi \) there is proper phase-matching if \( \psi = 0 \).

The FWHM (full width half maximum) bandwidth decreases geometrically as the number of series interferometers increases, this is evident from Fig. 8.3. The bandwidth is twice the value of \( \theta \) which makes \( I_o/I_1 = 0.5 \) or where;

\[
\sin(\frac{\theta}{2} \cdot 2^N) - \sqrt{0.5} \cdot 2^N \sin \frac{\theta}{2} = 0
\]

(8.4)

For \( N = 1,2,3,4,5 \) and 6 the FWHM bandwidth is 180°, 82°, 40°, 20°, 10° and 5° respectively.

8.2.2 Series interferometers with linearly increasing phase retardation

If the phase retardation in successive interferometers increases linearly, the response would be sharper; but not as much as when it increases geometrically. Geometrically increasing phase retardation, however, require geometrically increasing electrode lengths. Consequently, for more than three sections such devices become impractical. Here, the performance of three and four interferometers in series, with the phase retardation increasing linearly from one interferometer to the next, are considered.
Fig. 8.3. Relative intensity of series interferometer/filter; $\theta$ is change in angle from the principal maxima.
8.2.2.1 Three interferometers in series

The output intensity of the first interferometer shown in Fig. 8.4 is given by (8.1). The resultant intensity, \( I_o \), after passage through the three interferometers, shown in Fig. 8.4 is:

\[
I_o^{1/2} = I_1^{1/2} \left( \cos \frac{\theta}{2} \cdot \cos \frac{2\theta}{2} \cdot \cos \frac{3\theta}{2} \right) \quad (8.5)
\]

Here \( I_1 \) is the input intensity, and \( N\theta \) is the phase retardation introduced in the \( N \)th interferometer. Setting \( x = \exp(j\theta/2) \), (8.5) becomes:

\[
\left( \frac{I_o}{I_1} \right)^{1/2} = \frac{1}{8} \left( x + x^{-1} \right) \left( x^2 + x^{-2} \right) \left( x^3 + x^{-3} \right)
\]

\[
\left( \frac{I_o}{I_1} \right)^{1/2} = \frac{1}{8} \left[ 2 + \frac{x^2(1 - x^6)}{(1 - x^2)} + \frac{x^{-2}(1 - x^{-6})}{(1 - x^{-2})} \right]
\]

\[
= \frac{1}{8} \left[ 1 + \frac{x^7 - x^{-7}}{x - x^{-1}} \right]
\]

Substituting the value of \( x \),

\[
\left( \frac{I_o}{I_1} \right) = \frac{1}{64} \left\{ 1 + \frac{\sin \frac{7\theta}{2}}{\sin \frac{\theta}{2}} \right\} \quad (8.6)
\]

The response of the above device is plotted in Fig. 8.4, with \( \psi = 0 \). The FWHM bandwidth relative to a single section interferometer is 0.28.

8.2.2.2 Four interferometers in series

The output intensity \( I_o \) of four interferometers in series, Fig. 8.4, with linearly increasing electrode lengths is,
Fig. 8.4. (a), (b) and (c) Modulator configurations; and (d) relative output intensity for the three configurations.
\[
\left( \frac{I_0}{I_1} \right)^{1/2} = \cos \frac{\theta}{2} \cdot \cos \frac{2\theta}{2} \cdot \cos \frac{3\theta}{2} \cdot \cos \frac{4\theta}{2}
\]

Here \( I_1 \) is the input intensity, and \( N\theta \) is the phase retardation introduced in the \( N \)th interferometer. Setting \( x = \exp(j\theta/2) \)

\[
\left( \frac{I_0}{I_1} \right)^{1/2} = \frac{1}{16} (x + x^{-1})(x^2 + x^{-2})(x^3 + x^{-3})(x^4 + x^{-4})
\]

\[
= \frac{1}{16} \left\{ 2 + \left( \frac{x^5 - x^{-5}}{x - x^{-1}} \right) \right\}
\]

Substituting \( x = \exp(j\theta/2) \)

\[
\left( \frac{I_0}{I_1} \right) = \frac{1}{256} \left\{ \frac{\sin \frac{5\theta}{2} + \sin \frac{11\theta}{2}}{\sin \frac{\theta}{2}} \right\}
\]  \hspace{1cm} (8.7)

This equation is plotted in Fig. 8.4 with \( \psi = 0 \). The FWHM bandwidth relative to single section interferometer is 0.19.

8.2.3 Experimental results

An integrated optical circuit with two interferometers in series was fabricated in \( Y \)-cut \( \text{LiNbO}_3 \). The waveguide and electrode pattern is shown in Fig. 8.5. The overall device length was 17960 \( \mu \)m, and the electrode lengths for the first and second modulators were 3712 \( \mu \)m and 7428 \( \mu \)m respectively including the push-pull effect. The measured results are shown in Fig. 8.6 - 8.9. The half-wave voltage \( V_\pi \) is \( \sim 8 \) V, and the theoretical value is given by,

\[
V_\pi = \left( \frac{\lambda}{n_e r_{33}} \right) \frac{d}{nL} = \frac{3.11}{\eta} \text{ volts}
\]  \hspace{1cm} (8.8)
Fig. 8.5. Two interferometers in series.
Fig. 8.6. Applied triangular voltage (20 V<sub>pp</sub>) and the output light intensity of two interferometers in series.

Fig. 8.7. Applied triangular voltage (30 V<sub>pp</sub>) and the output light intensity of two interferometers in series.
Fig. 8.8. Applied triangular voltage ($40 \text{ V}_{pp}$) and the output light intensity of two interferometers in series.
Fig. 8.9. Applied sinusoidal voltage (40 V $pp$) and the output light intensity of two interferometers in series.
with $\lambda = 0.6328 \ \mu m$, $n_e = 2.203$, $r_{33} = 30.8 \times 10^{-12} \ m/V$, $d = 6 \ \mu m$ and $L = 2 \times 1856 \ \mu m$. Comparing the measured and the theoretical values of $V_\pi$, the overlap integral $\eta \approx 0.4$. The extinction ratio is moderate; 75%. The side lobes are 0.25 of the main lobes in the output response. These are affected by the phase difference $\Delta \psi (= \psi_1 - \psi_2)$; where $\psi_1$ and $\psi_2$ are the intrinsic phases of the first and second modulators respectively. This point is elaborated below.

8.2.4 Effect of the intrinsic phase

The resultant output intensity $I_o$ after passage through two interferometers in series is,

$$ I_o/I_1 = \cos^2 \left( \frac{KL\psi}{2} \right) \cos^2 \left( \frac{K2LV + \Delta \psi}{2} \right) \quad (8.9) $$

Using the above measured value $KL\psi = 180^\circ$, or $KL \approx 22.5^\circ/volt$, where $K = (n_e^3 r_{33} \eta)/\lambda d$. Plots of $I_o/I_1$ using $\Delta \psi = 0^\circ$, $45^\circ$ and $90^\circ$ are shown in Fig. 8.10. As value of the $\Delta \psi$ increases, the peak value of the relative output intensity decreases, the passband broadens, and the sidelobes increase. A comparison of Fig. 8.9 and 8.10, indicates $\Delta \psi = 90^\circ$.

8.3 Multibrancl landfillterometer/filter [Ahmed and Young 1980a]

A response narrower than that of a Mach-Zehnder interferometer may be obtained with a multibrancl interferometer. Such an electrooptic filter, with voltage tunable response is shown in Fig. 8.11. The electrode lengths are $L$, $2L$, $3L$, $\ldots$, $NL$ on the successive branches. Alternatively, a narrowband
Fig. 8.10. Relative intensity $I_o/I_1$ as a function of voltage, with $\Delta \psi$ as parameter.
Fig. 8.11. Electrooptic multibranch interferometer.

Fig. 8.12. Variable length multibranch interferometric filter.
filter can be realized by incrementing the length of each successive branch by 
\( \lambda_g \ (= \lambda/n_z) \), as shown in Fig. 8.12. Here, \( \lambda_g \) and \( \lambda \) are the guide and the
free-space wavelengths respectively, and \( n_z \) is the longitudinal propagation
constant.

8.3.1 N branch interferometer

The resultant intensity of \( N \) overlapping waves, of the same frequency and
travelling in the same positive direction is given by [Hecht and Zajac 1979];

\[
E_0^2 = \sum_{r=1}^{N} E_{r} \exp(\imath \alpha_r) \quad \ldots \quad (8.10)
\]

\[
E_0^2 = (E_{o1} \exp(\imath \alpha_1) + E_{o2} \exp(\imath \alpha_2) + \ldots + E_{oN} \exp(\imath \alpha_N))
\]

\[
(E_{o1} \exp(-\imath \alpha_1) + E_{o2} \exp(-\imath \alpha_2) + \ldots + E_{oN} \exp(-\imath \alpha_N)) \quad (8.11)
\]

If the amplitude \( E_1 \) of the incident wave is split equally into \( N \) equal parts, then;

\[
E_1/N = E_{o1} = E_{o2} = \ldots = E_{oN} \quad (8.12)
\]

Furthermore, if the electrode lengths are such that the phase retardation in
the branches is 0, 2\( \theta \), 3\( \theta \), ..., \( N\theta \), then \( \alpha_1 = \theta \), \( \alpha_2 = 2\theta \), ..., \( \alpha_N = N\theta \).

Using (8.12) and values of \( \alpha \) in (8.11);

\[
E_0^2 = \frac{E_1^2}{N} \{ \exp(\imath \theta) + \exp(2\imath \theta) + \exp(3\imath \theta) + \ldots + \exp(N\imath \theta) \}
\]

\[
\times \{ \exp(-\imath \theta) + \exp(-2\imath \theta) + \exp(-3\imath \theta) + \ldots + \exp(-N\imath \theta) \}
\]
Summing the geometric series;

\[ E_0^2 = \frac{E_1^2}{N^2} \left( e^{j\theta} \frac{1 - e^{jN\theta}}{1 - e^{j\theta}} \right) \left( e^{-j\theta} \frac{1 - e^{-jN\theta}}{1 - e^{-j\theta}} \right) \]

\[ E_1^2 = \frac{E_1^2}{N^2} \left( 1 - e^{jN\theta} - e^{-jN\theta} \right) \]

\[ I_0 \cdot \frac{E_0^2}{I_1} = \frac{E_0^2}{E_1^2} = \frac{\sin^2(N\theta/2)}{N^2 \sin^2(\theta/2)} \tag{8.13} \]

Applying L'Hospital's rule to (8.13),

\[ \frac{I_0}{I_1} = \frac{d(1 - \cos N\theta)}{N^2 d(1 - \cos \theta)} + \frac{d(N \sin N\theta)}{N^2 d(\sin \theta)} \frac{\cos N\theta}{\cos \theta} = 1 \tag{8.14} \]

at \( N \theta = 2m\pi, m = 0, \pm 1, \pm 2, \ldots \) \tag{8.15}

The principal maxima occur at the above value of \( \theta_m \). This is to be expected since a lossless system is assumed, and at \( \theta = \theta_m \) there is proper phase-matching if \( \psi = 0 \).

The FWHM bandwidth is twice the value of \( \theta \) which makes

\[ \sin(N\theta/2) = (0.5)^{1/2} N \sin(\theta/2) \tag{8.16} \]

The computed response of a multibranch filter for \( N = 2, 3, \ldots, 6 \) with \( \psi = 0 \) is plotted in Fig. 8.13, and the bandwidths 180°, 112°, 82°, 65° and 54° respectively. A comparison of the FWHM bandwidth as a function of \( N \) for the series and the multibranch interferometers is made in Fig. 8.14, the bandwidth contraction with increasing \( N \) is more rapid in the former case. Although the
Fig. 8.13. Relative intensity of multibranched interferometer or a filter; $\theta$ is change in angle from the principal maxima.
Fig. 8.14. A comparison of the FWHM bandwidth of the series \((m=N)\) and the multibranch \((m=N-1)\) interferometers.
bandwidth contraction is less in the case of the multibranch filter, it does have the merit of requiring less "real estate".

8.3.2 Three branch interferometer with geometrically increasing phase retardation

A multibranch interferometer has sharp response compared to that of a single section two branch interferometer. A scheme for a three branch interferometer is depicted in Fig. 8.15. Here, the path lengths for the various branches differ by \( \lambda_g \), \( 2\lambda_g \) and \( 4\lambda_g \). Alternatively, for an electrooptic material, the electrode lengths are \( L \), \( 2L \) and \( 4L \). If amplitude \( E_i \) of the three incident waves is split equally, then:

\[
\frac{E_1}{3} = E_{01} = E_{02} = E_{03}
\]  

(8.17)

Furthermore, if the electrode lengths are such that the phase retardation in the branches is \( \alpha_1 = \theta \), \( \alpha_2 = 2\theta \) and \( \alpha_3 = 4\theta \), then from (8.11) and (8.17),

\[
E_o^2 = \frac{E_i^2}{9} (e^{j\theta} + e^{j2\theta} + e^{j4\theta})(e^{-j\theta} + e^{-j2\theta} + e^{-j4\theta})
\]  

(8.18)

With \( x = e^{j\theta} \).

\[
E_o^2 = \frac{E_i^2}{9} (3 + x + x^2 + x^3 + x^{-1} + x^{-2} + x^{-3})
\]  

(8.19)

\[
E_o^2 = \frac{E_i^2}{9} \left[3 + \frac{x^{1/2}(1 - x^3)}{x^{-1/2} - x^{1/2}}\right]
\]  

(8.20)

\[
= \frac{E_i^2}{9} \left[3 + \frac{3.5 - x}{x^{1/2} - x^{-1/2}}\right]
\]  

(8.21)
Fig. 8.15. A three branch interferometer with geometrically increasing electrode lengths.

Fig. 8.16. A four branch interferometer with geometrically increasing electrode lengths.
Substituting \( x = e^{j\theta} \)

\[
\frac{I_o}{I_i} = \frac{1}{9} \left[ 2 + \frac{\sin \frac{7\theta}{2}}{\sin \frac{\theta}{2}} \right]
\]

(8.22)

This equation is plotted in Fig. 8.17 with \( \psi = 0 \). The FWHM bandwidth relative to that of a single section interferometer is 0.42.

### 8.3.3 Four branch interferometer

Performance of a four parallel branch interferometer of Fig. 8.16 is described here. The successive branch length increases are \( \lambda_g, 2\lambda_g, 4\lambda_g \) and \( 8\lambda_g \). Alternatively, for an electrooptic material, the electrode lengths are \( L, 2L, 4L \) and \( 8L \), and the phase retardation introduced is \( \alpha_1 = 0, \alpha_2 = 2\theta, \alpha_3 = 4\theta, \) and \( \alpha_4 = 8\theta \). If \( E_1 \) is split so that,

\[
E_1/4 = E_{o1} = E_{o2} = E_{o3} = E_{o4}
\]

(8.23)

Then the resultant intensity using (8.11) and (8.23) is:

\[
E_o^2 = \frac{E_1^2}{16} (e^{j\theta} + e^{j2\theta} + e^{j4\theta} + e^{j8\theta})(e^{-j\theta} + e^{-j2\theta} + e^{-j4\theta} + e^{-j8\theta})
\]

(8.24)

Setting \( x = e^{j\theta} \),

\[
\frac{E_2}{E_o} = \frac{E_1^2}{16} (x + x^{-2} + x^4 + x^8)(x^{-1} + x^{-2} + x^{-4} + x^{-8})
\]

(8.25)

\[
= \frac{E_1^2}{16} \left\{ 4 - (x^5 + x^{-5}) + \sum_{n=1}^{7} (x^n + x^{-n}) \right\}
\]

(8.26)
\[
E_0^2 = \frac{E_i^2}{16} \{ 4 - (x^5 + x^{-5}) + \frac{x^{1/2}(1 - x^7)}{x^{1/2} - x^{-1/2}} + \frac{x^{-1/2}(1 - x^{-7})}{x^{1/2} - x^{-1/2}} \} \quad (8.27)
\]

Simplifying, and using \( x = e^{i\theta} \):

\[
\frac{I_0}{I_1} = \frac{1}{16} \left[ 3 + \frac{\sin \left( \frac{15\theta}{2} \right)}{\sin \left( \frac{\theta}{2} \right)} - 2 \cos \theta \right] \quad (8.28)
\]

This equation is plotted in Fig. 8.17 with \( \psi = 0 \). The FWHM bandwidth relative to a single section two branch interferometer is \( \sim 0.20 \).

### 8.3.4 Experimental results

A three branch interferometer shown in Fig. 8.18 was fabricated in Y-cut LiNbO\(_3\). The centre arm is the reference arm with the ground electrode on either side. The two modulating electrodes are 4752 \( \mu \)m and 9504 \( \mu \)m (=2x4752) long. The measured results are shown in Fig. 8.19 - 8.21, where the positive and negative going triangular waveform is the applied voltage. In Fig. 8.19, 8.20 and 8.21 is the response with voltage applied to the short electrode, the long electrode, and to both electrodes respectively. With voltage applied only to the short electrode, Fig. 8.19, the half wave voltage \( V_\pi = 7 \) \( \text{V} \), the intrinsic phase difference \( \psi_1 = 290^\circ \), and the extinction ratio is \( \sim 86\% \). The theoretical \( V_\pi = 2.43\text{V} \) with \( d = 4 \text{ \( \mu \)m} \), and \( L = 4752 \text{ \( \mu \)m} \), so the form factor \( \eta = 0.35 \). The output with voltage applied to the long electrode is shown in Fig. 8.20, from which \( V_\pi = 3.5 \text{ volts}, \psi_2 = 190^\circ \), and the extinction ratio \( = 65\% \). The output response of the complete device is shown in Fig. 8.21. The
Fig. 8.17. Comparison of N=2, 3 and 4 branch interferometers with $2^N L$ electrode lengths.
Fig. 8.18. A three branch interferometer.
Fig. 8.19. Applied triangular voltage on the short electrode of the three branch interferometer and the output light intensity.

Fig. 8.20. Applied triangular voltage on the long electrode of the three branch modulator and the output light intensity.
Fig. 8.21. Applied triangular voltage on both electrodes of the three branch interferometer and the output light intensity.
measured ratio, (FWHM bandwidth of complete device/FWHM bandwidth with voltage on short electrode), is 0.45: the theoretical value is 0.61. The sidelobes are about 0.16 of the main lobes. If the phase difference, $\Delta \psi (= \psi_1 - \psi_2)$, could be reduced, the side lobes would be suppressed.

The series and multibranch interferometers/filters narrow the passband, are voltage tunable, and would be useful for pulse shaping and pulse width modulation.
CHAPTER IX

ELECTROOPTIC ANALOGUE-TO-DIGITAL CONVERTER

9.1 Introduction

The signal measured in a physical system is usually in an electrical analogue form, but there is often a need to convert it into digital form. An analogue-to-digital converter (ADC) does this by sampling the analogue signal at regular intervals, and generating a series of pulses to represent the signal level. The ADCs are used in computing, communication and control systems. There is a growing need for high speed (> 100 MS/s) ADCs. The available ADCs use Si ICs which have been developed to their performance limits. For higher conversion rates, ADCs based on transferred electron devices, Josephson junctions, and optical systems are being explored.

Integrated optical ADC based on electrooptic diffraction [Wright et al. 1974], electrooptic deflection [Saunier et al. 1977], and electrooptic modulators [Taylor 1975] have been proposed, and these are reviewed below. All of the above ADCs require an array of detectors and comparators. A scheme that dispenses with the external comparators from the one using electrooptic Mach-Zehnder modulators is proposed. Design of a 4-bit comparatorless ADC is done. Experimental results on 1-bit comparatorless ADC are presented, and a three branch network used in it to apportion power is described.
9.1.1 Electrooptic diffraction ADC

An electrooptic diffraction ADC, Fig. 9.1, [Wright et al. 1974] uses variable threshold method and requires \( n \) comparators instead of \( 2^n - 1 \) called for in some schemes. While it is capable of fast synchronous operation, it does introduce an \( n \)-clock period delay. The output is in Gray code. The ADC is essentially an electrically controlled diffraction grating. Voltage applied to the periodic structure produces a periodic variation in refractive index to intensity modulate the diffracted beam into side orders, Fig. 9.1. The output is given by,

\[
I_m = j_m^2(\delta W)
\]

where \( j_m \) is \( m \)th order Bessel function.

9.1.2 Electrooptic phased-array deflector ADC

An EO phased-array deflector shown in Fig. 9.2 could work as an ADC [Saunier et al. 1977]. The linear EO effect in conjunction with variation in apodized electrodes, produces differential optical phase delay between adjacent light beams. The linear \( \phi \) slope is induced across aperture of the beam. A change in voltage induces phase slope to switch the beam.

9.1.3 Optical ADC with Mach-Zehnder modulators

Integrated optical ADC using Mach-Zehnder modulators has been proposed [Taylor 1975; Taylor et al. 1978]. Such an ADC uses the fact that the output intensity varies in a periodic fashion with the applied voltage \( V \);
Fig. 9.1(a). Electrooptic diffraction modulator.

Fig. 9.1(b). Sequential transfer characteristics and threshold levels.

Fig. 9.1(c) Experimental arrangement (Wright et al. 1974).
Fig. 9.2(a). Light deflector with apodized electrodes.

Fig. 9.2(b). Position of the deflected spot with drive voltage as a parameter (Saunier et al. 1977).
\( I_o = 0.5 I_1 \{1 + \cos(\phi + \psi)\} \) \hspace{1cm} (9.2)

where \( \phi = KLV \) and \( \psi \) is the intrinsic phase difference. An ADC with \( N=4 \) bits is discussed, its schematic and output is shown in Fig. 9.3.

The sinusoidal variation of the output intensity \( I_o \) of the modulators can be used to represent binary bits, because the bit value in a binary representation also varies periodically with the analogue signal. Thus, several modulator outputs can be combined in parallel to represent a digital word, and with the Gray code only one modulator per bit is required. Variation of the output intensity with the analogue signal \( V_a \) is shown in Fig. 9.3. The binary code obtained after detection, amplification, and comparison with a reference signal is shown too. A "0" is generated for \( I_o > I_t \), and "1" otherwise. The signal sampling could be performed by a mode-locked laser, which puts out a train of very narrow pulses [Taylor 1977]. With the Gray code, the modulator electrode lengths are \( L, L, 2L \) and \( 4L \). In general [Taylor 1979; Leonberger et al. 1979];

\( L_1 = L \) \hspace{1cm} (9.3)

\( L_N = 2^{N-2} L \), \( N = 2, 3, 4 \) \hspace{1cm} (9.4)

and

\( \phi_1 = KLV \), \( \psi_1 = \pi/2 \) \hspace{1cm} (9.5)

\( \phi_N = 2^{N-2} KLV \), \( \psi_N = 0 \), \( N = 2, 3, 4 \) \hspace{1cm} (9.6)

Where \( N = 1 \) corresponds to the MSB (most significant bit), \( N = 2 \) to the next-to-MSB, and \( N=4 \) to the LSB. Value of the intrinsic phase difference is \( \psi_1 = \pi/2 \) for MSB, and \( \psi_N = 0 \) for all other bits. A phase change \( \phi \) of \( \pi/2 \) for
Fig. 9.3(a). Schematic diagram of a 4-bit A/D converter.

Fig. 9.3(b). Intensity vs. voltage with Gray scale output (Taylor et al. 1978).
the LSB corresponds to a quantization step "q" in the analogue voltage 
\( V_a = V_m \sin(\omega t) \), to produce one bit change. There are \( 2^N \) quantization steps, 
so \( q = \frac{V_m}{2^{N-1}} \) and \( KL = \frac{\pi}{V_m} \). The Gray code for a given applied voltage and 
electrode lengths, gives one more bit of precision over the binary off-set 
code. The Gray code also minimizes error, since only a single channel bit 
changes with a change in the quantization step \( q \); whereas, with the off-set 
binary code several bits change.

The above ADC, although in theory very fast, would require the use of 
conventional electronics, such as, the sample and hold circuits and the 
comparators. The ultimate speed of this ADC is limited by the external 
electronics. If the output intensity of the modulators was a square wave 
instead of a sine wave, the comparators could be eliminated. In the following 
section a "Comparatorless A/D converter" is described. Another method to 
obtain square wave output, would be to use for each bit, a modulator followed 
by a bistable device. The bistable device could be an interferometer with 
feedback.

9.2 Comparatorless A/D converter

In order to utilize the possible high speed capabilities of ADCs 
employing waveguide modulators, the comparators would have to be eliminated 
[Ahmed and Young 1980b]. In Taylor's scheme each modulator output is 
translated into a two level square wave by the comparator. It is proposed 
that the sinusoidal variation of modulator output be altered to a square
variation by suitably combining the outputs of two or more modulators, Fig. 9.4. A square wave is approximated by its truncated Fourier series;

\[ v(x) = \frac{A}{2} + \frac{2A}{\pi} \left( \cos x - \frac{1}{3} \cos 3x + \frac{1}{5} \cos 5x \right) \] (9.7)

Let two additional modulators for each bit have outputs:

\[ I_2 = \frac{I_0}{6} \{ 1 - \cos (3KLV + \psi_2) \} \] (9.8)

\[ I_3 = \frac{I_0}{10} \{ 1 + \cos (5KLV + \psi_3) \} \] (9.9)

Then summing the output light intensities (not amplitudes) \( I_1, I_2 \) and \( I_3 \) of the three modulators, and neglecting static phase shifts (\( \psi \) 's):

\[ I = \sum_{i=1}^{3} I_i = \frac{I_0}{2} \left\{ \frac{23}{15} + \cos(KLV) - \frac{1}{3} \cos(3KLV) + \frac{1}{5} \cos(5KLV) \right\} \] (9.10)

A schematic of a single element of the proposed Comparatorless ADC is shown in Fig. 9.5. The three beams must not overlap before detection. The static phase shifts can be tuned, either with dc voltage on compensation electrodes or by film loading technique (Chapter IV). The previous schemes [Taylor 1979; Leonberger et al. 1979] also call for a dc voltage on an additional electrode to obtain \( \pi/2 \) phase difference between the intensity output for MSB and next-to-MSB. For each bit, it is necessary to apportion light intensity between the three modulators, so that relative strength of the three outputs is 1:1/3:1/5. For this purpose, either directional couplers or branching network can be employed. A design indicated 2 \( \mu \)m gap couplers. Because such narrow gap was difficult to realize, a branching network was used. The design and
Fig. 9.4. (a) Output light intensity variation of a single modulator; (b) Output of the comparator; and (c) Approximation of the rectangular wave by three sinusoids.
Fig. 9.5. Single cell of an A/D converter incorporating three modulators.
experimental results of a three branch network are presented, after a discussion of 4-bit comparatorless A/D converter.

9.3 Design parameters of a 4-bit ADC [Taylor 1979; Leonberger et al. 1979]

Various design parameters for a 4-bit electrooptic ADC, for sampling (S) rates of 1 GS/s and 2 GS/s, are considered. The electrode length and capacitance, the modulator driving power, as well as the optical power requirements are calculated. In addition, pulse jitter and width, and transit times are calculated.

9.3.1 Electrode length

A comparatorless ADC of Fig. 9.6, with N = 4 bits of precision, using Gray code requires for the Nth bit, an electrode length $l_1$, for the first harmonic modulator, Fig. 9.4 and 9.5;

$$l_1 = 2^{N-2} l_n$$

(9.11)

where $l_n$ is the electrode length to produce a phase shift of $\pi$ in the output. For the third and fifth harmonic modulators, the electrode lengths are $l_3 = 3l_1$, and $l_5 = 5l_1$ respectively.

For a Y-cut, X propagating LiNbO$_3$, at $\lambda = 0.6328 \ \mu$m, $r_{33} = 30.8 \times 10^{-12}$ m/V, and $n_e = 2.203$;

$$l_n = \frac{\lambda}{n_e^3 r_{33} \eta V_m} = 1.281 \ \text{mm}$$

(9.12)

Assuming a maximum modulating voltage $V_m = 10$V, with the coplanar electrode gap $d = 4 \ \mu$m, and the electrical and optical field overlap factor $\eta = 0.6$. 
Fig. 9.6. Schematic diagram of a 4-bit A/D converter.

Fig. 9.7. Equivalent circuit for the modulator circuitry.
Thus, for $N = 4$ bits of precision, length of the longest electrode is,

$$l_5 = 5(2^{N-2} \cdot l_{\pi}) = 20 \cdot l_{\pi} = 2.562 \text{ cm}$$  \hspace{1cm} (9.13)

Since the modulating voltage is applied to both arms of the modulator, the total modulator length is $l_5/2 = 1.281 \text{ cm}$. After allowing for compensation electrodes and the branching network, total device length is estimated to be $3.429 \text{ cm}$, see Fig. 9.6. Thus, several such devices could be accommodated on 2" diameter wafer.

### 9.3.2 Electrode capacitance

The electrode capacitance $C$, with electrode width equal to the electrode gap $d$, is given by [Taylor 1979],

$$C = 0.6 \varepsilon_0 (1+\varepsilon) L = 17.486 \text{ pF}$$  \hspace{1cm} (9.14)

where $\varepsilon = \sqrt{\varepsilon_1 \varepsilon_3} = 34.5 \text{ pF/cm}$ for the Y-cut LiNbO$_3$, and $\varepsilon_0 = 8.9 \times 10^{-2} \text{ pF/cm}$. The electrical power, $P_e$, dissipated in the modulator circuitry is,

$$P_e = \pi C V_m^2 B/2$$  \hspace{1cm} (9.15)

where $B$ is the -3 dB bandwidth. For a high speed ADC,

$$\frac{\text{Sampling rate (Samples/s)}}{\text{Max. signal freq., } f_m (\text{Hz})} = 3.3$$  \hspace{1cm} (9.16)

The above ratio ranges from 2 (the Nyquist rate) to 4. For a sampling rate of 1 GS/s, then $f_m = 300 \text{ MHz}$. Assuming the bandwidth $B = 2 f_m = 600 \text{ MHz}$, the power dissipation is:

$$P_e = \pi C V_m^2 B/2 \approx 1.65 \text{ W}$$  \hspace{1cm} (9.17)
The power dissipation determines the analogue signal amplifier requirements, and hence the size, weight and cost of the ADC. The power dissipation must be limited to prevent damage by overheating.

9.3.3 Signal sampling errors

Use of a mode-locked laser, putting out a train of pulses, as an optical power source has been suggested [Taylor 1977]. This is in lieu of the sample-and-hold circuits in the conventional high speed ADC, and like the sample-and-hold circuits, pulsed laser produces errors. The errors due to the fluctuation in sampling time (jitter), width of the optical pulse, and the optical transit time are calculated [Taylor 1979].

9.3.4 Jitter

The maximum error due to a pulse arriving at \( t_{i+1} = t_i + \Delta t + \delta t \) instead of \( t_{i+1} = t_i + \Delta t \), for a signal of the form \( V = V_m \sin(2\pi f t) \) is,

\[
|\delta V_{\text{max}}| = 2\pi f V_m \delta t_{\text{max}} 
\]

The condition that the error in an ADC be less than half the level spacing \( \Delta V/2 = V_m/2^N \) gives,

\[
\delta t_{\text{max}} < 1/(2^{N+1} \pi f_m) = 33.2 \text{ ps} 
\]

9.3.5 Electrooptic interaction duration error

For the above \( V \), at time \( t_i \), the error due to \( \Delta T \), the sum of the pulse width \( T_p \) and the optical transit time \( T_{\text{opt}} \) [Taylor 1979] is,
\[ \delta V = -V(t_1) + \frac{1}{\Delta T} \int_{t_1-0.5\Delta T}^{t_1+0.5\Delta T} V(t) \, dt \]

\[ = (\frac{\pi f m \Delta T}{6}) V(t_1) \]  

Again, with the condition the \( |\delta V|_{\text{max}} < 0.5 \Delta V \),

\[ \Delta T < \left( \frac{3/2 N-1}{\pi f m} \right)^{1/2} \approx 650 \text{ ps} \]

(9.21)

The optical transit time through the longest electrode \( l = 2.562 \text{ cm} \) for the LSB is,

\[ T_{\text{opt}} = \frac{n_e l}{c} = 188 \text{ ps} \]

(9.22)

where \( n_e = 2.203 \) is the guide index, and \( c = 29.97925 \times 10^9 \text{ cm/s} \) is the speed of light in free-space. Thus, the maximum allowable pulse width is \( T_p = \Delta T - T_{\text{opt}} = 462 \text{ ps} \).

### 9.3.6 Optical Power

In order to satisfy a given error probability criterion, sufficient optical power \( P_1 \) from the source is required. For instance, to correctly detect MSB (or the bit next-to-MSB), when there is simultaneous zero crossing of the output intensity of the LSB and the MSB (or the bit next-to-MSB) [Leonberger et al. 1979],

\[ P_1 > \frac{i_n N^{-1}}{\pi k_1 \gamma G} \]  

(9.23)

For Si avalanche photodiode (APD) \( i_n = \sqrt{\frac{4kT\Delta f}{R}} \), with \( R = 50 \Omega \), and \( \Delta f = 3 \text{ GHz} \);
because thermal noise dominates the shot noise. The unity-gain responsivity $\gamma = 0.2 \ \mu A/\mu W$, and the gain $G = 100$. The total source to guide, and guide to detector coupling factor $k_i = 0.005$, for end-fire coupling. The signal to rms noise level $x = 6$, corresponding to a 2-bit error probability of $10^{-9}$ assuming Gaussian statistics.

In [Taylor 1979] the power required by an optical receiver with Si APD, at a SNR of 21.5 dB, for $10^{-9}$ bit error rate, with $N = 4$, was estimated to be $-33$ dBm. This power was estimated by computer simulations, using the Monte Carlo calculation to determine the effect of the quantum errors on the ADC performance. Thus, for a mode-stabilized injection laser diode with 7 dBm output, a 40 dB loss through the device can be tolerated. The above design parameters are compared for 1 GS/s, and 2 GS/s integrated optical ADC in Table 9.1.

Table 9.1

<table>
<thead>
<tr>
<th>Bits</th>
<th>Sample Rate GS/s</th>
<th>Transit Time ps</th>
<th>Laser Pulse Width ps</th>
<th>Pulse Jitter ps</th>
<th>Peak Opt. Power mW</th>
<th>$L_{max}$ cm</th>
<th>C pF</th>
<th>Mod. Power Diss. W</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
<td>188</td>
<td>462</td>
<td>33</td>
<td>0.16</td>
<td>2.56</td>
<td>17.5</td>
<td>1.65</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>188</td>
<td>137</td>
<td>17</td>
<td>0.16</td>
<td>2.56</td>
<td>17.5</td>
<td>3.3</td>
</tr>
</tbody>
</table>
9.3.7 Measurement results

One bit of the ADC was produced by the circuit shown in Fig. 9.8. It was built in Y-cut, X propagating LiNbO$_3$. The waveguide width was 4 $\mu$m, and the electrode gap 6 $\mu$m. All the bifurcation angles were 1°. The three modulators had electrode lengths of 2 mm, 6 mm and 10 mm, including the push-pull effect.

End-fire technique was used to couple 0.6328 $\mu$m light, into and out of the crystal. Microscope objectives 100X and 40X were used at the input and the output respectively. The modulating voltage was applied via steel probes on the Al electrode pads. The output light was viewed on a paper screen, or else fed to a detector-amplifier-oscilloscope chain. The three output beams could not be discerned separately with a naked eye; the beams formed a thin narrow strip of light. This is to be expected, since the inter-waveguide spacing is only 16 $\mu$m. The three outputs were examined individually with a photodiode and used for diagnostics and evaluation. The three outputs were from the first, third and fifth harmonic modulators; these are shown in Fig. 9.9(a), (b) and (c) respectively. These are in response to a triangular voltage. For the first harmonic, the measured value of $V_1$ was 11.5V, at an extinction ratio of 81% and the intrinsic phase difference $\psi$ was estimated at $\sim 230^\circ$. The theoretical value of $V_\pi$ is,

$$ V_\pi = \frac{\lambda}{n_e r_{33}} \frac{d}{L} $$

$$ = \frac{0.6328}{(2.203)^3 30.8 \times 10^{-12}} \times \frac{6}{n(2 \times 1000)} = \frac{5.765}{n} \text{ volts} \quad (9.24) $$

where the electrode gap $d = 6$ $\mu$m, the total electrode length $L = 2 \times 1000$ $\mu$m,
Fig. 9.8. Waveguide structure and the electrode pattern of the comparatorless A/D converter.
FIRST HARMONIC $I_o$

0.2 V/DIV.

10 ms/DIV.

(a)

THIRD HARMONIC $I_o$

0.2 V/DIV.

10 ms/DIV.

(b)
Fig. 9.9. Applied triangular voltage and the output light intensity of the comparatorless A/D converter; (a) first harmonic, (b) third harmonic, (c) fifth harmonic, and (d) composite $I_o$. 
and \( \eta \) is the optical and electrical field overlap factor. Thus, \( \eta = 5.765/11.5 \approx 0.5 \). From Fig. 9.9(a) the extinction ratio \( \approx 81\% \). These could be improved by a better Li\(_2\)O compensation, use of a higher wavelength (1.15 \( \mu \)m) or else a narrower waveguide (3 \( \mu \)m). Also, symmetrical Y bifurcations with uniform edges would ensure equal signal split and reduce the dc component in the output.

The three harmonic outputs and the composite square wave are compared in Table 9.2. The harmonic beams were coalesced at the photodiode with a lens, and the output waveform is shown in Fig. 9.9(d). The result is approximately a square wave. The distortion is due to the differences in the intrinsic phases in the three modulators. The phase difference between the I and III harmonic is \( \sim 150^\circ \) instead of \( 180^\circ \) from Fig. 9(a) and (b). Better results could be obtained by using a Ta\(_2\)O\(_5\) film for phase matching, Chapter IV.

The branching network was used to apportion power in the 1:0.33:0.2 ratio. The measured ratio was \( \sim 1:0.5:0.1 \). A low frequency signal was used to test the circuit, but the device could operate 1-4 GHz, by making the electrodes travelling wave structures made up of properly terminated microstriplines to the harmonics.
Table 9.2

PERFORMANCE OF THE ADC

<table>
<thead>
<tr>
<th>Harmonic Modulator</th>
<th>Fig. 9.9</th>
<th>Meas. V̂ (Volts)</th>
<th>Theor. V̂ (Volts)</th>
<th>( \phi ) Relative I Harmonic</th>
<th>1/2 Cycles /30 Vpp</th>
<th>I (Arb. Units)</th>
<th>Extinction Ratio %</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>(a)</td>
<td>11.5</td>
<td>5.77</td>
<td>0°</td>
<td>2.6</td>
<td>.44</td>
<td>81</td>
</tr>
<tr>
<td>III</td>
<td>(b)</td>
<td>4.8</td>
<td>1.92</td>
<td>~150°</td>
<td>6.3</td>
<td>.24</td>
<td>64</td>
</tr>
<tr>
<td>V</td>
<td>(c)</td>
<td>3.8</td>
<td>1.15</td>
<td>~0°</td>
<td>8.0</td>
<td>.05</td>
<td>38</td>
</tr>
<tr>
<td>Composite</td>
<td>(d)</td>
<td>10.</td>
<td>5.77</td>
<td>-</td>
<td>1.3</td>
<td>.14</td>
<td>50</td>
</tr>
</tbody>
</table>

This could be done by forming coplanar lines of 4 μm width and gap, around 3 μm wide guides. Additionally, a fast APD in conjunction with a microwave amplifier would be required. Finally, replacement of the Y-branch modulators by the BOA devices (Chapter V) would simplify the electrode and waveguide structures.

9.4 Branching network

A power splitting network is required to feed the three Mach-Zehnders in the ADC. One option is to use couplers to apportion power, and another is to use a branching network. A design of the directional couplers [Marcatili 1969] indicated a gap of 2 μm between the parallel coupler arms. Such a narrow gap was considered impractical due to limitations of the mask and equipment, so a branching network shown in Fig. 9.10 was designed and evaluated.
For the ADC an intensity split in the proportion \(I_A : I_B : I_C = 1 : 1/3 : 1/5\) is required. This is approximated by the branching network of Fig. 9.10. The signal splits at the first bifurcation into \(I_2\) and \(I_4\), and then \(I_4\) divides at the second bifurcation into \(I_5\) and \(I_6\). The division of the signal is; 
\[I_3 : I_5 : I_7 = 1 : 1/4 : 1/4\]. A more accurate estimate of the signal split is made by taking into account the branching angle [Burns and Milton 1975], and the bending losses [Hutcheson et al. 1980].

### 9.4.1 Bending and branching losses

The bending losses of two straight, parallel, noncollinear waveguides are calculated. The output power, \(P_0\), after the second bend for the case shown in Fig. 9.11 is given by [Hutcheson et al. 1980],

\[
P_0 = P_1 |a_{12}|^2 |a_{23}|^2 \exp(-\gamma_0 L_0) \tag{9.25}
\]

where \(P_1\) is the input to the first bend, \(\gamma_0\) is the attenuation constant due to Rayleigh scattering and absorption, \(L_0\) is the length of the joining segment, and \(|a_{pq}|^2\) is the relative power coupling of the fundamental mode of the two straight waveguides \(p\) and \(q\). The power coupling coefficient for the configuration shown in Fig. 9.11 are given by [Hutcheson et al. 1980],

\[
|a_{12}|^2 = |a_{23}|^2 = \exp(-\beta^2 \chi_0^2 \sin^2 \theta/4) \tag{9.26}
\]

where \(\beta\) and \(\chi_0\) are the propagation constant and half width of the fundamental mode respectively. The bending angle \(\theta\) is,

\[
\theta = \tan^{-1} \left( \frac{\chi_B}{Z_s} \right) \tag{9.27}
\]
Fig. 9.10. (a) The branching network and (b) the A/D converter waveguide structure.

Fig. 9.11. Two parallel waveguides joined by a straight segment.
where $X_s$ is the transverse separation of the parallel guides, and $Z_s$ is the axial displacement. The absorption and the Rayleigh scattering losses (1.5 dB/cm) due to the path-length difference $L_o - Z_s$, which is small (0.0005 cm), for $X_s = 60 \mu m$ and $\theta = 1^\circ$, are neglected ($\gamma_0 = 0$). This is reasonable, since for the branching network (Fig. 9.13), and the ADC (Fig. 9.8), the relative outputs are more important than the absolute values. With $\theta = 1^\circ$, and using the representative parameters in [Hutcheson et al. 1980], the post-bending transmission power ratio $T_c$ from (9.25), for one carrier is; $T_c = 0.883$.

The branching loss for $\theta = 1^\circ$, from [Ranganath and Wang 1977; Burns and Milton 1975] are about 23% ($1-2A_j^2; A_j = 0.62$). That is, the post-branching power ratio, $T_B = 0.3844$. Using the above values of post-bending power ratio ($T_c = 0.883$), and post-branching power ratio ($T_B = 0.3844$), relative power output for the branching network and the ADC (Fig. 9.8), are given below.

\[
\begin{align*}
P_1 &= 1; P_4 = P_1 T_B = 0.384; P_3 = P_1 T_B T_C = 0.339; P_5 = P_4 T_B = 0.148; \\
P_7 &= P_4 T_B T_C = 0.130; P_A = P_3 T_C^2 = 0.264; P_B = P_5 = 0.148; P_C = P_7 T_C^2 = 0.102
\end{align*}
\]

The losses corresponding to the three modulators in Fig. 9.10(b) are ignored, because of the commonality. The calculated values are compared with the measured ones in Tables 9.3 and 9.4.
Table 9.3
THE BRANCHING NETWORK OUTPUT POWER RATIO

<table>
<thead>
<tr>
<th>Output</th>
<th>Calculated</th>
<th>Measured</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Absolute</td>
<td>P/P₃</td>
<td>Absolute</td>
<td>P/P₃</td>
</tr>
<tr>
<td>P₃</td>
<td>0.339</td>
<td>1.0</td>
<td>5.0</td>
<td>1.0</td>
</tr>
<tr>
<td>P₅</td>
<td>0.148</td>
<td>0.44</td>
<td>1.8</td>
<td>0.36</td>
</tr>
<tr>
<td>P₇</td>
<td>0.130</td>
<td>0.38</td>
<td>0.5</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Table 9.4
COMPARATORLESS ADC OUTPUT POWER RATIO

<table>
<thead>
<tr>
<th>Output</th>
<th>Desired</th>
<th>Calculated</th>
<th>Measured</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Absolute</td>
<td>P/Pₐ</td>
<td>Absolute</td>
<td>P/Pₐ</td>
</tr>
<tr>
<td>Pₐ</td>
<td>1.0</td>
<td>0.26</td>
<td>1.0</td>
<td>0.44</td>
<td>1.0</td>
</tr>
<tr>
<td>P₅</td>
<td>0.33</td>
<td>0.15</td>
<td>0.56</td>
<td>0.24</td>
<td>0.5</td>
</tr>
<tr>
<td>P₇</td>
<td>0.20</td>
<td>0.10</td>
<td>0.39</td>
<td>0.05</td>
<td>0.1</td>
</tr>
</tbody>
</table>

A T₁ diffused waveguide branching network was made in Y-cut LiNbO₃. Typical dimensions of the branching network are shown in Fig. 9.12. The
output intensities $I_3$, $I_5$ and $I_7$ were measured through a 0.5 mm diameter pin-hole. The measured values were: $I_3 = 5$, $I_5 = 1.8$, and $I_7 = 0.5$. Thus, the branching network splits light (1:0.36:0.1) fairly close to the calculated proportions; (1:0.44:0.38).
Fig. 9.12. The branching network.
CHAPTER X

SUMMARY AND CONCLUSIONS

An adjustment of the propagation constant of a diffused waveguide by film loading has been demonstrated. An application of integrated optical devices to HV measurement has been discussed. Series and multibranch interferometers/filters, and a comparatorless A/D converter (ADC) have been proposed and demonstrated.

The devices were fabricated by diffusing 500 Å of Ti at 1000°C for 6 hours into Y-cut X-propagating LiNbO$_3$. These were evaluated at 0.6328 μm, and the light was coupled through polished edges by the end-fire method. The waveguides were 4 μm wide, and the electrode gap 6 μm. The bifurcation angle for the Y-branch modulators was 1°, and all devices were 17960 μm long. An Al lift-off technique was developed to pattern the sputtered Ta loading film. Tests on devices with TM mode modulation on Z-cut LiNbO$_3$ indicated that Li$_2$O compensation is required to prevent change in n$_e$.

An input impedance equation for the transformation of an arbitrary complex impedance through a dissipationless, exponentially tapered transmission line was derived. It was used to obtain the characteristic equation for the propagation constant in a linearly graded optical waveguide. The equations were used to model, and to compute numerical results for a Ta$_2$O$_5$ film loading a Ti indiffused waveguide. An application of these results to alter the intrinsic phase $\psi$, of a Y-branch modulator was successfully carried out. A Ta$_2$O$_5$ loading film ~ 1200 Å thick, ~ 6400 μm long on one branch produced a change in $\psi$, of about 6°/minute of heating in O$_2$. This technique
can also be applied to the phase tuning of other length sensitive devices, e.g. a directional coupler.

Three Y-branch modulators with 88, 98 and 108 µm arm separation, and a two mode (BOA) modulator for HV measurement were built. For the former, \( V_\pi \approx 5.5 \) volts for \( L \approx 7200 \) µm, and \( d = 8 \) µm was measured. For the BOA modulator, corresponding to \( L = 9004 \) µm, and gap \( d = 12 \) µm, the measured \( V_\pi \approx 9 \) volts. The BOA modulator is the simpler of the two configurations. As it does not require a branching network, the size is smaller, and it requires only two electrodes. Because of wider electrode gap \( V_\pi \) is larger.

An expression for the output intensity \( I_o \) of a series interferometer/filter is,

\[
\frac{I_o}{I_1} = \left( \frac{\sin \frac{\theta}{2} \cdot 2^N}{2^N \sin \frac{\theta}{2}} \right)^2
\]

where \( 2^N L \) is the length of the \( N^{th} \) electrode. the FWHM bandwidth for \( N = 1, 2, 3, 4, 5, 6 \) is \( 180^\circ, 82^\circ, 40^\circ, 20^\circ, 10^\circ \) and \( 5^\circ \) respectively. A two section filter with \( L = 3712 \) µm and \( 2L = 7428 \) µm, and an electrode gap \( d = 6 \) µm was fabricated. It has a bandwidth of \( \sim 108^\circ \), as compared to a theoretical value of \( 82^\circ \). The output intensity \( I_o \) of a multibranch interferometer/filter is given by; \( I_o/I_1 = \frac{\sin^2(N\theta/2)}{N^2 \sin^2(\theta/2)} \) where \( NL \) is the length of the \( N^{th} \) electrode. The FWHM bandwidth, for \( N = 2, 3, 4, 5, 6 \) is \( 180^\circ, 112^\circ, 82^\circ, 65^\circ \) and \( 54^\circ \) respectively. The bandwidth of a three branch interferometer that was fabricated, was \( 90^\circ \); it agrees with the theoretical prediction \( (112^\circ) \). The series and multibranch interferometers/filter offer
the advantage of narrow output, and voltage tunability. The devices, however, are more complex, cover more area, and require a phase matching of the component modulators.

An elimination of the external comparators, which are the slowest component of the A/D converter (ADC) based on an interferometer, is proposed. Each comparator, is replaced by three modulators - the output of which is combined in a detector to produce a square wave. Design calculations indicate that such a device, for an analogue voltage 0-10V, with 4-bit precision, using the Gray code, is feasible on Y-cut LiNbO$_3$. The device would be 35 mm long, with 3 μm wide waveguides, and an electrode gap of 4 μm. A comparatorless ADC to produce 1-bit was fabricated. The output intensity of three component modulators was measured. For $L = 1000$ μm, and $d = 6$ μm, a $V_\pi = 11.5$ volts, and an extinction ratio = 81% were measured. The composite output resembled a square wave. By using electrodes that are essentially terminated microwave transmission lines, an ADC speed 1-2 GS/s could be realized. The proposed ADC is complex, covers wider area, is longer, and requires a phase matching of the component outputs. A three branch power splitting network was fabricated and tested, it split power in $1:0.36:0.1$ ratios. When more accurate fabrication techniques evolve, and the phase tuning methods are perfected, the comparatorless ADC will become reality.

Integrated optical devices will find practical applications, only if the source (or fibre) to waveguide loss, and the device attenuation are reduced. Some of the problems can be alleviated by operating at longer wavelengths, 1.3 - 1.5 μm. The benefits are that only one mode exists in a 4 μm waveguide, the optical damage threshold in LiNbO$_3$ is two orders of magnitude higher than
it is at 0.6328 µm, and the propagation loss of optical fibres is minimum, ~ 0.5 dB/km. Measured values of $V_\pi$ for the BOA modulator is lower than the one reported so far. As the BOA structure is simple, and it requires only a pair of electrodes, it could replace Y-branch modulators in various devices. It is particularly suited to the HV measurement application, as immersion in the HV electric field would modulate the light. For this application, a BOA modulator in Z-cut LiNbO$_3$ crystal, with TM mode modulation to utilize $r_{33}$ is suggested. An additional benefit of this crystal orientation is a greater overlap between the fibre and the waveguide fields [Fukuma and Noda 1980].

An interferometer composed of two straight, parallel, contiguous, but uncoupled waveguides, only one of which is electrooptic, is proposed for HV sensing. The non-electrooptic waveguide could be Ta$_2$O$_5$ superstrate waveguide, adjacent to a Ti diffused waveguide in LiNbO$_3$.

The output interference pattern in the vicinity of a waveguide is composed of radial and parallel lines. It may be possible, to gain information pertaining to the diffusion depth and profile, from this pattern.
REFERENCES


J.L. Jackel, 1980, "Ion exchange for optical waveguides in LiNbO$_3$ and LiTaO$_3$", Topical Meeting Integrated and Guided-Wave Optics (Optical Society of Amer., Wash., D.C.), paper WB4-1.


APPENDIX A

Solution of differential equation for the input impedance of exponentially tapered transmission line terminated in $Z_a$

Rewriting (3.16),

$$\frac{d^2u}{d\xi^2} + 2k \frac{du}{d\xi} + \beta^2 u = 0$$  \hspace{1cm} (A1)

From which the differential operator $D$ is,  \cite{Murray 1958},

$$D = -k \pm \sqrt{k^2 - \beta^2}$$

Let $p = -k$ \hspace{1cm} (A2)

and $q = \sqrt{k^2 - \beta^2}$ \hspace{1cm} (A3)

Then, solution of differential equation (A1) is,

$$u = c_1 \exp(p+q)\xi + c_2 \exp(p-q)\xi$$ \hspace{1cm} (A4)

where $c_1$ and $c_2$ are constants of integration.

Then,

$$\frac{du}{d\xi} = c_1(p+q) \exp(p+q)\xi + c_2(p-q)\exp(p-q)\xi$$ \hspace{1cm} (A5)

Substituting (A4) and (A5) in (3.10);

$$y = \frac{Z}{j\beta} \times \frac{1}{u} \times \frac{du}{d\xi}$$ \hspace{1cm} (3.10)

$$y = \frac{Z}{j\beta} \left[ \frac{c_1(p+q)\exp(p+q)\xi + c_2(p-q)\exp(p-q)\xi}{c_1\exp(p+q)\xi + c_2\exp(p-q)\xi} \right]$$

Using $c = c_2/c_1$ and simplifying
\[ y = \frac{Z}{j\beta} \left[ \frac{(p+q)\exp(qz) + c(p-q)\exp(-qz)}{\exp(qz) + c\exp(-qz)} \right] \]  

From Figs. 3.1 and 3.2, recalling that when,

\[ z = 0, \quad Z(0) = Z_1 \quad \text{and} \quad y = Z_{\text{in}}(0) = Z_a. \]

Using this in (3.21)

\[ Z_a = \frac{Z_1}{j\beta} \left[ \frac{(p+q) + c(p-q)}{1 + c} \right] \]

or,

\[ \frac{j\beta Z_a}{Z_1} = \frac{(p+q) + c(p-q)}{1 + c} = r \quad \text{(say)} \]

Then,

\[ c = \frac{(p+q) - r}{r - (p-q)} \]

Substituting (A8) in (A6),

\[ y = \frac{Z}{j\beta} \left[ \frac{(p+q)(r-p-q)\exp(qz) + (p-q)(p+q-r)\exp(-qz)}{(r-p+q)\exp(qz) + (p+q-r)\exp(-qz)} \right] \]

\[ y = -\frac{jZ}{\beta} \left[ \frac{(p+q)r-(p^2-q^2)\exp(qz) + ((p^2-q^2)-(p-q)r)\exp(-qz)}{(r-p+q)\exp(qz) + (p+q-r)\exp(-qz)} \right] \]

\[ \frac{j\beta y}{Z} = \left[ \frac{pr\sinh(qz) + qr\cosh(qz) - (p^2-q^2)\sinh(qz)}{r \sinh(qz) - p \sinh(qz) + q \cosh(qz)} \right] \]
\[
\frac{j\beta y}{z} = \left[ \frac{pr \tanh(q\xi) + qr - (p^2 - q^2) \tanh(q\xi)}{r \tanh(q\xi) - p \tanh(q\xi) + q} \right]
\]  

(A9)

Let \( \frac{18y}{Z} = \frac{N}{D} \)  

(A10)

Where \( N \) and \( D \) are numerator and denominator respectively in (A9).

\[
N = qr + (pr - p^2 + q^2) \tanh(q\xi)
\]  

(A11)

Using (A2), (A3) and (A7) in (A11),

\[
N = \beta \left[ \frac{Za}{Z_1} \sqrt{k^2 - \beta^2} + \left( -\frac{kZa}{Z_1} + j\beta \right) \tanh \xi \sqrt{k^2 - \beta^2} \right]
\]  

(A12)

Similarly,

\[
D = \sqrt{k^2 - \beta^2} + (\beta Za/Z_1 + k) \tanh \xi \sqrt{k^2 - \beta^2}
\]  

(A13)

Substituting (A12) and (A13) in (A10),

\[
\frac{18y}{Z} = \frac{\beta \left[ \frac{Za}{Z_1} \sqrt{k^2 - \beta^2} + \left( -\frac{kZa}{Z_1} + j\beta \right) \tanh \xi \sqrt{k^2 - \beta^2} \right]}{\sqrt{k^2 - \beta^2} + (\beta Za/Z_1 + k) \tanh \xi \sqrt{k^2 - \beta^2}}
\]

\[
\frac{y}{Z} = \frac{\frac{Za}{Z_1} \sqrt{\beta^2 - k^2} - (\frac{kZa}{Z_1} - j\beta) \tanh \xi \sqrt{\beta^2 - k^2}}{\sqrt{\beta^2 - k^2} + (\beta Za/Z_1 + k) \tanh \xi \sqrt{\beta^2 - k^2}}
\]
\[ \frac{\nu}{z} = \frac{\frac{Z_0}{Z_1} (\sqrt{\beta^2 - k^2} - \tan \phi \sqrt{\beta^2 - k^2}) + j \beta \tan \phi \sqrt{\beta^2 - k^2}}{\sqrt{\beta^2 - k^2 + \tan \phi \sqrt{\beta^2 - k^2}} + j \tan \phi \sqrt{\beta^2 - k^2}} \]  

(A14)

The propagation constant \( \beta = \frac{2\pi}{\lambda} \)

and from (15) \( k = \frac{1}{2L} \ln \frac{Z_2}{Z_1} \)

From [Jasik 1961] \( k = \frac{1}{2L} \ln \frac{Z_2}{Z_1} = \frac{2\pi}{\lambda_c} \)

Then,

\[ \beta^2 - k^2 = \left(\frac{2\pi}{\lambda}\right)^2 - \left(\frac{2\pi}{\lambda_c}\right)^2 \]

\[ \left(\frac{2\pi}{\lambda_g}\right)^2 = \left(\frac{2\pi}{\lambda}\right)^2 - \left(\frac{2\pi}{\lambda_c}\right)^2 \]  

(A15)

where \((\beta^2 - k^2)\) is defined to equal \( \left(\frac{2\pi}{\lambda_g}\right)^2 \)

\[ \therefore \lambda_g = \sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2} \]  

(A16)

This expression for "guide" wavelength is similar to the one for waveguide propagation, where \( \lambda, \lambda_c \) and \( \lambda_g \) respectively stand for free-space, cut-off and guide wavelength. Using (A15) in (A14), and \( y = Z_{in} \) from (3.6),
\[
\frac{Z_{in}}{Z(z)} = \frac{Z_a \left( \frac{1}{\lambda_g} - \frac{1}{\lambda_c} \tan\left(\frac{2\pi z}{\lambda_g}\right) \right) + j \frac{Z_1}{\lambda} \tan\left(\frac{2\pi z}{\lambda_g}\right)}{Z_1 \left( \frac{1}{\lambda_g} + \frac{1}{\lambda_c} \tan\left(\frac{2\pi z}{\lambda_g}\right) \right) + j \frac{Z_a}{\lambda} \tan\left(\frac{2\pi z}{\lambda_g}\right)}
\]

(A17)

where,

\[
z(z) = Z_1 \exp\left(\frac{g}{L} \ln \frac{Z_2}{Z_1}\right)
\]

\[
\lambda_c = 4\pi L \ln \left(\frac{Z_1}{Z_2}\right)
\]

\[
\lambda_g = \sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}
\]
Fig. B.1. Flow chart of FORTRAN IV program to compute change in the propagation constant of waveguide when loaded with film.
C PROGRAM: SEP18
C PROGRAM TO CALCULATE THE EFFECT OF A FILM ON PROPAGATION
C CONSTANT OF A DIFFUSED WAVEGUIDE
C
C NA, NF, NS ARE REFRACTIVE INDICES OF AIR, FILM AND SUBSTRATE
C NE, NO ARE EXTRAORDINARY AND ORDINARY REFRACTIVE INDICES
C MODE=1 FOR TE, 2 FOR TM

IMPLICIT REAL*8 (A-H,0-Z)
REAL*8 NA, NF, NS, K, NE, NO, N0, LHS, N1, N2, N1I, N1II
REAL*8 NEFF, LEFT, NX, NY, NZ
PI=3.14159265
NA=1.0
QA=NA*NA
NE=2.2030
NO=2.2868

DO 140 IF=1,1
DO 140 IW=1,4
W=2.5+FLOAT(IW-1)*0.5
W=3
P=0.
Q=0.
WAVLEN=0.6328
K=2.0*PI/WAVLEN

DO 140 MODE=1,1
NAME=0
IF (MODE-2) 130, 131, 131
130 NS=NE
NF=2.2134-FLOAT(IF-1)*.005
GO TO 132
131 NS=NO
NF=2.2038-FLOAT(IF-1)*.02
132 CONTINUE
QS=NS*NS
QF=NF*NF

DO 140 ID=1,14
D=7.0-FLOAT(ID-1)*0.5
TDK=2.*D*K

DO 140 IT=1,1
THICK=0.1000+FLOAT(IT-1)*0.05

DO 140 NAM=1,1
DN=0.001+0.002*FLOAT(NAM-1)
TOP=NS+DN
GX=TDK*DSQRT(2.0*NS*DN)

DO 30 NUM=1,2
IF (NUM-2) 32, 31, 31
C
JUNE 17, 1981. SOURCE: PP. 164, B. J. LEY.

C
HALF INTERVAL SEARCH METHOD FOR FINDING ROOTS

WRITE (6, 1)
TOP = DSQRT( 2.0*NS*DN+NS*NS)
IF (NUM-2) 92, 91, 91
91 T1 = N1II
T2 = T1 + 0.002
ACC = 0.00001
GO TO 93
92 T1 = TOP
T2 = T1 -.002
ACC = 0.00001
93 CONTINUE
TUP = T2
STEP = T2 - T1
CALL PROP1 (MODE, DN, QA, QF, QS, TDK, GX, K, T, T1, LHS, RHS)
FT1 = LHS - RHS
I = 0
3 I = I + 1
   IF (DABS(T1 - T2) - ACC) 10, 10, 20
20 T3 = (T1 + T2)/2.
   CALL PROP1 (MODE, DN, QA, QF, QS, TDK, GX, K, T, T3, LHS, RHS)
   FT = LHS - RHS
   WRITE (6, 4) T1, T2, T3, FT , LHS, RHS
4 FORMAT(6D15.7)
   IF (FT*FT1) 156, 10, 150
5 T2 = T3
   GO TO 3
6 CALL PROP1 (MODE, DN, QA, QF, QS, TDK, GX, K, T, T2, LHS, RHS)
   FT2 = LHS - RHS
   IF (FT*FT2) 156, 10, 150
156 T1 = T3
   GO TO 3
150 CONTINUE
C
WRITE (6, 151) T1, T2
151 FORMAT (' NO ROOT IN THE RANGE', 2D15.7, /)
T1 = TUP
T2 = T1 + STEP
TUP = TUP + STEP
GO TO 93
10 CONTINUE
C
WRITE (6, 7) T3, I
7 FORMAT (' THE ROOT =', D15.7, ' INDEX ', I5, ' AND WAS FOUND IN ', I15, ' ITERATIONS')
   IF (NUM-2) 34, 35, 35
34 N1II = T3
   GO TO 30
35 N1I = T3
IF (N1I-N1II) 260,260,261

260 NROOT=NROOT+1
T1=N1I
T2=N1I+STEP
GO TO 93

261 CONTINUE

NAME=NAME+1
IF (NAME .GT. 1) GO TO 600
WRITE(6, 300) NS,NE,NO,NF,WAVLEN,MODE,W
300 FORMAT(///,' NS=',F8.6,5X,'NE=',F8.6,5X,'NO=',F8.6,5X,'NF=',F8.6,5X,'WAVLEN=',F8.6,4X,'MODE=',I2,14X,'WIDTH=',F5.2)
332 WRITE(6,41)
41 FORMAT(///,'DN',8X,'T',8X,'D',8X,'CN',8X,'N1I',18X,'N1II',8X,' N2F',6X,'NZA',8X,'BFILM',8X,'BAIR',18X,'ALTER')
600 CONTINUE
30 CONTINUE
DO 800 IDE=1,2
IF (IDE-1) 811,811,812
811 NO=N1I
GO TO 813
812 NO=N1II
813 CONTINUE
NEFF=NS
INDEX=1
IF (INDEX.EQ.1) GO TO 140
C JUNE 17,1981. SOURCE: PP.164, B. J. LEY.
C HALF INTERVAL SEARCH METHOD FOR FINDING ROOTS
C
C WRITE (6,51)
51 FORMAT (6D15.7)

58 CONTINUE
CALL PROP2 (NO,K,NEFF,W,MODE,P,Q,T1,PI,LEFT,RIGHT)
FT1= LEFT-RIGHT
I=0
53 I= I+1
IF (DABS(T1- T2)- ACC) 60, 60, 70
70 T3= (T1+T2)/2.
CALL PROP2 (NO,K,NEFF,W,MODE,P,Q,T3,PI,LEFT,RIGHT)
FT= LEFT-RIGHT
C WRITE (6,54) T1, T2, T3, FT, LEFT, RIGHT
54 FORMAT (6D15.7)
55 T2= T3
GO TO 53
56 CALL PROP2 (NO,K,NEFF,W,MODE,P,Q,T2,PI,LEFT,RIGHT)
FT2=LEFT-RIGHT
IF (FT*FT2) 256, 60, 250
256 T1=T3
FT1=FT
GO TO 53
250 CONTINUE
C WRITE(6, 251) T1, T2
251 FORMAT(/' NO ROOT IN THE RANGE', 2D15.7)
T1=TUP
T2=T1+STEP
TUP=TUP+STEP
GO TO 58
60 CONTINUE
C WRITE(6, 57) T3, I
57 FORMAT(///, ' THE ROOT =', D15.7, ' INDEX ',
1 //, ' AND WAS FOUND IN ',
1I5, ' ITERATIONS')
IF (IDE-1) 801,801,802
801 ZF=T3
GO TO 804
802 ZA=T3
804 CONTINUE
800 CONTINUE
CN=ZF-ZA
BFILM=K*ZF
BAIR=K*ZA
ALTER= (BFILM-BAIR)*100./BAIR
333 WRITE(6,40)DN,THICK,D,CN,N1I,N1II,ZF,ZA,
1BFILM,BAIR,ALTER
40 FORMAT(/, F8.4,2F9.4,F10.4,F11.6,F12.6,F12.6,F11.6,
1F13.6,2X,2F11.6)
C WRITE(6, 700) N0,N1, N2, NX, NY, NZ
700 FORMAT(/, 6F10.6)
90 CONTINUE
140 CONTINUE
STOP
END
C SUBROUTINE TO COMPUTE EFFECTIVE MODE INDICES
C
SUBROUTINE PROP1 (MODE,DN,QA,QF,QS,TDK,GX,K,T,N1,LHS,RHS)
IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 NA, NF, NS, K, NE, NO, N0, LHS, N1, N2, N1I, N1II
IF (MODE-2) 100, 110, 110
100 ETA=1.
XI=1.
GO TO 120
110 ETA= QS/QF
XI= QF/QA
120 CONTINUE
Q1=N1*N1
FACT=DABS(QF-Q1)
VEE= TDK*DSQRT(Q1- QS)
VPLUS= VEE+1.
XJV= BESJ (GX, VEE)
XJVP= BESJ (GX, VPLUS)
XJVM= (2.0*VEE*XJV/GX)-XJVP
IF (XJV) 72, 71, 72
71 WRITE (6, 73)
73 FORMAT (//' XJV = 0 ')
72 CONTINUE
LHS=(XJVP-XJVM)-FLOAT(MODE-1)*2.0*GX*XJV/(TDK*TDK*QS)
82 B1= K* DSQRT(FACT)
   S= DSQRT(DABS((Q1- QA)/ FACT))
   RHS= ETA* 2.0* DSQRT(FACT)
   RHS= RHS* (XJVP*TDK/GX)
IF (NF-N1) 85,84,84
84 RHS=RHS*(XI* S- DTAN(B1* T))/(1.0+ XI* S* DTAN (B1*T))
GO TO 83
85 RHS=RHS*(XI* S+DTANH(B1* T))/(1.0+XI* S*DTANH(B1*T))
83 RHS=RHS*(1.0+(FLOAT(MODE-1)*2.*DN/DSQRT(QS)))
C WRITE(6,300)GX,VEE,XJVP,XJVM,XJV,B1,S
300 FORMAT(/,'***',7D15.7)
RETURN
END
C SUBROUTINE TO COMPUTE N2
C
SUBROUTINE PROP2 (NO,K,NEFF,W,MODE,P,Q,N2,PI,LEFT,RIGHT)
IMPLICIT REAL*8 (A-H,0-Z)
REAL*8 NA, NF, NS, K, NE, NO, N0, LHS, N1, N2, N1I, N1II
REAL*8 NEFF, LEFT
B2= K* DSQRT( NO*NO - N2*N2)
GAMS= K* DSQRT(N2* N2 - NEFF*NEFF)
IF (MODE-2) 210, 200, 200
200 ZETA= 1.
   GO TO 220
210 ZETA= (NO/NEFF)**2
220 CONTINUE
LEFT= B2* W
RIGHT= Q* PI
RIGHT= RIGHT+2.0* DATAN (ZETA* GAMS/ B2)
BAL=LEFT-RIGHT
RETURN
END