LOW-COMPLEXITY ITERATIVE DECODING FOR
BIT-INTERLEAVED CODED MODULATION

by

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ABSTRACT

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In this thesis, bandwidth-efficient transmission with bit-interleaved coded modulation (BICM) over fading channels is considered. The main focus of this work is on the design and analysis of iterative decoding schemes employing hard-decision feedback. Although suboptimum by nature, hard-decision feedback allows for low-complexity iterative decoders, which renders this approach advantageous for practical implementations. Two particular 16-ary modulation schemes with their corresponding decoders are considered for bandwidth-efficient transmission. The first scheme is 16-ary quadrature amplitude modulation (16QAM) with coherent iterative decoding (ID), so-called BICM-ID, which relies on (possibly imperfect) channel estimation. We analyze the reliability of the output of the demodulator, which is the inner component decoder in the iterative decoding scheme, and we propose the application of a metric truncation technique to improve the quality of the decision variable and thus the performance of hard-decision feedback iterative decoding. Simulation results for different variants of this metric truncation show notable gains in power efficiency, while decoding complexity is not increased. The second scheme we consider is so-called twisted absolute amplitude and differential phase-shift keying (TADPSK), which allows for iterative decoding without the need for channel estimation. We extend previous work on iterative decision-feedback decoding for TADPSK, so-called iterative decision-feedback differential demodulation (DFDM),
and propose a sliding-window DFDM (SWDFDM) module as inner component decoder. Similar to the case of 16QAM, the application of metric truncation yields significant performance improvements also for TADPSK transmission. Finally, we compare 16QAM and TADPSK transmission by means of simulations. Depending on the quality of channel estimation, TADPSK with low-complexity iterative DFDM is shown to outperform 16QAM BICM-ID in some cases.
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LIST OF ABBREVIATIONS AND SYMBOLS

Acronyms

BER       Bit error rate
BICM      Bit-interleaved coded modulation
BICM-ID   Bit-interleaved coded modulation – iterative decoding
CDM       Conventional differential demodulation
CSI       Channel state information
DAPSK     Differential amplitude and phase-shift keying
DFDD      Decision-feedback differential detection
DFDM      Decision-feedback differential demodulation
DPSK      Differential phase-shift keying
DSTM      Differential space-time modulation
GL        Gray labeling
ID        Iterative decoding
ML        Mixed labeling
MSDM      Multiple-symbol differential demodulation
MSPL      Modified set partitioning labeling
PEP       Pairwise error probability
QAM       Quadrature amplitude modulation
SISO      Soft-input soft-output
SNR       Signal-to-noise ratio
SWDFDM  Sliding-window decision-feedback differential demodulation
TADPSK  Twisted absolute amplitude and differential phase-shift keying

Operators and Notation

\((\cdot)^*\)  Conjugate

\((\cdot)^T\)  Transpose

\((\cdot)^H\)  Hermitian transpose

\(|\cdot|\)  Absolute value

\(||\cdot||\)  Frobenius norm

\(\mathcal{E}\{\cdot\}\)  Expectation

\(J_0(\cdot)\)  Zeroth order Bessel function of the first kind

\(\text{diag}\{\cdot\}\)  Diagonal matrix with diagonal entries of vector argument

\(I_N\)  Identity matrix of size \(N\)

\(R(c[i] = 1)\)  Assumed reliability

\(F\{c[i] = 1\}\)  Relative frequency

\(\text{cov}(a,b)\)  Covariance between variables \(a\) and \(b\)

\(\text{var}(a)\)  Variance of variable \(a\)

\(P(q; I)\)  A priori probability for variable \(q\)

\(P(q; O)\)  A posteriori probability for variable \(q\)
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Chapter 1 INTRODUCTION

In this chapter, we first present some background information. We briefly review the history of BICM and its extension – BICM-ID. We also discuss the combination of BICM with differential encoding. Then, we list the main contributions. Finally, the outline of this thesis is given.

1.1 Background

Recently, bit-interleaved coded modulation (BICM) has been introduced as a technique to achieve large coding gain with coherent detection in a fading environment [1], [2]. With BICM, the encoder and the modulator can be designed and implemented separately. Often, perfect channel state information (CSI) is assumed to be available at the receiver. More realistic models consider imperfect CSI, also known as partial CSI [3].

Iterative decoding for BICM, so-called BICM-ID, has been introduced in [4] and [5]. It has been shown to improve the performance of BICM. Ideally, soft information is exchanged between the component decoders, i.e., the demodulator, which is the inner
component decoder, receives soft feedback from the outer decoder. However, BICM-ID with soft feedback requires the use of highly complex soft-input soft-output (SISO) modules. A low-complexity alternative is BICM-ID with hard-decision feedback. Although the performance suffers with hard-decision feedback, the complexity of the demodulator is significantly reduced as simple demodulation and the standard Viterbi decoder can be applied. 16-ary quadrature amplitude modulation (16QAM) transmission and BICM-ID with both soft and hard decision feedback have been studied in [6]. It was found that the performance largely depends on the labeling of the signal constellations and that BICM-ID with hard-decision feedback suffers from convergence problems due to erroneous decision feedback symbols.

BICM can also be combined with different encoding, e.g., differential phase-shift keying (DPSK), which enables noncoherent detection without channel estimation. Similar to the case of coherent detection with CSI, iterative decoding is possible. Such iterative decoding was proposed in [7]. To keep decoding complexity low, hard-decision feedback decoding schemes were devised. The major difference of BICM-ID in the coherent case is the structure of the demodulator, which is referred to as differential demodulator. In analogy to decision-feedback differential detection (DFDD) for uncoded DPSK, cf. [8], [9], an increased observation window of \( N > 2 \) symbols is used to obtain decision variables in the demodulator, and the overall iterative decoder is referred to as iterative decision-feedback differential demodulator [7]. With this iterative decoding scheme, considerable performance improvements over non-iterative, conventional
differential demodulation (CDM) have been reported [7], [10]. Moreover, a simplified DFDM has been proposed in [10] to further reduce the complexity.

For improved bandwidth efficiency, DPSK can be extended to joint differential phase and amplitude modulation. This is known as differential amplitude and phase shift keying (DAPSK) [11]. However, poor reliability is found for the changes in amplitude. To combat this, a technique that uses differential phase encoding and redundant amplitude modulation is proposed in [12]. This is referred to as twisted absolute amplitude and differential phase-shift keying (TADPSK). Iterative DFDM for 16-ary TADPSK (T2AD8PSK) was presented and studied in [10]. In both [7] and [10], it was claimed that (TA)DPSK requires Gray labeling of signal points to achieve convergence of, and hence good performance with, iterative DFDM.

1.2 Contributions

The main contributions are as follows:

- We investigate 16QAM BICM-ID in a fading environment with possibly imperfect channel state information. Based on our findings, we propose a metric truncation technique, which is shown to considerably improve the performance of BICM-ID.
- We assess and quantify the performance improvement due to the proposed metric truncation for various channel scenarios and labelings of 16QAM signal constellation.
• We present a new demodulator module for iterative DFDM and 16-ary twisted absolute amplitude and differential phase-shift keying (T2AD8PSK), which significantly enhances the power efficiency of this scheme.

• We apply the metric truncation technique, developed for coherent BICM-ID, to iterative DFDM, and show that other labelings can yield better performance than Gray labeling.

• We evaluate the performance of the low-complexity iterative DFDM with T2AD8PSK for various metric truncations, labelings, window sizes $N$, and amplitude ratios.

• We compare the performance of 16QAM BICM-ID to T2AD8PSK with iterative DFDM.

1.3 Outline

This thesis is organized as follows. Chapter 2 presents the system model used. In Chapter 3, 16QAM and BICM with iterative decoding are discussed and analytical bounds are obtained assuming perfect and imperfect CSI. In Chapter 4, we propose metric truncation for BICM-ID and evaluate the resulting performance improvements. In Chapter 5, iterative DFDM with T2AD8PSK is investigated and a new decoder module as well as metric truncation are devised. A comparison of the two schemes is presented in Chapter 6. Finally, conclusions are drawn in Chapter 7.
Chapter 2  SYSTEM MODEL

In this chapter, the system model for 16QAM and T2AD8PSK transmission is introduced. The transmitter model is looked at first. Afterwards, the channel model and the channel estimation model are considered. The last section will be on the receiver model.

2.1 Transmitter Model

The block diagrams of the discrete-time transmitter models for 16QAM and T2AD8PSK are illustrated in Fig. 2.1. We consider the low-pass equivalents for all the signals and the channel. First, binary data is passed through a convolutional encoder of rate $k/n$, whose output symbols are interleaved. The interleaved bits, $c[i]$ ($i \in Z$: bit discrete-time index), are sent through the mapper. $l = \log_2(M)$ consecutive bits of $c[i]$ are grouped together to form $M$-ary data-carrying symbol $s[k]$ in case of 16QAM and $v[k]$ in case of T2AD8PSK transmission ($k \in Z$: symbol discrete-time index). This is referred to as bit-interleaved coded modulation (BICM) [2].
2.1.1 Modulation with 16QAM

According to Fig. 2.1(a), $s[k]$ is sent directly over the channel without further processing. Three different labelings are considered. They are Gray labeling (GL), Modified Set Partitioning labeling (MSPL), and Mixed labeling (ML) [6]. The labelings are given in Fig. 2.2.
2.1.2 Modulation with T2AD8PSK

According to Fig. 2.1(b), we can see that $v[k]$ is differentially encoded first before sending through the channel. For classical differential phase-shift keying (DPSK) modulation [13], $v[k] \in \{e^{j2\pi m/M} | m = 0, 1, ..., M - 1\}$ is taken from an $M$-ary PSK constellation and we obtain the channel input as

$$s[k] = v[k] \cdot s[k-1]$$

Information is carried in phase changes for DPSK. To incorporate both amplitude and phase in modulation for improved bandwidth efficiency, an alternative scheme called "differential amplitude and phase-shift keying" (DAPSK) has been proposed [11]. Even though DAPSK outperforms DPSK for high-rate uncoded transmission, it was found that information carried by the amplitude changes is not reliably transmitted for coded transmission and low-complexity iterative decoding.

An alternative scheme, called "twisted absolute amplitude and differential phase-shift keying" (TADPSK), offers a new way for mapping bits onto corresponding symbols [12]. The data-carrying symbols $v[k]$ are placed in $\alpha$ distinct concentric rings. The radius of each of the rings is labeled as $r_i, i = 0, ..., \alpha - 1$, where the innermost ring is $r_0$. Each ring has $\beta$ uniformly spaced phases, but with different phase offsets with respect to zero phase. The data-carrying symbols belong to the following set

$$v[k] \in \{r_m \mod \alpha e^{j2\pi m/\beta} | m = 0, ..., \alpha \beta - 1\}$$
There is a total of \( M = \alpha \beta \) different possible phases. This setup also assures that there is no two rings that have the same phase. The transmit symbols belong to the following set.

\[
s[k] \in \left\{ r_i e^{j2\pi m/\alpha} \mid i \in \{0,\ldots,\alpha - 1\} ; m \in \{0,\ldots,\alpha\beta - 1\} \right\}
\]  

(2.3)

Fig. 2.3 T2AD8PSK and T4AD4PSK.
Again, there is a total of \( \alpha \beta \) different possible phases, but each of the rings would have all \( \alpha \beta \) phases. To obtain the transmit symbol from the data-carrying symbol, we let

\[
v[k] = r_{l[k]} e^{j\phi_{m[k]}} \quad \text{and} \quad s[k-1] = r_{l[k-1]} e^{j\phi_{m[k-1]}} \quad [12].
\]

It follows that

\[
s[k] = r_{l[k]} e^{j\phi_{(m[k-1] + m[k]) \mod \alpha \beta}} \quad (2.4)
\]

Using this constellation, we can tell which signal it is just by the phase offset. The \( \alpha \) different amplitudes also allow us to tell which group of signals it is referring to. Redundant encoding of the information is observed, as differential phase encoding and absolute amplitude encoding are used. With constant ring ratio \( r = r_{i+1} / r_i, 0 \leq i \leq \alpha - 2 \), two examples are given in Fig. 2.3. Looking at the constellation for the data-carrying symbols, we observe a twisting feature of the signal points which completely suits the name of this scheme. The notation TaADpPSK is used to distinguish different constellations. For example, for two rings and eight phase offsets in each ring, we would use the notation T2AD8PSK.

For labeling of T2AD8PSK, we have one bit for the amplitude and three bits for the phase offset. We assign the first bit for the amplitude and the lower three bits for the phase offset. Since only one bit is available for the amplitude labeling, we designate a bit 0 for the inner ring and a bit 1 for the outer ring. As for the phase offset, three different types of labeling are used, namely Gray labeling, Ungerboeck labeling, and Mixed labeling. For Gray labeling, the two closest signal points to a signal point 0 would differ by one bit to 0. For Ungerboeck labeling, partitioning of the inner ring and the outer ring
signal points are done separately [14]. Once we have the Ungerboeck labeling of the
signal points, we can use it to obtain Mixed labeling. Consider the inner ring first, we
partition the eight signal points into two groups of size four, i.e. take the second
partitioning level from Ungerboeck labeling directly. We denote the signal points in first
group as “c₀c₂c₄c₆”, and the signal points in second group as “c₁c₃c₅c₇”. Then we
swap the last two signal points in each group to attain “c₀c₂c₆c₄” and “c₁c₃c₅c₇”.
Taking the two groups, we transform them back to the one ring constellation with
labeling “c₀c₁c₂c₃c₄c₅c₆c₇” [15]. The different labeling schemes are given in Fig. 2.4.

![Fig. 2.4 Labelings for T2AD8PSK.](image)

### 2.2 Channel Model

We consider a frequency-nonselective fading channel and assume that the channel
does not change significantly during one symbol interval \( T \). Also, the transmitter and
receiver filters have square-root Nyquist characteristics. Thus, we have discrete-time
channel model with the received signal sample
where $g[k]$ denotes the Ricean fading process and $n[k]$ denotes the noise process. $g[k]$ and $n[k]$ are correlated and uncorrelated zero-mean complex Gaussian random processes, respectively. The Ricean fading process $g[k]$ and the noise process $n[k]$ are also mutually uncorrelated. Using appropriate normalization, $g[k]$ has power $\mathbb{E}(|g[k]|^2) = 1$ and $n[k]$ has variance $\sigma_n^2 = N_0 / \bar{E}_s$, where $\mathbb{E}\{\cdot\}$ denotes expectation. $N_0$ is the single-sided power spectral density of the passband noise and $\bar{E}_s$ is the mean received energy per symbol. The direct component of the fading process is $g_d[k] = \mathbb{E}(g[k]) = e^{j2\pi f_D k} g_m$ and the scattered component of the fading process is $g_s[k] = g[k] - g_d[k]$, where $f_D$ is the Doppler shift of the direct component, $g_m$ is the magnitude of the mean of the fading process. The Ricean factor $K$ is defined as $K = |g_m|^2 / \sigma_s^2$, where $\sigma_s^2 = \mathbb{E}(|g_s[k]|^2)$ is the variance of the scattered component. When the Ricean factor $K$ is equal to zero, the channel becomes a Rayleigh-fading channel. Using Clarke fading model [16] with one-sided bandwidth $B_f$, the autocorrelation of the fading process is

$$R_g[k] = \mathbb{E}\{g^*[k]g[k + \kappa]\} = \frac{e^{j2\pi f_D k} \cdot K + J_0(2\pi B_f T \kappa)}{K + 1}$$

(2.6)

where $J_0(\cdot)$ is the zeroth order Bessel function of the first kind.
2.3 Channel Estimation Model

The 16QAM modulation scheme considered requires explicit channel estimation to obtain channel state information (CSI) for decoding. Often, perfect channel state information is assumed at the receiver. A more realistic model takes imperfect channel estimation into account. To obtain a channel model for the case of imperfect channel estimation, we follow the exposition in [3].

2.3.1 Rayleigh Fading

For Rayleigh Fading, we model $g[k]$ and its estimate $\hat{g}[k]$ as

$$g[k] = \alpha e^{j\theta} = X_1 + jY_1$$
$$\hat{g}[k] = \hat{\alpha} e^{j\hat{\theta}} = X_2 + jY_2$$

(2.7)

$X_1$ and $Y_1$ are i.i.d. Gaussian random variables with variance $\sigma_1^2$, and $X_2$ and $Y_2$ are i.i.d. Gaussian random variables with variance $\sigma_2^2$. $X_1$ and $X_2$ are independent. $Y_1$ and $X_2$ are also independent. The correlation coefficient between $X_1$ and $X_2$, and between $Y_1$ and $Y_2$ is defined as

$$\rho_{12} = \frac{\mathbb{E}\{X_1 X_2\}}{\sigma_1 \sigma_2} = \frac{\mathbb{E}\{Y_1 Y_2\}}{\sigma_1 \sigma_2}$$

(2.8)

To obtain the estimate $X_2$ from the true value $X_1$, we use $X_2 = cX_1 + z$, where $z$ is a Gaussian RV $\sim N(0,\sigma_z^2)$. $Y_2$ is obtained from $Y_1$ in the same way. $c$ and $\sigma_z^2$ can be calculated as

$$c = \frac{\rho_{12} \sigma_2}{\sigma_1}$$
$$\sigma_z^2 = (1 - \rho_{12}^2) \sigma_2^2$$

(2.9)
Since the power of the fading process is normalized to one, \( \sigma_1^2 = \frac{1}{2} \). In order to obtain \( \sigma_2^2 \), we need to introduce the power coefficient \( \rho \) and the power ratio \( r \). For the Rayleigh-fading case,

\[
\rho = \frac{\text{cov}(\alpha^2, \hat{\alpha}^2)}{\sqrt{\text{var}(\alpha^2) \cdot \text{var}(\hat{\alpha}^2)}} = \rho_{12}^2
\]

\[
r = \frac{\varepsilon \{\hat{\alpha}^2\}}{\varepsilon \{\alpha^2\}} = \frac{\sigma_2^2}{\sigma_1^2}
\]

where \( \text{cov}(\alpha^2, \hat{\alpha}^2) = 4 \rho_{12}^2 \sigma_1^2 \sigma_2^2 \), \( \text{var}(\alpha^2) = 4 \sigma_1^4 \), and \( \text{var}(\hat{\alpha}^2) = 4 \sigma_2^4 \). The power coefficient \( \rho \) is used to control the accuracy of the channel estimation. The value of \( \rho \) is between zero and one, with one excluded. The closer the value is to one, the better the channel estimation is. We can vary the power coefficient for different cases. Usually, the power ratio \( r \) is very close to one. We can assume the power ratio \( r \) to be one in all cases considered [3]. With this assumption, we can obtain \( \sigma_2^2 \) as desired.

### 2.3.2 Rician Fading

For the case of Rician fading, the actual and estimated fading would be as follows

\[
g[k] = \alpha e^{j\theta} = X_1 + j Y_1 + A_1 e^{j\beta_1}
\]

\[
\hat{g}[k] = \hat{\alpha} e^{j\hat{\theta}} = X_2 + j Y_2 + A_2 e^{j\beta_2}
\]

Unbiased estimation is considered, which implies that line of sight component is the same for both actual fading and estimated fading. With the assumption, we have \( A_1 = A_2 = A \), and \( \beta_1 = \beta_2 = \beta \). Everything from the Rayleigh fading case holds for the Rician fading...
case. However, different expressions are obtained for the power coefficient $\rho$ and the power ratio $r$.

$$\rho = \frac{\text{cov}(\alpha^2, \hat{\alpha}^2)}{\sqrt{\text{var}(\alpha^2) \cdot \text{var}(\hat{\alpha}^2)}} = \frac{\rho_\gamma (\rho_\gamma \sigma_2 + A^2)}{\sqrt{(\sigma_1^2 + A^2) (\sigma_2^2 + A^2)}}$$

$$r = \frac{\mathbb{E}\{\hat{\alpha}^2\}}{\mathbb{E}\{\alpha^2\}} = \frac{2\sigma_2^2 + A^2}{2\sigma_1^2 + A^2}$$

where $A = \sqrt{K / (1 + K)}$. In this case, the normalization is $A^2 + 2\sigma_1^2 = 1$. Depending on the Rician factor, we would be able to obtain $\sigma_1^2$ from the previous formula. Afterwards, the power ratio formula is used to obtain $\sigma_2^2$. $\rho$ can be again modified for different cases of imperfect channel estimations.

### 2.4 Receiver Model

The block diagram of the receiver model is illustrated in Fig. 2.5.

![Fig. 2.5 Receiver model.](image)

The received signal is first passed through a demodulator for bit-wise metric calculation. The demodulator is also called the inner decoder and it gives soft input to the deinterleaver. The deinterleaved output is then passed through an outer decoder, or just
decoder. The outer decoder can feed back either soft output or hard output to the demodulator. After passing through an interleaver, the demodulator would utilize this information for recomputing bit branch metrics and pass them again to the decoder. This information exchange is repeated in a number of iterations.

With this receiver model, we will consider the scenario of 16QAM transmission in Chapter 3 and 4. The iterative scheme mentioned above is used for coherent detection with imperfect CSI and referred to as BICM-ID. Noncoherent detection without CSI for T2AD8PSK transmission will be considered in Chapter 5. The iterative scheme of Fig. 2.5 is maintained and we refer to it as iterative DFDM.
Chapter 3  BICM - ITERATIVE DECODING

There are several different bit-interleaved coded modulation (BICM) receiver structures. In this chapter, we will first look at BICM without iterative decoding. Then, we extend it to BICM-iterative decoding (BICM-ID) with soft feedback. This is an optimum iterative decoding scheme, but it is very complex. A simplified scheme – BICM-ID with hard-decision feedback is discussed next. We will also obtain the analytical bit error rate bounds for BICM-ID with perfect and imperfect channel state information.

3.1 BICM without Iterative Decoding

BICM without iterative decoding is referred to as conventional BICM. It was originally proposed by Zehavi in 1992 [1]. The receiver for conventional BICM consists of three components: the demodulator, the deinterleaver, and the decoder. This is illustrated in Fig. 3.1.
The demodulator takes the received signals and applies the following suboptimum log-likelihood bit metric [1], [2]

\[
\bar{\lambda}(s'[k] = b) = \log \sum_{s[k] \in \mathcal{X}_b^i} P(r[k] | s[k]) \\
\approx \max_{s[k] \in \mathcal{X}_b^i} \log P(r[k] | s[k]) \\
= - \min_{s[k] \in \mathcal{X}_b^i} \| r[k] - g[k]s[k] \|^2
\]

where \( b \) can take on the value of zero or one, \( s'[k] \) indicates the \( i \)th bit of \( s[k] \), \( \mathcal{X}_b^i \) is all the possible symbols \( s[k] \) whose label has value \( b \) at bit position \( i \). Out of \( \mathcal{X}_b^i \), we select the symbol \( s[k] \) that has the minimum distance with respect to the received symbol \( r[k] \). The log-likelihood bit metrics are bit deinterleaved and the deinterleaved metrics form the input of the Viterbi decoder.

### 3.2 BICM with Iterative Decoding

Decoding can be performed iteratively. This is often called "bit-interleaved coded decoding".
modulation - iterative decoding”, or “BICM-ID” for short. In this section, we will look at BICM-ID with soft feedback first, followed by BICM-ID with hard-decision feedback.

3.2.1 BICM-ID with Soft Feedback

The BICM-ID receiver with soft feedback uses a soft-input soft-output decoder instead of a Viterbi decoder. The receiver is given in Fig. 3.2. Although the receiver still applies separate demodulation and decoding as in conventional BICM, the output of the soft-input soft-output decoder is fed back to the demodulator through an interleaver. The demodulator updates the bit-metrics using decoder feedback and passes them to the decoder again.

Using the notation in [17], $P(q; I)$ is the a priori probability for a variable $q$ and $P(q; O)$ is the a posteriori probability for a variable $q$. After the first decoding, $P(c'[k]; O)$ is fed back to the demodulator as $P(s'[k]; I)$. Instead of assuming equally likely symbols $s[k]$ as in the first iteration, the probability of the symbols $s[k]$, can be...
updated as follows

\[ P(s[k]) = \prod_{i=1}^{l} P(s'[k] = c'[k]; I) \] (3.2)

where \( c'[k] \) is the value of the \( i \)th bit of the label for \( s[k] \) according to the applied labeling.

The a posteriori bit probability can be calculated as follows

\[ P(s'[k] = b | r[k]) = \sum_{s[k] \in \mathcal{X}_b^i} P(s[k] | r[k]) \]
\[ = \sum_{s[k] \in \mathcal{X}_b^i} P(r[k] | s[k]) P(s[k]) \] (3.3)

Combining (3.2) and (3.3), the extrinsic a posteriori bit probability is obtained as [6]

\[ P(s'[k] = b; O) = \frac{P(s'[k] = b | r[k])}{P(s'[k] = b; I)} \]
\[ = \frac{\sum_{s[k] \in \mathcal{X}_b^i} P(r[k] | s[k]) P(s[k])}{P(s'[k] = b; I)} \]
\[ = \sum_{s[k] \in \mathcal{X}_b^i} \left[ P(r[k] | s[k]) \prod_{j \neq i} P(s'[k] = c'[k]; I) \right] \] (3.4)

In the first iteration, the a priori probability \( P(s[k]) \) is not available and all the symbols \( s[k] \) are assumed equally likely, which is the metric in (3.1) as in conventional BICM. Demodulation (3.4) and the soft-input soft-output decoding process is repeated in
a number of iterations. At the end of the last iteration, the final decoded outputs given by
the decoder are the hard decisions on $P(u'[k]; O)$.

### 3.2.2 BICM-ID with Hard-Decision Feedback

Although BICM-ID with soft feedback is a viable solution, it is often computationally too complex to implement soft-input soft-output decoding and demodulation as given in (3.4). An alternative solution is to keep the Viterbi decoder as in conventional BICM and to feed back only the binary decisions. This binary-decision feedback is also called hard-decision feedback, since distinct decisions are fed back instead of probabilities.

As mentioned in the previous section, the bit metric is the same as (3.1) for the first iteration. For further iterations, feedback decisions are used in the bit metric calculation. For example, to calculate the bit metric $\lambda(s^1[k] = 0)$, we use the substitution $s[k] = [0, s^2[k], ..., s'[k]]$, where $s^2[k], ..., s'[k]$ represents the decoding decisions from the previous round. If currently in the second round of decoding, $s^2[k], ..., s'[k]$ would represent the decoding decisions from the first round of decoding, and so on. The bit metric for this case would be

$$
\lambda(s^1[k] = 0) = \log P(r[k] | [0, s^2[k], ..., s'[k]])
= -\| r[k] - g[k][0, s^2[k], ..., s'[k]] \|^2
$$

(3.5)

Depending on the bit of interest, other bit metrics can be calculated likewise by inserting 0 or 1 at the bit location of interest and inserting feedback bits for the rest.
3.3 Analytical Bit Error Rate Bound for BICM-ID

For convolutional codes of rate $k/n$, the union bound of the bit error rate $P_b$ is as follows [6], [14]

$$P_b \leq \frac{1}{k} \sum_{d=0}^{\infty} W_f(d) f(d, \mu, \mathcal{X}) \quad (3.6)$$

$W_f(d)$ is the total input weight of error events at Hamming distance $d$. $W_f(d)$ can be calculated using several methods [2]. Pairwise error probability (PEP) is denoted by $f(d, \mu, \mathcal{X})$, as it depends on the Hamming distance $d$ between the input sequence and its estimate, the labeling $\mu$, and the signal set $\mathcal{X}$. $d_{min}$ is the minimum Hamming distance of the code.

A bound for the PEP, called the BICM Union Bound (UB) [6], is

$$f(d, \mu, \mathcal{X}) \leq l^{-d} \sum_{\mathcal{S}} 2^{-d} \sum_{\mathcal{U}} 2^{-d(l-1)} \sum_{x \in \mathcal{X}_e^S} \sum_{z \in \mathcal{X}_e^S} P(x \rightarrow z)$$

$$\leq \frac{1}{2\pi J} \int_{\alpha-j\infty}^{\alpha+j\infty} \left[\psi_{ub}(s)\right]^d \frac{ds}{s} \quad (3.7)$$

and $\psi_{ub}(s)$ is defined as

$$\psi_{ub}(s) = \frac{1}{l \cdot 2^l} \sum_{i=0}^{l} \sum_{x \in \mathcal{X}_i^c} \sum_{z \in \mathcal{X}_i^c} \Phi_{\Delta(x, z)}(s) \quad (3.8)$$

$c$ and $\hat{c}$ denotes the sequence of label positions and labeling maps. $c$ and $\hat{c}$ are the input sequences and its estimate. $x$ and $z$ are the two signal sequences. $\Delta(x, z)$ is the metric difference between the components $x$ and $z$ of the sequence $x$ and $z$. $\Phi_{\Delta(x, z)}$ then represents the Laplace transform of the probability density function of $\Delta(x, z)$.
If Gray Labeling is used, a new and reduced bound for the PEP, called the BICM Expurgated Bound (EX), is as follows

\[
    f(d, \mu, \mathcal{X}) \leq l^{-d} \sum_{k \in \mathcal{X}_k} \sum_{i} 2^{-d} \sum_{j} 2^{-d(l-i)} \sum_{x \in \mathcal{X}_k} P(x \rightarrow \hat{x})
\]

\[
    \leq \frac{1}{2\pi J} \int_{\alpha-j \rightarrow}^{\alpha+j \rightarrow} [\psi_{\text{ex}}(s)]^d \frac{ds}{s}
\]

and \(\psi_{\text{ex}}(s)\) is defined as

\[
    \psi_{\text{ex}}(s) = \frac{1}{l_1 \cdot 2^d} \sum_{i=1}^{l_1} \sum_{b=0}^{1} \sum_{x \in \mathcal{X}_b} \Phi_{\Delta(x, \hat{x})}(s)
\]

\(\hat{x}\) is defined as the unique nearest neighbor of \(x\). While this BICM bound provides us with a reduced form for Gray labeling, we wish to obtain a reduced BICM-ID bound for all labelings.

With iterative decoding, the performance of BICM-ID improves over iterations if it converges to a bound. To obtain this bound, we assume error-free feedback. We can partition the \(M\)-ary constellation into \(M/2\) different 2-ary (binary) constellations, as all other bits are known from the feedback. In other words, there are \(M/2\) number of 2-ary constellations, and only one of the \(M/2\) constellations has to be considered for each possible set of feedback bits. For each of the 2-ary constellations, the minimum inter-signal Euclidean distance is larger when compared to the original \(M\)-ary constellation. Since only two signal points are found in the reduced constellations, there is only one nearest neighbor for any signal point on any one of the constellations. This holds regardless of which labeling is used.
With error-free feedback, the reduced constellations can be used for each \( x \in \mathcal{X}_b^i \).

Since there is only one neighbor for \( x \) in the reduced constellation, \( \mathcal{X}_b^i \) has only one term \( \hat{z} \). \( \hat{z} \) is the single nearest neighbor of \( x \) and it only differs from \( x \) at the \( i \)th bit position. The bound for the PEP with error-free feedback is then

\[
f(d, \mu, \mathcal{X}) \leq \frac{1}{2\pi j} \int_{a-j\infty}^{a+j\infty} [\psi_{ef}(s)]^d \frac{ds}{s}
\]

and \( \psi_{ef}(s) \) is defined as

\[
\psi_{ef}(s) = \frac{1}{l \cdot 2^l} \sum_{i=1}^{l} \sum_{b=0}^{l-1} \sum_{x \in \mathcal{X}_b^i} \Phi_{\delta(x, z)}(s)
\]

From [6], \( \Phi_{\delta(x, z)}(s) \) is defined as the following for Rician fading channels with perfect CSI

\[
\Phi_{\delta(x, z)}(s) = \frac{1 + K}{(1 + K) - s(N_0 s - 1) \|x - z\|^2} \cdot \exp \left\{ \frac{K (s(N_0 s - 1) \|x - z\|^2)}{(1 + K) - s(N_0 s - 1) \|x - z\|^2} \right\}
\]

(3.13) reduces to the following for Rayleigh fading channels with perfect CSI

\[
\Phi_{\delta(x, z)}(s) = \frac{1}{1 + s(1 - N_0 s) \|x - z\|^2}
\]

From (3.10) and (3.12), the only difference is between \( \hat{z} \) and \( \tilde{z} \). Depending on the labeling, \( \hat{z} \) is not necessarily the same as \( \tilde{z} \), and generally \( \hat{z} \) is different from \( \tilde{z} \).
Also, (3.10) is only valid for BICM with Gray labeling. However, (3.12) is valid for BICM-ID with any labeling.

For imperfect CSI, we can still use (3.12) but with different value for \( \Phi_{\Delta(x,z)}(s) \).

We need to evaluate \( \Delta(x,z) \) first. From [3], we obtain \( \Delta(x,z) \) as the following

\[
\Delta(x,z) = 2(\alpha e^{j\Delta \theta} x, \hat{\alpha}(x-z)) - \hat{\alpha}^2 (|x|^2 - |z|^2) + 2(\hat{\alpha} e^{j\hat{\theta}}(x-z), n) \tag{3.15}
\]

\( \alpha e^{i\theta} \) is the fading coefficient and \( \hat{\alpha} e^{i\hat{\theta}} \) is the estimated fading coefficient. \( \Delta \theta \) is defined as \( \Delta \theta = \theta - \hat{\theta} \). The expression \((a,b) = \text{Re}(a^* b)\) also holds. With given \( \alpha, \hat{\alpha}, \) and \( \Delta \theta \), \( \Delta(x,z) \) is a conditional Gaussian random variable with mean \( \mu \) and variance \( \sigma^2 \) defined as the following

\[
\mu = 2(\alpha e^{j\Delta \theta} x, \hat{\alpha}(x-z)) - \hat{\alpha}^2 (|x|^2 - |z|^2) \tag{3.16}
\]

\[
\sigma^2 = 4 |\hat{\alpha} e^{j\hat{\theta}}(x-z)|^2 \cdot \frac{N_0}{2} = 2N_0 \hat{\alpha}^2 d_{xz}^2 \tag{3.17}
\]

\( d_{xz} \) is the Euclidean distance between \( x \) and \( z \).

Knowing \( \Delta(x,z) \) as a conditional Gaussian random variable, the Laplace transform of the probability density function of \( \Delta(x,z) \) conditioned on \( \alpha, \hat{\alpha}, \) and \( \Delta \theta \) is then

\[
\Phi_{\Delta(x,z)}(s|\alpha, \hat{\alpha}, \Delta \theta) = \exp(-s\mu + \frac{1}{2}s^2\sigma^2) \tag{3.18}
\]

Using (3.18), the Laplace transform of the probability density function of \( \Delta(x,z) \) is
\[ \Phi_{A(x,z)}(s) = \mathbb{E}\left\{ \Phi_{A(x,z)}(s | \alpha, \hat{\alpha}, \Delta \theta) \right\} \]
\[ = \int \int \int \Phi_{A(x,z)}(s | \alpha, \hat{\alpha}, \Delta \theta) \cdot p(\alpha, \hat{\alpha}, \Delta \theta) \, d\alpha \, d\hat{\alpha} \, d\Delta \theta \]

The expression (3.19) does not have a closed form. Monte Carlo method for numerical integration is used to calculate \( \Phi_{A(x,z)}(s) \).

\[ \Phi_{A(x,z)}(s) = \frac{1}{N} \sum_{n=1}^{N} \Phi_{A(x,z)}(s | \alpha_n, \hat{\alpha}_n, \Delta \theta_n) \]  

(3.20)

\( \alpha_n, \hat{\alpha}_n, \) and \( \Delta \theta_n \) are independent and identically distributed random samples.
Chapter 4  16QAM AND BICM-ID WITH HARD-DECISION FEEDBACK

In this chapter, we consider 16QAM and BICM-ID with hard-decision feedback. We will examine the soft output of the demodulator first. In particular, we analyze the reliability of the soft output, and as a result, we propose ways to improve the performance of BICM-ID. Then the performance improvement will be shown in the following subsection. We will also consider performance comparisons provided by computer simulation results.

4.1 Reliability Analysis

Considering the output of the demodulator, it is imperative to ensure its reliability in order to achieve desired performance [18]. If the reliability of the soft output is not assured, this could cause wrong decisions to be made by the decoder. Feedback bits would contain more errors and these erroneous bits would be used in the next iteration for
calculation. To minimize the wrong decisions, we need to consider the assumed reliability and the relative frequency. Suppose a large amount of binary data is sent, we would be able to obtain a large amount of soft outputs given by the demodulator module. Taking all the soft outputs with certain fixed value of probability, the bits that correspond to the probability are observed. Since a large amount of data is available, we would expect that the percentage of the bits being 1 would be equal to that certain fixed value of probability. For example, if we take all the soft outputs with probability 70%, we would expect 70% of the bits corresponding to those soft outputs to have the value of 1. The expected percentage is called assumed reliability. It is denoted by \( R(c[i] = 1) \). In practice, we measure the true percentage of the bits corresponding to those soft outputs having the value of 1. This true measure is called relative frequency. It is denoted by \( F\{c[i] = 1\} \).

It is desirable to check the correspondence between the assumed reliability and the relative frequency. In Fig. 4.1, an example is given, where the relative frequency \( F\{c[i] = 1\} \) is plotted as a function of the assumed reliability \( R(c[i] = 1) \). For 16QAM with hard-decision feedback, we plot \( \log_{10}(F\{c[i] = 1\}) \) over \( \log_{10}(R(c[i] = 1)) \). The setup is with eight-state, rate 1/2 code. The signal-to-noise ratio (SNR) is set to \( 10\log_{10}(\frac{E_s}{N_0}) = 11 \) dB. Gray labeling and Modified Set Partitioning labeling are observed in the graph. The iteration number is used as a parameter.
Fig. 4.1 Relative frequency vs assumed reliability for 16QAM.

We would expect the relative frequency $\log_{10}(F\{c[i]=1\})$ to converge to the assumed reliability $\log_{10}(R(c[i]=1))$ in the ideal case. Since the curves are symmetric with respect to $R(c[i]=1) = 0.5$, we focus on $R(c[i]=1) < 0.5$. Looking at the dashed and solid lines in the reliability graph, iteration 2 for Gray labeling and iterations 2 to 5 for Modified Set Partitioning labeling are considered. For Gray labeling, we observe some mismatch between the relative frequency $\log_{10}(F\{c[i]=1\})$ and the assumed reliability $\log_{10}(R(c[i]=1))$ for the second iteration. However, for Modified Set Partitioning labeling, the mismatch is more significant. The mismatch is substantial for
the second iteration, and gradually decreasing for latter iterations. Although the drop off in mismatch is observed over iterations, the mismatch is still considerable at iteration 5 when compared against Gray labeling. As an example, when the soft output has a probability of $10^{-5}$ for $c[i] = 1$, the true probability (relative frequency) is $10^{-1.5}$ for the second iteration and approximately $10^{-2.5}$ for the fifth iteration.

4.2 Metric Truncation

The mismatches show a bias of the soft outputs toward high reliabilities. The effects are more prominent for non-Gray labeling. In order to compensate for the bias, we propose to limit the range of bit metrics. We truncate the bit metric to a certain value if it exceeds or falls below some threshold. Choosing the lower threshold as $10^{-j}$ and the upper threshold as $(1 - 10^{-j})$ for $R(c[i] = 1)$ in iteration $j \geq 2$, we set the soft outputs to the threshold values if it falls below the lower threshold or exceeds the upper threshold. We will refer to this type of truncation as Truncation I. We can apply this truncation to both Gray labeling and Modified Set Partitioning labeling curves in Fig. 4.1. Looking at the dash-dotted and dotted curves, we observe that the truncation does not have much effect for Gray labeling. As for Modified Set Partitioning labeling, the truncated curves move closer towards the ideal case, where the relative frequency and the assumed reliability is the same, as iteration is increased. Since the truncated curves are closer to the ideal case, the reliability of the soft outputs is increased and the performance of the receiver is improved.
To further improve the scheme, we propose an additional truncation on top of the original truncation discussed above. As an additional indicator of reliability, we compare the soft outputs given by the demodulator in the current iteration with the hard decisions fed back from the previous iteration. For soft outputs that have values greater than 0.5, we would expect the hard-decision feedbacks for those outputs to be 1s. Likewise, for soft outputs that have values less than 0.5, we would expect the feedbacks for those outputs to be 0s. Sometimes the correspondences do not happen. In those cases, the soft outputs are not very reliable. To compensate for this, we propose a tighter truncation threshold for those soft outputs. When the soft outputs have values greater than 0.5 but with 0s as feedbacks, or when the soft outputs have values less than 0.5 but with 1s as feedbacks, we truncate the soft outputs according to the new threshold. We choose $10^{-j^{+1}}$ as the lower threshold and $(1-10^{-j^{+1}})$ as the upper threshold in iteration $j \geq 2$. If the soft outputs fall below the lower threshold or exceed the upper threshold, we simply assign the soft outputs to the threshold values. We will refer to this type of truncation as Truncation II. The performance gain provided by the tighter truncation can be seen in the following section.

4.3 Performance Evaluation

Using the BICM-ID scheme discussed in the last section, the BER performance under various conditions is assessed. Following the setup in [3] and [6], a 16QAM constellation is considered. Convolutional code with 8-state and rate 1/2 is used in the simulations. Each information block contains 20000 bits. Randomly generated bit
Truncation II is applied. Two different fading channels are considered. One is a Rayleigh fading channel and the other is a Rician fading channel with the Ricean factor $K = 3$ dB. From [3], we can set the power ratio $r$ to be equal to one.

4.3.1 Performance as Function of Decoding Iterations

We would like to investigate how the BICM-ID receiver would perform as function of decoding iterations. The BER performance is plotted in Fig. 4.2 as we consider 16QAM constellation with the receiver employing hard-decision feedback.
MSP labeling is applied on the symbols and Truncation II is used. Rayleigh fading channel with power correlation coefficient $\rho = 0.99$ is considered in this case.

For the first iteration where no feedback is available, the BER performance improves as signal-to-noise ratio (SNR) increases. From 7 dB to 11 dB, it drops from around $1.6 \times 10^1$ to $7 \times 10^{-3}$. We would expect the performance to improve once the feedbacks are incorporated in later iterations. For the second iteration, we observe that it gives a better result from around 8 dB and on. At 11 dB, the BER is approximately $10^{-4}$, which is an improvement over $7 \times 10^{-3}$ in the first iteration. The 3rd iteration curve outperforms the 2nd iteration curve as it reaches $4 \times 10^{-6}$ at 11 dB while the 4th iteration curve has a BER of $10^{-6}$ at 11 dB. With iterative decoding, the performance improves gradually over iterations. This kind of performance improvement can also be observed on a Rician fading channel with $K = 3$ dB. The dashed line represents the error-free feedback bound, where the feedbacks are assumed to be all correct. We can see that the performance moves closer and closer towards the error-free feedback bound as we use more iterations.

4.3.2 Improvement Through Metric Truncation

In this subsection, we would like to observe the performance improvement offered by applying metric truncation. Considering the 4th iteration, we would like to see the differences among these BER curves: curve without metric truncation, curve with
Fig. 4.3 Performance comparison of 16QAM BICM-ID without metric truncation, with Truncation I, and with Truncation II.

Considering the curve without metric truncation first, we can see that it outperforms the 1st iteration curve, where the feedback is not available, at around 9.5 dB. It is able to reach a BER of $5 \times 10^{-5}$ at 13 dB. For the curve with Truncation I, we can see that it does not give a better performance when compared to the curve without metric
truncation for SNR below 9.2 dB. However, the degradation is not that great. For SNR above 9.2 dB, the curve with Truncation I starts to perform better. A gain of approximately 2 dB can be observed for a BER of $10^{-4}$. For the curve with Truncation II, it performs even better. No degradation is observed. Comparing to the case without metric truncation, a gain of around 3 dB can be seen. Using the 1st iteration curve as a reference, we can see that the point of intersection is at 8 dB when Truncation II is applied, while the point of intersection for the curve without metric truncation is at 9.5 dB. Comparing the curve with Truncation II to the curve without metric truncation, the point of intersection is 1.5 dB less, which means that the performance gain can be achieved earlier. Similar results are found for Rician fading channel. In the following subsections, the term "metric truncation" implies that Truncation II is applied.

4.3.3 Performance for Different Labelings

So far we have only considered MSP labeling for the 16QAM constellation, it is interesting to examine the performance when different labelings are used.

*Rayleigh Fading Channel:* Considering a Rayleigh fading channel with power correlation coefficient $\rho = 0.99$, we plot the 4th iteration curves in Fig. 4.4. The solid lines represent the curves without metric truncation and the dashed lines represent the curves with metric truncation.

Looking at the two Gray labeling curves, the difference is negligible. We observe
that the metric truncation does not have much of an effect if Gray labeling is used. This corresponds with what we have discovered in Section 4.1. The Gray labeling curve has a BER of $10^{-4}$ at 12 dB. As for Mixed labeling, a more significant difference can be seen between the curves. Without metric truncation, the Mixed labeling curve sees its BER drops from $10^{-1}$ to $10^{-5}$ from 7 dB to 13 dB. If metric truncation is applied, the performance gain is around 2 dB. Comparing with the Gray labeling curves, the Mixed labeling curve without truncation gives better performance after SNR of 10 dB. With
truncation, the Mixed labeling curve outperforms the Gray labeling curve at a lower SNR – approximately 7 dB. Moreover, the curve with truncation is closer to the dash-dotted curve with soft feedback than the curve without truncation. A gap of around 0.7 dB can be seen. Finally, MSP labeling curve without truncation has inferior performance when compared against untruncated Gray labeling and Mixed labeling curves. However, considering truncation applied to each labeling, the MSP labeling curve is able to outperform the Gray labeling curve for SNR higher than 8.6 dB and outperform the Mixed labeling curve for SNR higher than 9.4 dB. For SNR higher than 9.4 dB, we can see that it is more advantageous to use MSP labeling if metric truncation is used. In addition, we can see that metric truncation closes the gap between the curve with hard-decision feedback and the dash-dotted curve with soft feedback. A gap of approximately 2 dB can be observed between the curve with soft feedback and the truncated curve with hard-decision feedback.

Rician Fading Channel: We would like to investigate how the labelings affect the performance in a Rician fading environment next. The 4th iteration curves are plotted in Fig. 4.5. The power correlation coefficient $\rho = 0.99$ is used. Solid curves indicate that metric truncation is not applied. Dashed curves indicate that metric truncation is applied.

Similar to Fig. 4.4, the two Gray labeling curves have negligible differences. If Gray labeling is used, the performance does not depend on metric truncation. From
Fig. 4.5 Performance of 16QAM BICM-ID over Rician fading channel with Gray labeling, Mixed labeling, and MSP labeling.

having a BER of $3 \times 10^{-2}$ at 5 dB, the curve drops down to having a BER of $8 \times 10^{-5}$ at 10 dB. As for Mixed labeling, metric truncation does provide a performance improvement. This improvement is present for SNR above 5 dB. The performance gain is 2 dB if a BER of $10^{-4}$ is used as a reference. Even without metric truncation, we can see that the Mixed labeling curve eventually outperforms Gray labeling curves for SNR above 9 dB. If metric truncation is enforced, this superior performance is observed for SNR above 6.3 dB. The gap between the dash-dotted curve with soft feedback and the curve with hard-
decision feedback is also reduced with metric truncation. Using a BER of $10^{-5}$ as a reference, the gap between the curve with soft feedback and the curve with truncation is less than 1 dB. As for MSP labeling, the performance improvement provided by metric truncation is seen starting at around 6 dB. Without metric truncation, the point of intersection between the MSP labeling curve and the Gray labeling curve is at a SNR of just above 10 dB. This point of intersection is seen at a lower SNR of 7.5 dB if metric truncation is enforced. In other words, with metric truncation applied, the performance gain provided by MSP labeling can be seen at a lower SNR. The performance of the curve with metric truncation approaches to that of the curve with soft feedback. A gap of around 1.5 dB can be seen between the two curves.

4.3.4 Comparison of BICM-ID With Realistic and With Error-Free Feedback

After looking at the performance difference between curves with and without metric truncation for different labelings, we would like to investigate how close the truncated curve is to the error-free feedback bound.

Rayleigh Fading Channel: The 4th iteration curves are plotted with dashed lines. The error-free feedback bounds are plotted with dotted lines. Rayleigh fading channel is considered in this case, and the power correlation coefficient $\rho$ is set to 0.99. Fig. 4.6 is then plotted according to the above setup.
Fig. 4.6 Comparison of 16QAM BICM-ID with realistic and with error-free feedback over Rayleigh fading channel with Gray labeling, Mixed labeling, and MSP labeling.

With Gray labeling, we can see that the 4th iteration curve is almost identical to the error-free feedback curve, i.e. it fully converges to the error-free feedback bound. With Mixed labeling, we do not observe such a behavior. Instead there is a 1 dB difference between the curves for BER below $10^{-4}$. Even though a gap still exists, the convergence is improved with metric truncation. If we consider SNR higher than 8 dB, although Mixed labeling curves outperforms Gray labeling curves, Mixed labeling curves
do not converge better than Gray labeling curves. With MSP labeling, a difference of greater than 2 dB can be seen between the 4th iteration curve and the error-free feedback bound. When metric truncation is applied, the performance is improved and the gap between the 4th iteration curve and the error-free feedback bound is reduced. Although we do not achieve the ideal bound in the 4th iteration, MSP labeling curve still outperforms both Gray and Mixed labeling curves for SNR greater than 9.4 dB.

**Rician Fading Channel:** We would like to see the closeness between the truncated curve and the error-free feedback bound in a Rician fading environment. We plotted the 4th iteration curves with the power correlation coefficient $\rho$ set to 0.99. Fig. 4.7 is plotted with dashed lines representing the 4th iteration curves and dotted lines representing the error-free feedback bounds.

From the plot, for the case of Gray labeling, the 4th iteration curve is almost identical to the error-free feedback bound. As in the case with Rayleigh fading channel, full convergence is observed. For Mixed labeling, the truncated curve gradually approaches the error-free feedback bound with increasing SNR. However, there is still a gap of 1 dB when the reference point is at a BER of $10^{-5}$. If perfect feedback is available, we can see that the Mixed labeling curve outperforms the Gray labeling curve with a performance gain of 3.5 dB. For MSP labeling, the gap between the truncated curve and the error-free feedback bound is greater compared to when Mixed labeling is used. The gap is around 3 dB. Assuming ideal feedback for both Gray labeling and Mixed labeling
curves, the MSP labeling truncated curve without ideal feedback still performs better for SNR greater than 8.5 dB. From observation, if we were looking for the best convergence, we would use Gray labeling over the other two labelings. On the other hand, if we were looking for the best performance above 8.5 dB, we would use MSP labeling over the other two labelings.
4.3.5 Summary

With iterative decoding, we see that the BER performance improves gradually over iterations for 16QAM BICM-ID with hard-decision feedback. Metric truncation significantly improves the reliability in subsequent iterations and thus, enhances the convergence of iterative decoding. As enhancement in convergence of iterative decoding is achieved, a substantial performance improvement follows. When either Mixed labeling or MSP labeling is used, the gap between the curve without metric truncation and the curve with metric truncation can be readily seen. Moreover, the curve with metric truncation is closer to the error-free feedback bound than the curve without metric truncation. The gap between the BER curve and the error-free feedback bound is reduced when metric truncation is applied. From the results we obtained, metric truncation serves as a way to improve the performance when non-Gray labeling is used for 16QAM BICM-ID with hard-decision feedback. Different from BICM-ID with soft feedback, metric truncation can be introduced with no increase in decoding complexity.
Chapter 5  ITERATIVE DFDM WITH TADPSK

In Chapter 4, we have considered 16QAM and BICM-ID for coherent detection with imperfect CSI at the receiver. In this chapter, we will focus on iterative decision-feedback differential demodulation (DFDM), which is similar to BICM-ID, but does not require channel estimation. We consider differentially encoded 16-ary transmission with TADPSK, i.e., T2AD8PSK, and devise a new demodulator module for iterative DFDM. We will also take a look at the soft output of the demodulator and give methods to improve its quality. Moreover, we will show performance comparisons by means of simulation results.

5.1 New Demodulator Module for Iterative DFDM

The decoder structure of iterative DFDM is equivalent to that of BICM-ID with coherent detection. The only difference lies in the demodulator module, which is also referred to as DFDM-module in iterative DFDM. In DFDM, each $N$ consecutive
received samples are jointly processed to obtain bit metrics to be passed to the outer decoder. Considering modulation with T2AD8PSK, we would like to calculate the bit metric with an observation window size \( N \). We thereby follow the approach in [19], [20], and [21], where DPSK modulation has been considered, and extend it to T2AD8PSK. First, we define the following vectors

\[
\begin{align*}
\mathbf{s}[k] &= [s[(N-1)] \ldots s[k] s[k]]^T \\
\mathbf{r}[k] &= [r[(N-1)] \ldots r[k] r[k]]^T \\
\mathbf{v}[k] &= [v[(N-2)] \ldots v[k] v[k]]^T
\end{align*}
\]  

The first vector \( \mathbf{s}[k] \) is the transmitted vector, which consists of \( N \) transmitted symbols. The second vector \( \mathbf{r}[k] \) is the received vector, which consists of \( N \) received symbols. The last vector \( \mathbf{v}[k] \) is the data-carrying vector, which consists of \( N-1 \) information symbols. The received vector \( \mathbf{r}[k] \) can be expressed as the following

\[
\mathbf{r} = \mathbf{S} \mathbf{g} + \mathbf{n}
\]  

where the fading vector, the noise vector, and the transmitted matrix are defined as

\[
\begin{align*}
\mathbf{g}[k] &= [g[(N-1)] \ldots g[k] g[k]]^T \\
\mathbf{n}[k] &= [n[(N-1)] \ldots n[k] n[k]]^T \\
\mathbf{S} &= \text{diag}\{s[m], m = N-1, N-2, \ldots, 0\}
\end{align*}
\]  

The multiple-symbol differential demodulation (MSDM) metric for the trial data-carrying vector \( \mathbf{\tilde{v}}[k] \) is obtained as

\[
\lambda = \mathbf{r}^H \mathbf{C}_r^{-1} \mathbf{r}
\]  

where \( \mathbf{C}_r = \mathbf{S}(\mathbf{S} + \delta_n^2 \mathbf{A}) \mathbf{S}^H \) with \( \mathbf{A} \) defined as \( \mathbf{A} = \mathbf{E}\{\mathbf{g} \mathbf{g}^H\} \) and
\[ A = \text{diag}\left\{ s[k-m]^{-2}, m = N-1, N-2, ..., 0 \right\} \]  The middle term \( C_g + \delta_n^2 A \) can be approximated by \( C_g + \delta_n^2 I_N \), where \( \delta_n^2 = \delta_n^2 \left( \frac{1}{\sum_{j=0}^{\infty} r_j} \right) \) and \( I_N \) is a \( N \times N \) identity matrix.

Equation (5.6) can be rewritten as

\[
\lambda = L^H \left( S^H \right)^{-1} L^H L S^{-1} r 
\]

(5.7)

where \( L^H L \) denotes the inverse of \( C_g + \delta_n^2 I_N \). When Cholesky decomposition is applied, \( L \) is a lower triangular matrix with the diagonal elements having the values \(-1/\sigma_{\epsilon}^{N-1-n}\), \( N-1 \geq n \geq 0 \) and the strictly lower triangular elements having the values \( p_{\phi-n}^{N-1-n}/\sigma_{\epsilon}^{N-1-n} \), \( N-1 \geq n \geq 0 \) and \( N-1 \geq \phi \geq n+1 \). \( p_{\phi}^{n} \) represents the \( \phi \)th coefficient of a \( n \)th order linear predictor and \( (\sigma_{\epsilon}^{n})^2 \) represents the prediction error variance. By exchanging the elements in \( S^{-1} \) and \( L \), we have the following

\[
\lambda = \left\| LR S^{-1} \right\|^2 
\]

(5.8)

where \( R = \text{diag}\left\{ r[k-m], m = N-1, N-2, ..., 0 \right\} \) and \( S^{-1} = \left[ \left( 1/s[k-(N-1)] \right) ... \left( 1/s[k] \right) \right]^T \).

After expanding out the terms, the MSDM metric for the trial data-carrying vector \( \bar{v}[k] \) is as follows

\[
\lambda_{\text{MSDM}}^{\bar{v}[k]} = \sum_{n=0}^{N-1} \frac{1}{(\sigma_{\epsilon}^{N-1-n})^2} \left| \frac{r[k-n]}{\bar{v}[k-n]} \right| - \sum_{\phi=0}^{N-1-n} \sum_{\xi=n+1}^{\phi+1} \frac{1}{\sigma_{\epsilon}^{N-1-n}} \left| \frac{\prod_{\xi=n+1}^{\phi+1} \bar{v}[k-\xi]}{\bar{v}[k]} \right| \left| \frac{r[k-\phi]}{\bar{v}[k]} \right|^2 
\]

(5.9)

This metric requires us to search for the best combination of symbols to make up the trial data-carrying vector \( \bar{v}[k] \) and it does not use the feedbacks that are available to us.
When feedback symbols are incorporated, the scheme becomes decision-feedback differential demodulation (DFDM). The DFDM metric for the trial data-carrying symbol $\tilde{v}[k-d]$, $0 \leq d \leq N-2$ is

$$
\lambda^{DFDM}_{\tilde{v}[k-d]} = \sum_{n=0}^{d} \frac{1}{(\sigma_e N^{-1-n})^2} \left| \frac{r[k-n]}{\tilde{v}[k-n]} - \sum_{\phi=n+1}^{N-1} p_{\phi-n} \right| \left( \prod_{\xi=n+1}^{\phi-1} \left| \frac{\tilde{v}[k-\xi]}{\tilde{v}[k-d]} \right| \right)^2
$$

(5.10)

Considering the DFDM metric, decision-feedback symbols are used in the computation of the metric. This scheme takes advantage of feedback symbols and also allows us to change the position of the trial symbol. By varying the position of the trial symbol, the performance of the receiver changes. To show this, we will plot the required SNR to achieve a BER of $10^{-3}$ as a function of the position $d$. In Fig. 5.1, the performance is best achieved by setting the trial symbol at the middle of the observation window with window size of 4, 6, and 8. The least desired performances are found at the ends and the performance improves gradually as the trial symbol moves closer and closer towards the center of the observation window. So for even $N$, $d$ can be set to $\frac{N}{2} - 1$.

From (5.10), we can see that we need to calculate all the different predictor coefficients and prediction error variances for $d+1$ linear predictors. The predictors also have different orders. To simplify, we would like to avoid using predictors of different orders. As we replace the upper limit of the second summation, we would have a constant window size of $\frac{N}{2}+1$ that slides across $v[k-(\frac{N}{2}-1)]$ within the whole
Fig. 5.1 Effect of trial symbol position on SNR.

observation window of size $N$. Once this is done, we also have to change the predictor coefficients $p_{\varphi,n}^{N-1,n}$ and the prediction error variances $(\sigma^2_{\epsilon} N^{-1,n})^2$ to $p_{\varphi,n}^N$ and $(\sigma^2_{\epsilon} N)^2$ correspondingly. Since $(\sigma^2_{\epsilon} N)^2$ is a constant, the sliding-window decision-feedback differential demodulation (SWDFDM) metric is as follows

$$
\lambda_{SWDFDM} = \sum_{n=0}^{d} \left| r[k-n] \right| \frac{N}{\sum_{\varphi=0}^{N-1} \sigma^2_{\epsilon} N \ |\hat{\nu}[k-n]|} - \sum_{\varphi=0}^{N-1} p_{\varphi,n}^N \left( \prod_{i=0}^{d-1} \frac{\hat{\nu}[k-\xi]}{|\hat{\nu}[k-\xi]|} \left( \prod_{\xi=d}^{\varphi-1} \frac{\hat{\nu}[k-\xi]}{|\hat{\nu}[k-\xi]|} \right) \frac{r[k-\varphi]}{|r[k-\varphi]|} \right)^2
$$

(5.11)
As before, \( N \) is even and \( d \) is set to \( \frac{N}{2} - 1 \). This metric is less complex than (5.10) while maintaining the use of feedbacks.

With the SWDFDM metric, we can define the bit metric expression. For a desired bit \( c[i] \) within a symbol, we can use the feedbacks that are available to us. We can use the rest \( l-1 \) feedback bits in the bit metric calculation. For the desired bit, we have to determine whether it is zero or one. Using the \( l-1 \) feedback bits, we can determine the two trial symbols, which only differs at the desired bit. If the desired bit is the first bit, the two trial symbols are obtained by the following

\[
\tilde{v}_b[k-d] = [b \ \tilde{c}[i+1] \ \tilde{c}[i+2] \ ... \ \tilde{c}[i+l-1]]
\]  

(5.12)

where \( \tilde{c}[] \) denotes the feedback bits and \( b \) can take on the value of zero or one. With this notation, we can define the log-likelihood-ratio bit metric as below

\[
\lambda_{SWDFDM}^{c[i]} = \lambda_{SWDFDM}^{0[k-d]} - \lambda_{SWDFDM}^{1[k-d]}
\]  

(5.13)

Since SWDFDM is used, we set \( N \) to be even and \( d \) to be \( \frac{N}{2} - 1 \).

### 5.2 Reliability Analysis

Considering T2AD8PSK with hard-decision feedback, it is essential to check the reliability of the demodulator output \( \lambda_{c[i]} \) [18]. We need to verify the correspondence between the assumed reliability \( R(c[i]=1) \) and the relative frequency \( F\{c[i]=1\} \). Consider the demodulator of the receiver using the SWDFDM metric discussed in the
Fig. 5.2 Relative frequency vs assumed reliability for T2AD8PSK.

Previous section, we plot the relative frequency $F\{c[i] = 1\}$ as a function of the assumed reliability $R(c[i] = 1)$ in Fig. 5.2. The signal-to-noise ratio is set to 14 dB. Eight-state, rate 1/2 code is used. We apply Gray labeling and Ungerboeck labeling to the signals points. The iteration number is treated as a parameter.

First, we consider the dashed and solid line curves in the figure. For Gray labeling, we see that iteration 2 and iteration 3 both demonstrate mismatch between the relative frequency and the assumed reliability. In the ideal case, we would expect the relative frequency to converge to the assumed reliability. We notice that the mismatch
decreases as more iterations are performed; iteration 2 has a greater mismatch than iteration 3. For Ungerboeck labeling, iterations 2 through 5 are shown. All four curves shown present a considerable mismatch. Although the mismatch diminishes as we take more iterations, the mismatch at the fifth iteration is still significant when compared against Gray labeling. To show the mismatch, we consider the soft output having a probability of $10^{-5}$ for $c[i] = 1$. Looking at the graph, the true probability is above $10^{-2}$ for the second iteration and around $10^{-3}$ for the fifth iteration.

5.3 Metric Truncation

To compensate for the bias, we truncate the bit metric if it exceeds or falls below some threshold. For iteration $j \geq 2$, the lower threshold is chosen as $10^{-j}$ and the upper threshold is chosen as $(1 - 10^{-j})$. When the soft output is below the lower threshold, it is set to the lower threshold value. When the soft output is above the upper threshold, it is set to the upper threshold value. We will refer to this type of truncation as Truncation I. After the truncation is performed (dash-dotted and dotted curves in Fig. 5.2), there is still not a big difference when Gray labeling is considered. However, looking the curves for Ungerboeck labeling, the improvement is more significant. The truncated curves are closer and closer to the ideal case as we increase the number of iterations. Since they are closer to the ideal case, a better performance can be achieved.

We can apply an additional truncation after the above described truncation is completed, in order to improve the reliability in subsequent decoding iterations. We can
truncate the soft outputs to new thresholds if the soft output values exceed 0.5 but have 0s as feedbacks, or if the soft output values fall below 0.5 but have 1s as feedbacks. For iteration $j \geq 2$, the lower threshold is chosen as $10^{-j/4}$ and the upper threshold is chosen as $(1-10^{-j/4})$. Once the soft outputs fall below or exceed the thresholds, we set the soft outputs to the threshold values. We will refer to this type of truncation as Truncation II. With this additional metric truncation, a further performance enhancement is achieved.

5.4 Performance Evaluation

In this section, we will investigate the BER performance of the proposed iterative scheme under various conditions. Convolutional code with 8-state and rate 1/2 is used as it offers good tradeoff between performance and complexity. Randomly generated bit interleavers are utilized and hard-decision feedback is employed. Each information block has 20000 bits. With fading rate of 0.01, we consider two different fading channels. Rayleigh fading channel and Rician fading channel with the Ricean factor $K = 3$ dB are considered. We will first look at how the performance changes over iterations. The improvement through metric truncation will be considered next. The performance is also influenced by the labeling that we use, so we will analyze the effects of different labelings. Moreover, we will examine how different window sizes and amplitude ratios affect the performance, and also provide a performance comparison of T2AD8PSK and 16DPSK. Finally, we will investigate the performance for different bit metrics.
Fig. 5.3 Performance of iterative DFDM with T2AD8PSK as function of decoding iterations. Truncation II is applied.

5.4.1 Performance as Function of Decoding Iterations

We look at the performance as function of decoding iterations first. We consider T2AD8PSK with ring ratio 1.8. Observation window size $N$ is set to 4 and Ungerboeck labeling is used. With Truncation II, we use the sliding-window decision-feedback differential demodulation (SWDFDM) metric discussed in this section. The BER performance over a Rayleigh fading channel is plotted in Fig. 5.3.
For the first iteration where no feedback is available, the curve starts with a BER of $3.5 \times 10^{-1}$ at 7 dB. As SNR is increased, the curve gradually decreases and reaches a BER of $4.5 \times 10^{-3}$ at 15 dB. For the second iteration, we feed back symbols. We observe performance gain over the 1st iteration after around 11 dB. The second iteration curve is able to reach a BER of $6 \times 10^{-5}$ at 15 dB. The third and fourth iteration curves provide even better performance gain over the 1st iteration curve. The fourth iteration curve has a BER of $10^{-5}$ at close to 14 dB. The error-free feedback bound is shown by the dashed line. This is the ideal situation where the feedbacks are all correct. As more iteration is considered, the curve moves closer to the error-free feedback bound. Similar type of behavior can be found on a Rician fading channel with $K = 3$ dB.

5.4.2 Improvement Through Metric Truncation

Performance improvement offered by metric truncation is considered in this subsection. Using the same setup as in the previous subsection, the 4th iteration curves are plotted in Fig. 5.4. We look at the cases where metric truncation is not applied, Truncation I is applied, and Truncation II is applied.

Considering the curve without metric truncation first, it outperforms the 1st iteration curve at about 11.4 dB. When Truncation I is applied, we can see that it provides a performance improvement over the curve without metric truncation for SNR greater than 11.6 dB. However, we observe a performance loss for lower SNR. We do not observe such behavior if we use Truncation II. For SNR below 10 dB, the curve with
Fig. 5.4 Performance comparison of iterative DFDM with T2AD8PSK without metric truncation, with Truncation I, and with Truncation II.

Truncation II corresponds with the curve without truncation. No performance loss is observed in this case. Comparing the point of intersection, we would also prefer to use Truncation II. We will use the term "metric truncation" to mean that Truncation II is applied in the following subsections.

5.4.3 Performance for Different Labelings

In this subsection, we would like to investigate how the BER curves vary for different labelings.
Fig. 5.5 Performance of iterative DFDM with T2AD8PSK over Rayleigh fading channel with Gray labeling, Mixed labeling, and Ungerboeck labeling.

Rayleigh Fading Channel: Observation window size is set to $N = 4$ and ring ratio is set to 1.8. Considering T2AD8PSK with receiver employing SWDFDM metric, we can plot the 4th iteration BER curves in Fig. 5.5. The solid lines represent the curves without metric truncation and the dashed lines are the curves with metric truncation applied.

Looking at the Gray labeling curves first, we can see that the curve without metric truncation and the curve with metric truncation do not differ a lot. Metric truncation does
no help us in this case. As for Mixed labeling, the performance gain can to seen to start at around 8 dB. It is able to approach $10^{-5}$ at 14 dB. A gain of 2 dB can be observed if we use BER of $10^{-4}$ as a reference. Comparing with Gray labeling, it is able to outperform the Gray labeling curves at around 11.5 dB. As for Ungerboeck labeling, we can see that without metric truncation, it does not give better performance when compared to Gray labeling and Mixed labeling. When metric truncation is applied, it outperforms the Gray labeling curve at 12.5 dB and it gives comparable result at 14 dB. Therefore, for SNR above 14 dB, both Ungerboeck and Mixed labelings can be used to provide better results than Gray labeling.

*Rician Fading Channel:* We would like to look at how the labelings affect the BER performance under a Rician fading environment next. Fig. 5.6 is plotted with ring ratio of 1.8. We employ SWDFDM metric with observation window size of $N = 4$. The 4th iteration BER curves without metric truncation are plotted by solid lines. The 4th iteration BER curves with metric truncation are plotted by dashed lines.

For Gray labeling, only negligible difference is found between the curve without metric truncation and the curve with metric truncation. In this case, metric truncation does not provide performance gain. For Mixed labeling, when metric truncation is applied, the curve starts to outperform the curve without metric truncation at 7 dB. The performance gain can be seen to increase gradually. Using a BER of $10^{-5}$ as a reference, we can see a gain of 2 dB. Comparing with the Gray labeling curves, Mixed labeling
curve with metric truncation is not a preferred choice for SNR below 10.6 dB. However, Mixed labeling curve with metric truncation starts to perform better for SNR above 10.6 dB. For Ungerboeck labeling, it does not provide better performance than the other two labelings when metric truncation is not used. We can see that with metric truncation applied to the Ungerboeck labeling curve, it is able to outperform the untruncated Mixed labeling curve at 10.5 dB and outperform the Gray labeling curves at 11.7 dB. With metric truncation, Mixed labeling and Ungerboeck labeling curves both give similar performance gain over the Gray labeling curve for SNR above 12 dB.
5.4.4 Comparison of Iterative DFDM With Realistic and With Error-Free Feedback

With metric truncation applied to the BER curves, we can compare them to the error-free feedback bounds.

Rayleigh Fading Channel: We plot the 4th iteration BER curves in Fig. 5.7. We consider a ring ratio of 1.8. T2AD8PSK is used. We employ SWDFDM metric and we
use window size of $N = 4$. The dashed lines represent the curves with metric truncation applied and the dotted lines represent the error-free feedback bounds.

Observing the Gray labeling curves first, we can see that the difference between the curve with metric truncation and the error-free feedback bound gradually decreases. The difference between the two remains fairly consistent after 11 dB. A gap of around 0.3 dB can be seen. As for Mixed labeling, we can see that its truncated curve is able to outperform the error-free feedback bound of Gray labeling. In other words, we can exceed the optimal performance of Gray labeling if we apply Mixed labeling. This performance gain is seen after 12 dB. As for Ungerboeck labeling, its truncated curve is also able to outperform the error-free feedback of Gray labeling. This occurs at 14 dB. With BER of $10^{-4}$ as a reference, a gap of around 1.3 dB can be seen between the truncated curve and the error-free feedback bound. Compare to the gap with Gray labeling and Mixed labeling, the gap with Ungerboeck labeling is larger. The convergence to the error-free feedback bound is better if Gray labeling is used. It is able to approach closer to the error-free feedback bound in the 4th iteration. If ideal feedback is available, Ungerboeck labeling would give the best performance, followed by Mixed labeling and Gray labeling.

**Rician Fading Channel:** We would like to compare the BER curves with metric truncation with the error-free feedback bounds for different labelings under a Rician fading environment. Window size is set to $N = 4$ and ring ratio is set to 1.8. SWDFDM
Fig. 5.8 Comparison of iterative DFDM with T2AD8PSK with realistic and with error-free feedback over Rician fading channel with Gray labeling, Mixed labeling, and Ungerboeck Labeling.

metric is considered. The 4th iteration curves and the error-free feedback bounds for different labelings are plotted in Fig. 5.8. The dashed lines are the 4th iteration curves with metric truncation and the dotted lines are the error-free feedback bounds.

For Gray labeling, we can see that the 4th iteration curve with metric truncation is very close to the error-free feedback bound for SNR above 10 dB. For Mixed labeling,
the 4th iteration curve with metric truncation is able to outperform the ideal bound for
Gray labeling at around 10.8 dB. However, the gap between the 4th iteration curve with
metric truncation and the error-free feedback bound is larger when compared to the gap
for Gray labeling. If convergence were desired, we would prefer to use Gray labeling
over Mixed labeling. For Ungerboeck labeling, the 4th iteration curve with metric
truncation also outperforms the error-free feedback bound for Gray labeling but at a
higher SNR. The performance gain occurs at around 11.8 dB. For SNR greater than 12
dB, both Mixed labeling and Ungerboeck labeling provide comparable performance
improvement over Gray labeling. If we have error-free feedback available to us,
Ungerboeck labeling would give us the best performance. Mixed labeling would come
second followed by Gray labeling.

5.4.5 Performance as Function of Observation Window Size

So far we have fixed the observation window size to be 4. We would like to
investigate how the observation window size affects the BER performance in this
subsection.

Rayleigh Fading Channel: We consider T2AD8PSK with ring ratio of 1.8.
SWDFDM metric and Ungerboeck labeling are used. Considering Rayleigh fading, the
4th iteration BER curve with metric truncation is plotted in Fig. 5.9. The solid lines
represent the 4th iteration curves with metric truncation. The dashed lines represent the
error-free feedback bounds.
Fig. 5.9 Performance of iterative DFDM with T2AD8PSK over Rayleigh fading channel with window size of 4, 6, 8, and 10.

With window size of 4, we can see that its 4th iteration curve with metric truncation provides the best performance for SNR below 12.8 dB. When the window size is increased to 6, the performance degrades by around 0.4 dB for SNR below 12.8 dB. When SNR is above 12.8 dB, a performance improvement over the curve with window size of 4 can be observed. A performance gain of around 0.5 dB is found. As the window size is increased to 8 and 10, a minor performance loss is present for SNR below 12.8 dB. For SNR above 12.8 dB, the BER performance for curves with window size of 8 and 10 offer a similar result when comparing to the curve with window size of 6. Thus
increasing the window size does provide a performance improvement but the improvement is reached with window size of 6 in this case. For SNR below 12.8 dB, we still prefer a window size of 4. Looking at the error-free feedback bounds, we can see that the least desired performance is with window size of 4. This is followed by the error-free feedback bound with window size of 6. Similar results are found for the error-free feedback bounds with window size of 8 and 10.

**Rician Fading Channel:** By varying the window size, the effect on the BER performance under a Rician fading environment is investigated. Ungerboeck labeling is used. Ring ratio is set to 1.8. With SWDFDM metric, the BER performance is plotted in Fig. 5.10. The 4th iteration BER curves with metric truncation are represented by the solid lines and the error-free feedback bounds are represented by the dashed lines.

For SNR below 8 dB, the 4th iteration curve with window size of 4 has similar performance as the 4th iteration curve with window size of 6. When SNR is above 8 dB, the performance of the curve with window size of 6 starts to degrade. A gap as large as 0.8 dB can be observed. However, the curve with window size of 6 begins to outperform the curve with window size of 4 when SNR reaches 11.9 dB. For the curve with window size of 8, it has comparable performance as the curve with window size of 6 up to 10 dB. It suffers a performance loss until 12.3 dB. For the curve with window size of 10, it is similar to the curve with window size of 8 but with a slight performance loss beginning at 10 dB. As for the error-free feedback bounds, we can see that the performance improves
as we increase the window size. The gap between the error-free feedback bounds narrows as window size is increased. If ideal feedback were available to us, we would prefer to use a window size of 10 rather than a window size of 4.

5.4.6 Performance as Function of Amplitude Ratio

In this subsection, we would like to see how the BER performance is affected by the amplitude ratio.
Rayleigh Fading Channel: Using SWDFDM metric, we apply Ungerboeck labeling to the signal points. Window size is set to 4. Metric truncation is applied. Fig. 5.11 is plotted with the 4th iteration BER curves. The solid lines are the 4th iteration curves with metric truncation. The dashed lines are the error-free feedback bounds.

Looking at the BER curve with amplitude ratio of 1.1, we can see that it does not provide a better performance when compared to the BER curve with amplitude ratio of
1.8. The gap between the curves can be as big as 0.8 dB. For SNR less than 9 dB, both curves give identical performance. If we increase the amplitude ratio from 1.8 to 2.5, the performance does not improve. The 4th iteration BER curve with amplitude ratio of 2.5 actually has the least desired performance among the three amplitude ratios considered. For SNR lower than 10 dB, it offers comparable performance as the curve with amplitude ratio of 1.1. For SNR higher than 10 dB, a performance loss is observed. The gap between the curve with amplitude ratio of 1.8 and the curve with amplitude ratio of 2.5 can be as large as 1.6 dB. Thus decreasing the amplitude ratio to 1.1 or increasing the amplitude ratio to 2.5 does not help us to improve the performance. The error-free feedback bound with amplitude ratio of 2.5 is outperformed by the 4th iteration curves with amplitude ratio of 1.1 and 1.8. So even with ideal feedback available to the curve with amplitude ratio of 2.5, we would still prefer to use amplitude ratio of 1.1 or 1.8 when SNR is above 13.8 dB.

*Rician Fading Channel:* We would like to consider how the amplitude ratio influences the BER performance in a Rician fading environment. With window size of 4, we use SWDFDM metric. Ungerboeck labeling is used. This is plotted in Fig. 5.12 with the solid lines representing the 4th iteration BER curves with metric truncation and the dashed lines representing the error-free feedback bounds.

When SNR is below 9 dB, the BER curve with amplitude ratio of 1.1 offers equivalent performance as the BER curve with amplitude ratio of 1.8. However, we
Fig. 5.12 Performance of iterative DFDM with T2AD8PSK over Rician fading channel with amplitude ratio of 1.1, 1.8, and 2.5.

observe a performance loss for the curve with amplitude ratio of 1.1 when SNR is above 9 dB. A gap as large as 0.8 dB can be found between the two curves. When amplitude ratio of 2.5 is considered, additional performance loss is seen. For SNR up to 10 dB, the curve with amplitude ratio of 2.5 gives similar performance when compared to the curve with amplitude ratio of 1.1. The gap between the curve with amplitude ratio of 1.8 and the curve with amplitude ratio of 2.5 can be as large as 1.3 dB. The error-free feedback bound with amplitude ratio of 2.5 is outperformed by the error-free feedback bounds with amplitude ratio of 1.8 and 1.1.
Fig. 5.13 Performance comparison of iterative DFDM with T2AD8PSK and 16DPSK over Rayleigh fading channel.

5.4.7 Performance Comparison of T2AD8PSK and 16DPSK

We have only considered T2AD8PSK thus far. It is interesting to compare T2AD8PSK with 16DPSK.

*Rayleigh Fading Channel:* Amplitude ratio of 1.8 is used for T2AD8PSK. With Ungerboeck labeling, we use SWDFDM metric. Window size is set to 4. Considering Rayleigh fading, the 4th iteration BER curves are plotted in Fig. 5.13. Solid lines are the
4th iteration curves without metric truncation. Dashed lines are the 4th iteration curves with metric truncation applied. Dotted lines are the error-free feedback bounds.

When metric truncation is not applied, comparing the 4th iteration BER curve for 16DPSK with the 4th iteration BER curve for T2AD8PSK, we can see that T2AD8PSK does perform better. A performance gain of around 1 dB can be found. For SNR less than 8 dB, the performances of the two curves are identical. The improvement increases gradually for SNR greater than 8 dB. At 14 dB, we can see that the 16DPSK curve has a BER of $5 \times 10^{-3}$ and the T2AD8PSK curve has a BER of $8 \times 10^{-4}$. When we apply metric truncation to the T2AD8PSK curve, a further performance improvement is found. The curve is now able to reach a BER of $1.5 \times 10^{-5}$ at 14 dB, which is better when compared to 16DPSK with metric truncation. For the error-free feedback bounds, the 16DPSK bound initially performs better than the T2AD8PSK bound. The difference between the two bounds becomes smaller and smaller as SNR approaches 12 dB. For SNR greater than 12 dB, the T2AD8PSK bound begins to offer a better performance than the 16DPSK bound.

**Rician Fading Channel:** We would like to compare the performance of T2AD8PSK with 16DPSK under a Rician fading environment. Window size of 4 and amplitude ratio of 1.8 are used. Ungerboeck labeling is applied. With SWDFDM metric, the results are plotted in Fig. 5.14. Solid lines represent the 4th iteration BER curves without metric truncation. Dashed lines represent the 4th iteration BER curves with metric truncation. Dotted lines represent the error-free feedback bounds.
For SNR less than 9 dB, the 4th iteration curve for 16DPSK has comparable performance as the 4th iteration curve for T2AD8PSK when metric truncation is not used. For SNR above 9 dB, the gap between the two curves can be as large as 1.5 dB. Performance improvement can be observed when T2AD8PSK is used instead of 16DPSK. At SNR of 12 dB, the 16DPSK curve without metric truncation has a BER of $5 \times 10^{-2}$ and the T2AD8PSK curve without metric truncation has a BER of $2 \times 10^{-3}$. When metric truncation is applied to the T2AD8PSK curve, more improvement is seen as BER reaches $2 \times 10^{-5}$ at 12 dB. Comparing to the 16DPSK curve with metric truncation, improvement
in performance is observed for T2AD8PSK as the 16DPSK curve is only able to reach a BER of $3 \times 10^{-4}$ at 12 dB. As for the error-free feedback bounds, the 16DPSK bound performs better than the T2AD8PSK bound. However, the gap between the two bounds narrows as SNR is increased.

5.4.8 Performance Comparison for Different Bit Metrics

Up to now we have considered SWDFDM metric. We would like to see the performance difference between the DFDM metric and the SWDFDM metric.

*Rayleigh Fading Channel:* Considering T2AD8PSK with amplitude ratio of 1.8, we use a window size of 4. Under a Rayleigh fading environment, the error-free feedback bounds are plotted in Fig. 5.15. Dotted lines are the error-free feedback bounds with SWDFDM metric (5.11) and solid lines are the error-free feedback bounds with DFDM metric (5.10).

For Gray labeling, we can see that the gap between the curve with SWDFDM metric and the curve with DFDM metric is not very large. A difference of 0.3 dB can be observed. For Mixed labeling, similar result is found. The performance improvement of the DFDM curve over the SWDFDM curve is approximately 0.4 dB. For Ungerboeck labeling, the performance improvement is around 0.3 dB. Regardless of the labeling, we do not suffer from a great performance loss if we use the SWDFDM metric instead of the DFDM metric, as the BER performance with SWDFDM metric is within 0.4 dB of the
Fig. 5.15 Performance comparison for DFDM and SWDFDM metrics over Rayleigh fading channel with Gray labeling, Mixed labeling, and Ungerboeck labeling.

BER performance with DFDM metric. Best BER performance is with Ungerboeck labeling. Next are with Mixed labeling and Gray labeling.

*Rician Fading Channel:* We would look at the BER performance difference between using the DFDM metric and the SWDFDM metric under a Rician fading environment. Window size of 4 and amplitude ratio of 1.8 are used. In Fig. 5.16, the error-free feedback bounds are plotted. Solid lines represent the error-free feedback
Fig. 5.16 Performance comparison for DFDM and SWDFDM metrics over Rician fading channel with Gray labeling, Mixed labeling, and Ungerboeck labeling.

bounds with DFDM metric (5.10). Dotted lines represent the error-free feedback bounds with SWDFDM metric (5.11).

First, we look at the Gray labeling curves. The DFDM curve provides a slight performance improvement over the SWDFDM curve. The gap between the two curves is around 0.3 dB. For Mixed labeling, we observe that the DFDM curve provides a performance improvement of about 0.4 dB over the SWDFDM curve. For Ungerboeck
labeling, the gap is around 0.4 dB. If SWDFDM metric is used instead of DFDM metric, we can see that the performance loss is within 0.4 dB for any labeling. Thus, no great performance loss is observed if we use the SWDFDM metric. Ungerboeck labeling provides the best performance, followed by Mixed labeling and Gray labeling.
Chapter 6  PERFORMANCE COMPARISON

In Chapters 4 and 5, we have considered two bandwidth-efficient 16-ary modulation schemes independently of each other. Whereas 16QAM BICM-ID requires channel estimation at the receiver, T2AD8PSK with iterative DFDM accomplishes detection without the need for channel estimation. A comparison of both schemes under the realistic assumption of imperfect CSI for 16QAM BICM-ID is the subject of this chapter.

6.1 Comparison with Metric Truncation

In this section, we would like to compare the BER performance of 16QAM with T2AD8PSK. For 16QAM, metric truncation is applied with hard-decision feedback and MSP labeling is used. For T2AD8PSK, the ring ratio is set to 1.8 and the observation window size is set to 4. Considering Ungerboeck labeling, SWDFDM metric is used with hard-decision feedback. Considering Rayleigh fading, the 4th iteration BER curves are plotted in Fig. 6.1. The solid lines are the 16QAM curves and the dashed lines are the T2AD8PSK curves.
Fig. 6.1 Performance comparison between 16QAM and T2AD8PSK when Truncation II is applied.

Comparing to the 16QAM curve with power correlation coefficient $\rho$ of 0.95, the first iteration T2AD8PSK curve without any feedback does not perform better for SNR above 11.5 dB. At 14 dB, the 16QAM curve with $\rho$ of 0.95 has a BER close to $10^{-4}$ while the first iteration T2AD8PSK curve can only obtain a BER of around $10^{-2}$. On the other hand, when we consider the fourth iteration T2AD8PSK curve, it performs better when compared to the 16QAM curve with $\rho$ of 0.95. It is able to approach close to a BER of $10^{-5}$ at 14 dB, which is an improvement over a BER of around $10^{-4}$ for 16QAM
with $\rho$ of 0.95. As we increase the number of decoding iterations, the T2AD8PSK curve with metric truncation can gradually outperform the 16QAM curve with $\rho$ of 0.95.

6.2 Comparison with Different Labelings for the Rayleigh Fading Channel

We would also like to compare the BER performance of 16QAM with T2AD8PSK with different labelings. The value of the power correlation coefficient $\rho$ is varied for 16QAM. For 16QAM, MSP labeling is applied as it offers better performance than the other two labelings. Considering hard-decision feedback, metric truncation is used. For T2AD8PSK, the observation window size is 4 and the ring ratio is 1.8. With SWDFDM metric, we employ metric truncation. With Rayleigh fading, the 4th iteration BER curves are plotted in Fig. 6.2. The solid lines represent the 16QAM curves. The dashed lines represent the T2AD8PSK curves.

Considering the 16QAM curve with power correlation coefficient $\rho$ of 0.97, it does not perform better when compared to the T2AD8PSK curve with Gray labeling for SNR less than 10.2 dB. Thus we would prefer to use T2AD8PSK without any explicit channel state information (CSI) rather than 16QAM with imperfect CSI ($\rho$ of 0.97) when SNR is less than 10.2 dB. For the T2AD8PSK curve with Mixed labeling, the performance improvement over the 16QAM curve with $\rho$ of 0.97 occurs for SNR below 9.3 dB. For the T2AD8PSK curve with Ungerboeck labeling, it falls between the
Fig. 6.2 Performance comparison between 16QAM and T2AD8PSK with Gray labeling, Mixed labeling, and Ungerboeck labeling over Rayleigh fading channel.

16QAM curves with $\rho$ of 0.95 and 0.97. It provides a performance gain when compared to 16QAM with imperfect CSI ($\rho$ of 0.95) but a performance loss when compared to 16QAM with imperfect CSI ($\rho$ of 0.97). Similar results are discovered for T2AD8PSK with Gray labeling above 10.2 dB and T2AD8PSK with Mixed labeling above 9.3 dB. If $\rho$ is 0.95 for 16QAM, we would prefer to use T2AD8PSK regardless of the labeling.
Fig. 6.3 Performance comparison between 16QAM and T2AD8PSK with Gray labeling, Mixed labeling, and Ungerboeck labeling over Rician fading channel.

6.3 Comparison with Different Labelings for the Rician Fading Channel

The BER performance of 16QAM and T2AD8PSK will be compared under a Rician fading environment in this subsection. For 16QAM, hard-decision feedback is used. We apply metric truncation and MSP labeling. For T2AD8PSK, we use SWDFDM metric. Applying metric truncation, we set the observation window size to 4 and ring ratio to 1.8. Fig. 6.3 shows the 4th iteration BER curves. The solid lines are the 16QAM curves and the dashed lines are the T2AD8PSK curves.
Considering the Gray labeling curve for T2AD8PSK first, it performs better when compared to the 16QAM curve with \( \rho \) of 0.89 for SNR less than 12.2 dB. In other words, T2AD8PSK with no channel state information (CSI) is a better choice than 16QAM with imperfect CSI (\( \rho \) of 0.89) when SNR is less than 12.2 dB. As for the Mixed labeling curve, the point of intersection with the 16QAM curve having \( \rho \) of 0.89 is at 12.7 dB. As for the Ungerboeck labeling curve, it is very close to the 16QAM curve with \( \rho \) of 0.89. Only a slight degradation in performance is observed. It is the preferred choice when compared to the 16QAM curve with \( \rho \) of 0.87. In fact, T2AD8PSK is the preferred choice regardless of the labeling if \( \rho \) is 0.87 for 16QAM.
Chapter 7  CONCLUSION

Some general comments are presented as concluding remarks in this chapter. This is followed by recommendations for future work.

7.1 General Comments

The design of bit-interleaved coded modulation – iterative decoding (BICM-ID) over Rayleigh and Rician fading channels is investigated. With BICM-ID, coding and modulation can be performed independently and decoding can be executed iteratively to improve the performance. Through the use of soft feedback, BICM-ID is able to outperform noniterative schemes. The complexity can be reduced by feeding back decisions instead of probabilities.

To represent a realistic model, the use of imperfect channel state information (CSI) is considered. The power correlation coefficient is the crucial parameter for the realistic model. Moreover, the reliability of the demodulator output is analyzed. To improve the
performance, metric truncation can be applied. Considering a 16QAM constellation, iterative decoding with metric truncation provides a great performance gain over conventional decoding schemes. When metric truncation is applied, the use of non-Gray labelings is a possibility as it offers better performance compared to Gray labeling. The convergence to the error-free feedback bound is improved with the aid of metric truncation.

Using a noncoherent detection scheme, a low-complexity iterative receiver is presented. Explicit CSI is not required for noncoherent detection. Employing an observation interval of $N > 2$ symbols, multiple-symbol differential demodulation (MSDM), decision-feedback differential demodulation (DFDM), and sliding-window decision-feedback differential demodulation (SWDFDM) are considered and bit metrics are obtained. The complexity is reduced with the use of SWDFDM. The use of twisted absolute amplitude and differential phase-shift keying (TADPSK) is considered as it allows whole information to be expressed by the phase offset and partial redundant information to be expressed by the amplitude. Furthermore, the demodulator output is examined to ensure reliability. Metric truncation can be used to provide a performance improvement.

By using iterative decoding, the performance of receiver employing SWDFDM with T2AD8PSK is examined. When non-Gray labelings are used, the performance can be improved by the use of metric truncation. The performance with Gray labeling can be
outperformed by the performance with non-Gray labeling if metric truncation is employed. Moreover, increasing the window size will lead to a performance gain for SNR above a certain threshold value. On the other hand, performance improvement is not observed when the amplitude ratio is increased. With the use of T2AD8PSK and metric truncation, the performance is greatly enhanced compared to conventional 16DPSK. Finally, a comparison between 16QAM and T2AD8PSK transmission is presented. Depending on the value of the power correlation coefficient, T2AD8PSK with low-complexity iterative DFDM is able to outperform 16QAM BICM-ID regardless of the labeling.

7.2 Future Work

The work presented in this thesis can be extended in several respects. First, it would be interesting to apply the presented metric truncation schemes to other iterative detectors with hard-decision feedback. Such detectors could be equalizers, multi-user detectors, or detectors for multiple antenna transmission. Second, the iterative DFDM receiver could be extended to differential space-time modulation (DSTM) with non-unitary matrices, which corresponds to TADPSK in the single antenna case considered in this thesis. Finally, low-complexity iterative decoding employing hard-decision feedback could be cast into a more general framework of iterative decoding with impulsive noise, where impulsiveness arises from erroneous feedback symbols. Hence, the application of detection techniques developed for impulsive noise to the problem at hand might be fruitful.
BIBLIOGRAPHY


