THE EFFECT OF CORONA ON WAVE PROPAGATION ON TRANSMISSION LINES

by

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ABSTRACT

Fast transients on power transmission systems, such as the ones produced by lightning and faults, are usually modelled by the Telegrapher's Equations which, because of the corona effect, are nonlinear. Although it has been long recognized that the method of characteristics of partial differential equations (PDE's) theory is the most adequate to tackle this problem, its previous applications have been very limited. A very general technique for the simulation of transients on lines with corona, based on the method of characteristics, is thus proposed in this thesis. This technique consists of representing the transmission lines by a system of first order quasilinear partial differential equations (PDEs) and of solving them on a characteristic system of coordinates by applying interpolation techniques.

A method of analysis and simulation is first developed by applying the technique of characteristics with interpolations to the 2x2 system of quasilinear PDE's representing a monophasic line with static corona. This method is further implemented on a computer. The numerical examples provided show that this method overcomes the problem of numerical oscillations which is often found at the tails of waves simulated by means of conventional methods based on constant discretization schemes. Another important feature of the developed method is that it requires substantially fewer discretization points than the conventional ones.

The developed method is then extended to the time domain analysis of multiconductor lines both, the linear ones and the quasilinear ones with
static corona. Most conventional methods for the analysis of multiconductor lines in the time domain are based, either directly or indirectly, on modal transformations from frequency domain analysis. One problem with this approach is, however, that these transformations usually introduce complex quantities which lack physical meaning in the time domain. The extension developed here maintains the analysis in the domain of the real numbers. In the case of transmission lines with corona, an additional problem of conventional modal transformations is that they presuppose linearity. The extension developed here avoids this shortcoming.
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1. INTRODUCTION.

1.1) Problem Statement.

Modern insulation design of transmission lines and of substations is increasingly becoming more dependent on digital simulations. Due to progress in research and development over the past number of years, the simulation methodology for linear lines has achieved a satisfactory level [27,28,53,81,95]. Often in practice, however, one has to deal with the corona effect which introduces a distributed nonlinearity in the transmission lines. Although the problem of wave propagation on lines with corona has been studied for a long time, progress has been slow and there is still much to do. A thorough account of the most relevant work in lines with corona, since its beginnings, is given in a survey paper by Carneiro [72].

The minimization of corona by overdesigning transmission systems would be extremely expensive and, furthermore, it may not be desirable. Corona tends to
attenuate the parts of a waveform that are above the corona threshold level at a much higher rate than the ones below it. In order to bring corona into design considerations, however, the techniques for the analysis of its nonlinear characteristics would have to be brought to a similar level of development as their linear line modelling counterparts. There are, at present, several difficulties. Firstly, the phenomenon of corona in itself is highly complicated and is still a very active field of research. Secondly, reliable results from field and laboratory are very difficult to obtain and, therefore, scarce. Thirdly, a nonlinear line theory is expected to be far more complicated than the linear one. The work reported in this thesis focuses on the third issue.

It was felt that a method for analyzing wave propagation on nonlinear transmission lines, which was based on as few assumptions as possible, would be a valuable tool for advancing the subject. Firstly, such a method would provide an insight into the required features of the corona models. This method would serve to test hypotheses, to be an aid in the design of experiments and in the interpretation of their results and to help to establish, through sensitivity analysis, which parameters of the corona models are relevant for transient studies. Finally, a design methodology would be established.

1.2) Background

Initial studies of corona go as far back as 1911. Between this year and 1929, Peek published the results of his pioneering research. Some of his results are still in effect, as for instance his well known corona inception law [5]. Progress since then has been slow. Some other important results are the determination of the hysteretic characteristic of corona by Ryan and Henline in 1924 [1] and the initial studies of wave distortion due to corona by Skilling and Dykes in 1937 [8]. In the 50's, Wagner, Lloyd and Gross conducted an extensive
research program on corona and its effects on wave propagation [10,11]. They made several tests on a short transmission line. Their results, published in 1954 [10], are often taken as benchmarks for new simulation methods. Wagner and Lloyd also did experiments aimed at the characterization of the phenomenon of corona [11]. It was perhaps due to the limitations of their measuring equipment that they concluded wrongly that corona was an essentially static phenomenon. In this context, a phenomenon or its representation is said to be static when the spatial charge is a function of the instantaneous voltage only and not of its rate of change with respect to time "$\partial v/\partial t$". If, on the other hand, the spatial charge also depends on $\partial v/\partial t$, the phenomenon or its representation is said to be dynamic. Recent experiments show that corona is dynamic [42]; however, in spite of the modern equipment and techniques, the effect of $\partial v/\partial t$ doesn't seem to be fully determined yet.

Wagner and Lloyd also proposed a finite difference technique to evaluate travelling wave distortion due to corona [11]. It seems that this was the first application of a digital computer to this type of problem. A further digital technique was proposed later, in 1965, by Stafford, Evans and Hingorani [23]. These authors suggested that further developments should make use of the characteristic curves of the transmission line partial differential equations (PDE's). These curves, known as characteristics, have as a property that, along them, the line PDE's turn into ordinary differential equations (ODE's). In 1970 Zielinsky presented a graphical method based on characteristics [30]. Its applicability was, however, very limited.

In the 70's and until the middle 80's, the emphasis of power transient research was directed to linear line models. Two complementary techniques emerged. One based on frequency domain methods [28,95] and, the other, on time
domain methods [27,37,53]. Several successful developments led to the
development of the Electromagnetic Transients Program (or EMTP) which has
become one of the most used programs of its kind [27]. Its availability, along
with the fact that the time domain is far better suited for dealing with
nonlinearities, has encouraged the development of EMTP compatible techniques to
simulate lines with corona. Among them are the one proposed by K. C. Lee [59],
the one by Semlyen and Wei–Gang [70,76], the one by Hamadami–Zadeh [68] and,
more recently, the one by Carneiro, Martí and Dommel [91,94].

Further progress in the topic requires a deep understanding of the corona
phenomenon. Some knowledge has been provided by the experiments conducted at
The Hydro–Québec Institute of Research (IREQ) [42] and at Electricité de France
(EDF) [43]. However, many more experiments are needed. As they are very
difficult and expensive to conduct, it may be convenient to coordinate them with
the development of mathematical models of the corona effect which are based
on the actual physical processes. An example of such a mathematical/physical
model is the one proposed recently by Abdel–Salam and Stanek [71]. The
simulation of transients, on the other hand, necessitates computationally less
intensive corona models. Simpler models that aim to preserve the features
relevant to transient wave propagation have been already proposed. Among them
are the static corona models by Gary, Dragan and Cristescu [85], the dynamic
model based on shells of spatial charge by Harrington and Afghahi [55], the
refined model of shells by Semlyen and Wei–Gang [70] and the dynamic model
based on the time delay of charge formation by Li, Malik and Zhao [87]. These
models should still be subjected to further tests that would establish their range
of validity. The tests, however, would require a very reliable and flexible method
for calculating nonlinear transients as well as a set of well documented line
experiments.
In addition to the line experiments conducted by Wagner, Lloyd and Gross, there are only few more that are well documented. Among them are the ones reported by Gary, Dragan and Cristescu [43], the ones by Ouyang and Kendall [35] and the ones by Inoue [44,65]. More experiments are needed and it is desirable that some of them involve waves with multiple peaks and/or reflections [48].

Concerning the methods for calculating nonlinear transients, in addition to the ones already mentioned before, few more have been proposed. These methods can be classified into the following three groups: 1) wavefront delay methods, 2) lumped element approximation methods and 3) finite difference methods. The wavefront delay methods are essentially extensions of early graphical techniques. Since they are not meant to be very accurate [85], they will not be given further consideration here. As for the lumped element methods, it is usually possible to establish their equivalence with a finite difference method. The research reported here is thus focused on finite differences. Variational methods, such as the finite element method, are often considered better alternatives to finite differences; however, their theory and techniques for handling the nonlinear hyperbolic PDE's arising from transmission lines with corona are still in an early stage of development [73]. It is important, nevertheless, to keep track of their progress. Other techniques that should be considered in future studies are the analytic ones that are currently being developed [90]. An example is the hodograph transformation which converts a homogeneous (lossless) nonlinear PDE into a linear PDE by exchanging the roles between dependent and independent variables [41]. A recent extension by Fusco and Manganaro [86] permits the application of the hodograph transformation to nonhomogeneous PDE's and, consequently, to the analysis of lossy lines with corona.
Lumped element methods are very popular, perhaps because it is relatively easy to derive models of lines with corona from the well known \( \pi \)-line models. In addition, charge/voltage relationships describing corona are often given as circuits involving capacitors, voltage sources, diodes and sometimes resistors. The traditional \( \pi \)-line models are often modified by adding lossless line sections as modelling elements [83]. Additional elements may be inserted to account for the frequency dependence of the line parameters [68,70]. Most of the EMTP compatible methods, including the abovementioned ones, belong to the lumped element category.

Concerning the finite difference methods, few more alternatives have been proposed in addition to the ones by Wagner and Lloyd and by Stafford, Evans and Hingorani. In 1978 and, later, in 1985 Inoue published his experimental results that have been mentioned earlier. Along with them, he proposed a finite difference method [44,65] in which the lines are considered lossless. A further limitation of Inoue's work is that both the experiment and the simulations seem to deal only with nondecaying wavetails. In 1981 Kudyan and Shih presented a finite difference method, also for lossless lines [50]. In their paper, they introduced some stability and convergence considerations. More refined finite difference schemes have been introduced by Gary, Dragan and Cristescu [56], by M. T. Correia de Barros [64] and by Li, Malik and Zhao [87]. In the first method, a discrete convolution is incorporated into the discretization scheme to account for frequency dependence. It seems that the authors didn't use recursive convolution [37,53] and it is thus possible that the computational performance of their method could be improved. In the second method, the line PDE's are turned into a differential-difference system. The resulting differential equations are integrated through Gear's method. In the third method, a hybrid discretization scheme based on forward and backward differences is applied, producing a
tridiagonal system of equations. The method is combined with the model of corona proposed also by the same authors [80].

Extensive research on numerical methods for the simulation of lines with corona was conducted at The University of British Columbia between 1987 and 1989 [72,78,91,94]. Several important results have come from this project. First, it was established that the corona shunt conductance losses have negligible effect on the propagation of transient waves [72]. Second, that the frequency dependence effects are negligible for line distances shorter than 10 km. A similar conclusion was reached by Gary [56]. Third, that the space discretization, either in the lumped or in the finite difference methods, causes artificial reflections whose magnitudes, under certain circumstances, become considerable [78,91]. In addition to this last problem, Janischewskyjji and Gela [48] have pointed out that most of the existing methods do not satisfactorily reproduce the slow decay of travelling wavetails.

1.3) Scope and Aims of the Research

It seems from the previous two sections that a new method of simulating propagation on lines with corona, which complements the already existing ones, is required. Such a method should enable the representation of corona as a distributed phenomenon. In addition, a desirable feature is that its extension to multiconductor lines does not require linear modal analysis or any other technique based on the linear superposition principle. In a preliminary study by this author, it became clear that the transmission line PDE's are quasilinear when corona is represented by a static model. As a recall, a PDE is said to be quasilinear when the highest order derivatives of its dependent variables (that is, the ones that determine the order of the equation) occur only to the first degree [15,33,45]. It became clear also that the method of characteristics is well suited for
representing distributed corona. This method is, in fact, regarded by many authors as the best one for handling quasilinear hyperbolic PDE's [41,45,79]. Apart from Zielinski's work, the characteristics haven't been adopted as a general tool for the analysis and simulation of lines with corona. The aim of the research reported here is, therefore, to develop a methodology for the analysis and simulation of transmission lines with corona that is based on the method of characteristics.

A further consideration made in the preliminary study was that the quasilinear line equations could develop shocks [33]. In this case, the method of characteristics would provide the most convenient way for detecting them [14], analyzing them [15] and handling them [13]. Apparently, the possibility of a shock on a line with corona hasn't been considered before. The closest hint of it, found by the author, is on a recent CIGRE report by Gary, Dragan and Cristescu [93] where the authors take note of the ambiguity that arises when applying the wavefront delay method to the crest of travelling waves.

In order to apply characteristics to the numerical analysis of transmission lines with corona, a decision has to be made as to whether the nonlinear line equations should be handled as a system of first order PDE's [14] or as an equation of higher order [45]. Since there is a general theory for quasilinear hyperbolic systems [15,33,39], the former approach is adopted here. It has the advantage that its extension to multiconductor lines is straightforward and that it doesn't require the use of the linear superposition principle. A further advantage of the adopted approach stems from the fact that nonlinear PDE's can always be turned into quasilinear systems of PDE's [15,39]. The methods developed in the thesis can thus be extended to analyze lines with dynamic corona in a rigorous manner. Because of time limitations, this very important issue was not pursued further here. It is proposed, instead, as a future project.
From all the above, the goals of the research reported in this thesis are stated as follows:

1. To propose, develop and implement a method to analyze the propagation of monopolar impulses on lossy monophasic lines with static corona.

2. To obtain, from the numerical applications of the implemented method, new ideas concerning the features that are required from the adopted corona models.

3. To propose and develop an extension of the method of characteristics for multiconductor lines with corona.

Finally, in passing, an alternative to the conventional methods for handling frequency independent models of multiconductor linear lines in the time domain is also proposed.
2. TRANSMISSION LINE MODELS FOR TRANSIENT STUDIES.

2.1) Underlying Theory of Transmission Line Modelling.

Consider a transmission line formed by the ground plane and a wire running horizontally at a height h. If one assumes that the wire and the ground are perfect conductors, that the air between them is a perfect dielectric and that none of the wavelengths involved are smaller than 4h, then the electric and magnetic fields are transversal (TEM) to the direction of the wire \([26,63]\). Maxwell's first equation (Faraday's Law) yields \([26]\):

\[
-\frac{\partial v}{\partial x} = L_g \frac{\partial i}{\partial t}
\]

........(2.1a)

where \(v\) is the voltage of the wire relative to the ground, \(i\) is the current carried by the wire and \(L_g\) is the line inductance per unit length. Because of its dependence on the sectional line geometry, this inductance is called geometric. A second relationship, companion of (2.1a), is obtained by applying Maxwell's Second
equation (Ampere's Law) to the ideal line [26]:

$$\frac{\partial i}{\partial x} = C_g \frac{\partial v}{\partial t}$$ \hspace{1cm} \text{(2.1b)}

where $C_g$ is the line capacitance per unit length which is also known as geometric because of its dependence on the sectional geometry of the line. In fact, $C_g$ and $L_g$ are inversely related as follows:

$$L_g C_g = \mu_0 \varepsilon_0$$ \hspace{1cm} \text{(2.1c)}

where $\mu_0$ and $\varepsilon_0$ are, respectively, the magnetic permeability and the electric permitivity of the air. These two constants are practically identical to the ones of the vacuum and either side of (2.1c) is equal to the inverse of the speed of light squared.

Equations (2.1a) and (2.1b) describe in fact an ideal (lossless) transmission line in which waves travel without distortion. Actual lines, however, consist of imperfect (lossy) conductors and dielectrics. In consequence, propagating waves usually suffer distortion. The standard practice to account for these losses is to modify (2.1a) and (2.1b) as follows [26]:

$$\frac{\partial v}{\partial x} = L \frac{\partial i}{\partial t} + R \cdot i$$ \hspace{1cm} \text{(2.2a)}

and

$$\frac{\partial i}{\partial x} = C \frac{\partial v}{\partial t} + G \cdot v$$ \hspace{1cm} \text{(2.2b)}

where $L$ is the line inductance, $C$ is the line capacitance, $R$ is the series resistance that accounts for the imperfect conductor losses and $G$ is the shunt conductance that accounts for the dielectric losses. All these parameters are in per unit length. Equations (2.2a) and (2.2b) are known as the Telegrapher's equations. In a strict sense, the $L$ and $C$ parameters differ somewhat from their geometric counterparts $L_g$ and $C_g$, because they must account now for the
penetration of the fields (magnetic and electric, respectively) into the imperfect conductors. Nevertheless, in the case of aerial lines, the value of C is very closely approximated to the one of \( C_g \) for a large range of frequencies and of ground resistivities which encompasses most cases of practical interest [51]. In addition to this, the shunt conductance of these lines is negligible [67]. The following equation can thus be used instead of (2.2b):

\[
\frac{\partial i}{\partial x} = C_g \frac{\partial v}{\partial t} \quad \text{(2.2c)}
\]

A more rigorous analysis of lossy transmission, lines via Maxwell's equations, is only possible after making certain simplifying assumptions concerning the distribution of the electric and magnetic fields. A number of these assumptions, known as quasi-transversal electromagnetic or quasi-TEM, lead to the following general expressions [2, 4, 6, 7]:

\[
\frac{dV}{dx} = (j\omega L + R) \cdot I \quad \text{(2.3a)}
\]

and

\[
\frac{dI}{dx} = (j\omega C + G) \cdot V \quad \text{(2.3b)}
\]

where \( V \) and \( I \) are the Fourier transforms of the actual voltage and current waveforms, respectively. Note that (2.3a) and (2.3b) are ordinary differential equations (ODE's) in the frequency domain. The quasi-TEM assumptions are justified for a wide range of frequencies and ground resistivities which comprises most power transient analysis and power line carrier applications [38, 40].

Despite the similarities between (2.3a) and (2.2a), these two equations are not equivalent. Since in the former equation the \( R \) and \( L \) parameters are functions of the frequency, its inverse Fourier transform would involve a convolution. For example, Radulet, Timotin and Tugulea have proposed the following expression as a time domain equivalent of (2.3a) [29]:

\[\text{(Expression for Radulet, Timotin, and Tugulea)}\]
\[
\frac{\partial v}{\partial x} = L \frac{\partial i}{\partial t} + \frac{\partial}{\partial t}\int_0^t r(t - \tau) \cdot i(\tau) \, d\tau
\]  \quad (2.4)

where \( r(t) \) is defined as the unit step resistance of the line. As for (2.3b), it is clear that its inverse Fourier transform yields (2.2c).

Equations (2.3a) and (2.3b) provide the basis for a frequency domain technique to calculate transmission line transients [28]. As for the time domain techniques, most of them are based on (2.2a) and (2.2c). The models derived from these equations are said to be frequency independent. A major reason for using (2.2a), instead of a convolution expression like (2.4), is that frequency independent models are simple and adequate for many practical situations [95]. In addition, frequency dependence features can be incorporated later on to the models by means of recursive convolution techniques [37, 53]. An important and apparently still unresolved issue, concerning the development of frequency independent line models, is the systematic determination of the values of the R and L parameters that better represent a transmission line for a given simulation situation. Another apparently unresolved issue is the establishment of criteria for determining which transient analysis cases require frequency dependent modelling.

The analysis leading to equations (2.2a) and (2.2c) is based on the assumption that transmission lines respond linearly. This is not the case, however, if a line is affected by corona for which equation (2.2c) has to be modified [56, 65]. In many practical studies, this modification results in the representation of the line capacitance by a function of the line voltage [44]. Due to the fact that this function is multivalued, it is considered here that corona cannot be represented adequately in the frequency domain. The approach adopted in this thesis is thus based on the time domain. Nevertheless, a frequency domain method is also employed here as an aid for the selection of parameters for
frequency independent line models. Section 2.2 thus provides a broad description of this method. Section 2.3 provides a summary of current time domain methods for the analysis of transients on transmission lines. Section 2.4 provides preliminary considerations for the analysis of transients on lines with corona. Section 2.5, finally, provides the remarks of this chapter.

2.2) Frequency Domain Transient Analysis of Lines.

It is customary to write (2.3a) and (2.3b) in the following form:

\[ \frac{dV}{dx} = Z \cdot I \] \hspace{1cm} (2.5a)
\[ \frac{dI}{dx} = Y \cdot V \] \hspace{1cm} (2.5b)

where Z is the line series impedance and Y is the line shunt admittance, both in per unit length. For a transmission line consisting of a thin wire above a semi-infinite ground plane, this impedance is made up of three terms:

\[ Z = j\omega L_g + Z_{cond} + Z_{gnd} \] \hspace{1cm} (2.5c)

The first one is the geometric impedance due to the magnetic flux in the air, the second one accounts for the effects of this flux inside the wire and the third one for the effects of this flux inside the ground. Details for the calculation of these terms are provided in references [20] and [74]. As for the admittance, it has been already mentioned that the shunt capacitance per unit length of an aerial line is practically identical to the geometric line capacitance; therefore:

\[ Y = j\omega C_g \] \hspace{1cm} (2.5d)

Equations (2.5a) and (2.5b) are further manipulated to produce the two following second order ODE's:
\[ \frac{d^2 V}{dx^2} = ZY \cdot V \quad \ldots \ldots (2.6a) \]

and

\[ \frac{d^2 I}{dx^2} = YZ \cdot I \quad \ldots \ldots (2.6b) \]

The solution of (2.6a) is readily obtained as follows:

\[ V(x) = C_1 \exp (-\gamma x) + C_2 \exp (\gamma x) \quad \ldots \ldots (2.7a) \]

where \( C_1 \) and \( C_2 \) are integration constants and

\[ \gamma = \sqrt{ZY} \quad \ldots \ldots (2.7b) \]

is the propagation constant of the line. The real part of this constant corresponds to the line attenuation and its imaginary part to the line phase delay, both are in per unit length. Whereas the term of (2.7a) with the positive exponential factor represents a voltage wave travelling forwards along the line, the other one with the negative exponential factor represents a voltage wave travelling backwards [24]. The respective amplitudes of these two travelling waves, \( C_1 \) and \( C_2 \) are determined from the line boundary conditions; that is, from the connections at both ends of the line section. As for the solution of (2.6b), the following expression is obtained by applying (2.7a) to (2.5a):

\[ I(x) = Y_C \left[ C_1 \exp (-\gamma x) + C_2 \exp (\gamma x) \right] \quad \ldots \ldots (2.7c) \]

where

\[ Y_C = \frac{1}{Z} \gamma \quad \ldots \ldots (2.7d) \]

or

\[ Y_C = \sqrt{\frac{Y}{Z}} \quad \ldots \ldots (2.7e) \]

is the characteristic admittance of the line.

Consider now a homogeneous line section of length \( l \). Let \( V_{in} \) and \( I_{in} \) be the voltage and current at the side of the line considered the input side; that is,
at $x = 0$. Let $V_{out}$ and $I_{out}$ be the voltage and current at the output side of the line; that is, at $x = L$. By applying expressions (2.7a) and (2.7c) to this line section, the following relationship between the aforesaid input and output variables can be obtained [18]:

$$
\begin{bmatrix}
I_{in} \\
I_{out}
\end{bmatrix} =
\begin{bmatrix}
A & B \\
B & A
\end{bmatrix}
\begin{bmatrix}
V_{in} \\
V_{out}
\end{bmatrix}
$$

(2.8a)

where $A$ and $B$ are defined as follows:

$$
A = Y_c \coth(\gamma L)
$$

(2.8b)

and

$$
B = -Y_c \cosech(\gamma L).
$$

(2.8c)

Expression (2.8a) provides a convenient frequency domain model for a homogeneous line section [54,95]. Although this model presupposes a pure sinusoidal excitation, its extension to transient analysis is possible through the use of the superposition principle of linear systems theory. In the method adopted here, first the excitation waveform is decomposed into its harmonic components, then the response of the line section to each of these components is obtained through expression (2.8a) and finally the total response is obtained by superposing the individual harmonic responses [28]. The processes of harmonic decomposition and of superposition are conveniently performed by the Fast Fourier Transform Algorithm (FFT). Since these processes involve the discretization of the frequency range, the undesirable effects of Gibbs oscillations and aliasing are inevitable. Nevertheless, these effects are reduced by applying the complex frequency concept which consists of replacing the imaginary frequency $j\omega$ by a complex variable whose real part is a damping factor and its imaginary part is the frequency itself [46].
An attractive feature of the above described frequency domain method is that the numeric error bound can be controlled. In fact, the number of harmonic components, the truncation frequency (that is, the highest harmonic frequency considered) and the damping factor determine this error bound [46,54]. An application example is provided next. A transient is simulated on a semi-infinite line consisting of an aluminum conductor of radius $r = 2.54$ cm at a height $h = 18.9$ m above a ground plane whose resistivity is $100$ $\Omega$m. Figure 2.1 shows the applied excitation impulse which is a 1.0 p. u. [$2/20\mu$s] double linear ramp; that is, a ramp with a maximum amplitude of one unit, which reaches this maximum value in 2 $\mu$s and which decays to 50 % of this maximum in 20 $\mu$s. Figure 2.1 also shows the impulse after it has travelled 500, 1000, 1500, 2000 and 3000 m along the line. The comparison of these different waveforms shows the smoothing effect on the corner of the impulse as it propagates. Fig. 2.1 simulation has been made with 256 samples spread over an observation time of 33 $\mu$s and with a damping factor that has been chosen for a maximum numeric error of 0.1 %. Note from this figure that only the first 30 $\mu$s of simulation
have been displayed. Since the aliasing error tends to increase in the last portion of the time scale, it is a common practice to display only the first 90% portion of the time scale [46].

The extension of the previously described simulation method to multiconductor lines is straightforward provided the superposition property holds. This extension is outlined as follows for an homogeneous line section. Consider the following multiconductor line equations in the frequency domain [54,67]:

\[
\frac{dV}{dx} = Z \cdot I \\
\frac{dI}{dx} = Y \cdot V
\]

where \( V \) is the vector of conductor voltages, \( I \) is the vector of conductor currents, \( Z \) is the matrix of series impedances per unit length of the line and \( Y \) is the matrix of shunt admittances of the line. The off-diagonal elements of these two matrices account for the mutual impedances and admittances between conductors, respectively. Note that vector and matrix quantities are denoted by bold letters. Let \( T_v \) be the matrix that diagonalizes the \( ZY \) matrix product as follows:

\[
T_v^{-1} \cdot ZY \cdot T_v = \Lambda
\]

where \( \Lambda \) is the diagonal matrix of eigenvalues of \( ZY \). Let \( \Gamma \) be the diagonal matrix whose nonzero elements are the square roots of the diagonal elements of \( \Lambda \) with positive or zero real part. The general solution of (2.9a) and (2.9b) can be expressed as follows in terms of \( \Gamma \) [54]:

\[
V(x) = C_1 \exp(-\psi x) + C_2 \exp(\psi x) \\
I(x) = -Y_C C_1 \exp(-\psi x) + Y_C C_2 \exp(\psi x)
\]
where
\[ \Psi = T_v \cdot \Gamma \cdot T_v^{-1} \] ...........(2.11c)

and
\[ Y_c = Z^{-1} \cdot \Gamma \] ............(2.11d)

Note that (2.11a) and (2.11b) are matrix generalizations of (2.7a) and (2.7c). Note further, from the comparison between (2.7c) and (2.11b), that \( Y_c \) is a matrix of characteristic admittances.

Consider now a multiconductor line section of length \( \lambda \) for which \( V_{in} \) and \( I_{in} \) represent the vectors of the voltages and currents at the input side of the line section and \( V_{out} \) and \( I_{out} \) represent the vectors of voltages and currents at the output side. The application of expressions (2.11a) and (2.11b) to this line section yields [54]:

\[
\begin{bmatrix}
I_{in} \\
I_{out}
\end{bmatrix} =
\begin{bmatrix}
A & B \\
B & A
\end{bmatrix}
\begin{bmatrix}
V_{in} \\
V_{out}
\end{bmatrix}
\] ...........(2.12a)

where
\[ A = Y_c \coth(\Psi x) \] ............(2.12b)

and
\[ B = -Y_c \cosech(\Psi x) \] ............(2.12c)

Expression (2.12a) provides the desired generalization of model (2.8a) for multiconductor transmission lines.

2.3) Transient Analysis of Lines in Time Domain Through the Method of Bergeron.

Several methods have been proposed for the analysis of transients on transmission lines in the time domain. Among them, the method of Bergeron is perhaps the most widely used one [27]. In addition, some of the most important
line models of the Electromagnetic Transients Program (EMTP) are based on this method [67]. For these reasons, this section focuses on the method of Bergeron.

Consider first a transmission line in which the resistive losses are neglected. After some elementary manipulations, equations (2.2a) and (2.2b) become:

\[ \frac{\partial^2 v}{\partial x^2} = LC \frac{\partial^2 v}{\partial t^2} \]  

\[ \frac{\partial^2 i}{\partial x^2} = LC \frac{\partial^2 i}{\partial t^2} \]  

Their general solution can be stated as follows [27]:

\[ v(x,t) = F_1(x - Ct) + F_2(x + Ct) \]  

\[ i(x,t) = (1/Z_c)F_1(x - ct) - (1/Z_c)F_2(x + Ct) \]

where \( c \) is the wave velocity

\[ c = \frac{1}{\sqrt{LC}}. \]

\( Z_c \) is the characteristic impedance of the line

\[ Z_c = \sqrt{LC} \]

and \( F_1 \) and \( F_2 \) are functions determined by the initial and boundary conditions of the line. Expressions (2.14a) and (2.14b) can be further combined yielding:

\[ v + Z_c i = 2F_1(x - ct) \]  

\[ v - Z_c i = 2F_2(x + ct) \]

Note that (2.15a) represents a constant wave travelling forwards; that is, in the direction defined as positive. Similarly, (2.15b) represents a constant wave.
Figure 22.- a) Lossless line section. b) Equivalent circuit for a lossless line section. c) Lossy line model.
travelling backwards; or, in the negative direction of the line. The left hand side terms of (2.15a) and (2.15b) are the Riemann invariants of equations (2.13a) and (2.13b) [60]. In the context of power line transient analysis, these two terms sometimes are called characteristics; although in this thesis this term is reserved for those curves of the x-t plane along which a PDE becomes an ODE. The solution of equations (2.13a) and (2.13b), at any point along the lossless line, can thus be expressed as a superposition of these invariants.

Consider now the lossless line section between nodes k and m, of length \( \Delta x \) which is illustrated in fig. 2.2a. According to expressions (2.15a) and (2.15b), the voltage and current at node k are related to their values at node m as follows:

\[
v_k(t) - Zc_i^k(t) = v_m(t - \tau) - Zc_i^m(t - \tau)
\]  
\[\ldots(2.16)\]

where \( \tau \) is the travel time of the line section:

\[\tau = \Delta x/c.\]

Since the right hand side of (2.16) is constituted of past values, it is referred to as the history term and is denoted by \( E_{hist,k} \) [27]:

\[
E_{hist,k} = v_m(t - \tau) - Zc_i^m(t - \tau)
\]  
\[\ldots(2.17a)\]

Relation (2.16) becomes thus:

\[
v_k(t) - Zc_i^k(t) = E_{hist,k}
\]  
\[\ldots(2.17b)\]

In an analogous manner, the following expression is established for node m;

\[
v_m(t) - Zc_i^m(t) = E_{hist,m}
\]  
\[\ldots(2.17c)\]

In this case, the history term \( E_{hist,m} \) is given as follows in terms of the past values of voltage and current at node k:
Expressions (2.17b) and (2.17c) constitute the basic lossless line model of the EMTP [27]. The circuit equivalent to these equations is shown in fig. 2.2b.

Figure 2.2b model can be applied to the simulation of a frequency independent lossy line by considering that the resistive losses are lumped at few points along its length [27]. Figure 2.2c illustrates this method as implemented in the EMTP [67]. Note from the figure that the losses are lumped at three points as follows: one fourth at one end, one half at the middle and one fourth at the other end of the line. The two lossless line sections between these points are represented by fig. 2.2b model. Numerical experiments have shown that fig. 2.2c line representation is accurate inasmuch as $R \cdot \Delta x \ll Z_c$ [67]. In order to meet this condition, a long transmission line can be subdivided into several line sections and each one can be represented by fig. 2.2c model. As an example, consider a 50 km transmission line whose parameters are: $L = 1.25 \text{ mH/km}$, $C = 9.664 \text{ nF/km}$ and $R = 11.74 \text{ \Omega/km}$. Consider that this line is terminated in a 360 $\Omega$ load which corresponds to the characteristic impedance when the resistive losses are neglected. A 1.0 p. u. $[\text{10/90\mu s}]$ double ramp impulse is applied at $x = 0 \text{ kms}$. Figure 2.3 shows this impulse as well as its waveshape after it has travelled 5.17, 10.34, 15.52 and 20.69 kms. One important aspect of frequency independent line models, which is apparent in this figure, is that they do not reproduce the gradual smoothing of a waveform corner as it travels. Figure 2.3 simulations are made with EMTP ©MicroTran by dividing the transmission line into 87 lossy line sections. Essentially the same results are obtained through a finite difference technique presented in chapter 4.

There is one development that permits us to account for losses as well as for frequency dependence effects by means of essentially the same model as for
lossless lines depicted in fig. 2.2b [53]. In this development, $Z_c$ as well as the history terms expressions (2.17a) and (2.17d) are modified. While in the lossless line case the $Z_c$ element consists of a pure resistance, in the frequency dependent case it is a network that approximates the characteristic impedance of the line within a large range of frequencies. As for the history terms, in the frequency dependent case they involve a convolution relationship. Fortunately, this convolution can be performed recursively [37,53].

![Figure 2.3.- Time domain transient simulation with a frequency independent line model.](image)

The abovementioned time domain models presuppose monophasic lines. Their extension to multiconductor lines is customarily performed by applying a modal transformation borrowed from the frequency domain techniques described in the previous section. One problem here is, however, that these transformations are normally complex and introduce imaginary quantities which lack physical meaning in time domain analysis. This problem is usually approached in two ways. One
way is to force the transformation to be real, either by assuming that the line is perfectly balanced or by neglecting the series resistive losses in their calculation. The other way is to calculate the complex transformation matrix at a suitable frequency and to discard its imaginary part [67].

Another problem related to the application of modal transformations in the development of multiconductor line models in the time domain is that these transformations are functions of frequency. In many practical cases, their approximation by a constant matrix provides acceptable results. There are cases, however, in which the frequency dependence effects of the modal transformation matrices have to be considered [81,95]. A time domain model that takes into account these effects has been developed for underground cables [81].

2.4) Preliminary Considerations for the Modelling of Transmission Lines with Corona.

When the voltage of a line conductor reaches a value for which the electric field in the neighborhood is higher than the dielectric strength of the air, many of the air molecules break and an ion layer is formed around this conductor. This is the corona effect whose description is often made by a diagram like the one shown in fig. 2.4, which is known as the Q−v curve. In this figure, segment AB corresponds to the region where Q and v maintain a linear relation. The slope of this segment is the geometric capacitance of the line. The point B where the Q−v relation ceases to be linear is called the corona inception point and $v_c$, the value of the voltage there, is called the corona inception voltage. As the conductor's voltage increases above $v_c$, the ions that accumulate around determine the shape of the segment BC. It is assumed for this segment that the voltage is increased monotonically from $v_c$ to its maximum value $v_{\text{max}}$. Segment CD corresponds to the region where the voltage decreases monotonically from the maximum value $v_{\text{max}}$ towards zero. The slope of this segment is
almost constant and it is usually in practice considered equal to the geometric line capacitance.

Assume that the corona generated spatial charges move in the transversal directions to the line conductor only. These movements do not affect the first of the Telegrapher's equations; that is, (2.2a). The effects on (2.2c), the second of the Telegrapher's equation, are considered next. Let $Q$ represent the line charge per unit length. From the law of conservation of charge [65]:

$$\frac{\partial i}{\partial x} = \frac{\partial Q}{\partial t} \quad (2.18)$$

Assume now that the corona effect can be considered static, as in the previously mentioned $Q-v$ curve. The charge depends thus on the voltage only:

$$Q = F(v) \quad (2.19)$$

On applying the chain rule [16], (2.18) becomes:
By introducing the following definition:

$$C_c = \frac{\partial Q}{\partial v} \quad \text{(2.21)}$$

equation (2.20) takes the same form as (2.2c); the only difference is that the capacitance $C_c$ is now a function of the line voltage. Note from the $Q$–$v$ curve that $Q$ is a double valued function. Its derivative with respect to the voltage should thus be evaluated by considering one branch of this curve at a time.

Equations (2.2a) and (2.20) represent a monophasic line with static corona. These two equations can be combined into a single second order equation by eliminating the current $i$ as follows. On differentiating (2.2a) with respect to $x$:

$$\frac{\partial^2 v}{\partial x^2} = \frac{L}{R} \frac{\partial^2 i}{\partial x \partial t} + R \frac{\partial i}{\partial x} \quad \text{(2.22)}$$

On differentiating (2.20) with respect to $t$:

$$\frac{\partial^2 i}{\partial x \partial t} = \left( \frac{\partial}{\partial v} C_c \left( \frac{\partial v}{\partial t} \right)^2 \right) \frac{\partial Q}{\partial v} \frac{\partial^2 v}{\partial t^2} \quad \text{(2.23)}$$

Now, the terms containing $i$ are eliminated from (2.22) by applying (2.20) and (2.23):

$$\frac{\partial^2 v}{\partial x^2} = R \left( C_c \frac{\partial v}{\partial t} \right)^2 + L \left( \frac{\partial}{\partial v} C_c \frac{\partial v}{\partial t} \right)^2 + L (C_c) \frac{\partial^2 v}{\partial t^2} \quad \text{(2.24)}$$

In practice, this single second order PDE is equivalent to the two equations (2.2a) and (2.20) [45]. Note in this equation the term involving the derivative of the capacitance with respect to the voltage and the square of the derivative of the voltage with respect to time. Due to the fact that the second order derivatives of $v$ occur in (2.24) to the first degree only, this equation is quasilinear. For ease of manipulation, (2.24) is expressed as follows:

$$\frac{\partial^2 v}{\partial x^2} = a \frac{\partial^2 v}{\partial t^2} + b \quad \text{(2.25a)}$$
where a and b are two variables defined as follows:

\[ a = \text{LC}_c \]  
\[ b = \text{R}(\text{C}_c)\frac{\partial^2 v}{\partial t^2} + \text{L}(\frac{\partial}{\partial v}) \text{C}_c\frac{\partial v}{\partial t} \]  

A set of ODE's are derived next from (2.25a).

Let the following definitions be introduced:

\[ p = \frac{\partial v}{\partial x} \]
\[ q = \frac{\partial v}{\partial t} \]
\[ r = \frac{\partial^2 v}{\partial x^2} \]
\[ s = \frac{\partial^2 v}{\partial x \partial t} \]
\[ w = \frac{\partial^2 v}{\partial t^2} \]

In terms of these definitions, equation (2.25a) becomes:

\[ r - a \cdot w - b = 0 \]

The total differentials of p and q are:

\[ dp = r \cdot dx + s \cdot dt \]
\[ dq = s \cdot dx + w \cdot dt \]

By applying (2.27) and (2.28) into (2.26) r and w are eliminated yielding:

\[ \frac{dp}{dx} - s \frac{dt}{dx} - a \frac{dq}{dt} + as \frac{dx}{dt} - b = 0 \]

On multiplying (2.29) by (dt/dx):

\[ -s[(\frac{dt}{dx})^2 - a] + \frac{dp}{dx} \frac{dt}{dx} - a \frac{dq}{dx} - b \frac{dt}{dx} = 0 \]

By imposing the following condition on (2.29):
\[
\left(\frac{dt}{dx}\right)^2 - a = 0 \quad \text{.........(2.31)}
\]

the term containing \( s \) is eliminated:

\[
\frac{dp}{dx} \cdot dt - a \cdot dq - b \cdot dt = 0 \quad \text{.........(2.32)}
\]

Condition (2.31) has two roots, each of which determines an equation hereafter called a characteristic equation. When the roots are real, each of the characteristic equations determines a family of curves in the \( x-t \) plane. Along any of these curves, which are known as characteristic curves or simply as characteristics, PDE (2.25a) is equivalent to the pair of ODE's constituted by (2.32) and by the following one [45]:

\[
dv = p \cdot dx + q \cdot dt \quad \text{.........(2.33)}
\]

This last equation is the total differential of the voltage \( v \) in terms of \( p \) and \( q \).

If in addition to being real, the roots of (2.31) are different, equation (2.25a) is said to be hyperbolic. These two roots would imply that there are always two different characteristics, one from each family, passing through each point of the \( x-t \) plane [45]. In consequence, the characteristics constitute a system of coordinates in which the ODE's (2.32) and (2.33) are equivalent to PDE (2.25a). It follows from (2.25b) and (2.31) that (2.25a) is hyperbolic as long as the corona capacitance defined as \( \partial Q/\partial v \) is positive. As in most studies of wave propagation on lines with corona, this assumption is adopted here. Nevertheless, the possibility of \( \partial Q/\partial v \) taking negative values is reviewed briefly in chapter 4.

From a physical point of view, the characteristics represent trajectories of the \( x-t \) plane along which wave disturbances propagate. The slope of a characteristic corresponds to the inverse of the local speed at which a wave disturbance propagates. The existence of two characteristic speeds at every point \((x,t)\), with equal magnitude and opposite sign, agrees with the fact that wave
propagation occurs in two directions. Consider now a double exponential impulse being injected at the beginning of a transmission line by means of a pure voltage source. This impulse is depicted in fig. 2.5 along with the corresponding positive slope characteristics. Note that the points A, B, C and D marked on this impulse are related to the ones on the Q–v curve of fig. 2.4. Since the line capacitance for segment AB is constant, the characteristics between points A and B' are parallel. As the waveform voltage increases from point B to C, the capacitance increases, the propagation velocity decreases and the characteristics diverge. As soon as the voltage waveform starts decreasing, right after point C, the propagation velocity increases suddenly and the characteristics in the neighborhood of point C' in fig. 2.5 tend to converge.

![Figure 2.5.- Double exponential impulse and characteristics for a transmission line with corona.](image)

A situation that can arise in nonlinear wave modelling is the crossing over of characteristics belonging to the same family. This situation, hereafter called a shock, is illustrated in fig. 2.5. Its occurrence implies that the model being employed doesn't provide a unique response any longer. Even though there are
physical phenomena that admit multivalued solutions, a shock is usually regarded as a breakdown of the used model which calls for further refinement [33]. As for the modelling of transmission lines with corona, it doesn't seem to be clear yet whether or not shock conditions can arise. The finalization of this issue would require a very specialized analysis beyond the scope of this thesis. As an alternative, the approach adopted here is to implement a shock detection mechanism in the developed simulation programs. It is clear, from the above analysis, that a simulation program based on characteristics would facilitate this implementation. In addition, in the event of a shock occurrence the characteristics provide a means to deal with it [13].

The quasilinear form of the above line equations is a direct consequence of the assumption of a static representation of corona. Because of this form, the theory of characteristics can be applied directly to transform these equations into an equivalent system of ODE's. Laboratory experiments indicate, however, that corona is a dynamic phenomenon. This means that, in addition to the voltage, it depends on \( \partial v/\partial t \) and even on higher derivatives of \( v \) with respect to \( t \) [56]:

\[
Q = F \left( v, \frac{\partial v}{\partial t} \right)
\]  
\[\text{(2.34a)}\]

and

\[
Q = F \left( v, \frac{\partial v}{\partial t}, \frac{\partial^2 v}{\partial t^2}, ... \right).
\]  
\[\text{(2.34b)}\]

If any of these two representations are adopted instead of (2.19), the resulting line equations are nonquasilinear. Since these equations can always be converted into a system of first order quasilinear PDE's [15], the theory of characteristics can still be applied to the analysis of transients on lines with dynamic corona.
2.5 Remarks.

An overview of current techniques for the analysis of transients on lines has been presented in this chapter as a background for the development of a method of analysis for transmission lines with corona. It has been determined here that, for this development, the time domain techniques are better suited than the frequency domain ones. Nevertheless, a method based on frequency domain techniques has been considered convenient as a tool for the construction of frequency independent time domain line models.

The analysis of a monophasic line under the assumption that corona can be modelled as a static phenomenon has produced a second order quasilinear PDE. The further application of the theory of characteristics of PDE's has converted this equation into an equivalent system of ODE's. From the numerical point of view, it is more convenient to deal with this equivalent system of ODE's than with the second order PDE directly [45]. In addition, the use of characteristics facilitates the detection and the handling of shocks [13].

Although the preliminary analysis of lines with corona presented here is based on their description by a second order PDE, this can be done also by handling the two Telegrapher's equations as a system of two first order PDE's [79]. An attractive feature of this alternate approach is the possibility of its extension to a system of equations with a larger number of first order PDE's. Such an extension is needed for the analysis and simulation of multiconductor lines as well of lines with dynamic corona.
3. CORONA MODELS FOR LINE TRANSIENT STUDIES.

3.1) Preamble.

The objectives of the research reported in this thesis are to propose, to develop and to implement a model of lines with corona by applying the method of characteristics of partial differential equations theory. The application of such a model requires a representation of the corona effect. To this end, a corona model is chosen from among the many that have been proposed in the specialized literature. It should be mentioned that the study of corona as well as its modelling still require further research.

This chapter discusses some aspects of the physics and modelling of corona as a background for the developments presented in the following chapters. A very general description of the physics of the phenomenon is thus provided in section 3.2. Section 3.3 follows with an overview of some of the most commonly adopted models for the analysis of transients on transmission lines.
Finally, section 3.4 provides the considerations for the selection of a corona model for this work, as well as suggestions for future research aimed at improving the representation of corona on transmission line transient studies.

3.2) Overview of the Phenomenon of Corona.

Corona consists essentially of the ionization of an electrically stressed region in the air or in any other gaseous dielectric that surrounds a conductor at a high voltage. It is generally accepted that this ionization is initiated by the free electrons that are being continuously produced, mostly, by natural radiation. These electrons are accelerated by the presence of an electric field and, as they travel, they undergo multiple collisions against the air molecules. Although the majority of these collisions are elastic, the few inelastic collisions are the ones that cause the ionization.

In an elastic collision the electron transfers some of its kinetic energy to the molecule. An inelastic collision, on the other hand, may result in the molecule being broken into a positive ion and an electron; this depends on the kinetic energy accumulated by the electron. If the molecule is not broken, part of the electron's kinetic energy is absorbed by one of the molecule's atoms, which is now said to be in a high energy state or to be excited. The electron, after this, may continue its travel or become attached to the molecule thus creating a negative ion. Excited atoms will eventually release the extra energy in the form of photons which may reach, later on, the conductor with the negative polarity (that is, the cathode) and extract electrons through the photoelectric effect [31].

The generation of free charges through electron–molecule collisions is considered to be the principal mechanism of ionization. For this reason, it is referred to as the primary ionization process [31,34]. The other mechanisms, which
are grouped as secondary processes, belong to one of the following two categories: 1.— cathodic electron emissions and 2.— ion molecule ionizing collisions. An example of the first category is the previously mentioned electron extraction through photoelectric effect. Other causes of cathodic electron emission are positive ion bombardment, Schottky effect and Malter effect [31].

The quantitative study of the ionization phenomena is performed through the following coefficients: the first ionization coefficient of Townsend, the generalized second ionization coefficient of Townsend and the attachment coefficient. They are denoted, respectively, as \(a\), \(\gamma\) and \(\eta\). The first one is related to the primary ionization process and is defined as the average number of ionizing collisions per unit distance of electron travel. The second one is related to all the secondary processes which, despite their basic differences, can be described by a single generalized coefficient [31]. The third coefficient, the one of attachment, is defined as the inverse of the mean distance travelled by an electron before becoming attached. The three coefficients are functions of the electric field intensity \(E\) and of the pressure \(p\). They are in fact often expressed as functions of \(E/p\) [34,92]. A derived coefficient is the one of effective ionization which is denoted by \(\bar{a}\) and is obtained by subtracting \(\eta\) from \(a\).

Consider now a region in which the electric field is uniform. If primary ionization and electron attachment were the only processes to be considered, the spatial distribution of electrons would be given by the following expression [89,92]:

\[
n = n_0 \exp(\bar{a}x_0) \tag{3.1}
\]

where \(n_0\) is the number of initial electrons at \(x = 0\) and \(n\) is the number of electrons at the distance \(x = x_0\). One can notice from (3.1) that if \(\bar{a}\) is greater than zero, the electron density increases with \(x\). From the definition of \(\bar{a}\), it
follows that for a self-sustained ionization the primary ionization coefficient must be greater than the attachment coefficient. When secondary processes are introduced into the analysis leading to (3.1), the following expression is obtained instead [31]:

\[ n = n_0 \frac{a e^{\alpha x_0} - \eta}{\alpha - \gamma [e^{\alpha x_0} - 1]} \]  

(3.2)

One important aspect of uniform fields is that a self-sustained ionization process leads, almost invariably, to a total breakdown [31]. As a matter of fact, the following breakdown criterion is obtained by equating the denominator of (3.2) to zero:

\[ \gamma [e^{\alpha x_0} - 1] = \alpha \]

Since the corona effect consists of a series of partial breakdowns, rather than a total one, its analysis requires the consideration of a nonuniform electric field. In this case, expressions (3.1) and (3.2) become [92]:

\[ n = n_0 e^{\int_0^{x_0} \alpha(x) dx} \]

(3.3)

and

\[ n = n_0 \frac{1 + \int_0^{x_0} a(x) e^{\int_0^{x_0} \alpha(z') dz'} dx}{1 - \int_0^{x_0} \gamma(x) e^{\int_0^{x_0} \alpha(z') dz'} dx} \]

(3.4)

Practical application of (3.3) and (3.4) to the analysis of corona would still require the following: 1. the introduction of additional spatial dimensions, 2. the determination of the functional dependence of \( \alpha \), \( \eta \) and \( \gamma \) on the electric field and 3. the determination of the spatial distribution of the electric field. Concerning the first point, since the present work focuses on transmission lines, the analysis can be confined to two dimensions. Any pair of coordinates that determines the plane transversal to the line is adequate. Concerning the second point, there are well-established empirical expressions for \( \alpha \) and \( \eta \). However, very
little is known about $\gamma$ [31,92]. As for the third point it can be said that, even without corona, electric field calculations are difficult [47]. Further complications arise with corona because of the presence of spatial charges.

Other important aspects of corona, which highlight its complexity, become apparent in macroscopic observations in the laboratory [34]. They show, for instance, that there are substantial differences between the corona near the anode and the one near the cathode. The former is called positive corona and the latter is called negative corona. Another important difference is the one between the fast occurring phenomenon and the slow occurring one. The first is in the order of microseconds or less, while the second is above this order. Fast positive corona, for instance, produces streamerlike avalanches, while fast negative corona produces featherlike ones [31,34]. These basic differences can be attributed to the direction of electron travel. In the first case, the electrons are being attracted towards the electrode and, in the second one, they are being repelled from it. As for the slow occurring corona, this is a more complicated phenomenon whose general description is provided in the following two paragraphs.

It is often possible to distinguish three stages in positive slow corona [31,34]. The first one starts as soon as the electric field surpasses the corona inception value; that is, the value at which the ionization coefficient becomes bigger than the attachment coefficient. This stage is characterized by the appearance of onset streamers that resemble the ones from fast positive corona. As the field is increased, the second stage commences. It is characterized by the disappearance of the streamers and, in their place, the appearance of a glow region in which the electric field is quasiuniform. Further increases in the field intensity brings the phenomenon into the third stage which is characterized by the appearance of long breakdown streamers and, if the field becomes strong enough,
For slow negative corona, three stages can also be detected [34]. The first one is marked by the appearance of Trichel pulses [31] whose frequency varies between 2 kHz and a few MHz and depends directly on the field intensity. As this intensity increases, the second stage may or may not appear. It is characterized by a glow which can be inhibited by the presence of electronegative gases; that is, of gases to which electrons can attach easily. Oxygen is one of these. The third stage, finally, is characterized by the appearance of negative streamers.

3.3) Corona Models.

It is clear from the previous section that corona is a very complicated phenomenon and that its modelling, on the basis of the actual physical mechanisms, would require a considerable effort. The models proposed so far, for practical applications to transmission lines, are therefore based on simplifying assumptions. The three most important ones are: 1.- The Deutsch assumption, 2.- The Kaptsov assumption and 3.- The Peek empirical formula for the corona inception value of the electric field. The Deutsch assumption states that the spatial charges have an effect on the magnitude of the electric field but not on its direction. The Kaptsov assumption states that the electric field intensity remains at the inception value on the surface of the conductor for as long as the ionization is ongoing. The Peek empirical formula, which provides the corona inception value of the field for cylindrical conductors, can be stated as follows [80]:

\[ E_c = 3 \times 10^6 m \delta (1 + \frac{3}{\sqrt{\delta r_0}}) \]

where \( m \) is a factor that accounts for the irregularities in the conductor's surface and for the polarity, \( \delta \) is the relative air density and \( r_0 \) is the conductor radius.
Peek's formula is used to calculate the corona inception voltage for cylindrical conductors. It is also applied to bundled conductors by taking the equivalent radius of the bundle as \( r_0 \).

Several models of corona have been proposed for line transient simulations. They range from the very simple to the very sophisticated [72]. Much research must still be done, however, to determine the features of the phenomenon that are relevant to transmission line transient analysis, as well as for establishing the degree of sophistication required from its models. A simplistic model, on the one hand, may not allow us to obtain dependable results from the transient simulations. A sophisticated model, on the other, could impose an insurmountable computational burden on the line simulations. The ideal model, thus, should provide the essential aspects of the phenomenon and, at the same time, be computationally efficient. In addition, another desirable feature is that the model can be derived entirely from line specifications.

![Diagram](image)

**Figure 3.1.**— Piecewise linear approximation of the \( Q-v \) curve by using one and two straight line segments from \( v_c \) to \( v_{max} \).
Perhaps the simplest model of corona is the one obtained by adding a constant value to the line capacitance when the line voltage is increasing and beyond the corona inception voltage $v_c$ and, as soon as the voltage starts decreasing or when it is lower than $v_c$, the added value is zero. Since this model is equivalent to the $Q-v$ curve approximation shown in fig. 3.1, it is known as piecewise linear. Note from the figure that this approximation can also include more than one segment between the inception voltage point $v_c$ and the point of maximum voltage $v_{max}$.

A further improvement in the corona representation is obtained by approximating the $Q-v$ curve segment between $v_c$ and $v_{max}$ by a generalized parabola which is, in its most general form, given by the following expression:

$$Q = k_1 + k_2 (1 + \frac{V}{V_c})^{k_3}$$

where $k_1$, $k_2$ and $k_3$ are parameters chosen to match measured $Q-v$ curves and $k_3$ is greater than 1.0. This model is known as parabolic. As an example, consider the model proposed by Inohue [44,65] which uses two parabolic segments between $v_c$ and $v_{max}$. This model unfortunately relies very strongly on measurements. Another parabolic model is the following one proposed by Gary, Cristescu and Dragan [85,93]:

$$Q = \begin{cases} 
C_g v & v \leq v_c, \frac{\partial v}{\partial t} > 0 \\
C_g v_c (\frac{v}{v_c})^{k_3} & v_c < v, \frac{\partial v}{\partial t} > 0 \\
C_g v & \frac{\partial v}{\partial t} \leq 0 
\end{cases} \quad \text{(3.5a)}$$

where $C_g$ is the geometric line capacitance. This model is proposed along with the following empirical formulas for $k_3$ which cover all cases of practical interest. For positive polarity on a single conductor:
\[ k_3 = 0.22r_0 + 1.2 \]  \text{\\ (3.5b)}

where \( r_0 \) is the conductor radius in centimeters as before. For positive polarity on a bundled conductor:

\[ k_3 = 1.52 - 0.15 \log_{10} n \]  \text{\\ (3.5c)}

where \( n \) is the number of subconductors in the bundle. For negative polarity on a single conductor:

\[ k_3 = 0.07r_0 + 1.12 \]  \text{\\ (3.5d)}

For negative polarity on a bundled conductor:

\[ k_3 = 1.28 - 0.08 \log_{10} n \]  \text{\\ (3.5e)}

Note that this model introduces a discontinuous jump in the total line capacitance given by the slope of the \( Q-v \) curve \( \partial Q/\partial v \).

The previously described model has been derived by applying a statistical curve fitting process to the vast amount of data that has been collected through several years of experimental research conducted at the Electricité de France research facilities [85]. Another approach in the development of corona models is to make use of spatial charge considerations. Harrington and Afghahi, for instance, have proposed a model in which the spatial charges are assumed to be concentrated on several hypothetical concentric shells [55]. These authors reported, however, that their algorithmic implementation of the multiple shell model was prone to numeric oscillations. They thus used a single shell for their line simulations [57,58]. A modification to the multiple shell model has been proposed by Semlyen and Huang [70]. Unfortunately, it requires either field or laboratory measurements for the determination of some of its parameters. Two additional models have been proposed more recently, one by Al-Tai et al. [83] and the other by Li, Malik and Zhao [87]. Both of them are based on spatial charge considerations and can be derived entirely from line information.
Often, the proposed corona models provide the spatial charge as a function of the local voltage only. These models are known as static and are equivalent to Q–v curve representations. Since the early days of research on corona it has been proposed, however, that the derivative of the local voltage with respect to time \( \partial v/\partial t \) has also an important influence in the evolution of the phenomenon and, consequently, in the propagation of transient waves on transmission lines. It has even been suggested that higher order derivatives of the voltage with respect to time could also be important [56]. Fairly recent experiments [42,77] indicate that, at least, the effect of \( \partial v/\partial t \) should be considered. The models that account for it are known as dynamic.

One of the most important features of corona, that are influenced by \( \partial v/\partial t \), is the inception voltage. In reality, corona is a probabilistic phenomenon whose initiation requires the presence of at least one free electron inside the region where the electric field has reached its inception value. This causes a time delay between the point at which the voltage (or the field) reaches the inception value and the point at which the ionization process starts. The concept of mean time delay can thus be established by statistical means. This concept provides an explanation to the fact that measured inception voltages tend to be bigger for fast rising impulses than for slow rising ones [87]. Another feature of corona that is affected by the rate of growth of the voltage is the formation of spatial charge. This effect can be attributed to the mobility of the ions which can be neglected only in the fast occurring phenomena.

A dynamic model has been proposed by M. M. Suliciu and I. Suliciu on the basis of the similarities between corona and viscoplastic phenomena [52]. One drawback of this model, as of many others, is the requirement of experimental information. The required information must include, in this case, different q–v...
curves obtained for different rates of rise of voltage in order to convey the
dynamic features of the phenomenon. The model parameters have to be obtained,
in addition, through a fairly complicated identification process whose application
requires a great deal of expertise [68]. A more convenient approach seems to be
the one proposed by Li, Malik and Zhao. In their corona model, the dynamic
features from both the inception voltage and the formation of spatial charge are
accounted for by introducing a time delay parameter [87].

An ambitious model, in which the three assumptions mentioned at the
beginning of the section are removed, has been proposed by Abdel-Salam and his
coworkers [71,84]. This model simulates the principal mechanisms of corona.
Spatial charges are represented by longitudinal lines of charge. Many of these
lines, in the order of the hundreds, are assumed to be distributed around the
conductor throughout the entire ionization space. The conductors are also
represented by lines of charge which are obtained through the well known charge
simulation method [62]. In order to keep the computations within manageable
limits, the model has been implemented for cylindrical symmetry only. Even in
this case the amount of computation is considerable; consequently, its application
to line transient simulation is not yet deemed feasible. It is suggested here,
nevertheless, that this model is invaluable as an aid for laboratory experiments as
well as for providing benchmarks for the more simple corona models.

There are, finally, two important aspects in the modeling of corona for
transient wave propagation that have received scant consideration. They are the
corona effect on multiconductor lines and the behavior of corona during excitation
impulses having more than one peak above the corona threshold. Concerning the
first aspect, it seems that only recently has a first approximation been proposed
[85]. It consists of, first, assuming that the spatial charge is axially symmetrically
distributed around the coronating conductor, then, to form the matrix of potential coefficients of Maxwell and, finally, to invert this matrix to obtain the self and mutual capacitances. An advantage of using the potential coefficients matrix, as an intermediate step in the calculation of the line capacitances, is that the assumed distribution of charge only affects the self potential coefficient corresponding to the coronating conductor. As for the second aspect, Köster and Weck have quoted experimental evidence [49] which leads to the following Q-v representation: after a maximum peak, the effective capacitance takes the line geometric value for as long as successive peaks remain below \( v_{\text{max}} \); that is, the voltage reached by the maximum peak. If this peak is surpassed, the Q-v curve continues along its parabolic segment from the point at which \( v = v_{\text{max}} \).

3.4) Final Considerations and Recommendations.

An overview of the physics of corona and of the techniques to model its effects on transmission lines has been presented in the previous two sections. It is apparent that the two issues are highly specialized. It is also apparent that further progress is required for the successful application of corona models to practical analysis and design. Concerning the research work reported here, a model of corona is needed in the implementation of the proposed transmission line model. This model has been selected from among the ones described in section 3.3. In order to keep the transmission line equations strictly quasilinear, the selection has been restricted to the static models. The parabolic one proposed by Gary, Cristescu and Dragan is the most convenient one for the purposes of the thesis. Firstly, because it has been obtained from experimental data, secondly, because its implementation from some given line information is straightforward and, thirdly, because the supplementary formulas (3.5b) to (3.5e) cover all cases of practical interest.
Future research work on the corona effect and its modelling should put a special emphasis on experimental aspects. In addition, in the opinion of this author, a preliminary stage consisting of computational simulations is highly desirable. A study of sensitivity on the line models, for instance, could help determine the relevance of the different features of corona to propagation phenomena. It could help, in addition, determine the effect of other factors that are unrelated to corona on the measured waves, as for instance, the effect of conductors' sagging.

In addition to the field experiments and to their computer simulation, more studies should be conducted on scaled down transmission lines, as the ones reported in references [75] and [97]. One advantage of scaled down lines is that one can eliminate from them many of the factors that corrupt the corona experiments on actual transmission lines. Wave propagation measurements also provide valuable information about the corona effect itself. Reference [97] provides q-v curves derived from measured travelling waves. A collection of these curves, obtained at different points along the line, would show the effect of the shape of the voltage impulse on the corona characteristic. This effect is very closely related to the dynamic features of the phenomenon of corona. Finally, it is recommended that further laboratory experiments of the corona effect be performed in coordination with studies on an advanced model like the one proposed by Abdel-Salam et al. [84].
4. QUASILINEAR MODELS FOR MONOPHASIC LINES WITH CORONA.

4.1) Preamble.

As in most studies of lines with corona, in this chapter the Telegrapher’s Equations are adopted as a basis of mathematical line models. However, instead of combining them into a single second order PDE as in chapter 2, they will be treated as a 2x2 system of first order quasilinear equations. This approach has the advantage that the techniques developed can be extended to PDE systems of higher dimensions. These extensions are needed to deal with multiconductor lines and with nonlinearities that do not belong to the quasilinear category [15].

The Telegrapher’s Equations do not take into account the frequency dependence effects caused by the Skin Effect of the line conductors and of the ground plane. Whereas this is acceptable for most studies of lightning and arcing impulses, where the line sections involved are usually less than 10 km long [94], more general situations may require that frequency dependence be accounted for.
This may be done through numeric convolutions [29,56] which, in order to
decrease the computational burden of the models, should be performed recursively
[37,53].

4.2) Systems of Equations for Monophasic Transmission Lines.

Equations 2.2a and 2.2b can be put in the form of a 2x2 system of PDEs:

\[ \frac{\partial}{\partial t} U + \frac{\partial}{\partial x} A U + BU = 0, \]  \hspace{1cm} (4.1)

where:

\[ U = \begin{bmatrix} \nu \\ i \end{bmatrix}, \]  \hspace{1cm} (4.2a)

\[ A = \begin{bmatrix} 0 & 1/C \\ 1/L & 0 \end{bmatrix}, \]  \hspace{1cm} (4.2b)

and

\[ B = \begin{bmatrix} 0 & 0 \\ 0 & R/L \end{bmatrix}. \]  \hspace{1cm} (4.2c)

By assuming a static model of corona, where the capacitance "C" is a function
of the voltage "v" only, equation 4.1 becomes quasilinear. The two eigenvalues of
A are:

\[ \lambda_1 = +i\sqrt{\frac{1}{LC}} \]  \hspace{1cm} (4.3a)

and

\[ \lambda_2 = -i\sqrt{\frac{1}{LC}} \]  \hspace{1cm} (4.3b)

Since \( \lambda_1 \) and \( \lambda_2 \) are distinct and real, (4.1) is a strictly hyperbolic system [60].
Associated with the eigenvalues are the following matrices of right and left eigenvectors, respectively:

\[ E_r = \begin{bmatrix} 1 & 1 \\ \frac{1}{\sqrt{C/L}} & -\frac{1}{\sqrt{C/L}} \end{bmatrix} \] ..................................(4.4)

and

\[ E_\lambda = \begin{bmatrix} 1 & \sqrt{L/C} \\ 1 & -\sqrt{L/C} \end{bmatrix} \] ..................................(4.5)

\( \lambda_1 \) and \( \lambda_2 \) are equal to the characteristic directions obtained in chapter 2 for the second order PDE. They thus correspond to the wavefront velocities. Since the second row elements of \( E_r \) have the dimensions of an admittance and the ones in the second column of \( E_\lambda \) have those of an impedance, the following two definitions are introduced:

\[ Y_W = \sqrt{C/L} \] ..................................(4.6a)

\[ Z_W = \sqrt{L/C} \] ..................................(4.6b)

Notice that \( L \) and \( C \) are evaluated at the frequency considered representative of the study in which equation 4.1 is to be applied. \( Z_W \) differs, thus, from the surge impedance as defined in [22], for instance. As it also differs from the characteristic impedance, as defined elsewhere, it will be called here "the wave impedance". Similarly, \( Y_W \) will be called "the wave admittance".

Equation (4.1) is turned next into an equivalent 2x2 system of ordinary differential equations (ODE's). First, let (4.1) be multiplied by the first row of \( E_\lambda \):
\[ \frac{\partial v}{\partial t} + Z \frac{\partial i}{\partial t} + \lambda_1 \frac{\partial v}{\partial x} + Zw \frac{\partial i}{\partial x} + \lambda_1 Ri = 0. \]

Then, let the terms in the same dependent variables be grouped:

\[ (\frac{\partial v}{\partial t} + \lambda_1 \frac{\partial v}{\partial x}) + Zw(\frac{\partial i}{\partial t} + \lambda_1 \frac{\partial i}{\partial x}) + \lambda_1 Ri = 0. \]

Finally, by restricting this equation to the curves of the \(x-t\) plane defined by

\[ \frac{dx}{dt} = \lambda_1, \] (4.7)

the following ODE is obtained:

\[ \frac{dv}{dt} + Zw \frac{di}{dt} + Ri \frac{dx}{dt} = 0. \] (4.8)

A second ODE:

\[ \frac{dv}{dt} - Zw \frac{di}{dt} + Ri \frac{dx}{dt} = 0. \] (4.9)

is obtained by multiplying (4.1) by the second row of \(E\) and by restricting it to the curves in the \(x-t\) plane defined by the following expression:

\[ \frac{dx}{dt} = \lambda_2. \] (4.10)

The two families of curves defined by (4.7) and (4.10) are the characteristic curves or simply, characteristics of (4.1). As the strict hyperbolicity of (4.1) guarantees that two characteristics pass through each point of the \(x-t\) plane, these curves can be taken as a new coordinate system where (4.8) and (4.9) form a 2x2 system equivalent to (4.1) [15]. The main advantage of this new formulation lies in the availability of powerful techniques to deal with systems of ODE's.

In the lossless case
and equation 4.1 is said to be homogeneous. Its solution leads to the Riemann invariants:

\[ v + Z_w i = k_1 \]

and

\[ v - Z_w i = k_2, \]

where \( k_1 \) and \( k_2 \) are two constants. These are the invariants encountered in section 2.3. A numerical method of approximating the solution of the lossy line equations that removes assumption 4.11 is presented in the next section. The method is extended later, in section 4.4, to the quasilinear case for the simulation of transmission lines with corona.

4.3) Monophasic Line Linear Model Based on Characteristics.

Consider the terminated transmission line depicted in figure 4.1a and the corresponding \( x-t \) plane of coordinates of fig. 4.1b, in which the boundaries of the wave propagation problem have been represented. It is assumed here that the solution has been determined at the initial boundary line (i.e., at \( t = 0 \) in fig. 4.1b) from the initial conditions of the transmission line. This initial solution can be extended to neighboring points of the initial boundary through the techniques described as follows and, furthermore, to any point of the region delimited by the additional boundaries at \( x = 0 \) and at \( x = l \) of fig. 4.1b by their consecutive application. In addition to the usual assumptions of time and distance invariance of the line parameters, the one of linearity is made in this section. It thus follows that the characteristics are straight lines. A further implication of linearity is that the characteristics can be obtained beforehand, since the integration of (4.7) and (4.10) is independent from that of (4.8) and (4.9).
Figure 4.1. - Transmission line initial-boundary problem, a) Scheme of a terminated transmission line, b) $x$-$t$ plane with the line problem boundaries.

Figure 4.2. - Extension of the solution from points D and E to F.
Let D and E be two points of the x-t plane where the solution is known. They are represented in fig. 4.2, along with their characteristics whose intersections define two additional points F and F'. Only F however, in the direction of the t-coordinate increase, is of interest for wave propagation analysis. Let \((X, T)\) and \((X+\Delta x, T)\) be the respective coordinates of D and E. Thus, from fig. 4.2, the coordinates for F are:

\[(X+\Delta x/2, T+\Delta t/2),\]

where:

\[\Delta x/\Delta t = \lambda\] .......(4.12)

Wave behavior between points D and F is described by equation 4.8, which can be approximated as follows:

\[(V_f - V_d) + Z_w(I_f - I_d) + \frac{R\Delta x}{4}(I_f + I_d) = 0,\] .......(4.13)

where \(V_d\) and \(I_d\) are the values of voltage and current at point D, and \(V_f\) and \(I_f\) are the corresponding ones at point F. Note that, in the third term of (4.8), the current is approximated by the average of its values at the end points D and F. In the same way, (4.9) is approximated between E and F by the following expression:

\[(V_f - V_e) - Z_w(I_f - I_e) - \frac{R\Delta x}{4}(I_e + I_f) = 0,\] .......(4.14)

where \(V_e\) and \(I_e\) are the respective values of voltage and current at point E. Equations 4.13 and 4.14 can be rearranged, respectively, in the following form:

\[V_f + \left(Z_w + \frac{R\Delta x}{4}\right)I_f = V_d + \left(Z_w - \frac{R\Delta x}{4}\right)I_d\] .......(4.15)

and
Further manipulation of (4.15) and (4.16) yields the following formulas:

\[ V_f = \left(\frac{V_d + V_e}{2}\right) + \left(\frac{I_d - I_e}{2}\right)Z_2 \]  \hspace{1cm} (4.17)

\[ I_f = \left(\frac{V_d - V_e}{2Z_1}\right) + \left(\frac{I_d + I_e}{2}\right)Z_2/2Z_1 \]  \hspace{1cm} (4.18)

where

\[ Z_1 = Z_W + \frac{R \Delta x}{4} \]  \hspace{1cm} (4.19)

and

\[ Z_2 = Z_W - \frac{R \Delta x}{4} \]  \hspace{1cm} (4.20)

Note that expressions 4.15 and 4.16 lead to the EMTP lossy line model depicted in figure 2.2c which was obtained by intuitive and empirical means [27,67]. Expressions 4.15 and 4.16 provide, thus, an additional justification for it. Expressions 4.17 and 4.18, on the other hand, provide \( V_f \) and \( I_f \) explicitly in terms of known values. They are therefore adopted here for algorithmic computations. Even though the points D and E of figure 4.2 have been selected with the same \( t \)-coordinate, the solution extension can be performed with any pair of solved points, provided they are not along the same characteristic. Extensions are also possible for points on any of the two additional boundary lines at \( x = 0 \) and at \( x = 1 \). Each of these points, however, has to be at the intersection of the boundary with a characteristic that passes through a solved point. The characteristic provides one equation and the other one, required for solving the two unknowns of voltage and current, is derived from the boundary conditions. The technique is illustrated next through two elementary examples.
Consider a point S at the intersection of the boundary at \( x = 0 \) and a characteristic passing through the solved point E as in fig. 4.3a. Let the coordinates of E be \((\Delta x/2, T)\). The ones for S are thus \((0, T+\Delta t/2)\) and (4.9) is approximated as follows:

\[
I_S = \frac{(V_S - V_e + I_e Z_2)}{Z_1},
\]

where \( Z_1 \) and \( Z_2 \) are as in (4.19) and (4.20). If the boundary condition is that of an ideal voltage source whose waveform is a function "f" of time only, the second equation is:

\[
V_S = f(T+\Delta t/2)
\]

and point S is entirely determined.

Consider now the point L of fig. 4.3b at the intersection of the boundary line at \( x = l \) and a characteristic passing through a solved point D whose coordinates are \((l-\Delta x/2, T)\). The approximation of (4.8) yields:

\[
V_1 + Z_1 I_1 = V_d + Z_2 I_d.
\]

If the boundary condition is that of a pure resistive load \( R_l \), \( V_1 \) and \( I_1 \) are related as in the following expression:

\[
V_1 = R_l \cdot I_1
\]

which, along with (4.21), determines point L solution.

In order to apply the above extension techniques to transient simulation of a line, like the one of fig. 4.1a, a mesh of characteristics has to be constructed in the \( x-t \) plane. The construction is as follows: first, the transmission line is divided into equal length sections by a number of points. For each of them, there
Figure 4.3.— Extension of the solution to boundary points. a) From a known point E to a point S on the source boundary. b) From a known point D to a point L on the load boundary.

Figure 4.4.— a) and b), two possible meshes of characteristics for linear transient simulation.
is another corresponding point, hereafter called the initial value point, on the boundary line of initial values of the \(x-t\) plane. This is illustrated in figure 4.4a. Next, the characteristics of each initial value point are plotted in the direction of the \(t\)-coordinate increase and, as soon as one of them intersects one of the additional boundaries (either the one at \(t = 0\) or the one at \(t = 1\)) the plot is continued along the second characteristic of the intersection point. Finally, the plot is continued until it spans the observation time required for the simulation. Figures 4.4a and 4.4b show two possible mesh constructions. The one shown in fig. 4.4b was adopted here, since it is equivalent to the one required for nonlinear transient simulations.

Figure 4.5 shows the simulation of a 10/90 \(\mu\)s double ramp impulse traveling along a 50 km long transmission line which is terminated in a 360 \(\Omega\) load. The line parameters are \(L = 1.25\) mH/km, \(C = 9.664\) nF/km and \(R = 11.74\) \(\Omega\)/km. The line was divided for the simulation into 174 sections. The plots correspond to the travelling wave obtained at 0, 5.17, 10.34, 15.52 and 20.69 kms distance from the source. The same results were obtained with EMTP MicroTran (see fig. 2.3). Two of the main features of frequency independent models of lossy lines are apparent in fig. 4.5. The first is that the corner of the double ramp is preserved. The second is that the width of the pulse broadens as it travels along the line.

Concerning the simulation of linear transients by the method of characteristics, there are three issues that should be addressed; namely, convergence, accuracy and number of line subdivisions. The three of them are inter-related. The convergence of discrete approximations of linear hyperbolic PDEs is guaranteed by the Courant–Friederichs–Hilbert (CFL) condition [3] which,
for equation 4.1, can be written as follows:

\[
\frac{\Delta x}{\Delta t} \leq \frac{1}{\sqrt{LC}}.\]

Note that expression 4.12 is in fact the equal sign option of (4.22). It thus follows that the method of characteristics fulfills implicitly the CFL condition. Regarding the accuracy, it has been shown [27] that the fig. 2.3 model is an accurate representation of a lossy line inasmuch as:

\[R\Delta x \ll Z.\]  \hspace{1cm} (4.23)

It has been shown through numerical experiments [67] that when the value of \(Z\) is ten times larger than that of \(R\Delta x\), the accuracy is acceptable for engineering applications. Furthermore, it is always possible to meet condition 4.23 by increasing the number of line subdivisions. In this respect, it can also be said that (4.23) determines a lower bound for the number of line subdivisions. Another factor to consider is the specification of a maximum sampling interval \(\Delta t\), since it determines through (4.12) the maximum length \(\Delta x\) of the line sections and, therefore, the minimum number of subdivisions that are required.

4.4) Quasilinear Model for a Line with Static Corona.

When the line capacitance is a function of voltage, the eigenvalues of \(A\) in (4.1) are also functions of the voltage and the characteristics become curved lines in the \(x-t\) plane. In consequence, the integration of (4.7) and (4.10) cannot be done independently from that of (4.8) and (4.9). The extension of solutions to neighboring points can be done, nevertheless, through iterations. Two techniques have been proposed to do this [14]. In one of them, for each pair of solved points that are not on the same characteristic, each iteration yields a better approximation to the coordinates and the solution of the crossing point of the
Figure 4.5.— Simulation of a linear double ramp traveling wave.

Figure 4.6.— a) Regular grid for nonlinear line calculation. b) Finding the interpolation points.
characteristics in the $t$-coordinate direction. Since the characteristics are curved, it
is clear that this technique produces an irregularly distributed grid of solved
points. In the other technique [9], a regular grid of points is chosen beforehand.
The solution points are then forced to coincide with the nodes of the regular
grid by means of interpolations. The first technique, hereafter called "pure
characteristics", is a complement to analytic studies. The latter one, hereafter
called "characteristics with interpolation", is more suitable for engineering
applications and, for this reason, it is adopted in this section.

Consider a regular grid as shown in fig. 4.6a. Usually in practice, the time
step $\Delta t$ is determined by the minimum time resolution needed for an adequate
description of the important features, such as rise time, of the particular
waveform excitation under study. From this value, the distance step $\Delta x$ is fixed in
such a way that the ratio $\Delta x/\Delta t$ is at least equal to the largest possible $A$
eigenvalue or wave speed. Suppose now that the solution is known at the node
points on the line $t = T$ of fig. 4.6a. Let D, E and F be three of these points,
whose respective values of voltage and current $V_d, I_d, V_e, I_e, V_f$ and $I_f$ are to
be used in determining the point G solution; that is, $V_g$ and $I_g$. Let the
characteristics be approximated, between lines $t = 0, t = \Delta t, t = 2\Delta t$, etc., by
straight segments. Let $\lambda_d, \lambda_e, \lambda_f$ and $\lambda_g$ represent the positive eigenvalues of $A$
at points D, E, F and G. The following steps are then applied:

1. Assume $\lambda_e$ initially as the eigenvalue of $A$ at G. Let $\Delta x_{d''}^e$ be the distance
from point $D''$, where the G-characteristic with positive slope intersects the
line $t = T$, to point E and $\Delta x_{ef''}$ the distance from point E to point F'',
where the G-characteristic with negative slope intersects $t = T$. Their values
are calculated as follows (see fig. 4.6b):

$$\Delta x_{d''}^e = \lambda_g \Delta T$$
2. Obtain first order estimates of current and voltage at points D" and F" through the following linear interpolation expressions:

\[ V_{d''} = V_{d}k_1 + V_e(1-k_1) \]
\[ I_{d''} = I_{d}k_1 + I_e(1-k_1) \]
\[ V_{f''} = V_{f}k_1 + V_e(1-k_1) \]
\[ I_{f''} = I_{f}k_1 + I_e(1-k_1) \]

where \( k_1 = \Delta x_{d''}e / \Delta x \).

3. Obtain a first order estimate of the voltage and current of point G, from the following expressions:

\[ V_g - V_{d''} + Z_{Wd''}(I_g - I_{d''}) + (I_g + I_{d''})R\Delta x_{d''}e/2 = 0 \]
\[ V_g - V_{f''} + Z_{Wf''}(I_g - I_{f''}) + (I_g + I_{f''})R\Delta x_{ef''}/2 = 0 \]

where \( Z_{Wd''} \) and \( Z_{Wf''} \) are the wave impedances evaluated at D" and F".

4. Obtain from \( V_g \), the eigenvalue \( \lambda_g \) of A at point G. Obtain two new distances \( \Delta x_{d''}e \) and \( \Delta x_{ef''} \) from the following expressions:

\[ \Delta x_{d''}e = \Delta T(\lambda_g + \lambda_{d''})/2 \]
\[ \Delta x_{ef''} = \Delta T(\lambda_g + \lambda_{f''})/2 \]

5. Obtain second order estimates of \( V_{d''}, \ I_{d''}, \ V_{f''} \) and \( I_{f''} \) through the following quadratic interpolation expressions [32]:

\[ V_{d''} = V_e - k_1(V_f - V_d) + k_2(V_d + V_f - 2V_e) \]
\[ I_{d''} = I_e - k_1(I_f - I_d) + k_2(I_d + I_f - 2I_e) \]
\[ V_{f''} = V_e + k_3(V_f - V_d) + k_4(V_d + V_f - 2V_e) \]
\[ I_f'' = I_e + k_3(I_f-I_d) + k_4(I_d+I_f-2I_e) \]

where \( k_1 = \Delta x_d''/\Delta x \), \( k_2 = (k_1)^2/2 \), \( k_3 = \Delta x_{ef''}/\Delta x \) and \( k_4 = (k_3)^2/2 \).

6. Obtain a second order estimate of \( V_g \) and \( I_g \) from the expressions:

\[
V_g - V_d'' + (I_g-I_d'')(Z_{W_d''}+Z_{W_g})/2 + (I_g+I_d'')R\Delta x_d''e/2 = 0
\]

\[
V_g - V_f'' + (I_g-I_f'')(Z_{W_f''}+Z_{W_g})/2 + (I_g+I_f'')R\Delta x_{ef''}/2 = 0
\]

7. Repeat steps 4 to 6, until the desired convergence is attained. The convergence criterion used here is that the relative difference between the new value of \( V_g \) and the one from the previous iteration is equal or less than \( 10^{-6} \).

The extension of the solution to boundary points is similar to the previously described procedure. In much the same way as in the linear algorithm, a general boundary condition is described by a difference equation which is incorporated into the iterative process along with the approximation of the corresponding propagation equation. Very complex boundary conditions can be handled by combining the above described procedure code with a general time domain transient simulation package, such as the EMTP.

As an example, the iteration method is applied to the transmission line depicted in figure 4.7a, whose linear parameters are \( C_0 = 9.664 \, \text{nF/km}, \) \( L = 1.25 \, \text{mH/km} \) and \( R = 11.743 \, \Omega/km \). The model used for corona is the static one proposed by Gary, et. al. in [85] and given by equations 3.10 and 3.11. For a conductor radius \( r = 2.54 \, \text{cm} \), expression 3.11 yields a value of \( \eta = 1.7588 \).

Figure 4.7b shows the distortion of a 2/20 \( \mu \)s double ramp impulse with a 1.0 p. u. initial amplitude as it propagates. The corona inception voltage \( v_0 \) has been taken at 0.3 p. u. The plotted waveforms correspond to \( x = 0 \) (the source
Figure 4.7.— Application of the method of characteristics with interpolations. a) Transmission line diagram, b) Distortion of a linear double ramp travelling along the line depicted in fig 4.7a with corona, c) Partial view of the map of characteristics.
side), \( x = 571 \text{ m}, x = 1142.9 \text{ m}, x = 1714.3 \text{ m} \) and \( x = 2285.7 \text{ m} \). The line has been made long enough, \( x = 9 \text{ km} \), so that the reflections from the load side do not play any role in the simulation during the observation time \( t = 30 \mu \text{s} \). Figure 4.7c shows a portion of the map of characteristics corresponding to fig. 4.7b. The same simulation is made for a shorter line length (\( l = 3 \text{ km} \)) and a load impedance equal to the surge impedance:

\[
Z_s = \sqrt{\frac{L}{C_0}}
\]

\[
= 360.0 \ \Omega
\]

The resulting waveforms are shown in figure 4.8 where one can observe that this load termination produces some reflection.

One of the important differences between linear and nonlinear lines is illustrated by fig. 4.9, where the same parameters as for the simulation of fig. 4.7b have been assumed. The line is made long enough to exclude the reflections from the load end during the simulation time and the travelling waveform at \( x = 3 \text{ km} \) is obtained and plotted in the figure. If the line was open at the measuring point, instead, one would expect from experiences with linear lines that the previous waveform would double. Figure 4.9 presents both, the simulation result and the double of the original waveform. One can notice there that the former wave is smaller than the latter one.

4.5) Performance of the Proposed Quasilinear Method.

A problem with many of the methods of discretization of the line equations is that the chosen step lengths, \( \Delta X \) and \( \Delta T \), provoke numeric oscillations [23] which could result in a large ripple at the tail of the simulated propagating wave. Such methods, therefore, are not adequate for studying phenomena that involve reflections. With the method proposed here the numeric
Figure 4.8.— Simulation of a double ramp on a line terminated in its linear surge impedance.

Figure 4.9.— Transient wave at 3 km distance from the source for both, open-ended and semi-infinite line. The double of the semi-infinite line response is included as reference.
oscillations are minimized. It can be, in fact, observed that figs. 4.7b, 4.8 and 4.9 are practically free of them. Two additional simulations are made for the line of fig. 4.7a with a line length of \( I = 3 \text{ km} \). The first one with the load side open ended and the second one with a shortcircuited load. These terminations are chosen in order to produce large reflections as a test for the proposed method. Figure 4.10a shows the results of the first simulation. One can notice there the reflected wave with the same polarity as the incident wave which is characteristic of open ended lines. Figure 4.10b shows, on the other hand, the reflected wave with polarity opposite to the one of the incident wave due to the shortcircuited termination.

In order to show the effect of the discretization step length on the accuracy of the method of characteristics with interpolation, fig. 4.10a simulation is repeated with different numbers \( N \) of line sections ranging from \( N = 12 \) to \( N = 270 \). Figures 4.11, 4.12 and 4.13 show the respective results for 270, 18 and 12 line sections. These plots should be compared among themselves and with fig. 4.10a plot which was obtained with 105 line sections. Figures 4.10a and 4.11 are practically identical. They both predict almost the same magnitudes for the overvoltage peaks and the wavefront delays. The finer resolution of the latter figure is apparent only after a detailed examination; see for example the second peak of the waveform obtained at the distance \( x = 571 \text{ m} \) from the source side. The comparison of figs. 4.12 and 4.13 with fig. 4.10a shows that even the coarsest discretizations yield acceptable results. This is remarkable, especially for fig. 4.13 simulation, where the time step \( \Delta T = 0.8689 \mu s \) implies that the initial wavefront is described by two samples only. The corresponding distance interval of \( \Delta X = 250 \text{ m} \) contrasts favorably with the more common figure of \( \Delta X = 10 \text{ m} \) that is normally required by other methods in similar simulation conditions [65].
Figure 4.10.— Simulations involving reflections.
a) Open ended line, b) Short circuited line end.
Figure 4.11.- Open ended line transient simulation with 270 line segments.

Figure 4.12.- Open ended line transient simulation with 18 line segments.
The abovementioned features of the proposed method can be attributed in part to its iterative nature. The number of iterations that are required in the nonlinear part of the wavefront is usually three and a maximum of four has been detected. The first implementation showed convergence difficulties at the points where the corona capacitance model had a discontinuous jump; that is, at the point at which the wavefront voltage reaches the corona inception voltage and at the wavecrest where \( \frac{\partial v}{\partial t} \) changes sign. The largest differences found at these points were below 1%. All the convergence difficulties at the crest and most of them at the corona inception point disappeared when the jumps were replaced by fast but continuous transitions. They were implemented by performing linear or quadratic interpolations inside a distance interval \( \Delta X \). This approach has the added advantage that finer discretizations automatically provide better approximations of discontinuities. As for the physical possibility of discontinuous jumps of the capacitance, this is an issue that requires further consideration. More about it is mentioned in the next section.

The possibility of a shock condition developing at the crest of a wave on a line with corona has been a concern throughout the research reported here. Nevertheless, such a condition has not been detected so far. The map of characteristics of fig. 4.7c, for instance, shows that some of these curves become closer as the time increases. The effect is more evident in fig. 4.14 which is a continuation of fig. 4.7c. Although it seems there that some characteristics are merging, a detailed review of the numerical data shows that they only approach each other asymptotically.

Figure 4.15a shows the results of one of the field tests performed by Wagner, Gross and Lloyd [11]. To establish a comparison between the actual experiment and a simulation, a 1.3/6.2 \( \mu \)s double exponential impulse with an
Figure 4.13.- Open ended line transient simulation with 12 line segments.

Figure 4.14.- Map of characteristics in which some of the curves belonging to the same family seem to merge.
amplitude $V = 1560 \text{ kV}$ is considered. This excitation is injected at the point $x = 0$ of a line whose parameters are assumed to be the following: $L = 1.73 \mu\text{H/km}$, $C = 7.8 \text{ pF/km}$ and $R = 11.35 \Omega/\text{km}$. The parameters correspond to a conductor radius $r = 2.54 \text{ cm}$, a conductor mean height above the ground $h = 18.9 \text{ m}$, an earth resistivity $\rho = 30 \Omega\text{m}$ and a frequency $f = 192.3 \text{ kHz}$. The frequency is chosen as the inverse of four times the time to peak of the excitation impulse. The excitation impulse is plotted in fig. 4.15b, along with the simulated travelling waves at $x = 655.6 \text{ m}$, $x = 1311.25 \text{ m}$ and $x = 2185.4 \text{ m}$. These waves compare well with the experimental curves. It can be observed that the model reproduces the transfer of energy from the wavefront to the wavetail. There are noticeable differences between the two sets of curves at the nonlinear portion of the wavefront. They are attributed here mostly to the following factors:

1. In the simulation corona is represented by a static model instead of by a dynamic model.
2. In the real line the conductor height varies between towers from $15.24 \text{ m}$ to $26.2 \text{ m}$. In addition to the line impedance variations, the corona inception voltage is affected too.
3. The real line is a polyphasic system consisting of two horizontal three phase lines in parallel and with their respective ground wires, whereas the line model is monophasic.
4. The earth resistivity is an estimate only.

4.6) Observations and remarks.

It is shown in this chapter that the method of characteristics of the partial differential equations theory is well suited for the analysis and simulation of transients on transmission lines with corona. The implementation presented here
Figure 4.15.— Comparison between measurements and a computer simulation. a) Measurements performed by Wagner, Gross and Lloyd and reported in ref. [11]. b) Simulation of Wagner, Gross and Lloyd's experiment.
solves some of the problems found with other methods; namely: numerical oscillations, very fine discretization requirements and not predicting well the evolution of travelling wave tails. The advantages, however, come at the price of an increase in the number of computations. If the average number of iterations required is three, one could say that the proposed method takes four times the number of calculations required by a non-iterative one (one time for the first estimate plus one for each iteration). This is by considering that both methods have the same discretization level. The computational increase, nevertheless, is compensated for by the ability of the method to perform well with much coarser discretizations. In addition to the aforesaid features of the proposed method, it should be said that its major advantage perhaps is the possibility of extensions in a rigorous manner for the handling of dynamic corona and of multiconductor lines.

The examples of the chapter suggest at least two points to consider in the elaboration of further models of the corona effect. The first one is concerned with the discontinuous jumps of the line capacitance implied by several corona models. The second one is concerned with travelling waves with multiple crests above the corona inception voltage.

From a physical point of view, a discontinuous jump of the capacitance involves an instantaneous transfer of energy. At the corona inception voltage, it may be possible that the actual capacitance makes a sharp but continuous transition. See for instance fig. 4.16a which was obtained experimentally by Köster and Weck [49]. As for the wavecrest, a Q-v curve like the one illustrated in fig. 4.16b would be required to keep the transition continuous. It is interesting to note that such a characteristic has been reported in some experimental studies [12,97]. A rounded corner, on the other hand, like the one depicted in fig. 4.16c
would introduce a negative value of $\frac{\partial q}{\partial v}$ in addition to the jump. This negative value would make the line equations elliptic which implies that, if they still have physical meaning, they do not describe wave propagation anymore.

The analysis of propagation of waves with more than one peak above the corona inception voltage requires the corona model to be able to represent the physical phenomena ongoing between peaks. In this chapter, the simulations that involved reflections were made by assuming the geometric capacitance as the corona capacitance at the intermediate voltages below a prior maximum peak value. This assumption is suggested by experimental observations made by Köster and Weck [49].

From all the above, it is apparent that further progress in the subject requires more experimentation aimed at the determination of the features of the corona effect that are relevant to wave propagation, especially to the dynamic ones. Meanwhile, the author considers that the method proposed here is a valuable tool for the analysis of data from experimental lines.
Figure 4.16.— a) Corona capacitance obtained experimentally by Köster and Weck and reported in ref. [42]. b) Q–v curve form for a continuously varying corona capacitance. c) Rounded form of the q–v characteristic. The slope between P₁ and P₂ is negative.
5. LINEAR AND QUASILINEAR ANALYSIS OF MULTICONDUCTOR TRANSMISSION LINES.

5.1) Multiconductor Linear Lines.

The following equations describe a linear multiconductor transmission line under the assumption of frequency independent parameters [17,18]:

\[
\frac{\partial V}{\partial x} = L \frac{\partial I}{\partial t} + RI \\
\frac{\partial I}{\partial x} = C \frac{\partial V}{\partial t}
\]

\[(5.1a) \quad \quad \quad (5.1b)\]

where \( L \), \( R \) and \( C \) are, respectively, the matrix of series inductances, the matrix of series resistances and the matrix of shunt capacitances, all in per unit length, and \( V \) and \( I \) are the vectors (or column matrices) of voltages and currents. If the line has \( n \) conductors, the order of \( L \), \( R \) and \( C \) is \( nxn \).

Let (5.1a) and (5.1b) be written in the following form:
which constitutes a system of 2n first order partial differential equations (PDE's) with \( x \) and \( t \) as independent variables. By definition, (5.2) is hyperbolic if the matrix

\[
A = \begin{bmatrix}
0 & C^{-1} \\
L^{-1} & 0
\end{bmatrix}
\]

is algebraically similar to a diagonal matrix whose nonzero elements are real [60]. In order to show that this is indeed the case, the inverse of (5.3)

\[
A^{-1} = \begin{bmatrix}
0 & L \\
C & 0
\end{bmatrix}
\]

is analyzed as follows. It will also be shown that the eigenvalues and eigenvectors of (5.3) and (5.4) are related to the ones of the LC matrix product.

Let \( T_V \) be the matrix that diagonalizes the LC product in the following form [69]

\[
T_V^{-1} \text{LC} \ T_V = \Lambda
\]

According to the conventional modal theory of transmission lines [19], \( T_V \) corresponds to the matrix whose columns are the voltage modes of a transmission line without resistive losses. The nonzero elements of the diagonal matrix \( \Lambda \) are the inverses of the squared speeds of the corresponding modes [18]. The physics thus requires the diagonal elements of \( \Lambda \) to be real and positive. Transmission line matrices \( L \) and \( C \) are real and symmetric. Since it is assumed here that they are positive definite, the LC eigenvalue problem

\[
\text{LC} \ V = \lambda V
\]
is equivalent to the generalized eigenvalue one

$$\mathbf{C} \mathbf{V} = \lambda \mathbf{L}^{-1} \mathbf{V}$$

which is known to have real eigenvalues [25]. In practice, as the diagonal elements of \( \mathbf{LC} \) are positive and much bigger than the off-diagonal ones, the Gershgorin theorem [82] can be used to show on a case by case basis that the eigenvalues are positive.

Another important matrix is \( \mathbf{T}_i \), the one of column current modes, which diagonalizes the \( \mathbf{CL} \) product; that is:

$$\mathbf{T}_i^{-1} \mathbf{CL} \mathbf{T}_i = \mathbf{\Lambda} \quad \text{(5.6)}$$

\( \mathbf{T}_v \) and \( \mathbf{T}_i \) are related as follows [19]:

$$\mathbf{T}_v^T = \mathbf{T}_i^{-1} \quad \text{(5.7)}$$

Two additional matrices from conventional modal theory are still required. They are denoted here by \( \mathbf{L}' \) and \( \mathbf{C}' \) and arise as a direct consequence of the diagonalization of \( \mathbf{LC} \) product. From (5.5),

$$\mathbf{T}_v^{-1} \mathbf{L} (\mathbf{T}_i \mathbf{T}_i^{-1}) \mathbf{C} \mathbf{T}_v = \mathbf{\Lambda} \quad \text{(5.8)}$$

By defining:

$$\mathbf{L}' = \mathbf{T}_v^{-1} \mathbf{L} \mathbf{T}_i \quad \text{(5.9a)}$$

and

$$\mathbf{C}' = \mathbf{T}_i^{-1} \mathbf{C} \mathbf{T}_v \quad \text{(5.9b)}$$

(5.8) becomes:

$$\mathbf{L}' \mathbf{C}' = \mathbf{\Lambda} \quad \text{(5.9c)}$$

It can be shown that \( \mathbf{L}' \) and \( \mathbf{C}' \) are diagonal matrices [19,54].
Let $E_L$ and $E_R$ be the two $2n \times 2n$ matrices such that:

\[ E_L E_R = 1 \]  \hspace{1cm} \text{(5.10)}

and

\[ E_L A^{-1} E_R = \begin{bmatrix} \Gamma_1 & 0 \\ 0 & \Gamma_2 \end{bmatrix} \]  \hspace{1cm} \text{(5.11)}

where $1$ denotes the unit matrix and $\Gamma_1$ and $\Gamma_2$ are two diagonal $n \times n$ matrices. $E_L$ is called the (left hand side) matrix of row eigenvectors, while $E_R$ is called the (right hand side) matrix of column eigenvectors. Let it be assumed that $E_R$ has the following form:

\[ E_R = a \begin{bmatrix} T_V M & T_V M \\ T_I N & -T_I N \end{bmatrix} \]  \hspace{1cm} \text{(5.12)}

where $M$ and $N$ are proportionality matrices to be determined later. According to (5.10), $E_L$ must have the following form:

\[ E_L = a \begin{bmatrix} M^{-1} T_V^{-1} & N^{-1} T_I^{-1} \\ M^{-1} T_V^{-1} & -N^{-1} T_I^{-1} \end{bmatrix} \]  \hspace{1cm} \text{(5.13)}

with $a = (1/\sqrt{2})$.

On applying (5.12) and (5.13) to (5.11):

\[ E_L A^{-1} E_R = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \]

where

\[
\begin{align*}
P_{11} & = \frac{1}{2}(M^{-1} T_V^{-1} L T_I N + N^{-1} T_I^{-1} C T_V M) = \Gamma_1 \\
P_{12} & = \frac{1}{2}(-M^{-1} T_V^{-1} L T_I N + N^{-1} T_I^{-1} C T_V M) = 0 \\
P_{21} & = \frac{1}{2}(M^{-1} T_V^{-1} L T_I N - N^{-1} T_I^{-1} C T_V M) = 0
\end{align*}
\]
By further applying (5.9a) and (5.9b) in the above expressions:

\[
P_{11} = \frac{1}{2}(M^{-1}L_{\nu}^{-1}N + N^{-1}C\tilde{M}) = \Gamma_1 \quad \text{(5.14a)}
\]

\[
P_{12} = \frac{1}{2}(M^{-1}L_{\nu}^{-1}N + N^{-1}C\tilde{M}) = 0 \quad \text{(5.14b)}
\]

\[
P_{21} = \frac{1}{2}(M^{-1}L_{\nu}^{-1}N - N^{-1}C\tilde{M}) = 0 \quad \text{(5.14c)}
\]

\[
P_{22} = \frac{1}{2}(M^{-1}L_{\nu}^{-1}N - N^{-1}C\tilde{M}) = 0 \quad \text{(5.14d)}
\]

it is seen then, that if \(E_r\) and \(E_L\) are of the forms given by (5.12) and (5.13):

\[
\Gamma_1 = -\Gamma_2 \quad \text{(5.15)}
\]

and

\[
M^{-1}L_{\nu}N = N^{-1}C\tilde{M} \quad \text{(5.16)}
\]

By further choosing \(M = 1\), (5.16) is satisfied by the following diagonal form of matrix \(N\):

\[
N = \sqrt{L^{-1}C'} \quad \text{(5.17)}
\]

and (5.14a) becomes:

\[
P_{11} = \sqrt{L'C'} = \Gamma_1 \quad \text{(5.18)}
\]

It is straightforward to check that:

\[
(\Gamma_1)^2 = (\Gamma_2)^2 = \Lambda \quad \text{(5.19)}
\]

which also agrees with (5.9c). Expression (5.19) shows that the eigenvalues of \(A^{-1}\) are real, and this implies that the ones of \(A\) are real too. Due to the fact that \(N\) has the dimensions of an admittance and that \(N\) and \(N^{-1}\) will play important roles later on, the following definitions are introduced:

\[
Z_w = \sqrt{L'C^{-1}} \quad \text{(5.20)}
\]

and
In addition to providing support to the assumption that (5.2) is hyperbolic, the above results provide a convenient way to obtain the eigenvalues and eigenvectors of the system. First, because the diagonalizing process is applied to the nxn matrix LC instead of to the 2nx2n matrix $A^{-1}$ and, second, because the new eigenvalues/eigenvectors are related to the more familiar ones from the conventional modal theory [54]. Summarizing:

$$E_r = \left( \frac{1}{\sqrt{2}} \right) \begin{bmatrix} T_v & T_v \\ T_i & -T_i \end{bmatrix}$$ ...........(5.22a)

$$E_L = \left( \frac{1}{\sqrt{2}} \right) \begin{bmatrix} T_i^{-1} & Z_W T_i^{-1} \\ T_v^{-1} & -Z_W T_i^{-1} \end{bmatrix}$$ ...........(5.22b)

and

$$\begin{bmatrix} \Gamma_1 & 0 \\ 0 & \Gamma_2 \end{bmatrix} = \begin{bmatrix} \Gamma_1 & 0 \\ 0 & -\Gamma_1 \end{bmatrix}$$ ...........(5.22c)

where

$$\Gamma_1 = \sqrt{\lambda}$$ ...........(5.22d)

or

$$\Gamma_1 = \sqrt{L^1 C^1}$$ ...........(5.22e)

Expression 5.2 can be transformed to a more convenient form by left-multiplying it by $E_L$, yielding:

$$\frac{\partial \mathbf{V}_m}{\partial t} + \Gamma \frac{\partial \mathbf{V}_m}{\partial x} + Z_W \frac{\partial \mathbf{1}_m}{\partial t} + \Gamma \frac{\partial \mathbf{1}_m}{\partial x} + \Gamma \mathbf{T}_v^{-1} R_l = 0$$ ...........(5.23a)

and

$$\mathbf{Y}_w = \sqrt{L^{-1} C}$$ ...........(5.21)
where:

\[ V_m = T_v^{-1}V, \]  
\[ I_m = T_i^{-1}I \]

and

\[ \Gamma = (\Gamma_1)^{-1} = \sqrt{L^{-1}C^{-1}} \]

Note that \( V_m \) and \( I_m \) correspond to the vectors of voltage and current in the modal domain from conventional modal theory with the difference that the present analysis is based on the real matrix \( LC \) rather than on the complex \( ZY \) one. Note also that \( \gamma_j \), the \( j \)-th diagonal element of \( \Gamma \), is the velocity of the \( j \)-th mode.

Expressions (5.23a) and (5.23b) define \( 2n \) pairs of equations in the following form:

\[ (\frac{\partial V_m}{\partial t} + \gamma_j \frac{\partial V_m}{\partial x}) + Z_w (\frac{\partial I_m}{\partial t} + \gamma_j \frac{\partial I_m}{\partial x}) + \gamma_j (\sum_{k=1}^{n} R'_j k I_k) = 0 \]  
\[ \text{expression (5.27a)} \]

and

\[ (\frac{\partial V_m}{\partial t} - \gamma_j \frac{\partial V_m}{\partial x}) - Z_w (\frac{\partial I_m}{\partial t} - \gamma_j \frac{\partial I_m}{\partial x}) - \gamma_j (\sum_{k=1}^{n} R'_j k I_k) = 0 \]  
\[ \text{expression (5.27b)} \]

where the single sub-index \( j \) denotes the \( j \)-th element in the case of the column matrices \( V_m \) and \( I_m \) and the \( j \)-th diagonal element in the case of the diagonal matrices \( \Gamma \) and \( Z_w \). \( R'_j k \) is the element on \( j \)-th row and \( k \)-th column of matrix \( R' \), which is defined as follows:

\[ R' = T_v^{-1}R \]

Equations (5.27a) and (5.27b) become:
\[ \frac{dV_{mj}}{dx} + Z_{wj} dI_{mj} + \left( \sum_{k=1}^{n} R_{jk}^{i} I_{k} \right) dx = 0 \] ...........(5.28a)

and

\[ \frac{dV_{mj}}{dx} - Z_{wj} dI_{mj} + \left( \sum_{k=1}^{n} R_{jk}^{i} I_{k} \right) dx = 0 \] ...........(5.28b)

along their respective characteristics, namely:

\[ \frac{dx}{dt} = \gamma j \] ...........(5.28c)

and

\[ \frac{dx}{dt} = -\gamma j \] ...........(5.28d)

Expressions (5.28a), (5.28b), (5.28c) and (5.28d) should be compared with their monophasic counterparts; i.e., (4.8), (4.9), (4.7) and (4.10). It can be said, in the form of a summary, that the linear transformation defined by \( E_L \) turns the multiconductor line equations 5.2 into a system of 2n first order ODE's.

### 5.2) Numerical Solution of the Linear Multiconductor Line Equations.

Consider the two lines parallel to the x-axis shown in figure 5.1. Consider further that the solution of the multiconductor line equation is known along the line \( t = T \) and that it is to be extended to the point \( G \) on the line \( t = T + \Delta T \). According to the results of the previous sections, there are 2n characteristics passing through \( G \) which provide an equal number of equations in the form of (5.28a) or (5.28b). These equations are sufficient to determine the 2n unknowns at point \( G \) in terms of the data at the points where the characteristics intersect the line \( t = T \). In case of two or more equal eigenvalues, which implies that their characteristics become one, the associated equations will still be different inasmuch as the eigenvectors are linearly independent. Consider now that the solution along \( t = T \) has been produced numerically and, therefore, is determined only at a finite number of points. It is clear that these points will
Figure 5.1.— Characteristics of a multiconductor linear line passing through a point in the $x$–$t$ plane.

Figure 5.2.— Positive and negative slope characteristics intersecting the line $t = T$. 
not necessarily coincide with the characteristic intersection points and, furthermore, it may not be possible to coordinate the different characteristic meshes in order to produce a useful distribution of solution points in the region of the x—t plane delimited by the initial—boundary condition lines. A solution to this problem is proposed next. It consists of the use of interpolations in much the same way as was done in chapter 4 to generate a regular grid of characteristics.

Let \( \gamma_j \) denote the j—th positive eigenvalue of \( A \) with \( j = 1, \ldots, n \). Let \( \gamma_j \) denote its negative value which is also an eigenvalue of \( A \). Suppose that the positive eigenvalues are ordered as follows:

\[
0 < \gamma_n \leq \gamma_{n-1} \leq \ldots \leq \gamma_1
\]

Suppose also that the solution points are equally spaced along a segment of the line \( t = T \) and that \( \Delta X \) is the spacing between two successive points. Figure 5.1 depicts three of these points denoted by D, E and F. Let \( \Delta T \) be determined as follows:

\[
\Delta T = \frac{\Delta X}{\gamma_1}, \quad \ldots \ldots (5.29)
\]

Note that such \( \Delta T \) is the smallest one possible. Let \( \Gamma_j \) denote the characteristic with slope \( \gamma_j \) and \( \Gamma_{-j} \) the one with slope \( \gamma_{-j} \). Note also from fig. 5.1 that the point \( G \) on \( t = T + \Delta T \) is chosen in such a way that the \( \Gamma_{+1} \) characteristic passes through D and that the \( \Gamma_{-1} \) one passes through F. Equations 5.28a and 5.28b can be approximated along \( \Gamma_{+1} \) and \( \Gamma_{-1} \), respectively, as follows:

\[
V_{m1}^G - V_{m1}^D + Z_w(I_{m1}^G - I_{m1}^D) + \frac{\Delta X}{2} \sum_{k=1}^{n} R_{kk}^*(I_{mk}^G + I_{mk}^D) = 0 \quad \ldots \ldots (5.30a)
\]

and

\[
V_{m1}^G - V_{m1}^F + Z_w(I_{m1}^G - I_{m1}^F) + \frac{\Delta X}{2} \sum_{k=1}^{n} R_{kk}^*(I_{mk}^G + I_{mk}^F) = 0 \quad \ldots \ldots (5.30b)
\]

where \( V_{m1}^D, V_{m1}^E, V_{m1}^F \) and \( V_{m1}^G \) represent the values of the first component of the modal voltage vector at the points D, E, F and G, respectively, \( I_{m1}^D, I_{m1}^E, \ldots, I_{m1}^G \).
and I, the values of the first component of the modal current vector at points D, E and F and R\textsubscript{jk} is the element on the j-th row and k-th column of the matrix \( R' \), which is defined as follows:

\[
R' = T_v' RT_1
\]

Consider now any other two characteristics \( \Gamma_+ j \) and \( \Gamma_- j \), where \( j = 2, \ldots, n \). Along them (5.28a) and (5.28b) become:

\[
\begin{align*}
V_m^{G} - V_m^{D} &+ Z_w (I_m^{G} - I_m^{D}) + \frac{\Delta X}{2} \sum_{k=1}^{n} R_{jk} (I_m^{G} + I_m^{D}) = 0 \quad \ldots (5.32a) \\
V_m^{G} - V_m^{F} &- Z_w (I_m^{G} - I_m^{F}) - \frac{\Delta X}{2} \sum_{k=1}^{n} R_{jk} (I_m^{G} + I_m^{F}) = 0 \quad \ldots (5.32b)
\end{align*}
\]

where \( D_j \) and \( F_j \) are the intersection points of the \( \Gamma_+ j \) and \( \Gamma_- j \) characteristics with the line \( t = T \). It can be seen in fig 5.2 that \( D_j \) and \( F_j \) are at a distance \( \Delta X_j \) from point E given by:

\[
\Delta X_j = \gamma_j \Delta T \quad \ldots \quad (5.33)
\]

The evaluation of the dependent variables at points \( D_j \) and \( F_j \) can be done through interpolation. From the quadratic interpolation formulas of section 4.4, \( V_m^{D_j} \) and \( V_m^{F_j} \) become:

\[
\begin{align*}
V_m^{D_j} &= a_1 V_m^{D} + a_2 V_m^{E} + a_3 V_m^{F} \quad \ldots (5.34a) \\
V_m^{F_j} &= a_3 V_m^{D} + a_2 V_m^{E} + a_1 V_m^{F} \quad \ldots (5.34b)
\end{align*}
\]

where:

\[
\begin{align*}
a_1 V_m^{D} &= r_j + (r_j)^2 / 2 \quad \ldots (5.34c) \\
a_2 V_m^{D} &= 1 - (r_j)^2 \quad \ldots (5.34d) \\
a_3 V_m^{D} &= (r_j)^2 / 2 - r_j \quad \ldots (5.34e)
\end{align*}
\]
\[ r_j = \Delta X_j / \Delta X = \gamma_j / \gamma_1 \]

The same formulas apply for the other dependent variables \( I_{m_1}^D, I_{m_2}^D, ..., I_{m_n}^D \), \( I_{m_1}^F, I_{m_2}^F, ..., I_{m_n}^F \). Equations (5.30a) and (5.32a) with the interpolation formulas incorporated into them can be put into the following more convenient form:

\[ V_m^G + (Z + \Delta X R) V_m^G = a_1 [ V_m^D + (Z + \Delta X R) V_m^D ] \]
\[ a_2 [ V_m^E + (Z + \Delta X R) V_m^E ] \]
\[ a_3 [ V_m^F + (Z + \Delta X R) V_m^F ] \] \hspace{1cm} (5.35a)

where:

\[ \Delta X = \text{diag} (\Delta X_1, \Delta X_2, ..., \Delta X_n) \] \hspace{1cm} (5.35b)
\[ a_1 = \text{diag} (1, a_{12}, ..., a_{1n}) \] \hspace{1cm} (5.35c)
\[ a_2 = \text{diag} (0, a_{22}, ..., a_{2n}) \] \hspace{1cm} (5.35d)
\[ a_3 = \text{diag} (0, a_{32}, ..., a_{3n}) \] \hspace{1cm} (5.35e)

In the same manner, equations (5.30b) and (5.32b) with the interpolation formulas lead to the following expression:

\[ V_m^E - (Z + \Delta X R) V_m^E = a_3 [ V_m^D - (Z + \Delta X R) V_m^D ] \]
\[ a_2 [ V_m^E - (Z + \Delta X R) V_m^E ] \]
\[ a_1 [ V_m^F - (Z + \Delta X R) V_m^F ] \] \hspace{1cm} (5.35f)

Expressions (5.35a) and (5.35f) provide the solutions of voltages and currents of point G in terms of the known values of these variables at points D, E and F. Note that the addition of these two expressions yields an explicit solution for the voltage at G, while their subtraction yields an expression for the unknown currents at G without the unknown voltages. An additional feature of (5.35a) and (5.35f) is that the matrices involved in them are constant; in consequence, their numerical implementation is computationally very efficient.
5.3) Multiconductor Quasilinear Transmission Lines.

The relatively simple expressions obtained in the previous two sections for multiconductor linear lines were possible because the eigenvectors of the system were constant. In the quasilinear case they are variable and, furthermore, they cannot be determined beforehand as they depend on the solution. Consider now that the elements of matrix \( C \) of (5.2) depend on the components of the voltage vector \( V \). For convenience, (5.2) will be represented as follows:

\[
\frac{\partial U}{\partial t} + A\frac{\partial U}{\partial x} + BU = 0 \tag{5.36a}
\]

where

\[
U = \begin{bmatrix} V \\ 1 \end{bmatrix} \tag{5.36b}
\]

\[
A = \begin{bmatrix} 0 & C^{-1} \\ L^{-1} & 0 \end{bmatrix} \tag{5.36c}
\]

and

\[
B = \begin{bmatrix} 0 & 0 \\ 0 & L^{-1}R \end{bmatrix} \tag{5.36d}
\]

Note that (4.1) is a special case of (5.36) in which \( A \) and \( B \) are 2x2 matrices. Let \( E_r, E_L \) and \( \Gamma \) be the right eigenvectors matrix, the left eigenvectors matrix and the diagonal matrix of eigenvalues of \( A \), respectively. They have the forms given by expressions (5.22a), (5.22b) and (5.22c); however, the sub-matrices \( T_v, T_i, Z_w \) and \( Y_w \) depend now on some of the components of \( U \). Let \( U_m \) be the vector of dependent variables in modal domain; that is

\[
U = E_r U_m \tag{5.37}
\]

The substitution of (5.37) into (5.36a) yields [60];
\[ E_r \frac{\partial U_m}{\partial t} + A \frac{\partial U_m}{\partial x} + \left( \frac{\partial E_r}{\partial t} + A \frac{\partial E_r}{\partial x} \right) U + BU = 0 \]

By applying (5.11):

\[ E_r \left( \frac{\partial U_m}{\partial t} + \Gamma \frac{\partial U_m}{\partial x} \right) + \left( \frac{\partial E_r}{\partial t} + A \frac{\partial E_r}{\partial x} + B \right) U = 0 \]

and by multiplying by \( E_r^{-1} \):

\[ \frac{\partial U_m}{\partial t} + \Gamma \frac{\partial U_m}{\partial x} + \left( E_r^{-1} \frac{\partial E_r}{\partial t} + \Gamma E_r^{-1} \frac{\partial E_r}{\partial x} + E_r^{-1} B \right) U = 0 \]  

Expression 5.38 represents 2n equations of the form:

\[ \frac{\partial U_{mj}}{\partial t} + \gamma \frac{\partial U_{mj}}{\partial x} + E_{Lj} \left( \frac{\partial E_r}{\partial t} + \Gamma E_r^{-1} \frac{\partial E_r}{\partial x} + E_r^{-1} B \right) U = 0 \]  

where \( E_{Lj} \) is the j-th row of \( E_L \). Along the curve

\[ \frac{dx}{dt} = -\gamma_j \]  

equation (5.38) becomes:

\[ \frac{dU_{mj}}{dt} + \sum_{k=l}^{2n} b_{jk}^t U_{mk} = 0 \]  

where \( b_{jk}^t \) is the element on the j-th row and k-th column of the matrix \( B^t \), which is defined as follows:

\[ B^t = (E_L \frac{\partial E_r}{\partial t} + \Gamma E_L \frac{\partial E_r}{\partial x} + E_L B)E_r \]  

Note that a consequence of having variable transformation matrices, \( E_r \) and \( E_L \) is that the partial derivatives of \( E_r \) with respect to \( x \) and \( t \) are present in each of the 2n equations of the form (5.39). An alternate approach to the solution of (5.2) is presented as follows [15]. Let (5.36a) be left-multiplied by \( E_L \):

\[ E_L \frac{\partial U}{\partial t} + E_L A \frac{\partial U}{\partial x} + E_L BU = 0 \]
On applying (5.11), the following expression is obtained

\[ E \frac{\partial U}{\partial t} + \Gamma E \frac{\partial U}{\partial x} + E_LBU = 0 \]  

(5.41)

which contains 2n equations of the form

\[ E_L \left( \frac{\partial U}{\partial t} + \gamma_j \frac{\partial U}{\partial x} \right) + E_LBU = 0 \]  

(5.42)

They are sufficient for determining the 2n unknown dependent variables. The parenthesized part of (5.42a) can be interpreted as the differentiation of U in the direction given by:

\[ \frac{dx}{dt} = -\gamma_j \]  

(5.42b)

where (5.42b) defines a curve in the x-t plane which is hereafter denoted by \( \Gamma_j \) (with \( j = 1, 2, \ldots, 2n \)). Along this curve (5.42a) becomes:

\[ E_L \left( \frac{dU}{dt} \right) + E_LBU = 0 \]  

(5.42c)

An interpretation of expression (5.41) is illustrated in figure 5.3. Let G be a point on the line \( t = T + \Delta T \); the 2n characteristics passing through it are curved now. They intersect the line \( t = T \) at the points denoted by \( D_1, D_2, \ldots, D_{2n} \). The 2n equations of the form (5.42c) relate the solution at these points to the one at point G.

In the linear case the first n characteristics were mirror images of the additional ones. In the quasilinear case, however, this will not always be true. The lack of symmetry, along with the variability of the eigenvalues and eigenvectors of \( A \), makes the nonlinear case much more complicated than the linear one that was discussed in sections 5.1 and 5.2. One consequence of this is that a more explicit form of (5.41), which is obtained by applying the definitions of \( E_L, \Gamma, U \),
and B given by (5.22b), (5.22c), (5.36b) and (5.36d), is cumbersome. For this reason further development is based on (5.41).

![Figure 5.3](image)

**Figure 5.3**— Characteristics of a multiconductor quasilinear line passing through a point in the x–t plane.

### 5.4) Proposed Numerical Solution of the Quasilinear Multiconductor Line Equations.

From the two alternate ways for dealing with the quasilinear multiconductor line equations presented in the previous section, the second one which led to (5.41) seems to be the most convenient one for numerical implementation. Let the eigenvalues of A be denoted now by $\gamma_j$ (with $j = 1, 2, ..., 2n$) and let the following order be assumed among them:

$$\gamma_{n+1} \leq \gamma_{n+2} \leq ... \leq \gamma_{2n} < 0 < \gamma_{n} \leq \gamma_{n-1} \leq ... \leq \gamma_{1}$$

The integration of (5.42c) along a segment of $\Gamma_j$ between $D_j$ and $G$ can be performed by appealing to the mean value theorem, yielding:
where the tilde denotes the mean value of the corresponding variable. A numerical approximation of (5.43) is obtained by replacing each mean value by its two end point average:

\[
\frac{1}{2}(E_{Lj}^G + E_{Lj}^D)(U^G - U^Dj) + \frac{\Delta T}{4}(E_{Lj}^G + E_{Lj}^D)B(U^G + U^Dj) = 0
\]

(5.44)

Further algebraic manipulation leads to the following expression:

\[
(E_{Lj}^G + E_{Lj}^D)(1 + \frac{\Delta T}{2}B)U^G = (E_{Lj}^G + E_{Lj}^D)(1 - \frac{\Delta T}{2}B)U^Dj
\]

(5.45)

Suppose now that the intersection points \(D_1, D_2, \ldots, D_{2n}\) are always between the solution points \(D\) and \(F\), as indicated in fig 5.3. \(U^Dj\) in expression (5.45) can thus be obtained as follows by means of quadratic interpolation:

\[
U^Dj = a_1jU^D + a_2jU^E + a_3jU^F
\]

(5.46a)

where

\[
a_1jV^D_mj = (r^2_j)/2 - r^2_j
\]

(5.46b)

\[
a_2jV^D_mj = 1 - (r^2_j)
\]

(5.46c)

\[
a_3jV^D_mj = r^2_j + (r^2_j)/2
\]

(5.46d)

and

\[
r^2_j = \gamma_j(\Delta T/\Delta X)
\]

(5.46e)

Similarly, \(E_{Lj}^Dj\) is determined as follows:

\[
E_{Lj}^Dj = a_1jE_{Lj}^D + a_2jE_{Lj}^E + a_3jE_{Lj}^F
\]

(5.47)

By applying (5.46a) and (5.47), (5.45) becomes:

\[
(E_{Lj}^G + a_1jE_{Lj}^D + a_2jE_{Lj}^E + a_3jE_{Lj}^F)(1 + \frac{\Delta T}{2}B)U^G =
\]

\[
a_1j(E_{Lj}^G + a_1jE_{Lj}^D + a_2jE_{Lj}^E + a_3jE_{Lj}^F)(1 - \frac{\Delta T}{2}B)U^D +
\]
The 2n equations defined by (5.48) can be grouped into the following matrix expression:

\[ E'B'U^G = a_1E'B''U^D + a_2E'B''U^E + a_3E'B''U^F \]  

where

\[ a_1 = \text{diag} (a_{11}, a_{12}, \ldots, a_{1n}) \]  

\[ a_2 = \text{diag} (a_{21}, a_{22}, \ldots, a_{2n}) \]  

\[ a_3 = \text{diag} (a_{31}, a_{32}, \ldots, a_{3n}) \]  

\[ E' = E_L + a_1E_L + a_2E_L + a_3E_L \]  

\[ B' = 1 - \frac{\Delta T}{2}B \]  

\[ B'' = 1 - \frac{\Delta T}{2}B \]

Expression 5.49a can be used to determine \( U^G \) by means of an iterative process, such as the one proposed next.

1. Initialize by assuming

\[ E_L^G = E_L^E \]

and

\[ \Gamma^G = \Gamma^E \]

2. Assume that \( \Gamma \), the diagonal matrix of mean eigenvalues, is equal to \( \Gamma^G \) and obtain the initial matrices of interpolation coefficients \( a_1, a_2 \) and \( a_3 \) by applying (5.46b), (5.46c), (5.46d) and (5.46e).

3. Obtain an initial estimate of \( U^G \) by applying (5.49a).

4. Evaluate matrix \( A \) at point \( G \) by using the most recent estimate of \( U^G \).
5. Obtain new $E^G_L$ and $\Gamma^G$ from the new matrix $A$.

6. Obtain the diagonal elements for a new matrix of mean eigenvalues $\tilde{\gamma}$ as follows:

$$\tilde{\gamma}_j = (\gamma^G_j + a_1 \gamma^D_j + a_2 \gamma^F_j + a_3 \gamma^F_j) \quad (j = 1, 2, \ldots, 2n)$$

7. Obtain new matrices of interpolation coefficients $a_1$, $a_2$, and $a_3$ by applying the new $\tilde{\gamma}_j$'s to expressions (5.46b), (5.46c), (5.46d), and (5.46e).

8. Obtain a new $U^G$ by applying (5.49a).

9. If the new $U^G$ and its previously estimated value are equal within a preselected tolerance, convergence has been attained; otherwise, take the new $U^G$ as the most recent estimate of $U^G$ and repeat the process from step 4 onwards.

5.5 Remarks.

The extension of the method of characteristics to multiconductor lines proposed in this section provides an attractive alternative, even in the linear case. One problem of the more traditional approaches for handling these lines in the time domain stems from the fact that they based on the eigenvalue/eigenvector analysis of the $ZY$ matrix product. This introduces imaginary terms that lack physical meaning in time domain. By using the modes of the $LC$ product, instead, the analysis is confined here to the real domain.

As for the analysis of transients on multiconductor lines with corona, most methods so far proposed resort to constant modal transformation matrices as well as to the linear superposition principle. The removal of these two assumptions is made possible in the method proposed here at the expense of an increase in the number of computations. It is believed here, nevertheless, that its numerical implementation will prove valuable as a principal method in practical
analysis as well as in providing benchmarks for simpler techniques.
6.1.- Preamble.

The application of the method of characteristics of PDE theory to the analysis of lines with corona had been already suggested by some researchers [23,30,52]. As the actual progress towards this end previously has been very limited [30], the research work reported in this thesis was devoted to this issue. As a result, a new methodology for the analysis and simulation of transients on transmission lines with corona, based on the aforesaid theory, has been proposed and developed.

First, a new method for the analysis and simulation of transients on monophasic lines with and without corona has been proposed, developed and implemented on a computer. The effectiveness of this method has been further established through numerical tests on its computer implementation. Included among these tests is the simulation of a field experiment. Also, a new method
for the analysis and simulation of multiconductor linear lines has been proposed and developed as an extension of the method for monophasic lines. Finally, a new method for the analysis and simulation of multiconductor linear lines with corona has been proposed and developed, also as an extension of the aforesaid monophasic method. The following section provides a summary of the main features of these three new methods as well as of the main results of this thesis. Section 6.3 provides suggestions for future research in the area of transient simulations on transmission lines. Section 6.4, finally, provides the concluding remarks for this thesis.

6.2.— Summary of Results.

The method of characteristics of PDE's provides in principle a better way for dealing with transients on nonlinear lines, such as the ones affected by corona, than more conventional methods based on constant discretization steps [23,30]. The author considers that the results presented in this thesis establish the practicality of this method. The constant discretization steps of conventional methods introduce numerical errors in the form of artificial reflections which are often observed as large oscillatory waves superimposed on the tail of the simulated travelling waves [78]. In the method of characteristics, the sizes of the discretization steps are adjusted in a way which minimizes these artificial reflections. The step size variations, however, result in the irregular distribution of the solution points throughout the x-t plane of coordinates. Nevertheless, this undesirable effect is overcome by combining the method of characteristics with an interpolation process [14].

The approach adopted here consists of expressing the line equations as a system of first order PDE's and applying the method of characteristics with interpolations. A major advantage of this approach is that it performs equally
well with the multiconductor line case as with the monophasic one. In the latter case, the resulting method is closely related to Bergeron's method on which the EMTP is based [67]. It has been shown here, in fact, that the application of the method of characteristics with interpolations to a monophasic lossy linear line results in a well known line model of the EMTP (see fig. 2.1). [67].

The absence of errors caused by reflections is readily apparent in all the simulated transients on monophasic lines with corona that are presented in this thesis. This feature represents an advantage over more conventional methods based on fixed discretizations wherein numerical errors are bound to occur. Most of the simulations presented are done for a double linear ramp excitation, because its sharp corner provides a more stringent test than the rounded peaks from other more commonly used waveforms. The absence of artificial reflections permitted the simulation of travelling waves with multiple peaks above the corona threshold. As it is observed in the examples provided in chapter 4, these multiple peaks can arise from wave reflections in the line.

The computer implementation of the method proposed for monophasic lines with corona performs very well with coarse discretization schemes. This feature has been tested by simulating a monophasic line case with different discretization levels. The widest step length that was used allowed only one point to represent the wavefront. It is remarkable that, even with such a low discretization level, the implemented method delivers meaningful results. To produce equivalent results, other more conventional methods require step sizes that are five to ten times smaller.

In addition to the previously described simulations, a field experiment conducted by Wagner and Evans [10] has been reproduced by means of the aforesaid computational method. The simulation results are in satisfactory
agreement with the experimental ones, particularly with respect to the evolution of the wavetail as it propagates. Most other methods are not able to reproduce accurately the tails of waves propagating on a line with corona [48].

A new method for the simulation of multiconductor linear lines, based on the technique of characteristics with interpolations, has been developed in this thesis. Although the discovery of this method was a byproduct of the analysis of multiconductor lines with corona, it is an important result in its own right. Many conventional methods for analyzing multiconductor linear lines in the time domain make use of frequency domain modal transformations [67]. The problem is, however, that these transformations introduce complex quantities which lack physical meaning in the time domain. The new method developed here confines the analysis to the real domain.

A new simulation method has also been developed in this thesis for multiconductor lines with corona. It consists of applying the method of characteristics with interpolations to the quasilinear system of PDE’s which is obtained when representing transmission lines under the assumption that corona is a static phenomenon. Most other methods are based on conventional modal analysis which poses two particular problems. First, complex quantities are introduced; and second, the line is presumed to respond linearly. This new method avoids these two shortcomings. Another important feature, is that the method can be extended, in a rigorous manner, to multiconductor lines with dynamic corona.

The abovementioned new methods proposed here are based on an eigenvalue/eigenvector analysis. Although there are some basic differences between this analysis as applied here and the conventional line modal analysis [19], it has been shown in this thesis that the two of them are related. This relationship provides a computationally efficient way to derive the new eigenvector
transformations from the conventional modal ones.

6.3.— Future Research Recommendations.

A number of future research topics became apparent throughout the development of this project. These topics came out partly from existing needs in the field of transient analysis in general, from needs in the subject of transient analysis of lines with corona and from new possibilities opened by the techniques developed in this thesis. The author proposes that these topics be dealt with by means of research projects described as follows.

The issues related to general line simulations, which require further development, can be grouped in one project. The aim of the main project would be the further development of the methods based on characteristics with interpolations for their application to practical linear line analysis. The first stage of this project would consider the development of a systematic way for selecting frequency independent parameters of lines and for determining the cases that require frequency dependent modelling. The second stage of this project could consist of the numerical implementation of the multiconductor linear line method that is developed in this thesis and, perhaps, its incorporation into a transient simulation package like the EMTP. A third stage of this project could be the incorporation of frequency dependence modelling capabilities into the method of characteristics with interpolations. It is possible, however, that this last stage deserves to be considered another project by itself.

Concerning the issues related to the analysis of lines with corona, it is proposed here that one project be devoted to the practical implementation of the method developed in this thesis for multiconductor lines with static corona. The first stage of this project would consist of the algorithmic implementation of this
method and of the refinement of this implementation through numerical tests and comparisons with suitable field tests. The second stage of this project would consist of the incorporation of this resulting algorithm into a host transient simulation program, possibly the EMTP.

A second project proposed here for transmission lines with corona would be concerned with the implementation of frequency dependent and of dynamic corona features on the quasilinear methods proposed in this thesis. The first stage of this project would be the incorporation of frequency dependent features into these two methods, the monophasic and the multiconductor one. The second stage would consist of the development of a simulation method for monophasic lines with dynamic corona and its implementation in a computer program. The third stage would consist of applying this computer program to sensitivity studies aimed at determining the need for dynamic corona modelling for practical analysis of transmission lines. It would be desirable that this third stage included comparisons with field tests; however, these comparisons are subject to the availability of suitable field test results. The results of this third stage would indicate whether the next two stages are required or not. The fourth stage would consist of the extension of the method developed in the second stage to multiconductor lines. The fifth stage, finally, would consist of the implementation of frequency dependent features into the methods of simulation of lines with dynamic corona.

The need for results from field experiments on actual lines, in addition to the ones that are available from the specialized publications [10,44,43], has been stressed in several sections of this thesis. Hence, this author considers that a research project consisting of an extensive program of field experiments is essential for the advancement of the techniques for the analysis of transients on
lines with corona. Although the required experiments are very difficult and expensive to perform, it is proposed here that the use of computer simulations can help to increase their effectiveness and, consequently, to decrease the cost of a field experimental project. It is therefore proposed here that the methods of analysis developed in this thesis be used for the design of field experiments on transmission lines with corona, for the interpretation of their results and for the determination of the effects that other factors unrelated to corona have on these results.

It is highly desirable that another research project consisting of laboratory experiments on corona be conducted in coordination with the abovementioned field experiments. This author proposes that, in much the same manner as suggested above for the field tests, laboratory experiments on short conductor sections and on electrodes could be made more effective through the application of an advanced corona simulation model; e.g., the one proposed by Abdel-Salam [84]. This author proposes also that a laboratory experimental project include transient propagation experiments on scaled down transmission lines [97].

6.4.- Concluding Remarks.

A new methodology for the analysis of lines with corona has been proposed in this thesis. The application here of this methodology in the development of specific methods has demonstrated its usefulness in the analysis of power transmission lines with or without corona. This methodology overcomes some important technical problems of conventional methodologies. The author believes that this methodology is useful for other fields of application outside the power analysis field, such as the analysis of radiofrequency transmission lines.
Much research work has still to be done in the area of electromagnetic wave propagation on lines with corona in both aspects, the experimental and the analytical, before a practical methodology can be established. Nevertheless, the author believes that the methodology of analysis presented in this thesis offers valuable tools for this pursuit.
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