AN ANALYSIS OF TEACHING PROCESSES IN MATHEMATICS EDUCATION FOR ADULTS

by

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ABSTRACT

This study explored the teaching processes in mathematics education for adults and how they are shaped by certain social and institutional forces. Teaching processes included the selection and ordering of content to be taught; the choice of such techniques as lectures or groupwork; the expectations, procedures and norms of the classroom; and the complex web of interactions between teachers and learners, and between learners themselves. The study addressed three broad questions: (1) What happens in adult mathematics classrooms? (2) What do these phenomena mean for those involved as teachers or learners? and (3) In what ways do certain factors beyond the teachers' control affect teaching processes?

The theoretical framework linked macro and micro approaches to the study of teaching, and offered an analytical perspective that showed how teachers' thoughts and actions can be influenced and circumscribed by external factors. Further, it provided a framework for an analysis of the ways in which teaching processes were viewed, described, chosen, developed, and constrained by certain "frame" factors.

The study was based in a typical setting for adult mathematics education: a community college providing a range of ABE-level mathematics courses for adults. Three introductory-level courses were selected and data collected from teachers and students in these courses, as well as material that related to the teaching and learning of mathematics within the college. The study used a variety of data collection methods in addition to document collection: surveys of teachers' and adult learners' attitudes, repeated semi-structured interviews with teachers and learners, and extensive ethnographic observations in several mathematics classes.

The teaching of mathematics was dominated by the transmission of facts and procedures, and largely consisted of repetitious activities and tests. Teachers were pivotal in the classroom, making all the decisions that related in any way to mathematics education. They rigidly followed the set textbooks, allowing them to determine both the content and the process of mathematics education. Teachers claimed that they wished to develop motivation and responsibility for learning in their adult students, yet provided few practical opportunities for such development to occur. Few attempts were made to encourage students, or to check whether they understood what they were being asked to do. Mathematical problems were often repetitious and largely irrelevant to adult students' daily lives. Finally, teachers "piloted" students through problem-solving situations, via a series of simple questions, designed to elicit a specific "correct" method of solution, and a single correct calculation. One major consequence of these predominant patterns was that the overall approach to mathematics education was seen as appropriate, valid, and successful. The notion of success, however, can be questioned.

In sum, mathematics teaching can best be understood as situationally-constrained choice. Within their classrooms, teachers have some autonomy to act yet their actions are influenced by certain external factors. These influences act as frames, bounding and constraining classroom teaching processes and forcing teachers to adopt a conservative approach towards education. As a result, the cumulative effects of all of frame factors reproduced the status quo and ensured that the form and provision of mathematics education remained essentially unchanged.

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Why learn math? Well, I'll tell you.....[I was] standing on the jobsite with a fellow who was the layout man for doing the carpentry work, eh. And we had a big curve to do in the front of the building, and he worked that out....He just took a piece of wood, he measured how long the perimeter was, and then just bent it, and it bent in the curvature, and I thought this was the most incredible thing, I said, "Geez, this guy must be an engineer, you know, I mean this is incredible. He's a genius!" And I realized that all he was doing was basic math.....I want to be able to do it like that.

Construction Worker, March 1994

This study concerns mathematics education for adults and seeks to explain why the teaching of mathematics takes the form that it does. In particular, it focuses on several mathematics classrooms in a typical Adult Basic Education (ABE) setting and explores how such education is viewed by those involved as teachers or learners. Further, the study examines the teaching of mathematics in light of its social context, and investigates how teaching processes are shaped by social and physical resources and constraints.

My interest in the teaching of mathematics to adults arose from three sources. First, as an adult literacy/numeracy teacher, I realized how many adults regarded themselves as innumerate and avoided both numerical data and arithmetic calculations whenever they could. Of particular concern was that these people were often disenfranchised by their lack of mathematical skills from taking an active and informed role in decisions that involved either numerical data or computational skills. Second, given how widespread the problem of innumeracy is seen to be, the paucity of research on adult numeracy or mathematics education for adults is startling. Although extensive research has been conducted in the corresponding field of adult literacy, few of the approaches, assumptions, topics, or questions that have marked this research, or the insights, applications, or policies that it has generated,

have been translated into research on adult numeracy. Third, such published discussions that do exist often cite the predominant methods of teaching mathematics in schools as the biggest contributor to poor mathematical ability. As one noted mathematician puts it: "School mathematics is simultaneously society's main provider of numeracy and its main source of innumeracy" (Steen, 1990, p. 222). I was interested in determining whether the teaching of mathematics in adult education reproduced that of its school counterpart.

In this chapter I introduce the elements of my study. I first sketch some of the background to my study of mathematics education. Next, I provide some definitions of numeracy, discuss some of the consequences of innumeracy, and explain what steps innumerate adults can take to improve their mathematical skills. To outline the specific focus of this study, I provide a justification for, and a statement of, my research questions. Finally, I outline the structure of this dissertation.

Background

The mathematical abilities of adults regularly give cause for concern to government bodies, business and community leaders, and adult and mathematics educators throughout the industrialized world. There is a strong consensus, amongst these groups, that the mathematical skills, awareness, and understanding of adult learners, whether high-school leavers or college graduates, have deteriorated alarmingly in recent years. Adults "know less, understand less, have little facility with simple [mathematical] operations, and find difficulty in solving any but the shortest and simplest of mathematical problems" (Barnard & Saunders, 1994).

So what? Millions of people appear to function perfectly well without ever needing to use much of the mathematics that they remember from school. No one claims to be particularly disadvantaged by a lack of mathematical abilities. In addition, many people see mathematics as an esoteric subject having little to do with their everyday lives. Indeed, mathematics commonly represents a body of ultimately abstract, objective and timeless truths, far removed from the concerns and values of humanity. If mathematics seems so tangential to everyday life, why is it such a problem if so many people can't do math very well?

Primarily it is a problem because of the societal and individual consequences of innumeracy. Numeracy--mathematical ability--is commonly recognized as a major determinant for job and career choices, and a key to economic productivity and success in modern, industrial societies. Numeracy, then, functions as "cultural capital." Hence, the extent of mathematical ability operates as a social filter, and access to social effectiveness and privilege is restricted to those with sufficient mathematical ability.

It doesn't start out that way. Indeed, numeracy is one of the major intended outcomes of schooling, and mathematics occupies a central position in virtually every K–12 school curriculum. But somehow, mathematics teaching fails to produce numerate adults. As Western society has become increasingly informationally and technologically saturated, the innumerate are increasingly disadvantaged--confused and manipulated by numbers, unable to critically assess assumptions and logical fallacies, and unable to participate as effective and informed citizens. For example, how often are adults prepared to take statistical information and their stated conclusions at face value? How many of us feel skilled enough to look beyond the numbers to interpret what the statistics mean? Of particularly concern is the underlying pattern of inequity in adult numeracy; surveys of mathematical abilities show that performance is lower especially among working class, women, Hispanic,

and Afro-American learners. So, mathematical ability is important if only because it is capable of empowering so many.

Why are adults' mathematical abilities as low as they are? It has been proposed that the primary contributor is the poor teaching in school mathematics classrooms (Frankenstein, 1981; Paulos, 1988). Traditionally, mathematics education is taught as an abstract and hierarchical series of objective and decontextualized facts, rules, and answers. Further, predominant teaching methods use largely passive, authoritarian, and individualizing techniques that depend on memorization, rote calculation, and frequent testing (Bishop, 1988). Knowledge is thus portrayed as largely separate from learners' thought processes, and mathematics education is experienced as a static, rather than dynamic process. Adults who do wish to upgrade their mathematical skills have access to a variety of courses run by local public sector educational bodies. It is unclear, however, if these courses are, in any way, adult-oriented, or merely reproduce the curricula and teaching methods so common in traditional K-12 mathematics. Given the rapid decline in adult numeracy, the nature of its social consequences, and the apparent inadequacy of current educational approaches to remedy it, this study of the teaching processes in adult mathematics classrooms is both timely and necessary.

Adult Numeracy

To be numerate is to function effectively mathematically in one's daily life, at home, and at work. Being numerate is one of the major intended outcomes of schooling, and mathematics occupies a central place in the school curriculum.

Indeed, mathematics is "the only subject taught in practically every school in the world" (Willis, 1990, p. 16). However, despite this privileged position of mathematics education, there is much evidence that the mathematical abilities of many adults in Britain and North America do not equip them to function effectively in their daily lives (Cockcroft, 1982; Kirsch, Jungeblut, Jenkins, & Kolstad, 1993; Paulos, 1988; Statistics Canada, 1991). For example, Statistics Canada report that 38% of Canadians surveyed in 1991 did not "possess the necessary skills to meet most everyday numeracy requirements" (1991, p. 11).

There are few published works that deal exclusively with adult numeracy. Occasionally, books (e.g., Dewdeney, 1993; Paulos, 1988; Tobias, 1978; Zaslavsky, 1994) are published where the authors condemn the current state of adults' mathematical ability and suggest some alternatives for both the mathematics profession and the public. Although generating some concern at the time of their publication, these works are rarely discussed in either the mathematics education or adult education literature, and their impact on mathematics education for adults is unknown. What is clearer, however, is that overwhelmingly these books concentrate on "innumeracy" as opposed to "numeracy," and in so doing, focus on the negative rather than the positive aspects of individual mathematical ability. This suggests the implicit classification of those who are numerate as "good" or "worthy," and those who are innumerate as somehow "bad" or "inadequate." Given the implications of merit in that classification, it is useful, first, to consider some definitions of numeracy. What, in practical terms, does it mean to be numerate? And, alternatively, what are some of the consequences of innumeracy at both personal and societal levels? I will discuss both of these, and finally, describe what opportunities exist for those adults who wish to improve their mathematical abilities.

Definitions of Numeracy

Public discussion about the mathematical ability of adults is usually couched

within the context of debates about adult literacy. Indeed, numeracy and literacy are

often linked. For example, the Crowther Report (1959) describes numeracy as "the

mirror image of literacy," and one noted mathematician introduces his survey of

contemporary approaches to mathematics education in the USA by stating that

"numeracy is to mathematics as literacy is to language" (Steen, 1990, p. 211). Further,

common definitions of literacy often include some reference to arithmetic skills, and

numeracy as a concept is often considered a part of the wider concept of literacy. For

example, UNESCO defines a literate person as one who

> can engage in all those activities in which literacy is required for effective
> functioning of his/her group and community and whose attainments in
> reading, writing, and arithmetic make it possible for him [sic] to continue to
> use these skills for his own and the community's development. (UNESCO,
> 1962)

More extensive definitions of numeracy are provided in the Cockcroft Report

on mathematics teaching in Britain (1982). Cockcroft discusses a range of definitions

from a broad conception—including familiarity with the scientific method, thinking

quantitatively, avoiding statistical fallacies—to narrower ones such as the ability to

perform basic arithmetic operations. Cockcroft uses the word "numerate" to mean

the possession of two attributes:

> 1. An "at-homeness" with numbers, and an ability to make use of
> mathematical skills which enables an individual to cope with the practical
> mathematical demands of everyday life, and

> 2. An appreciation and understanding of information which is presented in
> mathematical terms, for instance in graphs, charts or tables or by reference to
> percentage increase or decrease. (p. 11)

There are some noteworthy aspects of this definition. First, both attitudes and

skills are considered important. Second, being practical is the criterion by which

skills are considered important; the relevant context is provided by the demands of

the person's everyday life. Third, the appreciation of numerical information is
considered important as well as the use of mathematical techniques.

Hope (quoted in Keiran, 1990) provides a broader definition of numeracy; one
more in tune with peoples' everyday demands than with the narrower interests of
mathematicians. After reviewing several research studies and curriculum guides,
Hope identified a set of quantitative tasks that everyone should be able to perform.
These included such tasks as handling money and calculating costs, reading recipes,
planning renovations, using technical instruments and devices, understanding
simple statistics, working with graphs, and using scoring schemes in leisure
activities and games. Hope then determined five categories of essential mathematical
"understandings and competencies" that he considered essential for these tasks:
knowing how to use mathematics to solve problems, knowing how to perform
calculations, knowing how to measure, knowing how to work with space and shape,
and knowing how to analyze and interpret quantitative data and arguments based
on this information.

Other, broader, definitions of numeracy are beginning to emerge as
educational research documents changes in school practices during the 1970's and
1980's. A "broad" approach regards teaching mathematics more as the development
of heuristic or problem-solving skills than as the transmission of a body of concepts,
facts, and skills (Baker & Street, 1994). For example, Mason, Burton, and Stacey
(1985) describe numeracy as the ability to "think mathematically," which involves the
processes of conjecturing, specializing, generalizing, convincing, explaining, and
describing--seen as essential to solving mathematical problems.

Whatever the exact definition used, many authors claim that the kinds of
mathematical skills needed by people to function effectively in daily life are
changing, and are likely to continue to change. The need for certain mathematical
skills such as arithmetic or algebraic computation is decreasing due to the

availability of calculators and computers, while other mathematical ideas such as estimation or those associated with probability and statistics are assuming greater importance (National Research Council, 1989). This continuing change in the mathematical needs of adults highlights the need for some discussion of the consequences of innumeracy.

Consequences of Innumeracy

In general, innumeracy is not considered as socially unacceptable as its counterpart illiteracy. One often hears statements about peoples' mathematical inadequacies, spoken without any apparent embarrassment: "I've never been able to work out how much to tip"; "I never check my change from the store"; or "I'm a people person, not a numbers person." One mathematician, John Allen Paulos, claims that part of peoples' lack of concern about their mathematical ignorance is because the consequences of innumeracy are not as "obvious as other weaknesses" (1988, p. 4).

However, regardless of popular opinion, there are several consequences of innumeracy among adults. At an individual level, there are restrictions on freedom of access to further education and training, and to higher-paying jobs. Most institutions of higher education formally require that in order to be accepted, applicants demonstrate their mathematical ability by passing certain standard examinations such as the GCSE (in Britain) or those examinations that lead to certificates of high school completion (in North America). Once enrolled in higher education, students are often required to take further mathematics courses before they can register in courses in particular disciplines (e.g., science, medicine, or economics).

Numeracy can also improve the prospects for non-university level students. Researchers have found that a knowledge of algebra and geometry can make the difference between a low score and a high score on most standard entry-level tests for the civil service, and for many industrial occupations (Cockcroft, 1982; Tobias, 1978). A final individual consequence of innumeracy is that adults suffering from "a low level of confidence in their constructive skills and critical insights [tend] to be dependent on the views of the 'expert' or 'professional' for their opinions" (Evans, 1989, pp. 212-213).

On a societal level, the consequences of innumeracy include the loss of industrial production (in both quantity and quality), a waste of resources, the production of inaccurate or useless information, and a diminution in active citizenship (Thorstad, 1992). From a purely business perspective, numeracy and its related thinking skills are increasingly required by employers, particularly in the fast-growing high-technology fields of computers, environmental science, and biotechnology. According to the Workforce 2000 study (Johnston & Packer, 1987), the proportion of jobs requiring the equivalent of four years of high school mathematics will be 60% greater in the 1990's than it was in the 1970's, while the proportion where only rudimentary math skills are used will decline by as much as half. In such circumstances, merely being able to remember a few mathematical formulae--one commonly-accepted definition of mathematical ability--will no longer be enough. In a world of rapidly changing technologies, incomplete and uncertain information, and unpredictable events, all workers must be able to do more than competently apply a given mathematical formula; they must know when to apply these procedures, and which ones to use.

Another aspect of innumeracy in a social context is the underlying pattern of inequity in adult numeracy; surveys of mathematical abilities indicate that

performance is lower among women, working class, Hispanic, and Afro-American learners. Easley and Easley (cited in Willis, 1990) argue that

> elitist attitudes about mathematics, and acute inequities in mathematics learning, have become part of what oppress many groups who are educationally disadvantaged on the basis of their gender, class, or race. Mathematics is powerful, but much of the power of school mathematics resides not in the mathematics but in the myth of mathematics, in the meritocratic prestige of mathematics as an intellectual discipline. Knowledge is power, particularly when that knowledge has high cultural value and is exclusive. (p. 17)

Improving Numeracy

In British Columbia, if adults choose to improve their mathematical knowledge and abilities, they have two main options. They can either undertake a process of individual, self-directed learning, purchasing one of several standard "refresher texts," or they can enroll in remedial mathematics courses offered by adult education providers in their communities. Both the refresher texts and the organized courses cover much the same curriculum--both are designed to help adults prepare for and pass one of the standardized examinations (such as Math 10, 11, or 12, the General Education Development Test, or the Adult Basic Education Provincial Diploma) necessary for entrance to further education.

Almost all of the locally-provided mathematics education is organized and controlled by the public education sector. For example, within the Acton area, both the Acton School Board and the community college system offer a variety of "math upgrading" courses to adults at several centres. Most adults in these courses are trying to obtain one of four certificates (Dogwood, Adult Dogwood, College Provincial, and GED) equivalent to high school completion. This pattern of provision is repeated in most urban areas across North America.

Much of this provision is intended to supply opportunities for "lifelong learning," best described as "the opportunity for individuals to engage in purposeful and systematic learning throughout their lives" (Faris, 1992, p. 6). Within Canada, the federal discussion paper Learning Well...Living Well calls for the development of a lifelong learning structure and an associated learning culture that includes the provision of mathematics education for adults (Canada, 1991).

For educators, this is quite significant, given the current dearth of information about the teaching of mathematics to adults. Central to the concept of lifelong learning are certain widely-held assumptions about, and practices within, adult education, that are built on ideas and theories about how adults learn and should be taught. These ideas include: teaching must be problem-centered, it must emanate from the participants' experience of life and develop the individual socially, participants must exert definite influence on the planning of the course and the conduct of the teaching, and techniques used must be based on an interchange of experience (Knowles, 1980). Although adult education policies within Canada acknowledge the goals of lifelong learning (see, for example, Faris (1992), in relation to adult education within British Columbia); how far the practice of adult mathematics education meets its ideals is currently undocumented.

In addition, there is no published research in North America "relating to the unique aspects of teaching math to adults" (Gal, 1993, p. 14). Similarly, there is little international research in this area despite UNESCO's recognition of numeracy as a key component of literacy. Some studies have been published in Australia (Foyster, 1990), Britain (ALBSU, 1983; Cockcroft, 1982; Harris, 1991; Sewell, 1981), and Sweden (Höghielm, 1985; Löthman, 1992), but, overall, this work has been sporadic and unconnected.

Given this background, my study focuses on the provision of mathematics education to adults within a typical adult education setting. In the next section I present a justification for, and a statement of, my research questions.

Research Questions

In the background to my study, I have shown that a degree of numeracy is considered a necessary skill for adults in order for them to be engaged citizens and productive workers; further, I have shown that adults who wish to improve their mathematical abilities have access to a variety of "upgrading" courses. Despite this, the mathematical abilities of many adults continue to cause concern in many industrialized countries. Nevertheless, little research has been conducted on adult numeracy or the teaching of mathematics to adults.

Much of the published material about innumeracy and the learning of mathematics by adults is written from the viewpoint of government or industry leaders (e.g., National Research Council, 1989) or university professors of mathematics (e.g., Paulos, 1988; Willoughby, 1990). These viewpoints overwhelmingly reflect either policy-making and managerial perspectives or the academic research interests of the profession. Further, they are often based on narrow technical and instrumental models of education that ignore much adult learning theory and the importance of such issues as self-concept, motivation, values, attitudes, and intentions in learning. What is missing from the published literature are the voices of those most intimately involved in mathematics education--adult teachers and the learners in their courses. Eisenhart (1988) has identified that

"the meanings encoded in the language of mathematics--in the way it is presented to, and used by, students--have not been a focus in mathematics education" (p. 111). She encourages researchers to seek answers to these questions, and to "use sociocultural theories to help interpret their findings" (p. 111).

If both the curricula and teaching practices used in adult mathematics classrooms are based solely on those of school-based education, then there is a strong possibility that adults are expected to repeat the approach to mathematics education that they faced when they were children. This situation may persist in spite of the myriad studies which have repeatedly identified that exposure to inappropriate curricula and poor teaching practices in mathematics education is a key source of adult innumeracy.

In an attempt to address this, my study explores the teaching processes in mathematics education for adults. In particular, it examines how mathematics education is viewed by those involved in it, and how such education is shaped by certain social and institutional forces. It seeks descriptive accounts of teaching processes in mathematics education from those missing perspectives, and relates those accounts to the ways in which teaching processes are influenced by external factors. By concentrating on descriptive accounts of teaching, I have been able to access "the specifics of action and of meaning-perspectives of actors [which are] often those...overlooked in other approaches to research" (Erickson, 1986, p. 124).

I have used the term teaching processes generically to refer to all that "goes on" in the classroom. Thus, teaching processes include the selection of content to be taught; the choice of such techniques as lectures or groupwork; the expectations, procedures and norms of the classroom; and the complex web of interactions between teachers and learners, and between learners themselves.

In an effort to illuminate the realities of mathematics education for adults, my study will consider the following broad questions:

(1) What happens in adult mathematics classrooms?

(2) What do these phenomena mean for those involved as teachers or learners?

(3) In what ways do certain factors beyond the teachers' control affect teaching processes?

Structure of the Dissertation

In Chapter Two, I present, first, a survey of the literature on teaching in general, and on mathematics in particular; and second, my theoretical framework derived from a synthesis of the literature on teaching. In this chapter, I also discuss some of the implications of combining macro and micro approaches to social research.

In Chapter Three, I present the methodological design of the research. I provide, in turn, details of the data sources, data gathering methods and procedures, and data analysis and interpretation procedures.

In Chapter Four, I describe the background elements to my study. I discuss the settings where mathematics education takes place, the people involved in those settings, and the work that they do.

In Chapter Five, I focus specifically on the teaching processes in mathematics classrooms. I describe situations and episodes that are both typical and common in mathematics education in general, and on the teaching of algebra in particular. Here I include data from both my own observations, and from the perspectives of those involved as teachers and students.

In Chapter Six, I analyze these teaching processes using concepts from my theoretical framework. I identify certain frame factors in adult mathematics education and examine their effects on teachers' thoughts and actions.

Finally, in Chapter Seven I summarize my study and discuss certain of its limitations. I also provide certain recommendations both for further research and for improving the teaching of mathematics to adults.

Teaching is a social and political process, and therefore is subject to social and political influences. Consequently, a thorough explanation of teaching processes must have a conceptual framework that relates teaching processes to decisions taken in the social and political arena. Such relationships are, however, often invisible and unarticulated by the people involved. Rather than pinpoint them accurately, a researcher can discern and record their effects by both observing teaching processes as they unfold in their natural settings and by examining them from the perspectives of those involved as teachers or learners.

In this chapter I describe the theoretical tools I used to so investigate teaching processes in mathematics education. First, I examine the literature on mathematics education for adults. Next, I turn to the adult education research literature, and then to the wider research on teaching in general. I categorize this research into three paradigms, and, for each, provide a brief overview of research on teaching in general, and on the teaching of mathematics in particular. During this discussion I introduce the two general domains of research which inform my own study: research on frame factors and on teachers' thinking. These two domains can be seen as representing quite different approaches to the examination of social reality, and indicate that, if considered separately, teaching processes can be regarded as both independent of, and dependent upon, their broader social context. I next explore the differences between these approaches in terms of the macro/micro dichotomy and the related issue of structure and agency, and I indicate how this study, which combines both macro and micro approaches, resolves these issues. Finally, I provide a model of the theoretical framework of this study and briefly discuss the elements of this model.

Despite the wealth of information available on the mathematical abilities of, and education for, school children of various ages, relatively little exists on that for adults. Within the English-speaking world, only Cockcroft (1982), ALBSU (1983), Statistics Canada (1991), and Kirsch, Jungeblut, Jenkins, and Kolstad (1993) provide detailed studies. Cockcroft's (1982) survey was based on observations of almost 3,000 British adults taking a test on their everyday or "practical" mathematics skills. The findings were supported by further evidence from a study conducted a year later, using data from the National Child Development Study that was based on interviews from 12, 500 23-year olds (ALBSU, 1983). The Canadian report (Statistics Canada, 1991) was based on interviews with, and the testing of, almost 10,000 adults. Finally, the most comprehensive study (Kirsch et al., 1993) was based on interviews with, and surveys of, over 25,000 US adults. All four studies reported that a significant proportion of adults had problems with numerical calculations and cited difficulties in their everyday lives arising from these problems.

These studies provide valuable information on adults' mathematical abilities, and on their attitudes towards, beliefs about, and uses of mathematics. Much of this research, however, has had little discernible impact on mathematics education for adults, which continues to be based on research on the learning of mathematics by schoolchildren (Faris, 1992; Gal, 1992). Many adult educators make strong distinctions between adult and pre-adult education. Löthman (1992), in particular, identifies those distinctions relating to mathematics education. Two major distinctions that affect teaching processes in mathematics education for adults are those that concern adults' beliefs about and attitudes towards mathematics, and their everyday uses of mathematics.

Beliefs and Attitudes about Mathematics

One key skill required of an adult educator is to determine the existing concepts, beliefs and attitudes held by adult learners (Brookfield, 1986). This is no less important in mathematics education than elsewhere. Several authors (e.g., Buxton, 1981; Paulos, 1988; Quilter & Harper, 1988) stress that recognizing and acknowledging adults' beliefs and attitudes about mathematics is key to encouraging learning. There is evidence to suggest that long before many children leave school they have adopted a view of mathematics as a cold, mechanical subject with little relation to "real life" (Paulos, 1988). Such children do their best to avoid mathematics wherever possible, and manifest anxiety when faced with even simple arithmetic problems. As these children grow into adults they "manage to organize their lives so [that] they make virtually no use of mathematics" (Steen, 1990, p. 215).

In discussing the beliefs and attitudes that adults have about mathematics, several commentators (Buxton, 1981; Michael, 1981; Paulos, 1988; Tobias, 1978) have developed the concept of "mathematics anxiety." This has been described most rigorously by Michael (1981) as "a psychological state engendered when a person experiences (or expects to experience) a loss of self-esteem in confronting a situation involving mathematics" (p. 58). Much of the discussion of mathematics anxiety locates the problem and seeks to remedy it at the individual level (for example, by suggesting that sufferers keep journals or work through self-paced learning material). There are few suggestions for practical classroom activities.

Löthman's study also found that adults were able to learn mathematics better if they could relate what they were learning to their everyday lives. Consequently, I now examine studies on the relevance of mathematics to peoples' daily lives.

Because most research on the learning of mathematics is based on research in school-based education, mathematics education tends to be defined in terms of a school situation. However, in recent years, there have been a number of studies of the use of mathematics in the work and everyday life by adults of specific occupations and cultures (e.g., Carraher, Carraher, & Schliemann, 1985; Lave, Murtagh, & de la Rocha, 1984; Millroy, 1992; Scribner, 1985). For example, Millroy (1992) studied the uses of mathematics by a group of carpenters in order to document the ideas that were "embedded" in their everyday woodworking activities. She found that, although the carpenters had received very little formal mathematics education, they demonstrated tacit mathematical understanding in their actions. They were fluent with, and made extensive use of, such conventional mathematical concepts as congruence, symmetry, and proportion, and such skills as spatial visualization and logical reasoning.

These studies are part of an emerging area of study in mathematics education that adopts a more anthropological approach in order to explore and describe the mathematics that is created in different cultures and communities. D'Ambrosio (1991) uses the term "ethnomathematics" to describe "the art or technique of understanding, explaining, learning about, coping with, and managing the natural, social, and political environment by relying on processes like counting, measuring, sorting, ordering, and inferring" (1991, p. 45). Ethnomathematics, which links cultural anthropology, cognitive psychology, and mathematics, can challenge the dichotomy between "practical" and "abstract" mathematical knowledge. It forces learners to consider others' thinking patterns, to re-examine what has been labeled "non-mathematical," and to reconceptualize what counts as mathematical knowledge (Frankenstein & Powell, 1994).

The work of several distinguished mathematics educators can illuminate these different understandings of mathematics. Ascher (1991) looks at the mathematical ideas in the spatial ordering and numbering system used by the Inca people in South America. Gerdes (1988) focuses on the mathematics "frozen" in the historical and current everyday practices of traditional Mozambican craftsmen, whose baskets, weavings, houses, and fish-traps often demonstrate complex mathematical thinking, as well as the most efficient solutions to construction problems. Pinxten (1983) examines spatial concepts in the cultural traditions of the Navajo people. Unlike Western people who tend to regard the world statically and atomistically, the Navajo have a more dynamic and holistic worldview which fundamentally influences their notions of such geometrical concepts as points, distance, and space.

Turning to the dominant culture within North America, Lave (1988) considers the mathematical experiences inherent in common workplace and domestic activities. In one example, she compares adults' abilities to solve arithmetic problems arising while grocery-shopping in a supermarket with their performance on similar problems in a pencil-and-paper test. The participants' scores on the arithmetic test averaged 59%; in the supermarket they managed to make 98%--virtually error free. Lave argues that test-taking and grocery-shopping are very different activities, and people use different methods in different situations to solve what can be seen as similar arithmetic problems.

Drawing upon the mathematical traditions present in different cultures, and basing mathematical activities on adults' day-to-day experiences of their social and physical environments broadens the traditional and often narrow approach of much mathematics education. Furthermore, this brings the learning of mathematics "into contact with a wide variety of disciplines, including art and design, history, and social studies, which it conventionally ignores. Such a holistic approach...serve[s] to

augment rather than fragment [learners'] understanding and imagination" (Joseph,
1987, p. 27).

Although these studies show that different forms of mathematics are generated by different cultural groups, they can still be seen as the result of broadly similar activities. Bishop (1988) identifies six fundamental mathematical activities which he regards as universal, necessary, and sufficient for the development of mathematical knowledge:

> *Counting*: the use of a systematic way to compare and order discrete objects [involving] body- or finger-counting, tallying, using objects...to record, or [using] special number words or names.
>
> *Locating*: exploring one's spatial environment and conceptualizing and symbolizing that environment with models, diagrams, drawings, words, or other means.
>
> *Measuring*: quantifying amounts for the purposes of comparison and ordering, using objects or tokens as measuring devices.
>
> *Designing*: creating a shape for an object or for any part of one's spatial environment.
>
> *Playing*: devising and engaging in games and pastimes, with more or less formal rules.
>
> *Explaining*: finding ways to represent relationships between phenomena. (pp. 182-183)

All of these anthropological studies document the distinctive character of the mathematical skills and procedures used in work and everyday life, as compared with those taught in school mathematics. They also highlight the success of such procedures when used in particular contexts. However, the use of these procedures outside of the specific situations where they normally occur is problematic. For example, street vendors who successfully perform many relevant calculations daily in their heads find "similar" calculations to be performed with pencil and paper, outside the context of the street market, exceedingly difficult, and make many more errors (Carraher et al., 1985).

Sewell (1981) studied this phenomenon in considerable detail. Her study relied upon interview data designed to cover four areas: a discussion of selected situations related to shopping and household tasks, in which mathematics might be involved; a discussion of other matters such as reading timetables and using calculators; attitudes to mathematics; and background information. Initially, 107 adults, chosen to reflect the range of expected mathematical abilities and of occupation, were interviewed. Next, follow-up interviews of greater length were conducted with about half of those who had taken part in the first stage. Those interviewed were invited to answer a series of questions about a range of mathematical situations. Of these questions, some involved calculations, others required an explanation of method but no calculation, and others required the explanation of information expressed in mathematical terms.

Sewell's findings indicate that there are many adults who are unable to cope confidently and competently with any everyday situation that requires the use of mathematics. Further, she found that the need to use mathematics could induce feelings of anxiety, helplessness, fear, and guilt. These feelings were especially marked among those with high academic qualifications, and who, consequently, felt that they ought to have a confident understanding of mathematics. Further findings included a widespread sense of inadequacy amongst those who felt they either had not used the proper method, obtained the exact answer, or performed with sufficient speed when solving mathematical problems.

These studies of adults' attitudes towards, and their daily uses of, mathematics hold rich information about how people learn and relate to mathematics; information that can have many implications for adult educators. In particular, research on teaching mathematics can be informed by what these studies reveal about learning. I now turn to the adult education research literature to examine how these issues have been studied.

Given the wealth of information on adults' attitudes towards, and daily uses of, mathematics, research on the teaching of mathematics to adults is, surprisingly, almost non-existent. Indeed, specific adult education research on teaching in general is limited; teachers and teaching are not the main focus of discussions about adult education practice. For example, the most recent <u>Handbook of Adult and Continuing Education</u> (Merriam & Cunningham, 1989) cites no references to either teachers or teaching in its subject index. Further, of the key surveys of the developments in adult education research, theory, and practice (Jensen, Liveright, & Hallenback, 1964; Long, 1983; Peters, Jarvis, & Associates, 1991) only that of Long contains any discussions of research on teaching. Although the adult education literature includes a variety of approaches to teaching (e.g., Apps, 1991; Beder & Darkenwald, 1982; Brookfield; 1986; Conti, 1985; Daloz, 1986; Galbraith, 1990; Hayes, 1989; Johnson, 1993; Pratt, 1992, in press; Renner, 1993; Rogers, 1986; Seaman & Fellenz, 1989; Wlodkowski, 1986), many are written simply as guides for practitioners and concentrate on describing strategies and tactics for improving adult learning.

When describing the teaching/learning process, adult educators tend to focus largely on learners and their learning; several authors (e.g., Apps, 1991; Brookfield, 1986; Knowles, 1980; Knox, 1990) pragmatically define teaching or the teaching process solely as the process of facilitating or helping adults learn. This view has led to further empirical research that has contributed towards developing principles of good teaching practice (e.g., Ampene, 1972; Beno, 1993; Beder & Carrea, 1988, Conti, 1985, Conti & Fellenz, 1988; James, 1983; Suanmali, 1981). To take one example, Conti (1985) sought to synthesize the work on adult learning into some central

principles and then designed a research instrument to examine the extent to which these principles were exemplified in practical settings. Much of this work, however, although adding to the corpus of research on teaching, has remained theoretically plurative. In addition, these studies tend to regard adult education in general, and because few are based on empirically collected data in adult classrooms, downplay the influence of subject-matter or situational context. Yet, as Anyon (1981) and Stodolsky (1988) show, these factors can strongly influence teaching practices.

Turning to the published studies of the teaching of mathematics to adults (e.g., Buerk, 1985; Buxton, 1981; Frankenstein, 1987, Höghielm, 1985; Kogelman & Warren, 1978, Löthman, 1992), only those of Löthman and Höghielm fully consider the teaching of mathematics in formal settings. (The others describe particular courses set up for specific groups of people, or to tackle specific issues.) Löthman's study makes several theoretical and methodological contributions to my own study, and I discuss it more fully later.

Höghielm (1985) investigated mathematics teaching in Swedish municipal adult schools to determine the extent to which teaching was in accordance with certain principles of adult education. These principles were codified in a Swedish Government Bill as "the most appropriate ideals for the teaching of adults." They include:

> Teaching must emanate from the participants' experiences of life, [it] must develop the individual socially, [it] must be problem-oriented, the techniques must be based on an interchange of experience, participants must exert definite influence on the planning of the course and the conduct of teaching, and evaluation must comprise a mutual (teacher-participant) measurement of course content and planning. (Höghielm, 1985, pp. 207-208)

Apart from these latter two studies, a pragmatic approach is common (at least within North America) to adult education research in general. Further, the adult education field has also suffered from a lack of theoretical sophistication and rigor. A recent collection of different perspectives towards adult education research

(Garrison, 1994) identifies how the field of adult education has suffered from a lack of overall focus. As Blunt notes,

> Meetings of adult education researchers and their discussions about how research ought to be conducted...and disseminated...are characterized by division [and] disagreement....The differences...also extend to disagreements over what research problems ought to be identified as priorities and the usefulness of the research results produced to date. (1994, p. 168)

What is certain about recent adult education research is the diversity of its methods, approaches, topics for study, and purposes. Rubenson (1982, 1989) has identified how North American adult education research has focused more on pragmatic program needs and pedagogical concerns than it has on theory development or policy-related issues. Also, much of the research has been dominated by a narrow reliance on a psychological approach, rather than on anthropological, historical, philosophical, or sociological approaches; consequently, adult education research in general has not been well informed by these different approaches or disciplines. Much current adult education research appears to be unconcerned with developing a stronger theoretical base or with drawing upon research in other disciplines.

The result of such a pragmatic and atheoretical approach within adult education is that it hasn't contributed substantially to the wider field of educational research, or indeed, to wider economic and political issues and questions. Further, it has attempted to deal, on an individualistic and local basis, with the effects of social and political processes, but has not effectively addressed the causes of them in any meaningful way. Given that adult education is heavily influenced by social and political forces, this seems an unexplored opportunity for the field.

As a source for theoretical exploration for the exploration of teaching processes, then, the adult education literature is barren. Consequently, I have to now turn to the wider literature on teaching in general.

Research on teaching has gone through several periods of change. During the past 80 or so years, this research has become successively more comprehensive and complex in its foci of study, theoretical sophistication, and methodological rigor. Rosenshine (1979) and Medley (1979) both present historical overviews of research on teaching, describing the changes in terms of cycles or phases. For Rosenshine, research on teaching initially focused on teacher personality and characteristics, then on teacher-student interactions, and finally on student attention and subject content. Medley presents a similar view, and categorizes research as focusing first on characteristics of effective teachers, then on the methods they used, next on teacher behaviors and classroom climate, and finally on teachers' competencies.

In many ways, these stages can be seen as representing different paradigms. In each stage, different schools of thought, assumptions, and conceptions have been dominant, which has led, in turn, to different goals, starting points, methods, and interpretations for research (Shulman, 1986). Three distinct paradigms can be discerned in the development of research on teaching, which I have chosen to label as behaviorist, structuralist, and interpretivist. Although these developments in approach broadly correspond to historical periods, they are not uniquely tied to them. For example, although the positivist period saw a predominant focus on psychological research models, much psychological research is currently being conducted in the more interpretive tradition, influenced by recent developments in cognitive psychology. I now discuss each of the three stages or paradigms in turn, describing a general overview of its research foci and key ideas on teaching, followed by more specific examples of research on teaching mathematics.

Until the 1970s, almost all research on teaching was behaviorist and empirical in nature and based on the positivist perspective. Textbooks on research strategies (e.g., Kerlinger, 1973; Travers, 1970) regarded educational research as an "objective" enterprise, and concentrated on describing appropriate research methods designed to formulate and verify particular hypotheses. It is not surprising, therefore, to discover that reports of studies on teaching from this period were substantially empirical and used such techniques as experiments and surveys to produce solely quantitative data. For example, Gage's (1963) handbook of research on teaching contained no section on participant observational research. Further, Dunkin and Biddle's (1974) comprehensive survey of studies on teaching contains only reports of research that employ quantifiable measures; it mentions no others from more qualitative or interpretivist perspectives (Shulman, 1986).

Such research tended to concentrate on the development of normative laws or models about educational goals, content, and methods of instruction, and was primarily based on psychological perspectives, particularly that of behaviorism. Consequently, theories and models about teaching in this period were principally derived from individualistic approaches, and were bound to, or reduced to, phenomena about learning and cognition (Lundgren, 1979).

One key emphasis of this paradigm of research is the question of how knowledge about learning can affect teaching practices. For example, both Thorndike (1923) and Skinner (1968) argued that ideas about teaching should be based on theories about learning. Because so much of this work is conducted from a purely behaviorist perspective, it therefore focuses on the outcomes of learning rather than on how learning occurs. Consequently, most research studies have been designed to investigate what changes in teaching could produce measurable benefits

in student learning. For specific examples of this, I now turn to the research on teaching mathematics.

Behaviorist Research on Teaching Mathematics

Within what I am calling the behaviorist paradigm, the research on teaching mathematics has been based almost exclusively on theories or assumptions about how children learn. A key emphasis has been its pragmatic focus on what makes such teaching more efficient or effective; namely, what improves student achievement. Also, most of the studies in this paradigm on teaching mathematics have focused solely on the behaviors of teachers rather than on those of learners and/or on the lesson content. They have sought to illuminate student learning only in light of teachers' actions; the "culture" of the classroom, lesson content, or student behavior or understanding have been of little consideration.

Perhaps because of its predominantly behaviorist approach, most research on the teaching of mathematics within this paradigm has tended to isolate a particular "variable" and determine its effects as it was experimentally controlled. Reviewing several studies gives a flavor of the research: how children learn numerical operations (Bell, Fischbein, and Greer, 1984); the stages of children's learning (Donaldson, 1978); time spent on task (Peterson, Swing, Stark & Waas, 1984); and seatwork (Anderson, 1981). In general, by focusing on the learning of individuals, such research has attempted to search for clusters of common characteristics from which to generalize about particular types of teachers or learners, and to offer predictions for successful ways of teaching mathematics (Nickson, 1992).

Romberg and Carpenter (1986), in a summary of reviews of recent research studies on the teaching of mathematics in this behaviorist paradigm, identified several overall conceptual aspects that concern them about this research. First, they

found that much research suffered from inadequate conceptualization, and was

theoretically weak and haphazard in its choice of which teaching behaviors to study.

Researchers used "different labels for the same behavior, or the same label for

different behaviors, [and] different coding procedures which yield[ed] different

frequencies" (p. 860).

Second, lacking substantive theories of teaching, researchers tended to focus

on methodological questions. As most research was of an experimental design,

researchers then concentrated on "improving research designs, providing better

operational definitions of variables, or devising more adequate procedures for

counting behaviors, and better techniques of statistical analysis" (p. 860). Such a

concentration not only limited the kinds of problems addressed but also the ways in

which they were conceptualized.

Third, most studies were regarded as being too "global" in that they

disregarded the content of lessons. For example, researchers tended to ignore the

specific content of what was being taught to specific sorts of students, or assumed

that it lay outside the scope of inquiry. Romberg and Carpenter describe an earlier

study (Romberg, Small, & Carnahan, 1979) that "located hundreds of studies that

assessed the effectiveness of almost every conceivable aspect of teaching behavior,

but found few models of instruction that included a content component" (p. 861).

Fourth, researchers tended to categorize student learning as the dependent

variable. Further, in order to operationalize notions of students' achievements and

attitudes, researchers relied overwhelmingly on standardized achievement tests.

However, as Romberg and Carpenter say

> Such tests have serious problems. They rarely reflect what was taught in any
> one teacher's classroom; when used with young, bilingual, or lower
> socioeconomic status children, they may yield biased results; and at best, they
> indicate only the number of correct answers produced by a student, not how
> a problem was worked....Their use merely compounds the problems when
> there is a lack of concern for the content being taught. (p. 861)

Romberg and Carpenter also discuss four major findings of their review of research. The first concerns the variability of teaching practices. As they describe it, "Every day is different in every classroom [and] every classroom is different from every other classroom" (p. 861). The variability extends to teachers' and students' behaviors, texts, time allocated, and content coverage. However, despite these several variations, the dominant pattern of teaching practices, in a wide range of classrooms, was "to emphasize skill development via worksheets, not to select activities that encourage discussion and exploration" (p. 862).

The second finding concerns the time available for instruction. Repeatedly, studies showed that, while there were limits on the amount of time available for mathematics instruction, those teachers who consistently devoted less time to teaching mathematics than did their colleagues experienced poorer student achievement. Further, studies showed that the time available for mathematics is most effective when it is well-used in terms of its content coverage, episodic nature, and interactive engagement. "Students should be engaged in activities that are reasonable and intentional....Lessons and units should have a...start, a development, a climax, and a summary....Finally, [students should] be...interacting with ideas" (p. 863).

The third finding was that student learning was increased if teachers devoted part of each lesson towards increasing students' comprehension of skills and concepts. If teachers helped students relate new ideas to past and future ideas, then both student engagement and achievement was increased. This process was also increased if students were required to work in small groups. Those students who studied in small groups were found to be not only more cooperative and less competitive than their peers, but also to have a greater comprehension of how ideas were linked (Noddings, 1985; Weissglass, 1993).

A fourth finding concerns classroom management. Although the primary purposes of teachers' behaviors were to cover the assigned content and get their students to learn something, they were also designed to maintain classroom order and control. For example, teachers would occasionally adapt materials not to increase students' potentials for learning, but to better manage their classroom. Teachers would thereby curtail the time available for students to invent, explore, and apply mathematical relationships. Further, the teachers' approach to textbooks was also significant. Throughout most studies, teachers would promote the textbook "as the authority on knowledge and the guide to learning" (p. 867). Although teachers could have departed from the syllabus, Romberg and Carpenter found that they chose to do so only to increase their classroom control.

In sum, the behaviorist approaches to research on teaching mathematics and teaching regarded it, in general, as a predominantly one-to-one activity between a teacher and a student. Missing from this approach was any adequate conceptualization of education that linked teaching with more cultural, political, and social factors. In addition, there was no research focus on the specific nature of occurrences and events, let alone the meanings that these events had for the people involved. These two areas were developed more in subsequent research approaches. Next I consider a paradigm that sought to illuminate the links between education and social influences.

The Structuralist Paradigm

In the late 1960s and early 1970s, educational researchers began to adopt more sociological approaches in their studies of teaching. This research tended to fall into two contrasting positions about how issues were approached and interpreted: the

consensus and conflict perspectives. Briefly, the consensus perspective is based upon the notion that "societies cannot survive unless their members share at least some perceptions, attitudes and values in common" (Rubenson, 1989a, p. 53). Education is regarded as, first, an agent of socialization into the broadly-accepted values of society, and, second, a means of selection of individuals for particular societal roles based upon performance and achievement. Here, inequality is seen as inevitable, and both necessary and beneficial to society.

Alternatively, the conflict perspective, with its roots in the work of Marx, Durkheim, and Weber, questions whether inequality must be inevitable or necessary. By focusing more on the interests of various groups and individuals within society (rather than on society as a unified whole), conflict theorists emphasize "competing interests, elements of domination, exploitation and coercion" (Rubenson, 1989, p. 54). The conflict perspective also promoted critical analysis of the roles and functions of education in society.

Interweaving education and its function in society was hardly new; in 1916, Dewey identified educational institutions as promoting and reproducing the dominant values of society. However, the radical critics of the 60s and 70s challenged the dominant liberal view of education as merely offering opportunities for individual development, social mobility, and a redistribution of political and economic power. They argued instead that the main function of education is to reproduce the dominant cultural and political ideology, its forms of knowledge, and the social division of labor (Aronowitz & Giroux, 1993). Before turning to particular studies on teaching mathematics that adopt this approach, I first outline some analytic tools which commentators have used to describe how society influences education.

To radical critics, education has several other functions that are not expressed in curricular content, and which often remain invisible to those involved. For example, within North America, several authors promote the idea that schools and other educational institutions exist to "colonize" students into accepting the culture, values, norms, purposes, and goals of the dominant class. This view is fully explored theoretically in the work of Apple (1979), Bowles and Gintis (1976), Carnoy and Levin (1985) and the early work of Giroux (1981).

Most of these views about the roles and functions of educational institutions in society are based on the earlier work of Althusser (1971) and Gramsci (1971), both of whom emphasized how educational institutions transmit and maintain society's dominant ideologies. In particular, they both identified how the needs of the dominant culture shape the provision and form of education to produce "hegemonic" knowledge and ways of thinking. From a slightly different perspective, Bourdieu (1977) argues that education is better understood in terms of more general stratifying processes. In contrast to Althusser and Gramsci, Bourdieu regards educational institutions less as agents of state control, and more as relatively autonomous bodies that are indirectly influenced by more powerful economic and political institutions. He maintains that various types of "capital"—either economic (money, objects), social (positions, networks), cultural (skills, credentials), or symbolic (legitimating codes)—are distributed unequally based on social class. For each class, there is a distinct culture--"habitus"--which is the collection of largely unconscious perceptions, choices, preferences, and behaviors or members of that class. Children learn within their habitus, acquire capital from their parents and from peers, acquire academic credentials (one form of cultural capital), and then, in turn, exchange this for other forms of capital. Thus, educational credentials become

one of the key media for the purchasing and exchanging of one kind of capital for another.

By seeking to align actual classroom processes with the ways that education functions within society most of these radical researchers have emerged using a largely structuralist (or "macro") approach. This approach assumes that societal influences determine classroom behavior. As such, these researchers have focused on large-scale theoretical explanations of the relationship between schooling and society (e.g., Bowles & Gintis, 1976), or certain aspects of social structures (such as gender, ethnicity, or class) as if they were causal variables (e.g., Young, 1971). In studies such as these, the freedom of action that people have within classroom situations, or the meanings they make about those situations, are largely downplayed or even ignored.

As an alternative to these large-scale approaches, other researchers have considered small-scale studies of individual schools, teachers, or specific classroom interactions (e.g., Ball, 1981; Donovan, 1984; Hammersley & Woods, 1984). These "micro" approaches have typically focused on individual actors, regarding them as autonomous actors in situations and subject to few outside constraints. These approaches are more concerned with the subjective meanings that actors hold about the particular situations in which they find themselves, and the human actions and interactions that take place there.

These two approaches have tended to be polarized and regarded as incompatible; researchers have, in general, adopted either one approach or the other. There have been few attempts to reconcile the macro-micro issue, or to design research that bridges both perspectives. Hargreaves (1985) notes that although the macro-micro issue has been the subject of a great deal of theoretical debate, it has not resulted in much empirical research.

However, a comprehensive examination of any social phenomenon--such as

teaching--cannot be limited to a set of either external (macro) or internal (micro)

explanations or theories. Teaching can neither be reduced to psychological principles

or laws of learning, nor can it be seen as simply determined by contextual factors. To

be thorough, a study must attempt to bridge these two perspectives and incorporate

both macro and micro approaches. Dahllöf (1977) summarizes some characteristics

of what such a model and a methodology would include:

> Data are curriculum-related, reflecting the goals and intentions of the
> instructional program as well as the ambitions of the teacher.
>
> Data are related to basic patterns of teaching...and reflect the cumulative
> character of the teaching process and its long term effects.
>
> Data mirror the teaching process as a continuous change of perceptions and
> behaviors over time towards certain goals.
>
> The analysis [considers] that the teaching of a certain curriculum unit
> generally run[s] through a series of phases like presentation, training, and
> control--each phase with its own characteristic pattern [of] communication
> and interaction.
>
> Data are dynamic...in that they relate in a meaningful way to the restrictions
> that are imposed upon most teaching situations by frame factors like space
> and time, [and] they try to describe and do justice to the role played by
> students in the teaching situation and its different phases. (p. 406-407)

"Frame factor" theory is one particularly useful tool of analysis that meets

Dahllöf's criteria and integrates both the macro and micro approaches. Because it

bears on the theoretical framework developed for this study, it warrants detailed

examination here.

Frame Factor Theory

Frame factor theory (Bernstein 1971, 1975; Dahllöf, 1971; Lundgren, 1977,

1981) analyzes the ways in which teaching processes are chosen, developed, and

constrained by certain frames. In contrast to research in the behaviorist paradigm

that investigated teaching processes by examining how changes in teachers' behavior affected student learning, frame factor theory is more concerned with exploring how teachers' actions are limited by external forces.

Briefly, a frame is "anything that limits the teaching process and is determined outside of the control of the teacher" (Lundgren, 1981, p. 36). Examples of frames include the physical settings of teaching, curricular factors such as the syllabi or the textbooks used, and organizational influences such as the size of class or the time available for teaching. Frame factor theory claims that teaching processes are governed by "the possible scope of action which exists in a given situation" (Lundgren, 1983, p. 150). The frames mark out the limits that teaching processes have; the actual teaching is conducted within those limits.

The concept of frames as a constraint on teaching processes was first developed by Bernstein (1971, 1975) and Dahllöf (1971). Bernstein refers to a frame in the "form of the context in which knowledge is transmitted and received...the specific pedagogical relationship of the teacher and [the] taught" (1971, p. 50). He explains that frames refer to the degree of control that teachers and learners "possess over the selection, organization, and pacing of the knowledge transmitted and received in the pedagogical relationship" (p. 50). Dahllöf describes frames more broadly, extending Bernstein's earlier notion to include the decisions made about teaching that are outside of the teacher's and the student's control. Dahllöf's usage therefore links the macro- and micro- aspects of analysis in a way that Bernstein's does not.

Lundgren (1972) conducted a study of students grouped by ability in Swedish high-school classrooms using Dahllöf's definition of frames. He developed a model of three types of frame factors: the goals or objectives of teaching a particular subject area, the sequence of content units (lessons) through which the goals were to be achieved, and the time needed by students to master the content. Each student

needed different amounts of time to learn new material, and this was related to what

content was being taught and how it was taught. Lundgren found that, in order to

deal with those situations in which there was insufficient time to teach the required

content to all the students, teachers created a "steering group" of students. When

having to choose whether to continue with a particular topic area or whether to

move on to the next, even though not all of the students had fully learned the

existing material, teachers would base their decision upon the demonstrations of

ability from those students in the steering group.

Following his study, Lundgren further developed the notion of frame factors.

Recognizing that any society and the educational systems it promotes are

inextricably linked, he argued that because the cultural, political, economic, and

social structures of society have an effect on education, they can be regarded as

frames, and therefore studied in research on teaching situations. Institutions such as

schools and colleges

> promote learning in terms of postulated knowledge, skills, attitudes, and
> values. Legislation and rules prescribe the form of this institution, while the
> available resources in terms of personnel, teaching aides and composition of
> students determine how the actual teaching corresponds to the formal goals
> and regulations. (1979, p. 20)

Hence, for Lundgren, frames are the realization of fundamental structural

conditions. In his own studies he identified that time, curricula, regulations,

personnel, teaching aids, and the composition and size of classes act as the most

visible frames that govern and constrain the teaching processes. In his later work

(noted in Elgström and Riis, 1992), Lundgren has also included more conceptual

constraints in his notion of frame factors. Thus, personal competencies, attitudes,

values, and beliefs can also be regarded as frame factors.

Linking of the minutiae of classroom activity with larger social processes is

integral to frame factor theory. Stable patterns of classroom interactions can be

discovered by studying teaching processes, and then seen as "realizations of

underlying rules that shape and steer the process....As society is governed by certain rules for interpersonal relations and by social perceptions, teaching is governed by frames and perceptions that functionally form the rules for the participants" (Torpor, 1994, p. 2375).

Since then, particularly in Swedish educational research, frame factor theory has been regularly applied to classroom studies (e.g., Englund, 1986; Gustafsson, 1977; Kallós & Lundgren, 1979; Pedro, 1981) at both preschool and high school levels. It is ideal for research that seeks to analyze teaching processes in terms of their links with more structural elements. The factors governing, steering, and controlling teaching processes are always subject to change, so, as Torper (1994) puts it, "Frame factor theory with its wide scope and its ambition to encompass the deep structures of society, is well suited to the task of analyzing these processes" (p. 2376).

Structuralist Research on Teaching Mathematics

Although the structuralist orientation to research on teaching is theoretically rich, empirical studies of teaching from this approach are more rare. However, Lerman (1990) and Anyon (1981) each provide a specific example from mathematics education. Lerman initially identified several predominant views in general society about mathematics and their possible influence on mathematics education, and then conducted a field study among mathematics teachers to explore some of the issues arising from his theoretical perspective. He found that teachers' conceptions of mathematics clearly affected their teaching.

Anyon studied mathematics teaching in five schools at different socio-economic levels and found that, although all the schools used the same textbooks, the teaching differed dramatically. Teachers in the two working-class schools focused on procedure without explanation or attempts at helping students

understand. Teachers in the middle-class school attempted more flexibility and made some efforts towards developing student understanding. At the "professional-level" school, teachers emphasized discovery and experience as a basis for the construction of mathematical knowledge. Finally, teachers at the "executive-class" school extended the discovery approach, and used enhanced instruction on problem-solving and encouraged students to justify their answers to demonstrate their mastery of the concepts.

Although structuralist studies of teaching mathematics are infrequent, similar approaches to mathematics education in general are more common. Several mathematics educators (e.g., Evans, 1989, Fasheh, 1982; Frankenstein, 1981, 1987, 1989; Mellin-Olsen, 1987) are interested in the "culture" and values that are transmitted in traditional mathematics education. They note that the curricula and commonly used teaching methods are designed to reproduce the existing economic, status, and power hierarchies, and socialize learners into accepting the status quo. To these educators in particular, the traditional mathematics curriculum consists of an abstract and hierarchical series of objective and decontextualized facts, rules, and answers. Much of this curriculum covers a fixed body of knowledge and core skills largely unchanged for centuries. It is based on the assumption that learners absorb what has been covered by repetition and practice, and then become able to apply this knowledge and these skills to a variety of problems and contexts.

Further, they regard teaching methods in traditional mathematics education as using largely authoritarian and individualizing techniques that depend on memorization, rote calculation, and frequent testing (Bishop, 1988). These methods convince learners that they are stupid and inferior if they can't do simple calculations, that they have no knowledge worth sharing, and that they are cheating if they work with others. When education is so presented as a one-way transmission of knowledge from teachers, mathematics can be regarded merely as collections of

facts and answers. Knowledge is seen as largely separate from learners' thought

processes, and education is experienced as a static, rather than a dynamic, process.

As Frankenstein describes it, much mathematics teaching is

> based on what Freire calls "banking" methods: "expert" teachers deposit knowledge in the blank minds of students; students memorize the required rules and expect future dividends. At best, such courses make people minimally proficient in basic math and able to get somewhat better paying jobs than those who can't pass math skills competence tests. But they do not help people learn to think critically or to use numbers in their daily lives. At worst, they train people to follow rules obediently, without understanding, and to take their proper place in society, without questioning. (1981, p. 12)

Consequently, many learners of mathematics find themselves in classes in

which little effort has been made to place the subject matter in any meaningful

context. For many, mathematics remains a mystery unrelated to other subjects or

problems in the real world; they often come to regard mathematics as a subject

largely irrelevant to their own lives.

Other critics of traditional mathematics education have questioned its aims

and purposes. In general, two rationales are given for why mathematics should be

taught: (1) Mathematics is necessary for personal life and a prerequisite for many

careers; and (2) Mathematics improves thinking, because it trains people to be

analytical, logical and precise, and it provides mental exercise. Of course, these

rationales do not specify what mathematics should be taught, merely that some

mathematics should be. One could expect, therefore, that mathematics education

would differ substantially from place to place. It is surprising, then, that one

researcher discovered there was little diversity in mathematics classrooms the world

over (Willis, 1990).

Ernest (1990) notes that the aims of mathematics education in any location are

often discussed in isolation from any social and political content. Arguing that

education in society reproduces its social structure, he distinguishes three groups

who have distinct aims for mathematics education: mathematics educators,

mathematicians, and representatives of business and industry. To these, Howson and Mellin-Olsen (1986) add further categories of parents, employers, and those in higher education.

These authors claim that the aims of mathematics education are not decided on rational or educational grounds but on the basis of the power of these groups to effect change. For example, Ernest (1990) explains the changes in British mathematics education during the 1960s as a result of a struggle between certain groups he calls the "Industrial Trainers" (who emphasized a "back-to-basics" approach involving drills and rote learning), the "Old Humanists" (who were proponents of mathematics for its own sake, stressing its logic, rigor, and beauty), the "Public Educators" (who saw mathematics as a means to empower students to critically examine the uses of, and political and social issues surrounding, mathematics), the "Technological Pragmatists" (who believed in teaching mathematics through its applications and emphasized practical problems and utilitarian problem-solving skills), and the "Progressive Educators" (who emphasized student-centered teaching, active learning, creativity and self-expression).

The authors studying the aims of mathematics education argue that it is the form rather than the content which conveys those social aims. The ways that mathematics is taught "can emphasize and reinforce the values and relationships that underlie what is produced, how it is produced, and for whose benefit" (Cooper, 1989, p. 151). Cooper also quotes two earlier researchers (Stake and Easley, 1978) who found that teachers in their study

> saw science and mathematics as "heavily-laden with social values", and recognized that scientific and mathematical knowledge "may function more and more as a behavioral badge of eligibility for employment" and...wanted help in inculcating the work ethic values they saw as important in present society (p. 152).

Each of these authors considers only mathematics education for children and their arguments cannot be necessarily applied to adult mathematics education.

Education for children and adult education differ significantly in most Western countries. Löthman (1992), in particular, highlights the differences between adult and childrens' mathematics education and between how children and how adults wanted to be taught mathematics. She found that adults, in particular, wanted to be able to use the mathematics they learned, and, therefore, wished to be taught by practical methods. She therefore argues that there should be substantial differences between the classroom practices and the course content used in adult settings, and those used in childrens' education.

However, in most mathematics classes for adults, the curriculum appears to follow that of school-based mathematics education, itself largely determined by the requirements of college entrance boards. Within British Columbia, many adult learners of mathematics are "following the same curriculum and using the same materials as their youthful colleagues" (Faris, 1992, p. 30). Thus, it is possible that the teaching processes in mathematics education for adults closely resemble those in K-12 education.

All of these authors show how dominant views of mathematics affect how mathematics is considered and taught. In this way, the dominant conceptions of mathematics can be seen as frame factors influencing and restraining teaching processes in mathematics education. These examples from mathematics education, frame factor theory, and the structuralist approach to research on teaching all share a concern to explain how education functions in relation to social production, and how, in turn, social and political influences surface in educational settings. Lacking in this approach is much consideration for people as autonomous actors in situations. The structuralist paradigm tends to view people as being passively socialized into an institutional framework rather than "participating in their own conceptual constructions of the world and [their] own fate as a project" (Sharp & Green, 1975, p. 5). The third paradigm seeks to respond to this somewhat

functionalist approach to teaching by foregrounding the roles of autonomous actors within social structures.

The Interpretivist Paradigm

For many researchers, the structuralist paradigm is overly deterministic. For them, teaching is not merely the result of external factors but is also heavily influenced by what teachers think and do. Some researchers (e.g., McLaren, 1989, Willis, 1977) identified a need to document specific details of classroom interactions in order to understand how immediate and local circumstances reflected broader structural forces. Other researchers, disenchanted with structuralism, but even less captivated with the predominantly behaviorist approach to studying teaching, began to focus more on the specific nature of educational occurrences and events and the meanings that these have for the people involved. Their studies adopted qualitative or "interpretive" perspectives and studied teaching from ethnographic, participant observation, case study, symbolic interactionist, phenomenological, or constructivist approaches. While these several approaches differ from each other slightly, they all share a central research interest in discovering the meanings that people (whether participants or researchers) make about aspects of human life and human interactions.The rich variety of this research can be gleaned from considering the work of, for example, Fox (1983), Samuelowicz and Bain (1992), and Pratt (1992), and the studies in Marton, Hounsell, and Entwhistle (1984).

Rather than consider the general character and overall distribution of educational events and situations, interpretive studies of teaching focus more on the specifics of particular situations or events. Such studies deliberately focus on the perspectives of the people involved, and seek their meanings and interpretations

about their situations. By concentrating on specific situations and actions, and on the "local" meanings actors give to these, qualitative research has attempted to uncover the "invisibility of daily life" (Erickson, 1986, p. 121). Typical of the research in the interpretivist paradigm are studies concerning teachers' thinking. This includes such foci as teachers' beliefs about students and teaching, their thought processes while planning instruction, and the kinds of decisions they make during teaching. Because this research also informs the theoretical framework for my own study, it warrants close examination here.

Teachers' Thinking

A large part of the context of teaching consists of the thinking, planning, decision-making, and actions of teachers. Researchers from all three paradigms agree that teachers' classroom behaviors are substantially affected by their thinking, and deliberate teaching requires choices as to what and how to teach. The term "teachers' thinking" refers to those mental processes of teachers that involve perception, reflection, problem-solving, and the manipulation of ideas, and is concerned with how knowledge itself is acquired and used (Calderhead, 1987).

This research regards teachers as active and autonomous agents in teaching situations and seeks to explore new ways of conceptualizing and understanding teaching. Research has focused on, for example, the nature of teachers' knowledge (Zeicher, Tabachnik, & Densmore, 1987), the differences in the use of knowledge between novice and expert teachers (Berliner, 1987), teachers' conceptions (Pratt, 1992, Samuelowicz & Bain, 1992), teachers' planning (Clark & Yinger, 1979), teachers' thoughts, decisions, and behaviors (Shavelson & Stern, 1981), and teachers' theories and beliefs about students, teaching, learning, and subject matter (Clark & Peterson, 1986).

Clark and Peterson (1986) have developed a model that relates teachers' thoughts to their actions, considering such aspects as teachers' thoughts, decisions, theories, and beliefs. Their model is based on an interpretive perspective that addresses such questions as, for example, differences in meaning regarding learners' achievements, and regarding the teacher's role in classroom interactions. The model consists of two domains involved in the teaching process: teachers' thought processes and teachers' actions and their observable effects. The domain of teachers' thought processes includes teachers' planning, interactive thoughts and decisions, and theories and beliefs (about teaching, learning, students, and subject matter). The domain of teachers' actions and effects includes teachers' classroom behavior, students' classroom behavior, and student achievement. In both domains, the elements are seen as inter-related and their relationships as cyclical and reciprocal rather than linearly causal. For example, in the "action" domain, teacher behavior is seen as affecting student behavior, which, in turn, affects both teacher behavior and student achievement. Student achievement can cause teachers to behave differently towards the student, which then, in turn, affects student behavior and student achievement.

Clark and Peterson identify a difference concerning the domains which has implications for research. Teachers' behavior, and its effects (e.g., student behavior, and student achievement scores) are observable phenomena. In contrast, because teachers' thought processes occur "inside teachers' heads," they are unobservable, and hence must be investigated by a more interpretive approach. Further, until fairly recently, the relationship between the two domains was considered unidirectional and causal; they followed a "process-product" model that assumed a causal chain between teachers' thinking, teachers' classroom behavior, learners' classroom behavior, and, finally, learners' achievement. However, these domains are now seen as interacting in the reciprocal and cyclical way described above. Teachers' thinking affects their actions, which in turn, influence their subsequent thinking. This

reciprocity suggests that by examining the two domains together, teaching processes can be more fully understood.

Interpretivist Research on Teaching Mathematics

Within the interpretivist paradigm, there have been attempts to draw some teaching implications from recent research in cognitive science, particularly that concerning constructivism (e.g., Resnick, 1987) or metacognition (e.g., Schoenfeld, 1985, 1987). Much of this research has focused on the belief that learners construct knowledge rather than passively absorb what they are told. This has significant implications for a subject such as mathematics, which has enjoyed a rather unusual status as a fixed body of knowledge and core skills.

Views on the nature of mathematics range from "a discipline characterized by accurate results and infallible procedures" (Thompson, 1992, p. 127) somewhat "akin to a tree of knowledge [where] formulas, theorems, and results hang like ripe fruits to be plucked" (Steen, 1988, p. 611) to a human activity that "deals with ideas. Not pencil marks or chalk marks, not physical triangles or physical sets, but ideas" (Hersh, 1986, p. 22). The two poles of this range have been categorized severally as "Euclidean" and "Quasi-empirical" by Lakatos (1978), "Platonic" and "Aristotelian" by Dossey (1992), "Absolutist" and "Fallibilist" by Lerman (1990), and, perhaps most simply as "external" and "internal" by Polya (1963). Despite their appellation, the poles correspond broadly to a view of mathematics either as fixed, certain, value-free, abstract, and unchallengeable, or as dynamic, relative, constructed, and negotiable.

For over 2,000 years, mathematics has been dominated by an absolutist view, which regarded it as "a body of objective truths, far removed from the affairs and values of humanity" Ernest (1991, p. xi). However, in the past 20 years, mathematics

has undergone a "Kunhian revolution," in which several philosophers (e.g., Lakatos,
1976; Davis & Hersh, 1980) have regarded mathematics as more "fallible and changing, and like any other body of knowledge, the product of human inventiveness" (Ernest, 1991, p. xi). This philosophical shift has a significance that goes far beyond mathematics. For, as Ernest maintains, "mathematics is understood to be the most certain part of human knowledge, its cornerstone. If its certainty is questioned, the outcome may be that human beings have no certain knowledge at all." (p. xi).

Mathematics, as a school subject, has been largely unchanged for many years. Indeed, since the commercial and navigational needs of fifteenth century Europe began to demand an educational provision to improve arithmetic skills, much of the mathematics taught in formal settings has remained unaltered. National systems of education (that included mathematics as a school subject) were founded in France and Prussia at the beginning of the nineteenth century, in England some 50 years later, and in North America shortly after that (Howson, 1990). Within those systems, the mathematical curricula gradually expanded from commercial arithmetic to include successively algebra, geometry, trigonometry, and finally, in the early twentieth century, calculus. Since then, within North America, there has been "constant reform rhetoric but little actual reform of the school mathematics curriculum" (Stanic & Kilpatrick, 1992, p. 407).

Within a particular topic area--such as, for example, that of algebra--there has also been little change in how schools have approached it during the past century. Kieran (1992) lists the topics covered in beginning algebra courses in the early 1900s as including: "the simplification of literal expressions, the forming and solving of...equations, the use of these techniques to find answers to problems, and practice with ratios, proportions, powers, and roots" (p. 391). These topics are identical to those in a beginning algebra course in the 1990s (see Appendix 10).

Mathematics teaching, traditionally, has been based on the assumption that learners absorb what has been covered by repetition and practice, and that they then become able to apply this knowledge and these skills to a variety of problems and in a variety of contexts. Recent research, however, has revealed that the commonly-used techniques do not work as well as anticipated. In fact, learners use mathematical procedures depending on context and environment, rather than, as is commonly thought, on the mathematical nature of the problems they wish to solve (Boaler, 1993; Lave, 1988). The implications of this for the teaching and learning of mathematics are only beginning to be explored.

For example, within the USA, there has recently been some movement away from an overly abstract approach towards one that teaches mathematics more contextually. Modern approaches are designed to reflect the demands of real life problems and prepare learners for the mathematical requirements they might meet in their everyday lives. Recent calls for reform in mathematics education have focused on the need to promote institutional practices that facilitate what is called "meaningful learning" (National Council of Teachers of Mathematics, 1989; National Research Council, 1989). This approach is spelled out more fully in two recent NCTM documents (1989, 1991) that encourage teachers to develop school mathematics curricula and activities around promoting and enhancing mathematical understanding and skills rather than concentrating on imitation or recall. However, it is too early to determine their effect on teaching practices and mathematical ability and understanding in either school-based or adult mathematics education.

How mathematics is regarded has a special significance for educators. For, if mathematics is a body of infallible, objective truths, then it has no special concern with social responsibility. Educational concerns such as the transmission of social and political values and the role of education in the distribution of wealth and power are of no relevance to mathematics. Alternatively, if mathematics is a fallible

human construct, then it is not a finished product but a field of human creation and
invention. Hence, mathematics education must include opportunities for learners to study mathematics in "living contexts which are meaningful and relevant to them" (Ernest, 1991, p. xii), to create their own mathematical knowledge, and to discuss the social contexts of the uses and practices of mathematics.

Further, notions of what mathematics is also affect how mathematics is taught. An "externalist" view of mathematics education would stress the mastery of existing concepts and procedures; an "internalist" view would concentrate on providing "purposeful activities that grow out of problem situations, requiring reasoning and creative thinking, gathering and applying information, discovering, inventing, and communicating ideas, and testing those ideas through critical reflection and argumentation" (Thompson, 1992, p. 128).

The impact of predominant worldviews of mathematics on educational practices have also interested ethnographic and anthropological researchers. In particular, these researchers have focused on the social context of mathematics education and the "culture" that is transmitted by mathematical activities, both in and out of school. Eisenhart (1988) surveys the recent research conducted from an ethnographic perspective. In particular, she draws attention to the work of Cole and Scribner (1974) and Lave (1982, 1985) who, instead of studying the learning of formal mathematics in schools, have instead tried to understand mathematical problem-solving outside of schools. This work, says Eisenhart, "is predicated on the idea that by understanding existing, 'natural' knowledge and beliefs, researchers can bridge the gap between subjects' capabilities and the capabilities that researchers or teachers believe students should have" (p. 111). Both Cole and Scribner's, and Lave's work focuses on the mathematics used by adults, and I shall discuss it in more detail below.

Other recent studies on the teaching of mathematics that fall within the interpretivist paradigm have concentrated on the knowledge, beliefs, and attitudes of teachers (e.g., Ernest, 1989; Thompson, 1984, 1992) and learners' actual thought processes during mathematics education (e.g., Cobb, 1986; Desforges & Cockburn, 1987; Neuman, 1987). These studies recognize that teachers and learners perceive and interpret teaching situations differently and hence, they attempt to identify these separate interpretations. Much of this work is also based on a phenomenographic approach (Marton, 1981) that seeks the "insider" interpretations and meanings of those involved. For example, Neuman (1987) explored learners' ways of thinking about numbers and arithmetic to see if learning was improved if teachers used learners' concepts as a basis for teaching. In particular, Neuman was primarily interested in discovering what children's initial concepts about numbers were and sought these "meanings" through a series of interviews with children.

Löthman (1992) was also interested in discovering what students actually learned. She drew distinctions (based on the work of Bauersfeld, 1979) between the subject content that is meant (i.e., what is contained within the course syllabi and textbooks), taught (i.e., contained in the teacher's thinking and approach), and learned (i.e., what the learners perceive they have learned). Löthman regarded teaching as ideal when all three parts combined, but recognized that the different backgrounds and perceptions of participants influenced their perceptions so that they interpreted teaching in different ways. Consequently, she noted that a dynamic social process is developed in the classroom and affects how mathematics education is regarded and constructed differently by different people.

Her study focused on the conceptions of mathematics education held by two groups of learners--one of adults, one of high school children--who were studying equivalent mathematics coursework. Her purpose was to "describe conceptions of mathematics education in connection with a concrete educational course [in order to]

find patterns and structures and catch their importance for the actors and the

education" (p. 140). She categorized her results into four "pictures" of different, but

related, conceptions that "showed teaching and learning as an entirety":

> MATHEMATICAL TRADITIONS, consisting of the conventional dwelling of mathematical problems in relation to the students' experiences.
>
> MATHEMATICAL STRATEGIES, consisting of the students' ways of understanding, reflecting on, and solving mathematical problems.
>
> MATHEMATICAL REASONINGS, consisting of the students' ways of discussing, analyzing, and judging mathematical information.
>
> MATHEMATICAL APPLICATIONS, consisting of the students' ways of understanding and practicing mathematical concepts outside school. (p. 148)

Mathematical traditions played an integral part in all the pictures. Löthman

found that both the adult group and the high-school group preferred "strong rules

and formal dispositions of problems" (p. 148). The mathematical strategies of the

adult group showed that they used a range of procedures due to their practical

experiences of calculation and their earlier education. Mathematical reasonings

differed between the two groups. Adults preferred to know why they were doing

something (such as a problem-solving technique) before they did it; schoolchildren

merely wanted a rapid and expedient model. Finally, because of their greater

experience, adults were able to see the practical uses of mathematical applications

far more clearly than were high-school learners. Löthman further found differences

between adult learners' and their teachers' conceptions of mathematics education.

They were "pointing in two directions. [The learner] was aiming at comprehension

and [the teacher] was aiming at procedure" (p. 147). Löthman claims that this

difference comes partly from the adult learners' previous mathematics education

and partly from their "experiences of different occupations....These experiences

convinced them of the necessity of understanding" (p. 148).

What is meant by teaching for "comprehension" or "mathematical

understanding," or promoting "meaningful learning" can be drawn from examples of

research in of instructional situations. The work of Skemp (1976), Richards (1991), and Brown, Collins, and Duguid is helpful here. Skemp (1976) discusses the distinction between "relational" and "instrumental" learning. Instrumental learning involves being able to follow rules without ever developing the true ability to synthesize. In contrast, relational learning means knowing both what to do and why. In a mathematical context, this is the difference in being able to solve a textbook word problem through application of series of rules versus an on-the-spot reckoning of currency exchange while bargaining in a foreign country.

Similarly, Richards (cited in Cobb, Wood, Yackel, & McNeal, 1992) developed the notions of "school mathematics" and "inquiry mathematics". School mathematics, which corresponds to instrumental learning, is best characterized as the transmission of knowledge from the teacher to passive students. Here, teachers establish not only the content of what is to be learned, but also how it is to be regarded and interpreted. Students, in order to be successful, must adopt the teachers' interpretations of the content. For them, learning mathematics becomes the acceptance of others' norms rather than an active construction of knowledge. Further, because teachers promote mathematics as having its own internal logic and meanings, mathematics for students is "reduced to an activity that involves constructing associations between signifiers that do not signify anything beyond themselves" (Cobb et al., p. 587). In this way, teachers enculturate students into what Lave (1988) calls "folk beliefs" about mathematics. These include

> the conviction that it is impermissible to use any methods other than the standard procedures taught in schools to solve school-like tasks and that the use of these procedures is the rational and objective way to solve mathematical tasks in any situation whatsoever. (Cobb et al., p. 589)

On the other hand, Richards maintains that inquiry mathematics actively seeks to promote a deeper understanding. Teachers, rather than regarding themselves as the sole validators of what counts as legitimate mathematical activity and learning, encourage and guide students to propound and discuss their own

interpretations and insights. In this way, teachers promote the notion of mathematics
more as a legitimated set of interpretations of "activities that were intrinsically explainable and justifiable" (Cobb et al., 1992, p. 594) rather than as a set of acontextual and fixed rules and procedures.

Richards observed that because teachers tended to follow either one set of practices or the other, the school and inquiry mathematics approaches took on the character of traditions. Cobb et al. (1992) found this school/inquiry dichotomy too simplistic and argued that, regardless of their tradition, teachers "initiated their students into particular interpretive stances [where] students learned which mathematics activities were acceptable, which needed to be explained or justified, and what counted as a legitimate explanation or justification" (p. 597). In the classrooms Cobb studied, regardless of the approach of the teacher, students would generally experience an activity as meaningful if it made sense to them within the classroom context rather than with reference to their individual, beliefs, values, and purposes.

Brown, Collins, and Duguid (1989) further investigate this phenomenon in their contrast of "authentic" and "inauthentic" mathematical activity. To them, authentic mathematical activity is the "ordinary [mathematical] practices of the culture" (p. 34) which is coherent, meaningful, and purposeful only when it is socially- and contextually-situated. In contrast, common school mathematical activities are not authentic because they prevent students from engaging with everyday culture and impose a more "school" culture. As Brown et al. say, "although students are shown the tools of many academic cultures in the course of a school career, the persuasive cultures that they observe [and] in which they participate...are the cultures of school life itself" (p. 34). In other words, classroom activities take place within a school, rather than an everyday culture. The activities that students are asked to perform within this culture of school are attributed to other cultures--

such as those of mathematicians--and yet, they "would not make sense or be endorsed by the cultures [and practitioners] to which they are attributed" (p. 34).

In sum, the interpretivist paradigm seeks not to articulate causal relationships between teaching and its effects, nor to illuminate the interweaving of social and political influences in the minutiae of the classroom; instead it seeks to discover the meanings which participants ascribe to a situation. In terms of this study, the interpretivist research about teachers' thinking is especially topical. Teachers have theories and beliefs that influence their perceptions, decision-making, planning, and actions. These, in a reciprocal and cyclical way affect, and are affected by, learners. The notion that knowledge may be "constructed" and "interpreted" rather than "fixed" and "transmitted" is particularly intriguing in the teaching of mathematics. Uncovering teachers' own ideas, theories, beliefs, and "hidden" mental activities in mathematics classrooms can prove fruitful in light of actual classroom experiences.

Summary & Discussion

Research on teaching has tended to fall into one of three distinct paradigms and, within each paradigm, examined only "macro" or "micro" aspects of teaching. Macro studies have focused on large-scale general features of society such as organizations, institutions, and culture and used experimental and quantitative methods to derive explanations about the effects of these external influences on teaching. Alternatively, micro studies have focused on the more personal and immediate aspects of teaching and used descriptive or interpretive frameworks and methods of inquiry to understand the internal meanings and perspectives of

participants. Most of this research has been theoretically dichotomous and,

consequently, difficult to compare. Part of the difficulty arises from the

epistemological distinctions between the different approaches. Indeed, in many

ways, each paradigm can be seen as reflecting different positions on the macro-micro

issue.

In addition, most research on teaching has focused predominantly on the

relationship between teaching and learning; indeed, it has largely viewed teaching

solely as the promotion of learning. Such a view has tended to reduce social

phenomena, such as classroom behaviors and processes, to the behaviors of single

individuals. Further, discussions of teaching have tended to regard curriculum

(what to teach) and instruction (how to teach) as separate, and, as Doyle (1992)

describes, "work in each domain has gone on if the other did not exist" (p. 486).

Such separations are, to me, artificial and reductionist. They ignore the social

nature of much classroom behavior and assume divisions between external and

internal perspectives, between teaching and learning, and between curriculum and

pedagogy. Although teaching and learning can be regarded as separate (although

linked) processes, they can also be studied as one pedagogic or "teaching/learning"

process involving content, activity, and people (Löthman, 1992; Lundgren, 1981;

Pask, 1976). Further, curriculum and pedagogy can also be related: "A curriculum is

intended to frame or guide teaching practice and cannot be achieved except during

acts of teaching. Similarly, teaching is always about something so it cannot escape

curriculum. Teaching practices, in themselves, imply curricular assumptions and

consequences" (Doyle, 1992, p. 486). As the structuralist paradigm was too

determinist or functionalist, the interpretive paradigm, in regarding all social life as

ultimately explicable in terms of the actions and intentions of individuals, has gone

too far in the opposite direction. The reconciliation of the two implies that observed

behavior and its effects need to be viewed both in the context of the meanings and motivations of actors and within wider social contexts and influences.

Clark and Peterson (1986) accept that a complete discussion of teaching processes cannot only concern what teachers' think, it must also include an understanding of constraints and opportunities that impinge upon them. As they see it, "teachers' actions are often constrained by the physical setting or by external influences such as the school, the principal, the community, and the curriculum" (p. 258). In addition, teachers' actual thought processes can be similarly constrained. For example, teachers may have (or think they have) less opportunity to plan their lessons in the ways that they wish because certain decisions have already been made by the institution in which they work. In this way, research on teachers' thinking can appear to overlap with the more structuralist frame factor theory.

Hence, research that attempts to combine both theories must overcome certain issues—the relationship between macro and micro approaches and the related issue of structure and agency—to which I now turn.

Structure & Agency

Structure and agency are concepts which attempt to explain actions in social settings as the effects of large scale structural forces or policies (structure) or small scale individual, voluntary actions and patterns of behavior (agency). As traditionally conceived, structure and agency are regarded as competing explanations of social reality. Hence, attempts to combine them lead to ontological (how social processes are generated and shaped), epistemological (what counts as knowledge), and methodological (how research should be conducted) problems.

There is considerable overlap between notions of structures and macro phenomena in that they both refer to the reproduction of patterns of power and social organization (Layder, 1994). Within educational settings, "structural" studies have focused on explanations of how educational practices and processes are situated in, and determined by, broad social structures. In these studies, it is considered unnecessary to gather the perspectives of actors because they can be deduced from determining the effects of social structures. Similarly, micro analyses overlap with a concern for agency. "Agency" studies have supposed that events and actions are produced by largely autonomous individuals; they therefore have concentrated on eliciting the actors' intentions, meanings, and actions about situations.

The difficulty with this dichotomy is that it makes the assumption...that social life exists on different levels (Shilling (1992). Both approaches are, therefore, limited and fail to capture the totality of social life in general, or educational settings in particular. People do not exist on different levels, so separating social life into hierarchical levels "makes it difficult to conceptualize change as a dynamic process involving both structure and human agents" (Shilling, 1992, p. 70). As Marx put it, in describing human activity: "Men make their own history...not under circumstances chosen by themselves, but under circumstances directly encountered, given, and transmitted from the past" (1950, p. 225). Consequently, research on social settings-- such as classroom teaching--that attempts to synthesize micro and macro approaches needs to include both empirical evidence of actual events in particular settings with particular actors (micro and more agency-driven explanations), and supra- individual theories that provide a context for those events (macro and more structurally-driven explanations).

As Giddens (1976, 1984) reminds us in his theory of structuration, people are, at the same time, creators of social systems and also created by them. In other words,

instead of viewing structure and agency as separate phenomena, structuration theory stresses their inter-connection. Thus, structuration theory has potential for empirical application in research. For example, it highlights how a study of teaching can consider the intermediary aspects between structure and agency both within and outside the classroom. Therefore, it is particularly useful for looking at teaching as a social and political process. "Between the rules, negotiations, and bargainings of classroom interaction, and the dynamics of the capitalist economy, or the relative autonomy of the state, lie a whole range of intermediary processes and structures" (Hargreaves, 1985, p. 41). More practically, structuration theory suggests that researchers use multi-strategy approaches to achieve a dense theoretical and empirical coverage of the topic, initiate and develop theory from fieldwork, recognize that all activities are contextually-situated and all situations are the product of human actions, and seek the relevance of pieces of empirical data to wider theoretical issues.

This discussion of the macro-micro duality and the relationship between structure and agency inform my own study. I now describe the theoretical framework for my study and discuss how it addresses these issues.

Theoretical Framework

In adopting a social theory, rather than a psychological approach, my study is based on and develops the conceptual frameworks, theories, and methodologies of other researchers. Further, it provides a way of looking at the relationship between

social interaction in mathematics classrooms and the reproduction of the major
structural principles which characterize society.

Because it adopts a social theory perspective in examining teaching processes, this study must situate its conceptual sights on local, observable phenomena, and also explore openly the various forces constraining the educational activities within the classroom. This involves examining factors that may influence teaching (and teachers) which are not necessarily obvious or apparent to those involved. Although teachers are the main source of classroom teaching activity, they are also, themselves, part of a wider context. Thus, a study of only their choices and behaviors would be incomplete; a thorough study of teaching processes must attempt to examine these contextual factors as well. Of course it is not possible to isolate and examine these factors; one can only discern their effects by studying what teachers do, and what they say about themselves and why they're doing it. To clarify how my theoretical framework for my research incorporates all of these considerations, I now describe it in detail.

The conceptual framework of this study combines Clark and Peterson's model of teachers' thinking and frame factor theory to construct a lens for observing teaching processes. This combination allows me to examine the process of socialization that takes place in mathematics classrooms as well as how mathematical meaning and knowledge are formed and developed. This study concerns the whole pedagogic process of teaching and learning--a process I have chosen to name, in recognition that teachers are its chief initiators, as teaching processes. By teaching processes I do not mean only the selection of content to be taught, or the choice of such techniques as lectures or discussions or whether to use group-work. I also include in my definition the expectations, rules, procedures, and norms of the classroom, as well as the complex web of interactions between teachers

and learners, and between learners themselves. In other words, studying teaching

processes means developing an understanding of "what goes on" within classrooms.

Asking "What is going on here?" may seem at first a trivial question, but as Erickson (1986) points out: "Everyday life is largely invisible to us (because of its familiarity...and contradictions. We do not realize the patterns in our actions as we perform them....The fish would be the last creature to discover water" (p. 121). Hence, asking "What is going on?" can problematize the commonplace and make the invisible visible, as is appropriate in a social theory approach.

To understand what is going on, one also needs to address "local" meanings and the differing perspectives of those involved. Teachers and learners regard teaching from different perspectives, and in different ways. Also, what appears to be happening may be misleading. Events that look the same may be entirely different and have distinctly different local meanings. In other words, the question "What is going on?" can be extended to include the question "What is going on for whom?"

Further, to understand why teaching processes take the shape that they do, one can also identify the forces that are acting on them, and in what ways. Teaching is subject to many forces, some of which can be traced to the political, cultural, and social structures in society. Consequently, to examine fully classroom teaching processes, a research study must seek to relate them to the political and social structures of society.

Thus, this study links two strands of educational research. A model of teachers' thoughts and actions--which examines both the internal mental processes of teachers as they plan, conduct, and evaluate their teaching, and their subsequent observable behavior--is connected to frame factor research, which examines how teaching processes are affected by external factors. In this way I examine how

teaching processes are viewed from the perspectives of those involved as well as consider how teaching processes are influenced by other factors.

A Model for Understanding Teaching Processes

As I outlined above, by teaching processes I mean "what goes on" in adult mathematics classrooms. Teaching processes include the selection and ordering of the content to be taught; the expectations, rules, and procedures of the classroom; and the nature and quality of interactions between teachers and learners, and between learners themselves. To understand why teaching processes take the shape that they do, one needs to identify what forces are acting upon them, and in what ways.

Applying frame factor theory to a study of mathematics education for adults, one can determine several constraining factors. Within any society, the institutional framework of adult education provision, the particularities of educational settings, and the mathematical curricula chosen to be presented in those settings all affect teaching processes in mathematics education for adults. Further, the effects of those factors can be seen as interacting with the life and professional experiences of adult teachers and their learners. In other words, social and cultural norms and values affect the settings in which adults can learn mathematics, the mathematics they are expected to learn, and the ways they and their teachers experience and regard mathematics education.

These factors are not isolated, however; they act upon and react with each other. Further, the relationships between factors are dynamic rather than static, and constellate uniquely in every classroom and every setting. For example, adult

learners of mathematics in a community drop-in literacy center are markedly different from their counterparts in university-level settings. As such, they expect to learn different mathematical skills and knowledge from their more academic counterparts. These differences and expectations are also not constant; they change over time, and affect (to differing degrees) the individuals concerned, the institutions they attend, and the mathematics they study.

Consideration of these issues requires a framework for understanding teaching processes and the forces that shape and constrain them. A model of the factors I have identified and their interrelationships is portrayed in Figure 1. The two groups--the worldview of mathematics and the institutional framework, each influenced by social structures--represent possible frame factors. The third group--experiences of teachers and learners--represents a way of observing the effects of these frame factors on teaching processes. The solid lines represent the relationships between elements that are the focus of this study. The dashed lines represent other links that exist but are not dealt with in this study. I now discuss each element in turn.
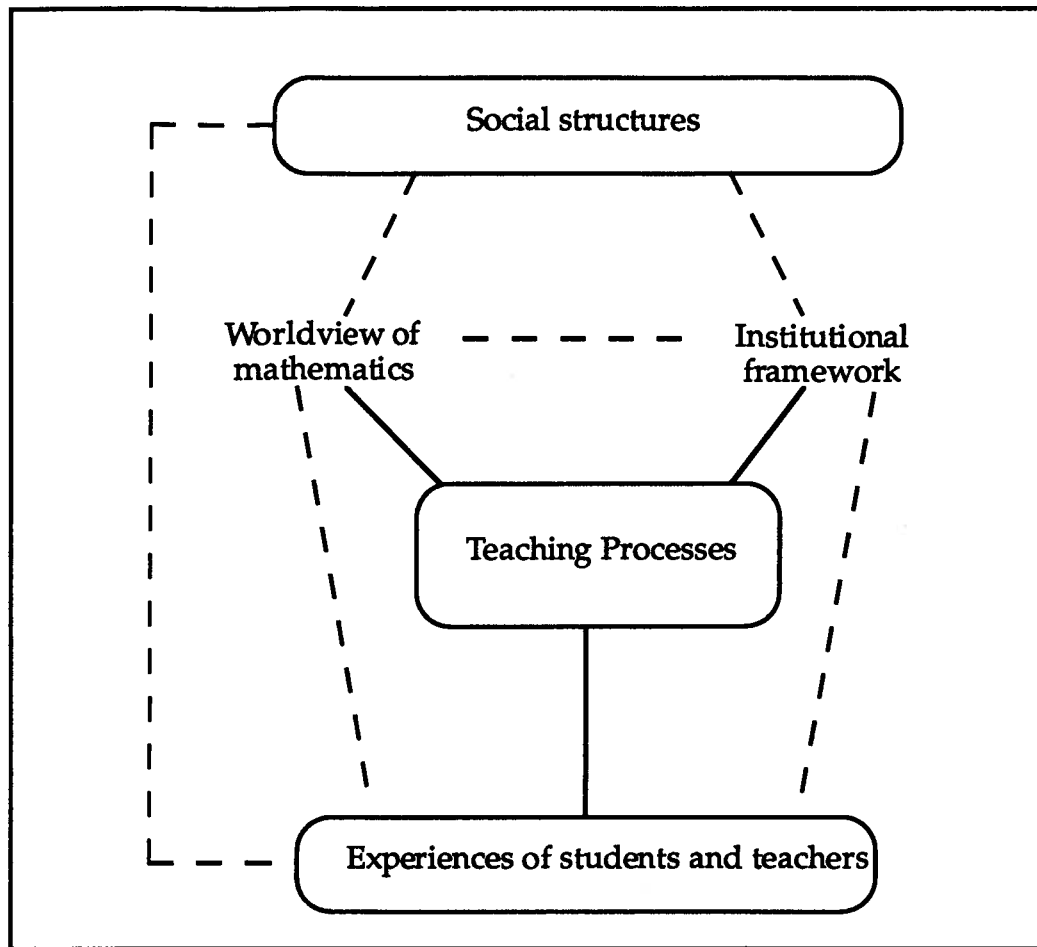
Figure 1: Understanding Teaching Processes

Social Structures

As can be seen in Figure 1, social structures are not direct influences on teaching processes; rather, they are mediated through the worldview of mathematics, the institutional framework, and the experiences and perceptions of teachers and learners. However, given that their effects are so widespread and so readily apparent, I feel it necessary to briefly explain them here.

Following Giddens (1984), I regard social structures as the rules and resources that people draw upon as they produce and reproduce society in their activities.

Hence, social structures are both the medium and the outcome of social activity rather than a system of relationships operating "above" people. In this way, social structures affect, but do not determine, human activity; they are more the result of a "process of creative interpretations by individuals who are engaged in a vast number of concerted interactions with each other" (Sharp & Green, 1975, p. 19). Practically, then, analyses of subjective meaning need to be supplemented with some description of the actual social structures within which people live and act.

For a study of teaching, this implies that teachers' thoughts and actions should be situated within a context of social and physical rules, resources, and constraints. Although teachers may not perceive these resources and constraints, they nevertheless are bound by them. Teachers' working situations, freedom of action, and thinking are all shaped and limited by social structures. Within the classroom, it is clear that teachers have far more power to act, direct others, and access facilities and resources than do students. Hence, this unequal distribution of power has considerable significance in explaining the differences in perspectives on teaching processes, and classroom behaviors between teachers and learners.

The Worldview of Mathematics

A worldview is the set of presuppositions or conceptions of a phenomenon that is held by a particular society or group. In encompassing all the different views of people in that group, the worldview reflects their specific cultural, social, and historical contexts. Hence, the notion of a worldview emphasizes the shared and social basis of knowledge; knowledge is present in the society into which individuals are socialized, and it is a resource shared by members of that society. Knowledge is seen not as a collection of "content," but more in the "style or pattern of

thought....The social basis of knowledge lies in the categories of meaning used to
think or perceive or understand the world" (Dant, 1991, p. 18).

As I described earlier, the worldview of mathematics, common in all industrialized countries, is that it is a logical and impersonal branch of knowledge consisting of objective truths and "theories about quantity, space, and pattern [and] the study of abstract symbolic structures used to deal with these theories" (Davis, 1992, p. 134). Mathematics is regarded as an influential and privileged subject in most schools, and possession of mathematical knowledge has a high value in many cultures (Willis, 1990). As Dossey (1992) notes, "Perceptions of the nature and role of mathematics...have had a major influence on the development of...[mathematics] curriculum, instruction, and research" (p. 39). Hence, how mathematics is conceptualized affects how it is taught. As it lies outside the control of the teacher, the worldview of mathematics can act as a frame factor in the mathematics classroom.

Institutional Framework

Institutional factors can be of two kinds: organizational factors such as the overall provision of education within an area and the physical structures of, and administrative systems in, educational institutions; and curricular factors that specify what is to be taught and in what way, and the textbooks and teaching materials to be used. In both cases, these factors, because they lie outside the control of the teacher, act as frames.

Organizational Factors. In British Columbia, almost all of the locally-provided mathematics education is organized and controlled by the public education sector. For example, within the Acton area, both the Acton School Board and the Community College system offer a variety of "math upgrading" courses to

adults at several centers. Most adults in these courses are trying to obtain one of four certificates (Dogwood, Adult Dogwood, College Provincial, and GED) equivalent to high school completion. This pattern of provision is repeated in most urban areas across North America.

Much of this provision is based on the furnishing of opportunities for "lifelong learning," best described as "the opportunity for individuals to engage in purposeful and systematic learning throughout their lives" (Faris, 1992, p. 6). Central to this concept of lifelong learning are certain widely-held assumptions about, and practices within, adult education that are built on ideas and theories about how adults learn and should be taught. These ideas include: teaching must be problem-centered, it must emanate from the participants' experience of life and develop the individual socially, participants must exert definite influence on the planning of the course and the conduct of the teaching, and techniques used must be based on an interchange of experience (Knowles, 1980).

Another set of organizational factors that can act as frames on teaching includes the physical structures of colleges and their classrooms, and administrative arrangements within particular institutions such as the size of the class, the time available for teaching, and the evaluation system. Each of these limits, but does not determine, teaching processes.

Curricular Factors. A second set of institutional factors that can act as a frame on teaching concerns the process of codifying an area of knowledge into an academic discipline and appropriate curricula. When any subject matter (such as mathematics) is taught in a formal setting, it becomes a discipline by its choice of content, teaching methods, homework assignments and evaluation procedures. Thus, the teaching of mathematics becomes bounded by, and negotiated between, the inherent qualities of the subject and the goals and dynamics of the institutions in which the teaching takes place. In the case of mathematics, these negotiations become visible through

two processes: setting the aims of mathematics education, and codifying its subject matter into textbooks.

I described earlier how the aims for mathematics education can affect its teaching. Briefly, I explained how the aims for mathematics education were, in general, set by certain dominant groups within society, and yet did not reflect, at least overtly, any social or political content. This ensured that mathematics was not seen as a tool for questioning dominant attitudes within society, and its method of presentation reflected this: it tended to be taught in an authoritarian and hierarchical way.

A second process through which curricular factors influence teaching concerns the use of textbooks. In many ways, textbooks are the most central and defining feature of mathematics education. The content and structure of most mathematics courses are determined by the content and structure of the set textbooks. In many ways, the textbooks are the curriculum, codified. Romberg and Carpenter (1986), in their survey of research on teaching and learning mathematics found that "the textbook was seen as the authority on knowledge and the guide to learning" (p. 867) in all of the studies they surveyed. They concluded that many teachers "see their job as covering the text" and that mathematics was "seldom taught as scientific inquiry [but rather] presented as what the experts had found to be true" (p. 867).

These conclusions are supported by Höghielm's study of adult mathematics classes in Sweden, in which he found that teaching practices "were organized on a 'cramming' basis [where] the teachers play the part of living textbooks" (1985, p. 207). It is not unrealistic to expect teachers to use some textbook or other; it is common practice in most classrooms. However, as Bishop (1988) asks, "Whose are these books? Who writes them, and for whom, and why?" (p. 10).

Basing teaching largely on the textbook has several consequences. First, it sustains and promotes a "top-down" approach to education. Mathematics textbooks are written by experts who purport to know what learners need, and the order and methods they need to learn it. Second, they make no distinction between what different learners bring with them to the classroom. By treating learners as impersonal or generalizable, textbooks privilege content over process. They encourage the teaching of subject matter, rather than the teaching of people. Finally, mathematics in textbooks is presented in a supposedly value-free and decontextualized way. Mathematical knowledge is seen as impersonal; learners are not encouraged to make their own meanings, or find their own significance.

Institutional factors, then, can also have a large influence on teaching. The physical structures act as tangible constraints; the curricular factors act more conceptually, limiting what counts as legitimate knowledge or as an approved way of teaching and learning.

Experiences of Students and Teachers

Regardless of the different settings for adult mathematics education and the different curricula that exist in these settings, two groups of people are affected by and in turn affect, these factors. Adult teachers and their students are each the focus of the factors that limit classroom teaching processes, and, simultaneously, the agents of change on those factors. To my knowledge, there is no published research in North America that focuses on the teachers of mathematics to adults. However, as was mentioned earlier, adult education theories often stress the centrality of the participants to the teaching processes. In practice, this means that effective adult education teaching should relate to participants' needs and interests (Brookfield, 1986). Earlier, I discussed studies that had considered the needs and interests of

those adult learners engaged in mathematics education. I showed how adults' attitudes towards, and their daily uses of, mathematics influenced both how they approached their mathematics education and how successful they were at learning. Consequently, these attitudes and expectations, which lie outside of the teachers' control, can also be seen as frame factors affecting teaching processes.

Summary

These, then, are the elements of my model for studying teaching processes. In this chapter I have described a theoretical framework and its constituent elements that have been designed to investigate and analyze teaching processes from a social theory perspective. The elements of this model will be animated through the participants' classroom interactions in a variety of ways. I intend to observe, record, and analyze these interactions using diverse and multiple methods in order to achieve a dense, theoretical, and empirical coverage of the topic. In the following chapter, I explain my methodology.

In this chapter I describe the methodology I used in my study. I first discuss my selection of a specific research site and study participants. I next describe my data collection and analysis procedures, and conclude with a brief discussion of certain issues concerning the "criteria of soundness" of my study..

Selection of Research Site and Participants

I needed to contextualize my study of teaching processes in actual mathematics classroom situations. I could not, of course, gather data from all of the many providers of formal mathematics education, even in as geographically-compact an area as the Lower Mainland region of British Columbia. It seemed appropriate to base my study in a local setting where such provision commonly took place, and within that setting, to choose courses that typically reflected the overall provision.

Selection of Site

Within the Acton area, both the local School Boards and the Community College system provide a range of mathematics education courses to adults. Each system's provision is comparable: their courses are of similar levels of difficulty, and are offered at broadly similar dates and times. However, one institution in the

Community College system offered easy access and was already known to me. I had conducted some initial research in Acton College's mathematics department and regarded it as an ideal site. It contained a range of informants, and it provided a high probability of finding a rich mix of the teaching processes and frame factors that I wished to study.

The selected college offered a range of mathematics courses for adults during both the daytime and the evening. Their provision was organized into five distinct levels, corresponding broadly to grade levels 9 through 12, and an introductory calculus course. Their most mathematically-basic courses, corresponding to grade 9, are two half-courses (050 and 051) and a combined course (050/051) which provide "a review of basic math skills and a study of metric measurement and introductory algebra and geometry" (AC, 1992). They are deliberately designed "for the student who has never studied academic mathematics before or who is lacking a good foundation in basic algebraic skills" (AC, 1993, p. 59).

I chose to study the three sections of these courses (050, 051, and 050/051) offered during the 1994 Spring term. Each course was to be taught by a different instructor and was expected to recruit between 15 - 20 students. The courses I chose were typical of the college's mathematics provision and their curriculum was supposedly designed to reflect a balance between the formal and practical mathematical needs of learners.

Selection of Participants

The participants in the study were of two types: (a) teachers of mathematics to adults at the chosen college, and (b) students in the three introductory mathematics courses.

Teachers. All of the eight teachers in the mathematics department were interviewed at the beginning of the term. The reasons for interviewing all of the teachers were twofold. First, as those most concerned with teaching processes, teachers have both the power to effect change, and are among those most affected by, the factors involved. I was concerned to ensure that I gained as much data as possible on the teachers' understandings of teaching processes, the constraints that they feel, and the reasons for their choices in relation to the specific and concrete situations of their teaching. Consequently, I determined that the fullest understanding of teaching processes at the college could only be gained by interviewing all the teachers involved.

The second reason for interviewing all of the teachers was more pragmatic. At the outset of my data collection, decisions about who would teach which course had not yet been finalized. Therefore, by interviewing all eight teachers, I was able to ascertain the approaches that they were taking to planning their courses before they had begun. Subsequently, when teaching assignments had been decided, I focused on the three teachers of the introductory courses more specifically.

Students. All of the adult students attending any of the three introductory mathematics courses during the first two weeks of term were given a copy of the survey protocol together with an explanatory letter that invited them to participate in further stages of the research. (Copies of the explanatory letter and the survey protocol are attached as Appendices 1 and 2.) Thirty-two students completed the survey, all of whom indicated their willingness to participate further.

From these 32 respondents I selected 15 (5 from each section) to interview further. Interviewees were largely selected on the basis of several demographic characteristics (viz., gender, age, ethnic origin, and employment). My intention in using these criteria for selection was not to obtain a sample of participants that purported to be in any way representative of the wider population of adult

mathematics students. Instead, it was intended to help me appreciate the range of characteristics of the group of adult learners who had enrolled in the basic mathematics courses. I was interested in exploring whether adults from different backgrounds had different attitudes and approaches towards their mathematics education, or experienced teaching processes in different ways.

Data Collection Procedures

The study used multiple data collection procedures, combining those from both qualitative and quantitative approaches. My data were gathered through surveys of learners' demographic characteristics and attitudes, extensive participant observation, repeated semi-structured interviews, and document collection. Each specific procedure is described more fully below, but first, I explain the reasons for using such a variety of methods.

Using Multiple Methods

The principle use of multiple methods was to add methodological rigor to the study. Although qualitative and quantitative data collection are often seen as coming from contradictory notions of reality, here my avowed post-empiricist approach assumes that there is one "truth" to a situation. Although this truth may not be exactly "captured" by any one means, both quantitative and qualitative methods can be seen as different ways of examining the same phenomenon, and obtaining a closer correspondence with the truth. Therefore, findings that have been derived

from more than one method of investigation can be viewed with greater confidence and with a greater claim to validity. Denzin (1970, p. 301) describes this combination of multiple methods as "methodological triangulation" and adds that "the flaws of one method are often the strengths of another, and by combining methods, observers can achieve the best of each, while overcoming their unique differences" (p. 308).

A second reason for combining both qualitative and quantitative methods concerned the differing perspectives between the researcher (outsider) and the participants (insiders). Quantitative methods, such as the survey I used, were oriented to my own specific concerns (in this case, learners' attitudes towards mathematics). Alternatively, the more qualitative interviews and observations were oriented more towards the participants' perspectives. Integrating both methods in one study combined the perspectives of both insiders and outsiders, and added strength to the findings.

A third reason concerned the need for triangulation in the data. Students and teachers view teaching from different standpoints. Teaching is what teachers do, whereas teaching is done to students. As such, they have little say in, or control over, those decisions that affect teaching. Consequently, how students and teachers regard, and respond to, teaching differs markedly. In order to capture those different perspectives fully, I needed to obtain data from both groups in several different ways. Triangulation using different methods, and from different perspectives, allowed me to better capture the totality of the phenomenon of mathematics teaching.

The final reason for using the particular mix of methods was that more quantitative data derived from the surveys of learners' attitudes could be used to supplement and focus some of the later qualitative data collection. Not only did the survey data provide information that could not have been readily gained by a

reliance on participant observation or semi-structured interviews, but it also enabled its use in the subsequent interviews with both teachers and learners.

I now provide an overview of the three phases of data collection, and then consider each method and procedure in turn. For each, I describe its use and discuss some of the implications of using it.

Overview

Data collection fell into three separate phases. In the first phase (approximately 4 weeks long), I was concerned with developing some understanding of the culture and ethos of the college in general, and of the three introductory mathematics classes in particular. Here, I distributed a survey to all learners, interviewed all of the teachers within the mathematics department about their perceptions and beliefs, and conducted a series of informal discussions and ethnographic observations (ranging from 45 minutes to 2 hours) within the three introductory mathematics classes. During this phase I conducted 8 interviews and 19 observations.

The second stage of data collection focused more specifically on one particular section of the syllabus: that content area described in the course text as "Introduction to Algebra." The choice of the specific content area on which to focus was based on the following criteria, drawn from Löthman (1992):

(a) It should consist of topics that are dealt with in the textbook, but can also be easily discussed in class.

(b) It should contain problems where the content can be derived from the learners' everyday experiences.

(c) It should contain learning tasks that require written or oral products and have alternative methods of solution.

(d) It should be possible to discuss and analyze the solutions to any problem.

The introductory courses all covered the content areas of geometry, percentages, and equations in addition to algebra; each of which would have met the above criteria. Algebra was finally chosen because of its expediency to the research. Course 050/051 was due to begin the algebra section during the earlier part of this phase of data collection; course 050 some three or four weeks later. This interval allowed me to complete this part of my data collection in one course before repeating the procedures in the next.

In this second phase (8 weeks), I interviewed learners in, and the teachers of, the introductory classes several times. These interviews focused on the specific teaching processes in the lessons concerning algebra. I also observed every lesson that covered the algebra content and made extensive field-notes of my observations. Several of these lessons were also videotaped, and the video-recordings used as the basis of "stimulated recall" interviews with the class teachers. During this phase I conducted 17 interviews with students, 9 interviews with teachers, and 27 observations.

In the third and final phase (4 weeks), I completed my observations of the three introductory classes, observing the concluding days of instruction in each course and their preparations for the end-of-term class examinations. I also interviewed all the students again, ascertaining their views on mathematics education in general, and the teaching in their particular course in detail. Finally, I held in-class group interviews with each of the three introductory courses. Group interviews were chosen as "a good way of getting insights [as] subjects can often stimulate each other to talk about topics" (Bogdan & Biklen, 1992, p. 100). Further,

group interviews can involve those who feel either reluctant, or that they lack sufficient authority, to speak (Lancy, 1993). In these group interviews I particularly encouraged responses from those individuals who had not been selected for the individual interview processes, or who had participated, but said little.

During this third phase of the research, I conducted 8 observations, 3 interviews with teachers, and 15 interviews with the students (both individually and as a group). I now describe each data collection method in more detail.

Survey

I distributed a simple survey, by hand, to all the learners enrolled in the three sections of the course 050/051 in the Spring 1994 term. This survey gathered demographic data of gender, age, ethnic origin, and occupation; students' attitudes towards mathematics; and an indication of willingness to further participate by agreeing to be interviewed (see Appendix 2).

The section in the survey on attitudes towards mathematics was based on two instruments devised by Aiken (1974) that measure participants' (a) enjoyment of mathematics, and (b) their perceived importance and relevance of mathematics to the individual and to society. Aiken's instruments each use a 12 question, 5 point Likert-type scale. In a discussion of the internal reliability of the two scales, Aiken (1974) found 10 items on the "Enjoyment" scale that had a correlation coefficient between item scores and total scores above 0.75, and 10 items on the "Value" scale with a similar correlation coefficient above 0.60. These 20 items were then randomly mixed to produce the survey protocol I used.

Interviews

Teachers. I initially conducted semi-structured interviews with all of the teachers in the department during January 1994. All interviews lasted about 1 hour, were tape-recorded, and later transcribed for subsequent analysis. My interviews concerned teachers' understandings of teaching processes, the factors they felt affected their teaching, and their reasons for choices they made in relation to specific and concrete situations they encountered in their teaching. (A copy of the interview protocol forms Appendix 3.) Some specific question areas concerned the planning of teaching, what problems teachers foresaw, their choice of course material and instructional strategies, and what they liked to know about the learners in their class. (Note: this, and all other interview protocols, were field-tested prior to data collection).

I further interviewed the teachers of the three introductory classes three more times during the term. The second and third interviews--each lasting about 15 - 20 minutes--took place immediately before and immediately after the lessons concerning the specific content area on which I focused. The fourth interviews-- which lasted about 45 minutes each--took place during the last week of instruction. These interview protocols form Appendices 4 and 5. Briefly, they covered such issues as:

(a) Before the lesson: Specific examples of how teachers chose lesson content and instructional strategies for the particular section of the syllabus, how the lesson fit into the overall course, what learner knowledge was considered a pre-requisite, and what learner problems were anticipated.

(b) After the lesson: Specific examples of what changes were made to the lesson, what mathematics difficulties showed up, how those difficulties were dealt with, and what happened that was unexpected.

(c) <u>At the end of term</u>: Specific examples of what changes were made to the teaching throughout the term, what mathematics difficulties showed up, how they were dealt with, and what happened that was unexpected.

<u>Stimulated recall</u>. Certain lessons in this phase had also been videotaped and the teachers were asked to participate in stimulated recall interviews: a means of collecting teachers' retrospective reports of their thought processes. Stimulated recall is a term used to describe a variety of interview techniques designed to gain access to others' thoughts and decision-making. Typically, it involves audio-taping or videotaping participants' behavior, such as counseling or teaching, in situ. Participants are then asked to listen to, or view, these recordings and describe their thought processes at the time of the behavior. It is assumed that the cues provided by the audiotape or videotape will enable participants to relive the episode to the extent of being able to provide, in retrospect, accurate verbal accounts of their original thought processes (Calderhead, 1981).

Stimulated recall has largely been used in three different areas: with learners, with teachers, and with other practitioners (such as doctors or counselors) engaged in skilled behavior. It has taken slightly different forms in these three different contexts. Bloom (1953), who pioneered its use, was interested in learners' thought processes during different learning situations. He played back audiotapes of lectures and discussions to university students and recorded their commentaries of their thoughts. These reported thoughts were later categorized according to their content and their relevance to the subject matter being studied. Kagan, Krathwohl, and Miller (1963) developed Interpersonal Process Recall: a form of stimulated recall using video-recordings as a means of increasing counselors' awareness of interpersonal interactions during counseling interviews. Elstein, Kagan, Shulman, Jason, and Loupe (1972) used stimulated recall in research on clinical decision-

making attempting to identify the thought processes of clinicians in simulated diagnostic situations. Leithwood and others (1993) studied the problem-solving processes of school superintendents during their meetings with senior administrative staff. In classroom-based research, stimulated recall has also been used in a variety of ways to investigate the thought processes and decision-making of teachers while teaching. Keith (1988) demonstrates the diversity found in a group of selected studies.

One factor which can influence the data collected by stimulated recall concerns the way in which participants are prepared for their commentary, and how they are instructed to comment. Calderhead (1981) notes that respondents can often identify, and hence comply with, the aims of the researcher. He also describes a study by McKay and Marland (1978) in which the researchers, although avoiding the imposition of any research model on the thoughts of teachers, provided detailed instructions before the videotaped lesson on the kinds of thoughts teachers were expected to recall. Calderhead claims that the provision of explicit instructions may have influenced the teaching itself, and the procedures may also have encouraged the teachers to place a greater degree of rationality on their behavior.

Mindful of these issues, in this study I merely asked the teachers to view the videotapes of their teaching and to comment on whatever they wanted. I gave them no instructions as to what to comment on, or how to comment. The teachers were given a remote control for the video playback machine and could stop the tape at any time they chose; they then commented (in whatever way they wanted) on the section of tape they had just viewed. These comments were audio-taped and later transcribed for subsequent analysis.

Students. I also interviewed four adult students in each of the three classes that I selected (a total of 12 people). These people were interviewed three times: first, immediately after the first lesson of the algebra section; second, immediately

after the end of the algebra section; and third, at the end of the term. The interview protocols form Appendices 6 and 7. The first and second interviews--which lasted about 15 - 20 minutes each--dealt with the specific content of the observed lesson. Typical questions covered such areas as the content of the lesson, the work that students were asked to perform, any difficulties they encountered, and if they found this lesson typical of others.

The third interviews--each lasting between 15 - 75 minutes--covered broader areas and included data from the preliminary survey of learners' attitudes towards mathematics. These interviews covered such areas as learners' experiences of mathematics education at school, and as an adult, their involvement in classroom activities during the course, and their attitudes towards the course content and teaching processes.

Observations

Direct observation in classrooms allowed me to study the teaching processes as they took place in their natural setting. I was able to gather such data as the form and content of verbal interaction between participants, non-verbal behavior, patterns of actions and non-action, and references to the textbook and other instructional material. Further, by acting as an "involved interpreter" I was able to "understand the events that occur, not merely record their occurrence" (Anderson & Burns, 1989, p.138).

I observed the teaching in selected classes of all three sections of the introductory mathematics courses. Two sections of this course met twice each week (Tuesday/Thursday) and a combined section met four times each week (Monday - Thursday) for the full term (17 weeks). Each class was scheduled to run for two

hours (12:30 - 2:30 PM). I could not, of course, observe all the classes in each section (they met at the same time).

I carried out three sets of observations. The first, early in the term, allowed me to familiarize myself with the classroom and college settings. Here I observed the whole lesson several times in each course (2 hours each). During this period I was able to introduce myself to the participants, gain their trust and cooperation, and collect some general data about the physical layouts of the buildings and classrooms, typical events in the college and in the mathematics classes, and details about the participants (such as their dress, their relations with others, and their behaviors). Data collected in this way informed the structuring of the later interviews.

In the second set of observations, I focused specifically on one particular section of the syllabus (Introducing Algebra). Again I observed lessons in their entirety. I was able to observe how teachers introduced the chosen subject matter, and how they structured their lessons around it. Extensive field notes were taken for each observation. Events observed each time included: whether the lesson began on time, whether anyone was late, whether the teacher appeared to be following a lesson plan, the activities students were asked to perform, the students' attentiveness and participation, what things appeared to concern the students, and the evaluation procedures used.

Videotaping. I also videotaped some of these lessons using one camera to cover both the teacher and the students. In sum, 6 complete lessons were videotaped; three from each of courses 050 and 050/051. In videotaping the lessons I concentrated on supplementing my earlier ethnographic observations which had produced elaborate, though partial, field-notes. I tended to initially concentrate largely on the teachers as they moved around the classroom. However, as much of the lesson time was given over to individual student work, I could also focus the camera (with its in-built microphone) on the students' actions and utterances. The

videotaping of lessons therefore provided a richer and more detailed record than the earlier note-taking. In particular, video-recording captured the details of small movements and oral comments as well as larger physical movements and differences in behavior.

Some implications of using the video-recording equipment in the classroom concerned entry into the setting, the timing of taping segments, the visual point of focus, and the analysis of the data. I will deal with the data analysis issues in the next section of this chapter, but here, discuss each of the others in turn.

I was initially concerned that appearing in the classroom with the video-recording equipment--which although light and portable was, nevertheless, obvious and intrusive--would be seen as overly disruptive to the participants. Consequently, I took the equipment into the classrooms for several periods beforehand and practiced filming--without making any recording--so that participants could become acclimatized to the change. I also fully explained my purposes in taping, so that participants were aware that my intention was to capture what the teaching of mathematics "looked like" rather than a focus on one particular individual or group of individuals. As participants became used to the presence of the camera in their classroom, and were never aware when I was actually recording and when I wasn't, they tended, over time, to ignore the presence of the equipment and they did not appear to behave in specific ways for the camera. Consequently, I feel assured that the recordings I made are accurate renditions of episodes and situations in mathematics classrooms.

With respect to the timing of recording, I had initially planned to sample the total available time, and record 10 minute segments of classroom behavior. After initially trying this, I changed this approach to recording the entire class lesson. The physical movement of the camera between taping and not taping proved more

disruptive to the class than I anticipated, and I felt that by taping the whole lesson I was better able to capture data that matched my research questions.

Visual focus was also a point of concern. Initially, I tended to concentrate the camera on the teacher, but later, as I grew more adept as a camera operator, I moved it more around the room and was able to focus on both the teacher and the students. I also tried to capture multiple points of reference: sometimes trying to capture what was seen from the students' perspective (by taking one of the seats reserved for students), sometimes by setting the camera up to view the class as the teacher might see it, other times trying to move around the room focusing on odd segments of behavior or snatches of conversations.

Document Collection

The overall content of most ABE mathematics courses in British Columbia is determined largely by provincial curriculum guides. In addition, research has shown that the content and structure of the set textbooks also determines the specific classroom content and structure of many mathematics courses in both British Columbia (Faris, 1992) and elsewhere (Höghielm, 1985; Romberg & Carpenter, 1986). Therefore, I gathered copies of documents relevant to the teaching of mathematics to the adults in the study. These included the Provincial Update on Adult Basic Education Articulation which contains generic course outlines of all ABE courses in British Columbia, copies of specific course outlines and syllabi, course handouts, examination papers, and mathematics textbooks used as course texts. In addition, I gathered any relevant material that pertained to the adult students' uses of mathematics (e.g., the learner's notebooks and completed homework assignments). Finally, I was able to collect copies of the students' examination papers, after they had been marked by the teacher.

Data analysis is the process of bringing order, structure, and meaning to a mass of collected data. Within qualitative research, analysis consists of a search for general statements about relationships among categories of data. Much of this process consists of organizing the data, sorting and coding the initial data set, generating themes and categories, testing the emerging themes and concepts against the data, searching for alternative explanations, and writing the final report.

In this study, analysis of the data was ongoing and iterative, and guided throughout by my conceptual framework. Data analysis fell into two phases. The first phase of data analysis began almost as immediately as data collection, where concepts that had been identified in my theoretical framework began to appear in examples of classroom practices in the early data. This concurrent process of data collection and analysis enabled me to identify themes and patterns of teaching processes from both observations of classroom practices and from teachers' and students' comments. It also served as a check that sufficient and appropriately focused information was gathered before the completion of the data collection period.

Initial analysis of the observational data involved searching through the data to obtain categories and themes that would portray an overall understanding of the framework of the teaching processes. Here, I paid particular attention to the roles that teachers adopted, and the tasks that they asked students to perform. During this early analysis, I also made analytic notes about specific points to pursue with the teachers in subsequent interviews.

As the analysis proceeded, I modified my initial categories and themes to better refine the portrayal of teaching processes and introduce the influences of frame factors. As more data were collected, particular interpretations and concepts could be refuted or confirmed by checking them against the most recently collected data.

Once data collection had ceased, the second phase of data analysis began. Now, the concepts and categories that had been developed in the first phase formed the basis for a narrowing of focus, and the beginning of a process of abstracting and theorizing. I describe below the procedures I followed when analyzing different types of data.

Survey Data

The survey protocol contained 20 items that measured two different variables: enjoyment of, and perceived relevance of, mathematics. Respondents had indicated their attitude towards the questions on a 5-point Likert-type scale. Each person's survey responses were quantified and transferred into computer data. Data relating to each variable were separated and total scores for each of the variables were produced for each respondent. Histograms of the distributions of total scores for each variable were produced, and their means and standard deviations calculated. Finally, a correlation of the scores for each person was calculated and a scattergram of the distribution was produced.

Observational Data

I had made extensive fieldnotes throughout my observations. These notes were transcribed shortly after each observation and used to generate and test

conceptual themes and categories during this first phase of data analysis as described above. During the second phase, when the earlier categories and themes had become more "fixed" into theories, I reread the entire corpus of fieldnotes looking for recurrent patterns and examples that might challenge or disprove those theories.

Analysis of the video-recorded data proved more complicated. I found no guides or established procedures to help me. Existing guides to coding video data tended to require elaborate and intricate coding procedures and techniques, or focused instead on sophisticated micro-analysis of small segments of data. As my purpose was to capture a wider, classroom perspective, I adopted a less rigorous approach. I viewed the video-recordings repeatedly and took extensive field-notes during each viewing. Next, I used my conceptual framework and data obtained through other collection methods to discern some examples of the themes in the video data. I identified which segments of the tape corresponded to those themes and finally fully transcribed the audio tracks of these segments, noting specific behaviors in the margins of my transcribed notes.

Interview Data

Interview data were treated in much the same way as the observational fieldnotes. Each interview was tape-recorded and transcribed, and its transcription was checked against the recording, and amended where necessary for greater accuracy. The interview transcripts were then fully examined and used to search for, and refine, examples of themes and categories.

I had considered returning copies of the interview transcripts to each interviewee for them to clarify certain points, and comment on my initial interpretations. This proved impossible. Data collection was intensive and little time

was available (for either the researcher or the respondents) for the further interviews that such clarification would have necessitated. Further, because so many interviews were being conducted, the interview-transcription-checking process was also intensive, and each cycle took upwards of 3 weeks before the interview transcripts could have been returned to respondents. As so much of the data collected was contextually specific, I was afraid that the lengthy time difference would influence the ability of the respondents to adequately remember the earlier situations. In any case, I was interviewing the same people repeatedly, and could more easily verify their earlier responses in subsequent interviews.

Stimulated recall interviews. The data obtained through the stimulated recall interviews provided a means of collecting teachers' retrospective reports of their thought processes. As such, it provided a further type of data to be organized and interpreted; additional categories could be developed to analyze the kinds of thoughts that teachers report. However, such categories also reflected my interests as the researcher and, hence, differed from the interpretive frameworks of teachers.

I also identified several other issues relating to the interpretation of such data: Could stimulated recall reports be taken to reflect teachers' real thoughts during teaching? Did teachers' reasons for their behaviors constitute adequate explanations? Did the teachers censor or distort their thoughts in order to present themselves more favorably? I was unable to adequately answer these questions. However, my purpose was not to inquire too deeply into whether what the teachers said they were thinking corresponded exactly to what they were thinking at the time. I therefore resolved that the data I obtained through the stimulated recall interviews, together with the other data on teachers' thinking and behavior, enabled me to gain some picture of the types of decisions that teachers made, and the ways that they described their own actions.

These issues have also been identified in other discussions of the use of stimulated recall interviews. Calderhead (1981) identifies two types of factors that determine the significance or status of stimulated recall data. First, several factors may influence the extent to which people recall and report their thoughts. For example, viewing a videotape of one of their lessons can be, for many teachers, an anxiety-provoking experience which may influence their recall or the extent to which they report it. Additionally, Bloom (1953) suggests that each individual perceives a unique set of visual clues which may or may not be recorded by the researcher. Fuller and Manning (1973) make a similar point in suggesting that teachers viewing videotapes of their lessons are perceiving the lesson from a different perspective--as observers rather than actors. This, they claim, can affect participants' thinking. Additionally, they note that participants can be distracted by, for example, their physical characteristics.

A second category of factors concern those areas of a person's knowledge that have never been verbalized and may not be communicable in verbal form. Calderhead (1981) describes this as *tacit* knowledge which, although forming a large part of everyday cognitive ability, may have been developed through experience or trial and error, and cannot be verbalized during stimulated recall. For experienced teachers, much of their classroom behavior may be unthinking and automatic: they have long since forgotten the rationale for such behavior. It would seem unlikely that stimulated recall could reveal thoughts which occur at a low level of awareness or without any awareness whatsoever. Nisbett & Wilson (1977) argue that self-reporting of such higher-order cognitive processes is impossible and that data collected by stimulated recall is not the result of introspective awareness but the result of recalling of *a priori* causal theories which participants may regard as appropriate explanations for the outcome of their thoughts. In this way, they may not represent the actual decision-making processes involved.

Documents

Curriculum and textbook materials were analyzed in two ways. First I analyzed the content and style of curriculum guides and textbooks to examine what they specifically said about mathematics education. For example, questions I considered included: What do they say are the goals of mathematics education? Do they indicate how these goals can be achieved? What kinds of knowledge are represented as being important? Do textbooks encourage classroom activities that build on learners' experiences? Do they indicate how learners can use texts to help them learn? Selander (1990) describes a theory of pedagogic text analysis and offers methodological suggestions for such analysis. Briefly, his theory suggests that a proper understanding of textbook content involves the consideration of "background data: the social, political, and economic system in which a certain...written curriculum is situated;....the selection of facts and themes; the style of writing...and the combination of facts and explanations" (p. 147). This analysis helped me determine how much the curriculum guides and set texts are appropriate for adult learners.

Second, I considered the place and role of the textbooks within each course. I gathered data on how textbooks were perceived and used by adult learners and their teachers. For example, I sought to discover: How do the teachers and learners use their textbooks? How are textbooks discussed within the class? How does the textbook fit in to the course syllabus/lesson format/classroom activity? Analysis of this data helped me understand the use of textbooks from the participants' perspectives.

Finally, I considered the students' notebooks, homework assignments, and end-of-term examination papers. Careful inspection and analysis of these enabled

me to determine those areas of mathematics which students found easy, and those

where they had difficulty. Further, I was able to determine the areas where students did not appear to understand the work they were being asked to perform, whether they appeared to be aware of any such conceptual lack, and, if so, how they dealt with it.

Criteria of Soundness

It is necessary in descriptions of research to discuss certain issues that normally fall into the categories of validity, reliability, and generalizability. These categories, however, are more appropriate to research conducted from a positivistic approach, and are usually considered inappropriate (at least in their generally accepted form) for more qualitative studies (Lincoln & Guba, 1985; Merriam, 1988). That is not to say that the issues are any less important, but that they are conceptualized and described somewhat differently in qualitative research. Qualitative research still concerns itself with "issues of a studies' conceptualization and the ways in which data have been collected, analyzed, and interpreted" (Merriam, 1988, p. 165).

I examine these aspects more fully in a methodological coda to this study (Appendix 22) by considering several general standards of judging research propounded by Hammersley (1992) and Howe and Eisenhart's (1990). However, here, I identify and briefly describe these issues using Lincoln and Guba's (1985) notions of criteria of soundness—considered by them as appropriate constructs for

judging qualitative inquiry. Their four criteria concern the issues of credibility, transferability, dependability, and confirmability.

Credibility

Here, the goal is "to demonstrate that the inquiry was conducted in such a manner as to ensure that the subject was accurately identified and described" (Marshall & Rossman, 1989, p. 145). My study was based solely on data derived from extensive study in adult mathematics classrooms. Descriptions of that data were only considered within the parameters of those settings, the people in those settings, and the theoretical framework of this study. My initial findings were presented to the students and teachers of the classes I studied in both individual and group interview situations. Here, participants were able to examine some of my initial concepts and explain and clarify their own perspectives on them, and on teaching in general. Further, the study was, throughout, conducted under the watchful guidance of my doctoral research committee. All stages of research conceptualization, data collection, analysis, and report writing were recounted to, and discussed with, them. In this way, I can verify that the study was conducted in a credible manner.

Transferability

This second criterion refers to the researcher's ability to demonstrate that his findings can be transferred, or applied, to other contexts. I make few claims on the transferability of this research to other settings; the burden of applicability seems, to me at least, to rest with those who wish to make such a transfer. However, for those researchers who may wish to replicate such a study as this, I have, throughout my study, provided details of the theoretical parameters of my research, and my data

collection and analysis procedures methods. In addition, my research involved

triangulation of multiple sources of data. I gathered data in several situations, from

multiple informants, and chose data collection techniques that provided data from

several sources to "corroborate, elaborate, or illuminate" the research (Marshall &

Rossman, p. 146).

Dependability

Here the concern lies in accounting "for changing conditions in the

phenomenon chosen for study" (Marshall & Rossman, p. 146). As the classrooms and

settings I studied were constantly changing, I can make no sweeping claims for

dependability. However, my continued presence in the chosen settings over a period

of time allowed me to recognize and respond to any such changes. My data were

gathered over the complete lifespan of the phenomenon--a term-length course--and I

was able to observe the teaching in all of its different phases. Further, by constantly

relating the data to the theoretical framework, I minimized any effects that changing

conditions could have made on my study. Finally, I kept an "audit trail" of what data

was gathered and how it was gathered and can therefore account for both the

process and the product of my study.

Confirmability

Confirmability refers to the issue of whether the findings of the study can be

confirmed by others, and not overly biased by the "natural subjectivity of the

researcher" (Marshall & Rossman, p. 147). I attempted to do this in three ways: by

ensuring that the study's data and protocols are available for inspection, by

constantly ensuring that all aspects of the study were related to the conceptual

framework and the tenets of my chosen approach, and by ensuring that my methods of data collection were responsive, and sympathetic to, the study participants' own situations.

In this chapter I animate my research model (see Figure 1) and consider the background elements to my study: those frame factors that can influence or limit teaching processes. I first examine the institutional framework: the contexts and settings in which mathematics education takes place. I discuss, in turn, the places where such education happens: the College, its departments, and its classrooms. Next, I turn to the people involved, and discuss their backgrounds, experiences, and attitudes. Finally, I consider the work that these people do, by examining the curricula of mathematics courses and the key role played by the set textbooks.

Institutional Framework

Classroom research often ignores the context or settings in which education takes place. Yet such contexts can often have a major influence on educational processes. The institutional framework of educational establishments, their administrative and physical structures, and their rules and procedures can all affect teaching processes and how those processes are perceived.

In this section I discuss the first element of my model that can be seen as a frame factor: the institutional aspects of mathematics education. I first describe the College in terms of its physical and administrative structures, its overall course provision, and the services it offers to students. Next, I focus on the mathematics department itself and consider its goals and operation, the mathematics courses it

offers, and some of its departmental policies. Third, I describe the mathematics classrooms and how they are perceived by those who use them. Finally, I consider some of the ways that these settings affect the teaching and learning of mathematics.

College

Acton College (a pseudonym, abbreviated throughout as AC) is a publicly-funded post-secondary educational institution established in the 1960s from several existing adult education bodies and institutions. Its mission is to

> Provide adults with quality, student centered educational opportunities which promote and support lifelong learning, personal development, employability and responsible citizenship. The college welcomes all members of its culturally diverse and global community irrespective of ability or previous education, including those encountering barriers to their full participation in society. (AC, 1994, p. 1)

AC offers a wide variety of academic and vocational programs and courses to several thousand students each year. Four aspects of the College warrant discussion in terms of their apparent influence on teaching processes: its physical and organizational structures, its courses offerings, and its provision of student services.

Physical Structure

AC is a multi-storied concrete building situated on the side of a hill in an Acton suburb. Because of its hilly setting, the college can be entered from a variety of doorways on four of its levels. The doorways open onto a series of interlocking corridors that house all the instructional and administrative facilities. On the first floor, these corridors are all linked by outdoor patios where potted trees and shrubs are interspersed with bench seats. This area is close to the college cafeteria and is a

popular place for students to take a break between classes or pause to smoke, eat their lunches, or chat with friends.

Within the building, classrooms and offices for each program area are grouped together. For example, the several automotive training programs are all located in the basement. The Adult Basic Education division (which includes the mathematics department) is housed on the third floor. The internal layout of the college, although making access and movement easy, is confusing. There are few signs indicating which floor one is currently on or giving directions of where one may need to go. Also, the room numbering system is complex, and one can often be stopped by bewildered students wandering the corridors searching for a missing room. Despite this, the college has a friendly, if somewhat impersonal feel. It seems designed to be largely functional: a place to be used but not to be especially visited. "It's not a hangout sort of a place," as one student put it (L.3.10).

The college is a place for the purposeful. If you know what you're doing (and where you should be doing it), then the physical structure seems designed to help you; if you don't (or are unsure), then the structure is confusing.

Organizational Structure

Academically, AC is organized into three main divisions: Adult Basic Education (which includes the Mathematics Department), Career, and English as a Second Language. The first offers programs in academic and vocational upgrading and Adult Special Education. Instruction is offered in courses from a basic literacy level through to BC Provincial Adult Secondary School Completion (Grade 12). Students can attend courses on either a full-time (registered for 20 or more hours per week) or part-time basis, and, in most areas, can opt for either classroom-based education or an individualized, self-paced program of study.

AC's organizational structure naturally affects teachers more than it does students. For example, Departments and Divisions compete for limited financial resources which determine staffing levels and the scope of course provision. Despite this, however, teachers in the mathematics department thought that AC provided a good working situation. As one teacher put it, "normally, [it's] a good place to work....The environment is good...also the colleagues. We are like [a] big family...we just cooperate with each other" (T.1.4).

Overall, the college places few administrative restrictions on its teachers. There are no guidelines about <u>how</u> to teach, for example, although the college does expect its faculty to be familiar with the different learning needs and styles of adults. To aid this, it requires an "Instructor's Diploma" of its newer faculty and provides (voluntary) refresher "instructional development workshops." However, all the teachers in the mathematics department have either sufficient service to be exempt from the compulsory requirement or have a BC Teachers Certificate (which also exempts them). It offers refresher "instructional development workshops," but only one teacher had participated, and he found it of little use: "I suppose it was all-right...not much about math, though" (T.1.3).

The only administrative restriction mentioned by teachers concerned the recommended minimum class size. (Each class is supposed to have fifteen registered students by the third week of classes or it is canceled.) As the department head put it, "If there's enough...to run the class, once the term is underway [then] the college stays pretty much out of it" (T.1.1). Other teachers agreed, "There's supposed to be a hard and fast rule about [class size] but often it depends on the department and what other [courses] are being offered....[For instance], could students move to another section if this [class] was cut?" (T.1.8).

Teachers reported that the college administrative organization did not overly influence their day-to-day teaching. "Everything's changing all the time," said one

teacher, "although we still have a lot of freedom. The upper level [of administration] is so busy with all that's happening outside of...instruction [that] we're left alone" (T.4.2). Teachers regarded their isolation as advantageous. "I don't think anyone should be telling us how to teach," said one, "It's our job to know that" (T.1.8). Indeed, teachers appeared to relish being left alone and tried hard to maintain distance between themselves and the college administration.

Despite the current budgetary uncertainties, teachers appreciated what they called the "lower stress levels" of college teaching as compared to high-school teaching. One teacher listed the benefits of teaching in a college: "You don't have to deal with parents, [and] attendance isn't an issue. If a student doesn't want to come to class, we try to find out why...but it's not essential [that we do]" (T.1.1). "You don't have school district rules to contend with [so] it's much more relaxed here," agreed a second teacher. "You don't have to plan topics to the nth degree, there are few discipline problems [and] you don't need any classroom survival skills" (T.1.3).

In sum, we can see that teachers are relatively free to choose many of their classroom teaching processes without much interference from the college administration. Indeed, they prefer it that way. Even those factors (such as minimum class size) that could affect teaching are "negotiable." If anything, administrative factors served to encourage teachers to devote less time to teaching. As one teacher put it, "If there's any danger that this class may be [cut]...then I'm going to be a little bit cautious in terms of how much preparation I do" (T.1.8).

Course Provision

The mathematics courses offered by the college form part of a system of requirements for completion of the ABE Provincial Diploma. This Diploma, sanctioned by the BC Government's Ministry of Advanced Education and Job

Training in 1986, is awarded to any student who completes the requirements for secondary school graduation as laid out in the Provincial ABE Framework (see Appendix 8). The Diploma is recognized by colleges and universities throughout the province as an official credential for entry into university studies (Ministry of Advanced Education, Training, and Technology, 1992). The Diploma (and the ABE Framework) is overseen by a ministerial committee on Adult Basic Education comprised of representatives of those institutions that provide ABE courses throughout the province.

For students who study mathematics at AC, three aspects of course provision seem to affect them most: the admissions procedures, the financial cost of the courses, and the attendance requirements.

Admissions. As a "post-secondary institution committed to educating the adult learner," AC normally only accepts Canadian or landed immigrant students who are 18 years of age or older, or who are aged between 15 and 17 with no school-attendance in the past year. However, on occasions, "a small number of international students [are accepted] on a cost recovery basis" (AC, 1993, p. 8). Students who apply for admission must attend a pre-registration interview and "present details of their previous educational attainments" (p. 8). If prospective students have been away from an educational setting for three or more years, they are normally required to take (at cost) an assessment examination to determine their "appropriate placement level" for each of their chosen subjects (p. 8). Further, AC requires that its students must have "adequate English language skills to understand class lectures, take part in class discussions, and complete written assignments" (p. 7). Consequently, all students whose first language is not English must take an English Language Proficiency assessment before they are admitted. Hence, prospective students intending to study several subjects (as most do) are faced with an array of examinations and charges before their courses start.

The College admissions procedures also seem unduly complicated. "You need a degree just to get in here," said one teacher (T.1.7). In general, teachers thought that the college restrictions could be unduly harsh on students, given their often "unconventional" lives. One teacher spoke about such difficulties:

> I think the whole assessment procedure is a little too off-putting for some of our students. It's more of a hurdle than a help. We put too much credibility on assessment tests. Of course, you have to be careful: we do give [more] credibility to presented academic credentials than discussed academic credentials. But getting in can be a challenge in itself. The assessment costs $15 which can be...daunting for unemployed, broke people. If [students] are not sure what they want to do...or are just shopping around, then $15 for math and $15 for English or whatever can frighten them away. (T.1.3)

The admissions procedure certainly served to weed out the uncommitted. For anyone aged over 20 who wished to study full-time, the application procedure alone could cost almost $100. Additionally, acceptance into the college depends entirely on students' past academic performance or their achievement on subject-based "assessment" examinations. Those adult learners who pass the initial examinations and are admitted to the College have already been prepared to equate success and achievement with standardized and impersonal forms of assessment.

Finance. Tuition for each course costs a further $90. In addition, students are expected to purchase the textbooks and any required extra material. (For example, students in one of the introductory mathematics classes are required to buy a $15 geometrical construction set.) Many students were not paying for courses themselves. Several indicated that they were receiving financial aid, either from family members or from government grants or loans. "I could never afford to pay for this myself," said one publicly-funded student, dismayed at the costs he did incur, "I have to buy the books as it is" (L.3.4).

Obviously, the combined cost of the admissions procedure and the courses can influence the attitudes of students who enroll. Regardless of who is paying, enrolling for courses represents a sizable investment. Many students feel pressured

to attend all the sessions of those courses in which they enrolled, and try to pass these courses, if only to realize their investment. One student described: "I feel like this is my last chance....I totally wasted my time at school...and now I'm getting money for a second shot....I'm really lucky...many folks don't get this opportunity...so I'd better not blow it" (L.3.2).

Attendance. Once admitted to the college and enrolled in courses, students are expected to attend and participate in all of its sessions. "Successful completion of and progress through courses/programs is based on...class assignments, examinations, participation and attendance" (AC, 1993, p. 16). Indeed, students who fail to attend the first three classes of a course or who do not regularly attend throughout the term are asked to withdraw. However, the college recognizes that it has a responsibility to assist students in overcoming problems that affect their performance and attendance. It makes such assistance principally available from either the academic department involved (for instructional and learning problems) or the college's counseling service (for students' vocational and personal concerns).

Student Services

The college provides a variety of services to "help students with their studies and assist them in completing their goals and objectives" (AC, 1993, p. 20). These include an Assessment Centre, Bookstore, Cafeteria, Counseling Service, Daycare, Health Service, a Learning Centre, and Library. For mathematics assistance, students identified the Assessment Centre, Counseling Service, and the Learning Centre as most useful.

The Assessment Centre offers (for a fee) assessments of students' abilities in reading, writing, mathematics, typing, accountancy, or English language. These tests are designed to "help students determine their appropriate placement levels" and the

results "may be used in lieu of school transcripts for admission to courses" (AC, 1993, p. 20). The mathematics assessment is scheduled to take 1 hour and determines students' skills in basic arithmetic and algebra. Most students in the introductory mathematics courses had taken the assessment as part of the admissions process and found it useful. "It wasn't too hard," said one. "It showed where I needed help and told me which was the right course" (L.3.3). Most of these students had accepted the results of their assessments and enrolled in the courses at the suggested level. A couple of students, however, although having been "placed" in higher level courses, wanted to re-study the more basic material. "I've done all this before," explained one student, "when I was at school....But I thought that I'd better go over it again...to get my hand in" (L.3.1).

The Counseling Service provides a range of services: educational, career and vocational counseling; crisis, stress management, test and mathematics anxiety intervention; instruction in life skills; and services for disabled and international students. Most counseling is provided on an appointment-only basis, although the Service also provides a limited drop-in and emergency facility. The Counseling Service also operates a self-help resource centre for current and prospective students to obtain information about the college and its facilities and services for students.

The Learning Centre is a drop-in "learning support service...provid[ing] students with assistance with their studies" (AC, 1993, p. 24). Its services include "one-on-one tutoring, specialized small group workshops, audio tapes and listening carrels, computer software, study areas, course materials, makeup test services, and course-related worksheets for a variety of subjects" (including mathematics) (AC, 1993, p. 25). The Centre's regular staff includes a mathematics tutor; in addition, instructors from the mathematics department are each scheduled to provide two hours extra tutoring throughout the week. Finally, the learning centre is one of the few quiet places at AC in which students can study.

Teachers were very positive about the student services provided by the College, and saw them as one of the main benefits for students who choose to study at AC. "We offer as much as we can...certainly more than other [colleges]" said one teacher. "Funding for poor students, a counseling centre for those with learning disabilities, a learning centre which is like a study hall. There's a good math tutor there" (T.1.4). "Often, our students do not really know much about how to learn," said another teacher, "so the counseling centre and the other facilities can be a real benefit" (T.3.2). Teachers also saw their own work as fitting in to this network of facilities. As one put it,

> We help students get what they need....We do more than just teach. There are times when students come and try to sort out their lives with us. All sorts, from people who want to know what computer to buy, to people who are crying on your shoulder because they have larger problems. (T.1.2)

Most students, in contrast, although aware of the College facilities, found little reason to use them. As one put it, "I find I don't need them at the moment. I really just stay here as short a time as [possible]. I go home and do my work....Maybe next term when I'm taking more courses" (L.3.2). Another claimed that, "the library's OK, but...I don't use it much for math though....The teacher said not to bother with other books in case we got confused (L.3.6).

Perhaps to counteract the rather impersonal aspects of the College, some students appreciated the opportunities for more human problem-solving help:

> There's a guy at the Learning Centre...he's really helpful. I can go to him and he'll sit down and show me a way of doing it that's really simple. Then he lets me sit there and practice 'til I've got it right....He makes it look really easy. He does it to everybody...even the guys that are taking calculus. (L.3.4)

Overall, students compared AC favorably to their high-schools: "AC is much better...I learn faster" (L.1.3). Another agreed: "That's why I came here," she said,

> A lot of people told me it's good here, and they were right....I have a friend who took Math 11 at high-school and he took it again here, and he says that he learned a lot more here...in a quicker period as well. (T.3.4)

Students particularly appreciated the adult atmosphere at AC. "The teachers

[here] treat you like human beings," said one student.

> Like when I was at school, there was gangs and that....I mean, I was never part of that, but it's going on around you. That does something to your learning. Like, in school I was OK in math...I was a "C+." But since coming here, I've been an "A." So...it's the same work, but I'm learning faster. I'm in a better environment. I can concentrate more...it helps me out having people I can relate to around me. (L.1.3)

Departments

The Adult Mathematics department, together with the departments of Science, Humanities, Business, and Computer Studies are organized into an administrative section called College Foundations (CF), all part of the ABE Division. In this section students can only take semester-long, classroom-based courses. (There is a self-study program for students who wish to study on an individualized, self-study basis.)

The goal of the CF mathematics department is to "enable the adult student to study mathematics in an environment where the student can make progress and experience success" (AC, 1993, p. 58). Teachers in the department assumed the responsibility for setting an appropriate climate to meet that goal. Teachers described how such climate-setting involved not only their own classroom behavior but also the interpersonal relationships within the department. "People are very eager to help each other," said one teacher. "There's no professional jealousy...our personalities just sort of mesh" (T.1.7). "It all works for the students," explained a second. "If [teachers] are getting on together, then I'm not distracted and can focus on my teaching" (T.3.3).

Teachers also place expectations and responsibilities on students so that they can "make progress and experience success." Apart from the general college guidelines on appropriate student behavior, the department requires that

> The new student must enter the course appropriate to his/her background. Therefore when the student has not taken a mathematics course during the prior three years, an assessment is recommended. ESL students must be at the Upper Intermediate Level of English or higher. (AC, 1993, p. 58)

Thus, prospective students are obliged to fit into a pre-existing structure, irrespective of their wishes. The determination of which course is "appropriate to his/her background" rests with the department; students cannot enroll for whatever courses they choose. In this system, assessment of prior knowledge is key, both for those who have been away from school for three years (presumably most adults), and for those whose first language is one other than English. Nearly all of the students enrolled in the introductory mathematics classes fall into these categories.

The mathematics department has a good reputation among students and teachers. "You get a lot of support," explained one student. "People are tempted to wallow...but here you get a push...and there are deadlines you have to keep" (L.3.1). Some students, fearful of repeating their earlier bad experiences in math classrooms, had thought about enrolling in the self-paced program: "When I came in for my initial interview," said one student, "the teacher suggested that I give the classroom-based course a try. I'm glad I did...it's not as bad as I thought...and I need the discipline...someone to hold my nose to the grindstone" (L.3.6). "You're forced to pay attention," said a third student. "The teacher goes so fast that you can't afford to miss anything" (L.3.3).

Teachers identified how the mathematics department also had a good reputation with other provincial colleges. "It's been built up over the past number of years," said one teacher, "we hear from BCIT [and] UBC...that our students do really well in their courses" (T.1.6). "[BCIT] are very pleased with our program," agreed the

department head. "They tend to send, or encourage students to come here to get the

pre-requisites" (T.1.1). When asked why she thought that was, she continued,

> I think it's a number of things....They recognize that we have an adult focus...
> and we teach the sort of things that they want....We meet with them...and
> they say that they notice that their students are really weak in particular
> areas, so we take that into account when we're planning our courses....The
> amount of depth in our courses is sometimes determined by what the
> receiving institutions want....I mean, we want our students to be successful.
> (T.1.1)

Notice that, in terms of reputation, students mention comments made by

other students; the teachers refer to comments made by colleagues at other

participating institutions.

Courses

AC offers mathematics courses at three levels corresponding to academic

grades 10 - 12 (see Appendix 9). Within each level, there are three courses: two half-

courses and one "double-block" course which combines the curricula of the two half-

courses. The College calendar cautions that, "double-block classes are very intensive;

they are not recommended for students who have difficulty with mathematics or

who have an unduly heavy workload" (AC, 1993, p. 59).

My research focused on the three courses offered at the most basic (Grade 10)

level: 050, 051, and 050/051. The college calendar describes them briefly:

> Mathematics 050 and 051 are ABE Intermediate level mathematics courses
> designed for the student who has never studied academic mathematics before
> or who is lacking a good foundation in basic algebra skills. The content
> includes: a review of basic math skills, a study of measurement, and
> introductory algebra and geometry. Mathematics 050 must be taken before
> Mathematics 051. (AC, 1993, p. 59)

Further written information about each course is given to students during the

first meetings of each course. Usually, this information concerns the instructor's

name and phone number, a list of the set books and extra material required, the meeting dates for course sessions and what topics will be covered on what days, and the course assessment guidelines. Appendices 10 - 12 give details on the three courses on which my research focused. Notice that the handouts are all quite s' .dár: they follow the same layout and contain the same sorts of information described in similar ways. (Indeed, the handouts describing the information for all the courses offered by the department follow the same structure.)

Curiously, the neat regularity of this schedule allows for considerable flexibility amongst staff. The teaching of courses is shared among all eight of the full-time mathematics teachers in the department. One teacher explained how scheduling decisions were made: "The department head...sends round a schedule...usually it's what we did last term, and teachers can make any comments or requests on it. Then, if there are objections, conflicts...they can be discussed at a department meeting" (T.1.4). "The day instruction runs from 8:30 am to 4:30 pm," said one teacher. "We can usually pick the times we want to teach...but not which classes....We give our preferences but it's the head that decides" (T.1.8). However, according to some teachers, times at which specific courses are offered vary little from term to term. One teacher explained that

> Math 12 and 11, they're always offered early....Some of that has to do with the science classes.... Students taking those levels of math are also taking the physics, and biology and chemistry, so that you have to make it flexible for them. So, 050 and 051 get offered later, usually at 12:30. They're never offered early. (T.1.7)

Teachers appreciated this opportunity to influence their teaching schedules and supported the department policy of rotating teachers among its classes:

> To make teaching more efficient, we have to constantly change [and] revise our curriculum. It's the changing that makes teaching interesting and challenging and [keeps] us constantly awake...instead of teaching the same thing. Can you imagine talking about sine and cosine, sine and cosine, sine and cosine all the time? (T.1.4)

Although teachers never expressed preferences for teaching certain course levels, they did choose particular days or time slots to teach. So, as the times and days on which the courses were offered changed little from term to term, teachers could effectively choose which levels they wanted to teach. I also observed a departmental staff meeting where teachers chose which courses to offer during the coming summer session, and the times and days on which to offer them. Vacation decisions had already been agreed, and final decisions about course offerings were taken on the basis of who would be taking vacation (and, hence, who would be left to work) during which periods. Here, decisions were taken, not to fit the department policy of rotation, but to "fit in" with teachers' personal arrangements. "We try to rotate as much as possible," said one teacher, "but it depends on the schedule... whether everyone's schedule fits in" (T.1.4).

Policies

Other than the rotation of teachers, the department made few policies that affected how courses were taught. The only policy consistently mentioned by teachers concerned the standardized term-end examinations. "Each grade...has one test," explained one teacher. "One of us has responsibility for designing the test for all classes at that level. Then all students in that grade...take a general test" (T.1.5). This procedure serves to impose added conformity on teaching in each class which teachers claimed to find reassuring: "It stops people doing their own thing. If I'm not preparing the test for my class, I have to make sure that I cover all the areas properly" (T.1.7).

A second policy which somewhat affected the mathematics department concerned the end of term evaluation of courses by students. Students have a right (under the College regulations) to evaluate each course, and the mathematics

department has devised an evaluation form for such a use (see Appendix 13). The
department head explained how it should be distributed:

> The departmental assistant [should] give it out in each class before the end of
> term...so that [teachers] have a chance to respond to some of the things that
> the students say. Its a kind of A-B-C-D-E scale and then there are some spaces
> for written responses. It's completely anonymous, so the students can say
> what they like. (Field Notes, 931220)

However, I noticed that this form was never distributed (nor even mentioned)
in the three classes I observed. Further, I never observed any time allocated to an in-
class discussion of evaluation. "It's a real political issue," explained one teacher.
"Some people are scared stiff of it...so it's never gotten off the ground" (T.1.2). Others
had different explanations: "I'd like to do [the evaluation]," said one teacher, "but at
the end of term there's not enough time" (T.1.2). "I think that [it] would be too
intimidating," said another teacher, referring to in-class evaluations. "Students
would feel put on the spot" (T.3.3).

Evaluation was commonly forgotten. As one teacher put it, "Some [student]
will say something...such as 'Why do we need to do all of this homework?' and I'll
think 'Maybe I should discuss that with the class.' But usually I don't" (T.1.7).
However, most teachers, when asked, merely shrugged. "It's the way we do things, I
guess," said one. "If one [of us] doesn't do it, then there's no pressure on the rest of
us" (T.4.2).

Classrooms

AC's mathematics classrooms are situated close to each other (and the
teachers' offices) on the third floor of the college. Classrooms for the music, science,
and ESL departments are close by. Indeed, in the corridor outside the classrooms

one is constantly aware of the proximity of other departments. Trumpet solos or operatic scales can be regularly heard, the science labs emit curious chemical odors, and ESL students chatter to each other ceaselessly in other languages.

The department has sole use of two of their classrooms; the third is shared with neighboring departments. Each classroom is similar to the others: they are each about 10m x 7m in size, well-lit, and with centrally-controlled heat and air-conditioning. Each is linoleum-floored, and contains 10 or so rectangular (about 2m x 1m) wooden tables and 25-30 wooden chairs laid out in rows facing the teacher's desk and the blackboard. The length of one wall in each room is taken up entirely with desk-height windows that look out onto a concrete walkway and other classroom windows beyond. Two of the rooms also contain an overhead projector and screen set up in one corner next to the blackboard. The rooms look and feel like traditional college classrooms: anonymous, businesslike, formal.

Two of the classrooms have notice boards carrying a variety of posters. These have details of the College's health and counseling services, a notice advertising a long-past college event, the library opening hours, what to do if there's a fire, and advertisements for credit cards and magazine subscriptions. Only a few deal with mathematics. Of these, most seem to have been produced by textbook publishers and assure the reader that "MATH IS FUN" or detail "Six Steps to Problem Solving." A poster in one room has a large photograph of Albert Einstein over the caption, "Do not worry about your difficulties in mathematics. I can assure you that mine are greater." Another headed "MATHPHOBIA CAN COST YOU A CAREER!" lists jobs that people are supposedly unable to hold if they have mathphobia: "statistician, physicist, pilot, dental technician, accountant, surveyor, welder, chemist." Although these posters are presumably displayed for the benefit of students, I never saw any student stop to read them; nor were they ever referred to by the teachers. Indeed, by

their yellowish tinge and curled corners, the posters all looked as if they hadn't been
moved for some time.

Those who used the rooms differed in their reactions towards the room layout. Teachers were aware of how classroom layout could affect learning and teaching. "Some of the rooms are better set up for a kind of interactive approach," said one teacher. "Those with hexagonal tables are great for getting small-group work going. Some of our classrooms [have] just rectangular tables and it's much more difficult to do anything [other] than pairs" (T.1.1). Another teacher said that he preferred to have students working together but that the table layout did not encourage collaborative working: "We do our best with the rectangular tables, but the students know that [even though they sit together] they're not doing cooperative groupwork. I mean they don't get the same grade...so they're not that committed to each other" (T.1.6). I asked the teachers if they ever changed the table layout.

> [Another teacher] and I tried that one term...we spent quite a bit of time rearranging all the desks. The next morning the students had come in and rearranged them all the way they had been before. They obviously didn't want to work in small groups, I guess. (T.3.1)

Students, conversely, were largely unconcerned about room layout; their only comments about the rooms concerned their size. "I find...they're a bit too large," complained one student, "I sometimes can't hear what other people are saying" (L.3.3). A second student explained that, "classes always start out large...then get whittled down....So, you've got this huge room with only eight people in it" (L.3.1). I was interested to note that the layout of the two math-only rooms did not change throughout the whole term, whereas in the shared room, the layout changed regularly. Sometimes the tables would be in rows as in the other rooms, sometimes in a hollow U-shape, once in a solid block of tables, and once with the tables put together in pairs for groupwork. During my observations in this room, neither teachers nor students ever mentioned the change in the room layout. "Don't make no difference," said one student when asked directly, "math is math" (L.3.9). "I do like to

see everyone's face," said a second student, "but it's really only important in a classroom where you're going to discuss things...like psychology or English" (L.3.11).

I noticed that most students preferred to pick one seat and keep it throughout the term. "I like to sit near at the front," said one student, "I can concentrate better if there's not too many distractions....You know, people coming and going....If you get that big guy, Harry, in front of you, you can't see a thing" (L.3.2). "I'm supposed to wear glasses," admitted another student, "so I like to sit as close to the board as I can" (L.3.4). Being close to the board was clearly important: "I don't really care where I sit, so long as I can get a good look at the board. [The teacher] likes to write all over it, so you need to be able to see even the bottom corners" (L.3.6). Only one student said that it shouldn't matter where people sit. "We're there to learn," he said, "we shouldn't try to get as close to the board as possible" (L.3.4).

Experiences of Students and Teachers

Studies of teaching are often limited by focusing either solely on classroom practices and dynamics or solely on the backgrounds and experiences of learners or teachers. However, in reality, these two areas are interrelated and interact to affect classroom practices and influence interpretations of those experiences. In this section, I consider the second element of my model: the experiences of students and teachers involved in three introductory-level mathematics classes.

I first discuss details of students' backgrounds and experiences, their attitudes towards mathematics education, and their reasons for enrolling in a mathematics

course. Next, I consider certain characteristics of all eight teachers within the

mathematics department. Although my research focused most closely on three

classes, there are two reasons for obtaining data from all the teachers. First, the

mathematics department seeks to arrange a teacher rotation to ensure that all

teachers teach introductory level courses. Second, at the outset of data collection, no

decisions had been made about which courses would recruit enough students to

proceed, or which teachers would be teaching which courses.

Students

The beginning of the term is filled with uncertainty. Some students enroll in

courses and don't show up. Some come to classes for only a few sessions and then

leave; others attend without ever having registered. College policy recommends that

a minimum number of 15 students be registered in each course by the third week of

classes or the course is canceled. Consequently, during the early part of the Spring

term, there was considerable anxiety within the department that one or more courses

would not be allowed to proceed. However, by the College's deadline in mid-

January, 37 students had enrolled in the three sections of the introductory-level

mathematics courses. During the fourth week of instruction I administered a survey

protocol (Appendix 2) to the 32 students who were attendance that week. Their self-

descriptions of basic demographic data are summarized in Appendix 14.

Students' Backgrounds

Students in this study have a variety of backgrounds in terms of their gender,

age, ethnicity, and occupation. The literature identifies several other features of

students' backgrounds which can influence teaching: students' English language

fluency, other courses being taken, and students' previous experiences of mathematics education. I now discuss each of these background features in turn.

Gender. Of the 32 students surveyed, 20 were male and 12 female. The gender balance remained the same throughout the term even though some students dropped ou of classes and others joined. When the survey was re-administered in the final two weeks of term, the proportion was unchanged.

These figures represent all the students enrolled in the three introductory-level classes. When each course is considered separately, a difference in the gender balance appears. In the two classes that met only twice per week (050 and 051) the gender balance was almost even, while the four-day per week "double-block" class (050/051) contained only male students. "Pretty normal," said the double-block class teacher, "usually [a] lot more men. [They] have more time" (T.1.5).

Age. Students' ages ranged from 18 to 45 years with a mean of slightly over 24 years. The distribution of ages is shown in Figure 2. (Note: only 30 of the 32 students gave their ages in the demographic survey.)
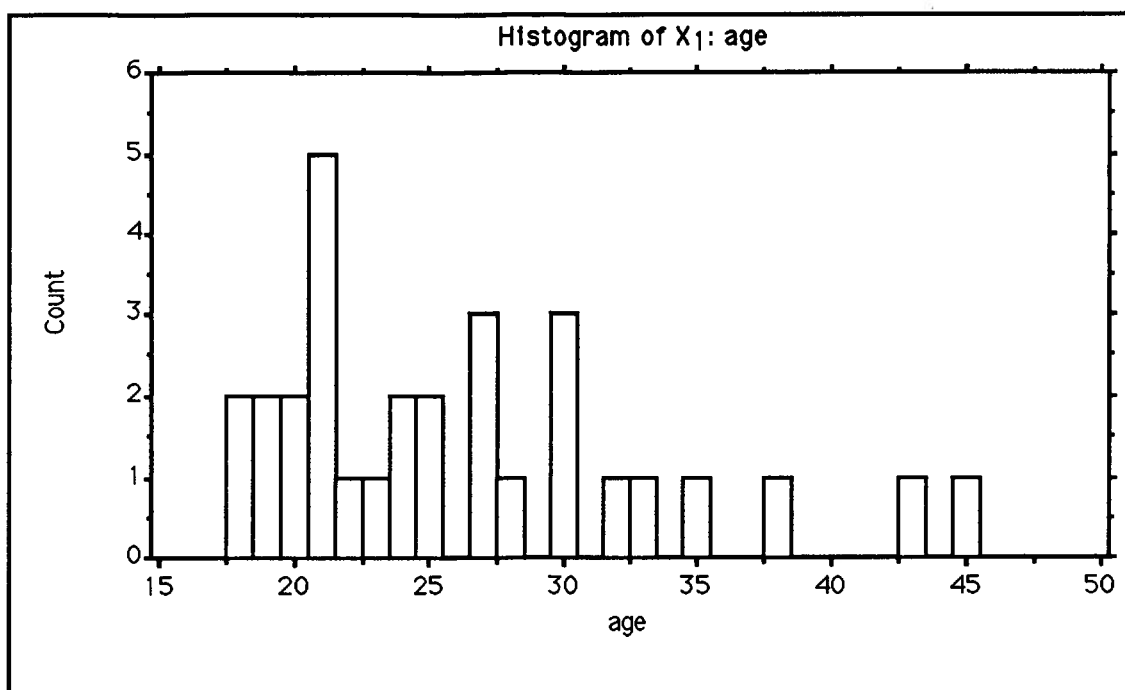
Figure 2: Distribution of Student's Ages

There was no marked difference between the range of ages of the male and female students. Further, with regard to the distribution of their students' ages, the three classes were similar: each class contained one or two students aged 19 years or younger, five or six students in their 20s, and two or three aged 30 years or over. Most students said that they liked the range of ages in their classes. "You don't feel so stupid," said one, "when you see guys in their 40s in the class" (L.3.6).

Ethnicity and English language ability. Only half of the students were either Canadian or part-Canadian. The others identified themselves as either First Nations members (3) or immigrants from Europe (5), Asia (4), Africa (3) or Central America (1). Most (23) students spoke English as their first language; the exceptions were the non-European immigrants, all of whom were also enrolled in college English classes.

Language ability was key for many students. Although the language used in the mathematics classes was not seen as "hard", it occasionally contained uncommon words, which, if not understood, could delay students' understanding of the

mathematics. Several of the immigrant students said that, although they had

previously studied mathematics in their native countries, they were re-taking Grade

10 mathematics in order to gain further familiarity with the English language. "They

wanted to put me in a higher grade," said one Chinese student when referring to her

initial placement interview, "but I said I want to go over [grade 10] again--to

revise...and to practice with the words" (L.1.1). Few of the foreign students identified

that they had much difficulty with spoken or written language in the math class.

"Sometimes the teacher goes a little fast," said one, "but I can read it later in the book"

(L.3.4). A couple of students said that they carried dictionaries with them to the

mathematics class, and would occasionally look up unfamiliar words. However, this

remained a private activity--often carried out under the desk (and out of sight of the

teacher).

Language difficulties were never addressed publicly in class, although fellow

students could often determine who was struggling. "I think a couple of the non-

English speakers are having difficulty," said one Canadian student. "They sit up at

the front and you can tell [that] it's not clicking" (L.2.1).

Language ability and use was also an issue for native English speakers.

Several students commented on the way language was used by teachers: "Some of

the teachers talk to you like you're a 12-year-old. Enunciating everything...like you're

stupid" (L.3.2). Another English-speaking student, describing a similar experience,

explained:

> One of my classmates said near the middle of the term, they're a bit peeved
> because she [the teacher] seemed to speak down to them all the time, but she's
> not really speaking down, and now this classmate has now said, "Oh, I'm glad
> she does do that, because it means she makes sure that you know." She [takes]
> great pains to make sure you understand...almost to the point of annoyance.
> But I don't mind that...she's just trying to help. (L.3.1)

Occupation and student status. Sixteen students said that they worked at

least part-time; the others were either full-time students or unemployed. Of the 16

workers, the eight men had jobs as cook, musician, taxi driver, clerk/cashier,
maintenance worker, and night-watcher. The eight women had jobs as arts
administrator, cook, waitress, clerk, hostess, and cashier.

Several students spoke about the experience of being working students:

> I work about 20 hours a week in a store. I finish here [college] at quarter to three and start work at three. Then work until 8 or 9 at night. Get home about 9:30; then I spend one hour to do my homework. (L.3.8)

> I'm a taxi-driver...and I usually work late afternoons [so that] I can come to class during the day. That's not so bad...but it's the homework. I try to do some while I'm at work...but usually I have to get up early to do it. When you've not gotten in until 2 or 3 am...It's hard. (L.3.11)

> I work at [a record store] stacking CDs. I'm only part-time so it's usually it's about 10 hours per week. Normally, they're very good about letting me have time off to come to class...sometimes I have to switch shifts with other staff. A couple of weeks ago there was stocktaking and that was hell. It was very difficult at school —I had a lot of papers to hand in for my other courses...and we had a math test, so it was very stressful. I couldn't miss any work in case I got laid off, so I had to miss a couple of classes. I'm just about caught up now, but it was pretty difficult. (L.3.1)

Other students who worked found that they needed to alter their working
arrangements to fit their school timetable: "Before I did the full-time school and full-
time work. After a while I found it's too much work. Now I just [work] on weekends,
in a restaurant" (L.1.1). Only one student indicated that her employer gave her time
off work to attend college: "I work in a glass shop...auto glass and window glass.
My boss, he gives me time off...no pay mind you...but I tell him when I want to
come and he lets me off early or changes my shifts" (L.3.6).

Of the 16 non-working students, 13 attended college full-time. They either
lived at home with (and were financially supported by) family members, had built
up sufficient savings to fund their time at the college, or funded their studies by
student loans or government grants. One young student described his financial
position:

> I get a grant to cover my fees. I could never afford all this by myself....Then there's the books...Social Assistance has to pay for that. You have to be 19, but

they will help for education. It's not that I want to go on it, but I'm going to have to to survive, to have an income to pay rent....My Mom's letting me off with the rent right now because I'm not of age, but when I'm of age she's going to expect it....I'll keep getting the funding as long as I pass [the courses]. (L.3.5)

Other courses. For most students, mathematics was only one part of their studies. All the students I interviewed were taking other courses at the college; all immigrants were studying English as a Second Language, and either computing or accountancy courses. Among all the students, mathematics, computing, and accountancy appealed because they were less language-based than other subjects. "I like the math class best," said one foreign student. "There's just one book...the language is easier [than in other courses]...and you don't have to speak in class" (L.3.7). Many foreign-born students had recently committed to attend college full-time and did not want to overtax their limited language abilities at this stage. "Sometimes [in mathematics] the words are hard," said one Afghani student, "but [there's] not much writing" (L.3.6).

Among the non-immigrant students, computing courses were also popular, and several students also studied Science and English Literature. These students were trying to gain their high school equivalency and said that they took the science courses to help them gain access into higher education courses at other colleges or local universities.

Students said that, in general, they preferred the mathematics classes to the others they were taking: "[Mathematics] is easier that way. You know what you have to do and when you have to do it" (L.3.3). Another added: "I can sit down to my math homework and know that there is an end to it...even though it might take me all night. With other subjects...like English, I never feel that" (L.3.9). For many students, part of mathematics' attractiveness as a subject was its "boundedness"—the way that it was treated as a fixed and permanent body of facts and procedures.

"Once you get the rule," said one student, "you're away....You don't have to think about what it means...or if it applies in every case....You know it does" (L.3.3).

Experiences of mathematics education. Almost all of the students were entering an adult mathematics class for the first time; indeed, for many it was their first experience of adult education. Several students remembered their childhood mathematics education: these were commonly described as unpleasant experiences. For some, math was just one part of an altogether negative school experience: "I detested school, period....Where I grew up, school was not a big pastime...there was a lot of...law bending and stuff. I was totally alienated at times....Not just the math, everything suffered" (L.3.3).

> When I was a kid, we moved around a lot. I didn't do very well...because we were always moving. I don't think that math was any worse than other subjects...I think you can wing it [in math] because...all other classes are dealing with language. (L.3.1)

For others, mathematics education was worse than for other subjects: "I can look back on it now...on my math courses...and a few of my teachers were duds and didn't make the course interesting at all...they had no enthusiasm" (L.3.2). Another added, "I could never get the hang of it....For some reason the math teachers were always the worst...shouting, moaning...sometimes hitting you....[In math] I've always done three months or so...then got kicked out" (L.1.3).

> In the old days, the [math] class would go at a really rapid pace. The teachers would go like a bat out of hell. You swam as fast as you could and if you...couldn't keep up you went flying over the waterfall....What got lost was...I never understood any of this stuff. (L.1.5)

For many students, learning mathematics at school was a process of sitting quietly and listening to the teacher, rather than one of asking questions. One described:

> The thing I remember most is that I was...pretty frightened to raise my hand and ask a question. For two reasons: many of the teachers were of the opinion...that children should be seen and not heard. If you're not listening

then that's why you didn't get it the first time. And secondly, if you make a fool of yourself as a child, other children, they're very cruel. (L.3.1)

Even foreign students had not really enjoyed their math classes in school: "It was just [a subject] you had to do," said one. "Not very interesting" (L.3.4). "[Math] was the same as now, but in my own language" agreed another. "When I was at school, sometimes I [found] it hard....I didn't [find] anything interesting in math because it was for me sometimes confusing and I didn't know anything" (L.3.12).

Algebra is one of the key topics in Grade 10 mathematics. For many of the students, the AC math class was the first time they had encountered it. "I sort of dropped the [math] class before we ever got there," said one. "I didn't know what algebra was...and it just seemed so foreign. I think I missed the middle steps to where you start algebra" (L.1.7). Another student expressed the view that any previous mathematics education could prove a hindrance:

> What my uncle told me about algebra, he said best....He said if you're just starting to learn it now, it's easier. He said if you...if you learn algebra before and [then] you learn it again, it's confusing. But if you just start learning it now, you should be OK. (L.1.2)

Students' Attitudes and Aims

Given their diverse backgrounds, the students had different attitudes towards mathematics and different reasons for learning it. I now consider each of these.

Attitudes towards mathematics. Students gave information about their attitudes towards mathematics in two ways: in a survey completed at the beginning of the term, and in personal interviews throughout the term.

The survey protocol (see Appendix 2) measured two dimensions of students' attitudes: (a) their enjoyment of mathematics, and (b) their perception of its value. The scale of scores for each dimension ran from 0 to 40, with larger scores indicating

greater enjoyment or greater perception of value. The distribution of scores for the two dimensions are presented in Appendix 21. In each figure, the horizontal axis (E-sum and V-sum respectively) refers to the score, the vertical axis (count) refers to the tally for each score.

The mean score for the first dimension (enjoyment of mathematics) is 24.5 (with a standard deviation of 7.4); the mean score for the second (perceived value) is 30 (with a standard deviation of 4.1). A comparison of the two sets of scores shows that students (as a group) were more likely to perceive a use for mathematics than to enjoy it. However, a correlation of the two sets of scores shows that students who scored highly on the enjoyment dimension scale also had higher scores on the value dimension scale. (There was a positive correlation between the scores of 0.195.)

The survey scores give only a limited picture of students' attitudes; their comments are more descriptive. Most students were strongly convinced of the value of mathematics. For them, mathematics occupied a central position in the world: "It [math] relates to life, right? I mean it all relates back. All this relates to something" (L.1.3). Another student described mathematics as, "the rules. You have to be precise...right on...no in-betweens. It's the logical way...the way things are. You're either right or you're wrong" (L.3.5). A third thought mathematics was, "the modern language. It is in everything. If you want to live...you want to live comfortably, you must know math." After a pause, she continued, "Even if you want to live not comfortable. If you want to live in this world, you have to know math" (L.1.8).

For other students, mathematics was a way of thinking: "It's like reasoning...the way of figuring out problems," said one (S.2.5). "It's very precise," said a second student, "It's black and white. You can often get in a tangle with words, because they can mean different things to different people. But if you want to prove something...you use math" (L.3.7). "It helps me to think," explained another student, a salesperson:

When I'm in the shop...I don't necessarily use [math] all the time because it's all computerized. But if something goes wrong, you have to know "Oh, that's not right." I mean I occasionally press the wrong buttons [on the computer] and I get...the readout and it's clearly wrong. You need to know [math] all the time. (L.3.2)

Students' views on mathematics not only referred to the topics they were studying. One student described

coming into a classroom before the last class was finished. There were all these squiggles and stuff all over the board. I didn't know what it was about. I asked the teacher and he said [calculus]. It looked really hard...I don't think we do it in this class, but I'm sure I will learn it one day. I mean, it's all going to be relevant or helpful...it's going to have some bearing sooner or later. (L.1.4)

Only a couple of students were unsure of the usefulness of mathematics in general, or of certain topics in particular. One student described how she was unsure of the usefulness of algebra:

I don't know if I ever will [use algebra]. For what I've been involved with I have used different types of math. Certainly I can see that.... But this [algebra]...I don't know how specifically I'll use it. I can't think of any uses right off. (L.1.7)

Another student also wondered why he should bother learning algebra:

It's just how it is, I guess. I need it on my transcript. That's the only reason I'm taking it....It's so ridiculous you've got to learn how to do it so you don't feel you got beaten by a ridiculous concept. If it wasn't mandatory, I bet people would be taking a lot of different kinds of math. I don't think many people take it 'cause they like it....I suppose that's why it's mandatory...otherwise no-one would take it. (L.3.2)

Reasons for learning mathematics. Students' stated reasons for learning mathematics were many and varied. Half of those I interviewed had clear reasons: they were trying to complete their high-school education. Whether they were Canadians who had left school before graduation or immigrants who needed to secure credentials that would be recognized in Canada, learning math and high-school completion were necessary for entrance to higher education or different (better-paying) jobs. One student explained that, for him, mathematics

is one of my pre-req[uisite]s. I'm thinking of getting into BCIT or some
university, either a radiology or nuclear medicine program. If I get my Math
10 then I can get my sciences, which is my key to get into the program. (L.3.3)

Other students cared less or were unsure about further education. "I like to continue [to take] other courses, higher and higher," said one, "but I don't have any [goal] in mind right now...so I'm just continuing my education" (L.3.4). "I want to start my own business doing massage," said another. "So I'm going to take massage therapy, which I need my biology for, and also Shiatzu, [for] which you need biology and chemistry" (L.3.8). Some students were less certain of their future direction generally and were looking to their studies to show them a way. "That's what I'm here to find out," said one student, "I get lots of ideas of things to do, but then there's lots of drawbacks to each one. So I'm just taking the courses and trying to think of what I actually want to do" (L.3.2).

Several students described their reasons for learning math more personally. For them "improving self esteem" or challenging themselves intellectually was more important than a career. Some talked about the embarrassment or fear of being identified as "math anxious." They described how they were tired of feeling unconfident or lazy. "I really didn't feel good about math...or about myself," said one, "I wanted to do something about it" (L.1.7). "I took the course because I found my brain was getting lazy," said another. "The more you exercise it, the less lazy it gets" (L.1.2).

Other students described different reasons. One student mentioned, "the horror of not being able to do basic mathematics, and not being able to admit it...is really depressing....It brings you down so that you feel like you shouldn't do anything" (L.1.5). A second student described how

> embarrassing it is to be my age and not know...45% of something...you don't
> have a bloody clue what it is. Everybody assumes because of your age that
> you know all of these things right off the bat. I have been doing manual labor
> all my life because of an embarrassment with not knowing math, too stupid to
> actually come, too embarrassed to come [back] to school. (L.1.5)

Finally, a couple of students said they were learning mathematics in order to help their children. One student who had already studied mathematics in China (her home country) explained that she came "back to learn math to help my kid. I teach them...times table, and I do some of their school math with them. It help me too" (L.1.1).

Teachers

In this section I describe some personal and professional characteristics of the teachers. After providing some brief demographic details, I focus on their educational and teaching experiences, and their attitudes about teaching, mathematics, and their students.

The College's adult mathematics department consists of seven full-time teachers including the department head (who teaches part-time). In addition, one teacher is shared between the mathematics department and the science department. Finally, several part-time instructors are used on an on-call basis. Of the eight full-time mathematics teachers, six are male. All the teachers are Canadian citizens, most by birth, although two are immigrants (from Hong Kong and England). The youngest teacher is 26 years old and the oldest is 56; most, however, are in their late 30s to late 40s.

Educational Experience

Teachers regard their work as a career; most have worked in their present jobs for over 10 years. Indeed, for the majority of teachers in the department, teaching

mathematics is the only work they have ever done. Two teachers came straight to the College as soon as they had completed their first university degree; the others had previously taught in other Canadian high-schools or colleges. Several teachers also indicated that they had, from time to time, tutored other people (usually children) in mathematics.

All the teachers have a minimum of a bachelors degree (a college requirement); most of these are in science-related subjects (mathematics, science, or science education). Four teachers have continued their formal education with post-graduate study and some also have a provincial teaching certificate which allows them to teach in any British Columbia secondary school. Two of the teachers have never studied mathematics at college level; their degrees are in music and general studies. Both of these teachers were hired because of existing relationships with the College, as either volunteer tutors or as ex-students. One described how he was hired:

> I had been an elementary school teacher [but left] after eight years of teaching to set up my own business. [After a while] I thought I should go back to teaching....I didn't really relish the thought of teaching young people anymore, I wanted to teach adults. So I applied here, which is my old alma mater, [to] teach English. They said, "Oh, you're one of our ex-students. It would be nice to have you on our staff. How about teaching mathematics first?" I didn't really feel qualified for the position but they said, "It's just general program math. I'm sure you could handle that." So I started teaching general program math, business math, that sort of thing. (T.1.7)

Although the teachers have, as a group, a reasonably strong background in mathematics or science education, they have markedly less training in adult education. Only two teachers have taken any formal courses in adult education. When asked about how they had learned to teach adults, most teachers said that they "picked it up as they went along." For example, "Early on, I sensed I had to change certain styles....I found topics didn't have to be pre-planned to the nth degree without any ten-second lags just for classroom management survival reasons. (T.1.3)

When I was a student myself I taught a small group of other students. I
perform[ed] well in mathematics, so...a group of students approached me
and asked me to help them, so I actually had some experience in teaching. It's
really informal...but I found it's very interesting. So that's a reason I tried
teaching, tried out teaching in this college. (T.1.4)

In any case, adult education training was not seen as important by teachers

because "there really wasn't any difference between teaching math to children and to

adults" (T.1.3). Even those teachers who had studied education at a postgraduate

level had found them of little value in their own subsequent teaching. Occasionally,

the courses had helped with teaching techniques:

I guess teaching...this is to quote a lot of UBC profs...it's like having a
shotgun and you try to get as many people as possible. So when I plan my
classes what I usually do is I try to aim for the middle ground, to present it so
I do not lose the lower students, but at the same time not lose the top
students. Depending on...class interaction...I could, you know, go higher or
lower. (T.1.2)

Only one teacher found his college mathematics education useful. He

described being told about

a meta-analysis of calculative research in the 1970s; the results were
fascinating. The commonly-held belief is that if people can work with
calculators [or] computers then there'll be some attrition of paper and pencil
skills with arithmetic. But the major finding of that analysis was that if
calculators are used at the same time as paper and pencil skills, the people
with calculators have a better ability not only in conceptualizing but also with
the paper and pencil skills. So it's a win-win situation for people with
calculators....I came back to the department and said, "We should be using
calculators," and everybody said, "Yeah, you're right." So, since then we have.
(T.1.3)

Most teachers claimed that they learned about teaching from their own

experiences of learning. Sometimes, their memories significantly color their

perceptions:

I would think that often how we teach is affected by how we have been
taught.... I have often thought about my teaching in this way. At university I
had to work very hard and very independently. I've referred [in class] to
working hard, working on your own...these are some of the things [that] I've
adopted....I think that affects my own personal philosophy of striving for
excellence...I'm a person who really likes to see excellence and organization.
What bothers me is students' lack of achievement and interest and lack of

organization in their own lives. They can come in and there's just total chaos in their notebooks, and I think if their notebooks look this way, what do their minds look like? (T.1.8)

A second teacher spoke of how his own experiences with learning mathematics affected his current teaching practices:

> I tend to give a lot of notes because I find that the textbooks are usually not given in the simple terms that...novices to math could use. So what I usually try to do is I break things down into...simpler terms and so on. I guess that's one of the things I picked up when I was going through education. (T.1.3)

Often teachers remembered unpleasant memories of their own math education. One spoke of his "experience with mathematics instructors [as] almost universally horrid....The tedium, the shiny polyester pants, the unchanged suits, the sweat stains, the jacket that never changes....I try hard never to be like that" (T.1.3). Another said:

> There's nothing worse than coming into a math or a science class [and] it's deadly silent, you don't know anybody else in the room, you have no idea what to expect. You're nervous or whatever, and some guy comes in and says, "Here's the course outline, here's the first chapter, get on with it." This is what I left school for, to get away from this stuff. (T.1.6)

Attitudes about Teaching

Perhaps because of the similarity in their backgrounds and experience, teachers also held similar views about teaching. In general, they thought that teaching mathematics was largely a matter of conveying fixed concepts and set procedures. As the content was established, their role became one of deciding how to convey that content. Teaching became an exercise in selecting the "most efficient strategies." Such a process could be influenced by students, but only occasionally, and only indirectly.

Teachers developed their favorite strategies with experience. "I've built up three or four different ways of approach, " said one teacher. "Of course, I have my

favorites which I will always use unless students tell me that it doesn't make
sense....For me, when you teach you're trying to sort through a whole garbage
dump and see what is appropriate" (T.1.2). "I like to get a feel of a group," explained
a second teacher, "get a feel of their learning attitudes...So I can tell which strategy is
best...to accommodate that, to help them achieve their learning goals" (T.1.4).

In general, however, teachers subjugated the learning needs of students to
their own need to "cover the material." While they acknowledged that students had
different learning styles, teachers didn't necessarily change their teaching approach
to accommodate students: "I'd like to say that it [teaching] depends on the type of
students...but it doesn't really. It's almost dictated by the length of the class and
what we have to cover...there's so much to get through" (T.1.6). "There's a lot of
pressure here to get through the material," agreed another teacher, "You can't always
do what might be best for the student" (T.1.8). A third added:

> I haven't the time to get into learning disabilities and stuff...I'm not really
> qualified. I mean there is a structure that should help students in that sense.
> My job is to make sure that we cover the material....If students are having
> difficulty they can come and see me after class...or go to the learning centre.
> (T.2.2)

Most teachers also thought that students should feel a sense of personal
accomplishment at the end of each course. "It's important that [students] meet their
goals, not only pass [the] test" explained one teacher (T.3.1). Teachers appreciated
that some students were already highly motivated. "They're here because they want
to be...they've got their own reasons...but they're very focused. It makes your job
wonderful, sometimes" (T.1.6). "The more you give them, the more they give back.
They respond if you try to make it interesting" (T.1.1).

Encouraging motivation among the less enthusiastic students was also
necessary, although these students were seen as requiring guidance towards "setting
realistic goals." "Many students are not prepared for the hard work that they have to
do," explained one teacher. "Sometimes it comes as quite a shock" (T.1.1). A second

teacher explained that adult students, often with "poor study habits...well, no study habits at all" are unprepared for the level of work expected of them.

> I always tell them, right at the beginning, "Listen this isn't going to be hard, but it's going to be fast... and if you are taking 6 classes...and have two kids at home, and you're working 20 hours a week, then you should really take a look at what you've bitten off." I try to tell them that the demands are great. (T.1.8)

"Encouraging students to take responsibility for their own learning," was seen as particularly important for adult learners. "When they leave this place they're going to be very much on their own at UBC or wherever," explained one teacher. "To get [them] ready for that means that we can't hold their hand the whole way through the term...we've got to get them on their own feet" (T.1.6). "It's impossible to cover everything," said another,

> I have a philosophy that there comes a point where the student has to make certain connections on their own. I tell them this, "I cannot foresee every possibility and difficulty you might have. You must come and tell me of [your] problems." Some do, some don't. But, you know they're adults...they should know what responsibility is. (T.1.2)

Another teacher compared teaching children and teaching adults. "When you're teaching children," he explained,

> you have some responsibility for the actual learning that the person's doing. When you're working with adults you're free of that responsibility. This [math education] is such a minor part of their lives...they've got more pressing problems in their lives than learning. The choice to learn is clearly their own....My responsibility is to remind them of that. (T.1.3)

Developing responsibility in learning affects how people teach. One teacher thought it crucial to understand students' backgrounds to ascertain their attitudes: "I like to find out who I've got here...where they're coming from. That's going to affect what I do. It tells me whether people are here because they've failed or because they haven't had it (T.1.8).

Other teachers said that they develop a sense of responsibility in students by encouraging them to ask questions: "What I like to do in the first 20 minutes or so,"

explained one teacher, "is deal with any questions that they have...from their own

work and study. I encourage everybody to ask questions...even if they're dumb

questions. It's important that they say what they don't understand" (T.1.3).

Encouraging a questioning approach in students was seen, by some teachers,

as time-consuming yet rewarding:

> I find that with the lower levels...more time seems to be spent on individual
> activity. I'll coach more...spend more time with each individual student...to
> get them to think...to understand it for themselves...so they can work more
> on their own in the future. (T.1.3)

Other teachers were less enthusiastic:

> Oh, it gets very hard to maintain your level of enthusiasm over the term. You
> can see some students just don't have a clue...even though you spend time
> after time with them. I find I get personally worn down. Especially at a level
> like 050 where there's a lot of attrition anyway. (T.1.8)

> Often, you think you've got it just right...the right climate and everything....
> Then students will come up with unexpected problems...perhaps financial or
> their cat is going to die...or they're going to have to take off for a week to go
> to Toronto because they don't want to be alone for Halloween. (T.3.3)

Aims of teaching. Teachers' attitudes about teaching influenced their

instructional goals and aims. Overwhelmingly, teachers said their general goals or

aims in teaching mathematics were to foster an understanding or enjoyment of

mathematics. Only one teacher described his goals purely in terms of completing the

course material. (For this teacher, the main goal was "helping students to get the

course done. This fulfills the requirements" (T.1.5).) The others spoke of "getting

students...to have a basic grip of what it's about...and liking it" (T.1.1), "getting

students to know how to do it and to understand it" (T.1.8). "The best enjoyment I

get," said one teacher, "is when some students come back and say 'I really liked that

course...I really felt I understood math for the first time.' It's great when that

happens" (T.1.2). However, encouraging such understanding was not always easy:

"Some people are impatient," said one teacher,

because that's the way that math has been taught to them, they don't have a lot of patience or tolerance. They want you to tell them what to do and when. They don't see that it's necessary [for them] to do some work to have some understanding. (T.1.8)

Here, it should be mentioned that, for teachers, understanding is seen as learning (and being able to reproduce) existing knowledge. The notion of understanding as "making meaning" was never mentioned. Teachers spoke of mathematics as either a fixed body of knowledge or as a way of thinking. They felt it important that students appreciate and understand the inherent logic and organization of mathematics: "Something I try to do," said one teacher, is get students "to appreciate the logic behind the steps...and why to go about it that way instead of another way" (T.1.2). "It's important for me to convey the thought of mathematics," said another, "not just teach them math that they can use" (T.1.4).

Several teachers also spoke about how they tried to encourage motivation, interest, "taking control of learning," and "independent learning" in their students. Fostering independence in students is seen as crucial: "If we send them out still dependent on walking into a classroom and sitting down and waiting for it all to happen, we're not doing them any favors at all," said one teacher (T.1.6). A second described how self-motivation was far more effective at encouraging learning among adult students than any specific teaching technique:

> It's that sense of responsibility...that learning is not my, but [the student's] responsibility. I try to achieve that sense of internal motivation in the student, and that's why I strongly believe that a zillion different instructional strategies work....but in the end, it's down to the student. Their responsibility, their interest, their desire. (T.1.3)

Often, encouraging motivation involved building on students' life experiences. One teacher described how students have

> already made some major changes in their lives...quit their jobs, or left their spouses or whatever. They've already made some major decisions; taken some responsibility....It's my job to remind them of that and to encourage them to see their learning as something [else] they can take responsibility for. (T.1.3)

Another teacher described a telling example of how "a sense that you <u>can</u> do it" and persistence in students can pay off:

> You see that guy, David, that guy who came in this morning. He's had to take the course about three times because his life was a mess, you know there was all sorts of things going on in the background. His writing was terrible, his reading was awful, but he succeeded....I must have spent hours with him, going over the work. [Although] most of it was his own determination to do it. Without that it wouldn't have worked. (T.1.6)

Additionally, teachers recognized that self-motivation was aided by a non-threatening atmosphere in the classroom. They spoke of trying to minimize the "math anxiety" that many adult students feel, by making their courses less threatening. To do this, they would tell jokes or encourage "banter" to "lighten the atmosphere." One teacher brings classical music tapes into his classroom to play as background music. Another, a skilled artist, uses cartoons about math to get students "laughing...it helps to break the atmosphere right at the beginning" (T.1.7). A third described how she would use "fun activities" to "make math seem more enjoyable":

> I call them "algebra adages" because we use them in the algebra level[1] . They have to solve a series of simple puzzles and each answer has a letter assigned to it, like a code. When they've solved one question they write the letter in the answer space...and when they've done all the questions the answers spell out a saying...an adage. They're a bit elementary...but even though they seem like children's activities...the adults enjoy them...and are not insulted. The [activities] certainly make doing algebra more fun. (T.1.8)

Talking about learning mathematics is also seen as crucial for dealing with any student anxiety:

> I ask them if they think they suffer from math anxiety. If I know that someone's really anxious about math, I'll try and jolly them a lot more. If [students] come into the class with a negative attitude towards math it affects their achievement. Like a self-fulfilling prophecy, "I can't do math." Well, if you keep telling yourself that, you won't be able to, that's for sure. (T.1.1)

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[1] One example of an "algebra adage" is included as Appendix 20.

I try to bring it out in the open, and say "Look, you might be the only person that said you were scared, but I'll let you know that there's at least two thirds of the people in here feel the same way....Of course, sometimes people aren't very comfortable talking in a group, so I try and get them on their own so we can talk about it there. (T.1.6)

Another teacher described how he would speak to a class about his own struggles with learning:

I tell [students] that when I left school I had a Grade 5 education in math. Right away I tell them that I'm on their side; I understand where they're coming from. And I explain to them that when I went back to school I had to change my lifestyle. I was really nervous about learning because it had been so long, and I wasn't sure I could do it. But the fact that you're there means that you want to be there...and that makes up for quite a bit. (T.1.7)

Attitudes about Teaching Mathematics

Teachers tended to regard mathematics as either a set of thinking skills or a fixed body of knowledge that transcends context. "Most of us who have ever taught math know it's a universal thing," as one teacher put it (T.1.2). Another described that "the important thing [in learning mathematics] is the ability to think and reason. So it's not so much being able to factor, but...[understand] the process behind factoring" (T.3.3). "Mathematics is everyday in our lives," said a third:

If you go to fill up your tank in the car, then you figure out how many litres you get...how much money you pay. Everyday you listen to news...you see percents...like unemployment is down, interest rates up....Also, mathematics trains our mind. Some of the things we may not use in our daily lives, but everybody has to think and everybody has to do some kind of mathematics. (T.1.5)

Despite the ubiquity of mathematics, teachers acknowledged that for many students, learning it was hard work. "A lot of mathematics is learning how to use other means other than a laboring--a pick and shovel--approach to thinking your way out of situations," said one teacher.

I often tell the students that if you want to be lazy, you look for the easy way
out. You can go out there and run a measuring tape between here and Mars,
or you can use your head and not have to do that labor. (T.1.3)

"Our job," said a second teacher, "is to bring [mathematics] down to a simple

level. We often expect our students to know [the same] things that we do...because

of their age. I've found that you sometimes can't get basic enough" (T.1.8).

For teachers, the "organization of mathematics" as a subject provided a sense

of mental discipline. One described his students at the beginning of the course:

> You won't believe how disorganized some are. They come into the classroom
> with no books, nothing to write with. God knows what the insides of their
> brains are like. You have to show them...first this, then that. The books help a
> lot because they're so well laid out. You can't do this chapter before you've
> done the previous ones. You can't understand this concept unless you've
> mastered those.... By the end of the course they're getting the idea. (T.1.5)

Another teacher gave a telling example of how to use a mathematical concept

to "train students' memories."

> The day before the test I will ask them to write π in 6 or 8 digits. Then I will
> say, "Tomorrow, on the test you will have to write π to 8 digits.' Then they
> will be worried, "How can I remember that? I remember right now, but
> tomorrow I forget everything." So, then I come up with some kind of memory
> aide, so they can remember. And they will. (T.1.4)

A third described how "in addition to explaining the structure of

mathematics," she tried to model "good organization and discipline" in her teaching.

For her, the emphasis was notably physical:

> I make sure that all material that I give them is on 8-1/2 x 11 paper. I make
> sure that I three-hole punch every sheet that I give them. Also, I give them
> special sheets to keep track of their records...so they can see [the records] at
> all times. I encourage...record-keeping and organization through this....I
> think some of them catch on to it; maybe those who do it well would have
> done it anyway, but I will sometimes come along and talk to somebody...if
> they're supposed to be working on something and this student hasn't got an
> idea where it is...I'll say "How about putting these in order and filing them at
> the end of the day so you know where they are?" Hopefully the organization
> that I bring in, the preparedness that I bring into the classroom, you know,
> that encourages some of them to do more of that. (T.1.8)

This emphasis on the neatness and order of mathematics is not the only characteristic that teachers highlight. One spoke of trying to foster more positive attitudes towards mathematics among students: "I love math, I think it's beautiful; the symmetry and the application and everything. If I can get even one or two students to develop an appreciation for the beauty of mathematics...I think I've been successful" (T.1.1).

Even in this, however, notice that the teacher regards mathematics as fixed; its qualities (whether of order, or of beauty) already exist and await discovery. The role, here, for the teacher is that of expert/guide, leading the way while also encouraging a notion of self-reliance.

Teachers commonly thought that they could increase learning by relating mathematics to students' interests and experiences. Most teachers claimed that they tried to make mathematics seem relevant to their students. "For example," said one, "in doing angular speed I use car examples. Or in mixture problems I'll talk about mixing drinks and things like that" (T.1.1). Another teacher described how he would introduce the concept of a slope of a line:

> I'll say, "Do we have anyone from the construction industry here? This question is like finding out the pitch of a roof." People will usually volunteer that sort of information. It's great when they do, because all of a sudden there's somebody else who is saying "This is my experience...this mathematics is relevant to my life." (T.1.7)

Overall, however, most teachers felt that, at the introductory level at least, the mathematical topics covered contained little of intrinsic interest for either them or their students. Teachers described how students would occasionally ask them, "What use is this?" or "Why do we have to learn this stuff?" "Got to do it...on the test," was one teacher's stock reply. Another would be equally as honest:

> I'll say, "I don't know. It's in the curriculum guide. I don't know why it's there." And I don't. Most people never use this. For example, set notation diagrams. No working mathematician, scientist, or engineer ever uses set notation diagrams. I guess it's...the concepts behind the idea of set notation that are important. (T.1.6)

A third teacher described how, if so asked, he would answer with this story:

> One time a youngster got a job. The job is to make carpets. Well, the manager only asked him to make one little corner of the carpet. Just one little piece. Other people are making other part[s]. So, everyday they are doing this. The lad is getting bored only doing the same piece day after day. So the youngster asks the manager, "I don't like this job. I'm getting bored, everyday just doing this or that. My hands are getting tired. Why [is] it like this?" The manager took the youngster and sent him a little bit away to look at the whole carpet. "See the whole picture....You are making a beautiful picture on the carpet. And the piece that you are making is the most beautiful thing." So, from that time on, the boy say, "Oh, I want to finish this beautiful carpet now." So, if he see only his part, very boring...cannot see the whole thing. But if you can see it in the future, the part he is doing is part of the beautiful carpet.
>
> So, I give them this illustration. If you can see it, in the future you will have your career, you will want to do this, this mathematics. This is part of it. (T.3.1)

Finally, teachers did not really expect their students to feel positively towards mathematics. As one put it, "Math can be very boring. Really dry, very boring. But the students don't seem to mind that. They seem to expect math to be boring" (T.1.6).

Teachers' Attitudes about Students

Irrespective of which classes they taught, teachers claimed that they disregarded students' characteristics and backgrounds. Teachers' only concern for students' characteristics seemed to be how much they affected a student's attitude. "Some of them haven't been to school in a long time," explained one teacher. "They forget some things. But their spirit...their attitude, is much better than younger students" (T.1.5). Adult students were also thought to be "more purposeful....Mostly it's their second chance of education," as one teacher put it. "They tend to be more serious about their study" (T.1.4).

Teachers recognized that students had different life experiences, and even "appreciated" them. "They have such interesting stories...I really enjoy hearing about

how they came to be here" (T.1.1). "The population of students is extremely diverse," said a second teacher:

> Many classes are often "United Nations." You'll have...in 30 students you'll have 30 countries represented. Often very recent arrivals. I find that doesn't make a difference [to teaching]. Whether they're an international student from Hong Kong paying a zillion dollars to sit in class...or whether they're a Native student or a Caucasian from the East Side...it doesn't matter. The only thing I look for is motivation, and that doesn't depend on where you come from. (T.1.3)

However, such differences were not allowed to intrude unnecessarily into the classroom. Most teachers agreed that the demographic make-up of the class did not materially affect how or what they taught. For teachers, students only differed in two areas: their "learning characteristics" and their goals.

Learning characteristics. Teachers thought that the adults in their classes had particular learning characteristics. As one teacher put it,

> Adult students seem to have poorer memory than children....Adult[s] enjoy more with logical reasonings. Everything they do, if they can reason, they can understand, they can perform better. Young children they have better memory so they don't necessarily want to reason that much. (T.1.4)

Then, there were those whom teachers saw as "problem" students. "Some of our students are people that other institutions don't want," (T.1.2) explained one teacher. "We get all sorts at this college," agreed another:

> Schizophrenics, manic depressives, people from penitentiaries, weirdos, wackos, prostitutes, everything. There are others of course...but we do tend to get more people who are down-and-outers and have problems, a lot of those. Sometimes, these people have been kicked around all their lives, and they feel as if everyone's out to get them....So you have to be able to spot them, in case there's any trouble. (T.1.7)

Problem students were also identified as those with certain learning difficulties. "We also get a fair bit of those," said one teacher:

> They are very dependent in their learning styles. I find that very irritating. Dependent, whining...the person who wants you to chew their lunch for them. One of the first things that comes to mind is...is that person going to create any problems for me because I've got to deal with someone who's miserable and whining all the time. (T.1.3)

Another teacher, when asked about learning difficulties he encountered, described students who do not wish to learn in the way that he chooses to teach:

> Sometimes there are students who do not want to do math the way that you show them. Last semester, I had one...he rejected doing the word problems by setting equations. He did it by arithmetic; got everything right. So this is a tough case to deal with. Because you have to convince him, otherwise he see[s] no point. Some students, they've been doing it their way for 10 years...you cannot change them in a week. (T.1.4)

Finally, teachers also gave special attention to those students who may be thinking about dropping the course:

> Certainly at the lower levels we tend to have very high attrition rates. A higher-risk population; older students. Often people in your class have attempted the course before. Like there's one fellow in this class...I must have had him in the same class in 1984. Here he is again. He's enthusiastic all-right but I'm sure he's taken six runs at it. For some of them it's an infinite loop. (T.1.3)

Appreciating the students' learning characteristics allowed the teachers to sometimes make allowances:

> Many of our students are single parents and working, trying to hold down part-time jobs while going to school...so we try to make allowances for that...we recognize that they can't always be here to take a test. So we have days for doing make-ups and giving extra time on tests for those that have a recognized learning disability. (T.1.1)

Knowing the students' educational history also allowed teachers to choose which students to focus on. As one teacher described:

> I know them and I know their marks. If I know their marks then I know who is good [student]. Usually the good ones I don't pay too much attention to. And the very poor one, I cannot spend too much time because he is holding [up] the class. So then when I teach... I teach the middle. (T.1.5).

> I try to baby my students a bit. I lead them in one particular direction...and encourage them to make the next logical step. Although in some cases you do have top students who are capable of seeing past something and going beyond that. But these I usually keep on a one-to-one level. (T.1.2)

Three teachers gave examples of how background information on students affects the language that they used in class: "If I have a lot of students who have

never done any algebra...sometimes you do....If they have no algebra at all, then I will probably talk to them differently. I'll talk more about the arithmetic background" (T.1.8).

Goals. Teachers also perceive that knowing students' ultimate goals affects their teaching. Most teachers thought that "almost all" the students were headed for higher education: "Most want to go on to some kind of post-secondary training, either university or at BCIT. In fact BCIT sends many students over here to get the pre-requisites" (T.1.1). "They apply for courses [at those institutions] and they go to some form of counseling and learn that they have to get certain requirements to get in...prerequisites like Math 11 or Math 12. So they come here for those subjects" (T.1.5).

One teacher spoke of how he might use information about students' career goals:

> If I get a lot of students interested in a certain area then I might use the terminology of that area. If say 15 out of 20 say they're going into landscape architecture...then I might talk a bit about the area of a yard, and...how many bits of rolled turf would fill this. (T.3.3)

"We also get a lot of students who didn't finish high-school math," said another:

> They need math for a particular course, so they come for that. There are [also] some who just want to change careers...and pick up things they've never done. We get some that are not happy with what they got previously. We get some international students who are here to complete their education....Everyone [is] looking for a change. (T.1.2)

One teacher felt that the name of the College department was a clue to understanding the students' goals:

> The name...College Foundations...it means preparation doesn't it? Not completion. So, for a lot of students, what they want is to say, "Look give me what I need to go on and get me out of here and on to the next step." (T.1.6)

Teachers tended to regard their students as individuals who had clear goals; consequently, they often simply presented material and relied on students to raise any problems. "I don't think you can break things down into this for this kind of group, because each person is so unique, said one teacher. "They know what they want, all they want from me is some ideas of how to do it...usually as speedily as possible" (T.3.2). Further, although much of the classroom time was spent on individual activity, teachers tended to focus their teaching on the "average level" of a group, arguing that those at the margins would raise problems upon encountering them. "I guess we're really striving for the medium [of a group]," said one teacher. "We're not going one way or the other...so we [can] cover everything" (T.1.2).

Overall, however, knowledge of students' backgrounds did not influence teaching significantly. "I don't really concern myself with why they're here," as one teacher put it. "They are here and I just try to pass something on to them" (T.1.2). "We're supposed to...pitch the course at the right level," said another teacher, "but in reality we have to do what's in the book, so it really doesn't make any difference" (T.1.6).

Curriculum

In this section, I discuss the curriculum of the mathematics courses. Rather than being a separate frame factor, the curriculum can be seen as a combination of two elements of my model: the worldview of mathematics (the main factor) and the institutional framework. I discussed the overall worldviews of mathematics earlier; here, I consider how these worldviews are translated, through the administrative

machinery of provincial articulation and college provision, into course curricula. I first describe the curriculum that is used as a basis for the college's introductory mathematics courses. Next, I consider the use and role of set textbooks. I detail the structure and layout of one book in particular (a set text for the courses on which my research focused most closely), and the uses of, and views towards this book held by both the teachers of those courses and by their students.

The curriculum for the mathematics courses at AC follows the province-wide guidelines as described in the Provincial Update on Adult Basic Education Articulation (BC Government, 1992). The provincial ABE program framework has four hierarchical levels: Fundamental, Intermediate, Advanced, and Provincial. Completion of the Provincial level leads to the award of the ABE Provincial Diploma which is recognized by the province's universities and degree-granting colleges as equivalent to secondary school graduation and is, therefore, accepted as a necessary credential for entry into university-level study. The mathematics courses at AC correspond to the three higher levels (Intermediate, Advanced, and Provincial). (A copy of an AC poster advertising their course and showing which courses correspond to which ABE level is attached as Appendix 15.)

My research focused on three courses at the Intermediate level (Math 050, 051, and 050/051). The ABE Articulation Guide details the topics to be covered at this level. The relevant section forms Appendix 16, but, briefly, it covers the following generic topics: measurement; ratio and proportion; percentages; geometry; algebra; charts, tables, and graphs; statistics, problem-solving, and trigonometry. Further, the Guide makes it clear that the goal of courses at this level is "to enable adult learners to acquire mathematical knowledge, skills, and strategies needed to enter appropriate higher level courses or to satisfy personal or career goals" (BC Government, 1992, p. 26).

Teachers were well aware of the existence, content, and goals of the
curriculum guide. One teacher described how the curriculum they follow "started
within the department originally, but now its a province-wide standard. Now, they
set the core curriculum and we follow it....As a department, we set our content to
match the provincial standard" (T.1.7). "It's a decision of all the department
instructors," agreed a second teacher, "everyone teaching this course has to cover this
material...because students are going on to the next course" (T.1.4).

Only rarely did teachers depart from the curriculum laid down for them. The
beginning of the introductory courses was one such instance. "We do a catch-up
thing here," explained one teacher.

> We do a lot more review of arithmetic in the early stages...which is not in the
> Articulation guide. It's a college decision...what we call Grade 10 is not really
> Grade 10. Our students definitely need to do some revision. (T.1.1)

Spending extra time on revision of topics required for (but not covered by) the
courses affected what teachers called the "pacing" of the courses. "You can set your
own pace," said one teacher. For example, "you can spend a little longer at the
beginning on review topics, and then speed up more towards the end" (T.1.4).
However, pacing was also problematic: "It's one of the frustrations of teaching,"
explained a second teacher. "We have a set curriculum that must be done in a set
time, [and] if I'm behind schedule then I have to rush to finish everything. I don't
have much choice" (T.1.2). "Inevitably we end up rushing," said a third. "We have so
much...too much to get through.....I try to look at alternative [ways of
teaching]...like going to the computer lab...but it's as if you never have enough time
for everything" (T.1.3).

Much of the reason for this lack of time was because teachers had chosen a
series of set texts that covered the curriculum and followed them meticulously. As
will be seen in the next section, teachers based their entire course structure,
individual lesson planning, evaluation procedures, and even teaching methods

around these books. Further, when the course timetable departed (for whatever reason) from the tight schedule of study laid down in the textbook, teachers tended to assign extra work for students so that they could quickly regain their place in the schedule.

The Textbook

The College's introductory (Grade 10) level mathematics courses are based on three textbooks all by the authors Keedy and Bittinger: Basic Mathematics, Introductory Algebra, and Introductory Geometry. The first several chapters of this last book have been photocopied by the college and sold to the students at a much reduced price.

These books form part of a coordinated series of textbooks, audio and video tapes, and computer software that covers the math curriculum from grades 10 to 12. (The three books identified cover the Grade 10 curriculum.) Each textbook comes in two editions. Teachers are provided (free) with the "Teacher Edition," while students purchase the "Student Edition" for $65 new ($45. 70 used) at the College bookstore. Both editions are large format (27 cm x 21 cm) softbacks that weigh over 500 gm. Each book contains between 600 - 650 pages and is almost 3 cm thick. They have brightly-colored covers with the authors' names, the book title, and the edition superimposed on a repeating, abstract, design. A description of the content of each follows.

Teacher Edition

The teacher edition of Basic Mathematics contains, in addition to the material in the student text, answers to all the problems and supplementary information "designed to help [the teacher] maximize the effectiveness of the text" (p. T-1). This supplementary information includes: a list of the review sections and objectives covered in each chapter test; a list of topics for which "Extra Practice Problem" sheets have been developed; suggested course guidelines and syllabi for different length courses; and a "pedagogical flowchart" designed to show how "students might use this text to enhance their learning process" (p. T-15). The suggested course guidelines contain fairly detailed instructions to teachers who plan to use the textbook. For example, teachers are advised to "follow [the daily time schedule] to the letter. Students in this course are procrastinators. They cannot be allowed to set the pace" (p. T-7). Notice how, right at the beginning of the book, students are stereotyped and categorized as incapable of making their own decisions. For example, all students are described as procrastinators, regardless of their actual characteristics.

Teachers are further cautioned to "expect at least 45% of the students to withdraw or fail" (p. T-7). The guidelines also describe a "typical daily class" consisting of "a period to answer questions about the preceding assignment, followed by a lecture about the new material" and concluding with a short quiz at the end of every day (p. T-7). Finally, the guidelines exhort the teachers that they "must give eight tests and a two-hour comprehensive final examination [to] all students."

Teachers also have access to the Instructor's Resource Guide which contains extra practice sheets for each textbook chapter, indexes to the associated audio and video tapes and software packages, test-making aids and transparency masters, and a set of short essays written to help the teacher think about particular issues related

to the learning of mathematics. These essays, written by university and college 146
teachers from the USA, deal with such topics as math anxiety, study skills, language
and applications, and using manipulatives in mathematics.

Student Edition

The student edition of the <u>Basic Mathematics</u> book is "intended for students
who do not have basic arithmetic skills" (p. ix). It is "appropriate for a one-term
course in arithmetic or pre-algebra," and covers a planned sequence of arithmetic
and algebraic concepts "designed to help today's students both learn and retain
mathematical concepts" (p. ix). It contains 12 chapters, each dealing with a specific
topic area (e.g., "Operations on whole numbers," "Multiplication and division:
Decimal notation," "Algebra: Solving equations and problems"). The chapters are
organized hierarchically so that the content of each chapter draws upon and adds to
the material presented in preceding chapters. The overall effect is that of a
systematic and sequential presentation of knowledge in which the students enter (as
if into a many-roomed mansion) at page 1 and are guided through the text by a
series of presentations, examples, exercises, and reviews until they complete the
"final examination" on page 547.

This systematic approach is deliberately stressed in the book's preface. Here,
the authors describe some distinctive features of their "approach and pedagogy that
[they] feel will help meet some of the challenges all instructors face in teaching
developmental mathematics" (p. xi). Under the heading "Careful Development of
Concepts" the authors describe how they

> have divided each section into discrete and manageable learning objectives.
> Within the presentation of each objective, there is a careful build up of
> difficulty through a series of developmental and follow-up examples. These
> enable students to thoroughly understand the mathematical concepts
> involved at each step. Each objective is constructed in a similar way, which

gives students a high level of comfort with both the text and the learning process. (p. ix)

Notice that learning mathematics is presented here as a series of discrete objectives, with the textbook functioning as the authority on content (what is to be learned), process (how it is to be learned), and evaluation (both how and which learning is to be assessed). Two aspects are notable by their absence: any discussion of the learning needs of adult students, and a recognition that different learners may have differing abilities or differing learning needs and styles. Finally, although this is an extract from the preface to the student edition, throughout the preface, students are consistently referred to in the third person.

The authors continue the explanation of their approach by describing how throughout the text they "present the appropriate mathematical rationale for a topic, rather than the mathematical 'shortcuts'" (p. ix). Such a presentation, they claim, prevents student errors (from "incorrectly remembered shortcuts") in this and future courses. The authors also claim to "include real-life applications and problem-solving techniques throughout the text to motivate students and encourage them to think about how mathematics can be used" (p. xi).

Finally, the authors claim to introduce a "five-step problem-solving process" to be used whenever a mathematical problem is to be solved. This process—"Familiarize, Translate, Solve, Check, and State the Answer"--is regarded as a heuristic device applicable in all situations. This can be understood as an emphasis on a universal approach to solving problems, that all problems can be solved "mathematically," and that there can only be one right answer. However, if students have difficulty with knowing which arithmetic "rules" to apply in different situations, the textbook offered little guidance. Although it provided numerous examples of the applications of those rules, it offered little towards why one would choose those rules in the first place.

Nevertheless, the text's carefully thought-out and articulated approach is

summarized in a short section at the beginning of the book headed "To The Student."

Here the authors lay out certain guidelines to students about using the book to help

them "succeed in basic mathematics." They describe the layout of the book and of

each chapter, how to "work through a section," the associated supporting material,

the best way to prepare for the tests, and other general study tips. In all instances,

the textbook is presented as the authority of content and process ("When you see an

instruction to 'Do exercises x-xx'...you should always stop and do these to practice

what you have studied because they greatly enhance the readability of the text" (p.

xx).

An example of how this approach looks in practice can be gleaned from

Chapter 12 of the text: "Algebra: Solving Equations and Problems." (I have chosen

this chapter, not because it differs substantially from the other chapters, but because

it contains the content of the lessons (an introduction to algebra) on which my

classroom observations focused.) As do the other chapters, Chapter 12 opens with a

cover page in a different color and of a different layout than the regular pages of the

text. It reads:

> In this chapter, we continue our introduction to algebra. We consider the
> manipulation of algebraic expressions. Then we use the manipulations to
> solve equations and problems. The review sections to be tested in addition to
> the material in this chapter are 9.6, 10.1, 11.2, and 11.4. (p. 505)

The introduction also provides the reader with a list of earlier sections that

can be reviewed in order to do the test exercises in the chapter. The "Introductory

Guide to the Student" (pp. xix-xx) stresses, "It's a good idea to restudy these sections

to keep the material fresh in your mind for the midterm or final examination" (p.

xix).

The cover page also contains a "real-life" application of the material to be

studied in the chapter. On the left of the page is a black-and-white photograph of a

mountain range with downhill ski slopes in the foreground. Next to it, in the centre
of the page, under the heading "AN APPLICATION" is written "The state of Colorado
is a rectangle whose perimeter is 1300 mi. The length is 110 mi more than the width.
Find the dimensions." Finally, on the right of the page, under the heading "THE
MATHEMATICS" the reader is advised to "Let w = the width of the state of Colorado.
The problem can be translated to the following equation: $2(w + 110) + 2w = 1300$" (p.
505). (Note: this "problem" is eventually re-presented and solved at the end of the
chapter.)

A short pre-test for the chapter immediately follows the introductory page.
This test is purposely designed to "diagnose student skills and place the students
appropriately within each chapter, allowing them to concentrate on topics with
which they have particular difficulty" (p. xii). The pre-test for Chapter 12 consists of
14 questions that cover the entire content of the chapter.

Chapter 12 is split into five sections, each dealing with a specific content area:
"Introduction to Algebra and Expressions, The Addition Principle, The
Multiplication Principle, Using the Principles Together, and Solving Problems." Each
of these sections is between four and eight pages in length and follows the same
broad format as the others. (I have attached Section 12.2 "The Addition principle" as
Appendix 17 to give an example of the text layout) . An introductory paragraph
outlines the topic of the section, describes the section "objective(s)," and introduces
key concepts and words (e.g., "equivalent equations" or "the addition principle").
The objectives are written as behavioral objectives for the students (e.g., "After
finishing this section 12.2 you should be able to solve equations using the addition
principle" (p. 515)). Often sections have more than one objective and these are
referred to repeatedly in the text by use of a color-boxed symbol. The purpose of this
is explained in the textbook's preface: "The symbol next to an objective appears next
to the test, exercises, and answers that correspond to that objective, so that you can

always refer back to the appropriate material when you need to review a topic" (p.
xix).

Key words and concepts are often highlighted in the text by use of bold face type or color boxes. Several worked examples of the principle being discussed follow the introduction, and students are encouraged to work through these themselves before trying any exercises in the text. There are two worked examples for section 12.2. As an example, here is the first:

Example 1   Solve $x + 5 = -7$

We have

$$x + 5 = -7$$
$$x + 5 - 5 = -7 - 5 \qquad \text{Using the addition principle, adding -5 on both sides or subtracting 5 on both sides}$$

$$x + 0 = -12 \qquad \text{Simplifying}$$

$$x = -12$$

We can see that the solution of $x = -12$ is the number -12. To check the answer we substitute -12 in the original equation.

Check:
$$\begin{array}{c|c} x + 5 & -7 \\ -12 + 5 & -7 \\ -7 & \text{True} \end{array}$$

The solution of the original equation is -12.

In Example 1, to get x alone, we used the addition principle and subtracted 5 on both sides. This eliminated the 5 on the left. We started with $x + 5 = -7$, and using the addition principle we found a simpler solution for $x = -12$, for which it was easy to "*see*" the solution. The equations $x + 5 = -7$ and $x = -12$ are equivalent.

Mathematics is presented here as an ordered application of rules to determine the right answer. Additionally, mathematics is presented as a series of operations on abstract entities; there is no attempt to contextualize the problem, for example by a "real life" application. The purpose here is to give students the rules to solve abstract problems so that they will then be able to apply this method to other situations.

One feature of the text is the use of "margin exercises" where short problems that pertain directly to the topic being presented are placed alongside the text. These margin exercises are very similar to the worked problems in the text, and students are similarly encouraged to do the margin exercises as they work through the text "to practice what [they] have just studied because they greatly enhance the readability of the text" (p. xx). It is worth noting that the authors could make a point here about how immediate practice can enhance retention and learning (which would be in keeping with their stated philosophy), but choose instead to emphasize "readability."

Alongside the problem solved above is margin exercise 1:

Solve using the addition principle: $x + 7 = 2$.

Under each margin exercise there is space for the students to write their answers and show their work. Each section concludes with an "exercise set" of 50-60 problems that cover the content of the section. These exercise sets conclude with "synthesis exercises" which are designed to "encourage students to synthesize several learning objectives or to think through and provide insight into the present material" (p. xi). As can be seen from Appendix 17, the synthesis questions for Section 12.2 ask students to solve such problems as "$x + x = x$" or "$x + 4 = 5 + x$."

Students are advised to

Do the assigned exercises as soon as you have completed a section. The exercises are keyed to the section objectives so that if you get an incorrect answer, you know that you should restudy the text section that follows the corresponding symbol. (p. xx)

Students can check their own work because the answers to all the margin exercises and the odd-numbered questions in the problem sets are given in the back of the book. Additionally, a solutions manual with the complete worked-out solutions is available (at cost) from the publisher.

Each chapter concludes with a summary and review of the important

formulae and concepts introduced in the chapter, review exercises, a chapter test,

and a review exercise that covers all the material in the chapters to date. Finally, each

chapter contains a short "Thinking It Through" exercise, which is intended to

"encourage students to think and write about key mathematical ideas that they have

encountered in the chapter" (p. xi). The "thinking it through" exercise for Chapter 12

asks students to:

Explain all possible errors in each of the following:

1.   Solve $4 - 3x = 5$
                                 $3x = 9$
                                    $x = 3$

2.   Solve $2x - 5 = 7$
                                   $2x = 2$
                                    $x = 1$

Other examples of "Thinking it Through" exercises from other chapters ask

students to "Discuss how decimal notation is defined in terms of fractional notation"

(p. 202), "List as many reasons as you can for using the metric system exclusively" (p.

430), and describe "In what way is a unit price a rate?" (p. 272).

A final feature of the textbook is the periodic inclusion of "Sidelights."

Sidelights are "additional and optional" half-page sections that cover topics relevant

to the study of mathematics, such as study tips, career opportunities involving

mathematics, "real world" applications of mathematics, and computer-calculation

exercises. A typical example occurs on page 132:

> ### Application of LCM's: Planet Orbits
>
> The earth, Jupiter, Saturn, and Uranus all revolve around the sun. The earth takes 1 year, Jupiter 12 years, Saturn 30 years, and Uranus 84 years to make a complete revolution. On a certain night you look at all the planets and wonder how many years it will take before they have the same position again. (Hint: To find out, you find the LCM of 12, 30, and 84. It will be that number of years.)

The language used throughout the textbook is a reflection of the authors' view that "students' perception of mathematics as a foreign language is a significant barrier to their ability to think mathematically" (p. xi). Throughout the book, the authors consistently try to use simple and commonly-used words. This, they claim, encourages "students to think through mathematical statements, synthesize concepts, and verbalize mathematics whenever possible" (p. xi). While this aim may be laudable, one must question the basis on which the authors feel justified in making assumptions about students' perceptions of mathematics.

The authors try to model their beliefs about the efficacy of their language in their presentation of concepts. However, it often seems as if they have first thought of the mathematics and then tried to translate it into a language that students can understand. For example, consider their description of "Division by Zero":

> We cannot divide any nonzero number b by 0. Consider $b \div 0$. We look for a number that when multiplied by 0 gives b. There is no such number because the product of 0 and any number is 0. Thus we cannot divide a nonzero number b by 0.

> On the other hand, if we divide 0 by 0, we look for a number r such that $0 \times r = 0$. But $0 \times r = 0$ for any number r. Thus it appears that $0 \div 0$ could be any number we choose. Getting any answer we want when we divide 0 by 0 would be very confusing. Thus we agree that division by zero is undefined.

> Division by zero is undefined. That is, $a \div 0$ is undefined for all real numbers a. But $0 \div a = 0$, when a is nonzero. That is, 0 divided by a nonzero number is 0. (p. 493)

Here we can see that an already complicated concept is further complicated by the rather tortuous explanation that the textbook presents. I observed one class where this concept was being presented and the teacher exhorted the students to "Never mind the explanation; just remember the rule" (Field Notes 940228).

The textbook is designed to be used in a variety of teaching situations. In the preface, the authors describe four: in a standard lecture format, a "modified" lecture format (where "the instructor stops lecturing and has the students do margin exercises"), a "no-lecture" class (where the "instructor makes assignments for the students to do on their own"), and in a learning laboratory or "other self-study situation" (p. xiii). For those classroom-situated formats, each section (which includes reading the text, doing the margin exercises, and starting the exercise set) is designed to take between 1 hour and 1 1/2 hours of the student's time.

As these four formats are described, they vary in the amount of time that the instructor is involved; no mention is ever made (except tangentially) of the length of time of, or quality of, any student engagement. Student activity within classroom settings is never discussed and no suggestions are made about how students could work together. Even in the "Introductory Guide to the Student" (pp. xix-xx) or in the Sidelight on "Study Tips" (p. 6) where one might expect some acknowledgment that learning mathematics could be a shared activity, there is none.

Overall, the predominant tone of these guides is that learning is an solitary process and that any difficulties are to be solved by an individual's concentration on "checking answers," "identifying sections that give special difficulty," "doing the margin exercises as soon as possible," "reviewing homework," "analyzing your situation to allow yourself time to do a lesson," "maintaining daily preparation," and "keeping one section ahead of the syllabus."

Finally, although the textbook has been purposely designed to be used in
several specific instructional situations, the content is regarded as fixed and
appropriate for all situations. Indeed, its goal is not the production of new
knowledge but the reproduction of already existing, and universally-applicable
knowledge.

I have described the layout and content of the book in some detail because it
figures so prominently in the content and structure of the courses I studied. As is
obvious, the textbook is meticulously planned to provide a comprehensive learning
guide for students in both classroom and self-taught settings. In the next section I
describe how first the teachers, and next, their students regarded and used the book
in a classroom setting.

Teachers' Use of Textbook

All the teachers in the department thought that the textbook series they used
represented "the best choice we had; certainly they're...the most appropriate" (T.1.2).
"Appropriateness" seemed to be the key feature for textbook selection. The series of
textbooks is deliberately chosen for its appropriateness toward both adult learners
and towards mathematics. One teacher described how it was particularly suitable
for adult learners: "It's simple and has examples they can read [so that] when they
do the questions they can...immediately practice" (T.1.5). Another thought it
appropriate because "students can write in it if they want. It makes it more of a
workbook" (T.1.3). A third "found that some [other] books really talk down to
students....the material and the language is geared to...a twelve-year-old...[Whereas]
the book we use has a language and problems that are geared much more to adults"
(T.1.1). When asked to cite an example of adult oriented questions, one teacher
identified a problem that asked students to calculate how many miles a salesperson

could drive if he could spend only a certain amount, and the car rental rates were so

much at a flat rate and so much extra per mile. "Very good problem, everyone have

to do this," said the teacher (T.3.2).

Teachers described additional strengths of the textbook: the breadth of

emphasis ("they cover everything in those books" (T.1.1)), and "the considerable

amount of review material in the earlier parts of the chapters" (T.1.3). Particularly at

the introductory level ("which is where the students have left off their schooling"),

the books were seen as providing a wealth of review material. The need for constant

review was seen as critical:

> You know [the students] need review. Even if, in say Chapter 4 we're having
> trouble, well the book will say, "You need to review these portions of Chapter
> 1." It pops up throughout the course...this constant review. It's the biggest
> thing in terms of working with adults. (T.1.3)

The teachers relied enormously on the text and saw it as a central part of their

teaching: "It's crucial...you may have the best lecturer or teacher in the world, but

once you go out of the classroom half of it disappears" (T.1.2).

Not only were the textbook's structure, content, and problems deemed

appropriate for adults, they were also seen as appropriate to mathematics as a

subject area. "Compared to other math textbooks, these [problems] are challenging

but not outrageous." (T.1.4) "The textbook is laid out well....The topics are presented

in an order you can easily follow" (T.1.7). Another teacher thought that the chosen

textbooks "handled topics in a consistent and intelligent fashion, so I don't have to

devote time to reinventing the wheel" (T.1.3). Only one was cautious, " I think

overall [the textbook] is quite nice. Whether [the students] understand it or not is

another thing" (T.1.2).

One teacher described how the textbooks were subject to regular review: "We

have a departmental committee and when we have a new course or a new book

comes out, or a new edition of a book that we are currently using comes out, we

have the committee look at it and see how it fits into the existing structure...and how

best to teach it" (T.1.2). This process was generally seen as ensuring both stability

and quality in choice of textbooks, "I tend to follow the chronology of textbooks

fairly closely, [and] because hundreds of minds have gone into that strategy of

establishing [the textbook we use], I've found that to be a consistent...indicator of

quality" (T.1.3). However, there were some reservations:

> I don't know, possibly [the textbook] was chosen because the price was right
> or because it came with a very good instructors' package or...it could be that
> they'd been using [earlier] editions and it was much easier to roll into that
> than it was to do a complete change. (T.1.6)

Using the same series of textbooks for all the courses, offered, in the teachers'

opinions, great benefits for the students and teachers. "It's nice in the sense that it's

uniform between instructors, if one student goes from one class to another we have

the same book" (T.1.2). "We used to have textbooks [from different series] but it

created problems because the materials don't match up exactly, so there were things

missing" (T.1.4). Using a coordinated series of textbooks also helps teachers move

from class to class. One of the classes I observed had four different teachers for their

first four sessions--a disconcerting beginning to any course--but throughout these

changes in personnel, the structured format of the textbook provided a reference

point for the class and their successive teachers. New teachers would ask "Where did

you get to last class?" The students would usually answer with the relevant page

number, and the teacher would be able to follow on from there. One teacher

described how he had been asked to teach a course at very short notice. "After a

week or so I was really floundering... I'd never taught this introductory level

before...but I was able to follow the book and the way that [it] laid it out until I got

my bearings" (T.1.6).

As it was, no-one ever checked the students' opinions of the textbook, or

whether they thought that its content and layout met their needs. Teachers assumed

that because students could have an opportunity to comment on the course at the

end of term, they would make known their views about the textbook at that time. As one teacher put it, "We have an instructor questionnaire...or a class questionnaire at the end. It has some objective questions, rating on a scale of 1 to 5 the textbooks, the instructor etc. It seems the students are generally extremely positive on those things" (T.1.3). (It is worth noting that such questionnaires were never distributed in any of the classes I studied.)

All the teachers said that they used the textbook's order of topics to structure their courses. Indeed, several instructors wrote their course outlines showing which numbered sections of the textbook (rather than the topics they covered) were to be covered on which dates (see Appendix 11). Although there was no college or departmental mandate about structuring the courses around the textbooks, such a policy had been agreed by the teachers. One teacher described that "it's a general recommendation...not a written policy....It's something that most people prefer to do" (T.1.4).

The department head described the advantages of following the textbook so closely: "It's easier for the students to keep track of where they are. At one time we jumped around the book a fair bit, and it was harder on the students" (T.1.1). Also, one teacher felt that "if people pick up a book, particularly at a lower level, it makes sense to them to do page 100 before page 200" (T.1.6). Teachers also described how the textbook represented the end result of a systematic process: "Hundreds of minds have gone into establishing it....and a lot of the [authors'] work is a consensus from all the letters...from across North America...the major users of their texts. There's a tremendous amount of input from throughout the field" (T.1.3).

Overall, the format of the textbook and the order in which it presented ideas was seen as "common sense...you can't do this before that. For example, the students need to know how to add and subtract real numbers before they start on algebra" (T.1.4). This "logical" order was seen as one that had been established after extensive

consultation: "The authors, usually they said 'After teaching this topic, then continue on to this next one.' They already have thought about this. So, if we follow this then we don't have any problems" (T.1.5). " The textbook's into what, the seventh edition? They should have got all the bugs worked out of it by now" (T.1.6).

Finally, many teachers stressed that changing the textbook's order of topics created more work for them: "I'm basically lazy....I don't have a burning desire to...reorder things. I have other things to do with my time" (T.1.6). Most teachers agreed that the curriculum they had to cover was fairly intensive and they didn't feel they had much leeway to alter either the topics or their sequence. "There's a lot of pressure to make sure we cover everything. We can't afford to get too far off course," said one (T.1.1). Another teacher agreed:

> I try as much as possible, given the time constraints, to maybe look at other alternatives...maybe try to sometimes break into fun-type activities and so on, but, you know, it's like...you're always behind the eight-ball trying to finish everything. (T.1.2)

A third teacher identified a trend she saw developing:

> I do see a certain level of people becoming, the longer they're here, becoming more and more relaxed and less conscientious about putting themselves out. I mean they start with page 1 of the book and go doggedly through to the end, never deviating. (T.1.8)

As the format of the textbook largely dictates the order of the course, so it also dictates how topics are presented, the form of instruction, and how students are evaluated. One teacher described how he introduced topics: "I usually follow the book...there are times when I might jump around a bit, but the way the books are designed, it's quite important to take things the way the book does them" (T.1.8). This approach was common. Another teacher explained how, "for adult students, jumping around is not very accommodating. They tend to get lost....If they cannot make a class, they'll...if you jump a lot around they're lost" (T.1.4). Only one teacher questioned following the textbook so closely: "Spending so much time on [certain]

sections can be a problem....some people are being bored while other people are having a very difficult time" (T.1.3).

The textbook also had a large influence on instructional techniques. When beginning a new topic, one teacher described how "I always look at the book, and get students to read it through with me. Then we look at the example and then the margin questions. Sometimes I just show them [how to do] the margin questions and they have the example and then they do the exercises" (T.3.2). The emphasis here is on learning how to follow a procedure in the same way that the book describes it; students were rarely encouraged to ask why they were following a particular procedure. For example, early in the term, I observed one teacher introduce the concept of Lowest Common Multiples to her class by describing "the correct way" to approach it:

> "Before you do anything, look at the book...preview...preview means look at. What, then how. How to solve equation, how to find LCM. Do you know what LCM means?" he asks. No one answers; he waits about 3 seconds, then says "Lowest common multiple." The students all write this down. The teacher continues, "And, of course, why. Why we do this? A lot of whys are too difficult. We don't do them in this class. Wait for higher class. I would never ask you 'Where?', 'When?', [or] 'Who?' Like 'Who invented zero?' Not important. So, always look in textbook. The important names are highlighted. (Field Notes 940104)

Although following the book so closely was the predominant pattern of instruction ("most people prefer to do it that way" (T.1.4)), some teachers felt the license to deviate: "Sometimes we feel that we can teach that topic in this way although the book does it differently, or sometimes we can skip it" (T.1.5). Also,

> I like to do some extras that aren't in the book...and sometimes there are things that I don't think really need to be covered so I leave them out. I'll say 'This isn't in the course, it isn't in the book, it won't be on the test, but I think this is really neat.' (T.1.1)

However, this varied and less-structured approach was more prevalent in the more advanced courses; teachers of the introductory level courses teachers tended to

follow the book more closely. As one described it: "We're really teaching people

how to think as much as what to think, so we don't have as much license" (T.1.8).

Following the textbook so closely ensured that different classes often covered the same content in almost the same way. For example, I observed two classes dealing with an introduction to algebra. These classes differed markedly as to teachers, students' backgrounds and experience, class sizes, meeting times, and classroom layout. Despite this, however, each class had exactly the same structure of mathematical content: each covered the content in the allotted (according to the textbook) seven days; five days to cover sections 12.1 - 12.5 (one day per section), one day for review, and one day for the chapter test. Within each topic section, the instructional plan for each day was also remarkably similar: 30 minutes for student problems from the previous session's homework, 20 minutes for the instructor to present new material, 10 minutes for student practice on the margin exercises, 15 minutes break, and 30 minutes for the students to do as much of the problem set as they could before the end of the allotted period. It's clear that the teachers of these classes were following the "suggested course guidelines" (Teacher's Edition, p. T-7) to the letter.

In their presentation of content, teachers also rarely deviated from the textbook. One teacher described how the textbook "usually presents material, and the students they just write it all down. Repeat what's in the book" (T.1.2). Only one teacher mentioned this as problematic: "The textbooks usually fail in that respect. They say 'A rational number can be expressed as A over B,' as if that makes sense to the average person" (T.1.2). This teacher found it necessary in his teaching to go beyond the textbook's narrow definitions.

> Like an example is the commutative law. People say "Commutative law, a + b = b + a." They've just memorized it. But I would look beyond that and say "Well, the word commutative, what does it mean? What's the root word?" And they'll say "Commute" right? And I'll say "OK, what does commute mean?....Moving around," and so from there we kind of look at the word and

say "Oh yeah, I know that word. I know what it means." So it's not from some
law. They can see where it comes from. (T.1.2)

Of course, not every teacher indicated that they followed the textbook so slavishly. I observed several instances where teachers, noticing that many of their students did not seem to understand the textbook's explanation of a concept, would try to reframe it in their own words, often with more success. However, most teachers mentioned that they placed the textbook's presentation of concepts centrally in their teaching, and they only added to it or deviated from it when they found it necessary. Another teacher described how he sometimes deviated from the textbook presentation:

> I show them the way that makes sense to me. Usually it's the way the book does it; sometimes it's not. I really don't care if they follow the textbook method or my method or some method of their own, as long as what they do is logical. (T.1.2)

Just as the mathematical procedures of the book were rarely questioned by teachers, neither was the content ever publicly challenged, even when it was inappropriate or posed problems for the students. For example, although the courses were held in a Canadian institution, the teachers chose a series of textbooks published, and written for students, in the USA. The textbooks use US examples (e.g., finding the area of the state of Colorado) and US systems of measurement (which differ from those in Canada). Again, the teachers did not regard this as problematic:

> I don't think it's not suitable. I suppose there's a cultural thing...I mean the [textbook authors] talk a lot about baseball as if everybody knows what baseball is....But I think even if we had a Canadian textbook there'd still be some cultural things....It's the nature of our population. A lot of Canadian students are not native North Americans....Although it bugs me that they're still using feet and yards and pounds in there. But that's just my own bias. (T.1.6)

This unchallenging approach to the textbook was common amongst the teachers. During one observation, the teacher directed the class to notice a particular

question in the book which mentioned a measurement in feet and inches. "You have
to be bilingual in Canada, because you will see inches in the real world" (T.1.3).

Teachers were also loath to criticize the textbook even when it was incorrect. Twice I observed incidents in different classes where the book was shown to be faulty:

> The teacher turns to the class. "What problems from the homework?" "Number 55," one student shouts. A second student says "The book answer is wrong." The other students look interested. The teacher writes the problem on the board, solves it himself, and then looks up the answer in the back of the book. [The answer given in the book is wrong] "Oh, they're usually right," the teacher says. "But sometimes mistakes slip through. I'll look out for that next time." (Field Notes, 940217)

> The teacher is going over the Chapter test he's just handed back. There's some consternation that some of the answers in the book are incorrect, and that students have lost marks on what they think are correct answers. Brendan, one of the students, asks "Who makes the problems up?" The teacher replies "They drop like manna from the sky." [He's being sarcastic but no-one laughs.] The same student asks again "No, who makes the textbooks up? Students?" This time the teacher is more forthcoming. "No they're used on students. Yes. They pick up the mistakes." "The textbook has lots of mistakes," one student tells me later, "the answers in the back are often wrong." (Field Notes, 940324)

Despite these incidents, neither teacher involved offered any criticism of the textbook when interviewed at the end of the term. "Oh, I like the book," said one, "It's a good guide for the students" (T.4.2).

Only amongst themselves did teachers feel free to criticize the textbook content. For example, during one faculty meeting I observed one teacher who said, to the general agreement of his colleagues, "Problems in the books are stupid. For example [they ask] 'You are now 3 times what you were 6 years ago. How old will you be in 2 years time?'" (Field Notes, 940302). However, despite this, no teacher I interviewed claimed to alter any of these seemingly "stupid" questions, and I never observed any of the teachers deviate from the way the textbook worded a problem.

All the teachers explained that they used the various types of exercises in the textbook or the associated supplementary material as the only assessment tools for measuring student learning. Again, the department head provided a rationale: "Well, we need to test what's in the curriculum guide. That's what tells us what to teach. We've chosen the textbook to follow the guide, so if we use the tests that are provided in the textbook, then we know that we are testing appropriately" (T.1.1). The textbooks certainly contained enough exercises and problems on which to base tests:

> The problem sets at the end of each section...there are hundreds. I ask students to do lots of them. I think that you can never do enough for practice....Although I don't take everything in and mark it, because it's so open to abuse. I mean the answers are in the back so anyone can get 100%. There's no point in using the textbook for that. (T.1.6)

However, I often observed classes where the teacher, when assigning homework, either did not ask students to tackle the harder "synthesis" or "skill-building" questions at the end of each problem set...or played down their importance: "'The second question from last night's homework...is synthesis question,' the teacher says. 'The synthesis question is a harder one--a challenge question. If you cannot do this, do not worry'" (Field Notes, 940217).

For marked assignments, most teachers used the test material that came with the Teachers' Supplementary Material package. "There's one of these for every level...sometimes five or six forms of the test, so it's very easy if you need one at short notice. I find that more and more people are using the ready-made tests" (T.1.7).

Although most teachers spoke about the ease of access to a supply of test material, few made many comments about the content of the tests. Only one teacher was critical: "I feel that sometimes the tests seem to test just the very simplest concept and give very easy questions" (T.1.8). This was felt to be a disservice to the

students: "We're not preparing them or challenging them to work through multi-
step questions, or asking them synthesis- or analysis- type questions" (T.1.8).

Despite this concern, teachers rarely altered the tests that were provided for them. Any extra worksheets that the teachers developed were only given to the students for revision or extra practice.

> I have always designed a lot of my own worksheets, over and above the textbook. As good as [the book] is, and as much practice as it has, I find that it doesn't push the students enough....I leave my sheets in the program development room so other teachers can use them. I know they do. (T.1.7)

One teacher did speak of how, in addition to the textbook-prepared questions, she "sometimes add[s] a bonus question. Just to encourage the students that are interested [and those] that...pay attention when I go off on one of my flights of fancy. The keen ones try and do them...they're usually quite challenging questions" (T.1.1).

Occasionally, students would bring in other mathematics books. At the introductory level this was not encouraged. "Sometimes, students they go to the library, pick up references, other books. Then they want to start reading it from beginning, unless you tell them 'Too confusing, Stick to course book.' If [teachers] choose the textbook, [they] satisfy and also [keep] in line with course objectives" (T.1.5). Additionally, teachers were aware of the high cost of mathematics textbooks: "We try to make good use of the book since the student pays sixty-some dollars" (T.1.4).

Finally, it is worth noting here that at no time, during either my observations or during the interviews, did any of the teachers use or ever refer to the associated material that went with the textbook--the audio- and video-tapes and computer software. Indeed, the copy of this material, although accessible in the teachers' resource room, was never touched during the time I spent at the college.

Students had much less to say about the textbook than did their teachers. For students, it was just part of the course: "I mean, it's the teacher's job to choose all that stuff, isn't it? What book to use, what we do..." (L.3.3). Overall, students enjoyed using the textbook: "I love the textbook, as a matter of fact....I like the way they have laid it out, there are lots of good examples" (L.1.5), and "That book is a marvelous thing...once you understand all the different processes of it" (L.3.6) were typical comments. Students particularly appreciated the opportunity to use the textbook as reference material:

> What you don't pick up in class...you can go home and then find out about it in the book....If you forget a concept, rather than just be lost all of a sudden, you can go to your textbook and it quite easily explains the concept. It's the backup, that's what I like. (L.1.5)

Other students described how the textbook helped them learn: "First it'll present some ideas, then you have to do the margin exercises, then you'll get some more information. So it breaks it down into steps...makes it easier to learn. I like that" (L.2.1). "The key ideas are in a different ink, right?...and it explains a bit about each one, too. I go through and highlight them. Then I can see them easier when I go back to it" (L.3.2).

> Students also appreciated that the answers were in the back of the book:
>
> What helps me in the class sometimes is doing the set questions and then checking the answers in the back. So, if I get one wrong I can find out the answer then try different ways to see how I can get that answer. (L.1.3)
>
> Another student described his approach to the textbook problems:
>
> I don't think you can get enough practice....First I'll do all of the exercises, then I'll go on to the chapter test...and I'll sort of try it out again, but once I've depleted those I can't go back again because I already know the answers, I know where it's going to go. (L.3.1)

Finally, although many students didn't write in their books, preferring to resell them at the end of term, some did: "It gives you these margin examples and

space underneath to put in your answers. Then I can compare what I do with what the book says" (L.1.5). "I write in comments that [the teacher] says or I expand on what's there in my own words....Or I underline...I scribble all over my book. I make a total mess out of it." (L.3.1). Sometimes, students found themselves writing down comments that the teacher had made only to find them in the book. "She [the teacher] will say something and I'll write it and then I'll think 'Oh, it's already down there.' Because she's...obviously she's been reading it out of the book" (L.3.2).

During my observations I never saw any instructor refer to the introductory section of the textbook, or the extra tips on problem-solving, although each describes, in some detail, how students can use the book to enhance their learning. Only one student described how an instructor had "stress[ed] that we should read that part to explain the mechanics of the book" (L.1.5). However, most students had not read the introductory sections, largely because, they claimed, no-one had directed their attention to it: "I didn't know they were there. We just started on page 1" (L.3.4).

Of those students that had read the introduction, few could remember much:

Oh, yeah, I read that....Never thought twice about it....Well now you mention it, I think we did discuss it once. How you can just pull a chapter out, do that chapter, get it all down, and then move on. You can jump around in the book, I know that...it doesn't have to follow consecutively. (L.3.3)

Only one student found the introduction directly helpful: "You know, right in that very first part of the book it says that students who are having trouble are not doing their margin questions. Well, that was me" (L.2.1).

Also, few students had read the Sidelights--those "optional" sections that "covered topics relevant to the study of mathematics." Students preferred to concentrate on the regular text. "Oh, I saw them, but I just skipped it and did the exercises" (L.1.5).

They're not much use...they're often too abstract or irrelevant. Like I was reading this at lunch time, and it said 'This will be handy in the future.' Well, we don't care about the future, we want to be able to solve the problems right today. We'll worry about the future tomorrow. (L.2.2).

Overall, students saw the textbook as "what the course was about" (L.3.1). They spoke of how instructors used the textbook:

Mostly he [the teacher] starts a new section by having us read the first couple of pages. Then he'll work a problem or two on the board. Then it'll be our turn to do the problems in the book. He walks around and checks our work. If there's a lot of difficulty, he might do another problem on the board to show us where we're going wrong. Then we have to finish the problems for homework. (L.2.2)

All the students described how the homework they were given came from the book, and how it was seen as central:

We get lots of homework, and this homework is in the books...it's designed so that each part of the homework has one lesson, and if you do the homework you get the lesson.... Because it pounds it into you...like 36 questions, you know, later...you've got that little concept together. (L.1.5)

Occasionally students spoke of how teachers would supplement the textbook's way of presenting concepts or procedures. "He [the teacher] will go through the book, but then show us an easier way. Explain things in like two, three kinds of solving problems...and the easier way we can choose" (L.3.4).

Students rarely voiced critical opinions about the book. For them, it represented unchallengeable knowledge. Any difficulty they had must stem from within themselves: "It's a good book, but sometimes I get confused....It's like spaghetti in my mind. I have to go over and over it" (L.2.2). Students' only criticisms of the textbook concerned its explanations. These were perceived as either being too complicated...

If you try to understand their explanations it gets very confusing. Even since the first chapter when there is, you know, simple addition, you know, add 5 and 5 kind of thing. They can somehow write it out so that...you don't think you can add 5 and 5. So I ignore their explanations and go on to the examples. (L.2.7)

...or too concise:

> You need to get used to the format. Sometimes, they do sort of hop a step and you miss it out and you think, "How the hell have they come to that conclusion?" So sometimes things are missed out in my opinion. (L.3.1)

Summary & Discussion

In this chapter I have described the contexts and settings in which mathematics education takes place. I considered, in turn, the places where such education happens, the people involved, and the work that they do. Before I describe how these elements combine in practice, I summarize and briefly discuss the main points of this chapter.

First, it is evident that the college's physical and organizational structures and general *ethos* provide a disciplined, if somewhat anonymous, framework to the mathematics education provided there. Students are encouraged to be serious about, and committed towards, their learning; those who are most purposeful receive most of the benefits. The provision of student services ensures that students with academic and personal problems can obtain help should they request it. However, the structure of such provision predetermines that these problems are defined at an individual level; students are not encouraged to seek collective or structural solutions to their problems. Further, although students are given the appearance of choice, in practice their choices are limited. When they apply, students can elect to study whichever subjects they like. However, they must first prove their academic ability in each subject they select. Once admitted, they can only enroll in those courses (or ones at a "lower" level) that the department sanctions, regardless of their

wishes. It is worth noting here that this process equates success with passing academic examinations and privileges assessment over understanding; factors that may also significantly influence classroom teaching.

At AC, mathematics education is just one part of a greater administrative whole, afforded no special significance nor linked in any permanent way with other areas of provision. In fact, it is remarkable how little the mathematics department has to do with either the college administration or those departments covering other subject areas. Teachers in the department appeared to like "being left alone." In their contacts with others, whether college administrators, other teachers outside of the department, students, or parents, teachers sought to minimize any influence upon them. "I like to just come here and do my job," said one teacher. "What matters is what goes on in the classroom" (T.4.2). The close (and closed) nature of the inter-relationships within the mathematics department encouraged a collegial bonding between the mathematics teachers, which teachers saw as benefiting classroom teaching. However, this close working relationship also encouraged the avoidance of decisions that could cause dissent within the department, and precluded much reassessment of course curricula. Interestingly, the only contacts that teachers did allow to influence their teaching were with colleagues in other academic institutions. Teachers had developed both informal and semi-formal links with institutions which the more academically-inclined students were expected to attend. Indeed, there was some evidence that teachers changed the emphasis of their classroom teaching to better meet the needs of these receiving institutions, although many students were not destined for further education.

Classrooms were set up in traditional fashion—an ordered array of tables with chairs all facing the blackboard and the teacher's desk. In this way, the classroom setting for mathematics education for adults replicates that for high-school students. Although teachers professed to be aware of the effect of room layout on learning,

they did nothing to change the design. Students claimed their learning was unaffected by room design; "being close enough to the board" was the only significant aspect mentioned. Students liked to be able to see, and copy down, all that the teachers wrote. Indeed, they seemed to revere all written mathematical explanations whether on the blackboard or in the textbook.

The college's approach to curricula may also affect classroom teaching processes. For example, the overall course provision and the individual course curricula follow already established and governmentally-approved guidelines. Hence, the teachers' role in choosing and ordering course content became one of merely selecting appropriate texts that conformed to existing guidelines as closely as possible. Teachers based their entire course structure, individual lesson planning, evaluation procedures, and even teaching methods around the set textbooks. Following the textbook so closely resulted in different classes often looking surprisingly similar in terms of their course schedules, lesson content, and teaching methods. Teachers rarely deviated from this approach even when it posed difficulties. For example, students entered the courses with a wide range of mathematical abilities and often needed substantial opportunities to review their previous learning. Hence, the teachers had to rearrange their previously planned schedules (based closely on that of the set textbooks) to allow for this revision period and then rush through the rest of the course material in an attempt to "keep up with the book." Consequently, teachers often felt they had inadequate time to cover the course material.

Teaching may also be influenced by teachers' backgrounds and experiences. In general, the mathematics teachers' previous educational experiences led them towards valuing the subject of mathematics as paramount. For these teachers, both the learners and the processes of teaching and learning were subjugated to the nature and demands of the course content. Teachers viewed themselves as content

specialists and concentrated their teaching on its efficient delivery. They regarded (and hence, portrayed) mathematics as a system of hierarchically-ordered knowledge consisting of fixed concepts and established procedures. For them, learning mathematics was a matter of discipline and "hard work," best accomplished by memorization and repeated practice. As the content was already determined (and therefore, unchangeable), teachers, instead, focused on developing the motivation of students' towards learning it. Adult students were regarded as "being in charge of their own learning" and therefore having sole responsibility for learning (or not learning). Teachers' decisions about classroom activities were predicated upon the assumption that anyone could learn the content given sufficient motivation and opportunity to practice. Any persistent learning problems that the students may have were considered inappropriate for classroom remedy and best dealt with by "the learning disability folks."

A focus on individuality was apparent in several areas of the mathematics teachers' work. I indicated earlier that these teachers took little active part in the wider aspects of college life, minimized any influence by others, and preferred to be "left alone." Within the classroom, teachers also promoted an individual approach. First, mathematics was portrayed as a subject distinct, and separate, from other areas of knowledge. Second, it was seen as unsuitable for discussion or collaborative work. Doing or learning mathematics was never presented as a communal activity and teachers made no suggestions about how students could work together. Ironically, although focusing on individuality, teachers subtly ignored the differing interests and goals of their students, preferring to treat them as one homogenous group.

However, the demographic details of the students indicate a notable variety in their backgrounds in terms of age, gender, ethnicity, language ability, employment, previous educational experiences, and reasons for studying mathematics. About half of the students attended college full-time and had clear

goals towards pursuing higher education. Their studies (in mathematics as well as in other subjects) formed a large part of their lives. Other students, often older than full-time students, were much less academically-oriented. They tended to work, at least part-time, and although they perceived education as "a way out" of their current economic positions, they tried to fit mathematics courses into the rest of their lives. For all of these students, regardless of the group they were in, studying mathematics was intimately related to their everyday lives. For full-time students, mathematics had to compete with (and therefore, relate to) other subjects. For working students, the time spent on attending courses or doing homework had to be fitted around work schedules.

I expected teachers, when planning their courses, to take such diversity into account, and I was surprised when they did not. In general, teachers largely ignored the students' backgrounds and attitudes. Even though teachers had access to details about the students' backgrounds and interests through an information sheet they distributed for students to complete at the beginning of each course (see Appendix 19), teachers rarely used such details. "They're all the same to me," said one teacher. "They're here to learn math" (T.1.4). Teachers' only concern appeared to be how the students' personal characteristics affected their motivation. Adult learners were thought to be, in general, more serious and purposeful than younger learners. Teachers also ignored the students' experiences and goals, tending to treat everyone as if they were intended for further education. Here, "developing student autonomy" was crucial. Because college students were expected to "be in charge of their own learning," teachers felt that everyone should be taught in a way that they thought would benefit students in college-level mathematics courses. This meant, in practice, emphases on the academic (rather than the practical) aspects of mathematics, and on the memorization of set facts and procedures (rather than the development of an exploratory or questioning approach).

This approach was based on the one promoted in the textbooks which 174
presented mathematical knowledge in a structured and sequential way. Each chapter and section, based on "discrete and manageable learning objectives," contained presentational material and a wealth of exercises to "enable students to thoroughly understand the concepts at each step." The textbook thus functioned as an authority on both course content and the process of learning. However, two aspects were notable by their absence from the textbook: any discussion of the learning needs of adult students, and a recognition that different learners may have differing abilities or learning styles.

The textbook, although comprehensive in its treatment of mathematical topics, was sometimes faulty. Some of the given answers were incorrect and some of the procedures unnecessarily unwieldy. Further, the textbook was structured on the assumption that most learning difficulties can be overcome by repetition and practice. It provided numerous examples of applications and rules but little explanation of why one might choose these rules in the first place. It described "common student difficulties" with mathematics as due to "incorrectly remembered shortcuts," an inability to understand how mathematics can be used, and a lack of motivation. Yet it provided little support for those (or other) difficulties other than repeated practice of "application" problems. The "real life" problems were unlikely and seemed designed to provide opportunities for further calculation rather than to encourage an understanding of the uses of mathematics.

When dealing with problem-solving, both the textbook and the teachers promoted the existence of a universal approach, the notion that all problems can be solved, and that, always, there is only one "right" answer. Further, teachers made few attempts to contextualize problems; mathematics was presented as a series of operations on abstract entities. Their purpose seemed to be to give students rules to solve abstract problems so that these rules could then be applied to "real life"

situations. In this way, mathematics was presented as the reproduction of existing knowledge. A second outcome was the privileging of the fixed knowledge over student understanding. What was to be learned was the content already contained in the textbook; facts that already "exist." The meanings and understandings that students made of these facts are downplayed. Indeed, the students' role in learning became one of merely remembering pre-determined content to pass pre-determined tests specifically designed around the textbook "facts."

The textbook periodically included additional sections designed to encourage students to think about mathematical ideas and real world applications. However, these sections appeared to have been designed with little thought for the needs and interests of adult learners. In any case, both students and teachers generally ignored these sections.

Having described the places where education happens, the people involved, and the work that they do, I now consider how these elements combine. In the next chapter, I examine in detail what takes place in mathematics classrooms by focusing on specific classroom episodes and activities.

Teachers and learners in this study regard teaching in markedly different ways. For teachers, classroom teaching is the main aspect of their job. Choosing <u>what</u> to teach and <u>how</u> to teach it are areas where teachers can develop both their personal teaching styles and their professional expertise. Further, classroom teaching is the one aspect of teachers' work where, regardless of other institutional changes around them, teachers can maintain both their autonomy and their authority.

Learners, on the other hand, view teaching quite differently. They are "the taught," the recipients of teaching. As such, they have little say in, or control over, the decisions that affect classroom teaching. At the beginning of each course, the course outline and content is presented to students in such a way that, if they choose to remain in the course, they tacitly accept. Further, the teaching methods are even less a realm for student influence. How the course is to be taught is never directly presented to students, and it, too, is seen as given. Hence, students have little opportunity to affect or even discuss different possibilities about the teaching and learning of mathematics. As a consequence, students find it difficult to distinguish <u>what</u> is taught in their mathematics classes from <u>how</u> it is taught. For students, mathematics becomes a mixture of facts, rules, procedures, expectations and requirements totally determined by others.

In the previous chapter, I discussed some of the elements of mathematics education: the settings where such education takes place, the people involved, and the work they did. Now, I consider, in detail, how these elements combine by examining what actually takes place within mathematics classrooms. For clarity, my discussion follows the process of education from the teachers' perspective: an orderly progression of planning, doing, and evaluating. First, I consider the planning

of teaching (both before and during the courses). Second, I describe common classroom episodes and activities and how they are perceived by those involved. Third and fourth, I discuss the roles played, respectively, by homework and assessment. Finally, I consider, in detail, how these specific roles and activities manifest in one particular part of the introductory courses: that concerned with an introduction to algebra. In each section I analyze these classroom situations and activities (and how they are described) to show how they are influenced by the elements discussed in the previous chapter.

Planning Teaching

Mathematics teaching begins with some teacher planning before the term starts. "Coming up with a course outline schedule...and working out how to fit the course material into the given time frame," is one teacher's description of his planning (T.1.4). A second teacher told how she would "walk through the whole course [in her mind] dealing with the mechanics....Consider what assignments I'm going to give, planning when I'm going to give my tests. [Planning is] all to do with the scheduling...fixing the course outline" (T.1.8).

As the courses follow already established curriculum guides and base their content around set textbooks, course objectives and material are already largely fixed and require little adjusting prior to the start of a course. "If we follow the textbook," explained one teacher, "then we know we are on [the] right track to meet our goal" (T.1.5). "I'm quite happy to have the content and the structure of the

courses all laid out and planned for me," said another teacher, "so I can focus all my time on...motivation" (T.1.3).

In structuring their courses, teachers closely followed the structure of the textbook. "Generally, we do things sequentially," explained one teacher. "The way these books are designed it's important to take things in the order that they appear" (T.1.8). "That's why I don't give out a lot of notes," explained another teacher, " I tell them [that] they've just bought a wonderful set of notes for $70" (T.1.4).

Despite its necessity, teachers said that they rarely spent much time on course planning. Partly this apparent lack of preparation is due to the teachers' experience. Most of the mathematics teachers had worked at AC for over 10 years and had, therefore, already taught all of the courses before and were aware of what was required in planning specific courses. In any case, teachers claimed that course planning required similar activities regardless of the course involved.

Furthermore, teachers claimed that the uncertainty about which courses they would be teaching also affected their planning. Several teachers mentioned that they had, on several occasions, spent a lot of time on course planning only to find an under-enrolled course canceled. Then, as one teacher put it, "the course outline I so carefully prepared went straight into the wastebasket" (T.1.6). "The biggest problem is enrollment," agreed a second. "If only four or five students, then the course can't run. So this affects your schedule, other instructors' schedules, the whole department. Perhaps you even have to teach in the evening" (T.1.5). Obviously, teachers viewed such a change as quite disruptive, given their statements about ideal scheduling.

Class size affected teachers' choices in both practical and psychological ways. The level of student enrollment affected teachers' course planning once the course had begun. As one teacher commented,

I get really affected if there's only a few in the class. Psychologically it's just so different when you go into a group that's only got 12 and you're wondering, "Why are there so few?" and when you talk to them they're thinking, "Where is everybody?" You don't get as charged up...it's flat. (T.4.2)

"What am I going to do if ten people show up at the first session and only three the next?" asked another teacher (T.1.3). "There's a big difference between teaching 18 or 30 [students]," explained a third.

If there's 30, I might think, "This assignment will be terrible to mark if I want to get it back to students by the next day." So, I might use something else...or just give it to them as a worksheet. (T.1.8)

To prevent any potential drop-outs, teachers said that they would often spend the first few course sessions creating a welcoming environment and ensuring that students were aware of, and prepared for, the course requirements and expectations. "[It's] important we start with [the] right attitude," said one teacher. "Make [sic] the rest of term go easy" (T.1.5).

Day-to-day decisions about teaching largely involved choosing how to teach. Here, teachers were concerned about how best to introduce the subject-matter. "I think about how I can present the material to make it as interesting...as I can," one teacher said (T.1.1). "I think of teaching like a [musical] performance," she continued, "If I can make a really interesting performance, hopefully the students will enjoy it." These "performances" were more improvised than rehearsed; teachers appeared never to plan the day's lesson to any great extent. Their reliance on "making it up as you go along" was most noticeable when they were asked about what problems they foresaw at the beginning of every course. Most said that they didn't think much about problems in advance. As one teacher explained: "You kind of develop a sense of dealing with things as they come up....It comes with experience" (T.1.3).

Because the rhythm of the course followed the pre-planned outline so closely, teachers rarely had to make decisions about when to move from topic to topic. However, because, as they put it, "One topic is built on another," they often needed

to check the students' comprehension of mathematical topics before they could proceed. This sometimes provoked a conflict for teachers between the need to move on (to keep up with the schedule) and the need to ensure a certain level of understanding.

This conflict was the most serious planning problem that teachers mentioned. Their prime concern was in ensuring that they covered all of the course work in the required time by "keeping to the schedule." "The pace is pretty hectic," explained one teacher. "We have so much to get through to cover the core" (T.1.7). Teachers' concern to cover all of the course material often led them to set a fast pace early in the course. After an initial two-week "honeymoon" period, teachers tended to assign students more work --often double the amount--that they would assign later in the term. "Got to keep up," I heard one teacher say to her students. "if we fall behind the schedule, we'll never catch up" (Field Notes, 940113).

The pressure of time--having to cover all the material in the allotted time--limited educational choices for students. Teachers felt that, ideally, how they taught should be in some way related to the learning needs of students. But, as one teacher put it, "it isn't really. I mean the [students] have all got different learning styles and preferences and we should tailor our work to them. But there isn't time" (T.1.6).

Planning within the time constraints was a particular feature in the introductory courses. Here, teachers concentrated on "individual activity...assigning material out of the text or other worksheets" (T.1.5). "There's a lot to get through," said another teacher, "and each topic builds on the previous [one], so [students] have to have the work of one class done for the next class. It doesn't require much explanation but much practice" (T.1.8). "I give a lot more time to individual activity at this level," said a third. "I assign material out of the textbook...and see how they handle it" (T.1.3).

Teachers' schedules don't always allow them to deal with student queries
immediately. As one student explained,

> Sometimes, [the teacher] will come in late, and there have been [students]
> here early waiting for him….Then if you have a problem with something
> that's happened in class you can't talk to him straightway because he has
> another class to get to and so he has to leave right after. (L.1.7)

"He does have a seminar day," said another student of his teacher. "But it's
usually on a day we don't have class…so you can't always go to them. I mean, we
have other lives don't we? You can't always make a special trip out here just because
it's his seminar day" (L.1.7).

Teachers also spoke of their planning being frustrated by "typical student
mistakes." "Each year, different class…but same mistakes," said one teacher. "Slow
you down" (T.2.1). "You know where students will have difficulties," said another
teacher,

> I've taught these classes often enough now that I know what problems are
> going to come up. I have a joke going in all my classes, about the standard
> problems that students encounter. Like 'This is course 061, most made
> mistake number 1' and so on. I'll write it on the board and we'll laugh. I try to
> introduce the students to the mistakes that are most common. So far we're up
> to mistake number 23….It's often frustrating, when you want to get on…but
> you can't. (T.1.1)

However, despite teachers' awareness of which parts of the course would
cause the most difficulty to students, it seemed that they never altered their
prepared timetables accordingly. Any changes to the prepared schedule were made
on a day-to-day basis, and then with the imputation that students were "slow" and
they needed to work faster to "catch up."

In order to see the results of these planning decisions, I now describe some
common teaching situations.

As can be seen, the curriculum of mathematics courses at AC is established, before courses begin, as an ordered and segmented hierarchy of mathematical facts and procedures. Similarly, the expectations and requirements for students are also set in advance. Students are expected to master sequentially one concept or skill after another, while demonstrating their computational proficiency. Hence, the work of teachers becomes that of covering the material in an ordered and efficient manner and providing regular opportunities for students to demonstrate their new-found competencies.

Just as the textbook provided the content of the mathematics lessons, so also did it form the teaching of those lessons. Much of the textbook was organized into a show--drill--test pattern and teachers found themselves adopting this pattern for their teaching. In all of the classes I visited throughout the term the sequence of lesson activities was the same: first, teachers would deal with any problems from the previous homework. Then they would introduce new material, giving a brief explanation which linked it with the previous lesson. Next, teachers would work through a couple of examples of textbook problems based on the new material. Students would then be assigned to complete several "practice" problems by themselves. Here, teachers would circulate among students, checking their work and answering questions. Next, teachers would select a couple of problems that were causing students particular difficulty, and solve them on the blackboard, encouraging students to copy down the "the correct method of solution." Finally, teachers would assign further problems for students to work on in the remainder of the lesson. Here, teachers would tend not to circulate amongst the students but sit at their desks, usually completing other work. Students encountering difficulties were

expected to approach the teacher with their problems. Homework for each lesson

consisted of the completion of this last problem set often with additional work set by the teacher.

Considering each of these stages separately would be repetitious. Hence, I discuss three episodes common to each lesson: starting the lesson, teaching new material, and working with individual students. Although each episode is particular, and described uniquely, it is typical of similar episodes and activities in other classes. For each episode, I give a description of the situation (drawn from my observational field notes) followed by (in italics) the teacher's comments on the situation, made when they watched video-recordings of the situations.

Starting the Lesson

At 12:30 PM (the scheduled start time for the lesson) five students (out of 10 enrolled) are working individually at their desks. They all seem to be sitting in the same seats as they have all term. Two female students are talking quietly about some aspects of the course; it appears as if one is helping another with her homework. The teacher enters the room 5 minutes later. The students who have been talking together fall silent and take their seats. The teacher doesn't speak to, or even look at, the students but goes straight to his table, gets some books and papers out of his briefcase and lays them out. A sixth student enters the room, right behind him, and hovers near the door. The teacher looks up and waves at her, again without speaking. She comes up to him and asks for a copy of a sample test that he has previously handed out. After finding a copy and giving it to her, the teacher turns to the board and writes out the plan for the day's lesson:

1. Applications: using algebra in real life
2. A review of Chapter 12

3. Remember: <u>Test</u> Chapter 12, Tuesday April 5 184

     Assignment 3 due next Tuesday

A few students glance up from their work, but still nobody speaks. They all seem to be waiting for him to start.

Teacher: What I do at the beginning of the class is I always write an agenda on the right corner, as per Education Psychology 361, UBC 1977, an advance organizer for the students, and just topics, some preliminary discussion of what we're going to do and then we do it, rather than sort of going into a world with no boundaries and no explanations....That's a habit that right from the beginning I've done. I mean it's like three words on the board and I may talk about it for a few moments, because ...I can tell, and I remember as a student it was always so frustrating ...particularly in a world like math, like you're into the system but you can't see the perimeters or the horizons, and so you get this horrible amorphous feeling, like, "What am I doing? Where am I? What is this? Where is the handhold?" It's like climbing a rock face with no handholds. Where am I? What is this? It's just those obvious questions [that are] never addressed.

He turns to the class and explains what he's written. "Next week, you'll be starting an extract from a new book so remember to bring that with you to class. OK, today we look at Applications. Using algebra in life. Hopefully, we'll see some kind of...situations where we can actually apply some of the work we've been...studying. The other thing we'll do is spend a bit of time reviewing Chapter 12. I've got an exercise sheet...with the answers, which you won't have of course, but then.... It'll give you a chance to practice some of the techniques that we've been working on over the last few classes. OK, a couple of things, important things, to remember: Test chapter 12, Tuesday April 5. And the other thing is...Assignment 3, some of you have handed it to me already...Assignment 3 is due by next Tuesday."

Next, he tells students," Let's look at some applications. Turn to page 538 in the textbook, question 32. Take a minute and read over question 32...just as an example." While the students are riffling through their books, looking for the right page, he writes "Page 538, Question 32" on the board. One student speaks up: "Do you want us to solve it?" "Err, not right now," the teacher replies. "We're going to take a minute to look at it." After a short pause, he continues, "Sure, take a shot at it, see if you can solve it. If you can't, that's fine. We'll be looking at it together in a minute." Most of the students start writing. One female student gets up and walks to the front with her textbook and asks about Assignment 3 [a test on Chapter 11 of the textbook, assigned at the last lesson]. She's been absent for a couple of recent lessons and is uncertain about how to do some of the problems in the assignment. The teacher turns towards her and they begin discussing her uncertainties.

Teacher: Yeah, the behavior of Helen here is pretty...it's common and it's always frustrating. You're in the middle of the class and, you know, you're getting things going and then they come up with tactics of delay, the assignments are late, tests are..."Oh, I just realized I'm missing something," or "Oh, was this due yesterday?" ...this kind of problem. And then you have to get involved in this endless dance. Yeah, I can see I looked pissed off here. Just sort of tolerating the endless dripping water torture here.

"Let's look at number 1 in the section test," the teacher says to the student. "Because if you can do number 1, you should be able to do all the others. Can you remember what to do when the questions look like this?" The student doesn't know. "Err, I'm not sure," she mumbles. "Well, we did these on the previous page," continues the teacher, turning back the page in the book. "Look, like this. What do you do in that case?" The student still doesn't know and doesn't say anything. "What do you do in each one of those?" asks the teacher again. "Err, you multiply them," guesses the student. "By what?" asks the teacher, again getting no response.

He waits for several seconds and then says, "When something's sticking to the variable....We were working on this yesterday...last class. The way you get rid of it, you multiply it by its reciprocal....Remember that? We were talking about that last class. You multiply it by the reciprocal."

Teacher: Yeah, I'm helping her. It's not...yeah, it's not a major issue. I mean...I always have to rethink it whenever it happens, I mean so what? You're there to help them learn math and if you've got the time you do it. Some of the teachers...test people on the course outline and test them twice during the term and keep the results and then use the results with the student to say you passed the course outline, which said you read it and you understand the question correctly that the test was on April the 15th. You know, I've gotten in a record that you were cognizant of this....Just another way of helping them be responsible.

If you multiply 3 times 1/3 what do you get?" asks the teacher. "One...three... one," says the student. "You get one," says the teacher. "Which one is it?" "One," agrees the student. " Good," says the teacher. "So, if we multiply 1/3 by 3/1 we get 3/3 which equals 1. So, in other words the nice thing about multiplying by the reciprocal is that it removes the thing that's sticking to the variable. So, in other words there's just one step to that question....How about a question that's like this? X + 5 = 14. How would you get rid of that 5?" After a moment's thought the student says, "subtract." "Yes, subtract," says the teacher. "In this case we subtract 5 from both sides, that gives us X = 9. Now, on this page, what makes these different is that we have two-step problems to do. These earlier ones were all one-step problems, these here need two steps. OK?"

Teacher: I think here we're definitely dealing with something more valuable....I see it as an opportunity to help...one particular student with math. All right, it might not be the appropriate time for the rest of the class, but this is the time for her. It was good to grab her then, also because, as you can see, the rest of them are all working away anyway, I glanced

and I could see that because they were attending to some material then I was fine to.... You *know, if people were sitting there twiddling their thumbs then I'd dismiss her, but...but because I could see that people were working away...quite self-motivated, that's fine, then I'm willing to entertain that.*

They continue working through several problems in the section test for over 5 more minutes. During this time, the teacher does most of the talking and the writing; the student merely watches him, her chin resting on her hands. She is also largely silent, only responding to direct questions. The other students are still working. A couple are wearing earphones attached to personal stereos; no-one speaks. "OK, you see how it goes," the teacher finally tells the student and she goes and sits back down. As she does this, another student raises his hand...

Teacher: It's interesting how long this takes. I'm surprised. It...I vaguely remember her coming up and helping her. It seemed like it took a minute. Like it took half a minute, but this looks like it's taken several minutes. I really believe that at this level, a paced class like this is...I prefer a total self-paced approach at this level, frankly....But I think having it as a...lecture milieu, this is the problem. Not the problem, but this is the reality. That's what you end up doing anyway. You end up working individually with students. That's the reality. When the content gets pretty heavy and ability seems to homogenize a bit more, then I think that a, teacher on hind legs approach is more effective.

Students said that they liked the way that instructors started each class by dealing with the problems from the previous homework. "I never have any difficulty in class, because it's fresh in my mind," said one student.

> But when you go home sometimes...perhaps you're a little tired, you just draw a complete blank and forget something that's so obvious....When you come in next day it's almost guaranteed that someone in class will ask that question that you were confused on...and he shows it all out and shows you exactly where you went wrong. (L.2.1)

In the previous two lessons, the class has been working on solving algebraic equations using the addition and multiplication principles. Today's lesson extends that work into the translation of problems into algebraic expressions and equations and the solution of such equations. Students had been asked to work through a set of these problems for homework. The teacher asks the class to read a particular problem from the textbook:

> A 480-m wire is cut into three pieces. The second piece is three times as long as the first. The third piece is four times as long as the second. How long is each piece?" (p. 538).

He turns to the class. "OK, let's look at this problem. If you have solved it, great. If you haven't, that's OK. Has anybody got an answer?" Three or four students shout out (different) answers. One answer is correct; one clearly incorrect: the numbers are out by a factor of at least 10x. The teacher ignores most answers and selects the one that is obviously incorrect. "OK, I think we might be a little bit off track here. It might be a bit extreme. Let's have a look at it more closely. Big numbers usually mean one small mistake. Let's take a look at it."

Notice here how the teacher in ignoring other answers, focuses particularly on the obviously incorrect answer. In this way he subtly reinforces his teacher-centeredness as someone who already knows which answers are correct and whose role is to tell students whether their answers are right or wrong. Further, in what follows, the teacher never attempts to explore how students reached their own (differing) solutions, but presents a series of problem-solving "hints" and a step-by-step process for solving this specific problem. In this way, he is reminding students that not only does he know the right answer, but also the "right" way to solve the

problem. By failing to ask students to examine their own methods, he reinforces the view that such analysis is unnecessary.

The teacher is reading a list of problem-solving hints that he's previously written on the board:

1. Read the problem over several times.

2. Gain an understanding of the problem situation.

"To solve problems...you have to get in there and start groveling," he tells the students. "You need to make sketches...make notes. Invent a simpler problem." As he says these, he writes "sketches, notes, simpler problem" on the board under hint #2. "These are all tips," he continues. He says that his first reaction to a problem is often that he doesn't understand it so he has to read it through several times; sometimes he also finds it helpful to make a sketch of the problem.

Here, the problem he's trying to solve concerns a length of wire so he draws a straight line on the board and writes 480m underneath it, saying, "A picture is worth a thousand words." The problem concerns the length divided into three unequal pieces, so he draws two perpendicular lines through the original line cutting it into three approximately similar lengths.

"What shall we do next?" he asks the class. One student says, "We should call one of the pieces 'X,' probably the first piece, and then we can find measures for the others." The teacher does that, labeling the pieces 'X', '3X', and '12X', while the other students look on, perplexed.

Teacher: I was amazed that...I was really quite taken aback that Nick came up with the orthodox approach, just out of the blue. I wasn't anticipating that at all. I was anticipating ...what I was looking for...what I thought might happen was just sort of a guessing...groveling, I call it...which is fine with problem solving. But surprisingly, Nick came up with this, "Call the smaller piece X, the next 2X because it's two times the size,

make an equation." I mean he gave me the classic solution, which was actually nice in one sense but in another sense I was kind of hoping it wouldn't come up so immediately. So I could say, "Oh, here's another way of approaching it."

"Why can't you call them X, Y, and Z?" asks one student. "Good point," says the teacher.

> If you don't know something, you call it 'X.' If you don't know a whole bunch of things you might call them X, Y, Z, Q, whatever, the first letter of your name, anything. But here, if we call them X, Y, or Z or whatever, we can't add them. Remember, at our stage here, what we've be working with for a few weeks is...we've only been working with one variable, usually X, and here, we're going to have to cook our work so that we have the same variable.

> Now, you will reach the stage, when you hit 061, the next class, you'll start working with equations that have 2 variables in them because sometimes there's two things you don't know. Actually in this case there's three things we don't know: each of the three pieces. But the nice thing about this question is that we know the three pieces in relation to each other. Yes, we could say X, Y, Z. But, if we went X + Y + Z = 480 and then we came back to it to solve it, we'd be in a lot of trouble. There could be a million of them. We could cook up a whole load of Xs, Ys, and Zs that could fit in there....Don't worry about it; we're going to use that, although not in 050. In some situations it's actually easier to use more than one variable. In fact, my preference would be to use more than one; that's what I'd prefer.

> Anyway, we're stuck with one variable, X, right now. And because we're told that the second is three times the first, and the third is four times the second, we're getting this kind of situation. So, we call them X, 3X and 12X. But, that's a really good point. This is the point that you're reaching the understanding part. Your anxiety levels get going and your stomach starts churning, a galvanic skin response. If you put electrodes on, you'd be sweating. All this kind of stuff happens at this point.

Teacher: This is basically a lesson that I teach about three or four times in a term with every class, looking at steps and strategies for problem solving. I mean when you're dealing with applications in algebra you've got problem solving. It's essentially a lesson that I get to work several times, but particularly in this environment it works the best....Personally, I just really enjoy it, because it's a creative part of math. It's not just learning technique and executing strategies but it's a fun part where you're trying to...pick up answers and actually doing artwork and all sorts of things. So, I like it, it's a fun part.

The teacher continues, "Now you've come very nicely into the third stage which is '3. Translate the problem into an equation.'"

He writes this on the board. Then says, "No, we haven't actually done that yet," and rubs out what he's just written, and replaces it with

3. Find Unknowns.

4. Translate into an equation.

After he writes this, he turns and stage-whispers the words to the students, putting his hand in front of his mouth as if he's passing on a secret and doesn't want anyone else to know. The students (particularly the non-English speakers) look perplexed by this behavior but write down the hints, nonetheless. By his gesture, the teacher is making something more complicated than is necessary.

"You may have found this problem easy," says the teacher. "If you found it simple, that's OK." The student who had previously spoken shouts out, "Well don't ask me the same question 3 weeks from now [which is the date of the final test] because I won't know it." The teacher ignores this and continues:

> You can get an idea of how to solve a problem even if it's simple for you. For me there's problems I can solve and there's problems I go "Huh?" and then there's a bunch in between where you feel that you can maybe get somewhere with. Well, it's a lot like that for everyone. There's some you can do, some you can't do, and a whole bunch in the middle that you might be able to handle if you're on a good day...if you're on a roll. But that's the way it goes.

> OK, translate into an equation. Is there any way we can get an equation out of this? Get an equals sign?

"Add them up," suggests one of the students who has already spoken. Other students nod and shout out "16X." "OK, good," says the teacher.

> I'll just show you the step before this. Because we've got 12 apples and 3 apples and one apple we can add them up and get 16 apples. You don't have to write this step if you see it. So now, we've got $16 X = 480$, so we can just divide by 16 and we've got $x = 30$. We're laughing all the way to the bank. So, now what have we got here? What are we asked to find? We're asked to find the length of each piece. So, X stands for 30. So the first piece is 30, the second, 90 and the third, 360. So, there's our three pieces.

The students copy this down into their books. "Is that it, then? Is that the answer?" asks one student. The others look at the teacher in anticipation. "What do you think?" says the teacher. Nobody answers and most look confused. I think they're unclear about what he's asking, but nobody wants to say so. After several seconds of silence the teacher, ignoring the question, continues, "A couple more stages we need to deal with here," and he writes two more hints on the board:

5. Solve.

6. Check.

The students dutifully copy these down. "By 'Check,'" the teacher says,

I mean, does my answer make sense? You can check your algebra. You can check your arithmetic by using a calculator. That's good. But I'm not talking about these kind of checks. I mean, "Does my answer make sense? That's the best kind of answer you can keep in mind. It's a thinking check. It's a question you can keep on the back burner all the time. It's like steering a car. Does my answer make sense? Does my answer make sense? Does what I'm doing make sense. My gut reaction when I look at this problem is that what we've done does make sense. The answer is in the right ballpark. Remember when you're solving a problem you're rolling up your sleeves and you're groveling. You're working to the point where you've got an equation like this, and then you're like a machine. Just bang, bang...out pops the answer. Just getting to that point, you have to think a little differently, a bit rubbery...just be a little flexible."

Teacher: It's striking me again how long this is taking, discussing these topics here, I'm really surprised, I'm really sensing there's quite a lot of actual time. I didn't get the sense of time while in the middle of this process, but going through this, you know, it felt to me like about...10 or 15 minutes but it's always longer. I'm aware from looking at a watch afterwards and seeing that it took a lot longer to go through this, but it always feels like about 15 minutes, it always feel much shorter. It's a long process.

The teacher has now finished solving the problem, largely by himself, at the board. It's taken 20 minutes from when he began. At no time did he ask the students about the answers they got, how they had solved the problem, or if they had any other questions about either the problem or his solution. Although the teacher

intended this part of the lesson to get across some of the key stages in problem-solving, he failed to make this explicit and, hence, risked confusing the students. Further, he compounded any confusion by using his solution of the initial problem as a vehicle for a short lecture on problem-solving and ignoring the students' interest in finding the right answer. Here, too, one can notice how the language that the teacher uses adds to some students' confusion. In his choice of words and his discourse patterns, the teacher seems untroubled that not every student may understand him, and, hence, that they may also not understand the mathematics he is trying to teach. Indeed, from his following comments one can see that he is also unconcerned by students' inattention (even though it may be due to a lack of understanding).

He now tells the students to turn back to their books and do several more problems. "OK turn back to page 534 and try questions..."

Teacher: I know I was having a good time with the class. It's interesting, I've had occasionally students saying on evaluation forms, one class, whenever you get a comment more than once then I know it's for real...if it's a criticism. In one class I had two people say, "I can't hear you," and I think what's happening is I'm speaking too rapidly which is a common problem for me. When I speak rapidly I guess some of the words collapse in on themselves. If I get too excited about a topic. But it's something I've consciously worked with, actually, the last few years....Particularly because I can see how Jim was not understanding what I was saying and my difficulty with Tania too. It's an ESL environment that I'm in so I've got to really...concentrate on that issue [of] slower speech. You know, I'm not concerned if students aren't listening to me, if they're beavering away ahead of me that's fine.

The problem solving, which [is a] lesson...I use many times, is not an instructor-focused activity. I can see now why I feel tired after a class. After a couple of classes I feel like I've had a day's work...looking at that I can see why. I put an incredible amount of energy into my teaching I'm concentrating a lot on what I'm doing. I'm quite a physical person too,

forever moving around ...not just sort of sitting there opting out, you know. I mean I'm very involved with what I'm doing and what the students are doing.

That's the reason why I use a blackboard, I used occasionally an overhead first, but I found it incredibly constraining, I found it such a frustrating...it would be a lot cleaner to use an overhead, I suppose, just your hands would get dirty, but also that's why I wear sort of average clothing, I don't, you know, I don't wear a suit...because they're just covered in chalk every day. I find it just physically too constraining, I'm really uncomfortable being chained to a little box like that, hunched over, even though you are facing people while you're writing. With a blackboard you have to turn all the time, but I find just the physical...the physical size of a blackboard much more...it's more expansive. I want to wave my arms more, you know, and those little overhead boxes you're stuck with this...it's just...it's never suited my personality.

I think it gives the impression that I'm involved with the math on the board. It's something that you're working at while you're doing it, which I think, you know, sends a different picture to students than you coming in with lots of overheads and saying this is how it is. In many ways, to me it seems, that I'm sort of personifying struggle, you know? And I think that sends a good message to students, that...sometimes you've got to struggle with this a bit, but you've got to get involved in it. That's probably what I'm sensing is when I've used the blackboard, you can do that kind of digging away.

Yeah, I'm acting. On purpose, too...to create that sense of activity. I mean you go by a math class and you can hear the...thinking going on, [but] where's the behavior to observe? But there's tremendous cognitive activity often. And yet people...you know, for two hours they may not move. I've given tests and sat there for three and a half hours and a person's just sat there, but you know, there's a stack of work beside them; the amount of cognitive activity has been incredible. But yeah, sometimes...I take it on myself to behave at least in a visibly observable fashion so....If it's going to be teacher-focused I think that moving around helps a bit in that setting.

Notice here how the teacher talks about using classroom equipment as a prop for his preferred teaching style. In general, mathematics teachers ignored the equipment in their classrooms and few thought that it had any influence on how they taught or their degree of teacher-centeredness. Only one other teacher deliberately based her teaching around the classroom equipment:

> We don't have the [black]boards in our classrooms that I like....In the previous campus we had boards that wrapped around half the room and I could get 16 people up. I like to get the whole class up there and they can all do an example on the board. I can't do that now...it's a shame. (T.1.8)

This teacher claimed that although some students found this initially threatening, "with a bit of encouragement they come to enjoy it...and it does help them learn" (T.1.8).

Students seemed to appreciate the teacher-focused dynamics and said that they liked the rhythm and the balance of being shown how to do a problem followed by individual work. "I like the way he shows us," said one student. "He shows us the proper way and then we can follow it for ourselves" (L.1.2). "We do get a lot of problems to solve," said one student, "but that's how you learn isn't it? You have things explained to you many times...and then through repetition you get it" (L.1.3). "It gets a bit tedious," said a second student. "It's almost depreciating [sic] the way he goes over and over some of the answers...there's never much excitement or enthusiasm" (L.1.7). "He could just show us one example of each type, and explain the rule," said a second student. "Sometimes it gets ridiculous how many questions we go over. I mean...the point is to learn how to do them, right? Not just find the answer to every one" (L.3.2).

Notice here how little is required of students. Their role is cast as that of audience, perhaps appreciating a good performance, maybe even the themes, but confined to be passive observers of others. Of course, as one can appreciate music without being able to play an instrument, one can also appreciate mathematics

without doing any. However, the students did not enroll in a class merely to appreciate mathematics, but to develop their mathematical understanding and skills—to be doers instead of watchers.

In choosing how to teach, teachers looked for "ways of organizing material" that would not delay the course schedule but also kept them engaged as teachers. "Looking for new ways of approaching things keeps me interested," as one teacher put it (T.1.3). Another teacher described how he would

> Pick out things that [he thought] would be confusing or difficult [for students] and rearrange them into some pattern that is sensible....Break it down into little steps...and make mental notes about things that might cause confusion or common mistakes that people would make...or things I can relate to the real life of people....So that I remember to mention that [to the students]. (T.1.2)

The centrality of the textbook is also notable. Students are subtly reminded that the course content concerns a mathematics that is removed from their (indeed, anyone's) experiences, rather than a mathematics that concerns their understandings or confusions. Teachers, intentionally or not, reduced the students' opportunities to influence their education. Teachers who pursued only their own perspective as to how to enliven the course content or deal with anticipated confusion again cast the students in a passive role.

Teachers claimed they made mathematics seem more relevant to students, by either giving "practical, real-life examples" or by linking the course content to the student's experiences, interests and goals. I observed one teacher, waving his hands in the air, describe the principle that the product of two negative numbers is positive.

> [The teacher says] "Let's consider [an example]. Look at the changes in temperature and time.... For example, negative times, the temperature's dropping, minus whatever's in the past, so your answer is the temperature is falling." The students look bewildered. (Field Notes, 940120)

However, merely making mathematics "relevant" does not ensure understanding. Another teacher explained how students would often ignore a common-sense interpretation and assume that whatever was produced by a mathematical procedure must be accurate. He described his "London Drugs test":

That's when you go in and buy some batteries and a birthday card and a packet of writing paper and you come out and they scan it and they say that's $485. What are you going to do? You say "Excuse me, there's an error here." So do students do that when they get an answer that's clearly wrong with their calculators? No, no, no! They just write the answer down. (T.1.6)

The confusion here arose because students were expected to develop some intuitive understanding of mathematics while being drilled and tested in repeated calculations and procedural applications. If we assume that those "practical real-life examples" do relate in some way to students' lives (for example, by relating to shopping for commonplace items), the "formal" mathematics done using those examples does not relate at all. It is always abstract and presented as an application of a universal principle; formal mathematics masquerades as the real thing.

Surprisingly, students felt that teachers' attempts to make mathematics interesting or relevant was unnecessary. Only one appreciated his teacher's efforts:

> I bring my bike in to the classroom in case it gets stolen....and the teacher sometimes refers to it. Like when we did geometry he was showing us all the angles....He'll use [another student's] briefcase and try to turn it into a workable example. (L.3.3)

Other students were unconcerned. "Just depends on the chapter," said one. "If like me and learned math in other country, then perhaps more examples better. Sometimes the language is tricky [so we] need plenty of practice (L.1.1). Another student asked:

> Maybe we could have more interesting problems?...Often we'll have to do 30 or so questions, and they're all really simple. For example we had a test on percents, right? And there wasn't many questions but they were all "Work out

4% of this and 20% of that." There was nothing about sales tax or compound interest or anything interesting. (L.1.6)

Even in these instances, when the content (tangentially) acknowledged their lives, students were still cast in a passive role. The extent of their activity was in generating the questions that formed the focus of the activity and copying down the teacher's working of the solution. Often students remained silent during an entire classroom episode. Teachers may have talked aloud (sometimes to the class, sometimes to themselves) as they solved problems, but any discussion between students and teacher was minimal. Students were rarely asked to volunteer the next step in the solution or the answer to an arithmetic calculation, or even say if they "understood" anything better at the end.

Teachers claimed that they used this question-asking period as an informal check on the class's level of understanding: "If they ask too many questions I know, 'OK, this is not sinking in.' So I would shift gears accordingly" (T.1.2). However, shifting gears usually meant postponing the introduction of a new topic until students had completed a dozen or so more problems from the textbook. I noticed too that after this further revision period, teachers would move on to the next topic without any further checks for understanding.

Teachers would ask comprehension questions of students in class. As one teacher put it, "Most of the time I lecture, [and] lead them along. But, sometimes I stop, ask question. See what they remember" (T.1.2). A second teacher described that, once he had demonstrated how to solve a particular sort of problem on the board, he would, "put another one up there, just like the first, and say 'Right, you tell me how to do it'" (T.1.7). However, this teaching behavior was not common and I observed few attempts to actively involve the students in any ways of checking for understanding.

Indeed, teachers appeared to want to involve the students as little as possible. "There's not really a lot of choices that students can make," explained one teacher. "Most of what we do is already planned for us...and I choose the rest" (T.1.6). In general, teachers confused student involvement with student participation in class. As one teacher described it, students "become involved by asking questions in class. I tell them at the beginning of term that I've never said a question was stupid" (T.1.1). Of course, asking questions is quite threatening for some: "The international students, for example, [suffer] a loss of face to ask questions, to admit that [they] don't understand something," said one teacher (T.1.7). "There are some students who are just too shy," agreed another. "They'll come to you afterwards and say, 'I could have told you that but I was too afraid to put up my hand.' Well, that's the price they pay" (T.1.2).

Here we notice that it becomes the students' responsibility to raise issues that they don't understand--a situation that perhaps expects too much of students returning to education for the first time in several years. Students' often negative memories of their school mathematics education (e.g., being publicly embarrassed) may not encourage them to broadcast their ignorance. It is also remarkable that students, while not expected to know much mathematics, are expected to know how to learn despite their lack of previous opportunity to practice this skill.

The only area in which students seemed to have any choice concerned whether they would have a break in the middle of their lesson. "I usually give them this choice," explained one teacher. "Either to have a 15 minute break or to work right through and finish earlier" (T.1.1). Most teachers followed the same procedure. "Usually they'll work right through," explained another teacher. "Then they can get out early. Most of them have other classes and it gives them a bigger break between classes" (T.1.5). Teachers, who structured lessons so that they can leave the room whenever they like, clearly preferred to work straight through.

One teacher said that "Basically, the only choice students get in my class is

whether they're going to take this course seriously, whether they're going to work at

it or not work at it. They have that decision" (T.1.8). Another agreed: "They can

choose whether they come to class; they can choose whether to do the homework.

Really, that means...they can choose whether to be successful" (T.1.7). Overall,

teachers felt that,

> There's a tendency on the part of students, especially the younger ones to feel that it's [the teacher's] class and [the teacher] is in control. They have a stake in it, but in terms of decisions they would have to defer to the instructor. There might be a contradiction there with them expecting their money's worth, but I don't think so. I think there is a difference between covering the content and dealing with what [the students] want to get out of the course. I mean the course is going to run my way because I'm the teacher and it's my classroom and my course. (T.1.6)

Teachers' reluctance to use group-work offers a good example of how their

views of the subject influence their teaching. Although students had consciously

chosen to study with others in a classroom setting, once there, they were largely

required to work by themselves. By assigning students to work together, teachers

could have tackled the problem of teaching students of different abilities (one of

their concerns). However, at no time did I observe any group work and no teacher

said that they regularly used group work or cooperative learning in their classes.

When they did so, they chose groupwork as a way of dealing with a large-sized

class. "When our class is bigger we try to group the better students to help the slow

students," said one teacher (T.2.1). "When size is small, cooperative learning not

necessary," agreed another. "I can take care of each one" (T.1.5). In other cases,

choosing to work in groups seemed to be left up to the students: "I do encourage

them to work in groups," said one teacher,

> But a lot of people don't like to. I tell them that, "Two or three heads are better than one" and that they can generate ideas and enthusiasm from each other, but usually they go off and work on their own. (T.1.7)

Despite this last comment, all of the other teachers regarded group-work

solely as a classroom management technique rather than as valuable for any other

purpose. In any case, teachers thought that mathematics did not lend itself to group work. As one put it, "You can't say, 'Here's factoring; the two of you go and figure it out'" (T.1.2).

Working with Individual Students

After the presentation of any new material, teachers assigned problems for students to work on. Here, teachers thought that they acted as a "coach," or as one who encouraged "autonomy" or "responsibility." As one teacher described it: "I've got to instill some responsibility into them. They've got to learn to stand on their own two feet. [So] I'll give them some problem sets and then go around and check how they're doing" (T.1.6). "I find students at this level are very dependent," said another,

> Some of them want you to chew their lunch for them....So I find that if I give them plenty of individual work, the good ones will go right ahead and not bother you...and the ones who have twenty questions...I can spend time with them without slowing the others down. (T.2.1)

Let us now consider a specific (although typical) example of an episode that occurred about midway through a 2-hour class:

Everyone sat, working quietly by themselves at their assigned algebra problems. The teacher was marking test papers from another class. One student started waving his hand frantically in the air, but said nothing. Nobody noticed him for a minute or two. Eventually the teacher looked up, sighed, went across to the student, and began the following exchange:

Teacher: Norm, which one are you doing? Number 10? ["A 58 inch board is cut into two pieces. One piece is 2 inches longer than the other. How long are the pieces?"]

Norm: Yes.

T: OK, let's be careful here. This is a little different here. This piece is 2 inches longer than the other. It's not two times as long. That's what you've got there: two times.

N: Oh, OK. Yeah.

T: What do you think you should do there?

N: Two inches longer. That's what I did. Fifty-eight divided by two is 29, right? So you get one board is 27 inches long and one board is 31 inches.

T: Those are 4 inches apart though. That one's only got to be 2 inches longer.

N: Oh, OK.

T: Try the other way. Get your board down...58. OK, take your chain saw....Brrrm...cut it into two.

N: Into two...which is 29 inches.

T: Hold on. No, you don't know that. That's if they're equal length. They're not equal length. One piece is 2 inches longer than the other.

N: Yeah. Probably 2 inches this way.

T: It's off center somehow. Yeah, if one of them's X, the other isn't two times X. It's....What would it be?

N: If one's X...er...We know one is X, then the other one is...two times. No, no, 2 inches longer. It's 2 plus...

T: Exactly, 2 plus X.

N: X plus 2, right?

T: The other one would be X plus 2.

N: Right, X plus 2.

T: OK, so if one of them is X, the other is X plus 2. So?

N: So X equals X plus 2, right?

T: Mmm.

Teacher: This is a bit unusual in that I did give so much explanation here. Although on this topic with the problem solving and algebra, I felt really comfortable with that amount of discussion, because it is a topic that people can phase in and phase out of and come back and pick up on. But it's sort of like drawing a lot of spirals, you have to go over and over and

over and over and over, and...with some forward progress. But I think the problem with self-paced environment is it's the motivation problem and people are so vulcanized they can't...they don't have the motivation, but I think... this kind of setting right now I think is about the best.

N: But then you have two variables that we don't know what's going on.

T: Yeah. Well, what did we do in this [other problem] situation? What did we do here? Each of the three pieces added together gives you a grand total. I think we have the same kind of situation here. The grand total in this case is 58.

N: I once worked in a woodwork factory, you know? Where we did something similar. Like for a bookcase, right? We'd have to measure half of 58 inches, so we'd make a line right down the middle...with a pencil. We'd add 2 inches to one board, and the other board we'd leave alone.

T: You'd leave it alone, eh? You didn't want to cut it ahead of time though? You'd mark it.

N: Yeah, you'd mark it.

T: Well, anyway....Let's get back to the problem.

Teacher: I notice whether I'm working with people individually or in...with the group, you can tell by their body language that,...expecting someone to follow you in a long linear fashion is absurd. The...the topic in this case, it was lucky in the topic in this case, the problem solving, it's a lot of meandering anyway and so people, if they're tuning in and tuning out [they are] like a...like a microscope focusing in and out. That's OK, you can actually carry with the topic, but even individually you're basically kind of hinting and giving tips, it's not a long linear, you know, this is how you start, middle, finish. That's useless in terms of education it's...it's a prodding, coaching, steering type of process, because I see Norm is focusing on his own stuff here quite a bit. Then out of all I've said and done probably one or two things are enough to prod him that he says, " Oh yeah, I see, it's...," and off he goes. Because you sure know with the other way when you've gone through all the labor, when you go through that kind of labor and then the person asks you the same question again.

N: So, see here. From here to here is 58 inches. Was it 58? Half of 58 inches is 29 inches.

T: OK, yeah. We're stuck on that half business. Hold on...Errm... If one of the pieces is X, the other one is two bits bigger. You've already identified it as "X plus 2." The relationship is.....You add the two of them together. That'll equal 58, the grand total. This is the situation we want to solve.

N: Yeah.

T: So, if we go through the steps. 2X plus 2 equals 58. 2X equals 56. Divide by 2. So we get one of the things being 28...not 29, 28. If X is 28, what would the length of the other one be?

N: If X is 28...

T: If that's 28, what is X plus 2?

N: Oh, because that's 2 inches longer, right?

T: What should it be?

N: 28 plus 2 is...30.

T: Yep, yep. That would be 30. I know it's a bit of a formal...kind of formally setting it up in that fashion. But it's a way of approaching it. By dividing by 2, you're getting an approximation. I mean 29 is pretty close to what the two of them were...

N: Yeah.

T: But it's not getting it dead on, unfortunately.

N: No.

T: You try that next one yourself. Remember, take a read through the problem several times first.

Teacher: I think in 050 classes in the future I'm just going to have lots of material, sign it out, maybe give a few tips and just work with people one on one. Because I think although they don't get a pile of immediate instructor intervention, they don't need it. What they need is help occasionally. I think when you're dealing with some arcane trigonometry, circular trigonometry, then you need to pass a lot of gas. But just to get people through a few things...in this environment, this is much better, just the one on one and have people working and then just queuing up and take a number, and me saying, "I'll be there with you in a few minutes."

In another interview, this same teacher acknowledged that students often had unpleasant experiences of mathematics education which could have affected their learning. He saw his role as counteracting any negative memories:

> I think I'm responsible for the students learning in whatever way [they can].
> Partly because of their backgrounds where they've been let down or didn't fit,
> or whatever their background they haven't succeeded. It's usually not their
> own fault [but] the system wasn't able to fit around them. (T.1.6)

This method of allowing students to work individually had its flaws. One teacher described how she felt that some teachers just gave work "to keep the students busy. In my opinion they're not helping the students at all...they're just left alone.... Sometimes, the teachers say 'Do numbers 1 to 35' and then the teacher just sits at the front doing their own work...perhaps marking papers from another class" (T.1.8).

No teacher ever speculated on how they might teach different parts of the course in different ways; teachers adopted a standard format and stuck with it. Further, they rationalized their choices as being in the students' best interests. "I like to get a feel of [the students'] learning attitudes," one teacher said. "In the first class I get a feel for their background...and then try and accommodate that...with my teaching...to help them maximize their learning" (T.1.4).

Throughout these episodes, students worked very much alone, rarely visibly interacting with their classmates. However, on occasions, students attempted to make their activity more communal. I noticed that when teachers were working with individual students, those students sitting nearby often stopped their own work and looked over to see what was happening. Sometimes they would even leave their seats and go and stand around the teacher observing the discussion. They seemed to enjoy the social nature of the situation: it gave them an opportunity to watch (and sometimes, to participate in) a discussion about mathematics. By describing how they solved (or failed to solve) a problem, students showed how the making of

mistakes was a common, rather than an individual, experience. In this way, students often made the practice of mathematics seem more human. The teachers neither encouraged nor discouraged this behavior among students. In fact, they didn't even mention it, merely allowing it, once started, to continue until students got tired and returned to their own seats.

Homework

Homework formed a key part of mathematics coursework. After the presentation and review of new material during classroom time, the homework required students to demonstrate their mastery of that new material. In addition, time for homework was built in to every lesson. Teachers finished their classes by assigning homework for the next class, and started the subsequent class by dealing with any student difficulties concerning the homework. In this way, teachers claimed to check the students' comprehension of new material before proceeding.

Without exception, homework in each class was chosen from the textbook. Each textbook section (usually covered in one class session) concluded with a section test of 40 - 60 problems based on the section material. These tests formed the homework that students were to complete before the next lesson. Depending on the time available, teachers would usually allocate the first 15 or so problems in these tests as an in-class exercise and tell students to complete the rest for homework.

In general, teachers expected students to complete all of the problems in the test, thinking that, "the more you do the better you get" (T.1.6). I observed one teacher hand out a practice test. "

Oh, God. More of this stuff?" asks one student. "There's always more," replies the teacher. "In fact, I've got about 350 books of this stuff. I can always get more for you. There's an endless amount of problems. They're good for you. (Field Notes, 940402)

Often the problems were repetitive and their number excessive. Students would commonly look to see how many problems were being assigned for homework, and, if there was a large number, they would groan. Several times, teachers would then reduce the number of problems to be solved. "OK, only do the odd numbered ones," said one teacher I observed. "No better still, do all those up to number 30" (Field Notes, 940131).

When teachers curtailed the number of problems to be solved for homework, they commonly asked students to omit those problems that occurred at the end of the test. However, this part contained the "skill maintenance" problems that tested students' comprehension of earlier parts of the course together with the most recent course material. By removing those problems, the teachers were also removing an opportunity for student revision and the development of more relational mathematical thinking.

Teachers expected that homework should take students approximately the same time to complete as the time spent in the classroom: on average, an hour or so each evening. As one teacher described it: "The general gut feeling I have in...assigning homework is...for every hour in [class] there'll be an hour out" (T.2.1). However, students said that they regularly took much longer; several described how they would often spend up to twice that time. Homework took so long for some students because they found the material difficult; others were less challenged by the content but overwhelmed by the amount of work they were required to do. These students diligently tried to complete all the homework problems for every lesson. One student described that he often spent two or more hours every night on his homework:

I don't take any shortcuts. I don't use my book...I write every question out.
That takes a bit longer.... And when I'm doing my homework I use the long
method...you know, show all the steps. But I think it helps me you know...to
get right down to the meat.... I think the more questions you do the better
you learn the course. (L.3.9)

The amount of homework was clearly a problem for many students. "It's as if
he [the teacher] doesn't know that we have other lives," said one student. "I have to
go to work when I leave here, and sometimes I don't get home until late at
night....Then, if I have to do 50 of these stupid problems....Well, sometimes I don't"
(L.3.6). Homework was seen as repetitious as well as excessive. "Often it seems as if
the problems are too many," said a second student. "I don't see why we should be
doing a lot that [are] all the same....If you can do the first ones OK, then why do lots
more?" (L.3.8). However, in general, students said that they usually completed all of
the homework they were assigned, feeling that, as one student described it, "the
more you do, the better you get" (L.3.4).

Students' commitment towards completing their homework was remarkable
in light of the realization that such work was largely voluntary and rarely marked.
However, teachers would regularly advise students that homework, although not
compulsory, was a necessary part of the course. One teacher explained that he
would tell students, "You don't have to do the homework...but you're not going to
succeed if you don't do it...so it's up to you" (T.1.4). Further, teachers regarded
homework as the part of the course where students could exercise some choice about
learning. I observed one teacher, when writing up a homework assignment on the
board, tell students,

This is where you have to be in charge of your own learning. Some of these
problems are for extra practice—you don't have to do them, they're strictly
optional. I won't be collecting them in, but at the beginning of next class I'll
ask for any questions. I'll deal with them and then we'll move on. (Field
Notes, 940111)

The decision not to require or mark homework was clearly one that the
teachers had deliberately chosen to suit their own needs; this seemed largely so as to

avoid the volume of marking that would ensue. "We have enough to do as it is," said one teacher. "If we had to mark all the homework as well, we'd never get out of here" (T.4.1). Teachers also questioned the validity of testing "when the answers were in the book." "It's so open to abuse," explained one teacher. "They can look up the answer and then work backwards from there. What's the point in marking that?" (T.1.6). "It doesn't check their understanding," agreed a second teacher. "It's much better to give them a mark for in-class tests" (T.1.3).

Teachers, invariably, used homework for checking student comprehension of the course material. Each class I observed devoted the first 20 - 50 minutes of class time (out of a total 2 hours) to problems that students had encountered in the previous lesson's homework. Here, any difference in time taken depended solely on how many questions the students asked; I observed no teacher curtail this part of their lesson, despite their concern with keeping to the schedule. One teacher explained the benefits of spending so long on students' problems: "Each time before we begin the new lesson I [en]sure they understand the contents [of the lesson] before....If they don't understand it [then] they ask questions. Then we can go on, and I know that they are comfortable "(T.1.5).

Thus, the responsibility to raise any difficulties lay solely with students; these problems were the only vehicle for students to raise any conceptual difficulties. Teachers assumed that students would ask questions about anything they didn't understand. However, this was not always the case. As mentioned above, students' confidence was often shaky. Several students indicated that they did not always choose to show their lack of understanding. "If I've already asked several questions," said one, "I'm going to be wary about raising my hand. He [the teacher] might think I don't know anything" (L.3.5). A second student explained that, "I'm kind of shy...you don't want to bring something up because you may sound stupid" (L.3.2). Often students would wait for others to ask questions about common difficulties.

"Sometimes I'll hold back a bit just to hear if my question happens to pop up ahead of me, " explained another student (L.3.9).

Teachers commonly worked through these homework problems on the board for everyone to follow. Occasionally they would use different methods of solution from those suggested in the book. One teacher explained that , "I like to show them my way of doing problems. Textbook has its way, often not clear. If I show my way, students can learn what suits them" (T.1.5). Without exception this blackboard explication approach predominated. Even when teachers saw its weaknesses, they still followed the same approach. As one explained:

> The danger with that approach [is that] once you've said, "Here's how you do it, here's an example, now you tell me how to do it." They do their homework, they know how to do it. But when it comes to the test and they read the instructions, they'll say, "We never did that. What's that mean?" They forget. You've got to constantly review material to get them to see the relationships between what they're being asked to do with what they've done. (T.1.7)

Most students appreciated these additional explanations that teachers gave, particularly when they found the textbook unclear. Any confusion over the multiplicity of methods was mitigated by the slow, explanatory approach that teachers adopted. As one student described:

> The book will often give you all the steps...sometimes too many. That's where the problems start because...you're not going to do that in real life or it'd take you...six hours to solve anything....So, you get confused because you're trying to condense it down too far....What [the teacher] does is give you the in-between point where he gets rid of stuff so it isn't too long and then he adds a couple of steps that'll help you out....Like he'll add a couple of brackets to make it a little bit easier...and I'll see where I went wrong and then I'll follow his procedure. (L.3.7)

Overwhelmingly, students welcomed the time devoted to the solving of their problems. For many, this was the best part of their classroom work. "I really like it," explained one student. "[The teacher] will sometimes spend an hour dealing with people's problems....That's really good. He makes sure that everyone understands

something before he moves on" (L.3.1). "Another thing [the teacher] does...is get us feeling good about what we do know," said a second student.

> So, we feel OK about asking questions....Like if we don't understand something...maybe we've already covered it, but we've forgotten it...and you're afraid to bring it up because it was a couple of sections back. [The teacher] doesn't mind if we go back. He'll whip right back to that one, no problem at all. (L.3.10)

Students generally recognized that although they might find parts of the course easy, others might not. Additionally, most students thought they would encounter difficulties later in the course. "I'm doing OK on this part," explained one student, "but I know it's going to get more challenging ...and I'll have some problems" (L.2.1).

However, those students who were familiar and confident with the material were frustrated by the time spent dealing with homework problems. "I can't see why [the teacher] has to go over 20 problems, all much the same," said one student. "It's...the way of doing it that's important...not the answer to every little question. Why can't [the teacher] go over one problem and then the rule and get [students] to try and do it themselves?" (L.3.2). "Maybe he can teach more and let us do the homework at home. That way he could go a little faster," said another (L.1.1). "It is a bit boring in the foundations part...doing 25 subtraction problems, but it'll get more interesting later, I'm sure" (L.1.5). One of the other students suggested, "Maybe the slower students can go and see the teacher afterwards [to] get some help...some extra instructions" (L.1.2). But again, this would likely increase anxiety for the less confident students who might be embarrassed to ask for help for fear of admitting their lack of knowledge. "I've wanted to go to see the teacher [after class]", said one student, "but it makes me look stupid...because I didn't remember. Not because it's hard...it's not really, it's just that I forgot how to do it" (L.3.5). Interrupting the teacher during class was also difficult for many students. One student, discussing his teacher, said that,

She's hard to challenge. She presents this sort of quite ebullient, happy front but you can't...stop her in mid-flow. Nobody feels comfortable to say "Now just hold on a minute, can we go over that bit more slowly or can you show us again?" (L.3.4)

These faster-learning students often found strategies to deal with their boredom during the homework part of the lesson. Some doodled or did work for other courses. "That's when I try to do the reading for my English homework," said one student. "I keep my ears peeled for the [math] questions I got wrong. Usually someone else will have a difficulty with them. Then I pay attention" (L.1.3). Over the term one student adopted the practice of regularly arriving 30 minutes after the class had begun. "No point in coming for the homework bit, " he explained. "I usually get them all right...so I might as well spend the time at home" (L.3.3).

Assessment

The mathematics department is located in the College Foundations section of the College. It is not surprising to find that assessment plays so large a role in mathematics courses given that courses in this section are primarily designed for students with academic aspirations who are encouraged to gauge success by the passing of examinations. The course outlines handed out in the first few sessions of each course indicate the importance of assessment. Each outline not only specified how assessments contributed to the evaluation of students' coursework, but also when such assessments would be held, and what they would cover.

Mathematics courses used several forms of student assessment: monthly tests, a final examination, and "term work" (a category covering a pot-pourri of

marks based on attendance, spot quizzes, and occasional hand-in homework
assignments). Each assessment form contributed to an overall course mark for
students: the monthly tests (in total) contributed 60-65% of the total mark, the final
examination 25%, and the term work 10-15%.

These forms of assessment (and the relative weight of their marks) were
standard across the department. "We use the same methods [of assessment] in all
our courses," explained the department head. "We've found that the students prefer
it...they know what's expected of them and it makes their work easier" (T.1.1). "The
department likes a lot of assessment," agreed a teacher. "They demand a bunch of
assignments coming in...it's always been a popular device....It's a sort of a check to
motivate the students" (T.1.3). "Assessment very important," said another teacher.
"It's department policy....Students will not do work unless it is assigned...[so] we
need to push them...and set regular tests" (T.1.4). Notice here, that teachers (who
had a great deal of autonomy in deciding the format and timing of assessment)
presented the needs or wishes of others as justifications for their own choices.

Teachers set assessment tests regularly throughout the term. The final
examination—on the whole term's material—was held in the last week of classes.
Before that, approximately at the end of every month, teachers planned "chapter
tests" on the course material covered since the last test. The course schedules
detailed when these tests would be, and most teachers regarded the testing dates as
sacrosanct. "It's better in the long run if we don't change the dates [of the tests],"
explained one teacher. "It wouldn't be fair to the students. You see, they know when
they are, and prepare for them" (T.1.8).

Some teachers said they would occasionally let students have some say over
the choice of grading structure or test dates, although I rarely observed this in
practice. "I always explain my grading system to the students," said one teacher.

I tell them 25% for final exam is fixed, but chapter tests are 60%, homework and attendance 15%. Any objections?' Sometimes they say that, 'Test is too heavy, 60% too much for five tests.' So I say if they want they can have more tests and each one is worth less. But it's hard because of the time frame...you cannot have ten tests, no room for you to do that. (T.1.4)

In only one course I studied (050/051, the "double-block" course) would the teacher allow any leeway over the test dates. "Sometimes the students not ready," he said. "I give them an extra day or two to practice" (T.3.1). Not only would this teacher be flexible about the test date, but he would let the students (as a whole) decide when it should be. "If they do not feel ready...I let them put a test off for one or two days," he explained (T.4.1). His students welcomed both the leeway and the opportunity to choose. As one described it:

> Sometimes we'll have a debate in class about whether the test's on Wednesday or Monday. He [the teacher] is very good like that. Sometimes we've spent so much time on people's homework problems that it's set us a day or so behind...and we've not covered all of the material. So people are nervous about that and want some extra time to revise. He only gives us a couple of choices of dates...but it's nice to be asked. (L.3.2)

Generally, students accepted the regularity of testing, and, as the term progressed, grew to appreciate it. As one student described:

> It's like a regular check on us...how well we're doing....And I don't feel like I'm going through a test and feeling as if I don't know the material. As a matter of fact, I never felt more confident...because...you never know what's going to be asked...but there's material here that we've worked on...and we know the technique of the test. We know the general outline of it. (L.3.9)

Students unable to take the monthly tests on whatever day they were held were allowed to take "make-ups" during the following two weeks; several students opted for this. In order to take the "make-up" option, students were expected to bring a doctor's note or other evidence of justifiable absence on the original test date. However, teachers were generally forgiving of those that could not provide such evidence, and often allowed students to take their make-up tests without any proof. "What good would it do to stop them taking it?" asked one teacher. "They're usually the ones who are having most difficulty, and they often need the most

encouragement" (T.4.3). Students could not, however, retake a test to improve their

mark. "It's department policy," explained one teacher. "They only get one crack at

it...if they don't do too well, then they have to try harder next time" (T.1.6).

Both the final examination and the chapter tests followed the same format.

Each was designed to be completed by students in one class period (2 hours) and

consisted of between 20 - 30 questions. These questions tested students' memories of

"important properties and formulae" and their abilities to apply these concepts in

solving simple mathematical problems. When they finished they handed in their

answer sheets and were then allowed to leave. Although they could take up to 2

hours to complete the tests, most were finished well in advance. (I observed four

"chapter tests" in different classes, and, in each case, only one student (out of 8 - 10)

had not finished by the end of the allotted time.) Test-taking was a singularly

individual activity: students worked alone (without the help of either the textbook

or their notes) and were barred from talking to each other during the test period.

This last point was strongly enforced by teachers. As one put it: "The whole validity

of testing would be in jeopardy if students could talk to each other" (T.1.3).

Because each course followed its set textbook so closely, teachers naturally

looked to the textbook and its associated supplementary material to provide ready-

made tests. As one teacher described,

> After 15 years [of teaching] I'm not looking for ways of making my life more
> miserable and more stressful. If something appears to be working well, that's
> fine. I'm quite happy to have content, structure, testing etc. all laid out and
> planned... so that I can focus all my time on working with students. (T.1.3)

Both the student edition of the textbook and the Instructor's Resource Guide

included examples of such tests, and teachers would select one from these examples,

photocopy it and distribute it at the beginning of the appropriate lesson. Each test

normally contained between 20 - 30 items; tests on the more advanced material

contained fewer questions. As an example, the Chapter 12 test from the Basic

Mathematics book (used in all three of the basic courses) is attached as Appendix 18.
One can see that it contains 18 questions, of increasing complexity, dealing with the content of Chapter 12: an introduction to algebraic expressions and the use of addition and multiplication principles to solve problems.

Students generally regarded the substance of the test as unproblematic. One or two, however, questioned the relevance or usefulness of the tests. "They're not very challenging," said one. "I wish they could relate a bit more to my life" (L.3.11).

Some teachers were similarly critical of the tests provided in the textbook. "They seem to test just the very simplest concept and give very easy questions," said one. "I sometimes feel that we're not preparing our students if we are not challenging them to work through multi-step questions, and use a more problem-solving approach" (T.4.3). In a department meeting I observed teachers discussing this point more fully:

> One teacher says, "We should be preparing them for the outside." He explains that he thinks that the math they teach should have some relevance [though he doesn't say for whom] and that they should test students' "practical abilities" more. Another teacher claims that how the students feel about taking the test is crucial: "There's so much math anxiety." A third teacher says, "We're teaching a skill, a way of life. Does the chapter test mean that much?...Finally, a fourth teacher claims that, "Problems in the books are stupid. For example, 'You are now 3 times what you were 6 years ago.'" After these comments the discussion peters out. I get the feeling that this is an area that is regularly visited, and regularly abandoned without much progress. (Field Notes, 940302)

The validity of these criticisms can be judged by examining some of the test problems themselves. For example, consider the problems presented in the chapter 12 test. Of the 18 problems, 12 contain only variables or numerals and ask students to solve simple algebraic equations (e.g., "8 - Y = 16"). The remaining six are similar, although written out in words (which students are expected to first translate into an algebraic expression, then solve). None of the 18 require a multi-stepped approach or check students' understanding of the concepts they are applying. Further, the

problems are of doubtful relevance to everyday uses of mathematics. For example,

question 12 (the only one that refers to a "real-life" situation) tells students:

> A movie theater had a certain number of tickets to give away. Five people got the tickets. The first got 1/3 of the tickets, the second got 1/4 of the tickets, and the third got 1/5 of the tickets. The fourth person got eight tickets, and there were five tickets left for the fifth person. Find the total number of tickets given away. (p. 542)

Despite their criticisms of the textbook, teachers were reluctant to abandon its

sample tests. "They're just so easy to use," said one teacher.

> For example, I have a test to give tomorrow and I really haven't had time to prepare one. So, I'll look in the book...or we have a file drawer in the teachers' resource room which contains five or six different tests for each subject at each grade. I'll do that. (T.1.8)

"It takes a lot of time to draw up your own tests," explained a second teacher.

> If I'm typing math notation into the computer it could take me 4 or 5 hours designing a test, doing all the symbols and stuff...like superscripts. It's a lot of work. So, I tend to use the prepared ones. (T.1.6)

Additionally, teachers would commonly reuse old tests. Towards the end of

term, I observed one teacher give out a sample final exam paper. He told the class

that,

> It's from 1984...but don't worry. It's not changed much. The only difference is that you can use calculators now--the world has moved on....Oh, and don't do numbers 24, 25, 29, and 30. We haven't covered those in this class. (Field Notes, 940418)

The centrality of assessment became clear to me as I observed how teachers

referred to the tests. Both in interviews and in their conversations with students,

teachers would often refer to test dates as a check on course progress if not

comprehension. Several times, I observed teachers saying that, for example, "A test

will be held next week...so we're going to have to race through this next couple of

sections" (Field Notes, 940322). One teacher, interviewed at the end of the term,

spoke proudly of how he had "kept to all of the test days" when reviewing his

teaching over the term (T.4.1). Some students found this amazing. As one put it: "It's always amazing how he just magically ends the chapter just as we're going to have a test, according to the schedule. How does he do that?" (L.2.2)

Teachers would refer to tests as "practice for real life." One teacher I interviewed spoke clearly about "how students are going to have to take a lot of tests in their future lives, so they may as well start doing them properly now" (T.1.7). In her classroom, she placed great emphasis on upcoming tests, and reminded students that although tests "weren't a cause for anxiety...they should prepare for them carefully." By careful preparation, she explained, she meant, "Paying attention to such issues as complete revision, tidy note-taking...and going to bed early" (Field Notes, 940224).

Teachers varied in their approaches to the different forms of assessment. However, the final examination was seen as fundamental by everybody. "We always use exams," said one teacher, "that's the main way....It's not the best way...but nobody's come up with a better method to my knowledge" (T.1.2). "I'm embarrassed to say it's all test and assignment evaluation," agreed another teacher. "Some instructors include an attendance section and others put a couple of bonus questions in, but they aren't part of the curriculum so I suppose they shouldn't be there" (T.1.1). "The final exam is absolutely critical," said a third. "It's a closed book comprehensive exam and it's essential for demonstrating mastery. At any level, you need to have a comprehensive test at the end [of the course] that covers all the course material" (T.3.2). In agreeing, a second teacher criticized mathematics courses in other institutions for their lack of such an examination:

> I think that's what's lacking in some ABE programs...they have no comprehensive examination at the end. I mean, it's easy to demonstrate mastery if you say, "Unit one. 2 + 2 = 4. OK, here's test number one: '2 + 2 = ?' And everyone gets it right. 100% mastery. Easy. "OK, now unit 2." There's nothing...it's ridiculous. (T.2.1)

"That's what makes our program so attractive," said the department head. "Other institutions are not quite so...rigorous as we are. The receiving institutions don't like it if the students do open book exams.... They don't like [students] to get too independent" (Field Notes, 940115).

The regular monthly tests too were also regarded as necessary. "They are so important," explained one teacher. "They keep the students on their goal...and give them...and me, regular checks on how well they are doing" (T.1.5). Indeed, because of this, most teachers said that they would prefer the courses to contain more tests, even though it would take more time. "We have little time as it is," said another teacher, "But maybe if the tests were shorter we could have them more often" (T.1.8).

Teachers criticized their own reliance on established tests:

> We don't get much written work from our students....In school they have to hand something in every day and it gets marked. That's the ideal....But here, [students] don't hand anything in, unless it's the occasional quiz, so students don't really find out until test time...which is too late. (T.1.8)

Yet, remarkably, they never created any alternatives and used tests regularly.

Most of the students I interviewed agreed that testing was a necessary, if unpleasant, part of the course. "I mean, that's how we learn, isn't it?" asked one. "The teachers...and the book tell us stuff. So then we have to show them we know it" (L.3.2).

Students, without exception, were anxious about the tests. At the beginning of all three courses I observed, after the teacher had explained the course timetable, students asked, "Does it show us when the tests are?" Later, in the first few sessions of the courses, students would ask repeatedly about the tests: How would they would be structured? Were they going to be difficult? Were notes or calculators allowed?

Students' dread of assessment often affected their coursework. One described
that,

> What I've been doing is focusing too much on the course where the test is
> coming up and then letting the other work ride. The tests increase my anxiety
> something fierce! Oh, just incredibly so.....When it came to this one, I was
> mentally a wreck....And so I couldn't concentrate on it. (L.2.2).

A second student, interviewed at the end of the term, described the
atmosphere of the classroom throughout the term as "generally pleasant...well,
really quite good. But," he continued, "when it got to test time, it changed. Students
would be really uptight and nervous. It was generally awful" (T.3.10).

Teachers' approached "term work" (marks based on attendance, spot quizzes,
and occasional hand-in homework assignments) differently than they did the final
and monthly exams. Some teachers would assign special take-home assignments;
others would require students to periodically hand-in their normal homework
assignments. However, some teachers saw this as redundant or ineffective as a
method of assessment. One described her term work as not giving "any more
information than ...from the chapter tests" (T.1.3). Teachers were also aware that
students could copy the work of others and hence, as another teacher put it, "get
credit for other people's work." "You never know how they're doing," she went on,
"by just putting a tick on their homework" (T.1.8). Teachers also questioned the role
of "term work" as a motivator for students. "It doesn't work," said one.

> With the home assignments...you're marking correct the people who you've
> marked correct anyway on the chapter tests....So it means that those
> [students] who understand the material get even better marks, and those who
> don't...well, they don't do the home assignments anyway. (T.2.1).

Some teachers developed what they called "mini-quizzes" which they gave in-
class on a regular basis. One teacher described the benefits: "It allows me to check
their understanding [because] it's based on the material in the textbook but it's not
copied out of it. I've built up quite a supply of these quizzes over the years and I just

recycle them every so often" (T.1.7). Another teacher felt that testing students

unexpectedly helped them learn:

> These little mini-quizzes of mine are randomly assigned; they never know
> when to expect one. And also...I think it breeds a little bit more
> cohesiveness...like the Dunkirk spirit. We're all suffering under this dreaded
> little mini-quiz that's coming along...so it's getting class involvement. Of
> course, students say they don't like it that much, but they _do_ say they like
> what it's doing to their study habits, so that's the point. (T.1.6)

Finally, some teachers merely used "classroom attendance" as a means of

allocating "term work" marks. Here, term work was largely used as a device for

teachers to increase the marks of borderline students. As one teacher described: "If

they [have a score of] 53, 54 but they have to have a 55 [to pass the course], I will do

something to juggle the term mark" (T.1.8).

Although some assessment was seen as necessary, given the teachers' (and

students') views towards it I expected the teachers to have developed more creative

and less threatening ways of gauging student understanding. However, as was

mentioned earlier, assessment in the math classroom was seen as "practice for real

life." Is it realistic to assume that these students, indeed, most students, use

mathematics primarily for regular pencil-and paper problem-solving and tests of

their memories?

Having described some common features, episodes, and activities of

mathematics classrooms in general, I now turn to one specific content area to

illuminate these features more clearly. All the basic mathematics courses conclude

their sections on arithmetic with an introductory section on algebra. I now focus

specifically on the lessons that cover this part of the course.

The algebra material in the introductory math courses fits into the overall course goals of covering part of the Provincially-approved Grade 10 Math Curriculum. It is intended to prepare students to take a word problem, translate it into an algebraic equation and solve it. The material follows Chapter 12 of the textbook and covers: an introduction to algebraic expressions, including the idea of equivalency of expressions; explanation of the distributive law of multiplication over addition and subtraction; factoring; collecting like terms; the additive and multiplicative principles; translating problems into algebraic expressions; and using the additive and multiplicative principles to solve problems. I now describe the classroom activities for each of the four days of instruction, and the day after the chapter test.

Day One

The algebra section of the 050/051 double-block course starts midway through a lesson. The teacher has spent the first 40 minutes of the lesson handing back marked copies of the previous chapter test and dealing with several questions on the test that had caused problems. Here, the teacher selected the problems to work on and proceeded to solve them on the board, rarely asking for any student input. The students were merely required to copy down the teacher's solution and his working.

"Any more questions about the test?" the teacher asks the class when he's finished the set of problems he's chosen. No-one answers. The teacher picks up his textbook: "OK, now we come to the last chapter for this book. That's 'Introduction to

algebra.'" The students put away their test papers, pull out their textbooks and turn
to the appropriate chapter.

In the first part of the lesson we go over the test. We'll take 20 to 30 minutes to explain some of the things they missed. Now, it is introduction to algebra. There's a big gap between arithmetic and algebra. But they are already started on it. Although Chapter 11 was about signed numbers they were algebraic numbers. But here we introduce equations, which will be very useful, and we have a lot of applications. I think the students will enjoy it. When they solve some of the problems by means of arithmetic they find it rather difficult. When they learn to use algebra, they find there is a better way.

This lesson fits in quite well into the overall course. This course is fundamental knowledge for 061/071, so they have some basic idea of what the algebra is about. In fact 050 and 051 lay a good foundation, give some good background before they go to 061/071, which is Math 11.

The teacher appears to read from the book:

The topic is 'Algebra: Solving equations and problems.' Now we can use algebra to solve equations...to solve a lot of the problems, by means of an equation. We have learned some equations. Remember in solving percent problems we learned some equations. An equation is a sentence...a mathematical sentence...which is an open sentence. The open sentence means you are going to fill in some numbers...to make it closed. Now this sentence ...we use variables, 'unknowns.' In algebra we use a lot of these unknowns...we call algebra expression. The algebra expression would be something like this: 'X + 2' or you may have 'A - 7' or '4B'.

As he says these expressions, he writes them on the board. The students copy down the expressions into their notebooks. "Now by the way, what does '4B' mean?" asks the teacher. Without waiting for a reply he continues, "Four times B. These letters, these are algebra expressions. They mean some numbers, but these..." Here, he underlines the 'X,' 'A,' and 'B' on the board. He continues,

These are variables. But sometimes, these letters can mean only one number. OK, for instance, if I have, say, 'B,' it stands for the day of your birthday...that's only one day. My birthday, it's the 12th...12th of certain month. Birthday. So sometimes a variable stands for just one number. Other times, a variable it stands for any other number...many, many different numbers. I can use 'X' to stand for the wages we earn. X can mean $20 per hour, your wage...it can be $18, or my wage can be $23 per hour. Your wage can be $30.

Until now, the students have either been looking bewildered or writing in their notebooks. At the mention of a wage of $30 per hour, a couple look up at each other and laugh. The teacher continues,

> So, this is a variable. When you have a variable, the value....Suppose I have an expression. We want to find "10H." 10H is an expression. H stands for the number of hours I work, and 10 is the rate. Each hour I get $10. So I can find out, "Suppose I work 5 hours, how much do I get?" Closed. Before you do a lot of the deductions, you evaluate, "10H equals 10 times 5...equals 50."

> Now, notice the symbol we have. When we come to the numbers 10 times 5, I use a dot, it means multiplication. When you have 10H...when I write 10H I did not write any symbol. Because there would be no confusion. Now if you come to 10 times 5 you have to put some symbol for multiplication. Otherwise, if you don't put some symbol in here it becomes 105. So, you put a dot...or use a bracket to mean multiplication. Ten times five equals 50. This 50 is called a "value." The value of this expression '10H.' You say the value of 10H is 50 when H equals 5.

> Now, suppose we say...another week I work 12 hours. The case will be different. What is the value of 10H now? H equals 12. Ten times 12 equals what?

One student shouts out "120." "Good," says the teacher, writing '120' on the board. "Now this 120 is called what?" "A value," says the first student. "A value," repeats the teacher. "Now this 10H is called an expression and its value is 120. And we are doing evaluation. Sometimes the question will ask you 'to evaluate.' Evaluate means to do something exactly as I am doing." He turns to his textbook, "Now here we have some of the expressions that the question wants us to evaluate. I'm going to let you do some. Turn to page 507 and do those three questions, the margin questions. First one, "To evaluate A + B for A = 38 and B = 26." Let's try those three questions. He sits down and starts marking his attendance list. The he turns to his textbook and starts copying down the answers to the problems he's assigned the students.

> *I was hoping that the people in that situation would have discovered that dealing with variables is the same as constants, but no...they're treating it as a different. So they'll need to do some more work....It's basically a matter of just working on it and working on it, like water dripping on stone...eventually you'll get something happening.*

For five minutes the students work in silence. Then the teacher gets up and walks around the class, leaning over the students' shoulders as they write and marking their work with a red pen. No-one seems to be having any difficulty. The class has been going for over an hour, so the teacher walks to the centre of the room and announces, "We can do the rest after the break." Four students get up and leave; the teacher follows. The remaining three students gather round one desk and start talking about the test papers handed back at the beginning of the lesson.

After the break, the teacher tells the students to continue solving the margin questions they had started before the break. He gives them some more questions to work on. After a few moments he starts moving around the class again from student to student checking their work and helping them if they have any difficulties. To one student he says, "This one you do in your head but you write down 16, right? The answer is right, but you must show your working."

They have to master the operation of the integers, because this time they are exposed to new kinds of numbers, different from the natural numbers. They have to have a concrete knowledge of the negative numbers. Chapter 11 gives them a thorough knowledge of that. Some of them may be mixed up with the addition and subtraction. When they subtract they're finding it more difficult.

I find that I teach more to the people that I know have difficulty. I have to, to teach them properly. I don't really change what I do in the classroom for people having difficulty; I encourage them and expect them to come outside of class. I tell them that I have a seminar, to come and see me in their spare time. But they may have other appointments. I suppose that in the class I do give them more time to practice. Then I can give individual help. This class is particularly small; so I can spend more time with each person.

Students seem appreciative of the teacher's efforts and often try to engage him in conversation about the problems they are working on, even if they have no difficulties:

Student: So this would be a minus X here, is that right?

Teacher: Minus X. That is correct. Minus X.

S: And this one? This next one?

T: This is 2 times minus 6.

S: Two times...?

T: Two times X. When X is...

S: Oh, OK.

T: It is positive times negative.

S: So that is a negative?

T: Negative...that is right.

S: So that is 2 times minus 6...which is negative 12.

T: Negative 12, that is correct. Good, do the next one now.

The teacher continues moving around the class, checking students' work. If the work is correct the teacher makes a large check mark on the student's paper. If it's incorrect, the teacher stops and explains where the errors occur. Students appeared to welcome these interruptions and would often ask the teacher to check their working on a problem, even though they thought they had obtained the correct answer. Students often seemed to be unsure if their method of solution was correct:

S: Look here at number 3. I tried working it backwards; I don't know if it's right.

T: OK. You've got 2N - 3 = 7. So you said, "It's the 3 you've got to get over first, because that's the one that's floating around." Good. OK. So, you've got 2N = 7 + 3. So that's 10. Good. So you've got 2N = 10. Then your last step is to multiply by the reciprocal...great. You want to pry that 2 off. Thus N = 10 over 2. You're absolutely right. Or you can just simply call it 5, that would be easier. Good...tick.

S: So that's how all these are done?

T: Yeah. Go for it.

S: Oh. I've always thought that the negative ones would be different...that they would have a negative answer.

T: No, they're just the same. Take them one at a time. You've got the general strategy down. Now get some practice.

If a student's work is incorrect, the teacher talks his way through a solution of the problem for students to follow. For example:

T:	These are right. This one here.... You have 5 times 4 point 8.

S:	Five times 4 point 8?

T:	Yes, you have a decimal point...it's not 4 times 8. It's 4.8.

S:	Oh, yeah. OK. A decimal.

T:	24.

S:	24, right.

T:	You get the idea how to do it?

S:	Yeah.

T:	This next one. [Evaluate 3(X +Y) and 3X + 3Y when X = 5 and Y = 7.] Should give you 36. If you add them first... 5 plus 7 gives you...what?

S:	Huh?

T:	Five plus 7 is what?

S:	12.

T:	Good. What is 12 times 3?

S:	I did add them first, then I stopped. It seemed kind of the wrong way.

T:	So what is 12 times 3?

S:	36.

T:	36 is right. Good, you get the idea now. You can do the rest on your own.

He turns to the rest of the class. "Some of you may have finished those. Keep on with the margin problems on page 509, 510. He continues going around the class, for a further 20 minutes, marking work and giving advice.

When I assign the exercises I ask them to do the odd numbers and then finish the odd numbers at home; then the even numbers we can practice in the class. So, in the classroom I check on them and if they are ready to do the work at home and I ask them to go home to do it. Some of the students have done the hardest questions, the ones right at the end of the exercise, even though I didn't assign them to do. Some like to do the hard questions; sometimes they ask for the harder ones, the optional questions. I don't expect them to do these kinds of question in this introductory algebra; I only want them to master the basic technique.

The teacher closes the class by assigning homework: "For tonight you can work on the exercises for section 12.1...just do the odd numbers. You do all the odd

numbers...and also you read sections 12.1 and 12.2. OK, I see you tomorrow."

There's an immediate buzz of activity. The students get up and stretch. Some start

talking to each other; most pack their bags as quickly as they can and leave. Within 1

minute, the classroom is empty.

I always assign homework. On the course outline, I tell them that for every class session they must do a certain number of questions for homework. So they know that every day we cover so much, and every night they must do so much. This becomes a habit; every day they must set aside an hour or so to do their homework. Homework should take them on average about one hour. Some may spend more time. That is general for all of the homework that I give. Tonight, they'll be expected to read 12.1 and do the exercises and read 12.2. Then tomorrow, I will give some examples and lecture about the material.

Having a small class has advantages and disadvantages. One advantage is that you can have more personal contact, I can give more individual help. A smaller class maybe affects [students'] attitudes. The small class, it seems so lonely and if more people don't come, they might feel it's a little bit boring. I feel when we have a small class...it's as if I'm a musician, I want to perform something. If the audience is small, then I feel nobody is listening; only a few people appreciate what I'm doing. Although having a small class doesn't make me change what I do, to try and get more response. Even though there is only one student, I'd still do the same. Whether we have 10 students or 20 students, we still have to cover certain material.

Day Two

In the second and third lessons, the teacher introduces further topics related

to algebra: equivalence of expressions, the distributive laws, and the addition and

multiplication principles. These topics are introduced in the same way, and in the

same order, as in the textbook. The teacher starts each class as usual, by dealing with

student problems from the homework. Students shout out the number of a problem

which they have found difficult, the teacher then reads out the problem and solves it

on the board.

Twenty minutes (and 12 problems) into the second lesson all the homework

problems have been dealt with. The teacher then opens his textbook and turns to the

class:

OK, now we introduce something new. What we have been doing is called "expressions," and we have been evaluating expressions. To evaluate means to put a value into an expression. Now when you do this, you use something which appears in mathematics very often...which is called "substitution." let me show you how we substitute. If you look at the questions you have done...for example, number 4. It says "Evaluate A over B, given that A = 200 and B = 8." Then to find the value of A/B, we substitute the numbers for A and B. So, A is changed to 200; B is changed to 8. Because this is the substitution. Two hundred divided by 8 would be what? Twenty-five. Now, let's do one more. [He turns to the book, and reads out another problem.]

Number 5. "Evaluate 10P over Q where P = 40 and Q = 25." We substitute those. P will be replaced by 40. So 10P...that means multiply...10 times 40... divided by Q which equals 25. So after you make the substitution there should be no letters, only numbers. The numbers are called "constants." Now we can turn around the operation. Ten times 40 equals 400, divide by 25, turns out to be 16. We refer to evaluation this way: we substitute what is given into the expression.

Now, when we do something like here, [textbook problem] number 6, that gives you two kinds of expressions and asks you to evaluate, and they turn out to be the same value. Look at number 6: there are two kinds. One is '1 times X', the other one is 'X'. No matter what the X is, it will always be the same. Can you see it? When X = 3, this is 1 times 3 and this is 3. One times 3 is always equal to 3. Not only 3, it can be negative, can be any number. So 1 times any number is just that number. If for any of these, these two expressions are the same we say they are "equivalent" expressions. If two expressions are equivalent we can put an equals sign in between them. So, in this case, 1 times X = X. This has a special name..."identity." In fact, it is multiplicative identity. We will learn two identities. One is for addition; another one for multiplication. The one for multiplication is 1. Because any number multiplied by one is just that number. The additive identity would be zero; any number add zero would be...that number.

He continues in this vein for a further 10 minutes. During this period the students sit and watch him, sometimes copying down what he's writing. They look listless and bored; rarely are they asked any questions. The teacher next assigns some problems from the book, and he walks around checking on their work.

His explanations of some concepts have proved inadequate. Several times, the teacher has to explain the purpose of learning the distributive law to students. For example, he says to one student:

The reason behind this exercise is to give you a feel that the distributive law is valid. In other words if you have 10 (X + Y) that's the same thing as 10X + 10Y. You can take the 10 and multiply it through the brackets. It's the distributive law...that's what we're emphasizing here. That's that handy law

that we talked about earlier. It's one you need...particularly to get variables out of brackets. It's really handy. You'll use it a lot. Later on there'll be several places that you'll find yourself using the distributive law.

Sometimes, students in difficulty will try to get the teacher to solve their problems. For example:

S: I'm stuck here with this one. Where you have to divide.

T: OK. The problem is the 2X here. That's 2 times X. Now, remember in the problem that you're dealing with...

S: So, we should get rid of it, right...the 2? Or the X.

T: The question as posed, says that the second one is 2 more than, not 2 times...

S: Oh.

T: So, it's $X + X + 2 = 58$. There's a little bit of a difference between this question and that one. The numbers...they're not identical but they're similar. OK?

S: But one piece is twice as long as the other.

T: Yeah, in that case, yeah. But, in the other, one is 2 more than...

S: It's 2 inches longer...

T: Yeah, so one involves addition and one multiplication.

S: That would be 2X then, and that one would be $X + 2$.

T: $X + X + 2$.

S: That would be...X...X...$X + 2$

T: OK, that's the long piece, $X + 2$. The short piece is X. We're going to get that one, X, and that one, $X + 2$, and we're going to add them, $X + X + 2$.

S: So you're going to have to share it out then, into 2?

T: No, because you're adding. One plus one is two. One X plus one X is two Xs.

S: Oh, so that's $1 + 1 + 2 = 4X$.

T: Where did you get the 4X from?

S: You add them all up.

T: No. Remember you have to collect like terms. So count up all the Xs.

S: One X and one X is 2X.

T: Right. And you've still got this 2. So that's 2X + 2. So now all you have to do is solve this equation. You've got the idea.

This was a fairly typical lesson. I gave a lecture, and I had the question answering period. Also the students did their own work and I gave them help. That's usually the style of my lessons.

Day Three

The third lesson continues with the addition and multiplication principles. Some of the problems involve working with fractions—an added complication for students. One student is having particular difficulty with this problem:

$$5/3 + 2X/3 = 25/12 + 5X/4 + 3/4$$

"OK," says the teacher,

Let's take this left hand X to the right hand side. Then we will get a positive number. Remember, I like positive numbers. You can think of it this way. Five quarters is more than one, and two-thirds is less than one. So take the smaller one over here, it will give you a positive number. So, 5X/4 - 2X/3 ...and these two fractions they move to the other side. So, 5/3 minus 25/12 minus 3/4. See?

The student keeps quiet. The teacher writes the modified equation (5/3 - 25/12 - 3/4 = 5X/4 - 2X/3) in the student's notebook saying, "You must remember not to leave anything behind. Like when you move house. You don't want to leave any desk behind, or any suitcase. So, count." He points to each fraction in the original equation. "One, two, three, four, five. You have five boxes. Now, count here." He points to the equation he's just written:

One, two, three, four, five. All accounted for. Now, I'm going to show you the way using the multiplication principle. It says that you can multiply the whole equation. The whole equation. The left hand side has three terms, every term; the right hand side has two terms, every term multiplied by the same number. So what number will that be? We want to clear the whole denominator. What can we use?

"Twelve?" suggests the student. "Very good," says the teacher. "How did you pick 12?" "It's the LCM," says the student. "Good, the LCM," says the teacher.

> The lowest common multiple. Here it is 12. So we multiply everything by 12, it will clear the denominator. So when you multiply you use the distributive law. So, we get '20 - 25 - 9 = 15X - 8X.' If we simplify we get '-14 = 7X.' So, divide by 7. We get '-2 equals X' so X = -2. You see? You need to clear the denominators, then it becomes easy. Keep on with the next one.

> *My main goal, the main plan, I stay with. I change it slightly, depending on how many problems the students had. I can change and switch my teaching from time to time. When [students] ask questions I know which part they don't understand, then I can adjust. Here, their problem is that when we introduce the theory of solving equations, we use the division principle and the multiplication principle. Sometimes they mix them up. When we get to some simplified steps, they just transpose a term to the other side and then when we get to using the multiplication principle, they also change the side. So that's why they have to know these principles. When we simplify those steps, we have those principles in mind. I am adding the terms to both sides or I am multiplying two numbers to both sides. I give them lots of practice to do this. That is normal. I tell them that without practice they won't get the knowledge.*

There are few other problems with the mathematics. Saddiq, a foreign-born student has some trouble with the language used in one of the questions:

S: This one here. I get confused. "Subtracted" means take away?

T: Yes, subtraction in English...means minus. The problem with subtraction is that you have to get them in the right order. When the word "from" is used: "If 5 is subtracted from 3 times a number"...it means that 5 is subtracted from 3 times it. From..."If 5 is subtracted from....It comes...if you said 2 is subtracted from 8 that would be like this: 8 - 2. When you see the words "subtracted from" it means that this number goes after this one.

S: Oh, yes. Thank you.

T: Unfortunately in English there's several synonyms for subtraction...and you've got to get the order right because subtraction is not commutative. You get them in the wrong order and your answer's wrong.

> *As usual, some students are very good and some are a little bit lacking, especially those who haven't done math for a long, long time. Some of them, they haven't touched mathematics for three or four years or even more, and some of the students, they may not have formal education. We have three students, they said they only learned mathematics in their community. I don't know what kind of community they have...they come from different countries. When they say they've learned it from the community, I don't know what they mean. I assume they mean they didn't go to a formal school.*

After the break, a student who I don't recognize comes into the class. The
teacher explains to me that, although this student is officially registered for the class,
he doesn't attend with any frequency, and, hence, is "having problems." As the other
students are working individually, the teacher has time to deal with this student's
difficulties. They start by discussing the distributive law:

S: What about when it gets more complicated, like here where there are 2 terms
       inside the brackets?

T: Well, there's actually 3 things inside the bracket...but it doesn't matter how
       many terms there are, you distribute it through.

S: So that would be like this? [He points to an example in the textbook?]

T: Yeah, that's it...you've got the idea.

S: Oh, OK. Well how about factoring?

T: Oh, that's another topic. Well, OK. That's like doing this in reverse. In
       factoring, you're trying to take things out. Let's look at an example, 6Y + 18.
       You just look at these 2 terms and say, "What can I pull out of both of these?"
       What goes evenly into this, and also evenly into that? What's going to be the
       biggest number that goes into both of them? Here it looks like 6 is the biggest
       number we can pull out of these. That would leave a Y behind, and a what?

S: What?

T: If we pull a 6 out of here, out of this 18. What do we have left?

S: 3?

T: That's right. Because 6 x 3 = 18. Now, with these questions, you're doing it in
       reverse. I'd go through these ones first and then tackle these others.

S: OK. What about 'collecting like terms'?

T: What allows you to collect like terms is, in fact, the distributive law. What you
       do...well the way you would do it ordinarily is just to add these numbers in
       front together. Nine apples plus 10 apples is 19 apples, that's basically all
       there is to it. What allows us to do that is the distributive law. If we factor out
       the apples, 10 + 9 = 19.

S: What about if there was a "Z" in the middle of it?

T: No, you can only collect those terms that are the same. So if there was
       anything else, you'd have to leave it in there.

S: Oh, OK. It's just like doing things in reverse.

T: Exactly.

S: OK, what else? I've got to make sure I've got all this.

T: Yeah, you're going through an awful lot here. The trick is to do lots of practice. The book can help you, and all the worksheets that you've got too. All these exercise sets would be good to go through.

S: So with these ones [he points to his book,] you just pull out the factors and then add them up?

T: Well, these ones are double-barreled. What you have to do here….In sections 2 and 3 you just had one step. These have 2 steps. What I suggest you do is take these numbers out first and get rid of whatever is free-floating. Then you can try and isolate the X by multiplying by the reciprocal or whatever. OK, I'll show you the steps. We've got $5X + 6 = 31$. So first step is we take away 6 from both sides. The reason I'm subtracting 6 is that I want to get rid of this positive 6 here. I'm showing you the steps but you can do some of these in your head. So we've got $6 - 6$. So $5X = 25$. So, that's always your first step: to get rid of any loose numbers that are floating around. Then your last step is to get anything sticking to the variable. Divide by 5. $X = 5$. These are all 2-step questions.

S: OK, what about fractions. If there's fractions in there?

T: Well, we spent a fair amount of time in the last few classes working on these. Basically, the trick with fractions is to clear them first. Look at this: $7X/2 + X/2 = 3X + 5X/2$. OK, there's lots of fractions here. But if you clear the fractions first you can make it look a lot simpler. It looks so awful right now with all of those fractions. Clearing fractions? The way you do it is multiply both sides by the lowest common denominator. So in this case what you do is multiply by 2, that's the lowest common denominator. You put a bracket around everything and you multiply it all by 2. What'll happen is that the fractions will disappear. So 2 times $7X/2$ gives 7…

S: Oh, it's a whole number.

T: That's right. So we get 7X and X, and over here we get 6X and 5X. See? Because everything's been multiplied by 2. The fractions have gone.

S: Wow.

T: Now what we do is gather up the like terms. Collect like terms. This expression has variables on both sides so it has everything in it. Now what we need to do is gather all of the variables on one side. I like putting it on the left but it doesn't matter which side. So we've got 11X on this side so we subtract it from both sides. Which gives us $8X - 11X$ which equals $-3X$.

S: So that's the answer?

T: Not quite there. We haven't isolated the X. We still have something sticking to it, but we want to get X all by itself. What do we do?

S: Er… take away 3? Minus 3?

T:	Well, from Section 3, what we were studying is…you divide by it. Or
multiply by its reciprocal. Whenever anything is sticking to the variable you
divide by it or multiply by its reciprocal. OK, -3 divided by -3 is 1, or you can
do it by multiplying by -1/3, which is the reciprocal. If we do that we get -3 x
-1/3 = -3/3 = -1 which is what we had in the first place. OK, I think maybe
you should try some of these yourself. There's lots…we've gone through tons
of work here.

S:	I'll just see if there's anything more. I guess…

T:	Well, we're working on this exercises right here now. Numbers 12 and 13.
That's what we're working on right now, so you might want to….That'll give
you a lot more practice.

*This student hasn't come to class for some time. It's very interesting thing, you
know: in geometry, he was one of the students who was always at the top of the class, when
he does algebra he is close to the bottom. He does things very fast and a little bit rough. Not
so careful. Other difficulties involve how to translate from words to symbols, that's one of the
difficulties I find he has. Yet in geometry he knew how to use those tools, compass and
straight edge, to do it really well. Some students [are] like that: some prefer algebra; some
better at geometry. It all depends. Most though, no difference. They don't find the algebra
any harder than the arithmetic, or don't find the geometry any harder.*

Day Four

The class starts as usual by dealing with the problems from the previous

lesson's homework. Today, this period is taking longer than usual. After 45 minutes,

the teacher is still working his way through all the problem questions.

T:	Any more?

S1:	Number 43.

T:	Number 43. "4Y - 4 + Y + 24 = 6Y + 20 - 4Y." Now, let's simplify this side first
[pointing to the left-hand-side of the equation]. Four Y plus Y is equal to
what?

S2:	5Y.

T:	Good. Now, negative 4 plus 24 equals 20. And this side, 6Y minus 2Y is 4Y,
plus 20. So, now we have 5Y + 20 equal to 2Y + 20. Can you see something we
can cancel?

S3:	Both sides have a '20.' Can we cancel them?

T: Cancel 20? Let us see. If we cancel 20, it means we take this 20 from this side
      to the other side and it becomes minus 20. So, 20 plus minus 20 equals zero.
      So, we can cancel: 5Y = 2Y. Now, we are looking for a number so that five
      times that number equals twice that number. Is there any number?

Silence. The students look blankly at the blackboard.

T: Which number multiplied by 5 would be the same as the same number
      multiplied by 2?

S2:   Ten.

T: Zero. Can you see? Zero is alright. If zero is multiplied by 5 it is the same as
      zero multiplied by 2. If we follow the way we are doing, we group the Y to
      one side. So take the 2Y to the other side. So, 5Y - 2Y = 0. Left hand side equals
      to 3Y equals zero. Now you can ask, 'Three multiplied by a number equals
      zero. What is that number?'

S1:   Zero?

T: That is right.

S4:   Well, I got that. I got the answer right, but I didn't understand it. Because of
      the 3Y on one side.

T: Well, 3 times zero equals zero. The solution is zero. You can do it the way we
      have done, or you can divide both sides by 3. let's divide by 3. Three Y
      divided by 3 is Y, zero divided by 3 is...what?

No-one answers.

T: Zero. So, Y = 0.

S4:   Would you carry on with that last step when you write the test?

T: No, not necessary. Here, if you can see 3Y = 0, then you can put Y = 0. Any
      more?

They continue for a further 10 minutes until there are no more homework

problems left. "By this time I think we have mastered the basic technique...how to

solve equations," says the teacher.

      Now we learn how we can use this technique...put this into application. The
      next section will be 'Solving problems.' When we solve problems we have to
      learn how to translate from words to equations. There are some key words to
      learn. When we want to add numbers together what words do we use for the
      answer?

"Sum," says one student reading from the book. "Sum, yes," says the teacher. "When we subtract we call the result 'difference.' For multiplication, 'product'; and for division, 'quotient.' These words are very important; we will use them a lot. There are other words we can use in the book. For addition we can say 'plus,' 'more than,' or 'increased by.' For subtraction there are...." The teacher continues to read out the lists of words from the textbook; the students following in their own books.

When he's finished reading the list of words, the teacher says, "Now, consider this. One half of my salary, my income...goes to my mortgage on my house. Suppose my income is $200, one half of it is $100. So, we divide by 2. One half means divide by 2. One quarter means divide by 4. Now, let us do some translations. Some examples. Look at the book." He reads out the first three examples from page 531 of the textbook ("Translate to an algebraic expression: 1. Twice (or two times) some number); 2. Seven less than some number; 3. Eighteen more than a number"), each time writing the answer on the board. "Now it is your turn," he tells the students. "You do the next ones." He reads out the textbook examples and waits for students to shout out the answers. Here, the same 2 or 3 students provide all the answers. The teacher appears not to be bothered by this; he never invites individual students to answer his questions, or asks those who are clearly confident to keep quiet.

The teacher next proceeds to work through two textbook examples (numbers 6 and 7 on page 533) showing how algebra can be used to solve the problems. He follows the textbook, step by step, translating the word problems into algebraic identities, then solving them. Throughout this period he rarely asks for any student input. "This is how we use algebra," he says. "We can solve problems like this using algebra." He has concentrated on the computational aspects of the problem but has never explained <u>why</u> one would pick this particular method.

The book is very helpful here; it is written for the students. So if you look at the textbook, it has example and then it has the margin questions. The margin question is almost identical to the example except the numbers are changed. So I look at the example and then see whether the student would be able to do it. So sometimes I use the examples in the book

and then I do the marginal question; sometimes I just show them the margin questions and they have the example to read and do the exercise.

After the break the teacher says "I'll let you do a few questions from the exercise set on page 537. Do numbers 20, 22, and 24. They're all even numbers; we'll save the odd numbers for homework." After a minute or two he starts making his rounds. Students appear to be having difficulties. The problems they have been given ask them merely to translate word problems into an equation (e.g., #20: "43% of some number"). Students want to continue working on the problems to solve them. The teacher doesn't like this:

T: You should leave this. This is not solving a problem, it's just translate.

S: I understand that, but if you were to go on to solve it wouldn't you actually divide it?

T: You have no equation to solve. There's no equation.

S: OK.

T: Only translate to an expression. That's it.

As he checks the students' work he tells them to continue with the odd numbered problems in the set. One student in particular is having difficulty. The teacher wants to show him how to solve the problems.

T: Number 21, I'll show you how to do it. "What number added to 60 is 112?" Don't put so many Xs in. The number you don't know you call X. So 60 + X = 112. Is means equals. That's the way to get the equation.

S: Oh, OK. I see...52.

T: That's correct...52 + 60 = 112. Good

The teacher prefers to have students write out their solutions in his approved manner. He checks the work of one student who has merely written down the (correct) answer to the problem: "A consultant charges $80 an hour. How many hours did the consultant work to make $53, 400?" "You get the right answer, but you should say how," the teacher explains. "I show you the way. We say $80 per hour,

but you don't know how many hours. Call it X. So 80X = 53,400. This is the right

way. Solve for X; X = 53,400/80. Which is 667.5 hours. This is the way we write it

out. So you can see how you got the answer."

After a further 15 minutes, the teacher says "You're all doing really well. You

are very good at translations." He's about to move on to something else but one

student shouts out, "Number 19." The teacher stops and reads the problem aloud:

"Number 19. "Translate the product of 97% and some number to an algebraic

expression."

T: OK. Let's look at this. To find the product...you must have 2 numbers. You must have 2 numbers to multiply. So here, we want to find the product of 97% and some number. Here, some number...we don't know what number yet...it's a variable. We can use any letter. I like to use X. So one of them is X, the other is 97%. Here, we do have 2 numbers. So to find the product we multiply. We multiply 97% and X. Suppose we said, "Find the product of 6 and 4." How would we write the product? We would say 6 times 4 or write it like this: "6 x 4", or "6 . 4". How about the product of 7 and X?

S1:   7X.

T: Yes, 7X. What about the product of 40% and X?

Silence

T: We could write it like this: '40% . X'. What is the product? Can anyone see it?

S2:   I would think 97X plus some number.

T: Plus. Plus should be a sum. There is no sum.

S2:   OK...Say if I had one dollar, and my product would be a dollar and ten. So it would be 97 times whatever X would be, plus Y, it would seem to me.

T: But the product here is between the two numbers. When you say product it means multiply.

S2:   Yes, I know. But the word 'and' means plus.

T: This 'and' in here, it doesn't mean adding.

S2:   Oh.

T: Let me give you another example. Suppose we say...the difference of your money and my money. You have $40 and I have $15. Listen to my question.

What is the difference between your money and my money. Notice I use the
word 'and." Now what is the difference?

S2: The difference is between 40 and 15.

T: Tell me what is it?

S2: 25.

T: 25, that is correct. You are not adding though.

S2: No, I was subtracting.

T: · You were subtracting. Yes. Even though the word "and" was there, you were subtracting. You were finding the difference. Here we have to find the product. To find the product we have to have two numbers. One number <u>and</u> another number. This "and" just tell these two numbers. Does it ring a bell?

S2: No. But that's OK.

T: When you come to questions like these. Where we are asked to find the product. Like "What is the product of 2 and 5?" Tell me what the answer should be?

S3: 10.

T: That is right. Two times 5. Why did you multiply?

S2: Because it asked for a product.

T: OK, it asked for a product...so we multiply. The "and" means we multiply those two numbers.

S2: OK. I think I get it now.

T: Now, I'd like you to do two more examples, and then you'll be able to do your homework. Look through examples 8 and 9.

The students continue working on their own for a further 5 or so minutes.

Then its the end of class. Students start putting on their coats. The teacher says, "OK,

finish the exercise set for homework and do some revision of this chapter. Tomorrow

is the test. We have had a good class...you will do well on the test."

This section on algebra, I think it's terrific. I think the students have learned something and this is their first time exposed to algebra. Some of them, they didn't know what algebra was, and I think they did pretty well. It is a good foundation for them to move to the next step. I'm very pleased with what they have done. By the way they worked their problems and also when I gave them individual help, I can tell they can master the basic techniques to solve equations. Of course they have to improve, but in this short time they have already moved to know some basic algebra.

The students really have the spirit to learn, they're really committed to the course. For all my classes, I want to develop this spirit. Through my own enthusiasm—I love the material, I love math—and also I expect them to want to learn something so that their time is well spent A student will feel satisfied when they learn something. If they learn something then they feel happy, and as their teacher, if they're happy, I'm happy. So, day to day activities will build up this spirit. Sometimes we talk about the application of mathematics, its use in the future, and sometimes they may see immediately. Other times they say, "Why we learn this?" I tell them, "This equation you may not use for your whole life." But I give them some illustration, and say, "Maybe you don't see the application right away, but looking at the future, you will see it.

After the Test

The test for the Algebra section of the courses contained 18 questions covering all of the material presented in Chapter 12 of the textbook (see Appendix 18). Most students did much better on the test than they expected: in total, 15 students completed the test, and 12 of these scored over 90%.

Their high test-scores influenced the students' attitudes towards the algebra section. "I can't believe how well I did," said one. "It's my best mark. I really feel chuffed" (L.3.5). Other students clearly associated "doing well on the test" with their understanding of, and enjoyment towards, the algebra course material. Said one student:

> I just didn't think I knew what's going on, but the questions on the test were surprisingly easy. And...I found the algebra enjoyable too....Finding out I could solve those problems is pretty darn enjoyable, I tell you! I don't know, it's just kind of neat the way everything always works out. Doing [algebra] is kind of like accounting. You know, you get all these figures and everything, and then at the bottom line they balance. It's the same with algebra, you've got all this amazing junk and then the thing'll actually work out. And I like that, the way it's like a puzzle, and you boil it down to one specific answer and it's not ... it's not like English where it's a translation, it's always exactly that number, you know, so I like that a lot. (L.2.2)

All but one of the students I interviewed said that the algebra section, although initially feared, was one of the more enjoyable aspects of the course. One student described her initial reaction towards algebra:

I was really nervous about it, just because algebra, the word all by itself, creates panic. Like what is it? I don't know. I didn't know what it was and it just seemed so foreign. I never understood...the middle steps...from like arithmetic...to where you start algebra. They seemed to be missing, and I panicked. (L.1.7)

However, later in the course, she described the algebra section as: "So easy...and I've done so well. I really think I can go on now to the next course" (L.3.8).

Most students attributed their success to the teaching. "I would never have been able to do it without him," explained one student. "He made it seem so easy" (L.3.9). "It's his manner," explained another,

> His technique is to make you feel relaxed, and when you're relaxed then you can take in a lot more. And he's extremely good at...if you've got a problem with a particular point, he'll pound that into your head...he'll take time out in the class to handle anybody's individual problems. (L.2.4)

A third student added that,

> He gives a lot of praise. I can tell that the teacher cares if I learn by the way he acts. Like...he always gives comments, good comments like..."Oh, you made a mistake but you can correct it." He makes you feel good. He says, "Oh you did well." He'll never say anything bad, you know? Never say anything to hurt you, like, "Oh, you didn't do anything right." He does say, "You made a mistake," but he'll always show you a way of doing it right. (L.2.2)

Apart from the teacher's attitude, the students also appreciated the way that he taught. Several students described what they liked about his approach:

> He's really good, he relaxes everybody and...he's got a neat technique, because what he does is...he doesn't scare you into doing your home assignments, he makes you feel ashamed if you don't, you know, because he's ... he's such a nice guy. If you say, "I'm not going to do it tonight," and then you say, "Ah but I'll let him down. He'll feel bad." That's the way...his technique, and it works extraordinarily good, at least in my case. (L.1.5)

> He's actually a fairly good teacher, I think he really puts it across, and...he's open to other possibilities of how to do it. If something has to be a certain way, then he doesn't make it really technical. He tries to word it in a way that you can relate it to something else and grab onto the idea and then he goes through the examples. He's not so heavy-duty...I can see what he's saying. (L.1.7)

> He gives us plenty of time in the class to do the exercises...to practice. He teaches a little bit, then comes round and checks us. I like that...if we are

making mistakes he can stop us and tell us where we go wrong. Maybe show us better way. (L.2.6)

He also gives us lots of homework, and the homework in these books is designed so that each part of the homework covers one lesson. If you do the homework you get the lesson. Because it pounds it into you, and 36 questions later, you've kind of got that little concept together. (L.2.5).

Paradoxically, because students had found the algebra section so easy, they were also more critical of the teaching than in other sections of the course. Most criticism revolved around the number of problems they were given to solve. As one student described, "It gets a little boring doing...25 problems, all the same" (L.2.3). "The teacher's going pretty slow already," said a second student. "I wish he could speed up a bit...maybe give some extra instructions to the other students that can't...like the guy [sat] next to me or...the other people that don't catch on as fast" (L.2.5). "I'd like a faster pace," agreed a third student.

> But that might just be because I'm doing better than I thought and not have any problems. The level of the course probably is for people who are having a great trouble with the stuff he's teaching, whereas I seem to know it. God knows how, but it's there. (L.3.7)

> I think too much time is spent in the class on the exercises. Maybe he can teach it a little bit more, then let us to do the homework at home. Maybe that's the time we can go a little bit faster. But I don't know, maybe another classmate, they think that's a good speed for them. (L.3.1)

The only student I interviewed who found the algebra difficult was one of the few who scored poorly (67%) on the test.

> It's a lot more difficult than the stuff we were doing. It just gets confusing remembering the rules of when you bring things back and forth from side to side, like on either side of the equals sign. And negatives and positives...and fractions of X and stuff like that, that gets a little confusing sometimes...and can be really frustrating when you don't get it. Also, if you have too many different variables, like too many different Xs and Ys and Zs and stuff like that, it can get frustrating. (L.3.11)

This student could also find little use for algebra:

> I haven't had to use any of it yet. I suppose it might be useful in sciences and chemistry and stuff like that... finding out unknowns and dividing things and

stuff, I guess. But, to me...in my life I can't see it being of any use whatsoever.
(L.3.11)

Conversely, those students who were more successful could find algebra more useful. As one student described:

> I'll tell you what I noticed though: I tried to work things out mathematically without using algebra, and it was a lot harder. I kind of did the calculation and then tried to make the algebra suit the answer. No good. Now, I do it the opposite way around. Instead of thinking of it mathematically, I'm thinking of it algebraically and then kind of trusting the outcome rather than trying to do it the other way around....I think that's probably why I'm starting to understand the problems a little bit better. (L.2.1)

The students' test papers illuminated those areas where students found difficulty. Most students lost marks for either omitting questions entirely or insufficient attention to detail (e.g., several students, although calculating correctly omitted the minus signs when copying their solutions). However, several test questions were commonly answered incorrectly:

12. Translate to an algebraic expression: five less than the product of two numbers.

17. Solve: $X + 2 = 2 + X$

18. The width of a rectangle is 7/8 of the length. The perimeter of the rectangle becomes 68 cm when the length and width are each increased by 2 cm. Find the length and width.

The first problem (#12) produced such incorrect answers as

$$X . X = Z - 5$$

$$2 X - 5 \text{ (twice)}$$

$$X - 5 \times 2$$

$$A . B = C - 5$$

$$Y \times 5 - X$$

$$X^2 - 5$$

The second problem (#17) was commonly answered as

$X = 0$ (4 times)

$0 = 0$

$X = 2$

"No solution" (twice)

The third problem (#18) caused the most difficulty. Only two students succeeded in obtaining the correct answer. Most students, although beginning to conceptualize the problem, and, in some cases, going so far as to draw a diagram of the rectangle, did not proceed to attempt a solution. It is noteworthy that this is the only problem of the 18 in the test that required students to do more than simply calculate. That so few students attempted to solve it must call into question the ability to understand the applications of the algebra rules and procedures that students were supposedly learning.

Discussion

This consideration of the examples of teaching situations and episodes show how, in their algebra lessons, students were subjected to a ritualized and compartmentalized approach to mathematics. Algebra was presented as either generalized arithmetic or as a set of procedures for problem-solving, where students were asked merely to master and perform rituals for manipulating symbols. Further, the algebra they studied was broken down into a hierarchical series of smaller rituals (such as "removing parentheses," "collecting like terms," or "simplifying"). Students were shown how to perform these rituals with no mention of, or concern for, either

the larger goals of the course or the demands of the "real world." Students were expected to perform these rituals in the "correct" way and in the "correct" order with little consideration for the meanings of either the symbols themselves or the operations to be made on them.

Algebra could, however, be taught in other ways, reflecting changes in either instructional approach or in content. Rather than be presented as generalized arithmetic or as a set of procedures, algebra could be presented as the study of relationships among quantities. Alternatively, rather than being treated as a subject in isolation, algebra could be located within the larger framework of mathematics as an area of knowledge, or within the broader context of the course. I now consider each of these, in turn.

Algebra as the study of relationships. Algebra is commonly presented as the study of particular formulas to solve particular problems. For example, the textbook asked students, "The state of Colorado is a rectangle whose perimeter is 1300 mi. The length is 110 mi. more than the width. Find the dimensions" (p. 535). To solve this problem, students were first introduced to the formula, "2l + 2w = P where l = the length, w = the width, and P = the perimeter" of a rectangle. They were then asked to substitute "w + 110" for l and 1300 for P and solve the resulting equation. Here, the use of the variables l, w, and P are static: they have one fixed value. Alternatively, algebra could be regarded as the study of variables that vary. For example, students could be asked to investigate such problems as, "What happens to 1/x as x gets larger?" Here, students are not asked to solve any problem for a value of x; there is no value of x given. In this approach, the variable x is presented, not as an unknown to be found, but as a parameter that changes.

Algebra as part of a wider program of study. Mathematics is often presented as if it has an inherent logical structure which can be ordered (and taught) in a simple-to-complex hierarchy of knowledge. This presumes that in order to

understand algebra, one must first be conversant with, and proficient in, basic arithmetic operations and the real number system. However, breaking the mathematics curriculum into a hierarchy of discrete pieces that can be taught independently assumes that everything worth considering can be placed into one of these "bits." This assumption is questionable: most practicing mathematicians would agree that mathematics is much more than collections of facts or rules to be memorized. It also includes, for example, such processes as problem solving, problem posing, conjecturing, justifying, and convincing (NCTM, 1989). A study of algebra does not necessarily have to start at a particualr point. All that is necessary is the recognition that variables can represent a wide variety of quantities, that variables can be classified and related according to their characteristics, and that these relationships can be communicated in a variety of ways (Howden, 1990).

Therefore, students could be asked to regard algebra within the context of the broader goals of both the mathematics courses and the other non-mathematics courses they were taking. By considering algebra as a subject within a larger context, students can see how it fits into some coherent whole. This approach also recognizes that students already have some knowledge (however rudimentary) of other branches of mathematics; knowledge that can be developed. Most research on learning indicates that people do not learn in an hierarchical, systematic fashion. Rather, they bring their existing understanding and interpretations to a learning situation and fit any new knowledge into already existing schemas. Indeed, "there is compelling evidence that students learn mathematics well only when they construct their own mathematical understanding" (National Research Council, 1989).

Such a constructivist approach could be applied to individual classroom activities. For example, groups of students could be presented with a problem (or design one for themselves), and be asked to discover how it could be solved. By thinking and talking about solving problems, students could gain a deeper

understanding of the problem, which, in turn, could suggest possible methods of solution.

Algebra in society. A third approach to teaching algebra could be based on encouraging students to consider what it means to learn and do algebra in society. Students could be asked to regard their mathematics classroom as a place of inquiry and exploration. Here, algebra would be taught, not as an isolated series of rituals, but as skills and knowledge that are connected to a rich variety of interesting social problems and situations. Students are often unaware that the seemingly abstract mathematics they are presented with in their textbooks has itself been socially, culturally, and historically determined, that it reflects any values whatsoever, or that it is often the result of former controversies about fundamentals or priorities.

Further, students could be encouraged to see mathematics as a social activity; one that requires as much social interaction and interpersonal communication as subjects such as English or history. By interacting with others in a mathematics classroom, students might start to do their own mathematics instead of being mere passive receptors of others' ideas about mathematics. Also, by interacting with others, students would learn ways of behaving around mathematics: what others think is important; what values teachers and other learners place on mathematical ability, on homework, on competition, and on cooperation; and what happens if they try, or don't try, to solve the problems they face. Finally, by interacting with others, students would learn more about how to deal with uncertainty. In particular, they learn how to make guesses or propose conjectures, how to exchange ideas, how to develop arguments to test and advance claims, and how to construct proofs. Most importantly, they would learn to explore those mathematical situations that are not obvious or clearly defined--precisely those they face in real life.

When considering the classroom interactions discussed earlier, notice how the teachers used largely technical terms or phrases without considering what meanings

the students may make of these terms. For example, students were asked repeatedly to translate such expressions as "3 more than twice some number" into an algebraic expression. Although they may, through practice, have become able to derive "2x + 3" almost automatically, they were never asked to consider such an expression in any meaningful context. Three more than twice what? By following this approach, teachers ensure that the skills developed by students remained isolated. With a contextualized approach, teachers could highlight typical misconceptions, discuss topics, and ask students to explain the links between the (more familiar) language of the word problems and the more abstract language of algebra. In addition, students could be asked to discuss problems orally, compose their own problems, or make conjectures or predictions.

Finally, teachers could focus more specifically on the areas in which students commonly have difficulty. The examples of classroom episodes given earlier yield several common problem areas: negative numbers, the application of the distributive law, working with fractions, confusing the multiplicative and additive principles, and translating word problems into symbolic (algebraic) form. I now take two of these examples, review how they are currently approached, and suggest specific alterations to the instructional strategies that could better meet students' needs.

Negative numbers. Students often have difficulty in determining any difference between the number "negative 3" and the operation "subtract 3" both of which can be (and are) written as "-3." Further confusion can ensue if these two uses of the "-" sign are combined, as in the question that asks students to solve "-(-3)." Students' difficulties often stem from their poor understanding of the concept of negative numbers.

The textbook introduces the concept of negative numbers in a discussion of

the real number system (p. 471ff). Negative integers are described as being

"opposite" to the natural numbers 1,2,3 and so on:

> For the number 1, there will be an *opposite* number -1.
> For the number 2, there will be an *opposite* number -2.
> For the number 3, there will be an *opposite* number -3,
> and so on. (p. 471)

This explanation is then extended to include the concept of rational numbers.

For example, "all numbers that can be named in the form a/b, where a and b are

integers and b is not 0" (p. 472). The section concludes with a discussion of "the

number line," irrational numbers, absolute values, and the real number system.

Here, notice that merely the inclusion of the concept of negative numbers with

several other concepts is itself bewildering. Students can be forgiven for confusing

the distinctions between what makes a number negative, opposite, rational, or real.

Further, the use of the minus sign "-" both as an indicator of a negative number and

as the operation of subtraction is particularly confusing when students are asked to

solve such problems as: "$-3 - X = 2X + 1$."

The concept of negative numbers also could be introduced by taking students'

previous knowledge into account by asking them to think of examples of the use of

negative numbers in their own lives. Students need not learn concepts as

formalizations, studying their properties before they can then use them in situations

that are meaningful. In this way, students can see that they are already familiar with

negative numbers and the process of subtraction. For example, most adult students

are familiar with the ideas of debts and losses, and could be asked to prepare a

budget chart showing their weekly income and expenses. Here the notions of

negative numbers and of subtraction are presented as contextualized, meaningful,

and relevant to students' lives.

<u>Word problems</u>. Further examples of students' difficulties in learning mathematics showed in their attempts to translate and solve word problems. The difficulties appeared to arise here because there are no set rules for students to follow; they must instead demonstrate some intuitive understanding of the problem and be able to reason appropriately.

The textbook introduces word problems as central to the study of algebra:

> Many kinds of problems require the use of equations in order to be solved effectively. The study of algebra involves the use of equations to solve problems....In algebra we translate problems to equations. The different parts of an equation are translations of word phrases to algebraic expressions. (p. 507/531)

It then presents a series of word problems based on supposedly "real-life" situations which the students are asked to translate to an algebraic equation. For example,

> The total amount spent on women's blouses in a recent year was $6.5 billion. This was $0.2 billion more than was spent on women's dresses. How much was spent on women's dresses?

> Abraham Lincoln's 1863 Gettysburg Address refers to 1776 as "Four *score* and seven years ago." Write an equation and find what a score is.

> A storekeeper goes to the bank to get $10 worth of change. The storekeeper requests twice as many quarters as half dollars, twice as many dimes as quarters, three times as many nickels as dimes, and no pennies or dollars. How many of each coin did the shopkeeper get? (pp. 537-538)

The textbook problems seemed artificial and repetitive. They did not build in difficulty, had little intrinsic interest, and never related to each other. Further within the classroom setting, students were expected to solve the problems individually and were never encouraged to hazard possible methods of solutions or even discuss the problems with others. In this approach, learning algebra was a matter of repeatedly solving simple problems so that one could mechanistically perform the appropriate operations (e.g., "collecting like terms"). Teachers made little attempt to provide any conceptual underpinning (e.g., examining why one would conduct

these operations in the first place) or to link this work with anything else the students were being asked to perform.

This approach runs counter to recent research on adult learning. Most adults learn more successfully when they can determine the use and relevance of what they are being asked to do. Further, given widespread use of calculators, knowing <u>when</u> to perform certain calculations has become a more important skill than knowing <u>how</u>.

Therefore, students could alternatively be asked to compose their own problems, either based on those in the textbook, or derived from their own experience. It is not only the particular examples that are important here, but also the "making" and "doing" of mathematics by students. Students asked to explore and determine the relationships between phenomena could become capable of developing a greater understanding of the underlying concepts and principles and demonstrate a greater reasoning ability than those asked to solve already determined problems.

Students could also be introduced to algebra by being given opportunities to derive rules and determine patterns. For example, students could be asked to determine the prices of various goods offered at a local department store's "15% off" sale. Alternatively, students could be asked to conduct physical investigations to, say, study the concept of a function. For example, classroom experiments to determine the relationship between the release height and the rebound height of a dropped rubber ball could provide a set of data pairs which could be graphed and then used to find an equation that fits the relation, as well as providing a memorable and engaging activity. In each of these situations, students, by so deriving it, could better understand the rationale for finding algebraic expressions for the relationships between variables.

Several key themes can be identified from this consideration of actual episodes and activities in mathematics classrooms. First, decisions about teaching are made, almost without exception, by teachers; the learners' influence is minimal. Teachers made choices about course planning, the pattern and pacing of classroom activities, homework, and assessment with little consideration for the needs and interests of their learners. The overall goal for most teachers was to "cover the assigned material" without losing too many students along the way.

Within the classroom, the teacher's role was crucial. Teachers subtly reinforced the idea that mathematics is a difficult subject, full of "tricks" and best tackled by "motivation," hard work, and repeated practice on one's own. Teachers adopted a largely teacher-centered approach: they assumed that their own attitudes were common or preferred, they rarely asked students any questions or attempted to foster a spirit of discovery, they "helped" the students to find right answers, they seldom checked student comprehension, they focused on errors, and they used complicated and often idiomatic language, that often confused students (particularly non-native English speakers) to explain their points. Although the courses took place in a classroom--a social setting--teachers tended to work mostly with individual students, and so limited the opportunities for student interaction and discussion.

Second, the range of choices that teachers made was limited. Most of the decisions about the structure and content of each course were already made before the course began. The overall curriculum followed provincial standards; the form and content of each course and individual lessons replicated the structure and content of the set textbook; even the teaching of those lessons was largely based upon the textbook's style of show--drill--test. The only apparent decisions that

teachers made concerned their own personal teaching style, if and when the class would have its break, and which assignments would be used for assessment. Here the teachers appeared to make decisions largely to suit themselves, again regardless of the needs of students, although they often described their decisions as being "in the students' best interests."

Third, the teacher and the textbook assumed the roles of ultimate authorities on mathematical knowledge. Mathematical knowledge was transmitted through either the textbook or in the teachers' explanations, never as a subject to be created or investigated. Indeed, students were given few opportunities to explore mathematics, and, when those opportunities occurred by chance, they were largely ignored by teachers. Consequently, being successful in mathematics came to mean being adept at calculation but having largely unexamined ideas about those calculations. Within each course, success was almost totally determined by regular assessment tests, with their form and content taken directly from the textbook. Teachers repeatedly stressed that such tests were a preparation for the future, regardless of the specific aims of the students.

Fourth, students were assigned and, subsequently assumed a passive role in their own education. Students entered the class, often cowed by their lack of mathematical ability as well as the academic environment, and were forced to take part in a series of activities that, although based on mathematics, often seemed meaningless and irrelevant to them. Students largely accepted this, believing that teachers knew the most appropriate ways to increase learning, and that the mathematics would "get more interesting later on." Despite differences in their background, experience, and ability, all students were required to do the same work, and because little time was given over for discussion, they got few opportunities to explore how the mathematics they are learning could relate to their everyday lives. Although they had little mathematics education with which to compare it, they came

to accept and even enjoy the repetitious pattern of the class activities. Students occasionally attempted to make their learning more social but these attempts were often ignored or undeveloped by teachers.

Teachers placed great stock in the motivation, responsibility, and autonomy of students. Students were handed the responsibility of "being in control of their own learning" although, in practice, this meant little more than coming regularly to class, doing all of the homework, and asking questions when they didn't understand something.

Piloting

Given the range of options open to the teachers, it was surprising to find that their choice of teaching behavior was so limited. Without exception, they assumed the roles of interpreters of, or guides to, existing knowledge, and steered the students through the course material and the accompanying exercises much as a pilot steers a boat into a port. In both situations the destination is already fixed; the pilot's task is to choose a route that minimizes difficulties and circumvents potential hazards.

In much of their classroom behavior, teachers consistently chose to do that which asked the least of students. In their choice of problems, teachers regularly selected the easier ones at the beginning of the problem sets, and ignored the more difficult ones. In their presentation of new material to students, teachers normally adopted an expository approach that permitted few opportunities for student questions. Finally, in their discussions with individual students, teachers phrased their questions in such a way that students could best answer them with simple, often one-word, answers. Hence, teachers piloted the students towards both the

answer and the approved method of solution, by simplifying the problems so that students could solve them by answering a chain of simple questions.

The effect of such piloting on students was twofold. First, students liked the approach: the problems they were asked to solve appeared more manageable, they were given help and guidance in solving those problems, and they felt successful when such problems were solved. Overall, students felt that they were learning mathematics successfully, and they appreciated the care that they thought teachers took in making the learning of mathematics as simple and enjoyable a task as possible. In this way, piloting can be seen to be a successful teaching strategy. Students completed the problems successfully, and, in turn, passed the required course examinations. Teachers were satisfied because their teaching approach seemed successful: sufficient numbers of students passed the courses, students appeared to enjoy their learning, and demonstrated sufficient "understanding" of mathematics, and the teacher's role as content expert was never threatened. Further, the teachers' notions of accountability (to either the college as employer, or to the students as clients) was never questioned or examined.

However, if one takes a closer look at this success, one can discern a second effect of piloting on students; one that raises some serious questions about education. What actually, were students being "trained" to do? What meta-learning took place? What understanding was developed? Students were, in practice, subjected to a teaching treatment that involved them as little as possible. They were trained to mechanically perform a series of routines, with little classroom time being devoted to developing any understanding about what they were doing or why they were doing it. Further, students were always asked to perform other people's mathematics, rather than do their own. The mathematics asked of students ignored their daily lives and often appeared irrelevant and overly repetitious. Finally, students were encouraged to view learning in general, and learning mathematics in

particular, as an exercise over which they had little control or influence. Course structures, lesson content, learning methods, assessment methods were all decided for students; learning was portrayed as a passive acceptance of other people's ideas. Perhaps, most seriously, students were prevented from reflecting on the value and application of what they were learning. If students were never asked to employ their knowledge to any purpose (other than that of taking mathematics examinations), how could they develop any meta-learning abilities?

How could there be such a discrepancy between teachers' intentions and the results of their actions? Let us now turn to a deeper analysis of teaching processes and consider the effects of frame factors.

In the previous two chapters, I described common teaching practices in mathematics education drawn both from my own observations and from the perspectives of the participants I interviewed. In this chapter, I discuss why these teaching processes take the shape that they do. I first summarize the predominant patterns of teaching and suggest possible explanations for them. Selecting my choice of the most coherent and persuasive explanation, I reintroduce concepts from my theoretical framework and use them as tools for analyzing teaching processes. Specifically, I identify how mathematics teaching becomes trapped between certain frames and discuss the resulting classroom effects. I conclude with a brief commentary on the educational success of the teaching processes identified in this analysis.

Revisiting Teaching Processes

The descriptions in the previous two chapters demonstrated how, in general, mathematics teachers focused on teaching the syllabus rather than the students. Indeed, the overwhelming concern of all the teachers was to "cover the required curriculum." Mathematics teaching was akin to inculcation: students were required to engage in repetitious activities to practice set facts and procedures until they could demonstrate their abilities on a test specially designed for such a purpose. Teachers dominated the classroom, making all the decisions that related in any way

to mathematics education. As part of this, teachers rigidly followed the set
textbooks, and allowed them to determine both the content and the process of the courses. Teachers claimed that they wished to develop motivation and responsibility for learning in their adult students, yet provided few practical opportunities for such development to occur. In reality, teachers used these notions of motivation and responsibility to sidestep some of their own obligations as teachers; this placed the onus for learning entirely on learners. Those students who did not succeed were "blamed" for a lack of sufficient motivation. Further, although the teachers and the textbook both sought to develop "mathematical understanding," in essence, this meant nothing more than the ability to reproduce textbook definitions and single rule procedures outside of any contextual application. In addition, the mathematical problems that students were asked to solve were often repetitious and largely irrelevant to their daily lives. Finally, teachers adopted a stance of "piloting" students through problem-solving situations via a series of simple questions designed to elicit a specific "correct" method of solution, and a single correct calculation.

These prevailing patterns could be discerned, to some degree, in every classroom episode and activity. Indeed, their presence was signal: a constant reminder of the inviolability of mathematics education. That these trends appeared so often, and so commonly, regardless of teacher or lesson topic, indicates that they were not mere vaguely similar clusters of isolated incidents. What seem, at first glance, to be simple acts of classroom planning and management appear on closer inspection to be products of other influences that reflect deeper issues about authority and legitimacy. Teaching is not a neutral activity. It is, rather, situated within a nexus of relationships permeated with values about individuality, knowledge, and society that themselves reflect larger cultural, economic, and political issues.

In my earlier review of research, I discussed both structural and agency approaches towards the study of teaching. Each of these approaches can provide explanations as to why the teaching processes in this study took the shape they did. One "structural" reading of the prevailing patterns of teaching would be that teachers are agents of social control. Educational efforts, therefore, would seek to embody not only content, but also values favorable to dominant social forces. In such a view, teachers would be seen as part of a process designed to inculcate appropriate knowledge, values, behaviors, and skills required by the prevailing socio-economic system. According to this explanation, the teaching processes that stress those traits considered necessary in a compliant workforce (e.g., obedience, respect for authority, conformity, uniformity, and productivity) are thus promoted and reinforced.

Alternatively, an "agency" approach would reject this overly deterministic view, and focus instead on the nature of teachers' thoughts and ideas as central. In this explanation, teachers' behavior is understood to shape teachers' ideas (about mathematics, teaching, learning, and their students); teaching practices are chosen on the basis of those ideas. Thus, teachers who regard mathematics as fixed and infallible and who think that learning it requires hard work, memorization, and repetition would plan and arrange their teaching to reflect those ideas.

Of course, there can be no one single comprehensive explanation. Each of those outlined above sounds plausible yet does not account for all of the episodes and examples identified in this study. For example, the teacher as an agent of social control explanation, although connecting the larger cultural and social structures to the minutiae of classroom interactions, explains neither how this connection functions nor the relative autonomy of teachers in their own classrooms. Further, the explanation that focuses on teachers' beliefs as central does not account for the apparent similarity in different teachers' behaviors, the durability of certain teaching

practices over time, or the influence of external processes (such as the demand of the
"receiving institutions" or the necessity of regular testing or assessment).

For me, the explanation that is most coherent and persuasive, that best fits the data, and that accords with structuration theory is that teaching can be best regarded as "situationally-constrained choice" (Cuban, 1993). In this explanation, teachers are regarded as having, within their classrooms, some autonomy to act, but their actions are also influenced by certain external factors. These factors act as frames, influencing, bounding, and constraining teaching processes. To clarify, in the context of this study, consider how little control each teacher had over the following issues (each of which affects classroom teaching): the number of students in each class, what extra help students receive, the length of each lesson and the number of lessons each week, the choice of textbooks, the testing and grading procedures, and the overall course content. Alternatively, teachers could decide the following: the arrangement of classroom furniture, the grouping of students for instruction, the patterns of classroom communication, the nature and frequency of teacher-student interactions, actual pedagogical processes, and the use of instructional tools. Decisions about these latter aspects offered opportunities for teachers to act autonomously and make their classrooms unique and distinct.

To analyze the data from this perspective, it is useful to return to the theoretical model proposed earlier for understanding teaching processes (see Figure 1) and to assess the influences of certain frame factors. Considering the common activities and episodes of teaching through the lens of this model can help identify the constraining role of certain factors by illuminating their effects.

There are both "conceptual" and "physical" frame factors, each of which "limit the teaching process and [are] determined outside the control of teacher and students" (Lundgren, 1981, p. 36). In this study, the frames include: the worldview of mathematics as reflected in the set curriculum, the syllabus, and the textbooks; and institutional factors such as the overall educational contexts, the size and structure of buildings and classrooms; physical equipment in those rooms; and organizational arrangements such as the admissions procedures, the size of the class, the number of lessons each week, assessment procedures, and the time available for teaching. Of course, these frames do not operate in isolation, nor are they purely causal. Rather, the frames mark out the limits that teaching can take; the actual teaching is conducted within those boundaries. Together, these frame factors influence a cascade of pedagogical decisions. In this research, some of these factors were direct influences on classroom management and behavior; others were more subtle and served to reinforce social norms. Finally, they influenced what was considered a successful educational result--i.e., what being "mathematically educated" meant within the college program and post-secondary education system.

The most influential frame limiting teaching processes in this study was the predominant worldview of mathematics. Since schooling began in North America, the mathematics curriculum has been based on the predominant worldview of mathematics as a fixed, formal, and hierarchical system of infallible concepts (Kamens & Benavot, 1992). Essentially, this worldview holds that, unlike other areas of knowledge, mathematics is not "verifiable by reference to experience" (Lakatos, 1976, p. 2). Hence, what can be called "informal" mathematics--such as its everyday uses and applications--do not count as mathematics per se. Additionally, such a

worldview does not acknowledge or take into account how mathematical knowledge changes and grows nor how it develops in each individual. In sum, a conventional mathematics curriculum, based on this worldview, does not present mathematics as an active tool for knowing and interpreting the everyday world but, instead, promotes mathematics as an abstract discipline removed from human contexts and the practicalities of everyday life. In this worldview, the value of mathematics is perceived to be inherent. Furthermore, because of mathematics' infallibility, people's perceptions of (and beliefs about) it are regarded as essentially irrelevant: it simply exists, and is a necessity to learn.

The effects of this worldview of mathematics are enhanced by the second set of frame factors—institutional frames. Institutional approaches to education value the attainment of prescribed objectives above all else, emphasize individual achievement based on merit, and value managerial concerns of technical control and efficiency over the social contexts of teaching and learning. Further, these kinds of educational institution often disregard the societal aspects of the production of certain kinds of knowledge. As is evident from the magnitude of the social costs of innumeracy mentioned in Chapter 1, ignoring these connections can prove problematic.

Different educational institutions reflect different contexts about educational goals, but in any event, they all "process" knowledge societally by enhancing and legitimating "particular types of cultural resources which are related to unequal economic or social forms" (Apple, 1979, p. 36). For example, mathematics is commonly regarded as "high-status" knowledge, the possession of which is necessary for the technological and economic furtherance of societies, as well as individual advancement in such societies. Of course, to be most efficient, societies require that not everyone possess such knowledge to the same degree. Hence educational institutions can be part of a system that produces only sufficient numbers of people with sufficient mathematical knowledge. In this way, they serve

to reproduce a society which establishes unequal roles for people and unequal distribution of knowledge.

In this study, the evidence of institutional frames began outside the college with the requirements of provincial standards and their emphasis on programs that permitted transferability among, and to, other institutions. Provincial standards prescribe generic topic outlines and goals for each of the college's four levels of mathematics coursework. Even without the influence of fixed notions about the subject area stemming from the worldview of mathematics, the pressures of such institutional frames would contribute dramatically to shape the teaching processes.

This explains, at least in part, the college's focus on selection, attainment and testing, credentialling, subject-centered education, an "academic" (as opposed to a practical) focus, and objective standards tied to the needs of "receiving" institutions (all of whom share the same worldview). Thus, the college emphasized regular testing as a measure of determining students' capabilities, and required students to pass the appropriate academic examinations as both an entry to, and an exit from, their chosen courses and programs of study. Whilst this did not have an immediate effect on day-to-day teaching processes, it did acclimatize the students to the notion that educational progress was meaningfully measured in superficial tests, and not by other means.

These two types of frame factors not only affected the curricula of mathematics, but also its pedagogical approaches. In my study, teachers appeared to assume that the predominant worldview of mathematics was the only view, and also unquestioningly accepted their teaching function within the educational system. In essence, this established their roles as primarily transmitters of a fixed body of knowledge. Teaching that fixed knowledge efficiently was paramount, and that, too, was understood in a traditional, conventional way as best achieved by a pattern of constant practice, drills, and repeated testing. Hence, even before classroom

instruction began, teachers' overall approach to mathematics education was already circumscribed by the contexts in which they taught.

The influence of these frame factors on teaching processes were fundamental. Teachers, as the sole originators of classroom activities, were trapped between two pressures: covering a set curriculum in a fixed time and doing so in a manner that displayed student progress (and thus ensured the courses' continuance). As a result, teachers adopted a restricted approach to both curriculum and pedagogy. For example, the setting of the curriculum was affected by the closed and close nature of the interpersonal relationships within the mathematics department which encouraged a collegial bonding. However, such close working relationships also encouraged a form of "group think," and prevented the making of decisions that could challenge the status quo or cause dissent, and precluded much self-examination or reassessment of the curriculum and teaching approaches.

In pedagogic terms, this restricted approach featured the teacher as pivotal to all other elements. This was evident even in the actual physical patterns of movement within the classroom. Students were, without exception, expected to sit singly in rows, facing the blackboard and the teacher's desk, for the duration of class. By contrast, the teacher assumed complete freedom of movement within the room. Although for the bulk of the classroom period teachers stood, walked, lectured, and wrote on the blackboard in the front quarter of the room, when they did interact with the students physically, it was always on an individual rather than a small group basis. Sometimes, teachers circulated among students at their desks; other times, individual students approached the teacher (who was seated at the teacher's desk, often working on projects unrelated to the current class). Such patterns of movement again subtly reinforced the centrality and authority of the teacher, discouraging (and even precluding) student interaction, and kept the pacing and rhythm of the class in the teacher's control.

In fact, there are five particularly notable classroom effects which reveal the overlapping influences of the frames: an overall reliance on the textbook; the privileging of the syllabus of mathematics over the students; the portrayal of mathematics as a set of isolated skills, separate from other areas of knowledge and everyday life; and foci on testing and motivation.

Reliance on Textbook

In order to reach the provincial- and college-set standards and goals, the department selected course material (a series of textbooks that covered all four levels of coursework). Once teachers had selected a uniform and efficient set of texts, they were loath to depart from the structure of the books in any way. Within their individual courses, although teachers may have not felt that the chosen book described a mathematical concept in an ideal way, or they did not agree with the textbook's structure and order of the topics, they did not feel the license to depart from their previous, collectively-agreed upon decision. Such reluctance to deviate from the text virtually forced teachers into set approaches, which, in the interest of efficiency, were teacher- and text- centered and discouraged student participation. Thus, the selection and ordering of course content involved social and ideological choices. Specifically, these choices appeared to be made on the basis of technological efficiency (i.e., what approach will cover the most content as effectively as possible), and cost to the student.

The texts selected, however, in turn limited the instructional approach. First, they presented mathematical knowledge as uni-directional, undeviating, and hierarchical. Therefore, the text book-driven approach, and the teachers' dependence on it, reinforced the idea that learning mathematics is a process of absorbing existing knowledge and acquiring disconnected skills, rather than one of inquiring

scientifically or intuitively about the world. This top-down approach reinforced the attitude that others (whether the textbook authors, teachers, or mathematics educators) know both what learners need to know and how they need to learn it. As we have seen, these three groups promote the notion that mathematics is best learned in a repetitive, drilled cycle of rule explication–practice–assessment.

A second, more subtle, narrowing of the instructional approach occurred from reliance on the textbooks in that mathematical knowledge was not presented as the product of any social activity. It appeared, instead, as an body of absolute, timeless, and universal truths far removed from the concerns and values of humanity. In fact, what we commonly regard as mathematics has long been a subject for debate and dissent. Mathematics, like all knowledge, is constructed by people through interaction with others in specific social, historical, and cultural contexts. However, such a view was entirely absent from the textbook.

Finally, throughout the textbooks, what was presented as "mathematically interesting" (e.g., the content of mathematical problems) also tended to reinforce and reify specific cultural and political values and perspectives and ignore others. For instance, the text made constant references to financial transactions, property-owning, and the dimensions of buildings in both the problem sets and in the explanatory notes. One textbook problem asked students to compute the size of $1,000,000 given the dimensions of a $1 bill. Others involved, for example, calculating how much money should be paid back on a loan at a specified rate of interest, determining how many consumer items could be purchased for a specified sum of money, or calculating the size of property. Even the introductory guide "To the Student" contained an exhortation to learners to consider the amount of time they allotted to study as important a criterion as the location of any property they might wish to buy. Linking the learning of mathematics with the buying of real

estate in this way suggested that both are possible, desirable, and appropriate goals for students.

Of course, teachers claimed that they used set textbooks differently depending on the level of course. In the lower level courses, teachers said they followed the textbook religiously; in the more advanced courses, they claimed to use them more as a guide, often supplementing them with other books and teacher-prepared material. Teachers explained this difference in use as relating to the different needs of students in different grade levels. Students in the more advanced courses, they explained, are expected, and therefore have "to be trained" to work more independently (for example, by deciding themselves how much work they need to do to fully understand a concept). In contrast, students in the basic courses "haven't learned how to do that yet" and "need to be shown what to do and how to do it." Therefore, students in the basic courses were encouraged to work independently in the sense of working alone, but not in the sense of having the responsibility of choosing their own level of work or ways of working. In this way they were prevented from exploring the doing of mathematics in favor of following a pattern of working that others had set.

Privileging the Syllabus over the Students

As a consequence of their reliance on textbooks, teachers were concerned to cover all of the material contained within the books, regardless of students' needs. This choice often led to severe time pressure to "cover [the] vast amount of material in such a limited time." All teachers, regardless of the courses they taught, described that they had too little time to cover the assigned material. Curiously, no teacher phrased this as having too much material to cover in the time available; the amount of course material was seen as permanently fixed, only the timetable was subject to

change. Despite this, teachers never made any attempt to limit the work that had to be covered (although much of it was repetitious and could easily have been reduced), or to spend the available time more productively.

In general, teachers dealt with time constraints by sticking religiously to the course timetable, and putting the onus on the students to raise any issues or problems. Once prepared, the course timetable was sacrosanct, and teachers were strongly averse to departing from it in any way. Course timetables were usually prepared before the courses began and remained unchanged regardless of the number of students in the class, or their levels of understanding. The teachers' principal concern was to cover of all of the assigned material; sticking to the timetable was the only way that they could ensure that this was accomplished. Again, the learning needs of the students were clearly subjugated to the orderly progression of the curriculum; if students were having difficulty it was their own responsibility to speak out. Further, if students fell behind for any reason, it was up to them to "catch up." (This in itself is rather remarkable when the time available for the basic mathematics courses in the College is compared with that in other institutions. Schoolchildren studying the same curriculum spend 2 - 4 hours more each week in their math classes.)

Further, the pressure (as they perceived it) on teachers to effectively "pilot students through" a set curriculum towards undeviating and universally desired goals produced another classroom norm. Within the logic circumscribed by the worldview of mathematics and institutional frames, fixed "expert" knowledge took precedence over students. Thus, student diversity was regarded as educationally irrelevant. Indeed, teachers mostly ignored the students' backgrounds, experiences, interests, and needs and tended to treat them all alike. As one teacher put it: "They're all the same to me, they're here to learn math." Further, teachers regarded all students (despite their stated preferences) as if they wished to continue with their

mathematics education so that they could enter other colleges. This perception

among teachers influenced their classroom teaching: teachers stressed the academic

value and uses of mathematics rather than its practical relevance, they rationalized

any curriculum alterations brought about by the needs of the "receiving institutions"

as being in the students' interests, and they reduced learning difficulties to

"problems with motivation." In addition, by treating all students as if they were

alike, teachers were better able to encourage (and thereby generate) certain

personality attributes to become identified with success: obedience, motivation,

perseverance, diligence, and an non-inquisitive approach. This is not mere opinion.

In actuality, students were given no opportunities to influence any classroom

decisions or any curricular arrangements; instead, they were expected to follow the

teachers' instructions and "work hard."

One example of how the syllabus was privileged over students was shown in

a teacher's classroom explanation of why she required students to complete so many

similar homework problems: "They're going to need to know how to do this stuff

when they go to _____. They might as well learn it properly here." To the students,

she explained that, "If you're having any difficulties with a type of problem, just take

a few more runs at it. If you find it easy, then the practice is always useful." After the

class she explained: "It's challenge and reward....They get the experience of

struggling and then being successful. If they can already do the work, then there's no

problem with them having a thorough background...they're happy about being so

successful." What is noteworthy is her method and level of assurance about its

efficacy, even when faced with students who are not interested in furthering their

mathematics education.

Such confidence perhaps stems from teachers' unexamined preconceptions

about students. As my interviews indicated repeatedly, it seemed that teachers'

previous experiences were uppermost in how they viewed their work. Most teachers

had taught in K-12 schools and often favorably compared teaching in the college.

"There are no attendance problems...you don't have to deal with parents, or plan lessons to the nth degree," was one teacher's summation. Further, most teachers had only ever taught mathematics, and so had little knowledge of teaching other subjects. In addition, few teachers had much adult education experience: they "picked it up as they went along." Perhaps this explains their commonly-held belief that "there's no difference between teaching math to adults and teaching math to children," except that, in adult education, teachers are "free of the responsibility for [student] learning."

Technical and Isolated Mathematics

In the texts, mathematics was presented as a fragmented and hierarchical series of isolated parts, with little relation to a greater whole. Topics (such as "arithmetic" or "algebra") were broken up into discrete chunks (e.g., "addition and subtraction" or "the multiplication principle") and taught separately. This arrangement duplicated the organization of mathematics as a high-school subject, where topics are similarly partitioned into "grade levels" and taught sequentially. Even in adult classes, trigonometry (a "Grade 11" subject), could only be approached after a display of sufficient mastery of "Grade 10" arithmetic, algebra, and geometry. (It is worth noting here that 6th Century Hindu mathematicians were able to develop trigonometry without ever using algebraic principles.) This method of segmenting and sequencing college mathematics led to the incorrect assumption that there truly is a strict partitioning and order to mathematics as a subject. Consequently, the acquisition of information and the ability to demonstrate mastery of certain skills became ends in themselves.

Within the mathematics courses themselves, teachers' approaches seemed to reinforce mathematics presented as taught separate from, and little influenced by, other areas of knowledge. Further, the highly-structured and sequential nature of the selected textbooks required students to learn mathematical concepts and skills in isolation even from other areas of mathematics, and influenced teachers to teach such material in highly specialized ways. Specialization, in turn, required a teacher-centered "revelatory" set of teaching procedures rather than more student centered, exploratory approaches.

Part of this revelatory, teacher-centered approach was a marked under-utilization of instructional aids. Mathematics teachers seemed to abhor the use of any equipment other than a blackboard that might help their adult learners, although it was freely available. (Indeed, the textbook is published as one part of a coordinated series of audio and video tapes and computer software.) In general, teachers preferred to limit their teaching to directly presenting and explaining concepts; for this purpose, teachers found the blackboard sufficient. It is worth noting that this exemplifies how the teachers decided what was appropriate in the classroom based only on their own needs. Although students may have been helped by the use of devices such as audio-tapes or computer programs, such devices were never deliberately used, nor was their use ever encouraged. During my observations I never saw teachers refer to the textbook's supportive material and the College's copies of these tapes and computer disks were kept locked away in the teachers' resource rooms.

Just as alternative methods and tools were underutilized, so too was the social nature of the classroom. Although the great majority of the teaching took place within a social setting, teachers rarely took advantage of the social possibilities of the classroom, and directed much of their behavior towards students in isolation from their peers. Further, teachers never encouraged any classroom interaction between

students themselves, and regarded any such collaboration as potential cheating. In addition, although learning mathematics is a process that requires interaction and dialog, students were never asked to perform any mathematics socially. The problems they were given required no collaboration or collective inquiry. Students were never asked to work with their peers, to discuss different methods of solution, or to see a problem from more than one perspective.

This approach to teaching mathematics presented it as separate from real life. In the latter, situations are often discussed, and problems solved, in collaboration with others. By contrast, classroom mathematics was rarely used as a tool for making sense of, or solving, the problems of the world outside of the classroom. Reinforcing this distinction between formal "classroom" mathematics and informal "real-life" mathematics prevented students from encountering and dealing with examples and practices of mathematics in their own ways, or in ways that were appropriate to their own lives. When students faced a mathematical problem in the classroom, they were encouraged to disregard their own experience, intuition, and existing problem-solving skills (which they would be expected to use if such a problem occurred in the real world), isolate themselves, and carefully follow memorized procedures. Even more significantly, teachers downplayed students' comments about life and recast them as comments about mathematics. For example, several times I heard teachers tell students, who were complaining that their work was unfamiliar and difficult, that, "You've been doing algebra for some time now. You just didn't know you were."

Finally, the contrast between the mathematics and non-mathematics classrooms reinforced the notion that mathematics is a body of knowledge separate and distinct from other areas of knowledge. Although the math classrooms were adjacent to those of other departments, mathematics education remained undisturbed by such proximity. Indeed, when other areas intruded (as, for example,

in the moving of classroom furniture), such intrusion was either ignored or the 274 previous order was quietly restored. This preoccupation, and the association of "normalcy" with order and precision is noteworthy. Such tidiness seems to reify the ideal of isolated, intact, and technically pure forms of knowledge.

Focus on Testing

The college is part of an educational system that measures success by the passing of examinations and, hence, requires its students to be tested regularly. Students' achievements in the basic mathematics courses were measured only by scores on standardized tests; students had five or six 2-hour chapter tests and one final 2-hour examination during their 15-week course. The mathematics teachers supported this policy and ensured a degree of standardized testing in their own courses. Although there were several sections of each course (each taught by a different teacher), only one teacher had the responsibility for setting the final examination for all the courses at that level. This policy served to impose an added conformity on teaching. "It stops people doing their own thing," as one teacher described it. It also thwarted the possibility of teachers relating mathematical material--even to the extent of a single question or section of the final exam--to the actual students present in a particular classroom.

In addition, by mirroring the style of presentation in the textbook, teachers ensured that their lessons involved, indeed were built around, significant testing of the students, using problem sets from the textbook. Homework for students between each class consisted of completing the textbook problem set for the particular topic area they had just studied. Further, at the end of each month (or at the completion of each textbook chapter), students had a "chapter test" to complete in class. Such tests asked students merely to reproduce the most recently presented pieces of

knowledge, and rarely to link it to anything they had learned earlier. Consequently, current learning was seldom integrated with previous learning. Even when consolidating questions were included in the coursework, their prominence was downplayed. Each exercise set in the textbook (which formed the basis of each lesson's homework) did contain some "synthesis questions" which were designed to require students "to put together objectives of the [current] section or preceding sections of the text" (p. 8). However, these questions were always placed at the end of the chapter tests (usually, they formed the final four out of 60 questions) and were described as "extra and optional."

The nature of all these tests, however, reinforced a narrow definition of success. Students were rarely presented with, or encouraged to use, several concepts or skills at once. Indeed, almost all of the courses' "problems" were single rule exercises which demanded little inquiry or investigation. It was not surprising, therefore, to find that students had most difficulty with the exceptions to the one-rule problems; e.g., those that demanded some understanding of relationships between mathematical procedures, or those where they were asked to apply the appropriate mathematical procedures to a variety of problem settings.

Focus on Motivation

One final classroom effect of the frame factors concerned the teachers' focus on motivation as a pedagogical approach. The cumulative effects of privileging syllabus, teacher, and texts over students resulted in the teachers' perception that they had minimal responsibility for providing meaningful learning experiences apart from encouraging students to experience success by repeatedly completing problem sets. Indeed, the inviolate and canonical primacy of mathematics as a content-pure subject allowed it to "float," as it were, above the mundane and

complicated tasks of classroom management. And, because their students were

adults (and hence "in charge of their own learning"), teachers regarded any roles

other than "pilots" as non-essential. Constrained as they were by the pressures of the

subject and the institutional requirements for regular assessment, the teachers' only

avenue for exercising their professionalism lay in developing students' motivation

towards their education.

Instead, teachers encouraged students to "take greater control of their own

learning" and adopt a "businesslike and efficient" approach to their own education.

Largely, this meant, as one teacher described it, that students should "decide that

they want to learn...want to be in this class...and are prepared to do what it takes to

be successful." In this way, teachers placed the onus of responsibility for learning

firmly on the students' shoulders. Any difficulties the students had, were, therefore,

the students' own responsibility. Indeed, when students would volunteer examples

of where they found a particular piece of mathematics difficult, teachers would often

steer the discussion away from an examination of mathematics and towards the

student's own inability to understand it. At these times, students were portrayed as

faulty; the syllabus or the teaching never so.

Teachers assumed that competent students should experience no difficulty in

understanding concepts presented in everyday and idiomatic English, even when it

became obvious that difficulties with the language were causing difficulties with

learning mathematics. Teachers argued that their purpose was solely to teach

mathematics, and that if students had language difficulties then they should remedy

those difficulties elsewhere.

Of course, no teacher concerned with effective delivery of content wants to be

"held up by difficulties." Nevertheless, in this study, teachers appeared not to

consider how student learning could be enhanced by any group discussion of

difficulties. At no time did I witness any classroom attempts to discuss common

learning difficulties or any discussion of study skills. Further, teachers never asked
students to justify their answers, or clarify or explain their ideas or problem-solving
processes. Because teachers never examined <u>how</u> students were thinking, they lost
opportunities or contexts for students to develop their reasoning skills. Instead, the
emphasis on the need for individualized effort, single-rule techniques, drills and
repetitive testing seemed to guide the teachers' actions. Indeed, the predominant
theme in how teachers exhorted their students to succeed involved an unvarying
emphasis on individual motivation.

Curiously, students themselves appeared to accept this. By the time students
had entered the classroom, they had already emerged from a series of assessment
and application processes, and invested a substantial (to them) amount of money.
Furthermore, they were, in most cases, first time adult students, approaching the
mathematics classroom with little self-confidence. For most of these students,
algebra was a novel experience which they approached with much apprehension
and some conservatism. Most had left or "virtually dropped out" of school before the
curriculum reached algebra, and they were encountering it for the first time and did
not know, therefore, what to expect.

For that very reason, many students said that they preferred the mathematics
courses to the others that they were taking, largely because they thought that math
was straightforward. For many students, mathematics was "bounded": a fixed body
of facts and procedures. The attitude that math was "the rules" was widespread
among students. "Once you get the rule, you're away," said one student, "You don't
have to think about what it means." Students also appreciated that math problems
had a definite answer: "You're either right or you're wrong," was a common
attitude. "Even if it takes me all night," explained another student of his math
problems, "I know there's an end to it." This comforting "solidity" of mathematics

was particularly attractive to non-English speakers. For them, mathematics was an even more popular option because it was less language-based than other subjects.

These attitudes created a particularly receptive climate for the teacher's fact-and-procedure approach. Students welcomed the discipline that the math courses provided. They felt that they "needed to be pushed" and were critical of those teachers who, they felt, didn't so extend them. Students also largely identified their lack of math ability as a product of their earlier schooling, although they tended to blame themselves for their "laziness" rather than criticize their former teachers or school system. In this way, students also began to internalize the necessity of motivation, and the validity of tussling with calculations and abstractions as a way to learn. Students would repeatedly struggle in isolation with difficult or unfamiliar concepts, convinced that if they went over the problems one more time, they would understand what they were being asked to do.

This emphasis on motivation subtly evidenced the effects of a number of frames. For several teachers, encouraging students to work independently, and not to rely on others (including the teacher), was essential. Teachers regularly stressed that the skills of independent work were particularly desirable when learning mathematics. One teacher described doing mathematics: "So often, you're stuck with a problem...you've tried different ways and got nowhere...and then you've got to reach down, deep inside and pull up the answer from within yourself." Although this advice is sound it cannot help in a situation where the students are struggling in isolation with meaningless procedures and never having the opportunity for validating their meaning with others. Especially revealing in this comment is how the teacher configured her role as one who must keep an individual student "going," even though students were pursuing an approach to learning mathematics or re-encountering materials that were largely identical to the ones which they had already "failed" (in high school).

Again, this speaks to the teachers' almost evangelical belief in the power of motivation. This is admirable, given the considerable research that indicates how motivation is key to adult learning. Indeed, as a means to increase motivation, successful teaching in adult education often includes a variety of learning tasks and attempts to demonstrate the usefulness and relevance of any new material to learners' existing goals, interests, and understandings. However, as has been shown, in these classrooms such tasks and demonstrations were absent, and students were expected to learn in one single, set, and approved way.

And yet, enhanced motivation was still the expected result. In sum, the conflation of the frames affecting the teaching processes resulted in a certain circular logic. The prevailing worldview of mathematics and the constraints of institutional frames shaped attitudes and preconceptions about mathematics, teaching, and students. Although teachers did have autonomy to act within these constraints, assumptions about each of these areas reinforced for teachers the notion that mathematics is best taught to adults in highly teacher-centered ways, within an abstract, computational, and academic context. To counter the overwhelming odds against an adult actually succeeding in learning meaningful mathematical applications in such a context, teachers rightly identified the necessity for high student motivation. Their observations were astute; their attempts to create such motivation, however, were meagre.

In the end, however, the teaching processes, influenced by these frames produced a "product": that is, some measurement of what it meant to be "mathematically educated." How successful was the product? By conventional accounts, the college mathematics programs appeared successful. Sufficient numbers of students enrolled, and sufficient numbers of those students passed the final examinations. However, these notions of success were limited: students were never asked to demonstrate mastery of any but the simplest and most mechanical procedures, nor to apply any logical reasoning skills, or consider how they could improve their learning. The dominant notion of success (e.g., passing the test) was, in fact, at odds with much current thinking about mathematics education (see for example, the recommendations of the National Council of Teachers of Mathematics (NCTM, 1989). Why was it like this?

First, and fundamentally, a narrow notion of success is supported by a widely held preconception about mathematics testing: that it reveals a particularly useful form of intelligence. Many people believe "in the primacy of mathematical thinking and of the assumption that ability or training in mathematics will transfer to other areas" (Smith, 1994, p. 61). Often, this belief is translated into standardized tests (e.g., the Scholastic Aptitude Test), which use mathematical ability as a primary instrument to judge admissions to college programs. Those who design and use this test believe that skill in mathematical reasoning transfers to other areas that require logical reasoning. However, according to Smith (1994), "no empirical evidence exists to support such a claim...especially the proposition that the learning of mathematics will facilitate...the learning of the logic of physics or...of economics" (p. 62).

Nevertheless, in this study, provincial standards upheld this preconception about mathematics testing, and prescribed generic topic outlines for the college's four levels of coursework. The goals for the mathematics courses that I studied (all at the Provincial "intermediate" level), were "to enable adult students to acquire mathematical knowledge, skills, and strategies to enter higher level courses or to satisfy personal or career goals." Note here the presence of dual goals: courses were intended to satisfy students' academic or personal goals. Achieving those goals was measurable, within the provincial and college systems, only through written assessments at both the onset and conclusion of each course. Naturally, because of the teachers' policy of standardizing all their end-of-course tests, such assessments precluded measuring anything other than a replication of the textbook's facts and procedures.

Thus, it is questionable whether it was possible for students to become thoroughly mathematically educated within this configuration of frame factors. The implications of this go beyond the immediate concern of the college program, however. Because of the way academic mathematics was recognized as a canon of established rules and knowledge, and was taught in a way that emphasized a fragmented, rather than a linked and coherent approach, a dichotomy was set up between how math was understood, taught, and used in classroom situations versus its function in adult students' actual lives.

Obviously, the cumulative effects of all of these frames act on both teachers and learners, limiting the ways each think and behave. For example, teachers' ideas about mathematics education and the practices that flow from those ideas are considered normal and natural. Because of this assumption that the way things are is the "commonsense" view, it is little open to question or analysis by learners or teachers. In this way, everyday thinking is conducted within narrow, fixed, and unquestioned limits. Teachers do not subscribe to any particular ideology, and yet,

they all support an approach that centers around compliance, and regards conflict as
negative. Further, they all regard most classroom problems (and solutions) as located within, and deriving from, individuals. Hence, the commonsense view of individual responsibility and compliance is promoted rather than the cultivation of more collective and critical views.

Thus, mathematics education, as commonly practiced in this institution, acts to reproduce the status quo and creates and recreates "forms of consciousness that enable social control to be maintained without the necessity of dominant groups having to resort to overt mechanisms of domination" (Apple, 1979, p. 3). Of course, neither mathematics education itself, the specific subject content, the institutions in which it is conducted, nor the teachers of it operate in such an overly deterministic or purposeful way. This supposes a conscious manipulation by those in power. Rather, the constituent parts of mathematics education (its settings, actors, and forms of knowledge) combine in a hegemonic way; that is, together, they portray the commonly adopted approaches as the only, or the commonsense, approach. As one student described it, when asked to imagine how his mathematics education could be different: "Well, it can't be [different] can it? I mean, this is what it is...this is what math is like."

This study explored the teaching processes in mathematics education for adults and how teachers make pedagogical choices within circumstances shaped by certain social and institutional forces. In this concluding chapter I provide a summary of the research, identify some limitations of my study, discuss some of its implications, and suggest some recommendations both for further research and for improving the teaching of mathematics to adults.

Summary of Study

Purpose

This study was designed to answer three broad questions: (1) What happens in adult mathematics classrooms? (2) What do these phenomena mean for those involved as teachers or learners? and (3) In what ways do certain factors beyond the teachers' control affect teaching processes?

I sought observational data and descriptive accounts of teaching processes and related them to the ways in which such processes are framed by certain societal forces. Teaching processes were regarded as including the selection of content taught; the choice of techniques such as lectures or groupwork; the expectations, procedures and norms of the classroom; and the complex web of interactions between teachers and learners, and between learners themselves.

Theoretical Framework

An attempt to more fully understand education as a social phenomenon was made possible by examining teaching processes in their cultural contexts. The theoretical framework of this study linked a macro and a micro approach to the study of teaching, and offered an analytical perspective for considering how teachers' thoughts and actions may be influenced and circumscribed by factors beyond their control. Further, it provided a framework for an analysis of the ways in which teaching processes were viewed, described, chosen, developed, and constrained by certain "frame" factors.

Methods

The methodological approach was chosen in order to portray the teaching processes in mathematics classrooms in dynamic rather than static terms, and to provide an in-depth description of classroom situations, episodes, and behaviors, and the meanings that these had for the people involved. The approach allowed me to get as close to my subject of interest as I possibly could, partly through direct observation of events in natural settings, and partly by access, through interviews, to the specific meanings that the observed events had for those involved.

The study was based in a typical setting for adult mathematics education: a community college providing a range of ABE-level mathematics courses for adults. Three introductory-level courses (each taught by different teachers) were selected and data was collected from teachers and students in these courses, as well as material that related to the teaching and learning of mathematics within the college. The study used a variety of data collection methods in addition to document

collection: surveys of teachers' and adult learners' attitudes, repeated semi-
structured interviews with teachers and learners, and extensive ethnographic
observations in several mathematics classes. Several lessons were video-recorded
and later used as the basis for "stimulated recall" interviews with the teachers
concerned. All interviews were tape-recorded and transcribed for subsequent data
analysis. The complete data set was then coded and initial concepts and categories
from the theoretical framework were linked into broader themes and patterns to
develop increasingly complex concepts and assertions. Finally, the data set was
again systematically searched for both disconfirming and confirming data to support
all claims and assertions.

Summary of Results

The study showed that the mathematics teaching in this study can thus be
understood as situationally-constrained choice. Within their classrooms, teachers
have some autonomy to act yet their actions are influenced by certain external
factors, and these influences act as frames, bounding and constraining teaching
processes. The pressure of such frame factors as the worldview of mathematics and
the institutional and administrative concerns with credentialling and testing led to
the teachers adopting a conservative approach towards both mathematics and
towards education.

This restricted approach can be seen in the everyday episodes and activities in
mathematics classrooms. Here, three key patterns could be identified. First, the
instructional approaches used by teachers were narrow and limited. Only one

method of learning mathematics was promoted: learn a rule, then apply it repeatedly until its use becomes almost automatic. This pedagogical approach was followed rigorously by teachers who, without exception, structured their lessons into a cyclic pattern of presentation, practice, and assessment: an approach parallel to that of the textbook. Largely, this was because the teacher and the set textbooks assumed the position of the ultimate authorities of mathematical knowledge in regard to both content (what is to be taught) and process (how it should be taught). Mathematical knowledge was transmitted only through either the textbook's or the teachers' explanations, and was never presented as a subject to be created or investigated. Indeed, students were given few opportunities to explore mathematical concepts for themselves; when those opportunities occurred by chance, they were largely ignored by teachers. Consequently, students assumed that being successful in mathematics meant being adept at calculations regardless of knowing the reasons for making those calculations in the first place. Within each course, achievement was almost totally determined by regular assessment tests, with their form and content taken directly from the textbook. Teachers repeatedly stressed that such tests were essential preparation (either academically or vocationally) for the future, regardless of the specific goals that students may have had.

Second, within the classroom, the teacher's role and approaches were pivotal. Almost all decisions about classroom activities were made by teachers; learners' influences were minimal. Further, although it was not their stated intention to do so, teachers appeared to make their choices with little consideration for the needs and interests of their learners. The overall goal for most teachers was to cover the assigned material without losing too many students along the way. Although teachers appeared to make their decisions largely to suit themselves, they often described their decisions as being "in the students' best interests."

Teachers apparently felt confident enough to assume that their own attitudes about teaching and mathematics were common or preferred. In practice, this meant that teachers rarely asked students for their own views or interpretations or fostered a spirit of discovery, they "helped" the students to find right answers, they seldom checked student comprehension, they focused on students' errors, and they used complicated and idiomatic language which often confused students (particularly non-native English speakers).

Partly because they did not use student-centered approaches, teachers subtly reinforced the idea that mathematics is an intrinsically uninteresting subject, full of "tricks," and best tackled by memorization of rules, dogged motivation, and repeated practice. As such, teachers positioned themselves into a "piloting" role. That is, they "steered" the students through complex, decontextualized material towards both the answer and the approved method of solution by asking chains of simple questions. Although the courses took place in classrooms--a social setting--teachers tended to work mostly with individual students, fostered competition among students, and limited opportunities for student interaction and discussion.

Third, teachers tended to regard their students homogeneously and treat them as if they were essentially alike. Despite differences in their backgrounds, experiences, expectations, abilities and interests--all rich resources for enhancing learning--the students were all required to perform the same work in the same sequence and at the same pace. If teachers considered students' backgrounds at all, it was only to focus on mathematical ability rather than the personal experiences, needs, and interests of students. Students' aspirations were also largely discounted, except where they involved some form of continuing in education. Indeed, teachers so consistently assumed that the students in their courses were proposing to further their mathematical studies, that they would often explain the purpose of a current mathematical activity in terms of its future usefulness (e.g., by suggesting that,

regardless of its evident irrelevance in the present, students would "need this stuff
later on").

There were several consequences of these predominant patterns. The teachers' overall approach to mathematics education was seen as appropriate, valid, and successful. Freed from any substantial challenge, the teachers assumed that their teaching "worked." Sufficient numbers of students successfully completed the course to satisfy the college, its mathematics department, and the students. This notion of success must be tempered, however, by a consideration of its nature. In its simplest terms, success meant achieving high enough marks (55%) on the monthly chapter tests and the final examination; tasks students were prepared for by the regular sequence of drills and testing throughout the term. For those students who regularly attended classes and did all of the required homework, it was hard not to be successful. However, a broader notion of success (and one in keeping with many teachers' stated goals) would also include a greater enjoyment of, appreciation for, and understanding of, mathematics. By this definition of success, the teaching appeared to "work" less well. Although most students who attended regularly also completed the course successfully, their understanding of mathematics cannot be said to have greatly improved. So, students were successful in passing the test, but less successful in understanding what they were doing, or applying mathematical techniques to a variety of problems.

What is noteworthy is that these broader notions of success, although mentioned by teachers, were not promoted or deliberately encouraged. As is clear from the key patterns, there was little exploration of either the broader implications, or uses, of mathematics; teachers concentrated instead on only teaching rules and procedures. Enhanced enjoyment, understanding, and appreciation was tangential to the "real thing." In this, teachers concentrated on developing students' motivation and their potential for "taking control of their own learning."

An additional consequence is that adult learners were given a "double message" about responsibility for their own education. Initially, students were presented with an inflexible set of rules, content, expectations, methods, and classroom norms. Students could, in no way, influence any of these. In spite of this, teachers then informed them that personal motivation and taking sole responsibility for learning were the keys to successfully completing the course. Even more ironically, over time, with the socialization of the class, students came to regard this passive role as normal, and completely internalized these notions. Most importantly, they also internalized the simple notion of success as "passing the test," and regarded their test scores as evidence of increased understanding.

Teaching is embedded in, and constrained by, multiple contexts. These contexts, permeated with cultural, economic, and political values, can be seen as frame factors limiting and constraining the teaching within them. In this study, the teaching processes I observed were influenced by several frame factors, including the worldview of mathematics, the administrative and physical structures of the institutions, and the codifying of mathematics into curricula and textbooks. These frames shaped and limited not only the actions and behaviors of teachers and students but also their thoughts and approaches towards mathematics education. Overall, the cumulative effects of all frame factors reproduced the status quo and ensured that the form and provision of mathematics education remained essentially unchanged.

The first limitation concerns the design of this study, which, although not explicitly, nevertheless followed the case study approach in social science. One concern with case study analysis is in its ability to provide much basis for generalization. This is particularly important for a study, such as this, which attempts to link the minutiae of classroom behaviors and interactions with larger social forces. The appropriate response here is to emphasize that case studies are not designed to be "samples of one" and, therefore, generalizable to other cases. Rather, case study results can be seen as more generalizable to theoretical propositions (Yin, 1989).

A second limitation concerned the data collection procedures. Although I conducted an extensive number of observations and interviews, it was not possible, except in a few instances, to clarify the interpretations of individual teachers and learners about specific instances of classroom interactions, to the extent that all ambiguity was removed. In this way, it could be argued that the data collection procedures privileged "breadth" over "depth," resulting in a lack of rigor. In response, I can point to repeated observations and interviews that produced similar findings, so that although "exact" interpretations could not be determined, the "partial" interpretations I did collect were replicated in others' responses. Hence, the results can be seen as more compelling because they came from a variety of informants.

A third limitation also involved data collection. Although repeated extensive interviews were held with the mathematics teachers, similar interviews with students were not possible. Partly this arose through the limited time available to the researcher who collected all the data himself. In addition, students in the

mathematics courses had many constraints on their time, and consequently, were not always available for interviews. The number of interviews with students had to be reduced (from 15 to 12) when the interviewees failed (several times) to show up.

Specific data collection procedures also produced a fourth limitation. When observing some of the classes, I used a video camera to record the detailed classroom interactions of teachers and students. As only one camera was being used, the whole classroom could not be captured on video, and an element of selection was inevitable. At first I chose to follow the teachers' movements and record their interactions with students. Later, I abandoned this approach and recorded segments of students working alone. Hence, video data was gathered wherever the camera was pointed and not in the other parts of the classroom. In this way, the research can be seen to be biased by the researcher's own subjectivity, thus possibly under-recording the full range of teaching processes.

A final limitation of the study concerns the overall impact of teaching on the adult learners of mathematics. The study was conducted in three one-term courses, and hence, concluded with the end of term. The long term effects of the course on the students (such as their subsequent uses of, and attitudes towards, the mathematics they studied) were not able to be determined. This is particularly important when one considers the "narrow" notion of success that was promoted by teachers (and accepted by learners). A longer study would have been able to examine whether these notions of success were still maintained by students subsequent to the course.

In this section I briefly discuss the implications of this study and suggest recommendations for further research and for improving the teaching of mathematics to adults.

Implications for Further Research

There are several implications for further research that arise from this study. First, greater attention needs to be given to the implications of the macro/micro dualism in social research. In this study I attempted to combine both macro and micro approaches to the study of teaching, and yet, have given only a superficial consideration to the links between these levels. I thoroughly examined the literature on teaching for studies that had adopted a similar approach but, although I found several rich theoretical discussions, there were correspondingly few empirical studies. It is clear that more empirical research is urgently needed on the links between micro analyses of classroom interactions and the macro analyses of educational policy-making. In particular, instead of explaining teaching processes only from the perspective of autonomous individuals, such studies should be conducted from a perspective that regards individuals as formed by their context and the way in which they view the world around them. For example, a learner should be seen not as someone with some inherent characteristics, but instead as one who plays a role in interpreting, and acting in, a particular context.

Second, within the field of education (particularly so within adult education), empirical research on teaching is woefully insufficient. Although there have been several reforms in (high-school) education during the past century, most of the

research on these reforms have focused on the changes in curriculum rather than
changes in pedagogy. As a result, "the 'how' of teaching has been neglected from the 'what'" (Cuban, 1993, p. 284). Further, the published research on teaching that does exist is largely atheoretical and designed only to determine what can improve learning. Specifically, it does not address the links between classroom interactions and larger social forces. Consequently, those with policy responsibilities for teaching have a set of

> mistaken assumptions about how teaching is shaped and how instructional changes can be encouraged. They have tended to invest tremendous amounts of money and energy in developing new texts and materials, and they seem to believe that if these are given to teachers, real changes in teaching will occur. (Darling-Hammond, 1990, p. 144)

Hence, further research could explore the links between classroom teaching and wider socio-economic concerns. In addition, researchers could explore the "climate" of classrooms investigating, for example, such issues as the patterns of communication and other relationships between teachers and students or students' perspectives on teaching.

Third, research on the concept of frame factors can be further extended. The initial work of both Dahllöf and Lundgren originally expressed only the "materialistic" nature of frame factors. However, Lundgren has more recently included the notion of "conceptional" constraints in his concept of frames (Elgström & Riis, 1992). Thus, such factors as personal competencies, attitudes, values, and beliefs of different kinds can also be regarded as potential frames. Following on the work of Lundquist (1987), Elgström and Riis call these conceptual constraints "ideational structures" and examine their role in the selection of curricula. These extensions of frame factors can be used in further examinations of classroom interactions. Further, if this wider definition of frame factors is adopted, then research can examine how, if at all, such factors can be altered by, or "negotiated" between, the people involved.

Fourth, although much adult education practice concerns ABE and hence involves the teaching of particular subjects (such as English, Mathematics, or Social Sciences), there is hardly any research on the teaching (or learning) of particular subjects. The assumption is that teaching and learning in adult education is similar regardless of the subject being taught. However, as this study has demonstrated, what is being taught can profoundly influence how it is taught. Hence further study needs to be conducted on the interrelations between teaching processes and subject matter.

Finally, published research on mathematics education for adults is minimal. Apart from those studies cited in Chapter Two, no others (to my knowledge) exist. Yet, there are myriad topics for investigation. For example, the huge amount of research about mathematics education for children could be examined for topics that would interest adult education researchers. Further, research that takes into account the unique aspects of adult education could examine: instructional areas in numeracy that are particularly difficult for adults, different teaching methods for developing numeracy skills in adults, factors affecting the transfer and generalization of numerical skills from the classroom to everyday practices, how adults' everyday experiences and knowledge can be used to facilitate learning, the impact of mathematics teaching on adult learners' beliefs and attitudes, and the extent of learning mathematical concepts (e.g., the extent to which adult learners are able to apply what they have learned in a variety of contexts and the endurance of this knowledge over time).

Implications for Mathematics Teaching

This study of teaching processes was based on what actually happened rather than what I would have liked to see happening. Consequently, when discussing

implications for the improvement of teaching, it is important to avoid the existential fallacy of assuming that knowledge of what is can inform what should be. However, based on my study, I can make several suggestions that, if considered or adopted, may improve the teaching of mathematics to adults.

First, the curriculum and the environment in which much adult mathematics education takes place is not conducive to effective learning, and needs to be altered. Although designed for K–12 education, the Standards for Teaching Mathematics (NCTM, 1989) offer some suggestions as to how this could be achieved. Briefly, they suggest that mathematical tasks should engage students' interests and intellect, and should provide opportunities for students to deepen their understanding of the mathematics being studied and its applications. They also recommend that teachers seek, and help students seek, connections to previous and developing knowledge by providing a range of individual, small-group, and whole-class work. Finally, they suggest that teachers orchestrate classroom discourse in a way that promotes investigation and growth of mathematical ideas, and use, and encourages students to use technological tools in their mathematical investigations.

A second area of improving mathematics education involves taking its social context into account. Several research studies indicate that the learning of mathematics is enhanced if teachers relate mathematics to both the everyday social and physical environment of learners and to the wider society of which they are a part. For example, by relating mathematics more to the real worlds of adult students, teachers could seek, acknowledge, and address students' attitudes towards mathematics as a means of encouraging meta-learning and appreciation of the value of mathematics. The teachers in my study appeared to perceive no value in exploring the diversity of the students as a pedagogical technique. This reluctance to acknowledge the differing experiences of adult students seems a peculiar approach, and one inconsistent with most research on adult learning. Instead, the teachers

could acknowledge the "adultness" of their students. For example, teachers could use
the backgrounds, experiences, and situations of their students for examples of
"mathematics-in-use," or ask students to derive mathematical problems from their
own experiences.

Teachers could also adopt a more socially-aware approach to mathematics.
Instead of treating it as an ahistorical and asocial body of absolute, timeless, and
universal truths, teachers could show how mathematics, like all knowledge, is
constructed and validated by people individually and through interaction with
others in specific social, historical, and cultural contexts. Instead of selecting
problems that appeared to be either far removed from the concerns and values of
humanity, mathematics could be used as a way of making sense of, or solving, the
problems of the world outside of the classroom. Alternatively, teachers could
explore the examples given by the NCTM (1989) or the possibilities and suggestions
offered by the radical educators and ethnomathematicians discussed earlier. Here,
teachers could choose mathematical problems that focused on, for example, job loss,
changes in wages and benefits, or the percentages of federal budgets spent on
welfare payments and military spending, instead of, as they do, those that reinforced
the dominant cultural and political values and perspectives (such as calculating how
much money should be paid back on a loan at a specified rate of interest,
determining how many consumer items could be purchased for a specified sum of
money, or calculating the size of property).

Third, teachers could better utilize the knowledge that already exists about
mathematics learning. For example, there is a wide consensus among researchers
and leading mathematics educators that powerful mathematics learning cannot be
achieved through traditional rote methods. As two noted mathematicians say:

> Most people do not develop conceptual understanding when they are
> presented with low-level, discrete skills and algorithmic procedures to
> imitate, procedures in which they are then given "drill and practice" and on
> which they are subsequently tested. Furthermore, most children learn to

dislike mathematics when it is taught this way. Such a model simply does not work. (Davis & Maher, 1993, p. 3)

Ideas about learning mathematics that consider the experiences of, and relevance for, learners are hardly new. For example, earlier this century, Thorndike (1923) studied the learning of arithmetic and algebra. He reasoned that solving problems in school ought to be for the sake of solving of problems in real life, and he argued that the only mathematics worth teaching was that which was practical and useful. He examined a variety of sources, such as encyclopedias and almanacs, to see how much algebra was actually incorporated into real-life settings. His conclusions were mostly negative: only a fraction of algebra that was then taught was ever applied in daily living. Specifically, he felt that only a few algebraic topics were worth considering: the idea of symbolism, the ability to read formulas, the ability to evaluate and solve formulas, and the ability to read graphs. Thorndike criticized the techniques behind much mathematics teaching:

> The faith in indiscriminate reasoning and drill was one aspect of the faith in general mental discipline, the value of mathematical thought for thought's sake and computation for computation's sake being itself so great that what you thought about and what you computed with were relatively unimportant. (1923, p. 96)

Thorndike also drew attention to the "bogus" and "fantastic" problems that were used in algebra textbooks which he found were usually generated from, and organized by, the algebraic techniques involved rather for than their use in everyday life. Is it very much different today?

Instead, teachers could change their current focus from what Skemp (1976) calls "instrumental understanding"–being able to follow rules–towards developing "relational understanding"–knowing both what to do and why. Such relational understanding often involves using a multiplicity of rules and mathematical concepts. Skemp argues that student learning is increased if common one- or two-step problems in mathematics are supplemented by mathematical activities that

require students to do more than merely remember and perform a few sequential arithmetical operations.

Finally, teachers could examine interactions in mathematics classrooms not only in educational and pedagogical terms, but also as social experiences. Students are usually the least powerful actors within these social settings, and their experiences can illuminate the effects of social forces. Within the classroom, students not only learn mathematics but also classroom norms about how to behave, how to learn, how to react to the demands of teaching and assessment, how to please teachers, as well as what they need to do to pass the course.

As my study shows, the conflation of frame factors can render the social experience of the mathematics classroom strangely contradictory. On the one hand, students' actual life experiences are treated as irrelevant, while on the other, the orientation of the teacher, the text, and the teaching methods all reinforce values which are often not appropriate for, or shared by, the students. Students are neither encouraged to reflect on wider contexts and applications of mathematics, nor on their own learning processes. Learning mathematics in this way becomes the acceptance of, and obedience to, the authority of others, rather than a process of discovery, awakening, or understanding. Students are encouraged to see mathematics, neither as an integral part of daily life, nor as a part of a complex web of knowledge, activities, and values, but as a subject separate from other areas of human knowledge, bounded by the textbook, and incapable of being questioned or challenged. It would seem that this research affirms how, consciously and unconsciously, a variety of frames rigidify to ensure just such a result.

As this study has shown, the inviolability of mathematics as a subject, and the ways in which it is encountered, presented, and wrestled into memory using a fragmented, single-rule approach are potentially quite far-reaching. In many ways, mathematics acts as a social filter within society, limiting access to higher education

and better-paying jobs. Possession of mathematical knowledge is seen as governing learners' future occupational and economic roles. Consequently, the hegemonic role of mathematics education can be a crucial subject for instilling the values that society regards as necessary in its workforce: individualism, passivity, obedience to authority, and competition. In fact, for most students, the regard for these four values, more than any mathematical content, may be what is most profoundly retained when they leave the classroom at the end of term.

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Appendix 1

Teaching Processes in Adult Mathematics Education

Subject Consent Form

This research is part of a thesis for a doctoral degree at the University of British Columbia. The investigators are (a) Tom Nesbit, a doctoral student in adult education (822-2946), and (b) Professor Kjell Rubenson, the research supervisor (822-4406). The purpose of the research is to examine mathematics education for adults and how such education is perceived by those involved as either teachers or learners. The research will benefit those involved in adult mathematics education by adding to what is known about adult mathematics education and improving the quality of mathematics teaching to adults.

As a potential participant you need to be aware of the following six (6) points:

1. The research is being conducted in the adult mathematics department at VCC King Edward Campus during the Spring 1994 term. The research will involve those students and teachers in the adult mathematics courses 050, 051, and 050/051.

2. The research will involve three data collection procedures involving participants: survey, interview, and observation. Each is described below. The research does not involve any new or non-traditional procedure whose efficacy has not been proved in controlled studies.

(a) <u>Survey</u>. All students enrolled in the courses 050, 051, and 050/051 during the Spring 1994 term will be given a short survey to complete in class. The survey will ask for demographic data, their attitudes towards mathematics, and an indication of their willingness to take part in the interview stage of the research. It will take 15 minutes to complete.

(b) <u>Interview</u>. All the instructors and a sample of the students will be interviewed at least twice during the term. Each interview will last 30-45 minutes and will be tape-recorded for later analysis.

(c) <u>Observation</u>. The researcher will observe several classes in the three courses. At least one class in each course will be videotaped for later analysis. Each class lasts 2 hours.

A total of 80 minutes will be required of those participants who consent to be interviewed; a total of 20 minutes of those who are not interviewed.

3. You will be not be offered any monetary compensation.

4. You have the <u>right to refuse to participate or withdraw</u> from this research at any time without jeopardizing your involvement in these or subsequent courses.

5. <u>Giving your name is entirely voluntary</u>. Everybody's identity will be kept strictly confidential. All data that refers to individuals by name will use pseudonyms and will be destroyed on completion of the project.

6. If you agree to participate in this project, you should sign this consent form
in the space below indicating that you have given your <u>consent to participate</u> and
that you have received a copy of this form.

If there are any questions concerning any of the above, please do not hesitate
to contact either the research supervisor or myself at the numbers listed above.

Thank you for your cooperation,

Tom Nesbit

I consent to participate in this study and acknowledge that I have received a
copy of this form.

Signature_____

Name   _____

Appendix 2

Attitude Questionnaire

Directions: Draw a circle around the letter(s) that show how closely you agree with each statement. SD (Strongly Disagree), D (Disagree), N (Neither Agree nor Disagree), A (Agree), SA (Strongly Agree).

---

1. Mathematics helps develop a person's mind and teaches them to think.   SD   D   N   A   SA

2. I am interested and willing to use mathematics outside college and at my work.   SD   D   N   A   SA

3. Mathematics has contributed greatly to science and other fields of knowledge.   SD   D   N   A   SA

4. Mathematics is needed in designing almost everything.   SD   D   N   A   SA

5. Mathematics makes me feel uneasy and confused.   SD   D   N   A   SA

6. I enjoy going beyond the assigned work and trying to solve new problems in mathematics.   SD   D   N   A   SA

7. I am interested and willing to acquire further knowledge of mathematics.   SD   D   N   A   SA

8. Mathematics is less important to people than art or literature.   SD   D   N   A   SA

9. Mathematics is dull and boring because it leaves no room for personal opinion.   SD   D   N   A   SA

10. I have never liked mathematics and it is my most dreaded subject.   SD   D   N   A   SA

11. Mathematics is not important for the advance of civilization and society.   SD   D   N   A   SA

12. Mathematics is enjoyable and stimulating to me.   SD   D   N   A   SA

13. Mathematics is needed to keep the world running.   SD   D   N   A   SA

14. Mathematics is a very worthwhile and necessary subject.   SD   D   N   A   SA

15. I have always enjoyed studying mathematics.   SD   D   N   A   SA

16. I would like to develop my mathematical skills and study this subject more.   SD   D   N   A   SA

17. Mathematics makes me feel uncomfortable and nervous.   SD   D   N   A   SA

18. An understanding of mathematics is needed by artists and writers as well as scientists.   SD   D   N   A   SA

19. Mathematics is not important in everyday life.   SD   D   N   A   SA

20. There is nothing creative about mathematics; it's just memorizing formulas and things.   SD   D   N   A   SA

---

21. Your occupation?..   22.

Age?....................

23. Ethnic Origin?..   24. Gender?
....

25. (Voluntary) I agree to participate further with this research by being interviewed later in the term. My name
is..

Teacher Interview Questions (1)

1. How did you come to be teaching mathematics to adults at Acton College (AC)?

2. How does teaching at AC compare with other places in which you've taught?

3. What is your educational background?

4. What training have you had in teaching (a) mathematics, (b) adults?

5. How do you think teaching math to adults is different from teaching it to children?

6. What are you trying to achieve in your teaching?

7. When you start planning your teaching where do you start?

8. What problems do you foresee?

9. How do you select/order your course/lesson content?

10. How do you choose which instructional strategies to use?

11. How do you check for understanding?

12. How do you relate what you're teaching to the students' interests/ experiences/goals?

13. What do you like to know about the students in your class?

14. Why do you think students enroll on your courses? What do they want to do after they're finished?

15. How do you encourage student involvement?

16. What can students in your class decide about?

17. What use do you make of set texts?

18. How do you evaluate your classes/ learners?

19. How is your class evaluated?

20. What do you find difficult in your teaching?

21. What would you change if you could?

22. Is there anything else that affects your teaching?

Teacher Interviews (2) & (3)

The second (and third) interviews with the teachers took place immediately before and immediately after the observed lessons (topic area) and focused on:

(a) <u>Before the lesson (topic area)</u>: Specific examples of how teachers chose lesson content and instructional strategies for the particular section of the syllabus. For example, such questions as:

> - What will the lesson (topic area) deal with?
> - How does the lesson (topic area) fit into the overall course?
> - What learner knowledge do you consider a pre-requisite?
> - What learner problems do you anticipate?
> - How will you deal with those problems?
> - What will you watch for?
> - What will get you to change your lesson?
> - Will you assign any homework? What?
> - How long do you expect the homework to take?
> - What instructional strategies will you use?

(b) <u>After the lesson (topic area)</u>

> - What did you think about the lesson (topic area)?
> - Did you follow the plan you'd made?
> - Did you make any changes at all? Why?
> - Did any of the students have problems? What?
> - What did you do with students' problems?
> - In what ways is this a typical/good/bad group?
> - Is there anything else that you'd like to tell me about the lesson (topic area)?

Teacher Interviews (4)

The fourth interviews took place with the class teachers of 050, 051, and 050/051 at the end of the term.

Typical questions were:

1. Overall, how do you think the course went?

2. Did the course go as you planned?

3. Did you make any changes to what you had planned? What?

4. Did the students find the course easy/hard?

5. Is this a typical group?

6. Did any students have difficulties? Which? What difficulties?

7. What did you do about those difficulties?

8. Did every student who started the course complete it?

9. What happened to those who dropped out? Did you do anything about them?

10. Were you satisfied with the textbook you used? Would you make any changes next time?

11. Did anything surprise you about the course?

12. Apart from what we've covered, is there anything else that you'd like to say about the course?

Student Interviews (1) & (2)

The first (and second) interviews with students dealt with the specific content of the observed lesson (topic area). Questions included:

1. Do you like this class? What do you like about it?

2. What was this lesson (topic area) about?

3. Can you tell me what you thought about it?

4. What actual tasks did you do?

5. Would you have liked to have been asked to do different tasks? What?

6 What notes did you keep during the lesson (topic area)?

7. How did you feel during the lesson (topic area)?

8. Did you think about anything else during the lesson (topic area)? What?

9. What difficulties did you have? Do you think anyone else had difficulties?

10. If you (or anyone else) had difficulties, how did the instructor help with those difficulties?

11. What would you have liked the instructor have done to help with those difficulties?

12. What was the most important thing that you learned in this lesson (topic area)?

13. How will you use this math in the future?

14. In what ways was this lesson (topic area) like other lessons (topics)in the course?

15. Is the pace of the lesson about right/too fast/too slow?

16. Did you have any homework to do?

17. How much time did you spend on your homework?

18. How much time would you like to spend learning math?

19. Is this part of the course more difficult than other parts?

20. Did you understand everything in the lesson (topic area)?

21. What did you think about how the textbook dealt with the subject content?

22. In what ways is learning math important for you?

23. How do feel you're doing in the course generally?

24. Do you get enough feedback about how you are doing?

25. Apart from what we have covered is there anything else that you'd like to tell me about this lesson (topic area)?

Student Interviews (3)

a. Them

How many courses are you taking? How many hours/week is that?

Do you work? At what? How many hours/week?

What are you going to do after the course finishes?

Why is learning math important to you?

Did you study math as a child? What was it like?

Apart from this course, have you ever studied math as an adult? Where? What was it like?

Think about when you use math in your daily life (e.g., at work/home). Can you describe a couple of situations?

How does the math you're learning in the course help you in those situations?

How have your attitudes about math changed while you've been taking the course?

b. Acton College

Why are you taking math courses at AC? Did you consider taking courses anywhere else?

Why did you enroll in this course in particular?

How is studying math at AC different/better/worse than other places?

How do you see the course fitting in with the rest of your life?

How much do the courses cost you? How are you paying for your studies?

What (if any) courses are you taking at AC ? Other institutions?

How does AC differ from other institutions?

What AC facilities (e.g., library, counseling/tutorial services/learning laboratory) have you used? How often?

Do you like the classroom the math course is in? Would it make any difference to you if it were laid out differently?

c. The course

Think about a typical lesson from the course. What comes to mind? Can you describe it?

Do you find the course hard/easy?

Which sections of the course do you find harder or easier than other sections?

If you ever have any difficulties do you feel you can raise them in class? With the instructor after class? Have you ever raised them?

Did you ever get any extra help from elsewhere? Where?

How was the planning of the course done? (such as...) How much time was spent planning the course?

Did you feel involved in the course planning?

Does the course meet at a convenient time for you? How could it be organized so that it is more convenient?

How did the instructor involve you or use your experience in the course? To what extent did s/he involve other students

How did other students contribute to your learning?

What were the main teaching methods used? What do you like/dislike about them? Do you wish the instructor had used different methods? Which?

What did you like/dislike about the textbooks that were used? Specifically what did you find most/least helpful?

Did you ever read the extra sections in the textbook? (such as the preface, the sidelights, the study help)

What notes did you keep during the course?

Was the instructor available to meet with you outside of the class?

Did you ever go to see the instructor outside of class times? Why? Was s/he helpful?

Do you find the instructor generally helpful/not helpful? What does s/he do that is particularly helpful/not helpful?

Is there anything that you wish the instructor would differently or do less/more of?

Did you get enough information about the course? About other courses? Where did you get such information from?

Do you find the course material relevant to you?

Does the instructor do anything to make it more relevant? What?

Do you ever work with the other students in the course? How?

Do you understand the math you're doing? How does the instructor check that you understand?

How has your work been evaluated? Do you think the evaluation system is fair?

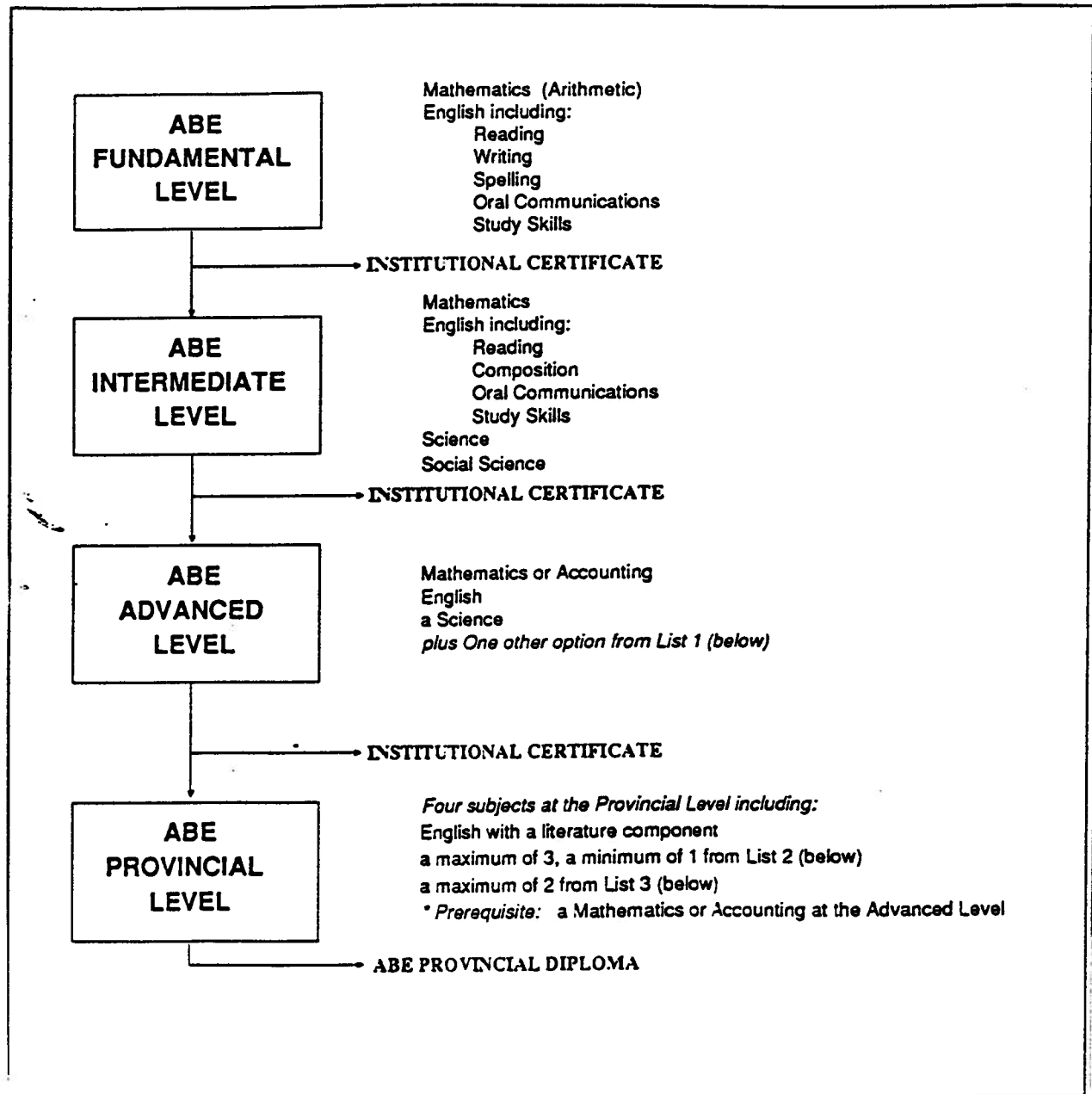Do you think the amount of work you had to do was reasonable?

Did you usually have homework to do? Was it helpful? Was it of a reasonable amount?

How do you think you're doing in the course?

What surprised you about the course or about the teaching? What surprised you about your learning?

Apart from what we've talked about is there anything else that you'd like to say about the course?

ABE Program Framework

ABE FUNDAMENTAL LEVEL	Mathematics (Arithmetic)
	 English including:
	     Reading
	     Writing
	     Spelling
	     Oral Communications
	     Study Skills

→ **INSTITUTIONAL CERTIFICATE**

ABE INTERMEDIATE LEVEL	Mathematics
	 English including:
	     Reading
	     Composition
	     Oral Communications
	     Study Skills
	 Science
	 Social Science

→ **INSTITUTIONAL CERTIFICATE**

ABE ADVANCED LEVEL	Mathematics or Accounting English a Science *plus One other option from List 1 (below)*

→ **INSTITUTIONAL CERTIFICATE**

ABE PROVINCIAL LEVEL	*Four subjects at the Provincial Level including:* English with a literature component a maximum of 3, a minimum of 1 from List 2 (below) a maximum of 2 from List 3 (below) * *Prerequisite:* a Mathematics or Accounting at the Advanced Level

→ **ABE PROVINCIAL DIPLOMA**

Tentative Mathematics Schedule - Spring Term 1994

NO.	SEC.	TIMES	DAYS	INSTRUCTOR	ROOM
:					
050	871	12:30 pm - 2:30 pm	T/Th		3247
050/051	881	12:30 pm - 2:30 pm	MTWTh		3094
051	871	12:30 pm - 2:30 pm	M/W		3247
061	871	8:30 am - 10:30 am	M/W		3247
	872	12:30 pm - 2:30 pm	M/W		3242
	891	6:00 pm - 8:00 pm	T/Th		3247
061/071	881	8:30 am - 10:30 am	MTWTh		3092
	882	12:30 pm - 2:30 pm	MTWTh		3092
071	871	10:30 am - 12:30 pm	T/Th		3094
	872	2:30 pm - 4:30 pm	T/Th		3094
	891	8:00 pm - 10:00 pm	T/Th		3247
083	871	8:30 am - 10:30 am	T/Th		3247
	872	10:30 am - 12:30 pm	M/W		3246
	891	6:00 pm - 8:00 pm	M/W		3247
083/093	881	8:30 am - 10:30 am	MTWTh		3094
	882	10:30 am - 12:30 pm	MTWTh		3092
093	871	10:30 am - 12:30 pm	M/W		3247
	872	2:30 pm - 4:30 pm	T/Th		3092
	891	8:00 pm - 10:00 pm	M/W		3247
096/097	881	10:30 am - 12:30 pm	MTWTh		3247

Course Outline: Math 050

TERM: Spring, January 5 to April 21st, 1994

CLASS TIMES: T/Th 12:30 - 2:20 pm Room 3247
INSTRUCTOR: _ ` Phone: ._._ Office: 3219

TEXTBOOK: *BASIC MATHEMATICS by Keedy/Bittinger* (6th Edition)
METRICS MANUAL by W. Ko & W. Wilson

SUPPLIES: Calculator

COURSE GOALS: As one half of the ABE Intermediate Level Algebraic Mathematics, Math 050 is intended to give the student a good foundation in elementary algebra and _ measurement.

COURSE CONTENT: The algebra section of the course includes: words and symbols used in algebra, signed numbers, variable expressions, exponents, equations, ratio and proportion, problem solving. The measurement section includes: metric measurement, perimeter, area, volume.

COURSE EVALUATION:

Distribution of Marks		Letter Grade Scale	GPA
Term Work	10%	A (88-100%)	4
Term Tests	65%	B (76-87%)	3
Final Exam	25%	C⁺ (70-75%)	2.5
Total	100%	C (61-69%)	2
		*C- (55-60%)	1.5
		D (48-54%)	1
		F (0-47%)	0

N Ceased to attend
W deadline: one month before final or for double blocks, 2 weeks before final. Does not affect GPA.

COURSE REQUIREMENTS:
1) Course work includes:
   - assigned textbook exercises and supplementary worksheets
   - hand-in assignments, quizzes, tests, and final exam
   Note: Marks will be lost if assignments are late or incomplete; Tests must be written in class on date scheduled.
2) To be successful in the course it is important to:
   - attend classes regularly and keep-up with assigned homework
   - ask questions in class and get extra help in seminars, office hours, and the Learning Centre.

A minimum of "C-" is required to proceed to Math 051

Math 050 - Tentative Course Outline

Week	Month	Date	Topic
Week 1	January	4	Introduction
		6	Review of whole numbers and fractions
Week 2		11	Review: Common Fractions (Ch.2)
		13	Review: Common Fractions (Ch.3)
Week 3		18	Review: Decimal Fractions (Ch.4,5)
		20	Review: Ratio and Proportion (Ch. 6)
Week 4		25	**Test: Review Chapters 1 - 5**
		27	Review: Percents (7.1-7.4)
Week 5	February	1	Review: Percents (7.5,7.8)
		3	8.1,8.2
Week 6		8	**Test, Review Chapters 6,7** –
		10	8.3,8.4
Week 7		15	11.1,11.2
		17	11.2
Week 8		22	11.3
		24	11.4
Week 9	March	1	11.5
		3	**Test, Ch. 8 and Ch. 11**
Week 10		8	12.1
		10	12.2
Week 11		15	12.3
		17	12.4
Week 12		22	12.5
		24	Metrics Manual
Week 13		29	Metrics Manual
		31	**Test, Ch. 12**
Week 14	April	5	Metrics Manual
		7	Metrics Manual
Week 15		12	Metrics Manual
		14	**Test, Metrics**
Week 16		19	Review
		21	**Final Exam**

COURSE OUTLINE: MATH 051 - SECTION 871

TERM: Spring 1994 (Jan. 5 - Apr. 25)

CLASS TIME: M/W 12:30 - 2:20 (Room 3247)
Seminars TBA

INSTRUCTOR: ..... - ¨   Phone: .   Office: 3235
Office Hours: TBA

TEXTBOOK: *INTRODUCTORY ALGEBRA* by Keedy/Bittinger (6th Edition)
INTRODUCTORY GEOMETRY by Ruth Behnke (VCC Booklet)

SUPPLIES: calculator, ruler, compass, protractor

COURSE GOALS: The goals for Math 051 are to complete the requirements for ABE Intermediate
Level Mathematics in both algebra and geometry. The course builds on the
elementary algebra concepts developed in Math 050 and enables ABE students to
acquire the basic mathematical knowledge and skills required for vocational or
career programs that require Math 10 equivalency.

COURSE CONTENT: Math 051 consists of Chapters 1 - 3(4) of *Introductory Algebra* and Units 1 - 6
of *Introductory Geometry*. Algebra topics covered include: integers and rational
numbers, solving equations and problems, polynomials. Geometry topics include:
a study of plane figures, congruence and constructions, angle relationships and
measurements, parallel lines, circles and polygons, congruent and similar
triangles, Pythagorean' Theorem, basic trigonometric ratios.

COURSE EVALUATION:

Distribution of Marks		Letter Grade Scale	
		A	88-100
Quizzes	20%	B	76-87
Term Tests	55%	C+	70-75
Final Exam	25%	C	61-69
Total	100%	C-	55-60
		D	48-54

COURSE REQUIREMENTS:

1) Course work includes:
   - assigned textbook exercises and supplementary worksheets
   - hand-in assignments, quizzes, tests, and final exam

2) To be successful in the course it is important to:
   - attend classes regularly and keep up with assigned work
   - ask questions in class
   - get extra help in seminars, office hours and the Learning Centre

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A minimum of "C-" is required to proceed to Math 061.

COURSE SCHEDULE: Math 051 - Section 871
                     Spring 1994 (Jan. 5 - Apr. 25)

Note: Dates are tentative - advance notice will be given of necessary changes.

WEEK	SECTIONS COVERED	TESTS
Jan. 5	Introduction, R.1 - R.4	
Jan. 10, 12	R.5, 1.1 - 1.6	
Jan. 17, 19	1.7 - 1.8, 2.1 - 2.3	
Jan. 24, 26	2.4 - 2.5	Jan. 24, Chpt. 1 T
Jan. 31, Feb. 2	2.6 - 2.8	Jan. 31, Chpt. 2 Q
Feb. 7, 9	3.1 - 3.2	Feb. 9, Chpt. 2 T
Feb. 14, 16	3.2 - 3.6	Feb. 16, Chpt. 3 Q
Feb. 21, 23	3.7 - 3.8, 4.1 - 4.2	
Feb. 28, Mar. 2	4.5, 1.1 - 1.3	Feb. 28, Chpt. 3T
Mar. 7, 9	1.4, 2.1 - 2.3	
Mar. 14, 16	3.1 - 3.4	Mar. 14, Unit 1/2 Q
Mar. 21, 23	4.1 - 4.2	Mar. 23, Unit 1-3 T
Mar. 28, 30	4.3 - 4.4, 5.1 - 5.3	
Apr. 6	5.3	Apr. 6, Unit 4 Q
Apr. 11, 13	5.4, 6.1 - 6.3	
Apr. 18, 20	Review	Apr. 18, Unit 4-6 T
Apr. 25	Final	FINAL EXAM

TESTING POLICY

1)   **Tests must be written in class on the date scheduled.**

2)   If you fail to write a test at the scheduled time, the following **Make-Up Test Policy** applies:
     You must submit a signed and dated statement when you return, giving your reason
     for absence and requesting permission for a make-up test. If the instructor
     considers your reason for absence to be legitimate, youw ill be permitted to write a
     make-up test in Room 3094 on either of the last two Fridays in the term (April 7
     and April 14).

3)   **IF YOU HAVE A LOW MARK ON A TEST, YOU WILL NOT BE ABLE TO DO A RE-WRITE.**

COURSE OUTLINE : MATH 051 - 881

TERM   Spring 1994 (Jan 4 - Apr 27)

CLASS TIMES: MTWTh 12:30 p.m. - 2:20 p.m. Room 3094

INSTRUCTOR: _ Phone: Office: 3232

TEXTBOOKS: Introductory Algebra (6th Edition) by Keedy/Bittinger (Ch. 1-4)
                 Introductory Geometry - VCC Booklet - Ruth Behnke

SUPPLIES: 3-ring binder, ruler, compass, protractor, scientific calculator

COURSE GOALS: The goals for Math 051 are to complete the requirements for ABE Intermediate Level Mathematics in both algebra and geometry. The course builds on the elementary algebra concepts developed in Math 050 and prepares students for the next higher level courses, Math 061 and 071 (ABE Advanced Level). The course enables ABE students to acquire the basic mathematical knowledge and skills required for vocational or career programs that require Math 10 equivalency.

COURSE CONTENT: Math 051 consists of chapters 1 - 4 of Introductory Algebra and units 1 - 7 of Introductory Geometry. Topics covered in algebra include: integers and rational numbers, solving equations and problems, polynomials, and factoring. The geometry topics include: a study of plane figures, congruence and constructions, angle relationships and measurements, parallel lines, circles and polygons, congruent and similar triangle, Pythagoras' Theomrem, basic trigonometric ratios, and co-ordinate geometry.

COURSE EVALUATION:

Distribution of Marks		Letter Grade Scale	GPA
		A (88-100%)	4
Term Work	10%	B (76-87%)	3
Term Tests	65%	C+ (70-75%)	2.5
Final Exam	25%	C (61-69%)	2
Total	100%	*C- (55-60%)	1.5
		D (48-54%)	1
		F (0-47%)	0
		N Ceased to attend	
		W deadline: two weeks before final. Does not affect GPA.	

COURSE REQUIREMENTS: 1) Course work includes:
            - assigned textbook exercises
            - hand-in assignments, tests, and final exam
           Note: Marks will be lost if assignments are late or incomplete; Tests must be written in class on date scheduled.
       2) To be successful in the course it is important to:
            - attend classes regularly and keep-up with assigned homework
            - ask questions in class and get extra help in seminars, office hours, and the Learning Centre.
    A minimum of "C-" grade is required to proceed to Math 061

COURSE SCHEDULE: Math 050/051 - Section 881
Spring, 1994 (Jan 4 - Apr 27)

<u>Note</u>: Dates are tentative - advance notice will be given of necessary changes.

<u>WEEK</u>	<u>SECTIONS COVERED</u>	<u>TESTS</u>
1 Jan 4 - 6	Sec 1.1 - 1.8; 2.1 - 2.7;3.1 - 3.6	
2 Jan 10 - 13	Sec 4.1 - 4.4; 5.1 - 5.5; 6.1 - 6.3	Th Jan 13 Chapters 1-5
3 Jan 17 - 20	Sec 7.1 - 7.8; 8.1 - 8.2	
4 Jan 24 - 27	Sec 8.3 - 8.4;Metrics 1 - 3	T Jan 25 Chapters 6,7
5 Jan 31 - Feb 3	Metrics 4 - 5; Sec 11.1 - 11.3	Th Feb 3 Metrics
6 Feb 7 - 10	Sec 11.4 - 11.5; Sec 12.1	
7 Feb 14 - 17	Sec 12.2 - 12.4	M Feb 14 Chapter 11
8 Feb 21 - 24	Sec 12.5; Review	W Feb 23 Chapter 12
9 Feb 28	050 Final exam	M Feb 28 050 FINAL
Mar 1 Mar 1 - 3	<u>BEGINING OF MATH 051</u> Introductory Algebra Sec 1.1 - 1.8	
10 Mar 7 - 10	Sec 2.1 - 2.8	Th Mar 10 Chapters 1 & 2
11 Mar 14 - 17	Sec 3.1 - 3.6	
12 Mar 21 - 24	Sec 3.7; Omit 3.8; Sec 4.1 - 4.2 Geometry Sec 1.1 - 1.4	W Mar 23 Chapters 3 & 4
13 Mar 28 - 31	Geometry Sec 2.1 - 2.3; Sec 3.1 - 3.3	
14 Apr 4 Apr 5 - 7	EASTER HOLIDAY --NO CLASSES Geometry Sec 3.4; Sec 4.1 - 4.4	Th Apr 7 Units 1 - 3
15 Apr 11 - 14	Geometry Sec 5.1 - 5.4; Sec 6.1 - 6.2	
16 Apr 18 - 21	Geometry Sec 6.3; Sec 7.1 - 7.3 Review	T Apr 19 Units 4 - 6
17 Apr 25	FINAL EXAM 051	M Apr 25 FINAL EXAM
Apr 27	Exam Results & Final Marks	

ADULT BASIC EDUCATION DIVISION - MATHEMATICS DEPARTMENT

STUDENT QUESTIONNAIRE

Instructor: _____   Date: _____

Course: _____   Section: _____

This questionnaire is designed to guide your instructor in improving his/her teaching effectiveness. For Part A please mark the letter from A to E that most closely expresses your view on each item. Leave the item blank if you are unable to respond. Questions in Part B will enable you to make additional comments.

RATING SCALE: Strongly Agree   Strongly Disagree

                        A   B   C   D   E

PART A: MY INSTRUCTOR ...

1.	Has made course goals and objectives clear	A B C D E
2.	Is well-prepared for class	A B C D E
3.	Uses class time in a productive way	A B C D E
4.	Appears to be competent and knowledgeable in this subject area	A B C D E
5.	Is enthusiastic about and stimulates interest in this subject	A B C D E
6.	Presents material clearly and logically	A B C D E
7.	Uses a variety of teaching methods	A B C D E
8.	Assigns work that is helpful in learning course material	A B C D E
9.	Speaks clearly and distinctly	A B C D E
10.	Clearly outlines the course requirements and methods of evaluation	A B C D E
11.	Evaluates students often enough so they know how they are progressing	A B C D E
12.	Gives well-designed tests and quizzes that enable students to show what they have learned	A B C D E
13.	Uses a fair marking system	A B C D E
14.	Encourages students' questions and comments	A B C D E
15.	Encourages students to think critically and to work independently	A B C D E

16. Treats students with consideration and respect   A   B   C   D   E

17. Is patient and tries to help students feel confident in their work   A   B   C   D   E

18. Is interested in students' progress and takes into account their abilities, needs, and interests   A   B   C   D   E

19. Is available to give students extra help during scheduled office hours and seminars   A   B   C   D   E

20. Provides information about student services and resources (e.g., library, learning centre, counselling, health services, etc.)   A   B   C   D   E

PART B: COMMENT ITEMS ...

21. Describe strengths of this course/instructor.

_____

_____

_____

22. Describe weaknesses of this course/instructor.

_____

_____

_____

23. What changes would you recommend for this course/instructor?

_____

_____

_____

24. Any other comments?

_____

_____

_____

Appendix 14

Student Demographic Data

	Occupation	Age	Ethnicity	Gender
	050 Class			
1	student	21	Native	m
2	student	24	•	m
3	student	21	Salvadoran	m
4	cook	28	German/Yugoslavian	f
5	unemployed	18	Canadian	m
6	student	35	Vietnamese	f
7	hostess	18	Italian	f
8	•	27	Thai	f
9	cook	38	Canadian	m
10	waitress	32	Chinese	f
	050/051 Class			
11	student	21	white	m
12	student	43	Canadian	m
13	cook	21	Canadian	m
14	musician	27	Canadian	m
15	taxi driver	45	Irish/Native	m
16	maintenance worker	30	white	m
17	student	19	Afghani	m
18	student	20	Chinese	m
19	cashier	22	Afghan	m
	051 Class			
20	clerk	•	Polish	f
21	student	21	Canadian	f
22	nightwatcher	20	Canadian	m
23	cashier	25	Ethiopian	f
24	student	19	Canadian	m
25	student/musician	30	Scottish	m
26	arts administrator	25	Canadian/British	f
27	student	27	First Nations	f
28	student	30	Canadian/Irish	f
29	clerk/cashier	33	anglo	m
30	none	24	Canadian/German	f
31	student	23	Caucasian	m
32	•	•	white	m

(Key: • indicates no response given)

UPGRADE ... MATH

- Do you need to **upgrade your Math** to qualify for entrance to career, vocational, technical, or college/university programs?

- Would you like to **brush up on forgotten Math skills?**

- Would you like a "second chance" to **prove that you can do Math?**

Take some MATH courses at .

"Easing Into Math" ... **MATH 031**
ABE Fundamental Level
A brush up course in artihmetic skills and introduction to reasoning and problem solving skills.

Calculators are used in all courses.

Beginning Algebra ... * **MATH 050/051**
ABE Intermediate Level   (Math 10 Equivalency)
A review of basic Math skills and a study of metric measurement and introductory algebra and geometry.

Computer-assisted instruction available for some topics.

Intermediate Algebra ... * **MATH 061/071**
ABE Advanced Level   (Math 11 Equivalency)
A review of basic algebra skills and a study of topics in intermediate algebra.

Advanced Algebra & Trigonometry ... * **MATH 083/093**
ABE Provincial Level   (Math 12 Equivalency)
A review of intermediate algebra and a study of topics in pre-calculus algebra and trigonometry.

"Easing Into Calculus" ... **MATH 096/097**
An introductory calculus course designed to ease the transition to post-secondary calculus.

---

* These courses can also be taken on a Self-Paced Program: start anytime, choose the hours that suit your schedule, and proceed at your own pace.

Contact: **Math Department** (Group Instruction)   Office 3235   Phone:
   or   **BTSD Department** (Self-Paced Program)   Office 2089   Phone: .

REGISTER for the Spring Term 1992 (January 6 - April 27)

DAYTIME / EVENING CLASSES
Course Schedule available at
Counselling Centre (Rm 3002) or Admissions (4ᵗʰ Floor)

Mathematics

Mathematics: Intermediate Level

Goal Statement

The goal of Intermediate Mathematics is to enable adult learners to acquire mathematical knowledge, skills, and strategies needed to enter appropriate higher level courses or to satisfy personal or career goals.

Generic topic outline

1. Measurement
   a. S.I. units of length, area, volume (solid & liquid), mass (weight)
   b. conversion between common units
   c. calculation of areas & perimeters of triangles, squares, rectangles, and composite figures
   d. given the formulae, calculation of volume and surface areas of rectangular prisms, cubes, and cylinders
   e. solution of application problems
   f. (optional) S.I. units of time and temperature
   g. (optional) conversion between S.I. and Imperial measurements

2. Ratio & Proportion
   a. ratio as simplest form of the relationship between two numbers or quantities
   b. direct ratio
   c. proportion as a statement of equivalence between two ratios
   d. determine whether a proportion is true
   e. solution of practical problems using direct proportion to find the unknown
   f. (optional) inverse ratios; joint, inverse, and combined proportions; application problems

3. Per Cent
   a. review Fundamental Level per cent requirements
   b. application of percentage

4. Geometry
   a. plane figures (including quadrilaterals, polygons)
   b. constructions:
      i. perpendicular to a line
      ii. bisect an angle
      iii. bisect a line
      iv. construct an angle equal to another
      v. construct a circle
      vi. construct a triangle
      vii. construct angles of 30°, 60°, 45°
   c. angular relationships & measurements
      i. name angles (acute, obtuse, etc.)
      ii. identifying angles (supplementary, complementary, etc.)
      iii. use of a protractor
   d. triangles: congruency & similarity
   e. Pythagoras' Theorem (square roots)
   f. parallel & perpendicular lines
      i. recognize
      ii. construct

5. Algebra
   a. operations with signed numbers
   b. order of operations
   c. solution of 1st degree equations in one variable using the addition, multiplication properties and removing parentheses
   d. substitution into formulas
   e. manipulation of simple formulas such as $D = ST$
   f. manipulation to solve for a required variable and then substitute

6. Charts, Tables & Graphs
   a. interpret and construct bar, line & pie graphs from tabulated data

7. Statistics
   a. calculate and use mean, median, mode & range

8. Problem Solving
   a. apply word-problem solving techniques to all topics

Plus: ONE of A, or B, or C. *

A 9. Algebra
   a. glossary of algebraic terminology
   b. monomials and polynomials: add, subtract, multiply and divide polynomials by monomials
   c. powers, exponents: $a^m \cdot a^n$, $a^m \div a^n$, $(a^m)^n$, where m and n are integers
   d. scientific notation
      i. write numbers in scientific notation & vice versa
      ii. multiply & divide numbers in scientific notation
   e. Cartesian coordinate system: name axes, plot points
   f. graph linear equations through tabulation
   g. factor out common terms, including exponents, numbers & variables

A 10. Trigonometry
   a. sine, cosine and tangent with right-angle triangles

A 11. Problem-solving
   a. apply word-problem solving techniques

B 9. Additional material pertaining to specific vocations

C 9. Additional material in preparation for other non-algebraic Mathematics options at the Advanced Level.

* *(Selection A prepares the student for the Advanced Level Algebraic Mathematics or Advanced Level Developmental Mathematics. Selection B is intended for students exiting the ABE structure at the Intermediate level. Selection C is self-explanatory.)*

12.2 The Addition Principle

a Solving Equations Using the Addition Principle

Consider the equation

$$x = 7.$$

We can easily "see" that the solution of this equation is 7. If we replace x by 7, we get

$$7 = 7, \quad \text{which is true.}$$

Now consider the equation

$$x + 6 = 13.$$

The solution of this equation is also 7, but the fact that 7 is the solution is not as obvious. We now begin to consider principles that allow us to start with an equation and end up with an equation like $x = 7$, in which the variable is alone on one side and for which the solution is easy to find. The equations $x + 6 = 13$ and $x = 7$ are **equivalent.**

> Equations ...

One principle that we use to solve equations concerns the addition principle, which we have used throughout this text.

> The ...
>
> If an ...
>
> is true ...

When we use the addition principle, we sometimes say that we "add the same number on both sides of an equation." Now we can add negative as well as positive numbers.

We can also subtract the same number on both sides. This is true since we can express every subtraction as an addition. That is, since

$$a - c = b - c \quad \text{means} \quad a + (-c) = b + (-c),$$

the addition principle tells us that we can "subtract the same number on both sides of an equation."

1. Solve using the addition principle:

$$x + 7 = 2.$$

Solve.

2. $8.7 = n - 4.5$

3. $y + 17.4 = 10.9$

▶ **EXAMPLE 1**   Solve: $x + 5 = -7$.

We have

$$x + 5 = -7$$
$$x + 5 - 5 = -7 - 5 \qquad \text{Using the addition principle, adding } -5 \text{ on both sides or subtracting 5 on both sides}$$
$$x + 0 = -12 \qquad \text{Simplifying}$$
$$x = -12.$$

We can see that the solution of $x = -12$ is the number -12. To check the answer, we substitute -12 in the original equation.

Check:

$$\frac{x + 5 = -7}{-12 + 5 \mid -7}$$
$$-7 \mid \text{TRUE}$$

The solution of the original equation is -12.   ◀

In Example 1, to get x alone, we used the addition principle and subtracted 5 on both sides. This eliminated the 5 on the left. We started with $x + 5 = -7$, and using the addition principle we found a simpler equation $x = -12$, for which it was easy to "*see*" the solution. The equations $x + 5 = -7$ and $x = -12$ are equivalent.

DO EXERCISE 1.

Now we solve an equation with a subtraction using the addition principle.

▶ **EXAMPLE 2**   Solve: $-6.5 = y - 8.4$.

We have

$$-6.5 = y - 8.4$$
$$-6.5 + 8.4 = y - 8.4 + 8.4 \qquad \text{Using the addition principle, adding 8.4 to eliminate } -8.4 \text{ on the right}$$
$$1.9 = y.$$

Check:

$$\frac{-6.5 = y - 8.4}{-6.5 \mid 1.9 - 8.4}$$
$$-6.5 \quad \text{TRUE}$$

The solution is 1.9.   ◀

Note that equations are reversible. That is, if $a = b$ is true, then $b = a$ is true. Thus, when we solve $-6.5 = y - 8.4$, we can reverse it and solve $y - 8.4 = -6.5$ if we wish.

DO EXERCISES 2 AND 3.

EXERCISE SET 12.2

a Solve using the addition principle. Don't forget to check!

1. $x + 2 = 6$

2. $x + 5 = 8$

3. $x + 15 = -5$

4. $y + 9 = 43$

5. $x + 6 = -8$

6. $t + 9 = -12$

7. $x + 16 = -2$

8. $y + 25 = -6$

9. $x - 9 = 6$

10. $x - 8 = 5$

11. $x - 7 = -21$

12. $x - 3 = -14$

13. $5 + t = 7$

14. $8 - y = 12$

15. $-7 + y = 13$

16. $-9 + z = 15$

17. $-3 + t = -9$

18. $-6 + y = -21$

19. $r + \dfrac{1}{3} = \dfrac{8}{3}$

20. $t + \dfrac{3}{8} = \dfrac{5}{8}$

21. $m + \dfrac{5}{6} = -\dfrac{11}{12}$

22. $x + \dfrac{2}{3} = -\dfrac{5}{6}$

23. $x - \dfrac{5}{6} = \dfrac{7}{8}$

24. $y - \dfrac{3}{4} = \dfrac{5}{6}$

1. ___
2. ___
3. ___
4. ___
5. ___
6. ___
7. ___
8. ___
9. ___
10. ___
11. ___
12. ___
13. ___
14. ___
15. ___
16. ___
17. ___
18. ___
19. ___
20. ___
21. ___
22. ___
23. ___
24. ___

25. $-\frac{1}{5} + z = -\frac{1}{4}$

26. $-\frac{1}{8} + y = -\frac{3}{4}$

27. $7.4 = x + 2.3$

28. $9.3 = 4.6 + x$

29. $7.6 = x - 4.8$

30. $9.5 = y - 8.3$

31. $-9.7 = -4.7 + y$

32. $-7.8 = 2.8 + x$

33. $5\frac{1}{6} + x = 7$

34. $5\frac{1}{4} = 4\frac{2}{3} + x$

35. $q + \frac{1}{3} = -\frac{1}{7}$

36. $47\frac{1}{8} = -76 + z$

SKILL MAINTENANCE

37. Add: $-3 + (-8)$.

38. Subtract: $-3 - (-8)$.

39. Multiply: $-\frac{2}{3} \cdot \frac{5}{8}$.

40. Divide: $-\frac{3}{7} \div \left(-\frac{9}{7}\right)$.

SYNTHESIS

Solve.

41. $\blacksquare - 356.788 = -699.034 + t$

42. $-\frac{4}{5} + \frac{7}{10} = x - \frac{3}{4}$

43. $x + \frac{4}{5} = -\frac{2}{3} - \frac{4}{15}$

44. $8 - 25 = 8 + x - 21$

45. $16 + x - 22 = -16$

46. $x + x = x$

47. $x + 3 = 3 + x$

48. $x + 4 = 5 + x$

49. $-\frac{3}{2} + x = -\frac{5}{17} - \frac{3}{2}$

50. $|x| = 5$

51. $|x| + 6 = 19$

TEST: CHAPTER 12

Solve.

1. $x + 7 = 15$

2. $t - 9 = 17$

3. $3x = -18$

4. $-\frac{4}{7}x = -28$

5. $3t + 7 = 2t - 5$

6. $\frac{1}{2}x - \frac{3}{5} = \frac{2}{5}$

7. $8 - y = 16$

8. $-\frac{2}{5} + x = -\frac{3}{4}$

9. $0.4p + 0.2 = 4.2p - 7.8 - 0.6p$

ANSWERS

10. _____

11. _____

12. _____

13. _____

14. _____

15. _____

16. _____

17. _____

18. _____

Solve.

10. The perimeter of a rectangle is 36 cm. The length is 4 cm greater than the width. Find the width and the length.

11. If you triple a number and then subtract 14, you get $\frac{1}{3}$ of the original number. What is the original number?

12. Translate to an algebraic expression: Nine less than some number.

SKILL MAINTENANCE

13. Multiply:

$(-9) \cdot (-2) \cdot (-2) \cdot (-5).$

14. Add:

$\frac{2}{3} + \left(-\frac{8}{9}\right).$

15. Find the diameter, the circumference, and the area of a circle when the radius is 70 yd. Use 3.14 for π.

16. Find the volume of a rectangular solid when the length is 22 ft, the width is 10 ft, and the height is 6 ft.

SYNTHESIS

17. Solve: $3|w| - 8 = 37.$

18. A movie theater had a certain number of tickets to give away. Five people got the tickets. The first got $\frac{1}{3}$ of the tickets, the second got $\frac{1}{4}$ of the tickets, and the third got $\frac{1}{5}$ of the tickets. The fourth person got eight tickets, and there were five tickets left for the fifth person. Find the total number of tickets given away.

<u>COLLEGE FOUNDATIONS DIVISION</u>   Date: _____

<u>MATHEMATICS DEPARTMENT</u>

Course: _____ ; Section: _____ ; Instructor: _____

_____ _____ ; _____ ; _____
Last Name (Print)   First Name   Registration No.   Age

_____ ; Telephone: _____ ; _____
Address   Home   Work

1) What is the last Mathematics course you have taken? _____

   Where? _____ ; When? _____

   Mark obtained: _____ ; Do you feel this mark "fairly" reflect
                                          your achievement? _____

2) Did you write a Mathematics assessment before registering for this course? _____

   Mark(s) Obtained: Arithmetic _____ % ;   Basic Algebra _____ %
                     Basic Geometry _____ % ; Inter. Algebra _____ %

   Course recommended: _____

3) Why are you taking Mathematics? Is it a required subject? _____

   _____

   _____

4) Do you plan to continue on in Mathematics? _____

   To what level? _____

5) What is your vocational/educational goal? _____

   _____

6) Do you have a job? _____ ; Full-time _____ or Part-time _____

   Specify hours of work: _____

7) How many other courses are you taking this semester? _____

   If any, list: _____

8) Additional information: (Indicate any physical handicaps, transportation or other
   difficulties, relevant information you feel the instructor should be aware of)

   _____

   _____

   _____

9) Briefly describe your attitude towards Mathematics. What do you like most about
   it? What do you like least about it?

   _____

   _____

   _____

Why Did Everybody Hate The Diaper Thief?

Simplify any expression below and find your answer in the corresponding answer column. Write the letter of the exercise in the box that contains the number of the answer. Keep working and you will discover the answer to the title question.

(S) $5x + 2 + 3x$
(F) $3 + 7x + 8$
(E) $9 + 6x + 2x$
(I) $4x + 7 + 4$
(O) $9x + 3 + 7x + 4$
(T) $x + 3x + 6$
(A) $4x + 7 + x$
(Y) $9 + x + 1 + 2x$

(I) $3t + 4v + 5t$
(A) $7t + 6 + 3v + 6v$
(S) $6v + 5t + 8v + 2t$
(H) $3t + 9v + 4t + 9v$
(E) $t + 5v + 6 + 7t$
(O) $8 + 4v + 9t + v$
(T) $3t + v + t + 7v$
(W) $2v + 8 + t + 7$

---

(15) $4x + 11$
(5) $7x + 11$
(13) $5x + 7$
(28) $8x + 9$
(11) $3x + 10$
(17) $8x + 2$
(23) $16x + 7$
(29) $4x + 6$

(2) $8t + 5v + 6$
(21) $7t + 18v$
(10) $4t + 8v$
(26) $7t + 9v + 6$
(18) $t + 2v + 15$
(32) $9t + 5v + 8$
(19) $8t + 4v$
(7) $7t + 14v$

---

(1) $7x + 11y + 8$
(24) $12y + 11$
(8) $12x + 6y + 4$
(9) $9x + 12y + 1$
(12) $9x + 13y$
(16) $15x + 7y + 12$
(3) $x + 16y + 9$
(20) $7x + 7y + 15$

(27) $8u + 5z + 15$
(6) $14z + 12$
(31) $7u + 16z$
(4) $14u + 12z + 9$
(25) $12u + 11z$
(22) $7u + 11z + 9$
(14) $14u + 4z + 9$
(30) $2u + 3z + 19$

(I) $4y + 3x + 2y + 9x + 4$
(E) $3 + 7x + 7y + 8x + 9$
(H) $5x + 8 + 3y + 2x + 8y$
(T) $6y + 9 + y + 7x + 6$
(X) $1 + 8x + 3y + x + 9y$
(L) $x + 7y + 9 + 3y + 6y$
(P) $2y + 7 + y + 9y + 4$
(B) $5x + 6y + 3x + 7y + x$

(N) $3z + 6u + 8z + 9 + u$
(T) $4 + 3z + 7z + 8 + 4z$
(E) $5u + 3z + 9 + 9z + 9u$
(C) $z + 6 + 4z + 9 + 8u$
(B) $9 + 6u + 3z + 8u + z$
(O) $2u + 4 + 3z + 6 + 9$
(L) $5u + 7z + 6u + u + 4z$
(G) $2z + 8z + 3u + 6z + 4u$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32

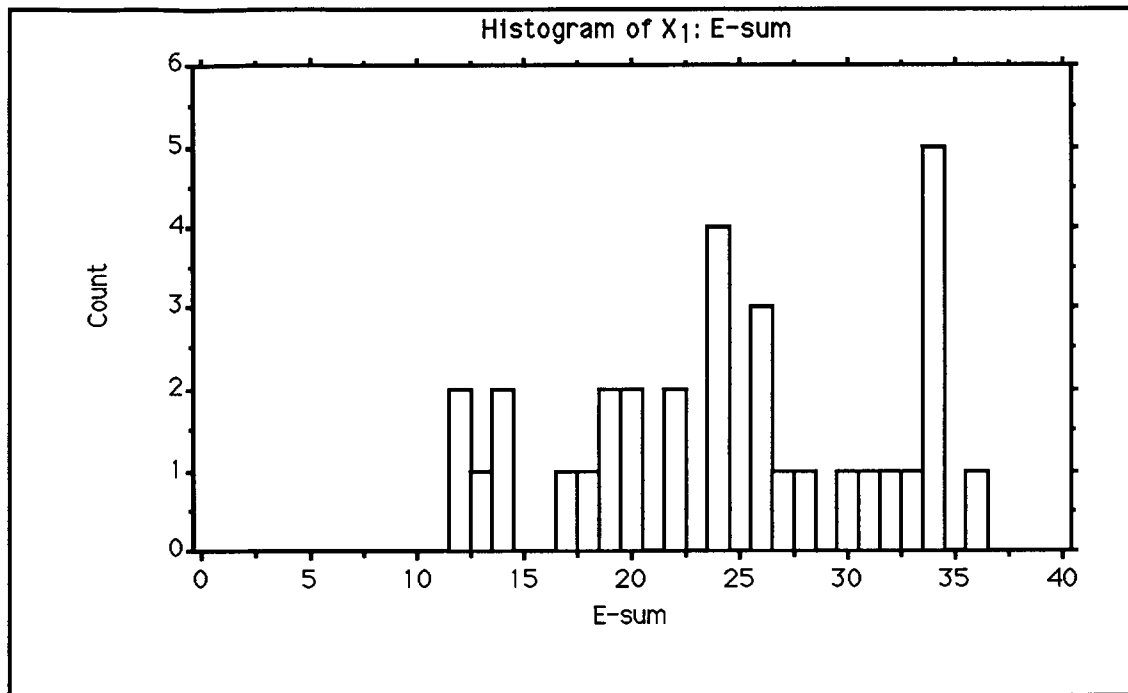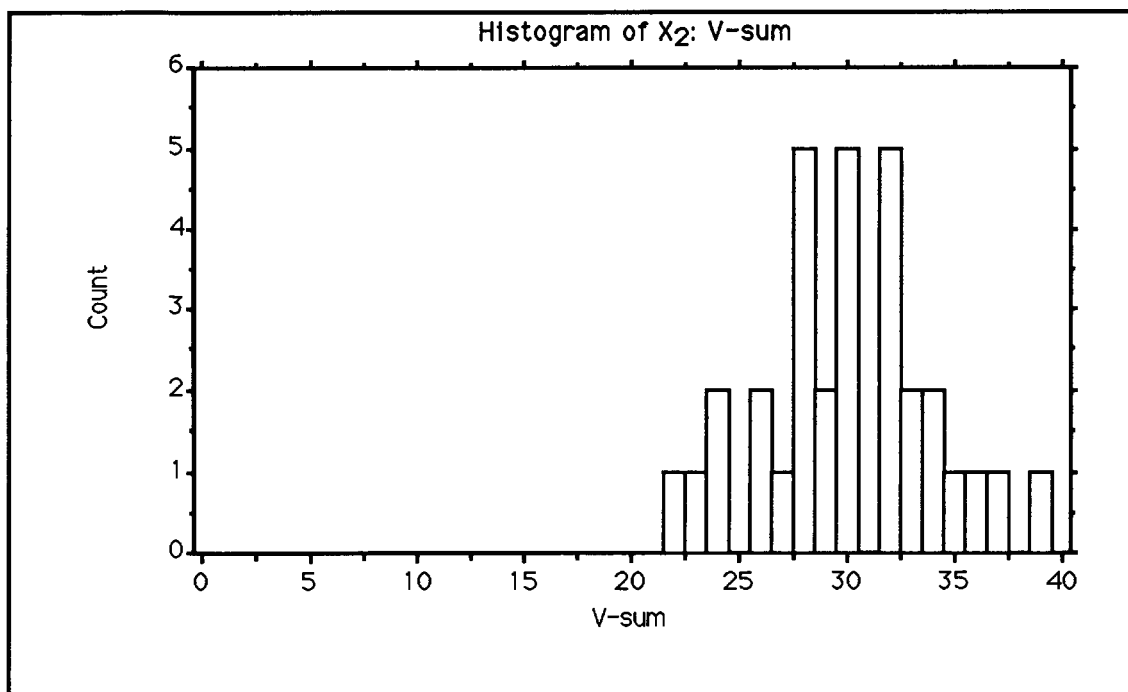Figure 3: Distribution of Student's Enjoyment of Mathematics Scores

Figure 4: Distribution of Student's Perceived Value of Mathematics Scores

Appendix 22

A Methodological Coda

In the body of my report I briefly discussed certain criteria of soundness that are applicable to judging research. Here, I extend that discussion by first considering several general standards propounded by Howe and Eisenhart's (1990). Additionally, I consider the additional criterion of relevance (Hammersley, 1992) which can be further broken into two elements--importance and newsworthiness-- which relate respectively to the importance of the topic for particular audiences, and the contribution of its findings more generally.

Howe and Eisenhart (1990) suggest five general standards that can be applied to post-empiricist research:

(a) There must be a match between the research questions and the data collection and analysis techniques.

(b) The collection and analysis techniques must be correctly applied.

(c) The subjectivity of the researcher must be made explicit so as to clarify the research approach and findings.

(d) The researcher must explain why some conclusions were reached and how disconfirming evidence was sought.

(e) The researcher must be able to demonstrate the value of the research for educational practice, and have adhered to the proper ethical canons.

In order to demonstrate how this study meets these standards and fulfills the criteria, I first consider Lincoln and Guba's notions of criteria of soundness:

credibility, transferability, dependability, and confirmability. Then I consider the

additional criterion of relevance (Hammersley, 1992) which is broken into two

elements--importance and newsworthiness--which relate, respectively, to the

importance of the topic for particular audiences, and the contribution of its findings

more generally.

Credibility

Here, the goal is "to demonstrate that the inquiry was conducted in such a

manner as to ensure that the subject was accurately identified and described"

(Marshall & Rossman, 1989, p. 145). Throughout my research, I sought the assistance

and guidance of both fellow adult education professionals (some of whom had

mathematics education experience) and a fellow doctoral student (at another

University)--who had extensive knowledge of both college and adult education

settings--as "peer debriefers" who helped me explore and make explicit all aspects of

my research. Further, the research was, throughout, conducted under the watchful

guidance of my doctoral research committee. All stages of research

conceptualization, data collection, analysis, and report writing were reported to, and

discussed with, them.

Further, my study was based solely on data derived from extensive study in

adult mathematics classrooms. Descriptions of that data were only considered

within the parameters of those settings, the people in those settings, and the

theoretical framework of this study. My initial findings were presented to the

students and teachers of the classes I studied in both individual and group interview

situations. Here, participants were able to examine some of my initial concepts and

explain and clarify their own perspectives on them, and on teaching in general. By

doing this, I was able to meet the challenge of what Giddens (1976) calls the "double

hermeneutic." In this, the problem lies in the researcher (who brings his own

interests, purposes, and values to the interpretation) being able to accurately depict

the interpretations of others (who are likewise influenced). For Giddens, social

inquiry "deals with a universe which is already constituted within frames of

meaning by social actors themselves, and reinterprets these within its own

theoretical schemes" (p. 162). This notion of a "circle of interpretation" requires post-

empiricist researchers, at least, "to establish an important role for independent

evidence--that is for reality itself--in the process of distinguishing knowledge from

opinion and good from bad research" (Smith, 1993, p. 76). These several points

enabled me to verify, to my own and others' satisfaction, that the study was

conducted in a credible manner.

Transferability

This second criterion refers to the researcher's ability to demonstrate that his

findings can be transferred, or applied, to other contexts. I make few claims on the

transferability of this research to other settings; the burden of applicability seems, to

me at least, to rest with those who wish to make such a transfer. However, for those

researchers who may wish to replicate such a study as this, I have provided essential

details. First, I have fully stated and explained the theoretical parameters of my

research, and used them as guides in the subsequent aspects of the research. For

example, my choice of which data to collect and the collection methods employed

was guided by the concepts and models described in my theoretical framework.

Further, the setting I chose for my study was, in my opinion, typical of those

institutions that provide formal mathematics education to adults, and typical of the

classrooms that one finds in those settings. In this way, future researchers who

design research using these same parameters can determine any possible

transferability. Finally, my research involved triangulation of multiple sources of

data. I gathered data in several situations, from multiple informants, and chose data collection techniques that provided data from several sources to "corroborate, elaborate, or illuminate" the research (Marshall & Rossman, p. 146).

Dependability

Here the concern lies in accounting "for changing conditions in the phenomenon chosen for study" (Marshall & Rossman, p. 146). As the classrooms and settings I studied were constantly changing, I can make no sweeping claims for dependability. However, my continued presence in the chosen settings over a period of time allowed me to recognize and respond to any such changes. My data was gathered over the complete lifespan of the phenomenon--a term-length course--and I was able to observe the teaching in all of its different phases. Further, by constantly relating the data to the theoretical framework, I minimized any effects that changing conditions could have made on my study. Finally, I kept an "audit trail" of what data was gathered and how it was gathered and can therefore account for both the process and the product of my study.

Confirmability

Confirmability refers to the issue of whether the findings of the study can be confirmed by others, and not overly biased by the "natural subjectivity of the researcher" (Marshall & Rossman, p. 147). I attempted to do this in three ways: by ensuring that the study's data and protocols are available for inspection, by constantly ensuring that all aspects of the study were related to the conceptual framework and the tenets of my chosen approach, and by ensuring that my methods

of data collection were responsive, and sympathetic to, the study participants' own situations.

First, as I described earlier, the study was, throughout, reported on and discussed with several knowledgeable others. Further, I have also kept my total data set and my data collection procedures and protocols easily available for others to examine. All completed survey forms, interview transcripts, audio- and video-tapes, and field-notes are easily retrievable in both paper and computer disk form. I also kept a research log that recorded each research design decision and the reasons behind it.

Second, throughout all aspects of my study, I assiduously checked (and rechecked) the data against my conceptual framework. I also deliberately searched for negative instances and disconfirming or alternative theories that would challenge and test my own interpretations.

Third, because this study was conducted in a natural setting and sought the viewpoints of participants in that setting, I was concerned to ensure that all aspects of the study purpose and methodology were fully explained to the participants. I also fully explained certain ethical issues (e.g., those that related to participation and confidentiality) to participants, and gained their informed consent before proceeding with any data collection. Research participants were, throughout, treated with respect, and their cooperation was sought (and granted) in all aspects of the research. Further, what participants say and do must be regarded with what Silverman (1993) calls "contextual sensitivity." That means that a researcher must ensure the accuracy of all the behavior and speech they describe. Further, when drawing conclusions or making interpretations, researchers should remember that people say and do things in response to particular circumstances--such as being observed or being asked questions. Hence, research participants are "positioned"--as indeed, are researchers--and the positions of both are relative and ever-changing.

Relevance

In a discussion of the importance of demonstrating relevance of a research study, Hammersley (1992) advises that researchers "consider the question of audience" (p. 73). He goes on to identify two categories of audience: fellow researchers, and groups of practitioners who work in the area related to the research focus. In the concluding section of this chapter, I demonstrate the relevance of the research for the adult and mathematics education research communities, as well as for practitioners--those who teach mathematics to adults.

First, this research seeks to add to the existing research on adult innumeracy. In one of the rare adult education journal articles on adult numeracy, Coben (1992) notes that "much research on maths/numeracy focuses on children" (p. 15) and there is a lack of relevant research on the mathematics learning of adults. She cites as reasons for this:

> The difference between the culture and ethos of academic research and that of adult numeracy; the perceived difficulty of mathematics as the subject-matter of numeracy; the lack of a forum in which adult numeracy issues might be discussed and through which research findings might be disseminated. (p. 15)

Second, the research draws upon and seeks to develop research in school mathematics education. For example, one of the recently produced Professional Standards for Teaching Mathematics stresses that mathematics teachers should engage in "ongoing analysis of teaching and learning by observing, listening to, and gathering other information about students to assess what they are learning [and] examining effects of tasks, discourse, and learning environment on students' mathematical knowledge, skills, and dispositions" (NCTM 1991, p. 4). Additionally, in their survey of research on teaching and learning mathematics, Romberg and Carpenter (1986) suggest several areas for future research:

It should be carried out in typical classroom environments....and bring together notions about the classroom, the teacher, and the students' roles in that environment; dynamic models should be constructed that capture the way meaning is constructed in classroom settings; and mathematical content should be included in such models. (p. 868)

Third, the research attempts to extend existing knowledge on attitudes and beliefs in mathematics education—an area where further research is needed (McLeod, 1992). In recent publications, both the National Council of Teachers in Mathematics (1989) and the National Research Council (1989) have reaffirmed the centrality of considering affective issues in mathematics education in order to transform peoples' beliefs and attitudes about mathematics.

Finally, the research seeks to contribute to improving the quality of mathematics teaching to adults. Although there are several published examples of "innovative" or non-traditional approaches to teaching mathematics to adults (e.g., Buerk, 1985; Frankenstein, 1987), few are based explicitly on research. In addition, although descriptions of particular courses and programs dealing with "mathematics anxiety" in adults (e.g., Buxton, 1981; Kogelman & Warren, 1978; Tobias, 1978) stress the importance of uncovering and dealing with adults' attitudes and beliefs towards mathematics, they stop short of offering suggestions for improving practice in mathematics classrooms.

In sum, this study attempts to be both newsworthy and important. It draws upon and adds to several areas of educational research while providing both theoretical interest and practical application. It advances what is known about adult numeracy and the teaching of mathematics to adults. It builds upon existing educational research about mathematics education and develops research methodology in mathematics education. Finally, it advances what is known about the role of attitudes and beliefs in mathematics education, and contributes to improving the quality of mathematics teaching to adults.