GENERALIZED ECONOMIC MODEL, RISK ANALYSIS FRAMEWORK AND DECISION SUPPORT SYSTEM FOR THE ANALYSIS AND EVALUATION OF CAPITAL INVESTMENT PROJECTS

by

AHMED MOHAMED ABDEL-AZIZ
B.Sc., Zagazig University, Egypt, 1986
M.Sc., Zagazig University, Egypt, 1992

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

in

THE FACULTY OF GRADUATE STUDIES
DEPARTMENT OF CIVIL ENGINEERING

We accept this thesis as conforming
to the required standard

The University of British Columbia
May 2000

In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the head of my department or by his or her representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Department of Civil Engineering

The University of British Columbia
Vancouver, Canada

Date April, 28, 2000

DE-6 (2/88)

ABSTRACT

The goal of this thesis is to contribute to the knowledge base for making high quality decisions about capital investment projects. To achieve this goal, a generalized economic model, risk analysis framework, and decision support system for the evaluation and risk analysis of capital investment projects are developed.

A detailed investigation of the characteristics of capital investment projects for which the economic model, risk analysis framework and decision support system are to be built is made. A framework called "requirement structure" is introduced in order to investigate the characteristics of a project during its life cycle. A number of infrastructure transportation projects developed under alternative procurement modes in the U.S., U.K., and Canada are used in this study.

Capital investment projects are a feature of several industries, markets and business sectors. These markets and business sectors have different characteristics and require the use of a variety of methods in preparing estimates and forecasts. An objective of the thesis deals with modeling such diversity. To achieve this objective, a generalized economic model is developed with a multipurpose hierarchical network-based time function structure. One concept behind the generalized model is that of cash flow classification. A classification represents a domain, e.g. maintenance or finance, and possesses the properties and methods of that domain. This allows a cash flow of a classification type to inherit its domain's properties and methods. Another concept behind the generalized model is shape functions, which allows the variables of the generalized model to change over time according to a selected pattern. More importantly, shape functions serve in converting an estimate into an expenditure flow. The model structure is organized in four components reflecting four classification domains in capital investment projects namely, capital expenditure, revenue, operation and maintenance, and project financing. The basic elements in a component are called constructs. Each construct represents a cash flow that has the same classification type of the component and consequently inherits its properties and methods. With the generalized economic model, a project economic structure can be formulated with any required properties and methods. The generalized model embraces a broad range of periodic and cumulative cash flows and performance measures such as net present value, internal rate of return, total costs (e.g. total construction cost), life cycle cost, total revenues, debt service coverage ratio, loan life cover ratio, and benefit cost ratios.

To model the uncertainties inherent in the estimates of variables and economic indicators of capital investment projects, a risk analysis framework is introduced. The framework uses an analytical two- and four-moment approach that directly derives the four moments of the performance measures in the generalized model regardless of how complicated their economic structure might be. The framework reduces the necessity of computing intermediate moments as in other moment approaches. A rigorous and expanded derivation for the four moments of a system function is introduced for the framework in order to enhance the accuracy over the standard moment approach. Considerable flexibility in terms of several types of methods, e.g. percentile values, moments, and full probability distribution is introduced for modeling the uncertainty of variables in the generalized model. This provides flexibility over the simulation risk analysis approach that works only with full probability distributions. Pearson and Schmeiser-Deutsch distribution families are used to qualify/fit a distribution model for a performance measure based to its moments.

A practical implementation of the generalized model and the risk analysis framework through a decision support system, called Evaluator, is presented. The system has three components: data, model, and interface/dialogue components and makes use of existing software tools.

Two examples are presented in order to validate the output of the system and to show application of the system to a transportation project. Decision makers in the public and private sector should find the system to be an effective tool to assist in making decisions regarding the procurement, investment, financing, and risk allocation of capital investment projects.

Table of Contents

PART II: DESIGN

PART III: IMPLEMENTATION

List of Figures

List of Tables

Chapter 1

Introduction

1.1 General

Economic modeling and risk analysis are two processes that constitute an important body of knowledge necessary for performing appraisal studies of capital investment projects. Decisions in the public and private sectors depend largely on the findings of such appraisal studies. A number of methods and techniques have been developed to address these processes. The purpose of this thesis is to make further contributions to the development of these methods and techniques, in order to improve the quality of appraisal studies and consequently the quality of decisions. Specifically, this thesis describes a generalized economic model, a risk analysis framework and a decision support system developed for the economic evaluation and risk analysis of capital investment projects at the appraisal stage of a project's life cycle.

The next section overviews the scope of the thesis. This is followed by a section that describes some of the characteristics of capital investment projects and the problem issues raised in the design of economic models for such projects. Research objectives and methodology are then introduced and supplemented by some of the major thesis findings. Finally, a guide to the chapters of the thesis is introduced in the last section.

1.2 Scope Definition

An investment for the development of a project can be defined as:

> *"A long-term commitment of economic resources made with the objective of producing and obtaining net gains in the future. The main aspect of the commitment is the transformation of liquidity - the investor's own and borrowed funds- into productive assets, represented by fixed investment and net working capital, as well as the generation of liquidity again during the use of these assets"* (Behrens and Hawranek 1991).

Capital investment projects as defined in this thesis embrace this definition and include those projects that feature large capital needs for initial development, substantial recurring and replacement costs, an extended service life, and exposure to many uncertainties during all phases of the project life cycle. Infrastructure projects are typical examples of capital investment projects, and generally include several types of projects as follows (World Bank 1994, Dias 1994):

1. Public utilities: e.g. power, gas, water and sanitation systems, and telecommunications;
2. Public works: e.g. roads, bridges, airports, and urban transportation; and
3. Social works: e.g. education and health care facilities.

The realization of such capital investment projects can be arranged through several delivery systems. Traditionally, public infrastructure has been delivered by the public sector through public funds. Recently, and especially in the past decade, several revenue-generating infrastructure projects have been delivered by the private sector through public-private partnerships (PPP) - a system that provides for more involvement of the private sector in the development, financing, marketing and operation of public infrastructure (Augenblick and Custer 1990, Price Waterhouse 1993, UNIDO 1996). Examples of these projects are the Fixed Link or Channel Tunnel (U.S. $ 16 billions) between UK and France; Northumberland Strait Crossing

Project - Confederation Bridge (Cdn \$ 840 million, US\$ 661 in 1992) between New Brunswick and Prince Edward Island, Canada; the Second Severn Crossing (£ 300 million, US\$ 549 million in 1992) in UK; and, the State Route 91 Median Improvement project (US\$ 126 million) in California, USA. The business environment of capital investment projects and the PPP delivery system will be emphasized in the thesis.

The delivery of capital investment projects whether through the private sector, public sector or a public-private partnership, passes through several stages that start with the appraisal stage in which the viability of the project or the investment is assessed by the interested parties. These parties may include government or public sector agency, private sector developer (promoter), lenders, and investors (e.g. equity investors). Capital investment projects under the PPP delivery system typically feature the interaction of these four parties. Project appraisals represent a crucial requirement for these parties before making decisions on whether or not to undertake the project.

A total project appraisal comprises several types of appraisals including (Frohlich et al. 1994, Behrens and Hawranek 1991): commercial, technical, environmental and economic. Generally, a technical appraisal involves an analysis to determine if the project design, development and operation will be technically sound, completed on time and within budget, and according to the required standards and specifications. Therefore, the technical appraisal analyzes, for example, the various technologies that will be employed, time and cost estimates and allocations, and resource requirements. A commercial appraisal seeks an analysis of the market for a proposed project in terms of: potential customers; prices, production and demand; competitiveness; and, industry regulations. The main output of the commercial analysis is the formulation of a demand function(s) essential for making demand and revenue forecasts for the proposed project. An environmental study is an analysis required to determine the impact of a project both during construction and after on the ecological (air, water, soil) and socio-economic environment. Thus, it deals with determining how environmentally sound the project is and how responsive the design, development, and operation of the project is to necessary environmental requirements.

The economic[1] appraisal, the fourth appraisal type, draws on and integrates the information from the other appraisals in order to perform an economic analysis and evaluation for use by the decision-makers. The general objective of the economic analysis for the private and public parties to a project is to assess the economic viability of the project under the prevailing and future conditions of the project. Specifically, the objectives of economic analysis are: to determine whether the project will generate an acceptable return; to determine the most attractive project alternative that will achieve the greatest economic return; to determine the most critical variables that will affect the investment; to determine the impact of different proportion of debt to equity in financing a project and whether the project can service its debt; and, to determine the flow of financial resources required during a project life cycle. The output of the analysis can further be used by the public sector to decide on which delivery system to use when procuring the project, to assess the bids submitted in a competitive tender, to check the strength of project cash flows and determine whether contributions will be required (UNIDO 1996). Economic analysis and evaluation of capital investment projects will be emphasized throughout the thesis.

Essential to the economic analysis of a project are an economic model and an analysis of risks. Nearly all the frameworks that have been developed or proposed to date for risk analysis and management of projects have a risk analysis stage in which economic models and quantitative

[1] The focus in this thesis is on the investment or financial analysis of a project, mainly from the perspective of the private sector, but also for use by the public sector in assessing projects that are required to be self-financing. For such an analysis, the focus is on actual or realizable cash flows, not imputed cash flows. It is recognized that other analyses such as benefit cost analyses would be conducted by government in order to account for imputed flows as well as other societal value systems. Having said the foregoing, the words economic, financial and investment analyses have tended to be used as synonyms throughout the thesis.

risk analysis techniques are employed (APM 1997; ICE et al. 1998; PMI 1996; Chapman and Ward 1997; Thompson and Perry 1992; Hertz and Thomas 1983; Perry and Hayes 1985; Tummala and Burchett 1999; Al-Bahar and Crandall 1990). A project economic model is an economic structure depicting all cash flows (costs and benefits) to be experienced during all the phases of a project's life cycle. Attached to the structure is a set of one or more performance measures (economic indicators) that use these cash flows to provide a value or values with which a conclusion or a statement on a project's viability can be made. Risk analysis is performed on the economic model to provide a picture of the behavior of the performance measures under conditions of uncertainty, with the objective being to provide for better conclusions on project viability and for the establishment of risk management strategies.

To help achieve the objectives of economic analysis for a project, the formulated economic model and the chosen risk analysis technique are generally integrated together as an essential component of a decision support system (DSS). A DSS is an interactive computer-based system that is intended to help decision-makers identify and solve problems and make decisions in strategic and tactical situations based on mathematical models and data. The main functions of a DSS are to facilitate in the formulation of alternatives, the analysis and interpretation of their impacts and the selection of an appropriate option for implementation (Sage 1991). A decision support system has three primary components: a data management component; a model management component; and, a dialogue management component (Pearson and Shim 1995).

The development of a generalized and versatile economic model, risk analysis framework and decision support system for the analysis and evaluation of capital investment projects represents the main theme of the thesis.

1.3 Problem Definition

The thesis problem statement is introduced in this section by way of a project example that highlights the main issues and problem areas which need to be addressed when appraising capital investment projects. The example is for a toll highway to be developed, financed, and operated by the private sector in a public-private partnership with a government.

A technical appraisal for the toll highway would involve, as explained earlier, an analysis of the design, construction, operation and maintenance (O&M) of the project. Part of the analysis would cover the construction phase in terms of the following:

- Construction Methods: to decide on the construction methods to develop the project;
- Planning: to identify all project segments (e.g. work packages) and the order in which they are to be accomplished, i.e. logic that links them;
- Scheduling: to determine the duration of each project segment and its times (e.g. early and finish times) using the logic devised in the planning stage (e.g. critical path method);
- Resource Estimating: to determine for the resources (material, labor and equipment) of each project segment their quantity, pricing (unit prices, wages), escalation rates and production rates; and,
- Cost Estimating: to apply to each project segment the appropriate cost estimating method (e.g. preliminary and detailed) and calculate total cost of the segment and the project.

Part of the technical appraisal would cover the operation and maintenance phases as follows:

- O&M Methods: to decide on the methods used for operation and maintenance, e.g. tolling system such as conventional vs. intelligent transportation system ITS (e.g. automatic vehicle identification AVI);
- Planning and Scheduling: to decide on the O&M tasks, their type, order, duration and timing; and,
- Resource and Cost Estimating: to decide for each O&M task how cost will be estimated, and the relevant quantities, prices and escalation rates for the required resources.

A commercial appraisal for the toll highway would involve an analysis to determine likely revenue sources and forecasts for the project during its operational life. Relevant analyses would include:

- an analysis of the classes of vehicles that will use the highway and traffic volumes;
- an analysis of traffic growth for a planned time horizon;
- an analysis of traffic volume to be captured by the toll highway using, for example, an urban transportation modeling system or transportation choice models (e.g. Stated Preference Technique); and,
- an analysis of tolling rates and adjustment mechanism (e.g. due to inflation rate changes).

Part of the appraisal work involves carrying out a pure financial analysis to determine whether the project would attract the necessary finance if placed in the market. This analysis include:

- sources and methods of financing (debt and equity);
- type of debt, likely amount , and method of drawing (money advances);
- debt currency and applicable exchange rates;
- type of interest (e.g. floating and fixed) and applicable rates;
- methods of repayment and term of debt (maturity); and,
- insurance and guarantees required by lenders.

Findings from the technical, commercial, and financial appraisals provide essential input to the economic analysis of the toll highway. All the information from the various analyses are integrated in an economic model to determine the value of one or more project performance measures. These measures include, for example, total costs (e.g. total construction cost), total revenues and associated cash flow function(s), life cycle cost (LCC), net present value (NPV), internal rate of return (IRR), benefit-cost ratios (B/C) and debt-service-cover-ratio (DSCR). The analysis at this level is generally referred to a deterministic, fixed-value, analysis.

A sensitivity analysis and/or risk analysis is then performed on the measures to determine their behavior under conditions of uncertainty. How comprehensive is the sensitivity and/or risk analysis applied to the measure depends on how detailed the model structure in representing the project. For example, the economic model may accept total revenues as mere single sums for each year during the operation period of a project. The uncertainty around any of these single revenue sums might then be modeled, say, by a normal distribution. Alternatively, if the model is comprehensive in representing project revenues, then the uncertainty in each of the model parameters that model a single revenue sum, e.g. initial volume of demand, traffic growth rate, toll rate and inflation rate, might be modeled separately.

Generally, project appraisal requires the investigation of several alternatives or scenarios under, for example, different design methods, different construction methods, different financing methods and different tolling methods. The economic structure for the scenarios may require the development of several economic models that reflect the methods required by the specified alternatives or scenarios. Further, sensitivity and risk analyses would be revised to deal with the parameters of each model. That would all be required in timely manner so as to inform decision makers of the appropriate alternatives or scenarios and their relevant results.

All capital investment projects are required to pass through the foregoing appraisal processes. Since the economic analysis integrates the other analyses and since the appraisal process involves several issues as described above, the structure of the economic model and the risk analysis framework need to address and deal with the issues raised in the appraisal process. These issues can be described as challenges facing the designers of economic models and risk analysis frameworks, and can be summarized as follows:

1. ***Several Industries and Methods***

   There are several project types, and depending on the type of project several industries, markets and/or sectors (e.g. transportation, maintenance, finance) need to be consulted for carrying out the various appraisals of a project. Since each industry, market or sector has its own business practices and calculation methods (e.g. transportation demand methods and finance mechanisms), the model should be equipped with such methods.

2. ***Multiple Phases and Sub-Phases***

   When appraising a project, the analysis usually should be based on an economic model that mostly closely replicates the market place. A project usually includes several segments or phases such as design, construction, O&M, revenue, and finance. Some phases may be arranged in terms of work packages where each package has its own duration, logic and timing. Other phases, e.g. finance and revenue, have its own characteristics that the model needs to reflect in its structure.

3. ***Cash Flow Characteristics***

   Cash flows can be described in three forms. The first is direct in terms of an assignment of discrete values at discrete points of time. The second is indirect where an estimate e.g. total cost of a work package, is distributed over the work package duration according to a specified pattern. The third is driven where an estimate computed according to a specified method is converted into cash flow distributed over time following the way the parameters of the estimating method actually change over time. The economic model should be capable of dealing with such cash flow methods.

4. ***Time Variables***

   Most project variables change their values over time, such as resource prices; escalation, interest and exchange rates; and, project demand. Further, work package quantities may have consumption rates or patterns that can predict the amount of work done at a specified period of time. The model needs to include several rate functions and patterns and needs to allow its variables to change over time following any of such patterns.

5. ***Several Measures***

   There are several performance measures that can be used in the assessment of a project economic viability. Depending on the objectives of decision-makers, a project appraisal may be required to carry out evaluation and analysis on more than one the performance measures. The model structure should be equipped to deal with multiple measures.

6. ***Uncertainty***

   Models and their parameters are mere abstracts of reality. Similarly, estimates of costs, revenues, and financial indicators represent the best estimates that can be made at the time of analysis. Uncertainty is inherent in all such estimates. Risk analysis frameworks need to model the uncertainty of the estimates and the variables making these estimates.

7. ***Several Alternatives***

   Economic models should be able to formulate several project alternatives or scenarios in order to investigate the project under varying assumptions or conditions.

The foregoing issues describe how detailed and general an economic model needs to be in order to realistically model capital investment projects. Further, there is a need to address the above issues when appraising capital investment projects in order to improve the quality of appraisal studies and consequently the decisions based on them. Current support systems with their underlying economic models and risk analysis frameworks lack the ability to address both the detailed and generality aspects of capital investment projects as described in the foregoing seven issues or problem areas. The current thesis tries to fill this gap, which is the main thesis contribution, by developing a generalized economic model, risk analysis framework and decision support system for the appraisal of capital investment projects.

1.4 Research Objectives and Methodology

The main objectives of the thesis are:

1. To develop an understanding of the characteristics of capital investment projects;
2. To develop a generalized economic model that can reflect the detailed and generality aspects of capital investment projects;
3. To develop a risk analysis framework that deals with the detailed aspects of the generalized economic model; and
4. To develop a prototype decision support system implementing both the generalized economic model and the risk analysis framework.

1.4.1 Capital Investment Projects

The study explores the characteristics of capital investment projects in its first objective and uses an analysis framework, called requirements structure, which classifies and links the requirements in a project to the essential elements of a project structure (e.g. construction and operation) the execution of which takes a project into full development until the end of its life cycle[2]. The output of this analysis is a description of the requirements attached to the development (e.g. design, construction), finance, operation, revenues, and liabilities of a project. The accomplishment of such analysis required an extensive knowledge about how projects are executed. The study was fortunate that it had access to a set of five capital investment projects in terms of their tender and/or contractual documents including contractual agreements between government and developers, request for proposals (RFPs), request of expressions of interest (EOIs) and government acts legislated specifically for the projects. The set of projects studied represents large revenue-generating infrastructure transportation projects, developed under the

[2] Part of the PPP characteristics can be reviewed in: Russell A. D., and Abdel-Aziz, A. M. (1997)."Public-Private Partnerships and Public Infrastructure." 1st International Conference on Construction Industry Development: Building the Future Together, Dec. 9-11, Singapore.

public-private partnership delivery system, as examples of capital investment projects. In addition to the contractual documents, the study was supplemented by an extensive literature search and insights from work done by the author for the use of public-private partnership in the procurement of educational infrastructure[3].

The fulfillment of the first objective in understanding capital investment projects represents a first stage in systems development, i.e. system analysis (Kendal and Kendal 1999). This stage helps to identify the characteristics of such projects and consequently helps in the delineation of the structure of the proposed economic model and decision support system.

1.4.2 Generalized Economic Model

The generalized economic model developed in this study is a multipurpose hierarchical network-based time function structure. In its simplest form, the economic model provides the amount of money spent/acquired at any given point of time. In its complicated forms, the model's time functions can be arranged to serve a specific purpose; for example, to compute financial and economic performance indicators at any given point of time.

One concept behind the generalized economic model is the formation of cash flow *classifications*. A classification represents a domain, such as maintenance, and possesses the *properties* and *methods* of that domain. This allows a cash flow of that classification type to inherit the domain's properties and methods. Properties of a classification cover, for example, time units/intervals, duration, and logic; and methods depending on the classification type include cost estimating methods, demand and revenue modeling methods, financing instruments

[3] Abdel-Aziz, A. M. (1998). "Public-Private Partnerships In Infrastructure Development: The North East Burnaby High School Case Study." *Rep.* Public-Private Partnership Advisory Committee, Economic Partnership Branch, Ministry of Employment and Investment, Ministry of Finance, B.C., Canada. .

(e.g. syndicated term loans, private placement bonds). Typically, each method has a number of variables and one or more mathematical expressions.

The "structure" of the generalized economic model consists of four *components* reflecting four classification domains in capital investment projects namely, capital expenditure, revenue, operation and maintenance, and project financing. The basic elements in a component are called constructs, which are called work packages for the capital expenditure component and streams for the other components. Each construct of a component represents a cash flow that has the same type of classification representing the component.

With the generalized economic model's concept and structure, a component (e.g. revenue component) in the model is represented by a time function formulated by all of the cash flow time functions representing the constructs of that component. Arranging the components' time functions, then, serves to compute a specific performance measure as explained earlier.

The generality aspect of the economic model is reflected by the richness of the properties and methods of each of the four classifications represented in the model structure. Along with the earlier survey on the characteristics of capital investment projects, an extensive literature search was made to acquire the properties and calculation methods of each of the four classification domains and to acquire the knowledge about the economic modeling of projects. Numerous properties and methods of the four classifications were then employed in the hierarchical time-function structure of the generalized economic model.

The concept of classifications has served in addressing the 7th problem issue in economic model design (to model several alternatives). Any alternative can be formulated since the properties and methods used in the scenario will be selected from those of the model classifications without the need to build new models. The challenge was in dealing with the different types of properties and methods of each classification knowing that each new construct would have its own values of such properties and would have any of the available methods; the final model time function would have to combine all such differences before processing any performance measure. While several of the estimating methods and economic indicators used in building the generalized model are not new, the contribution stems from refining and assembling them in a consistent manner and making them available in a generalized modeling framework suitable for the economic and risk analysis of investment projects.

1.4.3 Risk Analysis Framework

Given the detailed aspects of the generalized economic model in representing any required project alternative or scenario and computing any of the mentioned performance measures, it is necessary to develop a framework for risk analysis that can deal with such details. In probabilistic estimating and subject to the available information about a variable, it is possible to deal with either a limited number of percentiles, the first two moments, the first four moments, or a full probability distribution for the variable.

One approach or framework is to use Monte Carlo simulation, however, this will always require the use of full probability distributions to model the uncertainty of the variables. This might not be practical or feasible at all times particularly at the appraisal stage of a project. Further, the processing time for simulation of highly detailed economic structure of an alternative might be significant. Another approach is to use an analytical method that derives the four moments in all the hierarchical levels of an economic model until it reaches the top level that represent a performance measure (Ranasinghe 1990; Russell and Ranasinghe 1992). This approach, however, with the detailed aspects of the generalized model means that the four moments will need to be calculated several times within each construct (e.g. moments of labor cost, moments of material cost, ...etc), then for each construct of the total number of constructs that may be added to a component, then to each of the four components, then finally to the performance measure. This approach, while possible, seems prohibitive in terms of calculation time needed for highly detailed project representation, which is usually the case in investment projects.

The framework adopted in this study, instead, uses an analytical approach that derives the four moments directly for the performance measure function that represents the hierarchical cash flow structure of the generalized economic model. This serves to derive the uncertainty of any required function in the hierarchical structure, i.e. any project performance measure modeled through the economic model regardless of the form or the mathematical expression of the measure. The uncertainty of a performance measure is derived through the uncertainties of the variables used in the measure in two steps. The first derives the four moments of the performance measure and the second determines the distribution of the measure.

The uncertainty of a variable is modeled in a probabilistic manner using 16 uncertainty modeling methods defined in three categories: (1) two and four moments; (2) three and five percentiles (Pearson and Tukey 1965, Keefer and Bodily 1983, Pfeifer et al. 1991); and, (3) full probability distributions (Bury 1999, Evans et al. 1993). This provides a general capability to model the uncertainty of a variable according to the data available for modeling the uncertainty. The four moments of any performance model are obtained by approximating the measure by a multivariate Taylor series expansion and taking expectations to derive the four moments of the measure using the moments of the variables (Kottas and Lau 1982; Siddall 1972; Russell and Ranasinghe 1992). Using this approach an expanded formulation is developed for deriving moments of performance measures in the risk analysis framework. Finally, a probability distribution for any performance measure is determined using its four moments and the characteristics of the four-parameters systems of frequency curves (Elderton and Johnson 1969; Hahn and Shapiro 1994; Johnson et al. 1994; Ord 1972; Schmeiser and Deutsch 1977).

1.4.4 Decision Support System[4]

The decision support system (DSS) represents the implementation part of the generalized economic model and risk analysis framework; the system has been named Evaluator. The system is designed in line with the three components of a DSS, i.e. data, model, and the interface components. The system's database uses the Jet database engine of Microsoft (Visual Basic 1998) and holds the data, methods, results, and graphs for any project defined in the system. The model base represents the generalized economic model and the risk analysis framework and is built using Mathcad programming (Mathcad 1998); graphs are processed using Excel (Excel 1996). The interface is designed using the development environment of Visual Basic (Visual Basic 1998). The interface is the managing module of the system and interacts with the other parts of the system using the Object Linking and Embedding (OLE) protocol in the Microsoft environment. The system works through windows and menus.

With the system it is possible to experiment with a project during its appraisal by forming any required alternative or scenario, maintaining, reproducing, and processing information for the alternatives and obtaining results in several formats, e.g. tables and graphs, all through the interface only. Figure 1.1 shows the summarized analysis menu through which any desired processing could be chosen. Figure 1.2 shows part of the data for a debt stream that uses a syndicated term loan as a financing method in the financing classification (component). This figure shows the level of detail supported by the generalized economic model in representing capital investment projects. Figure 1.3 shows a typical output of Evaluator for the probabilistic analysis of a performance measure - in this case total cost of a group of work packages.

---

[4] Abdel-Aziz, A. M., and Russell, A. D. (1999). "Decision Support System for Infrastructure Project Appraisal and Risk Analysis." The Canadian Society of Civil Engineers, 27^{th} Annual CSCE Conference, June 2-5, Regina.

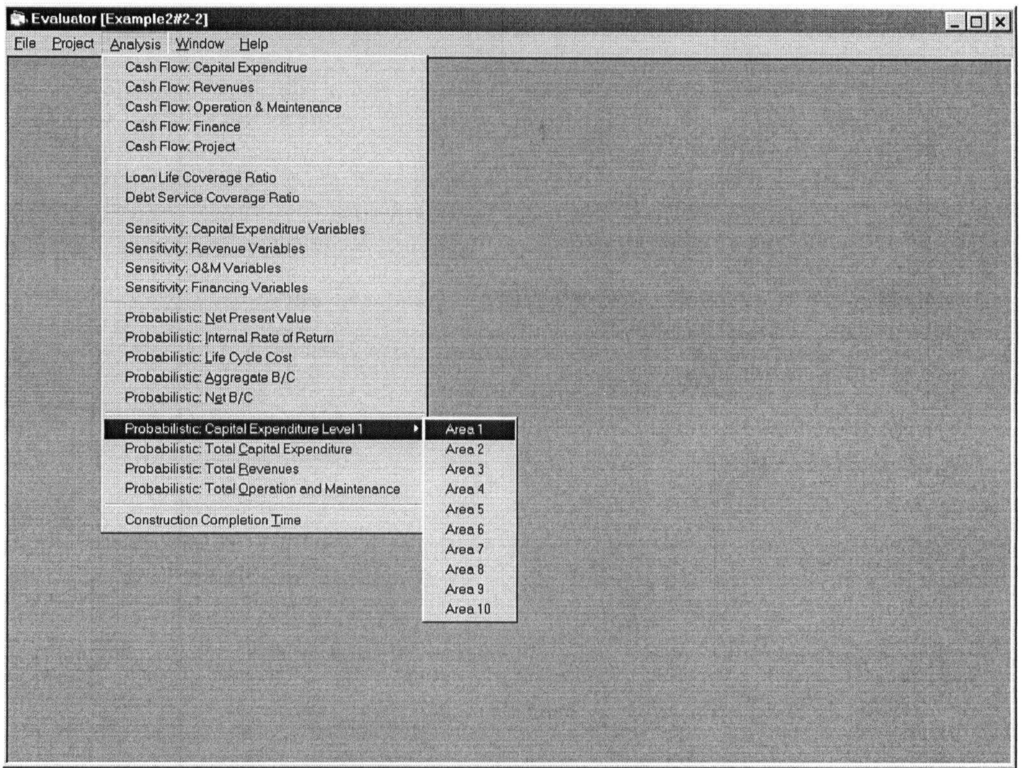


Figure 1.1: Evaluator's analysis menu

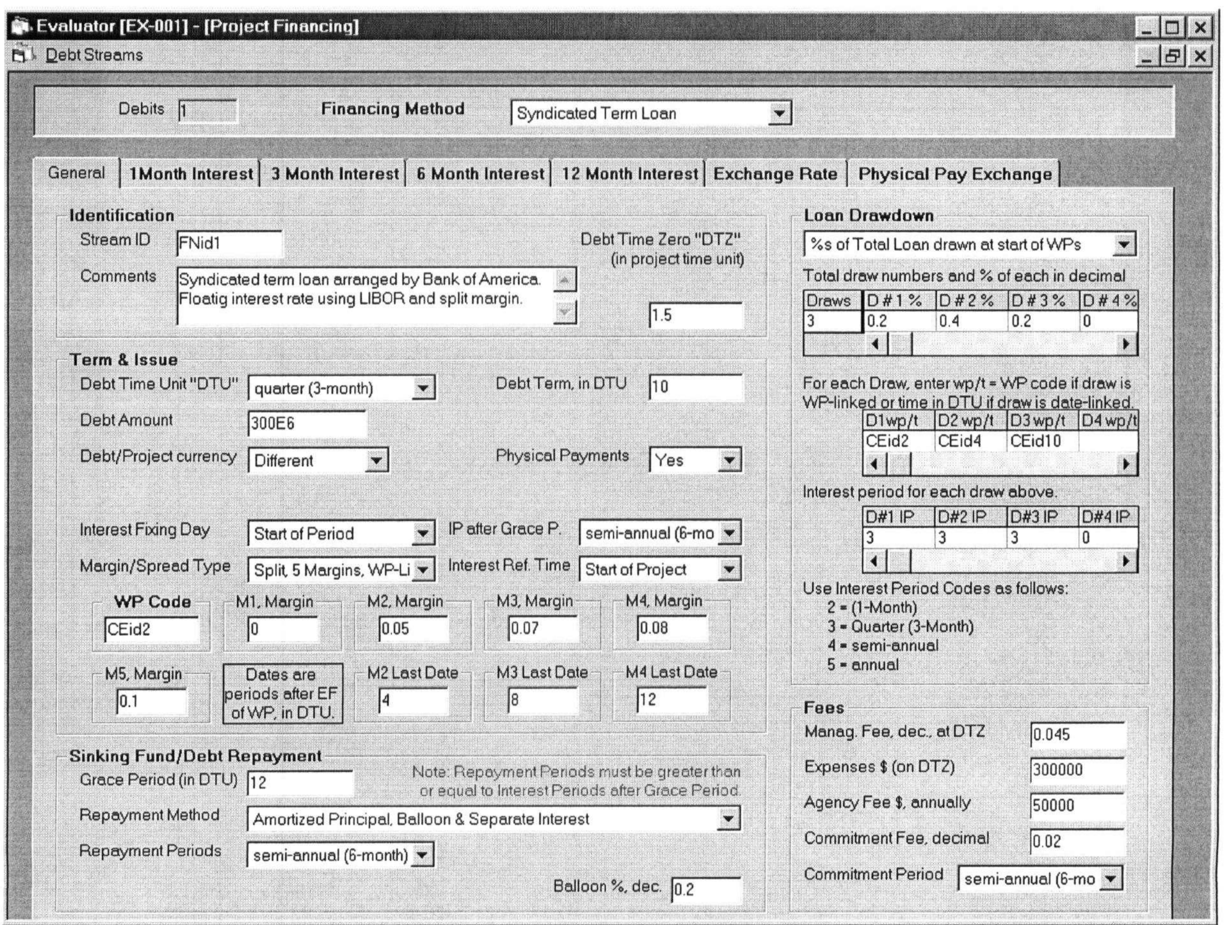


Figure 1.2: A debt stream using Syndicated term loan financing method

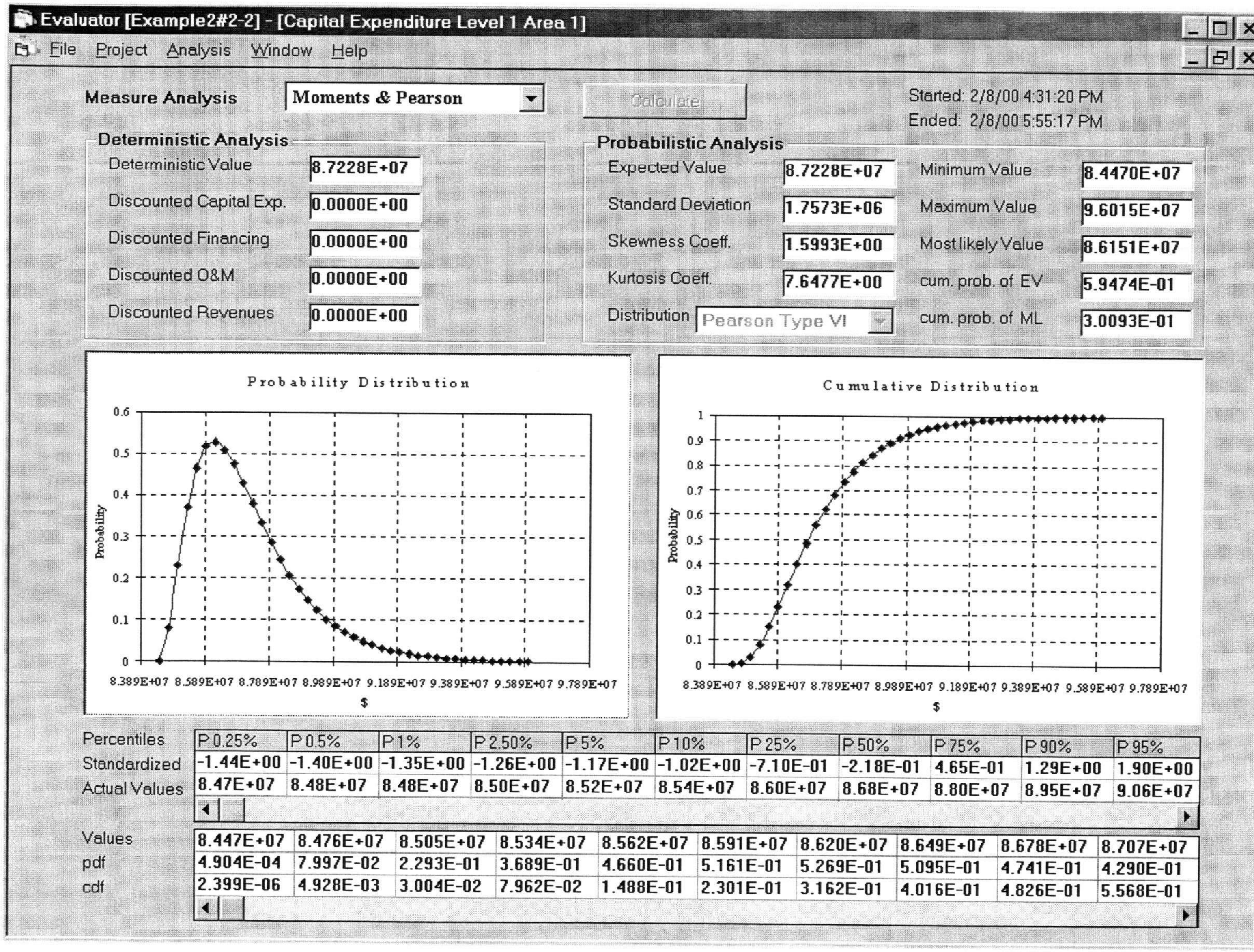


Figure 1.3: Evaluator's probabilistic output for the total cost of a group of work packages

1.6 Research Outline

The thesis is structured in a system development life cycle sequence as shown in Fig. 1.4. "Part I" is the analysis part that establishes the real characteristics and criteria of capital investment projects which should be reflected when building economic models and support systems. Therefore, chapter two provides a detailed analysis of a number of real infrastructure revenue-generating projects developed under the PPP delivery system. To explain the key features of capital projects, the requirement structure is introduced as an analysis framework. The structure has three dimensions, namely, rights, obligations and liabilities; each dimension consists of a number of attributes. Project possession (ownership) and revenues are the attributes covered for the rights dimension. Development, operation and financing are the attributes of the obligation dimension. General liability, risks and taxes are the attributes of the liability dimension. The analysis of such attributes explains much about the characteristics of capital projects.

Chapter three is the point of departure for the work in this thesis; the chapter covers three main themes. The first establishes, based on the attributes of the requirement structure, that the properties and calculation methods used in the business environment of the eight attributes of the requirement structure should be the basis for the design of economic models. A review of previous work by other researchers on economic models and support systems is then introduced in the second part of the chapter. Finally, the chapter concludes with a list of some of the characteristics that should be recognized by the proposed model and support system.

"Part II" is the design part, where the generalized economic model and the risk analysis framework are introduced. Chapter 4 presents a detailed description of the concept, structure and mathematical formulations of the economic model and its performance measures. The chapter starts with a general description of the classifications, methods, properties and shape functions

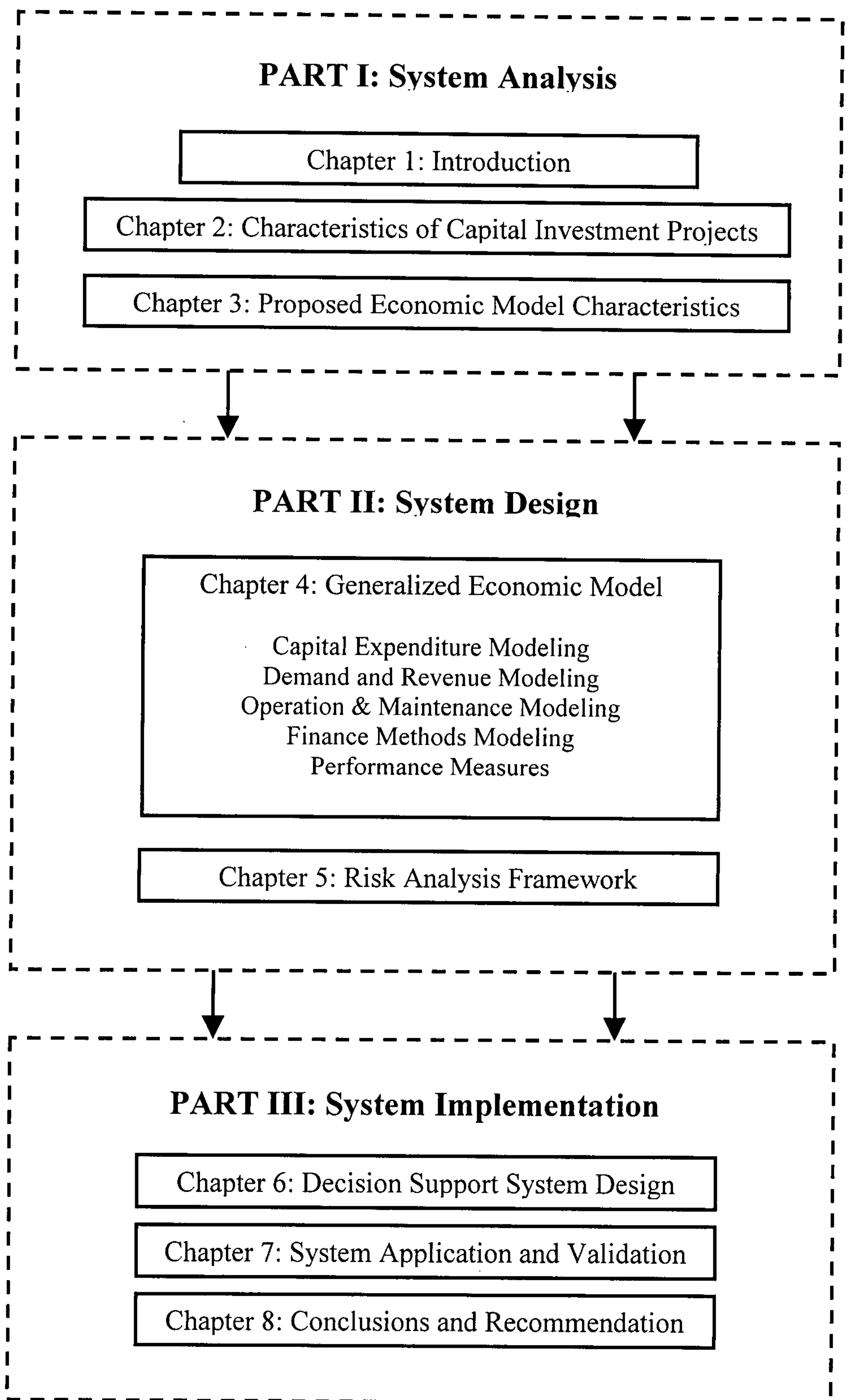


Figure 1.4: Thesis Guide

used by the model. Then, four subsections are introduced to provide a detailed description of the four components that comprise the model. A review of the methods used by the industry of each component is introduced followed by a development of the model formulations for that component. Formulations of the model performance measures are then introduced.

Chapter 5 describes the risk analysis framework used by the decision support system. A detailed description of the analytical framework is introduced explaining its three main parts: modeling uncertainty of a variable, four moments of a performance measure, and distribution of the performance measure. A review of uncertainty modeling methods is introduced with an explanation of their implementation in the framework. The second part derives the first four moments of a performance measure using a multivariate Taylor series expansion. The third part explains how a probability distribution of a performance measure is determined using its four moments of the measure and the characteristics of the Pearson and Schmeiser-Deutsch distribution families. The framework integrates the three parts for risk analysis.

"Part III" is the implementation part of the thesis where the design, testing and application of the decision support system are introduced followed by conclusions. Chapter 6 describes the design process of the system. It covers the three parts of the system; its database, model base and interface. Chapter 7 presents a simple example to verify the system output. This is followed by a detailed example of a highway project in order to demonstrate the generality and flexibility of the generalized economic model, risk analysis framework and the decision support system. Finally, conclusions and future work are described in Chapter 8.

Chapter 2

Characteristics of Capital Investment Projects

2.1 Introduction

A cash flow model generally reflects the expenditures and revenues that are relevant to the party using the model. For example, if maintenance activities of a PPP project are the responsibility of government under a project agreement, then maintenance costs will not be seen on the cash flow model of the private sector developer. Instead, they will be included in the government cash flow model. Similarly, if land costs are subsidized by government, then these costs may not be included in a private developer's cash flow model as part of his ownership costs, unless the government requires otherwise. These requirements vary among project delivery systems and among projects. Thus, as a prerequisite to developing an economic model and support system for such projects to study their business environment in order to establish the various requirements which the model and the system will have to address.

In delivering an infrastructure project, government may adopt a conventional delivery system or engage in a partnership with a private sector developer as an alternative delivery system (PPP). For the latter, several, if not all aspects of the project life cycle are covered in the project documents. These include contractual agreements, legislative acts/regulations, request for proposals and call for expressions of interest. These documents explain all of the requirements that have to be addressed in the developers' proposals. Consequently, before submitting proposals developers will carry out project appraisals as outlined in the previous chapter and build models that express these requirements in cash flow elements if they can be quantified, or highlight them along with other non-financial aspects of the project (Lopes and Flavell 1998) for further assessment and possible negotiation if they are qualitative. Developers would do most of the same steps even for unsolicited proposals (Bederman and Trebilcock 1997). Based on its requirements government too would develop its own economic models to appraise the project and to evaluate the bids submitted. It may require the developer to submit a detailed cash flow as well ("Highway 104" 1995). Project lenders and investors would similarly appraise a project under the given requirements, particularly those that are related to revenues and guarantees.

In conclusion, the study of project requirements as established by government or proposed by developers in an unsolicited bid constitute an essential step to identify the information necessary for project economic analysis and more importantly for the development of economic models. This chapter presents the results of a study of the characteristics of a number of PPP revenue generating infrastructure transportation projects as representatives of capital investment projects. PPP projects are emphasized since they typically embrace all phases in a project life cycle and experience inputs from nearly all project participants including government, developer, lenders and investors.

The following section provides a brief background on PPP followed by a description of a structure proposed to organize project requirements. Then, a brief general description of the projects reviewed in the study is given followed by a detailed description of the attributes of the requirements structure. While transportation projects are emphasized, the requirements structure is broadly applicable to a diverse range of projects. Finally, the chapter ends with a set of conclusions relating to the use of the proposed requirements structure in the analysis of public-private partnerships and in building economic models.

2.2 Public-Private Partnerships (PPP)

2.2.1 PPP Arrangements

The conventional delivery system for infrastructure involves government assuming full responsibility for financing, development (planning, acquisition, design, and construction), operation and maintenance of projects. The private sector is involved in this process through the provision of consulting, design, and/or construction services. For this delivery system government usually acts as a provider and deliverer of services through delivery methods designed to facilitate rigorous standards and control and to promote indicators of measurable performance. This approach is not intended to realize a financial reward or speculative gain for government, and the services are not withheld from those who cannot afford them (Flynn 1997; Baldry 1998). Financing such projects has traditionally been done using pay-as-you-go financing and debt financing (Robinson and Leithe 1990; Feldman et al. 1988). With pay-as-you-go financing, funds to pay for infrastructure costs are secured directly from government current revenues such as taxes, fees and user charges, interest earnings, and grants. Debt financing, on the other hand, requires government to tap credit markets to raise the necessary funds through issuance of debt, e.g. general obligation and revenue bonds and revolving loans.

Despite the use of rigorous standards and control, Baldry (1998) explained that: "In practice, however, the history of performance of such projects [public sector projects] in general, and certain notable projects in particular, has indicated a less than satisfactory performance resulting in substantial cost and time overruns, inappropriate project outcomes and significant secondary effects in terms of disruption and frustration of operational and strategic activity." Baldry explained further that: "A service which is provided monopolistically by an arm of government, and which is free at the point of consumption, is divorced completely from the market economy, resulting in inefficiencies of delivery with consequent cost and time penalties."

A Public-private partnership (PPP) is an alternative project delivery system that provides for an increased involvement of private sector organizations in the delivery of functions that were previously the exclusive domain of government. This delivery system represents a viable alternative to the conventional approach as it provides a solution for the financial problem expressed by increased government debt and a capability to increase the efficiency of the delivery of public infrastructure in terms of time, money, quality and management (Kay 1993; Blaiklock 1992). For the developing countries, reducing budget deficits and government debt appears to be a strong motivation for using PPP arrangements (World Bank 1994; Augenblick and Custer 1990). For developed countries, e.g. the U.K., the objective of using PPP is directed more at harnessing the entrepreneurial, financial and management skills of the private sector in the provision of infrastructure ("New Roads" 1989; "Paying" 1993; Blaiklock 1992; Winfield 1996).

A Public-private partnership (PPP) can be defined as a contractual arrangement between the public and private sectors for the development of public infrastructure where the two parties share resources, risks and rewards appropriately for the successful implementation of the project (Price Waterhouse 1993; CCPPP 1998; IBI 1995). Specifically, PPP is a partnership that defined the responsibilities for project design, construction, financing, operation and maintenance. The contractual arrangement usually takes the form of a concession or franchise; a concession "is the award of a right or license to build, own and operate a public infrastructure for a given period" (Blaiklock 1992). According to the allocation of responsibilities, a PPP arrangement can take several forms/modes, for example, Build-Operate-Transfer (BOT), Build-Own-Operate (BOO), Build-Own-Operate-Transfer (BOOT), Build-Transfer-Operate (BTO), Lease-Develop-Operate

(LDO), Buy-Build-Operate (BBO), and design-build-finance-operate (DBFO) (Price Waterhouse 1993; UNIDO 1996; Walker and Smith 1995; CCPPP 1996). The BOT arrangement is the most referred to approach; it is defined as (UNIDO 1996):

> *"A contractual arrangement whereby a private sector entity undertakes the construction, including design and financing, of a given infrastructure facility and the operation and maintenance thereof. The private sector entity operates the facility over a fixed term during which it is allowed to charge facility users appropriate fees and other charges not exceeding those proposed in its bid and incorporated in the project agreement to enable the private sector entity to recover its investment and operation and maintenance expenses in the project, plus a reasonable return thereon. At the end of fixed term the private sector entity transfers the facility to the government agency or to a new private entity through public bidding."*

The BOOT arrangement differs mainly from the BOT arrangement in that the private sector entity (developer) owns the facility during the term of agreement. In BOO arrangements, the developer owns the facility in perpetuity. Financing for PPP projects is usually raised by the developer using both equity and debt markets. Equity comes in part through project developers and mainly through equity investors. Debt is usually is raised through "Project financing" instruments such as syndicated term loans and private placement bonds (see chapter four).

The PPP approach has received much attention worldwide and several acts have been legislated promoting this alternative delivery system. Examples include the New Roads and Street Works Act ("New" 1991) and Private Finance Initiative in the U.K. (Moore 1994); the Virginia Public-Private Transportation Act of 1995 and its implementation guidelines ("Public" 1995), and the Minnesota Toll Road Enabling Legislation ("Toll" 1993) in the U.S.A. As of 1998, Public Works Financing reports in its database 2208 infrastructure concessions worth $1.1 trillion for various types of projects and services. "Of the 2,208 projects, 795 have been awarded since 1985 by governments in 64 countries for development and operation of $335 billion worth of power, road, rail, airport, water, institutional buildings, and other infrastructure facilities" (PWF 1998).

However, the PPP delivery system has not been successful in several projects because of political, social and financial reasons. For example, Texas High Speed Rail, a $5.6 billion 50-year concession project, was cancelled in 1994 because the developers were unable to raise financing for the environmental studies ($170 million) resulting in a loss of approximately $40 million by the developers ("Franchise" 1991). TH212 in Minnesota was cancelled in 1996 after one of the communities affected by the road project vetoed the project after the signing of the initial agreement ("TH 212" 1996). In Washington State, Substitute House Bill 1006 (SHB1006) was enacted in 1993 to allow for BOOT/BTO project procurement. A number of demonstration transportation projects were initiated ("Public" 1994). Due to a political change from Democratic to Republican coupled with a public outcry over the prospect of tolls, SHB1006 was amended in 1995 by SHB 1317 and followed by Substitute Senate Bill 6044 which dramatically affected the demonstration projects resulting in the cancellation of some.

The literature regarding project and implementation aspects of PPP arrangements has been growing. Some best practice guidelines have been published which provides general description of the implementation process of PPP arrangements (World Bank 1990; UNIDO 1996; CCPPP 1996; Price Waterhouse 1993; ACPPP 1998; Merna and Smith 1996a). Other literature explains the analysis process regarding the evaluation and negotiation of proposals and appropriateness of a project for PPP procurement (Tiong and Alum 1997a, 1997c; Ngee et al. 1997; Ashley et al. 1998; Dias and Ioannou 1996). Still other literature covers general issues dealing with financing, risks and guarantees of BOT projects (Tiong 1990b; Tam 1995; Levy 1996; Walker and Smith 1995; Shen 1996; Merna and Smith 1996b). General contractual and financial aspects of BOT projects have also been treated (McCarthy and Tiong 1991; Tiong and Alum 1997b; Haley 1992). Critical success factors for PPP projects have also been examined (Tiong et al. 1992; Tiong 1995a, 1995b, 1997; Tiong and Yeo 1993; Keong et al. 1997; Blaiklock 1992).

2.2.2 PPP Project Company

Early in the implementation process of a PPP project, a consortium of private sector companies is formed either to prepare an unsolicited proposal for a project of interest or to review an issued RFP and if warranted carry out appraisal study on the project and submit a proposal. The consortium usually enters into a preliminary consortium agreement in order to submit a solicited/unsolicited proposal to government. The consortium agreement represents the initial step before the establishment of a PPP Project Company.

A PPP project company, sometimes referred to as developer, promoter, concessionaire or owning company, is the company which, in a typical BOT project will ultimately be responsible for project development, design, construction, finance, operation and maintenance of the project. A project company is usually a special purpose-company formed as a partnership or joint venture (Clough and Sears 1994; Beidleman et al. 1990). The company usually includes partners such as large engineering and construction firms, equipment suppliers, operation and maintenance companies, and equity investors (e.g. investment banks). Figure 2.1 shows typical participants to a PPP.

A project company assembled for the final realization of a PPP project will have to enter into several contractual agreements as shown in Figure 2.1. These agreements include, for example, a development agreement (sometimes called concession agreement, omnibus agreement), a construction agreement, an operation and maintenance contract, financing contracts (loan agreement), insurance contracts, and supply and off-take contracts (McCarthy and Tiong 1991; Walker and Smith 1995; Pyle 1997; UNIDO 1996; Merna 1996a; Payne 1996). Several of these agreements are mandatory on the project company as being required as a satisfaction to the government or as a condition to obtain finance from project lenders.

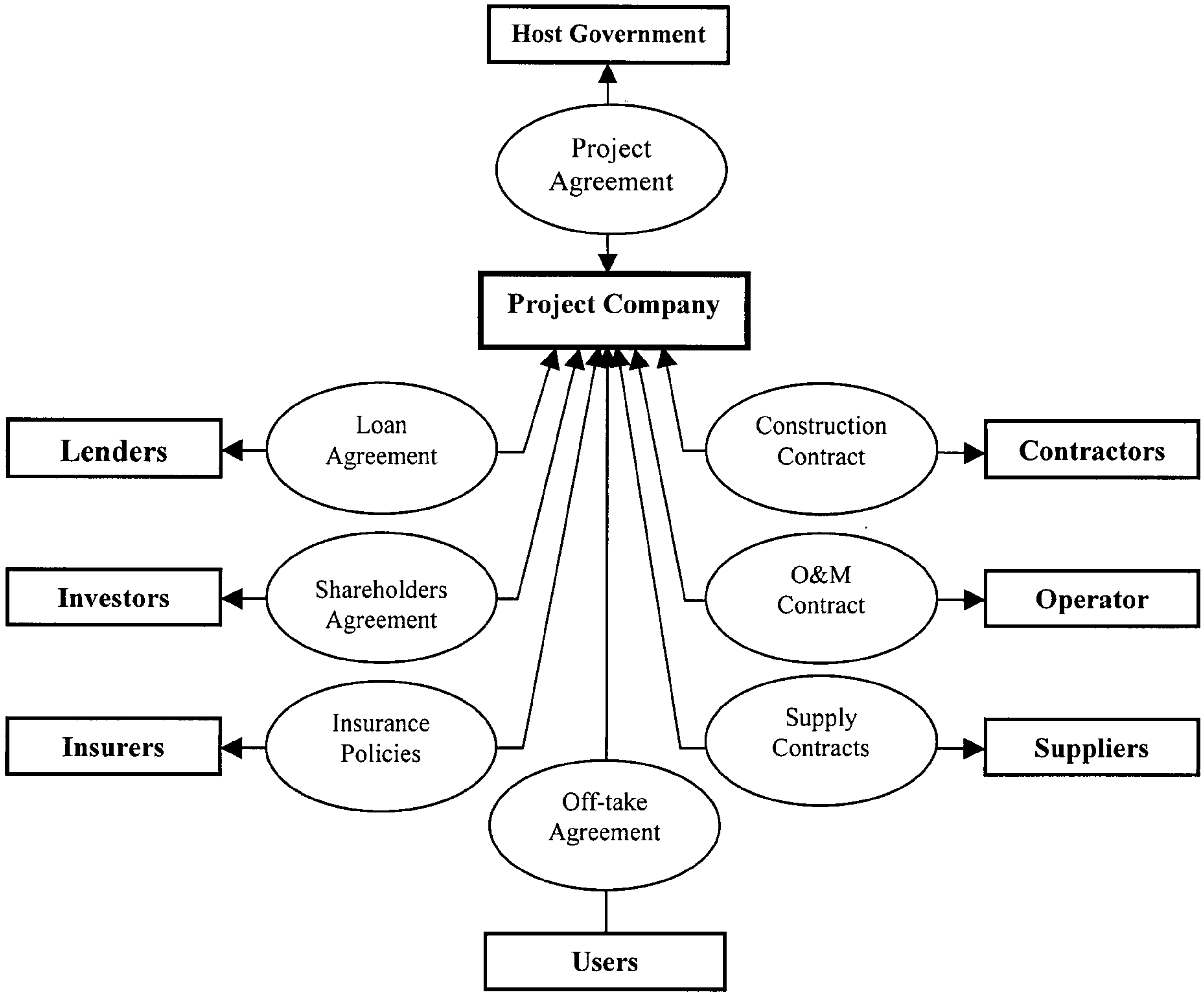


Figure 2.1: Project company structure and agreements, adapted from McCarthy and Tiong 1991, UNIDO 1996, Haley 1992, and Merna and Smith 1996a.

Several authors have examined various facets of the skill sets and factors that lead to the successful promotion and winning of PPP projects by a project company. For example, McCarthy and Tiong (1991) explained that a project company would have to play several roles during the term of a project including roles as consultant, sponsor, contractor and equity holders. "Strength of consortium" was one of six critical success factors in winning a BOT contract as explained by Tiong et al. (1992) and Tiong (1996). Dias and Ioannou (1996) introduced "Desirability Model, DM", a multiattribute evaluation model that assesses the capability of a private sector company to become a promoter for a project as well as the attractiveness of a project to be promoted by a given company. DM included nine attributes relating to company competence and which were grouped under three categories: internal organization characteristics; production capability; and, financial resources and constraints. The diversity of participants to a PPP shown in Figure 2.1 helps to explain the range of skill sets and roles required by a project company to carry out and manage a PPP project. These roles and responsibilities can be usefully grouped under four categories of functions: (1) project company, (2) project, (3) investment, and (4) government.

1. Concession company related functions
   - Assess private company technical, financial and legal resources
   - Perform a project needs-assessment
   - Select qualified partners and form the concession company
   - Draft contractual agreements with various parties (e.g. contractors and suppliers)
   - Prepare technical, financial, and operating proposals
   - Carry out project administration
   - Perform company and shareholder administration

2. Project related functions regarding design, construction, and operation
   - Carry out project studies (e.g. technical and environmental)
   - Develop project conceptual design and related construction methods
   - Conduct value engineering, construction method studies and constructability reviews
   - Prepare operation and maintenance plans for facility management
   - Develop construction management framework (e.g. planning, scheduling and control)
   - Perform quality management

3. Investment related functions regarding cost, economic analysis and financing
   - Estimate capital, O&M, administration, and management costs
   - Conduct commercial studies (e.g. supply, demand, competition, and tolls)
   - Seek and evaluate various sources for project financing
   - Administer debt and prepare security package
   - Conduct inflation analysis
   - Perform economic evaluation (e.g. NPV, IRR, benefit-cost analysis, cash flows)
   - Perform risk analyses project viability

4. Government related functions
   - Negotiate government support to obtain all project approvals and legislation
   - Negotiate concession terms and conditions with government
   - Negotiate various government support and guarantees
   - Seek support to obtain financing at favorable terms
   - Seek guarantees on project revenues, minimum demand, and no-second facilities
   - Seek subsidization for market imperfections
   - Manage stakeholder involvement process

2.2.3 PPP Project Evaluation

Project evaluation is an essential stage for all participants involved in a PPP project. Evaluation involves, as explained in the first chapter, several types of appraisals including technical, environmental, commercial and financial appraisals. For the provision of an infrastructure project, government will check, in a preliminary feasibility/appraisal study, the viability of the project under several delivery systems, e.g. conventional and PPP. Later, if a procurement decision is made to adopt a PPP approach, government during the selection process of a private developer will evaluate the submitted proposals against stated criteria with the overall objective being to achieve a technically sound, cost effective, and financially attractive solution. Evaluation criteria normally reflect the technical, environmental, commercial and financial aspects of the project, and generally include price and non-price criteria. Generally, decisions in the selection process may be based on the value of particular criterion (e.g. net present value, initial user charges) or on a weighting/scoring system of several criteria (Merna 1996; Tiong and Alum 1997a). For example, Tiong and Alum (1997a) explained a differentiation between the evaluation criteria as (1) MUST criteria for which a developer must comply if it is to continue in the process and (2) WANT criteria for which a weight or number of points is attached in a scoring system.

Several studies analyzed the relative importance of the technical solution (including environmental issues) and the financial package (including commercial aspects) of a proposal when evaluating the submitted bids in competitive tendering (Tiong 1995b;Tiong and Yeo 1993). The ability to provide an attractive financial package was judged to be critical under the conditions that the project is technically certain, the level of tolls to be charged are the government's main concern, competition is keen, and project financing is uncertain (Tiong 1995a; Tiong and Alum 1997b; Merna 1996b).

The economic/financial criteria that are generally of concern to a government when appraising a project or evaluating proposals in the selection process are included in the list below (Tiong and Alum 1997a, 1997c; Moles and Williams 1995). The comprehensive evaluation of projects involves the treatment of risk and uncertainty, and hence the evaluation of these criteria generally involves sensitivity and risk analysis. The criteria, grouped under four categories, are:

1. Cost aspects
   - Acquisition costs
   - Development costs (e.g. design and construction)
   - Operation and maintenance costs
   - Life cycle cost
2. Financial aspects
   - Equity amount and debt/equity ratio
   - Sources of debt
   - Interest rates
   - Debt drawdown and repayment schedules
   - Currency of debt and repayments
   - Financial charges (e.g. management and syndication fees)
   - Financial commitments and security package
3. Commercial aspects
   - Initial service charges (e.g. tolls and tariff)
   - Adjustment mechanism of service charges (e.g. due to inflation or demand changes)
   - Length of project/concession period
   - Demand forecasts
   - Projected revenues
   - Lease payments
4. Investment aspects
   - Net present value
   - Rates of return on equity and total capital
   - Benefit/cost ratios
   - Payback period
   - Insurance policies
   - Project cash flows

Project lenders and investors would look at several of the foregoing criteria particularly those that cover the cost, commercial and investment aspects. The commercial aspects receive the greatest attention since project financing is generally raised based on the merits of future project cash flows, the robustness of the demand forecasts (Nevitt and Fabozzi 1995), and debt service cover ratios. Lenders would be much more interested in performing sensitivity and risk analyses on the project (Woodward 1995) since they are the main source of finance for capital projects and their funds would be exposed if the project experiences completion risk, cost overrun risk, and lower than expected demand.

Developers, before submitting a solicited/unsolicited proposal, would also evaluate all of the foregoing criteria together with sensitivity and risk analyses in order to check the viability of the project. Typically developers would have to prepare for the prospective lenders and equity investors a project information memorandum detailing the findings from a project appraisal/feasibility study along with results of project sensitivity and risk analyses in order to explain the merits of the project and receive the required credits and funds (Nevitt and Fabozzi 1995; Rhodes 1993; McDonald 1982).

2.2.4 PPP Project Risks

Risk analysis, identification and management is of major concern to government, developers, lenders and investors when appraising a PPP project. PPP project risks receive special attention since these projects involve exposing large capital investments before any revenues/return are obtained in the extended life cycle of the project. Among a large list of PPP project risks, the major risks that receive considerable attention by all project participants particularly the lenders and investors, an increase in capital costs, delay in construction completion, partially completed project/construction and less than expected demand or revenues represents. To hedge against these risks, several security packages, insurance packages and guarantees are usually required. A considerable body of literature have been devoted to the analysis and identification of PPP project risks in general and those identified for particular projects (Tiong 1990a, 1995c; "Paying" 1993; Wang et al. 1999; Moles and Williams 1995; Woodward 1995; UNIDO 1996; Merna and Smith 1996a; Merna and Adams 1996; Stein and Pote 1997; Yeung 1997; Beidleman et al. 1990; Hurst 1996). In general, the literature tends to classify PPP risks into two broad categories:

1. General (or Country or Global) risks: i.e. those risks that are related to a country's *political, economic and legal* environment.

   This include, for example, risk of expropriation and nationalization or cancellation of the concession, change in laws and regulation (e.g. discriminatory taxation regimes), restriction on repatriation of revenues or profits, currency inconvertibility risk, fluctuation of foreign exchange rates, devaluation risk, and inflation risk.

   These risks are generally outside the management and control of a private sector developer and the estimation of their impact is usually problematic. These risks can affect the whole project as an investment and lead to a total loss, reduce demand for the output of the project, or lead to the deterioration of project cash flows and consequently the viability of the project. Lenders will not provide any credits to a project if they are not sufficiently comfortable with the political stability of the country of the project.

   Considerable negotiation is the norm in PPP projects to address several of the above risks, particularly if government does not provide for their allocation or management (e.g. by exchange rate guarantee) when it issues an RFP. Along with negotiation, some political insurance vehicles can be used to deal with such risks. Insurance products can be obtained, usually at high premiums, to reduce the effect of political and exchange rate risks. Examples include those provided by the Overseas Private Insurance Corporation (OPIC), the Multilateral Insurance Guarantee Agency (MIGA), International Finance Corporation (IFC) and the World Bank (UNIDO 1996; Hurst 1996). Also, capital market instruments such as swaps, options and futures can be obtained to hedge against the movements in the currency and interest rates thus providing some risk relief to project developers (Coopers & Lybrand 1987).

2. Specific (or elemental) project risks: i.e. risks that are related to project *construction, operation, finance, and revenues.*

   The major risks in this category include construction cost overruns, completion delay risk, interest rate risk, demand/revenue risk (volume and/or price), supply risk (volume and/or price), and force majeure risks.

Construction cost overruns and completion delay risks pose critical risks since they affect project return - the whole investment could be lost, particularly if the project is not completed. Demand risks affect project cash flow, which can directly affect project return and the ability to repay project debt on time. Interest rate fluctuation can pose a risk of raising project total cost, which can lead to a decrease in return if service charges (e.g. toll rates) or project/concession periods are not adjusted accordingly.

Several of the risks under this risk category are within the control of, and usually allocated to, the private sector developer who may generally distribute such risks through secondary contracts with its participants. Insurance and bond packages are usually used for the comfort and security of government and lenders; RFPs may stipulate specific coverage during construction and operation. Negotiation is usually involved for force majeure risks particularly for any event that is not covered by insurance. The effect or impact of risks in the specific risk category can generally be analyzed and quantified through a formal risk management process (APM 1997; ICE et al. 1998; PMI 1996).

The success of a PPP project depends to large extent on the allocation and management of the general and specific risks; and the rational in the allocation usually calls for a particular risk to be allocated to the party most able to control or influence it.

2.3 Government Requirement Structure

2.3.1 Requirement Structure Description

Government requirements under public-private partnership arrangements cover all of the contractual, technical and financial aspects of a project. Each requirement may be described by some specific aspects and consequences that need to be considered carefully under PPP arrangements. An example of a government requirement is project ownership; government may provide for specific forms such as public ownership, private ownership, or both where transfer of ownership may occur during the term of development. The domain of ownership can be the whole project or it may be defined for individual parts of the project, e.g. real property (land), facility (improvements), movable and immovable properties, and intellectual property rights. The domain of ownership may have consequences for both governments and developers in terms of tax treatments or on the availability of rights or licenses to use a technology after project transfer to government. During the appraisal stage, where possible, government requirements would be converted into financial terms in a cash flow model and the requirements or their effects on a project scenario analyzed. Therefore, the use of PPP calls for governments to address the range of conditions that they may stipulate for each requirement and the consequences of each.

Based upon a detailed study of several PPP projects and acts (explained below), a useful structure for describing the key features of a PPP project during its life cycle has three major dimensions: rights; obligations; and, liabilities. These dimensions along with explanatory attributes are shown in figure 2.2. The rights dimension describes the various rights given by government to the private entity in return for carrying out a specified set of obligations. Possession of the facility and access to revenues constitute the primary attributes of the rights dimension. Obligations represent the promises that the developer and the government agree to be bound to under the agreement. Obligations can be described by three attributes: development obligations (e.g. planning, design, construction, environmental); operating obligations (e.g. operation and maintenance); and, financing obligations. The liabilities dimension covers the most controversial issues in PPP negotiations and includes three attributes: general liability (e.g. tort or third party liability and facility damage); risk liabilities; and, tax liabilities.

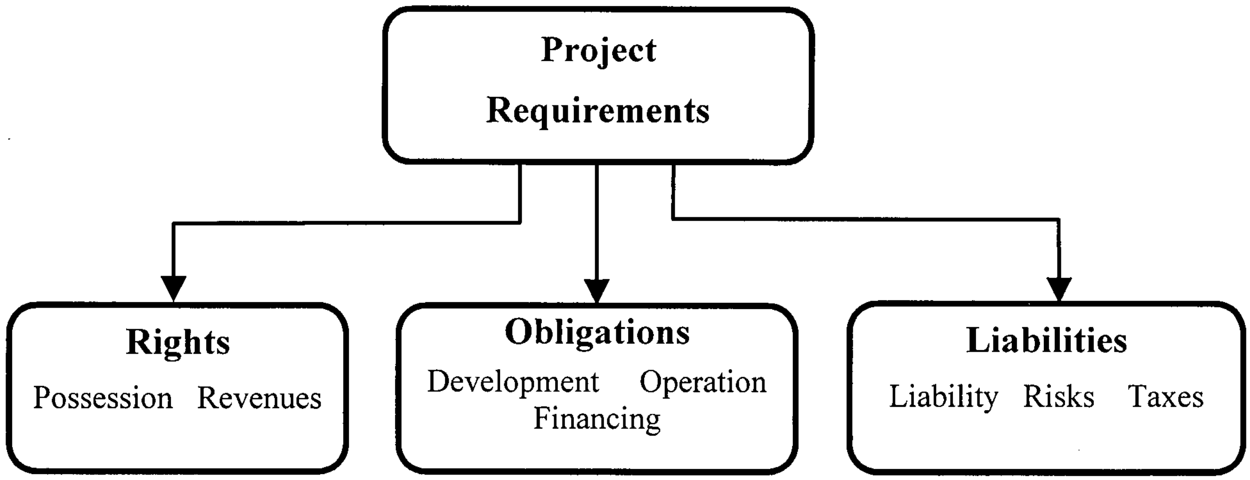


Figure 2.2: Requirement structure

A starting point is that all requirements and associated attributes belong to, or are the responsibility of, government, as is ownership of a facility. Under PPP, selected or all attributes of a requirement can be temporarily or permanently assigned to another party. Therefor, various allocations of the attributes of the requirement structure can be assembled which in turn leads to the spectrum of procurement modes commonly associated with PPP (e.g. BOT, BTO). These allocations provide for various agreement titles such as development, franchise and omnibus.

The following subsection explains the projects and acts used for investigating government requirements. Following this description, a subsection is devoted to each of the attributes identified under the three requirement dimensions. Description of an attribute starts with a general summary of findings followed by a detailed description of main terms and conditions.

2.3.2 Projects and Acts Considered

The Channel Fixed Link, UK/France (BOT)

The Fixed Link is a twin bored tunnel rail link with associated service tunnel under the English Channel between England and France. The approximately 50 km link was developed at a cost of £ 9 billions. The invitation to promoters (equivalent to a RFP) was issued in 1985 ("Invitation" 1985) with no prior call for expressions of interest. Agreement with the successful developer was reached in 1986 ("Concession" 1986), and the project was legislated in U.K. by the Channel Tunnel Act in 1987 ("Channel" 1987). The project was inaugurated in 1994. The promoter, Eurotunnel, consisted of a consortium of British and French engineering and construction companies and banks: Channel Tunnel Group and France-Manche.

Second Severn Bridge, UK (BOOT, DBFO)

The Second Severn Bridge is a 920-m. cable-stayed bridge and two 2000 m. approach viaducts over the Severn Estuary between England and Wales with a total cost of £ 300 million. Following a Notice and Invitation for Prospective Tenderers ("Second" 1988) (equivalent to a call for expressions of interest) and Tender Invitation ("Second" 1989), the project was arranged as a DBFO (Design-Build-Finance-Operate); however, it was known also as a BOOT project. The project promoter, Severn River Crossing, SRC, a joint venture between UK's John Laing and France's GTM Entrepose, expanded after awarding the project in April 1990 to include Bank of America and Barclays' De Zoete and the concession agreement was signed in 1990. Project approval by Parliament came in November 1991 in the form of the Severn Bridges Act in 1992 ("Severn" 1992). Construction started in 1992 and ended in 1996. Along with the provision of a new crossing, the government required the promoter to take over the responsibility for the existing crossing over the Severn Estuary and inherit its debt ("Severn" 1988, 1989).

Highway 104 Western Alignment, Nova Scotia, Canada (BOT)

The Western Alignment is a 45-km four-lane highway which forms part of Highway 104 (Trans Canada Highway) in Nova Scotia. The total capital cost of the project is Cdn $ 113 million. The Request for Proposals, issued in 1995, was followed by six addenda ("Highway" 1995). The legislation required for the project forms the Western Alignment Act ("An Act" 1995), W-A Act. This Act provided for the creation of the Western Alignment Corporation as a single-purpose corporate vehicle, not a public authority or crown corporation. This corporation was created to assist the developer, Atlantic Highways Corp, a subsidiary of Canadian Highways International Corp., in contracting with the Province for the realization of the project.

Northumberland Strait Crossing Project NSCP, New Brunswick/PEI, Canada (BOOT)

The NSCP bridge crosses the Northumberland Strait between New Brunswick and Prince Edward Island, Canada. The estimated cost of the 13.5-km bridge was about Cdn $ 840 million although the actual cost was in excess of this. After receiving unsolicited proposals for the project, the government issued a CFEI in 1987 ("Northumberland" 1987) followed by a call for proposals and six addenda in 1988 ("Northumberland" 1988). The project was legislated by the Northumberland Strait Crossing Act ("An Act" 1993a) and financial closing with the developer, Strait Crossing Inc., was made in 1993 after a number of environmental assessments and challenges in the courts. A number of 39 separate agreements and 400 documents were executed including a development agreement, a construction contract, a project security agreement, a project trust agreement, an operation agreement, and a regional agreement (FHWA 1996).

State Route 91 Median Improvement, California, US (BTO)

The State Route 91 (SR91) median improvement is one of four demonstration projects in California authorized by Assembly Bill 680 ("Assembly" 1989). These projects were proposed by the private sector after issuance of Guidelines for Conceptual Project Proposals ("Guidelines" 1990) by the California Department of Transportation, Caltrans. The SR 91 development franchise agreement signed in 1991 was granted to the developer, California Private Transportation Corporation (CPTC) with final approval of the agreement being contingent on meeting environmental requirements. The agreement was amended and restated in 1993 ("Amended" 1993). The project represents a 10-mile (16-km) all-electronic tolled new four express lanes within the center median of the State Route 91. Construction started in 1993 and the project opened in December 1995. Estimated cost of the project was U.S. $126 million.

US Acts

After California's initiatives as explained by Assembly Bill 680, several States enacted similar legislation for PPP projects. In Minnesota, Toll Road Enabling Legislation 1993 ("An Act" 1993b), TREL, was enacted to provide for the development of BOOT/BTO projects through the TRANSMART program ("Request" 1995). Highway TH 212, proposed following the initiation of TRANSMART, was first signed by the government ("TH 212" 1996). However, during a 30-day voting period required by the TREL Act for community approval, one of four cities on the proposed highway voted against it. In Virginia, the Public-Private Transportation Act ("Public" 1995) was enacted to provide further refinements for the implementation of PPP projects following the earlier Virginia Highway Corporation Act of 1988 ("Virginia" 1988) and the Qualifying Transportation Act of 1994 ("Qualifying" 1994).

While the above projects receive detailed description in the following subsections, only the major requirements included in the US acts, i.e. those of Minnesota and Virginia, are considered in the description.

2.3.3 Rights Dimension

2.3.3.1 Rights Dimension: Possession Attribute

The investigation of government requirements for this attribute has emphasized the types of properties and related government requirements for the possession and transfer of property. Several types of properties have been mentioned in the selected projects and acts. These include:

1. Land or real property needed for the project;
2. Improvements or the facility the developer agreed to construct on the land (e.g. highway, bridge, structure, movable and immovable properties, plant, equipment);
3. Airspace premises (e.g. over and under the right-of-way); and,
4. Intangible properties needed for the development, operation, and ownership (e.g. intellectual property rights, patent rights, project documents, reports, drawings, plans and specifications).

Generally, not all of these properties have been explicitly identified and defined in the RFPs or agreements. Except for the U.S. experience, governments seemingly dislike to explicitly state that the developer will be the owner of the project. All-encompassing statements which treat the transfer of all property at the expiration of the agreement, such as with the Second Severn Bridge (BOOT), are typically featured in the agreements. When lease agreements are made for land or right-of-way, the reversion of the improvement (facility) may be written explicitly such as with the NSCP (work is deemed to be a fixture to the land) or implied to occur with the reversion of the land at the end of the lease such as with the Channel Tunnel. The two BOOT projects, Severn Bridge and the NSCP, leased the land to the developer at a minimal rent. For the Channel and Western Alignment BOT projects, the first provided land at cost and the second was free. Intangible properties such as intellectual property rights were a subject of transfer for the Channel Tunnel. However, for the Western Alignment, the RFP stated that it was to be under government possession at all times.

Table 2.1 provides a summary of the relevant characteristics of the possession attribute. Projects were generally required to be transferred or revert at no charge to the government at the end of the agreement. While this transfer requirement might be common in PPP projects, exceptions can be found. For example, the Texas High Speed Rail project had a requirement that at termination the government had the option to purchase the facility at its fair market value ("Franchise" 1991).

Channel Fixed Link

The Channel tender invitation explained that the chosen promoters would benefit from a concession to construct and operate the Link for a period of time, and the rights of the promoters would expire when the concession was terminated. The governments required the Link to be kept in the public domain ("Concession" 1986). The term Fixed Link was an all-encompassing term, defined to include a twin bored tunnel rail link with associated service tunnel, together with the terminal areas and dedicated facilities for control of, access to, an egress from, the tunnels. The term also included plant, machinery, movable and immovable equipment and railway shuttle rolling stock. Land, referred to as Operational Land and Construction Site Land, for the project was provided by the governments after compulsory acquisition and/or agreement and was leased to the developer. The agreement required the promoters to pay in respect of such lands the cost of acquisitions for land acquired after the agreement, market value for land acquired before the agreement, and the cost of vesting the foreshore and bed of the sea in the British Minister.

Upon expiration or termination of the agreement, the Fixed Link will be handed over to the two governments. Immovable property will revert to government and land leases will end.

Project or Act	PPP Mode	Possession Characteristics	Acquisition by	Property Transfer at termination	Property Reversion at termination
Channel Tunnel	BOT	Land and facility in public domain, land leased to developer at cost	Government, at cost to developer	Intellectual property rights, movable property	Immovable property; land leases to end
Second Severn Bridge	BOOT	Land leases, no explicit clause of private ownership. Known as DBFO contract.	Government, at peppercorn cost to developer	All property, rights, liabilities	N/A
Highway 104 Western Alignment	BOT	Public possession at all times of facility, all improvements, patents, intellectual property	Government, at no cost to developer	N/A	Project to revert at expiration
Northumberland Strait Crossing	BOOT	Ground lease, work is fixture to land; own means to own leasehold interest in facility	Government, at peppercorn cost to developer	Work becomes absolute property of Government at termination of lease.	N/A
SR 91 and AB 680 California	BTO	Construction lease term, transfer of property at completion to Government, operation lease term. Airspace option lease.	Developer. Eminent domain is available, all at developer's cost	N/A	Surrender of real property and facility
PPTA Virginia 93	BOOT	Developer to acquire, construct, improve, maintain, and/or operate a facility.	Developer/Gov. Cost by developer	Project to be dedicated to the public entity involved	N/A
TREL Minnesota 95	BTO/ BOOT	Agreement to provide for any mode or procurement (BTO or BOOT) or operation	Developer/ Gov. Gov. sell, lease, or donate Right-of-way	Facility titles transfers to Government after expiration under BOOT	Facility titles under BTO transferred before operation and reverts at end of lease

Table 2.1: Rights Dimension: Possession-attribute characteristics for selected projects and acts (N/A: not applicable).

In respect of intellectual property rights, the government required the developer to grant a non-exclusive royalty free license to use or sub-license any intellectual property which will be vested in the government for purposes of construction and operation of the Fixed Link after the expiration of the agreement.

Second Severn Bridge

The tender invitation ("Second" 1989) stated that both crossings would be highways for which the Secretary of State is the highway authority. The Severn Bridges Act ("Severn" 1992), S-B Act, granted the British Secretary of State the right to construct the new bridge and to delegate all relevant functions and power to levy tolls to a private promoter according to a concession agreement. The S-B Act authorized the acquisition of lands and to grant a lease or other interest in or right over any land according to a concession agreement. Such lease was to be provided at a peppercorn (insignificant) rent as mentioned by the Tender Invitation.

The tender invitation provided for the transfer by the promoter of both crossings to the government at the end of the concession period. The S-B Act provided for the transfer of all property, rights, and liabilities of the concessionaire with no explicit definition of property.

Highway 104 Western Alignment

The RFP explained that the project as developed will be part of the public highway system and the ownership of the project facility at all times is vested in the Province of Nova Scotia. All the needed lands were acquired by the government, at its cost, and made available to the project. The RFP provided for the project facility to include the road and all improvements, buildings, erections and structures, and all chattels, machinery, equipment, materials, tools, forming part thereof or used in the construction or operation. The first addendum provided for construction equipment not to be part of the facility during project operation upon request from one of the RFP respondents. Along with public ownership of the facility, the RFP provided for the exclusive use and possession of the government of all project materials and information and their related patents, copyrights and other industrial and intellectual property rights including trade secrets. With such prior possession of the project by government, no transfer clause was included in the RFP.

Northumberland NSCP

The federal government explained as one of its objectives that the project be financed, designed, constructed, operated and maintained by the developer under a long-term subsidy agreement. The 1988 NSCP Proposal call explained that the development agreement would include a ground lease and a schedule of requirements, terms and conditions. The project facility was described in the NSCP Proposal Call to include collectively the lands, the work complete in all respects, with all operation and maintenance systems in place, and any other improvements or structures located on the lands. The work means all improvements and all appurtenances, which the developer agreed to construct on the lands.

For project possession, the proposal call states that "the work shall be fixtures to the lands and shall become the absolute property of the Landlord [Minister of Public Works] without compensation upon the expiration or termination of this lease". In the first addendum, the government explained further its intention by stating that the contract with the developer would be to build, to own, and to operate the facility for 35 years, after which it would be transferred at a nominal amount to the government. The second and third addenda explained that the nominal amount was meant to effect that the facility will revert to the government after the 35 year ownership period, and that alternative private sector ownership could be considered and be the subject of negotiation after selection. 'Own' was defined by the third addendum to mean: "to own the leasehold interest in the facility". In the sixth addendum the government stated that for purposes of financing and taxation, the project was a private sector venture.

State Route 91

AB680 ("Assembly" 1989) authorized Caltrans to enter into agreements for the construction by and lease to private entities of transportation projects. The Bill mentioned three types of lease: lease of rights-of-way (real property); lease of airspace over or under state highways; and, lease of the facility (private transportation project). Lease terms would be up to 35 years during which private entities would charge fees for the use of the facilities. Facilities would be state-owned at all times and revert to the state after expiration of the lease term at no charge.

In the SR 91 agreement ("Amended" 1993) Caltrans agreed to lease all of its rights, title to and interest in the real property, together with all improvements including the facility for a "construction lease" term and a 35-year "operating lease" term. Caltrans made available its power of eminent domain to be used in right-of-way acquisitions if requested by the developer (CPTC). Acquisitions would be made at all times at CPTC's cost. The SR 91 agreement provided for Caltrans to issue a notice of acceptance after the facility achieved substantial completion. On the acceptance date, the construction lease term would expire, CPTC would transfer title to Caltrans and the operating lease term would start. Upon expiration of the operating lease term, CPTC would be required to surrender the real property and the facility.

Several grants and rights were identified in the SR agreement. A 1.5-mile Absolute Protection Zone was defined to protect CPTC's franchise rights and economic viability. Under this provision, Caltrans would not finance, grant or convey any franchise to any party other than

CPTC for the development or operation of a public transportation project within the protection zone, unless the proposed facility did not represent economic competition to the project. CPTC was given the right of first offer and first refusal with respect to the development and operation of any commercial airspace improvement, over, under, on or within the State Transportation Facility, State Route 91 right-of-way, in Orange County, California. Such airspace rights could run up to 99 years. Further, CPTC was granted an option for the development of three phased extensions to the current facility to be exercised during the term of the agreement.

2.3.3.2 Rights Dimension: Revenues Attribute

The revenue attribute is the second right assigned by governments to developers. The investigation emphasized the requirements in connection with toll and revenue arrangements along with other rights that may be given to developers; a summary is given in Table 2.2. Typically, BTO and BOOT modes as implemented in US projects and acts, provide for freedom in toll setting and application of congestion pricing (except for the Virginia act, Table 2) while setting up caps on the rates of return. Governments, in general, tend to control the term of agreement through statements directed at early termination if debt or revenues are satisfied (Western Alignment, Severn Bridge) and with an indirect statement if rates of return are met (SR91 and US acts). Generally, governments seem to have two objectives: (1) control the amount of revenue generated by the project and (2) control the amount of revenue the developer is entitled to earn. Both works to achieve the equity[1] principle used in evaluating finance methods for public projects (Blackburn and Dowall 1991, Robinson and Leithe 1990).

[1] Equity is determined in two ways. The first holds that those who benefit from a service should pay for it. A user's ability to pay for a service is the second equity principle (Robinson and Leithe 1990).

Project or Act	PPP Mode	Revenue Characteristics	Toll Characteristics
Channel Tunnel	BOT	Term 55 years extended to 65. 'No second facility' guarantee. 'No interference in operation' guarantee.	Service levels and tariffs at promoters' discretion.
Second Severn Bridge	BOOT	Term 30 years. May be terminated early if entitled revenues are met, and may be extended to account for traffic levels.	Initial setting by developer adjusted to inflation. Toll may be adjusted to account for actual traffic flow.
Highway 104 Western Alignment	BOT	Term to continue until debt retired. Return to developer to come from construction, O&M, return on debt.	Tolls set to $3 per car, $2 per axle for truck, $4 for recreational vehicles. Toll adjusted for inflation and debt coverage ratio.
Northumberland Strait Crossing	BOOT	Term 35 years. Revenue floor established. No cap on revenues. Revenue distribution mechanism established. Subsidy Cdn$42 million/year indexed to inflation.	Pre-established tolls adjusted to 75% of CPI and adjusted to reflect changes in taxes and insurance premiums.
SR 91 and AB 680 California	BTO	Term 35 years. Reasonable return on investment with base return of 17% adjusted to T bonds. Incentive return, an additional 6%maximum. Excess revenues to State Fund.	Freedom in establishing and changing tolls. Congestion pricing.
PPTA Virginia 93	BOOT	A reasonable maximum rate of return. Excess revenues to State Fund or developer to reduce debt.	User fees to be established by the parties at negotiation. Tolls to achieve reasonable rate of return.
TREL Minnesota 95	BTO/ BOOT	A reasonable rate of return to be established. Residual revenues to developer.	Variable tolls are allowed based on time of day (congestion pricing).

Table 2.2: Rights Dimension: Revenue-attribute characteristics for selected projects and acts

Channel Fixed Link

Governments through the Channel Invitation and Concession Agreement offered political guarantees not to intervene in the conduct and operation of the Link and not to terminate the promoters' right to construct or operate a Link provided that the concession terms are adhered to. The developer was given commercial freedom in setting tunnel tariffs. The Agreement stated that "The Concessionaires will be free to determine their tariffs and commercial policy and the type of service to be offered". Earlier in the Channel Invitation ("Invitation" 1985), the government explained that the duration of the concession would consider the type of project selected and would be sufficient to allow repayment of debt and permit a reasonable return on equity. The concession period was initially set by the agreement as 55 years. Due to delays and cost overruns the concession was extended 10 years by the governments involved (Huot 1995).

Second Severn Bridge

The S-B Act provided the authority to levy tolls on both bridges to be exercised by the concessionaire. The government in the Tender Invitation required promoters to state the initial tolls required by each class of traffic proposed. Further, the invitation required the basis for any subsequent adjustment of tolls due to inflation, the index of inflation to be used and its weighting in the adjustment formula, components of cost to which it would be applied, minimum toll increases, and the time period between adjustments. The Tender Invitation allowed for differential tolls by day or date provided road safety was not impacted.

The government allowed for additional proposals for adjusting the toll level (and/or concession period) in order to take account of actual traffic flows diverging from the bid assumptions. For this case, detailed information was required regarding the mechanism for adjusting toll levels and/or the concession period, the traffic demand assumptions, and the upper limit for the concession period. As enacted later in the U.K., specifications for maximum tolls to be levied on new roads have been described in the New Roads Act ("New" 1991). This act provides for specifying maximum tolls if the road consists of a major crossing for which there is no reasonably convenient alternative. Toll periods in this act may end on a specific date, or be determined by the achievement of specific financial objectives, or passage of specified number of vehicles, or the earlier or later of specified dates.

The S-B Act provides the concessionaire with the power to levy tolls for a maximum of 30 years. The S-B Act provided, however, for early termination of this right if the revenue requirement had been met, i.e. the toll income received is equal to or greater than the amount the concessionaire is entitled to receive by the concession agreement.

Highway 104 Western Alignment

The W-A Act granted the Corporation the right to collect tolls and this became the responsibility of the selected respondent. The RFP provided for initial tolls and any proposed mechanism for increasing the initial tolls during the concession period to be established during negotiation of the Omnibus Agreement. However, the RFP stated that tolls would be sufficient to (a) pay the debt incurred to build the facility, (b) establish an operating and maintenance reserve, and (c) provide for required repair and rehabilitation work. The government included with the RFP a study of the current and future traffic volume and revenue forecasts for a range of toll road options. Beck, president of Canadian Highways, explained that tolls were initially set at Cdn $3 per car, Cdn $2 per axle for trucks and Cdn $4 for recreational vehicles (Beck 1997). Further, he explained that if debt service coverage were not met, tolls were to be adjusted automatically and that tools would be adjusted for inflation.

Government required that the Omnibus Agreement term be limited to the length of time toll revenues were needed to repay all the money borrowed or made available to pay for construction as well as to pay for any reserve requirements as mentioned previously. The government reinforced through the first addendum that the selected respondent must earn its return from the construction contract, operating contract, and a return on any debt, which the respondent chooses to hold. In the RFP addendum the, the government stated further that: "DOTC will give no assurance or guarantee that a fair market rate of compensation will be achieved by the Selected Respondent within the Concession Period to be fixed in the Omnibus Agreement."

Northumberland NSCP

The NSCP Act allowed the government to make regulations prescribing tolls for the use of the crossing. Toll collection was the responsibility of the developer and tolls were to be adjusted annually for 75% of the consumer price index (CPI). As explained in the proposal call and its first and fifth addenda, a toll revenue floor was established to be the greater of either $8 million in 1988 dollars or the actual toll revenues experienced by the ferry service in the full year preceding the date of substantial completion of the facility. It was explained also that toll rates may be increased by more than the permitted 75% of the CPI should toll revenues be lower than the established floor and additionally if tax changes or insurance premiums result in cost increases. Short falls in toll revenues were to be recouped in the succeeding year. With no explicit cap on toll revenues, the government required a separate account for toll revenues where the distribution of revenues would follow certain priorities. The toll distribution priorities included (1) payment of insurance premiums on a $150 million accident policy, (2) payment of interest and capital for the financing secured against toll revenues, (3) payments into a facility repair and maintenance fund, and (4) payment of the balance to the developer.

State Route 91

Caltrans entitled CPTC to establish, levy and collect tolls, fees and charges for the use of the facility. Toll adjustments and arrangements were at the discretion of CPTC without prior approval or evaluation of Caltrans. Further, CPTC was authorized by the SR 91 agreement to implement a congestion pricing arrangement to respond to dynamic traffic flows and to maintain the highest levels of service. According to the daily demand patterns toll rates move from \$0.5, \$1.0, \$1.5 and \$2.0 in four time zones with the rate being \$0.25 for off-peak hours. Rates for Monday to Thursday differ from those for Friday and the weekend (PWF 1995) while High-occupancy vehicles (HOV) pay no tolls. However, the SR 91 agreement provides for tolling HOVs after two years of operation if the debt coverage ratio is not met.

While Caltrans established no cap or control on toll rates, it established a 17% base return rate (BRR) for use in discounting calculations; this rate is to be adjusted annually, and upward only, according to the average yield on 5-year U.S. Treasury Bonds. CPTC is entitled to a reasonable return on investment (ROI), comprised of a base ROI and an incentive ROI for any fiscal year. CPTC is entitled to retain the available cash in any fiscal year as a base ROI whenever the base NPV calculated using BRR is less than zero. The incentive ROI is implemented to encourage CPTC to modify and improve the facility to maximize the number of vehicle occupants travelling during peak demand periods on the combined facility, SR 91. An incentive return rate gives 20 basis points (0.2%) increase on the base return rate for each 1% increase in the annual peak hour vehicle occupant volume; however, incremental increases may not exceed six hundred basis points (6.0%) for any fiscal year. If the base NPV is equal to or greater than zero, CPTC will share available cash for the fiscal year with Caltrans only if the total NPV calculated at the incentive return rate is less than zero; otherwise excess revenues will be directed to the State Highway Fund.

2.3.4 Obligations Dimension

2.3.4.1 Obligations Dimension: Development and Operation Attributes

This section deals with government requirements under the first three obligation attributes. For all projects, emphasis was placed on the spectrum of functions that governments require developers to be responsible for (planning, design, construction, environment, operation and maintenance); and the power governments have in project review, inspection and approvals.

Generally, the projects studied showed that all project functions are the responsibility of the developer unless government suggested or required certain functions to be its responsibility such as maintenance, traffic management, and police services. For example maintenance was highly encouraged to be provided by the government for the SR 91(BTO), Western Alignment (BOT), and the Minnesota TREL Act (BOOT/BTO). Bylaws required by developers (e.g. for traffic management) were generally subject to government approvals and could not compromise safety.

Under traditional procurement arrangements, governments have an active involvement in all project functions. For PPP, governments seek to maintain a role in those functions for which they have a responsibility for the public at large. The investigation showed that governments would provide for (1) the appointment of a representative or agent and consultants or independent engineers, (2) the default and substituted entity clauses in project agreements to handle cases of defaults by the proponent, and (3) the monitoring functions during development and operation.

Supervision and approval duties may undergo more scrutiny in public-private partnerships. Supervision provides for checking compliance with standards and specifications, and takes place while work progresses. Approval provides for accepting the work after it has been reviewed or checked. Approval may hinder the progress of work if it takes time to be done. Government will generally carry out both processes and promoters will seek strategies to speed them up, for example by having an independent engineer perform such functions, as in the NSCP case.

Generally, the approval process has been substituted or replaced by one or more processes dealing with inspection/monitoring, quality control and quality assurance QC/QC, with the possible role of an independent engineer who may provide (1) approval of design and construction as in the NSCP project, (2) quality control services during construction as in the Western Alignment project, and (3) review of performance during the design and construction as in the Channel Tunnel and Severn Bridge (BOOT) project. The SR91 project provided for government approval of design and inspection of construction and operation. Generally, however, governments provide for final inspection of completed work before they accept the work or authorize operation such as in the Channel Tunnel, Western Alignment, and SR91 projects. Table 2.3 summarizes the major characteristics of the obligation dimension.

The projects examined demonstrate that governments may direct or authorize changes to the work at their discretion as in the Western Alignment project or based on pre-agreed reasons such as in the Channel Tunnel and Northumberland Crossing projects where reasons included safety, defense, security, the environment, errors and omissions, or non-conformity. Time and cost adjustments as a consequence of changes may be subject to negotiations as with the Severn Bridge project, or added to the capital or operational costs as with the SR91 project.

Project or Act	PPP Mode	Development and Operation Attributes Characteristics		Financing Characteristics	
		Proponent Responsibilities	Government Responsibilities and Power	Proponent Responsibility	Government Support
Channel Tunnel	BOT	All development, operation, and maintenance functions. Bylaws[1] subject to approval.	Supervise and right to change. PM reviews compliance. Government to inspect completed work before operation is authorized.	Financing all phases, and raised based on rights conferred by agreement	No support. No financial guarantees.
Second Severn Bridge	BOOT	All development, operation, and maintenance functions. Bylaws[1] subject to approval.	Right to change. Agent monitors design, construction and O&M.	Financing all phase, inherit debt on existing crossing.	Promoter to operate 2nd facility
Highway 104 Western Alignment	BOT	All functions. QC/QA program. Maintenance management plan.	Right to change. Monitoring and QA. Provide regular maintenance services at cost.	Financing all phases. No lien to be made on the WA.	$62 million. Heavy truck to use WA.
Northumberland Strait Crossing	BOOT	All development and O&M. Environmental compliance. Regional industrial benefits.	Monitors and right to change. Independent engineer approves design, construction, and progress payments.	No mortgage of land and facility. Equity 10% or $75 million. $10 million prime cost for fisheries.	Cdn $ 42 million annual subsidy indexed to inflation.
SR 91 and AB 680 California	BTO	All development and O&M. Bylaws[1] subject to approval.	Approval of design. Right to change. Monitoring all phases. May provide at cost traffic management, maintenance, and police.	Financing all phases and based on rights in tolls, agreement, lease, project.	No support at all. Reimbursed for any service.
PPTA Virginia 93	BOOT	All development, operation, and maintenance functions. Bylaws[1] subject to approval.	Review and approve plans and specifications. Inspections of construction	May pledge property interest in the project.	May make grants or loans.
TREL Minnesota 95	BTO/ BOOT	All development, operation, and maintenance functions.	Provide maintenance and police services at cost. Regular inspections.	May pledge interest in the facility, agreement, income.	May issue bonds for toll facility.

[1] Bylaws required by proponents.

Table 2.3: Obligation-dimension for the selected PPP projects and related acts

Channel Fixed Link

The Agreement explained the developer's obligations to develop the Link in terms of design, construction, operation, and maintenance. The C-T Act provided for the concessionaires to make bylaws regulating the operation and use of the tunnel system, which were the subject of approval by the two governments.

Promoters were required to carry out an environmental impact assessment in both the UK and France. Promoters were required to be aware of the procedures of the International Maritime Organization (IMO) before starting the development such that no permanent structure (e.g. ventilation shafts, artificial islands) would hamper the freedom and safety of navigation. Other requirements included provisions for facilities and installations for policing the tunnel and for frontier controls (customs, immigration, and animal health checks) which the Concessionaires would pay for but which would be organized and performed by the two governments.

For the supervision of construction and operation, the governments authorized an intergovernmental commission and safety authority for the performance of these functions and required the concessionaire to comply with its directions. However, no strict approval process was mentioned in the agreement. The governments provided for their inspection of the completed work before they would authorize operation. An independent project manager, "Maitre d' Oeuvre", was appointed to review the performance during design and construction. The agreement explained that the concessionaire could proceed with the works relating to the 'Avant Projet' (project outline drawings and documentation list) unless the governments raised an objection to such Avant Projet. Huot (1995) noted that government imposition of the latest innovations, safety and other regulations, after the start of construction led to severe design changes and increased costs.

Second Severn Bridge

Through the Tender Invitation and the S-B Act, government required the promoter to take responsibility for the design, construction, maintenance and operation of the second crossing as well as for the maintenance and operation of the existing crossing. Other specific requirements included quality assurance, navigational requirements and environmental aspects.

The S-B Act gave a power of temporary prohibition or restriction of traffic to be exercisable by the concessionaire. The New Roads Act ("New" 1991) provided for similar power and for all highway functions to be exercisable by the concessionaire except for the power to make schemes, regulations, or give directions under the Road Traffic Regulations Act of 1984, U.K.

The government explained in the tender invitation for this project that special procedures would replace the normal technical approval arrangements. The invitation explained the appointment of a consulting engineer to work as a government agent to monitor design and construction, audit the promoter's quality assurance system, and possibly the maintenance and operation of the works. Further requirements explained that the promoter was required to employ the services of a designer under a formal contractual relationship such that the contract would ensure the designer was sufficiently independent from the promoter. That was required to enable the designer to comply with government requirements, check the promoter's proposed construction methods, materials, and each element of work. Along with that, the detailed design was required to be checked by an independent checker. The government agent was to receive certificates of satisfactory completion from the designer and the checker.

Design changes were allowed such that if the government issued a change, then the implications for the promoter's program and financial adjustment would be subject to negotiation. If the promoter issued a change, then it would be subject to the agent's approval with no financial adjustment to the promoter, who also would bear the consequences of any delay.

Highway 104 Western Alignment

The respondent obligations as set in the RFP included design, construction, operation and maintenance, repair and rehabilitation. The government required an environmental management plan for the facility. The government set a 20-month objective for completion and a guaranteed maximum price for design and construction. Also, it required a marketing plan to maximize the use of the facility. Approvals and permits were the respondent's responsibility. The government was prepared to provide maintenance, repair and rehabilitation services. Beck (1997), president of Canadian Highway International Corporation, the developer, explained that an annual maintenance agreement was signed with the government for regular maintenance services.

The RFP explained that the government at any time might direct or authorize changes in the work to be performed. The government through the RFP reserved its rights to undertake its own quality assurance activities. However, it was stressed that quality control and quality assurance (QA/QC) for the development, design, and construction were the responsibility of the respondents. The first addenda explained that members of the developer could perform QC, however QA had to be performed by an independent material testing firm and laboratory. The respondent was to be responsible for any corrective action due to non-compliant test results.

For the operation and maintenance of the road, the government required the preparation of a road maintenance management plan explaining performance specifications, maintenance functions, and how the respondent would perform such functions. The government would evaluate periodically the performance of the respondent according to this plan.

Northumberland NSCP

The developer's obligations included all the development and operation functions including design, construction, operation and maintenance. A service life of 100 years was a design requirement. Extensive environmental reviews and assessments of the biophysical (e.g. air, marine and terrestrial life) and socioeconomic (e.g. labor issues, regional benefits and affected businesses) consequences of the crossing were required for the project and the developer was required to comply with all the requirements. A fixed crossing was considered to pose a threat of delaying the clearance of ice from the Strait. It was thought that such an "ice-out" could delay the start of the fishing season and could reduce the local temperature which in turn could delay the spring planting of crops (FHWA 1996). All designs were assessed against a 2-day delay in ice-out in any year over a period of 100 years. Developers were required to comply with this maximum ice-out delay among other requirements, which were addressed by the developer in its commitment to develop a plan for the management of all environmental aspects of the project in the Final Registration for the project ("Northumberland" 1993).

Among the other obligations, the developer was required to maximize the economic and industrial benefits to the Atlantic region regarding businesses, employment, purchasing (material, equipment, supplies, and services), and technology amongst others. The regional benefit agreement signed for the project included several covenants on the developer such that 70% of all materials, 96% of labor, at least Cdn $20 million of engineering work after closing, and 75% of all marine workers had to be procured from the Atlantic Provinces region (FHWA 1996).

Monitoring performance during design, construction, commissioning, operation and maintenance, was one of the government's roles for the project. The government at no cost to itself could request changes to the work if the reason of change was due to errors, omissions, or non-compliance on the part of the developer. Time for additional work resulting from changes authorized by the government would be negotiated. An independent engineer was appointed for the review of design, construction, operation and maintenance procedures. Work approvals were the subject of negotiation. The government wished to retain the right to approve construction work and progress payments. A compromise was reached where the independent engineer would approve construction work and monitor the cost to complete of major work items (Pirie 1996).

State Route 91

Caltrans through the SR 91 agreement required CPTC to design, develop, acquire, construct, install and operate the project transportation facility. Along with allocating such responsibilities to CPTC, Caltrans offered to assist CPTC in preparing and presenting documents required to obtain any permits and approvals needed for the project. For the operation of the project CPTC was responsible for performing the administrative, toll collection and traffic management activities. The AB680 and the SR 91 agreement encouraged CPTC to pursue possible contracts with Caltrans to perform traffic management activities and maintenance, and with the California Highway Patrol for police services.

Environmental studies for the facility were CPTC's responsibility. CPTC was required to prepare all documents for environmental clearance and analysis in order to obtain all of the necessary permits and approvals. Final approval of the project and commencement of construction were contingent on meeting requirements of the California Environmental Quality Act.

The SR91 agreement explained that Caltrans had the right to review and approve the design prior to commencement of construction. The approval process was limited to validating that the design was in accordance with the Caltrans design standards cited in the agreement, and provided for Caltrans objections or approvals within twenty-one days. Construction of the facility was required to be in accordance with standards and specifications described in the agreement. Caltrans provided for overseeing CPTC' compliance with such standards.

2.3.4.2 Obligations Dimension: Financing Attribute

For all of the projects studied, emphasis was placed on the promoter's financing responsibilities, the security used in raising finance, and the form of government support to the project, if any. Generally, for the projects and acts examined, government provided for all financing risks to be carried by the developer. Further, no financial guarantees were provided. However, support was provided in terms of (1) a direct subsidy as in the NSCP project, (2) operation of existing facilities as in the Severn Bridge, and (3) establishing a policy in favor of the facility such as in the Western Alignment. For purposes of calculating capital, operating and maintenance costs, governments generally require developers to maintain reserve funds such as a working capital reserve fund, maintenance and capital improvement reserve fund, and a debt service fund.

To enable lenders to provide finance or credit support governments generally, allow the developer to use an umbrella of security instruments that cover the developer's interests in and rights under development, lease, and any project related agreements; tolls, income and project revenues; and, all developer's shares. However, as explained below for BOT and BOOT projects and as shown in Table 2.3, governments restrict the use of the project land and facility (i.e. improvements) as security. This restriction is imposed as a government requirement even for a project for which the developer has private possession/ownership, i.e. BOOT, such as the Northumberland NSCP project, and the Severn Bridge project.

However, there are cases where such restrictions may be relaxed until the occurrence of a stated condition or phase such as in BTO procurements. For example, the SR 125 franchise agreement ("Development" 1991a) explained that the financing assignment used as debt security might cover the developer's interests in all or any portion of "(i) the franchise documents, (ii) the project [toll highway, real property on which such toll highway will be located, personal property and intangible property], (iii) project revenues and/or (iv) any other property or rights (including operating rights) of developer." It was explained that such a financing assignment should not be made in a manner that precludes passing of the project title to Caltrans on the title transfer date before the start of operation. A similar assignment was made for the Mid State Tollway project ("Development" 1991b), however, it covered only the real property of the project. Both the SR 125 and Mid State Tollway projects were under the AB680 bill ("Assembly" 1989).

The Virginia Act ("Public" 1995) provides the power necessary to the project developer/operator such that it could acquire, construct, improve or operate the facility. The act stated that the operator may "... secure any financing with a pledge of, security interest in, or lien on, any or all of its property, including all of its property interest in the qualifying transportation facility". Similar provisions for the use of facility as security were included in the cancelled Texas High Speed Rail franchise agreement ("Franchise"1991).

Channel Fixed Link

Governments through the terms of the Channel Invitation ("Invitation" 1985) and the Concession Agreement (1986) ruled out all support from public funds or government guarantees and required financing to meet all construction and likely cost overruns and delays. In its White Paper ("The Channel" 1986), the U.K. government explained that for the evaluation of proposals, solid financing commitments coupled with the ability to attract financing were the final test for the evaluation, which was best met by Eurotunnel's proposal. Financing was entirely the responsibility of the promoters and was to be raised based on the rights conferred in the agreement to the promoter. The amount of equity capital was left to the determination of the promoters, however, it was expected to be substantial.

The Channel Invitation explained that full information on the promoter's anticipated capital structure, proposed time for calls on the various markets, and expected amounts to be raised on each of these markets were required by government. Further, as evidence of the robustness and viability of proposals, a detailed financial plan and a cash flow forecast along with related assumptions were required from the promoters. Detailed annual financial forecasts up to ten years after repayment of debt were also required including assessment of costs, traffic, measures of profitability, and related assumptions. Promoters were required to show the sensitivity of the project's economics to variations in traffic flow, cost overrun, delays in completion, and changes in interest and exchange rates. Similar analyses were performed by the promoters in the project information memorandum presented to the lenders in preparation to raise credits (Roger 1990).

Holiday et al. (1991) reported through Eurotunnel reports that initial construction costs for the project was £ 2.3 billion in 1985. In 1990, total project cost was £ 7.608 billion, of which £ 4.208 billion for construction and £ 3.4 billion for corporate, inflation, and financing costs. Project cost at completion in 1994 was £ 9 billion, as reported in the Eurotunnel web site[2]. Finance was raised through several debt and equity tranches. Equity was in four tranches totaling £ 1.589 billion. Debt finance, £ 7.123 billion, was arranged though a large syndicate of lenders from around the world. Total debt and equity finance, therefore, was £ 8.712 billion raised between 1987 and 1990 and was advanced in several currencies (Holiday 1990).

[2] http://www.eurotunnel.co.uk. and http://www.channeltunnel.co.uk

Second Severn Bridge

The Tender Invitation (1989) required the promoter to finance both the existing and new crossings and inherit an estimated debt of £ 122 million for the existing bridge. The government required that "proposals involve no material risk on financial grounds regarding the completion of the second crossing to time and specification, the acquisition of the concession and existing crossing, and the operation and maintenance of both crossings."

Financing of the Second Severn Crossing involved a fully underwritten facility of £ 340 million arranged by the Bank of America in 1992. For the lenders the commercial viability of the project was very promising because of the higher traffic levels and congestion on an existing Severn crossing (5 km from the Second Severn Crossing) and because the project company would operate both bridges, creating a monopolistic structure that attracted lenders. Moreover, as mentioned in the tender documents for this project, adjustments for project tolls were allowed during the concession period based on changes in inflation rates.

Highway 104 Western Alignment

The government required the project to be entirely self-financing apart from Cdn $ 29 million under the SHIP Agreement (Canada–Nova Scotia Strategic Highways Improvement Program), which was raised to Cdn $ 55 million by the second addendum. The corporation was to borrow money without recourse to the government. The government explained that "it will not guarantee any debt incurred by the selected respondent or corporation". The government established a policy whereby all heavy trucks, except for local traffic, would use the Western Alignment.

The W-A act (1995) explained that the corporation could borrow money based on its own credits, and could secure its borrowings against any or all of its assets and undertakings and the revenue arising from the collection of tolls. The act explained that "no debt of the corporation constitutes any lien or other charge on the Western Alignment". Beck (1997) explained that Cdn \$62 million toll revenue bonds were used to finance the project.

A detailed project cash flow model and pro-forma financial statements were required reflecting forecasts and estimates for each year of the concession period. For evaluation purposes, government required the preparation of two sets of financial statements with accompanying cash flow models for two sets of toll revenue forecasts provided with the RFP. Assumptions for both sets included a 2.35% inflation rate, a 20-month completion period, a 35-year concession period, Cdn \$ 650,000 annual maintenance cost, and an 8.25% yield on 30-year Canada Bonds.

Northumberland NSCP

The government explained in the proposal call and the NSCP Act that its annual subsidy to the existing ferry service would be Cdn \$42 million (1992 prices). This subsidy was provided to reduce the government's cost to maintain its obligation for continuous communication with PEI. The annual subsidy was to continue for 35 years commencing with the operation of crossing and indexed 100% to the consumer price index.

The government in the sixth addendum insisted that investors should be aware that the project was a private sector venture and the subsidy should be considered as income to support toll revenues. The goal of the federal government was to have its participation "off book". However, the Auditor General of Canada subsequently ruled that the NSCP project financing had to be considered a debt obligation 'on balance sheet' of the federal government (TFPPP 1996). This subsidy was used by the developer to raise about Cdn \$660 million. Pirie (1996), Vice President of Strait Crossing Inc., explained that based on the subsidy, real-rate bonds paying a yield of

4.5% plus the annual inflation rate were issued and were taken up mainly by pension funds. Later the developer negotiated a reinvestment strategy for the bond proceeds to maximize the use of the loan considering the project's anticipated drawdown schedule.

Equity for the project, as explained by the fifth addenda was required to be the lesser of 10% of total project cost (including direct and indirect cost, interest during construction, contingencies, start-up costs and working capital) or Cdn \$ 75 million. Instead of requiring the deposit of equity up-front in a trust account, the government allowed the developer to pay in equity pro-rata (supported by a letter of credit) with debt proceeds during the course of the project. The developer was also required to designate a 'Prime Cost Sum' of Cdn \$ 30 million (reduced to Cdn \$10 million) for disbursement at the government's discretion for fisheries compensation.

Along with the above requirements, the government required that the agreement between the developer and his financier include provisions to reflect that progress payments be disbursed only after progress certificates were signed and issued by the developer and an independent consultant. Pirie (1996) explained that during negotiations the involvement of the independent consultant was kept to major work items only.

The government emphasized in the RFP and first addenda that neither the crossing nor the lands could be mortgaged or pledged as collateral by the developer in any way and were incapable of seizure by the developer's creditors. By the third addenda the government explained that it would permit some form of mortgage or pledge to the extent necessary to permit the placement of the required mezzanine financing (subordinated loan). However, by the fifth addenda the government emphasized its earlier restriction and added that "lenders will have available an assignment of cash flow security through the trust accounts and certain insurance proceeds".

State Route 91

Financing was the responsibility of CPTC. Caltrans explained in the SR 91 agreement that it had no responsibility to meet any debt incurred by CPTC for the development and operation of the facility. Caltrans explained in the proposal guidelines ("Guidelines" 1990) that the development had to be performed and completed at no cost to the State. All services provided by Caltrans were to be reimbursed by the developers. This included reimbursement for optional services requested by the developer (e.g. traffic projection, maintenance, police services, etc.), and reimbursement for non-optional services performed to protect the State's interest (e.g. costs associated with proposal selection, review right-of-way acquisition, design and construction oversight and technical activities, etc.). CPTC was required to maintain a number of reserve funds for working capital, major maintenance, capital improvements, and debt service.

Financing and debt security instruments referred to in the SR 91 agreement as leasehold mortgages were made based on CPTC's interest in the agreement, the lease, the project facility, and the tolls and profits of CPTC. Rights of leasehold mortgagees were subject to the provisions of the SR 91 agreement. The agreement stated that no CPTC default would be grounds for termination by Caltrans of the agreement or the lease until all remedies raised by Caltrans in a default notice in a cure period were met by CPTC or its leasehold mortgagees. Equity paid by the CPTC was $19 million. Taxable finance raised by CPTC included $35 million 17-year institutional debt, $65 million 14.5-year variable rate term loans, and a $7 million subordinated loan from the Orange County Transportation Authority (PWF 1995).

2.3.5 Liabilities Dimension

2.3.5.1 Liabilities Dimension: General Liability Attribute

The general liability attribute is the first attribute in the liability dimension. Table 2.4 summarizes the general characteristics of the general liability, risk, and tax attributes of this dimension for the projects examined. Governments generally require developers to maintain liability insurance policies sufficient to: (1) insure coverage of tort liability (claims arising on account of personal injury or death or damage to real or personal property) to third parties, users, and employees; (2) protect against physical loss or damage to the facility in order to ensure continued use of the facility; and, (3) provide protection against business interruption (loss of income or earnings due to an insured peril such as delay in start-up/completion). Other policies may be required particularly if government provides support or will carry risk if the project is not completed such as in the Northumberland Strait Crossing project. Exceptions, however, can be made to relieve a developer from part of the liability coverage as in the SR91 project.

Channel Fixed Link

The two governments required the promoters to be liable for damage caused to users of the Link and third parties. Two insurance programs were required, one during construction and one at start of operation which had to be renewable on a one, two, or three year basis. The risks to be insured included (1) physical damage to the Fixed Link, (2) tort liability to third parties, and (3) delay in start up and interruption of operations resulting from facility physical loss or damage. Such requirements proved to be invaluable when in November 1996 a fire erupted in the freight shuttle train and caused serious damage to the concrete lining. As a consequence the tunnel was closed for 16 days with revenue losses per day in the order of £ 1 million. The damage cost was approximately £ 230 million and insurance coverage repaid about 98% of the cost (Bennette 1997).

Project or Act	PPP Mode	Liability Attribute	Risk Attribute	Taxes Attribute
Channel Tunnel	BOT	By developer for all phases. Insurance: third parties, facility damage, start up delay and business interruption.	All by developer. Time extension for FM[1]. Government compensation for defense interruption.	All taxes by developer.
Second Severn Bridge	BOOT	By developer for all phases. Insurance: third parties and facility damage.	All by developer. Performance bond. Time extension for FM. Government retains war repairs.	Capital allowance for corporation tax. Value added taxes on construction.
Highway 104 Western Align.	BOT	By developer for all phases. Insurance: third parties, facility damage and business interruption.	All by developer. Performance and ML[2] bonds. Time extension for FM.	All taxes by developer. The non-public corporation is exempted.
Northumberland Strait Crossing	BOOT	By developer for all phases. Insurance: third parties, facility damage and business interruption.	All by developer. Performance, ML bonds, and others. Time extension for FM[1]. Government retained certain FM repairs.	All taxes by developer. Provinces to give municipal and provincial property tax exemption.
SR 91 and AB 680 California	BTO	By developer before acceptance date, and shared thereafter. Insurance to third parties and facility damage.	All by developer. Performance and ML bonds. Time extension for FM. Government retained certain FM repairs.	All taxes by developer. Developer may depreciate the improvements. Franchise fees to be reduced by taxes.
PPTA Virginia 93	BOOT	By developer for all phases. Insurance: third parties and facility damage.	All by developer. Delivery of performance and ML bonds.	N/A.
TREL Minnesota 95	BTO/ BOOT	N/A. In TH 212[3] (BTO) the developer was to carry all liabilities and government to carry liabilities from developer's compliance with design and construction standards defined in the agreement.	All by developer. In TH 212, included performance and ML bonds acceptable to lender or completion guarantees, time extension for FM.	N/A. In the TH 212, developer pays all types of taxes.

[1]FM: Force Majeure. [2]ML: Material and Labor Bond. [3] proposed based on TREL legislation.

Table 2.4: Liabilities-dimension for the selected projects and acts

Second Severn Bridge

During construction the government required full contractor's all risk, third party, and employer's liability coverage. During operation, the invitation required insuring both crossings against all loss or damage. No explicit coverage was mentioned for liabilities of third parties during operation. However, the tender invitation required the promoter to indemnify the government against any liabilities to third parties arising out of development and operation of the two crossings.

Highway 104 Western Alignment

The RFP required the corporation to indemnify and hold harmless the government against any and all claims, damages, losses, liabilities, costs and expenses arising out of the performance or non-performance by the corporation in relation to the design, construction, maintenance and operation of the facility. The RFP required the respondent to maintain throughout the concession period a liability insurance coverage acceptable to the government. The fourth addendum described the insurance and bonding requirements during construction and operation to include (1) coverage for all risks of property damage to the facility and (2) coverage to protect against all claims of liability arising out of property damage, bodily injury including death and personal injury.

Northumberland NSCP

To protect itself from being required to pay the subsidy payments and operate the ferry service or complete or repair the work (i.e. double payments), the government took certain precautions. A very expensive insurance coverage pre and after completion was required in the sixth addendum to (1) preserve the work (property) against all risks of physical damage, (2) to pay damages arising from claims from third parties for injury, death or loss of property, and (3) to reimburse the government the cost of the subsidy or of providing ferry service if the completion date was not met. Pirie (1996) explained that Strait Crossing Inc. managed during negotiations to change the insurance limits so that they were based on the maximum foreseeable loss rather than the full replacement cost/value required by the government.

State Route 91

Caltrans through the AB680 and the SR91 agreements provided for CPTC, the developer, to be protected and indemnified by the Tort Claim Act. Reasons and explanations for this protection included (1) Caltrans authority and obligation to supervise and provide specifications and operational requirements for the design, construction, and maintenance of the project, (2) Caltrans to hold title to the real property and facility, and (3) the designation of the facility to be deemed part of the state highway system. This enabled savings to CPTC which otherwise would have been reflected in the toll rates.

Before the transfer of title to Caltrans, the agreement provided for CPTC to bear the risk of injury, loss or damage to the facility. Third party claims, except those that arise out of CPTC fault, were carried by Caltrans if these included claim that arise out of fault of Caltrans, any non-negligent actions taken or omitted by CPTC in compliance with any Caltrans permits or regulations, or design and construction which conforms to the standards in the agreement. The same also applied after the acceptance date. However, Caltrans also assumed further the tort claims arising out of any act or omission in connection with traffic management and maintenance activities for which it is responsible. CPTC was required to maintain throughout construction and operation, bodily injury and property damage liability coverage of at least $50 million general aggregate per year.

2.3.5.2 Liabilities Dimension: Risk Attribute

Governments in general seek procurement by PPP in order to transfer more risks to the private sector than can be done using conventional procurement arrangements ("Paying" 1993). Three categories of requirements can be distinguished from the RFPs and agreements for the projects studied: (1) risks related to the developer's obligations including financing, (2) risks related to the developer's rights particularly those related to revenues, and (3) force majeure risks. An overview of the risk attribute for the selected projects is outlined in Table 2.4.

For the first category of risks, governments generally allocate all development and operation risks to the developer in clear wording in the RFPs and agreements. Emphasis was placed on explaining that government monitoring, inspection, and quality assurance processes did not relieve the developer from his responsibility for the work. This is different than government directed changes to the work for which time and cost consequences may be negotiated as explained earlier. Further, for the allocation of risk, governments usually require completion guarantees, performance bonds, and labor and material payment bonds with amounts that vary according to each project's circumstances. As PPPs are vehicles used to derive private finance, financial risks are the responsibility of the developers.

The second category under the risk-attribute deals with revenue risks. The general requirement is for developers to carry all such risks with no guarantees. Governments, may provide (1) adjustments for facility rate and/or term of agreement to account for some risks such as inflation, and actual traffic growth rates as in the Severn Bridge, and (2) policies to protect the developer's revenues from competing facilities through a 'no second facility' guarantee as in the Channel Tunnel, or 'absolute protection zone' for the SR91 project.

The third risk category deals with force majeure risks. The definition of force majeure varied among the projects studied. It is helpful to categorize force majeure risks in general to include (1) war actions - including war, invasion, acts of foreign enemy, and nuclear events (2) civil actions - including riots, insurrection, act of terrorism, sabotage, and strikes, (3) government actions - including expropriation, changes in law, interference by civil or military authorities, and (4) natural catastrophes - including floods, earthquakes, unforeseeable geological conditions, chemical contamination, and epidemics.

When force majeure risks are realized, governments in general provide developers with a time extension for the performance of their obligations. Cost consequences, however, vary among projects and may more usefully be considered along with the insurance coverage for facility physical damage and loss generally required from developers. Governments in general provide no financial compensation for force majeure risks except for war actions, as defined above, for which the government provides compensation or retains the risk and carries the cost of repairs, such as in the Severn Bridge (BOOT), Channel Tunnel (BOT), and Northumberland Crossing (war and extreme catastrophes) (BOOT). In SR91 (BTO), for all force majeure risks government will restore land and reinforcements to restore the weight-bearing capacity of the real property.

Channel Fixed Link

Both the Channel Invitation and Concession Agreement emphasized that the Link would be constructed and operated at the promoters' own risk without recourse to the governments. For force majeure risks or exceptional circumstances, the agreement explained that the time allowed for the performance of obligation would be extended accordingly. However, no compensation would be made to the concessionaires due to interruption of construction or operation based on such risks. However, if interruption occurred based on national defense, the concessionaires would be compensated. If these conditions/risks lead to the termination of the concession, "no compensation will be made to the concessionaires but the Principals may pay to the Concessionaires such amount which takes account of the net financial benefits, if any, to the Principals resulting therefrom". As mentioned earlier, insurance coverage was required for physical loss or damage to the facility arising from civil actions and natural catastrophes.

For financial and revenue risk, along with the requirement for no-recourse to government funds, governments gave concessionaires the freedom to determine their tariffs and commercial policy. Further, the government undertook not to facilitate the construction of another fixed link whose operation would commence before the end of 2020.

Second Severn Bridge

Through the Tender Invitation, the government required all design and construction risks to be allocated to the developer. The government explained that its agent appointed to monitor design, construction, operation and maintenance would not relieve the promoter of any of his responsibilities. The government transferred all geo-technical risks arising from physical conditions and artificial obstructions to the developer. A substantial on-default performance bond and/or parent company guarantee was required from the developer.

Basically, the developer was responsible for the care of works including cost of repairs from any causes except for force majeure risks for which compensation and time extension would be allowed. These force majeure risks did not include natural catastrophes. Insurance for physical loss or damage of the crossing was required as mentioned earlier.

Revenue risks related to changes from initial traffic volume and traffic growth forecasts were transferred to the developer. Further, the government stated that it would not be liable for any loss of revenues arising from any closure of approach roads to the crossings. The government provided traffic records on the existing crossing and projections for future levels of traffic. However, it assumed no liability from the use of such projections ("Second" 1988, 1989). Given the provisions dealing with toll adjustment and the variable concession period, which account for the actual traffic flows, the cost of such risks to the developer was reduced.

Highway 104 Western Alignment

The government stated in the RFP that the developer must assume all project risks and for that it is entitled to earn a fair market rate of return commensurate with the risks assumed. A guaranteed maximum price of design and construction was required along with an objective of 20 months to open the road. Performance and labor and material bonds in the amount of 50% of the maximum price were required for the construction phase. A performance bond was also required for the operation phase in an amount of 50% of the annualized contract for operation and maintenance.

For force majeure risks, the RFP explained that an extension of time for completion of the road would only be allowed for the affected activities on the critical path of the project. No time extension for force majeure would be made unless it was filed within seven days of its first occurrence. An "All Risk" property insurance policy was required from the developer.

For financing and revenue risks, the government stated that it would not guarantee any debt incurred by the respondent nor the corporation, forecasted traffic levels, and any factors that might impact revenues or costs. However, it covenanted all heavy trucks to use the highway.

Northumberland NSCP

The government explained in the RFP that the developer must bear all project risks during both the construction and operation periods with the exception of legal challenges and regulatory impediments risk (delays and cost increases directly attributable to government actions). The government explained that its inspection and independent check of the work did not relieve the developer of his responsibility for the work.

The government through the NSCP proposal call ("Northumberland" 1988) and its addenda required the developer to provide a security package that assured the completion of the facility, assured the specified level of operating performance, assured the specified condition at the time of turnover, and assured the interim funding of the ferry service. Pirie (1996) explained that the security package against completion risk and cost overrun included along with parent company guarantees, a cover of Cdn $ 200 million performance bond and a Cdn $20 million labor and material bond along with a Cdn $ 73 million letter of credit for cost overrun risk. Further, the developer agreed to pay the operating cost of the ferry in case of completion delay.

Through the terms of the NSCP proposal call and the first addendum, the government relieved the developer from its responsibilities for the normal operation of the facility and completion of the facility in four force majeure cases: (1) acts of the Queen's enemies, (2) government retroactive legislation, (3) earthquakes in excess of design criteria, and (4) a catastrophic event. A catastrophic event was defined as an event which damages the facility and renders it inoperable. Under such circumstances and where the government was bound by its constitutional obligation for "continuous communication", the government required the developer to provide as part of the security package reimbursement of an amount equivalent to the subsidy paid during the period of time the government assumed responsibility and operated the crossing service.

Pirie (1996) explained that during negotiations with the government, force majeure risks were replaced by what was defined as 'Project Risk Event'. Project risk events are retained by the government under the occurrence of acts of war, acts of government, extreme weather conditions, earthquakes beyond certain standards, and nuclear event; while the normal force majeure risks were carried by the developer. 'Project delay event' was another negotiated concept that described events beyond the developer's reasonable control such as contaminated material, third party strike or walkout. The realization of such delay events could, subject to negotiations, provide the developer with time extension and toll adjustments.

State Route 91

In addition to liability requirements for facility damage and tort, Caltrans required CPTC in the SR 91 agreement to furnish payment and performance bonds or completion guarantees. Yet, with no specific amounts requested, Caltrans required such bonds to be acceptable to CPTC's lenders. For events of force majeure, CPTC's time to perform its obligations would be extended by an equal amount. Where the force majeure event damaged or destroyed all or any part of the real property, Caltrans would be obligated to restore the land, grading and reinforcements necessary to restore the weight-bearing capacity of the real property immediately prior to such event. However, the agreement explained that failing to restore the land should not be considered default if Caltrans had also declined to restore the land on the state transportation facility (SR 91 is a median improvement to State Route 91 that includes an adjacent SR 91 free highway).

Strong protections were provided by the SR 91 agreement for CPTC against Caltrans' default, event of loss, and change in law. In the agreement Caltrans stated that it would not grant, nor convey to any other party other than CPTC, and would not finance with public funds, the development of a transportation facility that might present economic competition to the project within the absolute protection zone. Failure of the application or performance of representations, warrants, and obligations would constitute a default by Caltrans, while failure to comply with covenants or requisition of title or requisition of use would constitute an event of loss. Both entitled CPTC to remedies, compensation, and/or termination of the agreement and the lease.

The agreement explained that under a "change in law" that adversely impairs CPTC's exercise of its property, franchise and other contract rights, CPTC could elect to close the project and seek payment by Caltrans of all unrecovered costs at the date of calculation (capital and operating costs, interest on debt, distribution to equity investors minus total revenues at that date). Caltrans stated in the agreement that it would protect and defend CPTC against any challenges to the validity or enforceability of the acts and challenges to the enforceability of the agreement.

2.3.5.3 Liabilities Dimension: Taxes Attribute

The treatment of taxes is the third attribute under the liabilities dimension. Generally, governments require developers to be familiar with all tax rulings (e.g. corporate, income, and property taxes) that might apply to their proposed business structure. Further, governments make no representations or warrants to the tax consequences or accuracy of the developers' proposed business structure. Summarized in Table 2.4 are the tax attributes of the projects examined.

Governments, according to the circumstances for each project, may provide for certain vehicles to support project development. These vehicles may include exemptions for certain types of taxes such as the exemption of property tax in the Northumberland NSCP project (BOOT), capital allowance such as in the Severn Bridge (BOOT), or creation of a corporate body with special characteristics such as in the Western Alignment (BOT).

Channel Fixed Link

The Channel Invitation ("Invitation" 1985) explained that the principle of territoriality of taxation will be applied where each country will apply its normal laws to the construction, maintenance and operation of that part of the project falling within its jurisdiction. The requirement for the levying of taxes was set also in the Concession Agreement (1986) requiring that "all duties and taxes levied or to be levied, including taxes on immovable property, will be liabilities of the Concessionaires and will be applied according to the provisions of national law".

Second Severn Bridge

The promoter as mentioned in the tender invitation was to be treated as "trading" for corporation tax purposes and would be able to claim capital allowance for construction expenditure. The Tender invitation explained that value added taxes were payable on construction and exempted for project tolls. The invitation mentioned that local authority rates would be excluded.

Highway 104 Western Alignment

The RFP explained that the government made no representation or warrants concerning the tax or legislative consequences of any structure used by the respondent. Further, it explained that the respondents must satisfy themselves about the consequences of the provisions of Canadian and Provincial tax laws. The government did not entertain special tax concessions.

The Western Alignment Corporation, not a public authority or crown corporation, was created by the W-A Act to assist the RFP respondent in the realization of the project (development and finance). The W-A Act stated that neither the corporation nor its property was liable to taxation including income tax under any enactment. The government required the RFP respondents to satisfy themselves as to the tax status of the corporation.

In the first addenda, August 1995, to the RFP the government was asked about sales taxes (GST) and it emphasized that "each respondent is responsible for obtaining its own advice as to all tax matters" and added "if necessary the corporation will be declared an agent of the Crown in relation to its toll collection activities".

Northumberland NSCP

The government explained that the development was designed as a private sector venture and the developer's corporate structure was required to comply fully with both the letter and the spirit of the Income Tax Act of Canada in order to be accepted. The developer was required to satisfy itself and make appropriate allowances in regard to all taxes of every nature and kind that may be imposed on the facility, improvements, equipment, or any property brought on lands. The government explained that special tax concessions would not be entertained. A potential increase in sales tax liability was considered a business risk which must be assumed by the developer. The Provinces of New Brunswick and Prince Edward Island were considering exempting the crossing facility from municipal and provincial property taxes.

State Route 91

The SR91 agreement explained that all taxes imposed on the real property and the project were the sole responsibility of CPTC as part of its capital and operating costs; despite the fact that the real property and the project were to be considered property of Caltrans at all times. The agreement, however, provided for franchise fees (base, variable, and excess) to be reduced by the amount of taxes after title transfer to Caltrans. CPTC was concerned about depreciating the project, after title transfer to Caltrans, and was advised it could depreciate the improvements.

2.4 Requirements Structure: Conclusions

The above investigation explained key parts of the scope of government requirements for each of the eight attributes of the requirement structure as summarized in Tables 2.1 to 2.4. The output or benefits of the requirement structure can be summarized in three points.

- The requirement structure is a framework through which the characteristics and requirements of a project during its life cycle can be usefully studied and analyzed in a comprehensive manner.

  The requirement structure was useful in explaining how governments implemented BOT, BOOT, DBFO and BTO procurement modes for a number of PPP projects. Alternatively, the structure can be used to identify arrangements under several PPP modes and assist in formulating other modes. The structure shows that for the traditional public procurement all eight attributers are under the responsibility of the government, while under build-own-operate or full privatization all eight attributes are the responsibility of the private developer. Between these two extremes are a number of responsibility allocations which lead to other PPP procurement modes.

  The structure can be used by government in negotiation with developers such that the eight attributes can be assigned to achieve a balance between the rights, obligations, and liabilities of the private developer.

- Categorizing the requirements through the eight attributes should help in distinguishing the features of each attribute such that their key characteristics and variables can be treated quantitatively and/or qualitatively in economic models and/or analytical frameworks and the uncertainty or risk surrounding them can be analyzed. That represents the main theme of the following chapter and the rest of this thesis.

A framework based on the requirement structure assists in making decisions about the details of the eight attributes and on which allocation of the attributes will result in the best value for the public at large. For example, it can be used to help gauge what the benefits would be (e.g. reduction in facility rates as a requirement in the revenue attribute) if ownership of the facility (possession attribute) were in the private sector hands during the concession period?

- Clarity with which the terms and conditions in tender documents, or specifically the eight attributes of the requirements structure, need to be emphasized for PPP projects.

The range of terms and conditions of each attribute as explained by the requirement structure, the large number of supporting documents to RFPs, the long negotiation process between government and developers, and the involvement of several stakeholders in PPP decisions suggest that the rights, obligations and liabilities in PPP projects need to receive more analysis in general and particularly when appraising and preparing for PPP projects. The clear identification and articulation of government requirements is needed in order for developers to carry out properly their economic appraisals of projects and to respond with proposals that fit the requirements, reduce the amount of time spent in negotiations, and reduce the amount of RFP supplemental materials. Specific questions that need to be addressed under the various dimensions are identified below.

Rights Dimension

A clear statement of possession requirements in general and with each property type is needed. Items that should be addressed include:

1. Types of project properties (land, improvements, airspace, intangible property);
2. Type of possession permitted for each type (e.g. public, private, lease);
3. Properties, if any, that can be taken as a security instrument;
4. Clear title statement during the different phases and terms of agreement;
5. Who will carry the responsibility for the acquisition of land and right-of-way and its related costs (e.g. government, developer, or both); and,
6. Properties which are the subject matter of reversion, transfer, or dedication at the expiration of the agreement or at default.

While many of the revenue terms are kept for the negotiation phase, explicit statements regarding project revenues are needed for the following:

1. Term of agreement: type (e.g. fixed, variable);
2. Term of agreement: measure, (e.g. NPV, IRR pre and after tax, specified amount of revenues, specified number of vehicles);
3. Types of revenues permitted to the developer (tolls, charges);
4. Treatment of collateral revenues (e.g. revenues from airspace improvements)
5. Toll types allowed (e.g. direct, shadow, congestion, or at developer's discretion);
6. Toll setting authority (e.g. developer, government, or both);
7. Toll adjustment mechanism (e.g. formula for inflation, traffic demand, debt ratios);
8. Toll caps (e.g. maximum toll rates allowed);
9. Base returns allowed: measure and value (e.g. NPV, IRR, specified revenues);
10. Incentive returns allowed and related performance measure (e.g. achieving specified use of the facility, vehicle occupants, number of cars); and,
11. Excess revenues, their measures and their distribution (e.g. shared, or allocated).

Obligations Dimension

The obligation dimension represents the purpose of the PPP venture and the core of the requirement structure. Explicit requirements have to be set for two issues. The first is the developer's extent of obligations and responsibilities. The second relates to the extent and terms of the government's power in performing inspection/supervision, approval, and the right to request changes. Public-private partnership acts, RFPs, and agreements have to consider details for the obligation requirements, some of which include the following:

1. Description of project functions for which the developer is responsible (e.g. planning, permits, acquisitions, design, construction, operation, maintenance, environmental assessments and compliance);
2. Project functions the government prefers, or is required, to perform (e.g. traffic management, maintenance, police services);
3. Statement of the applicable standards and specifications
4. Extent of government monitoring, inspection, and approval processes, and right to make changes;
5. Statement of quality control and quality assurance systems and the responsibility for performing such activities (e.g. developer, independent consultant, government); and,
6. Processes for addressing time and cost effects resulting from changes made by government (e.g. allocated to capital/operating costs, to be negotiated).

While general statements are provided by government regarding project financing, it is important that CFEIs and RFPs treat the following:

1. Financial risks, if any, that may be absorbed by the government, (e.g. interest rate);
2. Type of financial support or guarantees that might be provided; and,
3. Type of security instruments permitted (e.g. project revenues and rights).

Liabilities Dimension

Explicit statements are needed by government to explain its requirements regarding project general liabilities, risks and taxes. They should cover the following:

1. Types of liability coverage (e.g. facility damage, tort and business interruption);
2. Responsible party for each liability during project development and operation;
3. Amounts of each insurance coverage required during construction and operation
4. Types and amounts of project bonds needed for construction and operation;
5. Extent and conditions, if any, of government liability (e.g. due to developers compliance with government specifications and standards);
6. Statement regarding the allocation of risks in relation to the developer's obligations;
7. Explicit definition of force majeure risks;
8. Time and cost consequences of force majeure risks; and,
9. Statements about tax policies, exemptions or allowances for the project.

Chapter 3

Proposed Economic Model Characteristics

3.1 Introduction

This chapter sets out in the following section some of the concepts that should be used in developing generalized economic models that fully reflect the attributes of the requirements structure described in Chapter 2. In light of these concepts, a review is made of the major economic models and support systems that have been developed to date by other authors and organizations for the appraisal of projects. The characteristics of this thesis' proposed model and support system are then introduced at the conclusion of the chapter.

3.2 Economic Model Underlying Concepts

An economic model can be described as a generalized model if it attains the following: (1) its structure directly or indirectly supports, and differentiates between, the several attributes of a project requirements structure; (2) the structure can support for each attribute a generalized representation covering the calculation methods across several industrial sectors (e.g. power and transportation); (3) its structure is able to include any number of operations (e.g. work tasks, activities, or phases) in a project life cycle; and (4) its structure is flexible enough to formulate several performance measures (i.e. not restricted to one economic indicator). These four characteristics are described below.

The requirements structure was used in the previous chapter to explain the various characteristics and aspects of capital investment projects, as exemplified by PPP projects. Capital investment projects are realized, as explained, by satisfying those *obligations* represented by the requirements in the development (e.g. design, and construction), operation and financing attributes, and by acquiring those *rights* represented by the possession and revenue attributes. The successful realization of projects, however, also involves addressing and analyzing project *liabilities* as represented by the general liabilities, risks and tax attributes.

When building an economic model for project appraisal, the design of the model should convert or interpret all the requirements of the foregoing attributes into cash flow elements. The degree to which a model represents all of the attributes is in turn a measure of how comprehensive is the model's capacity to represent the real economic life of a project. In other words, the rights, obligations and liabilities attributes of a project should be integral to the design of the economic model, independent of the level of detail sought in representing the project life cycle for a specified project. Therefore, the most important objective in phasing a project, e.g. in two phases as engineering and operational (Willmer 1991) or four phases as conceptual, design, construction and operation (Meyer and Cressman 1984), should be to distinguish between these phases in terms of the calculation methods and work tasks for each phase.

So the challenge becomes how to represent or interpret the requirements of the different project attributes in the form of cash flow elements when designing economic models. It is asserted herein that the model design should include two essential structural elements for each attribute: operations (e.g. number of work tasks, activities, or cost/revenue items) and methods (e.g. estimating and forecasting methods). Once these operations and methods are defined for each attribute, a project life cycle may be modeled appropriately since a project phase that may be represented by an attribute can be distinguished from other phases in a project life cycle. The two structural elements are discussed further below.

The first structural element represents the set of operations, their duration and logic necessary to represent requirements of each attribute. Operations for the ***development obligation-attribute*** represent, for example, the tasks used in planning, preparation, design, and construction of a project. Tasks for the ***operation obligation-attribute*** cover the administrative, operation, maintenance and utility-services activities in a project. The same applies to the ***financing obligation-attribute*** where financing operations can be represented by several bonds and loans used by a project. Tasks for the ***revenue rights-attribute*** cover all potential revenue streams in a project. The same goes for the ***possession rights-attribute*** where the expenditures attached to the various properties in a project may represent ownership tasks. Similarly, the liabilities attributes can be represented in a model as expenditure tasks required for the fulfillment of the project.

The second structural element represents the methods used in the estimation or calculation of the requirements of each of the eight attributes. A generalized model design should support a smorgasbord of various methods used in estimation. Each of the obligation attributes, for example, may have particular methods of calculation that can be found in the business environment of the attribute such as construction cost estimating methods, maintenance cost estimating methods, and project financing methods. Similarly, for the rights attributes, the spectrum of available demand analysis methods and service charge (e.g. toll) methods all need be recognized in the design of the model.

An economic model can be structured to represent the expenditures and revenues in a project life cycle in one or a combination of three arrangements. The first arrangement comes through direct assignment of cash outflows and discrete inflows at specific points of time or project periods without reference to how these net sums were obtained (this is referred to later as crude or aggregated estimating methods). This structural arrangement offers very little opportunities for risk analysis on the economic viability of a project. The second arrangement calls for the identification of packages for different phases of the project life to which expenditures/revenues can be attached and distributed over the duration of the relevant package (this is referred to later as semi-detailed estimating methods). The third arrangement provides for the inclusion of the methods used to derive a calculation/estimate along with their variables in the model design or structure (this is referred to later as detailed estimating methods). This arrangement provides the greatest opportunity for analyzing the project through risk analysis.

While various business sectors (e.g. power, water, or transportation) may share some methods in common, often there are differences between sectors. Therefore, an economic model should allow for a "specialized" representation of a particular attribute so that the characteristics of the operations and methods of this attribute in a particular business sector can be treated. For example, a model that includes in its structure most of the transportation demand/revenue methods can be categorized to have a specialized revenue attribute that reflects the transportation sector. Alternatively, an economic model can have a "generalized" representation of a particular attribute if its structure reflects the characteristics of the operations and methods of this attribute across several business sectors, thus providing an open architecture where several methods can be added to the model.

Further examples of the specialization/generalization of attribute methods include the following. The representation of the finance attribute of a project in an economic model may reflect only the bond instrument of the financial sector. However, when it reflects all the other instruments, e.g.

syndicated loans, term loans, and private placement bonds, it can be said to have a generalized representation of the finance attribute. Similarly, cost-estimating methods from the conceptual through detailed (Clough and Sears 1991) for the development attribute of a project can be utilized across several sectors or project types, and therefore they constitute generalized representations.

The difficulty in developing generalized representation of an attribute lies in the necessity of acquiring the methods used by the several sectors or project types for the calculation of a particular attribute. One approach adopted herein to assist in this task is enriching the specialized representation of an attribute by crude and semi-detailed methods such that it becomes a generalized representation.

The final point to address in model design is the flexibility of the model structure. With a flexible model structure, several economic performance measures can be modeled based on, or using, the same structure of the attribute design. For example, a model can be structured to calculate the total construction cost from the sum of the estimate of construction work packages. These same work packages should be available for the calculation of, or use by, any other performance measure such as net present value, internal rate of return, benefit-cost ratios, and loan-life-cover-ratio. With flexibility, (1) several performance measures can be built in a model without any repetition of calculations and without redundant model structure, and, (2) only one-time input to the model can be used to provide all the estimates of the performance measures in the model.

3.3 Previous Models and Systems for Project Appraisals

To date, several economic models and computer-based support systems have been developed to help decision-makers at the appraisal stage of capital investment projects. This section reviews the most significant project appraisal models and systems in light of the concepts presented in the previous section.

Since the 1970s the World Bank has been relying on economic models and risk analysis techniques for use in project evaluation. Pouliquen (1970) and Reutlinger (1970) explained the World Bank's formulation and use of economic models, sensitivity analysis and risk analysis using Monte Carlos simulation for the appraisal of projects. The models were built by the World Bank, as a lender, for the appraisal of projects as a basis to assist in making lending decisions.

The models explained by Pouliquen and Reutlinger for the appraisal of road projects in Africa were constructed using a number of concise mathematical statements/ equations to determine an economic indicator such as net present value or internal rate of return. Figure 3.1 and Table 3.1 shows a flow chart and mathematical formulation for a road project appraisal as referred to by the Reutlinger (1970). In terms of operations and calculation methods, the model differentiated between construction, revenue and maintenance operations. An aggregated cost sum was used for construction, an exponential method for revenue/savings calculations and a linear method for maintenance cost calculations. However, the model made no use of a network structure to logically link operations and was not generalized enough to handle other road projects. New models would have to be constructed to include other methods for construction, revenue and maintenance, to handle other project operations or attributes (e.g. financing), and to calculate other measures such as completion time, cost-benefit ratios, and debt-service-cover ratio.

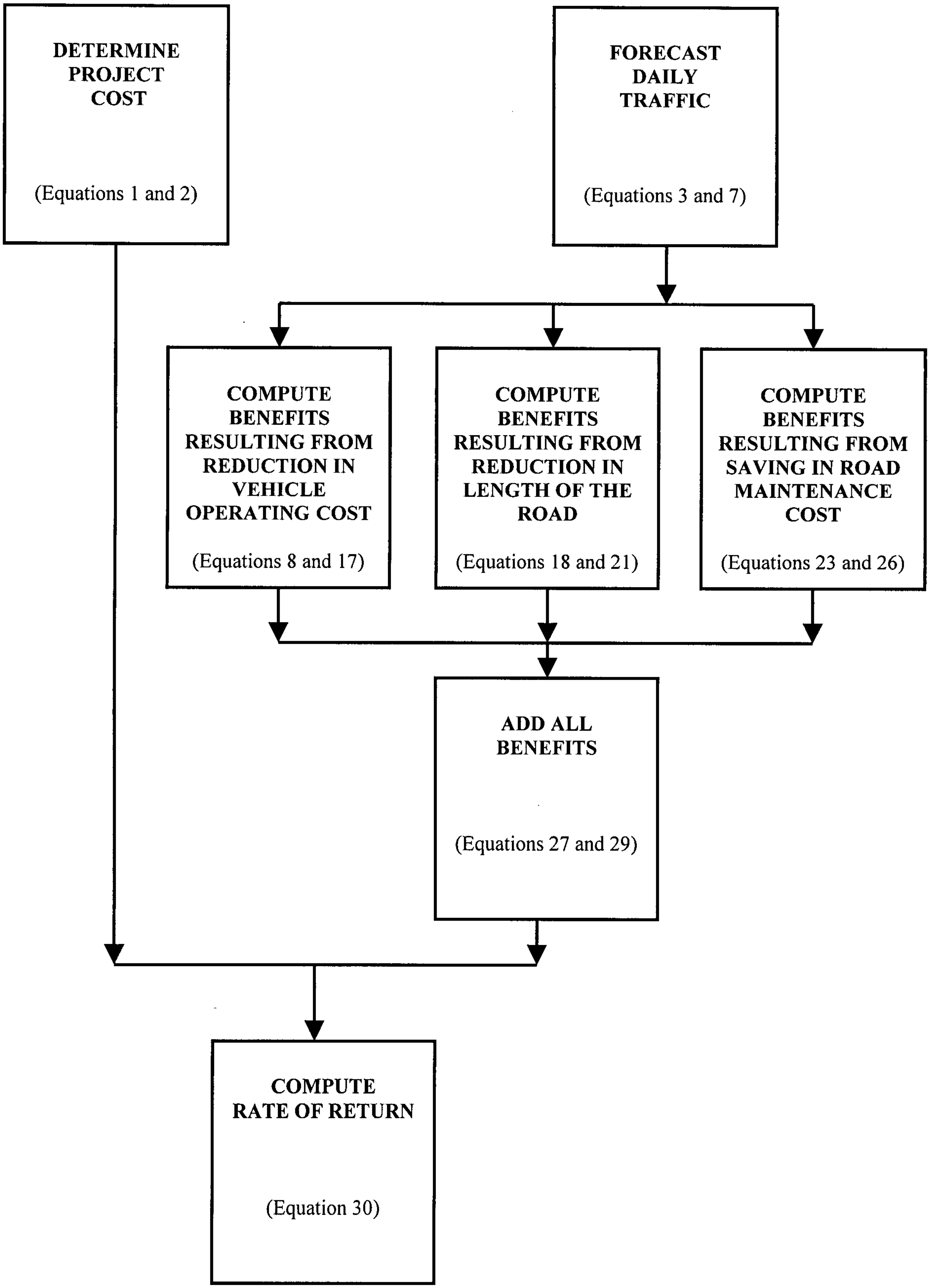


Figure 3.1: Flow chart for road project appraisal, Reutlinger (1970)

Table 3.1: Road Project Appraisal, Reutlinger (1970)

1.	(Project Cost)	= **(Pavement Base)** + **(Sub-Base/Shoulders)** + **(Earth Works)** + **(Borrow Materials)** + **(Others)**
2.	(Cost)$_t$	= (Project Cost) / **(Construction Time)** (n) if $t \le$ **(Construction Time)** = C if $t >$ **(Construction Time)**
3.	(Traffic Cars)$_t$	= (1 + **Traffic Growth Cars)**^{t} × **(Initial Traffic Cars)**
4.	(Traffic Trucks)$_t$	= (1 + **Traffic Growth Trucks)**^{t} × **(Initial Traffic Trucks)**
5.	(Traffic Buses)$_t$	= (1 + **Traffic Growth Buses)**^{t} × **(Initial Traffic Buses)**
6.	(Traffic Trailers)$_t$	= (1 + **Traffic Growth Trailers)**^{t} × **(Initial Traffic Trailers)**
7.	(Traffic Special)$_t$	= (1 + **Traffic Growth Special)**^{t} × **(Initial Traffic Special)**
8.	(Cost Saving p.v.m. Car)	= **Cost p.v.m. Old Road Car - Cost p.v.m. New Road Car**
9.	(Cost Saving p.v.m. Truck)	= **Cost p.v.m. Old Road Truck - Cost p.v.m. New Road Truck**
10.	(Cost Saving p.v.m. Bus)	= **Cost p.v.m. Old Road Bus - Cost p.v.m. New Road Bus**
11.	(Cost Saving p.v.m. Trailer)	= **Cost p.v.m. Old Road Trailer - Cost p.v.m. New Road Trailer**
12.	(Cost Saving p.v.m. Special)	= **Cost p.v.m. Old Road Special - Cost p.v.m. New Road Special**
13.	(Operating Cost Saving Car)$_t$	= (Cost Saving Car) × (Traffic Cars)$_t$ × **(Miles)** × 365
14.	(Operating Cost Saving Truck)$_t$	= (Cost Saving Truck) × (Traffic Trucks)$_t$ × **(Miles)** × 365
15.	(Operating Cost Saving Bus)$_t$	= (Cost Saving Bus) × (Traffic Buses)$_t$ × **(Miles)** × 365
16.	(Operating Cost Saving Trailers)$_t$	= (Cost Saving Trailer) × (Traffic Trailers)$_t$ × **(Miles)** × 365
17.	(Operating Cost Saving Special)$_t$	= (Cost Saving Special) × (Traffic Special)$_t$ × **(Miles)** × 365
18.	(Mileage Cost Saving Car)$_t$	= (Traffic Cars)$_t$ × **(Cost Travel Old Road Car)** × **(Reduction Miles)** × 365
19.	(Mileage Cost Saving Truck)$_t$	= (Traffic Trucks)$_t$ × **(Cost Travel Old Road Truck)** × **(Reduction Miles)** × 365
20.	(Mileage Cost Saving Bus)$_t$	= (Traffic Buses)$_t$ × **(Cost Travel Old Road Bus)** × **(Reduction Miles)** × 365
21.	(Mileage Cost Saving Trailer)$_t$	= (Traffic Trailers)$_t$ × **(Cost Travel Old Road Trailers)** × **(Reduction Miles)** × 365
22.	(Mileage Cost Saving Special)$_t$	= (Traffic Special)$_t$ × **(Cost Travel Old Road Special)** × **(Reduction Miles)** × 365
23.	(Traffic Units)$_t$	= (Traffic Cars)$_t$ × 2(Traffic Trucks)$_t$ × 2(Traffic Buses)$_t$ × 3(Traffic Trailers)$_t$ × (Traffic Special)$_t$
24.	(Maintenance Cost Old Road)$_t$	= **(a + b)** × (Traffic Units)$_t$, where a and b are constants
25.	(Maintenance Cost New Road)$_t$	= **(c + d)** × (Traffic Units)$_t$, where c and d are constants
26.	(Maintenance Cost Saving)$_t$	= (Maintenance Cost Old Road)$_t$ - (Maintenance Cost New Road)$_t$ × **(Miles)**$_t$
27.	(Total Operating Cost Saving)$_t$	= (Operating Cost Saving Car)$_t$ + (Operating Cost Saving Truck)$_t$ + (Operating Cost Saving Bus)$_t$ + (Operating Cost Saving Trailer)$_t$ + (Operating Cost Saving Special)$_t$
28.	(Total Mileage Cost Saving)$_t$	= (Mileage Cost Saving Car)$_t$ + (Mileage Cost Saving Truck)$_t$ + (Mileage Cost Saving Bus)$_t$ + (Mileage Cost Saving Trailer)$_t$ + (Mileage Cost Saving Special)$_t$
28.	(Benefits)$_t$ =	when $t >$ (Construction Time) then = (Total Operating Cost Saving)$_t$ = (Total Mileage Cost Saving)$_t$ + (Maintenance Cost Saving)$_t$ when $t <$ (Construction Time) then = C
29.	Calculate r such that $\sum (1 + r)^{-t}$ (Cost)$_t$ = $\sum (1 + r)^{-t}$ (Benefits)$_t$, $t = 1, \ldots, n$	

Notes: Traffic refers to Average Daily Traffic; Cost p.v.m. is cost per vehicle mile; any variable followed by subscript t indicates amount per year. User specified data is in **boldface.**

Clark and Chapman (1987) explained British Petroleum's (BP International) development of decision support systems for risk analysis of the time and cost of the construction phase of several projects including North Sea oil production platforms, oil processing facilities in refineries, and projects in chemical, mineral, and communication industries. The structure of the cost analysis model included decomposing total cost into cost line items through which risks could be identified and propagated to determine the total cost distribution. Time analysis incorporated the use of activities in a CPM network where risks can be identified for each activity. The technique used for risk analysis was the Controlled Interval and Memory (CIM) method (Chapman and Cooper 1983a; Cooper et al. 1985; Cooper and Chapman 1987; Chapman and Ward 1997). The structure of the cost model did not support the use of different construction cost estimating methods and did not treat time dependent costs.

Thompson and Willmer (1987), Willmer (1991), Thompson and Perry (1992) and Thompson (1993) explained some features of CASPAR (Computer Aided Simulation for Project Appraisal and Review). CASPAR was used to analyze part of the costs of the Channel Tunnel and was applied to the modeling and appraisal of a number of build-operate-transfer (BOT) projects. The model structure underling CASPAR was designed to simulate the interaction between time and money during a project life cycle which was considered to consist of two phases - engineering and operation. The structure of the model is network-based made of inter-related activities to which costs and revenues can be attached as a lump sum and which can be made discrete at specific periods of time or spread uniformly over the duration of the activity, or as unit cost/revenue per unit of time or unit of quantity. Each of the two life cycle phases in the model can have up to seven cost centers (e.g. production, administration, marketing) to which the activity costs can be assigned for the purpose of structural analysis. Different inflation factors can be applied to each cost center.

CASPAR was implemented in two separate programs, one for time analysis, and the other for cost analysis. CASPAR has a special way of dealing with sensitivity and risk analyses. It defines a number of risk variables (a maximum of twenty) which are defined in terms of various elements (maximum twenty one in the cost program and fourteen in the time program) of the original deterministic data. A design delay risk variable, for example, may contain all those activities involving design where their duration and/or costs can be added to the risk variable. As stated by Willmer (1991): "In the simplest case, the duration of one activity may define a risk variable. In more complicated cases activity duration, costs, and resource quantities may all be combined in one variable". Each risk variable can be defined on a percentage range change. In sensitivity analysis, all data elements within a risk variable will be altered to the same extent when the risk variable changes in its range. In probability analysis and using the cost program, a triangular or uniform distribution can be defined over the range of each of the twenty risk variables. In the time program, a triangular distribution can be assigned to each activity duration to determine the effect on the whole project network. CASPAR then relies on Monte Carlo sampling for risk analysis processing. CASPAR supports NPV, IRR, and payback period.

While CASPAR possesses some useful capabilities in terms of its network-based structure and the patterns of costs and revenues, its structure has several limitations. The model does not distinguish between the two phases treated in terms of the different calculation methods that usually apply to these two phases. Network activities can be designated as belonging to either of the two phases and assigned costs or revenues in an aggregated or unit cost/price form as described above. CASPAR structure does not model financing, revenue and maintenance methods. Further, the use of risk variables in the way it is described above means several risk elements (time and cost) are analyzed using the same range of variation or distribution of the risk variable that represent these elements. The CASPAR cost and time programs are not combined.

Since 1983 the United Nations Industrial Development Organization have been developing computer software for project appraisal which has resulted in the introduction of COMFAR III Expert (Computer Model for Feasibility Analysis and Reporting) for the financial and economic appraisal of industrial and non-industrial investment projects (UNIDO 1994). This program has been licensed to several users including development financing institutions, investment banks and industrial development corporations in 120 countries. COMFAR models the planning horizon of a project in two phases - a construction phase and production phase. Project costs are defined as sub-items under two categories: fixed investment costs and production costs. Project revenues are defined as sub-items/products under a sales program category. Cost and revenue sub-items have the same duration as of the phase to which they belong. A unit cost estimating method is used for all sub-items where quantities and costs/prices are defined at discrete periods during the duration of the relevant phase. Unlike the other systems described so far, COMFAR provides for the inclusion of sources for project finance including equity, long-term loans and short term finance. The model has the capability to use up to 20 currencies for cost/revenue calculations.

COMFAR lacks three important elements which are essential for functionality and generality. First, it does not have a network structure that can define logic connecting the various cash flow streams. Second, it does not address the spectrum of calculation methods used in the estimation of projects. Third, it does not allow probabilistic risk analysis to be carried out on the variables of the model - only sensitivity analysis can be performed.

Despite their weaknesses, CASPAR, COMFAR, BP's models and the World Bank's models represent distinguished systems for the economic appraisal of capital investment projects. Explained in the literature, however, are other works that contain a general description of economic models with an implementation of sensitivity and/or risk analysis attached to the models. These applications included to large extent crude or aggregated models, with the goal being to explore the behavior of, or gain insights on, some specific problem types or areas. For example, Wahdan et al. (1995) and Wahdan (1995) developed an economic model for the analysis of public private partnership projects. The model had a pre-defined set of work packages with which the project economic structure must fit, it used an aggregated cost method, it recognized a demand calculation method for revenue calculation, it did not support a network structure, and it included a preliminary loan calculation but it did not support financial calculation methods. Ngee et al. (1997) explained the development of an automated mechanism for dynamic negotiation between government and developers. The mechanism is a spreadsheet that uses crude estimates of cost, concession period, and level of tariff. The purpose of the mechanism was to simplify project cash flow calculations during the final negotiation and reach a balance between risk and return. Salem and Ariaratnam (1999) developed a decision support system for life cycle cost analysis of construction/rehabilitation road projects. The model had a highly crude/coarse structure, did not distinguish between life cycle processes and calculation methods, did not support a network structure, and did not support indicators other than life cycle cost. Other models introduced to date for specific purposes include a cash flow model for risk analysis of power plants (Chee and Yeo 1995), a model for the evaluation of oil and gas projects (Skjong and Lereim 1988), and a model for the economic evaluation of chemical plants (Westerterp and Vrijland 1984).

To large extent, the above models and systems can be categorized as semi-detailed models in which the differentiation between project phases was not clear and the methods used in calculations (e.g. financing and demand) were largely crude and semi-detailed methods.

3.4 Characteristics of the Proposed Model and Support System

To satisfy the objective of this thesis of building a generalized economic model and support system for the risk analysis and evaluation of capital investment projects necessitates that the concepts elaborated upon previously should be employed in order to obtain a model which can provide a realistic representation of these projects. Therefore, the proposed generalized model and support system should recognize the following characteristics:

1. Differentiation between project attributes in rights, obligation and liabilities

   Regardless of the number of phases required to represent a project life cycle, the model should be able to distinguish between the various requirements of a project during its life cycle.

2. Recognition of attribute characteristics: Multiple Operations

   Each attribute should be capable of being represented in one or more phases in a project life cycle. Each phase should be represented by one or more operations or tasks. These tasks should be logically linked - both internal to a phase, and between phases, using a network structure.

3. Recognition of attribute characteristics: Methods

   The structure of the model should be able to include crude, semi-detailed and detailed calculation methods in its treatment for the various attributes. Crude and semi-detailed methods should be structured so that the model can address several types of projects in case the sector-specific detailed methods required for the project at hand are not supported by the model. Detailed methods of project financing and construction cost estimating should be included in the model.

4. Flexibility of the model structure

   The flexibility required of the model structure can be described as follows:

   - The ability to formulate any project alternative or scenario with any number of operations and methods, or indirectly with any number of phases;
   - The ability to model several performance measures which would assist different types of decision makers;
   - The ability to link together project operations;
   - The ability to support hierarchical representation (aggregation levels) of costs/revenues;
   - The ability to model project variables over time; and
   - The ability to perform sensitivity and risk analyses on any performance measure.

5. Effective risk analysis framework

   The risk analysis framework should support:

   - The ability to model the risk/uncertainty of any variable in a way that reflects the available amount of information about the variable in question; and
   - The ability to derive the statistics and probability distribution of performance measures.

6. Flexible decision support system

   The decision support system represents the implementation of the generalized model and risk analysis framework. The major capabilities of the system should include: (1) a flexible user interface; (2) the ability to formulate any alternative or scenario; and (3) the ability to maintain, manipulate and reproduce data and results for any alternative.

By satisfying the foregoing characteristics the proposed generalized economic model and risk analysis framework would contribute for improved quality in decisions for capital investment projects. This would fill the gaps in the current state-of-the-art systems and models as explained earlier in the chapter, namely:

- the inability to distinguish between the requirements of the different project phases as to the functional estimating and forecasting methods used in each phase (e.g. construction estimating methods and demand forecasting methods);
- the inability to provide a link between the phases that might be required in the calculation of performance measures (e.g. a link between financing drawdown and capital expenditure required, and a link between O&M costs and project demand);
- the inability to aggregate costs into groups for further analyses (e.g. to determine deterministically and probabilistically the total cost of a group of work packages);
- the lack of any terms and methods (e.g. drawdown methods, repayment methods, interest rate types, debt fee calculations) for the financial instruments mainly used in financing capital investment projects (e.g. syndicated term loans and private placement bonds, floating rate notes); and
- the inability to model the uncertainty of individual project variables in a comprehensive manner (as in CASPAR) and the inability to use different probabilistic methods in the modeling process (i.e. not only using full probability distributions).

The following three chapters elaborate on these characteristics and describe the design of a generalized economic model and risk analysis framework, and their implementation in the form of a decision support system called Evaluator.

Chapter 4

Generalized Economic Model

4.1 Introduction

This chapter presents the design of a generalized economic model for the analysis and evaluation of capital investment projects. The design is built on the characteristics required of an economic model for project appraisal as outlined in the previous chapter. The model is a multipurpose hierarchical time function structure. The model structure integrates the properties and methods of the various industries and markets in capital investment projects through what is introduced herein as components, classifications and shape functions. It is hoped with the structure of the generalized model that enables detailed modeling of capital investment projects to provide a contribution to the economic modeling of such projects that can fill the gaps of the previous models. A general description of the concepts and structure of the model is given in the next section. The four sections that follow it provide a detailed description of the components of the model structure. Model performance measures are explained in the final section. Equations that are part of the general model are numbered sequentially. Other equations numbered with the letter E as a prefix are used for explanatory purposes and are not part of the model.

The risk analysis framework and decision support system (DSS) are introduced in chapters 5 and 6 respectively. However, as the DSS represents the implementation of the model and framework, the dialogue (interface) part of the DSS which deals with the input and output of data and results is used in this chapter to illustrate details of the economic model components developed herein.

4.2 Generalized Economic Model

4.2.1 Model Structure and General Concepts

A project cash flow can be used to represent any of (1) "expenditures" for the design, construction, maintenance, replacement or operation of a project; (2) "revenues" earned from operation, subsidies, or collateral raised from side businesses to the project (e.g. leasing air space in a project right-of-way); or, (3) "financial funds" advanced, or repaid, for the project. The basic concept behind a cash flow is a relationship between time and money which can be expressed as a function, $f_c\ (t,\ \boldsymbol{x})$, where t is time and $\boldsymbol{x}$ is a vector that contains references to the variables in the cash flow. The value obtained by a cash flow function at any time t depends on (1) the methods used in the calculation of that cash flow using the $\boldsymbol{x}$ vector and (2) the value at time t of each variable in $\boldsymbol{x}$.

In cost/revenue calculations, infrastructure cash flows may share common *properties* and common *methods*. Properties are specifications such as type of cash flow (e.g. construction cash flow and maintenance cash flow) while methods deals with computational procedures (e.g. revenue estimating methods). These properties and methods can be used to form classifications of cash flows, with each classification having its own particular properties and methods. A new cash flow formed for an economic analysis can become a member of a particular classification, and thereby inherit the properties and methods of the classification. A classification can be represented by a model function, $f_C\ (t,\ \boldsymbol{X})$ where t is time and $\boldsymbol{X}$ is a matrix of all the $\boldsymbol{x}$ vectors in the classification. By forming classifications, the following benefits can be achieved:

- Classifications provide an avenue to model the eight attributes of the rights, obligations and liabilities dimensions of a project and to differentiate between these dimensions. An attribute can be represented by a classification and consequently the properties and methods of the classification can be used to represent the attribute operations and methods. For example, a development attribute can be represented by a classification that includes those properties and methods commonly found in design and construction operations. Consequently, a construction operation (e.g. work package or activity) can be represented by a cash flow of the "development classification" and estimates for that operation will be made using those properties and methods of the classification.

- The whole life cycle of a project can be represented by the classifications representing the eight attributes. Differentiation between life cycle phases can be reasonably made since each phase would be represented by one or more classifications. For example, the engineering phase in a two-phase life cycle (e.g. engineering and operation) would be represented by cash flows of the finance and development classifications that represent the finance and development attributes.
- Any number of operations (e.g. construction work packages or revenue streams) in a project life cycle can be introduced to an economic model since the addition or deletion of an individual operation, which is represented by a cash flow that inherits its properties and methods of its classification, will not affect the other parts of the model.
- Methods of a particular classification can include the crude, semi-detailed and detailed methods used in the calculation or estimation of a relevant attribute (e.g. financing). Therefore, a classification can have a specialized as well as generalized representation covering the attribute calculation methods.

The structure of the generalized economic model is built on classifications as defined above. The model structure as shown in Fig. 4.1 is represented by four inter-related components: capital expenditure (CE); operation and maintenance (OM); revenue (RV); and, financing (FN). A component is the "physical" representation of a particular classification in terms of its properties, methods and $\boldsymbol{X}$ matrix, and corresponds to a classification function $f_C\ (t,\ \boldsymbol{X})$. Therefore, the generalized model includes four classifications representing the attributes of the requirement structure. As shown in the sketch below, the capital expenditure, operation and maintenance, revenue, and finance classifications represent the development, operation, revenue, and finance attributes respectively. Methods for the possession, general liabilities and tax attributes are represented in the capital expenditure and operation classifications assuming they are represented by crude and semi-detailed methods. The risk analysis framework handles the risk attribute separately.

Classifications		**Requirement Structure Attributes**
Capital Expenditure	→	Development, Possession, Liabilities, Tax
Operation & Maintenance	→	Operation; Possession, Liabilities, Tax
Revenue	→	Revenue
Finance	→	Finance

Each of the four components is considered to have basic "constructs" that are called work packages for the CE component and streams for the other components. A construct, *i*, Figure 4.2, is the "physical" representation of the inherited properties and methods and the $\boldsymbol{x}$ vector that is part of $\boldsymbol{X}$, and represents a cash flow function $f_c(t,\ \boldsymbol{x}_i)_i$. Any number of constructs can be added to a component. Since each construct's cash flow belongs to the component classification, cash flow calculations for the construct can be formed using any of the methods (crude, semi-detailed or detailed) that are included in the component classification. This provides for the component cash flow function, $f_C\ (t,\ \boldsymbol{X})$, to be formulated or integrated from the cash flows of the constructs in that component, knowing that each construct may be different than others in the component in terms of properties' value and method of calculation. This provides for a flexible model structure that is not affected by the addition of deletion of any construct to a model component.

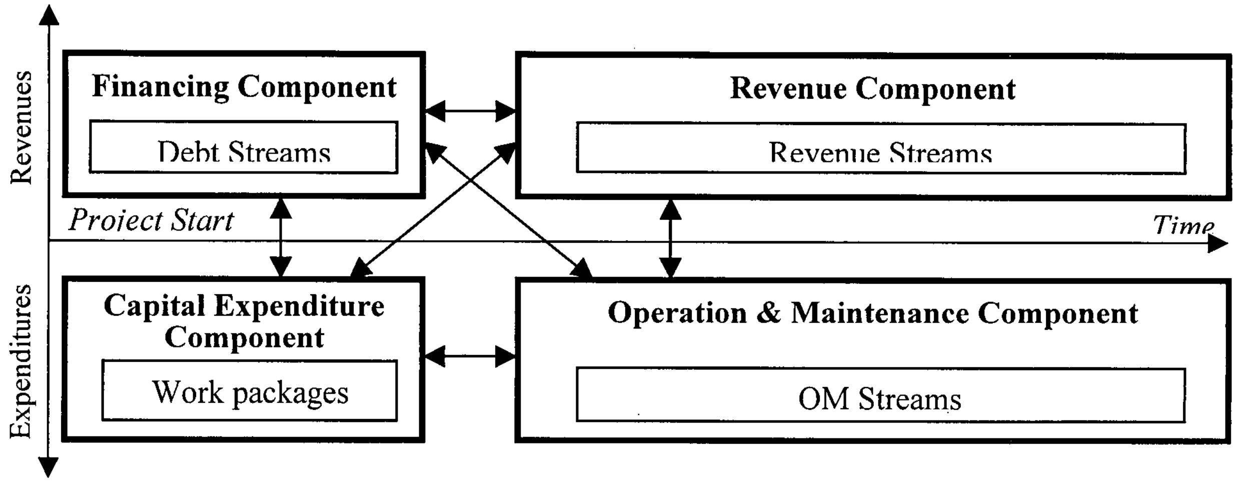


Figure 4.1: Economic model - Interrelated components and basic constructs

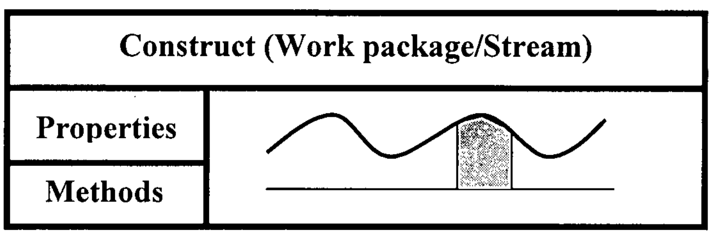


Figure 4.2: A construct has properties and methods

Let *m, p* and *q* be the number of constructs in the relevant component and *i* be a construct number in the component, then:

$$f_{CE}(t, X_{CE}) = \sum_{i=1}^{m} f_c^{CE}(t`, x_{CE\,i})_i \tag{4.1}$$

$$f_{RV}(t, X_{RV}) = \sum_{i=1}^{q} f_c^{RV}(t`, x_{RV\,i})_i \tag{4.2}$$

$$f_{OM}(t, X_{OM}) = \sum_{i=1}^{p} f_c^{OM}(t`, x_{OM\,i})_i \tag{4.3}$$

In the above formulations and throughout the model, t is global time defined in terms of global/ project time unit (GTU) and referenced to project start (Fig. 4.1), and $t`$ is construct local time defined in terms of construct local time unit (LTU) and referenced to construct start. Time conversion from $t$ in the left side of the equation to $t`$ in the right side is made accordingly and will be described below. The subscripts CE, RV, and OM of $\boldsymbol{x}$, $\boldsymbol{X}$ and construct cash flow function refer to a classification/component type and have been omitted later for clarity with $\boldsymbol{x}$ and $\boldsymbol{X}$, as has the subscript i which refers to a specific construct/cash flow in a component.

The FN component has its own constructs, properties, methods and $\boldsymbol{X}$ matrix. However, because of the discrete nature of the flow of financial funds, the cash flow of the i-th debt stream is represented by an information vector $\boldsymbol{TRIF}_i$ instead of a cash flow function $f_c^{FN}(t, \boldsymbol{x})_i$.

The concept of cash flow classifications of properties and methods, which is behind the formulation of components and constructs, represents the main concept behind the structure of the generalized economic model. Each of the four components and their classifications will be described later in detail. The following subsections, however, describe a set of properties and methods that are common and essential to all components.

Unless otherwise noted throughout the model formulations, all flows and usage rates are assumed to be continuous. However, allowance is made for discrete flows in the four model components particularly the FN component. Discounting of continuous and discrete cash flows is performed using continuous compounding (Remer et al. 1984; Tanchoco et al. 1981). Continuous compounding gives flexibility in dealing with any cash flow pattern (Park and Sharp 1990) which is suitable for dealing with the various types of cash flows in capital investment projects.

4.2.2 Component Common Properties

Time Unit Characteristics

Time characteristics of a project are defined in terms of several properties. Two time unit properties are defined and each can assume any value of the following time periods/intervals: one-month, three-month (quarter), six-month (semi-annual) or twelve-month (annual). The first time unit property refers to project time, t, that uses project/global time unit (GTU) which can be used to refer to project duration, a constructs' early start time, and elapsed times after project start. The second time unit property refers to local time of a construct, t', which uses local time unit (LTU) to define the duration of the construct or to delineate the time scale for the cash flow pattern of the construct. With LTU each construct can have its own value of the time unit property. For example, a revenue stream can be defined in annual periods while another revenue stream can be specified in month periods. The CE component, however, is an exception where the same time unit value must be shared by all of the CE constructs, i.e. work packages, for scheduling purposes.

Reference Characteristics

Since continuous cash flow patterns are used in the generalized model, a reference property is used to define the origin of calculations for the cash flows. This property can take project (global) start or construct (local) start as a reference value for calculation. Nearly all the model variables that will be defined later have an attached project or local reference. When a variable in a construct has a continuous function with time that uses project start as a reference, then GTU is used as the time unit for the time scale of the function. When the reference is local, i.e. construct start, then LTU is used. The reference used for all model variables will be detailed later when describing the model components.

Since several time unit values are involved in model calculations, conversion from GTU to LTU and from LTU to GTU is required. The conversion is made simple by a conversion factor that divides the month equivalent of the first time unit by the month equivalent of the second time unit:

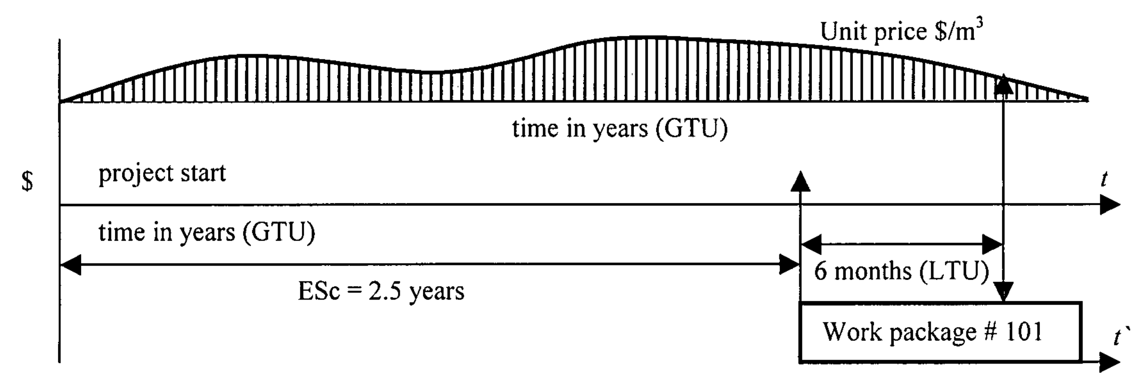
Global to local time conversion (GtoL) = GTU (in months) / LTU (in months) (E4.1)

Local to global time conversion (LtoG) = LTU (in months) / GTU (in months) (E4.2)

As an example, consider converting from a project time t defined in annual periods to a construct local time $t`$ defined in quarters. Then GtoL would be equal to 12/3 = 4. Where $ESc$ is construct start time in GTU, then the construct local time $t`$ is

$$t` = (t - ESc) \cdot \mathrm{GtoL} \tag{E4.3}$$

In another example that considers the reference property as well as time unit property, a work package construct may use month-period as a LTU and the work package starts at ESc = 2.5 years (GTU in years) as shown in the sketch below. If in the construct a unit price of material $\$/m^3$ is defined as a continuous function with global reference and global time unit, then the spot rate of the unit price of material will be determined at all times by converting LTU to GTU, where LtoG = 1/12.



Therefore, the unit price of material at a time $t`$ = 6 months will be obtained by setting the unit price function at that time to 3 years using

$$t = t` \cdot \mathrm{LtoG} + Esc \tag{E4.4}$$

Network Characteristics

A project operation can be described by a time span (duration), logic and lead/lags time with other operations, therefore *duration, lead/lag* and *logic* represent three *properties* included in the generalized model for component constructs. These three properties integrate together to provide the generalized model with a network-based structure. These properties delineate the domain of application for the cash flow of a relevant construct where the cash flow time function has a value only within the construct duration. These properties are further detailed later.

Contract Characteristics

Since capital investment projects may be procured through the alternative PPP delivery system, a contract period is usually defined for developers in the concession or development agreements as explained in Chapter 2. This contract period can be referenced to start from the signing of contracts or from an established future date. Therefore, *contract period* and its *contract reference* are another two *properties* in the generalized model. For example, the Northumberland Strait Crossing has a 35 year concession period starting from "Date Certain" (construction completion). These properties define the domain of application for the RV and OM streams to be limited to the end of the contract period, if it is given (e.g. 35-year concession); otherwise calculations consider the whole duration of each cash flow (e.g. 45-year revenue stream).

Figure 4.3 illustrates an interface window for project identification in Evaluator showing some of the above properties. The list box in front of "Project/Global Time Unit" allows the choice of a time period as a GTU for the project. "Contract Duration" is illustrated with a "Contract Duration Reference" to satisfy the properties under the contract characteristics above. Figure 4.4 illustrates the use of LTU for the CE component along with information explaining that the constructs of the other components will have their own time units.

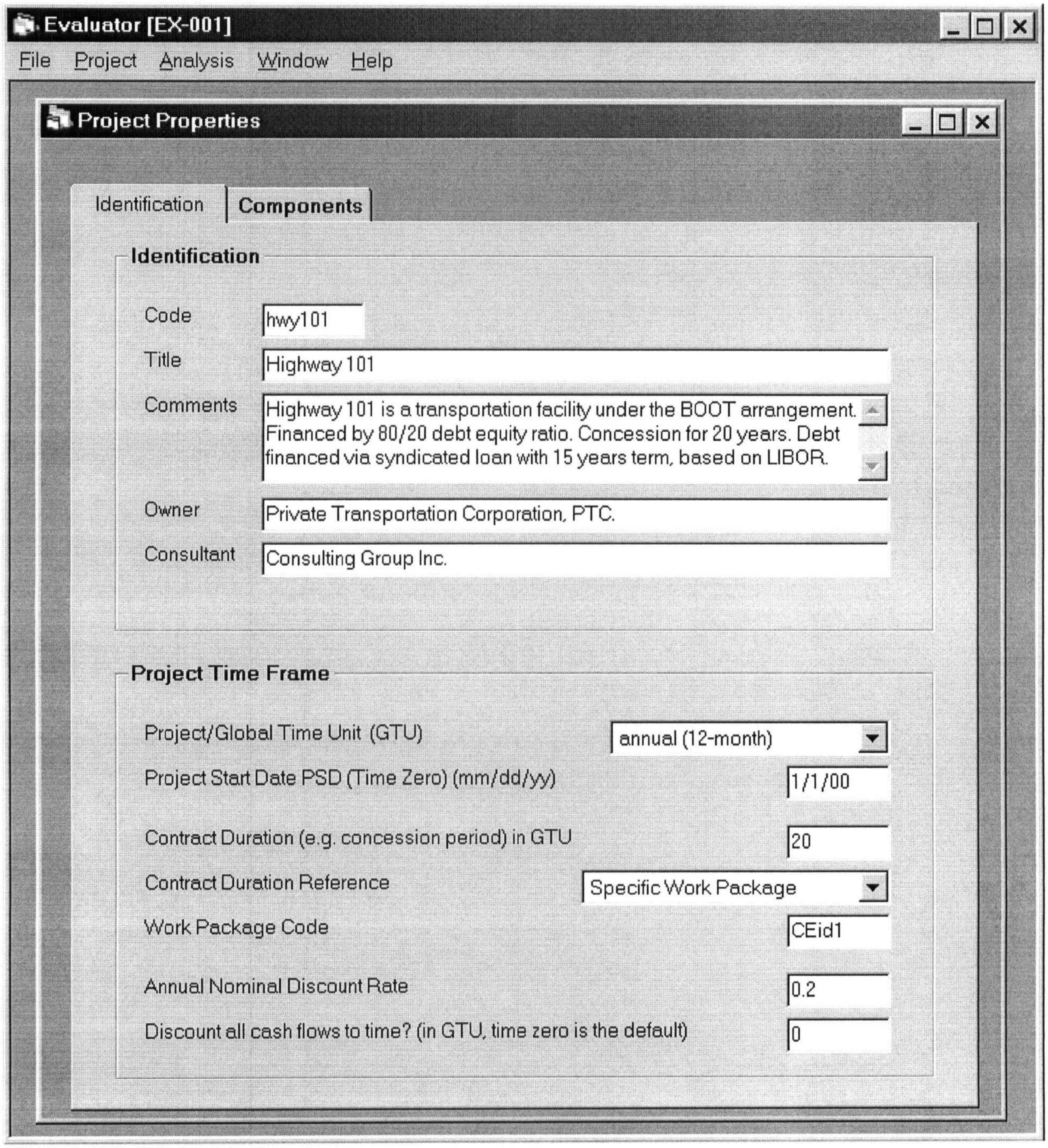


Figure 4.3: Project identification and common properties

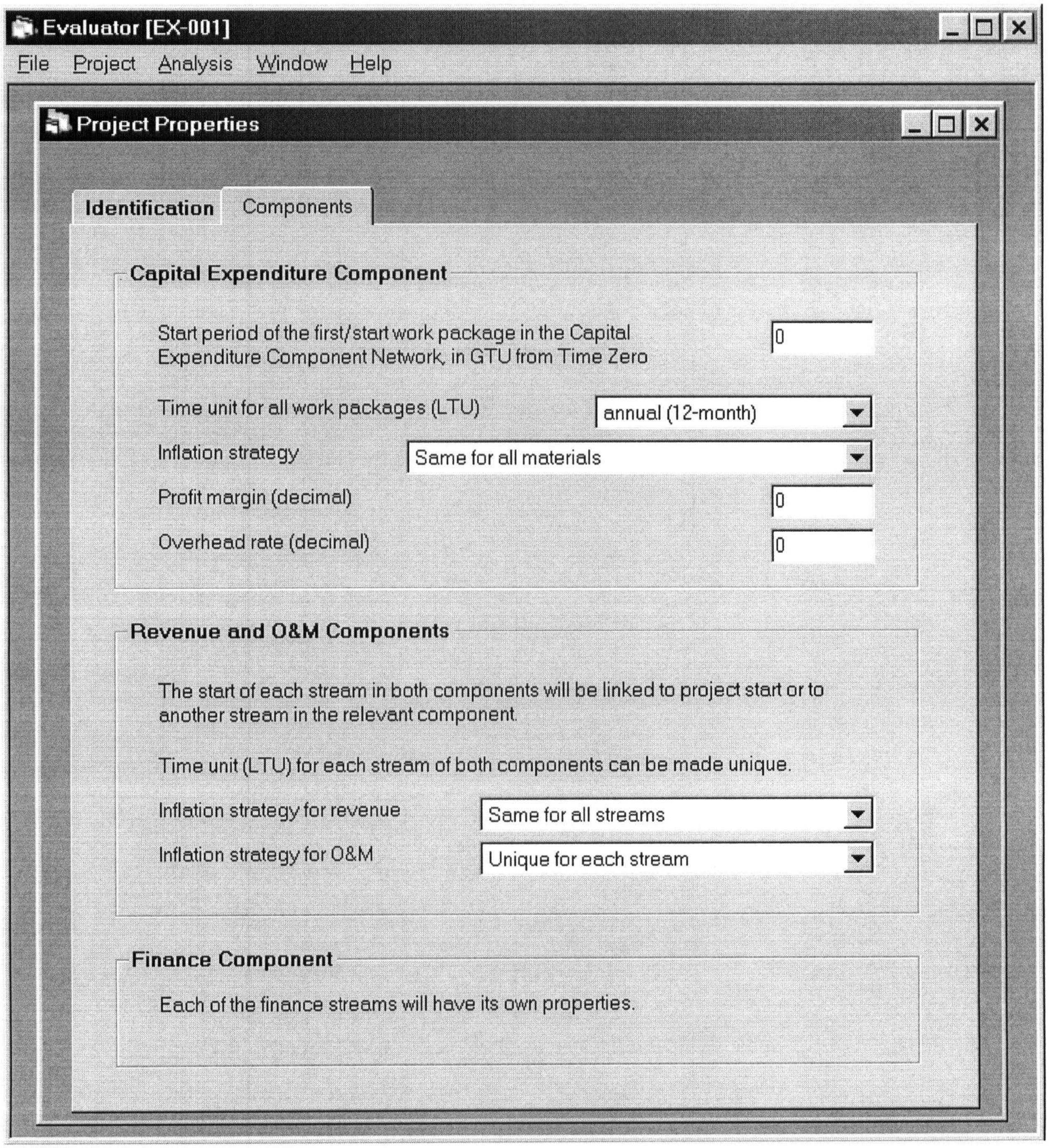


Figure 4.4: Component properties

4.2.3 Components Common Methods: Shape Functions

Component classifications have two categories of methods: *component-specific* and *common* methods. Component-specific methods represent a group of methods unique to each classification type and will be described later for each component. The "general" methods are a number of methods common to all classifications and are described herein as "shape functions", $f_S\,(t``, \boldsymbol{y})$, where $t``$ is either local or global time and $\boldsymbol{y}$ is a vector of sub-variables.

Shape functions represent patterns of change of a variable over time. In the generalized economic model, any variable Z in a cash flow vector $\boldsymbol{x}$ can be represented by a shape function and described by function notation, $Z(t`, \boldsymbol{y})$ or $Z(t``, \boldsymbol{y})$. In model formulations, $\boldsymbol{y}$ will be dropped for clarity. With $t`$ the variable is expressed in terms of a local time unit and local reference. With $t``$ the variable can have either local time ($t`` = t`$) or global time ($t`` = t$) that is defined in terms of a global time unit and global reference. Some variables in the model, as explained later, are local only, such as productivity and work quantities; others such as unit costs, inflation rates and floating interest rates may experience both local and global domains.

The main use of shape functions in the generalized model is to give all model variables, particularly those representing rates, the ability to change over time. This allows improvements to economic analyses and economic models that mainly express variables as average fixed values over time, as explained in the previous chapter. The other use of shape functions is to represent semi-detailed calculation methods. Semi-detailed methods were described before as methods that can represent aggregated sums of cost/revenues spread over time according to a specific pattern such as a uniform flow of expenditures.

To maintain generality, a total of 32 shape functions in two categories "rate functions" and "area functions", as described below, have been included in the economic model. These functions may experience several characteristics or forms such as step, ramp, decay, and exponential growth.

4.2.3.1 Rate Functions

Rate functions include a number of time-related functions. The mathematical expressions for these functions are described in Table 4.1 along with a description of the sub-variables or parameter variables that are included in the $\boldsymbol{y}$ vector of the shape function. Some of these functions have been used by others to describe general cash flow patterns in economic analysis (Remer et al. 1984; Tanchoco et al. 1981; Almond and Remer 1979; Park and Sharp-Bette 1990).

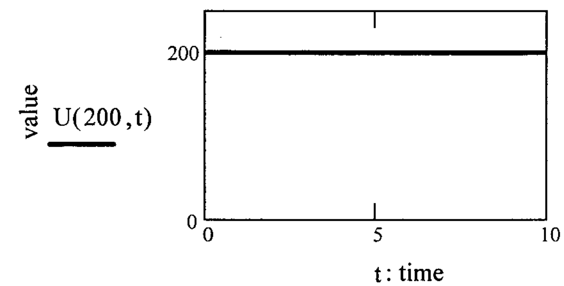
The value assumed by any single variable (e.g. inflation) in a construct cash flow calculation at a given time is obtained using its shape function expression and depends on the *local/global* domain of the variable. It is worth mentioning that the mathematical forms of shape functions may depend on several sub-variables. For example, the "step uniform" pattern could be defined using nine parameter variables where five rates can be defined along with four time limits as shown on the step uniform figure in Table 4.1. This provides for a refined level of detail in modeling any single variable in the generalized model.

4.2.3.2 Area Functions

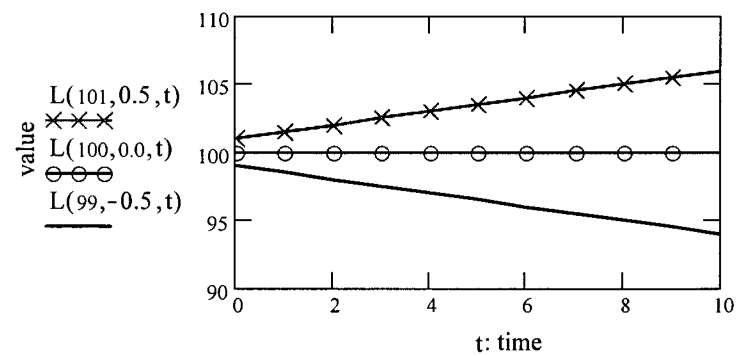
Area functions are functions that model situations in which the total value of a variable is constrained regardless of time or of how the variable will change over time. Area functions can only use a *local* reference as compared to rate functions that can use *global/local* reference.

Table 4.1: Rate Functions

<table>
<tr><th>Pattern</th><th>Shape Function: $f_s(t, y)=$</th><th>Form</th></tr>
<tr><td>Uniform</td><td>Qu

Where U(Qu,t):
Qu: value per time unit</td><td>
</td></tr>
<tr><td>Linear</td><td>$Qs + Qr \cdot t$

Where L(Qs,Qr,t):
Qs, Qr: start value, rate

Notes: The function following the value of Qr can take several forms such as ramp, decay and uniform.</td><td>
</td></tr>
<tr><td>Step Uniform</td><td>R1 if $0 \le t < T1$ R4 if $T3 \le t < T4$
R2 if $T1 \le t < T2$ R5 if $t \ge T4$
R3 if $T2 \le t < T3$

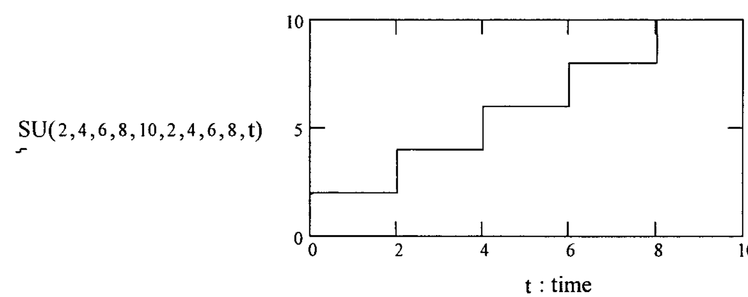
Where SU(R1,R2,R3,R4,R5,T1,T2,T3,T4,t):
R1 to R5: rate values
T1 to T4: up-to time values</td><td>
</td></tr>
</table>

Table 4.1: Rate Functions (continued)

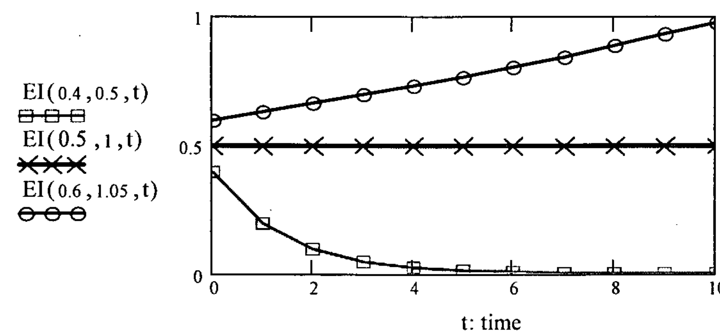
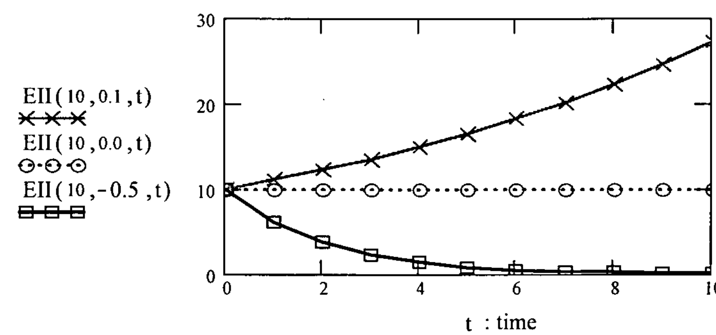
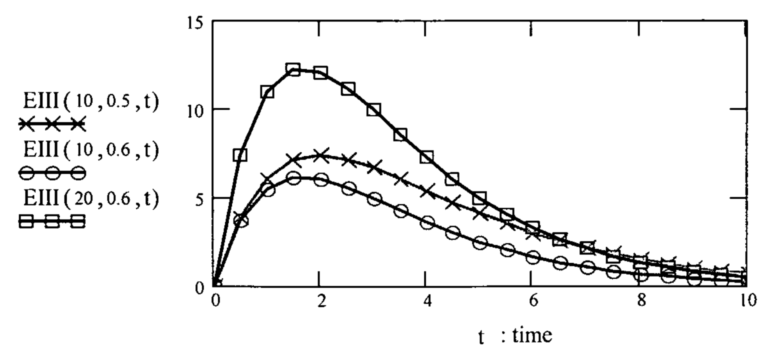
Pattern	Shape Function: $f_s(t, y)$	Form
Exponential I	$Qs \cdot Qr^{t}$ Where EI(Qs,Qr,t): Qs, Qr: start value, rate value Notes: The function can be uniform (Qr=1), decreasing (0<Qr<1), increasing (Qr>1).	
Exponential II	$Qs \cdot e^{Qr \cdot t}$ Where EII(Qs,Qr,t): Qs, Qr: start value, rate value Notes: The function can take several forms, e.g. uniform, decrease and increase patterns. Qr can be positive or negative.	
Exponential III	$Qs \cdot t \cdot e^{-Qr \cdot t}$ Where EIII(Qs,Qr,t): Qs, Qr: magnifier, rate value Notes: Peak at t = (1/Qr). Max. value = Qs/Qr·e Qr>0.	

Table 4.1: Rate Functions (continued)

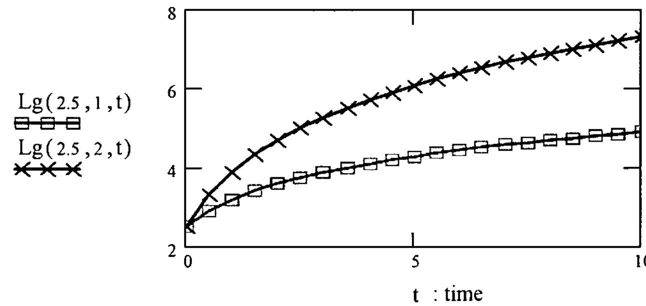
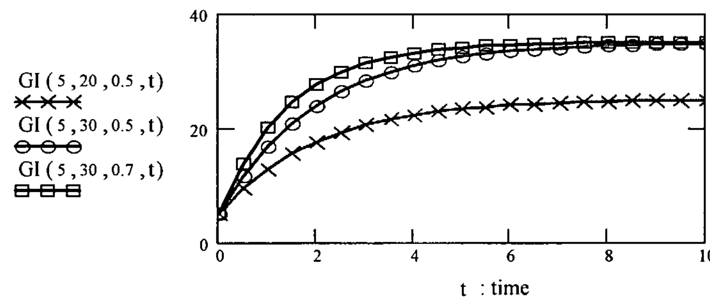
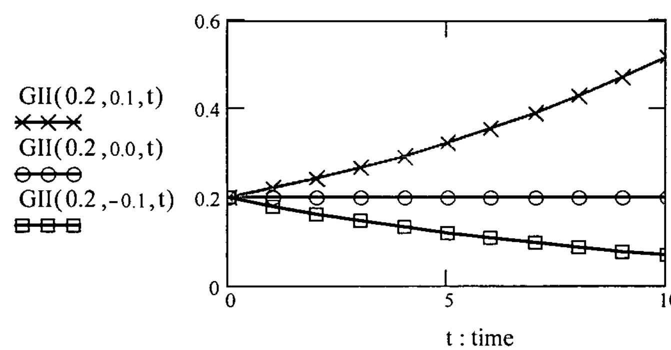
Pattern	Shape Function: $f_s(t, y)$	Form
Logarithmic	$Qs + Qr \cdot \ln(t + 1)$ Where Lg(Qs,Qr,t): Qs, Qr: start value, rate value	
Growth I	$Qs + Qa \cdot \left(1 - e^{-Qr \cdot t}\right)$ Where GI(Qs,Qa,Qr,t): Qs, Qa, Qr: start value, amplitude, rate value	
Growth II	$Qs \cdot (1 + Qr)^{t}$ Where GII(Qs,Qr,t): Qs, Qr: start value, rate value	

Table 4.1: Rate Functions (continued)

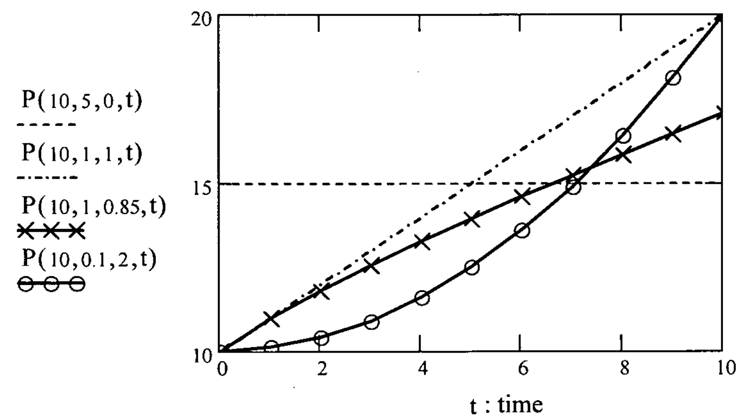
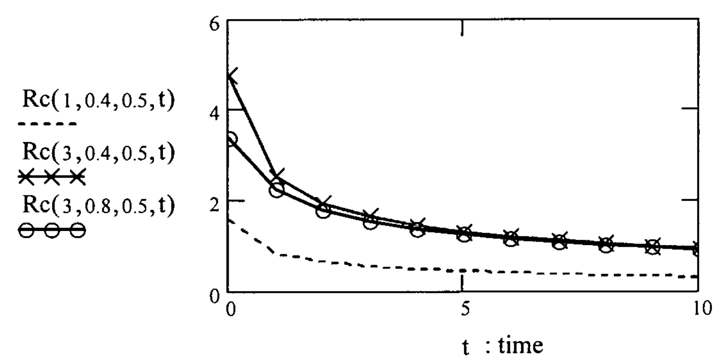
Pattern	Shape Function: $f_s(t, y)$	Form
Power	$Qs + Qm \cdot t^{Qr}$ Where P(Qs,Qm,Qr,t): Qs, Qm, Qr: start value, magnifier, rate value Notes: Uniform (Qr=0), root (0<Qr<1), linear (Qr=1) Parabola (Qr>2)	
Reciprocal	$Qo / (t + Qr)^{Qm}$ Where Rc(Qo,Qr,Qm,t): Qo, Qr, Qm: base value, rate, magnifier Notes: Initial value = $Qo/ (Qr^{Qm})$ Qr>0, Qm >0	

Table 4.1: Rate Functions (continued)

<table>
<tr><th>Pattern</th><th>Shape Function: $f_s(t, y)$</th><th>Form</th></tr>
<tr><td>Polynomial</td><td>

$$Qo + Q1\cdot t + Q2\cdot t^2 + Q3\cdot t^3 + Q4\cdot t^4$$

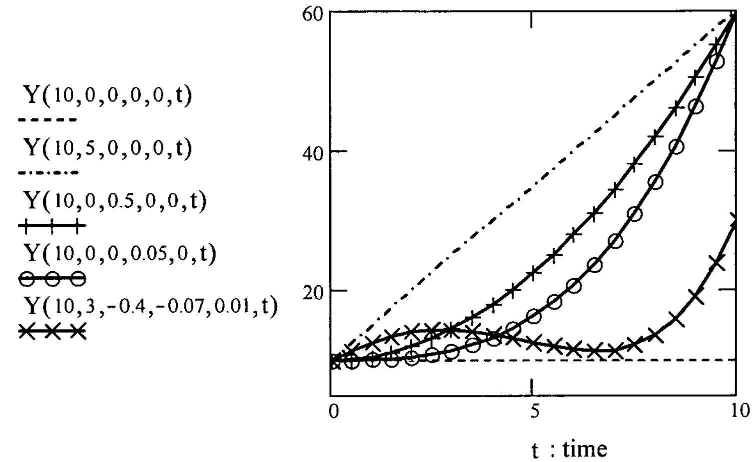
Where Y(Qo,Q1,Q2,Q3,Q4,t):

Qo,Q1,Q2,Q3, Q4: parameter variables

Notes:

Uniform, linear, quadratic, cubic, 4th degree

</td><td>

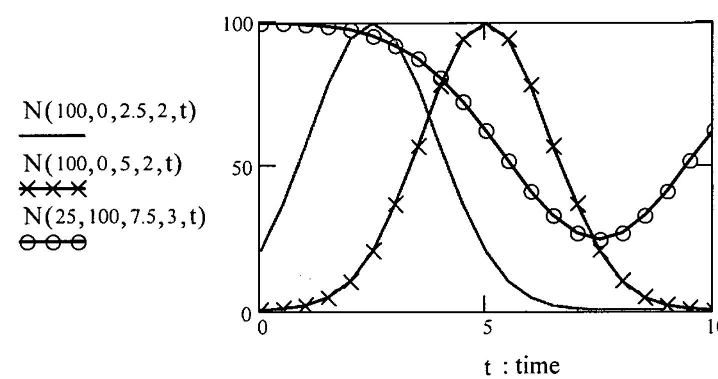


</td></tr>
<tr><td>Normal II</td><td>

$$Qs + (Qm - Qs)\cdot \exp\left[-\frac{(t-\mu)^2}{\sigma^2}\right]$$

Where N(Qm,Qs,μ,σ,t):
Qm: maximum value
Qs: start value
μ: time at maximum value
σ: shape factor

</td><td>



</td></tr>
</table>

Table 4.1: Rate Functions (continued)

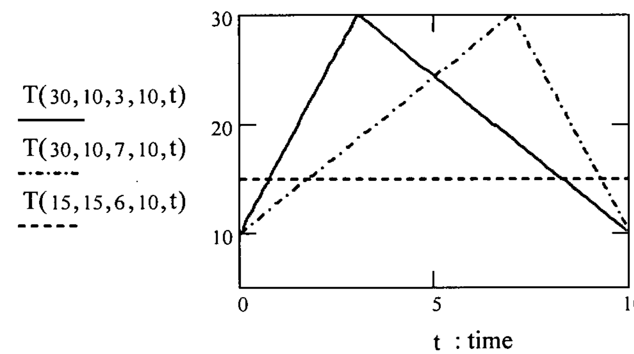
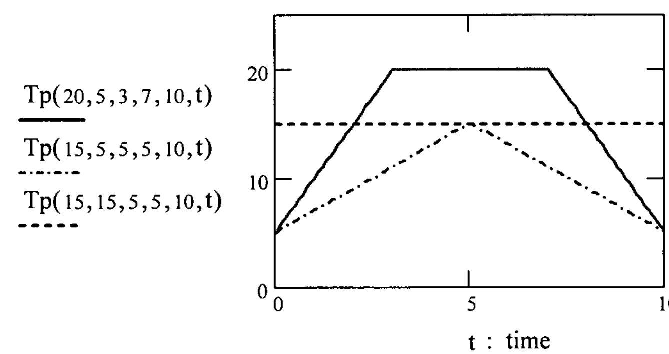
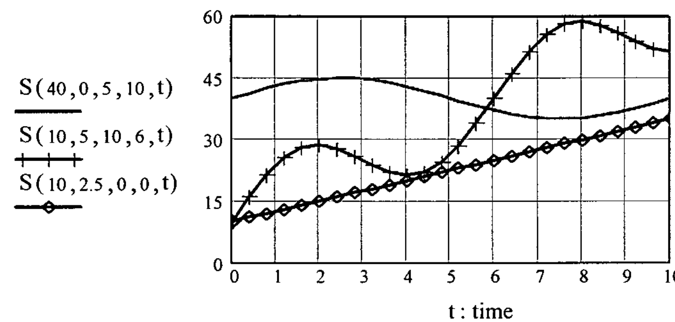
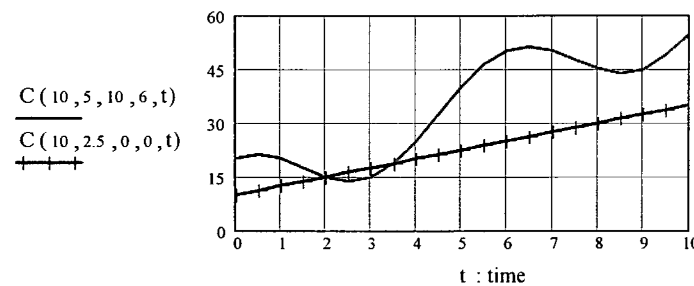
Pattern	Shape Function: $f_s(t, y)$	Form
Triangular	$Qs + \frac{Qm - Qs}{c} \cdot t \quad \text{if } (0 \le t < c) \;\&(c \ne 0)$ $Qs + \frac{Qm - Qs}{(1 - z)} - \frac{(Qm - Qs) \cdot t}{(1 - z) \cdot b} \quad \text{if } c \le t \le b$ Where T(Qm,Qs,c,b,t): Qm, Qs, c: max. value, start value, time at max. value Notes: z = c / b , b is total time (duration). Should only be used with local reference system.	
Trapezoidal	$Qs+(Qm-Qs) \cdot t/c \quad \text{if } (0 \le t \le c)\&(c \ne 0)$ $Qs + (Qm - Qs) \quad \text{if } (c \le t < d)$ $Qs + \left[\frac{Qm - Qs}{1 - z} - \frac{(Qm - Qs) \cdot t}{(1 - z) \cdot b}\right] \quad \text{if } d \le t \le b$ Where Tp(Qm,Qs,c,d,b,t): Qm, Qs: max. value (mv), start value c, d : time at start and end of maximum value (mv) Note: z = d / b ; b is total time (duration). Should only be used with local reference system.	

Table 4.1: Rate Functions (continued)

Pattern	Shape Function: $f_s(t, y)$	Form
Sinusoidal	$Qs + Qr \cdot t + Qa \cdot \sin\left(\frac{2 \cdot \pi}{Qc} \cdot t\right)$ Where S(Qs,Qr,Qa,Qc,t): Qs, Qr: start value, rate value Qa, Qc: amplitude, cycle length	
Linear Cosine	$Qs + Qr \cdot t + Qa \cdot \cos(2 \cdot \pi \cdot t/Qc)$ Where C(Qs,Qr,Qa,Qc,,t): Qs, Qr: start value, rate value Qa, Qc: amplitude, cycle length	

The mathematical expressions of *area functions,* Table 4.2, are derived by integrating the relevant *rate function* over a total duration *b,* which represents the duration of a construct, and equating the results to the total value of the variable over that duration, i.e.

$$Qt := \int_0^b f_S(t,y)\,dt \qquad \text{(E4.5)}$$

For example, the linear function L [$= f_S(t, y)$] is derived as follows. Letting Qs and Qr be the start and rate values, respectively and letting Qt be the total value of the variable, i.e.

$$f_S(t, y) = Qs + Qr \cdot t \quad \text{(see Table 4.1)} \qquad \text{(E4.6)}$$

$$Qt := Qs \cdot b + \frac{Qr \cdot b^2}{2} \qquad \text{(E4.7)}$$

$$L(Qt, Qr, b, t) := \frac{-Qr \cdot b^2 + 2 \cdot Qt}{2 \cdot b} + Qr \cdot t \quad \text{(see Table 4.2)} \qquad \text{(E4.8)}$$

The following example helps explain the use of rate and area functions. If 10000 mhrs are needed to finish a work package in 50 days, then a variable for labor input can be described as 200 mhrs/day (25 laborers/day) using a "uniform" *rate function* (see the solid line in Figure 4.5) which means that if the duration is extended to 60 days then another 2000 mhrs would be needed. Alternatively, using the "uniform" *area function* then 10000 mhrs will be fixed; that is 166.67 mhrs/day with 60 days - see the dashed line in Figure 4.5. Further, if a minimum number of laborers is needed in a day and the work force builds up as work progresses, then a "Normal" *area function* can be used with 10000 mhrs as a total value, 2000 mhrs as a base value (i.e. at least 4 laborers/day in 8-hrs day), and 60 days as duration. Thus, 37.024 mhrs/day (5 laborers), 107.045 mhrs/day (13 laborers), 355.89 mhrs/day (44 laborers) will be assigned for the 1st, 2nd and 3rd day respectively, Figure 4.5.

Figure 4.6 illustrates how the shape functions have been implemented into the system. Material inflation in a work package is described using Step Uniform with 5 rates and global reference. Area functions have the word "Total" attached to their names in the system.

Table 4.2: Area functions

<table>
<tr><th>Pattern</th><th>Shape Function: $f_s(t, y)$</th><th>Form</th></tr>
<tr><td>Uniform</td><td>Qt / b

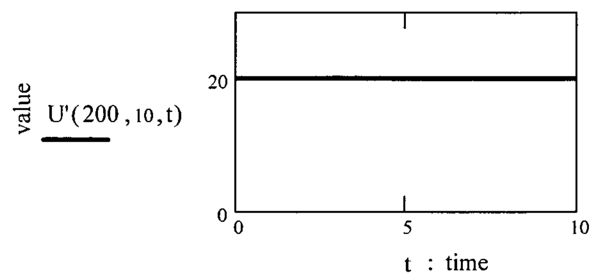
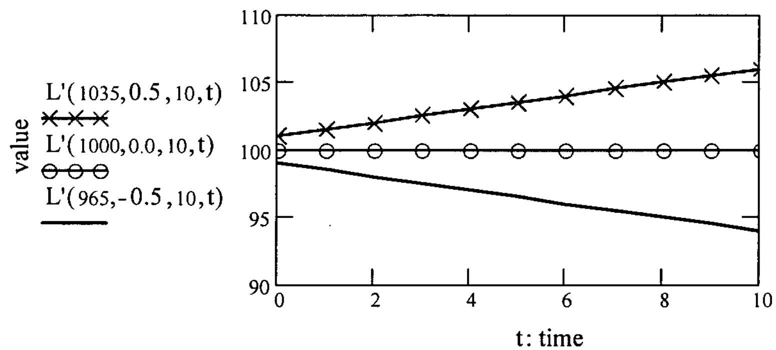
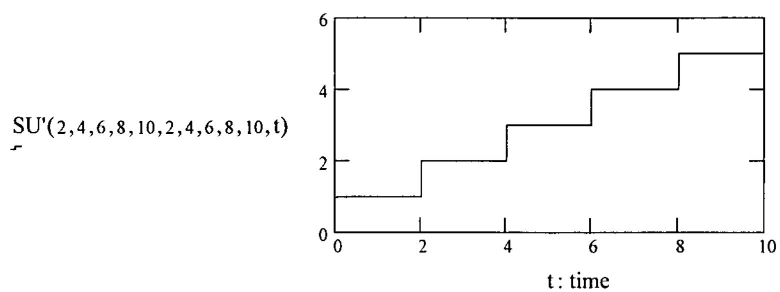
Where U`(Qt,b,t):
Qt: total value
b*: total time (duration)</td><td>
</td></tr>
<tr><td>Linear</td><td>$$\frac{1}{2}\cdot\frac{\left(-Qr\cdot b^2 + 2\cdot Qt\right)}{b} + Qr\cdot t$$
Where L`(Qt,Qr,b,t):
Qt: total value
Qr: growth rate
b*: total duration</td><td>
</td></tr>
<tr><td>Step Uniform</td><td>A1/T1 if 0≤t<T1   A4/(T4-T3) if T3≤t<T4
A2/(T2-T1) if T1≤t<T2   A5/(b-T4) if T4≤t
A3/(T3-T2) if T2≤t<T3
Where SU`(A1,A2,A3,A4,A5,T1,T2,T3,T4,b,t):
A1 to A5: area values
T1 to T4: time values; b*: total duration</td><td>
</td></tr>
</table>

Table 4.2: Area functions (continued)

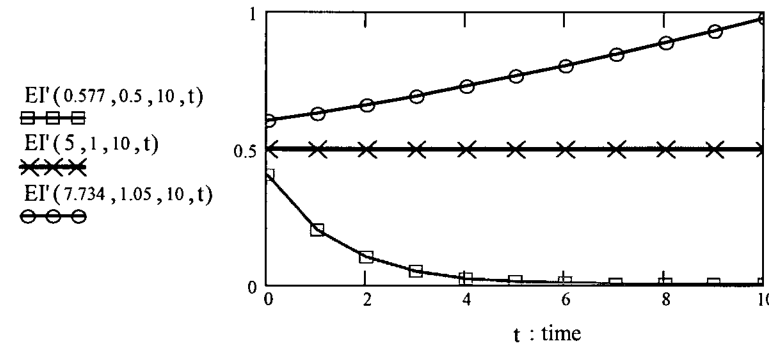
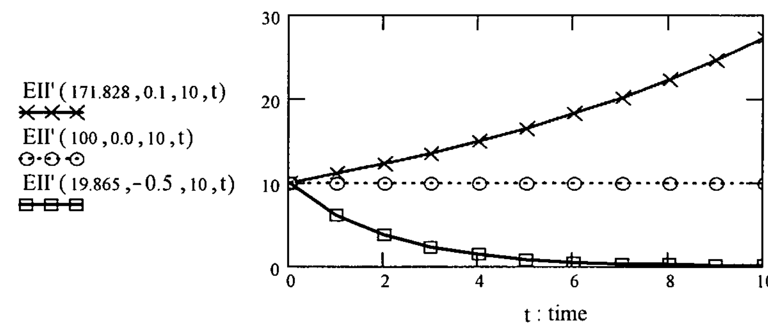
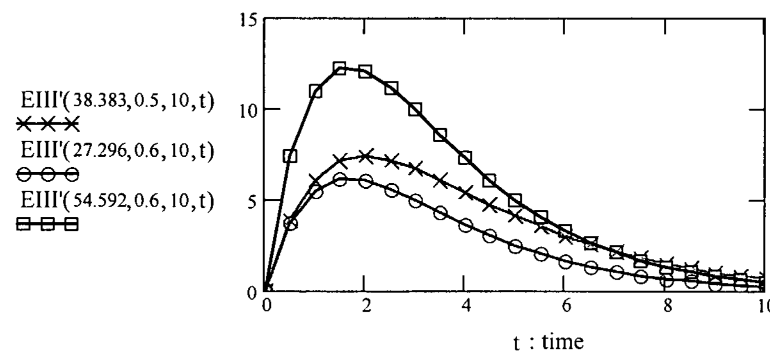
Pattern	Shape Function: $f_s(t, y)$	Form
Exponential I	$Qt \cdot \ln(Qr) \cdot Qr^{t} / (Qr^{b} - 1)$ Where EI`(Qt,Qr,b,t): Qt, Qr: total value, rate b^{*}: total duration	
Exponential II	$Qt \cdot Qr \cdot e^{Qr \cdot t} / (e^{Qr \cdot b} - 1)$ Where EII`(Qt,Qr,b,t): Qt, Qr: total area, rate value b^{*}: total time (duration)	
Exponential III	$\dfrac{-Qt \cdot Qr^{2}}{\exp(-b \cdot Qr) \cdot b \cdot Qr + \exp(-b \cdot Qr) - 1} \cdot t \cdot \exp(-Qr \cdot t)$ Where EIII`(Qt,Qr,b,t): Qt, Qr: total area, growth rate b^{*}: total time (duration) Time at maximum value = 1/Qr; maximum value = $\dfrac{-Qt \cdot Qr \cdot e^{-1}}{\exp(-b \cdot Qr) \cdot b \cdot Qr + \exp(-b \cdot Qr) - 1}$	

Table 4.2: Area functions (continued)

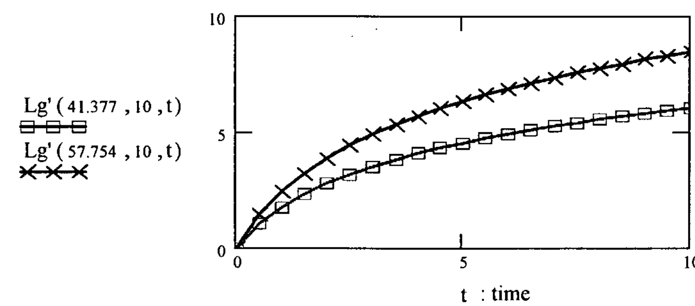
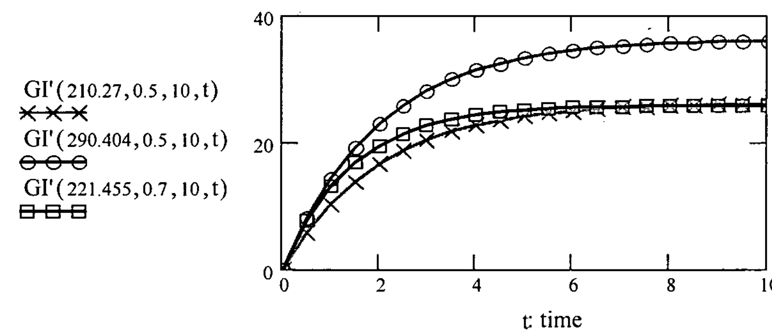
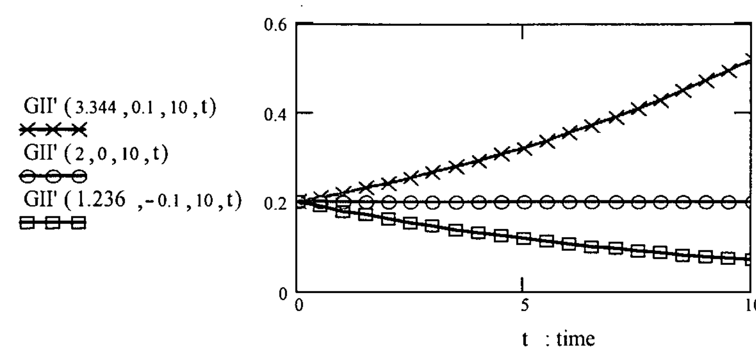
Pattern	Shape Function: $f_s(t, y)$	Form
Logarithmic	$Qt \cdot \ln(t+1) / [\ \ln(b+1) \cdot b - b + \ln(b+1)\ ]$ Where Lg`(Qt,b,t): Qt: total value b^*: total time (duration)	
Growth I	$\dfrac{Qt \cdot Qr}{b \cdot Qr + \exp(-b \cdot Qr) - 1} \cdot (1 - \exp(-Qr \cdot t))$ Where GI`(Qt,Qr,b,t): Qt, Qr: total value, growth rate b^*: total time (duration)	
Growth II	$\dfrac{Qt \cdot \ln(1 + Qr)}{\exp(b \cdot \ln(1 + Qr)) - 1} \cdot (1 + Qr)^t$ Where GII`(Qt,Qr,b,t): Qt, Qr: total value, growth rate b^*: total time (duration)	

Table 4.2: Area functions (continued)

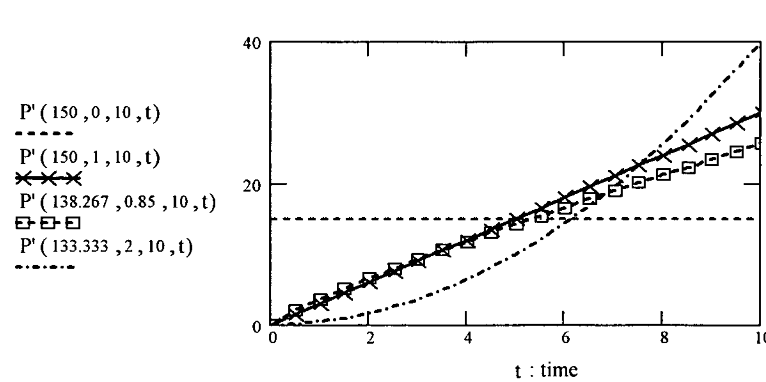
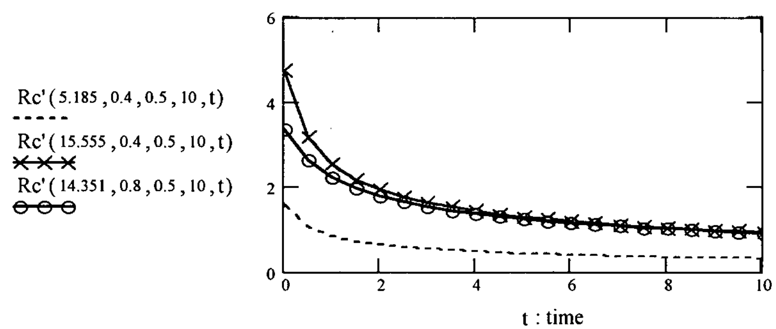
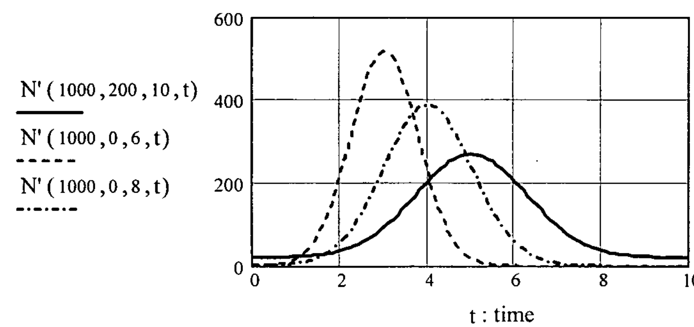
Pattern	Shape Function: $f_s(t, y)$	Form
Power	$Qt \cdot (Qr+1) \cdot t^{Qr} / b^{Qr+1}$ Where P`(Qt,Qr,b,t): Qt, Qr: total area, rate value b^*: total time (duration)	
Reciprocal	$$\frac{-Qt \cdot (Qm-1) \cdot (t+Qr)^{-Qm}}{(b+Qr)^{(-Qm)} \cdot b + (b+Qr)^{(-Qm)} \cdot Qr - Qr^{(1-Qm)}}$$ Where Rc`(Qt,Qr,Qm,t): Qt, Qr, Qm: total value, rate, magnifier b^*: total duration. Initial value = $$\frac{-Qt \cdot (Qm-1) \cdot Qr^{-Qm}}{(b+Qr)^{(-Qm)} \cdot b + (b+Qr)^{(-Qm)} \cdot Qr - Qr^{(1-Qm)}}$$	
Normal I	$$\frac{Qb}{b} + (Qt - Qb) \cdot \left[\frac{1}{\sigma \cdot \sqrt{2 \cdot \pi}} \cdot \exp\left[\frac{-1}{2} \cdot \left(\frac{t-\mu}{\sigma} \right)^2 \right] \right]$$ where: $\mu := 0.5 \cdot b$ and $\sigma := 0.5 \cdot b / 3.9$ Where N`(Qt,Qb,b,t): Qt, Qb: total area, base area b^*: total duration, Peaks at $0.5 \cdot b$, total area under curve is 99.99 (z=3.9).	

Table 4.2: Area functions (continued)

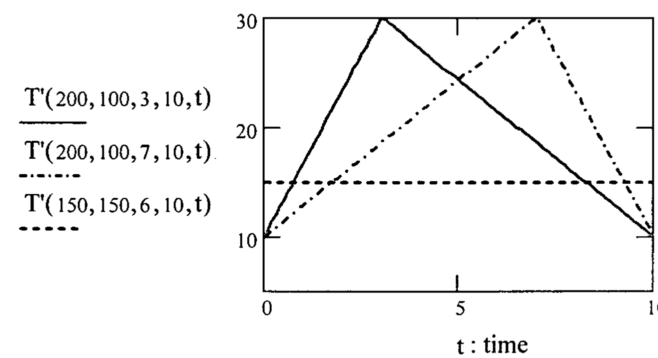
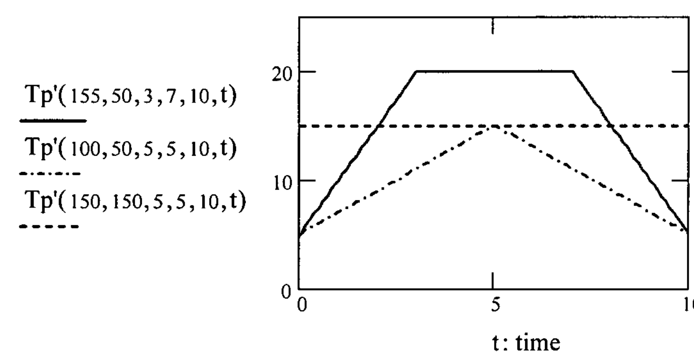
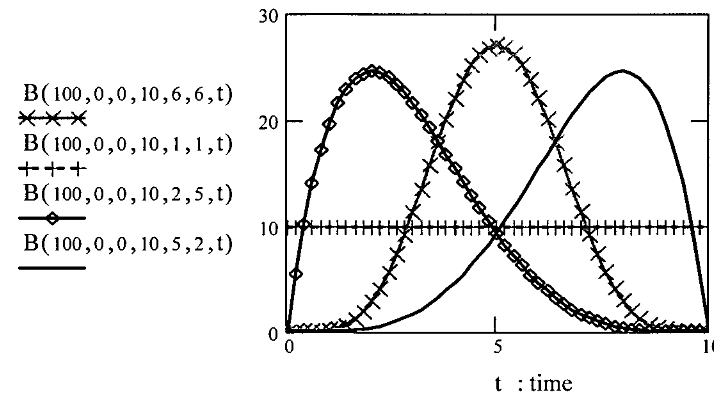
Pattern	Shape Function: $f_s(t, y)$	Form
Triangular	$\frac{Qb}{b} + \frac{2\cdot(Qt - Qb)}{z\cdot b^2}\cdot t$ if $(0 \le t < c)$ & $(c \ne 0)$ $\frac{Qb}{b} + \left[\frac{2\cdot(Qt - Qb)}{(1 - z)\cdot b} - \frac{2\cdot(Qt - Qb)\cdot t}{(1 - z)\cdot b^2}\right]$ if $c \le t \le b$ Where T`(Qt,Qb,c,b,t): Qt, Qb, c: total area, base area, time at max. b^*: total time (duration) z = c / b; maximum value = 2·(Qt-Qb)/b + (Qb/b)	
Trapezoidal	$(Qb/b)+(w\cdot t/c)$ if $(0 \le t < c)$ & $(c \ne 0)$ $(Qb/b)+w$ if $(c \le t < d)$ $(Qb/b)+\left(\frac{w}{1-z} - \frac{w\cdot t}{(1-z)\cdot b}\right)$ if $(d \le t \le b)$ Where Tp`(Qt,Qb,c,d,b,t): Qt, Qb: total area, base area (included in total area) c,d: start and end time of maximum value b^*: total duration. z = d / b; w=2·(Qt-Qb)/((d-c)+b) Maximum value = 2·(Qt-Qb)/((d-c)+b) + (Qb/b)	

Table 4.2: Area Functions (continued)

Pattern	Shape Function: $f_s(t, y)$	Form
Beta Pattern	$$\left[\frac{\Gamma(\lambda 1+\lambda 2)}{\Gamma(\lambda 1)\cdot\Gamma(\lambda 2)}\cdot\left(\frac{t-\mu 1}{\mu 2-\mu 1}\right)^{\lambda 1-1}\cdot\left(1-\frac{t-\mu 1}{\mu 2-\mu 1}\right)^{\lambda 2-1}\cdot\frac{Qt-Qb}{\mu 2-\mu 1}\right]+\frac{Qb}{\mu 2-\mu 1}$$ Where B(Qt,Qb,μ1,μ2,,λ1,λ2,t): Qt,Qb: total value, total base value (included in Qt) μ1,μ2**, λ1,λ2: start time, total time (duration), first shape factor, second shape factor. Notes: Uniform (λ1=λ2=1), normal (λ1=λ2= value) Skewed left (λ1>λ2), skewed right (λ1<λ2)	

* b is a total time representing total duration, e.g. duration of a work package. The implementation of area functions in Evaluator does not require the specification of b as an argument to area functions; b is already known by the system from the construct duration.

** The values of μ1 and μ2 as being arguments to the Beta function should be within the duration of the construct in which the function is used. μ2 can be assigned the same value as the duration of the construct. If μ2 is larger than the duration of the construct, Evaluator will use the duration instead of μ2 in the computation.

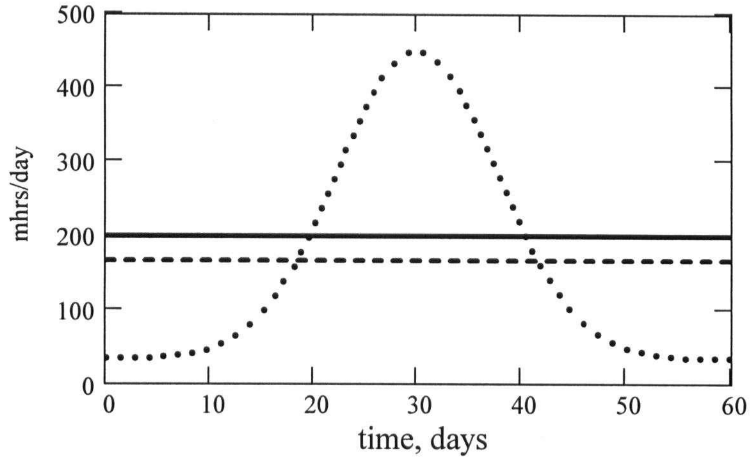


Figure 4.5: Shape functions

—— Uniform, - - - Uniform (a), ...Normal I (a)

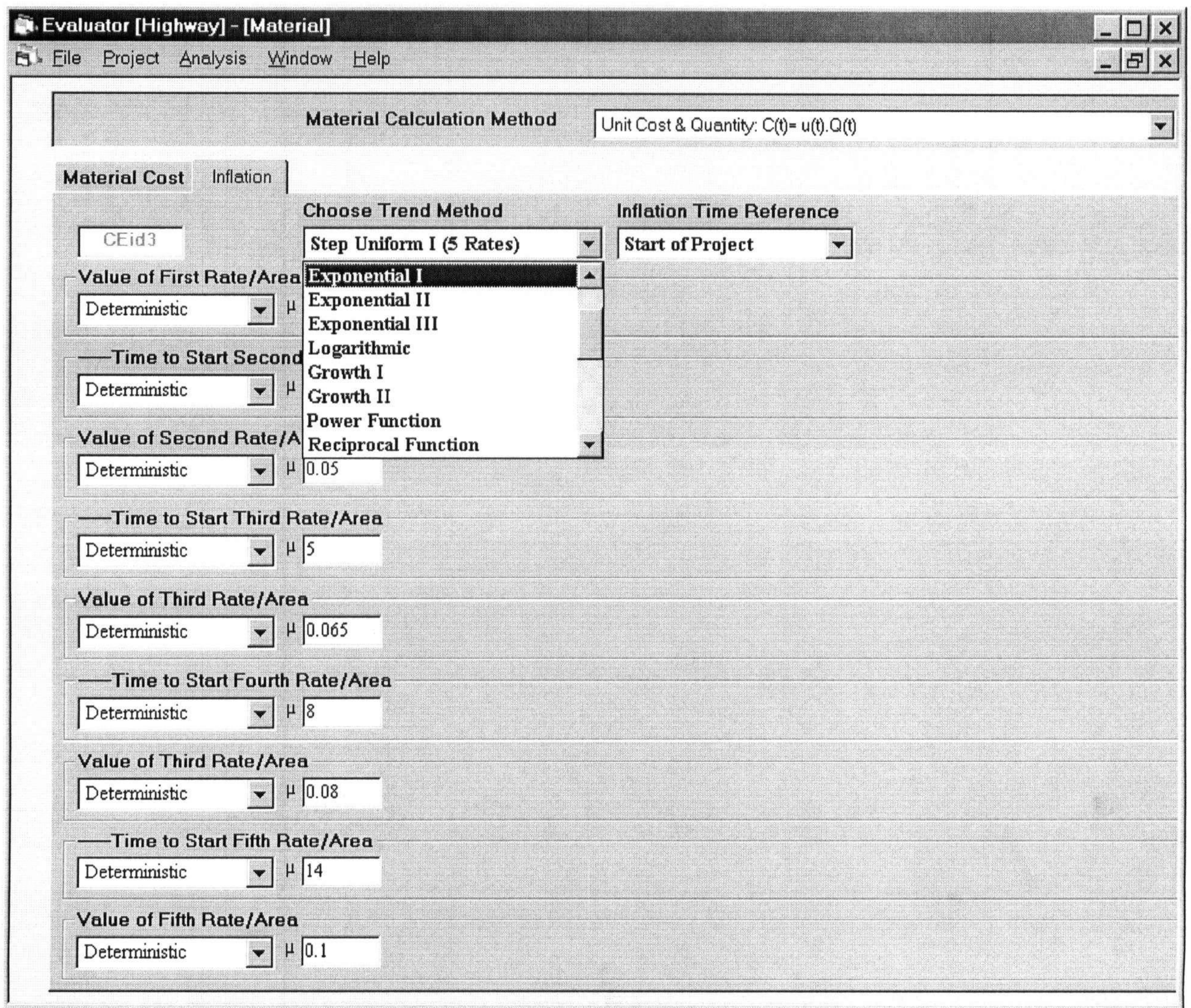


Figure 4.6: Shape function and global reference of material inflation of a work package

4.3 Capital Expenditure Component

Before a capital investment project becomes a productive asset, i.e. before start of operation, a large sum of money must be put in place to develop the project. This sum of money is called the *total investment cost*, which is composed of three components: fixed-capital investment, pre-production expenditure and working capital investment (Humphreys 1991; Peters and Timmerhaus 1991; Behrens and Hawranek 1991; Frohlich et al. 1994). The first two cost parts represent the capital required to develop, construct and equip the project. The foregoing authors explained in detail several cost items and categories of costs for these two components of the total investment cost, using the following classification (Behrens and Hawranek 1991):

1. Land (e.g. purchase and transfer, legal charges) and site preparation and development
2. Civil works, structures and buildings (including engineering and design costs)
3. Plant machinery and equipment
4. Incorporated fixed assets (e.g. industrial property rights and technical know-how, patents and license fees)
5. Pre-production expenditures (e.g. business set-up costs, human resources, pre-investment studies, project and site management, marketing costs)

Working capital investment represents the amount of capital needed to get the project started and meet its current obligations. Humphreys (1991) identified the following working capital items:

1. Raw materials inventory
2. Work-in-progress inventory (semi-finished goods) and finished-products inventory
3. Supplies for product manufacture
4. Taxes payable
5. Accounts receivable
6. Cash or equivalent on hand for salaries, wages, etc.
7. Accounts payable.

The different types of cost items described above for the total investment cost cover the costs needed to meet the requirements of the *development, possession, general liabilities, and tax* attributes as explained previously in Chapter 2. The capital expenditure component (Figure 4.1) is the vehicle through which cost items of the total investment cost can be estimated and modeled for cash flow analysis.

4.3.1 CE Component Structure and Properties

The structure of the CE component is made of constructs called work packages linked together via the *critical path method* (CPM) with finish-to-start relationships and lead/lag times (Clough and Sears 1991). A work package possesses time and cost characteristics using which the cost of the work package can be estimated, distributed over a period of time and presented for cash flow analysis. In a coarse representation, the total capital expenditure cost can be defined in a few number of work packages (e.g. three) linked together by CPM as shown in level 1 in Figure 4.7. Alternatively, at a higher level of detail, the total capital expenditure can be defined in several number of work packages detailing all items of cost, and linked together by CPM as shown in level 4 in Figure 4.7. The cost of any group of work packages can be combined to provide an aggregate or lump sum cost of the group; two or more groups can further be combined to yield a higher lump sum. The CE component provides four group/aggregation levels as shown in Figure 4.7. Detail/coarse representation, however, can better be defined in terms of the type of cost estimating method selected for cost calculation of a work package, as detailed later.

The CE component has *time unit, network* and *reference* properties, which can be inherited by any new work package where each work package may have its own values of such properties. The same time unit property, however, must be shared by all work packages. If the component uses, for example, a year as a time unit, then all work packages must use the same time unit (LTU). While CE component work packages use a LTU, the early and finish times of work packages, *ESc* and *EFc*, and defined in global time units GTU (see Eqs. E4.1, E4.2).

Figure 4.8 shows three windows that explain the implementation of all such properties in the support system. Through the "capital expenditure" window any work package can be added or deleted. The "scheduling" window shows the duration, predecessors, and lead/lag times for work package *Ceid3*. Work package time unit is shown in the "Project Properties" window.

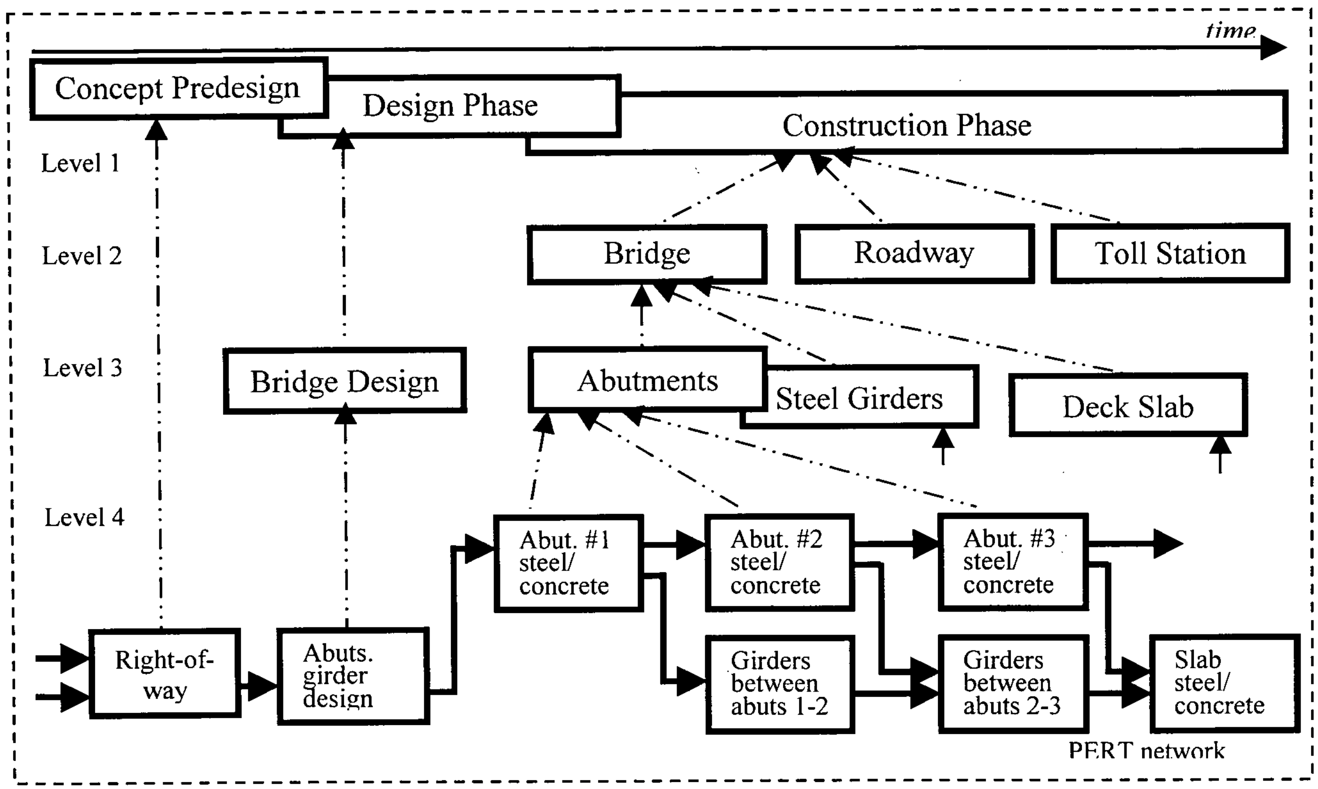


Figure 4.7: Capital expenditure component structure

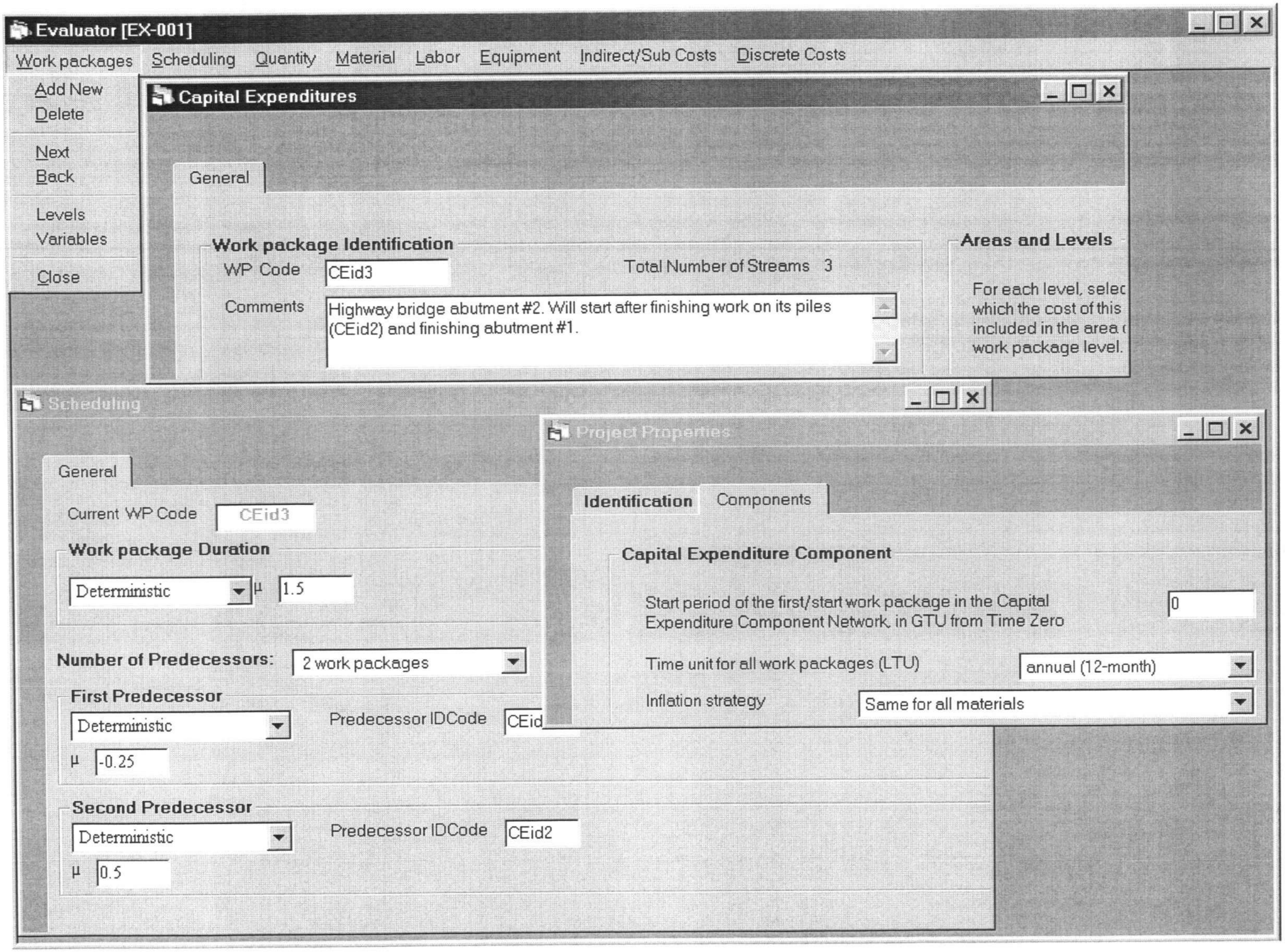


Figure 4.8: Time and network properties in the CE component

Any item of cost or expenditure must be attached or made within a CE work package. Each work package represents a cash flow function $f_c\ (t, \boldsymbol{x})$ where $\boldsymbol{x}$ refers to the work package variables that are used in cost and cash flow calculations. These variables can be categorized under material, labor, equipment, indirect or sub-contracting, capital forecasting, discrete, inflation, and network variables. The variables that are price-variables (e.g. unit costs and wages) can have their *reference property* set to either a global or local reference; inflation variables can be local, global or null (i.e. no inflation); capital forecasting variables and non-price-variables (e.g. quantity and productivity) have a local reference only; however, discrete and network variables may have no reference. Table 4.3 summarizes some of the properties of these variables. Figure 4.6 shows the specification of inflation of material cost of a work package using a global reference system.

Table 4.3: Properties of work package variables

Variable*	Definition	Shape Functions	Time Reference	Function Input** (e.g.)	Function Output (e.g.)
$M(t')$	Total material cost per time unit	Area/rate	Local	\$ or \$/day	\$/day
$C_m(t'')$	Unit cost of material	Rate	Global/local	$\$/m^3$	$\$/m^3$
$Q(t')$	Quantity per unit of time	Area/rate	Local	m^3 or m^3/day	m^3/day
$U_l(t')$	Labor usage/input per time unit	Area/rate	Local	mhrs or mhrs/day	mhrs/day
$P_l(t')$	Labor productivity/	Rate	Local	m^3/mhr	m^3/mhr
$W_l(t'')$	Labor unit cost per labor time unit	Rate	Global/local	\$/mhr	\$/mhr
$C_l(t'')$	Labor unit cost per unit of production	Rate	Global/local	$\$/\ m^3$	$\$/\ m^3$
$S(t)$	Subcontracted/indirect cost	Area/rate	Local	\$ or \$/day	\$/day
$X(t')$	Capital forecasting	A/R/other	Local	\$ or \$/day	\$/day
$\theta(t'')$	Inflation, 6 variables	Rate	Global/local/ No reference	N/A	N/A

* Variables subscripted to *l* (labor) have equivalents referring to equipment and are subscripted to *e*

** Depending on the type of function, arguments to a function may vary, e.g. total value or value per unit of time.

As explained previously, each work package has a cash flow vector $\boldsymbol{x}$ that refers to the above variables; and typically, each work package may have its own values for such variables. Economic analysis may be performed, however, with the assumption that values of some variables are the same throughout the calculation. For example, a variable for concrete unit price $C_m(t``)$ may be considered to have the same value (e.g. 1000 $\$/m^3$) across some work packages and different values (e.g. 2000 $\$/m^3$) for others. Sensitivity analysis on total cost of construction with respect to the 1000 $\$/ m^3$ concrete unit price will need to check on which work packages have this value of this variable before performing such analysis.

For this purpose, the CE classification has three properties that can be set to determine the treatment of the CE variables. The first property refers to *common* variables where if a variable is *common* then all work packages that use this variable will use the same value and *shape* function method in all cash flow calculations - see Figure 4.9. The second property refers to *special* variable where only those identified work packages that have this special variable will have the same value and shape function method in their cash flow calculation - see Figure 4.9. The third property refers a treatment of inflation variables. The CE classification provides several strategies to deal with inflation as shown in Figure 4.10, where except for the *unique* treatment, the same value and *shape* function method will be used according to the treatment:

- *Unique*: each work package will have its values of the inflation variables;
- *Same-for-material or -labor or -equipment;*
- *Same-for-indirect or -discrete or -capital forecasting;*
- *Same-for-all-work packages;*
- *Same-for-project*: all three component CE, RV and OM will have same value and *shape* function method across all project work packages and streams.

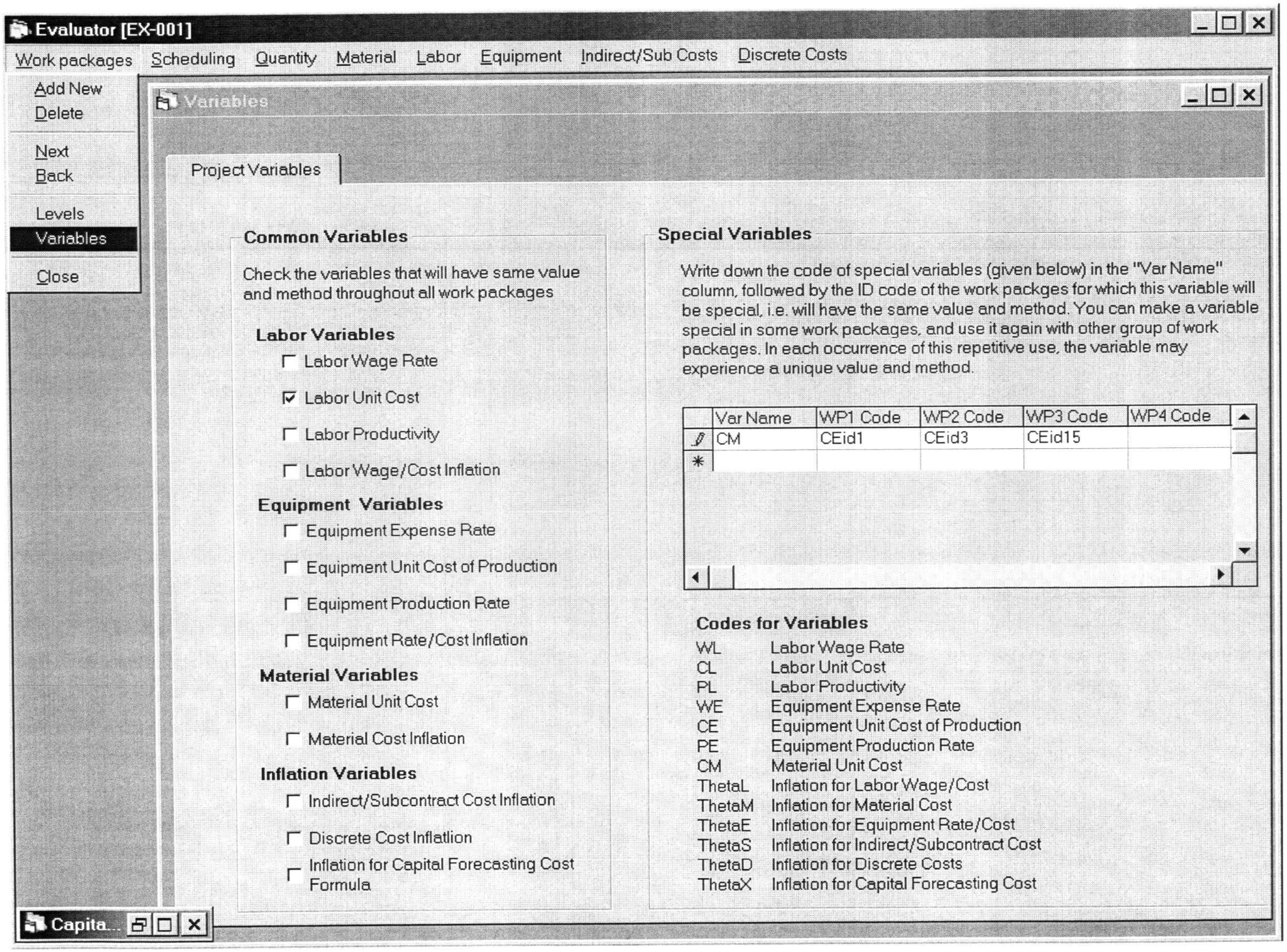


Figure 4.9: Capital expenditure common and specific variables

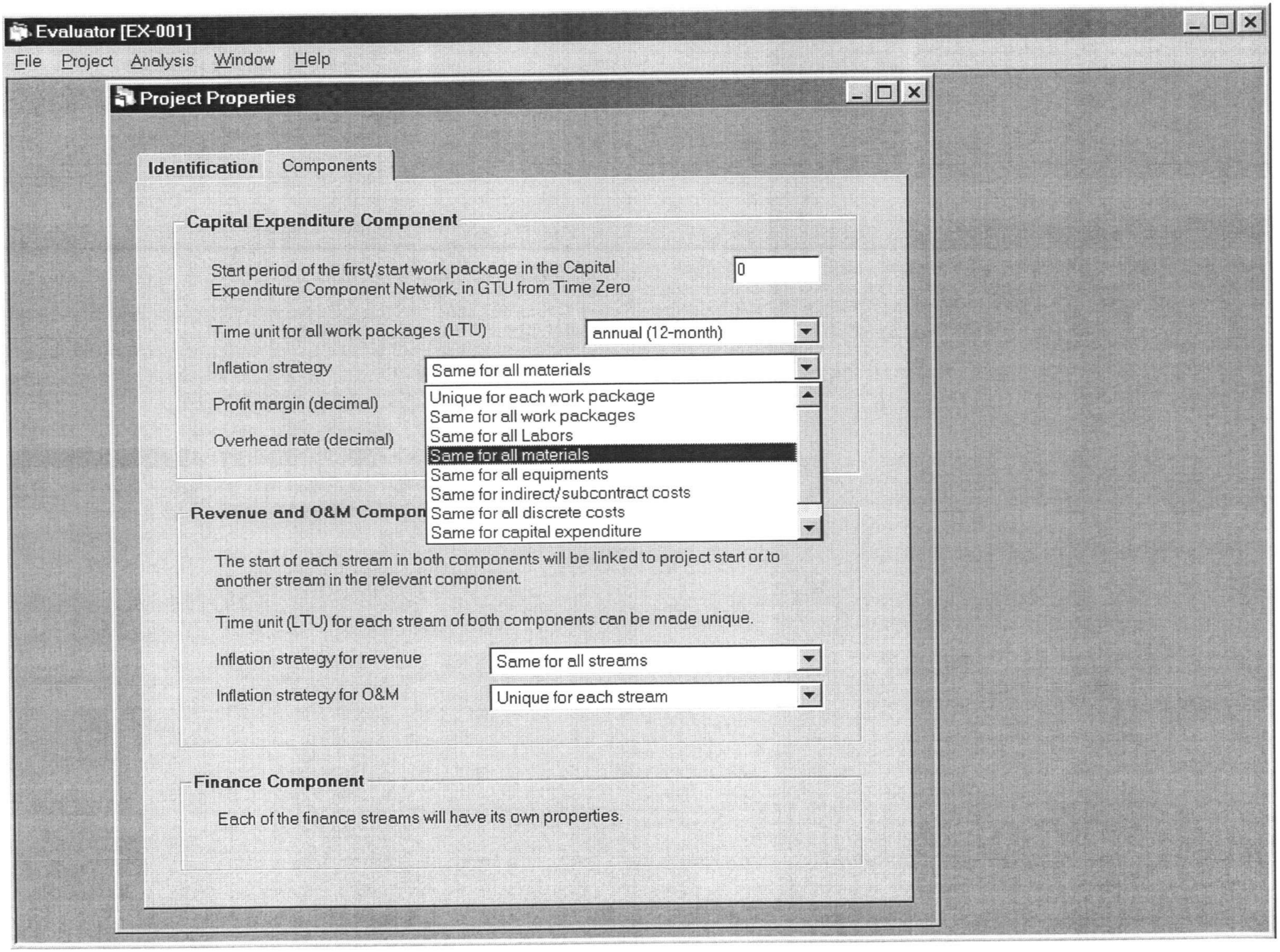


Figure 4.10: Inflation strategies for the CE, RV and OM components

4.3.2 CE Component Methods and Cash Flow Formulation

The CE component performs all of its cost calculations using semi-detailed (preliminary), detailed methods (Clough and Sears 1991; Humphreys 1991; PMI 1996; Hendrickson and Au 1989), and crude methods as represented in the CE classification. The cost of a work package can be distributed over its duration in two ways. The first way is to distribute cost according to a selected pattern of the shape function. The second is indirect where the variables comprising the cost estimate are allowed to change over time using shape functions and the distribution pattern of the total cost of the work package is derived from its variables. Therefore, while a work package cash flow function represents the expenditure flow needed to produce a quantity of work, it represents a cost estimating method as well. The following sub-sections describe how a work package cash flow function $f_c(t, \boldsymbol{x})$ is derived using the methods in the CE classification.

4.3.2.1 CE Semi-detailed Methods

Semi-detailed methods represent preliminary methods that arrive at a cost estimate based on experience, judgment and historical records. In the CE classification, semi-detailed methods are referred to as "gross" methods that distribute the total cost of a work package over its duration using a particular chosen pattern without regard to the individual items (e.g. material, labor, equipment) that contribute to total cost estimate. A cash flow function of a CE work package i can be described as follows:

$$f_c^{CE}(t`,\boldsymbol{x}) = X(t`) \cdot e^{\int_0^{t``} \theta_X(\underline{t})\mathrm{d}\underline{t}} \tag{4.4}$$

where $t`$ and $t``$ are as defined before; $X(t`)$ is a constant dollar capital expenditure variable, explained below, and has *local* reference; and, $\theta_X(t``)$ is an inflation variable for X and can have *global/local* reference and can take any form of *rate* functions in Table 4.1.

Semi-detailed methods representing *X(t`)* can be categorized in two parts. The first part of the methods is represented by the *area functions* described in Table 4.2. Therefore, the total cost of a work package must be defined in advance where the shape/pattern of the chosen area function is used to distribute the total cost over the work package duration.

The following are examples that help to explain the use of area functions in modeling capital expenditure. A preliminary estimate may distribute the cost of a project over its duration using a distribution or loading profile that divides the duration of a project into a number of periods where each period is loaded with a percentage of the total cost. This loading scheme was used by Texas TGV Corporation for their proposal for Texas High Speed Rail ("Texas" 1991). The corporation loaded the costs for one phase of the project as 5%, 10%, 15%, 40% and 30% over five years for engineering costs; 60% and 40% for right-of-way; 20%, 40% and 40% for fixed capital; 10%, 50% and 40% for rolling stock; and 40% and 60% for the cost of stations. Similar loading profiles called disbursement profiles were explained by Bacon and Besant-Jones (1996) and Merrow et al. (1990) from the World Bank, see Table 4.4.

The above two loading profile examples can be readily modeled by the *step uniform area* function in Table 4.2. However, the step uniform area function as shown in Table 4.2, which is implemented in the generalized model, can have up to five areas or totals representing the cost in a work package. If the total cost is to be divided in more than five areas, then another one or more work packages will have to be defined. For example, to model in the generalized economic model the loading profile that has 11 disbursements in Table 4.4, three work packages will be needed and linked together by finish-to-start relationships with zero lead times.

Table 4.4: Disbursement profiles for project cost in current price terms, Merrow et al. 1990

Annual Disbursements (of totals) in years	*Implementation periods (years)*									
	11	10	9	8	7	6	5	4	3	2
1	0.02	0.03	0.03	0.04	0.05	0.10	0.15	0.20	0.25	0.55
2	0.03	0.04	0.07	0.10	0.15	0.20	0.25	0.30	0.50	0.65
3	0.06	0.08	0.12	0.15	0.20	0.25	0.35	0.40	0.25	
4	0.01	0.12	0.15	0.25	0.25	0.25	0.20	0.10		
5	0.20	0.20	0.20	0.20	0.20	0.15	0.05			
6	0.20	0.20	0.20	0.15	0.10	0.05				
7	0.15	0.15	0.12	0.07	0.05					
8	0.10	0.10	0.08	0.04						
9	0.08	0.05	0.03							
10	0.04	0.03								
11	0.02									
SUM	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

The second part of the semi-detailed methods for *X(t`)* represents *CE component-specific* methods. These methods represent a number of capital forecasting cash flow models that were developed based on historical data from building and civil projects. Using these models, total project cumulative cash flow is described by an *S* form or pattern. For this case, total work package/project cost and duration are specified along with a number of parameters. The CE classification includes five of such models referred to in the literature among a body of work in capital expenditure modeling. The models are based on the logit transformation and s-curve formulas described in Table 4.5 (Kenley and Wilson 1986, 1989; Hudson 1978; Berny and Howes 1982; Miskawi 1989; De La Mare 1979). Capital forecasting models in Table 4.5 can be considered as shape functions that have a vector of variables ***y***, which represents the parameter variables in each of the five models. The expressions provided in Table 4.5 are the cumulative forms of the five models. The first derivative of these forms represents the expenditure function for each capital forecasting model, which usually takes a bell shape.

Table 4.5: Capital forecasting functions

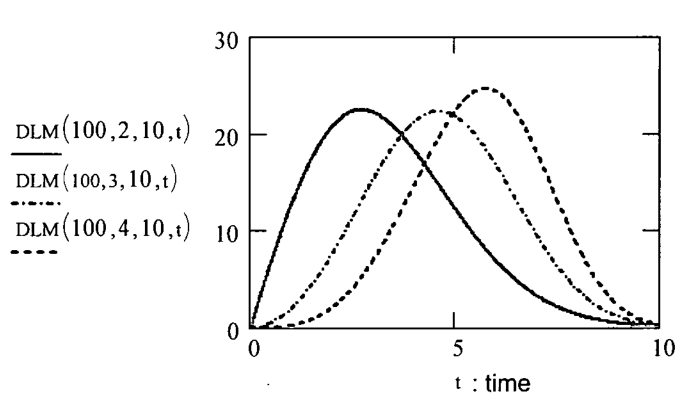
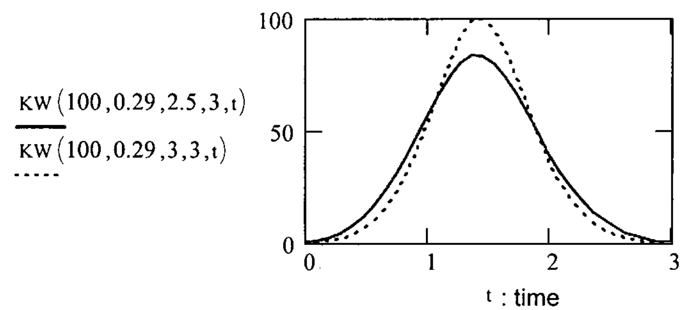
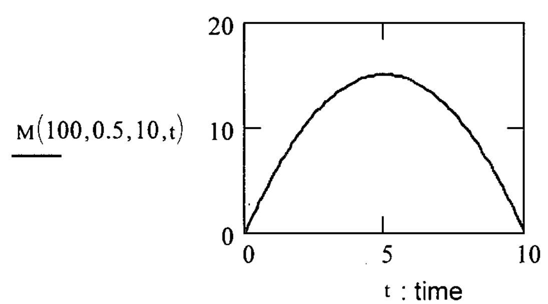
Pattern	Capital forecasting Function: $f_s(t, y)$	Form
CE Formula 1	$d'\left[Qt - Qt\cdot\exp\left[-t^{\beta}\cdot\left(\frac{1}{\eta}\right)^{\beta}\right]\right]$ where: $\eta = b\cdot 6.90776^{\left(\frac{-1}{\beta}\right)}$ d' : derivative w.r.t (t) Where DLM(Qt,β,b,t): Qt, β, b : total value, shape factor, total time (duration) De La Mare (1979) Very early & very steep rises in capital expenditure (β=2) Very late & very slow rises in capital expenditure (β=4)	
CE Formula 2	$d'\left(Qt\cdot\frac{F(t/b,\alpha,\beta)}{1+F(t/b,\alpha,\beta)}\right)$ where: $F(t,\alpha,\beta) = e^{\alpha}\cdot\left(\frac{t}{1-t}\right)^{\beta}$ Where KW(Qt,α,β,b,t): Qt, α, β : total value, shape parameters b: total time (duration). d`: derivative w.r.t. (t). Kenley and Wilson (1986, 1989)	
CE Formula 3	$d'\left[Qt\cdot\left[\frac{3^{t}}{2}\cdot\sin\left[\pi\cdot\frac{(b-t)}{2\cdot b}\right]\cdot\sin\left(\frac{\pi\cdot t}{b}\right)\cdot\ln\left(\frac{t+0.5\cdot b}{\alpha+t}\right) - \frac{2\cdot t^{3}}{b^{2}} + \frac{3\cdot t^{2}}{b}\right]\right]$ Where M(Qt,α,b,t): Qt, α, b: total value, shape parameter, total time (duration) Miskawi (1989) Developed to determine percentage completion of a project. Modified here to include Qt such that cost would be in proportion to percentage complete. α = (0.02:0.97)	

Table 4.5: Capital forecasting functions (continued)

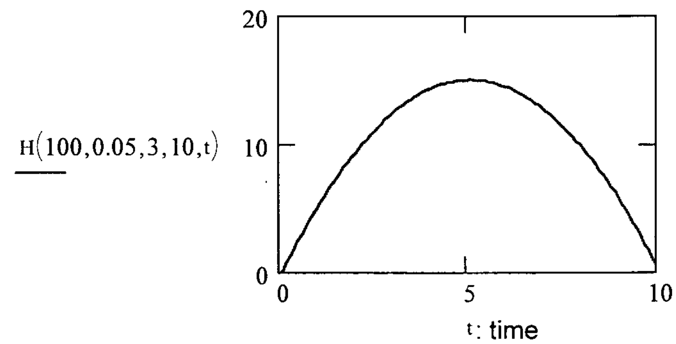
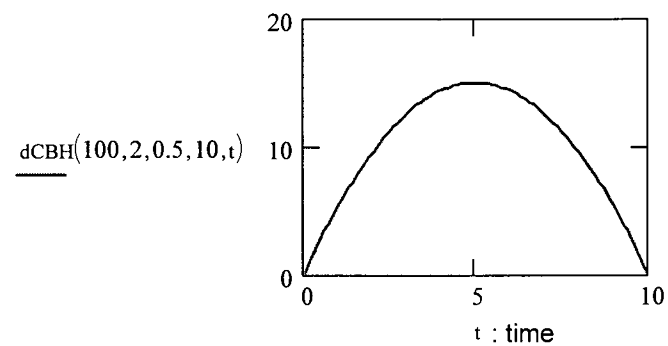
Pattern	Shape Function: $f_s(t, y)$	Form
CE Formula 4	$d`\left[Qt\cdot\left(\frac{t}{b}+\alpha\cdot\left(\frac{t}{b}\right)^2-\alpha\cdot t-(6\cdot\left(\frac{t}{b}\right)^3-9\cdot\left(\frac{t}{b}\right)^2+3\cdot\left(\frac{t}{b}\right))/\beta\right)\right]$ where d` is derivative w.r.t (t) Where H(Qt,α,β,b,t): Qt, α, β: total value, shape parameters b: total time (duration) Hudson (1978)	
CE Formula 5	$d`\left(Qt\cdot\frac{t}{b}\cdot\left(\alpha\cdot\left(1-\frac{t}{b}\right)\cdot(t-\beta)\right)\right)$ where d`: derivative w.r.t (t) Where dCBH(Qt,α,β,b,t): Qt, α, β: total value, shape parameters b: total time (duration) Berny and Howes (1978)	

Figure 4.11 shows that semi-detailed methods, called "holistic" in the interface, can be used for cost calculation of a work package. In the "capital forecasting" window, the De La Mare capital forecasting method is used and therefore two parameters must be specified as in Table 4.5, the first is the total cost of the work package and the second is the shape factor β. The order of these parameters is the same order used to explain the variables in the $\boldsymbol{y}$ vector in Tables 4.1,4.2, and 4.5. As shown on Figure 4.11, capital forecasting has no reference as it is considered *local* by default.

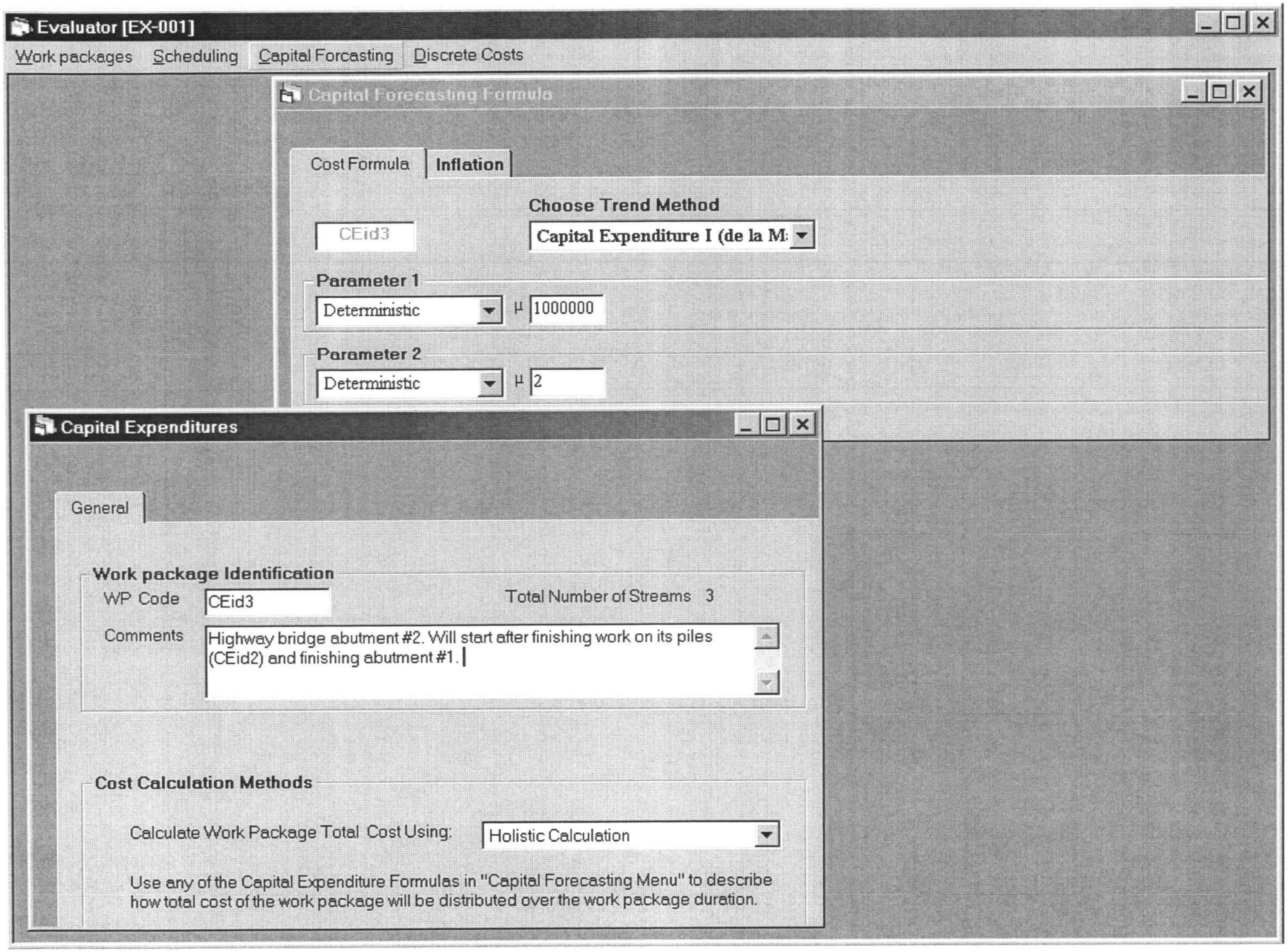


Figure 4.11: Work package cost calculation using capital forecasting methods

4.3.2.2 CE Detailed Methods

The second category of CE classification methods represents detailed cost estimating. An item of work is decomposed into its basic four elements - material, labor, equipment and subcontracted/indirect costs. The cost of the work item is then determined by aggregating the costs of its elements. Detailed estimating is generally referred to as definitive or bottom-up estimating (Clough and Sears 1991; Humphreys 1991; PMI 1996). Each of the basic elements may have its own methods of calculation; the CE classification, therefore, contains several methods for each of the four elements, as detailed below.

Equation 4.4 describes how a work package cash flow function is derived using semi-detailed methods for preliminary estimating. A cash flow function, however, for a decomposed estimating approach a CE work package can be written as follows:

$$f_c^{CE}(t`,\boldsymbol{x}) = f_{cm}(t`,\boldsymbol{x})\cdot e^{\int_0^{t``}\theta_m(\underline{t})\mathrm{d}\underline{t}} + f_{cl}(t`,\boldsymbol{x})\cdot e^{\int_0^{t``}\theta_l(\underline{t})\mathrm{d}\underline{t}} + f_{ce}(t`,\boldsymbol{x})\cdot e^{\int_0^{t``}\theta_e(\underline{t})\mathrm{d}\underline{t}} + f_{cs}(t`,\boldsymbol{x})\cdot e^{\int_0^{t``}\theta_s(\underline{t})\mathrm{d}\underline{t}} \tag{4.5}$$

where $t`$ and $t``$ are as defined before. $\theta_m(t)$, $\theta_l(t)$, $\theta_e(t)$, and $\theta_s(t)$ are inflation variables for the money-related variables (described below) in the material, labor, equipment and subcontracted elements respectively and they can be represented by any form of *rate* function in Table 4.1. The functions $f_{cm}(t`, \boldsymbol{x})$, $f_{cl}(t`, \boldsymbol{x})$, $f_{ce}(t`, \boldsymbol{x})$ and $f_{cs}(t`, \boldsymbol{x})$ are for cash flow calculations of the material, labor, equipment and subcontracted items respectively. The CE classification methods used for each of these functions are described below. All the variables in these functions are modeled by *shape* functions.

a) Material Estimating Methods

The material element f_{cm} *(t`, x)* of the decomposed estimate of a work package can be estimated through semi-detailed and detailed methods. Semi-detailed material estimating is where gross total material cost is defined and distributed over the work package duration using an *area* function or where gross total material cost per unit of time is defined using a *rate* function over the duration of the work package. *M(t`)* as shown in Eq. (4.6) below represents material cost gross estimating methods, uses *area/rate* functions and a *local* reference system.

Detailed material estimating corresponds to unit cost estimating methods. These methods depend on the use of unit cost/price C_m*(t``)* and work quantity *Q(t`)* to determine the total material cost of a work item. Alternatively, the quantity of work can be obtained through the labor input $U_l$*(t`)* required by the work item and the productivity of labor $P_l$*(t`)*. Both methods are described in Eq. (4.6) below. $C_m$*(t``)* can have a *local/global* reference while *Q(t`)*, $U_l$*(t`)* and P_l*(t`)* can only have a *local* reference. C_m*(t``)* and P_l*(t`)* are modeled by *rate* functions while *Q(t`)* and U_l*(t`)* are modeled by *rate/area* functions.

A work package material cost cash flow function can be described as follows:

$$f_{cm}(t`,x) = \begin{cases} M(t`) \\ C_m(t``) \cdot Q(t`) \\ C_m(t``) \cdot P_l(t`) \cdot U_l(t`) \end{cases} \tag{4.6}$$

where *M(t`)* the total cost of material per unit of time, C_m*(t``)* is the unit cost of material (e.g. $\$/m^3$); *Q(t`)* is the work package scope/quantity placed per unit of time; $U_l$*(t`)* is the labor usage/input per unit of time (e.g. mhrs/day); and P_l*(t`)* is the labor productivity in placing a unit of quantity (e.g. m^3/mhr).

For example, if a work package needs 500 m^3 of concrete to be placed in 10 days, and where the price of 1 m^3 is \$1000, then the material cost of the work package is \$500,000. Assume that the price will remain constant during the work package duration, i.e. *uniform rate* function (Table 4.1). Further, assume that large quantities will be placed in the first four days and the placement rate will tail off over the remaining duration. This placement profile can be represented by an *exponential III area* function (Table 4.2). Then the material cash flow function Eq. (4.6) will be as shown in Figure 4.12. If it is assumed that the placement rate was constant over the work package duration, a *uniform area* function would be used and the cash flow function would be the dashed line in Figure 4.12. In both cases, the total cost by the end of the work package is \$500,000. However, their discounted values would be different.

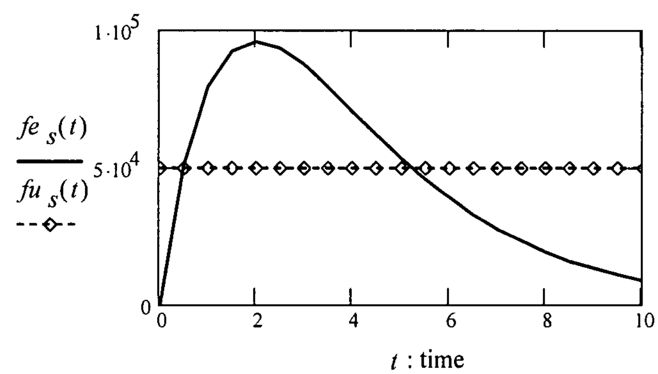


Figure 4.12: Material cost cash flow function

b) Labor and Equipment Estimating Methods

Both the labor cost f_{cl} *(t`, x)* and equipment cost $f_{ce}$ *(t`, x)*, as defined in Eq. (4.5), use similar calculation methods and variables. These methods, described in Eq. (4.7) below, fall into three categories: gross estimating methods, cost per unit of time and cost per unit of production.

The gross labor methods determine the labor cost in a work package in terms of total labor cost that is distributed over the work package duration using a particular pattern or in terms of gross total labor cost per unit of time. Gross methods are represented by the *area/rate* functions and modeled by $H_l(t`)$ in Eq. (4.7) using a *local* reference system.

The second category determines labor cost per unit of time in terms of labor unit cost per unit of time $W_l(t``)$ (e.g. hourly labor-wage rate) multiplied by the labor usage/input $U_l\,(t`)$ (e.g. number of man hours per unit of time). Alternatively, where rate of work $Q(t`)$ and productivity of labor $P_l(t`)$ are available, $U_l\,(t`)$ can be evaluated (i.e. $Q(t`) = U_l\,(t`) \cdot P_l(t`)$) as shown in Eq. (4.7). $W_l(t``)$ can have a *local* or *global* reference while $U_l(t`)$, $Q(t`)$ and $P_l(t`)$ can have only *local* reference. $W_l(t``)$ and $P_l(t`)$ are modeled by *rate* functions while $Q(t`)$ and $U_l(t`)$ are modeled by *rate/area* functions.

The third category for labor cost determines labor cost in a work package in terms of labor unit cost $C_l(t``)$, i.e. labor cost per unit of production, and rate of work $Q(t`)$. The quantity of work can be obtained alternatively through the labor input $U_l(t`)$ required by the work item and the productivity of labor $P_l(t`)$ as shown in Eq. (4.7). $C_l(t``)$ can have either *local* or *global* reference while $U_l(t`)$, $Q(t`)$ and $P_l(t`)$ can only have a *local* reference system. $C_l(t``)$ and $P_l(t`)$ are modeled by *rate* functions while $Q(t`)$ and $U_l(t`)$ are modeled by *rate/area* functions.

Using the above three categories of methods, labor cost cash flow function can be expressed as follows:

$$f_{cl}\,(t`,\boldsymbol{x}) = \begin{cases} H_l(t`) \\ W_l(t``)\cdot U_l(t`) \\ W_l(t``)\cdot Q(t`)/P_l(t`) \\ C_l(t``)\cdot Q(t`) \\ C_l(t``)\cdot P_l(t`)\cdot U_l(t`) \end{cases} \tag{4.7}$$

where, $H_l(t`)$ is a total labor cost per unit of time; $W_l(t``)$ is unit labor cost per unit of labor time (e.g. \$/mhr); $U_l(t`)$ is labor usage per unit of time (e.g. mhrs/day); $Q(t`)$, is work package scope/quantity placed per unit of time; $C_l(t``)$ is labor cost per unit of quantity; and $P_l(t`)$ is labor productivity in placing a unit of quantity (e.g. m^3/mhr).

The expressions for the equipment component make use of the same cost estimating methods just described for the labor component. Thus, the equipment cash flow function f_{ce} *(t`, x)* is similar to the labor cash flow function in Eq. 4.7, but with all variables subscripted to *e*.

c) Subcontracted/Indirect Estimating Methods

Subcontracted costs are expenditures made to subcontractors for performing a specific portion of the project. Indirect costs generally represent those expenditures made to cover overheads (e.g. job and office), supervision, construction expenses (e.g. utilities, temporary facilities, permits, taxes), and contingencies (Humphreys 1991; Peters and Timmerhaus 1991). Although these costs are general project expenses and usually not assignable to a specific work package they can be represented in the generalized model from within work packages. This provides for these costs to be either reflected as part of a direct cost work package, or to stand alone as if one or more work packages dedicated solely to indirect costs.

The subcontracted/indirect element of a work package cash flow is represented by gross methods that determine the estimate in terms of cost distributed over the work package duration in a particular pattern or in terms of gross total cost per unit of time. Gross methods are represented by the *area/rate* functions and modeled by *S(t)* using a *local* reference system. Therefore, the subcontracted/indirect cash flow of a work package is represented as

$$f_{cs}(t`, \boldsymbol{x}) = S(t`) \tag{4.8}$$

4.3.2.3 CE Crude Methods: Discrete Costs

The preceding sections determined work package cash flow in terms of semi-detailed and detailed calculation methods that assumed expenditures were made as continuous functions of time. However, some expenditures may have to be made as discrete sums (e.g. procurement of major permanent equipment item or construction equipment). Therefore, the generalized model provides for discrete costs to be added to any work package, independent of whether the other cash flow component is formulated by semi-detailed or detailed methods.

The discrete cost $\mathbf{D}_j^{CE}$ is represented as follows: Let $\mathbf{Dv}_j^{CE}$ and $\mathbf{Dt}_j^{CE}$ represent constant dollar value and local time respectively for the j-th of n discrete costs in a work package, and let $\theta_d(t)$, modeled by *rate* functions, be the discrete-cost inflation variable. Then,

$$\mathbf{D}_j^{CE} = \mathbf{Dv}_j^{CE} \cdot e^{\int_0^{t^*} \theta_d(t)dt} \Bigg|_{\substack{t^*=\mathbf{Dt}_j^{CE} \text{ if } \theta_d \text{ is in local time} \\ t^*=\mathbf{Dt}_j^{CE}+ESc \text{ if } \theta_d \text{ is in global time}}} \tag{4.9}$$

4.3.2.4 CE Component Cash Flow

It is useful to summarize the flexibility offered in modeling CE component cash flows. Any work package cash flow function $f_c(t, \mathbf{x})$ can be formulated either through a semi-detailed function using Eq. (4.4) or through a detailed function using Eq. (4.5). The detailed function is represented through material, labor, equipment and indirect cost components as per Eqs. (4.6,7,8). Both the semi-detailed and detailed formulation of a work package can be supplemented by the discrete formulation given in Eq. (4.9).

$f_c\ (t,\ \boldsymbol{x})$ represents the essential information for any work package in the CE component. By aggregation, all work package cash flow functions integrate to form the CE component cash flow function as described in Eq. (4.1).

Evaluator, the decision support system developed as part of this thesis, has implemented the foregoing cash flow formulation. Figure 4.11 depicts the semi-detailed cost estimating approach. Figure 4.13 illustrates the detailed cost representation, where the top menu in the window shows material, labor, equipment, and indirect cost menus. Figure 4.13 shows the quantity window as well, where total quantity is assumed to be distributed using a *uniform area* function with no time reference as it is considered as *local* by default with quantity. Figure 4.14 illustrates the material and labor cost calculation windows for a work package. The top bars in these two windows allow the selection of a material and labor calculation method from those described in Eqs. (4.6,7). According to the method selected in these two windows, appropriate tabs are shown to request the information required for the selected calculation method. Figure 4.15 shows the input windows for the indirect and discrete costs in a work package. Note that each of the material, labor, indirect, and discrete cost windows has a tab for describing the *inflation* function, which can be unique for each of them.

In Evaluator, any number of work packages can be used to represent the capital expenditure of a project. However, in terms of Evaluator's performance, the reduction in the number of work packages and the use of semi-detailed methods contribute to faster processing of the performance measures. It is suggested to use the detailed methods for the elements of costs or work packages that require more analysis in the appraisal stage of a project. The more the number of work packages, particularly with the cost elements represented as risk variables as explained later, the more the time it takes Evaluator to do processing.

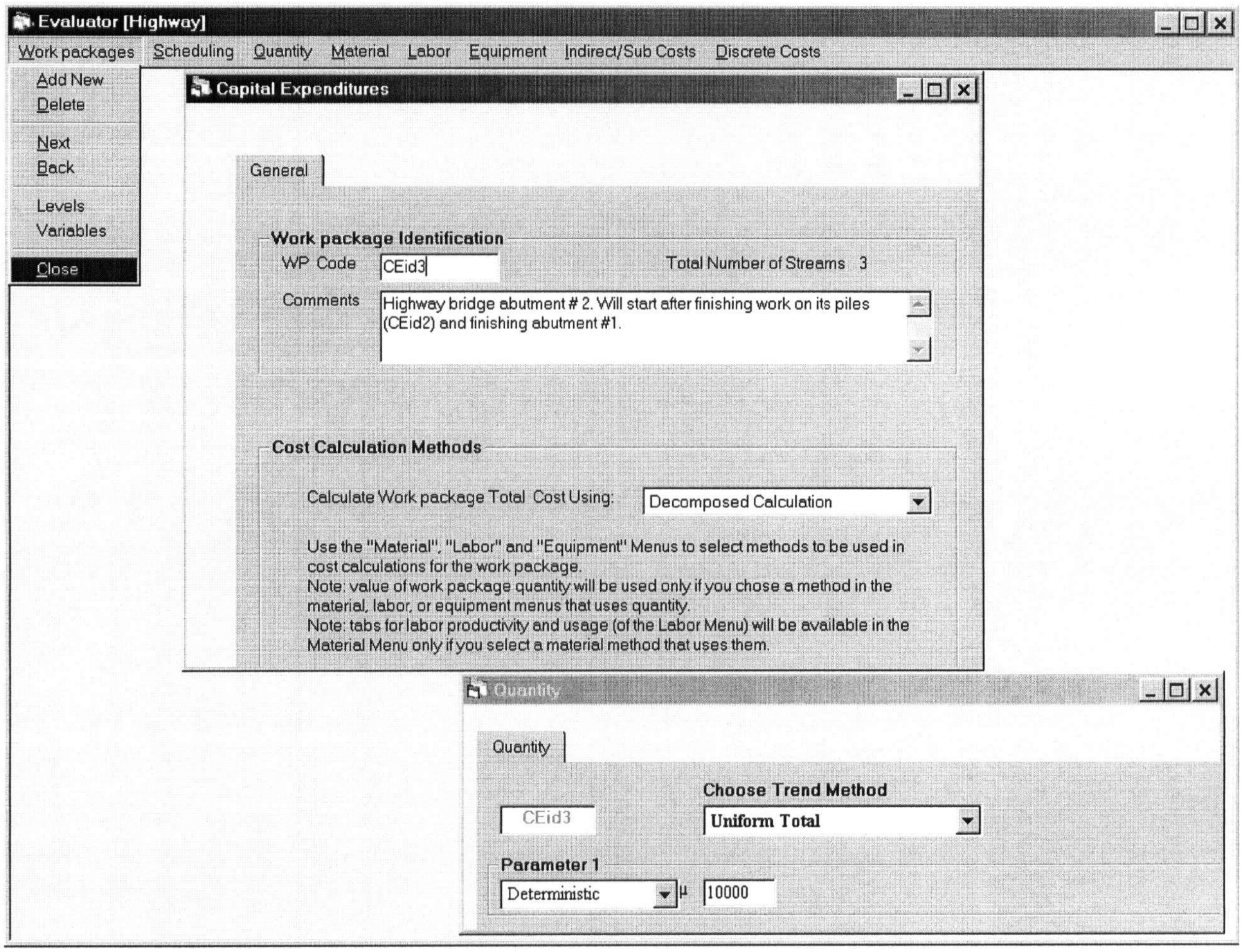


Figure 4.13: Work package cost calculation using decomposed methods and quantity window

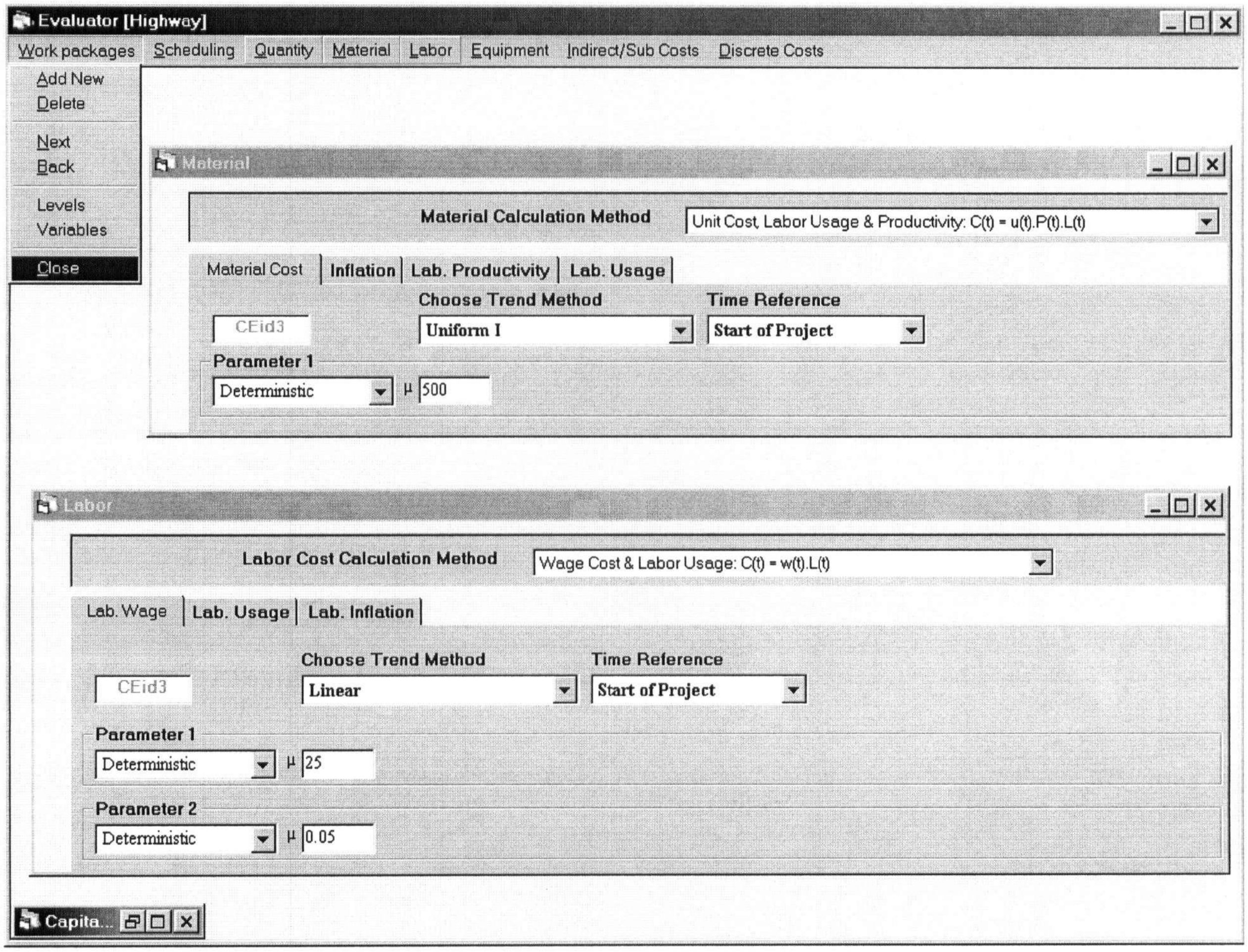


Figure 4.14: Work package material and labor cost window methods

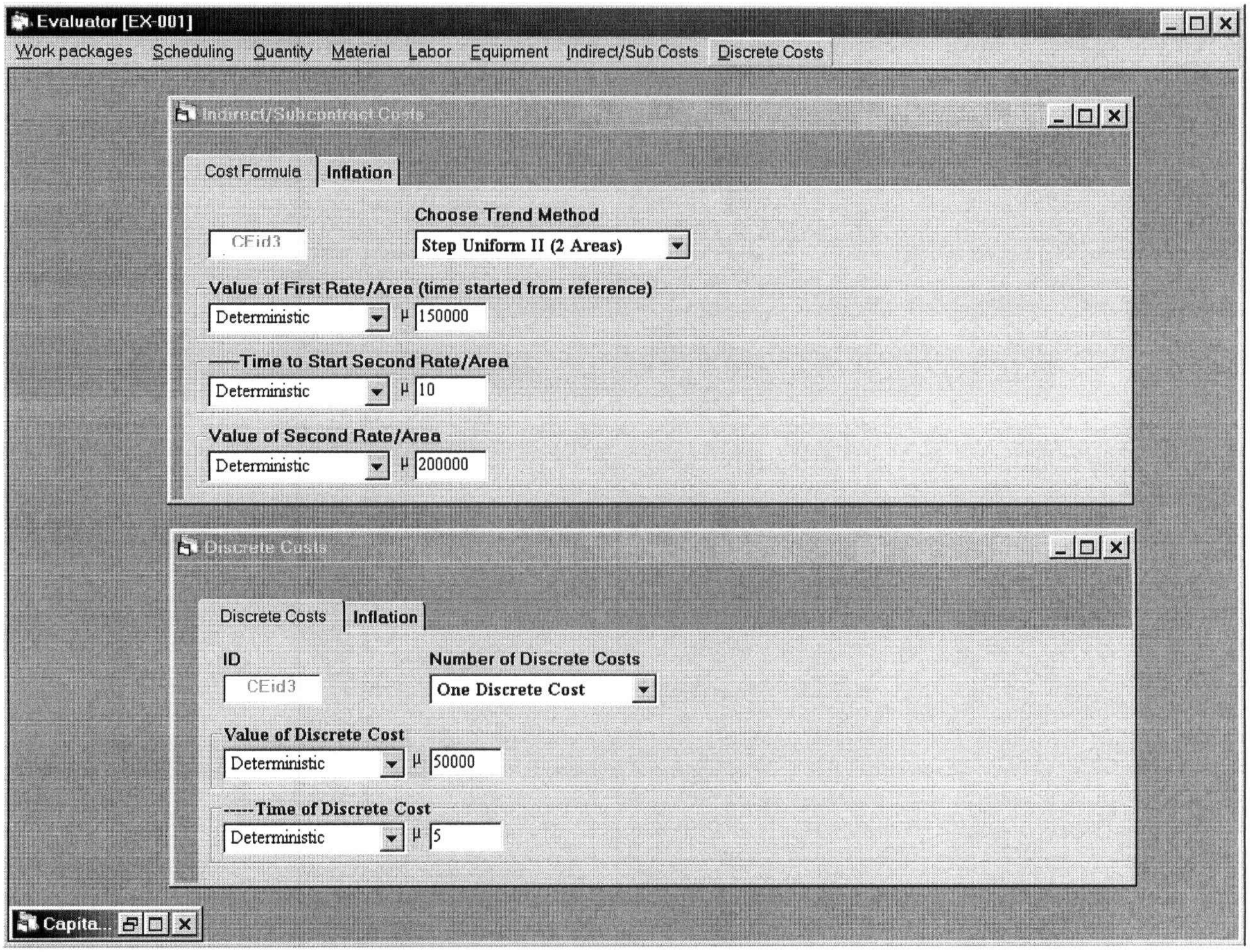


Figure 4.15: Work package indirect and discrete cost methods

4.3.3 CE Component Aggregation Levels

As explained early for the structure of a the CE component (section 4.3.1), a coarse or refined representation of capital expenditure cost can be modeled through the number of work packages used in the representation. However, the method used for cost estimating of a CE work package can contribute to how detailed or coarse the representation is. A work package can represent capital expenditure in a coarse level (e.g. total construction cost) using semi-detailed methods or in a refined level (e.g. cost of a single abutment) using detailed methods.

The cost of a group of work packages can be combined/aggregated in what is called a CE area. The CE component supports four aggregation levels each of which can contain several CE areas. A work package can belong to only one CE area per level. Two or more CE areas in a level can be aggregated as well but in a higher level. This reflects the concept of a work breakdown structure (WBS) (Mansuy 1991). Therefore, with reference to Figure 4.7 and a model that only includes the work packages in the fourth level, using the cost of "Abut. #1" and other fourth level work packages, it is possible to aggregate and get cash flows for "abutments" in the third level, "bridge" in the second level, and "construction phase" in the first level.

Using the work packages' cash flow function, the cash flow function of CE area A is:

$$f_a^{CE}(t,X)=\sum_{i=1}^{n} f_c^{CE}(t`,x_{A_i})_{A_i} \tag{4.10}$$

in which n is the number of work packages in the CE area A, A is a vector that contain work packages in the area, and t and $t`$ are as defined before.

Figure 4.16 illustrates how the system implements the levels and shows how areas are defined in the CE component. Figure 4.17 shows how work package *Ceid1* can belong to any or all of the three aggregation levels as exemplified by Figure 4.7 before.

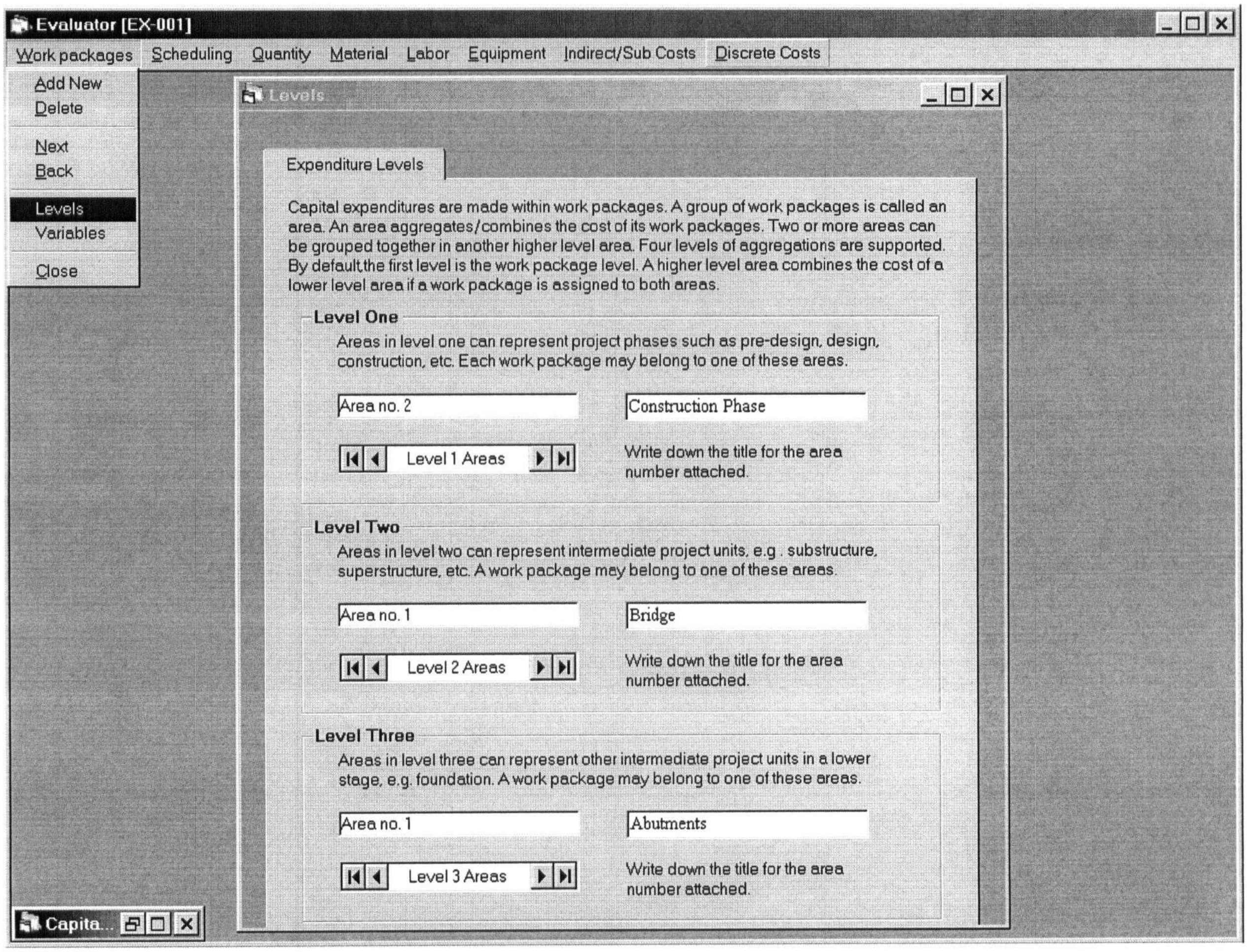


Figure 4.16: Capital expenditure component areas and levels

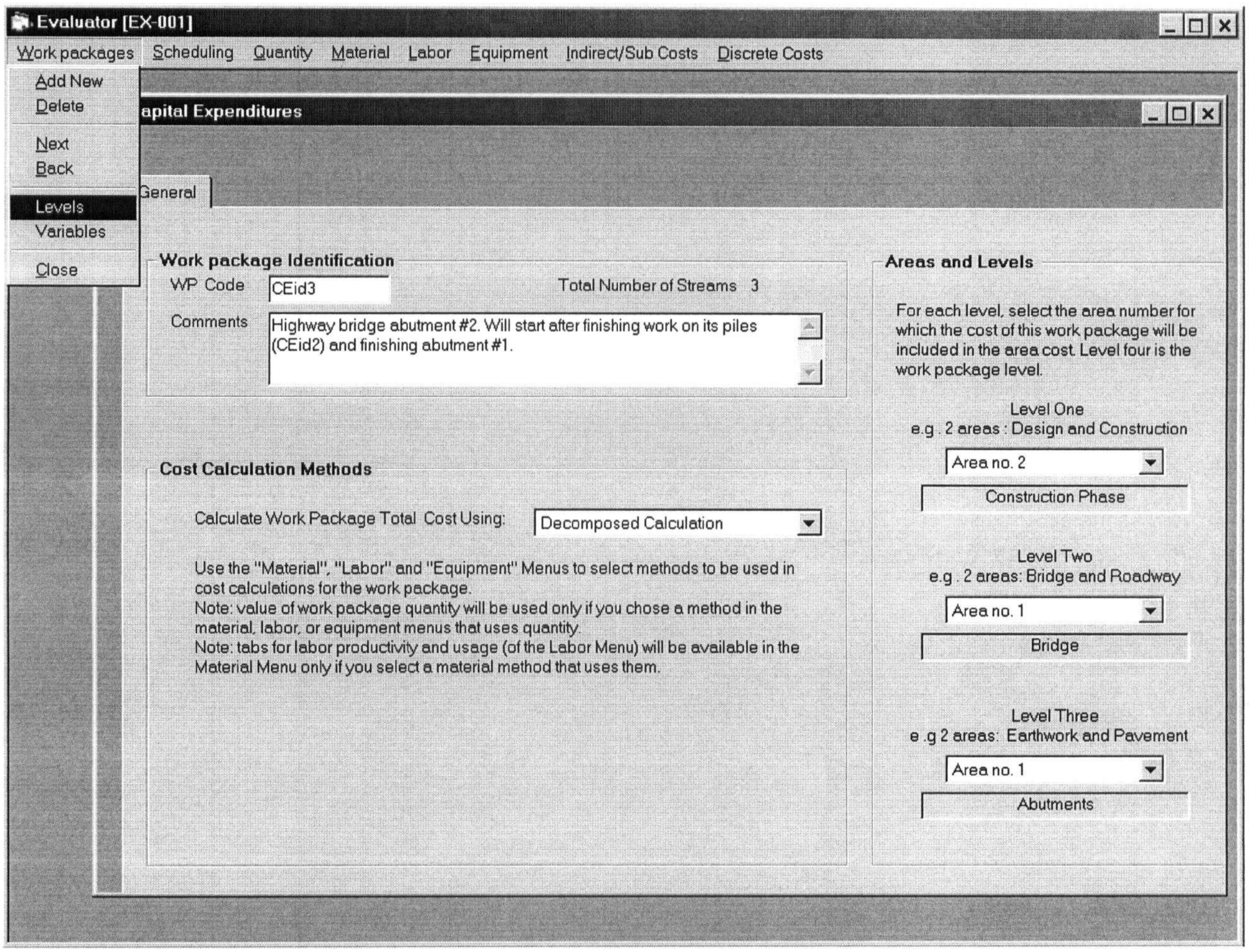


Figure 4.17: A work package may belong to any or all levels in the CE component

4.3.4 CE Component Cumulative and Discounted Costs

A cumulative cash flow $F_c^{CE}(t`,x)$ of a work package at any local time $t`$ can be obtained as follows, where r is the total number of discrete costs in a work package:

$$F_c^{CE}(t`,\boldsymbol{x}) = \int_0^{t`} f_c^{CE}(\underline{t},\boldsymbol{x})\,d\underline{t} + \sum_{j=1}^{r} \mathbf{D}_j^{CE} \Big|_{\text{for each } \mathbf{Dt}_j^{CE} \leq t`} \tag{4.11}$$

in which $\underline{t}$ is a dummy variable.

Consequently, cumulative cash flow $F_{CE}(t,\boldsymbol{X})$ of the CE component at any *global* time t can be obtained as follows, where $\overline{t - ESc_i}$ is the time elapsed in work package i and converted from GTU to LTU (explained before) and m is the total number of work packages:

$$F_{CE}(t,\boldsymbol{X}) = \sum_{i=1}^{m} F_c^{CE}(\overline{t - ESc_i}, \boldsymbol{x}_i)_i \Big|_{\text{for each } t \geq ESc_i} \tag{4.12}$$

Similarly, cumulative cash flow $F_a^{CE}(t,\boldsymbol{X})$ of a CE area in any of the three levels in the CE component can be obtained as follows, where A is a vector of n work packages in the CE area,

$$F_a^{CE}(t,\boldsymbol{X}) = \sum_{i=1}^{n} F_c^{CE}(\overline{t - ESc_{A_i}}, \boldsymbol{x}_{A_i})_{A_i} \Big|_{\text{for each } t \geq ESc_{A_i}} \tag{4.13}$$

Discounted cost $df_c^{CE}(Td,\boldsymbol{x})$ of a work package can be obtained as follows where all cash flows are discounted to time Td which is measured in GTU with respect to the *global* reference system; y is the nominal annual discount rate representing the client's minimum acceptable rate of return (MARR) and $\overline{y}$ is y converted from annual time periods to LTU; wd is the duration of the work package in LTU; $\overline{tb}$ is the time before start of the work package (ESc), converted from GTU to annual time unit; and, $\overline{tn}$ is the time elapsed in the work package, converted from GTU to LTU:

$$df_c^{CE}(Td,\mathbf{x}) = e^{-y\cdot\overline{tb}} \cdot \left[\int_0^{wd-\overline{tn}} f_c^{CE}(\underline{t}+\overline{tn},\mathbf{x})\cdot e^{-\overline{y}\cdot\underline{t}}\mathrm{d}\underline{t} + \sum_{j=1}^{r} \mathbf{D}_j^{CE} \cdot e^{-\overline{y}\cdot(\mathbf{Dt}_j^{CE} - \overline{tn})} \Big|_{\text{for each } \mathbf{Dt}_j^{CE} \geq \overline{tn}} \right]$$

(4.14)

in which $\overline{tb} = \begin{cases} ESc - Td & \text{if } Td < ESc \\ 0 & \text{otherwise} \end{cases}$ (4.15)

and $\overline{tn} = \begin{cases} Td - ESc & \text{if } Td \geq ESc \\ 0 & \text{otherwise} \end{cases}$ (4.16)

Consequently, discounted cost of the CE component cash flows can be obtained as follows:

$$df_{CE}(Td,\mathbf{X}) = \sum_{i=1}^{m} df_c^{CE}(Td,\mathbf{x}_i)_i \tag{4.17}$$

Similarly, discounted cost of an area in a CE component level can be obtained as follows:

$$df_a^{CE}(Td,\mathbf{X}) = \sum_{i=1}^{n} f_c^{CE}(Td,\mathbf{x}_{A_i})_{A_i} \tag{4.18}$$

Figure 4.18 illustrates Evaluator's output screen for the total capital expenditure cash flow of the CE component. The window shows the periodical cash flow graph that implements Eq. (4.1) and the cumulative cash flow graph that implements Eq. (4.12), both drawn with annual periods. The attached table in the window shows the cumulative cash flows at the end of the selected annual intervals during the development period. The table and graphs show negative values since they represent expenditures.

The generalized economic model described thus far permits the addition of any number of work packages in the CE component, the selection of any *detailed* or *semi-detailed* calculation method, the selection of any *rate/area* function, and the choice of values for the *properties* of each work package.

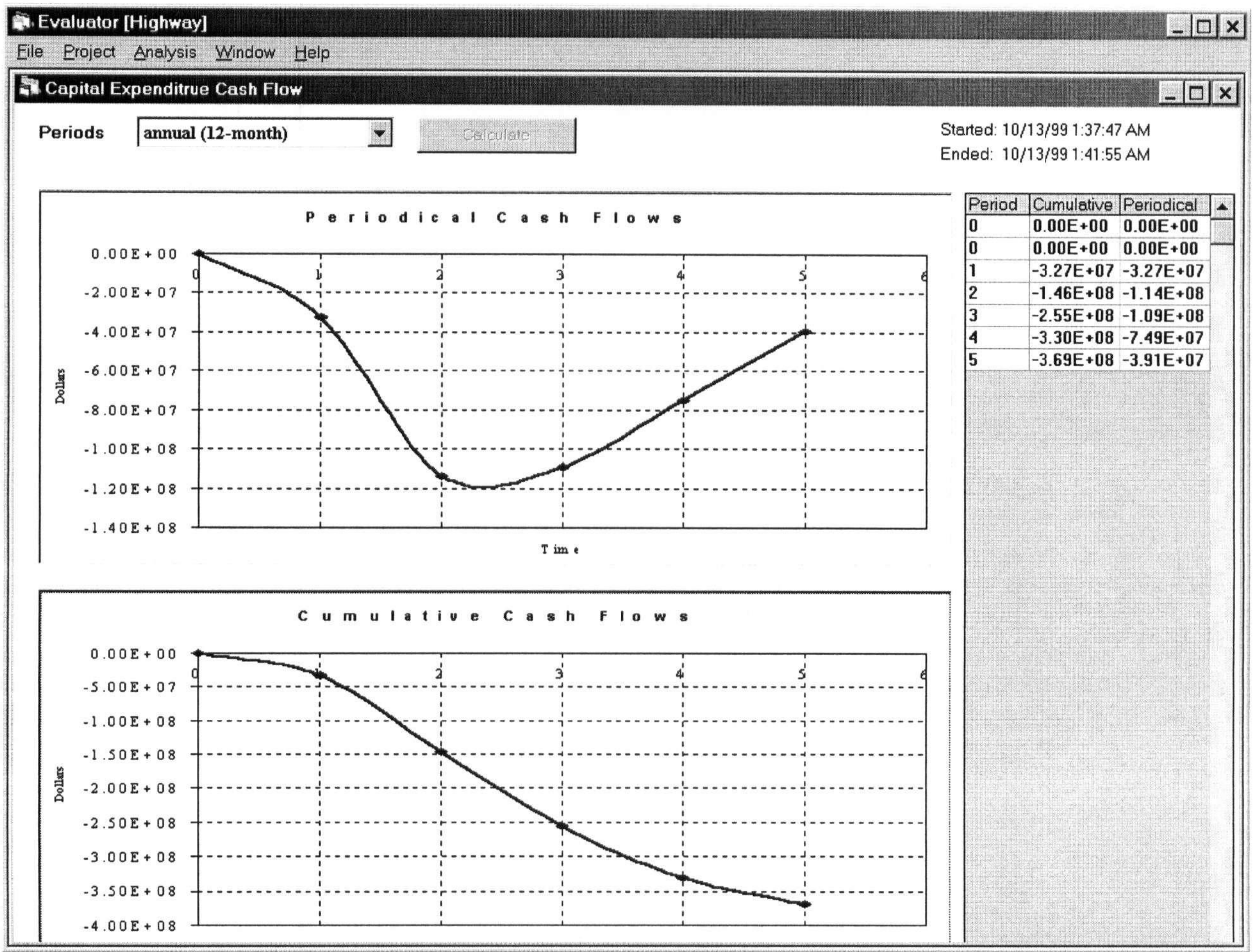


Figure 4.18: Total capital expenditure cash flow

4.4 Revenue Component

Market analysis helps in the determination of the scope of an investment or project, the possible production programs, and the likely revenue forecasts of the project. The analysis generally covers an assessment of three essential market ingredients for a proposed project: market potential, market volume, and market share. Market potential represents the total estimated demand (e.g. number of vehicles) for a product per time unit, while market volume (size) refers to the part of the market potential that is satisfied by the various producers. Market share is the relation between a company/project current volume and market volume (Frohlich 1994).

The process that determines the three essential market ingredients for a project generally includes the following analyses: market area delineation and location analysis, demand analysis, and supply analysis (Myers and Mitchell 1993; Fanning and Winslow 1988; Bailey et al. 1977). Demand analysis is concerned with how much demand might be experienced by a proposed project. Generally, demand analysis involves determination of the major market demand factors (e.g. population, income, and employment), studying historical market volumes (e.g. traffic volumes) and trends (e.g. growth rates), preparation of a detailed estimate of the current market potential and market volume, and projection of future market volumes (Behrens and Hawranek 1991; Wincott and Mueller 1995). Supply analysis involves mainly an analysis of competitive supply, study of past and current supply and forecast of future supply. Market analysis also entails an investigation of historical and future relationships between supply and demand for a certain product or facility.

Because market analysis generally furnishes projections about the future supply and demand of a service or product, various forecasting techniques are typically used in market analysis. Behrens and Hawranek (1991) explained some demand forecasting techniques, including trend (extrapolation) method, consumption-level method (including income and price elasticity of demand), end-use (consumption coefficient) method, and regression models. Generally these forecasts include several assumptions, and data that may include macroeconomic (e.g. gross domestic product) and microeconomic (e.g. regional and local population) factors. Consequently, demand projections are usually plagued with uncertainties that generally need to be addressed in commercial as well as economic appraisals.

The findings of market analysis represent key information for the commercial appraisal; these findings include, for example, market structure, volume and growth rates, demand for various products, demand patterns, demand factors, and the pricing structure of the likely products. These findings are essential for deriving demand and revenue function(s) for the economic analysis of a proposed project (e.g. they are employed in the project economic model) and consequently the determination of the likely revenues during a project life cycle.

As explained previously with the *revenue attribute* of the *requirement structure* of a project, revenues can be accumulated from direct operation of a project following its projected demand (i.e. based on market analysis), or from indirect or collateral sources such as government subsidies and side businesses to the project (e.g. advertising, parking, and leasing air space in a project right-of-way).

The generalized economic model structure in Figure 4.1 provides for a revenue component that possesses properties and methods through which various types of revenues as well as demand and revenue functions (as being derived from market analysis) can be modeled and represented for cash flow analysis. These properties and methods are the subject of the following subsections.

4.4.1 RV Component Structure and Properties

The structure of the RV component, as shown in Figure 4.19, is made of constructs called revenue streams. A stream has time, demand and revenue characteristics where various types of revenues can be modeled for cash flow analysis.

The RV component has *time unit, network* and *reference* properties, which can be inherited by any new revenue stream in the component. Each stream may have its own values of such properties. Unlike the CE component, each revenue stream in the RV component may have its own local value of the *time unit property*. For the *network* properties, each revenue stream can have its own duration and lead/lag times. The duration of each revenue stream is defined in its LTU. With no explicit network within the RV component, the logic property of a revenue stream allows each stream to have either an *absolute* start linked to the project start or a *contingent* start linked to the finish of any selected work package in the CE component or to the finish of another revenue stream in the RV component, as shown in Figure 4.19. Finish-to-start relationships with lead/lag (positive or negative) times are used for the logic property. Absolute stream start and lead/lag times are always defined in terms of the project global time unit GTU. The implementation of these stream properties is shown in Figure 4.20.

Each revenue stream represents a cash flow $f_c^{RV}(t, \boldsymbol{x})$ where $\boldsymbol{x}$ refers to the revenue stream variables. As described below, these variables represent demand, volume, service charge, and inflation variables. Each stream in the RV component can have its own inflation variable thus allowing revenue analysis in current dollar terms. The inflation variable of a stream can have either of *global*, *local* or *null* reference. Strategies to deal with inflation of all streams include (Figure 4.10): *unique* for each revenue stream, *same-for-all-streams*, and *same-for-project.*

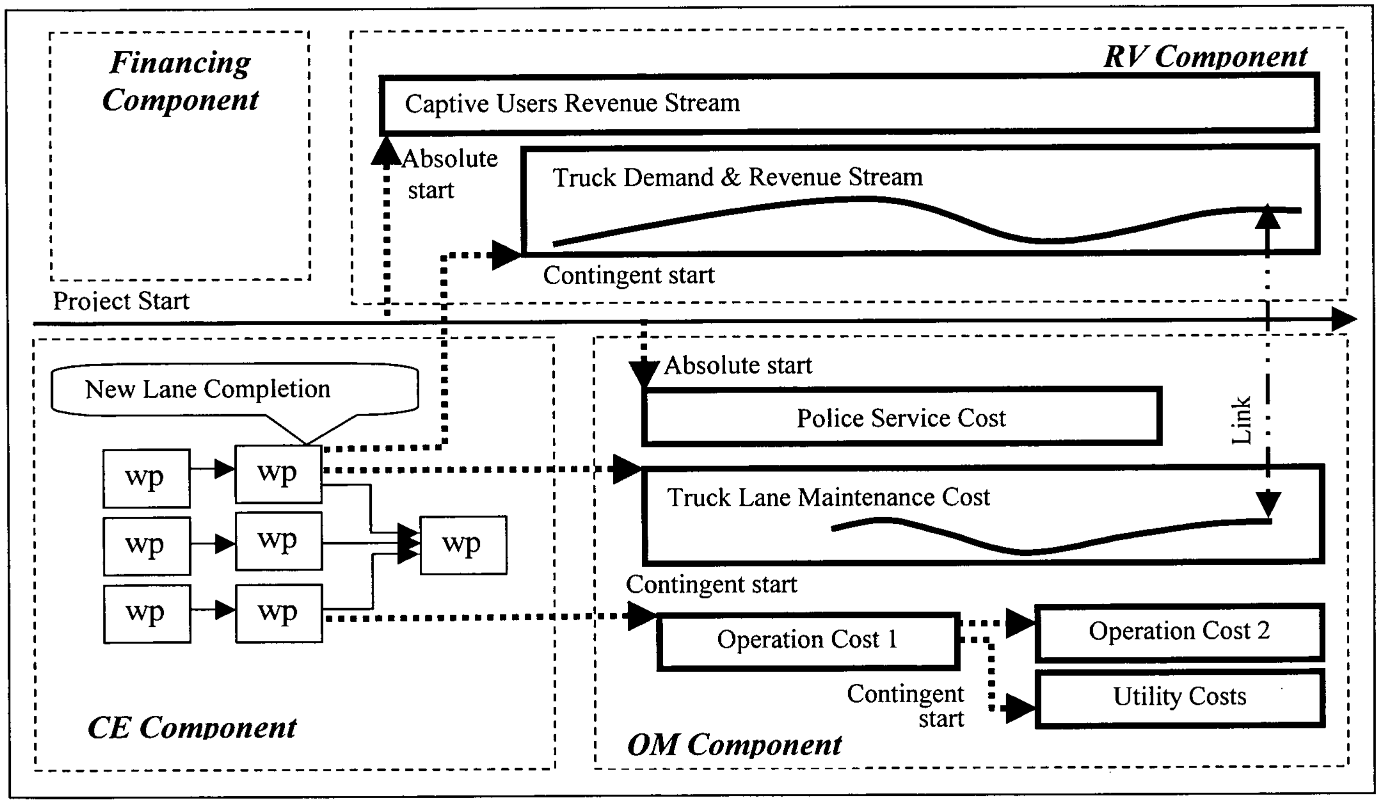


Figure 4.19: Structure of the revenue and operation components

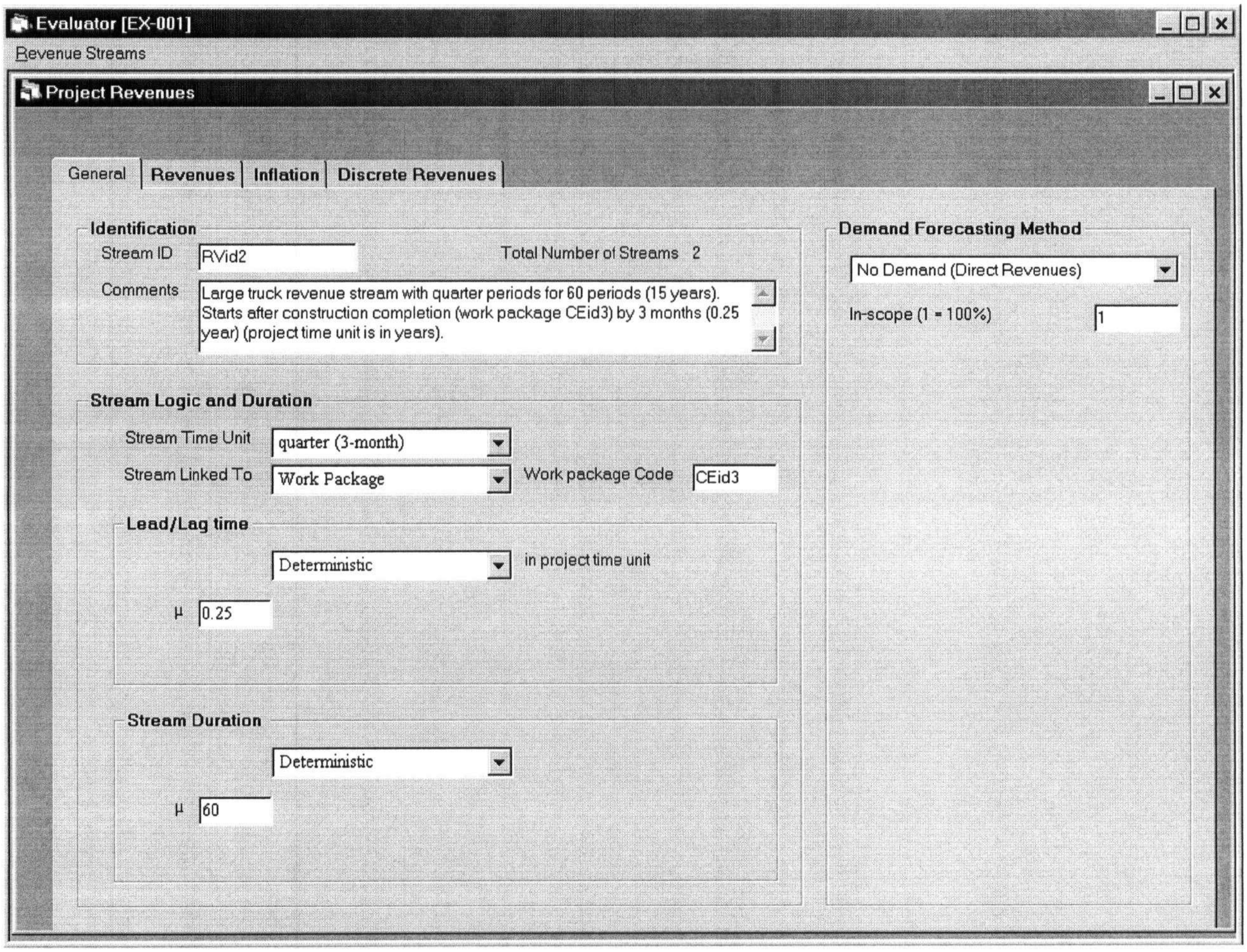


Figure 4.20: Properties of a revenue stream

4.4.2 RV Component Methods and Cash Flow Formulation

The RV classification includes crude, semi-detailed and detailed methods. The following sub-sections describe the formulation of revenue stream cash flow function using these methods.

4.4.2.1 RV Semi-detailed Methods

Semi-detailed methods in the RV classification represent preliminary "gross' methods that can estimate revenues as an aggregated total sum or total per unit of time, with no regard to how these revenues were derived or to their contributing factors. Semi-detailed methods are represented by the *area functions*, where the total amount of revenues sought from a revenue stream are modeled as a total sum along with a number of assumptions used to describe the pattern of the total revenues as selected from Table 4.2. Semi-detailed methods can also be represented by *rate functions* as well, where revenues are modeled based on assumptions about how they are going to change over time using Table 4.1; e.g. using an initial revenue value and a revenue growth rate in a linear function. By using any of the area/rate functions, the generalized model is able to benefit from experience or historical records about the characteristics (e.g. revenues and growth rates) of revenue streams similar to the one being modeled.

Where $RV(t``)$ is a constant dollar revenue variable, $\eta$ is a scope parameter variable limiting the amount of revenues considered in cash flow calculation, $\theta_{rv}(t``)$ is a revenue inflation variable, and $t`$ and $t``$ are as defined before, the gross revenue function can be expressed as:

$$f_c^{RV}(t`,\boldsymbol{x}) = \eta \cdot RV(t``) \cdot e^{\int_0^{t``} \theta_{rv}(\underline{t})\mathrm{d}\underline{t}} \tag{4.19}$$

The revenue variable $RV(t``)$ can correspond to any pattern of the *area/rate* functions in Tables 4.1 and 4.2, and can have either *global* or *local* reference (with *area* functions a *local* reference must be used). $\theta_{rv}(t``)$ may take any pattern of the *rate* functions in Table 4.1 and can have a *global, local* or *null* reference system.

As an example, consider the Northumberland Strait Crossing project which receives an annual government subsidy of Cdn $42 million in 1992 dollars, adjusted in accordance with the consumer price index, for 35 years starting from the "Date Certain", May 31, 1997 (Pirie 1996; " Northumberland" 1988). A RV stream can model this type of revenue with a GTU equal to a year, a revenue stream duration of 35 years, and, "absolute" start at year 5 (1997). The "method" would be: *RV(t)* modeled by a *uniform* shape function of $42 million per year with *local* reference; and, $\theta_{rv}(t``)$, as, say, a *linear* shape function with an initial inflation rate of 0.035 and growth rate of 0.0015 per year with *global* reference.

Figure 4.20 shows how semi-detailed methods are chosen for a revenue stream by selecting " No Demand (Direct Revenues)". Figure 4.21 shows the revenues defined as *sinusoidal rate* function.

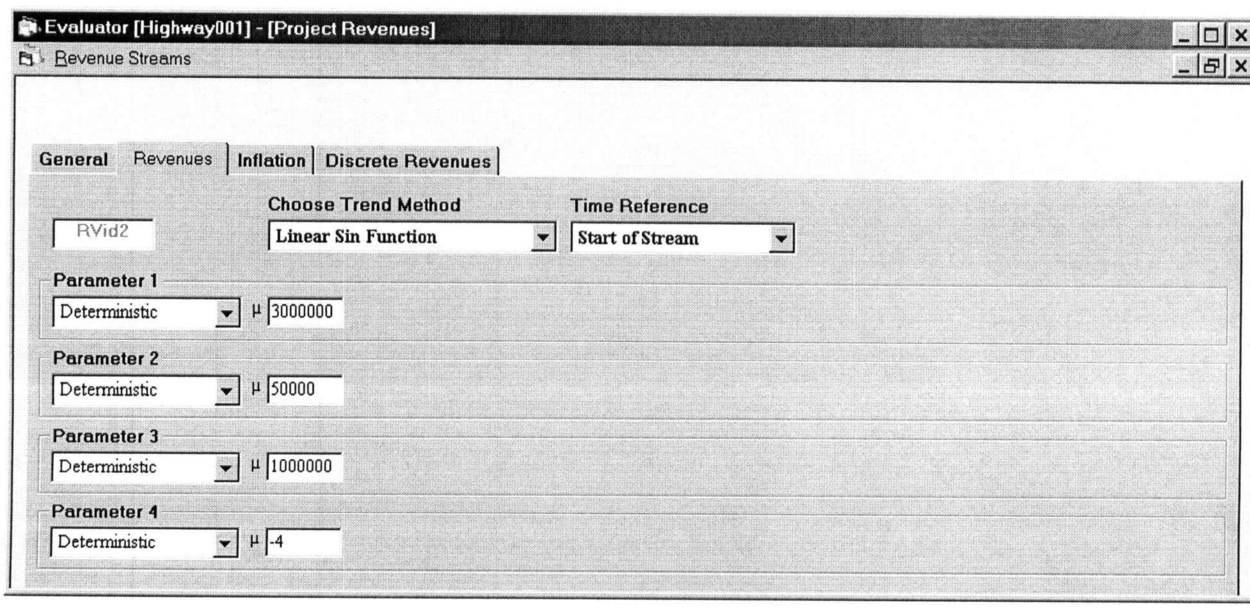


Figure 4.21: Aggregated revenues modeled as sinusoidal rate function, see Table 4.1

4.4.2.2 RV Detailed Methods

The second category of RV classification methods represents detailed methods through which revenue $f_c^{RV}(t^{`},x)$ at any time is expressed as a function of project demand and service charges at that time. Therefore, the final form of the revenue function of a revenue stream in the generalized model may be expressed as,

$$f_c^{RV}(t^{`},x) = R(t^{``},d) \cdot f_{cD}^{RV}(t^{``},x) \cdot e^{\int_0^{t^{``}} \theta_{rv}(\underline{t})d\underline{t}} \qquad (4.20)$$

in which $R(t^{``},d)$ is a service charge variable, $f_{cD}^{RV}(t^{``},x)$ is a demand function and $\theta_{rv}(t^{``})$ is an inflation variable, and $\underline{t}$ is a dummy variable. Inflation can be modeled by any *rate* function (Table 4.1) and can have a *global, local* or *null* reference. Service charge and demand functions are described below.

a) Service Charge Rate Variable

Service charge variable is the rate paid for using a project or facility. $R(t^{``},d)$ can be modeled to change with time using any pattern of the *rate* functions in Table 4.1. $R(t^{``},d)$ can have either a *global* or *local* reference and therefore service rates, e.g. tolls, can be set to change over the duration of a revenue stream using, for example, a *step uniform* pattern where up to five rates can be defined for five periods within the duration of the stream. $R(t^{``},d)$, however, can have an additional reference called *demand* reference where instead of having the rates change as a function of time, rates can change with the demand value d as obtained from the relevant demand function of the stream. Thus, in Table 4.1 instead of having t as argument of the *rate* functions, d (demand value) will be used as input to obtain the rate value. The generalized model uses $t^{``}$ in t he rate function $R(t^{``},d)$ to determine the value of demand at that time, then uses this demand as input to the rate function to determine the appropriate rate.

The new *demand* reference for the service charge function helps in cases where service rates are established based on the amount of demand for the service. Transportation projects procured under an alternative PPP delivery system may have their tolls paid to their private developers not by end users but by government according to the volume of traffic using the transportation facility; e.g. "shadow tolls" ("Paying" 1993). According to FHWA (1998), under tolls payments are made where "... traffic is divided into two to four "bands" representing different levels of annual traffic volumes with different per-vehicle payments attached to each. The lower bands have higher per-vehicle payments, while higher bands have lower per-vehicle payments. In all cases, the top band must be zero so that the government's liability is capped in the event of higher-than-expected traffic." Traffic bands may increase over time to match growth in traffic, as shown in Figure 4.22. Bands can be constructed for different classes of vehicles (e.g. vehicles 5.2 meters in length) and payments can be indexed to inflation. A *step uniform* rate function (Table 4.1) can be used to model such a shadow tolling case in which categories of traffic volumes are defined along with their established toll rates, such that if the expected traffic volume falls in a given category, the toll rate of this traffic category is used in cash flow calculations.

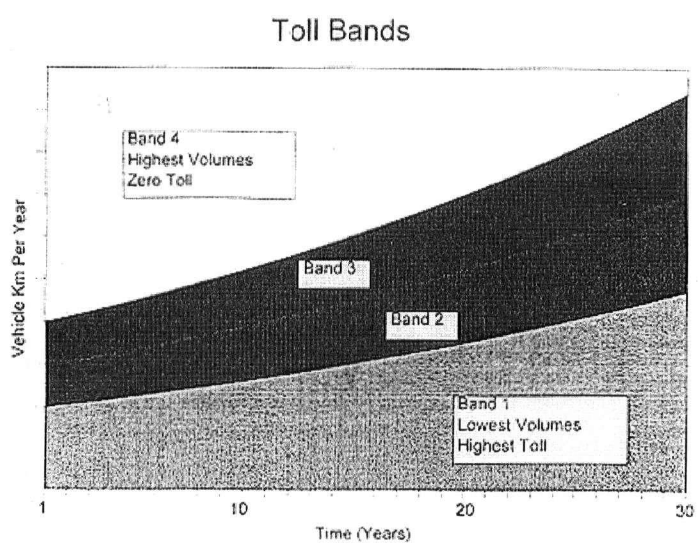


Figure 4.22: Toll bands for shadow toll structure, FHWA (1998)

Several transportation projects in the UK that were awarded to private developers after the Private Finance Initiative are being paid for through shadow tolls by the Highway Agency and Department of Transport, UK. These projects include the A74(M)/M74 motorway (£214 million) in Scotland (the contract was singed in April 1997); and four other projects signed between January and March of 1996 including the M1-A1 link road near Leeds, the A1(M) widening between Alconbury and Peterborough, the A419/A417 between Swindon and Gloucester, and the A69 between Carlisle and Newcastle-upon-Tyne. Payments to the private sector are made based on traffic volume subject to a maximum figure above which no further payments are made.

b) Total Volume of Demand

Estimates of future demand captured by a project (i.e. market share) can be derived from the total volume of demand (i.e. market volume in the study area) through a number of methods as will be described later. Forecasting the total volume of demand $TV(t``, \boldsymbol{x})$ occurring at future time $t``$ requires the use of several forecasting methods. As described herein these methods are called independent trend methods and dependent-trend methods, and both methods appear in the RV classification. Letting $V(t``)$ represent a general demand factor/variable, $G(t``)$ a demographic/socioeconomic variable, and b_0 to b_4 parameter variables, then:

$$TV(t``,\boldsymbol{x}) = \begin{cases} V(t``) \\ b_0 + b_1 \cdot G(t``) + b_2 \cdot G(t``)^2 + b_3 \cdot G(t``)^3 + b_4 \cdot G(t``)^4 \\ b_0 + b_1 \cdot ln(G(t``)+1) \\ b_0 \cdot e^{b_1 \cdot G(t``)} \end{cases} \tag{4.21}$$

The first category of methods for forecasting total volume of demand is represented by $V(t``)$ in Eq. (4.21) and describes independent-trend methods that forecast future demand volume by extrapolation from historical past data. In transportation planning, this category is described as simplified technique by Meyer and Miller (1984). $V(t``)$ can take any pattern of the *rate/area* functions in Table 4.1 with either a *local* or *global* reference.

The deficiency with trend methods is that the relationship established between future demand and time assumes that all other factors and relationships that could affect demand are constant over time. However, trend methods can be helpful if historical data exists and can be coupled with a sound judgement about future trends. This is important when there is no time to perform a better demand analysis and when speed is of concern for decision-makers. For example, the request for proposals for the Second Severn Crossing ("Second" 1988,1989) included information for the prospective bidders about traffic volume, annual average daily traffic and annual growth rates for the existing Severn Crossing between 1967 and 1989. This information could be used in a simple trend analysis to extrapolate likely future traffic volumes, providing bidders with a reasonable assessment of demand.

Within Evaluator, this category of demand forecasting trend methods is called "Trend Analysis Demand", as shown in Figure 4.23. Unlike Figure 4.20, the "Total Demand" and "Service Rate" tabs in Figure 4.23 are used for revenue calculation. Note that the tabs change with the change of the demand method.

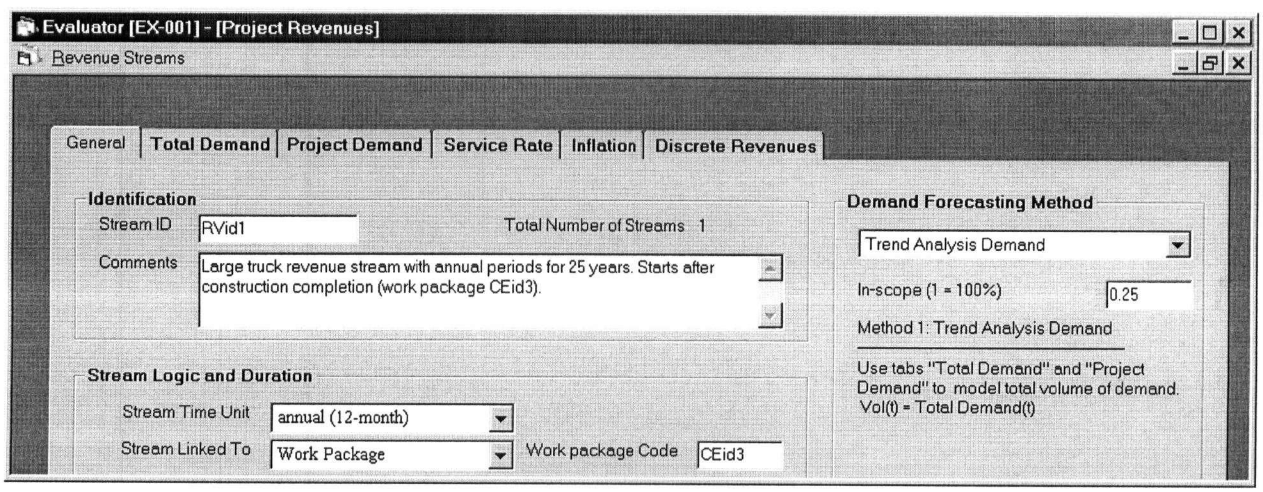


Figure 4.23: Unit rate revenues using trend analysis for demand forecasting

The second category of demand forecasting methods represents three dependent-trend methods as shown in Eq. (4.21): polynomial regression, logarithmic and exponential methods. They describe situations in which future volumes of demand are estimated based on their correlation or linkage to general or local demographic and socioeconomic indicators or variables, e.g. population, in the market area. Therefore, demand at any future time is established based on the estimated value of the relevant indicator at that time. In the three above methods, a demographic or socioeconomic variable is represented by $G(t``)$ which by itself can take any pattern of the *rate* functions in Table 4.1 with a *local* or *global* reference.

For example, future demand for the Highway 104 Western Alignment ("Highway" 1995) was modeled by regression models considering past and current traffic volumes as dependent variables while population, Gross Domestic Product (GDP), and other economic variables were treated as independent variables. GDP gave the best model fit for traffic volumes with a correlation of more than 0.98. Thus, future traffic volumes are obtained based on predictions of the GDP. For instance, using data from the project RFP and the polynomial regression relationship in Eq. (4.21) set to first degree with the b_0 coefficient set to -899 and the b_1 coefficient set to 0.0148 along with a *rate* function $G(t``)$ for GDP, then total future demand could be forecasted at any time based on the GDP[2] value at that time.

Demand dependent-trend methods are implemented in Evaluator as shown in Figure 4.24. In this figure a fourth degree polynomial method is selected to model future traffic and the coefficients represent parameters needed by Eq. (4.21). Figure 4.25 shows the "Demand Factor" tab through which indicators such as GDP is modeled - in this case a fourth degree polynomial (Table 4.1).

[2] Arthur Andersen LLP prepared a complete analysis of economic and demographic growth trends in Nova Scotia in support of the traffic and revenue forecasts prepared by Steer Davies Gleave of UK, the project traffic consultant.

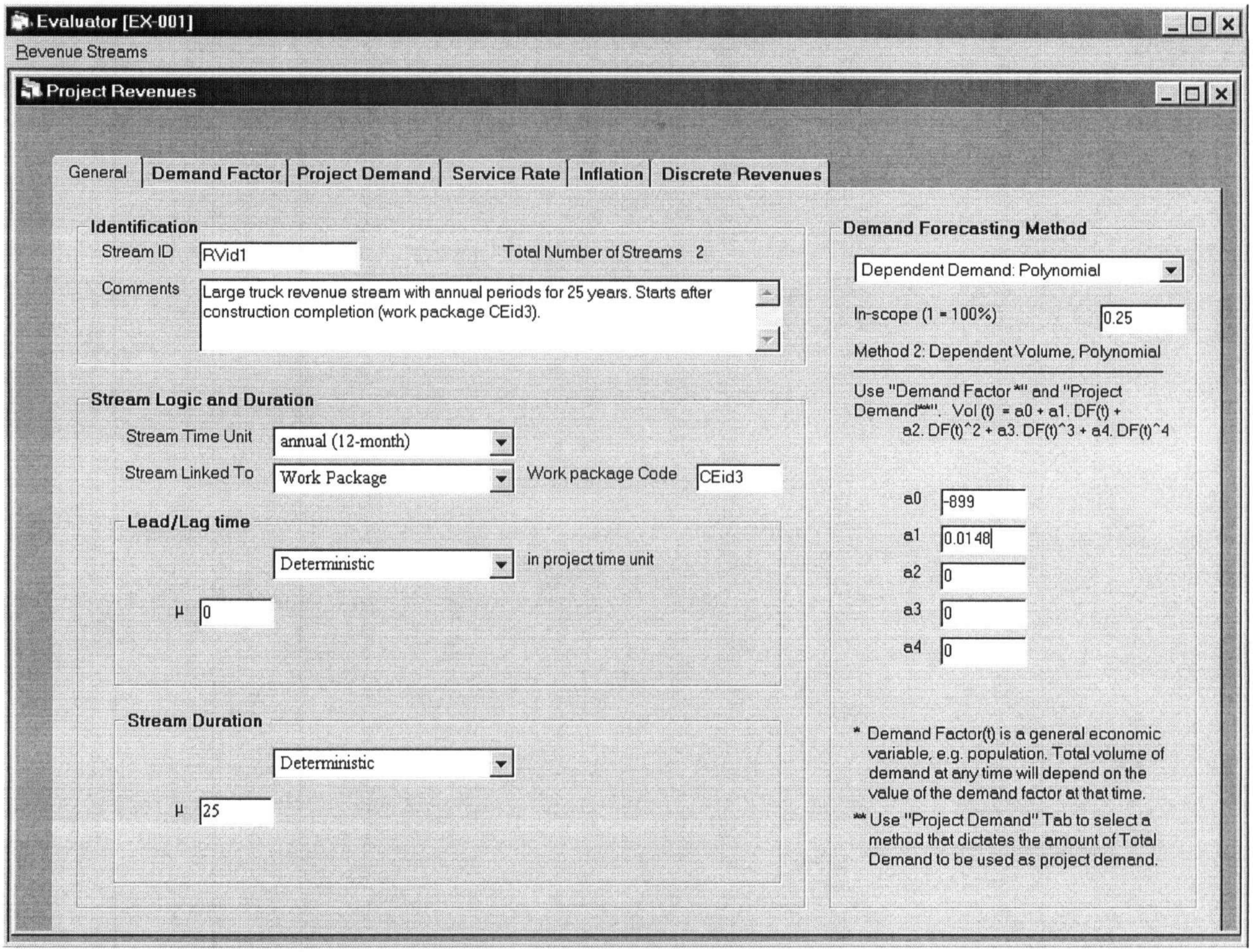


Figure 4.24: Forecasting demand using polynomial regression and demographic indicator

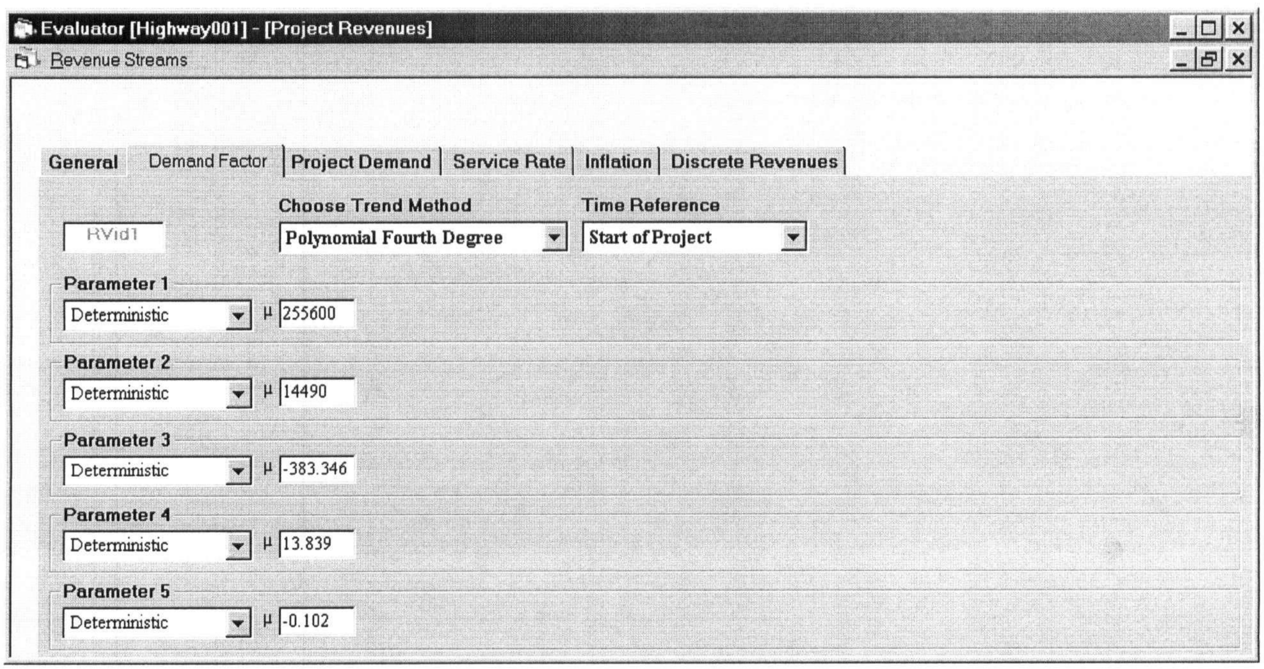


Figure 4.25: GDP (demographic indicator) modeled as a fourth degree polynomial

c) Project Demand Function

Total volume of demand can be modeled as described above by any of the methods in Eq. (4.21). Demand captured by a project (i.e. market share) represents part of the total forecasted demand. The RV classification includes some methods to determine project share of demand.

A project demand function can be modeled as in Eq. (4.20) through five methods in the RV classification. Letting *TV(t)* represent the total volume of demand (Eq. 4.21); η a scope variable limiting the total volume to any required percentage level; ε an elasticity of demand coefficient; *Ut(t)* a utility function representing the difference in utility between two alternatives (Eq. 4.23) in which LOS_i are level of service parameter variables, described below; *R(t``,d)* service charge as described before; *Ro* a base service charge, then

$$f_{cD}^{RV}(t``,\boldsymbol{x}) = \begin{cases} \eta \cdot TV(t``,\boldsymbol{x}) & \text{(a)} \\ \eta \cdot TV(t``,\boldsymbol{x}) \cdot \left(R(t``,d)/Ro\right)^{\varepsilon} & \text{(b)} \\ \eta \cdot TV(t``,\boldsymbol{x}) \cdot \left[1+\varepsilon \cdot \left(R(t``,d)-Ro\right)/Ro\right] & \text{(c)} \\ \eta \cdot TV(t``,\boldsymbol{x}) \cdot e^{-\varepsilon \cdot \left(R(t``,d)-Ro\right)} & \text{(d)} \\ \eta \cdot TV(t``,\boldsymbol{x}) \cdot \left[1-1/\left(1+e^{Ut(t``)}\right)\right] & \text{(e)} \end{cases} \tag{4.22}$$

where $Ut(t``) = a_0 + a_1 \cdot \left(R(t``,d)-Ro\right) + a_2 \cdot LOS_2 + a_3 \cdot LOS_3 + a_4 \cdot LOS_4$ (4.23)

These project demand methods represent three categories: simplified trend methods (Eq. 4.22a), elasticity-based methods (Eq. 4.22b, c, d), and individual choice-based methods (Eq. 4.22e).

i. Simplified Trend Methods

Method (a) in Eq. (4.22) is a simplified method which forecasts project demand at any time to be equal to the total volume of demand as obtained by Eq. 4.21, and weighted to the extent provided by the scope factor η. For example, the cars traffic volume can be defined for the study area of a project and only η of this traffic (i.e. in-scope) will be considered as the project market share.

Using simplified methods, project revenues can be obtained directly by multiplying demand by a unit rate. For example, for the Second Severn Crossing project independent-trend methods could be used to forecast demand for the project based on the usage data provided in the RFP; and, by specifying toll rates for the forecasted demand of each traffic class, revenues can be obtained.

ii. Elasticity Based Methods

Methods (b,c,d) in Eq. (4.22) are elasticity-based methods (Behrens and Hawranek 1991), which can be used to forecast project demand based on knowledge of how the variation of some variables in a project demand function might affect demand volume. "The elasticity of demand with respect to a certain variable (such as fare or headway) is defined as the rate of change of demand with respect to that variable, normalized by the current levels of demand and the variable in question" (Meyer and Miller 1984).

The elasticity coefficient ε of a general demand function D with respect to a demand variable x (e.g. toll rate) represents the linear sensitivity of demand to changes in the demand variable:

$$\varepsilon = \frac{\partial D}{\partial x} \cdot \frac{x_o}{D_o} \tag{E4.9}$$

Meyer and Miller (1984) explained that demand is said to be elastic if the absolute value of elasticity is greater than 1, that is where a 1 percent change in the demand variable results in a greater than 1 percent change in demand. Demand is said to be inelastic if a 1 percent change in the demand variable results in less than a 1 percent change in demand. Demand possesses unit elasticity if the absolute value of elasticity is 1. For example, transit demand is generally inelastic with respect to fares with a value around -0.3; that is, for instance, a 1% increase in fares may result in 0.3% decrease in demand patronage (Goodwin and Williams 1985; USDOT 1980).

Eq. E4.9 assumes that the demand function is available and differentiable with respect to x, that might not be available at an early analysis stage. Elasticity of demand, however, can generally be approximated by observing the demand at two operating points (D_0, x_0) and (D_1, x_1), or by observing how the demand changes at the change of a demand variable. A number of expressions for elasticity can be described. Method (b) in Eq. (4.22) assumes that elasticity is constant over the range of the variable and is expressed by (Meyer and Miller 1984):

$$\varepsilon = \frac{ln(D_1) - ln(D_0)}{ln(x_1) - ln(x_0)} \tag{E4.10}$$

Therefore, knowing the values of demand and service variable at an operating point (e.g. D_0 *and* x_0 or D_1 *and* x_1), the demand at another point D_2 can be obtained using Eq. 4.22(b) when the service variable changes to x_2, which means constant elasticity.

Method (c) in Eq. (4.22) assumes that demand will be linear and thus the elasticity value will change at different operating points (Meyer and Miller 1984). Thus, the elasticity at an operating point (D_0, x_0) can be given as (Ortuzar and Willumsen 1994; Meyer and Miller 1984):

$$\varepsilon = \frac{D_1 - D_0}{x_1 - x_0} \cdot \frac{x_0}{D_0} \tag{E4.12}$$

Consequently, linear demands calculated by Eq. 4.22(c) requires that if the elasticity is calculated at an operating base point then the demand and service charge value at that base point should be used in the calculations; if the base point changes then both the elasticity, demand, and service charge at the new point should be used instead. Behrens and Hawranek (1991) in conjunction with the demand method (c) explained that elasticity can be given by:

$$\varepsilon = \frac{D_1 - D_0}{x_1 - x_0} \cdot \frac{x_0 + x_1}{D_0 + D_1} \tag{E4.13}$$

The elasticity for method (d) in Eq. (4.22) can be given by:

$$\varepsilon = \frac{ln(D_2) - ln(D_1)}{x_1 - x_2} \tag{E4.14}$$

As explained for the elasticity-based methods (Eq. 4.22b,c,d), a reference or base operating point is needed at which the elasticity ε and demand are given. The reference/base operating point in the project demand function (Eq. 4.22) is given by *TV(t``)* and *Ro*. To model *TV(t``,x)* such that it reflects the base/reference demand in conjunction with *Ro*, the total demand function (Eq. 4.21) needs to be modeled by the *uniform rate* function (Table 4.1) such that its value will represent a constant base demand for any future demand forecasting. Since future service charge, *R(t``,d)*, is considered the service variable that affect demand in Eq. 4.22, then its future value should be modeled carefully. The service charge variable of a revenue stream changes according to the selected *rate* function independent of the inflation variable of the stream. If it is required to have the service charge change according to inflation rate, then (1) the stream inflation variable should be set to the *null reference* system, and (2) the service charge is adjusted or rearranged to consider inflation using any of the *rate* functions in Table 4.1.

Assuming the linear demand elasticity (Eq. E4.12) traffic demand for the Highway 104 Western Alignment ("Highway" 1995) showed that the toll demand elasticities ranged from -0.3 ($2 to $3) to -0.45 ($4 to $5) for cars and from -0.15 ($3 to $6) to -0.9 ($12 to $15) for trucks.

The implementation of elasticity-based demand forecasting methods is explained in Figure 4.26, which explains the "Project Demand" window with method (b) of Eq. (4.22). The value of *Ro* in methods (b) and (c) must not be zero. A default value greater than zero is given to the system. Figure 4.27 shows how base/reference demand is modeled by a *uniform rate* function for use in elasticity-based demand forecasting. If the service charge rate is modeled to change following demand (e.g. shadow tolls), then Eq.4.22(a) will only be used in revenue calculations since all other methods in Eq. 4.22 will lead to circular calculations; in these methods demand depends on the service charge. This is emphasized on the notes on the window in Figure 4.26.

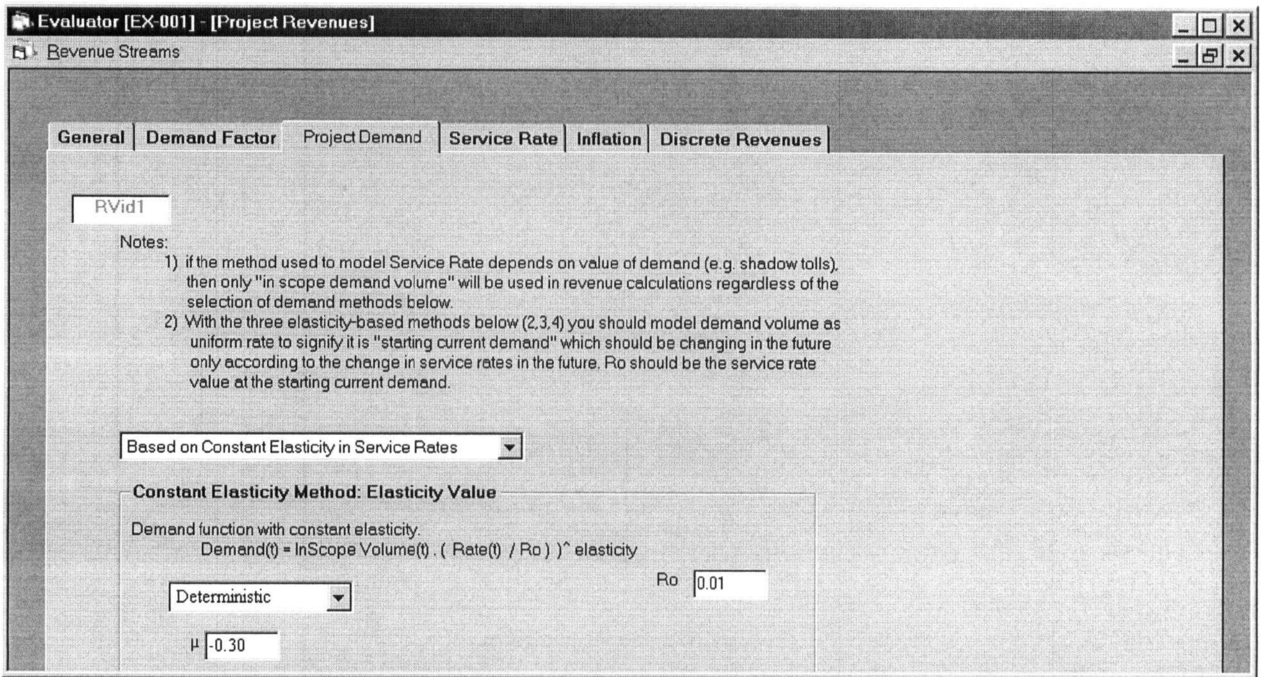


Figure 4.26: Forecasting project demand by elasticity-based methods

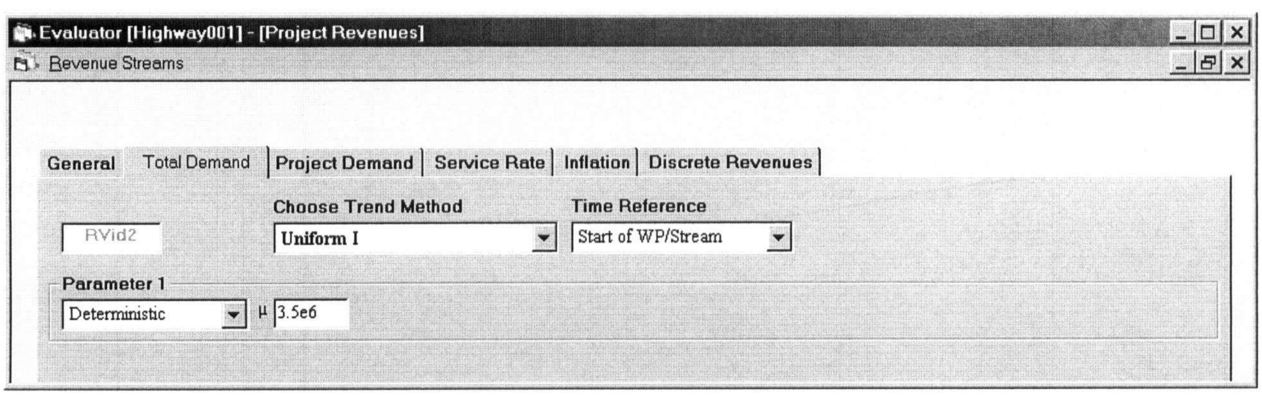


Figure 4.27: Uniform rate function for modeling base current demand

iii. Individual Choice Based Methods

The simplified trend and elasticity based forecasting methods just described can model project demand in general without being restricted to a specific type of project.

In transportation planning the above methods have been used in analyzing traffic demand, and are referred generally as "simplified techniques". Other techniques used in analyzing demand for transportation planning purposes include two broad classes which have some overlap: the urban transportation modeling system (UTMS) and individual choice models (Kroes and Sheldon 1988; Fowkes and Wardman 1988; Meyer and Miller 1984; Ortuzar and Willumsen 1994; Pearmain and Swanson 1990; Wardman 1988).

While the simplified techniques described above serve as generalized methods for modeling demand, the RV classification can be specialized further to embrace the individual choice model, which is generally referred to as "Stated Preference" (SP) method. Stated preference techniques were developed for conducting marketing research in the early 1970s and they have received increased attention in transport in the UK and around the world (Kroes and Sheldon 1988).

Individual discrete choice models attempt to establish demand based on the observed choices of individual travelers followed by summation over all travelers in order to obtain an aggregate demand prediction. This assumes that faced with a choice amongst several alternatives "the probability of individuals choosing a given option is a function of their socioeconomic characteristics and the relative attractiveness of the option" (Ortuzar and Willumsen 1994). The choice of an option by an individual is assumed therefore to depend on the relative utilities of the various available alternatives and that the individual will choose the alternative with the maximum utility.

The attractiveness of an alternative to an individual traveler is described by a utility function. A linear additive utility function is usually described in the form of (Fowkes and Wardman 1988):

$$U_{im} = \sum_{j} \beta_{ijm} \cdot X_{ijm} \tag{E4.15}$$

in which U_{im} is the utility value perceived by individual i for alternative m, X_{ijm} is the jth relevant attribute assumed to influence travel behavior of individual travelers, and the β_{ijm} are coefficients/parameters which reflect utility weights of the relevant attributes. The attributes may reflect variables such as cost/income/toll rates, travel time, waiting time, number of lanes, frequency of use or general level of service variables. The attributes are referred to as *generic* or *alternative-specific* variables (Meyer and Miller 1984); a generic attribute/variable (X_{ijm}) is one which will have the same attribute coefficient /parameter (β_{ijm}) value in all the alternatives. An alternative specific variable is an attribute/variable for which its coefficients will be different for different alternative utility functions. Since some of the variables such as perceptions and tastes of individual travelers may not be observed and included in the utility function, the above utility function is complemented by another part called the random portion or measurement error, ε_{im}.

$$U_{im} = \sum_{j} \beta_{ijm} \cdot X_{ijm} + \varepsilon_{im} \tag{E4.16}$$

The probability P_{i1} of an individual i choosing alternative 1 rather than alternative 2 is assumed to reflect the individual's utility maximization among the alternatives

$$P_{i1} = P[U_{i1} + \varepsilon_{i1} \geq U_{i2} + \varepsilon_{i2}] \tag{E4.17}$$

The probability of an alternative being chosen over other alternatives depends on the probability distribution of the random utility portion ε_{im}. An assumption of multinomial normal distribution for the ε's generated the *probit model*, and an assumption of independently and identically distributed ε's with a Gumbel distribution generated the *multinomial logit model* (Ortuzar and Willumsen 1994; Meyer and Miller 1984).

The multinomial logit model is commonly used with the stated preference technique and the probability of individual i choosing alternative m among several alternatives n ($k = 1,2...n$) using this model is (Meyer and Miller 1984)

$$P_{im} = \frac{e^{U_{im}}}{\sum_{k} e^{U_{ik}}} \tag{E4.18}$$

The binary form of the logit model for choosing between two alternatives can be described in any of following forms (Wardman 1988):

$$\begin{aligned} P_{i1} &= \frac{e^{U_{i1}}}{e^{U_{i1}} + e^{U_{i2}}} = \frac{e^{U_{i1}-U_{i2}}}{1 + e^{U_{i1}-U_{i2}}} \\ &= 1 - \frac{1}{1 + e^{U_{i1}-U_{i2}}} \end{aligned} \tag{E4.19}$$

A stated preference experimental design is employed to survey individual travelers about what they would choose to do given a number of hypothetical situations. These situations are designed to reflect decision contexts (hypothetical or real), alternatives (travel options designed through different levels of the utility attributes/variables, e.g. using \$2 toll and 2 lanes against \$3 toll and 4 lanes), and responses (ratings, ranking, or choices) (Ortuzar and Willumsen 1994). Analytical analysis is performed on the results of the survey data to establish the relative effect of each attribute on the overall utility; that is the analysis determines the preference weights (coefficients) of the attributes. The analysis is usually performed using the maximum likelihood method, regression analysis, and monotonic analysis of variance MONANOVA (Pearmain and Swanson 1990; Ortuzar and Willumsen 1994).

Using the resultant attribute weights it becomes possible to determine the probability of choosing a particular alternative using the above binary form of multinomial logit models. However, forecasting demand requires the probability of all individuals in a zone choosing a particular alternative rather than the probability of a single individual choice. To cope with this, several aggregation techniques have been suggested including *naive* aggregation, *classification with naive* aggregation, and *sample enumeration* (Meyer and Miller 1984; Ortuzar and Willumsen 1994). With naive aggregation the individual choice model is treated as if it were an aggregated model by using zonal/area average values for the utility function attributes in order to compute an average zonal probability. The aggregation error in the naive method is improved if the population is classified into homogenous groups before performing the aggregation. The sample enumeration method is the best of the three methods in terms of reducing aggregation errors.

The stated preference method is represented in the generalized model by method (e) in Eq. (4.22), which corresponds to one of the forms of the binary logit model as described above. The utility function in this method, and as described in Eq. (4.23) includes up to four *generic* utility attributes that may represent any level of service (*LOS*) variable (e.g. number of lanes, travel time) and one of these attributes is reserved for service charge (e.g. toll rate). According to the form of the binary model in Eq. (4.22), the values of the attributes of the utility function should represent the difference between values of alternative one minus alternative two, i.e. $U_1 - U_2$, (e.g. utility of a new toll highway minus that of an existing highway) which represents the 2nd and 3rd forms in Eq. (E4.19). The attributes coefficients a_1 to a_4 in the utility function must be the same for both alternatives since the attributes are generic; a_0 should represent the difference between the *alternative-specific constants* in the two alternatives, or represents one of these *alternative-specific constants* in case the other is zero. Both naive and classified aggregations can be used to obtain the probability of choosing the first alternative using Eq. (4.22).

For the Highway 104 Western Alignment project ("Highway" 1995), estimates for the total traffic volumes in the future were obtained as explained before using dependent-trend methods utilizing a regression model and the Gross Domestic Product (GDP) (Eq. 4.21). Project demand estimates were obtained through the stated preference technique using the binary logit model presented in Eq. (4.22). Utility functions for using the new alignment were constructed using attribute variables that included toll value, time saving, number of lanes, and method of toll payment. The logit model gave the probability (market share) of the likely transfers to the new alignment. Therefore, for an alternative that has a $2 toll, an 11-minute time saving, 4 lanes, and a cash payment (against smart cards), the traffic transferring (market share) would amount to 71.5% of the total available traffic volume for cars. Using the market share percentage coupled with estimates of the total future traffic volume, estimates were given for the traffic captured by the highway in the period between 1995 and 2030. Several forecasts of project traffic volumes under different scenarios were explored for this study period.

The structure adopted for the RV component can be very useful in modeling aspects like this one. Moreover, unlike the Highway 104 analysis that assumed constant scenarios when determining probability of using a road or its market share, the generalized model allows the scenario to change according to the future change in the service rate (toll) thus affecting the probability of using the road or its market share during the operational period. Further, the generalized model facilitates modeling future traffic volumes following the methods describe earlier (e.g. according to changes in GDP). Thus, using the model developed, project participants such as government or project developers would have a chance to experiment with different scenarios for tolls, demand and demand variables to determine the effect of changes in those factors as well as others on project viability. Such experimentation would be of assistance in setting, for example, an appropriate concession period.

The implementation of the individual choice method is shown in Figure 4.28. The window shows the SP method selected to model project demand along with its attributes and coefficients.

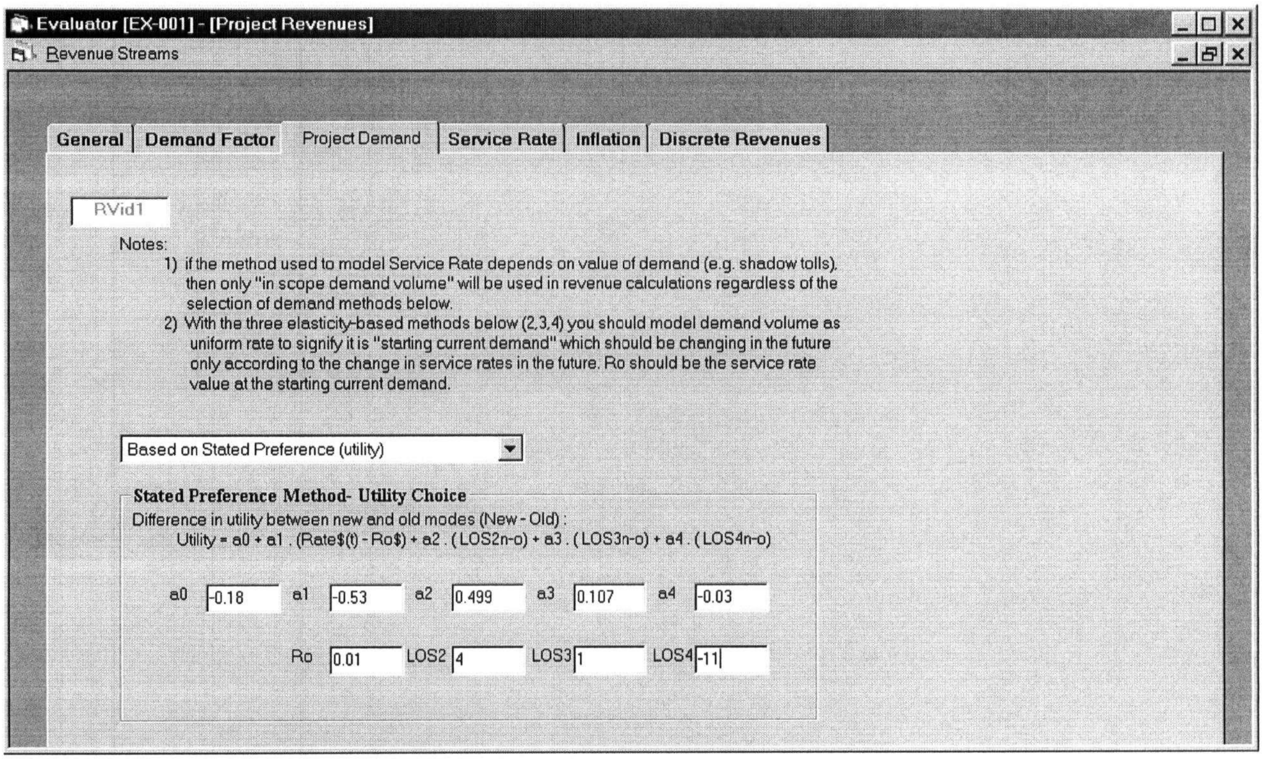


Figure 4.28: Project demand by stated preference technique

4.4.2.3 RV Crude Methods: Discrete Revenues

A revenue stream can allow revenues to be received as discrete payments during the revenue period in the same manner as explained with the CE component.

A discrete revenue $\mathbf{D}_j^{RV}$ is represented as follows: Let $\mathbf{Dv}_j^{RV}$ and $\mathbf{Dt}_j^{RV}$ represent constant dollar revenue value and local time of discrete revenue j of n discrete revenues in a revenue stream, and let $\theta_{rv}(t)$, modeled by *rate* functions, be the discrete-revenue inflation variable. Then,

$$\mathbf{D}_j^{RV} = \mathbf{Dv}_j^{RV} \cdot e^{\int_0^{t^*} \theta_{rv}(t)\mathrm{d}t} \Bigg|_{\substack{t^* = \mathbf{Dt}_j^{RV} \text{ if } \theta_{rv} \text{ is in local time} \\ t^* = \mathbf{Dt}_j^{RV} + ESc \text{ if } \theta_{rv} \text{ is in global time}}} \tag{4.24}$$

4.4.3 RV Component Cumulative and Discounted Revenues

Summarizing so far, any revenue stream cash flow function $f_c\ (t, \mathbf{x})$ can be formulated either through a semi-detailed function using Eq. (4.19) or through detailed function using Eq. (4.20). The detailed function by itself is represented through total demand and project demand in Eqs. (4.21,22). Both semi-detailed and detailed formulations of a revenue stream can be supplemented by the discrete formulation presented in Eq. (4.24).

By aggregation, all revenue stream cash flow functions integrate to form the RV component cash flow function as described in Eq. (4.2).

From the above formulation for a revenue stream cash flow function and discrete cost formulation, a cumulative cash flow $F_c^{RV}(t`,\boldsymbol{x})$ of a revenue stream at any local time $t`$ can be obtained as follows, where r is the total number of discrete revenues in a revenue stream:

$$F_c^{RV}(t`,\boldsymbol{x})=\int_0^{t`} f_c^{RV}(\underline{t},\boldsymbol{x})\,\mathrm{d}\underline{t}+\sum_{j=1}^{r}\mathbf{D}_j^{RV}\Big|_{\text{for each }\mathbf{Dt}_j^{RV}\leq t`} \tag{4.25}$$

Consequently, cumulative revenue cash flow $F_{RV}(t,\boldsymbol{X})$ of the RV component at any *global* time t can be obtained as follows, where $\overline{t-ESc_i}$ is the time elapsed in a revenue stream i and converted from GTU to LTU (explained before) and m is the total number of revenue streams:

$$F_{RV}(t,\boldsymbol{X})=\sum_{i=1}^{m}F_c^{RV}(\overline{t-ESc_i},\boldsymbol{x}_i)_i\Big|_{\text{for each }t\geq ESc_i} \tag{4.26}$$

Discounted value $df_c^{RV}(Td,\boldsymbol{x})$ of a revenue stream can be obtained as follows where all cash flows are discounted to time Td which is measured in GTU; y is the nominal annual discount rate representing the client's minimum acceptable rate of return (MARR) and $\overline{y}$ is y converted from annual time periods to LTU; wd is the duration of a revenue stream in LTU; $\overline{tb}$ is the time before start of a construct (see Eq. 4.15), converted from GTU to annual time unit; and, $\overline{tn}$ is the time elapsed in a construct (see Eq. 4.16), converted from GTU to LTU. Thus:

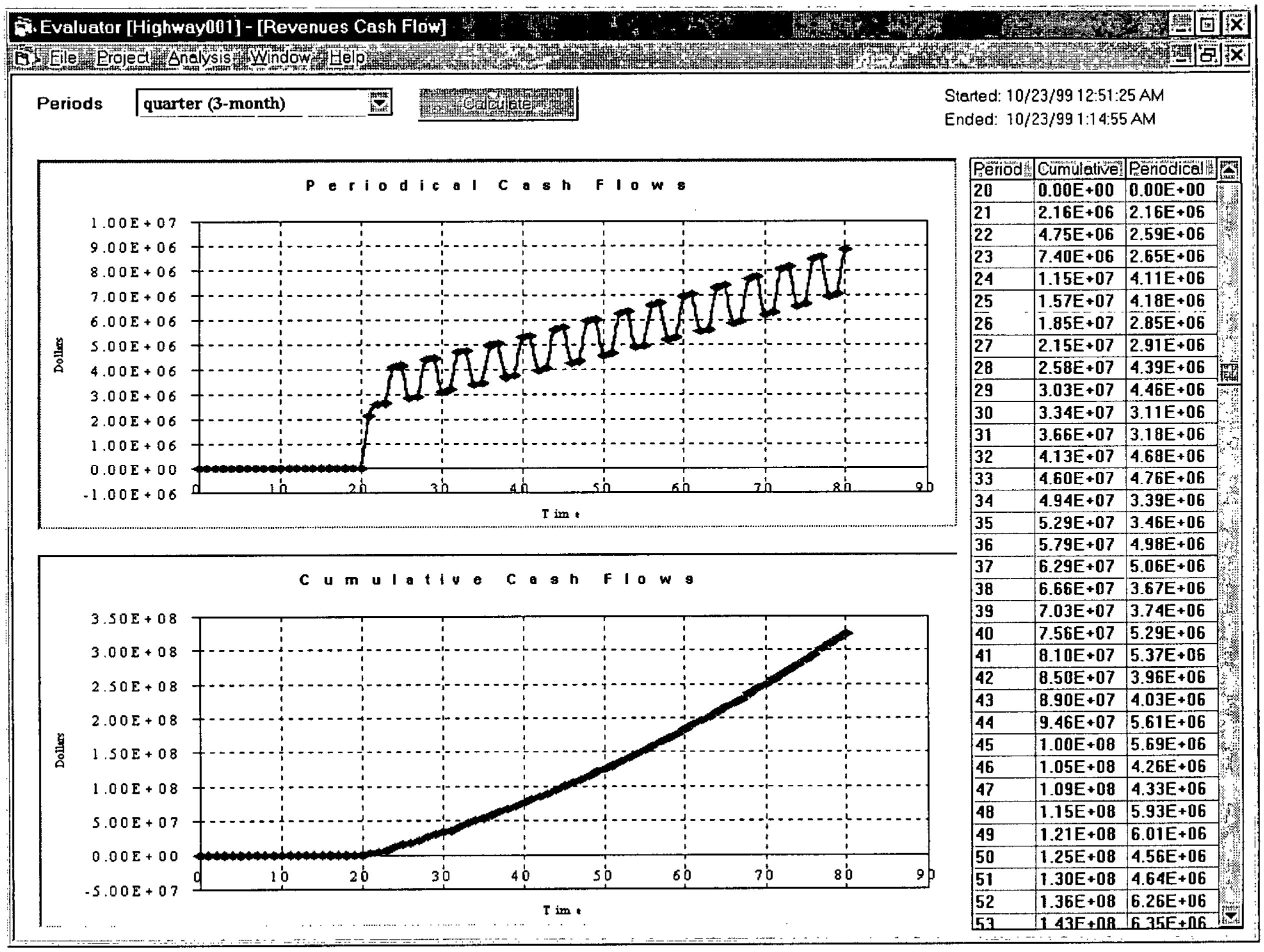
$$df_c^{RV}(Td,\boldsymbol{x})=e^{-y\cdot\overline{tb}}\cdot\left[\int_0^{wd-\overline{tn}} f_c^{RV}(\underline{t}+\overline{tn},\boldsymbol{x})\cdot e^{-\overline{y}\cdot\underline{t}}\,\mathrm{d}\underline{t}+\sum_{j=1}^{r}\mathbf{D}_j^{RV}\cdot e^{-\overline{y}\cdot(\mathbf{Dt}_j^{RV}-\overline{tn})}\Big|_{\text{for each }\mathbf{Dt}_j^{RV}\geq\overline{tn}}\right] \tag{4.27}$$

Consequently, discounted value of the RV component cash flows can be obtained as follows:

$$df_{RV}(Td,\boldsymbol{X})=\sum_{i=1}^{m} df_c^{RV}(Td,\boldsymbol{x}_i)_i \tag{4.28}$$

Figure 4.29 illustrates Evaluator's output screen for the total revenue cash flow. The window shows the periodical cash flow graph that implements Eq. (4.2) and the cumulative revenue cash flow graph that implements Eq. (4.25), both drawn with 3-month or quarterly. The attached table in the window shows the cumulative cash flows at the end of the selected quarterly periods since the start of the project.

The cash flows for the RV component have been obtained through the generalized economic model that allows the addition of any number of revenue streams in the RV component, the selection of any *detailed* or *semi-detailed* revenue forecasting method, the selection of any *rate/area* function for revenue variables, and the choice of values for the *properties* of each revenue stream.



Period	Cumulative	Periodical
20	0.00E+00	0.00E+00
21	2.16E+06	2.16E+06
22	4.75E+06	2.59E+06
23	7.40E+06	2.65E+06
24	1.15E+07	4.11E+06
25	1.57E+07	4.18E+06
26	1.85E+07	2.85E+06
27	2.15E+07	2.91E+06
28	2.58E+07	4.39E+06
29	3.03E+07	4.46E+06
30	3.34E+07	3.11E+06
31	3.66E+07	3.18E+06
32	4.13E+07	4.68E+06
33	4.60E+07	4.76E+06
34	4.94E+07	3.39E+06
35	5.29E+07	3.46E+06
36	5.79E+07	4.98E+06
37	6.29E+07	5.06E+06
38	6.66E+07	3.67E+06
39	7.03E+07	3.74E+06
40	7.56E+07	5.29E+06
41	8.10E+07	5.37E+06
42	8.50E+07	3.96E+06
43	8.90E+07	4.03E+06
44	9.46E+07	5.61E+06
45	1.00E+08	5.69E+06
46	1.05E+08	4.26E+06
47	1.09E+08	4.33E+06
48	1.15E+08	5.93E+06
49	1.21E+08	6.01E+06
50	1.25E+08	4.56E+06
51	1.30E+08	4.64E+06
52	1.36E+08	6.26E+06
53	1.43E+08	6.35E+06

Figure 4.29: Total revenues cash flow

4.5 Operation and Maintenance Component

Total capital investment cost, as modeled by the CE component, constitutes one part of a complete cost estimate of capital investment projects. The other part that completes the estimate is operating or production cost which represents the expenses needed for keeping a project running and producing during its life cycle. Generally, operating costs consist of two main elements that can be categorized under manufacturing costs and general expenses (Humphreys 1991; Peters and Timmerhaus 1991). Table 4.6 shows a comprehensive list of operating cost under these two categories; the same cost elements and categories have been used by several authors (Behrens and Hawranek 1991; Frohlich et al. 1994; Humphreys 1991; Peters and Timmerhaus 1991). Cost categories under manufacturing cost include: direct production costs (material and labor); indirect production costs (overhead and depreciation); contingencies; and, distribution costs. Cost categories under general expenses include: marketing costs and administrative expenses. Operating costs are generally estimated based on total annual costs and/or cost per unit of end product (Behrens and Hawranek 1991; Humphreys 1991; Peters and Timmerhaus 1991). Operating cost items can further be divided into variable[3] and fixed costs.

The above operating cost elements cover the costs needed to meet the requirements of the *operation, possession, general liabilities, and tax* attributes of the requirement structure. The OM component is a platform that has properties and methods for estimating and modeling these operating costs.

[3] "Variable costs change roughly in proportion to the variations in the level of production. Typical variable costs include materials, production labor and utilities. Variable costs can be divided further into: proportional costs, which change proportionally with the volume of production (for example, raw material); degressive costs, which change at a lower rate than the volume of production (for example, maintenance and repair); progressive costs, which change at a higher rate than the volume of production (for example, overtime); and regressive costs, which decrease with an increase in the volume of production (for example, maintenance costs of unutilized machines).
Fixed costs remain unchanged regardless of changes in the level of activity, and include mainly overhead and depreciation charges, the latter only if the calculation is time-based. Fixed costs also include long-term contractual services, rents, and administrative salaries.

Table 4.6: Components of total operating cost (Humphreys 1991)

I. Operating cost or manufacturing cost
  A. Direct production costs
    1. Materials
      a. Raw materials
      b. Processing materials
      c. By-product and scrap credit
      d. Utilities
      e. Maintenance materials
      f. Operating supplies
      g. Royalties and rentals
    2. Labor
      a. Direct operating labor
      b. Operating supervision
      c. Direct maintenance labor
      d. Maintenance supervision
      e. Payroll burden on all labor charges
  B. Indirect production costs
    1. Plant overhead or burden
      a. Administration
      b. Indirect labor
      c. Purchasing, receiving, and warehousing
      d. Personnel and industrial relations
      e. Inspection, safety, and fire protection
      f. Automotive and rail switching
      g. Accounting, clerical, and stenographic
      h. Communications - telephone mail, and teletype
      i. Plant custodial and protective
      j. Plant hospital and dispensary
      k. Cafeteria and clubrooms
      l. Recreational activities
      m. Taxes on property and operating licenses
      n. Insurance -property, liability
      o. Nuisance elimination - waste disposal and pollution control
    2. Depreciation
  C. Contingencies
  D. Distribution costs
    1. Containers and packages
    2. Freight
    3. Operation of terminals and warehouses
II. General expense
  A. Market of sales costs
    1. Direct
      a. Salespersons' salaries and commissions
      b. Advertising and promotional literature
      c. Technical sales service
      d. Samples and displays
    2. Indirect
      a. Sales supervision
      b. Travel and entertainment
      c. Market research and sales analysis
      d. District office expenses
  B. Administrative expenses
    1. Salaries and expenses of officers and staff
    2. General accounting, clerical, and auditing
    3. Central engineering and technical

Table 4.6: Components of total operating cost (Humphreys 1991) (continued)

4. Legal and patent
5. Research and development
a. Own operations
b. Sponsored, consultant, and contract work
6. Contributions and dues to associations
7. Public relations
8. Financial
a. Debt management
b. Maintenance of working capital
c. Credit functions
9. Communications and traffic management
10. Central purchasing and activities
11. Taxes and insurance

4.5.1 OM Component Structure and Properties

The structure of the OM component, as shown in Figure 4.19, is similar to that of the RV component, and is made of constructs called OM streams. The OM component has *time unit, network* and *reference* properties. Each OM stream may have its own local value of the time unit property and therefore each stream may have a specific local time unit LTU.

For the *network* properties, each OM stream can have its own duration and lead/lag times. The duration of each stream is always defined in its LTU. Similar to the RV component, the logic property of an OM stream provides each stream to have either an *absolute* start linked to project start or a *contingent* start linked to the finish of any selected work package in the CE component or to the finish of another OM stream in the OM component using *finish-to-start* relationship, as shown in Figure 4.19. Absolute stream start and lead/lag times are always defined in terms of the project global time unit GTU. Absolute start of a stream refers to the number of time units (in GTU) elapsed after the start of project (i.e. project time zero).

Each OM stream represents a cash flow $f_c^{OM}(t`, \boldsymbol{x})$ where $\boldsymbol{x}$ refers to the OM stream variables. These variables may represent dollar values, quantities, and inflation. They are described along below with their *reference* properties. Each stream in the OM component can have its own inflation variable thus allowing the analysis in current dollar terms. The inflation variable of a stream can have either of *global*, *local* or *null* reference. Similar to the RV component, three strategies are used to deal with inflation of all streams, these include (Figure 4.10): *unique* for each revenue stream, *same-for-all-streams*, and *same-for-project*.

4.5.2 OM Component Methods

The OM classification includes crude, semi-detailed and detailed methods. Several of these methods can aid in the estimate of OM costs, such costs can be categorized, for example, as periodical (e.g. annual), per-unit of production, or as variable and fixed costs. This section describes how an OM stream cash flow function is derived using the RV classification methods.

Letting $\theta_{om}(t``)$ represent an OM inflation modeled by shape functions, $S$ a scope parameter variable, and $t`$ and $t``$ as defined before, then:

$$f_c^{OM}(t`,\boldsymbol{x}) = f_{co}(t``,\boldsymbol{x}) \cdot e^{\int_0^{t``} \theta_{om}(t)\mathrm{d}t} \tag{4.29}$$

where

$$f_{co}(t``,\boldsymbol{x}) = \begin{cases} OM(t``) \cdot S & \text{(a)} \\ A(t``) \cdot S \cdot f_{cD}^{RV}(t``,\boldsymbol{x}) & \text{(b)} \\ A(t``) + B(t``) \cdot S \cdot f_{cD}^{RV}(t``,\boldsymbol{x}) & \text{(c)} \\ OR(t``) \cdot S \cdot OQ(t`) & \text{(d)} \end{cases} \tag{4.30}$$

The methods in Eq. (4.30) and their variables are described in the following subsections.

4.5.2.1 OM Semi-detailed Methods

Semi-detailed methods in the OM classification represent preliminary "gross' methods as modeled and represented by *OM(t``)* in Eq. (4.30)(a). Semi-detailed methods consider operating costs as being aggregated total sum or total per unit of time (periodic costs), which are not related to the production capacity of a facility or to the quantity of a raw material consumed. For cash flow analysis, *OM(t``)* can assume any pattern of the *rate/area* functions in Tables (4.1,4.2) *OM(t``)* can be expressed in a global or a local reference system, but, if *area* functions are used, then only *local* reference should be used. Fixed operating costs in an OM stream can be modeled by *area* functions since they are not related to production and since the fixed costs will not change if the duration of the stream changes. Time-based operating costs can be modeled by *rate* functions, since any extension to stream duration will induce an increase in these costs.

For example, with reference to Table 4.6, indirect costs and administrative expenses can be estimated as (1) total sum distributed over an OM stream using, say, a *uniform area* function (Table 4.2) or a *linear area* function if they are likely to increase or decrease; or, (2) estimated as cost or rate per unit of time and modeled using, say, *uniform* or *linear rate* functions (Table 4.1). As an example, the agreement for State Route 91 ("Amended" 1993) identified several OM expenditures such as routine maintenance costs; utility service fees; personnel salaries; supervision and overhead; police services; and, professional services. All of these expenditures can be considered as time-based costs and therefore estimated as periodic costs and modeled in separate OM streams such that each cost item could have its own properties and *rate* function. If the cost items are considered to have fixed values over the operating life of the facility, then *area* functions could be used.

Figure 4.30 and 4.31 illustrate the implementation of the "gross" methods called "Aggregated Methods"; the "O&M Cost" tab shows operating costs modeled by *Normal II* rate function.

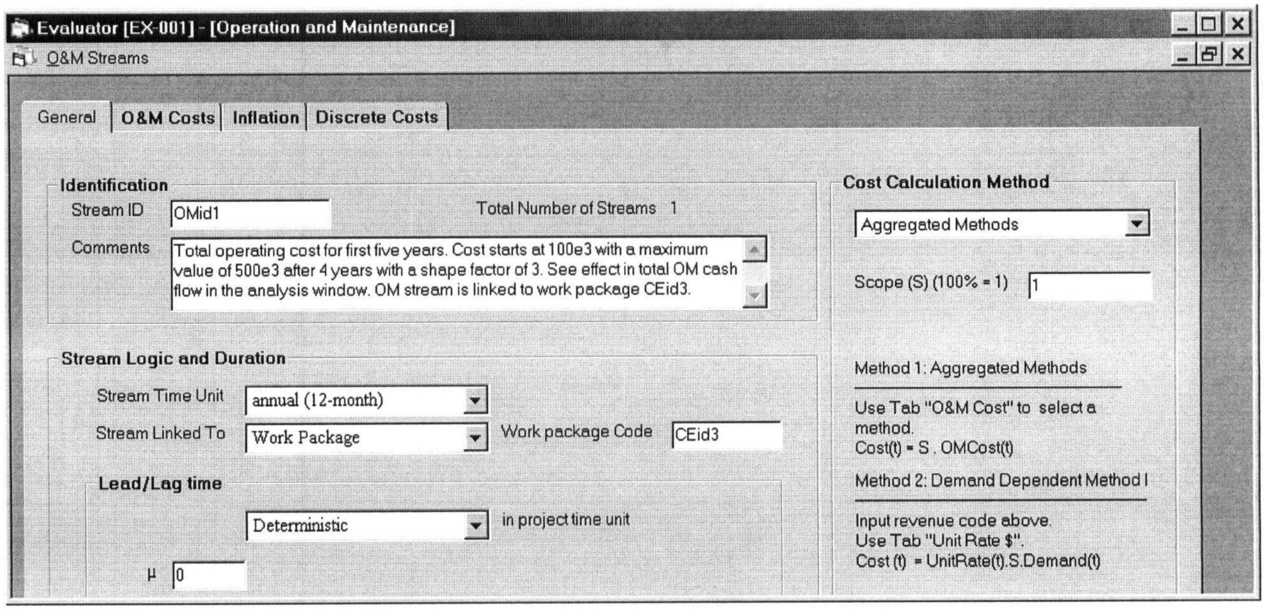


Figure 4.30: "Aggregated Methods" for operating cost

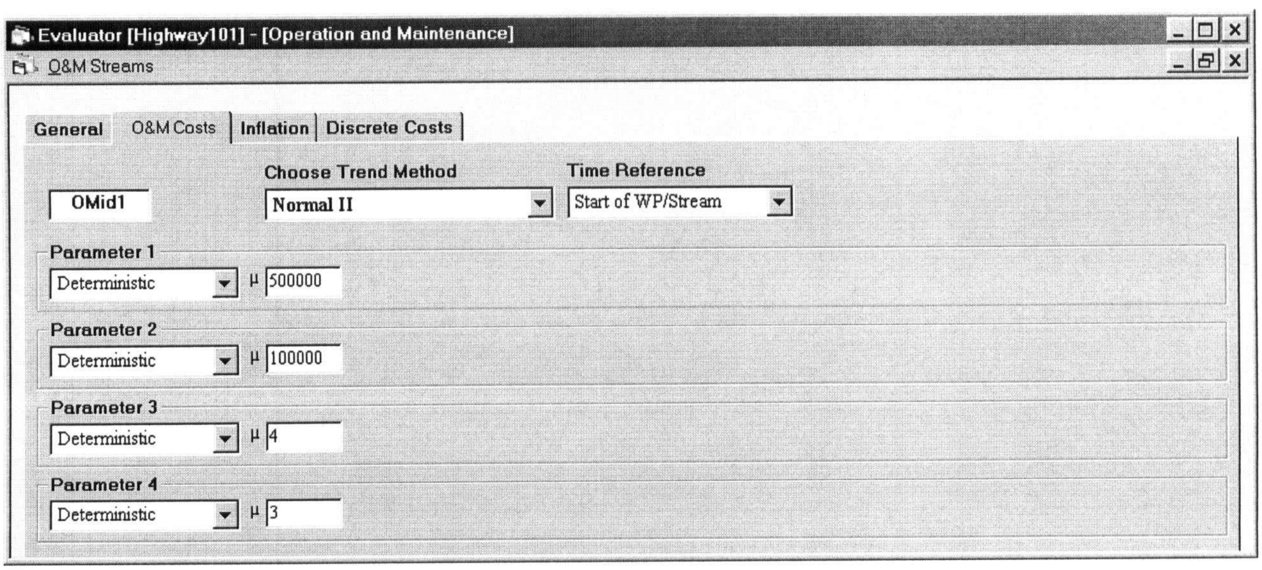


Figure 4.31: Normal *rate* function for marketing cost. *Parameter1:* maximum cost, *Parametr2:* starting annual costs, *Parameter3:* time of maximum cost within the stream duration, *Parameter4:* shape factor (see Table 4.1)

4.5.2.2 OM Detailed Methods

Detailed methods represent operating costs that are estimated based on either (1) cost per unit of production methods, or (2) unit cost method. Cost per unit of production methods are described as methods (b) and (c) in Eq. (4.30). For these two methods production is modeled by the *demand* function $f_{cD}^{RV}(t``,x)$ of a revenue stream in the RV component (Eq. 4.21). Thus, linking the O&M costs in an OM stream to the production (e.g. number of kilowatt-hours) or demand of a revenue stream in the RV component. The parameter variables $A(t``)$ in Eq. (4.30)(b) and $B(t``)$ in Eq. (4.30)(c) represent the unit cost of production (e.g. $ per kilowatt-hours) in these methods which can assume any pattern of the *rate/area* functions and can be referenced either by *global* or a *local* reference. $A(t``)$ in Eq. (4.30) (c) can be considered as an initial periodic cost that can be modeled by *rate/area* functions with a *global* or a *local* reference system. Using *area* functions for any variable must be accompanied by a *local* reference. The parameter variable S in these methods is used to limit the amount of production considered to a required value.

The variable operating costs identified in Table 4.6, such as raw materials, fuel, maintenance and labor, can be modeled by these detailed methods to reflect the effect of change in production or demand level on operating costs. For example, McNeil and Hendrickson (1982), for transportation projects, estimated an empirical model for the annual maintenance expenditure for roadways. The model considers both traffic volume on the road and pavement age in deriving OM costs. An OM stream can model this maintenance cost using Eq. (4.30) (c) with the OM stream linked to a RV stream so that traffic volume can be obtained from the demand function.

If the unit cost of a basic input item per unit of production, e.g. labor $/mhr/kilowatt-hour is used with production (e.g. kilowatt-hours), S can be used as a factor that converts production to the man-hour requirements (i.e number of man-hours per kilowatt-hour). The implied assumption here is that the relationship between labor requirement and production is linear.

The second category of operating cost methods is related to the unit cost method. This method is modeled by equation (d) in Eq. (4.30). In this method *OR(t``)* represents a unit cost variable that is modeled by *rate/area* functions with a *global/local* reference. *OQ(t`)* represents a quantity variable modeled by *area/rate* functions with a *local* reference. *S* is a scope variable. The unit cost method can be used to estimate several cost items in Table 4.6 such as utilities, raw materials and labor costs. The method depends only on the quantity (e.g. number of man-hours) of work required in an OM stream and the unit cost per unit of quantity (e.g. $ per mhr). In Evaluator, this method is called "Quantity Dependent" method.

Figure 4.32 illustrates the implementation of the "detailed" methods. "Initial Cost" and "Unit Rate" tabs in the window are used to model the initial periodic cost *A(t``)* and cost per unit of production *B(t``)* respectively (Eq. 4.30c). These are called "Demand Dependent Methods II" in the system. In this window, demand is obtained from the *Rvid2* revenue stream.

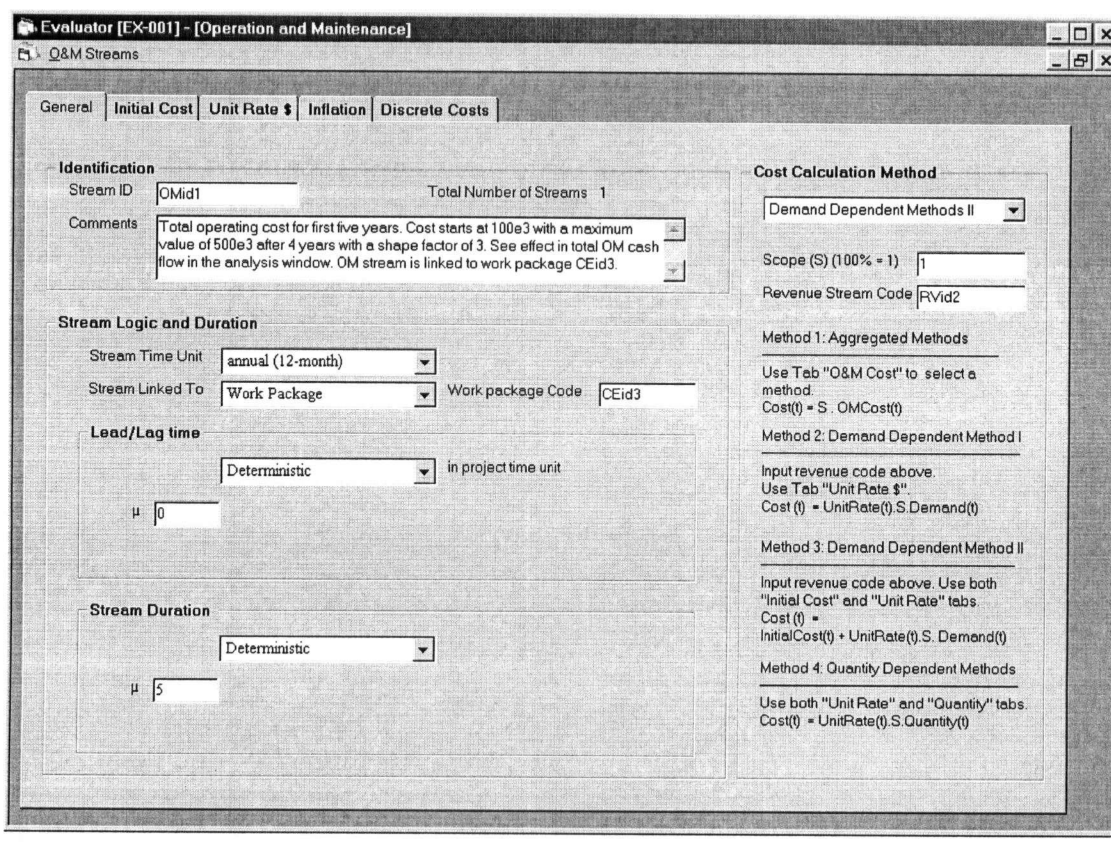


Figure 4.32: Detailed methods in the OM component

4.5.2.3 OM Crude Methods: Discrete Costs

The preceding sections described the cash flow function of an OM stream in terms of semi-detailed and detailed calculation methods, which relied on expenditures being made as continuous functions of time. However, since operating costs may include discrete expenditures, for example, major repair costs, the generalized model provides for discrete costs to be added to any OM stream in the same manner as discrete costs are treated in the CE and RV components.

The discrete cost $\mathbf{D}_j^{OM}$ is represented as follows: Let $\mathbf{Dv}_j^{OM}$ and $\mathbf{Dt}_j^{OM}$ represent constant dollar value and local time of discrete cost j of n discrete costs in an OM stream. Further let $\theta_{om}(t)$ be a discrete-cost inflation variable modeled by *rate* functions with *global/local* reference. Then,

$$\mathbf{D}_j^{OM} = \mathbf{Dv}_j^{OM} \cdot e^{\int_0^{t^*} \theta_{om}(t)\mathrm{d}t} \Bigg|_{\substack{t^* = \mathbf{Dt}_j^{OM} \text{ if } \theta_{om} \text{ is in local time} \\ t^* = \mathbf{Dt}_j^{OM} + ESc \text{ if } \theta_{om} \text{ is in global time}}} \tag{4.31}$$

4.5.3 OM Component Cumulative and Discounted Costs

The above cash flow formulation $f_c^{OM}(t, \boldsymbol{x})$ represents the essential part for any OM stream in the OM component. All of the operating stream cash flow functions can be integrated together to form the OM component cash flow function as described earlier in Eq. (4.3).

A cumulative cash flow $F_c^{OM}(t`,\boldsymbol{x})$ of an OM stream at any *local* time $t`$ can be obtained as follows, where r is the total number of discrete costs in a stream:

$$F_c^{OM}(t`,\boldsymbol{x}) = \int_0^{t`} f_c^{OM}(t,\boldsymbol{x})\,\mathrm{d}t + \sum_{j=1}^{r} \mathbf{D}_j^{OM} \Bigg|_{\text{for each } \mathbf{Dt}_j^{OM} \le t`} \tag{4.32}$$

Consequently, cumulative cash flow $F_{OM}(t,\boldsymbol{X})$ of the OM component for *global* time t can be obtained as follows, where $\overline{t-ESc_i}$ is the time elapsed in stream i and converted from GTU to LTU, ESc is the start time of stream i and m is the total number of streams:

$$F_{OM}(t,\boldsymbol{X})=\sum_{i=1}^{m} F_c^{OM}(\overline{t-ESc_i},\boldsymbol{x}_i)_i\Big|_{\text{for each } t\geq ESc_i} \tag{4.33}$$

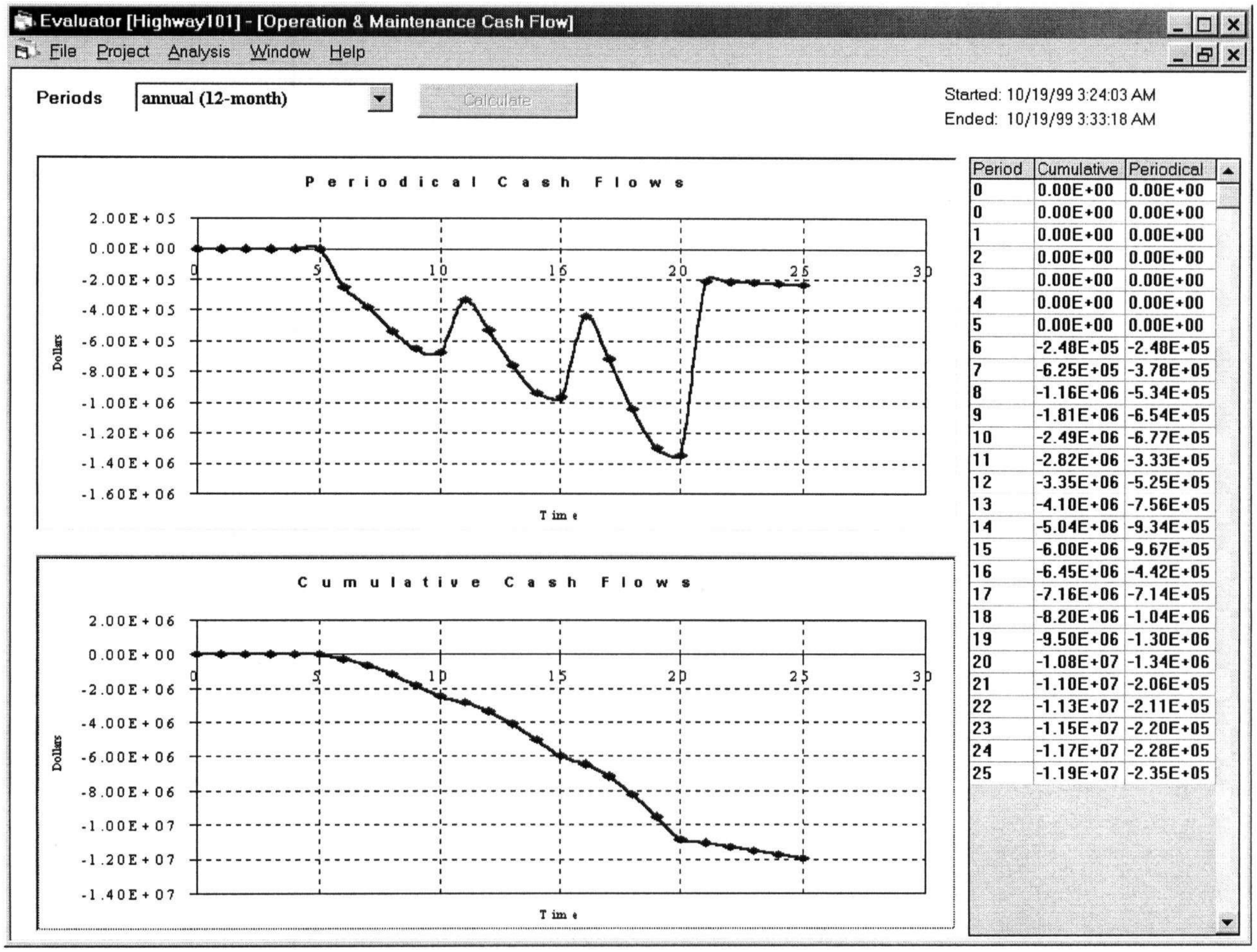
Discounted cost $df_c^{OM}(Td,\boldsymbol{x})$ of an OM stream can be obtained as follows where all cash flows are discounted to time Td which is measured in GTU with *global* reference; y is the nominal annual discount rate and $\overline{y}$ is y converted from annual time periods to LTU; wd is the duration of the stream in LTU; $\overline{tb}$ (Eq. 4.15) is the time before start of the stream, converted from GTU to annual time unit; and, $\overline{tn}$ (Eq. 4.16) is the time elapsed in the stream, converted from GTU to LTU. Thus:

$$df_c^{OM}(Td,\boldsymbol{x})=e^{-\overline{y}\cdot\overline{tb}}\cdot\left[\int_0^{wd-\overline{tn}} f_c^{OM}(\underline{t}+\overline{tn},\boldsymbol{x})\cdot e^{-\overline{y}\cdot\underline{t}}\mathrm{d}\underline{t}+\sum_{j=1}^{r}\mathbf{D}_j^{OM}\cdot e^{-\overline{y}\cdot(\mathbf{Dt}_j^{OM}-\overline{tn})}\Big|_{\text{for each } \mathbf{Dt}_j^{OM}\geq\overline{tn}}\right] \tag{4.34}$$

Consequently, discounted cost of the OM component can be obtained as follows:

$$df_{OM}(Td,\boldsymbol{X})=\sum_{i=1}^{m} df_c^{OM}(Td,\boldsymbol{x}_i)_i \tag{4.35}$$

Figure 4.33 illustrates Evaluator's output screen for the total operation and maintenance cash flow. The window shows the periodical cash flow graph that implements Eq. (4.3) and the cumulative cash flow graph that implements Eq. (4.32), both drawn with annual periods. The table in the window shows the cumulative and periodic OM cash flows at the end of the selected annual time unit during the project life cycle.



Period	Cumulative	Periodical
0	0.00E+00	0.00E+00
0	0.00E+00	0.00E+00
1	0.00E+00	0.00E+00
2	0.00E+00	0.00E+00
3	0.00E+00	0.00E+00
4	0.00E+00	0.00E+00
5	0.00E+00	0.00E+00
6	-2.48E+05	-2.48E+05
7	-6.25E+05	-3.78E+05
8	-1.16E+06	-5.34E+05
9	-1.81E+06	-6.54E+05
10	-2.49E+06	-6.77E+05
11	-2.82E+06	-3.33E+05
12	-3.35E+06	-5.25E+05
13	-4.10E+06	-7.56E+05
14	-5.04E+06	-9.34E+05
15	-6.00E+06	-9.67E+05
16	-6.45E+06	-4.42E+05
17	-7.16E+06	-7.14E+05
18	-8.20E+06	-1.04E+06
19	-9.50E+06	-1.30E+06
20	-1.08E+07	-1.34E+06
21	-1.10E+07	-2.06E+05
22	-1.13E+07	-2.11E+05
23	-1.15E+07	-2.20E+05
24	-1.17E+07	-2.28E+05
25	-1.19E+07	-2.35E+05

Figure 4.33: Periodical and cumulative cash flow for the OM component streams

4.6 Financing Component

4.6.1 Background and Project Financing

It is at the appraisal stage, during a pre-feasibility or feasibility study, that analyses are performed as to how a project will be financed, on the availability of financial resources and on the terms and conditions of the likely financial instruments. Financial resources for capital investment project can generally be arranged in three forms of capital: equity capital, debt financing and mezzanine financing (Nevitt and Fabozzi 1995; Spate 1997; and, MCFL 1996). Equity represents the capital injected by the owners of the company/project and other equity investors; it is usually advanced in terms of common and preferred stock/shares. Equity capital is usually called risk capital since equity investors rank the lowest in terms of distribution of repayments and in terms of the claims on project assets in case of failure. Debt capital represents the capital raised through commercial banks, institutional investors (pension funds and insurance companies), capital markets, export credit agencies, and bilateral and multilateral agencies (e.g. World Bank). It can generally take the form of several types of loans and bonds. Senior debt capital ranks high in terms of priority of distribution of repayments; it has the lowest rank of risk among the other sources of capital. Mezzanine financing, called subordinated debt or quasi-equity, represents capital raised mainly through the same sources for senior debt capital, using similar forms (e.g. subordinated loans). Mezzanine financing is junior to debt capital and senior to equity capital in terms of payment and risk. Mezzanine financing receives higher interest rates than senior debt. Debt capital represents the largest portion of financing for a project.

In traditional commercial bank activities for debt financing, financial transactions are typically designed to meet the financial requirements of a borrower in light of its creditworthiness. Assessment of borrower creditworthiness represents an essential function which lenders must do to protect against financial vulnerability. The primary sources of a lender's funds are usually *risk-averse short-term* or *on-demand* deposits from private customers, companies and other banks. Lenders use these deposits to provide loans to governments, other banks, companies, and personal borrowers in terms of generally *committed long-term* facilities (Wall and Mitchell 1980). Lenders would be vulnerable to liquidity problems if they were unable to meet the obligations of their depositors. Therefore, before any money is lent credit assessment and transaction design are performed in way to ensure to the extent possible that a facility will be repaid when due and that the return is commensurate with the risks assumed. (Nevitt and Fabozzi 1995; and, Willingham 1990). As explained by Nevitt and Fabozzi (1995) lenders can not afford to take any risk (e.g. equity risk) in a project other than the lending or credit risk. A lenders' credit risk represents a loss to the lenders if the borrower is unable to repay debt, which is unlike a market risk that represents the loss if the transaction cannot be sold in the market place except at discount (Willingham 1990).

Credit assessment by commercial banks is generally performed in terms of three operations: analysis of a company's business industry and country; analysis of a company's management and organization, annual reports, and ownership; and, analysis of a company's financial capacity and ability to generate cash from its activities or from the sale of assets in order to repay its debt. Following credit assessment a financial transaction would be designed to fit the needs of the borrower. Lenders generally consider the following key elements in the structure of the transaction: purpose, amount, term, repayment, security, control and monitoring, structure, and pricing (Willingham 1990). A similar credit assessment is performed to assess the creditworthiness of borrowers when issuing bonds in the capital markets. Independent bond rating agencies, such as Standard & Poor and Moody's, are usually consulted to assess the creditworthiness of an issuer of a bond (borrower) or to the assess the bond issue itself and consequently the risks associated with the issue. The rating consider elements such as: likelihood of default, nature of obligation (e.g. senior, subordinated, and secured), and protection afforded by the obligation under bankruptcy (Scarlett 1990; Nevitt and Fabozzi 1995; and, Brealey et al. 1991). The rating is described by letter notation and credit categories. For example, a rating from BBB to AAA represents investment grade issues (high creditworthiness), B- to BB+ represents distinctly speculative issues (low creditworthiness), and D to CCC+ represents predominately speculative (substantial risk or in default). Following the rating, the interest rate on the bond issue can be established.

Since traditional financing lending mechanisms or instruments put corporate assets (debt security) at risk in case of failure of a company sponsored project, other instruments have been developed to provide protection for a company while providing protection to the lender. Most important is the instrument of "project finance". US Accounting Standard FASB 47 defines the project finance concept as follows:

> *"The financing of a major capital project in which the lender looks principally to the cash flows and earnings of the project as the source of funds for repayment and to the assets of the project as collateral for the loan. The general credit of the project entity is usually not a significant factor, either because the entity is a corporation without other assets or because the financing is without direct recourse to the owner(s) of the entity"* (McKechnie 1990).

The general characteristics that characterize project finance can be described as follows (Nevitt and Fabozzi 1995; McKechnie 1990; MCFL 1996; and, Sapte 1997):

- A project under "project finance" is segregated into a separate special purpose company such that it becomes possible to isolate the cash flows (both inflows and outflows) of the proposed undertaking. The objective is to allow for financial independence of the project and the protection of other assets of the developers (usually subsidiaries of parent companies) in case of financial troubles. A detailed description of such a project company was given in Chapter 2 (Sec. 2.2.2). All the projects developed under the PPP delivery system described in Chapter two (Sec. 2.3.2) were procured through the development of a project company in order to raise the necessary project finance.
- Project finance is generally a *non-recourse* borrowing that contributes to raising capital that will not affect the credit standing or be on the balance sheet (off-balance sheet financing) of the sponsors. While the objective of using project finance to the private sector sponsors is to obtain *non-recourse* financing such that the lenders will be concerned exclusively with project cash flows and project assets, it is appropriately called *limited recourse financing* since the lenders will require credit support (e.g. guarantees and take-or-pay contracts) from sponsors and third parties who may benefit from the project. Therefore, credit support for project finance transactions depends heavily on several guarantees and contractual commitments between project participants and third parties such as government agencies, contractors, suppliers, and purchasers (e.g. take-or-pay contracts).
- Project finance is generally called cash flow lending. Lenders consider the main debt security is the projected future cash flows. Therefore, detailed analysis of a project's performance throughout its life is central for project finance approval. In infrastructure BOT projects, lenders will not even consider the assets of the project itself as security since the ownership and the physical assets will be transferred to the government by the end of the project agreement; i.e. under project failure lenders will not be able to sell the

assets (UNIDO 1996). Therefore, proper cash flow modeling and risk analysis are essential elements in project financing. As stated by Nevitt and Fabozzi (1995): "EBITDA is the mother's milk of project financing. EBITDA refers to earnings before interest, taxes, depreciation and amortization. EBITDA is the cash flow available to pay interest and debt principal."

- With project finance, developers, particularly in privately promoted infrastructure project, are required to contribute equity capital to the project. Some projects however, received minimal equity. The Dartford Crossing project in the UK was arranged with £1000 pinpoint equity while debt amounted to £166.4 million (PWF 1998; Levy 1996).
- In BOT projects, project financing provides for project private sector sponsors to be liable up to the amount of their equity contribution to the project while lenders carry part of the risk since they are lending without full recourse (UNIDO 1996). Lenders for project financing of several BOT projects, however required that the projects be rated by S&P or Moody's before they agreed to the financing. For example, bonds for the Northumberland Strait Crossing received a AAA rating by both rating companies (Pirie 1996).

Project finance represents an essential financing technique for capital investment projects particularly those to be developed under a public-private partnership arrangement. All the projects investigated in Chapter 2 had their capital raised through project finance. Davis (1996) identified several power, pipeline, telecommunication, and toll road projects that were financed by project finance instruments. However, and in general, equity, debt and mezzanine financing are required for capital investment projects. UNIDO (1996), as shown in Table 4.7, explained the likely financial capital requirements for BOT projects during a project life cycle.

Table 4.7: Sequencing of a BOT financial package (UNIDO 1996)

Activity	**Type & Source of Financing**
Pre-investment and development costs	Risk capital from project sponsors
Bidding and procurement	Risk capital from project sponsors Possible support from government
Financial structuring and development of security package	Equity capital from project sponsors
Agreements with institutional and other investors	Equity capital from institutional and other investors
Agreements with equipment suppliers	Long-term loans from export credit agencies for equipment purchase
Agreements with prime contractor and subcontractors on cost of construction	Short-term loans form commercial banks to finance construction
Financial restructuring as completion of construction approaches	Long-term loans form non-bank financial institutions and specialist investment funds
Financial closing Start of construction	Drawdown of equity and loan funds
Operation	Working capital from the project company Short-term loans from commercial banks

The FN component is the means through which the *financing attribute* of a project requirement structure can be modeled and represented for cash flow calculations. The properties and methods of the FN classification include five common instruments used in project financing; namely, general term loan, syndicated term loan, general bond, private placement bond, and floating rate note (Nevitt and Fabozzi 1995; McDonald 1982; Rhodes 1993; Howcroft and Solomon 1985; Brealey et al. 1992; Gelbard 1996; Lund et al. 1984; Nash 1990; and Ugeux 1981; Spate 1997; MCFL 1996). The following subsections describe the FN component's properties and methods.

4.6.2 FN Component Structure and Properties

The structure of the FN component, as shown in Figure 4.34, is made of constructs called debt streams. A debt stream inherits the properties and methods of the FN classification. Any number of debt streams can be defined in the component, and a stream can represent any of five financing instruments/vehicles identified previously. Figure 4.35 summarizes the properties and methods used with such instruments.

Similar to the other components, the FN component has its own constructs, properties, methods and $\boldsymbol{X}$ matrix. However, because of the discrete nature of the flow of financial funds, the cash flow of the i-th debt stream is represented by information matrices instead of a cash flow function. The properties and methods of each FN stream are used to prepare the information matrices, $\mathbf{TRIFt}_i$ and $\mathbf{TRIFv}_i$ of the FN component. The two matrices represent the time and value characteristics of the four main parts of any debt stream: *tranches*[3], *repayments*, *interest*, and *fees*. A decomposition of the matrices for each debt stream i is as follows (matrices and their subscripts are used throughout the following description of the FN component). $\mathbf{Tt}_{i,j}$ and $\mathbf{Tv}_{i,j}$ are the time and value of each tranche j of m tranches in a stream. $\mathbf{Rt}_{i,k}$ and $\mathbf{Rv}_{i,k}$ are the time and value of each repayment k of n repayments. $\mathbf{It}_{i,z}$ and $\mathbf{Iv}_{i,z}$ are the time and value of each interest payment z of p payments. $\mathbf{Ft}_{i,w}$ and $\mathbf{Fv}_{i,w}$ are the time and value of each fee payment w of q fees.

Variables used in a debt stream's calculations and financial cash flows include fixed interest rate, floating interest rates, and exchange rates. These variables are referenced in the $\boldsymbol{x}$ vector of a debt stream and used in preparing the debt stream information matrices. Floating interest and exchange rates can have *local/global* references.

[3] A tranche refers herein to a single advance of money, representing portion of a loan amount, made to a borrower.

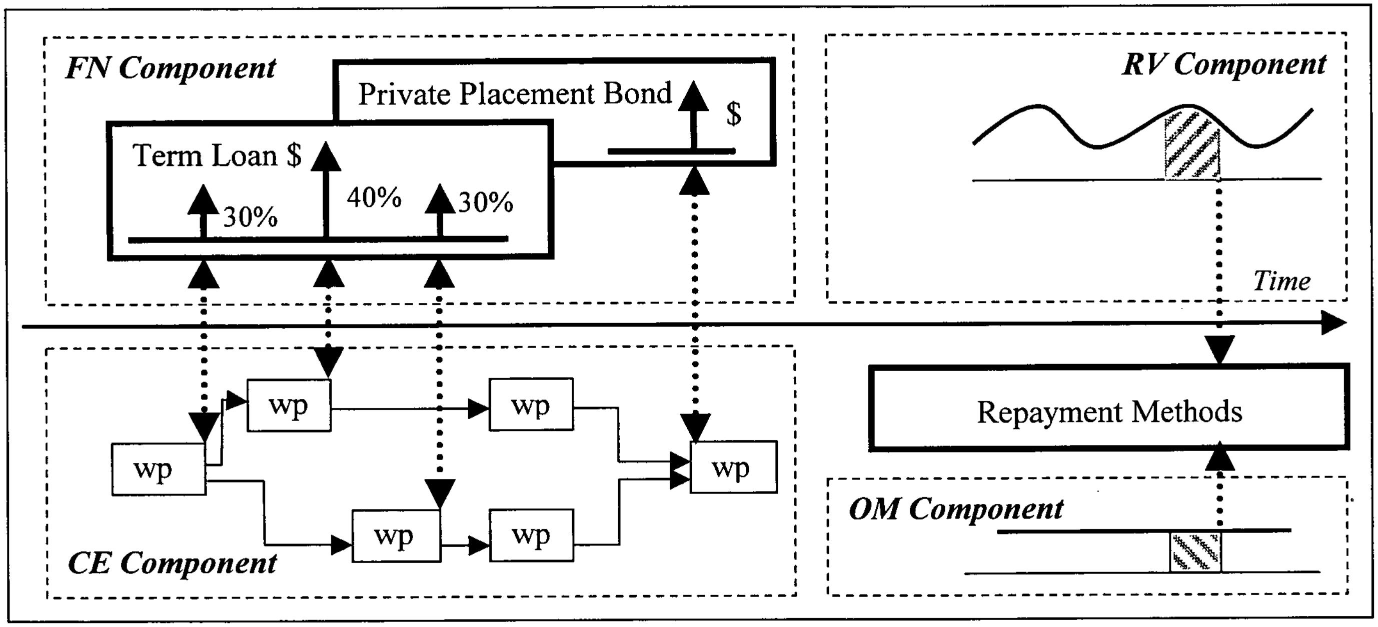


Figure. 4.34: Financing component structure

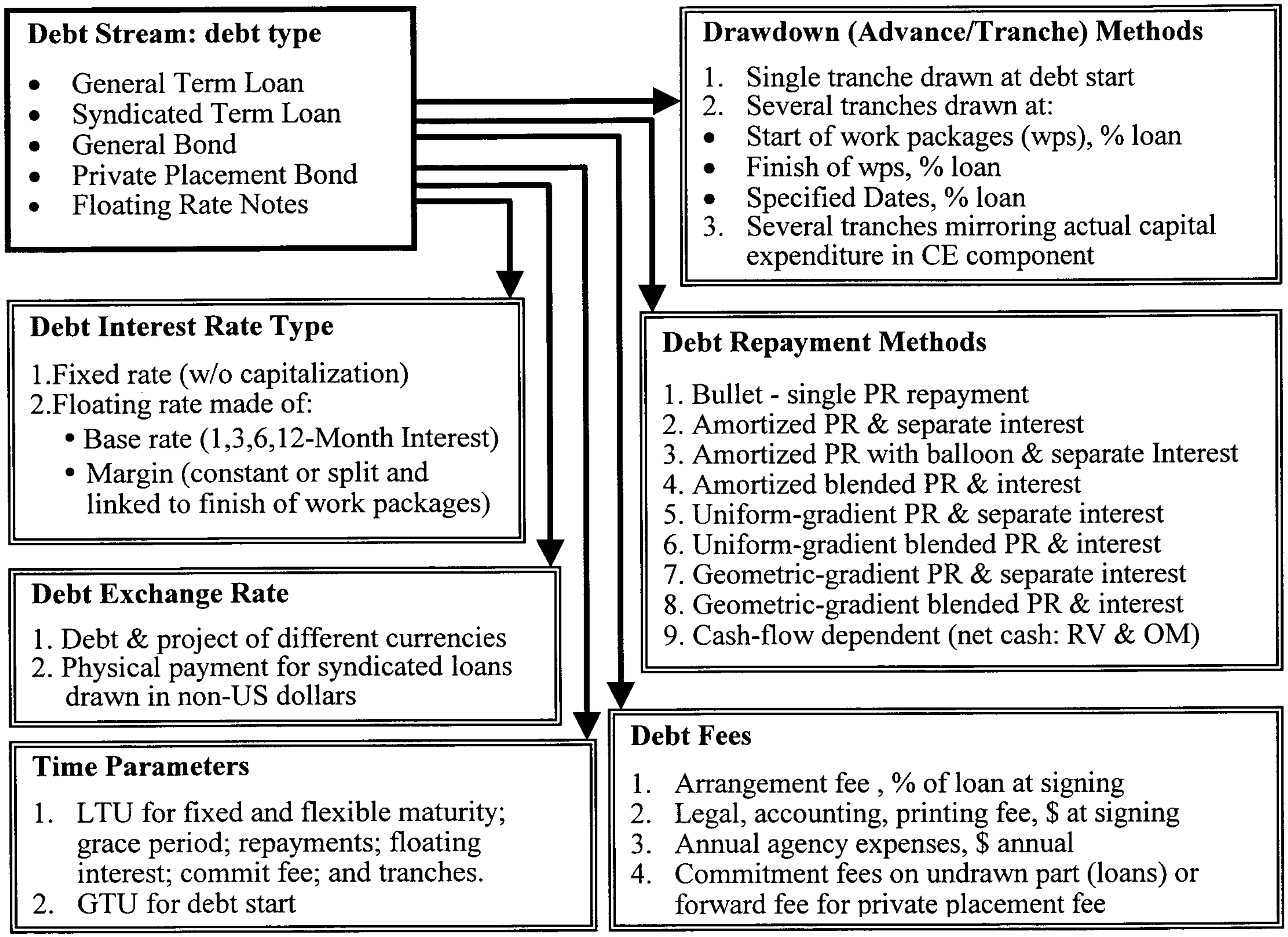


Figure 4.35: Properties and methods of the FN classification

Like the other components in the generalized model, the FN component has *time unit, network* and *reference* properties which can inherited by any new debt stream. Each debt stream has a number of *local time unit properties.* The main LTU is called *debt time unit*, which is used when referring to specific local times during the term (duration) of the credit agreement. The other local time units as described later are used to describe the frequency with which funds flow for *tranches*, *repayments*, *interest*, and *fees*. With respect to the *network property*, while there is no sequential order between debt streams, the *network property* establishes the time the debt stream will begin relative to the start of the project.

Figure 4.36 illustrates the main elements of a debt and will be referred to frequently. With reference to this figure, *debt start time* (*ds*) is measured in *global time unit* GTU. Debt *term* (*DT*) represents the life span of the debt until its final maturity and is measured from the debt start in *debt time unit*. Each debt has a *grace period* (*GP*) at the end of which repayments of debt become due in the *repayment period.* Grace period is measured in *debt time unit.* Repayment period is a derived period and uses *repayment time unit* that defines the frequency of repayments.

4.6.3 FN Component Common Methods

As mentioned above, any debt can generally be described in terms of four parts: *tranches* representing dollar advances/withdrawals to a borrower; *repayments* representing amounts required to retire the principal value of the debt; *interest* representing cost (i.e. price not fees) of lending money by a lender; and, *fees* representing administrative costs for managing the debt by the lender. Since many of the methods under these four parts can be used with the five instruments in the FN component, the following subsection provides a general description of the methods under these four parts. Then, a description if given for the five financial instruments in the model.

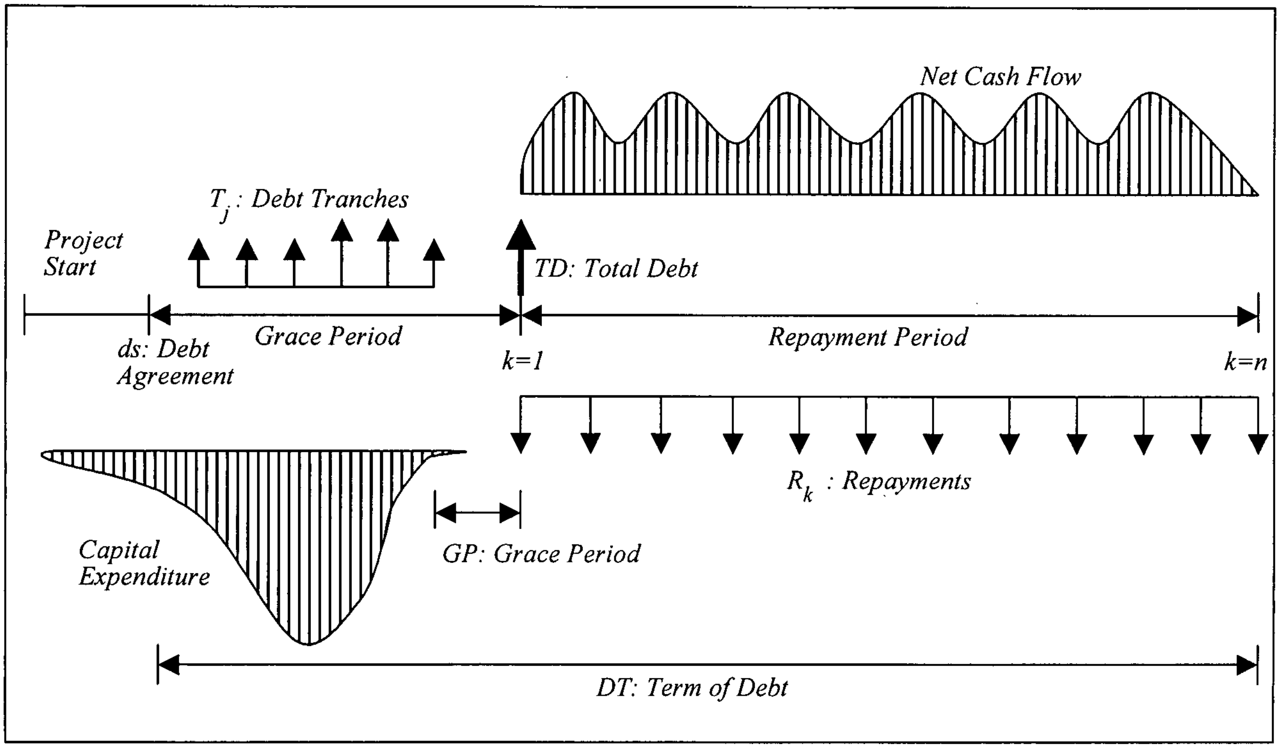


Figure 4.36: Description of debt elements, interest and fees are not shown

4.6.3.1 Debt Drawing Methods

Debt *tranches*, as illustrated in Figure 4.36, represent dollar advances or drawings and can be made through three methods that can be selected according to the type of financial instrument used. A description of drawing method/schedule under these methods is as follows:

1. Percentage of capital expenditure

   This method provides a debt drawdown profile for the *general* and *syndicated term loans* that is designed to match the cash flow of the capital expenditure component as derived in Eq. (4.1) and shown in Figure 4.18. With this method, it is assumed that several tranches can be drawn at equal time intervals over a time period defined by the early start time of a CE work package and early finish time of the same or another work package in the CE component. The total loan represents a constant percentage δ of the total capital expenditure in the defined time period. Each tranche represents the fraction δ of the capital expenditures of the preceding interval as determined through the cumulative capital expenditure function, Eq. (4.13). Thus, for tranche *j*, a tranche value $\mathbf{Tv}_{i,j}$ drawn at time $\mathbf{Tt}_{i,j}$ (end of interval) using the cumulative CE component cash flow function $F_{CE}(t``, X_{CE})$ as described by Eq. (4.13) is

   $$\mathbf{Tv}_{i,j} = \delta \cdot [\, F_{CE}(\mathbf{Tt}_{i,j}, X_{CE}) - F_{CE}(\mathbf{Tt}_{i,j-1}, X_{CE})\,] \quad \text{where } \mathbf{Tt}_{i,0} = 0 \qquad (4.36)$$

2. Percentages of specified total loan amount

   With this method a total debt amount is specified for the *general* and *syndicated term loans,* and tranches are defined as percentages of the total loan to be drawn at either of:

   (a) early start of specified work packages,
   (b) early finish of specified work packages, or
   (c) specified dates after *debt start* time using *debt time unit.*

3. Single tranche/drawdown

   With this method the total debt amount is specified and advanced in a single tranche. The drawdown date is specified as follows:

   (a) With the *general* and *syndicated term loans*, an elapsed time from *debt start* time is specified in *debt time unit*,
   (b) With *private placement bonds*, the time is specified as in (a), and
   (c) With the *general bond* and *floating rate notes*, debt is drawn at *debt start* time, i.e. zero elapsed time in *debt time unit.*

Using the above methods of drawdown the total number of tranches *Tnum* is computed for method 1 above, and directly specified for methods 2 and 3. With method 1, therefore, *Tnum* and tranche schedule time $\mathbf{Tt}_{i,j}$ can be derived in GTU as follows; where EFc_b is the early finish of work package b in the CE component, ESc_a is early start of another or same work package a, $\overline{tu}_i$ is the selected time period between drawdowns converted into project time unit GTU, $\overline{t}^{\backprime}$ is debt local time converted into GTU, and ds is the start time of debt in GTU:

$$Tnum_i = (EFc_b - ESc_a)/\overline{tu}_i \Big|_{\text{workpackages } a \text{ and } b \text{ are defined in debt } i} \tag{4.37}$$

$$\mathbf{Tt}_{i,j} = \begin{cases} ESc_a + j \cdot \overline{tu}_i & \text{(method 1)} \\ ESc_{a^\backprime} & a^\backprime \text{ is specified work package for tranch } j \text{ (method 2a)} \\ EFc_{a^\backprime} & a^\backprime \text{ is specified work package for tranch } j \text{ (method 2b)} \\ \overline{t}^{\backprime} & \overline{t}^{\backprime} \text{ is specified time for tranch } j \text{ (method 2c)} \\ \overline{t}^{\backprime} \text{ or } ds_i & ds \text{ is start time of debt } i \text{ (method 3)} \end{cases} \tag{4.38}$$

Figure 4.37 shows, in the right side of the window, how the first method above (percentage of capital expenditure) is implemented in the system. The percentage δ is specified followed by two work packages in the CE component and the time interval at which tranches are to be drawn. Figure 4.38 shows how the second method (percentages of specified total loan amount) is implemented, where percentages of the total loan are specified, followed by the work packages at the early start of which the relevant tranches will be drawn.

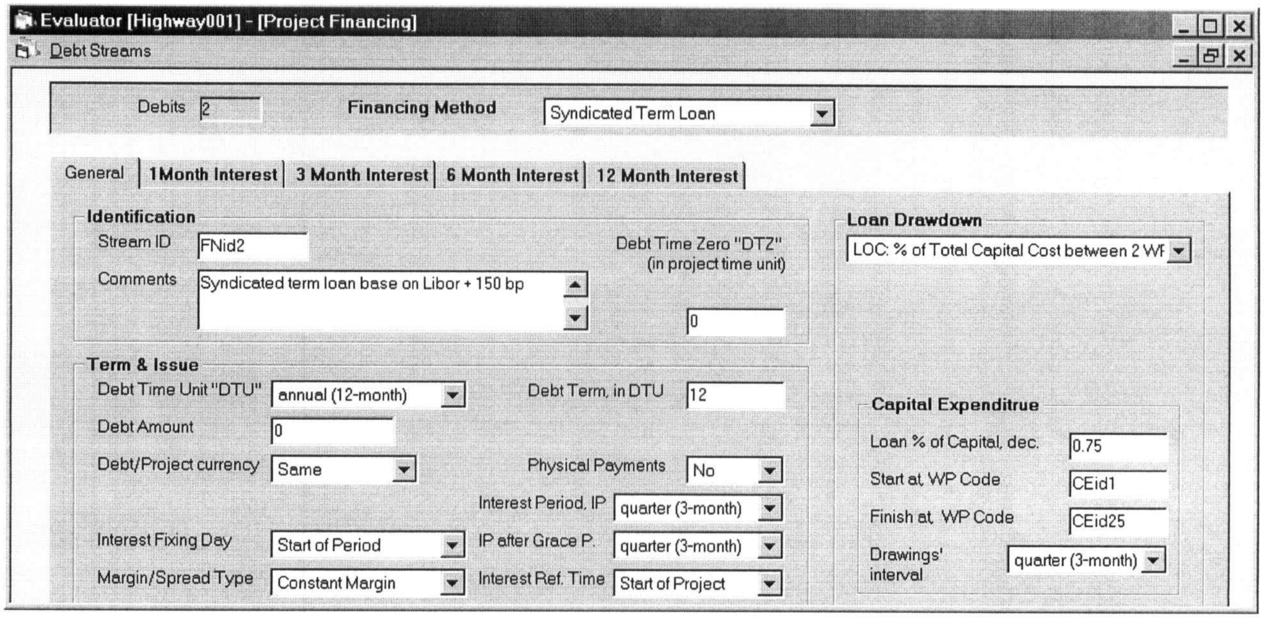


Figure 4.37: Debt drawdown as a percentage of capital expenditure

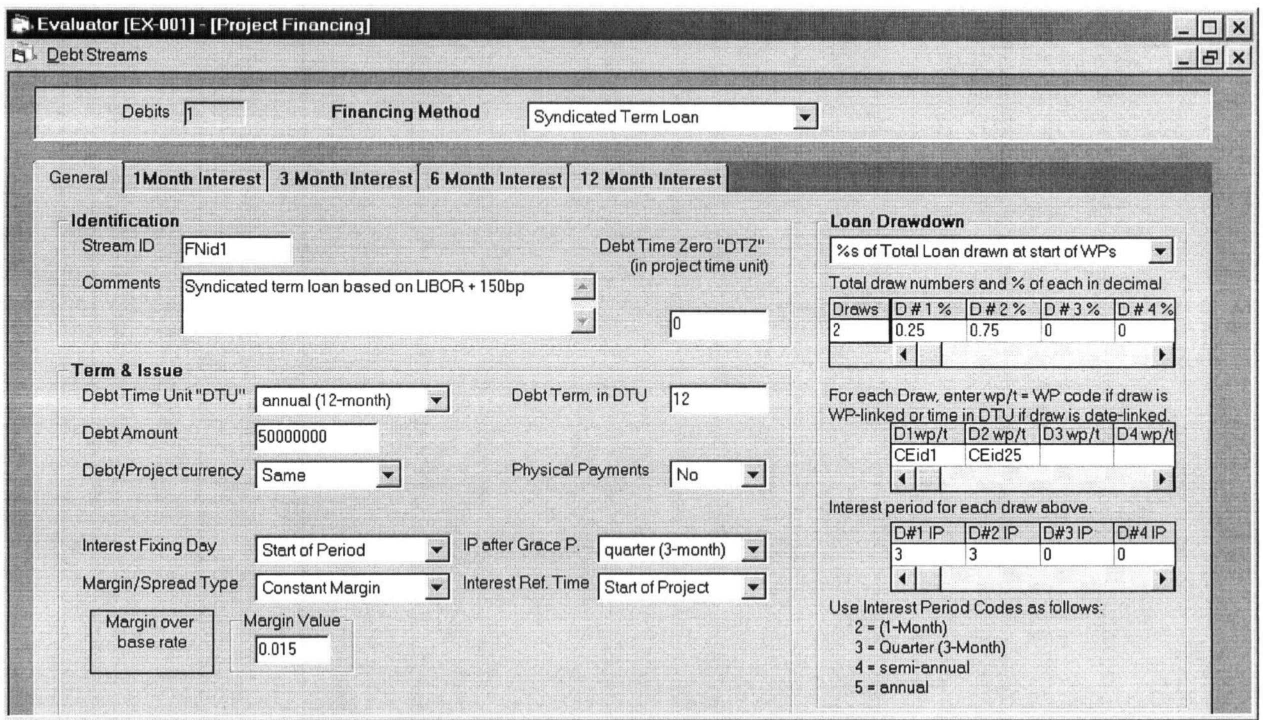


Figure 4.38: Debt drawdown at start of specified work packages

4.6.3.2 Debt Repayment Methods

A debt stream as represented in the generalized model can be described as shown in Figure 4.36. In this figure debt stream tranches are shown together with the total amount of debt *(TD)* outstanding at the end of the grace period and repayments occurring over the length of the repayment period. As can be noted in this figure the first repayment of debt ($k=1$) starts at the end of the grace period. According to the financial instrument that is represented by the debt stream, nine debt *repayment* methods can be used, as identified in Figure 4.35.

The calculation of repayment values of the debt *principal* amount depends on: (1) value of the outstanding debt at the end of the grace period, (2) number of repayments and repayment intervals, (3) repayment method, and (4) the interest rate during the repayment period.

Total debt outstanding at the expiration of a grace period can be obtained according to whether or not interest during the grace period is capitalized. If interest is not capitalized (i.e. being paid during the grace period), then total debt at the end of the grace period will be equal to the summation of the tranches of debt as determined by the three debt drawdown methods previously described (e.g. Eq. 4.36). Consequently, for a number of tranches m, total amount of debt i is

$$TD_i = \sum_{j=1}^{m} \mathbf{Tv}_{ij} \tag{4.39}$$

If interest is capitalized during the grace period, then the interest value will be included in the total debt outstanding at the end of the grace period. For fixed-interest-rate *general term loan*, in the generalized model, the total amount of debt outstanding will be

$$TD_i = \sum_{j=1}^{m} \mathbf{Tv}_{ij} \cdot (1+int_i)^{tl_j} \Big|_{tl_j = \overline{GP}_i - \mathbf{Tt}_{ij}} \tag{4.40}$$

in which int_i is a fixed interest rate effective per project time unit (GTU) (see Eq. 4.55), tl_j is the time length in GTU between drawdown time $\mathbf{Tt}_{i,j}$ (in GTU) of tranche j and grace period date $\overline{GP}_i$ (converted from debt time unit to GTU) (that is why int_i is an effective rate per GTU).

The number of repayments and repayment dates can be determined for a debt instrument using debt *term* DT_i, grace period GP_i, and repayment time interval ru converted into debt time unit $\overline{\overline{ru}}_i$ and into global time unit $\overline{ru}_i$, where

$$Rnum_i = (DT_i - GP_i) / \overline{\overline{ru}}_i \tag{4.41}$$

$$\mathbf{Rt}_{i,k} = ds_i + \overline{GP}_i + (k-1) \cdot \overline{ru}_i \tag{4.42}$$

In Eq. 4.41, both debt term and grace period are defined in debt time unit, and time conversion is made according to the repayment time interval. Since the *repayment period*, as shown in Figure 4.36, may include a fraction of time, the number of repayments is considered to be the floor value (i.e. integer value) of the difference between the debt term and grace period divided by the repayment time interval.

The interest used in repayment calculations is the effective interest rate per the repayment time interval as obtained below in Eq. 4.55. Table 4.8 describes nine methods that can be used to derive repayment $\mathbf{Rv}_{i,k}$ of n repayments in a debt stream i. Repayment expressions in Table 4.8 are derived for the economic model such that repayments, in annuity due style, start at the time the total debt TD becomes due and continue at end of periods. The methods are:

1. Amortized repayments of debt principal with separate interest payments
2. Amortized repayments of debt principal and balloon with separate interest payments
3. Uniform gradient repayments of debt principal with separate interest payments
4. Geometric gradient repayments of debt principal with separate interest payments
5. Bullet Repayment of debt principal with separate interest
6. Amortized repayments of blended principal and interest
7. Uniform gradient repayments of blended principal and interest
8. Geometric gradient repayments of blended principal and interest
9. Principal repayments as percentage of net cash flow with separate interest payments

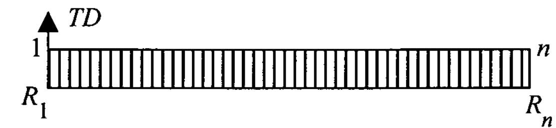
Table 4.8: Repayment methods (Notes: R_k is repayment value, TD is total debt, n is total number of repayments, i is interest rate)

<table>
<tr><th>Repayment Methods and Expressions</th><th>Repayments Profiles</th></tr>
<tr><td>

1. Amortized repayments of debt principal with separate interest payments

$R_k = TD / n$

</td><td>



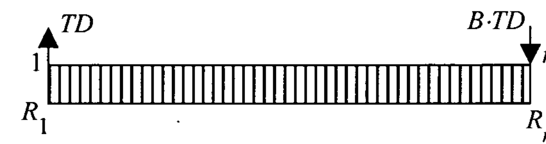
</td></tr>
<tr><td>

2. Amortized repayments of debt principal and balloon with separate interest payments

$R_k = \frac{(1-B)\cdot TD}{n}$ at $k < n$

$R_k = \frac{(1-B)\cdot TD}{n} + B\cdot TD$ at $k = n$

</td><td>



B: balloon percentage

</td></tr>
<tr><td>

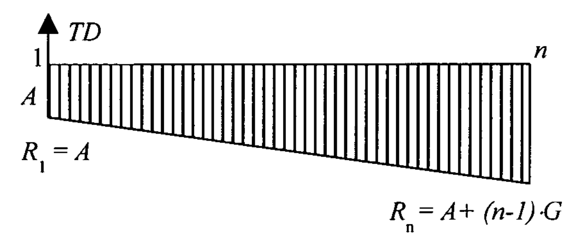
3. Uniform gradient repayments of debt principal with separate interest payments

$R_k = A + (k-1)\cdot G$

where:

$A = \frac{G\cdot n - G\cdot n^2 + 2\cdot TD}{2\cdot n}$ such that $TD = \left[\sum_{k=1}^{n} A + (k-1)\cdot G\right]$

</td><td>



G: uniform gradient

</td></tr>
<tr><td>

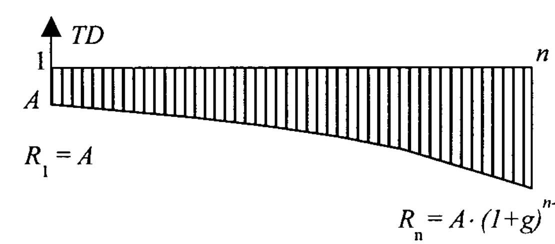
4. Geometric gradient repayments of debt principal with separate interest payments

$R_k = A\cdot(1+g)^{k-1}$

where:

$A = \frac{TD\cdot g}{(1+g)^n - 1}$ such that $TD = \sum_{k=1}^{n} A\cdot(1+g)^{k-1}$

</td><td>



g: geometric gradient

</td></tr>
</table>

Table 4.8: Repayment methods (Notes: R_k is repayment value, TD is total debt, n is total number of repayments, i is interest rate) (continued)

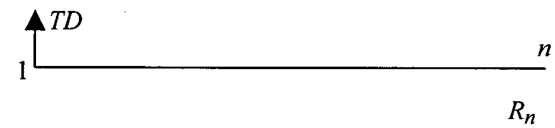
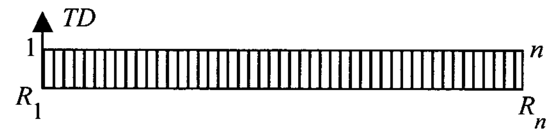
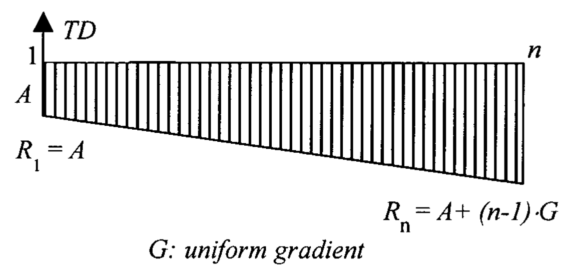
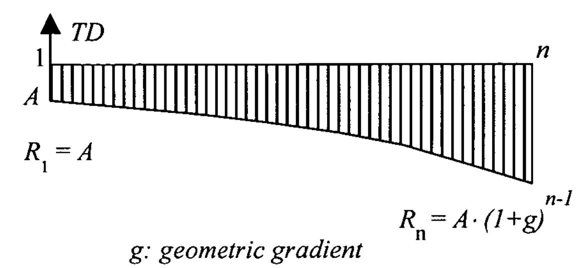
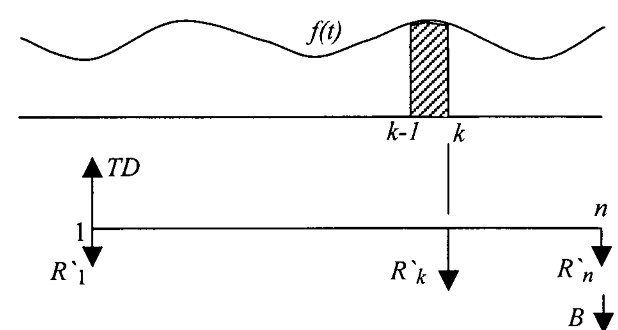
Repayment Methods and Expressions	Repayments Profiles
5. Bullet Repayment of debt principal with separate interest $R_n = TD$	
6. Amortized repayments of blended principal and interest $R_k = A = \dfrac{TD \cdot i \cdot (1+i)^{n-1}}{(1+i)^n - 1}$ such that $TD = \sum_{k=1}^{n} A \cdot (1+i)^{1-k}$	
7. Uniform gradient repayments of blended principal and interest $R_k = A + (k-1) \cdot G$ where: $A = TD / n$ if $i = 0$ and $G = 0$ $A = \dfrac{G \cdot n - G \cdot n^2 + 2 \cdot TD}{2 \cdot n}$ if $i = 0$ and $G \neq 0$ $A = \dfrac{G \cdot n \cdot i^2 + (1+i)^n \cdot TD \cdot i^2 - (1+i)^n \cdot G \cdot i + G \cdot i + G \cdot i \cdot n + G - (1+i)^n \cdot G}{i \cdot (1+i) \cdot \left[(1+i)^n - 1\right]}$ if $i \neq 0$ Such that $TD = \sum_{k=1}^{n} \dfrac{A + (k-1) \cdot G}{(1+i)^{k-1}}$	

Table 4.8: Repayment methods (Notes: R_k is repayment value, *TD* is total debt, *n* is total number of repayments, i is interest rate) (continued)

Repayment Methods and Expressions		**Repayments Profiles**
8. Geometric gradient repayments of blended principal and interest		
$R_k = A + (1+g)^{k-1}$		
where: $A = TD / n$	if $i = g$	
$A = \dfrac{TD \cdot (g-i) \cdot (1+i)^{n-1}}{(1+g)^n - (1+i)^n}$	if $i \neq g$	
Such that $TD = \sum_{k=1}^{n} \dfrac{A \cdot (1+g)^{k-1}}{(1+i)^{k-1}}$		
9. Principal repayments as percentage of net cash flow with separate interest payments		
$R_k = R'_k$	if $k \leq n$	
$R_n = R'_n + B$	if $k = n$	
where: $R'_k = h \cdot \int_{t_{k-1}}^{t_k} f(t)\,dt$	*h is a percentage of net cash flow*	
$B = TD - \sum_{k=1}^{n} R'_k$		

a) Special Repayment Clauses

The net cash flow required by method nine (principal repayments as a percentage of net cash flow with separate interest payments) in Table 4.8 is determined through a link between the FN component and the RV and OM components. The net cash flow at repayment period k is determined as follows using the cash flows of the RV and OM components Eqs. (4.26, 4.33) and the repayment *time unit*, where NC_k is the net cash flow of period k

$$NC_k = [F_{RV}(\mathbf{Rt}_{i,k}, X_{RV}) - F_{RV}(\mathbf{Rt}_{i,k-1}, X_{RV})] - [F_{OM}(\mathbf{Rt}_{i,k}, X_{OM}) - F_{OM}(\mathbf{Rt}_{i,k-1}, X_{OM})] \tag{4.43}$$

The first repayment method (amortized principal with separate interest) has three properties that can be used only with the *general* and *syndicated term loans* and *private placement bonds*. These properties reflect three contractual clauses that are generally found in debt agreements particularly with floating-interest-rate syndicated loans (McDonald 1982; Rhodes 1993; Nevitt and Fabozzi 1995). These clauses are reflected in the generalized model and are described as follows:

1. Optional or Accelerated Prepayments

   An optional prepayment clause provides the borrowers with the right to prepay all or part of the outstanding principal amount of the debt at the end of any interest period or equivalently at the repayment dates of the debt principal (McDonald 1982). Therefore, in debt stream i, if the *net cash flow* as described by Eq. (4.43) at the time of a repayment k exceeds the principal repayment value $\mathbf{Rv}_{i,k}$ (Method 1) and the interest payments made during the period by α, then repay γ of the excess amount (difference between net cash amount and repayment value) along with the value of the required repayment; i.e. the new repayment value would equal to

$$\mathbf{Rv}_{i,k} = \begin{cases} \mathbf{Rv}_{i,k} + \gamma \cdot (NC_k - \mathbf{Rv}_{i,k} - \sum\limits_{z} \mathbf{Iv}_{i,z}) & \text{if } \dfrac{(NC_k - \mathbf{Rv}_{i,k} - \sum\limits_{z} \mathbf{Iv}_{i,z})}{\mathbf{Rv}_{i,k}} > \alpha \\ & z \,\forall\, \mathbf{It}_{i,z} : \mathbf{Rt}_{i,k} < \mathbf{It}_{i,z} \le \mathbf{Rt}_{i,k-1} \\ \mathbf{Rv}_{i,k} & \text{otherwise} \end{cases} \tag{4.44}$$

in which, net cash flow has been adjusted to account for z interest payments between the current principal repayment k and the preceding debt principal repayment $(k\text{-}1)$. When exercising the optional repayment clause, repayments are calculated first as in Table 4.8 (method 1) then adjusted later in accordance with Eq.(4.44). This process is followed for all principal repayments while acknowledging that separate interest payments made between principal repayments (e.g. between k and $k+1$) will be based on the outstanding principal amount that is adjusted for the optional prepayments in the preceding periods.

2. Recapture Clause

McKechnie (1990) explained that under project financing the circumstances might change given either surplus revenues or shortfalls in revenues. Therefore, credit agreements might stipulate repayment terms that require the payment of the greater of an absolute dollar amount or a fixed percentage of the net cash flow after interest of the preceding repayment period. Therefore, under this clause repayments are first calculated as in Table 4.8 (method 1). Then the payment due is computed as follows, where γ is the required percentage to be paid of the net cash flow:

$$\mathbf{Rv}_{i,k} = \begin{cases} \gamma \cdot (NC_k - \mathbf{Rv}_{i,k} - \sum_z \mathbf{Iv}_{i,z}) & \text{if } (NC_k - \mathbf{Rv}_{i,k} - \sum_z \mathbf{Iv}_{i,z}) > \mathbf{Rv}_{i,k} \\ & z \forall \mathbf{Iv}_{i,z} : \mathbf{Rt}_{i,k} < \mathbf{It}_{i,z} \leq \mathbf{Rt}_{i,k-1} \\ \mathbf{Rv}_{i,k} \quad \text{otherwise} & \end{cases} \tag{4.45}$$

As with the optional prepayments, the process for calculating repayment values is followed for all principal repayments acknowledging that separate interest payments due between principal repayments (e.g. between k and $k+1$) are based on the outstanding principal amount adjusted for the captured net cash flow amount in the preceding periods.

3. Flexible Maturity Clause

Although rarely used or referred to in the literature, a flexible maturity clause provides borrowers with an opportunity to mitigate the risks of adverse fluctuations in project revenues that affect the ability to repay debt on time. With this clause, if a debt is not retired completely by the target maturity (debt term), it will continue to be outstanding and accrue interest until it is paid during or before an extended maturity. This has been implemented in the generalized model as follows. Repayments of the debt principal are calculated based on the target maturity. If the net cash flow at a debt principal repayment date is below the required repayment value (Table 4.8 method 1), a borrower will pay γ of the available net cash flow at the repayment date and continue with such repayments until the end of target maturity. Then, on the same basis the borrower will continue with the repayments of principal and interest until the loan is retired. During both the target and final maturity, the outstanding principal will be adjusted with the possibility of a balloon payment at the final maturity if the debt is not retired completely during the extended maturity. This is described in equation form as follows:

$$\mathbf{Rv}_{i,k} = \begin{cases} \mathbf{Rv}_{i,k} & \text{if } (NC_k - \mathbf{Rv}_{i,k} - \sum_z \mathbf{Iv}_{i,z}) > \mathbf{Rv}_{i,k} \\ \gamma \cdot (NC_k - \mathbf{Rv}_{i,k} - \sum_z \mathbf{Iv}_{i,z}) & \text{otherwise} \\ & z \,\forall\, \mathbf{Iv}_{i,z} : \mathbf{Rt}_{i,k} < \mathbf{It}_{i,z} \le \mathbf{Rt}_{i,k-1} \end{cases} \qquad (4.46)$$

The characteristics of the flexible amortization schedule are useful particularly when using an alternative PPP delivery system. For example, the Mexico City-Cuernavaca toll road was financed in 1994 by US$ 265 million, peso-exchange-rate-linked bonds raised in the US Rule 144A private placement market. The bonds had a target amortization of 5.5 years (January 2000). However, if the net cash flow was insufficient due to revenue and/or exchange rate fluctuations, the bonds would continue to be outstanding until the final maturity of 7 years (July 2001) (Euroweek 1994; Project & Trade Finance 1995).

b) Repayment Methods Summary

The nine repayment methods described in Table 4.8 are used in combination with the five debt instruments in the generalized model. Feasible combinations are as follows:

1. General term loan: Methods 1-9 (fixed interest rate); Methods 1-5, 9 (floating rate)
2. Syndicated term loan: Methods 1-5, 9
3. General bond: Methods 1-5
4. Private placement bond: Methods 1-9
5. Floating rate note: Methods 1-5

The matching of repayment methods to debt instruments is based on type of interest rate where blended principal and interest cannot work with floating interest rate instruments. The system, Evaluator, implements the foregoing matching by default as shown in Figure 4.39. As shown in the repayment frame of the window in Figure 4.39, an amortized-principal-repayment with separate interest is selected with semi-annual repayments and *flexible maturity clause*. The γ in Eq. (4.42) is the "x %, dec." in the window. The window shows the required final maturity "New Term" that has 14 year which is greater than the debt target term "Debt Term" of 12 years.

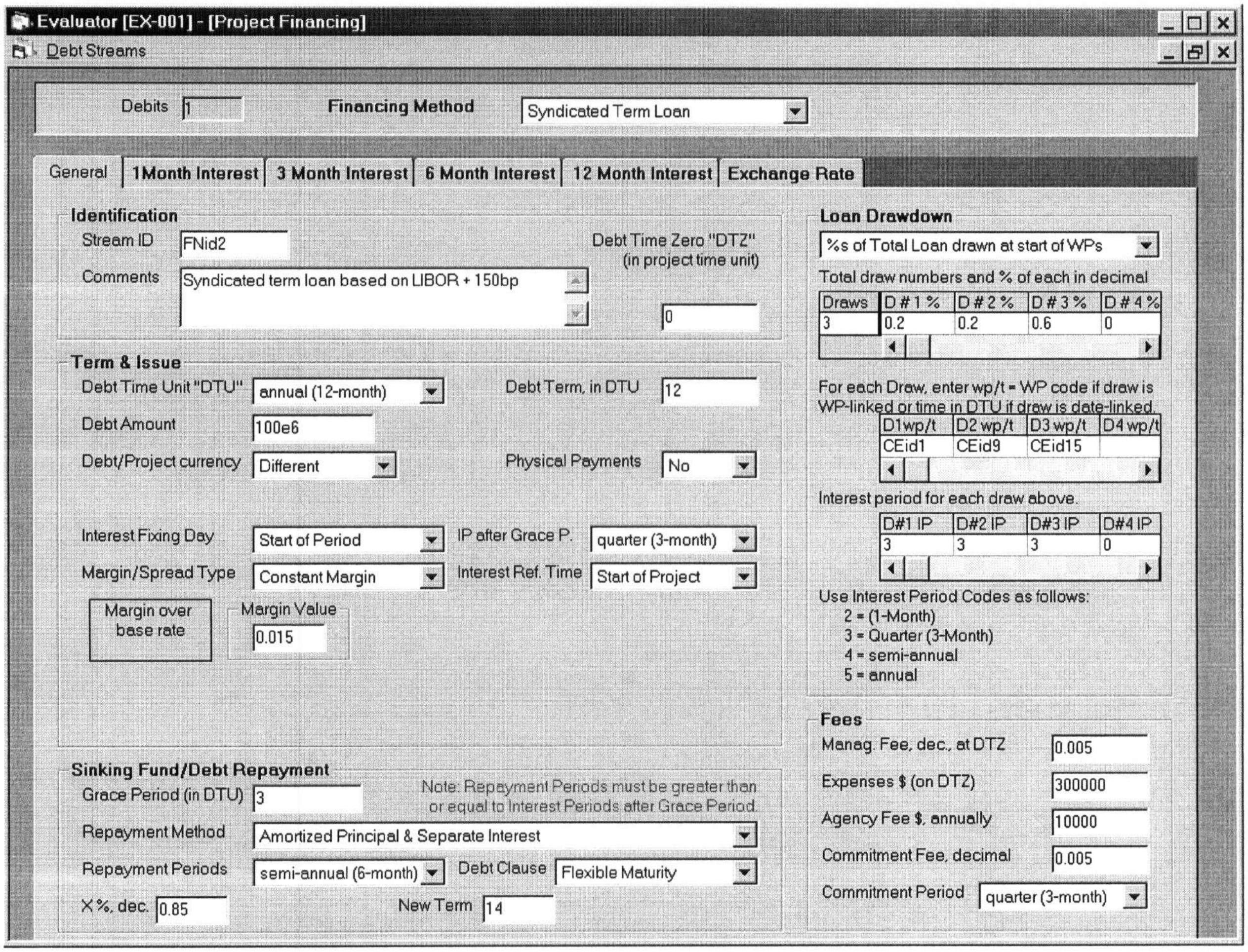


Figure 4.39: Amortized repayment methods with flexible maturity clause

4.6.3.3 Debt Interest Rate Methods

Interest rate determination represents one part in pricing a debt; the other part is administrative fees. Both parts complement each other in order to determine the yield required for the lenders in a transaction. The credit risk assessment process discussed earlier determines the interest rate a borrower will pay in a transaction which can be either fixed or floating rate of interest. Interest payments are usually paid during the grace period on the amounts borrowed. However, interest can be capitalized to the end of the grace period such that the future worth of the debt becomes the basis for determining the flows during the repayment period (Table 4.8). The following describes how interest *rates*, *payments* and *dates* are calculated.

a) Fixed Interest Rate

A fixed rate of interest is the rate established by lenders at the start/signing of a debt or credit agreement and remains unchanged during the term of debt. Fixed interest rates are usually quoted as nominal interest rate per year. Additional attributes include a compounding period and payable period such that an effective interest rate per payment period can be determined.

<u>Fixed Interest Rate For General and Private Placement Bonds</u>

Interest for *general bonds* and *private placement bonds* is usually paid separately during a bond term and with a fixed rate called the *coupon rate*. Most such bonds are payable semi-annually. For example, "Canada 13 3/4 of 1994" is a Government of Canada bond that pays an annual 13 3/4 % coupon rate on the face value of the bond ($1000) semi annually, i.e. $68.75 every half year (Brealey et al. 1992).

In the generalized model, interest payments to be made during and after the *grace period* of a bond are based on interest rate int_i calculated based on a coupon annual nominal rate r_i and a payable period K_i selected as any of one-month ($K_i = 12$), three-month ($K_i = 4$), semi-annual ($K_i = 2$), or annual ($K_i = 1$), where:

$$int_i = r_i / K_i \tag{4.47}$$

Consequently, interest payments $\mathbf{Iv}_{i,z}$ before and after the grace period can be calculated based on interest rate int_i . However, interest payment calculations before and after the bond grace period are different unless the bond is repaid in a bullet payment (single repayment at end of term). Before the grace period, interest payments $\mathbf{Iv}_{i,z}$ depend on the single tranche $\mathbf{Tt}_{i,j}$ ($j = 1$) representing the total debt:

$$\mathbf{Iv}_{i,z} = \mathbf{Tt}_{i,j} \cdot int_i \tag{4.48}$$

The number of interest payments $Inum_i$ and interest payment dates $\mathbf{It}_{i,z}$ before the grace period are determined using the bond's single tranche drawdown time $\mathbf{Tt}_{i,j}$ (where $j = 1$) and grace period $\overline{GP}_i$ (converted to GTU) and the payable period $\overline{K}_i$ (converted to GTU),

$$Inum_i = (\overline{GP}_i - \mathbf{Tt}_{i,j}) / \overline{K}_i \tag{4.49}$$

$$\mathbf{It}_{i,z} = \mathbf{Tt}_{i,j} + (z) \cdot \overline{K}_i \tag{4.50}$$

In a similar way, interest payments $\mathbf{Iv}_{i,z}$ are determined after the grace period, however, using the outstanding amount of the debt principal $DO_{i,k}$ as the base of calculations. Debt repayment dates $\mathbf{Rt}_{i,k}$ and outstanding debt $DO_{i,k}$ are calculated at repayment dates $\mathbf{Rt}_{i,k}$ (using repayment time interval). Interest payments $\mathbf{Iv}_{i,z}$, however, are calculated at interest payment dates $\mathbf{It}_{i,z}$ (using payable time period). Generally, debt agreements require that $\mathbf{It}_{i,z}$ must be made less than or equal to $\mathbf{Rt}_{i,k}$, i.e. made within the interval between two successive repayments (k and $k+1$), see

Figure 4.40. Therefore,

$$\mathbf{Iv}_{i,z} = DO_{i,k} \cdot int_i \Big|_{\substack{\mathbf{Rt}_{i,k} < \mathbf{It}_{i,z} \\ \mathbf{Rt}_{i,k} < \mathbf{It}_{i,z} \leq \mathbf{Rt}_{i,k+1}}} \quad \text{k where } \mathbf{Rt}_{i,k} \text{ is maximum repayment time before } \mathbf{It}_{i,z} \tag{4.51}$$

where

$$DO_{i,k} = DO_{i,k-1} - \mathbf{Rv}_{i,k} \Big|_{DO_{i,1} = TD_i - \mathbf{Rv}_{i,1}} \tag{4.52}$$

After the grace period, interest payment dates $\mathbf{It}_{i,z}$ are determined using *debt term DT*, grace period *GP*, $\overline{GP}_i$ (converted to GTU), interest payable period $\overline{K}_i$ (converted to GTU), and $\overline{\overline{K}}_i$ (converted to debt time unit):

$$Inum_i = (DT_i - GP_i) / \overline{\overline{K}}_i \tag{4.53}$$

$$\mathbf{It}_{i,z} = ds_i + \overline{GP}_i + (z) \cdot \overline{K}_i \tag{4.54}$$

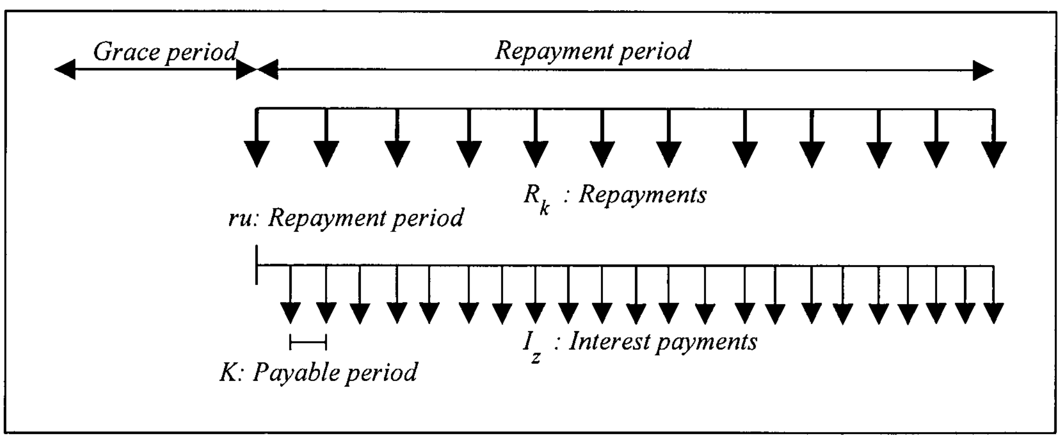


Figure 4.40: Bond interest/coupon rates during repayment period

While interest is generally paid separately from principal payments in bond instruments, on rare occasions private placement bonds may experience borrowers who prefer to blended payments (principal plus interest) after the grace period. This is referred to as *annuity* or *mortgage* style arrangement with repayments made at a predefined time interval. Repayments in this case are calculated with effective interest rate as in Eq. (4.55).

Interest Rate For Fixed-Rate General Term Loans

The effective interest rate per payment period int_i is calculated based on the nominal interest rate per year r_i, the number of payment periods (payable periods) per year K_i, and number of interest periods per payment period c_i as follows (Park and Sharp-Bette 1990):

$$int_i = \left(1 + \frac{r_i}{c_i \cdot K_i}\right)^{c_i} - 1 \tag{4.55}$$

For example, if r is 10% per annum compounded quarterly (i.e. 4 times a year) and payable semi-annually ($K = 2$), then $c = 4/2 = 2$, and the effective semi-annual interest rate is 5.0625%.

The above effective interest rate equation is a general one. Depending on whether or not interest is capitalized and which repayment method is used, the effective rate is adjusted during the term to reflect the definitions of payment periods K. This is explained as follows.

For fixed-rate *general term loans* with separate interest payments during the debt grace period GP (i.e. interest is not capitalized) and after the grace period, separate principal and interest repayment methods (methods 1-5 and 9 in Table 4.8), the effective interest rate is calculated using Eq. (4.55) with no adjustments.

If interest is capitalized during the grace period GP, it will be accumulated with the principal value (see Eq. 4.40) using the effective interest rate calculated using Eq. (4.55), but with K_i reflecting GTU since the time length to the end of GP of each tranche is defined in GTU. After the grace period, and using blended principal and interest repayment methods, the effective interest rate from Eq. 4.55 is used for repayment calculations, but with K_i reflecting the repayment time interval ru as per Eq. 4.41. In the above cases nominal interest rate per year r and compounding period are the definition of K changes.

For the case of repaying private placement bonds through blended interest and principal payments after the grace period *GP*, the effective interest rate from Eq. 4.55 is used instead of Eq. (4.47). However, since bonds uses coupon annual rate and payable periods, the compounding period is considered to be annual since the nominal interest rate r is per year. And, the payment period K in Eq. 4.55 reflects in this case the repayment intervals. (Note that interest before *GP* is calculated for private placement bonds based on Eq. 4.47).

Following the determination of the effective interest rate in Eq. (4.55), calculations for interest payments $\mathbf{Iv}_{i,z}$ and interest times $\mathbf{It}_{i,z}$ for the fixed-rate *general term loan* are made in accordance with equations (4.48-4.54) for before and after the grace period. However, two points need to be emphasized. First, unlike bonds, several tranches are encountered for a single loan. Calculations for each tranche proceed as above. However, a final step involves aggregating all the tranches such that the $\mathbf{Iv}_{i,z}$ and $\mathbf{It}_{i,z}$ vectors include all the interest times and values obtained from each single tranche in the loan. When interest payment dates of different tranches are similar, their interest payment values are aggregated together to avoid repeated dates. Second, unlike bonds, the last interest payment before the expiration of *GP* may be made with a partial length of the payable interest period (see tranche 2 in Figure 4.41 for an example). Therefore, the final aggregated interest payment $\mathbf{Iv}_{i,p}$ is computed as, where p is the final interest payment:

$$\mathbf{Iv}_{i,p} = \sum_{j=1}^{m} \mathbf{Tv}_{i,j} \cdot [(1 + int_i)^{tl_j} - 1] \Big|_{tl_j = \overline{GP}_i - \mathbf{It}_{i,j-1}} \tag{4.56}$$

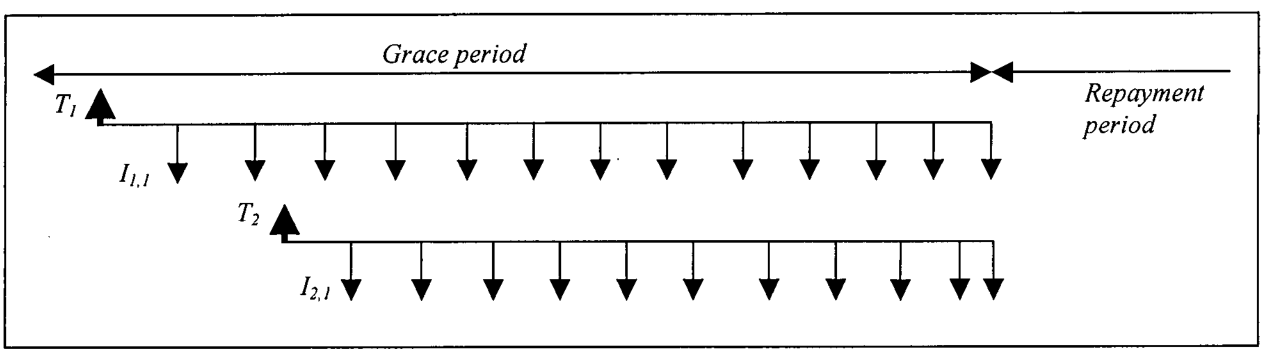


Figure 4.41: Interest payments on loan Tranches during grace period

b) Floating Interest Rate

A floating interest rate as defined by Nevitt and Fabozzi (1995) is: "an interest rate which fluctuates during the term of a loan and which is adjusted upwards or downwards during the term of a loan in accordance with some index of short-term rates." The short-term rate represents the main part of the floating interest rate; the other part is called margin or spread which is added to the short term rate to form the floating rate.

Short-term Rate

As mentioned earlier banks borrow short and lend long. The short-term rate represents the quoted rate in the inter-bank market or a country's benchmark rate such as US Treasury Bills. The inter-bank market is a market through which banks place deposits with each other at rates called bid and/or offered rates. These deposits are usually of short term ranging from one to twelve months.

For example, the international inter-bank market is commonly referred to as the Euro inter-bank deposits market and is denominated in Eurocurrency deposits. Bee (1981) defines the Eurocurrency market as "a market in deposits among banks located outside of the country in whose currency the deposits are denominated. For example, a Eurodollar is most generally defined as a U.S. dollar account held in a bank (including a branch of a United States bank) located outside of the United States." Lenders of *syndicated loans* usually fund their loan participation during the life of the loan by borrowing in the inter-bank deposit market. Deposit maturities are generally one to twelve months and commonly 3 and 6 months. Therefore, the *floating interest rate* on syndicated loans is usually determined as a margin/spread over the prevailing *short term rate* in the inter-bank market as quoted from time to time. Quotations for short-term rates can be made at the start or expiration of the interest periods during the life of the floating rate facility.

The most commonly cited short-term rate is Libor which is the London Inter-bank Offered Rate. Floating rate debts (e.g. *syndicated loans* and *floating rate notes*) are usually based on Libor, U.S. prime rate, or the local inter-bank market rates such as Sibor (Singapore Inter-bank Offered Rate) and Pibor in Paris. Figure 4.42 illustrates the variation in Libor rates on six-month deposits for four main currencies in the inter-bank market between 1980 and 1995. While Libor-based instruments enjoy its lower rate compared to long-term fixed rate instruments, the variation noted in Figure 4.42 in and between currencies can be dramatic on a project economy. Table 4.9 shows the average annual Libor rate for different deposit maturities ranging from overnight to one year.

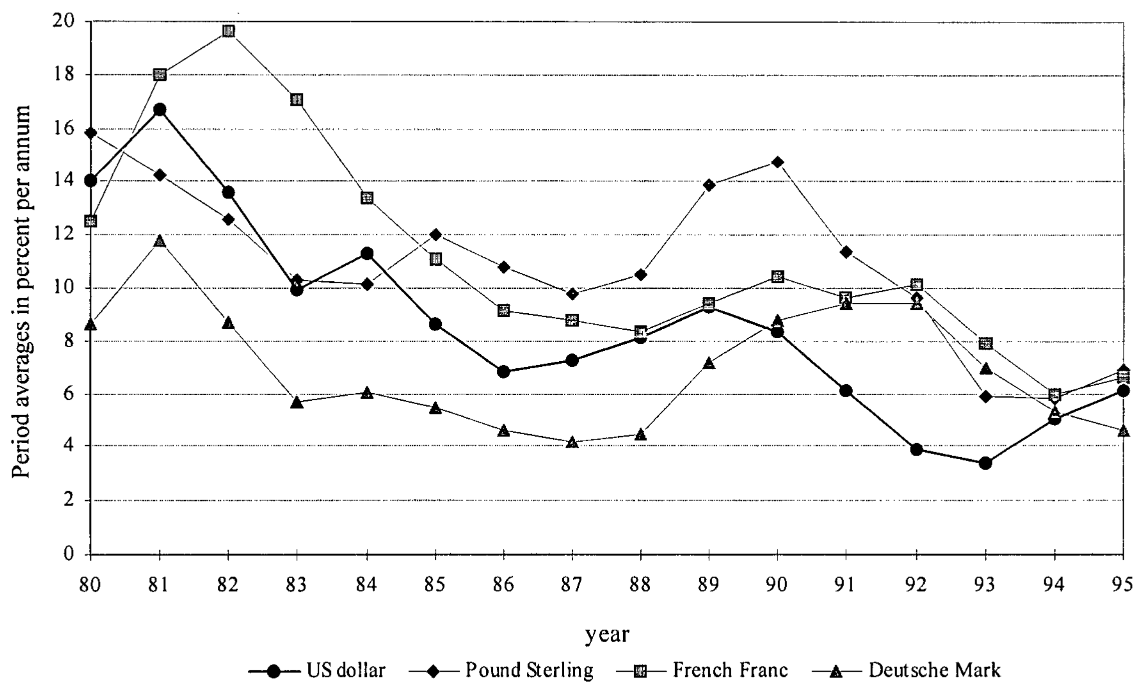


Figure 4.42: London Inter-bank Offered Rates on six-month deposits (Pound Sterling rates relate to the Paris market, Pibor), (compiled from International Monetary Fund 1997)

Table 4.9: London Inter-bank Offered Rates on US dollar deposits, period averages in percent per annum, (International Monetary Fund 1997)

year	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95
Overnight	16.56	19.70	12.61	9.32	10.38	8.04	6.97	6.63	7.56	9.21	8.13	5.78	3.60	3.05	4.24	5.90
Seven-day	13.48	16.84	12.77	9.47	10.53	8.20	6.95	6.82	7.68	9.26	8.20	5.87	3.66	3.08	4.31	5.93
One-month	13.96	16.79	12.95	9.53	10.64	8.23	6.94	6.99	7.81	9.24	8.29	5.90	3.72	3.16	4.46	5.97
Three-month	14.19	16.87	13.29	9.72	10.94	8.40	6.86	7.18	7.98	9.28	8.31	5.99	3.86	3.29	4.74	6.04
Six- month	14.03	16.72	13.60	9.93	11.29	8.64	6.85	7.30	8.13	8.27	8.35	6.08	3.90	3.41	5.07	6.10
One year	13.44	16.13	13.69	10.18	11.82	9.11	6.95	7.61	8.41	9.31	8.45	6.29	4.20	3.64	5.59	6.24

Margin/Spread

The spread or margin is a fixed percentage added to the floating short-term rate, e.g. Libor or U.S. prime, in order to form the interest rate the borrower pays in a floating rate facility. Generally, spread percentage is in the range of 1/2% to 2% over the funding basis (Fisher 1979). Table 4.10 shows the spread percentage expressed as basis points (1% = 100 basis points bp) compiled by OECD (1997) for international syndicated loans. Several factors contribute to the determination of a margin value including credit assessment of the borrower, amount to be borrowed, term and repayment method (McDonald 1982; Rhodes et al. 1993).

Table 4.10: Spreads (in basis points) on international bank loans, OECD (1997)

	1991	1992	1993	1994	1995	1996
OECD area	80	85	78	59	43	51
Non-OECD	78	87	103	113	117	99
General average	79	85	81	64	50	56

Spread percentage is usually fixed over the life of the loan facility. However, a spread may split during the loan life. Lenders for the Channel tunnel, at the time of arranging the syndicate were able to provide the loan facility starting at a margin of 1.25% for the main facility (FFR 50 billion) and 1.75% for the standby part of the facility (20% of the main facility) (Andrews 1987, ENR 1987). The Bank of America on its syndicated loan for the Dartford bridge in the UK required a margin in the range of 3/4% to 1.25% over prime (Levy 1996). The Elcogas power project in Spain, a $772 million 335 megawatt integrated gasification combined cycle power plant, was financed in 1994 by a syndicated term loan of $640 million based on Mibor (Madrid Inter-bank Offered Rate). The floating rate was made at Mibor plus 80 bp for the pre-completion period, 100 bp for the first 9 months following completion, 125bp for five years following the 9-month period, and then 140bp (Project & Trade Finance 1994).

Floating Interest Calculations

A debt facility may have one or more tranches $\mathbf{Tv}_{i,j}$ drawn at $\mathbf{Tt}_{i,j}$ using any of the methods explained before (Eq. 4.36, 4.38). Under a floating rate facility, interest payments $\mathbf{Iv}_{i,z}$ to be made at $\mathbf{It}_{i,z}$ for each tranche of debt $\mathbf{Tv}_{i,j}$ are calculated based on the short-term interest rate (e.g. Libor) plus the spread/margin percentage.

For a *syndicated term loan,* each tranche can rollover at a different interest period other than the preceding interest period used in the tranche. For example, a borrower may request that a debt tranche be advanced starting in 3-month interest period (i.e. an interest payment will be made at the end of the 3-month period based on 3-month interest rate) and request at the end of the interest period to use either the 3-month interest period or to change it to 6-month period (i.e. the following interest payment will be made after 6 months using a 6-month interest rate) or to other interest periods as allowed in the agreement (McDonald 1982; Rhodes et al. 1993). With *floating rate notes*, only one interest period is selected for the whole term of debt, and the number of coupons is prepared based on the selected interest period, i.e. the number of 3-month rate coupons will double that of 6-month rate coupons (Ugeux 1981).

In the financing component of the generalized model, following the above practice, each tranche of a floating rate debt can have its own interest rate selected as 1-month, 3-month, 6-month, or 12-month. To reduce the computation burden, however, under the a *syndicated term loan,* roll over of a single debt tranche to different interest periods is not permitted, i.e. an interest rate for a tranche will be used throughout the tranche calculations. The short-term rate of the floating interest rate is represented in the generalized model by the *rate functions*, which means that forecasting the rate over the time of the debt can be made according to any of the rate functions in Table 4.1. Short-term rates can use the *local/global* reference where they can be defined using either LTU or GTU.

The spread/margin for a floating rate facility can be chosen from between three methods:

1. Constant margin throughout the debt term.
2. Split with a maximum of five margins linked to specific work package in the CE component.
3. Split with a maximum of five margins linked to specific dates using the debt time unit.

Therefore, the floating interest rate used with tranche j of debt i before the grace period can be defined as follows, where *sint* is the annual rate of the short-term rate (e.g. annual percentage value of interest of 3-month Libor) used with tranche j and *mr* is the margin value obtained at time $t``$ using any of the three methods above:

$$int(t``,\mathbf{X})_{i,j} = sint(t``)_{i,j} + mr(t``)_i \qquad (4.57)$$

Calculations for the interest payment dates $\mathbf{It}_{i,z}$ of a single tranche follows that of Eqs. (4.49) before the expiration of the debt grace period, but using the short-term interest period instead of the payable period K_i in these equations.

Calculation of interest payments $\mathbf{Iv}_{i,z}$ at the end of each interest period z (i.e. at interest payment time $\mathbf{It}_{i,z}$) is based on four elements: (1) the annual rate of the floating interest rate (Eq. 4.57), (2) the principal amount of tranche j, $\mathbf{Tv}_{i,j}$, (3) the number of calendar days elapsed in the relevant interest period (e.g. 91 days in 3-month floating rate), and (4) the number of days in year[4] (Howcroft and Solomon 1985, Wiseman 1990; McDonald 1982; Rhodes et al. 1993):

$$\mathbf{Iv}_{i,z} = \frac{\mathbf{Tv}_{i,j} \cdot int(t``,\mathbf{X})_{i,j} \cdot \overline{\overline{d}}}{360} \Bigg| \begin{array}{ll} t``=\mathbf{It}_{i,z} & \text{if interest quotation at end of interest period} \\ t``=\mathbf{It}_{i,z-1} & \text{if interest quotation at start of interest period} \\ \overline{\overline{d}}=\mathbf{It}_{i,z}-\mathbf{It}_{i,z-1} & \text{elapsed time converted to days} \end{array} \qquad (4.58)$$

[4] The Euro-market adopts the 360-day calendar year. The US market adopts the 365-day year (McDonald 1982).

Under *syndicated term loans* that involve several tranches, consolidation of borrowings is made at the expiration of the grace period such that all borrowings after the grace period roll over simultaneously (McDonald 1982; Rhodes et al. 1993). Therefore, in the generalized model aggregation of interest payments is made as explained before and the final interest payment $\mathbf{Iv}_{i,p}$ at the expiration of the grace period is calculated as per Eq. (4.56) where

$$\mathbf{Iv}_{i,p} = \sum_{j=1}^{m} \frac{\mathbf{Tv}_{i,j} \cdot int(t``,X)_{i,j} \cdot \overline{\overline{d}}_j}{360} \qquad (4.59)$$

$$\left| \begin{array}{ll} t``=\mathbf{It}_{i,p}=\overline{GP}_i & \text{if interest quotation at end of interest period} \\ t``=\mathbf{It}_{i,p-1} & \text{if interest quotation at start of interest period} \\ \overline{\overline{d}}_j=\overline{GP}_i-\mathbf{It}_{i,p-1} & \text{elapsed time converted ot days} \end{array} \right.$$

After the grace period, calculation for the interest payment dates $\mathbf{It}_{i,z}$ follows that of Eq. (4.54) but using the short-term interest period instead of the payable period K_i in the equations. Following the consolidation of tranches at the expiration of the grace period, debt is rolled over using one interest period $int(t``,X)_i$ without links to any tranche. Interest payments are then calculated based on the amount of principal debt outstanding $DO_{i,k}$ calculated at debt repayment dates $\mathbf{Rt}_{i,k}$; outstanding debt is calculated as in Eq. (4.52). Repayments must be made at the end of any interest period, i.e. not within the duration of an interest period. This means that the repayment period must be greater then or equal to the selected interest period. Interest payments $\mathbf{Iv}_{i,z}$ can be calculated as follows:

$$\mathbf{Iv}_{i,z} = \frac{DO_{i,k} \cdot int(t``,X)_i \cdot \overline{\overline{d}}}{360} \qquad (4.60)$$

$$\left| \begin{array}{ll} \mathbf{Rt}_{i,k}<\mathbf{It}_{i,z} & k \text{ where } \mathbf{Rt}_{i,k} \text{ is maximum repayment time before } \mathbf{It}_{i,z} \\ t``=\mathbf{It}_{i,z} & \text{if interest quotation at end of interest period} \\ t``=\mathbf{It}_{i,z-1} & \text{if interest quotation at start of interest period} \\ \overline{\overline{d}}=\mathbf{It}_{i,z}-\mathbf{It}_{i,z-1} & \text{elapsed time converted to days} \end{array} \right.$$

The implementation of floating interest rates is illustrated in Figure 4.39. As shown on the "Loan drawdown" frame, three loan tranches are defined as percentages of the total loan, and tranches are to be advanced at the start of the relevant work packages. Then, interest periods are defined through codes for each loan tranche. In the "Term & Issue" frame, the interest quotation as to the start or end of interest period is selected under "Interest Fixing Day" and on the same line the interest period to be used after the grace period is selected under "IP after Grace P." along with the *local/global* reference in the next line under "Interest Ref. Time". Using these interest periods and the reference time, short-term interest rate *shape functions* can be defined through the *Tabs* at the top of the window. The margin/spread is then selected, with the *constant margin* being selected in this example. Both the short-term rates and the margin are used in determining the floating interest rate at any time during the interest and repayment calculations.

4.6.3.4 Debt Fees

Fees in a debt transaction represent the second critical part for the pricing of a credit facility. Fees together with the interest generate the required yield by the lenders on their capital. Fees come in five types, serve different purposes, and are charged at different times during the term of a debt. These five types are described and represented in the generalized models as follows:

1. Management Fee

   The Management fee represents the *arrangement* fee paid by a borrower to the arranger (bank) for a term loan facility, e.g. *syndicated loans*, or the *underwriting/commission/ agent* fee with respect to bond issues. This fee is payable on the signing date or within 30 days of signing the agreement. In the generalized model it is paid on signing date, i.e. *debt-start* date.

The management fee is payable as a percentage of the total debt amount and it varies with the type of debt and debt amount. With term loans, the fee is in the range of 0.25% to 2% of the loan amount (Wiseman 1990; Fisher 1979). With private placement bonds, this fee is in the range of 1/2% to 7/8% for issues of $5 million to $25 million, 3/4% to 1% for issues of $25 million to $50 million, and 3/8% to 3/4% for issues over $50 million (Nevitt and Fabozzi 1995). Roger (1990) mentioned that the initial commission paid by Eurotunnel or the bank arranger of the Channel Tunnel was 9/8% of the total amount of credits.

Management fee, MF_i, is implemented in the generalized model as shown in Figure 4.39 under "Commission" in the "fees" frame. Where total debt is TD_i as obtained by Eq. (4.39) and the management fee percentage is fm_i, then management fee , MF_i for debt i at debt start date ds is computed as follows:

$$MF_i = TD_i \cdot fm_i \tag{4.61}$$

2. Expenses

   Out-of-pocket expenses represent legal expenses, and accounting, printing, advertisement, and administrative costs used in the preparation and execution of the debt facility. These fees are payable at the signing of debt as a total dollar amount. Gelbard (1996) explained that for a $100 million offer under the *private placement 144A Rule* these expenses would be in the range of US$300,000 to US$450,000; for regular *private placement*, the expenses would be in the range of US$85,000 to US$140,000; and for US public offers the expenses would be in the range of US$380,000 to US$520,000.

3. Agency Fee

The agency fee represents the annual fee paid to the agent bank for the operational, maintenance, and surveillance work, e.g. quotation of Libor rates, distribution of repayments to the lenders, and arranging debt tranches, during the term of the debt (Rhodes et al. 1993; Gelbard 1996). This fee is payable at signing or within 30 days of signing the credit agreement and is payable on annual bases in the form of a lump sum payment. Figure 4.39 shows the implementation of the agency fee in the support system.

4. Commitment Fee

A commitment fee is usually charged with term loans to compensate the lenders of their obligation or commitment to lend money to a borrower for a predetermined period of time. It is usually charged as an annual percentage 1/4% to 1/2%, on the undrawn portion of the credit facility and is payable periodically, e.g. semi-annually, in arrears based on the actual number of days elapsed on the undrawn amounts (Willingham 1990; McDonald 1982; Bee 1980; Fisher 1979).

Rhodes (1993) explained that, in general, most market practitioners would set the level of commitment fee at 50% of the margin/spread required in the transaction. It may be waived if the draw down period is very short, e.g. 30 to 60 days. Roger (1990) mentioned a 1/8% commitment fee for the Channel Tunnel.

Figure 4.43 depicts the cash flow stream of commitment fee payments. Where CF_i is the commitment fee charged at a annual percentage fe_i, $\overline{cu}$ is the commitment fee payable period (converted to GTU), ds is the debt start time, and TD_i is the total debt, then the number of fee payments, fee payment dates and fee values may be expressed as:

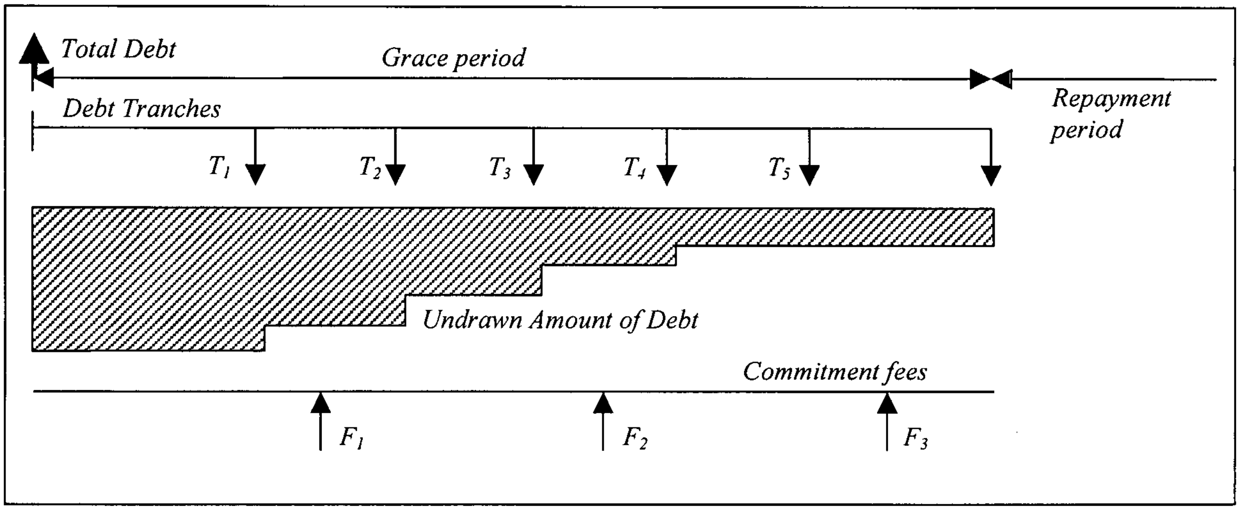


Figure 4.43: Commitment fee payments

$$fnum_i = (\overline{GP}_i - ds_i)/\overline{cu}_i \tag{4.62}$$

$$\mathbf{Ft}_{i,w} = ds_i + w \cdot \overline{cu}_i \tag{4.63}$$

$$\mathbf{Fv}_{i,w} = \sum_j \left(\frac{\mathbf{Tv}_{i,j} \cdot fe_i \cdot \overline{d1}_{i,j}}{360} \right) + \left(TD_i - \sum_j \mathbf{Tv}_{i,j} \right) \cdot \frac{fe_i \cdot \overline{d2}_{i,j}}{360} \Bigg| \begin{array}{lll} j: & \text{where } \mathbf{Tt}_{i,j} \text{ is maximum tranche} & \\ & \text{time before } \mathbf{Ft}_{i,w} & \\ \overline{d1}_{i,j} = \mathbf{Tv}_{i,j} - ds_i & & \text{in days} \\ \overline{d2}_{i,j} = \mathbf{Ft}_{i,w} - ds_i & & \text{in days} \end{array} \tag{4.64}$$

5. Forward Commitment Fee

   As with the commitment fees on term loans, private placement bonds carry a forward commitment fee as compensation to the lenders when the payout or drawdown of funds is several months in the future. Unlike the commitment fees described above, the forward commitment fee is paid upon signing of the agreement as a percentage in the range between 0.25% to 1% of the face value of the debt (Lund et al. 1984; Shapiro and Wolf 1972).

4.6.3.5 Debt Currency and Exchange Rates

Treatment of the exchange rate between debt currency and project currency, and the exchange rate used with syndicated loan *physical payments* are described here.

The debt currency can be different than the project currency. Each debt stream in the generalized model can have its own currency that is different than the currencies of the other debt streams and different than the project currency. As shown in Figure 4.39 a selection can be made as to whether or not to make the debt currency the same as the project currency. If the currencies are different, the "Exchange rate" tab becomes visible as shown in the figure. An exchange rate can be represented by any of the *rate functions* (Table 4.1) thereby allowing it to change over the term of the debt. Exchange rates can have either of a *local* or a *global* reference.

In the generalized model, an exchange rate is defined as the rate of exchange $Ex(t``)_i$ of loan currency to project currency satisfying the conversion

$$Project\ currency = Loan\ currency_i / Ex(t``)_i \tag{4.65}$$

For example, \$US 1 loan currency would equal to \$Can 1.49253 project currency with a US to Canadian dollar exchange rate Ex of 0.67.

In the generalized model two steps are involved for treating a project with exchange rates. First, if a debt is defined in a currency different than the project currency, then all tranches advanced through any of the three-drawdown methods (Sec. 4.6.3.1) will be in the loan currency. For example, if the first method is used (i.e. where debt is a percentage of capital costs), debt tranches values are obtained from the capital expenditure component in the project currency then converted to loan currency. This step is necessary in order to match all calculations of debt tranches, repayments, interest, and fees to the debt currency. For example, if the floating interest rate is defined as Libor on Euro-sterling, then *syndicated loan* or *floating rate note* must be expressed in *Pound Sterling* such that debt tranches, outstanding debt and interest payment would be all for the same currency. The second step is performed after the debt calculations are made and involves converting all the resulting values for debt tranches, repayments, interest, and fees to the project currency such that cash flow and discounted value calculations are performed in the project currency.

The second property described in this section deals with an exchange rate $Exc(t``)_i$ used for the *physical payments* for *syndicated term loans*. It is defined as the exchange rate from loan currency to US$ currency, satisfying the conversion:

$$US\$ \ currency = Loan \ currency_i / Exc(t``)_i \tag{4.66}$$

As explained by McDonald (1980): "Since most multi-currency loan agreements are essentially based upon an ultimate agreement to lend dollars (but with the option for the borrower to instruct the banks to convert the dollar into another currency), the amount of optional currencies must always be calculated by reference to the exchange rate prevailing between the base currency and the currency chosen at the beginning of the interest rate... A calculation is then made at the beginning of each interest payment to ensure that if the spot rate between the base and optional currency fluctuates from one interest period to the next, only the proper dollar equivalent will be outstanding in each subsequent interest period." Syndicated term loan agreements usually include a clause for maintenance of loan advances. Rhodes et al. (1993) explained that an mount will be paid at the end of each interest period either by the lender to the borrower or vise versa depending on the movement of the exchange rate between the base dollar value and the optional loan currency; such an amount is waived from both sides if it is within a agreed upon percentage.

The generalized model implements this property as shown in Figure 4.39. In the "Term & Issue" frame a choice can be made as to whether or not to allow physical payments in debt calculation. If required a "Physical Pay Exchange" tab becomes visible such that the exchange rate is defined as a *shape function* (Table 4.1) along with the waiver percentage. For interest calculations before the grace period, and where interest is not capitalized, tranche values $\mathbf{Tv}_{i,j}$ are checked at each interest payment date $\mathbf{It}_{i,z}$ using the value of the exchange rate at the start and end of the interest period. The same process is carried out after the grace period. However, the check is now made on the outstanding debt $DO_{i,k}$ at each interest payment date $\mathbf{It}_{i,z}$. In both cases, if the value changes by more than the defined percentage, a physical payment $PP_{i,z}$ is be computed according to the movement of the exchange rate before the grace period as:

$$PP_{i,z} = \frac{\mathbf{Tv}_{i,j}}{Exc(t1``)_i} \cdot Exc(t2``)_i - \mathbf{Tv}_{i,j} \Bigg|_{\substack{j & \textit{for each tranche } j \\ t2``=\mathbf{It}_{i,z} & \textit{time at end of interest payment } \mathbf{It}_{i,z} \\ t1``=\mathbf{It}_{i,z-1} & \textit{time at start of interest payment}}} \tag{4.67}$$

and after the grace period

$$PP_{i,z} = \frac{DO_{i,k}}{Exc(t1``)_i} \cdot Exc(t2``)_i - DO_{i,k} \Bigg|_{\substack{\mathbf{Rt}_{i,k} < \mathbf{It}_{i,z} & k \textit{ where } \mathbf{Rt}_{i,k} \textit{ is maximum repayment time before } \mathbf{It}_{i,z} \\ t2``=\mathbf{It}_{i,z} & \textit{time at end of interest payment } \mathbf{It}_{i,z} \\ t1``=\mathbf{It}_{i,z-1} & \textit{time at start of interest payment}}} \tag{4.68}$$

4.6.4 FN Component Project Financing Instruments

As explained earlier the five common financial instruments used in project financing are general term loans, syndicated term loans, general bonds, private placement bonds, and floating rate notes. Details of methods and properties used with debt drawdown, repayment, interest, and fees were described in the previous section. Using these methods and properties, this section describes the five financial instruments and their implementation as part of the financing component of the generalized model.

4.6.4.1 General Term Loans

A term loan represents a loan that has a maturity of more than one year from the day it is committed and is generally repayable according to a specified schedule. Term loans are usually of medium-maturities in the range of 5 to 10 years (Nevitt and Fabozzi 1995; Cunningham 1981). Commercial banks represent the main providers of term loans. Insurance companies provide term loans as well and generally with longer maturities than commercial banks (Brealey et al. 1992).

Term loans can be arranged in several forms according to the purpose for which a loan is required. Therefore, term loans can be in the form of construction financing (bridging financing or interim financing) that is extended for the period of construction and repaid at completion through a "takeout commitment" or long-term permanent financing. Other forms of term loans include permanent financing, a combination of construction and permanent financing, mortgage loans, capital expenditure and working capital financing, stand-by facilities, and project financing (Willingham 1990; Summerfield 1981; Nevitt and Fabozzi 1993).

Repayment of term loans can be in several forms. In fact, all the nine repayment methods described early in Table 4.8 can be used in term loans to match repayments with the anticipated revenues (Willingham 1990; Nevitt and Fabozzi 1993).

Term loans can be arranged as fixed interest rate loans or as floating rate loans based on Libor, US prime or other primes. Floating rate term loans would carry longer maturities since bank returns in a floating transaction would fluctuate with the base short-term rate of the floating rate thus reducing the exposure in lending long term fixed rate; borrowers on the other side sustain all the risk of rising rates and enjoy all the benefits of falling rates (Nevitt and Fabozzi 1993). All types of fees except for the forward commitment as described before are charged on term loans.

Figure 4.44 shows the implementation of term loans with fixed interest rates:

- In the "identification" frame *Debt Time Zero* is used to position the debt stream on the project time line using GTU.
- The "loan Drawdown" frame is used to select a drawdown method.
- The "Term & Issue" frame is used to define the total amount of the loan, and its term, currency, and type of interest to be used and whether or not it will be capitalized.
- In the "Fixed Interest Rate" frame the interest rate is defined in terms of its value, payment frequency and compounding period.
- The "Sinking Fund/Debt Repayment" frame is used to define when repayments will start (grace period) and how the loan will be repaid in terms of the repayment method, the repayment interval, and whether or not a debt clause is used (optional repayment in this case). All nine-repayment methods can be used along with the three debt clauses described before. If a bullet repayment is used, then the grace period must set equal to the term of the loan.
- The "fees" frame describes the different types of fees applicable with term loans.

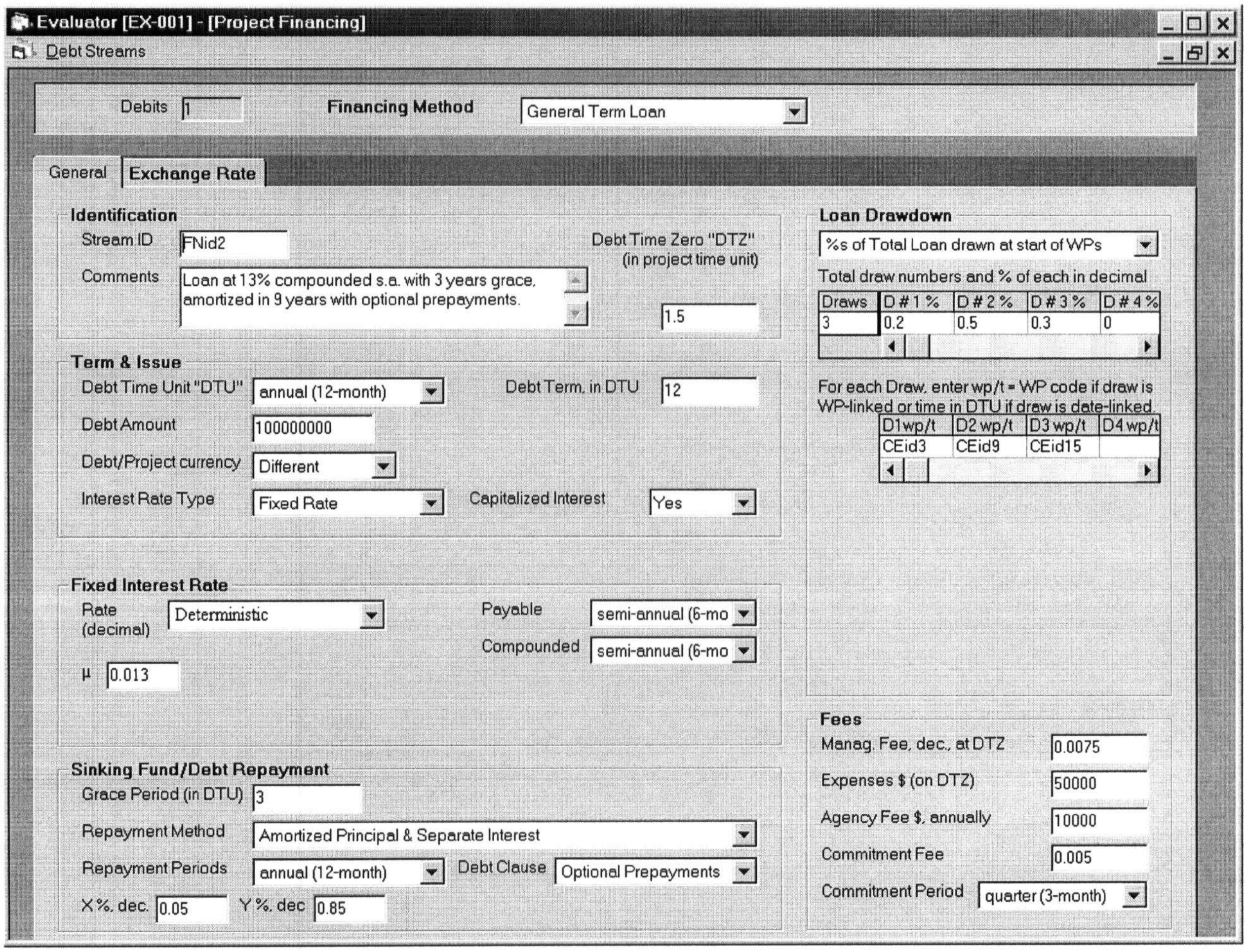


Figure 4.44: General term loan with fixed interest rate

Figure 4.45 shows the implementation of term loans with floating interest rates. The same features described above for the fixed rate loans in Figure 4.44 are used with floating rate loans, but with the following differences:

- Floating interest rate is selected in the "Term & Issue" frame and therefore the frame changes to include specific terms for floating rates. For example, fixing/quotation day is selected (start of period in this case), margin type and values are defined, and reference time (*global/local*) is defined such that at any time during the loan term, all floating rates use the same reference.
- The "Loan Drawdown" frame includes a table through which the interest period for each tranche before grace period can be defined. Tabs in the top of the window (e.g. 3 Month Interest) are used to define for each interest period a *rate function* that predicts the time variation of interest rates. For example, if quarter (code=3) and semi-annual (code=4) periods are used then "3 Month Interest" and "6 Month Interest" tabs must be used to define the annual 3-month and 6-month rates respectively. After the grace period, interest will rollover using the selected interest period "IP after Grace P." in the "Term & Issue" frame. This rate is defined (if not defined already for prior *GP* calculations) using the interest rate tabs at the top of the window.
- The "Sinking Fund/Debt Repayment" frame issues note warning against the selection of a repayment period less than the selected interest period after the grace period. For example, if a quarterly repayment period is selected while an annual interest period is selected for after-grace-period interest calculations, errors will occur in the computations of interest values (see. Eq. 4.60). Repayment methods used with floating rate loans are methods 1 to 5 and 9 in Table 4.8; i.e. the blended principal and interest payments cannot be used and the system will not show them by default. If a bullet repayment method is selected, then the grace period must be set equal to the loan term.

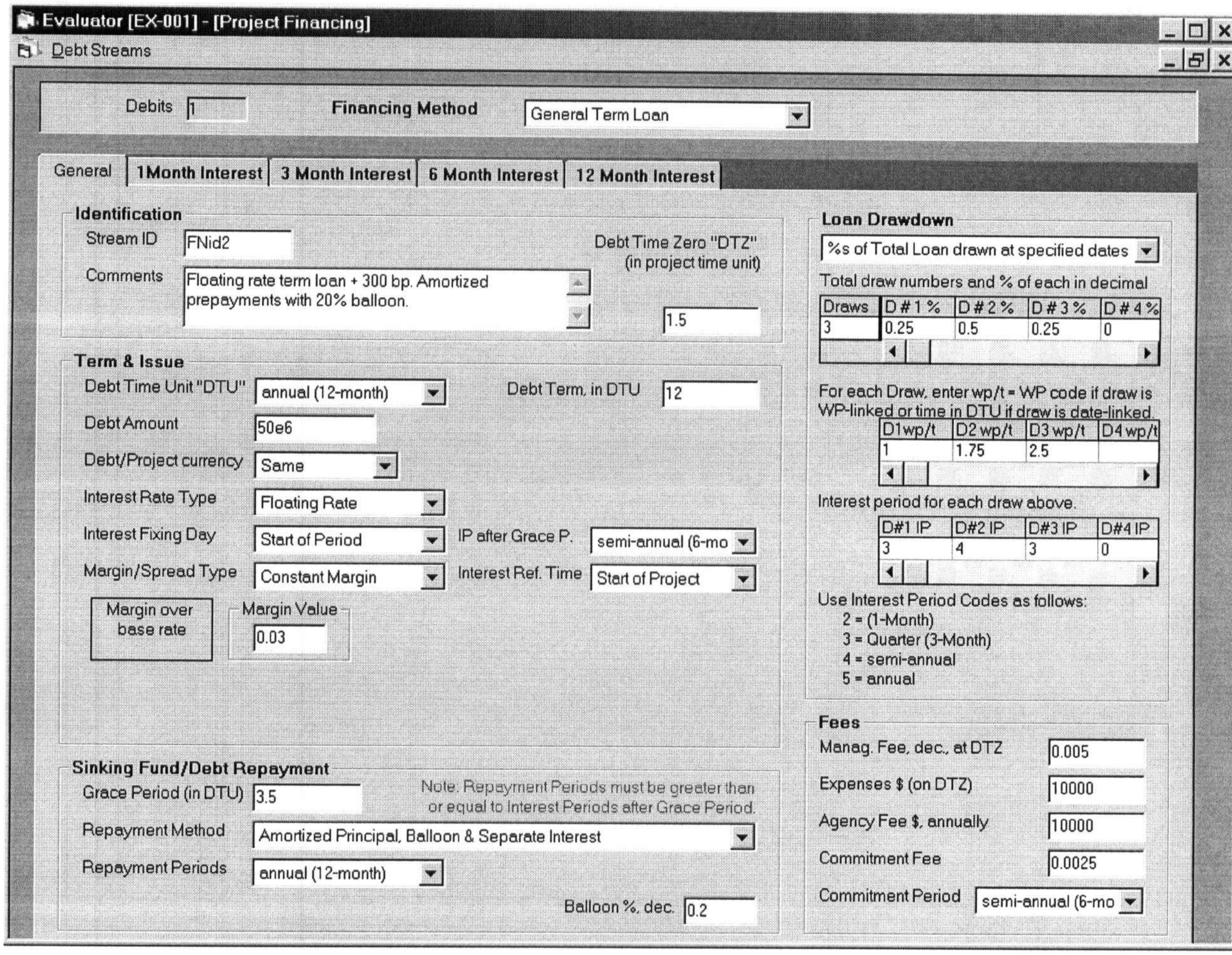


Figure 4.45: General term loan with floating rate of interest

4.6.4.2 Syndicated Term Loans

"A syndicated credit facility is one in which a number of banks undertake to provide a loan or other support facility to a customer on a pro rata basis under identical terms and conditions evidenced by a single credit agreement" Nevitt and Fabozzi (1995). Syndicated credit facilities can be structured in several forms including term loan facility, standby facility, standby letter of credit, revolving credit facilities, performance bonds, and a set of hybrid instruments such as revolving underwriting facilities (RUFs), note issuance facilities (NIFs), and multi-option facilities (MOFs) (Wiseman 1990). Syndicated term loan facilities are generally used for project financing of large capital investment projects.

The lending arrangement of syndicated loan transactions is usually formed by a structure composed of management group lenders called underwriters and general syndication lenders called participants. Lenders in both groups may be invited from all over the world through the syndication market. On average underwriters generally commit between \$25 million to \$100 million each while participants usually lend between \$1 million to \$20 million (Wiseman 1990, McDonald 1982). For example, the syndicated facility of the Channel Tunnel included more than 250 lenders, with 27% originating from France and UK, 39% rest of Europe, 23% Japan, and 11% rest of the world (Roger 1990).

The syndicated loan market represents the most significant source for raising large amounts of debt in the international market (Nevitt and Fabozzi 1993). For example, the facility for the Channel Tunnel was £ 8.712 billion advanced in several currencies raised between 1987 and 1990 (Holiday 1990). Figure 4.46 shows the amount of syndicated loans raised between 1972 and 1996 (OECD 1996), while Table 4.11 shows the distribution of currencies used in syndicated loans - note the domination of US dollars in this market.

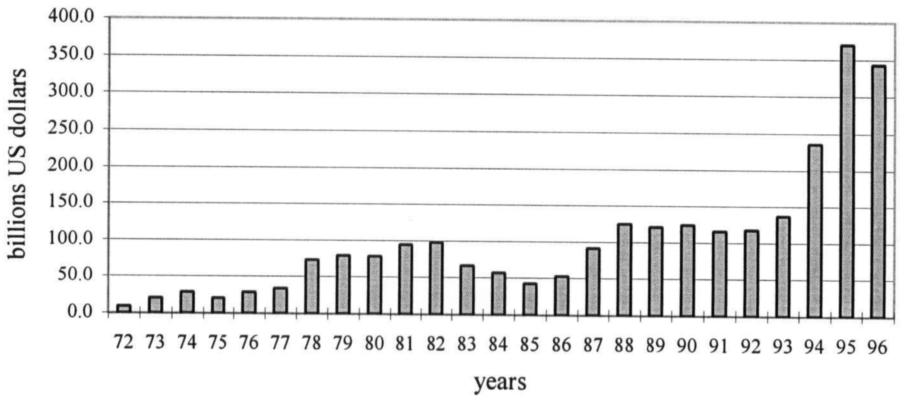


Figure 4.46: Syndicated loans in the international market, (OECD 1996)

Table 4.11: Currency distribution of syndicated loans (OECD 1997)

Currency	1990	1991	1992	1993	1994	1995	1996
US dollar	58.9	84.5	75.4	81.0	80.7	76.8	74.3
Pound sterling	17.5	4.2	1.9	2.2	8.6	11.7	12.6
French franc	-	-	0.7	1.9	1.6	1.5	4.8
Deutschmark	6.7	2.1	1.8	3.2	1.1	4.1	4.5
Swiss franc	0.1	0.6	0.3	0.4	0.1	0.1	0.5
ECU	8.7	3.9	15.0	6.4	3.9	3.8	0.2
Japanese yen	1.7	1.1	1.4	0.7	0.2	0.2	0.2
Other	6.4	3.6	3.5	4.2	3.8	1.8	2.9
Total %	100	100	100	100	100	100	100
US $ billion*	127.1	117.7	118.6	140.2	243.1	378.6	353.4

* *Currencies of denomination converted into US dollar at constant (end of 1990) exchange rates*

Syndicated facilities for a borrower can be arranged as 'fully underwritten', or 'partially underwritten' or 'best efforts' basis (Howcroft and Solomon 1985, Bee 1981, McDonald 1982). A fully underwritten offer commits lenders, in principle, to the full amount of the facility. Thus, the borrower has minimum risk, as the financing will be secured to the full amount needed. In partially committed offers, lenders commit to a portion of the facility and do their best efforts to raise the rest of the facility in the syndication market. This leaves the borrower with a risk of not raising the full amount. This risk is maximized if the offer is totally on a best efforts basis. The Second Severn Crossing described in Chapter two experienced a fully underwritten facility arranged by the Bank of America in 1992. The syndicated loan raised through project financing was for £340 million (US $ 599 million, 1992 dollars) and was fully underwritten by four banks: Bank of America, Lloyds Bank, Banque National de France and Credit Agricole. The underwriting and syndication structure included several brackets: underwriters and lead managers (10 banks), managers (3 banks) and co-managers (5 banks). Bank of America, the arranger, was also the agent bank (Rhodes et al. 1993).

Syndicated loans can have a flexible drawdown profile that suits a borrower's needs. Pricing is based on (1) a floating interest rate, usually Libor or US prime, (2) a management fee to be distributed among the underwriters and participants, (3) a commitment fee, (4) an agency fee, and (5) out-of-pocket expenses. Maturities are in the range of 10 years or more (Nevitt and Fabozzi 1995). Syndicated facilities can be arranged as a multi-option facility where 3 or 4 currencies can be outstanding at one time. Physical payments as described before are therefore required to keep the facility at the original dollar amount (Howcroft and Solomon 1985, Rhodes et al. 1993). To hedge against exchange rate movements, borrowers may seek currency swap instruments for which they pay a premium (Coopers and Lybrand 1987; Nevitt and Fabozzi 1995).

Figure 4.47 shows the implementation of the syndicated loan in the generalized economic model. Features of this financial instrument resemble those made with the floating rate term loan as shown in Figure 4.45. The only addition is the "Physical Payments" as shown in Figure 4.47 in the "Term & Issue" frame and tab. A description of some methods and properties of a floating rate loans is as follows:

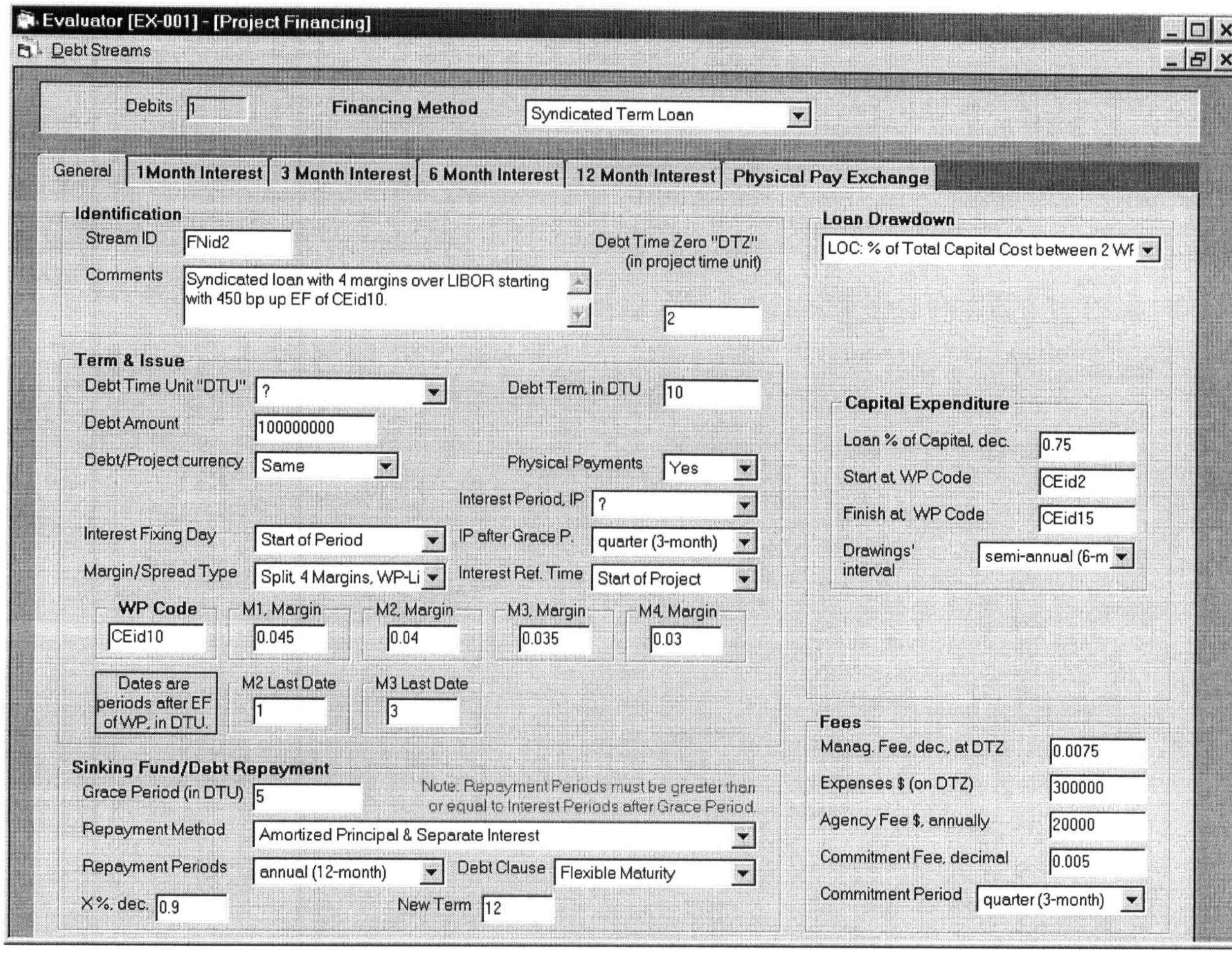


Figure 4.47: Syndicated term loan

- Unlike the drawdown in several tranches as shown earlier in Figure 4.45, when selecting the drawdown profile the loan as "LOC: % of Total Capital Cost" or as "Single Drawdown" (method 1 and 3 under Section 4.6.3.1) only one interest period will be used. This is defined under "Interest Period, IP" in the "Term & Issue" frame in Figure 4.47.
- As explained before, the margin/spread of floating rates may be split and linked to a work package, as shown in Figure 4.47, or linked to specified dates. With reference to Figure 4.47, four margins are used such that the first is applicable to the early finish *EFc* of the work package specified, the second is applicable up to one LTU after *EFc*, the third is applicable up to three LTUs after *EFc* (i.e applicable for two time units after the second margin application time), and the fourth margin is applicable after that to the end of the term. If margins were linked to specific times (the third method as described before) each margin would be applicable up to its specified time.
- Repayment of syndicated loans and floating-rate term loans can arranged through methods 1-5 and 9 in Table 4.8. If the first method (amortized principal with separate interest) is selected, any of the three debt clauses described in Section 4.6.3.2 and Eqs. (4.44 - 4.46) can be used. For example, in Figure 4.47 amortized principal is selected with flexible maturity and therefore a new term must be specified along with the percentage to be applied for repayment calculations as described in Eq. (4.46).
- If bullet repayment method is selected, then the grace period must be set equal to the loan term defined in the "Term & Issue" frame.

4.6.4.3 General Bonds

A bond is a negotiable security used to raise money in the capital market and can be defined as: "a certificate evidencing indebtedness - a legal contract sold by an issuer promising to pay the holder its face value plus amounts of interest at future dates" (Fisher 1979). A *bond* generally refers to debt secured by mortgages on some assets such as property, plant and equipment. If unsecured, a bond is referred to generally as a *debenture*. A bond or debenture with maturity shorter than 15 years is referred to as a *note* (Brealey et al. 1992; Nevitt and Fabozzi 1995).

Bonds usually carry a fixed coupon/interest rate, as described in this section. Bonds, however, can carry a floating coupon rate as in *floating rate notes*, which are described later. Bonds can be issued to the public at large but also may be placed privately with a number of investors (e.g. insurance companies) as in *private placement bonds*, described in the next section.

Bonds may be issued in the domestic market of a country and become subject to the regulations and registration aspects of the securities exchange commission of that country. Domestic bonds are issued in the local currency by the local underwriters to investors located in the country. Alternatively, a borrower can issue bonds in the international Eurobond market and the foreign bond market. Eurobonds could be of any currency, issued outside the borrower's domestic market and sold simultaneously in several counties to international investors. Bonds issued in the local currency of a country in the domestic market of that country by a foreign borrower are called foreign bonds (Scarlett 1990; Brealey et al. 1992; Nevitt and Fabozzi 1995).

Bonds, in general, provide maturities longer than what can be achieved in bank lending and syndicated loans. Another advantage, however, is the fixed rate of interest that protects borrowers from the fluctuation of the interest rates. Coupon rates are generally paid semi-annually. However, Eurobonds generally use annual payments. In the Eurobond market, the funds available for borrowers are more limited than in the syndicated loans market. Unlike the flexibility offered by term and syndicated loans where debt tranches can be matched to a borrowers needs, the drawdown of funds for a bond issue is restricted to a single/up-front tranche. Bullet repayment of bond issues is common in the bond market. However, repayment of a bond issue is usually made by means of a sinking fund that retires a stated percentage of the principal value of the issue each year over a number of years after an interest-only period (grace period) with the possibility of a balloon payment at maturity (Fisher 1979; Scarlett 1990; Spate 1997; Nevitt and Fabozzi 1995; Brealey et al. 1992).

In the generalized model, a general bond possesses the following characteristics as shown in Figure 4.48:

- Loan drawdown is made by a single tranche drawn at the signing of the agreement, i.e. "Debt Time Zero" in the "Identification" frame.
- The total bond issue must be defined along with the currency if they are different as well as the term in the "Term & Issue" frame.
- The coupon rate per annum and the payable period are defined in the "Fixed Interest Rate" frame. The payable period can be specified as 1-month, quarterly, semi-annually and annually.
- Five repayment methods can be used with any bond issue. These methods are the first five methods of Table 4.8 reflecting the bullet and separate interest and repayment methods. If a bullet repayment is used, no grace period will be required as it is replaced by the defined term for the bond.
- Three types of fees are used with bonds. These are shown in the "Fees" frame. No commitment fees of any type are used with general bond issues.

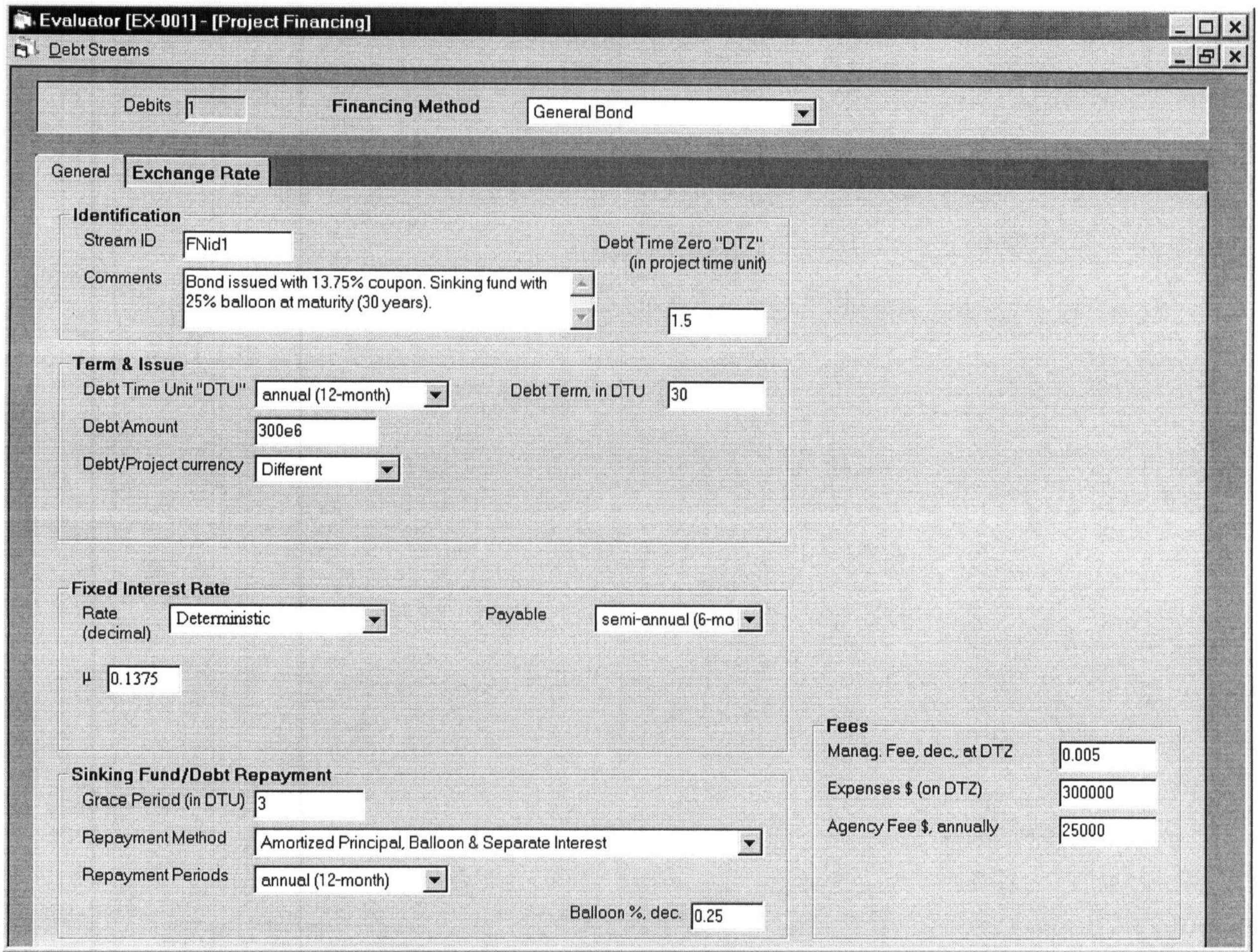


Figure 4.48: General bond issue

4.6.4.4 Private Placement Bonds

Private placement bonds refer to the raising of capital through the sale of long-term securities to a limited number of sophisticated investors rather than to the public at large. Investors in private placements are mainly institutional investors, such as life insurance companies and pension funds, which have the sophistication to understand the credit standing of a borrower. Private placements embrace several advantages over the public market in terms of: (1) no registration is required with a securities commission which protects the borrowers' information, (2) direct access to the investors in the market and speed in raising the money, (3) substantial reduction in fees, and (4) suitability to borrowers of special needs (e.g. for project financing) that are not offered by the public market (Carey et al. 1993; Bostwick 1996; Lund et al. 1984).

Unlike public issues, private placement bonds are not sold in a secondary market. In the US, private placement bonds must be purchased and kept (buy-and-hold) by the investor for at least two years before reselling to another sophisticated investor, under what is called Regulation D ("Regulation" 1983). In 1990, the US enacted Rule 144A to allow direct resale and since that time the private placement market has been working under these two regulations. In 1995, the US private and public market amounted to $624.9 billion of which 28.8% were in private placements (Gelbard 1996).

The size of an issue under a private placement is in the range of $10 million to $500 million under Regulation D and between $100 million to $1 billion under R144A, with matuities from 3 to 30 years (Gelbard 1996). While private placements follow the general practice of the public market, take down of a private placement issue can be delayed several months in the future; in the R144A delayed drawdowns are rare (Gelbard 1996). A forward commitment fee (as described before) might be required for a delayed drawdown (Shapiro and Wolf 1972; Lund et al. 1984). A private placement can be structured in terms of several tranches. Ball (1995) reports that some private issues have been made in 2 to 5 tranches, however, with each priced differently (e.g. 2 tranches priced at 80bp and 85 bp over US treasuries) with different maturities.

The interest on private placement bonds is usually fixed since most of the life insurance companies who dominate this market prefer investments of cash flows that matches their long-term fixed-rate liabilities (Carey et al. 1993). Interest/coupon rates of private issues are usually higher than public issues by 10 to 50 basis points because of the lack of liquidity in the private placement market (Brealey et al. 1992; Gelbard 1996). While the coupon rate of a private issue is fixed during the term, at the time of commitment the fixed rate is determined as a margin/spread over the yield on U.S. Treasury security corresponding to the average life of the bond issue.

Unlike the public bond market, borrowers in private placements may enjoy a tailored amortization schedule. However, investors usually prefer the bullet maturity. After an interest-only period (grace period), sinking fund provisions generally call for mandatory equal annual payments to retire the issue over a number of years until maturity; escalating sinking funds with balloon payments at maturity can also be found. The sinking fund may take the form of blended principal and interest payments as in mortgages or annuity style and this may provide for a longer maturity (Lund et al. 1984). Optional prepayments may be allowed without penalty/premium such that an issuer may "double-up" the required repayment up to a maximum percentage of the original issue (Nash 1990). Flexible maturity can be achieved as well under private placements. As mentioned before, the Mexico-Cuernavaca toll road was financed in 1994 by a R144A private placement issue of US265 million that carried a coupon of 9.25% peso-exchange-rate linked notes. The issue carried a target amortization schedule maturing in January 2000 with final maturity in July 2001 to mitigate risks due to fluctuations of toll revenues and the exchange rate.

Private placements are less expensive than public issues. Gelbard (1996) explain that fees (management/underwriter, expenses, and ongoing agency) for a $100 million issue are around $435,000 for a private placement (Regulation D), and $1.08 million to $1.26 million for the R144A and public issues. Unlike public issues which are usually underwritten with a firm commitment by an underwriter, private placements in general are sold through an agent on a 'best efforts' basis (Carey et al. 1993). This contributes to the reduction of fees associated with general private placements. Since R144A placements are generally underwritten their fees approach that of the public issues.

In the generalized model, private placement bonds have the following features as shown in Figure 4.49:

- Unlike general bonds, the drawdown of the single tranche of private placement can be delayed as defined in the figure under "Issue Date". This delay is measured from "Debt Time Zero".
- More so than with general bonds, repayment of a private placement issue can be made through the "mortgagee" or "annuity" style arrangement and therefore all blended principal and interest repayment methods can be used for the private placement. Thus, the nine methods in Table 4.8 can be used for repaying a private placement bond. The three debt clauses described before can be used as well (e.g. flexible maturity as shown in the figure).
- Interest used in the repayment calculations with blended repayment methods has its payable period adjusted as described in Eq. 4.55. Interest payments before the grace period of the private issue are calculated based on Eq. 4.47.
- As shown in Figure 4.49, a forward commitment fee is used to define the premium a borrower must pay to have a delayed drawdown. Three fees can be defined as well.

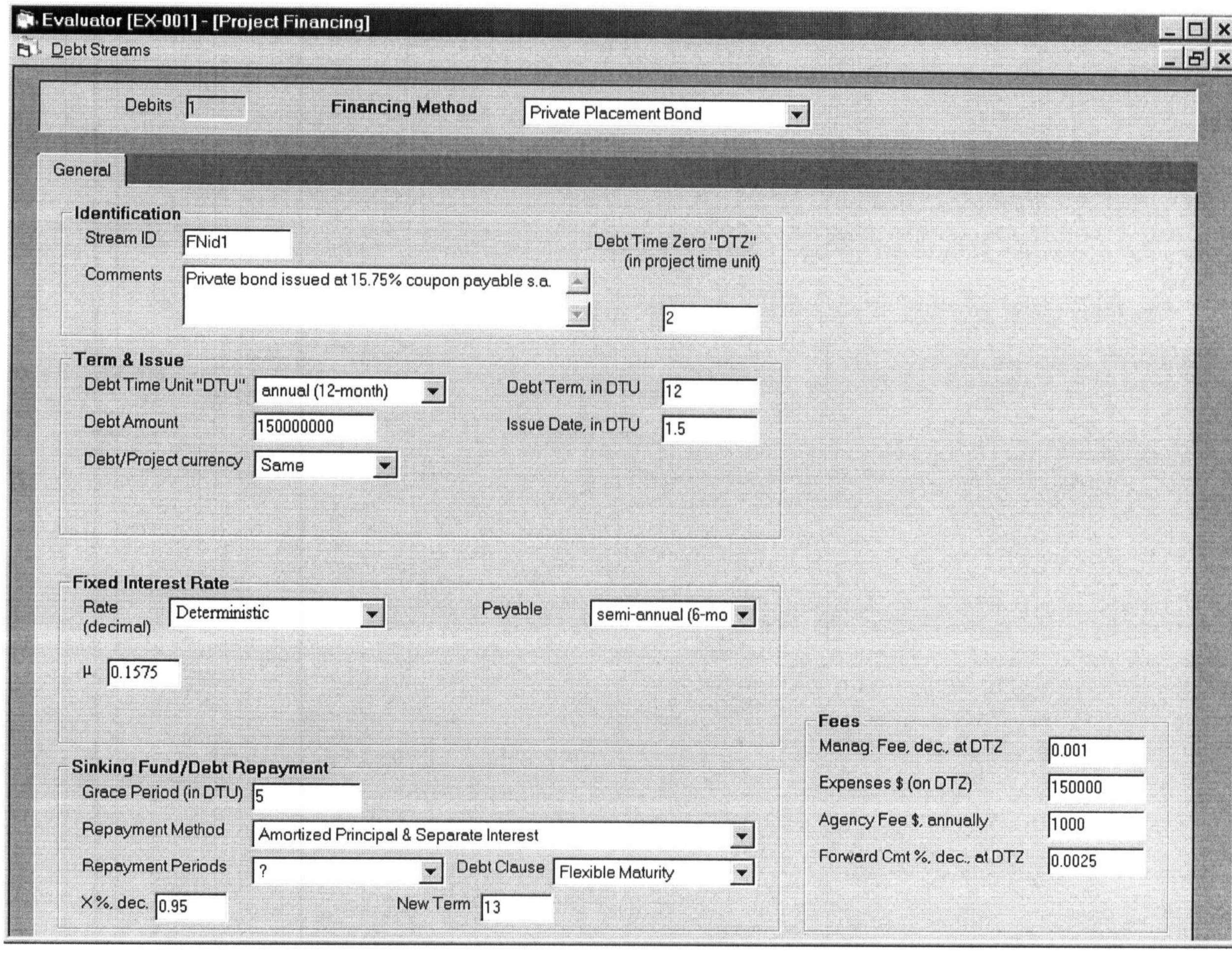


Figure 4.49: Private placement bond

4.6.4.5 Floating Rate Notes

As defined by Nevitt and Fabozzi (1995): "a floating rate note issue has no fixed rate of interest. The coupon is set periodically according to a predetermined formula tied to short-term rates in the appropriate market." Originally, the floating rate concept was limited to bank loans. It was introduced in the capital markets through the Eurobond market issuing floating rate notes in 1970. Later it was introduced to the domestic markets (Ugeux 1981).

Floating rate notes (FRNs) can be issued in any major currency, but the US dollar is the dominating currency. Maturities of FRNs are in the range of 5 to 15 years. The coupon rate is usually linked to a benchmark or reference short-term rate such as Libor and Treasury Bills. In the US domestic market, the coupon rate is set usually at a margin/spread over the three-, or six-month Treasury Bill rates. In the Euro market coupon rates are usually set at a margin/spread over offered rate Libor, bid rate called Libid[5] (London Inter-bank Bid Rate), or the arithmetic mean between the bid and offered rates, called Limean, rounded up to the nearest 1/16% (Scarlett 1990). The margin/spread is usually added to the reference rate to form the coupon rate. Quotations of the reference rates are usually made at the start of the relevant interest period chosen for the issue. Accrued interest is paid at the end of the interest period for which quotations of the new rates for the next interest period are made (usually quotations are made on the second business day prior to the beginning of the new interest period) (Ugeux 1981). The norm with FRNs is to set the coupon rate at a margin/spread over the reference rate. However, some issues (called reverse floaters) may carry a formula that moves the coupon rate in a reverse order to Libor (Nevitt and Fabozzi 1995); e.g. coupon rate = 16% – 1.5 · 3-month Libor.

[5] Libor is the rate at which major London banks offer to lend money to each other while Libid is the rate at which these banks offer to borrow from each other.

FRNs usually have a minimum rate (floor rate) where it will be used in case the coupon rate falls below it (Ugeux 1981; Scarlett 1990). As with general bond issues, repayment of FRNs is made through bullet repayments or through amortized repayments of the issue using sinking funds.

In the generalized model, floating rate notes have the following features as shown in Figure 4.50:

- Drawdown of a FRN is made in a single tranche at "Debt Time Zero". Unlike, private placements, delayed payments are not used with FRNs and, therefore, no commitment or forward commitment fee is used with FRNs.
- Short-term interest rate is defined under "Interest Period, IP". According to the selected interest interval, the relevant tab in the top of the window in the figure must be used to define the change of the short-term rate over the term of the issue. Spread/margin to be added to the short-term rate is a constant throughout the term. Interest will be paid at the end of each interest period according to the sum of the short-term rate at the end of the interest period plus the specified margin. A minimum coupon rate can be defined.
- Quotation date for the short-term rate under the "Interest Fixing Day" should be set to start of period since this is the norm for FRN issues.
- In the "Sinking Fund/Debt Repayment" the grace period needs to be defined, as repayments will start after that period. For the case of a bullet repayment, the grace period must be set equal to the term of the issue. Five repayment methods can be used to reflect bullet repayment and separate principal and interest payments (Table 4.8). Repayment periods defined under "Repayment Periods" must be larger than or equal to the selected interest period.
- Interest period after the grace period ("IP after Grace P.") must be set equal to the "Interest Period, IP" before the grace period since FRNs must have one interest period
- The three fee types used with FRNs are defined in the "Fees" frame in the figure.

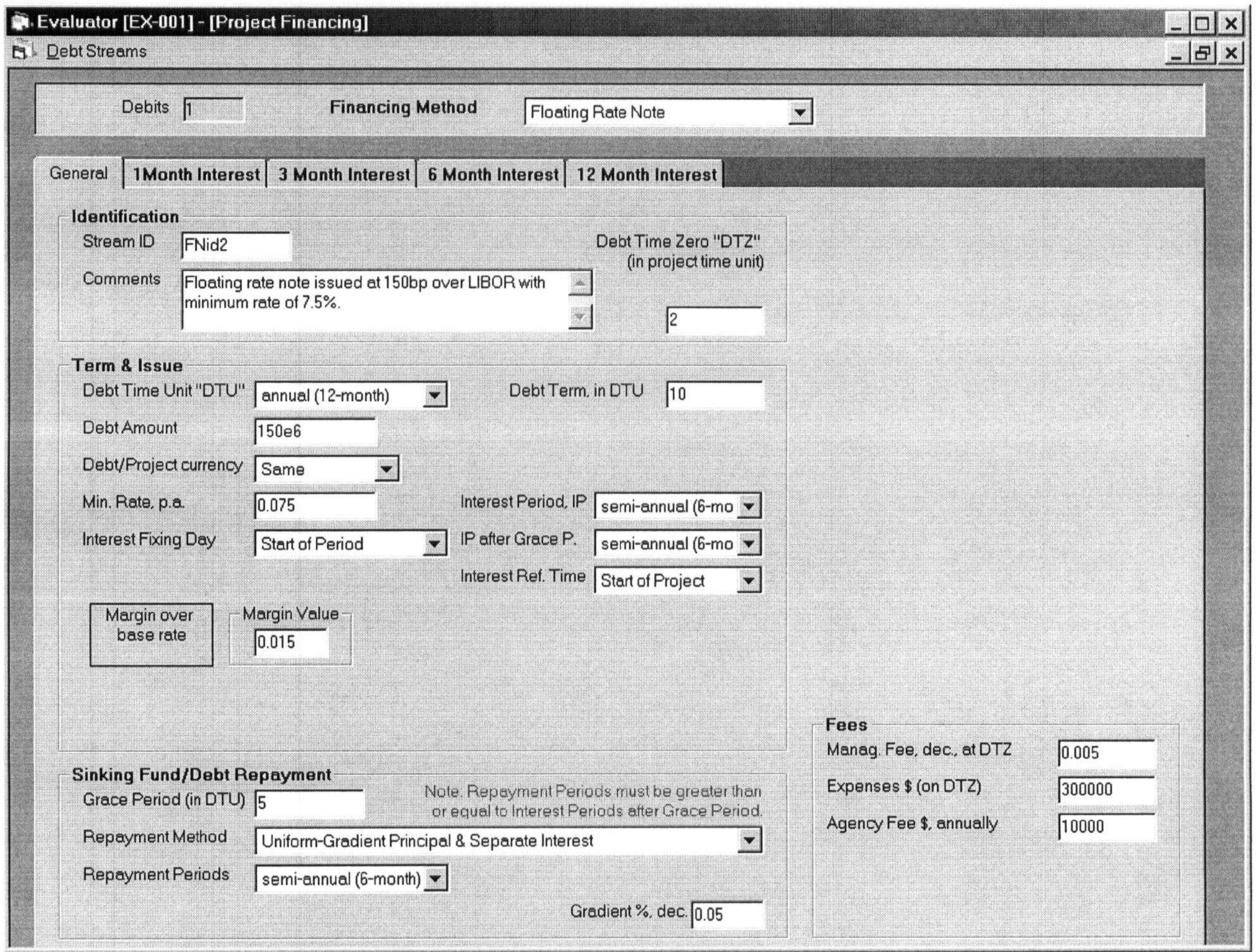


Figure 4.50: Floating rate note

4.6.5 FN Component Discounted Flows

The previous section described the properties and methods used in the FN classification through which any debt stream can represent any of the five financial instruments described above. The properties and methods information of each stream are used to prepare the time and value information matrices of the FN component $\mathbf{TRIFt}_i$ and $\mathbf{TRIFv}_i$. These are decomposed for each debt stream i in Section 4.6.2 to represent the four main parts of a debt stream: *tranches* ($\mathbf{Tt}_{i,j}$ and $\mathbf{Tv}_{i,j}$), *repayments* ($\mathbf{Rt}_{i,k}$ and $\mathbf{Rv}_{i,k}$), *interest* ($\mathbf{It}_{i,z}$ and $\mathbf{Iv}_{i,z}$), and *fees* ($\mathbf{Ft}_{i,w}$ and $\mathbf{Fv}_{i,w}$).

Figure 4.51 shows the results of financial calculations for the above matrices of two debt streams, a general bond and a syndicated loan. It shows dates and values of the debt streams. As can be noted, the syndicated loan has 3 tranches, with interest rates calculated based on 6-month periods while repayments are made on annual basis.

Using these information matrices, discounted cost of an FN stream and the FN component can be obtained as follows, where all cash flows are discounted to time Td which is measured in GTU with *global* reference; y is the nominal annual discount rate and $\bar{y}$ is y converted from annual time periods to GTU; wd is the duration of the stream in LTU; $\overline{tb}$ in Eq. 4.15 is the time before start of the stream, converted from GTU to annual time unit; and, $\overline{tn}$ in Eq. 4.16 is the time elapsed in the stream, converted from GTU to LTU. Thus:

$$df_c^{FN}(Td,\boldsymbol{x}) = \sum_{j=1}^{m} \mathbf{Tv}_j \cdot e^{-\bar{\bar{y}}\cdot(\mathbf{Tt}_j - Td)}\Big|_{\mathbf{Tt}_j \geq Td} - \sum_{k=1}^{p} \mathbf{Rv}_k \cdot e^{-\bar{\bar{y}}\cdot(\mathbf{Rt}_k - Td)}\Big|_{\mathbf{Rt}_k \geq Td} - \sum_{z=1}^{p} \mathbf{Iv}_z \cdot e^{-\bar{\bar{y}}\cdot(\mathbf{It}_z - Td)}\Big|_{\mathbf{It}_z \geq Td} - \sum_{w=1}^{r} \mathbf{Fv}_w \cdot e^{-\bar{\bar{y}}\cdot(\mathbf{Ft}_w - Td)}\Big|_{\mathbf{Ft}_w \geq Td} \tag{4.69}$$

$$df_{FN}(Td,\boldsymbol{X}) = \sum_{i=1}^{n} df_c^{FN}(Td,\boldsymbol{x}_i)_i \tag{4.70}$$

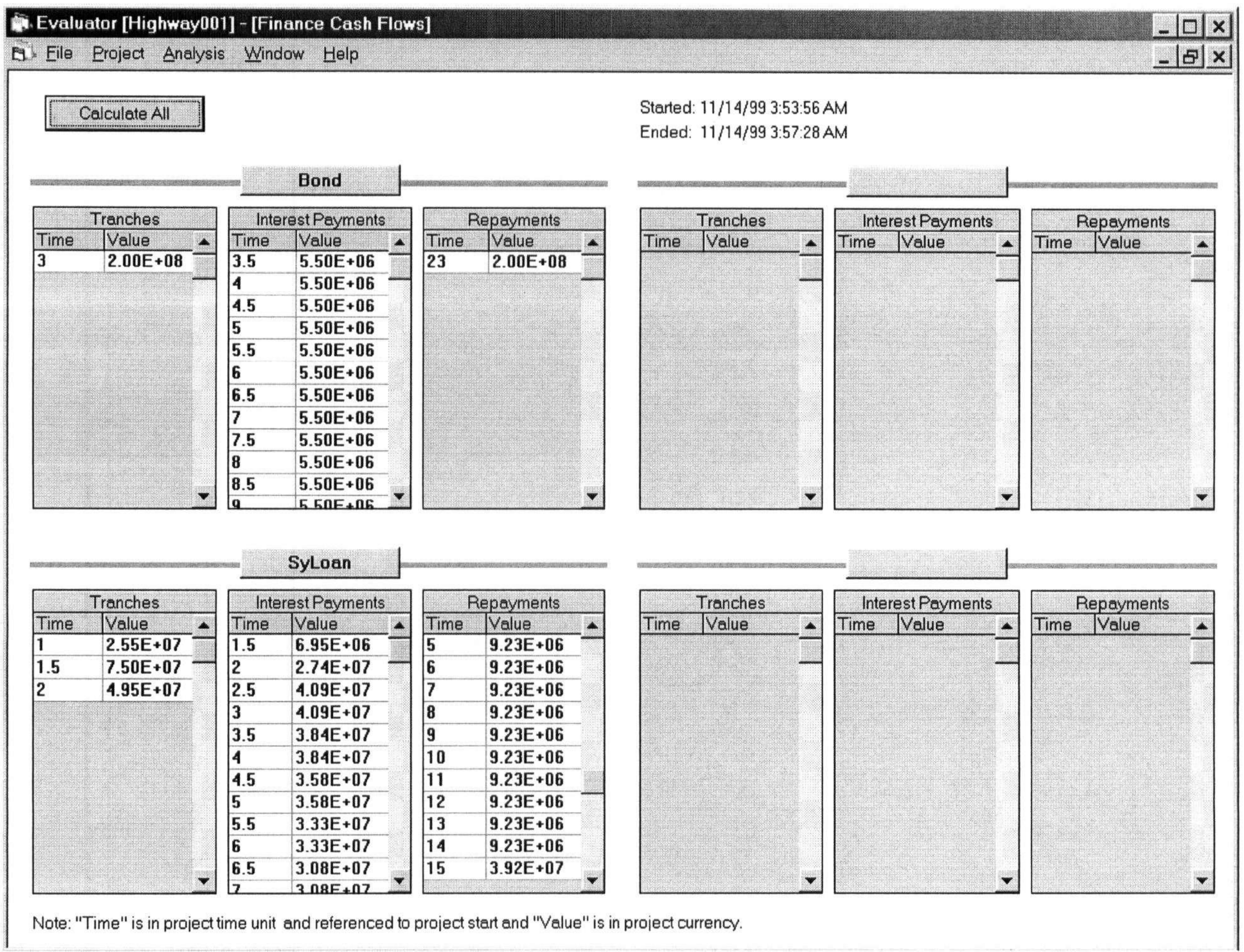


Figure 4.51: Tranches, interest payments and principal repayments of two debt streams

4.7 ECONOMIC MODEL CASH FLOWS AND PERFORMANCE MEASURES

This section summarizes the formulation of the generalized economic model. Acknowledging the various properties and estimating methods in the four domains capital expenditure, revenue, operation and maintenance, and finance of a capital investment project, as stated earlier, the overall objective was to derive a very generalized and flexible model with which to formulate:

1. A construct's cash flow and discounted cash flow
2. A component's cash flow and discounted cash flow
3. A project's cash flow
4. A component's performance measures
5. A project's performance measures.

4.7.1 Constructs, Components and Project Cash Flows

The previous four sections described the properties and methods (crude, semi-detailed, and detailed) of the four classifications representing the four domains of capital investment projects. Using these classifications equations for the cash flow and discounted cash flow of each construct (work package or stream) of a component as well as the component itself were derived, as summarized in Table 4.12.

Table 4.12: Construct and component cash flow modeling equations (equations between square brackets are supporting equations)

Component	Construct (Work package / stream)			Component		
	Cash Flow	Cumulative Cash Flow	Discounted Cash Flow	Cash Flow	Cumul-ative	Discounted Cash Flow
Capital Expenditure	(4.4), (4.5) [(4.6), (4.7), (4.8), (4.9)]	(4.11)	(4.14)	(4.1)	(4.12)	(4.17)
Revenue	(4.19), (4.20) [(4.21), (4.22), (4.24)]	(4.25)	(4.27)	(4.2)	(4.26)	(4.28)
Operation & Maintenance	(4.29) [(4.30), (4.31)]	(4.32)	(4.34)	(4.3)	(4.33)	(4.35)
Finance	(4.36) To (4.63)		(4.69)			(4.70)

A total project cash flow and cumulative cash flow can be derived from the cash flows of the capital expenditure, revenue, and operation and maintenance components (Eqs. 4.1, 4.2, and 4.3) as follows, where *Td* is time in project time unit, GTU:

$$f_{prj}(Td)=f_{RV}(Td,\boldsymbol{X}_{RV})-f_{CE}(Td,\boldsymbol{X}_{CE})-f_{OM}(Td,\boldsymbol{X}_{OM}) \tag{4.71}$$

$$F_{prj}(Td)=F_{RV}(Td,\boldsymbol{X}_{RV})-F_{CE}(Td,\boldsymbol{X}_{CE})-F_{OM}(Td,\boldsymbol{X}_{OM}) \tag{4.72}$$

Figure 4.52 illustrates the implementation of project cash flows in the generalized model. Periodic and cumulative cash flows (Eqs. 4.71 and 4.72) can be tabulated and graphed to any required interval (1-month, 3-month, 6-month, and 12-month) as shown in the figure.

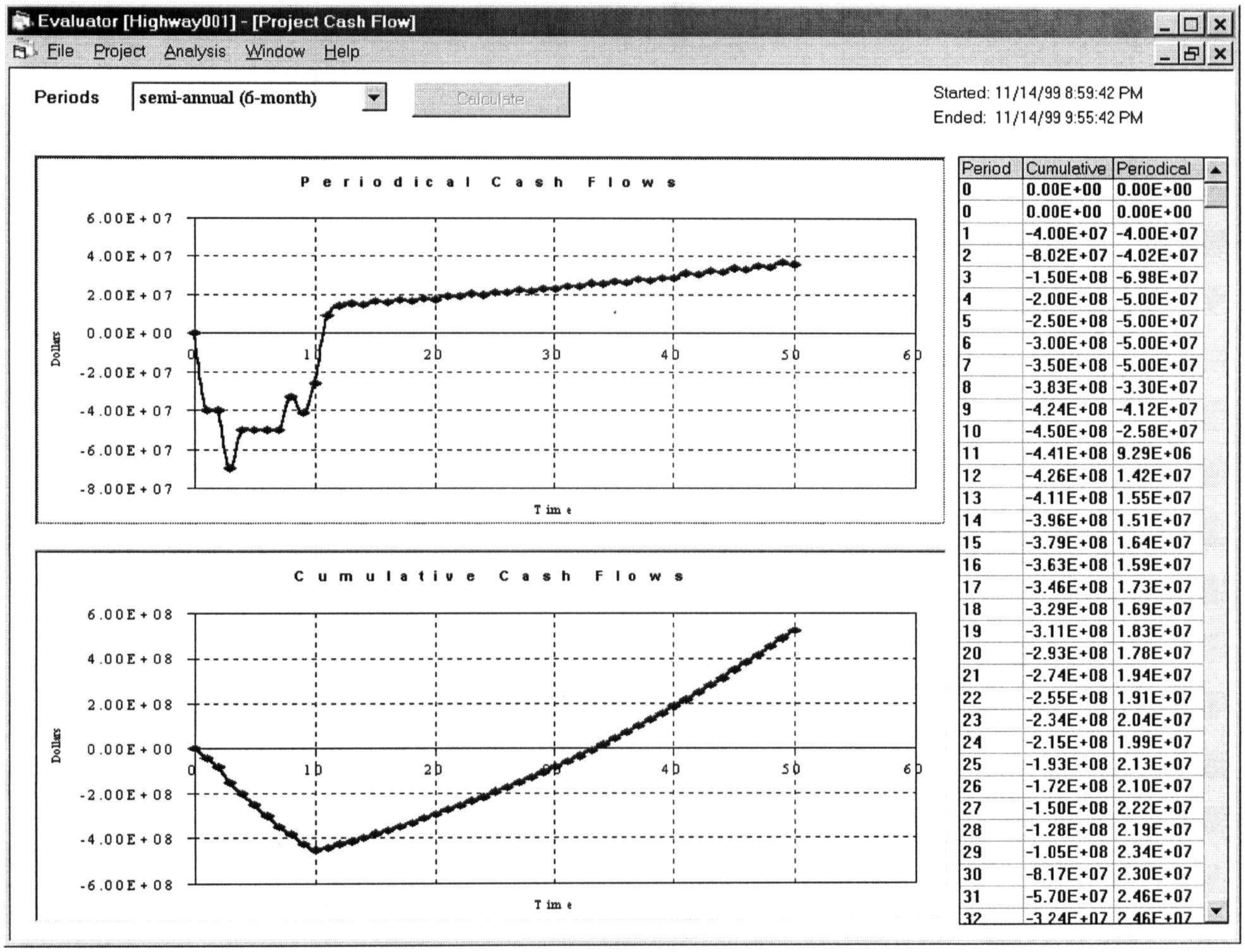
4.7.2 Components Performance Measures

In the CE component, Eq. 4.11 represents a function $F_c^{CE}(t`,\boldsymbol{x})$ that provides the cumulative cash flow of a work package at any time within the work package duration. Using this cumulative cash flow function, the total cost of any CE area *A* which represents a group of work packages in the CE component (Section 4.3.3) and the total cost of all work packages in the CE component can be obtained respectively as follows, where *wd* is work package duration, and *n* and *m* are the number of work packages in area *A* and the CE component respectively:

$$T_a^{CE}(\boldsymbol{X})=\sum_{i=1}^{n}F_c^{CE}(wd_{A_i},\boldsymbol{x}_{A_i})_{A_i} \tag{4.73}$$

$$T_{CE}(\boldsymbol{X})=\sum_{i=1}^{m}F_c^{CE}(wd_i,\boldsymbol{x}_i)_i \tag{4.74}$$

Similarly, the total revenues of all the streams in the RV component and the total costs of all the streams in the OM component can be obtained using the cumulative cash flow function of a stream in the two components, i.e. Eq. 4.25 and Eq. 4.32 respectively. Therefore, where *sd* is a stream duration in the RV or OM component, then total revenues and total OM costs are:



Period	Cumulative	Periodical
0	0.00E+00	0.00E+00
0	0.00E+00	0.00E+00
1	-4.00E+07	-4.00E+07
2	-8.02E+07	-4.02E+07
3	-1.50E+08	-6.98E+07
4	-2.00E+08	-5.00E+07
5	-2.50E+08	-5.00E+07
6	-3.00E+08	-5.00E+07
7	-3.50E+08	-5.00E+07
8	-3.83E+08	-3.30E+07
9	-4.24E+08	-4.12E+07
10	-4.50E+08	-2.58E+07
11	-4.41E+08	9.29E+06
12	-4.26E+08	1.42E+07
13	-4.11E+08	1.55E+07
14	-3.96E+08	1.51E+07
15	-3.79E+08	1.64E+07
16	-3.63E+08	1.59E+07
17	-3.46E+08	1.73E+07
18	-3.29E+08	1.69E+07
19	-3.11E+08	1.83E+07
20	-2.93E+08	1.78E+07
21	-2.74E+08	1.94E+07
22	-2.55E+08	1.91E+07
23	-2.34E+08	2.04E+07
24	-2.15E+08	1.99E+07
25	-1.93E+08	2.13E+07
26	-1.72E+08	2.10E+07
27	-1.50E+08	2.22E+07
28	-1.28E+08	2.19E+07
29	-1.05E+08	2.34E+07
30	-8.17E+07	2.30E+07
31	-5.70E+07	2.46E+07
32	-3.24E+07	2.46E+07

Figure 4.52: Project cash flows

$$T_{RV}(\boldsymbol{X}) = \sum_{i=1}^{m} F_c^{RV}(sd_i, \boldsymbol{x}_i)_i \tag{4.75}$$

$$T_{OM}(\boldsymbol{X}) = \sum_{i=1}^{m} F_c^{OM}(sd_i, \boldsymbol{x}_i)_i \tag{4.76}$$

4.7.3 Project Performance Measure

Using the discounted cash flows of the four components (Eqs. 4.17, 4.28, 4.35, and 4.70), several project performance measures can be obtained at any time Td measured in project time unit GTU. Normally all discounted value calculations are performed at time zero, i.e Td = 0. However, in the generalized model it is possible to set Td to a value such that only the cash flow values after time Td will be used in the calculation (see "Discount all calculations to time?" in Figure 4.3). The performance measures include:

- Life cycle cost LCC

$$LCC(\boldsymbol{X}) = df_{CE}\,(Td,\boldsymbol{X}_{CE}) + df_{OM}\,(Td,\boldsymbol{X}_{OM}) \tag{4.77}$$

- Benefit-cost ratios BC

  The "aggregate" BC_g and "netted" BC_n ratios (Park and Sharp-Bette 1990) are derived where benefits are represented by the discounted revenues, initial costs are represented by the discounted cost of the CE component and the annual costs are represented by the discounted cost of the OM component:

$$BC_g(\boldsymbol{X}) = \frac{df_{RV}\,(Td,\boldsymbol{X}_{RV})}{df_{CE}\,(Td,\boldsymbol{X}_{CE}) + df_{OM}\,(Td,\boldsymbol{X}_{OM})} \tag{4.78}$$

$$BC_n(\boldsymbol{X}) = \frac{df_{RV}\,(Td,\boldsymbol{X}_{RV}) - df_{OM}\,(Td,\boldsymbol{X}_{OM})}{df_{CE}\,(Td,\boldsymbol{X}_{CE})} \tag{4.79}$$

- Net present value NPV

$$NPV(\boldsymbol{X}) = df_{RV}\,(Td,\boldsymbol{X}_{RV}) + df_{FN}\,(Td,\boldsymbol{X}_{FN}) - df_{CE}\,(Td,\boldsymbol{X}_{CE}) - df_{OM}\,(Td,\boldsymbol{X}_{OM}) \tag{4.80}$$

- Internal rate of return IRR

$$IRR(\boldsymbol{X}) = y\Big|_{NPV(\boldsymbol{X})=0} \quad \text{and } y \text{ is the discount rate used with } NPV \tag{4.81}$$

- Loan-life-cover-ratio *LLCR*

Loan life cover ratio can be defined as: "the present value of the cash flow before debt service costs divided by the present value of the debt service costs over the life of the debt" (DCFL 1996). This ratio measures a project's future cash flow relative to the amount of debt outstanding. The minimum *LLCR* typically required by lenders ranges between 1.4 to 1.6 (DCFL 1996). At any time *Td, LLCR* can be defined as follows, where all the discounted calculations are performed at a discount rate selected for the cover ratio (discount rate other than that used in NPV calculations):

$$LLCR(Td,\mathbf{X})=\frac{df_{RV}\ (Td,\mathbf{X}_{RV})-df_{OM}\ (Td,\mathbf{X}_{OM})-df_{CE}\ (Td,\mathbf{X}_{CE})}{df_{FN}\ (Td,\mathbf{X}_{FN})} \qquad (4.82)$$

The flexible structure of the FN component helped in deriving the *LLCR* equation to cover all debt streams in a project, which used to be a complicated process as mentioned by DCFL (1996) if more than one debt stream is involved.

- Debt-service-cover-ratio *DSCR*

Debt service cover ratio can be defined as "cash flow before debt service costs divided by debt service costs" (DCFL 1996). *DSCR* is calculated generally semi-annually or annually. The value typically sought for toll roads is around 1.4 and for power projects around 1.2 (DCFL 1996; Nevitt and Fabozzi 1995). *DSCR* can be calculated as follows for a period *Td-Td`* in GTU, where *j* is the number of debt streams, and *n* and *p* are the number of repayments and interest payments respectively in stream *i*:

$$DSCR(Td,\mathbf{X})=\frac{[F_{RV}(Td,\mathbf{X})-F_{RV}(Td`,\mathbf{X})]-[F_{OM}(Td,\mathbf{X})-F_{OM}(Td`,\mathbf{X})]}{\sum_{i=1}^{j}\sum_{k=1}^{n}[\mathbf{Rv}_k]_i\big|_{Td`<[\mathbf{Rt}_k]_i\leq Td}+\sum_{i=1}^{j}\sum_{z=1}^{p}[\mathbf{Iv}_z]_i\big|_{Td`)<[\mathbf{It}_z]_i\leq Td}} \qquad (4.83)$$

- Construction completion time

  Early starts and finish times of any work package in the CE component are determined using the critical path to the work package, the duration of the work package and the *finish-to-start* lead/lag times between the work packages on the critical path to the work package. Construction completion time is determined as the early finish time of the last work package that has no successor in the *CPM*-network of the CE component. Letting j represent the last work package in the CE component, n the number of work packages on the critical path to j, $\overline{wd}_{j,i}$ the duration converted to GTU of work package i on the critical path to j, $\overline{fs}_{j,i}$ the finish-to-start lead/lag time converted to GTU of work packages i on the critical path to j, and h the early start of the start work package that has no predecessor in the *CPM*-network (see "Start Period of the first/start work package" in Figure 4.4), then the early finish of work package j can be expressed as follows:

$$EFc_j = h + \sum_{i=1}^{n} \left(\overline{wd}_{j,i} + \overline{fs}_{j,i} \right) \tag{4.84}$$

The foregoing explained the set of performance measures modeled in the generalized model. Two points needs emphasizing here. First, the calculation for any performance measure acknowledges the contract duration property (see "Contract Duration" in Figure 4.3 and Section 4.2.2) that can be defined for a project scenario. For example, if a revenue stream has an early finish larger than the contract duration, then the discounted cost of the stream will consider only the time period up to the specified contract duration.

Second, the evaluation of the foregoing performance measures depends on the values of the variables used in modeling a project alternative or scenario and is referred to as deterministic fixed-value analysis. As the value of a variable can be described by various methods, the

"Deterministic" method/option can be used to assign a variable with a constant value throughout all analyses. For example, the lead/lag time and duration of the OM stream in Figure 4.32 were defined as deterministic and labeled by μ. If any or all variables in the generalized economic model are modeled probabilistically (detailed in the next chapter) and a deterministic and/or probabilistic analysis is required for the foregoing performance measures, then the *expected value* of the probabilistic variables will be used in all computations.

4.8 Brief Summary

This chapter introduced a generalized economic model that contributes to filling demonstrable gaps in the economic modeling of capital investment projects. To cope with the various characteristics of different industries, the concept of classification was introduced in order to model the properties and methods of four domains in capital investment projects namely, capital expenditure, revenue, operation and maintenance and financing. The economic model was developed with a multipurpose hierarchical time function structure that integrates the four classification domains. The structure is made of four components and each component is made of constructs (e.g. work packages, revenue streams, and debt streams). A component represents a classification domain and a construct represents a cash flow time function. Each new construct inherits the properties and methods of its classification and its cash flow can use any of such properties and methods. Each variable in a method, or alternatively in the model, can change its value over time through the use of shape functions. With this model structure, several cash flows and performance measures were derived.

Chapter 5

Risk Analysis Framework

5.1 General

The generalized economic model, as presented in the previous chapter, described a complex economic structure comprised of four components representing the properties and methods of four domain areas in capital investment projects. The model translates the estimating methods used in each construct of a component into a cash flow profile. The model then integrates the cash flows of the constructs and the four components into several performance measures. The formulation of the performance measures in Eqs. 4.73 to 4.84 represent complete project *system functions* the evaluation of which provides a value or set of values with which a conclusion or a statement on project viability can be made and financing and investing decisions can be taken.

Given complete and accurate information on the values or inputs to an economic model, conclusions and decisions based on a project *system function* can be made with *certainty*, a situation that rarely occurs in capital investment projects. Systems can have several sources of uncertainty that lead to the uncertainty in the outcomes. Therefore, conclusions and consequently any decision based on the system performance are reached under conditions of risk or uncertainty. Bury (1999) and Ang and Tang (1975; 1990) explained several sources of uncertainty in models and systems including: data uncertainty due to the inherent variability of the quantities/variables being measured or estimated (e.g. duration of a construction task) in the system; statistical uncertainty due a limited sample size or limited amount of information about the variable being measured; and, model uncertainty since models represent an abstract representation of the actual world or real life. It is clear that the analysis of investment projects which involves making forecasts of events far into the future involves high levels of uncertainty of the first type. Figure 5.1, for example, shows how forecast revenues deviated from actual revenues on the Dallas North Tollway (Dedeitch et. al 1993), thereby illustrating the uncertainties involved in modeling traffic demand which forms the basis for revenue estimates.

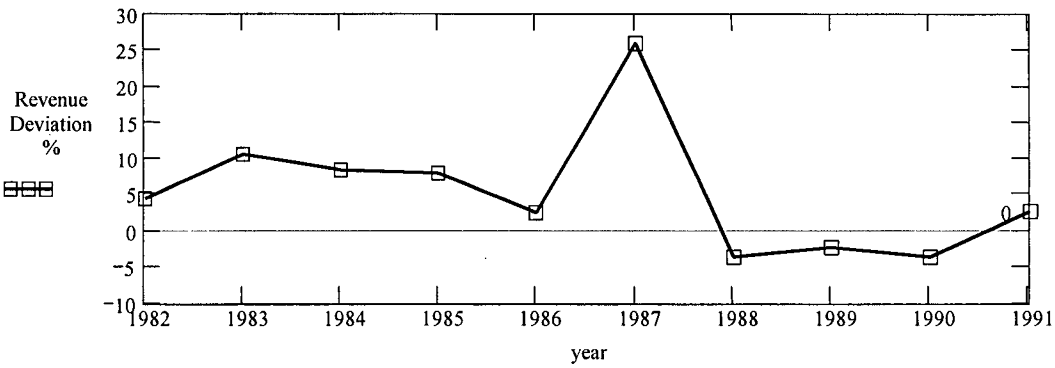


Figure 5.1: Deviation between forecasted and actual revenues, (Dedeitch et. al 1993)

Recognizing that uncertainties exist, appraisal studies usually involve sensitivity and probabilistic risk analyses as an essential complementary part to economic models. Risk analysis is performed on the economic model to provide a picture of the behavior of the performance measures under conditions of uncertainty, with the objectives being to achieve more informed decisions on the project and to establish appropriate risk management strategies. To date, several *project risk management* frameworks have been developed by a number of researchers to address the risk and uncertainty in projects. The main steps of these frameworks include risk identification, risk analysis/quantification, and risk response (APM 1997; ICE et al. 1998; PMI 1996; Chapman and Ward 1997; Thompson and Perry 1992; Hertz and Thomas 1983; Perry and Hayes 1985; Tummala and Burchett 1999; Al-Bahar and Crandall 1990).

The objective of this chapter is to present an analytical two and four moment risk analysis framework for the quantification of the uncertainty of performance measures. The framework provides considerable flexibility in modeling the uncertainty of the variables in the generalized model. Since the triangular distribution is commonly used in risk analysis by simulation, the third and fourth moment of this distribution are introduced in order to use them in the framework. A rigorous and expanded derivation for the four moments of a system function is introduced in order to enhance their accuracy to more than that of the standard four-moment approach. Pearson and Schmeiser-Deutsch distribution families are used to qualify/fit a distribution model for a performance measure based on its moments. The framework models the uncertainty of all the performance measures in Eqs. (4.73-4.84). The next sections describe the structure of the risk analysis framework. The last section is devoted to sensitivity analysis.

5.2 Analytical Risk Analysis Framework

The objective of the risk analysis framework is to derive the probabilistic characteristics of the generalized model performance measures given knowledge of the probabilistic characteristics of the variables in the model. In such probabilistic estimating, a performance measure is considered as a random variable or *system function* formulated using *base* or *primary* random variables.

Generally, it is difficult to obtain the exact probability distribution of a system function except for special and simple cases (Hahn and Shapiro 1967; Siddall 1972; Ang and Tang 1975). Therefore, approximate methods are typically adopted to provide probability distributions and/or probabilistic characteristics of the system function. The risk analysis framework presented herein adopts the analytical two and four-moment approaches to derive the probabilistic characteristic (moments) of the performance measures. An expanded formulation, however, is derived for the four moments of a system function to enhance the accuracy of such moments. The framework then automatically determines a probability distribution by fitting the moments to one of the distributions in the Pearson and Schmeiser-Deutsch (S-D) systems of frequency curves. Unlike previous literature that uses percentile tables to fit the four moments to a Pearson distribution, the framework developed herein uses the moments of a system function to mathematically determine which Pearson distribution to use and then to obtain its parameters, which provides higher generality and accuracy than using the tables.

Since the generalized model is highly detailed, the risk analysis framework assumes that no correlation exists between model variables, but it does not assume they are independent. Correlation is a significant element but its inclusion would have made the framework much more complicated since the number of correlation coefficients would be extremely large and would further increases with any new construct added to the model. Therefore, if the variables of a project are judged to be highly correlated with most of the correlations being of the same sign, then the framework could produce results that lead to wrong interpretations particularly if the project economic cash flow structure is highly disaggregated. As explained by Pouliquen (1970) for risk analysis of some projects appraised by the World Bank:

"Therefore the choice of the level of aggregation requires a trade-off between the advantages of clarity of judgment and of avoiding the hazards of disaggregation. It is a difficult choice and one often guided by the availability of time. Because we believe that the influence of correlations on the outcome of the analysis is more important than the influence of the shape of any particular distribution, we have usually opted for as little disaggregation as possible."

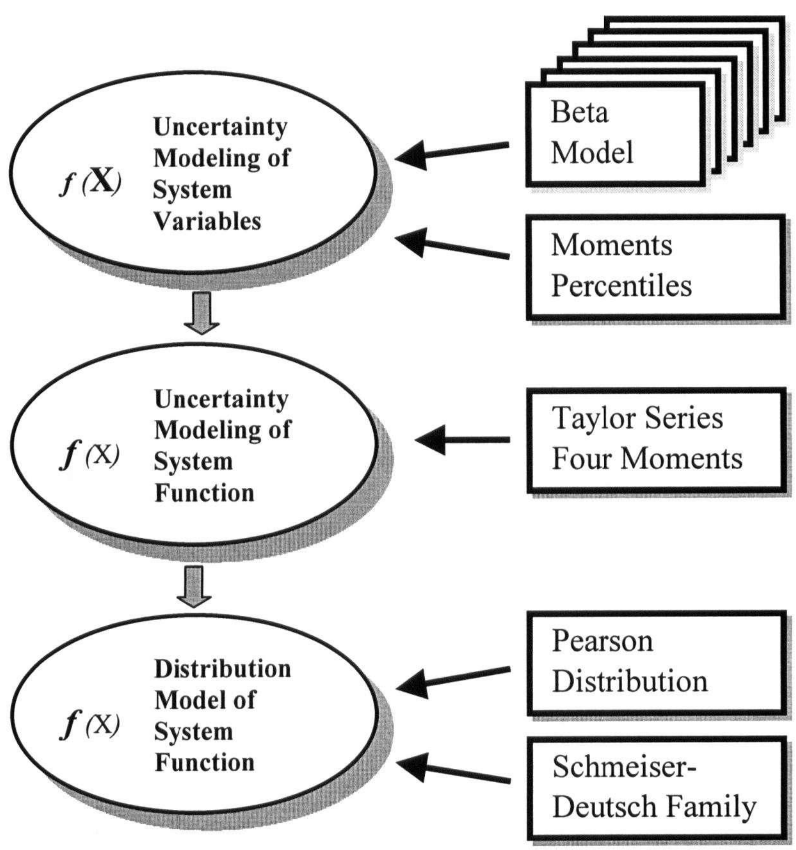


Figure 5.2: Probabilistic risk analysis framework

5.2.1 Probabilistic Characteristics of Model Variables

As explained in the formulation of the generalized economic model, all model variables in the four components are included in the constructs' $\boldsymbol{x}$ vectors. Since most of these variables (as used in estimating and cash flow calculations) are represented by *rate* or *area* functions, each of the variables is represented by a number of sub-variables $\boldsymbol{y}$ describing the relevant *rate/area* function $f_S(t, \boldsymbol{y})$ as shown in Tables 4.1, 4.2 and 4.5. Uncertainty modeling in the risk analysis framework proceeds to cover not only all of the variables in the $\boldsymbol{x}$ vectors, but also all of the sub-variables in the $\boldsymbol{y}$ vectors. This provides the analyst with the capability to model the uncertainty in each and all sub-variables in the formulation of the four components (see Table 4.12 and Eqs. 4.73-4.84).

The uncertainty of a variable can be modeled in a probabilistic manner in three approaches (Ang and Tang 1990): (1) objectively based on a set of observed data, (2) subjectively based on judgments of decision-makers or experts; or, (3) combination of both observed and subjective information, i.e. a Bayesian approach. In economic risk analysis, which concerns time, cost and economic performance of projects, the three types can generally be found, however, subjective probability is commonly used due to the general lack of objective data about future events. Modeling uncertainty in this case involves a probability elicitation framework along with the quantification of uncertainties using distribution models. The objective of elicitation frameworks is to produce accurate, calibrated and coherent subjective probabilities (Ranasinghe and Russell 1993; Cooper and Chapman 1987;Tversky and Kahneman 1974; Mak and Raftery 1992).

Quantification of uncertainty can be made in several ways according to the amount of information available about the variable in question. To ensure the generality aspect of the economic model, the risk analysis framework embraces 16 probabilistic models defined in four categories, as defined below:

1. <u>Full specification of probability distributions</u>

Several statistical distribution models can be used to describe the probabilistic behavior of an uncertain variable. Distribution models such as Normal, Log Normal, Beta, and uniform distributions have been used in the risk analysis of duration, cost, traffic and revenue data of projects (Mulholland and Christian 1999; Wall 1997; Cottrell 1999; Lam and Tam 1998; Pouliquen 1970; Touran and Wiser 1992; Chau 1995; AbouRizk and Halpin 1992; AbouRizk et al. 1993). Generally, the choice of a distribution function may be motivated by the properties of the process being modeled which suggests the form of the distribution a priori, or it can be determined empirically from observed data. Bury (1999) explained, for example, that the Gamma model can be postulated for a random variable if it is considered to be the sum of a small number of underlying causes (Gamma reproductive property), and when the number of causes increases, the variable approaches the Normal model (using the Central Limit Theorem). Similarly, Bury (1999) explained that the Beta model would be a good candidate for cases (e.g. cost variables and task completion times) in which unlimited tails of a distribution are not appropriate.

Eleven probability distributions are included in the risk analysis framework for modeling any variable or sub-variable in the generalized model. It is assumed that a distribution has been tested for fit and its parameters estimated (Bury 1999). With the distributions described below, the expected value μ'_1 describes the central, average or mean value of the variable, and the second moment (variance) μ_2 describes the dispersion or variability of the values of variable around the expected value. The third central moment μ_3 describes the skewness (or asymmetry) characteristic of the variable (for example, the skewness coefficient [$\sqrt{\beta 1} = \gamma_1 = \mu_3 / \mu_2^{3/2}$] is of a zero value for the normal distribution and the uniform distribution, i.e. symmetrical, and a value of 2 for the exponential distribution, i.e. positively skewed), and the fourth central moment μ_4 describes the kurtosis characteristics conveying a description about the peakedness of the data about the mean value (for example, the kurtosis coefficient [$\beta_2 = \gamma_2 = \mu_4 / \mu_2^{\ 2}$] for the normal distribution has a value of 3, for the uniform distribution a value of 1.8, and for the exponential distribution a value of 9).

The distributions, their shapes and parameters, and moments are presented in Table 5.1. These distributions are:

- Two and four parameter Beta distributions
- Two and three parameter Gamma distributions
- Two and three parameter Log Normal distributions
- Normal distribution
- Exponential distribution
- Gumbel distribution
- Chi-Squared distribution
- Uniform distribution

These distributions include those distributions that can be described with lower and/or upper limits (thresholds) (e.g. four parameter Beta), which are generally useful in modeling the time, cost, and economic variables in capital investment projects. Other than the few cases explained by Bury (1999) for choosing a specific probability distribution, the literature on economic risk analysis does not suggest choosing one distribution model over another. According the available data and information about the behavior of a variable, a distribution model can be selected to model the variable.

The risk analysis framework as implemented in the decision support system can accept either (1) the parameters of a distribution and then estimates the four moments correspondingly, or (2) the expected value, variance and distribution type and then estimates the third and fourth moments correspondingly. Therefore, each of the above eleven distribution in Table 5.1 is represented twice in the system.

Table 5.1: Four moments of probability distributions in the risk analysis framework, (Bury 1999; Evans et al. 1993; Johnson et al. 1994)

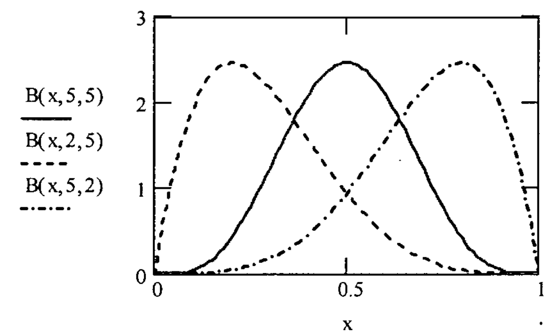
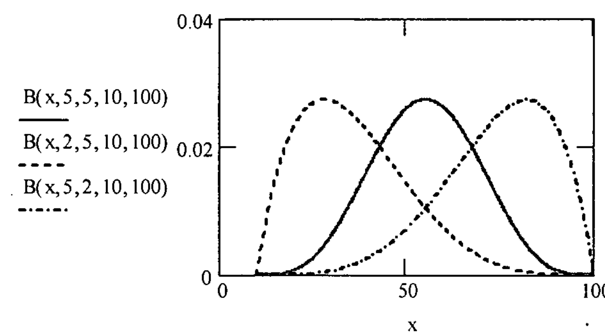
Distribution Model and Parameters	First Four Moments μ, μ_2, μ_3, μ_4	Distribution Form
Notes: γ_1: Skewness coefficient where γ_2: kurtosis coefficient where	$\mu_3 = \gamma_1 \cdot \mu_2^{3/2}$ $\mu_4 = \gamma_2 \cdot \mu_2^{2}$	
1. Beta, Two Parameters $\frac{\Gamma(\lambda 1+\lambda 2)}{\Gamma(\lambda 1)\cdot\Gamma(\lambda 2)}\cdot x^{\lambda 1-1}\cdot(1-x)^{\lambda 2-1}$ λ1, λ2: Shape parameters λ1, λ2 > 0 $0 \le x < \le 1$	$\mu = \frac{\lambda 1}{\lambda 1+\lambda 2}$ $\mu_2 = \frac{\lambda 1\cdot\lambda 2}{(\lambda 1+\lambda 2)^2\cdot(\lambda 1+\lambda 2+1)}$ $\gamma_1 = \frac{2\cdot(\lambda 2-\lambda 1)}{(\lambda 1+\lambda 2+2)}\cdot\sqrt{\frac{\lambda 1+\lambda 2+1}{\lambda 1\cdot\lambda 2}}$ $\gamma_2 = \frac{3\cdot(\lambda 1+\lambda 2+1)\cdot\left[2\cdot(\lambda 1+\lambda 2)^2+\lambda 1\cdot\lambda 2\cdot(\lambda 1+\lambda 2-6)\right]}{\lambda 1\cdot\lambda 2\cdot(\lambda 1+\lambda 2+2)\cdot(\lambda 1+\lambda 2+3)}$	
2. Beta, Four Parameters $\frac{\Gamma(\lambda 1+\lambda 2)}{\Gamma(\lambda 1)\cdot\Gamma(\lambda 2)}\cdot\left(\frac{x-\chi 1}{\chi 2-\chi 1}\right)^{\lambda 1-1}\cdot\left(1-\frac{x-\chi 1}{\chi 2-\chi 1}\right)^{\lambda 2-1}\cdot\frac{1}{\chi 2-\chi 1}$ λ1, λ2: Shape parameters χ_1, χ_2: Location parameters λ1, λ2 > 0 and $\chi_1 \le x \le \chi_2$ PERT: χ_1 (pessimistic); χ_2 (optimistic); λ1, λ2 (1.59, 4.41 +ve skew) or (4.41,1.59 –ve skew), Bury (1999).	$\mu = \chi 1 + (\chi 2-\chi 1)\cdot\frac{\lambda 1}{\lambda 1+\lambda 2}$ $\mu_2 = \frac{(\chi 2-\chi 1)^2\cdot\lambda 1\cdot\lambda 2}{(\lambda 1+\lambda 2)^2\cdot(\lambda 1+\lambda 2+1)}$ γ_1, γ_2 as in Beta 2-parameter	

Table 5.1: continued (Bury 1999; Evans et al. 1993; Johnson et al. 1994)

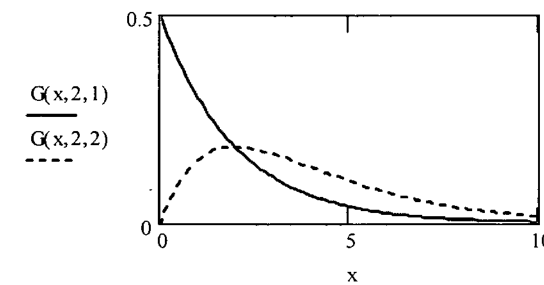
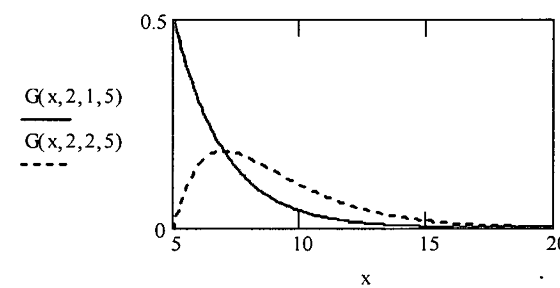
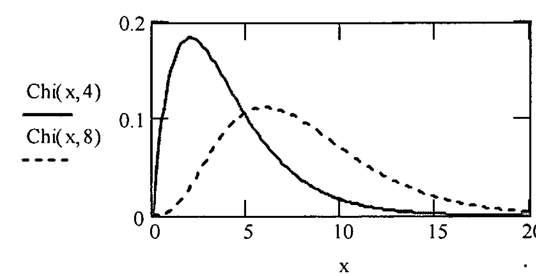
Distribution Model and Parameters	First Four Moments μ, μ_2, μ_3, μ_4	Distribution pdf Form
3. Gamma Two Parameters $\frac{1}{\sigma\cdot\Gamma(\lambda)}\cdot\left(\frac{x}{\sigma}\right)^{\lambda-1} exp\left(-\frac{x}{\sigma}\right)$ σ : Scale parameter λ : Shape parameter $\sigma, \lambda > 0$ $x \geq 0$	$\mu = \sigma\cdot\lambda$ $\mu_2 = \sigma^2\cdot\lambda$ $\gamma_1 = \frac{2}{\sqrt{\lambda}}$ $\gamma_2 = 3\cdot\left(1+\frac{2}{\lambda}\right)$	
4. Gamma Three Parameters $\frac{1}{\sigma\cdot\Gamma(\lambda)}\cdot\left(\frac{x-\chi}{\sigma}\right)^{\lambda-1} exp\left(-\frac{x-\chi}{\sigma}\right)$ σ : Scale parameter λ : Shape parameter χ : Location parameter $\sigma, \lambda > 0$ and $x \geq \chi$	$\mu = \chi + \sigma\cdot\lambda$ $\mu_2 = \sigma^2\cdot\lambda$ γ_1, γ_2 as in Gamma 2-parameters	
5. Chi-Squared Distribution [same as Gamma (2par.) with λ=ν/2 and σ=2] $\frac{1}{\sigma\cdot\Gamma(0.5\cdot\nu)}\cdot\left(\frac{x}{2}\right)^{0.5\cdot\nu-1} exp\left(-\frac{x}{2}\right)$ ν : parameter for degrees-of-freedom $\nu > 0$ $x \geq 0$	$\mu = \nu$ $\mu_2 = 2\cdot\nu$ $\gamma_1 = 2^{3/2}\cdot\nu^{-1/2}$ $\gamma_2 = 3 + (12/\nu)$	

Table 5.1: continued (Bury 1999; Evans et al. 1993; Johnson et al. 1994)

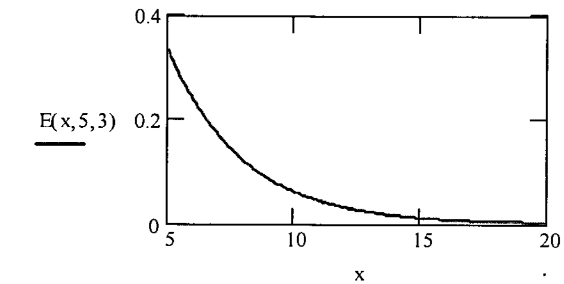
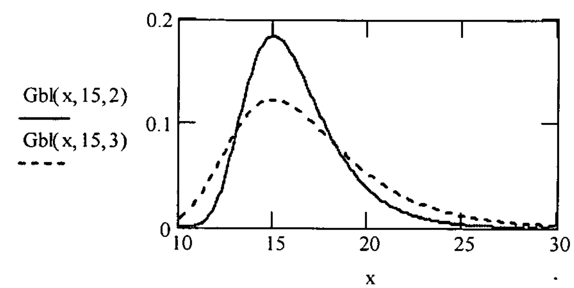
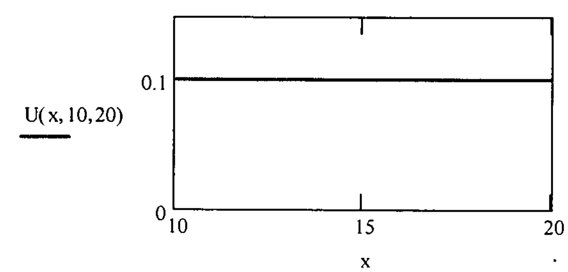
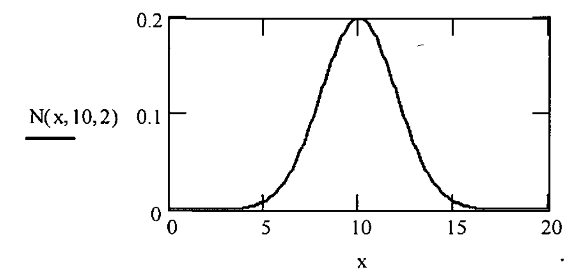
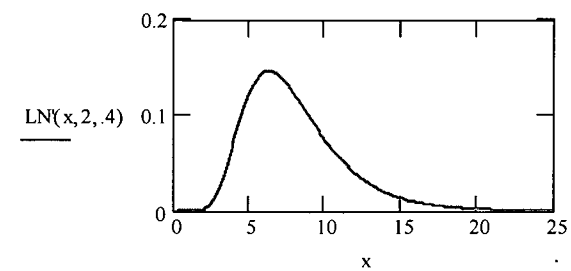
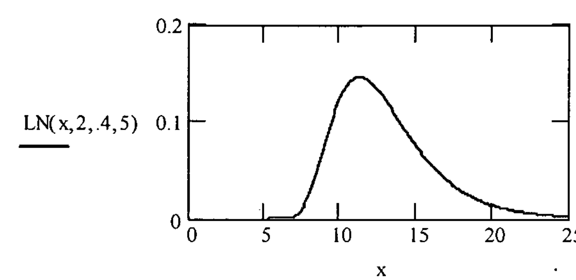
Distribution Model and Parameters	First Four Moments μ, μ_2, μ_3, μ_4	Distribution pdf Form
6. Exponential Distribution $\frac{1}{\sigma} exp\left(-\frac{x-\chi}{\sigma}\right)$ χ : Location parameter σ : Scale parameter $\chi \geq 0$, $\sigma > 0$ $x \geq \chi$	$\mu = \chi + \sigma$ $\mu_2 = \sigma^2$ $\gamma_1 = 2$ $\gamma_1 = 9$	
7. Gumbel Distribution $\frac{1}{\sigma} exp\left(-\frac{x-\xi}{\sigma} - exp\left(-\frac{x-\xi}{\sigma}\right)\right)$ ξ : Location parameter σ : Scale parameter $\xi < \infty$, $\sigma > 0$ $x > -\infty$	$\mu = \xi + 0.57722 \cdot \sigma$ $\mu_2 = 1.64493 \cdot \sigma^2$ $\gamma_1 = 1.139547$ $\gamma_1 = 5.4$	
8. Uniform Distribution $\frac{1}{max - min}$ min, max: Location parameters $0 \leq min < max$ $min \leq x \leq max$	$\mu = (min + max) / 2$ $\mu_2 = (max - min)^2 / 12$ $\gamma_1 = 0$ $\gamma_2 = 1.8$	

Table 5.1: continued (Bury 1999; Evans et al. 1993; Johnson et al. 1994)

Distribution Model and Parameters	First Four Moments μ, μ_2, μ_3, μ_4	Distribution pdf Form
9. Normal Distribution $\frac{1}{\sigma\sqrt{2\pi}} exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$ μ : Location parameter σ : Scale parameter $\mu < \infty$, $\sigma > 0$ $x > -\infty$	$\mu = \mu$ $\mu_2 = \sigma^2$ $\gamma_1 = 0$ $\gamma_1 = 3$	
10. Log-Normal Two Parameters $\frac{1}{x\sigma\sqrt{2\pi}} exp\left(-\frac{1}{2}\left(\frac{ln(x)-\xi}{\sigma}\right)^2\right)$ ξ : where $exp(\xi)$ is scale parameter σ : Shape parameter $-\infty < \xi < \infty$, $\sigma > 0$ and $x > 0$	$\mu = exp(\xi + \sigma^2/2)$ $\mu_2 = exp(2 \cdot \xi + \sigma^2) \cdot [exp(\sigma^2) - 1]$ $\gamma_1 = c^3 + 3 \cdot c$ $\gamma_1 = c^8 + 6 \cdot c^6 + 1.5 \cdot c^4 + 16 \cdot c^2 + 3$ where: $c^2 = [exp(\sigma^2) - 1]$	
11. Log-Normal Three Parameters $\frac{1}{(x-\chi)\sigma\sqrt{2\pi}} exp\left(-\frac{1}{2}\left(\frac{ln(x-\chi)-\xi}{\sigma}\right)^2\right)$ ξ : where $exp(\xi)$ is scale parameter σ : Shape parameter χ : Location parameter $-\infty < \xi < \infty$, $\sigma > 0$ and $x > \chi$	$\mu = \chi + exp(\xi + \sigma^2/2)$ $\mu_2 = exp(2 \cdot \xi + \sigma^2) \cdot [exp(\sigma^2) - 1]$ $\gamma_1 = c^3 + 3 \cdot c$ $\gamma_1 = c^8 + 6 \cdot c^6 + 1.5 \cdot c^4 + 16 \cdot c^2 + 3$ where: $c^2 = [exp(\sigma^2) - 1]$	

2. Two and four moments

Along with the above distribution models, direct specification of the expected value and the second to fourth central moments can be made in order to describe probabilistic characteristics of any variable. Working directly with the moments of random variables may be motivated by the insufficiency of information to postulate a probability model; a desire to work with descriptors of a random variable rather than with a complete distribution, or as a necessity under the analytical risk analysis approach. Generally, estimates for the four moments can be made subjectively by expert or experienced analyst, or, computed after specification of a probability model for a variable.

The uncertainty of a variable in the risk analysis framework can be defined directly by the first four moments or alternatively by the first two moments. If only two moments are used, the framework considers the third and fourth moments to have zero value.

3. Three and five percentiles

The third category of methods for modeling uncertainty describes two methods for estimating an approximate mean and variance for a random variable using three or five percentile values as a substitute for the probability distribution of the random variable.

Based on an observation regarding the ratio of the distances between a number of symmetrical percentage points (e.g. 5^{th} and 95^{th} percentiles) to the standard deviation for a set of distributions in the Pearson family, Pearson and Tukey (1965) suggested that useful approximations for the expected value and standard deviation could be obtained. The distance between the symmetrical points may be expressed as:

$$h\%\ \text{distance} = \frac{(100-h)\% - h\%}{\sigma} \qquad (5.1)$$

The authors found that there was constancy for this ratio particularly for the 5 and 2.5% distances (around 3.25 and 3.92, respectively) among the set of Pearson distributions in the study. The first approximations for the standard deviation using these points were:

$$\hat{\sigma}_{0.05} = \frac{[95\%]-[5\%]}{3.25} \tag{5.2}$$

$$\hat{\sigma}_{0.025} = \frac{[97.5\%]-[2.5\%]}{3.92} \tag{5.3}$$

To apply the above two approximations to a several of the distributions in the Pearson family, the approximation was corrected by a factor Δ that included the 50% percentile in the approximations to cover distributions that have their skewness β1 and kurtosis coefficients β2 in a β1β2 area represented by β1: from 0.0 to 4.0 and β2− β1−1= 0.0 to 15.0. The correction factor Δ can be expressed as

$$\Delta = [95\%]+[5\%]-2\cdot[50\%] \tag{5.4}$$

The standard deviation is obtained as the maximum of the standard deviations calculated using the 5% and the 2.5% percentiles version of approximations and an iterative process that starts with the values of standard deviation calculated by Eqs. 5.2 and 5.3:

$$\hat{\sigma}'_{0.05} = \frac{[95\%]-[5\%]}{max\left\{3.29-0.1\cdot\left[\dfrac{\Delta}{\hat{\sigma}'_{0.05}}\right]^2, 3.08\right\}} \tag{5.5}$$

$$\hat{\sigma}'_{0.025} = \frac{[97.5\%]-[2.5\%]}{max\left\{3.98-0.138\cdot\left[\dfrac{\Delta}{\hat{\sigma}'_{0.025}}\right]^2, 3.66\right\}} \tag{5.6}$$

The second central moment, then, can be calculated as follows:

$$\mu_2 = \left[max\left(\hat{\sigma}'_{0.05},\ \hat{\sigma}'_{0.025}\right)\right]^2 \tag{5.7}$$

The approximation suggested by Pearson and Tukey (1965) for the expected value was

$$\mu = [50\%]+0.185\cdot\Delta \tag{5.8}$$

As mentioned by Pearson and Tukey (1965), the error in the approximation for the expected value is not more than 0.1% for a large part of the $\beta 1 \beta 2$ area mentioned above, and for the standard deviation the error is less than 0.5% in the same area. Keefer and Verdini (1993), and Keefer and Bodily (1983), after comparing several approximations for the expected value and standard deviations, found that the Pearson and Tukey approximations were far more accurate than other approximations. Further, they suggested that an effective approximation for the standard deviation could be obtained through using Eq. 5.5 only without the iteration process and without the need to elicit the hard to obtained 2.5% and 97.5% percentile values. Pfeifer et al. (1991) reached similar conclusions on the accuracy of the Pearson and Tukey approximations. Lau et al. (1998) produced similar approximations that relied on the availability of several percentile values (seven to eight values), which improved the accuracy of the approximation after being compared to the Pearson and Tukey approximations.

In the risk analysis framework, the expected value and second central moment (variance) can be calculated using the Pearson and Tukey approximations if percentile values are selected as the method for modeling the uncertainty of a variable. Given the appropriate three percentiles, Eqs. 5.5 and 5.8 are used for the approximation. Given five percentiles, Eqs. 5.7 and 5.8 are used for the approximation, and the iteration process implied in the approximation has been implemented. Under both methods, the third and fourth central moments have to be specified directly since no approximations using percentiles are available in the literature for these higher moments.

4. <u>Triangular distribution</u>

In the absence of data, a minimum amount of information can be used in constructing what is called the *Triangular* distribution in order to represent the probability distribution or probabilistic behavior of a random variable. Like the distributions in the first category, the triangular distribution is commonly used in risk analysis. However, its validity has been subject to investigation since Chau (1995a,b) explained that the probability of exceeding the most likely value using this distribution is higher than what experts estimated subjectively.

The distribution is defined by three estimates: a minimum (pessimistic) L, most likely M, and maximum (optimistic) H values of a random variable:

$$f(x) = \begin{cases} f_{LM}(x) = \dfrac{2\cdot(x-L)}{(H-L)\cdot(M-L)} & \text{if } L \le x < M \\ f_{MH}(x) = \dfrac{2\cdot(H-x)}{(H-L)\cdot(H-M)} & \text{if } M \le x \le H \end{cases} \tag{5.9}$$

For this distribution model, the expected value and variance can be computed as follows (Ang and Tang 1975), see Figure 5.3:

$$\mu = (L + M + H)/3 \tag{5.10}$$

$$\mu_2 = (L^2 + M^2 + H^2 - LH - MH - ML)/18 \tag{5.11}$$

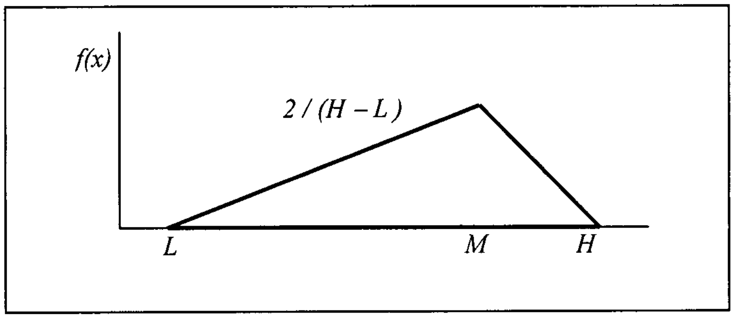


Figure 5.3: Triangular distribution

However, to use the triangular distribution for a four-moment analysis in the risk analysis framework, its third and fourth moments must be derived. The *k-th* central moment of a random variable X that has an expected value μ is defined as:

$$\mu_k = E(X-\mu)^k = \int_x (x-\mu)^k \cdot f(x)dx \tag{5.12}$$

The third and fourth moments μ_3 and μ_4 can be obtained using this definition, the distribution model in Eq. 5.9, and the expected value as computed in Eq. 5.10. The third moment can be expressed as

$$\mu_3 = E(X-\mu)^3 = \int_L^M (x-\mu)^3 \cdot f_{LM}(x)dx + \int_M^H (x-\mu)^3 \cdot f_{MH}(x)dx \tag{5.13}$$

which finally leads to:

$$\mu_3 = \frac{-1}{270} \cdot (L + H - 2 \cdot M) \cdot (M + L - 2 \cdot H) \cdot (M + H - 2 \cdot L) \tag{5.14}$$

Similarly, the fourth moment can be obtained as follows:

$$\mu_4 = E(X-\mu)^4 = \int_L^M (x-\mu)^4 \cdot f_{LM}(x)dx + \int_M^H (x-\mu)^4 \cdot f_{MH}(x)dx \tag{5.15}$$

$$\mu_4 = \frac{1}{135} \cdot \left(L^2 + H^2 + M^2 - L \cdot M - L \cdot H - H \cdot M\right) \tag{5.16}$$

For example, if L =10, M=20, and H=30, then symmetrical close to normal values would be obtained: $\mu = 20$, $\mu_2 = 16.667$, $\mu_3 = 0$, $\mu_4 = 666.667$, $\sqrt{\beta 1} = \gamma_1 = 0$, and $\beta 2 = \gamma_2 = 2.4$.

If L =10, M=20, and H=40, then a positively skewed distribution would be obtained: $\mu = 23.33$, $\mu_2 = 38.889$, $\mu_3 = 74.074$, $\mu_4 = 3.63E{+}3$, $\sqrt{\beta 1} = \gamma_1 = 0.305$, and $\beta 2 = \gamma_2 = 2.4$.

If L =0, M=20, and H=30, then a negatively skewed distribution would be obtained: $\mu = 16.667$, $\mu_2 = 38.889$, $\mu_3 = -74.074$, $\mu_4 = 3.63E{+}3$, $\gamma_1 = -0.305$, and $\beta 2 = \gamma_2 = 2.4$.

The above four categories described a total of 16 methods with which to model the uncertainty of a random variable. These methods have been implemented in the decision support system. The following examples describe how the risk analysis framework addresses all time, cost, revenue and finance variables in the hierarchical structure of the generalized economic model. While the examples show the flexibility and ability of the framework to model any variable or parameter variable, it is up to the analyst to chose which variable to model as probabilistic and with which method. The analyst should be aware of the consequences of his choices.

All time variables such as duration and lead/lag times between constructs of the four components as shown deterministically in Figures 4.8 and 4.20 can be modeled probabilistically, as shown in Figure 5.4. In this figure, the duration of the CEid3 work package is modeled as *4 Parameter Beta* distribution, while the lead/lag time of one predecessor is modeled as a *Normal* distribution and the other is represented by a *Triangular* distribution.

In the generalized model, Cost variables can also be described probabilistically. Figure 5.5 shows a simple example showing the breadth of the framework in modeling the cost of a work package. The work package is estimated using the aggregated method of Eq. 4.4 (Section 4.3.2.1) that distributes the total cost of a work package over its duration according to any of the area functions/profiles in Table 4.2- in this case "Exponential III". This cost profile requires two variables, the total cost and the growth rate that determines the shape of the profile. These two variables are the "Parameter1" and "Parameter2" in Figure 5.5. The total cost (parameter 1) is modeled as *Log Normal* distribution with a minimum threshold value of $70·E6, expected value μ = $100·E6, and standard deviation $\sqrt{\mu_2}$ = $15·E, see Figure 6.6. While these three estimates can be defined directly to the system, Figure 5.5 shows how the distribution of the total cost is defined instead by its parameters: scale parameter $\xi = 17.105$, shape parameter $\sigma = 0.472$, and location parameter χ = $70·E6. The growth rate (parameter 2) in Figure 5.5 is modeled as a *Uniform* distribution with a minimum value of 1.9 and maximum value of 2.5; i.e. it has $\mu = 2.2$, and standard deviation $\sqrt{\mu_2} = 0.173$. In Table 4.2, the graph of the "Exponential III" function shows 3 shapes obtained based on the values of the parameters. Therefore, the choice of a probabilistic method for a variable, as described in the above example, should be made with care.

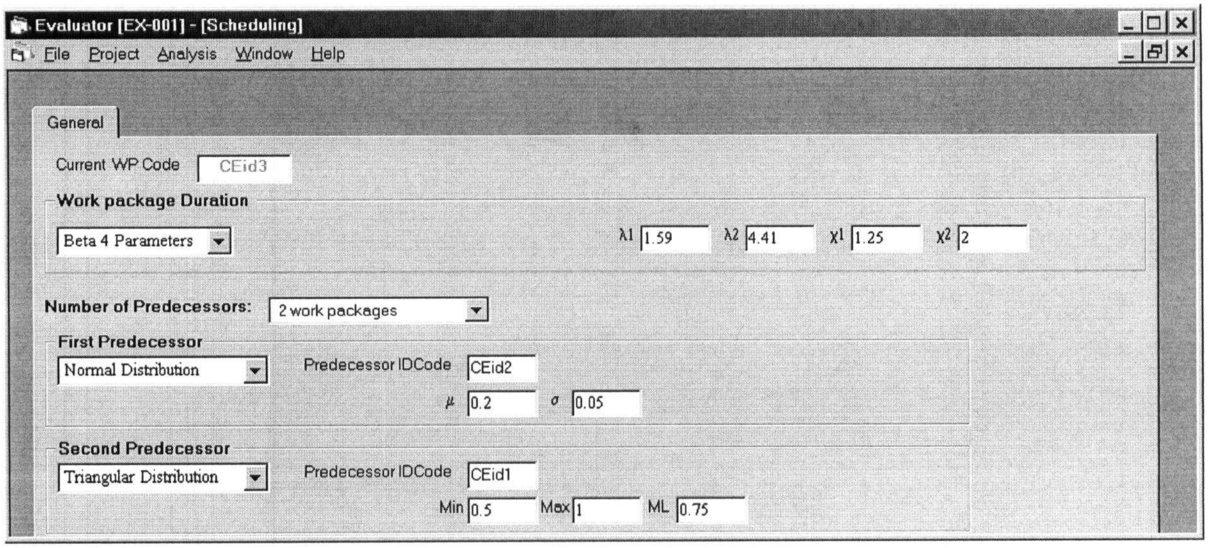


Figure 5.4: Modeling uncertainty of time variables

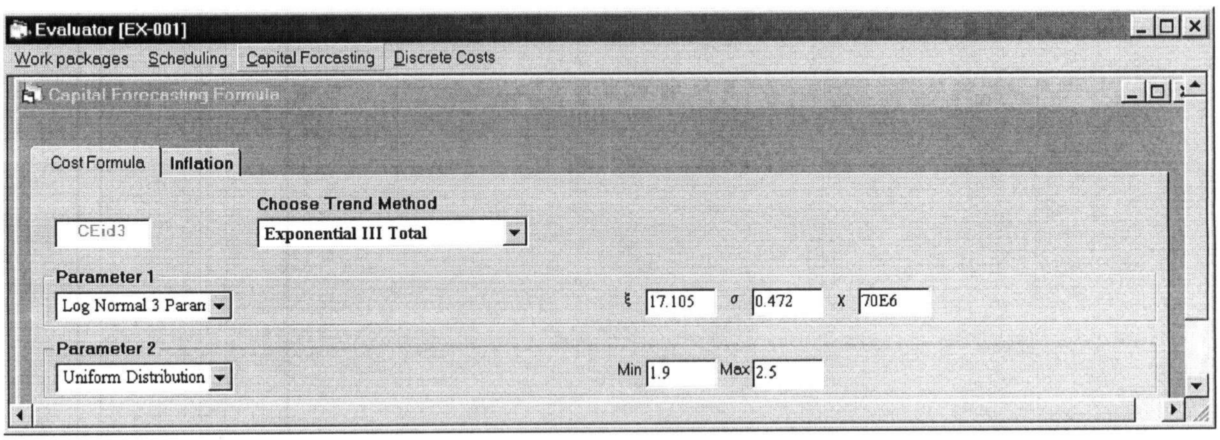


Figure 5.5: Modeling the uncertainty of a work package cost

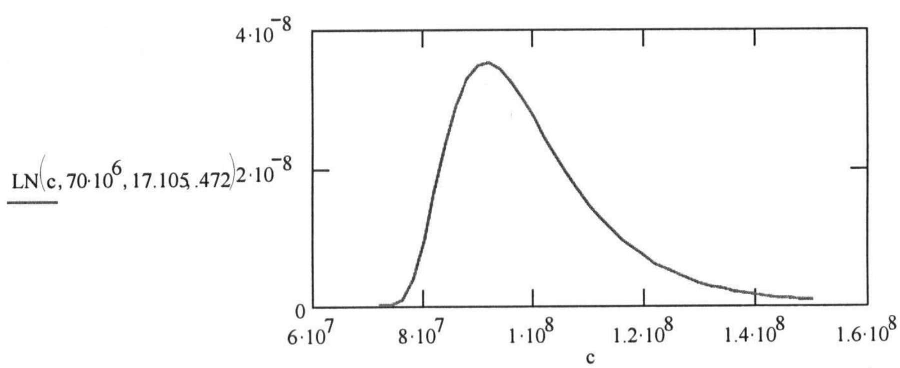


Figure 5.6: Uncertainty of total cost (Parameter1) modeled as Log Normal distribution

Similarly, _Revenue variables_ can be modeled as risk variables, including all service charge and demand variables in all revenues streams. This allows for the detailed evaluation of revenues under conditions of uncertainty (Lam and Tam 1998). Figure 5.7 shows how a revenue stream service charge (e.g. toll rate) can be modeled using a *step uniform rate function* (Table 4.1) where four tolls are used in four time periods and where both the rates and times are modeled deterministically and probabilistically using either the direct specification of moments or a specific distribution. In Figure 5.8 total traffic demand of a revenue stream is modeled as a linear rate function (see Eq. 4.21) where both the initial traffic volume (Parameter 1) and the traffic growth rate (Parameter 2) are modeled using Normal distributions. The value of demand captured by this revenue stream is modeled in Figure 5.9 using the constant elasticity method (see Eq. 4.22) in which the elasticity value itself is modeled using a Normal distribution.

Finance variables can be modeled probabilistically as well. Figure 5.10 shows how a 6-month floating rate Libor of a syndicated term loan is modeled as a sinusoidal rate function (Table 4.1) with the initial interest rate modeled using a Normal distribution (μ=6%, σ=0.5%), annual growth of interest rate modeled using a Normal distribution (μ=1%, σ=0.05%), extreme increase/decrease to the base rate (amplitude) modeled by a Triangular distribution, and a cycle length in which one peak and one drop interests are predicted every 4 years modeled using a Normal distribution (μ=4, σ=0.5). This is explained further in Figure 5.11.

Discrete variables can be modeled probabilistically as well. Figure 5.12 shows an O&M stream in which the values of two discrete costs are modeled as Triangular and 3-Parameter Gamma distributions with the application time of these discrete costs modeled as risk variables using Normal distributions. The Gamma model parameters are estimated for the discrete cost based on an assumption of minimum threshold cost χ of \$3·E6, expected value μ of \$5·E6 and standard deviation $\sqrt{\mu_2}$ of \$1·E6 (see Table 5.1); this gives the Gamma model as shown in Figure 5.13.

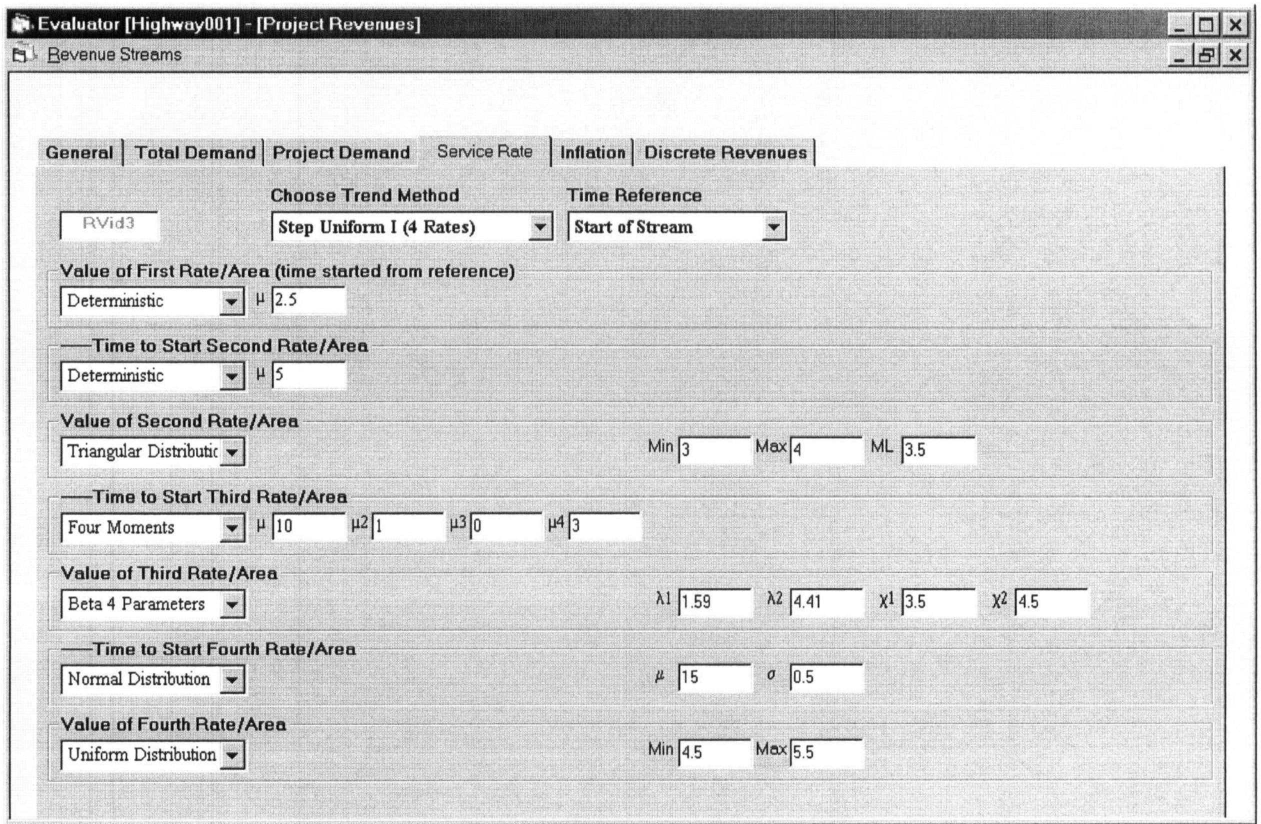


Figure 5.7: Modeling uncertainty of service rate in a revenue stream

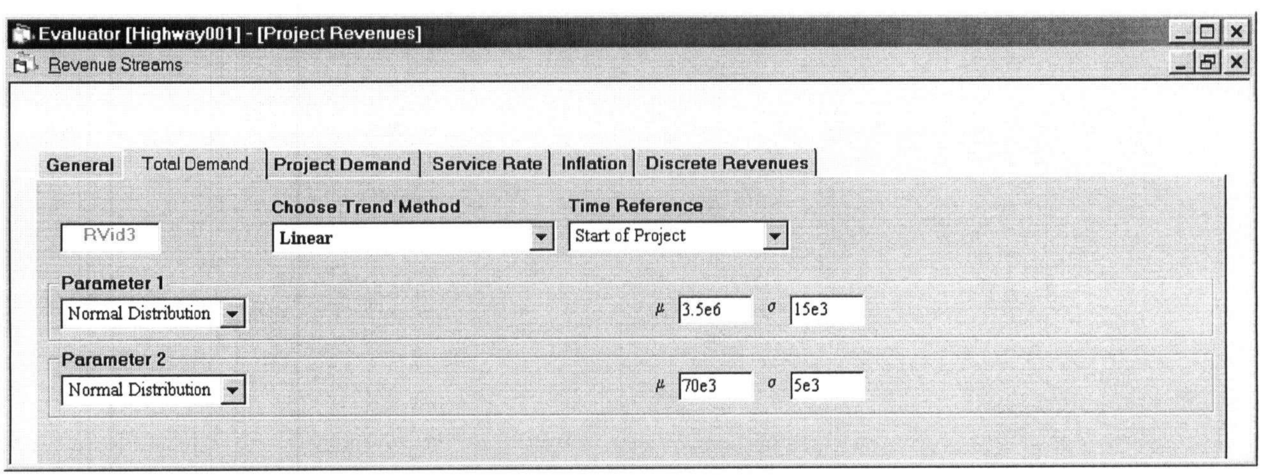


Figure 5.8: Linear demand modeled as an uncertain. Initial demand (Parameter 1) and growth rate (Parameter 2) are modeled with Normal distributions

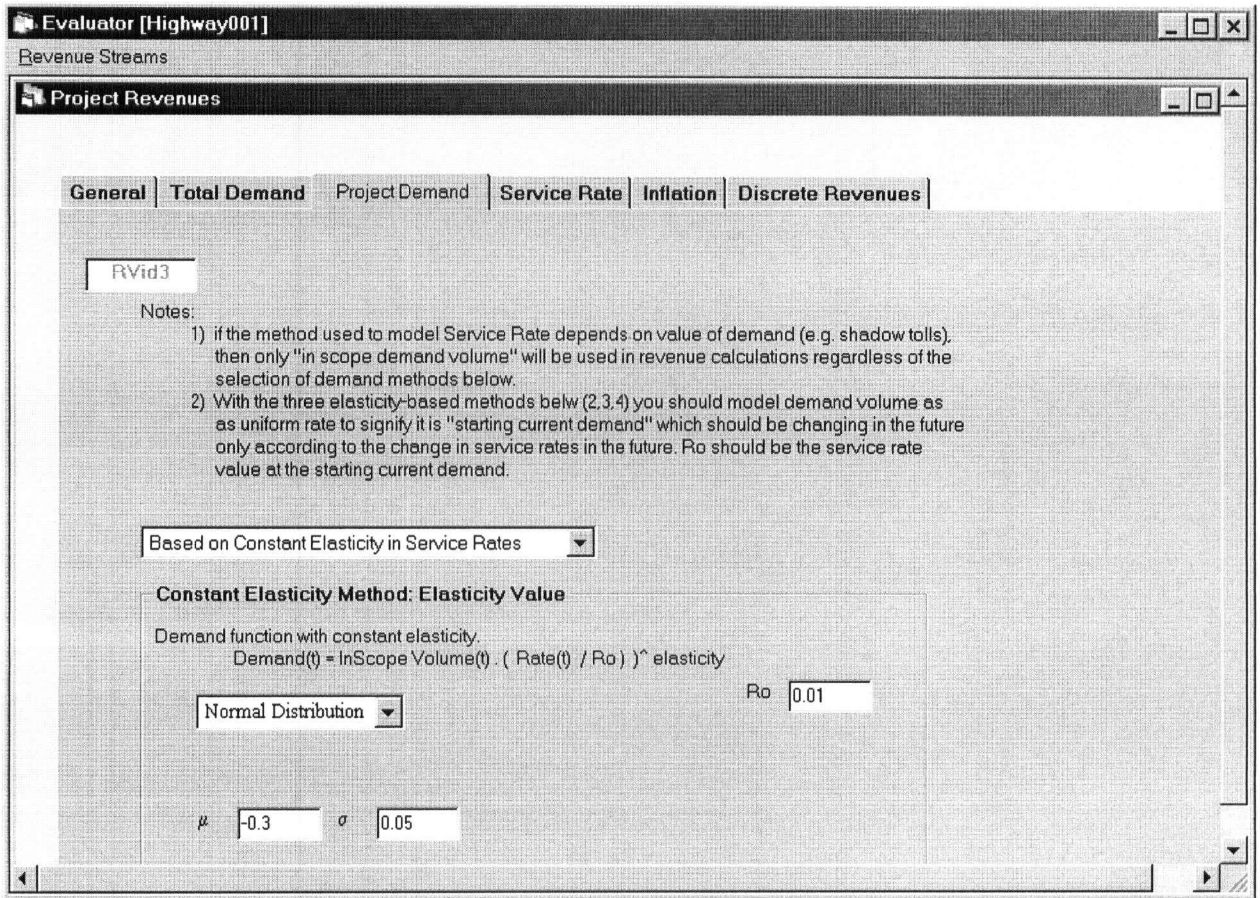


Figure 5.9: Elasticity of demand modeled using a Normal distribution

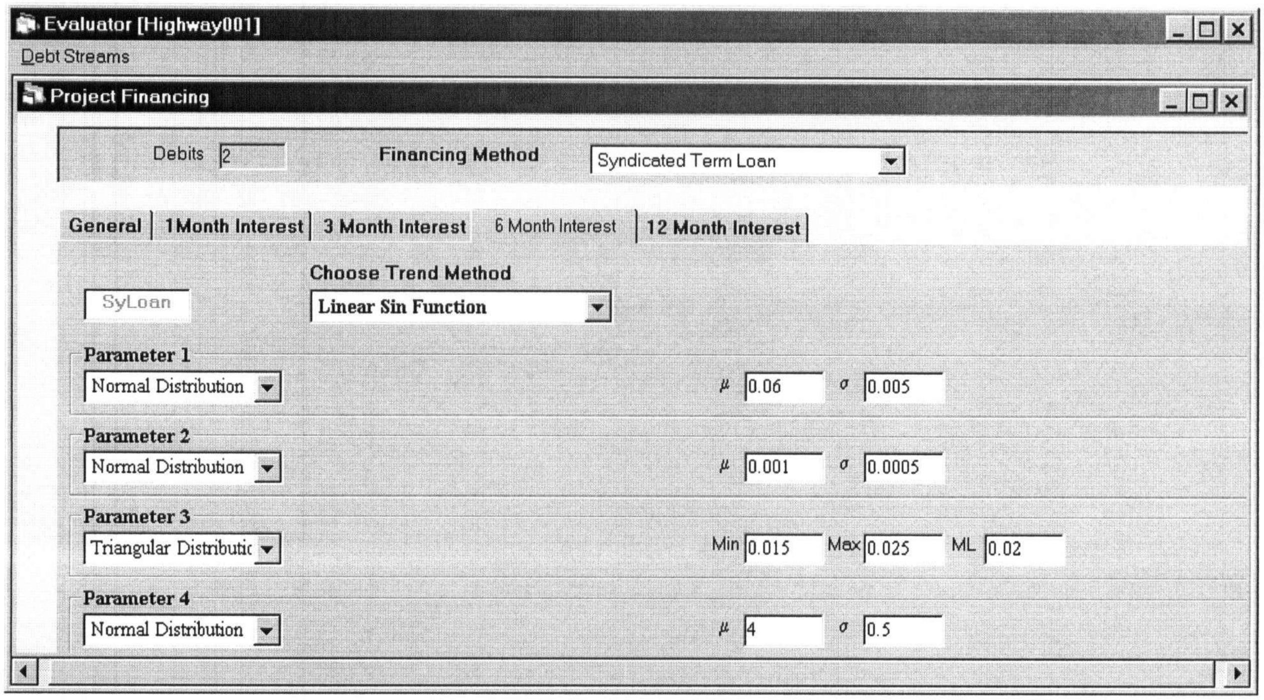


Figure 5.10: 6-month floating interest rate described as a sinusoidal function with uncertain parameter variables (see Table 4.1)

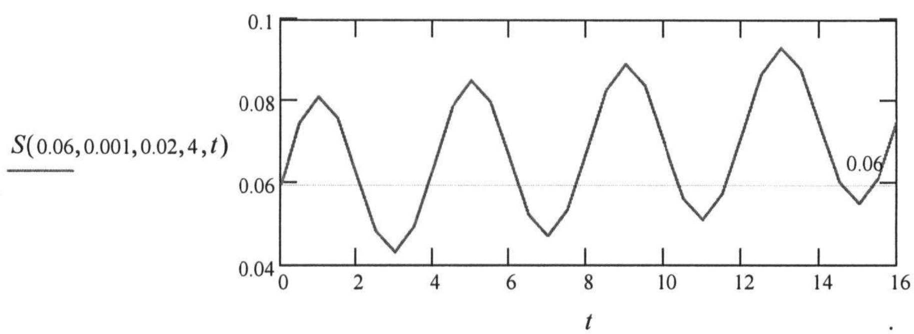


Figure 5.11: 6-month floating interest rate described as a sinusoidal function. Expected values as shown in Figure 5.10 are: starting rate 0.06, annual increase 0.001, amplitude value 0.02, and cycle length 4 years

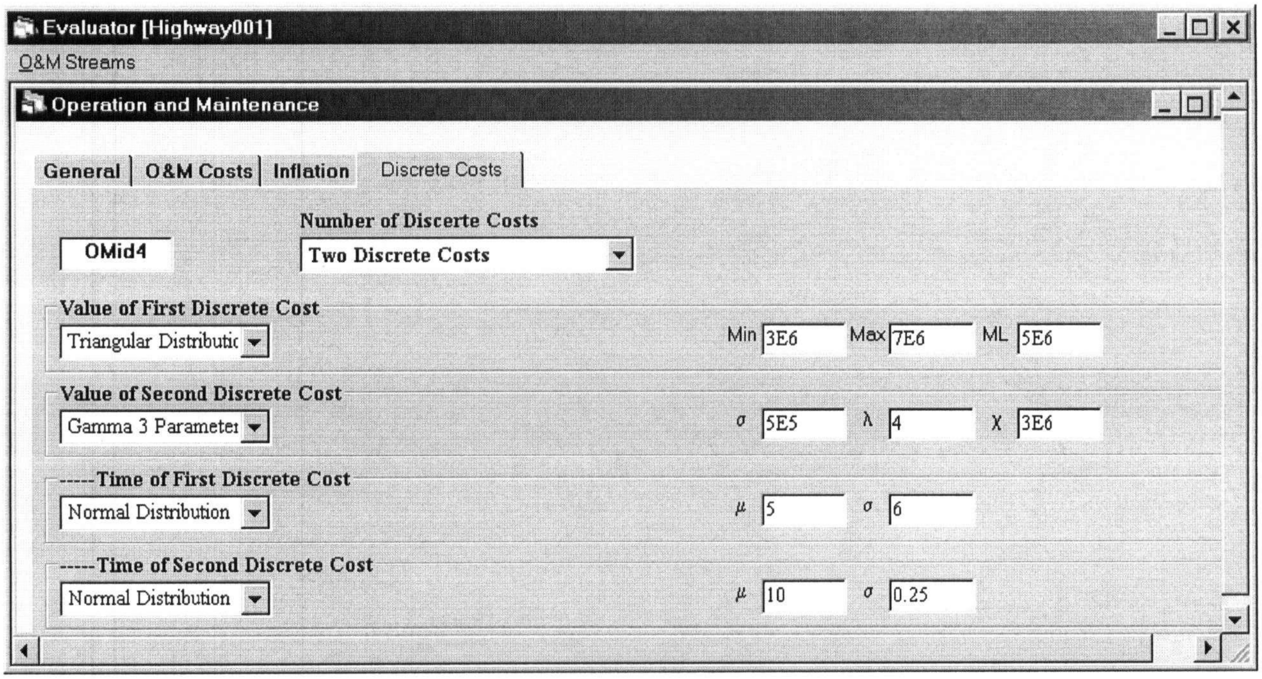


Figure 5.12: Discrete costs and their application times modeled as risk variables in an OM stream

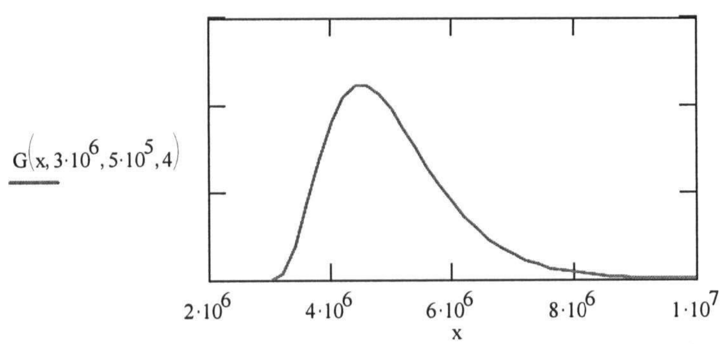


Figure 5.13: Discrete cost modeled as Gamma model

5.2.2 Four Moments of Performance Measures

5.2.2.1 Background

Following Hahn and Shapiro (1967), Siddall (1972) and Bury (1975), the first four moments of a system function (e.g. performance measure) can be obtained using the first four moments of the primary variables in the system function. The moment approach involves expanding the system function by a multivariate Taylor series, generally up the second order, around the means of the primary variables. Then, by taking expectations of the expanded system function, the four moments of the system function can be obtained.

Kottas and Lau (1978, 1980a, 1982) derived expressions for the four moments for the sum, difference, and product of two random variables. By disaggregating a system function into such basic mathematical operations, and then repeatedly applying the four moment equations to these operations, the moments of complicated system functions could be computed. In their derivation, dependency between two variables was described by functional relationships such as a linear and quadratic relationship. Moments higher than the fourth order would be required in the derivation and Kottas and Lau suggested the use of Pearson's family of distribution to get such moments.

Ranasinghe and Russell (1992) and Russell and Ranasinghe (1992) in a process called *variable transformation method* simplified the formulation of the four moments of a system function by reducing the covariance terms of approximate four moment expressions for transforming the system function into uncorrelated space. The method starts by transforming the correlated primary variables of the system function into uncorrelated space and obtaining new four moments for the variables. The system function is then transformed to the uncorrelated space (Der Kiureghian and Liu 1986) and together with the moments of the transformed variables is used to obtain the four moments of the system function.

5.2.2.2 The Four Moments

The risk analysis framework developed herein performs both two and four moment analyses. The four moments of the system function have been derived with the objective of retaining more terms of higher (third and fourth) moments of the primary variables in the approximation of the second, third and fourth moments of the system function, i.e. more than what Hahn and Shapiro (1967), Siddall (1972), and Ranasinghe and Russell (1992) had in their approximations. Unlike these previous approximations, the current derivation includes all the terms contained in the expansion of the moment equations, with the goal being to obtain better approximations of the system function moments. Unlike Kottas and Lau (1982) and Ranasinghe and Russell (1992), the system function is not disaggregated into several components. This reduces the requirements to obtain the interim four moments of the disaggregated components before calculating the moments of the system function.

The four moments of a general system function are derived in Appendix A. Two cases are considered in the derivation. The first assumes the variables of the system function are independent, i.e. assumes variables have zero correlation, and therefore cross moments in the moments derivation convert to moments of random variables, where applicable (see Eq. A.8 in Appendix A). In the second case variables are considered to be uncorrelated, but necessarily independent, and therefore in the derivation the terms that may lead to covariance or cross moments are ignored. While the derivation could be expanded to include terms of correlation between variables, the non-correlation assumption is thought to be sufficient for complex system functions such as those of the generalized model (e.g. Eqs. 4.73-4.84) and therefore is implemented in the development of the decision support system. Complex models would require a prohibitive number of correlation coefficients, which are usually difficult in their elicitation and determination. Further studies should be given to correlation in complex systems.

For the uncorrelated case, the four moments of a system function have been approximated as follows: Letting Z represent a system function $f(\boldsymbol{x})$ in which $\boldsymbol{x}$ is a vector of random variables x_i where $i = 1,2, \ldots, n$ (i.e. $x_1, x_2, \ldots, x_n$); $f(\bar{x})$ is Z calculated at the mean values of $\boldsymbol{x}$; f_i' and f_i'' are the 1st and 2nd partial derivatives of Z with respect to the i-th variable in $\boldsymbol{x}$; $\mu 1'_i$, $\mu 2_i$, $\mu 3_i$, and $\mu 4_i$ are the expected value, and 2nd to 4th central moments of the variable i; $\mu'_1(Z)$, $\mu_2(Z)$, $\mu_3(Z)$ and $\mu_4(Z)$ are the expected value, 2nd to 4th central moments of Z respectively,

$$\mu_1'(Z) = f(\bar{\mathbf{x}}) + \frac{1}{2}\cdot\sum_{i=1}^{n} f_i''\cdot\mu 2_i \tag{5.17}$$

$$\mu_2(Z) = \sum_{i=1}^{n} f_i'^2\cdot\mu 2_i + \frac{1}{4}\cdot\sum_{i=1}^{n} f_i''^2\cdot\mu 4_i - \frac{1}{4}\cdot\left(\sum_{i=1}^{n} f_i''\cdot\mu 2_i\right)^2 + \sum_{i=1}^{n} f_i'\cdot f_i''\cdot\mu 3_i \tag{5.18}$$

$$\begin{aligned}\mu_3(Z) = {} & \sum_{i=1}^{n} f_i'^3\cdot\mu 3_i + \frac{1}{4}\cdot\left[\sum_{i=1}^{n} f_i''\cdot\mu 2_i\right]^3 + \frac{3}{2}\cdot\sum_{i=1}^{n} f_i'^2\cdot f_i''\cdot\mu 4_i \\ & - \left[\sum_{i=1}^{n} f_i''\cdot\mu 2_i\right]\cdot\left[\frac{3}{2}\cdot\sum_{i=1}^{n} f_i'^2\cdot\mu 2_i + \frac{3}{2}\cdot\sum_{i=1}^{n} f_i'\cdot f_i''\cdot\mu 3_i + \frac{3}{8}\cdot\sum_{i=1}^{n} f_i''^2\cdot\mu 4_i\right]\end{aligned} \tag{5.19}$$

$$\begin{aligned}\mu_4(Z) = {} & \sum_{i=1}^{n} f_i'^4\cdot\mu 4_i - \frac{3}{16}\cdot\left[\sum_{i=1}^{n} f_i''\cdot\mu 2_i\right]^4 - \left[\sum_{i=1}^{n} f_i''\cdot\mu 2_i\right]\cdot\left[2\cdot\sum_{i=1}^{n} f_i'^3\cdot\mu 3_i + 3\cdot\sum_{i=1}^{n} f_i'^2\cdot f_i''\cdot\mu 4_i\right] \\ & + \left[\sum_{i=1}^{n} f_i''\cdot\mu 2_i\right]^2\cdot\left[\frac{3}{2}\cdot\sum_{i=1}^{n} f_i'^2\cdot\mu 2_i + \frac{3}{2}\cdot\sum_{i=1}^{n} f_i'\cdot f_i''\cdot\mu 3_i + \frac{3}{8}\cdot\sum_{i=1}^{n} f_i''^2\cdot\mu 4_i\right]\end{aligned} \tag{5.20}$$

The moments derived in the standard four-moment approach of Hahn and Shapiro (1967), Siddall (1972), and Ranasinghe and Russell (1992) can be considered as a subset of the current derivation. For purposes of comparison, the standard approach for the case of uncorrelated-variables expresses the moments as follows:

$$\mu_2(Z) = \sum_{i=1}^{n} f_i'^2 \cdot \mu 2_i + \sum_{i=1}^{n} f_i' \cdot f_i'' \cdot \mu 3_i \tag{5.21}$$

$$\mu_3(Z) = \sum_{i=1}^{n} f_i'^3 \cdot \mu 3_i \tag{5.22}$$

$$\mu_4(Z) = \sum_{i=1}^{n} f_i'^4 \cdot \mu 4_i \tag{5.23}$$

The fourth moment of the standard approach changes to the following under the independent variable case:

$$\mu_4(Z) = \sum_{i=1}^{n} f_i'^4 \cdot \mu 4_i + 6 \cdot \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} f_i'^2 \cdot f_j'^2 \cdot \mu 2_i \cdot \mu 2_j \tag{5.24}$$

5.2.3 Probability Model of Performance Measures

The third and final step of the risk analysis framework involves making probabilistic statements or determining the likelihood of specific values of the system function, e.g. the probability of negative return (as mentioned above). When no probability distribution is known about the performance measure, commonly, the application of either *Chebyshev's inequality* or the *Gauss-Camp-Meidall inequality* has been used to estimate the probability of the performance measure falling in a specific range defined in terms of the standard deviation of the performance measure (Hillier 1963; Wagle 1967; Spooner 1974; Diekmann 1983; Kottas and Lau 1978; Park and Sharp 1990). The *central limit theorem* (*CLT*) has also been used assuming that the system function will be normally distributed (Park and Sharp 1990; Hillier 1963; Wagle 1967). As well, the Log Normal and Gamma distributions are used to represent the system function if it is bounded from one end, and when bounded from two ends the Beta distribution is often used.

Since probability distributions can be described by four or more parameters, e.g. their location (mean), dispersion (variance), skewness and kurtosis, families of distributions have been developed such that, given values for four parameters, a specific distribution can be characterized or selected from within the family. The more the family is able to attain several shapes, the more useful it is for representing a range of random phenomena. Therefore, instead of assuming a specific probability distribution for a system function or applying certain rules (e.g. *CLT*) or inequalities, it becomes more desirable to use the first four moments of a system function to fit a distribution within a family of distributions. The common versatile distribution families or systems of frequency curves are the Pearson distributions, Schmeiser-Deutsch (S-D) system, and Johnson distributions (Hahn and Shapiro 1967; Ord 1972; Johnson et al. 1994; Schmeiser and Deutsch 1977).

The fitting process or characterization can provide a density function for the performance measure with which the probabilities associated with the various levels of a performance measure can be determined. The risk analysis framework applies such a method and determines the probability distribution of the performance measures of the generalized economic model by means of their four moments and the characteristics of the Pearson and Schmeiser-Deutsch (S-D) systems of frequency curves as explained in the next subsections.

5.2.3.1 Pearson Distribution Family

The Pearson family of distributions encompasses density functions of a wide variety of shapes, and includes several common probability distributions such as the Normal, LogNormal, Beta, Gamma, Exponential and Uniform (Johnson et al. 1963; Elderton and Johnson 1969; Hahn and Shapiro 1994; Johnson et al. 1994; Ord 1972).

Frequency curves, $y = f(x)$, in the Pearson family of distributions include three main distribution types (Type I, IV, VI) and a set of ten transition types (e.g. Normal, Type II, III, V, VII, ... XII). These distribution types are obtained as solutions to the following differential equation,

$$\frac{dy}{dx} = \frac{-y(x + b_1)}{b_0 + b_1 x + b_2 x^2} \tag{5.25}$$

where the parameters b_0, b_1, and b_2 are functions of the central moments of $f(x)$. The roots of the denominator in the equation determine the solution of the differential equation and consequently the distribution type. Therefore, defining the criterion k as follows (Elderton and Johnson 1969)

$$k = \frac{b_1^2}{4 b_0 b_1} = \frac{\beta_1 (\beta_2 + 3)^2}{4(2\beta_2 - 3\beta_1 - 6)(4\beta_2 - 3\beta_1)} \tag{5.26}$$

if $k < 0$ then Type I is obtained, $0 < k < 1$ then Type IV, and $k > 1$ then Type VI. While this covers the whole range of k, transitional types are obtained as simpler limiting cases; e.g. if $k = 0$ then the Normal distribution is obtained, if $k = \infty$ then Type III, and if $k = 1$ then Type V (Elderton and Johnson 1969; Ord 1972; Johnson et al. 1994). Figure 5.14 shows these distribution types against the criterion k. Figure 5.15 shows ($\beta 1$,$\beta 2$) plane for the various types of Pearson distribution in terms of the most common types. In this plane, Type I (Beta), IV, and VI occupy regions in the plane; the Normal, exponential, and uniform are represented by a single point; and, Type V (Log Normal) and Type III (Gamma and Chi-square) are represented by curves (Hahn and Shapiro 1967). The impossible area in the ($\beta 1$,$\beta 2$) plan is where $\beta 1 - \beta 2 - 1 < 0$.

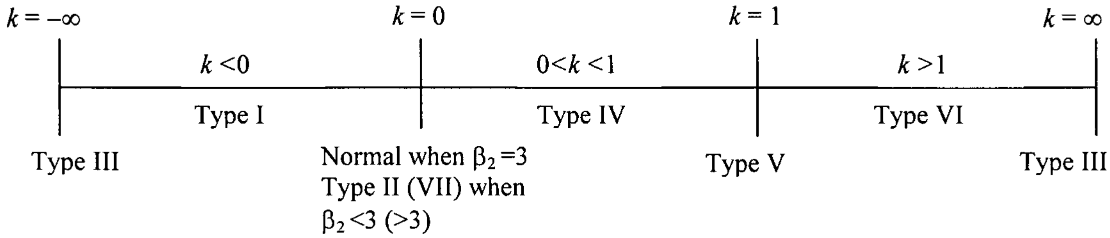


Figure 5.14: Pearson distribution types against the criterion k (Elderton and Johnson 1969)

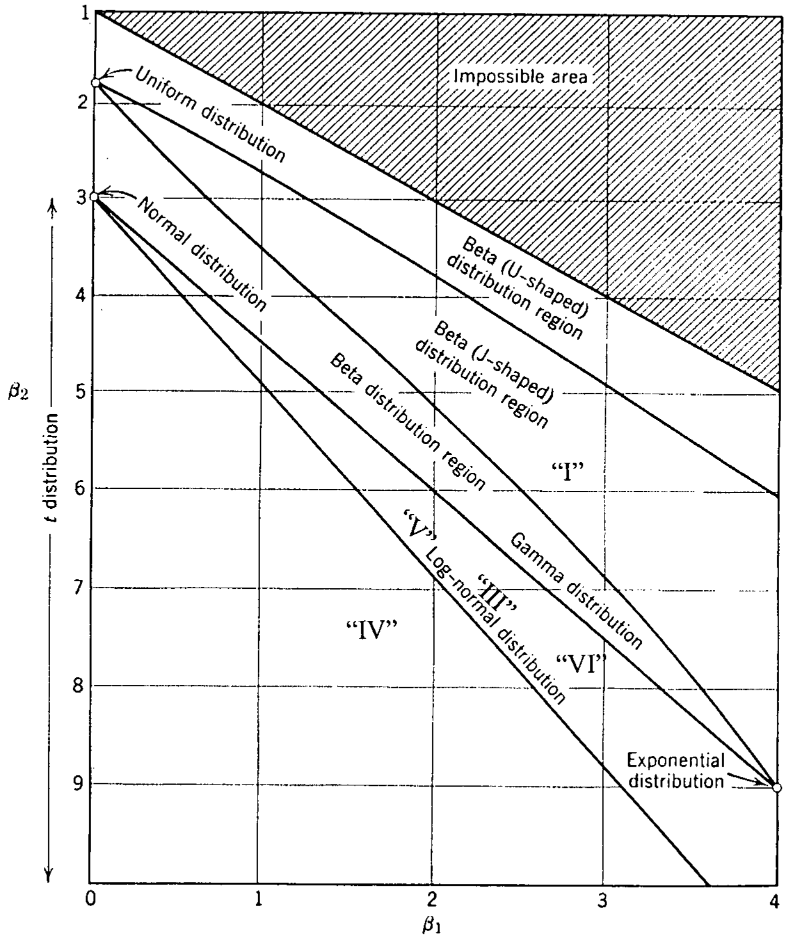


Figure 5.15: Regions in (β1, β2) plane for various distributions, adapted and modified from (Hahn and Shapiro 1967).
Note: skewness coefficient [$\sqrt{\beta 1} = \gamma_1 = \mu_3 / \mu_2^{3/2}$]
Kurtosis coefficient [$\beta_2 = \gamma_2 = \mu_4 / \mu_2^{2}$], see section 5.2.1.

Provided that the first four moments (and consequently $\sqrt{\beta 1}$ and $\beta 2$) of a random variable or system function exist, a particular Pearson distribution can be characterized or positioned in the ($\beta 1$,$\beta 2$) plane using the *k* criterion. However, as explained by Hahn and Shapiro (1967), fitting a distribution by its skewness and kurtosis coefficients does not necessarily ensure an adequate fit since a distribution's shape is not uniquely defined by these coefficients.

A common method for the characterization of a Pearson distribution is to use the double-entry tables compiled by Johnson et al. (1963). Given estimates of $\sqrt{\beta 1}$ and $\beta 2$, fifteen standardized percentage points X_p described as the median and the lower and upper 0.25, 0.5, 1.0, 2.5, 5.0, 10.0 and 25.0 percentage points (i.e. from 0.25th percentile to the 99.75th percentile) can be obtained directly and/or through interpolation methods. The actual values x_p of the random variable (or system function) can then be determined using the first two moments of the system function and the standardized percentile values as follows:

$$x_p = \mu + X_p \cdot \sigma \tag{5.27}$$

The characterization of a Pearson distribution through the tables developed by Johnson et al. (1963) have been used in the risk analysis literature by Kottas and Lau (1978, 1980a, 1980b, 1982), and Russell and Ranasinghe (1992). While the use of such double entry tables ($\sqrt{\beta 1}$ and $\beta 2$) simplifies the determination of the 15 percentile values of a system function, implementing the Johnson et al. (1963) approach in a computerized environment would require the storage of a large number of tables and the use of interpolation methods described by Johnson et al. (1963). (Note: The tables were compiled for $\sqrt{\beta 1}$ increasing by 0.1 and $\beta 2$ increasing by 0.2 within the possible region). Implementation of these tables has not been pursued in this thesis. Rather, the characterization of a Pearson distribution based on the work of Elderton and Johnson (1969) has been used for the risk analysis framework developed in this thesis. Under this approach, given values of the expected value and 2nd to 4th central moments, the k criterion is computed, distribution type is selected based on k, and the parameters of the selected distribution are then estimated. This provides the basis for selection of a frequency distribution which can then be used directly for any required probabilistic estimate. Originally, this characterization method was once unpopular due to the amount of work required for the computation of the parameters of each distribution type in the Pearson family (Kottas and Lau 1978).

As implemented, the risk analysis framework includes the three main types of Pearson family as well as the common transitional types. The mathematical expressions of such distributions and the computation of their parameters are explained in Table 5.2 (note, a distribution type can take several shapes). Given these expressions, the risk analysis framework carries out the following:

1. Calculates the moment coefficients ($\beta 1,\beta 2$), the k criterion, and selects a distribution based on k (see Figure 5.14);
2. Estimates the selected distribution's parameters and applicable range, thus a Pearson frequency distribution becomes available for any required probabilistic estimate;
3. Uses the characterized Person distribution to determine 15 standardized percentile values (commonly used in probabilistic estimating) and their actual values using Eq. (5.27);
4. Calculates the probability for 41 points within the range of the selected distribution; and,
5. Calculates cumulative probability for the 41 points.

Figure 5.16 shows the implementation of the various Pearson distribution types in the support system. For the example shown, a Pearson distribution has been used to represent the performance measure, total capital expenditure (Eq. 4.74), based on its four moments. The right frame shows the four moments (Eqs. 5.17-5.20), the skewness and kurtosis coefficients ($\sqrt{\beta_1}$, β_2), and the fitted Pearson distribution (Pearson Type I in this case). The graphs in the figure show the density and cumulative distributions of the performance measure. The first table at the bottom of the window shows the standardized percentiles and their actual values. Standardized values can be compared directly to those in Johnson et al. (1963) tables. The second table shows the 41 cost values within the range of the performance measure, and the density and cumulative probabilities of such values (these 41 values cannot be obtained from the mentioned tables).

Table 5.2: Pearson distributions and calculations, compiled from Elderton and Johnson (1969)

Equation with Origin at Mean	**Remarks and Parameter Calculation**
<u>**Pearson Type I**</u>	
$y = y_e \cdot \left(1 + \frac{x}{A_1}\right)^{m_1} \cdot \left(1 - \frac{x}{A_2}\right)^{m_2}$	$k<0$; $-A_1<x<A_2$; shapes may be: bell, U, J, twisted J; skew
	$r = \frac{6 \cdot (\beta_2 - \beta_1 - 1)}{(6 + 3\beta_1 - 2\beta_2)}$
	$m_{1(2)} = \frac{1}{2} \cdot \left(r - 2 \pm r(r+2)\sqrt{\frac{\beta_1}{\beta_1 (r+2)^2 + 16(r+1)}} \right)$
	$A_1 + A_2 = a_1 + a_2 = \frac{1}{2} \cdot \sqrt{\mu_2} \cdot \sqrt{\beta_1 (r+2)^2 + 16(r+1)}$
	$\frac{m_1 + 1}{A_1} = \frac{m_2 + 1}{A_2}$
	$y_e = \frac{1}{A_1 + A_2} \cdot \frac{(m_1+1)^{m_1} (m_2+1)^{m_2}}{(m_1 + m_2 + 2)^{m_1+m_2}} \cdot \frac{\Gamma(m_1 + m_2 + 2)}{\Gamma(m_1+1)\,\Gamma(m_2+1)}$
<u>**Pearson Type IV**</u>	
$y = y_o \cdot \left(1 + \left(\frac{x}{a} - \frac{v}{r}\right)^2\right)^{-m} \exp\left(-v \cdot \operatorname{atan}\left(\frac{x}{a} - \frac{v}{r}\right)\right)$	$0<k<1$; unlimited range; bell shaped; skew
	$r = \frac{6 \cdot (\beta_2 - \beta_1 - 1)}{(2\beta_2 - 3\beta_1 - 6)}$
	$m = (r+2)/2$
	$v = \frac{-r(r-2)\sqrt{\beta_1}}{\sqrt{16(r-1) - \beta_1 (r-2)^2}}$
	$a = \sqrt{\frac{\mu_2}{16}} \cdot \sqrt{16(r-1) - \beta_1 (r-2)^2}$
	$F(r,v) = \exp(-0.5\, v\, \pi) \int_0^{\pi} \sin(\phi)^r e^{v\phi} d\phi$
	$y_o = 1/[a \cdot F(r,v)]$

Table 5.2: continued.

Equation with Origin at Mean	Remarks and Parameter Calculation

Pearson Type VI

$$y = y_e \cdot \left(1+\frac{x}{A_1}\right)^{-q_1} \cdot \left(1+\frac{x}{A_2}\right)^{q_2}$$

$k>1$; $-A_2<x<\infty$ if $\mu_3>0$; bell and J shaped; skew.

$$r = \frac{6 \cdot (\beta_2 - \beta_1 - 1)}{(6 + 3\beta_1 - 2\beta_2)}$$

$$a = \frac{1}{2} \cdot \sqrt{\mu_2} \cdot \sqrt{\beta_1 (r+2)^2 + 16(r+1)}$$

$$q_2(-q_1) = \frac{r-2}{2} \pm \frac{r(r+2)}{2} \sqrt{\frac{\beta_1}{\beta_1 (r+2)^2 + 16(r+1)}}$$

$$A_1 = \frac{a(q_1 - 1)}{(q_1 - 1) - (q_2 + 1)}, \quad A_2 = \frac{a(q_2 + 1)}{(q_1 - 1) - (q_2 + 1)}$$

$$y_e = \frac{(q_2+1)^{q_2} (q_1 - q_2 - 2)^{q_1 - q_2} \Gamma(q_1)}{a(q_1 - 1)^{q_1} \Gamma(q_1 - q_2 - 1) \Gamma(q_2 + 1)}$$

Normal

$$y = \frac{1}{\sigma\sqrt{2\pi}} \cdot \exp\left(-\frac{x^2}{2 \cdot \sigma^2}\right)$$

$k=0$; $\beta_1=0$; $\beta_2=3$; $\sigma=\sqrt{\mu 2}$; unlimited range; symmetrical.

Pearson Type II

$$y = y_o \cdot \left(1 - \frac{x^2}{a^2}\right)^m$$

$k=0$; $\beta_1=0$; $\beta_2<3$; $-a<x<a$; bell shaped; U shaped if $\beta_2<1.8$.

$$m = \frac{5\beta_2 - 9}{2(3 - \beta_2)}$$

$$a^2 = (2\mu_2 \beta_2)/(3 - \beta_2)$$

$$y_o = \frac{1}{a\sqrt{\pi}} \cdot \frac{\Gamma(m+1.5)}{\Gamma(m+1)}$$

Pearson Type VII

$$y = y_o \cdot \left(1 + \frac{x^2}{a^2}\right)^{-m}$$

$k=0$; $\beta_1=0$; $\beta_2>3$;unlimited range; bell shaped; symmetrical.

$$m = \frac{5\beta_2 - 9}{2(\beta_2 - 3)}$$

$$a^2 = (2\mu_2 \beta_2)/(\beta_2 - 3)$$

$$y_o = \frac{1}{a\sqrt{\pi}} \cdot \frac{\Gamma(m)}{\Gamma(m-0.5)}$$

Table 5.2: continued.

Equation with Origin at Mean	Remarks and Parameter Calculation
Pearson Type III	
$y = y_e \cdot \left(1 + \frac{x}{A}\right)^p \cdot \exp(-\gamma \cdot x)$	$2\beta_2=6+\beta_1$; $-A<x<\infty$; bell and J shaped; unlimited range in one direction. $\gamma = 2\mu_2 / \mu_3$, $p = \frac{4}{\beta_1} - 1$, $a = \frac{2\mu_2^2}{\mu_3} - \frac{\mu_3}{2\mu_2}$, $y_e = \gamma \cdot \frac{(1+p)^p}{\exp(p+1)\Gamma(p+1)}$
Pearson Type V	
$y = y_e \cdot \left(1 + \frac{x}{A}\right)^{-p} \cdot \exp\left(\frac{p-2}{1+x/A}\right)$	k=1;$0<x<\infty$; bell shaped; unlimited range in one direction. Note: origin at start of curve. $p = 4 + \frac{8 + 4\sqrt{4+\beta_1}}{\beta_1}$, $\gamma = (p-2)\sqrt{\mu_2 (p-3)}$, $A = \gamma/(p-2)$, $y_e = \frac{(p-2)^p}{\gamma \cdot \exp(p-2)\Gamma(p-1)}$

In developing the system, several examples were made to check the accuracy of the computations. In each check, percentile values of each distribution was checked against the tables of Johnson et al. (1963) for given four moments. Since Pearson transition types are selected at $k = 0$, $k = 1$, and $k = \infty$ ($2\beta 2= 3\beta 1+6$), the risk analysis framework chooses a transition distribution (Figure 6.14) when these values are approached within ±5%. For example, if k comes between 0.95 and 1.05 it is assumed to be 1 and Type V is selected. If the value of $3\beta 1+6$ comes between 0.95 and 1.05 of $2\beta 2$, then Type III is selected.

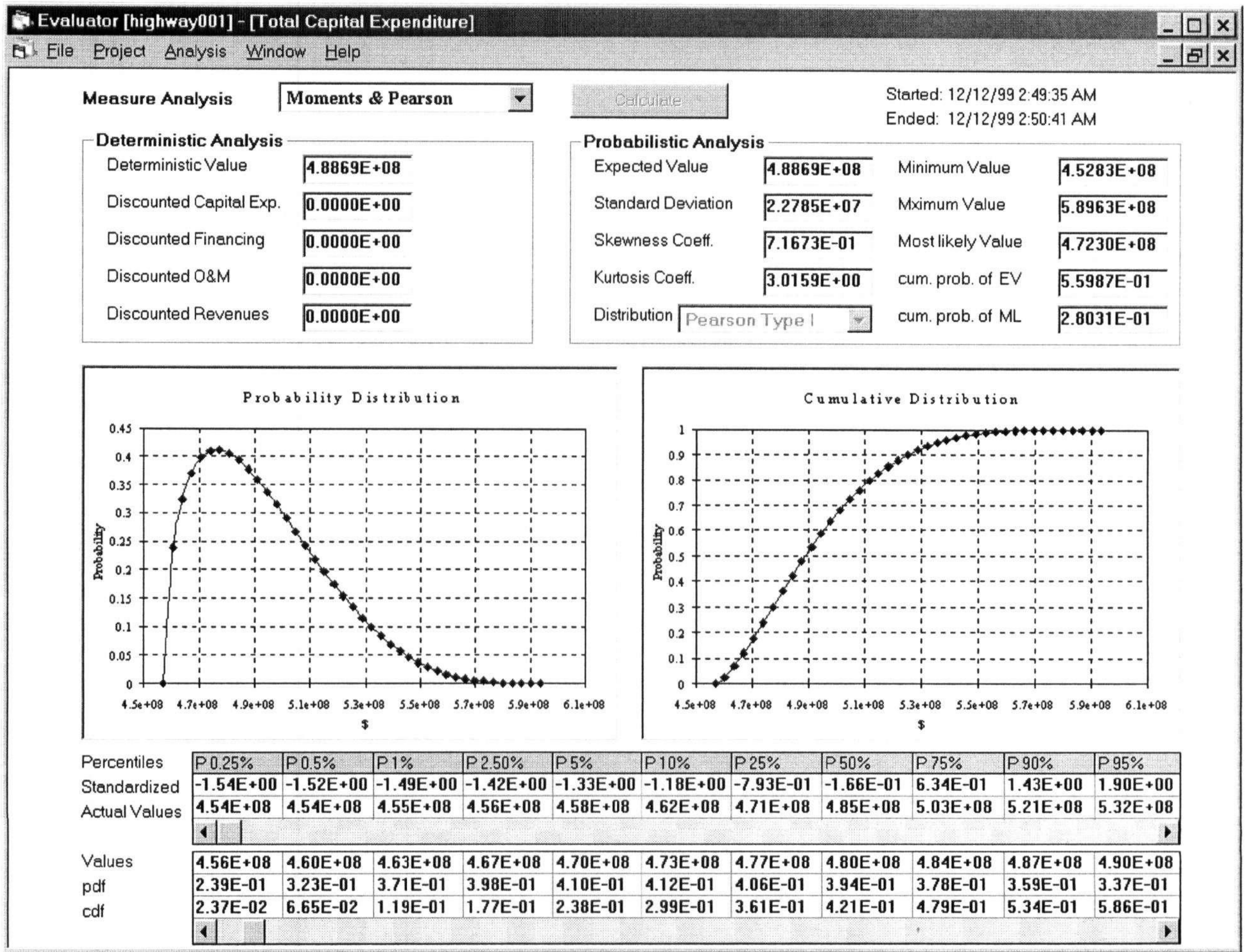


Figure 5.16: Total capital expenditure fitted to a Pearson distribution- Type I

5.2.3.2 Schmeiser–Deutsch Distribution Family

Schmeiser and Deutsch (1977) developed the S-D distribution family that, unlike the Pearson family, has a single functional form from which several shapes can be obtained such as the Exponential, Bernoulli and Uniform distributions. It is a four-parameter distribution family that can take any feasible $\beta_1\beta_2$ combination as in the Pearson family. The density and cumulative distribution functions are as follows (Schmeiser and Deutsch 1977):

$$f(x) = \left(\frac{1}{\lambda_1 \lambda_2}\right) \cdot \left|\frac{\lambda_1 - x}{\lambda_2}\right|^{(1-\lambda_3)/\lambda_3} \qquad \text{for all } x \in \left[\lambda_1 - \lambda_2 \lambda_4{}^{\lambda_3}, \lambda_2 (1-\lambda_4)^{\lambda_3}\right] \tag{5.28}$$

$$F(x) = \begin{cases} \lambda_4 - \left(\dfrac{\lambda_1 - x}{\lambda_2}\right)^{1/\lambda_3} & \text{if } \lambda_1 - \lambda_2 \lambda_4{}^{\lambda_3} \le x \le \lambda_1 \\ \lambda_4 + \left(\dfrac{x - \lambda_1}{\lambda_2}\right)^{1/\lambda_3} & \text{if } \lambda_1 \le x \le \lambda_1 + \lambda_2 (1-\lambda_4)^{\lambda_3} \end{cases} \tag{5.29}$$

in which λ_1 to λ_4 are parameters; λ_1 and λ_2 determine the location and spread of the distribution while λ_3 and λ_4 determine the shape in terms of skewness and peakedness. Symmetric S-D distributions correspond to $\lambda_4 = 0.5$; for $\lambda_3 > 1$, skew is to the right for $\lambda_4 < 0.5$ and to the left for $\lambda_4 > 0.5$; and, for $\lambda_3 < 1$, the direction of skew is reversed.

Schmeiser and Deutsch (1977) explained that given the first four moments of a random variable defined as μ, σ^2, μ_3, and μ_4, and its skewness and kurtosis coefficients as α^*_3 and α^*_4, the parameters λ_1 to λ_4 can be determined using the following process:

- Given the k^{th} raw moment, for λ_1=0, as

$$\begin{aligned} E\left[X^k \middle| \lambda_1 = 0\right] &= \int_{-\infty}^{\infty} x^k f(x|\lambda_1 = 0) dx \\ &= \lambda_2^k \cdot \left((-1)^k \cdot \int_0^{\lambda_4} (\lambda_4 - p)^{k\lambda_3} dp + \int_{\lambda_4}^{1} (p - \lambda_4)^{k\lambda_3} dp\right) \\ &= \lambda_2^k \cdot h(\lambda_3, \lambda_4) \end{aligned} \tag{5.30}$$

the skewness and kurtosis coefficients α_3 and α_4 can be expressed as functions of λ_3 and λ_4 only; and therefore, by solving the nonlinear programming problem

$$\text{Minimize } (\alpha_3 - \alpha^*_3)^2 + (\alpha_4 - \alpha^*_4)^2 \tag{5.31}$$

$$\text{Subject to } 0 \leq \lambda_4 \leq 1 \tag{5.32}$$

then λ_3 and λ_4 could be obtained.

- Then, λ_2 and λ_1 could be obtained as follows

$$\lambda_2 = \sigma \cdot \left(\frac{(2\lambda_3 + 1)(\lambda_3 + 1)^2}{(\lambda_3 + 1)^2 \left[\lambda_4^{2\lambda_3 + 1} + (1 - \lambda_4)^{2\lambda_3 + 1} \right] - (2\lambda_3 + 1) \left[(1 - \lambda_4)^{\lambda_3 + 1} - \lambda_4^{\lambda_3 + 1} \right]^2} \right)^{1/2} \tag{5.33}$$

$$\lambda_1 = \mu - \left(\frac{(1 - \lambda_4)^{\lambda_3 + 1} - \lambda_4^{\lambda_3 + 1}}{(\lambda_3 + 1)} \right) \cdot \lambda_2 \tag{5.34}$$

Along with the primary development to facilitate digital computer simulation (Schmeiser and Deutsch 1977) (the inverse cumulative distribution has a closed form function), the S-D distribution family has been suggested by Kottas and Lau (1980b) as relevant to solve some stochastic inventory problems. Similarly, the S-D distributions have been used by Ravichandran (1993) in his decision support system for stochastic cost-volume-profit analysis. In both these cases, the Pearson family was suggested for use along with the S-D family. The risk analysis framework implements the S-D distribution family such that the probabilistic characteristics of a performance measure can be described either by Pearson or S-D distributions.

Because of the single functional form and straightforward parameter determination, the S-D family as explained by Schmeiser and Deutsch (1977) has two characteristics which may affect its usefulness. The first involves the S-D density function which at the mode, $f(x=\lambda_1)$, may assume three values: zero if $0 \leq \lambda_3 < 1$, one for $\lambda_3 = 1$, and infinity for $\lambda_3 > 1$. The second characteristic is the truncated tails of the S-D distributions, which represents a great problem for applications that are sensitive to the tail of the distribution, as in most engineering problems (but not necessarily economic problems). Schmeiser and Deutsch (1977) stated that: "the family presented here has application when no common distribution is indicated or when a coarse approximation to a common distribution is adequate." Figure 5.17 shows how the total capital expenditure cost of Figure 5.16 (using Pearson distribution) is described using the S-D distribution family. Note that while the first four moments are identified the density function attains irregular shape with infinite value at the modal value since in this example $\lambda_3 = 2.163$ (Eqs. 5.31 and 5.32) which is greater than 1.

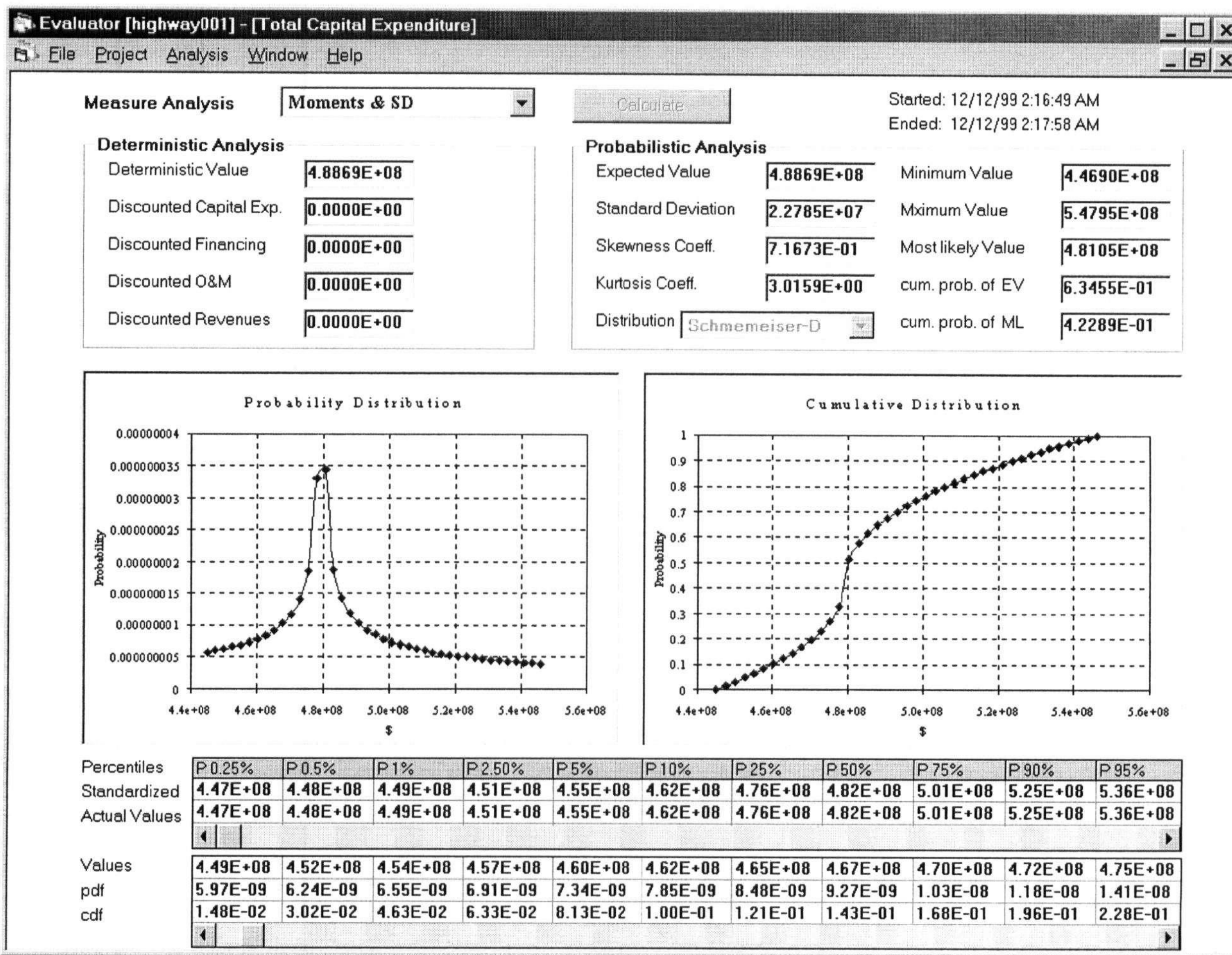


Figure 5.17: Total capital expenditure fitted by S-D distribution

While the SD frequency distribution has an irregular shape, decisions may be made based on the SD cumulative distribution. A comparison between Figures 5.16 and 5.17 for the Pearson and SD distributions shows that for the same measure modeled by these distributions, the measure has comparable range in terms of its minimum and maximum values, the actual percentile values, and the cumulative probability of the expected value. One advantage of the SD distribution that may be appreciation in the elicitation of probabilistic estimates of random variables is that the four moments of the variable can be determined based on three percentile values only, one of them is the mode and its cumulative distribution value. The process, as described by Schmeiser and Deutsch (1977), is direct and simple and uses the raw moments in Eq. (5.30).

5.2.4 Comparison of the Approximated Moments For Exact Cases

The following are two examples designed to test the approximated four moments and Pearson distribution against "known" exact moments and distributions of some performance measures. The approximated moments are those in Appendix A of the independent variables derivation case. The first example shows simple case that describes the functioning of the approximated moments; the second example describes more about the accuracy of the approximated moments.

5.2.4.1 Example 1: Summation of Gamma Variables

Regarding the *reproductive property* of the Gamma Distribution, Bury (1999) stated that: "if k independent Gamma Variables X_i with shape parameters λ_i and a common scale parameter σ are summed as

$$T = a \cdot \sum_{i=1}^{k} X_i \tag{5.35}$$

then T is again a Gamma variable with scale parameter $a\sigma$ and shape parameter $\lambda = \sum_i \lambda_i$."

Assume that T is comprised of three independent Gamma variables having the same scale parameters σ and different shape factors λ_i as shown in Table 5.3, then the moments and moment coefficients of the three variables can be estimated using the formulas in Table 5.1 and their values would be as in Table 5.3.

Table 5.3: Characteristics of three Gamma variables

	λ	σ	μ'_1	$\mu2$	$\mu3$	$\mu4$	$\gamma1=\sqrt{\beta1}$	$\gamma2=\beta2$	c.v.
X_1	1	10	10	100	2000	$9\cdot10^4$	2	9	1
X_2	1.25	10	12.5	125	2500	$1.218\cdot10^5$	1.7888	7.8	0.8944
X_3	1.5	10	15	150	3000	$1.575\cdot10^5$	1.6329	7	0.8164

Letting a=2, the moment characteristics of the summation T in Eq. 5.35 can be (1) computed exactly given the parameter values as explained above by Bury (1999), or (2) estimated approximately using the derivation of the four moments for the independent case in Appendix A (Eqs. A.11, A.13, A.21, A.37) since the variables are independent. Table 5.4 shows these two cases of estimates and shows a zero deviation between the exact and the approximate estimates.

Table 5.4: Characteristics of the summation T of Gamma variables

	μ'_1	$\sqrt{\mu2}$	$\mu2$	$\mu3$	$\mu4$	$\gamma1=\sqrt{\beta1}$	$\gamma2=\beta2$	λ	σ
Exact	75	38.7298	$1.5\cdot10^3$	$6.0\cdot10^4$	$1.035\cdot10^7$	1.0327	4.6	3.75	20
Approximate	75	38.7298	$1.5\cdot10^3$	$6.0\cdot10^4$	$1.035\cdot10^7$	1.0327	4.6	3.75	20
Deviation	0	0	0	0	0	0	0	0	0

Figures 5.18 and 5.19 show an exact match of the cumulative and density distributions for the summation when T is characterized using (1) the exact moments and Gamma model (Table 5.1), and (2) the approximated moments and Pearson distribution (Table 5.2). When fitting the approximated moments to the Pearson family, Pearson Type III (i.e. Gamma; see Figures 5.14 and 5.15) is the best candidate since the criterion k would attain infinity $[2\cdot\beta2 = 3\cdot\beta1+6 = 9.2]$ as explained in Table 5.2.

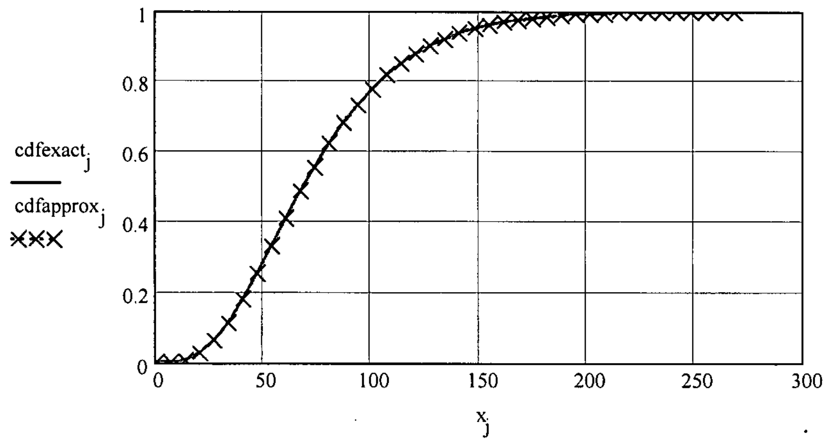


Figure 5.18: Cumulative distribution of Gamma and Pearson Type III for *T*

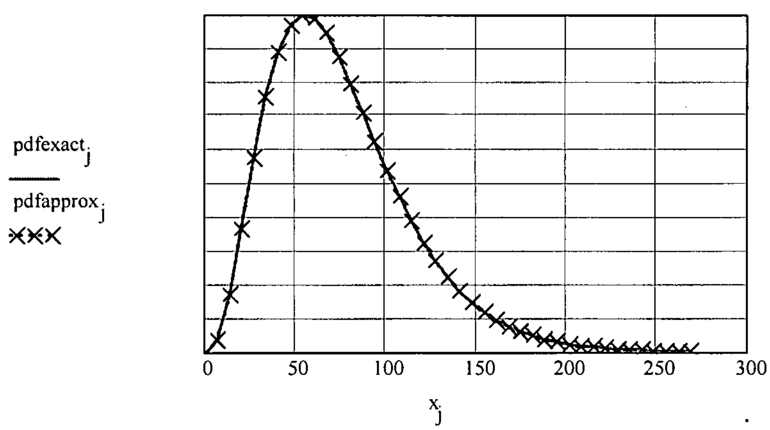


Figure 5.19: Density function of Gamma and Pearson Type III for *T*

The exact match is demonstrated by Table 5.5 which shows fifteen percentile values calculated by the exact Gamma distribution and the Pearson Type III along with the zero deviation between such percentile values.

Table 5.5: Percentile values of the exact Gamma and Pearson III of the summation variable

	P1%	P2.5%	P5%	P10%	P25%	P50%	P75%	P90%	P95%	P97.5%	P99%
Exact	14.38	19.31	24.46	31.58	46.61	68.45	96.30	126.92	147.91	167.77	192.88
Approx.	14.38	19.31	24.46	31.58	46.61	68.45	96.30	126.92	147.91	167.78	192.88
Dev. %	0	0	0	0	0	0	0	0	0	0.005	0

This example explained how the approximate moments and Pearson distribution were identical to the exact moments and Gamma distribution for the summation of Gamma variables. Similar results can be obtained for the case of independent normally distributed variables, but with the summation being normally distributed. The exact matches for both cases can be attributed to the simple form of the summation function which leads to a value of zero for all the second and higher derivatives of the system function (e.g. T) and therefore most of the terms of the derivation in Appendix A would be reduced to zero. Consequently, the moments of the uncorrelated case (Eq. 5.17 to 5.20) would give the same results as in the independent case.

5.2.4.2 Example 2: Product of Log Normal Variables

Bury (1999) explained that: "if k independent Log-Normal Variables X_i with parameters μ_i and σ_i are multiplied as

$$T = \prod_{i=1}^{k} b_i \cdot X_i^{a_i} \tag{5.36}$$

where a_i and b_i are constants, then T is again Log-Normal with parameters

$$\mu = \sum_{i=1}^{k} \ln(b_i) + \sum_{i=1}^{k} a_i \cdot \mu_i \quad \text{and} \quad \sigma^2 = \sum_{i=1}^{k} a_i^2 \cdot \sigma_i^2 \text{ ."} \tag{5.37}$$

The example presented here shows an application of this reproductive property through the comparison of the approximated four moments with the exact moments of a cost function of three Log Normal variables. Humphreys (1983) explained an *exponent estimating technique* that can be used to prepare preliminary estimates; i.e. cost estimates of a new plant or equipment can be obtained using the size and cost of similar plants or pieces of equipment raised to some power called a cost capacity factor. The method is usually called the *six-tenths (0.6) factor rule* where the power is set to 0.6 (Peters and Timmerhaus 1991). Therefore, the cost C_2 of a new plant of size Q_2 can be obtained using the cost C_1 and size Q_1 of similar plant as follows:

$$C_2 = C_1 \cdot \left(\frac{Q_2}{Q_1}\right)^{0.6} = C_1 \cdot Q_2^{0.6} \cdot Q_1^{-0.6} \tag{5.38}$$

If C_1, Q_1, and Q_2 are independent and log-normally distributed with known expected values and variance, then the parameters μ and σ, third and fourth moments, and moment coefficients can be estimated according to Table (5.1). The results are presented in Table 5.6

Table 5.6: Moment characteristics of the three cost variables

	μ'_1	$\mu2$	$\mu3$	$\mu4$	μ	σ	$\gamma1=\sqrt{\beta1}$	$\gamma2=\beta2$	c.v.
C_1	16 M$	2^2	3.0156	52.059	2.7648	0.1245	0.377	3.2537	0.125
Q_1	$1\cdot10^5$ t/yr	15000^2	$1.53\cdot10^{12}$	$1.7\cdot10^{17}$	11.5018	0.1492	0.4534	3.3677	0.15
Q_2	$2\cdot10^5$ t/yr	35000^2	$2.274\cdot10^{13}$	$5.2\cdot10^{18}$	12.191	0.1737	0.5304	3.5042	0.175

Following Bury (1999), C_2 should be log-normally distributed with exact parameters as in Eq. (5.37) and moments according to Table (5.1). The results of such calculations are shown in Table 5.7. Along with the exact results of C_2, Table 5.7 shows the moment characteristics of C_2 when estimated using the approximated moments derived in Appendix A for the independent case. Comparing the results shows the closeness of the values of the approximated and exact moments; where, the deviation of μ'_1 and μ_3 are in the range of less than 2% and of μ_2 and μ_4 in the range of less than 5%. Further, the table shows that the approximated moments are more accurate than the results using the standard moments (Eq. 5.21-5.24) when compared to the exact moments; μ_3 is highly underestimated under the standard approach.

Table 5.7: Characteristics of the product of Log-Normal variables for the C_2

	μ'_1	$\sqrt{\mu2}$	$\mu2$	$\mu3$	$\mu4$	$\gamma1=\sqrt{\beta1}$	$\gamma2=\beta2$	μ	σ
Exact	24.4233	4.5673	20.8605	54.0751	$1.557\cdot10^3$	0.5676	3.5781	3.1784	0.1854
Approximate	24.4243	4.4821	20.0891	53.4394	$1.4867\cdot10^3$	0.5935	3.6838	3.179	0.182
Deviation %	0.0039	-1.8661	-3.6975	-1.1757	-4.5188	4.5701	2.9538	0.0209	-1.8391
Standard	24.4243	4.4361	19.6786	14.5438	$1.3041\cdot10^3$	0.1666	3.367	3.1793	0.1552
Deviation %	0.0039	-2.874	-5.6657	-73.104	-16.248	-70.645	-5.8864	0.0314	-16.288

Fitting the approximated four moments to a Pearson distribution leads to a value for the k criterion equal to 0.9249 which approaches unity as a requirement for the use of Pearson Type V distribution (i.e. Log Normal). Figure 5.20 shows the cumulative distribution of C_2 characterized using the exact moments with the Log Normal model (Table 5.1) and the approximated moments with Pearson Type V (Table 5.2). The graph shows the closeness of the Pearson Type V to the exact Log Normal model, which is a result that is substantiated further by the percentile values of both cases in Table 5.8. While Pearson Type V, which is a transition Pearson distribution, is used because k approaches unity, Pearson Type III (i.e. Gamma, transition distribution) could be used since [$2 \cdot \beta 2 = 7.3677$)] approaches [$3 \cdot \beta 1 + 6 = 7.0567$]. Similarly, Pearson Type IV could be used since ($0 < k < 1$). The deviations (D%) for these cases in Table 5.7 show higher deviations at the lower tail for Type III and upper tail for Type IV when compared to Type V.

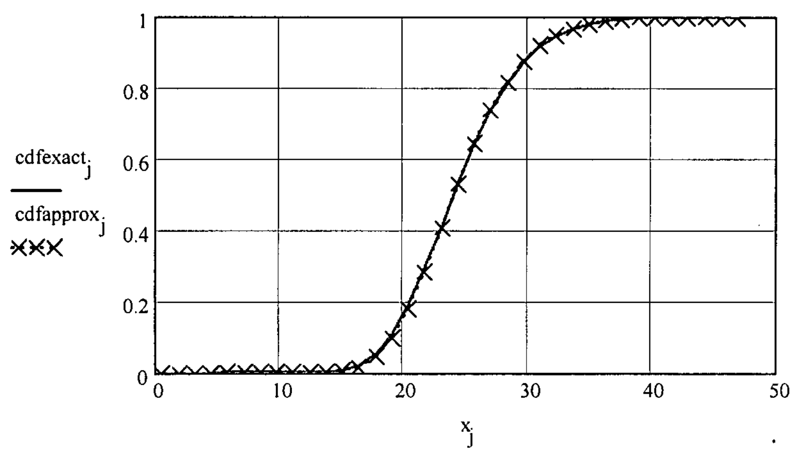


Figure 5.20: Cumulative distribution of Gamma and Pearson Type V for C_2

Table 5.8: Percentile values of the exact Log Normal and Pearson Type V for C_2

	P1%	P2.5%	P5%	P10%	P25%	P50%	P75%	P90%	P95%	P97.5%	P99%
Exact	15.597	16.693	17.697	18.930	21.185	24.007	27.205	30.446	32.567	34.526	36.952
Approx.	15.790	16.867	17.851	19.056	21.254	24.005	27.132	30.322	32.425	34.379	36.817
D%: V	1.237	1.044	0.870	0.663	0.325	-0.010	-0.268	-0.407	-0.437	-0.426	-0.364
D%: III	2.4234	1.588	1.0406	0.5786	0.1169	-0.099	-0.168	-0.225	-0.291	-0.3831	-0.537
D%: IV	1.1036	0.9564	0.8034	0.6077	0.2555	-0.139	-0.531	-0.973	-1.420	-2.1165	-3.673

5.3 Sensitivity Analysis of Performance Measures

Sensitivity analysis is one of the most common methods in risk analysis in investment appraisals (Lefley 1997; Jovanovic 1999). It is the simplest form of risk analysis where the output of a model or a system function can be studied under the effect of changes in the input data to the model. A sensitivity (spider or tornado) diagram is usually constructed showing how and to what extent individual variables are likely to influence the output of the system function. Prior to a comprehensive appraisal study, sensitivity analysis can help determine which variables or parameters should be studied further (Pouliquen 1970; Reutlinger 1970).

Traditionally, sensitivity analysis quantifies the effect of a change in an individual variable on the value of the performance measure while keeping all other variables fixed. Sensitivity (spider) diagrams show the relative effect of each of the variables in the analysis (Perry and Hayes 1985). Two-variable sensitivity analysis has been suggested which leads to the use of sensitivity tables, bivariate sensitivity diagrams, breakeven charts, and Iso-quant graphs, all of which help to explain the effect of change of a combination of two variables on the outcome of a system function (Jovanovic 1999; Flanagan et al. 1987; Lang and Merino 1993). However, along with individual and two-variable analyses, sensitivity analysis can be used to study the simultaneous effect of several variables on the outcome of a system function. This form of sensitivity analysis is referred to as scenario analysis (Eschenbach 1992, 1996).

The implementation of sensitivity analysis requires that the independent variables that comprise a system function changes within their likely ranges. Values for the system function are computed at intervals in the range of the relevant variable. As an alternative approach to sensitivity analysis, the necessity of computing the system function at all of the interval points for each variable can be reduced by linearizing the analysis (Russell 1999; Wahdan et al. 1995).

Linear sensitivity analysis is an approximate method that makes use of the generally called *error propagation* in the estimate of system functions approximated by a first-order multivariable Taylor series expansion (Chapra and Canale 1988).

A second-order multivariate Taylor series expansion is presented in Eq. (A.1) of Appendix A. A first order expansion can be written as follows, where $f(\mathbf{x})$ is a system function with vector of variables $\mathbf{x}$, and $f(\tilde{x})$ is f calculated at $\tilde{x}$ which represents base values of $\mathbf{x}$:

$$f(\mathbf{x}) - f(\tilde{\mathbf{x}}) = \sum_{i=1}^{n} f'(\tilde{x}_i) \cdot (x_i - \tilde{x}_i) \tag{5.39}$$

or

$$\Delta f(\tilde{\mathbf{x}}) = \sum_{i=1}^{n} f'(\tilde{x}_i) \cdot \Delta \tilde{x}_i \tag{5.40}$$

in which $\Delta f(\tilde{\mathbf{x}})$ is the change in the outcome of the system function due to the change of the base variables $\Delta \tilde{x}_i$. By normalizing the change in the system function and in the variables, then

$$\begin{aligned} \frac{\Delta f(\tilde{\mathbf{x}})}{f(\tilde{\mathbf{x}})} &= \sum_{i=1}^{n} f'(\tilde{x}_i) \cdot \frac{\tilde{x}_i}{f(\tilde{\mathbf{x}})} \cdot \frac{\Delta \tilde{x}_i}{\tilde{x}_i} \\ &= \sum_{i=1}^{n} S_i \cdot \frac{\Delta \tilde{x}_i}{\tilde{x}_i} \end{aligned} \tag{5.41}$$

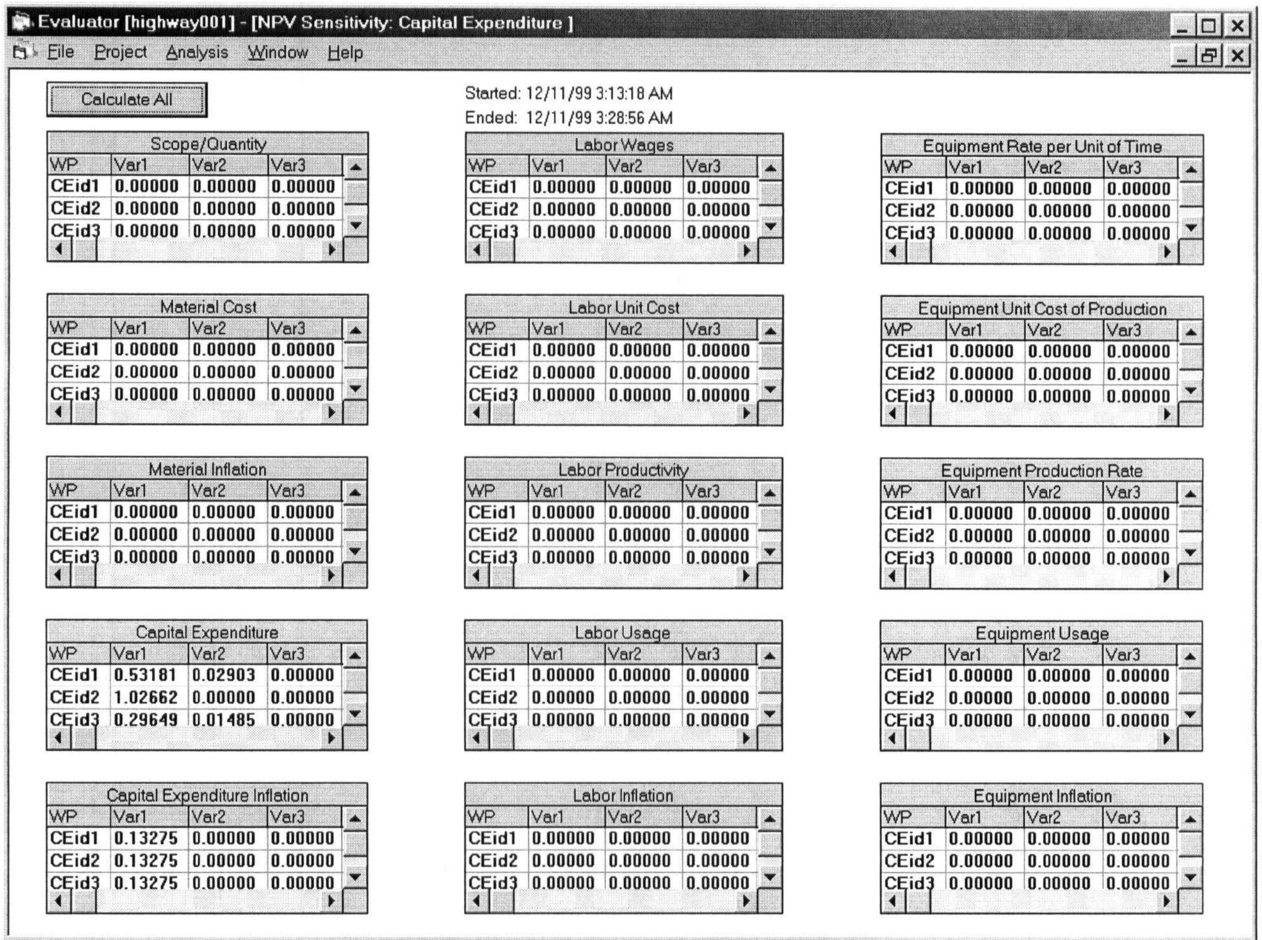
or

$$\delta f(\tilde{\mathbf{x}}) = \sum_{i=1}^{n} S_i \cdot \delta \tilde{x}_i \tag{5.42}$$

in which $\delta f(\tilde{\mathbf{x}})$ is the fractional change in the outcome of the system function due to the sum of the fractional changes of the base variables, $\delta \tilde{x}_i$. S_i is a normalized sensitivity coefficient of variable x_i and represent the slope of the sensitivity line of the relevant variable in a sensitivity diagram. Using the coefficients, S_i, the base variables of a system function can be compared directly to each other thereby helping to determine the most important variables or those which need more investigation, i.e. rather than relying on visual inspection of sensitivity or spider diagrams. The variables of greatest interest are those that have high sensitivity. The accuracy of analysis is, however, limited since it is based on a first-order approximation.

As explained in the generalized economic model, variables in the four components of the model are represented by *rate/area* functions $f_S\ (t,\ \mathbf{y})$ as per Tables 4.1 and 4.2. The risk analysis framework implemented in the support system Evaluator carries out sensitivity analysis only for the *net present value* of a project where the normalized sensitivity coefficients of all sub-variables of all of the variables of the four model components are computed using Eq. (5.42) with the derivatives and variables evaluated at the variable's deterministic or average value.

Figure 5.21 shows the *NPV* normalized sensitivity coefficients for the variables in the capital expenditure CE component. In this figure, the last two tables in the first column of tables show the sensitivity coefficients related to capital expenditure $X(t`)$ and its inflation $\theta_X(t``)$ (see Eq. 4.4). As explained earlier in Figure 5.5, the total cost of work package *CEid3* was estimated using an aggregated estimating method. $X(t)$ was represented by an "Exponential III" *area* function which required two variables: total cost (Parameter 1, modeled by a Log Normal distribution) and cost rate (Parameter 2, modeled by a Uniform distribution). In Figure 5.21, the row titled *CEid3* has 0.29649 and 0.01485 as the required linear sensitivity coefficients for the two parameter variables under "var1" and "var2" respectively. This means, for example, that for each 1% change in total cost of the *CEid3* work package there will be a 0.29% change in the net present value of the project. Similar windows to Figure 5.21 are included in the support system for the sensitivity coefficients of the RV, OM, and FN component variables.



Evaluator [highway001] - [NPV Sensitivity: Capital Expenditure]

File Project Analysis Window Help

Calculate All

Started: 12/11/99 3:13:18 AM
Ended: 12/11/99 3:28:56 AM

Scope/Quantity

WP	Var1	Var2	Var3
CEid1	0.00000	0.00000	0.00000
CEid2	0.00000	0.00000	0.00000
CEid3	0.00000	0.00000	0.00000

Labor Wages

WP	Var1	Var2	Var3
CEid1	0.00000	0.00000	0.00000
CEid2	0.00000	0.00000	0.00000
CEid3	0.00000	0.00000	0.00000

Equipment Rate per Unit of Time

WP	Var1	Var2	Var3
CEid1	0.00000	0.00000	0.00000
CEid2	0.00000	0.00000	0.00000
CEid3	0.00000	0.00000	0.00000

Material Cost

WP	Var1	Var2	Var3
CEid1	0.00000	0.00000	0.00000
CEid2	0.00000	0.00000	0.00000
CEid3	0.00000	0.00000	0.00000

Labor Unit Cost

WP	Var1	Var2	Var3
CEid1	0.00000	0.00000	0.00000
CEid2	0.00000	0.00000	0.00000
CEid3	0.00000	0.00000	0.00000

Equipment Unit Cost of Production

WP	Var1	Var2	Var3
CEid1	0.00000	0.00000	0.00000
CEid2	0.00000	0.00000	0.00000
CEid3	0.00000	0.00000	0.00000

Material Inflation

WP	Var1	Var2	Var3
CEid1	0.00000	0.00000	0.00000
CEid2	0.00000	0.00000	0.00000
CEid3	0.00000	0.00000	0.00000

Labor Productivity

WP	Var1	Var2	Var3
CEid1	0.00000	0.00000	0.00000
CEid2	0.00000	0.00000	0.00000
CEid3	0.00000	0.00000	0.00000

Equipment Production Rate

WP	Var1	Var2	Var3
CEid1	0.00000	0.00000	0.00000
CEid2	0.00000	0.00000	0.00000
CEid3	0.00000	0.00000	0.00000

Capital Expenditure

WP	Var1	Var2	Var3
CEid1	0.53181	0.02903	0.00000
CEid2	1.02662	0.00000	0.00000
CEid3	0.29649	0.01485	0.00000

Labor Usage

WP	Var1	Var2	Var3
CEid1	0.00000	0.00000	0.00000
CEid2	0.00000	0.00000	0.00000
CEid3	0.00000	0.00000	0.00000

Equipment Usage

WP	Var1	Var2	Var3
CEid1	0.00000	0.00000	0.00000
CEid2	0.00000	0.00000	0.00000
CEid3	0.00000	0.00000	0.00000

Capital Expenditure Inflation

WP	Var1	Var2	Var3
CEid1	0.13275	0.00000	0.00000
CEid2	0.13275	0.00000	0.00000
CEid3	0.13275	0.00000	0.00000

Labor Inflation

WP	Var1	Var2	Var3
CEid1	0.00000	0.00000	0.00000
CEid2	0.00000	0.00000	0.00000
CEid3	0.00000	0.00000	0.00000

Equipment Inflation

WP	Var1	Var2	Var3
CEid1	0.00000	0.00000	0.00000
CEid2	0.00000	0.00000	0.00000
CEid3	0.00000	0.00000	0.00000

Figure 5.21. Sensitivity coefficients of capital expenditure variables in the NPV formulation

Chapter 6

Decision Support System Design

6.1 Introduction and System Structure

As explained in the first chapter, development of a prototype decision support system (DSS) for the appraisal of capital investment projects was one of the objectives of this thesis. The current chapter describes a decision support system entitled Evaluator, which integrates the generalized economic model and the risk analysis framework presented previously.

Generally, a decision support system has three primary components: a data management component, a model management component, and a dialogue (interface) management component (Sage 1991). These components can be classified as having strong, moderate or weak capabilities, resulting in a general characterization of the DSS structure (Pearson and Shim 1995). Figure 6.1 illustrates the components of the current Evaluator DSS. Using Pearson and Shim's characterization, it can be described as having: a moderate model base since it supports multiple analyses (deterministic, probabilistic and sensitivity), multiple models (several performance measures and cash flows), and multiple data items (several shape functions, probability distribution models). However, the system does not have tools to build new mathematical expressions that may represent, for example, new estimating methods or new shape functions. Further, Evaluator has a moderate database component as the system interacts and functions through a database. However, it is unable to extract/import data from other sources. Finally, the system has a moderate dialogue component that has a flexible user interface that interacts with the database and the model base. These three components are briefly described in the following sections.

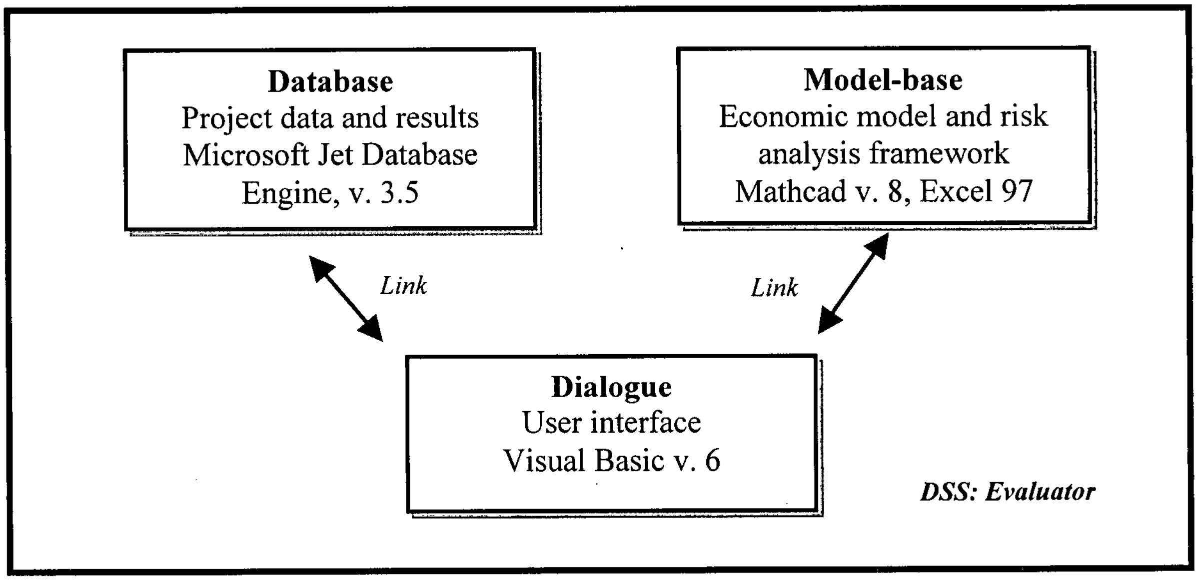


Figure 6.1: Components of the decision support system

6.2 System Database

The system uses the *Data controls* and *Data Access Objects* (DAO) of the Microsoft Jet database for the manipulation of project data and storing of results (Visual Basic 1998). Five databases have been designed: one for each component (CE, RV, OM, and FN); and, a fifth containing references to the methods (e.g. shape functions and probability models). Each of the four component databases is made of a number of linked tables that characterize all of the variables in the four components. For any new project, these databases hold a mixture of:

1. Raw data needed by the generalized economic model and the risk analysis framework, e.g. the $\boldsymbol{X}$ matrices of the four components as described in Chapter 4,
2. Processed or computed data, e.g. the four moments of a risk variable as calculated from a distribution model or percentile values as explained in Chapter 5, and
3. Information or results from the deterministic and probabilistic analyses.

The concept of using a classification where components can have constructs that inherit properties and methods of the classification as explained in Chapter 2 was implemented in the database structure, where records in the component databases represent complete knowledge about a construct's variables, properties, and methods. For example, assuming a single CE work package, variables held by the database are all those described in Table 4.3 which makes up the $\boldsymbol{x}$

vector of the work package. For each variable the data items include:

1. Shape function used and the number of its sub-variables (Tables 4.1 and 4.2),
2. Uncertainty model (e.g. probability distribution) of each sub-variable (Table 5.1),
3. Values of parameters of the uncertainty model of each sub-variable, and
4. Four moments of each sub-variable.

Properties held by the database for the work package include data items such as time characteristics, network characteristics, and reference (local/global) characteristics (see section 4.2.2). The methods held by the database for the work package include:

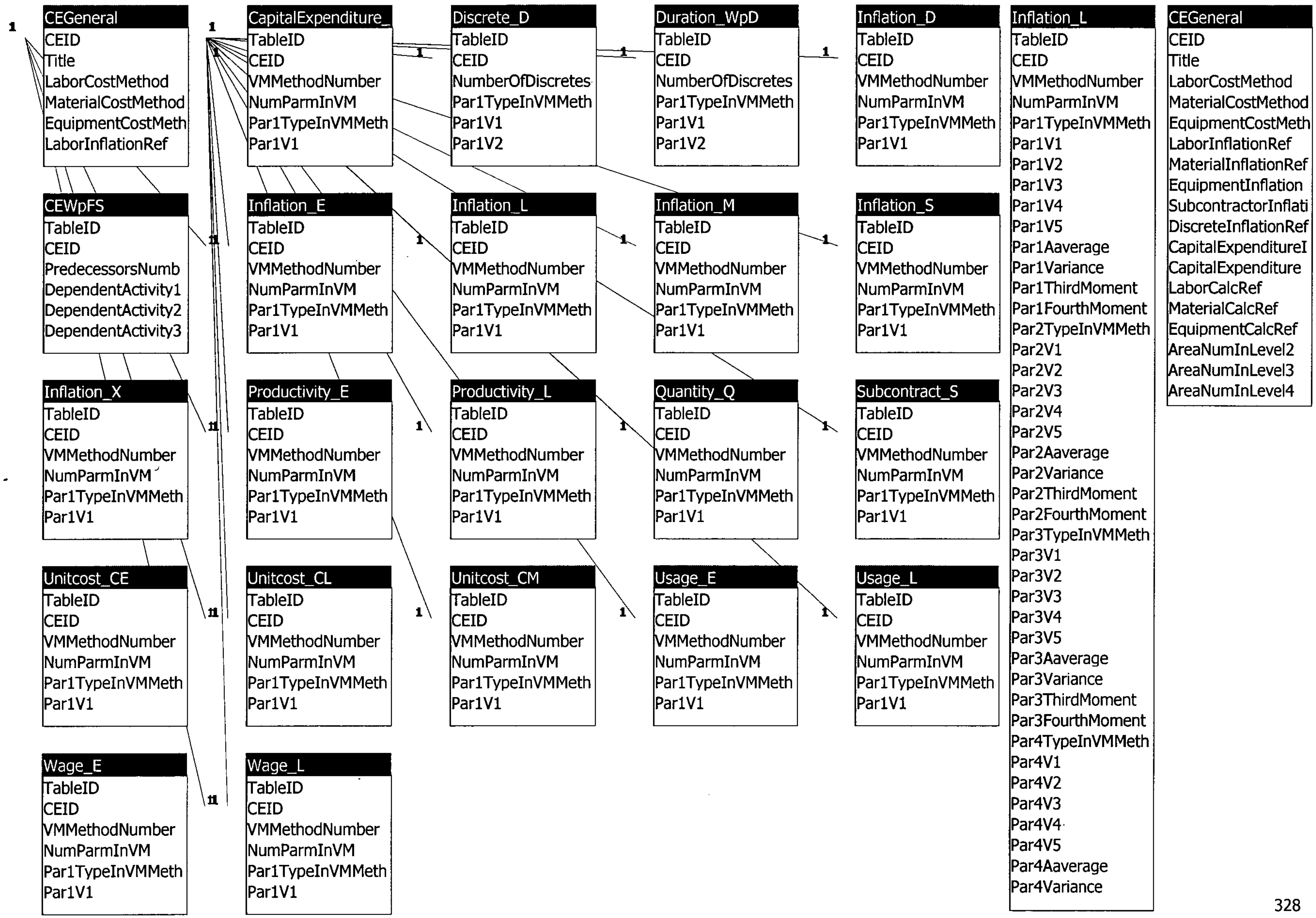
1. Cost estimating method, e.g. detailed and semi-detailed methods (see section 4.3.2), and,
2. If a detailed method is selected, then the method used for each of the material, labor, equipment, and subcontractor cost components in this work package.

Table 6.1 shows the relationships and table definitions in the capital expenditure database along with filed names of two tables. Each of these table definitions, expect the "CEGeneral", includes 103 fields. For example, the "Inflation_L" table represents data for the inflation of labor costs in work packages. The "CEID" field represents the work package code. The "VMMethodNumber" represent the shape function used to represent inflation. The "NumParmInVm" represent the number of parameters of the shape function. Each parameter is represented by 10 fields that hold up to five probability distribution parameters along with the first four moments and the distribution type.

6.3 System Model Base

The model base of the DSS represents both the generalized economic model and the risk analysis framework. The model base was implemented (coded) using Mathcad programming (Mathcad 1998). The formulations in Chapters 4 and 5 of the components' and constructs' cash flow functions, discounted functions, performance measures, derivatives and moments were modeled as subroutines in Mathcad modules. The DSS Mathcad modules can be categorized as component modules, project modules, and general modules.

Table 6.1: Table definitions for the capital expenditure database and field names for the "Inflation_L" and "CEGeneral" tables



Component modules describe the formulation of each of the four components and are represented by a set of linked Mathcad modules that contain:

1. Estimating (cost/revenue/financing) functions,
2. Performance measures on a component level,
3. Discounted estimating and performance measure functions,
4. Derivatives of the estimating functions using *centered-finite-divided-difference* methods (Chapra and Canale 1988), and,
5. The four moments of the component performance measures (see Chapter 5).

Project modules are five modules, the first three of which are linked to the components modules for cash flow and discounted functions, and include:

1. A module for deterministic and probabilistic analysis,
2. A module for sensitivity analysis, and
3. A module of all periodic and cumulative cash flow calculations.
4. Two modules for Pearson and S-D distribution family calculations (see Tables 5.2)

The general modules represent a collection of subroutines accessed and needed by all the modules in order to carry out specific functions such as:

1. Critical path method network calculations,
2. Shape (rate/area) functions and their calculations (see Tables 4.1, 4.2, and 4.5),
3. Four moment calculations for distribution models (see Table 5.1),

All of the calculations in the DSS model base implemented in Mathcad are performed using nested arrays/matrices where the rows represent constructs of a component and the columns represent data items. The data stored in these matrices represent the databases of the four components as described previously. Conversion from databases into matrices is implemented through subroutines coded in the system interface. The conversion was needed since Mathcad does not support database management routines. Similarly, results obtained from the model base are converted by the system interface into information which is stored in the project database.

While all the computation are performed in Mathcad modules, plots of the resulting periodic and cumulative cash flows as well as the probability distributions are performed with the support of Excel (Excel 1996) and stored in the system database.

6.4 System Interface

The interface management system of the DSS was designed using the development environment of Visual Basic v.6 (Visual Basic 1998). The interface is a *graphical user interface* that provides users a direct manipulation with the system through the use of graphical representations, forms, windows and menus. The interface represents the managing module of the DSS that handles two main functions:

1. Dialogue between the user and the system for input-output processing; and
2. Controls and interacts with the model base (Mathcad and Excel) and the database (Jet engine) as shown in Figure 6.1.

The interaction or link between the system interface and both Mathcad and Excel is carried out through the Object Linking and Embedding (OLE) protocol. Mathcad and Excel work as linked objects or automation servers that receive and process inputs and send back outputs to the system interface. Thus, a user of the DSS does not interact directly with either Mathcad or Excel. The interface controls this process through the OLE protocol.

Figure 6.2 shows a summarized, high level, data flow diagram illustrating how the system works in carrying out deterministic and probabilistic analyses. Rounded rectangles in the figure represent system processes (e.g. Mathcad modules). Open-side rectangles represent data storage (project databases). Single- and double-headed arrows represent flow of information in the system. In a deterministic analysis, data flows to compute a required performance measure at the component or project levels, and results are then stored in the system. In a probabilistic analysis, moments of the data are calculated first by the "Calculate: data uncertainty" process and then placed in storage. Then, in order to compute the properties of the performance measure of interest, the system proceeds to formulate the measure and its cash flows, compute its derivatives and moments, derive its distribution, plot the relevant graphs, and store the results at each stage.

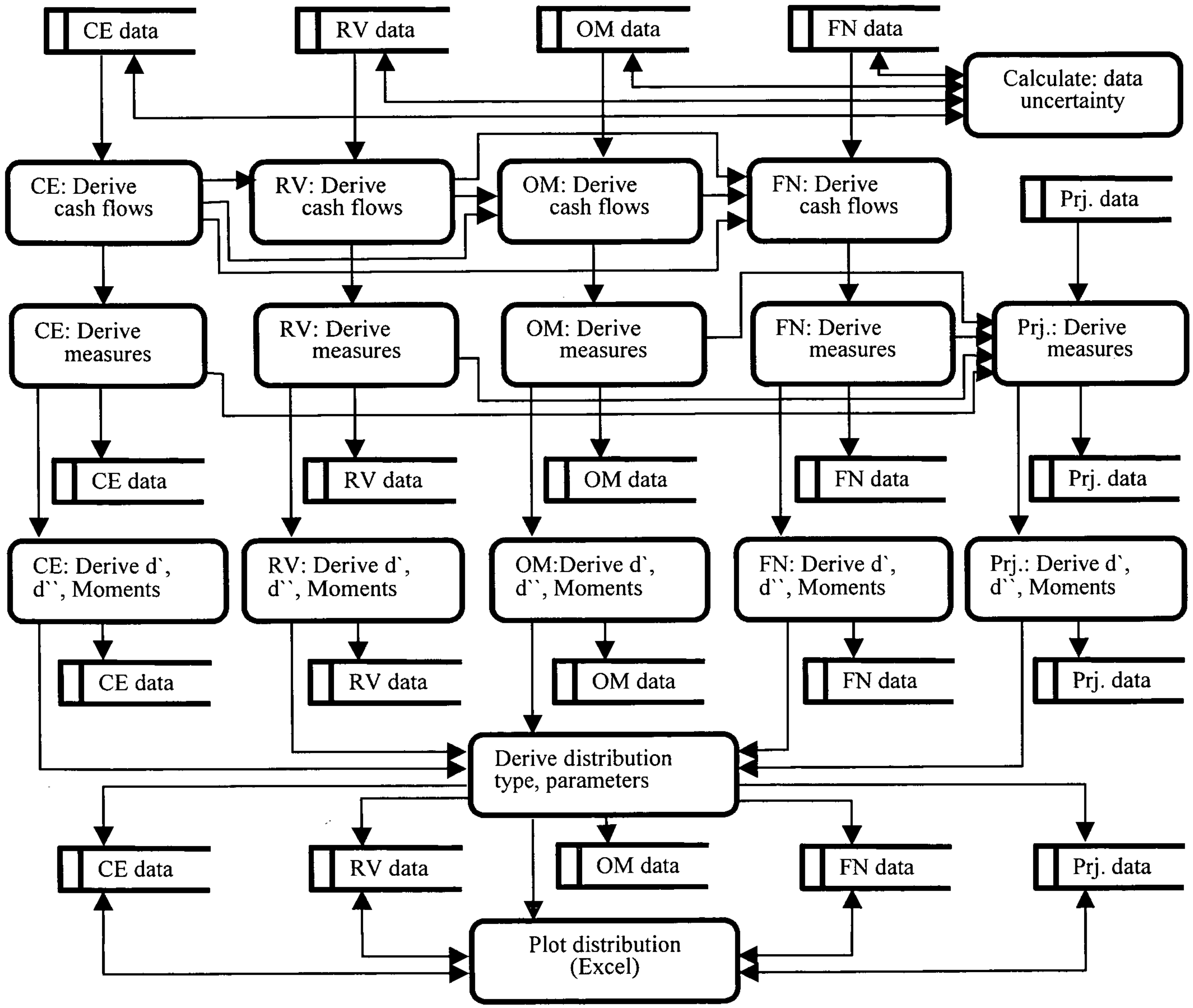


Figure 6.2: Data flow diagram of the decision support system

A new project is started by defining its name as shown in Figure 6.3. This triggers the system to build the new project databases and enable all the menus. Different scenarios for a capital investment project can be developed by saving an existing project into new names using the system "save as" menu (under File menu). This makes a copy of all existing data and information to the new project files. Each scenario, then, becomes a stand-alone project.

Figure 6.4 illustrates the "project" menu where any component can be selected to start adding its constructs, defining properties, and choose estimating methods. Because a large number of screens were used in Chapters 4 and 5 to illustrate how various aspects of the system functionality have been implemented, they are not repeated here.

Figure 6.5 illustrates the analysis menu. An analysis can be performed when the required data are available in the system (e.g. deterministic and probabilistic total capital expenditure needs only capital expenditure data to be defined in the system). This supports the analysis and evaluation of individual components in the appraisal stage of the project, which consequently may assist in making intermediate decisions during the appraisal stage.

While the system interface should function properly, some errors might happen. The system uses a large number of controls (Visual Basic language objects) in nearly all of its forms/windows and uses considerable number of tables in the databases. These all take a large amount of the computer memory. As shown in Figure 6.6, a run-time error occurs when trying to open the finance component because the other components were opened. It is advisable to close unnecessarily windows before start working in another one. One of the common errors that might happen when building a new construct in a component is: "This action has been cancelled by an associated object". This builds partial tables for the new construct and affects the integrity of the databases. Once it happens, it is advisable to delete the new construct and build it again.

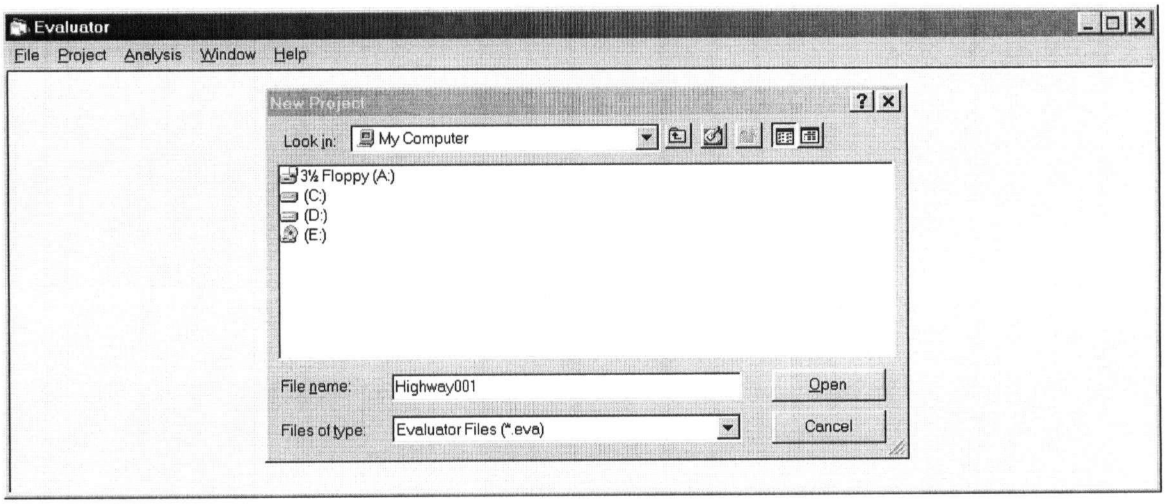


Figure 6.3: Building a new project in Evaluator

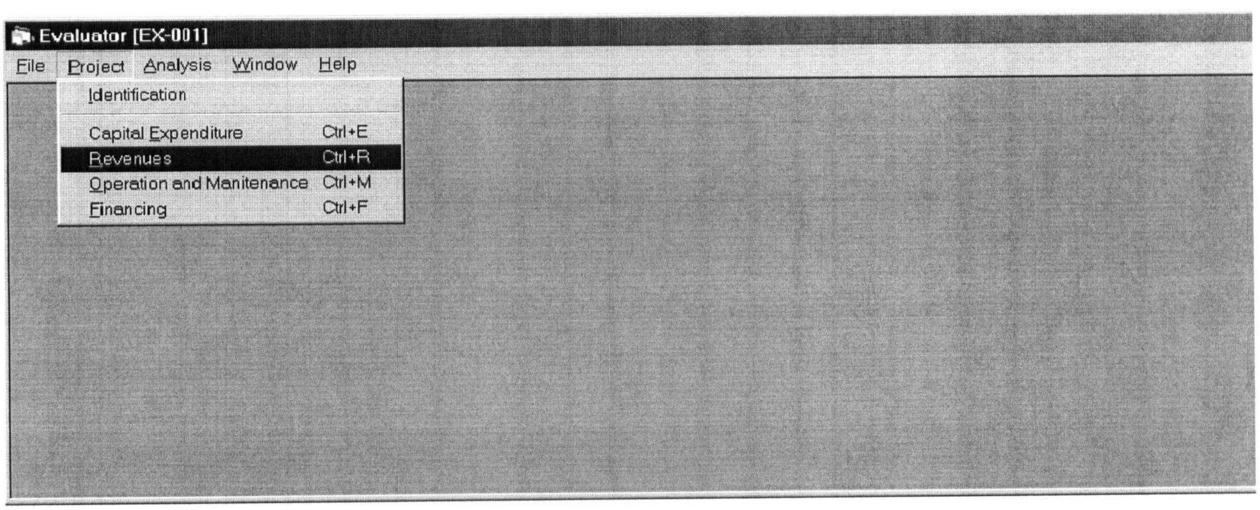


Figure 6.4: Selecting a project component to work with in the system

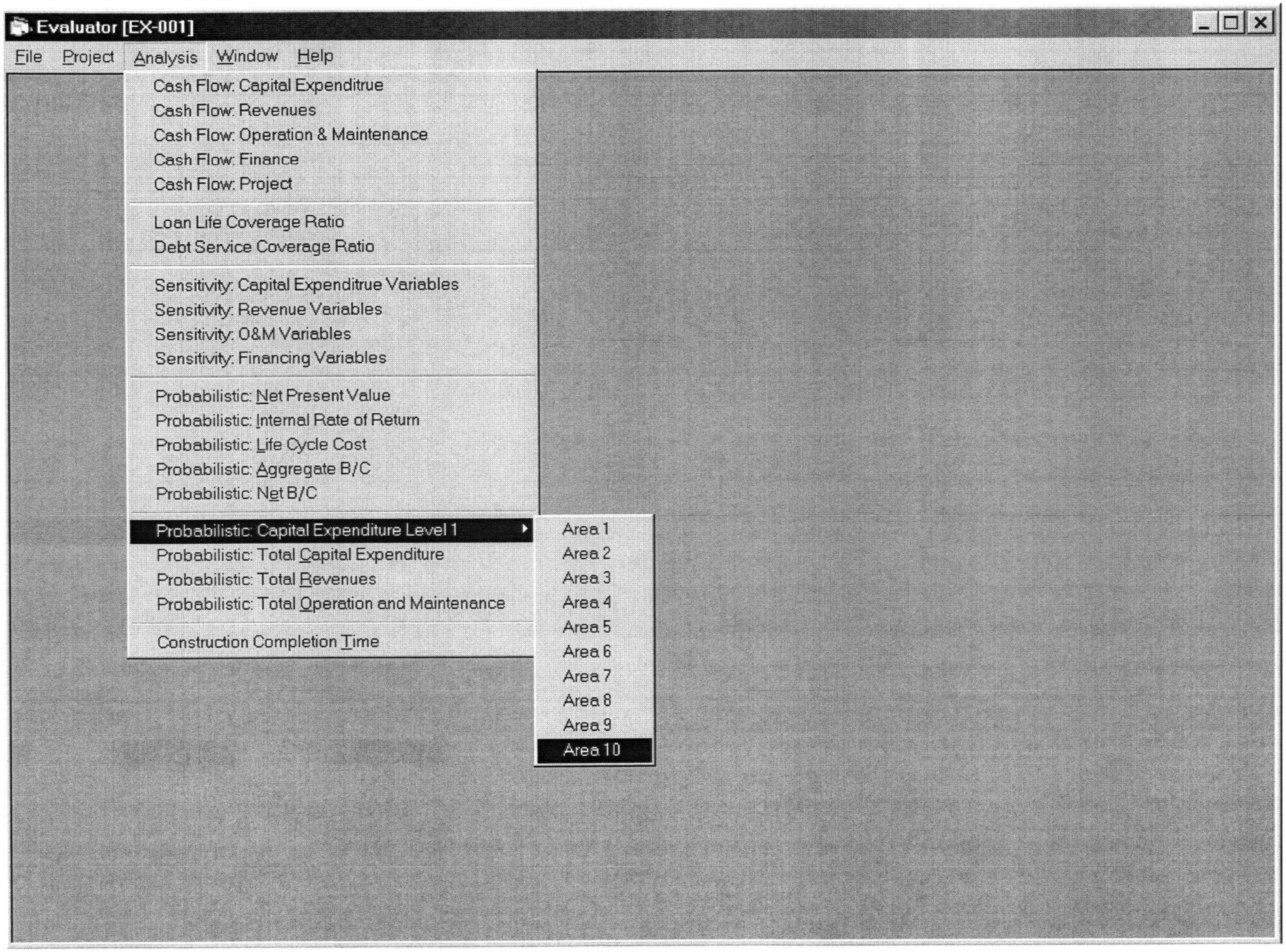


Figure 6.5: The analysis menu in the system

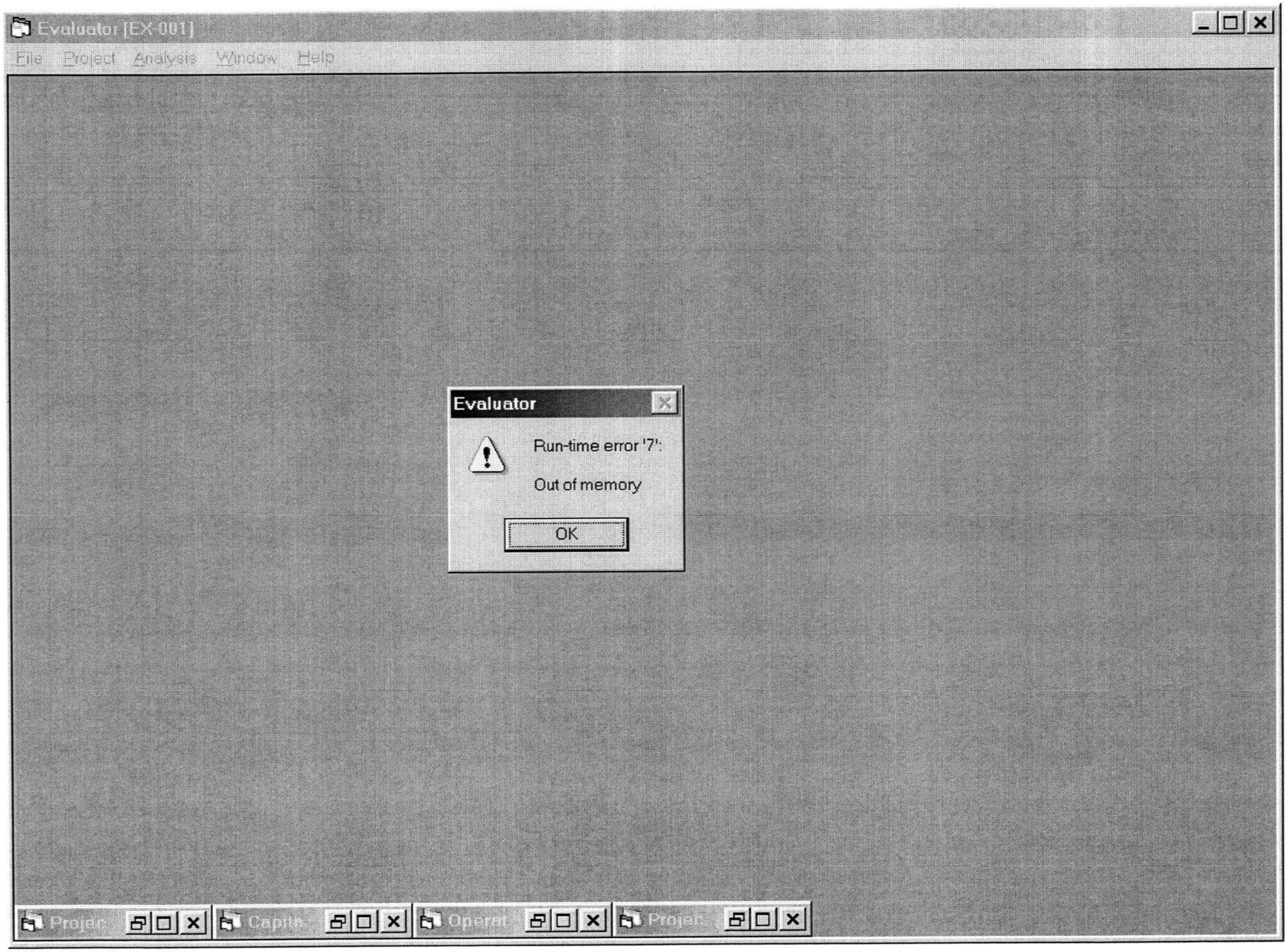


Figure 6.6: Too many forms opened at the same time triggers errors

Chapter 7

Decision Support System Validations and Applications

7.1 Introduction

The decision support system can be used by government or public authorities, private sector companies, developers, and financial institutions for the analysis and evaluation of capital investment projects. Using the system capabilities of estimating and financing methods, time-patterns, uncertainty models, and wide range of performance measures, individual or comparative project economic analyses can be conducted for:

1. Appraisal of a project: The essential use of the system is to formulate the economic structure of an investment project, compute economic and financial performance measures, and carry out sensitivity and probability analyses on the measures.
2. Building scenarios for project development plans: The economic structure of a project under alternative delivery systems (e.g. BOT) can be formulated as project scenarios where each scenario structure can reflect alternative design/construction technologies, alternative O&M plans, alternative financing methods, and/or alternative revenue structures.

This chapter provides two examples to demonstrate the functionality of the economic model, the risk analysis framework, and the Evaluator support system. The first example consists of a simple structure of cash flows for a general project. The results obtained from the DSS for this example are compared against explicit calculations in order to verify the accuracy of the formulation of the generalized model and the risk analysis framework and the integrity of the DSS. This example has been used to validate the economic model. The second example is a more elaborate one and was designed to illustrate several of the main features of the system.

7.2 Example 1: Validation of Model Performance Measures

Figure 7.1 illustrates cost and revenue cash flows of a simple engineering project, e.g. real estate development, during its life cycle. Cost cash flows are outflows required to pay for the land, design and construction costs. Revenue cash flow represents the inflow of money obtained from selling project units (e.g. condominium units). To support the development of the project a loan is obtained to cover 55% of the design and construction costs.

This project was also modeled explicitly in the form of a series of Mathcad sheets. The results of this formulation were compared to those obtained from the DSS (generalized model and risk analysis framework) in order to validate the performance of the DSS. The example uses semi-detailed aggregated estimating methods as commonly employed in primitive appraisal studies, in order to simplify the comparison of the explicit formulation with the DSS results.

The following sections describe the variables and performance measures used in the example project. The measures include:

1. Net present value
2. Internal rate of return on total capital
3. Internal rate of return on equity
4. Total capital expenditure
5. Total revenues
6. Construction completion time
7. Debt service cover ratio
8. Loan life cover ratio

Both deterministic and probabilistic analyses are used in the evaluation of the project. The probabilistic analysis, conducted through the DSS and its risk analysis framework, assumes that the variables are uncorrelated but necessarily independent.

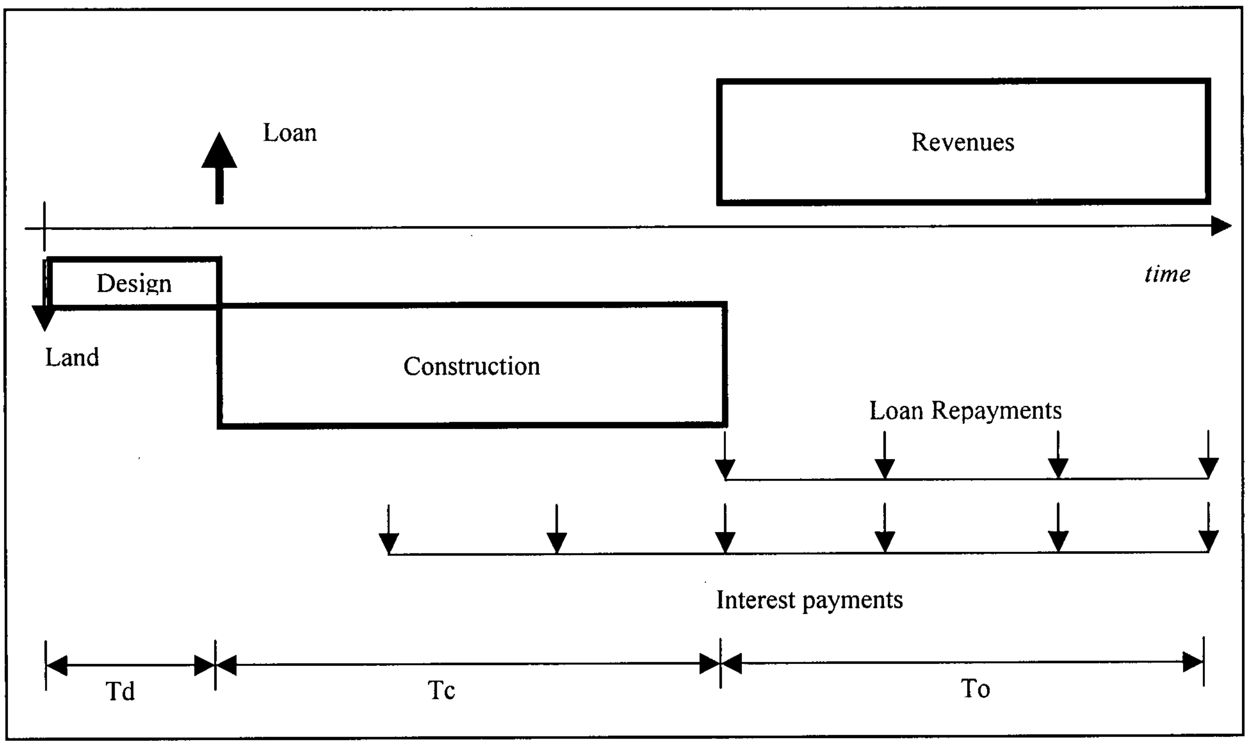


Figure 7.1: Cash flow structure of example project

7.2.1 Example Project Variables

For this example, three distribution models (Log Normal, Gamma, and Triangular) are used to describe the uncertainties of the variables. The moment characteristics of these models are shown in Table 7.1 as functions of the model parameters or first and second moments. (Table 7.1 is part of Table 5.1 that was used in developing the DSS). Table 7.2 shows the first four moments μx_i, $\mu 2x_i$, $\mu 3x_i$, and $\mu 4x_i$ of variables used in the formulation. Table 7.3 shows the distributions of such variables. Generally, the expected value and variance are defined for a variable along with a distribution model. Then the parameters and other moments are determined using the formulation in Table 7.1.

In detailing Table 7.2, land cost is defined as a fixed value. Design cost is estimated as a lump sum cost (aggregated method) to be distributed uniformly (area shape function) over the duration of the design and both are described as Log Normal variables. Construction cost and duration are treated in a manner similar to design cost and duration - i.e. Log Normal models. Inflation for construction cost is described as a Triangular distribution with its moments calculated following the derivation of Chapter 5. Project revenues are calculated using the unit price estimating method. The selling price of a unit is defined as Triangular distribution with the number of units deterministic. Uncertainty of revenue duration or absorption period is described using a Gamma model. Total revenues are assumed to be distributed uniformly over the revenue duration. Debt is defined as a general loan with a 3-year term and which is advanced in a single drawdown. Loan repayments start after 1.5-year grace period. Uncertainty in the interest rate is described using a Gamma distribution. Interest payments are paid during and after the grace period separately from loan repayments as shown in Figure 7.1. As illustrated in Table 7.3 all cost and duration variables are positively skewed, signifying a tendency to escalate. However, the selling price per unit is treated as being negatively skewed, signifying a tendency to fall.

Table 7.1: Explicit formulation of the example project (Mathcad sheet)

A) Characteristics of general distribution models used in the example project (see Table 5.1 and Section 5.2.1)

Characteristics of the Log Normal Distribution Model:

$$LN(x,\mu,\sigma) := \frac{1}{x\cdot\sigma\cdot\sqrt{2\cdot\pi}}\cdot\exp\left[\frac{-1}{2}\cdot\left(\frac{\ln(x)-\mu}{\sigma}\right)^2\right]$$

$$LN'(x,\mu,\sigma) := \frac{1}{\sigma\cdot\sqrt{2\cdot\pi}}\cdot\int_0^x \frac{1}{t}\cdot\exp\left[\frac{-1}{2}\cdot\left(\frac{\ln(t)-\mu}{\sigma}\right)^2\right]dt$$

$$EXL(\mu,\sigma) := \exp\left(\mu+\frac{\sigma^2}{2}\right)$$

$$SDL(\mu,\sigma) := \sqrt{\exp\left(2\cdot\mu+\sigma^2\right)\cdot\left(\exp\left(\sigma^2\right)-1\right)}$$

$$\sigma L(\mu 1,\mu 2) := \sqrt{\ln\left(\mu 2+\mu 1^2\right)-2\cdot\ln(\mu 1)}$$

$$\mu L(\mu 1,\mu 2) := \ln(\mu 1)-\frac{\sigma L(\mu 1,\mu 2)^2}{2}$$

$$c(\mu 1,\mu 2) := \sqrt{\exp\left(\sigma L(\mu 1,\mu 2)^2\right)-1}$$

$$\gamma 1L(\mu 1,\mu 2) := c(\mu 1,\mu 2)^3+3\cdot c(\mu 1,\mu 2)$$

$$\gamma 2L(\mu 1,\mu 2) := c(\mu 1,\mu 2)^8+6\cdot c(\mu 1,\mu 2)^6+15\cdot c(\mu 1,\mu 2)^4+16\cdot c(\mu 1,\mu 2)^2+3$$

$$\mu 3L(\mu 1,\mu 2) := \gamma 1L(\mu 1,\mu 2)\cdot\mu 2^{1.5}$$

$$\mu 4L(\mu 1,\mu 2) := \gamma 2L(\mu 1,\mu 2)\cdot\mu 2^2 \qquad cv(\mu,\sigma) := \frac{\sigma}{\mu}$$

Characteristics of the Gamma Distribution Model:

$$G(x,\sigma,\lambda) := \frac{1}{\sigma\cdot\Gamma(\lambda)}\cdot\left(\frac{x}{\sigma}\right)^{\lambda-1}\cdot\exp\left(\frac{-x}{\sigma}\right)$$

$$G'(x,\sigma,\lambda) := \frac{1}{\Gamma(\lambda)}\cdot\int_0^{x\cdot\sigma^{-1}} z^{\lambda-1}\cdot\exp(-z)\,dz$$

$$EXG(\sigma,\lambda) := \sigma\cdot\lambda \qquad SDG(\sigma,\lambda) := \sqrt{\sigma^2\cdot\lambda}$$

$$\sigma G(\mu 1,\mu 2) := \mu 2\cdot\mu 1^{-1} \qquad \lambda G(\mu 1,\mu 2) := \mu 1^2\cdot\mu 2^{-1}$$

$$\gamma 1G(\lambda) := 2\cdot\lambda^{-0.5} \qquad \gamma 2G(\lambda) := 3\cdot\left(1+2\cdot\lambda^{-1}\right)$$

$$\mu 2G(\sigma,\lambda) := \sigma^2\cdot\lambda$$

$$\mu 3G(\mu 1,\mu 2) := \gamma 1G(\lambda G(\mu 1,\mu 2))\cdot\mu 2^{1.5}$$

$$\mu 4G(\mu 1,\mu 2) := \gamma 2G(\lambda G(\mu 1,\mu 2))\cdot\mu 2^2$$

Characteristics of the Triangular Distribution Model:

$$T(x,L,M,H) := \begin{cases} \text{return } \dfrac{2\cdot(x-L)}{(H-L)\cdot(M-L)} & \text{if } L\le x<M \\ \text{return } \dfrac{2\cdot(H-x)}{(H-L)\cdot(H-M)} & \text{if } M\le x\le H \\ 0 & \end{cases}$$

$$EXT(L,M,H) := \frac{L+M+H}{3}$$

$$\mu 2T(L,M,H) := \frac{1}{18}\cdot\left(L^2+M^2+H^2-L\cdot H-M\cdot H-M\cdot L\right)$$

$$\mu 3T(L,M,H) := \frac{-1}{270}\cdot(L+H-2\cdot M)\cdot(M+L-2\cdot H)\cdot(M+H-2\cdot L)$$

$$\mu 4T(L,M,H) := \frac{1}{135}\cdot\left(L^2+H^2+M^2-L\cdot M-L\cdot H-H\cdot M\right)^2$$

Table 7.2: Variables and parameter variables used in the example project (Mathcad sheet)

B) Characteristics of variables used in the example

Land value: $L := 10 \cdot 10^6$

Design variables (cost D, and duration Td):

D $\mu x_1 := 1.28 \cdot 10^6$ $\mu 2x_1 := \left(0.1 \cdot 10^6\right)^2$ $\mu 3x_1 := \mu 3L\left(\mu x_1, \mu 2x_1\right)$ $\mu 4x_1 := \mu 4L\left(\mu x_1, \mu 2x_1\right)$ $cv\left(\mu x_1, \sqrt{\mu 2x_1}\right) = 0.078125$ $D := \mu x_1$

$\sigma L\left(\mu x_1, \mu 2x_1\right) = 0.078006$ $\mu L\left(\mu x_1, \mu 2x_1\right) = 14.059328$

Td $\mu x_2 := 0.5$ $\mu 2x_2 := 0.025^2$ $\mu 3x_2 := \mu 3L\left(\mu x_2, \mu 2x_2\right)$ $\mu 4x_2 := \mu 4L\left(\mu x_2, \mu 2x_2\right)$ $cv\left(\mu x_2, \sqrt{\mu 2x_2}\right) = 0.05$ $Td := \mu x_2$

$\sigma L\left(\mu x_2, \mu 2x_2\right) = 0.049969$ $\mu L\left(\mu x_2, \mu 2x_2\right) = -0.694396$

Construction variables (cost C, duration Tc, inflation):

C $\mu x_3 := 30 \cdot 10^6$ $\mu 2x_3 := \left(7.5 \cdot 10^6\right)^2$ $\mu 3x_3 := \mu 3L\left(\mu x_3, \mu 2x_3\right)$ $\mu 4x_3 := \mu 4L\left(\mu x_3, \mu 2x_3\right)$ $cv\left(\mu x_3, \sqrt{\mu 2x_3}\right) = 0.25$ $C := \mu x_3$

$\sigma L\left(\mu x_3, \mu 2x_3\right) = 0.246221$ $\mu L\left(\mu x_3, \mu 2x_3\right) = 17.186396$

Tc $\mu x_4 := 1.5$ $\mu 2x_4 := 0.25^2$ $\mu 3x_4 := \mu 3L\left(\mu x_4, \mu 2x_4\right)$ $\mu 4x_4 := \mu 4L\left(\mu x_4, \mu 2x_4\right)$ $cv\left(\mu x_4, \sqrt{\mu 2x_4}\right) = 0.166667$ $Tc := \mu x_4$

$\sigma L\left(\mu x_4, \mu 2x_4\right) = 0.165526$ $\mu L\left(\mu x_4, \mu 2x_4\right) = 0.391766$

θ $\mu x_5 := EXT(0.025, 0.035, 0.045)$ $\mu x_5 = 0.035$ $\mu 3x_5 := \mu 3T(0.025, 0.035, 0.045)$ $\theta := \mu x_5$

$\mu 2x_5 := \mu 2T(0.025, 0.035, 0.045)$ $\sqrt{\mu 2x_5} = 4.082483 \cdot 10^{-3}$ $\mu 4x_5 := \mu 4T(0.025, 0.035, 0.045)$ $cv\left(\mu x_5, \sqrt{\mu 2x_5}\right) = 0.116642$

Revenue variables (selling price S, duration To, number of units N, and discount rate r):

S $\mu x_6 := EXT\left(1.5 \cdot 10^6, 1.7 \cdot 10^6, 1.8 \cdot 10^6\right)$ $\mu x_6 = 1.666667 \cdot 10^6$ $\mu 3x_6 := \mu 3T\left(1.5 \cdot 10^6, 1.7 \cdot 10^6, 1.8 \cdot 10^6\right)$ $S := \mu x_6$

$\mu 2x_6 := \mu 2T\left(1.5 \cdot 10^6, 1.7 \cdot 10^6, 1.8 \cdot 10^6\right)$ $\sqrt{\mu 2x_6} = 6.236096 \cdot 10^4$ $\mu 4x_6 := \mu 4T\left(1.5 \cdot 10^6, 1.7 \cdot 10^6, 1.8 \cdot 10^6\right)$ $cv\left(\mu x_6, \sqrt{\mu 2x_6}\right) = 0.037417$

To $\mu x_7 := 1.5$ $\mu 2x_7 := .25^2$ $\mu 3x_7 := \mu 3G\left(\mu x_7, \mu 2x_7\right)$ $\mu 4x_7 := \mu 4G\left(\mu x_7, \mu 2x_7\right)$ $cv\left(\mu x_7, \sqrt{\mu 2x_7}\right) = 0.166667$ $To := \mu x_7$

$\sigma G\left(\mu x_7, \mu 2x_7\right) = 0.041667$ $\lambda G\left(\mu x_7, \mu 2x_7\right) = 36$

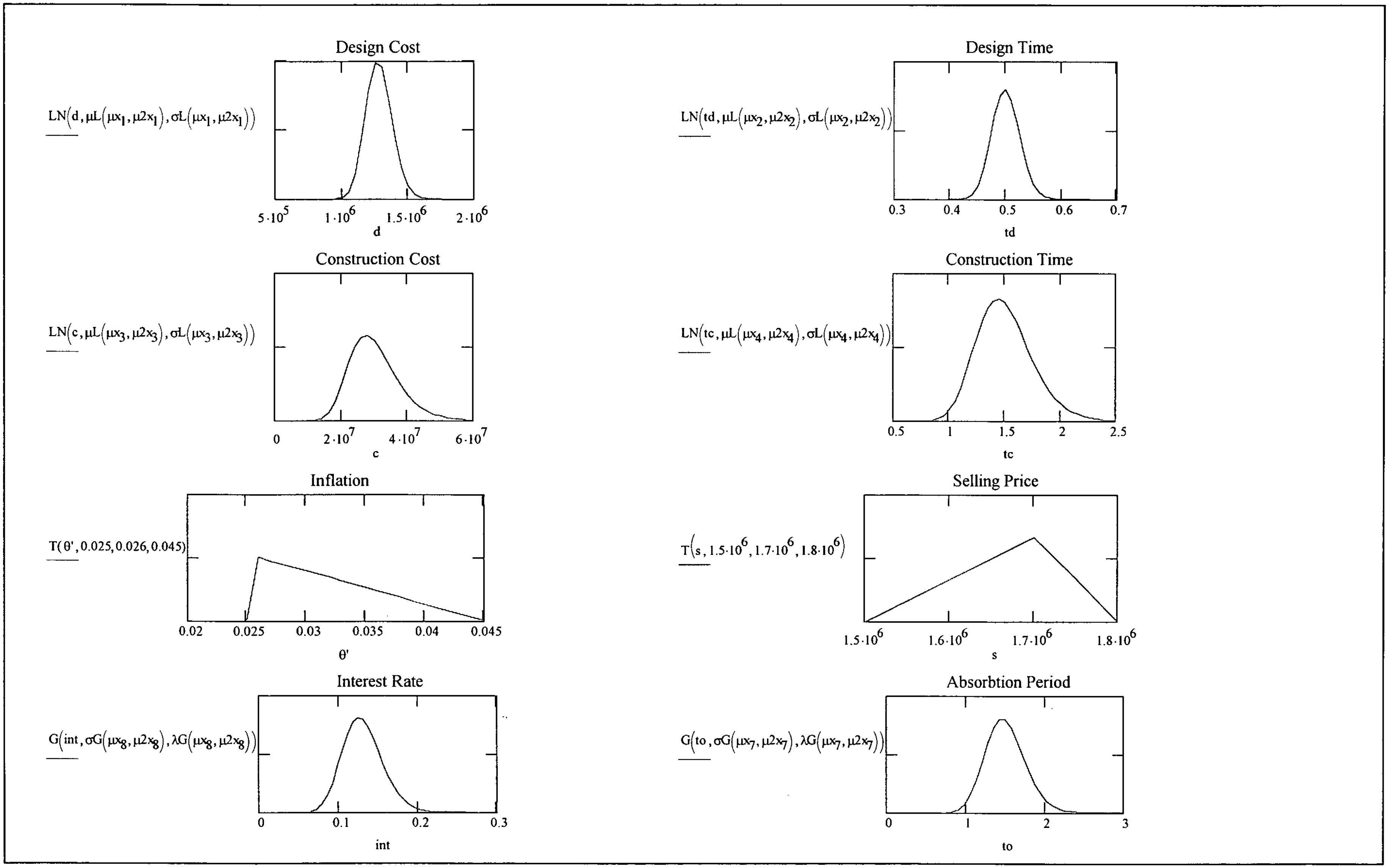
$N := 45$ $r := 0.17$

Loan variables (interest i, and loan value Tloan):

i $\mu x_8 := 0.13$ $\mu 2x_8 := .025^2$ $\mu 3x_8 := \mu 3G\left(\mu x_8, \mu 2x_8\right)$ $\mu 4x_8 := \mu 4G\left(\mu x_8, \mu 2x_8\right)$ $cv\left(\mu x_8, \sqrt{\mu 2x_8}\right) = 0.192308$ $i := \mu x_8$

$Tloan := 0.55 \cdot (D + C)$ $\sigma G\left(\mu x_8, \mu 2x_8\right) = 4.807692 \cdot 10^{-3}$ $\lambda G\left(\mu x_8, \mu 2x_8\right) = 27.04$

Table 7.3: Description of variables used in the example project (Mathcad sheet)



Figures 7.2 to 7.7 illustrate how the design, construction, revenue and loan variables and methods are defined in the DSS. Design and construction are defined as two work packages in Figures 7.2 and 7.3 respectively. Land cost is defined as a single discrete cost in the design work package and could have been modeled separately in another work package. In Table 7.2 using the first two moments of a variable and its distribution model, the 3rd and 4th moments of the variable and the parameters of its distribution model are obtained. While this can be readily used in the DSS, alternatively the parameters of the variables can be used to determine the four moments of the variable as shown in Figure 7.2, which is used to further validate the DSS. Construction inflation is defined as a "Uniform I" *rate function* where the inflation rate will be the same over the entire construction duration and its statistical moments will be calculated using the Triangular distribution as shown in Figure 7.3.

Revenues as shown in Figure 7.4 have a Gamma duration and are estimated using the trend method. The trend method selected is "Uniform Total", as shown in Figure 7.5. This method distributes revenues uniformly over the revenue duration. Selling price is constant across the revenue stream as is estimated using the "Uniform I" *rate function* with the value obtained from the Triangular distribution as shown in Figure 7.6.

Project debt is defined as a general term loan as shown in Figure 7.7. In this figure, all characteristics of the loan are defined in a single window. Unlike the explicit formulation of the performance measures, as described later, the DSS determines the number of loan repayments and interest payments using the term of the loan, grace period, and the interest and repayment intervals as explained in Chapter 4.

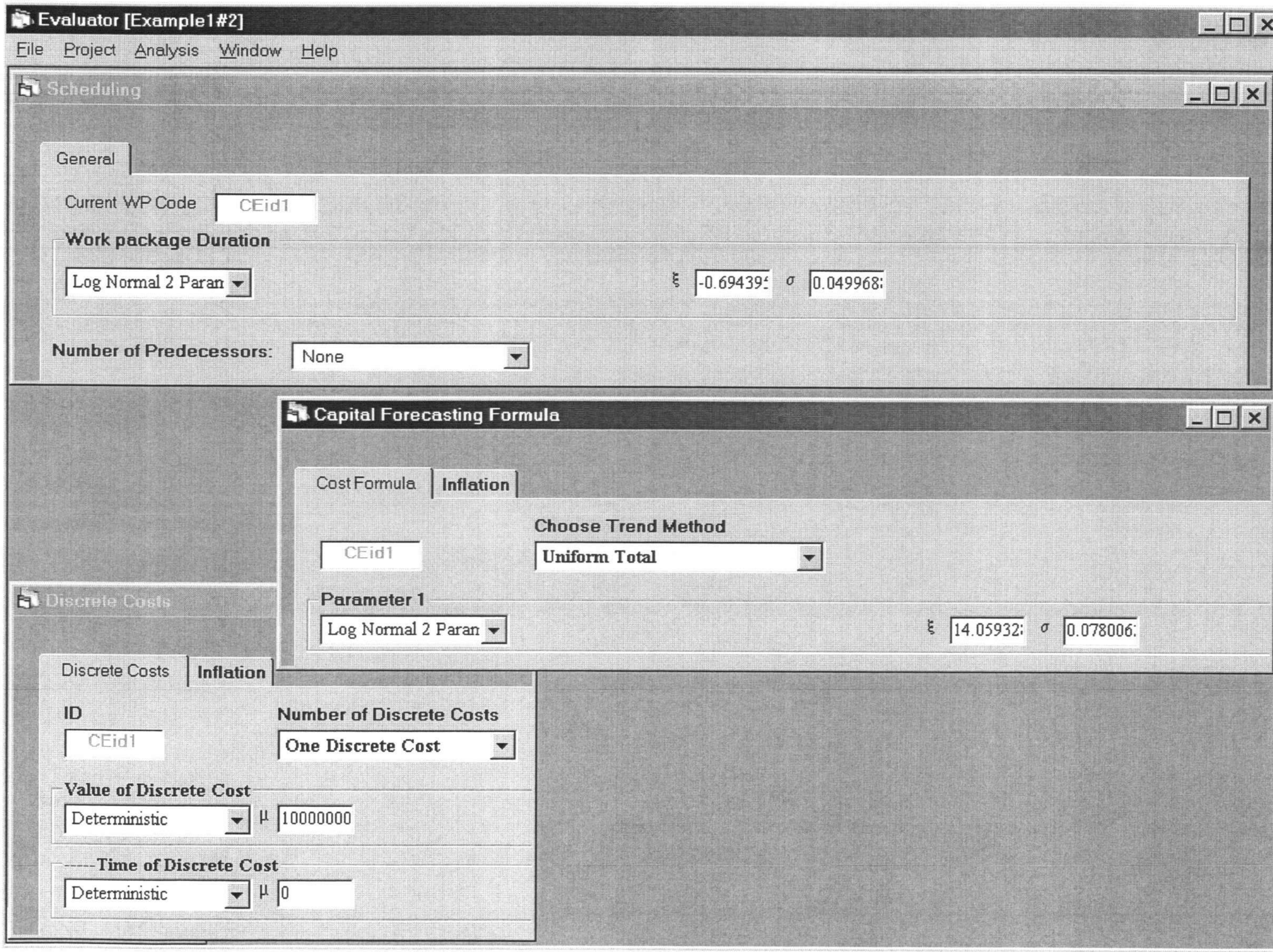


Figure 7.2: Design work package

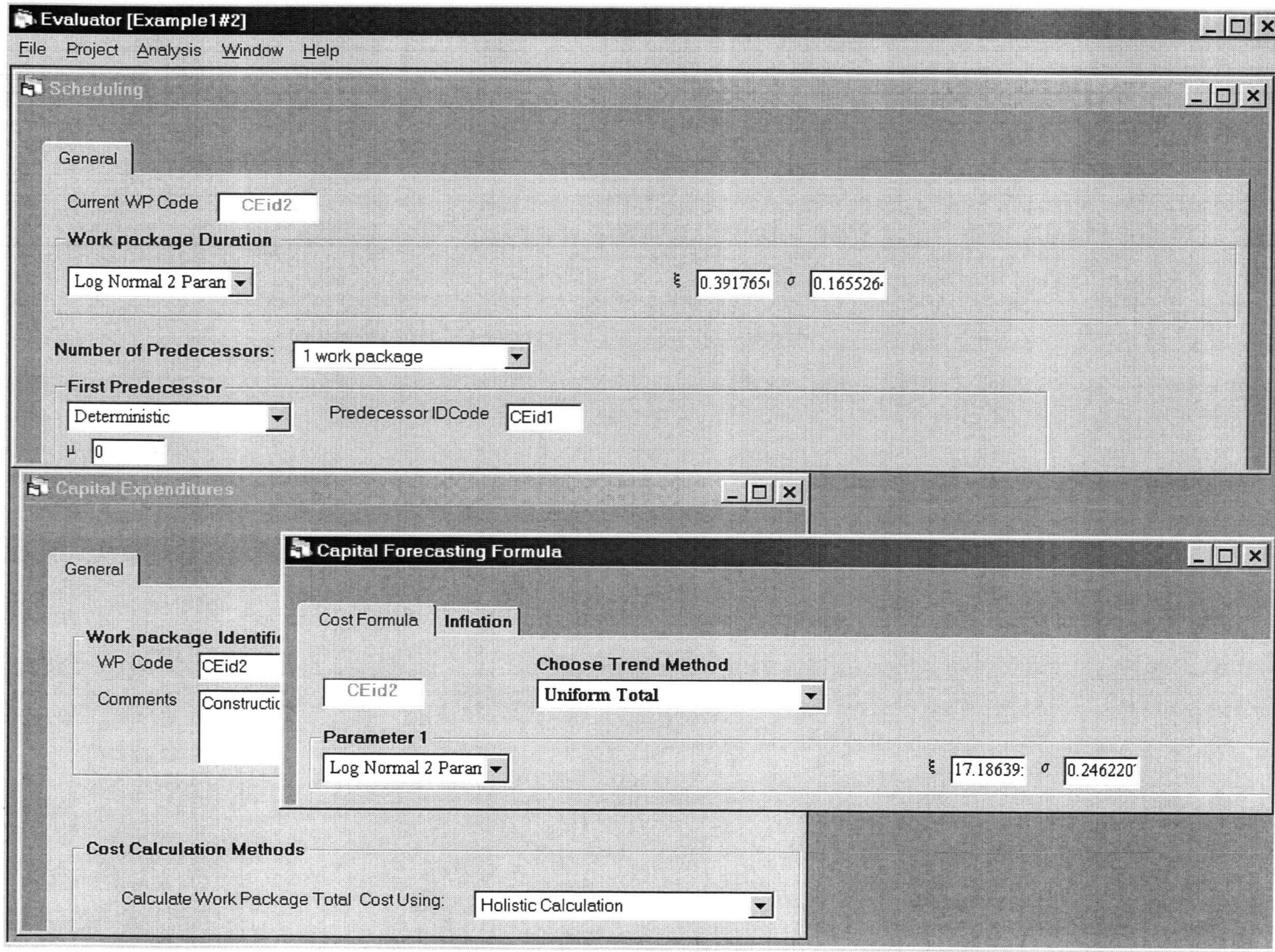


Figure 7.3: Construction work package

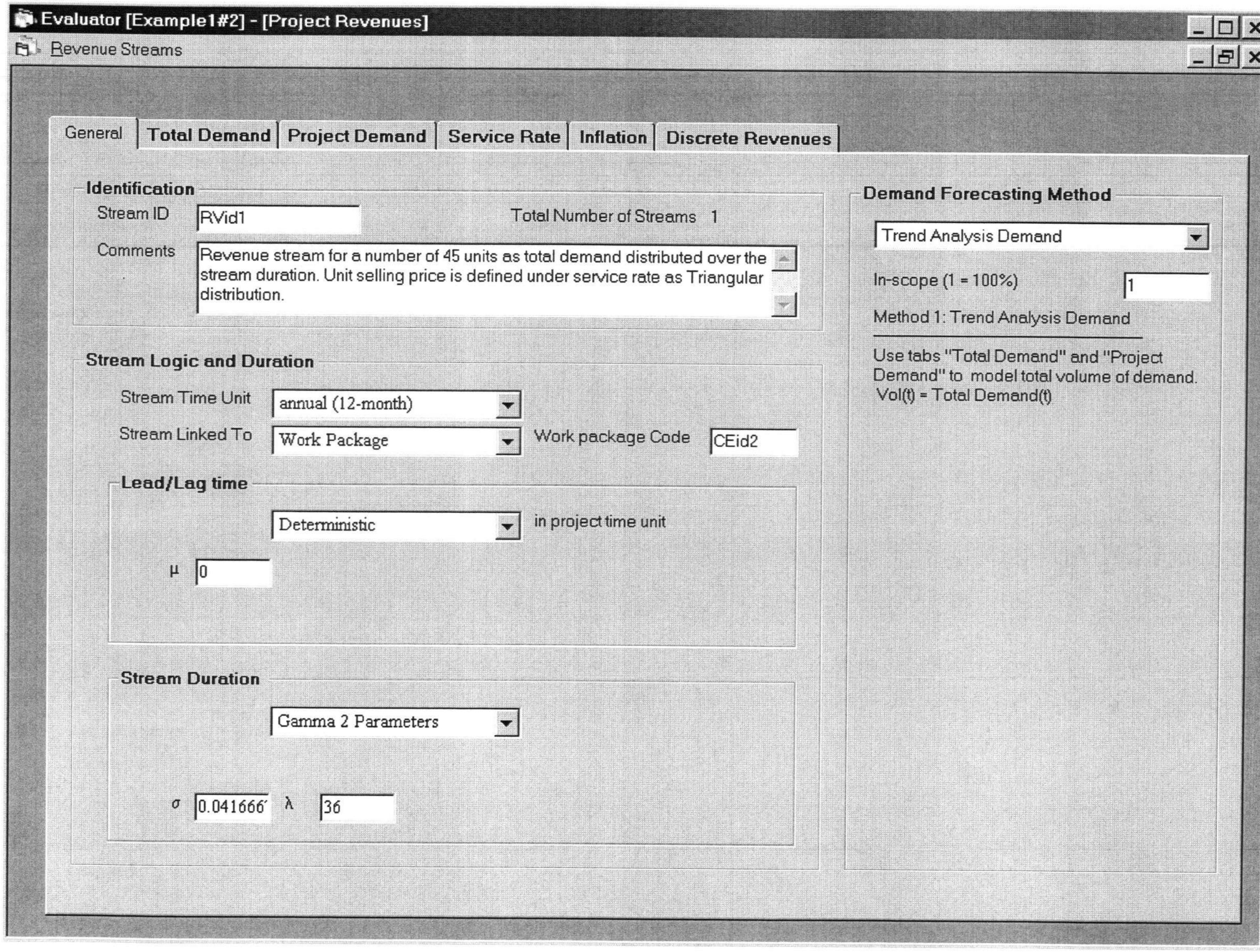


Figure 7.4: Revenue stream duration and calculation method

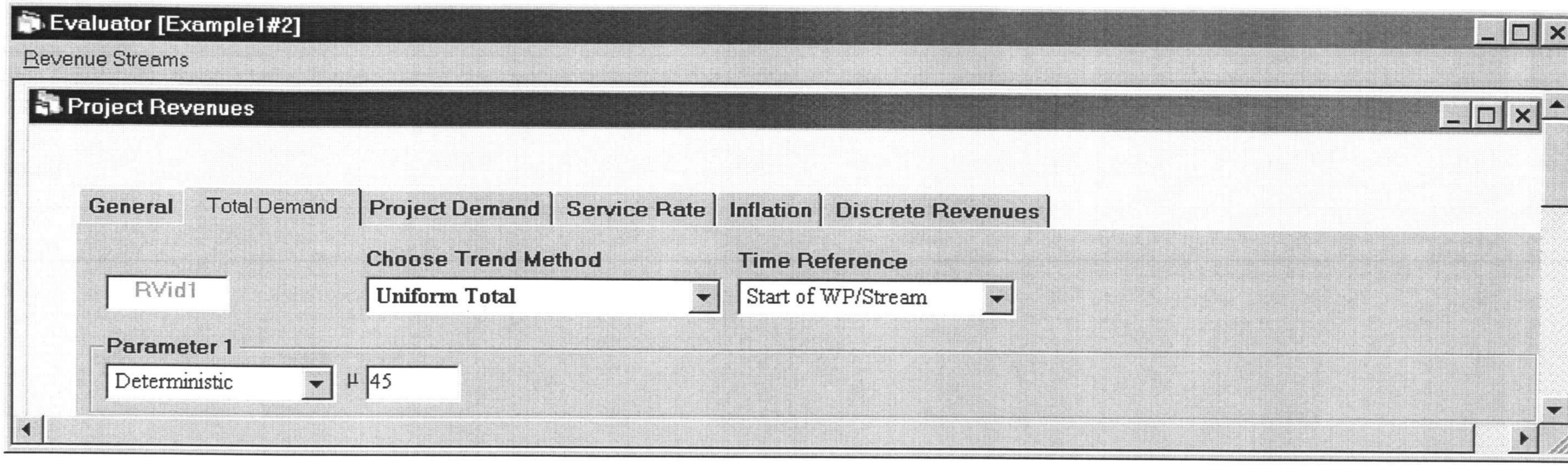


Figure 7.5: Total number of units for the project

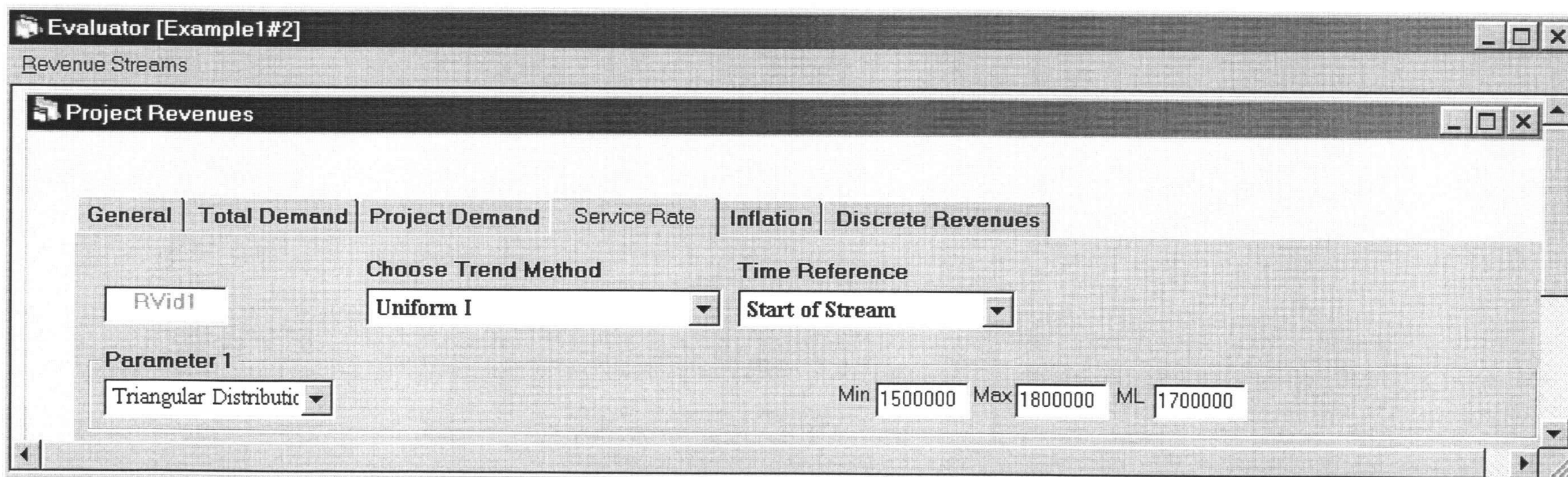


Figure 7.6: Unit selling price defined as Triangular distribution

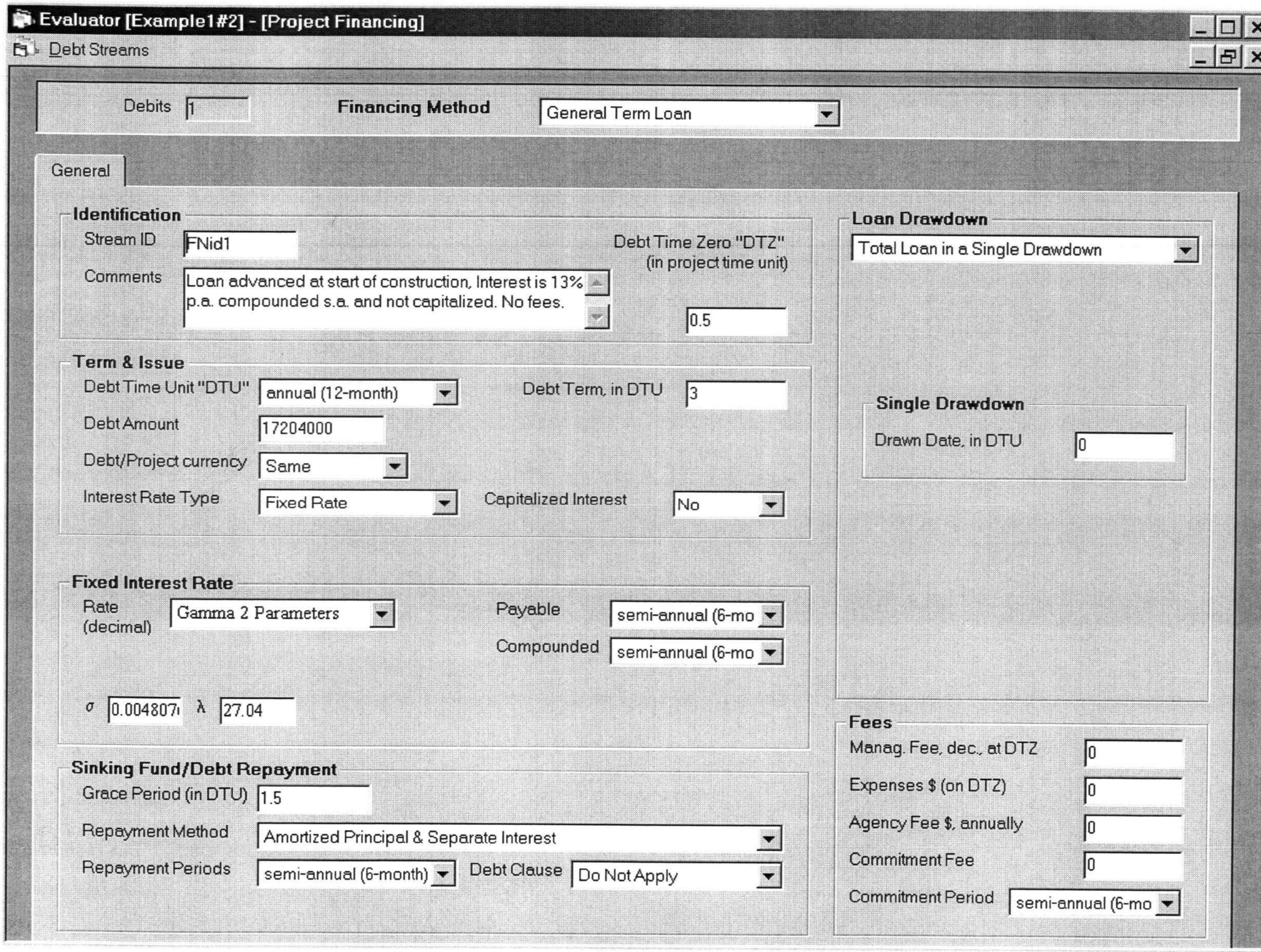


Figure 7.7: Drawdown, interest payments, and repayment of term loan for the project

7.2.2 Net Present Value

The five-page Table 7.4 illustrates the calculation Mathcad sheets that include the explicit formulation and risk analysis of the net present value (NPV) used to validate the DSS. The bottom of the first table page shows the deterministic value of NPV as determined based on the average values of the variables in Table 7.2. Pages two and three illustrate the first and second derivatives of NPV with respect to all the risk variables defined in Table 7.2. The derivatives are calculated using both the Mathcad derivative algorithm and the *centered-finite-divided-difference* method (used in the DSS). Mathcad uses Ridder's algorithm that follows the divided difference methods in order to determine an accurate first derivative within 7 or 8 significant digits. As shown, both algorithms give almost identical results. The second derivatives with respect to construction cost C and selling price S (fourth and seventh columns) should be equal to zero, however, they are different in both cases because of the accuracy or the zero tolerance (a value, e.g. 10^{-15}, after which a variable is considered zero) in Mathcad – clearly, both values should be considered as zero. Page 4 of Table 7.4 illustrates the calculation of the four moments of the NPV as per the equations in Chapter 5 and Appendix A for the uncorrelated variable case (see Eqs. 5.17-5.20). On the fifth table page, β1, β2 and the criterion *k* are computed (see section 5.2.3.1) which results in the Pearson Type I distribution being selected (see Table 5.2).

Figure 7.8 illustrates the DSS output for the NPV analysis – it shows an exact match between the DSS and the explicit formulation of Table 7.4. As shown in Figure 7.8, the Pearson Type I is selected shows a negatively skewed NPV distribution, in keeping with the NPV skewness coefficient. The cumulative distribution is further explained in the Figure 7.9 (a capability of the DSS). A further validation to these results was performed through a Monte Carlo simulation of 50000 iterations for NPV using Excel and @Risk and the formulation of Table 7.2 and NPV formulation in Table 7.4. Results are shown in Table 7.5.

Table 7.4: Calculation sheet for the risk analysis of NPV

C) Net present value formulation:

Capital expenditures and revenues

$$\mathrm{Design}(D, Td, r) := \int_0^{Td} \frac{-D}{Td} \cdot e^{-r \cdot t}\, dt \qquad \mathrm{Design}(D, Td, r) := \frac{-D}{Td} \cdot \left(\frac{e^{-r \cdot Td} - 1}{-r} \right)$$

$$\mathrm{Construction}(C, Tc, Td, \theta, r) := e^{(\theta - r) \cdot Td} \cdot \int_0^{Tc} \frac{-C}{Tc} \cdot e^{(\theta - r) \cdot t}\, dt \qquad \mathrm{Construction}(C, Tc, Td, \theta, r) := e^{(\theta - r) \cdot Td} \cdot \frac{-C}{Tc} \cdot \left[\frac{e^{(\theta - r) \cdot Tc} - 1}{\theta - r} \right]$$

$$\mathrm{Revenue}(S, Tc, Td, To, r) := e^{-r \cdot (Td + Tc)} \cdot \int_0^{To} \frac{S \cdot N}{To} \cdot e^{-r \cdot t}\, dt \qquad \mathrm{Revenue}(S, Tc, Td, To, r) := e^{-r \cdot (Td + Tc)} \cdot \frac{S \cdot N}{To} \cdot \left(\frac{e^{-r \cdot To} - 1}{-r} \right)$$

Loan calculations

$$ie(i) := \left(1 + \frac{i}{1 \cdot 2} \right)^{1} - 1 \qquad ie(i) = 0.065$$ effective interest rate per payment period, i.e. 6-month

$$\mathrm{Loan}(i, r) := \mathrm{Tloan} \cdot e^{-r \cdot 0.5} - \sum_{j=1}^{4} \frac{\mathrm{Tloan}}{4} \cdot e^{-r \cdot (1.5 + j \cdot 0.5)} - \mathrm{Tloan} \cdot ie(i) \cdot \sum_{j=1}^{3} e^{-r \cdot (0.5 + j \cdot 0.5)} - \frac{\mathrm{Tloan}}{4} \cdot ie(i) \cdot \left(3 \cdot e^{-r \cdot 2.5} + 2 \cdot e^{-r \cdot 3} + 1 \cdot e^{-r \cdot 3.5} \right)$$

$$\mathrm{Loan}(i, r) = 1.32969 \cdot 10^{6}$$

Net present value function at $r = 0.17$

$$\mathrm{NPV}(D, Td, C, Tc, \theta, S, To, i, r) := -L + \mathrm{Design}(D, Td, r) + \mathrm{Construction}(C, Tc, Td, \theta, r) + \mathrm{Revenue}(S, Tc, Td, To, r) + \mathrm{Loan}(i, r)$$

$$\mathrm{NPV}(D, Td, C, Tc, \theta, S, To, i, r) = 1.183759 \cdot 10^{7}$$ deterministic value of NPV based on average values

Table 7.4: continued.

D) Risk Analysis of NPV

First derivatives of NPV using Mathcad algorithm

$$d1Z_1 := \frac{d}{dD} NPV(D, Td, C, Tc, \theta, S, To, i, r)$$

$$d1Z_2 := \frac{d}{dTd} NPV(D, Td, C, Tc, \theta, S, To, i, r)$$

$$d1Z_3 := \frac{d}{dC} NPV(D, Td, C, Tc, \theta, S, To, i, r)$$

$$d1Z_4 := \frac{d}{dTc} NPV(D, Td, C, Tc, \theta, S, To, i, r)$$

$$d1Z_5 := \frac{d}{d\theta} NPV(D, Td, C, Tc, \theta, S, To, i, r)$$

$$d1Z_6 := \frac{d}{dS} NPV(D, Td, C, Tc, \theta, S, To, i, r)$$

$$d1Z_7 := \frac{d}{dTo} NPV(D, Td, C, Tc, \theta, S, To, i, r)$$

$$d1Z_8 := \frac{d}{di} NPV(D, Td, C, Tc, \theta, S, To, i, r)$$

Second Derivatives of NPV using Mathcad algorithm

$$d2Z_1 := \frac{d^2}{dD^2} NPV(D, Td, C, Tc, \theta, S, To, i, r)$$

$$d2Z_2 := \frac{d^2}{dTd^2} NPV(D, Td, C, Tc, \theta, S, To, i, r)$$

$$d2Z_3 := \frac{d^2}{dC^2} NPV(D, Td, C, Tc, \theta, S, To, i, r)$$

$$d2Z_4 := \frac{d^2}{dTc^2} NPV(D, Td, C, Tc, \theta, S, To, i, r)$$

$$d2Z_5 := \frac{d^2}{d\theta^2} NPV(D, Td, C, Tc, \theta, S, To, i, r)$$

$$d2Z_6 := \frac{d^2}{dS^2} NPV(D, Td, C, Tc, \theta, S, To, i, r)$$

$$d2Z_7 := \frac{d^2}{dTo^2} NPV(D, Td, C, Tc, \theta, S, To, i, r)$$

$$d2Z_8 := \frac{d^2}{di^2} NPV(D, Td, C, Tc, \theta, S, To, i, r)$$

$$d1Z^T = \begin{bmatrix} 0 & -0.958679 & -4.480597\cdot10^6 & -0.846164 & -6.354696\cdot10^6 & -3.108904\cdot10^7 & 28.271957 & -3.835157\cdot10^6 & -2.803235\cdot10^7 \end{bmatrix}$$ Mathcad Algorithm

$$dF1Z^T = \begin{bmatrix} 0 & -0.958679 & -4.480597\cdot10^6 & -0.846164 & -6.354696\cdot10^6 & -3.108904\cdot10^7 & 28.271957 & -3.835157\cdot10^6 & -2.803235\cdot10^7 \end{bmatrix}$$ Finite Difference

$$d2Z^T = \begin{bmatrix} 0 & 0 & 8.87555\cdot10^5 & -1.918283\cdot10^{-22} & 1.215302\cdot10^6 & -4.282482\cdot10^7 & 2.310672\cdot10^{-19} & 4.252614\cdot10^5 & 0 \end{bmatrix}$$ Mathcad Algorithm

$$dF2Z^T = \begin{bmatrix} 0 & 0 & 8.875519\cdot10^5 & 8.278423\cdot10^{-16} & 1.215302\cdot10^6 & -4.282533\cdot10^7 & -2.682209\cdot10^{-13} & 4.252609\cdot10^5 & 0 \end{bmatrix}$$ Finite Difference

Table 7.4: continued.

First and second derivatives of NPV using finite difference method $ss := 0.0001$ (used for step size)

$$dF1Z_1 := \frac{NPV(D\cdot(1+ss),Td,C,Tc,\theta,S,To,i,r) - NPV(D\cdot(1-ss),Td,C,Tc,\theta,S,To,i,r)}{2\cdot D\cdot ss}$$

$$dF1Z_2 := \frac{NPV(D,Td\cdot(1+ss),C,Tc,\theta,S,To,i,r) - NPV(D,Td\cdot(1-ss),C,Tc,\theta,S,To,i,r)}{2\cdot Td\cdot ss}$$

$$dF1Z_3 := \frac{NPV(D,Td,C\cdot(1+ss),Tc,\theta,S,To,i,r) - NPV(D,Td,C\cdot(1-ss),Tc,\theta,S,To,i,r)}{2\cdot C\cdot ss}$$

$$dF1Z_4 := \frac{NPV(D,Td,C,Tc\cdot(1+ss),\theta,S,To,i,r) - NPV(D,Td,C,Tc\cdot(1-ss),\theta,S,To,i,r)}{2\cdot Tc\cdot ss}$$

$$dF1Z_5 := \frac{NPV(D,Td,C,Tc,\theta\cdot(1+ss),S,To,i,r) - NPV(D,Td,C,Tc,\theta\cdot(1-ss),S,To,i,r)}{2\cdot\theta\cdot ss}$$

$$dF2Z_1 := \frac{NPV(D\cdot(1+ss),Td,C,Tc,\theta,S,To,i,r) - 2\cdot NPV(D,Td,C,Tc,\theta,S,To,i,r) + NPV(D\cdot(1-ss),Td,C,Tc,\theta,S,To,i,r)}{(D\cdot ss)^2}$$

$$dF2Z_2 := \frac{NPV(D,Td\cdot(1+ss),C,Tc,\theta,S,To,i,r) - 2\cdot NPV(D,Td,C,Tc,\theta,S,To,i,r) + NPV(D,Td\cdot(1-ss),C,Tc,\theta,S,To,i,r)}{(Td\cdot ss)^2}$$

$$dF2Z_3 := \frac{NPV(D,Td,C\cdot(1+ss),Tc,\theta,S,To,i,r) - 2\cdot NPV(D,Td,C,Tc,\theta,S,To,i,r) + NPV(D,Td,C\cdot(1-ss),Tc,\theta,S,To,i,r)}{(C\cdot ss)^2}$$

$$dF2Z_4 := \frac{NPV(D,Td,C,Tc\cdot(1+ss),\theta,S,To,i,r) - 2\cdot NPV(D,Td,C,Tc,\theta,S,To,i,r) + NPV(D,Td,C,Tc\cdot(1-ss),\theta,S,To,i,r)}{(Tc\cdot ss)^2}$$

$$dF2Z_5 := \frac{NPV(D,Td,C,Tc,\theta\cdot(1+ss),S,To,i,r) - 2\cdot NPV(D,Td,C,Tc,\theta,S,To,i,r) + NPV(D,Td,C,Tc,\theta\cdot(1-ss),S,To,i,r)}{(\theta\cdot ss)^2}$$

$$dF2Z_6 := \frac{NPV(D,Td,C,Tc,\theta,S\cdot(1+ss),To,i,r) - 2\cdot NPV(D,Td,C,Tc,\theta,S,To,i,r) + NPV(D,Td,C,Tc,\theta,S\cdot(1-ss),To,i,r)}{(S\cdot ss)^2}$$

$$dF2Z_7 := \frac{NPV(D,Td,C,Tc,\theta,S,To\cdot(1+ss),i,r) - 2\cdot NPV(D,Td,C,Tc,\theta,S,To,i,r) + NPV(D,Td,C,Tc,\theta,S,To\cdot(1-ss),i,r)}{(To\cdot ss)^2}$$

$$dF2Z_8 := \frac{NPV(D,Td,C,Tc,\theta,S,To,i\cdot(1+ss),r) - 2\cdot NPV(D,Td,C,Tc,\theta,S,To,i,r) + NPV(D,Td,C,Tc,\theta,S,To,i\cdot(1-ss),r)}{(i\cdot ss)^2}$$

Table 7.4: continued.

E) Four Moments of the net present value:

$n := 8$

$Z_\mu := NPV(\mu x_1, \mu x_2, \mu x_3, \mu x_4, \mu x_5, \mu x_6, \mu x_7, \mu x_8, r)$ (NPV calculated at the mean values) $Z_\mu = 1.183759 \cdot 10^7$

$$EX_Z := NPV(\mu x_1, \mu x_2, \mu x_3, \mu x_4, \mu x_5, \mu x_6, \mu x_7, \mu x_8, r) + \frac{1}{2} \cdot \sum_{j=1}^{n} d2Z_j \cdot \mu 2x_j$$

$EX_Z = 1.188878 \cdot 10^7$

$$\mu 2_Z := \sum_{i=1}^{n} (d1Z_i)^2 \cdot \mu 2x_i + \frac{1}{4} \cdot \sum_{i=1}^{n} (d2Z_i)^2 \cdot \mu 4x_i + \frac{-1}{4} \cdot \left(\sum_{i=1}^{n} d2Z_i \cdot \mu 2x_i \right)^2 + \sum_{i=1}^{n} d1Z_i \cdot d2Z_i \cdot \mu 3x_i$$

$\sqrt{\mu 2_Z} = 6.876679 \cdot 10^6$

$\mu 2_Z = 4.728871 \cdot 10^{13}$

$$\mu 3_Z := \sum_{i=1}^{n} (d1Z_i)^3 \cdot (\mu 3x_i) + \frac{1}{4} \cdot \left(\sum_{i=1}^{n} d2Z_i \cdot \mu 2x_i \right)^3 + \frac{3}{2} \left[\sum_{i=1}^{n} (d1Z_i)^2 \cdot d2Z_i \cdot \mu 4x_i \right] + \left[\sum_{i=1}^{n} (d2Z_i) \cdot \mu 2x_i \right] \cdot \left[\frac{-3}{2} \cdot \sum_{i=1}^{n} (d1Z_i)^2 \cdot \mu 2x_i \right] \ldots$$
$$+ \left[\sum_{i=1}^{n} (d2Z_i) \cdot \mu 2x_i \right] \cdot \left[\frac{-3}{2} \cdot \sum_{i=1}^{n} (d1Z_i) \cdot d2Z_i \cdot \mu 3x_i \right] + \left[\sum_{i=1}^{n} (d2Z_i) \cdot \mu 2x_i \right] \cdot \left[\frac{-3}{8} \cdot \sum_{i=1}^{n} (d2Z_i)^2 \cdot (\mu 4x_i) \right]$$

$\mu 3_Z = -2.059637 \cdot 10^{20}$

$$\mu 4_Z := \left[\sum_{i=1}^{n} (d1Z_i)^4 \cdot (\mu 4x_i) + \frac{-3}{16} \cdot \left(\sum_{i=1}^{n} d2Z_i \cdot \mu 2x_i \right)^4 + \left[- \sum_{i=1}^{n} (d2Z_i) \cdot \mu 2x_i \right] \cdot \left[2 \cdot \sum_{i=1}^{n} (d1Z_i)^3 \cdot \mu 3x_i \right] + \left[\sum_{i=1}^{n} (d2Z_i) \cdot \mu 2x_i \right]^2 \cdot \left[\frac{3}{2} \cdot \sum_{i=1}^{n} (d1Z_i)^2 \cdot \mu 2x_i \right] \right] \ldots$$
$$+ \left[\sum_{i=1}^{n} (d2Z_i) \cdot \mu 2x_i \right]^2 \cdot \left[\frac{3}{2} \cdot \sum_{i=1}^{n} (d1Z_i) \cdot d2Z_i \cdot \mu 3x_i \right] + \left[- \sum_{i=1}^{n} (d2Z_i) \cdot \mu 2x_i \right] \cdot \left[3 \cdot \sum_{i=1}^{n} (d1Z_i)^2 \cdot d2Z_i \cdot \mu 4x_i \right] + \left[\sum_{i=1}^{n} (d2Z_i) \cdot \mu 2x_i \right]^2 \cdot \left[\frac{3}{8} \cdot \sum_{i=1}^{n} (d2Z_i)^2 \cdot (\mu 4x_i) \right]$$

$\mu 4_Z = 6.675718 \cdot 10^{27}$

Table 7.4: continued.

F) Distribution for NPV:

$\gamma 1 := \dfrac{\mu 3_Z}{\mu 2_Z^{\,1.5}}$ $\gamma 2 := \dfrac{\mu 4_Z}{\mu 2_Z^{\,2}}$ $\beta 1 := \gamma 1^2$ $\beta 2 := \gamma 2$

$\gamma 1 = -0.633366$ $\gamma 2 = 2.985265$ $\beta 1 = 0.401152$ $\beta 2 = 2.985265$

$$k\_criterion := \frac{\beta 1 \cdot (\beta 2 + 3)^2}{4 \cdot (4 \cdot \beta 2 - 3 \cdot \beta 1) \cdot (2 \cdot \beta 2 - 3 \cdot \beta 1 - 6)}$$

$k\_criterion = -0.271376$ k is less than zero, then Pearson Type I is selected

Table 7.5: Comparison between DSS and simulation results for NPV

NPV	Mean	Variance	Skewness	Kurtosis
DSS	11888780	4.72887E+13	-0.633366	2.985265
Simulation	11888880	4.72896E+13	-0.585630	3.754189
Deviation %	-0.00084	-0.00182	8.15109	-20.481

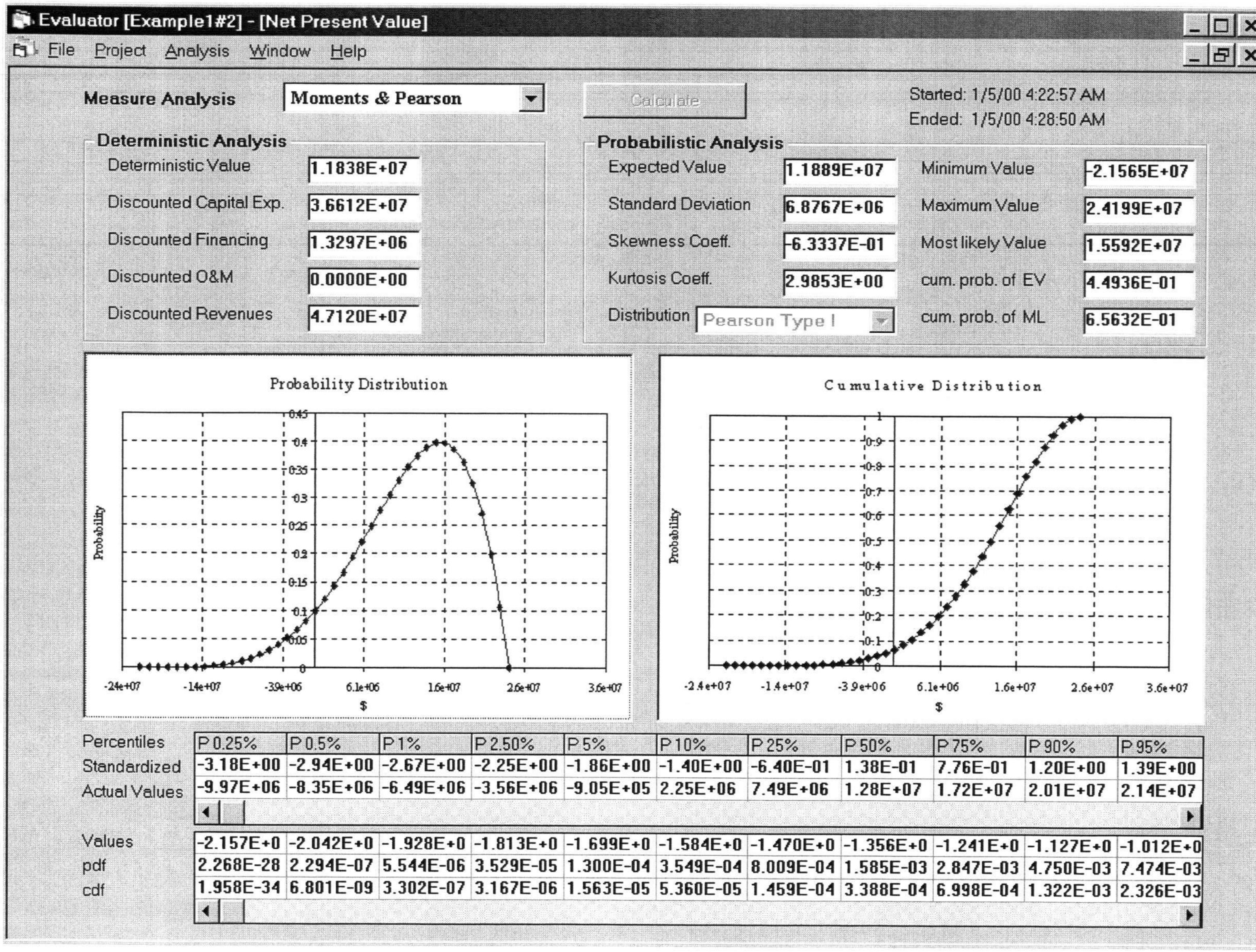


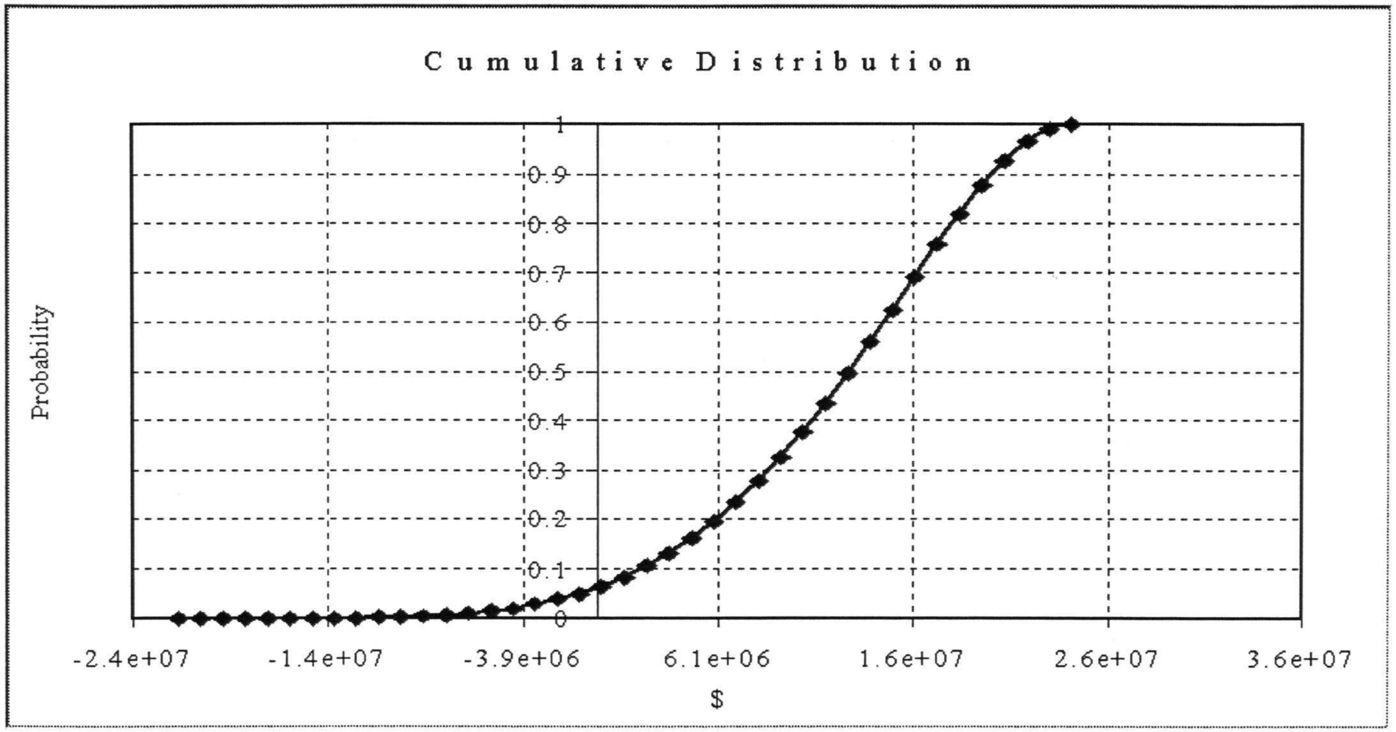
Figure 7.8: Net present value results for the example project (NPV values in the 2nd table has a power of 7 unseen)

Evaluator [Example1#2] - [Net Present Value]

File Project Analysis Window Help

Measure Analysis: Moments & Pearson — Calculate

Started: 1/5/00 4:22:57 AM
Ended: 1/5/00 4:28:50 AM



Percentiles	P 0.25%	P 0.5%	P 1%	P 2.50%	P 5%	P 10%	P 25%	P 50%	P 75%	P 90%	P 95%
Standardized	-3.18E+00	-2.94E+00	-2.67E+00	-2.25E+00	-1.86E+00	-1.40E+00	-6.40E-01	1.38E-01	7.76E-01	1.20E+00	1.39E+00
Actual Values	-9.97E+06	-8.35E+06	-6.49E+06	-3.56E+06	-9.05E+05	2.25E+06	7.49E+06	1.28E+07	1.72E+07	2.01E+07	2.14E+07

Values	-2.16E+07	-2.04E+07	-1.93E+07	-1.81E+07	-1.70E+07	-1.58E+07	-1.47E+07	-1.36E+07	-1.24E+07	-1.13E+07	-1.01E+07
pdf	2.27E-28	2.29E-07	5.54E-06	3.53E-05	1.30E-04	3.55E-04	8.01E-04	1.58E-03	2.85E-03	4.75E-03	7.47E-03
cdf	1.96E-34	6.80E-09	3.30E-07	3.17E-06	1.56E-05	5.36E-05	1.46E-04	3.39E-04	7.00E-04	1.32E-03	2.33E-03

Figure 7.9: Cumulative distribution of the net present value

Table 7.5 and Figure 7.10 show near exact match for the expected value and variance of the simulation and DSS analyses on NPV. The coefficients of skewness and kurtosis experienced differences that can be attributed to (1) the simulation experiment assumes totally uncorrelated and independent variables which is unlike the DSS that assumes uncorrelated variables but not independent, and (2) the derivation of the four moments of system functions in Appendix A does not include moments higher than the fourth moment of the variables. However, the comparison of both the simulation and analytical DSS results can be considered reasonable as seen from the cumulative distributions in Figure 7.10.

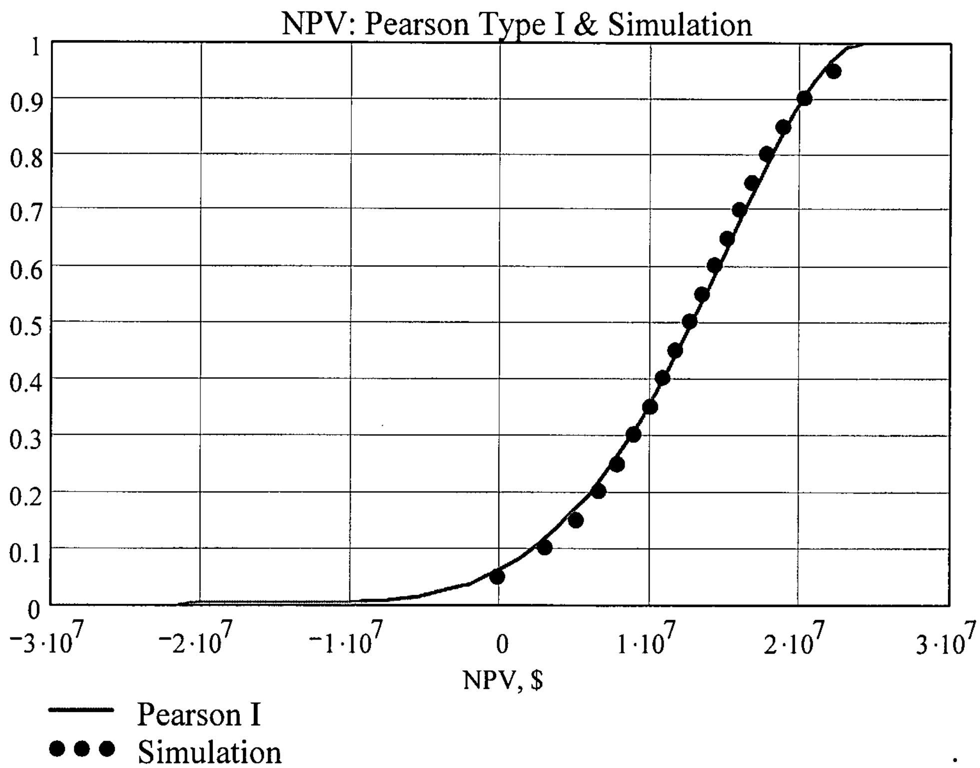


Figure 7.10: Comparison between DSS and simulation for NPV distribution

7.2.3 Internal Rate of Return on Equity and Total Capital

The formulation and risk analysis of the internal rate of return on equity (IRRe) and total capital (IRRtc) of the example project using Mathcad are detailed in Appendix B. The results of this explicit formulation are shown in Table 7.6. Since the criterion k approaches zero, the distribution of IRRe should be Pearson Type VII. However, Pearson Type IV could be used as well since $0<k<1$. The distribution of IRRtc should be Pearson Type III since $2\cdot\beta 2$ [6.52] approaches $3\cdot\beta 1+6$ [6.55] and k is highly negative. When processed in the DSS, the results for IRRe and IRRtc, as shown in Figures 7.11 and 7.12, corresponds to those obtained through the explicit formulation.

Table 7.6 Moment characteristics of internal rate of return on equity and total capital (Explicit Formulation)

	At mean	μ'_1	$\sigma = \sqrt{\mu 2}$	$\mu 3$	$\mu 4$	$\gamma 1 = \sqrt{\beta 1}$	$\gamma 2 = \beta 2$	k criterion
IRRe	0.42379	0.44439	0.16598	-7.416E-4	2.5312E-3	-0.16217	3.33472	0.03369
IRRc	0.30439	0.31147	0.09279	-3.426E-4	2.4189E-4	-0.42883	-3.26292	-5.5829

7.2.4 Total Cost, Total Revenues, and Construction Completion Time

The detailed formulation of total capital expenditure, total revenues, and construction completion time are explained in Chapter 4. Appendix B provides the explicit Mathcad formulation and the results are summarized in Table 7.7.

Table 7.7 Moment characteristics of total cost, revenues, and construction completion time (Explicit Formulation)

	At mean	μ'_1	$\sigma = \sqrt{\mu 2}$	$\mu 3$	$\mu 4$	$\gamma 1 = \sqrt{\beta 1}$	$\gamma 2 = \beta 2$	k
Total Cost	4.2625E7	4.2626E7	7.839E6	3.683E20	1.53E28	0.7642	4.052	1.425
Revenues	7.5E7	7.5E7	2.806E6	-6.75E18	1.488E26	-0.305	2.4	-0.049
Completion	2.00	2.00	0.25125	7.88E-3	0.0135	0.4973	3.388	5.665

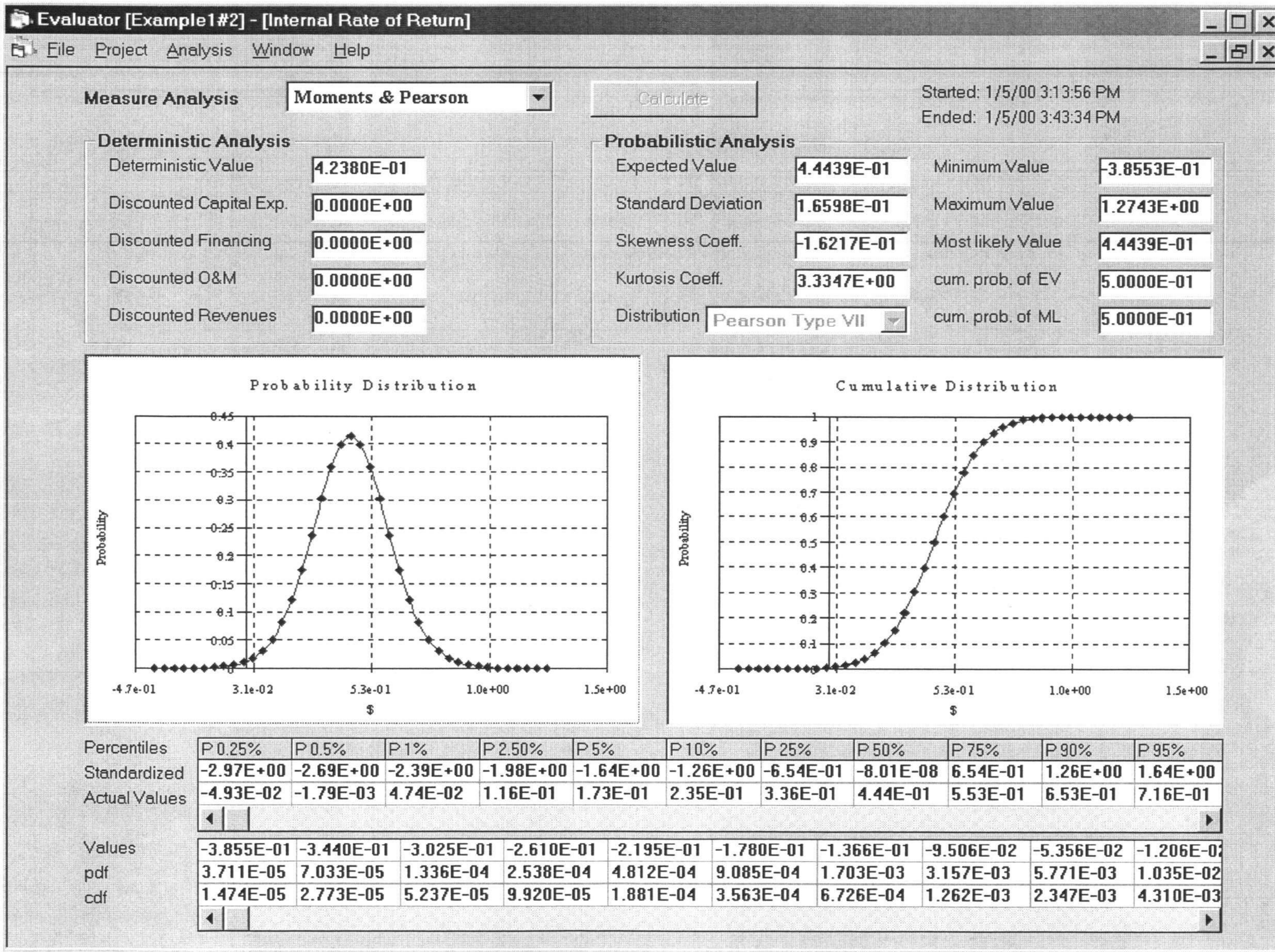


Figure 7.11: Internal rate of return on equity

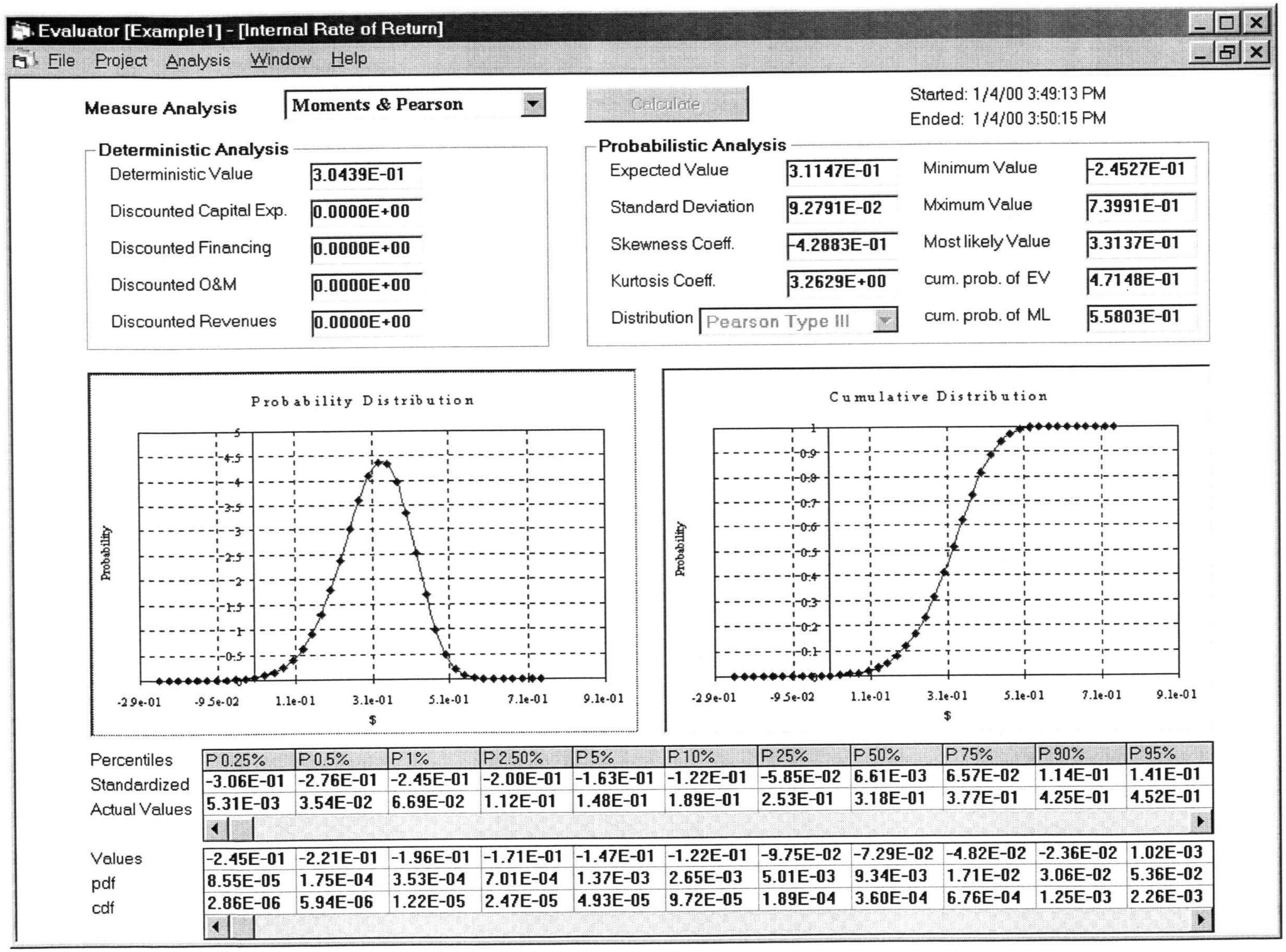


Figure 7.12: Internal rate of return on total capital

Figures 7.13, 7.14, and 7.15 illustrate the total capital expenditure, total revenue, and completion time determined using the DSS. Identical results are obtained for the moment characteristics as for the explicit formulation. A Pearson Type VI distribution is fitted to total cost since $k>1$; as shown in Figure 7.13 the distribution is positively skewed resulting in a slight probability of large cost overruns. Total revenue was modeled using a negatively skewed Pearson Type I distribution. The negative skew reflects the negatively skewed selling price modeled by a Triangular distribution as described in Tables 7.2 and 7.3. Completion time was assigned a positively skewed Pearson Type III distribution (i.e. Gamma model) reflecting the summation of the Log Normally distributed and positively skewed design and construction durations.

7.2.5 Debt Service and Loan Life Cover Ratios

Appendix B shows the explicit calculation of the debt service and loan life cover ratios (DSCR, LLCR) at 6-month intervals. A discount rate of 10% is used with the LLCR. The results are the same as those obtained by the DSS. Figure 7.16 shows the number and values of loan tranches, interest payments, and principal repayments as determined in the DSS using the information given about the loan in Figure 7.7. Figures 7.17 and 7.18 tabulate the values of the DSCR and LLCR at 6-month intervals. DSCR values in these figures are linked for simple representation.

7.2.6 Sensitivity Analysis Of Project Variables

Figures 7.19, 7.20 and 7.21 illustrate the DSS normalized sensitivity coefficients for NPV as a function of the variables used for describing the project. Similar results are obtained in Appendix B for the explicit formulation. The analysis shows that the NPV is most sensitive to the selling price and construction cost. Figure 7.19 shows the sensitivity coefficient for inflation in the construction work package. Note that design cost in the first work package was expressed in current dollars- hence the sensitivity coefficient of inflation for work package CEid1 is 0.

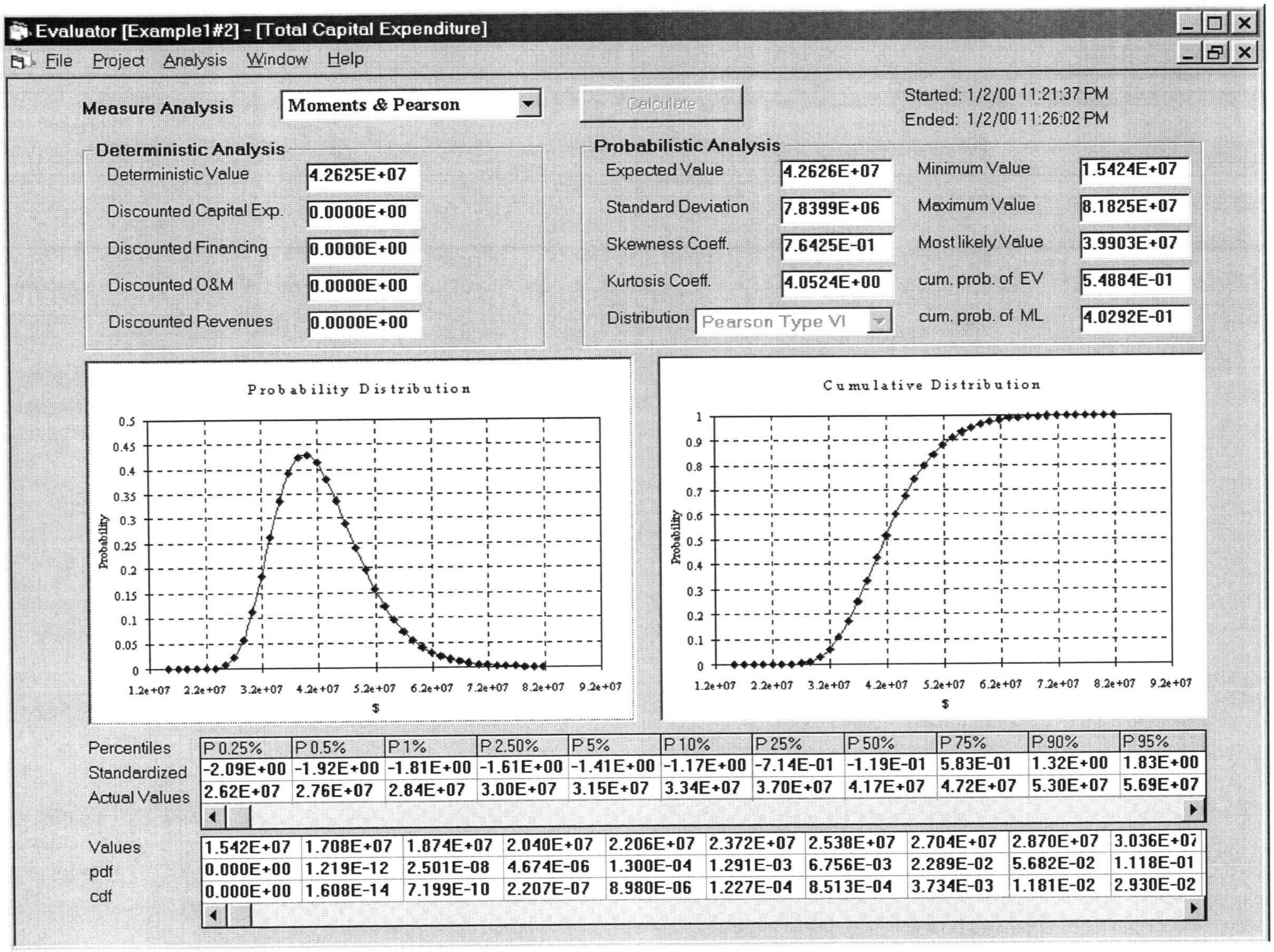


Figure 7.13: Total capital expenditure analysis

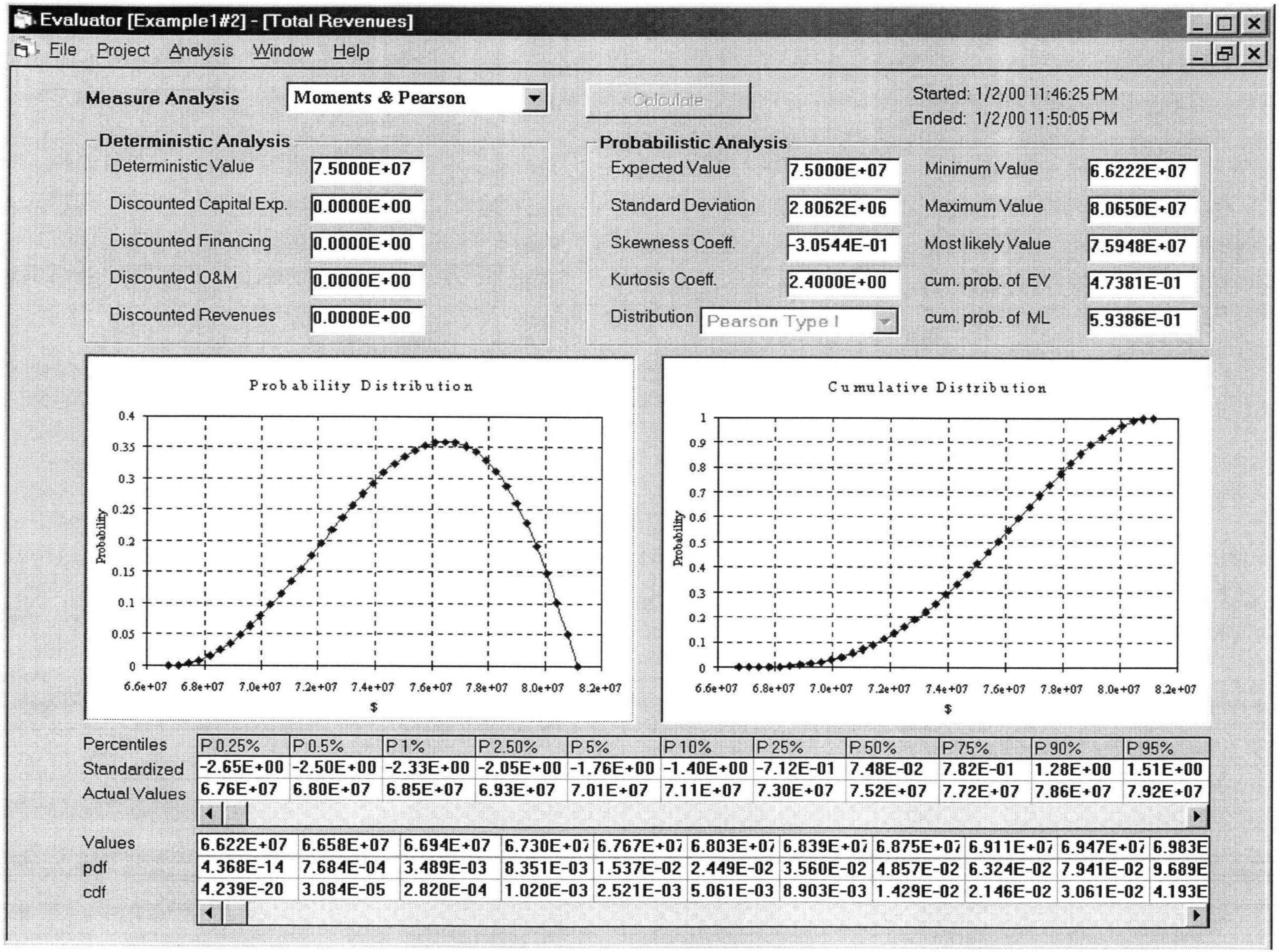


Figure 7.14: Total revenue analysis

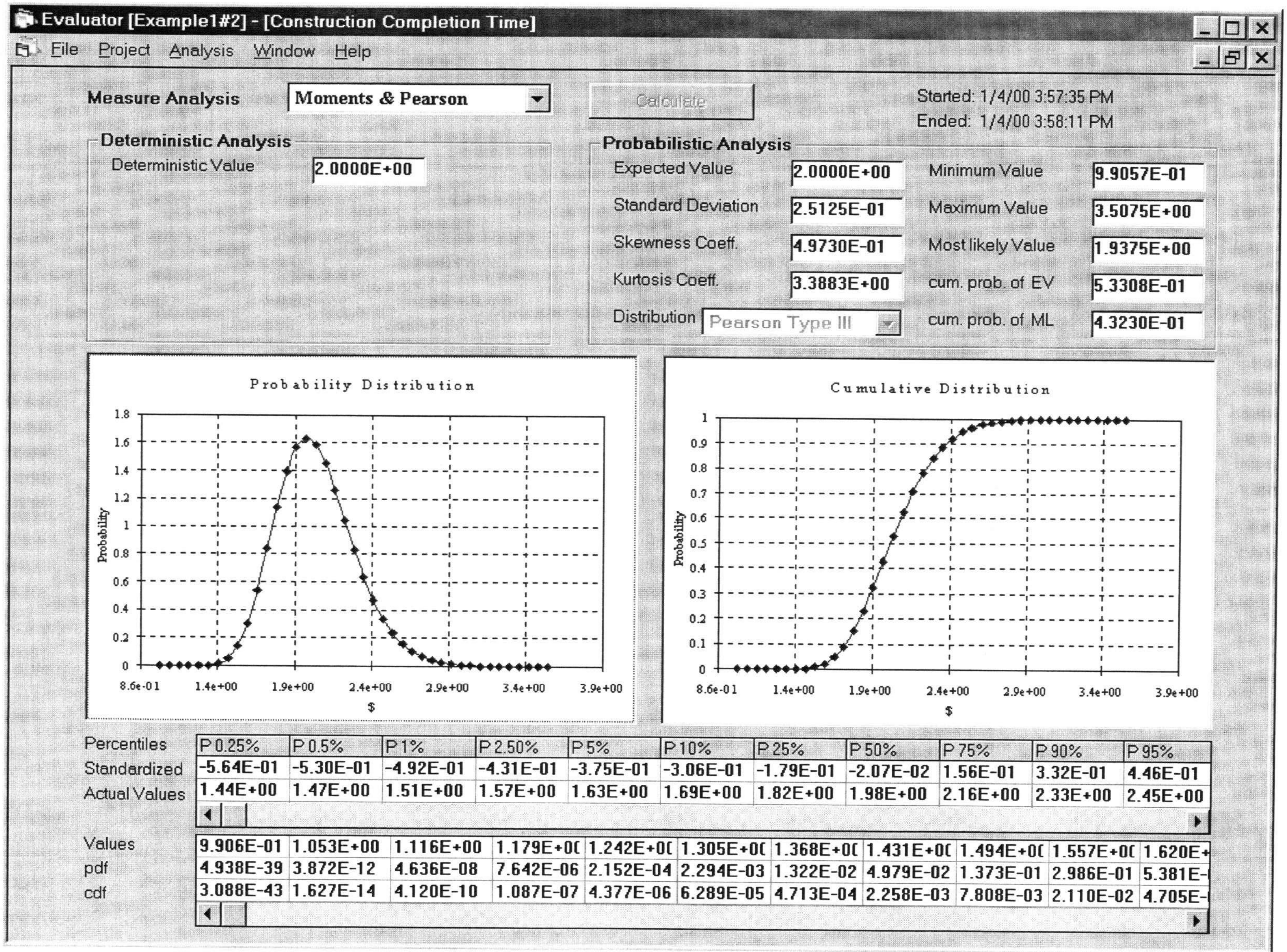


Figure 7.15: Construction completion time analysis

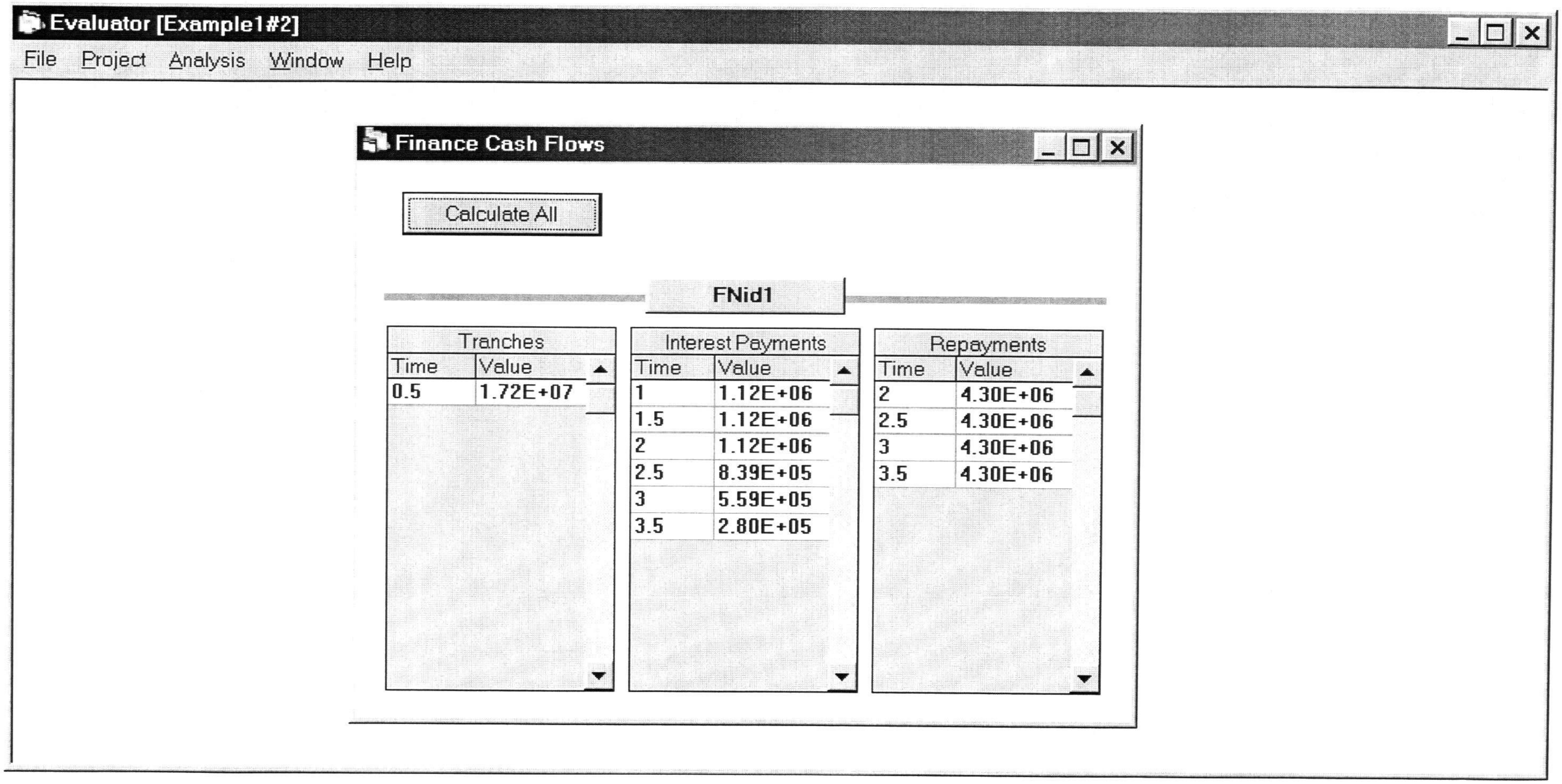


Figure 7.16: Debt elements for the example project

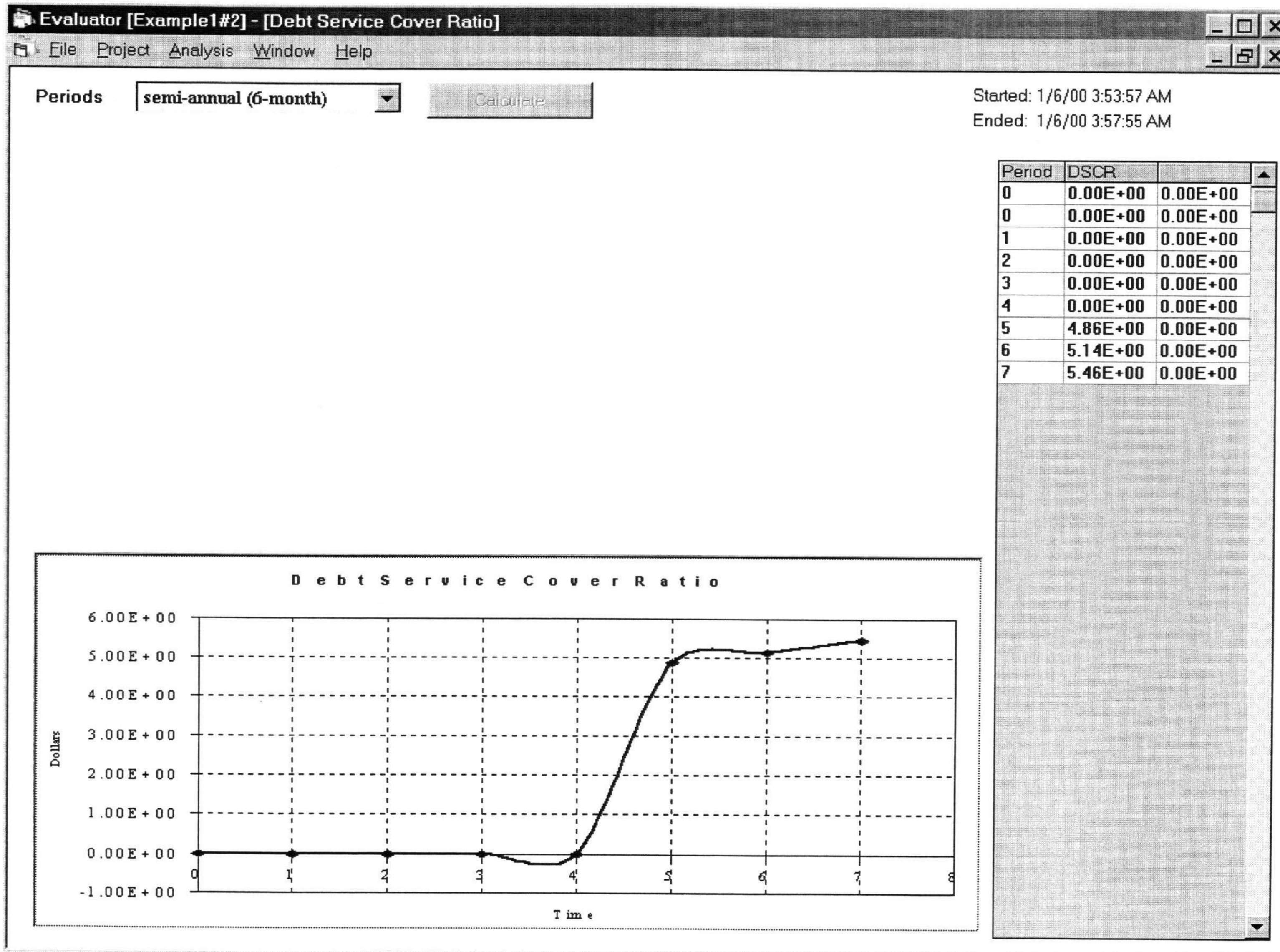


Figure 7.17: Debt service cover ratio for the example project

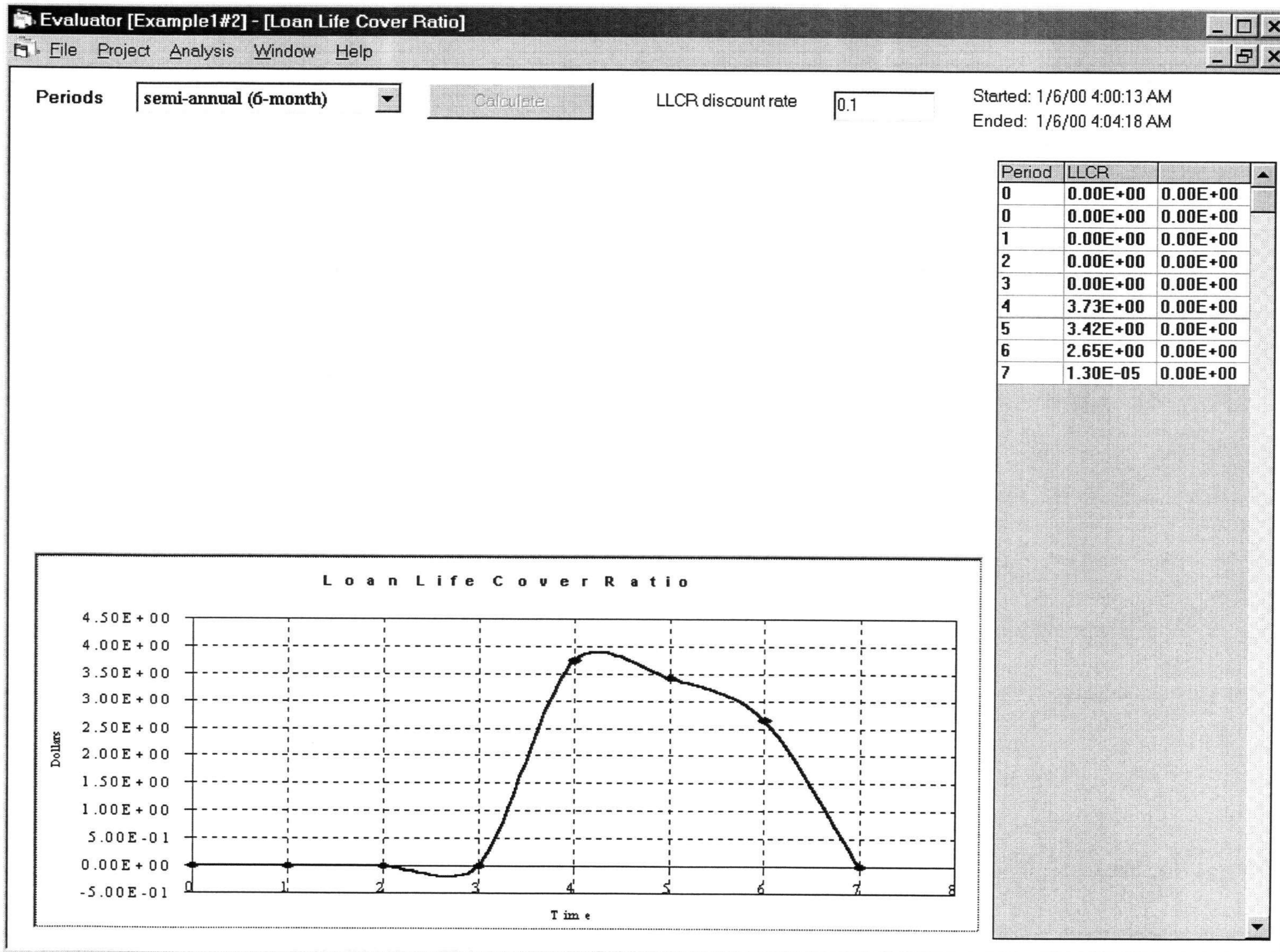


Figure 7.18: Loan life cover ratio of the example project

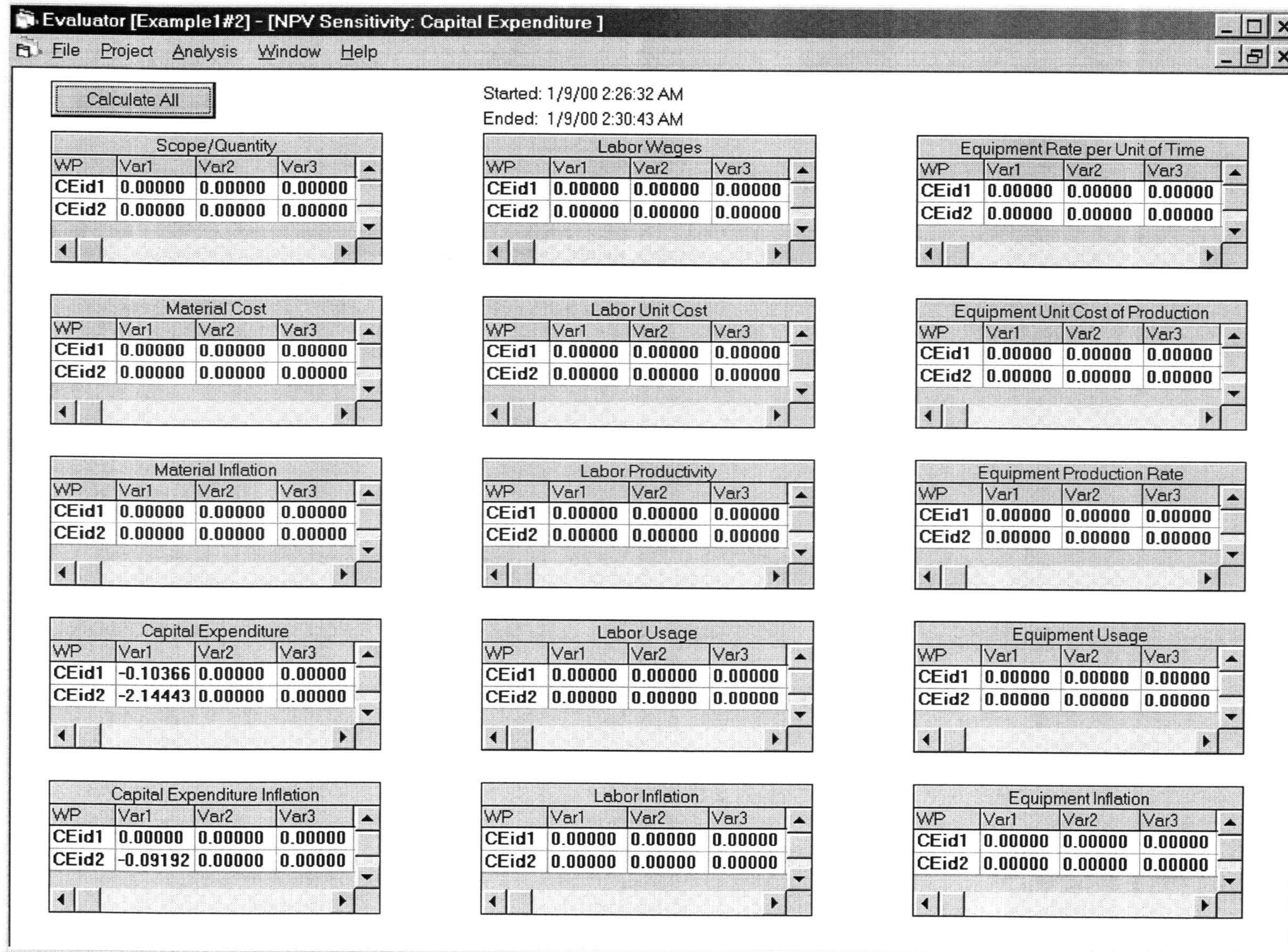


Figure 7.19: Normalized sensitivity coefficients of the capital expenditure variables

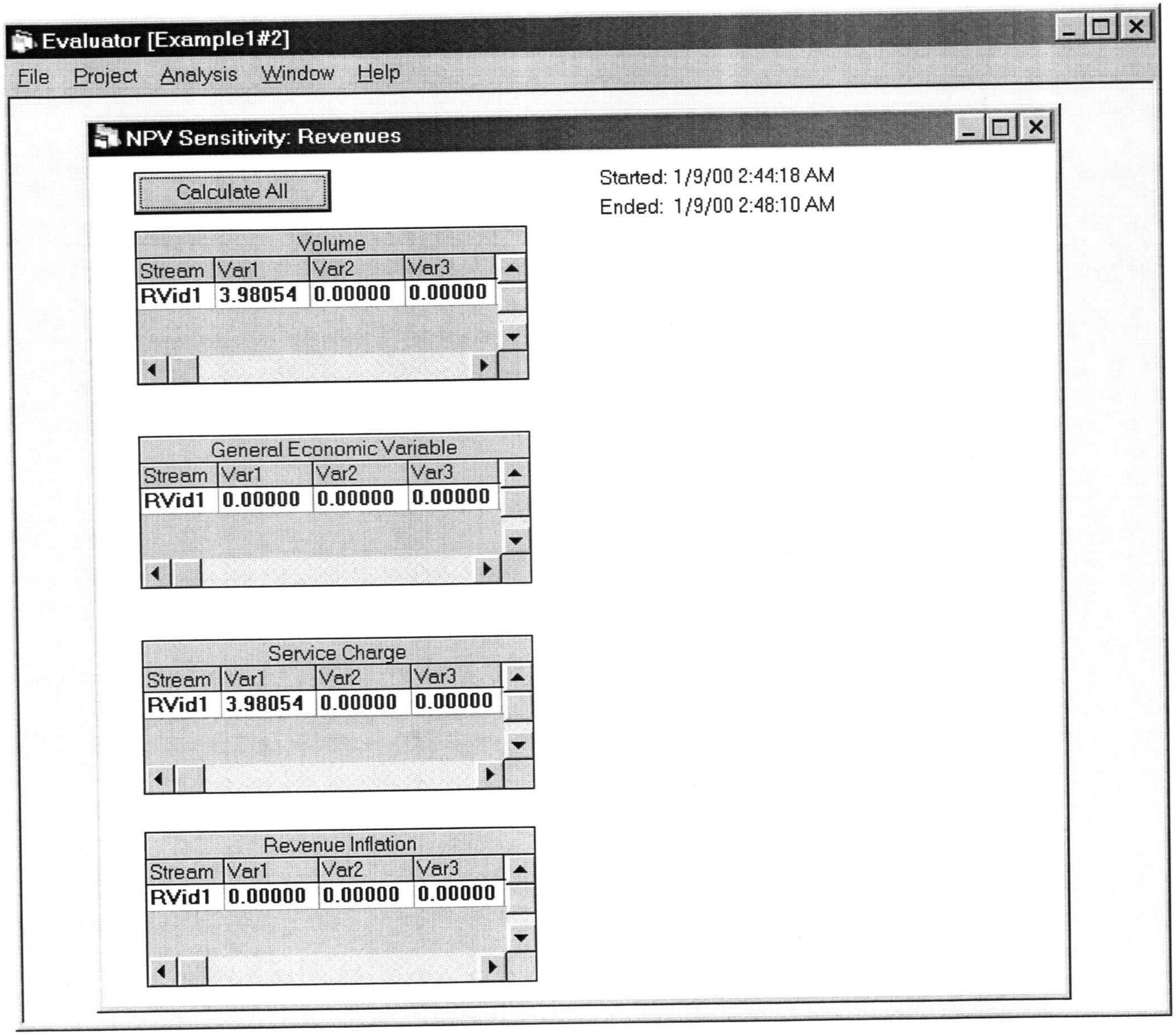


Figure 7.20: Normalized sensitivity coefficients of revenue variables

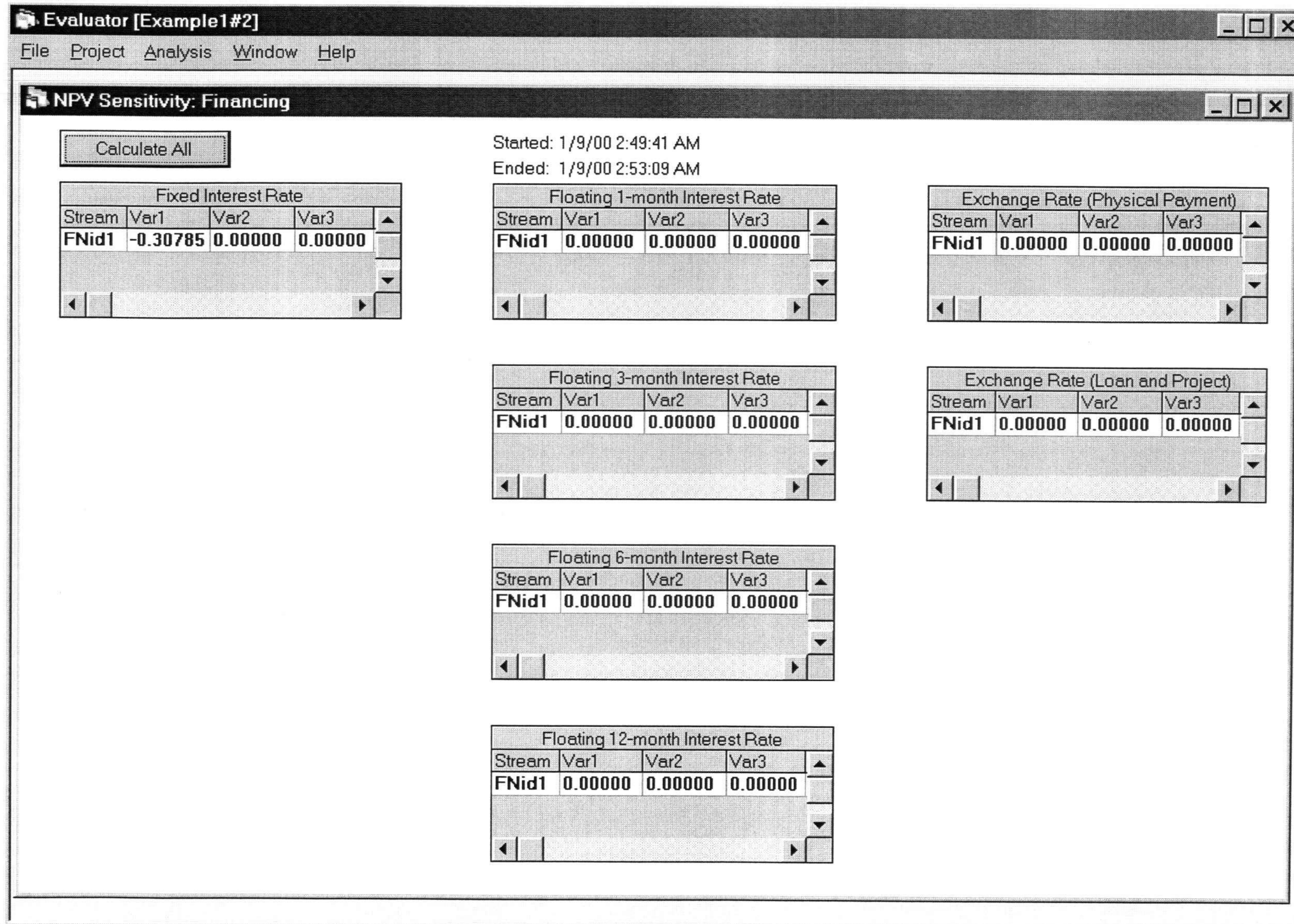


Figure 7.21: Normalized sensitivity coefficients of revenue variables

7.3 Example 2: DSS Modeling and Analysis Features

The DSS is used in this example to demonstrate how it can handle the economic appraisal of a highway project under different assumptions of information in the capital investment, operation, revenue and financing portions of the project. Data for the example project reflect real data obtained from a highway project in Eastern Canada; however, some assumptions were made since complete information was not available. The project is a 45-km 4-lane divided highway offered under the PPP delivery system. The example is presented in the form of three cases studies reflecting detailed, semi-detailed and detailed analyses of the project.

7.3.1 Case 1: Preliminary Appraisal Study and Deterministic Analysis

For a preliminary analysis of the project, a simple economic model may be constructed to give a quick deterministic fixed-value analysis to both government and developers. Figure 7.22 shows an economic structure of the highway project that starts in 1995 (project time zero). Total investment cost, O&M costs and revenues, along with the duration of work packages and cash flow streams are explained in Table 7.8. For the preliminary analysis aggregated methods are assumed for project costs and revenues; expenditure profiles for the cost of design, road construction and road structures are illustrated in Figures 7.23 to 7.25.

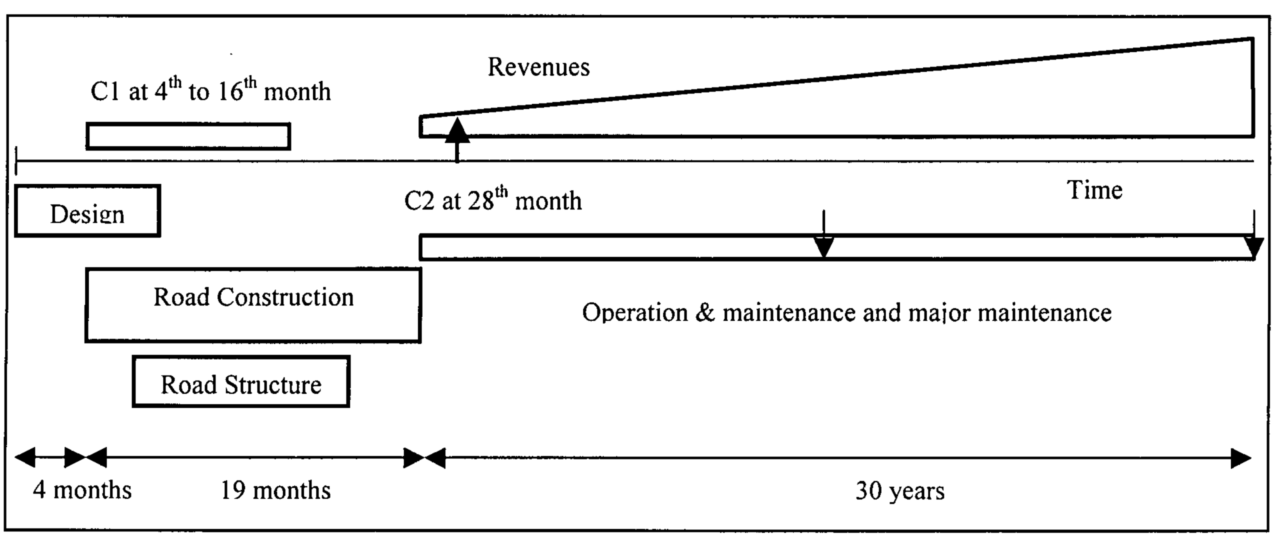


Figure 7.22: Economic structure of the highway project. C1 and C2 are contributions.

Table 7.8: Constant dollar costs and revenues for the preliminary analysis

Total Investment Cost:	
Design	\$13 million in 8 months
Construction	\$100 million in 19 months, divided as follows
Road	\$84.75 in 19 months, overlapped by 4 months with design
Structures	\$15.25 in 11 months, overlapped by 1 months with design
Operation & Maintenance Cost:	
O&M Costs	\$2.9 million per year for 30 year after construction
Major Maintenance	\$17 million each 15 years after construction completion
Project Revenues:	
Toll Revenues	\$7.777158 million estimated at start of project
Annual Increase	\$0.393529 million per year (5.06% from initial)
Inflation	2.35% affects construction, O&M, and revenues

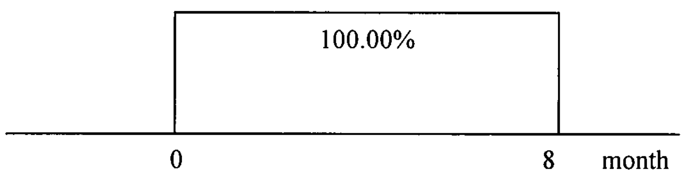


Figure 7.23: Design cost profile

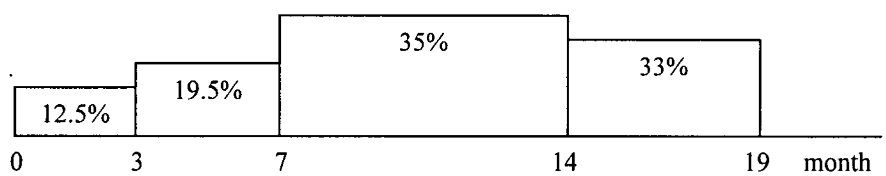


Figure 7.24: Road construction cost profile, see Figure 7.26

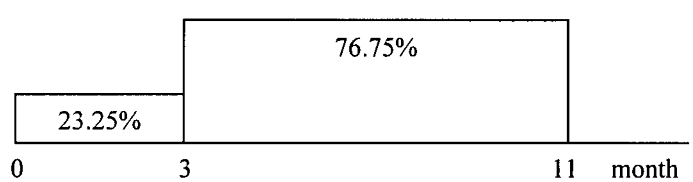


Figure 7.25: Road structures cost profile

In the DSS, the above information is defined in a new project, where

- In the "Identification" menu,

  1. The project is set to an annual global time interval, work packages are set to local month intervals, while each O&M and revenue stream is set to annual time intervals.
  2. The discount rate is set to 8.25%, which reflect the yield on 30-year Government of Canada Bonds in 1995. The use of this rate reflects a government requirement in the RFP to compare between proposals, to emphasize that a company should earn its return from construction contracts, and/or to compare the project to 25-year investments in the market although Canada Bonds are highly rated and guaranteed.

3. The concession period is set to 30 years from completion of construction.
4. The inflation rate is set to be same for all project work packages and streams, and is defined within each construct with global reference using a *uniform rate function.*

- In the "Capital Expenditure" component menus,
  1. Four work packages are defined for the design, road construction, road structure, and finish work packages.
  2. Cost profiles are defined using the *Step Uniform area function* and semi-detailed estimating methods (see section 4.3.2). Figure 7.26 illustrate how the road construction work package (described in Figure 7.24 above) is modeled in the DSS.
  3. Work packages are linked to each other using CPM logic - see Figure 7.26.

- In the "Revenue" component menus,
  1. Revenues are defined using the *linear rate shape function* with global reference since revenues are estimated from the start of the project. Revenues from the start of the stream will be accounted for during calculations (note that revenue reference is global). Revenue function is an aggregation of underlying traffic forecasts predicted from project start (detailed in the later case studies).
  2. The start of the RV stream is linked to the completion of construction.

- In the "Operation and Maintenance" component menus,
  1. O&M cost is defined using the *uniform rate function* of the semi-detailed methods (see section 4.5.2); major maintenance costs are defined using discrete costs in the OM stream (see section 4.5.2.3).
  2. The start of O&M is linked to the completion of construction.

By carrying out the analysis using the DSS, the preliminary results of project measures shown in Table 7.9 provide for a barely acceptable undertaking using the 8.25% discount rate. It would be difficult for the private sector to approach such a project particularly with a payback period that extends almost to 14 years, as shown in Figure 7.27. Among the several strategies to enhance the economics of the project would be to extend the concession period beyond 30 years, to reduce the number of lanes in order to reduce investment cost, or to provide a contribution to the project. Such strategies can be readily investigated and analyzed by the DSS. In the actual development of this project, however, the government provided a $55 million dollar contribution in two tranches: $29 million distributed over one year and $26 million in a single sum (see C1 and C2 in Figure 7.22). Both contributions are added in a single RV stream without inflation.

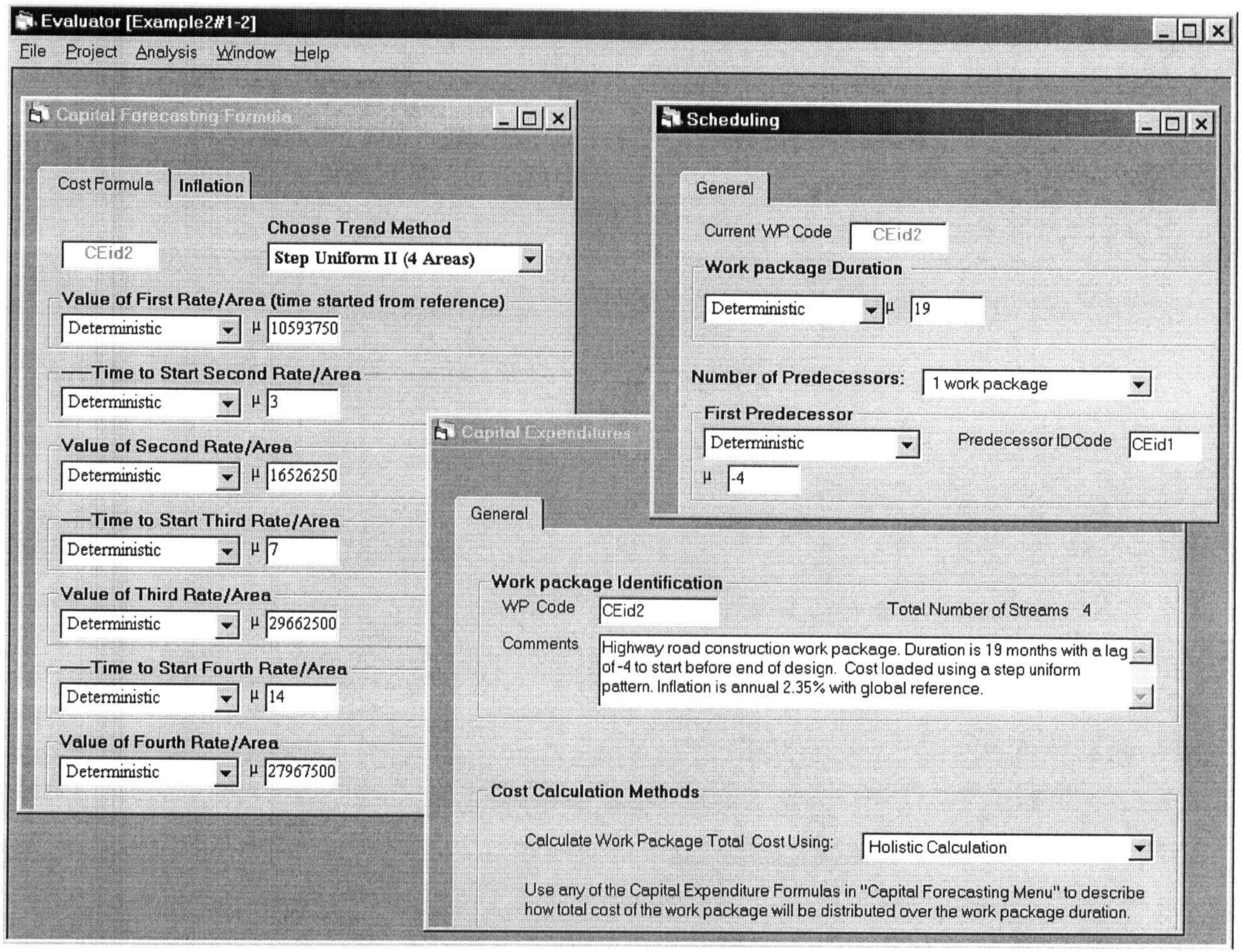
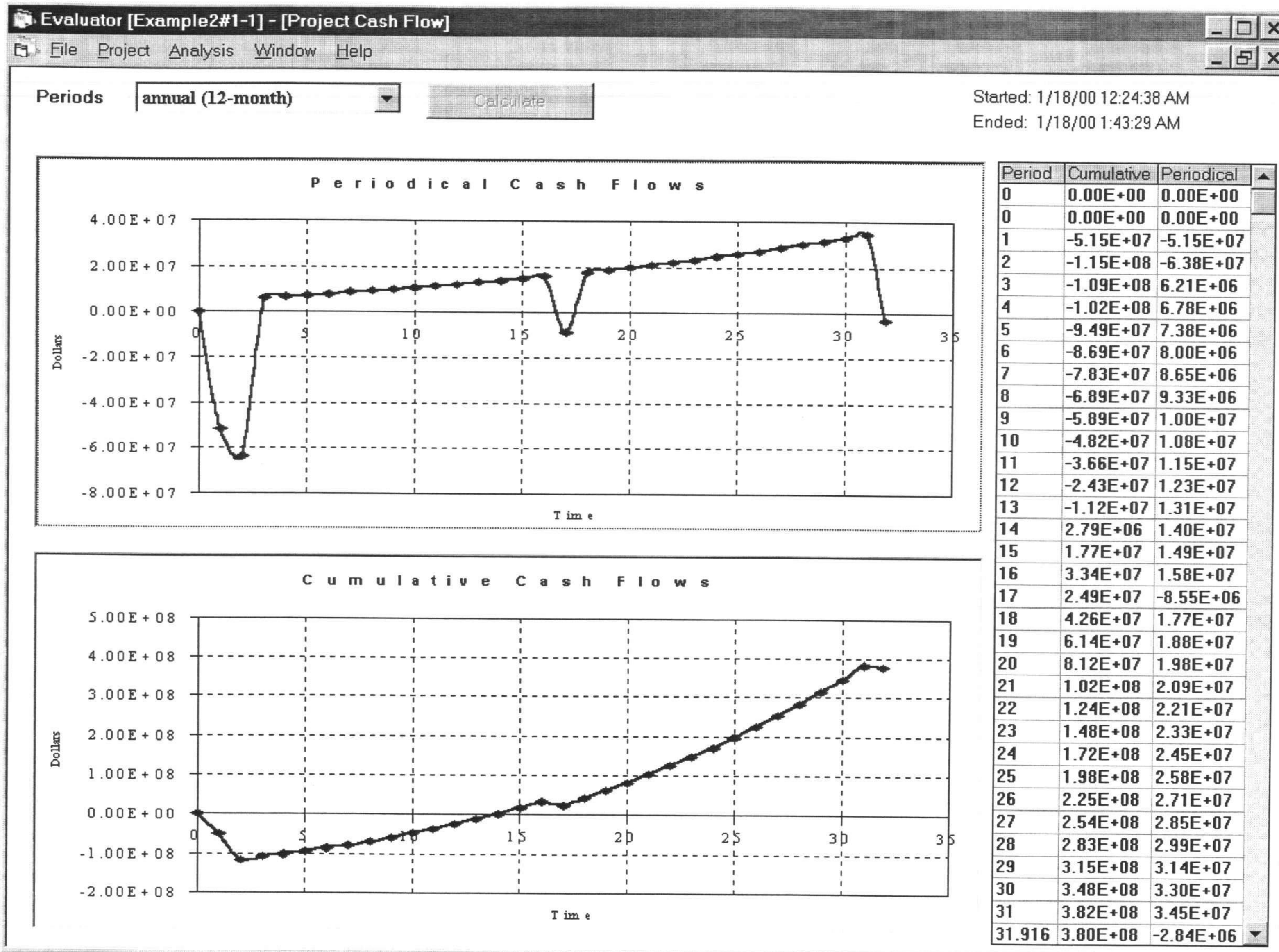


Figure 7.26: Modeling the cost and time of the road construction work package

Table 7.9: DSS results for the preliminary study

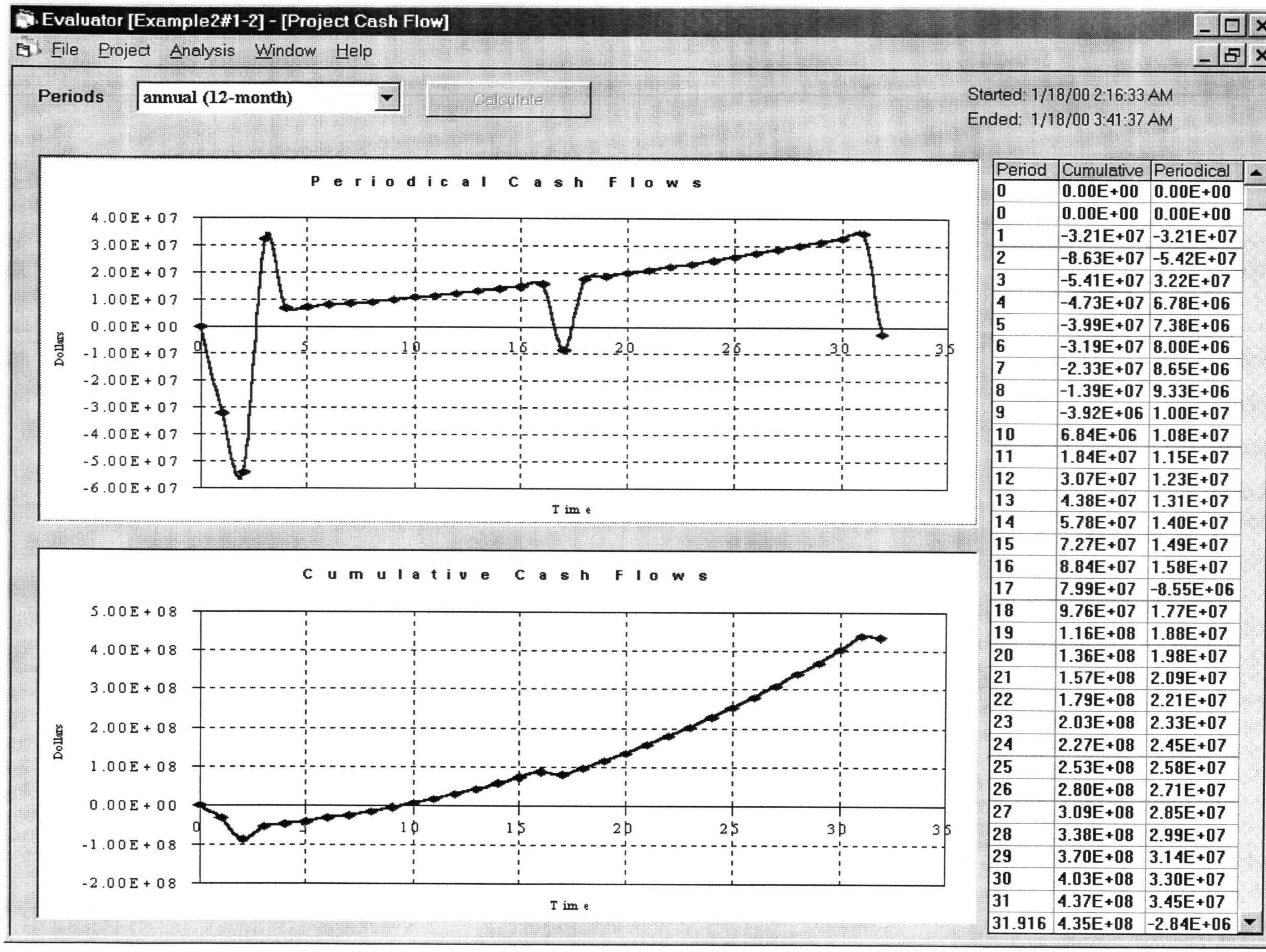
Project Measures	NPV	IRR On Total Capital	B/C Aggregated	B/C Net	Total Current $ Investment	Total Current $ Revenues
No Contribution	$9.076 million	8.86%	1.06	1.086	$116 million	$689 million
With Contribution	$57.604 million	13.91%	1.3805	1.543	$116 million	$744 million

With the contribution, project measures as obtained from the DSS are significantly improved, as shown in Table 7.9. The payback period is now reduced to 9 years, as shown in Figure 7.28. The effect on revenue cash flows before and after the contribution can be seen in Figures 7.29 and 7.30. The cash flow table in Figure 7.30 has year one reflecting the part of C1 that was received during the first year; year two revenues accounted for the rest of C1 and 1month of tolls.



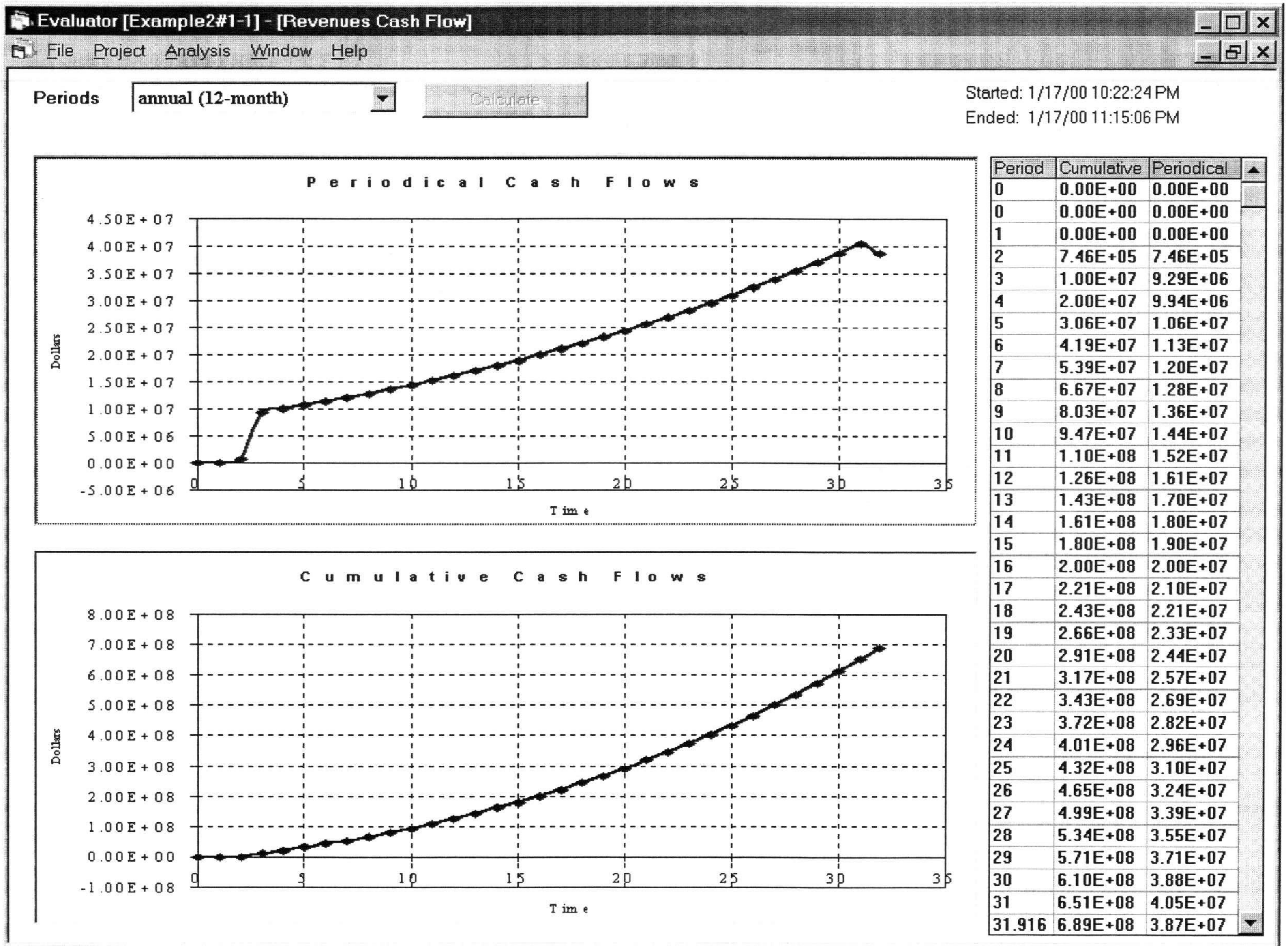
Period	Cumulative	Periodical
0	0.00E+00	0.00E+00
0	0.00E+00	0.00E+00
1	-5.15E+07	-5.15E+07
2	-1.15E+08	-6.38E+07
3	-1.09E+08	6.21E+06
4	-1.02E+08	6.78E+06
5	-9.49E+07	7.38E+06
6	-8.69E+07	8.00E+06
7	-7.83E+07	8.65E+06
8	-6.89E+07	9.33E+06
9	-5.89E+07	1.00E+07
10	-4.82E+07	1.08E+07
11	-3.66E+07	1.15E+07
12	-2.43E+07	1.23E+07
13	-1.12E+07	1.31E+07
14	2.79E+06	1.40E+07
15	1.77E+07	1.49E+07
16	3.34E+07	1.58E+07
17	2.49E+07	-8.55E+06
18	4.26E+07	1.77E+07
19	6.14E+07	1.88E+07
20	8.12E+07	1.98E+07
21	1.02E+08	2.09E+07
22	1.24E+08	2.21E+07
23	1.48E+08	2.33E+07
24	1.72E+08	2.45E+07
25	1.98E+08	2.58E+07
26	2.25E+08	2.71E+07
27	2.54E+08	2.85E+07
28	2.83E+08	2.99E+07
29	3.15E+08	3.14E+07
30	3.48E+08	3.30E+07
31	3.82E+08	3.45E+07
31.916	3.80E+08	-2.84E+06

Figure 7.27: Project cash flows for the preliminary base case of the highway project



Period	Cumulative	Periodical
0	0.00E+00	0.00E+00
0	0.00E+00	0.00E+00
1	-3.21E+07	-3.21E+07
2	-8.63E+07	-5.42E+07
3	-5.41E+07	3.22E+07
4	-4.73E+07	6.78E+06
5	-3.99E+07	7.38E+06
6	-3.19E+07	8.00E+06
7	-2.33E+07	8.65E+06
8	-1.39E+07	9.33E+06
9	-3.92E+06	1.00E+07
10	6.84E+06	1.08E+07
11	1.84E+07	1.15E+07
12	3.07E+07	1.23E+07
13	4.38E+07	1.31E+07
14	5.78E+07	1.40E+07
15	7.27E+07	1.49E+07
16	8.84E+07	1.58E+07
17	7.99E+07	-8.55E+06
18	9.76E+07	1.77E+07
19	1.16E+08	1.88E+07
20	1.36E+08	1.98E+07
21	1.57E+08	2.09E+07
22	1.79E+08	2.21E+07
23	2.03E+08	2.33E+07
24	2.27E+08	2.45E+07
25	2.53E+08	2.58E+07
26	2.80E+08	2.71E+07
27	3.09E+08	2.85E+07
28	3.38E+08	2.99E+07
29	3.70E+08	3.14E+07
30	4.03E+08	3.30E+07
31	4.37E+08	3.45E+07
31.916	4.35E+08	-2.84E+06

Figure 7.28: Project cash flows after government contribution



Period	Cumulative	Periodical
0	0.00E+00	0.00E+00
0	0.00E+00	0.00E+00
1	0.00E+00	0.00E+00
2	7.46E+05	7.46E+05
3	1.00E+07	9.29E+06
4	2.00E+07	9.94E+06
5	3.06E+07	1.06E+07
6	4.19E+07	1.13E+07
7	5.39E+07	1.20E+07
8	6.67E+07	1.28E+07
9	8.03E+07	1.36E+07
10	9.47E+07	1.44E+07
11	1.10E+08	1.52E+07
12	1.26E+08	1.61E+07
13	1.43E+08	1.70E+07
14	1.61E+08	1.80E+07
15	1.80E+08	1.90E+07
16	2.00E+08	2.00E+07
17	2.21E+08	2.10E+07
18	2.43E+08	2.21E+07
19	2.66E+08	2.33E+07
20	2.91E+08	2.44E+07
21	3.17E+08	2.57E+07
22	3.43E+08	2.69E+07
23	3.72E+08	2.82E+07
24	4.01E+08	2.96E+07
25	4.32E+08	3.10E+07
26	4.65E+08	3.24E+07
27	4.99E+08	3.39E+07
28	5.34E+08	3.55E+07
29	5.71E+08	3.71E+07
30	6.10E+08	3.88E+07
31	6.51E+08	4.05E+07
31.916	6.89E+08	3.87E+07

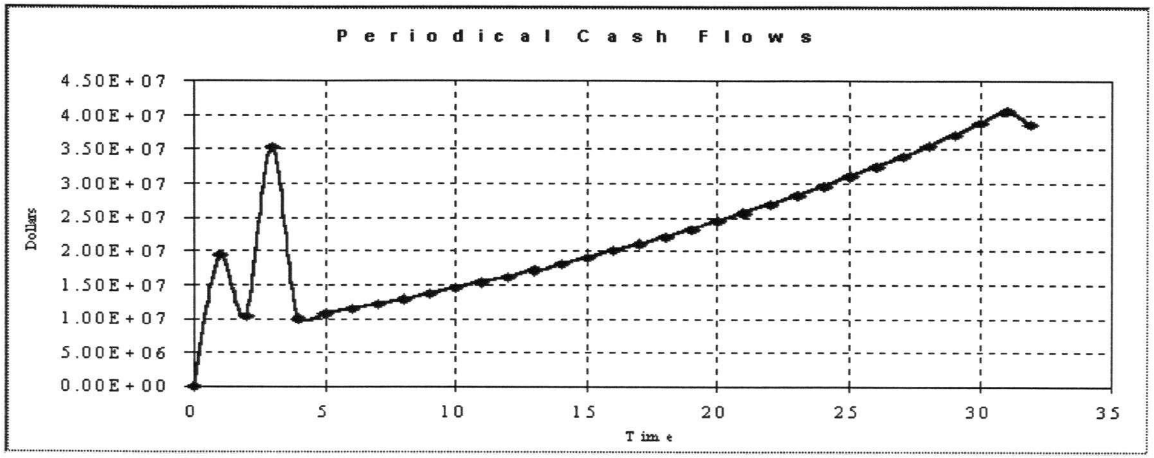
Figure 7.29: Revenue cash flow before government contribution

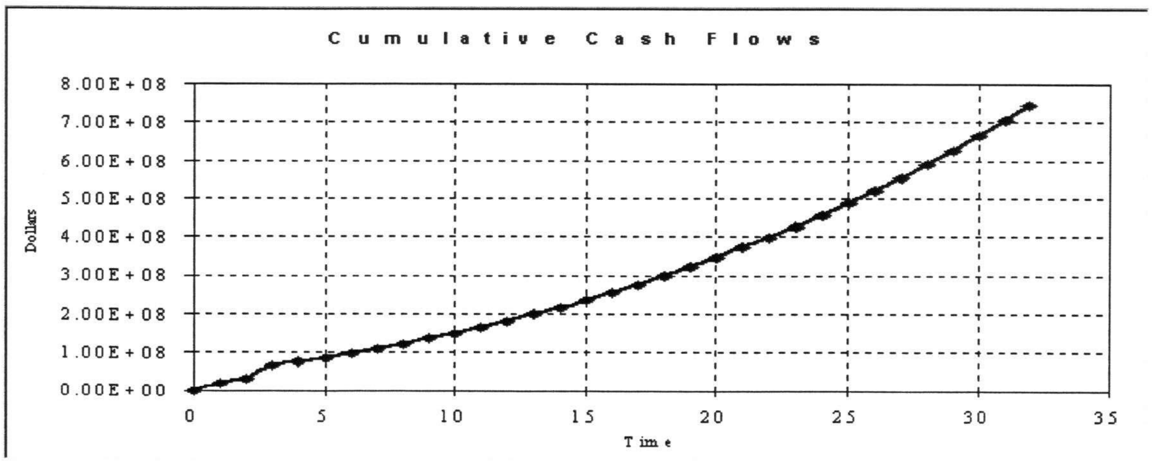
Evaluator [Example2#1-2] - [Revenues Cash Flow]

File Project Analysis Window Help

Periods annual (12-month) Calculate

Started: 1/19/00 5:04:29 AM
Ended: 1/19/00 5:14:04 AM





Period	Cumulative	Periodical
0	0.00E+00	0.00E+00
0	0.00E+00	0.00E+00
1	1.93E+07	1.93E+07
2	2.97E+07	1.04E+07
3	6.50E+07	3.53E+07
4	7.50E+07	9.94E+06
5	8.56E+07	1.06E+07
6	9.69E+07	1.13E+07
7	1.09E+08	1.20E+07
8	1.22E+08	1.28E+07
9	1.35E+08	1.36E+07
10	1.50E+08	1.44E+07
11	1.65E+08	1.52E+07
12	1.81E+08	1.61E+07
13	1.98E+08	1.70E+07
14	2.16E+08	1.80E+07
15	2.35E+08	1.90E+07
16	2.55E+08	2.00E+07
17	2.76E+08	2.10E+07
18	2.98E+08	2.21E+07
19	3.21E+08	2.33E+07
20	3.46E+08	2.44E+07
21	3.72E+08	2.57E+07
22	3.98E+08	2.69E+07
23	4.27E+08	2.82E+07
24	4.56E+08	2.96E+07
25	4.87E+08	3.10E+07
26	5.20E+08	3.24E+07
27	5.54E+08	3.39E+07
28	5.89E+08	3.55E+07
29	6.26E+08	3.71E+07
30	6.65E+08	3.88E+07
31	7.06E+08	4.05E+07
31.916	7.44E+08	3.87E+07

Figure 7.30: Revenue cash flow after government contribution

7.3.2 Case 2: Semi-Detailed Appraisal Study and Probabilistic Analysis

This case study shows an application and illustration of the capabilities of the generalized model in providing a more detailed representation of the life cycle of the highway project example. A semi-detailed economic structure is shown in Figure 7.31 (compare with the simplified structure in Figure 7.22). Table 7.10 presents the more detailed estimates of project costs and revenues. Project estimates are disaggregated where the road-construction work package is expanded into five work packages, the road-structure work package is expanded into 4 work packages, the revenue stream is expanded into 3 streams, and the O&M stream is expanded into 2 streams.

Table 7.10: Constant-dollars costs and revenues for semi-detailed analysis (overlapped work packages are shown in Figure 7.31)

Total Investment Cost:	
Design	$13 million in 8 months
Construction	
Road Construction	100% = $84.75 million
Clearing & Grubbing	15%, in 5 months, with -4 months overlap
C&F, Rock Blasting, Compaction	20%, in 6 months, with -3 months overlap
Road Sub-Base Layer	10%, in 7 months, with -1 month overlap
Road Base Layer	10%, in 7 months, with -6 months overlap
Road Asphalt Pavement	45%, in 7 months, with -3 months overlap
Road Structures	100% = $15.25 million
Culverts	13%, in 4months, with -5 months overlap
Tunnels	18%, in 4 months, with -5 months overlap
Interchanges	49%, in 8 months, with -2 months overlap
Bridges	20%, in 8 months, with -2 months overlap
Operation & Maintenance Cost:	
Maintenance Costs	$0.65 m. /yr for 30 years after construction
Major Maintenance	$11.3 million each 10 years after construction
Toll Operation	$2.259 m. /yr for 30 years after construction
Project Revenues:	
Total AADT Traffic	See Table 7.11 for actual traffic data
In-scope Traffic	82.8% for cars and 90.0% for trucks
Vehicle Class	75% cars, 6.6% small trucks, 18.4% large trucks
Cars Toll Rate	$3.00, with annual adjustment for inflation
Trucks Toll Rate	$2.00 per axle, to be adjusted for inflation

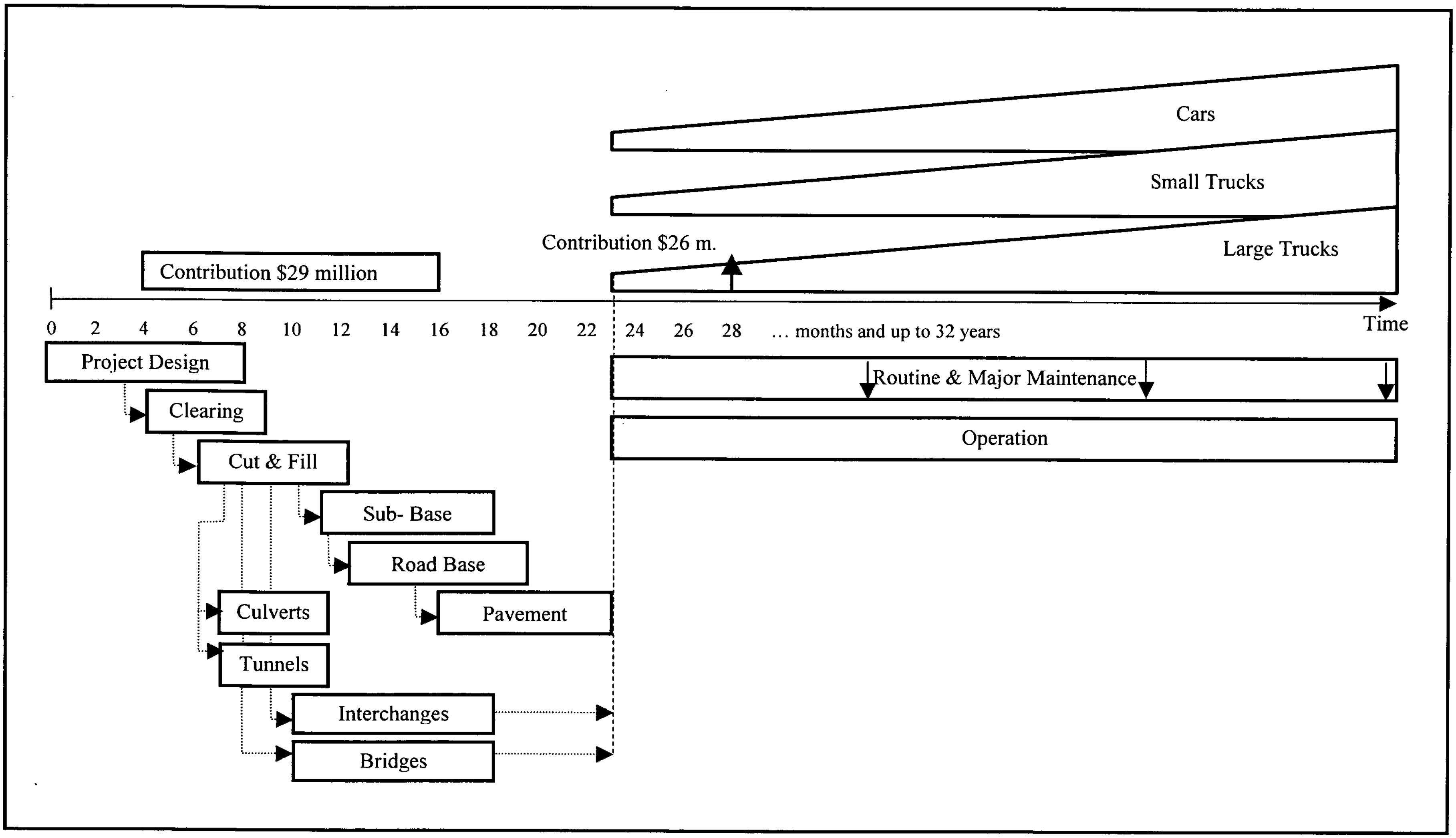


Figure 7.31: Semi-detailed economic structure for the highway project

a) Total Investment Cost

Data in Table 7.10 along with the identified work packages in Figure 7.31 are modeled via the DSS into a new copy of the highway project, where:

1. Total cost of each of the ten work packages is loaded into the work package using the *Uniform area function,* i.e. using unit cost estimating (see section 4.3.2).
2. *Finish-to-start* relationships are defined for the work packages to reflect the network in Figure 7.31 and the duration and lag times in Table 7.10.

b) Traffic Demand and Project Revenues

Unlike the total aggregated revenues used in the first case, project revenues are disaggregated into car, small truck and large truck revenue streams. Revenue forecasts for each stream are obtained using traffic volume and toll rate function for each stream as required when using detailed methods (section 5.2.2.2). Table 7.11 presents the Average Annual Daily Traffic (AADT) data for the study area of the highway project from 1970 to 1994 (reported in the RFP of the actual project). As in simple trend methods, linear and polynomial regression models can be used to construct forecasting formulas for future traffic. A regression analysis on the data in Table 7.11 lead to the following forecasting formulas:

Table 7.11: AADT on the highway example project

Year	Time	AADT	Year	Time	AADT	Year	Time	AADT
1970	0	2880.0	1979	9	4530.0	1988	18	6030.0
1971	1	3020.0	1980	10	4620.0	1989	19	6400.0
1972	2	3330.0	1981	11	4500.0	1990	20	6530.0
1973	3	3510.0	1982	12	4450.0	1991	21	6520.0
1974	4	3800.0	1983	13	4890.0	1992	22	6600.0
1975	5	3990.0	1984	14	4720.0	1993	23	6750.0
1976	6	3820.0	1985	15	5100.0	1994	24	6900.0
1977	7	4190.0	1986	16	5410.0			
1978	8	4380.0	1987	17	5630.0			

$$AADT = 2918.615 + 165.115 \cdot t \quad \text{linear regression} \tag{7.1}$$

$$AADT = 3066.154 + 126.627 \cdot t + 1.604 \cdot t^2 \quad \text{polynomial } 2^{nd} \text{ order} \tag{7.2}$$

$$AADT = 2983.128 + 172.926 \cdot t - 3.319 \cdot t^2 + 0.317 \cdot t^3 \quad \text{polynomial } 3^{rd} \text{ order} \tag{7.3}$$

Figure 7.32 overlays these models against the actual data of Table 7.11. While the correlation coefficients R^2 are larger than 0.97, Figure 7.33 illustrates the diverse behavior of the three models in forecasting future traffic. Generally, trend analysis would be used in short term forecasting. Lacking other objective reasons to choose a forecasting model, a pessimistic analysis of revenues would apply the linear or 2^{nd} order polynomial assumption. A very optimistic analysis would go for the 3^{rd} degree polynomial. In the DSS, any of these models can be used and the linear model is selected for use. Since Eq. 7.1 starts its application from 1970 while the project analysis starts at 1995 (which is the reference for all project dates), Eq. 7.1 is rearranged to start its application from year 25 (i.e. 1995):

$$AADT \qquad = 7046.5 + 165.115 \cdot t \tag{7.4}$$

Future traffic volumes for cars, small trucks and large trucks are obtained from the annual equivalent of AADT in Eq. 7.4 through the application of the in-scope traffic factors and vehicle class percentages in Table 7.10 assuming they will be the same for future traffic. Therefore, traffic volume for each stream is

$$\begin{aligned}
&\textit{Cars} && = [5284.8975 + 123.836 \cdot t] \cdot 365 \cdot 0.828 && (7.5)\\
& && = [1.928988E6 + 4.520023E4 \cdot t] \cdot 0.828 && (7.6)\\
&\textit{Small Trucks} && = [465.0709 + 10.8975 \cdot t] \cdot 365 \cdot 0.9 && (7.7)\\
& && = [1.697509E5 + 3977.5875 \cdot t] \cdot 0.9 && (7.8)\\
&\textit{Large Trucks} && = [1296.5612 + 30.38116 \cdot t] \cdot 365 \cdot 0.9 && (7.9)\\
& && = [4.73245E5 + 1.108912E4 \cdot t] \cdot 0.9 && (7.10)
\end{aligned}$$

Revenues for each stream are then obtained using the appropriate equation (Eqs. 7.6, 7.8, 7.10) the toll rates for each vehicle class as specified in Table 7.10.

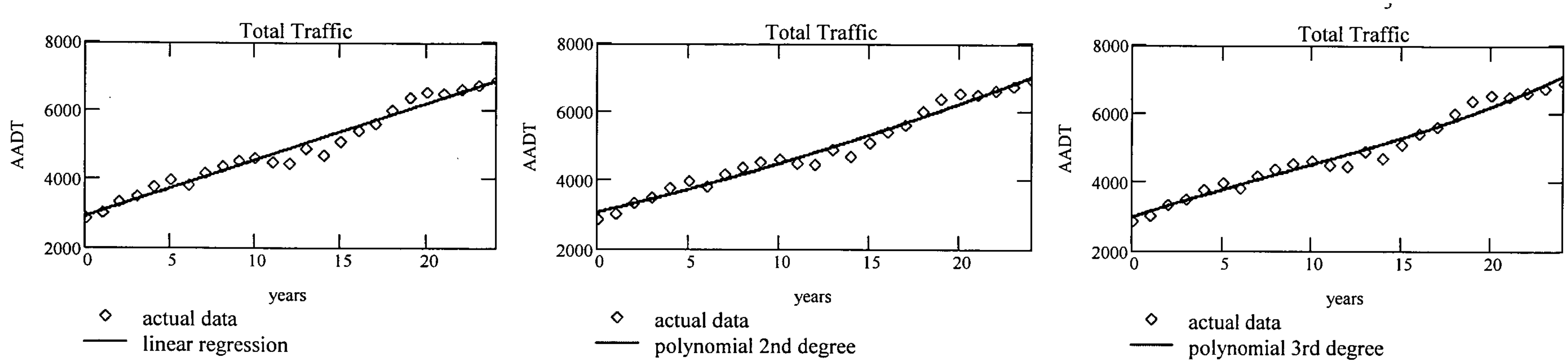


Figure 7.32: Actual AADT on highway project example and forecasting models (linear and 2[nd] and 3[rd] degree polynomials)

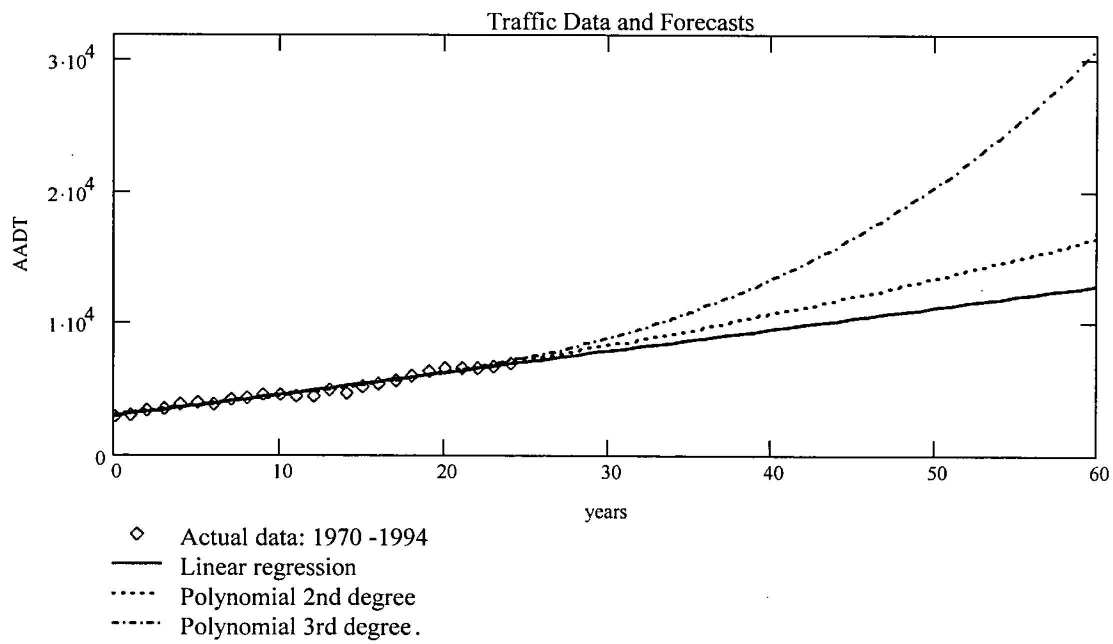


Figure 7.33: Actual and forecasted AADT for the life of the highway project, project starts in year 24

c) Project Operation and Maintenance

For the operation and maintenance of the project during its 30 year concession period and unlike the preliminary analysis, O&M costs are divided into two streams, one for operation costs and two other for maintenance costs. Both streams are defined using the *uniform rate function* of the semi-detailed methods (see section 4.5.2). As shown in project cash flow graphs in Figure 7.27, the application of the *major maintenance* costs affects revenues dramatically and therefore, major maintenance in the current analysis are scheduled every 10 years instead of every 15 years, which reduces the effect on project revenues.

d) Project Inflation Variables

In the first case, the inflation rate was set at 2.35% annually and considered to affect all project costs and revenues equally. In the current case, inflation is set to be different for each cash flow component except for the capital investment cost, which remains at 2.35%. For O&M costs, the *maintenance stream* is to inflate at a constant of 1.5% plus an annual increase of 0.04% annually; the *operations stream* is assumed to inflate at the constant rate of 2.35%. For toll revenue streams, inflation is assumed to change in a sinusoidal pattern starting at a rate of 2.35% at project start along with an annual increase of 0.05%, the sinusoidal amplitude is set at 0.3% and the cycle length is assumed to be 10 years, as shown in Figure 7.34.

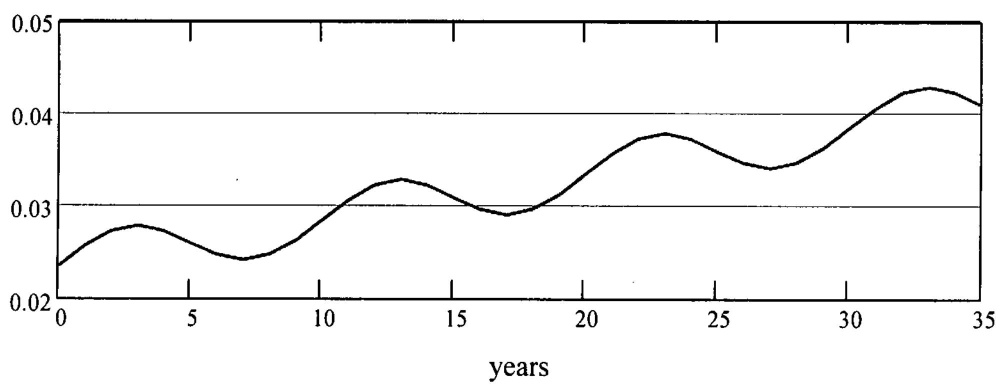


Figure 7.34: Toll inflation modeled as sinusoidal function during project life cycle

e) Project Financing and Financing Measures

Since the highway project is being pursued under the PPP delivery system, project financing by the private developers will be needed either through debt or debt and equity. The actual financing of this project came though two private placement bond issues; the first amounted to $51 million for a 30-year term with a 10.52% coupon, and the second amounted to $10 million for a 15-year term at a 11.203% coupon. Similar arrangements are used to finance the example in the DSS. Since the project experiences low traffic volume, revenues during the early operating years are not sufficient to recover the debt. Therefore, the first bond is structured to have a long grace period of 10 years with the repayment of principle to increase at $0.1 million per year during the repayment period. The second bond issue is structured to be a straight bond with sinking fund repayments to start after a 5-year grace period. The first bond is issued at the start of construction, i.e. the 4th month in Figure 7.31, followed after 8 months by the second bond issue.

Figure 7.35 shows interest payments and principal repayments for both bond issues as processed by the generalized economic model (section 4.6.2). As shown in the figure and as planned, repayments of the first bond "FNid1" increases at 0.1E6 to match the slow build up of project revenues in Figure 7.30. Figures 7.36 and 7.37 show the project's ability to repay debt bonds; the figures illustrate and tabulate debt-service-cover-ratio DSCR and loan-life-cover-ratio. DSCR starts with negative values during the construction two-year period then rises sharply during the 3rd year due to the second government contribution. DSCR then performs unsatisfactorily until the 10th year when a ratio in excess of 1.4 is achieved, a condition commonly required in road projects. DSCR has sharp negative values in years 12 and 22 because of the requirements for major maintenance as rearranged in this case. Unlike DSCR, which considers period-by-period analysis, LLCR in Figure 7.37 considers the remaining revenues in a project life; LLCR in the figure experiences ratios of more than 1.6 as generally required by lenders in road projects.

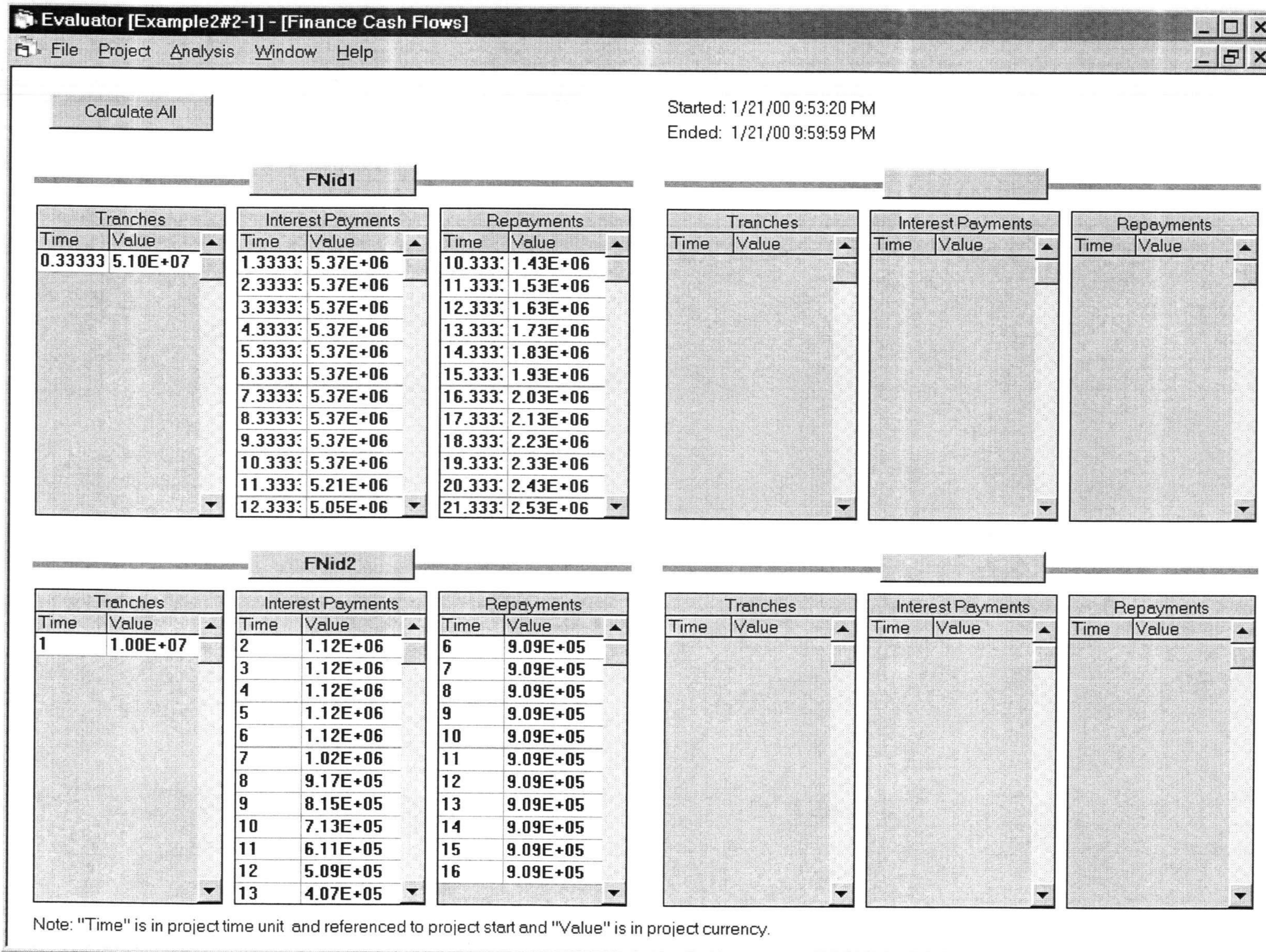


Figure 7.35: Bond cash flow payments for the highway project

Evaluator [Example2#2-1] - [Debt Service Cover Ratio]

File Project Analysis Window Help

Periods annual (12-month) Calculate

Started: 1/30/00 4:30:15 AM
Ended: 1/30/00 4:46:36 AM

Period	DSCR	
0	0.00E+00	0.00E+00
0	0.00E+00	0.00E+00
1	-7.25E+00	0.00E+00
2	-8.22E+00	0.00E+00
3	5.21E+00	0.00E+00
4	1.28E+00	0.00E+00
5	1.36E+00	0.00E+00
6	1.26E+00	0.00E+00
7	1.35E+00	0.00E+00
8	1.44E+00	0.00E+00
9	1.54E+00	0.00E+00
10	1.37E+00	0.00E+00
11	1.47E+00	0.00E+00
12	-1.25E-01	0.00E+00
13	1.72E+00	0.00E+00
14	1.87E+00	0.00E+00
15	2.03E+00	0.00E+00
16	2.21E+00	0.00E+00
17	2.74E+00	0.00E+00
18	2.95E+00	0.00E+00
19	3.17E+00	0.00E+00
20	3.43E+00	0.00E+00
21	3.73E+00	0.00E+00
22	9.21E-01	0.00E+00
23	4.45E+00	0.00E+00
24	4.90E+00	0.00E+00
25	5.39E+00	0.00E+00
26	5.95E+00	0.00E+00
27	6.59E+00	0.00E+00
28	7.32E+00	0.00E+00
29	8.18E+00	0.00E+00
30	9.22E+00	0.00E+00
31	0.00E+00	0.00E+00

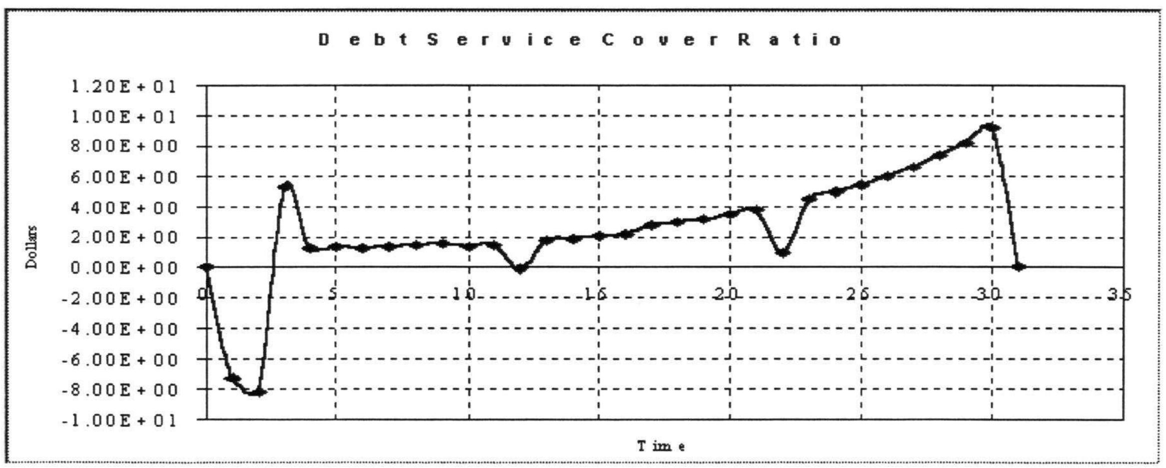


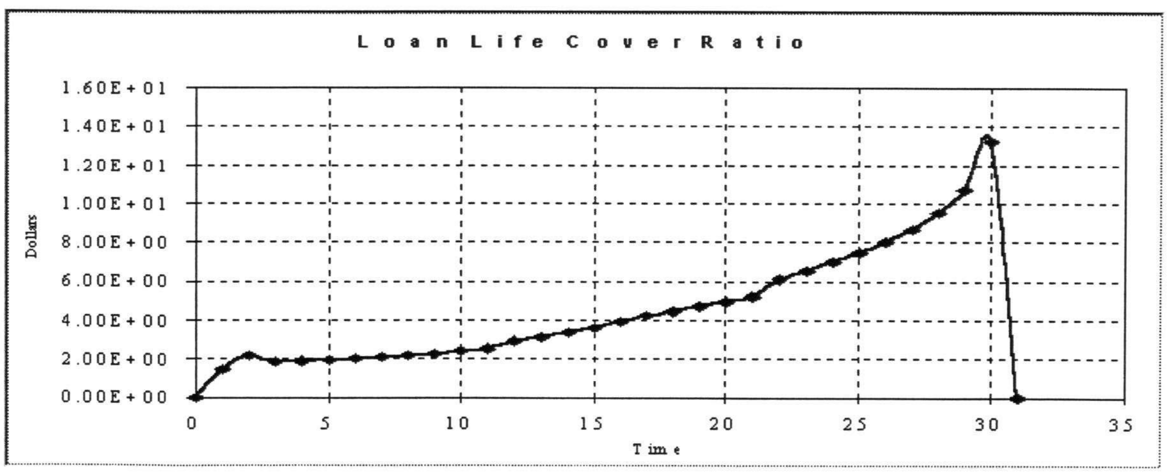
Figure 7.36: Debt service cover ratio for the highway project

Evaluator [Example2#2-1] - [Loan Life Cover Ratio]

File Project Analysis Window Help

Periods annual (12-month) Calculate LLCR discount rate 0.1

Started: 1/30/00 4:48:35 AM
Ended: 1/30/00 5:02:38 AM



Period	LLCR	
0	0.00E+00	0.00E+00
0	0.00E+00	0.00E+00
1	1.43E+00	0.00E+00
2	2.16E+00	0.00E+00
3	1.84E+00	0.00E+00
4	1.91E+00	0.00E+00
5	1.97E+00	0.00E+00
6	2.03E+00	0.00E+00
7	2.12E+00	0.00E+00
8	2.21E+00	0.00E+00
9	2.29E+00	0.00E+00
10	2.38E+00	0.00E+00
11	2.53E+00	0.00E+00
12	2.94E+00	0.00E+00
13	3.15E+00	0.00E+00
14	3.37E+00	0.00E+00
15	3.62E+00	0.00E+00
16	3.90E+00	0.00E+00
17	4.21E+00	0.00E+00
18	4.44E+00	0.00E+00
19	4.69E+00	0.00E+00
20	4.94E+00	0.00E+00
21	5.21E+00	0.00E+00
22	6.08E+00	0.00E+00
23	6.50E+00	0.00E+00
24	6.95E+00	0.00E+00
25	7.44E+00	0.00E+00
26	8.00E+00	0.00E+00
27	8.66E+00	0.00E+00
28	9.48E+00	0.00E+00
29	1.07E+01	0.00E+00
30	1.33E+01	0.00E+00
31	0.00E+00	0.00E+00

Figure 7.37: Loan life cover ratio for the highway project

f) Uncertainty Modeling of Project Variables

Table 7.12 explains the modeling of uncertainty for project time, cost and revenue variables of Table 7.10. These variables cover almost all of the work packages and streams in Figure 7.31. In modeling the uncertainty of variables, use has been made of distribution models that allow a lower threshold for the variable in question as characterized, for example, in the 3-Parameter Log-Normal distribution or through the use of lower and upper thresholds which form part of the 4-Parameter Beta and Triangle distributions. Threshold values are used to limit the range in which a variable may change. Unlike the first example (section 7.2) where the models are described to the DSS by their parameters, the uncertainty models in Table 7.12 are defined by the expected value, variance and threshold values, which are used by the DSS to compute the parameters of the distribution and obtain its third and fourth moments (see Table 5.1). Figure 7.38 illustrates the range and shape of distributions of some variables in Table in 7.12. As can be recognized from Table 7.12, the risk analysis framework of the DSS can model almost any variable defined in a work package or a stream; e.g. the initial value and annual growth of AADT of a revenue streams, and the duration of, and overlap times between, work packages.

g) Project Measures and Risk Analysis

Using the DSS, a deterministic analysis of the major project measures provides a NPV of \$55.328 million, a total current dollar investment cost of 115.86 million, and total revenues of \$771.98 million. The minor differences of the results from those in Table 7.9 of the first case study are due to the assumptions made for the current case. Figure 7.39 shows the uncertainty of the total expenditure cost (design and construction) as being fitted to Pearson Type VI distribution. From this figure, \$114 million is the most likely value with a cumulative probability of 30.3% while \$115 million is an expected value with a probability 59%. If the project is awarded at its expected value, then there is a 41% chance of a cost overrun.

Table 7.12: Modeling the uncertainty of project variables

Total Project Investment:		
Cost	***Duration*** (in months)	***Overlap*** (in months)
Design		
$13 million [deterministic]	Beta 4 [$\mu = 8$; $\mu_2 = (0.5)^2$; $\chi 1 = 7$; $\chi 2 = 10$]	0
Clearing and Grubbing		
$12.7125 million [deterministic]	Beta 4 [$\mu = 5$; $\mu_2 = (0.2)^2$; $\chi 1 = 4$; $\chi 2 = 6.5$]	4 [deterministic]
C&F, Rock Blasting		
Log Normal 3 [$\mu = \$16.95E6$; $\mu_2 = (1.4E6)^2$; $\chi = \$14.5E6$]	Beta 4 [$\mu = 6$; $\mu_2 = (0.2)^2$; $\chi 1 = 5.5$; $\chi 2 = 7$]	Triangle [-4, -3, -2]
Sub-Base Layer		
Log Normal 3 [$\mu = \$8.475E6$; $\mu_2 = (0.15E6)^2$; $\chi = \$8.0E6$]	Log Normal 3 [$\mu = 7$; $\mu_2 = (0.8)^2$; $\chi = 6$]	Triangle [-3, -1, -0.5]
Base Layer		
Log Normal 3 [$\mu = \$8.475E6$; $\mu_2 = (0.15E6)^2$; $\chi = \$8E6$]	Log Normal 3 [$\mu = 7$; $\mu_2 = (0.9)^2$; $\chi = 6$]	Triangle [-6.5, -6, -3]
Asphalt Pavement		
Log Normal 3 [$\mu = \$38.1375E6$; $\mu_2 = (0.95E6)^2$; $\chi = \$36E6$]	Log Normal 3 [$\mu = 7$; $\mu_2 = (0.25)^2$; $\chi = 6$]	Normal [-3, 0.15]
Bridges		
Log Normal 3 [$\mu = \$3.0E6$; $\mu_2 = (0.18E6)^2$; $\chi = \$2.6E6$]	Triangle [7.5, 8, 10]	Triangle [-3, -2, -1]
Operation & Maintenance Cost:		
Major Maintenance	Maintenance Inflation: Initial and Growth Rate	
Triangle [$10E6, $11.3E6; $15E6]	1.5% [deterministic], Triangle [0.0003, 0.0004, 0.0005]	
Project Revenues:		
Cars Initial Traffic Volume	Cars Annual Growth	
Beta 4 [$\mu = 1.92E6$; $\mu_2 = (3.1025E5)^2$; $\chi 1 = 2.55E5$; $\chi 2 = 2.48E6$]	Normal [$\mu = 4.52E4, \sigma = 1.095E3$]	
Small Trucks Initial Traffic Volume	Small Trucks Annual Growth	
Beta 4 [$\mu = 1.69E5$; $\mu_2 = (7.3E3)^2$; $\chi 1 = 1E5$; $\chi 2 = 2E5$]	3.977E3 [deterministic]	
Large Trucks Initial Traffic Volume	Large Trucks Annual Growth	
Beta 4 [$\mu = 4.73E5$; $\mu_2 = (1.825E4)^2$; $\chi 1 = 3E5$; $\chi 2 = 5.25E5$]	Normal [$\mu = 1.108E4, \sigma = 547.5$]	
Toll Inflation Initial Rate, Amplitude, Cycle Length	Toll Inflation Annual Growth Rate	
2.35%, 0.3%, 10 years [deterministic]	Triangle [0.00045, 0.00055, 0.00055]	

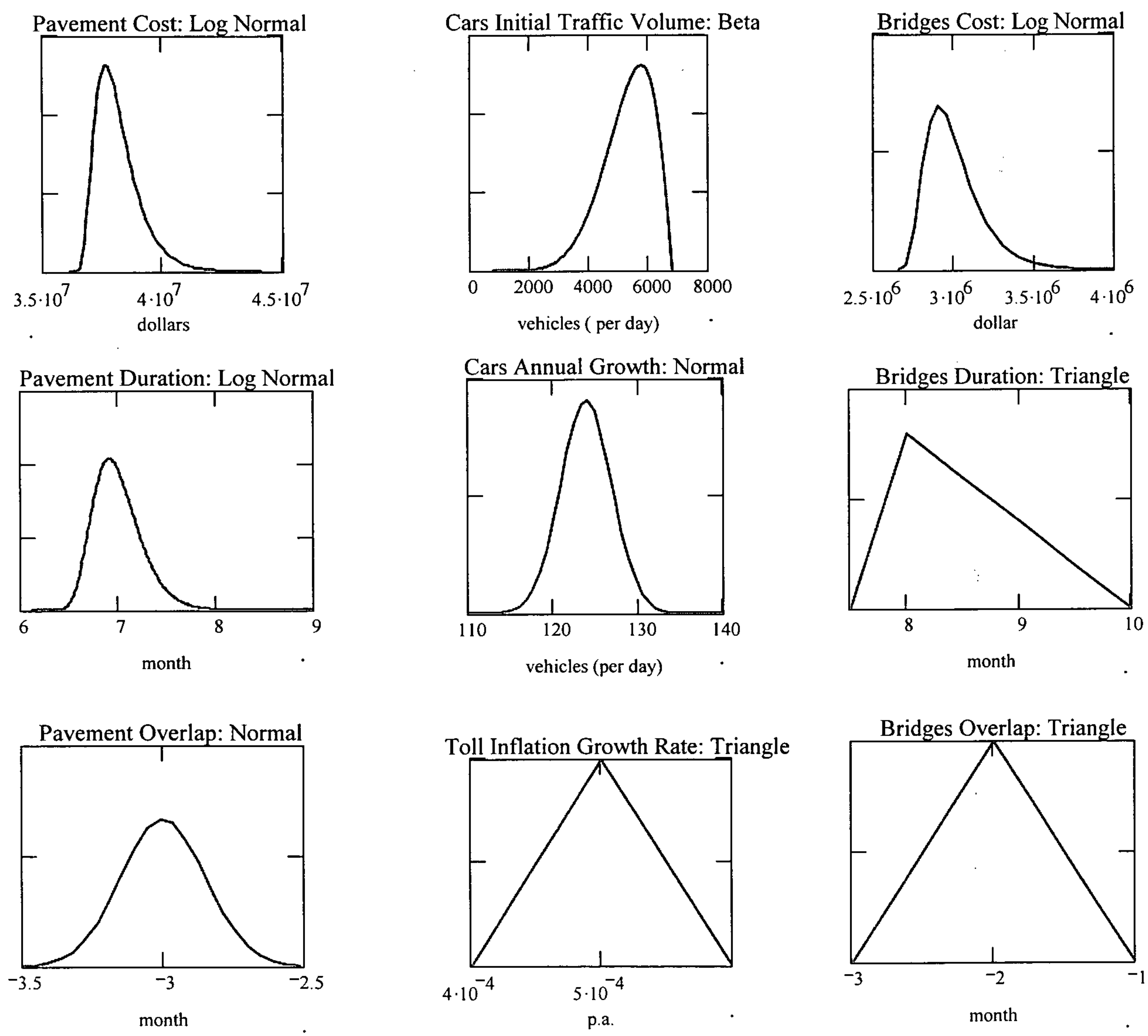
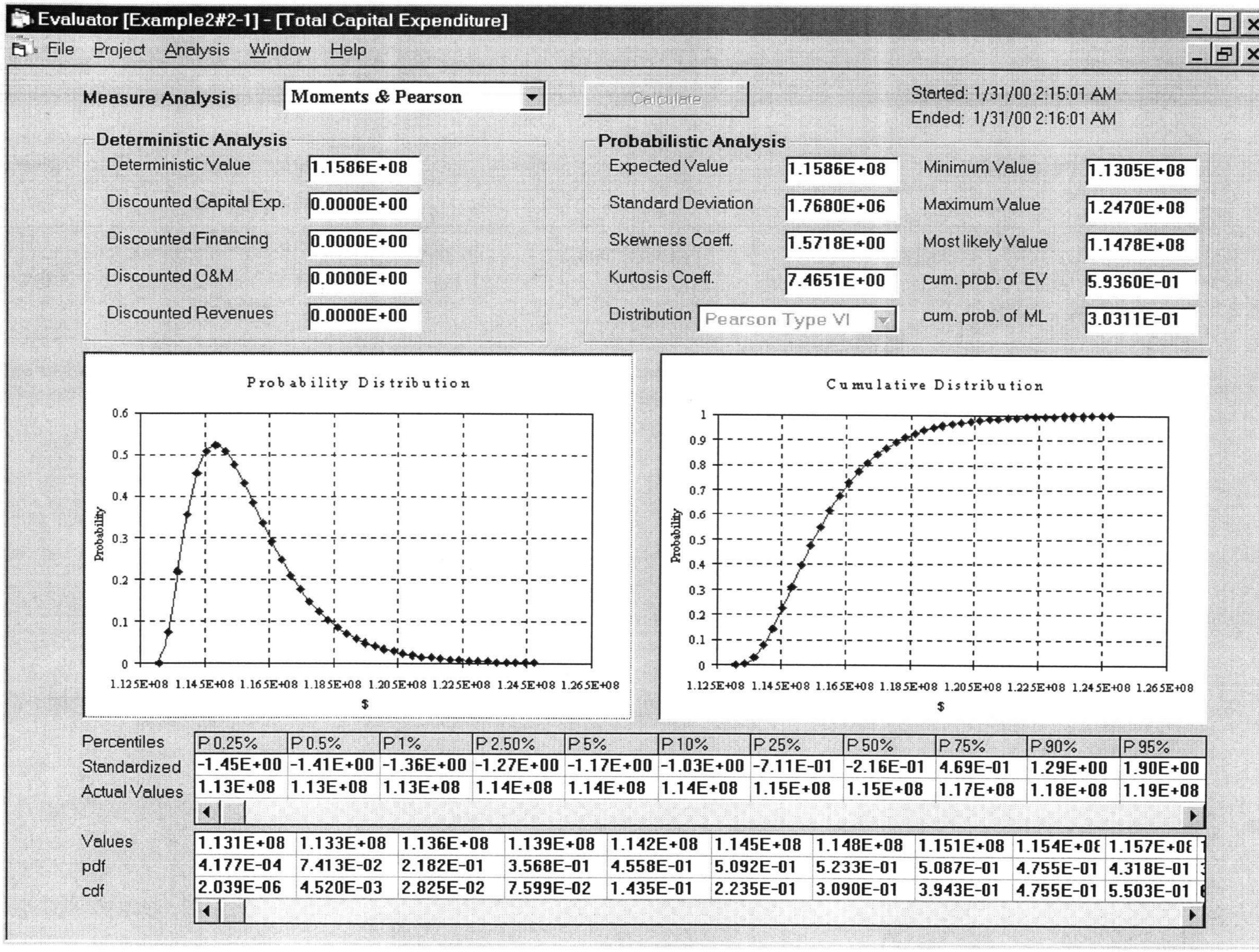


Figure 7.38: Distribution models of selected variables in the highway project example



Percentiles	P 0.25%	P 0.5%	P 1%	P 2.50%	P 5%	P 10%	P 25%	P 50%	P 75%	P 90%	P 95%
Standardized	-1.45E+00	-1.41E+00	-1.36E+00	-1.27E+00	-1.17E+00	-1.03E+00	-7.11E-01	-2.16E-01	4.69E-01	1.29E+00	1.90E+00
Actual Values	1.13E+08	1.13E+08	1.13E+08	1.14E+08	1.14E+08	1.14E+08	1.15E+08	1.15E+08	1.17E+08	1.18E+08	1.19E+08

Values	1.131E+08	1.133E+08	1.136E+08	1.139E+08	1.142E+08	1.145E+08	1.148E+08	1.151E+08	1.154E+0[illegible]	1.157E+0[illegible]
pdf	4.177E-04	7.413E-02	2.182E-01	3.568E-01	4.558E-01	5.092E-01	5.233E-01	5.087E-01	4.755E-01	4.318E-01
cdf	2.039E-06	4.520E-03	2.825E-02	7.599E-02	1.435E-01	2.235E-01	3.090E-01	3.943E-01	4.755E-01	5.503E-01

Figure 7.39: Risk analysis of total cost of total design and construction cost

Figure 7.40 illustrates the uncertainty of construction completion time, which is expressed in the project global time units (annual), not local work package time units (month). The results in the figure show that the deterministic and expected completion times are equal to 1.9444 years (23.328 months); both values are the same since the completion time is a linear equation. The expected completion time has a probability of achievement of 56.76% for the Pearson Type I distribution fitted to the moments of completion time. In the actual implementation of this project, the government required a completion of the project in 20 months from the data of signing of agreement (the 4^{th} month in Figure 7.31), i.e. 24 months from project start data (time zero in Figure 7.31, which is the date by which the developer is selected). On the cumulative graph and the percentile table of the completion time, there is 75% chance to achieve this target date, or alternatively a 25% chance of not meeting target completion. The actual construction of this project took 19 months i.e. 23 months from project start date, which is close to the expected completion time and within the government's requirement.

Figure 7.41 illustrates risk analysis on the total current dollar project revenues over the 30-year concession period. Revenues are negatively skewed with a Pearson Type I distribution. The realization of the deterministic $55.3 million NPV depends on the realization of the revenues forecasted for the project, that is the deterministic $771.98 million in Figure 7.41. This amount of revenues is close to the expected revenue value and has a probability of 44.56% of being achieved, or alternatively, the project has a 55.44% probability of getting more revenues than expected. Government may stipulate the maximum amount of revenues to be realized by a developer where any surplus would go to the public authority involved in the project or shared with the developer as an incentive; in that case, the project NPV and IRR would be checked against the stipulated amount and Figure 7.41 would explain the probability of achieving such an amount.

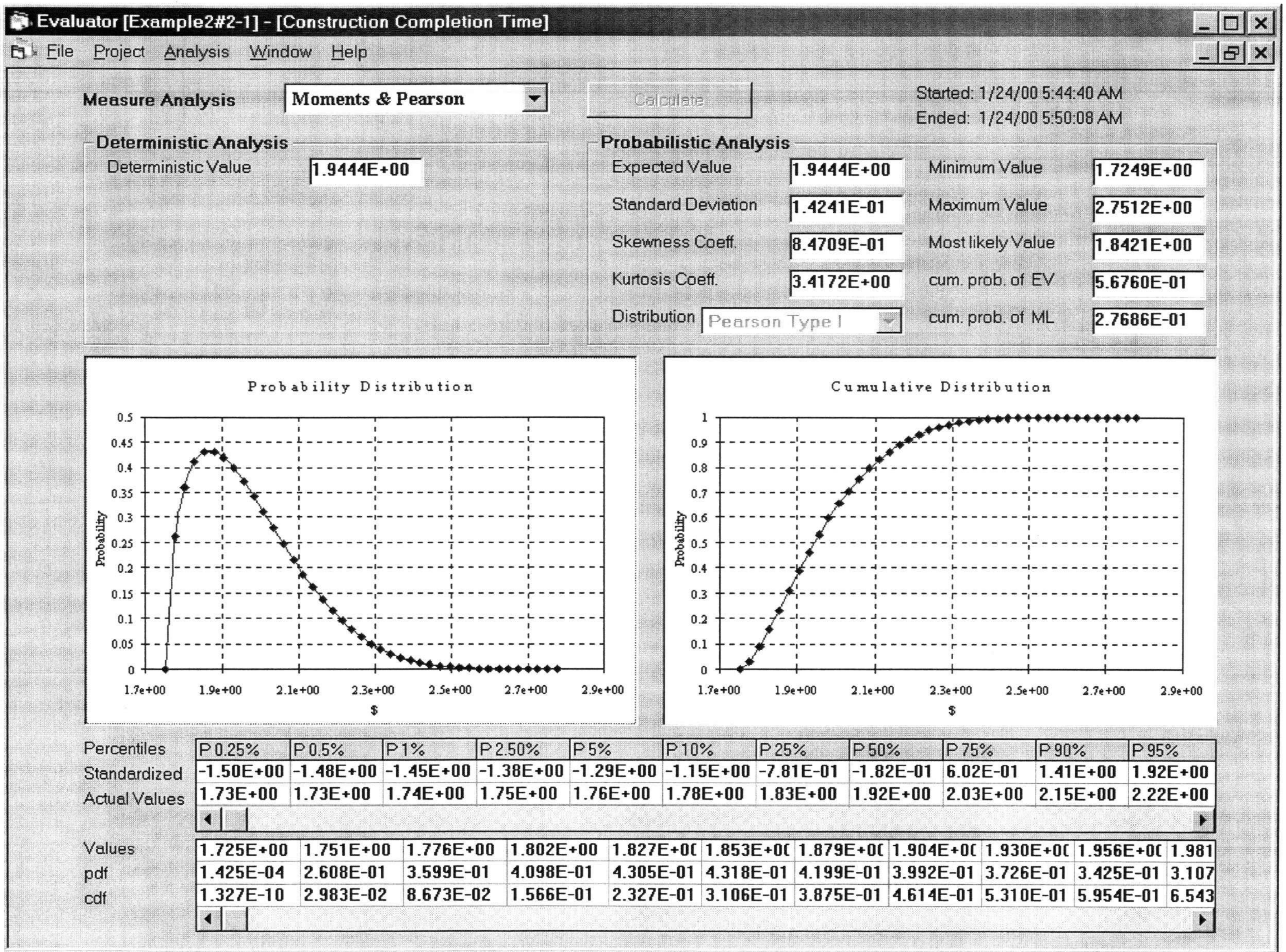
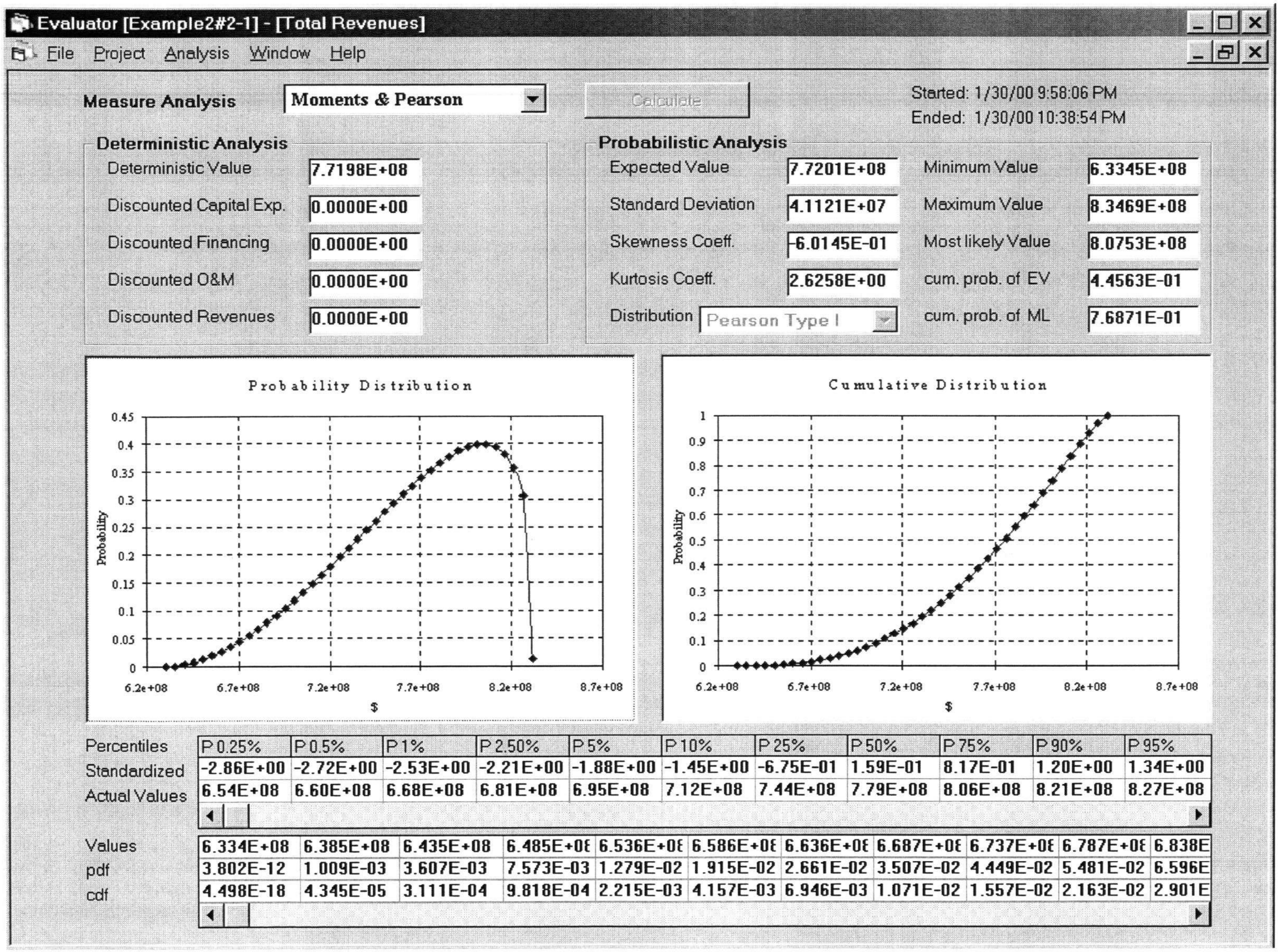


Figure 7.40: Probability distribution of construction completion time for the highway project



Percentiles	P 0.25%	P 0.5%	P 1%	P 2.50%	P 5%	P 10%	P 25%	P 50%	P 75%	P 90%	P 95%
Standardized	-2.86E+00	-2.72E+00	-2.53E+00	-2.21E+00	-1.88E+00	-1.45E+00	-6.75E-01	1.59E-01	8.17E-01	1.20E+00	1.34E+00
Actual Values	6.54E+08	6.60E+08	6.68E+08	6.81E+08	6.95E+08	7.12E+08	7.44E+08	7.79E+08	8.06E+08	8.21E+08	8.27E+08

Values	6.334E+08	6.385E+08	6.435E+08	6.485E+0ٴ	6.536E+0ٴ	6.586E+0ٴ	6.636E+0ٴ	6.687E+0ٴ	6.737E+0ٴ	6.787E+0ٴ	6.838E
pdf	3.802E-12	1.009E-03	3.607E-03	7.573E-03	1.279E-02	1.915E-02	2.661E-02	3.507E-02	4.449E-02	5.481E-02	6.596E
cdf	4.498E-18	4.345E-05	3.111E-04	9.818E-04	2.215E-03	4.157E-03	6.946E-03	1.071E-02	1.557E-02	2.163E-02	2.901E

Figure 7.41: Risk analysis of current-dollar total revenues for the highway project

Figure 7.42 illustrate the uncertainty of the net present value of the project calculated at the 8.25% discount rate. The NPV lies in a range between $18.19 million and $71.75 million with a coefficient of variation of 19.3795%. The expected value of $55.3 million has a probability of 44.5% on the Pearson Type I distribution fitted for the NPV moments. With the NPV having such a large positive amount and no chance of negative values, the project would be judged beneficial under the uncertain circumstances described in Table 7.12 and Figure 7.38 for the various project cost and revenue variables. As with project revenues, government may stipulate the NPV on a project and use it in comparing several bids, as in the case of the Second Severn Crossing ("Second" 1988, 1989). Figure 7.41 would be a tool through which government can check on the NPV fixed-value deterministic analysis of a project, NPV statistics, and probabilities of realizing various NPV values before making decisions on the project. Developers would use the information similarly to investigate the NPV likely range and whether the probability attached to a required NPV value would satisfy the risk perception of the decisions makers.

The first and second case studies presented deterministic and probabilistic analysis for different levels of detail of the total investment cost, revenues, operation and maintenance and financing at the appraisal stage of a project. The two cases show the capabilities of the generalized economic model and the risk analysis framework in modeling capital investment projects. The next case shows the capabilities of the DSS in modeling higher levels of detail in the cost and revenues of projects.

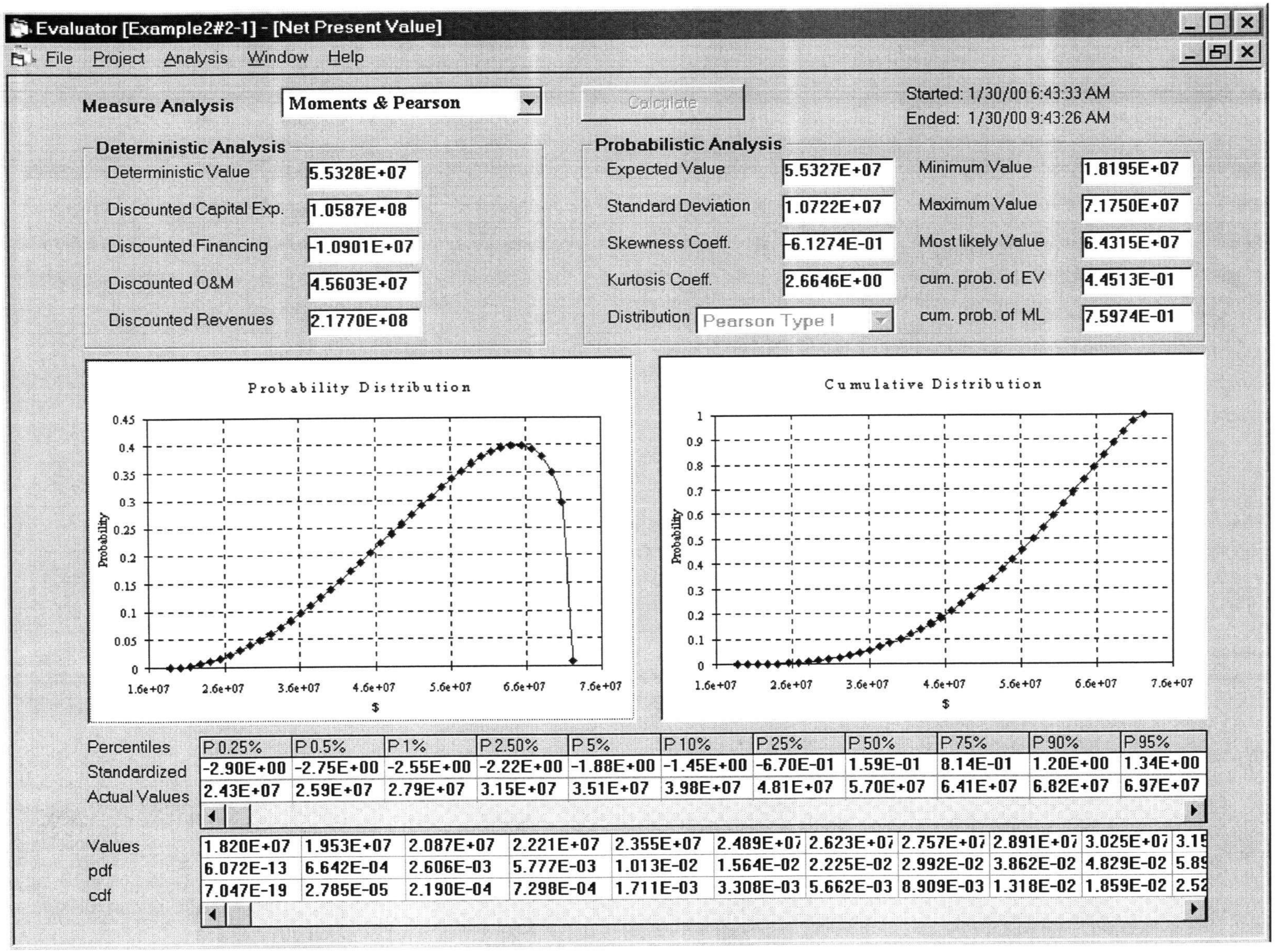


Figure 7.42: Risk analysis of net present value of the highway project

7.3.3 Case 3: Detailed Modeling and Sensitivity Analysis

This case study extends the application of the DSS to show the capabilities of the generalized economic model in treating detailed project cost and revenue estimates. The case covers for te the highway project example some aspects of capital expenditure estimating, demand forecasting, sensitivity analysis for the NPV, and cost aggregation levels.

a) Detailed Cost Estimating

In Figure 7.22 the economic structure of the total investment cost was represented by three work packages (design, road construction, and road structures). In Figure 7.31 the economic structure of the investment cost was expanded into ten work packages detailing the cost of the three earlier work packages. For example, the road structures work package was divided into culverts, tunnels, interchanges, and bridges. Following the structure of the generalized model, it is possible to further expand any of the work packages into smaller work packages. In this case study, the bridge work package is divided into four work packages representing four bridges in the project and one of these bridges "B1" is further divided into ten work packages representing major cost centers in bridge construction estimating[1], as shown in Figure 7.43.

Table 7.13 explains the cost of the ten work packages of the bridge as being divided into labor, equipment, material, and subcontracted/indirect cost items. Table 7.14 details further the estimate of the labor and equipment cost for the "structural excavation" and "deck slab" work packages by explaining the variables used in the unit-cost estimating method, i.e. quantity, production rate (used here interchangeably with productivity rate), and wage rate. This represents the lowest and most detailed level in the DSS at which risk analysis can be carried out.

[1] Bridge cost estimate and bridge network are due to Clough and Sears (1991) with some modifications.

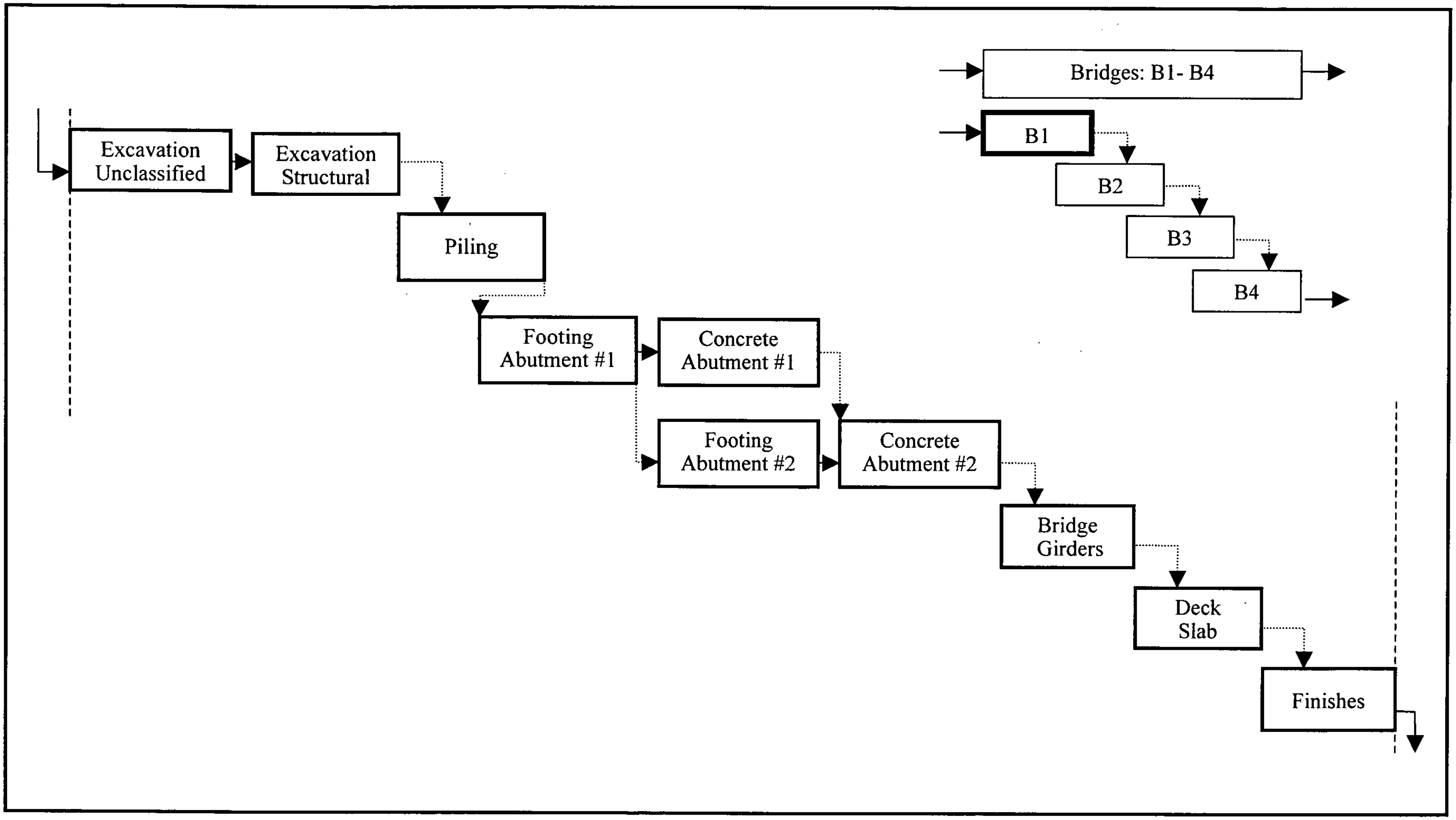


Figure 7.43: Work packages and network for bridge "B1" of the highway project example

Table 7.13: Cost items in a single bridge of the highway project and estimated duration

Work packages	Labor Cost	Equipment Cost	Material Cost	Subcontract/ Indirect Cost	Total Cost	Duration (Month)
Excavation, Unclassified	6254.35	2144.84	0.00	0.00	8399.16	0.35
Excavation, Structural	5265.59	1351.35	0.00	1895.35	8512.29	0.35
Piles	37962.54	38752.56	106340.85	0.00	183055.95	0.5
Footing Abutment #1	4005.54	1004.85	11365.20	17444.54	33820.13	0.35 (0.25 overlap)
Footing Abutment #2	4007.27	1004.85	11365.20	17444.54	33821.86	0.35
Concrete Abutment #1	41489.91	10433.12	29014.18	17444.54	98381.74	0.35
Concrete Abutment #2	41489.91	10433.12	29014.18	17444.54	98381.74	0.35
Girders	8572.41	5987.52	93735.18	0.00	108295.11	0.34
Deck Slab	28017.99	3613.99	15949.39	69776.44	117357.82	0.5
Finishes	0.00	0.00	0.00	61288.92	61288.92	0.66
Total	177065.48	74726.19	296784.18	202738.88	751314.74	

Table 7.14: Detailed labor and equipment cost for the structural excavation

<u>**Excavation, Structural Work Package**</u>

Quantity = 120 cy

Labor Cost:

Crew wage	= 351.04 \$/hr	= 2808.31 \$/day	= 84249.39 \$/month
Production rate	= 8 cy/hr	= 64 cy/day	= 1920 cy/month
Labor cost	= Quantity· Crew wage / Production rate		
	= \$5265.587		

Equipment Cost:

1 cy Backhoe rate	= 90.09 \$/hr	= 720.72 \$/day	= 21621.6 \$/month
Equipment cost	= Quantity· Backhoe rate / Production rate		
	= \$1351.35		

<u>**Deck Slab Work Package**</u>

Quantity = 200 sy

Labor Cost:

Crew unit cost = 140. \$/sy

Labor cost = Crew unit cost · Quantity = \$28017.99

Material Cost:

Material unit cost = 79.74 \$/sy

Material cost = Material unit cost · Quantity = \$15949.40

Equipment Cost:

Equip.lump sum = \$3613.99

Figure 7.44 shows the how the variables of the "excavation, structural" work package ("CEid13") are modeled in the DSS. In this figure, the top of the window shows the detailed estimating menus available. The labor and equipment cost windows shown open depict one of the detailed methods for cost estimating of labor and equipment costs (see Eq. 4.7). Estimates of quantity, wage rates and production rates in Table 7.14 represent the input to these estimates in the DSS. The time reference for labor wage is *local* and therefore estimated in units of months, as per the work package local time unit. Quantity and productivity have local reference by default, and cannot be made global. Any of the variables describing the work package can change over time using any of the *rate* and/or *area* functions. Wage and production rates are modeled with a *Uniform I rate function* - i.e. their values remain constant over the duration of the work package. The work package quantity is made *Uniform Total* - i.e. the extension of the work package duration will not affect the total quantity or scope of the work package. Any of the parameter variables of the *shape functions* in the work package can be modeled by any of the risk models in Chapter 5. As shown in Figure 4.4, labor and equipment costs can be assigned their own inflation rates. However, for this example identical rates have been used.

Figure 7.45 shows the NPV normalized sensitivity coefficients for the capital expenditure variables. Each variable has a sensitivity coefficient according to whether it represents total cost (e.g. in aggregated method), cost item (e.g. total labor cost of a work package), or a variable of a cost estimate method (e.g. productivity). This can be explained further as follows, (reference is made to Figure 7.45, work package identification is in Table 7.15):

- "Scope/Quantity" table: Sensitivity coefficients in this table are for the variables representing the quantity of a work package or representing the variables making the *shape* function used for work package quantity. For example, with reference to "CEid20", the sensitivity coefficient of the 200 sy of the deck slab work package in Table 7.14 is –0.00264 which means that a 1% increase in quantity will reduce the NPV by 0.00264%.

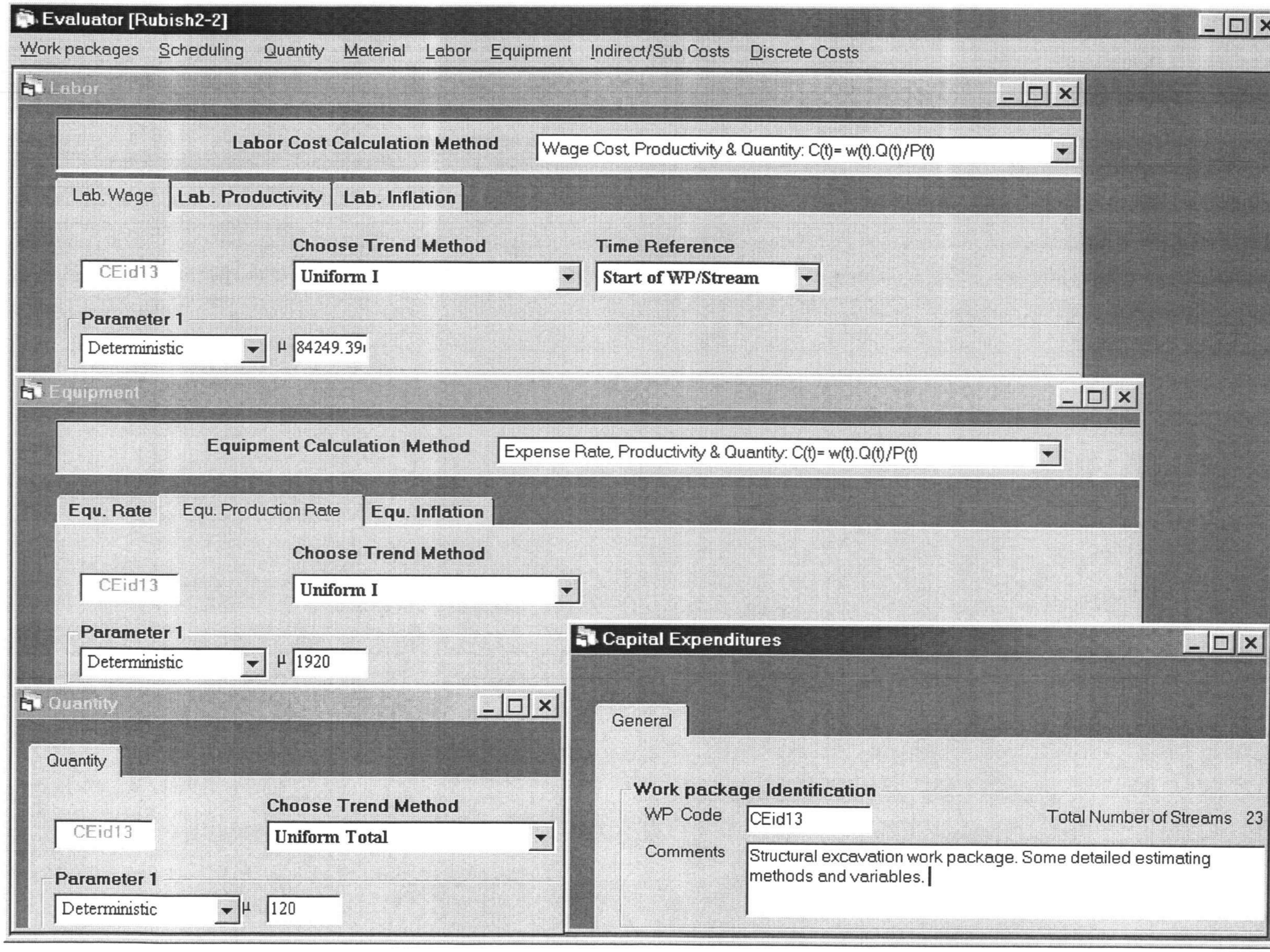


Figure 7.44: Detailed estimating of the "excavation, structural" work package

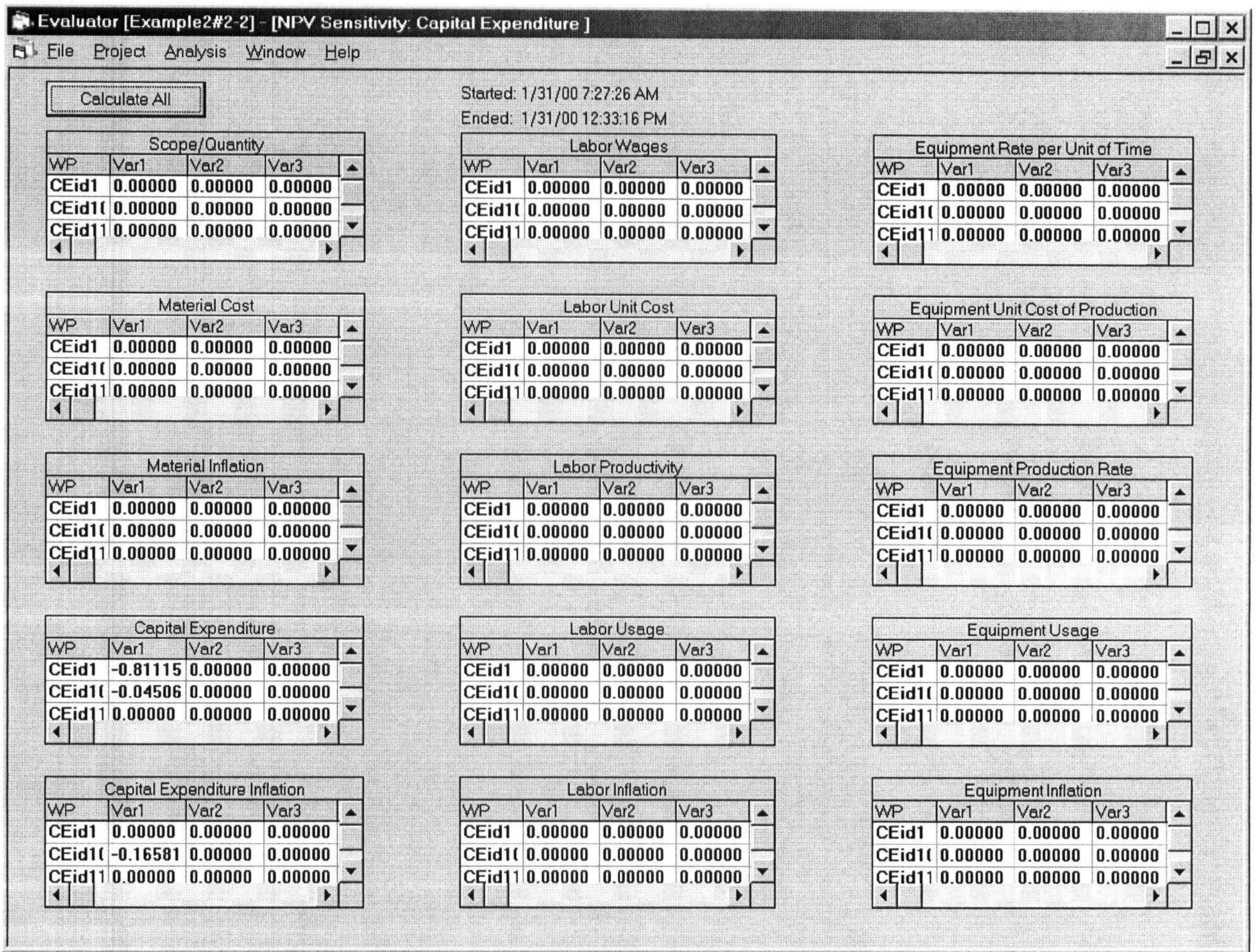


Figure 7.45: NPV normalized sensitivity coefficients for some capital expenditure variables

Table 7.15: Work packages in the highway example project

Work Package	Detail	Predecessor	Work Package	Detail	Predecessor
CEid1	Design	-	CEid13	Excavation, Structural	CEid12
CEid2	Clearing and Grubbing	CEid1	CEid14	Piles	CEid13
CEid3	Cut & Fill	CEid2	CEid15	Footing Abutment #1	CEid14
CEid4	Sub Base Layer	CEid3	CEid16	Concrete Abutment #1	CEid15
CEid5	Base Layer	CEid4	CEid17	Footing Abutment #2	CEid16
CEid6	Pavement	CEid5	CEid18	Concrete Abutment #2	CEid17
CEid7	Culverts	CEid3	CEid19	Bridge Girders	CEid18
CEid8	Tunnels	CEid3	CEid20	Deck Slab	CEid19
CEid9	Interchange	CEid3	CEid21	Finishes	CEid20
CEid10	Bridge #2	CEid21	CEid22	Bridge #3	CEid10
CEid11	End, last work package in the CE network	CEid6, CEid7, CEid8, CEid9, CEid23	CEid23	Bridge #4	CEid22
CEid12	Excavation, Unclassified	CEid3			

The unit price of the concrete in this work package has it sensitivity coefficient in the "Material Cost" table. The order of Var1, Var2...VarN in the sensitivity tables in Table 7.45 is the same order of the variables of each *shape* function in Tables (4.1, 4.2 and 4.5).

- All of the inflation tables in Figure 7.45 have the same sensitivity coefficient since the CE component was set for this case to have the same inflation rate across all work packages (see inflation property in Figure 4.4). The coefficient is –0.16581 meaning that for each 1% increase in inflation the NPV would be reduced by 0.16581%.

- "Capital Expenditure" table: Sensitivity coefficients in this table are for the variables representing the capital expenditure variable *X(t)* (see Eq. 4.4) for an aggregated semi-detailed cost of a work package or the variables of the *shape* function representing *X(t)*. For example, the "road asphalt pavement" work package in Table 7.10 and Figure 7.31 has a total aggregated cost of $38.1375 million (=33.75% of total project cost or 45% of road construction cost) which, following "CEid6" in the sensitivity table, has a significant effect on the project NPV. That is, for each 1% increase in the total pavement cost the NPV is expected to fall by 2.2% from its original value.

- The "Labor Wages" table contains the sensitivity coefficients for the labor wage ($ per unit of time) as used in the labor cost of a work package. For example, the structural excavation work package "CEid13" has crew wage of $84249.3 per month in Table 7.14, this have a negative sensitivity coefficient of –00032. "Labor Unit Cost" table continues the sensitivity work and can explain the coefficients of the labor unit cost ($ per unit of production) or total aggregated cost of labor in a work package. For example, the sensitivity coefficient of "CEid16" represents the total labor cost of the "concrete abutment #1" work package in Table 7.13. The same description can be used with the equipment cost tables in Figure 7.45.

- Similarly, the "Labor Productivity" table shows the sensitivity coefficients of the productivity variables. For example, the crew production rate for the structural excavation in Table 7.14 has a sensitivity coefficient of 0.00032 ("CEid13").

Figure 7.45 contains the sensitivity coefficients of the variables used in representing all the work packages of the single "B1" bridge in Figure 7.43, as well as all the cost items and variables of the other work packages that comprise the whole capital expenditure component in Figure 7.31.

b) Detailed Traffic Demand

For the first case of the highway project, revenues are defined in Table 7.1 by an aggregate revenue stream with a linear function to describe time variation of demand. In Table 7.10 in the second case, revenues are decomposed into three revenue streams (cars, small trucks and large trucks) with traffic demand being modeled using simple regression of the historical AADT as shown in Figures 7.32 and 7.33. In Figure 7.33 the three forecasting formulas (linear, 2nd and 3rd order polynomials) fit the actual data quite well but differed significantly in forecasting the future. The linear model was selected simply because it fit the historical records well and provided a conservative estimate of future use; factors that might affect traffic growth or propensity of travelers to use the new highway were not part of the forecasting formula. The current case explains how future traffic demand is modeled using the "Stated Preference" technique (see Chapter 4). The car revenue stream is the subject of interest for this case study.

In the RFP for the actual project it was explained that the AADT is highly correlated to the Gross Domestic Product GDP of Canada. Table 7.15 shows an account of the values of AADT and GDP. A simple regression for this relationship can be modeled as follows, where GDP is expressed in millions of dollars:

$$AADT = -898.553 + 0.015 \cdot GDP \tag{7.11}$$

Table 7.15: AADT (vehicles-day) and GDP (in millions of dollars)

Year	Time	AADT	GDP
1970	0	2880.0	252299
1975	5	3990.0	320035
1980	10	4620.0	381992
1985	15	5100.0	438450
1990	20	6530.0	503659
1991	21	6520.0	494530
1992	22	6600.0	497789
1993	23	6750.0	510946
1994	24	6900.0	528484

The equivalent of Eq. 7.11 for the annual in-scope car traffic (see Table 7.10 for the 82.8% in-scope traffic and the 75% of traffic is cars) can be expressed as follows for actual GDP values:

$$\textit{Cars Traffic} = [\,-2.4598{\cdot}10^{5} + 4.0403{\cdot}10^{-6} \cdot GDP\,] \cdot 0.828 \qquad (7.12)$$

Forecasting GDP, however, requires considerable analysis regarding the several factors that might affect the economy of a country or a region. The actual project was subject to this type of analysis and the RFP included future estimates of GDP. Figure 7.46 illustrates the actual GDP values in Table 7.15 (in this figure, years 0 to 24 correspond to 1970 to 1994). For the current example, two models are used to model the actual data. The first is linear as shown in Figure 7.46. While it fits the actual data well during the first 25 years, it deviates from the future estimates contained in the RFP. The second model is exponential rate function (Growth II rate function in Table 4.1), which seemed to reasonably fit the data as shown in Figure 7.46:

$$GDP(t) = 5.283{\cdot}10^{6} \cdot (1 + 0.03)^{t} \qquad \text{starts in 1995} \qquad (7.13)$$

in which, t is years from project start and 0.03 is a 3% corresponds to a 3% effective annual growth rate; see Figure 7.47(a,b).

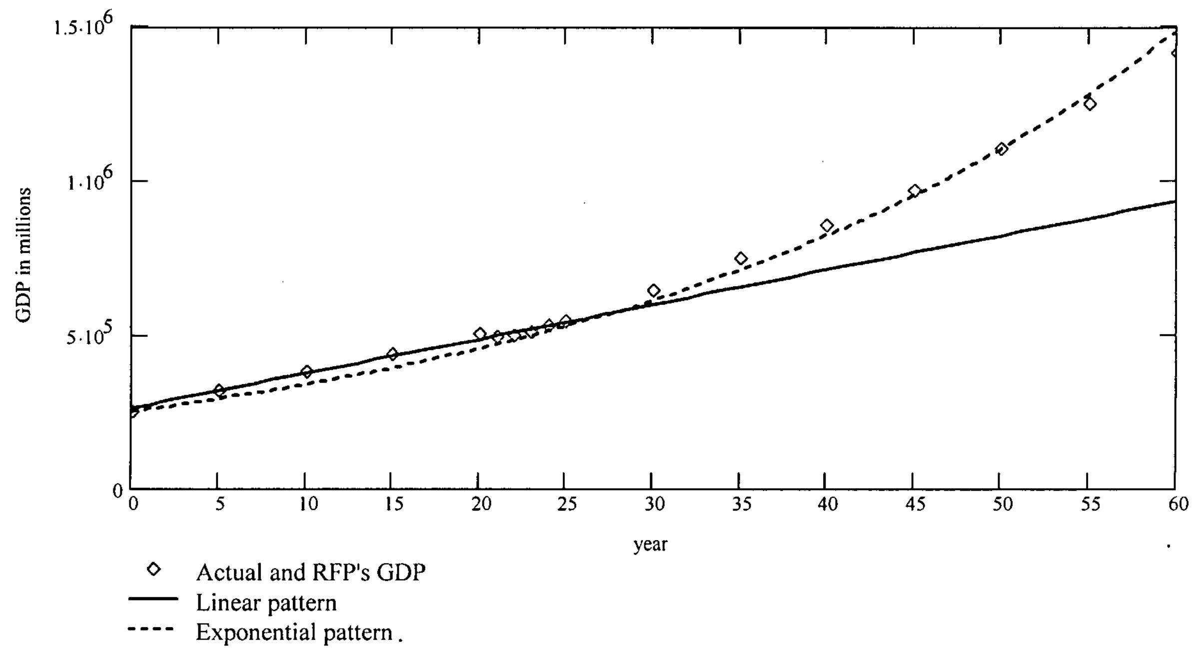
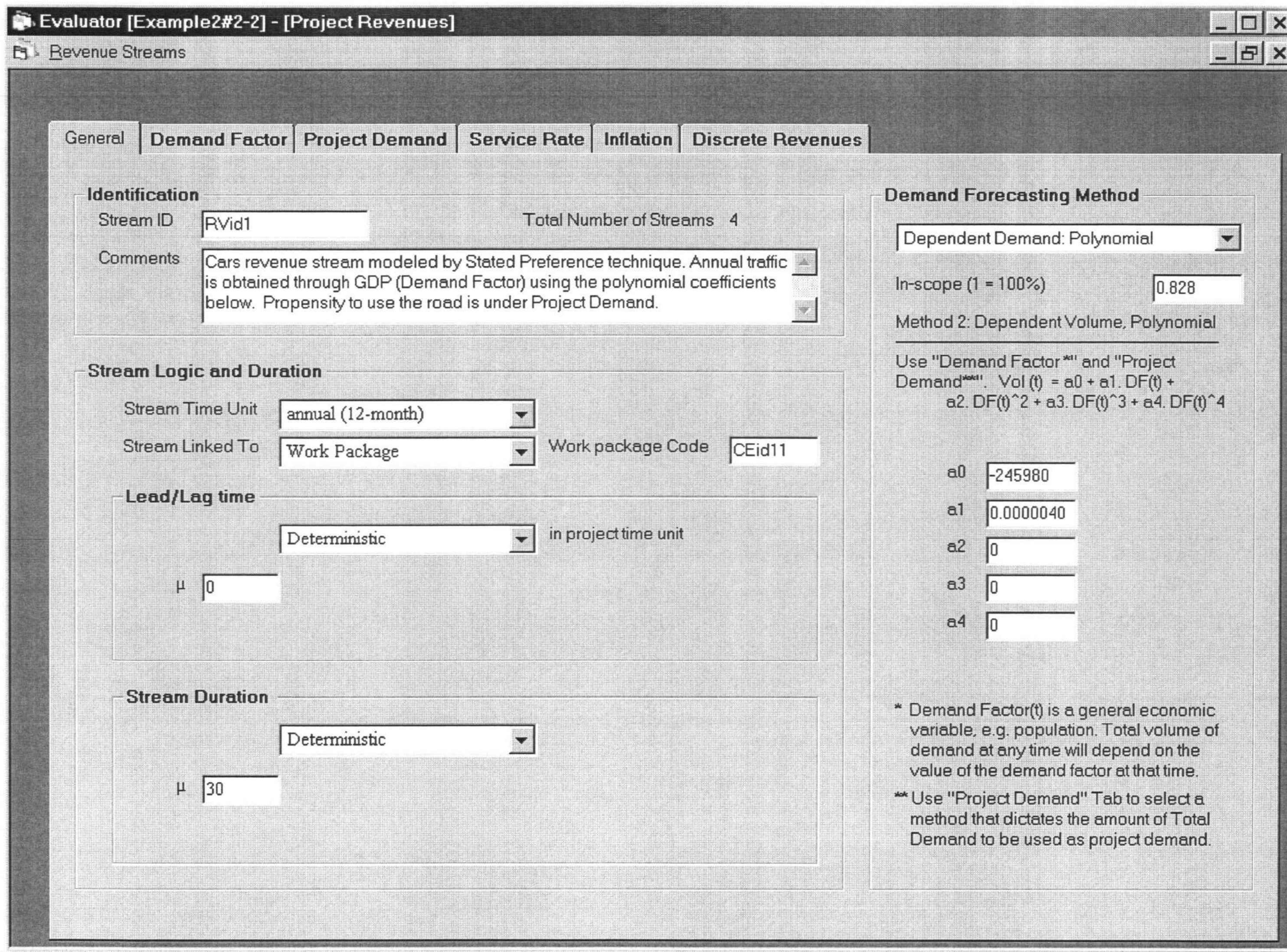
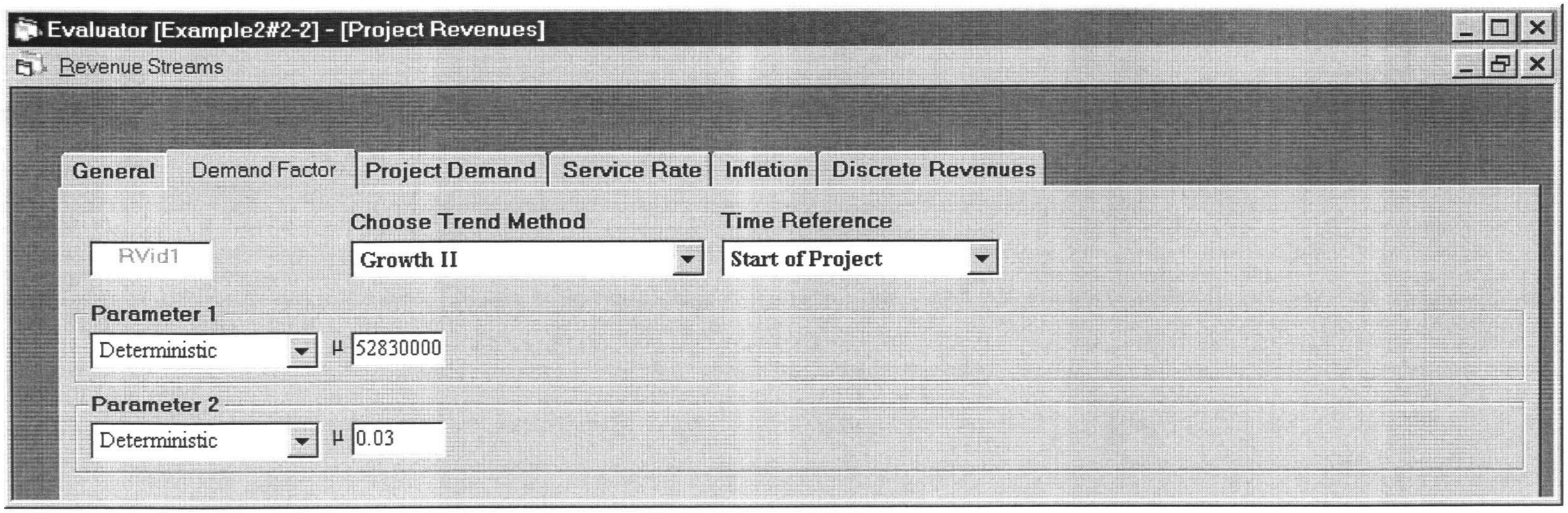


Figure 7.46: GDP models and RFP estimates



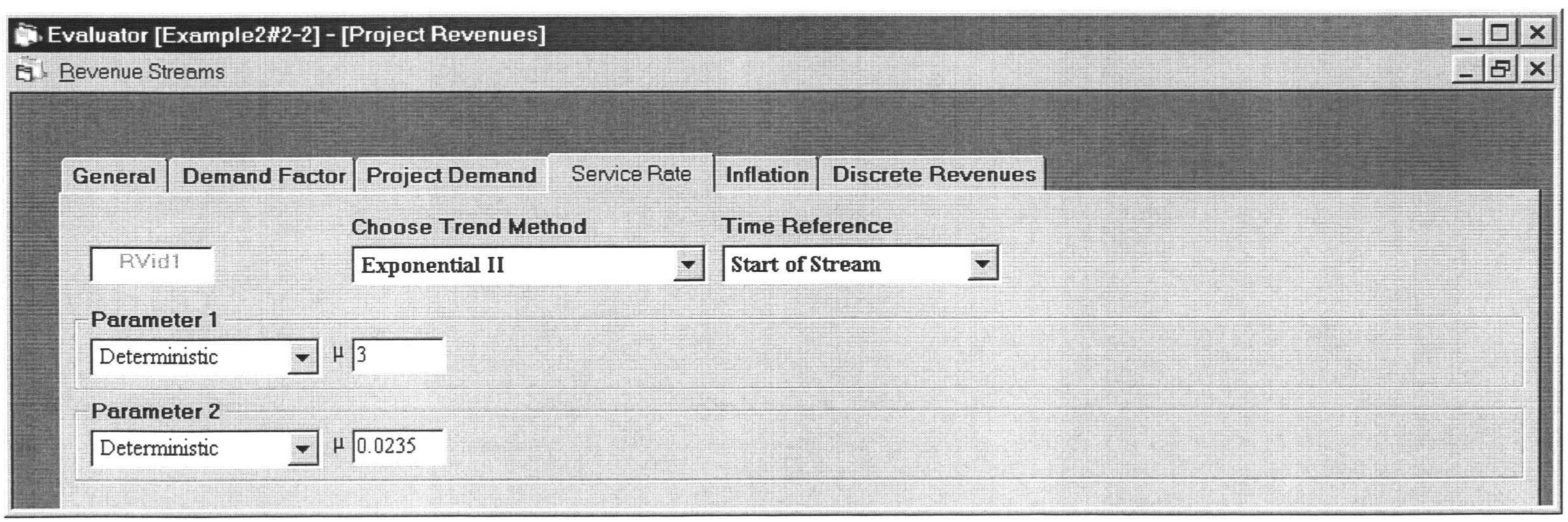
(a)

Figure 7.47: Modeling cars revenue stream using stated preference technique. (specifying the parameters of the annual cars traffic in Eq. 7.12)



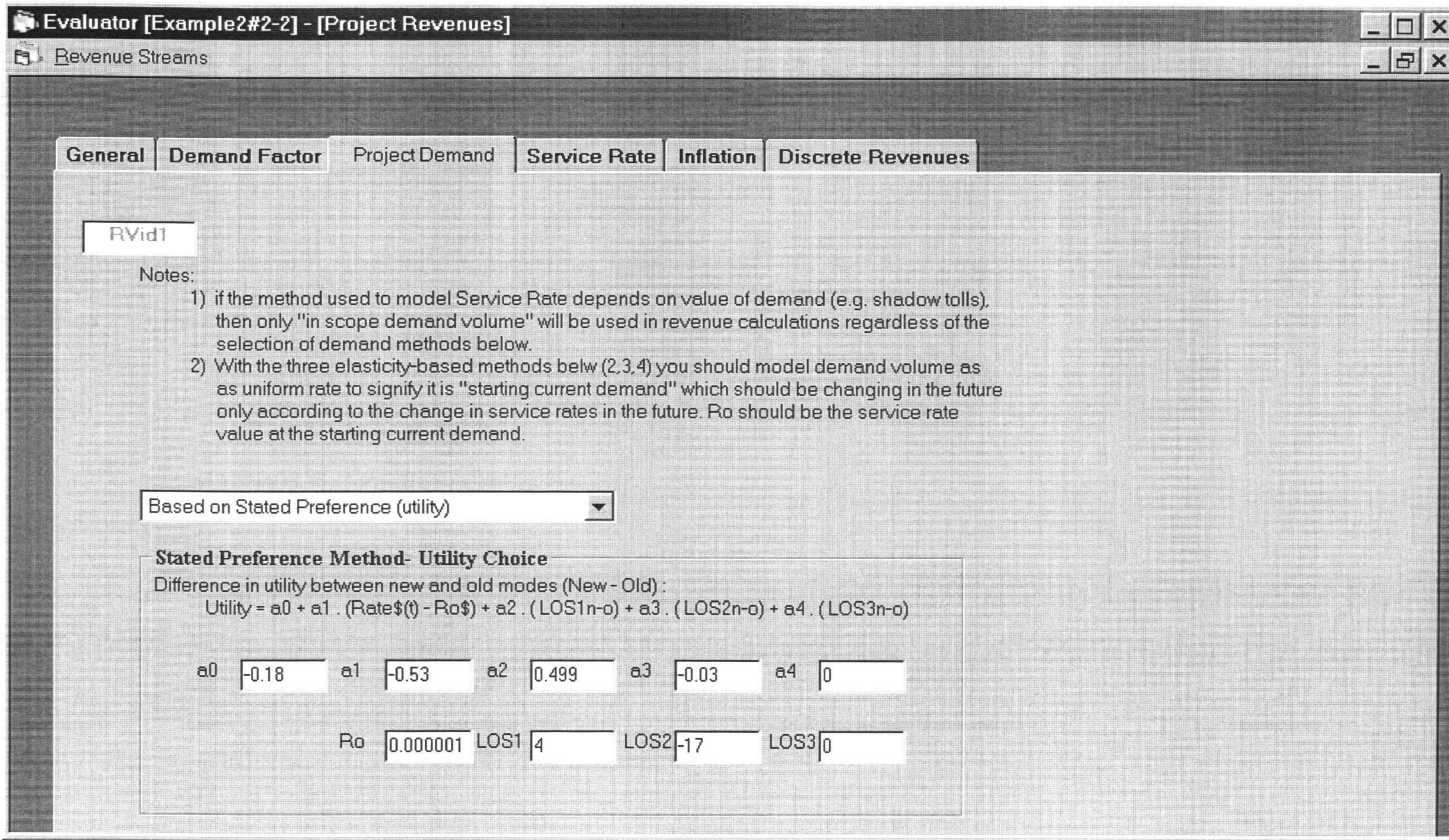
(b)

Figure 7.47: Continued (specifying the parameters of the GDP function in Eq. 7.13, see also Table 4.1)



(c)

Figure 7.47: Continued (specifying the exponential rate function of the toll rate, see Table 4.1)



(d)

Figure 7.47: Continued (specifying the parameters of the utility function Eq. 7.14, see also Section 4.4.2)

Generally, not all future traffic will be directed to the new highway. Project market share normally depends on several variables such as toll level, number of lanes, time saving, safety on the road, and other factors. For the actual project, the propensity for using the new highway, or its market share, was determined through the stated preference technique. Utility functions for the new highway were formulated for each class of vehicle and through the application of the *logit* mode (section 4.4.2), the project's market share was determined. The utility function of the new highway for car users in the example project is the same as for the actual project with minor modification (the actual utility function has another attribute for payment type):

$$Utility = a_o + a_1 \cdot (number\ of\ lanes) + a_2 \cdot (toll\ charge) + a_3 \cdot (time\ saving) \qquad (7.14)$$

in which, a_o to a_3 are the parameters obtained from the regression analysis performed on the results of a stated preference technique survey conducted for the project. The value are: $a_o = -0.18$, $a_1 = 0.499$, $a_2 = -0.53$, and $a_3 = -0.03$. The number of lanes on the new highway is four; the toll charge for cars is \$3; and, the time saving is estimated to be 17 minutes. These parameters and values are defined for the car revenue stream as shown in Figure 7.47(d); tolls are defined in Figure 7.47(c). Finally, the propensity for using the new highway, or its market share, is illustrated in Figure 7.48, which assumes that all variables in the utility function are constant except the toll charge.

Using the cars revenue stream utility function as described in Eq. 7.13, two scenarios can be explored in modeling future traffic. The first scenario provides for future toll rate increases to affect the market share as shown in Figure 7.48. This scenario considers only the change in tolls to affect the use of the highway, i.e. it assumes that the other parameters in Eq. 7.13. This scenario leads to \$18.43 million for NPV and \$591 million for revenues.

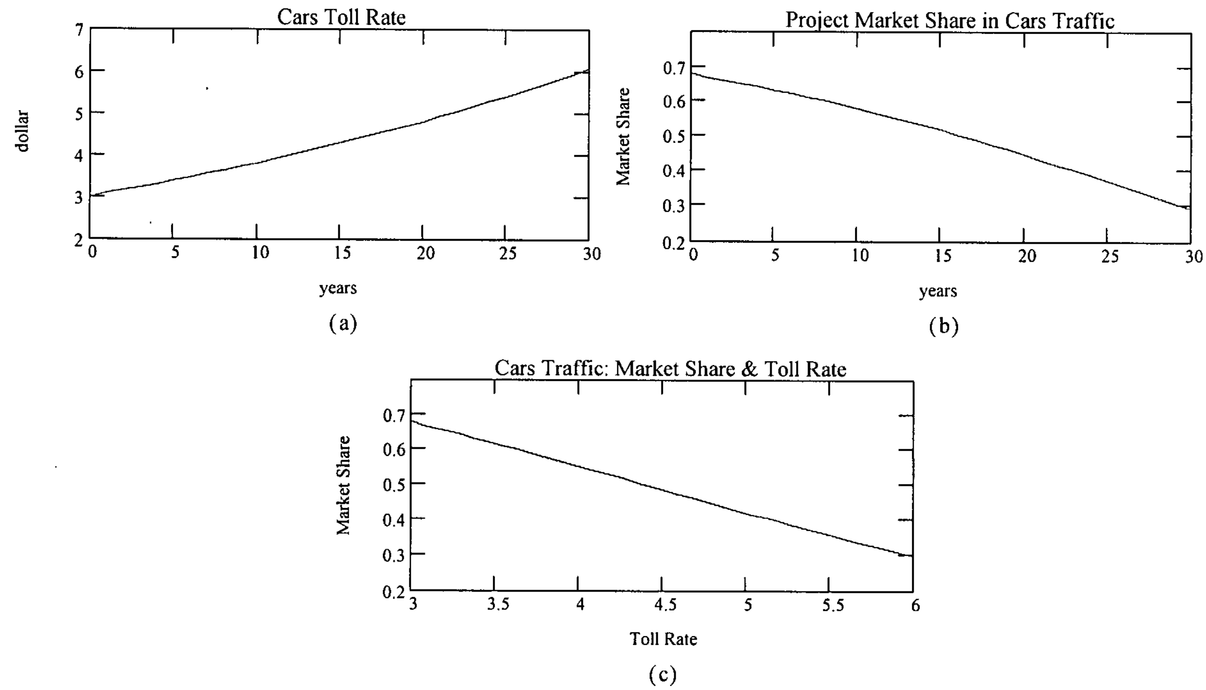


Figure 7.48: Relationship between car traffic, toll rate and time, the first scenario.

The second scenario in modeling future traffic considers the results of the utility function and the *logit* model to give a market share value that will be constant throughout the duration of the revenue stream. Therefore, for a \$3 toll the project market share will be at 67.6% and this share will be constant during the stream duration regardless of toll increases. This scenario produced a NPV of \$34.02 million and total revenues of \$690.0 million. Traffic and revenue forecasts in the RFP of this project reflect a utilization of this modeling assumption for the car and truck revenue streams; i.e. constant market share during project life.

To explain the effect of changes in the variable values of the revenue function on the NPV of the highway project example, a sensitivity analysis was carried out for the first of the two future traffic modeling scenarios above, with the results being as shown in Figure 7.49. These results can be elaborated further upon as follows:

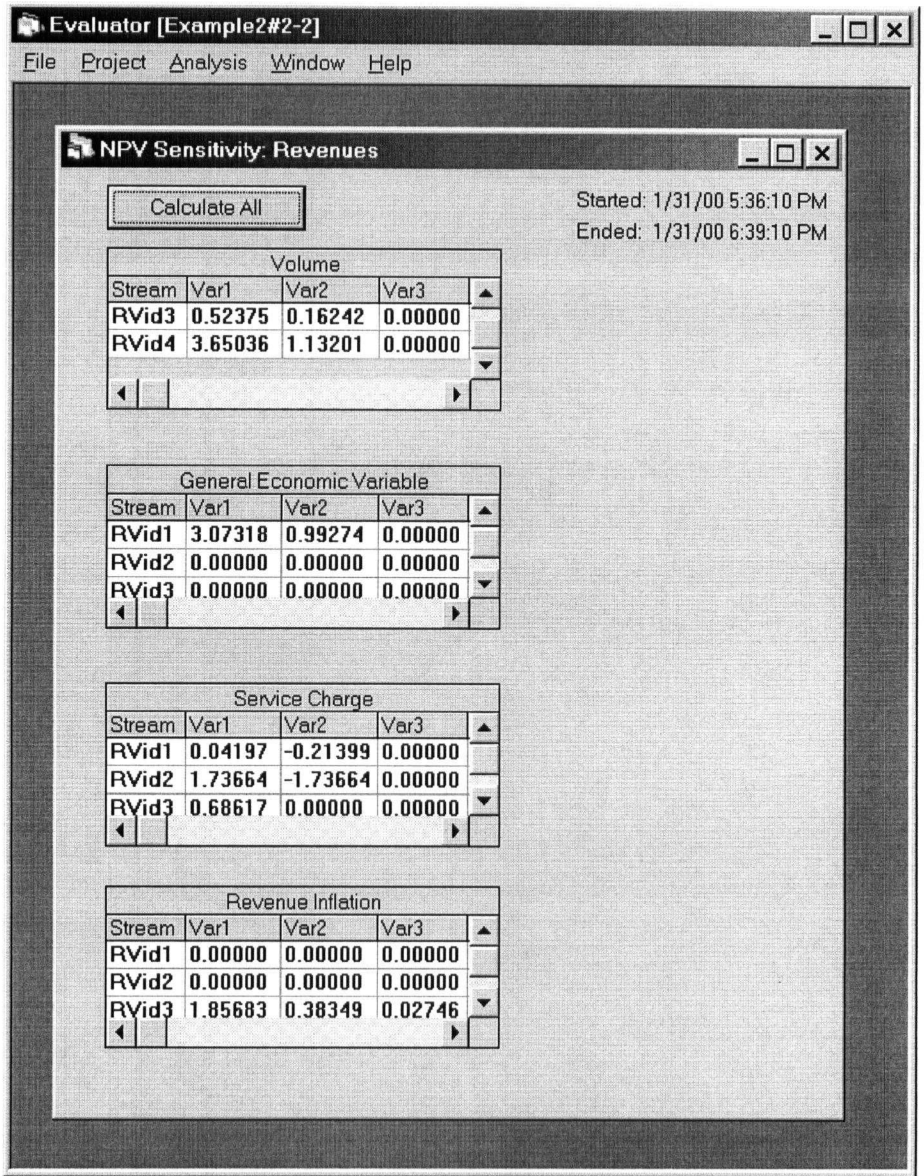


Figure 7.49: NPV normalized sensitivity coefficients for some revenue variables

- "Volume" table- This table shows the sensitivity coefficients of the variables used in modeling the traffic volume of each revenue stream. For example, future traffic volume of the "large trucks" revenue stream ("RVid4") was modeled by Eq. 7.10 as a linear model using an initial traffic volume ("Var1") and annual growth ("Var2"). The 3.65 coefficient for "Var1" means that for each 1% increase in the forecasted initial large trucks volume there will be a 3.65% increase in the NPV of the project. Similarly, for a 1% change/increase in the annual growth amount "Var2", a 1.13% increase in the NPV will occur. The "small trucks" revenue stream ("RVid3") has significantly lower sensitivity values than that of the large trucks traffic volume.

- "General Economic Variable" table- Coefficients in this table correspond to the variables used in modeling the economic indicators. For example, future traffic for the cars revenue stream ("RVid1") was modeled by Eq. 7.12 as a function of the future forecasts of GDP. GDP in Eq. 7.13 was modeled by an exponential function of an initial GDP forecast ($5.283 \cdot 10^6$) and an annual growth percentage (0.03). Therefore, for each 1% change/increase in the initial GDP "Var1" there will be 3.07% increase in the NPV of the project as shown in the sensitivity table. Similarly, if there is an increase in the annual growth percentage of GDP, a 0.99% increase in NPV will be expected.

- "Service Charge" table- Coefficients in this table are for the variables used in modeling the service charge or toll rate for each revenue stream. Since the toll rate for the cars revenue stream "RVid1" was modeled by an exponential function, two variables are used- one for the initial toll ($3) and one for the growth rate (0.0235; note the rate follows the local/global reference of its shape function). Therefore, for "RVid1" there are two sensitivity coefficients, 0.04 for the initial toll rate and –0.2 for the annual growth percentage. Despite the fact that increasing tolls generally increases revenues and NPV, these coefficients are not of great significance unlike the sensitivity for the traffic volume variables. The reason for this difference can be attributed to the traffic modeling scenario adopted for the analysis where increasing tolls would deviate cars drivers away from the new highway during the duration of the revenue stream (see Figure 7.48c). When the analysis was made using the second traffic-modeling scenario, toll increases had a higher sensitivity coefficient of 0.88 for toll rate affecting positively the NPV, see Figure 7.50.

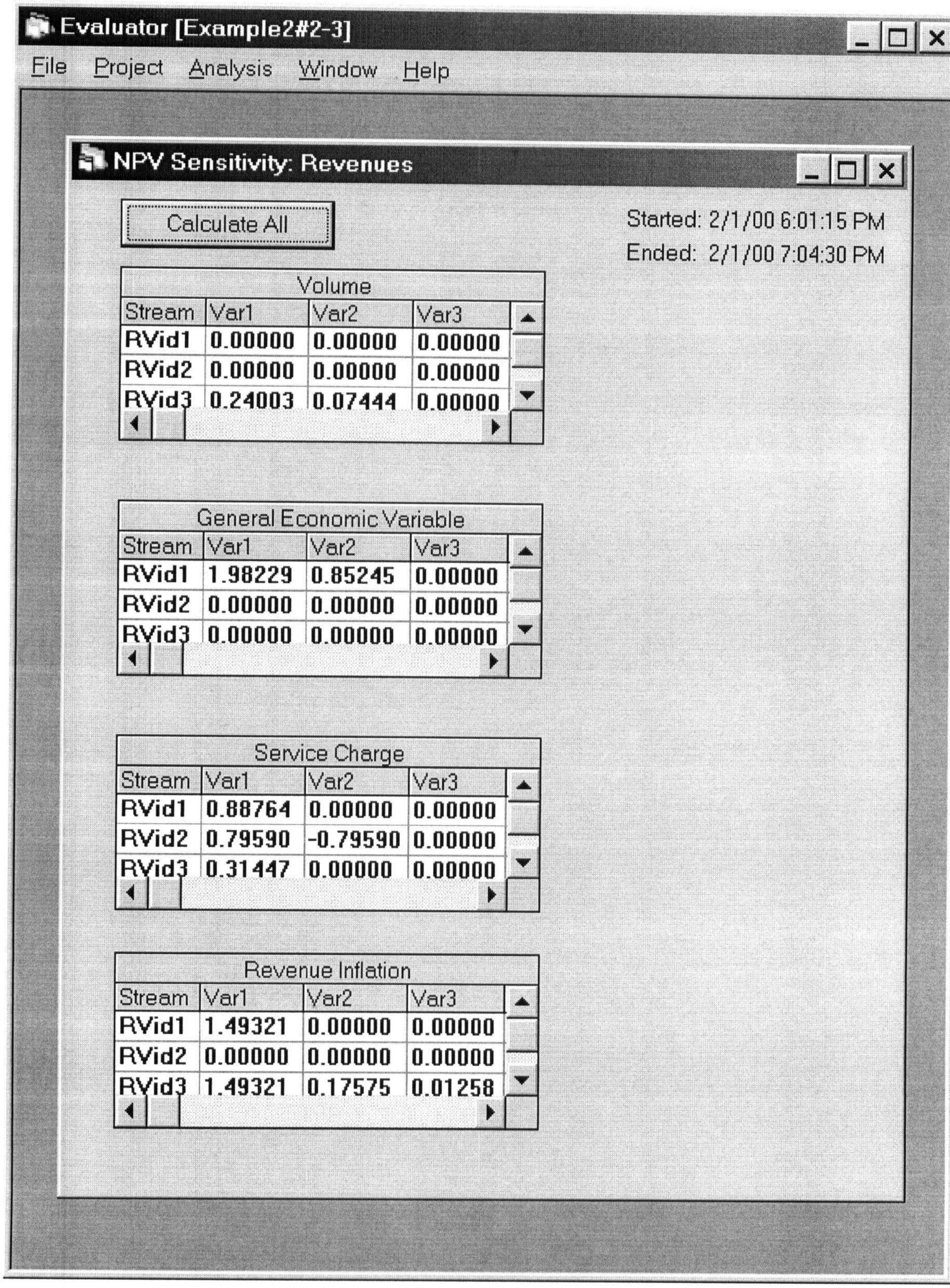


Figure 7.50: NPV sensitivity coefficients for some revenue variables (second revenue scenario)

The "Service Charge" table reflects also the coefficients of variables representing total aggregated revenues. For example, the first government contribution to the project ($29 million), which is represented by the "RVid2" revenue stream has a 1.736 sensitivity coefficient explaining the significant effect of the government's contribution on NPV. The second coefficient "Var2" reflect the duration over which the contribution is distributed; if the duration extends, then the discounted revenues will decrease and the NPV will fall leading finally to the negative sign of the sensitivity coefficient (-1.736).

- "Revenue Inflation" table: Coefficients in this table in Figure 7.49 are for the variables of the shape function representing inflation. The cars revenues stream "RVid1" has zero values since under the first traffic modeling scenario, toll increases at the same value of inflation and therefore no separate inflation rate is defined for the stream (see Figure 7.47c in which tolls increase at 2.35% annually). In Figure 7.50, with the second traffic modeling scenario, an inflation variable of 2.35% is used for the stream and therefore there is a sensitivity coefficient of 1.49, meaning that for every 1% increase in the cars revenue' inflation, there will be a 1.49% increase in NPV.

  The "small trucks" revenue stream has, as explained in the second case study, a *sinusoidal* inflation function and therefore "RVid3" has four sensitivity coefficients, one for each parameter variable of the function (see Table 4.1).

As a summary to the modeling of revenues of a project, the three case studies explained how the generalized economic model and the DSS could formulate different types of revenue functions, from the aggregated revenues in case one throughout to choice/utility models in case three. While not done, the analysis of the third case study could be extended further through a probabilistic analysis using the DSS. What is important to note is the considerable flexibility and modeling power offered by the DSS and the economic model contained there in.

c) Cost Aggregation

Figure 7.31 shows a semi-detailed structure of the highway project example. The bridges work package is detailed further into the several work packages shown in Figure 7.43. In Figure 7.31, the clearing, cut & fill, sub-base, road base, and pavement represent the work packages that describe construction of the road (different form design and road structure work pakcages). The uncertainty of the total cost of road construction can be obtained from the uncertainties of its work packages. Therefore, each of the road construction work packages are assigned to a CE area - in this case CE area 1 - as shown for the pavement work package in Figure 7.51. Table 7.10 shows the cost of the road construction work packages and Table 7.12 shows some of variables in these work packages.

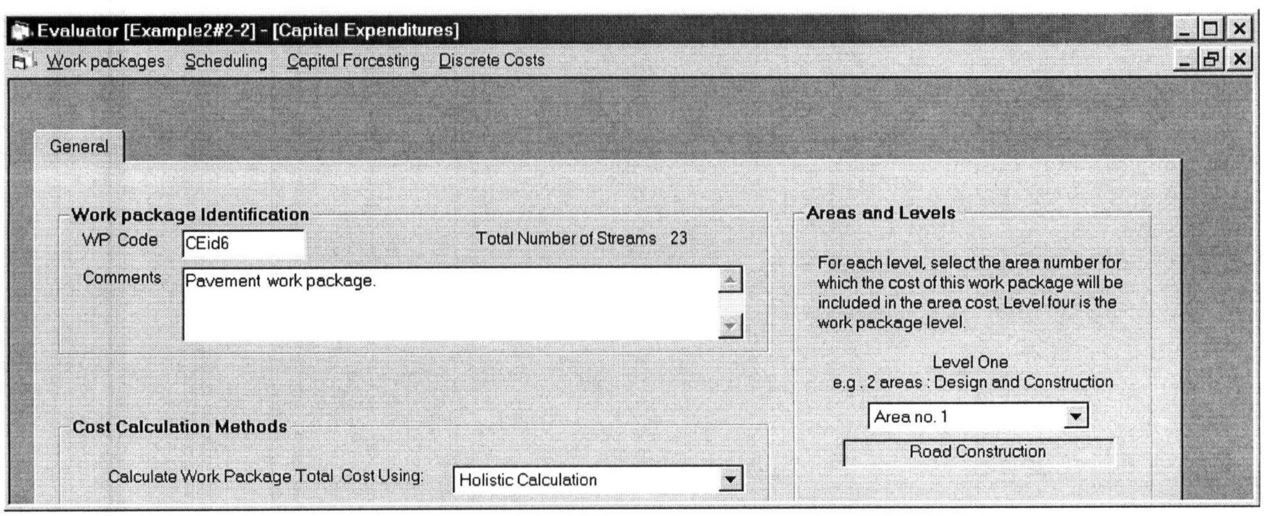


Figure 7.51: Pavement work package assigned to road construction CE area 1

Figure 7.52 shows a positively skewed Pearson Type VI distribution for the road construction cost. The expected current dollar value in the figure is $87.228 million with a probability of 59.47%. With the expected and standard deviation values the coefficient of variation is small and equals to 0.02014. With the small coefficient of variation, road construction has a range of cost between $84.4 million to $96 million. While these figures may build confidence in the original estimate, Figure 7.52 can be used to arrange cost contingency for the road construction estimate.

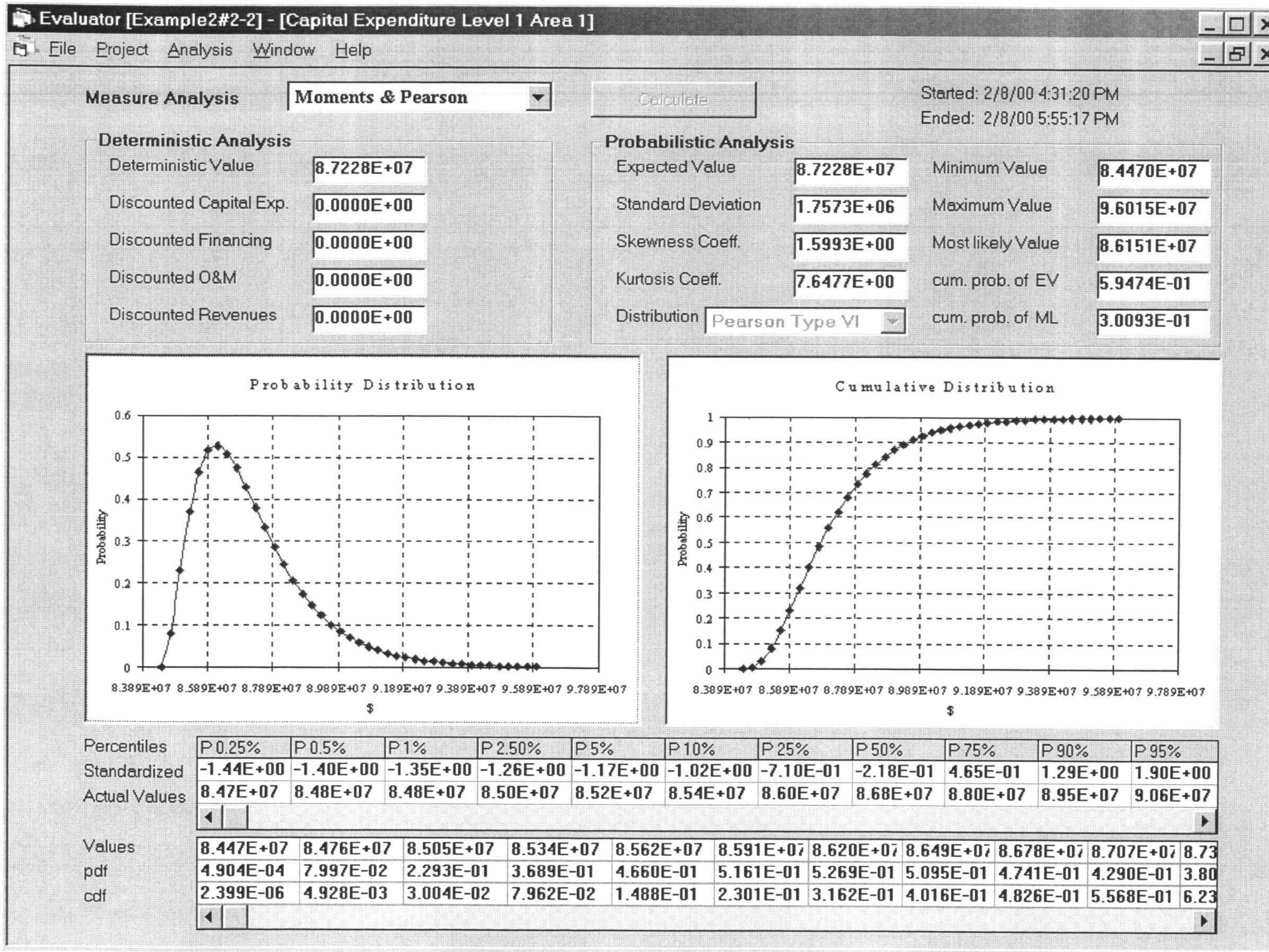


Figure 7.52: Probabilistic characteristics of the total cost of the road construction work packages- CE area 1

7.4 Summary

This chapter presented two examples explaining the functionality of the decision support system and its underlying generalized economic model and risk analysis framework. The second example explained how detailed the generalized economic model is in representing various levels of detail in the economic structure of a project. Table 7.17 explains the structure of the work packages used in representing the three case studies of the second example. The example shows the abilities of the DSS that can be used by both public and private developers in formulating the economic structure of a project under alternative delivery systems, evaluating the project through several periodic and cumulative cash flows and performance measures, modeling the uncertainty of project variables and analyzing the uncertainty of the performance measure.

Table 7.17: Work packages in the three case studies of the highway example project

Case 1 Work Package	Code	Case 2 Work Package	Code	Case 3 Work Package	Code
Design	CEid1	Design	CEid1	Design	CEid1
Road Construction	CEid2	Clearing and Grubbing	CEid2	Clearing and Grubbing	CEid2
		Cut & Fill	CEid3	Cut & Fill	CEid3
		Sub Base Layer	CEid4	Sub Base Layer	CEid4
		Base Layer	CEid5	Base Layer	CEid5
		Pavement	CEid6	Pavement	CEid6
Road Structure	CEid3	Culverts	CEid7	Culverts	CEid7
		Tunnels	CEid8	Tunnels	CEid8
		Interchange	CEid9	Interchange	CEid9
		Bridges (B1,B2,B3,B4)	CEid10	B1: Excavation, Unclassified	CEid12
				B1: Excavation, Structural	CEid13
				B1: Piles	CEid14
				B1: Footing Abutment #1	CEid15
				B1: Concrete Abutment #1	CEid16
				B1: Footing Abutment #2	CEid17
				B1: Concrete Abutment #2	CEid18
				B1: Bridge Girders	CEid19
				B1: Deck Slab	CEid20
				B1: Finishes	CEid21
				Bridge B2	CEid10
				Bridge B3	CEid22
				Bridge B4	CEid23
End work package	CEid4	End work package	CEid11	End work package	CEid11
Total	4		11		23

Chapter 8

Conclusions and Recommendations

8.1 Conclusions

8.1.1 Capital Investment Projects

The first objective of the research was to understand capital investment projects. This objective has been pursued through an extensive literature search and a detailed study of the contractual documents of a set of five capital investment projects. The requirement structure has been introduced to carry out this objective. The main results and conclusions are as follows:

- The requirement structure is a framework through which the characteristics and requirements of a project during its life cycle can be usefully studied and analyzed in a comprehensive manner. The structure can be used to identify arrangements under several public-private partnership modes and assist in formulating other project delivery systems.
- The requirement structure has three dimensions (rights, obligations and liabilities) comprising eight attributes (ownership/possession, revenue, development, operation, finance, liability, tax and risk). Categorizing the requirements through the eight attributes helps in distinguishing the features of each attribute such that their key characteristics and variables can be modeled and treated quantitatively and/or qualitatively in economic models and the uncertainty or risk surrounding them can be quantified and analyzed.
- The clarity with which the terms and conditions in the tender documents of public-private partnership projects need to be emphasized.

8.1.2 Generalized Economic Model

The second thesis objective was to develop a generalized economic model. The model has been developed as a hierarchical network-based time function structure. The main results and conclusions from the development of the generalized economic model are as follows.

- The hierarchical time function structure of the generalized model allows any number of cash flows to be added to the economic structure of a project when modeling its life cycle. While previous models provide for a similar ability, the structure of the generalized economic model provides for each cash flow to be modeled as an entity that can have its own properties and calculation method. Further, the time function structure allows flexibility in the formulation of several periodic and cumulative cash flows, and of several time, cost, economic and financial performance measures from the same model structure. These include, for example, net present value, internal rate of return, benefit-cost ratios, life cycle cost, debt service coverage ratio, loan life coverage ratio, total investment cost, total revenues, total O&M cost, and completion time.
- The model has the ability to transform an estimate into expenditure flow. In the model, an estimate, e.g. cost or revenue, can be assigned directly to a point of time or distributed over time using a specified profile. While these two assignment procedures can be found in previous models, the generalized model has a unique feature. The model assembles a large set of estimating methods and allows an estimate to be computed using one of these methods and simultaneously distributes the estimate over time in a way that reflects how the variables of the calculation method actually change over time.
- The concept of cash flow classifications is a useful contribution for modeling the properties and methods (crude, semi-detailed and detailed) of four domains in capital investment projects. Methods of a domain can be added to a classification, thereby becoming an integral part of the model's structure.

- The *rate* functions allow any variable in the economic model to change over time. The *area* functions, however, are introduced and derived in the thesis to address situations in economic modeling in which total value of a variable is constrained.
- The generalized economic model can be considered to have specialized representation of three attributes of a project requirement structure. The first attribute is the development attribute; properties and methods of the capital expenditure classification include those properties and methods commonly found in the construction industry for time and cost computation. The second attribute is the revenue attribute; the revenue classification has a specialized representation for the properties and methods used in transportation demand/revenue modeling. The third attribute is the finance attribute; the finance classification has detailed properties and methods of five financial instruments commonly used in project financing of capital investment projects. Further to these specialized representations, the operation and maintenance component has a unique ability allowing the O&M costs to be a function of demand as modeled in the revenue component. None of the previous economic models and systems has such a wide range of methods in a single model structure as in the generalized economic model.
- The network-based structure as used in the generalized model, as well as in CASPAR (Thompson and Willmer 1987; Thompson 1993), is an important feature that allows the integration between time and money when modeling a project life cycle.

8.1.3 Risk Analysis Framework

The third objective of the thesis was to develop a risk analysis framework. The framework adopted in this study is an analytical four-moment approach that can obtain the four moments of a function through the moments of the variables used by the function. Some results and conclusions are as follows.

- The virtue of the analytical approach as compared to the traditional simulation approach is in its ability to handle different types of information (percentiles, moments, and distributions) for modeling the uncertainty of variables. Further, if compared to other analytical approaches, the current approach eliminates the need to calculate intermediated moments as in Ranasinghe (1990) and Russell and Ranasinghe (1992).
- The risk analysis framework is useful in modeling the uncertainty of any variable in the hierarchical structure of the generalized economic model, e.g. modeling all the parameter variables of the rate and area functions. This provides for a more comprehensive treatment of uncertainty of variables over that introduced in other models as in CASPAR (Thompson and Willmer 1987; Thompson 1993).
- The four moments derived during the study for a system function (e.g. any of the performance measure of the generalized economic model) contribute to enhance the accuracy of the moment estimates when compared to other standard approaches.
- The Pearson and Schmeiser-Deutsch families of frequency curves are used by the framework to model the uncertainty of a performance measure. However, unlike previous frameworks, the risk analysis framework fits the moments of a measure mathematically to a Pearson distribution by determining which distribution of the family is suitable and then determines its parameters. This improves the accuracy of probability values, which otherwise could have been obtained by interpolation from the Pearson tables.

8.1.4 Decision Support System

The last objective of the thesis was to develop a decision support system. This objective has been pursed through implementing the generalized economic model and the risk analysis framework into Evaluator, the system developed for the study. Some results are as follows.

- The system advances the state-of-the-art for the appraisal of capital investment projects.

- Through a flexible user interface the system has the ability to formulate any project alternative or scenario utilizing the capabilities of the generalized economic model and to analyze project risks utilizing the capabilities of the risk analysis framework.
- Through its database, the system has the ability to maintain, manipulate and reproduce data and results for any project alternative or scenario.
- The system was validated through a detailed example through which the deterministic and probabilistic results for several performance measures were compared to those obtained from an explicit formulation of the measures. Identical results were obtained. The derived four moments of a system function received another validation by comparing the results of two examples to exact solutions of the problems investigated.
- Governments and private-sector developers should find the system to be an effective economic and risk analysis tool to assist in making decisions regarding the procurement, investment and financing of investment projects. Scenarios of a project under different delivery systems can be formulated, evaluated and analyzed using the system. At the appraisal stage of a project, the system facilitates the treatment of: construction issues such as initial costs, cost overrun, escalation and delay; revenue issues such as tolling rate and mechanism, demand parameters, subsidies and total revenues; operation and maintenance issues such as length of concession and cost forecasts; financing issues such as type of financing, interest rates, and repayment methods; and overall project issues such as equity amount, performance measures and project risks.

As explained in Chapter 3, previous models and systems were generally highly aggregated, unable to fully address or model the uncertainty of all project variables, and unable to realistically model the detailed characteristics of capital investment projects. The current research concluded with the development of: (1) a generalized economic model that has the ability to formulate any required economic structure of a project, the generality to model a project at several levels of detail, and the capability to distinguish and recognize various properties and methods of several industries and business sectors, (2) a risk analysis framework that has the ability, through several types of methods, to model the uncertainty of any project variable, and determine the probabilistic characteristics of several performance measures and (3) a decision support system that represent a practical tool for the appraisal of investment projects

8.2 Recommendations For Future Work

8.2.1 Capital Investment Projects

The requirement structure with its eight attributes provides a useful framework to study the characteristics of capital investment projects. It is recommended that the framework be used to study the characteristics of power, water supply, and waste water treatment projects in order to enrich the knowledge base about these projects which would provide valuable information for the economic modeling and decision making of such projects.

8.2.2 Economic Models

Some recommendations for future work in economic modeling are as follows.

- The four classifications of the generalized economic model included several properties and methods of the four domains they represent. It is recommended to introduce a specialized representation for the power industry water supply and waste water treatment industry projects in the revenue and operation and maintenance classifications. Modeling demand for transportation projects through the urban transportation modeling system (UTMS) should be included in future classifications.
- The tax and liabilities attributes have been represented in a general way in the generalized economic model. It is recommended that a thorough treatment should be given to these attributes, e.g. through building new classifications for such attributes.

- The network-based structure of the generalized economic model implements the critical path method using finish-to-start relationships with positive and negative lag times between constructs. While this should be sufficient at the appraisal stage, it is recommended that other types of relationships be implemented in the model structure.

8.2.3 Risk Analysis Framework

Some recommendations for developing risk analysis frameworks are as follows.

- The four moments derived in the study assumed no correlation between the variables of a system function. While the derivation could have been extended to include the correlation terms, the inclusion of correlation in the risk analysis framework would have required the elicitation of a prohibitive number of correlation coefficients. While the variable transformation technique of Russell and Ranasinghe (1992) provided a framework to deal with correlation, a correlation matrix for all the variables would have to be elicited. It is recommended to study a treatment of correlation in complex/detailed systems (Kendall et al. 1994) with consideration being given to a bounding approach. This will significantly increase the accuracy of the results if correlated variables are encountered.
- While the framework is equipped with almost all of the methods used in modeling the uncertainty of a variable, it does not provide assistance in eliciting the probabilistic estimates of the variables. Estimates (percentiles, moments, distributions) have to be pre-elicited, then given to the system or the risk analysis framework. It is recommended that elicitation frameworks be developed for probabilistic variables in future DSSs.
- The risk analysis framework provided for modeling the uncertainty of a performance measure through either the Pearson or the Schmeiser-Deutsch systems of frequency curves. However, the irregular shape of the distributions in the Schmeiser-Deutsch distribution family necessitates further studies for economic risk analysis. This family has the ability to determine the four moments of a variable if its modal value is known along

with any two other percentile values (Schmeiser and Deutsch 1977).

8.2.4 Decision Support Systems

Some recommendations for developing decision support system are as follows.

- It is recommended that the decision support system be taken into the selection of a project alternative or scenario amongst several alternatives (Sage 1991). The current support system facilitates building and processing individual scenarios; however, the decision maker would is then responsible for comparing these scenarios according to stated goals or objectives and making a selection from them.
- The implementation of the generalized economic model and risk analysis framework in a prototype decision support system utilized software such as Mathcad, Visual Basic, and Excel and the Object Linking and Embedding OLE protocol. It is recommended that future development of systems pursue the use of single powerful language such as Visual C++, this would reduce the overhead that comes with the use of multiple of software, e.g. use of computer memory and processing control. In the current system, once processing of a performance measure starts it is difficult to stop it until it ends.
- While the classifications introduced for the generalized economic model provides a theoretical avenue for developing *new* properties and methods for addition to the model structure, the implementation of such an avenue in the current system, Evaluator, is not possible. Mathcad, for example, requires the direct specification of the mathematical expression of a function before starting to use it. It is not possible to formulate the function outside of Mathcad and then send it to be part of a Mathcad processing sheet. Using a powerful computer language would provide an avenue to code and implement the addition of new properties and methods to the data structure of the system in real time such that they could be used in the modeling of new projects.

Bibliography

"Amended and Restated Development Franchise Agreement: State Route 91 Median Improvements, Orange and Riverside Counties, California." (1993) Department of Transportation, California.

"An Act Respecting the Northumberland Strait Crossing - Chapter 43." (1993a). Government of Canada, Ottawa.

"An Act to Provide for the Financing, Construction and Operation of the Western Alignment of Highway 104 - Bill No. 10." (1995). Nova Scotia, Canada.

"An Act: Toll Road Enabling Legislation: Chapter No. 211." (1993b). State of Minnesota.

"Assembly Bill No. 680 - Chapter 107 - State Transportation Facilities." (1989). California, July 10, U.S.A.

"Channel Tunnel Act 1987- Chapter 53." (1987). Department of Transport, U.K.

"Concession Agreement - The Channel Fixed Link." (1986). HMSO Cmnd 9769, Her Majesty Stationery Office, London, U.K.

"Development Franchise Agreement for a Privatized Transportation Project by and between California Transportation Venture Inc., and the State of California Department of Transportation (SR 125)." (1991a), Department of Transportation, California.

"Development Franchise Agreement for a Privatized Transportation Project by and between California Toll Road Company, and the State of California Department of Transportation (Mid State Tollway)", (1991b), Department of Transportation, California.

"Franchise Agreement between the Texas High-Speed Rail Authority and the Texas High Speed Rail Corporation" (1991), State of Texas.

"Guidelines for Conceptual Project Proposals for Toll Revenue Transportation Projects" (1990). California Department of Transportation, Office of Privatization, U.S.A.

"Highway 104 Western Alignment Request for Proposals", (1995), Department of Transportation and Communication, Nova Scotia, Canada.

"Invitation to Promoters for the Development, Financing, Construction, and Operation of a Channel Fixed Link Between France and the United Kingdom." (1985). Department of

Transport, U.K.
"New Roads and Street Works Act 1991." (1991). Department of Transport, U.K.

"New Roads By New Means - Bringing in Private Finance- a Consultation Paper." (1989). Presented to the Parliament by the Secretary of State for Transport by Command of Her Majesty, U.K.

"Northumberland Strait Crossing Project – Call for Expression of Interest." (1987). Public Works Canada.

"Northumberland Strait Crossing Project - Proposal Call." (1988). Public Works Canada.

"Northumberland Strait Crossing Project- Final Registration." (1993). Strait Crossing Inc., prepared for NB Department of the Environment, Fredericton, New Brunswick.

"Paying for Better Motorways: Issues for Discussion - Green Paper." (1993). Presented to Parliament by the Secretaries of State of Transport, for Scotland and for Wales, Cmnd 2200.

"Public-Private Initiatives in Transportation - New Partners - Request for Proposals." (1994). Washington State Department of Transportation, U.S.A.

"Public-Private Transportation Act of 1995 - Implementation Guidelines." (1995). Virginia Department of Transportation, Virginia, U.S.A.

"Qualifying Transportation Facilities Act of 1994." (1994). Virginia Department of Transportation, Virginia, U.S.A.

"Request for Proposals for TRANSMART: Minnesota's Toll Facilities Public-Private Partnership Initiatives Program." (1995). Minnesota Department of Transportation.

"Regulation D - Under the Securities Act of 1933 - Rules Governing the Limited Offer and Sale of Securities Without Registration." (1983). Bowne & Co., Inc., USA.

"Severn Bridges Act 1992- Chapter 3." (1992). Department of Transport, U.K.

"Second Severn Crossing - Information for Prospective Tenderers." (1988). Department of Transport, London, U.K.

"Second Severn Crossing - Tender Invitation: Volume I: Administration, Contractual and Financial Aspects." (1989). Department of Transportation, U.K.

"Texas TGV - Franchise Application to Construct, Operate, Maintain and Finance a High-Speed

Rail Facilty." (1991). Texas High-Speed Rail Authority, Texas.

"TH 212 development agreement: a public-private partnership between the Minnesota Department of Transportation, TH 212 Community Highway Association, and Interwest/DLR Infrastructure Group, LLC." (1996). Minnesota Dept. of Transportation.

"The Channel Fixed Link – White Paper." (1986). HMSO Cmnd 9735, U.K.

"Virginia Highway Corporation Act of 1988." (1988). Virginia Department of Transportation, Virginia, U.S.A.

@Risk - Advanced Risk Analysis for Spreadsheets. (1997). Palisade Corporation, NY.

AbouRizk, S., and Halpin, D. (1992). "Statistical Properties of Construction Duration Data." *J. Construction Engineering and Management,* v. 118, v. 3, pp. 525-544.

AbouRizk, S., Halpin, D., and Wilson, J.R. (1993). "Fitting Beta Distribution Based on Sample Data." *J. Construction Engineering and Management,* v. 120, v. 2, pp. 288-305.

Advisory Committee on Public-Private Partnerships (ACPPP). (1998). "The Province of British Columbia's Best Practices Guide to Public-Private Partnerships." *Rep.,* Ministry of Employment and Investment, British Columbia.

Ashley, D., Bauman, R., Carroll, J., Diekmann, J., and Finlayson, F. (1998). "Evaluating Viability of Privatized Transportation Projects." *J. of Infrastructure Systems,* v. 4, n. 3, pp. 102-110.

Al-Bahar, J. F., and Crandall, K. (1990). "Systematic Risk Management Approach for Construction Projects." *J. of Construction Engineering and Management*, v. 116, n. 3, pp. 533-546.

Almond, B., and Remer, D. (1979). "Models for Present Worth Analysis of Selected Industrial Cash Flow Patterns." Engineering and Process Economics, 4, 455-466.

Andrews, S. (1987). "Can Eurotunnel Dig Itself out of a Hole", *Institutional Investor*, v. 21, n. 11, pp 264-272.

Ang, A. H., and Tang, W. H. (1975). *Probability Concepts in Engineering and Design Voulme I - Basic Principles.* John Wiley, New York.

Ang, A. H., and Tang, W. H. (1990). *Probability Concepts in Engineering and Design Voulme II - Decision, Risk, and Reliability.* Copyright the authors, Arikan, Y. (1994). "Build Operate Transfer Model for New Power Plants for Turkey." *IEEE,* pp. 1089-1092.

Association of Project Management (APM). (1997). "PRAM - Project Risk Analysis and Management Guide." The APM Group Ltd., UK.

Augenblick, M., and Custer, B. S. (1990). "The Build, Operate, and Transfer ("BOT") Approach to Infrastructure Projects in Developing Countries.", *Working Papers, WPS 498*, Policy, Research, and External Affairs, The World Bank, Washington, DC.

Aybers, M. N., and Sahin, B. (1990). "Nuclear Power Costs in the Build, Operate, Transfer Approach." Kerntechnik, v. 55, n. 1, pp. 56-59.

Bacon, R., Besant-Jones, J., and Heidarian, J. (1996)."Estimating Construction Costs and Schedules", *Rep. World Bank Technical Paper No. 325, Energy Series,* The World Bank, Washington, D.C.

Bailey, J. B., Spies, P. F., and Weitzman, M. K. (1977). "Market Study + Financial Analysis = Feasibility Report." *The Appraisal Journal,* v. 45, pp. 550-577.

Baldry, D. (1998). "The Evaluation of Risk Management in Public Sector Capital Projects." *Int. J. of Project Management*, v. 16, n. 11, pp. 35-41.

Ball, M. (1995). "Easy Pickings in US Private Market." *Corporate Finance,* v. 122, pp. 32-37.

Bee, R. P. (1981). "Syndication of International Loans." *Bank Credit,* Herbert B. Prochnov, ed., Harper & Row Publishers, New York.

Beck, J. M. (1997)., "Highway 407ETR- Express Toll Route, Highway 104 Western Alignment : Innovative Highway Development Through Public Private Partnerships", *Annual Conference of the Canadian Society of Civil Engineering*, pp. 89-96.

Bederman, N., and Trebilcock, M. (1997). "Unsolicited Bids for Government Functions." *Rep.,* The Canadian Council for Public-Private Partnerships, and Center for the Study of State & Market at The University of Toronto.

Behrens, W., and Hawranek, P. M. (1991). *Manual for the Preparation of Industrial Feasibility Studies,* United Nations Industrial Development Organization (UNIDO), Vienna.

Beidleman, C. R., Fletcher, D., and Veshosky, D. (1990). "On Allocating Risk: The Essence of Project Finance." *Sloan Management Review*, Spring, pp. 47-55.

Bennette, S. (1997). "Traffic Returns as Fire Probe Continuous." *International Railway Journal*, v. 37, n 3, pp. 21-30.

Berny, J., and Howes, R. (1982). "Project Management Control Using Real-time Budgeting and Forecasting Models." *Construction Management and Economics*, 2, 19-40.

Beidleman, C. R., Fletcher, D., and Vesbosky, D. (1990). "On Allocating Risk: the Essence of Project Finance." *Sloan Management Review,* Spring 1990.

Blackburn, S. J. and Dowall, D. E. (1991). "The Tools for Financing Infrastructure." Financing Public Infrastructure in Mexico City : Problems and Solutions, Lincoln Institute of Land Policy, Cambridge, Massachusetts.

Blaiklock, M. (1992). "Financing Infrastructure Projects as Concessions." *Financing Transport Infrastructure, Proc. of Seminar L held at the PTRC Transport, Highways and Planning,* University of Manchester, Institute of Science and Technology, England, 14-18 September, pp. 211-223.

Bostwick, L. (1996). "The SEC Response to Internationalization and Institutionalization: Rule 144A Merit Regulation and Investors." *Law and Policy in International Business,* v. 27, pp. 423-445.

Brealey, R., Myers, S., Sick, G., and Giammarino, R. (1992). *Principles of Corporate Finance,* McGraw-Hill Ryerson Ltd., Toronto.

Bury, K. (1975). *Statistical Models in Applied Science*, John Wiley & Sons, New York.

Bury, K. (1999). *Statistical Distributions in Engineering*, Cambridge University Press, Cambridge, UK.

Canadian Council for Public-Private Partnerships (CCPPP) (1998). "Public-Private Partnerships - Canadian Project and Activity Inventory 1998." *Rep.,* The Canadian Council for Pubic-Private Partnerships.

Canadian Council for Public-Private Partnerships (CCPPP) (1996). "Best Practice Guidelines - Initiating Contracts and Contracting with the Private Sector" *Rep.,* The Canadian Council for Pubic-Private Partnerships.

Carey, M., Prowse, S., Rea, J., and Udell, G. (1993). "The Economics of Private Placements: A New Look." *Financial Markets, Institutions and Instruments,* v. 2, n.3, pp. 1-67, Blackwell Publishers, Cambridge, MA, USA.

Chapman, C. B., and Cooper D. F. (1983a). "Risk Engineering: Basic Controlled Interval and Memory Models." *J. of Operational Research Society*, v. 34, n. 1, pp. 51-60.

Chapman, C. B., and Cooper D. F. (1983b). "Risk Analysis: Testing some Prejudices." *European Journal of Operational Research*, v. 14, pp. 238-247.

Chapman, C. B., and Ward, S. (1999). *Project Risk Management- Processes, Techniques and Insights.* John Wiley & Sons.

Chapra, S. C., and Canale, R. P. (1988) *Numerical Methods for Engineers.* McGraw Hill.

Chau, K. W. (1995a). "Monte Carlo Simulation of Construction Costs Using Subjective Data." *Construction Management and Economics,* v. 13, pp. 369-383.

Chau, K. W. (1995b). "The Validity of the Triangular Distribution Assumption in Monte Carlo Simulation of Construction Costs: Empirical Evidence from Hong Kong." *Construction Management and Economics,* v. 13, pp. 15-21.

Chee, T. S., and Yeo, K T. (1995). "Risk Analysis of a Build-Operate-Transfer (B.O.T) Power Plant Project." *95 Engineering Management Conference, IEEE*, pp. 275-280.

Clark, P., and Chapman, C. B. (1987). "The Development of Computer Software for Risk Analysis: A Decision Support System Development Case Study." *European J. Operational Research*, 29, 252-261.

Clough, R. H., and Sears, G. A. (1994). *Construction Contracting, Sixth Edition.* John Wiley & Sons. Inc., New Yourk.

Clough, R. H., and Sears, G. A. (1991). *Construction Project Management, Third Edition.* John Wiley & Sons. Inc., New Yourk.

Cooper, D. F., and Chapman, C. B. (1987). *Risk Analysis for Large Projects- Models, Methods and Cases.* John Wiley & Sons.

Cooper, D. F., MacDonald, D. H., and Chapman, C. B. (1985). "Risk Analysis of a Construction Cost Estimate." *Project Management,* v. 3, n. 3, pp. 141-149.

Coopers & Lybrand (1987). *A Guide to Financial Instruments.* Euromoney PLC, UK.

Cottrell, W. D. (1999). "Simplified Program Evaluation and Review Technique." *J. of Construction Engineering and Management,* v. 125, n. 1, pp. 16-22.

Cunningham, J. (1981). "Term Loans." *Bank Credit,* Herbert B. Prochnov, ed., Harper & Row Publishers, New York.

Davis, H. A. (1996). *Project Finance - Practical Case Studies.* Euromoney PLC, London.

De La Mare, R. F. (1979). "Modeling Capital Expenditure." Eng. and Process Economics, 4, 467-477.

Dedeitch, B. P., Machemehl, R. B., Euritt, M. A., Harrison, R., and Walton, C. M. (1993). "Reliability of Toll Road Revenue Forecasts for Selected Toll Roads in the United States." *Rep.* Center for Transportation Research at the University of Texas as Austin and Texas Department of Transportation.

Dias, A. J. (1994). "A Managerial and Financial Study on the Involvement of Private-Sector Companies in the Development, Construction, Operation, and Ownership of Infrastructure Projects," Ph.D. thesis, The University of Michigan, Published by UMI, A Bell & Howell Company, Ann Arbor, MI, USA.

Dias, A. J., and Ioannou, P. (1996). "Company and Project Evaluation Model for Privately Promoted Infrastructure Projects." *J. of Construction Engineering and Management,* v. 122, n. 1, pp. 71-82.

Diekmann, J. E. (1983). "Probabilistic Estimating: Mathematics and Applications." *J. of Construction Engineering and Management,* v. 109, n. 3, pp 297-308.

Dir Kiureghian, A., and Liu, P. (1985). "Structural Reliability under Incomplete Probability Information." *J. of Engineering Mechanices,* v. 112, n. 1, pp. 85-104.

Elderton, W., and Johnson, N. (1969). *Systems of Frequency Curves,* Cambridge University Press, NY.

ENR (1987). *"UK Ready to Bore to France."* Sept. 10.

ENR (1988). *"Hard Chalk Slows Channel Bore: Owners Conquer Tunnel Funding, but contractors Struggle to Overcome Delays."* Nov. 3, pp 32-40.

ENR (1992). *"Money is out there for Projects that Make Lots of Sense : Lenders Want Long-term Revenues and Controllable Risks."* May 4.

ENR (1995). "Eurotunnel's Financial Peril." v 234, n 16, pp 12.

Eschenbach, T. G. (1992). "Quick Sensitivity Analysis for Small Projects and Feasibility Studies." *AACE Transactions,* pp. L.6.1-L.6.8.

Eschenbach, T. G. (1996). "Risk Management Through Sensitivity Analysis." *AACE Transactions,* pp. D&RM.4.1-D&RM.4.6.

Euromoney (1994). "Thailand Syndicated Loans: Some Pressures on Margins." March, p 29.

Euromoney (1997a). *Power Project Documentation,* Asia Law & Practice, Euromoney Publications, UK.

Euromoney (1997b). *The Life and Death of an Infrastructure Project,* Asia Law & Practice, Euromoney Publications, UK.

Euroweek (1994). "Lehman Draws Praise for Price, Structure of Cuernavaca Financing". July, Euroweek, London, UK.

Evans, M., Hastings, N., and Peacock, B. (1993). *Statistical Distributions*, John Wiley & Sons, Inc., NY.

Excel 97 reference manual. (1996). Microsoft Corporation, USA.

Fanning, S., and Winslow, J. (1988). "Guidelines for Defining the Scope of Market Analysis in Appraisal Assignments." *The Appraisal Journal,* v. 56, pp. 466-476.

Federal Highway Administration (FHWA) (1996). "FHWA Study Tour of Northumberland Strait Crossing Project NSCP." *Rep.,* U.S. Department of Transportation.

Federal Highway Administration (FHWA) (1998). "The Selective Use of Shadow Tolls in the United States." *Rep.,* Prepared by Urs Greiner, Inc., and Public Financial Management, Inc. for the U.S. Department of Transportation; online version at: *http://www.fhwa.dot.gov/innovativefinance/shadtoll.htm.*

Feldman, R. D., Mudge, R., and Rubin, K. I. (1988). *Financing Infrastructure - Tools for the Future.* Executive Enterprise Publications Co. Inc.

Fisher, F. G. (1979). *The Eurodollar Bond Market.* Euromoney Publications, UK.

Flanagan, R., Kendell, A., Norman, G., and Robinson, G. (1987). "Life Cycle Costing and Risk Management." *Construction Management and Economics,* v. 5, pp. S53-S71.

Flynn, N. (1997). *Public Sector Management, Third Edition.* Prentice Hall/ Harvester Wheatsheaf, Great Britain.

Fowkes, T., and Wardman, M. (1988). "The Design of Stated Preference Travel Choice Experiements." *J. Transport Economics and Policy*, 22(1), 27-44.

Frohlich, E. A., Hawranek, P. M., Lettmayr, C. F., and Pichler, J. H. (1994). *Manual for Small Industrial Businesses - Project Design and Appraisal,* United Nations Industrial Development Organization (UNIDO), Vienna.

Gelbard, S. (1996). "Institutional Private Placements and Other Financing Alternatives." *Private placements*, R. C. Nash, R. R. Plumridge and R. Stevenson, eds., Practicing Law Institute, NY., 205-248.

Goodwin, P. B., and Williams, H. C. (1985). "Short Notes and Research Communications- Public Transport Demand Models and Elasticity Measures: An Overview of Recent British Experience." *Transportation Res. B.,* v. 19B, n. 3, pp. 253-259.

Hahn, G., and Shapiro, S. (1967). *Statistical Models in Engineering*, John Wiley & Sons, Inc., New York.

Haly, G. (1992). "Private Finance for Transportation and Infrastructure Projects: a View." *Int. J. of Project Management,* v. 10, n. 2, pp. 63-68.

Handley, P. (1994). "Thailand : Bitterly, Kumagai Gumi bows out of the Bangkok Expressway." *Institutional Investor*, v. 28, n. 4.

Hendrickson, C., and Au, T. (1989). *Project Management for Construction: Fundamental Concepts for Owners, Engineers, Architects, and Builders (Civil Engineering, Engineering Mechanics).* Prentice Hall Inc.

Hertz, D. B., and Thomas, H. (1983). *Risk Analysis and its Application*, John Wiley & Sons, Chichester.

Hillier, F. S. (1963). "The Derivation of Probabilistic Information for the Evaluation of of Risky Investments." *Management Science,* v. 9, n. 1, pp. 69-80.

Holiday, I., Marcou, G., and Vickerman, R. (1991). *The Channel Tunnel - Public Policy, Regional Development and European Integration.* Belhaven press, London.

Howcroft, B. and Solomon, C. (1985). *Syndicated Lending by Banks*. Bangor Occasional Papers in Economics, n. 32, University of Wales Press.

Hudson, K. W. (1978). "DHSS Expenditure Forecasting Method." *Chartered Surveyor: Building and Quantity Surveying Quarterly*, 5, 42-45.

Humphreys, K. (1991) *Jelen's Cost and Optimization Engineering.* McGraw Hill, Inc.

Huot, J. (1995). "The Eurotunnel Financial Black Hole." *AACE Transactions*, pp. D&RM.5.1-D&RM.E.5.

Hurst, P. (1996). "Identifying and Controlling Risks." *Projects Procured by Privately Financed Concession Contracts, v2.* A. Merna and N. Smith, eds., Asia Law & Practice, Hong Kong, pp. 27-43.

IBI (1995). "Public-Private Partnerships in Municipal Infrastructure." *Rep.,* prepared by IBI Group for Canada Mortgage and Housing Corporation.

International Monetary Fund. (1997). *"International Financial Statistics", Rep.,* IMF, several issues, 1987, 1992, 1995,1996,1997.

Institution of Civil Engineers (ICE), Faculty of Actuaries (FA), and Institute of Actuaries (IA). (ICE et al.) (1998). "RAMP - Risk Analysis and Management of for Projects." Thomas Telford, London, UK.

Johnson, N., Kotz, S., and Balakrishnan, N. (1994). *Continuous Univariate Distributions - Volume 1*, John Wiley & Sons, Inc., New York.

Johnson, N. L., Nixon, E., and Amos, D. E. (1963). "Table of Percentage Points of Pearson Curves, for Given $\sqrt{\beta_1}$ and β_2 Expressed in Standard Measrure." *Biometrica,* v. 50, n. 3, pp. 459-498.

Jovanovic, P. (1999). "Application of Sensitivity Analysis in Investment Project Evaluation Under Uncertainty and Risk." *International J. of Project Management,* v. 17, n. 4, pp. 217-222.

Tversky, A., and Kahneman, D. (1974). "Judgment Under Uncertainty: Heuristics and Biases. " *Science,* v. 185, pp. 1124-1131.

Kay, J. (1993). "Efficiency and Private Capital in the Provision of Infrastructure." *Infrastructure Policies for the 1990s,* Organization for Economic Co-operation and Development, OECD.

Keefer, D., and Bodily, S. (1983). "Three-Point Approximations for Continuous Random Variables." *Management Science*, 29(5), 595-609.

Keefer, D., and Verdini, W. (1993). "Better Estimation of PERT Activity Time Parameters." *Management Science*, v. 39, n. 9, pp. 1086-1091.

Kendall, K. E., and Kenall, J. E. (1999). *Systems Analysis and Design.* 4th ed., Prentice Hall, New Jersey.

Kendall, M. G., Stuart, A., Ord, J. K., and O'Hagan, A. (1994). *The Advance Theory of Statistics, Vol. 1,* Halsted Press, New York.

Kenley, R. and Wilson, O. D. (1986). "A Construction Project Cash Flow Model- an Idiographic Approach." *Construction Management and Economics*, 4, 213-232.

Kenley, R., and Wilson, O. D. (1989). "A Construction Project Net Cash Flow Model." *Construction Management and Economics*, 7, 3-18.

Keong, C. H., Tiong, R. K., and Bring, J. A. (1997). "Conditions for Successful Privately Initiated Infrastructure Projects." *Proc. of the Institution of Civil Engineers - Civil Engineering*, v. 120, n. 2, pp. 59-65.

Kottas, J. and Lau, H. (1980a). "A Review of the Statistical Foundation of a Class of Probabilistic Planning Models." *Computer & Operation Research*, v.7, pp. 277-284.

Kottas, J. and Lau, H. (1980b). "The Use of Versatile Distribution Families in Some Stochastic Inventory Calculations." *J. of Operational Research Society*, v.31, pp. 393-403.

Kottas, J. and Lau, H. (1982). "A Four-Moments Alternative to Simulation for a Class of Stochastic Management Models." *Management Science*, v.28, n.7, pp.749-758.

Kreydieh, A. (1995). "Risk Management in BOT Project Financing", M.S. Thesis in Civil Engineering, MIT.

Kroes, E., and Sheldon, R. (1988). "Stated Preference Methods- an Introduction." *J. Transport Economics and Policy*, 22(1), 11-25.

Lam, W. H., and Tam, M. L. (1998). "Risk Analysis of Traffic and Revenue Forecasts for Road Investment Projects." *J. of Infrastructure Systems,* v. 4, n. 1, pp. 19-27.

Lang, H. J., and Merino, D. N. *The Selection Process of Capital Projects.* John Wiley & sons. New York.

Lau, H., Lu, A. H., and Ho, C. (1998). "Improved Moment-Estimation Formulas Using More Than Three Subjective Fractiles." *Management Science,* v. 44, n. 3, pp. 346-351.

Lawson, G. (1994). "Will the Private Sector Fund Road." *Transport Financing, Proc. of Seminar L held at the PTRC European Transport Forum*, University of Warwick, England.

Lefley, F. (1997). "Approaches to Risk and Uncertainty in the Appraisal of New Technology Capital Projects." *International J. of Production Economics,* v. 53, pp. 21-33.

Levy, S. M. (1996). *Build, Operate, Transfer: Paving the Way for Tomorrow's Infrastructure.* John Wiley & Sons.

Logan, F. D. (1980). "*Term Loan Agreements." International Financial Law,* Robert S. Rendell, ed., Euromoney Publications, UK.

Lopes, M. D., and Flavell, R. (1998). "Project Appraisal - a Framework to Assess Non-financial Aspects of Projects during the Project Life Cycle." *Int. J. of Project Management*, v. 16, n. 4, pp. 223-233.

Lund, H., Sibert, R., and Chamberlain, P. eds., (1984). *Private Placements.* Euromoney PLC, London, U.K.

McCarthy, S. C., and Tiong, R. L. (1991). "Financial and Contractual Aspects of Build-Operate-Transfer Projects." *Int. J. of Project Management,* v. 9, n. 4, pp. 222-227.

Mansuy, J. (1991). "Work Breakdown Structure: A Simple Tool for Complex Jobs." *Cost Engineering*, 33(12), 15-18.

Mathcad Professional user's guide; release 8. (1998). MathSoft, Inc., Cambridge, MA.

McDonalds, R. (1982). *International Syndicated Lending*, Euromoney PLC, London.

Macquarie Corporate Finance Limited (MCFL) (1996). *Project Finance: The Guide to Financing Transport Projects,* Euromoney Publications.

Mak, S., and Raftery, J. (1992). "Risk Attitude and Systematic Bias in Estimating and Forecasitn." *Construction Management and Economics,* v. 10, pp. 303-320.

Mckechnie, G. (1990). "Project Finance (Limited Recourse Finance)", *International Finance and Investment: Multinational Corporate Banking,* Brian Terry, ed., the Chartered Institute of Bankers, London.

McNeil, S., and Hendrickson, C. (1982). "A Statistical Model of Pavement Maintenance Expenditure." *Transportation Research Record*, 846, 71-76.

Merna, T. (1996a). "Concession Contracts." *Projects Procured by Privately Financed Concession Contracts, v1.* A. Merna and N. Smith, eds., Asia Law & Practice, Hong Kong, pp. 1-21.

Merna, T. (1996b). "Preparation and Evaluation of Concession Contract Packages." *Projects Procured by Privately Financed Concession Contracts, v1.* A. Merna and N. Smith, eds., Asia Law & Practice, Hong Kong, pp. 23-40.

Merna, T., and Adams, C. (1996). "Appraising Risks in Concession Projects." *Projects Procured by Privately Financed Concession Contracts, v1.* A. Merna and N. Smith, eds., Asia Law & Practice, Hong Kong, pp. 89-102.

Merna, T., and Njiru, C. (1998). *Financing and Managing Infrastructure Projects.* Asia Law & Practice, Hong Kong.

Merna, A., and Smith, N. (1996a). *Guide to the Preparation and evaluation of Build Own Operate Transfer Project Tenders.* Asia Law & Practice, Hong Kong.

Merna, A., and Smith, N. (eds.)(1996b). *Projects Procured by Privately Financed Concession Contracts.* Asia Law & Practice, Hong Kong.

Merrow, E., Shangraw, R., Kleinberg, S., Unterkofler L., Madaleno, R., Ziomkoski, E., and Shcroeder, B. (1991)."Understanding the Costs and Schedules of World Bank Supported Hydroelectric Projects", *Rep. World Bank Energy Series Paper No. 31,* The World Bank, Washington, D.C.

Meyer, E. R., and Cressman, K. R. (1984). "The Cost Engineer's Niche in the Overall Project." *AACE Transactions*, N.4.1-N.4.6.

Meyer, M., and Miller, E. (1984). *Urban Transportation Planning A Decision – Oriented Approach*, McGraw-Hill Book Company, NY.

Miskawi, Z. (1989). “An S-curve Equation for Project Control.” *Construction Management and Economics*, 7, 115-1224.

Moles, P., and Williams, G. (1995). "Privately Funded Infrastructure in the UK: Participants' Risk in the Skye Bridge Project." *Transport Policy*, v. 2, n. 2, pp. 129-134.

Moore, J. (1994). "Private Funding for Roads in the United Kingdom." *New Ways of Managing Infrastructure Provision*, Organization for Economic Co-operation and Development, OECD, occasional paper n. 6, Paris.

Mulholland, B., and Christian, J. (1999). "Risk Assessment in Construction Schedules." *J. of Constrution Engineering and Management,* v. 125, n. 1, pp. 8-15

Myers, D., Mitchell, P. (1993). "Identifying a Well-Founded Market Analysis." *The Appraisal Journal,* v. 61, pp. 500-508.

Nash, R. C. (1990). "Institutional Private Placement: A Legal Overview." *Introduction to Private Placements,* Michael G. Wolfson, Practising Law Institute.

Nevitt, P., and Fabozzi, F. (1995). *Project Financing*, Euromoney Publications, London, U.K.

Ngee, L., Tiong, R. L., and Alum, J. (1997). "Automated Approach to Negotiation of BOT Contracts." *J. of Computing in Civil Engineering,* v. 11, n. 2, pp. 121-128

OECD (1996). *"International Capital Markets Statistics 1950-1995"*, *Rep.,* Organisation for Economic Co-operation and Development, OECD, Paris.

OECD (1997). *"Financial Market Trends"*, Several issues (n. 45 Feb. 1990, n. 51. Feb. 1992, n. 60 Feb. 1995, n. 63 Feb. 1996, and n. 66 March 1997), Organization for Economic Co-operation and Development, OECD, Paris.

Ord, J. K. (1972). *Families of Frequency Distributions*, Griffin, London, U.K.

Ortuzar, J., and Willumsen, L. (1994). *Modeling Transport*, John Wiley & Sons Ltd., New York.

Park, C., and Sharp-Bette, G. (1990). *Advanced Engineering Economics*, John Wiley, New York.

Parsons, N. (1996). "Slow Roads to the Bank." *Asiamoney*, v. 7, n. 9, pp. 35-45.

Payne, H. (1996). "Concession Agreements." *Projects Procured by Privately Financed Concession Contracts, v1.* A. Merna and N. Smith, eds., Asia Law & Practice, pp. 75-88.

Pearmain, D., and Swanson, J. (1990). "The Use of Stated Preference Techniques in the Quantitative Analysis of Travel Behavior." *Proc., IMA Conference in Transport Planning and Control*, University of Cardiff, Wales, (Sept.), 411-420.

Pearson, E. S. and Tukey, J. W. (1965). "Approximate Means and Standard Deviations Based on Distances between Percentage Points of Frequency Curves." *Biometrika*, v. 52, pp. 533-546.

Pearson, J. M., and Shim, J. P. (1995). "An Empirical Investigation into DSS Structures and Environments." *Decision Support Systems*, v. 13, pp. 141-158.

Perry, J. G., and Hayes, R. W. (1985). "Risk and its Management in Construction Projects." *Proc., Institution of Civil Engineers,* UK, Part I, v. 78, pp. 499-521.

Peters, M. S., and Timmerhaus, K. D. (1991). *Plant Design and Economics for Chemical Engineers.* McGraw-Hill, Inc.

Pfeifer, P. Bodily, S., and Frey, Jr. S. (1991). "Pearson-Tukey Three-Point Approximations versus Monte Carlo Simulation." *Decision Sciences*, v. 22, pp. 74-90.

Pirie, J. D. (1996). "Public Private Partnering - Project Financing and Development", *Proc., 1st Structural Specialty Conference,* The Canadian Society of Civil Engineers, Edmonton, Alberta, 17-31.

Pouliquen, L. Y. (1970). "Risk Analysis in Project Appraisal", *Rep. World Bank Staff Occasional Papers Number Eleven,* The John Hopkins University Press, Baltimore.

Price Waterhouse (1993). "A Guide to Public-Private Partnerships in Infrastructure - Bridging the Gap between Infrastructure Needs and Public Resources", *Rep. Public-Private Partnership Services*, Price Watherhous, Toronto, Ontario.

Project & Trade Finance (1994). "The Risk Behind Elcogas.", September, London, UK.

Project & Trade Finance (1995). "Mexico Toll Road Pays Up.", January, London, UK.

Project Management Institute (PMI). (1996). "PMBOK Guide - The Guide to the Project Management Body of Knowledge." Project Management Standards Committee, USA.

Public Works Financing (PWF) (1995). “91 Express Lanes Opened.” *PWF*, v. 91, December.

PWF (1998). "1998 International major projects survey: public-private partnerships in infrastructure development" *Public Works Financing,* October, V. 122.

Ranasinghe, M. (1990). "Analytical Method for Quantification of Economic Risks During Feasibility Analysis For Large Engineering Projects," Ph.D. thesis, The University of British Columbia, Vancouver.

Ranasinghe, M., and Russell, A. D. (1992). "Analytical Approach for Economic Risk Quantification of Large Engineering Projects; Validation." *Construction Management and Economics*, v. 10, pp. 45-68.

Ranasinghe, M., and Russell, A. D. (1993). "Elicitation of Subjective Probabilities for Economic Risk Analysis: An Investigation." *Construction Management and Economics*, v. 11, pp. 326-340.

Ravichandran, R. (1993). "A Decision Support System for Stochastic Cost-Volume-Profit Analysis." *Decision Support Systems,* v. 10, pp. 379-399.

Remer, D. S., Tu, J. C., Carson, D. E., and Ganiy, S. A. (1984). “The State of the Art of Present Worth Analysis of Cash Flow Distributions.” *Engineering Costs and Production Economics*, 7(4), 257-278.

Reutlinger, S. (1970). *Techniques for Project Appraisal Under Uncertainty*, International Bank for Development and Reconstruction, The John Hopkins University Press, Baltimore.

Rhodes, T. (1993). *Syndicated Lending- Practice and Documentation*, Euromoney PLC, London.

Robinson, S. G. and Leithe, J. L. (1990). *Building Together: Investing in Community Infrastructure*. The National Association of Counties of the United States (NACo) and National Association of Home Builders (NAHB), NW, Washington D. C.

Roger, S. (1990). "Eurotunnel: An Innovative Approach to Project Financing," M.Sc. Thesis, Massachusetts Institute of Technology, USA.

Rubinstein, R. Y. (1981). *Simulation and the Monte Carlo Method.* John Wiley & Sons, NY.

Russell, A. D. (1995). *CIVL 522:Project and Construction Economics- Course Notes.* Department of Civil Engineering, The University of British Columbia, Vancouver.

Russell, A. D., and Ranasinghe, M. (1992). "Analytical Approach for Economic Risk Quantification of Large Engineering Projects." *Construction Management and Economics*, 10, 277-301.

Ryan, R. H. Jr. (1980). "International Bank Loan Syndication and Participation." *International Financial Law*, Robert S. Rendell, ed., Euromoney Publications, UK.

Sage, A. (1991). *Decision Support Systems Engineering*, John Wiley & Sons, Inc., New York.

Salem, O. M., and Ariaratnam, S. T. (1999). "Infrastructure Management: Decision Support and Life Cycle Cost Analysis." *Annual Conference of the Canadian Society for Civil Engineering,* Regina, Saskatchewan, June 2-5, 349-358.

Scarlett, S. (1990). "International Capital Markets - Bonds", *International Finance and Investment : Multinational Corporate Banking,* Brian Terry, ed., the Chartered Institute of Bankers, London.

Schmesier, B. W., and Deutsch, S. J. (1977). "A Versatile Four Parameter Family of Probability Distributions Suitable for Simulation." *AIIE Transactions*, v.9, n.2, pp. 176-182.

Shen, L., Lee R., and Zhang, Z. (1996). "Application of BOT Systems for Infrastructure Projects in China." *J. of Construction Engineering and Management,* v. 122, n. 4, pp. 319-323.

Siddall, J. S. (1972). *Analytical Decision-Making in Engineering Design*, Prentice-Hall, Inc.

Skjong, R., and Lerein, J. (1988). "Economic Risk of Offshore Field Development." *AACE Transacations,* pp. J.3.1-J.3.9.

Songer, A. D., Kiekmann, J., and Pecsok, R. S. (1997). "Risk Analysis for Revenue Dependent Infrastructure Projects." *Construction Management and Economics,* v. 15, pp. 377-382.

Spooner, J. E. (1974). "Probabilistic Estimating." *J. of the Construction Division,* v. 100, n. CO1, pp. 65-77.

Sapte, W. (1997). *Project Finance- The Guide to Financing Build-Operate-Transfer Projects Uses in PPP,* Euromoney Publications, London, UK.

Stein, W., and Pote, J. (1997). "Offtakers and Power Purchase Agreements" *Power Project Documentation,* Asia Law & Practice, Hong Kong.

Summerfield, H. G. (1981). "Commercial Bank Real Estate Lending." *Bank Credit,* Herbert B. Prochnov, ed., Harper & Row Publishers, New York.

Tam, C. M. (1995). "Features of Power Industries in Southeast Asia." *Int. J. of Project Management,* v. 13, n. 5, pp. 303-311.

Task Force on Public-Private Partnerships (TFPPP). (1996). "Building Partnerships- Report of the Task Force on Public-Private Partnerships." *Rep.,* Ministry of Employment and Investment, British Columbia.

Tanchoco, J., Buck, J., and Leung, L. (1981). "Modeling and Discounting of Continuous Cash Flows under Risk." *Engineering Costs and Production Economics*, 5, 205-216.

Thompson, P. A. (1993). *The Organization and Economics of Construction.* McGraw-Hill, U.K.

Thompson, P. A., and Perry, J. (1992). "Engineering Construction Risks - A Guide to Project Risk Analysis and Risk Management." *Rep.,* Thomas Telford, London, U.K.

Thompson, P. A., and Willmer, G. (1987). "CASPAR – A Program for Engineering Project Appraisal and Management". *Proc., Int. Conference in Civil and Structural Eng. Computing*, 1, (Dec.), 75-71.

Tiong, R. K. (1990a). "BOT Projects: Risks and Securities." *Construction Management and Economics,* v. 8, pp. 315-328.

Tiong, R. K. (1990b). "Comparative Study of BOT Projects." *J. of Management in Engineering*, v. 6, n. 1, pp. 107-122.

Tiong, R. K. (1995a). "Competitive Advantage of Equity in BOT Tender." *J. of Construction Engineering and Management*, v. 121, n. 3, pp. 282-289.

Tiong, R. K. (1995b). "Impact of Financial Package versus Technical Solution in a BOT Tender." *J. of Construction Engineering and Management*, v. 121, n. 3, pp. 304-311.

Tiong, R. K. (1995c). "Risks and Guarantees in BOT Tender." *J. of Construction Engineering and Management*, v. 121, n. 2, pp. 183-188.

Tiong, R. K. (1996). "CSFs in Competitive Tendering and Negotiation Model for BOT Projects." *J. OF Construction Engineering and Management*, v. 122, n. 3, pp. 205-211.

Tiong, R. K., and Alum, J. (1997a). "Evaluation of Proposals for BOT Projects." *Int. J. of Project Management,* v. 15, n. 2, pp. 67-72.

Tiong, R. K., and Alum, J. (1997b). "Financial Commitments for BOT Projects." *Int. J. of Project Management,* v. 15, n. 2, pp. 73-78.

Tiong, R. K., and Alum, J. (1997c). "Final Negotiation in Competitive BOT Tender." *J. of Construction Engineering and Management,* v. 123, n. 1, pp. 6-10.

Tiong, R. K., and Yeo, K. T. (1993). "Project Financing as a Competitive Strategy in Winning Overseas Jobs." *Int. J. of Project Management,* v. 11, n. 2, pp. 79-86.

Tiong, R. K., Yeo, K., and McCarthy, S. C. (1992). "Critical Success Factors in Winning BOT Contracts." *J. of Construction Engineering and Management*, v. 118, n. 2, pp. 217-228.

Tummala, V. M., and Bruchett, J. F. (1999). "Applying a Risk Management Process (RMP) to Manage Cost Risk for and EHV Transmission Line Project." *Int. J. of Project Management,* v. 17, n. 4, pp. 223-235.

Ugeux, G. (1981). *Floating Rate Notes.* Euromoney PLC, London, U.K.

United Nations Industrial Development Organization (UNIDO). (1994). *COMFAR III Expert – Computer Model for Feasibility Analysis and Reporting*, Vienna, Austria.

United Nations Industrial Development Organization (UNIDO). (1996). *Guidelines for Infrastructure Development through Build-Operate-Transfer (BOT) Projects*, Vienna, Austria.

United States Department of Transport (USDOT) (1980). "Patronage Impacts of Changes in Transit Fares and Services." *Rep.,* prepared by Mayworm, PD Lago, AM McEnroe, JM, U.S. Department of Transportation.

Visual Basic documentation manuals; release 6. (1998). Microsoft Corporation, USA.

Wagle, B. (1967). "A Statistical Analysis of Risk in Capital Investment Projects." *Operational Research Quarterly,* v. 18, pp. 13-33.

Wahdan, Y. M. (1995). "An Analysis Framework for Public-Private Partnership," Ph.D. thesis, The University of British Columbia, Vancouver.

Wahdan, Y. M., Russell, A. D., and Ferguson, D. (1995). "Public Private Partnerships and Transportation Infrastructure." *Proc., the 1995 Annual Conference of the Transportation Association of Canada,* Victoria, British Columbia.

Wahlstrom, B. (1994). "Models, Modelling and Modellers: an Application to Risk Analysis." *European J. of Operational Research,* 75(3), 477-487.

Walker, C., and Smith, A. J. (1995). *Privatized Infrastructure: the Build Operate Transfer Approach.* Thomas Telford, London, UK.

Wall, D. M. (1997). "Distributions and Correlations in Monte Carlo Simulation." *Construction Management and Economics,* v. 15, pp. 241-258.

Wall, D., and Mitchell, C. (1980). "The Eurodollar Market: Loans and Bonds.", *International Financial Law*, Robert S. Rendell, ed., Euromoney Publications, UK.

Wang, S. Q., Tiong, K. L., Ting, S. K., and Ashley, D. (1999). "Political Risks: Analysis of Key Contract Clauses in China's BOT Project." *J. of Construction Engineering and Management,* v. 125, n. 3, pp. 190-197.

Wardman, M. (1988). "A Comparison of Revealed Preference and Stated Preference Models of Travel Behavior." *J. Transport Economics and Policy*, 22(1), 71-91.

Westerterp, K. R., and Vrijland, M. S. (1984). "Plant Economics and Reactor Design." *AACE Transactions,* pp. M.5.1-M.5.3.

Willmer, G. (1991). "Time and Cost Risk Analysis." *Computers & Structures*, 41(6), 1149-1155.

Willingham, J. (1990). "Principles of Lending - Part I, II, III, IV, V", *International Finance and Investment : Multinational Corporate Banking,* Brian Terry, ed., the Chartered Institute of Bankers, London.

Wincott, R., and Mueller, G. (1995). "Market Analysis in the Appraisal Process." *The Appraisal Journal,* v. 63, pp. 27-32.

Winfield, B. (1996). "Procuring BOOT Projects." *Projects Procured by Privately Financed Concession Contracts, v2.* A. Merna and N. Smith, eds., Asia Law & Practice, pp. 1-8.

Wiseman, P (1990). "Syndicated Facilities." *International Finance and Investment: Multinational Corporate Banking,* Brian Terry, ed., the Chartered Institute of Bankers, U.K.

Woodward, D. G. (1995). "Use of Sensitivity Analysis in Build-Own-Operate-Transfer Project Evaluation." *Int. J. of Project Management,* v. 13, n.4, pp. 239-246.

World Bank (1994). "World Development Report 1994: Infrastructure for Development." Published for World Bank by Oxford University Press.

Yeung, F. A. (1997). "Documenting the Deal: Debt Components." *The Life and Death of an Infrastructure Project*, Thomas H. Pyle, ed., Asia Law & Practice, Hong Kong.

Appendices

Appendix A

The First Four Moments of A System Function

A.1 General

The derivation of the four moments of a system function (or a dependent random variable) in this Appendix uses a number of expansions for the power and product of summations. Let a, x, and y be three vectors of variables. Then:

$$\left[\sum_{i=1}^{n} a_i \cdot (x_i + y_i)\right]^2 = \sum_{i=1}^{n} a_i^2 \cdot (x_i + y_i)^2 + 2 \cdot \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} a_i a_j \cdot (x_i + y_i) \cdot (x_j + y_j) \tag{A.1}$$

$$\begin{aligned}\left[\sum_{i=1}^{n} a_i \cdot (x_i + y_i)\right]^3 &= \sum_{i=1}^{n} a_i^3 \cdot (x_i + y_i)^3 + 3 \cdot \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} a_i^2 a_j \cdot (x_i + y_i)^2 \cdot (x_j + y_j) \\ &+ 3 \cdot \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} a_i a_j^2 \cdot (x_i + y_i) \cdot (x_j + y_j)^2 \\ &+ 6 \cdot \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^{n} a_i a_j a_k \cdot (x_i + y_i) \cdot (x_j + y_j) \cdot (x_k + y_k)\end{aligned} \tag{A.2}$$

$$\begin{aligned}\left[\sum_{i=1}^{n} a_i \cdot (x_i + y_i)\right]^4 &= \sum_{i=1}^{n} a_i^4 \cdot (x_i + y_i)^4 + 4 \cdot \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} a_i^3 a_j \cdot (x_i + y_i)^3 \cdot (x_j + y_j) \\ &+ 4 \cdot \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} a_i a_j^3 \cdot (x_i + y_i) \cdot (x_j + y_j)^3 \\ &+ 6 \cdot \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} a_i^2 a_j^2 \cdot (x_i + y_i)^2 \cdot (x_j + y_j)^2 \\ &+ 12 \cdot \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^{n} a_i^2 a_j a_k \cdot (x_i + y_i)^2 \cdot (x_j + y_j) \cdot (x_k + y_k) \\ &+ 12 \cdot \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^{n} a_i a_j^2 a_k \cdot (x_i + y_i) \cdot (x_j + y_j)^2 \cdot (x_k + y_k) \\ &+ 12 \cdot \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^{n} a_i a_j a_k^2 \cdot (x_i + y_i) \cdot (x_j + y_j) \cdot (x_k + y_k)^2 \\ &+ 24 \cdot \sum_{i=1}^{n-3} \sum_{j=i+1}^{n-2} \sum_{k=j+1}^{n-1} \sum_{l=k+1}^{n} a_i a_j a_k a_l \cdot (x_i + y_i) \cdot (x_j + y_j) \cdot (x_k + y_k) \cdot (x_l + y_l)\end{aligned} \tag{A.3}$$

$$\left[\sum_{i=1}^{n} x_i \cdot \sum_{j=1}^{n} x_j^2\right] = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i \cdot x_j^2$$
$$= \sum_{i=1}^{n} x_i^3 + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} x_i \cdot x_j^2 + \sum_{i=2}^{n}\sum_{j=1}^{i-1} x_i \cdot x_j^2 \tag{A.4}$$

The expectation (expected value) E of a constant c, product of constant and random variable w, and sum of random variables in a vector x are as follows:

$$E(c) = c \tag{A.5}$$

$$E(c \cdot w) = c \cdot E(w) \tag{A.6}$$

$$E\left[\sum_{i=1}^{n} c_i \cdot x_i\right] = \sum_{i=1}^{n} [c_i \cdot E(x_i)] \tag{A.7}$$

The nomenclature and assumptions used in the derivation of the four moments of a system function or a dependent random variable Z are:

- $\boldsymbol{x}$ is a vector of variables i where $i = 1,2, \ldots, n$ (i.e. $x_1, x_2, \ldots, x_n$).
- $\mu_1`_i$, μ_{2_i}, μ_{3_i}, and μ_{4_i} are the expected value, and 2nd to 4th central moments of variable i.
- $\delta_i = (x_i - \mu_1`_i)$.
- Z is a system function $f(\boldsymbol{x})$ for any performance measure.
- $f(\overline{\boldsymbol{x}})$ is f calculated at the mean values of all the variables in $\boldsymbol{x}$.
- f_i' and f_i'' are the 1st and 2nd partial derivatives of f with respect to the i-th variable in $\boldsymbol{x}$.
- $f_{i,j}'$ is the second partial derivative of f with respect to i and j variables in $\boldsymbol{x}$.
- $\mu`_1(Z)$, $\mu_2(Z)$, $\mu_3(Z)$ and $\mu_4(Z)$ are the expected value, 2nd to 4th central moments of Z.
- Two cases are considered in the derivation. The first assumes the variables of the system function are statistically independent, i.e. assumes variables have zero linear correlation. Therefore cross moments converts to moments of random variables as follows:

$$E\{[x_i - \mu 1'_i]^r \cdot [x_j - \mu 1'_j]^s\} = E[x_i - \mu 1'_i]^r \cdot E[x_j - \mu 1'_j]^s \tag{A.8}$$

Only values of r and/or s up to the fourth order are considered in the derivation. If r or s = 1 in Eq. (A.8), then the expectation in Eq. (A.8) becomes zero since

$$E[x_i - \mu 1'_i]^1 = 0 \tag{A.9}$$

In the second case, variables are considered to be uncorrelated, but not necessarily independent, and therefore in the derivation the terms that may lead to covariance or cross moments (e.g. Eq. A.8) are ignored. No functional correlation is considered.

- Only moments up to the fourth order are considered in the derivation. If a term after expansion leads to higher moments requirements, then only that part that needs higher moments is ignored. For example, in the expansion of Eq. (A.2), if $(x_i + y_i)$ are squared, then the first term to the right of the equal sign leads to the sixth central moment and therefore is ignored. However, the other terms require the second and fourth central moments and therefore are considered in the derivation.

Using the above nomenclature, a system function $f(\boldsymbol{x})$ can be approximated by a multivariate Taylor series expansion of a second order about the mean values of $\boldsymbol{x}$ as follows

$$f(\mathbf{x}) = f(\overline{\mathbf{x}}) + \sum_{i=1}^{n} f'_i \cdot \delta_i + \frac{1}{2} \cdot \sum_{i=1}^{n} f''_i \cdot \delta_i^2 + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} f'_{i,j} \cdot \delta_i \cdot \delta_j \tag{A.10}$$

A.2 The Expected Value

The expected value of the system performance function can be obtained as follows (Eq. 5.17):

$$\begin{aligned} \mu'_1(Z) &= E[f(\mathbf{x})] \\ &= f(\overline{\mathbf{x}}) + \frac{1}{2} \cdot \sum_{i=1}^{n} f''_i \cdot \mu 2_i \end{aligned} \tag{A.11}$$

A.3 The Second Central Moment

The second central moment of the system performance function can be obtained as follows:

$$
\begin{aligned}
\mu_2(Z) &= E[(Z-\mu_1'(Z))^2] \\
&= E[(f(\mathbf{x})-E[f(\mathbf{x})])^2] \\
&= E\left\{\left(\sum_{i=1}^{n} f_i' \cdot \delta_i + \frac{1}{2}\cdot\sum_{i=1}^{n} f_i'' \cdot \delta_i^2 + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f_{i,j}' \cdot \delta_i \cdot \delta_j - \frac{1}{2}\cdot\sum_{i=1}^{n} f_i'' \cdot \mu 2_i\right)^2\right\} \\
&= E\{(H+I+J+K)^2\}
\end{aligned}
\tag{A.12}
$$

Then,

$$
\begin{aligned}
\mu_2(Z) &= E\left[H^2\right] + E\left[I^2\right] + E\left[J^2\right] + E\left[K^2\right] \\
&\quad + E[2HI] + E[2HJ] + E[2HK] + E[2IJ] + E[2HK] + E[2JK]
\end{aligned}
\tag{A.13}
$$

where under the assumption of statistical independence between variables (the first case),

$$E\left[H^2\right] = \sum_{i=1}^{n} f_i'^2 \cdot \mu 2_i \tag{A.14}$$

$$E\left[I^2\right] = \frac{1}{4}\cdot\sum_{i=1}^{n} f_i''^2 \cdot \mu 4_i + \frac{1}{2}\cdot\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f_i'' \cdot f_j'' \cdot \mu 2_i \cdot \mu 2_j \tag{A.15}$$

$$E\left[J^2\right] = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f_{i,j}'^2 \cdot \mu 2_i \cdot \mu 2_j \tag{A.16}$$

$$E\left[K^2 + 2IK\right] = -\frac{1}{4}\cdot\left(\sum_{i=1}^{n} f_i'' \cdot \mu 2_i\right)^2 \tag{A.17}$$

$$E[2HI] = \sum_{i=1}^{n} f_i' \cdot f_j'' \cdot \mu 3_i \tag{A.18}$$

The second moment is obtained as the summation of Eqs. (A.14) to (A.18). The other terms of Eq. (A.13) all drop out since they lead to either a value of zero or moments beyond the 4th level.

In case variables of the system function are uncorrelated (the second case) the following terms drop out of the above formulation: (1) second term of Eq. (A.15), and (2) Eq. (A.16). Therefore the formulation in this case reduces to:

$$\mu_2(Z) = \sum_{i=1}^{n} f_i'^2 \cdot \mu2_i + \frac{1}{4} \cdot \sum_{i=1}^{n} f_i''^2 \cdot \mu4_i - \frac{1}{4} \cdot \left(\sum_{i=1}^{n} f_i'' \cdot \mu2_i \right)^2 + \sum_{i=1}^{n} f_i' \cdot f_i'' \cdot \mu3_i \tag{A.19}$$

which corresponds to Eq. (5.18).

A.4 The Third Central Moment

The third central moment of the system performance function can be obtained as follows:

$$\begin{aligned} \mu_3(Z) &= E[(Z - \mu_1'(Z))^3] \\ &= E[(f(\mathbf{x}) - E[f(\mathbf{x})])^3] \\ &= E\left\{ \left(\sum_{i=1}^{n} f_i' \cdot \delta_i + \frac{1}{2} \cdot \sum_{i=1}^{n} f_i'' \cdot \delta_i^2 + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} f_{i,j}' \cdot \delta_i \cdot \delta_j - \frac{1}{2} \cdot \sum_{i=1}^{n} f_i'' \cdot \mu2_i \right)^3 \right\} \\ &= E\left\{ (H + I + J + K)^3 \right\} \end{aligned} \tag{A.20}$$

Then,

$$\begin{aligned} \mu_3(Z) = {} & E\left[H^3 \right] + E\left[I^3 \right] + E\left[J^3 \right] + E\left[K^3 \right] \\ & + E\left[3H^2 I \right] + E\left[3H^2 J \right] + E\left[3H^2 K \right] + E\left[3HI^2 \right] + E\left[3HJ^2 \right] + E\left[3HK^2 \right] + E\left[6HIJ \right] + E\left[6HIK \right] \\ & + E\left[3I^2 J \right] + E\left[3I^2 K \right] + E\left[3IJ^2 \right] + E\left[3IK^2 \right] + E\left[6IJk \right] \\ & + E\left[3J^2 K \right] + E\left[3JK^2 \right] \end{aligned} \tag{A.21}$$

where under the assumption of statistical independence between variables (the first case),

$$E\left[H^3\right] = \sum_{i=1}^{n} f_i'^3 \cdot \mu 3_i \tag{A.22}$$

$$\begin{aligned} E\left[I^3\right] &= \frac{3}{8} \cdot \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} f_i''^2 \cdot f_j'' \cdot \mu 4_i \cdot \mu 2_j + \frac{3}{8} \cdot \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} f_i'' \cdot f_j''^2 \cdot \mu 2_i \cdot \mu 4_j \\ &+ \frac{6}{8} \cdot \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^{n} f_i'' \cdot f_j'' \cdot f_k'' \cdot \mu 2_i \cdot \mu 2_j \cdot \mu 2_k \end{aligned} \tag{A.23}$$

$$E\left[J^3\right] = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} f_{i,j}'^3 \cdot \mu 3_i \cdot \mu 3_j \tag{A.24}$$

$$E\left[K^3 + 3IK^2\right] = \frac{1}{4} \cdot \left(\sum_{i=1}^{n} f_i'' \cdot \mu 2_i \right)^3 \tag{A.25}$$

$$E\left[3H^2 I\right] = \frac{3}{2} \cdot \left[\sum_{i=1}^{n} f_i'^2 \cdot f_i'' \cdot \mu 4_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} f_i'^2 \cdot f_i'' \cdot \mu 2_i \cdot \mu 2_j + \sum_{i=2}^{n} \sum_{j=1}^{i-1} f_i'^2 \cdot f_i'' \cdot \mu 2_i \cdot \mu 2_j \right] \tag{A.26}$$

$$E\left[6H^2 J\right] = 6 \cdot \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} f_i' \cdot f_j' \cdot f_{i,j}' \cdot \mu 2_i \cdot \mu 2_j \tag{A.27}$$

$$E\left[3H^2 K\right] = \left(\sum_{i=1}^{n} f_i'' \cdot \mu 2_i \right) \cdot \left[\frac{-3}{2} \cdot \sum_{i=1}^{n} f_i'^2 \cdot \mu 2_i \right] \tag{A.28}$$

$$E\left[3H I^2\right] = \frac{3}{4} \cdot \left[\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} f_i' \cdot f_i'' \cdot f_j'' \cdot \mu 3_i \cdot \mu 2_j + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} f_j' \cdot \cdot f_i'' \cdot f_j'' \cdot \mu 2_i \cdot \mu 3_j \right] \tag{A.29}$$

$$E\left[3H J^2\right] = 3 \cdot \left[\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} f_i' \cdot f_{i,j}'^2 \cdot \mu 3_i \cdot \mu 2_j + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} f_j' \cdot f_{i,j}'^2 \cdot \mu 2_i \cdot \mu 3_j \right] \tag{A.30}$$

$$E\left[3I^2 J\right] = \frac{3}{2} \cdot \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} f_i'' \cdot f_j'' \cdot f_{i,j}' \cdot \mu 3_i \cdot \mu 3_j \tag{A.31}$$

$$E[6HIK] = \left(\sum_{i=1}^{n} f''_i \cdot \mu 2_i\right) \cdot \left[\frac{-3}{2} \cdot \sum_{i=1}^{n} f'_i \cdot f''_i \cdot \mu 3_i\right] \tag{A.32}$$

$$E\left[3I^2K\right] = \left(\sum_{i=1}^{n} f''_i \cdot \mu 2_i\right) \cdot \left[\frac{-3}{8} \cdot \sum_{i=1}^{n} f''^2_i \cdot \mu 4_i + \frac{-3}{4} \cdot \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} f''_i \cdot f''_j \cdot f'_{i,j} \cdot \mu 2_i \cdot \mu 2_j\right] \tag{A.33}$$

$$E\left[3J^2K\right] = \left(\sum_{i=1}^{n} f''_i \cdot \mu 2_i\right) \cdot \left[\frac{-3}{2} \cdot \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} f'^2_{i,j} \cdot \mu 2_i \cdot \mu 2_j\right] \tag{A.34}$$

$$E\left[3IJ^2\right] = \frac{3}{2} \cdot \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} f''_i \cdot f'^2_{i,j} \cdot \mu 4_i \cdot \mu 2_j + \frac{3}{2} \cdot \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} f''_j \cdot f'^2_{i,j} \cdot \mu 2_i \cdot \mu 4_j \tag{A.35}$$

For the case where the variables of the system function are uncorrelated, the following terms drop out of the above formulation: Eqs. (A.23), (A.24), (A.27), (A.30), (A.31), (A.34) and (A.35); as do the second and third term of Eq.(A.26) and the double summation in Eq. (A.33). Therefore, the expression for the third moment in this case reduces to:

$$\begin{aligned} \mu_3(Z) = {} & \sum_{i=1}^{n} f'^3_i \cdot \mu 3_i + \frac{1}{4} \cdot \left[\sum_{i=1}^{n} f''_i \cdot \mu 2_i\right]^3 + \frac{3}{2} \cdot \sum_{i=1}^{n} f'^2_i \cdot f''_i \cdot \mu 4_i \\ & - \left[\sum_{i=1}^{n} f''_i \cdot \mu 2_i\right] \cdot \left[\frac{3}{2} \cdot \sum_{i=1}^{n} f'^2_i \cdot \mu 2_i + \frac{3}{2} \cdot \sum_{i=1}^{n} f'_i \cdot f''_i \cdot \mu 3_i + \frac{3}{8} \cdot \sum_{i=1}^{n} f''^2_i \cdot \mu 4_i\right] \end{aligned} \tag{A.36}$$

which corresponds to Eq. (5.19).

A.5 The Fourth Central Moment

The fourth central moment of the system performance function can be obtained as follows:

$$\mu_4(Z) = E[(Z-\mu_1'(Z))^4]$$
$$= E[(f(\mathbf{x}) - E[f(\mathbf{x})])^4]$$
$$= E\left\{\left(\sum_{i=1}^{n} f_i' \cdot \delta_i + \frac{1}{2}\cdot\sum_{i=1}^{n} f_i'' \cdot \delta_i^2 + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f_{i,j}' \cdot \delta_i \cdot \delta_j - \frac{1}{2}\cdot\sum_{i=1}^{n} f_i'' \cdot \mu 2_i\right)^4\right\} \tag{A.37}$$
$$= E\left\{(H+I+J+K)^4\right\}$$

Then,

$$\begin{aligned}\mu_4(Z) = {} & E\left[H^4\right] + E\left[I^4\right] + E\left[J^4\right] + E\left[K^4\right] \\ & + E\left[4H^3I\right] + E\left[4H^3J\right] + E\left[4H^3K\right] + E\left[6H^2I^2\right] + E\left[6H^2J^2\right] + E\left[6H^2K^2\right] \\ & + E\left[12H^2IJ\right] + E\left[12H^2IK\right] + E\left[12H^2JK\right] + E\left[12HI^2J\right] + E\left[12HI^2K\right] + E\left[12HJ^2K\right] \\ & + E\left[12HIJ^2\right] + E\left[12HIK^2\right] + E\left[12HJK^2\right] + E[24HIJK] + E\left[4HI^3\right] + E\left[4HJ^3\right] + E\left[4HK^3\right] \\ & + E\left[4I^3J\right] + E\left[4I^3K\right] + E\left[6I^2J^2\right] + E\left[6I^2K^2\right] + E\left[12I^2JK\right] + E\left[12IJ^2K\right] + E\left[12IJK^2\right] \\ & + E\left[4IJ^3\right] + E\left[4IK^3\right] + E\left[4J^3K\right] + E\left[6J^2K^2\right] + E\left[4JK^3\right]\end{aligned} \tag{A.38}$$

where under the assumption of statistical independence between variables,

$$E\left[H^4\right] = \sum_{i=1}^{n} f_i'^4 \cdot \mu4_i + 6 \cdot \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} f_i'^2 \cdot f_j'^2 \cdot \mu2_i \cdot \mu2_j \tag{A.39}$$

$$E\left[I^4\right] = \frac{6}{16} \cdot \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} f_i''^2 \cdot f_j''^2 \cdot \mu4_i \cdot \mu4_j + \frac{12}{16} \cdot \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^{n} f_i''^2 \cdot f_j'' \cdot f_k'' \cdot \mu4_i \cdot \mu2_j \cdot \mu2_k$$
$$+ \frac{12}{16} \cdot \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^{n} f_i'' \cdot f_j''^2 \cdot f_k'' \cdot \mu2_i \cdot \mu4_j \cdot \mu2_k + \frac{12}{16} \cdot \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^{n} f_i'' \cdot f_j'' \cdot f_k''^2 \cdot \mu2_i \cdot \mu2_j \cdot \mu4_k \tag{A.40}$$

$$E\left[J^4\right] = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} f_{i,j}'^4 \cdot \mu4_i \cdot \mu4_j + 6 \cdot \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^{n} f_{i,j}'^2 \cdot f_{j,k}'^2 \cdot \mu2_i \cdot \mu4_j \cdot \mu2_k$$
$$+ 6 \cdot \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^{n} f_{i,k}'^2 \cdot f_{j,k}'^2 \cdot \mu2_i \cdot \mu2_j \cdot \mu4_k + 6 \cdot \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^{n} f_{i,j}'^2 \cdot f_{j,k}'^2 \cdot \mu4_i \cdot \mu2_j \cdot \mu2_k$$
$$+ 12 \cdot \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^{n} f_{i,j}' \cdot f_{i,k}'^2 \cdot f_{j,k}' \cdot \mu3_i \cdot \mu2_j \cdot \mu3_k + 12 \cdot \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^{n} f_{i,j}'^2 \cdot f_{i,k}' \cdot f_{j,k}' \cdot \mu3_i \cdot \mu3_j \cdot \mu2_k$$
$$+ 12 \cdot \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^{n} f_{i,j}' \cdot f_{i,k}' \cdot f_{j,k}'^2 \cdot \mu2_i \cdot \mu3_j \cdot \mu3_k \tag{A.41}$$

$$E\left[K^4 + 4IK^3\right] = \frac{-3}{16} \cdot \left(\sum_{i=1}^{n} f_i'' \cdot \mu2_i\right)^4 \tag{A.42}$$

$$E\left[4H^3 I\right] = 2\cdot\left[\sum_{i=1}^{n}\sum_{j=1}^{n} f_i'^3 \cdot f_j'' \cdot \mu 3_i \cdot \mu 2_j\right] \tag{A.43}$$

$$E\left[4H^3 J\right] = 12\cdot\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f_i'^2 \cdot f_j' \cdot f_{i,j}' \cdot \mu 3_i \cdot \mu 2_j + 12\cdot\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f_i' \cdot f_j'^2 \cdot f_{i,j}' \cdot \mu 2_i \cdot \mu 3_j \tag{A.44}$$

$$E\left[4H^3 K\right] = \left[\sum_{i=1}^{n} f_i'' \cdot \mu 2_i\right]\cdot\left[-2\cdot\sum_{i=1}^{n} f_i'^3 \cdot \mu 3_i\right] \tag{A.45}$$

$$E\left[6H^2 I^2\right] = \frac{6}{4}\cdot\left[\sum_{i=1}^{n}\sum_{j=1}^{n} f_i'^2 \cdot f_j''^2 \cdot \mu 2_i \cdot \mu 4_j + 4\cdot\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f_i' \cdot f_j' \cdot f_i'' \cdot f_j'' \cdot \mu 3_i \cdot \mu 3_j\right] \tag{A.46}$$

$$E\left[4H^2 J^2\right] = 12\cdot\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f_i' \cdot f_j' \cdot f_{i,j}'^2 \cdot \mu 3_i \cdot \mu 3_j + 6\cdot\left[\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f_i'^2 \cdot f_{i,j}'^2 \cdot \mu 4_i \cdot \mu 2_j + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f_j'^2 \cdot f_{i,j}'^2 \cdot \mu 2_i \cdot \mu 4_j\right] \tag{A.47}$$

$$E\left[6H^2 K^2\right] = \left[\sum_{i=1}^{n} f_i'' \cdot \mu 2_i\right]^2 \cdot\left[\frac{3}{2}\cdot\sum_{i=1}^{n} f_i'^2 \cdot \mu 2_i\right] \tag{A.48}$$

$$E\left[12H^2 I J\right] = 12\cdot\left[\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f_i'' \cdot f_i' \cdot f_{i,j}' \cdot \mu 4_i \cdot \mu 2_j + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f_j'' \cdot f_i' \cdot f_{i,j}' \cdot \mu 2_i \cdot \mu 4_j + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f_i'^2 \cdot f_j'' \cdot f_{i,j}' \cdot \mu 3_i \cdot \mu 3_j\right] \tag{A.49}$$

$$E\left[12H^2 I\,K\right] = \left(-3\cdot\sum_{i=1}^{n} f''_i\cdot \mu2_i\right)\cdot\left[\sum_{i=1}^{n} f'^2_i\cdot f''_i\cdot \mu4_i + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f'^2_i\cdot f''_i\cdot \mu2_i\cdot \mu2_j + \sum_{i=2}^{n}\sum_{j=1}^{i-1} f'^2_i\cdot f''_i\cdot \mu2_i\cdot \mu2_j\right] \tag{A.50}$$

$$E\left[12H^2 J\,K\right] = \left(-12\cdot\sum_{i=1}^{n} f''_i\cdot \mu2_i\right)\cdot\left[\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f'_i\cdot f'_j\cdot f'_{i,j}\cdot \mu2_i\cdot \mu2_j\right] \tag{A.51}$$

$$E\left[12H\,I^2\,J\right] = 6\cdot\left[\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f'_i\cdot f''_i\cdot f''_j\cdot f'_{i,j}\cdot \mu4_i\cdot \mu3_j + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f'_j\cdot f''_i\cdot f''_j\cdot f'_{i,j}\cdot \mu3_i\cdot \mu4_j\right] \tag{A.52}$$

$$E\left[12H\,I^2\,K\right] = -3\cdot\left(\sum_{i=1}^{n} f''_i\cdot \mu2_i\right)\cdot\left[\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f'_i\cdot f''_i\cdot f''_j\cdot \mu3_i\cdot \mu2_j + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f'_j\cdot f''_i\cdot f''_j\cdot \mu2_i\cdot \mu3_j\right] \tag{A.53}$$

$$E\left[12H\,J^2\,K\right] = -6\cdot\left(\sum_{i=1}^{n} f''_i\cdot \mu2_i\right)\cdot\left[\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f'_i\cdot f'^2_{i,j}\cdot \mu3_i\cdot \mu2_j + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f'_j\cdot f'^2_{i,j}\cdot \mu2_i\cdot \mu3_j\right] \tag{A.54}$$

$$E\left[12H\,I\,J^2\right] = \frac{12}{2}\cdot\left[\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f'_i\cdot f''_j\cdot f'^2_{i,j}\cdot \mu3_i\cdot \mu4_j + \sum_{i=1}^{n}\sum_{j=1}^{n-1}\sum_{k=j+1}^{n} f'_i\cdot f''_i\cdot \mu3_i\cdot f'^2_{j,k}\cdot \mu2_j\cdot \mu2_k\right] \tag{A.55}$$

$$E\left[12H\,I\,K^2\right] = \left(\sum_{i=1}^{n} f''_i\cdot \mu2_i\right)^2\cdot\left[\frac{3}{2}\cdot\sum_{i=1}^{n-1} f'_i\cdot f''_i\cdot \mu3_i\right] \tag{A.56}$$

$$E[24H\,I\,J\,K] = \frac{-24}{4}\cdot\left(\sum_{i=1}^{n} f''_i \cdot \mu2_i\right)\left[\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f'_i \cdot f''_j \cdot f'_{i,j} \cdot \mu2_i \cdot \mu3_j\right] \tag{A.57}$$

$$E\left[4H\,I^3\right] = \frac{12}{8}\cdot\left[\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f'_j \cdot f''^2_i \cdot f''_j \cdot \mu4_i \cdot \mu3_j + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f'_i \cdot f''^2_i \cdot f''_j \cdot \mu3_i \cdot \mu4_j\right] \tag{A.58}$$

$$E\left[4H\,J^3\right] = 4\cdot\left[\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f'_i \cdot f'^3_{i,j} \cdot \mu4_i \cdot \mu3_j + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f'_j \cdot f'^3_{i,j} \cdot \mu3_i \cdot \mu4_j\right] \tag{A.59}$$

$$E\left[4I^3\,K\right] = \left(\frac{-1}{4}\cdot\sum_{i=1}^{n} f''_i \cdot \mu2_i\right)\cdot\left[\begin{array}{l} 3\cdot\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f''^2_i \cdot f''_j \cdot \mu4_i \cdot \mu2_j + 3\cdot\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f''_i \cdot f''^2_j \cdot \mu2_i \cdot \mu4_j \\ +\sum_{i=1}^{n-2}\sum_{j=i+1}^{n-1}\sum_{k=j+1}^{n} f''_i \cdot f''_j \cdot f''_k \cdot \mu2_i \cdot \mu2_j \cdot \mu2_k \end{array}\right] \tag{A.60}$$

$$E\left[6I^2J^2\right] = \frac{6}{2}\cdot\sum_{i=1}^{n}\sum_{j=1}^{n-1}\sum_{k=j+1}^{n} f''_i \cdot f'^2_{j,k} \cdot \mu2_i \cdot \mu2_j \cdot \mu2_k + 6\cdot\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f''_i \cdot f''_j \cdot f'^2_{i,j} \cdot \mu4_i \cdot \mu4_j \tag{A.61}$$

$$E\left[6\,I^2K^2\right] = \left(\sum_{i=1}^{n} f''_i \cdot \mu2_i\right)^2 \cdot\left[\frac{3}{8}\cdot\sum_{i=1}^{n-1} f''^2_i \cdot \mu4_i + \frac{3}{4}\cdot\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f''_i \cdot f''_j \cdot \mu2_i \cdot \mu2_j\right] \tag{A.62}$$

$$E\left[12I^2J\,K\right] = \left(-\sum_{i=1}^{n} f''_i \cdot \mu2_i\right)\left[3\cdot\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f''_i \cdot f''_j \cdot f'_{i,j} \cdot \mu3_i \cdot \mu3_j\right] \tag{A.63}$$

$$E\left[12IJ^2K\right] = \left(-3\cdot\sum_{i=1}^{n} f_i''\cdot\mu2_i\right)\left[\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f_i''\cdot f_{i,j}'^2\cdot\mu4_i\cdot\mu2_j + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f_j''\cdot f_{i,j}'^2\cdot\mu2_i\cdot\mu4_j\right] \tag{A.64}$$

$$E\left[4IJ^3\right] = 2\cdot\sum_{i=1}^{n}\sum_{j=1}^{n-1}\sum_{k=j+1}^{n} f_i''\cdot f_{j,k}'^3\cdot\mu2_i\cdot\mu3_j\cdot\mu3_k + 12\cdot\sum_{i=1}^{n}\sum_{j=1}^{n-2}\sum_{k=j+1}^{n-1}\sum_{m=k+1}^{n} f_i''\cdot\mu2_i\cdot f_{j,k}'\cdot\mu2_j\cdot\mu2_k\cdot f_{j,m}'\cdot\mu2_m\cdot f_{k,m}' \tag{A.65}$$

$$E\left[4J^3K\right] = \left(-\sum_{i=1}^{n} f_i''\cdot\mu2_i\right)\cdot\left[2\cdot\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f_{i,j}'^3\cdot\mu3_i\cdot\mu3_j\right] \tag{A.66}$$

$$E\left[6J^2K^2\right] = \left(-\sum_{i=1}^{n} f_i''\cdot\mu2_i\right)^2\cdot\left[\frac{3}{2}\cdot\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f_{i,j}'^2\cdot\mu2_i\cdot\mu2_j\right] \tag{A.67}$$

For the case where the variables of the system function are uncorrelated (the second case), most of the terms drop out of the above formulation. What remains include the following: the single summation in Eq. (A.39); all of Eqs. (A.42), (A.45), (A.48), and (A.56); and, the single summations in Eqs. (A.50) and (A.62). Therefore, the expression for the fourth moment in this case reduces to:

$$\begin{aligned}\mu_4(Z) = {} & \sum_{i=1}^{n} f_i'^4\cdot\mu4_i - \frac{3}{16}\cdot\left[\sum_{i=1}^{n} f_i''\cdot\mu2_i\right]^4 - \left[\sum_{i=1}^{n} f_i''\cdot\mu2_i\right]\cdot\left[2\cdot\sum_{i=1}^{n} f_i'^3\cdot\mu3_i + 3\cdot\sum_{i=1}^{n} f_i'^2\cdot f_i''\cdot\mu4_i\right] \\ & + \left[\sum_{i=1}^{n} f_i''\cdot\mu2_i\right]^2\cdot\left[\frac{3}{2}\cdot\sum_{i=1}^{n} f_i'^2\cdot\mu2_i + \frac{3}{2}\cdot\sum_{i=1}^{n} f_i'\cdot f_i''\cdot\mu3_i + \frac{3}{8}\cdot\sum_{i=1}^{n} f_i''^2\cdot\mu4_i\right]\end{aligned} \tag{A.68}$$

which corresponds to Eq. (5.20).

Appendix B

Calculation Sheets for the Example Project

This appendix contains the calculation sheets in Mathcad format (Mathcad 1998) for the explicit formulation and risk analysis of the performance measures of the first example project in Chapter 7. The Appendix includes the following calculations:

1. Distribution models and variables in the example project
2. Net present value and sensitivity analysis
3. Internal rate of return on equity
4. Internal rate of return on total capital
5. Total capital expenditure
6. Total revenues
7. Construction completion time
8. Debt service and loan life cover ratios

B.1 Distribution Models And Variables In The Example Project

Characteristics of the Log Normal Distribution Model:

$$LN(x,\mu,\sigma) := \frac{1}{x\cdot\sigma\cdot\sqrt{2\cdot\pi}}\cdot\exp\left[\frac{-1}{2}\cdot\left(\frac{\ln(x)-\mu}{\sigma}\right)^2\right]$$

$$LN'(x,\mu,\sigma) := \frac{1}{\sigma\cdot\sqrt{2\cdot\pi}}\cdot\int_0^x \frac{1}{t}\cdot\exp\left[\frac{-1}{2}\cdot\left(\frac{\ln(t)-\mu}{\sigma}\right)^2\right]dt$$

$$EXL(\mu,\sigma) := \exp\left(\mu+\frac{\sigma^2}{2}\right)$$

$$SDL(\mu,\sigma) := \sqrt{\exp\left(2\cdot\mu+\sigma^2\right)\cdot\left(\exp\left(\sigma^2\right)-1\right)}$$

$$\sigma L(\mu1,\mu2) := \sqrt{\ln\left(\mu2+\mu1^2\right)-2\cdot\ln(\mu1)}$$

$$\mu L(\mu1,\mu2) := \ln(\mu1)-\frac{\sigma L(\mu1,\mu2)^2}{2}$$

$$c(\mu1,\mu2) := \sqrt{\exp\left(\sigma L(\mu1,\mu2)^2\right)-1}$$

$$\gamma1L(\mu1,\mu2) := c(\mu1,\mu2)^3+3\cdot c(\mu1,\mu2)$$

$$\gamma2L(\mu1,\mu2) := c(\mu1,\mu2)^8+6\cdot c(\mu1,\mu2)^6+15\cdot c(\mu1,\mu2)^4+16\cdot c(\mu1,\mu2)^2+3$$

$$\mu3L(\mu1,\mu2) := \gamma1L(\mu1,\mu2)\cdot\mu2^{1.5}$$

$$\mu4L(\mu1,\mu2) := \gamma2L(\mu1,\mu2)\cdot\mu2^2 \qquad cv(\mu,\sigma) := \frac{\sigma}{\mu}$$

Characteristics of the Gamma Distribution Model:

$$G(x,\sigma,\lambda) := \frac{1}{\sigma\cdot\Gamma(\lambda)}\cdot\left(\frac{x}{\sigma}\right)^{\lambda-1}\cdot\exp\left(\frac{-x}{\sigma}\right)$$

$$G'(x,\sigma,\lambda) := \frac{1}{\Gamma(\lambda)}\cdot\int_0^{x\cdot\sigma^{-1}} z^{\lambda-1}\cdot\exp(-z)\,dz$$

$$EXG(\sigma,\lambda) := \sigma\cdot\lambda \qquad SDG(\sigma,\lambda) := \sqrt{\sigma^2\cdot\lambda}$$

$$\sigma G(\mu1,\mu2) := \mu2\cdot\mu1^{-1} \qquad \lambda G(\mu1,\mu2) := \mu1^2\cdot\mu2^{-1}$$

$$\gamma1G(\lambda) := 2\cdot\lambda^{-0.5} \qquad \gamma2G(\lambda) := 3\cdot\left(1+2\cdot\lambda^{-1}\right)$$

$$\mu2G(\sigma,\lambda) := \sigma^2\cdot\lambda$$

$$\mu3G(\mu1,\mu2) := \gamma1G(\lambda G(\mu1,\mu2))\cdot\mu2^{1.5}$$

$$\mu4G(\mu1,\mu2) := \gamma2G(\lambda G(\mu1,\mu2))\cdot\mu2^2$$

Characteristics of the Triangular Distribution Model:

$$T(x,L,M,H) := \begin{cases} \text{return } \dfrac{2\cdot(x-L)}{(H-L)\cdot(M-L)} & \text{if } L\le x<M \\ \text{return } \dfrac{2\cdot(H-x)}{(H-L)\cdot(H-M)} & \text{if } M\le x\le H \\ 0 & \end{cases}$$

$$EXT(L,M,H) := \frac{L+M+H}{3}$$

$$\mu2T(L,M,H) := \frac{1}{18}\cdot\left(L^2+M^2+H^2-L\cdot H-M\cdot H-M\cdot L\right)$$

$$\mu3T(L,M,H) := \frac{-1}{270}\cdot(L+H-2\cdot M)\cdot(M+L-2\cdot H)\cdot(M+H-2\cdot L)$$

$$\mu4T(L,M,H) := \frac{1}{135}\cdot\left(L^2+H^2+M^2-L\cdot M-L\cdot H-H\cdot M\right)^2$$

Characteristics of variables used in the example

Land value: $L := 10 \cdot 10^6$

Design variables (cost D, and duration Td):

D $\mu x_1 := 1.28 \cdot 10^6$ $\mu 2x_1 := (0.1 \cdot 10^6)^2$ $\mu 3x_1 := \mu 3L(\mu x_1, \mu 2x_1)$ $\mu 4x_1 := \mu 4L(\mu x_1, \mu 2x_1)$ $cv(\mu x_1, \sqrt{\mu 2x_1}) = 0.078125$ $D := \mu x_1$

$\sigma L(\mu x_1, \mu 2x_1) = 0.078006$ $\mu L(\mu x_1, \mu 2x_1) = 14.059328$

Td $\mu x_2 := 0.5$ $\mu 2x_2 := 0.025^2$ $\mu 3x_2 := \mu 3L(\mu x_2, \mu 2x_2)$ $\mu 4x_2 := \mu 4L(\mu x_2, \mu 2x_2)$ $cv(\mu x_2, \sqrt{\mu 2x_2}) = 0.05$ $Td := \mu x_2$

$\sigma L(\mu x_2, \mu 2x_2) = 0.049969$ $\mu L(\mu x_2, \mu 2x_2) = -0.694396$

Construction variables (cost C, duration Tc, inflation):

C $\mu x_3 := 30 \cdot 10^6$ $\mu 2x_3 := (7.5 \cdot 10^6)^2$ $\mu 3x_3 := \mu 3L(\mu x_3, \mu 2x_3)$ $\mu 4x_3 := \mu 4L(\mu x_3, \mu 2x_3)$ $cv(\mu x_3, \sqrt{\mu 2x_3}) = 0.25$ $C := \mu x_3$

$\sigma L(\mu x_3, \mu 2x_3) = 0.246221$ $\mu L(\mu x_3, \mu 2x_3) = 17.186396$

Tc $\mu x_4 := 1.5$ $\mu 2x_4 := 0.25^2$ $\mu 3x_4 := \mu 3L(\mu x_4, \mu 2x_4)$ $\mu 4x_4 := \mu 4L(\mu x_4, \mu 2x_4)$ $cv(\mu x_4, \sqrt{\mu 2x_4}) = 0.166667$ $Tc := \mu x_4$

$\sigma L(\mu x_4, \mu 2x_4) = 0.165526$ $\mu L(\mu x_4, \mu 2x_4) = 0.391766$

θ $\mu x_5 := EXT(0.025, 0.035, 0.045)$ $\mu x_5 = 0.035$ $\mu 3x_5 := \mu 3T(0.025, 0.035, 0.045)$ $\theta := \mu x_5$

$\mu 2x_5 := \mu 2T(0.025, 0.035, 0.045)$ $\sqrt{\mu 2x_5} = 4.082483 \cdot 10^{-3}$ $\mu 4x_5 := \mu 4T(0.025, 0.035, 0.045)$ $cv(\mu x_5, \sqrt{\mu 2x_5}) = 0.116642$

Revenue variables (selling price S, duration To, number of units N, and discount rate r):

S $\mu x_6 := EXT(1.5 \cdot 10^6, 1.7 \cdot 10^6, 1.8 \cdot 10^6)$ $\mu x_6 = 1.666667 \cdot 10^6$ $\mu 3x_6 := \mu 3T(1.5 \cdot 10^6, 1.7 \cdot 10^6, 1.8 \cdot 10^6)$ $S := \mu x_6$

$\mu 2x_6 := \mu 2T(1.5 \cdot 10^6, 1.7 \cdot 10^6, 1.8 \cdot 10^6)$ $\sqrt{\mu 2x_6} = 6.236096 \cdot 10^4$ $\mu 4x_6 := \mu 4T(1.5 \cdot 10^6, 1.7 \cdot 10^6, 1.8 \cdot 10^6)$ $cv(\mu x_6, \sqrt{\mu 2x_6}) = 0.037417$

To $\mu x_7 := 1.5$ $\mu 2x_7 := .25^2$ $\mu 3x_7 := \mu 3G(\mu x_7, \mu 2x_7)$ $\mu 4x_7 := \mu 4G(\mu x_7, \mu 2x_7)$ $cv(\mu x_7, \sqrt{\mu 2x_7}) = 0.166667$ $To := \mu x_7$

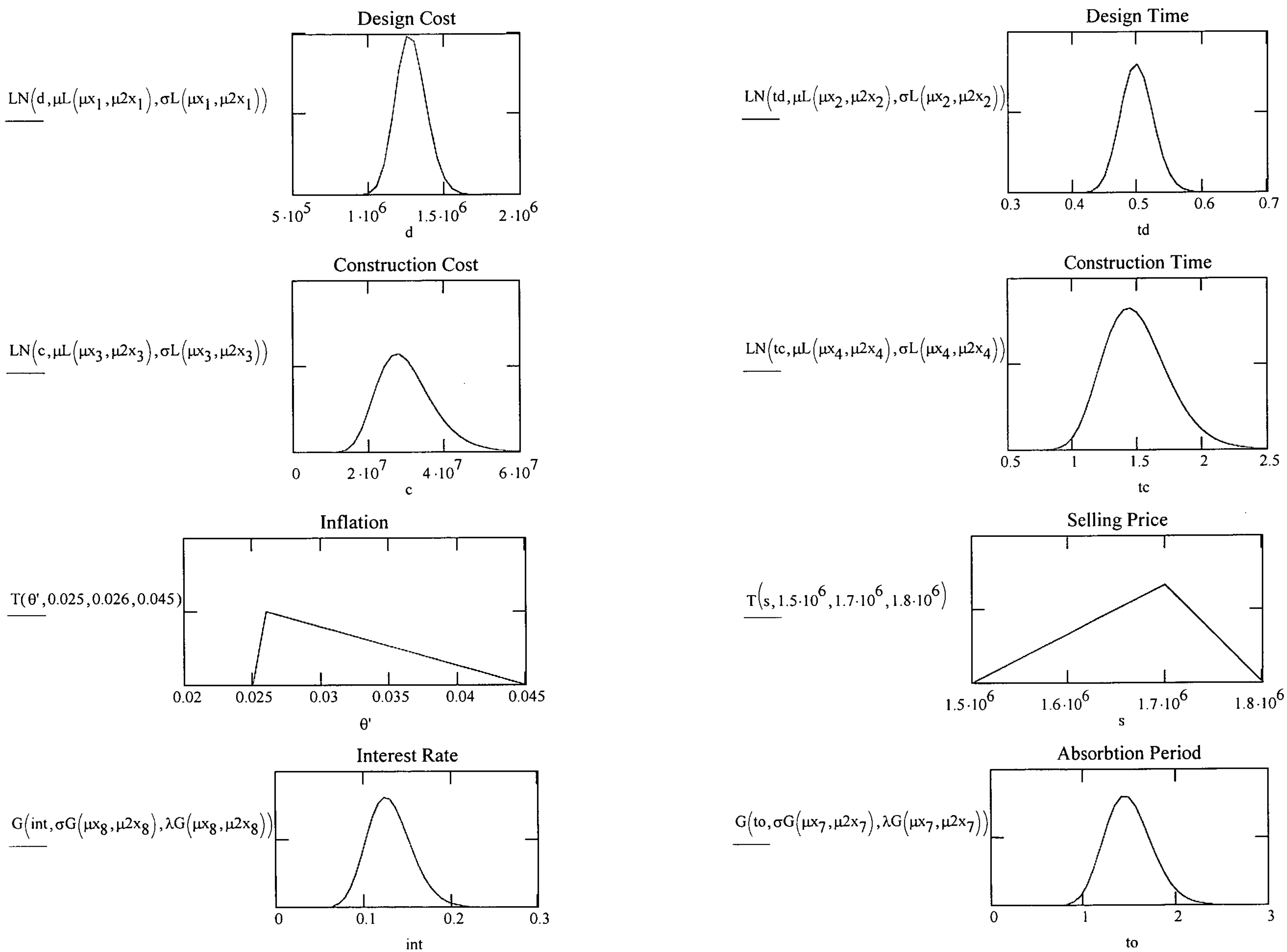
$\sigma G(\mu x_7, \mu 2x_7) = 0.041667$ $\lambda G(\mu x_7, \mu 2x_7) = 36$

$N := 45$ $r := 0.17$

Loan variables (interest i, and loan value Tloan):

i $\mu x_8 := 0.13$ $\mu 2x_8 := .025^2$ $\mu 3x_8 := \mu 3G(\mu x_8, \mu 2x_8)$ $\mu 4x_8 := \mu 4G(\mu x_8, \mu 2x_8)$ $cv(\mu x_8, \sqrt{\mu 2x_8}) = 0.192308$ $i := \mu x_8$

$Tloan := 0.55 \cdot (D + C)$ $\sigma G(\mu x_8, \mu 2x_8) = 4.807692 \cdot 10^{-3}$ $\lambda G(\mu x_8, \mu 2x_8) = 27.04$


Design Cost
LN(d, μL(μx1, μ2x1), σL(μx1, μ2x1))
5·10^5
1·10^6
1.5·10^6
2·10^6
d
Construction Cost
LN(c, μL(μx3, μ2x3), σL(μx3, μ2x3))
0
2·10^7
4·10^7
6·10^7
c
Inflation
T(θ', 0.025, 0.026, 0.045)
0.02
0.025
0.03
0.035
0.04
0.045
θ'
Interest Rate
G(int, σG(μx8, μ2x8), λG(μx8, μ2x8))
0
0.1
0.2
0.3
int
Design Time
LN(td, μL(μx2, μ2x2), σL(μx2, μ2x2))
0.3
0.4
0.5
0.6
0.7
td
Construction Time
LN(tc, μL(μx4, μ2x4), σL(μx4, μ2x4))
0.5
1
1.5
2
2.5
tc
Selling Price
T(s, 1.5·10^6, 1.7·10^6, 1.8·10^6)
1.5·10^6
1.6·10^6
1.7·10^6
1.8·10^6
s
Absorbtion Period
G(to, σG(μx7, μ2x7), λG(μx7, μ2x7))
0
1
2
3
to

B.2 Net Present Value Formulation, Risk Analysis and Sensitivity Analysis

NPV formulation

Capital expenditures and revenues

$$\mathrm{Design}(D, Td, r) := \int_0^{Td} \frac{-D}{Td} \cdot e^{-r \cdot t}\, dt \qquad \mathrm{Design}(D, Td, r) := \frac{-D}{Td} \cdot \left(\frac{e^{-r \cdot Td} - 1}{-r} \right)$$

$$\mathrm{Construction}(C, Tc, Td, \theta, r) := e^{(\theta - r) \cdot Td} \cdot \int_0^{Tc} \frac{-C}{Tc} \cdot e^{(\theta - r) \cdot t}\, dt \qquad \mathrm{Construction}(C, Tc, Td, \theta, r) := e^{(\theta - r) \cdot Td} \cdot \frac{-C}{Tc} \cdot \left[\frac{e^{(\theta - r) \cdot Tc} - 1}{\theta - r} \right]$$

$$\mathrm{Revenue}(S, Tc, Td, To, r) := e^{-r \cdot (Td + Tc)} \cdot \int_0^{To} \frac{S \cdot N}{To} \cdot e^{-r \cdot t}\, dt \qquad \mathrm{Revenue}(S, Tc, Td, To, r) := e^{-r \cdot (Td + Tc)} \cdot \frac{S \cdot N}{To} \cdot \left(\frac{e^{-r \cdot To} - 1}{-r} \right)$$

Loan calculations

$$ie(i) := \left(1 + \frac{i}{1 \cdot 2} \right)^{1} - 1 \qquad ie(i) = 0.065$$

effective interest rate per payment period, i.e. 6-month

$$\mathrm{Loan}(i, r) := \mathrm{Tloan} \cdot e^{-r \cdot 0.5} - \sum_{j=1}^{4} \frac{\mathrm{Tloan}}{4} \cdot e^{-r \cdot (1.5 + j \cdot 0.5)} - \mathrm{Tloan} \cdot ie(i) \cdot \sum_{j=1}^{3} e^{-r \cdot (0.5 + j \cdot 0.5)} - \frac{\mathrm{Tloan}}{4} \cdot ie(i) \cdot \left(3 \cdot e^{-r \cdot 2.5} + 2 \cdot e^{-r \cdot 3} + 1 \cdot e^{-r \cdot 3.5} \right)$$

$$\mathrm{Loan}(i, r) = 1.32969 \cdot 10^{6}$$

Net present value function at $r = 0.17$

$$\mathrm{NPV}(D, Td, C, Tc, \theta, S, To, i, r) := -L + \mathrm{Design}(D, Td, r) + \mathrm{Construction}(C, Tc, Td, \theta, r) + \mathrm{Revenue}(S, Tc, Td, To, r) + \mathrm{Loan}(i, r)$$

$$\mathrm{NPV}(D, Td, C, Tc, \theta, S, To, i, r) = 1.183759 \cdot 10^{7}$$

deterministic value of NPV based on average values

NPV risk analysis

First derivatives of NPV using Mathcad algorithm

$$d1Z_1 := \frac{d}{dD}NPV(D,Td,C,Tc,\theta,S,To,i,r)$$

$$d1Z_2 := \frac{d}{dTd}NPV(D,Td,C,Tc,\theta,S,To,i,r)$$

$$d1Z_3 := \frac{d}{dC}NPV(D,Td,C,Tc,\theta,S,To,i,r)$$

$$d1Z_4 := \frac{d}{dTc}NPV(D,Td,C,Tc,\theta,S,To,i,r)$$

$$d1Z_5 := \frac{d}{d\theta}NPV(D,Td,C,Tc,\theta,S,To,i,r)$$

$$d1Z_6 := \frac{d}{dS}NPV(D,Td,C,Tc,\theta,S,To,i,r)$$

$$d1Z_7 := \frac{d}{dTo}NPV(D,Td,C,Tc,\theta,S,To,i,r)$$

$$d1Z_8 := \frac{d}{di}NPV(D,Td,C,Tc,\theta,S,To,i,r)$$

Second derivatives of NPV using Mathcad algorithm

$$d2Z_1 := \frac{d^2}{dD^2}NPV(D,Td,C,Tc,\theta,S,To,i,r)$$

$$d2Z_2 := \frac{d^2}{dTd^2}NPV(D,Td,C,Tc,\theta,S,To,i,r)$$

$$d2Z_3 := \frac{d^2}{dC^2}NPV(D,Td,C,Tc,\theta,S,To,i,r)$$

$$d2Z_4 := \frac{d^2}{dTc^2}NPV(D,Td,C,Tc,\theta,S,To,i,r)$$

$$d2Z_5 := \frac{d^2}{d\theta^2}NPV(D,Td,C,Tc,\theta,S,To,i,r)$$

$$d2Z_6 := \frac{d^2}{dS^2}NPV(D,Td,C,Tc,\theta,S,To,i,r)$$

$$d2Z_7 := \frac{d^2}{dTo^2}NPV(D,Td,C,Tc,\theta,S,To,i,r)$$

$$d2Z_8 := \frac{d^2}{di^2}NPV(D,Td,C,Tc,\theta,S,To,i,r)$$

$$d1Z^T = \begin{bmatrix} 0 & -0.958679 & -4.480597\cdot10^6 & -0.846164 & -6.354696\cdot10^6 & -3.108904\cdot10^7 & 28.271957 & -3.835157\cdot10^6 & -2.803235\cdot10^7 \end{bmatrix}$$ Mathcad algorithm

$$dF1Z^T = \begin{bmatrix} 0 & -0.958679 & -4.480597\cdot10^6 & -0.846164 & -6.354696\cdot10^6 & -3.108904\cdot10^7 & 28.271957 & -3.835157\cdot10^6 & -2.803235\cdot10^7 \end{bmatrix}$$ Finite difference method

$$d2Z^T = \begin{bmatrix} 0 & 0 & 8.87555\cdot10^5 & -3.928213\cdot10^{-21} & 1.215302\cdot10^6 & -4.282482\cdot10^7 & 2.497487\cdot10^{-19} & 4.252614\cdot10^5 & 0 \end{bmatrix}$$ Mathcad algorithm

$$dF2Z^T = \begin{bmatrix} 0 & -4.547474\cdot10^{-13} & 8.875519\cdot10^5 & -8.278423\cdot10^{-16} & 1.215301\cdot10^6 & -4.282594\cdot10^7 & -2.682209\cdot10^{-13} & 4.252616\cdot10^5 & 0 \end{bmatrix}$$ Finite difference

NPV four moments

$n := 8$

$Z_{\mu} := NPV\left(\mu x_1, \mu x_2, \mu x_3, \mu x_4, \mu x_5, \mu x_6, \mu x_7, \mu x_8, r\right)$ (NPV calculated at the mean values)

$Z_{\mu} = 1.183759 \cdot 10^{7}$

$$EX_Z := NPV\left(\mu x_1, \mu x_2, \mu x_3, \mu x_4, \mu x_5, \mu x_6, \mu x_7, \mu x_8, r\right) + \frac{1}{2} \cdot \sum_{j=1}^{n} d2Z_j \cdot \mu 2x_j$$

$EX_Z = 1.188878 \cdot 10^{7}$

$$\mu 2_Z := \sum_{i=1}^{n} \left(d1Z_i\right)^2 \cdot \mu 2x_i + \frac{1}{4} \cdot \sum_{i=1}^{n} \left(d2Z_i\right)^2 \cdot \mu 4x_i + \frac{-1}{4} \cdot \left(\sum_{i=1}^{n} d2Z_i \cdot \mu 2x_i\right)^2 + \sum_{i=1}^{n} d1Z_i \cdot d2Z_i \cdot \mu 3x_i$$

$\sqrt{\mu 2_Z} = 6.876679 \cdot 10^{6}$

$\mu 2_Z = 4.728871 \cdot 10^{13}$

$$\mu 3_Z := \sum_{i=1}^{n} \left(d1Z_i\right)^3 \cdot \left(\mu 3x_i\right) + \frac{1}{4} \cdot \left(\sum_{i=1}^{n} d2Z_i \cdot \mu 2x_i\right)^3 + \frac{3}{2} \cdot \left[\sum_{i=1}^{n} \left(d1Z_i\right)^2 \cdot d2Z_i \cdot \mu 4x_i\right] + \left[\sum_{i=1}^{n} \left(d2Z_i\right) \cdot \mu 2x_i\right] \cdot \left[\frac{-3}{2} \cdot \sum_{i=1}^{n} \left(d1Z_i\right)^2 \cdot \mu 2x_i\right] \ldots$$
$$+ \left[\sum_{i=1}^{n} \left(d2Z_i\right) \cdot \mu 2x_i\right] \cdot \left[\frac{-3}{2} \cdot \sum_{i=1}^{n} \left(d1Z_i\right) \cdot d2Z_i \cdot \mu 3x_i\right] + \left[\sum_{i=1}^{n} \left(d2Z_i\right) \cdot \mu 2x_i\right] \cdot \left[\frac{-3}{8} \cdot \sum_{i=1}^{n} \left(d2Z_i\right)^2 \cdot \left(\mu 4x_i\right)\right]$$

$\mu 3_Z = -2.059637 \cdot 10^{20}$

$$\mu 4_Z := \left[\sum_{i=1}^{n} \left(d1Z_i\right)^4 \cdot \left(\mu 4x_i\right) + \frac{-3}{16} \cdot \left(\sum_{i=1}^{n} d2Z_i \cdot \mu 2x_i\right)^4 + \left[-\sum_{i=1}^{n} \left(d2Z_i\right) \cdot \mu 2x_i\right] \cdot \left[2 \cdot \sum_{i=1}^{n} \left(d1Z_i\right)^3 \cdot \mu 3x_i\right] + \left[\sum_{i=1}^{n} \left(d2Z_i\right) \cdot \mu 2x_i\right]^2 \cdot \left[\frac{3}{2} \cdot \sum_{i=1}^{n} \left(d1Z_i\right)^2 \cdot \mu 2x_i\right]\right] \ldots$$
$$+ \left[\sum_{i=1}^{n} \left(d2Z_i\right) \cdot \mu 2x_i\right]^2 \cdot \left[\frac{3}{2} \cdot \sum_{i=1}^{n} \left(d1Z_i\right) \cdot d2Z_i \cdot \mu 3x_i\right] + \left[-\sum_{i=1}^{n} \left(d2Z_i\right) \cdot \mu 2x_i\right] \cdot \left[3 \cdot \sum_{i=1}^{n} \left(d1Z_i\right)^2 \cdot d2Z_i \cdot \mu 4x_i\right] + \left[\sum_{i=1}^{n} \left(d2Z_i\right) \cdot \mu 2x_i\right]^2 \cdot \left[\frac{3}{8} \cdot \sum_{i=1}^{n} \left(d2Z_i\right)^2 \cdot \left(\mu 4x_i\right)\right]$$

$\mu 4_Z = 6.675718 \cdot 10^{27}$

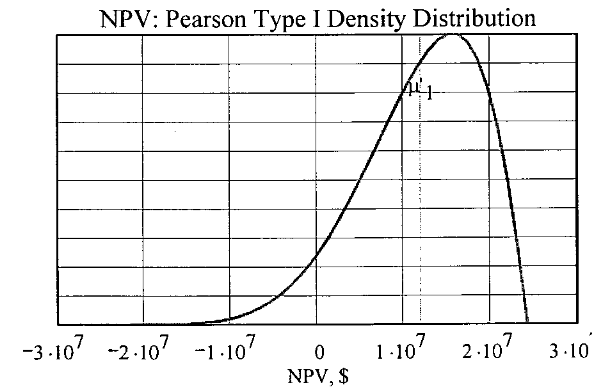
NPV distribution

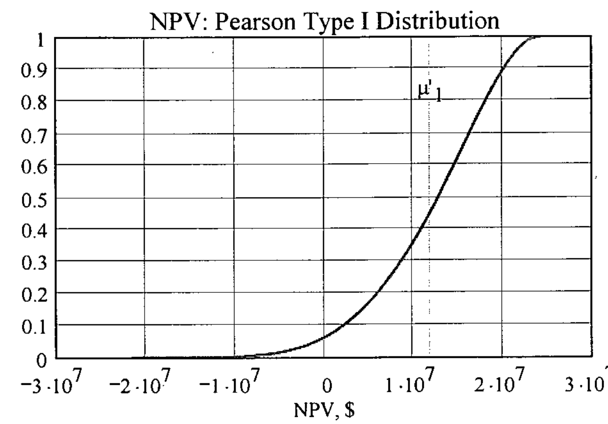
$\gamma 1 := \dfrac{\mu 3_Z}{\mu 2_Z^{\;1.5}}$ $\quad\gamma 2 := \dfrac{\mu 4_Z}{\mu 2_Z^{\;2}}$ $\quad\beta 1 := \gamma 1^2$ $\quad\beta 2 := \gamma 2$

$\gamma 1 = -0.633366$ $\quad\gamma 2 = 2.985265$ $\quad\beta 1 = 0.401152$ $\quad\beta 2 = 2.985265$

$$k\_criterion := \frac{\beta 1\cdot(\beta 2 + 3)^2}{4\cdot(4\cdot\beta 2 - 3\cdot\beta 1)\cdot(2\cdot\beta 2 - 3\cdot\beta 1 - 6)}$$

$k\_criterion = -0.271376$ k is less than zero, then Pearson Type I should be selected





NPV sensitivity analysis

Normalized sensitivity coefficients of design and construction variables

D $\quad d1Z_1\cdot\dfrac{D}{Z_\mu} = -0.103662$ $\quad$ Td $\quad d1Z_2\cdot\dfrac{Td}{Z_\mu} = -0.189253$

C $\quad d1Z_3\cdot\dfrac{C}{Z_\mu} = -2.144434$ $\quad$ Tc $\quad d1Z_4\cdot\dfrac{Tc}{Z_\mu} = -0.805235$

θ $\quad d1Z_5\cdot\dfrac{\theta}{Z_\mu} = -0.09192$

Normalized sensitivity coefficients of revenue variables

S $\quad d1Z_6\cdot\dfrac{S}{Z_\mu} = 3.980535$ $\quad$ To $\quad d1Z_7\cdot\dfrac{To}{Z_\mu} = -0.485972$

Normalized sensitivity coefficients of finance variables

$i := 0.13$ $\quad d1Z_8\cdot\dfrac{i}{Z_\mu} = -0.30785$

B.3 Internal Rate Of Return On Equity

Design and construction discounted functions

$$\mathrm{Design}(D, Td, r) := \int_0^{Td} \frac{-D}{Td} \cdot e^{-r \cdot t}\, dt \qquad \mathrm{Design}(D, Td, r) := \frac{-D}{Td} \cdot \left(\frac{e^{-r \cdot Td} - 1}{-r} \right)$$

$$\mathrm{Construction}(C, Tc, Td, \theta, r) := e^{(\theta - r) \cdot Td} \cdot \int_0^{Tc} \frac{-C}{Tc} \cdot e^{(\theta - r) \cdot t}\, dt \qquad \mathrm{Construction}(C, Tc, Td, \theta, r) := e^{(\theta - r) \cdot Td} \cdot \frac{-C}{Tc} \cdot \left[\frac{e^{(\theta - r) \cdot Tc} - 1}{\theta - r} \right]$$

$$\mathrm{Revenue}(S, Tc, Td, To, r) := e^{-r \cdot (Td + Tc)} \cdot \int_0^{To} \frac{S \cdot N}{To} \cdot e^{-r \cdot t}\, dt \qquad \mathrm{Revenue}(S, Tc, Td, To, r) := e^{-r \cdot (Td + Tc)} \cdot \frac{S \cdot N}{To} \cdot \left(\frac{e^{-r \cdot To} - 1}{-r} \right)$$

Loan calculations

$$ie(i) := \left(1 + \frac{i}{1 \cdot 2}\right)^{1} - 1 \qquad ie(i) = 0.065$$

effective interest rate per payment period, i.e. 6-month

$$\mathrm{Loan}(i, r) := Tloan \cdot e^{-r \cdot 0.5} - \sum_{j=1}^{4} \frac{Tloan}{4} \cdot e^{-r \cdot (1.5 + j \cdot 0.5)} - Tloan \cdot ie(i) \cdot \sum_{j=1}^{3} e^{-r \cdot (0.5 + j \cdot 0.5)} - \frac{Tloan}{4} \cdot ie(i) \cdot \left(3 \cdot e^{-r \cdot 2.5} + 2 \cdot e^{-r \cdot 3} + 1 \cdot e^{-r \cdot 3.5}\right)$$

$$\mathrm{Loan}(i, r) = 1.32969 \cdot 10^{6}$$

NPV and IRR formulations

$$\mathrm{NPV}(D, Td, C, Tc, \theta, S, To, i, r) := -L + \mathrm{Design}(D, Td, r) + \mathrm{Construction}(C, Tc, Td, \theta, r) + \mathrm{Revenue}(S, Tc, Td, To, r) + \mathrm{Loan}(i, r)$$

$$\mathrm{NPV}(D, Td, C, Tc, \theta, S, To, i, r) = 1.183759 \cdot 10^{7}$$

deterministic value of NPV based on average values

$$irr' := 0.25$$

$$\mathrm{IRR}(D, Td, C, Tc, \theta, S, To, i) := \mathrm{root}(\mathrm{NPV}(D, Td, C, Tc, \theta, S, To, i, irr'), irr')$$

$$\mathrm{IRR}(D, Td, C, Tc, \theta, S, To, i) = 0.423796$$

IRR risk analysis

First derivatives of IRR using Mathcad algorithm

$$d1Z_1 := \frac{d}{dD} IRR(D, Td, C, Tc, \theta, S, To, i)$$

$$d1Z_2 := \frac{d}{dTd} IRR(D, Td, C, Tc, \theta, S, To, i)$$

$$d1Z_3 := \frac{d}{dC} IRR(D, Td, C, Tc, \theta, S, To, i)$$

$$d1Z_4 := \frac{d}{dTc} IRR(D, Td, C, Tc, \theta, S, To, i)$$

$$d1Z_5 := \frac{d}{d\theta} IRR(D, Td, C, Tc, \theta, S, To, i)$$

$$d1Z_6 := \frac{d}{dS} IRR(D, Td, C, Tc, \theta, S, To, i)$$

$$d1Z_7 := \frac{d}{dTo} IRR(D, Td, C, Tc, \theta, S, To, i)$$

$$d1Z_8 := \frac{d}{di} IRR(D, Td, C, Tc, \theta, S, To, i)$$

Second derivatives of IRR using Mathcad algorithm

$$d2Z_1 := \frac{d^2}{dD^2} IRR(D, Td, C, Tc, \theta, S, To, i)$$

$$d2Z_2 := \frac{d^2}{dTd^2} IRR(D, Td, C, Tc, \theta, S, To, i)$$

$$d2Z_3 := \frac{d^2}{dC^2} IRR(D, Td, C, Tc, \theta, S, To, i)$$

$$d2Z_4 := \frac{d^2}{dTc^2} IRR(D, Td, C, Tc, \theta, S, To, i)$$

$$d2Z_5 := \frac{d^2}{d\theta^2} IRR(D, Td, C, Tc, \theta, S, To, i)$$

$$d2Z_6 := \frac{d^2}{dS^2} IRR(D, Td, C, Tc, \theta, S, To, i)$$

$$d2Z_7 := \frac{d^2}{dTo^2} IRR(D, Td, C, Tc, \theta, S, To, i)$$

$$d2Z_8 := \frac{d^2}{di^2} IRR(D, Td, C, Tc, \theta, S, To, i)$$

$$d1Z^T = \begin{bmatrix} 0 & -3.03249 \cdot 10^{-8} & -0.086334 & -2.099291 \cdot 10^{-8} & -0.228531 & -0.741581 & 4.801283 \cdot 10^{-7} & -0.151718 & -0.59905 \end{bmatrix}$$ Mathcad Algorithm

$$d2Z^T = \begin{bmatrix} 0 & 2.66748 \cdot 10^{-15} & -8.446231 \cdot 10^{-3} & 4.530555 \cdot 10^{-16} & 0.188171 & -0.596166 & -4.516144 \cdot 10^{-13} & 0.091432 & -0.02578 \end{bmatrix}$$ Mathcad Algorithm

Four moments of IRR on equity:

$n := 8$

$Z_{\mu} := \mathrm{IRR}\left(\mu x_1, \mu x_2, \mu x_3, \mu x_4, \mu x_5, \mu x_6, \mu x_7, \mu x_8\right)$ (IRR calculated at the mean values) $\qquad Z_{\mu} = 0.423796$

$$EX_Z := \mathrm{IRR}\left(\mu x_1, \mu x_2, \mu x_3, \mu x_4, \mu x_5, \mu x_6, \mu x_7, \mu x_8\right) + \frac{1}{2} \cdot \sum_{j=1}^{n} d2Z_j \cdot \mu 2x_j \qquad EX_Z = 0.444395$$

$$\mu 2_Z := \sum_{i=1}^{n} \left(d1Z_i\right)^2 \cdot \mu 2x_i + \frac{1}{4} \cdot \sum_{i=1}^{n} \left(d2Z_i\right)^2 \cdot \mu 4x_i + \frac{-1}{4} \cdot \left(\sum_{i=1}^{n} d2Z_i \cdot \mu 2x_i\right)^2 + \sum_{i=1}^{n} d1Z_i \cdot d2Z_i \cdot \mu 3x_i$$

$\sqrt{\mu 2_Z} = 0.165985$

$\mu 2_Z = 0.027551$

$$\mu 3_Z := \sum_{i=1}^{n} \left(d1Z_i\right)^3 \cdot \left(\mu 3x_i\right) + \frac{1}{4} \left(\sum_{i=1}^{n} d2Z_i \cdot \mu 2x_i\right)^3 + \frac{3}{2} \cdot \left[\sum_{i=1}^{n} \left(d1Z_i\right)^2 \cdot d2Z_i \cdot \mu 4x_i\right] + \left[\sum_{i=1}^{n} \left(d2Z_i\right) \cdot \mu 2x_i\right] \cdot \left[\frac{-3}{2} \cdot \sum_{i=1}^{n} \left(d1Z_i\right)^2 \cdot \mu 2x_i\right] \ldots$$
$$+ \left[\sum_{i=1}^{n} \left(d2Z_i\right) \cdot \mu 2x_i\right] \cdot \left[\frac{-3}{2} \cdot \sum_{i=1}^{n} \left(d1Z_i\right) \cdot d2Z_i \cdot \mu 3x_i\right] + \left[\sum_{i=1}^{n} \left(d2Z_i\right) \cdot \mu 2x_i\right] \cdot \left[\frac{-3}{8} \cdot \sum_{i=1}^{n} \left(d2Z_i\right)^2 \cdot \left(\mu 4x_i\right)\right]$$

$\mu 3_Z = -7.416094 \cdot 10^{-4}$

$$\mu 4_Z := \left[\sum_{i=1}^{n} \left(d1Z_i\right)^4 \cdot \left(\mu 4x_i\right) + \frac{-3}{16} \left(\sum_{i=1}^{n} d2Z_i \cdot \mu 2x_i\right)^4 + \left[-\sum_{i=1}^{n} \left(d2Z_i\right) \cdot \mu 2x_i\right] \cdot \left[2 \cdot \sum_{i=1}^{n} \left(d1Z_i\right)^3 \cdot \mu 3x_i\right] + \left[\sum_{i=1}^{n} \left(d2Z_i\right) \cdot \mu 2x_i\right]^2 \cdot \left[\frac{3}{2} \cdot \sum_{i=1}^{n} \left(d1Z_i\right)^2 \cdot \mu 2x_i\right]\right] \ldots$$
$$+ \left[\sum_{i=1}^{n} \left(d2Z_i\right) \cdot \mu 2x_i\right]^2 \cdot \left[\frac{3}{2} \cdot \sum_{i=1}^{n} \left(d1Z_i\right) \cdot d2Z_i \cdot \mu 3x_i\right] + \left[-\sum_{i=1}^{n} \left(d2Z_i\right) \cdot \mu 2x_i\right] \cdot \left[3 \cdot \sum_{i=1}^{n} \left(d1Z_i\right)^2 \cdot d2Z_i \cdot \mu 4x_i\right] + \left[\sum_{i=1}^{n} \left(d2Z_i\right) \cdot \mu 2x_i\right]^2 \cdot \left[\frac{3}{8} \cdot \sum_{i=1}^{n} \left(d2Z_i\right)^2 \cdot \left(\mu 4x_i\right)\right]$$

$\mu 4_Z = 2.531245 \cdot 10^{-3}$

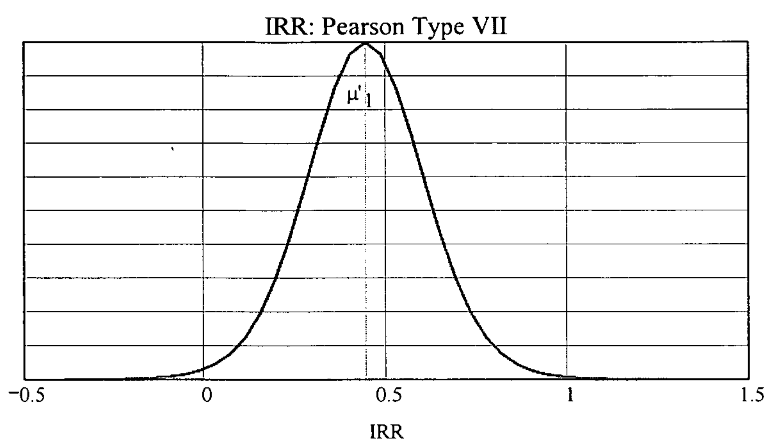
Pearson distribution of IRR on equity

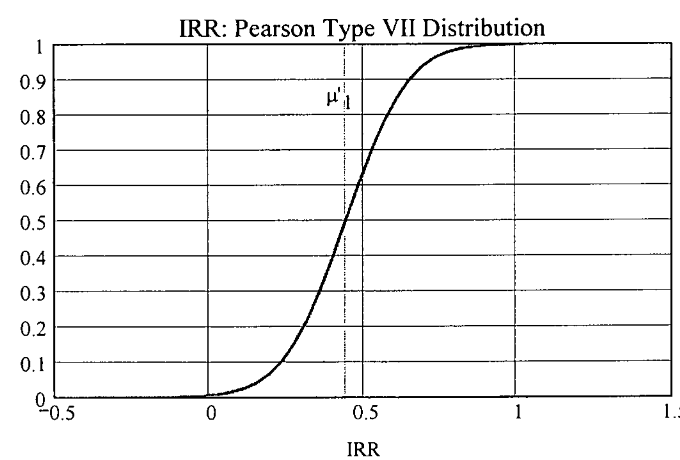
$\gamma 1 := \frac{\mu 3_Z}{{\mu 2_Z}^{1.5}}$ $\quad \gamma 2 := \frac{\mu 4_Z}{{\mu 2_Z}^{2}}$ $\quad \beta 1 := \gamma 1^2$ $\quad \beta 2 := \gamma 2$

$\gamma 1 = -0.16217$ $\quad \gamma 2 = 3.334726$ $\quad \beta 1 = 0.026299$ $\quad \beta 2 = 3.334726$

$$k\_criterion := \frac{\beta 1 \cdot (\beta 2 + 3)^2}{4 \cdot (4 \cdot \beta 2 - 3 \cdot \beta 1) \cdot (2 \cdot \beta 2 - 3 \cdot \beta 1 - 6)}$$

$k\_criterion = 0.033692$ Since k and $\beta 1$ approach zero while $\beta 2 > 3$, then Pearson Distribution VII is selected. Type IV could be used as well. Type VII is a special case of Type IV.





B. 4 Internal Rate Of Return On Total Capital

$$\mathrm{Design}(D, Td, r) := \int_0^{Td} \frac{D}{Td} \cdot e^{-r \cdot t}\, dt \qquad \mathrm{Design}(D, Td, r) := \frac{-D}{Td} \cdot \left(\frac{e^{-r \cdot Td} - 1}{-r} \right)$$

$$\mathrm{Construction}(C, Tc, Td, \theta, r) := e^{(\theta - r) \cdot Td} \cdot \int_0^{Tc} \frac{C}{Tc} \cdot e^{(\theta - r) \cdot t}\, dt \qquad \mathrm{Construction}(C, Tc, Td, \theta, r) := e^{(\theta - r) \cdot Td} \cdot \frac{-C}{Tc} \cdot \left[\frac{e^{(\theta - r) \cdot Tc} - 1}{\theta - r} \right]$$

$$\mathrm{Revenue}(S, Tc, Td, To, r) := e^{-r \cdot (Td + Tc)} \cdot \int_0^{To} \frac{S \cdot N}{To} \cdot e^{-r \cdot t}\, dt \qquad \mathrm{Revenue}(S, Tc, Td, To, r) := e^{-r \cdot (Td + Tc)} \cdot \frac{S \cdot N}{To} \cdot \left(\frac{e^{-r \cdot To} - 1}{-r} \right)$$

$$\mathrm{NPV}(D, Td, C, Tc, \theta, S, To, r) := -L + \mathrm{Design}(D, Td, r) + \mathrm{Construction}(C, Tc, Td, \theta, r) + \mathrm{Revenue}(S, Tc, Td, To, r)$$

$$\mathrm{NPV}(D, Td, C, Tc, \theta, S, To, r) = 1.05079 \cdot 10^{7}$$

$irr' := 0.15$

$\mathrm{IRR}(D, Td, C, Tc, \theta, S, To) := \mathrm{root}(\mathrm{NPV}(D, Td, C, Tc, \theta, S, To, irr'), irr')$

$\mathrm{IRR}(D, Td, C, Tc, \theta, S, To) = 0.30439$

IRR risk analysis

First derivatives of IRR

$$d1Z_1 := \frac{d}{dD} IRR(D, Td, C, Tc, \theta, S, To)$$

$$d1Z_2 := \frac{d}{dTd} IRR(D, Td, C, Tc, \theta, S, To)$$

$$d1Z_3 := \frac{d}{dC} IRR(D, Td, C, Tc, \theta, S, To)$$

$$d1Z_4 := \frac{d}{dTc} IRR(D, Td, C, Tc, \theta, S, To)$$

$$d1Z_5 := \frac{d}{d\theta} IRR(D, Td, C, Tc, \theta, S, To)$$

$$d1Z_6 := \frac{d}{dS} IRR(D, Td, C, Tc, \theta, S, To)$$

$$d1Z_7 := \frac{d}{dTo} IRR(D, Td, C, Tc, \theta, S, To)$$

Second derivatives of IRR

$$d2Z_1 := \frac{d^2}{dD^2} IRR(D, Td, C, Tc, \theta, S, To)$$

$$d2Z_2 := \frac{d^2}{dTd^2} IRR(D, Td, C, Tc, \theta, S, To)$$

$$d2Z_3 := \frac{d^2}{dC^2} IRR(D, Td, C, Tc, \theta, S, To)$$

$$d2Z_4 := \frac{d^2}{dTc^2} IRR(D, Td, C, Tc, \theta, S, To)$$

$$d2Z_5 := \frac{d^2}{d\theta^2} IRR(D, Td, C, Tc, \theta, S, To)$$

$$d2Z_6 := \frac{d^2}{dS^2} IRR(D, Td, C, Tc, \theta, S, To)$$

$$d2Z_7 := \frac{d^2}{dTo^2} IRR(D, Td, C, Tc, \theta, S, To)$$

$$d1Z^T = \begin{bmatrix} 0 & -1.49489\cdot10^{-8} & -0.0642 & -1.15864\cdot10^{-8} & -0.11701 & -0.41698 & 3.16727\cdot10^{-7} & -0.07425 \end{bmatrix}$$

$$d2Z^T = \begin{bmatrix} 0 & 0 & 0.0179 & 0 & 0.08664 & -0.45169 & -2.03244\cdot10^{-13} & 0.03658 \end{bmatrix}$$

IRR four moments

$n := 7$

$Z_{\mu} := \mathrm{IRR}\left(\mu x_1, \mu x_2, \mu x_3, \mu x_4, \mu x_5, \mu x_6, \mu x_7\right)$ (IRR calculated at the mean values) $Z_{\mu} = 0.30439$

$$EX_Z := \mathrm{IRR}\left(\mu x_1, \mu x_2, \mu x_3, \mu x_4, \mu x_5, \mu x_6, \mu x_7\right) + \frac{1}{2}\cdot\sum_{j=1}^{n} d2Z_j\cdot\mu 2x_j$$

$EX_Z = 0.31147$

$$\mu 2_Z := \sum_{i=1}^{n}\left(d1Z_i\right)^2\cdot\mu 2x_i + \frac{1}{4}\cdot\sum_{i=1}^{n}\left(d2Z_i\right)^2\cdot\mu 4x_i + \frac{-1}{4}\cdot\left(\sum_{i=1}^{n} d2Z_i\cdot\mu 2x_i\right)^2 + \sum_{i=1}^{n} d1Z_i\cdot d2Z_i\cdot\mu 3x_i$$

$\sqrt{\mu 2_Z} = 0.09279$

$\mu 2_Z = 8.61017\cdot 10^{-3}$

$$\mu 3_Z := \sum_{i=1}^{n}\left(d1Z_i\right)^3\cdot\left(\mu 3x_i\right) + \frac{1}{4}\cdot\left(\sum_{i=1}^{n} d2Z_i\cdot\mu 2x_i\right)^3 + \frac{3}{2}\cdot\left[\sum_{i=1}^{n}\left(d1Z_i\right)^2\cdot d2Z_i\cdot\mu 4x_i\right] + \left[\sum_{i=1}^{n}\left(d2Z_i\right)\cdot\mu 2x_i\right]\cdot\left[\frac{-3}{2}\cdot\sum_{i=1}^{n}\left(d1Z_i\right)^2\cdot\mu 2x_i\right] \ldots$$
$$+ \left[\sum_{i=1}^{n}\left(d2Z_i\right)\cdot\mu 2x_i\right]\cdot\left[\frac{-3}{2}\cdot\sum_{i=1}^{n}\left(d1Z_i\right)\cdot d2Z_i\cdot\mu 3x_i\right] + \left[\sum_{i=1}^{n}\left(d2Z_i\right)\cdot\mu 2x_i\right]\cdot\left[\frac{-3}{8}\cdot\sum_{i=1}^{n}\left(d2Z_i\right)^2\cdot\left(\mu 4x_i\right)\right]$$

$\mu 3_Z = -3.42609\cdot 10^{-4}$

$$\mu 4_Z := \left[\sum_{i=1}^{n}\left(d1Z_i\right)^4\cdot\left(\mu 4x_i\right) + \frac{-3}{16}\cdot\left(\sum_{i=1}^{n} d2Z_i\cdot\mu 2x_i\right)^4 + \left[-\sum_{i=1}^{n}\left(d2Z_i\right)\cdot\mu 2x_i\right]\cdot\left[2\cdot\sum_{i=1}^{n}\left(d1Z_i\right)^3\cdot\mu 3x_i\right] + \left[\sum_{i=1}^{n}\left(d2Z_i\right)\cdot\mu 2x_i\right]^2\cdot\left[\frac{3}{2}\cdot\sum_{i=1}^{n}\left(d1Z_i\right)^2\cdot\mu 2x_i\right]\right] \ldots$$
$$+ \left[\sum_{i=1}^{n}\left(d2Z_i\right)\cdot\mu 2x_i\right]^2\cdot\left[\frac{3}{2}\cdot\sum_{i=1}^{n}\left(d1Z_i\right)\cdot d2Z_i\cdot\mu 3x_i\right] + \left[-\sum_{i=1}^{n}\left(d2Z_i\right)\cdot\mu 2x_i\right]\cdot\left[3\cdot\sum_{i=1}^{n}\left(d1Z_i\right)^2\cdot d2Z_i\cdot\mu 4x_i\right] + \left[\sum_{i=1}^{n}\left(d2Z_i\right)\cdot\mu 2x_i\right]^2\cdot\left[\frac{3}{8}\cdot\sum_{i=1}^{n}\left(d2Z_i\right)^2\cdot\left(\mu 4x_i\right)\right]$$

$\mu 4_Z = 2.41897\cdot 10^{-4}$

IRR distribution

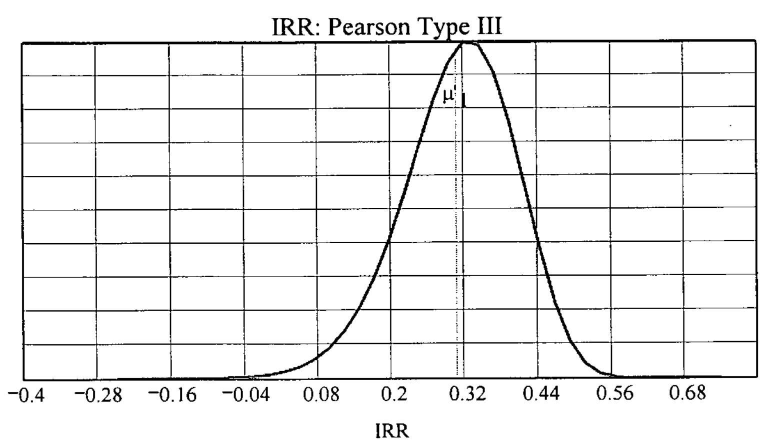
$$\gamma 1 := \frac{\mu 3_Z}{{\mu 2_Z}^{1.5}} \qquad \gamma 2 := \frac{\mu 4_Z}{{\mu 2_Z}^{2}} \qquad \beta 1 := \gamma 1^2 \qquad \beta 2 := \gamma 2$$

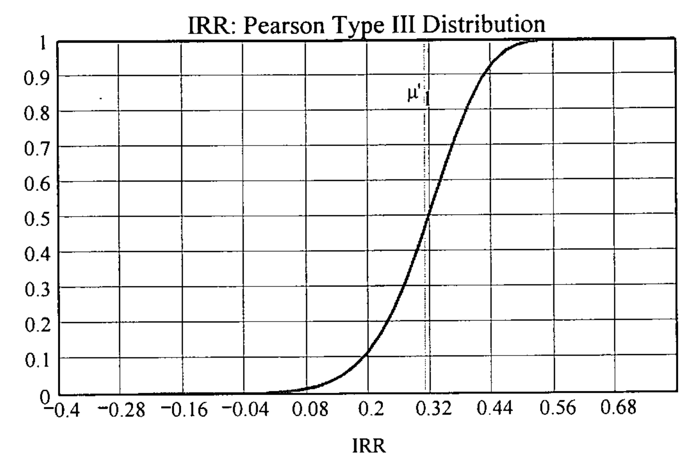
$\gamma 1 = -0.42883$ $\gamma 2 = 3.26292$ $\beta 1 = 0.18389$ $\beta 2 = 3.26292$ $2 \cdot \beta 2 = 6.52584$

$3 \cdot \beta 1 + 6 = 6.55168$

$$k\_criterion := \frac{\beta 1 \cdot (\beta 2 + 3)^2}{4 \cdot (4 \cdot \beta 2 - 3 \cdot \beta 1) \cdot (2 \cdot \beta 2 - 3 \cdot \beta 1 - 6)}$$

$k\_criterion = -5.58299$ since 2*β2 [6.525] approaches 3*β1+6 [6.55] Pearson Distribution Type III is selected





B. 5 Total Capital Expenditure

$$TC(D, Td, C, Tc, \theta) := L + \int_0^{Td} \frac{D}{Td}\,dt + e^{\theta \cdot Td} \cdot \int_0^{Tc} \frac{C}{Tc} \cdot e^{\theta \cdot t}\,dt \qquad TC(D, Td, C, Tc, \theta) := L + D + e^{\theta \cdot Td} \cdot \frac{C}{Tc} \cdot \frac{e^{\theta \cdot Tc} - 1}{\theta}$$

$$TC(D, Td, C, Tc, \theta) = 4.26252 \cdot 10^{7}$$

Risk analysis of total capital expenditure

First Derivatives of TC

$$d1Z_1 := \frac{d}{dD} TC(D, Td, C, Tc, \theta)$$

$$d1Z_2 := \frac{d}{dTd} TC(D, Td, C, Tc, \theta)$$

$$d1Z_3 := \frac{d}{dC} TC(D, Td, C, Tc, \theta)$$

$$d1Z_4 := \frac{d}{dTc} TC(D, Td, C, Tc, \theta)$$

$$d1Z_5 := \frac{d}{d\theta} TC(D, Td, C, Tc, \theta)$$

Second Derivatives of TC

$$d2Z_1 := \frac{d^2}{dD^2} TC(D, Td, C, Tc, \theta)$$

$$d2Z_2 := \frac{d^2}{dTd^2} TC(D, Td, C, Tc, \theta)$$

$$d2Z_3 := \frac{d^2}{dC^2} TC(D, Td, C, Tc, \theta)$$

$$d2Z_4 := \frac{d^2}{dTc^2} TC(D, Td, C, Tc, \theta)$$

$$d2Z_5 := \frac{d^2}{d\theta^2} TC(D, Td, C, Tc, \theta)$$

$$d1Z^T = \begin{bmatrix} 0 & 1 & 1.09708 \cdot 10^{6} & 1.04484 & 5.53341 \cdot 10^{5} & 3.93872 \cdot 10^{7} \end{bmatrix}$$

$$d2Z^T = \begin{bmatrix} 0 & 0 & 3.83979 \cdot 10^{4} & 0 & 1.29676 \cdot 10^{4} & 5.53689 \cdot 10^{7} \end{bmatrix}$$

Total cost four moments

$$n := 5$$

$$Z_\mu := TC\left(\mu x_1, \mu x_2, \mu x_3, \mu x_4, \mu x_5\right) \qquad Z_\mu = 4.26252 \cdot 10^{7}$$

$$EX_Z := TC\left(\mu x_1, \mu x_2, \mu x_3, \mu x_4, \mu x_5\right) + \frac{1}{2} \cdot \sum_{j=1}^{n} d2Z_j \cdot \mu 2x_j \qquad EX_Z = 4.26261 \cdot 10^{7}$$

$$\mu 2_Z := \sum_{i=1}^{n} \left(d1Z_i\right)^2 \cdot \mu 2x_i + \frac{1}{4} \cdot \sum_{i=1}^{n} \left(d2Z_i\right)^2 \cdot \mu 4x_i + \frac{-1}{4} \cdot \left(\sum_{i=1}^{n} d2Z_i \cdot \mu 2x_i\right)^2 + \sum_{i=1}^{n} d1Z_i \cdot d2Z_i \cdot \mu 3x_i$$

$$\sqrt{\mu 2_Z} = 7.83987 \cdot 10^{6}$$

$$\mu 2_Z = 6.14635 \cdot 10^{13}$$

$$\mu 3_Z := \sum_{i=1}^{n} \left(d1Z_i\right)^3 \cdot \left(\mu 3x_i\right) + \frac{1}{4} \cdot \left(\sum_{i=1}^{n} d2Z_i \cdot \mu 2x_i\right)^3 + \frac{3}{2} \cdot \left[\sum_{i=1}^{n} \left(d1Z_i\right)^2 \cdot d2Z_i \cdot \mu 4x_i\right] + \left[\sum_{i=1}^{n} \left(d2Z_i\right) \cdot \mu 2x_i\right] \cdot \left[\frac{-3}{2} \cdot \sum_{i=1}^{n} \left(d1Z_i\right)^2 \cdot \mu 2x_i\right] \ldots$$
$$+ \left[\sum_{i=1}^{n} \left(d2Z_i\right) \cdot \mu 2x_i\right] \cdot \left[\frac{-3}{2} \cdot \sum_{i=1}^{n} \left(d1Z_i\right) \cdot d2Z_i \cdot \mu 3x_i\right] + \left[\sum_{i=1}^{n} \left(d2Z_i\right) \cdot \mu 2x_i\right] \cdot \left[\frac{-3}{8} \cdot \sum_{i=1}^{n} \left(d2Z_i\right)^2 \cdot \left(\mu 4x_i\right)\right]$$

$$\mu 3_Z = 3.68266 \cdot 10^{20}$$

$$\mu 4_Z := \left[\sum_{i=1}^{n} \left(d1Z_i\right)^4 \cdot \left(\mu 4x_i\right) + \frac{-3}{16} \cdot \left(\sum_{i=1}^{n} d2Z_i \cdot \mu 2x_i\right)^4 + \left[-\sum_{i=1}^{n} \left(d2Z_i\right) \cdot \mu 2x_i\right] \cdot \left[2 \cdot \sum_{i=1}^{n} \left(d1Z_i\right)^3 \cdot \mu 3x_i\right] + \left[\sum_{i=1}^{n} \left(d2Z_i\right) \cdot \mu 2x_i\right]^2 \cdot \left[\frac{3}{2} \cdot \sum_{i=1}^{n} \left(d1Z_i\right)^2 \cdot \mu 2x_i\right]\right] \ldots$$
$$+ \left[\sum_{i=1}^{n} \left(d2Z_i\right) \cdot \mu 2x_i\right]^2 \cdot \left[\frac{3}{2} \cdot \sum_{i=1}^{n} \left(d1Z_i\right) \cdot d2Z_i \cdot \mu 3x_i\right] + \left[-\sum_{i=1}^{n} \left(d2Z_i\right) \cdot \mu 2x_i\right] \cdot \left[3 \cdot \sum_{i=1}^{n} \left(d1Z_i\right)^2 \cdot d2Z_i \cdot \mu 4x_i\right] + \left[\sum_{i=1}^{n} \left(d2Z_i\right) \cdot \mu 2x_i\right]^2 \cdot \left[\frac{3}{8} \cdot \sum_{i=1}^{n} \left(d2Z_i\right)^2 \cdot \left(\mu 4x_i\right)\right]$$

$$\mu 4_Z = 1.53089 \cdot 10^{28}$$

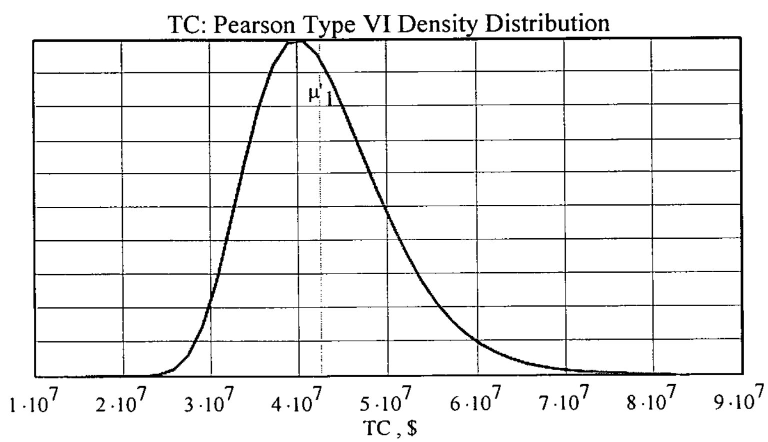
Total cost distribution

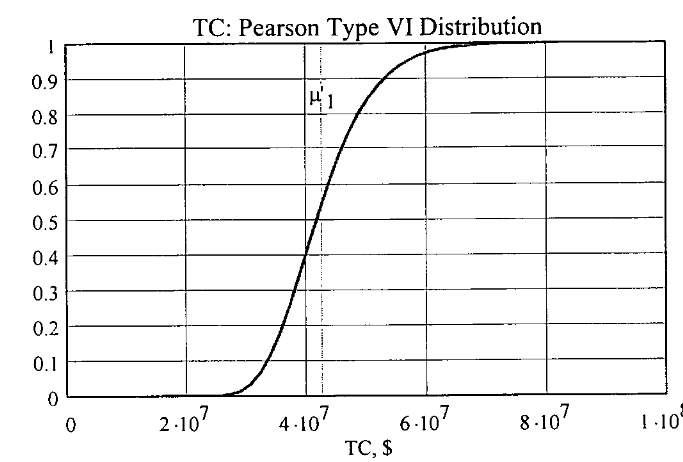
$\gamma 1 := \dfrac{\mu 3_Z}{{\mu 2_Z}^{1.5}}$ $\quad\quad$ $\gamma 2 := \dfrac{\mu 4_Z}{{\mu 2_Z}^{2}}$ $\quad\quad$ $\beta 1 := \gamma 1^2$ $\quad\quad$ $\beta 2 := \gamma 2$

$\gamma 1 = 0.76425$ $\quad\quad$ $\gamma 2 = 4.05236$ $\quad\quad$ $\beta 1 = 0.58408$ $\quad\quad$ $\beta 2 = 4.05236$

$$k\_criterion := \frac{\beta 1 \cdot (\beta 2 + 3)^2}{4 \cdot (4 \cdot \beta 2 - 3 \cdot \beta 1) \cdot (2 \cdot \beta 2 - 3 \cdot \beta 1 - 6)}$$

$k\_criterion = 1.4251$ $\quad\quad$ k is larger than 1, then Pearson Distribution Type VI should be selected





B.6 Total Revenues

$TR(S) := S \cdot N$

Risk analysis of total revenues

First derivative of TR

$d1Z_6 := \frac{d}{dS} TR(S) \qquad d1Z^T = (0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 45)$

Second derivatives of TR

$d2Z_6 := \frac{d^2}{dS^2} TR(S) \qquad d2Z^T = (0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0)$

Four moments of total revenues $\quad n := 6$

$Z_\mu := TR(\mu x_6) \qquad Z_\mu = 7.5 \cdot 10^7$

$EX_Z := TR(\mu x_6) + \frac{1}{2} \cdot \sum_{j=1}^{n} d2Z_j \cdot \mu 2x_j \qquad EX_Z = 7.5 \cdot 10^7$

$$\mu 2_Z := \sum_{i=1}^{n} (d1Z_i)^2 \cdot \mu 2x_i + \frac{1}{4} \cdot \sum_{i=1}^{n} (d2Z_i)^2 \cdot \mu 4x_i + \frac{-1}{4} \cdot \left(\sum_{i=1}^{n} d2Z_i \cdot \mu 2x_i \right)^2 + \sum_{i=1}^{n} d1Z_i \cdot d2Z_i \cdot \mu 3x_i$$

$$\sqrt{\mu 2_Z} = 2.80624 \cdot 10^6$$

$$\mu 3_Z := \sum_{i=1}^{n} (d1Z_i)^3 \cdot (\mu 3x_i) + \frac{1}{4} \cdot \left(\sum_{i=1}^{n} d2Z_i \cdot \mu 2x_i \right)^3 + \frac{3}{2} \cdot \left[\sum_{i=1}^{n} (d1Z_i)^2 \cdot d2Z_i \cdot \mu 4x_i \right] + \left[\sum_{i=1}^{n} (d2Z_i) \cdot \mu 2x_i \right] \cdot \left[\frac{-3}{2} \cdot \sum_{i=1}^{n} (d1Z_i)^2 \cdot \mu 2x_i \right] \ldots$$
$$+ \left[\sum_{i=1}^{n} (d2Z_i) \cdot \mu 2x_i \right] \cdot \left[\frac{-3}{2} \cdot \sum_{i=1}^{n} (d1Z_i) \cdot d2Z_i \cdot \mu 3x_i \right] + \left[\sum_{i=1}^{n} (d2Z_i) \cdot \mu 2x_i \right] \cdot \left[\frac{-3}{8} \cdot \sum_{i=1}^{n} (d2Z_i)^2 \cdot (\mu 4x_i) \right]$$

$$\mu 3_Z = -6.75 \cdot 10^{18}$$

$$\mu 4_Z := \left[\sum_{i=1}^{n} (d1Z_i)^4 \cdot (\mu 4x_i) + \frac{-3}{16} \cdot \left(\sum_{i=1}^{n} d2Z_i \cdot \mu 2x_i \right)^4 + \left[- \sum_{i=1}^{n} (d2Z_i) \cdot \mu 2x_i \right] \cdot \left[2 \cdot \sum_{i=1}^{n} (d1Z_i)^3 \cdot \mu 3x_i \right] + \left[\sum_{i=1}^{n} (d2Z_i) \cdot \mu 2x_i \right]^2 \cdot \left[\frac{3}{2} \cdot \sum_{i=1}^{n} (d1Z_i)^2 \cdot \mu 2x_i \right] \right] \ldots$$
$$+ \left[\sum_{i=1}^{n} (d2Z_i) \cdot \mu 2x_i \right]^2 \cdot \left[\frac{3}{2} \cdot \sum_{i=1}^{n} (d1Z_i) \cdot d2Z_i \cdot \mu 3x_i \right] + \left[- \sum_{i=1}^{n} (d2Z_i) \cdot \mu 2x_i \right] \cdot \left[3 \cdot \sum_{i=1}^{n} (d1Z_i)^2 \cdot d2Z_i \cdot \mu 4x_i \right] + \left[\sum_{i=1}^{n} (d2Z_i) \cdot \mu 2x_i \right]^2 \cdot \left[\frac{3}{8} \cdot \sum_{i=1}^{n} (d2Z_i)^2 \cdot (\mu 4x_i) \right]$$

$$\mu 4_Z = 1.48837 \cdot 10^{26}$$

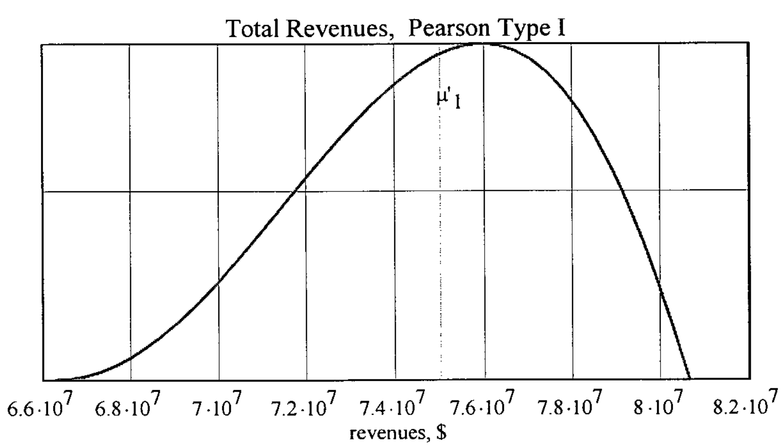
Distribution for TR

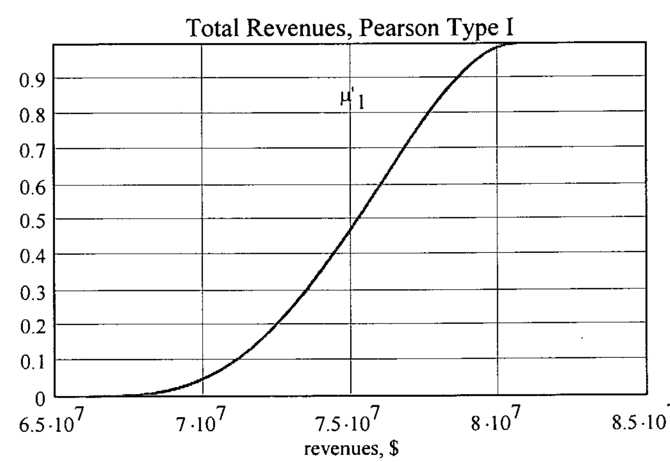
$$\gamma 1 := \frac{\mu 3_Z}{{\mu 2_Z}^{1.5}} \qquad \gamma 2 := \frac{\mu 4_Z}{{\mu 2_Z}^{2}} \qquad \beta 1 := \gamma 1^2 \qquad \beta 2 := \gamma 2$$

$$\gamma 1 = -0.30544 \qquad \gamma 2 = 2.4 \qquad \beta 1 = 0.09329 \qquad \beta 2 = 2.4$$

$$k\_criterion := \frac{\cdot\, \beta 1 \cdot (\beta 2 + 3)^2}{4 \cdot (4 \cdot \beta 2 - 3 \cdot \beta 1) \cdot (2 \cdot \beta 2 - 3 \cdot \beta 1 - 6)}$$

$k\_criterion = -0.04931$ since $\beta 1 < 0$, then Pearson Distribution Type I should be selected





B.7 Construction Completion Time

$$\mathrm{CTime}(Td, Tc) := Td + Tc$$

Risk analysis of completion Time

First derivatives of CTime

$$d1Z_1 := 0$$

$$d1Z_2 := \frac{d}{dTd}\mathrm{CTime}(Td, Tc)$$

$$d1Z_3 := 0$$

$$d1Z_4 := \frac{d}{dTc}\mathrm{CTime}(Td, Tc)$$

$$d1Z^T = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$d2Z^T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Second derivatives of CTime

$$d2Z_1 := 0$$

$$d2Z_2 := \frac{d^2}{dTd^2}\mathrm{CTime}(Td, Tc)$$

$$d2Z_3 := 0$$

$$d2Z_4 := \frac{d^2}{dTc^2}\mathrm{CTime}(Td, Tc)$$

Completion time four moments

$n := 4$

$Z_{\mu} := \mathrm{CTime}(\mu x_2, \mu x_4)$ (Ctime calculated at the mean values) $Z_{\mu} = 2$

$$EX_Z := \mathrm{CTime}(\mu x_2, \mu x_4) + \frac{1}{2} \cdot \sum_{j=1}^{n} d2Z_j \cdot \mu 2x_j \qquad EX_Z = 2$$

$$\mu 2_Z := \sum_{i=1}^{n} (d1Z_i)^2 \cdot \mu 2x_i + \frac{1}{4} \cdot \sum_{i=1}^{n} (d2Z_i)^2 \cdot \mu 4x_i + \frac{-1}{4} \cdot \left(\sum_{i=1}^{n} d2Z_i \cdot \mu 2x_i \right)^2 + \sum_{i=1}^{n} d1Z_i \cdot d2Z_i \cdot \mu 3x_i \qquad \sqrt{\mu 2_Z} = 0.25125$$

$$\mu 3_Z := \sum_{i=1}^{n} (d1Z_i)^3 \cdot (\mu 3x_i) + \frac{1}{4} \left(\sum_{i=1}^{n} d2Z_i \cdot \mu 2x_i \right)^3 + \frac{3}{2} \cdot \left[\sum_{i=1}^{n} (d1Z_i)^2 \cdot d2Z_i \cdot \mu 4x_i \right] + \left[\sum_{i=1}^{n} (d2Z_i) \cdot \mu 2x_i \right] \cdot \left[\frac{-3}{2} \cdot \sum_{i=1}^{n} (d1Z_i)^2 \cdot \mu 2x_i \right] \ldots$$
$$+ \left[\sum_{i=1}^{n} (d2Z_i) \cdot \mu 2x_i \right] \cdot \left[\frac{-3}{2} \cdot \sum_{i=1}^{n} (d1Z_i) \cdot d2Z_i \cdot \mu 3x_i \right] + \left[\sum_{i=1}^{n} (d2Z_i) \cdot \mu 2x_i \right] \cdot \left[\frac{-3}{8} \cdot \sum_{i=1}^{n} (d2Z_i)^2 \cdot (\mu 4x_i) \right]$$

$$\mu 3_Z = 7.88718 \cdot 10^{-3}$$

$$\mu 4_Z := \left[\sum_{i=1}^{n} (d1Z_i)^4 \cdot (\mu 4x_i) + \frac{-3}{16} \cdot \left(\sum_{i=1}^{n} d2Z_i \cdot \mu 2x_i \right)^4 + \left[- \sum_{i=1}^{n} (d2Z_i) \cdot \mu 2x_i \right] \cdot \left[2 \cdot \sum_{i=1}^{n} (d1Z_i)^3 \cdot \mu 3x_i \right] + \left[\sum_{i=1}^{n} (d2Z_i) \cdot \mu 2x_i \right]^2 \cdot \left[\frac{3}{2} \cdot \sum_{i=1}^{n} (d1Z_i)^2 \cdot \mu 2x_i \right] \right] \ldots$$
$$+ \left[\sum_{i=1}^{n} (d2Z_i) \cdot \mu 2x_i \right]^2 \cdot \left[\frac{3}{2} \cdot \sum_{i=1}^{n} (d1Z_i) \cdot d2Z_i \cdot \mu 3x_i \right] + \left[- \sum_{i=1}^{n} (d2Z_i) \cdot \mu 2x_i \right] \cdot \left[3 \cdot \sum_{i=1}^{n} (d1Z_i)^2 \cdot d2Z_i \cdot \mu 4x_i \right] + \left[\sum_{i=1}^{n} (d2Z_i) \cdot \mu 2x_i \right]^2 \cdot \left[\frac{3}{8} \cdot \sum_{i=1}^{n} (d2Z_i)^2 \cdot (\mu 4x_i) \right]$$

$$\mu 4_Z = 0.0135$$

Completion time distribution

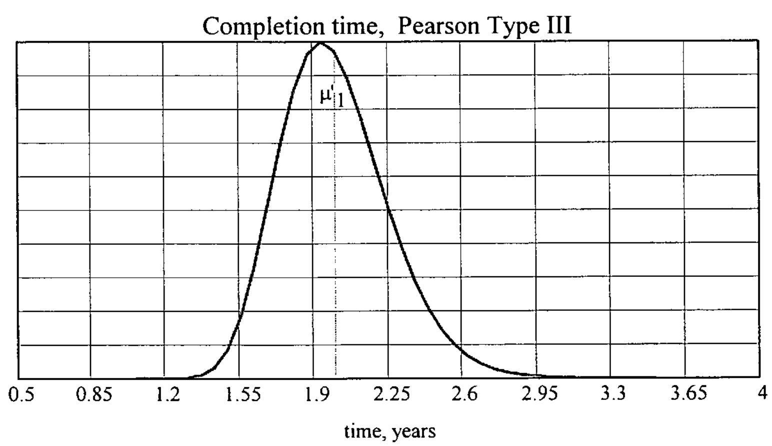
$\gamma1 := \frac{\mu3_Z}{\mu2_Z^{1.5}}$ $\gamma2 := \frac{\mu4_Z}{\mu2_Z^{2}}$ $\beta1 := \gamma1^2$ $\beta2 := \gamma2$

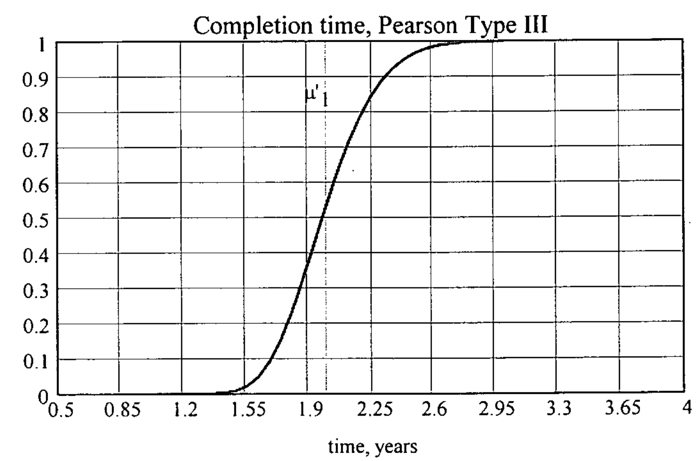
$\gamma1 = 0.4973$ $\gamma2 = 3.38835$ $\beta1 = 0.24731$ $\beta2 = 3.38835$

$$k\_criterion := \frac{\beta1\cdot(\beta2+3)^2}{4\cdot(4\cdot\beta2-3\cdot\beta1)\cdot(2\cdot\beta2-3\cdot\beta1-6)}$$

$2\cdot\beta2 = 6.77669$ $6 + 3\cdot\beta1 = 6.74193$

$k\_criterion = 5.66517$ since 2*β2 approaches [3*β1+6] , then Pearson Distribution Type III (Gamma distribution) should be selected





B.8 Debt Service And Loan Life Cover Ratios

Debt repayments and interest payments

(4 loan repayments)

$\mathrm{Tloan} = 1.7204 \cdot 10^{7}$ $\quad \mathrm{RepaymentValue} := \frac{\mathrm{Tloan}}{4}$ $\quad \mathrm{RepaymentValue} = 4.301 \cdot 10^{6}$

(6 interest payments, 'ie' is effective rate)

$IP_1 := \mathrm{Tloan} \cdot ie(i)$ $\quad IP_2 := \mathrm{Tloan} \cdot ie(i)$ $\quad IP_3 := \mathrm{Tloan} \cdot ie(i)$

$IP_4 := \frac{3}{4} \cdot \mathrm{Tloan} \cdot ie(i)$ $\quad IP_5 := \frac{2}{4} \cdot \mathrm{Tloan} \cdot ie(i)$ $\quad IP_6 := \frac{1}{4} \cdot \mathrm{Tloan} \cdot ie(i)$

revenues:

$\mathrm{Rev} := S \cdot N$ $\quad \mathrm{Rev\_6\_month} := \frac{\mathrm{Rev}}{3}$

$\mathrm{Rev} = 7.5 \cdot 10^{7}$ $\quad \mathrm{Rev\_6\_month} = 2.5 \cdot 10^{7}$

Debt service ratio after start of operation $\quad y := 0.1$

$\mathrm{DSC\_5} := \frac{\mathrm{Rev\_6\_month}}{\mathrm{RepaymentValue} + IP_4}$ $\quad \mathrm{DSC\_5} = 4.864102$ $\quad$ period 5 (at 2.5 yr):

$\mathrm{DSC\_6} := \frac{\mathrm{Rev\_6\_month}}{\mathrm{RepaymentValue} + IP_5}$ $\quad \mathrm{DSC\_6} = 5.143895$ $\quad$ period 6 (at 3 yr):

$\mathrm{DSC\_7} := \frac{\mathrm{Rev\_6\_month}}{\mathrm{RepaymentValue} + IP_6}$ $\quad \mathrm{DSC\_7} = 5.457842$ $\quad$ period 7 (at 3.5 yr):

Loan life cover ratio after start of operation $\quad y := 0.1$

$$\mathrm{Debt\_4} := \left(\frac{\mathrm{Tloan}}{4} + IP_3\right) + \left(\frac{\mathrm{Tloan}}{4} + IP_4\right) \cdot e^{-y \cdot 0.5} + \left(\frac{\mathrm{Tloan}}{4} + IP_5\right) \cdot e^{-y \cdot 1} + \left(\frac{\mathrm{Tloan}}{4} + IP_6\right) \cdot e^{-y \cdot 1.5}$$

$$\mathrm{Rev\_4} := \int_0^{1.5} \frac{S \cdot N}{To} \cdot e^{-y \cdot t}\,dt \qquad \mathrm{LL\_4} := \frac{\mathrm{Rev\_4}}{\mathrm{Debt\_4}} \qquad \mathrm{LL\_4} = 3.734682$$

$$\mathrm{Debt\_5} := \left(\frac{\mathrm{Tloan}}{4} + IP_4\right) + \left(\frac{\mathrm{Tloan}}{4} + IP_5\right) \cdot e^{-y \cdot 0.5} + \left(\frac{\mathrm{Tloan}}{4} + IP_6\right) \cdot e^{-y \cdot 1}$$

$$\mathrm{Rev\_5} := \int_0^{1} \frac{S \cdot N}{To} \cdot e^{-y \cdot t}\,dt \qquad \mathrm{LL\_5} := \frac{\mathrm{Rev\_5}}{\mathrm{Debt\_5}} \qquad \mathrm{LL\_5} = 3.421278$$

$$\mathrm{Debt\_6} := \left(\frac{\mathrm{Tloan}}{4} + IP_5\right) + \left(\frac{\mathrm{Tloan}}{4} + IP_6\right) \cdot e^{-y \cdot 0.5}$$

$$\mathrm{Rev\_6} := \int_0^{0.5} \frac{S \cdot N}{To} \cdot e^{-y \cdot t}\,dt \qquad \mathrm{LL\_6} := \frac{\mathrm{Rev\_6}}{\mathrm{Debt\_6}} \qquad \mathrm{LL\_6} = 2.6456$$

$$\mathrm{Debt\_7} := \left(\frac{\mathrm{Tloan}}{4} + IP_6\right)$$

$$\mathrm{Rev\_7} := \int_0^{0} \frac{S \cdot N}{To} \cdot e^{-y \cdot t}\,dt \qquad \mathrm{LL\_7} := \frac{\mathrm{Rev\_7}}{\mathrm{Debt\_7}} \qquad \mathrm{LL\_7} = 0$$