A SYNTHESIZED APPROACH FOR ESTIMATING LIQUEFACTION-INDUCED DISPLACEMENTS OF GEOTECHNICAL STRUCTURES

by

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A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

in

THE FACULTY OF GRADUATE STUDIES
DEPARTMENT OF CIVIL ENGINEERING

We accept this thesis as conforming to the required standard.

THE UNIVERSITY OF BRITISH COLUMBIA

OCTOBER, 2001

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Date October 11, 2001
ABSTRACT

Liquefaction has caused many failures of buildings, bridges, and dams during earthquakes. Damage is caused by the soil displacements that follow the initiation of liquefaction. The potential for liquefaction-induced failures in seismic areas is a widespread concern since many structures were built when liquefaction and its effects were not well understood. Damage observed in a number of earthquakes, including 1971 San Fernando, 1989 Loma Prieta, and 1995 Hyogoken Nanbu (Kobe), has prompted a re-examination of a number of these structures. Many are being retrofitted at great expense. A key factor in such examinations is the magnitude and pattern of displacements arising from soil liquefaction.

A total stress dynamic approach is presented for estimating these displacements from seismically induced liquefaction. The approach is derived from widely accepted assumptions for evaluation of liquefaction triggering, flow slide potential, and limited displacements. These different evaluations are combined into a single analysis while eliminating some of the inherent simplifications in current procedures.

An explicit finite difference model is used with the earthquake motion applied to the base. Triggering of liquefaction in each element is continuously assessed by weighting each cycle of shear stress. Postliquefaction stiffness and strength properties are assigned to an element when sufficient cycles of shear stress have accumulated. Elements continue to liquefy and respond to inertia loads as the shaking proceeds, causing the displacements to increase with the duration of shaking.

The proposed method is used to evaluate case histories and geotechnical structures. The approach is found to give reasonable predictions of displacements. The importance of
various input parameters is investigated. Both residual strength and the character of the earthquake motion are found to be critical variables.

Limitations of the method are also reviewed. The desirability of using a more fundamental and complex effective stress model to supplement this total stress approach is discussed. Such a combination of analyses may help identify key characteristics of the liquefaction response, particularly when the flow of pore water is a potential concern.
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LIST OF SYMBOLS AND ABBREVIATIONS

\( \alpha_\sigma \) Angle between major principal stress and deposition direction of sand

\( \beta_c \) Fraction of critical damping at frequency \( f_c \)

\( \varepsilon_v \) Volumetric strain

\( \varepsilon_v^e \) Elastic volumetric strain

\( \varepsilon_v^p \) Plastic volumetric strain

\( \phi \) Friction angle

\( \phi_{cv} \) Friction angle at constant volume conditions

\( \gamma \) Total shear strain

\( \gamma^e \) Elastic shear strain

\( \gamma^p \) Plastic shear strain

\( \gamma_r \) Residual shear strain

\( \eta \) Stress ratio, where \( \eta = \tau_{\text{max}} / \sigma_m' \)

\( \eta_{cv} \) Stress ratio corresponding to \( \phi_{cv} \)

\( \nu \) Poisson's ratio

\( \rho \) Mass density

\( \sigma_f \) Total stress on failure plane

\( \sigma_f' \) Effective stress on failure plane

\( \sigma_m' \) Mean normal effective stress

\( \sigma_m \) Mean normal total stress

\( \sigma_{vo} \) Initial vertical effective stress

\( \sigma_{xx} \) Lateral normal total stress

\( \sigma_{xx}' \) Lateral normal effective stress

\( \sigma_{yy} \) Vertical normal total stress

\( \sigma_{yy}' \) Vertical normal effective stress

\( \sigma_i' \) Major (most compressive) principal effective stress

\( \sigma_{io}' \) Initial major principal effective stress

\( \sigma_i' \) Intermediate principal effective stress

\( \sigma_j' \) Minor principal effective stress

\( \tau \) Shear stress

\( \tau_{\text{cyc}} \) Cyclic shear stress on horizontal plane
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<td>$\tau_{cliq}$</td>
<td>Value of $\tau_{cyc}$ required to trigger liquefaction in current half cycle</td>
</tr>
<tr>
<td>$\tau_f$</td>
<td>Maximum shear stress at failure</td>
</tr>
<tr>
<td>$\tau_{max}$</td>
<td>Maximum shear stress at a point</td>
</tr>
<tr>
<td>$\tau_{st}$</td>
<td>Initial static shear stress on horizontal plane</td>
</tr>
<tr>
<td>$\tau_{xy}$</td>
<td>Shear stress on horizontal plane</td>
</tr>
<tr>
<td>$\tau_{xy\text{liq}}$</td>
<td>Value of $\tau_{xy}$ required to trigger liquefaction in current half cycle</td>
</tr>
<tr>
<td>$\tau_{15}$</td>
<td>Value of $\tau_{cyc}$ required to trigger liquefaction in 15 cycles</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Dilation angle</td>
</tr>
<tr>
<td>AP</td>
<td>Air pluviated</td>
</tr>
<tr>
<td>$b$</td>
<td>Intermediate principal stress parameter where $b = (\sigma_2 - \sigma_3)/(</td>
</tr>
<tr>
<td>$B^e$</td>
<td>Elastic bulk modulus</td>
</tr>
<tr>
<td>$c$</td>
<td>Cohesion</td>
</tr>
<tr>
<td>CPT</td>
<td>Cone penetration test</td>
</tr>
<tr>
<td>CRR</td>
<td>Cyclic resistance ratio</td>
</tr>
<tr>
<td>$\text{CRR}_1$</td>
<td>CRR corresponding to one cycle</td>
</tr>
<tr>
<td>$\text{CRR}_{15}$</td>
<td>CRR corresponding to 15 cycles</td>
</tr>
<tr>
<td>CSR</td>
<td>Cyclic stress ratio $\tau_{cyc} / \sigma_{vo}$</td>
</tr>
<tr>
<td>$\text{CSR}_{15}$</td>
<td>Equivalent cyclic stress ratio consisting of 15 uniform cycles</td>
</tr>
<tr>
<td>$D_{50_{15}}$</td>
<td>Average $D_{50}$ within $T_{15}$ in mm</td>
</tr>
<tr>
<td>$D_h$</td>
<td>Horizontal displacement</td>
</tr>
<tr>
<td>$D_r$</td>
<td>Relative density</td>
</tr>
<tr>
<td>D/S</td>
<td>Downstream</td>
</tr>
<tr>
<td>$E_i$</td>
<td>Initial elastic modulus for hyperbolic model</td>
</tr>
<tr>
<td>EQ</td>
<td>Earthquake</td>
</tr>
<tr>
<td>$f_i$</td>
<td>Frequency corresponding to fundamental response mode</td>
</tr>
<tr>
<td>$F_{15}$</td>
<td>Average fines content within $T_{15}$ in percent</td>
</tr>
<tr>
<td>$f_c$</td>
<td>Frequency corresponding to damping constant $\beta_c$</td>
</tr>
<tr>
<td>$g$</td>
<td>Acceleration due to gravity</td>
</tr>
<tr>
<td>FC</td>
<td>Fines content as percentage passing #200 sieve</td>
</tr>
<tr>
<td>$\text{FS}_L$</td>
<td>Factor of safety against liquefaction where $\text{FS}<em>L = \text{CRR}</em>{15}/\text{CSR}_{15}$</td>
</tr>
<tr>
<td>$G_{dyn}$</td>
<td>Equivalent shear modulus of nonliquefied soil during cyclic loading</td>
</tr>
<tr>
<td>$G_i$</td>
<td>Initial shear modulus for hyperbolic model</td>
</tr>
<tr>
<td>$G_{liq}$</td>
<td>Postliquefaction shear modulus during loading</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$G_{\text{max}}$</td>
<td>Shear modulus at small strain</td>
</tr>
<tr>
<td>$G_{\text{st}}$</td>
<td>Equivalent elastic shear modulus for static loading</td>
</tr>
<tr>
<td>$G^e$</td>
<td>Elastic shear modulus</td>
</tr>
<tr>
<td>$G^p$</td>
<td>Plastic shear modulus</td>
</tr>
<tr>
<td>GWT</td>
<td>Groundwater table</td>
</tr>
<tr>
<td>HCT</td>
<td>Hollow cylinder torsional test</td>
</tr>
<tr>
<td>HF</td>
<td>Hydraulic fill</td>
</tr>
<tr>
<td>$I_A$</td>
<td>Arias intensity</td>
</tr>
<tr>
<td>$k$</td>
<td>Coefficient of permeability</td>
</tr>
<tr>
<td>$K_\alpha$</td>
<td>CRR correction factor for static bias</td>
</tr>
<tr>
<td>$K_b$</td>
<td>Bulk modulus number for hyperbolic model</td>
</tr>
<tr>
<td>$K_c$</td>
<td>Ratio of $\sigma'_i/\sigma'_j$ after consolidation</td>
</tr>
<tr>
<td>$K_e$</td>
<td>Elastic modulus number for hyperbolic model</td>
</tr>
<tr>
<td>$K_m$</td>
<td>CRR correction factor for earthquake magnitude</td>
</tr>
<tr>
<td>$K_{\sigma}$</td>
<td>CRR correction factor for effective confining stress</td>
</tr>
<tr>
<td>$K_{2\text{max}}$</td>
<td>Modulus coefficient for estimating $G_{\text{max}}$</td>
</tr>
<tr>
<td>$m$</td>
<td>Bulk modulus exponent for hyperbolic model</td>
</tr>
<tr>
<td>MRF</td>
<td>Modulus reduction factor equal to $G_{\text{dyn}}/G_{\text{max}}$</td>
</tr>
<tr>
<td>$M_w$</td>
<td>Moment magnitude of earthquake</td>
</tr>
<tr>
<td>$n$</td>
<td>Elastic modulus exponent for hyperbolic model</td>
</tr>
<tr>
<td>$N_{1.60}$</td>
<td>Blowcount from SPT corrected to $\sigma'_{vo} = 96$ kPa and 60% efficiency</td>
</tr>
<tr>
<td>$N_{1.60cs}$</td>
<td>$N_{1.60}$ blowcount corrected to clean sand conditions</td>
</tr>
<tr>
<td>$N$</td>
<td>Cumulative number of loading cycles</td>
</tr>
<tr>
<td>$N_{eq}$</td>
<td>Number of cycles of $\tau_{15}$ that are equivalent to one-half cycle of $\tau_{\text{cyc}}$</td>
</tr>
<tr>
<td>$N_{\text{liq}}$</td>
<td>Number of loading cycles of $\tau_{\text{cyc}}$ required to trigger liquefaction</td>
</tr>
<tr>
<td>$P_a$</td>
<td>Atmospheric pressure</td>
</tr>
<tr>
<td>pga</td>
<td>Peak ground acceleration</td>
</tr>
<tr>
<td>$r_u$</td>
<td>Excess pore pressure ratio where $r_u = (u - u_0) / \sigma'_{vo}$</td>
</tr>
<tr>
<td>$R$</td>
<td>Horizontal distance to nearest seismic energy source in km</td>
</tr>
<tr>
<td>$R_f$</td>
<td>Failure ratio for hyperbolic model</td>
</tr>
<tr>
<td>RS</td>
<td>Rayleigh stiffness proportional damping</td>
</tr>
<tr>
<td>RMS</td>
<td>Rayleigh mass + stiffness proportional (combined) damping</td>
</tr>
<tr>
<td>$S$</td>
<td>Ground slope in percent</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>$S_{qs}$</td>
<td>Quasi-steady state strength from undrained laboratory tests</td>
</tr>
<tr>
<td>$S_{qs-ss}$</td>
<td>Minimum undrained strength after strain softening</td>
</tr>
<tr>
<td>$S_r$</td>
<td>Mobilized residual strength</td>
</tr>
<tr>
<td>$S_{ss}$</td>
<td>Steady state strength from undrained laboratory tests</td>
</tr>
<tr>
<td>$S_u$</td>
<td>Representative undrained strength prior to liquefaction for use in analysis</td>
</tr>
<tr>
<td>$S_{u-peak}$</td>
<td>Peak strength before strain softening from undrained laboratory tests</td>
</tr>
<tr>
<td>SPT</td>
<td>Standard penetration test</td>
</tr>
<tr>
<td>SS</td>
<td>Simple shear test</td>
</tr>
<tr>
<td>$T_{15}$</td>
<td>Total thickness of saturated layers in metres with $N_{1.60} &lt; 15$</td>
</tr>
<tr>
<td>$u$</td>
<td>Pore pressure</td>
</tr>
<tr>
<td>$u_0$</td>
<td>Initial pore pressure at start of earthquake</td>
</tr>
<tr>
<td>UHRS</td>
<td>Uniform hazard response spectra</td>
</tr>
<tr>
<td>U/S</td>
<td>Upstream</td>
</tr>
<tr>
<td>$V_s$</td>
<td>Shear wave velocity</td>
</tr>
<tr>
<td>WP</td>
<td>Water pluviated</td>
</tr>
</tbody>
</table>
ACKNOWLEDGEMENTS

Returning to the status of full-time student after ten years in the work force is both difficult and indulgent. I owe my gratitude to the many people who have given support and made this experience worthwhile:

- To my wife Lisa, who is an endless source of encouragement and understanding.
- To my daughter Eaven Michelle, who brings joy and wonder to both of us.
- To my extended family, to Don and Doris Beaty, and to Charles and Marie Richardson, for their support, patience, and interest.
- To my advisor Peter Byrne, for his encouragement and trust, and for his passion to pursue a fundamental understanding of engineering and geomechanics.
- To the Vancouver geotechnical engineering community, for the many opportunities they provided to advance my research and for their constructive reviews. To Ernie Naesgaard and his quiet enthusiasm for geotechnical analysis, and to John Howie for our many interesting discussions.
- To Les Harder and Ross Boulanger, who provided invaluable encouragement and advice in my decision to pursue these studies.
- And to God, who gives us all life and opportunity.
CHAPTER 1 — Introduction

Liquefaction produces a dramatic change in the material properties of soils. The soil may weaken and the average stiffness may drop by an order of 100 or more. If liquefaction occurs during earthquake loading, the overall structure responds not only to the change in properties but also to the on-going inertial loads. The reduced capacity of the soil may persist for some time after the earthquake and in some cases it may further degrade. It is no surprise that liquefaction is a significant cause of large earthquake-induced deformations in soil structures.

The effects of liquefaction are varied. Most embankment dam failures during earthquakes have been attributed to liquefaction (Fell et al., 2000). Bridges and buildings have been destroyed or suffered significant structural damage as shown in Figure 1-1.
Foundations are compromised by liquefaction and lateral spreading. Embankments and levees fail. Widespread damage to lifelines, including gas and water pipelines, roadways, and sewer works, can result from relatively modest ground deformations as shown in Figure 1-2. Devastating landslides occur. The consequences of liquefaction are not only long lasting, but impact a community’s immediate response to an earthquake.

Damage to a modern city can be substantial. The 1989 Loma Prieta earthquake ($M_w = 7.1$) caused an estimated $8$ billion in overall losses (all dollar amounts are US$). Damage amounting to $20$ billion was caused by the 1994 Northridge earthquake ($M_w = 6.4$) and losses resulting from the 1995 Hyogoken Nanbu (Kobe) earthquake ($M_w = 7.2$) were greater than $100$ billion. An annualized earthquake loss of $4.4$ billion has been estimated.
just for the United States, with nearly 40% of this risk occurring in just the Los Angeles area (FEMA, 2000). This is a minimum estimate of risk as it does not include long-term losses or damage to lifeline infrastructure. The impacts of liquefaction and ground failure were excluded from these annualized estimates but would be expected to add considerable cost.

Concerns regarding liquefaction have been heightened by our improved understanding of seismology. Local and near field effects have increased seismic design levels in many areas. Relatively dense sands that were previously thought safe from liquefaction are now considered potentially liquefiable. More fine-grained soils are now recognized to be susceptible to liquefaction-type behaviour (Boulanger et al., 1998). While the anticipated deformations might be less for dense sands than for loose sands, reliable estimates are still required to evaluate sensitive structures.

The importance of seismically induced liquefaction has been demonstrated in many earthquakes. Descriptive accounts of liquefaction have been noted as early as 1783 (Hobbs, 1907). Two major earthquakes of 1964, in Niigata (Seed & Idriss, 1967; Kawasumi, 1968) and in Alaska (Seed, 1968; Ross et al., 1969), were instrumental in advancing our knowledge as well as raising awareness of the importance of liquefaction. A select number of additional liquefaction events are outlined in Table 1-1.

The observations in Niigata were particularly instructive for the built environment. Significant structural distress, including damage to 25% of the modern concrete structures, occurred primarily on the reclaimed ground of old river courses consisting of loose sands. It was also noted that “the maximum intensity of earthquake motion was not so strong as to cause a vibration damage of any importance in the old town area of somewhat consolidated ground” (Kawasumi, 1968). Two-thirds of the damage to concrete buildings consisted of
### Table 1-1.  A limited list of liquefaction-induced deformation events

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Liquefaction/Deformation Event</th>
</tr>
</thead>
</table>
| 1995 Kobe (Hyogoken-Nanbu) | Widespread damage to river levees and embankments  
                           | Displacement of waterfront and quay walls  
                           | Damage to building foundations  
                           | Failure or damage to 4 small reservoir embankments |
| 1994 Northridge  | Pipe breakage and pavement disruption  
                           | Flow slide of Tapo Canyon tailings dam |
| 1989 Loma Prieta | Damage to apartment buildings in Marina District |
| 1985 Central Chile | Extensive damage to mine tailing dams |
| 1979 Izu-Oshima | Slide in Mochikoshi tailings dam |
| 1971 San Fernando | Flow slide in upstream slope of Upper San Fernando dam  
                           | 2 m displacement of Lower San Fernando dam  
                           | Large lateral spread on gentle slope |
| 1968 Tokachi-Oki | Failure of Kona Numa railway embankment |
| 1965 Chile       | Failure of numerous tailings dams |
| 1964 Niigata     | Bearing failure of buildings and bridge piers  
                           | Failure of railway embankment and quay walls  
                           | Widespread building damage due mostly to liquefaction |
| 1964 Anchorage   | Massive landslides damage structures and waterfront  
                           | More than 250 bridges compressed or buckled |
| 1957 San Francisco | Flow slide into Lake Merced |
| 1940 El Centro   | Failure of Solfatara Canal dike |
| 1925 Santa Barbara | Failure of Sheffield dam |

Sources: Bardet et al. (1995), Bolt (1993), National Research Council (1985), Seed (1987), and Stewart et al. (1994).
settling, tilting, or overturning without any damage to the superstructure. Much of the remaining damage resulted from differential foundation movements.

Liquefaction can be dramatic in consequence but rather simple in underlying cause. Soil is composed of both solid particles and pore spaces with the arrangement of solid particles forming the soil skeleton. Pore spaces are typically filled with some combination of water and air. Saturated soils are those with essentially no air in the pore spaces. While the bulk stiffness of air is very low, and the stiffness of water with even a small amount of free air is also low, water by itself can be considered nearly incompressible. Saturated soils are therefore very resistant to volume change unless the pore fluid is allowed to flow into or out of the soil.

The granular skeleton of a soil will initially contract as it strains under shear loading. This tendency for contraction will be resisted by the near incompressibility of the pore fluid if the soil is saturated. The resulting increase in pore water pressure can greatly reduce the intergranular effective stress. Liquefaction may manifest itself as the strain-softening behaviour seen in monotonic loading or by occurrences of 100% excess pore pressure ratio, $r_u$, during earthquake or cyclic loading. Either form of liquefaction involves a significant loss in shear stiffness of the soil.

Given the on-going occurrence of liquefaction that results in deformations and damage, and the increased awareness of seismic hazard, the need for reliable and practical analytical tools is clear.

1.1 Liquefaction Analysis: State of Practice

Two questions must be addressed in most assessments of liquefaction:

1. Will liquefaction occur in significant zones?
2. If liquefaction does occur, what displacements will result?
The accepted practice in North America for addressing these questions is often a three-phase total stress procedure as outlined below (NAVFAC, 1997; Babbitt & Verigin, 1996).

1.1.1 Triggering evaluation

Liquefaction due to seismic loading is usually predicted by comparing estimates of cyclic shear stress with the anticipated resistance to liquefaction:

\[ \text{FS}_L = \frac{\text{CRR}}{\text{CSR}} \]

where \( \text{FS}_L \) is the factor of safety against liquefaction, \( \text{CRR} \) the cyclic resistance ratio, and \( \text{CSR} \) the cyclic stress ratio.

\( \text{CSR} \) is a measure of the anticipated cyclic loading. It is often defined as an average shear stress ratio using the following formula (Seed & Idriss, 1971; Seed & Harder, 1990):

\[ \text{CSR} = 0.65 \frac{\tau_{cyc,max}}{\sigma'_{vo}} \]

where \( \tau_{cyc,max} \) is the maximum shear stress on a horizontal plane due to cyclic loading and \( \sigma'_{vo} \) is the initial vertical effective stress. \( \text{CSR} \) is estimated by dynamic ground response analysis or a simple formula. The simple formula requires only a knowledge of the peak horizontal ground acceleration, soil unit weights and layer thickness, and the water table depth (Youd & Idriss, 2001). Dynamic analysis options include the equivalent linear technique. Many variations of the original SHAKE program (Schnabel et al., 1972), such as SHAKE91 (Idriss & Sun, 1992), are available for one-dimensional columns. Two-dimensional structures may be evaluated using programs such as FLUSH (Lysmer et al., 1975) or QUAD4M (Hudson et al., 1994). Although these dynamic methods estimate the
cyclic shear stress history through the entire earthquake, only the peak value is typically used when assessing liquefaction.

CRR can be estimated directly from laboratory tests or indirectly by empirical correlation with in situ tests. An empirical approach developed for the Standard Penetration Test (SPT) by Professor Seed (Seed & Idriss, 1971; Seed et al., 1985) is commonly used. This approach has recently been formalized by MCEER (Youd & Idriss, 1998, 2001) along with a similar correlation for the Cone Penetration Test (CPT). While the original 1998 MCEER report is more detailed, the later 2001 publication contains updated information.

1.1.2 Flow slide evaluation

Displacements of an embankment or foundation are often small when there is no liquefaction and may be estimated using preliquefaction properties. The potential for displacements becomes a concern as the extent of liquefaction increases.

The possibility of a flow slide is usually evaluated with limit equilibrium techniques. A flow slide is considered to occur when the available strength following liquefaction is less than the strength required to maintain equilibrium under gravity loading. Large changes in geometry may be required to reduce the driving stresses and establish equilibrium, and the resulting deformations are often termed a flow slide. The susceptibility to large deformations is found by assigning residual or postliquefaction strengths to the liquefied zones and appropriate strengths to the nonliquefied material. If the factor of safety is less than or near one, large deformations are assumed.

1.1.3 Displacement evaluation

Large but limited displacements may occur even if the structure has sufficient strength following liquefaction to maintain overall stability. These displacements accumulate during
the earthquake and are a direct result of the continuing inertia loading. Two approaches are available to estimate these displacements: empirical equations and mechanics-based analyses.

Empirical relationships are valuable tools as they directly include the intangible and uncertain aspects of field response. They are derived from field observations but are limited to specific topographic and material conditions. They are often impractical for evaluating two-dimensional effects, such as those caused by embankments, structures and foundations, or by site remediation such as densification. Examples of empirical relationships include those by Youd et al. (1999); Hamada et al. (1987); Bardet et al. (1999a); and Rauch et al. (1999). Empirical equations do provide a convenient database for calibrating other approaches.

Mechanics-based methods approximate soil behaviour using numerical models. These models attempt to capture the physics of dynamic soil response, although their success is limited by inherent simplifications. These methods often require some knowledge of soil properties, such as stiffness, strength, and unit weight.

The simplest and most common method for estimating displacements was first proposed by Newmark (1965). This approach uses an extremely simple mechanical representation: the displacing soil is modeled as a rigid block translating on an inclined plane as shown in Figure 1-3. The earthquake motion is applied to the underlying plane. The available strength along the shear surface is represented by a yield acceleration. This yield value for typical structures is often estimated from limit equilibrium analyses. Relative displacements begin when the input acceleration exceeds the prescribed yield acceleration. Cumulative displacements are easily computed through double integration procedure. Although the effect of strength loss can be included in the analysis, the system is treated as rigid so any effect of stiffness changes on the response is not considered. These stiffness
effects include amplification of motions due to structural response, the reduction in stiffness with liquefaction, and the saw tooth or ratchet-type displacements that occur after liquefaction.

An extension of the Newmark approach to include the structural response of moderately high embankments was proposed by Makdisi and Seed (1978). Pseudo-dynamic approaches to the Newmark integration are also available for one-dimensional (Byrne, 1991) and two-dimensional (Jitno & Byrne, 1994) situations. These methods approximately include the reduced postliquefaction stiffness, although the earthquake is simplified to a series of velocity pulses. Additional techniques and variations are reviewed by Rathje and Bray (1999) and referenced by Lin and Hynes (1998).

1.1.4 Discussion

Current practice has many advantages, including wide experience and the relative simplicity of procedure and input. But there are many disadvantages including the simple evaluation of triggering, the crude modeling of postliquefaction mechanics, the reliance upon one-dimensional representations of structural response, and the disjointed nature of the three-phase procedure.
Chapter 1 — Introduction

The development and application of more sophisticated analysis approaches is relatively recent. Large displacement analyses to evaluate postliquefaction deformations at dams were first used in 1989 (Finn, 1999). These sophisticated analysis tools now include total stress and simple effective stress approaches (e.g., Moriwaki et al., 1998; Inel et al., 1993) to a number of more complex effective stress methods (e.g., Prevost, 1985; Finn et al. 1986; Beaty & Byrne, 1998a; Wang & Makdisi, 1999).

While some of these tools are gaining acceptance, they are generally beyond the state of practice. Total stress methods are less complex and are closer to common procedures. This class of tools will likely be adopted first into the state of practice. However, there are many hurdles. The California Division of Safety of Dams notes that many of the new analysis methods “have been formulated logically and combine the work of many researchers, but have not been substantiated by observed performance of existing dams and must therefore be viewed with conservative skepticism” (Babbitt & Verigin, 1996).

Due to the complexity of the problem, the objective of these analyses is focused: to predict general deformation patterns and approximate estimates of displacement magnitudes. Although a well-formulated analysis can provide reasonable predictions of the magnitude and possible variation of displacement, the results must always be checked and calibrated against experience. Imprecision results from simplifications in the constitutive model and, perhaps more importantly, uncertainty in the material properties and stratigraphy. Rather than precise predictors of displacement, these analysis tools are best used for understanding the potential behaviour of a structure and evaluating the relative effectiveness of remedial measures. These qualifications apply equally well to the analysis approach that is the basis of this dissertation.
1.2 **Research Objectives**

This thesis has three main goals:

3. To develop and calibrate a practical analytical tool for estimating displacements due to earthquake-induced liquefaction.

4. To provide preliminary guidelines on input parameters and identify critical variables.

5. To make observations regarding the method and analyses that may be useful to future applications.

Emphasis in developing the practical analytical tool is given to eliminating many of the weaknesses inherent in the current state of practice while maintaining the ease of understanding and interpretation. In particular, the analytical method should adequately represent the basic stress-strain and strength response of both liquefied and nonliquefied material, should synthesize the triggering/flow/displacement aspects of the analysis into a cohesive and rational whole, and should be formulated for use in a two-dimensional numerical model. To achieve these goals, the following philosophy was followed in developing the proposed approach:

1. Rely upon common and well-understood interpretations of liquefaction and postliquefaction behaviour.

2. Build upon accepted practice while bridging the gap between state of practice and the more fundamental and advanced analyses.

3. Maintain simplicity. Allow for complexity when and if necessary.

1.3 **Dissertation Overview**

The presentation of this research begins in Chapter 2 with an overview of typical sand behaviour. An emphasis is given to observations from laboratory testing and includes a
discussion of residual strength, a critical and uncertain parameter in any postliquefaction analysis. The simple analytical approach is proposed in Chapter 3, followed by a more detailed discussion of the required input parameters in Chapter 4. The following chapter shows the application of the approach to various structures and case histories. Lessons learned from these analyses are discussed. A summary, conclusions, and thoughts for future research are given in Chapter 6. The organization of this presentation is intended to stress the simple yet fundamental basis of the approach and its overall usefulness as a practical analytical tool.
CHAPTER 2 — Overview of Sand Behaviour and Liquefaction

As with any numerical or mechanics-based approach, the proposed analysis tool is only an approximation of actual soil behaviour. It is worthwhile to review the complex stress-strain behaviour of sand as it applies to the liquefaction-deformation problem before presenting the analysis procedure. Such an understanding is necessary to select material properties and evaluate results even for analyses that use simple constitutive models.

Key aspects of sand behaviour include the stiffness and strength prior to liquefaction, the resistance to triggering of liquefaction, and the postliquefaction stiffness and strength. While the focus here is on undrained behaviour, the response of sand is controlled by the effective stresses on the soil skeleton. The primary difference between undrained, drained, or partially drained response is simply the constraint imposed by the pore water. This is of critical importance given the tendency of the sand skeleton to contract or dilate while shearing.

This chapter reviews the response of sand to monotonic loading supplemented with observations from cyclic testing and empirical evaluations. It concludes with a brief discussion of mobilized residual strength. An overview of key factors impacting the deformation analysis is presented. Many of the interpretations highlight the laboratory testing performed by Prof. Y. P. Vaid and students at the University of British Columbia (e.g., Vaid & Thomas, 1995; Uthayakumar & Vaid, 1998; Vaid & Sivathayalan, 2000). While this chapter is primarily a literature review, several new data compilations and interpretations have been developed and are presented.
2.1 Introduction

Liquefaction of saturated sands has been loosely defined as any process leading to a loss of shear strength or to large strains due to transient or repeated loadings (National Research Council, 1985; Jitno, 1995). For monotonic loading, strain softening following an initial peak strength is often described as liquefaction. This is illustrated by the lower two stress-strain curves in Figure 2-1. Strain-softening results from an increase in pore pressure due to shear-induced contraction. Regions of contractive and dilative behaviour in monotonic loading are delineated by a constant stress ratio as shown on Figure 2-1(b). The stress ratio $\eta$ is defined as $\tau_{\text{max}}/\sigma'_{m}$ where $\tau_{\text{max}}$ is the maximum shear stress and $\sigma'_{m}$ is the mean effective normal stress. The stress ratio defining regions of contraction and dilation is related to the constant volume friction angle $\phi_{cv}$ where $\eta_{cv} = \sin\phi_{cv}$.

Figure 2-1. Illustration of stress-strain behaviour for undrained monotonic loading.
Several measures of mobilized shear strength are needed to describe undrained monotonic loading: initial peak $S_{u\text{-}\text{peak}}$, quasi-steady state $S_{qss}$, and steady-state $S_{ss}$. These strengths are indicated on Figure 2-1. $S_{qss}$ occurs during monotonic loading as the sand changes from a contractive to a dilative response at phase transformation. $S_{ss}$ corresponds to the stress condition where dilation is curtailed and the sand strains at constant volume. The term $S_{qss\text{-}ss}$ will be used to indicate the minimum undrained strength after strain softening regardless of whether it is a steady state or quasi-steady state strength. $S_{u\text{-}\text{peak}}$ is defined here as the initial peak strength that occurs prior to strain softening.

The strength at phase transformation is associated with a significant modulus change for both strain hardening and strain softening samples. The stiff behaviour observed in much of the contractive region can dramatically soften once the sand begins to dilate. For numerical analysis purposes, the strength at phase transformation for strain hardening soils is analogous to the $S_{u\text{-}\text{peak}}$ from strain softening samples. For simplicity in this thesis, $S_{u\text{-}\text{peak}}$ will refer to both the initial peak strength for strain softening samples and the strength at phase transformation for strain-hardening samples.

Liquefaction during cyclic loading is also caused by shear-induced contraction of the sand skeleton. For cyclic loading, the triggering or onset of liquefaction in a looser sand coincides with a significant softening of the sample. Very high pore pressures may be generated at stress reversals following the initial onset of liquefaction and produce a temporary but complete loss of effective normal stress and shear strength. In practical terms, the occurrence of large shear strains during cyclic loading is generally described as liquefaction even if some effective stress is maintained. Typical liquefaction response during cyclic loading is shown schematically in Figure 2-2.
Liquefaction is sometimes defined as the condition of zero effective stress when the pore pressure ratio $r_u$ equals 100%. The use of the term liquefaction in this thesis is somewhat broader and is used to describe the beginning of the soft liquefied response. A saturated sand that has liquefied is either strain softening or has a softened loading response controlled by its tendency to dilate at higher stress ratios. The pore pressure ratio $r_u$ will typically vary and will often be less than 100%. Liquefaction usually applies to loading that is predominantly undrained, although the behaviour may occur in conjunction with significant flow of pore water. Postliquefaction refers to the typical softened stress-strain

![Diagram of Liquefaction](image)

Figure 2-2. Illustration of liquefaction due to cyclic loading.
response of a sand after triggering.

The mobilized residual strength $S_r$ is the strength available after liquefaction as observed in field case histories (Stark et al., 1998). The minimum undrained strength $S_{qss}$ from laboratory tests is often assumed to represent the residual strength. An important consideration with $S_r$ is the shear strain required to achieve this strength. Unacceptable deformations may occur even if a suitable strength is eventually reached. Strain-compatibility is also a concern since significant loads may transfer to stiffer soil elements before the full $S_r$ is mobilized. A reliable analysis must account for such effects. A further concern is whether the strength determined from undrained laboratory tests are applicable to field response.

2.2 Characteristic Response

The stress-strain response of sand is a function of many parameters including strain level, density, confining stress, fabric, anisotropy, initial static bias, principal stress rotation, fines content, and drainage conditions. Although most studies investigating these effects have focused on monotonic loading, there is a corresponding impact on cyclic loading.

2.2.1 Nonlinear stress-strain response

A chief feature of the stress-strain response of sand is its nonlinearity. Typical drained and undrained response is illustrated in Figure 2-3 and Figure 2-4. The stress-strain curve for monotonic loading up to the peak shear stress is often described by a truncated hyperbola (Kondner & Zelaski, 1963; Duncan & Chang, 1970; Matsuoka & Nakai, 1977). The hyperbolic shape is applied to either drained or undrained loading. Typical hyperbolic parameters are available in the literature (Wong & Duncan, 1974; Byrne et al., 1987).
Cyclic stress-strain behaviour is also nonlinear and has a significant component of strain-dependent hysteretic damping as suggested by Figure 2-2d. An early analytical approach for addressing nonlinear behaviour was developed by Seed and Idriss (1970). They used cyclic test data to develop relationships between cyclic shear strain and secant shear modulus and damping. These relationships were used in an iterative linear analysis to approximate nonlinear response (Schnabel et al. 1972; Lysmer et al., 1975). Many modulus and damping relationships have been proposed since that time for various material types (e.g., Hardin & Drnevich, 1972; Seed et al., 1986; Sun et al., 1988; Idriss, 1990; Boulanger et al., 1998). A common relationship for sands is shown in Figure 2-5.

2.2.2 Void ratio and confining stress

Figure 2-3 and Figure 2-4 illustrate the effects of dilation and contraction caused by shear strain. Dilation increases the peak drained strength due to the work done as the element expands against its boundary stresses (Bishop, 1954). For undrained response, the pore fluid...
resists the contraction or dilation of the sand skeleton resulting in a change of pore pressure. The rate of dilation depends on many factors. For a given sand, the effects of relative density and confining stress appear to be critical. Mineralogy, angularity, and grain size are also significant contributors.

The importance of density to strength and stiffness has been well established. Figure 2-6 shows undrained response from simple shear tests of air pluviated (AP) Syncrude sand. Increasing density corresponds to stiffer and stronger behaviour. While the impact on the minimum undrained strength is clear and dramatic, this figure also shows an influence on the initial peak undrained strength. $S_{u\text{-peak}}/\sigma'_{vo}$ varies from 0.13 for $D_r = 34\%$ to 0.20 for $D_r = 70\%$.

A similar effect occurs with confining stress (Riemer & Seed, 1997). Sand dilates more easily under low confining stress and contracts more readily under high confining stress. A loose sample under low confining stress may behave like denser sand under high confining

![Figure 2-5. Example modulus reduction and damping curves for sand. (Data from Seed & Idriss, 1970; Idriss, 1990)]
stress. This is illustrated in Figure 2-7 and Figure 2-8.

Density is critical to the strength of liquefied sand. The steady state strength $S_{ss}$ is often considered a unique function of void ratio. $S_r$ estimated from field studies is typically plotted as a function of penetration resistance, an indicator of relative density. But laboratory testing shows the quasi-steady state strength $S_{qss}$ is a function of both void ratio and initial effective confining stress (Vaid & Sivathayalan, 1999). The test results summarized in Figure 2-9 illustrate that while $S_{qss}$ increases as void ratio decreases, it is not a unique function of $e$. The $S_{qss}$ versus $e$ relationships appears to normalize with respect to $\sigma_{vo}$.

2.2.3 Anisotropy and stress path

Both stiffness and strength are functions of loading direction. This has been seen in
Chapter 2 — Overview of Sand Behaviour and Liquefaction

Water pluviated (WP) Fraser River sand
(Loosest WP state)

Figure 2-7. Undrained response in triaxial compression and extension.
(Figure after Vaid & Sivathayalan, 1999)

Water pluviated (WP) Fraser River sand
in undrained simple shear.
Loosest WP state ($D_r = 25-36\%$).

Figure 2-8. Undrained response as a function of confining stress.
(Figure after Sivathayalan, 1994).

comparisons of triaxial compression and extension tests (Riemer & Seed, 1997; Vaid & Sivathayalan, 1999; Figure 2-7) as well as hollow cylinder torsion (HCT) tests (Vaid & Sivathayalan, 1999; Figure 2-10). Compression tests are typically performed with the major principal stress acting parallel to the direction of deposition (i.e., aligned with gravity). The major principal stress in an extension test is oriented perpendicular to the deposition direction (i.e., parallel to the bedding planes), while the major principal stress direction in an HCT test may be varied to investigate different orientations. Anisotropy impacts the deformation analysis since it affects the available strength and stiffness and may influence the mode of failure.

Water-pluviated (WP) samples loaded in undrained compression tend to be strain hardening. Although their maximum undrained strengths are typically greater than their drained strengths, loose samples may require substantial strains to develop this strength. Samples loaded in extension are softer and more prone to strain soften, while loading in
simple shear appears to be intermediate. Unfortunately, loading similar to simple shear is common in geotechnical structures. Similar stress conditions can be caused by many types of loading including foundation bearing pressures, gravity loading of slopes or embankments as illustrated in Figure 2-11, and earthquake motions.

The variation of $S_{qss}$ strength with direction of loading may be approximated by an interpolation function as shown by Equation 2.1. The strength in any direction is estimated from the strength in the compressive direction $S_{0^\circ}$, the strength in the simple shear direction $S_{45^\circ}$, and an interpolation factor $F_{a\sigma}$.

$$S_{a\sigma} = S_{45^\circ} + F_{a\sigma} \times (S_{0^\circ} - S_{45^\circ})$$

Eqn. 2.1
$F_{\alpha \sigma}$ can be estimated from Figure 2-12. This figure was derived from three groups of HCT tests performed on loose sands (Uthayakumar & Vaid, 1998). The test samples had relative densities of 30% to 40%, values of intermediate principal stress parameter $b$ of 0 and 0.5, and initial mean effective stresses of 200 kPa and 400 kPa. The curves are based on $S_{qss}$ strengths for tests that strain softened. The mobilized resistance at 5% shear strain was used for orientations that did not strain soften. The curve in Figure 2-12 is described by Equation 2.2. Although a somewhat complicated expression, this form has the advantage of a zero slope for $\alpha_\sigma$ equal to 0° and 90°.

$$F_{\alpha \sigma} = \frac{0.39 \times \cos 2\alpha_\sigma}{1 - 0.585 \times \cos 2\alpha_\sigma - 0.025 \times \cos^2 2\alpha_\sigma}$$  Eqn. 2.2

Anisotropy affects the initial peak strength as well as the minimum undrained strength $S_{qss-ss}$. For the set of HCT results used to develop Figure 2-12, the initial peak strength varies from about $S_u/\sigma'_{vo} = 0.33 - 0.38$ in compression to $S_u/\sigma'_{vo} = 0.20 - 0.22$ in extension.
2.2.4 Initial shear stress

Sivathayalan (2000) presents data for the $S_{qss-ss}$ strength of loose Fraser River sand as a function of the initial shear stress or static bias and the loading direction $\alpha_\sigma$. When $S_{qss-ss}$ was normalized by $\sigma'_i$, the initial major principal effective stress, it was found to be independent of the initial shear stress as defined by $K_c$. $S_{qss-ss}/\sigma'_i$ does vary with $\alpha_\sigma$ as shown in Figure 2-13. The variation of strength with $\alpha_\sigma$ is in general agreement with the trend already presented in Figure 2-12.

Since it is common to normalize $S_{qss-ss}$ with respect to $\sigma'_i$, data in Figure 2-13 were converted for this study and shown in Figure 2-14. While $S_{qss-ss}/\sigma'_i$ is independent of $K_c$, $S_{qss-ss}/\sigma'_o$ varies greatly with initial shear stress. The differences are smallest for loading in the compressive direction and largest in the extensive direction. This is significant to deformation analyses since the static bias within many slopes and embankments is variable and can be relatively large. Using $S_{qss-ss}/\sigma'_o$ ratios developed from $K_c = 1$ conditions could

![Figure 2-13](image1.png)  
Fraser River sand (WP)  
$D_r = 19\% - 24\%$  
$\sigma'_o = 200$ kPa  

![Figure 2-14](image2.png)  
Fraser River sand (WP)  
$D_r = 19\% - 24\%$  
$\sigma'_o = 200$ kPa  

Figure 2-13. Effect of static bias and $\alpha_\sigma$ on $S_{qss-ss}/\sigma'_i$ from HCT. (Figure after Sivathayalan, 2000)  
Figure 2-14. Effect of static bias and $\alpha_\sigma$ on $S_{qss-ss}/\sigma'_o$ from HCT.
produce low predictions of the \( S_{qss} \) strength if the initial horizontal shear stress is large. As suggested by Sivathayalan (2000), \( S_{qss}/\sigma'_{io} \) may give improved estimates of the initial minimum undrained strength.

Figure 2-15 presents data on the initial peak strength \( S_{u\text{-peak}} \) from this same set of tests. These strengths have been normalized to \( \sigma'_{vo} \) and plotted against the initial static shear stress in Figure 2-16. It is clear that \( S_{u\text{-peak}} \) and \( S_{u\text{-peak}}/\sigma'_{vo} \) are both functions of initial shear stress. Figure 2-16 shows the effect on \( S_{u\text{-peak}}/\sigma'_{vo} \) becomes significant as the initial shear stress approaches the peak shear stress.

The effect of static bias on the preliquefaction stiffness should be secondary to most deformation analyses that include significant liquefaction. This effect will not be considered here. The impact of static bias on cyclic resistance will be discussed in Section 2.2.7.

![Figure 2-15](image1.png)

**Figure 2-15.** Effect of static bias and \( \alpha_\sigma \) on \( S_{u\text{peak}} \) from HCT. (Figure after Sivathayalan, 2000).

![Figure 2-16](image2.png)

**Figure 2-16.** Effect of static bias and \( \alpha_\sigma \) on \( S_{u\text{peak}}/\sigma'_{vo} \) from HCT.
2.2.5 Fabric

The orientation of particles and particle contacts within the sand matrix is called the fabric (Oda, 1972; Oda et al., 1978). The effect of fabric on the stress-strain response in simple shear is shown in Figure 2-17. Samples of Syncrude sand were prepared at essentially the same void ratio using three different methods: moist tamping (MT), air pluviation (AP), and water pluviation (WP). \( S_{qs} \) from these tests varies by an order of magnitude, with the moist-tamped sample having the lowest strength in this case and the water-pluviated sample the highest strength. The effect of fabric should disappear with strain as the sample becomes remolded, although the strains required to achieve this may be intolerably large.

Many geotechnical structures may have fabrics consistent with water pluviation. These include hydraulic fill dams, some tailings impoundments, water-dumped backfill behind quay walls, and some alluvial or delta deposits. Vaid and Sivathayalan (1999) compared test results from undisturbed frozen samples and from water-pluviated samples of Fraser River.
sand. The frozen samples were obtained from alluvial deposits in the Fraser River delta. Similar stress-strain behaviour was seen in monotonic undrained simple shear. Good agreement was also obtained for cyclic liquefaction resistance. Vaid and Sivathayalan concluded that water pluviation appears to closely replicate the fabric of water-deposited in situ sands.

Much of the field database on residual strength appears to be derived from water-deposited material (Wride et al., 1999), although other types of fabrics do occur in engineering structures. A lightly compacted fill may be liquefiable and have a structure similar to moist tamping. The fabric of landslide deposits or residual soils may be unique. Care should be taken before using relationships derived from water-pluviated sands to structures with other types of fabrics. Figure 2-17 suggests that such assumptions can be very unconservative.

### 2.2.6 Principal stress rotation

Significant plastic shear strains and pore pressures may occur during the rotation of principal stresses even if the shear stress \( \tau \) or stress ratio \( \tau/\sigma' \) is held constant (Gutierrez et al., 1991). The response to rotation is a function of both the stress history and the direction of rotation (Shibuya & Hight, 1987; Sivathayalan, 2000). The resulting strains can be large for some stress paths, such as undrained rotation from a compressive direction \( (\alpha_\sigma = 0^\circ) \) to an extensive direction \( (\alpha_\sigma = 90^\circ) \) at a large stress ratio. A further complication to developing a constitutive model that includes principal stress rotation is noncoaxiality: the increments of plastic strain are not in the same direction as the principal stresses (Gutierrez et al., 1991; Wijewickreme & Vaid, 1993). Coaxiality is a common assumption in plasticity.

Although the effect of principal stress rotation can be significant, it is often considered secondary and not typically included in stress-deformation analyses.
2.2.7 **Cyclic strength and triggering**

The resistance of sand to liquefaction under cyclic loading has been investigated in the laboratory and through empirically based studies of liquefaction field events. Laboratory studies reveal the triggering of liquefaction to be a function of cyclic stress ratio $\tau_{cyc}/\sigma'_{vo}$ and number of load cycles.

Cyclic strength data from undisturbed frozen samples of natural deposits was compiled as part of this thesis. Figure 2-18 shows data from both simple shear and triaxial laboratory tests of samples that were frozen in situ. The cyclic resistance ratio, CRR, from each test was normalized by the interpolated value of CRR to cause liquefaction in 15 cycles, CRR$_{15}$. This normalization allows results from different sites to be combined even if the tests were performed at different relative densities or stress levels. The data follows a fairly narrow trend after normalizing. The best-fit curves shown on Figure 2-18 assume a linear relationship on a log-log plot. Except for the Duncan Dam sand that had a fines content of 10%, all of the tests were performed on clean sands with fines contents less than 5%. Additional information is provided in Appendix B.

The normalized data on Figure 2-18 suggests the cyclic strength curve may become steeper as the relative density increases. Figure 2-19 shows the estimated values of CRR$_1$/CRR$_{15}$ plotted against relative density for each series of tests. The trend shown on this figure is tentative for several reasons. Difficulties in defining a triggering criterion for dense sands, as discussed in Appendix B, affect the value of CRR$_1$ for these sands. The estimated CRR$_1$/CRR$_{15}$ ratio also depends upon the assumed shape of the best-fit curves. Except for the Niigata Station sand, relatively few tests produced liquefaction in less than 4 cycles of loading: Niigata Showa Bridge sand had 2 tests, Duncan dam sand had 0 tests, Massey sand had 0 tests, and Kidd II sand had 1 test. Surprisingly, the tests on the denser deposits provide
the most data on liquefaction in small numbers of cycles. The estimate of $\frac{\text{CRR}_i}{\text{CRR}_{15}}$ depends to some extent on the number and distribution of test results from each series.

A common empirical relationship for cyclic resistance that was developed at UC Berkeley (Seed et al., 1984) relates CRR for a magnitude 7.5 earthquake to corrected SPT blowcount $N_{1.60}$. Figure 2-20 shows the updated curve recently published by MCEER (Youd

![Diagram](image_url)

**Figure 2-18.** Normalized cyclic strength from frozen samples.
& Idriss, 2001). Many other curves have been proposed, including those that use other in situ tests or address the probability of liquefaction (Youd & Idriss, 1998, 2001; Cetin et al, 2000).

The curves shown in Figure 2-20 are for a specific set of conditions: $\sigma'_{vo} = 96$ kPa, level ground, and a Magnitude 7.5 earthquake. Modifying them to other situations is usually done through the following equation:

$$CRR = K_a \cdot K_\sigma \cdot K_m \cdot CRR_{chart}$$  \hspace{1cm} \text{Eqn. 2.3}

$K_a$ corrects for initial horizontal shear stress. $K_\sigma$ reduces the CRR for effective confining stresses greater than 100 kPa. $K_m$ is the magnitude scaling factor and accounts for differences in the anticipated number of load cycles for different earthquake magnitudes. Since a magnitude 7.5 earthquake is considered to have approximately 15 significant cycles

![Trend Line: CRR1 / CRR15 = 1.3 - 0.5*D_r + 0.7*D_r^2](image)

Note: CRR1 / CRR15 values for Kidd II, Massey, and Duncan dam required significant extrapolation of test data.

Figure 2-19. Estimated values of CRR1 / CRR15 versus relative density.
of loading (Seed & Harder, 1990), using a $K_m = 1.0$ provides an estimate of CRR$_{15}$. The MCEER workshop suggested relationships for each of these factors (Youd & Idriss, 1998, 2001). Although each of these corrections includes substantial uncertainty, $K_a$ is the most problematic. While the original MCEER report provides a $K_a$ correction chart, it does not recommend it for general use (Youd & Idriss, 1998).

2.2.8 *Pore pressure response*

Pore pressure rise and liquefaction are caused by plastic volumetric contraction under shear loading. Changes in pore pressure can be directly related to increments of plastic shear

![Figure 2-20. CRR versus $N_{1-60}$ from MCEER. (Figure from Youd & Idriss, 2001)](image-url)
strain by a few simple observations. If the volumetric response of a saturated soil due to a
reduction in mean effective stress is assumed elastic, then a change in pore pressure $\Delta u$ under
constant mean total stress can be directly related to an increment of elastic volumetric strain
$\Delta \varepsilon^e_v$:

$$-\Delta u = \Delta \sigma'_m = B^e \Delta \varepsilon^e_v$$  \hspace{1cm} \text{Eqn. 2.4}

where $B^e$ is the elastic bulk modulus of the soil skeleton. The net volumetric strain for
undrained conditions is trivial due to the constraint of the pore water. To satisfy this
condition, an increment of elastic volumetric strain must balance any increment of plastic
volumetric strain $\Delta \varepsilon^p_v$ due to contraction:

$$\Delta \varepsilon_v = \Delta \varepsilon^e_v + \Delta \varepsilon^p_v \equiv 0 \Rightarrow \Delta \varepsilon^p_v = -\Delta \varepsilon^e_v$$  \hspace{1cm} \text{Eqn. 2.5}

Since the plastic volumetric strain is assumed to result solely from shear induced
contraction, it is a simple matter to relate the plastic volumetric strain to the plastic shear
strain $\gamma^p$:

$$\sin \psi = \frac{\Delta \varepsilon^p_v}{\Delta \gamma^p}$$  \hspace{1cm} \text{Eqn. 2.6}

where $\psi$ is known as the dilation angle and is defined by this ratio. Combining these three
equations gives a simple relationship between $\Delta u$ and $\Delta \gamma^p$:

$$\Delta u = (B^e \cdot \sin \psi) \cdot \Delta \gamma^p$$  \hspace{1cm} \text{Eqn. 2.7}

Equation 2.7 reveals that elastic bulk modulus and dilation angle are dominant factors
in pore pressure response. Additional discussion is provided in Appendix A and includes the
derivation of a relationship between $\Delta u$ and the increment of total shear strain $\Delta \gamma$. 
Typical pore pressure response to uniform cycles of shear stress and no static bias was shown in Figure 2-2. Pore pressures increase with each cycle of loading until the initiation of liquefaction at \( r_u = 100\% \). \( r_u \) is the ratio of excess pore pressure generated by the cyclic loading to the initial effective confining stress. An \( r_u \) of 100\% is associated with the loss of intergranular effective stress. Seed et al. (1976) compiled data from cyclic simple shear tests of sands at various relative densities and noted the relationship between \( r_u \) and cycles of loading \( N \) show in Figure 2-21.

One effect of the generated pore pressure is its impact on stiffness. Figure 2-22 shows an approximate relationship between the cyclic ratio and the reduction in stiffness due to an increase in pore pressure. The cyclic ratio is defined as the number of uniform loading cycles \( N \) that have been imparted to the sand element divided by the number of these load cycles required to trigger liquefaction \( N_{\text{liq}} \). The reduction in stiffness is estimated using Equation 2.8 where \( G_o \) is the initial stiffness at \( r_u = 0\% \) and \( G \) is the reduced modulus after pore pressure increase. Figure 2-22 and Equation 2.8 are based on the observation that both \( G_{\text{max}} \) and the initial tangent shear modulus from monotonic loading tests often vary as the square root of the effective confining stress (Byrne et al., 1987). As such, this relationship applies primarily to these specific tangent stiffness values.

\[
G = G_o \left(1 - r_u\right)^{0.5} \quad \text{Eqn. 2.8}
\]

For comparison, cyclic data from undisturbed samples of Fraser River sand were plotted in Figure 2-23 to show the observed reduction in secant stiffness. These observations agree well with the approximate relation for tangent modulus.
Figure 2-21. Typical pore pressure generation in sands due to cyclic loading. (Data from Seed et al., 1976).

Figure 2-22. Estimated reduction in $G_{\text{max}}$ due to increase in $r_u$. 

Recommended equation from Seed et al. (1976):

$$r_u \approx \frac{1}{2} \sin^{-1} \left( \frac{2}{N_{\text{liq}}} \left( \frac{N}{N_{\text{liq}}} \right)^{1/4} - 1 \right)$$
Softening due to pore pressure rise prior to liquefaction is likely a secondary effect in liquefaction-deformation analyses with significant zones of liquefaction, especially considering the much larger drop in stiffness that occurs after liquefaction is triggered in looser sands. Figure 2-23 suggests there may be relatively little change in average stiffness until a sand nears liquefaction (e.g., \( G/G_0 \approx 0.65 \) at \( N/N_{\text{lq}} \approx 0.7 \) in Figure 2-23).

Empirical correlations for CRR are typically based on the assumption of no softening. Including this softening due to pore pressure rise in a triggering analysis may produce erroneous results if the resistance to liquefaction is based on the empirical correlations. The MCEER workshop recommends that pore pressures be ignored when using the simplified procedure for evaluating liquefaction (Youd & Idriss, 2001).

Although the generation of pore pressure leads to liquefaction of a soil element, pore pressure dissipation can affect the triggering of liquefaction. If water is able to flow quickly from a zone then liquefaction may be delayed or prevented. On the other hand, water flowing

![Figure 2-23. Estimated change in stiffness due to increase in \( r_u \).](image)
into an element from an adjacent zone could hasten liquefaction. This will be discussed in more detail in Section 2.2.10.

Unloading of a soil after the triggering of liquefaction is often accompanied by a dramatic increase in pore pressure. A complete loss of effective stress at $r_u = 100\%$ can occur at shear stress reversals. This is demonstrated by the cyclic simple shear test in Figure 2-24.

![Figure 2-24](image-url) *Postliquefaction response of Nevada sand.* *(Figures after Bardet, 1997)*
High pore pressures coincide with a temporary but complete loss of shear strength. Lateral total stresses are large and consistent with a hydrostatic stress state.

2.2.9 Postliquefaction stiffness

While the shear stiffness for loading prior to liquefaction is related to increasing stress ratio $\tau/\sigma'$, the postliquefaction shear stiffness $G_{liq}$ results from the tendency for dilation at nearly constant stress ratio as shown in Figure 2-1. Sand can support additional load with strain due to the resulting drop in pore pressure and increase in normal effective stress. $S_{ss}$ is reached when the sand no longer has a tendency to dilate, although redistribution of pore pressure or cavitation of the pore fluid may occur before this steady state strength is reached. The mechanism of dilation that produces the postliquefaction strain hardening results in a shear modulus that can be much softer than the preliquefaction stiffness.

Equation 2.9 indicates that postliquefaction stiffness should primarily be a function of the elastic bulk modulus $B^e$ and dilation angle $\psi$. This was further investigated in Appendix A where the following approximate relationship for $G_{liq}$ was found:

$$G_{liq} = \frac{\Delta \tau}{\Delta \gamma} \sim \frac{\Delta \sigma'_m}{\Delta \gamma} \approx B^e \cdot \sin \psi$$  \hspace{1cm} \text{Eqn. 2.9}

Equation 2.9 suggests the postliquefaction stress strain response should be concave upward as $B^e$ increases with the rising effective stress. This is seen in the test data shown in Figure 2-25. Sands eventually reach a relatively constant $G_{liq}$ at higher shear stress. It is clear that the initial response at low shear stress may be quite soft. Figure 2-25 also suggests that $G_{liq}$ is not a function of the initial effective confining stress.

Figure 2-25 demonstrates the importance of anisotropy even to postliquefaction stiffness. Vaid and Sivathayalan (1997) present similar data from simple shear tests that suggests the corresponding $G_{liq}$ may be even softer than the curves shown for extension. It is
surprising that this anisotropy is maintained despite excursions through $r_u = 100\%$. Although these occurrences of very low effective stress and the corresponding shear strains are expected to modify much of the original fabric, an anisotropic structure appears to remain. This assessment, however, is complicated by the differing boundary conditions of the tests and the difficulties in testing postliquefaction loading cycles.

Density is seen to affect $G_{\text{iq}}$ through its importance to both $B_e^\psi$ and $\psi$. Vaid and Sivathayalan (1997) present data from Fraser River sand (WP) that shows $G_{\text{iq}}$ in simple shear increasing from 125 kPa to 900 kPa, a factor of 7, as the relative density increases from 28\% to 60\%. These test results are shown in Figure 2-26.

![Figure 2-25. Postliquefaction stress-strain response of Fraser River sand. (Figures after Vaid & Sivathayalan, 1999)](image-url)
Vaid and Thomas (1995) present data that indicates the type of loading that causes $r_u = 100\%$, either cyclic loading or monotonic loading followed by unloading, does not affect the magnitude of $G_{iq}$. Further, Vaid and Sivathayalan (1997) contend that the tangent shear modulus in the dilative portion of a monotonic test is essentially the same as the shear modulus obtained from loading a sample from an $r_u = 100\%$ state following cyclic loading. This is demonstrated by the simple shear test results shown in Figure 2-27. This is fortuitous since it allows a standard monotonic loading test to provide valuable information on the postliquefaction stiffness.

Liquefied sands under cyclic loading experience high pore pressures during reversals of shear stress. The stress-strain response immediately after stress reversal is often extremely soft and weak as shown by the simple shear results in Figure 2-24. The tendency of the sand to dilate and stiffen with further strain appears related to its strain history, with the region of

![Figure 2-26. Effect of relative density on postliquefaction response in simple shear. (Figure from Vaid & Sivathayalan, 1997)](image_url)
very low stiffness a function of the previous shear strains (Vaid & Thomas, 1995; Sivathayalan, 2000).

Density, anisotropy, strain path, and shear stress are seen to affect the postliquefaction stiffness of a given sand. The stiffness should also vary by the type of sand, with mineralogy, gradation, and angularity having some impact on the tendency to dilate.

Although the loading modulus of liquefied soils is soft, the unloading modulus is relatively stiff. For the last two load cycles shown on Figure 2-24, the average unloading modulus between $\tau=5$ kPa and $\tau=20$ kPa is approximately 3400 kPa, while the corresponding loading modulus is only 400 kPa. The ratio is about 8.5, or nearly an order of magnitude. This ratio drops to about 6 if the loading modulus that corresponds to the more linear portion of the loading curve is used, a value of about 550 kPa. Figure 2-28 shows similar results in

![Figure 2-27. Monotonic versus postcyclic response in simple shear.](Figure after Vaid & Sivathayalan, 1997)
triaxial compression. The loading modulus for the linear portion of the loading curve is approximately 2800 to 3800 kPa, while the unloading modulus is about 40000 to 47000 kPa. The ratio in this case is about 12 to 14.

This difference in moduli can produce a ratcheting effect when there is a static bias, as shown in Figure 2-28. Strain accumulates with each cycle of load even though the full strength is not achieved. Similar response has been inferred from field recordings of earthquakes and from centrifuge tests (Dobry & Abdoun, 1998). This behaviour is currently the focus of much research and may account for many instances of lateral spreading (Elgamal & Yang, 2000).

2.2.10 Drainage

The coefficient of permeability \( k \) might be \( 10^{-3} \) to \( 10^{-5} \) m/s for clean sand and \( 10^{-5} \) to \( 10^{-7} \) m/s for silty sand (Kulhawy & Mayne, 1990). This is sufficient to allow predominantly

![Figure 2-28](image)

*Figure 2-28. Postliquefaction stress-strain response of Syncrude sand.*

*(Figure after Sivathayalan, 2000)*
drained behaviour in clean sands under many types of loadings. But drainage even in clean sands and gravels becomes less certain with abrupt loadings such as earthquakes. True undrained conditions may exist for some time, or substantial flow of pore water may occur during or after the earthquake. Transient flow may temporarily increase or decrease the pore pressure in a zone. CRR and $S_r$ may be enhanced or degraded. Preferential flow paths can greatly affect local drainage. Barriers to drainage may lead to catastrophic consequences by forming weakened zones or water lenses (Kokusho et al., 2000).

Water flowing from loose zones into dense zones with lower pore pressures can significantly reduce the strength of the denser material or increase its liquefiability (Eliadorani, 1999; Vaid & Sivathayalan, 1999). The potential impact is illustrated in Figure 2-29, which compares the response of loading at zero volumetric strain and loading with a controlled water inflow. Vaid and Sivathayalan conclude the drastic change in

Figure 2-29. Undrained loading versus controlled expansion [water inflow]. (Figure after Vaid & Sivathayalan, 1999)
behaviour is not simply due to the change in density since the corresponding volumetric
strains are only 1%. Rather, it occurs from the interplay between the rate of shear-induced
contraction or dilation and the rate of water inflow. Similar behaviour has been observed
from biaxial tests, where monotonic loading is applied in one direction and cyclic loading in
an orthogonal direction (Meneses et al., 1998). The cyclic loading essentially creates a pore
pressure generator that greatly affects the monotonic response in the orthogonal direction.

Although undrained behaviour is commonly assumed in analyses, this assumption can
be unconservative. Atigh demonstrated the importance of drainage through analysis of a
slope using the UBCSAND effective stress constitutive model (Atigh & Byrne, 2000). The
effects of drainage may be loosely approximated in a less sophisticated analysis by selecting
material properties that account for the anticipated behaviour.

Granular soils deposited by man or nature tend to be layered or laminated. Low
permeability layers may impede drainage causing a loose zone or even free water to form
below the base of the layer. The importance of this behaviour has been effectively
demonstrated by Kokusho et al. (2000) using a shaking table. Silt layers were found to
greatly affect the deformations in these simple tests. The formation of water films is a
particular concern for stratified sites. The need for adequate site characterization with careful
identification of the stratigraphy is clear.

2.2.11 Fines content

This discussion of fines content is limited to its influence on two parameters critical to
a liquefaction-deformation analysis: CRR and mobilized residual strength $S_r$. Empirical
correlations for CRR and $S_r$ are typically formulated for clean sand conditions. These
correlations are then adjusted to account for fines content (FC), but these correction factors
are uncertain even for nonplastic fines. The influence of plastic fines is even less well defined (Seed, 1987; Ishihara, 1993; Youd & Idriss, 1998, 2001).

Common empirical correlations relate CRR or \( S_r \) to penetration resistance. Fines content likely affects all three parameters, although the relative effect is uncertain (Youd & Idriss, 2001). For convenience, the measured penetration resistance is typically modified to account for fines content (e.g., \( N_{1-60} \) is corrected to \( N_{1-60cs} \)).

The MCEER workshop recommended a fines content correction for CRR to be used with the clean sand base curve of Figure 2-20. This correction for \( N_{1-60} \) is given by Equation 2.10 and Table 2-1. This recommendation is somewhat different than shown by the curves in Figure 2-20.

\[
\text{FC Correction for CRR: } \quad N_{1-60cs} = \alpha + \beta \times N_{1-60} \quad \text{Eqn. 2.10}
\]

The fines content correction for residual strength \( S_r \), as proposed by Seed and Harder (1990), is given in Equation 2.11 and Table 2-2.

\[
\text{FC Correction for } S_r: \quad N_{1-60cs} = N_{1-60} + \Delta N_{corr} \quad \text{Eqn. 2.11}
\]

**Table 2-1. Coefficients for MCEER CRR fines content correction**

<table>
<thead>
<tr>
<th>Fines Content FC (%)</th>
<th>( \alpha )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>FC ( \leq 5% )</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>5% ( &lt; FC &lt; 35% )</td>
<td>( e^{(1.76 - 190/FC^2)} )</td>
<td>0.99 + FC(^{1.5}/1000 )</td>
</tr>
<tr>
<td>35% ( \leq FC )</td>
<td>5</td>
<td>1.2</td>
</tr>
</tbody>
</table>
Table 2-2. $S_r$ fines content correction for $N_{1-60}$
(Seed and Harder, 1990)

<table>
<thead>
<tr>
<th>Fines Content FC (%)</th>
<th>$\Delta N_{corr}$ (blows/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>1</td>
</tr>
<tr>
<td>25%</td>
<td>2</td>
</tr>
<tr>
<td>50%</td>
<td>4</td>
</tr>
<tr>
<td>75%</td>
<td>5</td>
</tr>
</tbody>
</table>

Recent evaluations of the empirical database for CRR provide further information on
the effect of fines content (Cetin et al., 2000). These new studies are preliminary but suggest
the standard corrections may result in CRR values that are too high.

It is interesting that of the 17 case histories in the Seed and Harder (1990) database for
$S_r$, no more than 5 (29%) are from clean sand sites (representative FC $\leq$ 5%). The average
fines content of the entire database is roughly 25%, and four of the sites have FC values of
40% or more (Wride et al., 1999). For comparison, the 1984 Seed database for liquefaction
triggering contains 74 sites (Seed et al., 1984). Of the 63 sites with gradation information, 36
(57%) are from clean sand sites and 27 (43%) have fines contents greater than 5%. The
average fines content of the siltier sites is 22%. Although the evidence is not conclusive,
fines content may be one indicator of susceptibility to large postliquefaction deformations
(R.B. Seed, personal communication with P.M. Byrne).

In contrast, empirical correlations for displacement by Youd et al. (1999) show greatly
decreasing displacement with increasing fines content (Section 5.1.4). Youd notes that silts
tend to have lower residual strengths than sands or silty sands at a given density, but alluvial
silts are generally laid down in a moderately dense state (Youd & Gilstrap, 1999). These
moderately dense silts will tend to be dilative at low confining stresses. Youd cautions that loose silts may be prone to catastrophic flow failure.

2.2.12 Mixing

Large strains and localized shearing may cause mixing of layered soils. A uniform soil tends to have a higher void ratio than a graded soil at the same relative density. Mixing two relatively uniform soil layers will result in a more broadly graded soil, although its void ratio will reflect the density of the premixed components. Since well-graded soils are prone to have lower void ratios than uniform soils, mixing will create a soil with a tendency to compact and strain soften if sheared undrained. Mixing was identified as a critical component of the flow failure at the Mufulira Mine in Zambia in 1970 (Byrne & Beaty, 1998). Hydraulic fill dams and tailings deposits may be prone to mixing due to their layered stratigraphy. Although large strains are likely required to mix soils, it may be a crucial factor in progressive instability or in situations where liquefied sands have flowed for great distances.

2.2.13 Cyclic versus monotonic liquefaction

Much of our understanding of $S_{qss}$ has been developed from monotonic loading. Vaid and Sivathayalan (1999) have shown that strain softening initiated by cyclic loading results in essentially the same value of $S_{qss}$ as that which occurs from monotonic loading. This is shown in Figure 2-30.

Sivathayalan (2000) also concludes that $S_{qss}$ is a strength that occurs only after an initial strain softening as shown in Figure 2-31. Once a liquefied undrained sample starts dilating, as will occur after $r_u = 100\%$ in cyclic loading, it continues to gain strength until either cavitation or $S_{ss}$ is reached. This is a reasonable finding since a dilating sand does not pass through a phase transformation state as occurs during the initial strain softening.
The initial stress-strain response shown in Figure 2-31 appears to have a lower strength than the same sample after stress reversal. However, this conclusion is uncertain. The initial response may have strain hardened through dilation had a larger strain been imposed.

### 2.3 Mobilized Residual Strength

The mobilized residual strength is the strength that is developed after liquefaction as inferred from field case histories (Stark et al., 1998). It may be different than strengths determined in the laboratory as it includes the intricate behaviour of real soil deposits subject to complex load paths and drainage conditions. An appropriate measure of residual strength is critical to any analysis approach. It may be estimated using several techniques:

1. Directly from testing of undisturbed samples,
2. Indirectly from penetration resistance tied to back analysis of field case histories (e.g., Seed & Harder, 1990), and
3. From a theoretical understanding developed from laboratory testing.

---

![Figure 2-30](image1.png)

**Figure 2-30.** Equivalence of $S_{qss}$ from monotonic and cyclic loading. (Figure after Vaid & Sivathayalan, 1999)

![Figure 2-31](image2.png)

**Figure 2-31.** Postliquefaction response before and after stress reversal. (Figure after Sivathayalan, 2000)
Regardless of the approach used to estimate strengths, it is helpful to have a framework of behaviour that is derived from laboratory testing. A commonly used framework is critical state soil mechanics.

2.3.1 Aspects of critical state soil mechanics

The critical or steady state of sand may be defined as the condition that occurs at large shear strain where further increments of shear strain occur without change in volume or shear stress (Roscoe et al., 1958; Poulos, 1981; Been et al., 1991). Critical state generally refers to drained loading while the steady state is found from undrained tests (Sivathayalan, 2000). The undrained strength associated with this deformation state is known as the steady state strength, $S_{ss}$, and is a function of the effective stress on the failure plane $\sigma_f$ at failure:

$$S_{ss} = \sigma_f \times \tan(\phi_{cv})$$  
Eqn. 2.12

where $\phi_{cv}$ is the constant volume effective friction angle, often about 29° to 33° for sand regardless of loading path. According to critical state soil mechanics, $\sigma_f$ is solely a function of void ratio.

An element sheared to large strain might be expected to have its steady state strength controlled only by void ratio. The influence of fabric, anisotropy, or initial confining stress is clear in laboratory tests but would likely be erased by the large strains possible in situ. But it is clear from the test results presented above that strain as high as 30% is often insufficient to reach this state. In addition, sufficiently undrained conditions may not occur in the field. Significant changes to void ratio may also occur. Although a useful and appealing framework, a simple critical state approach may not adequately describe many liquefaction problems.
2.3.2 Relationships for $S_r$

The most commonly used relationship for $S_r$ was presented by Seed and Harder (1990). It relates the residual strength estimated from 17 case histories to representative values of $N_{1.60}$. The data and curves from Seed and Harder are shown in Figure 2-32 along with the relationship for $S_r$ proposed by Idriss (1998). Idriss recommends using the median value of $N_{1.60cs}$, the value of $N_{1.60}$ corrected to clean sand conditions.

Estimating a single $S_r$ value to represent the complex configuration and behaviour of a case history is a difficult and uncertain process. Wride et al. (1999) has compiled a descriptive list describing the various estimates of $S_r$ in the empirical database. It appears that

![Figure 2-32. Mobilized residual strength estimated from case histories.](image-url)
simple analyses such as limit equilibrium or Newmark were typically used. Uniform isotropic strengths would be assumed over fairly large zones. The importance of momentum to the estimate of mobilized strength is approximately considered in some of the estimates.

The data shown in Figure 2-32 is also somewhat of a worst-case collection. Liquefied sites with minimal displacement are more likely to go unnoticed or escape rigorous evaluation. Sites with large deformations may be affected by drainage considerations or mixing that may not occur at all sites. The $S_r$ estimates tend to reflect the strength at the end of sliding. The strength that can be mobilized with strain immediately after triggering, and perhaps well into the earthquake, may be substantially greater.

The lower bound curve is likely conservative for many applications, and something closer to the Idriss curve is considered state of practice. Site-specific test data on undisturbed samples may provide additional guidance, but interpretation of these tests should consider the valuable lessons contained in the empirical database. The lessons include the potential effects of drainage, mixing, and stress path. Some of the difficulties in using laboratory strength estimates were demonstrated during the re-evaluation of the Lower San Fernando dam (Seed et al., 1988).

The data used to develop this curve is primarily from shallow sites. All but one of the 17 sites have a representative $\sigma'_v$ of less than 190 kPa (Stark & Mesri, 1992). These 16 sites have an average $\sigma'_v$ of only 100 kPa. Care should be exercised when applying this curve to a deep deposit.

2.3.3 Relationships for $S_r/\sigma'_v$

Laboratory data for minimum undrained strength, $S_{qss}$, has been shown to normalize with respect to the initial effective vertical stress, at least for cases where the vertical stress is also the major principal stress (Section 2.2.2 and Section 2.2.4). Although this strength is
rarely a true steady-state strength for water pluviated sands, it is often assumed to approximate the residual strength. One justification for using the $S_{qs-st}$ in deformation analyses is that large strains are generally required before substantially higher strengths are achieved.

Since the minimum undrained strength from laboratory tests has been found to normalize, it has been suggested that the mobilized residual strength developed from case histories might also normalize. Stark and Mesri (1992) estimated the average $\sigma'_{vo}$ at the middle of the liquefied layer for each of the case histories in the residual strength database. They then converted the empirical $S_r$ estimates to $S_r/\sigma'_{vo}$. The converted data from Stark and Mesri is shown in Figure 2-33.

![Figure 2-33](image_url)  
**Figure 2-33.** Mobilized residual strength ratio estimated from case histories.
The trend curves plotted on Figure 2-32 were modified to $S_r/\sigma'_{vo}$ conditions by recognizing that many of the sites in the case history database are fairly shallow (Byrne & Beaty, 1999). An approximate $\sigma'_{vo}$ of 96 kPa (2000 lb/ft$^2$) was used to normalize these curves as shown in Figure 2-33. This normalization is expedient as it equates the curves shown on Figure 2-32 and Figure 2-33 at a useful confining stress. While the modified Seed and Harder curve no longer forms a true lower bound, the modified Idriss curve serves as a reasonable average. These curves can be approximated by the following simple equations:

\[
\frac{S_r}{\sigma'_{vo}} = 0.025 \cdot e^{0.16 \cdot N_{1-60cs}} \quad \text{Eqn. 2.13}
\]

Modified Seed-Harder Lower Bound (for $N_{1-60cs} > 6$):

\[
\frac{S_r}{\sigma'_{vo}} = 0.0778 - 0.0277 \cdot N_{1-60cs} + 0.0025 \cdot (N_{1-60cs})^2 \quad \text{Eqn. 2.14}
\]

It is clear from Figure 2-33 that these curves are not well constrained for $N_{1-60cs}$ blowcounts above about 12. The relationships developed by Stark and Mesri (1992) suggest smaller strength ratios at the higher blowcounts.

Undisturbed frozen samples have also been recovered from a number of sites where penetration testing was performed (B.C. Hydro, 1992; Robertson et al., 2000b). This allows a direct relationship between $N_{1-60cs}$ and a laboratory estimate of $S_{qs-ss}/\sigma'_{vo}$ in simple shear as shown in Figure 2-34. These laboratory results are in general agreement with the case history interpretations except for the Mildred Lake results that fall well below the general trend. Robertson et al. (2000b) suggests this may be due to the high $\sigma'_{vo}$ of about 500 kPa for the Mildred Lake sand. There may be a trend of decreasing $S_{qs-ss}/\sigma'_{vo}$ at higher values of $\sigma'_{vo}$. An initial investigation of this potential effect is presented by Yoshimine et al. (1999).
At first glance, the importance of high $\sigma'_{vo}$ is also suggested by the Duncan dam results in Figure 2-34 (B.C. Hydro, 1992; Byrne et al., 1994; Pillai & Byrne, 1994). The undisturbed samples were tested at confining stresses of 200, 400, and 600 kPa. An increase in the initial stress from 200 to 600 kPa corresponded to an increase in the $N_{1-60cs}$ of 12 to 15.5. But the tests showed no corresponding increase in $S_r/\sigma'_{vo}$. The lack of improvement in $S_r/\sigma'_{vo}$ for this case may be an artifact of testing undisturbed frozen samples in a simple shear device. The Duncan dam strengths are from postcyclic monotonic loading that begins at a low shear stress level. Slippage of the smooth porous stone during this loading can prevent the full strength from being realized in the test. The strength ranges shown on the figure are likely low estimates (Y.P. Vaid, personal communication, March, 1999).

![Figure 2-34](image)
Given these concerns regarding the effect of $\sigma'_vo$ on the values of $S_r/\sigma'_vo$, care should be taken when applying Figure 2-34 to deep deposits. As with Figure 2-32, most of the data in this figure is for relatively shallow sites.

There is further ambiguity in comparing the laboratory and case history estimates in Figure 2-34. The plotted results for Mildred Lake, Fraser River, and J Pit only represent those samples that strain softened. The test results reflect a low estimate of the average strength since many samples did not strain soften. For example, only 3 of the 8 simple shear tests performed on the Mildred Lake sand were strain softening.

Strengths determined from standard laboratory tests are applicable only to undrained loading. $S_{qss-ss}/\sigma'_vo$ ratios are most valid when the postliquefaction response is primarily monotonic. This type of loading may occur during earthquakes at locations with a high static shear bias or when widespread liquefaction occurs during a single, very large earthquake pulse. $S_{qss-ss}/\sigma'_vo$ can represent a conservative estimate of the initial undrained strength, although this may not be the case if applied to locations with confining stresses greater than tested in the laboratory.

The modified Idriss curve described by Equation 2.13 again provides a reasonable average trend for the data.

As noted above, there is reasonable agreement between the laboratory and case history estimates of $S_r/\sigma'_vo$ shown on Figure 2-34. However, the validity of this agreement is uncertain. Difficulties in interpreting the laboratory test results have already been described. It is also unclear if the mobilized strengths from case histories should be expected to normalize. These strengths often reflect very high strains and may be affected by pore pressure migration (Section 2.2.10) or mixing (Section 2.2.12).
2.3.4 Factors affecting $S_r$

Many of the factors that affect the residual strength of a given sand have been discussed above. These include void ratio, anisotropy and loading direction, confining stress, fabric, drainage, and mixing. Many of these parameters are uncertain or may even change during an earthquake. These factors can be significant, but their importance to the observed response of case histories is difficult to determine with any degree of confidence.

2.4 Conclusion

This chapter presented an overview of typical sand behaviour as it relates to the liquefaction-deformation problem. The stress-strain behaviour of saturated sand is a function of many variables. Density, confining stress, and nonlinearity have long been recognized as important. Anisotropy, fabric, and drainage may be just as critical. Other factors discussed include principal stress rotation, fines content, and the importance of initial shear stress.

Key conclusions and contributions include the following:

1. Samples loaded monotonically in compression are generally dilative with an undrained strength greater than their drained strength. Samples loaded in extension tend to have much lower strengths, and samples loaded in simple shear are intermediate.

2. Soil fabric is vitally important. Samples prepared by water pluviation appear to simulate the behaviour of in situ sands deposited by water. Soils prepared by moist tamping, which might simulate a lightly compacted soil, can have engineering properties that are less desirable than water-pluviated soils. It is important not to apply empirical relations or laboratory results derived from one type of fabric to a soil with a different fabric without careful consideration.
3. A simple interpolation relationship was developed to approximately describe the change in $S_{qss}$ with loading direction.

4. Data on initial static bias was presented in a modified form. The importance of initial shear stress to $S_{qss}$ and $S_{u-peak}$ was found to be most significant for loading in the extensive direction and least significant in the compressive direction. The effect on $S_{u-peak}$ increases in importance as the initial shear stress approaches $S_{u-peak}$.

5. The relationship between shear stiffness and number of load cycles was investigated for saturated, nonliquefied sand. The results indicate stiffness may drop gradually until the sand nears liquefaction. This change in stiffness prior to liquefaction can likely be ignored in many analyses that involve significant liquefaction.

6. Cyclic strength data from frozen undisturbed samples at five locations was compiled. The data was found to normalize reasonably well and typical cyclic strength relationships were presented.

7. Although the mobilized residual strength $S_r$ is the value inferred from case histories, it is sometimes estimated through laboratory testing. Fabric, drainage conditions, density, stress conditions, and stress path must all simulate field conditions for laboratory strengths to be valid.

8. Laboratory tests for $S_r$ or $S_r/\sigma'_{vo}$ are often difficult to interpret and relate to field behaviour. Reasonable agreement was found between $S_r/\sigma'_{vo}$ estimated from case histories and values determined in the laboratory from undisturbed strain-softening samples. This agreement may be fortuitous, since the values from the laboratory tests tended to be low estimates of $S_r/\sigma'_{vo}$ and it is not clear if the mobilized residual strengths from case histories should be normalized.
9. $S_r$ values estimated from case histories are in some ways a worst-case estimate. The strength that can be mobilized immediately after triggering, and perhaps well into the earthquake, may be substantially greater.

10. Local drainage and mixing of soil layers may be critical factors in the occurrence of large displacements.

To accurately include all of the effects presented in this chapter into a numerical model would be daunting. Simple models can still be valuable tools as long as their limitations are understood. At the very least an analysis should capture the two phases of any liquefaction-deformation problem: the preliquefaction phase where the material is relatively stiff and strong, and the postliquefaction phase where liquefied elements may be quite soft and weak while loading but stiff in unloading. Such an analytical tool was developed for this thesis and will be presented in the next chapter.
CHAPTER 3 — Synthesized Approach: Description

A simple numerical approach to the liquefaction-deformation problem was developed for this thesis. The goal of this approach is to build upon the current state of practice while removing some of its limitations. Perhaps the most significant improvement in this method is the synthesis of the three main aspects of the problem into a single interdependent analysis: triggering, flow slide, and limited deformation.

The method relies upon standard two-dimensional techniques in numerical modeling. The analysis is performed in the time domain with the earthquake motion applied to the base of the model. Liquefaction is evaluated independently and continuously in each element as a function of its developing stress history. The properties of liquefied elements are adjusted at the instant of triggering to reflect the anticipated weak and soft behaviour. Liquefaction occurs first in the most susceptible elements and progressively spreads as the earthquake continues. The computed deformations are affected by the interaction of the liquefied zones with the ongoing seismic loading. The approach is a total stress procedure that assumes undrained behaviour, although any anticipated effects of drainage can be approximated through the choice of material properties.

A list of key input variables is given in Appendix C and a flow chart of the coded routine in Appendix D.

The synthesized approach was motivated from the need of the local geotechnical engineering community to have improved but practical tools for estimating liquefaction induced displacements. The basic approach was initially formulated by Prof. Byrne, with continued development pursued by both the author and Prof. Byrne at the University of British Columbia. An early version of this approach was first presented in a report to the
Seymour Falls Dam Review Board in September of 1998 (Beaty & Byrne, 1998b). Aspects of the approach have since been presented at several conferences (Beaty & Byrne, 1999a,b,c; 2000; 2001). Other engineers that worked directly with Prof. Byrne during the early development and have published their work include Dharmasetia (2000) and Kostaschuk et al. (1999, 2000). The description of the synthesized approach presented in this chapter represents the contributions of the author with significant assistance from the creativity and guidance of Prof. Byrne.

3.1 Numerical Framework

The framework of the synthesized approach is the numerical solution technique. FLAC, or Fast Lagrangian Analysis of Continua, is the commercial finite-difference continuum code that was used for this thesis (Itasca, 1998). The approach can be easily adapted to other codes or to a finite element formulation. General two-dimensional structures are discretized into elements, masses are lumped at nodal points, and constitutive relationships and properties are assigned to the elements. The earthquake motion is applied to the base of the model. An example finite difference grid is shown in Figure 3-1.

An explicit solution scheme was used where the dynamic equations of equilibrium are satisfied for each nodal mass at every timestep. This approach maintains dynamic

![Example of finite difference grid with contours of displacement.](image-url)
equilibrium throughout the imposed motion so the effects of material softening, dynamic inertia, gravity loads, and imposed stress changes are directly considered: each element strains and deforms to maintain dynamic equilibrium.

Each timestep actually includes a two-step calculation process. Resultant forces on the nodal masses are first estimated from the current element stresses ($F_{\text{elements}}^i$), body forces due to gravity ($g_i$), any applied loads ($F_{\text{applied}}^i$), and the effective force due to viscous damping ($F_{\text{damping}}^i$). The equation of motion is evaluated at each node to estimate its change in velocity due to the resultant force as shown in the following equation:

$$\frac{dV_i}{dt} = \frac{g_i \cdot \text{mass}_{\text{node}} + F_{\text{applied}}^i + F_{\text{damping}}^i + \sum F_{\text{elements}}^i}{\text{mass}_{\text{node}}}$$

Eqn. 3.1

The new nodal velocity is considered to act over a full timestep and results in a displacement of the node. In essence, the calculation of displacement increments is shifted by a half timestep from the calculation of stresses. New element stresses are computed from the new strains and the process begins for the next timestep.

The explicit scheme requires very small timesteps to maintain numerical stability, often less than 0.0001 second. These small timesteps are required so that each mass can be treated as an independent system over each timestep. Numerical problems develop if the response of one node is physically able to propagate as a wave to an adjacent node before the next solution increment. Although long solution times may be required, the explicit formulation easily permits large strain behaviour and nonlinear response to be included. The large strain formulation in FLAC uses a Lagrangian approach where the grid coordinates are updated every timestep in response to displacements.
3.2 The Synthesized Approach

A basic formulation of the synthesized approach will be presented in this section. This version is simple, requires relatively little programming, and is sufficient for many applications. Some useful enhancements will be presented in Section 3.3.

3.2.1 Preseismic analysis

The synthesized approach begins with the model at static equilibrium. Construction processes, groundwater changes, or other significant loading changes may be modeled as part of the static analysis. Since initial displacements are often unimportant, the primary benefit of modeling the construction or other significant loading is in the estimation of initial stresses. Overly detailed static analyses may not be warranted since these estimates are often uncertain.

Use of complex constitutive models during the static phase of the analysis may be unnecessary for similar reasons. A linear elastic-plastic constitutive model with stress dependent parameters will often give suitable results. Concerns regarding initial stresses may be best handled through limited sensitivity studies.

3.2.2 Seismic analysis: preliquefaction

The analysis prior to liquefaction is similar to the equivalent linear method of SHAKE (Schnabel et al., 1972; Idriss & Sun, 1992) or FLUSH (Lysmer et al., 1975). A linear elastic-plastic constitutive model is used in conjunction with Rayleigh viscous damping. Elastic moduli are estimated from the maximum shear modulus $G_{\text{max}}$ and a modulus reduction factor, MRF (Section 2.2.1). A schematic showing the stress strain behaviour of a nonliquefied element is shown in Figure 3-2.

There are a few significant differences between the synthesized and equivalent linear approaches. Appropriate values of MRF and damping must be selected prior to an analysis.
since the synthesized approach is not iterative. An iterative synthesized approach is not currently feasible due to lengthy solution times. The synthesized approach also incorporates a strength cap. Stress increments that attempt to exceed this strength will induce plastic flow. This cap is necessary to permit reasonable estimates of deformations. Since equivalent linear methods allow stresses to exceed realistic strength values, estimates of liquefaction triggering may differ between the two methods.

The third significant difference between the two methods is the formulation of viscous damping. While the equivalent linear approach employs frequency independent damping, many numerical analysis codes rely upon Rayleigh viscous damping for dynamic problems. Unfortunately, Rayleigh damping is frequency dependent. Three different forms of Rayleigh damping are available in FLAC: mass proportional (RM), stiffness proportional (RS), and combined (RMS). RS is the preferred type for the synthesized approach as discussed in Section 4.3.2.

Preliquefied properties are maintained in each element and are not modified until that

Figure 3-2. Stress-strain behaviour of nonliquefied elements.
element liquefies. For the sake of simplicity, and for the reasons discussed in Section 2.2.8, this thesis does not consider changes in stiffness due to pore pressures prior to liquefaction. This softening effect may be significant to the response in some situations, such as when relatively little liquefaction is predicted. As a result, this particular formulation is most appropriate for situations where the occurrence of liquefaction will govern the displacement response. The potential dissipation of pore pressure prior to triggering was also ignored, although this enhancement may be useful in some situations.

3.2.3 Seismic analysis: triggering of liquefaction

The initiation of liquefaction is evaluated by tracking the dynamic shear stress history on the horizontal plane $\tau_{xy}$ within each element. The cyclic pulse $\tau_{cyc}$ is computed at every timestep and is defined as the difference between the current shear stress and the static bias, or $\tau_{cyc} = |\tau_{st} - \tau_{xy}|$. The static bias $\tau_{st}$ is the $\tau_{xy}$ that exists prior to dynamic loading. The irregular shear stress history caused by the earthquake is interpreted as a succession of half cycles with the contribution of each half cycle to triggering determined by its maximum value of $\tau_{cyc}$. This definition of cyclic loading is shown schematically in Figure 3-3.

A cumulative damage approach is used to combine the effects of each half cycle that is similar to the work of Lee and Chan (1972) and Seed et al. (1975). This approach converts the nonuniform $\tau_{cyc}$ history into an equivalent series of uniform stress cycles. For convenience, the amplitude of this uniform history is set equal to $\tau_{15}$, the value of $\tau_{cyc}$ required to cause liquefaction in 15 cycles. This is approximately the number of cycles in a magnitude 7.5 earthquake (Seed et al., 1984) and corresponds to the triggering chart of Figure 2-20. This transformation of one stress pulse of $\tau_{cyc}$ to an equivalent number of cycles of $\tau_{15}$ employs the following procedure:
Chapter 3 — Synthesized Approach: Description

1. A weighting curve is used to find the number of uniform cycles of $\tau_{cyc}$ that will cause liquefaction. This required number of cycles is termed $N_{liq}$. The process of finding $N_{liq}$ from a weighting curve is shown in Figure 3-4. The weighting curve is based on cyclic strength data from laboratory tests.

2. Recognizing that one pulse of $\tau_{cyc}$ is equivalent to one-half cycle of loading, $N_{liq}$ is multiplied by 2 to find the required number of half cycles of $\tau_{cyc}$ to cause liquefaction.

3. $N_{eq}$ is the number of cycles of $\tau_{15}$ that are equivalent to one-half cycle of $\tau_{cyc}$. $N_{eq}$ can be computed from the following equation:

$$N_{eq} = \frac{15}{2 \times N_{liq}}$$  \hspace{1cm} Eqn. 3.2

To illustrate, an $N_{liq}$ of 4 as shown in Figure 3-4 means the soil element will liquefy if a total of 4 full cycles of $\tau_{cyc}$ are applied. The current half cycle of $\tau_{cyc}$ has therefore supplied $1/(2 \times 4) = 1/8$ of the loading necessary to initiate liquefaction. Since a total of
15 cycles of \( \tau_{15} \) will also liquefy the soil element, the current half cycle of \( \tau_{\text{cyc}} \) is equivalent to \( 15/8 = 1.875 \) cycles of \( \tau_{15} \).

The effect of each half cycle can be combined once it has been converted into \( N_{eq} \). This is done as the earthquake proceeds by simply adding together the \( N_{eq} \) values from each half cycle of loading since the beginning of the earthquake to the current time. Liquefaction is triggered when the \( \sum N_{eq} \) exceeds 15.

The weighting curves derived from undisturbed frozen samples and shown in Figure 2-18 assume a linear relationship between the logarithm of \( \frac{CRR}{CRR_{15}} \) and the logarithm of \( N_{eq} \). This assumption produces a reasonable match to the data and gives a consistent application of the cumulative damage approach (Idriss, 1998). Using this equation form, the weighting curve can be represented by Equation 3.3:

\[
\log \left( \frac{CRR}{CRR_{15}} \right) = -\frac{\log(CRR_{1}/CRR_{15})}{\log(15)} \times \left( \log(N_{eq}) - \log(15) \right)
\]

Eqn. 3.3

Figure 3-4. Use of weighting curve.
The log-log relationship allows the normalized weighting curve to be defined by only one point: the value of CRR\textsubscript{i}/CRR\textsubscript{15}. The log-log curve also simplifies some of the calculations. For example, an equation relating \(N_{eq}\) to \(\tau_{cyc}\) for any half cycle can be derived from Equation 3.2 and Equation 3.3:

\[
N_{eq} = 0.5 \cdot \left( \frac{\log(\tau_{cyc})}{\log(CRR_{i}/CRR_{15})} \right)
\]

Eqn. 3.4

An idealized stress-strain curve showing the onset of liquefaction in loose sand is shown in Figure 3-5. The softening associated with liquefaction begins while the sample is being loaded and not at the end of a half cycle (i.e., when \(\tau_{cyc} = 0\)). The synthesized approach attempts to capture this behaviour by anticipating the value of \(\tau_{cyc}\) that will trigger liquefaction. Liquefied behaviour is imposed at the instant this required stress is achieved rather than waiting until the end of the half cycle.

The value of \(\tau_{cyc}\) that will trigger liquefaction at the end of the current half cycle is

Figure 3-5. Idealized stress-strain behaviour including liquefaction.
termed $\tau_{\text{cliq}}$. It is a straightforward process to estimate $\tau_{\text{cliq}}$ from the weighting curve. The cumulative amount of loading that an element has received at the end of a half cycle is given by $\Sigma N_{\text{eq}}$. The number of additional cycles of $\tau_{15}$ necessary to trigger liquefaction is simply $15 - \Sigma N_{\text{eq}}$. It follows that $\tau_{\text{cliq}}$ is the cyclic stress pulse for one-half cycle that is equivalent to $15 - \Sigma N_{\text{eq}}$ cycles of $\tau_{15}$. Computing $\tau_{\text{cliq}}$ is just the inverse of the process used to find $N_{\text{eq}}$ from one-half cycle of $\tau_{\text{cyc}}$.

An equation for $\tau_{\text{cliq}}$ can be derived from Equation 3.4 by setting $N_{\text{eq}} = 15 - \Sigma N_{\text{eq}}$. Solving for $\tau_{\text{cyc}}$ gives the following expression:

$$
\tau_{\text{cliq}} = \tau_{15} \cdot \left(30 - 2 \sum N_{\text{eq}}\right) \frac{\log(CRR_{1}/CRR_{15})}{\log(15)}
$$

Eqn. 3.5

The above discussion assumes the critical loading for triggering of liquefaction is similar to a simple shear loading. $\tau_{xy}$ was chosen as the liquefaction indicator to maintain consistency with common practice. Early analyses of liquefiable structures focused on vertically propagating shear waves that induce cyclic changes in horizontal shear stress (Seed & Idriss, 1971). Common triggering charts rely on the same assumption (Youd & Idriss, 2001). This focus has been maintained despite the more complex loading that occurs in two-dimensional structures (Seed et al., 1973; Seed & Harder, 1990). While the maximum shear stress $\tau_{\text{max}}$ could be used instead of $\tau_{xy}$, principal stress rotation and anisotropy would also need to be considered. This would substantially complicate the triggering evaluation.

The triggering of liquefaction in simple shear is more likely to occur when the cyclic pulse and the static bias are in the same direction. This occurs because liquefaction is fundamentally related to stress ratio $\tau/\sigma'$ (Beaty & Byrne, 1998). The use of $\tau_{\text{cyc}}$ to evaluate triggering in the synthesized approach is simply a convenience that reflects the total stress formulation.
It is possible to consider the direction of $\tau_{xy}$ in the synthesized approach by using $\tau_{xy}$ rather than $\tau_{cyc}$ to assess liquefaction. Triggering is assumed to occur when $|\tau_{xy}| = |\tau_{xy}^{\text{liq}}| = |\tau_{xy}| + |\tau_{eq}^{\text{liq}}|$ where $\tau_{xy}^{\text{liq}}$ is the value of $|\tau_{xy}|$ necessary to trigger liquefaction in the next half cycle. Using $\tau_{xy}^{\text{liq}}$ instead of $\tau_{eq}^{\text{liq}}$ avoids the unrealistic possibility of triggering liquefaction while the element is unloading below $\tau_{st}$. Since it is possible for $\Sigma N_{eq}$ to greatly exceed 15 without satisfying the $\tau_{xy}^{\text{liq}}$ criterion, it is prudent to impose a second criterion based solely on $\Sigma N_{eq}$. Other rational criteria can also be used to address these concerns.

A simple example may be helpful in demonstrating this approach to triggering. A sand element is assumed to have a $\tau_{cyc}$ of 20 kPa, a $\tau_{eq}$ of +10 kPa, and a weighting curve defined by a CRR$_1$/CRR$_{15}$ ratio of 1.5. The element is subjected to several cycles of loading as shown in Figure 3-6. The calculated values of $N_{liq}$ and $\tau_{eq}^{\text{liq}}$ shown in Table 3-1 can be derived directly from the weighting curve, but more simply from Equations 3.4 and 3.5.

![Figure 3-6. Triggering of liquefaction using synthesized approach.](image-url)
Table 3-1. Example calculations for triggering of liquefaction

<table>
<thead>
<tr>
<th>Cycle No.</th>
<th>Compute at Start of Half Cycle</th>
<th>Compute at End of Half Cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\Sigma N_{eq})</td>
<td>(\tau_{ciq}) (kPa)</td>
</tr>
<tr>
<td>1/2</td>
<td>0</td>
<td>33.3</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>33.1</td>
</tr>
<tr>
<td>1-1/2</td>
<td>5.2</td>
<td>31.2</td>
</tr>
<tr>
<td>2</td>
<td>12.7</td>
<td>25.1</td>
</tr>
</tbody>
</table>

3.2.4 Seismic analysis: postliquefaction

Several changes are made to an element when it liquefies. The loading stiffness is reduced and a residual strength \(S_r\) is specified at the instant of triggering. Plastic flow occurs whenever stress increments try to exceed \(S_r\). The constitutive model is modified so that unloading increments use a stiffer modulus than loading increments as shown in Figure 3-7.

![Figure 3-7. Bilinear response of liquefied element.](image)
The stiff unloading modulus is also used for reloading after a partial unload (i.e., the previous unloading did not result in a stress reversal). The soft loading modulus is again used once the shear stress exceeds the value at the start of the partial unload cycle.

While the postliquefaction loading stiffness is initially nonlinear (Section 2.2.9), the simplest version of the synthesized approach assumes a linear loading relationship of similar stiffness. This stiffness can be defined by the following:

$$G_{lq} = \frac{S_r}{\gamma_r}$$  \hspace{1cm} \text{Eqn. 3.6}

where $\gamma_r$ is the shear strain required to mobilize the residual strength from a condition of zero shear stress.

This bilinear model permits the accumulation of displacement due to ratcheting (Section 2.2.9) while still maintaining an uncomplicated elastic-plastic formulation. Although simple, the bilinear model can reasonably capture the characteristic undrained behaviour shown on Figure 3-5.

A feature of liquefied behaviour commonly observed in cyclic laboratory tests is the occurrence of high pore pressures at stress reversals (Section 2.2.8). This is simulated in the analysis by imposing a hydrostatic stress state whenever a liquefied element has a shear stress reversal. The lateral stresses are set equal to the vertical stress and the horizontal shear stress is removed: $\sigma_{xx} = \sigma_{yy}$, $\sigma_{zz} = \sigma_{yy}$ and $\tau_{xy} = 0$. This momentarily removes all shear stress from the element as is the case with $r_u = 100\%$. A force imbalance will generally occur and the grid will strain in response.

The viscous damping in liquefied elements should also be reduced. Liquefied elements can have a large hysteretic damping component caused by either plastic flow or ratcheting behaviour. Combining this hysteretic damping with a viscous component based on the
original damping parameters may result in an excessively large amount of damping for liquefied zones.

3.2.5 Comparison with laboratory data

Although simple, the basic synthesized approach incorporates much of the key behaviour observed in laboratory element tests. Results from a cyclic direct simple shear test on medium Nevada Sand are shown in Figure 3-8 (Bardet, 1997). It is difficult to define the onset of liquefaction from this test as the element appears to transition into liquefied behaviour over about one cycle of loading. This transition starts about a half cycle before Point 1, as shown in Figure 3-8, and continues until about Point 3. For simplicity, the triggering of liquefaction is assumed to occur at Point 1 as this marks the start of the first strongly dilative response. This is shown in Figure 3-8(b) by the increase in effective stress between Point 1 and Point 2 accompanied by a soft stress-strain response.

Unloading from Point 2 is contractive and stiff and is accompanied by an increase in pore pressure. The minimum effective stress, Point 3, occurs near the stress reversal. Although $r_u$ does not reach 100% on this transition cycle, it essentially reaches this value on later cycles. Loading from point 3 is dilative and soft, with a typical concave stress-strain curve to Point 4. Unloading is again steep and contractive, and the process of dilation and contraction continues.

While the approach shown in Figure 3-7 captures much of the postliquefaction behaviour of Figure 3-8, there are some key differences. A direct comparison is shown in Figure 3-9. The brief transition into liquefied behaviour is not accounted for in the synthesized approach as liquefied properties are assigned immediately upon triggering. The bilinear model also produces a relatively quick increase in shear stress following a stress
Figure 3-8. Example of cyclic simple shear response.
(Base figures after Bardet, 1997)
reversal. As a result, the basic model is most appropriate for situations where strains accumulate primarily in one direction as shown in Figure 2-28. Another notable difference is the area of the postliquefaction stress strain loop. The concave shape of the test data reduces this area and results in less hysteretic damping than with the bilinear model. This difference should be smaller for elements having a pronounced one-sided response, such as those with a significant static bias.

3.3 Additional Features

Each of the following additional features has been included in the model and is used in the subsequent analyses unless noted.

3.3.1 Anisotropy

Anisotropy affects many areas of undrained sand response including stiffness, peak strength, and minimum undrained strength (Sections 2.2.3 and 2.2.9). The effect on minimum undrained strength is perhaps the most significant. The synthesized approach uses a

![Figure 3-9. Comparison of VELACS test results with basic synthesized model. (Base figure after Bardet, 1997)]
simplified method of addressing anisotropy by making the current value of residual strength $S_r$ a function of the major principal stress direction. An interpolation function was proposed in Figure 2-12 to facilitate this calculation. The direction of loading will change during an earthquake due to the inertial loads, stress redistribution after liquefaction, and geometry changes. The $S_r$ value is regularly updated to reflect these changes.

The postliquefaction stiffness also varies with loading direction as shown in Figure 2-25. This effect was approximated in the synthesized approach by adjusting the postliquefaction stiffness for loading direction using the same factor applied to $S_r$.

This approach is a simple but approximate method for including anisotropic stiffness and strength. An anisotropic plasticity formulation would be more rigorous although substantially more complicated. It is possible, perhaps, for the computed stress direction in the simplified approach to remain oriented in a nonrealistic direction. For example, the high strength associated with an initially compressive direction may prevent strains from developing along a softer or weaker direction. The strain direction in a truly anisotropic element may be somewhat different than for the approximate isotropic element. This could affect the rotation of principal stresses. However, the method does appear to produce reasonable results. It is prudent to check the predicted stress direction pattern for reasonableness, especially if unexpected results are encountered.

3.3.2 Symmetric loading

The simple bilinear model shown on Figure 3-7 produces a relatively rapid increase in shear stress after a stress reversal. As discussed above, this representation has two drawbacks. It tends to over predict the hysteretic damping and it is most useful in situations where the strains occur in one direction. The soft response near stress reversals has been discussed by Vaid and Sivathayalan (1997) and Shamoto et al. (1998).
An alternative loading model was developed to represent the response of liquefied soil to symmetric loading as seen in Figure 2-24. A softened loading modulus is imposed immediately after a stress reversal. This softened modulus is used whenever the liquefied element is loading but is passing through a region of strain space that it has already traversed. This is most easily visualized for simple shear loading as demonstrated in Figure 3-10. The maximum and minimum shear strains that have occurred in the horizontal plane are continuously updated. The softened loading modulus is used whenever the current shear strain is within these limits. The loading modulus switches to the original $G_{\text{eq}}$ when a loading increment increases the current strain beyond its previous limits. Reasonable agreement is seen with the behaviour of Figure 3-10 and Figure 2-24.

This simple logic for simple shear loading is extended to general two-dimensional loading by assessing the strains in a number of prescribed directions rather than just the horizontal orientation.

Laboratory tests of denser sands suggest a maximum limit to the shear strain that can

![Figure 3-10. Modified model for symmetric loading.](image-url)
accumulate during symmetric loading (Seed et al., 1984; Shamoto et al., 1998). This has been termed the limiting shear strain and is dependent on relative density. According to the Seed chart, sands with SPT blowcounts below 10 ($D_r < 40$ to 50%) appear to have a virtually unlimited strain potential, while denser sands with blowcounts above about 25 ($D_r = 70$ to 80%) have limiting shear strains less than about 10%. It may be desirable to limit the maximum strain of the low modulus zone in denser sands to the limiting strain, although it is unclear if this limit also applies to one-sided loading. Inflow of water into dense sand would likely increase the strain limit.

### 3.3.3 Strain-based trigger

The soft and weak nature of liquefied elements will cause shear stresses to be transferred to stiffer nonliquefied elements. This redistribution may increase the potential for these elements to liquefy under cyclic loading. But it is also possible for these elements to liquefy from the increased load through a monotonic response regardless of the ongoing cyclic loading. Test data suggests elements trigger monotonically at shear strains of less than 1 to 2% (Figure 2-6 to Figure 2-8, Figure 2-10). Shear strains of 4 to 5% coincide with strain softening behaviour or a soft dilative response. While the synthesized approach primarily addresses liquefaction caused by cyclic loading, it is a simple matter to impose an additional liquefaction criterion based on accumulated shear strain to approximately address triggering from monotonic response.

Since shear strains of 3 to 5% would be unlikely to occur in an analysis without substantial plastic flow, using such strain values as a trigger criterion is not unreasonable. However, strain estimates are also a function of the relative size of elements. Coarse grids may give differing strain estimates than fine grids since the strains are smoothed over a larger
area. Appropriate strain criteria depend not only on laboratory test results, but also on the element size and deformation pattern.

Ideally, a strain-based trigger would be unnecessary. An increase in shear stress due to stress redistribution should cause an element to liquefy more easily when using the cumulative damage approach described in Section 3.2.3. However, the need for a strain-based trigger criterion results from two simplifications in this approach. First, cyclic shear stresses are evaluated only on the horizontal plane. And second, the undrained shear strength $S_u$ is specified in a simple manner.

There are two aspects to this first simplification. The horizontal plane considered in the triggering evaluation may not be the plane of maximum shear stress. Stress redistribution may increase the shear stresses on an inclined plane but have relatively little effect on $\tau_{xy}$. Any change in $\tau_{xy}$ is a function of the direction of the imposed stresses and the constraints of equilibrium as described by a Mohr’s circle. Failure leading to large strains and liquefaction may occur without a significant change in the horizontal shear stress.

The second aspect of this simplification is similar but applies even when there is no stress redistribution. An element may initially have a high shear stress on an inclined plane due to the static loading. An increase in $\tau_{xy}$ due to the cyclic earthquake loading will also cause an increase in the maximum shear stress. Failure can occur on an inclined plane while $\tau_{xy}$ and $\tau_{cyc}$ are still relatively low as shown in Figure 3-11. Further increases in $\tau_{xy}$ with cyclic loading can occur only if the principal stresses rotate. This may not happen to a sufficient degree in some elements, or the required rotation may cause a delay in liquefaction. The problem is more significant when $\tau_{st}$ and $\tau_{cyc}$ are in the same direction. Adding a strain-based trigger is a simple way of moderating the effects of inclined shear stresses.
A second reason for imposing a strain-based trigger criterion results from the simple way the synthesized approach addresses the undrained strength prior to liquefaction. While $S_u$ is related to the shear stress required to liquefy the element in the next half cycle, $\tau_{sylq}$, the synthesized approach considers the two strengths as completely unrelated items. The cyclic strength specified for small numbers of loading cycles could be well above the specified $S_u$. This may delay triggering when using a cumulative damage approach since the peak $\tau_{xy}$ from large stress cycles will be capped to the specified $S_u$ value. Since stress pulses that try to exceed $S_u$ will induce plastic flow, one method of addressing this problem is with strain-based trigger criteria. Additional discussion of this concern is given in Section 4.3.4 along with an alternative solution.

### 3.3.4 Postliquefaction stress reversal

While the triggering calculations are based on changes in $\tau_{xy}$, this simple definition for loading and unloading is not always sufficient for postliquefaction behaviour. Soil elements

![Example Mohr's circle for element with peak $\tau_{max}$ on inclined plane.](image-url)
near slopes may be under large shear stresses despite relatively low values of $\tau_{xy}$. Changes in horizontal shear stress in conditions such as these often have more to do with stress rotation than with stress loading or unloading. This is shown by the Mohr's circle diagram in Figure 3-12. While changes in $\tau_{xy}$ will affect the diameter and rotation of the Mohr's circle, the sample remains under a substantial shear stress.

Under these conditions, it may not be reasonable to modify the shear stiffness or impose $r_u = 100\%$ conditions based solely on changes in $\tau_{xy}$. It is not clear if such loading even leads to a complete loss of effective stress. Although this loading condition might be considered similar to a simple shear test, the response of the lateral stresses is very different. Horizontal stresses in a simple shear device will rise to meet the vertical stress, but this will not occur at all locations adjacent to a slope. An inadequate definition for loading, unloading, and stress reversal may result in excessive displacement predictions near slope boundaries.

An alternative definition for postliquefaction stress reversal has been implemented. The magnitude and direction of $\tau_{\text{max}}$ is computed. An increase in $\tau_{\text{max}}$ is considered a loading

![Figure 3-12. Example Mohr's circle for element with static bias.](image-url)
increment. The routine remembers the peak $\tau_{\text{max}}$ and its corresponding direction for the current half cycle. A stress reversal is assumed to occur whenever the current $\tau_{\text{max}}$ is less than the peak $\tau_{\text{max}}$ and the direction of principal stresses has changed by at least 45° from the peak $\tau_{\text{max}}$ condition. The peak $\tau_{\text{max}}$ and its direction are updated immediately upon stress reversal. This definition for stress reversal is demonstrated in Figure 3-13. Stress Path A does not experience a stress reversal, while Stress Path B is shown at the instant of stress reversal.

The critical assumption behind this definition for stress reversal is that the plane of peak $\tau_{\text{max}}$ is the critical plane for the current half cycle of loading. This might be partially justified on the basis of induced anisotropy. A stress reversal is assumed when the direction of loading on this plane has reversed. This definition is seen to collapse to the simple $\tau_{xy}$ loading criteria for cases where $\sigma'_{xx} = \sigma'_{yy}$.

![Figure 3-13. Alternative definition for postliquefaction stress reversal.](image)
3.4 Postearthquake Settlement

The focus of the synthesized approach and this thesis is to estimate displacements caused by earthquake loading and due to changes in stiffness and strength resulting from liquefaction. But it is also possible to extend the approach to consider postearthquake settlements due to consolidation during pore pressure dissipation. While some fundamental effective stress models might be capable of estimating settlements directly from the constitutive behaviour, element state, and the predicted flow of pore water, the simple undrained formulation of the synthesized approach lacks the necessary complexity.

The synthesized approach can impose postearthquake volumetric strains in liquefied elements by incrementally reducing their internal normal stresses. Each drop in normal stress causes the element to strain inward until equilibrium is again achieved. Volumetric strains can be imposed as vertical axial strains by assigning a Poisson’s ratio of zero and adjusting only the vertical normal stress. This is illustrated in Figure 3-14. The two-dimensional grid will respond to the consolidating zones and the resulting displacements will be estimated.

Figure 3-14. Illustration of imposed volumetric strain.
Fortunately, guidelines for anticipated volumetric strains can be obtained in the literature (Tokimatsu & Seed, 1987; Ishihara & Yoshimine, 1992). These guidelines typically relate the expected volumetric strain to relative density or $N_{1-60}$ as well as a measure of the earthquake loading such as the factor of safety against liquefaction $FS_L$. They are developed from laboratory data as well as limited field observations.

This simple analysis approach is based on two assumptions about the consolidation process. First, volumetric strains occur primarily in the vertical direction. Sufficient strength and lateral total stress is assumed to exist to prevent lateral strains caused directly by consolidation. Secondly, it is assumed that most of the consolidation occurs under high pore pressures and low strengths. The initial state of the sand particles is assumed to be very collapsible and much of the consolidation occurs with little increase in effective stress. The rate of consolidation decreases once a less collapsible state is achieved. This should be accompanied by a drop in pore pressure and a significant increase in strength. Since much of the consolidation is assumed to occur under high pore pressures, the residual strength and postliquefaction stiffness of the liquefied zones may be used through this part of the analysis.

The internal normal stress should be reduced gradually to avoid an artificial dynamic response. These dynamic effects are also minimized by using a two-stage solution process for each increment of loading. Large cohesive and tensile strengths are temporarily assigned during the initial solution phase after an increment of loading. After equilibrium is achieved, these strengths are reduced for the second phase.

The total change in $\sigma_{yy}'$ that is required to induce $\Delta \varepsilon_v$ in an unrestrained element is easily computed from the bulk modulus as shown in the following equation:

$$\Delta \sigma_{yy}' = 3 \cdot B^e \cdot \Delta \varepsilon_v \quad \text{for} \quad \nu = 0$$  

Eqn. 3.7
Using this relation for an incremental analyses in large strain mode will somewhat underpredict the imposed strains due to the gradual reduction in element size. An approximate correction for this effect is included in the following equation:

\[
\Delta \sigma_{yy}' = \frac{3 \cdot B^e \cdot \Delta \varepsilon_v}{1 - \Delta \varepsilon_v/2} \quad \text{for} \quad \nu = 0 \quad \text{and large strain mode} \quad \text{Eqn. 3.8}
\]

### 3.4.1 Importance of stiffness

Simple tests of this technique to calculate postearthquake settlements produce exact results for an unrestrained element. Somewhat variable predictions occur when the method is applied to liquefied zones bounded by non-liquefied elements. Settlement of the consolidating zones may be restrained by the stiffer boundaries. This effect can extend well away from the edges. One might expect this edge effect to be relatively limited if much of the consolidation occurs at relatively low effective stress and strength. Using a large bulk modulus in liquefied zones during the first solution phase tends to produce a more uniform contraction within the liquefied areas. The second phase should use a softer modulus such as \(G_{liq}\).

The dependence of predicted volumetric strain on stiffness is demonstrated by a simple analysis. A flat section of ground is evaluated as shown in Figure 3-15. A central portion 120 m long and 21 m high is assumed to liquefy. The shear modulus of the liquefied material varies from about 960 kPa at the top to 1840 kPa at the bottom. Postearthquake volumetric strains of 5% are imposed. The importance of stiffness to predicted settlements was evaluated by performing four analyses with different stiffness values in the first solution phase: the bulk and shear moduli were modified by factors of 0.1 to 100.

The effect of stiffness on predicted volumetric strain is shown in Figure 3-15. For the low stiffness analysis shown in Figure 3-15(b), the effect of the stiffer edges is seen to extend
over most of the liquefied zone. But using stiffer initial moduli results in a more uniform strain distribution as shown in Figure 3-15(c). Table 3-2 shows that while the low stiffness analysis results in only 65% of the expected volumetric strain, increasing the stiffness results in over 97% of the desired strain.

Stiffness also affects the predicted surface displacements as shown in Figure 3-16. Although the settlement near the center of the liquefied zone is unaffected by stiffness, there

Figure 3-15. Effect of stiffness on predicted volumetric strain.
Table 3-2. Effect of modulus on average $\varepsilon_v$ in liquefied zone

<table>
<thead>
<tr>
<th>Stiffness</th>
<th>Average $\varepsilon_v$</th>
<th>% of imposed $\varepsilon_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal (imposed) volumetric strain</td>
<td>5.00%</td>
<td>—</td>
</tr>
<tr>
<td>$G_{\text{modified}} = G_{\text{liq}} * 0.1$</td>
<td>3.25%</td>
<td>65%</td>
</tr>
<tr>
<td>$G_{\text{modified}} = G_{\text{liq}} * 1.0$</td>
<td>4.41%</td>
<td>88%</td>
</tr>
<tr>
<td>$G_{\text{modified}} = G_{\text{liq}} * 10.$</td>
<td>4.86%</td>
<td>97%</td>
</tr>
<tr>
<td>$G_{\text{modified}} = G_{\text{liq}} * 100.$</td>
<td>4.92%</td>
<td>98%</td>
</tr>
</tbody>
</table>

are large differences as one moves towards the edge of liquefaction. While it is tempting to use softened moduli to shorten the solution time, these analyses demonstrate that soft moduli may give poor displacement predictions. Shear moduli in the range of 10,000 to 20,000 kPa were sufficient to produce reasonably uniform volumetric strains for this analysis.

A review of the predicted volumetric strains should be made after an analysis. Liquefied zones with relatively smooth variations in stiffness may produce the most realistic estimates. It may be prudent to use isotropic stiffness properties during this phase as the variation of stiffness with loading direction may lead to undesired results. While the actual extent of the edge effect is uncertain, adjusting the stiffness provides a method for evaluating realistic variations.

3.4.2 Importance of shear stress

An additional uncertainty in this semi-empirical approach is the effect of postearthquake shear stress on the expected settlements. If a liquefied element is supporting a substantial shear load then the sand grains are experiencing a significant effective normal stress due to dilation. Liquefied elements under little shear load will have a much lower effective stress. Elements with little effective stress may consolidate to a greater degree since
Figure 3-16. Variation of surface settlement with modulus.

the sand grains can more easily move and compact as the effective stress increases. Using settlement relationships derived from level ground conditions may overpredict the anticipated response.

This effect of shear stress on consolidation might be particularly significant for estimating settlements beneath large raft foundations. Uniform liquefaction in a uniform layer beneath the foundation may still cause differential settlements due to changes in shear stress beneath the foundation. One might be able to address this problem by developing relationships that relate anticipated volumetric strain to the ratio of $\tau_{\text{max}}/\sigma'_0$ where $\tau_{\text{max}}$ is estimated at the end of the earthquake. Shamoto et al. (1998) provide an interesting look at this problem.
3.5 Comments on Base Isolation

One of the strengths and potential weaknesses of the synthesized approach is its ability to predict base isolation. The current state of practice relies on equivalent linear analyses to estimate zones of liquefaction. These methods do not include the effect of liquefaction on the continuing dynamic response of the structure. But a liquefied layer that is soft and weak may prevent much of the earthquake energy from being transmitted to higher layers. It will also create a reflecting boundary that affects the loading of lower layers. The importance of liquefaction to site response has been observed during earthquakes, such as at Kobe Port Island (Elgamal et al., 1996) or the Wildlife Site (Zeghal & Elgamal, 1994). Some isolation effects have also been noted in effective stress analyses (Popescu et al., 1998).

Base isolation is directly modeled by the synthesized approach. Isolation can occur when a liquefied layer is failing at the residual strength or when it is shearing at very low stiffness after a stress reversal. Some isolation may occur simply due to the drop in loading stiffness upon liquefaction. The impact of base isolation on the overall response is related to the relative size of the isolating layer. It is possible for liquefaction of a deep continuous layer to dramatically reduce the number of zones predicted to liquefy in the upper layers.

There is a tendency for analysts to assign low values of strength and stiffness to liquefied zones. Just using the empirical \( S_r \) charts to develop strengths may give values less than average expected strengths (Section 2.3.2). While this is considered conservative in many simple evaluations, it may not be prudent in a more sophisticated analysis. Lower bound strengths or stiffnesses can overpredict the base isolation effect and could result in lower displacements. The base isolation effect may be intensified in the synthesized approach since it assumes an abrupt change into liquefied behaviour, with its reduced stiffness and strength, and neglects any transition.
Base isolation must be carefully evaluated when predicted by an analysis. Displacements, surface accelerations, and postearthquake settlements are all affected. One option for evaluating its effect is to run the analysis without imposing $G_{\text{liq}}$ or $S_r$ at liquefaction. This permits the importance of base isolation on triggering to be seen. Residual strengths or volumetric strains may then be assigned to these liquefied zones for a displacement comparison. Increasing the CRR, $G_{\text{liq}}$, or $S_r$ in a sensitivity study might also be helpful. Transition behaviour could be modeled in a simple way by delaying the onset of liquefied behaviour after triggering. $K_\sigma$ and $K_{\sigma f}$ factors should be carefully selected. For example, the minimum $K_\sigma$ values recommended by MCEER may not give the largest displacement predictions (Youd & Idriss, 2001).

3.6 Comments on Use of Supplementary Effective Stress Analyses

Some analyses will benefit from using the synthesized approach in combination with a more complex effective stress analyses. While the synthesized approach has a number of advantages, including the simple way that each aspect of the analysis is addressed and the relative ease that predictions may be understood and interpreted, there may be instances when it is appropriate to perform analyses that are more complex. For example, the evaluation of sites where pore pressure migration is a concern, such as with stratified deposits or locations that rely upon engineered drainage, may benefit from fully coupled effective stress analyses. While the synthesized approach can provide significant insight into the behaviour of such sites, the need to approximate the effect of pore pressure migration through the specification of material properties is a significant drawback.

Advantages of using the synthesized approach in conjunction with fully coupled effective stress analyses include the following:
1. Results from the synthesized approach can be used to tailor the effective stress analyses. This will allow the more complex analyses to focus on important details of the behaviour, improving the overall efficiency of the analysis program.

2. The insights learned from the synthesized approach can assist the process of interpreting the effective stress results.

3. The synthesized approach provides a tie into the simpler class of analyses where experience and judgment is more fully developed.

The substantial analytical effort required to perform effective stress analyses can be minimized if the same mesh and material characterizations are used for both analyses. UBCSAND is one effective stress model that can work well in conjunction with the synthesized approach (Puebla et al., 1997; Beaty & Byrne, 1998).

3.6.1 **UBCSAND effective stress model**

UBCSAND is an elastoplastic effective stress formulation based on an assumed hyperbolic relation between stress ratio and plastic shear strain. The constitutive model represents the behaviour of the soil skeleton with the effect of any pore fluid introduced through its volumetric stiffness. The model is fully coupled so that effects of pore water flow are rationally included. As a plasticity model, it includes such features as a yield surface, a non-associative flow rule, and definitions for loading, unloading, and hardening. Details of the model development and applications to monotonic liquefaction are provided by Puebla et al. (1997) and Puebla (1999). Atigh and Byrne (2000) have applied the model to evaluating the effects of barrier layers and pore water inflow.

An early extension of the UBCSAND model to earthquake loading was presented by Beaty and Byrne (1998). The model was used to simulate monotonic and cyclic element tests as well as field observations from the Wildlife Site seismic array. These comparisons with
laboratory tests and field response show promise, although the need for further improvement is indicated. The model is currently being modified to better address the complex loading produced by earthquakes. Recent attention is focused on improving its ability to represent principal stress rotation, plastic flow on unloading, and postliquefaction stress-strain response.

3.7 Comments on Solution Time

One of the difficulties in using an explicit formulation for dynamic analysis is the small timesteps required for the time domain solution. The size of the timestep required for numerical stability is dependent on many factors. In FLAC, the element size, density, stiffness, and the amount of stiffness-proportional Rayleigh viscous damping are used to estimate the critical timestep. Consideration should be given to these factors when preparing the numerical model. For example, a very high bulk modulus in portions of the model can greatly increase the necessary solution time. But a sensitivity study might show that a softer modulus is reasonable and results in similar predictions.

The analyses reported in Chapter 5 of this thesis were performed on a 450 MHz Pentium 3 computer. Computation times with this computer are frequently about 6 to 24 hours per analysis for typical structures. This is roughly the solution time required for the embankments analyzed in Chapter 5. Very small, simple structures may be analyzed in a matter of a few minutes. Some complex structures may require several days or more to complete an analysis even after careful selection of input properties.

3.8 Comparison to Other Analytical Approaches

Tools to predict displacements of earthen structures and foundations due to liquefaction have received a lot of research attention over the past 20 years (Sections 1.1.3 and 1.1.4).
Most of the development focuses on empirical relationships, adaptations of the simple Newmark method, or on sophisticated constitutive models. However, a few methods have been published that incorporate a similar philosophy to the synthesized approach. Two methods are briefly described below. A limited comparison of displacement predictions from the two methods with the synthesized approach will be given in Section 5.2.8.

3.8.1 Approach of Inel, Roth, and de Rubertis (1993)

The method proposed by Inel et al. (1993) relies upon the prediction of pore pressures due to cyclic loading. As with the synthesized approach, FLAC is used to perform a two-dimensional time domain analysis. An elastic-plastic Mohr-Coulomb model is used where the shear modulus $G_{\text{dyn}}$ is made a function of the mean effective stress $\sigma'_m$. Frictional strengths are specified for all cohesionless materials, saturated or unsaturated.

Pore pressure generation is estimated using a cumulative damage approach and weighting curve. Increments of pore pressure are estimated at the end of each loading cycle based on the current number of accumulated cycles. Pore pressure dissipation is included and impacts the estimate of cumulative damage. The assigned pore pressure reaches the initial vertical effective stress $\sigma'_v$ at the instant of triggering ($r_u = 100\%$) and produces a condition of zero shear stiffness and strength. Subsequent strain hardening results only from pore pressure dissipation, and this is offset by continuing pore pressure generation due to cyclic loading.

There are significant differences between this method and the synthesized approach. Dissipation of pore pressures prior to liquefaction is a feature that may be useful and important in some situations (Section 2.2.8). But the use of frictional strengths for saturated materials may underpredict, or overpredict, the available shear strength. Liquefaction due to
monotonic loading is not considered (Section 3.3.3), nor is anisotropy (Sections 2.2.3 and 3.3.1).

The postliquefaction behaviour is also modeled rather simply. The soft dilative response demonstrated in laboratory tests, with a potential for an $S_r$ cap, is represented solely through the generation and dissipation of high pore pressures. Displacements due to ratcheting will not be predicted well (Section 2.2.9 and 3.2.4). The postliquefaction model is a vital component of an analysis since it will affect the loading and triggering of nearby elements as well as the accumulation of strains within liquefied zones.

3.8.2 Approach of Moriwaki, Tan, and Ji (1998)

The method proposed by Moriwaki et al. (1998) has fundamental similarities to the synthesized approach. FLAC is also used to perform a time domain analysis with the Mohr-Coulomb elastic plastic constitutive model. Excess pore pressures are incrementally generated using a cumulative damage approach. The shear modulus is degraded as a function of $r_u$ from its initial value of $G_{max}/2$ to a residual shear modulus reflecting liquefied conditions. The shear strength is reduced at triggering to the residual strength. Pore pressure dissipation or redistribution is not considered. The stress-strain model is illustrated in Figure 3-17.

This modified equivalent linear formulation is similar to the synthesized approach, although the degradation of shear modulus with $r_u$ is likely a secondary effect (Section 2.2.8). The ability to specify a postliquefaction shear modulus and residual strength also provides flexibility in representing element response and is consistent with the accepted understanding of liquefied behaviour. However, limitations of this method as compared to the synthesized approach include the following items. The stress-strain formulation does not capture postliquefaction strains due to ratcheting (Section 2.2.9 and 3.2.4) and it overpredicts
damping for symmetric loading conditions (Section 3.3.2). Liquefaction due to monotonic loading is not considered (Section 3.3.3), nor is anisotropy (Sections 2.2.3 and 3.3.1).

3.9 Conclusion

A simple but powerful analytical tool has been developed that uses a two-dimensional finite difference program for estimating displacements due to liquefaction. The fundamental goal of the approach is to assess the initiation of liquefaction and to assign appropriate stiffness and strength properties to liquefied elements. These computations are performed in the time domain, allowing liquefaction and the resulting deformations to evolve as the seismic loading continues. The synthesized approach is founded on the assumption that the dramatic change in stiffness and strength in liquefied elements will govern the overall displacement response.

The bilinear postliquefaction model is capable of representing the ratcheting type behaviour that is expected at locations with moderate static bias. An extension of this model was developed to better represent the response to symmetric loading where the static bias is
small. Anisotropy of residual strength and stiffness has been included in an uncomplicated but effective manner. Additional enhancements include improvements to the definition of postliquefaction stress reversal and the ability to trigger liquefaction due to monotonic loading. A technique was also presented for estimating deformations due to consolidation of liquefied elements after the earthquake.

While the synthesized approach provides a rational but simple procedure for estimating liquefaction-induced displacements, selecting appropriate input properties is a challenging task. Chapter 4 presents a discussion of the primary input properties. Potential difficulties and possible approaches for selecting preliminary values are presented.
CHAPTER 4 — Synthesized Approach: Input Parameters

The material properties required for the synthesized approach are similar to those needed for many current numerical analysis procedures. The properties are not esoteric but have a simple physical interpretation. Unfortunately, as in any liquefaction or deformation analysis, this does not mean they are easy to determine.

A large amount of high quality testing of truly undisturbed samples is desirable, but rarely feasible or even possible. Difficulties in sampling and limitations in laboratory testing detract from these efforts. For projects of moderate scale, material properties are often based on a combination of in situ investigation, empirical correlations, and judgment supplemented with limited index and mechanical testing. Many of these empirical correlations are based on significant experience and field observation. Correlations from observed field behaviour may even address processes that cannot be easily or reliably modeled in the laboratory. Such processes might include the effects of three-dimensional earthquake loading coupled with a complex pattern of pore pressure redistribution.

This section provides an overview of the required input parameters. Selected correlations and laboratory data are provided for general discussion. A good degree of judgment and reliance upon experience, particularly as expressed in the literature, will always be required to select suitable material properties. A sensitivity study of key parameters is a necessity since the ideal of accurate characterization is unattainable.

4.1 Site Characterization

A priority for any serious evaluation should be a thorough site investigation and in situ testing program. SPT tests are common but must be performed with care. Energy
measurements are highly desirable. Wide experience, familiarity, and the ability to retrieve samples have all added to the longevity of this tool. But uncertainties and potential inaccuracies in SPT tests make it an inexact tool for site characterization. It is most reliably used in combination with other tests.

Seismic CPT tests are invaluable for defining stratigraphy, a crucial concern, and estimating the $G_{max}$ distribution. The repeatability of the CPT and its ability to discern stratigraphy may be its biggest technical advantages. Other tools such as the pressuremeter, Becker hammer, and a selection of geophysical devices also have their uses. Regardless of the tools selected, the main objective is to identify the important aspects of stratigraphy and to characterize the critical properties of the deposits.

The remainder of this thesis uses $N_{1.60}$ from the SPT test as an indicator of relative density. This is primarily for convenience as it simplifies the discussion. The use of several tools will typically be required to adequately characterize a deposit.

### 4.1.1 Representative $N_{1.60}$

Characterizing a site by penetration resistance is a difficult task due to the wide scatter of observations within any zone of reasonable size. When empirical charts for triggering (Figure 2-20) or residual strength (Figure 2-32 or Figure 2-34) are used, it is important to characterize the site in a manner similar to that used in the empirical database. Much of the variation in penetration resistance, at least on a small scale, is directly included in the empirical estimates. The data in Figure 2-20, Figure 2-32, and Figure 2-34 appear to rely upon median or average values of $N_{1.60}$ within critical zones, although this is unclear except for the case of the Idriss $S_r$ curve (Idriss, 1998).

Average values of $N_{1.60}$ may not give the best characterization of a site. Yoshimine et al. (1999) suggest using the mean minus one standard deviation ($84^{th}$ percentile) to
characterize a soil when using the CPT tip resistance for estimating the undrained strength. Popescu et al. (1997, 1998) have performed Monte Carlo simulations of two sites using effective stress analyses to evaluate the triggering of liquefaction. The 1997 analyses consider an undensified hydraulically placed sand characterized by CPT tests. The 1998 study evaluates a natural deposit characterized by SPT tests. For each site, one set of deterministic analyses was performed that assumed the penetration resistance was laterally uniform but increased linearly with depth. In a second set of stochastic analyses the authors varied the penetration resistance by element in accordance with statistical distributions derived from the SPT and CPT tests. Comparable behaviour could be achieved between the two sets of analyses only when the deterministic analyses used a penetration resistance that was much less than the average value. The authors concluded that an 80th percentile value gave a good to relatively conservative estimate of the liquefaction triggering response for the two test sites (i.e., 80% of the SPT or CPT measurements at any depth were greater than the value chosen for analysis). However, the authors cautioned that the appropriate percentile may depend on the variability of the soil deposit and the seismic loading. A simple parametric study suggested the appropriate percentile of penetration resistance may drop towards the median value as the actual variability of the soil decreases (Popescu et al., 1998).

Popescu et al. (1998) suggest that pockets of loose zones within the soil may initiate liquefaction. The degree of variability within the soil deposit was also found to have a significant effect on the liquefaction response. The authors warn against using too low of an estimate of penetration resistance due to the potential for attenuating the incoming seismic waves.

Characterizing a deposit with a blowcount that reflects the looser fraction is also supported by considerations of postliquefaction deformation. Although significant strains
may occur within both loose and dense materials, the critical shear surface will tend to follow the weaker zones. Loose zones will also consolidate more than denser zones. This causes them to be a larger source of pore water at high pressure that can degrade the response of neighboring zones.

One method of addressing this concern while still maintaining some consistency with the empirical charts would be to vary the assigned $N_{1.60}$ within stratigraphic units based on observed variations from the in situ testing. This will likely produce an earlier and perhaps more extensive liquefaction than if a median or average value of $N_{1.60}$ is used. Although the effect of pore pressure redistribution cannot be directly considered by a total stress approach, load shedding will occur as the looser zones liquefy. This transfer of load to denser elements will increase their potential for liquefaction. A distributed $N_{1.60}$ may also alleviate some concerns for overprediction of base isolation. Clusters of dense elements will allow motions from the earthquake to bypass the weaker liquefied zones.

There are a few potential drawbacks to using a variation in $N_{1.60}$. The need to look at a variety of potential distributions only adds to the computational effort. The importance of this can be seen by noting the concave upward shape of the $S_r$ versus $N_{1.60}$ curve. This means the average $S_r$ in a soil unit will be greater if a distributed $N_{1.60}$ is used compared to a uniform $N_{1.60}$. However, the predicted response will depend on how the looser zones are aligned and not on a simple average value. It may be desirable to use an $N_{1.60}$ distribution whose mean is less than the mean determined from in situ testing.

A second difficulty is that the $N_{1.60}$ distribution will not be completely random on a small scale. Loose sands in water deposited material might be expected to occur in subhorizontal pockets. The scale of these pockets in comparison to the grid size should be considered and may require detailed investigation and statistical evaluation (Popescu et al.,
A third difficulty or uncertainty involves using a variable $N_{1.60}$ in combination with the empirical charts for liquefaction resistance and residual strength that already include the effect of density variations. The potential for using a distributed $N_{1.60}$ in the synthesized approach is a topic for future research.

### 4.2 Input Parameters – Static Analysis

The main goal of the static analysis is simply to estimate a reasonable stress distribution for the start of the earthquake. Suitable analyses range from the simple to the complex. Linear elastic-plastic constitutive relations may be appropriate, although more complex hyperbolic stress-strain relationships can also be used. The steady state groundwater conditions can be estimated using a flow analysis or, if the water table is known, imposed in a simple manner as suggested by Itasca (1998).

The initial stress conditions cannot be precisely known or predicted. Although reasonable global estimates can be made, local conditions will vary due to uncertain and variable material properties as well as creep. Stress history, including previous earthquakes and groundwater fluctuations, will change stresses from initial conditions. Keeping this uncertainty in mind, it is still desirable for the analysis to include a few basic features: a plasticity-based constitutive model, shear and bulk stiffness that are functions of confining stress, and a simple modeling of the construction sequence.

Four properties are critical to the static analysis: shear modulus $G$, bulk modulus $B^e$, friction angle for drained conditions $\phi'$, and density.

#### 4.2.1 Stiffness

It is common to perform static analyses using a nonlinear constitutive model. Hyperbolic formulations, as discussed in Section 2.2.1, are common and typical parameters
are available in the literature (Wong & Duncan, 1974; Byrne et al., 1987). The basic hyperbolic equations for stiffness used in this thesis are as follows (modified from Byrne et al., 1987):

Initial elastic modulus:  
\[ E_i = K_e \times P_a \times \left( \frac{\sigma_m}{P_a} \right)^n \]  
Eqn. 4.1

Bulk modulus:  
\[ B^e = K_b \times P_a \times \left( \frac{\sigma_m}{P_a} \right)^m \]  
Eqn. 4.2

Initial shear modulus:  
\[ G_i = \frac{3B^e E_i}{9B^e - E_i} \]  
Eqn. 4.3

Tangent shear modulus:  
\[ G = G_i \left( 1 - R_f \frac{\tau_{\text{max}}}{\tau_f} \right)^2 \]  
Eqn. 4.4

where \( E_i \) is the initial elastic modulus of the hyperbolic curve, \( K_e \) is the elastic modulus number, \( n \) is the elastic modulus exponent, \( B^e \) is the bulk modulus, \( K_b \) is the bulk modulus number, \( m \) is the bulk modulus exponent, \( G_i \) is the initial shear modulus of the hyperbolic curve, \( R_f \) is the failure ratio, and \( \tau_f \) is the maximum shear stress at failure.

A simpler approach may be appropriate in many instances. A linear elastic-plastic model may be used. An appropriate shear modulus \( G_{st} \) can be developed by applying a reduction factor to \( G_{\text{max}} \). This factor is based on two considerations. First, \( G_{\text{max}} \) reflects the stiff unloading-reloading response at small strain. The initial shear modulus \( G_i \) observed in conventional monotonic tests may be only 1/3 to 1/4 of this value (Byrne et al., 1987). The softening that occurs with increasing strain will further reduce the average secant modulus by perhaps 1/2 to 1/3 depending on the strain level. Assuming an average \( G_{st} \) equal to 10% to
20% of $G_{max}$ should be sufficient for many analyses. Alternatively, the softening that occurs with increasing strain can be approximated from the hyperbolic parameters and the stress conditions at the start of each load increment.

Methods for estimating $G_{max}$ are discussed in Section 4.3.1. Since $G_{max}$ is a function of confining stress, the estimate for $G$ should be frequently updated in the analysis to reflect changes in mean stress.

A bulk modulus $B^e$ that is equal to the initial shear modulus $G_i$ is generally appropriate for static analyses. This corresponds to an initial Poisson’s ratio $\nu$ at low strain of about 0.125. Since $G \approx 1/3 \ G_i = 1/3 \ B^e$, the average $\nu$ including the reduction in shear modulus with strain becomes 0.35.

**4.2.2 Friction angle $\phi'$**

Although laboratory tests are preferred, many empirical correlations are available for initial estimates of friction angle. The Electric Power Research Institute (EPRI) has compiled several common relationships in their manual on foundation design (Kulhawy & Mayne, 1990). The choice of friction angle should consider the effects of stress level, relative density, and material type. EPRI provides some general guidance that suggests silty materials will have lower values of $\phi'$ and gravels will have higher values. Gradation, angularity, and mineral type can be important.

Trends and typical values from several correlations are shown in Figure 4-1. The correlations are assumed to provide values of peak $\phi'$ as anticipated in a triaxial compression test, although the basis for the correlations are not always well defined. The importance of relative density and stress level is clearly shown.
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Notes:
The correlations are assumed to provide estimates of $\phi'$ as anticipated in a triaxial compression test. The Bolton and Schmertmann relationships are as summarized in the EPRI manual. The Schmertmann relationship is for $\sigma'_vo > 10$ to $20$ kPa. The Schmertmann relationship does not apply to the $\sigma'_vo = 1000$ kPa case. The Schmertmann relationship is modified from N to $N_{1-60}$. $D_r$ in the Bolton relationship was converted to an approximate $N_{1-60}$. The Bolton curves assume $\phi'_vr = 32^\circ$ and a mean stress at failure of $\sigma'_vo$.

Figure 4-1. Comparison of selected correlations between $N_{1-60}$ and friction angle.
4.3 Input Parameters – Preliquefaction

Five additional properties are critical to the preliquefaction phase: shear modulus $G_{\text{dyn}}$, bulk modulus $B'$, viscous damping coefficients $\beta_c$ and $f_c$, and the undrained strength $S_u$.

4.3.1 Stiffness

The strain-dependent nature of the shear modulus is approximated through an appropriate reduction of $G_{\text{max}}$:

$$G_{\text{dyn}} = \text{MRF} \times G_{\text{max}}$$  \hspace{1cm} \text{Eqn. 4.5}

$G_{\text{max}}$ is often estimated from in situ determinations of shear wave velocity or through correlations to penetration resistance. An example correlation modified from Seed et al. (1986) is shown:

$$G_{\text{max}} \approx 22 \times K_{2\text{max}} \times P_a \times \left(\frac{\sigma'_m}{P_a}\right)^{1/2}$$  \hspace{1cm} \text{Eqn. 4.6}

$$K_{2\text{max}} \approx 20 \times (N_{1-60})^{1/3}$$  \hspace{1cm} \text{Eqn. 4.7}

where $P_a$ is the atmospheric pressure in the chosen units and $\sigma'_m$ is the mean effective confining stress.

The modulus reduction factor (MRF) can be estimated from equivalent linear analyses. SHAKE analyses are a practical choice since they are simple and quick to perform (Idriss & Sun, 1992). The modulus reduction and damping curves may be based on published trends, as discussed in Section 2.2.1, or more rarely on laboratory testing. Available techniques include resonant column or bender elements (Bray et al., 1999) for low strain estimates and cyclic triaxial or torsional tests for higher strain values. Recent research using high-accuracy
transducers permits triaxial or torsional tests to be used for both high and low strain levels (Yasuda & Matsumoto, 1993; Tatsuoka et al., 1991).

Although representing two-dimensional response with a collection of one-dimensional columns is a crude approximation, the results can still be useful in many situations. A couple of studies in the literature shed some light on the potential problems. Seed and Harder (1990) compared analyses of the Lower San Fernando dam using SHAKE (Jong, 1988) and FLUSH. Seed found that SHAKE significantly underestimates both the acceleration and cyclic shear stress near the crest and upper faces of the embankment. However, relatively good agreement was obtained for cyclic shear stress near the base of the shells. Boulanger et al. (1993) provide a comparison of SHAKE estimates versus recorded motion at two locations of Puddingstone dam. The $M_w = 6.1$ Whittier Narrows earthquake caused a peak ground acceleration of about 0.07 g that was amplified to 0.19 g at the crest and 0.18 g at the middle of the downstream face. SHAKE was found to give fair agreement with the observed spectral acceleration values at the downstream face. Poor agreement was achieved at the crest surface where two-dimensional effects are more critical. Since the synthesized approach will include two-dimensional response, and all that is needed are rational estimates of $G_{dy}$, using SHAKE to obtain these estimates is considered reasonable for many situations.

The bulk modulus for saturated soils should reflect a Poisson's ratio $\nu$ of near 0.5. A value of $B^e = 50 \times G_{dy}$ corresponds to $\nu = 0.49$ and is often appropriate. The Poisson's ratio of unsaturated soils at low strains will typically be much less, perhaps on the order of $\nu = 0.1$ (Byrne et al., 1987). Assuming a $B^e \approx G_{max}$ for unsaturated soils should often be sufficient.

4.3.2 Damping

One advantage of using an equivalent linear analysis to estimate the MRF is that a consistent value of average damping $\beta_c$ is also provided. The difficulty in applying this $\beta_c$ to
a FLAC model is determining the appropriate center frequency $f_c$ and deciding which formulation of Rayleigh damping gives the best results.

The frequency $f_c$ associated with $\beta_c$ can be estimated by simulating a SHAKE column using FLAC and calibrating the results. Methods of calibration might include matching the peak CSR in key elements, the equivalent number of cyclic shear cycles, or the Fourier spectra of the shear stress histories. Examples of a calibration are provided in Section 5.1.1.

Recognizing that the fundamental frequency $f_i$ is the critical response frequency for many geotechnical structures can simplify the process. Assuming $f_c = f_i$ and performing some level of sensitivity study should give reasonable results in many cases. $f_i$ can be estimated in an explicit analysis by assigning the preliquefaction dynamic properties and imposing an initial velocity distribution that is roughly similar to the expected fundamental mode shape. Histories of velocity or shear stress can be monitored at key locations. Although the first few cycles of response may be somewhat erratic, the fundamental response mode will soon become clear.

The dependence of Rayleigh damping on frequency is illustrated in Figure 4-2. Both stiffness proportional (RS) and combined damping (RMS) have characteristics useful for a deformation analysis. RMS damping produces a relatively uniform damping ratio over a frequency range of about $2/3 f_c$ to $3/2 f_c$. This is particularly advantageous during the triggering portion of the analysis. But response at frequencies outside this range may receive much higher damping. Large deformations are often associated with low frequencies that may be much less than $2/3 f_c$. Deformations often occur as result of plastic flow with a corresponding energy loss through hysteresis. Adding a large viscous damping component to these motions can be unconservative.
RMS damping can be a particular problem if there is a thick nonliquefied crust displacing atop a liquefied zone. While the liquefied zone will be assigned low damping, and FLAC will suspend the stiffness component of RMS damping in zones that are plastic, the nonliquefied crust will still respond with its full initial damping. Since the crust is essentially displacing as a rigid body, using its velocity to estimate the damping is not realistic. This can result in a significant restraining force if the displacements occur at long period.

In contrast, the RS formulation has a linear relationship between frequency and damping. While it does not have a zone of near constant damping, the decrease in damping with decreasing frequency has advantages for the deformation problem. The biggest shortcoming to the RS approach is a large decrease in the allowable timestep and a big increase in the solution time. It may be worthwhile for many projects to evaluate both RS and RMS damping. RMS damping may be appropriate for further sensitivity studies if the

Figure 4-2. Frequency dependence of Rayleigh viscous damping.
relative differences are small.

There are instances when using RS damping, the $\beta_c$ from SHAKE, and $f_c = f_i$ will result in too much viscous damping. Some structures, such as the near the crest of embankments, may have significant response in higher modes. The second mode for a homogeneous deposit at level ground conditions has a response frequency that is three times higher than $f_i$:

$$f_n = \frac{(2n-1) \times V_s}{4H}$$  

Eqn. 4.8

Taking $f_c = (f_i + f_2)/2$ for this case instead of $f_c = f_i$ would double the value of $f_c$. This appears very conservative since the higher mode response is rarely as important as the fundamental mode.

SHAKE analyses may overpredict damping in locations of significant static bias. SHAKE assumes level ground conditions and relatively symmetric stress-strain loops. But many of the load cycles may be dominated by the stiff unload-reload response in locations with static bias. Accounting for this response can reduce the overall average damping. Another concern is with large values of $\beta_c$. These generally correspond to stress-strain loops that have become quite soft at higher stresses. Since FLAC models this soft response and the resulting energy loss through plastic flow, a large value of $\beta_c$ from SHAKE may be too large for use in FLAC. Given these concerns, it is prudent to evaluate a range of damping in any analysis. Using RS damping with an $f_c$ of $f_i$ to $1.5 \times f_i$ appears to be a reasonable initial range. This range can also be expressed as a change in the damping ratio from $\beta_c$ to $2/3 \times \beta_c$. A larger reduction in $\beta_c$ may be warranted when $\beta_c$ is large.

4.3.3 Weighting curve and CRR$_{15}$

The weighting curve may be obtained from cyclic testing of undisturbed samples or approximated from published correlations and curves. Data from undisturbed frozen samples
was compiled as part of this thesis and shown to normalize reasonably well with CRR\textsubscript{15}. This data covers a wide range in relative densities and includes both cyclic simple shear and cyclic triaxial tests. Typical average curves were developed that have CRR\textsubscript{1}/CRR\textsubscript{15} ratios of about 1.6 and 1.8. This data is summarized in Appendix B and shown on Figure 2-18.

The normalized weighting curves are scaled to the anticipated material response through the estimated value of CRR\textsubscript{15}. In typical usage, CRR\textsubscript{15} is a function of relative density or penetration resistance, fines content, and confining stress. The value can be obtained from laboratory testing but is more frequently estimated from correlations to in situ tests. The MCEER triggering chart (Youd & Idriss, 2001) can be used to relate N\textsubscript{1-60} to CRR\textsubscript{15} as shown in Figure 2-20 and discussed in Section 2.2.7. The triggering chart corresponds to a Magnitude 7.5 earthquake, which is considered to have the equivalent of about 15 uniform cycles of motion at the given CRR (Seed & Harder, 1990; Youd & Idriss, 2001). Although CRR\textsubscript{15} can be obtained directly from laboratory data, the empirical chart has the advantage of incorporating field response such as the effect of the earthquake loading in all three directions.

The MCEER approach includes corrections for confining stress $K_\alpha$, initial static bias $K_\sigma$, and fines content. The magnitude correction $K_m$ is not needed for the synthesized approach since it directly considers the effect of duration and number of loading cycles. The MCEER charts for $K_\sigma$ and $K_\alpha$ are provided for reference in Figure 4-3 and Figure 4-4. The older 1998 curve for $K_\sigma$ is also shown in Figure 4-3 since this curve was used for the analyses presented in Chapter 5.

$K_\alpha$ is a difficult parameter to use with confidence. The chart provided by MCEER shows wide bands of potential behaviour. The MCEER workshop participants noted the lack of consensus on the effects of static shear stress on liquefaction resistance and the difficulty
in evaluating liquefaction beneath slopes. They concluded that “general recommendations for the use of $K_\alpha$ by the engineering profession is not advisable at this time.” The chart was not included in the updated summary paper (Youd & Idriss, 2001). A thorough discussion of $K_\alpha$ is given by Harder and Boulanger in the workshop report (Youd & Idriss, 1998).

The possible effects of static bias should be considered when evaluating analysis results. It may be appropriate to use some correction factor in certain cases. Values of $K_\alpha$ that increase triggering resistance should be used with care. The earthquake loading that is not aligned with the slope may induce liquefaction and defeat any supposed benefit from the

![Figure 4-3. $K_\sigma$ curves from MCEER workshop.](image)

![Figure 4-4. $K_\alpha$ curve from MCEER workshop.](image)
static shear stress. Initial studies of this possibility have been made by Boulanger and Seed (1995). A further difficulty with the $K_a$ correction is that the appropriate factor is likely stress path dependent. Since most laboratory cyclic tests rely upon uniform loading cycles, and earthquake loading is often highly non uniform, values derived from the laboratory may not always be appropriate to field situations.

Cetin, Seed, & Der Kiureghian (2000) are currently developing new correlations between $N_{1-60}$ and $CRR_{15}$. The curves are anticipated to be improvements and should be widely used once finalized. Enhancements include an explicit evaluation of the probability of triggering, a smaller correction for fines content, and a more rational $K_a$ relationship that is not capped at 1.0. Professor Seed has also noted that the MCEER triggering chart may be unconservative when used in conjunction with a site response analysis. This bias will be removed from the revised correlations (Seed, R.B.; Conference Presentation; March 31, 2001).

4.3.4 Undrained strength

The preliquefaction undrained strength $S_u$ is required in the synthesized approach because elements may fail even if they have not liquefied due to cyclic loading. For example, liquefaction at the toe of a slope may induce failure along a complete shear surface that includes many nonliquefied elements. Appropriate strengths must be assigned to all zones along this surface if reasonable estimates of displacement are to be made. But selecting an appropriate value of $S_u$ can be challenging. While a high strength may provide too much resistance to deformation, a strength that is too low can reduce the ability of an element to transfer dynamic shear stresses to higher elements, or may affect the triggering calculations by reducing the peak stress that can occur in an element (Section 3.3.3).
Chapter 4 — Synthesized Approach: Input Parameters

As discussed in Sections 2.2.2 to 2.2.4, the undrained strength $S_u$ is a function of density, stress level, initial shear stress, and principal stress direction. Undrained simple shear tests can be very useful in defining an appropriate strength range. A combination of triaxial compression and extension tests may also serve well. It may be necessary in some situations to directly model the anisotropic behaviour of $S_u$.

A further complication is the effect of dilation. Although it can produce very high mobilized strengths in denser sands, cavitation and pore water inflow make such strengths unreliable. For these reasons, the undrained strength is typically limited to a maximum of the drained strength in analyses.

One method of estimating the preliquefaction $S_u$ in denser sands is given in Equations 4.9 and 4.10. This approach uses the drained strength parameters and assumes the mean stress at failure is equal to the initial vertical effective stress. This stress condition occurs when the maximum shear stress is on the horizontal plane. Using the sine of the friction angle in Equation 4.9 causes the estimate of $S_u$ to equal the maximum mobilized shear stress under the assumed conditions. The use of the sine is appropriate, although it gives a strength that is significantly lower than the common assumption based on the tangent as shown in Figure 4-5.

$$S_u \leq \sigma'_y \times \sin \phi' \approx 0.5 \text{ to } 0.6 \sigma'_y$$

Eqn. 4.9

A modified form of the equation is used if both a cohesion and friction angle are used to define the drained strength:

$$S_u \leq \sigma'_{vo} \times \sin \phi' + c \times \cos \phi'$$

Eqn. 4.10

These equations give a practical upper limit for $S_u$ in many types of analyses. Such high strengths are most suitable for dense sands. It may also be appropriate for looser sands
loaded in compression, although the strain required to reach such a high strength might be excessive. The limited data presented in Chapter 2 suggests values of $S_u/\sigma'_{vo}$ of only 0.15 to 0.35 for loose sands ($D_r = 30\%$ to $45\%$). This data reflects consolidation stresses between 50 and 400 kPa. The lower end of this strength range corresponds to loading in the extension or simple shear directions, and the upper end to compressive loading. Low strengths such as these should only be used in conjunction with a strain-based criterion for liquefaction. A modest increase in strength may also be required in zones that have a significant initial shear stress. This is supported by the data relating static bias to $S_u/\sigma'_{vo}$ shown on Figure 2-16.

As mentioned earlier in this section, even a carefully selected value of $S_u$ may unintentionally inhibit liquefaction in some areas of the structure. The problem occurs when the undrained strength is too close to the maximum shear stress caused by static loading. The Mohr’s circle may not be able to increase sufficiently in size to permit a significant $\tau_{cy}$ when the loading cycle is aligned in the same direction as the static bias. A comparison of the

![Figure 4-5. Comparison of $\sin\phi'$ with $\tan\phi'$.](image-url)
initial Mohr's circle and $S_u$ has been built into the method. Critical locations can be identified and the $S_u$ increased if desired. A precise evaluation is not possible since the rotation of stresses during cyclic loading will affect the maximum $\tau_{cyc}$ that can occur. This potential problem with $S_u$ should also be reduced if a strain-based liquefaction criterion is used.

### 4.3.5 Earthquake loading

The earthquake record is one of the most critical parameters in a nonlinear dynamic analysis. Records are often selected based on spectral ordinates, earthquake magnitude, fault type, and foundation conditions at the site of the recording. Analyses summarized in Chapter 5 demonstrate that duration and the detailed character of the record are equally important for the liquefaction-deformation problem. Even the orientation of the input motion, positive or negative, can have a large impact on the predictions. Analyses should always investigate the importance of earthquake record direction.

It is prudent that a suite of carefully selected records be evaluated for any deformation analysis. Selecting records based solely on response spectra may also be misleading. A more distant earthquake of greater magnitude but lower spectrum may be more damaging due to its increased duration. The relative duration of earthquakes may be compared by preparing plots of Arias intensity (Idriss, 1978; Dobry et al., 1978). These plots can be developed from a simple integration:

$$ I_A(t) = \frac{\pi}{2g} \int_0^t a^2(t) \, dt $$  \hspace{1cm} \text{Eqn. 4.11} 

Arias has shown this integral to be a measure of the energy of the accelerogram (Dobry et al., 1978). The plot of $I_A(t)$ gives an indication of the time variation of energy and can be used to estimate duration. A common definition for duration is the time interval between $I_A$ values that are 5% and 95% of the final $I_A$ value. The Army Corps of Engineers (USACE,
2000) cites guidelines for expected durations, although these were originally published by Dobry, Idriss, and Ng (1978) and may be outdated.

Many projects have potential seismic sources that are only a short distance away. Near field effects can have a significant impact on the predicted displacements. One characteristic of near field records is commonly called fling. The fling is a form of directivity effect and occurs when the fault rupture propagates toward a site. If the fault rupture speed and wave transmission speed are similar, much of the seismic energy arrives at the site in a very short time interval. This presents itself as a large loading pulse early in the record that typically has a period 0.5 to 2 seconds or more. This large pulse occurs in a direction normal to the fault due to the radiation pattern of shear waves generated by the fault rupture. Although fling records are normally of shorter duration, the large initial pulse may itself cause significant liquefaction and displacement. The research developed by Somerville et al. (1997) is a valuable resource on near-field effects. Other local effects, such as those caused by topography or sediment-filled basins, may also need to be considered (Somerville, 1998; Somerville, 2000; USACE, 2000).

The selected earthquake records will generally require modification to give reasonable agreement with the target response spectrum. Uniform scaling, frequency domain modification (e.g., Naumoski, 1985), or time domain modification (e.g., Abrahamson, 1992) may be used. The U.S. Army Corps of Engineers provides an interesting discussion on selecting and modifying appropriate time histories (USACE, 2000).

The question of whether an earthquake record should be modified to fit the target response spectrum or scaled to match in the period range of interest has been somewhat controversial. Anderson et al. (1998), in a study involving 22 scaled and modified records, have shown that modified records are appropriate for use at soil sites. Open discussion during
the Second International Conference on Earthquake Geotechnical Engineering led to the conclusion that using modified records is appropriate (P.M. Byrne, Personal Communication, June 1999). The main point of contention at the conference was whether the modifications should be performed in the frequency or time domain. Chapter 5 shows that the technique used to modify the record can have a large impact on predicted displacements.

High frequency components in the input motions have been known to create spurious oscillations in predicted response (Itasca, 1998). The ability of a numerical mesh to adequately transmit motion at a given frequency is a function of the element size and stiffness. In essence, good results have been obtained as long as there at least 8 to 10 elements to describe a full wavelength at the maximum frequency (Kuhlemeyer & Lysmer, 1973). The allowable maximum frequencies can be estimated from the following equation (Itasca, 1998):

$$f_{\text{max}} \leq \frac{C}{10 \times \Delta l} \quad \text{Eqn. 4.12}$$

where $\Delta l$ is the element length, $C$ is the wave transmission speed, and $f_{\text{max}}$ is the maximum allowable frequency in the input motion. For shear waves,

$$C_s = \sqrt{\frac{(G_{\text{dyn}} \text{ or } G_{\text{liq}}) / \rho}{\rho}} \quad \text{Eqn. 4.13}$$

Frequencies higher than the allowable should be filtered from the input motion. As a minimum, the mesh should be fine enough to transmit frequencies that are several times greater than the fundamental response frequency of the structure. While this is achievable prior to liquefaction, it is difficult or impossible to achieve once liquefaction occurs or when substantial zones of plastic flow are occurring. Stiffer zones are capable of transmitting higher frequencies than softer zones. If unexpected noise or oscillations are predicted by an
analysis, it may be useful to analyze both a filtered and unfiltered record to assess the importance of the higher frequencies.

The effect of vertical accelerations on geotechnical structures is often not considered. Limited analyses in Chapter 5 suggest their effect may often be minor. The development of spectral attenuation relations for vertical motions has largely been ignored. Recent work by Bozorgnia, Campbell, and Niazi likely represents the state of the art (Bozorgnia et al., 2000; Bozorgnia & Niazi, 1998).

A further consideration is the effect of the geotechnical structure and the flexible foundation on the base motion. Earthquake recordings are generally measured at surface locations. But the motions that occur at depth can be significantly affected by the stratigraphy of a site. SHAKE can be used to estimate the response that would occur beneath a soil column using a motion recorded on a bedrock outcropping. This response at depth, typically at the bedrock-soil interface, is called the “within” motion.

The importance of using a “within” motion for analysis was demonstrated by Mejia and Boulanger (1993) through their analysis of Stafford dam and its response to the 1989 Loma Prieta earthquake. Stafford dam is an compacted earthfill dam approximately 24 metres high and founded on about 12 metres of alluvium. The authors used SHAKE to estimate an appropriate “within” motion at two locations along the base of the embankment. The original input motion was measured at a rock outcrop near the dam. Subsequent predictions of crest motion using FLUSH were found to be significantly affected by whether or not a “within” motion was used at the base. The greatest response in terms of peak acceleration and response spectrum was predicted when the original outcrop record was used as the FLUSH input motion.
4.4 Input Parameters – Postliquefaction

4.4.1 Residual strength

Selecting appropriate values of residual strength is problematic for almost any liquefaction-deformation analysis. Fell et al. (2000), in a paper on risk assessment of dams, consider residual strength to possess the greatest uncertainty in estimating deformations due to liquefaction.

The curves and data presented in Figure 2-32 and Figure 2-34 are regarded as the state of practice and should be suitable for many applications. They should be appropriate for simple shear conditions given the relative importance of this loading condition to the behaviour of slopes and embankments. Use of strength ratios can produce very low estimates of $S_r$ near the surface. The analyses in Chapter 5 assume a minimum $S_r$ of 3 kPa in these elements. The maximum $S_r$ in any element was taken as the drained strength regardless of which figure was used.

These strength figures are derived primarily from water pluviated sands at relatively low confining stresses of about 100 kPa. There is uncertainty when applying these strengths to large confining stresses (Section 2.3.3) or to other fabrics such as poorly compacted material (Section 2.2.5). The degree of anisotropy in loosely compacted soils may also differ from that observed in tests on water pluviated sands (Section 2.2.3). The strengths in Figure 2-32 and Figure 2-34 may be low rather than average estimates (Sections 2.3.2 and 2.3.3). This has the potential of being unconservative in a deformation analysis due to the importance of strength to base isolation (Section 3.5).

This latter concern is related to an interesting question. Might the appropriate strength in a liquefied element vary with time? Consider an element that liquefies during a large loading pulse as shown in Figure 4-6. The initial loading after liquefaction might be similar...
to an undrained monotonic loading test and exhibit an $S_{qss}$ strength. Subsequent load cycles could be associated with a much higher strength as indicated by many laboratory tests (e.g., Figure 2-28). Depending on the permeability and stratification of the material, the available strength might eventually degrade due to pore pressure redistribution and mixing. This final strength might be similar to those back calculated from case histories. The potential for strength degradation is suggested in a few case histories where the large displacements began after the end of earthquake shaking, such as at the Lower San Fernando dam or the Mochikoshi Tailings dam No. 2 (Wride et al., 1999).

Considering this potential range in strength further complicates a simple total stress analysis. It may be best to consider several possibilities in an analysis. For example, one set of analyses can be performed assuming one or more of the trends shown on Figure 2-32 or Figure 2-34. A second analysis might use the expected values of $S_{qss} / \sigma_{vo}$ from undrained laboratory tests, or perhaps even assume the peak undrained strength $S_u$ is still appropriate.

![Figure 4-6. Illustration of potential drop in postliquefaction strength.](image-url)
after liquefaction. This second analysis can also consider a delayed strength reduction to the empirical values. Displacement predictions using different strength formulations might be evaluated in terms of potential risk. Although probabilities could be assigned to the different strengths to develop a risk estimate, selecting suitable probabilities would be difficult.

Uncertainty regarding residual strength also raises questions regarding the importance of anisotropy. Anisotropy is clearly significant in undrained laboratory tests. It therefore seems likely that the initial strength following liquefaction will be anisotropic. Including this initial anisotropy in an analysis may even be necessary to predict the appropriate displacement patterns. But the degree of anisotropy associated with the low values of mobilized residual strength is uncertain. The effects of pore water inflow and mixing are likely significant regardless of loading direction. Given these concerns, it seems appropriate to consider anisotropy whenever the strength is considered to be primarily from undrained response. Isotropic \( S_r \) values may be more suitable for evaluations when the potential for additional strength degradation is being considered.

Section 2.2.4 also suggests that using an \( S_r/\sigma'_o \) ratio instead of \( S_r/\sigma'_v \) may give improved estimates of the initial postliquefaction strength. But this refinement may be unwarranted when estimating residual strengths from Figure 2-34. While \( S_r/\sigma'_o \) may give a more realistic variation in strength across a structure, there are considerable uncertainties in magnitude of strength obtained from Figure 2-34. In addition, the effect and variation of principal stress direction has not been considered in the evaluation of the case histories.

### 4.4.2 Postliquefaction stiffness

The shear stiffness for loading following liquefaction is a function of density, stress direction, and strain as discussed in Section 2.2.9. The initial portion of the stress-strain curve from \( r_u = 100\% \) may be very soft. This gradually stiffens until a nearly linear stress-strain
relation is achieved at higher shear stresses. The synthesized approach, in an effort to maintain as much simplicity as possible, represents this curved loading path with a linear relationship of comparable average stiffness. The shear modulus is defined in Equation 3.6 where \( G_{liq} = S_r / \gamma_r \). The residual strain \( \gamma_r \) is selected to approximate the soft stress strain response of postliquefaction loading.

The postliquefaction loading stiffness can be estimated from element tests or approximately inferred from the response of centrifuge tests or seismic downhole arrays (Dobry & Abdoun, 1998). Useful information on \( G_{liq} \) can also be obtained from the dilative portion of standard monotonic tests as discussed in Section 2.2.9.

Magnitudes of \( G_{liq} \) and \( \gamma_r \) were estimated from laboratory data presented in Section 2.2.9 and are summarized in Table 4-1. \( G_{liq} \) was determined directly from the more linear portion of the stress-strain curve that occurs at higher shear stress. The corresponding \( S_r \) was approximated using the Idriss curve shown on Figure 2-32 and an estimate of the clean sand blowcount \( N_{1-60cs} \) derived from the relative density and fines content. A simple conversion from \( D_r \) to \( N_{1-60} \) was based on the interpretation of Kulhawy and Mayne (1990):

\[
N_{1-60} \approx (60 + 25 \times \log(D_{50} \text{ in mm})) \times D_r^2
\]

Eqn. 4.14

The amount of shear strain required to mobilize the computed value of residual strength for each test was found by dividing this \( S_r \) by \( G_{liq} \) as summarized in Table 4-1. For example, the simple shear tests compiled on Figure 2-26 lists a \( G_{liq} \) of 275 kPa for a relative density of 39%. Using Equation 4.14, this relative density value was converted to an approximate \( N_{1-60} \) of 7. Since the fines content is zero, \( N_{1-60cs} \) is also equal to 7. The Idriss curve of Figure 2-32 suggests an expected residual strength of 7.3 kPa for this value of \( N_{1-60cs} \). \( \gamma_r \) was then computed by dividing the estimate of \( S_r \) by \( G_{liq} \), or \( \gamma_r = 7.3/275 = 2.7\% \).
Table 4-1. Estimates of $G_{liq}$ and $\gamma_r$ from laboratory tests

<table>
<thead>
<tr>
<th>$D_r$ (%)</th>
<th>Estimated $N_{1-60cs}$</th>
<th>$G_{liq}$ (kPa)</th>
<th>$S_r$ (kPa)</th>
<th>$\gamma_r$ (%)</th>
<th>Sand Type</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple shear:</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>28%</td>
<td>4</td>
<td>125</td>
<td>4.5</td>
<td>3.6%</td>
<td>1</td>
<td>Figure 2-26</td>
</tr>
<tr>
<td>39%</td>
<td>7</td>
<td>275</td>
<td>7.3</td>
<td>2.7%</td>
<td>1</td>
<td>Figure 2-26</td>
</tr>
<tr>
<td>50%</td>
<td>12</td>
<td>520</td>
<td>16</td>
<td>3.1%</td>
<td>1</td>
<td>Figure 2-27</td>
</tr>
<tr>
<td>60%</td>
<td>17</td>
<td>900</td>
<td>36</td>
<td>4.0%</td>
<td>1</td>
<td>Figure 2-26</td>
</tr>
<tr>
<td>63%</td>
<td>14</td>
<td>550</td>
<td>22</td>
<td>4.0%</td>
<td>2</td>
<td>Figure 2-24 (last 2 cyc.)</td>
</tr>
</tbody>
</table>

Triaxial compression:

| 20%       | 2                        | 1200             | 3.3         | 0.3%           | 1         | Figure 2-25 |
| 40%       | 8                        | 2400             | 8.6         | 0.4%           | 1         | Figure 2-25 |
| 50%       | 12                       | 2750             | 16          | 0.6%           | 3         | Figure 2-28 (1st cycle) |
| 50%       | 12                       | 3800             | 16          | 0.4%           | 3         | Figure 2-28 (2nd cycle) |
| 50%       | 12                       | 5400             | 16          | 0.3%           | 3         | Figure 2-28 (3rd cycle) |
| 60%       | 17                       | 3500             | 36          | 1.0%           | 1         | Figure 2-25 |

Triaxial extension:

| 20%       | 2                        | 300              | 3.3         | 1.1%           | 1         | Figure 2-25 |
| 40%       | 8                        | 1300             | 8.6         | 0.7%           | 1         | Figure 2-25 |
| 60%       | 17                       | 1800             | 36          | 2.0%           | 1         | Figure 2-25 |

Sand Type:
1. Fraser River sand: water pluviated; natural alluvial sand; $D_{50} = 0.3$ mm; fines content = 0% (Vaid & Thomas, 1995).
2. Nevada sand: air pluviated; $D_{50} = 0.1$ mm; fines content = 7.7% (Arulmoli et al., 1992).
3. Syncrude J-Pit sand: undisturbed frozen sample; age = 0.1 year; tailing sand; $D_{50} = 0.2$ mm; fines content = 12% (Sivathayalan, 2000).

Notes:
$S_r$ is derived from the estimate of $N_{1-60cs}$ and the Idriss curve of Figure 2-32.
$G_{liq}$ is an upper bound estimate derived from the linear portion of the stress-strain curve.
$\gamma_r$ is an approximate value equal to $S_r/G_{liq}$.

These estimated values are simple approximations of the magnitude of $\gamma_r$. Appropriate values of $\gamma_r$ for any situation depend on a number of factors, including the choice of $S_r$, the effects of anisotropy and stress level dependence, and the importance of the softer portion of the loading curve at low shear stress levels.
This initial soft response can be significant to the average stiffness, particularly at lower confining stress. The estimated values of $S_r$ listed in Table 4-1 often occur within this curved portion of the stress-strain response and below the linear portion used to estimate $G_{liq}$. It is unclear how much $\gamma_r$ should be increased to account for this effect. A comparison of Figure 2-24 and Figure 2-28 also suggests the importance of strain path to the initial stiffness. Although uncertain, increasing $\gamma_r$ by 1% or 2% to account for this initial curved response does not appear unrealistic. Based on this limited data, $\gamma_r$ values in the range of 4% to 6% appear reasonable for loose to medium dense sands in simple shear loading when used in conjunction with the Idriss curve of Figure 2-32.

For each test shown in Table 4.1, the laboratory sample was significantly stronger than suggested by the Idriss $S_r$ value. One might expect a softer stress-strain response if the laboratory strength had been as low as the Idriss $S_r$ value. But the intention of the $\gamma_r$ estimated from Table 4.1 is not to provide a precise estimate of the average postliquefaction stiffness. The intention is to suggest a value of suitable magnitude that gives a reasonable estimate of typical stiffness. The appropriateness of this range in $\gamma_r$ and its effect on displacements will be evaluated by back analysis of field observations in Chapter 5.

Different $\gamma_r$ values may be suitable if the selected residual strength differs significantly from the Idriss relationship. This can occur when a strength ratio is used as shown in Figure 2-33. It may be better in these instances to directly specify a $G_{liq}$ that is appropriate over a range of $\sigma'_{vo}$. Dividing the $S_r$ from the Idriss curve by the corresponding value of $\gamma_r$ is one approach to estimating $G_{liq}$. For example, if the representative $N_{1-60cs}$ gives an $S_r$ of 16 kPa from the Idriss curve, then a reasonable estimate for $G_{liq}$ might be $16/0.05 = 320$ kPa. A stiffer value might be used at higher confining stress levels where the importance of the initially soft zone would be less pronounced.
Chapter 4 — Synthesized Approach: Input Parameters

It is interesting that the estimates of $\gamma$ are not greatly affected by density. While the stiffness increases with density, there is a compensating increase in the residual strength. The loading mode or direction appears to have a much larger influence. It is not clear why the loading in extension is stiffer than in simple shear, but may be related to the differing boundary conditions between the tests and the difficulties in testing postliquefaction loading cycles.

Unloading follows a stiffer stress-strain path than loading. The limited information reviewed in Section 2.2.9 suggests the unloading modulus is an order of magnitude stiffer than the linear portion of the loading modulus.

4.4.3 Postliquefaction damping

The postliquefaction stress strain behaviour of liquefied elements includes significant hysteretic damping, either through ratchet behaviour or through plastic flow at the residual strength. Some amount of viscous damping is needed for the smaller stress cycles, particularly at the end of the earthquake. These will tend to stay within the linear unloading-reloading portion of the stress-strain loop with no hysteretic damping as discussed in Section 3.2.4. To lessen the amount of excess damping, the viscous component in liquefied elements should be small. A $\beta_c$ on the order of 2% is appropriate.

4.5 Corroboration of Properties

Probably the best way to corroborate the material properties is to compare predicted performance with field observations. This is particularly important when there is limited laboratory testing. For example, there are a number of possibilities for evaluating analyses of embankments. The estimated peak crest acceleration may be compared to the trends observed by Harder et al. (1998) and shown in Figure 4-7. This comparison provides an approximate
confirmation of damping, stiffness, and possibly undrained strength. Unfortunately, the predicted crest acceleration may not be very sensitive to rather large changes in these variables.

Another possible test is to compare the computed fundamental period with observations such as compiled by Oner (1998) for small strain loading. Swaisgood (1998) has also developed a database of seismically induced deformations at embankment dams. This information can provide an indispensable comparison, but only if the predicted zones of liquefaction are limited and relatively small. Although not always possible, it is worthwhile

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**Figure 4-7.** Peak accelerations observed at embankment dams. (Figure from Harder et al., 1998).
to make these comparisons with dams that are as close in character to the analyzed structure as possible.

4.6 Conclusion

This chapter provides an overview of key properties for the synthesized approach. Any analysis technique is useless unless reliable input properties can be developed. Selecting input parameters will always require significant judgment and must consider the unique conditions of the site. A combination of empirical, laboratory, and in situ testing will generally provide the best characterization. Applying the synthesized approach to case histories is one method of improving judgment regarding input properties, and several examples are provided in Chapter 5.

The synthesized approach itself does not require a fixed set of input parameters or empirical relations. The approach may be modified and adjusted to reflect the understanding and concerns of a particular site. Several empirical correlations were discussed that may be useful at some sites.

Although the synthesized approach is a significant extension of common practice, it still includes major simplifications to the actual behaviour of soil. These simplifications, coupled with the typical and substantial uncertainties in characterizing soil deposits and defining material properties, create the need to perform some level of parametric or sensitivity evaluation in any analysis. Variations in residual strength and earthquake records should always be evaluated. Other parameters, such as residual strain and damping, should be routinely studied. Chapter 5 will explore the potential importance of these and other variables.
CHAPTER 5 — Synthesized Approach: Applications

Several applications of the synthesized approach are described in this chapter. Analyses are presented for a hypothetical lateral spreading site, the Upper and Lower San Fernando dam case histories, and the Elsie Lake Main dam located on Vancouver Island, Canada. Comparisons are made between predicted and observed behaviour as captured in empirical relations and case histories. Comparisons are also made with results from other analysis procedures. Particular attention is paid to parametric studies. These sensitivity analyses serve to evaluate the importance of selected input variables and give a sense for the overall stability of the approach.

The primary objectives of this section are to demonstrate the utility of the synthesized approach, to provide an assessment of its strengths and limitations, and to make general observations regarding the input parameters, analyses, and results.

5.1 Lateral Spreading

Liquefaction-induced displacements at sites of gentle slope are often described as lateral spreading. These displacements occur despite the relatively small driving stresses due to gravity. Lateral spreading is generally not a flow problem. Displacements accumulate during the earthquake primarily due to cyclic mobility or ratcheting (Section 2.2.9). Observed movements compiled by Bartlett and Youd show that displacements of 1 m to 3 m are common for sites with slopes less than 6% (Bartlett, 1998). 83% of the observed displacements are less than 3 m and 67% are between 1 and 3 m. Of the 253 measurements of lateral spreading in this database, the maximum observed displacement is 5.4 m.
Lateral spreading is an ideal case for initial testing and parametric evaluation of the synthesized approach. The large database of observed behaviour permits evaluation of the predicted displacements and trends. The finite-difference model is simple and consists of a single column of elements with boundary conditions that simulate an infinite slope. One-dimensional response is ensured by requiring the displacements predicted along the two vertical edges to be identical. The effect of ground slope is approximated by inclining the direction of gravity.

Analyses of an idealized slope using the synthesized approach are presented below. Particular attention is given to sensitivity studies of various input parameters. These analyses are an update of previously published work (Beaty & Byrne, 1999a).

5.1.1 Analysis description

A 10-metre-thick deposit of loose clean sand is assumed to overlie competent material. The density is characterized by an $N_{1-60}$ of 10 while the underlying material has a shear wave velocity of 600 m/s. Ground water is at a depth of 2 m. Ground slopes of 0% to 6% are evaluated to cover the range of the Youd et al. (1999) database. A detailed list of model parameters is given in Table 5-1.

Appropriate values of modulus reduction factor and damping were estimated by calibrating the FLAC model to a SHAKE equivalent linear analysis. The final values are shown in Figure 5-1. The modulus reduction and damping curves from Figure 2-5 were used in the SHAKE analysis. The center frequency associated with the SHAKE damping ratio was taken as the fundamental response frequency $f_i$ of the column prior to liquefaction.

The values of MRF and damping were verified by simulating a SHAKE analysis using FLAC with Rayleigh stiffness proportional damping. Figure 5-2 provides a comparison of
### Table 5-1. Lateral spread model parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Base Analysis</th>
<th>Sensitivity Analyses</th>
</tr>
</thead>
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<td><strong>Geometry:</strong></td>
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<tr>
<td>Height</td>
<td>10 m</td>
<td></td>
</tr>
<tr>
<td># Elements</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Depth to GWT</td>
<td>2 m</td>
<td></td>
</tr>
<tr>
<td>Ground slope</td>
<td>0% to 6%</td>
<td></td>
</tr>
<tr>
<td><strong>Material parameters:</strong></td>
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<td></td>
</tr>
<tr>
<td>Density</td>
<td>19 kN/m$^3$</td>
<td></td>
</tr>
<tr>
<td>$N_{1-60}$</td>
<td>10</td>
<td>6 to 14</td>
</tr>
<tr>
<td>$K_{2\text{max}}$</td>
<td>20 $(N_{1-60})^{1/3}$</td>
<td></td>
</tr>
<tr>
<td>$B^c / G_{\text{dyn}}$ (saturated)</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>MRF</td>
<td>0.15 – 0.7</td>
<td></td>
</tr>
<tr>
<td>$\beta_c$</td>
<td>3% to 16%</td>
<td></td>
</tr>
<tr>
<td>$f_c$</td>
<td>1.9 Hz</td>
<td>2.85 Hz</td>
</tr>
<tr>
<td>Rayleigh damping type</td>
<td>Stiffness (RS)</td>
<td>Combined (RMS)</td>
</tr>
<tr>
<td><strong>Triggering parameters:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{CRR}<em>1 / \text{CRR}</em>{15}$</td>
<td>1.6</td>
<td></td>
</tr>
<tr>
<td>$\text{CRR}<em>{15}$ versus $N</em>{1-60\text{cs}}$</td>
<td>Figure 2-20</td>
<td></td>
</tr>
<tr>
<td>$K_\sigma$</td>
<td>0.98 - 1.0</td>
<td></td>
</tr>
<tr>
<td>$K_\alpha$</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td><strong>Postliquefaction parameters:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_r$</td>
<td>$S_r / \sigma'_{vo} = 0.124$</td>
<td>Various $S_r / \sigma'_{vo}$ &amp; $S_r$</td>
</tr>
<tr>
<td>$\gamma_r$</td>
<td>4%</td>
<td>1% to 20%</td>
</tr>
<tr>
<td>$G_{\text{liq(unload)}} / G_{\text{liq(loaded)}}$</td>
<td>10.</td>
<td></td>
</tr>
<tr>
<td>$\beta_c$ (liquefied zones)</td>
<td>$\approx 2%$</td>
<td></td>
</tr>
<tr>
<td>Postliquefaction model</td>
<td>Symmetric</td>
<td>Bilinear</td>
</tr>
</tbody>
</table>

The shear stress histories at depths of 3.5 m and 7.5 m. Good agreement is obtained at both depths. FLAC tends to overpredict the stress peaks by a small amount, particularly for the lesser stress cycles. This may result primarily from two causes. A simplified damping distribution was selected for FLAC that has an overall damping lower than the SHAKE average. The variation of damping with frequency in FLAC will also affect the shear stresses.
The Fourier spectra of these stress histories are compared in Figure 5-3. The frequency content of the SHAKE and FLAC results is similar except near the fundamental frequency $f_i$ of 1.9 Hz. The overprediction by FLAC near $f_i$ is likely due to the smaller average damping. Overall, the agreement between the FLAC and SHAKE results is good. Further refinement is not warranted due to the inherent uncertainties and simplifications in these analyses.

The weighting curve used for the triggering evaluation was derived from Figure 2-18 and had a CRR$_1$/CRR$_{15}$ of 1.6. The estimate of CRR$_{15}$ was based on $N_{1-60}$ and the MCEER recommendations (Youd & Idriss, 1998). The residual strength ratio $S_r/\sigma'_{vo}$ was approximated from the modified Idriss curve of Figure 2-34. An $S_r/\sigma'_{vo}$ of 0.5 was assumed to approximate a drained shear strength. The postliquefaction stress-strain response used the symmetric model described in Section 3.3.2.

![Figure 5-1](image_url)  
Comparison of MRF and Damping from SHAKE and FLAC analyses.
Figure 5-2. Comparison of shear stress histories from FLAC and SHAKE.
Figure 5-3. Comparison of Fourier Spectra from shear stress histories.
The Kagel Canyon 360 record from the 1994 Northridge earthquake ($M_w = 6.7$) was selected as the input motion. The recording is from a free-field site on weathered sandstone and has a peak ground acceleration of 0.43 g and a fault distance of 10.6 km (SMDB, 1999). The acceleration trace is plotted in Figure 5-4. The “within” record computed by the SHAKE analysis was applied as a velocity history at the base of the FLAC column. The record was applied in both a positive and negative sense to investigate the importance of direction. The positive direction corresponds to the peak acceleration pulse of 0.43g occurring in a downslope direction and the peak velocity pulse of 0.51 m/s occurring in an upslope direction. The analysis was performed for 45 seconds of record. This lengthy solution time was selected to allow the liquefied column to respond to the longer period motion at the end of the earthquake.

5.1.2 Analysis results

Figure 5-5 shows typical displaced shapes from the base analyses described in Table 5-1. Large strains are distributed throughout the liquefied zones. This was generally true for all analyses, although there were a few cases where the strains occurred predominantly in one zone. The analysis at 6% slope is seen to have less liquefaction than for the gentler slope. This was noticed in several but not all of the parametric analysis sets. Displacement predictions versus slope for the base analyses are summarized in Figure 5-6.

5.1.3 Sensitivity studies

A limited set of parametric studies was performed to evaluate the importance of key variables. All analyses described below use the base analyses parameters except as indicated. The range of parameters investigated is shown in Table 5-1. Results of these analyses are summarized below.
Figure 5-4. Kagel Canyon 360 record from Northridge earthquake.
Figure 5-7 presents the estimates of displacement versus slope predicted by all analyses. A somewhat erratic nature is seen for some of the predicted displacement trends. Two of the factors leading to this scatter is the prediction of flow slides for low residual strengths and the variable effect of base isolation as discussed in Section 3.5.

Base isolation can be especially pronounced for the infinite slope case. In general two-dimensional analyses, there may be zones of material at any depth that either do not liquefy or that liquefy later in the earthquake. These zones provide a path for some of the earthquake energy to bypass any liquefied pockets. The scatter seen in these infinite slope analyses, and

![Diagram of displaced shapes](image)

**Figure 5-5.** Selected displaced shapes from base analyses.
the possible influence of base isolation, reinforces the need to perform some level of parametric study in any analysis.

**Effect of earthquake direction**

The importance of earthquake direction is clearly seen in Figure 5-6. Displacements changed by a factor of 2 by simply reversing the loading direction. This observation is reinforced by Figure 5-7. It is clear that applying the Kagel Canyon motion in a negative sense produces consistently larger displacements. On average, the positive direction gives displacements that are only 2/3 as large as the negative direction for slopes ≥ 0.5%.

This effect on displacements may be related to the number of elements that liquefy. For the base analyses with slopes ≥ 0.5%, an average depth of 7 m liquefies when the earthquake is applied in a negative direction. But this decreases to only 1 m to 3 m when the earthquake is applied in the positive direction.

The presentation of results from the sensitivity study will focus on the earthquake in

---

**Figure 5-6.** Final relative displacement from base analyses.

**Figure 5-7.** Final relative displacement from all analyses.
the negative direction since this gives the larger and more critical estimates of displacement. This simplifies the presentation of results, although the earthquake in the positive direction might give different trends.

**Effect of slope**

Figure 5-6 reveals a general trend of increasing displacement with increasing slope. The collection of curves on Figure 5-7 follow a similar pattern, except for a few that become quite steep at larger slopes. These steep curves reflect an impending flow slide where a low value of residual strength is not sufficient to support the driving stresses due to gravity.

Two common empirical relationships, one by Hamada et al. (1987) and the other by Youd et al. (1999), suggest nearly the same relationship between slope and displacement as shown in Equations 5.1 and 5.2. The trend with slope does not depend on any of the other variables included in the empirical relationships, such as layer thickness, gradation parameters, or earthquake magnitude and distance.

Hamada et al.:

\[ D_h \propto (\text{slope})^{1/3} \]  
Eqn. 5.1

Youd et al.:

\[ D_h \propto (\text{slope})^{0.343} \]  
Eqn. 5.2

Comparisons between the Youd et al. trend and the predictions from the synthesized approach can be made by normalizing the displacement estimates. Since most of the sensitivity studies using the synthesized approach were performed for a range of slope values, the predictions from each set of analyses can be normalized by dividing each displacement by the corresponding prediction at 2% slope. This comparison of trends is shown in Figure 5-8 and Figure 5-9. These figures include all of the results from the base and sensitivity analyses described in this section, except for those that used the earthquake in the positive direction, investigated variations in \( \gamma \), or resulted in flow slides.
There is good correspondence between these empirical correlations and the synthesized approach as shown in Figure 5-8 and Figure 5-9. The base analysis reveals a slower increase in displacement with slope than suggested by the empirical relations for slopes greater than about 1%. However, there is good agreement with the average trend from all of the synthesized analyses as shown in Figure 5-9. This agreement is encouraging but not conclusive. The sensitivity studies included in the average trend cover a wide range in properties, not all of which are equally likely or valid. In addition, the Kagel Canyon record may not be representative of the empirical database. Evaluating a large suite of earthquake records of varying magnitudes would give an improved comparison.

Figure 5-8 and Figure 5-9 show a tendency for the synthesized approach to predict larger than expected normalized displacements at slopes of about 1%. This is partly due to the simple way that lateral spreading is modeled. The only way for energy to dissipate in this model is through hysteretic and viscous damping. Radiation damping does not occur due to
the infinite slope boundary conditions and the reflecting boundary at the base. But the average viscous damping may be small since little viscous damping is assigned to liquefied elements. This can lead to relatively slow energy dissipation at the end of an earthquake if there is substantial liquefaction.

Low average damping is a concern for gentle slopes where the column is likely to experience stress reversals even for lesser stress cycles. This produces a ratchet effect that can result in significant ongoing displacements at the trailing end of an earthquake. The displacement predictions are still increasing after 45 seconds of motion for many of the analyses reported in this section. It is not clear how much of this additional displacement is realistic. The longer period motion at the end of the earthquake may contain significant energy at the fundamental frequency of the liquefied column.

Figure 5-10 shows the average displacement increase that occurred between 30 and 45 seconds from all analyses. Although somewhat arbitrary, this time interval occurs well after the strong motion of the earthquake record as shown on Figure 5-4. A significant change in displacement is seen for gentle slopes less than 2%. The analyst should be aware of this potential behaviour, although it is unlikely to be a concern for a general two-dimensional model with limited zones of liquefaction. The average trend of displacement versus slope using the predictions at 30 seconds is shown in Figure 5-11. These results produce a better agreement with the empirical relationships.

Effect of residual strength

Values of $S_r/\sigma'_w$ were varied from a very low value of 0.06, which is one-half of the modified Idriss value, up to high of 0.5, which corresponds to the assumption for drained strength. Figure 5-12 shows the predicted trends are reasonable, with higher strengths producing lower displacements. The main exception is with the lowest $S_r/\sigma'_w$ ratio. This case
produced smaller than expected displacements until a flow slide was predicted at a slope of 4%. This reduction in displacements is likely the result of base isolation. For slopes of 0.5% to 2%, only 2 or 3 elements liquefied when $S_r/\sigma'_{vo} = 0.06$ compared to 7 or 8 when $S_r/\sigma'_{vo} = 0.124$.

The significance of using a residual strength ratio ($S_r/\sigma'_{vo} = 0.124$) versus a uniform residual strength ($S_r = 11.7 \text{ kPa}$) is shown in Figure 5-13. This $S_r$ value is obtained from the Idriss curve shown on Figure 2-32. For comparison, this strength ratio converts to $S_r$ values of 5.3 kPa to 13.2 kPa. The $S_r/\sigma'_{vo}$ case is expected to produce larger displacements since most of the column is assigned a lower strength and stiffness. Only the lowest two elements have strengths greater than the $S_r$ of 11.7 kPa. The importance of this can be seen in Figure 5-13 where the displacement from the $S_r$ analysis exceeds the $S_r/\sigma'_{vo}$ prediction at the steepest slope.
Strengths at low confining stresses may not follow a strict stress ratio. Allowing the strength to drop to nearly 5 kPa in the $S_r/\sigma'_v$ case may be unrealistic. These low strength values near the top of the column may increase the displacements, although the $S_r$ and $S_r/\sigma'_v$ predictions are still within about 40%.

The choice of residual strength will influence the degree of base isolation that occurs. This can be seen by comparing the number of zones that liquefy to the residual strength ratio as shown in Figure 5-14. Higher residual strengths allow for a greater extent of liquefaction to occur.

**Effect of $\gamma_r$**

The importance of postliquefaction stiffness to displacement was evaluated across a very wide range of $\gamma_r$. The results for $\gamma_r = 1\%$ to 20\% are shown in Figure 5-15. Although the effect is significant, there is not a direct 1 to 1 relationship between stiffness and displacement: doubling the residual shear strain does not double the displacements. In this
case, doubling $\gamma_r$ tends to increase the displacement by about 0.4 m for most of the range. Postliquefaction stiffness may have its biggest effect when displacement predictions are relatively small.

Two estimates of residual strength ratio were used in the evaluation of $\gamma_r$: $S_r/\sigma'_{vo} = 0.124$ and $S_r/\sigma'_{vo} = 0.5$. The latter value was prompted by the many undrained laboratory tests that suggest dilating soils have high strengths (Section 2.2.13). The larger strength yields significantly less displacement until a very high value of $\gamma_r$ is reached.

**Effect of relative density**

Neither the empirical relationship of Hamada et al. (1987) or Youd et al. (1999) directly considers the effect of density on the resulting displacements. But density will influence many variables in the synthesized approach, including the estimates of $G_{dyn}$, $\tau_{15}$, $S_r$, and possibly $\gamma_r$. To simplify this evaluation, the impact of density was limited to its effect on
triggering resistance and $S_r/\sigma'_{vo}$. The residual strength ratio was defined by the modified Idriss curve on Figure 2-34.

Uniform $N_{1.60}$ distributions of 6, 10, and 14 were analyzed. An additional analysis was also performed where the $N_{1.60}$ varied from 14 at the bottom to 6 at the top with an average value of 10. Results are shown in Figure 5-16. If the flow slide predictions in the $N_{1.60} = 6$ material are ignored, the influence of $N_{1.60}$ is seen to be significant but modest. The displacement estimates varied over a range of about ±30%.

The case with nonuniform $N_{1.60}$ is perhaps the most interesting. The predicted displacements for gentle slopes are somewhat lower than for the uniform $N_{1.60} = 10$ analysis due to the increased strength at the base. The displacements become quite large for steeper slopes as the low strength of the $N_{1.60} = 6$ zones begins to dominate the response.

**Effect of postliquefaction model**

Two models were discussed for representing the postliquefaction stress-strain behaviour in Section 3.3.2: a simple bilinear model and a more complex version that was...
used in the base analyses and captures the extremely soft behaviour at stress reversals. The difference in displacement estimates between these two models is expected to be largest when the static bias is small and significant stress reversals are likely. This trend is confirmed in Figure 5-17 where displacement estimates using the bilinear model are presented.

The choice of postliquefaction model might be expected to affect the trend of slope versus displacement. This is shown in Figure 5-18. No clear conclusions can be drawn due to the limited comparison, although for this case the symmetric model appears to give the better trend.

Predicted stress-strain behaviour for the two postliquefaction models is shown in Figure 5-19. The symmetric loading model provides a good match to the behaviour seen in Figure 2-28.

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**Figure 5-17.** Effect of postliquefaction model on displacement.

**Figure 5-18.** Effect of postliquefaction model on normalized displacement.
Effect of damping

Two variations of viscous damping were evaluated. The first variation increases the center frequency $f_c$ for the Rayleigh stiffness (RS) damping by a factor of 1.5. This effectively reduces the viscous damping by a factor of 2/3. The original $f_c$ was based on the fundamental frequency $f_i$ of the column prior to liquefaction. Using an $f_c > f_i$ may be appropriate in some situations where higher mode response is significant. The second

Figure 5-19. Comparison of stress-strain response at 3.5 m depth and 0.5% slope.
variation uses a combined mass and stiffness formulation (RMS). A comparison of results is shown in Figure 5-20.

Displacements from the two RS formulations are seen to agree well despite the large change in damping. This may be due in part to the large number of zones that liquefy early in the earthquake. These liquefied zones are assigned low damping ratios that reduce the importance of the original damping specification.

The RMS formulation differs significantly from the RS estimates and the predicted trend with slope is somewhat erratic. At 6% slope the displacements are about 40% lower than the RS estimates. This may be due to the tendency for RMS to impose high damping on long period motion as indicated by Figure 4-2. While the $f_i$ of the column prior to liquefaction is about 1.9 Hz, this reduces to only 0.3 Hz when $G_{1q}$ is assumed in the lower 8 m. The response frequency may drop even further when the column is failing at the residual strength. This change in frequency can be seen in the comparison of the base input and surface predictions for acceleration and velocity shown in Figure 5-21.

![Figure 5-20. Effect of damping parameters on displacement.](image)
This large decrease in response frequency can effectively increase the RMS damping by a factor of 3.5. While the liquefied zones may still have little viscous damping, the damping associated with the nonliquefied elements can become very large. For the RMS analysis at 6% slope there were five elements that remained non-liquefied for at least 14 seconds into the earthquake.
Effect of site conditions

The initial analyses reported by Beaty and Byrne (1999) include the simulation of 40 m of alluvium beneath the 10 m column. This was approximated by modifying the base motion using SHAKE to reflect the new stratigraphy. This change more than doubled the displacements and indicates the importance of site conditions to the predicted response. It also suggests that the frequency content and character of the earthquake are important.

Effect of element size

Mesh density is a difficult modeling variable. Fine meshes may require extremely long solution times and may not accommodate large strain behaviour. But coarse meshes can impose significant approximations in material representation, stresses, and strain response. The base analyses were repeated using 0.5-metre-high elements instead of the original 1 m elements. Although both of these meshes might be considered somewhat coarse, they are believed to be common element sizes for many analyses of engineering structures. Both sets of analyses gave similar results as shown in Figure 5-22.

5.1.4 Comparison to empirical predictions

The empirical correlation of Hamada et al. (1987) is very simple. It does not consider potentially significant parameters such as earthquake magnitude, intensity of motion, or relative density. The correlation relates displacement to the thickness of the liquefied layer and the ground slope:

\[ D_h = 0.75 \times (\text{thickness})^{1/2} \times (\text{slope})^{1/3} \]  
Eqn. 5.3

A plot of displacement versus slope for an 8-metre-thick liquefied layer is given in Figure 5-23. Estimates from the synthesized approach are repeated from Figure 5-7 for comparison. These displacements are generally 1/2 to 2/3 of those given by the Hamada
correlation. Although this might be considered good agreement in the uncertain world of displacement prediction, there are several contributing factors to the discrepancy. These include the simple nature of the Hamada correlation, the observation that many of the synthesized analyses generated less than 8 m of liquefaction, the choice of $y_r = 4\%$, and the apparent sensitivity of displacement predictions to the character of the input motion.

The empirical correlation of Youd et al. (1999) attempts to improve the displacement estimate by addressing more factors:

$$\log(D_h) = -17.614 + 1.581M_w - 1.518\log(R + 10^{0.89M_w - 5.64}) - 0.011R$$
$$+ 0.343\log S + 0.547\log T_{15} + 3.976\log(100 - F_{15})$$
$$- 0.923\log(D_{50_{15}} + 0.1)$$  \hspace{1cm} \text{Eqn. 5.4}

where $M_w$ is the moment magnitude, $R$ is the horizontal distance to nearest seismic energy source in km, $S$ is the ground slope in percent, $T_{15}$ is the total thickness of saturated layers in

\[Figure 5-22. \quad \text{Effect of element size on displacement.}\]

\[Figure 5-23. \quad \text{Comparison of Hamada et al. (1987) with synthesized approach.}\]
metres with $N_{1,60} < 15$ and with depth $< 20$ m, $F_{15}$ is the average fines content within $T_{15}$ in percent, and $D_{50,15}$ is the average $D_{50}$ within $T_{15}$ in mm.

This correlation has several features that contrast with the synthesized approach. The value of $N_{1,60}$ is not critical to the estimate of $D_h$, although blowcounts less than 15 are apparently needed before displacements are a concern. The displacement estimates are very sensitive to gradation as specified by $D_{50,15}$ and $F_{15}$. These gradation factors are not directly considered by the synthesized approach. While fines content and gradation will affect many properties including triggering resistance and residual strength, there is not a consensus on how to fully address their impact in selecting material properties. Laboratory testing of undisturbed samples may provide site-specific guidance. Many analyses rely upon the simple corrections for fines content described in Section 2.2.11.

Bartlett and Youd (1995) and Youd et al. (1999) provide allowable limits for each of these six variables. While these limits are based on the overall range of parameters within the database of observations, it is easy to select an acceptable set of parameters that are very different from any combination within the database. This occurs because the vast majority of observations in the database come from only two events: the 1964 Niigata and the 1983 Nihonkai-Chubu earthquakes. Since predictions using this equation are very sensitive to the values of $F_{15}$ and $D_{50,15}$, several estimates of displacements were made using combinations of these parameters as they are found in the database. The selected values are summarized in Table 5-2. The gradation values are the average reported for sloping ground conditions from the stated earthquake. Values of $F_{15}$ and $D_{50,15}$ corresponding to Fraser River delta sand were also used for comparison (Robertson et al., 2000b). The remaining parameters used in the displacement estimates were $M_w = 6.7$, $R = 10.6$ km, $T_{15} = 8$ m, and slopes from 0.1% to 6%.
Table 5-2. Gradation parameters for estimating $D_h$ using Youd et al. (1999)

<table>
<thead>
<tr>
<th>Sand</th>
<th>$D50_{15}$ (mm)</th>
<th>$F_{15}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1964 Niigata EQ</td>
<td>0.325</td>
<td>9</td>
</tr>
<tr>
<td>1971 San Fernando EQ</td>
<td>0.064</td>
<td>59</td>
</tr>
<tr>
<td>1979 Imperial Valley EQ</td>
<td>0.066</td>
<td>62</td>
</tr>
<tr>
<td>1983 Nihonkai-Chubu EQ</td>
<td>0.350</td>
<td>1</td>
</tr>
<tr>
<td>Fraser River delta</td>
<td>0.200</td>
<td>1</td>
</tr>
</tbody>
</table>

The computed displacements are shown in Figure 5-24. The estimates differ by a factor of more than 20 as the gradation is changed. It is interesting that the sands from the two Japanese earthquakes give much higher displacements than the sands representing the two California earthquakes. These Japanese sands were coarser and cleaner than the California sands. The Fraser River delta sand, finer than the Japanese sands but still having a low fines content, gave the highest displacement predictions.

![Figure 5-24. Displacement estimates from Youd et al. (1999).](image)
Chapter 5 — Synthesized Approach: Applications

It is difficult to make a comparison between the synthesized approach and this empirical correlation due to the large range in predictions. The common correction for fines content serves only to modify the representative value of \( N_{1-60} \), something that is not considered in the empirical approach. But the above parametric study suggests that modest changes in representative blowcount would not decrease the displacement estimates from the synthesized approach by a factor of 20 unless liquefaction was completely curtailed.

The comparison is made more difficult because the proper choice of \( R \) for the empirical correlation is not a clear one. The reported value of 10.6 km is based on the actual distance for the Kagel Canyon recording. However, Bartlett and Youd (1995) suggest that \( R \) should be based on the anticipated pga of the site using an attenuation chart provided in their paper. An \( R \) of perhaps 1 km is appropriate for a pga of 0.43 g. This change in \( R \) would increase the empirical predictions of displacement by 10 fold.

The selection of \( R \) is further complicated by the amount of amplification that may occur at the site. The SHAKE analyses, which do not consider liquefaction, suggest the possibility of significant amplification. But liquefaction might produce deamplification if triggering occurs early in the earthquake. The FLAC analyses suggest the peak ground acceleration might only be 0.2 to 0.3 g. Given all these variables and their significant effect on the estimated displacements, it is not clear how the Youd, Hansen, and Bartlett equation should be best applied.

Figure 5-25 directly compares the synthesized and empirical estimates. The empirical estimates from Youd et al. (1999) assume \( R = 10.6 \) km and are the same as shown in Figure 5-24. There is reasonable agreement between these approaches for the cleaner sand material, although the estimates from the synthesized approach are consistently larger. While
the Youd, Hansen, and Bartlett correlation is a major contribution, ongoing efforts to improve the empirical methods may allow for improved comparisons (Bardet et al., 1999b).

5.1.5 Summary

A series of parametric studies was made of lateral spreading on an infinite slope using the synthesized approach. The method gave displacement estimates that are relatively insensitive to modest changes in blowcount, damping, residual strength, $\gamma_r$, and element size for the selected earthquake applied in its critical direction. Displacement estimates are generally within 30% to 60% of average predictions despite the wide range in properties that were analyzed. The results also indicate the importance of site conditions below the liquefied soil, the direction of loading, and possibly the frequency content of the earthquake to the predicted displacements.

The analyses suggest that viscous damping using a Rayleigh stiffness proportional formulation may be preferable to combined mass and stiffness damping. The symmetric
loading model for postliquefaction stress-strain behaviour was found to produce reasonable behaviour, particularly for conditions of little static bias.

A comparison was made to the Hamada et al. (1987) and the Youd et al. (1999) empirical procedures. Fair agreement was obtained, although a precise comparison with the Youd et al. procedure is difficult. The average trend of slope versus predicted displacement was found to compare well with the empirical relationships. The predicted displacements were also within the common range of observed values (Bartlett, 1998).

The scatter in the predicted displacements reflects the inherent uncertainty in any analysis approach to the liquefaction-deformation problem. Even if a perfect representation of soil behaviour could be achieved in a numerical model, the uncertainty in input parameters makes precise displacement estimates impossible. However, trends observed in these analyses suggest the synthesized approach can give reasonable and relatively stable estimates of displacement. Results may differ for other structures or earthquake loading, and some level of parametric study is always warranted.

5.2 Upper San Fernando Dam

The Lower and Upper San Fernando dams are located in southern California, roughly 30 km north of downtown Los Angeles. The dams were built as part of the Los Angeles Aqueduct system with construction beginning in 1912 for the lower dam and 1921 for the upper dam. Both dams were constructed using variations of the hydraulic fill (HF) method. Although there were slight differences in placement technique and borrow source for the two dams, the character of the hydraulic fill in both dams was apparently similar (Seed et al., 1973). An aerial view showing the relative location of the dams is given in Figure 5-26. Analyses of the Lower San Fernando dam will be provided in the following section.
The Upper San Fernando dam (USFD) was completed in 1922 using a semi-hydraulic fill technique (Seed et al., 1973). The dam was constructed on about 15 to 18 m of alluvium overlying bedrock. The embankment material is believed to have been hauled to the borrow area in wagons, dumped into a pond between containment dikes, and dispersed by hydraulic jetting. This method yielded a central clayey core with highly stratified shells consisting of sand, silty sand, and clay. The sandy layers have a representative fines content of about 25% (Harder et al., 1989). Unfortunately, detailed records or photographs of the construction are not available.

The upper dam is approximately 21 m high, although it was not constructed to its full-intended height. Instead, a 5.5-metre-high rolled fill section was placed on the upstream portion of the hydraulic fill, leaving a 30.5-metre-wide bench on the downstream slope. This

Figure 5-26. Aerial view of the Upper and Lower San Fernando dams. (Photograph from Steinbrugge Collection)
gives the dam a wide profile for its height. The slopes of the dam are 2.5:1. A representative cross section as developed by Seed et al. (1973) is shown in Figure 5-27.

5.2.1 Observed seismic response

The San Fernando earthquake of February 9, 1971 occurred on a thrust fault and had a magnitude $M_w$ of 6.6. Although the epicentral distance was about 11 km, the dam was located very near the extreme western edge of the observed surface faulting. Indications of surface rupture were suspected within the reservoir of the lower dam. Peak ground accelerations at the site were estimated by Seed to be about 0.55 g to 0.6 g (Seed et al., 1973).

Displacement observations of the upper dam made after the earthquake are shown on Figure 5-29. Additional displacement data is provided in Harder et al. (1989). The crest moved downstream up to 1.5 m and settled vertically up to 1.0 m (Harder et al., 1989).

Several longitudinal cracks were noted running the full length of the upstream face near the reservoir level (Serff et al., 1976; Figure 5-29). These cracks suggested two or three well-defined slip surfaces. A 0.6-metre-high pressure ridge was also observed at the

Figure 5-27. Representative cross-section of Upper San Fernando dam.
Figure 5-28. Displacements following 1971 San Fernando earthquake, USFD. (Figure from Serff et al., 1976)

Figure 5-29. Crest of Upper San Fernando dam after 1971 earthquake. (Photograph from Steinbrugge Collection)
downstream toe. Tension cracks were seen in the central and upstream portions of the outlet conduit at the base of the embankment. Evidence of compression failure in the conduit was also seen near the downstream end. The magnitude of movement in these distressed areas was much less than the measured embankment displacement. Seed inferred that most of the deformation probably occurred within the embankment section and was not limited to a well-defined slip surface at depth (Seed et al., 1973).

The occurrence of liquefaction was suggested by sand boils in fill below the toe and by increased water levels in the three standpipe piezometers. Water overflowed from two of these instruments located near the centerline of the dam and within or near the predominantly clayey core. A sinkhole was also observed in the downstream shell above a crack in the outlet conduit.

5.2.2 Seismic loading

The only earthquake recordings near the dam site consist of seismoscope traces at the crest and abutment of the lower dam. Although seismoscope traces do not directly contain any time information, R.F. Scott was able to approximate a time history from the abutment record by recognizing that small regular waves in the seismoscope recording were actually a peculiarity of the instrument and could be related to time (Seed et al., 1973). This inferred time history contains some unusual low frequency components. Seed developed an alternative motion by modifying the controversial recording from the Pacoima dam abutment. Peak acceleration pulses were first reduced to a maximum of 0.9 g and then the entire record was scaled to a pga of 0.6 g. It is this record that is commonly used in back analyses of the San Fernando dams and is adopted here (Seed & Harder, 1990; Inel et al., 1993; Moriwaki et al., 1998).
The modified Pacoima dam record is shown in Figure 5-30. Although the input motion appears reasonable, it is unfortunate that the actual seismic loading is not better defined. Analyses of the Elsie Lake Main dam in Section 5.4 clearly demonstrate the importance of input motion to displacement predictions. Relatively minor differences in the character of the motion can have a pronounced effect on the displacement response.

The large velocity pulse early in the modified Pacoima dam record is due to near-field directivity and radiation pattern effects. These near-field effects diminish quickly off the edges of a dip slip fault (Somerville et al., 1997). The location of the dam relative to the fault rupture is not clearly defined in the available references. If the fault rupture extended beneath the dam site, the pronounced fling in the modified record is appropriate although its magnitude is uncertain. Improving the estimate of ground motion and the amplitude of any near-field pulse would require detailed study by a seismologist.

As the dam crest and fault strike are roughly parallel, any near-field pulse or fling will be oriented in the transverse direction of the dam. This velocity pulse was assumed to occur in the downstream direction for the base analysis. This orientation implies the geologic block above the fault moved up and over the lower block during this pulse.

5.2.3 $N_{1.60}$ characterization

Representative blowcounts for the hydraulic fill shells are given in Table 5-3. The values are based on SPT tests performed during April and May 1971 as reported by Harder et al. (1989). Two values are provided: $N_{1/3}$ and median $N_{1.60}$. $N_{1/3}$ is a measure of the looser fraction and is the value of $N_{1.60}$ where a third of the observations are less. The blowcount data is shown in Figure 5-31. The wide scatter demonstrates the difficulty in characterizing sites and the supports the use of representative $N_{1.60}$ values less than the median. For
Figure 5-30. Modified Pacoima dam earthquake record.
Table 5-3. $N_{1-60}$ of hydraulic fill shells, USFD

<table>
<thead>
<tr>
<th>Distance below crest * (m)</th>
<th>Thickness (m)</th>
<th>$N_{1-60}$ (median)</th>
<th>$N_{1/3}$</th>
<th>$N_{1-60}^{**}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.0 - 14.6</td>
<td>7.6</td>
<td>9</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>14.6 - 18.6</td>
<td>4.0</td>
<td>15</td>
<td>11</td>
<td>13</td>
</tr>
<tr>
<td>18.6 - 22.2</td>
<td>3.6</td>
<td>15</td>
<td>8</td>
<td>13</td>
</tr>
</tbody>
</table>

Notes:  
* Crest at elevation 371.23 m (1218 ft).  
** From Harder et al. (1989).

![Figure 5-31. SPT Blowcounts for USFD.](Figure from Harder et al., 1989)
comparison, the average relative density of the shells following the earthquake was about 54% (Seed et al., 1973).

The values shown in Table 5-3 have not been corrected to equivalent clean sand conditions. These corrections were applied using the relationships described in Section 2.2.11. The Seed and Harder (1990) recommendations for \( S_r \) suggest a correction of \( \Delta N_{corr} = +2 \) for a representative fines content of 25%. This yields \( N_{1/3cs} \) values of 9, 13, and 10 for use with the empirical \( S_r \) and \( S_r /\sigma'_{so} \) charts. The fines correction for CRR varies with blowcount (Youd & Idriss, 2001). The \( N_{1/3} \) values of 7, 11, and 8 are corrected to \( N_{1/3cs} \) values of 12.1, 16.6, and 13.2 when used with Figure 2-20.

One difference between the proposed characterization and that developed by Harder et al. (1989), also shown in Table 5-3, is the correction applied for postearthquake volumetric strain. The 1989 values assume the vertical settlements measured after the earthquake were entirely caused by volumetric strain within the hydraulic fill. An average \( \varepsilon_v \) of about 4.5% was estimated from settlements of the crest and downstream shell. This was converted by Harder et al. into a blowcount correction of \( \Delta N_{\varepsilon_v} = -4.5 \) by estimating the change in relative density. The Harder values of \( N_{1-60} \) in Table 5-3 have already been reduced by 4.5 blows/foot from the measured values.

This correction factor contains significant uncertainty and may be high. The relative settlement of saturated and unsaturated hydraulic fill was unknown as was the settlement within the alluvial foundation. Some of the vertical settlement was likely from kinematic movement of the slide mass as demonstrated by the analyses of Section 5.2.6. A volumetric strain of 4.5% also appears high when compared to the work of Tokimatsu and Seed (1987) or Ishihara and Yoshimine (1992). It also seems likely that soils under significant shear stress
will consolidate less after an earthquake than those at level ground conditions (Section 3.4.2; Shamoto et al., 1998).

Seed et al. (1988) performed a similar correction for the looser material within the downstream shell of the Lower San Fernando dam. A volumetric strain of about 1.2% was estimated to have been caused by the earthquake and subsequent consolidation from 1971 to 1985. This was converted into a blowcount correction of $\Delta N_{e_v} = -2$. Although the upper and lower dams are similar in many ways, the blowcount correction to account for consolidation may be affected by differences in the amount of liquefaction as well as the shear strain history during the earthquake. Considering the evaluation of the lower dam and the difficulties in estimating volumetric strains at the upper dam, an appropriate correction for the upper dam is likely between $\Delta N_{e_v} = -2$ and $\Delta N_{e_v} = -4.5$. A factor of $\Delta N_{e_v} = -3$ was assumed for the blowcount estimates developed for this thesis. This correction has already been applied to the median $N_{1-60}$ and $N_{1/3}$ blowcounts shown in Table 5-3 (i.e., these blowcounts have already been reduced by 3 to account for consolidation).

### 5.2.4 Static analysis description

The material zoning, geometry, and phreatic surface were based on the representative section developed by Seed et al. (1973) and shown in Figure 5-27. The finite difference grid and $N_{1/3}$ distribution is shown in Figure 5-32 and Figure 5-33.

The initial static analysis was performed using a hyperbolic stress-strain model. The embankment was constructed in layers and then the reservoir load and phreatic surface were added in stages. The material properties used in the analysis were based on the testing and data evaluation performed during the 1973 study. The values of $K_e$ and $R_f$ have been revised from those developed by Seed to reflect a modification in the form of the hyperbolic relationship. A simple correlation for bulk modulus was also used (Byrne et al., 1987) instead
of the complex expression developed by Seed et al. (1973). The properties selected for the static analysis are listed in Table 5-4.

5.2.5 Dynamic analysis description

Properties for the base dynamic analysis are given in Table 5-5. The relative density of the hydraulic fill is characterized by the $N_{1/3}$ values. This was considered reasonable due to the significant difference between $N_{1/3}$ and the median $N_{1-60}$ as well as the relatively large proportion of material that was less than or near $N_{1/3}$.

The $K_{2max}$ values in the hydraulic fill sand and alluvium were estimated from cross-hole seismic surveys by Seed et al. (1973). Although $K_{2max}$ is not strictly applicable to the clay core, an approximate value was estimated from the relation proposed by Seed. MRF and
Table 5-4: Material properties for static analysis, USFD

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Hydraulic Fill &amp; Clay Core</th>
<th>Rolled Fill</th>
<th>Alluvium</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unit weight:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Moist</td>
<td>kN/m³</td>
<td>18.9</td>
<td>21.0</td>
<td>—</td>
</tr>
<tr>
<td>Saturated</td>
<td>kN/m³</td>
<td>19.2</td>
<td>22.0</td>
<td>20.3</td>
</tr>
<tr>
<td><strong>Strength:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cohesion</td>
<td>kPa</td>
<td>0</td>
<td>125</td>
<td>0</td>
</tr>
<tr>
<td>( \phi' )</td>
<td>°</td>
<td>37°</td>
<td>25°</td>
<td>37°</td>
</tr>
<tr>
<td>Failure ratio, ( R_f )</td>
<td></td>
<td>0.84</td>
<td>0.9</td>
<td>0.79</td>
</tr>
<tr>
<td><strong>Stiffness:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( K_e )</td>
<td></td>
<td>380</td>
<td>160</td>
<td>250</td>
</tr>
<tr>
<td>( n )</td>
<td></td>
<td>0.52</td>
<td>0.76</td>
<td>0.80</td>
</tr>
<tr>
<td>( K_b )</td>
<td></td>
<td>230</td>
<td>100</td>
<td>150</td>
</tr>
<tr>
<td>( m )</td>
<td></td>
<td>0.26</td>
<td>0.38</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Damping values were developed from a FLAC-SHAKE calibration similar to the one discussed in Section 5.1.1. Due to the wide profile of the dam, a single column was evaluated at the midsection of the downstream berm. The SHAKE values for MRF and damping are shown in Figure 5-34. The center frequency \( f_c \) associated with the damping value was taken as the fundamental frequency of the dam \( f_t \) developed from the preliquefaction stiffness values. The “within” motion computed by the SHAKE analysis was used as the base motion for the FLAC analyses.

The undrained strength \( S_u \) for the saturated hydraulic fill, rolled fill, and alluvium zones was assumed equal to the drained strength following the assumptions of Section 4.3.4. The drained strength parameters for the rolled fill were reduced to zero cohesion and a friction angle of 30°. This reduction was made to reflect the extensive cracking that was observed.
Table 5-5. Parameters for base dynamic analysis, USFD

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Hydraulic Fill</th>
<th>Clay Core</th>
<th>Rolled Fill</th>
<th>Alluvium Upper/Lower</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Material parameters:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_{1-60cs}$</td>
<td>9 / 13 / 10</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$K_{2\text{max}}$</td>
<td>30.</td>
<td>30.</td>
<td>52.</td>
<td>40. / 110.</td>
</tr>
<tr>
<td>MRF</td>
<td>0.1 – 0.5</td>
<td>0.1 – 0.5</td>
<td>0.5</td>
<td>0.55</td>
</tr>
<tr>
<td>$\beta_c$</td>
<td>6% - 20%</td>
<td>6% - 20%</td>
<td>6%</td>
<td>6%</td>
</tr>
<tr>
<td>$f_c$</td>
<td>0.9 Hz</td>
<td>0.9 Hz</td>
<td>0.9 Hz</td>
<td>0.9 Hz</td>
</tr>
<tr>
<td>Damping type</td>
<td>RS</td>
<td>RS</td>
<td>RS</td>
<td>RS</td>
</tr>
<tr>
<td>$B^e / G_d\text{yn (saturated)}$</td>
<td>10.</td>
<td>10.</td>
<td>10.</td>
<td>5.</td>
</tr>
<tr>
<td>$S_u$</td>
<td>$0.6\sigma'_{vo}$</td>
<td>$0.24\sigma'_{vo}$</td>
<td>$0.5\sigma'_{vo}$</td>
<td>$0.6\sigma'_{vo}$</td>
</tr>
<tr>
<td><strong>Triggering parameters:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CRR$<em>1 / \text{CRR}</em>{15}$</td>
<td>2.2</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>CRR$<em>{15}$ versus $N</em>{1-60cs}$</td>
<td>Figure 2-20</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Strain-based trigger</td>
<td>None</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$K_\sigma$</td>
<td>Figure 4-3a</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$K_\alpha$</td>
<td>1.0</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td><strong>Postliquefaction parameters:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_r / \sigma'_{vo}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- simple shear</td>
<td>0.17</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>- compression</td>
<td>0.5</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$\gamma_r$</td>
<td>4%</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$G_{liq(unload)} / G_{liq(load)}$</td>
<td>10.</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$\beta_c$ (liq. zones)</td>
<td>2%</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Postliquefaction model</td>
<td>Symmetric</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

within this zone (Harder et al., 1989). The value for the clay core was estimated from a large number of small torvane tests (Seed et al., 1973).

Liquefaction was permitted throughout the saturated hydraulic fill material except for the triangular elements on the upstream face. Liquefaction was not permitted in the rolled fill section, the clayey core, or the denser alluvium foundation. The upper alluvium zone is
apparently less dense than the lower alluvium, but liquefaction was not considered based on the evaluation and analysis of Seed et al. (1973).

The shape of the weighting curve is the same as shown on Figure 2-18 although the specified curve is somewhat steeper. A \( \text{CRR}_1 / \text{CRR}_{15} \) value of 2.2 was estimated from cyclic triaxial testing of Shelby tube samples after isotropic consolidation (Seed et al., 1973). \( \text{CRR}_{15} \) is estimated from the MCEER chart in Figure 2-20 and gives reasonable agreement with the cyclic triaxial testing of isotropically consolidated samples. These test results, after multiplying by 0.6 to approximate simple shear conditions, give a \( \text{CRR}_{15} \) of about 0.15. This includes an approximate and slight reduction of 4% to account for densification of the sand due to the earthquake (after Seed et al., 1988). Using the \( N_{1/3} \) blowcounts, a fines content of

![Figure 5-34. MRF and Damping for USFD.](image-url)
The residual strength ratio in simple shear was estimated from back analyses of the dam. Limit equilibrium techniques were used by Harder et al. (1989) to estimate $S_r$ values of 23.9 to 33.5 kPa. Similar analyses performed by Beaty and Byrne (2001) produced estimates of 19.2 to 31.1 kPa. These strengths primarily reflect the base of the downstream shell, although some liquefied material in the upstream shell was also included in the critical shear surface. These strength values were then converted to strength ratios by estimating the average $\sigma'_vo$ along the critical shear surface. The range in $S_r/\sigma'_vo$ estimates is 0.12 to 0.18 for Harder et al. and 0.10 to 0.16 for Beaty and Byrne. The higher end of each range corresponds to the assumption that $S_r$ was mobilized in all saturated hydraulic fill zones outside of the core. As significant liquefaction of the hydraulic fill is anticipated in the analysis, a value of $S_r/\sigma'_vo = 0.17$ is assumed. This ratio is preferred for the base analysis over the empirical curves of Figure 2-34 since it is site specific. For comparison, the modified Idriss curve of Figure 2-34 would give $S_r/\sigma'_vo$ values of 0.11, 0.12, and 0.20 for the $N_{1/3}$ characterization of the hydraulic fill zones.

5.2.6 Analysis results

A substantial amount of liquefaction is estimated by the base analysis as shown in Figure 5-35a. Much of the upstream shell is predicted to liquefy along with most of the lower blowcount material in the downstream shell. The $N_{1/3cs} = 13$ layer in the downstream shell did not liquefy. This was probably due to increased triggering resistance as well as some reduction in cyclic stresses due to the underlying liquefaction. In general, the estimated extent of liquefaction is greater than predicted by the analysis of Seed et al. (1973) and shown in Figure 5-35b.
Plots of estimated displacement and shear strain are presented in Figure 5-36. Most of the embankment is seen to be moving in the downstream direction, including the crest, which agrees well with observations. The change in the direction of displacement, from downstream to upstream, occurs on the upstream face just below the reservoir level. Again, this approximately conforms to the location of observed longitudinal cracks. Zones of significant shear strain are indicated by the shaded regions on Figure 5-36c. These zones suggest a fairly well defined shear mechanism through much of the dam. This is particularly true within the downstream shell.

Figure 5-35. Comparison of liquefied zones, USFD.
Figure 5-36. Displacement and strain predictions for base analysis, USFD.
A comparison of observed and simulated displacements is shown on Figure 5-37. The estimated pattern of horizontal surface displacement agrees well with the observations. This particular set of input properties yields a somewhat lower magnitude of displacement across the downstream berm but is still in reasonable agreement. The downstream slope is predicted to deform more than was observed. This suggests the actual strains were distributed within

Figure 5-37. Comparison of observed and predicted displacements, USFD.
the downstream shell rather than concentrated at the base as predicted by the analysis. This may reflect the tendency of the analysis to overpredict base isolation. Specifying a random distribution of blowcount within soil units rather than a simple uniform characterization might result in a more distributed strain pattern as discussed in Section 4.1.1.

The extensive liquefaction predicted in the upstream shell results in the large displacements estimates upstream of the crest. These predictions are likely too high. Although the reservoir was not lowered for some time after the earthquake, primary descriptions of the dam response do not mention any significant displacements of the upstream slope (Seed et al., 1973; Serff et al., 1976).

The estimated pattern of vertical displacements shown in Figure 5-37 does not match the observations quite as well. The analysis does not capture the reduced vertical displacement over the core, or the increase in settlement near the downstream slope. However, the magnitudes of the settlements are generally similar, especially near the crest. These vertical displacements are due only to kinematic movements and do not include postearthquake consolidation. The analysis also predicts a pressure bulge at the downstream toe, although only about half as high as was observed.

The predicted areas of liquefaction are more extensive than those originally estimated by Seed et al. as shown in Figure 5-35b. This is especially true along the upstream face and beneath the downstream slope. Seed considered the effects of initial static shear stress on liquefaction and based the triggering resistance for zones beneath the slope on anisotropically consolidated cyclic triaxial tests. These tests indicate a substantial increase in liquefaction resistance beneath the slopes that was not considered in the base analysis. Any anticipated improvement in triggering resistance due to a $K_a$ factor is often unreliable due to earthquake loading that will occur in the longitudinal direction. However, since the near-field pulse for
this earthquake is oriented in the transverse direction, there is at least the possibility of improvement in cyclic strength due to $K_a$ effects.

Cyclic triaxial testing might not be the most appropriate test for evaluating $K_a$. Figure 4-4, which relies more heavily on simple shear and torsional simple shear tests, suggests relatively little $K_a$ correction for soils with relative densities similar to the hydraulic fill. However, Harder and Boulanger note the importance of site specific testing (Youd & Idriss, 1998).

There is good reason to suspect that liquefaction did not occur near the upstream face. It is likely that extensive liquefaction would have caused large displacements of the slope. The containment dikes along the upstream and downstream faces may have been more compact than the internal fill, although their thickness and character is uncertain. It is possible they were too thin at the upper dam to enhance the stability. The material near the outer edges of the hydraulic fill tends to be coarser than near the core, although it is not clear if this would have improved the performance. Elements near the surface might be more likely to achieve their full drained strength after liquefaction due to the low confining stresses. This may reduce the potential for large but shallow displacements if liquefaction should occur near the slope. Permitting liquefaction to occur right to the upstream face, and specifying the residual strength as a strength ratio, probably led to the overprediction of displacements of the upstream face.

Predicted stress-strain response of liquefied elements in the downstream shell is shown in Figure 5-38. These curves suggest the downstream shell responded in a nearly monotonic mode following liquefaction. The ratcheting effect and the soft zone associated with symmetric loading are not very significant for these elements. Although these effects were
more important in the upstream shell behaviour, they appear to have had a small effect on the general displacement prediction of downstream movement.

Time histories of horizontal displacement relative to the model base are shown in Figure 5-39. Large displacements begin at the start of widespread liquefaction and continue for about 7 seconds. Much of the displacement is predicted to occur within only 3 seconds. It is reasonable to assume that such displacements would occur under primarily undrained conditions. This supports the use of an anisotropic strength ratio for the initial strength of liquefied elements at this dam.

It is interesting that much of the liquefaction occurs about 3 to 4 seconds into the earthquake. This coincides with the falling limb of the velocity pulse shown on Figure 5-30. Many of the elements in the downstream shell are predicted to liquefy at about 3.2 to 3.4
seconds. The rising limb of the velocity pulse increases the velocity of the dam in a
downstream direction since few elements have liquefied. As the base velocity quickly drops,
newly liquefied zones within the hydraulic fill have a reduced ability to slow the velocity of
the dam section. The dam deforms almost monotonically until its kinetic energy is dissipated
through plastic flow. This is demonstrated by the velocity histories in Figure 5-40. For this
case, it appears the near-field velocity pulse may be critical to the prediction of deformations.
Uncertainty regarding the exact nature of this pulse affects the results of this back analysis,
especially with respect to displacement magnitudes.

Figure 5-39. Selected time histories of relative horizontal displacement, USFD.
5.2.7 Sensitivity studies

A limited set of parametric analyses was performed to evaluate the influence of key variables. A brief presentation of the results is given below. Comparisons are based primarily on the estimates of final displacement. A summary of displacement predictions at the crest and top of downstream slope is given in Table 5-6 and Table 5-7 at the end of this section.

Effect of damping

Two analyses were performed to evaluate assumptions for damping as described below and shown in Figure 5-41.

1. USFTBf15 reduces the viscous damping by 33% by increasing the center frequency $f_c$ from a value of $f_i$ (0.9 Hz) to $1.5f_i$ (1.35 Hz). This rather large change in damping produced only a nominal increase in the displacement predictions.
2. USFTBms substitutes Raleigh mass and stiffness (RMS) damping for stiffness only (RS) damping. As anticipated, the mass formulation has a tendency to reduce displacements. Displacements of the downstream berm were reduced by about 30% from the base analysis.

Effect of residual strength ratio

Assumptions for the residual strength ratio $S_r/\sigma'_{vo}$ were varied in two analyses as described below and with results shown in Figure 5-42.

3. USFTBid bases the residual strength ratio in simple shear on the $N_{1/3}$ characterization and the modified Idriss curve of Figure 2-34. This residual strength ratio might be selected for a typical preliminary analysis of the dam. This produced a significant drop in $S_r/\sigma'_{vo}$ in much of the hydraulic fill. While a uniform $S_r/\sigma'_{vo} = 0.17$ was used for the
base analysis, USFTBid used values of 0.11, 0.12, and 0.20 in the zones with $N_{1/3}$ of 7, 8, and 11.

4. USFTBiso uses the same strength ratio as the base analysis, but does not consider the effects of anisotropy.

Both analyses result in much higher displacements, approximately double the base analysis predictions. This highlights the importance of residual strength to displacement predictions. The effect of anisotropy is particularly interesting. Although much of the shearing surface is near simple shear conditions at the end of the earthquake, the effect of loading direction is still important to the overall response. Anisotropy appears to be a key variable for analyses of undrained conditions.

Anisotropy may also impact the process of estimating residual strength from back analyses of case histories. Anisotropy might be most important for cases with modest
displacements that are likely associated with undrained conditions. The Upper San Fernando dam is one such history. Although the residual strength that was estimated from this dam was assumed appropriate for simple shear conditions, a good representative value of simple shear strength may be somewhat lower.

**Effect of residual strength**

Although the initial undrained strength following liquefaction may be best described as a strength ratio, it is customary to use isotropic residual strengths derived from back analyses in deformation evaluations. The effect of using an isotropic $S_r$ was assessed in two analyses as described below and with results shown in Figure 5-43.

5. USFTBsr3 assumes an isotropic residual strength in liquefied elements based on $N_{1/3}$ and the Idriss curve of Figure 2-32. This gives residual strength values of 10.1, 19.2, and 11.8 kPa for the $N_{1/3}$ values of 7, 11, and 8. These estimates are lower than
determined from back analysis as discussed in Section 5.2.5 and would be considered very conservative. Not surprisingly, the analysis predicts a flow slide of the downstream shell and crest.

6. USFTBsr3m is similar to USFTBsr3 except the median $N_{1.60}$ is used to develop the residual strength. This gives $S_r$ values of 13.9 and 36.3 kPa for $N_{1.60}$ values of 9 and 15. These strength estimates are somewhat higher than determined from back analysis. Triggering in this analysis is still based on the $N_{1/3}$ value. Predicted results are shown in Figure 5-43. The resulting displacements are reasonable but somewhat lower than either the observations or the base analysis.

These analyses suggest that using a residual strength from the Idriss curve of Figure 2-32 is reasonable, although basing this strength on the $N_{1/3}$ values was overly conservative in this case. The optimum value of $S_r$ based on the Idriss curve would use a blowcount characterization somewhere between the $N_{1/3}$ and median $N_{1.60}$. The possibility that the upper dam may have failed given a modest reduction in residual strength should be kept in mind.

**Effect of blowcount characterization**

The effect of blowcount characterization is shown in Figure 5-44. These two analyses assume one of the alternative characterizations from Table 5-3 with anisotropic residual strength ratios based on the modified Idriss curve of Figure 2-34.

7. USFTBnm assumes the median $N_{1.60}$ characterization.

8. USFTBnh assumes the Harder et al. (1989) characterization.

It is not surprising that the alternative $N_{1.60}$ characterizations have a significant impact on displacements. Both alternatives assume higher values of $N_{1.60}$ that increase both the triggering resistance and the residual strength. For example, the $S_r/\sigma'_{vo}$ used for the lower
hydraulic fill in the downstream shell was 0.17 for the base analysis, but increased to 0.38 for the USFTBnm analysis and to 0.28 for the USFTBnh analysis. Comparing these results with analysis USFTBgd and Figure 5-42 suggests the optimum blowcount characterization to use with the modified Idriss curve of Figure 2-34 would lie closer to the $N_{1/3}$ characterization than to the median.

Effect of residual strain

Two additional values of residual strain $\gamma_r$ were analyzed as described below and shown in Figure 5-45.

9. USFTBg8 increased the residual strain from 4% to 8%.

10. USFTBg12 further increased $\gamma_r$ to 12%.

The displacement predictions were found to be almost completely insensitive to these changes in $\gamma_r$. 

Figure 5-44. Effect of blowcount characterization, USFD.
Effect of liquefaction beneath slopes

As discussed in Section 5.2.6, there is some reason to suspect that liquefaction did not occur beneath the sloping faces. The effect of prohibiting liquefaction in these areas is described below and shown in Figure 5-46.

11. USFTBns does not allow triggering of liquefaction beneath the slopes. The affected elements are shown in Figure 5-46a.

12. USFTBnst is identical to USFTBns except liquefaction will trigger in zones beneath the slopes if the maximum shear strain exceeds 4%.

The analyses show that the estimate of peak undrained strength beneath the slopes is sufficient to significantly decrease the displacements of both the upstream and downstream shells. Final displacements are reduced by more than 50%. However, large strains will still occur in these zones. If strain softening is considered, as in USFTBnst, then the resulting
displacements of the downstream shell are nearly as large as if liquefaction is triggered by cyclic loading.

Figure 5-46. Effect of liquefaction beneath slopes, USFD.

(a) Location of elements beneath slopes where liquefaction is not permitted.

(b) Predicted displacements.

Figure 5-46. Effect of liquefaction beneath slopes, USFD.
**Effect of weighting curve**

The weighting curve for the base analysis has a CRR$_1$/CRR$_{15}$ ratio of 2.2. The effect of this curve was evaluated in two analyses.

13. USFTBw1 assumes a CRR$_1$/CRR$_{15}$ ratio of 1.6 as suggested by Figure 2-18.
14. USFTBw2 assumes a CRR$_1$/CRR$_{15}$ ratio of 1.9.

Figure 5-47 reveals that the choice of weighting curve has almost no effect on the displacements. This may be an earthquake-specific conclusion. The large loading pulse early in the modified Pacoima dam record will easily induce liquefaction in many zones regardless of modest changes in the triggering resistance.

**Effect of undrained strength $S_u$**

The undrained strength prior to liquefaction has been modeled rather simply. An isotropic cohesion is assumed that is based on the drained strength parameters. As this
strength may be too high for some loading directions, the following analysis was also performed.

15. USFTBsu reduces the $S_u / \sigma'_{wo}$ from 0.6 to 0.4 in all non-liquefied hydraulic fill shell zones.

This reduction in strength increases the displacements of the upstream shell but reduces the downstream shell displacements as shown in Figure 5-48. This is likely associated with a moderate base isolation effect caused by the reduced strengths. This is suggested by the reduced number of liquefied elements in the downstream shell (from 24 to 20) and from the slight delay in the average time of liquefaction at the base of the downstream shell (from 3.3 seconds to 4.6 seconds). This delay is likely the more important due to the significance of the early velocity pulse to the displacements.

![Figure 5-48. Effect of undrained strength $S_u$, USFD.](image)
**Effect of earthquake direction**

As there is some uncertainty with respect to the direction of earthquake loading, in particular the near-field velocity pulse, the analysis was also performed with the velocity pulse oriented in the upstream direction. Results are shown in Figure 5-49.

16. USFTB- applies the earthquake motion in a negative direction. The earthquake direction has a modest effect on the displacements as shown in Figure 5-49. The displacements of the downstream shell are reduced by about 30%.

**Effect of model formulation**

Two formulations of the synthesized approach were evaluated as described below and shown in Figure 5-50.

17. USFTB$sxy$ uses the horizontal shear stress to determine loading, unloading, and stress reversals for liquefied elements. In contrast, the base analysis uses the direction and

![Graph showing displacement vs. horizontal distance](image)

**Figure 5-49.** Effect of earthquake loading direction, USFD.
magnitude of the maximum shear stress to make these determinations as described in Section 3.3.4. The more sophisticated approach used in the base analysis had the biggest impact on the upstream shell displacements. The logic based on $\tau_{xy}$ produces horizontal displacements that are about 50% larger for the upstream shell.

18. USFTBos uses the simpler bilinear model for postliquefaction loading while the base analysis uses the symmetric loading option described in Section 3.3.2. As expected, the symmetric loading option made very little difference to the displacement predictions due to the significant static bias.

**Effect of upper alluvium**

The base analysis assumes that significant liquefaction will not occur in the alluvium foundation. This characterization was based on conclusions from the initial Seed et al. (1973) study. The 1973 testing program and dynamic analysis indicated that significant liquefaction

![Graph showing displacement vs. horizontal distance](image)

Figure 5-50. Effect of model formulation, USFD.
did not occur within the alluvium. This was supported by their interpretation of field observations that suggested little deformation occurred within the foundation.

This simple characterization of the foundation may not be appropriate. For example, the blowcounts and shear wave velocity measurements reported in the Seed et al (1973) study indicate the upper alluvium layer shown on Figure 5-27 might be liquefiable. This is supported by the $N_{1.60}$ values from borings A-1, B-1, and C-1 as shown in Figure 5-31. Although a detailed evaluation of the penetration resistance was not done, data in the 1973 report indicates the upper alluvium zone could have a representative $N_{1.60}$ that is similar to the overlying hydraulic fill. The following analysis was performed to investigate the effect of this layer.

19. USFTBua assumes an $N_{1/3}$ value of 11 in the upper alluvium zone. The $CRR_1 / CRR_{15}$ in this zone was assumed to be the same as for the overlying hydraulic fill. Predicted displacements are shown in Figure 5-51. These estimates give a somewhat better match to observed values than the base analysis predictions, particularly for the upstream slope where the displacements may have been smaller than predicted by the base analysis. This change in displacement of the upstream shell may have resulted from a base isolation effect due to liquefaction in the upper alluvium. The number of liquefied elements in the upstream hydraulic fill decreased from 38 in the base analysis to 29 in USFTBua.
Figure 5-51. Effect of upper alluvium, USFD.
### Table 5-6. Selected displacements from parametric study, USFD

<table>
<thead>
<tr>
<th>Analysis name</th>
<th>Top of D/S Slope</th>
<th>Crest</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizontal (m)</td>
<td>Vertical (m)</td>
</tr>
<tr>
<td>OBSERVED</td>
<td>2.2</td>
<td>-0.4</td>
</tr>
<tr>
<td>Base Analysis</td>
<td>1.63</td>
<td>-0.04</td>
</tr>
<tr>
<td>1 USFTBf15</td>
<td>1.72</td>
<td>-0.13</td>
</tr>
<tr>
<td>2 USFTBms</td>
<td>1.14</td>
<td>-0.07</td>
</tr>
<tr>
<td>3 USFTBid</td>
<td>3.44</td>
<td>-0.13</td>
</tr>
<tr>
<td>4 USFTBiso</td>
<td>2.89</td>
<td>-0.12</td>
</tr>
<tr>
<td>5 USFTBnm</td>
<td>0.22</td>
<td>-0.01</td>
</tr>
<tr>
<td>6 USFTBnh</td>
<td>0.33</td>
<td>0.00</td>
</tr>
<tr>
<td>7 USFTBg8</td>
<td>1.63</td>
<td>-0.05</td>
</tr>
<tr>
<td>8 USFTBg12</td>
<td>1.70</td>
<td>-0.04</td>
</tr>
<tr>
<td>9 USFTBsr3</td>
<td><em>fs</em></td>
<td><em>fs</em></td>
</tr>
<tr>
<td>10 USFTBsrn</td>
<td>1.10</td>
<td>-0.04</td>
</tr>
<tr>
<td>11 USFTBns</td>
<td>0.63</td>
<td>0.01</td>
</tr>
<tr>
<td>12 USFTBnst</td>
<td>1.30</td>
<td>-0.05</td>
</tr>
<tr>
<td>13 USFTBw1</td>
<td>1.69</td>
<td>-0.10</td>
</tr>
<tr>
<td>14 USFTBw2</td>
<td>1.67</td>
<td>-0.09</td>
</tr>
<tr>
<td>15 USFTBsSu</td>
<td>1.11</td>
<td>-0.12</td>
</tr>
<tr>
<td>16 USFTBsu</td>
<td>1.15</td>
<td>-0.08</td>
</tr>
<tr>
<td>17 USFTBsxy</td>
<td>1.85</td>
<td>-0.06</td>
</tr>
<tr>
<td>18 USFTBos</td>
<td>1.61</td>
<td>-0.05</td>
</tr>
<tr>
<td>19 USFTBua</td>
<td>1.73</td>
<td>-0.04</td>
</tr>
</tbody>
</table>

Note: *fs* indicates flow slide in the downstream direction.
Table 5-7. Percentage change in displacements from base analysis, USFD

<table>
<thead>
<tr>
<th>Analysis name</th>
<th>Top of D/S Slope</th>
<th>Crest</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizontal (% change)</td>
<td>Vertical (% change)</td>
</tr>
<tr>
<td>OBSERVED</td>
<td>35%</td>
<td>900%</td>
</tr>
<tr>
<td>Base Analysis (BA)</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>1 USFTBf15</td>
<td>6%</td>
<td>190%</td>
</tr>
<tr>
<td>2 USFTBms</td>
<td>-30%</td>
<td>62%</td>
</tr>
<tr>
<td>3 USFTBid</td>
<td>112%</td>
<td>204%</td>
</tr>
<tr>
<td>4 USFTBiso</td>
<td>78%</td>
<td>164%</td>
</tr>
<tr>
<td>5 USFTBnm</td>
<td>-86%</td>
<td>-79%</td>
</tr>
<tr>
<td>6 USFTBnh</td>
<td>-80%</td>
<td>-94%</td>
</tr>
<tr>
<td>7 USFTBg8</td>
<td>0%</td>
<td>2%</td>
</tr>
<tr>
<td>8 USFTBg12</td>
<td>5%</td>
<td>-1%</td>
</tr>
<tr>
<td>9 USFTBs13</td>
<td>fs</td>
<td>fs</td>
</tr>
<tr>
<td>10 USFTBsr17</td>
<td>-32%</td>
<td>-7%</td>
</tr>
<tr>
<td>11 USFTBns</td>
<td>-61%</td>
<td>-124%</td>
</tr>
<tr>
<td>12 USFTBnst</td>
<td>-20%</td>
<td>21%</td>
</tr>
<tr>
<td>13 USFTBw1</td>
<td>4%</td>
<td>138%</td>
</tr>
<tr>
<td>14 USFTBw2</td>
<td>3%</td>
<td>107%</td>
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<tr>
<td>15 USFTBs15</td>
<td>-32%</td>
<td>167%</td>
</tr>
<tr>
<td>16 USFTB-</td>
<td>-29%</td>
<td>78%</td>
</tr>
<tr>
<td>17 USFTBsx17</td>
<td>14%</td>
<td>30%</td>
</tr>
<tr>
<td>18 USFTBos</td>
<td>-1%</td>
<td>13%</td>
</tr>
<tr>
<td>19 USFTBua</td>
<td>7%</td>
<td>-13%</td>
</tr>
</tbody>
</table>

Note: *fs* indicates flow slide in the downstream direction.
5.2.8 **Comparison with other analytical approaches**

Two similar analytical approaches by Inel, Roth, and de Rubertis (1993) and Moriwaki, Tan, and Ji (1998) were discussed in Sections 3.8.1 and 3.8.2 respectively. The cited references also provide predictions of the Upper San Fernando dam response. While the analyses differ in both formulation and material properties, a simple comparison of results is helpful to gauge the techniques.

Limited displacement results were presented by Inel et al. (1993) as shown in Figure 5-52. This approach predicts moderate displacements of both the upstream and downstream shell. However, the crest is predicted to move somewhat upstream and the magnitude of the downstream shell displacements appears too small. This method does capture the sense of vertical displacements across the berm better than the synthesized approach.

Roth, a co-author of the paper, has stated that this technique gives improved displacement results when the interpreted seismoscope record is used (W. Roth, Personal Communication, September 1999). However, it is not clear if the unusual character of this record will lead to reliable predictions as discussed in Section 5.2.1. This does not diminish

![Figure 5-52. Deformed mesh of USFD from Inel et al. (1993).](image-url)
any criticism of the modified Pacoima dam record and its suitableness. Rather, it supports the importance of the earthquake loading for estimating liquefaction-induced displacements.

Displacement results presented by Moriwaki et al. (1998) are shown in Figure 5-53. This analysis results in much larger displacements, particularly for the downstream berm and slope. The crest is predicted to move upstream along with a minor bulging of the upstream face. One reason for the large displacement estimates is the prediction of nearly complete liquefaction of saturated hydraulic fill in both the upstream and downstream shells. In addition, the large displacement predictions were likely affected by the choice of residual strength. An $S_r$ value of only 19.2 kPa was used. This value is at the low end of the estimates from the limit equilibrium back analyses.

The predictions from the synthesized approach compare favorably with these alternative techniques. In particular, the overall displacement mode consisting of a relatively uniform downstream shift of the crest and downstream shell is best captured by the synthesized approach, at least for the modified Pacoima dam earthquake record.

5.2.9 Summary

The synthesized approach was applied to the response of the Upper San Fernando dam

![Figure 5-53. Deformed mesh of USFD from Moriwaki et al. (1998).](image-url)
to the 1971 San Fernando earthquake. The approach produces a reasonable simulation of the observed behaviour. In particular, the tendency for the dam crest and downstream shell to suffer large but limited displacements in the downstream direction was reproduced. Predicted displacements of the upstream shell appear to be high. While the downstream deflections from the base analysis are somewhat low, an improved match may be obtained with a small reduction in residual strength. Major uncertainties in the analyses include the appropriateness of the earthquake record and the suitability of the idealized cross section. A favorable comparison was also made between the synthesized approach and predictions from other analytical methods.

A number of parametric analyses were performed. Residual strength, blowcount characterization, postliquefaction anisotropy, and whether or not liquefaction is triggered near the slopes were found to have a large impact on the predicted displacements. The undrained shear strength \( S_u \), earthquake direction, use of \( \tau_{xy} \) for evaluating stress reversals, and use of MS Rayleigh damping all had a more modest impact on displacements. Parameters with little if any effect on displacements were the symmetric loading logic, weighting curve, residual strain \( \gamma_r \), and a drop in viscous damping of 33%. Due to the nonlinear nature of the analysis, the importance of many of these parameters is likely a function of the structure being analyzed.

There are indications that the predictions were affected by some degree of base isolation along the bottom of the downstream shell. The actual shear strains may have been more distributed throughout the shell rather than concentrated at its base as was predicted by the analyses. It is possible that an earthquake record with a different character or the use of a nonuniform blowcount within each hydraulic fill unit would have produced a more distributed strain pattern.
It appears the synthesized approach would have provided a reasonable method for evaluating the response of the dam prior to the earthquake. A difficulty in such an analysis would have been the selection of an appropriate residual strength. Displacement predictions are highly sensitive to this value, yet it is rarely known with confidence. Selecting appropriate input motions would have been a further difficulty.

At least three questions arise when selecting values of residual strength. First, should $S_r$ or $S_r/\sigma'_{vo}$ be used? Laboratory tests suggest the initial undrained strength may be best represented by anisotropic $S_r/\sigma'_{vo}$ or $S_r/\sigma'_{v0}$ ratios (Sections 2.2.4 and 4.4.1). These analyses evaluate both $S_r$ and $S_r/\sigma'_{vo}$ and found that either can give reasonable results. The inclusion of anisotropy was found to greatly improve the predicted response of the upstream slope. Additional analyses that support the use of $S_r/\sigma'_{vo}$ as an initial postliquefaction strength and $S_r$ as a final mobilized strength have been published (Beaty & Byrne, 2001).

Second, should the residual strength be based on a median representation of penetration resistance or some lower value? And third, should an average strength relationship such as the Idriss curve be used or a lower measure such as the Seed-Harder Lower Bound curve (Figure 2-32 and Figure 2-34)? These questions have added importance at this dam given the large difference between the median $N_{1.60}$ and $N_{1/3}$.

Estimates of $S_r$ and $S_r/\sigma'_{vo}$ are summarized in Table 5-8 for the lower hydraulic fill zone in the downstream shell. The choice of penetration resistance and empirical curve can produce a wide range in strength estimates. The analyses suggest the optimum $S_r/\sigma'_{vo}$ in simple shear for the synthesized approach is somewhat higher than the value given by the modified Idriss curve of Figure 2-34 and the $N_{1/3}$ characterization. For this case history, using a median $N_{1.60}$ along with the modified Idriss curve or the Seed-Harder Lower Bound curve of Figure 2-34 may have produced displacement estimates that are low and misleading.
Table 5-8. Residual strength estimates in lower HF zone of downstream shell, USFD

<table>
<thead>
<tr>
<th>Description</th>
<th>( S_r/\sigma'_{vo} ) in simple shear</th>
<th>( S_r ) (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Median Ni=60cs=17</td>
<td>Median Ni=1-60cs=17</td>
</tr>
<tr>
<td>Back analysis</td>
<td>0.17</td>
<td>0.17</td>
</tr>
<tr>
<td>Range from back analysis</td>
<td>0.11 - 0.17</td>
<td>0.11 - 0.17</td>
</tr>
<tr>
<td>Modified Idriss *</td>
<td>0.12</td>
<td>0.38</td>
</tr>
<tr>
<td>Modified Seed-Harder LB *</td>
<td>0.05</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Note: * From Figure 2-32 and Figure 2-34.

Use of isotropic \( S_r \) values based on the modified Idriss curve of Figure 2-32 and a median \( N_{1-60} \) produced low but fairly reasonable estimates of displacements. This approach to developing strengths may be considered standard practice, although many engineers will introduce conservatism by selecting an \( N \) value lower than the median or using a curve closer to a lower bound measure. An isotropic \( S_r \) derived from the modified Idriss curve and the \( N_{1/3} \) characterization resulted in a flow slide.

It appears that the optimum value of isotropic \( S_r \) or anisotropic \( S_r/\sigma'_{vo} \) for use in the synthesized approach during the earthquake can be obtained with the Idriss curve of Figure 2-32 or the modified Idriss curve of Figure 2-34. For the Upper San Fernando dam, the appropriate \( N \) value to use with these curves lies somewhere between the estimates of \( N_{1/3} \) and median \( N_{1-60} \).
5.3 Lower San Fernando Dam

The Lower San Fernando dam (LSFD) was about 44 m in height, founded on up to 11 m of alluvium, had a crest length of 634 m, and original slopes of 2.5:1. Much of the embankment was constructed using a common hydraulic fill technique (Seed et al., 1973). The embankment material was apparently excavated in the borrow area by water jetting and transported to the dam in wooden troughs or by pumping. The material was discharged along the outside edges of the dam with the coarser material settling rapidly near the containment dikes. This method yielded a central clayey core with highly stratified shells consisting of sand, silty sand, and clay. The silty sand layers have a representative fines content of about 25% (Seed et al., 1988).

The upper part of the dam consisted of a compacted fill. A rolled fill berm with a 4.5:1 slope was added to the downstream face in 1940. This berm created a 6-metre-wide bench at an elevation 15 m below the crest. A representative cross section of the dam as developed by Seed et al. (1973) is shown in Figure 5-54.

![Figure 5-54. Representative cross-section of Lower San Fernando dam.](image-url)
5.3.1 Observed seismic response

The San Fernando earthquake occurred on February 9, 1971, had a magnitude $M_w$ of 6.6, and an epicentre about 11 km from the dam. Peak ground accelerations at the site were estimated to be about 0.55 to 0.6g (Seed et al., 1973). Additional description of the earthquake including a representative time history is provided in Section 5.2.1.

The response of the lower dam was dominated by the near catastrophic slide of the upstream face and crest. The 11 m of freeboard prior to the earthquake were reduced to a fragile 1.5 m. The slide was about 550 m in length and involved much of the upstream face and crest. The slide mass extended 45 to 60 m into the reservoir. 80,000 people that were downstream of the dam were immediately ordered to evacuate (Gates, 1972). Photographs of the damage are shown in Figure 5-55.

An extensive field investigation program including trenching, boring, and sampling was performed after the earthquake. Liquefaction near the base of the upstream shell was identified as being responsible for the slide (Seed et al., 1973). Large blocks of intact fill were transported by the liquefied soil. An evaluation of the crest seismoscope record indicates the slide began about 20 to 30 seconds after the end of earthquake shaking (Seed et al., 1988). Seed concludes that the slide may have moved rather gradually, requiring about 40 seconds for displacement with an average velocity of about 1.5 m/s.

Seed et al. (1973) provided several cross-sections through the deformed dam. Based on these sections, the crest appears to have moved downward as much as 20 m or so. Upstream movement of the parapet wall appears to have been nearly as great.
Figure 5-55. Lower San Fernando dam after 1971 earthquake.
(Photographs from Steinbrugge Collection)
5.3.2 \( N_{1-60} \) characterization

The \( N_{1-60} \) characterization adopted in this study was based on the work of Seed et al. (1988). It considers numerous SPT tests performed in the downstream shell in 1971 and 1985. Both the median and \( N_{1/3} \) characterizations were developed as discussed in Section 5.2.3 for the upper dam. The blowcounts include a correction of \( \Delta N = -2 \) to account for postearthquake consolidation as developed by Seed et al. (1988). The various blowcount estimates for the upstream shell are shown in Table 5-9. These estimates have been reduced by \( \Delta N = -1 \) from the values determined for the downstream shell to approximately account for the difference in confining stresses during consolidation (Seed et al., 1988). This difference is due to the presence of the reservoir and the later construction of the downstream berm. The median and \( N_{1/3} \) values in Table 5-9 are the averages of separate evaluations made from the 1971 and 1985 data.

As with the upper dam, \( N_{1/3} \) values are assumed for the base analysis. The scatter in SPT values is shown in Figure 5-56. For comparison, Seed et al. (1973) found the average relative density of the shells after the earthquake to be about 50% to 54%. Seed et al. (1988)

<table>
<thead>
<tr>
<th>Distance below crest * (m)</th>
<th>Thickness (m)</th>
<th>( N_{1-60} ) (median)</th>
<th>( N_{1/3} )</th>
<th>( N_{1-60}^{**} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.6 – 27.1</td>
<td>5.5</td>
<td>15</td>
<td>12</td>
<td>16</td>
</tr>
<tr>
<td>27.1 – 32.3</td>
<td>5.2</td>
<td>11</td>
<td>9</td>
<td>11.5</td>
</tr>
<tr>
<td>32.3 – 36.9</td>
<td>4.6</td>
<td>21</td>
<td>17</td>
<td>21</td>
</tr>
<tr>
<td>36.9 – 44.2</td>
<td>7.3</td>
<td>11</td>
<td>9</td>
<td>11.5</td>
</tr>
</tbody>
</table>

Notes: * Crest at elevation 348.98 m (1145 ft). ** From Seed et al. (1988).
estimates that consolidation due to the earthquake may have increased the relative density by an increment of about 4% (i.e., from 46% to 50% up to 50% to 54%).

5.3.3 Static analysis description

The material zoning, geometry, and phreatic surface were based on the representative section developed by Seed et al. (1973) and shown in Figure 5-54. The finite difference grid and $N_{1/3}$ distribution is shown in Figure 5-57.

Static analyses were performed using a hyperbolic model as discussed in Section 4.2.1. The embankment was constructed in layers and the reservoir added in stages. The material
properties were based on the testing and data evaluation performed during the 1973 study. A simplified relation for bulk modulus was used (Byrne et al., 1987). The properties selected for the static analysis are listed in Table 5-10.

5.3.4 Dynamic analysis description

The properties for the base dynamic analysis are given in Table 5-11. Liquefaction was permitted throughout the saturated hydraulic fill sand zone but not in the other embankment zones or the foundation. Liquefaction was also prohibited in the triangular elements along the upstream face. The input motion was the modified Pacoima dam record as shown in Figure 5-30. The record was scaled to a pga of 0.55 g as assumed by Seed and Harder (1990).

\[ K_{2max} \] values in the hydraulic fill sand and alluvium were estimated from cross-hole seismic surveys by Seed et al. (1973). A simple relation was used to estimate \( G_{max} \) for the clay core: \( G_{max} = 2300 \times S_u \) (Seed et al., 1973). MRF and damping values were developed from a FLAC-SHAKE calibration similar to the one discussed in Section 5.2.5. Three SHAKE columns were evaluated, one column each through the upstream shell, crest, and downstream shell. A simpler distribution of MRF and damping was selected for the lower dam than for the upper dam: uniform values were assigned to each zone. The center frequency \( f_c \) associated with the damping values was taken as the fundamental frequency of the dam \( f_i \) determined with the preliquefaction stiffness properties. The “within” motion

![Figure 5-57. Finite difference grid and N_{1/3} distribution of liquefiable zones, LSFD.](image)
Table 5-10. Material properties for static analysis, LSFD

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>HF Sand</th>
<th>HF Clay Core</th>
<th>HF Ground Shale</th>
<th>Rolled Fill</th>
<th>Rolled Berm</th>
<th>Alluvium</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Moist</td>
<td>Saturated</td>
<td>HF Ground Shale</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Unit weight:</strong></td>
<td></td>
<td>kN/m$^3$</td>
<td>kN/m$^3$</td>
<td></td>
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</tr>
<tr>
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<td>19.8</td>
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<tr>
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<td>22.0</td>
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<td>20.4</td>
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<tr>
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<tr>
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<tr>
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<td>0.98</td>
<td>0.82</td>
<td></td>
<td></td>
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<tr>
<td>$\phi'$</td>
<td>$^\circ$</td>
<td>37$^\circ$</td>
<td>25$^\circ$</td>
<td>38$^\circ$</td>
<td></td>
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<td></td>
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<tr>
<td>$K_e$</td>
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<td>200</td>
<td>300</td>
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</tr>
<tr>
<td>$n$</td>
<td></td>
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<td>0.76</td>
<td>0.41</td>
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<td></td>
</tr>
<tr>
<td>$K_b$</td>
<td></td>
<td>270</td>
<td>120</td>
<td>180</td>
<td></td>
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</tr>
<tr>
<td>$m$</td>
<td></td>
<td>0.27</td>
<td>0.38</td>
<td>0.2</td>
<td></td>
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</table>

computed by the SHAKE analysis at the bottom of the upstream shell column was used as the base motion for the FLAC analyses.

The undrained strength $S_u$ for the hydraulic fill and alluvium zones was assumed equal to the drained strength following the assumptions of Section 4.3.4. The undrained strength of the rolled fill was assumed equal to $0.6\sigma_{vo}$. The strength of the clay core, $S_u = 0.3\sigma_{vo}$, was estimated from a large number of small torvane tests (Seed et al., 1973).

The weighting curve was derived from cyclic triaxial testing performed on isotropically consolidated tube samples of the hydraulic fill (Seed et al., 1973). Additional testing was performed on 9 tube samples and 2 carved samples from the lower dam in 1985 (Seed et al., 1988). The CRR$_{15}$ from these later tests might be 10% less than the earlier results, but overall


Table 5-11. Parameters for base dynamic analysis, LSFD

<table>
<thead>
<tr>
<th>Parameter</th>
<th>HF Sand</th>
<th>HF Clay Core</th>
<th>HF Ground Shale</th>
<th>Rolled Fill/Berm</th>
<th>Alluvium Upper/Lower</th>
</tr>
</thead>
</table>

**Material parameters:**

- $K_{2max}$
  - HF: 43
  - Clay: —
  - Ground: 52
  - Shale: 55/48
  - Rolled: 52/105
- MRF
  - HF: 0.14
  - Clay: 0.66
  - Ground: 0.26
  - Shale: 0.26/0.14
  - Rolled: 0.23
- $\beta_c$
  - HF: 18%
  - Clay: 10%
  - Ground: 14%
  - Shale: 14%
  - Rolled: 15%
- $f_c$
  - HF: 0.83 Hz
  - Clay: 0.83 Hz
  - Ground: 0.83 Hz
  - Shale: 0.83 Hz
  - Rolled: 0.83 Hz

**Damping type**
- HF: RS
- Clay: RS
- Ground: RS
- Shale: RS
- Rolled: RS

**$B'$ / $G_{dyn}$ (saturated)**
- HF: 10
- Clay: 10
- Ground: 10
- Shale: 10
- Rolled: 10

**$S_u$**
- HF: 0.6$\sigma'_{vo}$
- Clay: 0.3$\sigma'_{vo}$
- Ground: 0.6$\sigma'_{vo}$
- Shale: 0.6$\sigma'_{vo}$
- Rolled: 0.6$\sigma'_{vo}$

**Triggering parameters:**

- $CRR_1 / CRR_{15}$
  - HF: 2.1
- $CRR_{15}$ versus $N_{1-60cs}$
  - HF: Figure 2-20
- Strain-based trigger
  - HF: 4%
- $K_{\sigma}$
  - HF: Figure 4-3a
- $K_a$
  - HF: 1.0

**Postliquefaction parameters:**

- $S_r / \sigma'_{vo}$
  - HF: 0.17
  - Clay: —
  - Ground: —
  - Shale: —
  - Rolled: —
  - Alluvium: —

- $\gamma_r$
  - HF: 4%
  - Clay: —
  - Ground: —
  - Shale: —
  - Rolled: —

- $G_{lq(unload)} / G_{lq(load)}$
  - HF: 10
  - Clay: —
  - Ground: —
  - Shale: —
  - Rolled: —

- $\beta_c$ (liq. zones)
  - HF: 2%
  - Clay: —
  - Ground: —
  - Shale: —
  - Rolled: —

- Postliq. model
  - HF: Symmetric
  - Clay: —
  - Ground: —
  - Shale: —
  - Rolled: —
  - Alluvium: —

The two test programs were in good agreement. The weighting curve derived from these tests has a $CRR_1 / CRR_{15}$ of 2.1.

The $CRR_{15}$ used in the base analysis is estimated from the MCEER chart in Figure 2-20. Using the $N_{1/3}$ blowcounts from Table 5-9 ($N_{1/3} = 12, 9, 17, 9$) and a fines content of 25%, Figure 2-20 gives $CRR_{15}$ estimates of 0.19, 0.15, 0.26, and 0.15. These values can be compared with the cyclic triaxial tests of hydraulic fill samples that were
isotropically consolidated to 1 kg/cm² (98.1 kPa). The test results, after multiplying by 0.6 to approximate simple shear conditions, give a CRR₁₅ of about 0.14. This includes an approximate reduction of 4% to account for densification of the sand due to the earthquake (Seed et al., 1988). The MCEER estimate for the N₁/₃ = 9 zones agrees well with the test results. It is not entirely clear, but the test results may reflect something closer to an average strength of the hydraulic fill. If this is the case, the MCEER chart appears to overpredict the average cyclic strength even though a low estimate of blowcount was used.

The residual strength for this dam has been estimated by both limit equilibrium analyses and laboratory testing (Seed et al., 1988). Limit equilibrium analyses at the start of sliding suggest the strength of liquefied material at the initiation of failure may have been as high as 38 kPa. The initial strength may have been even higher since there was apparently little movement during the earthquake. The extent and mobilized strength of the nonliquefied toe region was identified as a significant uncertainty in this strength estimate (Seed et al., 1988). The precise size and distribution of the liquefied zones prior to sliding is an additional unknown.

Analyses of the final displaced geometry suggest residual strengths of only 14 to 24 kPa. Uncertainties in this estimate include the inertia effects associated with the velocity of the failing mass. The upper end of the range also includes a large but uncertain 70% reduction in strength for the liquefied soil that displaced into the reservoir.

These strengths were converted to strength ratios by estimating the average σ'₀ of the liquefied zones. An approximate strength ratio of up to 0.16 was estimated for the start of failure and 0.06 to 0.10 for the final geometry. These final strength ratios appear to be quite low, especially when compared to the estimated values from the upper dam (0.11 to 0.17). Since these strength ratios are based on the final geometry of the failed upstream shell, the
mixing of clean and silty fill materials during the large strains may be one cause of these low strength values. The hydraulic fill is highly stratified and mixing with an associated loss of strength is anticipated. Trenches and borings through the upstream shell after the earthquake found that soil in liquefied zones showed considerable disturbance (Seed et al., 1973). Pore pressure migration may have also affected the final strength values.

A summary of undrained triaxial compression tests from selected hydraulic fill zones was presented by Seed et al. (1988). An attempt was made to test samples representative of the looser hydraulic fill material. Eleven tests were performed on sandy silt samples (FC = 43% to 85%) and five tests on silty sand samples (FC = 4% to 22%). Seven of the tests were performed on undisturbed carved samples from an exploration shaft and the remainder on tube samples from boreholes. Seed et al. (1988) concluded there was little difference in results between the carved and tube samples.

The shear strengths were corrected for the effects of sample disturbance and earthquake consolidation to give the expected undrained strength at the time of the earthquake. The corrections were made using relationships between steady-state strength and void ratio developed from reconstituted samples. These corrections for changes in void ratio were very significant, sometimes requiring the strength from the laboratory test to be reduced by an order of magnitude.

The average corrected strengths, after neglecting two tests with high strength values, are about 41.8 kPa for the silt samples (FC > 43%) and 65.7 kPa for the sand samples (FC < 22%). The strengths reported by Seed et al. (1988) may be interpreted as quasi-steady state values and converted to approximate strength ratios. This was done for this study and the results are shown in Figure 5-58. The average $S_{qss}/\sigma'_{vo}$ for the silt samples was found to be 0.12 and for the sand samples was 0.21. These appear to be low values for initial
undrained strength from compressive tests, which are generally dilative with little or no strain softening as discussed in Section 4.3.4. There is also a large scatter in results as shown in Figure 5-58. A similar scatter was found for the $S_{qss,ss}$ results.

The stress-strain response for these laboratory tests was not provided in the summary report by Seed et al. (1988), which makes the interpretation of the reported strengths more difficult. There is also a good deal of uncertainty in these estimates due to the large correction for changes in void ratio. The use of a compressive orientation rather than simple shear direction for loading also decreases the usefulness of these strengths to the deformation problem.

![Figure 5-58](image)

Figure 5-58. Estimates of $S_{qss,ss}/\sigma'_{vo}$ from triaxial compression tests, LSFD.
Estimates of mobilized residual strength are summarized in Table 5-12. The strengths from the limit equilibrium analyses are assumed to represent the critical hydraulic fill zone at the base of the embankment, although they may actually reflect a more average response. Also shown for comparison are strengths estimated from the Idriss curve on Figure 2-32 and the modified Idriss curve of Figure 2-34.

As the initial response of the liquefied material may be considered essentially undrained, and the duration of the earthquake was fairly short, it is reasonable to use a strength ratio for the initial strength of the liquefied zone. Based on Table 5-12, the strength ratios estimated from the modified Idriss curve on Figure 2-34 are reasonable choices for the base analysis. The value in the critical zone is somewhat lower than the 0.17 ratio used for the upper dam, although one might expect slightly greater strengths at the lower dam due to its higher $N_{1/3}$ blowcount.

### Table 5-12. Summary of residual strength estimates for upstream shell, LSFD

| $S_r/\sigma_v'$ estimates: | \(
| N_{1/3} | \text{Median} | \text{Limit Equilibrium} | \text{Laboratory} | Figure 2-32 | \text{Figure 2-34} |
|---|---|---|---|---|
| Start of Sliding | End of Sliding | Laboratory | $N_{1/3}$ | Median |
| 12 | 15 | ? | ? | ? | 0.24 | 0.38 |
| 9 | 11 | ? | ? | ? | 0.14 | 0.20 |
| 17 | 21 | ? | ? | ? | 0.52 | 0.6 |
| 9 | 11 | < 0.16 | 0.06 – 0.10 | 0.12 – 0.21 | 0.14 | 0.20 |

| $S_r$ estimates (kPa): | \(
| N_{1/3} | \text{Median} | \text{Limit Equilibrium} | \text{Laboratory} | Figure 2-32 | \text{Figure 2-34} |
|---|---|---|---|---|
| Start of Sliding | End of Sliding | Laboratory | $N_{1/3}$ | Median |
| 12 | 15 | ? | ? | ? | 22.5 | 36.3 |
| 17 | 21 | ? | ? | ? | 50.0 | 57.4 |
| 9 | 11 | < 38 | 14 – 24 | 42 – 66 | 13.9 | 19.2 |
5.3.5 Analysis results

Two features distinguish the actual response of the Lower San Fernando dam: (1) limited displacements during and immediately after the earthquake, and (2) a near catastrophic failure of the upstream slope following the earthquake. A useful analysis should predict the relatively small displacements at the end of shaking while still giving a strong indication of the potential failure that did occur.

Figure 5-59 shows predictions of liquefaction within the hydraulic fill. Widespread triggering is predicted by the synthesized approach in the critical zone at the base of the embankment. This is the zone in the upstream shell that is believed to be responsible for the slide (Seed et al., 1973). The predicted zones of liquefaction agree well with an evaluation performed by Seed and Harder (1990) using the equivalent linear program FLUSH. The synthesized approach predicts somewhat less liquefaction in the upstream shell and somewhat more in the downstream shell. The results are quite close given the differences in the analyses: the inclusion of base isolation effects in the synthesized approach, the use of a $K_a$ factor in the FLUSH evaluation, and the more detailed prediction of MRF and damping in the FLUSH analysis.

Both analyses may have overestimated the liquefaction response of the downstream shell. There was apparently little field evidence for significant liquefaction in this zone except for the upper hydraulic fill near the core (Seed et al., 1988). The original Seed et al. (1973) analysis predicts little liquefaction at the base of the downstream shell as shown in Figure 5-59. But this analysis appears to include a significant $K_a$ correction and does not consider the apparent variation in density within the hydraulic fill zone.
Although a liquefaction criterion based on shear strain was included in the synthesized approach, it resulted in the liquefaction of only 5 elements. Four of these elements were at the upstream toe.

Figure 5-60 shows displacement and strain estimates from the base analysis. The dam is predicted to be stable at the end of the earthquake with significant but limited

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**Figure 5-59. Comparison of liquefied zones, LSFD**

- a) Liquefied zones for base analysis using synthesized approach.
- b) Liquefied zones from analyses of Seed and Harder (1990).
- c) Liquefied zones from analyses of Seed et al. (1973).
deformations. The crest is estimated to move about 0.8 m upstream and settle about 1.6 m. Maximum displacements are 4.0 m. The top of the downstream berm moves 0.7 m laterally. This is generally consistent with expectations. Figure 5-60c suggests a relatively well-defined pattern of strain. This figure indicates the primary shear zone would intersect the downstream face at roughly the reservoir elevation. This is reasonably consistent with the observed geometry of the slide.

Figure 5-60. Displacement and strain predictions for base analysis, LSFD.
5.3.6 Sensitivity studies

A limited number of parametric analyses were performed to determine the effect of key variables and to discover if other reasonable combinations of input parameters would also produce realistic results. Selected results from these studies are summarized in Table 5-13. While the base analysis predicts significant deformation, and the magnitude of the

<table>
<thead>
<tr>
<th>Analysis name</th>
<th>Time (sec)</th>
<th>Status</th>
<th>Max</th>
<th>Midheight U/S Face</th>
<th>Crest Centerline</th>
<th>Top of D/S Berm</th>
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<tr>
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<td>4.0</td>
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<td>0.0</td>
<td>0.8</td>
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<td>1 LSFTBsrn</td>
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<td>S</td>
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<td>1.2</td>
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<td>0.3</td>
</tr>
<tr>
<td>2 LSFTBsi3</td>
<td>13.6</td>
<td>OD</td>
<td>34</td>
<td>24</td>
<td>0.6</td>
<td>10</td>
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<td>3 LSFTBsim</td>
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<td>O</td>
<td>22</td>
<td>16</td>
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<tr>
<td></td>
<td>24.0</td>
<td>M</td>
<td>30</td>
<td>18</td>
<td>0.4</td>
<td>9.0</td>
</tr>
<tr>
<td>4 LSFTBt</td>
<td>16.1</td>
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<td>2.5</td>
<td>1.7</td>
<td>0.0</td>
<td>0.5</td>
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<tr>
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<td>16.1</td>
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<td>1.2</td>
<td>1.1</td>
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<td>0.4</td>
</tr>
<tr>
<td>6 LSFTBus3</td>
<td>16.1</td>
<td>O</td>
<td>19</td>
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<td>8.0</td>
</tr>
<tr>
<td></td>
<td>24.0</td>
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<td>24</td>
<td>16</td>
<td>0.0</td>
<td>8.8</td>
</tr>
<tr>
<td>7 LSFTBusm</td>
<td>16.1</td>
<td>O</td>
<td>5.4</td>
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<td>3.2</td>
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<tr>
<td></td>
<td>24.0</td>
<td>M</td>
<td>6.3</td>
<td>6.3</td>
<td>0.0</td>
<td>3.7</td>
</tr>
<tr>
<td>8 LSFTB-</td>
<td>20.0</td>
<td>S-M</td>
<td>8.6</td>
<td>4.6</td>
<td>0.2</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Status legend:

$S$ Response appears generally stable with little or no plastic response.
$M$ Marginal stability or localized instability. May include significant plastic response.
$O$ Significant ongoing displacements are occurring.
$D$ Finite difference mesh is too distorted to continue analysis.
predictions should not give comfort to any analyst, it did not necessarily indicate the likelihood of a catastrophic failure. A limited set of sensitivity studies should make such a possibility clear.

**Effect of residual strength ratio**

The base analysis estimates the residual strength ratio based on an $N_{1/3}$ characterization of the dam. The importance of blowcount characterization is evaluated in the following analysis:

1. LSFTBsrn assumes an $S_r/\sigma'_{vo}$ based on the median $N_{1-60}$ and the modified Idriss curve of Figure 2-34. These strength ratios are shown in Table 5-12. The triggering resistance is still based on the $N_{1/3}$ values. This increase in strength caused a reduction in predicted displacements of about 40% to 50%. The greater strength also led to an increase in the number of liquefied elements in the upstream shell from 33 to 43.

2. LSFTBs$\text{si3}$ uses an isotropic $S_r$ based on the $N_{1/3}$ blowcount and the Idriss curve of Figure 2-32.

3. LSFTB$\text{sim}$ is similar to LSFTB$\text{si3}$ except the $S_r$ is based on the median $N_{1-60}$.

Both analyses resulted in a major slump or flow slide of the upstream face and crest as indicated in Table 5-13. This is not surprising since the chosen residual strengths are derived from case history observations after large displacements have been observed. These strengths may include effects of pore pressure redistribution or mixing that are not included in laboratory-based estimates.
LSFTBsim appears to be reaching a stable geometry at displacements much less than observed. Several factors may contribute to the low displacement predictions. The residual strengths based on median $N_{1/60}$ and the Idriss curve may be somewhat high. The solution is really one of marginal stability and the mesh may be too coarse to provide accurate predictions of this condition (Zettler et al., 1999). Many of the elements are also highly distorted and may no longer give accurate predictions. Regardless, both LSFTBsim and LSFTBsi3 indicate considerable deformations and distress.

While LSFTBsim gave displacement estimates that were somewhat low, the final displacements for LSFTBsi3 could not be determined. Minor and periodic remeshing was required to allow the analysis for LSFTBsi3 to proceed to 13.6 seconds. At this point the mesh was very distorted and the analysis was halted. While the results are useful and indicate the large displacements that did occur, the analyses demonstrate the difficulty in accurately predicting very large displacements. This has implications for some remediation schemes that rely upon increasing the available freeboard to achieve safety criteria. Determining an appropriate freeboard requirement could be troublesome if large deformations are predicted.

The displaced shape for LSFTBsi3 is shown in Figure 5-61 together with the predictions for the base analysis. Although the displacements for LSFTBsi3 are not final values, the general pattern agrees well with the observed deformations. While it appears that a residual strength ratio produces a better prediction of the initial response of the dam, the empirically derived strength values correctly indicate the potential of a massive slide.

**Effect of triggering resistance**

The laboratory test data indicates a lower average $CRR_{15}$ than was developed from the $N_{1/3}$ blowcount and the MCEER chart (Figure 2-20). This discrepancy was evaluated in the following analysis:
c) Displaced shape at 24 seconds for LSFTBus3.

Figure 5-61. Predictions of displaced shape, LSFD.
4. LSFTBt assumes a uniform $CRR_{15}$ of 0.14 before the $K_o$ correction. The reduced $CRR_{15}$ led to an almost complete liquefaction of the downstream shell hydraulic fill and increased the number of liquefied elements in the upstream shell from 33 to 39. Surprisingly, this led to a modest reduction in displacements of about 30% to 40% as shown in Table 5-13. Key elements in the upstream shell trigger somewhat earlier for the reduced $CRR_{15}$ and may have impacted the estimated response. $CRR_{15}$ may be a good parameter to vary in routine sensitivity studies.

**Effect of dilatant zone adjacent to slopes**

The importance of stronger material along the upstream and downstream faces that is more resistant to liquefaction was investigated by three analyses. It has been suggested that such a zone was responsible for the initial stability of the upstream shell at the end of the earthquake (Seed et al., 1988). The width and existence of this zone is uncertain. An assumed location was defined as shown in Figure 5-62. The bottom width of this zone was derived from the assumptions used by Seed et al. in performing their limit equilibrium analyses. An $N_{1.60}$ blowcount of 25 was arbitrarily assigned to these zones.

Three analyses evaluate the effect of these stronger zones in conjunction with reasonable assumptions for residual strength. Liquefaction is permitted in the strengthened areas when the maximum shear strain of an element exceeds 4%. Due to the assumed high

![Figure 5-62. Assumed location of dilatant zones, LSFD.](image-url)
blowcount, the residual strength assigned to many of these elements is limited to the drained strength.

5. LSFTBur3 assumes the strengthened zones as discussed above. Triggering resistance and residual strength ratios are based on the assumed blowcount and Figure 2-34. An $S_r/\sigma_{w0}$ of 0.6 is assigned to any dilatant zone that liquefies. The presence of the dilatant zone causes a reduction in displacements of 50% or more as shown in Table 5-13.

6. LSFTBus3 is similar to LSFTBur3, except an isotropic residual strength is assigned to liquefied zones based on the $N_{1/3}$ blowcount and the Idriss curve from Figure 2-32. The $S_r$ in the dilatant zones is limited to the minimum of the drained strength or 57.4 kPa. Despite the dilatant zone, the analysis predicts significant sliding of the upstream face and crest. The final deformed shape, which is marginally stable, is shown in Figure 5-61c.

7. LSFTBusm is similar to LSFTBus3, except the median $N_{1-60}$ blowcount is used to develop $S_r$. Although significant sliding of the upstream face and crest is predicted, the final displacements are only about 30% to 50% of the LSFTBus3 estimates. The upstream dilatant zone also caused a large reduction in predicted displacements as compared to LSFTBsim.

While LSFTBus3 and LSFTBusm both predict significant displacements, they may not strongly indicate the large slide that did occur. Several factors may have contributed to these low displacement predictions. The assumed size and character of the dilatant zones may be inappropriate. The drained strengths assumed in these zones after liquefaction may be too high for simple shear loading. Pore pressure inflow from the interior of the embankment may have also degraded the strength in these zones.
A final consideration is the possibility that residual strengths from the Idriss curve are somewhat high. The residual strength of the zone near the dam base using $N_{1/3}$ agrees well with the lower range of the Seed et al. (1988) back analysis as shown in Table 5-12. However, the average $S_r$ through the upstream shell hydraulic fill is closer to the upper end of the range from back analysis. The average $S_r$ using the median $N_{1.60}$ is well above the range for end of sliding conditions. Since Seed et al. assumed only one residual strength for the entire upstream shell, even the $N_{1/3}$ values are probably too high.

A particular concern about assigning individual residual strengths to the 4 hydraulic fill layers is the strength assigned to the densest layer. For the $N_{1/3}$ case, this layer was assigned a residual strength of 50 kPa based on its clean sand blowcount of 19. However, this layer is only 4.6 m thick and lies between the two loosest hydraulic fill layers. It seems unlikely this entire layer could maintain a high residual strength due to the tendency of pore pressure inflow from below and above. A substantial reduction of strength in this zone would likely improve the predictions.

**Effect of earthquake direction**

As there is some uncertainty with respect to the direction of earthquake loading, in particular the near-field velocity pulse, the analysis was also performed with the velocity pulse oriented in the upstream direction.

8. LSFTB- applies the earthquake motion in a negative direction. This essentially doubled the displacement predictions of the upstream shell and crest as shown in Table 5-13, although the dam is still stable at the end of the analysis.
5.3.7 Summary

The earthquake record and residual strength are again seen to be critical parameters in the estimation of displacements. The analyses suggest it may be sensible to evaluate modest changes in triggering resistance during routine parametric studies.

Perhaps the most interesting conclusion is that using a residual strength ratio $S_r/\sigma'_{w0}$ for the initial strength of the liquefied elements correctly predicts the moderate but limited displacements that are believed to have existed at the end of the earthquake. Although using an isotropic $S_r$ in these elements overpredicts the response during the earthquake, it correctly indicates the potential for a large slump of the upstream shell and crest.

The predictions that consider a denser zone near the edges of the hydraulic fill did not conclusively indicate the potential for a near catastrophic slump. While the character of this zone is very uncertain, its inclusion in the analysis may have led to inappropriate conclusions regarding the stability of the dam. Using a lower bound estimate for residual strength, as originally recommended by Seed and Harder (1990), may have improved these estimates. The analyses also suggest that the residual strength assigned to a denser layer sandwiched between two loose layers should not be based solely on its penetration resistance.

The analyses demonstrate the difficulty in accurately predicting very large displacements. This may be important in situations where very large but limited deformations are tolerable.

Overall, it appears the synthesized approach would have been a reasonable method for evaluating the response of the dam prior to the earthquake using a current understanding of input parameters.
5.4 Elsie Lake Main Dam

Elsie Lake Main dam (ELMD) is a zoned earthfill embankment constructed in 1957-1958. It is located in central Vancouver Island about 30 km from Port Alberni. The dam has a maximum height of 31 m, a crest length of 190 m, and original slopes of 1.9:1 upstream and 2.1:1 downstream. The analyzed section includes a downstream rockfill berm with a slope of 2.5:1. A representative cross section is shown in Figure 5-63. The shells are sandy gravels and the core is a broadly graded glacial till. Although the dam was built using modern compaction techniques, the degree of compaction achieved in the upper parts of the dam was apparently variable.

A series of analyses were performed to estimate the seismically induced deformations related to a layer of medium density in the upstream shell. The analyzed cross section was one of several proposed remediation schemes. The characterization of the dam was based on information supplied by BC Hydro and their consultants (Byrne & Beaty, 2001).

Although this dam is not a case history, and its response to the design earthquake is not known, analysis predictions will be checked for reasonableness by comparing them to behaviour observed at other dams. These analyses also provide an additional set of sensitivity studies with clear implications on the importance of earthquake loading to displacement predictions.

5.4.1 Static analysis description

The characterization of the dam was based primarily on SPT and Becker Hammer values along with limited gradation tests. A detailed blowcount distribution was developed for the dam by consultants to BC Hydro. The distribution consisted of 32 different zones. $N_{1,60}$ values assigned to these zones varied from 10 to 50. Much of the dam appears to be dense with representative blowcounts greater than 30.
The finite difference mesh used in the analysis is shown in Figure 5-64. The location of looser zones with $N_{1-60} \leq 30$ is also shown. These looser zones make up a small portion of the dam, primarily in the core material near the crest, in a limited band in the upper half of the upstream shell, and in the bedding layer for the upstream riprap. The layers with blowcounts of 20 to 30 within the upstream shell are the focus of these analyses.

The static analysis was performed using a simple stress-dependent linear elastic plastic model as discussed in Section 4.2.1. The embankment was constructed in layers and the reservoir load added in stages. Material properties were supplied directly by BC Hydro and...

Figure 5-63. Representative cross section for ELMD.

Figure 5-64. Finite difference mesh with $N_{1-60}$ for ELMD.
their consultants or estimated from the N_{1-60} distribution, empirical relations, and information in the literature. The properties selected for the static analysis are listed in Table 5-14.

$G_{\text{max}}$ for the non-berm portion of the dam was estimated from the empirical formulas described in Section 4.3.1 and repeated below. The stiffness of the gravel shells was further increased by a factor of 1.5 as suggested by the work of Seed et al. (1986). $G_{\text{max}}$ for the rockfill berm was estimated from geophysical and laboratory data presented by Yasuda and Matsumoto (1994) for rockfill at Miho and Shichigashuku dams. The equivalent elastic shear modulus for static loading $G_{\text{st}}$ was estimated as shown in Table 5-14.

$$G_{\text{max}} \approx 22 \times K_{2\text{max}} \times P_a \times \left( \frac{\sigma_m'}{P_a} \right)^{1/2}$$  
Eqn. 4.6

$$K_{2\text{max}} \approx 20 \times (N_{1-60})^{1/3}$$  
Eqn. 4.7

<table>
<thead>
<tr>
<th>Table 5-14. Material properties for static analysis, ELMD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parameter</strong></td>
</tr>
<tr>
<td><strong>Unit weight:</strong></td>
</tr>
<tr>
<td>Moist kN/m$^3$</td>
</tr>
<tr>
<td>Saturated kN/m$^3$</td>
</tr>
<tr>
<td><strong>Strength:</strong></td>
</tr>
<tr>
<td>Cohesion kPa</td>
</tr>
<tr>
<td>$\phi'$ (avg.) °</td>
</tr>
<tr>
<td>$\phi'$ (range) °</td>
</tr>
<tr>
<td><strong>Stiffness:</strong></td>
</tr>
<tr>
<td>$K_{2\text{max}}$ —</td>
</tr>
<tr>
<td>$G_{\text{max}}$ kPa</td>
</tr>
<tr>
<td>$G_{\text{st}}$ kPa</td>
</tr>
<tr>
<td>$B$ kPa</td>
</tr>
</tbody>
</table>
The assigned friction angle was a function of both confining stress and density. An attempt was made to follow the trends observed in empirical relationships, such as shown in Section 4.2.2, while maintaining conservative estimates. The range in assigned $\phi'$ is shown in Table 5-14.

The shells have a representative fines content of 10% and the core a fines content of 35%. Only the blowcounts in the core were adjusted for fines content.

5.4.2 Dynamic analysis description

Properties for the base dynamic analysis are given in Table 5-15. $G_{\text{max}}$ values were estimated as discussed in Section 5.4.1. MRF and damping values were developed from a FLAC-SHAKE calibration similar to the one discussed in Section 5.3.4. Two SHAKE columns were evaluated to consider both the upstream and downstream shells. An MRF of 0.20 and viscous damping of 11% was used in the interior of the dam for the base analysis. The MRF was increased to 0.65 at the surface and the damping was decreased to 3% in accordance with the SHAKE analyses. The center frequency $f_c$ was based on the fundamental frequency of the dam, $f_1 = 1.85$ Hz. The SHAKE analyses showed significant second mode response in the upper half of the dam. An $f_c$ of 3.7 Hz was used in this region.

The undrained strength $S_u$ was assumed equal to the drained strength following the assumptions of Section 4.3.4. Although deriving $S_u$ from $\sigma'_v$ and $\sin \phi'$ is considered reasonable for denser dilative soils, it does include the possibility of negative pore pressures due to dilation. These pore pressure may be required to achieve the assumed $S_u$ due to the low initial lateral stresses in much of this steep structure.

Although very large undrained strengths may be temporarily achieved in dense saturated material due to high negative pore pressures, these strengths are not reliable and are generally not considered in displacement predictions. Somewhat low values of friction angle
Table 5-15. Parameters for base dynamic analysis, ELMD

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Shell</th>
<th>Transition</th>
<th>Bedding</th>
<th>Core</th>
<th>Berm</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Material parameters:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MRF</td>
<td>≥ 0.20</td>
<td>≥ 0.20</td>
<td>≥ 0.20</td>
<td>≥ 0.20</td>
<td>≥ 0.20</td>
</tr>
<tr>
<td>$\beta_c$</td>
<td>≤ 11%</td>
<td>≤ 11%</td>
<td>≤ 11%</td>
<td>≤ 11%</td>
<td>≤ 11%</td>
</tr>
<tr>
<td>$f_c$ (Hz)</td>
<td>1.85; 3.7</td>
<td>1.85; 3.7</td>
<td>1.85; 3.7</td>
<td>1.85; 3.7</td>
<td>1.85; 3.7</td>
</tr>
<tr>
<td>Damping type</td>
<td>RS</td>
<td>RS</td>
<td>RS</td>
<td>RS</td>
<td>RS</td>
</tr>
<tr>
<td>$B^e / G_{dy}n$ (saturated)</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$S_u / \sigma_{vo}$</td>
<td>0.58-0.70</td>
<td>0.61-0.66</td>
<td>0.56-0.61</td>
<td>0.60-0.63</td>
<td>&gt; 0.64</td>
</tr>
<tr>
<td><strong>Triggering parameters:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$CRR_1 / CRR_{15}$</td>
<td>2.5</td>
<td>—</td>
<td>2.5</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$CRR_{15}$ versus $N_{1-60cs}$</td>
<td>Figure 2-20</td>
<td>—</td>
<td>Figure 2-20</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Strain-based trigger</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$K_\sigma$</td>
<td>Figure 4-3a</td>
<td>—</td>
<td>Figure 4-3a</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$K_\alpha$</td>
<td>1.0</td>
<td>—</td>
<td>1.0</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td><strong>Postliquefaction parameters:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_r$ (kPa)</td>
<td>≤ 57.4</td>
<td>—</td>
<td>≤ 11.8</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$\gamma_r$</td>
<td>2%</td>
<td>—</td>
<td>2% - 5%</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$G_{liq(unload)} / G_{liq(load)}$</td>
<td>10</td>
<td>—</td>
<td>10</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$\beta_c$ (liq. zones)</td>
<td>2%</td>
<td>—</td>
<td>2%</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Postliq. model</td>
<td>Bilinear</td>
<td>—</td>
<td>Bilinear</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

have already been assumed that reflect a guarded estimate for the amount of dilation that will occur. Actual friction angles may be substantially greater, particularly at low confining stresses. However, the very large strains that may occur along failure planes could cause these strengths to drop to a residual value.

Liquefaction was permitted in any zone with $N_{1-60cs} \leq 30$. Although the compacted core material was assumed to have an $N_{1-60}$ of only 25, this material was considered nonliquefiable after its blowcount was corrected for a fines content of 35%. This assessment introduces some uncertainty into the results. The fines content is based on only one sample.
A recent empirical study also suggests that sands with high fines contents may be more liquefiable than previously believed (Cetin et al, 2000).

The CRR$_{15}$ was estimated from the MCEER chart shown in Figure 2-20. The weighting curve for the base analysis was assumed to have a CRR$_1$/CRR$_{15}$ of 2.5. This value was based on an interpretation made by Idriss (1998) of cyclic tests on undisturbed samples of dense Niigata Station Sand.

Liquefied zones were assigned isotropic residual strengths based on the Idriss curve of Figure 2-32. The residual strength in many zones was reduced to the estimated undrained strength $S_u$. This reduction occurred in areas of low confining stress and was intended to limit the residual strength to a maximum of the drained strength. Although the residual strengths proposed by Idriss are considered state of practice, they are not lower bound. There is a potential for fabric effects (Section 2.2.5) or pore pressure inflow (Section 2.2.10) to result in lower strengths. Fabric effects may be a particular concern for this material since the lightly compacted condition may be similar to a moist tamped fabric.

A residual shear strain $\gamma_r$ of 2% was assigned to liquefied elements in the shell and the upper portion of the bedding. A $\gamma_r$ of 5% was used for the lower bedding with $N_{1.60} = 10$.

5.4.3 Seismic loading

The selection of earthquake time histories were governed by the following recommendations of BC Hydro:

1. Records should be from crustal earthquakes occurring in plate boundary regions.
2. Earthquakes should be either strike-slip or thrust events.
3. Earthquake magnitudes should be approximately in the range of M6.75 to M7.5.
4. Time histories should be recorded on rock.
5. The response spectrum shape should generally match the target acceleration spectrum as shown in Figure 5-65.

6. Scaling of time histories should be less than a factor of two in the period range of interest.

A large number of near field recordings from various earthquakes were screened for possible use. Four records were selected that generally meet the above guidelines. Additional information on the individual records and the methods used for spectral matching is given in Section 5.4.5.

The TCU078 E-W record from the 1999 Chi-Chi (Taiwan) earthquake was modified by SYNTH and used for the base analysis. SYNTH is a program for adjusting a record in the frequency domain to match a response spectrum (Naumoski, 1985). The acceleration response spectrum is shown in Figure 5-65 and the acceleration history in Figure 5-66.

The record was baseline corrected by applying a high pass filter at 0.08 to 0.062 Hz (T = 12.5 to 16 seconds). High frequencies were removed with a low pass filter at 8 to 10 Hz. It is the removal of high frequencies that drops the peak ground acceleration from the target of 0.55 g down to 0.39 g. This should not affect the predicted response of the dam as the fundamental period is about 0.55 seconds.

5.4.4 Analysis results

Although only a small fraction of the upstream shell is considered liquefiable, most of these zones are predicted to liquefy in the base analysis as shown in Figure 5-67. The layers with $N_{1-60}$ values of 20 and 25 are estimated to trigger within the first 8 seconds of the earthquake. Much of the denser overlying zone with $N_{1-60}$ of 30 is also predicted to liquefy, although this occurs much later in the earthquake. This delay may result from both the higher
relative density as well as attenuation of the earthquake motion due to the underlying liquefied layers.

Figure 5-65. Response spectra for earthquake record TCU078Esyn, ELMD.

Figure 5-66. Acceleration time history for earthquake record TCU078Esyn, ELMD.
Predicted deformations and strains are shown in Figure 5-68. Estimated crest displacements for the base analysis are approximately 1.2 m horizontal in an upstream direction and 0.6 m vertical. The maximum embankment displacement, excluding surface displacement along the upstream bedding/riprap and crest, is about 2.0 m. The deformations are governed by the upstream movement of a block that includes the crest and has the $N_{1,60} = 20$ layer in the upstream shell as its base. Displacements at the crest are greatly affected by strains within this looser layer.

Figure 5-67. Prediction of liquefied zones for base analysis, ELMD
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Figure 5-68. Displacement and strain predictions for base analysis, ELMD.
The stress-strain response near the center of the liquefied $N_{1-60} = 20$ layer is shown in Figure 5-69. While much of the strain occurs as plastic flow at the residual strength, the importance of strain accumulation due to ratcheting is also clear. The relative significance of plastic flow versus ratchet behaviour is dependent on the static shear stresses, dynamic loading, and the residual strength. The relatively high residual strength at ELMD increases the importance of ratcheting despite the significant static shear stresses. This curve demonstrates the usefulness of including ratchet-type behaviour in the postliquefaction model.

5.4.5 Sensitivity studies

A set of parametric studies were performed to develop a range of displacement predictions and to identify areas of greatest uncertainty. The primary focus of the analyses is the effect of the earthquake record on the predicted displacements. Selected results are presented below.

Figure 5-69. Stress-strain response of liquefied element, ELMD.
**Effect of earthquake record and spectral matching technique**

Four earthquake records and three techniques for spectral matching were used. Table 5-16 provides basic information on the earthquake records and Table 5-17 gives a list of the modified records used for the analyses. The MRF and damping values for the FLAC analyses were revised by repeating the SHAKE analyses described in Section 5.4.2 with the new earthquake motions. SHAKE analyses were not performed for every record since the required revisions were found to be minor. Table 5-17 lists the source of MRF and damping for each analysis.

Three different methods were used to adjust the earthquake records to the target response spectrum: uniform scaling of the time history, frequency-domain matching using SYNTH (Naumoski, 1985), and a method of modifying the earthquake in the time domain that was developed by Abrahamson (1992). The factor for uniform scaling was selected to give an approximate match between the target response spectrum and the input motion near the fundamental period of the dam. The potential for moderate increases in the response

---

**Table 5-16. Earthquake records for dynamic analysis, ELMD**

<table>
<thead>
<tr>
<th>Earthquake and Record</th>
<th>M&lt;sub&gt;w&lt;/sub&gt;</th>
<th>Fault Type</th>
<th>Site</th>
<th>Distance (km)</th>
<th>Fault</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999 Chi-Chi (Taiwan) EQ TCU078 E-W</td>
<td>7.6</td>
<td>Thrust</td>
<td>Stiff soil</td>
<td>7.1</td>
<td>8.3</td>
</tr>
<tr>
<td>1999 Chi-Chi (Taiwan) EQ TCU129 E-W</td>
<td>7.6</td>
<td>Thrust</td>
<td>Stiff soil</td>
<td>11.9</td>
<td>2.2</td>
</tr>
<tr>
<td>1994 Northridge (California) EQ Pacoima Dam D/S 265</td>
<td>6.7</td>
<td>Blind thrust</td>
<td>Rock</td>
<td>42</td>
<td>8.0</td>
</tr>
<tr>
<td>1992 Landers (California) EQ Lucerne Valley 270</td>
<td>7.3</td>
<td>Strike-slip</td>
<td>Rock</td>
<td>19.3</td>
<td>1.1</td>
</tr>
</tbody>
</table>
Table 5-17. Modified earthquake records, ELMD

<table>
<thead>
<tr>
<th>Earthquake Record</th>
<th>Spectral Matching</th>
<th>LP Filter (Hz)</th>
<th>Source of MRF and Damping</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>TCU078 E-W:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TCU078Esca</td>
<td>Scaled by 0.8</td>
<td>8 - 10</td>
<td>TCU078Esyn</td>
</tr>
<tr>
<td>TCU078Esyn</td>
<td>SYNTH</td>
<td>8 - 10</td>
<td>SHAKE</td>
</tr>
<tr>
<td>TCU078Ersp</td>
<td>RSPMatch</td>
<td>15 - 20</td>
<td>SHAKE</td>
</tr>
<tr>
<td><strong>TCU129 E-W:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TCU129Esyn</td>
<td>SYNTH</td>
<td>8 - 10</td>
<td>TCU078Esyn</td>
</tr>
<tr>
<td><strong>Pacoima Dam D/S 265:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PDD265sca</td>
<td>Scaled by 1.26</td>
<td>None</td>
<td>SHAKE</td>
</tr>
<tr>
<td><strong>Lucerne Valley 270:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LV270sca</td>
<td>Scaled by 1.2</td>
<td>15 - 20</td>
<td>SHAKE</td>
</tr>
<tr>
<td>LV270syn</td>
<td>SYNTH</td>
<td>15 - 20</td>
<td>LV270rsp1</td>
</tr>
<tr>
<td>LV270rsp1</td>
<td>RSPMatch</td>
<td>15 - 20</td>
<td>SHAKE</td>
</tr>
<tr>
<td>LV270rsp2</td>
<td>RSPMatch (T = 0 to 1 sec)</td>
<td>15 - 20</td>
<td>LV270rsp1</td>
</tr>
</tbody>
</table>

period of the dam with liquefaction was considered during the scaling. Prof. D. L. Anderson at the University of British Columbia performed the time-domain adjustments using the computer program RSPMatch. This program was obtained directly from Abrahamson and likely follows the method described in Abrahamson (1992) as referenced in USACE (2000).

A somewhat different approach was used to match the Lucerne Valley LV270rsp2 record to the target spectrum. While the other records modified with SYNTH or RSPMatch used a target spectrum defined for periods of 0 seconds to at least 8 seconds, LV270rsp2 matched the target spectra only between the periods of 0 and 1 second. This abbreviated range should cover the critical response periods of the structure. Longer periods were left
unmodified to preserve the near-field character of the original record that includes a significant long period velocity pulse.

Response spectra and time histories for each earthquake record are provided in Appendix E. In general, both the SYNTH and RSPMatch programs produce records with acceleration spectra that are very close to the target. There are some minor deviations, particularly at periods that are between the spectral ordinates used in the matching process. As expected, the scaled records give the poorest overall match to the target spectra. Each of the records was low pass filtered to remove high frequency components. This caused the peak ground acceleration of several records to drop below the target spectrum.

Although each of the analyzed records meets the earthquake loading requirements of Section 5.4.3, time histories are quite different in character as shown in Appendix E. This is reflected in the predictions for displacement as summarized in Figure 5-70 and Table 5-18. Displacement estimates are very sensitive to the source earthquake record, the method of spectral matching, and the direction the record is applied to the model.

Considering the predictions only for the critical earthquake direction, horizontal crest displacements range from 0.3 to 1.7 m (average = 0.9), vertical crest displacements range from 0.1 to 0.8 m (average = 0.4), and maximum dam displacements range from 0.4 to 2.9 m (average = 1.3). The earthquake record affects these displacements by factors of up to 8. The character of the earthquake motion is clearly a critical parameter.

The importance of character can be seen in the analyses based on the Lucerne Valley record. Four different approaches were used to match the Lucerne Valley record to the target spectrum. These approaches result in very different motions as shown in Figure 5-71. Much of the difference between the records is related to the near-field velocity pulse and how it is
Figure 5-70. Displacement predictions versus earthquake, ELMD.
Table 5-18. Selected displacements from sensitivity study of input motion, ELMD

<table>
<thead>
<tr>
<th>Analysis Parameters</th>
<th>Estimated Displacement (metres)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EQ Direction</td>
</tr>
<tr>
<td>TCU078Esca</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>-</td>
</tr>
<tr>
<td>TCU078Esyn</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>-</td>
</tr>
<tr>
<td>TCU078Ersp</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>-</td>
</tr>
<tr>
<td>TCU129Esyn</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>-</td>
</tr>
<tr>
<td>PDD265sca</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>-</td>
</tr>
<tr>
<td>LV270sca</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>-</td>
</tr>
<tr>
<td>LV270syn</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>-</td>
</tr>
<tr>
<td>LV270rsp1</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>-</td>
</tr>
<tr>
<td>LV270rsp2</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>-</td>
</tr>
</tbody>
</table>

Notes:  
“Crest Centerline” refers to the center of the crest prior to construction of berm.  
“Dam Max” excludes displacements in riprap and bedding elements and the top two rows of elements at crest.
Figure 5-71. Velocity time histories for Lucerne Valley 270 records.
treated during the matching process, although there are also significant differences in the frequency and magnitude of the lesser velocity peaks. These four versions of the same record give displacement predictions that differ by factors of 3 to 5.

Figure 5-72 shows the effect these records had on liquefaction within the upstream shell. A moderate impact is seen on both the extent and timing of liquefaction. The figure also shows there is not a direct correspondence between displacement and liquefaction.

A better understanding of the four analyses can be gained from Figure 5-73, a comparison of displacement histories versus input motion. The displacements are from the upstream face directly beneath the riprap bedding layer. The history points are located at elevations that are 33%, 70%, and 95% of the height of the dam.

Figure 5-72. Liquefaction versus time for Lucerne Valley 270 records, ELMD.
Figure 5-73. Displacement histories from Lucerne Valley 270 records, ELMD.
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The displacements for LV270sca, LV270rsp1, and LV270rsp2 are controlled by the near-field velocity pulse. This pulse gradually rises to its peak and gives the dam a large upstream velocity. Then the base quickly changes direction. It is this abrupt change in loading that leads to much of the permanent displacement. The displacements from these three analyses appear to be a function of the size of this velocity pulse.

The response of the dam to the LV270syn record is fundamentally different. This record retains little of the original velocity pulse, although it still has a pulse sufficient to impact the early displacements. But the base motions after the velocity peak are of a frequency and amplitude to significantly increase the displacement predictions. This can also be seen in Figure 5-72 where a significant increase in liquefaction occurs between 14 and 20 seconds.

While the four LV270 records were of very different character, the TCU078 E-W records are relatively similar as shown in Figure 5-74. This greater similarity may be due to the spectral shape of the original record that was quite similar to the target spectrum. The lack of a near-field velocity pulse may have also improved the agreement.

Displacement predictions for the TCU078 E-W records are also more consistent. Estimates shown on Table 5-18 for the earthquakes applied in a positive direction differ by less than a factor of 2. Differences in displacement response appear to occur throughout the earthquake as shown in Figure 5-75. These differences are caused by relatively subtle variations in the earthquake record. The variations are perhaps most significant between 25 and 35 seconds as shown on Figure 5-74 and Figure 5-75.

Design earthquakes are typically selected with an emphasis on the target response spectrum. These analyses show that acceleration spectra are relatively insensitive indicators
Figure 5-74. Velocity time histories for TCU078 E-W records.
of permanent displacements for geotechnical structures. This is not surprising since response spectra define the single maximum acceleration, velocity, or displacement response of a linear elastic structure during an earthquake. Target spectra have proved useful for evaluating buildings and other structures that are expected to experience relatively limited plastic deformations. Additional criteria are needed for developing earthquake motions when significant plastic deformations can occur with each load cycle.
The effect of duration and loading cycles is often specified solely through earthquake magnitude. The earthquakes for the ELMD analyses were required to fall within a magnitude range of about 6.75 and 7.5. This range, while reasonably narrow, still permits significant differences in earthquake loading and duration.

The USACE (2000) suggests that duration be specified as part of the design earthquake requirements. The importance of duration and the cumulative energy of the earthquake motion can be seen in a plot of Arias intensity $I_A$ as discussed in Section 4.3.5. Such a plot is shown in Figure 5-76. This figure reveals that while displacements are related to duration and intensity, the time varying characteristics of an earthquake motion are equally important. Arias intensity and duration may still be useful for screening potential earthquake records. It is unfortunate that such parameters are not included in common attenuation relationships.

The necessity of using a number of earthquake records for nonlinear analyses is strongly supported by these analyses and is a common recommendation. The U.S. Army Corps of Engineers (USACE) suggests that at least five earthquake records be evaluated to cover a range in time domain characteristics when evaluating concrete hydraulic structures (USACE, 2000).

As suggested in Section 4.5, a general check of the analyses can be made by comparing the estimated peak crest accelerations to the observed values reported by Harder et al. (1998). This comparison is shown in Figure 5-77. The range of estimates from the analyses appears to be reasonable when compared to observations, although the predictions tend to be somewhat lower than the average for observed motions. This may be related to the selected values for damping and undrained strength.

Unfortunately, there is also some error in the estimated values. The plotted peak accelerations were obtained from one point on the crest surface with the values recorded
Final horizontal crest displacement are provided in parentheses next to earthquake designation.

TCU078rsp (1.7 m)  
TCU078syn (1.2 m)  
TCU129syn (0.7 m)  
TCU078sca (1.0 m)  
LV270sca (1.2 m)  
LV270syn (1.0 m)  
LV270sca (0.7 m)  
LV270sca (0.3 m)  
PDD265sca (0.4 m)

Figure 5-76. Arias intensity $I_A$ versus time, ELMD.

every 0.02 seconds through the earthquake. There is commonly some noise in acceleration predictions that will affect the results. This noise cannot be easily filtered from the recorded history since it occurs at a much higher frequency than the Nyquist frequency of the acceleration history (i.e., 25 Hz).

It may be preferable to use an average acceleration from several crest points. The base analysis was repeated in order to obtain these additional histories. While the peak crest
acceleration for the base analysis reported in Figure 5-77 was 0.55 g, two adjacent crest nodes had peak crest accelerations of 0.71 g and 0.60 g. When the histories from these three nodes were averaged together, the peak crest acceleration was found to be 0.56 g, or very near the reported value. This example does suggest the large scatter that may be included in Figure 5-77.

Figure 5-77. Comparison of predicted crest accelerations with case histories, ELMD.
Effect of vertical earthquake motion

It is standard practice to consider only the horizontal earthquake motion in a liquefaction and displacement analysis. Vertically propagating shear waves are considered the critical loading for liquefaction. All of the analyses thus far have considered only a horizontal base motion. Analysis TCU078Esyn-Vert was performed to evaluate the effect of vertical motions on the displacements.

The original vertical recording from TCU078 E-W was used after scaling to a peak acceleration of 0.545 g using a factor of 3.25. The appropriateness of the imposed loading is uncertain since a target UHRS spectrum was not developed for vertical motion. However, the impact on displacements was relatively small. Crest movements increased by about 15% while the peak dam displacement increased by about 20% as shown in Table 5-19.

Table 5-19. Selected displacements from additional sensitivity studies, ELMD

<table>
<thead>
<tr>
<th>Analysis Name</th>
<th>EQ Direction</th>
<th>U/S Riprap</th>
<th>Dam Max</th>
<th>Crest Centerline Horiz.</th>
<th>Crest Centerline Vert.</th>
</tr>
</thead>
<tbody>
<tr>
<td>TCU078Esyn (Base Analysis)</td>
<td>+</td>
<td>3.7</td>
<td>2.0</td>
<td>1.2</td>
<td>0.6</td>
</tr>
<tr>
<td>TCU078Esyn-Vert</td>
<td>+</td>
<td>4.9</td>
<td>2.4</td>
<td>1.3</td>
<td>0.7</td>
</tr>
<tr>
<td>TCU078Esyn-2/3h</td>
<td>+</td>
<td>3.2</td>
<td>1.7</td>
<td>1.3</td>
<td>0.6</td>
</tr>
<tr>
<td>TCU078Esyn-γr</td>
<td>+</td>
<td>4.7</td>
<td>2.5</td>
<td>1.2</td>
<td>0.7</td>
</tr>
<tr>
<td>TCU078Esyn-wc</td>
<td>+</td>
<td>3.9</td>
<td>2.1</td>
<td>1.2</td>
<td>0.6</td>
</tr>
<tr>
<td>TCU078Esyn-Su</td>
<td>+</td>
<td>7.5</td>
<td>3.7</td>
<td>2.2</td>
<td>1.9</td>
</tr>
<tr>
<td>TCU078Esyn-Noliq</td>
<td>+</td>
<td>2.6</td>
<td>1.1</td>
<td>0.9</td>
<td>0.5</td>
</tr>
<tr>
<td>TCU078Esyn-0.7g</td>
<td>+</td>
<td>6.0</td>
<td>3.3</td>
<td>2.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Notes: “Dam Max” is the peak displacement excluding surface elements. Riprap, bedding, and top two rows of crest elements are not included in this estimate. Analysis TCU078Esyn-Su was performed in small strain mode.
Effect of damping

The impact of viscous damping was assessed with a series of four analyses. The only difference between these analyses and the base analysis was a change in the viscous damping: a factor of 0.5, 0.75, 1.5, or 2.0 was applied to $\beta_c$. This resulted in viscous damping coefficients of 5.5% to 22% within the interior of the embankment. Results from these analyses are shown graphically in Figure 5-78.

A regular trend is seen between damping and displacement. Changing the damping by a factor of 2 causes a 30% to 40% change in displacements with a somewhat lesser effect on accelerations. Although these changes are significant, they occur only after a very large change in damping. Displacement predictions appear to be much less sensitive to damping than to the earthquake record.

Figure 5-78. Effect of damping on selected response values, ELMD.
Damping can wield a strong influence on acceleration response. The predicted crest accelerations are compared to observed values from Harder et al. (1998) as shown on Figure 5-79. The values for ELMD are plotted against the nominal peak base acceleration of 0.545 g. The predicted accelerations are in reasonable agreement with the range of observed values. The damping used for the base analysis appears to produce a peak crest acceleration
near the average trend from observations. It is also clear that predictions of crest acceleration are not highly sensitive to damping. While the data from Harder et al. (1998) can provide a general check for an analysis, a reasonable comparison can still be achieved over a wide range in damping.

*Effect of preliquefaction stiffness*

Preliquefaction stiffness influences the response of the dam through its importance to the fundamental period. The base analysis prior to liquefaction has a fundamental period of about 0.55 seconds. Due to the shape of the response spectrum, a moderate increase in stiffness should increase the response. This interpretation may be simplistic since a nonlinear structure does not have a true fundamental period. However, the $f_i$ based on the preliquefaction elastic stiffness is expected to be a good indicator of the primary response frequencies prior to widespread liquefaction.

This assumption was evaluated for Analysis LV270rsp1 by developing Fourier spectra of the crest accelerations. Using the recorded motion between 0 and 10 seconds, which is prior to liquefaction in the upstream shell, the Fourier spectrum has predominant peaks at periods of about 0.5 and 0.8 seconds. This is similar to the fundamental period of 0.6 seconds for this earthquake. For the time frame of 10.5 to 15 seconds, which is after significant liquefaction within looser zones as shown in Figure 5-72, the spectral peaks are at 0.7 and 0.9 seconds. There may be some shift due to the modest amount of liquefaction, but the predominant period of the input motion has also lengthened.

The effect of preliquefaction stiffness was evaluated in a series of four analyses. Each analysis applied a uniform factor to the preliquefaction stiffness values of the base analysis: 0.5, 0.75, 1.5, or 2.0. Selected results are shown graphically in Figure 5-80. A smooth trend is seen between stiffness and the selected displacements. The displacements are relatively
insensitive to stiffness, with the observed effect less than 20%. The general trend is as expected, with an increase in stiffness causing an increase in response.

The effect on the peak crest acceleration is somewhat variable. This may be due in part to noise in the acceleration prediction. The unexpected increase in acceleration for the factor = 0.5 case is due to a single acceleration spike.

**Effect of section height**

Although most dams are true three-dimensional structures, the synthesized approach simplifies the response to a two-dimensional plane strain section. The choice of section height is important since it also affects the prediction of fundamental frequency. Sections that are shorter than the maximum are sometimes analyzed to approximately include the stiffening effects of three-dimensional response.
Analysis TCU078Esyn-2/3h assumes a section height of 18.4 m, about 2/3 of the base analysis height. Approximately 50% of the dam is taller than this reduced height. This analysis gives a slightly larger crest displacement than the full height section as shown in Table 5-19. This small change in displacement is consistent with the analyses that varied preliquefaction stiffness. The short section has a fundamental period of about 0.38 seconds. This same fundamental period can be achieved with the original section by increasing its stiffness by a factor of about 2. Figure 5-80 suggests that such a change in stiffness and fundamental period should increase the crest displacements by 10% to 20%.

Miscellaneous effects

A number of additional sensitivity studies were performed. Selected results are summarized below and in Table 5-19.

1. TCU078Esyn-\( \gamma_r \) assumes a residual strain \( \gamma_r \) of 10%\(^1\) in the upstream shell, a five-fold increase over the base analysis. This changed the crest displacements by less than 0.1 m. The decrease in postliquefaction stiffness had a larger effect near the upstream face with the peak dam displacement increasing by about 25%.

2. TCU078Esyn-wc evaluates the influence of the weighting curve. The base analysis assumes a high \( \text{CRR}_1 / \text{CRR}_{15} \) ratio of about 2.5. This analysis reduces this ratio to 1.5 as suggested by Figure 2-18. This large change produced a slight but inconsequential increase in displacements.

3. TCU078Esyn-\( S_u \) assumes a 20% reduction in the undrained shear strength of nonliquefied saturated elements. This reduction also affects the values of \( S_r \) in shallow

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\(^1\) This analysis produced erroneous results in the upstream riprap and bedding when a \( \gamma_r \) of 10% was used in the bedding. The reported results assume a \( \gamma_r \) of 2% in the bedding. The initial results were apparently related to a problem in FLAC 3.4 that affects elements that experience large volumetric strains due to tension failure while in large strain mode. This problem has been corrected in FLAC 4.0.
liquefied zones since these strengths have been limited to a maximum of $S_u$. The results in Table 5-19 suggest that reducing the undrained strength of all saturated zones by 20%, an arbitrary but large amount, could double or even triple the predicted displacements.

4. TCU078Esyn-Noliq is similar to the base analysis except liquefaction is not permitted within the shells. Zones within the upstream shell having $N_{1-60}$ values of 20 to 30 have been reassigned values of 40. Table 5-19 shows that the occurrence of liquefaction in the upstream shell increases the maximum dam displacement by nearly 50%. Crest displacements are less influenced by liquefaction, with this analysis showing a reduction of only 15% to 25% as compared to the base analysis.

5. Analysis TCU078Esyn-0.7g evaluates an increase in the design loading requirements. The target UHRS was scaled from a pga of 0.545 g to 0.7 g. This 28% change in the base acceleration produced a 65% increase in displacement as shown in Table 5-19. This large jump in response results directly from the nonlinear nature of the analysis.

5.4.6 Displacements including consolidation

Settlements resulting from densification or consolidation of the soil skeleton are not directly considered by the synthesized approach. Data presented by Tokimatsu and Seed (1987) for saturated clean sands suggest the volumetric strain for non-liquefied zones should be less than about 0.5%. The data also suggest the strains in the liquefied zones might be 1% to 2%. However, since the relative size of the liquefied area is small, and considering the significant static shear stresses that are supported by this zone following the earthquake (Section 3.4.2), consolidation of the liquefied zones may not have a great influence on crest settlements. Since the $N_{1-60}$ values for most of the dam are high, and many zones may have
relatively little increase in pore pressure due to the earthquake, the average volumetric strain in saturated elements might be significantly less than 0.5%.

Tokimatsu and Seed (1987) also present data and recommendations for consolidation of dry sand. Assuming a representative $N_{1-60}$ for the downstream shell of about 40, and estimating the effective cyclic strain using the SHAKE results, the average volumetric strain in the downstream shell is estimated to be less than about 0.15%. This strain estimate has already been doubled to include the effects of biaxial loading.

Tokimatsu and Seed (1987) suggest the error in estimation of settlements is likely greater than 25% to 50%. A reasonable estimate of crest settlement due to densification might be something less than 0.5% of the dam height. This suggests an additional 0.0 to 0.2 m of crest settlement.

A more precise evaluation of crest settlement might be made using the procedures described in Section 3.4. Volumetric strain estimates could be developed for each zone depending on its blowcount and estimated $r_u$ or equivalent cyclic strain. However, such an analysis would still contain significant uncertainties. The effort is likely unwarranted given the relatively small settlements indicated by this simpler approach.

Estimated crest displacements including consolidation are summarized in Table 5-20. These estimates are based primarily on the set of nine analyses summarized in Table 5-18. Only the earthquake direction that gives the greatest crest displacement is considered. Both the horizontal and vertical displacement estimates have been increased by 0.1 m to approximately account for vertical motions as suggested by analysis TCU078Esyn-Vert.

5.4.7 Comparison to case history observations

One method of corroborating the predicted response is to compare it with observed behaviour from case histories. Case histories also provide a wealth of information beyond
Table 5-20. Crest displacements including consolidation estimates, ELMD

<table>
<thead>
<tr>
<th>Description</th>
<th>Estimated Crest Displacement (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizontal</td>
</tr>
<tr>
<td>Maximum (9 EQ analyses)</td>
<td>1.8</td>
</tr>
<tr>
<td>Average + 1 standard deviation (9 EQ analyses)</td>
<td>1.5</td>
</tr>
<tr>
<td>Average (9 EQ analyses)</td>
<td>1.0</td>
</tr>
<tr>
<td>Base analysis</td>
<td>1.3</td>
</tr>
<tr>
<td>Base analysis with no liquefaction</td>
<td>1.0</td>
</tr>
</tbody>
</table>

just the observed displacements, such as the extent and depth of cracking and the variability in typical response. A number of displacement observations from recent earthquakes are compiled below and in Table 5-21 for comparison with the ELMD estimates. An attempt was made to select case histories that were as close as possible to the conditions and loading at ELMD. However, the differences are still significant and make a direct comparison somewhat difficult.

A number of dams experienced severe shaking during the 1989 Loma Prieta earthquake. Although the earthquake was a magnitude 7, the duration of strong motion was only 7 to 10 seconds. The response of five dams is summarized below as described by Harder (1991): Austrian, Lexington, Guadalupe, Newell, and Elmer J. Chesbro Dams. Harder did not mention any observations of liquefaction at these dams, although pore pressure increases were noted for Austrian Dam.

Austrian Dam is about twice the height of Elsie Lake Main Dam (56 m) and has flatter slopes (2.5:1 to 3.5:1). The fill is generally a clayey sandy gravel. The reservoir at the time of the earthquake was quite low, about 30 m below the crest. The dam is located directly above the fault rupture and experienced an estimated peak ground acceleration of 0.55 to 0.6 g. The
earthquake resulted in a relative horizontal crest displacement of about 0.5 m and a maximum vertical crest displacement of about 0.8 m. These displacement values are not absolute measures as the survey monuments were not tied into a stationery benchmark.

Lexington Dam is located about 10 km from Austrian Dam, has a maximum height of about 62 m, and slopes of 5.5:1 upstream and 3:1 downstream. The dam has a sandy and gravelly clay core with random shell zones of clayey sands and silts. The peak transverse acceleration measured on both the left abutment and the crest was 0.45 g. The reservoir was about 30 m below the crest at the time of the earthquake. The shaking produced about 0.08 m

Table 5-21. Crest displacements of embankment dams following earthquakes

<table>
<thead>
<tr>
<th>Dam</th>
<th>Height (m)</th>
<th>U/S Slope</th>
<th>D/S Slope</th>
<th>Freeboard (m)</th>
<th>EQ Mag.</th>
<th>pga</th>
<th>Crest Disp. (m)</th>
<th>Horiz</th>
<th>Vert</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austrian</td>
<td>56</td>
<td>2.5:1 - 3.5:1</td>
<td>2.5:1 - 3.5:1</td>
<td>30</td>
<td>7</td>
<td>0.55 - 0.6*</td>
<td>0.5**</td>
<td>0.8**</td>
<td></td>
</tr>
<tr>
<td>Lexington</td>
<td>62</td>
<td>5.5:1</td>
<td>3:1</td>
<td>30</td>
<td>7</td>
<td>0.45</td>
<td>0.08</td>
<td>0.26</td>
<td></td>
</tr>
<tr>
<td>Guadalupe</td>
<td>43</td>
<td>2.5:1 (B)</td>
<td>2.5:1</td>
<td>24</td>
<td>7</td>
<td>0.4 - 0.45*</td>
<td>0.05</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>Newell</td>
<td>55</td>
<td>?</td>
<td>?</td>
<td>15</td>
<td>7</td>
<td>0.4 - 0.45*</td>
<td>n/s</td>
<td>n/s</td>
<td></td>
</tr>
<tr>
<td>Elmer J. Chesbro</td>
<td>29</td>
<td>2:1 - 3:1 (B)</td>
<td>2:1</td>
<td>21</td>
<td>7</td>
<td>0.4 - 0.45*</td>
<td>0.02</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>Pantabangan</td>
<td>107</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>7.8</td>
<td>0.35 - 0.5 *</td>
<td>0.70</td>
<td>0.26</td>
<td></td>
</tr>
<tr>
<td>Masiway</td>
<td>25</td>
<td>2.33:1</td>
<td>2:1</td>
<td>7</td>
<td>7.8</td>
<td>0.35 - 0.5 *</td>
<td>?</td>
<td>0.92</td>
<td></td>
</tr>
</tbody>
</table>

Legend:
* = estimated  
** = uncertain  
n/s = no significant displacements noted  
(=) = unclear if reported value is maximum displacement  
(B) = slope has buttress
of downstream lateral movement and 0.3 m of settlement.

Guadalupe Dam is about 43 m in height, has 2.5:1 slopes, a substantial upstream buttress, and experienced estimated peak ground accelerations of 0.4 to 0.45 g. The dam section is nearly homogeneous. The reservoir at the time of the earthquake was about 24 m below the crest. Induced displacements were up to 0.05 m upstream and 0.2 m vertical.

Newell Dam is 55 m high and generally consists of clayey material with an upstream zone of dirty rockfill. The reservoir was about 15 m below the crest. Peak ground accelerations were estimated at 0.4 to 0.45 g. No significant crest movements were noted.

Elmer J. Chesbro Dam is a 29 m high dam with an upstream slope of 2:1 to 3:1 and a downstream slope of about 2:1. A downstream berm extends to half the dam height. The dam section is essentially homogeneous. The reservoir was about 21 m below the crest. Peak ground accelerations were estimated at 0.4 to 0.45 g. The earthquake resulted in about 0.1 m of crest settlement and 0.02 m of lateral movement.

The response of two dams to the 1990 Philippine earthquake has also been described (Ashford et al., 1998): Pantabangan and Masiway Dams. The earthquake had a magnitude of 7.8 ($M_s$) with surface rupture approximately 10 km from the dams. The peak ground acceleration at the dams is unknown but has been estimated to be between 0.35 and 0.5 g (Ashford et al., 1998).

Pantabangan Dam is a zoned earthfill dam with a height of 107 m. A horizontal deflection of 0.7 m toward the reservoir was noted. The maximum crest settlement was 0.26 m.

Masiway Dam is a 25-metre-high zoned earthfill embankment with slopes of 2.33:1 and 2:1. The reservoir at the time of the earthquake was 7 m below the crest. The maximum
observed crest settlement was 0.92 m. The dam is founded on conglomerate with lenses of sandstone and shale.

A graphical comparison of the vertical settlements is shown in Figure 5-81. The ELMD estimates include the effects of earthquake record, consolidation, and vertical motions as summarized in Table 5-20. Predicted displacements for ELMD tend to be somewhat larger than for the case history observations, although there is general agreement. A comparison is also shown of crest settlement normalized to the height of the dam. It is sometimes

![Graphical Comparison of Crest Settlements](image_url)

Figure 5-81. Comparison of crest settlements with case histories, ELMD.
convenient to express settlement in terms of vertical strain, although this is not always appropriate. The displacements caused by the liquefied layers at ELMD are not a function of the dam height as shown in Figure 5-68.

Predictions of horizontal displacement tend to be significantly larger than these case history observations as shown in Figure 5-82. Although closer agreement between the predictions and observed trends might be desired, the differences shown on Figure 5-81 and Figure 5-82 are not unexpected. Only seven case histories were selected from two earthquakes. Five of the case histories were from the Loma Prieta earthquake, which appears to have been a less severe earthquake than the motions used for evaluating ELMD. There are also significant differences between the embankments in the case history database and ELMD. The large amount of freeboard for the Loma Prieta dams, the steep upstream slope at ELMD, and the occurrence of liquefaction at ELMD are just a few of the differences. Some conservative but reasonable assumptions were also made in developing the ELMD model,

Figure 5-82. Comparison of horizontal crest displacements with case histories, ELMD.
such as those used for estimating friction angle.

5.4.8 Summary

Analyses of the Elsie Lake Main Dam were performed and the results compared to case history observations. Peak crest accelerations (Figure 5-77 and Figure 5-79) and crest displacements (Figure 5-81 and Figure 5-82) show reasonable agreement with observed trends.

Nine variations of four earthquake records were analyzed. These records produce a wide range in displacements even though each record meets the specified loading requirements. Displacement predictions varied by factors of up to 8.

At least four critical factors were identified for selecting earthquake motions: response spectra, duration of strong motion, energy of record (e.g., $I_A$), and character of the record in the time domain. The importance of both intensity and duration was shown in Figure 5-76, while the influence of the earthquake character can be seen in Figure 5-73 and in the influence of earthquake direction. Earthquake motions should always be analyzed twice, once in either direction, since it is not always clear which earthquake direction is most critical. Near-field effects can also be significant.

One implication of these observations is that response spectra are not the only or necessarily the best measure of the input motion. It is common to have more than one earthquake scenario for evaluation of a structure. One scenario may represent a near field but lower magnitude earthquake, while a second may be for a distant but large magnitude earthquake. The distant earthquake is often ignored if their anticipated response spectrum is sufficiently below the spectra for the near-field event. Unfortunately, it is not always clear when a distant event can be neglected due to the effects of duration and frequency content of these events.
The method of spectral matching can have a large impact on the character of the record. The spectral matching technique may have the biggest effect on records having distinctive features in the time domain such as a near-field pulse. The importance of earthquake character to analyses will increase as the consideration of near-field effects becomes a part of standard practice.

A reasonably large set of earthquake motions should be evaluated in any nonlinear displacement analysis. Selection of appropriate records can be improved if both duration and intensity are explicitly considered. Criteria based on a combination of these two parameters might be useful. An Arias intensity spectra could be a helpful criterion, although it will remain impractical unless attenuation relationships for this parameter are developed. The use of parameters such as duration or intensity for loading criteria would be improved if such parameters were included in standard attenuation relationships.

Additional analyses suggest that $S_u$, the undrained strength of nonliquefied elements, can have a large effect on displacements. Viscous damping was seen to have a moderate influence. The importance of including ratchet-type behaviour in the postliquefaction model was also demonstrated. The vertical earthquake motions, weighting curve, preliquefaction shear stiffness, section height, and postliquefaction stiffness were found to have relatively little effect in limited analyses. Displacement estimates appear to be most sensitive to the input earthquake loading.

5.5 Applications by Engineering Profession

One of the main objectives of this research was to develop an analytical tool that would have immediate and practical use within the engineering community. The success of this effort is perhaps best evaluated by the interest of the local engineering community. A number
of consulting and engineering organizations have used the synthesized approach on a variety of projects since its initial development. Examples of its use include the following:

- **Dams:**
  - Seymour Falls dam (North Vancouver, BC)
  - Keenleyside Power dam (Castlegar, BC)
  - Coquitlam dam (Coquitlam, BC)
  - Elsie Lake dams (Vancouver Island, BC)

- **Bridge/Road structures:**
  - George Massey tunnel (Delta/Richmond, BC)
  - Okanagan bridge (Kelowna, BC)
  - Pitt River bridge (Port Coquitlam, BC)
  - Second Narrows bridge (Vancouver, BC)

- **Waterfront structures:**
  - Canada Place cruise ship terminal (Vancouver, BC)
  - VanTerm container terminal (Vancouver, BC)

- **Building foundations:**
  - An early, limited version of the postliquefaction model was used as an aid in evaluating the response of buildings founded on a firm crust overlying liquefied material (Naesgaard, et al., 1998).

### 5.6 Conclusion

The synthesized approach was demonstrated by application to four geotechnical structures including two well-documented case histories:
1. A hypothetical lateral spread site and its predicted response to the 1994 Northridge earthquake ($M_w = 6.7$). Results were compared to observed trends as represented by the empirical relationships from Hamada et al. (1987) and Youd et al. (1999).

2. The Upper San Fernando dam and its response to the 1971 San Fernando earthquake. Displacement results were compared to available survey data. The extent of liquefaction and the prediction of displacement were also compared with previous analyses (Seed et al., 1973; Inel et al., 1993; Moriwaki et al., 1998).

3. The Lower San Fernando dam and its response to the 1971 earthquake. Displacement predictions were qualitatively compared with the observed failure of the upstream slope.

4. The Elsie Lake Main dam and its potential response to a magnitude 6.75 to 7.5 design earthquake. Results for peak acceleration and crest displacement were compared to observations made at other dams.

These four sets of analyses demonstrate the usefulness of the synthesized approach. Reasonable predictions of displacement magnitude and pattern were obtained using common assumptions for material properties. The utility of the method is also evident from the range of ways it has been used by the engineering profession. The synthesized approach has assisted in the evaluation of dams, bridges, tunnels, and waterfront structures. Potential use in the evaluation of building foundations has also been investigated.

A primary focus of this chapter was the presentation of detailed sensitivity studies. Although the importance of input parameters will vary with the structure being analyzed, an attempt was made to identify those variables with the greatest influence on the displacement results. Many of the sensitivity analyses are qualitatively summarized in Table 5-22. This
Table 5-22. Qualitative summary of sensitivity studies

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Relative Effect on Displacements as Observed in Sensitivity Studies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lateral Spread</td>
</tr>
<tr>
<td>Earthquake:</td>
<td></td>
</tr>
<tr>
<td>Record</td>
<td>—</td>
</tr>
<tr>
<td>Direction</td>
<td>mod</td>
</tr>
<tr>
<td>Vertical component</td>
<td>—</td>
</tr>
<tr>
<td>Preliquefaction:</td>
<td></td>
</tr>
<tr>
<td>Damping: $f_c$</td>
<td>low</td>
</tr>
<tr>
<td>Damping: RS versus RMS</td>
<td>mod</td>
</tr>
<tr>
<td>Stiffness $G_{dyn}$</td>
<td>—</td>
</tr>
<tr>
<td>Undrained strength $S_u$</td>
<td>—</td>
</tr>
<tr>
<td>Triggering:</td>
<td></td>
</tr>
<tr>
<td>Weighting curve</td>
<td>—</td>
</tr>
<tr>
<td>CRR15</td>
<td>—</td>
</tr>
<tr>
<td>Postliquefaction:</td>
<td></td>
</tr>
<tr>
<td>Stiffness: $\psi$</td>
<td>mod</td>
</tr>
<tr>
<td>Residual strength: Anisotropic $S_r/\sigma_{vo}'$</td>
<td>mod</td>
</tr>
<tr>
<td>Residual strength: Isotropic $S_r$</td>
<td>—</td>
</tr>
<tr>
<td>Residual strength: $S_r/\sigma_{vo}'$ versus $S_r$</td>
<td>mod</td>
</tr>
<tr>
<td>Residual strength: Anisotropy</td>
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</tr>
<tr>
<td>$\tau_{xy}$ versus $\tau_{max}$ stress reversal</td>
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</tr>
<tr>
<td>Symmetric versus bilinear stress-strain</td>
<td>mod-high</td>
</tr>
<tr>
<td>Other:</td>
<td></td>
</tr>
<tr>
<td>Element size</td>
<td>low</td>
</tr>
<tr>
<td>Site/foundation conditions</td>
<td>high</td>
</tr>
<tr>
<td>Total number of analyses:</td>
<td>174</td>
</tr>
</tbody>
</table>

Note: The effect on displacements has been categorized as either low, moderate, or high.
table is subjective and intended only to highlight those variables that appear to have the most impact on the displacement predictions for these analyses. While a large number of analyses were performed, not every parameter was evaluated in detail or in more than one structure. Nevertheless, the analyses suggest certain trends and strongly support the need to do sensitivity studies as part of any analysis.

Parameters that showed a large effect on the displacement predictions include the direction and character of the horizontal earthquake record, the formulation and choice of residual strength, the inclusion of postliquefaction anisotropy, and the undrained strength $S_u$. The use of the bilinear versus symmetric model for postliquefaction stress-strain response was also found to be important in situations with low static bias. How the stratigraphy is characterized can be extremely important, particularly as it affects the selection of residual strength.

Parameters with a relatively low importance in these analyses include the vertical earthquake motion, the amount of RS viscous damping, the weighting curve, and the residual strain $\gamma_r$. MRF and damping values derived from SHAKE analyses appear to be appropriate when used in conjunction with $f_c$ values that are near the fundamental frequency of the dam. Rayleigh RMS damping is less suitable than RS damping.

The character of the earthquake record is a critical factor. Both the USFD and ELMD analyses show the importance of near-field velocity pulses. Even the method of spectral matching can be have a large influence on the predicted displacements, although the significance of this effect may be a function of the character of the original record. Selection of appropriate design records can be improved if duration criteria are specified along with response spectra, magnitude, faulting type, and the potential for near-field effects. Parameters
such as Arias Intensity $I_A$ might also be useful. A reasonably large set of earthquakes should always be evaluated due to the importance of the time domain character.

The importance of earthquake record to liquefaction-induced displacement may have a bearing on empirical relationships for displacement such as the Youd et al. (1999). Much of the Youd et al. database is for data from only two Japanese earthquakes. While the displacement observations would have been made at a number of sites, it is possible that the specific characteristics of these two earthquakes may be influencing the derived empirical relationships.

The residual strength is a second factor of critical importance. While an anisotropic $S_r/\sigma_v'$ may be preferred for an initial undrained strength, reasonable results were also obtained with isotropic $S_r$ values. It may be prudent to perform analyses with both sets of strength formulations since the $S_r$ values obtained from back analysis may include the effects of partial drainage and mixing. The importance of using both types of strengths was demonstrated by the LSFD analyses.

Using a low estimate of relative density or penetration resistance, such as $N_{1/3}$, for selecting residual strength appears to be warranted when using the Idriss curve of Figure 2-32 or the modified Idriss curve of Figure 2-34. The analyses show that values based on $N_{1/3}$ are sometimes conservative, although often more appropriate than strengths based on median $N_{1-60}$ values.

The synthesized approach has some drawbacks due to its relative simplicity. These include a rather crude representation of the preliquefaction soil behaviour as well as a tendency to overpredict the effects of base isolation. Its sensitivity to the value of residual strength is cause for some concern. This is a critical parameter in many postliquefaction
analyses and its importance to the displacement predictions is expected. Unfortunately, it is not a parameter that can be easily or confidently selected.

The relative stability and reasonableness of the predictions suggest the synthesized approach can be a valuable tool for comparing the relative effectiveness of remediation measures. Guidance can be obtained not only for the extent of remediation but also for its optimal location.

The scatter in the predicted displacements reflects the inherent uncertainty in any analysis approach to the liquefaction-deformation problem. Even if a perfect representation of soil behaviour could be achieved in a numerical model, the uncertainty in input parameters and loading makes it impossible to precisely estimate the likely displacements. However, the trends observed in these analyses indicate the synthesized approach can give reasonable and relatively stable estimates of displacement. While the method has limitations due to its simplicity, this lack of complication is also one of its strengths. Its similarities to common practice make it a reasonable analysis choice for many liquefaction-deformation problems.
A new analytical tool for evaluating the liquefaction-deformation problem was developed and termed the synthesized approach. The basis of this approach, details of its implementation, a discussion of relevant input parameters, and applications of the tool for demonstration and verification were presented.

6.1 Summary

6.1.1 Chapter 1 — Introduction

An overview of the liquefaction-deformation problem and its importance was given in Chapter 1. The current state of practice for analysis was discussed and objectives were defined for developing and presenting a new approach:

1. To develop and calibrate a practical analytical tool for estimating displacements due to earthquake-induced liquefaction. The tool should build upon accepted practice while bridging the gap to more fundamental and advanced analyses.

2. To provide preliminary guidelines on input parameters and identify critical variables.

3. To make observations regarding the method and analyses that may be useful to future applications.

6.1.2 Chapter 2 — Overview of Sand Behaviour and Liquefaction

A review of fundamental sand behaviour and liquefaction was provided in Chapter 2. A detailed understanding of this behaviour is required to identify key aspects of the liquefaction-deformation problem as well as to understand the potential limitations of the analysis method. Important factors in sand behaviour include the density, stress level, stress
path and anisotropy, fabric, static bias, drainage, nonlinearity, and the effect of mixing at large strains.

Several new or updated relationships were developed from published data:

1. Curves relating CRR/CRR\textsubscript{15} to the number of cycles to liquefaction were developed from cyclic tests of undisturbed frozen samples of natural sand deposits (Figure 2-18, Appendix B). The normalized CRR values are found to give similar trends within a reasonable amount of scatter despite the variety in sand deposits and test conditions.

2. A tentative relationship between CRR\textsubscript{1}/CRR\textsubscript{15} and relative density was estimated from these same cyclic tests (Figure 2-19). The slope of the cyclic strength curve is observed to become steeper as the relative density increases.

3. A relationship was derived from HCT test results for estimating the variation in $S_{qss-ss}$ as a function of the direction of the major principal stress (Figure 2-12; Equations 2.1, 2.2).

4. HCT test results were reinterpreted to better demonstrate the importance of static bias and principal stress direction on values of $S_{qss-ss}/\sigma'_vo$ (Figure 2-14) and $S_{u-peak}/\sigma'_vo$ (Figure 2-16).

5. The change in secant stiffness versus cyclic ratio, $N/N_{liq}$, was estimated from cyclic tests of undisturbed frozen samples of Fraser River sand (Figure 2-23). A significant drop in average stiffness was not seen until the sample neared liquefaction (e.g., $G/G_o \approx 0.65$ at $N/N_{liq} \approx 0.7$).

6. Estimates of $S_{qss-ss}$ from monotonic and postcyclic tests of undisturbed frozen samples were plotted against the corresponding values of $N_{1-60cs}$ (Figure 2-34). This figure also shows estimates of $S_r/\sigma'_vo$ from case histories as developed by Stark and Mesri (1992).
Chapter 6 — Summary and Conclusion

An average trend line is proposed that is consistent with the Idriss curve for $S_r$ versus $N_{1-60cs}$ (Figure 2-32).

6.1.3 Chapter 3 — Synthesized Approach: Description

Based on the description of sand behaviour provided in Chapter 2, a total stress dynamic approach was developed in Chapter 3 for estimating the displacements from seismically induced liquefaction. The approach is derived from widely accepted assumptions for evaluating the triggering of liquefaction, flow slide potential, and limited displacements. These different evaluations are combined into a single analysis while eliminating some of the inherent simplifications in current procedures. The recommended procedure is as follows:

A two-dimensional analysis is performed using a finite difference or finite element mesh (Section 3.1). The preliquefied response in each element is represented in a manner similar to the equivalent linear method (Section 3.2.2). Triggering of liquefaction is evaluated separately in each element by accumulating the effects of each shear stress cycle (Section 3.2.3). Postliquefaction properties and behaviour are assigned to each element at the instant of its triggering (Section 3.2.4). The postliquefaction stress-strain model is formulated to include the softened and potentially weak response of liquefied soils. The model also incorporates the ratcheting behaviour that allows for accumulation of strain with loading cycles even for cases of high residual strength. The very soft response observed at shear stress reversals can be included (Section 3.3.2). A simple interpolation method was developed for approximating anisotropic response after liquefaction (Section 3.3.1). Other details of the synthesized approach are discussed, including the usefulness of using an additional liquefaction criterion based on shear strain (Section 3.3.3).

A method for incorporating postliquefaction settlements due to consolidation was developed (Section 3.4). One-dimensional contraction is imposed on liquefied elements.
through a two-stage stress reduction technique. The method was demonstrated and gave reasonable results, although the predictions near the boundaries of liquefied areas were found to be sensitive to the bulk stiffness of the liquefied zones.

The majority of the information presented in this chapter is a new contribution in its present form. As the method was derived from the state of practice, many of the components of the synthesized approach have similarities with other methods that have been proposed to address this problem. Development of this approach has also been greatly influenced by the guidance, encouragement, and creativity of Prof. Peter Byrne. Aspects of the synthesized approach that possess the most innovation include the following:

1. Triggering of liquefaction occurs while the sample is being loaded rather than at the end of a shear stress cycle or during unloading. Triggering in this case is defined as the onset of softened and potentially weak behaviour.

2. Imposing two triggering criteria for liquefaction, one based on cyclic shear stress and the second based on accumulated shear strain.

3. The formulation of two simple postliquefaction stress-strain models. The bilinear model captures the ratcheting-type behaviour anticipated for elements with a static bias. The symmetric loading model includes both the ratcheting effect and the greatly softened response that can occur as the shear stress reverses its direction.

4. Representing the effects of \( r_u = 100\% \) at postliquefaction stress reversals by imposing a hydrostatic total stress state.

5. Developing a criterion for shear stress reversal based on \( \tau_{max} \) and the change in principal stress direction.

6. Formulating an approach for imposing one-dimensional consolidation in liquefied elements.
6.1.4 Chapter 4 — Synthesized Approach: Input Parameters

Although the input parameters required for this approach are relatively simple and intuitive, their selection can still be a difficult process. Chapter 4 provides an overview and discussion of the model parameters, including the following:

- Representative $N_{1-60}$ (Section 4.1.1)
- Stiffness for static analysis (Section 4.2.1)
- Friction angle (Section 4.2.2)
- Preliquefaction dynamic stiffness (Section 4.3.1)
- Damping (Section 4.3.2)
- Cyclic weighting curve (Section 4.3.3)
- Cyclic resistance ratio, $CRR_{15}$ (Section 4.3.3)
- Undrained strength $S_u$ (Section 4.3.4)
- Earthquake loading (Section 4.3.5)
- Residual strength (Section 4.4.1)
- Postliquefaction stiffness (Section 4.4.2)
- Postliquefaction damping (Section 4.4.3)

New interpretations or contributions in this chapter include the following:

1. A compilation was prepared of various empirical correlations and general guidelines relevant to the selection of input parameters for the liquefaction-deformation analysis were discussed.

2. Published correlations for friction angle were plotted together on Figure 4-1 to demonstrate the importance of $N_{1-60}$ and confining stress to friction angle.
3. The effect of viscous damping on displacement predictions was discussed. The advantage of using Raleigh RS (stiffness proportional) damping in FLAC was advanced.

4. The use of the MCEER triggering chart (Figure 2-20) was suggested for estimating $CRR_{15}$. Although $CRR_{15}$ can be obtained directly from laboratory data, the empirical triggering chart has the advantage of incorporating field response such as the effect of three-dimensional earthquake loading.

5. The concept was proposed that the strength that can be mobilized immediately after liquefaction might correspond to truly undrained conditions. This strength may be represented by the steady state strength $S_{ss}$, the quasi-steady state strength $S_{qss}$, or the drained strength. The initial strength appropriate for many analyses might be an anisotropic $S_{qss}/\sigma'_{vo}$ value based on undrained laboratory tests. However, this strength could degrade with time and strain due to pore pressure redistribution, void ratio changes, or mixing. Such degraded strengths are likely represented in the case history database of mobilized residual strength as compiled on Figure 2-32. Therefore, analyses that rely upon $S_{qss}/\sigma'_{vo}$ strengths must also address the $S_r$ values of Figure 2-32.

6. Estimates of the shear strain $\gamma_r$ required to mobilize $S_r$ were derived from limited laboratory tests as summarized in Table 4-1 and discussed in Section 4.4.2.

Many of the properties required by the synthesized approach can be estimated for preliminary analyses using the common empirical relations. In many cases, it may be desirable to perform sensitivity studies of the key variables before expending great effort on developing precise estimates. A combination of empirical, laboratory, and in situ testing will generally provide the best characterization.
6.1.5 Chapter 5 — Synthesized Approach: Applications

The synthesized approach was demonstrated in Chapter 5 with four applications: 1) a hypothetical lateral spread site, 2) the response of the Upper San Fernando dam to the 1971 earthquake, 3) the response of the Lower San Fernando dam to this same earthquake, and 4) an evaluation of the Elsie Lake Main dam to a magnitude 6.75 to 7.5 design earthquake. The predictions for the two case histories were compared directly to the observed behaviour, while the response of the Elsie Lake Main dam and the lateral spread site were compared to trends based on empirical relations or other case histories. Reasonable predictions of displacement magnitude and pattern were obtained for each analysis.

An extensive set of sensitivity studies was performed with each analysis. The observed trends indicate the synthesized approach can give reasonable and relatively stable estimates of displacement. The importance of the input parameters to the displacement predictions will vary with the structure being analyzed. However, observations from these analyses should provide some guidance in designing sensitivity studies for future analyses. Parameters having a large impact on the displacement predictions of Chapter 5 include the following:

- direction and character of the horizontal earthquake record,
- method of spectral matching of earthquake record,
- residual strength, including whether \( S_r \) or \( S_r/\sigma'_{vo} \) is used,
- postliquefaction anisotropy of stiffness and strength,
- undrained strength \( S_u \),
- and the postliquefaction stress-strain model, symmetric versus bilinear, for situations of low static bias.

The displacement predictions of Chapter 5 appear to be relatively insensitive to the following parameters:
• vertical earthquake motion,
• amount of RS viscous damping,
• preliquefaction stiffness $G_{dy}$,
• weighting curve,
• and residual strain $\gamma_r$.

The value of CRR$_{15}$ may have a more moderate effect. The criteria used for defining stress reversals may be important beneath slopes.

Overall, the synthesized approach was found to give reasonable predictions of liquefaction-induced displacements for the cases studied. The method appears to be a useful blend of sophistication and approximation. The analyses and interpretations presented in this chapter are original contributions.

6.2 Additional Findings

A brief summary of additional findings, conclusions, and key background information is provided below. Emphasis is given to observations regarding residual strength and earthquake record since these appear to be critical parameters to the prediction of displacement. Although most of the conclusions should be general and apply to analyses at many other sites, the unique behaviour of any given site must always be considered. Many of these conclusions should also be relevant to other analysis techniques of similar sophistication.

6.2.1 $N_{1.60}$ characterization

1. The value of $N_{1.60}$ used to characterize a deposit should typically be less than the median (Section 4.1.1). Using a value of $N_{1/3}$ to characterize a zone was proposed,
where \( N_{1/3} \) is defined as the value of \( N_{1-60} \) where 1/3 of the measurements in the zone are smaller than \( N_{1/3} \) and 2/3 are greater.

2. The analyses of Chapter 5 suggest that an \( N_{1/3} \) characterization is reasonable, although it can give low estimates of residual strength when used with the Idriss curve of Figure 2-32 or the modified Idriss curve of Figure 2-34.

### 6.2.2 Earthquake record

1. The character of the earthquake record may be the most critical factor affecting the displacement predictions.

2. The method used to match the earthquake record to the target response spectrum can have a large influence on the predictions, although the relative effect may be a function of the character of the original record.

3. Selection of appropriate design records can be improved if duration criteria are specified along with magnitude, faulting type, target response spectrum, and the potential near-field effects. Parameters such as Arias Intensity \( I_A \) might also be useful criteria. Inadequate specification of the design earthquake can lead to a very wide scatter in the possible displacement predictions.

4. A reasonably large set of earthquake records should always be analyzed due to the importance of the time domain character of each record. A plot of Arias Intensity \( I_A \) versus time was found to be a useful approach for initial comparison of earthquake records.

5. Near-field velocity pulses can control the displacement response.

6. Input motions should always be analyzed twice, once in either direction, since the predicted response can vary greatly depending on the sense of the record. It is not
always clear from a plot of the input motion which direction will give the largest displacements.

6.2.3 Preliquefaction material parameters

1. MRF and damping values derived from SHAKE analyses appear to be appropriate for use in the synthesized approach.

2. Care must be taken to ensure the motions associated with large displacements are not unduly damped. For this reason, Rayleigh RS (stiffness-proportional) damping is more suitable to the liquefaction-deformation analysis than Rayleigh RMS (mass and stiffness proportional) damping.

3. The center frequency for Rayleigh RS viscous damping can often be assumed equal to the fundamental frequency of the geotechnical structure or deposit prior to liquefaction.

4. The effect of stiffness degradation due to excess pore pressures prior to liquefaction is likely a secondary effect in cases with significant liquefaction.

5. $K_\alpha$ correction factors for the effect of static bias on the triggering of liquefaction should only be used with careful consideration. Increases in triggering resistance due to $K_\alpha$ should be viewed skeptically due to the importance of out-of-plane motions.

6.2.4 Postliquefaction material parameters

1. The mobilized residual strength of a given sand is affected by many factors, including void ratio, anisotropy and loading direction, stress path, confining stress, fabric, drainage, and mixing. Many of these parameters are not known with certainty or may even change during an earthquake.

2. Laboratory tests must adequately simulate these factors, including fabric, void ratio, stress path, and drainage, for the laboratory estimates of residual strength to be valid representations of field behaviour.
3. Local drainage, barriers to pore water flow related to stratigraphy, and the mixing of different soils during large strains may be critical factors in the occurrence of large displacements.

4. The minimum undrained strength from laboratory tests, $S_{qss-ss}$, has been found to normalize with respect to the initial effective major principal stress $\sigma'_{so}$ (Sivathayalan, 2000). It is common to normalize this strength with respect to the initial vertical stress $\sigma'_v$.

5. It is unclear if the mobilized residual strengths estimated from case histories should be expected to normalize with $\sigma'_{so}$ or $\sigma'_v$. These strengths often reflect very high strains that may be affected by pore pressure migration, density changes, or mixing.

6. Laboratory tests for $S_r$ or $S_r/\sigma'_v$ are often difficult to interpret and relate to field behaviour (Section 2.3.3). Reasonable agreement was found between values of $S_r/\sigma'_v$ estimated from case histories and the values determined from laboratory tests of undisturbed strain-softening samples as shown in Figure 2-34. This agreement may be fortuitous since the plotted laboratory values were low estimates of $S_r/\sigma'_v$.

7. $S_r$ values estimated from case histories are in some ways a worst case estimate. The strengths that can be mobilized immediately after triggering of liquefaction, and perhaps well into the earthquake, may be substantially greater.

8. The initial response of sand immediately after triggering might be assumed to be similar to undrained laboratory tests. If this is the case, then an anisotropic strength based on $S_{qss-ss}/\sigma'_v$ may be appropriate immediately after triggering. This strength may later degrade due to pore pressure migration and mixing. The modified Idriss curve relating $S_r/\sigma'_v$ to $N_{1-60cs}$ on Figure 2-34 appears to be an appropriate strength measure for the initial postliquefaction undrained strength for use in many analyses.
9. While an anisotropic $S_r/\sigma'_w$ may be preferred for the initial undrained strength, reasonable results were also obtained using isotropic $S_r$ values for the analyses of the Upper and Lower San Fernando dams.

10. Residual strengths less than the modified Idriss value on Figure 2-34 may be required in situations where a lower bound estimate is appropriate. Deep deposits may also require a reduction in the $S_r$ value.

11. The $S_r$ values shown on Figure 2-32 were estimated from case histories, often after significant displacements had occurred. These values may include the effects of pore water flow, barrier layers, and mixing that are not easily incorporated into laboratory testing. If analyses are performed using $S_r/\sigma'_w$ ratios from either laboratory testing or Figure 2-34, consideration should be given to the possibility that the residual strength will degrade to the values shown on Figure 2-32. The importance of this degradation was shown in the analyses of the Lower San Fernando dam.

12. Soil fabric can have a significant influence on the undrained response. It is important not to apply empirical relations or laboratory results derived from one type of fabric to a soil with a different fabric without careful consideration. For example, the empirical database for residual strength appears to be derived primarily from water-deposited material. Strengths estimated from this database may not be appropriate for soils that were placed by dry dumping or light compaction.

6.2.5 Pore pressures and drainage

1. Flow of pore water out of an element could delay the onset of liquefaction, while flow into an element from adjacent zones could hasten the triggering of liquefaction.

2. Assuming undrained behaviour in an analysis can produce predictions of the anticipated displacements that are low. The effects of drainage may be approximated in
the synthesized approach by selecting material properties that account for the anticipated behaviour.

6.2.6 General observations

1. Imprecision in liquefaction-deformation analyses results from simplifications in the numerical modeling and constitutive relationships as well as uncertainty in the material properties and stratigraphy. Rather than precise predictors of displacement, even sophisticated analysis tools are best used for understanding the potential behaviour of a structure, evaluating the relative effectiveness of remedial measures, and obtaining approximate estimates of the magnitudes of potential displacements.

2. The relative stability and reasonableness of the displacement predictions in Chapter 5 suggest the synthesized approach can be a valuable tool for comparing the relative effectiveness of remediation measures. Guidance can be obtained not only for the extent of remediation but also for the optimal location.

3. In some cases, the synthesized approach may be best used in conjunction with a more fundamental fully coupled effective stress analysis. These complex analysis tools can be used to study detailed effects such as the flow of pore water and its effect on strength, stiffness, and the triggering of liquefaction. The synthesized approach can provide a more conventional interpretation where experience and judgment are more fully developed.

4. The importance of base isolation in any prediction should be carefully evaluated. The synthesized approach can overpredict the effects of base isolation if a low strength is specified in liquefied zones, or if liquefaction is assumed to begin too soon or too abruptly. It is possible for liquefaction of a deep continuous layer to dramatically reduce the number of zones predicted to liquefy in the upper layers. This can lead to a
significant reduction in the estimated displacements. While this may be a reasonable prediction, but its occurrence and significance should always be carefully considered. Residual strength is a critical parameter in many postliquefaction analyses and its importance to the displacement predictions is expected. Unfortunately, it is not a parameter that can be easily or confidently selected.

5. Input properties will generally rely upon a combination of in situ investigation, empirical correlations, and judgment supplemented with limited index and mechanical testing. While laboratory tests provide valuable site-specific information, correlations developed from field behaviour may include processes than cannot be easily or reliably modeled in the laboratory. Such processes include the effects of three-dimensional earthquake loading coupled with a complex pattern of pore pressure redistribution.

6. The analyses of the Lower San Fernando dam demonstrate the difficulties in accurately predicting the occurrence of very large displacements. While the analyses clearly show the dam was at risk for significant distress, an exact prediction of the displacements could not be easily obtained. This finding is likely true for many finite element or finite difference based analyses. This has implications for some remediation schemes for water retaining dams that rely upon increasing the available freeboard to achieve safety criteria.

7. While the inherent simplicity of the synthesized approach is desirable, it also introduces some limitations. Many of these difficulties are common to most liquefaction-deformation analyses. For example, static bias is a hard parameter to address in a simple manner. Stress redistribution and out-of-plane earthquake loading complicate the effects of static bias. The potential for the synthesized approach to
exaggerate the base isolation effect or localize the shear strains can be overcome to some degree through sensitivity study and careful selection of material properties.

### 6.3 Future Development

There may be instances when adding further complication to the basic synthesized approach is warranted. Possible advancements include the following:

1. Adopt a slower transition into liquefied behaviour as observed in many laboratory tests of medium dense sands.

2. Improve modeling of the preliquefaction response. This might include the use of a nonlinear model with proper hysteretic damping and the inclusion of anisotropic stress-strain behaviour.

3. Consider the effects of pore pressure generation on the preliquefaction stiffness. Although this should often be a secondary effect, there may be instances where it becomes important.

4. Consider both pore pressure generation and dissipation in the preliquefaction response. Unfortunately, it may be difficult to rationally consider the flow of pore fluid in an analysis that is founded on total stress assumptions.

5. Investigate the potential advantages and difficulties with using a statistically distributed value of penetration resistance to characterize soil units.

It may often be best to leave the synthesized approach in its most simple but useful form. Many of the more detailed concerns can best be studied with a fully coupled effective stress model. A model that can be conveniently used with the synthesized approach is currently being developed at the University of British Columbia: UBCSAND (Section 3.6.1, Puebla et al., 1997; Beaty & Byrne, 1998).
6.4 Conclusion

A key objective of this thesis is to develop and demonstrate a practical analytical tool for evaluating the liquefaction-deformation problem. While this tool should incorporate a modern understanding of liquefaction behaviour, it is principally intended to be an extension of common practice. The assumptions, parameters, and techniques required to use this approach should be generally understood by many practitioners.

The descriptions and analyses described in this thesis have presented such an analytical tool. The formulation of the method is reasonably simple and founded on basic mechanics. The simplicity of the synthesized approach when compared to fundamental effective stress formulations is one of its greatest strengths. Although simple, many of the observed features of liquefied behaviour are still represented in its formulation. The effect of these features can be evaluated by modifying input values that are relatively intuitive. There is a wealth of experience in selecting many of the required input parameters and with interpreting similar but less sophisticated analyses.

The integration of the liquefaction, flow slide, and limited displacement problems into a modern numerical analysis code allows for complex and sophisticated evaluations of geotechnical structures. This synthesis permits the interplay between the structural dynamic response, the liquefying zones, and the imposed earthquake base motion to be rationally considered. The critical importance of earthquake character is an example of the lessons that can be learned from this model. Perhaps the most valuable asset of the synthesized approach is its capability of improving our understanding of the liquefaction-deformation response at specific sites of concern.
REFERENCES


References


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APPENDIX A — Relationship Between $\Delta \varepsilon_v^p$ and $\Delta \gamma$

The importance of dilation and contraction to undrained response is clearly seen when evaluating the elastic and plastic strain components. The total volumetric strain during undrained loading is trivial due to the stiffness of the pore fluid:

$$\Delta \varepsilon_v = \Delta \varepsilon_v^p + \Delta \varepsilon_v^e \approx 0$$  \hspace{1cm} Eqn. A.1

Simple elasticity theory assumes the increments of elastic shear strain occur at constant volume. Elastic volumetric strain is due only to changes in mean stress:

$$\Delta \varepsilon_v^e = \frac{\Delta \sigma_m'}{B^e}$$  \hspace{1cm} Eqn. A.2

Plastic volumetric strain is assumed to result solely from shear induced dilation or contraction, with negligible plastic strains due to changes in mean stress. The rate of dilation can be expressed as a dilation angle, $\psi$, and is defined as follows:

$$\sin \psi = \frac{\Delta \varepsilon_v^p}{\Delta \gamma^p} \quad \text{or} \quad \Delta \varepsilon_v^p = \sin \psi \cdot \Delta \gamma^p$$  \hspace{1cm} Eqn. A.3

Substituting gives the following relation between changes in mean effective stress and plastic shear strain. If changes in pore pressure $u$ due to changes in mean total stress $\sigma_m$ are ignored, then the relation also applies to $\Delta u$.

$$\Delta u = -\Delta \sigma_m' = -\left(B^e \cdot \sin \psi \right) \cdot \Delta \gamma^p$$  \hspace{1cm} Eqn. A.4

As with volumetric strain, any increment of shear strain can be decomposed into its elastic and plastic parts. The relative amount of plastic shear strain is a function of both the

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elastic and plastic shear moduli, $G^e$ and $G^p$. From elasticity theory, the change in shear stress $\tau$ is directly related to the increment of elastic shear strain $\Delta \gamma^e$:

$$\Delta \tau = G^e \cdot \Delta \gamma^e \quad \text{Eqn. A.5}$$

Noting that plastic shear strains in sand result from changes in stress ratio (Beaty & Byrne, 1998a), the following relationship exists:

$$\Delta \left( \frac{\tau}{\sigma'_m} \right) = \frac{G^p}{\sigma'_m} \cdot \Delta \gamma^p \quad \text{Eqn. A.6}$$

After differentiation, this can be simplified to the following:

$$\Delta \tau = G^p \cdot \Delta \gamma^p + \frac{\tau}{\sigma'_m} \cdot \Delta \sigma'_m \quad \text{Eqn. A.7}$$

Combining Equation A.5 and Equation A.7, substituting $\Delta \gamma^e = \Delta \gamma - \Delta \gamma^p$, solving for $\Delta \gamma^p$, and then substituting into Equation A.4 produces the following relation:

$$\Delta \sigma'_m = \frac{G^e}{G^e + G^p + B^e \cdot \sin \psi} \cdot \frac{\tau}{\sigma'_m} \cdot B^e \cdot \sin \psi \cdot \Delta \gamma \quad \text{Eqn. A.8}$$

The key parameters controlling the pore pressure response during undrained shear loading are seen to be $G^e$, $G^p$, $B^e$, $\psi$, and the current stress ratio $\tau/\sigma'_m$. $G^e$, $G^p$, $B^e$, are all functions of density and confining stress (Byrne et al., 1987) while the dilation angle can be related to the stress ratio (Puebla et al., 1997). It is interesting to note that at higher stress ratios, such as would occur during dilation after liquefaction, both $\sin \psi$ and $G^p$ become small. This reduces the influence of the quotient in Equation A.8 at these stress ratios.

Equation A.8 also sheds some light on postliquefaction stiffness. Loading after liquefaction occurs at nearly constant stress ratio. If $G^e$, $G^p$, and $B^e$ are assumed to
increase at the same rate with increasing normal stress, and \( \sin \psi \) is assumed to remain relatively constant, then an approximate form of Equation A.8 can be rewritten for postliquefaction loading:

\[
\Delta \sigma'_m \propto \left( B^e \cdot \sin \psi \right) \Delta \gamma
\]

Eqn. A.9

Expressing this in terms of the postliquefaction shear modulus gives the following relation:

\[
G_{liq} = \frac{\Delta \tau}{\Delta \gamma} \propto \frac{\Delta \sigma'_m}{\Delta \gamma} \propto B^e \cdot \sin \psi
\]

Eqn. A.10

As with Equation A.4, the elastic bulk modulus and dilation angle are seen to be critical parameters. As postliquefaction loading produces an increase in effective stress, both \( B^e \) and \( G_{liq} \) will increase with loading. This is one factor in the initially concave stress-strain response observed in liquefied soils. Shear strains and increasing effective stresses will eventually decrease and curtail the dilation angle \( \psi \).
CRR data from high quality samples obtained by in situ freezing is summarized below. Five sites are represented: 2 from Niigata, Japan and 3 from British Columbia, Canada. The tested sands were all naturally deposited. The tests cover a wide range in relative densities from 16% to 85%. The results are presented here to provide a convenient set of data. Test results from Meike Elementary School in Niigata, Japan are also presented by Yoshimi et al. (1989) and may be used to extend this data set.

It is important to ensure consistent interpretations when combining CRR data from various sources. Loose sands tend to abruptly reach a state of liquefaction and common criteria for defining liquefaction will likely produce similar interpretations. Denser sands may display a gradual transition into what is generally considered liquefaction. This may cause different but typical criteria to produce very dissimilar interpretations of the same test.

Common criteria for defining the onset of liquefaction in a cyclic test are based on measures of strain. The occurrence of 5% double amplitude (DA) axial strain in a triaxial test is probably the most common benchmark. An alternative criterion based on $r_u$ approaching 100% is an appropriate triggering definition for the synthesized analysis. The subsequent accumulation of strain after $r_u \approx 100\%$ is a function of the relative density and loading direction and can be modeled through the postliquefaction stiffness and strength.

There may be little difference between the 5% DA and the $r_u$ criteria for looser samples, but this does not appear to be the case for dense sand. These sands may require several loading cycles after $r_u$ approaches 100% before this strain level is reached. Data from both a 2.5% and 5% DA criteria were reported for several of the tests on denser sands. Interpretations based on both criteria are presented for comparison.
The best-fit curves to the data assume a linear relationship between log(CRR/CRR_{15}) and log(N_{liq}).

**B.1 Niigata Station Sand (Yoshimi et al., 1984)**

The Niigata Station sand was a dense, very clean, uniform fine sand. Yoshimi et al. (1989) describe the sand as coming from a natural deposit. Additional description is given in Table B-1. The sand was tested using cyclic triaxial equipment with an effective consolidation stress of 98 kPa. The limited data presented by Yoshimi suggests the samples may have reached an $r_u$ of 100% well before the liquefaction criteria of 5% DA axial strain. Yoshimi also provided results corresponding to 2.5% DA strain that suggest a somewhat flatter relationship between $N_{liq}$ and CRR as shown in Table B-2 and Figure B-1.

The values of CRR and $N_{liq}$ reported in Table B-2 were scaled from Yoshimi et al. (1984). The data was also reported in Yoshimi et al. (1989) with some differences. The 1989 data suggests a somewhat steeper curve.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average relative density</td>
<td>87%</td>
</tr>
<tr>
<td>$N_{1-60}$</td>
<td>39</td>
</tr>
<tr>
<td>$D_{50}$</td>
<td>0.29 mm</td>
</tr>
<tr>
<td>$D_{10}$</td>
<td>0.18 mm</td>
</tr>
<tr>
<td>Fines content</td>
<td>0%</td>
</tr>
</tbody>
</table>

Note: SPT blowcount converted to $N_{60}$ using Seed et al. (1984).
Table B-2. CRR data for Niigata Station sand

<table>
<thead>
<tr>
<th>( \text{N}_{\text{liq}} )</th>
<th>CRR</th>
<th>( \text{CRR}_{15} )</th>
<th>( \frac{\text{CRR}}{\text{CRR}_{15}} )</th>
<th>( D_r )</th>
<th>( \sigma'_{30} ) (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Liquefaction criteria = 2.5% DA strain:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td>1.35</td>
<td>0.610</td>
<td>2.21</td>
<td>—</td>
<td>98</td>
</tr>
<tr>
<td>0.8</td>
<td>1.31</td>
<td>0.610</td>
<td>2.15</td>
<td>—</td>
<td>98</td>
</tr>
<tr>
<td>1</td>
<td>0.89</td>
<td>0.610</td>
<td>1.46</td>
<td>—</td>
<td>98</td>
</tr>
<tr>
<td>1.8</td>
<td>0.89</td>
<td>0.610</td>
<td>1.46</td>
<td>—</td>
<td>98</td>
</tr>
<tr>
<td>2</td>
<td>1.03</td>
<td>0.610</td>
<td>1.69</td>
<td>—</td>
<td>98</td>
</tr>
<tr>
<td>2.4</td>
<td>0.8</td>
<td>0.610</td>
<td>1.31</td>
<td>—</td>
<td>98</td>
</tr>
<tr>
<td>7.5</td>
<td>0.66</td>
<td>0.610</td>
<td>1.08</td>
<td>—</td>
<td>98</td>
</tr>
<tr>
<td>14.5</td>
<td>0.69</td>
<td>0.610</td>
<td>1.13</td>
<td>—</td>
<td>98</td>
</tr>
<tr>
<td><strong>Liquefaction criteria = 5% DA strain:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.8</td>
<td>1.35</td>
<td>0.916</td>
<td>1.47</td>
<td>—</td>
<td>98</td>
</tr>
<tr>
<td>7.3</td>
<td>1.31</td>
<td>0.916</td>
<td>1.42</td>
<td>—</td>
<td>98</td>
</tr>
<tr>
<td>10</td>
<td>0.89</td>
<td>0.916</td>
<td>0.97</td>
<td>—</td>
<td>98</td>
</tr>
<tr>
<td>12.5</td>
<td>1.02</td>
<td>0.916</td>
<td>1.11</td>
<td>—</td>
<td>98</td>
</tr>
<tr>
<td>13.5</td>
<td>0.89</td>
<td>0.916</td>
<td>0.97</td>
<td>—</td>
<td>98</td>
</tr>
<tr>
<td>17.5</td>
<td>0.79</td>
<td>0.916</td>
<td>0.86</td>
<td>—</td>
<td>98</td>
</tr>
<tr>
<td>38</td>
<td>0.63</td>
<td>0.916</td>
<td>0.68</td>
<td>—</td>
<td>98</td>
</tr>
<tr>
<td>90</td>
<td>0.68</td>
<td>0.916</td>
<td>0.74</td>
<td>—</td>
<td>98</td>
</tr>
</tbody>
</table>

Note: Data scaled from plots in Yoshimi et al. (1984).

Figure B-1. \( \frac{\text{CRR}}{\text{CRR}_{15}} \) for Niigata Station sand
B.2 Niigata - Showa Bridge (Yoshimi et al., 1989)

Alluvial sand from the Showa bridge site in Niigata was tested in a cyclic triaxial device by Yoshimi et al. (1989). There was a significant difference between the 2.5% DA and 5% DA liquefaction criteria as was seen with the Niigata Station sand. Results from both criteria are reported below.

Table B-3. Description of Niigata – Showa Bridge sand (Yoshimi et al., 1989)

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average relative density</td>
<td>64%</td>
</tr>
<tr>
<td>N_{1-60}</td>
<td>23</td>
</tr>
<tr>
<td>D_{50}</td>
<td>0.30 mm</td>
</tr>
<tr>
<td>D_{10}</td>
<td>0.20 mm</td>
</tr>
<tr>
<td>Fines content</td>
<td>0.2%</td>
</tr>
</tbody>
</table>

Note: SPT blowcount converted to N_{1-60} using Seed et al. (1984).
Table B-4. CRR data for Niigata – Showa Bridge sand

<table>
<thead>
<tr>
<th>$N_{liq}$</th>
<th>CRR</th>
<th>CRR$_{15}$</th>
<th>CRR/CRR$_{15}$</th>
<th>$D_r$</th>
<th>$\sigma'_3$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Liquefaction criteria = 2.5% DA strain:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.488</td>
<td>0.277</td>
<td>1.762</td>
<td></td>
<td>98</td>
</tr>
<tr>
<td>3</td>
<td>0.319</td>
<td>0.277</td>
<td>1.152</td>
<td></td>
<td>98</td>
</tr>
<tr>
<td>20.5</td>
<td>0.25</td>
<td>0.277</td>
<td>0.902</td>
<td></td>
<td>98</td>
</tr>
<tr>
<td>88</td>
<td>0.221</td>
<td>0.277</td>
<td>0.798</td>
<td></td>
<td>98</td>
</tr>
<tr>
<td><strong>Liquefaction criteria = 5% DA strain:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td>0.488</td>
<td>0.304</td>
<td>1.605</td>
<td></td>
<td>98</td>
</tr>
<tr>
<td>6</td>
<td>0.319</td>
<td>0.304</td>
<td>1.049</td>
<td></td>
<td>98</td>
</tr>
<tr>
<td>25.5</td>
<td>0.25</td>
<td>0.304</td>
<td>0.822</td>
<td></td>
<td>98</td>
</tr>
<tr>
<td>94</td>
<td>0.221</td>
<td>0.304</td>
<td>0.727</td>
<td></td>
<td>98</td>
</tr>
</tbody>
</table>

Note: Scaled from Yoshimi et al. (1989).

Figure B-2. CRR/CRR$_{15}$ for Niigata – Showa Bridge sand
B.3 Duncan Dam Sand (B.C. Hydro, 1992)

Duncan dam is located in southeastern British Columbia, Canada. The sand from the dam foundation is fine, moderately dense, and angular to subangular with about 10% fines. It was deposited in a glaciofluvial or alluvial fan environment (Little et al., 1994). The tested samples were obtained at a depth of 12 to 17 m below the downstream toe of the dam. Additional description is provided in Table B-5.

Cyclic simple shear and triaxial tests were performed. A liquefaction criterion of 4% single amplitude (SA) shear strain was used for simple shear and 2.5% SA axial strain for triaxial. The strain criteria are absolute measures and include any strain offset that may occur during loading. The criteria for the two tests are approximately equivalent. The strains observed in the tests tend to increase rapidly as liquefaction is approached. CRR results are provided in Table B-6 and Figure B-3.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average relative density</td>
<td>56%</td>
</tr>
<tr>
<td>N&lt;sub&gt;1-60&lt;/sub&gt;</td>
<td>≈ 10</td>
</tr>
<tr>
<td>D&lt;sub&gt;50&lt;/sub&gt;</td>
<td>0.17 – 0.22 mm</td>
</tr>
<tr>
<td>e&lt;sub&gt;max&lt;/sub&gt;</td>
<td>1.153</td>
</tr>
<tr>
<td>e&lt;sub&gt;min&lt;/sub&gt;</td>
<td>0.720 – 0.768</td>
</tr>
<tr>
<td>Fines content (minus #200)</td>
<td>10%</td>
</tr>
</tbody>
</table>
Appendix B — Cyclic Resistance Ratio from Frozen Samples

Table B-6. CRR data for Duncan dam sand (B.C. Hydro, 1992)

<table>
<thead>
<tr>
<th>N\text{liq}</th>
<th>CRR</th>
<th>CRR\text{15}</th>
<th>CRR/CRR\text{15}</th>
<th>D_r</th>
<th>\sigma'_v or \sigma'_30 (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Cyclic simple shear:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.145</td>
<td>0.135</td>
<td>1.074</td>
<td>51%</td>
<td>196</td>
</tr>
<tr>
<td>29</td>
<td>0.122</td>
<td>0.135</td>
<td>0.904</td>
<td>34%</td>
<td>196</td>
</tr>
<tr>
<td>6</td>
<td>0.161</td>
<td>0.135</td>
<td>1.192</td>
<td>53%</td>
<td>196</td>
</tr>
<tr>
<td>16</td>
<td>0.148</td>
<td>0.149</td>
<td>0.993</td>
<td>65%</td>
<td>392</td>
</tr>
<tr>
<td>4.5</td>
<td>0.173</td>
<td>0.149</td>
<td>1.161</td>
<td>64%</td>
<td>392</td>
</tr>
<tr>
<td>8.5</td>
<td>0.160</td>
<td>0.149</td>
<td>1.074</td>
<td>56%</td>
<td>392</td>
</tr>
<tr>
<td>12</td>
<td>0.144</td>
<td>0.145</td>
<td>0.993</td>
<td>57%</td>
<td>589</td>
</tr>
<tr>
<td>4.5</td>
<td>0.175</td>
<td>0.145</td>
<td>1.207</td>
<td>60%</td>
<td>589</td>
</tr>
<tr>
<td>44</td>
<td>0.127</td>
<td>0.145</td>
<td>0.876</td>
<td>60%</td>
<td>589</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Cyclic triaxial:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>0.130</td>
<td>0.160</td>
<td>0.812</td>
<td>?</td>
<td>200</td>
</tr>
<tr>
<td>31</td>
<td>0.160</td>
<td>0.160</td>
<td>1.000</td>
<td>64%</td>
<td>200</td>
</tr>
<tr>
<td>5</td>
<td>0.175</td>
<td>0.160</td>
<td>1.094</td>
<td>56%</td>
<td>200</td>
</tr>
<tr>
<td>7</td>
<td>0.175</td>
<td>0.157</td>
<td>1.115</td>
<td>51%</td>
<td>1200</td>
</tr>
<tr>
<td>23</td>
<td>0.151</td>
<td>0.157</td>
<td>0.962</td>
<td>62%</td>
<td>1200</td>
</tr>
<tr>
<td>9</td>
<td>0.162</td>
<td>0.157</td>
<td>1.032</td>
<td>32%</td>
<td>1200</td>
</tr>
</tbody>
</table>

Figure B-3. CRR/CRR\text{15} for Duncan dam sand
Appendix B — Cyclic Resistance Ratio from Frozen Samples

B.4 Fraser River – Massey Sand (Wride & Robertson, 1997)

The Massey site is located along the southern arm of the Fraser river in southwestern British Columbia, Canada. The alluvial sand is loose, fine, and subrounded. Cyclic simple shear tests were performed with a strain criteria of 4% SA. Additional information is provided in Table B-7, Robertson et al. (2000b), and Vaid et al. (1998).

Table B-7. Description of Fraser River – Massey sand (Wride & Robertson, 1997; Robertson et al. 2000b)

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative density</td>
<td>24-40%</td>
</tr>
<tr>
<td>N1-60</td>
<td>10</td>
</tr>
<tr>
<td>D$_{50}$</td>
<td>0.20 mm</td>
</tr>
<tr>
<td>D$_{10}$</td>
<td>0.14 mm</td>
</tr>
<tr>
<td>$e_{\text{max}}$</td>
<td>1.1</td>
</tr>
<tr>
<td>$e_{\text{min}}$</td>
<td>0.7</td>
</tr>
<tr>
<td>Fines content (minus #200)</td>
<td>&lt;5%</td>
</tr>
</tbody>
</table>
Table B-8. CRR data for Fraser River – Massey sand
(Wride & Robertson, 1997)

<table>
<thead>
<tr>
<th>N_{liq}</th>
<th>CRR</th>
<th>CRR_{15}</th>
<th>CRR/CRR_{15}</th>
<th>D_r</th>
<th>\sigma'_v (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.125</td>
<td>0.102</td>
<td>1.225</td>
<td>40%</td>
<td>123</td>
</tr>
<tr>
<td>6</td>
<td>0.108</td>
<td>0.102</td>
<td>1.059</td>
<td>37%</td>
<td>118</td>
</tr>
<tr>
<td>7</td>
<td>0.100</td>
<td>0.102</td>
<td>0.980</td>
<td>33%</td>
<td>111</td>
</tr>
<tr>
<td>8</td>
<td>0.107</td>
<td>0.102</td>
<td>1.049</td>
<td>26%</td>
<td>132</td>
</tr>
<tr>
<td>9</td>
<td>0.109</td>
<td>0.102</td>
<td>1.069</td>
<td>26%</td>
<td>112</td>
</tr>
<tr>
<td>9</td>
<td>0.108</td>
<td>0.102</td>
<td>1.059</td>
<td>29%</td>
<td>112</td>
</tr>
<tr>
<td>13</td>
<td>0.112</td>
<td>0.102</td>
<td>1.098</td>
<td>34%</td>
<td>134</td>
</tr>
<tr>
<td>18</td>
<td>0.105</td>
<td>0.102</td>
<td>1.029</td>
<td>36%</td>
<td>118</td>
</tr>
<tr>
<td>20</td>
<td>0.090</td>
<td>0.102</td>
<td>0.882</td>
<td>24%</td>
<td>111</td>
</tr>
<tr>
<td>25</td>
<td>0.098</td>
<td>0.102</td>
<td>0.961</td>
<td>36%</td>
<td>118</td>
</tr>
<tr>
<td>31</td>
<td>0.095</td>
<td>0.102</td>
<td>0.931</td>
<td>26%</td>
<td>132</td>
</tr>
</tbody>
</table>

Note: Two samples with D_r \geq 60\% were excluded.
B.5 Fraser River – Kidd II Sand (Wride & Robertson, 1997)

The Kidd II site is north of the Massey site in the Fraser river delta, British Columbia, Canada. The alluvial sand is loose, fine, and subrounded. Additional information is provided in Table B-9, Robertson et al. (2000), and Vaid et al. (1998). Cyclic simple shear tests were performed with a strain criteria of 4% SA. CRR results are shown in Table B-10 and Figure B-5.

Table B-9. Description of Fraser River – Kidd II sand
(Wride & Robertson, 1997; Robertson et al. 2000b)

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative density</td>
<td>16-33%</td>
</tr>
<tr>
<td>$N_{1-60}$</td>
<td>13</td>
</tr>
<tr>
<td>$D_{50}$</td>
<td>0.20 mm</td>
</tr>
<tr>
<td>$D_{10}$</td>
<td>0.14 mm</td>
</tr>
<tr>
<td>$e_{\text{max}}$</td>
<td>1.1</td>
</tr>
<tr>
<td>$e_{\text{min}}$</td>
<td>0.7</td>
</tr>
<tr>
<td>Fines content (minus #200)</td>
<td>&lt;5%</td>
</tr>
</tbody>
</table>
Table B-10. CRR data for Fraser River – Kidd II sand
(Wride & Robertson, 1997)

<table>
<thead>
<tr>
<th>$N_{liq}$</th>
<th>CRR</th>
<th>$CRR_{15}$</th>
<th>$CRR/CRR_{15}$</th>
<th>$D_r$</th>
<th>$\sigma'_{vo}$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.118</td>
<td>0.101</td>
<td>1.168</td>
<td>16%</td>
<td>138</td>
</tr>
<tr>
<td>6</td>
<td>0.110</td>
<td>0.101</td>
<td>1.089</td>
<td>22%</td>
<td>153</td>
</tr>
<tr>
<td>12</td>
<td>0.094</td>
<td>0.101</td>
<td>0.931</td>
<td>18%</td>
<td>154</td>
</tr>
<tr>
<td>12</td>
<td>0.119</td>
<td>0.101</td>
<td>1.178</td>
<td>33%</td>
<td>138</td>
</tr>
<tr>
<td>38</td>
<td>0.089</td>
<td>0.101</td>
<td>0.881</td>
<td>21%</td>
<td>137</td>
</tr>
</tbody>
</table>

Note: One sample with $D_r = 56\%$ was excluded.

Figure B-5. $CRR/CRR_{15}$ for Fraser River – Kidd II sand
APPENDIX C — Input Variables for Synthesized Approach

Key input variables for the synthesized approach are given below. The description is taken from the program documentation so some of the variable names correspond to naming conventions in the FLAC macro language (Itasca, 1998). Many of the variables are simple program control parameters or less significant material properties and are typically left at the default values. Some of the minor program options have not been described in the thesis, but should be self-explanatory.

Listing of notes and input variables from UBCTOT6t.FIS:

```plaintext
;***TO USE:
; 1. Define grid and perform steady-state solution.
; 2. Call UBCTOT6t.FIS
; 3. Initialize input variables.
; 4. Modify program control variables if desired.
; 5. Define undrained strengths.
; 6. Run dyprop fish function.
; 6. Setup dynamic analysis and perform solution.

;***NOTES on UBCTOT6t:
; 1. Requires FLAC3.4 (or better?).
; 2. Only affects zones using the Mohr-Coulomb model.
; 3. Does not allow liquefaction in zones with zero pore pressure.
; 4. Uses CRR vs (Nl)60cs from MCEER 1997. CRR limited to max of 0.6 for (Nl)60cs > 30.26.
; 5. Uses fines correction for triggering from MCEER 1997 ($fctrig=1).
; 6. Uses fines correction for Sr from Seed-Harder 1990 if ex_2 or ex_3<0 ($fcsr=1).
; 7. Uses Ksigma from 1997 MCEER Workshop report.
; 8. Limits Tauxy15 to a minimum of $taul5fac*SPa.
; 9. Limits Sr to a minimum of $Srfac*SPa & $Srmin*Sr-compression.
; 10. Limits Sr to a maximum of the drained strength: Sr <= cohesion*cos(\phi) + Sigy'*sin(\phi)
; 11. Bulk modulus assumptions:
     Un saturated: B = Gmax
     Non-liquefied, saturated: B = ex_9(i,j)*Gdyn
     Post-liquefied: B = $bratioliq*(Gliq in simple shear)
     Note: Review B if use config gw + water bulk.
; 12. Triggering is based on Sxy since initiation of strain-softening is a function of stress, not cyclic amplitude, when there is a static bias. Sxy-trig is found by combining the cyclic amplitude required for trigger with Sxy-static. Liquefaction generally occurs when the cycle counter (ex_13) equals 1 or slightly higher. Some situations occur where ex_13 can get very large and the zone does not trigger. Use ex_11 to set N liqmax, the max allowable value of ex_13 before the zone is forced to trigger regardless of Sxy.
; 13. Anisotropy of Sr and Gliq defined by values of ex_2, ex_3, and
```
Appendix C — Input Variables for Synthesized Approach

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$Srdrained. Possible to define anisotropic Gliq while using isotropic Sr.

14. There are two methods for including a "factor of safety" against liquefaction. Both methods factor the liquefaction resistance, making it easier to liquefy when $FS > 1.

- $FSN modifies the number of cycles required to trigger.
- $FSCRR modifies Tauxy15.

In general, only one method should be used.

***REQUIRED INPUT VARIABLES:

- $Pa = Atmospheric Pressure (DEFAULT = 100. kPa)
- ex_1 = (Nl)60 blowcount
- ex_2 = Residual Strength (Sr) or Ratio (Sr/Esyy) in compression
  - If < 0, uses built-in curve specified by $Srcurve and multiplies by abs(ex_2). Sr/s' or Sr based on $Srtype.
- ex_3 = Residual Strength (Sr) or Ratio (Sr/Esyy) in simple shear
  - If < 0, uses built-in curve specified by $Srcurve and multiplies by abs(ex_3). Sr/s' or Sr based on $Srtype.
- ex_4 = Residual strain in decimal for simple shear (if $Gliq = 0.0)
  - Residual stiffness in simple shear (if $Gliq = 1)
- ex_5 = K2max
  - >= 0, K2max = ex_5
  - =0, K2max estimated from 20*((Nl)60)^1/3 (Tokimatsu&Seed,1987)
  - <0, K2max estimated from abs(ex_5)*20*((Nl)60)^1/3
- ex_6 = Shear Modulus Reduction Factor (G/Gmax)
- ex_7 = Trigger flag
  - 0 if neither dynamic properties or triggering is performed.
  - 1 if dynamic properties set, but triggering is not evaluated.
  - 2 if dynamic properties set, triggering evaluation performed.
  - 3 if dynamic properties set, but trigger time defined by ex_8.
- ex_8 = Dytime for imposed trigger (only if ex_7=3).
- ex_9 = B/Gdyn for saturated, non-liquefied elements (Default=10,pr=0.45).
- ex_10 = Fines content (<#200) in decimal.
- ex_11 = Value of Nliq_max (DEFAULT = 100.).

***PROGRAM CONTROL VARIABLES (Can modify values after call UBCTOT6+.fis):

---Primary:
- $biased = Toggles between assumption of symmetric or one-sided loading.
  - 0 DEFAULT. Assumes post-liq loading is one-sided.
  - 1 Uses a very low stiffness (slack) zone immediately following a post-liq stress reversal. The cumulative post-liq shear strain is monitored in equally-spaced directions as defined by $esector. The current strain in each direction is compared with the corresponding maximum and minimum values. If the current strain does not exceed the max and min limits in ANY direction, then a soft G is used (G = $Greduce*Gliq). If the current strain is greater than the max or less than the min, then G=Gliq.
  - 2 Same as 1, except G is reduced only if strain limits are exceeded in directions which are within +/- 10 degrees of the current maximum increment of shear strain.
- $CRRn1 = Defines ratio of (CRR at N=1)/(CRR at N=15) where N=# of cycles to liq at CRR. CRR v N curve is assumed linear on a log-log plot (see Idriess, 1998).
  - 2.0 DEFAULT.
  - Example values:
    - 2.5 - Frozen samples of dense sand (Idriess 1999; Yoshimi 1984).
    - 2.2 - Upper San Fernando Dam (Seed et al. 1973).
    - 2.1 - Lower San Fernando Dam (Seed et al. 1998).
Appendix C — Input Variables for Synthesized Approach

2.0 - Compiled data from Byrne (St. Louis 1991).
"Average" for N < 20.
1.5 - Tentative value for loose sands.

$Exectime = Time interval in secs between execution of TRIGGER routine.
=> 0.001 DEFAULT.

$FSCRR = Factor of safety against liquefaction applied to CRR value.
CRR = CRRold / $FSCRR.
=> 1.0 DEFAULT.

$FSN = Factor of safety against liquefaction applied to cycle counter.
sum(N) at trig = 1.0 / $FSN
=> 1.0 DEFAULT.

$Gliq = Determines if residual strain or stiffness is input in ex_4.
=> 0 DEFAULT. Residual strain where Gliq = Sr/ex_4.
=> 1 Residual stiffness, Gliq, in simple shear.

$SRev = Base post-liq stress reversal on "Sxy" or "Taumax + direction".
=> 0 DEFAULT. Base on reversal of Sxy.
=> 1 Base on Taumax + direction. Stress reversal is assumed
when direction of current max shear stress is > 45 deg
from direction of peak shear stress since last reversal.

$Srtype = Determines type of residual strength specified.
=> 0 DEFAULT. Sr/Esyy values are specified.
=> 1 Sr values are specified.

$Trigger = Turns TRIGGER function on and off.
=> 0 DEFAULT. Trigger subroutine not executed.
=> 1 Trigger subroutine executed.

---Other:

$Alphamax = Max value of alpha used in Kalpha relationship (for $Kalpa=0).
=> 0.3 DEFAULT. $Alphamax may be reduced, but not increased.

$anitime = Defines time in secs between Sr updates due to anisotropy.
=> 0.005 DEFAULT.

$bratioliq = Ratio of B/Gliq-ss for saturated, liquefied elements.
=> 50. DEFAULT. (pr=0.49 at ss)

$esector = For $biased=1-2. The range of strain directions from 0 to 90
degrees is divided into a number of equal sectors as defined
by $esector. If more than 12 sectors are desired, then
the value of $esector must be changed in init_tot_var.
=> 12 DEFAULT. (Must use AT LEAST 5).

$fcsr = Determines whether fines correction applied to Sr estimates.
For ex_2 or ex_3 < 0.
=> 1 DEFAULT. Use Seed-Harder 1990 correction.
=> 0 No correction.

$fctrig = Determines whether fines correction is applied to CRR.
=> 1 DEFAULT. Use MCEER 1997 correction.
=> 2 Use Seed-Harder 1990 Sr correction.
=> 0 No correction.

$Greduce = For $biased=1-2. G = $Greduce*Gliq in slack zone.
=> 0.05 DEFAULT.

$HydStr = Defines whether hydrostatic stress state is imposed at every
post-liq stress reversal, or just on first post-liq reversal.
=> 0 DEFAULT. Imposed every post-liq reversal.
=> 1 Imposed only on first post-liq reversal.

$Kalpa = Defines assumption for Kalpa factor.
=> 1 DEFAULT. Kalpa = 1.0.
=> 0 Kalpa defined by APPROX representation of MCEER 1997.
Use with caution. Should only use for sigvo' < 3tsf.
=> -1 Same as $Kalpa=0, except Kalpa <= 1.0.

$Ksigmafac = Factors Ksigma correction: Ksigma-mod = -1-(1-Ksigma)*$Ksigmafac.
=> 1 DEFAULT. Ksigma not modified.

$liqdamp = Fraction of critical damping assigned to liquefied elements.
Fc not changed. Assumes Rayleigh damp: M+S, Mass, or Stiff.
=> 0.02 DEFAULT. (2% of critical)

$nhsteps = Defines # of $Exectime steps used to collapse stresses to
hydrostatic upon post-liq reversal. Increase number of steps if problems occur due to rapid stress change.

$\texttt{Nmethod}$ = Defines method used for estimating trigger time.

- $\texttt{0}$ DEFAULT. Triggering evaluated continuously. Elastic constants and strength modified at triggering.
- $\texttt{1}$ Triggering evaluated after each half cycle. Elastic constants, strength, and stress are modified at trigger.

$\texttt{Noliq}$ = Sets maximum value of N1-60cs for which liquefaction is permitted. Zones with N1-60cs > $\texttt{Noliq}$ are non-liquefiable.

- $\texttt{30}$. DEFAULT.

$\texttt{Srcurve}$ = Determines Sr vs (N1)60cs curve for ex_2 or ex_3 < 0.

- If $\texttt{Srtype}=0$, then Sr/sigy' values are estimated by assuming original curve is appropriate for ITSF (Byrne and Beaty, 1999).
- $\texttt{0}$ DEFAULT. Use Idriss curve (1998).
- $\texttt{1}$ Use Seed-Harder lower bound (1990).

$\texttt{Srdrained}$ = Determines if post-liq cohesion is set to anisotropic Sr or to isotropic undrained strength.

- $\texttt{0}$ DEFAULT. Anisotropic Sr as specified in ex_2 and ex_3.
- $\texttt{1}$ Isotropic Sr as specified in ex_2. ex_3 used to define anisotropic Glq.
- $\texttt{2}$ Isotropic Su as specified by cohesion and phi prior to earthquake. ex_2 and ex_3 used to define anisotropic Glq. Strength not limited by $\texttt{Srfac}$.

NOTE: Glq always affected by $\texttt{Srfac}$, $\texttt{Sd}$, and $\texttt{Srmin}$.

$\texttt{Srfac}$ = Defines minimum value of residual strength: Sr >= $\texttt{Srfac}$*$\texttt{Pa}$.

- $\texttt{0.03}$ DEFAULT.

$\texttt{Srmin}$ = Defines minimum value of residual strength for anisotropic Sr: Sr >= $\texttt{Srmin}$*$\texttt{Sr-compression}$.

- $\texttt{0.06}$ DEFAULT.

$\texttt{StrnTrg}$ = Adds strain-based trigger in addition to cycle-based trigger.

- $\texttt{0}$ DEFAULT. Strain-based trigger not performed.

$\texttt{Sucheck}$ = Increases value of pre-liq Su to reduce chance of liquefaction being curtailed due to failure on a non-xy plane. Based on assumption that sigm and sigy do not change during loading.

- Under these assumptions, makes sure that zone has enough strength to permit liquefaction in 3 cycles. Change in Su applied only to cohesion.

NOTE: Cohesion_required / Cohesion_original stored in ex_20.

- $\texttt{0}$ DEFAULT. Su not modified.
- $\texttt{1}$ Su modified where required (limited by $\texttt{Sulim}$ and 1.0).

$\texttt{Sulim}$ = Limits the increase in Su due to $\texttt{Sucheck}$.

- $\texttt{1.3}$ DEFAULT.

$\texttt{Tau15fac}$ = Defines minimum value of Tauxy15. Tauxy15 >= $\texttt{Tau15fac}$*$\texttt{Pa}$.

- $\texttt{0.05}$ DEFAULT. (5 kPa, 100 psf)

$\texttt{UnldFac}$ = Determines when to switch from loading to unloading moduli for post-liq. Switches at $\texttt{UnldFac}$*$($peaksxy or $\texttt{peaktaumax}$).

- $\texttt{0.99}$ DEFAULT.

$\texttt{UnldMod}$ = Defines unload modulus for liquefied soil:

- Glq (unload)=Glq(load)*$\texttt{UnldMod}$.

- $\texttt{10}$. DEFAULT.

$\texttt{UnldModSft}$ = Defines unloading stiffness of liquefied zones near stress reversal (see $\texttt{UnldSft}$): G = $\texttt{UnldModSft}$ * Glq(load).

- $\texttt{1.0}$ DEFAULT.

$\texttt{UnldSft}$ = Post-liq unload modulus is softened when shear stress is less than $\texttt{UnldSft}$*$($peaksxy or $\texttt{peaktaumax}$).

- $\texttt{0.2}$ DEFAULT.
APPENDIX D — Flow Chart of Key Operations in Synthesized Approach

INITIAL MODEL SETUP (Manual)

1. Set up grid.
2. Perform static analysis.
   a. construction sequence
   b. loading sequence
   c. groundwater
3. Call UBCTOT FISH routine.
   a. Runs init_tot_var subroutine to initialize variables
   b. Loads DYPROP and TRIGGER subroutines
4. Initialize key input/material properties.
   a. N1-60
   b. Sr or Sr/\sigma'vo in compression (or use built-in curves)
   c. Sr or Sr/\sigma'vo in simple shear (or use built-in curves)
   d. K2max or stiffness factor
   e. MRF
   f. Trigger flag
   g. B^{e}/G^{dy}n for nonliquefied elements
   h. Fines content
5. Modify program control and miscellaneous input variables.
6. Define undrained strengths in saturated zones.
7. Call DYPROP routine for element setup.

\[\text{ELEMENT SETUP} \quad \text{(Performed by DYPROP subroutine)}\]

1. Check pore pressure for liquefiability.
2. Estimate G_{max} from \sigma'_{m}.
3. Compute G_{dyn} = MRF * G_{max}.
4. Set bulk modulus.
5. Compute CRR1 from 1997 MCEER curve, N_{1-60}, and fines content.
6. Compute K_{s} from 1997 MCEER (optional).
7. Compute K_{a} from 1997 MCEER (optional).
8. Compute \tau_{15} = K_{a} * K_{s} * CRR1 * \sigma'_{vo} (\tau_{15} > \$\text{tau15fac} * P_{d}).
9. Estimate S_{r} and G_{liq} in compression and simple shear.
   a. Use specified S_{r} or S_{r}/\sigma'vo
   b. Or built-in Seed & Harder (1990) lower bound
   c. Or built-in Idriss (1998)
   d. S_{r} > \$Sr_{min} * S_{r} in compression
   e. S_{r} \leq S_{u}
10. Adjust S_{u} for liquefiability concerns (optional).
PREPARE DYNAMIC ANALYSIS (Performed manually)

1. Set up dynamic analysis.
   a. Define input motion
   b. Define damping
2. Initialize displacements and velocities of nodes to zero.

TRIGGERING AND POSTLIQUEFACTION CALCULATIONS
(Performed by TRIGGER subroutine)

If liquefied or liquefiable:
1. Compute axial and shear strains.
2. Compute $\alpha_o$, the direction of $\sigma'$.

If liquefiable but not yet liquefied:
1. Perform weighting curve calculations.
2. Check $\tau_{\text{liq}}$ criteria for triggering.
3. Check strain-based criteria for triggering.
4. Check max $\Sigma N_{\text{liq}}$ criteria for triggering.

At Triggering of Liquefaction:
1. Adjust strength to anisotropic $S_r$.
2. Initial peak $\tau_{\text{max}}$ or $\tau_{\text{cy}}$ for stress reversal.
3. Adjust viscous damping.
If Liquefied Element:
1. Compute smoothed value of $\alpha_\sigma$.
2. Adjust $S_r$ for smoothed $\alpha_\sigma$.
3. Check/update strain limits and adjust $G$ for symmetric loading.
4. Check/update peak $\tau_{max}$ or $\tau_{xy}$ for stress reversal.
5. Impose hydrostatic stresses if stress reversal.
6. Adjust $G$ based on load/unload/reload criteria and stress reversal.
7. Adjust modulus as approach stress reversal.
APPENDIX E — Response Spectra and Earthquake Plots for ELMD Analyses

This appendix contains plots of response spectra and time histories for the earthquake motions used in the analyses of the Elsie Lake Main Dam. Further details are provided in Section 5.4. The time histories are plotted to the same scale to permit easier comparison between the earthquake records. The only exceptions are the velocity and displacement plots for the Lucerne Valley 270 records that required significantly different scales.

Although all of these records meet the criteria for the design earthquake, their character in the time domain varies greatly.

Figure E-1. Response Spectra at 5% Damping for TCU078 E-W records.
Figure E-2. Response Spectra at 5% Damping for TCU129 E-W record.

Figure E-3. Response Spectra at 5% Damping for Pacoima Dam D/S 265 record.
Figure E-4. Response Spectra at 5% Damping for Lucerne Valley 270 records.
Figure E-5. Time histories of TCU078Esca (scaling factor = 0.8).
Figure E-6. Time histories of TCU078Esyn (modified by SYNTH).
Figure E-7. Time histories of TCU078Ersp (modified by RSPMatch).
$pga = 0.50 \, g$

$pgv = 0.35 \, m/s$

$pgd = 0.22 \, m$

Figure E-8. Time histories of TCU129Esyn (modified by SYNTH).
Appendix E — Response Spectra and Earthquake Plots for ELMD Analyses

\[ pga = 0.54 \, g \]

\[ pgv = 0.39 \, m/s \]

Figure E-9. Time histories of PDD265sca (scaling factor = 1.26).
Figure E-10. Time histories of LV270sca (scaling factor = 1.2).
Appendix E — Response Spectra and Earthquake Plots for ELMD Analyses

Figure E-11. Time histories of LV270syn (modified by SYNTH).
Figure E-12. Time histories of LV270rsp1 (modified by RSPMatch).
Appendix E — Response Spectra and Earthquake Plots for ELMD Analyses

Figure E-13. Time histories of LV270rsp2 (modified by RSPMatch).