FINITE ELEMENT MODEL
FOR CYCLIC LOADING OF CONCRETE FILLED STEEL TUBE PILE
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We accept this thesis as conforming
to the required standard

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A non-linear finite element analysis is presented to predict the lateral response of a concrete-filled steel tube (CFT) pile subjected to cyclic loading. The computer program CFTPILE was developed as a part of this study. The pile is modeled as a beam element on a non-linear soil medium, which is able to resist compression only. Information on the soil-pile interaction and the stress-strain relationships for concrete and steel is required in order to calculate the response of the pile. The soil-pile interaction is assumed to follow a P-y curve proposed by Yan and Byrne. The formation of gaps between pile and soil is taken into account in the analysis. Tomii and Sakino’s confined concrete model is adopted to determine the stress-strain relationship for the concrete inside steel tube. Steel is assumed to be elasto-perfectly plastic. Numerical examples are presented for a CFT and a Hollow section pile.

Variability of pile capacity caused by the uncertainty in soil properties is investigated using the computer program RELAN and a response surface involving six random variables. The cumulative distribution function for maximum pile capacity is calculated for cases with and without correlation between the soil variables.
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1 INTRODUCTION

A pile using a concrete filled steel tube (CFT) has many advantages when compared to a hollow steel tube or a reinforced concrete member. For example, a hollow steel tube section has to meet certain limitation of diameter-thickness ratio (D/t) in order to have enough rotation capacity without triggering local bucking. On the other hand, the CFT member shows a significant yielding plateau in the load-deflection curve as a beam member. A test reported by Prion and Boehme (1994) shows a ductile behaviour of CFT members indicating the fact that neither concrete nor steel tubes are individually able to absorb significant amounts of energy under cyclic loading. The concrete inside the steel tube keeps it from buckling locally and the external steel casing prevents premature spalling and crushing of the concrete, providing a ductile behaviour that is superior to a hollow steel tube.

Research has been done with regard to the behaviour of hollow steel tube piles under earthquake load. When the pile is subjected to an earthquake excitation, it is required to have high-energy absorption capacity in order to resist the earthquake load in a ductile manner. This is provided by filling the steel tube pile with concrete.

The objective of this work is to present a non-linear finite element analysis to determine the lateral response of a CFT pile subjected to static cyclic loading. The model presented in this study is based on the analysis program HYST (Foschi, 2000), which was developed to calculate hysteresis loops of shear connectors in a non-linear medium, namely wood. Using the model, a CFT pile analysis program called CFTPILE (Appendix B), was developed as part of this work. The pile response to lateral cyclic head loading
has a non-linearity, which is caused by the elasto-plastic properties of the steel tube, the non-linear concrete behaviour, the non-linear soil-pile interaction and the formation of gaps between soil and pile.

Stress-strain relationships for steel and concrete were defined in this analysis to calculate stresses at each point over the cross-section and along the member. The soil-pile interaction was determined using P-y curves developed by Yan and Byrne (1992).

The analysis required the calculation of the tangent stiffness matrix at each step of loading. This was done numerically using a beam finite element.
To find the lateral response of a CFT pile subjected to cycling loading, the lateral load associated with a given displacement of the pile cap is obtained by a non-linear finite element analysis.

A beam element model is shown in Fig.1.
Each node at the ends of the element has five degrees of freedom: \( w, w', w'', u \) and \( u' \). \( u \) is the axial displacement in the x-direction at the centroid of the cross section, and \( w \) is the lateral displacement in the y-direction. \( w', w'' \) and \( u' \) are their derivatives with respect to \( x \). The displacement \( u(x) \) and \( w(x) \) are assumed to be, respectively, a cubic polynomial and a fifth-order polynomial. These functions can be written in matrix form using the vectors of shape functions and their derivatives, and the degrees-of-freedom vectors \( \mathbf{a} \):

\[
\begin{align*}
\mathbf{u}(x) &= \mathbf{N}_0^T \mathbf{a} \\
\mathbf{u}'(x) &= \mathbf{N}_1^T \mathbf{a} \\
\mathbf{w}(x) &= \mathbf{M}_0^T \mathbf{a} \\
\mathbf{w}'(x) &= \mathbf{M}_1^T \mathbf{a} \\
\mathbf{w}''(x) &= \mathbf{M}_2^T \mathbf{a}
\end{align*}
\]

Eq. 1

\[
\begin{bmatrix}
\mathbf{u} \\
\mathbf{w}
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 & n4 & n5 & 0 & 0 & 0 & n9 & n10 \\
n11 & n12 & n13 & 0 & 0 & m6 & m7 & m8 & 0 & 0
\end{bmatrix} \begin{bmatrix}
a1 \\
a2 \\
a3 \\
a4 \\
a5 \\
a6 \\
a7 \\
a8 \\
a9 \\
a10
\end{bmatrix}
\]

Eq. 2

\( \text{where } a1 \sim a5 : \text{Degrees-of-freedom at } i \text{ node (} w_i, w'_i, w''_i, u_i, u'_i \text{)} \)

\( a6 \sim a10 : \text{Degrees-of-freedom at } j \text{ node (} w_j, w'_j, w''_j, u_j, u'_j \text{)} \)

\( n4 \sim n10, m1 \sim m8 : \text{Shape functions. These are presented in Appendix A.} \)
Using the assumption of plane section remaining plane, the strain $\varepsilon$ along the element can be expressed as

$$
\varepsilon = \frac{\partial u}{\partial x} - y \frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2
$$

Eq. 3

where $y$ is a distance from the centroid to the point at which the strain is determined.

In order to formulate the equilibrium equation the principle of virtual work is adopted.

$$
\delta W = \delta W_i - \delta W_e = 0
$$

Eq. 4

where the internal work is

$$
\delta W_i = \int_{Vs} \sigma_s(\varepsilon) \cdot \delta \varepsilon \cdot dV + \int_{Vc} \sigma_c(\varepsilon) \cdot \delta \varepsilon \cdot dV
$$

$$
= \int_{Vs} \sigma_s(\varepsilon) \cdot \delta a^T [(N_1 - y M_2) + M_1 M_1^T a] \, dV
$$

$$
+ \int_{Vc} \sigma_c(\varepsilon) \cdot \delta a^T [(N_1 - y M_2) + M_1 M_1^T a] \, dV
$$

Eq. 5

Here, $\sigma_s(\varepsilon)$ : Steel stress in the member

$\sigma_c(\varepsilon)$ : Concrete stress in the member

$Vs$ : Volume of steel tube only

$Vc$ : Volume of concrete only.

The external work is, on the other hand,
\[ \delta W_e = - \int_0^L p(|w|) \left( \frac{\partial w}{\partial x} \right) \delta a^T \mathbf{M}_0 \cdot dx + F \cdot \delta a^T \mathbf{M}_0(\mathbf{x}=L) + P \cdot \delta a^T \mathbf{N}_0(\mathbf{x}=L) \]  

Eq. 6

Here,  
- \( F \) : Lateral load applied at \( x = L \) in the y-direction  
- \( P \) : Axial load applied at \( x = L \) in the x-direction  
- \( p \) : Soil reaction  
- \( L \) : Pile length

Since \( \delta a \) is arbitrary, the principle of virtual work results in a system of non-linear equations equivalent to a vector \( \psi \) being zero at the solution \( a \).

\[
\psi = \int_{V_e} \sigma_e \varepsilon \cdot [(\mathbf{N}_1 - y \mathbf{M}_2) + \mathbf{M}_1 \mathbf{M}_1^T \mathbf{a}] \, dV
\]

\[
+ \int_{V_e} \sigma_e \varepsilon \cdot [(\mathbf{N}_1 - y \mathbf{M}_2) + \mathbf{M}_1 \mathbf{M}_1^T \mathbf{a}] \, dV
\]

\[
+ \int_0^L p(|w|) \left( \frac{\partial w}{\partial x} \right) \mathbf{M}_0 \cdot dx - F \cdot \mathbf{M}_0(\mathbf{x}=L) - P \cdot \mathbf{N}_0(\mathbf{x}=L) = 0
\]  

Eq. 7

To find the solution, an iteration procedure using the Newton-Raphson method is applied to make the vector \( \psi \) approach zero with an allowable tolerance.

Using the Newton-Raphson method the solution vector \( a \) is obtained by

\[
a = a^* + [\nu \psi^*]^{-1} \cdot (-\psi^*)
\]  

Eq. 8

where \([\nu \psi^*]\) is the tangent stiffness matrix.
In detail,

\[ \{ \psi_i \} = \int_{0}^{L} \left[ \sigma_s(\varepsilon) \left[ N_1 - y \cdot M_2 + M_1 M_1^T a \right] \right]_i \cdot b_s(y) \cdot dy \cdot dx \]

\[ + \int_{0}^{L} \left[ \sigma_s(\varepsilon) \left[ N_1 - y \cdot M_2 + M_1 M_1^T a \right] \right]_i \cdot b_c(y) \cdot dy \cdot dx \]

\[ + \int_{0}^{L} p(|w|) \left( \frac{w}{|w|} \right) M_{0i} \cdot dx - F \cdot M_{0i(x=L)} - P \cdot N_{0i(x=L)} \]  

Eq. 9

\[ R, r, b_s(y) \text{ and } b_c(y) \text{ are shown in Fig.2.} \]

The element of the matrix \([\psi_i] \) in the \( i \) th row and \( j \) th column is represented by
Then Eq. 8 is solved to find a new vector $\mathbf{a}$ for each step of displacements. Convergence is achieved when Eq. 11 and Eq. 12 are satisfied.

For the residual force vector $\{\psi\}$,

$$\sum_{i=1}^{\text{NEQ}} \psi_i^2 \leq \text{Tol1}$$

Eq. 11

For the displacement correction vector $\{\mathbf{a} - \mathbf{a}^*\}$,

$$\sum_{i=1}^{\text{NEQ}} (a_i - a_i^*)^2 \leq \text{Tol2}$$

Eq. 12

where NEQ is the number of equations and $^*$ indicates the values at the previous iteration.
Toll1 and Toll2 are specified tolerances.

Integrations are conducted by a Gaussian integration scheme involving a coordinate transformation from x, y to normalized coordinates ξ and η. These have a range from −1 to 1.

2.2 MODELLING OF THE SOIL

Soil-pile interaction is represented using a P-y curve which was proposed by Yan and Byrne(1992) in order to predict pile response to lateral pile head loading. This curve has a nonlinear relationship shown in Fig.3.

Fig.3 Normalized P-y curve
The relationship is given by

\[
\frac{P}{E_{\text{max}} \cdot D} = \alpha \cdot \left( \frac{y}{D} \right)^\beta
\]

Eq. 13

where

- \( P \): Soil reaction (force/unit length)
- \( D \): Pile diameter
- \( y \): Lateral pile deflection
- \( E_{\text{max}} \): Soil maximum Young’s modulus
- \( \beta \): Value of about 0.5
- \( P/(E_{\text{max}}D) \) and \( y/D \) are percentages.

\( \alpha \) is a function of soil relative density and can be expressed as

\[
\alpha = 5 \cdot (D_r)^{-0.8}
\]

Eq. 14

where \( D_r \) is the relative density in percentage.

The normalized P-y curve has an initial linear portion with a slope of 45°. The intersecting point \((X_0, Y_0)\) with the power function from Eq.13 can be found by
\[ X_0 = Y_0 = \frac{Y}{D} \left( \% \right) = \alpha \cdot \left( \frac{1}{1 - \beta} \right) \]  

Eq. 15

This is required to avoid the infinite slope which Eq.13 implies at \( y = 0 \). In Eq.13, \( E_{\text{max}} \) increases with soil depth.

When the pile head displacement is reversed during cyclic loading, a gap between the pile and the soil is formed. This gap is a function of depth. It is assumed that the soil is not able to take any tensile load in order to model the soil gapping. Fig.4 shows the load-displacement relationship for soil used in the program CFTPILE.

![Load-displacement relationship for soil](image)

Fig.4 Load-displacement relationship for soil
In the numerical procedure, the program keeps track of a previous state soil displacement, $D_0$ (shown in Fig.4), at each side of the pile. $D_0$ is associated with the maximum displacement, $w_0$, achieved along the backbone curve. With an initial slope, $K$, $D_0$ is defined as

$$D_0 = w_0 - p_0 / K$$  \hspace{1cm} \text{Eq. 16}$$

Note that the initial slope $K$ is equal to the maximum Young's modulus of soil, $E_{\text{max}}$.

The following algorithm can find a soil reaction $p(w)$ for a new $w$.

1. \text{if } (w \leq D_0) \rightarrow p = 0
2. \text{if } (w > D_0) \rightarrow p = \min \{ p_1 = K(w - D_0), p_2 = p(w) \}
3. \text{if } (p = p_2) \rightarrow \text{update } D_0 : D_0 = w - p / K
4. \text{if } (p = 0) \text{ or } (p = p_1) \rightarrow D_0 \text{ unchanged}  \hspace{1cm} \text{Eq. 17}$$

The displacement $D_0$, at either side of the pile and as a function of depth, gives the magnitude of the gap along the pile.

2.3 MODELLING OF THE CONCRETE

Confined concrete can be defined as that which is restrained in the directions at right angle to the applied stress. If the compression zone of a concrete beam or column is confined by closely spaced steel stirrup ties or steel casing, the ductility of the concrete is enhanced and large ultimate curvatures may be reached (Kent and Park, 1971). When
concrete is subjected to cyclic compressive loading it has been assumed that an “envelope” curve exists and that this envelope curve is approximately the same as the complete stress-strain curve obtained under monotonically increasing strain. This assumption was shown to be true for confined concrete, as well as for plain, unconfined concrete (Shah, Fafitis and Arnold, 1983). It was also shown that the similarity of monotonic and cyclic stress-strain envelopes indicated that the specimens subjected to unloading and reloading cycles experienced very little or no strength degradation due to cycling.

When it comes to the degradation of elastic modulus in concrete subjected to cyclic load, concrete shows a gradual decrease of its elastic modulus after it reaches its peak stress, but it is not significant when compared to the one for unconfined concrete. It is known that the degradation is a direct consequence of volumetric expansion. Confining pressure to concrete gives a lower amount of expansion and this results in a lower amount of stiffness degradation during unloading.

In a CFT pile the tube confines the concrete inside the steel tube. However, the behaviour is somewhat different from that for the concrete confined by closely spaced steel stirrup ties. The steel tube undergoes axial loads and bending moments, as well as providing confinement, while steel stirrup ties in a reinforced concrete member mainly provides confining pressure. In order to take these differences into account, Tomii and Sakino (1979) proposed a model to determine the stress-strain relationship for the concrete in CFT.

In this paper the model proposed by Tomii and Sakino, which is shown in Fig.5, is adopted to calculate the stresses in the concrete.
A parabolic part of the curve (A-B) was represented by Eq. 18,

$$\frac{\sigma_c}{f'_c} = 2 \left( \frac{\varepsilon_c}{\varepsilon_{cb1}} \right) - \left( \frac{\varepsilon_c}{\varepsilon_{cb1}} \right)^2$$

Eq. 18

where

$$\varepsilon_{cb1} = 0.012354165 \times \sqrt{f'_c}$$

$f'_c$ : Compressive strength of concrete core in GPa

$\sigma_c$ : Concrete stress at given strain $\varepsilon_c$ in GPa

$\varepsilon_{cb1}$ : The strain when the stress in concrete reaches $f'_c$
After reaching the maximum stress $f'_c$, the stresses are assumed to decrease following a multi-linear function, shown in Fig.5, to a minimum $\sigma_1$. This minimum, for confined concrete, depends on the ratio $D/t$ according to Eq.19.

$$\sigma_1 = (1.6 - 0.025 \times D/t) \cdot f'_c$$

Eq. 19

where

$\sigma_1$: The stress at the strain, $\varepsilon_{cb3}$

$D$: Diameter of the CFT pile

$t$: Thickness of the pile tube

In Fig.5, $\varepsilon_{cb2}$ and $\varepsilon_{cb3}$ are constant values. ($\varepsilon_{cb2} = 0.005, \varepsilon_{cb3} = 0.015$)

The bond between the steel section and the concrete is assumed to be perfect. Tensile stresses in concrete are ignored. Therefore, the concrete stress is present only when the strain is compressive. In order to calculate the stress in concrete during unloading and reloading, the assumption is made that the unloading and reloading response is linear with a slope equal to the initial tangent modulus.

Fig.6 shows the stress-strain relationship for concrete, which is used in the program CFTPILE. Let us assume that, at a point along the member, the strain is $\varepsilon_1$ for the first time as shown in Fig.6. If we cut out a micro-cube at that point, as shown in Fig 7, the original cube is compressed by the amount of $\varepsilon_1$ as shown in Fig 6 and Fig 7.
Fig. 6 Stress-strain relationship for the concrete model

Fig. 7. Behaviour of concrete micro-cube
Now let us assume that unloading starts and the current strain arrives at the value of \( \varepsilon_2 \) at which there is no stress in the concrete. As shown in Fig.6 the elastic strain recovery is \( \varepsilon_1 - \varepsilon_2 \) and \( \varepsilon_2 \) can be referred to as the residual strain when the load is removed. From the point of \( \varepsilon_2 \), if the strain follows the path \( \varepsilon_2 \rightarrow O \), the concrete starts to undergo tension and it develops cracks as soon as the strain becomes smaller than \( \varepsilon_2 \), in order to agree with the assumption that the tensile stress in concrete is ignored. Then the stress in the concrete will be zero through the path \( O \rightarrow \varepsilon_3 \rightarrow O \rightarrow \) until the strain reaches \( \varepsilon_2 \) again where the cracks are closed. Now reloading starts as the strain becomes greater than \( \varepsilon_2 \). Let us assume that we reach the point \( \varepsilon_4 \). After unloading, we reach the point \( \varepsilon_5 \), which is the new residual strain. If the loading is reversed, again, the cracks which have existed since the first load reversal was made will open again and the concrete can not take any load. The whole loop will be repeated for subsequent strain cycles.

2.4 MODELLING OF THE STEEL

The stress-strain relationship in steel tube is assumed to be elasto-perfectly plastic. The relationship is shown in Fig.8, in which \( E \) is the modulus of elasticity and \( \sigma_y \) is the yield stress. Knowing the previous state of stress \( \sigma_0 \) at \( \varepsilon_0 \), the new stress, \( \sigma(\varepsilon) \), in the steel tube is obtained by the following algorithm.
\( F(\varepsilon) = \sigma_0 + E \cdot (\varepsilon - \varepsilon_0) \)

\[
|F(\varepsilon)| \leq \sigma_y \rightarrow \sigma(\varepsilon) = F(\varepsilon) \\
|F(\varepsilon)| > \sigma_y \rightarrow \sigma(\varepsilon) = \sigma_y \frac{F(\varepsilon)}{|F(\varepsilon)|}
\]

Eq. 20

Fig. 8 The stress-strain relationship for steel
3 NUMERICAL EXAMPLES

A pile with a length of 30,000mm and an outside diameter of 1500mm is considered for an example. The surrounding soil is assumed to be dense sand having a relative density of 75%. $E_{\text{max}}$ of the soil is assumed to be 0.12 GPa at the depth of 4,000 mm, 0.2 GPa at the depth of 10,000 mm, 0.28 GPa at the depth of 20,000 mm. These values are based on the fitted curve for $E_{\text{max}}$ shown in the Yan and Byrne (1992) reference. A linear interpolation is used for values of $E_{\text{max}}$ at depths other than those mentioned above. The values of $E_{\text{max}}$ at different depths are illustrated in Fig.9.

![Fig.9 Maximum soil Young's modulus $E_{\text{max}}$](image)
For the material properties of concrete and steel, the steel is assumed to have an elastic modulus of 200 GPa with a yield stress $\sigma_y = 0.25$ GPa, and the concrete to have an initial tangent stiffness of 30.18 GPa with a compressive strength of 0.03 GPa. A cyclic displacement with a maximum value of 15 mm is enforced to the pile cap. The displacement history is shown in Fig. 10.

![Displacement History](image)

The displacements were divided in 640 individual steps. No axial load was applied and tolerances for the residual force vector and displacement correction vector were $1 \times 10^{-3}$. Numerical integration is conducted using 5 Gaussian points in the x-direction and 16 in the y-direction. Fig. 11, 12 and 13 show the results of calculation for different conditions. These figures show calculated hysteresis loops for the pile-soil system. Maximum force-displacement curves for a CFT and a Hollow section with the thickness of 30 mm are illustrated in Fig. 11.
Fig. 11 Comparison between CFT and Hollow section (thickness=30mm)

Fig. 12 Force-displacement curve for CFT pile (thickness=10mm)
Fig. 13 and Fig 13 show a maximum force-displacement curve for either a CFT or a Hollow section with a wall thickness of 10mm.

It is observed that the pile shown in Fig.11, with wall thickness of 30 mm, mainly remains in the elastic region, while the one with thickness of 10 mm (Fig.12,13) goes far beyond the elastic limit, resulting in the presence of a lateral force at a displacement of zero. In other words, it can be explained that the pile in Fig.12 has a residual displacement although the lateral force is removed. Also, this accounts for the curves in Fig.11 showing that the last loading cycle follows the same force-displacement path as the previous one, resulting in overlapping. The maximum force increases up to 8% when the pile with a 30 mm tube is filled with concrete, while there is a 23% increment for the pile with 10 mm wall thickness. Because of the small strains developed in the section of
the 30 mm pile, the pile remains mainly elastic and the contribution of the concrete is smaller than in the case of the thinner tube. Fig.14 shows the lateral displacements along the CFT pile with the 10 mm tube when the displacement history reaches the maximum value of 15 mm.

<table>
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<td>0.00</td>
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Fig.14 Lateral displacement along the CFT pile (thickness = 10mm)

It is seen that only the upper third of the pile undergoes deformations, with largest curvatures and moments about 10m below the ground line.
1 UNCERTAINTY IN SOIL PROPERTIES AND ITS INFLUENCE ON VARIABILITY OF MAXIMUM LOAD

In this work an attempt was made to investigate how the uncertainty in soil properties can influence the variability in maximum load. For a specific displacement history, there are many factors that determine the maximum lateral force, such as pile material properties, pile size, and soil properties. These factors are considered to be uncertain, which plays a role in the reliability of the structure. Good quality control in fabrication will yield a small coefficient of variation, such as 0.05, for the geometric parameters of the structure, and 0.1 for the material (Geschwindner, 1994). On the other hand, the variation in soil properties is considered to be relatively large. Also, soil shows a larger effect on the maximum force than the other variables such as material properties of the steel and the concrete.

In order to discuss the variability of maximum load applied to the pile cap, during the deformation history shown in Fig. 10, a performance function is defined as

\[ G = F_{\text{max}} - F \]

where

\( F_{\text{max}} \) is the maximum force, a random variable, obtained from CFTPILE for specific values of the soil and pile properties.

\( F \) is any load level.
Entering different levels of $F$, the performance function can be used to calculate the probability of $G < 0$ or $F_{\text{max}} < F$. These probabilities are the coordinates of the cumulative distribution function for $F_{\text{max}}$.

For the purpose of an example, let us assume that the soil properties are the only random variables to be taken into account, having a coefficient of variation (COV) of 0.25 for the soil maximum Young’s modulus $E_{\text{max}}$ and COV = 0.1 for the depth at which the $E_{\text{max}}$ is taken. The same cyclic displacement shown in Fig.10 is applied to the pile cap. Using the same pile and soil properties (See Fig.9) used in the previous numerical example, the soil properties are given as follows.

$$E_{1\text{m}} = 0.12 \text{ GPa at the depth of } Z_{1\text{m}} = 4,000 \text{ mm}$$
$$E_{2\text{m}} = 0.2 \text{ GPa at the depth of } Z_{2\text{m}} = 10,000 \text{ mm}$$
$$E_{3\text{m}} = 0.28 \text{ GPa at the depth of } Z_{3\text{m}} = 20,000 \text{ mm}$$

Where the subscript $m$ indicates the variable mean value. All variables are assumed to have a Normal distribution. These variables and their related statistics are summarized in Table 1.

<table>
<thead>
<tr>
<th>Variable No.</th>
<th>Variables</th>
<th>Mean Value</th>
<th>COV</th>
<th>Mean + 2σ</th>
<th>Mean-2σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$E_{\text{max}1}$</td>
<td>0.12</td>
<td>0.25</td>
<td>0.18</td>
<td>0.06</td>
</tr>
<tr>
<td>2</td>
<td>$E_{\text{max}2}$</td>
<td>0.2</td>
<td>0.25</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>$E_{\text{max}3}$</td>
<td>0.28</td>
<td>0.25</td>
<td>0.375</td>
<td>0.125</td>
</tr>
<tr>
<td>4</td>
<td>$Z_1$</td>
<td>4,000</td>
<td>0.1</td>
<td>4,800</td>
<td>3,200</td>
</tr>
<tr>
<td>5</td>
<td>$Z_2$</td>
<td>10,000</td>
<td>0.1</td>
<td>12,000</td>
<td>8,000</td>
</tr>
<tr>
<td>6</td>
<td>$Z_3$</td>
<td>20,000</td>
<td>0.1</td>
<td>24,000</td>
<td>16,000</td>
</tr>
</tbody>
</table>

Table 1. Random variables and their related statistics.
Using these $N = 6$ random variables, a response surface for the maximum force $F_{\text{max}}$ is constructed by

$$F_{\text{max}} = a_1 + a_2 \cdot E_1 + a_3 \cdot E_1^2 + a_4 \cdot E_2 + a_5 \cdot E_2^2 + a_6 \cdot E_3 + a_7 \cdot E_3^2$$
$$+ a_8 \cdot Z_1 + a_9 \cdot Z_1^2 + a_{10} \cdot Z_2 + a_{11} \cdot Z_2^2 + a_{12} \cdot Z_3 + a_{13} \cdot Z_3^2$$  
Eq. 1

In order to find the constants, $a_i \sim a_{13}$, 13 numerical analyses were conducted changing the variables, using the mean and mean $\pm 2 \cdot \sigma$ values for each of those six variables. First, an analysis was done using the variable means, and then each variable, in turn, was changed to mean $+ 2 \cdot \sigma$ and then to mean $- 2 \cdot \sigma$. This generated $2N+1$ data sets, or 13 in this case. Since the six variables were assumed to have a Normal distribution, about 95% of the population fell within two standard deviations on either side of the mean. As a result of the 13 analyses, 13 values of $F_{\text{max}}$ were obtained, which are $F_{\text{max}1} \sim F_{\text{max}13}$, allowing calculation of the response surface coefficients $a_i \sim a_{13}$.

Using the response surface for $F_{\text{max}}$, a cumulative distribution function was constructed by the software RELAN (2001).

The calculated cumulative distribution functions for $F_{\text{max}}$ are shown in Fig.15 ~ 17. In each figure, two curves are drawn for different conditions. One is for the case that there is no correlation between the variables $E_{\text{max}}$. The other is for the case that there are certain correlations between those. These correlations are expressed using a Correlation Coefficient $\rho$ and, for the example, they were assumed as shown in the following Table 2.
Table 2. Correlation Coefficients

<table>
<thead>
<tr>
<th>Related Variables</th>
<th>Correlation Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 and 2 ($\rho_{12}$)</td>
<td>0.8</td>
</tr>
<tr>
<td>2 and 3 ($\rho_{23}$)</td>
<td>0.8</td>
</tr>
<tr>
<td>1 and 3 ($\rho_{13}$)</td>
<td>0.7</td>
</tr>
</tbody>
</table>

- Variable 1, 2 and 3 are referred to in Table 1

Fig.15 shows the cumulative distribution function for the CFT pile with 30 mm of thickness.

Since the pile mainly remains in the elastic region, as explained previously, the variation in the soil properties plays the main role in the variability of maximum lateral force.
Fig. 16 shows how the correlation between the variables influences the soil Young’s modulus.

Due to the correlation between the variables, it is more likely to happen that the variables 1, 2 and 3 have their maximum or minimum values at the same time. This accounts for the curve with correlations in Fig. 15 showing lower probabilities at higher levels of force and higher probabilities at lower levels of force. However, this trend does not appear for the thinner wall tube (Fig. 17 and 18).

Fig. 17 and 18 show, respectively, the cumulative distribution function for the CFT pile with 10 mm thickness and for the Hollow section with 10 mm thickness.
Fig. 17 Cumulative distribution function (CFT, thickness=10mm)

Fig. 18 Cumulative distribution function (Hollow, thickness=10mm)
The maximum force-displacement relationship for the piles shown in Fig. 17 and 18 involve some plastic behaviour as we have seen previously. Not only the soil property variation but also the plastic behaviour in the pile influence, in this case, the variability in maximum load.

It may be concluded that variability in soil properties has a substantial effect on the variability of the maximum load, but that correlations of soil properties with depth of the pile may not be important to the same extent.
5 CONCLUSIONS AND FUTURE RESEARCH

A non-linear finite element analysis, CFTPILE, has been presented for the calculation of CFT pile response with application of static cyclic loading to the pile cap. It was also investigated how the maximum force is influenced by the uncertainty in soil properties using the computer program RELAN.

Although only static cyclic loading was dealt with in this paper, the approach can also be used to find the pile structure response under dynamic excitation such as an earthquake. Fig.19 shows a pile subjected to an earthquake with ground acceleration $a(t)$.

\[
\begin{align*}
F(\Delta) & \quad \leftarrow \quad M \cdot \ddot{\Delta} \\
M & \quad \leftarrow \quad M \cdot a(t) \\
a(t) & \quad \rightarrow
\end{align*}
\]

Fig.19 Horizontal forces acting on mass
If a structure is assumed to have a mass $M$ at the pile cap and a displacement $\Delta$ caused by earthquake with acceleration $a(t)$ at the base, the equation of motion will be

$$M \ddot{\Delta} + F(\Delta) = -M \cdot a(t)$$  \hspace{1cm} \text{Eq. 17}$$

This approach can be used only if the free-field soil displacement is assumed to be uniform or independent of depth, an assumption which may have to be corrected for long piles. This may be a topic for future research. However, this thesis has made a contribution to the calculation of the force $F(\Delta)$, which changes with the earthquake demand $\Delta$. 

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REFERENCES


APPENDIX A

Beam Finite Element Shape Functions
Shape functions for $w$

$M_o(1, \xi) = (8 - 15\xi + 10\xi^3 - 3\xi^5) / 16$

$M_o(2, \xi) = (5 - 7\xi - 6\xi^2 + 10\xi^3 + \xi^4 - 3\xi^5)(\Delta / 32)$

$M_o(3, \xi) = (1 - \xi - 2\xi^2 + 2\xi^3 + \xi^4 - \xi^5)(\Delta^2 / 64)$

$M_o(4, \xi) = 0$

$M_o(5, \xi) = 0$

$M_o(6, \xi) = (8 + 15\xi - 10\xi^3 + 3\xi^5) / 16$

$M_o(7, \xi) = (-5 - 7\xi + 6\xi^2 + 10\xi^3 - \xi^4 - 3\xi^5)(\Delta / 32)$

$M_o(8, \xi) = (1 + \xi - 2\xi^2 - 2\xi^3 + \xi^4 + \xi^5)(\Delta^2 / 64)$

$M_o(9, \xi) = 0$

$M_o(10, \xi) = 0$

Shape functions for $w'$

$M_1(1, \xi) = (-15 + 30\xi^2 - 15\xi^4)(2 / 16\Delta)$

$M_1(2, \xi) = (-7 - 12\xi + 30\xi^2 + 4\xi^3 - 15\xi^4) / 16$

$M_1(3, \xi) = (-1 - 4\xi + 6\xi^2 + 4\xi^3 - 5\xi^4)(\Delta / 32)$

$M_1(4, \xi) = 0$

$M_1(5, \xi) = 0$

$M_1(6, \xi) = (15 - 30\xi^2 + 15\xi^4)(2 / 16\Delta)$

$M_1(7, \xi) = (-7 + 12\xi + 30\xi^2 - 4\xi^3 - 15\xi^4) / 16$

$M_1(8, \xi) = (1 - 4\xi - 6\xi^2 + 4\xi^3 + 5\xi^4)(\Delta / 32)$
\( M_j(9, \xi) = 0 \)
\( M_j(10, \xi) = 0 \)

Shape functions for \( w'' \)

\[
M_2(1, \xi) = \frac{(60\xi - 60\xi^3)}{4/16\Delta^2}
\]
\[
M_2(2, \xi) = \frac{(-12 + 60\xi + 12\xi^2 - 60\xi^3)}{2/16\Delta}
\]
\[
M_2(3, \xi) = \frac{(-4 + 12\xi + 12\xi^2 - 20\xi^3)}{16}
\]
\( M_2(4, \xi) = 0 \)
\( M_2(5, \xi) = 0 \)
\[
M_2(6, \xi) = \frac{(-60\xi + 60\xi^3)}{4/16\Delta^2}
\]
\[
M_2(7, \xi) = \frac{(12 + 60\xi - 12\xi^2 - 60\xi^3)}{2/16\Delta}
\]
\[
M_2(8, \xi) = \frac{(-4 - 12\xi + 12\xi^2 + 20\xi^3)}{16}
\]
\( M_2(9, \xi) = 0 \)
\( M_2(10, \xi) = 0 \)

Shape functions for \( u \)

\( N_0(1, \xi) = 0 \)
\( N_0(2, \xi) = 0 \)
\( N_0(3, \xi) = 0 \)
\[
N_0(4, \xi) = \frac{(2 - 3\xi + \xi^3)}{4}
\]
\[
N_0(5, \xi) = \frac{(1 - \xi - \xi^2 + \xi^3)}{(\Delta/8)}
\]
\[ N_0(6, \xi) = 0 \]
\[ N_0(7, \xi) = 0 \]
\[ N_0(8, \xi) = 0 \]
\[ N_0(9, \xi) = \frac{(2 + 3\xi - \xi^3)}{4} \]
\[ N_0(10, \xi) = \frac{\Delta}{8} \]

Shape functions for \( u' \)
\[ N_1(1, \xi) = 0 \]
\[ N_1(2, \xi) = 0 \]
\[ N_1(3, \xi) = 0 \]
\[ N_1(4, \xi) = \frac{(-3 + 3\xi^2)}{2\Delta} \]
\[ N_1(5, \xi) = \frac{(-1 - 2\xi + 3\xi^2)}{4} \]
\[ N_1(6, \xi) = 0 \]
\[ N_1(7, \xi) = 0 \]
\[ N_1(8, \xi) = 0 \]
\[ N_1(9, \xi) = \frac{3 - 3\xi^2}{2\Delta} \]
\[ N_1(10, \xi) = \frac{-1 + 2\xi + 3\xi^2}{4} \]
APPENDIX B

CFTPILE

Source Code
DIMENSION A(10000)

* INPUT
UNIT FOR GEOMETRIC VARIABLES AND MATERIAL PROPERTIES : MM, GPA
SP: BEAM SPAN
NCROSS: BEAM CROSS-SECTION TYPE
  IF CROSS-SECTION IS RECTANGULAR ENTER 0
  IF CROSS-SECTION IS CIRCULAR ENTER 1
  IF CROSS-SECTION IS ANNULAR ENTER 2
  if cross-section is circular tube with concrete
    inside enter 3
D: DIAMETER
DIN: INSIDE DIAMETER
E: STEEL YOUNG'S MODULUS
SY: STEEL YIELD STRESS
CNT: CONCRETE COMPRESSIVE STRENGTH
NLAYER: NO. OF LAYER
NPLAY: THE NO. OF LAYERS WITH DIFFERENT PROPERTIES
NEMAX: SOIL MAX. YOUNG'S MODULUS
DPT: SOIL DEPTH WHERE THE NEMAX IS TAKEN
ITYP: LAYER TYPE
IBEG: THE NO. OF THE FIRST ELEMENT IN THE LAYER
IFIN: THE NO. OF THE LAST ELEMENT IN THE LAYER
TL: THE LAYER THICKNESS
NGX: THE NO. OF GAUSS POINT IN THE X-DIRECTION
NGY: THE NO. OF GAUSS POINT IN THE Y-DIRECTION
FX: AXIALLY APPLIED LOAD
NNBC: THE NO. OF NODES WITH SUPPORT CONDITIONS
NBC: THE NODE NO.
KSC: NO. OF SUPPORT CONDITION AT THE NODE
IBC: CODES FOR THE SPECIFIED SUPPORT CONDITION
  FOR W = 1
  FOR W' = 2
  FOR W'' = 3
  FOR U = 4
  FOR U' = 5
NSPF: THE NODE NO. WITH THE SPECIFIED DISPL.
TOLF: TOLERANCE FOR THE OUT-OF-BALANCE VECTOR
TOLX: TOLERANCE FOR THE LENGTH OF THE CHANGE IN DEFORMATION VECTOR

OPEN (UNIT=2, FILE=NAME, STATUS='OLD')
READ (2, 40) TITLE
READ (2, *) SP
READ (2, *) NCROSS
IF (NCROSS.EQ.0) READ (2, *) D, BO
IF (NCROSS.EQ.1) READ (2, *) D
IF (NCROSS.EQ.2) READ (2, *) D, DIN
IF (NCROSS.EQ.3) READ (2, *) D, DIN
if (ncross.eq.3) read (2,*) cnt
READ (2, *) E, SY
read(2,*) medium
READ (2, *) NLAYER, NPLAY
read(2,*) alpha, beta, nemax
do 451 i=1, nemax
read(2,*) emax(i), dpt(i)
451 continue
NELEM=0
DO 460 I=1, NLAYER
  READ (2, *) ITYP(I), IBEG(I), IFIN(I), TL(I)
  NELEM1=IFIN(I)-IBEG(I)+1
  NELEM=NELEM+NELEM1
  DELTA(I)=TL(I)/NELEM1
460 continue
READ (2,*) NGX,NGY
CALL GAUSS (NGX, EGX, HGX, IERR)
CALL GAUSS (NGY, EGY, HGY, IERR)
READ (2,*) FX
READ (2,*) NNBC
IF (NNBC.EQ.0) GO TO 480
DO 470 I=1,NNBC
READ (2,*) NBC(I),KBC(I),(IBC(I,J),J=1,KBC(I))
470 CONTINUE
480 READ (2,*) NLDIS
IF (NLDIS.EQ.0) READ (2,*) NSPF
IF (NLDIS.GT.0) READ (2,*) (ILAY(I),I=1,NLDIS)
READ (2,*) TOLX, TOLF
CLOSE (2)
490 WRITE (*,500)
500 FORMAT(/' ENTER NAME OF FILE WITH INPUT DISPLACEMENT HISTORY'/)
READ (*,40) NAME1
OPEN (UNIT=3,FILE=NAME1,STATUS='OLD')
READ (3,*) NSTEP
READ (3,*) (A(I),I=1,NSTEP)
CLOSE (3)
WRITE (*,510)
510 FORMAT(/' ENTER STEP NUMBER AT WHICH TO SHOW SHAPE'/)
READ (*,*) NSTP
WRITE (*,520)
520 FORMAT(/' ENTER 1 TO SHOW COMPREHENSIVE INTERMEDIATE RESULTS'/
1 ' ENTER 0 TO SKIP'/)
READ (*,*) NSCRC
WRITE (*,521)
521 FORMAT(/' ENTER 1 TO SHOW SUMMARY OF INTERMEDIATE RESULTS'/
1 ' ENTER 0 TO SKIP'/)
READ (*,*) NSCR
DO 530 I=1,NLAYER
DO 530 J=IBEG(I),IFIN(I)
530 IDISP(J)=0
IF (NLDIS.EQ.0) GO TO 550
DO 540 I=1,NLDIS
DO 540 J=IBEG(ILAY(I)),IFIN(ILAY(I))
540 IDISP(J)=1
550 DO 580 I=1,NGY
IF (NCROSS.EQ.0) B(I)=B0
IF (NCROSS.EQ.1) B(I)=DSQR(1.0D0-EGY(I)**2)
IF (NCROSS.EQ.2) GO TO 560
IF (NCROSS.EQ.3) GO TO 560
GO TO 580
560 RATD=DIN/D
IF (DABS(EGY(I)).GE.RATD) GO TO 570
B(I)=DSQR(1.0D0-EGY(I)**2)-DSQR(RATD**2-EGY(I)**2))
GO TO 580
570 B(I)=DSQR(1.0D0-EGY(I)**2)
580 CONTINUE
WRITE (*,590)
590 FORMAT(/' ENTER NAME OF OUTPUT FILE'/)
READ (*,40) NAME2
WRITE (*,600)
600 FORMAT(/' ENTER MULTIPLICATION FACTOR FOR CALCULATED LOADS'/)
READ (*,*) FLOAD
OPEN (UNIT=4,FILE=NAME2,STATUS='UNKNOWN')
P1 = 0.0D0
WRITE(4,130) P1, P1
C
C * MAIN PROGRAM, SIZE OF VECTORS
C
43
NEQ = (NELEM + 1) * 5
LHB = 10
NA = NEQ * LHB

* INITIALLY, SET CONVERGED VECTORS TO ZERO

DO 610 I = 1, NEQ
   X0C(I) = 0.0D0
610   R(NEQ - 1) = FX

DO 650 II = 1, NLAYER
   DO 640 I = IBEG(II), IFIN(II)
      DO 630 IX = 1, NGX
         D0PC(I, IX) = 0.0D0
         D0NC(I, IX) = 0.0D0
         DO 620 IY = 1, NGY
            SOC(I, IX, IY) = 0.0D0
            EPSOC(I, IX, IY) = 0.0D0
            rc(i, ix, iy) = 0.0d0
620         CONTINUE
630      CONTINUE
640   CONTINUE
650 CONTINUE

* OBTAINS SHAPE FUNCTIONS AT INTEGRATION POINTS

DO 670 II = 1, NLAYER
   CALL SHAPES (N0L, NIL, M0L, M1L, M2L, NGX, EGX, DELTA(II))
   DO 660 I = 1, 10
      DO 660 J = 1, NGX
         Nl(II, I, J) = N1L(I, J)
         M0 (II, I, J) = M0L (I, J)
         M1 (II, I, J) = M1L (I, J)
         M2 (II, I, J) = M2L (I, J)
660   CONTINUE
670 CONTINUE

IF (NLDIS.EQ.0) KA = (NSPF - 1) * 5 + 1
OPEN (UNIT=8, FILE='SHAPEP', STATUS='UNKNOWN')
OPEN (UNIT=9, FILE='SHAPEN', STATUS='UNKNOWN')
OPEN (UNIT=7, FILE='SHAPEW', STATUS='UNKNOWN')

* STARTS SOLUTION FOR EACH STEP

IF (NSCRC.EQ.0) WRITE (*, 680)
680 FORMAT (/ ' ENTER STEP AT WHICH SWITCHING TO SHOWING'/',
         1 ' COMPREHENSIVE INTERMEDIATE RESULTS'/)
IF (NSCRC.EQ.0) READ (*, *) NSTPR

DELTAX = 0.0D0
DELTAF = 0.0D0
A0 = 0.0D0
CS = 0.0d0
CS2 = 0.0d0
DCSDEP = 0.0D0
R0 = 0.0D0

DO 1490 NST = 1, NSTEP
   IF (NSCRC.EQ.0.AND.NST.EQ.NSTPR) NSCRC = 1
   NSINT = 1
   A1 = A(NST)
   AA = A1

STARTS ITERATIONS WITHIN THE STEP A0 TO AA
C Sets vectors to the initial values for the step, starting from
C the last converged values
C
NDE=0
NITER=0
IF (NSCRC.EQ.1) WRITE (*,700) NST,NDE,A0,A1,AA
700 FORMAT( ' DATA=',I4,' NDE=',I3/
1 ' A0=',F8.3,' A1=',F8.3,' AA=',F8.3/) 
DO 710 I=1,NEQ
X0(I)=X0C(I)
710 CONTINUE
DO 740 JL=1,NLAYER
DO 730 IE=IBEG(JL),IFIN(JL)
DOP(IE,IX)=D0PC(IE,IX)
DON(IE,IX)=D0NC(IE,IX)
730 CONTINUE
740 CONTINUE
C
C OBTAIN THE REACTION FORCES AND STRESSES FOR THE CURRENT VECTOR (X0)
C
DO 810 JL=1,NLAYER
Q00=Q0(ITYP(JL))
Q10=Q1(ITYP(JL))
Q40=Q4(ITYP(JL))
DMAX0=DMAX(ITYP(JL))
XK0=XK(ITYP(JL))
PMAX0=PMAX(ITYP(JL))
zl=0.0d0
ISTRT(JL)=1
750 DO 810 JL=1,NLAYER
Q00=Q0(ITYP(JL))
Q10=Q1(ITYP(JL))
Q40=Q4(ITYP(JL))
DMAX0=DMAX(ITYP(JL))
XK0=XK(ITYP(JL))
PMAX0=PMAX(ITYP(JL))
zl=0.0d0
IF (JL.EQ.1) GO TO 752
DO 751 I=1,JL-1
zl=zl+T(I)
751 CONTINUE
752 CONTINUE
DO 800 IE=IBEG(JL),IFIN(JL)
XIE=(IE-1)*5+I
DO 760 IE=IBEG(JL),IFIN(JL)
W=0.0D0
EPS1=0.0D0
EPS2=0.0D0
EPS3(JL,IX)=0.0D0
750 CONTINUE
DO 800 IE=IBEG(JL),IFIN(JL)
DO 760 IE=IBEG(JL),IFIN(JL)
W=0.0D0
EPS1=0.0D0
EPS2=0.0D0
EPS3(JL,IX)=0.0D0
XIE=zl+(IE-IBEG(JL)+1)*DELTA(JL)-(1.0+EGX(IX))*DELTA(JL)/2.0D0
C xie : the depth under consideration, measured from the bottom
C interpolate to find Emax along the pile
C
DO 761 I=0,NMAX
EPS1(I)=EPS1(I+1)
EPS2(I)=EPS2(I+1)
EPS3(I)=EPS3(I,IX)
45
slop=(emax(i+1)-emax(i))/(dpt(i+1)-dpt(i))
temax=emax(i)+(xie-dpt(i))*slop
go to 762
else
end if
761 continue
762 continue
770 DO 770 K=1,10
   W=W+M0(JL,K,IX)*XE(K)
   EPS1=EPS1+N1(JL,K,IX)*XE(K)
   EPS2=EPS2+M2(JL,K,IX)*XE(K)
   EPS3(IE,IX)=EPS3(IE,IX)+M1(JL,K,IX)*XE(K)
   IF (IDISP(IE).EQ.0) WT=W
   IF (IDISP(IE).EQ.1) WT=W-AA
   AW=DABS(WT)
   IF (WT.GE.0.0D0) D0=D0PC(IE,IX)
   IF (WT.LT.0.0D0) D0=D0NC(IE,IX)
   IF (WT.GE.0.0D0) SWO(IE,IX)=1.0D0
   IF (WT.LT.0.0D0) SWO(IE,IX)=-1.0D0
   CALL PYSOIL(AW,ALPHA,BETA,D,TEMAX,D0,P,DPDW)
   C
   771 IF (WT.GE.0.0D0) DOP(IE,IX)=D0
   IF (WT.LT.0.0D0) DON(IE,IX)=D0
   PO(IE,IX)=P
   DPDW0(IE,IX)=DPDW
   DO 780 IY=1,NGY
      Y=EGY(IY)*D/2.0D0
      EPS=EPS1-Y*EPS2+(EPS3(IE,IX)**2)/2.0D0
      ST0=S0C(IE,IX,IY)
      EP0=EPS0C(IE,IX,IY)
      rO=rc(ie,ix,iy)
      CALL STRESS (ST0, EP0, EPS, E, SY, S, DSDEPS)
   IF(NCROSS.EQ.3) THEN
      CALL CONCT(NCROSS,D,DIN,RO,EPS,CNT,CS,DCSDEP)
   ELSE
      ENDIF
   C
   779 rr(ie,ix,iy) = r0
      cs0(ie,ix,iy) = cs
      dcsde0(ie,ix,iy) = dcsdep
      SO(IE,IX,IY)=S
      EPS0(IE,IX,IY)=EPS
      DSDE0(IE,IX,IY)=DSDEPS
   780 CONTINUE
   790 CONTINUE
   800 CONTINUE
   810 CONTINUE
   C
   * FOR EACH ELEMENT, CONSTRUCT TANGENT STIFFNESS MATRIX AND
   C
   * RIGHT HAND SIDE, AND ADD TO GLOBALS
   C
   DO 820 I=1,NA
      C(I)=0.0D0
   820 CONTINUE
   DO 900 JL=1,NLAYER
      DO 890 IE=IBEG(JL),IFIN(JL)
         DO 860 I=1,10
            CE(I,J)=0.0D0
            DPDW=DPDW0(IE,IX)
            CE(I,J)=CE(I,J)+DGX(IY)*DPDW*M0(JL,I,IX)*M0(JL,J,IX)*
            DELTA(JL)/2.0D0
            FACI=N1(JL,I,IX)+M1(JL,I,IX)*EPS3(IE,IX)
            FACJ=N1(JL,J,IX)+M1(JL,J,IX)*EPS3(IE,IX)
            DO 830 IY=1,NGY
               Y=EGY(IY)*D/2.0D0
               830 CONTINUE

46
DSDEPS=DSDE0(IE,IX,IY)
S=S0(IE,IX,IY)
dcsdep = dcsde0(ie,ix,iy)
FAC1=FACJ-J*M2(JL,J,IX)
FAC2=FACI-J*M2(JL,I,IX)
CE(I,J)=CE(I,J)+HGX(IX)*HGY(IY)*(DSDEPS*FAC1*FAC2*
B(JY)+S*M1(JL,IX)*M1(JL,J,IX)*B(IY))*(DELTA(JL)*
D)/4.0D0
+ hgx(IX)*hgy(IY)*(dcsdep*fac1*fac2*bc(iy)+cs0(i,ie,
ix,iy)*ml(jl,ix)*ml(jl,j,ix)*bc(iy))*(delta(jl)
*dn/4.0d0)
830 CONTINUE
840 CONTINUE
850 CONTINUE
860 CONTINUE
870 CONTINUE
880 CONTINUE
890 CONTINUE
900 CONTINUE

C
902 DO 910 I=1,NEQ
910 PSI(I)=0.0D0
DO 960 JL=1,NLAYER
920 DO 940 IE=IBEG(JL),INJN(JL)
940 PSIE(I)=0.0D0
DO 930 IX=1,NGX
950 P=P0(IE,IX)
960 SW=SW0(IE,IX)
PSIE(I)=PSIE(I)+HGX(IX)*P*SW*M0(JL,IX)*DELTA(JL)/
2.0D0
FAC1=N1(JL,IX)+M1(JL,IX)*EPS3(I,IE,IX)
DO 920 IY=1,NGY
930 Y=EGY(IY)*D/2.0D0
S=S0(IE,IX,IY)
FAC2=FACI-J*M2(JL,I,IX)
PSIE(I)=PSIE(I)+HGX(IX)*HGY(IY)*S*FAC2*B(IY)*
(DELTA(JL)*D)/4.0D0
+ hgx(IX)*hgy(iy)*cs0(i,ie,ix,iy)*fac2*bc(iy)*
(delta(jl)*dn/4.0d0)
920 CONTINUE
930 CONTINUE
940 CONTINUE
950 CONTINUE
960 CONTINUE
970 CONTINUE
980 CONTINUE

C
C * Enforces W displacement and then introduces support conditions
C
IF (NLDIS.GT.0) GO TO 1030
IF (KA.EQ.1) GO TO 1000
RM(I)=R(I)-PSI(I)
1030 CONTINUE
1000 CONTINUE
47
K2=KA-1
IF (K1.LE.0) K1=1
DO 990 J=K1,K2
II=(J-1)*(LHB-1)+KA
RM(J)=RM(J)-C(II)*(AA-XO(KA))
990 C(II)=0.0D0
1000 IF (KA.EQ.NEQ) GO TO 1020
K1=KA+1
K2=KA+LHB-1
DO 1010 I=K1,K2
II=(K2-1)*(LHB-1)+I
RM(I)=RM(I)-C(II)*(AA-XO(KA))
1010 C(II)=0.0D0
1020 KK=(KA-1)*(LHB-1)+KA
C(KK)=1.0D0
RM(KA)=AA-XO(KA)
C
1030 IF (NNBC.EQ.0) GO TO 1100
DO 1090 IB=1,NNBC
DO 1080 J=1,NNBC(I)
K=(NNBC(I)-1)*5+IBC(IB,J)
IF (K.LE.0) GO TO 1050
K1=K-LHB+1
K2=K-1
DO 1040 I=K1,K2
II=(I-1)*(LHB-1)+K
RM(I)=RM(I)+C(II)*X0(K)
1040 C(II)=0.0D0
1050 IF (K.EQ.NEQ) GO TO 1070
K1=K+1
K2=K+LHB-1
DO 1060 I=K1,K2
II=(K-1)*(LHB-1)+I
RM(I)=RM(I)+C(II)*X0(K)
1060 C(II)=0.0D0
1070 KK=(K-1)*(LHB-1)+K
C(KK)=1.0D0
RM(K)=X0(K)
1080 CONTINUE
1090 CONTINUE
1100 CONTINUE
C Computes magnitudes of the out-of-balance vector,
C and checks convergence if NITER > 0
C
RMC=0.0D0
DO 1110 I=1,NEQ
RMC=RMC+RM(I)**2
1110 CONTINUE
IF (NITER.EQ.0) then
RMP = RMC
ncount=0
rmcp=rmc
endif
IF (NITER.EQ.0) GO TO 1113
C * Checks convergence
C
if(rmcp.lt.rmc) then
ncount=ncount+1
pause
endif
KF=0
KM = 0
IF (RMC.LT.TOLF) KF = 1
IF (RMAX.LT.TOLX) KM = 1
IF (NSCRC.EQ.1) WRITE(*,1150) NST,NSINT,NDE,NITER,
1 RMP,RMC,KF,
1 RMAX,KM
1150 FORMAT(/' DATA =',I5,' INTERMEDIATE STEP=',I4,' NDE=',I4,
1 ' NITER =',I3,' INIT. BAL. VECTOR=',E12.5,
1 ' CURRENT BAL. VECTOR=',E12.5,' KF=',I2/
1 ' CURRENT DELTA(X) = ',E12.5, ' KM=',I2)
c IF (NSCRC.EQ.1) PAUSE
IF (KF.EQ.1.AND.KM.EQ.1) GO TO 1190
C
C 1113 CALL DECOMP (NEQ, LHB, C, IERROR)
IF (IERROR.EQ.1) THEN
IF (NSCRC.EQ.1) WRITE (*,1130)
1130 FORMAT(/' DECOMPOSITION FAILED'/)
C
GO TO 1170
END IF
CALL SOLV (NEQ, LHB, C, RM)
C
C * Finds length of displacement correction vector
C
RMAX=0.0D0
DO 1140 JL=1,NLAYER
DO 1140 IE=IBEG(JL),IFIN(JL)
JA=(IE-1)*5+1
RMAX = RMAX + RM(JA)**2
IF (IE.EQ.IFIN(NLAYER)) THEN
JA = IE*5 + 1
RMAX = RMAX + RM(JA)**2
END IF
1140 CONTINUE
C
NITER = NITER + 1
DO 1160 I=1,NEQ
X0(I)=X0(I)+RM(I)
1160 CONTINUE
C IF (NITER.EQ.1000) GO TO 1170
if(ncount.eq.10) go to 1170
rmcp=rmc
go to 750
C
1170 AA=(A0+AA)/2.0D0
NDE=NDE+1
C
IF (NDE.GT.11) THEN
WRITE (*,1180) NST,NSINT,NDE-1,NITER,A0,AA
1180 FORMAT(/' CANNOT FIND SOLUTION AT STEP NST=',I4/
1 ' NSINT=',I4,' NDE=',I4,' NITER=',I4/
1 ' A0=',E12.5,' AA=',E12.5)
go to 1495
END IF
go to 690
C
1190 IF (AA.EQ.A1) GO TO 1260
IF (AA.NE.A1) GO TO 1200
C
C * Convergence is achieved at AA not equal to A1
C * NSINT is a counter for the number of intermediate, converged steps
C
1200 DO 1210 I=1,NEQ
1210 X0C(I)=X0(I)
DO 1250 II=1,NLAYER
DO 1240 IE=IBEG(II),IFIN(II)
DO 1230 IX=1,NGX
   DOPC(IE,IX)=D0P(IE,IX)
   DONC(IE,IX)=DON(IE,IX)
DO 1220 IY=1,NGY
   SOC(IE,IX,IY)=S0(IE,IX,IY)
   EPSOC(IE,IX,IY)=EPS0(IE,IX,IY)
   rc(ie,ix,iy) = rr(ie,ix,iy)
1220 CONTINUE
1230 CONTINUE
1240 CONTINUE
1250 CONTINUE
C
A0=AA
AA= Al
NSINT=NSINT+1
if(nsint.eq.20) go to 1490
IF (RMAX.GT.DELTAX) DELTAX = RMAX
IF (RMC.GT.DELTAF) DELTAF = RMC
NDE=0
flag=0.0d0
GO TO 690
C
C * Convergence has been achieved at AA = Al
C
1260 A0=AA
DO 1270 I=1,NEQ
1270 X0C(I)=X0(I)
DO 1310 II=1,NLAYER
DO 1300 IE=IBEG(II),IFIN(II)
DO 1290 IX=1,NGX
   DOPC(IE,IX)=D0P(IE,IX)
   DONC(IE,IX)=DON(IE,IX)
DO 1280 IY=1,NGY
   SOC(IE,IX,IY)=S0(IE,IX,IY)
   EPSOC(IE,IX,IY)=EPS0(IE,IX,IY)
   rc(ie,ix,iy) = rr(ie,ix,iy)
1280 CONTINUE
1290 CONTINUE
1300 CONTINUE
1310 CONTINUE
IF (RMAX.GT.DELTAX) DELTAX = RMAX
IF (RMC.GT.DELTAF) DELTAF = RMC
C
C * Computes the load (FY0 for fixed layers, FY1 for moving layers)
C
IF (NLDIS.GT.0) GO TO 1350
FY0=0.0D0
DO 1340 JL=1,NLAYER
   DO 1330 IE=IBEG(JL),IFIN(JL)
      DO 1320 IX=1,NGX
         FY0=FY0+HGX(IX)*P0(IE,IX)*SW0(IE,IX)*DELTA(JL)/2.0D0
      1320 CONTINUE
   1330 CONTINUE
1340 CONTINUE
GO TO 1390
C
1350 FY1=0.0D0
FY0=0.0D0
DO 1380 JL=1,NLAYER
   DO 1370 IE=IBEG(JL),IFIN(JL)
      DO 1360 IX=1,NGX
         IF (IDISP(IE).EQ.1) FY1=FY1+HGX(IX)*P0(IE,IX)*SW0(IE,IX)*
            DELTA(JL)/2.0D0
      1360 CONTINUE
   1370 CONTINUE
1380 CONTINUE
GO TO 1390
DELTA(JL)/2.0D0

CONTINUE

CONTINUE

CONTINUE

C

IF (NLDIS.EQ.0) WRITE (4,130) AA, FY0*FLOAD
IF (NLDIS.GT.0) WRITE (4,130) AA, FY0*FLOAD
IF (NSCR.EQ.1) WRITE (*,1400) NST,NSINT,NITER, RMAX, KM, RMC, KF
1 FORMAT (' FINISHED DATA No. ',I5,' TOTAL STEPS =',I4,
1 ' NITER LAST STEP =',I3/
1 ' DELTA(X) = ',E12.5,' KM=',I2,' DELTA(F) = ',E12.5,' KF=',I2)
IF (NLDIS.GT.0) WRITE (*,1420) NST,AA,FYO*FLOAD,FY1*FLOAD
IF (NLDIS.EQ.0) WRITE (*,1410) NST,AA,FYO*FLOAD
1410 FORMAT(' DATA=',I5,' A=',E14.6,' F=',E14.6/)
1420 FORMAT(' DATA=',I5,' A=',E14.6,' F0=',E14.6,' F1=',E14.6/)

C Store shape if the step is as specified
C

IF (NST.NE.NSTP) GO TO 1490
DO 14 80 JL=1,NLAYER
ZL=0.0D0
IF (JL.EQ.1) GO TO 1440
DO 1430 I=1,JL-1
ZL=ZL+TL(I)
1430 CONTINUE
1440 CONTINUE
DO 1470 IE=IBEG(JL),IFIN(JL)
DO 1450 IX=1,NGX
XIE=ZL+(IE-IBEG(JL)+1)*DELTA(JL)-(1.0+EGX(IX))*DELTA(JL)/
1 2.0D0
WRITE (8,130) DOPC(IE, (NGX+1-IX) ) ,XIE
WRITE (9,130) -D0NC(IE,(NGX+1-IX)),XIE
1450 CONTINUE
IEE=(IE-1)*5+1
XIEE=ZL+(IE-IBEG(JL))*DELTA(JL)
WRITE (7,130) X0(IEE),XIEE
IF (JL.EQ.NLAYER.AND.IE.EQ.IFIN(JL)) GO TO 1460
GO TO 1470
1460 XIEE=ZL+(IE+1-IBEG(JL))*DELTA(JL)
IF (JL.EQ.NLAYER.AND.IE.EQ.IFIN(JL)) IEE=IE*5+1
IF (JL.EQ.NLAYER.AND.IE.EQ.IFIN(JL)) WRITE (7,130) X0(IEE),
XIEE
1470 CONTINUE
1480 CONTINUE
1490 CONTINUE
WRITE(*,1491) DELTAX, DELTAF
1491 FORMAT(' MAXIMUM DELTA(X) = ',E12.5,' MAXIMUM DELTA(F) = ',E12.5/)
$X4 = X^4$

$X5 = X^5$

**SHAPES FOR $W(X)$**

$M0(1,I) = (8.0D0 - 15.0D0 * X + 10.0D0 * X^3 - 3.0D0 * X^5) / 16.0D0$

$M0(2,I) = (5.0D0 - 7.0D0 * X - 6.0D0 * X^2 + 10.0D0 * X^3 + 4.0D0 * X^4 - 3.0D0 * X^5) * (D / 32.0D0)$

$M0(3,I) = (1.0D0 - X - 2.0D0 * X^2 + 2.0D0 * X^3 + X^4 + X^5) * (D / 64.0D0)$

$M0(4,I) = 0.0D0$

$M0(5,I) = 0.0D0$

$M0(6,I) = (8.0D0 + 15.0D0 * X - 10.0D0 * X^3 + 3.0D0 * X^5) / 16.0D0$

$M0(7,I) = (-5.0D0 - 7.0D0 * X + 6.0D0 * X^2 + 10.0D0 * X^3 - X^4 - 3.0D0 * X^5) * (D / 32.0D0)$

$M0(8,I) = (-5.0D0 - 7.0D0 * X + 6.0D0 * X^2 + 10.0D0 * X^3 - X^4 - 3.0D0 * X^5) / 64.0D0$

$M0(9,I) = 0.0D0$

$M0(10,I) = 0.0D0$

**SHAPES FOR $W'(X)$**

$M1(1,I) = (-15.0D0 + 30.0D0 * X^2 - 15.0D0 * X^4) * 2.0D0 / 16.0D0 * D$

$M1(2,I) = (-7.0D0 + 12.0D0 * X + 4.0D0 * X^3 - 15.0D0 * X^4) / 16.0D0$

$M1(3,I) = (1.0D0 - 4.0D0 * X + 6.0D0 * X^2 + 4.0D0 * X^3 - 5.0D0 * X^4) * D / 32.0D0$

$M1(4,I) = 0.0D0$

$M1(5,I) = 0.0D0$

$M1(6,I) = (-60.0D0 * X + 60.0D0 * X^3) * 4.0D0 / 16.0D0 * (D / 32.0D0)$

$M1(7,I) = (12.0D0 + 60.0D0 * X - 12.0D0 * X^2 - 60.0D0 * X^3) / 16.0D0$

$M1(8,I) = (1.0D0 - 4.0D0 * X - 6.0D0 * X^2 + 4.0D0 * X^3 + 5.0D0 * X^4) * D / (32.0D0)$

$M1(9,I) = 0.0D0$

$M1(10,I) = 0.0D0$

**SHAPES FOR $W''(X)$**

$M2(1,I) = (60.0D0 * X - 60.0D0 * X^3) * 4.0D0 / (16.0D0 * (D / 32.0D0))$

$M2(2,I) = (-12.0D0 + 60.0D0 * X + 12.0D0 * X^2 - 60.0D0 * X^3) / 2.0D0 / (16.0D0 * D)$

$M2(3,I) = (4.0D0 - 12.0D0 * X + 12.0D0 * X^2 - 20.0D0 * X^3) / 16.0D0$

$M2(4,I) = 0.0D0$

$M2(5,I) = 0.0D0$

$M2(6,I) = (60.0D0 * X + 60.0D0 * X^3) * 4.0D0 / (16.0D0 * (D / 32.0D0))$

$M2(7,I) = (12.0D0 + 60.0D0 * X - 12.0D0 * X^2 - 60.0D0 * X^3) * 2.0D0 / (16.0D0 * D)$

$M2(8,I) = (-4.0D0 - 12.0D0 * X + 12.0D0 * X^2 + 20.0D0 * X^3) / 16.0D0$

$M2(9,I) = 0.0D0$

$M2(10,I) = 0.0D0$

**SHAPES FOR $U(X)$**

$N0(1,I) = 0.0D0$

$N0(2,I) = 0.0D0$

$N0(3,I) = 0.0D0$

$N0(4,I) = (2.0D0 - 3.0D0 * X + X^3) / 4.0D0$

$N0(5,I) = (1.0D0 - X - X^2 + X^3) * D / 8.0D0$

$N0(6,I) = 0.0D0$

$N0(7,I) = 0.0D0$

$N0(8,I) = 0.0D0$

$N0(9,I) = (2.0D0 + 3.0D0 * X - X^3) / 4.0D0$

$N0(10,I) = (1.0D0 - X - X^2 + X^3) * D / 8.0D0$

**SHAPES FOR $U'(X)$**

$N1(1,I) = 0.0D0$

$N1(2,I) = 0.0D0$

$N1(3,I) = 0.0D0$

$N1(4,I) = (-3.0D0 + 3.0D0 * X^2) / (2.0D0 * D)$
Nl(5,I) = (-1.0D0 - 2.0D0*X + 3.0D0*X2) / 4.0D0
Nl(6,I) = 0.0D0
Nl(7,I) = 0.0D0
Nl(8,I) = 0.0D0
Nl(9,I) = (3.0D0 - 3.0D0*X2) / (2.0D0*DELTA)
Nl(10,I) = (-1.0D0 + 2.0D0*X + 3.0D0*X2) / 4.0D0

CONTINUE
RETURN
END

SUBROUTINE STRESS (STO, EPO, EPS, E, SY, S, DSDEPS)
IMPLICIT REAL*8 (A-H, O-Z)
S = STO + E * (EPS - EPO)
DSDEPS = E
SI = SY + 0.001D0 * E * EPS
S2 = -SY + 0.001D0 * E * EPS
IF (S.GE.SI) DSDEPS = 0.001D0 * E
IF (S.GE.S1) S = SI
IF (S.LE.S2) DSDEPS = 0.001D0 * E
IF (S.LE.S2) S = S2
RETURN
END

SUBROUTINE conct(NCROSS, d, din, r0, eps, cnt, cs, dcsdep)
IMPLICIT REAL*8 (A-H, O-Z)
DT = D/T
IF (DT.LT.64.0D0) DT = 64.0D0
IF (NCROSS.EQ.3) THEN
  t = (d - din) / 2.0D0
  cnt = cnt
  ceb1 = -0.012354165D0 * DSQRT(-cnt)
  ec = 2.0D0 * cnt / ceb1
  IF (eps.GT.0.0D0) THEN
    CS = 0.0D0
    DCSDEP = 0.0D0
  ELSE IF (eps.LE.r0) THEN
    CS = 0.0D0
    DCSDEP = EC
    GO TO 13
  ENDIF
  IF (eps.LT.r0) THEN
    CS = (1.6D0 - 0.025D0 * DT) * NT
    CS2 = (CEL + (CNT/CEI)**2) * EPS**2
  ELSE IF (eps.GT.0.05D0.AND.EPS.LE.0.015D0) THEN
    CS2 = (1.6D0 - 0.025D0 * DT) * CNT
    CS2 = CS2 + ((CNT - CS2) / (-0.015D0 + 0.005D0)) * (EPS + 0.005)
  ENDIF
  IF (EPS.LT.0.015D0) THEN
    CS2 = (1.6D0 - 0.025D0 * DT) * CNT
    CS2 = CS2 - CS2 * CS / EC
    IF (EPS.LE.0.005D0.AND.EPS.GE.0.015D0) THEN
      DCSDEP = 2.0D0 * CNT / (CNT - CEB1)**2 * EPS**2
    ENDIF
  ELSE IF (EPS.GT.0.0D0) THEN
    CS = CS2
    R0 = EPS - CS / EC
  ENDIF
ENDIF
GO TO 13
endif
if(eps.ge.-0.005d0.and.eps.lt.csl) dcsdep=0.0d0
if(eps.gt.-0.015d0.and.eps.lt.-0.005d0) then
  dcsdep=(-cnt+csl)/(-0.015+0.005)
endif
if(eps.lt.-0.005) dcsdep=0.0d0
cnt=-cnt
return
end

SUBROUTINE PYSOIL(AW,ALPHA,BETA,D,TEMAX,D0,P,DPDW)
IMPLICIT REAL*8(A-H,O-Z)
IF (AW.GE.D0) GO TO 763
P=0.0D0
DPDW=0.0D0
GO TO 771
763 IF(AW.GT.D0) GO TO 764
P=0.0D0
DPDW=TEMAX
GO TO 771
764 P=TEMAX*(AW-D0)
DPDW=TEMAX
x=(d/100.0d0)*alpha**(1.0d0/(1.0d0-beta))
if(aw.ge.x) then
  pub=(temax*d*alpha*((1.0d0/d)**beta)*aw**beta)/10.0d0
else
  pub=temax*aw
end if
if(p.lt.pub) go to 771
p=pub
if(aw.lt.x) then
  dpdw=temax
else
  dpdw=0.1d0*temax*d*alpha*((1.0d0/d)**beta)*beta*aw**(beta-1.0d0)
endif
d0=aw-p/temax
771 return
end

SUBROUTINE DECOMP (N, LHB, A, IERROR)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(2550)
IERROR=0
KB=LHB-1
TEMP=A(1)
IF (TEMP.LE.0.0D0) IERROR=1
IF (IERROR.EQ.1) RETURN
TEMP=DSSQRT(TEMP)
A(1)=TEMP
DO 10 I=2,LHB
  J1=J-1
  IJD=LHB+J-KB
  SUM=A(IJD)
  KO=1
  IF (J.GT.LHB) KO=J-KB
  DO 20 K=KO,J1
    JK=KB*K+J-KB
    TEMP=A(JK)
    SUM=SUM-(TEMP**2)
20    CONTINUE
10    CONTINUE
DO 60 J=2,N
  J1=J-1
  IJD=LHB+J-KB
  SUM=A(IJD)
  KO=1
  IF (J.GT.LHB) KO=J-KB
  DO 20 K=KO,J1
    JK=KB*K+J-KB
    TEMP=A(JK)
    SUM=SUM-(TEMP**2)
CONTINUE
IF (SUM.LE.0.0D0) IERROR=1
IF (IERROR.EQ.1) RETURN
A(IJD)=DSQRT(SUM)
DO 50 I=1,KB
   II=I+1
   KO=1
   IF (II.GT.LHB) KO=II-KB
   SUM=A(IJD+I)
   IF (I.EQ.KB) GO TO 40
   DO 30 K=KO,I1
      JK=KB*K+J-KB
      IK=KB*K+II-KB
      TEMP=A(JK)
      SUM=SUM-A(IK)*TEMP
   30 CONTINUE
40 A(IJD+I)=SUM/A(IJD)
50 CONTINUE
60 CONTINUE
RETURN
END

SUBROUTINE SOLV (N, LHB, A, B)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(2550), B(255)

* FORWARD SUBSTITUTION *
KB=LHB-1
TEMP=A(1)
B(1)=B(1)/TEMP
DO 20 I=2,N
   II=I-1
   KO=1
   IF (I.GT.LHB) KO=I-KB
   SUM=B(I)
   II=LHB*I-KB
   DO 10 K=KO,I1
      IK=KB*I-KB
      TEMP=A(IK)
      SUM=SUM-TEMP*B(K)
   10 CONTINUE
   B(I)=SUM/A(II)
20 CONTINUE

* BACKWARD SUBSTITUTION *
NL=N-1
LB=LHB*N-KB
TEMP=A(LB)
B(N)=B(N)/TEMP
DO 40 I=1,NL
   II=N-I+1
   NI=N-I
   KO=N
   IF (I.GT.KB) KO=NI+KB
   SUM=B(NI)
   II=LHB*N-KB
   DO 30 K=II,KO
      IK=KB*N+K-KB
      TEMP=A(IK)
      SUM=SUM-TEMP*B(K)
   30 CONTINUE
   B(NI)=SUM/A(II)
40 CONTINUE

SUBROUTINE GAUSS (N, E, H, IERR)
REAL*8 E (32), H(32)
M=(N-2) * (N-3) * (N-4) * (N-5) * (N-6) * (N-7) * (N-8)
M=M* (N-9) * (N-10) * (N-11) * (N-12) * (N-15) * (N-16) * (N-32)
IF (M.NE.0) GO TO 260
IERR=0
IF (N.EQ.32) GO TO 240
IF (N.EQ.16) GO TO 220
IF (N.EQ.15) GO TO 200
IF (N.EQ.12) GO TO 180
IF (N.EQ.11) GO TO 160
IF (N.EQ.10) GO TO 140
IF (N.EQ.9) GO TO 120
IF (N.EQ.8) GO TO 100
IF (N.EQ.7) GO TO 80
IF (N.EQ.6) GO TO 60
IF (N.EQ.5) GO TO 40
IF (N.EQ.4) GO TO 20
IF (N.EQ.3) GO TO 10
E(1)=0.577350269189626D0
E(2)=-E(l)
H(1)=1.0D0
H(2)=H(1)
RETURN
10 E(1)=0.774596669241483D0
E(2)=0.0D0
E(3)=-E(l)
H(1)=0.555555555555556D0
H(2)=0.888888888888889D0
H(3)=H(1)
RETURN
20 E(1)=0.861136311594053D0
E(2)=0.339981043584856D0
H(1)=0.347854845137454D0
H(2)=0.652145154862546D0
DO 30 I=1,2
E(5-I)=-E(I)
30 H(5-I)=H(I)
RETURN
40 E(1)=0.906179845938664D0
E(2)=0.538469310105683D0
E(3)=0.0D0
H(1)=0.239268850561899D0
H(2)=0.478628670499366D0
H(3)=0.568888888888889D0
DO 50 I=1,2
E(6-I)=-E(I)
50 H(6-I)=H(I)
RETURN
60 E(1)=0.932469514203152D0
E(2)=0.661209386466265D0
E(3)=0.238619186083197D0
H(1)=0.171324492379170D0
H(2)=0.360761573048139D0
H(3)=0.467913934572691D0
DO 70 I=1,3
E(7-I)=-E(I)
70 H(7-I)=H(I)
RETURN
80 E(1)=0.949107912342759D0
E(2)=0.74153118559394D0
E(3) = 0.405845151377397D0
E(4) = 0.000000000000000D0
H(1) = 0.129484966168870D0
H(2) = 0.279705391489277D0
H(3) = 0.381830500501190D0
H(4) = 0.417959183673469D0
DO 90 I = 1, 3
E(8-I) = -E(I)
90 H(8-I) = H(I)
RETURN
100 E(1) = 0.960289856497536D0
E(2) = 0.796666477413627D0
E(3) = 0.525532409916329D0
E(4) = 0.183434624956500D0
H(1) = 0.101228536290376D0
H(2) = 0.222381034453374D0
H(3) = 0.31370645877887D0
H(4) = 0.362683783378362D0
DO 110 I = 1, 4
E(9-I) = -E(I)
110 H(9-I) = H(I)
RETURN
120 E(1) = 0.968160239507626D0
E(2) = 0.836031107326636D0
E(3) = 0.613371432700590D0
E(4) = 0.32453423403809D0
E(5) = 0.000000000000000D0
H(1) = 0.081274388361574D0
H(2) = 0.180648160694857D0
H(3) = 0.260610696402935D0
H(4) = 0.312347077000030D0
H(5) = 0.33023935501260D0
DO 130 I = 1, 4
E(10-I) = -E(I)
130 H(10-I) = H(I)
RETURN
140 E(1) = 0.973906528517172D0
E(2) = 0.865063366688985D0
E(3) = 0.679409568299024D0
E(4) = 0.43395394129247D0
E(5) = 0.14874338981631D0
H(1) = 0.06671344308680D0
H(2) = 0.14945134815081D0
H(3) = 0.21908632515982D0
H(4) = 0.269266719309996D0
H(5) = 0.295524224714753D0
DO 150 I = 1, 5
E(11-I) = -E(I)
150 H(11-I) = H(I)
RETURN
160 E(1) = 0.978228659146057D0
E(2) = 0.887062597608950D0
E(3) = 0.730152005574049D0
E(4) = 0.519096129206812D0
E(5) = 0.269543155952345D0
E(6) = 0.000000000000000D0
H(1) = 0.055668567116174D0
H(2) = 0.125580369464905D0
H(3) = 0.186290210927734D0
H(4) = 0.233193764591990D0
H(5) = 0.262804544510247D0
H(6) = 0.272925086777901D0
DO 170 I = 1, 5
E(12-I) = -E(I)
170 H(12-I) = H(I)
RETURN
<table>
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<tr>
<th>I</th>
<th>E(I)</th>
<th>H(I)</th>
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<tr>
<td>1</td>
<td>0.981560636426719D0</td>
<td>0.047175336386512D0</td>
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<td>2</td>
<td>0.904117256370475D0</td>
<td>0.106939325995318D0</td>
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<tr>
<td>3</td>
<td>0.7699026198998180D0</td>
<td>0.160078328543346D0</td>
</tr>
<tr>
<td>4</td>
<td>0.587314989981080D0</td>
<td>0.203167426723066D0</td>
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<tr>
<td>5</td>
<td>0.365017954286617D0</td>
<td>0.233492536538355D0</td>
</tr>
<tr>
<td>6</td>
<td>0.24047805813403D0</td>
<td>0.249147045813403D0</td>
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DO 190 I = 1, 6
E(I+1) = -E(I)
H(I+1) = H(I)
RETURN

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<th>I</th>
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<th>H(I)</th>
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<tr>
<td>7</td>
<td>0.987992518020485D0</td>
<td>0.030753241996117D0</td>
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<td>8</td>
<td>0.93725373292400706D0</td>
<td>0.070366047488108D0</td>
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<td>9</td>
<td>0.848206853410427D0</td>
<td>0.107159220467172D0</td>
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<td>10</td>
<td>0.724417731360170D0</td>
<td>0.139570677926154D0</td>
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<tr>
<td>11</td>
<td>0.570972172608539D0</td>
<td>0.166269205816994D0</td>
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<tr>
<td>12</td>
<td>0.394151347077563D0</td>
<td>0.186161000015562D0</td>
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<tr>
<td>13</td>
<td>0.201194093997435D0</td>
<td>0.198431485327112D0</td>
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<tr>
<td>14</td>
<td>0.000000000000000D0</td>
<td>0.202570241925561D0</td>
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DO 210 I = 1, 7
E(I+1) = -E(I)
H(I+1) = H(I)
RETURN

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<th>E(I)</th>
<th>H(I)</th>
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<tr>
<td>15</td>
<td>0.989400934991650D0</td>
<td>0.027152459411754D0</td>
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<td>16</td>
<td>0.9372733922400706D0</td>
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<td>17</td>
<td>0.848206853410427D0</td>
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<td>18</td>
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<td>19</td>
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<td>20</td>
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DO 230 I = 1, 8
E(I+1) = -E(I)
H(I+1) = H(I)
RETURN

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<th>I</th>
<th>E(I)</th>
<th>H(I)</th>
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<tbody>
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<td>0.0997263861849482D0</td>
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<td>23</td>
<td>0.985611511545268D0</td>
<td>0.0985611511545268D0</td>
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<td>24</td>
<td>0.96476255587506D0</td>
<td>0.096476255587506D0</td>
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<td>25</td>
<td>0.93490607597774D0</td>
<td>0.0934906075977774D0</td>
</tr>
<tr>
<td>26</td>
<td>0.896321155766052D0</td>
<td>0.096321155766052D0</td>
</tr>
<tr>
<td>27</td>
<td>0.849367613732570D0</td>
<td>0.0849367613732570D0</td>
</tr>
<tr>
<td>28</td>
<td>0.794483795967942D0</td>
<td>0.0794483795967942D0</td>
</tr>
<tr>
<td>29</td>
<td>0.732182118740290D0</td>
<td>0.0732182118740290D0</td>
</tr>
<tr>
<td>30</td>
<td>0.663044266930215D0</td>
<td>0.0663044266930215D0</td>
</tr>
</tbody>
</table>
E(11)=0.506899908932229D0
E(12)=0.421351276130635D0
E(13)=0.331868602282128D0
E(14)=0.239287362252137D0
E(15)=0.144471961582796D0
E(16)=0.048307665687738D0
H(1)=0.007018610009471D0
H(2)=0.016274394730906D0
H(3)=0.025392065309262D0
H(4)=0.034273862913021D0
H(5)=0.042835898022227D0
H(6)=0.050998059262376D0
H(7)=0.05868493478536D0
H(8)=0.065822222776362D0
H(9)=0.072345794108849D0
H(10)=0.07819389578707D0
H(11)=0.083311924226947D0
H(12)=0.08765209304404D0
H(13)=0.091173878695764D0
H(14)=0.093844399080805D0
H(15)=0.095638720079275D0
H(16)=0.096540088514728D0
DO 250 I=1,16
E(33-I)=-E(I)
250 H(33-I)=H(I)
RETURN
260 WRITE (*,270)
270 FORMAT(' WRONG CHOICE FOR NUMBER OF GAUSS INTEGRATION POINTS'/)
IERR=1
RETURN
END