MINE WASTE DUMP INSTABILITY

by

KAREN MOFFITT

B.A.Sc., The University of Toronto, 1996

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF APPLIED SCIENCE

in

THE FACULTY OF GRADUATE STUDIES

Department of Civil Engineering

We accept this thesis as conforming to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

April 2000

© Karen Moffitt, 2000
In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the head of my department or by his or her representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Department of CIVIL ENGINEERING

The University of British Columbia
Vancouver, Canada

Date April 25, 2000

http://www.library.ubc.ca/spcoll/thesauth.html

4/29/00
Abstract

Advancements in coal mining methods and rates have led to the production of larger volumes of waste rock. Consequently, large waste rock dumps in B.C. are currently being constructed by end-dumping to heights of up to 400 m with face angles of 37-38°. Segments of the foundation slopes underlying these mine dumps are frequently as steep as 30°. These developments have resulted in an increase in the frequency and size of waste dump failures that have not been adequately predicted or explained in terms of conventional slope stability analysis. Failure is often rapid with runout distances of up to 2 km causing increased concern within the industry over potential impacts to the environment and risks to the safety of personnel, equipment and infrastructure. A comprehensive review of documented field behaviour carried out for the purpose of this study has indicated common patterns of deformation that suggest the concept of a unique 'double wedge' mode of failure responsible for these large runout events.

This study focuses on the development of a numerical model capable of capturing the commonly observed patterns of deformation, and investigating the development of the ensuing failure mechanism. Stress-deformation numerical analyses carried out using the computer code FLAC have yielded a good correlation with observed field behaviour and provided significant insight into the coincident stresses and deformations within the dump. Analyses indicate that while creep effects in the waste dump could cause significant effects on the magnitudes of displacement, the overall dump stability is governed by the strength of the foundation soils underlying the toe region. Consideration of both the stress and velocity fields within the framework of non-associated plasticity leads to factors of safety for various dump heights and foundation slopes that are approximately 66% less than predicted from limit equilibrium analysis. The results suggest potential inadequacies of conventional limit equilibrium analysis techniques when applied to the stability and design of waste dumps on steeply sloping terrain.
# TABLE OF CONTENTS

Abstract

List of Tables

List of Figures

Acknowledgements

Chapter 1 Introduction
  1.1 Background
  1.2 Objectives

Chapter 2 Mine Dump Characteristics
  2.1 Types
  2.2 Construction Procedures
  2.3 Material Segregation

Chapter 3 Mine Dump Behaviour
  3.1 Observed Deformations
  3.2 Failure Records
  3.3 Double Wedge Failure Mechanism

Chapter 4 Material Properties
  4.1 Waste Rock
    4.1.1 Mineralogy
    4.1.2 Gradation
    4.1.3 Shear Strength
    4.1.4 Deformation Properties
  4.2 Foundation Soils
    4.2.1 Gradation
    4.2.2 Shear Strength
    4.2.3 Deformation Properties
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Summary of Conditions of Failure</td>
<td>15</td>
</tr>
<tr>
<td>4.1</td>
<td>Components of Waste Rock</td>
<td>19</td>
</tr>
<tr>
<td>5.1</td>
<td>Summary of Waste Dump Geometry</td>
<td>39</td>
</tr>
<tr>
<td>5.2</td>
<td>Factors of Safety</td>
<td>44</td>
</tr>
<tr>
<td>6.1</td>
<td>Material Properties</td>
<td>67</td>
</tr>
<tr>
<td>6.2</td>
<td>Stress Ratios Acting on Failure Surface</td>
<td>71</td>
</tr>
<tr>
<td>7.1</td>
<td>Factors of Safety (Limit Equilibrium)</td>
<td>83</td>
</tr>
<tr>
<td>7.2</td>
<td>Factors of Safety (Non-Associated Flow Rule)</td>
<td>84</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Coalfields in British Columbia</td>
<td>3</td>
</tr>
<tr>
<td>2.1</td>
<td>Typical Mine Dump Configurations</td>
<td>5</td>
</tr>
<tr>
<td>2.2</td>
<td>Dump Construction Procedures</td>
<td>7</td>
</tr>
<tr>
<td>2.3</td>
<td>Dumping Methods</td>
<td>8</td>
</tr>
<tr>
<td>2.4</td>
<td>Material Segregation</td>
<td>9</td>
</tr>
<tr>
<td>3.1</td>
<td>Typical Cross-Section of an Active Dump</td>
<td>12</td>
</tr>
<tr>
<td>3.2</td>
<td>Dump Height vs. Base Slope for Failure Events</td>
<td>14</td>
</tr>
<tr>
<td>3.3</td>
<td>Profiles of Failure Planes</td>
<td>16</td>
</tr>
<tr>
<td>3.4</td>
<td>Double Wedge Mechanism</td>
<td>18</td>
</tr>
<tr>
<td>4.1</td>
<td>Range of Grain Size Distributions for Waste Rock</td>
<td>20</td>
</tr>
<tr>
<td>4.2</td>
<td>Results of Large-Scale Triaxial Testing of Rockfill</td>
<td>22</td>
</tr>
<tr>
<td>4.3</td>
<td>Estimation of Equivalent Strength (S) of Rockfill</td>
<td>24</td>
</tr>
<tr>
<td>4.4</td>
<td>Estimation of Equivalent Roughness of Rockfill</td>
<td>25</td>
</tr>
<tr>
<td>4.5</td>
<td>Nonlinear Strength Envelopes</td>
<td>26</td>
</tr>
<tr>
<td>4.6</td>
<td>$\phi'$ vs. log of Confining Stress</td>
<td>26</td>
</tr>
<tr>
<td>4.7</td>
<td>Contours of $\phi'$ Throughout Mine Dump</td>
<td>27</td>
</tr>
<tr>
<td>4.8</td>
<td>Variation of Shear Modulus with Confining Pressure</td>
<td>27</td>
</tr>
<tr>
<td>4.9</td>
<td>Range of Grain Size Distributions for Foundation Soils</td>
<td>29</td>
</tr>
<tr>
<td>4.10</td>
<td>Results of Triaxial and Direct Shear Tests on Foundation Soils</td>
<td>30</td>
</tr>
<tr>
<td>4.11</td>
<td>Idealized Relation of Dilation Angle and Shear Strain</td>
<td>32</td>
</tr>
<tr>
<td>5.1</td>
<td>Basic Explicit Calculation Scheme</td>
<td>34</td>
</tr>
<tr>
<td>5.2</td>
<td>Mohr-Coulomb Model</td>
<td>37</td>
</tr>
<tr>
<td>5.3</td>
<td>Typical Mine Dump Profile</td>
<td>39</td>
</tr>
<tr>
<td>5.4</td>
<td>Mobilized Base Friction Parallel to Foundation Slope</td>
<td>41</td>
</tr>
<tr>
<td>5.5</td>
<td>Deformed Dump Profile</td>
<td>43</td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>6.1</td>
<td>Mohr-Coulomb Criterion for Yield and Shear Band Development</td>
<td>47</td>
</tr>
<tr>
<td>6.2</td>
<td>Plastic Strain Increment Vectors</td>
<td>49</td>
</tr>
<tr>
<td>6.3</td>
<td>Simple Shear Testing of Soil</td>
<td>52</td>
</tr>
<tr>
<td>6.4</td>
<td>Results of Simple Shear Testing on Leighton Buzzard Sand</td>
<td>53</td>
</tr>
<tr>
<td>6.5</td>
<td>Passive Retaining Wall Experiments</td>
<td>55</td>
</tr>
<tr>
<td>6.6</td>
<td>Trajectories of Zero-Extension</td>
<td>55</td>
</tr>
<tr>
<td>6.7</td>
<td>Definition of Stress Orientation Angle, ( \theta )</td>
<td>60</td>
</tr>
<tr>
<td>6.8</td>
<td>Simple Shear Test</td>
<td>62</td>
</tr>
<tr>
<td>6.9</td>
<td>Numerical Results for Simple Shear Tests</td>
<td>66</td>
</tr>
<tr>
<td>6.10</td>
<td>Single Element Simple Shear Test Using FLAC</td>
<td>67</td>
</tr>
<tr>
<td>6.11</td>
<td>Effect of Dilation on the Maximum Stress Ratio On the Failure Plane</td>
<td>68</td>
</tr>
<tr>
<td>6.12</td>
<td>Rotation of Principle Stresses During Simple Shear</td>
<td>69</td>
</tr>
<tr>
<td>6.13</td>
<td>Change in Stress Parallel to Failure Plane</td>
<td>69</td>
</tr>
<tr>
<td>6.14</td>
<td>Influence of Dilation Angle on the Mobilized Friction Angle at Failure</td>
<td>71</td>
</tr>
<tr>
<td>6.15</td>
<td>Stress State Softening</td>
<td>73</td>
</tr>
<tr>
<td>7.1</td>
<td>Mobilized Friction Angle in the Foundation Layer</td>
<td>78</td>
</tr>
<tr>
<td>7.2</td>
<td>State of Stress Within the Foundation</td>
<td>81</td>
</tr>
<tr>
<td>7.3</td>
<td>Mobilized Friction Angle vs. Plastic Shear Strain in the Foundation Elements</td>
<td>82</td>
</tr>
<tr>
<td>7.4</td>
<td>Factor of Safety vs. Foundation Slope</td>
<td>85</td>
</tr>
</tbody>
</table>
Acknowledgements

I owe a debt of gratitude to Dr. Peter Byrne for his support and contributions throughout this study. Our discussions continually challenged and motivated me, collectively providing me with insights that shaped the direction of my research.

This project was initiated by Golder Associates (Burnaby) and I would like to thank all the individuals at Golder for their help and support while carrying out my research. I especially would like to thank Terry Eldridge, in particular, for providing me with such a challenging area of study. I would also like to thank Terry and others at Golder for providing me with all the necessary resources to carry out my research.

In addition, I gratefully acknowledge the financial support provided by both Golder Associates and Fording Coal.
1.0 Introduction

1.1 Background

The development of coal mine deposits in the Rocky Mountain region of Western Canada in recent decades has pushed the frontier of coal mining activities into more difficult topographic locations. Coal seams typically occur interbedded with siltstones, sandstones and mudstones. The open pit mining methods employed to access the coal seams necessitate the removal of large quantities of waste rock, which must be disposed of economically and safely. Currently, at the 10 active coal mines in the Canadian Rockies, 30 million tonnes of coal are produced annually with an associated production of 200 to 250 million tonnes of waste rock (Dawson, 1995). As haulage costs are sensitive to the proximity of waste dumps to pit areas, they must be located as close as possible to the open pits where the waste rock is generated. As a result, mined rock structures are currently being constructed by end dumping to heights in excess of 400m. Although mine planners generally attempt to take advantage of the gentler topographic slopes near the base of mountains, segments of the foundations underlying B.C. waste dumps are commonly as steep as 30°.

The disaster at Aberfan, Wales in 1966 first brought the problem of waste dump instability to public attention. This slide involved approximately 130,000 m$^3$ of spoil material flowing downslope resulting in the loss of 144 lives, 116 of those were children between the ages of 7 to 10 years (Bishop, 1973). Aberfan triggered widespread attention to mine dump instability but, to date, a thorough understanding of mine dump behaviour still does not exist. Recent failure events at B.C mine dumps have involved failure volumes upward to 30x10$^6$ m$^3$ indicating that our understanding is lagging far behind developments in mining technology which are allowing for more innovative mine dump design practices.

Records of failure events within B.C. substantiate widespread beliefs that the increased height and steeper foundation topography of modern B.C. waste dumps have resulted in a dramatic increase in the frequency and volume of failure events. Failure is often rapid with runout distances of up to 2 km and failure volumes upward to 30 million m$^3$ causing increased concern within the industry over potential impacts to the environment and risks to the safety of personnel, equipment and infrastructure. In the last 25 years, 50 high runout failure events have been
documented in B.C. alone and a recent survey indicates that over 30% of the coal mine waste dumps surveyed have experienced instability associated with a runout exceeding 100m (Piteau, 1991). These records are testimonial to the need for further investigation into the mechanics of dump instability. Unlike more conventional engineered fill structures such as earth dams and embankments, material is placed without any compaction. Therefore, any localized slip or failure is more likely to develop into a flow slide. Considering the frequency and consequences associated with failure, the disproportionate amount of attention this problem has received is surprising. Current design and operating guidelines incorporate 'safe fail' procedures but a lack of understanding of the cause and development of failure within large waste rock dumps is an impediment to good design and safe construction practices.

Monitoring data and careful observations carried out on dumps worldwide have provided significant insights into dump behaviour. The most widely published and accepted theory is commonly referred to as the 'double wedge' failure mechanism in which failure is believed to occur along planar surfaces that define two wedge shaped blocks within the rockfill. The theory is based on observations of common patterns of pre-failure deformations and post-failure dump profiles observed at mine dumps in western Canada and elsewhere. Although dump geometry and soil properties differ from region to region, similar observations observed in Australia and South Africa give valuable insight into the mechanics of dump deformation in western Canada and will therefore be included in this study. The proposed mechanism of deformation and failure is largely governed by the available strength of the foundation soils underlying the toe region of the dump. Pore pressures generated either by rapid construction of the dump or by subsequent shear movements will control the strength of the foundation soils. For a particular dump geometry, a limiting value of foundation strength is required to ensure dump stability. Campbell (1986) has adapted this theory and developed a limit equilibrium analysis technique to be employed on mine dump geometry and soil conditions typical of mine dumps in western Canada. This theory is at the heart of state-of-the-art practice in mine dump stability analysis and will be discussed in detail in subsequent chapters.
Figure 1.1 Coalfields in British Columbia
1.2 Objectives

The purpose of this study is to develop a detailed overview.

1. Present a review of published data pertaining to:
   - Field behaviour
     - Patterns of typical ongoing dump deformations
     - Pre-failure patterns of deformation
   - Laboratory model testing
   - Records of failure events to assess trends in dump geometry, site conditions and climate
   - Double wedge mechanism of mine dump failure

2. Determine representative engineering properties to be used in the analysis procedures.

3. Examine the validity of the 'double wedge' mechanism of failure by applying stress-deformation numerical analysis techniques using the computer code FLAC. The aim of the analysis is to simulate Campbell's 'double wedge' mechanism of failure while capturing the pre-failure patterns of deformation. Strength and stiffness parameters to be used in the FLAC analysis will be determined from a literature review of the engineering properties of mined waste rock and typical foundation soils. Model behaviour will be compared with the results of a comprehensive review of field behaviour involving both stable and unstable dumps on steep foundations. Factors of safety for various dump heights and foundation slopes will be determined and compared with the results of double wedge limit equilibrium analysis.
2.0 Mine Dump Characteristics

Mine dumps are very unique structures, not only due to their function, but also their location, geometry and type of construction. Each of these characteristics influences the behaviour of the structure. In order to present field and analytical observations concerning the behaviour and stability of mine dumps it is necessary to describe the variations in these key characteristics as well as their potential effects on mine dump behaviour.

2.1 Types

To minimize the degree of material rehandling and haulage costs, waste dumps are typically located in the mountainous terrain of the mine site. Three different waste dump configurations have been developed for this topographic environment, as shown below in Figure 2.1.

Figure 2.1 Typical mine dump configurations
The focus of this report is the stability and deformations associated with the sidehill fill construction. For economic necessity, this type of dump is designed to accommodate waste rock at, or near mountaintops. Consequently, they are the largest and most common type, accounting for over 70% of B.C. coal mine dumps (Piteau, 1991). High volume, catastrophic failures are often associated with sidehill fills due to the limited amount of confinement and steep toe slopes during intermediate stages of construction.

Wrap-around dumps are a modified version of sidehill fills adapted to accommodate waste rock from successively lower pit elevations as the mine develops. The wrap-around provides increased toe support and containment of any weak materials placed on the face of the high dump resulting in a significantly more stable configuration.

Valley fills are constructed parallel to topographic contours and are attractive due to the three-dimensional confinement provided by the valley walls. Disadvantages of this type of construction are related to the impact of stream flow on stability. If stream flow is disrupted, ponding may result upstream of the dump. More critically, the dump must be designed to withstand the 200-yr. flood event within the watercourse in which it is located. Design considerations must include an investigation of the potential for overtopping of the dump in the event of a large flood. Overtopping would result in slope failures and downstream sedimentation.

2.2 Construction Procedures

Sidehill fills are constructed by dumping waste material from the dump crest. Most commonly, the dump crest is built out in a direction perpendicular to topographic contours by the gradual accretion of material on the dump face (Figure 2.2a). In the preliminary stages of development, designers often take advantage of gullies or other topographic features to maximize 3-D confinement. Construction continues and the dump toe eventually establishes itself on the gentle slopes of the valley floor. The most unstable configuration exists at intermediate stages of development when the toe is founded on relatively steep slopes (30-35°) and the effects of 3-D confinement are negligible.

Once a section of the dump has advanced onto the valley floor, the dump crest may be advanced parallel to topographic contours, increasing stability during the critical intermediate stages of construction. Figure 2.2b shows a plan view of the dump as it is advanced in this manner. Section A-A depicts the development of the dump profile.
as the crest advances. Material is built up in a gradual manner, initially loading the lower flatter slopes progressing upward. The toe is never founded on the steep upper slopes and therefore stability during construction is increased although overall dump stability for the final dump profile is unchanged.

![Diagram of dump construction procedures]

**Figure 2.2** Dump Construction Procedures a) parallel and b) perpendicular to topographic contours

Two different types of dumping are used depending on the condition of the dump (Figure 2.3). The common type of dumping procedure is end dumping in which the track backs right up the dump face and discharges material directly down the dump face. In rare situations where monitoring data suggests accelerating trends of crest displacement or if increased cracking is observed on the dump platform, issues of safety necessitate the use of ‘push’ dumping. This procedure involves the truck stopping short of the crest and discharging material onto the dump platform to be pushed over the crest by a bulldozer.
2.3 Material Segregation

The conglomerate waste rock material disposed of in B.C. coal mine dumps is typically a dry mixture ranging from silt to boulder sized particles with a maximum particle size on the order of 1-2m. Dumping from the crest causes significant material segregation as the material rolls down the dump face. The resulting variation of grain size with elevation through the dump has important consequences on dump behaviour and stability.

Documentation of visual observations of dump faces invariably reports that the overall gradation appears progressively coarser with depth. The crest area generally contains a high proportion of fine materials while naturally segregated coarse materials settle beyond the toe. The coarse materials blanketing the lower slopes are eventually covered over by ongoing construction forming a continuous permeable drainage layer at the dump/foundation contact. The natural rock drain provides an excellent conduit for transmitting runoff and preventing the piezometric surface from rising into the dump.

Drained soil conditions are necessary to ensure dump stability and are an inherent assumption of design and stability calculations. Significant research has been carried out to assess the flow capacity of the underlying rock drain as adequate flow capacity is necessary to prevent the generation of any excess pore pressures below the dump foundation contact. An understanding of the amount and nature of segregation is also imperative in determining the potential for the migration of fines and subsequent obstruction or blinding of the underlying drainage layer. The
results of laboratory scale experiments reported by Golder (1987) are summarized in Figure 2.4 depicting the grain size distributions of materials tested at various elevations through a model dump. The variation from crest to base is in agreement with field observations, material sampling and photographic analysis. The material at the base of the dump has a much higher proportion of coarse material necessary for drainage while the progressively finer grain sizes toward the crest impede the downward migration of fines through the dump, preventing blinding or obstruction of the rock drain.

Several factors have been observed to affect the nature and extent of segregation. Nichols (1986) carried out more detailed laboratory scale experiments modeling dump construction under a variety of conditions to determine their effects. The length of slope or gradation of waste rock has been shown to have negligible effects on the amount or extent of segregation. In contrast, model dump construction using both end dumping and push dumping construction methods revealed significant differences. End-dumped material gains momentum by sliding out of the truck shovel causing the particles to roll rather than slide down the dump face. The angular momentum developed by an individual rock is much greater than the frictional resistance of the material causing the particles to roll until the slope flattens or they collide with particles of comparable size. Therefore, end dumping results in the collection of coarser material on foundation slopes beyond the dump toe and a stable dump face formed at an angle less than the friction angle of the waste rock.
In contrast, push-dumped material slides down the dump face and coarser materials tend to get hung up in the fine aggregate at the slope crest resulting in an oversteepened, unstable dump profile. This condition is quite frequent in the field and can lead to mass instability. To quantify the difference in segregation due to dumping methods, Nichols (1986) finds that an average of 75% of the largest particle size rolled beyond the dump toe for the end-dumped construction but only 40% for push dumping. The smaller proportion of coarse material reaching the slopes beyond the dump toe may have adverse effects on the continuity or thickness of the underlying drainage layer. In the past, push dumping has been used when crest displacement rates are abnormally high but this recent investigation suggests it only aggravates any problems of potential instability.

Another form of material segregation takes place in dump construction, although it is not as influential to dump behaviour. As the conglomerate mixture is dumped from the crest, subtle layers of coarse and fine materials form parallel to the dump face. These alternating layers of coarse and fine materials are believed to play a role in the formation of melt lines on dump faces (Campbell, 1997) which will be discussed in the following chapter.
3.0 Mine Dump Behaviour

3.1 Observed Deformations

As displacement monitoring of waste dumps is strictly limited to the crest area, visual observations of dump deformations are key in attempting to understand the mechanics of potential instability. With the recent increase in the frequency and size of failures, mine operators have put more emphasis on the importance of visual observations in assessing stability. Records of monitoring data and visual observations of typical ongoing dump deformations have indicated common patterns characteristic of large dumps founded on relatively steep (>15°) foundation slopes. The foundation soils mantling these slopes typically consist of a thin veneer of colluvium underlain by bedrock. While a significant portion of typical dump deformations is the result of compaction under self-weight, monitoring data gives evidence of common patterns of internal shear straining characteristic of dumps constructed in these conditions.

As material is end-dumped from the crest, the dump face generally forms at an angle of 37-38 degrees. Upon temporary suspension of dumping activities, the dump profile undergoes subtle but observable changes (Campbell, 1986). Figure 3.1 presents a cross-section of a surveyed active dump (Golder, 1987). Internal straining results in a slight flattening of the upper portion of the dump face and steepening of the lower portion. While the average slope of the dump face remains at approximately 37 degrees, angles as steep as 41-42 degrees have been observed in the lower third portion. Continued straining typically divides the dump face into two distinct regions, each exhibiting unique patterns of deformation. The boundary between these two regions may be visible by the formation of a discontinuity or 'step' parallel to the crest on the middle third of the dump face. While this discontinuity is rarely visible, the boundary is also emphasized by preferential snow melt and the formation of a horizontal melt line in this region, visible at the majority of large B.C. dumps during winter months. Horizontal lineations form on the upper portion of the face and are indicative of the large strains taking place within the dump. These horizontal bands are markedly absent in the lower portion of the dump face where vertical striations are commonly visible and form as a result of surface raveling due to oversteepening (Golder, 1987).
Figure 4.2 Results of large-scale triaxial testing of rockfill (Leps, 1970)

The relationship of friction angle and normal stress plots as a straight line on a semi-log plot emphasizing the large variation in friction angle at low and intermediate confining pressures. Available test data actually suggests that the 'average' rockfill line curves upward as normal pressure decreases from 10psi to 1 psi but this relationship has not yet been defined. Results of triaxial testing of crushed granite carried out by Seed and Goodman indicate friction angles as high as 67 degrees at extremely low values of confining pressure.

Relationships have been proposed by Leps, classifying rockfill according to density, particle strength and gradation into 3 broad categories:

- Weak – low density, poorly graded particles (≈ 3.5-17.4 MPa strength)
- Average (≈17.4 -70 MPa strength)
- Strong – high density, well graded strong particles (≈70-210 MPa)

Based on the proportions of constituent components in the waste rock and their unconfined compressive strengths (Table 4.1), a conservative weighted average of the U.C.S has been taken to be 70 kPa, allowing for wide variations
Deformations in the crest area commonly consist of a high scarp set well back from the crest or a series of scarps varying in height and width. Fig. 3.1 shows vectors of crest displacement inclined at approximately 50-60 degrees to the horizontal. The shallower inclination of movement vectors relative to the steep inclination of the scarps results in the opening of tension cracks visible on many dump platforms. Cracks and scarps are indicators of normal dump deformation and are useful in ensuring proper monitor locations. Unless monitor readings show rapidly accelerating displacement rates, cracks and scarps are regularly filled over with waste to maintain smooth road access to the crest and to control water infiltration.

![Cross Section of Complete Slope](image)

**Figure 3.1** Typical cross-section of an active dump (Golder, 1987)

Typically, if records of displacement vs. time show that crest movements are occurring at a rate in excess of 0.5m/day, the dump operations are temporarily suspended at that location and moved to an alternative area of the dump. This cut-off rate of 0.5m/day for crest movements is considered to be a conservatively safe limit (Tassie, 1998). Crest displacement rates of up to 4 to 5 m/day on a dump in excess of 200 m in height were commonly experienced without resulting in failure (Golder, 1987).

Little data is available regarding magnitudes of toe movements associated with periods of high crest displacement rates. End-dumping of waste rock from the crest makes displacement monitoring at the toe region infeasible due to the impact of large rolling boulders on monitoring equipment. Documentation of field observations generally
reports little or no toe movement of stable dumps experiencing large crest displacements although conflicting opinions exist (Golder, 1987). Based on observations of a large B.C. dump, Gold (1986) reports large toe movements corresponding to periods of high crest displacements. In contrast, Campbell’s observations of modest steepening of the lower portion of the dump face coupled with surface raveling validates the theory that the toe remains intact while the crest and face deforms. A general trend in the literature does suggest that the toe remains intact although the limited available data proves inconclusive.

3.2 Failure Records

Several different modes of mine dump failures have been described in detail by Caldwell and Moss (1981). This study is concerned solely with large volume events involving foundation failure at shallow depth below the dump/foundation contact. These events are typically described as ‘explosive’ by eyewitnesses and result in long runout paths with obvious implications to personnel safety and mining operations. Approximately 50 high runout events have been documented in BC during the last 25 years. Dump failure results in rapid loading of wet, saturated materials along the lower portion of the runout path causing excess pore pressures able to sustain mobility to distances exceeding 3000m. In order to prevent or predict catastrophic failure, a greater understanding of the mechanics of instability is required.

Broughton (1992) prepared an extensive failure database including specific details for 44 mine dump failures which have occurred between 1966 and 1991. Large volume events have been reviewed to investigate trends in pre-failure deformation patterns and site conditions. Relatively few similarities exist between all failure cases. Dumps heights are all in excess of 100m on relatively steep topography. Foundation profiles are typically concave downward in profile with foundation soils consisting of only a thin veneer of colluvial silty sands and gravels. Each failure record reports a period of increased crest settlement preceding failure. High scarps form on the dump platform along with tension cracking. In one case, on a 300 m high dump, the crest dropped 50-60 m prior to failure.

Documentation of failures from various sources validates these basic conditions exist in virtually all failure cases in the province. Figure 3.2 prepared by Golder (1987) presents a plot of dump height vs. foundation slopes for several mobile failure events. The chart indicates that failure is unlikely to occur on dumps less than 150m in height with foundation slopes less than 15 degrees.
Eyewitness accounts of failure formation generally report the formation of a face bulge in the lower third portion of the dump face prior to failure. This bulge appears to grow in size over the days or hours preceding failure. Over half of the failure cases presented by Broughton (1992) report face bulging prior to failure. Although some failures do not report observations of bulging, it is important to note that routine measurement and observation of this type of deformation has not been carried out to date. It is difficult to find a safe, suitable vantagepoint that provides a consistent view of the dump face in profile.

Failure is always preceded by an interval of several hours to a few days during which crest displacements develop at progressively faster rates. Truck drivers and other operators are often the first to notice increasing crest movements due to the increased effort required to maintain platform surfaces. In addition, monitoring records of crest displacement vs. time provide an adequate warning of impending failure.

The increased crest displacement and face bulging prior to failure indicates that failure is preceded by accelerating trends of the typical ongoing deformations described in Section 3.1. This suggests a connection between the mechanism responsible for the acceptable, ongoing deformation of an active, stable dump and that responsible for failure. The dump profile immediately prior to failure is basically an exaggerated version of Figure 3.1. In order for displacements to accelerate to failure, external conditions must exist which are capable of inducing mass instability.
A review of the database (Broughton, 1992) was carried out to investigate site conditions prevalent at failure sites. Failures were generally attributed to one or a combo of various adverse site conditions as shown in Table 3.1 below.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Frequency in Database (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High loading rates</td>
<td>60</td>
</tr>
<tr>
<td>Foundation seepage pressures</td>
<td>60</td>
</tr>
<tr>
<td>Intense rainfall</td>
<td>40</td>
</tr>
<tr>
<td>Poor waste rock quality</td>
<td>36</td>
</tr>
<tr>
<td>Frozen foundation soils</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 3.1. Summary of Conditions at Failure

It is clear that there is no single contributing factor responsible for these large volume events. It is important to note that each of the factors listed above is capable of contributing to the generation of excess pore pressures within the foundation. As stability is dependent on the strength provided by the foundation soils, excess pore pressure generation is widely thought to be the primary contributing factor to mass instability.

A vast amount of literature is available documenting the behaviour of spoil pile failures in Australia’s Bowen Basin. Spoil piles are supported on flatly dipping clay foundations not typical to mine dump sites in B.C. but do provide insight into the possible failure mechanism. Failures are not explosive and therefore post-failure deformation patterns provide further clues as to the mechanism of instability.

A number of spoil pile failures were extensively studied using accurate surface surveying and subsurface instrumentation to determine the kinematics of the failure process. Results are consistent with the observations of Dunbaven (1981) and Chowdbury (1986) as well as observations of Blight (1969) of failures of gold mine spoil piles in South Africa. Figure 3.3 depicts a comparison of the failure surfaces of 5 sites surveyed and indicates the geometric similarities between the failures.
The failure plane is bilinear with a back failure plane inclined at approximately 62 degrees and a narrow basal plane parallel to the foundation. Instrumentation reveals that for the greater part of the deformation process, the mass effectively moves as 2 distinct blocks. As displacements become large, the upper wedge appears to undergo multiple rupturing to overcome the kinematic singularity at the knee of the failure. This rupturing occurs as well-defined planes parallel to the back plane. Shear movements along the three slip planes often result in a characteristic berm or gentle s-shape on the slope face. A rill of foundation material is pushed up by the toe block.

3.3 Double Wedge Failure Mechanism

Conventional slope stability analyses involving limit equilibrium principles assume that the factor of safety is the save value along all segments of the potential failure surface. From the observations of dump deformations, both in B.C. and elsewhere, patterns and magnitudes of shear displacement are incompatible with this assumption. Differential shear displacements on the dump platform are commonly on the order of meters, indicating that the shear strength of this material is fully mobilized. Even with these large displacements in the crest region, the dump remains stable suggesting that the shear strength along segments of the potential failure surface at lower levels within the dump was only partially mobilized.

Consistent with observations of dump deformations previously discussed, the potential failure mass below the face of the dump consists of two wedge-shaped zones. The upper "active" wedge is in the active Rankine state which
implies that the shear strength parameters within this zone are fully mobilized. The active wedge retains its position on the dump face by virtue of the support provided to it by the toe wedge.

Various small-scale laboratory model tests have been carried out by Dunbaven (1980), Blight (1981) and Eckersley (1990). Model tests allow for a more intensive investigation of the deformations within the interior of the dump. Models closely simulated typical geometry and material conditions present at mine dumps in Australia and South Africa. The results of these tests have validated the concept of the 'double wedge' failure mechanism and have given insight into the sequence of failure. Dunbaven (1980) concluded that failure initiates by the simultaneous development of two slip planes, one through the weak basal layer and the second through the granular slope cutting the surface just behind the crest. At this stage, a broad shear zone exists between the two planes dividing the face of the dump into toe and crest regions. As movement of these two regions along their respective slip planes ensues, a distortion in the shear zone takes place, tilting of the lower half of the dump face away from the toe (Figure 3.4). A third slip lane then forms within the shear zone between the first two planes with no further shear distortion in that zone. This is followed by movement along the three slip planes with negligible volume change within the toe and crest regions.

Laboratory scale model tests were performed by Campbell (1987) to simulate geometry and soil conditions typical of waste dumps in western Canada. Model behaviour was consistent with the double wedge mechanism and provided the framework for the double wedge limit equilibrium analysis (Figure 3.4).
The active wedge is assumed to be in an active Rankine state which implies that internal strains have resulted in full mobilization of the shear strength parameters of waste rock contained in the active wedge as well as along its boundaries. This active wedge is assumed to be in a state of limiting equilibrium, and the unknown forces along its boundaries are calculated. The mobilized friction along the base corresponding to limiting equilibrium of the toe wedge is calculated. Trial and error calculations are carried out, varying the inclinations of the active wedge boundaries as well as the position at which these wedge boundaries intersect the foundation. The highest value of the mobilized base friction is then determined. The factor of safety is the defined as ratio of the tangent of the internal friction angle to the tangent of the mobilized base friction angle (Campbell, 1986).

This method of stability analyses is required as part of the feasibility assessment for a proposed waste rock dump. It is believed to be more appropriate than other methods of stability analyses that are in common usage to provide an accurate indication of the factor of safety.
4.0 Material Properties

The material properties to be used in the stability analysis are determined from available data of actual soil conditions. Where this data is unavailable, material properties are estimated from a review of published data for similar materials. Since the effects of potential excess pore pressure generation are beyond the scope of this study, all material is assumed to be fully drained.

4.1 Waste Rock

4.1.1 Mineralogy

Waste rock generated from open pit coal mines in Western Canada typically consist of varying proportions of sandstone, siltstone, mudstone, and shale. Mudstone and shale being the weakest components in the conglomerate waste rock exist in relatively small proportions.

<table>
<thead>
<tr>
<th>Rock Type</th>
<th>Typical Proportions within waste rock(%)</th>
<th>Unconfined Compressive Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandstone</td>
<td>50</td>
<td>110-170</td>
</tr>
<tr>
<td>Siltstone</td>
<td>30</td>
<td>60-110</td>
</tr>
<tr>
<td>Mudstone/shale</td>
<td>20</td>
<td>30-80</td>
</tr>
</tbody>
</table>

Table 4.1 Components of waste rock (after Golder, 1987)

4.1.2 Gradation

As discussed in Chapter 2, material gradation varies from crest to toe. Figure 4.1 depicts the grain size distribution of coarse waste rock as well as fine waste material typically found in the top 10% of the dump. Between these two bounds is the grain size distribution of rockfill materials for which published triaxial testing data is available.
An average value of the density of the waste material is 1900 kg/m$^3$. The effects of the variation in gradation throughout the waste dump on material density will be neglected and this average value will be used in the analysis.

4.1.3 Shear Strength

Understanding of rockfill behaviour in general has lagged far behind knowledge of the stress-strain behaviour of soils due to the cost of large-scale testing equipment capable of handling prototype-sized pieces of rock. Recurring problems of instability at mine dumps in Australia and elsewhere have necessitated large-scale testing of mine spoil for which significant published data exists. Unfortunately, inherent differences in the constitution of these mine waste materials renders this data inapplicable to the cohesionless waste rock generated at open pit coal mines in Western Canada.

In recent decades, construction of increasingly high rockfill dams has led to the development of large-scale testing equipment for rockfill. The range of grain-size distributions of this rockfill falls between the limits of the coarse and fine grained material within waste dumps (Figure 4.1). Published test data will be used for the purpose of this study to estimate the shear strength parameters of the mined waste rock.
The shear strength of both the waste rock and foundation soils is defined according to the Mohr-Coulomb failure criterion,

$$\tau = c' + \sigma_n \tan \phi$$  \hspace{1cm} (4.1)

The waste rock is modeled as purely frictional. Field observations indicate that the higher fines content in the top 10% results in an apparent cohesion responsible for the near vertical scarps that form on the dump platform. The magnitude of this value is unknown and will be neglected in this analysis as it is not considered to have any influence on the overall dump stability.

To date, conventional slope stability analyses carried out on waste rock dumps generally assume a uniform friction angle for the entire waste dump equal to the angle of repose (37-38°). This assumption implies that face stability is critical, and provides no explanation for the propagation of deep-seated failure mechanisms as discussed in Chapter 3. Insight gained from large-scale testing of rockfill suggests that this assumption is a gross oversimplification of material strength. Published data reveals a non-linear failure envelope indicating higher values of $\phi'$ at low stresses (Leps, 1970, Marsal, 1967). The friction angle of the rockfill is shown to be inversely proportional to the log of effective normal stress across the failure plane.

The effects of nonlinearity are increased when stability is assessed using finite difference modeling as it allows for stress redistribution and progressive failure. The shape of the failure envelope is of particular interest at the low stresses present at shallow depth below the dump face and toe where stress ratios are relatively high. Empirical approaches will be used to develop a representative non-linear failure envelope for the waste rock.

**Leps (1970)**

Leps (1970) has compiled an extensive database of published data of individual large scale triaxial tests on gravels and rockfill. This data is for rockfill material ranging from gravel to 8-in rock specimens covering a variety of 15 different rockfill sources (Fig. 4.1). Results are presented below as a plot of friction angle vs. the normal pressure across the failure plane (Fig. 4.2).
in the quality of the waste rock. Based solely on the UCS, this classifies the waste rock as 'strong'. The waste rock particle strength may be high but the end-dumped method of construction results in a poorly graded structure of relatively low density. Compaction of the waste rock matrix occurs only under the weight of added materials. Values of $\phi'$ have been shown to be extremely sensitive to density as opposed to crushing strength, particularly in the stress range of interest. Therefore it seems reasonable to assume that the function relating friction angle and confining stress would fall within the regions defined as 'weak' to 'average' rockfill.

This method of shear strength estimation is very rough and it is important to note its shortcomings. As material is only classified into three broad categories, it does not address the effects of relative density, material gradation, crushing strength and particle angularity.

**Barton and Kjaernsli (1981)**

The empirical relationship proposed by Barton and Kjaernsli (1981) is widely accepted as a means for developing a preliminary estimate of the peak drained friction angle of rockfill. The method is based on similarities between rockfill, interfaces and rock joints observed in the results of large scale triaxial and plane strain tests on rockfill as well as direct shear tests performed on 130 rock joint specimens. In each case, failure is resisted by strongly stress-dependent friction angles.

While Leps' approach involves classifying rockfill into very broad categories, this method is more thorough and material specific. The friction angle of a given rockfill is estimated from knowledge of:

- Uniaxial compressive strength
- $D_{50}$ particle size
- Degree of particle roundedness
- Porosity following compaction

These properties are used to determine an equivalent roughness term, $R$, and an equivalent strength, $S$, for a given rockfill related to the friction angle by the following equation

$$\phi' = R \log\left(\frac{S}{\sigma'_{R}}\right) + \phi_0$$

(4.2)
The equivalent strength is strongly dependent on the unconfined compressive strength of the rock with modifications to account for scale effects. The rockfill size is defined by $D_{50}$ although shear strength differences do exist between a widely and narrowly graded rockfill with equal $D_{50}$. Differences are generally due to the different porosities attainable and this is accounted for in the estimation of the equivalent roughness term. Figure 4.3 below is used to determine the value of the equivalent strength and was developed by trial and error fitting of numerous triaxial data with equation 4.2 to account for particle size effects. Microfractures that typically exist in larger rock specimens lead to the s-shaped curve relating crushing strength to particle size.

![Figure 4.3 Estimation of Equivalent Strength (S) of Rockfill (Barton and Kjaernsli, 1981)](image)

The equivalent roughness parameter is dependent on the source of the rockfill from quarried rock (angular) to fluvial deposits (smooth). The more angular the rockfill particles, the higher the equivalent roughness, and hence, the friction angle. The value of equivalent roughness to be used in the estimation of $\phi$ can be determined from Figure 4.4 below. This figure is a result of trial and error and back analysis of numerous triaxial and plane strain tests on rockfill materials.
For the case of mine waste rock,

D<sub>30</sub>=8mm (from Figure 4.1)

UCS=70 MPa (from Table 4.1)

Porosity≈0.35

Origin- angular quarried rock

From Figures 4.3 and 4.4, S=29 MPa and R=6. Considering a 200m high dump, the estimated friction angle ranges from 36° to 49° (Figure 4.7).

The parameters used in the estimation process were chosen conservatively. The failure envelope is essentially coincident a curve midway between the 'weak' to 'average' rockfill as defined by Leps classification system. The waste rock at most sites is loosely packed but high quality and therefore, this curve seems appropriate.
Figure 4.5  Non-linear strength envelopes

Figure 4.6  \( \phi' \) vs log of confining stress
4.1.4 Deformation Properties

The purpose of the analysis is the investigation of failure patterns within the waste dump model and therefore, the elastic deformation properties of the materials being modeled are not of prime concern. Ideally, for representative deformation modeling, the model would be calibrated to wireline extensometer and GPS data of an actual field case. Instead, estimates of the moduli for the rockfill will be based on empirical relationships.

The shear modulus of the rockfill is dependent on the level of confining stress as shown below.

\[
\begin{align*}
\tau & = \sigma_x^1 \leq \sigma_x^2 \leq \sigma_x^3 \\
\gamma & = \sigma_x^1 \\
\sigma_x^2 & = \sigma_x^3 \\
\end{align*}
\]

**Figure 4.7** Contours of $\phi'$ throughout mine dump

**Figure 4.8** Variation of shear modulus with confining pressure
The following empirical relationship proposed by Seed and Idriss (1970) is used to estimate $G_{\text{max}}$:

$$G_{\text{max}} = 22(k_2)_{\text{max}} Pa \left( \frac{\sigma'_m}{Pa} \right)$$

$(k_2)_{\text{max}}$ is estimated by the following relation:

$$(k_2)_{\text{max}} = (15 + D_r)F$$

$F$ is a constant ranging from 1.5 to 2 for gravel. Using $F=1.5$ and $D_r=50\%$ gives $(k_2)_{\text{max}}=100$. Due to the level of strain anticipated in the model, the value of the shear modulus, $G$, to be used in the analysis will be taken as $G_{\text{max}}/10$.

The bulk modulus is related to the shear modulus through Poisson’s ratio according to the following relation:

$$B = \frac{2G(1+\nu)}{3(1-2\nu)}$$

With Poisson’s ratio is taken as 0.33,

$$B = 2.67G$$

### 4.2 Foundation Soils

#### 4.2.1 Gradation

Foundation conditions at mine dump sites throughout B.C are broadly similar. Bedrock typically exists close to the ground on the steep upper slopes. Colluvial soils derived from the sedimentary units of bedrock mantle the slopes to a depth of approximately 1-2 m. Typical grain size distributions for these soils are shown below in Figure 4.9.
The thin veneer of colluvial soils underlying most foundations are generally classified as a dense, broadly graded silty, gravelly sand. An appropriate value for the density will be taken as 2000 kg/m$^3$. 

Figure 4.9 Range of grain size distributions for colluvial soils (Golder, 1987)
4.2.2 Shear Strength

The results of direct shear and triaxial tests performed on typical foundation soils is shown below in Figure 4.10.

![Figure 4.10 Results of triaxial and direct shear tests on foundation soils (after Golder, 1987)](image)

Note the lack of data in the low stress range (<100 kPa). The strength of the soils underlying the toe region of the dump has a large impact on overall stability. Sands typically exhibit higher values of friction angle at low confining pressure. The effect of confining pressure on the frictional strength of the foundation soils is not known and therefore, a uniform friction angle will be used for the analysis.

The range of friction angles is between 29 to 41°. An upper bound estimate of 40° will be used. The initial estimate of the friction angle has little consequence on the analysis as the material strength is incrementally reduced until failure ensues to determine the factor of safety. The factor of safety will be determined based on an average foundation strength of 35°. In reality, the failure envelope will be slightly curved, especially at the low values of confining pressure present in the foundation soils underlying the dump toe. Little data exists for tests at low confining pressure and therefore a uniform friction angle will be used throughout the entire foundation.
4.2.3 Deformation Properties

The shear modulus, $G$, is approximated by equations 4.2 in the same manner as for the rockfill. The relative density of the foundation soils is considerably greater than the rockfill. The shear modulus is determined as:

$$G = 220 P_a \left( \frac{\sigma_m}{P_a} \right)^{0.5}$$  \hspace{1cm} (4.6)

Using equation 4.3 with $v=0.33$,

$$B = 2.67G$$  \hspace{1cm} (4.5*)

For soils, the dilation angle is generally significantly lower than the friction angle of the material. Vermeer and de Borst (1984) observe that values for the dilation angle are approximately between 0 and 20 degrees whether the material is soil, rock or concrete. For a dense sand, a typical value is 15°. In general, the limits for the dilation angle are defined between the bounds $0 < \psi < \phi/2$. The foundation soils are relatively dense and therefore an initial dilation angle of 17.5° ($\psi = \phi/2$) is not unreasonable.

Model runs carried out by building the dump up in one layer will use a constant dilation angle independent of the level of plastic shear strain. For simplicity, the default value of $\psi=0$ is generally assumed in an analysis of overall stability. Since the analysis will study the influence of the flow rule (associated vs. non-associated) on stability, the dilation angle will be varied between the limits of $0 < \psi < \phi$. When the model waste dump is built up in layers to simulate the actual construction process, the strain softening model will be used to model the decrease in dilation with increasing strain as shown below in Figure 4.11.
Figure 4.11  Idealized relation of dilation angle and shear strain
5.0 Analysis

5.1 FLAC

FLAC is a two-dimensional explicit finite difference program developed primarily for use in geotechnical and mining applications. Stress-deformation numerical analysis performed using the computer code FLAC has several advantages over conventional analysis procedures used to assess slope stability. These simple limit equilibrium approaches examine a discrete pattern of potential failure planes based on user input that is not necessarily representative of the most critical failure surface. By reducing material strength within a given slope stability problem, FLAC will find the most critical failure mechanism and identify the failure plane directly. The strength at failure may then be compared with the actual material strength to determine the factor of safety. In addition, FLAC is able to generate plots of virtually any parameter at any stage of the analysis. This allows for a more thorough understanding of the progress of failure as well as the associated deformations and stresses throughout the dump where field monitoring is not possible.

A model is constructed in FLAC by adjusting the finite difference grid to meet the specific problem geometry. Materials are represented by two-dimensional quadrilateral elements within the grid, which are assigned respective material properties. Elements deform according to the linear or non-linear stress-strain relation in response to applied loading. In large-strain mode, the material being modeled can yield and flow and the grid can deform and move with the material that is represented. In fact, the explicit lagrangian calculation scheme employed in FLAC is particularly well suited for the analysis of large-strain problems in situations where physical instability may occur. For this reason, it is perfectly suited for the analysis of instability mechanisms in mine waste dumps.
A basic representation of the solution scheme used in FLAC is shown in Figure 5.1 below.

Even though a static solution is required, the dynamic equations of motion are included in the formulation. This enables FLAC to follow the physically unstable processes taking place within a mine dump without resulting in numerical instability. Velocities and displacements are first determined from the stresses and forces using the equations of motion. The velocities are then used to determine the strain rates and new stresses are, in turn, determined from the strain rates. Each cycle around the loop (Figure 5.1) represents one timestep. The timestep is chosen to be small enough so that the velocities are 'frozen' during each cycle. The newly calculated stresses do not affect the velocities until the subsequent time step and therefore, neighboring elements do not affect one another during the period of calculation. As long as the timestep for this explicit method is small enough for the calculation wave speed to always keep ahead of the physical wave speed, the assumption of 'frozen' velocities is justified. The equations will always operate on known values that are fixed for the duration of the calculation. After several cycles of the loop, disturbances can propagate across several elements just as they would propagate physically. As a consequence of the necessarily small timestep, many steps must be taken to establish equilibrium or steady state flow. As a result, the explicit formulation used in FLAC is best suited for ill-behaved systems.

The objective in FLAC is to achieve the steady state (either equilibrium or steady-flow) in a numerically stable way with minimal computational effort. The main disadvantage of the explicit finite difference method is the small timestep and the number of steps required to establish equilibrium (or steady-state flow). For the investigation of
instability of mine dumps, the advantages far outweigh the added time necessary for convergence. Whereas finite
element methods typically combine the element matrices into a large global stiffness matrix, it is relatively efficient
to regenerate the finite difference equations at each step. Memory requirements are thus always at a minimum.
Since matrices are never formed, large displacements are accommodated without additional computing effort. No
iterations are necessary when computing stress from strains in an element if the constitutive law is non-linear. The
lagrangian calculation scheme and the mixed-discretization zoning technique used in FLAC ensure that plastic
collapse and flow are modeled very accurately.

In the analysis of physical instability, numerical instability is often a problem. As physical instability develops in
real life situations, strain energy within the system is converted into kinetic energy, which radiates away from the
source and dissipates. In most finite element methods, inertial terms are not involved and therefore they must use
some numerical procedure to treat physical instabilities. This may prevent numerical instability but may have an
effect on the path that the solution takes. FLAC solves the dynamic equations of motion during each timestep,
incorporating inertial terms which handle the generation and dissipation of kinetic energy in a realistic manner. A
local non-viscous damping term is incorporated into the equation of motion that produces a damping force on each
node that is proportional to the magnitude of the unbalanced force.

Nodes within the grid are accelerated according to the finite difference form of Newton's second law of motion
modified to account for the effect of damping:

\[
\ddot{u}^{(t + \Delta t/2)} = \ddot{u}^{(t - \Delta t/2)} + \left( \sum F_i^{(t)} - (F_{d})_i \right) \frac{\Delta t}{m_n}
\]

where,

\[(F_{d})_i = \alpha \left| \sum F_i^{(t)} \right| \text{sgn}(\dot{u}_i^{(t - \Delta t/2)}) \]

\(F_d\) is the damping force, \(\alpha\) is a constant (0.8) and \(m_n\) is a fictitious nodal mass. The amount of damping varies from
point to point throughout the system depending on the value of the unbalanced force. The direction of the damping
force is such that energy is always dissipated. Since FLAC is designed to supply the static solution to a problem, the
nodal masses may be regarded as relaxation factors in the motion equation. The 'fictitious' nodal masses in eq. 5.1
are adjusted for optimum speed of convergence without influencing the magnitudes of the gravitational forces. The
best convergence is obtained when the local values of the critical timestep are equal, or equivalently, when the

35
natural response periods of all parts of the system are equal. As a result, the timestep is set to unity and the nodal masses are scaled to obtain this value.

The realistic manner in which FLAC treats physical instabilities and the subsequent generation and dissipation of kinetic energy makes this method ideal to study the development and propagation of failure zones in the waste dump model. FLAC has been used extensively in research areas related to the studies of the process of localization and evolution of shear bands in frictional materials. Grid resolution must be very fine to capture the development of the shear zones within the waste dump model. If an implicit solution scheme typical of most finite element programs were employed, memory requirements would be too excessive to allow for the number of zones required. In addition, FLAC's formulation involves no algorithm to bring the stress of each element to the yield surface. The plasticity equations are solved during each calculation step making it more efficient than some FEM codes for modeling plastic flow. The mixed discretization scheme is used for accurate modeling of plastic collapse loads and plastic flow. This scheme is believed to be physically more justifiable that the 'reduced integration' scheme commonly used with finite elements (Cundall, 1991).

Although beyond the scope of this study, future requirements of deformation modeling are easily facilitated using FLAC. As stability analysis is the focus of this study, thorough calibration of model deformations to actual field data was not necessary. If proper calibration were to be carried out, FLAC would be able to predict displacements throughout the dump per interval of crest advance. In addition, coupled groundwater flow-mechanical stress calculations could be performed to assess strength loss effects in the foundation due to excess pore pressure generation. The generation and dissipation of excess pore pressures due to construction loading or subsequent shearing action within the foundation layer could then be modeled.

5.2 Stress-Strain Relations

The Mohr-Coulomb model is generally used in soil mechanics applications. The granular nature of the materials to be modeled makes it particular well-suited for this study. The strength of the materials is defined by its friction angle according to the Mohr-Coulomb failure criterion:

$$\tau = \sigma_n \tan \phi$$  \hspace{1cm} (5.2)
The material will deform elastically corresponding to the linear-elastic portion of the stress-strain curve (Figure 5.2). Once the yield stress is reached, plastic flow takes place. The yield stress is dependent on only the major and minor principal stresses. The analysis assumes an elastic, perfectly plastic solid in plane strain, which conforms to the Mohr-Coulomb yield criterion.

![Mohr Coulomb Model](image.png)

**Figure 5.2** Mohr Coulomb Model a) Yield and potential functions b) stress-strain curve c) volumetric strain

The total strain increment may be decomposed into elastic and plastic parts, with only the elastic part contributing to the stress increment by means of an elastic law. Volumetric and shear deformations in the elastic region are determined from the values of the bulk and shear moduli of the material, respectively. For plastic flow, a shear yield function and non-associated flow rule are used.

The failure envelope is defined by the Mohr-Coulomb yield function as follows:

\[ f = \sigma_1 - \sigma_3 N_\phi \]

where,

\[ N_\phi = \frac{1 + \sin \phi}{1 - \sin \phi} \]  

(5.3)
The non-associated flow rule specifies the direction of the vector of the plastic strain increment as that normal to the potential surface. Therefore, the yield function (f) and plastic potential (g) are defined separately.

\[ g = \sigma_1 - \sigma_3 N_\psi \]

where,

\[ N_\psi = \frac{1 + \sin \psi}{1 - \sin \psi} \]  

The failure envelope and plastic potential are shown above in Figure 5.2a). The flow rule is termed 'associated' if the yield function and plastic potential coincide, i.e. \( \psi = \phi \).

If the stresses corresponding to the calculated total strain increment lie above the yield function in the generalized stress space, plastic deformation takes place. Only the elastic part of the strain increments contribute to the stress increment. The stress increment is then corrected by using the plastic flow rule to ensure that the stresses lie on the yield surface.

To define the material within the Mohr-Coulomb model, the following parameters are required: density, cohesion, friction angle, bulk modulus, shear modulus, dilation angle. Parameters used in the analysis have been defined in Chapter 4. Stress-dependent material properties for the waste rock are inputted using the built-in FISH programming language.
5.3 Mine Dump Model

5.3.1 Geometry

The FLAC analysis will be carried out on a generic mine dump model with the geometry shown below in Figure 5.3.

![Figure 5.3 Typical mine dump profile](image)

Both the foundation slope angle, $\beta$, and the waste dump height, $H$, will be varied to assess their influence on stability. The review of failure cases (Figure 3.2) indicates that failure is unlikely for dumps with heights less than 50m or slope angles less than 15°. To carry out a comprehensive stability study encompassing a large proportion of typical geometries observed to have failed, 9 different cases will be analysed.

<table>
<thead>
<tr>
<th>Case</th>
<th>Slope angle, $\beta$ ($^\circ$)</th>
<th>Waste dump height, $H$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>15</td>
<td>50</td>
</tr>
<tr>
<td>II</td>
<td>15</td>
<td>100</td>
</tr>
<tr>
<td>III</td>
<td>15</td>
<td>200</td>
</tr>
<tr>
<td>IV</td>
<td>20</td>
<td>50</td>
</tr>
<tr>
<td>V</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>VI</td>
<td>20</td>
<td>200</td>
</tr>
<tr>
<td>VII</td>
<td>25</td>
<td>50</td>
</tr>
<tr>
<td>VIII</td>
<td>25</td>
<td>100</td>
</tr>
<tr>
<td>IX</td>
<td>25</td>
<td>200</td>
</tr>
</tbody>
</table>

*Table 5.1 Summary of waste dump geometry*
5.3.2 Methodology

Once the model geometry has been inputted, the foundation material is brought to equilibrium under pre-construction stresses. In order to avoid geometry problems due to large distortions and buckling in elements at the toe area, the model will be run in small-strain mode. The dump is constructed in one lift and brought to equilibrium elastically to minimize inertial effects. The waste rock strength is then dropped incrementally to values determined from Equation 4.2. The model will reach equilibrium and the foundation strength is dropped from its initial value of 40° to induce failure. The factor of safety is determined as the ratio of the average value of the foundation strength, determined from triaxial testing data (35°), to the critical foundation strength resulting in instability.

To investigate the influence of sequential loading, a model run will be carried out constructing the model dump in lifts to simulate the construction process (see Figure 2.2a).

5.4 Failure Mechanism

There is typically a relatively thin layer of colluvial silty, gravelly sands underlying waste rock dumps in western Canada. The shear strength of this soil is less than that of the waste material and therefore the stress ratios acting in this layer may control overall dump stability. Field and laboratory observations indicate that failure takes place within the foundation at shallow depth. Several failure records report a rill of foundation material being pushed up ahead of the dump toe.

Failure is induced in the FLAC model by incrementally dropping the strength of the foundation soils. At a relatively high friction angle (φ=40°), a 100 m high dump on a 25° foundation slope is stable and the values of mobilized friction angle acting parallel to the foundation slope are shown below in Figure 5.4.
The average value of mobilized friction angle at the base of the model acting parallel to the foundation slope is 31.1°. The curve is not linear but rather concave upward with more rapidly increasing values of $\phi_m$ closer to the dump toe. The mobilized friction increases from a value of approximately 29° at the location of the back failure plane to 36° directly underneath the dump toe. It is evident that any decrease in available friction within the foundation soil will result in an initial failure underneath the dump toe, which may or may not propagate along the foundation layer.

Field observations summarized in Chapter 3 are somewhat vague on the issue of toe movements associated with periods of increasing deformations preceding failure but generally suggest that there is little or no toe movement associated with periods of accelerating crest movements. Visual observations of failure events typically report changes in the dump profile followed by an explosive type failure at the toe upon which the dump will obtain high mobility flowing downslope. In contrast, the pattern of mobilized friction along the foundation (Figure 5.4) suggests that failure would initiate at the toe, preempting the development of large strain deformations within the dump.

It is important to note that the model is analyzed with a linear foundation slope whereas sidehill fills are generally constructed on foundation profiles where the foundation slope may be significantly shallower directly underneath the dump toe than at greater depth. The average slope angle may be 25° but the effect of the non-linear foundation
profile often results in much shallower foundation segments directly under the dump toe on the order of 15°, significantly reducing the magnitudes of shear stresses in the toe region. In addition, a uniform value of friction angle for the foundation soils has been used. Granular materials typically exhibit increasing friction angles with decreasing confining stress. The low confining stresses present in the toe region of the dump will result in higher values of the friction angle. An average value of 35° has been determined from triaxial testing. This value has been averaged across a broad range of stresses although triaxial testing data does not include data in the stress range below approximately 100 kPa. This stress range is of particular interest as the high stress ratios acting in the toe region may effectively lead to overall instability.

The pattern of $\phi^*$ depicted in Figure 5.4 suggests that the generation of excess pore pressures within the foundation soils would result in significant toe movements prior to general failure. Field observations contradict this phenomenon. Failure records reveal that the toe holds while significant movement in the foundation takes place at greater depth below the waste rock pile. This results in a characteristic ‘toe bulge’ recorded by eyewitnesses preceding failure. The foundation soils are purely frictional consisting of medium dense colluvial silty sands and gravels. The effect of a non-linear foundation profile and strength envelope may effectively ‘hold’ the toe while the dump deforms.

As foundation strength in the FLAC model is incrementally dropped to 35 degrees, significant shear deformations begin to occur. At 35°, the two shear planes within the waste rock characteristic of the double wedge mechanism have formed (Figure 5.5). The toe begins to spread and a pronounced step forms on the dump face.
The two pronounced shear zones within the waste rock form at angles similar to those predicted from laboratory model tests. Vectors of crest displacement approach 60° at the dump crest, shallowing out to approximately 50° at the location of the back failure plane, in agreement with the results of crest surveying (see Figure 3.1).
A plot of contours of velocity in the y-direction taken during the analysis are shown in Figure 5.6. The purpose of this figure is to depict the location and orientation of velocity discontinuities in the rockfill. The lighter shading in the diagram denotes higher vertical velocities. The figure provides a sharp contrast between the active and passive wedges and the location of the back failure plane. The upper wedge is actively deforming, restrained by the stable toe wedge. Once the foundation strength is exceeded, the lower wedge begins to slide downslope resulting in overall instability. The back failure plane is evident as a velocity discontinuity, bounding the active wedge which is experiencing relatively high vertical velocities responsible for crest settlement. A more subtle discontinuity is visible indicating the boundary between the active and passive wedges.

### 5.5 Factor of Safety

The mode of failure observed in the numerical analysis is in agreement with the double wedge failure mechanism as described by Campbell (1986). For design and analysis purposes, the factor of safety for this specific mode of failure is of prime importance. FLAC model runs have been carried out for the 9 cases described in Section 5.3.1. As discussed, failure is induced within the model waste dump by an incremental lowering of the frictional strength of the foundation soils until instability results. As with the limit equilibrium double wedge analysis, the factor of safety is determined as the ratio of the shear strength of the foundation soils to the shear stress acting along the foundation failure plane underlying the toe wedge. The factors of safety for the various dump configurations are shown below in Table 5.2.

<table>
<thead>
<tr>
<th>Slope angle, $\beta$</th>
<th>Dump Height, H (m)</th>
<th>$\phi_m$ (along foundation)</th>
<th>Factor of Safety (FLAC)</th>
<th>Factor of Safety (Double Wedge)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15°</td>
<td>50</td>
<td>30°</td>
<td>1.21</td>
<td>1.85</td>
</tr>
<tr>
<td>15°</td>
<td>100</td>
<td>30°</td>
<td>1.21</td>
<td>1.79</td>
</tr>
<tr>
<td>15°</td>
<td>200</td>
<td>31°</td>
<td>1.17</td>
<td>1.73</td>
</tr>
<tr>
<td>20°</td>
<td>50</td>
<td>33°</td>
<td>1.08</td>
<td>1.53</td>
</tr>
<tr>
<td>20°</td>
<td>100</td>
<td>35°</td>
<td>1.00</td>
<td>1.49</td>
</tr>
<tr>
<td>20°</td>
<td>200</td>
<td>37°</td>
<td>0.93</td>
<td>1.45</td>
</tr>
<tr>
<td>25°</td>
<td>50</td>
<td>37°</td>
<td>0.93</td>
<td>1.29</td>
</tr>
<tr>
<td>25°</td>
<td>100</td>
<td>39°</td>
<td>0.86</td>
<td>1.27</td>
</tr>
<tr>
<td>25°</td>
<td>200</td>
<td>41°</td>
<td>0.81</td>
<td>1.24</td>
</tr>
</tbody>
</table>

**Table 5.2** Factors of Safety
The factors of safety for typical mine dump configurations determined from the FLAC analysis are quite alarming. The results suggest that dumps founded on slopes of 25° will not be stable, irrespective of dump height. Factors of safety determined from limit equilibrium double wedge analysis are on the order of 1.5 times the value of the factors of safety determined from the FLAC analysis. Double wedge analysis was carried out with uniform friction angles for the waste rock estimated as the average of the friction angle used in the FLAC analysis over the stress range of interest. The results should therefore differ, but only slightly. Another more fundamental difference between the two analysis techniques must be responsible for the dramatic disparity between the calculated factors of safety.

To investigate the reasons for the differences, a more in-depth look at the manner with which FLAC handles failure and plastic flow is necessary and will be explored in the following Chapter.
6.0 Shear Band Orientation

6.1 Plasticity Theory

Observed shear behaviour of sand, both in the field and laboratory, indicate that a failure surface or shear band develops within a soil mass when it is sheared to its yield stress level as first proposed by Coulomb in 1776. Up to the onset of failure, shear strains are observed to be uniform throughout the soil mass. Once the yield stress is reached, all subsequent shear strains occur within the shear band and the soil on either side behaves as a rigid body (Roscoe, 1970). Mine waste dump instability, as discussed in the previous chapter, is a result of the formation of a continuous failure surface or shear zone within the sandy foundation layer. Shear strains will be concentrated within the foundation layer, bounded on either side by the passive toe wedge and bedrock foundation. While the numerical analysis procedures employed for the purposes of this study are not able to accurately model the thickness of shear zones, their orientation and associated stresses are true to the theory on which they are based. It is this theory concerning the plastic flow rule and resulting shear band orientation that will be examined in this section. It will be shown that the orientation of the failure surface relative to the direction of the principal stresses is extremely sensitive to the assumptions made concerning the plastic flow rule employed in the analysis. In other words, an approach must be taken to predict the orientation of this failure surface that is both theoretically and physically admissible.
The Mohr-Coulomb yield criterion is generally used to describe the state of stress at failure in a soil mass (Figure 6.1 below).

For a cohesionless soil, this yield criterion states that the maximum stress ratio that can exist within a given soil mass is defined by $\tan\phi$, existing on a plane inclined at $\theta = 45^\circ + \phi/2$ to the minor principal stress direction. The direction of slip corresponding to the plane of maximum stress obliquity is commonly assumed, and is the basis of the limit equilibrium analysis described in Section 2.3.

Early use of plasticity theory in soil mechanics was based on the Mohr-Coulomb yield criterion and its associated flow rule. This theory assumes:

1. The Coulomb yield function is the plastic potential for the soil, so that the strain-rate components are related to the yield function by the associated flow rule of plasticity.
2. The principal axes of the stress and strain-rate tensors coincide.
The resulting strain rate vectors are plotted in Figure 6.1. The coaxiality of principal stress and strain rate directions allows for the superposition of stress and strain rate characteristics. It is evident that when the Mohr-Coulomb yield envelope is used to define the limit between states of elasticity and of continuing irreversible deformation, the assumptions of classical plasticity theory result in excessive volumetric expansion. An increase in volume is predicted of a magnitude far greater than that observed experimentally. The specific rate of energy dissipation is determined from the scalar product of the stress traction vector and velocity vector. If the normality criterion is upheld, the flow rule predicts an inadequate dissipation of mechanical energy.

From both an experimental and theoretical viewpoint, soil behaviour cannot be properly represented through the principals of associated plasticity. As a consequence, classical plasticity concepts have been extended to a 'non-associated' type in which the plastic potential and yield surfaces are defined separately. The Mohr-Coulomb failure envelope is defined by:

\[
\sigma_1 - \sigma_3 N \phi = 0
\]

\[
N \phi = \frac{1 + \sin \phi}{1 - \sin \phi}
\]

If the assumption of coaxiality between the axes of stress and strain is preserved, the shear or plastic potential is defined as:

\[
g = \sigma_1 - \sigma_3 N \nu
\]

where

\[
N \nu = \frac{1 + \sin \psi}{1 - \sin \psi}
\]

\[
\sin \psi = \frac{\delta \varepsilon_p^p}{\delta \gamma^p} = \frac{\dot{\varepsilon}_p^p + \dot{\varepsilon}_3^p}{\dot{\varepsilon}_3^p + \dot{\varepsilon}_1^p}
\]

The plastic strain rates are denoted by means of a superimposed dot. The superscript \( p \) is used to indicate plastic strains. Significant experimental data exists to verify the coaxiality of principal stress and strain rate directions, discussed in the following section.
The implication of the non-associated flow rule is easily understood by means of characteristics to define the stress and strain distributions within a soil mass. Stress characteristics are the curves on which the stress ratio \( T/\sigma_T \) is a limiting maximum. The basic equation of stress characteristics is:

\[
\frac{dy}{dx} = \tan \left[ \xi \pm \left( \frac{\pi - \phi}{4} \right) \right]
\]  

This equation gives two families of stress characteristics, each curve being inclined at \( \pi/4 - \phi/2 \) to the direction, \( \xi \), of the major principal stress, \( \sigma_1 \) (Figure 6.2b). To define the strain field, velocity characteristics, or curves of the zero extension line field are defined by:
The two families of velocity characteristics within a soil mass are equally disposed about the major principal stress direction at an angle of \( \psi/2 \) (Figure 6.2b). Only for an associated flow rule \( (\psi=\phi) \) do the stress and velocity characteristics coincide. For a non-associated flow rule, the two sets of characteristics diverge by an angle of \( (\phi-\psi)/2 \). Considering the statics and kinematics of the problem separately, the resulting slip surface direction will correspond to either a stress characteristic or a velocity characteristic, respectively. Several researchers (Roscoe, 1970, James and Bransby 1971) contend that the slip surface corresponds to a velocity characteristic while others (Meyerhof, 1971 and Rowe, 1971) hold that slip surfaces coincide with stress characteristics. If analysis only considers the statics of the problem without any consideration for the velocity field, the slip surface is predicted to form along a stress characteristic (plane of maximum stress obliquity), resulting in a Coulomb slip surface. The kinematic requirements suggest that the slip surface forms along a velocity discontinuity (line of zero extension), oriented an angle of \( \theta_k=45^\circ+\psi/2 \) with the minor principal stress direction, resulting in the Roscoe slip surface.

The orientation of the failure surface relative to the principal stress directions has a significant impact on the values of the shear and normal stresses acting on it at failure. If the Coulomb shear band forms, failure occurs along a plane of maximum stress obliquity where:

\[
\theta_c = \frac{\pi}{4} + \frac{\phi}{2}
\]  

(6.3)

The corresponding ratio of shear to normal stress acting on this plane is:

\[
\left( \frac{\tau}{\sigma} \right) = \tan \phi
\]  

(6.4)

If the shear band forms at an angle of

\[
\theta_k = \frac{\pi}{4} + \frac{\psi}{2}
\]  

(6.5)
to the direction of the minor principal stress, the ratio of shear-normal stresses acting along that discontinuity at failure is related to the dilation angle by the following equation:

\[
\frac{\tau}{\sigma} = \frac{\cos \psi \sin \phi}{1 - \sin \psi \sin \phi}
\]  

(6.6)

For material shearing at constant volume (\(\psi = 0\)), the shear stress at failure reduces to:

\[
\tau = \sigma_n \sin \phi
\]

(6.7)

Significant experimental evidence exists to support the Roscoe orientation and will be discussed in the following section. A theoretical formulation of shear band formation and orientation using the non-associated coaxial flow rule will also be reviewed to assess the critical orientations of the shear planes relative to the principal stress directions.

6.2 Experimental Observations

In recent years, a significant amount of laboratory research has been carried out to assess the validity of the Coulomb shear band orientation sparked by the pioneering work of Roscoe (1970).

The objective of Roscoe’s experimental research was to develop an understanding of the stress-strain behaviour of soils so that reliable predictions can be made for the load deformation characteristics in practical problems and at all working loads, not only those at the instant of failure. To this end, laboratory testing of soils was conducted using the simple shear device, shown schematically below in Figure 6.3. The simple shear apparatus allows for the rotation of the axes of principal stress and strain during shear making it more applicable to actual field conditions (Roscoe, 1970). Other testing procedures such as triaxial testing force the axes to coincide with the boundaries of the soil sample.
From the load cell readings, it is possible to determine the magnitudes and directions of the principal stresses, and therefore of stress increments at any stage of test. The corresponding data for strains and strain rates can be determined from the shear strain and vertical strain. One of the main features of the simple shear device is that there can be no linear strain in the horizontal x-direction, and therefore, horizontal lines are lines of zero extension.

The denser the sample, the greater the expansion and the larger will be the peak strength compared to the ultimate (critical) strength. While in the condition between peak and critical states, Roscoe found that the sand is unstable and will tend to fail in the thinnest possible zone, band or surface which is approximately 10 grains thick. Once these rupture bands have formed, the strains become concentrated within them and the neighboring material on either side behaves as a rigid body.

Roscoe's results have shown some significant insights which have led to an adjustment of the current rules of plasticity as applied to soils. Figure 6.4 shows that for monotonically increasing stresses, the principal axes of strain rate and of stress coincide as the sand is sheared, except for the earliest stages of the test before the sample developed its minimum void ratio. This coincidence of axes of strain rate and of stress corresponds to the behaviour expected of a plastic material.

Figure 6.3 Simple shear testing of soil
Radiographic testing before and after loading confirmed that the failure planes are always horizontal. The rupture surface that develops when the yield stress is reached, develops along a line of zero-extension rather than along a plane on which the ratio of the shear stress to the normal stress is a maximum.

The common assumption that the horizontal plane in the apparatus is a plane of maximum obliquity appears from the results to be quite unwarranted at any stage of the test. If the assumption were made that this is the plane of maximum obliquity, the peak stress ratio $\sigma_1/\sigma_3$ in the tests would be underestimated. The alternative assumption that the horizontal plane in a direct shear test is a plane of maximum shear stress would be justified for a medium-loose, but not for a dense sample in a drained test (Roscoe, 1970).
To be able to adequately predict the orientation of the slip surface relative to the principal stress directions, Roscoe introduced the dilation parameter defined as:

\[ \frac{\dot{\gamma}}{\varepsilon} = \frac{\dot{\varepsilon}_1 + \dot{\varepsilon}_3}{\varepsilon_1 - \varepsilon_3} = \sin \psi \]  

(6.8)

Where \( \varepsilon_1 \) and \( \varepsilon_3 \) are the major and minor principal strain rates respectively. Lines of zero extension in the simple shear apparatus were determined to be inclined at an angle of \( 45 - \psi/2 \) to the major principal stress direction. As the rupture surface forms along lines of zero extension, it follows that the rupture surface is inclined at an angle of \( 45 - \psi/2 \) rather than \( 45 - \phi/2 \) as commonly believed. When the sample is shearing at constant volume, the sample is no longer dilating and the stresses on the shear band now correspond to the plane of maximum shear stress.

To examine these findings for a practical problem, James and Bransby (1970) investigated the stress and strain fields through an experimental investigation of the passive failure of an initially vertical, rough, plane wall which is rotated about its toe into a mass of dry sand. Measurements were made of the distribution of the normal and shear stresses on the wall while strain fields are determined from observations by an x-ray technique measuring the positions of lead shot buried in the sand mass. From this, the form of the rupture surface mechanism in the sand is determined.
Figure 6.5 Passive Retaining Wall Experiments

a) contours of shear strain at various stages of wall rotation
b) rupture zones determined from radiographic testing (after James and Bransby, 1970)

Figure 6.6 Trajectories of zero-extension ($\alpha, \beta$) and principal compressive strain ($\xi$) determined from radiographic testing (after James and Bransby, 1970)
The pattern of strain (Figure 6.5a) indicates that deformation becomes concentrated into narrow bands of high deformation, or rupture surfaces, with little or no strain between them. Rupture surfaces were also detected by the x-ray technique (Figure 6.5b). The high shear deformation causes the sand to suffer sufficient density change in the rupture surfaces so that images of the rupture surfaces show up clearly as dark bands on the radiographs.

The measurement of the strains in the soil by x-ray technique also allowed for the computation of the trajectories of principal strain and the lines of zero extension. The three families of trajectories are plotted independently based on the measured displacement of the lead shot during the test (Figure 6.6). As the principal compressive strain increment direction is approximately horizontal at the sand surface, the angle of intersection of the $\alpha$-zero extension trajectories and the level surface of the sand will be $45^\circ - \phi/2$. In dense sand, this angle is $35^\circ$, a value similar to the angle observed on the radiographs of the intersection of the rupture surfaces and the sand surface. Thus the data tend to confirm the conclusion of James (1965) that rupture surfaces coincide with trajectories of zero extension.

As the value of $\phi$ for the material used in the above tests was determined as $49^\circ$, the stress characteristics would be inclined at an angle of $20.5^\circ$ to the sand surface. Therefore, the orientations of the rupture surfaces at the sand surface do not correspond with the stress characteristics of conventional theory. Rupture surfaces are a phenomenon associated with deformation patterns while stress characteristics are derived from considerations of the stresses alone (James, 1965). Thus there is no fundamental reason why rupture surfaces must coincide with stress characteristics. With coaxiality of the principal stress and strain-rate axes, stress characteristics and zero extension lines diverge by an angle of $(\phi - \nu)/2$.

At the outset of these results, Roscoe concludes that the most important fundamental error made in conventional methods of analysis, in which a shape is assumed for the rupture surface, is that the limiting maximum value of the stress ratio acts on this surface.

The results also question the customary method of interpreting the Mohr-Coulomb criterion. The criterion is generally thought of as one concept but is actually two:

a) sand will fail at that point when the greatest stress ratio reaches its limiting value $(\tau/\sigma)_{\text{max}} = \tan \phi$. 

56
b) The two planes through the point along which rupture surfaces develop are those upon which \((\tau/\sigma)_{\text{max}}\) is attained.

If this were true, then the rupture surfaces that would develop in the wall problem would all be parallel to the stress characteristics. Results confirm that rupture surfaces coincide with the directions of zero-extension. The results of the above data can only be made compatible if the Mohr-Coulomb criterion is restated in the form:

\[
\text{in any element within a soil mass a rupture surface will develop along a plane which is a direction of zero-extension at the instant when the stress ratio } \tau/\sigma \text{ on any other plane through that element attains a value corresponding to a point on the Mohr-coulomb envelope.}
\]

(Roscoe, 1970)

\(\tau\) and \(\sigma\) are now the shear and normal stresses respectively on this second plane and do not refer to the rupture surface.

\[
\text{in carrying out conventional stability analyses in which a failure plane is assumed, it is desirable to think of the failure surfaces as being zero-extension lines rather than being planes of maximum stress ratio... The relevant stress ratio to apply in the analysis to the assumed rupture surface is that which occurs on a zero extension line.}
\]

(Roscoe, 1970)

This conclusion led to the adaptations to classical plasticity theory used in the numerical analysis. There is now significant evidence that the velocity and stress characteristics do not coincide. Roscoe brought the phenomenon to general attention although James (1965) was the first to show experimentally that rupture surfaces (i.e. velocity discontinuities) coincide with the directions of zero extension in sands. Even earlier, Geiringer (1930) showed theoretically that this should be the case for a perfectly plastic non-dilating material. Rupture surfaces observed in the Cambridge research occur along lines of zero extension, recognized as the velocity characteristics of a non-associated flow rule material. Subsequent experimental research has been carried out although a debate still exists over the direction of plastic flow of a cohesionless soil. Research does indicate that the grain size may influence the
formation and orientation of the rupture surface. The research detailed here was carried out on a relatively coarse sand with a particle size of approximately one millimeter deemed appropriate to predict the behaviour of the coarse sand and gravel colluvium underlying waste rock dumps. Whereas experimental research on fine sands has indicated that rupture surfaces appear to form between the limits of the Coulomb and Roscoe orientations (Arthur et al, 1964).

6.3 Theoretical Formulation

It has already been established in the previous discussions that rupture surfaces or shear bands form in cohesionless media when the peak stress ratio is achieved. Experimental evidence suggests that the shear band may not form along a plane of maximum stress obliquity but rather, a plane of zero-extension, especially in the case of a coarse sand. It is necessary to evaluate the theoretical validity of each hypothesis and determine the treatment of shear band formation and propagation in the FLAC analysis carried out for the purpose of this study.

Theoretical validation of the coincidence of rupture surfaces with planes of zero extension has been presented by Lee and Herrington, and Davis (1968). The theoretical formulation presented by Vermeer (1990) will be summarized and presented here. The case of simple shear will be analysed using Vermeer’s closed form solution and compared with a simple shear test carried out using FLAC.

The general equation for an elasto-plastic model is given by:

\[
\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^P = D^{-1} \left[ \frac{1}{h} \frac{\delta g}{\delta \sigma} \frac{\delta f^T}{\delta \sigma} \right] \sigma
\]  

(6.9)

where 
- \( h \) = hardening modulus
- \( g \) = plastic potential function
- \( f \) = yield function

and

\[
\varepsilon = \begin{pmatrix} \varepsilon_{xx}, \varepsilon_{yy}, \gamma_{xy} \end{pmatrix}^T
\]
\[
\sigma = \begin{pmatrix} \sigma_{xx}, \sigma_{yy}, \sigma_{xy} \end{pmatrix}^T
\]

(6.10)
The superimposed dot indicates a rate and the superscripts denote either the elastic or plastic increment of strain. The analysis carried out for this study is only concerned with plane strain conditions, and therefore, \( \varepsilon_{zz} = 0 \). The out-of-plane stress is assumed to be the intermediate principal stress. The focus is on the behaviour at the point of failure, corresponding to the onset of shear banding and therefore Poisson’s ratio will be neglected and taken as zero. This gives the elasticity matrix as:

\[
D = \begin{bmatrix}
2G & 0 & 0 \\
0 & 2G & 0 \\
0 & 0 & G \\
\end{bmatrix}
\]  

(6.11)

where \( G \) is the shear modulus.

The inverse form of the elastoplastic relationship of equation 6.9 reads

\[
\sigma = \left[ D - \frac{1}{h + d} ab^T \right] \dot{\varepsilon}
\]  

(6.12)

where;

\[
a = D \frac{\delta g}{\delta \sigma} = G \left( 2 \frac{\delta g}{\delta \sigma_{xx}}, 2 \frac{\delta g}{\delta \sigma_{yy}}, \frac{\delta g}{\delta \sigma_{xy}} \right)^T
\]  

(6.13a)

\[
b = D \frac{\delta f}{\delta \sigma} = G \left( 2 \frac{\delta f}{\delta \sigma_{xx}}, 2 \frac{\delta f}{\delta \sigma_{yy}}, \frac{\delta f}{\delta \sigma_{yy}} \right)^T
\]  

(6.13b)

\[
d = \frac{\delta f^T}{\delta \sigma} D \frac{\delta g}{\delta \sigma}
\]  

(6.13c)

As we are concerned only with the behaviour at peak conditions, the hardening modulus will be disregarded yielding,

\[
\sigma = M \dot{\varepsilon}
\]

\[
M = D - \frac{1}{d} ab^T
\]  

(6.14)
The Mohr-Coulomb criterion is used to define a failure surface in stress space by defining the failure envelope and plastic potential. Equations 5.3 and 5.4 are rearranged and expressed in terms of $\sigma_{xx}$ and $\sigma_{yy}$ to give:

\[
\begin{align*}
    f &= \tau^* + \frac{1}{2} \left( \sigma_{xx} + \sigma_{yy} \right) \sin \phi \\
    g &= \tau^* + \frac{1}{2} \left( \sigma_{xx} + \sigma_{yy} \right) \sin \psi
\end{align*}
\] (6.15)

Where $\tau^*$ is the radius of the Mohr stress circle

\[
\tau^* = \sqrt{\frac{1}{4} \left( \sigma_{xx} - \sigma_{yy} \right)^2 + \sigma_{xy}^2}
\] (6.16)

With the yield function and plastic potential defined, vectors $a$ and $b$ can be determined:

\[
\begin{align*}
    a &= D \frac{\delta g}{\delta \sigma} = G \begin{pmatrix} \frac{\sigma_{xx} - \sigma_{yy}}{2\tau^*} + \sin \psi, & -\frac{\sigma_{xx} - \sigma_{yy}}{2\tau^*} + \sin \psi, & \frac{\sigma_{xy}}{\tau^*} \end{pmatrix}^T \\
    b &= D \frac{\delta f}{\delta \sigma} = G \begin{pmatrix} \frac{\sigma_{xx} - \sigma_{yy}}{2\tau^*} + \sin \phi, & -\frac{\sigma_{xx} - \sigma_{yy}}{2\tau^*} + \sin \phi, & \frac{\sigma_{xy}}{\tau^*} \end{pmatrix}^T
\end{align*}
\] (6.17)

\[d = G(1 + \sin \phi \sin \psi)\]

\[\text{Figure 6.7 Definition of the stress orientation angle, } \theta\]
The x-axis will be taken to be in the yet unknown direction of the shear band (Figure 6.7). \( \theta \) is then defined as the angle between the x-axis and the minor principal stress

\[
\begin{align*}
\cos \theta &= (\sigma_{xx} - \sigma_{yy}) / 2 \tau^* \\
\sin \theta &= \sigma_{xy} / \tau^*
\end{align*}
\]

Substituting equation 6.18 into equation 6.17 gives:

\[
\begin{bmatrix}
\cos \theta + \sin \psi \\
-\cos \theta + \sin \psi \\
\sin \theta
\end{bmatrix} = \frac{a G}{\sin \theta}
\begin{bmatrix}
\cos \theta + \sin \phi \\
-\cos \theta + \sin \phi \\
\sin \theta
\end{bmatrix}
\]

\[
d = G(1 + \sin \psi \sin \phi)
\]
6.3.1 Behaviour in simple shear

A simple shear test, as presented by Vermeer (1990) will be considered in order to obtain insight into the performance of the elastoplastic model and the shear-band mechanism. The predicted behaviour will be compared with a one-element shear test carried out with FLAC.

Although deformations tend to be heterogeneous in simple shear devices, the sample is assumed to deform uniformly

![Simple Shear Test Diagram](image)

**Figure 6.8 Simple Shear Test**

Equation 6.14 is used to compute the evolution of the stresses during the tests. For a simple elastic-perfectly plastic model defined by equation 6.14, each stress-strain curve begins with an elastic region. As soon as the yield condition is satisfied, the stress circle touches the failure envelope and the sample yields plastically. The non-linear elastoplastic response is computed from equation 6.14 or written in full:

\[
\begin{align*}
\sigma_{xx} &= M_{11}\epsilon_{xx} + M_{12}\epsilon_{yy} + M_{13}\gamma_{xy} \\
\sigma_{yy} &= M_{21}\epsilon_{xx} + M_{22}\epsilon_{yy} + M_{23}\gamma_{xy} \\
\sigma_{xy} &= M_{31}\epsilon_{xx} + M_{32}\epsilon_{yy} + M_{33}\gamma_{xy}
\end{align*}
\]  

(6.20)
In a simple shear test, the vertical stress is kept constant for the duration of the test while increasing shear stresses are applied. In addition, no strain is allowed in the horizontal direction and therefore,

\[
\varepsilon_{yy} = 0 \quad \varepsilon_{xx} = 0
\]  

(6.21)

Substituting these conditions into equation 6.20 allows us to solve for:

\[
\varepsilon_{yy} \left( \frac{M_{23}}{M_{22}} \right) \dot{\gamma}_{xy}
\]

Substituting equations 6.21 and the condition that \(\varepsilon_{xx}=0\) into equation 6.20 yields:

\[
\sigma_{xx} = M_{22}^{-1} \left( -M_{12}M_{23} + M_{13}M_{22} \right) \dot{\gamma}_{xy}
\]

(6.22a)

\[
\sigma_{xy} = M_{22}^{-1} \left( -M_{32}M_{23} + M_{33}M_{22} \right) \dot{\gamma}_{xy}
\]

(6.22b)

Or using equations 6.14 and 6.19:

\[
\sigma_{xx} = -2G^* \sin 2\vartheta (\cos 2\vartheta + \sin \psi) \dot{\gamma}_{xy}
\]

(6.23a)

\[
\sigma_{xy} = G^* (\cos 2\vartheta + \sin \psi)(\cos 2\vartheta + \sin \phi) \dot{\gamma}_{xy}
\]

(6.23b)

where;

\[
G^* = \frac{G}{2 \sin^2 2\vartheta + (\cos 2\vartheta + \sin \psi)(\cos 2\vartheta + \sin \phi)}
\]

Two cases of simple shear will now be considered. The first case simulates the behaviour of a normally consolidated soil sample in which the horizontal stresses are initially much smaller than the vertical stresses. In the second case, the horizontal stress will initially be much greater than the vertical stress.
Case I: Normally consolidated

If the horizontal stress is initially 25% of the vertical stress, the shear stress and horizontal stress will increase monotonically with shear strain. $\theta$, as defined by Figure 6.7, initially starts off at zero and increases to its ultimate value. To achieve steady state conditions, the stresses must all eventually become stationary. Referring to equation 6.23:

$$\sigma_{xx} = -2G^* \sin \theta (\cos \theta + \sin \psi) \gamma_{xy} = 0$$

$$\sigma_{xy} = G^*(\cos \theta + \sin \psi)(\cos \theta + \sin \phi) \gamma_{xy} = 0$$

In order to meet this requirement,

$$\cos \theta + \sin \psi = 0$$

And therefore, $\theta = \theta_r$

Case II: Overconsolidated

The initial horizontal stress in this test is taken as 4 times the vertical stress. Since the vertical stress is initially the minor principal stress, $\theta$ begins at an initial value of 90°. In this case, $\sigma_{xy} = 0$ at both peak and residual strength. It follows from equation 6.23b that

$$(\cos \theta + \sin \psi)(\cos \theta + \sin \phi) = 0$$

Which yields the solutions $\theta = \theta_r$ and $\theta = \theta_c$. Vermeer (1990) carried out the simple shear numerical analysis for a dense cohesionless soil with the parameters:

- $G = 10$ MPa
- $\phi = 40^\circ$
- $\psi = 10^\circ$
The results of the Vermeer’s analysis are shown in Figure 6.9. The overconsolidated sample corresponds to curve B, which shows both the peak behaviour, and softening down to residual strength conditions. The peak in curve B corresponds to $\theta=\theta_C$ while the residual strength corresponds to $\theta=\theta_R$. The initially large parallel stress decreases to reach the same steady-state value as for the normally consolidated sample. The decrease in parallel stress after peak conditions results in the softening behaviour shown by the curve.

Using equations 6.15 and 6.18, and the requirement that $f=0$, it is possible to formulate a general equation for the shear-normal stress relationship along the failure plane at both peak and residual strength.

\[
\frac{\sigma_{xy}}{-\sigma_{yy}} = \frac{\sin 2\theta \sin \phi}{1 + \cos 2\theta \sin \phi}
\]

Inserting the values of $\theta_R$ and $\theta_C$,

\[
\begin{align*}
\left( \frac{\sigma_{xy}}{-\sigma_{yy}} \right)_{peak} &= \tan \phi \\
\left( \frac{\sigma_{xy}}{-\sigma_{yy}} \right)_{residual} &= \tan \alpha \\
\tan \alpha &= \frac{\cos \psi \sin \phi}{1 - \sin \psi \sin \phi}
\end{align*}
\]
The peak stress ratio for the overconsolidated sample ($\theta=\theta_c$) corresponds to $\tan\phi$. As discussed in section 6.1, for a non-dilating soil, the residual strength corresponds to $\sin\phi$. For an associated flow rule where $\psi=\phi$, the residual ratio would also be $\tan\phi$. In reality, however, the dilation angle is not equal to the friction angle, but is significantly smaller.

![Figure 6.9 Numerical results for simple shear tests (Vermeer, 1990)](image-url)
A strain-controlled simple shear test has been carried out on a single element in FLAC (Figure 6.10). A constant velocity was applied parallel to the upper boundary forcing the element to fail in simple shear. The behaviour of the element in simple shear is carried out for comparison with Vermeer's theoretical results as well as to assess the effects of the dilation angle on the values of peak and residual stress. Initial horizontal and vertical stresses used in the analysis for both normally consolidated and overconsolidated cases are listed below in Table 6.1.

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Case I: normally consolidated</th>
<th>Case II: overconsolidated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dilation angle</td>
<td>0 - 35°</td>
<td>0-35°</td>
</tr>
<tr>
<td>$\sigma_{xx}$ (kPa) - initial</td>
<td>25</td>
<td>400</td>
</tr>
<tr>
<td>$\sigma_{yy}$ (kPa) - fixed</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

**Table 6.1 Material Parameters**
Figure 6.11 a) and b) Effect of dilation on the maximum stress ratio on the failure plane.
Figure 6.12 Rotation of principal stresses during simple shear

Figure 6.13 Change in stress parallel to failure plane
The results of the simple shear test carried out using FLAC are identical to the theoretical results described in the previous section (Fig. 6.9). If the initial horizontal stress is less than the normal stress, the failure plane will correspond to a plane of zero extension ($\theta_r=45^\circ+\psi/2$). The stresses acting on this failure plane are determined from equation 6.24c). It is only for an associated flow rule ($\psi=\phi$) that the stress and velocity characteristics coincide, that the plane of zero extension corresponds with the plane of maximum stress obliquity. For this special case, the effect of initial stresses disappears and the mobilized strength on the interface will correspond to $\tan\phi$.

In the case of the normally consolidated sample, the peak or ultimate strength is related to both the friction angle and dilation angle of the soil according to equation 6.24c). The simple shear test was carried out for a non-dilating material ($\psi=0$) and the changes in parallel stress and stress orientation angle, $\theta$, are shown in Figures 6.12 and 6.13 respectively. The parallel stress increases monotonically until it reaches the same value as the normal stress resulting in an ultimate value of $\theta_r=45^\circ$. According to equation 6.24c), the shear-normal relationship along the failure plane corresponds to $\sin\phi$. If $\phi=35^\circ$, $\alpha$ is equal to $35^\circ$ for $\psi=\phi$, $33.5^\circ$ for $\psi=\phi/2$, and $29.8^\circ$ for $\psi=0$.

For the overconsolidated sample, the stress orientation angle, $\theta$, starts off at $90^\circ$. As the sample is deformed in simple shear, the failure plane forms along a plane of maximum stress obliquity ($\theta_c=45^\circ+\phi/2$) at a peak strength corresponding to $\tan\phi$, regardless of the value of the dilation angle. If the dilation angle is any value less than the friction angle, the sample softens and the residual strength reaches the same value as that achieved at peak conditions for the normally consolidated sample. Considering a non-dilating sample ($\psi=0$), the stress orientation angle decreases from its initial value of $90^\circ$. At $\theta_c=45^\circ+\phi/2$, the peak strength condition in Figure 6.9 is achieved. Post-peak, the parallel stress continues to drop to equal the normal stress as the direction of the principal stresses aligns with the axes of principal strain until the stress orientation angle reaches its final value of $\theta_r=45^\circ+\psi/2$. The softening from peak conditions for the overconsolidated sample will be explained in the following section.

Figure 6.14 depicts the Mohr circle construction at constant volume shearing for 3 values of the dilation angle. As the dilation angle decreases, the circle becomes smaller with decreasing values of shear-normal on the failure plane.
\[
\tan \alpha = \frac{\cos \psi \sin \phi}{1 - \sin \psi \sin \phi}
\]

**Figure 6.14** Influence of Dilation angle on the mobilized friction angle at failure

<table>
<thead>
<tr>
<th>Friction angle</th>
<th>Dilation Angle, $\psi$</th>
<th>$\tan \alpha$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$35^\circ$</td>
<td>0</td>
<td>$\sin \phi$</td>
<td>$29.8^\circ$ ($\alpha_1$)</td>
</tr>
<tr>
<td>$35^\circ$</td>
<td>$\phi/2$</td>
<td>$\frac{\cos \psi \sin \phi}{1 - \sin \psi \sin \phi}$</td>
<td>$33.5^\circ$ ($\alpha_2$)</td>
</tr>
<tr>
<td>$35^\circ$</td>
<td>$\phi$</td>
<td>$\tan \phi$</td>
<td>$35^\circ$ ($\alpha_3$)</td>
</tr>
</tbody>
</table>

**Table 6.2** Stress ratios acting on the failure surfaces
6.5 Stress-State Softening

The results of the FLAC analysis are in agreement with the theoretical results obtained by Vermeer (1990). The softening effect from peak to residual behaviour for the overconsolidated sample is due to the non-associated nature of the material rather than any intrinsic material weakening. Peak behaviour corresponds to the Coulomb shear band orientation. After peak, the principal stress directions will want to align themselves with the principal axes of strain increment resulting in a lower stress ratio acting on the failure surface.

In a strain-softening material, material strength diminishes with increasing strain until constant volume shearing conditions are achieved. For a perfectly plastic material, softening occurs due to a decrease in the stress parallel to the shear band or, equivalently, a rotation of the principal stresses. In order to assess the process of stress-state softening, it is necessary to consider the stresses both inside and outside the shear band. Vermeer (1990) provides a thorough theoretical examination of a biaxial test to derive the change in parallel stress across the discontinuity, determined as:

\[ \delta \sigma_{xx} \propto (\cos \theta + \sin \psi) \gamma_{xy} \]  \hspace{1cm} (6.25)

This equation implies that generally a stress discontinuity will develop across the boundaries of a shear band. It is only for the Roscoe-band with \( \cos \theta + \sin \psi = 0 \) that there is no change of the parallel stress. Actually, the Roscoe-type and is a special case as all stress components remain stationary. The change of parallel stress becomes extreme for the Coulomb-type shear band. This softening effect takes place in any material for which the dilation angle is any value less than the friction angle.

Cundall (1990) gives a good physical interpretation of the softening behaviour of non-associated materials. Conventional thinking dictates that any softening effects are due to material softening in which the intrinsic material strength becomes weaker. In contrast, the softening effect described for the non-associated material can occur even when the intrinsic material strength is getting stronger (hardening) as has been determined from the numerical analysis and its theoretical formulation. Localization of strain into narrow regions in this sense is due to a change in stress state. The coaxiality of stress and strain increment causes the stress parallel to the failure plane to decrease. The static requirement that the normal and shear stress across the interface are equal is met as well as the kinematic requirement that the parallel strain across the interface is equal is also satisfied. The parallel stress across the
interface becomes discontinuous. The possibility for localization or shear banding exists when one or more stress components in an element are able to decrease with increasing stress.

This phenomenon has been termed 'stress-state softening', and can occur even when the intrinsic material is getting stronger (hardening). Just at or before failure, when the bifurcation condition is met (Cundall, 1990), the stress parallel to a potential shear band can decrease, causing a drop in mean stress and hence shear stress, since the stress state moves along the yield surface towards the origin.

![Stress-state softening diagram](image)

**Figure 6.15** Stress-state softening (after Cundall, 1990)

After the onset of shear banding, two different stress states exist at neighboring points in space – inside the band (I) and outside the band (O). Figure 6.15 shows typical Mohr's circles for these two points, for a cohesionless material.
Note that both states exhibit the same shear stresses and the same normal stresses, i.e. there is equilibrium across the band. However state (I) is at yield whereas state (O) has unladed into the elastic regime (Cundall, 1990).

6.6 Discussion

It has been established experimentally that a rupture or failure surface forms within a soil mass when sheared up to its peak strength. Subsequent shear strains take place within this narrow layer and the soil on either side behaves as a rigid body. The aim of any slope stability analysis technique is to predict the stresses acting on this failure surface and determine a factor of safety for the slope based on the available strength provided by the soil. A given slope is normally analysed by means of conventional limit equilibrium analysis to determine an estimate of the factor of safety. The engineer must not only rely on judgement in the determination of the location and orientation of the critical failure surface but must also accept the inherent assumption that the proposed failure surface represents a plane of maximum stress obliquity where the soil strength has been fully mobilized.

The application of stress deformation numerical analysis to the problem of slope stability allows the user to monitor the progress of failure throughout the slope. The stresses and strains within the soil mass are modeled by means of a constitutive relation and, if instability results, elements within the slope will deform and fail once the yield criterion is met. The factor of safety for the critical failure surface can then be determined.

The direction of plastic flow and the associated stresses acting on the failure surface has been discussed at length throughout this chapter. While limit equilibrium analysis is not concerned with deformation characteristics of the problem, the effect of the velocity field throughout a soil mass has been shown to significantly influence the direction of plastic flow, and consequently, the stresses acting on a potential failure surface. Only when the velocity and stress characteristics coincide, as is the case for an associated flow rule material, will failure be restricted to occur along the plane of maximum stress obliquity. Experimental data has established that soil behaviour may not always conform to the predicted behaviour of an associated flow rule material, and therefore, to apply classical plasticity theory to soil mechanics, modifications are necessary to bridge the gap between observed and predicted soil behaviour.
The extension of plasticity theory to a non-associated type was furthered by the experimental work of Roscoe (1970). The results of his research, both in simple shear testing of soils and model wall experiments, have had far reaching implications in the development of plasticity theory for soils:

1. Coaxiality of principal stress and strain increment
2. Stress and velocity characteristics do not coincide
3. Failure occurs along a plane of zero extension ($\tau = \sigma \tan \alpha$)

Coaxiality of principal stress and strain increment is an inherent assumption of associated plasticity, which is preserved in the theory for non-associated materials. The two theories differ in that the stress and velocity characteristics no longer coincide for a non-associated material. The plane of maximum stress obliquity and the plane of zero extension diverge by an angle determined to be $(\phi + \psi)/2$ where $\psi$ is defined as the dilation angle of the material. Both the simple shear testing by Roscoe and the model retaining wall experiments carried out by James and Bransby (1970) confirm this phenomenon and contend that the failure surface occurs along a plane of zero extension (velocity characteristic), oriented at an angle of $45 + \psi/2$ from the minor principal stress direction. More recent experimental data has shown that for coarser sands, the failure surface generally corresponds to the Roscoe orientation ($\theta_R = 45^\circ + \psi/2$) while finer sands tend to exhibit rupture planes anywhere between the limits of the Roscoe and Coulomb orientations ($45^\circ + \psi/2 < \theta < 45^\circ + \phi/2$).

The plasticity theory developed based on the experimental findings discussed above involves defining a yield surface and potential surface separately. Plastic strain increments are taken as perpendicular to the potential surface which is defined in terms of the dilation angle of the material. If the material is non-dilating, theory will predict only plastic shear strains and no volumetric strains. Plastic strain increments are taken as normal to the potential surface. It is important to note that the non-associated flow rule does not affect the yield stress but rather the direction of the plastic strain increments. The theoretical solution presented by Vermeer reveals that the resulting direction of plastic flow may correspond to either the Roscoe orientation or the Coulomb orientation depending on the initial stresses within the soil mass. If the stresses parallel to a potential shear band are less than the normal stresses at the point of failure, the theory will predict the failure surface to develop along a plane inclined at $\theta_R = 45^\circ + \psi/2$. If the stresses parallel to a potential shear band are initially greater than the normal stresses, the
Coulomb shear band will form. In the post-peak region, the Coulomb shear band exhibits 'stress-state' softening and the residual strength is equal to the peak strength for the Roscoe type band.

The non-associated coaxial flow rule developed based on the experimental and theoretical research presented is used to model plastic flow in FLAC. Soil behaviour predicted using FLAC is in agreement with the results obtained from a theoretical solution of behaviour in simple shear. Once the stresses within the material have reached the failure envelope, plastic strain increments are taken as perpendicular to a potential surface, determined from the dilatancy characteristics of the material. The resulting direction of slip may occur at either the Roscoe orientation or the Coulomb orientation depending on the initial stresses within the soil mass. If conditions are present to favour the formation of the Coulomb band, subsequent rotation of the principal stresses will result in a softening behaviour from peak to residual strength. At constant volume conditions, the stresses acting on either shear band will be identical.

Experimental work indicates that the thickness of a shear band is dependent on the internal features of the material and is usually on the order of 15* grain size. This is not built into FLAC’s constitutive models and therefore the bands in FLAC collapse down to the smallest possible width that can be resolved by the grid. The overall physics of shear banding is modeled correctly, the kinetic energy that accompanies band formation is released and dissipated in a physically realistic way.
7.0 Results

7.1 Effect of the Plastic Flow Rule

Slope stability analysis procedures generally assume that failure occurs only along a surface where the ratio of shear to normal stresses acting along that surface exceeds the strength of the soil. The most critical failure surface is found by trial and error and the factor of safety for the slope is determined as the ratio of the mobilized strength along the slip surface to the available shear strength of the soil. While limit equilibrium analysis procedures do not deal directly with concepts of plasticity, the underlying assumption that the failure plane is a plane of maximum stress obliquity is a fundamental assumption inherent in associated plasticity theory.

In the finite difference analysis carried out using the computer code FLAC, shear zones develop and propagate through the waste dump and foundation elements in response to the applied stresses. While the numerical model also satisfies limit equilibrium, consideration of the velocities and deformations provides a more thorough treatment of the mechanics of failure. A non-associated flow rule is used to model plastic flow in FLAC and therefore, the calculation of the plastic strain increments conforms to the theory presented in Chapter 6. The rationale for the use of a non-associated flow rule in combination with the Mohr-Coulomb failure criterion, particularly in the treatment of granular soils, has been discussed and substantiated by both theoretical and experimental evidence. The orientation of the failure planes is not limited to the direction of the plane of maximum stress obliquity and may be oriented anywhere between the limits of $\theta_b=45^\circ+\psi/2$ and $\theta_c=45^\circ+\phi/2$ to the direction of the minor principal stress.

The stresses in each element in a shear zone will have reached the failure envelope, but if the failure plane forms at any angle less than $\theta_c$, the shear stress at failure acting along a particular failure surface will not correspond to $\sigma_n\tan\phi$. This divergence from $\sigma_n\tan\phi$, explained within the framework of non-associated plasticity, is responsible for the differences between the factors of safety of the waste dump determined from limit equilibrium analyses and those determined from finite difference modeling.

The pattern of mobilized friction angle in the foundation elements is shown below in Figure 7.1 as determined from the FLAC analysis. The foundation elements have been modeled with both $\psi=0$ and $\psi=\phi$ to illustrate the effect of the flow rule on overall stability.
1 – maximum value of $\phi_m$ on any plane through the foundation elements

2 – mobilized friction angle on a plane parallel to the foundation slope (associated flow rule)

3 – mobilized friction angle on a plane parallel to the foundation slope (non-associated flow rule)

Figure 7.1 Mobilized friction angle in the foundation layer a) $\phi_{\text{foundation}}=40^\circ$ b) $\phi_{\text{foundation}}=35^\circ$

Figure 7.1a) depicts the pattern of mobilized friction angle for a stable dump with a relatively high foundation strength ($\phi=40^\circ$). Curve 1 represents the maximum mobilized friction angle on any plane through the foundation elements for both the $\psi=0$ and $\psi=\phi$ cases. This curve indicates that the full strength is mobilized along the entire
foundation underlying the toe wedge of the waste dump with the exception of elements directly underlying the dump toe irrespective of the dilation angle. Analysis presented in Chapter 5 has indicated that the critical failure mechanism corresponds to the double wedge and therefore, the direction of slip in the foundation is parallel to the foundation slope. The maximum mobilized friction angle in the foundation layer is compared with the value of mobilized friction angle acting on a plane parallel to the foundation slope ($\phi_{m25}$). Curves 2 and 3 of represent the pattern of $\phi_{m25}$ for both the associated and non-associated cases. These two curves are significantly lower than curve 1 indicating that the maximum friction angle is mobilized on a plane other than that corresponding to the foundation slope. For the high frictional strength of $\phi=40^\circ$ (Figure 7.1a), curves 2 and 3 generally coincide except in the elements directly below the dump toe where the associated flow rule materials mobilize higher values approaching the maximum mobilized friction angle.

When the friction angle is dropped to $35^\circ$, the full friction angle of $35^\circ$ is mobilized in the entire foundation underlying the toe wedge for both the $\psi=0$ and $\psi=\phi$ cases, and the effect of the flow rule on model behaviour becomes more pronounced (Figure 7.1b). If $\psi=\phi$, the dump remains stable and the value of $\phi_{m25}$ increases almost linearly, approaching a maximum value of approximately $34^\circ$ under the dump toe. If $\psi=0$, instability results as the relatively intact toe wedge of the waste dump begins to slide downslope. $\phi_{m25}$ is fairly uniform along the entire foundation at a value of approximately $30^\circ$. Although slip is occurring in the parallel to the foundation slope, this plane does not coincide with the plane of maximum stress obliquity. $\phi_{m25}$ is significantly less than the frictional strength of the soil. Whereas the associated flow rule restricts the direction of plastic flow to the plane of maximum stress obliquity, the results for $\psi=0$ indicate that the shear zone forming within the foundation slope corresponds with a velocity characteristic ($\theta_k$). This plane is oriented at an angle of $45^\circ-\psi/2$ to the direction of the major principal stress, and an angle of $(\phi-\psi)/2$ to the plane of maximum stress obliquity. The mobilized strength along this interface, $\tan \alpha$, may be determined from the Mohr circle construction (Fig. 7.2) or from:

$$\tan \alpha = \frac{\cos \psi \sin \phi}{1 - \sin \psi \sin \phi} \quad (6.24c^*)$$

When $\psi=0$, $\alpha=\sin\phi=29.84^\circ$ (curve 3 figure 7.1b).
The results presented in Chapter 5 assumed constant volume shearing conditions ($\psi=0$) as is generally the case in any stability analysis. The value of the dilation angle is not generally regarded as having a direct impact on the predicted factor of safety in the absence of any modeling of material weakening due to the effects of dilation. FLAC analysis indicates that the stability of the waste dump is sensitive to the value of the dilation angle. If the foundation elements are modeled as associated ($\psi=\phi$), the predicted factor of safety is increased by as much as 20%. Indeed, if all materials within the waste dump were to be modeled as associated, the results would be in close agreement with limit equilibrium.

It is important to note that the magnitude of the dilation angle used in the FLAC analysis has no effect on the yield stress or on the governing mechanism of failure. The effect of the dilation angle on the factor of safety however, requires further examination. The state of stress within an element of soil in the foundation layer is shown below in Figure 7.2 for a model run of a waste dump slope as determined from the FLAC analysis. The slope is at the point of incipient failure, the friction angle of the foundation soil in the model having been reduced to induce instability. The Mohr circle construction indicates that stresses within the foundation layer have reached the failure envelope and will indeed favour the formation of a shear zone oriented at $45^\circ-\psi/2$ to the major principal stress direction (Figure 7.2). The plane of maximum stress obliquity is quite steep, oriented at $(\phi-\psi)/2$ to the foundation slope. When the material is shearing at constant volume ($\psi=0$), failure progresses along this velocity characteristic and the relatively intact toe wedge of the waste dump begins to slide downslope.
A record of the mobilized friction vs. plastic shear strain during the analysis for a foundation element is shown in Figure 7.3. The data is for the model runs presented in Figure 7.1.
Figure 7.3 Mobilized friction angle vs. plastic shear strain in the foundation elements

The history of mobilized friction is taken after the waste dump has been constructed. The strength of the foundation soils is initially 40° and is then dropped to 35° to induce failure. The behaviour is similar to the simple shear behaviour presented in Section 6.4. The peak shear stress parallel to the shear band corresponds to $\tan \alpha$. Since the dilation angle has been taken as zero, $\tan \alpha = \sin \phi$ (from eq. 6.24c*).

7.2 Factor of Safety

A representative comparison of the factor of safety determined by FLAC with those determined by limit equilibrium analysis would require an associated flow rule ($\psi = \phi$). If a dilation angle equal to the friction angle of the waste rock is used in the FLAC analysis, large volumetric strains result in numerical instability. The waste dump model can therefore, not be analysed assuming an associated flow rule ($\psi = \phi$) using FLAC and only the results of limit equilibrium analysis are listed below in Table 7.1.
The above results are for a 100m high dump. Provided that all other modeling parameters are equal, the variation in the factor of safety determined by FLAC for different dump heights is due solely to the effects of the friction angle. The friction angle of the waste rock is inversely proportional to the confining stress and therefore, the average friction angle in a 200m high dump will be lower than that of a 50m high dump, exhibiting a correspondingly lower factor of safety. Limit equilibrium slope stability analyses carried out in practice commonly assume a uniform friction angle equal to the angle of repose for the waste rock ($\phi_{wr}=38^\circ$). Given the test data for rockfill of comparable grain size presented in Chapter 4, this value is believed to be overly conservative. As the dump is constructed, particles sliding out of the truck shovel gain momentum and roll down the dump face resulting in a flatter face angle. FLAC analysis carried out assuming a friction angle of 38° for the waste rock predicts shallow failures, parallel to the dump face and does not allow for the propagation of deep-seated failure mechanisms. Nevertheless, this value is commonly used in practice and the limit equilibrium results are included for the purpose of comparison.

FLAC analysis assuming an associated flow rule is not possible due to numerical instability. In addition, the use of such a high value of dilation (>40°) is unwarranted. The maximum dilation angle of a dense granular material, determined experimentally, is on the order of $\phi/2$ and generally ranges anywhere between the bounds of $0\leq\psi\leq\phi/2$. The waste rock is uncompacted and relatively loose and will have a correspondingly low value of dilation. Both field observations and the results of the FLAC analysis indicate that relatively large shear movements take place on the shear zones in the rockfill even when the dump has a high factor of safety indicating that the material will be shearing at constant volume. The foundation soils on the other hand are dense and would have an initially high value of dilation, approaching $\phi/2$. Model runs have been carried out using sequential loading simulating the

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\phi_{waste\ rock}$</th>
<th>$\phi_{m\ (fndn)}$</th>
<th>F.S</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>43</td>
<td>21.4</td>
<td>1.79</td>
</tr>
<tr>
<td>20</td>
<td>43</td>
<td>25.2</td>
<td>1.49</td>
</tr>
<tr>
<td>25</td>
<td>43</td>
<td>28.9</td>
<td>1.27</td>
</tr>
<tr>
<td>15</td>
<td>38</td>
<td>24.5</td>
<td>1.53</td>
</tr>
<tr>
<td>20</td>
<td>38</td>
<td>27.9</td>
<td>1.32</td>
</tr>
<tr>
<td>25</td>
<td>38</td>
<td>31.0</td>
<td>1.17</td>
</tr>
</tbody>
</table>

Table 7.1  Factors of safety (limit equilibrium)

The above results are for a 100m high dump. Provided that all other modeling parameters are equal, the variation in the factor of safety determined by FLAC for different dump heights is due solely to the effects of the friction angle. The friction angle of the waste rock is inversely proportional to the confining stress and therefore, the average friction angle in a 200m high dump will be lower than that of a 50m high dump, exhibiting a correspondingly lower factor of safety. Limit equilibrium slope stability analyses carried out in practice commonly assume a uniform friction angle equal to the angle of repose for the waste rock ($\phi_{wr}=38^\circ$). Given the test data for rockfill of comparable grain size presented in Chapter 4, this value is believed to be overly conservative. As the dump is constructed, particles sliding out of the truck shovel gain momentum and roll down the dump face resulting in a flatter face angle. FLAC analysis carried out assuming a friction angle of 38° for the waste rock predicts shallow failures, parallel to the dump face and does not allow for the propagation of deep-seated failure mechanisms. Nevertheless, this value is commonly used in practice and the limit equilibrium results are included for the purpose of comparison.

FLAC analysis assuming an associated flow rule is not possible due to numerical instability. In addition, the use of such a high value of dilation (>40°) is unwarranted. The maximum dilation angle of a dense granular material, determined experimentally, is on the order of $\phi/2$ and generally ranges anywhere between the bounds of $0\leq\psi\leq\phi/2$. The waste rock is uncompacted and relatively loose and will have a correspondingly low value of dilation. Both field observations and the results of the FLAC analysis indicate that relatively large shear movements take place on the shear zones in the rockfill even when the dump has a high factor of safety indicating that the material will be shearing at constant volume. The foundation soils on the other hand are dense and would have an initially high value of dilation, approaching $\phi/2$. Model runs have been carried out using sequential loading simulating the
construction process. The strain-softening model within FLAC was used to vary the dilation angle between an initial value of $\phi/2$ to zero at a prescribed value of plastic shear strain (see Figure 4.11). With a friction angle of 35° for the foundation, displacements at the toe of the advancing dump are sufficiently large to bring the soil to constant volume shearing conditions before the next increment of load is added. The resulting mechanism of failure and associated factor of safety for these model runs is identical to those corresponding to runs where the model was constructed. Therefore, when building the model up in one increment, a dilation angle of zero for the foundation soils is reasonable.

An alternate method of comparing factors of safety determined from both methods of analysis involves an adjustment of material strengths within the double wedge analysis to account for the non-associative behaviour of the soil. It has been established that the failure planes will propagate along planes of zero extension when the stress ratio on any other plane through the elements has reached the failure envelope. The relevant value of the friction angle to apply in the limit equilibrium analysis is $\alpha$ as determined from equation 6.24c*. The results of double wedge analysis carried out using a friction angle of $\alpha$ for the waste rock is shown below in Table 7.2. The value of mobilized base friction is compared with $\sin\phi$ as opposed to $\tan\phi$ to simulate the effects of the non-associated nature of the foundation soils. These values of mobilized base friction will be compared with the FLAC results presented in Chapter 5 where the factor of safety is determined by incrementally reducing the strength of the foundation soils until failure ensues. It is defined as the ratio of the shear strength of the foundation soils to the shear stress at failure.

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\phi_{wr}$</th>
<th>$\alpha_{\text{waste rock}} = \tan^{-1}(\sin\phi_{wr})$</th>
<th>$\alpha_m$ (fo)</th>
<th>$\phi_m$ (fo) = $\sin^{-1}(\tan\alpha_m)$</th>
<th>F.S</th>
<th>$\phi_f$ (fndn)</th>
<th>F.S</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>43</td>
<td>34.3</td>
<td>26.7</td>
<td>30.2</td>
<td>1.20</td>
<td>30</td>
<td>1.21</td>
</tr>
<tr>
<td>20</td>
<td>43</td>
<td>34.3</td>
<td>29.8</td>
<td>34.9</td>
<td>1.00</td>
<td>35</td>
<td>1.00</td>
</tr>
<tr>
<td>25</td>
<td>43</td>
<td>34.3</td>
<td>32.5</td>
<td>39.6</td>
<td>0.86</td>
<td>38</td>
<td>0.90</td>
</tr>
</tbody>
</table>

**Table 7.2** Factors of safety (non-associated flow rule)

The factor of safety determined from the modified double wedge analysis are within 5% of the FLAC results presented in Chapter 5. The close agreement confirms the contention that the difference between limit equilibrium and FLAC can be entirely accounted for by the non-associated flow rule used to model the materials in the FLAC
A comparison of the factors of safety for various foundation slopes determined from both limit equilibrium and FLAC analysis are shown below in Figure 7.4.

![Figure 7.4 Factor of safety vs. foundation slope](image)

7.3 Discussion

As described in Chapter 6, shear zones develop and propagate through both the foundation soils and the waste rock. The orientation of the failure surface at the base of the rock dump is limited by the dump geometry to the thin weaker foundation material overlying the bedrock. In contrast, shear zones forming within the rockfill are somewhat path dependent. Little discussion has been offered concerning the effect of the flow rule on the shear zones within the rockfill. FLAC analysis of the waste dump has shown that the formation of these shear planes is contingent on movements within the foundation and hence stability is largely governed by the mechanisms at work in the coarse sandy foundation layers. It is obvious that the peak and residual stresses acting on these discontinuities are also subject to the same principles described for the foundation soils. Their orientation will be somewhat controlled by both the grid size and its orientation. Grid dependency has been closely analyzed by experimentation with progressively finer mesh sizes and various grid orientations to obtain average results independent of grid orientation. The results are in close agreement with failure plane orientations observed in laboratory scale models as well as those determined from subsurface exploration of actual failed waste dumps.
Consideration of the velocity field within a waste rock dump model within the framework of non-associative plasticity indicates potential errors inherent in limit equilibrium analysis techniques. Experimental work on sands of comparative grain sizes has revealed that failure occurs along a velocity characteristic rather than the plane of maximum stress obliquity. FLAC analysis carried out assuming an associative flow rule (\(\psi = \phi\)) for the foundation materials yields a significantly higher factor of safety. Elements within the foundation are yielding but a continuous failure surface is prevented from forming since plastic flow is limited to the plane of maximum stress obliquity. When analysis is carried out assuming a more representative model of soil behaviour, the results are much less favourable. The yield criterion is again met within the foundation but now failure may occur along a plane of zero extension, oriented at (\(\phi - \psi\))/2 to the commonly assumed coulomb slip orientation. The factor of safety for a 50m mine dump on a 25 degree foundation angle predicted from the double wedge analysis is approximately 50% greater than that determined from the FLAC analysis.
8.0 Conclusions

The construction of mine waste dumps is an ongoing and necessary practice in Western Canada for the disposal of excess waste rock generated at open pit coal mines. The mines are typically located at or near mountaintops. As a result, segments of the foundation slope underlying waste dumps commonly exceed 25°. Typically, only a thin veneer of colluvial soils mantle the slopes to a depth of 1-2m underlain by bedrock of sedimentary origin.

The waste material disposed of in mine dumps consists of a conglomerate mixture of primarily sandstone and siltstone with smaller proportions of shale and mudstone ranging from silt to boulder sized particles. Material is end-dumped from the crest and the dump advances forward by the gradual accretion of material on the face. The momentum gained by material discharged from the truck shovel down the dump face results in a face angle slightly less than the angle of repose for the waste rock, generally 37° to 38° to the horizontal.

The end-dumped method of construction is unique to mine waste dumps. Unlike other man-made earth structures, compaction occurs only under the weight of added materials. This results in a relatively loose matrix of material in which localized failure may develop into overall instability. Any signs of potential instability are closely observed and monitored as equipment and personnel are constantly working at or near the dump crest during active dumping. Particle segregation due to end-dumping is an important aspect of dump construction contributing to the drainage characteristics and stability of the entire dump. Larger particles typically roll down the dump face and settle on the slope beyond the dump toe while finer material settles at the top of the dump face. The large, coarse particles form a continuous drainage layer at the dump/foundation contact ensuring that the piezometric surface does not rise into the dump. The particle gradation from crest to toe impedes the downward migration of fines which may result in obstruction or blinding of the drainage layer.

A review of observed field deformations indicates common patterns of deformation at the majority of dumps constructed on relatively steeply sloping terrain. Deformations consist of crest settlement resulting in the formation of near vertical scarps and tension cracks on the dump platform set back from the dump crest. Crest settlement is often accompanied by a slight flattening of the upper portion of the dump crest and slight steepening of the lower portion. A subtle 'step' has been observed on the middle third portion of the dump face.
These patterns of deformation are observed to accelerate over a period of several hours to several days preceding failure. The ongoing monitoring of stable and unstable dumps with similar patterns of deformation have led to the development of the double wedge theory of failure. The theory proposes that significant shear strain takes place on an inclined shear zone within the rockfill, dividing the dump face region into two wedge shaped zones. The stability of the upper wedge is dependent on the strength provided by the toe wedge. This theory has been substantiated by similar observations worldwide and furthered by laboratory scale models revealing the development of shear zones within model waste rock dumps. A limit equilibrium analysis technique for the double wedge failure mechanism is commonly used for design and stability purposes (Campbell, 1986).

Due to the severe implications of instability on personnel safety and infrastructure, a more in-depth study of the stresses and strains within the waste rock dump up to and at the point of failure have been facilitated by stress-deformation numerical analyses. Analysis has been carried out using the computer code FLAC which has been used extensively in the study of both slope stability and the study of shear band development in frictional media. Once the critical failure mechanism is established and understood, a review of construction practices, site conditions and design techniques may be more comprehensively evaluated in terms of their effects on the specific mode of failure. The scope of this study was limited to the mechanics of instability and associated factors of safety for various foundation slopes and dump heights. The stress-deformation numerical analyses carried out using the computer code FLAC have predicted patterns of deformation remarkably consistent with those observed in the field and laboratory. The model clearly indicates the sensitivity of the factor of safety to the available strength of the foundation soils underlying the toe wedge. Failure propagates along the foundation layer where the shear strength is significantly lower than the waste rock. Field observations of pre- and post-failure mine dump geometry often report a rill of foundation material pushed up ahead of the dump toe and the inclusion of foundation soils in the failure runout. These observations indicate that the failure surface does indeed extend into the foundation soils. As the strength of the foundation soils is reduced to a critical value within the FLAC model, shear planes form within the waste rock and foundation soils, separating the waste dump into two wedge shaped zones. The orientation and location of shear zones within the waste dump and foundation determined from the FLAC analysis are the same as those inferred from field observations and measured from laboratory scale models.
The results of the FLAC analysis differ significantly from limit equilibrium analysis in terms of the predicted factor of safety against failure. FLAC results presented in this study indicate a considerably lower factor of safety than that determined from double wedge analysis. Differences between the two methods of analysis are usually attributed to differences in the geometry of the failure surfaces being analyzed. In this case, the critical failure surface determined directly in FLAC is nearly identical to the failure surface analyzed with the double wedge method. With the same failure surface, material properties and constitutive relations, the factor of safety predicted from limit equilibrium is as much as 50% greater than values determined from FLAC analysis. Even assuming a friction angle of the waste rock equal to the angle of repose, limit equilibrium analysis predicts acceptable values of the factor of safety for foundation slopes of 25°. A literature review of rockfill strength indicates that a value of 38° is overly conservative and oversimplifies the effects of confining stress on strength. Using a more representative non-linear failure envelope for the waste rock widens the gap between the predicted factors of safety determined from both methods.

Large runout failure events at coal mine dumps in western Canada are a relatively common occurrence. In a recent survey, over 30% of the coal mine waste dumps surveyed have experienced instability associated with a runout exceeding 100m (Piteau, 1991). A review of failure records has revealed failures on waste dumps where the design factor of safety determined from limit equilibrium methods is acceptable. These failures are generally attributed to the generation of excess pore pressures within the foundation layer. During construction, site conditions are also present which may adversely affect the ability of the foundation soils to adequately dissipate pore pressures (i.e. rapid loading of saturated soils, groundwater recharge, seepage, etc.). Due to the high permeability of the thin layer of silty sands and gravels, it appears reasonable to assume fully drained conditions except in cases where the above conditions are extreme. The nature of the foundation soils makes the possibility of widespread excess pore pressures unlikely. Unfortunately this is not possible to confirm, as monitoring of pore pressures within the foundation is infeasible. The impact of large boulders rolling down the dump face during construction would effectively disable any monitoring equipment.

The numerical analyses carried out for this study predict lower factors of safety without any consideration of the effects of pore pressure generation within the foundation. The geometry of the failure mechanism is consistent with the double wedge mechanism as inferred from laboratory and field observations. The normal and shear stresses acting on the failure plane determined from FLAC analysis are in agreement with limit equilibrium results. The
difference is due to a more thorough treatment of both stresses and deformations within the FLAC model. When considering both the stress and velocity fields of a given problem, a plastic flow rule must be adopted to model the direction of plastic deformations. Although limit equilibrium analysis techniques only consider stresses and do not deal directly with concepts of plasticity, the treatment of the failure surface as a plane of maximum stress obliquity is an inherent assumption of associated plasticity theory. The use of an associated flow rule in combination with the Mohr Coulomb failure criterion implies a dilation angle equal to the friction angle of the material and limits plastic flow to the direction of the plane of maximum stress obliquity. For granular soils, the dilation angle typically ranges between the bounds of $0 \leq \psi \leq \phi/2$. To more accurately model plastic deformations, FLAC models plastic flow using a more representative non-associated flow rule developed based on experimental findings.

The dilation angles of both the foundation soils and waste rock are significantly less than the friction angles and therefore the stress and velocity characteristics of the problem do not coincide. In agreement with experimental findings, failure planes within a FLAC model do not necessarily coincide with a stress characteristic (plane of maximum stress obliquity) inclined at an angle of $\theta_{c}=45°+\psi/2$ to the minor principal stress direction. The theory on which the plastic flow formulation in FLAC is based has been substantiated by significant experimental evidence suggesting that shear zones form along a velocity characteristic whose orientation is dependent on the dilation characteristics of the material being modeled ($\theta_k=45°+\psi/2$) rather than the strength of the material. Although the stresses in each element along the shear zone have reached the failure envelope, the direction of plastic flow does not coincide with the plane of maximum stress obliquity. The stress ratio acting along the failure surface is determined from:

$$\tan \alpha = \frac{\cos \psi \sin \phi}{1 - \sin \psi \sin \phi} \quad (6.24c^*)$$

If the dilation angle is equal to the friction angle, velocity and stress characteristics coincide and $\alpha=\phi$. The value of $\alpha$ decreases with decreasing values of the dilation angle. At constant volume shearing conditions ($\psi=0$), the stress ratio is equal to $\sin \phi$. For stability and design purposes, a friction angle of $35°$ is typically used for the foundation soils. If the effects of dilation are neglected, the effective friction angle to be used in the limit equilibrium analysis is equal to $29.8°$. Sequential modeling of dump construction has indicated that successive displacements at the toe region of the advancing dump effectively bring the material to constant volume shearing conditions before the next
load increment is added. Therefore, effects of dilation are negligible and may be omitted in the analysis. The
difference in the factor of safety due to the non-associated flow rule is dramatic. The results of the numerical
modeling presented in chapter 7 indicate marginal stability for dumps whose factor of safety is as high as 1.49
determined from the double wedge analysis. Results have only been presented for a 100m high dump. Differences
in the factor of safety for larger dumps are due to the effects of confining pressure on the friction angle. Larger
dumps will exhibit lower friction angles at depth resulting in lower factors of safety. This difference can be upward
of 10 degrees over a 1MPa increase in stress.

Stress-deformation numerical analysis has provided significant insight into the failure process of mine waste dumps
in western Canada. The analysis has provided an understanding of the patterns of stress and strain within the waste
dump and foundation soils up to and at the point of failure. While the failure mechanism closely simulates field and
laboratory behaviour, results question the adequacy of conventional limit equilibrium methods used in the
determination of the factor of safety. The analysis has indicated that the foundation soils underlying waste dump
constructed on relatively steeply sloping terrain are subjected to stress ratios at or near the failure state. A review of
plasticity theory from both an experimental and theoretical viewpoint suggest that plastic flow takes place on a plane
inclined at $45^\circ+\psi/2$ to the minor principal stress direction when the stress on any other plane through that element
has reached the failure envelope. This effectively allows for failure along the foundation slope although the stress
ratio acting along this plane is less than the limiting maximum.

Conventional limit equilibrium approaches to slope stability determine the mobilized friction along a trial failure
surface and compare this value to the strength of the material. This approach yields an overly favourable estimate of
the factor of safety in light of the results presented in this study. Instead, the failure plane should be considered as
the plane of zero extension oriented at $45^\circ+\psi/2$ to the direction of the minor principal stress. The friction angle of
the material should be adjusted accordingly to determine a representative value of the factor of safety. Equation
6.24c* above should be used to determine the modified material strength to be used in the analysis.
Future Work

- The patterns of pre-failure dump deformations predicted by FLAC are similar to field observations although the magnitude of these deformations is not adequately captured. The large deformations are most likely a result of a combination of several complex mechanisms. As dump heights are upwards of 400 m, progressive particle crushing at depth may significantly influence crest settlement rates. In addition, materials generally exhibit time dependent deformation when subjected to high stress ratios. The numerical model predicts the formation of the back failure plane within the waste rock prior to the onset of foundation failure. Creep deformations along this and other planes of high stress ratio will also play a role in the magnitudes of pre-failure deformation. To adequately predict deformations, each mechanism of deformation would need to be examined and assessed individually. Initial calibration of the numerical model to shear and volumetric deformations would need to be followed up by consideration of the crushing strength of the particles and creep characteristics of the rockfill.

- To provide a more representative simulation of the failure process, a more thorough treatment of the material parameters is needed, particularly for the foundation soils. The strength of the foundation soils will be somewhat dependent on the confining stress although the relationship between the two has yet to be defined. The use of a constant value of friction within the entire foundation layer is a simplification that may have a significant effect on the model behaviour, particularly in the toe region where confining stresses are very low. The progress of failure will also be dependent on the site specific foundation profile which is generally concave downward with a decreasing slope angle towards the dump toe. All model runs have been carried out with a linear foundation profile. Failures typically involve the toe of the dump ‘hanging’ in until explosive failure. Stress ratios along the foundation determined from FLAC analysis indicate higher stress ratios at the toe of the dump. Using a constant friction angle and linear foundation profile, failure progresses from the toe back to the back failure plane through the rockfill.

- In addition to laboratory testing of material properties, future research would ideally involve a laboratory scale model testing to assess the actual conditions at failure. Once this has been carried out, a more thorough review of the theories of non-associated plasticity would be necessary.
Nomenclature

\begin{align*}
\phi & \quad \text{Friction angle} \\
\phi_m & \quad \text{Mobilized friction angle} \\
\gamma & \quad \text{Shear strain} \\
\gamma^p & \quad \text{Plastic shear strain} \\
\tau & \quad \text{Shear stress} \\
\sigma_n & \quad \text{Normal stress} \\
\sigma_m & \quad \text{Mean stress} \\
R & \quad \text{Equivalent roughness} \\
S & \quad \text{Equivalent strength} \\
B & \quad \text{Bulk modulus} \\
G & \quad \text{Shear modulus} \\
D & \quad \text{Relative density} \\
\nu & \quad \text{Poisson's ratio} \\
\psi & \quad \text{Dilation angle} \\
\varepsilon^p & \quad \text{Plastic strain} \\
\varepsilon_{vp} & \quad \text{Plastic volumetric strain} \\
\beta & \quad \text{Slope angle} \\
H & \quad \text{Dump height} \\
\theta & \quad \text{Stress orientation angle} \\
\theta_R & \quad \text{Roscoe shear band orientation} \\
\theta_C & \quad \text{Coulomb shear band orientation} \\
\xi & \quad \text{Direction of the major principal stress} \\
\lambda & \quad \text{Direction of the major principal strain rate} \\
\rho & \quad \text{Direction of the major principal stress rate} \\
\omega & \quad \text{Dilation angle}
\end{align*}
References


