

A PROBABILISTIC APPROACH FOR SEISMIC RISK
ASSESSMENT WITH UNCERTAIN INPUT PARAMETERS

by

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Abstract

The main objective of this thesis is the development of a procedure for analyzing the seismic reliability of a structure considering uncertainties in the factors affecting its performance. These uncertainties stem from different earthquakes records, from structural mass distributions, foundation characteristics, and the probabilistic nature of strength of materials. The method refers to building a database for the response of the structure for a range of uncertain parameters. This is then used in a reliability analysis to estimate the probability of failure of the structure in a given failure criterion. The software for the non-linear dynamic analysis of structure was CANNY and the reliability software was RELAN.

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List of Abbreviations

PGA	Peak Ground Acceleration
FORM	First Order Reliability Method
SORM	Second Order Reliability Method
SDOF	Single Degree of Freedom
MDOF	Multiple Degree of Freedom
PSHA	Probabilistic Seismic Hazard Analysis
MCS	Monte Carlo Simulation
NBCC	National Building Code of Canada

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Problem and Thesis Objective

The thesis will study a methodology by means of which a response database is first obtained with deterministic runs of a dynamic analysis software, and the application of a reliability analysis software to determine the reliability using the database results obtained a-priori. This approach is then compared with other approaches in the literature. This is followed by an example of a steel frame and the application of this approach to its reliability analysis.

1. Review of Previous Work

Structural design is a decision-making process in which a wide spectrum of requirements, expectations, and concerns needs to be properly addressed. Engineering design criteria are considered together with societal and client preferences, and most of these design objectives are affected by the uncertainties in the intervening variables and parameters. Therefore, realistic design frameworks must be able to incorporate multiple performance objectives and uncertainties from numerous sources. In this study, a multi-criteria based design framework for structural design under seismic conditions is explored. The emphasis is on reliability-based performance objectives and their interaction with design procedure. In the probabilistic response analysis, seismic loading uncertainties as well as modeling uncertainties are incorporated.

0.1 Multi-Criteria Optimal Design

To reach this objective, previously Beck et al. (1998) at Caltech Earthquake Engineering Research Laboratory, published a report titled "Performance-based optimal structural design methodology". In their approach, a general framework for multi-criteria optimal design was presented which was well-suited for performance-based design of structural systems operating in an uncertain dynamic environment.

The approach of Beck is valuable since it combines the design parameters and the performance parameters all together and defines a global optimization function for the system. Because of the iterative nature of this approach, it is practical when the researcher has access to massive computing resources. This is because the finite element analysis and design optimization and reliability analysis are closely combined together. In the approach given in this thesis, the researcher can easily implement the reliability analysis for a given system if the statistics of input parameters to the system and the system response to these values are available for a few samples. Following is a brief description

of this approach:

A decision theoretic approach was used, which was based on aggregation of preference functions for the multiple, possibly conflicting, design criteria. This allowed the designer to trade off these criteria in a controlled manner during the optimization. Reliability-based design criteria were used to maintain user-specified levels of structural safety by properly taking into account the uncertainties in the modeling and seismic loads that a structure might experience during its lifetime. Code-based requirements were also incorporated into this optimal design process. The methodology was demonstrated with two simple examples involving the design of a three-story steel-frame building for which the ground motion uncertainty was characterized by a probabilistic response spectrum, which is developed from available attenuation formulas and seismic hazard models.

In this process, analysis, evaluation and revision were repeated iteratively as long as it was necessary to find a design which was considered to give the best compromise solution to all the design criteria. Then this structural design process was converted into a performance-based multi-criteria optimization problem.

The design parameters, which were designated by a vector θ , were those parameters of the design which are selected to be varied during the search for an optimal design such as: geometric information for the structural members or cross-sectional dimensions. On the other hand, performance parameters, were designated by a vector \mathbf{q} , represent quantities related to the "performance" of the design, and can take the form of conventional structural parameters (e.g. stress, deflection, inter story drift) or other parameters (e.g. structural reliability, material cost. of the structural system). Obviously, the performance parameters, $\mathbf{q}(\theta)$, are functions of the current design parameters, θ .

Deterministic or (code-based) loads of structural performance parameters can be computed using a finite-element model of the structure. The construction cost can be

computed using a costing algorithm. On the other hand, reliability-based performance parameters, such as the uncertain peak lifetime inter story drift, must be analyzed using probabilistic analysis tools and a probabilistic seismic hazard model. In the next step called evaluation stage the objective was to obtain an overall design evaluation measure $\mu(\theta)$ for the design specified by the current value of the design parameter vector θ . This measure $\mu(\theta)$ served as an objective function which, at the revision stage, was used to determine improved. Since for evaluation of the design the designer may wish to impose many different design criteria, a multi-criteria decision methodology was required in which a design is quantitatively evaluated on the basis of each design criterion.

Since not every design criterion could be satisfied to its maximum extent simultaneously, a preference aggregation rule was given as a functional relationship, f , between the overall design evaluation measure, μ , and the individual preference values, $\mu_1, \mu_2, \dots, \mu_N$, for all of the design criteria. An optimal design for a given preference aggregation rule was given by a design parameter vector θ which maximizes:

$$\mu(\theta) = f(\mu_1(q(\theta)), \mu_2(q(\theta)), \dots, \mu_N(q(\theta))) \quad (1)$$

The function f can have several trade off strategies which are up to the decision maker to choose. The next step was to obtain an iterative revision of the design to achieve a better, and eventually an optimal, design such that for the sequence of $\theta^0, \theta^1, \dots, \theta^N$ we have:

$$\mu(\theta^0) < \mu(\theta^1) < \dots < \mu(\theta^N) \quad (2)$$

For considering the probability of structural failure, designated by $F(\theta)$ for a design corresponding to θ , there was a need to characterize the seismic hazard at the construction site by a set of ground motion parameters a (for example, peak ground acceleration, response spectrum ordinates, duration of motion, frequency content, etc.). In this case a

set of uncertain "seismicity" variables, which are designated here by a vector ϕ account for the uncertain regional seismic environment. For example, ϕ may include variables such as earthquake magnitude, fault dimensions, source parameters, epicentral distance, propagation path properties and local site conditions. The uncertain values of ϕ are described by a probability density function $p(\phi)$. For example, $p(\phi)$ might be chosen to model the probability of occurrence of an earthquake of a given magnitude and the probability of fault rupture at specific locations along a fault.

The required attenuation relationships were often derived by an empirical fit to the observed data. There is uncertainty associated with these attenuation models, even when ϕ is known, which is reflected by the scatter of the analyzed data about the mean or median model predictions. Therefore, the attenuation relationship should actually give a probabilistic description $p(\alpha | \phi)$ of the relation between the ground motion parameters α and the seismicity parameters.

Because of the presence of modeling errors, the structural model corresponding to a particular design θ will not accurately predict the response of the structure should it be built. These uncertainties mean that a failure probability corresponding to a design θ which is conditional on the ground motion parameters, designated by $F(\theta | \alpha)$, must be set up. This can be done using probabilistic analysis tools. For example, the effect of the uncertainty in the seismic excitation at the site can be treated using random vibration analysis if the ground motion is modeled as a stochastic process depending on the parameters.

Therefore the uncertainties in the seismic environment, ground motion modeling and structural modeling can be combined using the *total probability theorem* to determine the total failure probability given the occurrence of an earthquake event is:

$$f(\theta) = \int_{\phi} \int_{\alpha} F(\theta|\alpha) p(\alpha|\phi) p(\phi) \cdot d\alpha d\phi \quad (3)$$

The failure probability over the lifetime of the structure is then computed using an occurrence model for earthquake events. Assuming that the occurrences of earthquakes follow a Poisson arrival process, the probability that the structural safety requirements are not satisfied during the lifetime t years of the structure, is given by:

$$F_{life}(\theta, t) = 1 - e^{-\nu F(\theta)t} \quad (4)$$

where $F(\theta)$, given in (3), is the failure probability given the occurrence of an event, and ν is the expected number of events per year.

0.2 Reliability-Based Design Earthquake Identification

John W. van de Lindt (2000) at Michigan Technological University and John M. Niedzwecki at Texas A&M University have published a paper in ASCE Journal of Structural Engineering titled "A Methodology for Reliability-Based Design Earthquake Identification". Their study presented a methodology to rank the possible earthquake events through the estimation of a reliability importance parameter, which is a function of events magnitude, M , and site-to-source distance, R .

This paper is of interest for us since it considers the effect of site to source distance R . In our proposed approach, we are not directly considering the value of R . In addition, the earthquake magnitude is considered as a random variable. We claim that these two random variables can be embedded in the peak ground acceleration and the earthquake record that is applied to the system. In addition, in this report Monte Carlo simulation is

used for considering the probabilistic distribution of mass and damping of a structure. In this thesis, more advanced reliability techniques, which consider the gradient effect such as FORM and SORM, are considered. For more information and comparison please refer to Calculating the Probability of Failure or Non-Performance on page 34. The following gives more detail on this technique.

The M-R relation allows one to rank the importance of each design earthquake, not in terms of the acceleration of a SDOF oscillator, but rather in terms of the displacement-based reliability of a MDOF representation of the structure. A simulation procedure was introduced that couples a technique similar to Seismic Hazard Analysis with performance-based reliability estimates that consider inter-story drift criteria. The simulation procedure allowed for the inclusion of response spectra uncertainty and structural resistance uncertainties in the mass, stiffness and damping at each story level. The reliability importance factor leads to the definition of a Cumulative Earthquake Hazard Function. This function may be used as a basis in selecting the number of design earthquakes one wishes to use depending upon the desired complexity of the analysis. Two illustrative examples were presented.

The theoretical background for this is that, for engineering design purposes, it follows that only earthquakes with a magnitude greater than some threshold level M_{\min} , are of particular interest. From this point forward, events of interest will be those where $M > M_{\min}$ and can be assumed to be described using a Poisson process without any clumping of the events. It should be noted that this assumption neglects the occurrence of aftershocks. For small to moderate earthquakes the effect is negligible; however, for larger magnitude earthquakes there may be a significant effect on the reliability. The expected number of events per year will be equal to the Poisson crossing frequency, specifically

$$E[events] \simeq \nu \text{ year}^{-1} \quad (5)$$

In the present formulation, the expected number of events per year was assumed constant over the entire region. This simplification was employed because the methodology was the focus of this study. It is expected that the Gutenberg-Richter relation (Gutenberg and Richter 1956) will be used as a function of M in future application studies. The probability of occurrence conditional on an event can be expressed in terms of the conditional joint-probability mass function over M - R ; that is

$$P_{occurrence|event} = \int_M \int_R f_{MR}(m, r) dm dr \quad (6)$$

Then, the displacement-based probability of exceedance conditional on occurrence of M - R can be written in terms of a nonparametric survivor function, defined as the complement of the CDF of the maximum response, as

$$P_{event|occurrence} = 1 - F_{Y(M, R)}\{X\} = S_{Y(M, R)}\{X\} \quad (7)$$

where Y = vector describing the response; and X = vector describing the limit states. The unconditional probability of exceedance, $P_e(M, R)$, can be expressed as:

$$\begin{aligned} P_e(M, R) &= \nu P_{occurrence|event} P_{event|occurrence}(M, R) \\ &= \nu S_{Y(M, R)}\{X\} \int_M \int_R f_{MR}(m, r) dm dr \end{aligned} \quad (8)$$

Monte Carlo simulation (MCS) was employed to determine the $P_{e|occ}$ in (8). The structural

mass, W , structural stiffness, K , and structural damping, C , were assigned distributions and assumed to be independent. The load/response model consisted of a MDOF response spectrum analysis (RSA), or modal analysis.

Superposition of the modal maxima gives the total approximate maximum displacement as

$$Y_{max} = \sqrt{Y_{1,max}^2 + Y_{2,max}^2 + \cdots + Y_{n,max}^2} \quad (9)$$

The spectral acceleration at each frequency or structural period, S_a , can be a function of M and R , as well as several additional site/fault specific variables. This can be written functionally as

$$\ln(S_a) = g(M, R, \theta) + \epsilon \sigma_{\ln S_a} \quad (10)$$

in which g = median regression for the response spectrum in terms of M and R ; θ represents many different parameters including soil/rock type and fault characteristics; ϵ is a measure of spectral uncertainty; and $\sigma_{\ln(S_a)}$ = standard deviation of the spectral acceleration.

0.3 A Time Variant Approach to Performance-Based Engineering

In May 2000, J.W. van de Lindt and J.M Niedzwecki presented a paper at Structural Engineers World Congress 2000, Philadelphia, PA. The title was “A Time Variant Approach to Performance-Based Engineering” Their study presented an overview of an

approach that identifies a design earthquake in terms of a critical response spectrum at a site in order to estimate the time variant reliability of the structure against exceedance of a specified inter-story drift. This was accomplished by coupling a technique similar to Probabilistic Seismic Hazard Analysis (PSHA) with performance-based reliability. Once the critical response spectrum was identified in terms of earthquake magnitude, M , and site-to-source distance, R , Monte Carlo Simulation (MCS) is used to account for spectral scatter, e , and structural resistance uncertainty present in the mass, stiffness, and damping of the model. An illustrative example was included for clarity. This example combined MCS with response spectrum analysis to estimate the static reliability of the structure. Then, the return period of the design earthquake was determined by definition and the time interval of interest is modeled as Poisson with a mean proportional to the length of the interval, and the time variant reliability of the structure determined.

This is very similar to our approach since inter story drift is used as the performance criteria. However they obtained the critical response spectrum in terms of earthquake magnitude, M . In our approach broader range of earthquakes are available by giving the researcher the capability to directly modify the peak ground acceleration of an earthquake as well as different waveforms of earthquakes.

0.4 Seismic Performance Evaluation For Steel Moment Frames

Recently Yun, Seung-Yul, Hamburger, Ronald O., Cornell, C. Allin and Foutch, Douglas A. from Stanford University, published an article in the ASCE Journal of Structural Engineering (2001), "Seismic Performance Evaluation For Steel Moment Frames". They developed a performance prediction and evaluation procedure based on nonlinear dynamics and reliability theory. It featured full integration over the three key stochastic models: ground motion hazard curve, nonlinear dynamic displacement demand, and displacement capacity.

This method is similar to the approach in this thesis, where the nonlinear dynamic analysis of steel frames combined with displacement capacity is considered. In this method performance criteria, demand and capacity are considered more comprehensively; however in this thesis approach we focus on obtaining performance levels for a given inter story drift from available response data from a structure. Following is more detail on this approach:

A suite of uncertainty analysis was input to the procedure such as period, live load, material properties, damping, analysis procedure, and orientation of the structure. Two limit states were defined instead of the traditional single state. The procedure provided a simple method for estimating the confidence level for satisfying the performance level for a given hazard. The confidence level of a post- and a pre-Northridge 9-story building for a given hazard level was calculated using the procedure described in the paper. New steel moment frame buildings were expected to perform much better during major earthquakes than existing buildings designed and built with older technologies.

A systematic procedure for this approach is as follows:

1. Determine the performance objective to be evaluated. This requires the selection of one or more performance levels, that is, either Immediate Occupancy (IO) or Collapse Prevention (CP). These are defined as the post-earthquake damage state where only minor structural damage has occurred with no substantial reduction in building gravity or lateral resistance. Then define the appropriate hazard level, that is exceedance probability desired for this performance.
2. Determine the ground motion characteristics for the performance objective chosen. The ground motion intensity for each performance level should be chosen to have the same probability of exceedance as the hazard level of the design objective, e.g., 2/50 for the CP case.

3. Calculate the structural demand for each earthquake intensity. The demand is computed using standard methods of structural analysis. Either linear methods or nonlinear methods may be used. Once calculated, demand parameters such as the maximum inter story drift, Δ_{max} , are adjusted for bias inherent in the analytical procedure using the equation:

$$D = CB \Delta_{max} \quad (11)$$

where

CB = Analysis procedure-dependent bias coefficient

Δ_{max} = Maximum calculated inter story drift

The bias coefficients are calculated by performing a series of analysis, using representative building structures and the selected methodology, and by comparing the median of the results obtained to the median of results obtained from nonlinear time history analysis of the same structures for the same ground motions.

4. Determination of global and local collapse capacity and resistance factor. The resistance factors are a product of the integration (Cornell, et al., 2001) used to determine the total probability that demand will be greater than capacity. Resistance factors are given by the equation:

$$\phi = e^{\frac{k}{2b} \beta^2} \quad (12)$$

where k is the logarithmic slope of the hazard curve, i.e., a measure of the rate of change of ground motion intensity with probability of exceedance; b is a similar coefficient that represents the change in demand (for example inter story drift) as a function of ground motion intensity (set to unity for the default cases); and b is the standard deviation of the natural logarithm of the variation in capacity resulting from variability in ground motion

and structural characteristics.

5. Determine the factored-demand-to-capacity ratio λ . Once the demand is calculated and the demand and capacity factors are determined, the factored-demand-to-capacity ratio is calculated. The demand and analysis uncertainty factors, like the resistance factors, are products of the integration to obtain the total probability that demand is greater than capacity.

6. Evaluate the confidence level. The confidence in the ability of the building to meet the performance objective is determined, using the λ value determined in accordance with Step 5 above, by a back-calculation to obtain K_x from the equation

$$\lambda = e^{-\beta_{UT}(k_x - \kappa \frac{\beta_{UT}}{2b})} \quad (13)$$

where k and b are the coefficients previously described, β_{UT} is the logarithmic standard deviation of the distribution of both demand and resistance, considering all sources of uncertainty and K_x is the standard Gaussian variate associated with probability x of not being exceeded found in conventional probability tables, e.g., if $K_x = 1.28$ then $x = 90\%$. The values of the uncertainty coefficient β_{UT} used are dependent on a number of sources of uncertainty in the estimation of structural demands and capacities. Sources of uncertainty include, for example, the effective damping, the actual material properties, and the effective structural period and others each contain uncertainties. The uncertainty associated with each source (i) may be identified as β_{Ui} . Then

$$\beta_{UT} = \sqrt{\sum_i \beta_{Ui}^2} \quad (14)$$

7. Determine the confidence level. Once the confidence factor λ and the uncertainty coefficient β_{UT} are determined, the confidence level can be found.

0.5 Optimal Probabilistic Seismic Demand Models For Typical Highway Overpass Bridges

Mackie, K., and B. Stojadinovic from the Civil and Environmental Engineering department at UC Berkeley presented a paper at 12th European Conference on Earthquake Engineering titled "Optimal Probabilistic Seismic Demand Models For Typical Highway Overpass Bridges". In their report, a performance-based seismic design was founded on the ability to evaluate the performance of a structure in a given seismic hazard environment. In spring 2000, the Pacific Earthquake Engineering Research Center (PEER) was developing such a probability-based performance framework, one component of which was a seismic demand model. The objective was the development of an optimal probabilistic seismic demand model for typical highway overpass bridges. This demand model relates ground motion intensity measures, such as spectral displacement, to bridge Demand Parameters, such as column curvature ductility or drift ratio. An optimal model is defined herein as practical, sufficient, effective, and efficient. A parametric finite element model, representing an entire bridge portfolio, was used to compute values of bridge-specific Demand Parameters. Probabilistic seismic demand models were formulated by statistical analysis of the data produced during time history analysis of each bridge in the bridge portfolio under all ground motions. A representative relation between chosen Intensity and Demand Measure pairs forms the basis of the demand models were presented.

This paper provided insight into the selection of an optimal "Probabilistic Seismic Demand Model" for a class of real structures, in this case typical California highway

overpass bridges. Given the requirements that demand models need be practical, sufficient, effective, and efficient, the task of optimizing becomes complex. For the design parameters discussed in this paper applied to demand models in the bridge longitudinal and transverse directions, it was found that first mode spectral displacement is the optimal intensity measure when coupled with a variety of Engineering Demand Parameters. These models include local measures (maximum material stresses), intermediate measures (maximum column moment), and global measures (drift ratio). With a small tradeoff in efficiency, the use of period-independent Arias Intensity as the intensity measure was also acceptable.

The "Probabilistic Seismic Demand Model" described in this paper can be used directly by designers as structural demand hazard curves. They allow assessment of design parameter variations on structural performance. Cast as a component model in a performance-based earthquake engineering framework, provide the probabilistic relationship between ground motion intensity measures and structure-specific Engineering Demand Parameters.

The intensity measure side could be coupled with hazard models and the demand parameters to fragility models in order to achieve probabilities of exceedance of such economic variables such as repair cost, wherein the true value of "Performance-based earthquake engineering" lies.

1 Approach in the Code

In this section the approach in the National Building Code of Canada is discussed. The factors influencing seismic design are discussed one by one. The references to this section are “1995 National Building Code of Canada” and the “NBCC Part 4” as well as the Commentary J of this code.

1.1 Terminologies and Background Information

Earthquake engineering language sometimes contains contradictory definitions of widely used terms. This section defines the important seismic terms. Our definitions generally follow those given in the Earthquake Engineering Research Institute glossary of terms (Earthquake Spectra, November 1984), except that no size or scale is implied in the terms “zone” and “microzone”.

1.1.1 Seismic Zonation

Seismic zonation is the process by which areas are subdivided into seismic zones based on historical and predicted intensity of ground motion, which is expressed in terms of the peak horizontal ground acceleration or velocity. Seismic design requirements for structures are generally constant within a seismic zone. The acceleration-related seismic zones defined in the National Building Code of Canada (1990) are presented in Table 1.

SEISMIC ZONE Z _a	Range of Peak Horizontal Ground Acceleration (PGA), g, for 10% Probability of Exceedence in 50 years (475 years return period)
0	PGA <0.04
1	0.04 = PGA <0.08
2	0.08 = PGA <0.11
3	0.11 = PGA <0.16
4	0.16 = PGA <0.23
5	0.23 = PGA <0.32
6	0.32 =PGA

Table 1: Seismic Zones of National Building Code of Canada

Building codes seismic zonation of ground motion in Canada and the United States has been done using probabilistic models consisting of seismogenic zones, and recurrence relations for earthquakes within those zones, based primarily on historical records and limited geologic evidence of seismicity. Ground motion at any location is estimated using basic probabilistic procedures developed by Cornell (1968), based on the seismogenic zone activities and local attenuation functions. Shaking intensity is usually characterized as peak horizontal ground acceleration or velocity for rock or firm ground at a given probability of exceedance or return period. Figures 1 & 2 on page 18 display contours of peak horizontal ground acceleration and peak horizontal ground velocities for Canada, respectively. These contours have a probability of exceedance of 10% in 50 years.

1.1.2 Seismic Microzonation:

Seismic microzonation is a procedure for estimating the total seismic hazard from ground

shaking and related phenomena by taking into account the effects of local site conditions. For example, subsurface and topographic conditions can amplify or de-amplify the peak ground acceleration which would be expected for firm ground at a particular location, and these local effects would be reflected in a seismic microzonation map.

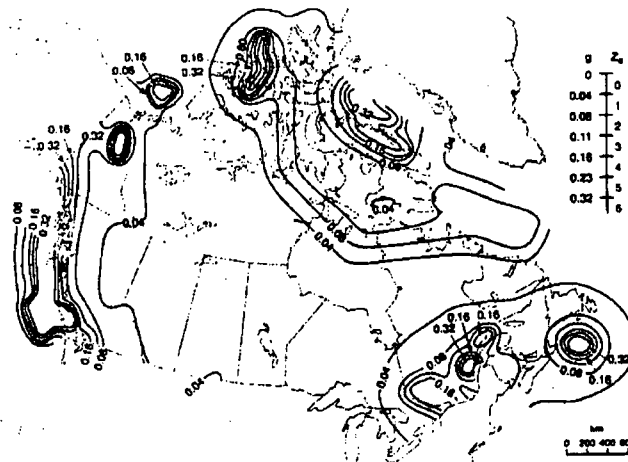


Figure 1 : Contours of peak horizontal ground acceleration in units of g, having a probability of exceedance of 10% in 50 years

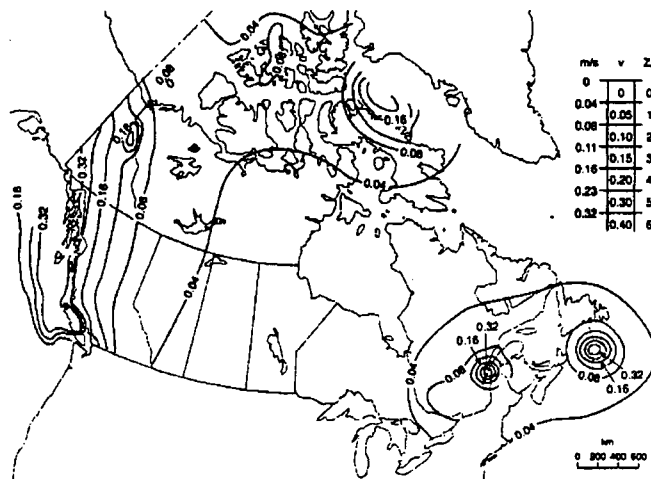


Figure 2 : Contours of peak horizontal ground velocities, in m/s, having a probability of exceedance of 10% in 50 years

1.1.3 Seismic Hazards

Seismic hazards are defined as those earthquake-related geologic processes that have the potential to "produce adverse effects on human activities" (Earthquake Spectra, November 1984), whether the threat is to life, constructed works, or real estate. For example, seismically-induced liquefaction is considered to be a seismic hazard, as are the associated ground displacements. However, fire caused by a gas main ruptured by liquefaction-induced ground displacement is not, since it is not strictly a geologic process.

Ground motion, the definitive characteristic of earthquakes, is a seismic hazard that causes damage to structures directly, by vibration, or indirectly, by inducing other seismic hazards such as liquefaction and landsliding (defined below). Since seismic zonation of the province is readily available for estimating the intensity of ground motions on firm ground, mapping of this hazard is not considered further in this report.

Although there is some overlap, the other primary seismic hazards can be grouped in six categories for mapping purposes:

1.1.4 Liquefaction

Liquefaction refers to the loss of shearing resistance or the development of excessive strains resulting from transient or repeated disturbances of saturated cohesionless soils. Liquefaction-induced horizontal ground movements can range from minor oscillations during ground shaking with no permanent displacement, to small permanent displacements, to lateral spreading and flow slides. Flow slides and submarine slope failures which are presumed to be caused by liquefaction are included in this category for convenience of analysis and mapping. Liquefaction can also induce vertical ground movements (settlement) by rearrangement of loose soils into a denser configuration.

1.1.5 Landslides

This hazard includes all types of seismically induced landslides (e.g., soil slumps, rock falls, debris flows, rock avalanches), except for those occurring directly as a result of ground liquefaction (see above).

1.1.6 Amplification of Ground Motion

The localized amplification of ground motion due to subsurface and/or topographic conditions at a site is considered to be a seismic hazard over and above the firm ground seismic motions of the area. Amplification of ground motion often occurs at sites overlain by thick, soft soil deposits, especially when the predominant period of the earthquake motions matches the predominant period of the ground. De-amplification of ground motions can also occur, but it is not considered to be a seismic hazard for obvious reasons.

1.1.7 Tsunamis and Seiches

This hazard includes waves in oceans, lakes, rivers or other bodies of water that are generated by tectonic subsidence or uplift, seismically-induced landslides or other seismic hazards.

1.1.8 Tectonic Subsidence or Uplift

Tectonic subsidence or uplift is the sudden relative elevation change of a large area of the earth's surface due to an earthquake. Historically, the impact of subsidence has been more

severe than uplift, especially where accompanied by flooding.

1.1.9 Ground Rupture

This category includes rupturing of the ground surface and/or the near-surface relative ground displacements that can occur during a seismic event.

1.2 Philosophy of Code

The objectives of the code are set to meet the following goals:

- Must prevent major failure and loss of life
- Building structures should be able to resist moderate earthquakes without significant damage
- Building structures should be able to resist major earthquakes without collapse (exit by occupants impossible due to failure of primary structure)

To achieve these goals the following load combinations are considered by applying the following factors to the applied loads on the structures. These factors are calibration factors to reach certain probability of failure given the uncertainty of input loads:

- $1.0 \text{ (Dead Load)} + 1.0 \text{ (Earthquake Load)}$
- $1.0 \text{ (Dead Load)} + 1.0 \text{ (Earthquake Load)} + 1.0 \text{ (Live Load)}$
for storage and assembly
- $1.0 \text{ (Dead Load)} + 1.0 \text{ (Earthquake Load)} + 0.5 \text{ (Live Load)}$
for other buildings

The following table shows a comparison between load factors for normal load conditions and earthquake load that are given in the code:

Factor	Normal Loading	Earthquake Loading
Dead Load (α_D)	1.25	1.0
Live Load (α_L)	1.5	1.0 or 0.5

Table 2: Load factors for normal load conditions and earthquake load

Earthquakes are rare events (design earthquake has 475-year return period), and there is no need for factoring up EQ load; however, other loads are represented by their expected values at time of earthquake

	Wind	Earthquake
Return Period	430 year wind (= 1.5 times 30 year wind)	475 years
Reduced Dead Load Factor	0.85	none

Table 3: Comparison of Earthquake loads with Wind Loads

1.2.1 Basic Shear Equation

Code procedure for seismic design is described by the following equations:

$$V = \frac{V_e}{R} U \quad (15)$$

$$V_e = v S I F W \quad (16)$$

where

V = base shear

V_e = equivalent lateral seismic force representing elastic response

R = force modification factor

$U = 0.6$, calibration factor

v = zonal velocity ratio

S = seismic response factor

I = importance factor for building

F = foundation factor

W = total expected weight of building

This equation approximates a linear elastic, response spectrum analysis of a single-degree-of-freedom (SDOF) structure with some modifications to account for higher modes of response for multi degree of freedom structures.

The value of each of the parameters should be calibrated to achieve a certain reliability level specified for the code. A brief description of each parameter is given in the following sections.

1.2.2 Zonal Velocity (v)

Zonal velocity v represents a base value for firm ground motion for the site and it is derived from consideration of the site's seismicity. Geological Survey of Canada has performed seismic risk calculations based on probabilistic studies by the Cornell method. The calculated values for are probabilities of annual exceedance, of 0.010, 0.005, 0.0021 and 0.001 with corresponding return periods, T_r , of 100, 200, 475 and 1000 years.

1.2.2.1 Geological Survey of Canada Data for v

These values are based on a “liquefaction potential” map, which combines information on soil behavior (i.e., liquefaction susceptibility) with regional seismicity data (i.e., the liquefaction “opportunity”) for an indication of the probability of liquefaction actually occurring.

Liquefaction potential has been estimated using a computer program developed in British Columbia called PROLIQ (Atkinson et al., 1986). This program combines the level ground liquefaction assessment developed by Seed (1979) with the probabilistic method of seismic risk assessment developed by Cornell (1968). The updated program is calibrated for local conditions and has been used in several studies to date.

The information required to produce a Level II liquefaction hazard map using PROLIQ to estimate liquefaction potential is described below.

Information required to estimate the liquefaction potential of a soil deposit includes the soil properties required for Seed's simplified, level ground, liquefaction assessment and the regional seismicity parameters used in Cornell's probabilistic seismic risk assessment. The following discussion pertains specifically to the use of PROLIQ, but generally applies to other methods of liquefaction potential estimation as well.

Seed's simplified method of liquefaction assessment is based on observations of the field performance of “level ground” soil deposits during past earthquakes. The Standard Penetration Test (SPT) resistance of a soil deposit is correlated to the minimum peak horizontal ground acceleration required to cause liquefaction for different magnitude (i.e., duration) earthquakes.

Soil properties required for the analysis include: soil strength or density, in the form of

SPT blow counts, or Cone Penetration Test (CPT) tip resistances converted to SPT values; soil types (i.e., sand, silt or silty sand, or "non-liquefiable" soil); soil depth, saturated and moist unit weight, and groundwater table elevation for calculation of overburden stress; and soil layer thickness.

The probabilistic Cornell method of seismic risk assessment models regional seismicity as a group of homogeneous seismogenic zones. Seismogenic zones may be represented either as a line representing a specific fault source, or as a geometric area when seismic activity is associated with a broad structure or fault sources that cannot be precisely identified.

For consistency in seismic hazard mapping, British Columbia seismicity should be modeled using the areal seismogenic zones incorporated in the National Building Code of Canada (NBCC, 1990). All liquefaction probability results calculated using PROLIQ should be checked against the global maximum using an earthquake magnitude versus maximum liquefaction distance curve (M-R curve).

Specific seismicity data required for the Cornell analysis includes: site location (longitude and latitude); regional attenuation function; geometry of surrounding seismic source zones; magnitude recurrence parameters for each seismic zone; and focal depth, if applicable. As discussed above, both area and line (i.e., fault) sources can be used in PROLIQ, but area source zones are more appropriate for British Columbia.

1.2.2.2 Mapping Parameters and Procedures

The Level II mapping parameter is "liquefaction probability", which is a quantitative estimate of liquefaction potential for a specific earthquake scenario, a particular ground motion return period, or due to any and all seismic events anticipated in a chosen length

of time (e.g., in a 50-year period). This section describes the basic procedures for map preparation using the computer program PROLIQ to estimate probability of liquefaction. Some of the procedures are general and will also apply to maps produced using other prediction models.

First, a specific seismic intensity or period of time for which the liquefaction potential map will apply must be selected to suit the objective of the map. For example, a map could show the probability of liquefaction occurring in a 50-year period, if that is the design life for the majority of structures in an area.

The degree of precision and accuracy in a Level II map is a function of the amount and quality of ground information, uncertainty in the seismic model, variability of the ground conditions and the level of effort put into map preparation. As a minimum, a basic Level II map can be produced by updating a Level I liquefaction susceptibility map with liquefaction potential values for each map unit. For example, the probability of liquefaction determined by PROLIQ for individual, representative geotechnical test holes can be plotted on the Level I map directly. Alternatively, a range of probabilities or “average” probability could be determined for each map unit from PROLIQ results for numerous test holes within that unit. With a sufficient number of PROLIQ results distributed fairly evenly across the map area, the analysis results could be contoured directly, with less guidance required from the Level I map unit boundaries. The latter method would produce a more precise map, though depending on the intent of the map, the level of effort required may not be worthwhile.

1.2.2.3 Liquefaction-Induced Permanent Ground Deformation

Liquefaction-induced permanent ground deformation (PGD) estimates are a further

refinement on prediction of liquefaction hazards. This is the parameter chosen for mapping referred to as "Level III" in this report. A Level III liquefaction hazard map is based on Level I and Level II maps insofar as a soil deposit must be susceptible to liquefaction and have liquefaction potential for estimates of PDG to have any validity.

A Level III liquefaction hazard map should indicate the location of liquefiable soils and the estimated magnitude of PGD for the seismic intensity specified on the map. For example, a map could be produced of predicted PGD magnitude for a peak horizontal ground acceleration with a return period of 1 in 475 years, which is the return period used as a reference level for the design of buildings in accordance with the 1990 National Building Code of Canada.

A Level III map will pertain to a specific seismic intensity, which can be represented by an earthquake event or ground motion return period. The chosen earthquake magnitude or ground motion return period will depend on the objective of the map. For example, a ground motion return period of 1 in 475 years may be desired if the map is to indicate hazards at the National Building Code of Canada ground motion return period for seismic design of most buildings. If a ground motion return period from a probabilistic seismic hazard assessment (PSHA) is used, it must be correlated to an earthquake magnitude and distance from the map area. A cumulative density function of earthquake magnitudes contributing to the return period can be output from a PSHA. The source to site (R) distance is then obtained using the PSHA attenuation function.

If the map area is small relative to the distance from the seismic source, then an average site-to-source distance may suffice for the PGD analysis. However, if the map area is large and/or the dominant seismic source relatively close, modeling results may be more reliable if the site-to-source distance is varied across the map area. The difference between estimated PGDs based on the average site-to-source distance and on actual distance(s) at the map periphery should be checked before the rest of the analysis are

commenced to determine if variation of the distance is warranted by the implied precision of the map.

1.2.3 Seismic Response Factor (S)

The seismic response factor (S) for a particular geographic location is a function of the following:

- fundamental period of the structure (T)
- velocity related seismic zone Z_v
- acceleration related seismic zone Z_a .

This factor represents the ideal elastic response of a 5% damped, single-degree-of-freedom system for the unit values of zonal velocity ratio v and weight W .

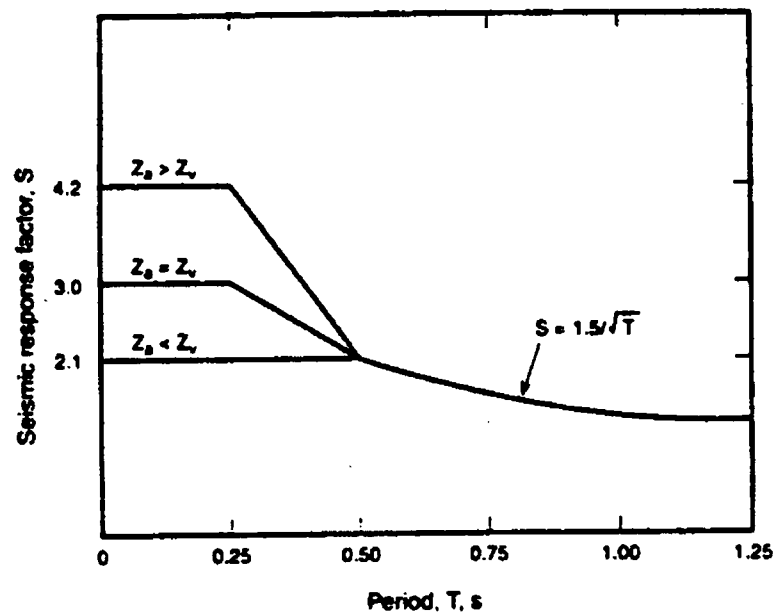


Illustration 3 - Seismic Response Factor vs Fundamental Period

1.2.4 Force Modification Factor (R)

The force modification factor, R , is assigned to different types of structural systems. It reflects design and construction experience as well as the evaluation of the performance of structures in major and moderate earthquakes. The value of R is an indication of the energy absorption capacity of structural system by damping and inelastic action through several load reversals. The value of R is considered to recognize the following characteristics:

- The capability of a structure to absorb energy within acceptable deformation and without failure.
- The existence of alternate load paths or redundancy of a structural system. This increases the energy absorption of the structure and reduces the risk of total failure should a member fail.
- A structure with an R value greater than 1.0 is considered capable of undergoing inelastic cyclic deformations.
- Buildings usually have nonstructural members that contribute to the load carrying system, which is not considered for structural analysis . Also buildings usually have higher damping values than the structural system considered for modeling.

1.2.5 Importance Factor (I)

This factor is 1.0 for normal buildings. For schools and post-disaster buildings, this value is 1.3. For structures that are designed for essential public services and post-disaster operation, such as fire and police stations, hospitals, telephone exchanges and fuel supplies, where the building operation after an earthquake is critical, the importance factor is 1.5

1.2.6 Foundation Factor (F)

The soil conditions at a site have a major influence on the type and amount of structural damage resulting from an earthquake. As the wave propagate from bedrock to the surface,

the soil may amplify the motions in selected frequency ranges around the natural frequencies of the structure. Also if a structure has some natural frequencies close to those of the layer, it may undergo even more intense shaking due to the development of a resonance between the structure and its foundation.

The NBC incorporate site effects by categorizing the wide variety of possible soil conditions into four types and assigning values to a foundation factor, F , depending on soil type and depth. The factor, F , reflects experience with these soil conditions in the field, and in an approximate way integrates the effect of possible soil amplification and soil-structure resonance into the estimation of the seismic design forces for buildings having no unusual structural characteristics. Sites underlain by deposits of very soft to soft fine-grained soils with depths greater than 15 m are assigned a foundation factor $F = 2.0$. This provision is based on the observation of large amplifications of incoming earthquake motions in the clay deposits of Mexico City during the September 1985 earthquake.

2 Reliability Based Seismic Analysis

In this section the reliability method for analyzing the seismic risk of a structure is discussed in detail. The approach begins by modeling the response of the structural system with an appropriate software. Next, performance criteria are defined for the problem. Similar to approaches discussed in Chapter 1, performance criteria can be written in terms of numerical values such as inter-story drift or specified stresses or deformations at connection locations. On the other hand, criteria can be formulated in terms of true or false values such as buckling or not of certain members. Also, performance criteria can have a wider definition including the cost associated with repairing the structure after damage.

In addition, similar to the approaches for reliability analysis mentioned in Chapter 1, the performance function may have multiple objective functions that are combined together to form a single combined performance function (refer to Multi-Criteria Optimal Design , page 2)

The next step is evaluating the response of the system under several operating or extreme conditions that the system may experience. These operating or extreme conditions are called load cases for simplicity. A load case may be given by linear combination of live and dead loads applied to a structure.

The goal is to build a database for all these load cases and the response of the system to these inputs. Obviously, the wider the range of input parameters and the more load cases recorded, the more comprehensive the resulting database. This database building can be achieved by means of appropriate structural software and automating the results.

The final step is to perform the reliability analysis using the above database based on sampling methods, which are explained more in detail in the section Calculating the Probability of Failure or Non-Performance below.

2.1 System Modeling

System models are mathematical sets of equations that describe the system's behavior.

For example the nonlinear dynamic response of a structure to seismic excitation obeys a vector differential equation as follows:

$$[M]\left\{\frac{\delta^2}{\delta t^2}\vec{u}(t)\right\} + [C]\left\{\frac{\delta}{\delta t}\vec{u}(t)\right\} + [K_L]\{\vec{u}(t)\} + \{\vec{F}_s(t)\} = -[M]\frac{\delta^2}{\delta t^2}x_g(t)\{\vec{e}\} \quad (17)$$

where

$$\left\{\frac{\delta^2}{\delta t^2}\vec{u}(t)\right\}, \left\{\frac{\delta}{\delta t}\vec{u}(t)\right\}, \{\vec{u}(t)\} = \text{respectively, vectors of mass accelerations,}$$

velocities, and displacements relative to the moving base.

[M] = global mass matrix

[C] = global damping matrix

[K_L] = linear stiffness matrix (beams and columns)

$\{\vec{F}_s(t)\}$ = nonlinear force vector such as from friction device elements

$\{\vec{e}\}$ = Vector with unit values corresponding to DOFs where inertia forces are present

$\frac{\delta^2}{\delta t^2}x_g(t)$ = ground acceleration (not a vector)

The solution of this time-dependent set of differential equations can be obtained by one of

several numerical methods for systems of differential equations. An example of "Time Step Integration Procedure" is described in the references (Filiatrault, 1988)

2.2 Defining Performance Criteria

All engineering design is performance-based, and structures have traditionally been designed for two performance levels

- a) serviceability
- b) failure

At service level loading, structures are designed to perform without damage and to maintain deflections below a level that would be troubling to occupants or supported systems. Structures are not specifically designed for failure level loads, however, they are proportioned such that under expected loading, the structure will provide an acceptable margin against the failure state. This basic approach is inherent in the strength design specifications, more recently termed Load and Resistance Factor Design (LRFD) in the United States and adopted over the last 25 years for all of the major material systems in Canada under the name of Limit State Design.

In earthquake engineering, important performance parameters include: the potential for loss of life, the cost of repairing sustained damage, and the amount of time the building is out of service for repair or, in extreme cases, replacement. While these parameters are meaningful to the public, and therefore can serve as a basis for selecting among building performance alternatives, they are not useful as a basis for design. An engineer cannot design for such performance specifications as a business interruption of two weeks or a repair cost that is 20% of replacement value. Therefore, as a prerequisite to practical implementation of performance-based engineering, corresponding relationships must be established among parameters that are meaningful to building users and design

professionals. Since a building can experience a wide spectrum of behavior states ranging from a complete absence of damage to complete collapse, establishing corresponding relationships is not a trivial task.

2.3 Calculating the Probability of Failure or Non-Performance

In the probabilistic approach, uncertainties are characterized by the probabilities associated with events. The probability of an event can be interpreted in terms of the frequency of occurrence of that event. When a large number of samples or experiments are considered, the probability of an event is defined as the ratio of the number of times the event occurs to the total number of samples or experiments. For example, the statement that the probability of the stress in a beam being less than a maximum stress means the following: From a large number of independent measurements of the stress under identical conditions, the number of times that the stress lies in the given range is equal to the probability times the total number of samples.

Probabilistic analysis is the most widely used method for characterizing uncertainty in physical systems, especially when estimates of the probability distributions of uncertain parameters are available. This approach can describe uncertainty arising from stochastic disturbances, variability conditions, and risk considerations. In this approach, the uncertainties associated with model inputs are described by probability distributions, and the objective is to estimate the output probability distributions.

On the other hand, sampling based methods do not require access to model equations or even the model code. These methods involve running a model for a set of sample points, and establishing a relationship between inputs and outputs, using the model results at the sample points. Some of the widely used sampling based sensitivity/uncertainty analysis methods are: (a) Monte Carlo and Latin Hypercube Sampling methods, (b) Fourier Amplitude Sensitivity Test (FAST) (c) reliability based methods, and (d) response surface methods.

2.3.1 Monte Carlo and Latin Hypercube Sampling Methods

Monte Carlo (MC) methods are the most widely used means for uncertainty analysis, with applications from engineering to finance. These methods involve random sampling from the distribution of inputs and successive model runs until a statistically significant distribution of outputs is obtained. They can be used to solve problems with physical probabilistic structures, such as uncertainty propagation in models or solution of stochastic equations, or can be used to solve non-probabilistic problems, such as finding the area under a curve. Monte Carlo methods are also used in the solution of problems that can be modeled by the sequence of a set of random steps that eventually converge to a desired solution. Problems such as optimization and the simulation of movement of fluid molecules are often addressed through Monte Carlo simulations.

Since these methods require a large number of samples (or model runs), their applicability is sometimes limited to simple models. In case of computationally intensive models, the time and resources required by these methods could be prohibitively expensive. A degree of computational efficiency is accomplished by the use of Modified Monte Carlo (MMC) methods that sample from the input distribution in an efficient manner, so that the number of necessary solutions compared to the simple Monte Carlo method is significantly reduced.

The Latin Hypercube Sampling is one such widely used variant of the standard Monte Carlo method. In this method, the range of probable values for each uncertain input parameter is divided into ordered segments of equal probability. Thus, the whole parameter space, consisting of all the uncertain parameters, is partitioned into cells having equal probability, and they are sampled in an “efficient” manner such that each parameter is sampled once from each of its possible segments. The advantage of this approach is that the random samples are generated from all the ranges of possible values, thus giving insight into the extremes of the probability distributions of the outputs.

2.3.2 Reliability Based Methods (FORM and SORM)

First- and second-order reliability methods (FORM and SORM, respectively) are approximation methods that estimate the probability of an event under consideration (typically termed “failure”). In addition, these methods provide the contribution to the probability of failure from each input random variable, at no additional computational effort. These methods are useful in uncertainty analysis of models with a single failure criterion.

For a model with random parameters

$$X = (X_1, X_2, \dots, X_n) \quad (18)$$

and a failure condition

$$g(X_1, X_2, \dots, X_n) < 0 \quad (19)$$

the objective of the reliability based approach is to estimate the probability of failure. In case of allowable drift exceedance, the failure condition can be defined as:

$$g(X) = C_R - C(X) < 0 \quad (20)$$

where C_R is a pre-specified maximum permissible capacity at a location of interest. If the

joint probability density function for the set \mathbf{X} is given by $f_{\mathbf{X}}$, then the probability of failure is given by the integral:

$$P_F = P\{g(\mathbf{X}) < 0\} = P\{C_R < C(\mathbf{X})\} = \int_{g(\mathbf{X}) < 0} f(\mathbf{X}) d\mathbf{X} \quad (21)$$

where the integration is carried out over the failure domain. The evaluation of this integral becomes computationally demanding as the number of random variables (the dimension of the integration) increases; in fact if \mathbf{m} is the number of function calls of the integrand per dimension, and \mathbf{n} is the dimension, the computation time grows as $\mathbf{m}^{\mathbf{n}}$.

FORM and SORM use analytical schemes to approximate the probability integral, through a series of the following steps:

- mapping the basic random variables \mathbf{X} , into a vector of standardized and uncorrelated normal variates \mathbf{U} , as $\mathbf{X}(\mathbf{U})$, resulting in in a performance function $G(\mathbf{U})$
- approximating the function $G(\mathbf{U})$ by a tangent plane (FORM) or a paraboloid surface (SORM) at a failure point \mathbf{U}^* closest to the origin
- calculating the probability of failure as a simple function of \mathbf{U}^*
- the probability of failure $P_f = \Phi(-\beta)$, where β is the closest distance between the origin and failure surface. Φ is the normal distribution function.

These methods are reported to be computationally very efficient compared to Monte Carlo methods, especially for scenarios corresponding to low probabilities of failure. Further, SORM is sometimes more accurate than FORM, but computationally more intensive, since it involves a higher order approximation of the performance function.

2.3.3 Response Surface Methods

The response surface methods involve the following steps:

- screening to determine a subset of important model input parameters
- making multiple runs of the computer model using specific values and pairings of these input parameters
- fitting a general polynomial model to the output data (for example using the method of least squares and a quadratic surface).

This fitted response-surface is then used as a replacement or proxy for the actual performance function, and all inferences related to sensitivity/uncertainty analysis for the original model are derived from this fitted model.

3 Proposed Methodology and A Numerical Example

In this section a real engineering problem is explained and its reliability is analyzed. In the first section the problem is described and the assumptions for this problem are clearly explained. Following these assumptions, the software to simulate this system is explained in detail. After modeling the system, the performance criteria are specified for the system and a database is generated based on the system characteristics and the performance criteria. The database is analyzed with the reliability analysis software called RELAN (Foschi, 2000).

3.1 Description of the Problem and Assumption

A two story steel frame is considered as an example for this approach. The frame width is 10 meters and the height of each floor is 4 meters. (Figure 4)

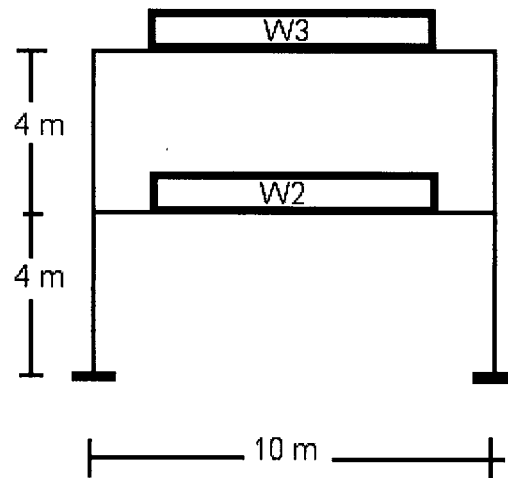


Figure 4: Outline of the steel frame

The columns are made of steel I-shape section. This section is 200 mm deep and the width of flanges is also 200 mm. The thickness of flange and the web is 10 mm.(Figure 5)

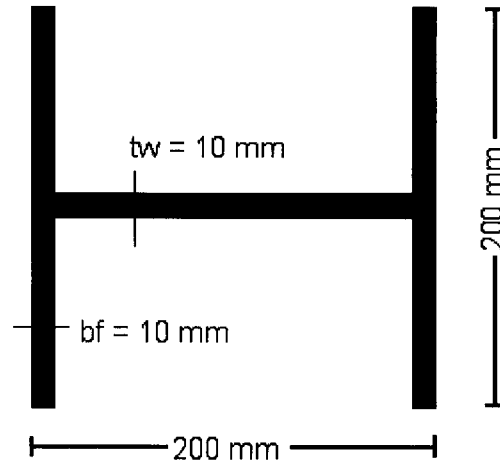


Figure 5: Cross Section of Columns

The behavior of steel is modeled with a bilinear hysteresis characteristics as follows:

Modulus of Elasticity	$E = 200 \times 10^6 \text{ [kN/m}^2\text{]}$
Yielding stress in Tension and Compression	$\sigma_s = 350 \times 10^3 \text{ [kN/m}^2\text{]}$
Parameters for stress-strain (force-displacement) relations	$\beta = 0.5$
	$\nu = 1.5$
	$\kappa = 0.01$
Parameters for unloading/reloading hysteresis rule	$\phi = 0$
	$\gamma = 0$
	$\theta = 0.75$

Table 4: Material Properties of Steel

More details of steel behavior are displayed in Figure 6 and Figure 7. For this hysteresis model, loading and unloading of steel is governed by a rule as per below:

➤ Rule 1: Elastic stage. When loading beyond elastic stage,

If $F > v f_{sy}$ go to rule 5 (if bilinear, $k = 1$) or rule 3 (if trilinear, $k > 1$);

If $F > v' f_{sy}$ go to rule 4 (if bilinear, $k' = 1$) or rule 2 (if trilinear, $k' > 1$).

➤ Rule 4: Tension yielding, $K_4 = \beta' K_s$. If unloading, go to rule 6.

➤ Rule 5: Compression yielding, $K_5 = \beta K_s$. If unloading, go to rule 7.

➤ Rule 6,7: Unloading. The unloading stiffness is determined as followings:

➤ if there is no yield in tension and compression $K_u = K_s$

➤ if there is yielding in either tension or compression

$$K_u = K_s \left(\frac{\kappa d_{sy} - \kappa' d_{sy}'}{d_m - d_m'} \right)$$

➤ If reloading over the unloading beginning point, go back to the preceding rule.

➤ If tension unloading crosses the horizontal axis and $F > \theta f_{sy}$, go to rule 9.

➤ If compression unloading crosses horizontal axis and $F < \theta f_{sy}'$, go to rule 8

➤ Rule 8: If loading over tension maximum point, go to rule 4.

If unloading, go to rule 6.

➤ Rule 9: If loading over compression maximum point, go to rule 5.

If unloading, go to rule 7.

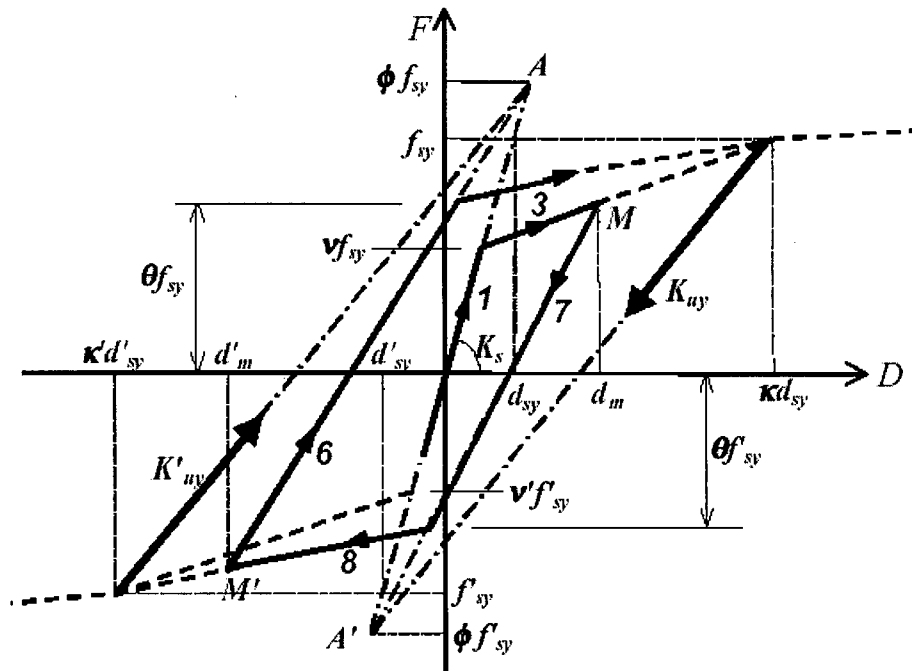


Figure 6: Steel Model Unloading before Yielding

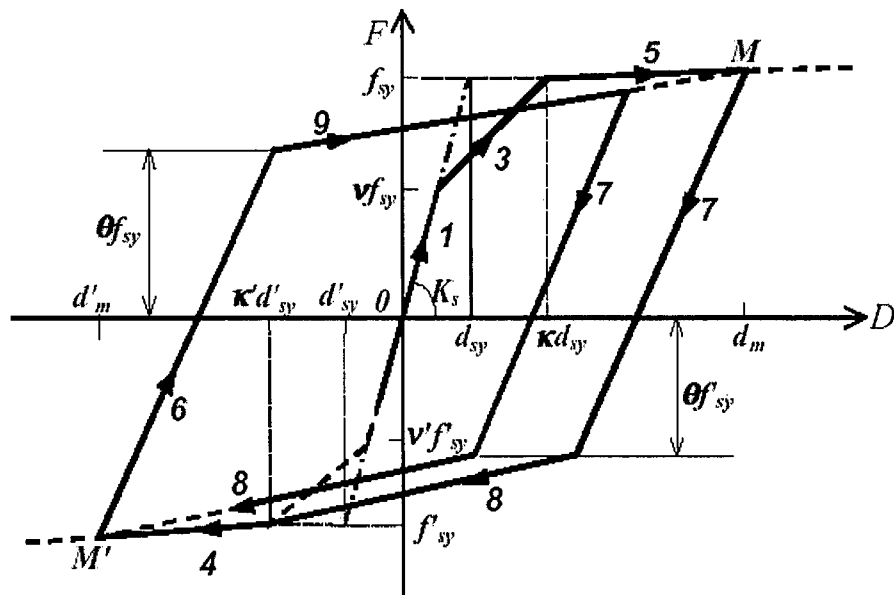


Figure 7: Steel Model Unloading after Yielding

3.2 Input Variables

In this section the input variables are explained. These are the random variables used for reliability analysis.

3.2.1 Distribution of Weights at each Level

The mass of the structure plays an important role in estimating the earthquake generated forces applied to the structure. The inertia forces applied to the structure are :

$$\{\vec{F}\} = m\{\vec{a}\} = \frac{W}{g}\{\vec{a}\} \quad (22)$$

In this example, the mass of each level is considered as a random variable. The weight of these masses is shown as W_3 and W_2 , which refer, respectively, to the weights at Level 3 (8 meters above the ground) and level 2 (4 meters above the ground). (Figure 4)

In this example, W_3 and W_2 , are independent random variables with normal distribution.

3.2.2 Distribution of Peak Ground Acceleration (PGA)

Peak Ground Acceleration or PGA is one of the most important factors in seismic response. The applied inertia forces during an earthquake are directly proportional the value and direction of peak ground acceleration (PGA). PGA is written in terms of g , where g is the gravity acceleration. The following distribution displayed in Table 8 is an estimate for actual earthquakes happening in Vancouver, Canada. This is based on earthquakes that have PGA of 0.23g with 475 years return period. Please refer to Appendix 1 for calculations of the distribution of PGA for actual earthquakes events.

3.2.3 Distribution of Earthquake Records

The type of earthquake is the next important factor in determining the seismic response of a structure . In this analysis several types of earthquakes have been considered that have

the same power spectral density function.

$$a(t) = h(t) \sum_{i=1}^N \sqrt{2S(\omega_i) \Delta \omega_i} \sin(\omega_i t + \theta_i) \quad (23)$$

θ_i is a random phase between 0 and 2π

ω_i corresponds to i^{th} frequency

$h(t)$ is an envelope modulation function introducing non-stationarity and is shown in Figure 8.

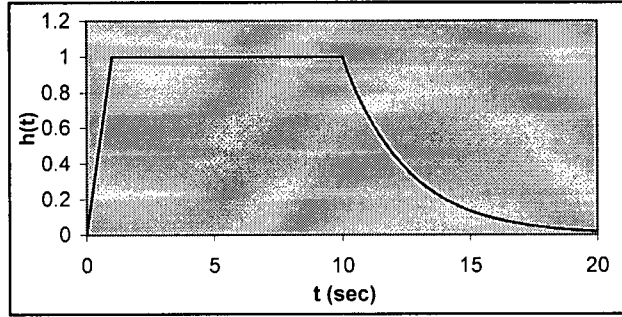


Figure 8: Shaping function $h(t)$ with an effective length of 10 sec

Five different records were simulated choosing five sequences of random phases, keeping $\Delta\omega = 2\pi$ and using $N=5000$ frequencies. These records were then modulated by the function given in Figure 8.

3.3 Performance Criteria

There are two performance criteria for this example. These are the inter-story drifts, called d_{32} and d_{21} , which, respectively, represent the drifts between levels 3 to 2 and levels 2 to 1. These G functions are described as follows assuming that the uncertainty with earthquake records can be represented with a log-normal distribution. This log-normal for the drift d_{32} has the following statistics:

$$\text{Mean} = \bar{d}_{32}$$

$$\text{Coefficient of Variation} = v_{d_{32}}$$

Databases were developed for the above quantities and are shown in Appendix 2. The same hold for the drift d_{21} .

Then:

$$G_{32} = \frac{H}{K} - d_{32} = \frac{H}{K} - \frac{\bar{d}_{32}}{\sqrt{1 + v_{d_{32}}^2}} e^{R_N \sqrt{\ln 1 + v_{d_{32}}^2}} \quad (24)$$

$$G_{21} = \frac{H}{K} - d_{21} = \frac{H}{K} - \frac{\bar{d}_{21}}{\sqrt{1 + v_{d_{21}}^2}} e^{R_N \sqrt{\ln 1 + v_{d_{21}}^2}} \quad (25)$$

For this example the value of K is assumed 200 and H is the story height, which is 4m. R_N is a random variable with a standard normal distribution, which then considers the effect of the different earthquakes.

Parameter	Type of Distribution	Parameters	
		Mean	Standard Deviation
R_N	Normal	0	1

Table 5: Implementing Distribution of Earthquake Records

3.4 Building the Database for d_{32} , v_{d32} , d_{21} , v_{d21}

Databases are generated by choosing combinations of input variables, with a certain grid. The range of input variables should be wide enough to enclose the statistical ranges used in the reliability analysis. Table 6 displays the input variables and their database values:

Random Variable Name	Grid Points
PGA [g]	0.1, 0.3, 0.5, 0.7, 1.0
W3 [kN]	0, 50, 100
W2 [kN]	0, 50, 100

Table 6: Random Variables Grid

The database contains then 45 possible combinations. Therefore, this led to 45 simulations for each earthquake record and a total of 225 (45×5) runs of the analysis program CANNY.

For each run, the maximum inter story drift for two floors were recorded in their absolute value.

3.5 Results of Reliability Analysis

This section presents the results of this reliability analysis using importance sampling

simulation and local interpolation of the database, as implemented in RELAN. This includes finding the region of interest in the database and then using reliability techniques within this region of interest. The database must, of course, have a range wide enough to include the region of interest.

Table 7 represents a possible combination of mean values of the weight of the 3rd level (W3) and 2nd level (W2).

Table 8 displays the statistics of the input variables given to the reliability analysis program.

Parameter Name	Type of Distribution	Distribution Constants	
		Mean	Standard Deviation
W ₃	Normal	20 [kN]	5 [kN]
W ₂	Normal	20 [kN]	5 [kN]

Table 7: One Possible Distribution of Weights in the Model

Parameter Name	Type of Distribution	Distribution Constants	
		Mean(g)	Standard Deviation(g)
PGA	Log normal	0.0865	0.04325

Table 8: Distribution of Peak Ground Acceleration

3.6 Results

For serviceability level, the value of H/200 was considered as the maximum allowable drift. Using the mentioned statistics, the following reliability indexes were calculated for inter story drift of two floors:

$$\beta_{32}=1.809 \text{ and } \beta_{21} = 2.544$$

The index β_{32} and β_{21} respectively represent the reliability of inter story drift of 2nd floor and 1st floor.

In Table 9, the mean value of mass for each floor was changed, while keeping the standard deviation constant. For each of these cases, the reliability indexes for the inter story drifts was calculated according to the given G functions. From this table we can see how different combination of masses can affect the safety levels. Also we may see how quickly the reliability indexes fall when the mass of second floor is increased by 10kN, while the same increase in the first floor causes a gradual change in the index levels.

In addition, by interpolating the results of Table 9, the analyst may suggest certain restriction on the floor masses to keep the reliability index reliability within a desired range.

<div>40 kN</div> <div>10 kN</div>	<div>40 kN</div> <div>20 kN</div>	<div>40 kN</div> <div>30 kN</div>	<div>40 kN</div> <div>40 kN</div>
$\beta_{32}=0.491$ $\beta_{21}=1.796$	$\beta_{32}=0.312$ $\beta_{21}=1.808$	$\beta_{32}=0.053$ $\beta_{21}=1.470$	$\beta_{32}=0.053$ $\beta_{21}=1.470$
<div>30 kN</div> <div>10 kN</div>	<div>30 kN</div> <div>20 kN</div>	<div>30 kN</div> <div>30 kN</div>	<div>30 kN</div> <div>40 kN</div>
$\beta_{32}=0.497$ $\beta_{21}=2.309$	$\beta_{32}=0.832$ $\beta_{21}=2.184$	$\beta_{32}=0.814$ $\beta_{21}=1.716$	$\beta_{32}=0.256$ $\beta_{21}=1.622$
<div>20 kN</div> <div>10 kN</div>	<div>20 kN</div> <div>20 kN</div>	<div>20 kN</div> <div>30 kN</div>	<div>20 kN</div> <div>40 kN</div>
$\beta_{32}=1.738$ $\beta_{21}=3.254$	$\beta_{32}=1.809$ $\beta_{21}=2.544$	$\beta_{32}=1.799$ $\beta_{21}=2.499$	$\beta_{32}=1.693$ $\beta_{21}=2.378$
<div>10 kN</div> <div>10 kN</div>	<div>10 kN</div> <div>20 kN</div>	<div>10 kN</div> <div>30 kN</div>	<div>10 kN</div> <div>40 kN</div>
$\beta_{32}=3.681$ $\beta_{21}=4.507$	$\beta_{32}=4.001$ $\beta_{21}=3.392$	$\beta_{32}=3.445$ $\beta_{21}=4.823$	$\beta_{32}=3.047$ $\beta_{21}=4.209$

Table 9: Reliability index values β for inter story drift $> H/200$ for the 2nd floor and the 1st floor

4 Conclusion

In this thesis an analytical approach for evaluating the seismic risk of a structure was discussed. A building was modeled by advanced structural analysis software to model the non linear dynamic response of the system. Then, the system was excited by a mother earthquake with a specified peak ground acceleration (PGA). In each run, the PGA of mother earthquake was scaled by certain value and the maximum values of inter story drifts were recorded. The mass of each floor was also changed to consider the effect of change mass at each level.

In order to consider the effect of different types of earthquakes, the structure was analyzed with few more earthquakes which have the same power spectral density and the same procedure was repeated for each quakes. This led to building a database that shows how the structure response to different ranges of earthquake records, floors mass distribution, and PGA.

For the reliability analysis, the statistics of each variable was given to the program (RELAN). However, in order to take into account different earthquake records, we assumed that the absolute value of inter story drifts follow a log normal distribution. This led to introducing a random variable with log normal distribution that has the same mean and standard deviation of inter story drifts for different earthquake records.

After the database was built, performance functions were defined for serviceability level. Then reliability indexes were calculated using importance sampling simulation and local interpolation of the databases. These indexes were calculated and compared for different mean values of masses at floor levels.

In this approach, the performance functions were defined after the database was built.

This gives the flexibility calculate the reliability index for some other performance criteria. As an example, we can calculate the reliability associated with failure (or maximum inter story drift of $H/50$) with the same database.

5 Future Research

In this thesis, the geometrical parameters of the structure and material properties were assumed to be constant. In order to have a better estimate of reliability, the statistics of strength of material and section dimensions can also be considered.

The database does not have to be necessarily obtained by the given computer program. We may use different computer software to build the database and then compare the reliability results. If it is possible, we may also build the database using lab tests, where there is a closer match to reality.

Comparing the results of different software with lab tests can also lead to benchmark how reliable different analysis software is. In this thesis, we assumed that the results of the analysis program are perfect. However, we may also consider the uncertainty with each software and implement that into the final results.

Appendix 1 – PGA Distribution of Individual Earthquakes

It is given that earthquakes in Vancouver, Canada have peak ground acceleration of 0.23g with a return period of 475 years. If the seismicity is such that the class of earthquakes have a mean arrival rate of $\alpha = 0.2$ (or an average of one every five years) we can write:

$$P_{\text{annual}}(a > a_D) = 1 - e^{-\alpha P_e(a > a_D)} = \frac{1}{475}$$

or

$$\alpha P_e(a > a_D) = -\ln\left(\frac{474}{475}\right)$$

$$P_e(a > a_D) = \frac{-1}{\alpha} \ln\left(\frac{474}{475}\right) = \frac{-1}{0.2} \ln\left(\frac{474}{475}\right) = 0.01054$$

Therefore if an earthquake happens, there is 0.01054 probability that it will have a PGA higher than 0.23g. This corresponds to reliability index or β of 2.3065 with normal distribution.

If the peak ground acceleration has Log normal distribution, with coefficient of variation of $v = 0.5$, we can write:

$$0.23 = \frac{a}{\sqrt{1+v^2}} e^{2.3065 \sqrt{\ln(1+v^2)}}$$

Therefore the average PGA of an earthquake with Log normal distribution and coefficient of variation 0.5 will be:

$$a = \frac{0.23 \sqrt{1+0.5^2}}{e^{2.3065 \sqrt{\ln(1+0.5^2)}}} = 0.0865$$

Appendix 2 – Results of Nonlinear Dynamic Analysis Software

Following is the result of 5 nonlinear dynamic analyses on the given structure. The maximum drifts (d32 and d21) are averaged for 5 different earthquakes, which share the same power spectral density. The other columns are the corresponding standard deviations and coefficient of variations for each variable combination.

simulation #	PGA(g)	W3(kN)	W2(kN)	Average		Standarad_dev		Average		Standarad_dev	
				d32max(m)	d32max(m)	v32	d32max(m)	d21max(m)	d21_max	v21	d21_max
1	1	100	100	0.25	0.03	0.12	0.12	0.01	0.1		
2	0.7	100	100	0.18	0.02	0.11	0.09	0.01	0.11		
3	0.5	100	100	0.14	0.01	0.1	0.07	0.01	0.1		
4	0.3	100	100	0.09	0.01	0.1	0.04	0	0.1		
5	0.1	100	100	0.03	0	0.12	0.01	0	0.11		
6	1	100	50	0.23	0.03	0.13	0.11	0.01	0.13		
7	0.7	100	50	0.17	0.02	0.1	0.08	0.01	0.11		
8	0.5	100	50	0.12	0.01	0.06	0.06	0	0.06		
9	0.3	100	50	0.08	0.01	0.08	0.04	0	0.08		
10	0.1	100	50	0.03	0	0.1	0.01	0	0.1		
11	1	100	0	0.22	0.03	0.12	0.1	0.01	0.13		
12	0.7	100	0	0.16	0.01	0.09	0.07	0.01	0.09		
13	0.5	100	0	0.12	0.01	0.08	0.06	0.01	0.09		
14	0.3	100	0	0.08	0.01	0.08	0.04	0	0.08		
15	0.1	100	0	0.03	0	0.1	0.01	0	0.1		
16	1	50	100	0.24	0.05	0.22	0.12	0.03	0.21		
17	0.7	50	100	0.17	0.04	0.2	0.09	0.02	0.18		
18	0.5	50	100	0.13	0.02	0.19	0.07	0.01	0.2		
19	0.3	50	100	0.08	0.01	0.17	0.04	0.01	0.18		
20	0.1	50	100	0.03	0.01	0.23	0.01	0	0.23		
21	1	50	50	0.19	0.02	0.13	0.09	0.01	0.13		
22	0.7	50	50	0.14	0.02	0.17	0.07	0.01	0.18		
23	0.5	50	50	0.11	0.02	0.19	0.05	0.01	0.19		
24	0.3	50	50	0.07	0.01	0.21	0.03	0.01	0.21		
25	0.1	50	50	0.02	0.01	0.26	0.01	0	0.26		
26	1	50	0	0.15	0.03	0.19	0.07	0.01	0.21		
27	0.7	50	0	0.11	0.02	0.19	0.05	0.01	0.2		
28	0.5	50	0	0.08	0.02	0.19	0.04	0.01	0.2		
29	0.3	50	0	0.06	0.01	0.23	0.03	0.01	0.23		
30	0.1	50	0	0.02	0.01	0.29	0.01	0	0.29		
31	1	0	100	0.09	0.02	0.18	0.07	0.01	0.17		
32	0.7	0	100	0.06	0.01	0.12	0.04	0.01	0.12		
33	0.5	0	100	0.05	0.01	0.12	0.03	0	0.12		
34	0.3	0	100	0.03	0	0.15	0.02	0	0.15		
35	0.1	0	100	0.01	0	0.14	0.01	0	0.18		
36	1	0	50	0.04	0.01	0.25	0.03	0.01	0.28		
37	0.7	0	50	0.03	0.01	0.24	0.02	0.01	0.24		
38	0.5	0	50	0.02	0	0.21	0.01	0	0.22		
39	0.3	0	50	0.01	0	0.24	0.01	0	0.24		
40	0.1	0	50	0	0	0.24	0	0	0.24		
41	1	0	0	0	0	0	0	0	0		
42	0.7	0	0	0	0	0	0	0	0		
43	0.5	0	0	0	0	0	0	0	0		
44	0.3	0	0	0	0	0	0	0	0		
45	0.1	0	0	0	0	0	0	0	0		

Appendix 3 – Applied Earthquakes

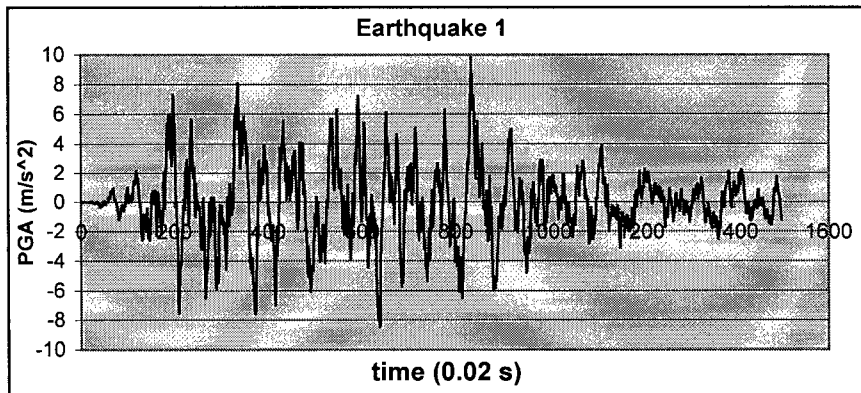


Figure 9: Earthquake 1 – time history

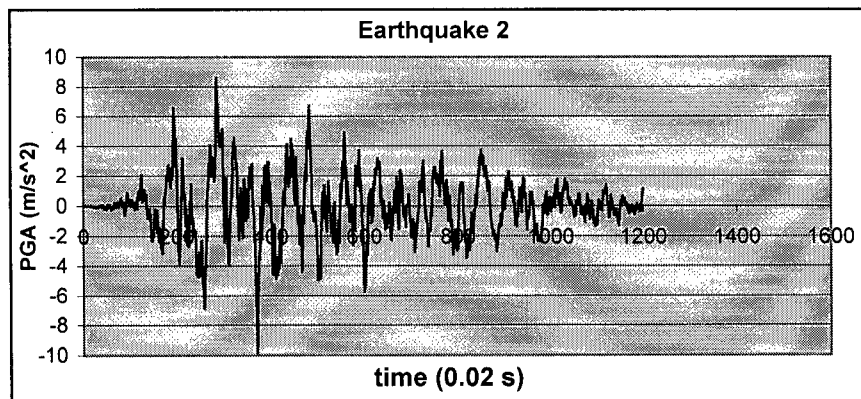


Figure 10: Earthquake 2 – time history

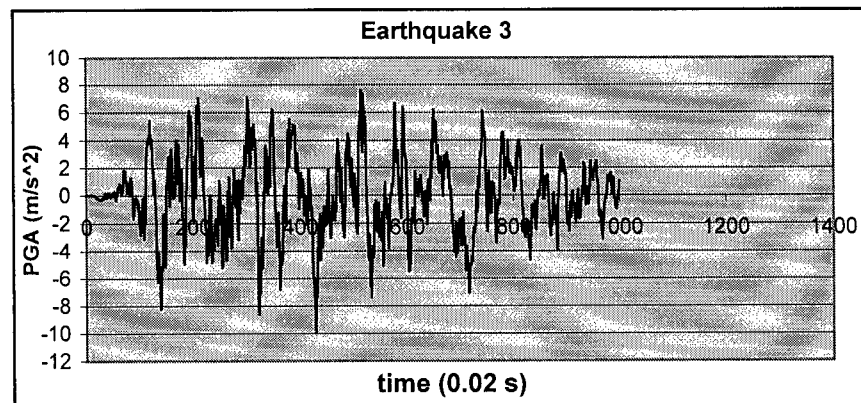


Figure 11: Earthquake 3 – time history

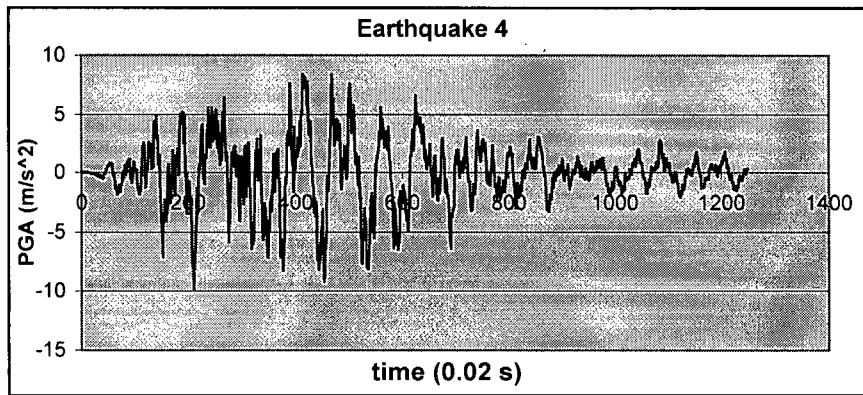


Figure 12: Earthquake 4 – time history

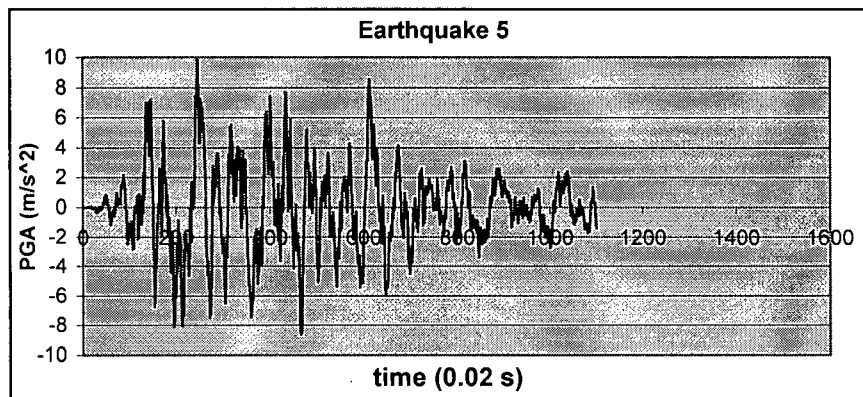


Figure 13: Earthquake 5 - time history

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