In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the head of my department or by his or her representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Department of Civil Engineering
The University of British Columbia
Vancouver, Canada

Date 27 August 2003
Abstract

A method for optimizing the operation of a system of two hydraulically separate reservoirs serving the same demand area for hydropower production is described. The reservoir system is assumed to be operated, and import and export decisions made, so as to maximize the value of energy produced while considering the value of water stored in the reservoirs at the end of the model time horizon. The optimization considers uncertain reservoir inflows, energy demands, and electricity prices, and is subject to physical and operational constraints. The proposed method consists of two cascaded models.

A longer-term monthly model based upon dynamic programming and linear programming is used to estimate the value of water stored in each reservoir as a function of the storage in both reservoirs, as well as the marginal values of water storage in the two reservoirs. Linear programming is used to evaluate the recursive equation in the dynamic program by making tradeoffs between releasing water, making energy trades, and keeping water in storage for the next month. The monthly energy value functions are input to the shorter-term model, which is based upon stochastic linear programming with recourse.

The shorter-term model allows for the planning of operations and the calculation of marginal water values over periods shorter than one month. The time horizon in the shorter-term model is divided into time steps that may be of variable duration. Uncertainty in the model is handled through a scenario tree. Scenarios describe the values assumed by the inflows, demands, and prices in each time step. Sub-periods allow for the consideration of on- and off-peak periods.

Application of the proposed model is made to a system based roughly on the two main river systems in the BC Hydro system—the Peace and Columbia. It is found that the marginal value of storage in the Columbia Reservoir is generally dependent upon the storage in both the Columbia and Peace Reservoirs, and vice versa. Regions of storage existed in which the marginal energy value in one reservoir was independent of storage in the second, although no general rules for identifying these regions were found.
# Table of Contents

Abstract .............................................................................................................................. ii  
Table of Contents ............................................................................................................. iii  
List of Tables .................................................................................................................... iv  
List of Figures ................................................................................................................... vi  
1 Introduction .................................................................................................................. 1  
2 Literature Review ........................................................................................................ 7  
  2.1 Introduction ............................................................................................................. 7  
  2.2 Techniques ............................................................................................................. 7  
     2.2.1 Dynamic Programming .................................................................................... 8  
     2.2.2 Linear Programming ....................................................................................... 17  
     2.2.3 Other Methods ............................................................................................... 23  
     2.2.4 Simulation ...................................................................................................... 28  
  2.3 Summary and Conclusions ...................................................................................... 29  
3 Stochastic Dynamic Programming and Linear Programming Based Model ........ 31  
  3.1 Introduction .......................................................................................................... 31  
  3.2 DP and LP Based Model Overview ........................................................................ 32  
     3.2.1 Terminology .................................................................................................... 32  
     3.2.2 Model Overview ............................................................................................ 34  
  3.3 Model Details ........................................................................................................ 37  
     3.3.1 Dynamic Programming Model ...................................................................... 37  
     3.3.2 Linear Programming Model .......................................................................... 46  
  3.4 Case Study ............................................................................................................. 55  
     3.4.1 Description ..................................................................................................... 55  
     3.4.2 Data ............................................................................................................... 56  
     3.4.3 Results .......................................................................................................... 61  
  3.5 Summary .............................................................................................................. 144  
4 Short-term Marginal Value Model .......................................................................... 150  
  4.1 Introduction .......................................................................................................... 150  
  4.2 STMVM Overview ............................................................................................... 151  
     4.2.1 Stochastic Linear Programming with Recourse ............................................. 153  
     4.2.2 Choice of Here-and-Now Variables ............................................................... 156  
  4.3 Model Details........................................................................................................ 157  
     4.3.1 Mathematical Formulation .......................................................................... 157  
     4.3.2 Solution Methodology ................................................................................... 166  
  4.4 Case Study .......................................................................................................... 167  
     4.4.1 Overview ....................................................................................................... 167  
     4.4.2 Scenario Definitions ...................................................................................... 168  
     4.4.3 Objective Function Values .......................................................................... 179  
     4.4.4 Peace Marginal Energy Values ..................................................................... 197  
     4.4.5 Columbia Marginal Energy Values ............................................................... 216  
     4.4.6 Summary ...................................................................................................... 235  
5 Summary and Conclusions .................................................................................... 243  
6 Literature Cited ........................................................................................................... 255
# List of Tables

Table 3-1: Average Monthly Inflows .................................................. 56
Table 3-2: Average Monthly Market Electricity Prices ............................. 57
Table 3-3: Average Monthly Electricity Demand .................................... 57
Table 3-4: Monthly Duration of HLH and LLH Periods ............................. 58
Table 3-5: Monthly Maximum Import and Export Transmission Limits .......... 58
Table 3-6: Monthly Minimum Import and Export Transmission Limits .......... 59
Table 3-7: Plant Related Limits ............................................................. 59
Table 3-8: Monthly Minimum Plant Discharge ....................................... 60
Table 3-9: Slopes of HK-Storage Function for Peace Plant ........................ 60
Table 3-10: Slopes of HK-Storage Function for Columbia Plant .................. 61
Table 3-11: Storage Intercepts for HK-Storage Functions ........................ 61
Table 3-12: Miscellaneous Data ........................................................... 61
Table 3-13: Minimum Plant Turbine Release Violation Penalties ................. 61
Table 3-14: Minimum and Maximum Monthly Base Case Columbia Marginal Energy Values ................................................................. 69
Table 3-15: Minimum and Maximum Peace Marginal Energy Values ............ 70
Table 3-16: Definition of Five-Scenario Cases ....................................... 79
Table 3-17: Unit Normal Function Values for Five-Scenario Cases .............. 80
Table 3-18: Minimum Peace Marginal Energy Values for Demands Only Scenarios ................................................................. 81
Table 3-19: Maximum Peace Marginal Energy Values for Demands Only Scenarios ................................................................. 82
Table 3-20: Minimum and Maximum Columbia Marginal Energy Values for Demands Only Scenarios ................................................................. 83
Table 3-21: Minimum and Maximum Peace Marginal Energy Values for Inflows Only Scenarios ................................................................. 85
Table 3-22: Minimum and Maximum Columbia Marginal Energy Values for Inflows Only Scenarios ................................................................. 86
Table 3-23: Minimum and Maximum Peace Marginal Energy Values for Perfect Positive Correlation Scenarios .............................................. 88
Table 3-24: Minimum and Maximum Columbia Marginal Energy Values for Perfect Positive Correlation Scenarios .............................................. 88
Table 3-25: Minimum and Maximum Peace Marginal Energy Values for Perfect Correlation—Positive for Demands and Inflows and Negative for Prices ............. 90
Table 3-26: Minimum and Maximum Columbia Marginal Energy Values for Perfect Correlation—Positive for Demands and Inflows and Negative for Prices ............. 90
Table 3-27: Minimum and Maximum Peace Marginal Energy Values for Perfect Correlation—Positive for Demands and Prices and Negative for Inflows ......................... 92
Table 3-28: Minimum and Maximum Columbia Marginal Energy Values for Perfect Correlation—Positive for Demands and Prices and Negative for Inflows ......................... 93
Table 3-29: Minimum and Maximum Peace Marginal Energy Values for Perfect Correlation—Positive for Inflows and Prices and Negative for Demands ......................... 94
Table 3-30: Minimum and Maximum Columbia Marginal Energy Values for Perfect Correlation—Positive for Inflows and Prices and Negative for Demands ......................... 94
Table 3-31: Modified Monthly Minimum Plant Discharge .......................... 136
Table 4-1: Length of Sub-Periods for January Studies ...................................................... 168
Table 4-2: Limits for January Studies .................................................................................. 169
Table 4-3: January Base Case Scenario-Dependent Parameters ........................................ 169
Table 4-4: Scenario-Dependent Demands for January Studies ........................................... 170
Table 4-5: Scenario-Dependent Inflows for January Studies ............................................. 170
Table 4-6: Scenario-Dependent Prices for January Studies .............................................. 171
Table 4-7: Length of Sub-Periods for May Studies ............................................................... 172
Table 4-8: Limits for May Studies ....................................................................................... 172
Table 4-9: May Base Case Scenario-Dependent Parameters ............................................. 173
Table 4-10: Scenario-Dependent Demands for May Studies ............................................ 173
Table 4-11: Scenario-Dependent Inflows for May Studies ................................................ 174
Table 4-12: Scenario-Dependent Prices for May Studies .................................................. 175
Table 4-13: Length of Sub-Periods for September Studies ................................................ 175
Table 4-14: Limits for September Studies ......................................................................... 176
Table 4-15: September Base Case Scenario-Dependent Parameters ................................ 176
Table 4-16: Scenario-Dependent Demands for September Studies .................................... 177
Table 4-17: Scenario-Dependent Inflows for September Studies ....................................... 177
Table 4-18: Scenario-Dependent Prices for September Studies ........................................ 178
List of Figures

Figure 3-1: Base Case January Storage Value Function ................................................. 63
Figure 3-2: Base Case April Storage Value Function ...................................................... 63
Figure 3-3: Base Case June Storage Value Function ...................................................... 64
Figure 3-4: Base Case October Storage Value Function .................................................. 64
Figure 3-5: Storage Value Function Slices for Columbia Reservoir Empty ............... 65
Figure 3-6: Storage Value Function Slices for Columbia Reservoir 50% Full .......... 66
Figure 3-7: Storage Value Function Slices for Columbia Reservoir Full .................. 66
Figure 3-8: Storage Value Function Slices for Peace Reservoir Empty ................. 67
Figure 3-9: Storage Value Function Slices for Peace Reservoir 50% Full ............. 67
Figure 3-10: Storage Value Function Slices for Peace Reservoir Full ................. 68
Figure 3-11: Peace Marginal Energy Values for Columbia Reservoir Empty ........ 71
Figure 3-12: Peace Marginal Energy Values for Columbia Reservoir 50% Full .... 72
Figure 3-13: Peace Marginal Energy Values for Columbia Reservoir Full ............. 72
Figure 3-14: Columbia Marginal Energy Values for Peace Reservoir Empty ......... 73
Figure 3-15: Columbia Marginal Energy Values for Peace Reservoir 50% Full .... 74
Figure 3-16: Columbia Marginal Energy Values for Peace Reservoir Full ......... 74
Figure 3-17: May Marginal Peace Energy Value Functions ........................................... 75
Figure 3-18: May Marginal Columbia Energy Value Functions .................................. 76
Figure 3-19: November Marginal Peace Energy Value Functions ....................... 77
Figure 3-20: November Marginal Peace Energy Value Functions—Detail ............. 77
Figure 3-21: November Marginal Columbia Energy Value Functions ................. 78
Figure 3-22: November Marginal Columbia Energy Value Functions—Detail ......... 78
Figure 3-23: Minimum Peace Marginal Energy Values for Demands Only Scenarios.... 81
Figure 3-24: Maximum Peace Marginal Energy Values for Demands Only Scenarios... 82
Figure 3-25: Minimum Columbia Marginal Energy Values for Demands Only Scenarios .... 83
Figure 3-26: Maximum Columbia Marginal Energy Values for Demands Only Scenarios 84
Figure 3-27: Maximum Columbia Marginal Energy Values for Coefficient of Variation of 0.40 ................................................................. 96
Figure 3-28: Maximum Columbia Marginal Energy Values for Coefficient of Variation of 0.25 ................................................................. 97
Figure 3-29: Maximum Columbia Marginal Energy Values for Coefficient of Variation of 0.10 ................................................................. 97
Figure 3-30: Minimum Columbia Marginal Energy Values for Coefficient of Variation of 0.40 ................................................................. 98
Figure 3-31: Minimum Columbia Marginal Energy Values for Coefficient of Variation of 0.25 ................................................................. 99
Figure 3-32: Minimum Columbia Marginal Energy Values for Coefficient of Variation of 0.10 ................................................................. 99
Figure 3-33: Maximum Peace Marginal Energy Values for Coefficient of Variation of 0.40 ................................................................. 100
Figure 3-34: Maximum Peace Marginal Energy Values for Coefficient of Variation of 0.25 ................................................................. 100
Figure 3-35: Maximum Peace Marginal Energy Values for Coefficient of Variation of 0.10 ................................................................. 101
Figure 3-36: Minimum Peace Marginal Energy Values for Coefficient of Variation of 0.40 ................................................................. 102
Figure 3-37: Minimum Peace Marginal Energy Values for Coefficient of Variation of 0.25 ................................................................. 102
Figure 3-38: Maximum Peace Marginal Energy Values for Coefficient of Variation of 0.10 ................................................................. 103
Figure 3-39: Maximum Columbia Marginal Energy Values for Demands Only Scenarios ................................................................. 105
Figure 3-40: Maximum Peace Marginal Energy Values for Demands Only Scenarios ................................................................. 106
Figure 3-41: Minimum Columbia Marginal Energy Values for Demands Only Scenarios ................................................................. 107
Figure 3-42: Minimum Peace Marginal Energy Values for Demands Only Scenarios ................................................................. 107
Figure 3-43: Maximum Columbia Marginal Energy Values for Inflows Only Scenarios ................................................................. 109
Figure 3-44: Maximum Peace Marginal Energy Values for Inflows Only Scenarios ................................................................. 109
Figure 3-45: Minimum Columbia Marginal Energy Values for Inflows Only Scenarios ................................................................. 110
Figure 3-46: Minimum Peace Marginal Energy Values for Inflows Only Scenarios ................................................................. 110
Figure 3-47: Maximum Columbia Marginal Energy Values for Prices Only Scenarios ................................................................. 111
Figure 3-48: Maximum Peace Marginal Energy Values for Prices Only Scenarios ................................................................. 112
Figure 3-49: Minimum Columbia Marginal Energy Values for Prices Only Scenarios ................................................................. 113
Figure 3-50: Minimum Peace Marginal Energy Values for Prices Only Scenarios ................................................................. 114
Figure 3-51: Maximum Columbia Marginal Energy Values for Perfectly Positively Correlated Demand, Inflow, and Price Scenarios ................................................................. 115
Figure 3-52: Maximum Peace Marginal Energy Values for Perfectly Positively Correlated Demand, Inflow, and Price Scenarios ................................................................. 115
Figure 3-53: Minimum Columbia Marginal Energy Values for Perfectly Positively Correlated Demand, Inflow, and Price Scenarios ................................................................. 116
Figure 3-54: Minimum Peace Marginal Energy Values for Perfectly Positively Correlated Demand, Inflow, and Price Scenarios ................................................................. 117
Figure 3-55: Maximum Columbia Marginal Energy Values for Demand and Inflow Perfectly Correlated and Price Perfectly Negatively Correlated Scenarios ................................................................. 118
Figure 3-56: Maximum Peace Marginal Energy Values for Demand and Inflow Perfectly Correlated and Price Perfectly Negatively Correlated Scenarios ................................................................. 119
Figure 3-57: Minimum Columbia Marginal Energy Values for Demand and Inflow Perfectly Correlated and Price Perfectly Negatively Correlated Scenarios ................................................................. 120
Figure 3-58: Minimum Peace Marginal Energy Values for Demand and Inflow Perfectly Correlated and Price Perfectly Negatively Correlated Scenarios ................................................................. 121
Figure 3-59: Maximum Columbia Marginal Energy Values for Demand and Price Perfectly Correlated and Inflow Perfectly Negatively Correlated Scenarios ................................................................. 122
Figure 3-60: Maximum Peace Marginal Energy Values for Demand and Price Perfectly Correlated and Inflow Perfectly Negatively Correlated Scenarios ................................................................. 123
Figure 3-61: Minimum Columbia Marginal Energy Values for Demand and Price Perfectly Correlated and Inflow Perfectly Negatively Correlated Scenarios ................................................................. 123
Figure 3-62: Minimum Peace Marginal Energy Values for Demand and Price Perfectly Correlated and Inflow Perfectly Negatively Correlated Scenarios ........................................... 124
Figure 3-63: Maximum Columbia Marginal Energy Values for Inflow and Price Perfectly Correlated and Demand Perfectly Negatively Correlated Scenarios ...................................... 125
Figure 3-64: Maximum Peace Marginal Energy Values for Inflow and Price Perfectly Correlated and Demand Perfectly Negatively Correlated Scenarios ...................................... 126
Figure 3-65: Minimum Columbia Marginal Energy Values for Inflow and Price Perfectly Correlated and Demand Perfectly Negatively Correlated Scenarios ...................................... 126
Figure 3-66: Minimum Peace Marginal Energy Values for Inflow and Price Perfectly Correlated and Demand Perfectly Negatively Correlated Scenarios ...................................... 127
Figure 3-67: Maximum Columbia Marginal Energy Values for Coefficient of Variation of 0.25 .......................................................................................................................... 129
Figure 3-68: Maximum Columbia Marginal Energy Values for Coefficient of Variation of 0.10 .......................................................................................................................... 130
Figure 3-69: Minimum Columbia Marginal Energy Values for Coefficient of Variation of 0.25 .......................................................................................................................... 131
Figure 3-70: Minimum Columbia Marginal Energy Values for Coefficient of Variation of 0.10 .......................................................................................................................... 131
Figure 3-71: Maximum Peace Marginal Energy Values for Coefficient of Variation of 0.25 .......................................................................................................................... 133
Figure 3-72: Maximum Peace Marginal Energy Values for Coefficient of Variation of 0.10 .......................................................................................................................... 133
Figure 3-73: Minimum Peace Marginal Energy Values for Coefficient of Variation of 0.25 .......................................................................................................................... 134
Figure 3-74: Minimum Peace Marginal Energy Values for Coefficient of Variation of 0.10 .......................................................................................................................... 135
Figure 3-75: Effect of Peace Ice Restrictions on Minimum Peace Marginal Energy Values .......................................................................................................................... 137
Figure 3-76: Effect of Peace Ice Restrictions on Maximum Peace Marginal Energy Values .......................................................................................................................... 138
Figure 3-77: Effect of Peace Plant Ice Restrictions on Minimum Columbia Marginal Energy Values .......................................................................................................................... 139
Figure 3-78: Effect of Peace Ice Restrictions on Maximum Columbia Marginal Energy Values .......................................................................................................................... 139
Figure 3-79: Effect of Peace Ice Restrictions on May Peace Marginal Energy Values .......................................................................................................................... 140
Figure 3-80: Effect of Peace Ice Restrictions on November Peace Marginal Energy Values .......................................................................................................................... 141
Figure 3-81: Effect of Peace Plant Restrictions on November Peace Marginal Energy Values—Detail .......................................................................................................................... 142
Figure 3-82: Effect of Peace Ice Restrictions on May Columbia Marginal Energy Values .......................................................................................................................... 142
Figure 3-83: Effect of Peace Ice Restrictions on November Columbia Marginal Energy Values .......................................................................................................................... 143
Figure 4-1: Two-Stage Recourse Problem .......................................................................................................................... 154
Figure 4-2: Multiple-Stage Recourse Problem .......................................................................................................................... 155
Figure 4-3: Objective Function Values for One-Scenario Base Cases .......................................................................................................................... 179
Figure 4-23: Difference in Peace Marginal Energy Values Between Base Case and Scenario-Dependent Case for No Non-Anticipative Constraints Assumption in May

Figure 4-24: Difference in Peace Marginal Energy Values Between Base Case and Scenario-Dependent Case for Non-Anticipative Turbine Release Constraints Assumption in May

Figure 4-25: Difference in Peace Marginal Energy Values Between Base Case and Scenario-Dependent Case for Non-Anticipative Ending Storage Volume Constraints Assumption in May

Figure 4-26: Difference in Peace Marginal Energy Values Between Base Case and Scenario-Dependent Case for No Non-Anticipative Constraints Assumption in September

Figure 4-27: Difference in Peace Marginal Energy Values Between Base Case and Scenario-Dependent Case for Non-Anticipative Turbine Release Constraints Assumption in September

Figure 4-28: Difference in Peace Marginal Energy Values Between Base Case and Scenario-Dependent Case for Non-Anticipative Ending Storage Volume Constraints Assumption in September

Figure 4-29: Difference in Peace Marginal Energy Value Under Non-Anticipative Turbine Release Assumption for January

Figure 4-30: Difference in Peace Marginal Energy Value Under Non-Anticipative Ending Storage Volume Assumption for January

Figure 4-31: Difference in Peace Marginal Energy Value Under Non-Anticipative Turbine Release Assumption for May

Figure 4-32: Difference in Peace Marginal Energy Value Under Non-Anticipative Ending Storage Volume Assumption for May

Figure 4-33: Difference in Peace Marginal Energy Value Under Non-Anticipative Turbine Release Assumption for September

Figure 4-34: Difference in Peace Marginal Energy Value Under Non-Anticipative Ending Storage Volume Assumption for September

Figure 4-35: Columbia Marginal Energy Value for One-Scenario Base Cases

Figure 4-36: Difference in Columbia Marginal Energy Values Between Base Case and Scenario-Dependent Case for No Non-Anticipative Constraints Assumption in January

Figure 4-37: Difference in Columbia Marginal Energy Values Between Base Case and Scenario-Dependent Case for Non-Anticipative Turbine Release Constraints Assumption in January

Figure 4-38: Difference in Columbia Marginal Energy Values Between Base Case and Scenario-Dependent Case for Non-Anticipative Ending Storage Volume Constraints Assumption in January

Figure 4-39: Difference in Columbia Marginal Energy Values Between Base Case and Scenario-Dependent Case for No Non-Anticipative Constraints Assumption in May

Figure 4-40: Difference in Columbia Marginal Energy Values Between Base Case and Scenario-Dependent Case for Non-Anticipative Turbine Release Constraints Assumption in May
Figure 4-41: Difference in Columbia Marginal Energy Values Between Base Case and Scenario-Dependent Case for Non-Anticipative Ending Storage Volume Constraints Assumption in May .......................................................... 224

Figure 4-42: Difference in Columbia Marginal Energy Values Between Base Case and Scenario-Dependent Case for No Non-Anticipative Constraints Assumption in September ......................................................... 225

Figure 4-43: Difference in Columbia Marginal Energy Values Between Base Case and Scenario-Dependent Case for Non-Anticipative Turbine Release Constraints Assumption in September ......................................................... 226

Figure 4-44: Difference in Columbia Marginal Energy Values Between Base Case and Scenario-Dependent Case for Non-Anticipative Ending Storage Volume Constraints Assumption in September ......................................................... 227

Figure 4-45: Difference in Columbia Marginal Energy Value Under Non-Anticipative Turbine Release Assumption for January .......................................................... 229

Figure 4-46: Difference in Columbia Marginal Energy Value Under Non-Anticipative Ending Storage Volume Assumption for January .......................................................... 230

Figure 4-47: Difference in Columbia Marginal Energy Value Under Non-Anticipative Turbine Release Assumption for May .......................................................... 231

Figure 4-48: Difference in Columbia Marginal Energy Value Under Non-Anticipative Ending Storage Volume Assumption for May .......................................................... 232

Figure 4-49: Difference in Columbia Marginal Energy Value Under Non-Anticipative Turbine Release Assumption for September ......................................................... 233

Figure 4-50: Difference in Columbia Marginal Energy Value Under Non-Anticipative Ending Storage Volume Assumption for September ......................................................... 234
1 Introduction

The problem of operating a reservoir, or system of reservoirs, in order to achieve some set of specified objectives is a fascinating and difficult problem. In order to appreciate some of the complexities, first consider the case of a single-purpose reservoir with natural inflows that is operated independently of any other reservoirs. In determining how to operate such a reservoir, decisions need to be taken as to how water should be stored and released in order to satisfy the stated objective. The main complication is that inflows to the reservoir are inherently uncertain. Thus, the decisions as to how much water to release must be made without knowing how much water will be flowing into the reservoir. The difficulty of the situation led one author to refer to the operation problem as “a game against nature” (Lindqvist, 1962).

If the reservoir in question is operated for hydropower production, additional complications arise. When the power plant is connected to an electricity grid with interconnections, energy transactions become possible, and thus part of the operation problem. The expanded problem consists of making decisions as to when, and how much, water to draft or store, and when, where, and what quantity of energy to buy or sell. The operating strategy, which provides answers to these questions, must meet the demand and respect all of the physical and operational constraints on the system. Uncertainty in the load and market factors, such as energy prices, becomes important factors to consider (Nash et al., 1999). Uncertainty in the inflow is important, as inflows affect the amount of power that can be generated. Uncertainty in the demand is important, as the firm demand must be met, regardless of cost. Uncertainties in the market prices are important as they determine the amount of energy that can be economically bought or sold on external markets.

The reservoir operation problem increases in difficulty again if the reservoir in question is not operated independently, but, rather, is one component of a system of reservoirs. All of the decisions just described for one reservoir must be made for each reservoir in the system. Determining the optimal operation of the system as a whole is different from determining the optimal operation of the individual components of the system. Furthermore, in most cases, the reservoir operation problem will be complicated by multiple, and generally, conflicting, objectives.

The difficulties that must be considered in scheduling the operation of a reservoir, or reservoir system, “well”—which is to say towards satisfying some goal or set of goals—have been part of the impetus for the development of computer models that can provide decision support in making the operating decisions. The literature reveals that such models can be descriptive simulation models, prescriptive optimization models, or some blend of the two. In addition, the literature (e.g., Perera and Codner, 1998; Turgeon and Charbonneau, 1998; Philbrick and Kitanidis, 1999) notes the relative lack of recent development of major new reservoir projects and the growing influence of a market-based approach in the hydropower industry, which together, exacerbate the pressure to operate reservoir systems as “well” as possible.
The British Columbia Hydro and Power Authority (BC Hydro), the electrical utility serving the majority of the province of British Columbia, Canada provides a good example of an electrical system dominated by hydropower. In the day to day operations of this system, the difficulties and uncertainties in inflows, demands, prices, etc. outlined above must be addressed. BC Hydro has developed two main computer models in order to provide decision support for the system operators.

The first model is a very detailed optimization model of the system on an hourly basis for the next week (Shawwash et al., 1999). Within this short “look ahead” period, loads, demands, prices, and inflows to the reservoirs are considered to be deterministic. The second model employed by BC Hydro is a long term stochastic dynamic programming (SDP) model for the main reservoir/hydro project on the Peace River, the largest and most flexible project in the BC Hydro system (Druce, 1989, 1990). The SDP model provides discounted present values for water stored in the reservoir, the slopes of which give marginal values for the value of storage. These values are taken as representative of the marginal value of energy throughout the system and are used in the short term optimization model just described, for purposes of trading off the value of energy generated in the next period against the value of the water if it is retained and stored in a reservoir for use in another period, and as decision support for marketing decisions. The SDP model can take into account uncertainty in inflows and future prices; this is done at a monthly time step.

To complement the existing modelling of a system such as that of BC Hydro, it is desirable to be able to consider more than the one large reservoir at a time (the BC Hydro system has one other major storage project in addition to that on the Peace River), as is done at present, in order to better understand the interactions between multiple reservoirs and generating plants. Further, it is desirable from an operating viewpoint to be able to include more detail, while maintaining the ability to consider uncertainty in inflows, demands, and prices in the intermediate period between the immediate short-term covered by the deterministic model and the longer term future addressed by the SDP. This thesis addresses these two needs, with the aim of filling an important gap in the modelling capability needed for decision support for the operation of a complex hydro system in today’s competitive, yet uncertain environment.

In this thesis, a method for optimizing the operation of a reservoir system for hydropower production, for both the long-term and intermediate-term, is developed. It is assumed that the reservoir system is to be operated to maximize the value of energy produced while taking into consideration the value of water remaining in the reservoirs at the end of the model time horizon. The model is developed for two non-hydraulically connected reservoirs that serve the same demand area. While this may seem limiting, aggregation techniques such as those developed by Turgeon and Charbonneau (1998) and Valdes et al. (1992) expand the potential range of application. Aggregation techniques can be used to form a single composite reservoir that simulates the behaviour of a set of reservoirs.
The method developed in this thesis is able to address uncertainty in market-related parameters such as energy prices, as well as in demands, in addition to the typically considered inflow uncertainty. The model allows the energy prices to be subdivided into several periods within the day, matching the current nature of energy markets in which energy prices vary within the day as well as across months. This ability to simultaneously consider uncertainty in energy market parameters and inflows for the operation of a multiple-reservoir hydropower system is vital, as the importance of uncertainty in the former can be of equal or greater importance than the latter (Kim and Palmer, 1997; Russell and Campbell, 1996). Models reported in the literature typically either ignore uncertainty in energy prices, fail to consider energy price differentiation within a day, or only consider a single reservoir. These simplified models fail to address the main problem actually faced by the operator of a two-reservoir hydropower system.

The proposed method involves the use of two cascaded models. A longer-term model is used to estimate the value of month end storage in the two reservoirs over a time horizon of several years, and to generate the marginal values of water stored in the reservoirs. The results produced by this model feed into a shorter-term model that is able to better deal with uncertainty in the first time step used in the longer-term model through the use of variable time-steps and scenario trees. The length of the time steps in each of the two models is variable. It is proposed that the time step used in the longer-term model be on the order of a month, and the time steps in the shorter-term model range from the order of a day for the first time step to the order of several weeks for the last time step.

Through the use of the two models, marginal energy values for two reservoirs that cover a time horizon stretching out several years, but which cover the near-term in sufficient detail are developed. These marginal energy values are crucial to price-based dispatching of a hydropower system, and can be input to an hourly dispatch model.

The longer-term model employs the optimization techniques dynamic programming (DP) and linear programming (LP). DP is employed to link the decisions made during one time interval with decisions during other time periods. DP deals with "stages" and "states"; here, the stages are on the order of months, and the states are vectors of the volume of water stored in the two reservoirs. A recursive equation that gives the optimal decision for a given starting state in a stage, in the form of the release from each reservoir, that maximizes the expected value of the sum of the value of the operation during the next time period and the value of being at the ending state for the next stage. In order to begin calculations, which move backwards in time, an assumption about being in any given state must be made. If this point is far enough in the future, its value is inconsequential in the results obtained by the DP.

Uncertainty in the DP model is handled through the use of scenarios, which describe possible values for the scenario-dependent inflows, demands, and energy prices. The number of possible scenarios can vary with the stage in the model. Associated with each scenario is an occurrence probability. For any given state in a given stage, the future can unfold in any one of the manners described by the set of scenarios for that stage. The model finds a turbine release policy that maximizes the expected value of system
operation over all scenarios—that is, for a given reservoir and stage, the single best
turbine release is found. Other variables, such as the ending storage for each reservoir,
spill from each reservoir, imports to the system, and exports from the system can vary
with the scenario. The values of these other variables are determined by the LP such that
the value of system operation under the scenario subject to the policy-specified turbine
release is maximized.

The LP model performs the tradeoffs between releasing water and making energy trades
during a stage and the value of keeping that water in storage until the next stage that are
contained within the recursive equation. The value of water stored beyond the current
time interval is represented in the LP by storage value curves, the slopes of which are the
marginal water values. Piecewise-linearization is used to model these terminal value
functions, which in general are dependent upon the month, as well as the storage in both
reservoirs. As the true hydropower production problem is not strictly linear, the use of
linear and piecewise linear relationships in the model requires that a series of closely
related LPs be solved successively until convergence is obtained.

The use of LP to evaluate the recursive DP equation departs from the traditional method.
Typically, in DP the optimal transition is from a discrete state in one stage to a discrete
state in the next stage. For the current problem, the discrete states would be vectors of
the volume of water stored in the two reservoirs. In the formulation outlined herein, the
ending storage volumes in the second stage are decision variables in the LP algorithm,
and are thus not restricted to discretized state values. The information generally
contained in the ending storage state vectors is contained in the piecewise-linear storage
value curves. In the storage value curves, the discretized storage states are the
breakpoints between linear segments. Between discretized storage volume points, the
marginal value of stored water is constant. The use of LP to evaluate the DP recursive
equation alleviates the problem of artificial spilling of water that can result from the
transition between discretized storage volume vectors. This problem can be particularly
acute if a coarse state space discretization is used to increase the speed of a typical DP
algorithm.

The monthly storage value functions, and the associated marginal values, produced by the
monthly DP and LP based model are input to the shorter-term model, which is based
upon a technique known as stochastic linear programming with recourse (SLPR). SLPR
is reported in the water resources literature as being very promising (Yeh, 1985) but has
not been widely used. The application of SLPR in the shorter-term model described
herein demonstrates its suitability for complicated hydropower reservoir operation
problems.

The shorter-term, SLPR based, model allows operations to be planned over time steps
shorter than those used in the longer-term model (typically on the order of one month)
and to generate short-term marginal values over each of these time steps. While the time
step employed in the DP and LP based model is appropriate for some types of studies, the
longer time step of necessity neglects variations in the scenario-dependent parameters
within the time step. The shorter-term variable time step model provides a way in which
these important departures from the mean values can be taken into consideration for both the calculation of marginal values and operation of the system. The shorter-term marginal values are of particular importance when operation of the reservoir system is constrained.

In SLPR uncertainty is handled through the use of a scenario tree. It is easiest to understand the idea of a scenario tree using the implied analogy. A scenario tree branches from a single root at the start of the modelled time horizon to leaves at the end of the time horizon being considered. The tree branches at locations where decisions must be made. A scenario in the tree is a direct path from the root to one of the leaves.

Consider the simple case of a model with one reservoir that is to be operated for two time periods, in which the inflow in each of the time periods can assume one of two discrete values. At the start of each time step a decision must be made as to the release from the reservoir. The scenario tree representing this simple problem has two branches growing out of the root at the start of the first period: one for the high inflow and one for the low inflow. The tree then branches again at the start of the second time period for both of the first period branches, giving four leaves at the end of time period two. There are thus four scenarios—direct paths from the root to a leaf—in the tree: high inflow in both time periods; high inflow in the first time period and low inflow in the second; low inflow in the first time period and high inflow in the second; and low inflow in both time periods. The scenario tree just described outlines all of the possible ways in which the future can unfold in the simple model.

The tree structure is also instructive in illustrating how decisions are made using SLPR. As described above, each branching point, or node, in the tree represents a place at which a decision must be made. At each node a probability is assigned to each branch, and the sum of the probabilities of all branches from a node must equal one. At each of these nodes a decision, in this case the quantity of water to release, must be made. This decision must be made prior to knowing in which one of the possible manners the future will unfold; therefore, the decision must be feasible over all possible evolutions of the future, and should be the optimal decision considering all possible outcomes. Such a decision, being made in the face of uncertainty is known as a “here-and-now” decision. Once the future has been revealed at the end of the time period, “recourse” decisions can be made based upon which of the possible futures occurred. Decisions of this latter type are referred to as “wait-and-see.” The decision making approach just described results in decisions that do not anticipate the future, and can therefore be implemented.

In the shorter-term, variable time step, SLPR-based model, scenarios in which reservoir inflows, demands, and energy market prices vary are defined by a scenario tree. The here-and-now decisions at the start of each time period are either the quantities of turbine release from the two reservoirs or the ending storage volume. The link between the shorter- and longer-term models is through the storage value curves generated by the longer-term model, which are used as terminal value functions in the shorter-term model.
Both the longer- and shorter-term models take into consideration that the energy produced is differentiated by when it is produced, with energy being more valuable during some portions of the day. This aspect of the models allows the division of generation into on- and off-peak times. Having the model recognize different energy products is particularly important with the inclusion of external market opportunities in the model. For instance, in some cases it may be optimal during one period of time to import energy during the lower priced period in the day and to export during the higher priced period.

The models presented in this thesis allow for the optimization of hydropower reservoir systems over a time horizon stretching from several hours or days in the future to years into the future. The selected approach, in which results from a monthly model feed in to a shorter-term variable time step model, allows sufficient detail to be considered such that the uncertain reservoir operating problem actually faced by the operator of a hydropower system, in which energy varies in price both within the day, between days, and between months, in which inflows and demands are also uncertain, and when generation can come from different sources, to be solved successfully. Typically, parts of the true operating problem have been ignored, either by dealing with only a single reservoir, or by not addressing uncertainty in all of the important parameters.

The final outputs from the models are marginal energy values for each period modelled, with the short-term considered at a finer time resolution, as well as operating decisions for each period in the shorter-term model. It is proposed that these short-term marginal values be used as input to a very-short term, very detailed, but deterministic reservoir operation model, with a time-step of an hour or less, and a total time horizon of a week or less, such as the one actually in use by BC Hydro (Shawwash et al., 1999).

The remainder of this thesis is organized in the following manner. Chapter 2 presents a review of the pertinent literature. The focus of the review of the application of systems analysis techniques to reservoir operations is on DP and LP, with particular attention given to reservoirs operated predominantly for hydropower production. Chapter 3 presents the DP and LP based model used for estimating the value of water stored behind hydropower dams over the longer-term at a monthly time step. In Chapter 4 the shorter-term, variable time-step, SLPR-based model is presented. Both chapters 3 and 4 contain case studies demonstrating the applicability and efficacy of the models developed to a hydropower system based upon two reservoirs with significant multiple-year storage from the BC Hydro system. Conclusions, recommendations for future research, and a summary of the work are presented in Chapter 5.
2 Literature Review

2.1 Introduction

In this chapter, the literature is reviewed for cases of the application of optimization and simulation techniques to the operation of reservoirs. The development of optimization techniques and their application to reservoir operations have been much intertwined (Yakowitz, 1982). As such, any review of the literature in this area must be limited in scope. The present review is focussed in several respects. Firstly, of the many available optimization techniques only the two that have been most widely used, dynamic programming (DP) and linear programming (LP), are considered in detail. A second restriction imposed upon the scope of the literature review is to place attention primarily on reservoir systems operated for hydropower production.

The operation of even one single-purpose reservoir is not a straightforward task. In operating such a reservoir, decisions must be made as to how water should be stored, or released, in order to satisfy some objective. The major complicating factor in the operation problem arises owing to the uncertainty of natural inflows into the reservoir. That is, decisions as to how much water should be released must be made without knowing how much water will flow into the reservoir as the result of naturally stochastic hydrological processes. The difficulty of the situation led one author to refer to the operation problem as “a game against nature” (Lindqvist, 1962). In addition to handling uncertainty in inflows and other important parameters, the reservoir operation problem can be complicated by multiple, often conflicting, objectives. When, as is often the case, a reservoir is part of a larger system, and so cannot be considered in isolation, the difficulty of the problem is further amplified.

The desire to operate a reservoir system “well”—where the meaning of well can be taken to be satisfying some set of objectives—has resulted in the expenditure of a great deal of effort in the development of tools and techniques to aid the decision maker in making operating decisions. Further, as noted in the literature (e.g., Perera and Codner, 1998; Turgeon and Charbonneau, 1998; Philbrick and Kitanidis, 1999) the relative lack of recent development of reservoir projects and the growing influence of a market-based approach by some reservoir operators are increasing the pressure to operate reservoir systems as well as possible. For this reason, it is useful to examine the literature to shed some light upon the techniques available to aid the reservoir system operator.

2.2 Techniques

As noted above, the body of previous work in which simulation and optimization techniques have been applied to reservoir operations is vast. Previous reviews of the literature have been conducted by Yeh (1985), Wurbs (1991, 1993), Reznicek and Cheng (1991), and Simonovic (1992). A literature review of DP techniques has been published
by Yakowitz (1982). The interested reader is referred to these earlier reviews for a more comprehensive examination of the topic.

The remainder of this chapter addresses particular systems analysis techniques applied to the optimization of reservoir operations. Dynamic programming is discussed first, followed by linear programming, other optimization techniques, and, lastly, simulation.

2.2.1 Dynamic Programming

Dynamic programming, introduced by Bellman (1957), is an approach to optimization that has been widely employed in the area of water resources. Wurbs (1993) notes that DP is one of the two optimization methods most widely applied to reservoir operation problems. DP is an attractive method for solving sequential decision processes in an efficient manner. The main benefits of DP are its ability to incorporate non-linear constraints and objectives, and the fact that the method can be used to handle stochastic inputs. The main limitation regarding DP is that the computational effort increases exponentially with the number of state variables in the problem, thus restricting the number of dimensions that can be considered practically; this limitation is commonly known as "the curse of dimensionality." Another drawback of DP cited by Wurbs (1993) is that as DP is not a precise problem solving algorithm, but rather an approach, commercial DP packages are not available, and each application must be custom built.

Despite the above limitations, Yeh (1985) states that DP is well suited to both short-term and long-term reservoir operation optimization problems; for the latter stochastic DP (SDP) is recommended, as compared to deterministic DP for the former. Literature reviews describing the application of DP to water resources problem in addition to Yeh (1985) include Yakowitz (1982), Reznicek and Cheng (1991), Simonovic (1992), and Wurbs (1993).

Applications of DP to the problem of optimizing reservoir operations are discussed below. Deterministic applications are discussed first, followed by stochastic applications.

2.2.1.1 Deterministic DP

Many applications of DP to reservoir operations problems assume that all model inputs are completely known, or deterministic. Yakowitz (1982) notes that the literature contains two justifications for applying deterministic methods: (i) in particular cases, flows are well-enough known that the assumption is justifiable; and, (ii) through applying deterministic methods to known inflows, knowledge of reservoir operations can be gained that will be useful in the future. Applications of deterministic DP are discussed next.

Becker and Yeh (1974) employ a combination of LP and DP (LP/DP) in order to optimize the operation of a multiple-reservoir system so as to minimize the loss of stored
potential energy. DP is used to select the optimal policy path, and LP is used for optimization between stages. Satisfactory ranges of ending reservoir levels must be specified. In the application of the model to the California Central Valley Project, the difference between on- and off-peak pricing is included in the formulation. In comparing LP/DP with successive LP and an optimal control algorithm, Grygier and Stedinger (1985) note that Bellman’s principle of optimality does not apply, meaning that a global optimum cannot be guaranteed; and, furthermore, concluded that LP/DP is dominated by the other two methods which are able to find more optimal solutions in less time.

Karamouz and Houck (1982, 1987) describe a method of combining deterministic DP, regression analysis, and simulation (DPR) to obtain reservoir operating policies, noting that there is no guarantee that DPR will converge to the optimal solution.

Efthymoglou (1985) uses deterministic DP to determine the optimal operation of a reservoir to produce hydroelectric energy in a hydro-thermal system. The objective is to minimize the thermal fuel cost over an annual period, with a monthly time step. Water values and short-run marginal costs are derived. The state variable is stored energy, summed over all reservoirs. The composite reservoir start-of-year and end-of-year energy must be the same. With regard to the deterministic nature of the model, Efthymoglou states: “Because of the stochastic nature of basic variables of the problem (demand, capacity availabilities, water inflows), the optimal solution could be considered in practice as providing operating decisions only for the first stage.”

Allen and Bridgeman (1986) apply deterministic DP to three separate hydropower scheduling problems at the instantaneous, hourly, and seasonal time scales. The latter two cases are of the most interest. For the hourly load dispatch problem the objective is to meet the forecast demand using water as efficiently as possible. Storage is the state variable. The seasonal load dispatch problem considers two reservoirs, the storage in each being a state variable, and the two releases as the decision variables. DP is used to minimize the cost of imported power and energy on an annual basis. Market prices are deterministic. With regard to the deterministic framework, Allen and Bridgeman (1986), state: “These model results are optimistic because the methodology of analyses assumes complete foreknowledge of future hydrology, while a real-time system operation must at best rely on forecasted hydrology for implementation of a flexible operational strategy.”

Braga et al. (1991) apply both deterministic DP and SDP to optimize the operation of a multiple-reservoir system for hydropower production. The deterministic DP model is used to calculate the value of water stored in each reservoir as a function of the reservoir storage quantities and the stage. The deterministic DP model is referred to as “an off-line, one time-only” problem.

### 2.2.1.2 Stochastic DP

In their review of the literature, Reznicek and Cheng (1991) found SDP to be one of the two stochastic techniques most often applied to reservoir operation optimization.
problems, and further that SDP has been applied to the greatest extent in longer-term studies in which variation in the inflows must be considered. Similarly, Yeh (1985) notes that SDP is particularly suitable for long-term reservoir operation optimization problems. However, in his review, Yeh (1985) also notes that the multiple-purpose and multiple-reservoir nature of many reservoir systems has presented problems for the application of SDP, due to the “curse of dimensionality”—the well-used phrase, coined by Bellman (1957), describing the exponential increase in computation with an increasing number of states. Reznicek and Cheng (1991) assert that SDP is unable to handle more than two state variables. Yakowitz (1982) cite the upper limit as two to three dimensions. Philbrick and Kitanidis (1999) place the limit at up to seven states. This latter estimate exceeds the number of states typically reported in the literature.

It is for these reasons that Yeh (1985) notes that decomposition is “essential” for the application of SDP to multiple reservoir systems, and that the application of stochastic methods has been to either single reservoir systems, or to multiple reservoir systems operated for a single purpose. Additionally, Yeh (1985) and Reznicek and Cheng (1991) report that in using SDP for multiple reservoir systems, the assumption of no cross-correlation between natural inflows into the system is typically made. Reznicek and Cheng (1991) state that, as a result, the results of such analyses must be considered “rough estimates of the real situation.”

Despite these limitations, SDP remains a very popular tool for including inflow uncertainty in the optimization of reservoir operations. Yakowitz (1982) traces the application of stochastic DP to reservoir operations back to Little (1955), and notes, interestingly enough, that this stochastic application preceded the first deterministic application. Applications of SDP, beginning with Little (1955), are discussed next.

Little (1955) applies SDP to a single-reservoir operation problem. In addition to the reservoir, the system is comprised of a known demand and a source of thermal generation. State variables are the reservoir level and the flow during the preceding stage. The objective is to minimize the expected cost of operation; towards this end, water is considered to be free, but a known cost function is used for thermal generation. Load curtailment is also handled as a cost. Inflows are assumed to follow a lag-one Markov process, and a time step of two weeks is used. Given the time step, Little (1955) states that “the assumption of complete independence is untenable for river flow. The flow this week is an indicator of next week’s flow, since a drainage basin charged with water will maintain river flow for some time.” The cost function is a power series of supplemental energy. Average head is used in computing hydroelectric generation. Application is made to the Grand Coulee reservoir on the Columbia River. Summer flows are high enough that the assumption is made that at some point the objective function is equal to zero. Quadratic and linear cost functions are used. Policies are obtained, and the performance of the system is simulated. A comparison of the policies thus obtained with the operation specified by a rule curve yielded a 1% reduction in cost. In some years the rule curve performed better. Linear cost operation was found to be less smooth than quadratic cost operation. Emphasis is placed on the use of non-linear relationships. In conclusion, Little (1955) states, presciently, that “It would seem that
expected value methods for the storage water problem are worthy of future development and of application to other models.”

Stage and Larsson (1961) investigate the problem of determining the incremental cost of hydropower. Their technique, the “incremental water value method” uses SDP. Lindqvist (1962) also uses the incremental water value method to determine the operating decisions for a reservoir system so as to minimize the operating cost of a hydro-thermal system. The stochastic element in the formulation is the inflow; all other elements are deterministic. The inflows in consecutive months are assumed independent of one another. The application is to an aggregated reservoir. The efficiency is assumed to be constant. Lindqvist (1962), in discussing the extension of the model to the multiple-reservoir case, notes that theoretically the incremental water value in one reservoir would depend upon the storage level in all reservoirs. He suggests, as an approximation, that the incremental water value of a reservoir could be expressed as a function of the storage in that reservoir and the sum of the storage in the rest of the system. Gjelsvik et al. (1992) also describe the use of the incremental water value method to minimize average operating costs with consideration given to the value of water remaining in storage at the end of the model time horizon.

Butcher (1971) describes the use of a stationary SDP model for the optimization of the operation of a single multi-purpose reservoir. The monthly inflows are modelled by a lag-one Markov process that is stationary from year to year. Flood control objectives are incorporated through the use of constraints that specify the maximum reservoir storage in each month such that a portion of the reservoir is available for flood reservation storage. Recreation objectives are taken into account by penalizing deviations from target levels. Water released for irrigation purposes is assumed to have a value in July through August, and all hydroelectric generation throughout the year is valued at a single price. Convergence is achieved in 30 months.

Takeuchi and Moreau (1974) employ a combination of LP and SDP to minimize the expected losses from a reservoir system operated for low-flow augmentation and water supply. The losses in one reservoir are a function of the storage in all reservoirs, and are specified by convex piecewise-linear functions. The LP component is used to make operating decisions within a stage—i.e., making the tradeoffs between immediate losses and the expected value of future losses. Simulation of the policies is used to find the required conditional probabilities in an iterative process.

Klemes (1977) investigates the problem of determining the necessary discretization of reservoir storage, and compares two types of storage representation schemes. In determining the required discretization, Klemes considers the probability of filling or completely drafting the reservoir.

Stedinger et al. (1984) introduce the use of the current inflow (as opposed to the previous period’s inflow) as a hydrologic state variable in SDP. The problem that they consider is one in which targets for release, generation, and irrigation are established, and deviations from these targets are penalized. It is found that the resulting policies, when used in
simulation, produce better outcomes, as measured by the objective function. Stedinger et al. (1984) also discuss the relative merits of stationary and non-stationary approaches. Pros and cons are cited for both approaches, but the main point made is that improved results can be obtained by using “better hydrologic state variables” in the analysis.

Goulter and Tai (1985) consider a stationary two reservoir problem, and investigate the effect of changing the discretization of the storage state variables on the gain of the system, and the amount of time to convergence to steady state. It is found that when too few storage states are used, states at the low end of the storage range can act as “trapping states”, causing the probability distributions to be unrealistically skewed. As the discretization of the storage states is increased, the gain increases.

Wang and Adams (1986) describe a two-step approach to the application of DP to the optimization of the operation of a reservoir. The method is centered on the idea of describing the time horizon by two phases. A stochastic DP model is used to describe the phase closest to the start of the model. A deterministic DP is used to find future steady state results that feed into the SDP.

Karamouz and Houck (1987) compare the results obtained using SDP with a lag-one Markov process describing the inflows to a combination of DP and regression. The application is to a single reservoir that is to be operated so as to minimize the losses from a piecewise exponential loss function, which depends only on the release. The time step is monthly. SDP is only found to have better performance when the reservoir, in comparison to annual inflows, is small. In addition, the SDP policy is found to be more sensitive to the discretization of the storage state. This result would seem to agree with that of Goulter and Tai (1985).

Tai and Goulter (1987) apply SDP and heuristics to solve a three-reservoir, “Y-shaped”, problem. The reservoirs are considered separately, with downstream reservoirs providing targets for upstream reservoirs. In each sub-problem, the inflow and storage are the state variables. The objective is to maximize revenue, where energy prices vary with the month. Inflows are described by a lag-one Markov process. In the application, the SDP results slightly under-perform the historical operation.

Druce (1989, 1990) applies SDP, and an associated simulation model, to operate a storage reservoir and downstream run-of-river reservoir so as to maximize expected net export revenue. Inflows and loads are stochastic through weather sequences. The time step is monthly. There are firm and interruptible energy markets, specified by price and quantity. In the formulation, perfect foresight is assumed for the current month, but not beyond. Reservoir storage is the state variable. Marginal water values are discussed and calculated. The model was developed as electric utilities were in the transition between energy conservation policies and profit maximization policies. There is no assumption of stationarity. Inflows are independent, not lag-one Markov, and the previous inflow is not a state variable. The state variables are the reservoir level and historical weather year. The model is run for longer than necessary to dampen end effects.
Johannesen and Flatabo (1989) describe a set of cascaded models for planning hydropower operation. For the long-term, the SDP water value method is used. The unit commitment problem is solved using forward dynamic programming.

Kelman et al. (1990) present the method known as sampling stochastic dynamic programming (SSDP). In SSDP inflow stochasticity is handled via streamflow sequences. The idea behind the method is to base the optimal decisions on joint consideration of all scenarios. The objective is to maximize benefits including a terminal value function for water remaining in storage. Application of the method is made to a one-reservoir system. Kelman et al. (1990) find that there is greater value in including hydrologic forecasts in the model when the value, as opposed to the quantity, of energy is maximized.

Braga et al. (1991) apply both deterministic DP and SDP to optimize the operation of a multiple-reservoir system for hydropower production. The SDP model makes use of the water storage values calculated by the DP model in determining the optimal releases to maximize system benefits. States in the SDP model are the previous monthly inflows and starting storage. The SDP model is applied to one reservoir at a time, with iterations until convergence. Application was made to a three-reservoir system. Inflows were discretized into ten states. Convergence is reported to have generally been achieved in three or four iterations. The time step in the application is one month.

Valdes et al. (1992) investigate the hydro-thermal coordination problem using an SDP model for an aggregated reservoir. State variables in the model are the energy storage and the previous inflow. The decision variable is the release; the stages are months; and the objective is to minimize the total cost of energy production. The results from this model are disaggregated at a daily time step. Several aggregation/disaggregation methods using LP are explored. In order to reduce the dimensionality of the problem, disaggregation is carried out in separate space and time steps, instead of in a single step, at the cost of sub-optimal results. It is found that it is better to disaggregate first in time, and then in space.

Karamouz and Vasiliadis (1992) propose Bayesian stochastic dynamic programming (BSDP) as a means to incorporate Bayesian decision theory into SDP. The method is applied to a single-reservoir problem in which the state variables are the current inflow, forecast inflow for the next period, and the current storage. Inflows are described by a lag-one Markov process. The objective is to minimize the expected loss. The stages are months, and the decision variables are the releases. The results of the method are compared to DP and SSDP. The effect of using Bayesian decision theory is to update prior probabilities to posterior probabilities as new information becomes available; in particular, the conditional occurrence probabilities of forecasts are updated given actual flows. The results, as measured by simulating the reservoir system operation based on the developed policies, indicate that BSDP outperformed both SSDP and DP. Kim and Palmer (1997) also apply BSDP in a formulation which has one reservoir and the current storage, current inflow, and snowmelt forecast as state variables. The reservoir is to be operated so as to maximize the revenue produced by generation. The energy prices and
loads are deterministic. The BSDP results are compared to SDP without hydrologic state variables. It is found that there is value in incorporating hydrologic state information into the problem formulation, and that this value increases with the storage capacity of the reservoir. With regard to the deterministic demand and price, the authors conclude: “The energy-related variables, such as energy demand and price, may be as significant as the hydrologic state variables.” Similarly, Kim and Palmer (1997) found that the effect of energy prices on operation increases with demand.

Tejada-Guibert et al. (1993) consider a stationary, infinite horizon, problem of optimizing the operation of a two reservoir system so as to maximize the expected benefits from hydropower production. The state variables are the two storage volumes, and one variable for the current period inflow, where it is assumed that the two inflows are perfectly correlated. One question addressed by Tejada-Guibert et al. (1993) is whether the use of multi-dimensional cubic spline interpolation is superior to multi-dimensional linear interpolation in evaluating the terminal value function; the conclusion is that cubic spline interpolation is better. The other major facet of this research involves the method used for determining policies in simulating the policies. The authors found that it was better to determine the policy release decisions by reoptimizing, using the terminal value function, within the simulation, than it was to interpolate within the policy tables. The two reasons cited for reoptimizing within each period in the simulation are: (i) the optimal policy for similar system states can be quite different; and, (ii) employing the actual current state of the system ensures that all constraints associated with these starting conditions will be respected.

Tejada-Guibert et al. (1995) investigate the value of incorporating hydrologic information into an SDP model of a two reservoir system. The decision variables are the releases from the two reservoirs. The results of the study show that when the benefit function is to maximize energy, the choice of hydrologic state variable is of little importance. Hydrologic state variables are found to be of greater importance for objective functions that penalize deviations from targets.

Russell and Campbell (1996) incorporate fuzzy logic into implicit SDP to investigate the problem of operating a reservoir so as to maximize the value of energy generated plus the value of water remaining in storage at the time horizon of the model. In the model, prices and inflows are both stochastic. The state variables are the storage, price, and inflow, and the release is the decision variable. Russell and Campbell (1996) conclude that the use of fuzzy logic does nothing to reduce the effect of the curse of dimensionality. With regard to the importance of uncertainty, Russell and Campbell (1996) conclude that foreknowledge of the prices is of greater importance than it is for the inflows.

Turgeon and Charbonneau (1998) employ SDP to produce an optimal reservoir operation, and associated marginal water values, for a reservoir system that is run to maximize the expected profits. Prices are given by deterministic piecewise-linear curves. The method employs aggregation/disaggregation to solve the problem. The entire reservoir system is first aggregated into a single reservoir, and SDP is used to determine the optimal operation. Another SDP model is then solved for each river system; in these problems
there is one state variable for the total storage in the river system under consideration and a second state variable for the storage in the remaining river systems. Finally, the operation of each river in all of the river systems is determined. The calculated marginal values are stated to be functions of the storage in all reservoirs. The loads used in the model are deterministic, in this regard Turgeon and Charbonneau (1998) state: "ideally the randomness of electricity demand should be taken into account." The results produced by the model cannot be guaranteed to be the global optimum.

Perera and Codner (1998) investigate computational methods to increase the speed with which SDP can find solutions. The problem considered by Perera and Codner is one of water supply, where the demand is considered deterministic. The objective is to maximize the value of release. Storage and inflow are the state variables. The stages are months. Two separate computational methods are suggested. The first is to limit streamflow cross-correlations to a narrow band. The second is to use a corridor approach to eliminate consideration of "infeasible and/or inferior storage volume combinations" in solving the recursive equation based on discrete differential DP.

Mo et al. (1998) describe a stochastic dynamic programming/stochastic dual dynamic programming (SDP/SDDP) model used to combine the operation of a reservoir system with the management of a portfolio of energy futures. The model includes a Markov process for spot prices. Convergence problems are experienced in the example. The model is able to calculate marginal water values. The objective of the model is to maximize expected benefits, with penalties included for revenue target shortfalls. The states in the model are reservoir storages, profit periods, trading periods, inflows, and prices.

Gjelsvik et al. (1999) describe a method employing both SDP and SDDP for incorporating spot price uncertainty into the operation, over the medium-term, of a hydro-thermal system. Independent stochastic processes are used for price and inflow. The demand is deterministic. Incremental water values are obtained for each reservoir. The terminal value function is evaluated using results from a longer-term aggregated model, making the terminal value function dependent on the total storage. The authors note that this assumption means that the calculated value of water may underestimate the value of water in well-regulated reservoirs.

Philbrick and Kitanidis (1999) discuss the limitations of applying deterministic techniques to the optimization of reservoir operations with stochastic inflows. Deterministic methods are stated to produce sub-optimal results unless the system under consideration is "certainty equivalent". Certainty equivalence occurs when the objective function is quadratic; system dynamics are linear; there are no inequality constraints; and uncertain inputs are independent parameters with normal distributions. Reservoir systems operated primarily for hydropower production are stated to be the reservoir operating problems that are closest to certainty equivalent, but still depart significantly from the requirements. When reservoirs are operated according to deterministically determined policies the average cost is greater (assuming minimization) than the average cost resulting from stochastically determined policies. When the actual inflow sequence
is very close to the assumed inflow sequence in the deterministic analysis, deterministic policies can outperform stochastic policies. Philbrick and Kitanidis (1999) assert that SDP is limited to models with up to seven state variables.

Lund and Guzman (1999) in a discussion of operating rules for reservoir systems, including those in series and parallel, note that SDP can be successfully used for making operating decisions. The problems with which they are concerned involve short-term operations for water supply, water quality, and energy production. The biggest drawbacks about the application of SDP stated by Lund and Guzman (1999) are “extreme computational demands” for problems with many state variables, and the description of streamflows by explicit probabilistic methods.

2.2.1.3 DP Approximation Methods

As noted above, the primary difficulty in applying DP to problems of reservoir operations is the “curse of dimensionality” that limits the number of state variables that can be included in a model. The result has been that the number of reservoirs and hydrological state variables that can be successfully modelled is quite limited. It comes as no surprise then, that a number of methods for lessening the effects of the curse of dimensionality have been developed for both deterministic and stochastic problems. These methods are discussed next.

Askew (1974) describes a method known as reliability-constrained DP. This method allows the operation of a reservoir to be optimized while taking into consideration the maximization of the discounted net benefits as well as a limit on the number of system failures over the model horizon. The ability to place restrictions on the number of system failures arises from the desire to be able to handle “noneconomic aversion to failure.” Chance-constraints are incorporated through the use of a penalty term in the objective function that takes system failures into account; the value of this penalty term is found through an iterative process. Inflows are described by probability density functions, and are considered independent between periods.

Heidari et al. (1971) describe a method known as discrete differential DP (DDDP), which is a generalization of state increment DP, introduced by Larson (1968). DDDP is stated to have been developed to ease the computational burden (computer memory and computer time) associated with traditional DP. DDDP is an iterative technique requiring an initially feasible reservoir trajectory. DP is then applied to a corridor surrounding the trial solution. The improved solution, if found, becomes the trial solution for the next iteration. The method continues until convergence. DDDP is found to be most effective for cases where the order of the state vector is equal to the order of the decision vector. Young (1967), in what Yakowitz (1982) notes as the first application of deterministic DP to reservoir operations, describes the use of a combination of DDDP and regression analysis used to produce reservoir operation rules. In the study, the reservoir is to be operated so as to minimize the losses given by a loss function. The operating rules are found by applying regression to the results produced by the DP.
Gal (1979) describes the “parameters iteration method,” which is a technique for approximating the optimal policy in the stochastic case. The idea behind the method is to assume that the recursive functions depend on the vector of state variables as well as a vector of parameters. Solving for the parameters allows the approximation of the optimal policy. The method is tested for a three-state problem, and is found to produce results comparable to those found by DP. Application of the method is then made to a water supply system, having several reservoir volumes and previous inflows as state variables.

Turgeon (1980) describes two possible approaches for dividing a computationally infeasible stochastic multi-reservoir weekly operation problem into smaller computationally feasible pieces. The first of these methods, termed the “one-at-a-time” method, breaks the original multi-state problem into a number of single state problems that can be solved by SDP. The second method, termed the “aggregation/decomposition” method, breaks the original multi-state problem into a number of problems with two state variables: one for the storage in the reservoir under consideration, and a second for the storage in the remainder of the system. Neither of the two models is able to guarantee a global optimum. The two techniques are applied to a system comprised of six reservoirs in parallel. The aggregation/decomposition method is found to obtain a more optimal operation policy.

Turgeon (1981) presents a method for decomposing the reservoir operation problem of \( n \) reservoirs in series into \( n-1 \) SDP problems of two state variables that can be solved by SDP. The two state variables are the storage in the reservoir under consideration and the amount of stored potential energy in the remainder of the system. The method is demonstrated on a system of four reservoirs in series.

Trezos and Yeh (1987) describe an analytical method that can be applied to DP problems under a limiting set of assumptions. For appropriate problems, the method provides an analytical solution, thus eliminating the computational problem associated with the curse of dimensionality. In order to apply the method the problem must have a quadratic objective function, linear system dynamics, and inflows that can be fully described using the first and second moments of the probability distributions. The process is iterative, requiring a feasible initial solution. The solutions at each iteration are obtained by quadratic programming. The method can be thought of as an extension of differential DP (DDP) (Jacobson and Mayne, 1970) to the case of stochastic inflows. Constraints in the model apply to the expected values of variables.

2.2.2 Linear Programming

Next to DP, linear programming (LP) is the optimization technique that has been applied most often to reservoir operations problems (Yeh, 1985; Wurbs, 1993; Simonovic, 1992). The application of LP to the water resources field is traced back to the early 1960s (Simonovic, 1992). Yeh (1985) reviews the “state-of-the-art” in LP models; his examination includes stochastic LP models, stochastic programming with recourse,
chance-constrained LP, and linear decision rules. Based on the review, it is Yeh's conclusion that LP is a useful tool for the optimization of reservoir operations. In particular, he cites the fact that, through the use of linearization techniques (e.g., piecewise-linearization and Taylor series expansion), LP can be used to successfully model non-linear constraints and objectives. Additional reasons for the popularity of LP cited by Yeh (1985) and Wurbs (1993) are: (i) it is a well-defined and easily-understood technique; (ii) the ability with which problems of relatively large dimension, as compared to other methods, can be solved; (iii) global optima are obtained; (iv) an initially feasible trial policy is not required; and, (v) commercial programs are widely available, so that the method does not need to be developed from scratch for each application.

The main disadvantage of LP, as stated by Wurbs (1993) is that the modelled problem must be strictly linear, thus, generally making it necessary to approximate the physical problem through linearization methods. An additional limitation of the method, cited by Yeh (1985) is that in some cases, for very large reservoir systems, decomposition techniques may still be required. One of Yeh's (1985) three recommended areas of future research with respect to the use of LP in the field of the optimization of reservoir operations is the development and implementation of decomposition techniques.

Linear programming is a broad field, within which many distinct branches have developed in addition to "traditional" LP. Typical applications of LP techniques to the optimization of reservoir operations problems selected from the literature are presented next. The previous work described is only presented as being representative of the field, not as an exhaustive review. The interested reader is referred to Yeh (1985), Wurbs (1993), Reznicek and Cheng (1991), and Simonovic (1992) for further examples of the use of LP in the water resources field.

Linear programming methods have been developed for addressing problems in which model parameters are uncertain, as well as models where all parameters are deterministic. As noted above, applications of LP in water resources date back to the 1960s. The focus in this section is on some of the more recently reported studies. Deterministic applications of LP are discussed first.

Becker and Yeh (1974) describe the use of a combination of LP and DP to optimize a reservoir system for energy production. The system is aggregated into an equivalent reservoir, with storage described in terms of energy. The objective is to minimize the loss of potential energy, LP is used to generate possible feasible stage transitions, and DP is used to choose the optimal transition. A combination of LP and DP is also used by Takeuchi and Moreau (1974) to optimize the operation of a reservoir system for irrigation low flow augmentation and water supply. LP is used to compute immediate economic losses, and the stages are linked by DP.

Grygier and Stedinger (1985) revisited the combination of LP and DP used by Becker and Yeh (1974) and concluded that the method was dominated—that is a solution with a higher objective function could be found in less time—by successive LP. Grygier and Stedinger (1985) also include an optimal control algorithm (OCA) in their comparison of
methods. The conclusion that they reach is that both successive LP and the OCA reach the global optimum of the problem considered. The advantage of LP is that it is fast to implement, owing to the existence of commercial LP codes; on the other hand, the OCA method is slower to implement, but finds results in less time.

Martin (1986) applies successive LP, in a deterministic context, to maximize the benefits of operating a multipurpose reservoir system on a daily time step. The decision variables in the model are the reservoir releases. The objective function is to minimize the penalties associated with deviations from targets. Non-linear terms are approximated by first-order Taylor series approximations and bounds are placed on the decision variables to ensure that the approximation remains valid. The algorithm is applied successively until convergence. In order to improve execution time, separate models are solved for each reservoir, and the results coupled; the expense of this approach is not having a guarantee of achieving a global optimum. Tao and Lennox (1991) apply deterministic successive LP to optimize the operation of the High Aswan Dam.

Barritt-Flatt and Cormie (1989) describe the use of deterministic LP to solve the problem of determining the optimal operation of reservoirs in a hydro-thermal electrical system so as to maximize net revenues, including the value of water in storage at the end of the model time horizon. Non-linear functions, such as those describing the relationship between head, generation, and discharge, are replaced with piecewise-linear functions.

Piekutowski et al. (1994) employ deterministic LP to schedule a reservoir system over the short-term so as to minimize the value of energy used, both as turbine discharge and spill, in meeting demand over the course of the study period. Incremental costs for each unit, as well as for the entire system, are produced. The problem is solved using a commercial LP package.

Christoforidis et al. (1995) use mixed integer linear programming to optimize the operation of a reservoir system, including energy trades, over the short-term so as to minimize the sum of energy costs and penalties for the violation of soft constraints. The energy generation functions of the power stations are described by piecewise-linear functions. The integer variables are used to describe the start-up and shut-down of units.

Lund and Guzman (1999), in a discussion of operating rules for reservoir systems, note that LP can be used for making operating decisions. The problems with which they are concerned involve short-term operations for water supply, water quality, and energy production. Lund and Guzman (1999) state that one advantage that LP has over simply applying "rules of thumb" is that it can be used to incorporate operational constraints into the analysis, with the obvious limitation that the constraints must be linear, or piecewise-linear.

The optimization of the operation of reservoir systems is dependent on a number of parameters that are inherently uncertain. The most readily apparent sources of uncertainty are reservoir inflows. In addition, when a reservoir system is to be operated for the generation of hydropower, energy demands and market prices for electricity are
also uncertain. A number of techniques have been developed that allow the analyst to account for the incorporation of these sources of uncertainty with an LP formulation. The use of mean values of uncertain parameters such as inflows generally results in overly optimistic policies—overestimated system benefits or underestimated system costs—that result from the failure to properly consider large-impact, low-probability, events (Reznicek and Cheng, 1991; Philbrick and Kitanidis, 1999). The exceptions to the above rule are those systems that are “certainty equivalent”; that is, for certainty equivalent systems, the results obtained using a deterministic model with the values of uncertain parameters taken at their mean values are equal to the results for a full stochastic examination. Certainty equivalence occurs for systems exhibiting the following properties: (i) the objective function is quadratic; (ii) the system dynamics are linear; (iii) there are no inequality constraints; and, (iv) the uncertain parameters are independent and normally distributed. It is because no reservoir system operation problems satisfy these requirements that stochastic techniques have been developed and employed (Philbrick and Kitanidis, 1999).

Linear programming approaches that take uncertainty into account include chance-constrained LP (CCLP), stochastic LP (SLP), stochastic LP with recourse (SLPR), and the combined use of LP and SDP (LP/SDP), all of which have been applied to reservoir operation optimization problems (Yeh, 1985; Reznicek and Cheng, 1991; Simonovic, 1992). Of the above methods, Reznicek and Cheng (1991) report that CCLP is the method that has been used most often. The main benefit and main drawback of CCLP, reported by Reznicek and Cheng (1991), respectively, are the lack of dimensionality problems as compared with SDP, and the inability to assess the impact of failure of the probabilistic constraints. Of the three areas of recommended future research in applying LP to the optimization of reservoir operations reported by Yeh (1985), two deal with uncertainty: addressing the dimensionality problem of stochastic LP; and, continuing the development and application of stochastic programming with recourse.

One manner of addressing uncertainty in model parameters, such as inflows, is to include constraints that are considered to be satisfied if they are only violated a certain percentage of the time. In chance-constrained programming (CCP), model constraints are specified along with the probability with which they must hold—or, conversely, the probability with which failure of a constraint is acceptable. Assuming that the cumulative density functions of the uncertain parameters are known, the probabilistic constraints can be written in deterministic form. If the remainder of the model only contains linear constraints and objectives, then the model can be solved using LP; in such cases the technique is known as chance-constrained linear programming (CCLP). The main drawback of CCLP outlined in Yeh (1985) is that violations of constraints are not explicitly penalized, nor can corrective actions to address the violations be taken. Finding the necessary data to specify the acceptable level with which constraints can be exceeded is another issue in employing CCP (Reznicek and Cheng, 1991). Despite these limitations, CCLP has been successfully applied to the optimization of reservoir operations. Reznicek and Cheng (1991) note that CCLP is one of the two most widely used techniques for including inflow uncertainty into the optimization of reservoir operations.
The use of CCLP in reservoir system optimization is traced back to ReVelle et al. (1969) by Reznicek and Cheng (1991). Datta and Houck (1984) employ chance constrained programming with linear decision variables to optimize the operation of a reservoir so as to minimize the expected deviations from storage and release targets. Inflows are described by conditional distribution functions relating the actual inflow to the forecast inflow. Time horizons that can be considered by the model range from several days to a month. Bhaskar and Whitlatch (1987) report the use of CCLP to generate monthly release policies for the operating of a multipurpose reservoir. In the same study, a combination of DP and regression is also used to determine operating rules. The two sets of resulting policies are compared using a simulation model. For the case studied, Bhaskar and Whitlatch (1987) find that the combination of DP and regression produced better policies, in that the average annual loss was lower while reliability levels were higher, than did the CCLP model.

Another approach that has been taken for dealing with input parameter uncertainty in LP is the use of scenarios. A scenario describes the values assumed by each of the uncertain parameters in each time step. LP methods have been developed that describe uncertainty using a finite number of scenarios, each having a discrete probability of occurrence.

A stochastic LP (SLP) model for optimizing the operation of a single reservoir is described by Loucks (1968). Inflows in the model are described by a lag-one Markov process. The objective function is to minimize a loss function that penalizes the deviation of the storage and release from targets. The model is formulated such that the decision variables are the joint probabilities of given starting reservoir volumes, inflows, and releases. Loucks (1968) notes that the formulation employed results in very large problems, as the problem size grows with the product of the discretized volumes, inflows, and times.

One technique that utilizes scenarios to describe uncertainty in model parameters is stochastic linear programming with recourse (SLPR). SLPR is a method that appears suitable for adding consideration of uncertainty to the LP framework in a manner that retains the positive features of LP. In fact, one of the conclusions reached by Yeh (1985) based on his review of the state-of-the-art of the application of LP techniques to water resources is that one of the areas in which future research should be directed is the continued development and application of SLPR. Reznicek and Cheng (1991) also discuss the potential of SLPR for incorporating uncertainty into the LP framework. Grygier and Stedinger (1985) recommend the “multiple futures” method, as described by Pereira and Pinto in an unpublished manuscript from 1984, as a promising means for incorporating inflow uncertainty into LP by describing future inflows using a decision tree. Based on the description, the multiple futures method appears to be equivalent to SLPR. A simplified version of SLPR, referred to as “two-stage linear programming” is briefly described by Loucks et al. (1981) for application to allocating flows from a river to different uses. Gjelsvik et al. (1992) apply two-stage stochastic programming to the problem of operating a reservoir system so as to minimize operating costs when the value of water at the end of the model time horizon is included in the analysis. The application
of Gjelsvik et al. (1992) employs Benders' method (Benders, 1962) to piece together subproblems. The authors state: "The two-stage stochastic programming approach has the drawback that each scenario is considered too optimistically since the future inflows are not really known."

With SLPR, decisions are divided into those that must be made at a given point in time, in the face of uncertainty—termed "here-and-now" decisions—and those that can be made in the future, after the values of the unknown parameters are known—termed "wait-and-see" decisions. The here-and-now decisions must take into account the fact that the uncertain parameters can assume a number of different possible values, and that for each one of these possible outcomes an optimal recourse wait-and-see decision will be made; this is done by basing the decision on the expected value over all of the possible outcomes. Constraints must be included in the model that enforce the fact that at each node, nothing is known about how the future will unfold. Failure to include these "non-anticipative" constraints is equivalent to assuming perfect foresight of the future. When uncertainty is described by a scenario tree, a here-and-now decision must be made at each node in the tree.

In a scenario tree, a scenario describes the outcome of the uncertain parameters at each branch in the tree from the root to a leaf; where a branch is a point at which decisions must be made, the root is the point in time when the first decision must be made, and a leaf is the point in time when all uncertainty has been revealed. Assuming that the problem for which the scenario tree represents uncertainty is completely linear, there is one linear programming problem associated with each scenario. The non-anticipative constraints link the individual linear programs for the individual scenarios into one large LP problem that can be solved as a typical LP problem. Alternatively, some type of aggregation/disaggregation method can be used. Yeh (1985) and Reznicek and Cheng (1991) note that applying SLPR can be difficult, as the size of the equivalent deterministic problem including all of the scenarios can be quite large.

Dupacova (1980) discusses the application of stochastic programming with recourse to water resources systems, noting in particular that the method can be used as a means of handling uncertain inflows. Dupacova states that while, from the viewpoint of the user, chance-constrained models may appear to be more attractive, their failure to consider the dependencies between the chance constraints makes it necessary to consider the use of alternative methods such as SLPR. In Dupacova (1980) the problem of operating a reservoir so as to minimize the penalties, described by piecewise-linear penalty functions, arising from deviating from storage targets is formulated, but not solved.

Fleten and Wallace (1998) employ SLPR to jointly address the problems of optimal reservoir operations for an aggregated reservoir operated for hydropower generation and risk management, through the use of financial instruments, of the generated revenue. Risk aversion is included in the model through the use of revenue targets and piecewise-linear penalties for failure to achieve these targets. The objective is to maximize the sum of the revenue less the penalty costs. The scenarios employed in the model are used to describe the uncertain inflows as well as the uncertain prices for electricity and the
financial instruments. The deterministic equivalent of the problem described by the scenario tree is solved.

A subset of linear programming problems can be termed "network flow" problems. Network flow problems can be described in terms of "nodes" and "arcs", where the arcs join nodes. In order to have a pure network programming problem, the only decision variables in the problem must be the flows along the arcs, and the only allowable constraints specify continuity of flow at the nodes. For problems that can be described by such a formulation, commercial LP solvers have algorithms that are able to find solutions in much less time than for more general LP formulations. Yeh (1985) notes that some water resources optimization problems have structures that allow network algorithms to be successfully applied. Wurbs' (1993) review of simulation and optimization models includes a section on network flow programming. It is noted by Wurbs (1993) that in some ways, a network flow model can be thought of as straddling the line between simulation and optimization. Given the speed of the technique, single-period or multiple-period, multiple-reservoir problems can be quickly solved. As with other LP methods, the network formulation allows for piecewise-linear penalty functions and objectives. In order to handle non-linearities, such as those associated with hydropower generation, Wurbs (1993) suggests the use of successive iterative algorithms.

A recent example of the application of a network formulation is described in Christoforidis et al. (1996), who address the problem of optimally scheduling a system of reservoirs for hydropower production over the medium- to long-term. The objective of the optimization is to minimize the sum of energy transaction costs and penalties for the violation of soft constraints. The time horizon in the model is divided into a deterministic stage and a stochastic stage. In the stochastic period the inflows are described by scenarios that have discrete occurrence probabilities. In the example problem, four scenarios are used to describe possible evolutions of the future. The model is solved using the interior point method, which is an alternative to the well-known simplex algorithm. Christoforidis et al. (1996) conclude that very large problems with similar structure can be solved using the interior point method.

### 2.2.3 Other Methods

Although dynamic programming and linear programming are the two optimization techniques employed most often in the optimization of reservoir operations, the use of other methods, including analytical techniques, genetic algorithms, goal programming, non-linear programming, optimal control algorithms, reliability programming, and stochastic dual dynamic programming have been reported in the literature. A selection of applications of the alternative methods is reviewed in this section. The techniques discussed, as well as the individual papers cited, are intended to be representative of those found in the literature. The review is not intended to be exhaustive.
2.2.3.1 Analytical Techniques

Included in the reservoir operation literature are examples of applying analytical solutions to optimization problems. Examples of the use of such techniques are presented in this section.

Gessford and Karlin (1958) use induction to obtain an optimal policy for the operation of a reservoir in a hydro-thermal system. The reservoir is to be operated so as to minimize the expected cost of supplying energy, where hydropower is considered to be free and the marginal cost of thermal power is non-decreasing, and inflows are described by a known probability function. In order to obtain a solution, it must be assumed that the reservoir has an infinite capacity, the turbine discharge has no upper bound, and head effects are not important.

Glimn and Kirchmayer (1958) employ numerical integration of non-linear differential equations, obtained using calculus, to optimize the operation of hydroelectric plants with variable head. In the demonstration problem it is necessary to assume that the reservoir is a vertical-sided tank, and that the relationship between discharge and generation can be described as the product of a constant, a quadratic function of net head alone, and a quadratic function of generation alone.

Morel-Seytoux (1999) uses calculus to calculate marginal water values, as continuous functions, for the case in which a multiple-reservoir system is operated so as to minimize costs, or maximize benefits. The problem is formulated in continuous time, implying that input data, such as inflows, can be described as continuous functions. The model is limited to a deterministic description of inflows.

The advantage in obtaining an analytical solution is that do so is fast. However, in order for analytical techniques to be applicable, the problem under consideration must meet stringent limiting properties. Typically, the required properties severely inhibit the applicability of the models. It is for this reason that analytical techniques have seen limited use for the optimization of reservoir operations.

2.2.3.2 Decomposition Methods

In recent years, several techniques have been proposed in the literature for addressing the very large problems that can result when stochastic parameters, such as inflow, demand, and energy price, are added to the solution of reservoir operation optimization problems. Typically, these techniques use decomposition to break the larger problem into smaller, more manageable pieces, the solutions of which are then reassembled to provide an approximation of the optimal solution to the underlying stochastic problem. Note that such methods cannot guarantee optimal solutions. Two recently discussed methods, stochastic dual dynamic programming (Pereira, 1989; Pereira and Pinto, 1991) and Dantzig and Infanger’s (1997) combination of importance sampling and Benders’
decomposition (Benders, 1962) are discussed briefly in order to provide an overview of such methods.

Pereira (1989) and Pereira and Pinto (1991) introduce the method known as stochastic dual dynamic programming (SDDP). SDDP was developed as a method for solving stochastic multiple-reservoir, multiple-stage, operation optimization problems, and represents a means by which stochasticity can be introduced into a problem without discretizing the state space. Since the state space is not discretized, SDDP can be used to solve problems with a large number of decision variables. A scenario tree representing all of the possible combinations of the random variables through time is used to model stochasticity. The idea behind the method is to approximate the "cost-to-go" function with a piecewise-linear function obtained from dual solutions of the single-stage sub-problems. The approximation "may be interpreted as Benders' cuts in a stochastic, multistage decomposition algorithm" (Pereira, 1989). Monte Carlo methods are used to select a subset of the scenarios on which to carry out forward simulations. The results of these simulations feed back into the cost-to-go functions, and optimization and simulation are run in sequence until convergence is achieved.

Pereira (1989) and Pereira and Pinto (1991) apply SDDP to the problem of hydro-thermal coordination of the Brazilian power system. Gorenstin et al. (1992) extend the work of Pereira (1989) and Pereira and Pinto (1991) by including the transmission network and an optimal power flow (OPF) model in the hydro-thermal coordination problem. Gorenstin et al. (1992) note that the use of SDDP allows the calculation of the expected marginal costs of various components of the model. Rotting and Gjelsvik (1992) apply SDDP to the problem of the seasonal scheduling of a reservoir system operated for hydropower production. The system is to be operated so as to minimize the thermal operating costs while taking the terminal value of water in storage into account. The inflows form the stochastic part of the model. The paper extends SDDP, as described by Pereira (1989) and Pereira and Pinto (1991), to some extent by using relaxation to solve the sub-problems. Gjelsvik et al. (1992) also apply SDDP to minimize the average reservoir operating costs, with consideration given to the value of water remaining in storage at the end of the model time horizon. Gjelsvik et al. (1999) describe a method employing both SDP and SDDP for incorporating spot price uncertainty into the operation, over the medium-term, of a hydro-thermal system. Some implementations of SDDP have exhibited numerical problems, such as failure to converge to a reasonable solution (e.g., Mo et al. 1998).

Dantzig and Infanger (1997) apply importance sampling and Benders' decomposition (Benders, 1962) for the optimization, and intelligent control, of a reservoir system that is to be operated so as to minimize expected costs. In the method, as in SDDP, a scenario tree is used to describe the uncertain inflows; errors in the control instrument readings are also described by the scenario tree. The time horizon of several hours is discretized into ten-minute increments, leading to a very large multi-stage decision problem. Importance sampling is used to evaluate a subset of all of the scenarios in the scenario tree, with the subset being chosen in the areas that are predicted to have the greatest effect on the objective function.
2.2.3.3 Genetic Algorithms

Genetic algorithms (GA) are an optimization technique based on the theory of natural selection. The idea behind the method is that a potential solution to a problem can be described by numerical "chromosomes", and that these chromosomes determine the "fitness", or objective function value, of the possible solution. Functions simulating the chance of survival, reproduction, and mutation of "individuals" are then applied to a "population", or possible solutions, over a number of "generations", or iterations. At the end of the iterations, the chromosomes of the individual with the highest fitness describe the optimal values of the decision variables.

Wardlaw and Sharif (1999) apply GAs to determine the optimal operation of a multiple-reservoir system for a deterministic problem of finite length. The objective of operating the reservoir system is to maximize system benefits less penalties for constraint violations. Penalties must be included in the objective function because in the GA implementation employed, the values of the decision variables generated are not checked for feasibility elsewhere; in alternative GA formulations only feasible sets of decision variables are generated. The results generated by the GA model for a problem with four release decision variables at twelve points in time, for a total of 48 decision variables, are compared with DDDP, and with LP. The GA model was found to be faster than DDDP, but slower than LP. The largest problem that Wardlaw and Sharif (1999) report solving has a total of roughly 400 decision variables. A drawback of the method is the need to calibrate the parameters that describe the operators on the chromosomes. A combination of GAs and Monte Carlo analysis is applied by Otero et al. (1995) to a multiple-reservoir system.

2.2.3.4 Goal Programming

Goal programming (GP) is a method that can be applied to multi-objective problems in which it is possible to establish a firm order in which the objectives are to be accomplished. Goals for higher-ranking objectives are fulfilled completely before lower ranked objectives are addressed.

Reznicek and Simonovic (1991) employ GP to optimize the operation of a reservoir so as to minimize a loss function based on storage and release targets. The authors state that the advantages of GP over an LP model in which conflicting objectives are weighed against one another are the lesser data requirements and simpler model formulation. Mohan and Keskar (1991) use goal programming to optimize the monthly operation of an irrigation supply reservoir so as to meet storage and release targets. The conclusion reached was that, at least for the system studied, release targets are superior to storage targets for determining the policies for the operation of a multi-purpose reservoir system.

An obvious limitation on GP is that most reservoir operation problems are complex enough that it is not possible to clearly articulate a set of ranked objectives with corresponding target values.
2.2.3.5 Non-linear Programming

Yeh (1985) discusses the use of non-linear programming (NLP) in his review, noting in particular the use of conjugate gradient and Lagrangian gradient methods. Simonovic (1992) includes quadratic programming, geometric programming, and separable programming as NLP techniques that have been applied. Impediments cited by Yeh (1985) and Simonovic (1992) to more widespread use of NLP methods are convergence problems, computer requirements, non-viability for non-separable problems, and the fact that they are difficult to apply to problems involving stochastic inflows. Reasons to use NLP include the ability to handle non-linear constraints and objectives, and the ability with which they can be linked to complex simulation models (Wurbs, 1993; Yeh, 1985).

Recent applications of NLP for the optimization of reservoir operations are by Tejada-Guibert et al. (1990) and Syalla (1994). Syalla (1994) uses a sub-gradient NLP technique for the optimization of a multiple-reservoir system that is to be operated so as to minimize the power deficit relative to a target, and to have this deficit be uniform through time. The model is deterministic, and employs a node-arc formulation. The benefit of directly including non-linear terms in the model is stressed. Syalla (1994) notes that while the model employed is reliable, the computational burden for larger problems is “a major drawback.” Tejada-Guibert et al. (1990) employ MINOS (Murtagh and Saunders, 1983) to operate a reservoir system so as to maximize the value, as opposed to the amount, of energy generated. The value of energy depends upon the time of year and time of day. As with other NLP methods, the optimal solution cannot be guaranteed to be the global optimum, and depends upon the initial estimates of the decision variables.

2.2.3.6 Optimal Control Algorithms

The use of optimal control algorithms (OCA) to optimize the operation of reservoir operations has been reported in the literature.

Grygier and Stedinger (1985) compare the performance of an OCA to a combination of LP and DP and to successive LP for optimizing the operation of a multiple-reservoir system, with a deterministic description of the inflows, over the medium-term. The objective is to maximize the value of power generated, taking into account the value of water in storage at the end of the model time horizon. The OCA cannot guarantee an optimal solution. Grygier and Stedinger (1985) find that, at least for simple systems, the OCA is significantly faster than successive LP, but requires much more time to implement.

Georgakakos et al. (1997) apply an OCA to optimize the operation of a reservoir system that is part of a combined hydro-thermal system. The objective function includes terms for thermal cost savings, dependable capacity, as well as storage bounds and targets. The time step of the model is hourly, and the time horizon is up to several years; Georgakakos
et al. (1997) report that the method is computationally efficient for such problems. The framework of the model is deterministic. The particular OCA employed, the Extended Linear Quadratic Gaussian control method, cannot guarantee a global optimum, and is dependent on an initial feasible solution.

Hayes et al. (1998) apply OCA to the optimal operation of a sub-system of the Tennessee Valley Authority hydroelectric system under water quality constraints.

2.2.3.7 Reliability Programming

Reliability programming can be described as an extension of chance-constrained programming (Reznicek and Cheng, 1991). The extension is that in reliability programming the probability with which constraints must be respected are decision variables, whereas in chance-constrained programming (CCP) they are input parameters. As a result of the extension, reliability programming contains non-linear relationships. Just as with CCP, reliability programming requires knowledge of the risk-loss function, which can be challenging to obtain (Reznicek and Cheng, 1991).

Srinivasan and Simonovic (1994) describe the application of reliability programming to a composite reservoir operated for multiple purposes, including hydropower production, placing emphasis on the incorporation of hydropower production into the reliability programming formulation. The reservoir is to be operated so as to maximize the value of hydropower produced less the losses described by a risk-loss function, arising from failing to meet required reliabilities for energy production and flood control. In the piecewise-linear model of the hydropower production function it is assumed that production is independent of the tailwater. In finding a solution, the algorithm cycles through non-linear search to find the probability limits and linear programming to solve the CCP. Rangarajan et al. (1999) also apply reliability programming to the problem of operating a reservoir for hydropower production while taking into account the losses arising from failure to meet the required load and provide the required flood control. The emphasis in Rangarajan et al. (1999) is placed on the determination of the risk-loss functions, and the sensitivity of the optimal model results to these functions.

2.2.4 Simulation

Although optimization models have received a great deal of attention in the research literature, in particular in the academic community, the institutions responsible for actual reservoir operations have been slow to use optimization, relying instead on simulation models (Yeh, 1985; Wurbs, 1993). Simulation models include widely available, general purpose models such as HEC-3 (Hydrologic Engineering Center, 1971) and HEC-5 (Hydrologic Engineering Center, 1979); custom-built project specific models developed in programming languages such as FORTRAN and C; and models constructed using general purpose software such as Excel, or modelling packages such as STELLA (Wurbs, 1993). Other simulation models employed by operating agencies such as the U.S. Army
Corps of Engineers include SSARR (U.S. Army Corps of Engineers, 1975), HYSSR (U.S. Army Corps of Engineers, 1985), and the Acres model (Sigvaldason, 1976; Bridgeman et al., 1989). Reasons cited for the preference of decision makers for simulation models over optimization models include: (i) the non-involvement of operators in optimization model development; (ii) the simplifications and abstractions required to apply optimization techniques to an actual system; (iii) the generally poor documentation of optimization models reported in the literature; and, (iv) the institutional constraints that impede reservoir operator-research interactions (Yeh, 1985; Simonovic, 1992; Wurbs, 1993; Russell and Campbell, 1996).

The line between simulation and optimization models can be a blurry one, in that some simulation models incorporate optimization techniques, and optimization model results are often interpreted through simulation (Yeh, 1985; Wurbs, 1993). The definition adopted here is that a simulation model can be thought of as a model having a primarily descriptive, as opposed to prescriptive, purpose—that is, the model is not to make recommendations on how a system should be operated, but is to model how the system would respond to specific operating rules. It is this difference in focus that allows simulation models to incorporate details that are of necessity ignored in optimization models, making them important for verifying the results of optimization (Yeh, 1985; Simonovic, 1992).

As this thesis is focused upon the application of optimization methods, the literature on simulation models, both project-specific and general, is not reviewed here. The interested reader is referred to the reviews of Wurbs (1993), Yeh (1985), and Simonovic (1992) for an in-depth treatment of this topic.

2.3 Summary and Conclusions

In this chapter the literature has been reviewed for applications of optimization techniques to the problem of reservoir operation optimization problems. Simulation has also been discussed briefly. The review of the literature reveals that the two most widely used optimization techniques are dynamic programming and linear programming. A number of other methods have been employed to a lesser extent.

The major drawback regarding DP is the exponential increase in computational effort associated with an increase in the number of state variables. Despite this limitation, SDP is one of the two most often applied techniques for the consideration of uncertainty in reservoir operation problems. One of the main advantages of DP is its ability to easily handle non-linearities.

Linear programming has also been widely applied, at least partly due to the widespread availability of LP solvers. The major drawback in the application of LP is the need for the problem to be completely linear; the use of linearization techniques can make this drawback less severe than it may initially appear. A number of techniques have been developed for incorporating uncertainty into LP formulations. One technique that shows
much promise, and has been recommended as a future research area by Yeh (1985), is stochastic linear programming with recourse.

From the review of the literature it can be concluded that DP and LP, particularly SLPR, remain very useful techniques for the optimization of reservoir operations. In the remainder of this thesis the use of DP and SLPR models for the optimization of a two-reservoir system operated to maximize the value of hydropower generated, while considering the value of water remaining in storage, is described.
3 Stochastic Dynamic Programming and Linear Programming Based Model

3.1 Introduction

The operator of a hydroelectric system with significant storage is faced with the problem of trading off the release of water to generate power at any given point in time with storing that water for potential release at a later date. In today’s environment of active markets for electricity trade, a comparison of the marginal value of the water in storage and the value of releasing water can be used in making the tradeoffs. In general, the marginal value of stored water in a reservoir is dependent upon the time of year, the volume stored in the reservoir, and the volumes stored in all other reservoirs in the system.

For example, the market electricity price for the current hour may be $50/MWh. At this price, the operator may want to consider generating electricity for export. Taking into account transmission losses and wheeling charges, the net price for exports that the operator would realize would be less, say $45/MWh. This energy price can be converted into a water price using the conversion rate between power production and turbine discharge. Supposing that the appropriate conversion rate is 1.5 MW/cms, the water price would be $30/cmsh, or $720/cmsd. The current export price for water could then be compared against the marginal value of water in storage. If the marginal value of water in storage exceeds the export water price, then a sale should not be made. On the other hand, if the export water price exceeds the marginal value of water in storage, then a sale could be profitably made. Similarly, a comparison could be made between an import water price and the marginal value of water in storage. If the import water price is less than the marginal value of water in storage, then the import could be profitably made. Conversely, if the import water price is greater than the marginal value of water in storage, then the purchase should not be made.

Another aspect involved in making the tradeoffs is weighing a certain price today against an uncertain marginal value in the future. The selected discount rate should reflect this fact. Uncertainty in the future marginal value arises because the estimate relies upon assumptions about future demands, inflows, and prices—all of which are inherently uncertain. For example, if inflows significantly exceed the forecast values, it may be impossible to operate a reservoir such that it does not spill. When a reservoir is spilling, the marginal value of water will be zero (and could actually be negative if spill causes damage) which will, in general, be less than the marginal value for non-spill conditions.

A model has been developed to estimate the marginal values of stored water over the medium- to long-term in order to aid the decision-maker in making the operating decisions discussed above. The purpose of the model is to generate storage water value curves (the slopes of which are the marginal water values) that can be used to value the water remaining in storage for models with a shorter time horizon. The model combines the use of backward-moving dynamic programming (DP) to link time periods together,
and the use of linear programming (LP) to determine the operation of the hydropower system and electricity trades within each time period. The LP also determines the end of period storage volumes.

Uncertainty is introduced into the model through scenarios. The scenario-dependent parameters are demands, inflows, and prices. The model is limited to the consideration of two reservoirs, which must be hydraulically separate from one another. The limited number of reservoirs that can be considered is a function of the well known “curse of dimensionality” that afflicts dynamic programming (DP). The apparently restricted range of applicability of the model can be extended using aggregate reservoirs, a technique that has been used in previous research (e.g., Turgeon and Charbonneau, 1998; Valdes et al., 1992). For example, the two reservoirs could be used to represent two separate river systems, each reduced into a single aggregate reservoir. Alternatively, one reservoir could represent either a single reservoir or the aggregate reservoir for a river system, and the second reservoir could represent an aggregate reservoir that replaces the remainder of the plants in the hydroelectric system.

The remainder of this chapter outlines the model and its application to two multiple-year storage facilities that are part of the BC Hydro reservoir system. The case study demonstrates the value of the model through its ability to calculate the marginal values for both reservoirs, as well as identifying regions of storage over which the reservoirs could be modelled individually while addressing the uncertainty in demands, electricity prices, and inflows.

### 3.2 DP and LP Based Model Overview

A model overview that will set a framework under which the model details can be discussed is presented in this section. In order to help elucidate the description of the model, terminology is presented first.

#### 3.2.1 Terminology

The DP model considers a **time interval** that is divided into an ordered set of **time periods**. The length of each time period in the set can differ. For example, one natural division of a time interval into a set of time periods is that of dividing a year into months. In this case, the time interval would be one year, there would be 12 time periods in the set, and the length of each time period would be the number of days in the month. The fact that the set is ordered means, for example, that January can be specified to chronologically precede February. In every **DP iteration** each time period in the time interval is considered. Continuing the example, one DP iteration would consider each of the 12 months in the year. The use of a recurring time interval divided into a set of fixed time periods means that the length of the time interval and time periods cannot vary with the DP iteration. Thus, in the example, each year must have the same number of days, as must each February. The DP algorithm is **backward moving**, meaning that the ordered set of time periods is moved through in anti-chronological order.
calculations start for the month of February, then the next time period considered is January, and the last time period considered in an iteration is March.

In standard DP terminology, the points at which decisions are made are known as stages. For the case here, the stages correspond to the start of each time period. In the example, a stage would correspond to the start of a month.

Uncertainty in the DP model is handled through scenarios. The number of scenarios in each time period can vary. A scenario is defined by the values specified for the scenario-dependent parameters. The scenario-dependent parameters are demands, inflows, and import and export prices. Scenario probabilities specify the probability with which each scenario is expected to occur. Note that since the scenarios are related to the time periods, the scenarios and scenario probabilities also recur in each time period in each time interval.

In the DP, the storage volume in each reservoir is described by discretized storage volumes. The discretized storage volumes for a reservoir are contained in the set of possible storage volumes at the beginning of each time period. The storage volume for a reservoir at the end of a time period is found by the LP algorithm, and is not restricted to the set of discretized storage volumes. In standard DP terminology, the values that can be assumed by a decision variable are known as states. In the current case, the discretized storage volumes are the states.

The LP algorithm is run for a time horizon, equal in length to one of the DP time periods, in order to evaluate the DP objective function. For example, the LP algorithm may be run for a time horizon equal to the number of days in the month of February. The time horizon in the LP can be divided into sub-periods, thus allowing the LP to consider time in greater detail than does the DP. For example, the time horizon in an LP may be divided into two sub-periods: one for the heavy load hours (HLH) and a second for the light load hours (LLH). Note that in this example, all of the monthly HLH would be grouped into one sub-period and all of the monthly LLH would be grouped into a second sub-period.

The outputs from the LP algorithm are an objective function value, and a policy. The objective function value is equal to the maximized scenario probability weighted sum of the export revenues, import and penalty costs, and the discounted terminal value of water remaining in storage. The policy is the vector of turbine releases that is optimal over all scenarios. Within each scenario, subject to the optimal turbine release, the values of ending storage and spill are found for each reservoir, and imports and exports for each sub-period, are calculated assuming perfect foresight within the scenario.

The storage values are derived from the objective function values. The storage value for a particular discretized state (in the two-reservoir case, the state is two-dimensional) is equal to the objective function value for that discretized state minus the minimum objective function value for any discretized state.
3.2.2 Model Overview

Data that must be input by the user include the time interval, the number of time periods, the length of each time period, the number of sub-periods, the length of each sub-period, and the period from which the run should commence. The user must provide the number of scenarios for each time period, and the scenario probability and scenario-dependent parameters for each scenario. In addition, all of the remaining sets and parameters outlined in sections 3.3.1.1 and 3.3.2.1 below must be provided.

The DP algorithm starts at some point that is far enough in the future such that the water remaining in storage at the end of the first time period can be considered valueless—i.e., the system operator is indifferent to the terminal storage values at the end of this time period. A storage value is found for each discretized state. In order to calculate the storage values, objective function values must be calculated for each discretized storage volume combination. Each objective function value is found by iteratively solving an LP until convergence. Iteration is required in the LP solution as the problem being modelled is non-linear.

The rate at which turbine discharge can be converted into power is a non-linear function of reservoir storage, and is approximated in the model by a piecewise-linear function. In the LP model, a single conversion rate is used for each reservoir for the entire time horizon. The rate used is an average of that for the known starting reservoir volume and that for the reservoir volume at the time horizon, which is one of the decision variables in the LP model. Convergence is recognized when the ending storage volume used in calculating the conversion rate and the optimal ending storage volume found by the LP fall within a specified tolerance of one another. The estimate of the ending storage volume used in the first iteration is equal to the starting storage volume, which is one of the discretized storage volumes. In subsequent iterations, the estimate of the ending storage volume is the ending storage volume found by the LP in the previous iteration.

The value of water in storage at the time horizon also depends upon the ending storage volumes. The value of water stored in each reservoir is a function of the ending storage volumes of both reservoirs. The storage value curves used in the LP model are functions of the estimated storage volumes at the end of the time horizon. When convergence is recognized, the forecast ending reservoir volumes (equal to the ending reservoir volumes found by the previous iteration of the LP) are equal to the ending reservoir volumes found by the LP.

Once the objective function value has been calculated for each discretized storage volume state, the storage values can be determined. The storage values are calculated by finding the minimum objective function value for any of the states, and subtracting this value from all of the objective function values. The reason for performing the subtraction is to remove a constant value from the optimization. The storage value can be thought of as the amount by which the value of a particular storage volume state exceeds the value of the worst state. The worst (in terms of objective function values) case will occur for the lowest discretized storage volumes.
After the storage values have been calculated, they are checked to ensure that convexity requirements are satisfied. Convexity problems may arise when the LP iterations end because a maximum number have been performed. If the convexity check fails, either the storage values, or the slopes of the line segments connecting storage values for the discretized storage volumes—the marginal storage values—are modified in order to restore convexity. At this point, the storage value table can be used to generate piecewise-linear curves that can be used to value the end of period storage when the DP steps backward in time by one period.

The storage value table consists of storage values for each possible combination of the discretized storage volume for the two reservoirs. The information in the table is a storage value that can be achieved for a given combination of discretized storage volumes. Thus, the storage value is associated with a pair of storage volumes. The table specifies a single storage value that is obtained when each of the two reservoirs is at a discretized storage volume; the storage value table does not indicate the respective contributions of the two reservoirs to the total storage value. However, the piecewise-linear curves that are used in the LP to provide the value of water remaining in storage at the end of the time horizon must be functions of a single storage volume.

Thus, two piecewise-linear storage value curves must be produced—one for each reservoir. Each curve gives the contribution of one reservoir towards the total storage value as a function of that same reservoir’s storage volume. As the total storage value is a function of the storage volume in both reservoirs, each curve is valid for a particular storage volume in the other reservoir. The requirement for these curves is that if a pair of end of time horizon storage volumes corresponds to a pair of discretized storage volumes, then the sum of the two piecewise-linear functions should equal the value in the storage value table corresponding to the two storage volumes. In other words, the results from the two piecewise-linear curves should correspond to the results of the storage value table.

The two piecewise-linear storage value functions are derived from the storage value table by leaving the marginal storage values in the storage value table unchanged, and altering the storage value intercepts. (A storage value intercept is the ordinate axis intercept on a plot of storage value versus storage in one reservoir for a specified storage in the second reservoir.) The storage value intercepts must be changed such that for the forecast ending storage volumes, the sum of the piecewise-linear functions is equal to the appropriate value in the storage value table. In this method, it is assumed that the two storage value intercepts for the two reservoirs are equal. The sum of the two revised storage value intercepts is set equal to the sum of the two unaltered storage value intercepts less the appropriate storage value table entry.

The above DP process is repeated for the next time period as the DP steps backwards. The newly calculated storage value curves become input to this next time period through the LP. The process is repeated for the set of time periods in the DP time interval. DP iterations continue until a convergence criterion is met.
The convergence criterion is based upon changes in the storage values between DP iterations. For each time period, the percentage change in the storage value from the previous iteration is calculated for each discretized storage volume state. The maximum percentage change for each time period is stored. In order for convergence to be recognized, all of the maximum percentage changes must be less than a specified tolerance.

Once convergence has been achieved (or a limiting number of DP iterations have been performed) one final DP iteration is performed. In this final step, during each time period, the optimal policy is stored for each discretized state. The policy specifies the turbine discharges, and the remainder of the optimal operation of the system over all scenarios, including the storage volumes at the time horizon, and spills, generation, imports and exports for each sub-period.

The manner in which scenarios are used has implications for the types of correlations that can be handled in the model. Correlations between scenario-dependent parameters—e.g., inflows to the two reservoirs—within a time period are handled through scenarios and scenario probabilities. The model does not allow for the correlation of a scenario-dependent parameter between time periods. That is, the inflow to a reservoir in one period cannot depend upon the inflow in the previous period; the demand in one period cannot depend upon the inflow in the previous period; nor can the price in one period depend upon the price in the previous period. This restriction is a limitation of the model, and a departure from the typical use (e.g., Little, 1955; Butcher, 1971; Karamouz and Houck, 1987; Tai and Goulter, 1987; and Karamouz and Vasiliadis, 1992) of a lag-one Markov process to represent reservoir inflows. Some recent models have also used a lag-one Markov process to represent energy prices on the spot market (Mo et al., 1998). The simplification has been made in order to reduce the number of state variables, and thus reduce the effects of the curse of dimensionality; similar assumptions have been made in other work (Druce, 1989, Druce 1990).

One additional difference between a typical DP model and the formulation employed here must be noted. Normally, DP is used to find the optimal transition from a state in one time step to a state in an adjacent time step. In the case of reservoir operations, the states would typically include discretized reservoir storage volumes. In the formulation employed here, the ending storage volumes are decision variables in the LP algorithm, and are not restricted to discretized values. The information generally contained in the ending storage states is contained in the piecewise-linear storage value curves. In the storage value curves, the discretized storage volumes are the breakpoints between linear segments. Thus, at each discretized storage volume the marginal value of stored water can potentially change. Between discretized storage volume points, the marginal value of stored water is a constant. The stepwise constant marginal values are used in the LP model in determining the optimal use of water over the time horizon.

When the end of period storage is restricted to discretized state values, the potential arises for forcing the "artificial" spilling of water. This problem would be particularly acute if,
in trying to increase the speed of the program, a coarse state space discretization was
used. Such a spill could occur if the inflow to a reservoir exceeded the maximum
discharge capacity of the turbines. Suppose that the inflow is higher than the turbine
capacity, the turbines are generating at maximum capacity with any excess generation
above domestic demand being exported, and that there is no spill. In this case, the
continuity equation would specify that the amount of water stored in the reservoir is
increasing with time. The additional storage volume is equal to the product of the time
step and the amount by which the inflow rate exceeds the turbine discharge rate. If this
additional storage volume is less than the increment in the discretized storage volumes,
then the next discretized state point cannot be reached. As a result, all of the additional
storage volume would need to be spilled. Such a spill would not be representative of
actual reservoir operations, and thus artificial. In the formulation employed here, the
additional storage volume could be stored, and the objective function would be greater by
the product of the marginal value of stored water and the additional storage volume.

This section has described the DP and LP based model in general terms. In section 3.3,
the details of the model are provided.

3.3 Model Details

In section 3.2, the DP and LP based model was outlined in general terms. In this section,
the details of the model, including a mathematical formulation, are presented. As implied
by the name, the DP and LP based model contains both DP and LP components. The DP
component operates at a higher level than the LP component, in that the LP component is
used to evaluate the DP objective function.

The uncertain natures of reservoir inflows, energy prices, and energy demands are
handled through the use of scenarios at the DP level. A scenario is defined by the values
specified for the scenario-dependent parameters—the energy demands, energy prices, and
reservoir inflows. The probability of occurrence of each scenario is defined by a scenario
probability. The scenario probabilities must sum to one for each stage. Correlations
between scenario-dependent parameters—e.g., inflows to the two reservoirs, or energy
demand and reservoir inflow—within a time period are handled through scenarios and
scenario probabilities. The model does not allow for the correlation of a scenario-
dependent parameter between time periods. That is, the inflow to a reservoir in one
period cannot depend upon the inflow in the previous period; the demand in one period
cannot depend upon the inflow in the previous period; nor can the price in one period
depend upon the price in the previous period. The simplification has been made in order
to reduce the number of state variables, and thus lessen the effects of the curse of
dimensionality.

3.3.1 Dynamic Programming Model
The use of dynamic programming in water resources problems is well established. For examples of the use of dynamic programming in the field, see Yeh (1985), Yakowitz (1982), Wurbs (1993), and Esogbue (1986).

Dynamic programming is used here to calculate the value of water and the marginal value of water for each reservoir, at the end of each period. In general, the value of stored water in each reservoir depends upon the amount of water stored in all reservoirs, as well as the time of year.

The mathematical formulation of the DP model is presented next.

### 3.3.1.1 Mathematical Formulation

Prior to presenting the mathematical formulation of the DP, some terminology is presented.

**Terminology**

Dynamic programming is best thought of as an approach to optimization, not as a particular technique. As a much-used optimization method, DP has developed its own terminology; this jargon is presented here.

DP is a method that can be used to make optimal decisions at a number of decision-making points. Often these points are times, but this is not a requirement. The points at which decisions must be made are referred to as stages. In DP, the decisions that are to be made are functions of some important model components. The decisions are of the form: “If, for a given stage, the values of the key model components have these particular values, then take that particular action.” The important model components are referred to as state variables. In DP, each state variable is typically discretized into a finite number of values. A vector composed of one discretized value for each state variable in the model is known as a state. In DP, it is common to refer to being in a particular state at a particular stage.

It is important to note that an optimal decision is found for each state. Associated with each optimal decision is an ending state; that is, an optimal decision defines the best transition from a particular state for a given stage to a particular state at an adjacent stage, as measured by an objective function. The optimal transition from one state to another is typically referred to as a policy. A policy simply specifies the action to take if a particular state occurs during a particular stage.

**Stages**
In section 3.2.1, it was outlined that in the DP model a time interval is divided into a set of time periods. The duration of the time periods can differ; for example, a time interval of one year may be divided into twelve monthly time periods. The stages in the model correspond to the beginning of the time periods.

**States**

In DP, optimal policies are found as functions of the state variables. The state variables used in the DP model are the storage volumes in the reservoirs. There is one state variable for each reservoir in the modelled system.

**Notation**

The sets, parameters, and state variables used in the DP model are defined next. Loucks et al. (1981) has been used as a guide for establishing some of the following notation.

**Sets**

Sets are used to index components of the model. The sets used are presented here.

Let the set of reservoirs in the system be represented by \( \{R\} \). In the model, \( \{R\} \) is limited to two hydraulically separate reservoirs.

Let the set of scenarios for stage \( t \) be represented by \( \{\Omega\} \). A scenario specifies values for the scenario-dependent demands, inflows, and prices.

Let \( \{D_r\} \) represent the set of discretized storage volumes for reservoir \( r \in \{R\} \).

Let \( \{G_r\} \) represent the set of discretized inflows for reservoir \( r \in \{R\} \).

Let \( \{J_r\} \) represent the set of releases for reservoir \( r \in \{R\} \).

**Parameters**

Let \( NR \) represent the number of reservoirs included in the model.

Let \( g_r \) represent the inflow volume for reservoir \( r \in \{1, \ldots, NR\} \) in stage \( t \); \( g_r \in \{G_r\} \).

Let \( j_r \) represent the reservoir release volume for reservoir \( r \in \{1, \ldots, NR\} \) in stage \( t \); \( j_r \in \{J_r\} \).

Let \( \omega_l \) represent the reservoir storage volume for reservoir \( r \in \{1, \ldots, NR\} \) under scenario \( \omega \in \{\Omega\} \) in stage \( t+1 \).
Let $n$ represent the number of stages to go until $T+1$, including the current period, $t$.

Let $NP$ represent the number of stages (time periods) in the time interval considered in one iteration.

Let $NS_r$ represent the number of discretized storage volumes for reservoir $r \in \{R\}$.

Let $\omega P'$ represent the probability of occurrence of scenario $\omega$ for stage $t$.

Let $SV_t^n$ represent the storage value for stage $t$.

Let $T$ represent the number of stages at which the objective function has been evaluated when convergence is recognized. The DP model is not concerned with operation of the reservoir system beyond stage $T+1$.

Let $\varepsilon$ represent the DP convergence recognition tolerance.

Let $\beta$ represent the discount factor for stage $t$.

Let $\omega \in \{\Omega_t\}$ represent a scenario for stage $t$.

**State Variables**

Let $k_r$ represent the reservoir storage volume for reservoir $r \in \{1, ..., NR\}$ in stage $t$; $k_r \in \{D_r\}$.

**Recursive Equation**

In dynamic programming, the aim is to find the policy maximizing the sum of the return from the current period and the return from the end of the current period until the model horizon.

Let $\omega B_{k_1, ..., k_{NR}, k_{t+1}}^{t, \alpha}$ represent the return during the current period given that the storage volume in reservoir $r$ for stage $t$ is $k_r$; the storage volume in reservoir $r$ for stage $t+1$ is $\alpha l_r$; the stage is $t$; and the scenario is $\alpha$. The return during the period is given by the sum of the net revenue generated through energy trades and penalties for constraint violations.

Let $\omega f_{t, n}(k_1, ..., k_{NR})$ represent the total value of system performance with $n$ periods to go until the horizon $T+1$, including the current period $t$, given that in period $t$ the initial storage volume in reservoir $r \in \{R\}$ is $k_r$ and that the scenario is $\alpha$. 

40
The objective function, which combines the return from the current period and the return from the end of the current period until the model horizon, is given as

$$f^a_t(k_1, ..., k_{NR}) = \max_{j_1, ..., j_{NR}} \left[ \sum_{\omega \in \Omega} \beta^\omega P_t \left( \omega^B_{k_1, ..., k_{NR}}, \omega^f_{j_1, ..., j_{NR}, t} + \omega^f_{f+1-t} (\omega l_1, ..., \omega l_{NR}) \right) \right]$$ (3-1)

$$\forall (k_1, ..., k_{NR}); (j_1, ..., j_{NR}) \text{ feasible for } (g_1, ..., g_{NR}), (k_1, ..., k_{NR}), \text{ and } (l_1, ..., l_{NR}), \omega_\alpha \text{ and subject to:}$$

$$k_i + g_i - j_i = l_j.$$ (3-2)

In equation (3-1), note that it is assumed that returns in the current period occur at the end of the period. Further, note that the total return for stage $t$ is dependent upon the total return for stage $t+1$. This dependence between stages has led to equations of the form of (3-1) being referred to as “recursive equations”.

The approach taken to evaluating (3-1) is presented below in section 3.3.1.2.

### 3.3.1.2 Solution Methodology

In section 3.3.1.1, a mathematical formulation of the DP model is presented. In this section, an approach to solving the model is presented.

**Algorithm**

Dynamic programming describes a general approach to decision making, not a specific technique. DP algorithms can be classified as either forward- or backward-moving. This categorization specifies the order in which stages are considered by the model. In forward-moving DP, the recursive equation is evaluated for stage $t$ prior to stage $t+1$, whereas the converse is true for backward-moving DP. For stochastic dynamic programming, only backward-moving DP is applicable (Yeh, 1985).

In the backward-moving DP algorithm employed here, the total returns for stage $t+1$ are calculated before the total returns for stage $t$. Consequently, an assumption needs to be made regarding the values of $\omega^f_{T+1} (k_1, ..., k_{NR})$ when calculations begin at stage $T$. The assumption used is that

$$\omega^f_{T+1} (k_1, ..., k_{NR}) = 0; \quad \forall \omega, k_1, ..., k_{NR}.$$ (3-3)

Equation (3-3) implies that at stage $T+1$, the system operator is indifferent to the volume stored in the system reservoirs. At this stage, the total value of system performance for all remaining periods is equal to zero under all scenarios. Obviously, stage $T+1$ occurs at some point far in the future. When the DP calculations begin, it is unknown how far into
the future stage \( T+1 \) lies. The recursive equation (3-1) is evaluated a sufficient number of times such that the assumption expressed in (3-3) can be held to be valid. Details of the criteria required for convergence are specified later in this section.

Evaluation of the iterative equation begins at stage \( T \). Under the assumption of equation (3-3), the iterative equation for stage \( T \) becomes

\[
\begin{align*}
   f_T(k_1, \ldots, k_{NR}) &= \max_{j_1, \ldots, j_{NR}} \left[ \sum_{a \in \Omega_i} \beta_{ij} \cdot \left( \omega B_{k_1, \ldots, k_{NR}, a_1, \ldots, a_{NR}, T} + \omega f_{T+1}^0(a_1, \ldots, a_{NR}) \right) \right] \\
   &= \max_{j_1, \ldots, j_{NR}} \left[ \sum_{a \in \Omega_i} \beta_{ij} \cdot \left( \omega B_{k_1, \ldots, k_{NR}, a_1, \ldots, a_{NR}, T} \right) \right].
\end{align*}
\] (3-4)

Following the evaluation of equation (3-4) for all discretized storage volume states \((k_1, \ldots, k_{NR})\), calculations proceed for stage \( t = T-1 \) using equation (3-1). Equation (3-1) is then used to successively evaluate stages until convergence recognition.

**Convergence**

Convergence should be recognized when equation (3-1) has been evaluated for a sufficient number of stages such that the assumption of equation (3-4) is valid. The assumption contained in equation (3-4) is that the system operator is indifferent to all ending states of the system. In order to determine if a sufficient number of stages have been evaluated, the concept of "storage values" is used. The storage value is the amount by which the recursive equation for a particular state in a given stage exceeds the minimum value, over all states, of the recursive equation for that stage. Thus, storage value, \( SV_i^n \), is defined as:

\[
SV_i^n(k_1, \ldots, k_{NR}) = f_i^n(k_1, \ldots, k_{NR}) - \min_{k_1, \ldots, k_{NR}} \left( f_i^n(k_1, \ldots, k_{NR}) \right).
\] (3-5)

Convergence is recognized when

\[
\max_{k_1, \ldots, k_{NR}} \left| SV_i^n(k_1, \ldots, k_{NR}) - SV_{i+1}^{n-NP}(k_1, \ldots, k_{NR}) \right| \leq \varepsilon;
\] \( \forall k_1, \ldots, k_{NR}, t \in \{1, \ldots, NP\} \) (3-6)

where \( \varepsilon \) is a specified tolerance. Equation (3-6) states that convergence is recognized when the maximum percentage change in storage value for each state in each stage within the iteration is less than a specified tolerance. An upper limit is set on the number of iterations that are to be performed. Thus, in some cases, the assumption of equation (3-4) may not strictly hold.

**Discussion**
The DP recursive equation defined by (3-1) differs in several respects from what one may term a “typical” DP problem (e.g., Loucks et al., 1981). These differences are described next.

**Ending State Discretization**

One important difference between a typical formulation and the one employed here has to do with the discretization of the ending states. Generally, the ending states are limited to the set of discretized starting states. Thus, a policy for a particular starting state would typically indicate which member of the finite set of feasible ending states is optimal. In the formulation used here, the ending state is not limited to a finite set of states; rather the values of the ending state variables are continuous.

To this point, little attention has been given to the two components of the recursive equation, or to the means by which the maximization in equation (3-1) is performed. The immediate return function, \( v_B \), takes into account the revenue obtained through the export of energy; the cost incurred through the import of energy; and the cost of any soft constraint violations. These costs and revenues should be those associated with the optimal operation of the reservoir system, subject to all appropriate constraints. The value of the system being in a particular state at the end of stage \( t \), \( v f_{t+1}(a_1, ..., a_{NR}) \), takes into account the value of water in storage. Stored water has value as it can be used in future periods.

The maximization in equation (3-1) is performed using linear programming. The objective function in the LP contains terms for the three components of the immediate return function. The LP objective function contains additional terms for the value of water remaining in storage at the end of the stage. Details of the LP model are presented in section 3.3.2. Linear programs employ continuous variables; hence the ending state variables cannot be limited to discretized states. The use of continuous ending state variables is discussed next.

For each starting state, \((k_t, ..., k_{NR})\), a linear program is solved in order to obtain a value for the total return function. The LP calculates the optimal operation of the reservoir system for the stage, starting from the given state, for each scenario. The optimal operation includes the reservoir operations and energy trades to make during the period, as well as the optimal ending values for the state variables at the end of the stage. In order to determine the optimal ending values of the state variables, it is necessary to specify functions giving the value of ending the stage with a particular state variable value. That is, functions providing the value of ending a stage with particular volumes of water stored in the reservoirs must be input to the LP. The types of functions that can be used in a linear program are limited to linear and piecewise-linear functions of the decision variables.

Piecewise-linear functions are used to specify the value of ending a stage with particular volumes of water stored in the reservoirs. The functions are generated by linearly
interpolating between the objective function values calculated for the states in the previous stage. That is, the objective function, as specified by equation (3-1) is first evaluated for all states, \((k_t, ..., k_{NR})\), in stage \(t+1\). The result is an objective function value for each discrete starting state. Piecewise-linear functions are then obtained by linearly interpolating between the discrete points. The intercepts of the objective function axes of the piecewise-linear functions for the two reservoirs are adjusted so as to avoid double counting in the objective function. These functions are used by the LP to assign a value to the ending states when equation (3-1) is solved for each discrete starting state in stage \(t\). The procedure is repeated as the DP proceeds backward through the stages in anti-chronological order. For the calculations at stage \(T\) (the stage at which recursion begins), the assumption is made that the value of system operation at stage \(T+1\) is equal to zero for all possible values of the state variables. This assumption is a restating of equation (3-3).

The maximization in equation (3-1) that is performed using LP contains a term for the immediate benefit and a term that accounts for the value of the state variables at the start of the next stage. Of these two terms, consider the latter, which is \(\tilde{m}_{f, l} (a_{l_1}, a_{l_2})\). The term states that the value of system operation from stage \(t+1\) until the horizon is a function of all of the state variables, which are the two reservoir storage volumes. In the LP, it is necessary to represent this term in the objective function. A difficulty arises, as in LP it is not possible to specify, using a piecewise-linear function, that the value of future system operation is a function of two reservoir volumes, each of which is a decision variable in the LP. As a result, an approximation is made.

The approximation used is:

\[
\tilde{m}_{f, l} (a_{l_1}, a_{l_2}) \approx \sum_{r \in \{R\}} \tilde{m}_{e, l+1, r} (a_{l_r})
\]

where \(\tilde{m}_{e, l+1, r} (a_{l_r})\) is a piecewise-linear function giving the future value of system operation solely dependent on the storage in reservoir \(r\).

Equation (3-7) states that the value of future system operation commencing from a particular state is approximated by a sum of piecewise-linear functions. The sum contains one piecewise-linear function for each reservoir. Each piecewise-linear curve gives the value of future system operation as a function of one reservoir—i.e., it is assumed that the value of future system operation in any reservoir can be assumed to be independent of the volume stored in all other reservoirs. In reality, the piecewise-linear function describing the storage value of one reservoir should be a function of the storage in the second reservoir. That is, for different ending levels of the second reservoir, there would be different piecewise-linear curves for the first reservoir. The piecewise-linear curves used depend upon forecasts of the ending volumes in the two reservoirs. Thus, for each starting state, the recursive equation must be repeatedly evaluated by solving instances of the linear program described in section 3.3.2 until the forecast and optimal ending volumes fall within a specified tolerance of one another.
As outlined in section 3.3.1.1, the DP described herein makes use of scenarios. A scenario is described by the values assumed by the scenario-dependent parameters—the demands, inflows, and energy prices. The occurrence probability of each scenario must be specified. The sum of the occurrence probabilities in each stage must sum to one:

\[
\sum_{a \in \{\Omega_{t}\}} p' = 1.0; \quad \forall t. \tag{3-8}
\]

In equation (3-1), optimal policies, in the form of a vector of turbine releases, are found for each state \(\{k_{1}, \ldots, k_{NIR}\}\). As explained previously, apart from the turbine release vector, which is the same over all scenarios, the reservoir system operation as given by spills, ending reservoir storage volumes, and energy trades, can vary with the scenario, and are found assuming perfect foresight within the scenario.

Typically (e.g., Loucks et al., 1981), one optimal ending state is found for each starting state. This optimal ending state is the one that maximizes the sum of the current period return, defined by start and end storage, and the probability weighted terminal value function. An additional dimension (or dimensions) would typically be added to the state in order to account for the scenario-dependent parameters. For example, in the case of a one-reservoir system, the system state may be described by a discretized reservoir volume and a discretized inflow value. In this instance, the policy would be a function of both the starting volume and an inflow.

The "curse of dimensionality" restricts the number of state variables that can be feasibly included. Here, the decision has been made to only use the reservoir volumes as state variables. Including additional state variables would severely affect the practicality of the algorithm for the two-reservoir case considered here. Having the ability to handle two reservoirs is vital to the present work, so other state variables have been excluded from the model. A consequence of this exclusion is that the future value of system performance is solely dependent upon the volume stored in the two reservoirs in the system. Or, to restate matters, the future value of system performance is not a function of the current demand, inflow, or price. This restriction is a limitation of the model. The purpose of the model is to estimate the value of water stored in each reservoir at some future point. Once calculated, these future water values are to provide end conditions to models with a shorter time horizon. Given that storage values are to be calculated for some point in the future, the restriction is justifiable. In an ideal case, the size of the state space could be expanded to include the reservoir storage volumes as well as the scenario-dependent parameters. However, the curse of dimensionality appears to preclude this possibility at present.

The preceding discussion raises a point that warrants elaboration. Because reservoir inflow is not a state variable, inflows cannot be assumed to follow a lag-one Markov process as is typical (e.g., Little, 1955; Butcher, 1971; Karamouz and Houck, 1987; Tai and Goulter, 1987; and Karamouz and Vasiladis, 1992). Similarly, Markov processes cannot be used for either the demands or prices. Thus, the assumption is made that the
demands, inflows, and prices in one stage are independent of the values in the preceding stage. This assumption is necessitated by the "curse of dimensionality". The scenarios can be defined in such a way as to capture cross-correlations between scenario-dependent parameters within a stage. For example, correlations between the inflows to two reservoirs can be captured, as can a correlation between inflow and price. Yeh (1985) notes that cross-correlations are generally ignored in SDP in order to reduce the dimension of the problem.

**Implementation**

The DP model described above has been implemented in a computer program written in AMPL (Fourer et al., 1993). The evaluation of the objective function is performed using a sub-routine that solves a series of linear programs. CPLEX (CPLEX Optimization, Inc., 1995) has been used as the LP solver. The linear programming sub-routine is described in section 3.3.2.

**3.3.2 Linear Programming Model**

As described in section 3.3.1, a linear programming (LP) routine is used to evaluate the dynamic programming (DP) recursive equation. This section describes the LP model.

**3.3.2.1 Mathematical Formulation**

In order to define the LP problem mathematically, the parameters, sets, and variables must first be defined. Following these definitions, the model constraints and objective are presented.

**Sets**

Sets are used to index parameters, variables, and constraints. Sets must be input to the LP. The sets in the LP model are specified below.

Let the set of reservoirs in the system be represented by \( \{R\} \). The membership of \( \{R\} \) is limited to two hydraulically separate reservoirs.

In order to differentiate between energy prices during different times within a period—e.g., heavy load hours and light load hours—the time horizon must be divided into sub-periods. Let the set of sub-periods be represented by \( \{U\} \).

Let the set of scenarios be represented by \( \{\Omega\} \).

**Parameters**
Model parameters are values that must be input to the LP, either through data files, or through pre-processing. The values of the decision variables found by the LP depend upon the parameters. The parameters in the LP model are specified below.

Let $\Delta t_u$ represent the length in days of sub-period $u \in \{U\}$.

Let $\theta p_u$ represent the market price, prior to discounting, of energy in $/MWh, under scenario $\omega$, during sub-period $u \in \{U\}$.

Let $\theta p r$ represent the occurrence probability of scenario $\omega$.

Let $\theta d_u$ represent the firm power demand, including losses, in MW, under scenario $\omega$, during sub-period $u \in \{U\}$.

Let $b$ represent the annual discount rate as a decimal fraction.

Let $\beta$ represent the discount factor: $\beta = (1 + b)^{\frac{1}{365}}$.

Let $q_r$ represent the maximum turbine discharge in cms from reservoir $r \in \{R\}$ and $q_r$ the minimum turbine discharge in cms from reservoir $r \in \{R\}$.

Let $s_r$ represent the maximum spill in cms from reservoir $r \in \{R\}$ and $s_r$ the minimum spill in cms from reservoir $r \in \{R\}$.

Let $o_r$ represent the maximum total plant discharge in cms from reservoir $r \in \{R\}$ and $o_r$ the minimum total plant discharge in cms from reservoir $r \in \{R\}$.

Let $v_r$ represent the maximum storage volume in cmsd in reservoir $r \in \{R\}$, and $v_r$ the minimum storage volume in cmsd in reservoir $r \in \{R\}$.

Let $v^0_r$ represent the initial storage volume in reservoir $r \in \{R\}$.

Let $g_r$ represent the maximum generation limit in MW for reservoir $r \in \{R\}$ and $g_r$ the minimum generation limit in MW for reservoir $r \in \{R\}$.

Let $m_u$ represent the maximum import limit in MW during sub-period $u \in \{U\}$, and $m_u$ the minimum import limit.

Let $x_u$ represent the maximum export limit in MW during sub-period $u \in \{U\}$, and $x_u$ the minimum export limit.
Let $\alpha HK_r$ represent the conversion factor between turbine discharge and power generation in MW/cms, under scenario $\omega$, for reservoir $r \in \{R\}$.

Let $y$ represent the decimal fraction by which the purchase price of energy should be increased above the market price to account for wheeling and losses.

Let $z$ represent the decimal fraction by which the sale price of energy should be decreased below the market price to account for wheeling and losses.

Let $\alpha i_{u,r}$ represent the inflow to reservoir $r \in \{R\}$, under scenario $\omega$, during sub-period $u \in \{U\}$.

**Variables**

The LP finds values of the decision variables that maximize the objective function, subject to the specified constraints. In other words, the decision variables are the results being sought from the model. The decision variables in the LP model are specified below.

Let $V_{r}^{0}$ represent the storage volume in cmsd in reservoir $r \in \{R\}$ at the start of the time period.

Let $\alpha Q_{u,r}$ represent the turbine discharge in cms from reservoir $r \in \{R\}$, under scenario $\omega$, during sub-period $u \in \{U\}$.

Let $\alpha S_{u,r}$ represent the spill in cms from reservoir $r \in \{R\}$, under scenario $\omega$, during sub-period $u \in \{U\}$.

Let $\alpha O_{u,r}$ represent the total plant discharge in cms from reservoir $r \in \{R\}$, under scenario $\omega$, during sub-period $u \in \{U\}$.

Let $^G_r^r$ represent the generation in MW from reservoir $r \in \{R\}$, under scenario $\omega$, during sub-period $u \in \{U\}$.

Let $^0 V_r$ represent the storage volume in reservoir $r \in \{R\}$, under scenario $\omega$, at the end of the time period.

Let $^0 M_u$ represent the power imported in MW, under scenario $\omega$, during sub-period $u \in \{U\}$.

Let $^0 X_u$ represent the power exported in MW, under scenario $\omega$, during sub-period $u \in \{U\}$.
**Constraints**

The non-linear problem of scheduling reservoirs for hydropower production is modelled using an iterative linear model. The model is subject to linear constraints that limit the values that can be assumed by the decision variables in the search for an optimal objective function value. The constraints in the LP model are specified below.

**Initial Volume Constraint**

The initial volume constraint specifies the value of the start of period volume for each reservoir, \( r \in \{R\} \):

\[
V_r^0 = v_r^0. \tag{3-9}
\]

**Continuity Constraint**

The continuity constraint specifies the conservation of mass equation for each reservoir, \( r \in \{R\} \), under each scenario \( \omega \in \{\Omega\} \):

\[
V_r^0 + \sum_{u \in \{U\}} (\Delta t_u (\omega i_{u,r} - \omega O_{u,r})) = V_r. \tag{3-10}
\]

**Load Balance Constraint**

The load balance constraint specifies that the demand must balance the net generation, where the net generation is equal to the generation plus the imports less the exports. The load balance constraint, under each scenario \( \omega \in \{\Omega\} \), for each sub-period, \( u \in \{U\} \), can be written as:

\[
\sum_{r \in \{R\}} (\omega G_{u,r} + \omega M_u = \omega d_u + \omega X_u. \tag{3-11}
\]

**Storage Limit Constraints**

The storage limit constraints specify the allowable operational range for each reservoir, \( r \in \{R\} \), under each scenario \( \omega \in \{\Omega\} \):

\[
v_r \leq \omega V_r \leq \omega v_r. \tag{3-12}
\]

**Turbine Limit Constraints**

The turbine limit constraints specify the allowable turbine discharge range for each reservoir, \( r \in \{R\} \), under each scenario \( \omega \in \{\Omega\} \), and sub-period, \( u \in \{U\} \):

\[
q_r \omega Q_{u,r} \leq q_r. \tag{3-13}
\]
Spill Limit Constraints

The spill limit constraints specify the allowable spill range for each reservoir, \( r \in \{ R \} \), under each scenario \( \omega \in \{ \Omega \} \), and sub-period, \( u \in \{ U \} \):

\[
S_r^u \leq \omega S_{u,r} \leq s_r.
\]  

(3-14)

In practice, the upper bound, \( s_r \), is not used.

Import Limit Constraints

The import limit constraints specify the allowable range of power imports, under each scenario \( \omega \in \{ \Omega \} \), for each sub-period, \( u \in \{ U \} \):

\[
m_u \leq \omega M_u \leq m_u.
\]  

(3-15)

In the objective function imports are priced in a step-wise manner—that is, a block of energy has a price associated with it. The most expensive block of energy corresponds to load curtailment, which can be viewed as an expensive, special energy import case. As such, in practice the upper bound, \( m_u \), is not used for the most expensive import block.

Export Limit Constraints

The export limit constraints specify the allowable range of power exports, under each scenario \( \omega \in \{ \Omega \} \), for each sub-period, \( u \in \{ U \} \):

\[
x_u \leq \omega X_u \leq x_u.
\]  

(3-16)

In the objective function exports are priced in a step-wise manner—that is, a block of energy has a price associated with it.

Total Plant Discharge Constraints

The total plant discharge constraints specify the allowable total plant discharge range for each reservoir, \( r \in \{ R \} \), under each scenario \( \omega \in \{ \Omega \} \), and sub-period, \( u \in \{ U \} \). The total plant discharge constraints can be handled as either “hard” or “soft.” The hard case is:

\[
o_r \leq \omega O_{u,r} \leq o_r.
\]  

(3-17)

In the soft constraint case, an additional variable is introduced. The variable \( O_{u,r} \), which represents the violation in cms of the minimum total plant discharge, \( o_r \), is added. The soft constraint formulation is:
In practice, the upper bound, $\bar{o}_r$, is not used.

**Generation Limit Constraints**

The generation limit constraints specify the allowable range of power generation, from a power plant at a given reservoir, $r \in \{R\}$, under each scenario $\omega \in \{\Omega\}$:

$$
\underline{o}_r \leq \omega O_{u,r} \leq \bar{o}_r.
$$

(3-18)

**Total Plant Flow Constraints**

The total plant flow constraints define total plant flow for each reservoir, $r \in \{R\}$, under each scenario $\omega \in \{\Omega\}$, and sub-period, $u \in \{U\}$, as the sum of the respective turbine discharge and spill flow:

$$
\underline{o}_r \leq \omega g_r \leq \bar{o}_r.
$$

(3-19)

**Power Generation Constraints**

The power generation constraints define the power generated from the power plant at reservoir $r \in \{R\}$, under each scenario $\omega \in \{\Omega\}$, during sub-period $u \in \{U\}$ as a function of the turbine discharge from plant $r$ during sub-period $u$:

$$
\omega G_{u,r} = \omega S_{u,r} + \omega Q_{u,r}.
$$

(3-20)

Note that the conversion factor is a function of the average storage volume—i.e., $\omega HK_r = \omega HK_r(v_r^0, v_r)$). The value of $\omega HK_r$ used in a solution of the linear program is calculated based upon a forecast reservoir ending volume.

**Scenario Probability Constraints**

The occurrence probabilities of the scenarios must sum to one:

$$
\sum_{\omega \in \{\Omega\}} \omega pr = 1.0.
$$

(3-22)

**Policy Decision Constraints**

The policy decision constraints specify that the policy decision variable must be the same over all scenarios. With the turbine release as the policy decision variable, the constraint for reservoir $r \in \{R\}$, under scenario $\omega_1$, where $\omega_1, \omega_2 \in \{\Omega\}$ and $\omega_1 \neq \omega_2$, during sub-period $u \in \{U\}$ are:
\[ a_h Q_{u,r} = a_h Q_{u,r} . \]  

(3-23)

**Non-negativity Constraints**

All variables are subject to the constraint that they cannot be negative:

\[ V_r, a Q_{u,r}, a S_{u,r}, a O_{u,r}, a G_{u,r}, a V_r, a M_u, a X_u \geq 0; \quad r \in \{ R \}; u \in \{ U \}; \omega \in \{ \Omega \}. \]  

(3-24)

**Objective Function**

The objective function seeks to maximize the value of operating the hydroelectric system, where the operation of the system is understood to include making energy trades. The objective function consists of four terms: the income generated through exporting energy; the expense incurred through importing energy; the value of water remaining in storage at the end of the time horizon; and the penalty costs associated with violation of soft constraints. The only soft constraint is the minimum total plant discharge, outlined in equation (3-18). The objective function could be modified to include penalties associated with spill, or spill above a certain level. Penalties for spill could be attributed to physical damage that may occur downstream, or to perceived damages that may accompany a spill. In order to understand the objective function, it is useful to discuss each of the four terms separately. The entire function is then presented.

The income generated through the export of energy is given by:

\[ \beta \cdot 24 \cdot \sum_{\omega \in \{ \Omega \}} \sum_{u \in \{ U \}} (a W_1(a X_u) \cdot \Delta t_u) . \]  

(3-25)

In equation (3-25), \( a W_1(a X_u) \) is a piecewise-linear function describing the revenue earned from the sale of energy. The slopes of the segments in \( a W_1(a X_u) \) are equal to the net export prices ($/MWh). The net export price is given by \( a p_u (1 - z) \); where \( a p_u \) is the market energy price, and \( z \) is a factor used to account for wheeling and losses.

Describing the export revenue as a piecewise-linear function of the power generated allows for a block price structure to be used. That is, an export price can be associated with a quantity of energy. For example, using two blocks in the structure would mean that all of the energy exported up to a certain limit would earn revenue at one rate, and all of the energy exported above this threshold would earn revenue at a lower rate. Note that in order to preserve the necessary convexity properties, the blocks of export prices must be in the shape of a descending staircase. Also note that the above formulation, in which \( \Delta t_u \) multiplies \( \sum_{u \in \{ U \}} a W_1(a X_u) \), implies that the power generation is constant over each sub-period.
The cost incurred through the import of energy is given by:

\[ P - 2A - X J K(X)^J \]  \hspace{1cm} (3-26)

In equation (3-26), \( a w_2(0 M_u) \) is a piecewise-linear function describing the cost of energy purchased. The slopes of the segments in \( a w_2(0 M_u) \) are the net import prices of energy (\$/MWh), where the net import price is given by \( a p_u(I + y) \). The parameter \( y \) is a factor used to account for the wheeling and losses associated with importing energy. Convexity requirements specify that the blocks of import prices must be in the shape of an ascending staircase. (The difference between this requirement and that for the export price blocks arises because the cost of purchasing energy is multiplied by negative one in the objective function.) The block pricing structure for import prices allows load curtailment to be modelled as an expensive import.

A benefit of including load curtailment in the model—essentially making the need to meet the fixed load a soft constraint—is that the DP algorithm does not need to be concerned with the possibility of “power-infeasible” cases. (The term “power-infeasible” is used to differentiate those cases in which infeasibility is caused by the load balance equation from those where infeasibility is caused by the continuity equation.) Without load curtailment in the model, if a reservoir starts a period empty, or nearly so, the sum of the generation from all of the available water plus the maximum imports may be less than the firm demand. In such a case, there would be no feasible ending states, as defined for equation (3-1), for the starting state. The DP algorithm would have to recognize, and handle this situation, which would prove difficult with the use of piecewise-linear curves representing the future value of system operation. Allowing load curtailment in the LP model alleviates this potential difficulty in the DP algorithm.

The value of water remaining in storage in a reservoir is given by:

\[ P - \sum_{0 \in \Omega} \sum_{0 \in \pi} w_3(0 V_r). \]  \hspace{1cm} (3-27)

In equation (3-27), the piecewise-linear function describing the value of water remaining in storage is a function of reservoir storage. In order for convexity requirements to be satisfied, the value of storage must decrease with increasing storage—that is the storage must have a decreasing marginal value. Note that while the above equation describes the value of water in storage as a function of a single reservoir storage, the actual dependency is on all reservoir storage volumes. The slopes of the segments used are only valid for forecast ending volumes. These estimates upon which the slopes are based, are updated in an iterative procedure, as described in section 3.3.1.2. The piecewise-linear functions \( w_3(V_r) \) in equation (3-27) are equivalent to the piecewise-linear functions \( e_i^{n-1} (l_r) \) in equation (3-7).

The final term included in the objective function accounts for the penalty associated with violating the minimum total plant discharge constraints. The cost of the penalty terms is:

\[ \beta \sum_{0 \in \Omega} \sum_{0 \in U} \sum_{r \in R} (w_4(0 O_{u,r}) \cdot \Delta t_u) \]  \hspace{1cm} (3-28)
Treating the total plant discharge constraint as a soft constraint introduces flexibility into the continuity equation; this flexibility is useful in the DP algorithm as it eliminates the need to consider "water-infeasible" cases. (The term "water-infeasible" is used to differentiate cases in which infeasibility is caused by the continuity equation from cases where the infeasibility is caused by the load balance equation.) Without flexibility in the continuity equation, the situation may arise where there is not enough water in the reservoir to meet a minimum plant discharge requirement. In such an instance, there would be no feasible ending states for a starting state. As discussed above, requiring the DP algorithm to recognize infeasible states would create difficulties with the use of LP to evaluate the value of future system operation.

The slopes of the segments in the piecewise-linear curves are the costs ($/cms) associated with violation of the minimum plant discharge constraint. A block cost model is used. As the costs of penalty violations are multiplied by negative one in the objective function, the shape of the block costs must be that of an ascending staircase.

The entire objective function comprised of the parts described by equations (3-25) to (3-28), summed over all scenarios, is:

$$\max \left\{ \beta \cdot \sum_{\alpha \in [2]} \sum_{t \in \{1\}} \left( 24 \cdot w_1(\theta X_u) - w_2(\theta M_u) \right) - \sum_{r \in [R]} w_{1,r}(\theta O_{a,r}) \right\} \cdot \rho r + \sum_{\alpha \in [L]} \sum_{r \in [R]} w_{3,r}(\theta V_r) \cdot \rho r \right\}.$$  

(3-29)

Note that it is assumed that revenues, costs, and penalty costs all occur at month end. The means by which the objective function described by (3-29) is optimized, subject to the constraints described by equations (3-9) to (3-24), is described in section 3.3.2.2.

3.3.2.2 Solution Methodology

The linear programming problem described by equations (3-9) to (3-29) is modelled using the language AMPL (Fourer et al., 1993). The main benefit of using AMPL is the ease with which sets can be defined and used. For example, sets of sub-periods and reservoirs can be defined, and parameters, variables, and constraints can then be indexed using these sets. The result is a much more compact notation than would result if it were necessary to individually specify each parameter, variable, and constraint. Obviously, having a more compact model facilitates making changes, and thereby maintenance of the model.

In AMPL, the model and data are specified individually. The model files specify a general LP model in terms of parameters, variables, constraints, and objective functions. The data files specify the values for the parameters and the objective function to use in solving a particular instance of the general model. Once a model has been developed, the input data can be modified without changes being made to the model itself.
AMPL is a modelling language, not a solver. When AMPL is run, provided that the model and data are valid, the data is reformatted to the specifications of the solver. AMPL can also be used to pass specific instructions to the solver. After the data have been properly reformatted, the solver is executed to find the optimal solution. The solver used to solve the linear program defined by (3-9) to (3-29) is CPLEX (CPLEX Optimization, Inc., 1995). CPLEX is primarily a linear programming solver, with the ability to handle integer and quadratic problems. Once the run of the solver is complete, control is returned to AMPL.

Another feature of AMPL that proved useful in model development is the ability to run in batch mode. In batch mode, a text file containing a number of AMPL commands can be specified as input. Batch mode allows another application to call AMPL and solve a particular instance of a model. This feature allows the DP computer program described in section 3.3.1 to call AMPL in order to evaluate the DP objective function values through use of the LP model described in section 3.3.2. Batch mode also makes it possible to iteratively solve the LP model until the forecast and optimal ending reservoir storage volumes agree within a specified tolerance.

3.4 Case Study

In this section, in order to demonstrate the ability of the model to address the uncertainty faced by the operator of a large hydropower system with considerable flexibility, the DP and LP based model described in sections 3.2 and 3.3 is applied to a test system. The test system is based upon a sub-system of the British Columbia Hydro and Power Authority (BC Hydro) system.

3.4.1 Description

The system used as a case study to demonstrate the DP and LP based model is derived from part of the BC Hydro system. The BC Hydro electric generation system, at the time of this study, included 29 hydroelectric plants, one conventional thermal station, and two combustion turbine stations. Approximately 75% of the total installed generation capacity of the BC Hydro system is at stations on two river systems: the Columbia and the Peace (BC Hydro, 1995). BC Hydro has two generating plants on the Peace River: G. M. Shrum and Peace Canyon. The Williston reservoir associated with the G. M. Shrum plant has significant over-year storage, whereas the Dinosaur reservoir associated with the Peace Canyon plant has some storage, but is largely run in hydraulic balance with G. M. Shrum. Similarly, BC Hydro generation facilities on the main-stem of the Columbia River include an upstream plant (Mica) with a reservoir having significant storage, and a downstream plant (Revelstoke) associated with a reservoir with less storage. Plants on other rivers in the Columbia basin contribute to the 75% figure cited above, but their contribution is significantly less than that of the Mica and Revelstoke stations. Together, the G. M. Shrum, Peace Canyon, Mica, and Revelstoke stations account for roughly 70% of the installed hydroelectric generation capacity in the BC Hydro system (BC Hydro, 1995).

In the test system used in the case study, generation on each of the two river systems has been modelled with a single plant. On the Columbia River, the "Columbia" aggregated plant replaces the Mica and Revelstoke plants. The "Peace" aggregated plant replaces the G. M. Shrum and Peace Canyon plants. By combining reservoirs, the effect of the curse of dimensionality, to which dynamic programming is subject, is diminished. The practice of aggregating reservoirs for this purpose has been widely reported in the literature (e.g., Turgeon and Charbonneau, 1998; Valdes et al., 1992).

The data for the aggregated Peace and Columbia plants are presented in the following section.

3.4.2 Data

The data for the case study are presented in this section. Data relating to the Columbia and Peace plants are based, respectively, upon data for the Mica and Revelstoke, and the G. M. Shrum and Peace Canyon facilities. As the intent of this work is to examine the relationship between the operation of two hydroelectric generating river systems at a gross scale, many of the details of the operation of the physical plants are neglected. The aim was to work with data that reasonably approximated the major characteristics of the underlying facilities. The above is to state that the Columbia and Peace plants, and the associated data, should be considered as conceptual only; there is not a direct correspondence between the data for them and the Mica, Revelstoke, G. M. Shrum, and Peace Canyon facilities.

3.4.2.1 Inflows

The average monthly inflows to the Columbia and Peace reservoirs are presented in Table 3-1.

<table>
<thead>
<tr>
<th></th>
<th>Peace Reservoir Inflow (cms)</th>
<th>Columbia Reservoir Inflow (cms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>300</td>
<td>150</td>
</tr>
<tr>
<td>February</td>
<td>220</td>
<td>140</td>
</tr>
<tr>
<td>March</td>
<td>220</td>
<td>140</td>
</tr>
<tr>
<td>April</td>
<td>430</td>
<td>280</td>
</tr>
<tr>
<td>May</td>
<td>2230</td>
<td>1100</td>
</tr>
<tr>
<td>June</td>
<td>3490</td>
<td>2190</td>
</tr>
<tr>
<td>July</td>
<td>2200</td>
<td>2170</td>
</tr>
<tr>
<td>August</td>
<td>1010</td>
<td>1510</td>
</tr>
<tr>
<td>September</td>
<td>780</td>
<td>820</td>
</tr>
<tr>
<td>October</td>
<td>810</td>
<td>450</td>
</tr>
<tr>
<td>November</td>
<td>480</td>
<td>270</td>
</tr>
<tr>
<td>December</td>
<td>330</td>
<td>180</td>
</tr>
</tbody>
</table>

Table 3-1: Average Monthly Inflows
3.4.2.2 Prices
The average monthly prices for heavy load hour (HLH) and light load hour (LLH) electricity assumed for the case study are presented in Table 3-2. The market is assumed to be deep enough such that transmission constraints will be reached prior to market constraints.

<table>
<thead>
<tr>
<th></th>
<th>HLH Price ($/MWh)</th>
<th>LLH Price ($/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>29.0</td>
<td>25.0</td>
</tr>
<tr>
<td>February</td>
<td>29.0</td>
<td>27.0</td>
</tr>
<tr>
<td>March</td>
<td>27.0</td>
<td>26.0</td>
</tr>
<tr>
<td>April</td>
<td>30.0</td>
<td>21.0</td>
</tr>
<tr>
<td>May</td>
<td>29.0</td>
<td>17.0</td>
</tr>
<tr>
<td>June</td>
<td>27.0</td>
<td>16.0</td>
</tr>
<tr>
<td>July</td>
<td>30.0</td>
<td>22.0</td>
</tr>
<tr>
<td>August</td>
<td>32.0</td>
<td>26.0</td>
</tr>
<tr>
<td>September</td>
<td>42.0</td>
<td>31.0</td>
</tr>
<tr>
<td>October</td>
<td>35.0</td>
<td>31.0</td>
</tr>
<tr>
<td>November</td>
<td>34.0</td>
<td>31.0</td>
</tr>
<tr>
<td>December</td>
<td>36.0</td>
<td>29.0</td>
</tr>
</tbody>
</table>

Table 3-2: Average Monthly Market Electricity Prices

3.4.2.3 Demands
The average HLH and LLH domestic demands that are assumed to be served by the Columbia and Peace generating stations is presented in Table 3-3.

<table>
<thead>
<tr>
<th></th>
<th>HLH Demand (aMW)</th>
<th>LLH Demand (aMW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>6100</td>
<td>4200</td>
</tr>
<tr>
<td>February</td>
<td>6000</td>
<td>4200</td>
</tr>
<tr>
<td>March</td>
<td>5600</td>
<td>3900</td>
</tr>
<tr>
<td>April</td>
<td>5000</td>
<td>3500</td>
</tr>
<tr>
<td>May</td>
<td>4800</td>
<td>3300</td>
</tr>
<tr>
<td>June</td>
<td>4700</td>
<td>3300</td>
</tr>
<tr>
<td>July</td>
<td>4700</td>
<td>3300</td>
</tr>
<tr>
<td>August</td>
<td>4800</td>
<td>3300</td>
</tr>
<tr>
<td>September</td>
<td>4800</td>
<td>3400</td>
</tr>
<tr>
<td>October</td>
<td>5100</td>
<td>3500</td>
</tr>
<tr>
<td>November</td>
<td>5700</td>
<td>4000</td>
</tr>
<tr>
<td>December</td>
<td>6000</td>
<td>4200</td>
</tr>
</tbody>
</table>

Table 3-3: Average Monthly Electricity Demand
3.4.2.4 Miscellaneous

In the model, each time period—month in this case study—is divided into HLH and LLH portions. It is assumed that the HLH period consists of 16 hours a day, Monday through Saturday; thus the LLH period includes 8 hours a day Monday through Saturday and all day Sunday. The number of HLH and LLH days in each month is shown in Table 3-4.

<table>
<thead>
<tr>
<th></th>
<th>HLH Period (days)</th>
<th>LLH Period (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>17.7</td>
<td>13.3</td>
</tr>
<tr>
<td>February</td>
<td>16.0</td>
<td>12.0</td>
</tr>
<tr>
<td>March</td>
<td>17.7</td>
<td>13.3</td>
</tr>
<tr>
<td>April</td>
<td>17.1</td>
<td>12.9</td>
</tr>
<tr>
<td>May</td>
<td>17.7</td>
<td>13.3</td>
</tr>
<tr>
<td>June</td>
<td>17.1</td>
<td>12.9</td>
</tr>
<tr>
<td>July</td>
<td>17.7</td>
<td>13.3</td>
</tr>
<tr>
<td>August</td>
<td>17.7</td>
<td>13.3</td>
</tr>
<tr>
<td>September</td>
<td>17.1</td>
<td>12.9</td>
</tr>
<tr>
<td>October</td>
<td>17.7</td>
<td>13.3</td>
</tr>
<tr>
<td>November</td>
<td>17.1</td>
<td>12.9</td>
</tr>
<tr>
<td>December</td>
<td>17.7</td>
<td>13.3</td>
</tr>
</tbody>
</table>

Table 3-4: Monthly Duration of HLH and LLH Periods

Given the division of time into HLH and LLH periods, parameters related to energy trade must also be differentiated on this basis. Table 3-5 presents the assumed upper transmission limits on imports and exports for the HLH and LLH periods of each month; the lower transmission limits are shown in Table 3-6.

<table>
<thead>
<tr>
<th></th>
<th>Max HLH Imports (aMW)</th>
<th>Max LLH Imports (aMW)</th>
<th>Max HLH Exports (aMW)</th>
<th>Max LLH Exports (aMW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>1620</td>
<td>1230</td>
<td>1900</td>
<td>1420</td>
</tr>
<tr>
<td>February</td>
<td>1500</td>
<td>1140</td>
<td>1720</td>
<td>1300</td>
</tr>
<tr>
<td>March</td>
<td>1740</td>
<td>1320</td>
<td>1720</td>
<td>1300</td>
</tr>
<tr>
<td>April</td>
<td>1950</td>
<td>1470</td>
<td>1600</td>
<td>1200</td>
</tr>
<tr>
<td>May</td>
<td>2010</td>
<td>1500</td>
<td>1660</td>
<td>1240</td>
</tr>
<tr>
<td>June</td>
<td>1950</td>
<td>1530</td>
<td>1600</td>
<td>1200</td>
</tr>
<tr>
<td>July</td>
<td>2100</td>
<td>1590</td>
<td>1840</td>
<td>1380</td>
</tr>
<tr>
<td>August</td>
<td>2100</td>
<td>1590</td>
<td>1840</td>
<td>1380</td>
</tr>
<tr>
<td>September</td>
<td>2040</td>
<td>1530</td>
<td>1780</td>
<td>1340</td>
</tr>
<tr>
<td>October</td>
<td>2100</td>
<td>1590</td>
<td>1900</td>
<td>1420</td>
</tr>
<tr>
<td>November</td>
<td>1860</td>
<td>1380</td>
<td>1840</td>
<td>1380</td>
</tr>
<tr>
<td>December</td>
<td>1620</td>
<td>1200</td>
<td>1900</td>
<td>1420</td>
</tr>
</tbody>
</table>

Table 3-5: Monthly Maximum Import and Export Transmission Limits
Table 3-6: Monthly Minimum Import and Export Transmission Limits

Limits related to turbine releases, spills, total plant releases, generation, and reservoir storage volume must be specified for each plant. Plant related limits are shown in Table 3-7 and Table 3-8.

<table>
<thead>
<tr>
<th></th>
<th>Peace Plant</th>
<th>Columbia Plant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max Turbine Discharge (cms)</td>
<td>2000</td>
<td>1500</td>
</tr>
<tr>
<td>Min Turbine Discharge (cms)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Min Spill (cms)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Max Reservoir Volume (cmsd)</td>
<td>500000</td>
<td>200000</td>
</tr>
<tr>
<td>Min Reservoir Volume (cmsd)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Max Generation (MW)</td>
<td>3400</td>
<td>3500</td>
</tr>
<tr>
<td>Min Generation (MW)</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3-7: Plant Related Limits

<table>
<thead>
<tr>
<th></th>
<th>Peace Plant Minimum Discharge (cms)</th>
<th>Columbia Plant Minimum Discharge (cms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>1600</td>
<td>85</td>
</tr>
<tr>
<td>February</td>
<td>200</td>
<td>85</td>
</tr>
<tr>
<td>March</td>
<td>200</td>
<td>85</td>
</tr>
<tr>
<td>April</td>
<td>200</td>
<td>85</td>
</tr>
<tr>
<td>May</td>
<td>200</td>
<td>85</td>
</tr>
<tr>
<td>June</td>
<td>200</td>
<td>85</td>
</tr>
<tr>
<td>July</td>
<td>200</td>
<td>85</td>
</tr>
<tr>
<td>August</td>
<td>200</td>
<td>85</td>
</tr>
<tr>
<td>September</td>
<td>200</td>
<td>85</td>
</tr>
</tbody>
</table>
In Table 3-8 note that an operational constraint on the Peace plant discharge is in effect for December and January. The effects of this constraint are explored below in 3.4.3.4.

The relationship between turbine release and generation is described by the "HK" factor, which takes into consideration efficiency, the specific weight of water, and the gross head—see (3-21). HK, which has units of MW/cms, is modelled as a piecewise-linear function of reservoir storage. The slopes of the segments for the Peace and Columbia plants are given, respectively, in Table 3-9 and Table 3-10. The storage intercepts for the two plants are given in Table 3-11.

<table>
<thead>
<tr>
<th>Range Number</th>
<th>Minimum Storage (cmsd)</th>
<th>Maximum Storage (cmsd)</th>
<th>Slope of HK vs. Storage (MW/(cms))²d</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
<td>50000.0</td>
<td>8.8395805668E-07</td>
</tr>
<tr>
<td>2</td>
<td>50000.0</td>
<td>100000.0</td>
<td>7.8826138426E-07</td>
</tr>
<tr>
<td>3</td>
<td>100000.0</td>
<td>150000.0</td>
<td>7.1926311364E-07</td>
</tr>
<tr>
<td>4</td>
<td>150000.0</td>
<td>200000.0</td>
<td>6.5153642709E-07</td>
</tr>
<tr>
<td>5</td>
<td>200000.0</td>
<td>250000.0</td>
<td>5.9569165768E-07</td>
</tr>
<tr>
<td>6</td>
<td>250000.0</td>
<td>300000.0</td>
<td>5.4044050367E-07</td>
</tr>
<tr>
<td>7</td>
<td>300000.0</td>
<td>350000.0</td>
<td>5.0455005931E-07</td>
</tr>
<tr>
<td>8</td>
<td>350000.0</td>
<td>400000.0</td>
<td>4.7493755667E-07</td>
</tr>
<tr>
<td>9</td>
<td>400000.0</td>
<td>450000.0</td>
<td>4.5268005924E-07</td>
</tr>
<tr>
<td>10</td>
<td>450000.0</td>
<td>500000.0</td>
<td>4.4196592180E-07</td>
</tr>
</tbody>
</table>

Table 3-9: Slopes of HK-Storage Function for Peace Plant

<table>
<thead>
<tr>
<th>Range Number</th>
<th>Minimum Storage (cmsd)</th>
<th>Maximum Storage (cmsd)</th>
<th>Slope of HK vs. Storage (MW/(cms))²d</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
<td>20000.0</td>
<td>3.9717525790E-06</td>
</tr>
<tr>
<td>2</td>
<td>20000.0</td>
<td>40000.0</td>
<td>3.5876743972E-06</td>
</tr>
<tr>
<td>3</td>
<td>40000.0</td>
<td>60000.0</td>
<td>2.9298473017E-06</td>
</tr>
<tr>
<td>4</td>
<td>60000.0</td>
<td>80000.0</td>
<td>2.5590057745E-06</td>
</tr>
<tr>
<td>5</td>
<td>80000.0</td>
<td>100000.0</td>
<td>2.4924027772E-06</td>
</tr>
<tr>
<td>6</td>
<td>100000.0</td>
<td>120000.0</td>
<td>2.2931980996E-06</td>
</tr>
<tr>
<td>7</td>
<td>120000.0</td>
<td>140000.0</td>
<td>2.0492320665E-06</td>
</tr>
<tr>
<td>8</td>
<td>140000.0</td>
<td>160000.0</td>
<td>1.8812578849E-06</td>
</tr>
</tbody>
</table>
Table 3-10: Slopes of HK-Storage Function for Columbia Plant

<table>
<thead>
<tr>
<th></th>
<th>Columbia Plant</th>
<th>Peace Plant</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>160000.0</td>
<td>180000.0</td>
</tr>
<tr>
<td>10</td>
<td>180000.0</td>
<td>200000.0</td>
</tr>
</tbody>
</table>

Table 3-11: Storage Intercepts for HK-Storage Functions

<table>
<thead>
<tr>
<th></th>
<th>Columbia Plant</th>
<th>Peace Plant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Storage Intercept</td>
<td>-553096.134848</td>
<td>-1773335.270998</td>
</tr>
</tbody>
</table>

Miscellaneous data for the case study are shown in Table 3-12. The curtailment cost is the cost of failing to supply demand. The loss factor takes into account transmission losses; for an import the loss factor is added to the cost, whereas it is subtracted for an export, as discussed in the paragraph following equation (3-25).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount rate (%)</td>
<td>8</td>
</tr>
<tr>
<td>Wheeling/loss factor (%)</td>
<td>7</td>
</tr>
<tr>
<td>Curtailment Cost ($/MWh)</td>
<td>1000</td>
</tr>
</tbody>
</table>

Table 3-12: Miscellaneous Data

Violation of the minimum plant release is penalized in the model, as shown in equations (3-28) and (3-29). The penalties are given in Table 3-13. The penalty rates in Table 3-13 equal the $1000/MWh curtailment cost for average HK values.

<table>
<thead>
<tr>
<th>Penalty for Violation of Minimum Plant Turbine Release ($/cmsd)</th>
<th>Peace Plant</th>
<th>Columbia Plant</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>42000</td>
<td>60000</td>
</tr>
</tbody>
</table>

Table 3-13: Minimum Plant Turbine Release Violation Penalties

3.4.3 Results

The results of applying the DP and LP based model described in sections 3.2 and 3.3 to the case study data specified in section 3.4.2 are presented in this section. Results are first presented in 3.4.3.1 for a base case in which there is only one scenario per month, with the scenario-dependent parameters assuming their mean values. This one-scenario case establishes a basis for comparing the marginal energy values for the Peace and Columbia reservoirs.

In section 3.4.3.2 eight cases covering different combinations of the scenario-dependent parameters are solved under the non-anticipative turbine release constraints specified by equation (3-23). In each of these eight cases there are five scenarios in the months of
May through October, and five scenarios in the months of November through April. In each of the cases the scenario-dependent parameters, and their correlations, during the months with five scenarios vary. The eight scenarios are: demands only; inflows only; prices only; demands, inflows, and prices all perfectly positively correlated; demands and inflows perfectly correlated and prices perfectly negatively correlated; demands and prices perfectly correlated and inflows perfectly negatively correlated; and inflows and prices perfectly correlated and demands perfectly negatively correlated. The above cases demonstrate the effect of uncertainty in the scenario-dependent parameters on the marginal energy values for the two reservoirs.

Section 3.4.3.3 presents results for the eight cases outlined in the previous paragraph under the assumption of perfect foresight. Perfect foresight is obtained by removing equation (3-23) from the model. A comparison of the marginal energy values for the two reservoirs is made between the results under the assumptions of non-anticipative turbine releases and perfect foresight in section 3.4.3.3.

In section 3.4.3.4 the effect of an operational constraint on the marginal energy values for the two reservoirs is examined. In all instances of the model solved in sections 3.4.3.1 through 3.4.3.3 the Peace plant releases are specified to have a minimum value during December and January that is significantly higher than the minimum in the other months. In section 3.4.3.4 a one-scenario case is solved for the case in which the December and January minimum Peace plant releases are the same as they are for the other months; no other model parameters change. A comparison is made between the results thus obtained and those in section 3.4.3.1.

3.4.3.1 One-Scenario Base Case

As described earlier in this chapter, the model can accommodate different scenarios for each time period—month in this case. Between scenarios, the scenario-dependent demands, inflows, and prices for each time period can vary. In later sections, scenarios employing different combinations of, and correlations between, the scenario-dependent parameters are explored. The scenario-dependent parameters in these later sections are symmetrically distributed about the mean value. In this section, a base case in which there is only one scenario in each month, and where the scenario-dependent parameters assume their mean values is studied. This base case establishes a comparison point for the multiple-scenario cases.

The storage value—that is the amount by which the objective function exceeds the objective function with both reservoirs empty, as defined in (3-5)—is a function of storage in both the Columbia and Peace reservoirs, as well as the month. The storage value functions for January, April, June, and October are shown, respectively, in Figure 3-1 through Figure 3-4.
Figure 3-1: Base Case January Storage Value Function

Figure 3-2: Base Case April Storage Value Function
There are several features to note in Figure 3-1 through Figure 3-4. First, it is apparent that the storage values, and hence the objective function values from which they are derived, are dependent upon the quantity of water stored in both reservoirs. Second, note that the maximum storage values for January and October are an order of magnitude greater than those for April and June, indicating that the value of having water in storage is very much dependent upon the time of year. Finally, note that in April the value of having both reservoirs empty is dramatically different than having either of the reservoirs empty and the other as much as 90% empty.

The three dimensional plots in the previous figures is useful for gaining an appreciation of the nature of the storage value surface. Further information is available by displaying
“slices” through the storage value surfaces. Figure 3-5 displays slices taken through the storage value functions for all months for an empty Columbia reservoir.

In Figure 3-5 observe that the storage value when the Peace reservoir is full is greatest in the early winter and late fall, is lowest in the spring and early summer, and is between these extremes during the late winter and late summer. These results agree with intuition, as they show that it is more valuable having the Peace reservoir full when in the low inflow/high demand/high price portion of the year, than it is during the high inflow/low demand/low price portion of the year.

The slopes of the line segments comprising the storage value vs. Peace reservoir storage in Figure 3-5 are the marginal values of water stored in the Peace reservoir; the decreasing marginal value of this water is apparent. The same features are evident in Figure 3-6 and Figure 3-7, which, respectively, present the storage value functions for the Columbia reservoir half-full and full.
In Figure 3-6 notice the increased storage value intercepts for a given month as compared with those in Figure 3-5; these increased intercept values demonstrate the value of having more storage in the Columbia reservoir. Also note that the curves start to flatten out at lower storage volumes for the Peace reservoir, illustrating that as the volume stored in the Columbia reservoir changes, the marginal value of water stored in the Peace reservoir is affected. These trends are further evident in Figure 3-7.

Just as the marginal value of water in the Peace reservoir can be affected by storage in the Columbia reservoir, the marginal water value in the Columbia reservoir can depend on storage in the Peace reservoir. Figure 3-8, Figure 3-9, and Figure 3-10, which present storage value functions, respectively, for the Peace reservoir empty, half full, and full, illustrate this fact.
Similar to the Peace reservoir, the worth of having the Columbia reservoir full is greatest in the early winter and late fall, is lowest in the spring and early summer, and is in between these extremes during the late winter and late summer. Again, these results agree with intuition, as they show that it is more valuable to have the Peace reservoir full when in the low inflow/high demand/high price portion of the year, than it is during the high inflow/low demand/low price portion of the year.

The dependence of the storage value on the storage in both reservoirs is evident in a comparison of Figure 3-8 and Figure 3-9. In Figure 3-9 the storage value intercepts for a
given month are greater than those in Figure 3-8. The increased intercept values demonstrate the value of having more storage in the Peace reservoir. The flattening of the curves at lower storage volumes for the Columbia reservoir, illustrate that as the volume stored in the Peace reservoir changes, the marginal value of water stored in the Columbia reservoir is affected. These trends are further evident in a comparison of Figure 3-9 and Figure 3-10.

![Figure 3-10: Storage Value Function Slices for Peace Reservoir Full](image)

As noted above, the slopes of the line segments in Figure 3-5 through Figure 3-10 give the marginal value of stored water, with units of $/cmsd. While these values can be of use in and of themselves, in the case of a hydroelectric system, the marginal values of stored energy, with units of $/MWh, are of greater use. The conversion of marginal water to marginal energy values is straightforward. From equation (3-21) recall that the generation of power is modelled as the product of the constant $HK$ with units of MW/cms and the turbine release in cms, and that $HK$ is assumed dependent on reservoir storage, but not on turbine release. Thus, the marginal energy value ($$/MWh) for a reservoir is equal to the marginal water value ($$/cmsd) in that reservoir divided by $HK$ (MW/cms) divided by the number of hours in a day (h/d). The $HK$ value used in the conversion should be representative of the range of reservoir storage over which the conversion is being performed. In the current case of converting the slope of a line segment from one of the storage value function slices, the representative $HK$ is equal to the average value of $HK$ associated with the reservoir storage at each end of the line segment.

Table 3-14 presents the range, over all of the storage states, of Columbia marginal energy values for each month for the one-scenario base case. It can be observed that, with the exceptions of May and June, there is a very considerable range between the minimum and maximum marginal energy values in each month. From Figure 3-5 through Figure 3-7 it is apparent that the high marginal Columbia energy storage values are associated
with low storage in both the Columbia and Peace reservoirs, while the low marginal Columbia energy storage values are associated with high storage in both reservoirs.

<table>
<thead>
<tr>
<th>Min Columbia Marginal Value ($/MWh)</th>
<th>Max Columbia Marginal Value ($/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>23.87</td>
</tr>
<tr>
<td>February</td>
<td>18.49</td>
</tr>
<tr>
<td>March</td>
<td>4.90</td>
</tr>
<tr>
<td>April</td>
<td>0.00</td>
</tr>
<tr>
<td>May</td>
<td>0.00</td>
</tr>
<tr>
<td>June</td>
<td>0.00</td>
</tr>
<tr>
<td>July</td>
<td>0.00</td>
</tr>
<tr>
<td>August</td>
<td>18.66</td>
</tr>
<tr>
<td>September</td>
<td>31.72</td>
</tr>
<tr>
<td>October</td>
<td>31.02</td>
</tr>
<tr>
<td>November</td>
<td>28.30</td>
</tr>
<tr>
<td>December</td>
<td>26.04</td>
</tr>
<tr>
<td></td>
<td>1084.66</td>
</tr>
<tr>
<td></td>
<td>1078.02</td>
</tr>
<tr>
<td></td>
<td>1067.13</td>
</tr>
<tr>
<td></td>
<td>948.87</td>
</tr>
<tr>
<td></td>
<td>48.39</td>
</tr>
<tr>
<td></td>
<td>75.02</td>
</tr>
<tr>
<td></td>
<td>1067.16</td>
</tr>
<tr>
<td></td>
<td>1143.96</td>
</tr>
<tr>
<td></td>
<td>1125.67</td>
</tr>
<tr>
<td></td>
<td>1117.20</td>
</tr>
<tr>
<td></td>
<td>1110.95</td>
</tr>
<tr>
<td></td>
<td>1098.45</td>
</tr>
</tbody>
</table>

Table 3-14: Minimum and Maximum Monthly Base Case Columbia Marginal Energy Values

In Table 3-14 note that for some spring and summer months the minimum marginal value of energy stored in the Columbia reservoir is equal to zero, indicating that there is water in excess of the turbine capacity. In reality, spills could potentially have negative value associated with them; these damages have not been considered in the model, possibly overestimating the minimum marginal energy values. Furthermore, note that with the exception of the months of May and June, the maximum marginal Columbia energy value is on the order of the curtailment cost of $1000/MWh. Recall from Table 3-1 that the freshet begins in May. This result suggests that if both reservoirs are at their respective minimums in any month other than the period between the onset of the freshet and one month following its onset, curtailment will be necessary.

Table 3-15 presents the range of Peace plant marginal energy values for each month for the one-scenario base case over all of the storage states. Similar to the Columbia, with the exceptions of May and June, there is a very large range between the minimum and maximum Peace marginal energy values.

<table>
<thead>
<tr>
<th>Min Peace Marginal Value ($/MWh)</th>
<th>Max Peace Marginal Value ($/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>24.97</td>
</tr>
<tr>
<td></td>
<td>1947.31</td>
</tr>
</tbody>
</table>
Table 3-15: Minimum and Maximum Peace Marginal Energy Values

<table>
<thead>
<tr>
<th>Month</th>
<th>Minimum Peace Marginal Energy</th>
<th>Maximum Peace Marginal Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>February</td>
<td>20.25</td>
<td>1065.03</td>
</tr>
<tr>
<td>March</td>
<td>9.11</td>
<td>1051.11</td>
</tr>
<tr>
<td>April</td>
<td>1.17</td>
<td>554.03</td>
</tr>
<tr>
<td>May</td>
<td>0.00</td>
<td>45.56</td>
</tr>
<tr>
<td>June</td>
<td>0.00</td>
<td>64.56</td>
</tr>
<tr>
<td>July</td>
<td>8.53</td>
<td>1102.03</td>
</tr>
<tr>
<td>August</td>
<td>30.84</td>
<td>1483.03</td>
</tr>
<tr>
<td>September</td>
<td>31.42</td>
<td>1694.98</td>
</tr>
<tr>
<td>October</td>
<td>30.93</td>
<td>1868.19</td>
</tr>
<tr>
<td>November</td>
<td>29.09</td>
<td>2034.20</td>
</tr>
<tr>
<td>December</td>
<td>27.52</td>
<td>2111.38</td>
</tr>
</tbody>
</table>

The maximum marginal Peace energy values for August through January are considerably higher than the maximum marginal Columbia energy value for the corresponding month. These differences are attributable to the penalty that accrues for failing to meet the minimum Peace plant release, as detailed in Table 3-8. For low Peace reservoir levels there is insufficient water to maintain the higher December and January minimum releases. Note that the effect of the penalty for violation of the minimum Peace plant flow restraint is felt in months other than December and January, as it is necessary to have stored the water required to maintain this minimum release ahead of time. For February, March, and July, the maximum marginal Peace energy value is attributable to load curtailment, and is comparable to the maximum marginal Columbia energy value for the respective month.

There is no consequential difference between the minimum marginal Peace and Columbia energy values. The minimum marginal Columbia energy values are, in general, slightly lower. This small difference is likely attributable to the fact that the ratio of average annual inflow to maximum reservoir storage is greater for the Columbia than it is for the Peace.

The marginal water values for Figure 3-5, Figure 3-6, and Figure 3-7 have been converted to marginal energy values using the appropriate HK factors. The results are presented, respectively, as Figure 3-11, Figure 3-12, and Figure 3-13. The decreasing marginal value of stored energy is apparent.
Figure 3-11: Peace Marginal Energy Values for Columbia Reservoir Empty

A comparison of Figure 3-11 and Figure 3-12 yields interesting results. In Figure 3-11, the effects of both the minimum Peace plant release penalty and curtailment cost are evident (i.e., the marginal Peace energy value is much greater than $1000/MWh), in at least some months, for Peace reservoir storage less than 150000 cmsd, and the effects of load curtailment are felt (i.e., marginal Peace energy value is much greater than market prices), at least in some months, for Peace reservoir storage less than 400000 cmsd. By way of comparison, in Figure 3-12, the effects of both the minimum Peace plant release penalty and curtailment cost are also felt, at least in some months, for Peace reservoir storage less than 150000 cmsd; the magnitude of the effects are lesser in Figure 3-12. The difference in magnitude is attributable to the additional 100000 cmsd of Columbia reservoir storage, which serves to reduce the curtailment cost component of the marginal Peace energy value. The effect of the additional Columbia reservoir storage is seen more dramatically in the 250000 cmsd Peace reservoir storage level above which curtailment costs are not felt in any month.
In comparing Figure 3-12 and Figure 3-13, neither the cost of load curtailment nor the minimum Peace plant release penalty costs are components of the marginal Peace energy value above Peace reservoir storage of 200000 cmsd. In fact, there is little or no curtailment cost component at any level of Peace reservoir storage with the additional Columbia storage.

Marginal Columbia energy value functions for the Peace reservoir empty, half full, and full, respectively are shown in Figure 3-14, Figure 3-15, and Figure 3-16.
Figure 3-14 contains interesting information. The marginal Columbia energy values are very similar in nature for the late fall and early winter. In these months, the effect of the Columbia reservoir storage on the marginal Columbia energy value becomes much stronger beyond 120000 cmsd; below this volume the marginal Columbia energy value curves are relatively flat. The nature of these curves reflects the fact that in these months, flows are on the receding limb of the inflow hydrograph, and the high demand season is approaching. Beyond the 120000 cmsd point, the curtailment cost component is reduced somewhat, but cannot be eliminated—that is, if the late fall and early summer months are started with the Peace reservoir empty, at some point load curtailment will be necessary. For the late winter and early spring months, with sufficient Columbia storage it is possible to avoid curtailment even with an empty Peace reservoir. During this period, the later into the year, the lower the Columbia reservoir storage at which the curtailment effect is lessened. For May and June only reduced curtailment effects are felt at very low Columbia storage. August, and in particular, July, demonstrate different characteristics as well, with the effect of Columbia storage on the marginal Columbia energy value decreasing with increasing storage as the reservoir heads towards full. In these months the peak of the freshet is past, yet the high demand season is still some time away.
When the Peace reservoir is half full as shown in Figure 3-15, the effect of curtailment costs on the marginal Columbia energy value is greatly diminished as compared to when the Peace reservoir is empty, as shown in Figure 3-14; the diminishment is further evident in Figure 3-16 for which the Peace reservoir is full. When the Peace reservoir is full, and the Columbia reservoir is approaching full, for some months the marginal Columbia energy value is equal to zero, indicating that water will be spilled.

In Figure 3-11 through Figure 3-16 there appear to be ranges of storage in both reservoirs over which the marginal energy values are not too dependent upon the reservoir storage; however, the vertical scales used in these figures obscure the details. A closer examination reveals that there are regions over which the marginal energy storage value in one reservoir is independent of the storage in the other reservoir, and may in fact not...
be all that dependent on the storage in the reservoir. Examples are shown below in Figure 3-17 through Figure 3-22.

In Figure 3-17, the slopes of the curves indicate the sensitivity of the marginal Peace energy value to Peace reservoir storage and the vertical distance between the curves indicates the sensitivity of the marginal Peace energy value to Columbia reservoir storage. Over a Peace reservoir storage range of 150000 to 300000 cmsd, the Peace marginal energy value is largely independent of the Columbia reservoir storage, for Columbia reservoir storage of at least 100000 cmsd. For example, for Peace reservoir storage of 300000 cmsd, the marginal Peace energy value is $31.75/MWh for the Columbia reservoir half full and $31.45/MWh for the Columbia reservoir full. Similarly, over this Peace reservoir storage range, the influence of Peace reservoir storage on the Peace marginal energy value is lower than that for other Peace reservoir storage.

As noted, the marginal Peace energy values for May are least sensitive to changes in the Columbia reservoir storage over a “mid-range” of Peace storage (150000 to 300000 cmsd) as well as when the Peace reservoir is full. In March through July a similar behaviour is observed.

Figure 3-17 contains another feature worth noting. In the figure, there are some cases in which the marginal Peace energy value is non-decreasing with increasing Peace reservoir storage. This discrepancy results from the maximum number of iterations being performed in the search for convergence recognition. The model handles non-convex storage value functions by applying an algorithm to smooth them such that they are convex.
In Figure 3-18, the slopes of the curves indicate the sensitivity of the Columbia marginal energy value to Columbia reservoir storage and the vertical distance between the curves indicates the sensitivity of the Columbia marginal energy value to Peace reservoir storage. Observe that when the Columbia reservoir storage is at least 140000 cmsd, the marginal Columbia energy value is essentially independent of Peace reservoir storage below 60% full. For Columbia reservoir storage of 140000 cmsd, the Columbia marginal energy value is $18.22/MWh when the Peace reservoir is empty and $17.90/MWh when it is 60% full.

The reduced dependence of the Columbia marginal energy value on Peace reservoir storage as the Columbia reservoir approaches full illustrated in Figure 3-18 is representative of the behaviour in all months.

Figure 3-19 displays the marginal Peace energy values for the month of November, while Figure 3-20 “zooms in” on the lower marginal Peace energy values shown in Figure 3-19.
In Figure 3-19, observe that for the Peace reservoir up to half full, the Peace reservoir exhibits little dependence on Columbia reservoir storage less than 60000 cmsd, relative to the other values. Additionally, note that for Peace reservoir storage of at least 350000 cmsd, the marginal Peace energy value does not exhibit much dependence on either Peace or Columbia reservoir storage at the scale of the figure.

Figure 3-20: November Marginal Peace Energy Value Functions—Detail

From the detail in Figure 3-20 observe that there is still some influence of both Peace and Columbia reservoir storage on the marginal Peace energy value. The reduced dependence of the Peace marginal energy values on Columbia reservoir storage as the Peace reservoir approaches full is demonstrated in both Figure 3-19 and Figure 3-20; this behaviour also occurs for the months of August through February.
Figure 3-21 presents the Columbia marginal energy value for November as a function of both Columbia and Peace reservoir storage. Figure 3-22 presents details at lower marginal Columbia energy values that cannot be readily seen in Figure 3-21.

Figure 3-21: November Marginal Columbia Energy Value Functions

In Figure 3-21 note that the marginal Columbia energy value is much less sensitive to Peace reservoir storage above half full than it is to Peace reservoir storage below half full over the entire range of possible Columbia reservoir storage. In addition, for lower Columbia reservoir storage, the marginal Columbia reservoir storage is less dependent on Peace reservoir storage levels equal to 60000 cmsd or less than it is for other values of Columbia reservoir storage. Figure 3-22 shows that there is still a slight dependence of the marginal Columbia energy value upon both Columbia and Peace reservoir storage.

Figure 3-22: November Marginal Columbia Energy Value Functions—Detail
In this section a one-scenario base case has been studied, establishing the behaviour of the storage values and marginal storage values for the mean values of the scenario-dependent demands, inflows, and prices. In the following section, scenarios in which the monthly scenario-dependent parameters vary symmetrically around their mean values are presented. By comparing the results in the following section with those in this section, the effect of varying the scenario-dependent parameters is determined.

3.4.3.2 Results for Five-Scenario Cases with Non-Anticipative Turbine Release

In this section, a number of five-scenario cases are studied. In each case, some subset of the scenario-dependent parameters assume specified values. As outlined above, the scenario-dependent parameters are the demands, inflows, and prices. The specification of the scenarios includes correlations between the scenario-dependent parameters. The seven five-scenario cases outlined in section 3.3 above were examined for coefficient of variation values of 0.10, 0.25, and 0.40. In the months of November through April there is only one scenario for all cases; in the months of May through October, there are either five scenarios or one scenario as indicated in Table 3-16.

<table>
<thead>
<tr>
<th>Case</th>
<th>Description</th>
<th>Demand</th>
<th>Inflow</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Demands Only</td>
<td>5 values</td>
<td>1 value</td>
<td>1 value</td>
</tr>
<tr>
<td>B</td>
<td>Inflows Only</td>
<td>1 value</td>
<td>5 values</td>
<td>1 value</td>
</tr>
<tr>
<td>C</td>
<td>Prices Only</td>
<td>1 value</td>
<td>1 value</td>
<td>5 values</td>
</tr>
<tr>
<td>D</td>
<td>Perfect Positive Correlation</td>
<td>5 values</td>
<td>5 values</td>
<td>5 values</td>
</tr>
<tr>
<td>E</td>
<td>Demands and Inflows Perfectly Correlated and Prices Perfectly Negatively Correlated</td>
<td>5 values</td>
<td>5 values</td>
<td>5 values</td>
</tr>
<tr>
<td>F</td>
<td>Demands and Prices Perfectly Correlated and Inflows Perfectly Negatively Correlated</td>
<td>5 values</td>
<td>5 values</td>
<td>5 values</td>
</tr>
<tr>
<td>G</td>
<td>Inflows and Prices Perfectly Correlated and Demands Perfectly Negatively Correlated</td>
<td>5 values</td>
<td>5 values</td>
<td>5 values</td>
</tr>
</tbody>
</table>

Table 3-16: Definition of Five-Scenario Cases

The values assumed by the scenario-dependent parameters under the five-scenario cases are dependent upon the mean value of the parameter and the coefficient of variation of the parameter. For the cases in which all of the scenario-dependent parameters assume five different values, the coefficient of variation is assumed to be the same for all of the parameters—that is, if the coefficient of variation for the demand is 0.10, then the coefficients of variation for the inflows and prices are also 0.10. Under a scenario, the scenario-dependent parameter is equal to the parameter mean plus the product of the parameter mean, the coefficient of variation, and the characteristic values for a unit normal function for the scenario. The unit normal characteristic values and the discrete occurrence probabilities of the scenarios are defined in Table 3-17. The values in Table 3-17 are from Kim and Palmer (1997).
Table 3-17: Unit Normal Function Values for Five-Scenario Cases

<table>
<thead>
<tr>
<th></th>
<th>Unit Normal Value</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Low</td>
<td>-1.72</td>
<td>0.107</td>
</tr>
<tr>
<td>Low</td>
<td>-0.76</td>
<td>0.245</td>
</tr>
<tr>
<td>Average</td>
<td>0.00</td>
<td>0.296</td>
</tr>
<tr>
<td>High</td>
<td>0.76</td>
<td>0.245</td>
</tr>
<tr>
<td>Very High</td>
<td>1.72</td>
<td>0.107</td>
</tr>
</tbody>
</table>

All of the cases studied in this section are for non-anticipative turbine releases—that is, equation (3-23) specifies that the turbine release must be the same over all of the scenarios. The effect of these non-anticipative constraints is to prevent the model from acting with perfect knowledge of the future.

**A Cases: Scenario-Dependent Demands Only**

In the scenario-dependent demands only cases, as shown in Table 3-16, the inflows and prices do not vary with the scenario, while the values assumed by the demand and the scenario probabilities, are defined by the characteristic values in Table 3-17. Perfect positive correlation is assumed between heavy load hour and light load hour demands. The cases studied are for coefficients of variation of 0.10, 0.25, and 0.40. The mean demands are shown in Table 3-3.

In order to assess the effect of varying the coefficient of variation, a comparison of the minimum and maximum marginal energy values, for both the Columbia and Peace reservoirs, is made.

For each coefficient of variation value, the monthly marginal values of energy stored in the Peace reservoir were calculated at an increment of 50000 cmsd from 0 to 500000 cmsd for the Peace reservoir and a 20000 cmsd increment in the Columbia reservoir between 0 and 200000 cmsd. For each month, and for each coefficient of variation value, the minimum Peace marginal energy values ($/MWh) are shown in Table 3-18, in which “CV” denotes the value of the coefficient of variation.
<table>
<thead>
<tr>
<th></th>
<th>CV = 0.00</th>
<th>CV = 0.10</th>
<th>CV = 0.25</th>
<th>CV = 0.40</th>
</tr>
</thead>
<tbody>
<tr>
<td>JAN</td>
<td>1947.31</td>
<td>1948.38</td>
<td>1948.89</td>
<td>1947.30</td>
</tr>
<tr>
<td>FEB</td>
<td>1065.03</td>
<td>1064.49</td>
<td>1064.52</td>
<td>1064.48</td>
</tr>
<tr>
<td>MAR</td>
<td>1051.11</td>
<td>1051.56</td>
<td>1053.48</td>
<td>1051.21</td>
</tr>
</tbody>
</table>

Table 3-19: Corresponding Maximum Peace Reservoir Marginal Energy Values ($/MWh) for Three Demands Only Scenarios, as Well as for the One-Scenario Base Case.
### Table 3-19: Maximum Peace Marginal Energy Values for Demands Only Scenarios

The data in Table 3-19 are presented graphically as Figure 3-24. Again, the seasonal shape of the maximum marginal Peace energy values in Figure 3-24 is representative of cases B through G as well.

<table>
<thead>
<tr>
<th>Month</th>
<th>APR</th>
<th>MAY</th>
<th>JUN</th>
<th>JUL</th>
<th>AUG</th>
<th>SEP</th>
<th>OCT</th>
<th>NOV</th>
<th>DEC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>554.03</td>
<td>570.20</td>
<td>639.19</td>
<td>557.86</td>
<td>45.56</td>
<td>76.06</td>
<td>210.63</td>
<td>53.72</td>
<td></td>
</tr>
<tr>
<td>Value</td>
<td>64.56</td>
<td>193.08</td>
<td>362.84</td>
<td>160.20</td>
<td>1483.03</td>
<td>1478.79</td>
<td>1466.76</td>
<td>1454.23</td>
<td></td>
</tr>
<tr>
<td>Value</td>
<td>1102.03</td>
<td>995.08</td>
<td>922.85</td>
<td>843.17</td>
<td>1694.98</td>
<td>1695.31</td>
<td>1707.04</td>
<td>1708.10</td>
<td></td>
</tr>
<tr>
<td>Value</td>
<td>1868.19</td>
<td>1860.66</td>
<td>1866.72</td>
<td>1867.49</td>
<td>2034.20</td>
<td>2034.06</td>
<td>2033.31</td>
<td>2034.11</td>
<td></td>
</tr>
<tr>
<td>Value</td>
<td>2111.38</td>
<td>2111.34</td>
<td>2111.22</td>
<td>2111.37</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note that the seasonal pattern of maximum marginal Peace energy values is independent of the coefficient of variation. However, the coefficient of variation value can affect the maximum marginal Peace energy value in a month by up to $300/MWh; in some other months, the range over all coefficient of variation values is less than $1/MWh. The influence of the coefficient of variation value is greatest for the April through July period. Note that the one-scenario base case can either under- or over-estimate the marginal Peace energy value over all of the other coefficient of variation values.**

For each coefficient of variation value the monthly Columbia marginal energy value has been calculated at an increment of 200000 cmsd from 0 to 200000 cmsd for the Columbia reservoir and at a 50000 cmsd increment in Peace reservoir storage between 0 and 500000 cmsd. For each month, and for each scenario, the minimum and maximum marginal Columbia energy values ($/MWh) are shown in Table 3-20.
Table 3-20: Minimum and Maximum Columbia Marginal Energy Values for Demands Only Scenarios

The minimum Columbia marginal energy values shown in Table 3-20 are presented graphically as Figure 3-25. The shape of the minimum Columbia marginal energy values in Figure 3-25 are representative of cases B to G.

Figure 3-25: Minimum Columbia Marginal Energy Values for Demands Only Scenarios

The seasonal pattern displayed by the minimum Columbia marginal energy values is similar to that for the minimum Peace marginal energy values. Again, in general, the minimum marginal energy value decreases with an increasing coefficient of variation. With the exception of the autumn months, the one-scenario base case sets an upper bound on the minimum Columbia marginal energy value. The range of difference explained by the coefficient of variation values only exceeds $10/MWh in the months of January,
February, August, and September. In some summer months, the minimum Columbia marginal energy value does not differ amongst the coefficient of variation values considered.

The maximum marginal Columbia energy values shown in Table 3-20 are presented graphically as Figure 3-26. The seasonal shape of the maximum Columbia marginal energy values in Figure 3-26 are representative of cases B to G.

![Maximum COL Marginal Energy Values for Demands Only Scenarios](image)

**Figure 3-26: Maximum Columbia Marginal Energy Values for Demands Only Scenarios**

As for the maximum Peace marginal energy values, the seasonal shape of the maximum Columbia energy value curve is independent of the coefficient of demand variations, and it is in the spring and summer that the maximum marginal energy values are most sensitive to the CV. At one extreme, over the range of coefficients of variation from 0.0 to 0.4, the monthly maximum value can vary by over $325/MWh; at the other extreme, the variation is less than $1/MWh.

The general observations to be taken from this section are that: the minimum marginal energy values are most sensitive to variation in demand over the autumn and winter; the maximum marginal energy values are most sensitive to variation in demand over the spring and early summer; there is the potential for the maximum, as compared to the minimum, marginal energy values to be more strongly affected by the coefficient of demand variation; for all but the autumn, the base case sets an upper bound on the monthly minimum marginal energy values; and the base case can set either a upper or lower bound on the monthly maximum marginal energy values.

**B Cases: Scenario-Dependent Inflows Only**

In these cases the demands and prices do not vary between scenarios in any month. There are five scenarios for the inflows in the months of May through October, and there is only one scenario in the months of November through April. The cases studied are for
coefficient of variation values of 0.10, 0.25, and 0.40. Table 3-1 defines the mean inflows, and the inflow values in the five scenarios are given by Table 3-17. Perfect positive correlation is assumed between inflows into the Columbia and Peace reservoirs. The minimum and maximum marginal energy values for the Peace and Columbia are given, respectively in Table 3-21 and Table 3-22.

<table>
<thead>
<tr>
<th></th>
<th>CV = 0.00</th>
<th>CV = 0.10</th>
<th>CV = 0.25</th>
<th>CV = 0.40</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
</tr>
<tr>
<td>FEB</td>
<td>20.25</td>
<td>1065.03</td>
<td>20.21</td>
<td>1064.48</td>
</tr>
<tr>
<td>MAR</td>
<td>9.11</td>
<td>1051.11</td>
<td>10.05</td>
<td>1051.13</td>
</tr>
<tr>
<td>APR</td>
<td>1.17</td>
<td>554.03</td>
<td>1.25</td>
<td>554.84</td>
</tr>
<tr>
<td>MAY</td>
<td>0.00</td>
<td>45.56</td>
<td>0.00</td>
<td>47.14</td>
</tr>
<tr>
<td>JUN</td>
<td>0.00</td>
<td>64.56</td>
<td>0.00</td>
<td>98.79</td>
</tr>
<tr>
<td>JUL</td>
<td>8.83</td>
<td>1102.03</td>
<td>10.34</td>
<td>1061.15</td>
</tr>
<tr>
<td>AUG</td>
<td>30.84</td>
<td>1488.03</td>
<td>30.26</td>
<td>1485.52</td>
</tr>
<tr>
<td>SEP</td>
<td>31.42</td>
<td>1649.28</td>
<td>31.11</td>
<td>1690.48</td>
</tr>
<tr>
<td>OCT</td>
<td>30.93</td>
<td>1868.19</td>
<td>30.61</td>
<td>1860.98</td>
</tr>
<tr>
<td>NOV</td>
<td>29.09</td>
<td>2034.20</td>
<td>29.63</td>
<td>2034.26</td>
</tr>
<tr>
<td>DEC</td>
<td>27.52</td>
<td>2111.38</td>
<td>27.52</td>
<td>2111.37</td>
</tr>
</tbody>
</table>

Table 3-21: Minimum and Maximum Peace Marginal Energy Values for Inflows Only Scenarios

For the coefficient of variation values studied, the minimum marginal Peace energy value is not affected to a great extent in absolute terms. In each month, the four values for the minimum Peace marginal energy value are within $5/MWh of one another. In comparison with case A, overall the coefficient of variation has less effect on the minimum Peace marginal energy values for case B; April, May, and June are exceptions. Depending upon the month, the minimum marginal Peace energy value for the one-scenario base case can be either an upper or lower bound.

The maximum Peace marginal energy value for the one-scenario base case can also be an upper or lower bound, depending upon the month. For the spring and early summer months, the maximum Peace marginal energy value increases with an increasing coefficient of variation—that is, when there is a wider range of possible inflows, the maximum marginal energy value is greater. Within a month, variation in the coefficient of variation can cause differences in the maximum Peace marginal energy value of more than $300/MWh or less than $1/MWh; for each of the months from November through March the spread in maximum marginal energy values is less than $5/MWh.

<table>
<thead>
<tr>
<th></th>
<th>CV = 0.00</th>
<th>CV = 0.10</th>
<th>CV = 0.25</th>
<th>CV = 0.40</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
</tr>
<tr>
<td>JAN</td>
<td>23.87</td>
<td>1084.66</td>
<td>24.41</td>
<td>1084.66</td>
</tr>
<tr>
<td>FEB</td>
<td>18.49</td>
<td>1078.02</td>
<td>18.91</td>
<td>1078.02</td>
</tr>
</tbody>
</table>
Table 3-22: Minimum and Maximum Columbia Marginal Energy Values for Inflows Only Scenarios

As with the minimum marginal Peace energy values, the minimum marginal Columbia values over all of the coefficient of variation values studied lie within a narrow band. In all months except for March, the range is less than $5/MWh. Also, as with the Peace values, there is less spread than for the case A values except for in the spring/early summer period. Depending on the month, the minimum Peace marginal energy value for the one-scenario base case can either be an upper or lower bound.

For the maximum Columbia marginal energy values, the one-scenario base case can set either an upper or lower bound. Similarly to the Peace, over the spring and early summer months the maximum Columbia marginal energy value increases with an increasing coefficient of variation. The range of coefficients of variation considered can account for a difference in the maximum Columbia marginal energy value of as much as $450/MWh, or as little as $0/MWh.

The general observations to be taken from this section are that: the minimum marginal energy values, in general, exhibit less sensitivity to a range of coefficients of inflow variation than for an equal variation in the coefficient of demand variation; over the spring and early summer months the maximum Columbia marginal energy values for both reservoirs increase with an increasing coefficient of variation; and, the base case can set either an upper or lower bound on the monthly minimum or maximum marginal energy values for either the Peace or Columbia.

C Cases: Scenario-Dependent Prices Only

In the C cases the demands and inflows do not vary between scenarios. For each month from November through April there is only one scenario for the price, and in the remaining months, there are five scenarios. Perfect positive correlation between heavy load hour and light load hour prices is assumed. The mean prices are defined in Table 3-2, and the prices for the five scenarios are defined by these average prices and the unit normal values specified in Table 3-17. The cases studied are for coefficient of variation values of 0.10, 0.25, and 0.40.
In this section, the non-anticipative turbine release constraints ensure that the turbine release is the same over all of the scenarios. As the only parameters that are changing with the scenario are the prices, the non-anticipative constraints ensure that the turbine release is independent of the energy prices. As the releases are the same for each scenario, so is the generation, as are the demands and inflows. Recall from the formulation presented in (3-1) to (3-29) that the HLH and LLH prices are only present in the objective function (3-29), where they are used to determine the export revenues and import costs. Since the generation and demand are the same under all scenarios, so will be the imports and exports, as well as the end of period storage volumes. Thus, the only difference between the objective function values under each scenario will be a linear function of the HLH and LLH energy prices. So, the probability weighted objective function value over all of the price scenarios is equal to the objective function value for the mean price. In other words, given the symmetrical distribution, for these prices only cases, the results are independent of the coefficient of variation, and are equal to the results for the one-scenario base case, reported in section 3.4.3.1.

**D Cases: Scenario-Dependent Demands, Inflows, and Prices—All Perfectly Positively Correlated**

For the D cases, each of the five scenarios for May through October has unique demands, inflows, and prices. There is only one scenario for the months of November through April. The assumption is made that demand, inflow, and price are all perfectly positively correlated—that is, when the demands are “Very Low”, as defined in Table 3-17, the inflows and prices are also “Very Low”; similarly, if the demands are “Very High”, so are the other two scenario-dependent parameters. Mean inflows, prices, and demands are respectively given by Table 3-1, Table 3-2, and Table 3-3. Perfect positive correlation between HLH and LLH prices is assumed, as is perfect positive correlation between HLH and LLH demands, and between Peace and Columbia inflows. The D cases are “good news/bad news cases”—if the demand is high, the inflows will also be high allowing for more generation, but the price of any imports will be high. If the demand is low, the inflows will be low, but the prices will also be low.

Coefficient of variation values of 0.10, 0.25, and 0.40 are studied. The purpose of studying these perfect correlation cases is to present an extreme case of possible results. The minimum and maximum marginal energy values for the Peace and Columbia are given, respectively in Table 3-23 and Table 3-24.

<table>
<thead>
<tr>
<th></th>
<th>CV = 0.00</th>
<th></th>
<th>CV = 0.10</th>
<th></th>
<th>CV = 0.25</th>
<th></th>
<th>CV = 0.40</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
</tr>
<tr>
<td>JAN</td>
<td>24.97</td>
<td>1947.31</td>
<td>24.43</td>
<td>1948.45</td>
<td>22.59</td>
<td>1949.20</td>
<td>5.33</td>
<td>1949.54</td>
</tr>
<tr>
<td>FEB</td>
<td>20.25</td>
<td>1065.03</td>
<td>17.32</td>
<td>1064.49</td>
<td>9.64</td>
<td>1064.69</td>
<td>0.73</td>
<td>1064.76</td>
</tr>
<tr>
<td>MAR</td>
<td>9.11</td>
<td>1051.11</td>
<td>3.96</td>
<td>1051.84</td>
<td>0.16</td>
<td>1054.75</td>
<td>0.05</td>
<td>1055.58</td>
</tr>
<tr>
<td>APR</td>
<td>1.17</td>
<td>554.03</td>
<td>0.17</td>
<td>580.21</td>
<td>0.00</td>
<td>684.41</td>
<td>0.04</td>
<td>714.44</td>
</tr>
<tr>
<td>MAY</td>
<td>0.00</td>
<td>45.56</td>
<td>0.00</td>
<td>97.84</td>
<td>0.00</td>
<td>303.52</td>
<td>0.00</td>
<td>362.82</td>
</tr>
<tr>
<td>JUN</td>
<td>0.00</td>
<td>64.56</td>
<td>0.08</td>
<td>198.52</td>
<td>0.00</td>
<td>403.56</td>
<td>0.00</td>
<td>392.98</td>
</tr>
</tbody>
</table>

87
Table 3-23: Minimum and Maximum Peace Marginal Energy Values for Perfect Positive Correlation Scenarios

With the exception of the late fall and early winter, the minimum Peace marginal energy values decrease with an increasing coefficient of variation. In the D cases, a higher coefficient of variation translates into a wider range of demands, inflows, and prices covered over the scenarios. So, the wider the range of scenario-dependent parameters, the lower the minimum Peace marginal energy value. The maximum range in monthly minimum Peace marginal energy values over the coefficient of variation values tested was approximately $20/MWh, while the minimum range was $0/MWh.

Over the range of coefficient of variation values tested, the monthly Peace maximum marginal energy value varies by as much as $340/MWh, or as little as less than $1/MWh. Over the spring and early summer, the maximum Peace marginal energy value increases with an increasing coefficient of variation. However, over the late summer and early fall, the maximum Peace marginal energy value decreases with an increasing coefficient of variation.

Table 3-24: Minimum and Maximum Columbia Marginal Energy Values for Perfect Positive Correlation Scenarios

For the Columbia, as for the Peace, with the exception of the late fall and early winter, the minimum marginal energy value decreases with an increasing coefficient of variation.
The range of coefficient of variation values studied results in a monthly range in the minimum marginal energy value of between $0/MWh and $22/MWh.

Over the range of coefficient of variation values tested, the monthly maximum Columbia marginal energy value can vary by as much as $415/MWh, or as little as less than $1/MWh. As with the Peace, during the spring and early summer, the maximum Columbia marginal energy value increases with an increasing coefficient of variation, while during the late summer and early fall, the maximum Columbia marginal energy value decreases with an increasing coefficient of variation.

From this section, the general observations that can be taken are: except for the late fall/early winter period, the minimum marginal energy values for both the Columbia and Peace decrease as the coefficient of variation increases; during the spring and early summer the maximum marginal energy values for both reservoirs increases with the coefficient of variation; and, during the late summer and early fall, the maximum energy values for the Columbia and Peace decrease with an increasing CV.

**E Cases: Scenario-Dependent Demands, Inflows, and Prices—Demands and Inflows Perfectly Positively Correlated and Prices Perfectly Negatively Correlated**

In the E cases there are five scenarios for the months of May through October, with each scenario in a month having unique demands, inflows, and prices. There is only one scenario for each of the months from November through April. The assumptions are that the demands and inflows are perfectly positively correlated and the prices are perfectly negatively correlated with the demands and inflows—for example, if the demands are “Low” as defined by Table 3-17 then the inflows are also “Low”, and the prices are “High”. Mean inflows, prices, and demands are given, respectively, in Table 3-1, Table 3-2, and Table 3-3. Perfect positive correlation between HLH and LLH prices is assumed, as is perfect positive correlation between HLH and LLH demands, and between Peace and Columbia inflows. The E cases are “generally good news” cases. If the demand is high, the inflows will also be high allowing for more generation, and the prices will be low if imports are required. On the other hand, if the demand is low, the inflows will also be low, but the prices will be high if there is any energy for export.

Coefficient of variation values of 0.10, 0.25, and 0.40 are studied. The same value of the coefficient of variation is assumed for each scenario-dependent parameter. The minimum and maximum marginal energy values for the Peace and Columbia are given, respectively, in Table 3-25 and Table 3-26.

<table>
<thead>
<tr>
<th></th>
<th>CV = 0.00</th>
<th></th>
<th>CV = 0.10</th>
<th></th>
<th>CV = 0.25</th>
<th></th>
<th>CV = 0.40</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
</tr>
<tr>
<td>JAN</td>
<td>24.97</td>
<td>1947.31</td>
<td>24.38</td>
<td>1948.45</td>
<td>22.64</td>
<td>1949.21</td>
<td>5.47</td>
</tr>
<tr>
<td>FEB</td>
<td>20.25</td>
<td>1065.03</td>
<td>17.26</td>
<td>1064.49</td>
<td>9.66</td>
<td>1064.69</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Table 3-25: Minimum and Maximum Peace Marginal Energy Values for Perfect Correlation—Positive for Demands and Inflows and Negative for Prices

The minimum Peace marginal energy value decreases with an increasing coefficient of variation value except for in the late fall and early winter. For the range of coefficient of variation values considered, the maximum range of minimum Peace marginal energy values in any month is $22/MWh, while the minimum range is $0/MWh.

The range, over the coefficient of variation values studied, of maximum Peace marginal energy values in a month ranges from a high of $340/MWh to a low of less than $1/MWh. Over the spring and early summer, the maximum Peace marginal energy value increases with an increasing coefficient of variation; whereas, during the late summer and early fall, the maximum Peace marginal energy value decreases with an increasing coefficient of variation.

Table 3-26: Minimum and Maximum Columbia Marginal Energy Values for Perfect Correlation—Positive for Demands and Inflows and Negative for Prices

Similarly to the Peace, the minimum Columbia marginal energy value decreases with an increasing coefficient of variation. In addition, the maximum marginal Columbia energy
value increases with an increasing coefficient of variation during the spring and early summer; whereas, during the late summer and early fall, the maximum marginal Columbia energy value decreases with an increasing coefficient of variation.

The range of variation in the monthly minimum Columbia marginal value attributable to the range of coefficients of variation tested is between $21/MWh and $0/MWh; this range is very similar to that for the Peace. The range, over the coefficient of variation values studied, of maximum marginal Columbia energy values in a month ranges from a high of $425/MWh to a low of less than $1/MWh.

From this section, the general observations are: except for the late fall/early winter period, the minimum marginal energy values for both the Columbia and Peace decrease as the coefficient of variation increases; during the spring and early summer the maximum marginal energy values for both reservoirs increases with the coefficient of variation; and, during the late summer and early fall, the maximum energy values for the Columbia and Peace decrease with an increasing CV. Note that these observations are the same as those for the D cases. The difference between cases D and E is that in case E, the prices are perfectly negatively correlated with the demands and inflows, while in case D all of the scenario-dependent parameters are perfectly positively correlated.

**F Cases: Scenario-Dependent Demands, Inflows, and Prices—Demands and Prices Perfectly Positively Correlated and Inflows Perfectly Negatively Correlated**

The cases considered here all have five scenarios for the months of May through October, with each scenario in these months having unique demands, inflows, and prices. The months of November through April each have a single scenario. It is assumed that the demands and prices are perfectly positively correlated and the inflows are perfectly negatively correlated with the demands and prices. For example, if the demands are “Very High” as defined by Table 3-17 then the prices are also “Very High”, and the inflows are “Very Low”. Perfect correlation between HLH and LLH prices is assumed, as is perfect positive correlation between HLH and LLH demands, and between Peace and Columbia inflows. The F cases studied here can be thought of as “bad news” cases—if the demand is high, the inflow will be low, and the prices will be high if imports are needed. Conversely, if the demand is low, there will be high inflow, but prices in the export market will be lower.

Coefficient of variation values of 0.10, 0.25, and 0.40 are considered. The same value of the coefficient of variation is assumed for each scenario-dependent parameter. The minimum and maximum marginal energy values for the Peace and Columbia reservoirs are given, respectively, in Table 3-27 and Table 3-28.
<table>
<thead>
<tr>
<th></th>
<th>CV = 0.00</th>
<th></th>
<th>CV = 0.10</th>
<th></th>
<th>CV = 0.25</th>
<th></th>
<th>CV = 0.40</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
</tr>
<tr>
<td>FEB</td>
<td>20.25</td>
<td>1065.03</td>
<td>17.29</td>
<td>1064.49</td>
<td>11.83</td>
<td>1064.69</td>
<td>7.82</td>
</tr>
<tr>
<td>MAR</td>
<td>9.11</td>
<td>1051.11</td>
<td>4.18</td>
<td>1051.88</td>
<td>2.88</td>
<td>1054.78</td>
<td>3.36</td>
</tr>
<tr>
<td>APR</td>
<td>1.17</td>
<td>554.03</td>
<td>0.10</td>
<td>581.51</td>
<td>0.88</td>
<td>685.87</td>
<td>1.68</td>
</tr>
<tr>
<td>MAY</td>
<td>0.00</td>
<td>45.56</td>
<td>0.00</td>
<td>100.40</td>
<td>0.43</td>
<td>306.42</td>
<td>0.65</td>
</tr>
<tr>
<td>JUN</td>
<td>0.00</td>
<td>64.56</td>
<td>0.00</td>
<td>201.98</td>
<td>0.00</td>
<td>404.14</td>
<td>0.29</td>
</tr>
<tr>
<td>JUL</td>
<td>8.53</td>
<td>1102.03</td>
<td>4.20</td>
<td>963.56</td>
<td>1.74</td>
<td>867.09</td>
<td>1.79</td>
</tr>
<tr>
<td>AUG</td>
<td>30.84</td>
<td>1483.03</td>
<td>27.61</td>
<td>1488.85</td>
<td>19.81</td>
<td>1457.18</td>
<td>10.67</td>
</tr>
<tr>
<td>SEP</td>
<td>31.42</td>
<td>1694.98</td>
<td>31.15</td>
<td>1703.29</td>
<td>30.48</td>
<td>1707.67</td>
<td>19.77</td>
</tr>
<tr>
<td>OCT</td>
<td>30.93</td>
<td>1868.19</td>
<td>30.79</td>
<td>1868.87</td>
<td>31.87</td>
<td>1865.36</td>
<td>25.24</td>
</tr>
<tr>
<td>NOV</td>
<td>29.09</td>
<td>2034.20</td>
<td>29.85</td>
<td>2033.95</td>
<td>30.75</td>
<td>2032.85</td>
<td>26.67</td>
</tr>
<tr>
<td>DEC</td>
<td>27.52</td>
<td>2111.38</td>
<td>27.57</td>
<td>2111.33</td>
<td>27.34</td>
<td>2111.14</td>
<td>23.49</td>
</tr>
</tbody>
</table>

Table 3-27: Minimum and Maximum Peace Marginal Energy Values for Perfect Correlation—Positive for Demands and Prices and Negative for Inflows

The maximum range of minimum Peace marginal energy values in any month is $20/MWh, while the minimum range is $0/MWh. The range, considering all months, over the coefficient of variation values studied is tighter than all previously considered cases, except for Case B, for which only the inflows varied. The minimum marginal Peace energy value decreases with increasing coefficient of variation for the late winter and the late summer and early fall.

The maximum Peace marginal energy value increases with an increasing coefficient of variation over the spring and early summer, while over the late summer and early fall the maximum Peace marginal energy value decreases with an increasing coefficient of variation. The range, over the coefficient of variation values studied, of maximum Peace marginal energy values in a month ranges from a high of $320/MWh to a low of less than $1/MWh.
As was the case for the Peace, the minimum Columbia marginal energy values have less spread, considering all months, over the coefficient of variation values studied for all cases except for B, the case with inflows as the only scenario-dependent parameters. The range of variation in the monthly minimum Columbia marginal value attributable to the coefficients of variation tested is between $18/MWh and less than $1/MWh. In addition, the minimum Columbia marginal energy value decreases with an increasing coefficient of variation for the late winter and the late summer.

The maximum Columbia marginal energy value increases with an increasing coefficient of variation over the spring and early summer, while over the late summer and early fall the maximum Columbia marginal energy value decreases with an increasing coefficient of variation; this was also true for the maximum Peace marginal energy value. The range, over the coefficient of variation values studied, of maximum Columbia marginal energy values in a month ranges from a high of $400/MWh to a low of less than $3/MWh.

The observations that can be taken for the F cases are: the minimum marginal energy value for both reservoirs decrease with an increasing coefficient of variation for the late winter and the late summer; the minimum marginal energy value curves are closer together for all cases studied so far except for the case with inflows as the only scenario-dependent parameters; during the spring and early summer the maximum marginal energy values for both reservoirs increases with the coefficient of variation; and, during the late summer, the maximum energy values for the Columbia and Peace decrease with an increasing coefficient of variation.

**G Cases: Scenario-Dependent Demands, Inflows, and Prices—Inflows and Prices Perfectly Positively Correlated and Demands Perfectly Negatively Correlated**

The G cases considered here have five scenarios in each of the months from May through October, with each scenario in these months having unique values for the demands, inflows, and prices. The months of November through April each have a single scenario. In this section it is assumed that the inflows and prices are perfectly positively correlated, and that both of these parameters are perfectly negatively correlated with the demands. By way of example, if the prices are “High”, as defined by Table 3-17, then the inflows are also “High” and the demands are “Low”. Perfect positive correlation between HLH and LLH prices is assumed, as is perfect positive correlation between HLH and LLH demands, and between Peace and Columbia inflows. The G cases are “good news/bad news cases”. If the demand is high, the inflows will be low, but the price of any necessary imports will be low. On the other hand, if the demand is low, the inflows will be high, and any exports will be sold at a high price.
Values for the coefficient of variation of 0.10, 0.25, and 0.40 are studied. The same coefficient of variation value is assumed for each scenario-dependent parameter. The minimum and maximum marginal energy values for the Peace and Columbia reservoirs are given, respectively in Table 3-29 and Table 3-30.

<table>
<thead>
<tr>
<th></th>
<th>CV = 0.00</th>
<th></th>
<th>CV = 0.10</th>
<th></th>
<th>CV = 0.25</th>
<th></th>
<th>CV = 0.40</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
</tr>
<tr>
<td>FEB</td>
<td>20.25</td>
<td>1065.03</td>
<td>17.20</td>
<td>1064.49</td>
<td>12.02</td>
<td>1064.70</td>
<td>7.83</td>
</tr>
<tr>
<td>MAR</td>
<td>9.11</td>
<td>1051.11</td>
<td>4.07</td>
<td>1051.88</td>
<td>2.90</td>
<td>1054.87</td>
<td>3.36</td>
</tr>
<tr>
<td>APR</td>
<td>1.17</td>
<td>554.03</td>
<td>0.09</td>
<td>581.57</td>
<td>0.86</td>
<td>688.89</td>
<td>1.68</td>
</tr>
<tr>
<td>MAY</td>
<td>0.00</td>
<td>45.56</td>
<td>0.00</td>
<td>100.54</td>
<td>0.00</td>
<td>312.37</td>
<td>0.065</td>
</tr>
<tr>
<td>JUN</td>
<td>0.00</td>
<td>64.56</td>
<td>0.00</td>
<td>202.53</td>
<td>0.00</td>
<td>409.78</td>
<td>0.29</td>
</tr>
<tr>
<td>JUL</td>
<td>8.53</td>
<td>1102.03</td>
<td>4.18</td>
<td>964.54</td>
<td>1.76</td>
<td>870.77</td>
<td>1.80</td>
</tr>
<tr>
<td>AUG</td>
<td>30.84</td>
<td>1483.03</td>
<td>27.48</td>
<td>1488.83</td>
<td>20.08</td>
<td>1458.31</td>
<td>10.67</td>
</tr>
<tr>
<td>SEP</td>
<td>31.42</td>
<td>1694.98</td>
<td>31.06</td>
<td>1703.29</td>
<td>30.88</td>
<td>1707.93</td>
<td>19.77</td>
</tr>
<tr>
<td>OCT</td>
<td>30.93</td>
<td>1868.19</td>
<td>30.70</td>
<td>1868.87</td>
<td>32.23</td>
<td>1865.48</td>
<td>25.24</td>
</tr>
<tr>
<td>NOV</td>
<td>29.09</td>
<td>2034.20</td>
<td>29.76</td>
<td>2033.95</td>
<td>31.02</td>
<td>2032.82</td>
<td>26.67</td>
</tr>
<tr>
<td>DEC</td>
<td>27.52</td>
<td>2111.38</td>
<td>27.52</td>
<td>2111.33</td>
<td>27.43</td>
<td>2111.13</td>
<td>23.49</td>
</tr>
</tbody>
</table>

Table 3-29: Minimum and Maximum Peace Marginal Energy Values for Perfect Correlation—Positive for Inflows and Prices and Negative for Demands

<table>
<thead>
<tr>
<th></th>
<th>CV = 0.00</th>
<th></th>
<th>CV = 0.10</th>
<th></th>
<th>CV = 0.25</th>
<th></th>
<th>CV = 0.40</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
</tr>
<tr>
<td>JAN</td>
<td>23.87</td>
<td>1084.66</td>
<td>24.10</td>
<td>1084.73</td>
<td>22.78</td>
<td>1084.98</td>
<td>12.90</td>
</tr>
<tr>
<td>FEB</td>
<td>18.49</td>
<td>1078.02</td>
<td>15.59</td>
<td>1078.00</td>
<td>9.96</td>
<td>1077.92</td>
<td>5.73</td>
</tr>
<tr>
<td>MAR</td>
<td>4.90</td>
<td>1067.13</td>
<td>2.54</td>
<td>1067.13</td>
<td>2.09</td>
<td>1067.13</td>
<td>2.08</td>
</tr>
<tr>
<td>APR</td>
<td>0.00</td>
<td>948.87</td>
<td>0.00</td>
<td>957.08</td>
<td>0.70</td>
<td>985.36</td>
<td>0.73</td>
</tr>
<tr>
<td>MAY</td>
<td>0.00</td>
<td>48.39</td>
<td>0.00</td>
<td>115.36</td>
<td>0.00</td>
<td>345.79</td>
<td>0.12</td>
</tr>
<tr>
<td>JUN</td>
<td>0.00</td>
<td>75.02</td>
<td>0.00</td>
<td>239.56</td>
<td>0.00</td>
<td>438.00</td>
<td>0.14</td>
</tr>
<tr>
<td>JUL</td>
<td>0.00</td>
<td>1067.16</td>
<td>0.00</td>
<td>954.36</td>
<td>0.03</td>
<td>856.72</td>
<td>0.94</td>
</tr>
<tr>
<td>AUG</td>
<td>18.66</td>
<td>1143.96</td>
<td>15.08</td>
<td>1132.18</td>
<td>7.54</td>
<td>1098.77</td>
<td>4.12</td>
</tr>
<tr>
<td>SEP</td>
<td>31.72</td>
<td>1125.67</td>
<td>31.09</td>
<td>1124.85</td>
<td>29.00</td>
<td>1122.30</td>
<td>14.15</td>
</tr>
<tr>
<td>OCT</td>
<td>31.02</td>
<td>1117.20</td>
<td>30.63</td>
<td>1117.54</td>
<td>31.95</td>
<td>1119.36</td>
<td>23.32</td>
</tr>
<tr>
<td>NOV</td>
<td>28.30</td>
<td>1110.95</td>
<td>29.77</td>
<td>1111.31</td>
<td>30.39</td>
<td>1112.64</td>
<td>25.73</td>
</tr>
<tr>
<td>DEC</td>
<td>26.04</td>
<td>1098.45</td>
<td>27.29</td>
<td>1098.65</td>
<td>26.90</td>
<td>1099.35</td>
<td>21.38</td>
</tr>
</tbody>
</table>

Table 3-30: Minimum and Maximum Columbia Marginal Energy Values for Perfect Correlation—Positive for Inflows and Prices and Negative for Demands

The minimum and maximum Peace marginal energy values for case G under the coefficient of variation values considered are virtually identical to those under case F, as are those for the minimum and maximum Columbia marginal energy values. The
observations for case F apply to case G as well. In both cases F and G the inflows and demands are perfectly negatively correlated, the difference between the two is that the prices are perfectly positively correlated with the demands in case F and the inflows in case G.

Comparing the results for cases A through G over the three coefficient of variation values yields the following observations. For the two cases in which the demands and inflows are perfectly positively correlated, as well as the demands only case, with the exception of the late fall and early winter, the minimum marginal energy values decrease with an increasing coefficient of variation. For the two cases in which the demands and inflows are perfectly negatively correlated, during the late winter and late spring, the minimum marginal energy values decrease with an increasing coefficient of variation.

It was found that with the exception of the demands only case, during the spring and early summer the maximum marginal energy values increase with the coefficient of variation. For the two cases in which the demands and inflows are perfectly positively correlated, during the late summer and early fall the maximum marginal energy values decrease with an increasing coefficient of variation. In the two cases in which the demands and inflows are perfectly negatively correlated and the demands only case, during the late summer, the maximum marginal energy values decrease with an increasing coefficient of variation.

A final point should be made with regard to the dependence of the marginal energy value in one reservoir on storage in the other reservoir as the storage in the first reservoir changes. Peace reservoir storage affects the dependence of the Peace marginal energy values on the Columbia reservoir storage to a greater extent than Columbia reservoir storage affects the dependence of the Columbia marginal energy values on Peace reservoir storage. For example, for a coefficient of variation value of 0.25, the ranges of Columbia storage over which the Columbia marginal energy values were least sensitive to changes in Peace storage did not change with the case for any month. Similarly, during August through February, the ranges of Peace reservoir storage over which the Peace marginal energy values were least sensitive to changes in Columbia storage did not change; however, changes with the case were observed for March through July.

**Comparison Over Coefficient of Variation Values**

Above, the manner in which the minimum and maximum marginal energy values for the Columbia and Peace vary with the coefficient of variation has been studied for each of the seven different scenario-dependent parameter cases. Below, a similar comparison over the seven cases is made for each coefficient of variation value.

Figure 3-27 plots the maximum Columbia marginal energy value for the one-scenario base case as well as for cases A to B and D through G for a coefficient of variation value of 0.40. In Figure 3-27 it can be seen that all cases have similar maximum Columbia marginal energy values over the fall and winter. Cases D to G—the cases in which there
is variation in all three of the scenario-dependent parameters—have very similar marginal values over the entire year. Case B, in which only the inflows vary, is similar to cases D to G over the fall, winter, and spring. During the fall, winter, and spring, the one-scenario base case is quite similar to case A, in which only the demands vary; while during the summer case B is very similar to the one-scenario base case.

Figure 3-27: Maximum Columbia Marginal Energy Values for Coefficient of Variation of 0.40

Figure 3-28 presents the maximum Columbia marginal energy values for each case for a coefficient of variation value of 0.25. For the months of January to March and September through December there is little difference in the maximum marginal Columbia energy values between the cases. Cases D through G—the cases in which all three scenario-dependent parameters vary with the scenario—all have very similar maximum marginal Columbia energy values. The maximum marginal Columbia energy values for case A, in which only the demands vary, are closer to those for cases D through G than they are to case B, in which only the inflows vary, or the one-scenario base case. Case B is closest to the one-scenario case. There are distinct differences between the results shown in Figure 3-27, for which the coefficient of variation is 0.40, and Figure 3-28.
Figure 3-28: Maximum Columbia Marginal Energy Values for Coefficient of Variation of 0.25

Figure 3-29 presents the maximum Columbia marginal energy values for each case for a coefficient of variation value of 0.10. In comparing the maximum Columbia energy values with those for the two higher coefficient of variation values shown in the previous two figures, note the extent to which variation amongst the cases is reduced in Figure 3-29. As was the case for a coefficient of variation value of 0.25, during the months of January to March and September through December there is little difference in the maximum marginal Columbia energy values between the cases; cases D through G, for which all of the scenario-dependent parameters vary, all have very similar maximum Columbia marginal energy values; the maximum Columbia marginal energy values for case A, which has scenario-dependent demands, are closer to those for cases D through G than they are to case B, which has scenario-dependent inflows, or the one-scenario base case; and case B is closest to the one-scenario case.
Figure 3-30 shows the minimum Columbia marginal energy values for a coefficient of variation value of 0.40. From Figure 3-30 it can be observed that: the one-scenario base case is closest to case B, which has scenario-dependent inflows; cases F and G are virtually identical, as are cases D and E; case A, which has scenario-dependent demands is more similar to cases D through G than it is to case B or the one-scenario base case; and, over most of the year, the one-scenario base case is least similar to cases D and E.

Figure 3-30: Minimum Columbia Marginal Energy Values for Coefficient of Variation of 0.40

Figure 3-31 plots the minimum marginal Columbia energy values for a coefficient of variation value of 0.25. In comparison to a coefficient of variation value of 0.40, some minimum marginal values are decreased, while others increase. However, the increase in the range, over the seven cases, of minimum marginal energy values that occurs for an increase in the coefficient of variation is obvious in a comparison of Figure 3-30 and Figure 3-31. As for the previous figure, the one-scenario base case and the scenario-dependent inflows case B are quite similar to one another; and cases D and E are virtually identical, as are cases F and G; case A, with scenario-dependent demands, is most similar to cases D through G. The one-scenario base case is least similar to case A.
Comparison of Minimum Columbia Plant Marginal Energy for CV = 0.25

Figure 3-31: Minimum Columbia Marginal Energy Values for Coefficient of Variation of 0.25

Figure 3-32 presents the minimum Columbia marginal energy values for a coefficient of variation of 0.10. Again, the effect of a reduced coefficient of variation on the similarity of the minimum Columbia marginal energy values for the cases is very noticeable through comparison with Figure 3-30 and Figure 3-31. Once again, case B, with scenario-dependent inflows, is most similar to the one-scenario base case, while case A, with scenario-dependent demands, is least similar to the one-scenario base case. Cases D through G, each with all three scenario-dependent parameters varying, are all virtually identical to one another.

Comparison of Minimum Columbia Plant Marginal Energy for CV = 0.10

Figure 3-32: Minimum Columbia Marginal Energy Values for Coefficient of Variation of 0.10

Figure 3-33 presents the maximum Peace marginal energy value for all of the cases studied for a coefficient of variation value of 0.40. In Figure 3-33 it can be observed that
over the months of January through March, and September through December all of the scenarios are very similar. Cases D through G are virtually identical to one another, and are also quite similar to case B, with its scenario-dependent inflows, over the spring. Over the spring and early summer, the one-scenario base case is closest to case A, which has scenario-dependent demands, while over the late summer, the one-scenario base case is closest to case B.

Comparison of Max PCE Marginal Energy Values for CV=0.40

![Figure 3-33: Maximum Peace Marginal Energy Values for Coefficient of Variation of 0.40](image)

In Figure 3-34 the maximum Peace marginal energy values for a coefficient of variation of 0.25 are presented for each case. As with the coefficient of variation value of 0.40, over the autumn and winter all of the cases have similar maximum Peace marginal values in any given month. During the spring and summer the one-scenario base case is most similar to case B, which has scenario-dependent inflows. Again cases D through G are all very similar.

Comparison of Max PCE Marginal Energy Values for CV=0.25

![Figure 3-34: Maximum Peace Marginal Energy Values for Coefficient of Variation of 0.25](image)
Figure 3-35 presents the maximum Peace marginal energy values for a coefficient of variation value of 0.10. A comparison of Figure 3-35 with Figure 3-33 and Figure 3-34 illustrates the effect of the coefficient of variation on the maximum Peace marginal energy value. As with a coefficient of variation value of 0.25, all scenarios are very similar over the autumn and winter, while over the spring and summer scenario B, which has scenario-dependent inflows, and the one-scenario base case are very close to one another, and cases D through G are also very similar to one another.

*Figure 3-35: Maximum Peace Marginal Energy Values for Coefficient of Variation of 0.10*

Figure 3-36 presents the minimum Peace marginal energy values for a coefficient of variation value of 0.40. Observe that the one-scenario base case is most consistent with case B, which has scenario-dependent inflows. Cases D and E are virtually identical, as are cases F and G. For the coefficient of variation value of 0.40, for all but the spring and early summer, the minimum Peace marginal energy values for the one-scenario base case and case B are greater than those for the three scenario-dependent parameters cases or case A, which has scenario-dependent demands.
In Figure 3-37 the minimum Peace marginal energy values for a coefficient of variation of 0.25 are presented. The effect of the reduced coefficient of variation on the minimum Peace marginal energy values is noticeable in a comparison with Figure 3-36. As was true for the case in which the coefficient of variation value is 0.40, the one-scenario base case and case B, with scenario-dependent inflows, are most similar, cases D and E are virtually identical, as are cases F and G. For the late winter, spring, and summer, the minimum marginal energy values for case B and the one-scenario base case are greater than those for cases D to G or case A, which has scenario-dependent demands.
Figure 3-38 further illustrates the effect of a reduced coefficient of variation and the similarity of the minimum Peace marginal energy values over the cases studied.

![Comparison of Min PCE Plant Marginal Energy Values for CV=0.10](image)

**Figure 3-38: Maximum Peace Marginal Energy Values for Coefficient of Variation of 0.10**

The comparison of the maximum and minimum marginal energy values over the seven cases for coefficient of variation values of 0.10, 0.25, and 0.40 yields some interesting insights.

Over the fall and winter period, there is little variation in the maximum Columbia marginal energy value over the seven different cases; the same is true for the maximum Peace marginal energy value. Throughout the remainder of the year, there is comparatively little variation in the Columbia marginal energy values over the four cases in which the three scenario-dependent cases all vary; again, this is also true of the maximum Peace marginal energy values. For both the Columbia and Peace reservoirs it was found that during the spring and summer as the coefficient of variation decreases, the case in which only the demands vary approached those for the cases in which all three of the scenario-dependent parameters vary. It was also found that during the spring and summer as the coefficient of variation decreases the maximum marginal energy value for the case in which only the inflows vary approaches that for the one-scenario base case. For all of the coefficient of variation values studied, it was found that, for both the Columbia and Peace, the one-scenario base case underestimates the maximum marginal energy value, with respect to the other cases, during the spring and early summer, and overestimates the maximum marginal energy value during the late summer.

For both reservoirs, and all coefficient of variation values studied, it was found that: the minimum marginal energy values under the one-scenario base case are closest to those under the inflows only case; the minimum marginal energy values under the two cases in which the demands and inflows are perfectly positively correlated are virtually identical, regardless of which of these two parameters is perfectly negatively correlated with the prices; the minimum marginal energy values under the two cases in which the demands

103
and inflows are perfectly negatively correlated are virtually identical, regardless of which of these two parameters is perfectly positively correlated with the prices; and, that as the coefficient of variation value decreases, the minimum marginal energy values under the one-scenario base case are furthest from those under the demands only case. For most of the year, when the coefficient of variation is 0.40, the one-scenario base case overestimates the minimum marginal energy values in both reservoirs as compared to the other cases. When the coefficient of variation is equal to either 0.10 or 0.25, the one-scenario base case overestimates the minimum marginal energy values for both reservoirs—with the exception of the inflows only case—in the late winter and late summer, and underestimates these values in the late fall and early winter.

The above results suggest that the minimum energy values are more sensitive to a variation in demand than they are to equal variation in the inflows. Recalling that the minimum energy values occur when both reservoirs are full, note that under healthy storage conditions, the marginal energy values are affected to a greater extent by a variation in the demand than by an equal variation in the inflow. Similarly, past the peak of the freshet, for the maximum marginal energy values—which occur under adverse storage conditions—variation in the demand was of greater influence on the marginal energy values than an equal variation in the inflows over all of the coefficient of variation values studied. In spring, leading up to, and including, the peak freshet month, it was found that for the two lower coefficient of variation values studied, the marginal energy values are more sensitive to demand variation than inflow variation. However, when the coefficient of variation value was 0.40, inflow variation was of greater importance than demand variation; this was confirmed for a CV value of 0.55.

3.4.3.3 Results for Five-Scenario Cases with Perfect Foresight

In this section, the results under the assumption of non-anticipative turbine releases from the previous section are compared against the case of operating without any non-anticipative constraints—that is, operating with perfect foresight. Perfect foresight is attained by removing constraint (3-23) from the model formulation. The same cases of scenario-dependent parameters, outlined in Table 3-16, are studied for coefficient of variation values of 0.10 and 0.25.

A Cases: Scenario-Dependent Demands Only

In the scenario-dependent demands only cases, as shown in Table 3-16, the inflows and prices do not vary with the scenario, while the values assumed by the demand, and the scenario probabilities, are defined by the characteristic values in Table 3-17. Perfect positive correlation is assumed between HLH and LLH demands. The cases studied are for coefficients of variation of 0.10 and 0.25; the mean demands are shown in Table 3-3.

For each coefficient of variation value studied, the monthly marginal values of energy stored in the Peace reservoir were calculated at an increment of 50000 cmsd from 0 to 500000 cmsd for the Peace reservoir and a 20000 cmsd increment in the Columbia reservoir between 0 and 200000 cmsd. The maximum monthly marginal energy values
for the Columbia and Peace are presented, respectively, as Figure 3-39 and Figure 3-40. In the figures, the symbol “NAQT” is used to represent the maximum marginal energy values under the non-anticipative turbine release constraints studied in section 3.4.3.2, and “NNA” represents the case of no non-anticipative constraints.

In Figure 3-39 observe that for all coefficient of variation values, during the late summer the maximum Columbia marginal energy values under the no non-anticipative constraints are greater than those for the non-anticipative turbine release constraints. During the spring and early summer the maximum Columbia marginal energy values under the perfect foresight case are exceeded by those for the non-anticipative turbine release constraints case.

As Figure 3-39 illustrates, the change of constraints regarding future knowledge on the problem can have a dramatic influence on the maximum marginal Columbia energy values in the spring and summer months. For a coefficient of variation of 0.10, the maximum marginal Columbia energy values under the no non-anticipative constraints in a month can be as much as $154/MWh lower, or as much as $95/MWh higher, than the corresponding value under the non-anticipative turbine releases; for a coefficient of variation value of 0.25, the corresponding values are $264/MWh and $180/MWh.
From Figure 3-40, observe that during the spring and early summer the maximum Peace marginal energy values are greater under the non-anticipative turbine release constraints case, while during the late summer, the values are greater under the no non-anticipative constraints case. For a coefficient of variation value of 0.25, the maximum Peace marginal energy values under the non-anticipative turbine release constraints in a month can exceed those under the no non-anticipative constraints by as much as $252/MWh, or be exceeded by as much as $124/MWh.
Figure 3-41 demonstrates that during the winter the minimum Columbia marginal energy values for the case of no non-anticipative constraints exceed those for the non-anticipative turbine release constraints case. When the coefficient of variation is 0.25, the minimum monthly Columbia energy values can change by as much as $15/MWh between the perfect foresight and non-anticipative turbine release cases.
Figure 3-42 demonstrates that during the winter and late summer, the minimum Peace marginal energy values for no non-anticipative constraints exceed those for non-anticipative turbine releases. During the fall the minimum Peace marginal energy values for non-anticipative turbine releases exceed those for perfect foresight. For a coefficient of variation value of 0.25, the difference in the monthly maximum Peace marginal energy value for the two assumptions can be as much as $15/MWh.

The general observations to be taken for the perfect foresight demands only cases are: during the spring and early summer the maximum marginal energy values for both reservoirs increase with an increasing coefficient of variation value; in the late summer, the maximum marginal energy values decrease with an increasing coefficient of variation; in the late summer the maximum marginal energy values are greater under the perfect foresight case, while in the spring and early summer the maximum marginal energy values are greater for the non-anticipative turbine releases; and, during the winter and late summer the minimum marginal energy values are greater in the perfect foresight case.

**B Cases: Scenario-Dependent Inflows Only**

In these cases the demands and prices do not vary between scenarios in any month. There are five scenarios for the inflows in the months of May through October and there is only one scenario in the months of November through April. The cases studied are for coefficient of variation values of 0.10 and 0.25. Table 3-1 defines the mean inflows, and the inflow values in the five scenarios are given by Table 3-17. Perfect positive correlation is assumed between inflows to the Columbia and Peace reservoirs. The maximum marginal energy values for the Columbia and Peace reservoirs are given, respectively, in Figure 3-43 and Figure 3-44, the corresponding minimums are given in Figure 3-45 and Figure 3-46.
In Figure 3-43 observe that for the inflows only case, with minor exceptions, the maximum marginal Columbia energy values under the non-anticipative turbine release constraints are greater than those under the no non-anticipative constraints case.

For the maximum Peace marginal energy values shown in Figure 3-44 there is no real pattern as to the effect of the differing constraints.
During the fall and early winter for both the Peace and Columbia, the minimum marginal energy values for the no non-anticipative constraints case exceed those for the non-anticipative turbine release case, with the converse being true for spring and early summer. The magnitude of these differences is not large, being on the order of $1/MWh.
The observations for the inflows-only scenarios under perfect foresight are: during the spring and early summer the maximum marginal energy values for both reservoirs increase with an increasing coefficient of variation value; with minor exceptions, the maximum Columbia marginal energy values are greater under the non-anticipative turbine releases; and, the minimum marginal energy values during the fall and early winter are greater for the perfect foresight case, while during the spring and early summer the minimum marginal energy values are greater for the non-anticipative turbine releases.

**C Cases: Scenario-Dependent Prices Only**

In the C cases the demands and inflows do not vary between scenarios in any month. For each month from November through April there is only one scenario for the price; in the remaining months, there are five scenarios. Perfect positive correlation between HLH and LLH is assumed. The mean prices are defined in Table 3-2, and the prices for the five scenarios are defined by these average prices, and the unit normal values specified in Table 3-17. The cases studied are for coefficient of variation values of 0.10 and 0.25.

In this section, the assumption of perfect foresight, in contrast to the previous section, allows the turbine releases to vary between scenarios. As a result, there are differences between case C and the one-scenario base case.

![Maximum COL Marginal Energy Values for Prices Only Scenarios](image)

**Figure 3-47: Maximum Columbia Marginal Energy Values for Prices Only Scenarios**

In Figure 3-47 observe that for the prices only case the differences in the monthly maximum Columbia marginal energy values between the non-anticipative turbine
releases and perfect foresight are minor. For a coefficient of variation value of 0.25, the maximum difference in the maximum monthly marginal value between the two cases is $2.18/MWh. During the fall and winter there are essentially no differences between the two cases, while in the spring and summer the maximum marginal Columbia energy values for the perfect foresight case exceed those for the assumption of non-anticipative turbine release.

![Max PCE Marginal Energy Values for Prices Only Scenarios](image)

**Figure 3-48: Maximum Peace Marginal Energy for Prices Only Scenarios**

For the maximum Peace marginal energy values, shown in Figure 3-48, the largest difference between maximum marginal energy values for the two cases is $2.27/MWh. During the spring, early summer, and early winter the maximum Peace marginal energy values are greater for the perfect foresight case for both coefficient of variation values, and the converse is true for February. For a coefficient of variation value of 0.25 there is little difference in the summer and fall values, while during this period for a coefficient of variation value of 0.10, the non-anticipative turbine release values are greater.

The effects of the assumptions regarding foresight on the minimum Columbia marginal energy values are shown in Figure 3-49. This assumption can affect the minimum monthly Columbia marginal energy value by up to $7.78/MWh for a coefficient of variation value of 0.25. During the spring and early summer there is no difference between the two cases, while in the late summer, the maximum Columbia marginal energy values under the assumption of non-anticipative turbine releases exceed those for the perfect foresight case. During the late fall and winter for a coefficient of variation value of 0.10 the maximum Columbia marginal energy values for the perfect foresight...
case exceed those for non-anticipative turbine releases, while the opposite is true for a coefficient of variation value of 0.25.

**Minimum COL Marginal Energy Values for Price Only Scenarios**

Figure 3-49: Minimum Columbia Marginal Energy Values for Prices Only Scenarios

Figure 3-50 illustrates the effect of assumptions regarding foresight on the minimum Peace marginal energy values. For a coefficient of variation value of 0.25 the assumption made can affect the minimum monthly Peace marginal energy value by up to $1.84/MWh. During the late spring and early summer the assumption made has no effect. Furthermore, for a coefficient of variation value of 0.25 the minimum Peace marginal energy values for all other periods, with the exception of July and the early spring, under the assumption of non-anticipative turbine releases exceed those under perfect foresight. For a coefficient of variation value of 0.10, the minimum marginal Peace energy values under the assumption of non-anticipative turbine releases exceed those under perfect foresight, with the exceptions of July and the late fall and early winter.
The observations to be taken for the prices only scenarios under the assumption of perfect foresight is that in comparison to the demands only and inflows only cases the changes in the maximum marginal energy values are minor for the prices only cases.

**D Cases: Scenario-Dependent Demands, Inflows, and Prices—All Perfectly Positively Correlated**

For these cases, each of the five scenarios for May through October has unique demands, inflows, and prices. There is only one scenario for the months of November through April. The assumption made is that demand, inflow, and price are all perfectly positively correlated—that is, when the demands are “Very Low”, as defined in Table 3-17, the inflows and prices are also “Very Low”; similarly, if the demands are “Very High”, so are the other two scenario-dependent parameters. Mean inflows, prices, and demands are respectively given by Table 3-1, Table 3-2, and Table 3-3. Perfect correlation between HLH and LLH prices is assumed, as is perfect positive correlation between HLH and LLH demands, and between Peace reservoir and Columbia reservoir inflows.

The maximum marginal energy values for the Columbia and Peace reservoirs are given, respectively, in Figure 3-51 and Figure 3-52, and the corresponding minimums are given in Figure 3-53 and Figure 3-54.
In Figure 3-51 observe that during the spring and early summer the maximum Columbia marginal energy values are greater under the case of non-anticipative turbine releases, while the converse is true during the late summer. For a coefficient of variation value of 0.25, the assumption regarding constraints can affect the maximum monthly marginal energy value by as much as $294/MWh.

Figure 3-51: Maximum Columbia Marginal Energy Values for Perfectly Positively Correlated Demand, Inflow, and Price Scenarios

Figure 3-52: Maximum Peace Marginal Energy Values for Perfectly Positively Correlated Demand, Inflow, and Price Scenarios
The observations for the maximum Peace marginal energy values are very similar to those for the Columbia. Again, for all coefficient of variation values studied, the maximum marginal energy values under the assumption of non-anticipative turbine releases are greater than those for perfect foresight during the spring and the early summer, with the opposite being true during the late summer. For a coefficient of variation of 0.25, the different constraints can affect the maximum monthly Peace marginal energy value by as much as $281/MWh.

Figure 3-53: Minimum Columbia Marginal Energy Values for Perfectly Positively Correlated Demand, Inflow, and Price Scenarios

As can be observed in Figure 3-53 during the winter and late summer the minimum Columbia marginal energy values are greater under the assumption of perfect foresight for all coefficient of variation values studied. In a similar manner, as shown in Figure 3-54, during the winter, spring, and summer the minimum Peace marginal energy values are greater under the assumption of perfect foresight for all coefficient of variation values studied.
The observations to be taken for the perfect positive correlation case under the assumption of perfect foresight are: the minimum marginal energy values decrease with an increasing coefficient of variation; during the spring and early summer the maximum marginal energy values increase with an increasing coefficient of variation, while during the later summer and early fall, the maximum marginal energy values decrease with an increasing coefficient of variation; during the spring and early summer the maximum marginal energy values are greater for the non-anticipative turbine releases, while during the late summer the maximum marginal energy values are greater under the assumption of perfect foresight; during the winter and late summer the minimum Columbia marginal energy values are greater for perfect foresight; and during the spring, winter, and summer the minimum Peace marginal energy values are greater for the assumption of perfect foresight.

**E Cases: Scenario-Dependent Demands, Inflows, and Prices—Demands and Inflows Perfectly Positively Correlated and Prices Perfectly Negatively Correlated**

In the E cases there are five scenarios for the months of May through October, with each scenario in a month having unique demands, inflows, and prices. There is only one scenario for each of November through April. The assumptions for the cases studied here are that the demands and inflows are perfectly positively correlated and the prices are perfectly negatively correlated with the demands and inflows—for example, if the demands are “Low” as defined by Table 3-17 then the inflows are also “Low”, and the prices are “High”. Mean inflows, prices, and demands are respectively given by Table
3-1, Table 3-2, and Table 3-3. Perfect correlation between HLH and LLH prices is assumed, as is perfect positive correlation between HLH and LLH demands, and between Peace and Columbia inflows. Coefficient of variation values of 0.10 and 0.25 are studied. The same value of the coefficient of variation is assumed for each scenario-dependent parameter.

The maximum marginal energy values for the Columbia and Peace are given, respectively, in Figure 3-55 and Figure 3-56, the corresponding minimums are given in Figure 3-57 and Figure 3-58.

![Maximum COL Marginal Energy Values for Inflows and Demands; Prices Scenarios](image)

**Figure 3-55: Maximum Columbia Marginal Energy Values for Demand and Inflow Perfectly Correlated and Price Perfectly Negatively Correlated Scenarios**

In Figure 3-55 observe that during the spring and early summer the maximum Columbia marginal energy values are greater under the case of non-anticipative turbine releases, while the converse is true during the late summer. These observations are the same as for case D. For a CV value of 0.25, the assumption regarding constraints can affect the maximum monthly Columbia marginal energy value by as much as $296/MWh.
The observations for the maximum Peace marginal energy values are very similar to those for the Columbia. Again, for all coefficient of variation values studied, the maximum marginal energy values under the assumption of non-anticipative turbine releases are greater than those for perfect foresight during the spring and the early summer, with the opposite being true during the late summer. For a coefficient of variation of 0.25, the different constraints regarding future knowledge can affect the maximum monthly Peace marginal energy value by as much as $284/MWh.
As can be observed in Figure 3-57 during the winter and late summer the minimum Columbia marginal energy values are greater under the assumption of perfect foresight for all coefficient of variation values studied. In a similar manner, as shown in Figure 3-58, during the winter, spring, and summer the minimum Peace marginal energy values are greater under the assumption of perfect foresight for all coefficient of variation values studied. These observations are again the same as those for case D, in which there is perfect positive correlation between all three scenario-dependent parameters.
Figure 3-58: Minimum Peace Marginal Energy Values for Demand and Inflow Perfectly Correlated and Price Perfectly Negatively Correlated Scenarios

The observations to be taken for the case of perfect positive correlation for the demands and inflows and perfect negative correlation for the prices under the assumption of perfect foresight are: the minimum marginal energy values decrease with an increasing coefficient of variation; during the spring and early summer the maximum marginal energy values increase with an increasing coefficient of variation, while during the later summer and early fall, the maximum marginal energy values decrease with an increasing coefficient of variation; during the spring and early summer the maximum marginal energy values are greater for the non-anticipative turbine releases, while during the late summer the maximum marginal energy values are greater under the assumption of perfect foresight; during the winter and late summer the minimum Columbia marginal energy values are greater for perfect foresight; and during the spring, winter, and summer the minimum marginal energy values are greater for the assumption of perfect foresight.

F Cases: Scenario-Dependent Demands, Inflows, and Prices—Demands and Prices Perfectly Positively Correlated and Inflows Perfectly Negatively Correlated

The cases considered here all have five scenarios for the months of May through October, with each scenario in these months having unique demands, inflows, and prices. The months of November through April each have a single scenario. It is assumed that the demands and prices are perfectly positively correlated and the inflows are perfectly negatively correlated with the demands and prices. For example, if the demands are “Very High” as defined by Table 3-17 then the prices are also “Very High”, and the inflows are “Very Low”. Perfect correlation between HLH and LLH prices is assumed,
as is perfect positive correlation between HLH and LLH demands, and between Peace and Columbia inflows. Coefficient of variation values of 0.10 and 0.25 are considered. The same value of the coefficient of variation is assumed for each scenario-dependent parameter.

The maximum marginal energy values for the Columbia and Peace are given, respectively, in Figure 3-59 and Figure 3-60; the corresponding minimums are given in Figure 3-61 and Figure 3-62.

As was true for the E cases, the observations for the F cases on the effects of the assumptions regarding foresight on the minimum and maximum marginal energy values for the two reservoirs are the same as those for case D, in which all three scenario-dependent parameters are perfectly positively correlated.

![Maximum COL Marginal Energy Values for Prices, Demands; Inflows Scenarios](image)

**Figure 3-59: Maximum Columbia Marginal Energy Values for Demand and Price Perfectly Correlated and Inflow Perfectly Negatively Correlated Scenarios**

In Figure 3-59 and Figure 3-60 observe that during the spring and early summer the maximum marginal energy values are greater under the case of non-anticipative turbine releases, while the converse is true during the late summer. For a coefficient of variation value of 0.25, the assumption regarding constraints on utilizing knowledge about the future can affect the maximum monthly Columbia marginal energy value by as much as $117/MWh and the maximum monthly Peace marginal energy value by up to $121/MWh.
Figure 3-60: Maximum Peace Marginal Energy Values for Demand and Price Perfectly Correlated and Inflow Perfectly Negatively Correlated Scenarios

As shown in Figure 3-61 during the winter and late summer the maximum Columbia marginal energy values are greater under the assumption of perfect foresight, and, as shown in Figure 3-62, during the winter, spring, and summer the maximum Peace marginal energy values are greater under the assumption of perfect foresight.

Figure 3-61: Minimum Columbia Marginal Energy Values for Demand and Price Perfectly Correlated and Inflow Perfectly Negatively Correlated Scenarios
The observations to be taken for the case of perfect positive correlation for the demands and prices and perfect negative correlation for the inflows under the assumption of perfect foresight are: the minimum marginal energy values decrease with an increasing coefficient of variation during the late summer, fall, and early winter, whereas they increase with an increasing coefficient of variation during the spring and early summer; during the spring and early summer the maximum marginal energy values increase with an increasing coefficient of variation, while during the later summer and early fall, the maximum marginal energy values decrease with an increasing coefficient of variation; during the spring and early summer the maximum marginal energy values are greater for the non-anticipative turbine releases, while during the late summer the maximum marginal energy values are greater under the assumption of perfect foresight; during the winter and late summer the minimum Columbia marginal energy values are greater for perfect foresight; and during the spring, winter, and summer the minimum Peace marginal energy values are greater for the assumption of perfect foresight.

**G Cases: Scenario-Dependent Demands, Inflows, and Prices—Inflows and Prices Perfectly Positively Correlated and Demands Perfectly Negatively Correlated**

The G cases considered here have five scenarios in each of the months from May through October, with each scenario in a month having unique values for the demands, inflows, and prices. The months of November through April each have a single scenario. For this section it is assumed that the inflows and prices are perfectly positively correlated, and that both of these parameters are perfectly negatively correlated with the demands. By
way of example, if the prices are "High", as defined by Table 3-17, then the inflows are also high and the demands are "Low". Perfect correlation between HLH and LLH prices is assumed, as is perfect positive correlation between HLH and LLH demands, and between Peace and Columbia inflows. Values for the coefficient of variation of 0.10 and 0.25 are studied. The same value of the coefficient of variation is assumed for each scenario-dependent parameter.

The maximum marginal energy values for the Columbia and Peace are given, respectively, in Figure 3-63 and Figure 3-64; the corresponding minimums are given in Figure 3-65 and Figure 3-66.

**Figure 3-63: Maximum Columbia Marginal Energy Values for Inflow and Price Perfectly Correlated and Demand Perfectly Negatively Correlated Scenarios**

Once more, the observations of the effects of the assumptions regarding foresight on the minimum and maximum marginal energy values for the two reservoirs for the G case are the same as those for case D, in which all three scenario-dependent parameters are perfectly correlated.

In Figure 3-63 and Figure 3-64 observe that during the spring and early summer the maximum marginal energy values are greater under the case of non-anticipative turbine releases, while the converse is true during the late summer. For a coefficient of variation value of 0.25, the assumption regarding constraints can affect the maximum monthly Columbia marginal energy value by as much as $118/MWh and the maximum monthly Peace marginal energy value by up to $125/MWh.
As shown in Figure 3-65 during the winter and late summer the minimum Columbia marginal energy values are greater under the assumption of perfect foresight, while, as shown in Figure 3-66, during the winter, spring, and summer the minimum Peace marginal energy values are greater under the assumption of perfect foresight.
The observations to be taken for the case of perfect positive correlation for the inflows and prices and perfect negative correlation for the demands under the assumption of perfect foresight are: the minimum marginal energy values decrease with an increasing coefficient of variation during the late summer, fall, and early winter, whereas they increase with an increasing coefficient of variation during the spring and early summer; during the spring and early summer the maximum marginal energy values increase with an increasing coefficient of variation, while during the later summer and early fall, the maximum marginal energy values decrease with an increasing coefficient of variation; during the spring and early summer the maximum marginal energy values are greater for the non-anticipative turbine releases, while during the late summer the maximum marginal energy values are greater under the assumption of perfect foresight; during the winter and late summer the minimum Columbia marginal energy values are greater for perfect foresight; and during the spring, winter, and summer the minimum Peace marginal energy values are greater for the assumption of perfect foresight.

Comparing the results for cases A through G over the two coefficient of variation values yields the following results. The variation in the maximum marginal energy values for the two reservoirs with the coefficient of variation was much less for the prices only case. For all but the prices only case, during the spring and early summer the maximum marginal energy values increase with the coefficient of variation. For the demands only case, during the late summer the maximum marginal energy values decrease with the coefficient of variation. For the four cases in which all three of the scenario-dependent parameters vary with the scenario, the maximum marginal energy values decrease with increasing coefficient of variation for the late summer and early spring. For the two cases in which the demands and inflows are perfectly positively correlated, the minimum
marginal energy values decrease with an increasing coefficient of variation. Similarly, for the two cases in which the demands and inflows are perfectly negatively correlated, the minimum marginal energy values decrease with an increasing coefficient of variation for the late summer through early winter, while they increase with an increasing coefficient of variation during the spring and early summer.

In terms of a comparison with the marginal values under the non-anticipative turbine releases and perfect foresight assumptions, the following results were found. For the demands only case, as well as for the four cases in which all three scenario-dependent parameters vary, the maximum marginal energy values during the late summer are greater under the assumption of perfect foresight; for these same cases, the maximum marginal energy values are greater under the assumption of non-anticipative turbine releases for the spring and early summer. With minor exceptions, the maximum marginal energy values for the inflows only case are greater in all months under the assumption of perfect foresight. For the demands only case, the minimum marginal energy values during the winter and late summer are greater under the assumption of perfect foresight. For the four cases in which all three scenario-dependent parameters vary, the minimum Columbia marginal energy values are greater under the assumption of perfect foresight during the winter and late summer. For these same four cases, the minimum Peace marginal energy values are greater under the assumption of perfect foresight during the spring, summer, and winter. For the inflows only case, the minimum marginal energy values during the fall and early winter are greater under the assumption of perfect foresight, while the opposite is true during the spring and early summer.

A final note should be made as to how the assumption regarding foresight affects the dependence of the marginal energy value in one of the reservoirs on the storage in the second reservoir with storage in the first reservoir. The region of Columbia reservoir storage over which the marginal Columbia energy values are least dependent on Peace reservoir storage do not change from the case of non-anticipative turbine releases in any month. However, in the months of March through June, the region of Peace reservoir storage over which the marginal Peace energy values are least dependent on Columbia reservoir storage change in the perfect foresight case.

**Comparison Over Coefficient of Variation Values**

Above, the manner in which the minimum and maximum marginal energy values for the Columbia and Peace reservoirs change with the coefficient of variation have been studied for each of the seven different scenario-dependent parameter cases under the assumption of perfect foresight. Below, a similar comparison over the seven cases is made for each coefficient of variation value for the perfect foresight case.

The monthly maximum Columbia marginal energy values for coefficient of variation values of 0.25 and 0.10 are presented as Figure 3-67 and Figure 3-68.
In Figure 3-67 it can be observed that during the fall and winter there is little difference in the maximum Columbia marginal energy values between scenarios. The maximum Columbia marginal energy values for cases D and E—the two cases for which the demands and inflows are perfectly positively correlated—are virtually identical throughout the year. Similarly, cases F and G—the two cases for which the demands and inflows are perfectly negatively correlated—are very similar throughout the year. During the spring and summer the maximum Columbia marginal energy values for case A—the case in which only the demands vary—are similar to those for the two cases for which the demands and inflows are perfectly positively correlated. During the late summer, maximum Columbia marginal energy values for case B—the inflows only case—are similar to those for the two cases in which the demands and inflows are perfectly negatively correlated. In the spring and summer, the maximum marginal energy values for the one-scenario base case are closest to those for the prices only case. During the spring and summer the maximum Columbia marginal energy values for the two cases in which the demands and inflows are perfectly negatively correlated are furthest from those for the one-scenario base case.

During the spring and early summer, the one-scenario base case provides the lowest maximum Columbia marginal energy values, and the cases in which the demands and inflows are negatively correlated provides the highest maximum Columbia marginal energy values. In the late summer, the one-scenario base case yields the highest maximum Columbia marginal energy values, and the cases in which the demand and inflows are perfectly negatively correlated yield the lowest maximum Columbia marginal energy values.

![Comparison of Max Columbia Plant Marginal Energy Values for CV = 0.25](image)

**Figure 3-67: Maximum Columbia Marginal Energy Values for Coefficient of Variation of 0.25**
The observations regarding the relationships between the maximum Columbia marginal energy values under the seven cases for Figure 3-68 are essentially the same as those for Figure 3-67, indicating that the relationships are not dependent on the coefficient of variation.

Comparing Figure 3-67 and Figure 3-68 the manner in which the differences between the maximum Columbia marginal energy values over the seven cases increase with the coefficient of variation is evident. A comparison of Figure 3-67 and Figure 3-68 and the corresponding figures for the non-anticipative turbine release case—Figure 3-28 and Figure 3-29—indicates that there is less variation between the maximum Columbia marginal energy values over the scenarios when there is perfect foresight. By way of example, the maximum and average variation in the maximum Columbia marginal energy values for a coefficient of variation of 0.25 for the non-anticipative turbine releases are $363/MWh and $81.1/MWh, while the corresponding values for the perfect foresight case are $284/MWh and $54.2/MWh.

The monthly minimum Columbia marginal energy values for coefficient of variation values of 0.25 and 0.10 are presented as Figure 3-69 and Figure 3-70.
Comparing Figure 3-69 and Figure 3-70 the manner in which the differences between the minimum Columbia marginal energy values over the seven cases increase with the
coefficient of variation is evident. Note that the minimum Columbia marginal energy values for the prices only case do not closely track those for the one-scenario base case.

A comparison of Figure 3-69 and Figure 3-70 and the corresponding figures for the non-anticipative turbine release case—Figure 3-31 and Figure 3-32—indicates that there is less variation between the minimum Columbia marginal energy values over the scenarios when there is perfect foresight. By way of example, for a coefficient of variation value of 0.25, the average monthly variation in the minimum marginal Columbia energy values was $4.7/MWh for the non-anticipative turbine release case, and $3.3/MWh for the perfect foresight case.

The monthly maximum Peace marginal energy values for coefficient of variation values of 0.25 and 0.10 are presented as Figure 3-71 and Figure 3-72.

In Figure 3-71 it can be observed that during the fall and winter there is little difference in the maximum Peace marginal energy values between scenarios. The maximum marginal energy values for cases D and E—the two cases for which the demands and inflows are perfectly positively correlated—are virtually identical throughout the year. Similarly, cases F and G—the two cases for which the demands and inflows are perfectly negatively correlated—are very similar throughout the year. During the spring and summer the maximum Peace marginal energy values for case A—the case in which only the demands vary—are similar to those for the two cases for which the demands and inflows are perfectly positively correlated. During the late summer, maximum Peace marginal energy values for case B—the inflows only case—are similar to those for the two cases in which the demands and inflows are perfectly negatively correlated. In the spring and summer, the maximum Peace marginal energy values for the one-scenario base case are closest to those for the prices only case. During the spring and summer the Peace marginal energy values for the two cases in which the demands and inflows are perfectly negatively correlated are furthest from those for the one-scenario case.

During the spring and early summer, the one-scenario base case provides the lowest maximum Peace marginal energy values, and the cases in which the demands and inflows are negatively correlated provide the highest maximum Peace marginal energy values. In the late summer, the one-scenario base case yields the highest maximum Peace marginal energy values, and the cases in which the demand and inflows are perfectly negatively correlated yield the lowest maximum Peace marginal energy values.

The above observations for the relationships between the maximum Peace marginal energy values for the different cases are the same as those that existed amongst the cases for the maximum Columbia marginal energy values.
Comparison of Max PCE Marginal Energy Values for CV=0.25

Figure 3-71: Maximum Peace Marginal Energy Values for Coefficient of Variation of 0.25

The observations regarding the relationships between the maximum Peace marginal energy values under the seven cases for Figure 3-72 are the same as those for Figure 3-71, which in turn are the same as those for the maximum Columbia marginal energy values, indicating that the relationships are not dependent on the coefficient of variation or the reservoir.

Comparison of Max PCE Marginal Energy Values for CV=0.10

Figure 3-72: Maximum Peace Marginal Energy Values for Coefficient of Variation of 0.10
Comparing Figure 3-71 and Figure 3-72, the manner in which the differences between the maximum Peace marginal energy values over the seven cases increase with the coefficient of variation is evident. A comparison of Figure 3-71 and Figure 3-72 and the corresponding figures for the non-anticipative turbine release case—Figure 3-34 to Figure 3-35—indicates that there is less variation between the maximum Peace marginal energy values over the scenarios when there is perfect foresight. By way of example, the maximum and average variation in the maximum Peace marginal energy values for a coefficient of variation of 0.25 for the non-anticipative turbine releases are $345/MWh and $86.7/MWh, while the corresponding values for the perfect foresight case are $246/MWh and $55.3/MWh.

The monthly minimum Peace marginal energy values for coefficient of variation values of 0.25 and 0.10 are presented as Figure 3-73 and Figure 3-74.

![Comparison of Min PCE Plant Marginal Energy Values for CV=0.25](image)

Figure 3-73: Minimum Peace Marginal Energy Values for Coefficient of Variation of 0.25
Comparing Figure 3-73 and Figure 3-74, the manner in which the differences between the minimum Peace marginal energy values over the seven cases increase with the coefficient of variation is evident. A comparison of Figure 3-73 and Figure 3-74 and the corresponding figures for the non-anticipative turbine release case—Figure 3-36 through Figure 3-38—indicates that there is less variation between the minimum Peace marginal energy values over the scenarios when there is perfect foresight. By way of example, for a coefficient of variation value of 0.25, the average monthly variation in the minimum Peace marginal energy values was $5.7/MWh for the non-anticipative turbine release case, and $2.4/MWh for the perfect foresight case. The corresponding figures for the minimum Columbia marginal energy values provide similar results.

The comparison of minimum and maximum marginal energy values for the two reservoirs over all of the cases for the three coefficient of variation values studied yield the following observations. Having perfect foresight—that is not enforcing any non-anticipative constraints—diminishes the variation between cases of the different assumptions regarding the scenario-dependent parameters for the marginal energy values. Again, as for the non-anticipative turbine releases, there is little variation in the maximum marginal energy values during the fall and winter, and the maximum marginal energy values under the two cases in which the demands and inflows are perfectly positively correlated are similar, as are the values for the two cases in which the demands and inflows are perfectly negatively correlated. In the spring and summer, the maximum marginal energy values for the demands only case approach those for the two cases in which the demands and inflows are perfectly positively correlated. In the spring and early summer the maximum marginal energy values under the one-scenario base case are the lowest, while during the late summer they are the highest. During the spring and
summer the maximum marginal energy values for the one-scenario base case are closest to those for the prices only case. Finally, in the spring and summer the maximum marginal energy values for the one-scenario base case are furthest from those for the two cases in which the demands and inflows are perfectly negatively correlated.

3.4.3.4 Winter Peace Operation Constraint vs. Relaxed Peace Winter Operation Constraint for Non-Anticipative Turbine Releases

The base case studied in section 3.4.3.1 used the minimum monthly Peace discharges specified in Table 3-8, which specifies minimum discharges in January and December that significantly exceed the allowable minimum in the other months. These higher minimum discharges are set for operational purposes. Setting a high minimum discharge during the freeze up of the river establishes an ice bridge at a stage established at the high flow; flows can be dropped below the level associated with this stage, but cannot be increased above it.

In this section, the effects of having to keep releases from the Peace high during December and January on the marginal energy values are briefly examined through a relaxation of the high minimum flow constraints. The effects are studied through a one-scenario case with identical parameters to the base case studied in section 3.4.3.1 except for the relaxed Peace plant minimum discharges specified in Table 3-31.

<table>
<thead>
<tr>
<th>Monthly Peace Plant Minimum Discharge (cms)</th>
<th>Monthly Columbia Plant Minimum Discharge (cms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>200</td>
</tr>
<tr>
<td>February</td>
<td>200</td>
</tr>
<tr>
<td>March</td>
<td>200</td>
</tr>
<tr>
<td>April</td>
<td>200</td>
</tr>
<tr>
<td>May</td>
<td>200</td>
</tr>
<tr>
<td>June</td>
<td>200</td>
</tr>
<tr>
<td>July</td>
<td>200</td>
</tr>
<tr>
<td>August</td>
<td>200</td>
</tr>
<tr>
<td>September</td>
<td>200</td>
</tr>
<tr>
<td>October</td>
<td>200</td>
</tr>
<tr>
<td>November</td>
<td>200</td>
</tr>
<tr>
<td>December</td>
<td>200</td>
</tr>
</tbody>
</table>

Table 3-31: Modified Monthly Minimum Plant Discharge

The effect of relaxing the Peace ice constraints on the minimum monthly Peace marginal energy values is shown in Figure 3-75, in which the “Base Case” is for the minimum Peace plant releases given in Table 3-8 and the “Relaxed Case” is for the minimum releases in Table 3-31.
It is immediately evident that by relaxing the Peace ice constraints, the minimum Peace marginal energy values are reduced in all months except for May and June where they were already equal to zero under the base case. With relaxation of the ice constraints, the minimum Peace marginal energy value is equal to zero in all months except for August through November. As the minimum Peace marginal energy values occur when both the Peace and Columbia reservoirs are full, it is apparent that the effect of having to maintain a high release in two winter months affects the Peace marginal value energy in all months except for May and June, even under the best possible storage situation.

The maximum Peace marginal energy values occur under the worst possible storage conditions—when both reservoirs are empty. The effect of relaxing the ice constraints on the maximum Peace marginal energy values is presented in Figure 3-76.
From Figure 3-76 it is apparent that relaxation of the ice constraints is very significant on the maximum Peace marginal energy values. The effect of relaxing the constraints is particularly marked in the months when the higher minimum Peace releases held—December and January—and in the preceding late summer and autumn months when the Peace marginal energy values were high, sending the price signal that it is valuable to store water in order to meet the minimum releases.

The two previous figures indicate that the higher minimum Peace plant releases that must be maintained during the winter raise the marginal Peace energy values above what they would be otherwise under all possible reservoir conditions in both the Peace and Columbia. Figure 3-77 and Figure 3-78, which present the effect of relaxing the ice constraints, respectively, on the minimum and maximum Columbia marginal energy values, show that the ice constraints also affect the Columbia marginal energy values.
Figure 3-77: Effect of Peace Plant Ice Restrictions on Minimum Columbia Marginal Energy Values

Figure 3-77 shows that relaxing the Peace ice constraints reduces the Columbia marginal energy values in all months in which they were non-zero in the base case. A comparison of Figure 3-77 and Figure 3-75 reveals that the effect of the ice constraints on the minimum Columbia marginal energy values is less than that for the minimum Peace marginal energy values.

Figure 3-78: Effect of Peace Ice Restrictions on Maximum Columbia Marginal Energy Values

As compared to the maximum Peace marginal energy values, the effect of the ice constraints on the maximum Columbia marginal energy values shown in Figure 3-78 is
slight. However, in all months except for February, March, and August, the ice constraints do raise the Columbia marginal energy value above what it would be otherwise; in February and March the maximum marginal energy values are unaffected, and in August there is a decrease.

The effects of the ice constraints on the Peace marginal energy values in May and November are shown, respectively, in Figure 3-79 and Figure 3-80 for the Columbia reservoir empty, half full, and full. The letter “R” is used in the legends of the Figures to indicate the relaxed ice constraints case.

Figure 3-79: Effect of Peace Ice Restrictions on May Peace Marginal Energy Values

As both minimum and maximum May Peace marginal energy values are reduced by relaxing the ice constraints, it is not surprising to see that relaxation of the constraints reduces the Peace marginal energy values in May regardless of the storage in the two reservoirs. Of particular note is that relaxation of the minimum Peace discharge constraints reduces the dependence of the marginal Peace energy values on the amount of water stored in the Columbia reservoir, as evidenced by the vertical distance between the curves for the relaxed case. This decoupling of the Peace marginal energy value and Columbia storage also occurs in June. In January through April and July there is a similar reduced dependence on the Columbia storage, provided that the storage in the Peace is above a certain level, which is between 100000 and 350000 cmsd depending upon the month.
Figure 3-80: Effect of Peace Ice Restrictions on November Peace Marginal Energy Values

Figure 3-80 indicates that during November the Peace marginal energy values are also reduced (with some minor exceptions) for all levels of storage in both reservoirs. The most significant effects of the relaxation of the ice constraints occur when storage in the Peace reservoir is less than 150000 cmsd; for greater storage amounts in the Peace reservoir, the effects of the relaxation are more muted. As for May, relaxing the ice constraints changes how both the Columbia storage and Peace storage affect the Peace marginal energy value. Figure 3-81 presents some of the obscured details in the lower right corner of Figure 3-80. Together, Figure 3-80 and Figure 3-81 show that the relaxation of the ice constraints can both increase and decrease the dependence of the marginal Peace energy values on both the Peace and Columbia storage. In contrast, in May, the relaxation of the ice constraints reduces the dependence of the Peace marginal energy values on the Peace reservoir storage as evidenced by the slope of the curves for the relaxed case as compared to those for the base case.
Effect of Peace Ice Constraints on November Marginal Peace Energy Values

Figure 3-81: Effect of Peace Plant Restrictions on November Peace Marginal Energy Values—Detail

Figure 3-82 and Figure 3-83 show, respectively, the effect of the ice constraints on the Columbia marginal energy values in May and November for the Peace reservoir empty, half full, and full.

Figure 3-82: Effect of Peace Ice Restrictions on May Columbia Marginal Energy Values

Regardless of the storage in the Columbia reservoir, the three May Columbia marginal energy curves for the relaxed case are closer to one another than they are for the base case, indicating that relaxation of the ice constraints reduces the dependence of the May Columbia marginal energy value on the Peace reservoir storage. No general conclusion can be drawn regarding how relaxing the ice constraints affects the dependence of the May Columbia marginal energy value on the Columbia reservoir storage—in some cases
The dependence increases, while in others it decreases. The results for the other Peace freshet months of June and July are similar to those for May.

**Figure 3-83: Effect of Peace Ice Restrictions on November Columbia Marginal Energy Values**

The effect of the relaxation of the ice constraints on the marginal Columbia energy values in November is very different than for May. In November, relaxation of the constraints does not, to any great extent, change the dependence of the Columbia marginal energy values on Peace storage. The behaviour of the other Peace non-freshet months is similar to that for November.

To summarize, it can be seen that the higher minimum Peace plant discharges in January and December have implications for both the Columbia and Peace marginal energy values. Relaxation of these minimums reduces both the minimum and maximum Peace marginal energy values in all months, with the greatest reduction in the maximum occurring for the period of July through January. In addition, the minimum Columbia marginal energy value is reduced in all months by a relaxation of the ice constraints. The less constrained operation has a reduced impact on the maximum Columbia marginal energy values, with the largest impacts occurring during May through July. The most interesting observation is that when the ice constraints are relaxed, the Peace marginal energy value is essentially decoupled from the Columbia reservoir storage for May and June. In other months provided that the storage in the Peace reservoir exceeds a certain threshold, the Peace marginal energy value is largely independent of the Columbia reservoir storage. Similarly, during the period of May through July, the Columbia marginal energy value is much less dependent on Peace reservoir storage, and becomes independent when there is sufficient storage in the Columbia reservoir.
3.5 Summary

In this chapter, a model has been proposed to estimate the storage values and marginal storage values of water over the medium- to long-term for a two-reservoir system. These marginal water values can be converted into marginal energy values which can be used by the operator of a hydroelectric system in making dispatch decisions while operating the system, and in making transactions on the energy markets, as well as for input to shorter-term models.

The model combines the use of backward-moving dynamic programming (DP) to link time periods together and the use of linear programming (LP) to determine the operation of the hydropower system and electricity trades within each time period, as well as the end of period storage for the two reservoirs. Uncertainty is introduced into the model through scenarios. The scenario-dependent parameters are demands, inflows, and prices. The model is limited to the consideration of two reservoirs, which must be hydraulically separate from one another. The limited number of reservoirs that can be considered is a result of the "curse of dimensionality" afflicting dynamic programming (DP). The apparently restricted range of applicability of the model can be extended using aggregate reservoirs, a technique that has been used in previous research (e.g., Turgeon and Charbonneau, 1998; Valdes et al., 1992). For example, the two reservoirs could be used to represent two separate river systems; each reduced into a single aggregate reservoir. Alternatively, one reservoir could represent either a single reservoir or the aggregate reservoir for a river system, and the second reservoir could represent an aggregate reservoir that replaces the remainder of the plants in the hydroelectric system.

In a case study, the model is applied to the two main river systems in the BC Hydro system: the Peace and the Columbia. The case studied approximates the actual hydro installations. BC Hydro has two generating plants on the Peace River: G. M. Shrum and Peace Canyon. The reservoir for the G. M. Shrum plant has significant over-year storage, whereas Peace Canyon is largely run in hydraulic balance with the upstream G.M. Shrum facility. Similarly, BC Hydro generation facilities on the main-stem of the Columbia River include an upstream plant with significant storage (Mica), and a downstream plant that is generally run in hydraulic balance with the upstream project (Revelstoke). In the case study, generation on each of the two river systems is represented by a single plant. On the Columbia River, the “Columbia” plant replaces the Mica and Revelstoke plants. The “Peace” plant replaces the G. M. Shrum and Peace Canyon plants.

The case study demonstrates the ability of the model to calculate the marginal values for two multiple-year storage reservoirs, while taking into account the uncertainty in hydrology, demand, and market prices faced by the operator of a large hydropower system with significant storage flexibility. The insights gained on the reservoirs examined in the case study are discussed below to illustrate both the flexibility of the model and its ability to provide critical decision support information for the operator of a complex hydropower operation. A key insight is that there are ranges of reservoir storage over which there is not much interdependence of the marginal value of storage in one reservoir with the storage in the second reservoir. The ability to identify such areas
allows more complex modelling of the two individual reservoirs within such storage ranges. The insights gained are discussed below.

In the case study the marginal energy value functions for the Peace and Columbia, which in general depend upon the storage in each of the two reservoirs, are determined under a number of different assumptions. First, a “base case” in which there is only one scenario in each month, and thus the scenario-dependent parameters—the demand to be met by the sum of Peace and Columbia generation, the inflows to the two reservoirs, and the energy prices—assume their mean values, was considered.

The model was also applied to a number of cases in each of which there were different assumptions regarding the correlations between the scenario-dependent parameters. In these cases, there was one scenario during the months of November through April, and five scenarios during the remaining months. During the months with five scenarios, the turbine release was specified to be independent of the scenario—that is, it was “non-anticipative”. The cases studied, described in terms of the scenario-dependent parameters, were: demands only, with inflows and prices assuming their mean values in all scenarios; inflows only; prices only; demands, inflows, and prices all perfectly positively correlated; demands and inflows perfectly positively correlated and demands perfectly negatively correlated; inflows and prices perfectly positively correlated and demands perfectly negatively correlated; and demands and prices perfectly positively correlated and inflows perfectly negatively correlated. The same coefficient of variation was assumed to apply to each of the scenario-dependent parameters, perfect correlation was assumed between the inflows to the two reservoirs, and perfect correlation was assumed for HLH and LLH energy prices and demands. The model was also applied to each of the seven cases just described under the assumption of perfect foresight—that is, allowing the turbine release to depend upon the scenario. Finally, a one-scenario case in which the operational ice constraints which restrict the minimum Peace release during December and January was examined.

For the base case it was observed that, in general, the marginal energy value for each of the two reservoirs is dependent on the storage in both reservoirs. However, in some months for particular storage ranges in a reservoir, the dependence of the marginal energy value for that reservoir is essentially independent of the storage in the other reservoir, providing that the storage in the second reservoir is in a particular range. For example, during May when the Peace reservoir is 60% full, the marginal Peace energy value varies by $0.30/MWh for a range of Columbia storage between half full and full. A second example, also for May, is that when the Columbia reservoir is 70% full, the marginal Columbia energy value only varies by $0.32/MWh over the lower 60% of storage in the Peace reservoir. These differences are small enough to be ignored for operational and trade decision purposes. The Columbia marginal energy values were found to display the least sensitivity to the entire range of Peace storage when the Columbia reservoir was approaching full in all months. The Peace marginal energy values for the months of August through February were found to be the least sensitive to changes over the entire range of Columbia storage when the Peace reservoir was approaching full. In the months of March through July, the Peace marginal energy values were found to display the least
sensitivity to changes over the entire range of Columbia storage when the Peace reservoir was in some mid-range, and in some of these months, when the Peace reservoir was full.

In the base case, for both the Peace and Columbia, in all months except for May and June there were very large differences between the minimum marginal energy values—occurring when both reservoirs are full—and the maximum energy values, which occur with both reservoirs empty. For the Columbia reservoir, the maximum marginal energy values were on the order of the curtailment costs—$1000/MWh in the case study. The maximum marginal energy values reflect both the curtailment costs and the cost of the ice constraint flows, making the maximum marginal Peace energy values greater than those for the Columbia for July through January. There was no great difference between the minimum Peace and Columbia marginal energy values. The minimum Columbia marginal energy values were, in general, slightly lower.

In the seven cases with non-anticipative turbine releases it was found that for the two cases in which the demands and inflows are perfectly positively correlated, as well as the demands only case, the minimum marginal energy values decrease with an increasing coefficient of variation except for in the late fall and early winter. Similarly for, the two cases in which the demands and inflows are perfectly negatively correlated, it was observed that during the late winter and late spring, the minimum marginal energy values decrease with an increasing coefficient of variation. With the exception of the demands only case, it was found that during the spring and early summer the maximum marginal energy values increase with the coefficient of variation. For the two cases in which the demands and inflows are perfectly positively correlated, it was observed that during the late summer and early fall the maximum marginal energy values decrease with an increasing coefficient of variation value. In the two cases in which the demands and inflows are perfectly negatively correlated as well as the demands only case, during the late summer, the maximum marginal energy values were found to decrease with an increasing coefficient of variation.

For the seven cases with non-anticipative turbine releases it was observed that the dependence of the Peace marginal energy values on the Columbia reservoir storage with Peace reservoir storage was affected by the case more than was the dependence of the Columbia marginal energy values on the Peace reservoir storage with Columbia reservoir storage.

It was observed that over the fall and winter period, there is little variation in the maximum Columbia marginal energy value over the seven different cases with non-anticipative turbine releases; the same is true for the maximum Peace marginal energy value. Throughout the remainder of the year, there is comparatively little variation in the Columbia marginal energy values over the four cases in which the three scenario-dependent cases all vary; again, this is also true of the maximum Peace marginal energy values. For both the Columbia and Peace, it was found that during the spring and summer as the coefficient of variation is reduced, the case in which only the demands vary approaches the cases in which all three of the scenario-dependent parameters vary. It was also found that during the spring and summer as the coefficient of variation is
reduced the case in which only the inflows vary approaches the one-scenario base case. For all of the coefficient of variation values studied, it was found that, for both the Columbia and Peace, the one-scenario base case underestimates the maximum marginal energy value, with respect to the other cases, during the spring and early summer, and overestimates the maximum marginal energy value during the late summer. For both reservoirs, and all coefficient of variation values studied, it was found that the minimum marginal energy values under the one-scenario base case are closest to those under the inflows only case. It was also observed that the minimum marginal energy values under the two cases in which the demands and inflows are perfectly positively correlated are virtually identical, regardless of which of these two parameters is perfectly negatively correlated with the prices, and that the minimum marginal energy values under the two cases in which the demands and inflows are perfectly negatively correlated are virtually identical, regardless of which of these two parameters is perfectly positively correlated with the prices. It was also noted that as the coefficient of variation value is reduced, the minimum marginal energy values under the one-scenario base case are furthest from those under the demands only case. For most of the year, when the coefficient of variation is 0.40, the one-scenario base case overestimates the minimum marginal energy values in both reservoirs as compared to the other cases. When the coefficient of variation is equal to 0.10 or 0.25, the one-scenario base case overestimates the minimum marginal energy values for both reservoirs—with the exception of the inflows only case—in the late winter and late summer, and underestimates these values in the late fall and early winter.

The results for the seven cases with non-anticipative turbine releases suggest that the minimum marginal energy values are more sensitive to a variation in demand than they are to an equal variation in the inflows. Recalling that the minimum marginal energy values occur when both reservoirs are full, note that under good storage conditions, the marginal energy values are affected to a greater extent by a variation in demand than by an equal variation in the inflow. Similarly, past the peak of the freshet, for the maximum marginal energy values—which occur under adverse storage conditions—variation in the demand is of greater influence on the marginal energy values than is an equal variation in the inflows over all of the coefficient of variation values studied. In spring, leading up to, and including, the peak freshet month, it was found that for the two lower coefficient of variation values studied, the marginal energy values are more sensitive to demand variation than inflow variation. However, when the coefficient of variation value is 0.40, inflow variation is of greater importance than demand variation; this was confirmed for a coefficient of variation value of 0.55.

For the seven cases with perfect foresight it was found that the variation in the maximum marginal energy values for the two reservoirs with the coefficient of variation was much less for the prices only case than it was for the others. For all but the prices only case, during the spring and early summer the maximum marginal energy values are found to increase with the coefficient of variation. For the demands only case, during the late summer the maximum marginal energy values are found to decrease with the coefficient of variation. For the four cases in which all three of the scenario-dependent parameters change with the scenario, the maximum marginal energy values are observed to decrease
with an increasing coefficient of variation for the late summer and early spring. For the two cases in which the demands and inflows are perfectly positively correlated, the minimum marginal energy values are found to decrease with an increasing coefficient of variation. Similarly, for the two cases in which the demands and inflows are perfectly negatively correlated, the minimum marginal energy values decrease with an increasing coefficient of variation for the late summer through early winter, while they increase with an increasing coefficient of variation during the spring and early summer.

In a comparison of the marginal values under assumptions of non-anticipative turbine releases and perfect foresight the following results were found. For the demands only case, as well as the four cases in which all three scenario-dependent parameters vary, the maximum marginal energy values during the late summer are found to be greater under the assumption of perfect foresight; for these same cases, the maximum marginal energy values are greater under the assumption of non-anticipative turbine releases for the spring and early summer. With minor exceptions, the maximum marginal energy values for the inflows only case are greater in all months under the assumption of perfect foresight. For the demands only case, the minimum marginal energy values during the winter and late summer are greater under the assumption of perfect foresight. For the four cases in which all three scenario-dependent parameters vary, the minimum Columbia marginal energy values are greater under the assumption of perfect foresight during the winter and late summer. For these same four cases, the minimum Peace marginal energy values are greater under the assumption of perfect foresight during the spring, summer, and winter. For the inflows only case, the minimum marginal energy values during the fall and early winter are greater under the assumption of perfect foresight, while the opposite is true during the spring and early summer.

For the seven cases with perfect foresight, the region of Columbia reservoir storage over which the marginal Columbia energy values are least dependent on Peace reservoir storage do not change from the case of non-anticipative turbine releases in any month. However, in March through June, the region of Peace reservoir storage over which the Peace marginal energy values are least dependent on Columbia reservoir storage do change in the perfect foresight case.

The comparison of minimum and maximum marginal energy values for the two reservoirs over all of the cases for the coefficient of variation values studied, under the assumption of perfect foresight, yield the following observations. Having perfect foresight—that is not enforcing any non-anticipative constraints—diminishes the variation between cases for the marginal energy values. Again, as for the non-anticipative turbine releases, there is little variation in the maximum marginal energy values during the fall and winter, and the maximum marginal energy values under the two cases in which the demands and inflows are perfectly positively correlated are similar, as are the values for the two cases in which the demands and inflows are perfectly negatively correlated. In the spring and summer, the maximum marginal energy values for the demands only case approach those for the two cases in which the demands and inflows are perfectly positively correlated. In the spring and early summer, the maximum marginal energy values under the one-scenario base case are the lowest, while
during the late summer, they are the highest. During the spring and summer the maximum marginal energy values for the one-scenario base case are closest to those for the prices only case. Finally, in the spring and summer the maximum marginal energy values for the one-scenario base case are furthest from those for the two cases in which the demands and inflows are perfectly negatively correlated.

It was found that relaxing the higher minimum Peace plant discharges in January and December reduces both the minimum and maximum Peace marginal energy values in all months, with the greatest reduction in the maximum occurring for the period of July through January. In addition, the minimum Columbia marginal energy value is reduced in all months by relaxation of the Peace ice constraints. The less constrained operation has a smaller impact on the maximum Columbia marginal energy values, with the largest impacts occurring during May through July. When the ice constraints are relaxed, the Peace marginal energy value is essentially decoupled from Columbia reservoir storage during May and June. In other months, provided that storage in the Peace reservoir exceeds a certain threshold, the marginal Peace energy value is largely independent of Columbia reservoir storage. Similarly, during the period of May through July, the Columbia marginal energy value is much less dependent on Peace reservoir storage, and becomes independent when there is sufficient storage in the Columbia reservoir.

The storage value curves calculated by the model described in this chapter serve as input for the shorter-term marginal value model described in the following chapter.
4 Short-term Marginal Value Model

4.1 Introduction

The previous chapter presented a method for estimating the values and marginal values of energy stored in a two-basin hydroelectric system. The appropriate time step for that model is on the order of one month. The motivation for developing the medium-term DP and LP based model was to provide energy values for a two-reservoir system that could be used to provide the value of storage at the end of a shorter-term model. Such a model is described in this chapter. The shorter-term model has been developed to allow reservoir operations, and energy trades, for a two-basin system to be planned over a period shorter than one month and to generate short-term marginal values over this period. These marginal values can be used in supporting both reservoir operation and energy marketing decisions.

While the monthly time step employed in the longer-term DP and LP based model is appropriate for longer-term planning and decision making, the time step neglects within-month variations in the scenario-dependent parameters. The shorter-term multiple-time step model provides a way in which these shorter-term departures from the monthly means can be taken into consideration for both the calculation of marginal values and operation of the system. The time steps in the model can be of variable length. For example, the model could be used with a total time horizon of one month. This month could be divided into a daily time step for three days (three time steps), a 3-day weekend time step, and a fifth time step could be used to model the balance of the days in the month. To summarize, the short-term marginal value model (STMVM) serves as a bridge between the very-short term (on the order of hours) and the medium-term (greater than one month into the future).

As was true for the DP and LP based model described in the previous chapter, uncertainty in the demands, inflows, and prices is taken into account by the shorter-term model. Uncertainty in all of these parameters is handled through the use of a scenario tree, which describes how the future can unfold.

It is easiest to understand the idea of a scenario tree using the implied analogy. A scenario tree branches from a single root at the start of the first time step considered by the model to leaves at the end of the last time step considered by the model. The tree branches at locations where a decision must be made—that is, at the start of each time step. A scenario in the tree is a direct path from the root to one of the leaves. Consider the simple case of a model with one reservoir that is to be operated for two time steps, in which the inflow in each of the time steps can assume one of two values. At the start of each time step a decision must be made as to the release from the reservoir. The scenario tree representing this simple problem has two branches growing out of the root at the start of the first step: one for high inflow in the first time step and one for low inflow in the first time step. The tree then branches again at the start of the second time step for both of the first period branches, giving four leaves at the end of time step two. There are thus
four scenarios—direct paths from the root to a leaf—in the tree: high inflow in both time steps; high inflow in the first time step and low inflow in the second time step; low inflow in the first time step and high inflow in the second time step; and low inflow in both time steps. The scenario tree just described outlines all of the possible ways in which the future can unfold in the simple model.

The tree structure is also instructive in illustrating how decisions are made using the technique of stochastic linear programming with recourse which is used in the STMVM described in this chapter. Each branching point, or node, in the tree represents a place at which a decision must be made. At each node a probability is assigned to each branch; the sum of the probabilities of all branches from a node must equal one. At each of these nodes a decision, such as the quantity of water to release, must be made. Note that this decision must be made prior to knowing in which one of the possible manners the future will unfold; therefore, the decision must be feasible over all possible evolutions of the future, and should in fact be the optimal decision considering all possible outcomes. Such a decision, being made in the face of uncertainty is known as a “here-and-now” decision. Once the future has been revealed at the end of the time period, corrective, or “recourse”, decisions can be made based upon which of the possible futures held. These latter decisions are referred to as “wait-and-see”. The decision making approach just described results in decisions that are “implementable”—that is they do not anticipate the future, and can therefore be implemented. It is proposed that the short-term marginal values produced by the model described here be used as input to a very-short term reservoir operation model, with a time-step of an hour or less, and a total time horizon of roughly one day (e.g., Shawwash et al., 1999; Piekutowski et al., 1993).

The STMVM and its application are described in greater detail in the remainder of the chapter. Application of the model to a two-reservoir system based upon the two major storage projects in the BC Hydro system in order to demonstrate its ability to produce the price signals required by an operator with significant storage and market opportunities while taking into account uncertainty in the demands and prices as well as the typically considered inflow uncertainty.

4.2 STMVM Overview

The STMVM has been designed for application to the hydroelectric system described in the previous chapter. The system consists of two, non-hydraulically linked, hydroelectric projects which serve the same load area. The system is assumed to be connected through tie lines to other regions making energy trades possible. The system is to be operated so as to maximize the value of its operation over the model time horizon—that is the sum of the net energy trade revenue earned over the modelled period, any penalties for constraint violation, and the value of water remaining in storage at the end of the model is to be as large as possible. The value of water remaining in storage is given by the storage value curves developed by the DP and LP based model described in the previous chapter. The maximization of the value of system operation is subject to physical and operational constraints. The maximization yields the marginal values of water for each reservoir for
The marginal energy values can then be directly calculated from the marginal water values.

The maximization is performed by selecting optimal values for the decision variables in the problem. The decision variables are the turbine releases, spills, total facility releases, reservoir storage volumes, generations, imports, and exports. The search for the optimal values of these decision variables must take into account the fact that some of the model parameters are uncertain. The parameters in the STMVM that are subject to uncertainty are the demands, inflows, and prices. The values that can be assumed by the uncertain parameters are described by a scenario tree, leading to the parameters being referred to as being "scenario-dependent". A scenario describes the values assumed by all of the scenario-dependent parameters for each time step in the model. The time steps can be of variable length, and each time step can be divided into sub-periods, for each of which the scenario-dependent parameters can assume different values. At the start of a time step all of the here-and-now decisions must be made, while recourse wait-and-see decisions can be made at the end of each time step. The here-and-now decisions consider all of the possible outcomes for the scenario-dependent parameters as described by the scenario tree.

The model is solved—that is, the values of the here-and-now and wait-and-see variables that maximize the value of system operation are found—by optimizing the stochastic linear programming problem with recourse (SLPR) described by the scenario tree. As the underlying problem is not strictly linear, the LP must be solved iteratively. The sources of non-linearity that must be addressed concern the functions which convert turbine discharge into power and the functions describing the value of water remaining in storage.

The relationship between turbine discharge and power production is approximated by a piecewise-linear function of storage. For each time step and each scenario, the reservoir storage used to calculate the relationship is assumed to be an average of that for the start and end of the period. As demonstrated in Chapter 3, the value of water remaining in storage in either reservoir at the end of the model time horizon in general depends upon the volume of water stored in both reservoirs. In order to provide the model with a piecewise-linear function for each reservoir that describes the value of water in storage as a function of the storage in that reservoir, estimates of the reservoir storage in the two reservoirs at the end of the model time horizon must be made. (These storage value functions and their derivation are described in Chapter 3.) However, the reservoir storage volumes upon which these two types of functions depend are decision variables. For the initial iteration, values of the storage in each reservoir under each scenario at the end of each time step are assumed. These assumed values are then used to calculate the conversion factors for each time step, reservoir, and scenario. The assumed values for the final time step are used to estimate the storage value functions. The model is then solved, and the marginal water values are compared with those from the previous iteration. If the difference in marginal value for either reservoir exceeds an allowable tolerance, another iteration is performed using the optimized reservoir storage levels to
estimate the conversion factors. The procedure is repeated until convergence or until a maximum number of iterations has been performed.

At the completion of a model run, the values of all the decision variables—turbine releases, spills, total facility releases, reservoir storage volumes, generations, imports, and exports—for each modelled time step are available, as are the marginal water and energy values. Of particular interest for decision support are the marginal energy values for each time step, and the values of the here-and-now variables for the first time step. The here-and-now decision variable values for the first time are of interest as they represent the optimal values, over the scenarios modelled, for the most pressing decisions. For example, if the here-and-now decisions are the turbine releases for the two reservoirs, the most pressing question is how much to release through the turbines for the first time step, which may be the coming day. The model provides results that can be used to help guide this decision for the two reservoirs. The optimal decisions for later time periods are of less immediate importance, as the model can be re-run with updated data before such decisions will need to be taken. The marginal energy values are of importance over all of the time steps as they can be used to support energy trade decisions and for input to models dealing with a time step on the order of one hour. Trade decisions are not limited to the first time step in the model. For example, if the first period represents the coming day, trades can be, and are, made beyond this period. The marginal energy values produced by the STMVM for subsequent time periods represent estimates incorporating the currently available information. As the model is run for subsequent periods, the marginal energy values will be recalculated and new energy trade decisions can be made.

Prior to presenting the details of the STMVM, the stochastic linear programming with recourse technique is introduced, and the division of decision variables into the here-and-now and wait-and-see categories is discussed.

4.2.1 Stochastic Linear Programming with Recourse

The STMVM makes use of the technique known as stochastic linear programming with recourse (SLPR). Yeh’s 1985 review of the optimization of water resources problems literature noted the application of SLPR to water resources problems as a promising area for future research. This section provides a brief introduction to SLPR.

Stochastic linear programming with recourse, for a two-stage problem, can be described mathematically as (Kall and Wallace, 1994):

\[
\min \{cx + Q(x)\} \quad (4-1)
\]

such that

\[
Ax = b, \ x \geq 0; \quad (4-2)
\]

where
\[ Q(x) = \sum_{\omega \in \Omega} pr(\omega) Q(x, \omega), \]  

(4-3)

\[ Q(x, \xi) = \min_x \{ q(\xi) y | W(\xi) y = h(\xi) - T(\xi) x, y \geq 0 \}, \]  

(4-4)

and \( h(\xi) = h_0 + H\xi = h_0 + \sum \xi_i \xi_i, T(\xi) = T_0 + \sum T_i \xi_i, \) and \( q(\xi) = q_0 + \sum q_i \xi_i. \) In (4-1) to (4-4) \( x \) are the “here-and-now” decision variables—i.e., those that must be made in the face of uncertainty—and \( y \) are “wait-and-see” decision variables—i.e., those that can be made once the uncertainty has been revealed. The function \( Q(x) \) is called the expected recourse function, and the function \( Q(x, \xi) \) is known as the recourse function. A scenario, \( \xi \), which is indexed by \( \omega \) over the set of scenarios \( \{ \Omega \} \), occurs with probability \( \omega pr. \)

Figure 4-1 illustrates a simple one-time step, two-stage, recourse problem for a case in which there are two scenarios—H and L. The here-and-now variables are determined at \( t-1 \), prior to knowing which of the two scenarios will occur, while the wait-and-see variables are determined at \( t \), with knowledge of which scenario has occurred.

In essence, (4-1) says that the function to be minimized by the LP is the product of the objective function coefficients and the decision variables plus some function of the decision variables. Given the LP framework, the function of the decision variables—the expected recourse function—must be linear. Equation (4-2) presents the usual LP formulation. Equation (4-3) states that the expected recourse function is equal to the probability-weighted average of the recourse function evaluated for each scenario. Equation (4-4), which presents the recourse function, contains both the wait-and-see and here-and-now decisions; however, at the point when the wait-and-see decisions are to be found, the here-and-now variables have already been determined, and thus have fixed values. In equation (4-4), note that the matrices and vectors are functions of the scenario.
The extensions required to write a multiple-stage SLPR problem are straightforward. For instance, the objective function can be written as (Dantzig and Infanger, 1997):

$$\min \{c_1 x_1 + \ldots + E(c_2 x_2 + \ldots + E(c_{T-1} x_{T-1} + E(c_T x_T)) \}$$

For the two-stage problem, (4-5) reduces to $$\min \{c_1 x_1 + E(c_2 x_2) \}$$ which is equivalent to (4-1), with $$c_1 x_1$$ replacing $$c x$$ and $$E(c_2 x_2)$$ replacing $$Q(x)$$.

Figure 4-2 illustrates the multiple-stage problem for a case in which there are two possible outcomes in each stage—H and L. A scenario consists of a path from the root of the tree to a leaf—i.e., S to either $$H_T$$ or $$L_T$$. For example, a scenario in which H always occurs would be $$(S, H_1, H_2, \ldots, H_{T-1}, H_T)$$, while a scenario in which L always occurs would be $$(S, L_1, L_2, \ldots, L_{T-1}, L_T)$$. The boxes indicate scenario groups for which the here-
and-now decisions at the previous stage must be the same for all scenarios in the group. For example, all scenarios \((S, H_1, \ldots)\) and \((S, L_1, \ldots)\)—that is all scenarios—must have the same here-and-now decision at \(t=0\). Similarly, all scenarios \((S, H_1, H_2, \ldots)\) and \((S, H_1, L_2, \ldots)\) must have the same here-and-now decision at \(t=1\).

The choice of here-and-now and wait-and-see variables is an important one. The following section examines the choice for the reservoir system operation problem under consideration.

### 4.2.2 Choice of Here-and-Now Variables

In the previous section, the concept of dividing decision variables into here-and-now and wait-and-see variables was presented. This section discusses the division of the STMVM decision variables into these two categories. The decision variables in the problem are the turbine flow, the spill, the total plant release, the end of time step storage, and the generation for each of the two reservoirs, as well as system imports and exports. The decision variables can be divided into water variables and power variables. The water variables are the turbine, spill, total plant flows, and the ending storage; the power variables are the generation, imports, and exports. It is not immediately obvious which variables should be here-and-now variables and which should be wait-and-see variables.

Considering the operation, including energy trades, of the two river basin hydroelectric system over one time step is of assistance in making the categorization.

To examine what the choice of here-and-now variable implies about the assumed operation of the system, consider an uncertain inflow which can assume either a high or a low value. Suppose that at the start of the time period a decision must be made as to what the ending storages for the time step will be—i.e., operation to achieve ending reservoir target levels. The continuity equations link the starting and ending storage, the inflow, and the turbine and spill flows. The starting and ending storage for the time step would then be fixed, while the inflows would vary between scenarios. Thus, any difference between inflows would need to be accommodated by the turbine and spill flows, meaning that these two are candidates for recourse variables. It is unlikely that spills would be decided upon at the start of the time step, so they should be recourse variables. If the turbine flows were also to be treated as here-and-now variables, then any difference in inflows between scenarios would need to be spilled, resulting in wasted water. So, if the end of time step storage is selected as a here-and-now variable, then it will be the only here-and-now variable from the set of water decision variables.

If the end of period storage is the only water decision to be made before the uncertainty is revealed, what does this mean with regard to the power variables? In the STMVM, as in the DP and LP based model described in Chapter 3, HK (which relates generation to turbine discharge) is assumed to depend upon storage only, not on turbine discharge, so it is determined based solely on the start and end of time step reservoir levels. These levels are found iteratively. So, if the end of time step storage is determined at the start of the time step, HK will be fixed, and generation during the period will only depend upon the turbine discharge. Therefore, generation must also be a recourse variable. Decisions
regarding imports and exports should be made in conjunction with generation decisions, making them recourse decisions as well.

Now, consider what would happen should turbine discharges be here-and-now variables. Again, to avoid spilling water, spills will not be here-and-now variables. If the discharges are specified, the ability to end at different storage levels must be maintained, so the end of time step storages will also be wait-and-see variables. The generation depends upon turbine discharge and end of period storage. Since the end of time step storages are recourse decisions, the generation should also be recourse variables. And, as energy transactions are decided upon in concert with generation, these will also be recourse decisions. Thus, a second possibility is to have turbine discharge as the here-and-now variables, and the remaining decision variables as wait-and-see variables.

Two potential here-and-now decision variables have been identified: end of time step storage levels and turbine flows. For the choice of either of these as the wait-and-see variables, the other, along with imports, exports, and generation will be recourse decisions. While it is believed that selecting the turbine flows as the here-and-now decision variables most closely represents the actual decision making used in operation of a hydroelectric system, both possible choices are explored in the case study in section 4.4.

With the concepts of SLPR and the choice of here-and-now and wait-and-see variables having been explored, the model details are presented in the following section.

4.3 Model Details

The preceding sections provide an overview of the STMVM. The hydroelectric system under consideration was described, and the concept of stochastic linear programming with recourse (SLPR) and choice of here-and-now and wait-and-see decisions were introduced. In this section a mathematical formulation of applying SLPR to the STMVM problem is presented, and the means of solving the model are discussed.

4.3.1 Mathematical Formulation

This section provides a mathematical formulation of the STMVM problem. As noted at the end of Chapter 3, the STMVM is essentially a multiple-period version of the LP component of the DP and LP based model. As such, much of the mathematical description contained in this section is the same as that in section 3.3.2.1.

In order to define the STMVM problem mathematically, the parameters, sets, and variables must be specified. After these have been defined, the model constraints and objective are presented.
**Sets**

Sets are used to index parameters, variables, and constraints. The sets in the model are specified below.

Let the set of reservoirs in the system be represented by \( \{R\} \). \( \{R\} \) is limited to two reservoirs, which must be hydraulically separate from one another.

In order to differentiate between energy prices during different times within a time step—e.g., heavy load hours and light load hours—each time step is divided into sub-periods. Let the set of sub-periods be represented by \( \{U_t\} \), where the \( t \) subscript indicates the time step \( t \in \{1, \ldots, T\} \).

Let the set of scenarios be represented by \( \{\Omega\} \). Let the set of possible vectors of scenario-dependent values be represented by \( \{SDV_t\} \), where the \( t \) subscript indicates the time step \( t \in \{1, \ldots, T\} \).

**Parameters**

Model parameters are values that are input to the model. The values of the decision variables found are dependent upon the parameters. These parameters are specified below.

Let \( T \) represent the number of time steps to be modelled.

Let \( \Delta_{t,u} \) represent the length in days of sub-period \( u \in \{U_t\} \) of time step \( t \in \{1, \ldots, T\} \).

Let \( \alpha_{t,u} \) represent the market price, prior to discounting, of energy in $/MWh, under scenario \( \alpha \), during sub-period \( u \in \{U_t\} \) of time step \( t \in \{1, \ldots, T\} \).

Let \( \alpha_{pr} \) represent the occurrence probability of scenario \( \alpha \).

Let \( \alpha_{d_{t,u}} \) represent the power demand, including losses, in MW, under scenario \( \alpha \), during sub-period \( u \in \{U_t\} \) of time step \( t \in \{1, \ldots, T\} \).

Let \( b \) represent the annual discount rate as a decimal fraction.

Let \( \beta_{t} \) represent the discount factor: \( \beta_{t} = (1 + b)^{\frac{\sum_{i=0}^{365} \alpha_{t,i}}{365}} \).

Let \( q_{r,t} \) represent the maximum turbine discharge in cms from reservoir \( r \in \{R\} \) and \( q_{r,t} \) the minimum turbine discharge in cms from reservoir \( r \in \{R\} \) during time step \( t \in \{1, \ldots, T\} \).
Let \( s_{r,t} \) represent the maximum spill in cms from reservoir \( r \in \{R\} \) and \( s_{r,t} \) the minimum spill in cms from reservoir \( r \in \{R\} \) during time step \( t \in \{1, ..., T\} \).

Let \( o_{r,t} \) represent the maximum total plant discharge in cms from reservoir \( r \in \{R\} \) and \( o_{r,t} \) the minimum total plant discharge in cms from reservoir \( r \in \{R\} \) during time step \( t \in \{1, ..., T\} \).

Let \( v_{r,t} \) represent the maximum storage volume in cmsd in reservoir \( r \in \{R\} \) and \( v_{r,t} \) the minimum storage volume in cmsd in reservoir \( r \in \{R\} \) during time step \( t \in \{1, ..., T\} \).

Let \( v^0_r \) represent the storage volume in reservoir \( r \in \{R\} \) at the start of the first time step.

Let \( g_{r,t} \) represent the maximum generation limit in MW for the plant at reservoir \( r \in \{R\} \) and \( g_{r,t} \) the minimum generation limit in MW for the plant at reservoir \( r \in \{R\} \) during time step \( t \in \{1, ..., T\} \).

Let \( m_{t,u} \) represent the maximum import limit in MW during sub-period \( u \in \{U_t\} \) of time step \( t \in \{1, ..., T\} \), and \( m_{t,u} \) the minimum import limit.

Let \( x_{t,u} \) represent the maximum export limit in MW during sub-period \( u \in \{U_t\} \) of time step \( t \in \{1, ..., T\} \), and \( x_{t,u} \) the minimum export limit.

Let \( ^\omega HK_{r,t} \) represent the conversion factor between turbine discharge and power in MW/cms, under scenario \( \omega \), for the plant at reservoir \( r \in \{R\} \) during time step \( t \in \{1, ..., T\} \).

Let \( y \) represent the decimal fraction by which the purchase price of energy should be increased above the market energy price to account for wheeling and losses.

Let \( z \) represent the decimal fraction by which the sale price of energy should be decreased below the market energy price to account for wheeling and losses.

Let \( d_{t,u} \) represent the inflow to reservoir \( r \in \{R\} \), under scenario \( \omega \), during sub-period \( u \in \{U_t\} \) of time step \( t \in \{1, ..., T\} \).

Let \( ^\omega SDV_t \) represent the vector of the scenario-dependent parameter values \( d_{t,u} \), \( d_{t,u} \), and \( p_{t,u} \) at time step \( t \in \{1, ..., T\} \),
**Variables**

The model finds values of the decision variables that optimize the objective function, subject to the specified constraints. That is, the decision variables are the results being sought from the model. The decision variables are specified below.

Let \( V_r^0 \) represent the storage volume in cmsd in reservoir \( r \in \{R\} \) at the start of the first time period.

Let \( \omega Q_{r,t,u} \) represent the turbine discharge in cms from the plant at reservoir \( r \in \{R\} \), under scenario \( \omega \), during sub-period \( u \in \{U_i\} \) of time step \( t \in \{1,...,T\} \).

Let \( \omega S_{r,t,u} \) represent the spill in cms from reservoir \( r \in \{R\} \), under scenario \( \omega \), during sub-period \( u \in \{U_i\} \) of time step \( t \in \{1,...,T\} \).

Let \( \omega O_{r,t,u} \) represent the total discharge in cms from reservoir \( r \in \{R\} \), under scenario \( \omega \), during sub-period \( u \in \{U_i\} \) of time step \( t \in \{1,...,T\} \).

Let \( \omega G_{r,t,u} \) represent the generation in MW from the plant at reservoir \( r \in \{R\} \), under scenario \( \omega \), during sub-period \( u \in \{U_i\} \) of time step \( t \in \{1,...,T\} \).

Let \( \omega V_r^\tau \), represent the storage volume in reservoir \( r \in \{R\} \), under scenario \( \omega \), at the end of time step \( t \in \{1,...,T\} \).

Let \( \omega M_{i,t,u} \) represent the power imported in MW, under scenario \( \omega \), during sub-period \( u \in \{U_i\} \) of time step \( t \in \{1,...,T\} \).

Let \( \omega X_{i,t,u} \) represent the power exported in MW, under scenario \( \omega \), during sub-period \( u \in \{U_i\} \) of time step \( t \in \{1,...,T\} \).

**Constraints**

The non-linear problem of scheduling reservoirs for hydropower production is modelled here using an iterative linear model. Iterations are required to handle the piecewise-linearization of the functions that relate generation, turbine release, and reservoir storage (HK as a function of reservoir storage) and the functions relating the terminal value functions to the storage in the two reservoirs at the end of the last time step in the model. Both functions are dependent on estimated reservoir storage volumes at the end of time steps. The model is subject to linear constraints. These constraints limit the values that can be assumed by the decision variables in the search for an optimal objective function value. The constraints in the STMVM are specified below.
Initial Volume Constraints

The initial volume constraints specify the storage volume of the start of the first time period for each reservoir, \( r \in \{R\} \):

\[
V_r^0 = v_r^0.
\]  

(4-6)

Continuity Constraints

The continuity constraints specify the conservation of mass for each reservoir, \( r \in \{R\} \), under each scenario \( \omega \in \{\Omega\} \), for each \( t \in \{1, ..., T\} \):

\[
V_{r,t-1} + \sum_{u \in \{U_t\}} (\Delta t_u \cdot (\omega_i_{r,t,u} - \omega O_{r,t,u})) = V_{r,t},
\]

(4-7)

where \( V_{r,0} = V_r^0 \).

Load Balance Constraints

The load balance constraints specify that the demand must balance the net generation, where the net generation is equal to the generation plus the imports less the exports. The load balance constraint, under each scenario \( \omega \in \{\Omega\} \), for each sub-period \( u \in \{U_t\} \) of time step \( t \in \{1, ..., T\} \), can be written as:

\[
\sum_{r \in \{R\}} (\omega G_{r,t,u} + \omega M_{t,u}) - \omega d_{t,u} = \omega X_{t,u}.
\]  

(4-8)

Storage Limit Constraints

The storage limit constraints specify the allowable operational range for each reservoir, \( r \in \{R\} \), under each scenario \( \omega \in \{\Omega\} \), for each time step \( t \in \{1, ..., T\} \):

\[
\underline{v_{r,t}} \leq \omega V_{r,t} \leq \overline{v_{r,t}}.
\]  

(4-9)

Turbine Limit Constraints

The turbine limit constraints specify the allowable turbine discharge range for the power plant at each reservoir, \( r \in \{R\} \), under each scenario \( \omega \in \{\Omega\} \), and sub-period \( u \in \{U_t\} \) of time step \( t \in \{1, ..., T\} \):

\[
\omega q_{r,t} \leq Q_{r,t,u} \leq \omega q_{r,t}.
\]  

(4-10)

Spill Limit Constraints
The spill limit constraints specify the allowable spill range for each reservoir, \( r \in \{R\} \), under each scenario \( \omega \in \{\Omega \} \), and sub-period \( u \in \{U_t\} \) of time step \( t \in \{1, ..., T\} \):

\[
s_{r,t}^{\omega} \leq S_{r,t,u} \leq s_{r,t}.
\]

(4-11)

In practice, the upper bound, \( \overline{s_{r,t}} \), is not used.

**Import Limit Constraints**

The import limit constraints specify the allowable range of power imports, under each scenario \( \omega \in \{\Omega \} \), for each sub-period \( u \in \{U_t\} \) of time step \( t \in \{1, ..., T\} \):

\[
m_{i,t,u}^{\omega} \leq M_{i,t,u} \leq m_{i,t,u}.
\]

(4-12)

In the objective function, imports are priced in a step-wise manner—that is, a block of energy has a price associated with it. The most expensive block of energy corresponds to load curtailment, which can be viewed as an expensive import. Thus, in practice the upper bound, \( \overline{m_{i,t,u}} \), is not used for the most expensive import block.

**Export Limit Constraints**

The export limit constraints specify the allowable range of power exports, under each scenario \( \omega \in \{\Omega \} \), for each sub-period \( u \in \{U_t\} \) of time step \( t \in \{1, ..., T\} \):

\[
x_{i,t,u}^{\omega} \leq X_{i,t,u} \leq x_{i,t,u}.
\]

(4-13)

In the objective function exports are priced in a step-wise manner—that is, a block of energy has a price associated with it.

**Total Plant Discharge Constraints**

The total plant discharge constraints specify the allowable total discharge range for each reservoir, \( r \in \{R\} \), under each scenario \( \omega \in \{\Omega \} \), and sub-period \( u \in \{U_t\} \) of time step \( t \in \{1, ..., T\} \). The total plant discharge constraints can be handled as either "hard" or "soft". The hard case is:

\[
o_{r,t}^{\omega} \leq O_{r,t,u} \leq \overline{o_{r,t}}.
\]

(4-14)

In the soft case, an additional variable is introduced. The variable \( O_{r,t,u}^{\omega} \), which represents the violation in cms of the minimum total plant discharge, \( o_{r,t} \), is added. The soft constraint formulation is:
In practice, the upper bound, \( O_r \), is not used.

**Generation Limit Constraints**

The generation limit constraints specify the allowable range of power generation in MW, from a power plant at reservoir, \( r \in \{R\} \), under each scenario \( \omega \in \{\Omega\} \), during time step \( t \in \{1,...,T\} \):

\[
\frac{o_r}{r,\omega} - \alpha O_{r,\omega}^i \leq \alpha O_{r,\omega} \leq \frac{o_r}{r,\omega}.
\]  
(4-15)

**Total Flow Constraints**

The total flow constraints define total flow from each reservoir, \( r \in \{R\} \), under each scenario \( \omega \in \{\Omega\} \), and sub-period \( u \in \{U\} \) of time step \( t \in \{1,...,T\} \), as the sum of the respective turbine discharge and spill flow:

\[
\hat{a}G_{r,\omega} = \hat{a}S_{r,\omega} + \hat{a}Q_{r,\omega}.
\]  
(4-17)

**Power Generation Constraints**

The power generation constraints define the power generated in MW from the power plant at reservoir \( r \in \{R\} \), under each scenario \( \omega \in \{\Omega\} \), during sub-period \( u \in \{U\} \) of time step \( t \in \{1,...,T\} \), as a function of the turbine discharge from the power plant at reservoir \( r \) during sub-period \( u \):

\[
\alpha G_{r,\omega} = \alpha Q_{r,\omega} \cdot \alpha HK_{r,\omega}.
\]  
(4-18)

Note that the conversion factor is a function of the average storage volume, but not the turbine discharge—i.e., \( \alpha HK_{r,\omega} = \alpha HK_{r,\omega}(V_{r,\omega}, \alpha V_{r,\omega}) \).

The values of \( \alpha HK_{r,\omega} \) used in a solution of the STMVM are pre-calculated based upon forecast reservoir ending volumes, and are updated iteratively.

**Probability Constraints**

The occurrence probabilities of the scenarios must sum to one:

\[
\sum_{\omega \in \{\Omega\}} \omega \mu = 1.0.
\]  
(4-19)

In addition, the occurrence probabilities of all branches leaving a node must also sum to one:
\[
\sum_{sDV} p_r = 1.0;
\]  
(4-20)

where \(\{SDV\}_t\) is the set of possible vectors of scenario-dependent values for time step \(t \in \{1, ..., T\}\).

**Policy Decision Constraints**

The policy decision constraints specify that the policy decision variable must be the same over each scenario group. With the turbine release rate over the time step as the policy decision variable, the constraints for reservoir \(r \in \{R\}\), under scenario \(\omega_t\), where \(\omega_t \in \{\Omega\}\), \(\omega_t \in \{\Omega\}^{\text{all} \, sDV_{r,t}} = \omega_2 \, sDV_{r,t}\) and \(\omega_t \neq \omega_2\), during sub-period \(u \in \{U\}\) of time step \(t \in \{1, ..., T\}\) are:

\[
\omega_0 \, Q_{r,t,u} = \omega_0 \, Q_{r,t,u}.
\]  
(4-21)

These constraints specify that the non-anticipative decision should be the same for all scenarios having common scenario-dependent parameter values during the previous time step. For the initial conditions of \(t=1\), the scenario-dependent parameters have the same values for all scenarios. Should the policy decision variable be the end of time step volume, the turbine release rate, \(Q_{r,t,u}\), in equation (4-21) should be replaced by \(V_{r,t,u}\).

**Non-negativity Constraints**

All variables are subject to the constraint that they cannot be negative:

\[
V_r, \, \omega Q_{r,t,u}, \, \omega S_{r,t,u}, \, \omega O_{r,t,u}, \, \omega O_{r,t,u}, \, \omega G_{r,t,u}, \, \omega V_{r,t}, \, \omega M_{r,t,u}, \, \omega X_{r,t,u} \geq 0; t \in \{T\}; r \in \{R\}; u \in \{U\}; \omega \in \{\Omega\}.
\]  
(4-22)

**Objective Function**

The objective function seeks to maximize the value of operating the hydroelectric system, the operation of which is understood to include making energy trades. The objective function consists of four terms: the revenue generated through exporting energy; the expense incurred through importing energy; the value of water remaining in storage at the end of the time horizon; and the penalty costs associated with violation of soft constraints. The cost of load curtailment is included as a special type of import. The only soft constraint in (4-6) to (4-22) is that for minimum total plant discharge. In order to understand the objective function, it is useful to discuss each of the four terms separately. The entire function is then presented.

The income generated through the export of energy is given by:
In equation (4-23), \(a_w_{1,t}(\alpha X_{t,u})\) is a piecewise-linear function describing the revenue earned from the sale of energy. The slopes of the segments in \(a_w_{1,t}(\alpha X_{t,u})\) are equal to the net export prices ($/MWh). The net export price is given by \(\alpha p_{t,u} (1 - z)\); where \(\alpha p_{t,u}\) is the market energy price, and \(z\) is a factor used to account for wheeling and losses.

Describing the export revenue as a piecewise-linear function of the power generated allows for a block price structure to be used. That is, an export price can be associated with a volume of energy. For example, using two blocks in the structure would mean that all of the energy exported up to a certain limit would earn revenue at one rate, and all of the energy exported above this threshold would earn revenue at a lower rate. Note that in order to preserve the necessary convexity properties, the blocks of export prices must be in the shape of a descending staircase. Also note that the above formulation, in which \(\alpha\) multiplies \(a_w_{1,t}\), \((\alpha X_{t,u})\), implies that the power generation is constant over each sub-period.

The cost incurred through the import of energy is given by:

\[
24 \cdot \sum_{\omega \in \Omega} \sum_{\omega \in U} \sum_{\omega \in \Omega} (\beta_{1,t} \cdot \alpha pr \cdot a_w_{2,t}(\alpha M_{t,u}) \cdot \Delta_{t,u})
\]

In equation (4-24), \(a_w_{2,t}(\alpha M_{t,u})\) is a piecewise-linear function describing the cost of energy purchased. The slopes of the segments in \(a_w_{2,t}(\alpha M_{t,u})\) are the net import prices of energy ($/MWh), where the net import price is given by \(\alpha p_{t,u} (I + y)\). The parameter \(y\) is a factor used to account for the wheeling and losses associated with importing energy. Convexity requirements specify that the blocks of import prices must be in the shape of an ascending staircase. (The difference between this requirement and that for the export price blocks arises because the cost of purchasing energy is multiplied by negative one in the objective function.) The block pricing structure for import prices allows load curtailment to be modelled as an expensive import.

The value of water remaining in storage in a reservoir is given by:

\[
\sum_{\omega \in \Omega} \sum_{\omega \in \Omega} \beta_{1,t} \cdot \alpha pr \cdot a_w_{3,t}(\alpha V_{r,T}).
\]

In equation (4-25), the piecewise-linear function describing the value of water remaining in storage is a function of reservoir storage. In order for convexity requirements to be satisfied, the value of storage must decrease with increasing storage—that is the storage must have a decreasing marginal value. Note that while the above equation describes the value of water in storage as a function of storage in a single reservoir, the actual dependency is, in general, on both reservoir storage volumes. The slopes of the segments used are only valid for forecast ending volumes. These estimates, upon which the slopes are based, are updated in an iterative procedure.

The final term included in the objective function accounts for the penalty associated with violating the minimum total discharge constraints. The constraints given by (4-6) to (4-
25) only contain one soft constraint: minimum total plant discharge. Therefore, the cost of the penalty terms is:

\[ \sum_{t \in \{1, ..., T\}} \sum_{u \in \{U\}} \sum_{w \in \{W\}} \beta_i \cdot w_{4,i} (\alpha O_{r,t,u}^i) \cdot \Delta_{i,t,u} \]  \hspace{1cm} (4-26)

The slopes of the segments in the piecewise-linear curves are the costs ($/cms) associated with violation of the minimum plant discharge constraint. A block cost model is used. As the costs of penalty violations are multiplied by negative one in the objective function, the shape of the block costs must be that of an ascending staircase.

The entire objective function comprised of the parts described by equations (4-6) to (4-26), summed over all scenarios, is:

\[ \text{max} \left( \sum_{t \in \{1, ..., T\}} \sum_{u \in \{U\}} \sum_{w \in \{W\}} \Delta_{t,u} \left( 24 \cdot \left( \alpha w_{1,t}(\alpha X_{t,u}^r) - \alpha w_{2,t}(\alpha M_{t,u}) \right) - \sum_{r \in \{R\}} w_{3,r}(\alpha O_{r,t,u}^i) \right) + \sum_{u \in \{U\}} \sum_{w \in \{W\}} w_{4,i} (\alpha O_{r,t,u}^i) \cdot \beta_i \right) \]  \hspace{1cm} (4-27)

The notation used in the description of the two time step and multiple time step stochastic linear programming problems with recourse given by (4-4) and (4-5) differs from that used in (4-6) to (4-27). The decision variables in (4-5) are \( x_1, x_2, ..., x_T \), while the decision variables in (4-6) to (4-27) are \( V_{r,t,u}^0, \alpha Q_{r,t,u}, \alpha S_{r,t,u}, \alpha O_{r,t,u}^i, \alpha G_{r,t,u}, \alpha O_{r,t,u}^i, \alpha Q_{r,t,u} \). The equivalence between the two is as follows. The \( x_t \) are all of the here-and-now decision variables that must be determined at the start of the first time step. The \( x_2 \) are all of the wait-and-see decision variables for the first time step and the here-and-now decision variables that must be determined at the start of the second time step. Similarly the \( x_{T-1} \) are all of the wait-and-see decision variables for time step \( T-1 \) and the here-and-now decision variables for time step \( T \). Finally, the \( x_T \) are all of the wait-and-see decision variables for time step \( T \). The \( c_i \) in (4-5) are equivalent to the parameters and piecewise-linear functions in (4-27).

The means by which the objective function described by (4-27) is optimized, subject to the constraints described by equations (4-6) to (4-22), is described in the following section.

### 4.3.2 Solution Methodology

The LP problem described by equations (4-6) to (4-27) is modelled using the language AMPL (Fourer et al., 1993). When AMPL is run, provided that the model and data are valid, the data is reformatted to the specifications of the desired solver. After the data have been properly reformatted, the solver is executed to find the optimal solution. The solver used in the STMVM is CPLEX (CPLEX Optimization, Inc., 1995).
4.4 Case Study

The test system used for the case study is the idealized sub-system of the BC Hydro system described in section 3.4.1. The sub-system is based upon the BC Hydro generation plants on the mainstem Columbia River (Mica and Revelstoke) and the Peace River (G. M. Shrum and Peace Canyon). Together, these projects account for roughly 70% of the BC Hydro hydroelectric generation capacity (BC Hydro, 1995). In the test system, generation on each of the two river systems is modelled by a single plant. The "Columbia" plant replaces the Mica and Revelstoke plants, and the "Peace" plant replaces the G. M. Shrum and Peace Canyon plants. As the intent of this work is to examine the relationship between the operation of two hydroelectric generating river systems at a system level, many of the details of the operation of the physical plants are neglected. The aim was to work with data that are a reasonable approximation of the major characteristics of the underlying facilities.

The physical, and much of the operational, data used here for the STMVM are the same as those for the DP and LP based model described in section 3.4. The minimum and maximum turbine releases, reservoir storage volumes, and plant turbine capacities are given by Table 3-7. The functions that are used to convert turbine discharge to power generation are given by Table 3-9, Table 3-10, and Table 3-11. The discount rate, the loss factor accounting for transmission and wheeling, and the curtailment cost are given in Table 3-12. The penalties for violation of the Peace minimum releases are given by Table 3-13.

The data for the demands, inflows, and prices vary with the month and scenario, while the import and export limits and Peace minimum discharge, vary with the month, and are given below in section 4.4.2.

The case study, which uses results from the DP and LP based model to evaluate the value of storage at the end of the modelled time horizon, demonstrates the extent to which the selected approach allows sufficient detail to be considered such that the uncertain reservoir operating problem actually faced by the operator of a hydropower system, in which energy varies in price both within the day, between days, and between months, in which inflows and demands are also uncertain, and when generation can come from different sources, to be solved successfully. Typically, parts of the true operating problem have been ignored, either by dealing with only a single reservoir, or by not addressing uncertainty in all of the important parameters. In order to illustrate the efficacy of the proposed method, and the critical pricing information for a two reservoir system which can be obtained from it, the case study is quite involved.

4.4.1 Overview

Using the STMVM, a series of problems based on the test system has been solved. The series was selected to represent different times of the year, initial storage conditions, scenario-dependent parameters, and assumptions regarding non-anticipative constraints.
Sets of problems have been solved for time horizons covering the months of January, May, and September. Each one of these sets includes problems under the assumptions of a one-scenario base case and three 125-scenario cases: one for each of the demands, inflows, and prices as the only scenario-dependent parameter. Each of the three scenario-dependent parameters is assumed to be normally distributed with a coefficient of variation of 0.10. Each of the 125-scenario cases is solved under the assumption of no non-anticipative constraints, non-anticipative turbine releases, and non-anticipative ending storage volumes. Finally, for each case so far described, nine individual problems, representing the combination of each reservoir starting at 10%, 50%, or 90% full, are solved.

The information obtained from the model that is of the most interest involves the objective function value and the marginal Peace and Columbia energy values. The manner in which these three items in the 125-scenario cases differ from those in the one-scenario base case, and the manner in which the marginal energy values vary for the 125-scenario cases under the two non-anticipative assumptions vary from those under the assumption of no non-anticipative constraints is presented after the definitions of the scenarios in the following section.

4.4.2 Scenario Definitions

In this section the parameters that define the scenarios are specified. Data are presented in turn for January, May, and September.

4.4.2.1 Data for January Scenarios

The first set of problems is for the month of January. In each of these problems, the time horizon is 31 days, divided into three time periods, with each time period being divided into HLH and LLH sub-periods. The length of these sub-periods is specified in Table 4-1.

<table>
<thead>
<tr>
<th>Time period</th>
<th>Length of HLH (days)</th>
<th>Length of LLH (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.571</td>
<td>0.429</td>
</tr>
<tr>
<td>2</td>
<td>1.142</td>
<td>0.858</td>
</tr>
<tr>
<td>3</td>
<td>15.988</td>
<td>12.012</td>
</tr>
</tbody>
</table>

Table 4-1: Length of Sub-Periods for January Studies

The first time step comprises January 1st, the next period comprises January 2nd through 3rd, and the final period is comprised of the balance of the month.
The minimum plant releases and minimum and maximum imports and exports are assumed to be the same in each January problem, and are shown in Table 4-2. The same limits are assumed to apply for all periods.

<table>
<thead>
<tr>
<th></th>
<th>HLH</th>
<th>LLH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum Peace Release (cms)</td>
<td>1600</td>
<td>1600</td>
</tr>
<tr>
<td>Minimum Columbia Release (cms)</td>
<td>85</td>
<td>85</td>
</tr>
<tr>
<td>Minimum Import (MW)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Maximum Import (MW)</td>
<td>1600</td>
<td>1250</td>
</tr>
<tr>
<td>Minimum Export (MW)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Maximum Export (MW)</td>
<td>1900</td>
<td>1400</td>
</tr>
</tbody>
</table>

Table 4-2: Limits for January Studies

The scenario-dependent data for the January one-scenario base case are given in Table 4-3.

<table>
<thead>
<tr>
<th></th>
<th>HLH</th>
<th>LLH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price ($/MWh) – Period 1</td>
<td>33</td>
<td>28</td>
</tr>
<tr>
<td>Price ($/MWh) – Period 2</td>
<td>26</td>
<td>22</td>
</tr>
<tr>
<td>Price ($/MWh) – Period 3</td>
<td>31</td>
<td>27</td>
</tr>
<tr>
<td>Demand (MW) – Period 1</td>
<td>6100</td>
<td>4200</td>
</tr>
<tr>
<td>Demand (MW) – Period 2</td>
<td>5900</td>
<td>4000</td>
</tr>
<tr>
<td>Demand (MW) – Period 3</td>
<td>6000</td>
<td>4100</td>
</tr>
<tr>
<td>Peace Inflow (cms) – Period 1</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>Peace Inflow (cms) – Period 2</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>Peace Inflow (cms) – Period 3</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>Columbia Inflow (cms) – Period 1</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>Columbia Inflow (cms) – Period 2</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>Columbia Inflow (cms) – Period 3</td>
<td>150</td>
<td>150</td>
</tr>
</tbody>
</table>

Table 4-3: January Base Case Scenario-Dependent Parameters

The scenario-dependent demands for the January 125-scenario cases with demands as the scenario-dependent parameters are defined in Table 4-4. For all of these cases the demand in each of the three sub-periods can assume one of five values. The five values come from applying the unit normal function values from Table 3-17 with a coefficient of variation of 0.10 to the sub-period demands given for the base case in Table 4-3.

<table>
<thead>
<tr>
<th></th>
<th>HLH Demand (MW)</th>
<th>LLH Demand (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1 Very High</td>
<td>7149</td>
<td>4922</td>
</tr>
<tr>
<td>Period 1 High</td>
<td>6564</td>
<td>4519</td>
</tr>
<tr>
<td>Period 1 Average</td>
<td>6100</td>
<td>4200</td>
</tr>
<tr>
<td>Period 1 Low</td>
<td>5636</td>
<td>3881</td>
</tr>
<tr>
<td>Period 1 Very Low</td>
<td>5051</td>
<td>3478</td>
</tr>
</tbody>
</table>
Period 2 Very High          | 6915 | 4688  
Period 2 High              | 6348 | 4304  
Period 2 Average           | 5900 | 4000  
Period 2 Low               | 5452 | 3696  
Period 2 Very Low          | 4885 | 3312  
Period 3 Very High         | 7032 | 4805  
Period 3 High              | 6456 | 4412  
Period 3 Average           | 6000 | 4100  
Period 3 Low               | 5544 | 3788  
Period 3 Very Low          | 4968 | 3395  

<table>
<thead>
<tr>
<th>Peace Inflow (cms)</th>
<th>Columbia Inflow (cms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1 Very High</td>
<td>352</td>
</tr>
<tr>
<td>Period 1 High</td>
<td>323</td>
</tr>
<tr>
<td>Period 1 Average</td>
<td>300</td>
</tr>
<tr>
<td>Period 1 Low</td>
<td>277</td>
</tr>
<tr>
<td>Period 1 Very Low</td>
<td>248</td>
</tr>
<tr>
<td>Period 2 Very High</td>
<td>352</td>
</tr>
<tr>
<td>Period 2 High</td>
<td>323</td>
</tr>
<tr>
<td>Period 2 Average</td>
<td>300</td>
</tr>
<tr>
<td>Period 2 Low</td>
<td>277</td>
</tr>
<tr>
<td>Period 2 Very Low</td>
<td>248</td>
</tr>
<tr>
<td>Period 3 Very High</td>
<td>352</td>
</tr>
<tr>
<td>Period 3 High</td>
<td>323</td>
</tr>
<tr>
<td>Period 3 Average</td>
<td>300</td>
</tr>
<tr>
<td>Period 3 Low</td>
<td>277</td>
</tr>
<tr>
<td>Period 3 Very Low</td>
<td>248</td>
</tr>
</tbody>
</table>

**Table 4-4: Scenario-Dependent Demands for January Studies**

The very high and very low values in each sub-period both have an occurrence probability of 0.107, the high and low values in each sub-period both have an occurrence probability of 0.245, and the average value has an occurrence probability of 0.296. It is assumed that within a time step there is perfect correlation between HLH and LLH demands, and that there is no relationship in demands between time steps.

The inflows and prices for these cases are the same as those in Table 4-3 for all scenarios.

The scenario-dependent inflows for the January 125-scenario cases with inflows as the scenario-dependent parameters are defined in Table 4-5. Perfect correlation between the Peace and Columbia inflows is assumed. For all of these cases the inflow in each of the three sub-periods can assume one of five values. The five values come from applying the unit normal function values from Table 3-17 with a CV of 0.10 to the inflows given for the base case in Table 4-3.
The very high and very low values in each sub-period both have an occurrence probability of 0.107, the high and low values in each sub-period both have an occurrence probability of 0.245, and the average value has an occurrence probability of 0.296. It is assumed that within a time step there is perfect correlation between HLH and LLH prices, and that there is no relationship in prices between time steps.

The demands and prices for these cases are the same as those in Table 4-3 for all scenarios.

The scenario-dependent prices for the January 125-scenario cases with prices as the scenario-dependent parameters are defined in Table 4-6. For all of these cases the price in each of the three sub-periods can assume one of five values. The five values come from applying the unit normal function values from Table 3-17 with a CV of 0.10 to the sub-period prices given for the base case in Table 4-3.

<table>
<thead>
<tr>
<th></th>
<th>HLH Price ($/MWh)</th>
<th>LLH Price ($/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1 Very High</td>
<td>38.68</td>
<td>32.82</td>
</tr>
<tr>
<td>Period 1 High</td>
<td>35.51</td>
<td>30.13</td>
</tr>
<tr>
<td>Period 1 Average</td>
<td>33.00</td>
<td>28.00</td>
</tr>
<tr>
<td>Period 1 Low</td>
<td>30.49</td>
<td>25.87</td>
</tr>
<tr>
<td>Period 1 Very Low</td>
<td>27.32</td>
<td>23.18</td>
</tr>
<tr>
<td>Period 2 Very High</td>
<td>30.47</td>
<td>25.78</td>
</tr>
<tr>
<td>Period 2 High</td>
<td>27.98</td>
<td>23.67</td>
</tr>
<tr>
<td>Period 2 Average</td>
<td>26.00</td>
<td>22.00</td>
</tr>
<tr>
<td>Period 2 Low</td>
<td>24.02</td>
<td>20.33</td>
</tr>
<tr>
<td>Period 2 Very Low</td>
<td>21.53</td>
<td>18.22</td>
</tr>
<tr>
<td>Period 3 Very High</td>
<td>36.33</td>
<td>31.64</td>
</tr>
<tr>
<td>Period 3 High</td>
<td>33.36</td>
<td>29.05</td>
</tr>
<tr>
<td>Period 3 Average</td>
<td>31.00</td>
<td>27.00</td>
</tr>
<tr>
<td>Period 3 Low</td>
<td>28.64</td>
<td>24.95</td>
</tr>
<tr>
<td>Period 3 Very Low</td>
<td>25.67</td>
<td>22.36</td>
</tr>
</tbody>
</table>

Table 4-6: Scenario-Dependent Prices for January Studies

The very high and very low values in each sub-period both have an occurrence probability of 0.107, the high and low values in each sub-period both have an occurrence probability of 0.245, and the average value has an occurrence probability of 0.296. It is assumed that within a time step there is perfect correlation between HLH and LLH prices, and that there is no relationship in prices between time steps.

The inflows and demands for these cases are the same as those in Table 4-3 for all scenarios.

The storage value curves that define the value of water in storage at the end of the month are calculated using the DP and LP based model under the assumption of inflows as the only scenario-dependent parameters, a coefficient of variation of 0.25, and non-anticipative turbine releases.
4.4.2.2 Data for May Scenarios

The second set of problems is for the month of May. In each of these problems the time horizon is 31 days, divided into three time periods, with each time period being divided into HLH and LLH sub-periods. The length of these sub-periods is specified in Table 4-7.

<table>
<thead>
<tr>
<th>Time period</th>
<th>Length of HLH (days)</th>
<th>Length of LLH (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.571</td>
<td>0.429</td>
</tr>
<tr>
<td>2</td>
<td>1.142</td>
<td>0.858</td>
</tr>
<tr>
<td>3</td>
<td>15.988</td>
<td>12.012</td>
</tr>
</tbody>
</table>

Table 4-7: Length of Sub-Periods for May Studies

The first time step comprises May 1st, the next period comprises May 2nd through 3rd, and the final period is comprised of the balance of the month.

The minimum plant releases and minimum and maximum imports and exports are assumed to be the same in each May problem, and are shown in Table 4-8. The same limits are assumed to apply for all periods.

<table>
<thead>
<tr>
<th></th>
<th>HLH</th>
<th>LLH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum Peace Plant</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>Release (cms)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum Columbia Plant</td>
<td>85</td>
<td>85</td>
</tr>
<tr>
<td>Release (cms)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum Import (MW)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Maximum Import (MW)</td>
<td>2000</td>
<td>1500</td>
</tr>
<tr>
<td>Minimum Export (MW)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Maximum Export (MW)</td>
<td>1700</td>
<td>1200</td>
</tr>
</tbody>
</table>

Table 4-8: Limits for May Studies

The scenario-dependent data for the May one-scenario base case are given in Table 4-9.

<table>
<thead>
<tr>
<th></th>
<th>HLH</th>
<th>LLH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price ($/MWh) – Period 1</td>
<td>32</td>
<td>19</td>
</tr>
<tr>
<td>Price ($/MWh) – Period 2</td>
<td>25</td>
<td>13</td>
</tr>
<tr>
<td>Price ($/MWh) – Period 3</td>
<td>34</td>
<td>20</td>
</tr>
<tr>
<td>Demand (MW) – Period 1</td>
<td>4800</td>
<td>3300</td>
</tr>
<tr>
<td>Demand (MW) – Period 2</td>
<td>4600</td>
<td>3100</td>
</tr>
<tr>
<td>Demand (MW) – Period 3</td>
<td>4900</td>
<td>3400</td>
</tr>
<tr>
<td>Peace Inflow (cms) – Period 1</td>
<td>2230</td>
<td>2230</td>
</tr>
<tr>
<td>Peace Inflow (cms) – Period 2</td>
<td>2230</td>
<td>2230</td>
</tr>
</tbody>
</table>
Table 4-9: May Base Case Scenario-Dependent Parameters

The scenario-dependent demands for the May 125-scenario cases with the demands as the scenario-dependent parameters are defined in Table 4-10. For all of these cases the demand in each of the three sub-periods can assume one of five values. The five values come from applying the unit normal function values from Table 3-17 with a coefficient of variation of 0.10 to the sub-period demands given for the base case in Table 4-9.

<table>
<thead>
<tr>
<th>Period</th>
<th>HLH Demand (MW)</th>
<th>LLH Demand (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1 Very High</td>
<td>5626</td>
<td>3868</td>
</tr>
<tr>
<td>Period 1 High</td>
<td>5165</td>
<td>3551</td>
</tr>
<tr>
<td>Period 1 Average</td>
<td>4800</td>
<td>3300</td>
</tr>
<tr>
<td>Period 1 Low</td>
<td>4435</td>
<td>3049</td>
</tr>
<tr>
<td>Period 1 Very Low</td>
<td>3974</td>
<td>2732</td>
</tr>
<tr>
<td>Period 2 Very High</td>
<td>5391</td>
<td>3633</td>
</tr>
<tr>
<td>Period 2 High</td>
<td>4950</td>
<td>3336</td>
</tr>
<tr>
<td>Period 2 Average</td>
<td>4600</td>
<td>3100</td>
</tr>
<tr>
<td>Period 2 Low</td>
<td>4250</td>
<td>2864</td>
</tr>
<tr>
<td>Period 2 Very Low</td>
<td>3809</td>
<td>2567</td>
</tr>
<tr>
<td>Period 3 Very High</td>
<td>5743</td>
<td>3985</td>
</tr>
<tr>
<td>Period 3 High</td>
<td>5272</td>
<td>3658</td>
</tr>
<tr>
<td>Period 3 Average</td>
<td>4900</td>
<td>3400</td>
</tr>
<tr>
<td>Period 3 Low</td>
<td>4528</td>
<td>3142</td>
</tr>
<tr>
<td>Period 3 Very Low</td>
<td>4057</td>
<td>2815</td>
</tr>
</tbody>
</table>

Table 4-10: Scenario-Dependent Demands for May Studies

The very high and very low values in each sub-period both have an occurrence probability of 0.107, the high and low values in each sub-period both have an occurrence probability of 0.245, and the average value has an occurrence probability of 0.296. It is assumed that within a time step there is perfect correlation between HLH and LLH demands, and that there is no relationship in demands between time steps.

The inflows and prices for these cases are the same as those in Table 4-9 for all scenarios.

The scenario-dependent inflows for the May 125-scenario cases with the inflows as the scenario-dependent parameters are defined in Table 4-11. Perfect correlation between the Peace and Columbia inflows is assumed. For all of these cases the inflow in each of the three sub-periods can assume one of five values. The five values come from applying the unit normal function values from Table 3-17 with a CV of 0.10 to the inflows given for the base case in Table 4-9.
<table>
<thead>
<tr>
<th></th>
<th>Peace Inflow (cms)</th>
<th>Columbia Inflow (cms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1 Very High</td>
<td>2614</td>
<td>1289</td>
</tr>
<tr>
<td>Period 1 High</td>
<td>2399</td>
<td>1184</td>
</tr>
<tr>
<td>Period 1 Average</td>
<td>2230</td>
<td>1100</td>
</tr>
<tr>
<td>Period 1 Low</td>
<td>2061</td>
<td>1016</td>
</tr>
<tr>
<td>Period 1 Very Low</td>
<td>1846</td>
<td>911</td>
</tr>
<tr>
<td>Period 2 Very High</td>
<td>2614</td>
<td>1289</td>
</tr>
<tr>
<td>Period 2 High</td>
<td>2399</td>
<td>1184</td>
</tr>
<tr>
<td>Period 2 Average</td>
<td>2230</td>
<td>1100</td>
</tr>
<tr>
<td>Period 2 Low</td>
<td>2061</td>
<td>1016</td>
</tr>
<tr>
<td>Period 2 Very Low</td>
<td>1846</td>
<td>911</td>
</tr>
<tr>
<td>Period 3 Very High</td>
<td>2614</td>
<td>1289</td>
</tr>
<tr>
<td>Period 3 High</td>
<td>2399</td>
<td>1184</td>
</tr>
<tr>
<td>Period 3 Average</td>
<td>2230</td>
<td>1100</td>
</tr>
<tr>
<td>Period 3 Low</td>
<td>2061</td>
<td>1016</td>
</tr>
<tr>
<td>Period 3 Very Low</td>
<td>1846</td>
<td>911</td>
</tr>
</tbody>
</table>

**Table 4-11: Scenario-Dependent Inflows for May Studies**

The very high and very low values in each sub-period both have an occurrence probability of 0.107, the high and low values in each sub-period both have an occurrence probability of 0.245, and the average value has an occurrence probability of 0.296. It is assumed that there is no relationship in inflows between time steps.

The demands and prices for these cases are the same as those in Table 4-9 for all scenarios.

The scenario-dependent prices for the May 125-scenario cases with the prices as the scenario-dependent parameters are defined in Table 4-12. For all of these cases the price in each of the three sub-periods can assume one of five values. The five values come from applying the unit normal function values from Table 3-17 with a CV of 0.10 to the sub-period prices given for the base case in Table 4-9.

<table>
<thead>
<tr>
<th></th>
<th>HLH Price ($/MWh)</th>
<th>LLH Price ($/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1 Very High</td>
<td>37.50</td>
<td>22.27</td>
</tr>
<tr>
<td>Period 1 High</td>
<td>34.43</td>
<td>20.44</td>
</tr>
<tr>
<td>Period 1 Average</td>
<td>32.00</td>
<td>19.00</td>
</tr>
<tr>
<td>Period 1 Low</td>
<td>29.57</td>
<td>17.56</td>
</tr>
<tr>
<td>Period 1 Very Low</td>
<td>26.50</td>
<td>15.73</td>
</tr>
<tr>
<td>Period 2 Very High</td>
<td>29.30</td>
<td>15.24</td>
</tr>
<tr>
<td>Period 2 High</td>
<td>26.90</td>
<td>13.99</td>
</tr>
<tr>
<td>Period 2 Average</td>
<td>25.00</td>
<td>13.00</td>
</tr>
<tr>
<td>Period 2 Low</td>
<td>23.10</td>
<td>12.01</td>
</tr>
<tr>
<td>Period 2 Very Low</td>
<td>20.70</td>
<td>10.76</td>
</tr>
</tbody>
</table>
The very high and very low values in each sub-period both have an occurrence probability of 0.107, the high and low values in each sub-period both have an occurrence probability of 0.245, and the average value has an occurrence probability of 0.296. It is assumed that within a time step there is perfect correlation between HLH and LLH prices, and that there is no relationship in prices between time steps.

The inflows and demands for these cases are the same as those in Table 4-9 for all scenarios.

The storage value curves that define the value of water in storage at the end of the month are calculated using the DP and LP based model under the assumption of inflows as the only scenario-dependent parameters, a coefficient of variation of 0.25, and non-anticipative turbine releases.

<table>
<thead>
<tr>
<th>Period 3 Very High</th>
<th>39.85</th>
<th>23.44</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 3 High</td>
<td>36.58</td>
<td>21.52</td>
</tr>
<tr>
<td>Period 3 Average</td>
<td>34.00</td>
<td>20.00</td>
</tr>
<tr>
<td>Period 3 Low</td>
<td>31.42</td>
<td>18.48</td>
</tr>
<tr>
<td>Period 3 Very Low</td>
<td>28.15</td>
<td>16.56</td>
</tr>
</tbody>
</table>

Table 4-12: Scenario-Dependent Prices for May Studies

4.4.2.3 Data for September Scenarios

The final set of problems is for the month of September. In each of these problems the time horizon is 30 days, divided into three time periods, with each time period being divided into HLH and LLH sub-periods. The length of these sub-periods is specified in Table 4-13.

<table>
<thead>
<tr>
<th>Time period</th>
<th>Length of HLH (days)</th>
<th>Length of LLH (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.571</td>
<td>0.429</td>
</tr>
<tr>
<td>2</td>
<td>1.142</td>
<td>0.858</td>
</tr>
<tr>
<td>3</td>
<td>15.417</td>
<td>11.583</td>
</tr>
</tbody>
</table>

Table 4-13: Length of Sub-Periods for September Studies

The first time step comprises September 1st, the next period comprises September 2nd through 3rd, and the final period is comprised of the balance of the month.

The minimum plant releases and minimum and maximum imports and exports are shown in Table 4-14. The same limits are assumed to apply for all periods.
### Table 4-14: Limits for September Studies

<table>
<thead>
<tr>
<th></th>
<th>HLH</th>
<th>LLH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum Peace Plant</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>Release (cms)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum Columbia Plant</td>
<td>85</td>
<td>85</td>
</tr>
<tr>
<td>Release (cms)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum Import (MW)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Maximum Import (MW)</td>
<td>2000</td>
<td>1500</td>
</tr>
<tr>
<td>Minimum Export (MW)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Maximum Export (MW)</td>
<td>1800</td>
<td>1300</td>
</tr>
</tbody>
</table>

The scenario-dependent data for the September one-scenario base case are given in Table 4-15.

### Table 4-15: September Base Case Scenario-Dependent Parameters

<table>
<thead>
<tr>
<th></th>
<th>HLH</th>
<th>LLH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price ($/MWh) – Period 1</td>
<td>50</td>
<td>35</td>
</tr>
<tr>
<td>Price ($/MWh) – Period 2</td>
<td>32</td>
<td>25</td>
</tr>
<tr>
<td>Price ($/MWh) – Period 3</td>
<td>42</td>
<td>31</td>
</tr>
<tr>
<td>Demand (MW) – Period 1</td>
<td>4800</td>
<td>3400</td>
</tr>
<tr>
<td>Demand (MW) – Period 2</td>
<td>4300</td>
<td>3100</td>
</tr>
<tr>
<td>Demand (MW) – Period 3</td>
<td>4800</td>
<td>3400</td>
</tr>
<tr>
<td>Peace Inflow (cms) – Period 1</td>
<td>780</td>
<td>780</td>
</tr>
<tr>
<td>Peace Inflow (cms) – Period 2</td>
<td>780</td>
<td>780</td>
</tr>
<tr>
<td>Peace Inflow (cms) – Period 3</td>
<td>780</td>
<td>780</td>
</tr>
<tr>
<td>Columbia Inflow (cms) – Period 1</td>
<td>820</td>
<td>820</td>
</tr>
<tr>
<td>Columbia Inflow (cms) – Period 2</td>
<td>820</td>
<td>820</td>
</tr>
<tr>
<td>Columbia Inflow (cms) – Period 3</td>
<td>820</td>
<td>820</td>
</tr>
</tbody>
</table>

The scenario-dependent demands for the September 125-scenario cases with the demands as the scenario-dependent parameters are defined in Table 4-16. For all of these cases the demand in each of the three sub-periods can assume one of five values. The five values come from applying the unit normal function values from Table 3-17 with a coefficient of variation of 0.10 to the sub-period demands given for the base case in Table 4-15.

<table>
<thead>
<tr>
<th></th>
<th>HLH Demand (MW)</th>
<th>LLH Demand (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1 Very High</td>
<td>5626</td>
<td>3985</td>
</tr>
<tr>
<td>Period 1 High</td>
<td>5165</td>
<td>3658</td>
</tr>
<tr>
<td>Period 1 Average</td>
<td>4800</td>
<td>3400</td>
</tr>
<tr>
<td>Period 1 Low</td>
<td>4775</td>
<td>3190</td>
</tr>
<tr>
<td>Period 1 Very Low</td>
<td>4435</td>
<td>3142</td>
</tr>
<tr>
<td>Period 2 Very High</td>
<td>5040</td>
<td>3633</td>
</tr>
<tr>
<td>Period 2 High</td>
<td>4627</td>
<td>3336</td>
</tr>
<tr>
<td>Period 2 Average</td>
<td>4300</td>
<td>3100</td>
</tr>
<tr>
<td>Period 2 Low</td>
<td>3973</td>
<td>2864</td>
</tr>
<tr>
<td>-------------------</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>Period 2 Very Low</td>
<td>3560</td>
<td>2567</td>
</tr>
<tr>
<td>Period 3 Very High</td>
<td>5626</td>
<td>3985</td>
</tr>
<tr>
<td>Period 3 High</td>
<td>5165</td>
<td>3658</td>
</tr>
<tr>
<td>Period 3 Average</td>
<td>4800</td>
<td>3400</td>
</tr>
<tr>
<td>Period 3 Low</td>
<td>4435</td>
<td>3142</td>
</tr>
<tr>
<td>Period 3 Very Low</td>
<td>3974</td>
<td>2815</td>
</tr>
</tbody>
</table>

**Table 4-16: Scenario-Dependent Demands for September Studies**

The very high and very low values in each sub-period both have an occurrence probability of 0.107, the high and low values in each sub-period both have an occurrence probability of 0.245, while the average value has an occurrence probability of 0.296. It is assumed that within a time step there is perfect correlation between HLH and LLH demands, and that there is no relationship in demands between time steps.

The inflows and prices for these cases are the same as those in Table 4-15 for all scenarios.

The scenario-dependent inflows for the September 125-scenario cases with the inflows as the scenario-dependent parameters are defined in Table 4-17. Perfect correlation between Peace and Columbia inflows is assumed. For all of these cases the inflow in each of the three sub-periods can assume one of five values. The five values come from applying the unit normal function values from Table 3-17 with a CV of 0.10 to the inflows given for the base case in Table 4-15.

<table>
<thead>
<tr>
<th></th>
<th>Peace Inflow (cms)</th>
<th>Columbia Inflow (cms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1 Very High</td>
<td>914</td>
<td>961</td>
</tr>
<tr>
<td>Period 1 High</td>
<td>839</td>
<td>882</td>
</tr>
<tr>
<td>Period 1 Average</td>
<td>780</td>
<td>820</td>
</tr>
<tr>
<td>Period 1 Low</td>
<td>721</td>
<td>758</td>
</tr>
<tr>
<td>Period 1 Very Low</td>
<td>646</td>
<td>679</td>
</tr>
<tr>
<td>Period 2 Very High</td>
<td>914</td>
<td>961</td>
</tr>
<tr>
<td>Period 2 High</td>
<td>839</td>
<td>882</td>
</tr>
<tr>
<td>Period 2 Average</td>
<td>780</td>
<td>820</td>
</tr>
<tr>
<td>Period 2 Low</td>
<td>721</td>
<td>758</td>
</tr>
<tr>
<td>Period 2 Very Low</td>
<td>646</td>
<td>679</td>
</tr>
<tr>
<td>Period 3 Very High</td>
<td>914</td>
<td>961</td>
</tr>
<tr>
<td>Period 3 High</td>
<td>839</td>
<td>882</td>
</tr>
<tr>
<td>Period 3 Average</td>
<td>780</td>
<td>820</td>
</tr>
<tr>
<td>Period 3 Low</td>
<td>721</td>
<td>758</td>
</tr>
<tr>
<td>Period 3 Very Low</td>
<td>646</td>
<td>679</td>
</tr>
</tbody>
</table>

**Table 4-17: Scenario-Dependent Inflows for September Studies**
The very high and very low values in each sub-period both have an occurrence probability of 0.107, the high and low values in each sub-period both have an occurrence probability of 0.245, and the average value has an occurrence probability of 0.296. It is assumed that there is no relationship in inflows between time steps.

The demands and prices for these cases are the same as those in Table 4-15 for all scenarios.

The scenario-dependent prices for the September 125-scenario cases with the prices as the scenario-dependent parameters are defined in Table 4-18. For all of these cases the price in each of the three sub-periods can assume one of five values. The five values come from applying the unit normal function values from Table 3-17 with a CV of 0.10 to the sub-period prices given for the base case in Table 4-15.

<table>
<thead>
<tr>
<th>Period</th>
<th>HLH Price ($/MWh)</th>
<th>LLH Price ($/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1 Very High</td>
<td>58.60</td>
<td>41.02</td>
</tr>
<tr>
<td>Period 1 High</td>
<td>53.80</td>
<td>37.66</td>
</tr>
<tr>
<td>Period 1 Average</td>
<td>50.00</td>
<td>35.00</td>
</tr>
<tr>
<td>Period 1 Low</td>
<td>56.20</td>
<td>32.34</td>
</tr>
<tr>
<td>Period 1 Very Low</td>
<td>41.40</td>
<td>28.98</td>
</tr>
<tr>
<td>Period 2 Very High</td>
<td>37.50</td>
<td>29.30</td>
</tr>
<tr>
<td>Period 2 High</td>
<td>34.43</td>
<td>23.90</td>
</tr>
<tr>
<td>Period 2 Average</td>
<td>32.00</td>
<td>25.00</td>
</tr>
<tr>
<td>Period 2 Low</td>
<td>29.57</td>
<td>23.10</td>
</tr>
<tr>
<td>Period 2 Very Low</td>
<td>26.50</td>
<td>20.70</td>
</tr>
<tr>
<td>Period 3 Very High</td>
<td>49.22</td>
<td>36.33</td>
</tr>
<tr>
<td>Period 3 High</td>
<td>45.19</td>
<td>33.36</td>
</tr>
<tr>
<td>Period 3 Average</td>
<td>42.00</td>
<td>31.00</td>
</tr>
<tr>
<td>Period 3 Low</td>
<td>38.81</td>
<td>28.64</td>
</tr>
<tr>
<td>Period 3 Very Low</td>
<td>34.78</td>
<td>25.67</td>
</tr>
</tbody>
</table>

Table 4-18: Scenario-Dependent Prices for September Studies

The very high and very low values in each sub-period both have an occurrence probability of 0.107, the high and low values in each sub-period both have an occurrence probability of 0.245, and the average value has an occurrence probability of 0.296. It is assumed that within a time step there is perfect correlation between HLH and LLH prices, and that there is no relationship in prices between time steps.

The inflows and demands for these cases are the same as those in Table 4-15 for all scenarios.

The storage value curves that define the value of water in storage at the end of the month are calculated using the DP and LP based model under the assumption of inflows as the
only scenario-dependent parameters, a coefficient of variation of 0.25, and non-anticipative turbine releases.

Having defined the parameter values for the case study, the series of STMVM runs described in section 4.4.1 were performed. The objective function values, Peace marginal energy values, and Columbia marginal energy values that result from these runs are presented in the remainder of this chapter.

### 4.4.3 Objective Function Values

In this section, the objective function values that result from applying the STMVM are examined. The objective function described by equation (4-27) consists of three parts—the net income from energy market transactions, the value of water remaining in storage at the end of the model horizon, and the penalty for violating minimum Peace release constraints.

The objective functions first examined are those for the respective one-scenario base cases for January, May, and September, which are presented in Figure 4-3.

![Objective Function Values for One-Scenario Base Cases](image)

**Figure 4-3: Objective Function Values for One-Scenario Base Cases**

In Figure 4-3, as well as in all of the objective function value charts that follow, Low C, Mid C, and High C, are defined, respectively, as 20000, 100000, and 180000 cmsd, and Low P, Mid P, and High P are defined, respectively, as 50000, 250000, and 450000 cmsd, these representing 10%, 50%, and 90% of full storage for the Columbia and Peace reservoirs.
From Figure 4-3 note that for each of the nine initial storage conditions, the objective function for September is greater than that for January, which in turn is greater than that for May. These results indicate that the value of having a given amount of storage heading into winter is greater than the value mid-way through the winter, which is greater than the value near the start of the freshet. Having insufficient water in storage heading into the winter, or during the winter, can lead to the high costs of curtailment and violating minimum plant releases. For each month, the decreasing marginal value of storage can be observed by comparing the objective function when one reservoir is at a given level and when the storage in the other reservoir assumes each of the low, mid, and high values.

### 4.4.3.1 Comparison of Objective Function Values Against Base Cases

In this section, the objective function values for the set of problems solved are compared against the objective function value for the one-scenario base case for the relevant month. This comparison shows how well the simpler one-scenario case approximates the results of each of the more complex 125-scenario cases. The comparison is made by subtracting the objective function for the 125-scenario case from the base case; the smaller the magnitude of this difference, the better the base case approximates the 125-scenario case.

**January**

In this section, the utility of employing the January base case to estimate the objective function is examined for three scenario-dependent parameters (demand, inflow, and price), three constraints regarding how to model the anticipative nature of the problem (no non-anticipative constraints, constraints for non-anticipative turbine releases, and constraints for non-anticipative ending storage volumes), and nine initial storage conditions. The parameters for the January scenarios are defined in Table 4-1 through Table 4-6.

The first non-anticipative assumption considered is that of no non-anticipative constraints, meaning that perfect knowledge of the scenario-dependent parameters is assumed when each decision is made.

The difference between the objective function values for the January base case and the January 125-scenario cases for each of the scenario-dependent parameters under the assumption of perfect foresight is presented in Figure 4-4.
From Figure 4-4, note that the departure from the base case objective function is larger when the demands are the scenario-dependent parameters than it is when the inflows are the scenario-dependent parameters. With the exceptions of when the one reservoir begins 10% full and the other begins either 10% or 50% full (i.e., for the three poorest initial storage conditions), the departures when the prices are the scenario-dependent parameters are larger than for scenario-dependent inflows. When the Columbia reservoir begins 10% full, and when the Columbia begins 50% full while the Peace begins 10% or 50% full, the departures are larger for scenario-dependent demands than they are for scenario-dependent prices; the converse is true for the other initial storage conditions. So, the base case does the best job, overall, of representing the case of scenario-dependent inflows. When the initial storage conditions, particularly in the Columbia, are poor, the base case does the worst job of representing the case of scenario-dependent demands, and when the initial storage conditions are healthy does the worst job of representing the scenario-dependent prices. The largest magnitude differences from the base case occur when the demands are the scenario-dependent parameters, and the initial storage conditions are poor.

The difference between the objective function values for the January base case and the January 125-scenario cases for each of the scenario-dependent parameters under the assumption of non-anticipative turbine releases is presented in Figure 4-5.
For non-anticipative turbine releases in January, departures from the base case objective function for scenario-dependent demands exceed those for scenario-dependent inflows, which equal or exceed those for scenario-dependent prices. Thus, the base case does the best job of representing the objective function for scenario-dependent prices, and the worst job for scenario-dependent demands. As was true for the perfect foresight case, the largest magnitude departures from the base case occur for scenario-dependent demands when the Columbia begins 10% full, and when the Columbia begins 50% full and the Peace begins 10% full. Note that for scenario-dependent prices the objective function values are the same under the assumption of non-anticipative turbine releases as under the base case. The reason for this result is that in the 125-scenario cases, the five prices are symmetrically distributed with a mean equal to that for the base case, and the prices only appear in the objective function.

The differences between the objective function values for the January base case and January 125-scenario cases for each of the scenario-dependent parameters under the assumption of non-anticipative ending storage are presented in Figure 4-6.
Difference Between Objective Functions for January Base Case and Scenario Dependent with Non-Anticipative Ending Storage Volume Constraints Case

Figure 4-6: Difference Between Base Case and Scenario-Dependent Case Objective Function Values for Non-Anticipative Ending Storage Volume Constraints Assumption in January

With the exception of when the Columbia begins 90% full and the Peace begins 10% full, the departures from the base case objective function for scenario-dependent demands equal or exceed those for scenario-dependent inflows, which, exceed those for scenario-dependent prices. Thus, the base case does the best job of representing the scenario-dependent prices, and the worst job under scenario-dependent demands. As was true for the other two anticipative assumptions, the largest magnitude departures from the base case occur for scenario-dependent demands when the Columbia begins 10% full, and when the Columbia begins 50% full while the Peace begins 10% full. Again, note that for scenario-dependent prices the objective function values are the same under the assumption of non-anticipative ending storage volumes as under the base case.

May

In this section, the utility of employing the May base case to estimate the objective function is examined for the same three scenario-dependent parameters, three anticipative assumptions, and nine initial storage conditions as for January. The parameters for the May scenarios are defined in Table 4-7 through Table 4-12.

The first non-anticipative assumption considered is that of no non-anticipative constraints. The differences between the objective function values for the May base case and the May 125-scenario cases for each of the scenario-dependent parameters under the assumption of perfect foresight are presented in Figure 4-7.
With the exception of when both reservoirs begin 10% full, the departures from the base case objective function for the scenario-dependent prices equal or exceed those for the scenario-dependent inflows; for the same exception, as well as when the Peace begins 90% full while the Columbia begins either 50% or 90% full, the departures for scenario-dependent demands are also less than those for scenario-dependent prices. Depending on the initial conditions, the departures can be greater for scenario-dependent demands than they are for scenario-dependent inflows, and vice versa. Overall, the largest magnitude departures are for scenario-dependent prices. The single largest departure from the base case again occurs for demands as the scenario-dependent parameters, but, as opposed to January, occurs when both reservoirs begin 90% full. Note that the largest magnitude departure is much smaller for May than for January under the assumption of perfect foresight.

The differences between the objective function values for the May base case and the May 125-scenario cases for each of the scenario-dependent parameters under the assumption of non-anticipative turbine releases are presented in Figure 4-8.
For scenario-dependent demands, the departures from the base case objective function are larger than they are for scenario-dependent inflows, with the exception of when the Columbia begins 90% full and the Peace begins either 10% or 50% full. For both scenario-dependent inflows and demands, the departures from the base case objective function are greater than they are for scenario-dependent prices. Again, this is due to the sole appearance of the prices in the objective function, and the symmetrical distribution with a mean equal to that used in the base case. In general, the base case does the best job of representing the objective function for scenario-dependent prices, and the worst job for scenario-dependent demands. Note that the largest magnitude differences from the base case objective function values occur for scenario-dependent demands, with the largest difference occurring when both reservoirs begin 90% full.

The difference between the objective function values for the May base case and the May 125-scenario cases for each of the scenario-dependent parameters under the assumption of non-anticipative ending storage volumes is presented in Figure 4-9.

With the exception of when both reservoirs begin 90% full, the objective function differences are larger for the inflow as the scenario-dependent parameter than for demand. Thus, the base case is a better estimate of objective function values for scenario-dependent demands than for scenario-dependent inflows. Note that the largest magnitude difference occurs for the inflows as the scenario-dependent parameter when both reservoirs begin only 10% full; the next largest departure occurs for the same initial storage case, with demands as the scenario-dependent parameters.

Figure 4-8: Difference Between Base Case and Scenario-Dependent Case Objective Function Values for Non-Anticipative Turbine Release Constraints Assumption in May
When the demands are the scenario-dependent parameters, under the assumption of non-anticipative ending storage volumes, the objective function differences are smaller than under the assumption of non-anticipative turbine releases; the converse is true for inflows as the scenario-dependent parameters.

Figure 4-9: Difference Between Base Case and Scenario-Dependent Case Objective Function Values for Non-Anticipative Ending Storage Volume Constraints Assumption in May

**September**

In this section, the utility of employing the September base case to estimate the objective function is examined for the same three scenario-dependent parameters, three anticipative assumptions, and nine initial storage conditions as for January and May. The parameters for the September scenarios are defined in Table 4-13 through Table 4-18.

The first non-anticipative assumption considered is that of no non-anticipative constraints. The difference between the objective function values for the September base case and the September 125-scenario cases for each of the scenario-dependent parameters under the assumption of perfect foresight is presented in Figure 4-10.
In Figure 4-10 observe that, with exceptions when the Columbia begins 10% full while the Peace begins either 10% or 90% full, the differences from the base case objective function are larger for scenario-dependent demands than for scenario-dependent inflows. The base case is a better estimate of the objective function for both scenario-dependent demands and inflows than for scenario-dependent prices when both reservoirs begin at least 50% full; when the Columbia begins 10% full and the Peace begins 90% full, the base case is a better estimate when the demands are the scenario-dependent parameters than when the prices are the scenario-dependent parameters. The largest magnitude difference occurs when the Columbia begins 50% full and the Peace begins 10% full, with the next largest difference also being for demands as the scenario-dependent parameters with the Columbia 90% full and the Peace 10% full. The next two largest departures occur for the same two initial storage conditions with the inflows as the scenario-dependent parameters.

The difference between the objective function values for the September base case and the September 125-scenario cases for each of the scenario-dependent parameters under the assumption of non-anticipative turbine releases is presented in Figure 4-11.
Figure 4-11: Difference Between Base Case and Scenario-Dependent Case Objective Function Values for Non-Anticipative Turbine Release Constraints Assumption in September

For non-anticipative turbine releases, departures from the base case objective function are greater for scenario-dependent demands than they are for scenario-dependent inflows, which are greater than those for scenario-dependent prices. As noted before, with prices only appearing in the objective function, and with the symmetrical distribution, the base case yields the same results as for the case with prices as the scenario-dependent parameters. Note that the largest magnitude departures occur for the demands when the Columbia begins 10% full and the Peace begins 50% full, and when the Columbia begins either 50% or 90% full while the Peace begins 10% full; these three initial conditions also produce the largest departures with inflows as the scenario-dependent parameters.

The difference between the objective function values for the September base case and the September 125-scenario cases for each of the scenario-dependent parameters under the assumption of non-anticipative ending storage volumes is presented in Figure 4-12.
Depending upon the initial storage conditions, the departures from the base case objective function can be greater for scenario-dependent demands than for scenario-dependent inflows or vice versa; departures are greater for scenario-dependent demands when both reservoirs begin at least 50% full, and when the Columbia begins 10% full and the Peace begins 90% full. The departures for both scenario-dependent inflows and demands are greater than those for scenario-dependent prices, as explained for the non-anticipative turbine releases. As was true for the non-anticipative turbine releases, the largest magnitude departures for both scenario-dependent demands and scenario-dependent inflows occur for the demands when the Columbia begins 10% full and the Peace begins 50% full, and when the Columbia begins either 50% or 90% full and the Peace begins 10% full.

A comparison of Figure 4-4 through Figure 4-12 yields additional insights. With scenario-dependent demands, regardless of the assumption regarding future knowledge, the departures from the base case are, generally, greater for January than for September, which are greater than those for May. That is, for scenario-dependent demands, the base case provides the best estimate of objective function values in May and the worst estimate in January. With inflows as the scenario-dependent parameters, departures from the base case objective function are generally greater for September than for May, which are greater than those for January regardless of the assumption regarding knowledge of the future. That is, for scenario-dependent inflows, the base case provides the best estimate of objective function values in September and the worst estimate in January. For prices as the scenario-dependent parameters, with either non-anticipative turbine
releases or ending storage volumes, for all months the departure from the base case objective functions is zero; this result occurs because prices appear only in the objective function, and the assumed distribution is symmetrical.

For demands as the scenario-dependent parameters, differences from the base case objective function for non-anticipative turbine releases are greater than or equal to those for non-anticipative ending storage volumes, which are greater than those for no non-anticipative constraints. With inflows as the scenario-dependent parameters, differences from the base case objective function for non-anticipative ending storage volumes are greater than those for non-anticipative turbine releases, which are greater than those for no non-anticipative constraints. For scenario-dependent prices, the departures from the base case objective functions are greater for no non-anticipative constraints than for either of the non-anticipative constraints, for both of which the departures are zero.

4.4.3.2 Expected Value of Perfect Information

In the previous section, comparisons were made between a number of STMVM runs for different scenario-dependent parameters, anticipative assumptions, and initial storage values for the months of January, May, and September. The comparison yielded information on how the objective function would change with respect to using an average value if the one point estimate for the scenario-dependent parameter was replaced with a five-point estimate based upon a normal distribution.

In this section, a comparison is made for each of the three months, nine initial storage conditions, and three scenario-dependent parameters between the objective function for the no non-anticipative constraints (perfect foresight) case and both of the non-anticipative turbine release and non-anticipative ending storage volume cases. The difference in the objective function values gives the expected value of perfect information (EVPI) of knowing the values of the scenario-dependent parameters. In this section the EVPI for each case considered is presented. The cases are presented by the month studied.

January

In this section, the EVPI is examined for three scenario-dependent parameters (demand, inflow, and price), two non-anticipative variables (turbine release and ending storage), and nine initial storage conditions. The parameters for the January scenarios are defined in Table 4-1 through Table 4-6.

The EVPI for January under the assumption of non-anticipative turbine releases is presented as Figure 4-13.
In Figure 4-13 note that, without exception, the EVPI for demands as the scenario-dependent parameters exceeds that for inflows as the scenario-dependent parameters. With the exceptions of when the Columbia reservoir begins only 10% full and when the Columbia begins 50% full while the Peace begins 10% full, the EVPI for prices as the scenario-dependent parameters exceeds that for inflows as the scenario-dependent parameters. Similarly, except for when the Peace begins 90% full and the Columbia begins either 50% or 90% full, the EVPI for demands as the scenario-dependent parameters exceeds that for the prices as the scenario-dependent parameters. Note that of all the situations considered, the highest EVPI value occurs when the demands are the scenario-dependent parameters, and the Columbia starts 50% full while the Peace starts 10% full.

The EVPI for January under the assumption of non-anticipative ending storage volumes is presented as Figure 4-14.
Figure 4-14: EVPI Under Non-Anticipative Ending Storage Volume Assumption for January

As shown in Figure 4-14, except for when the Columbia reservoir begins 90% full and the Peace reservoir begins 10% full, the EVPI with demands as the scenario-dependent parameters exceeds that with inflows as the scenario-dependent parameters. With the exception of when one of the two reservoirs begins 90% full and the other reservoir begins either 50% or 90% full, the EVPI with inflows as the scenario-dependent parameters exceeds that with prices as the scenario-dependent parameters. The EVPI with demands as the scenario-dependent parameters exceeds that with prices as the scenario-dependent parameters, except for when the Peace begins 90% full while the Columbia begins either 50% or 90% full, and when the Columbia begins 90% full while the Peace begins 10% full. As for the non-anticipative turbine releases, of all the situations considered, the highest EVPI value occurs when demands are the scenario-dependent parameters and the Columbia starts 50% full while the Peace starts 10% full.

May

In this section, the EVPI is examined for the same scenario-dependent parameters, non-anticipative constraints, and initial storage conditions as for January. The parameters for the May scenarios are defined in Table 4-7 through Table 4-12. The EVPI for May under the assumption of non-anticipative turbine releases is presented as Figure 4-15.
Figure 4-15: EVPI Under Non-Anticipative Turbine Release Assumption for May

In Figure 4-15, except for when one reservoir begins 10% full while the other begins 90% full and when the Columbia begins 90% full while the Peace begins 50% full, the EVPI with demands as the scenario-dependent parameters exceeds that with inflows as the scenario-dependent parameters. With the exceptions of when both reservoirs begin only 10% full and when the Peace reservoir begins 90% full while the Columbia reservoir begins 50% or 90% full, the EVPI with prices as the scenario-dependent parameters exceeds that with inflows as the scenario-dependent parameters. The EVPI with demands as the scenario-dependent parameters exceeds that with prices as the scenario-dependent parameters, with the exceptions of when one reservoir begins 10% full while the other begins 90% full as well as when the Columbia begins 90% full and the Peace begins 50% full. As was true for January, of all the cases considered, the largest EVPI occurs when the demand is the scenario-dependent parameter; however, in May this value occurs when both reservoirs begin 90% full.

The EVPI for May under the assumption of non-anticipative ending storage volumes is presented as Figure 4-16.
Figure 4-16: EVPI Under Non-Anticipative Ending Storage Volume Assumption for May

In Figure 4-16, without exception, the EVPI with inflows as the scenario-dependent parameters exceeds that with demands as the scenario-dependent parameters, and the EVPI with inflows as the scenario-dependent parameters exceeds that with prices as the scenario-dependent parameters. With the exceptions of when the Columbia begins 10% full while the Peace begins 90% full, and when the Peace begins 50% full while the Columbia begins either 50% or 90% full, the EVPI with demands as the scenario-dependent parameters exceeds that with prices as the scenario-dependent parameters. In contrast to non-anticipative turbine releases, the largest EVPI occurs when the inflows are the scenario-dependent parameters, and when both reservoirs begin only 10% full. In addition, note the increased value of having perfect information of the inflows, and the decreased value of having perfect information on the demands, with the ending storage volumes as the non-anticipative variables.

September

In this section, the EVPI is examined for the same scenario-dependent demands, inflows, and prices, non-anticipative turbine releases and ending storage volumes, and initial storage conditions as for January and May. The parameters for the September scenarios are defined in Table 4-13 through Table 4-18.

The EVPI for September under the assumption of non-anticipative turbine releases is presented as Figure 4-17.
Figure 4-17: EVPI Under Non-Anticipative Turbine Release Assumption for September

In Figure 4-17, without exception, the EVPI with demands as the scenario-dependent parameters exceeds that with inflows as the scenario-dependent parameters. With the exception of when one reservoir begins 90% full while the other reservoir begins either 50% or 90% full, the EVPI with inflows as the scenario-dependent parameters exceeds that with prices as the scenario-dependent parameters. Also, without exception, the EVPI with demands as the scenario-dependent parameters exceeds that with prices as the scenario-dependent parameters. Note that, for the cases studied, the largest magnitude EVPI occurs when demands are the scenario-dependent parameters and when the Columbia begins 10% full and the Peace begins 50% full; the EVPI is also high when the Peace begins 10% full and the Columbia begins either 50% or 90% full. Overall, perfect knowledge of the demands is the most valuable. The value of perfect knowledge of the prices, as compared to the value of perfect knowledge of the inflows, becomes greater for the three highest initial storage conditions.

The EVPI for September under the assumption of non-anticipative ending storage volumes is presented as Figure 4-18.
Figure 4-18: EVPI Under Non-Anticipative Ending Storage Volume Assumption for September

Figure 4-18 indicates that, with the exception of when the Peace reservoir begins 50% full, the EVPI with inflows as the scenario-dependent parameters exceeds that with demands as the scenario-dependent parameters. The EVPI with inflows as the scenario-dependent parameters exceeds that with prices as the scenario-dependent parameters except for when the Columbia reservoir begins 50% full while the Peace reservoir begins either 50% or 90% full. With the exception of when the Peace reservoir begins 90% full while the Columbia reservoir begins 10% full, the EVPI with demands as the scenario-dependent parameters exceeds that with prices as the scenario-dependent parameters. Although the EVPI is large for some initial storage conditions with inflows as the scenario-dependent parameters, the largest EVPI again occurs for demand as the scenario-dependent parameter, with initial storage of 10% in the Columbia and 50% in the Peace.

A comparison of the figures presenting the EVPI yields additional information. When the demands are the scenario-dependent parameters, for all three months examined, the EVPI with non-anticipative turbine releases equals or exceeds that with non-anticipative ending storage volumes. Conversely, when the inflows are the scenario-dependent parameters, for all three months, the EVPI with non-anticipative ending storage volumes exceeds that with non-anticipative turbine releases. Further, when prices are the scenario-dependent parameters, the EVPI with non-anticipative turbine releases equals that with non-anticipative ending storage volumes.

With the demands as the scenario-dependent parameters, and the turbine releases as the non-anticipative variables, with the exception of when both reservoirs begins only 10%
full, and when the Peace begins 90% full while the Columbia begins either 50% or 90% full, the EVPI for January is greater than that for May; with these same exceptions, the EVPI for September is greater than that for May; and with the exceptions of when the Columbia reservoir begins 10% full while the Peace begins either 10% or 90% full as well as when the Columbia begins 50% full while the Peace begins either 10% or 50% full the EVPI for September exceeds that for January. With either one or two fewer initial storage condition exceptions, these relationships also hold true for the ending storage volume as the non-anticipative variable. Thus, in general, with scenario-dependent demands, for a given initial storage condition, the EVPI for September is greater than that for January, which is greater than that for May.

With the inflows as the scenario-dependent parameters, and the turbine releases as the non-anticipative variables, with the exception of when the Columbia reservoir begins 10% full while the Peace begins either 50% or 90% full and when the Columbia begins 50% full while the Peace begins 10% full, the EVPI for May exceeds that for January; with the exception of when both reservoirs begin 10% full, and when one reservoir begins 90% full while the other begins either 50% or 90% full, the EVPI for September exceeds that for May; and when the Peace begins 10% full while the Columbia begins either 10% or 50% full, the EVPI for September exceeds that for January. With some minor differences in the exceptions, these relationships also prove true for the ending storage volume as the non-anticipative variable. Thus, in general, with scenario-dependent inflows, for a given initial storage condition, the EVPI for September is greater than that for May, which is greater than that for January.

With the prices as the scenario-dependent parameters, and the turbine releases as the non-anticipative variables, with the exception of when the Columbia begins 50% full while the Peace begins 90% full, and when the Columbia begins 90% full while the Peace begins either 10% or 90% full, the EVPI for May is greater than that for January; the EVPI for May is greater than that for September except for when the Peace begins 90% full while the Columbia begins either 50% or 90% full; with the exception of when the Columbia begins 50% full while the Peace begins 90% full, the EVPI for September exceeds that for January. These relationships also hold true with the ending storage volumes as the non-anticipative variables. Thus, in general, with scenario-dependent prices, for a given initial storage condition, the EVPI for May is greater than that for September, which is greater than that for January.

In this and the previous sections, the objective function values and the associated expected value of perfect information have been examined. The following sections consider the Peace marginal energy values.

### 4.4.4 Peace Marginal Energy Values

Some of the most important information to be produced by the STMVM are the marginal energy values for the Peace and Columbia reservoirs. These marginal values serve as price signals to indicate from which reservoir generation should preferentially come, and
to coordinate import and export opportunities. Marginal energy values for the Peace are presented in this section for the same months, scenario-dependent parameters, non-anticipative variables, and initial storage conditions as were presented for the objective function and the expected value of perfect information.

The Peace marginal energy values for the respective one-scenario base cases for January, May, and September are presented in Figure 4-19.

In Figure 4-19, for each of the initial storage conditions, except for a minor exception when the Columbia begins 50% full while the Peace begins 90% full, the Peace marginal energy value in September is greater than that for the same initial storage condition in January. Further, for each of the initial storage conditions, the Peace marginal energy value in January is greater than that for the same initial storage condition in May. So, for a given storage condition, it is more valuable to have water in storage in the Peace reservoir leading into the winter than it is in mid-winter, which in turn is more valuable than having water in storage near the beginning of the freshet.

4.4.4.1 Comparison of Peace Marginal Energy Value Against Base Cases

In this section, the Peace marginal energy values for the set of problems solved are compared to the Peace marginal energy value for the one-scenario base case for the relevant month. This comparison details how well the simpler one-scenario case approximates the value of energy in the Peace reservoir for the more complex 125-
scenario case. The comparison is made by subtracting the marginal energy value for the 125-scenario case from the base case; the smaller the magnitude of this difference, the better the base case approximates the 125-scenario case.

**January**

In this section, the utility of employing the January base case to estimate the Peace marginal energy value is examined for three scenario-dependent parameters (demand, inflow, and price), three constraints regarding how to model the anticipative nature of the problem (no non-anticipative constraints, constraints for non-anticipative turbine releases, and constraints for non-anticipative ending storage volumes), and nine initial storage conditions. The parameters for the January scenarios are defined in Table 4-1 through Table 4-6.

The first non-anticipative assumption considered is that of no non-anticipative constraints, meaning that perfect knowledge of the scenario-dependent parameters is assumed when each decision is made. The difference between the Peace marginal energy values for the January base case and the January 125-scenario cases for each of the scenario-dependent parameters under the assumption of perfect foresight is presented in Figure 4-20.

![Figure 4-20: Difference in Peace Marginal Energy Values Between Base Case and Scenario-Dependent Case for No Non-Anticipative Constraints Assumption in January](image)

From Figure 4-20 note that the difference in marginal energy values for the demand as the scenario-dependent parameter equals or exceeds that for the inflow as the scenario-
dependent parameter for all initial storage conditions. With the exception of when both reservoirs begin only 10% full, the difference in Peace marginal energy values for the price as the scenario-dependent parameter is greater than or equal to that when the inflows are the scenario-dependent parameters. With exceptions when the Columbia reservoir begins 10% full while the Peace begins either 10% or 50% full, and when the Columbia reservoir begins 50% full while the Peace begins 10% full, the difference in Peace marginal energy values with the price as the scenario-dependent parameter exceeds that for the demands as the scenario-dependent parameters. The exception when the Columbia reservoir begins 50% full and the Peace reservoir begins 10% full is the largest magnitude departure from the base case for any of the situations considered; the other two exceptions are the next two largest departures.

Thus, with the noted exceptions, the base case provides, over all of the scenarios, the worst estimate when prices are the scenario-dependent parameters, and the best estimate when inflows are the scenario-dependent parameters. Further, note that the base case does a very poor job of estimating the Peace marginal energy value for the demand as the scenario-dependent parameter when one reservoir begins only 10% full and the other begins either 10% or 50% full. The base case does the worst job of estimating the Peace marginal energy value with inflows as the scenario-dependent parameters when both reservoirs begin only 10% full. Conversely, the largest differences for prices as the scenario-dependent parameters occur when both reservoirs are relatively healthy in terms of storage.

The difference between the Peace marginal energy values for the January base case and the January 125-scenario cases for each of the scenario-dependent parameters under the assumption of non-anticipative turbine releases is presented in Figure 4-21.
Figure 4-21: Difference in Peace Marginal Energy Values Between Base Case and Scenario-Dependent Case for Non-Anticipative Turbine Release Constraints Assumption in January

In Figure 4-21 observe that with turbine releases as the non-anticipative variables the differences in Peace marginal energy values, for each initial condition, are greatest with the demand as the scenario-dependent parameter, and are equal with either the inflow or the price as the scenario-dependent parameter. So, the base case does the worst job of estimating the Peace marginal energy values when the demand is the scenario-dependent parameter. Again, note that when one reservoir begins only 10% full and the other reservoir begins only 10% or 50% full, the base case does a particularly poor job of estimating the Peace marginal energy value with the demand as the scenario-dependent parameter. When the storage in the system is relatively healthy, the base case can provide an adequate estimate of the Peace marginal energy value for any of the three scenario-dependent parameters.

The differences between the Peace marginal energy values for the January base case and the January 125-scenario cases for each of the scenario-dependent parameters under the assumption of non-anticipative ending storage volumes are presented in Figure 4-22.
Figure 4-22: Difference in Peace Marginal Energy Values Between Base Case and Scenario-Dependent Case for Non-Anticipative Ending Storage Volume Constraints Assumption in January

Note that, with minor exceptions when the Columbia reservoir begins 90% full and the Peace reservoir begins either 10% or 90% full, the difference in Peace marginal energy value is greater with the demand as the scenario-dependent parameter than it is for inflow as the scenario-dependent parameter. The difference in Peace marginal energy value with the price as the scenario-dependent parameter is either equaled or exceeded by that when the inflows or the prices are the scenario-dependent parameters. Again, note that when one reservoir begins only 10% full and the other begins only 10% or 50% full, the base case does a particularly poor job of estimating the Peace marginal energy value with the demand as the scenario-dependent parameter, with the largest magnitude departure occurring when the Columbia begins 50% full while the Peace begins 10% full.

May

In this section, the utility of employing the May base case to estimate the marginal Peace energy value is examined for three scenario-dependent parameters, three constraints regarding how to model the anticipative nature of the problem, and nine initial storage conditions. The parameters for the May scenarios are defined in Table 4-7 through Table 4-12.

The first non-anticipative assumption considered is that of no non-anticipative constraints, meaning that perfect knowledge of the scenario-dependent parameters is assumed when each decision is made.
The difference between the Peace marginal energy values for the May base case and the May 125-scenario cases for each of the scenario-dependent parameters under the assumption of perfect foresight is presented in Figure 4-23.

Figure 4-23: Difference in Peace Marginal Energy Values Between Base Case and Scenario-Dependent Case for No Non-Anticipative Constraints Assumption in May

Note that the largest difference between the base case and the Peace marginal energy values under any of the scenario-dependent parameters and initial storage conditions presented in Figure 4-23 is much smaller than the corresponding values for January as shown in Figure 4-20.

With minor exceptions when the Columbia begins 10% full while the Peace begins 50% full, and when the Columbia begins 50% full while the Peace begins 90% full, the difference in Peace marginal energy value when the inflow is the scenario-dependent parameter is greater than or equal to that with the demand as the scenario-dependent parameter. With an exception when both reservoirs begin only 10% full, the difference in Peace marginal energy value when the price is the scenario-dependent parameter equals or exceeds that when either the inflow or demand is the scenario-dependent parameter. So, in May with no non-anticipative constraints the base case does the worst job of estimating the Peace marginal energy value when the price is the scenario-dependent parameter, and the best job when the demand is the scenario-dependent parameter. The largest magnitude departure from the base case Peace marginal energy value occurs when both reservoirs begin 10% full with inflows as the scenario-dependent parameters; the next two largest departures occur for scenario-dependent prices, with the Columbia beginning 50% full while the Peace begins either 50% or 90% full.
The difference between the Peace marginal energy values for the May base case and the May 125-scenario cases for each of the scenario-dependent parameters under the assumption of non-anticipative turbine releases is presented in Figure 4-24.

![Difference Between Marginal Peace Energy Values for May Base Case and Scenario Dependent with Non-Anticipative Turbine Release Constraints Case](image)

Figure 4-24: Difference in Peace Marginal Energy Values Between Base Case and Scenario-Dependent Case for Non-Anticipative Turbine Release Constraints Assumption in May

With exceptions when both reservoirs begin 10% full and when the Columbia begins 10% full while the Columbia begins 90% full, the difference in Peace marginal energy value with demands as the scenario-dependent parameters exceed those when the inflows are the scenario-dependent parameters. When the scenario-dependent parameter is the price, the difference in Peace marginal energy values is exceeded when either the inflows or the demands are the scenario-dependent parameters. Thus, the base case does the best job of estimating the Peace marginal energy value when the price is the scenario-dependent parameter (for the assumed symmetrical distribution) and the worst job when the demand is the scenario-dependent parameter. The largest magnitude departure from the base case Peace marginal energy value occurs when both reservoirs begin 10% full and the inflows are the scenario-dependent parameters; the next largest departure is for scenario-dependent demands when both reservoirs begin 50% full.
The difference between the Peace marginal energy values for the May base case and the May 125-scenario cases for each of the scenario-dependent parameters under the assumption of non-anticipative ending storage volumes is presented in Figure 4-25.

Figure 4-25: Difference in Peace Marginal Energy Values Between Base Case and Scenario-Dependent Case for Non-Anticipative Ending Storage Volume Constraints Assumption in May

In Figure 4-25 note that with exceptions when the Columbia begins 50% full while the Peace begins either 10% or 90% full, the difference in Peace marginal energy value is greater or equal when the inflows, as compared to the demands, are the scenario-dependent parameters. When either the inflows or the demands are the scenario-dependent parameters, the difference in Peace marginal energy values exceeds that for when the prices are the scenario-dependent parameters. So, the base case does the worst job of estimating the Peace marginal energy values when the inflows are the scenario-dependent parameters, and the best job when the prices are the scenario-dependent parameters. Note that the largest departure from the base case Peace marginal energy value occurs when both reservoirs begin 50% full and the inflows are the scenario-dependent parameters; the next largest departure occurs for the same initial storage conditions when the demands are the scenario-dependent parameters.

September

In this section, the utility of employing the September base case to estimate the marginal Peace energy value is examined for three scenario-dependent parameters, three
constraints regarding how to model the anticipative nature of the problem, and nine initial storage conditions. The parameters for the September scenarios are defined in Table 4-13 through Table 4-18.

The first non-anticipative assumption considered is that of no non-anticipative constraints, meaning that perfect knowledge of the scenario-dependent parameters is assumed when each decision is made.

The difference between the Peace marginal energy values for the September base case and the September 125-scenario cases for each of the scenario-dependent parameters under the assumption of perfect foresight is presented in Figure 4-26.

![Difference Between Marginal Peace Energy Values for September Base Case and Scenario Dependent with No Non-Anticipative Constraints Case](image)

**Figure 4-26: Difference in Peace Marginal Energy Values Between Base Case and Scenario-Dependent Case for No Non-Anticipative Constraints Assumption in September**

With an exception when both reservoirs begin only 10% full, the differences in the Peace marginal energy values when the demands are the scenario-dependent parameters are equal to or greater than those when the inflows are the scenario-dependent parameters. The differences in the Peace marginal energy values when the prices are the scenario-dependent parameters are less than or equal to those when either the inflows or the demands are the scenario-dependent parameters, except for the three highest initial storage conditions. The base case does the best job of estimating the Peace marginal energy values when the prices are the scenario-dependent parameters, and the worst job when the demands are the scenario-dependent parameters. The largest magnitude departure from the base case Peace marginal energy value occurs when the Columbia begins 50% full and the Peace begins 10% full while the demands are the scenario-
dependent parameters. Note that for the three highest storage conditions, the base case does a good job of estimating the Peace marginal energy value for all three of the scenario-dependent parameters.

The differences between the Peace marginal energy values for the September base case and the September 125-scenario cases for each of the scenario-dependent parameters under the assumption of non-anticipative turbine releases are presented in Figure 4-27.

![Figure 4-27: Difference in Peace Marginal Energy Values Between Base Case and Scenario-Dependent Case for Non-Anticipative Turbine Release Constraints Assumption in September](image)

In Figure 4-27, with a minor exception when both reservoirs begin only 10% full, the differences in the Peace marginal energy values when the demands are the scenario-dependent parameters exceed those when the inflows are the scenario-dependent parameters. When the scenario-dependent parameter is the price, the difference in Peace marginal energy value is smaller than when either inflow or demand is the scenario-dependent parameter. The base case does the worst job of estimating the Peace marginal energy value when the demands are the scenario-dependent parameters, and the best job when the prices are the scenario-dependent parameters, as previously explained. The largest magnitude departure from the base case Peace marginal energy value occurs when the demands are the scenario-dependent parameters, and the Columbia begins 50% full while the Peace begins 10% full.

The difference between the Peace marginal energy values for the September base case and the September 125-scenario cases for each of the scenario-dependent parameters
under the assumption of non-anticipative ending storage volumes is presented in Figure 4-28.

**Figure 4-28: Difference in Peace Marginal Energy Values Between Base Case and Scenario-Dependent Case for Non-Anticipative Ending Storage Volume Constraints Assumption in September**

With exceptions when the Columbia reservoir begins only 10% full, the differences in Peace marginal energy values when the inflows are the scenario-dependent parameters are greater than or equal to those when the demands are the scenario-dependent parameters. When either the inflows or the demands are the scenario-dependent parameters, the differences in Peace marginal energy values are greater than or equal to those when the prices are the scenario-dependent parameters. The base case does the worst job of estimating the Peace marginal energy values when the inflows are the scenario-dependent parameters, and the best job when the prices are the scenario-dependent parameters. As was true for the other two assumptions regarding the anticipative constraints in September, the largest departure from the base case Peace marginal energy value occurs for demands as the scenario-dependent parameters, and with the Columbia beginning 50% full while the Peace begins 10% full.

From a comparison of Figure 4-20 through Figure 4-28 a number of observations can be made. For all three months when the demands are the scenario-dependent parameters, with minor exceptions at poor initial storage conditions, the differences in Peace marginal energy values for both non-anticipative variables are greater than those for no non-anticipative constraints. For non-anticipative turbine releases, the differences are greater than or equal to those with ending storage volumes as the non-anticipative variables. Thus, for the case of demands as the only scenario-dependent parameters, the base case
does the best job of approximating the Peace marginal energy values with perfect foresight, and the worst job with non-anticipative turbine releases.

With inflows as the scenario-dependent parameters, for January and September, with minor exceptions, the non-anticipative constraint cases have differences greater than those for perfect foresight, and the differences are greater under non-anticipative volumes than they are under non-anticipative turbine releases. Thus for these two months, the base case does the best job of approximating the Peace marginal energy values with perfect foresight, and the worst job with non-anticipative ending storage volumes. During May the relationships are not as clear.

When the prices are the scenario-dependent parameters, for all three months, the differences in marginal Peace energy value for perfect foresight are greater than or equal to those with either of the two non-anticipative variables, and the differences for the two non-anticipative variables are approximately equal. Thus, for prices as the scenario-dependent parameters, the base case does the best job of estimating the Peace marginal energy values for turbine releases and ending storage volumes as the non-anticipative variables, and the worst job when there is perfect foresight. Again, the reason for this is that the prices have a symmetrical distribution with a mean equal to that for the base case, and appear solely in the objective function. For all three months, when the Columbia starts off low, there is little to choose from in terms of differences between the three anticipative assumptions.

4.4.4.2 Comparison of Peace Marginal Energy Value Against Perfect Foresight Case

In the previous section, comparisons were made between a number of runs for different scenario-dependent parameters, non-anticipative assumptions, and initial storage volumes for the months of January, May, and September. The comparison yielded information on how the marginal Peace energy values change with respect to using an average value if the one point estimate for the scenario-dependent parameter is replaced with a five-point estimate based on a normal distribution.

In this section, a comparison is made for each of the three months, initial storage volumes, and scenario-dependent parameters between the Peace marginal energy value for the perfect foresight case and both the non-anticipative turbine release case and the non-anticipative ending storage volume case. The difference in the Peace marginal energy values gives the expected difference in the marginal Peace energy value with perfect information. The cases are presented by the month studied.

January

In this section, the difference in the marginal Peace energy value with perfect information is examined for three scenario-dependent parameters, two non-anticipative variables, and
nine initial storage conditions. The parameters for the January scenarios are defined in Table 4-1 through Table 4-6.

The difference in the Peace marginal energy value with perfect information for January under the assumption of non-anticipative turbine releases is presented as Figure 4-29.

![Figure 4-29: Difference in Peace Marginal Energy Value Under Non-Anticipative Turbine Release Assumption for January](image)

From Figure 4-29 observe that the difference in Peace marginal energy values with the demands as the scenario-dependent parameters are greater than or equal to those with inflows or prices as the scenario-dependent parameters, with minor exceptions when the initial storage conditions are healthy. With an exception when both reservoirs begin only 10% full, the difference in Peace marginal energy values with inflows as the scenario-dependent parameters are less than those with the prices as the scenario-dependent parameters. Thus, in order to accurately estimate the Peace marginal energy value, perfect knowledge of the demands is more valuable than perfect knowledge of either the inflows or prices, particularly for the cases when one reservoir begins 10% or 50% full while the second reservoir begins 50% full. Depending upon the initial storage conditions, perfect knowledge of either the inflows or prices can be of greater relative value in accurately estimating the Peace marginal energy value. The largest differences from the Peace marginal energy value under the assumption of perfect foresight occur for demands as the scenario-dependent parameters for the three lowest initial storage conditions.

The differences in the Peace marginal energy value with perfect information for January under the assumption of non-anticipative ending storage volumes are presented as Figure 4-30.
The magnitude of the departure of the marginal Peace energy value from that for the perfect knowledge case is larger for the demands as the non-anticipative constraints than it is for the inflows or prices as the non-anticipative constraints; this is particularly so when the reservoirs begin with the three worst initial storage conditions. Exceptions occur for the prices as scenario-dependent parameters when the Columbia begins 90% full, and for the inflows when both reservoirs begin 90% full. For the poorer initial storage conditions, the difference from the Peace marginal energy value with perfect foresight is larger for the inflows than it is for the prices. The prices cause more of a departure with more robust initial storage, where the magnitude of the differences is small. So, overall, perfect knowledge of the demands is of more value in calculating the Peace marginal energy values than perfect knowledge of either the inflows or prices. At lower storage conditions, perfect knowledge of the inflows is of more importance in calculating the Peace marginal energy values than perfect knowledge of the prices.

**May**

In this section, the difference in the expected Peace marginal energy value with perfect information is examined for three scenario-dependent parameters, two non-anticipative variables, and nine initial storage conditions. The parameters for the May scenarios are defined in Table 4-7 through Table 4-12.

The difference in the Peace marginal energy value with perfect information for May under the assumption of non-anticipative turbine releases is presented as Figure 4-31.
Difference Between Marginal Peace Energy Values for May Perfect Knowledge Case and Scenario Dependent with Non-Anticipative Turbine Release Constraints Case

\[ \text{Difference} = 0.20 \]

\[ \text{or} \quad 0 > 0.20 \]

\[ \text{between 0 and 0.40} \]

\[ \text{between -0.40 and -0.60} \]

\[ \text{between 0.20 and 0.40} \]

\[ \text{between -0.20 and -0.40} \]

\[ \text{between 0.40 and 0.60} \]

\[ \text{between -0.60 and -0.80} \]

\[ \text{between 0.60 and 0.80} \]

\[ \text{between -0.80 and -1.00} \]

### Figure 4-31: Difference in Peace Marginal Energy Value Under Non-Anticipative Turbine Release Assumption for May

With the exception of when both reservoirs begin only 10% full, the difference from the Peace marginal energy value under perfect knowledge with the demands as the scenario-dependent parameters are greater than or equal to those with the inflows as the scenario-dependent parameters. For the prices as the scenario-dependent parameters, the differences are greater than or equal to those for either inflows or demands; exceptions for scenario-dependent demands occur when the Peace begins 10% full while the Columbia begins either 10% or 50% full, and when both reservoirs begin 90% full, and exceptions for scenario-dependent inflows occur when both reservoirs begin 10% full. The largest departure from the perfect foresight Peace marginal energy value occurs for scenario-dependent inflows when the Columbia begins 90% full and the Peace begins 50% full. So, for May with non-anticipative turbine releases, perfect knowledge of the prices is generally the most important, and perfect knowledge of the inflows is least important, in terms of calculating the Peace marginal energy value. Note the difference in magnitude as compared to the differences in Peace marginal energy value for January.

The difference in the Peace marginal energy value with perfect information for May under the assumption of non-anticipative ending storage volumes is presented as Figure 4-32.
For May, under the assumption of non-anticipative ending storage volumes, the departures from the Peace marginal energy value as calculated with perfect knowledge for the inflows as the scenario-dependent parameters are, with an exception when both reservoirs begin 50% full, greater than or equal to those for the demands as the scenario-dependent parameters. With the prices as the scenario-dependent parameters the differences are larger than or equal to those for the demands, with exceptions when both reservoirs begin either both 10% full, or with the Columbia 50% full and the Peace 90% full. Overall, the differences with the prices as the scenario-dependent parameters are greater than or equal to those for the inflows, with the same two exceptions as well as when the Columbia begins 10% full while the Peace begins 50% full. The largest magnitude difference from the perfect foresight marginal Peace energy value occurs for scenario-dependent inflows when both reservoirs begin only 10% full. For May, perfect knowledge of the demands is of least importance for calculating the Peace marginal energy values, and the prices are, overall, of slightly more importance than the inflows.

**September**

In this section, the difference in the marginal Peace energy value with perfect information is examined for three scenario-dependent parameters, two non-anticipative variables, and nine initial storage conditions. The parameters for the September scenarios are defined in Table 4-13 through Table 4-18.
The difference in the Peace marginal energy value with perfect information for September under the assumption of non-anticipative turbine releases is presented as Figure 4-33.

![Difference Between Marginal Peace Energy Values for September Perfect Knowledge Case and Scenario Dependent with Non-Anticipative Turbine Release Constraints Case](image)

**Figure 4-33: Difference in Peace Marginal Energy Value Under Non-Anticipative Turbine Release Assumption for September**

With minor exceptions, the departures from the Peace marginal energy values as calculated with perfect knowledge when the demands are the scenario-dependent parameters are greater than or equal to those when the inflows or prices are the scenario-dependent parameters. Exceptions for the inflows occur when the Columbia begins 10% full while the Peace begins either 10% or 90% full, and when both reservoirs begin 50% full. Exceptions for the prices occur when both reservoirs begin 50% full. The differences when the inflows are the scenario-dependent parameters are greater than or equal to those for the prices, with exceptions when the Columbia begins 90% full while the Peace begins either 50% or 90% full. Also, note that for the three healthiest initial storage conditions, the differences are fairly minor. The largest magnitude departure from the perfect foresight case occurs for scenario-dependent demands when the Columbia begins 50% full while the Peace begins 10% full. So, for September, under the assumption of non-anticipative turbine releases, knowledge of the demands is generally of the most value for estimating the Peace marginal energy values, while knowledge of the prices is of the least value for this purpose.

The difference in the Peace marginal energy value with perfect information for September under the assumption of non-anticipative ending storage volumes is presented as Figure 4-34.
Figure 4-34: Difference in Peace Marginal Energy Value Under Non-Anticipative Ending Storage Volume Assumption for September

With minor exceptions, the differences from the marginal Peace energy value under perfect foresight when the demands are the scenario-dependent parameters are greater than or equal to those when the inflows or the prices are the scenario-dependent parameters. Exceptions for the inflows occur when both reservoirs begin 10% full and when the Columbia begins 10% full while the Peace begins 90% full, and exceptions for prices occur when the Columbia begins 10% full while the Peace begins 90% full, and when the Columbia begins 90% full while the Peace begins 50% full. Except for a minor exception when the Columbia begins 90% full and the Peace begins 50% full, the differences when the inflows are the scenario-dependent parameters are greater than those when the prices are the scenario-dependent parameters. The largest magnitude difference from the perfect foresight Peace marginal energy value occurs for scenario-dependent demands when the Columbia begins 50% full while the Peace begins 10% full. Thus, for September, under the assumption of non-anticipative ending storage, perfect knowledge of the demands is of the greatest value in calculating the Peace marginal energy values, while perfect knowledge of the prices is of the least value.

Several observations can be made through a comparison of Figure 4-29 through Figure 4-34. When demands are the scenario-dependent parameters, for both non-anticipative variables, perfect knowledge of the demands is of more value, overall, in calculating the Peace marginal energy values in January than in May or September, and in September than in May.
When the inflows are the scenario-dependent parameters, for both non-anticipative variables, perfect knowledge of the inflows is of more value, overall, in calculating the Peace marginal energy values in September than in January. For the non-anticipative turbine releases, perfect knowledge of the inflows is of more value, overall, in calculating the Peace marginal energy values in May than in January or September, while for the non-anticipative ending storage volumes the converse is true.

For the prices as the scenario-dependent parameters, perfect knowledge of the prices is of more value, overall, in calculating the Peace marginal energy values in May than in either September or January, and in September than in January.

For all three months, perfect knowledge of the demands is of equal or greater value in calculating the Peace marginal energy value when the turbine releases are the non-anticipative variables than for non-anticipative ending storage volumes. For January and September, perfect knowledge of the inflows is of equal or greater value in calculating the Peace marginal energy value when the ending storage volumes are the non-anticipative variables than for non-anticipative turbine releases; the converse is true in May. As for the demand, for all three months, perfect knowledge of the prices is of greater value in calculating the Peace marginal energy value when the turbine releases, as opposed to the ending storage volumes, are the non-anticipative variables.

### 4.4.5 Columbia Marginal Energy Values

Some of the most important information to be produced by the model are the marginal energy values for the Peace and Columbia reservoirs. These marginal values serve as price signals to indicate from which reservoir generation should preferentially come, and to coordinate import and export opportunities. Marginal energy values for the Columbia reservoir are presented in this section for the same months, scenario-dependent parameters, non-anticipative variables, and initial storage conditions as were presented for the Peace marginal energy values in section 4.4.4.

The Columbia marginal energy values for the respective one-scenario base cases for January, May, and September are presented in Figure 4-35.

In Figure 4-35, for each of the initial storage conditions, except for a minor exception when the Columbia begins 10% full while the Peace begins 90% full, the marginal Columbia energy value in September is greater than that for the same initial storage condition in January. Further, for each of the initial storage conditions, the Columbia marginal energy value in January is greater than the Columbia marginal energy value for the same initial storage condition in May. So, for a given storage condition, it is more valuable to have water in storage in the Columbia reservoir leading into the winter than it is in mid-winter, which in turn is more valuable than having water in storage near the start of the freshet. Apart from the difference in the exception, the same results were found for the Peace.
A comparison of the Columbia marginal energy values in Figure 4-35 and those for the Peace marginal energy values in Figure 4-19 reveals that in January when the Columbia reservoir is either 10% or 50% full, the Peace marginal energy value is lower than that for the Columbia, indicating that generation should be preferentially from the Peace. When the Columbia reservoir is 90% full, generation preferentially comes from the Columbia if the Peace is either 10% or 50% full, and from the Peace if the Peace is also 90% full.

In May, when the Columbia reservoir is 10%, or is 50% full while the Peace is 90% full, the Peace marginal energy value is lower than that for the Columbia, indicating that generation should be preferentially from the Peace. When the Peace reservoir is 10% or 50% full while the Columbia is at least 50% full, generation preferentially comes from the Columbia. If both reservoirs are 90% full, the marginal energy values for the two reservoirs are equal, meaning that generation from either reservoir is equally preferred.

In September, when the Peace reservoir is 10% full as well as when the Peace is 50% full while the Columbia is 90% full, generation should preferentially come from the Columbia. For the remaining initial storage conditions generation is preferentially from the Peace.

Thus, when the Columbia begins 10% full while the Peace begins either 50% or 90% full, and when the Columbia begins 50% full while the Peace begins 90% full, generation is preferentially from the Peace. When the Columbia begins 90% full while the Peace begins either 10% or 50% full, generation is preferentially from the Columbia. When both reservoirs begin 10% full or 90% full, as well as when one reservoir begins 10% full and the other begins 50% full, the generation preference depends upon the month.
4.4.5.1 Comparison of Columbia Marginal Energy Value Against Base Cases

In this section, the Columbia marginal energy values for the set of problems solved are compared against the Columbia marginal energy value for the one-scenario base case for the relevant month. This comparison shows how well the simpler one-scenario base case approximates the marginal value of energy in the Columbia reservoir calculated for the more complex 125-scenario case. The comparison is made by subtracting the marginal energy value for the 125-scenario case from that for the base case; the smaller the magnitude of this difference, the better the base case approximates the 125-scenario case.

January

In this section, the utility of employing the January base case to estimate the Columbia marginal energy value is examined for three scenario-dependent parameters (demand, inflow, and price), three constraints regarding how to model the anticipative nature of the problem (no non-anticipative constraints, constraints for non-anticipative turbine releases, and constraints for non-anticipative ending storage volumes), and nine initial storage conditions. The parameters for the January scenarios are defined in Table 4-1 through Table 4-6.

The first non-anticipative assumption considered is that of no non-anticipative constraints, meaning that perfect knowledge of the scenario-dependent parameters is assumed in making each decision. The differences between the Columbia marginal energy values for the January base case and the January 125-scenario cases for each of the scenario-dependent parameters under the assumption of perfect foresight are presented in Figure 4-36.
Figure 4-36: Difference in Columbia Marginal Energy Values Between Base Case and Scenario-Dependent Case for No Non-Anticipative Constraints Assumption in January

As shown in Figure 4-36, the departures for the Columbia marginal energy values from the base case for demands as the scenario-dependent parameters are greater than or equal to those for inflows as the scenario-dependent parameters. The largest difference from the Columbia marginal energy values for the base case occurs for scenario-dependent demands when the Columbia begins 50% full while the Peace begins 10% full. When the Columbia reservoir begins 90% full, or 50% full while the Peace reservoir begins either 50% or 90% full, the departures from the base case are greater for prices as the scenario-dependent parameters than for either the demands or inflows as the scenario-dependent parameters. When the Columbia reservoir begins 10% full, and when the Columbia reservoir begins 50% full while the Peace reservoir begins 10% full, the departures when the inflows or demands are the scenario-dependent parameters are greater than those when the prices are the scenario-dependent parameters. Thus, the base case provides a better estimate of the Columbia marginal energy values when the inflows are the scenario-dependent parameters, as opposed to the demands. For high initial Columbia storage, the base case gives a worse estimate for prices as the scenario-dependent parameters than for either the inflows or demands; the converse is true for poor initial storage conditions. These results are very similar to those found for the Peace marginal energy values.
The difference between the marginal Columbia energy values for the January base case and the January 125-scenario cases for each of the scenario-dependent parameters under the assumption of non-anticipative turbine releases is presented in Figure 4-37.

![Figure 4-37: Difference in Columbia Marginal Energy Values Between Base Case and Scenario-Dependent Case for Non-Anticipative Turbine Release Constraints Assumption in January](image)

From Figure 4-37, it is clear that the departure from the Columbia marginal energy values under the base case when the demand is the scenario-dependent parameter equal or exceed the differences when the inflows are the scenario-dependent parameters, which in turn exceed those for when the prices are the scenario-dependent parameters. The largest difference from the Columbia marginal energy values for the base case occurs for scenario-dependent demands when the Columbia begins 50% full and the Peace begins 10% full. Thus, the base case provides the best estimate of the Columbia marginal energy values when prices are the scenario-dependent parameters, and the worst estimate when the demands are the scenario-dependent parameters. Very similar results were found for the Peace marginal energy values. Again, recall that the base case perfectly represents the case with prices as the scenario-dependent parameters due to the symmetrical price distribution with mean equal to that in the base case, and the appearance of the prices solely in the objective function.

The differences between the Columbia marginal energy values for the January base case and the January 125-scenario cases for each of the scenario-dependent parameters under the assumption of non-anticipative ending storage volumes are presented in Figure 4-38.
With exceptions when one reservoir begins 10% full while the other reservoir begins either 50% or 90% full, the difference for the Columbia marginal energy values from the base case are greater when the inflow is the scenario-dependent parameter than for the demands. The largest difference from the Columbia marginal value base case occurs for scenario-dependent demands when the Columbia begins 50% full while the Peace begins 10% full. The differences when the inflows are the scenario-dependent parameters equal or exceed those for the prices as the scenario-dependent parameters; the same is true when the demands are the scenario-dependent parameters. Thus the base case provides the best estimate of the Columbia marginal energy values when prices are the scenario-dependent parameters, and, overall, the worst estimate when demands are the scenario-dependent parameters. (The exceptions between the demands and inflows show that for some initial storage conditions the base case provides a better estimate of the Columbia marginal energy values for the demands than for the inflows.) Similar results were found for the Peace marginal energy values, with the main difference being fewer exceptions between the demands and inflows for the Peace marginal energy values.
May

In this section, the utility of employing the May base case to estimate the marginal Columbia energy value is examined for three scenario-dependent parameters (demand, inflow, and price), three constraints regarding how to model the anticipative nature of the problem (no non-anticipative constraints, constraints for non-anticipative turbine releases, and constraints for non-anticipative ending storage volumes), and nine initial storage conditions. The parameters for the May scenarios are defined in Table 4-7 through Table 4-12.

The first non-anticipative assumption considered is that of no non-anticipative constraints, meaning that perfect knowledge of the scenario-dependent parameters is assumed when each decision is made. The differences between the marginal Columbia energy values for the May base case and the May 125-scenario cases for each of the scenario-dependent parameters under the assumption of perfect foresight are presented in Figure 4-39.

![Figure 4-39: Difference in Columbia Marginal Energy Values Between Base Case and Scenario-Dependent Case for No Non-Anticipative Constraints Assumption in May](image)

In Figure 4-39, observe that when the Columbia reservoir begins 90% full, and when the Columbia begins 50% full while the Peace is 90% full, differences from the base case Columbia marginal energy values under any of the three scenario-dependent parameters are small. For the other initial storage conditions, the differences when the demand is the scenario-dependent parameter exceed those when the price is the scenario-dependent parameter. Overall, the differences are greater for the inflows than they are for the prices or the demands; however, there are exceptions for both. Thus, the base case is the best
estimate for all of the scenario-dependent parameters when the Columbia reservoir begins 90% full. For lower initial storage conditions, the base case generally provides the best estimate of the Columbia marginal energy values for the prices as the scenario-dependent parameters, and generally the worst estimates for the inflows as the scenario-dependent parameters. The largest magnitude departure from the base case Columbia marginal energy value occurs for scenario-dependent inflows when the Columbia begins 50% full and the Peace begins 10% full. Comparing these results to those for the Peace indicates that the order of the prices and inflows and prices and demands is reversed.

The differences between the Columbia marginal energy values for the May base case and the May 125-scenario cases for each of the scenario-dependent parameters under the assumption of non-anticipative turbine releases are presented in Figure 4-40.

In Figure 4-40, note that when the Columbia reservoir begins 90% full, and when the Columbia begins 50% full while the Peace begins 90% full, the difference in the Columbia marginal energy value between the base case and each of the scenario-dependent parameters is very small, indicating that the base provides a good estimate of the Columbia marginal energy value in all of these cases. The same was true for the Peace marginal energy value. Of the remaining initial conditions, with an exception when both reservoirs begin 10% full, departures from the base case Columbia marginal energy values are greater for the demands as the scenario-dependent parameters than for the inflows. In turn, the departures are greater for the inflows than they are for the prices. Thus, the base case provides the best estimate of the Columbia marginal energy values when the prices are the scenario-dependent parameters, and the worst estimate when the
demands are the scenario-dependent parameters. The same results were found for the Peace marginal energy values.

The differences between the Columbia marginal energy values for the May base case and the May 125-scenario cases for each of the scenario-dependent parameters under the assumption of non-anticipative ending storage volumes are presented in Figure 4-41.

![Diagram](image)

**Figure 4-41: Difference in Columbia Marginal Energy Values Between Base Case and Scenario-Dependent Case for Non-Anticipative Ending Storage Volume Constraints Assumption in May**

In Figure 4-41, note that when the Columbia reservoir begins 90% full, and when the Columbia begins 50% full while the Peace begins 90% full, the difference in the Columbia marginal energy value between the base case and each of the scenario-dependent parameters is very small, indicating that the base case provides a good estimate of the Columbia marginal energy value in all of these cases. The same was true for the Peace marginal energy value. With minor exceptions, the departures from the base case Columbia marginal energy values with the inflows as the scenario-dependent parameters are greater than or equal to those when the demands are the scenario-dependent parameters, which in turn equal or exceed those for the prices as the scenario-dependent parameters. Thus, the base case is the best estimate of the Columbia marginal energy values when the prices are the scenario-dependent parameters, and generally the worst estimates when the inflows (and in some cases the demands) are the scenario-dependent parameters. Similar results were found for the Peace marginal energy values.
September

In this section, the utility of employing the September base case to estimate the Columbia marginal energy value is examined for three scenario-dependent parameters (demand, inflow, and price), three constraints regarding how to model the anticipative nature of the problem (no non-anticipative constraints, constraints for non-anticipative turbine releases, and constraints for non-anticipative ending storage volumes), and nine initial storage conditions. The parameters for the September scenarios are defined in Table 4-13 through Table 4-18.

The first non-anticipative assumption considered is that of no non-anticipative constraints, meaning that each decision is made with perfect knowledge of the scenario-dependent parameters. The difference between the Columbia marginal energy values for the September base case and those for the September 125-scenario cases for each of the scenario-dependent parameters under the assumption of perfect foresight is presented in Figure 4-42.

![Difference Between Marginal Columbia Energy Values for September Base Case and Scenario Dependent with No Non-Anticipative Constraints Case](image)

**Figure 4-42: Difference in Columbia Marginal Energy Values Between Base Case and Scenario-Dependent Case for No Non-Anticipative Constraints Assumption in September**

When one of the two reservoirs begins 90% full while the other begins either 50% full or 90% full, the base case provides a good estimate of the Columbia marginal energy value for all three of the scenario-dependent parameters. The same was true for the Peace marginal energy values.
For the remaining initial storage conditions, with minor exceptions when both reservoir begin 10% full and when one reservoir begins 10% full while the other begins 90% full, the departures from the base case Columbia marginal energy values are greater with the demand as the scenario-dependent parameter as compared to the inflow. The largest departure from the base case occurs for demands as the scenario-dependent parameters when the Columbia begins 50% full while the Peace begins 10% full. The departures with the inflows as the scenario-dependent parameters exceed those for the prices as the scenario-dependent parameters. Except for when the Peace begins 90% full while the Columbia begins either 10% or 50% full, the departures for scenario-dependent demands exceed those for scenario-dependent prices. Thus, the base provides a better estimate of the Columbia marginal energy values when the prices are the scenario-dependent parameters than when the inflows or demands are the scenario-dependent parameters. The base case is generally the poorest estimate for the demands as the scenario-dependent parameters, but can also be so for the inflows. Similar results, with fewer exceptions, were found for the Peace marginal energy values.

The difference between the Columbia marginal energy values for the September base case and those for the September 125-scenario cases for each of the scenario-dependent parameters under the assumption of non-anticipative turbine releases is presented in Figure 4-43.

![Graph showing the difference in Columbia marginal energy values between base case and scenario-dependent case](image)

Figure 4-43: Difference in Columbia Marginal Energy Values Between Base Case and Scenario-Dependent Case for Non-Anticipative Turbine Release Constraints Assumption in September
In Figure 4-43 observe that, with exceptions when the Columbia reservoir is initially 10% full, departures from the base case Columbia marginal energy values with demands as the scenario-dependent parameters are greater than or equal to those with inflows as the scenario-dependent parameters, which in turn exceed or equal those when the prices are the scenario-dependent parameters. The largest departure from the base case Columbia marginal energy value occurs for demands as the scenario-dependent parameters when the Columbia begins 90% full while the Peace begins 10% full. Thus, the base case provides the best estimate of the Columbia marginal energy value when the price is the scenario-dependent parameter, and the worst estimate when the demands (and in some cases the inflows) are the scenario-dependent parameters. The same results were found for the marginal Peace energy values.

The difference between the Columbia marginal energy values for the September base case and the September 125-scenario cases for each of the scenario-dependent parameters under the assumption of non-anticipative ending storage volumes is presented in Figure 4-44.

![Graph](image)

**Figure 4-44: Difference in Columbia Marginal Energy Values Between Base Case and Scenario-Dependent Case for Non-Anticipative Ending Storage Volume Constraints Assumption in September**

In Figure 4-44, the results indicate that the departures from the Columbia marginal energy values with inflows as the scenario-dependent parameters equal or exceed those for the demands, which in turn are greater than or equal to those for prices as the scenario-dependent parameters. The largest departure occurs for inflows as the scenario-dependent parameters when the Columbia begins 90% full while the Peace begins 10%
full. So, the base case provides the best estimate of the Columbia marginal energy values when the prices are the scenario-dependent parameters, and the worst estimate when the demands are the scenario-dependent parameters. For the Peace marginal energy values, the prices are the best estimate, and the inflows are the worst.

4.4.5.2 Comparison of Columbia Marginal Energy Value Against Perfect Foresight Case

In the previous section, comparisons were made between a number of STMVM runs for different scenario-dependent parameters, non-anticipative assumptions, and initial storage volumes for the months of January, May, and September. The comparison yielded information on how the Columbia marginal energy values change with respect to using an average value if the one point estimate for the scenario-dependent parameter is replaced with a five-point estimate based on a normal distribution.

In this section, a comparison is made for each of the three months, initial storage volumes, and scenario-dependent parameters between the Columbia marginal energy value for the perfect foresight case and both non-anticipative turbine releases and non-anticipative ending storage volumes. The difference in the Columbia marginal energy values gives the expected difference in the Columbia marginal energy value with perfect information. The cases are presented by the month studied.

January

In this section, the difference in the marginal Columbia energy value with perfect information is examined for three scenario-dependent parameters, two non-anticipative variables, and nine initial storage conditions. The parameters for the January scenarios are defined in Table 4-1 through Table 4-6.

The difference from the Columbia marginal energy value with perfect information for January under the assumption of non-anticipative turbine releases is presented as Figure 4-45.
From Figure 4-45, observe that the difference in Columbia marginal energy values with the demands as the scenario-dependent parameters are greater than or equal to those with inflows or prices as the scenario-dependent parameters. The largest departure from the Columbia marginal energy value for the perfect foresight case occurs for scenario-dependent demands when the Columbia begins 50% full while the Peace begins 10% full. With exceptions when both reservoirs begin 10% full, and when the Columbia reservoir is 50% full while the Peace reservoir is 10% full, the difference in Columbia marginal energy values with prices as the scenario-dependent parameters are greater than or equal to those with the inflows as the scenario-dependent parameters. So, perfect knowledge of the demands is of more importance in calculating the Columbia marginal energy values than perfect knowledge of the inflows or prices. Generally, perfect knowledge of the prices is of more importance than perfect knowledge of the inflows in calculating the Columbia marginal energy values, but the opposite can be true based upon the initial storage. For the Peace marginal energy values similar results were found, with the relative importance of the inflow or price depending on the initial storage conditions.

The difference in the Columbia marginal energy value with perfect information for January under the assumption of non-anticipative ending storage volumes is presented as Figure 4-46.

Figure 4-45: Difference in Columbia Marginal Energy Value Under Non-Anticipative Turbine Release Assumption for January
Figure 4-46: Difference in Columbia Marginal Energy Value Under Non-Anticipative Ending Storage Volume Assumption for January

From Figure 4-46, note that the importance of perfect knowledge of the scenario-dependent parameters is quite small when the Columbia begins 90% full, as well as when the Columbia begins 50% full while the Peace begins 90% full. For these higher storage conditions, perfect knowledge of the prices is of more importance in calculating the Columbia marginal energy values than perfect knowledge of either the demands or inflows. Conversely, for the lower initial storage conditions, perfect knowledge of the inflows and demands is of more importance in calculating the Columbia marginal energy values than perfect knowledge of the price. The largest departure from the Columbia marginal energy value for the perfect foresight case occurs for demands as the scenario-dependent parameters with the Columbia beginning 50% full while the Peace begins 10% full. With the exception of when both reservoirs begin only 10% full, perfect knowledge of the demands is or more importance than perfect knowledge of the inflows. Similar results were found for the Peace marginal energy values.

**May**

In this section, the difference in the Columbia marginal energy value with perfect information is examined for three scenario-dependent parameters, two non-anticipative variables, and nine initial storage conditions. The parameters for the May scenarios are defined in Table 4-7 through Table 4-12.
The difference from the Columbia marginal energy value with perfect information for May under the assumption of non-anticipative turbine releases is presented as Figure 4-47.

![Figure 4-47: Difference in Columbia Marginal Energy Value Under Non-Anticipative Turbine Release Assumption for May](image)

In Figure 4-47, when the Columbia reservoir begins 90% full, as well as when the Columbia begins 50% full while the Peace begins 90% full, perfect knowledge of the scenario-dependent parameters does not greatly affect the calculation of the Columbia marginal energy values. For the other storage values, with the exception of when both reservoirs begin 10% full, perfect knowledge of the demands is of more importance for calculating the Columbia marginal energy value than is perfect knowledge of the inflows. With the same exception, perfect knowledge of the prices is also of greater importance than perfect knowledge of the inflows. Perfect knowledge of the demands is of greater value in calculating the Columbia marginal energy values than perfect knowledge of the prices, with the largest departure from the perfect foresight case occurring for scenario-dependent demands when the Columbia begins 50% full while the Peace begins 10% full. By way of comparison, for the Peace marginal energy values, perfect knowledge of the prices was of greater importance than perfect knowledge of the demands. The other relationships, in terms of the importance of the scenario-dependent parameters, are the same as for the Peace marginal energy values; however for the Peace, perfect knowledge is of more importance at the higher initial Columbia storage conditions.

The difference from the Columbia marginal energy value with perfect information for May under the assumption of non-anticipative ending storage volumes is presented as Figure 4-48.
In Figure 4-48, as was true for the May non-anticipative turbine release case, perfect knowledge of the scenario-dependent parameters is of little value in calculating the Columbia marginal energy value when the Columbia reservoir begins 90% full, as well as when the Columbia reservoir begins 50% full while the Peace reservoir begins 90% full. For the remaining initial storage conditions, with the exception of when both reservoirs begin 50% full, perfect knowledge of the inflows is of more value than perfect knowledge of the demands in calculating the Columbia marginal energy value. For these same initial storage conditions, perfect knowledge of either the inflows or demands are of greater importance than perfect knowledge of the prices. So, perfect knowledge of the inflows is of the greatest value, and perfect knowledge of the prices is of the least value in calculating the Columbia marginal energy values. By way of contrast, for the Peace marginal energy values, perfect knowledge of the prices was overall of the most importance, and perfect knowledge of the demands was of the least value.

**September**

In this section, the difference in the Columbia marginal energy value with perfect information is examined for three scenario-dependent parameters, two non-anticipative variables, and nine initial storage conditions. The parameters for the September scenarios are defined in Table 4-13 through Table 4-18.
The differences in the Columbia marginal energy value with perfect information for September under the assumption of non-anticipative turbine releases are presented as Figure 4-49.

From Figure 4-49, observe that perfect knowledge of the demands, with minor exceptions when the Columbia begins 10% full while the Peace begins 90% full as well as when both reservoirs begin 50% full, is of more value in estimating the Columbia marginal energy values than perfect knowledge of the inflows. The largest departure from the perfect foresight Columbia marginal energy value occurs for scenario-dependent demands when the Columbia begins 90% full while the Peace begins 10% full. With exceptions when the Columbia begins 10% full while the Peace begins 90% full, and when one reservoir begins 50% full while the other begins 90% full, perfect knowledge of the inflows is of greater importance than perfect knowledge of the prices, in terms of estimating the Columbia marginal energy values. These same results were obtained for the corresponding Peace marginal energy values.
The difference from the Columbia marginal energy value with perfect information for September under the assumption of non-anticipative ending storage volumes is presented as Figure 4-50.

For September, with non-anticipative ending storage volumes, perfect knowledge of the inflows is of greater importance than perfect knowledge of the demands in terms of calculating the Columbia marginal energy values. The largest departure from the perfect foresight Columbia marginal energy values occurs for scenario-dependent inflows with the Columbia beginning 90% full while the Peace starts 10% full. Perfect knowledge of the demands is, in turn, of greater importance in calculating the Columbia marginal energy values than perfect knowledge of the prices. The relative importance of perfect knowledge of the inflows and demands is opposite that for the Peace marginal energy values; the relative importance is the same for the other scenario-dependent parameters as for the Peace marginal energy values.

Comparing Figure 4-45 through Figure 4-50 yields additional information. For all three months, when the demands are the scenario-dependent parameters, the differences in the Columbia marginal energy values as compared to those for the perfect foresight case are higher under the assumption of non-anticipative turbine release than under the assumption of non-anticipative ending storage volume. In contrast, for all three months, when the inflows are the scenario-dependent parameters, the differences in the Peace marginal energy values as measured against those for the perfect foresight case are higher under the assumption of non-anticipative ending storage volumes than under the assumption of non-anticipative turbine releases. For prices as the scenario-dependent
parameters, the differences in the Columbia marginal energy values as measured against those for the perfect foresight case are equal for the assumptions of non-anticipative turbine releases and non-anticipative ending storage volumes.

4.4.6 Summary

In this section, application of the STMVM has been made in performing a case study based upon the two major storage facilities in the BC Hydro system in order to demonstrate the efficacy of the model, as well as the insights that can be gleaned for a particular reservoir system through its use. The price signals generated by the model can be used to guide economic plant dispatch and to use the reservoir system in support of electricity trade. The results from the case study are summarized below.

In the case study, a series of problems have been solved based on the test system covering different times in the year, different scenario-dependent parameters, different assumptions regarding the anticipative nature of constraints, and different initial storage conditions. In all of these cases, the future is defined by 125 possible scenarios. For these 125-scenario problems, comparisons of the objective function value, Peace marginal energy value, and Columbia marginal energy value have been made against a one-scenario base case, as well as against the 125-scenario problem under the assumption of perfect foresight. The 125-scenario problems each assume a normal distribution, with a coefficient of variation of 0.10, for the scenario-dependent parameter.

For all of the situations studied, under the assumption of either non-anticipative turbine releases or non-anticipative ending storage volumes, when the prices are the scenario-dependent parameters the objective function value is the same as it is under the one-scenario base case. The reasons for this are that in the 125-scenario cases, the assumed prices have a symmetrical distribution with mean equal to that for the base case, and they appear solely in the objective function.

In January, for all three of the assumptions regarding the anticipative nature of the constraints, the base case was a much worse estimate of the objective function value when the Columbia begins 10% full and when the Columbia begins 50% full while the Peace starts 10% full, than for the other initial storage conditions. It was also found for January that, with one exception under the assumption of non-anticipative ending storage volumes when the Columbia begins 90% full and the Peace begins 10% full, the base case provides a better estimate of the objective functions for inflows, as opposed to the demands, as the scenario-dependent parameters. For the case of no non-anticipative constraints, the base case is closer to the demands as the scenario-dependent parameters than the prices, except for the four cases with the highest initial storage conditions. Also for the January no non-anticipative constraints case, the objective functions for prices as the scenario-dependent parameters are closer to the base case than are the inflows except for the three poorest initial storage conditions.
For all of the cases, the departures from the base case objective function values are smaller in May than they are for January or September. In May, under the assumption of non-anticipative turbine releases, the largest difference from the base case objective function occurs for demands as the scenario-dependent parameters when both reservoirs begin 90% full, while under the assumption of non-anticipative ending storage volumes, the largest difference occurs for scenario-dependent inflows for the case of both reservoirs beginning 10% full. For the case of perfect foresight, it was found that in May the prices are generally the furthest from the base case objective function values; depending on the initial storage conditions, either the demands or inflows can be further from the base case.

In September, it was found that with perfect foresight, generally the base case is a better estimate of the objective function for the inflows than it is for the demands. For the no non-anticipative constraints case, the base case is furthest from the prices only case when both reservoirs begin at least 50% full.

For all three months, differences from the base case objective function with inflows as the scenario-dependent parameters are larger when the ending storage volumes, as opposed to the turbine releases, are the non-anticipative constraints. Conversely, for demands as the scenario-dependent parameters, differences from the base case objective function are greater for turbine releases, as opposed to ending storage volumes, as the non-anticipative constraints.

With the demands and prices as the scenario-dependent parameters, the expected value of information (EVPI) is greater under the assumption of non-anticipative turbine releases than it is under the assumption of non-anticipative ending storage volumes. On the other hand, for inflows as the scenario-dependent parameters, the EVPI is greater for non-anticipative ending storage volumes than it is for non-anticipative turbine releases.

In January and September, for non-anticipative turbine releases, the EVPI with demands as the scenario-dependent parameters is greater than that with inflows as the scenario-dependent parameters. In May, the same trend exists; however, there are exceptions when the two reservoirs begin out of balance with one another.

In January, under the assumption of non-anticipative turbine releases, for poor initial storage conditions, the EVPI with inflows as the scenario-dependent parameters is greater than or equal to that for prices. In May, for non-anticipative turbine releases, with exceptions when the reservoirs both begin very full or very empty, the EVPI with prices as the scenario-dependent parameters is greater than or equal to that for inflows. In September, for non-anticipative turbine releases, except for the three cases with the highest initial storage conditions, the EVPI with scenario-dependent inflows is greater than or equal to that for scenario-dependent prices.

For January, with non-anticipative turbine releases, except for the three cases with the highest initial storage conditions, the EVPI with demands as the scenario-dependent parameters is greater than that for scenario-dependent prices. Similarly, for May under...
the assumption of non-anticipative turbine releases, the EVPI with demands as the scenario-dependent parameters is greater than that for scenario-dependent prices, although the exceptions differ. In September, under the assumption of non-anticipative turbine releases, the EVPI with scenario-dependent demands is greater than that for scenario-dependent prices.

Under the assumption of non-anticipative turbine releases for each of the three months examined, the largest EVPI occurs with demands as the scenario-dependent parameters. However, in each month the maximum EVPI occurs for a different initial storage condition: the Columbia 50% full and the Peace 10% full in January; the Columbia and Peace both 90% full in May; and the Columbia 10% full and the Peace 50% full in September.

For non-anticipative ending storage volumes, in January, with one exception, the EVPI for demands as the scenario-dependent parameters is greater than that for scenario-dependent inflows. In May, for non-anticipative ending storage volumes, the EVPI for inflows as the scenario-dependent parameters is greater than that for scenario-dependent demands; the same occurs in September with exceptions when the Peace reservoir begins 50% full. Also for non-anticipative ending storage volumes, in January, except for the three healthiest initial storage conditions, the EVPI for scenario-dependent inflows is greater than that for scenario-dependent prices. In May, with non-anticipative ending storage volumes, the EVPI for scenario-dependent inflows is greater than that for scenario-dependent prices; a similar result occurs for September, with exceptions for two of the three highest initial storage conditions. For January, with some exceptions, the EVPI for demands as the scenario-dependent parameters is greater than that for scenario-dependent prices; the same occurs for May and September.

For demands as the scenario-dependent parameters, for all three months examined, with minor exceptions for poor initial storage conditions, the base case provides a better estimate of the Peace marginal energy values under the assumption of no non-anticipative constraints than for either non-anticipative turbine releases or non-anticipative ending storage volumes; further, the base case provides a better estimate of the Peace marginal energy values under the assumption of non-anticipative ending storage volumes than under the assumption of non-anticipative turbine releases. With inflows as the scenario-dependent parameters, for January and September, with minor exceptions, the base case provides the best estimate of Peace marginal energy values under the assumption of perfect foresight and the worst estimate under the assumption of non-anticipative ending storage volumes. In May, the relationships are not as clear. For prices as the scenario-dependent parameters, the base case provides the same Peace marginal energy values as under the assumption of either non-anticipative turbine releases or non-anticipative ending volumes. The reasons for this result are that the prices appear solely in the objective function and the 125-scenario cases assume a symmetrical price distribution with a mean equal to that for the one-scenario base case.

In January, under the assumption of no non-anticipative constraints, the base case provides a better estimate of the Peace marginal energy values under the assumption of
scenario-dependent inflows than for scenario-dependent demands. For May, contrary results, with exceptions when the Columbia begins 10% full while the Peace begins 50% full as well as when the Columbia begins 50% full while the Peace begins 90% full, are found. In September, the results are very similar to those for January with an exception when both reservoirs begin 10% full.

Under the assumption of no non-anticipative constraints, for January with one exception when both reservoirs begin 10% full, the base case provides a better estimate of the Peace marginal energy values under the assumption of scenario-dependent inflows than for scenario-dependent prices. The same result occurs for May. In September, with exceptions when both reservoirs begin at least 50% full, the base case is closer to the Peace marginal energy values for scenario-dependent prices than for scenario-dependent inflows.

In January, under the assumption of no non-anticipative constraints, except for the three lowest initial storage conditions, the base case provides a better estimate of the Peace marginal energy values under the assumption of scenario-dependent demands than for scenario-dependent prices. The same result occurs for May, with the only exception being when both reservoirs begin 10% full. In September, with exceptions when both reservoirs begin at least 50% full, the base case provides a better estimate of the Peace marginal energy values under the assumption of scenario-dependent prices than for scenario-dependent demands.

For non-anticipative turbine releases, in January the base case provides a better estimate of the Peace marginal energy values for scenario-dependent inflows than for scenario-dependent demands. The results for May are similar to those in January, with exceptions when the Columbia begins 10% full while the Peace begins either 10% or 90% full. The results for September are also similar, with the only exception occurring when both reservoirs begin 10% full.

With non-anticipative ending storage volumes, for January, with exceptions when the Columbia begins 90% full while the Peace begins either 10% or 90% full, the base case provides a better estimate of the Peace marginal energy values for scenario-dependent inflows than for scenario-dependent demands. In May, with exceptions when the Columbia begins 50% full while the Peace begins either 10% of 90% full, the Peace marginal energy value for the base case is closer to that for scenario-dependent demands than that for scenario-dependent inflows. In September, with exceptions when the Columbia begins 10% full, the base case provides a better estimate of the Peace marginal energy values for scenario-dependent demands than for scenario-dependent inflows.

For both January and September, for all three of the anticipative assumptions, the largest departure from the base case Peace marginal energy value occurs for scenario-dependent demands when the Columbia begins 50% full and the Peace begins 10% full. In May, the largest departures occur with both reservoirs beginning 10% full and for inflows as the scenario-dependent parameters for both no non-anticipative constraints and non-anticipative turbine releases. For non-anticipative ending storage volumes the largest
difference from the May base case Peace marginal energy value occurs when both reservoirs begin 50% full and demands are the scenario-dependent parameters.

In January, for non-anticipative turbine releases, the Peace marginal energy values under the assumption of perfect foresight are closer to those for scenario-dependent inflows than those for scenario-dependent demands. The same results occur in May, with an exception when both reservoirs begin 10% full. Similar results are found for September, with additional exceptions when the Columbia begins 10% full while the Peace begins 90% full and when the Columbia begins 50% full while the Peace begins 50% full.

In January, for non-anticipative turbine releases the Peace marginal energy values with perfect foresight are closer to those for scenario-dependent prices than scenario-dependent demands, with exceptions when the Columbia begins 90% full while the Peace begins either 10% or 50% full. For non-anticipative turbine releases, the Peace marginal energy values with perfect foresight are closer to those for scenario-dependent demands than for scenario-dependent prices, with exceptions when the Peace begins 10% full while the Columbia begins either 10% or 50% full and when both reservoirs begin 90% full. With different exceptions, the results for September are the same as those for January.

For non-anticipative turbine releases, in January the Peace marginal energy values with perfect foresight are closer to those for scenario-dependent inflows than for scenario-dependent prices, with an exception when both reservoirs begin 10% full. The results are the same for May. In September, for non-anticipative turbine releases, the Peace marginal energy values with perfect foresight are closer to those for scenario-dependent inflows with exceptions when the Columbia begins 90% full while the Peace begins at least 50% full.

With non-anticipative ending storage volumes, in January the perfect foresight case provides a better estimate of the Peace marginal energy values for scenario-dependent prices than for scenario-dependent inflows. The results for May are opposite those for January, with exceptions when the Columbia begins 10% full while the Peace begins either 10% or 50% full and when the Columbia begins 50% full while the Peace begins 90% full. The results for September are the same as those for January with an exception when the Columbia begins 90% full while the Peace begins 50% full. Also in January, the perfect foresight case provides a better estimate of the Peace marginal energy values for scenario-dependent prices than for scenario-dependent demands with exceptions when the Columbia begins 90% full. The results for September are very similar to those for January, although the exceptions differ. In contrast, in May the Peace marginal energy values for perfect foresight are closer to those for scenario-dependent demands than scenario-dependent prices, with exceptions when both reservoirs begin 10% full and when the Columbia begins 50% full while the Peace begins 90% full. In January, with non-anticipative ending storage volumes, the base case Peace marginal energy values are closer to those for scenario-dependent inflows than for scenario-dependent demands, with one exception when both reservoirs begin 90% full. In contrast, for May, with an exception when both reservoirs begin 50% full, the perfect foresight Peace marginal
energy values are closer to those for scenario-dependent demands than they are for scenario-dependent inflows. The results for September are the same as those for January, although the exceptions differ.

For the case of scenario-dependent demands, overall the perfect foresight case provides the best estimate of Peace marginal energy values in May and the worst estimate in January for both non-anticipative variables. For scenario-dependent prices, for both non-anticipative variables, the perfect foresight case provides the best estimate in January and the worst estimate in May. For scenario-dependent inflows, for both non-anticipative variables, the perfect foresight case is a better estimate of the Peace marginal energy values in January than in September. For non-anticipative turbine releases, the perfect foresight case is better in September than in May, while for non-anticipative ending storage volumes, the perfect foresight case is better in May than in September.

For scenario-dependent demands and prices, for all three months, and for both non-anticipative variables, the Peace marginal energy values under the assumption of non-anticipative volumes are closer to those for perfect foresight than they are for non-anticipative turbine releases. The same results are found for inflows as the scenario-dependent parameters in May, but the opposite for January and September.

In January, for the case of perfect foresight, the base case provides a better estimate of the Columbia marginal energy value for scenario-dependent inflows than for scenario-dependent demands. With some exceptions, the same result is found for September. In May, with exceptions when the Columbia begins 10% full while the Peace begins either 50% or 90% full, the base case provides a better estimate of the Columbia marginal energy value for scenario-dependent demands than for scenario-dependent inflows. For the same assumption, in January, with an exception when both reservoirs begin 10% full, the base case provides a better estimate of Columbia marginal energy values for scenario-dependent inflows than for scenario-dependent prices. In May, with exceptions when the Columbia begins 10% full while the Peace begins either 50% or 90% full, the base case produces Columbia marginal energy values that are closer to those for scenario-dependent prices than scenario-dependent inflows; the same is true, without exceptions, for September. With exceptions when the Columbia begins 90% full while the Peace begins either 10% or 50% full, the January base case produces Columbia marginal energy values that are closer to those for scenario-dependent prices than scenario-dependent demands; the same is true for May without exceptions. In September, the base case produces Columbia marginal energy values closer to those for scenario-dependent prices than for scenario-dependent demands with exceptions when the Peace begins 90% full while the Columbia begins no more than 50% full.

In January, for non-anticipative turbine releases, the base case produces Columbia marginal energy values that are closer to those for scenario-dependent inflows than for scenario-dependent demands. Similar results are found in May, with exceptions when the Columbia begins 10% full while the Peace begins either 10% or 50% full. Similar results are also found for September, with exceptions when the Columbia begins 10% full while the Peace begins either 50% or 90% full.
For non-anticipative ending storage volumes, for January, the base case produces Columbia marginal energy values that are closer to those for scenario-dependent demands than for scenario-dependent inflows with exceptions when the Columbia begins 10% full while the Peace begins at least 50%, and when the Peace begins 10% full while the Columbia begins at least 50% full. For May, with exceptions when the Columbia begins 10% full while the Peace begins 90% full and when the Columbia begins 50% full while the Peace begins 10% full, the base case provides a better estimate of Columbia marginal energy values for scenario-dependent inflows than for scenario-dependent demands. For September, the base case provides a better estimate of the Columbia marginal energy values for scenario-dependent demands than for scenario-dependent inflows.

In January, for non-anticipative turbine releases, the Columbia marginal energy values under the assumption of perfect foresight are closer to those for scenario-dependent inflows than for scenario-dependent demands; the same is true in May except for when both reservoirs begin 10% full. In September, the same is true, with exceptions when both reservoirs begin 50% full as well as when the Columbia begins 10% full while the Peace begins 90% full. Further, for January, the perfect foresight case yields Columbia marginal energy values that are closer to those for scenario-dependent prices than those for scenario-dependent demands. The same is true for May. In September, the perfect foresight case yields Columbia marginal energy values that are closer to those for scenario-dependent prices than for scenario-dependent demands, with exceptions when the Columbia begins 10% full while the Peace begins 90% full. In January, for non-anticipative turbine releases, the Columbia marginal energy values for the perfect foresight case are closer to those for scenario-dependent inflows than those for scenario-dependent prices, with exceptions when both reservoirs begin 10% full and when the Columbia begins 50% full while the Peace begins 10% full. In May, the same is true except for when the Columbia begins 90% full, as well as when the Columbia begins 50% full while the Peace begins 90% full. In September, the assumption of perfect foresight produces Columbia marginal energy values that are closer to those for scenario-dependent prices than those for scenario-dependent inflows except for when the Columbia begins 10% full while the Peace begins 90% full, as well as when one reservoir begins 50% full while the other begins 90% full.

Under the assumption of non-anticipative ending storage volumes, for January, when the Columbia begins 90% full as well as when the Columbia begins 50% full while the Peace begins 90% full, there is little variation between the difference from the perfect foresight Columbia marginal energy values for the three scenario-dependent parameters, with the inflows being closest to the perfect foresight case and the prices being the furthest. For the other initial storage cases, the perfect foresight case is closest to that for scenario-dependent prices and the furthest from those for scenario-dependent demands.

For May, under the assumption of non-anticipative ending storage volumes, when the Columbia begins 90% full, as well as when the Columbia begins 50% full while the Peace begins 90% full, the Columbia marginal energy values for perfect knowledge are
close to those for all of the scenario-dependent parameters. For the remaining initial storage volumes, with the exception of when both reservoirs begin 50% full, the Columbia marginal energy values for the perfect foresight case are closer to those for scenario-dependent demands than those for scenario-dependent inflows. In addition, the Columbia marginal energy values for the perfect foresight case are closer to those with prices as the scenario-dependent prices than those for either scenario-dependent inflows or demands.

In September, under the assumption of non-anticipative ending storage volumes, the Columbia marginal energy values for the perfect foresight case are closer to those for scenario-dependent prices than those for either scenario-dependent demands or inflows. With the exception of when the Columbia begins 10% full while the Peace begins 50% full, the Columbia marginal energy values under the assumption of perfect foresight are closer to those for scenario-dependent demands than those for scenario-dependent inflows.

For all three months studied, for scenario-dependent inflows and prices the Columbia marginal energy values for non-anticipative turbine releases are closer to those for perfect foresight than are those for non-anticipative ending storage volumes. The opposite is true for all three months with scenario-dependent demands.
5 Summary and Conclusions

In this thesis, a method has been developed for determining "optimal" operations for a two-reservoir hydropower system in the face of uncertainty over a time horizon stretching from several hours to years into the future. The method is able to address uncertainty in market-related parameters such as energy prices, as well as in demands, in addition to the inflow uncertainty usually considered. In addition, the model allows the energy prices to be subdivided into several periods within the day, matching the products traded on energy markets in which electricity prices vary within the day as well as across months. This ability to simultaneously consider uncertainty in energy market parameters and inflows for the operation of a multiple-reservoir hydropower system is vital, as the importance of uncertainty in the former can be of equal or greater importance than the latter (Kim and Palmer, 1997; Russell and Campbell, 1996). Models reported in the literature typically either ignore uncertainty in energy prices, fail to consider energy price differentiation within a day, or only consider a single reservoir. These simplified models fail to address the main problem actually faced by the operator of a two-reservoir hydropower system.

The method developed involves the use of two models. A longer-term model is used to estimate the value of month-end storage in the two reservoirs over a time horizon of several years, and to generate the marginal values of water stored in the reservoirs. The results produced by this model feed into a shorter-term model that is able to better deal with uncertainty in the first time step employed in the longer-term model through the use of variable time-steps and scenario trees. It is proposed that the time step used in the longer-term model be on the order of a month, and the time steps in the shorter-term model range from the order of several hours to a day for the first time step to the order of several weeks for the last time step.

Through the use of the two models, marginal energy values for two reservoirs that cover a time horizon stretching out several years, but which cover the near-term in sufficient detail are developed. These marginal energy values are crucial to price-based dispatching of a hydropower system, and can be input to a deterministic hourly dispatch model such as those developed by Shawwash et al. (1999) and Piekutowski et al. (1993). The method developed helps to fill a key gap in the reservoir operations time frame between the very short-term future, which is usually modelled deterministically, and the longer-term in which inflow uncertainty is typically considered. The applicability of the method is demonstrated through application to a complex hydropower system that is operated in the current competitive energy markets.

The first of the two models employed in the method, while generalized, is designed to be used over a time increment on the order of a month covering a time horizon of several years, while taking into account uncertainty in all of the key reservoir operations drivers. This longer-term model generates the value of water stored in the two reservoirs, as well as the longer-term (e.g., monthly) marginal values of stored energy. The longer-term model employs both dynamic programming (DP) and linear programming (LP).
In the longer-term model, DP is employed to link the decisions made during one time interval with decisions during other time periods. DP deals with "stages" and "states", which in this model, respectively, are time steps on the order of a month, and vectors of the volume of water stored in the two reservoirs. Uncertainty in the longer term model is handled through the use of scenarios, the number of which can vary with the stage, which describe possible values for the scenario-dependent inflows, demands, and energy prices. An occurrence probability is associated with each scenario. For a state in a stage, the future can unfold in any one of the fashions described by the set of scenarios for that stage. A recursive equation giving the optimal decision for a given starting state in a stage, in the form of the release from each reservoir, which maximizes the expected net present value of the sum of the value of the operation during the next time period and the value of being at the ending state for the next stage is solved. Other variables, such as the ending storage for each reservoir, spill from each reservoir, imports to the system, and exports from the system can vary with the scenario. The values of these variables are determined such that the value of system operation under the scenario, subject to the policy-specified turbine release, is maximized. The optimal value of the turbine releases and the remaining non-policy variables are found using LP.

LP is used to evaluate the recursive equation in the DP model by trading off the value of releasing water, making energy trades during the period, and keeping water in storage beyond the current period, thus determining an optimal operating schedule. An optimal operation is found for each DP scenario. The value of water stored beyond the current time interval is represented in the LP by storage value curves, the slopes of which are the marginal water values. These terminal value functions, which are found, in general, to depend on the month and the storage in both reservoirs, are modelled using piecewise-linearization. Linearizing the underlying non-linear problem requires that a series of closely related LPs be solved successively until convergence.

The use of LP to evaluate the recursive DP equation departs from the traditional method. Typically in water resources applications of DP, the optimal transition is from a discrete state in one stage to a discrete state in the next stage. For the current problem, the discrete states would be vectors of the volume of water stored in the two reservoirs. In the formulation outlined in this thesis, the ending storage volumes in the second stage are decision variables in the LP algorithm, and are thus not restricted to discretized state values; nor are the reservoir releases restricted to discretized volumes. The use of LP to evaluate the DP recursive equation alleviates the problem of artificial spilling of water that can result from the transition between discretized storage volume vectors, which can be particularly acute if a coarse state space discretization is used to increase the speed of a typical DP algorithm. The value of future operation information generally evaluated at each ending storage state vector is contained in piecewise-linear storage value curves. In these storage value curves, the discretized storage states are the breakpoints between linear segments. Between discretized storage volume points, the marginal value of stored water is constant.

The monthly storage value functions produced by the longer-term DP and LP based model are input to the shorter-term model, which is based upon a technique known as
stochastic linear programming with recourse (SLPR). In his review of the water resources literature, Yeh (1985) reports that while SLPR appears to be a promising method, it has not been widely used. The application of this little-used methodology in the shorter-term model demonstrates its suitability for considering uncertainty in complicated hydropower reservoir operation problems.

The shorter-term, SLPR-based, model allows operations to be planned over time steps shorter than those used in the longer-term model, which should be on the order of a month, and to generate short-term marginal values over each of these time steps. While the time step employed in the DP and LP based model is appropriate for some types of studies, the longer time step of necessity neglects variations in the scenario-dependent demands, inflows, and prices within the time step. The shorter-term variable time step model provides a way in which these important departures from the mean values can be taken into consideration for both the calculation of marginal values and operation of the system. The shorter-term marginal values are of particular importance, as their departure from those of the longer-term model will be greatest, when operation of the reservoir system is constrained.

In SLPR, uncertainty is handled by a scenario tree. A scenario tree branches from a single root at the start of the modelled time horizon to leaves at the end of the time horizon being considered. The tree branches at locations, or nodes, where decisions must be made. A scenario in the tree is a direct path from the root to one of the leaves. The tree structure is instructive in illustrating how decisions are made using SLPR. At each node a probability is assigned to each branch, and the sum of the probabilities of all branches from a node must equal one. At each of these nodes a decision must be made. This decision must be made prior to knowing in which one of the possible manners the future will unfold; therefore, the decision must be feasible over all possible evolutions of the future, and should be the optimal decision considering all possible outcomes. Such a decision, being made in the face of uncertainty is known as a “here-and-now” decision. Once the future has been revealed at the end of the time period, “recourse” decisions can be made based upon which of the possible futures occurred. Decisions of this latter type are referred to as “wait-and-see.” The decision making approach just described results in decisions that do not anticipate the future, and can therefore be implemented.

In the shorter-term, variable time step, SLPR-based model a scenario tree is used to describe the scenarios in which reservoir inflows, demands, and energy market prices vary in each time period. The here-and-now decisions at the start of each time period are either the quantities of turbine release from the two reservoirs or the ending storage volume. The final outputs from the SLPR based short-term marginal value model (STMVM) are marginal energy values for each period modelled, as well as operating decisions for each period in the shorter-term model. It is proposed that these short-term marginal values be used as input to a very-short term deterministic reservoir operation model, with a time-step of an hour or less, and a total time horizon of a week or less, such as those described by Shawwash et al. (1999) and Piekutowski et al. (1993).
Together, the models presented in this thesis allow for the optimization of hydropower reservoir systems over a time horizon stretching from several hours to years into the future. The selected approach, in which results from a monthly model feed into a shorter-term variable time step model, allows sufficient detail to be considered such that the uncertain reservoir operating problem actually faced by the operator of a hydropower system, in which energy varies in price both within the day, between days, and between months, in which inflows and demands are also uncertain, and when generation can come from different sources, to be solved successfully. Typically, parts of the true operating problem have been ignored, either by dealing with only a single reservoir, or by not addressing uncertainty in all of the important parameters.

Demonstration of the ability of the proposed method is made through application of the two models in case studies based upon a sub-system of the British Columbia Hydro and Power Authority (BC Hydro) reservoir network. The sub-system is based upon the two main river systems in the BC Hydro system, with the case studies approximating the actual hydro installations. BC Hydro has two generating plants on the Peace River: G. M. Shrum and Peace Canyon. The reservoir for the G. M. Shrum plant has significant over-year storage, whereas Peace Canyon is largely run in hydraulic balance with G. M. Shrum. Similarly, BC Hydro generation facilities on the main-stem of the Columbia River include an upstream plant with significant storage (Mica), and a downstream plant that is generally run in hydraulic balance with the upstream project (Revelstoke). In the case study, generation on each of the two river systems is replaced by a single plant. On the Columbia River, the “Columbia” plant models the Mica and Revelstoke plants. The “Peace” plant models the G. M. Shrum and Peace Canyon plants.

A detailed application of the method was made to the BC Hydro sub-system to demonstrate and test the efficacy of the approach, as well as with the intent of determining relationships between the dependence of the marginal water value in a reservoir with the storage in another reservoir as well as in itself. It was hoped that these relationships would be evident, and, at least to some extent, be able to be generalized to other reservoir systems. It is for this reason that the models were applied many times for different combinations of the number of scenarios, months, scenario-dependent parameters, correlations between the scenario-dependent parameters, assumptions regarding the anticipative nature of the problem, and coefficient of variation values as described in the case studies in Chapters 3 and 4, and much description of the marginal values for the two reservoirs, objective and storage value functions, and expected value of perfect information was made. While the results did not turn out to be as readily generalized as desired, this fact demonstrates the importance of having a methodology that can be used to deliver key insights on the marginal values for specific reservoir configurations which can then be exploited in the operation of the system. The insights gained through application of the DP and LP based model as well as the STMVM to the BC Hydro two reservoir system include the following.

In general, dependence between the marginal energy value in one reservoir and the storage in both reservoirs was found to exist. However, the DP- and LP-based method was able to find regions of storage in which the marginal energy value in one reservoir is
independent, for practical operational purposes, of storage in the second reservoir. Being able to identify such regions is of great importance, as within them it would be possible to employ a separate marginal value model for each reservoir, increasing the detail with which each reservoir could be modelled.

The assumptions regarding the anticipative nature of the problem considered by the DP and LP based model (perfect foresight or non-anticipative turbine releases) and the manner in which the scenario-dependent parameters (demands, inflows, and prices) vary and are correlated were found to be of consequence in the marginal value results obtained. This result indicates the importance of using data and assumptions that most closely represent the actual system being modelled in application of the DP- and LP-based model to a real-world system, and the value of modelling uncertainty.

Through application of the DP- and LP-based model, the extent to which operational constraints, such as those for minimum releases from one reservoir, can have large magnitude impacts on the marginal energy values in both reservoirs for many seasons, and can increase the dependence of the marginal value of water in one reservoir upon storage in the second reservoir was observed. Obviously, this result demonstrates the importance of modelling such constraints correctly. The DP and LP based methodology is able to identify such situations, and quantify the benefits that would accrue from their relaxation.

From application of the SLPR-based model, it was found that the expected value of perfect information (EVPI) is affected by the assumption regarding which of the turbine release or end of period volume is the non-anticipative variable. Further, the EVPI is also impacted by the uncertainty in the scenario-dependent demands, inflows, and prices. In general, it was found that perfect knowledge of the prices is of the most importance when there is healthy storage in both reservoirs.

The two models employed in this thesis, one based on DP and LP, and the other based on SLPR, were found to work well for their respective purposes of calculating the longer-term and shorter-term marginal energy values in a two-reservoir system. Together, they provide the ability to discern key insights into the operation of a complex hydropower system under much uncertainty.

Future research in this area is warranted, and should be focussed in the following areas:
1. Larger reservoir systems should be considered, possibly through the use of aggregation and disaggregation techniques.
2. Greater emphasis should be placed upon the selection of realistic scenarios.
3. The impact of non-symmetrical probability distributions, particularly for prices, should be investigated.

Finally, at the risk of providing too much detail, the lessons learned from application of the two models to the case study based on the Peace and Columbia rivers in the BC Hydro system are outlined below. The point to be taken is that the two models detailed in this thesis provide the capacity to consider the uncertain operation problem faced by the
operator of a complex hydropower system in sufficient detail in order to determine how the marginal values interact with storage, what sources of uncertainty are important to consider, and how the marginal values in each of two reservoirs can behave differently with changes in model parameters.

Results from the case study for the longer-term DP and LP based model, run with a monthly time step, are discussed first. In the case study, the marginal energy values for the Peace and Columbia were determined under a number of different assumptions. First, a base case in which there was only one scenario in each month was considered. Next, the DP and LP based model was applied to a number of cases, in each of which there were different assumptions regarding the correlation between the scenario-dependent parameters. In these cases, there was one scenario during the months of November through April, and five scenarios during the remaining months. During the months with five scenarios, the turbine release was specified to be the "non-anticipative" variable. The cases studied, described in terms of the scenario-dependent parameters, were: demands only, with inflows and prices assuming their mean values in all scenarios; inflows only; prices only; demands, inflows, and prices all perfectly positively correlated; demands and inflows perfectly positively correlated and prices perfectly negatively correlated; inflows and prices perfectly positively correlated and demands perfectly negatively correlated; and demands and prices perfectly positively correlated and inflows perfectly negatively correlated. The same coefficient of variation was assumed to apply to each of the scenario-dependent parameters. Perfect correlation was assumed between the inflows to the two reservoirs, and perfect correlation was assumed for heavy load hour and light load hour energy prices. The model was then applied to each of the seven cases just described under the assumption of perfect foresight. Finally, a one-scenario case in which operational ice constraints restricting the minimum Peace release during December and January was examined.

From application of the DP and LP based model to the case study, several conclusions were reached:

1. Generally, the Peace marginal energy and water values are dependent on the storage in both the Peace and Columbia reservoirs, and the Columbia marginal energy and water values are dependent on the storage in both the Peace and Columbia reservoirs. However, there are exceptions, wherein the marginal energy value in one reservoir is, for practical purposes, independent of the storage in the second reservoir; these exceptions can occur when one reservoir approaches full, or in some mid-reservoir range.

2. With non-anticipative turbine releases and with perfect foresight, depending on the scenario-dependent parameter, case studied, and time of year, the minimum and maximum marginal energy values for both reservoirs can either increase or decrease as the coefficient of variation increases.

3. With non-anticipative turbine releases and a given Peace reservoir storage, the Peace marginal energy value depends upon storage in the Columbia reservoir to a greater extent than does the Columbia marginal energy value on storage in the Peace reservoir for a given Columbia reservoir storage.
4. With non-anticipative turbine releases, for all coefficients of variation studied, and for both the Peace and the Columbia reservoirs, the one-scenario base case underestimates the maximum marginal energy value in the spring and early summer, and overestimates the maximum marginal energy value in the late summer.

5. With non-anticipative turbine releases, as the coefficient of variation decreases, the minimum marginal energy values for both reservoirs with demands as the only scenario-dependent parameters are furthest, of all cases studied, from those for the one-scenario base case.

6. With non-anticipative turbine releases, the minimum marginal energy values, which occur under the highest storage conditions, are more sensitive to variation in demand than they are to equal variation in the inflow. In addition, for all coefficients of variation studied, and for both the Peace and the Columbia, the minimum marginal energy value for the one-scenario base case approaches that for inflows as the only scenario-dependent parameter.

7. With non-anticipative turbine releases, past the peak of the freshet, the maximum marginal energy values, which occur under the lowest storage conditions, are more sensitive to variation in the demand than to equal variation in the inflow.

8. With non-anticipative turbine releases, in the spring through the peak freshet month, for lower coefficient of variation values, the maximum marginal energy values are more sensitive to variation in demand than to equal variation in inflow; the opposite is true for higher coefficients of variation.

9. With perfect foresight, there is less variation in the marginal energy values for both reservoirs for the prices as the only scenario-dependent parameters than for the other cases studied.

10. With perfect foresight, depending upon the scenario-dependent parameters, the case studied, and the time of year, the minimum marginal energy values can either increase or decrease with an increasing coefficient of variation.

11. With perfect foresight: (i) during the freshet, the maximum marginal energy values increase with an increasing coefficient of variation, except for with prices as the only scenario-dependent parameters; and, (ii) when the demands and inflows are perfectly positively correlated the minimum marginal energy value decreases with an increasing coefficient of variation.

12. With inflows as the only scenario-dependent parameters, the maximum marginal energy values are greater under the assumption of perfect foresight than under the assumption of non-anticipative turbine releases. With demands as the only scenario-dependent parameters, as well as the four cases in which all three of the scenario-dependent parameters vary, the maximum marginal energy values can be greater under either of the assumptions depending upon the time of the year.

13. For all cases studied, with perfect foresight, the range of Columbia reservoir storage over which the Columbia marginal energy value is least dependent on the Peace reservoir storage is the same as for non-anticipative turbine releases. In contrast, for all cases studied, during the spring, through the peak of the freshet, with perfect foresight the range of Peace reservoir storage for which the Peace marginal energy value is least dependent on the Columbia reservoir storage differs from that for non-anticipative turbine releases.
14. For all cases studied, under the assumption of perfect foresight, the variation in marginal energy values over the cases studied is less than that for the assumption of non-anticipative turbine releases.

15. Removing the minimum Peace reservoir release constraints in December and January: (i) reduce the minimum and maximum Peace marginal energy values in all months, with the largest reductions occurring from July through January; (ii) reduce the Columbia marginal energy values in all months, particularly between May and July, although not to as great an extent as for the Peace; and, (iii) decouple the Peace marginal energy values from the Columbia reservoir storage and the Columbia marginal energy values from the Peace reservoir storage under all storage conditions in May and June, and in all other months, provided that a threshold Peace reservoir storage is exceeded.

Taken together, the above show that the DP and LP based method can be effectively employed to estimate the marginal energy values in a two-reservoir system. In general, dependence between the marginal energy value in one reservoir and the storage in both reservoirs was found to exist. The methodology is able to find regions of storage in which the marginal energy value in one reservoir is independent of storage in the second reservoir; however, the complexity of the relationships in the system is such that general conclusions cannot be drawn, emphasizing the benefits of being able to identify such regions using the DP and LP based model. Further, the assumptions regarding the anticipative nature of the problem (perfect foresight or non-anticipative turbine releases) and the manner in which the scenario-dependent parameters vary and are correlated were found to be of consequence in the results obtained, indicating the importance of using data and assumptions that most closely represent the actual system being modelled, in application of the DP and LP based model to a real-world system. Finally, the extent to which operational constraints, such as those for minimum releases, can have large magnitude impacts on the marginal energy values in both reservoirs for many seasons illustrates the importance of modelling such constraints correctly, and helps to quantify the benefits that would accrue from their relaxation.

The results from the STMVM case study are described next. In the case study, a series of problems are solved covering different times of the year, scenario-dependent parameters, non-anticipative constraints, and initial storage conditions. With the exception of the one-scenario base cases, the problems solved used 125 scenarios to describe the future. The 125-scenario problems assume a normal distribution, with a coefficient of variation of 0.10, of the scenario-dependent parameters.

From application of the STMVM to the case study, several conclusions were reached:

1. In January, with perfect foresight, (i) the objective function values for the base case are closer to those for the case with inflows as the only scenario-dependent parameters than to the case with demands as the only scenario-dependent parameters; (ii) the base case objective functions are closer to those for the case with demands as the only scenario-dependent parameters than to those with prices as the scenario-dependent parameters, with the exceptions of the four highest storage conditions; and, (iii) the objective functions under the base case are closer to those with prices as the
only scenario-dependent parameters than to those with inflows as the only scenario-dependent parameters with the exception of the three poorest storage conditions.

2. In May, departures from the base case objective functions are smaller than in either January or September.

3. In May, with perfect foresight, the base case objective functions are furthest from those with prices as the only scenario-dependent parameters.

4. In May, with non-anticipative turbine releases, the largest departure from the base case objective function occurs when both reservoirs are 90% full with demands as the scenario-dependent parameters; with non-anticipative ending storage volumes, the largest departure occurs for inflows as the scenario-dependent parameters when both reservoirs begin 10% full.

5. In September, for perfect foresight, the base case objective function values are closer to those with inflows as the only scenario-dependent parameters than they are to those with demands as the scenario-dependent parameters; and, the base case objective functions are furthest from those for the case with prices as the only scenario-dependent parameters when both reservoirs begin at least 50% full.

6. In all three months studied, departures from the base case with inflows as the scenario-dependent parameters are larger for non-anticipative ending storage volumes than they are for non-anticipative turbine releases.

7. With either the demands or the prices as the only scenario-dependent parameters, the expected value of perfect information (EVPI) with non-anticipative turbine releases is equal to or greater than the EVPI with non-anticipative ending storage volumes; whereas, with inflows as the only scenario-dependent parameters, the opposite is true.

8. In all three months studied, with non-anticipative turbine releases, the largest EVPI occurs with demands as the only scenario-dependent parameters. The storage levels for which the largest EVPI occurs is different in each month.

9. For all three months studied, with non-anticipative turbine releases: (i) generally the EVPI with demands as the scenario-dependent parameters equals or exceeds that with scenario-dependent prices, with exceptions occurring at high storage conditions; and (ii) generally the EVPI with demands as the scenario-dependent parameters equals or exceeds that with inflows as the scenario-dependent parameters

10. With scenario-dependent turbine releases, in January and September the EVPI for inflows as the scenario-dependent parameters exceed those for scenario-dependent prices for lower storage conditions. In May, the EVPI with prices as the scenario-dependent parameters exceed those for scenario-dependent inflows except for when the reservoirs are initially either very full or very low.

11. With non-anticipative ending storage volumes: (i) in January, the EVPI with demands as the only scenario-dependent parameter equals or exceeds that with inflows as the scenario-dependent parameters; the opposite is generally true for May and September; (ii) with non-anticipative ending storage volumes, in all three months, the EVPI for demands as the only scenario-dependent parameters generally equals or exceeds that for prices as the scenario-dependent parameters; and, (iii) with non-anticipative ending storage volumes, the EVPI for inflows as the only scenario-dependent parameters equal or exceed those for prices as the scenario-dependent parameters, except for cases of healthy storage in both reservoirs in January and September.
12. For all three months, the base case is generally a better estimate of Peace marginal energy values under the assumption of perfect foresight than under the assumption of non-anticipative ending storage volumes, which in turn are better than those for non-anticipative turbine releases.

13. In January and September, with inflows as the only scenario-dependent parameters, the base case is the best estimate of Peace marginal energy values with perfect foresight and the worst estimate under the assumption of non-anticipative ending storage volumes.

14. With perfect foresight: (i) in January and September, the base case is a better estimate of the Peace marginal energy value for inflows, as opposed to demands, as the scenario-dependent parameters; the opposite is true in May; (ii) in January and May, the Peace marginal energy values under the base case are closer to those for inflows, as opposed to prices, as the scenario-dependent parameters. In September, with the exception of the three healthiest storage conditions, the opposite is true; and, (iii) in January and May the Peace marginal energy values for the base case are closer to those with demands, as opposed to prices, as the scenario-dependent parameters, with some exceptions at low storage conditions. In September, except for the four highest storage conditions, the opposite is true.

15. For non-anticipative turbine releases, for all three months, with some exceptions, the base case Peace marginal energy values are closer to those with inflows, as opposed to demands, as the scenario-dependent parameters.

16. In May and September, the base case Peace marginal energy values are closer to those for demands, as opposed to inflows, as the scenario-dependent parameters under the assumption of non-anticipative ending storage volumes; the opposite is true in January.

17. In January and September, the largest departure from the base case Peace marginal energy values for all three anticipative assumptions occur for demands as the scenario-dependent parameters, with low initial storage. In May, the largest difference with either perfect foresight or non-anticipative turbine releases occurs with inflows as the scenario-dependent parameters and low initial storage, and occurs for non-anticipative ending volumes with demands as the scenario-dependent parameters and average initial storage.

18. With non-anticipative turbine releases: (i) the Peace marginal energy values for perfect foresight are closer, with some exceptions, to those for inflows, as opposed to demands, as the scenario-dependent parameters; (ii) in January and September, the Peace marginal energy values for perfect foresight are closer to those with prices, as opposed to demands, as the scenario-dependent parameters; the opposite is true in May; and, (iii) in January and September the Peace marginal energy values for perfect foresight are closer to those with inflows, as opposed to prices, as the scenario-dependent parameters. The opposite is true, with some exceptions when the reservoirs begin relatively full, in May.

19. In January and September with perfect foresight: (i) the Peace marginal energy values are closer to those with non-anticipative ending storage volumes when prices, as opposed to inflows, are the scenario-dependent parameters. The opposite is true in May, except for when the reservoirs begin relatively low; (ii) in January and September, the Peace marginal energy values are closer to those with non-anticipative
ending storage volumes when prices, as opposed to demands, are the scenario-
dependent parameters. The opposite is true in May; and, (iii) in January and
September, the Peace marginal energy values are closer to those with non-anticipative
ending storage volumes when the inflows, as opposed to demands, are the scenario-
dependent parameters. The opposite is true in May.

20. With demands as the only scenario-dependent parameters, the Peace marginal energy
values under the assumption of perfect foresight are closest to those for either non-
anticipative turbine releases or non-anticipative ending storage volumes in May, and
the furthest in January. The opposite was true with scenario-dependent prices. With
inflows as the only scenario-dependent parameters, the Peace marginal energy values
under the assumption of perfect foresight are closer to those for either non-
anticipative turbine releases or non-anticipative ending storage volumes in January
than for September; and, with non-anticipative turbine releases, the Peace marginal
energy values under the assumption of perfect foresight are closer to those in
September than those in May, with the opposite being true with non-anticipative
ending storage volumes.

21. With demands or prices as the scenario-dependent parameters, in all three months, the
Peace marginal energy values under the assumption of perfect foresight are closer to
those with the assumption of non-anticipative ending storage volumes than to those
for non-anticipative turbine releases. With inflows as the scenario-dependent
parameters, the same is true in May, and the opposite is true in January and
September.

22. With perfect foresight, for January and September, the Columbia marginal energy
values for the base case are closer to those with inflows, as opposed to either demands
or prices as the scenario-dependent parameters. The opposite is true for May.

23. With perfect foresight, in all three months, the base case Columbia marginal energy
values are closer to those with prices, as opposed to demands, as the scenario-
dependent parameters.

24. In January and September, with non-anticipative turbine releases, the base case
Columbia marginal energy values are closer to those with inflows, as opposed to
prices, as the scenario-dependent parameters. The opposite is true in May.

25. In January, with non-anticipative ending storage volumes, the base case Columbia
marginal energy values are closer to those with demands, as opposed to inflows, as
the scenario-dependent parameters, except for when the reservoirs are initially out of
balance. The same is true in September without the exceptions, and the opposite is
ture in May.

26. With non-anticipative turbine releases, in all three months (with some exceptions),
the Columbia marginal energy values under perfect foresight are closer to those with
inflows, as opposed to demands, as the scenario-dependent parameters. Similarly, the
Columbia marginal energy values under perfect foresight are closer to those with
prices, as opposed to demands, as the scenario-dependent parameters in all three
months. In January and May, the Columbia marginal energy values under perfect
foresight are closer to those with inflows, as opposed to prices, as the scenario-
dependent parameters. The opposite is true in September.

27. In January, with high initial storage conditions in the Columbia, the Columbia
marginal energy values for perfect foresight are close to those with each of the three
scenario-dependent parameters, with those for inflows being the closest, and those for prices being the furthest. For other initial storage conditions in January, the Columbia marginal energy values for perfect foresight are closest to those with prices as the scenario-dependent parameters, and furthest from those with demands as the scenario-dependent parameters.

28. In May, with high initial storage conditions in the Columbia, the Columbia marginal energy values for perfect foresight are close to those for all three scenario-dependent parameters, while for other initial storage conditions, are closest to those for prices, and furthest from those with inflows.

29. In September, the Columbia marginal energy values for perfect foresight are closest to those with prices as the scenario-dependent parameters, and furthest from those with scenario-dependent inflows.

30. In all three months, with either inflows or prices as the scenario-dependent parameters, the Columbia marginal energy values with perfect foresight are closer to those with non-anticipative turbine releases than those with non-anticipative ending storage volumes; the opposite is true for scenario-dependent demands.

31. In all three months, for all nine initial storage conditions considered, the reservoir from which generation should preferentially come are the same in the base case as in each of the 125-scenario cases studied. This result suggests that, with the reservoir discretization employed, the base case is sufficient for choosing which reservoir to run first. However, as the marginal energy values differ among the various cases considered, the marginal energy values produced by the case base cannot be used in making energy trade decisions.
6 Literature Cited


