Displacement-based Design of Concrete Tilt-up Walls

by

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Concrete tilt-up panels are commonly used in constructing lower-rising buildings in North America. Large window openings are sometimes required in the wall panels on the whole side of tilt-up buildings. Subject to severe earthquake, such a tilt-up panel is expected to experience inelastic deformation that has to be controlled. The relevant clause 21.7.1.2 is presented in the draft 2004 Canadian concrete code CSA Standard A23.3 regarding the displacement demand of concrete panel, which says: “Tilt-up Wall Panels shall be designed to the requirements of Clause 23 except that the requirements of Clause 21.7.2 shall apply to wall panels with openings when the maximum rotational demand on any part of the panel exceeds 0.02 radians.” The aim of this research is to develop a simple method to estimate the inelastic displacement demand of tilt-up wall panels with openings accounting for the influence of a flexible metal roof. The tilt-up panel with openings is modeled as a static frame simply supported at the ground and the roof is modeled as a simply-supported beam at the tilt-up walls. The first mode of the wall is assumed to be dominant over all other modes, and the wall panel can be idealized as a single degree of freedom system with equivalent stiffness and concentrated mass. And also, the first mode of the roof is assumed to be dominant over all other modes, so that the roof can be idealized as a single degree of freedom system with equivalent stiffness and concentrated mass. Hence, the lateral force resisting system in a typical tilt-up building is modeled as an idealized 2 degree of freedom system. The properties of typical tilt-up panels and metal roofs commonly used in the lower mainland of British Columbia are investigated, and the possible range of stiffness ratios and mass ratios between wall panel and roof are presented. The range of the stiffness ratios and the mass ratios is used in this research.

Program CANNY is used to perform simulations on computer with the idealized 2-DOF system. Total five earthquake records are used as excitations to the testing system. Three of them are modified to fit Vancouver acceleration spectrum of NBCC 2005, and two of them come from City of Los Angeles and Seattle. All of them are modified so that the earthquakes used in this research have a 2% probability of exceedance in 50 years.
The deformation of metal deck diaphragm developed by physical tests is studied. The severe pinching of the testing deck after it yields is realized to be critical over all other nonlinear dynamic responses. Heavy Pinching creates a small area enclosed by each force-displacement cycle of deck, which indicates very limited energy dissipation by deck yield. So, pinching of metal deck is the most important factor considered in the selection of hysteresis model for the nonlinear metal roof.

The simulation starts with linear elastic 2-DOF system. The yield strength of wall or roof used in the simulation of nonlinear 2-DOF system is determined by dividing the maximum force of wall or roof in the corresponding linear system by the force reduction factor (2 for concrete tilt-up panel). The simulations include 2 stages. During the first stage of test, the various factors that may affect the dynamic response of system, particularly the total elastoplastic displacement demand of tilt-up wall, are investigated. The parameters that were investigated include the force reduction factor of wall, the ductility of roof, the stiffness ratio of wall to roof, the mass ratio of roof to wall, the wall stiffness, the earthquake record. The equal displacement principal is verified for the current 2-DOF system by running simulations.

After first stage of simulations, an important discovery is presented, which is the roof drift or the roof force is reduced proportional to the force reduction of tilt-up wall panels due to the yield of wall panels. A simple formula is developed to estimate the maximum inelastic wall displacement demand according to the above discovery and the equal displacement principal. In the second stage of simulations, the formula is confirmed using the inputs derived from practical wall and roof. The linear solution of the corresponding 2-DOF system is presented using spectral acceleration of Code NBCC and mode extraction. The total elastic roof displacement and roof drift relative to wall with the corresponding linear system are needed to estimate the inelastic wall displacement demand in the nonlinear system. The thesis ends with summaries and conclusions of the research and suggestion for further work.
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CHAPTER 1

1. Introduction

1.1 Background of Research

Concrete tilt-up panels are widely used as loads bearing walls in constructing single storey industrial, commercial and recreational buildings. Attracted by its advantages of high strength, low cost, time-saving of construction, excellent durability to resist severe weathers and corrosive agents, both structural engineers and clients select tilt-up panels to construct the exterior walls of their low-rise buildings. Along with the acknowledgement of earthquake hazards and the development of seismic engineering theory, the tilt-up panel is required by Code to be efficient in resisting expected earthquakes in its service time.

During the lifetime of a tilt-up building located in a seismic active area, buildings are expected to experience few large severe earthquakes. Concrete tilt-up wall panels, together with roof diaphragm, are the primary element to resist lateral loads in tilt-up buildings. The solid panel and the panel with small openings are stiff and strong. They are believed to remain linear elastic through great earthquakes without any plastic deformations. So, the solid panel and panel with small openings can be designed as an elastic structural element to resist the maximum anticipated seismic load without yield of any sections.

It is quite often that large openings are required along the entire side of the building. Such examples are usually seen in the shopping mall with store fronts, and industrial warehouse or plant with front offices. It is uneconomical and impractical to design such a tilt-up building to
resist lateral loads caused by the maximum anticipated earthquake loads while its tilt-up panels with large openings remains linear elastic. The inelastic deformation at a specific section of panel can greatly dissipate seismic energy in tilt-up buildings and limit the damage to tilt-up panels. And thus a ductile structural design is advised for the tilt-up panels with big openings to resist critical earthquake as long as the inelastic deformation can be controlled under the capacity.

CSA standard in Clause 21.7 advises a moderate ductility, $R_d=2$, for the elastoplastic structural design of such a tilt-up panel. The relevant clause 21.7.1.2 is presented in the draft 2004 Canadian concrete code CSA Standard A23.3 regarding the displacement demand of concrete panel, which says: “Tilt-up Wall Panels shall be designed to the requirements of Clause 23 except that the requirements of Clause 21.7.2 shall apply to wall panels with openings when the maximum rotational demand on any part of the panel exceeds 0.02 radians.” Through this clause, CSA standard advises that the concrete tilt-up panel is not only designed for its strength but also for its deformation capability when it is expected to undergo earthquake loads or other severe lateral loads.

Hence, how to calculate the maximum rotational demand of any part of concrete tilt-up panel is therefore of central importance in the structural seismic design of a tilt-up wall panel with openings. This research is complementary to the relevant chapter of CSA standard about the maximum total elastoplastic displacement demand of a plastic-designed tilt-up wall panel with large openings through the earthquake. A simple method to evaluate the rotational demand of
the concrete sectional hinge in the tilt-up panel through the earthquake is developed as part of this research.

A tilt-up building is designed to contain a lateral load resisting system to resist any horizontal loads arising from wind and earthquakes. Concrete tilt-up wall and steel roof, composed of steel chord angles and shear diaphragm of steel deck, are commonly used as the lateral-force-resisting system in a single storey tilt-up building. Currently this system is modeled as a single degree of freedom system (See Figure 1-2) composed of a concentrated point mass sitting on a spring. The point mass is calculated as the total mass on roof plus the half of the mass of surrounding wall panels, and the stiffness of spring is calculated as the lateral point load exerted at the top of wall panel where a unit displacement is formed. The total elastoplastic displacement of the wall panel with the force reduction factor equal to 2 can be found according to the equal displacement principle developed by Newmark and Rosenblueth (1971), (See Figure 1-1).

For the current case, the concept of equal displacement principal can be explained as follows: the total elastoplastic displacement of a ductile tilt-up panel at roof level is equal to the total elastic displacement of the corresponding elastic panel subjected to the same excitation of earthquake loads.

In the Figure 1-1 are contrasted the elastic deformation process and in-elastic deformation process. $V$ in vertical coordinate axis is the maximum corresponding seismic force through
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the earthquake if the elastoplastic system is assumed to stay elastic with its original stiffness, \( V/\Delta_e \). This assumed elastic system is defined as the corresponding system to the original elastoplastic system. \( \gamma V/R \) in vertical coordinate axis is the designated yield strength of the elastoplastic system. It is expected by equal displacement principal that the total elastoplastic displacement, \( \Delta_y + \Delta_p \) of the initial system is approximately equal to the total elastic displacement, \( \Delta_e \) of the corresponding elastic system. Therefore the displacement demand of elastoplastic-designed wall panel with force reduction factor, \( R \) equal to 2.0, can be attained with the current single degree of freedom model in accordance with the equal displacement principal.

Great errors exist with the above method to calculate displacement demand of inelastic wall panels because the current model neglects the influence of flexible metal roof. The evidences can be referred to the observations and conclusions of simulations presented in Chapter 3 with a new model. The current model can not be executed in the displacement-based structural design of the lateral force resisting system of a tilt-up building. A new model is required more accurately to simulate the lateral force resisting system in a tilt-up building.

Accounting for the influence of flexible metal roof, the roof is modeled as an independent single degree of system sitting on the tilt-up wall. This is described in Chapter 2 in full details. The nonlinear dynamic analysis of the new model is performed with the aid of computer simulations. The observations and conclusions of the tests are presented in Chapter 3. Chapter 4 presents a simple formula developed to calculate the elastoplastic displacement demand of
ductile tilt-up wall panels based on the studies of simulation results. The accuracy of the formula is confirmed by performing extra simulation tests with variable setups of testing specimen. This research closes with Chapter 5, which is the summary of the all research.
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Seismic Load

\[ V, \Delta \varepsilon \]

\[ \frac{\gamma V}{R} \]

\[ \Delta_y \quad \Delta_y + \Delta_p \quad \text{Dis.} \]

Figure 1-1  Load-Deformation Relationships for an In-elastic System and Its Corresponding Elastic System

Figure 1-2  Single Degree of Freedom System Modeling Lateral Force Resisting System in a Tilt-up Building
2. Analytical Methods

This chapter starts with a description, in Section 2.1, of how the lateral force resisting system of a tilt-up building is modeled. Section 2.2 presents the tables of properties of typical tilt-up panels and metal roofs used in practice. Section 2.3 introduces Program CANNY, which was used for the nonlinear dynamic analysis, followed by a discussion of the hysteresis models used to simulate the force-displacement relationships in the idealized dynamic model. And then, presented is a description of the test performance conducted to investigate the in-elastic seismic behavior of tilt-up panels accounting for the influence of flexible roof diaphragm. This chapter closes with a description of the five recorded earthquakes used in the study.

2.1 Simplified Model

2.1.1 Lateral Force Resisting System

A typical tilt-up building consists of a metal roof and concrete tilt-up walls as the lateral force resisting system as shown in Figure 2-1(a). Under severe earthquake motions, it is not surprising that tilt-up panels with big openings undergo significant nonlinear deformations. This elastoplastic deformation of tilt-up panel should be controlled to an acceptable degree.

Subjected to a horizontal earthquake motion, the lateral force resisting system in a tilt-up building could be modeled as a complex dynamic system composed of a large number of elements with hundreds of degrees of freedom. It is difficult to calculate the displacement demand of tilt-up wall by performing a nonlinear analysis using such a model. Hence a
simplified model is developed for the easier analysis instead of a complex one. Consider the tilt-up panel with two big openings in Figure 2-2(a), constrained to move only in the direction of lateral seismic excitation. The static frame analysis problem has to be formulated with six degrees of freedom, 2 lateral displacements and 4 rotations, to determine the lateral stiffness of the tilt-up panel. In contrast, the structure has only a single degree of freedom, lateral displacement, for dynamic analysis if it is idealized with concentrated mass at the roof level.

The metal roof placed on the tilt-up wall at roof ends, shown in Figure 2-1(a), deforms only in the direction of earthquake motion. Hence it can be modeled as a simply supported I-beam with two flanges (steel chord angles at the edges of roof) and a web (diaphragm of roof deck). This beam, theoretically, can have infinite degrees of freedom with uniform distributed mass along the beam span. It is idealized as single degree of freedom system composed of a spring integrated with a concentrated mass if the first dynamic mode of this beam is dominant.

A general 2 degree of freedom dynamic system is therefore introduced to model the lateral force resisting system in tilt-up buildings. Referred to Figure 2-1(b), the tilt-up wall panel is modeled as a spring with a concentrated mass and the metal roof is modeled as another spring with a concentrated mass added on the top of the tilt-up panel. Subjected to horizontal earthquake motion, the nodes with concentrated mass can only move horizontally in plane with the constraint of the springs. The whole system is fixed to the ground. The second-order moment induced by gravity load acting with the lateral deflection of nodes is not included in the analysis. The equivalent stiffness and mass of the roof are described in Section 2.1.2, and
CHAPTER 2 Analytical Methods

those of tilt-up panel are described in Section 2.1.3.

2.1.2 Metal Roof

Equivalent Mass

As stated in Section 2.1.1, subjected to lateral loads, the roof diaphragm acts similar to a simply supported beam with uniform distributed mass. Consider the roof deformation graphically displayed in Figure 2-1(a) and the roof deforms similar to a curve of sin function in half period cycle. Hence this beam can be idealized as a single degree of freedom dynamic system with the overwhelming first mode. Theoretically, the first mode shape for a simply supported beam is expressed as:

\[ \phi(x)_b = \sin \left( \frac{\pi x}{L} \right) \]  

(2.1)

Wherein, \( x \) is the coordinate variable in the direction of beam span, and \( L \) is the length of beam span. The natural circular frequency corresponding to the first mode of beam is given as:

\[ \omega_b = \frac{\pi^2 EI}{L^2 \sqrt{m}} \]  

(2.2)

Wherein, \( m \) is the mass per unit length of beam, and \( EI \) is the flexural stiffness of beam.

If each end of beam is free, hinged, or clamped, the stiffness corresponding to \( nth \) mode is given as:

\[ K_n = \int_0^L EI(x)[\phi_n''(x)]^2 \, dx \]  

(2.3)

And the mass corresponding to \( nth \) mode is given as:

\[ M_n = \int_0^L m(x)[\phi_n(x)]^2 \, dx \]  

(2.4)
Wherein, \( EI(x) \) is a function of sectional flexural stiffness with respect to the coordinate variable \((x)\) in the direction of beam span, but \( EI(x) \) is constant for the current case. \( m(x) \) is a function of distributed mass over the beam, but it is constant for the current case.

The stiffness corresponding to the first mode of beam is attained by substituting Eq. 2.1 in Eq. 2.3 and performing mathematic integral over the length of beam span:

\[
K_1 = \frac{\pi^4}{2L^4} EI
\]

The mass corresponding to the first mode of beam is attained by performing mathematic integral over the length of beam span:

\[
M_1 = \frac{m \cdot L}{2}
\]

The “base shear” of beam attributed to the \( \text{nth} \) mode is given as:

\[
V_n = \Gamma_n A_n \int_0^L m(x) \psi_n(x) dx
\]

where

\[
\Gamma_n = \frac{L_n}{M_n} \quad L_n = \int_0^L m(x) \psi_n(x) dx
\]

\( A_n \) is the pseudo-acceleration corresponding to the \( \text{nth} \) mode.

Performing mathematic integral over Eq. 2.7 from 0 to \( L \) gives:

\[
V_{2n-1} = \frac{8mL}{(2n-1)^2 \pi^2} A_n \quad V_{2n} = 0 \quad \text{where } n=1, 2, 3...
\]

So, the effective mass of the \( \text{nth} \) mode is:

\[
M_{\text{eff}}^{2n-1} = \frac{8mL}{(2n-1)^2 \pi^2} \quad M_{\text{eff}}^{2n} = 0
\]

Substituting \( n=1, 2, 3 \) in Eq. 2.10 gives:
CHAPTER 2 Analytical Methods

\[ M_1^* = 0.81 mL \quad M_2^* = 0 \quad M_3^* = 0.09 mL \] (2.11)

From Eq. 2.11, the first mode account for 81% of total beam mass. Including all mode contribution using the rule of square root of sum of square pseudo and the acceleration of first mode give the equivalent mass of "beam" as:

\[ \tilde{M}_{beam} = \frac{\sqrt{\sum_{i=1}^{n} (M_i^* A_i)^2}}{A_i} \] (2.12)

For all metal roofs in our cases (roof size is from 200 ft \( \times \) 500 ft to 500 ft \( \times \) 200 ft), the natural period of "beam" beyond third mode is less than 0.2 second. Note that there is cut-off on the acceleration spectrum of NBCC 2005 for the period less than 0.2 second. The current tilt-up systems in our cases have the peak spectral acceleration of NBCC 2005 for the third mode. Considering the effective mass is zero for odd modes, Eq 2.12 can be conservatively re-arranged as:

\[ \tilde{M}_{beam} = \frac{\sqrt{(M_1^* A_1)^2 + (mL - M_1^*) A_1)^2}}{A_1} \] (2.13)

where, \( A_i \) is the peak acceleration in NBCC 2005 spectrum.

For simplicity, the total mass of "beam" is conservatively suggested as the equivalent mass of "beam".

The transverse panels move with the roof diaphragm by their firm connections in the direction of seismic exciation. The stiffness and strength of transverse panels are neglected conservatively in the calculations since they are much smaller than those of in-plane wall panels. However, partial mass of transverse panels should be added to the roof diaphragm. For
simplicity, the equivalent mass of roof, $M_{\text{eq}}$, should be the total mass on roof plus half mass of transverse wall. For some of cases, using half mass of transverse wall may underestimate the wall mass involved in seismic motions depending on geometry of transverse wall panels.

**Equivalent Stiffness**

As the roof diaphragm is modeled as a simply supported I-beam with a web representing the diaphragm of metal deck to take in-plane shear, the shear deformation in roof diaphragm should be included in the calculation of the equivalent roof stiffness. The influence of roof shear deformation is included by introducing the equivalent sectional flexural stiffness of roof diaphragm, $EI_{\text{eq}}$, calculated as shown below.

Consider a metal roof (See Figure 2-1(a)) is simply supported at both ends and subjected to a lateral uniformly distributed in-plane load, $w$, along the roof span. This roof is modeled as a simply-supported beam presented by Figure 2-1(c). With the assumption of only the steel chord angle placed at the edges of roof resisting in-plane moment, the flexural deflection in the middle of roof span, $\Delta_m$, is given as:

$$\Delta_m = \frac{5wL^4}{384EI} \tag{2.14}$$

Wherein, $I = A \times \left(\frac{D}{2}\right)^2$, $A$ is the sectional area of steel chord angle.

With the assumption of only the metal deck taking the in-plane shear, the shear deflection in the middle of roof span, $\Delta_s$, is given as:

$$\Delta_s = \frac{q_{\text{ave}}LF}{2 \times 10^6} = \frac{wL^2F}{8 \times 10^6 D} \tag{2.15}$$
Wherein, $q_{AVG}$ is the average shear in roof, $D$ is the width of roof, and $F$ is the flexibility factor corresponding to the shear stiffness of roof diaphragm. See CSSBI B-13-91, Design of Steel Deck Diaphragms, for more details.

With the existence of shear and flexural deformations, the roof diaphragm can be simulated as a spring, representing the flexural stiffness of roof diaphragm ($EI$), connected to another spring, representing the shear stiffness of roof diaphragm ($K_s$). The equivalent flexural stiffness of roof diaphragm, $EI_{eq_i}$, is calculated as follows:

$$\frac{1}{EI_{eq_i}} = \frac{1}{EI} + \frac{1}{K_s}$$

(2.16)

Re-arranging this formula, the equivalent flexural stiffness is

$$EI_{eq_i} = \frac{EI}{1 + \frac{EI}{K_s}}$$

(2.17)

Because the flexural stiffness of the roof diaphragm is inversely-proportional to the flexural deflection in the middle of roof span, $\Delta_m$, and the shear stiffness of roof diaphragm is inversely-proportional to the shear deflection in the middle of roof span, $\Delta_s$. The equivalent flexural stiffness of roof diaphragm is attained by substituting $\Delta_m$ and $\Delta_s$ into Eq. 2.17:

$$EI_{eq_i} = \frac{EI}{(1 + \frac{\Delta_s}{\Delta_m})}$$

(2.18)

The roof diaphragm is therefore modeled as a simply supported beam with the equivalent flexural stiffness, $EI_{eq_i}$. Accounting for the influence of shear deformation of roof diaphragm, the natural circular frequency of this beam is attained by substituting Eq. 2.14 in Eq. 2.2:

$$\omega_{n_{eq_i}} = \frac{\pi^2}{L^2} \sqrt{\frac{EI_{eq_i}}{m_{eq_i}}}$$

(2.19)

Wherein, $m_{eq_i} = M_{n_{eq_i}}/L$. 
In the end, the equivalent stiffness of roof diaphragm, $K_{B\text{-eqi}}$, is given as:

$$K_{B\text{-eqi}} = M_{B\text{-eqi}} \times \omega_{B\text{-eqi}}^2$$ \hspace{1cm} (2.20)

2.1.3 Tilt-up Wall Panel

Equivalent Stiffness

Consider a tilt-up panel with two openings shown in Figure 2-2(a), which takes in-plane seismic loads in tilt-up buildings. The huge lateral load caused by total roof mass acting with seismic excitation is transferred to the top of tilt-up wall. The whole wall panel is expected to sway in the direction of excitation, as shown by dashed line in Figure 2-2(a). The “sway” of panel with respect to the ground is the fundamental mode shape and dominant over all other modes. Therefore, the tilt-up panel can be modeled as a single degree of freedom system, a spring with a point mass sitting on the top. The roof diaphragm is modeled as another single degree of freedom system sitting on the top of tilt-up wall.

The panel with openings is modeled as a frame with beams and columns located at the centroid of the concrete sections, and rigid extensions to model the joint region as shown in Figure 2-2(b). Performing static frame analysis to the panel and accounting for the bending and shear deformations, the in-plane stiffness of the panel is equal to the lateral point load applied at roof level to cause a lateral unit displacement at the roof level. More details about frame analysis are presented in Appendix A.

The nonlinear stress-strain relationship of concrete should be included in this frame analysis.
CHAPTER 2 Analytical Methods

The concrete can be considered as a linear elastic material until it cracks in tension. Under the gravity and seismic loads, tilt-up wall panels undergo complex stress in the section induced by the combination of shear, moment, and axial force. Part of a concrete section may be in tension stress. As concrete is low in tension strength, cracks commonly occur at the tension side of the section, which decreases the sectional stiffness. Hence the post-cracking stiffness should be used in the frame analysis. To simplify the problem, an effective sectional stiffness of concrete was developed using Program Response 2000. The complete curves of sectional moment versus curvature are shown in Appendix B for the two concrete sections making up the beam and column elements in a typical tilt-up panel. The effective sectional stiffness, $EI_{eff}$, was calculated graphically using the equal area concept. Because the concrete tilt-up panel takes very limited gravity load in tilt-up buildings, the effective stiffness should be taken as 25% of the gross sectional stiffness ($I_{eff} = 0.25I_g$).

Equivalent Mass

It is suggested that half of the panel mass be used as the equivalent mass of idealized single degree of freedom system modeling the tilt-up panel subjected to a horizontal earthquake motions.

Consider a tilt-up panel shown in Figure 2-2(a), which is modeled as a static-indeterminate frame simply supported to the ground. Theoretically it can be idealized an assemblage of finite elements interconnected only at nodes. The fundamental period of this frame, $T_1$, is attained by performing finite element analysis with the aid of computer software; say E-tab or SAP 2000.
The generalized mass, \( M_w \), and the generalized stiffness, \( K_w \), for the first mode, are related:

\[
M_w = \frac{T_1^2 \cdot K_w}{4 \cdot \pi^2}
\]  

(2.21)

An alternate simple method to calculate the equivalent mass of tilt-up panel is introduced as follows. Consider a panel shown in Figure 2-2(a). Under the horizontal earthquake motions, no significant amplification of the ground motion at the top of the wall (See Papers by Freeman and Hawkins), the whole panel sways in the direction of excitation shown in Figure 2-3. Impart a unit acceleration at the ground level of panel, and the acceleration can be assumed evenly distributed through the height of tilt-up panel. Suppose the total beam masses \( M_{B1} \) and \( M_{B2} \) are concentrated at the beam level, respectively, and the total mass of column element, \( M_c \), is uniform distributed through the height \( H \) of column. Assume unit acceleration is applied at the ground level of tilt-up panel. The moment about the ground by the equivalent mass at the roof level acting with ground acceleration should be balanced by the moment produced by the distributed mass acting with the local acceleration. Therefore, the resulting mass at the roof level of panel, or equivalent mass of panel \( M_w \), is given as:

\[
M_w = 2 \cdot (1/2 \cdot M_c) + 1/2 \cdot M_{B2} + M_{B1}
\]

(2.22)

Contrasting to the total mass of panel \( 2M_c + M_{B2} + M_{B1} \), half of the total mass of panel is advised to be used as the equivalent mass for simplicity.

### 2.2 Typical Wall Panels and Metal Roofs

In the framework of setting up dynamic simulation tests, one of our interests is the case of all possible stiffness and mass of real roof and tilt-up panel that should be included in the tests.
Three typical tilt-up panels with different size of sections are selected to combine with six real metal roofs varying in their sizes.

Table 2-1 presents the values of equivalent stiffness, equivalent mass and natural period of three typical tilt-up panels with two openings corresponding to the first mode shape of dynamics. These tilt-up panels can be modeled as indeterminate static frames with the beam and column elements located at the centroid of their section. Shown in Figure 2-2, all the panels have a span of 8.1 m (from centroid of leg to centroid of leg) and a roof height of 6.75 m (from ground to centroid of upper beam), but have different sizes of beams and columns (See Table 2-1). Accounting for the influence of a light gravity load of 20 kN/m, the results are shown for including cracked sections and un-cracked sections in Table 2-1. Note that the influence of rigid extensions to model the joints in the frame is not included.

With the suggested methods to calculate $M_w$ and $K_w$ as shown in the previous sections, the period of tilt-up panel can be calculated as follows:

$$T_w = 2\pi\sqrt{M_w/K_w} \quad 2.23$$

<table>
<thead>
<tr>
<th>Wall Type</th>
<th>Wall Column Section</th>
<th>Wall Beam Section</th>
<th>Uncracked Wall (Elg)</th>
<th>Cracked Wall (Elc)</th>
<th>Wall Type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Wall Beam Section</td>
<td>Equivalent Stiffness</td>
<td>Equivalent Mass</td>
<td>Period</td>
<td>Equivalent Stiffness</td>
</tr>
<tr>
<td></td>
<td>(mm)</td>
<td>(kN/m)</td>
<td>(kg)</td>
<td>(second)</td>
<td>(kN/m)</td>
</tr>
<tr>
<td>Wall-1</td>
<td>600x190</td>
<td>900x190</td>
<td>4725</td>
<td>5043</td>
<td>0.205</td>
</tr>
<tr>
<td>Wall-2</td>
<td>900x190</td>
<td>1200x190</td>
<td>16614</td>
<td>7995</td>
<td>0.138</td>
</tr>
<tr>
<td>Wall-3</td>
<td>1200x235</td>
<td>200x235</td>
<td>45203</td>
<td>13016</td>
<td>0.107</td>
</tr>
</tbody>
</table>
Six typical roofs were selected with different sizes and configurations. All roofs were assumed to be subjected to 20 psf dead load and 10 psf live load. Standard deck profile was used with the same depth of 38 mm for all six roofs as well as 1800 mm joist spacing, 450 mm o.c. button punch. Also, the ratio of span of deck unit to average length of deck sheet (R) was assumed to be 1 for all roofs.

The roofs are different by geometric size, deck gauge and chord angle. For the roof with smaller size, such as Roof-1, Roof-2, Roof-3, and Roof-4 shown in Table 2-2, 22 gauge corrugated deck was used for normal shear area (80% of total roof area), 20 gauge corrugated deck was used for high shear area (20% of total roof area). For the larger roof, Roof-5 and Roof-6 shown in Table 2-2, 20 gauge corrugated deck was used for normal shear area (80% of total roof area), 18 gauge corrugated deck was used for high shear area (20% of total roof area).

After calculate $M_r$ and $K_r$ using the method as described in the former section, the period of roof can be calculated as follows:

$$T_r = 2\pi \sqrt{M_r/K_r}$$
Table 2-2 Properties of Metal Roof

<table>
<thead>
<tr>
<th>Deck Gauge</th>
<th>Roof Size (L by W)</th>
<th>A (chord)</th>
<th>I (chord)</th>
<th>Equivalent Stiffness</th>
<th>Total Mass</th>
<th>Period</th>
<th>Roof Type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ft</td>
<td>mm²</td>
<td>mm⁴</td>
<td>kN/m</td>
<td>kg</td>
<td>second</td>
<td></td>
</tr>
<tr>
<td>22/20</td>
<td>200x100</td>
<td>927</td>
<td>518000</td>
<td>12480</td>
<td>272200</td>
<td>0.928</td>
<td>Roof-1</td>
</tr>
<tr>
<td></td>
<td>100x200</td>
<td>927</td>
<td>518000</td>
<td>70790</td>
<td>272200</td>
<td>0.39</td>
<td>Roof-2</td>
</tr>
<tr>
<td></td>
<td>200x100</td>
<td>1850</td>
<td>1840000</td>
<td>15000</td>
<td>272200</td>
<td>0.846</td>
<td>Roof-3</td>
</tr>
<tr>
<td></td>
<td>100x200</td>
<td>1850</td>
<td>1840000</td>
<td>72960</td>
<td>272200</td>
<td>0.384</td>
<td>Roof-4</td>
</tr>
<tr>
<td>20/18</td>
<td>500x200</td>
<td>3060</td>
<td>4680000</td>
<td>14190</td>
<td>1361000</td>
<td>1.946</td>
<td>Roof-5</td>
</tr>
<tr>
<td></td>
<td>200x500</td>
<td>3060</td>
<td>4680000</td>
<td>91290</td>
<td>1361000</td>
<td>0.767</td>
<td>Roof-6</td>
</tr>
</tbody>
</table>

Table 2-3 presents the characteristic in the combinations of three uncracked wall panels with six metal roofs, and so does Table 2-4 except that the effect of post-cracking stiffness is included for the tilt-up wall panels. The roof stiffness and mass are calculated per panel width in a tilt-up building to match those of a couple of tilt-up panels because the lateral force resisting system composed of tilt-up wall and roof diaphragm can be simplified as a strip of roof diaphragm integrated with a couple of tilt-up panels placed at its ends. The equivalent stiffness and mass of a couple of tilt-up panels, $K_w$ and $M_w$, are calculated using the method presented in Section 2.1.3. The equivalent stiffness and mass of roof per panel width, $K_r$ and $M_r$, are calculated by the method described in Section 2.1.2. Modeling the tilt-up panels integrated with roof diaphragm as an idealized 2 degree of freedom system, the first and second mode period of this system can be evaluated by performing linear dynamic analysis (See Section 4.1).
## Table 2-3  Combination of Roof and Un-cracked Wall

<table>
<thead>
<tr>
<th>Wall Type</th>
<th>Roof Type</th>
<th>$K_w$ (2 Walls)</th>
<th>$M_w$ (2 Walls)</th>
<th>$K_r$ (per unit wall width)</th>
<th>$M_r$ (per unit wall width)</th>
<th>$K_r/K_w$</th>
<th>$M_r/M_w$</th>
<th>First Period</th>
<th>Second Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall-1</td>
<td>Roof-1</td>
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<td>10086</td>
<td>3685000</td>
<td>80378</td>
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<td>8.0</td>
<td>1.100</td>
<td>0.173</td>
</tr>
<tr>
<td></td>
<td>Roof-2</td>
<td>9450000</td>
<td>10086</td>
<td>10450000</td>
<td>40189</td>
<td>0.9</td>
<td>4.0</td>
<td>0.586</td>
<td>0.137</td>
</tr>
<tr>
<td></td>
<td>Roof-3</td>
<td>9450000</td>
<td>10086</td>
<td>44290000</td>
<td>80378</td>
<td>2.1</td>
<td>8.0</td>
<td>1.033</td>
<td>0.168</td>
</tr>
<tr>
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<td>10770000</td>
<td>40189</td>
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<td>4.0</td>
<td>0.582</td>
<td>0.135</td>
</tr>
<tr>
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<td>20950000</td>
<td>200945</td>
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<td>2.153</td>
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<td>53910000</td>
<td>80378</td>
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<td>0.162</td>
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<td>15990</td>
<td>3685000</td>
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<td>5.0</td>
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<td>10450000</td>
<td>40189</td>
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<td>0.452</td>
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<tr>
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<td>5.0</td>
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</tr>
<tr>
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<td>40189</td>
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<td>2.5</td>
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<td>12.6</td>
<td>2.007</td>
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<td>5.0</td>
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</tr>
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<td>44290000</td>
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</tr>
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<td>10770000</td>
<td>40189</td>
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Minimum= 0.9 1.5 0.408 0.100
Maximum= 43.2 19.9 2.153 0.186
### Table 2-4 Combination of Roof and Cracked Wall

<table>
<thead>
<tr>
<th>Wall Type</th>
<th>Roof Type</th>
<th>$K_w$ (2 Walls)</th>
<th>$M_w$ (2 Walls)</th>
<th>$K_r$ (per unit wall width)</th>
<th>$M_r$ (per unit wall width)</th>
<th>$K_w/K_r$</th>
<th>$M_r/M_w$</th>
<th>First Period</th>
<th>Second Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall-1</td>
<td>Roof-1</td>
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<td>3685000</td>
<td>80378</td>
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<td>8.0</td>
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<td></td>
<td>Minimum</td>
<td></td>
<td></td>
<td>0.2</td>
<td>1.5</td>
<td>0.468</td>
<td>0.160</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td></td>
<td></td>
<td>12.8</td>
<td>19.9</td>
<td>2.643</td>
<td>0.292</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 2-5 Summaries of Table 2-3 and Table 2-4

<table>
<thead>
<tr>
<th>ITEM</th>
<th>Uncracked Wall Minimum</th>
<th>Cracked Wall Minimum</th>
<th>Uncracked Wall Maximum</th>
<th>Cracked Wall Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_w$/$K_r$</td>
<td>0.9</td>
<td>43.2</td>
<td>0.2</td>
<td>12.8</td>
</tr>
<tr>
<td>$M_r$/$M_w$</td>
<td>1.5</td>
<td>19.9</td>
<td>1.5</td>
<td>19.9</td>
</tr>
<tr>
<td>$T_w$ (s)</td>
<td>0.107</td>
<td>0.205</td>
<td>0.196</td>
<td>0.396</td>
</tr>
<tr>
<td>$T_r$ (s)</td>
<td>0.384</td>
<td>1.946</td>
<td>0.384</td>
<td>1.946</td>
</tr>
<tr>
<td>$T_1$ (s)</td>
<td>0.408</td>
<td>2.153</td>
<td>0.468</td>
<td>2.643</td>
</tr>
<tr>
<td>$T_2$ (s)</td>
<td>0.1</td>
<td>0.186</td>
<td>0.16</td>
<td>0.292</td>
</tr>
</tbody>
</table>
2.3 Nonlinear Analysis

For an inelastic structural design of tilt-up panels, the maximum inelastic displacement demand should be investigated under the displacement capacity of tilt-up panel. In this research, nonlinear analysis of computer-based simulations is performed to develop a simple method of calculating the elastoplastic displacement demand of tilt-up panel. The preparations of these simulations are described in this section.

2.3.1 Program CANNY

CANNY is computer software to simulate the seismic behavior of structures through the existing or man-made earthquakes. It was originally developed by a Ph.D. student at the University of Tokyo in FORTRAN, and later it was re-written in C/C++ in 1995/1996, and became a general purpose computer program in the field of structures. The program can be used to perform non-linear dynamic analysis of three-dimensional reinforced concrete and steel frame structures subjected to the combinations of multiple components of earthquake motion in X, Y, Z directions and rotations. It also deals with time-varying external loads and static loads including gravity load, gradually increased load or cyclic load. Program CANNY was used here to simulate the nonlinear dynamic response of a general two degree of freedom system subjected to five typical earthquakes. For the current case of a generalized 2 degree of freedom system, it gives the natural mode periods, natural mode circular frequencies, mode shapes, the displacements at the nodes as well as the velocities and the accelerations, the seismic forces in two springs. Output includes the values at each time-historic step or the maximum and minimum values.
2.3.2 Hysteresis Models

Hysteresis Model for Concrete Tilt-up Wall Panel

Program CANNY includes a number of hysteresis models simulating non-linear force-displacement relationships. In this research, the hysteresis model, EL1, CL2 and SL2 were used to represent the linear and nonlinear behaviors of concrete tilt-up wall panel and metal roof.

The Model CL2, bilinear skeleton curve, is a uniaxial model, used to simulate the nonlinear force-displacement relationships of a concrete tilt-up panel. The force-deformation curve for the hysteresis model, CL2, is shown in Figure 2-4 with a small modification for the current case. The spring is initially linearly stretched in tension (see path 1). After concrete panel reaches its specified yield strength, the deformation continues to increase without increase of force (See path 2). While the ground motion reaches its peak acceleration in this excitation cycle and starts to change its loading direction, the panel picks up the load again when the load is lower than the yield strength. The panel starts to deform in the opposite direction but with a ever-lasting residual deformation (See path 3). The degradation of unloading stiffness and post-crack stiffness are neglected for the current case. It is because the concrete tilt-up wall panel is designed as equal strength for its both sides of section, the yield strengths of panel are equated for the both directions of loading. The pinching behavior is not functional in this model and the yield plateau is quite long due to close zero post-yield stiffness of panel, This model dissipates much of seismic energy after the panel yields. See the user manual of Program CANNY for more details.
Hysteresis Model for Metal Roof

The inelastic behavior of steel roof deck under cyclic loads was studied by Essa, Tremblay, and Rogers (2001) through a large number of physical tests. The observations of these tests are the basis for the selection of hysteresis model for the current roof diaphragm. A test plot showing the full cycle of curve of force against shear angle is copied as figure 2-6. As they are critical in the setups of simulation tests, some of the conclusions are summarized here as follows:

- The equivalent/critical damping ratio ranges between 4.1% and 6.1% for all diaphragms tested under quasi-static cyclic loading at 40% of the ultimate monotonic load.

- All tests exhibited strength degradation under cyclic loading following the peak load cycle. And the degradation occurred due to failures of some deck-to-frame connections. There also exists softening in steel deck after yielding, degradation of stiffness.

- The tests results showed clearly that the inelastic behavior, ductility, and energy dissipation characteristics were mainly dependent on the types of connections used. This indicates that the inelastic behavior, ductility and energy dissipation do not depend on the yielding of steel deck itself, which also supplies an explanation why the yielding plateau is so short. Yielding occurs due to the deformation at the connections not yielding of steel materials of deck itself. Therefore it is observed that the stiffness does not reduce much after yielding.

- Diaphragms with deck-to-frame fasteners made of either screws or welds showed a significant pinching behavior. Yield of roof does not dissipate much seismic energy. The full cycle of curve of force against shear angle includes little area.
Therefore the best hysteresis model to simulate the non-linear behavior of roof should include all the following aspects:

- The hysteresis model should include a degradation of stiffness after the roof yields, but post-yielding stiffness of metal deck is observed not reduce too much from its initial value before yield.
- The hysteresis model should include a gradual deterioration of strength with the increasing cycles of loadings after the roof yields.
- Heavy pinching behavior should be included in the model after the roof yields.
- The model should have symmetric shape of curve about the origin; there is no shift of the curve.
- Yielding plateau is very short and followed by reloading shortly.
- 5% damping coefficient is acceptable for the roof hysteresis model.

Currently no such hysteresis model in Program CANNY can exactly represent the nonlinear behavior of a metal roof under cyclic excitation. So a compromise had to be made in the selection of roof hysteresis model.

Consider the Load-Deformation Curve of testing a series of physical metal decks shown in Figure 2-6 by Essa, Tremblay and Rogers (2001). Only small area was enclosed by a full curve since the metal deck underwent a heavy pinching behavior after it yielded. Hence, not much seismic energy was dissipated by the yield of deck. Computer-Simulation tests were performed
to investigate Hysteresis Model SL2 for the metal roof. The result of Load-Deformation Curve of metal roof is presented in Figure 2-5. In simulations, the metal roof was idealized as a single degree of freedom system and hysteresis model (SL2) was used to simulate the Load-Deformation relationship of roof. The testing system was excited by an acceleration record of Earthquake La24. By the comparison of those curves shown in Figure 2-5 and Figure 2-6, the Load-Deformation curve developed by the computer-simulation tests has a narrow shape and encompasses a small area similar to the curve developed by testing physical deck. The small area enclosed indicates not much energy is dissipated by the yield of roof. Therefore, the hysteresis model (SL2) is believed appropriately to represent the nonlinear dynamic behavior of metal roof for the current study.

2.3.3 Test Procedure
The former section describes how the lateral force resisting system in a tilt-up building, composed of tilt-up panel and roof diaphragm, is modeled as a generalized two degrees of freedom system. This idealized system as shown in Figure 2-1(b) is composed of two springs and two node masses. The stiffness of the lower springs is equal to the lateral stiffness of wall panel and the stiffness of the upper springs is equal to the first mode stiffness of roof diaphragm. The mass of lower node is equal to half of the total mass of wall panel. The mass of upper node is equal to sum of the total mass on roof and half mass of transverse panels. The computer-simulation tests are performed with this system. The other inputs needed for simulations include force reduction factor (R) for tilt-up panel and roof, damping rate, and existing earthquake records.
For each testing specimen with specific configurations, two tests are carried out: linear simulation and nonlinear simulation. The linear simulation is performed with the spring elements remaining linear elastic force-deformation relationships. The hysteresis model, EL1, is used to simulate the linear force-deformation relationship for both springs modeling the idealized tilt-up panel and roof diaphragm. The purpose of this linear simulation is to obtain maximum seismic force demand of tilt-up panel and roof. And thus, the yield strength of tilt-up panel and roof can be established by dividing the maximum force demand by force reduction factor (R) for nonlinear simulation. As the force reduction factor (R) is assumed to be 2.0 for concrete shear walls according to NBCC Codes, the yield strength of tilt-up panel with the nonlinear hysteresis model (CL2) is obtained by dividing the maximum seismic force by 2.0.

Nonlinear simulation is performed with the same specimen used in the corresponding linear simulation except that the nonlinear hysteresis model (CL2) is used for the inelastic tilt-up panel instead of Hysteresis Model EL1. The roof diaphragm continues to remain linear elastic force-deformation relationships with the same hysteresis model (EL1). The maximum displacement demand of the tilt-up panel is obtained by performing nonlinear simulation. The ductility is studied in Chapter 3 instead of displacement because the total displacement demand of wall depends on too many parameters, such as magnitude of excitation, mass of wall and roof, stiffness of wall and roof, the force reduction factor for wall and roof. To simplify the problem, the displacement demand of wall is indirectly studied by investigating the ductility of wall.
For the next step of simulation tests, the strength capacity of roof diaphragm is specified less than the expected maximum seismic force, and thus the roof diaphragm as well as tilt-up wall undergoes nonlinear deformation. The displacement demand of tilt-up panel will be studied accounting for the influence of roof yield.

The observations and conclusions of the simulations are presented in Chapter 3. Before that, there are three words to be clarified for the meanings as follows: The “ductility of wall or tilt-up panel” is referred to the maximum ductility of wall or tilt-up panel. For simplicity, “ductility” is used instead of “maximum-ductility”. The “lateral force resisting system”, composed of tilt-up wall panel and roof diaphragm, is modeled as an idealized 2 degree of freedom dynamic system, for simplicity, this system will be named as “system” in the proceeding chapters. The “mass of wall” is referred to the half mass of tilt-up wall, which indicates the actual equivalent mass of wall panels involved in the seismic motion.

2.3.4 Inputs of Equivalent Mass and Stiffness

In order to perform simulation tests, a complete file of input data should be prepared. These data include existing earthquake records, force reduction factor for roof and wall, equivalent stiffness of wall and roof, and equivalent mass of wall and roof. Except the force reduction factor described in Section 3.3.4, the rest of them will be presented as follows:

The mass and stiffness used for the idealized tilt-up panel, modeled as the lower spring and the mass placed at the lower node (See Figure 2-1), are obtained from the practical full-scale of
tilt-up panel. The stiffness and mass of roof are determined proportional to those of roof. The ratio between roof and wall is within the range presented in Table 2-5 for the mass and stiffness. However, the presumptive equivalent stiffness and mass were used for the simulations of “fixing mass ratio levels and changing stiffness ratios of wall over roof”.

Prior to study the elastoplastic displacement of non-linear tilt-up wall integrated with flexible roof, the basic understandings of dynamic characteristics should be explored for a general 2 degree of freedom system by performing simulation tests with presumptive input data. For the observations of tests presented in Section 3.1.1, the input data were assumed as follows: the stiffness of wall was fixed to a constant, 2500 kip per feet. The total mass of roof and wall was set to 20 kip. These values are not attained from the real full-scale tilt-up panels as it is intended to develop the understandings about the dynamic characteristics of a generalized two degree of freedom vibration system.

Four real tilt-up panels are selected to calculate the equivalent stiffness and mass. The first specimen represents the type of a relatively rigid tilt-up panel, whose size is 8.5 m long by 8 m high by 0.14 m deep. The second tilt-up panel, representing the type of flexible tilt-up panel, is the same as the first one except with two openings of 5 meter long by 2.5 meter wide, located in the middle of panels. Standard hard grade 400 MPa reinforcing steel of 18M, placed at a spacing of 400 mm, is specified for both faces of panels. Static analysis, by which the lateral stiffness of panel is evaluated, does not take into account the rigid extensions of concrete joist zone and post-cracking stiffness. With the aid of commercial structural software, say SAP 2000,
the natural periods of these two panels are approximately 0.1 second and 0.38 second respectively representing the two extreme cases in the period scope of flexible panels (See Table 2-1) that could undergo in-elastic deformation in earthquake.

The whole simulations include two stages. The first stage of simulation tests is performed using the two typical tilt-up panels introduced above. The observations and conclusions are presented in Chapter 3. In the second stage, a simple method of estimating the total elastoplastic displacement demand of tilt-up panel is developed and confirmed by simulations with the test setups based on the other two real tilt-up panels, exactly used in practical structural design. See Chapter 4 for more details. In the first stage of simulations, tests are separated into several groups. In each group of tests, the influence of a specific factor on the inelastic displacement demand of flexible tilt-up panels is studied.

The stiffness and mass of an idealized roof diaphragm, modeled as the upper spring and the concentrated mass at upper node (See Figure 2-1), is proportional to those of panels in accordance with the indicative scope. The setups of mass and stiffness for the generalized model tends to contain all possible cases of tilt-up wall integrated with roof diaphragm. Referred to Table 2-4 for the possible range of mass and stiffness ratio between the idealized tilt-up panel and roof diaphragm, the roof is set 1, 5, 10, 15, and 20 times of the tilt-up panel in mass and stiffness. And thus, each case of mass and stiffness for the tilt-up panel specimen corresponds to 25 roof diaphragms with different equivalent mass and stiffness. The more cases maybe added for the roof diaphragm if necessary.
2.3.5 Damping

Rayleigh damping was used to simulate the damping effect of structural components, and other energy dissipation factors. It was set to 5\% for the first mode and the second mode. These values were selected to control the damping levels corresponding to the mode shape.

2.3.6 Earthquakes

Five earthquakes with a wide scope of mechanisms are selected for this study, which are named La24, Se32, VAN-1, VAN-2 and VAN-3. The diagrams of ground motion time-history and the spectra of those earthquakes as well as the updated acceleration design spectrum are presented in Figure 2-7, Figure 2-8 and Figure 2-9.

The time-history data of those earthquakes are recordings taken from the actual earthquakes, but are scaled for their intended region to the 2\% in 50 year probability of occurrence. Extreme ground motion time histories were selected for this study, such that non-linear behavior near the maximum allowable displacement limits could be observed.

- Seattle Based Record

One record used in this study was derived for the Seattle area (UBC, Zone 3). The 1985 Valparaiso (SE32), earthquake was scaled to the 2\% in 50 year hazard level. The Valparaiso record represents a large subduction earthquake occurring on the Cascadia plate interface, the Mendocino record represents a crustal earthquake that could occur on the Seattle fault. The
Olympia record is another type of subduction earthquake that could also occur on the Cascadia plate. It builds much more quickly than the Valparaiso earthquake, but has a smaller magnitude.

- **Los Angeles Based Record**

  One record used is for the Los Angeles area (UBC, Zone 4). 1989 Loma Prieta (LA24) was scaled to the 2% in 50 year hazard level, and it is a short and overall less powerful earthquake, but has a very lengthy sustained burst.

- **Capitola Based Record**

  One record is used for the Capitola area. Loma Prieta (V250LPCapNSacee12) was scaled to the 2% in 50 year hazard level and modified compatible with the uniform hazard spectra (UHS).

- **San Fernando Based Records**

  Two records are for the San Fernando area. Caltech (SFCT3162_50 and SFCT3172_50) were scaled to the 2% in 50 year hazard level and modified compatible with the uniform hazard spectra (UHS).
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Figure 2-1  (a) Tilt-up Building  (b) Model of Lateral Force Resisting System in a Tilt-up Building  
(c) Simply Supported Beam Modeling Roof Diaphragm Subjected to Lateral In-plane Loads.

Figure 2-2  (a) Tilt-up Panel with Openings  (b) Model of Tilt-up Panel with Openings
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Figure 2-3 First Mode of Tilt-up Panel with Two Openings

Figure 2-4 (a) Hysteresis Model CL2
(b) Hysteresis Model SL2
Hysteresis Model=SL2, EQ=La24, Mw=Mr, Kw=Kr

Figure 2-5 Load-Deformation Curve of Hysteresis Model SL2
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Figure 2-6 Load-Deformation Curves By Essa, Tremblay and Rogers.
Figure 2-7 Spectra of earthquakes (La24, Se32, VAN-1, VAN-2, and VAN-3)
Figure 2-8 Time-History of earthquakes (La24 and Se32)
Figure 2-9 Time-History of earthquakes (VAN-1 and VAN-2)
3. Discussion of Results

The computer simulations are performed with testing specimen including a number of factors that affect the dynamic response of tilt-up panels. These factors are stiffness ratios between concrete tilt-up wall and metal roof, mass ratios between wall and roof, force reduction factor for wall and roof, earthquake records, and ductility of roof. Of interest is how the ductility of tilt-up panels varies with these factors.

3.1 Ratio of Stiffness and Ratio of Mass

In the nonlinear simulation tests, either the stiffness ratio or the mass ratio between wall panel and roof are fixed, and the ductility of nonlinear wall panels are studied while changing the other ratio.

3.1.1 Fix Ratio of Mass

In this series of tests, the mass ratio of roof to wall was fixed at certain levels and the ratio of stiffness was varied between 1 and 40, which was in the range of possible stiffness value as discussed in Chapter 2. The wall stiffness was fixed to a constant value of 36524 kN/m. The total mass of roof and wall was set to 9072 kg. The total mass was shared by the two nodes in the ratios of 1, 3 and 9. Along with each fixed mass ratio, the stiffness ratio of wall to roof was varied from 1 to 25 with 45 data points in between. Only the La24 Earthquake record was used as an excitation for this phrase of study. The maximum seismic force in wall was attained by performing linear analysis with Program CANNY. The yield strength of wall panel was chosen
to be half of the maximum seismic force demand of wall. Alternatively to say, force reduction factor (R) was equal to 2 for the wall. (See Chapter 2 for more details). The roof remained linear elastic through the whole earthquake. The test results are plotted in Figure 3-1 and Figure 3-2.

Presented in Figure 3-1(b) is the ductility of wall panel versus the stiffness ratio of wall to roof. The ductility of the wall panel ranges from 0 to 45 when the stiffness ratio of wall to roof is larger than 2; but before this point, the ductility of wall panel tends to be a constant. This observation is explained as follows: the 2 degree of freedom system can be simplified as a single degree of freedom system if the roof diaphragm is considered very rigid over the wall. This simplified single degree of freedom system is composed of a spring, whose stiffness is approximately equal to the lateral stiffness of the wall panel, and a node mass, which is the total mass of the original system. Thus, the wall ductility attained with this simplified single degree of freedom model is approximately a constant if the force reduction factor is fixed for the wall.

Figure 3-1(a) presents the ductility of wall versus the first period of the system. The ductility ranges from 0 to 45. Beyond a period about 1.7 seconds, the ductility demand significantly increases and reaches a maximum value at a period of about 2.4 second. With the fixed mass ratio between the wall and the roof, the first period of the system varies with the variations of the stiffness ratio between the wall and the roof. As the wall stiffness is fixed and the stiffness ratio is changed by adjusting the roof stiffness, the variation of first period is related to the
change of roof stiffness. The first period increases when the roof is more flexible. Therefore, the curve of wall ductility versus the first period of the system presented in Figure 3-1(a) indicates that flexible roof produces significant influence on wall ductility.

Figure 3-2 presents the ductility of wall panel in both seismic loading directions expressed by positive and negative mathematical signs. In the cases that the mass ratios of roof to wall panel is equal to 3 and 9, the extreme asymmetry is observed for the negative and positive curves about horizontal coordinate axis. The ductility of wall panel is small when the system is subjected to a negative seismic excitation, and the ductility is extremely large when the system is subjected to a positive excitation. This observation is dominant when the first period of system is beyond a certain value or, in other words, when the roof is much more flexible than the wall. However Figure 3-2(c), for the case of which the wall mass is equal to the roof mass, does not present the described observation and the two curves are quite symmetric about the horizontal coordinate axis. It indicates the mass ratios between the wall and the roof also has influence on the ductility of wall. If more mass is added on the roof, it will create a larger ductility demand on the wall. But the ductility of wall is much less sensitive to the mass ratio than to the stiffness ratio.

The following conclusions can be made from the presented observations:

- The ductility of wall is very sensitive to the roof flexibility relative to the rigid wall or to the stiffness ratio between roof diaphragm and wall panels.
• If the tilt-up wall acts with a very flexible roof in earthquakes, the wall panel yields with very large ductility demands in either of the two loading directions, while it yields to a much less degree in the other loading direction.

The second conclusion could be explained as follows: It takes longer time and undergoes larger deformations for a flexible spring to transform the same amount of kinetic energy into a potential energy than a rigid spring. Consider the two degree of freedom vibration system shown in Figure 2-1(b), and a very flexible upper spring in the system can not catch the pace of a stiff lower spring. The upper spring is too flexible and it acts just like an isolator to block the transmission of seismic energy from the upper node to the lower spring. After the lower spring yields, kinetic energy accumulated in the upper node can not be efficiently dissipated through the yield of lower spring. In other words the very flexible upper spring promotes the inelastic displacement of the lower spring instead of decaying it. For the lateral force resisting system in a tilt-up building, composed of a tilt-up wall integrated with a metal roof, the very flexible roof not only creates long first period of system; but also produces great ductility demand in the wall panels.

3.1.2 Fix Ratio of Stiffness

These series of tests are performed using the equivalent stiffness and mass of the wall panel described in Chapter 2. The equivalent stiffness of the wall is 35960 kN/m and the equivalent mass is 7893 kg. The equivalent mass and stiffness of roof are proportional to those of the wall panel. The five Earthquake records are exerted as excitations on the system, which are La24,
Se32, VAN-1, VAN-2, and VAN-3. The maximum elastic seismic force demand of wall is attained by performing linear elastic analysis with each specific earthquake record. The yield strength of the wall is equal to half of the maximum force demand. Alternatively, the force reduction factor (R) is equal to 2 for the wall. To study the ductility of wall panel with the influence of elastic and in-elastic roof, the tests will be separated into two cases. For the first case, the roof is made to undergo inelastic deformation after it yields. For the second case, the roof remains linear elastic without yield.

It is believed that the roof deformation capacity is limited and the roof ductility is expected less than 2. So, the roof ductility has to be adjusted between 1 and 2 in simulations. If the roof ductility is out of this range, the iteration is required to achieve it by adjusting the force reduction factor for the roof. Firstly, perform linear simulation analysis with CANNY on the idealized 2-DOF system and find the maximum seismic force demand of roof. The yield strength of roof is equal to half of the maximum roof force. Secondly, perform nonlinear analysis using this yield strength for the roof and find the roof ductility demand. If the roof ductility is less than 2 and larger than 1, then the corresponding wall ductility is recorded. This single simulation is completed. Another simulation with new setups of specimen can be started. If the roof ductility is larger than 2 or less than 1, otherwise, the yield strength of roof has to be re-estimated by giving a new value to the force reduction factor of roof. And then, start additional simulations until the roof ductility is in the range between 1 and 2.

During the stage of studying the influence of mass ratio and stiffness ratio between wall and
roof, simulation tests include five parts corresponding to stiffness ratios (wall to roof) of 1, 2, 4, 10 and 20. These ratios are within the possible range of stiffness ratio of wall to roof (See Table 2-5). Some of the simulations were performed with the mass ratios out of the expected range as shown in Table 2-5. However, these test data are useful for developing the basic understanding of a general 2 degree of freedom dynamic system. With each fixed stiffness ratio, the first period of system varies only with the mass ratio between wall and roof. The observations of wall ductility are presented in Figure 3-4 to Figure 3-10.

**Elastic Roof**

Along with the elastic roof integrated in the testing system, the ductility demand of wall panel is presented in Figure 3-4 and Figure 3-5. In each plot of the figures are presented five curves of wall ductility versus first period of system marked by different symbols. Each curve corresponds to a different earthquake record. The curve is created by connecting some data points with the same stiffness ratio level and earthquake record. A single data point is created by a particular simulation with a specific testing specimen (including specific equivalent mass and stiffness of roof and wall, force reduction factor, earthquake record).

From the observations of presented curves in Figure 3-4 and Figure 3-5, the ductility demand of wall panel tends not to be sensitive to the type of earthquake except a shift of peak point in the curve, especially in the smaller period zone. And the wall ductility fluctuates about a constant value corresponding to each given stiffness ratio independent of mass ratios or the first period when the first period of system is beyond a certain value, say 0.5 second. With the
increase of stiffness ratio or roof flexibility, the ductility of wall panel tends to be more and more scattered and this constant, about which the wall ductility fluctuates, become larger and larger.

When the first period is less than a certain value, say 0.5 seconds, there emerges a pronounced rise of wall ductility demand. The turning-point on the curve depends on the characteristics of earthquake along with given values of equivalent stiffness and mass of tilt-up wall and metal roof.

In Figure 3-6(a) is presented the five curves of the mean wall ductility versus the stiffness ratio between wall panel and roof corresponding to different earthquakes. A data point on the curve corresponds to a certain stiffness ratio and earthquake record. It is created by averaging all the data of wall ductility corresponding to different mass ratios. In Figure 3-6(b) is presented the mean ductility of wall panel versus the stiffness ratio between wall panel and roof. A data point on the curve is produced by averaging all the data of wall ductility over all different mass ratios and five earthquake records, but each data of mean ductility of wall corresponds to a different stiffness ratio. A trend of linear relationship between the mean wall ductility and stiffness ratio could be expected by the observations of Figure 3-6. It is drawn to a conclusion that the ductility of tilt-up wall panel is linearly proportional to the stiffness ratio of wall panel to roof.

On the basis of the test observations described above, the following conclusions can be made:
• Firstly, the single degree of freedom system, which models the lateral force resisting system composed of a flexible tilt-up wall panel integrated with a flexible roof in a tilt-up building, is not applicable unless the roof could be considered very rigid relative to the wall. However the assumption of a rigid roof is not practical for most practical cases. The influence of a flexible roof can not be neglected in the estimation of the total displacement demand of a tilt-up wall panel when an inelastic structural design is executed for the wall panel.

• Secondly, the ductility of wall panel is much less sensitive to the mass ratio than the stiffness ratio between the wall panel and the roof. The ductility of wall panel seems apparently to fluctuate about a constant with a certain ratio of stiffness regardless of the variation of mass ratio. However the wall ductility is too scattered when the roof becomes more flexible or the first period of system is short. The ductility of wall panel could be approximately considered as a constant only if the stiffness ratio of wall panel to roof is less than 2 and the first period is larger than a value, say 0.5 second.

In-elastic Roof

The ductility of wall panel with the influence of in-elastic roof is presented in Figure 3-7, Figure 3-8, Figure 3-9 and Figure 3-10. In Figure 3-10 is plotted the ductility of wall panel versus the first period of the testing system. In this series of simulation tests, the testing system was excited by the earthquake record (La24). The displacement ductility demand of roof is adjusted between 1.5 and 2. To contrast the wall ductility for the case of elastic roof and that for the case of in-elastic roof, two tests are performed with the same setups for the testing
specimen except the roof ductility. The ductility of wall panel is presented in Figure 3-10 by a couple of curves, the one of them corresponds to an elastic roof and the other to an in-elastic roof. With each fixed stiffness ratio between wall panel and roof, the curves of wall ductility versus the first period of system are very similar to each other in shapes for the cases of elastic roof and in-elastic roof, except that the wall ductility declines for the case of in-elastic roof. Contrast the three plots in Figure 3-10, and the reduction of wall ductility increases with the increase of roof flexibility. But in the case of equal stiffness between wall and roof, the ductility of wall panel is basically not sensitive to the roof condition, yield or not yield.

Additional simulation tests are carried out with other earthquake records (La24, Se32, VAN-1, VAN-2 and VAN-3). The observations of wall ductility are presented in Figure 3-7, Figure 3-8 and Figure 3-9. Contrast the plots for the cases of elastic roof and the plots for the cases of in-elastic roof, and the wall ductility apparently decreases due to the influence of roof yield. The reduction of the wall ductility due to roof yield increases with the increase of roof flexibility. Moreover, some observations of wall ductility for the case of elastic roof are repeated for the case of in-elastic roof, such that the wall ductility basically is not sensitive to the mass ratio and the earthquake records. It fluctuates about a constant corresponding to a fixed stiffness ratio.

As per the above observations about the ductility of wall panel when it is integrated with an inelastic flexible roof in a tilt-up building, it is concluded that the dynamic motion of a tilt-up wall panel is reduced by the roof yield. Particularly, the wall displacement ductility demand is
reduced. Moreover, the influence of roof yield increases with the increase of roof flexibility.

As described in Section 3.2.1, a very flexible roof, which remains linear elastic in earthquake, acts just like an isolator to block the dissipation of seismic energy accumulated on roof through the yield of wall panel. The wall panel is subjected to a great ductility due to high seismic motion of roof. This phenomenon is aggravated with the increase of roof flexibility. However a wall panel integrated with a non-linear roof, on the contrary, undergoes a small ductility because the seismic energy in roof is dissipated by the yield of roof itself. No such a highly-active roof full of kinetic energy promotes the wall displacement any more. The observations of these series of tests also suggest that the more flexible the roof, the more the wall ductility is reduced by the roof yield. It is concluded that the roof yield helps a lot to reduce the seismic displacement demand of wall panel, especially for the case of a relatively rigid wall panel integrated with a very flexible roof. It is also advised that the ductility of wall panel will not be very different for the case of tilt-up wall integrated with a yield roof or not yield roof if the stiffness of wall panel and roof is close to each other.

3.2 Maximum Forces in Roof

For these series of simulations, the roof remained linear elastic through earthquakes. The seismic force in roof was studied when the tilt-up wall yielded or did not yield through earthquakes. A large number of simulation tests were performed including five different earthquakes and all possible mass ratios and stiffness ratios (See Table 2-4) between tilt-up wall and metal roof. The strength of the tilt-up wall was determined by assuming a force
reduction factor (R) equal to 2. The first two sample tilt-up panels described in Section 2.3.4 were used in the setups of tests. The results are presented in Figure 3-11 to Figure 3-15 according to earthquake records. There are six plots in each figure and in the each of first five plots of those figures is presented two curves of the maximum force of roof versus the first period. One of the curves is for the case of an elastic roof integrated with an inelastic wall, and the other curve is for the case of an elastic roof integrated with an elastic wall. Plot (f) in Figure 3-11 to 3-15 is the summary of the first five plots in each of those figures. Plot (f) presents the force ratio versus the first period. The force ratio is the maximum roof force, attained when the elastic wall acts with elastic roof, divided by the maximum roof force, attained when the corresponding inelastic wall acts with the same elastic roof.

Contrast the two curves presented in each plots of those figures, and regardless of the different earthquake records and the stiffness ratio between wall and roof, the two curves have the same shape. The vertical coordinate for the data point on the curve of elastic wall is about twice the value of the corresponding data point on the other curve of inelastic wall. From Plot (f) in each of figures, the seismic force of roof, attained in the case of that the elastic roof is integrated with an inelastic wall and the force reduction factor is equal to 2 for the tilt-up wall, is a little smaller than half of the corresponding roof force, attained in the case of that the same elastic roof is integrated with the same tilt-up wall except that the wall remain linear elastic through the same earthquake. This observation is explained in the following paragraphs:

Suppose a general 2 degree of freedom dynamic model as shown in Figure 3-16, and it is used
to model the lateral force resisting system in a tilt-up building. \( m_1 \) and \( m_2 \) are the equivalent
mass of tilt-up wall and the equivalent mass on roof involved in the seismic motion. \( k_1 \) and
\( k_2 \) are the equivalent lateral stiffness of tilt-up wall and metal roof. As shown in Figure 3-16(a)
is the case of that tilt-up wall remains linear elastic through earthquakes and as shown in Figure
3-16(c) is the case of that tilt-up wall yields in earthquakes and the force reduction factor is 2
for the wall. If the wall mass is neglected, those figures are converted to Figure 3-16(b) and
Figure 3-16(d) respectively.

Without the wall mass, the forces in the two springs are equal and the roof force \( (F/2) \) for the
case of in-elastic wall as shown in Figure 3-16(d) is exactly equal to half of the roof force \( (F) \)
for the case of elastic wall as shown in Figure 3-16(b). Consider to put the wall mass \( (m_1) \) back
in the system. For the case of elastic wall as shown in Figure 3-16(a), the roof force \( (F_e) \) is:

\[
F_e = F - m_1 a_i
\]  

3.6

\( a_i \) is the acceleration of wall mass induced by ground motion.

Eq. 3.6 indicates the maximum seismic force in the elastic roof is all the time smaller than the
maximum seismic force in the elastic wall. In the 2-DOF elastic system, it is not at the same
instant for both springs to reach their maximum forces. However, the accelerations of both
concentrated mass all the time are in the same direction. When either of two springs reaches its
maximum value of force, the other spring must be very close to its maximum value of force.
Hence, the maximum force in wall is always larger than the maximum force in roof if both wall
and roof remain linear elastic in earthquakes.
This concept was confirmed by performing simulations with linear elastic testing system. Two practical tilt-up panels, which are Wall-1 and Wall-3 presented in Table 2-1, were used in the setups of tests. Three earthquake records of VAN-1, VAN-2, and VAN-3 were used as excitations to the testing specimens. The stiffness ratios and the mass ratio were 1, 5, 10, 15 and 20 used in the tests. The results are presented in Figure 3-17(b) and Figure 3-18(b) by the curves of the force ratios versus the mass ratios. The force ratio is the ratio of the maximum force in the elastic roof over the maximum force in the elastic wall. The mass ratio is the product of dividing the roof mass by the wall mass.

It is observed that the roof force is always a little smaller than the wall force, but the difference between those forces gradually decreases with the increase of the mass ratio of roof over wall. Apparently, the wall force is very close to the roof force if the mass on roof is dominant over the mass of wall. Hence, the wall mass can almost be neglected as shown in Figure 3-16(b).

For the case of inelastic wall, the roof force is:

\[
F_{in-e} = \frac{F}{2} + |m_1a_1| \tag{3.7}
\]

Eq. 3.7 indicates that the maximum force of roof is always larger than the yield strength of wall. This observation can be explained as follows: As the maximum force of in-elastic wall is fixed to be half of the maximum force demand of the corresponding linear elastic wall, the maximum force of roof must occur during the time of wall yield. When the wall yields, the roof spring tends to contract itself and pull the both mass (m1 and m2 as shown in Figure 3-16(c)) towards
itself. The roof spring in the motion produces the force in the roof itself larger than the yield strength of the wall spring.

This concept is confirmed by performing simulations. The setups of tests are repeated as described above in this section except that the wall yields. The results are presented in Figure 3-17(a) and Figure 3-18(a) by the data points of force ratios versus mass ratios. The force ratio and the mass ratio contain the same meaning as defined above in this section.

It is observed that the maximum roof force is always a little larger than the maximum wall force except that the mass ratio between wall and roof is equal to one. This observation indirectly demonstrates the concept presented in Section 3.1. Much kinetic energy concentrated on roof is produced by the dominant roof mass relative to the small wall mass, and this energy can not be effectively dissipated by the wall yield since the very flexible roof spring acts as an isolator to block the energy transmission from roof to wall. Hence, the very large roof mass in seismic motion stretches the roof spring to extend and makes the roof spring pull the wall mass after the wall yields.

Take the ratio of the maximum force of roof for the case of elastic wall ($F_e$), as shown in Figure 3-16(b), over the maximum force of roof for the case of in-elastic wall ($F_{in-e}$) as shown in Figure 3-16(d), and the ratio is expressed as follows:

$$\frac{F_e}{F_{in-e}} = \frac{F - |m_1a_1|}{F/2 + |m_1a_1|}$$  \hspace{1cm} (3.8)
Consider Eq. 3.8, and the maximum roof force for the case of elastic wall is a little smaller than twice the roof force for the case of inelastic wall. Simulations were performed to confirm this theoretical conclusion. The setups of tests are the same as described before in this section. The results are presented in Figure 3-19 and Figure 3-20. Figure 3-19 presents the ratio of the maximum roof force for the case of elastic wall over the maximum force roof for the case of inelastic wall versus the first period of testing system. Figure 3-20 presents the ratio of the maximum roof force for the case of elastic wall over the maximum roof force for the case of inelastic wall versus the stiffness ratio of wall over roof. It is observed that the data points scatter between 1 and 2 and how the distribution of data points is depends on the stiffness ratio. Observe Figure 3-20 and the scattered range of data points increases with the increase of the stiffness ratio. Contrast the plots of Figure 3-19, Figure 3-20 and plot(f) in Figure 3-11 to Figure 3-15, and the data points scatter to a less degree between 1.5 and 2 in plot(f). The comparative observation demonstrates that the testing data are affected by the wall stiffness. More rigid tilt-up panel was used in the simulations to produce the data presented in plot(f) of Figure 3-11 to Figure 3-15. To summarize, the roof force ratio is between 1 and 2. This roof force ratio is calculated by dividing the maximum roof force, when the elastic roof is integrated with elastic wall in the linear 2-DOF system, by the maximum roof, when the same roof is integrated with inelastic wall in the corresponding nonlinear 2-DOF system.
According to the described observations, the yield of wall can reduce the seismic force of elastic roof approximately to the same degree as the wall force is reduced. Moreover, it is expected that the yield of roof can reduce the seismic force of elastic wall as well.

3.3 Force in Roof at the Time of Maximum Wall Displacement

In the simulations of nonlinear 2 DOF testing system, the wall yields and the roof remains linear elastic through earthquakes. At a certain instant, the wall reaches its maximum elastoplastic displacement demand; however, the maximum roof drift relative to the wall does not emerge at the time of maximum wall displacement demand. Of interest is the roof drift at this time. Since the roof remains linear elastic through earthquakes, the roof force is linear proportional to the roof drift at any instant. Hence, the roof force can be studied instead of the roof drift. Repeat the simulations with the same test setups using the two practical tilt-up panels (Wall-1 and Wall-3) described in Section 3.2 and the roof force ratios are taken by dividing the maximum roof force, for the case of that elastic wall acts with elastic roof, by the roof force at the time of maximum wall displacement demand. The results are presented in Figure 3-21 and Figure 3-22.

Figure 3-21 presents the roof force ratio versus first period and Figure 3-22 presents the roof force ratio versus the stiffness ratio of wall over roof. The results indicate the ratio varies from 1 to 4. But most of the ratios are located between 1.5 and 3. The data points for the case of flexible wall ($T_w=0.4$ second) have a little more scatter than the data points for the case of a
rigid wall \((T_w=0.2\ \text{second})\). Only for the case of equal stiffness between the wall and the roof, the data are a little larger than 2. From the observations, the roof force at the time of maximum elastoplastic wall displacement can be estimated by dividing the maximum roof force, attained from the linear analysis with fully elastic system (elastic wall and elastic roof), by the force reduction factor \((R=2)\) given for the wall. Since the roof drift relative to the wall is linear proportional to the roof force, the roof drift at the time of maximum elastoplastic wall displacement can be estimated by dividing the maximum roof drift, attained from the linear analysis with fully elastic system, by the force reduction factor \((R=2)\) given for the wall.

3.4 Ductility of Roof

The roof ductility is another factor for the ductility of wall panel. Another series of simulation tests were performed in order to investigate the ductility of wall panel with the changes of roof ductility. The roof ductility was controlled in the range between 1 and 2.5 by adjusting the force reduction factor \((R)\) for roof. The force reduction factor was 2 for the tilt-up wall panel. The earthquake, La24, was applied as an excitation to the testing model. The two tilt-up panels described in Section 2.3.4 were used in the setups of simulations. The stiffness ratio and the mass ratio between wall panel and roof were also fixed to a certain level corresponding to each testing specimen. There were four setups for the testing system. For the first setup, the ratio of roof stiffness over wall stiffness was 1 and the ratio of roof mass over wall mass was 15. For the second setup, the ratio of wall stiffness over roof stiffness was 10 and the ratio of roof mass over wall mass was 5. For the third setup, the ratio of wall stiffness over roof stiffness was 10 and the ratio of roof mass over wall mass was 200. For the third setup, the ratio of wall
stiffness over roof stiffness was 1 and the ratio of roof mass over wall mass was 200.

The output data are presented by four curves in Figure 3-23 as per the different setups of specimens, mass ratio and stiffness ratio. From the observations, the ductility of wall panel linearly declines with the increase of roof ductility when all other factors are fixed, such as mass ratio, stiffness ratio, force reduction factor for wall panel, earthquake. Contrast the four curves in Figure 3-23, and how fast the wall ductility declines is very sensitive to the stiffness ratio and not sensitive to the mass ratio. Moreover, the wall ductility is not sensitive to the variations of roof ductility if the stiffness of wall panel and roof are close to each other.

3.5 R for Roof

In these series of tests the ductility of wall and roof was investigated with three different R-values for the roof, which were equal to 1, 1.5, and 2.5 respectively. The Earthquakes, La24, was applied as the excitation to the testing system. The force reduction factor (R) for tilt-up wall was equal to 2. The two tilt-up panels described in Section 2.3.4 were used in the setups of simulations. The tests were performed with two stiffness ratios of wall over roof, 1 and 20. Along with each given stiffness ratio, the different testing specimen was configured by changing the mass ratio between roof and wall.

The ductility of wall and roof are presented in Figure 3-24 and Figure 3-25. Only the wall yielded and roof remained elastic when the force reduction factor was given 1 or 1.5 for the roof. Moreover, the ductility of wall was basically the same for those two cases although they
had different roof force reduction factors. On the contrary, only the roof yielded along with the elastic wall when the force reduction factor was equal to 2.5 for the roof and it was 2 for the wall. This observation is explained in Section 3.2 and it is repeated here as follows: In a generalized two degree of freedom vibration system (See Figure 2-1(b)), the seismic force of an elastic spring can be reduced by the yield of the other in-elastic spring. Moreover, how much force is reduced in the elastic spring depends on how much force is reduced in the in-elastic spring. Generally, the seismic forces of those two springs are reduced to the same degree by the yield of either of the two springs.

For the current case, when the force reduction factor (R) was 2 for the wall and it was 1.5 for the roof, only the wall yielded and the roof did not. It is because the seismic force demand of roof is reduced approximately by 2, which is the force reduction factor for the wall. Although the yield strength for the roof was given by dividing the maximum seismic force of roof attained in the linear analysis by 1.5 (the force reduction factor for the roof), the yield strength of roof is obviously larger than the maximum seismic force demand of roof in the nonlinear system in which the wall yields with the force reduction factor equal to 2. And thus, the roof remained elastic while the wall yielded. So was the case if the factor (R) was 2.5 for the roof and it was 2 for the wall.

3.6 Relatively Rigid Wall and Flexible Wall

These series of tests were carried out with the setups of three typical concrete tilt-up wall representing rigid wall, medium-rigid wall, and flexible wall. The Earthquakes (La24 and Se32)
CHAPTER 3 Discussion of Results

were applied as excitation to the testing specimen. The roof was set linearly elastic through earthquakes. R was 2 for the wall. The ratio of wall stiffness over roof stiffness was 1. The properties of the rigid wall were calculated based on a real solid tilt-up wall, whose size is 8 m long by 8.5 m high by 150 mm deep. The stiffness is 752100 kN/m, 135900 kN/m, and 35790 kN/m corresponding to rigid, mid-rigid, and flexible wall respectively. The first period of the testing system varied with the mass ratio of roof over wall. The stiffness ratios between the wall and the roof were fixed to 1 while the mass ratios varied from 1 to 40.

Figure 3-26 presents the curve of ductility of wall versus the first period of the testing system. Figure 3-26(a), corresponding to the earthquake (La24), presents three curves for the cases of rigid wall, medium-rigid wall, and ductile wall. Figure 3-26(b), corresponding to the excitation (Se32), presents two curves for the cases of rigid wall and flexible wall. It is observed that these curves approximately overlap each other. The ductility could be a constant if the first period of system is larger than a value, say 0.5 second. This observation indicates the ductility of wall is not very sensitive to the wall stiffness.

3.7 Earthquakes

In Figure 3-4, Figure 3-5, Figure 3-7, and Figure 3-8 the wall ductility demand for five different earthquakes are presented regardless whether the roof yields or does not yield, the wall ductility does not exhibit much different with different excitations for a particular stiffness ratio. It is only observed that the peak points of wall ductility on the curve shift corresponding to each different earthquake. This observation can be explained that the peak acceleration with
particular period and intensity in each earthquake creates a peak wall ductility corresponding to the testing system with a specific first period.
Figure 3-1 Ductility of Wall for Different Mass Ratios
Figure 3-2 Ductility of Wall in Both Loading Directions with Fixed Mass Ratio
Figure 3-4 Ductility of Wall Panel (Fix Level of Stiffness Ratio and Elastic Roof)
Figure 3-5 Ductility of Wall Panel (Fix Level of Stiffness Ratio and Elastic Roof)
Elastic Roof——5 Earthquakes

![Graph a](image)

(a) Mean Ductility of Wall Panel versus Kw/Kr (second)

Elastic Roof——Combine 5 Earthquakes

![Graph b](image)

(b) Mean Ductility of Wall Panel versus Kw/Kr

Figure 3-6  Mean Ductility of Tilt-up Wall versus Stiffness Ratio of Wall to Roof
Kw/Kr = 1

Kw/Kr = 2

Figure 3-7 Ductility of Wall Panel (Fix Level of Stiffness Ratio and In-elastic Roof)
Figure 3-8 Ductility of Wall Panel (Fix Level of Stiffness Ratio and In-elastic Roof)
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In-elastic Roof—5 Earthquakes

Figure 3-9  Mean Ductility of Tilt-up Wall versus Stiffness Ratio of Wall to Roof
Figure 3-10  Ductility of Wall Panel (Contrast the Influence of Elastic Roof and In-elastic Roof) (EQ=La24)
Figure 3-11 (a)-(e) Maximum Force in Roof (EQ=VAN-1)

(f) Maximum Force of Roof, Attained When Elastic Roof Integrated with Elastic Wall, Divided by Maximum Force of Roof, Attained When Elastic Roof Integrated with In-elastic Wall (EQ=VAN-1)
Figure 3-12 (a)-(e) Maximum Force in Roof (EQ=VAN-2)

(f) Maximum Force of Roof, Attained When Elastic Roof Integrated with Elastic Wall, Divided by Maximum Force of Roof, Attained When Elastic Roof Integrated with In-elastic Wall (EQ=VAN-2)
Figure 3-13 (a)-(e) Maximum Force in Roof (EQ=VAN-3)

(f) Maximum Force of Roof, Attained When Elastic Roof Integrated with Elastic Wall, Divided by Maximum Force of Roof, Attained When Elastic Roof Integrated with In-elastic Wall (EQ=VAN-3)
Figure 3-14 (a)-(e) Maximum Force in Roof (EQ=La24)

(f) Maximum Force of Roof, Attained When Elastic Roof Integrated with Elastic Wall, Divided by Maximum Force of Roof, Attained When Elastic Roof Integrated with In-elastic Wall (EQ=La24)
Figure 3-15 (a)-(e) Maximum Force in Roof (EQ=Se32)

(f) Maximum Force of Roof, Attained When Elastic Roof Integrated with Elastic Wall, Divided by Maximum Force of Roof, Attained When Elastic Roof Integrated with In-elastic Wall (EQ=Se32)
Figure 3-16 Model of 2 Degree of Freedom Dynamic System
Figure 3-17 Force Ratios versus Mass Ratios (Using Practical Tilt-up Panel)
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Figure 3-18 Force Ratios versus Mass Ratios (Using Practical Tilt-up Panel)
Figure 3-19 Roof Force Ratio by Dividing Maximum Force of Roof, Attained When Elastic Wall Integrated with Elastic Roof, by Maximum Force of Roof, Attained When Inelastic Wall Integrated with Elastic Roof.
Figure 3-20 Roof Force Ratio by Dividing Maximum Force of Roof, Attained When Elastic Wall Integrated with Elastic Roof, by Maximum Force of Roof, Attained When In-elastic Wall Integrated with Elastic Roof.
Figure 3-21 Ratios of maximum roof force for fully elastic system over the roof force at the time of maximum wall displacement demand for the in-elastic system.
Figure 3-22 Ratios of maximum roof force for fully elastic system over the roof force at the time of maximum wall displacement demand for the inelastic system.
Figure 3-23 Ductility of Wall Panel versus Ductility of Roof
Figure 3-24 Ductility of Wall and Roof with Changes of Force Reduction Factor for Roof
Figure 3-25 Ductility of Wall and Roof with Changes of Force Reduction Factor for Roof
Figure 3-26 Ductility of Wall with Different Stiffness of Wall
4. Estimation of Wall Displacement

The lateral force resisting system in a tilt-up building composed of tilt-up wall and flexible roof is modeled as a 2-DOF system. The maximum total roof displacement, roof drift relative to wall, and wall force of the linear elastic system are needed as inputs into the developed formula to estimate the plastic wall displacement demand of the corresponding nonlinear system. The linear solutions using modal and spectral analysis are presented for the specific 2-DOF elastic system. Because the concept of "equal displacement principle" is used to estimate the total roof displacement in the nonlinear system, applicability of the concept is investigated for the current 2-DOF system by running computer-based simulations. Then, a simple formula is developed to estimate the total wall displacement demand along with some sample calculations based on real tilt-up buildings. The chapter ends with the presentation of two modified formulas to estimate the total wall displacement. The comparison among those three formulas is made by running simulations.

4.1 Solution of Linear System

The method of calculating the equivalent stiffness \( K_w \) and \( K_r \) and mass \( M_w \) and \( M_r \) of tilt-up wall and metal roof is presented in Chapter 2. In the following, those 4 parameters are assumed to be given and the lateral force resisting system is idealized as a 2 degree of freedom dynamic system. The solution to that system is presented below:
CHAPTER 4  Estimation of Wall Displacement

Calculation of Natural Period of a 2-DOF Dynamic System.

The natural circular frequency of first mode, $\omega_1$, and the natural circular frequency of second mode, $\omega_2$, can be determined as follows:

$$\det\left[K - M \cdot \omega^2\right] = 0$$

Wherein,

$$M = \begin{pmatrix} M_w & 0 \\ 0 & M_r \end{pmatrix}, \quad K = \begin{pmatrix} K_{w1} & K_{r1} \\ K_{w2} & K_{r2} \end{pmatrix}$$

$$K_{w1} = K_r + K_w, \quad K_{w2} = -K_r, \quad K_{r1} = -K_r, \quad K_{r2} = K_r$$

Solve for the two real roots of the following equations:

$$(K_r + K_w - M_w \cdot \omega^2) \cdot (K_r - M_r \cdot \omega^2) - K_r^2 = 0$$

The natural period of first mode ($T_1$) and the natural period of second mode ($T_2$) can be calculated as follows:

$$T_1 = \frac{2\pi}{\omega_1}$$

$$T_2 = \frac{2\pi}{\omega_2}$$

Calculation of Natural Mode

The two mode shapes are shown in Figure 4-2b. The first mode shape ($\phi_1$) and the second mode shape ($\phi_2$) are:

$$\phi_1 = \begin{pmatrix} \phi_{w1} \\ \phi_{r1} \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} \phi_{w2} \\ \phi_{r2} \end{pmatrix}$$

The mode shapes are normalized by setting the mode vector equal to one at the wall.

With $\phi_{w1} = 1$, then $\phi_{w2}$ can be determined by solving for the root of the following equation:
CHAPTER 4  Estimation of Wall Displacement

\[ (K_{w1} - M_{w} \omega^2) \phi_{w1} + K_r \phi_{r1} = 0 \] 4.1.5

With \( \phi_{w2} = 1 \), then \( \phi_{r2} \) can be determined by solving for the root of the following equation:

\[ (K_{w1} - M_{w} \omega^2) \phi_{w2} + K_r \phi_{r2} = 0 \] 4.1.6

Calculation of Mode Mass and Stiffness

The mass of first mode \( (M_1) \) is

\[ M_1 = \phi_1^T M \phi_1 = \phi_{w1}^2 m_w + \phi_{r1}^2 m_r \] 4.1.7

The mass of second mode \( (M_2) \) is

\[ M_2 = \phi_2^T M \phi_2 = \phi_{w2}^2 m_w + \phi_{r2}^2 m_r \] 4.1.8

Calculation of Total Elastic Displacement of Wall and Roof

The two mode shapes and the equivalent single degree of freedom system representing each mode are shown in Figure 4-2b. The equivalent displacement of first mode \( (D_1) \) and second mode \( (D_2) \) are shown in Figure 4-2 (b-b) and Figure 4-2 (b-d). The contribution of first mode to the nodal displacements (wall displacement and total roof displacement) is calculated as follows:

\[ \mathbf{u}_1(t) = \Gamma_1 \phi D_1 \]

wherein

\[ L_1 = \phi_1^T M \phi_1 \]

\[ \Gamma_1 = \frac{L_1}{M_1} = \frac{\phi_{w1}^2 M_w + \phi_{r1}^2 M_r}{\phi_{w1}^2 M_w + \phi_{r1}^2 M_r} \] 4.1.11

\( \Gamma_1 \) is the modal participation factor of first mode, implying that it is a measure of how much
the first mode participates in the response. \( i \) is the influence vector and represents the displacements of the masses resulting from static application of a unit ground displacement. For the current 2-DOF system, \( i \) is equal to \( \begin{bmatrix} 1 \\ 1 \end{bmatrix} \). With the aid of NBCC seismic acceleration spectrum, the first mode displacement \( (D_1) \) can be found using the following equation:

\[
D_1 = \left( \frac{T_1}{2 \cdot \pi} \right)^2 a_1
\]

Wherein, \( a_1 \) is the spectral acceleration corresponding to the first mode period \( (T_1) \).

Repeat the above procedures, and the contribution of second mode to the nodal displacements (wall displacement and total roof displacement) is calculated as follows:

\[
u_2(t) = \Gamma_2 \phi_2 D_2 = \begin{bmatrix} \Gamma_2 \phi_2 D_2 \\ \Gamma_2 \phi_2 D_2 \end{bmatrix}
\]

Wherein,

\[
L_2 = \phi_2^T \mathbf{M} i = \left\{ \begin{array}{c} \phi_{w2} \\ \phi_{r2} \end{array} \right\} \begin{bmatrix} M_w & 0 \\ 0 & M_r \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \phi_{w2} M_w + \phi_{r2} M_r
\]

\[
\Gamma_2 = \frac{L_2}{M_2} = \frac{\phi_{w2} M_w + \phi_{r2} M_r}{\phi_{w2}^2 M_w + \phi_{r2}^2 M_r}
\]

\( \Gamma_2 \) is the modal participation factor of second mode. The second mode displacement \( (D_2) \) can be found using the following equation:

\[
D_2 = \left( \frac{T_2}{2 \cdot \pi} \right)^2 a_2
\]

where, \( a_2 \) is the spectral acceleration corresponding to the first mode period \( (T_2) \).

The square-root-sum-of-square (SRSS) rule is used for modal combination because the lateral force resisting system composed of tilt-up wall and metal roof possesses well-separated natural mode periods (See Table 2-3 and Table 2-4). The total linear elastic displacement of wall and
CHAPTER 4  Estimation of Wall Displacement

roof ($D_w$ and $D_{w+r}$) are calculated as follows:

$$D_w = \sqrt{\left(\Gamma_1 D_1 \phi_{w_1}\right)^2 + \left(\Gamma_2 D_2 \phi_{w_2}\right)^2} \quad 4.1.17$$

$$D_{w+r} = \sqrt{\left(\Gamma_1 D_1 \phi_{w_1}\right)^2 + \left(\Gamma_2 D_2 \phi_{w_2}\right)^2} \quad 4.1.18$$

The roof drift relative to the tilt-up wall can be calculated by subtracting the wall displacement from the total roof displacement relative to the ground. Because the maximum displacements of elements (tilt-up wall and metal roof) corresponding to each mode do not occur at the same time, the subtraction of displacements must be done before the combination of mode. The drift of roof relative to tilt-up wall is calculated as follows:

$$D_r = \sqrt{\left(\Gamma_1 D_1 \phi_{r_1} - \Gamma_1 D_1 \phi_{w_1}\right)^2 + \left(\Gamma_2 D_2 \phi_{r_2} - \Gamma_2 D_2 \phi_{w_2}\right)^2} \quad 4.1.19$$

4.2 Application of Equal Displacement Principle to Tilt-up buildings

The concept of “equal displacement principle” is used in the following chapter to estimate the total roof displacement in the nonlinear system, applicability of the concept is investigated for the current 2-DOF system. Equal Displacement Principle for the non-linear maximum displacement of structures was developed by Newmark and Rosenblueth. Newmark states: “the conclusion we had derived for single-degree in-elastic systems-that under a wide set of conditions their expected maximum deformation is approximately equal to that of an equivalent linear structure having the initial characteristics of the given system-may err seriously on the unsafe side for multi-degree systems.”

According to the equal displacement principle, for a single storey building with tilt-up wall and metal roof as its lateral force resisting system which can be modeled as a generalized 2 degree
of freedom dynamic system, it is expected that, in this idealized 2 degree of freedom system, the total displacement of elastic roof integrated with in-elastic wall is approximately equal to the total displacement of this roof integrated with the corresponding linear elastic wall. This principle is investigated by performing the following simulation tests. The tests are setup with given ratios of mass and stiffness under earthquake, VAN-1. The results are presented in Figure 4-1 by the data of roof displacement versus stiffness ratios of wall over roof. For comparison, in each plot of Figure 4-1 are shown two curves, one of them represents the roof displacement corresponding to the case of roof integrated with elastic wall while the other one represents the roof displacement corresponding to the case of roof integrated with in-elastic wall.

It is observed that the roof displacement corresponding to elastic wall is almost equal to that displacement corresponding to in-elastic wall, and for most of cases the former displacement is a little larger. More simulation tests are performed to include as many setups as possible, such that three earthquakes (VAN-1, VAN-2, and VAN-3) are used as the excitations, and two real tilt-up panels, representing relatively rigid tilt-up panel and flexible tilt-up panel, are selected to calculate the input data for the tests. The characteristics of those two panels are presented as Wall-1 and Wall-3 in Table 2-1 of Chapter 2. The results are presented in Figure 4-3 by the data points of roof displacement corresponding to in-elastic wall versus the displacement corresponding to elastic wall. From the comparison of those displacements, therefore, it is conservative if the displacement of roof, when it is integrated with the corresponding elastic wall, is used instead of the exact displacement of roof, when it is integrated with an in-elastic wall in a tilt-up building.
CHAPTER 4  Estimation of Wall Displacement

4.3 Proposed Method for Estimating Wall Displacement

There are two systems (linear elastic and nonlinear 2-DOF systems) used in this section. For clear illustration, those two systems are shown in Figure 4-2c. In each system, different symbols are used for the displacements at the nodes (wall and roof).

As stated in Chapter 2, the lateral force resisting system in a tilt-up building, composed of flexible tilt-up panel and flexible metal roof, is modeled as an idealized 2 degree of freedom system. Subjected to excitation of earthquake motion, the tilt-up panel is expected to undergo in-elastic deformation. At the time of maximum wall displacement demand, the total displacement of wall ($\Delta_w$) can be determined by subtracting the displacement of roof ($\Delta_r$) relative to the wall from the total displacement of roof ($\Delta_{w+r}$) relative to the ground. This is expressed by a formula as follows (See Figure 4-2a):

$$\Delta_w = \Delta_{w+r} - \Delta_r$$  \hspace{1cm} 4.3.1

The predicted wall displacement and the actual wall displacement are presented in Figure 4-4. All three values of $\Delta_w$, $\Delta_{w+r}$, $\Delta_r$ are attained from the simulation tests at the same instant when the wall reaches its maximum displacement demand. It is observed that all the data points is on the line, which indicates the predicted wall displacement calculated by Eq. 4.3.1 is exactly equal to the actual wall displacement.

According to the "Equal Displacement Principle", at the time of maximum elastoplastic wall displacement demand ($\Delta_w$), the total roof displacement ($\Delta_{w+r}$) can be approximately replaced
CHAPTER 4  Estimation of Wall Displacement  

by the total maximum roof displacement ($D_{w+r}$) attained with the corresponding fully linear elastic system. This replacement is conservative in terms of the research presented in Section 4.2.

According to the research presented in Section 3.2.3, at the time of maximum elastoplastic wall displacement demand ($\Delta_w$), the displacement of roof relative to the wall ($\Delta_r$) is approximately equal to the half of the maximum roof displacement relative to the wall ($D_r$) attained with the corresponding fully linear elastic system. Therefore, the roof drift ($\Delta_r$) in Eq. 4.3.1 can be approximately replaced by the maximum roof displacement relative to the wall ($D_r$) divided by R in the corresponding linear elastic system. These two substitutions lead to

$$\Delta_w = D_{w+r} - \frac{D_r}{R}$$  \hspace{1cm} 4.3.2

The simulation tests are performed to investigate how the estimation of wall displacement is calculated by Eq. 4.3.2. The two real wall panels, as presented in Table 2-1 in Chapter 2 (Wall-1 and Wall-3), are selected to calculate the input data for the tests. How to prepare the input data is presented in Chapter 3, and it is not repeated here. The three earthquakes (VAN-1, VAN-2, and VAN-3) will be used in the tests as the excitations. The force reduction factor, R, is given two for the wall panels.

$\Delta_w$, $D_{w+r}$, and $D_r$ are attained directly from simulations corresponding to each testing specimen. After giving the testing values of $\Delta_w$, $D_{w+r}$, and $D_r$ in Eq. 4.3.2, the comparison between the predicted wall displacement and actual wall displacement is presented in Figure 4-4 by five plots as per the stiffness ratios of wall to roof.
The cross line in Figure 4-4 indicates the predicted wall displacement is exactly equal to the actual wall displacement. It is observed that the distribution of data points are more scattered along with increase of the stiffness ratios of wall to roof. The predicted wall displacement is accepted with the presented accuracy, hence, Eq. 4.3.2 is recommended to estimate the total elastoplastic tilt-up wall displacement when it is integrated with an elastic flexible roof as the lateral force resisting system in a tilt-up building.

4.4 Example Case of a Typical Wall Integrated with Typical Roofs

$D_{w+r}$ and $D_r$, as described in Section 4.1, can be calculated using Mode-Extraction Method according to NBCC 2005. Subtract the capacity of lateral elastic wall displacement ($\Delta_{yw}$), calculated by performing a frame analysis for the wall with openings, from the total elastoplastic wall displacement demand ($\Delta_w$), and it leads to the in-elastic wall displacement demand ($\Delta_{iw}$):

$$\Delta_{iw} = \Delta_w - \Delta_{yw} \tag{4.4.1}$$

The sample calculations are made using Eq. 4.3.2 and Eq. 4.4.1. The force reduction factor ($R$) is 2 for the wall and the corresponding yield strength of the wall panel is 271 kN including the influence of rigid extension to model the joints in the tilt-up wall panel. The lateral plastic displacement capacity at the roof level of the panel is 0.051m if the in-elastic rotation first occurs in the column element right below the underside of the lower beam element (See Figure 2-2). Four typical roofs and one typical tilt-up panel used in CPCA Concrete Design Handbook by Gerry Weiler (1995) are selected for the sample calculations. The site location is Vancouver and site class is selected as C, D and E. The results are presented in Table 4-1 and Table 4-2.
corresponding to different site classes. Note that the yield strength of wall is assumed to be half of the maximum wall force demand with the corresponding linear system when the inelastic wall displacement is estimated. The actual wall strength listed in the table is only for comparison.

Table 4-1 Calculations of Wall Force (Site Class = C)

<table>
<thead>
<tr>
<th>Roof Size</th>
<th>First Period</th>
<th>System Mass</th>
<th>S1</th>
<th>Wall Force</th>
<th>Inelastic Wall Displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>ft(Width by Length)</td>
<td>s</td>
<td>kg</td>
<td>g</td>
<td>kN</td>
<td>m</td>
</tr>
<tr>
<td>100 x 200</td>
<td>1.03</td>
<td>6414</td>
<td>0.325</td>
<td>204</td>
<td>0.023</td>
</tr>
<tr>
<td>200 x 300</td>
<td>1.402</td>
<td>8024</td>
<td>0.266</td>
<td>209</td>
<td>0.045</td>
</tr>
<tr>
<td>100 x 300</td>
<td>1.821</td>
<td>9224</td>
<td>0.2</td>
<td>181</td>
<td>0.061</td>
</tr>
<tr>
<td>300 x 400</td>
<td>1.851</td>
<td>9899</td>
<td>0.194</td>
<td>188.4</td>
<td>0.062</td>
</tr>
</tbody>
</table>

Note: 1. The mass and wall force are transformed per panel.
2. 30lb per square feet for dead load plus live load
3. Assume wall yield and the force reduction factor is equal to 2
4. Assume site class is C
5. The wall displacement capacity is 0.051m

Table 4-2 Calculations of Wall Force (Site Class = E and D)

<table>
<thead>
<tr>
<th>Site Class E</th>
<th>S1</th>
<th>Wall Force</th>
<th>Inelastic Wall Displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>g</td>
<td>kN</td>
<td>m</td>
</tr>
<tr>
<td>0.618</td>
<td>389</td>
<td>0.07299</td>
<td>0.034</td>
</tr>
<tr>
<td>0.505</td>
<td>397</td>
<td>0.11496</td>
<td>0.061</td>
</tr>
<tr>
<td>0.377</td>
<td>341</td>
<td>0.14356</td>
<td>0.079</td>
</tr>
<tr>
<td>0.368</td>
<td>357</td>
<td>0.14591</td>
<td>0.081</td>
</tr>
</tbody>
</table>

From Table 4-1, all the wall forces are smaller than the yield strength of the tilt-up panel (271 kN). However, the plastic wall displacement demands for the last two cases are larger than the capacity (0.051 m). From Table 4-2, all the wall forces are larger than the yield strength of the tilt-up panel (271 kN). However, the wall fails for all the cases because the plastic wall displacement demands are much larger than the capacity (0.051 m). After the observations of these data, two conclusions are presented below:
For the current typical tilt-up panel, increasing size of roof not only increases the mass borne by single panel, but also increases the fundamental period of 2-DOF system. Observe the spectral curve and it is realized that the longer the natural period, the smaller the spectral acceleration when the period is beyond a certain value, say 0.25 second. Increase of roof size can reduce the spectral acceleration of this system. Hence, the seismic force demand in wall is not much sensitive to the size of roof because the increase of roof mass is counteracted by the decrement of the design spectral acceleration in the calculation of seismic wall force. The tilt-up panel can not yield even though it acts with a very large roof in the seismic motion.

The yield strength of tilt-up panel is never equal to half of the maximum seismic force demand in the corresponding fully linear elastic system. The force reduction factor is actually never exactly equal to 2 for the wall panel. From the observation of the data presented in Table 4-2, the tilt-up panels have failed in the over-large displacement demand before the force reduction factor for the tilt-up panel reaches the value of 2. The influence of over-strength must be included in the calculation of displacement demand of tilt-up panel.

The tilt-up wall panel is normally designed to be over-strength. Hence, a factor \( \gamma_w \) is used in Eq. 4.3.2 to include the influence of wall over-strength. The Eq. 4.4.2 becomes:

\[
\Delta_w = D_{w+r} - \frac{D_r}{(R/\gamma_w)}
\]

The method to calculate the wall over-strength factor \( \gamma_w \) is quite straightforward. For a
general 2-DOF system, the first mode accounts for more than 90% contribution to the base shear (wall force for the current case), the total elastic seismic wall force demand in the fully linear elastic system is calculated as follows:

\[ F_{E-w} = S_i \cdot g \cdot M_T \]  

4.4.3

Wherein, \( S_i \) is the design spectral acceleration factor corresponding to the first mode.

\( M_T \) is the system mass involved in the seismic motion and equal to sum of the total roof mass and half of the total wall mass.

The wall over-strength factor is:

\[ \gamma_w = \frac{R_{e-w}}{(F_w/R)} \]  

4.4.4

Wherein, \( R_{e-w} \) is the wall resistance and \( (F_w/R) \) is the wall force demand. \( R \) is 2 for the tilt-up panel.

Note: \( (R/\gamma_w) \) must be larger than 1.18, otherwise, the wall panel remains linear elastic through earthquakes. Generally, the resistance factor is 0.85 and the earthquake loading factor is 1.0. However, the nominal values of both resistance and load should be used in the calculation of tilt-up wall over-strength factor. Account for the influence of the wall resistance factor (0.85) and \( (R/\gamma_w) \) must not be less than 1.18.

For clear illustrations, the sample calculation of the wall elastoplastic displacement demand is performed using the same setups for Table 4-1 and Table 4-2 except that the influence of wall
over-strength is included by introducing the over-strength factor ($\gamma_w$). Assume the Site Class E. The results are presented in Table 4-3.

Table 4-3 Calculations of Wall Force (Site Class = E)

<table>
<thead>
<tr>
<th>Roof Size (Width by Length)</th>
<th>First Period (s)</th>
<th>System Mass (kg)</th>
<th>S1 (g)</th>
<th>Wall Force (kN)</th>
<th>$\gamma_w$</th>
<th>Inelastic Wall Displacement (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 x 200</td>
<td>1.03</td>
<td>6414</td>
<td>0.618</td>
<td>389</td>
<td>1.39432</td>
<td>0.04207</td>
</tr>
<tr>
<td>200 x 300</td>
<td>1.402</td>
<td>8024</td>
<td>0.505</td>
<td>397</td>
<td>1.36394</td>
<td>0.0773</td>
</tr>
<tr>
<td>100 x 300</td>
<td>1.821</td>
<td>9224</td>
<td>0.377</td>
<td>341</td>
<td>1.58934</td>
<td>0.06222</td>
</tr>
<tr>
<td>300 x 400</td>
<td>1.851</td>
<td>9899</td>
<td>0.386</td>
<td>357</td>
<td>1.51719</td>
<td>0.07436</td>
</tr>
</tbody>
</table>

Note: 1. The mass and wall force are transformed per panel.
2. 30lb per square feet for dead load plus live load
3. Assume force reduction factor is 2 for tilt-up panel and wall over-strength is considered
4. Assume site class is E
5. The wall displacement capacity is 0.051m

4.5 Modified Method to Calculate Wall Displacement

In Eq. 4.3.1, firstly, the displacement ($\Delta_r$) in roof relative to the wall, when the roof is integrated with the in-elastic wall, is replaced with the elastic displacement in roof ($D_r$) reduced by the force reduction factor (R), when the roof is integrated with the elastic wall. Account for the error induced by this substitution, and the factor, $F_2$, is introduced. Secondly, the total displacement of roof ($\Delta_{w+r}$), when the roof is integrated with the in-elastic wall, is replaced by the total displacement of roof ($D_{w+r}$), when the roof is integrated with the elastic wall. Account for the error induced by this substitution, and the factor, $F_1$, is introduced.

Include these two factors in Eq. 4.3.2 and lead to:

$$\Delta_w = D_{w+r} \cdot F_1 = \frac{D_r}{R} \cdot F_2$$

The factors, $F_1$ and $F_2$, are attained based on the simulation tests.

Option-1
The factor, \( F_1 \), is evaluated by dividing \( \Delta_{w+r} \) by \( D_{w+r} \). The values of \( \Delta_{w+r} \) and \( D_{w+r} \) are attained by running the simulation test with the same testing specimen. Average all the data of \( F_1 \), including all the cases of different mass ratios and three earthquakes, and the mean value is presented in Figure 4-6 (a) corresponding to each different stiffness ratio of wall over roof. To do so because it is believed that the dynamic displacement of the 2 degree of freedom system is not much sensitive to the type of earthquake and the mass ratio between the roof and the wall, but it is highly sensitive to the stiffness ratio.

The factor, \( F_2 \), is evaluated by dividing roof displacement relative to the wall at the time of maximum elastoplastic wall displacement (\( \Delta_r \)) by half of the maximum roof displacement in the corresponding linear elastic system (\( \frac{D_r}{R} \)). Repeat the procedure in the last paragraph, the mean values of \( F_2 \) corresponding to each different stiffness ratios of wall over roof is presented in Figure 4-6(b).

Simulation tests are performed and Eq. 4.5.1 is investigated with the values of \( \Delta_w \), \( D_{w+r} \), and \( D_r \) attained from the simulation tests. The data points of actual wall displacement versus the predicted wall displacement are presented by five plots in Figure 4-7 corresponding to each stiffness ratios of wall over roof. Investigate the distance from the data points to the straight linear representing the accurate estimation of the wall displacement, it is observed that the accuracy of estimation declines with the increase of the stiffness ratio of wall to roof. For the case of stiffness ratio equal to 15 and 20, the estimation is considered not conservative because half of the data points are placed above the line and some of them are too far away from the
line. This approach by introducing a mean value of factor $F_1$ and $F_2$ in Eq. 4.5.1 is no better than the method of using Eq. 4.3.2 without factors.

**Option-2**

A linear relationship is introduced between the factors ($F_1$ and $F_2$) and the stiffness ratio of wall to roof for the convenient calculation of those factors. See Figure 4-6, and it is more conservative than using the mean value of tests for the factors. The formulas to calculate those factors are presented as follows:

\[
F_1 = 1 - 0.005 \frac{K_w}{K_r} \quad 4.5.2
\]

\[
F_2 = 0.8 + 0.01 \frac{K_w}{K_r} \quad 4.5.3
\]

The simulation tests are carried out to investigate Eq. 4.5.1 with the new factors, $F_1$ and $F_2$. The plots of actual wall displacement versus the predicted wall displacement are presented in Figure 4-8. The same observations are repeated as described in Option-1. The estimation is still un-conservative for the case of large stiffness ratios of wall over roof. Eq. 4.3.2 is therefore recommended. And thus, derived from Eq 4.3.2, Eq. 4.4.2 is recommended in the estimation of wall displacement demand accounting for the influence of wall over-strength.

**4.6 Confirmation of Example Calculation by SAP2000**

Appendix C presents a sample calculation of total in-elastic tilt-up panel displacement demand.

The linear solution of the 2 degree of freedom mode is performed using dynamic mode analysis and NBCC acceleration spectrum. SRSS (square-root-of-sum-of-squares) rule is used
in the combination of modes. To confirm the linear part of the sample calculation in Appendix C, the commercial structural software, SAP2000, was used to perform linear dynamic analysis with the same model. Because SAP2000 does not supply spring element, 2 bending element were used with relatively infinite shear stiffness instead of 2 idealized springs in the model. The input data and results of SAP2000 are presented in Appendix D.

The input model of SAP is exactly as shown in Figure 2-1(b). The height of each pole is 1 meter and made of steel. To simplify the problem, the shear modulus of steel is set to be infinite. The cross section of the lower pole (wall spring) is square (0.096 m by 0.096 m). The lateral stiffness of wall spring is 16920 kN/m. The cross section of the upper pole (roof spring) is square (0.0728 m by 0.0728 m). The lateral stiffness of roof spring is 5616 kN/m. The roof mass and wall mass are 112.3 ton and 15.98 ton respectively. The NBCC 2005 spectrum was used. Assume the location of Vancouver and Site Class E.

The results done by manual calculation and SAP2000 are summarized in Table 4-4. Observe the data in the table, and the periods, wall displacement and roof displacement attained by manual calculation and SAP2000 are the same. However, the wall force demand calculated by SAP2000 is 728.1983 kN (for a couple of tilt-up panels), which is about 6.5% less than the value manually calculated in Appendix C. The error of manual calculation is created by assuming 100% contribution of first mode to the base shear (wall force).
To simplify the problem in Appendix C, the total mass is assumed to act only as the first mode in the calculation of base shear (wall force). Keep using the data in Appendix C and the calculation of base shear (wall force) using SRSS rule is presented as follows:

First the mode contribution to the base shear (wall force) is investigated:

The effective modal mass of the first mode is:
\[
M_1^* = \frac{(L_1)^2}{M_1} = \frac{(454730\text{kg})^2}{1730150\text{kg}} = 119515\text{kg}
\]

The effective modal mass of the second mode is:
\[
M_2^* = \frac{(L_2)^2}{M_2} = \frac{(11890\text{kg})^2}{16129\text{kg}} = 8765\text{kg}
\]

Check sum of effective modal mass as follows:
\[
M_1^* + M_2^* = 119515\text{kg} + 8765\text{kg} = 128280\text{kg}
\]
\[
M_w + M_r = 112300\text{kg} + 15980\text{kg} = 128280\text{kg}
\]

Those two sums of mass are equal, so the calculation is correct.

The percentage of first modal contribution to the base shear is:
The base shear contributed by first mode is:

\[ F_{w1} = M_1^*S_1g = 119515 \text{kg} \cdot 0.618 \cdot 9.81 \frac{N}{\text{kg}} = 724 \text{kN} \]

The base shear contributed by first mode is:

\[ F_{w2} = M_2^*S_2g = 8765 \text{kg} \cdot 0.846 \cdot 9.81 \frac{N}{\text{kg}} = 73 \text{kN} \]

Combining the base shears of the two modes as per SRSS rule leads to:

\[ F_w = \sqrt{F_{w1}^2 + F_{w2}^2} = \sqrt{724^2 + 73^2} \text{kN} = 728 \text{kN} \]
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Figure 4-1 Roof Displacement with the Cases of Elastic Wall and In-elastic Wall

Figure 4-2a Deformation of 2-DOF Model Subjected to Earthquake Loads
CHAPTER 4  Estimation of Displacement of Wall

Figure 4-2b Mode Shape of 2-DOF System

Figure 4-2c Node Displacements in Two 2-DOF Systems
Figure 4-3  Maximum Roof Displacement With the cases of elastic Wall and Inelastic Wall.
At Step Where Wall Reaches Maximum Displacement $EQ=VAN-1, VAN-2, VAN-3$ $Kw/Kr=1$

$Kw/Kr=10$

$Kw/Kr=15$

$Kw/Kr=20$

Figure 4-4  Predicted Wall Displacement versus Actual Wall Displacement at Step Where Wall Reaches Maximum Displacement
Figure 4-5  Actual Maximum Displacement of Wall versus Predicted Displacement of Wall by Eq. 4.3.2
Figure 4-6 Factor, F1 and F2
CHAPTER 4 Estimation of Displacement of Wall

Figure 4-7 Actual Maximum Displacement of Wall versus Predicted Displacement of Wall by Eq. 4.5.1
Figure 4-8 Actual Maximum Displacement of Wall versus Predicted Displacement of Wall by Eq. 4.5.1
Chapter 5

5. Conclusions

5.1 Summary and Conclusions

This research is complementary to the relevant chapter of CSA standard about the displacement demand of a plastic-designed tilt-up wall panel with large openings subjected to earthquake loads. Through this research a simple method used to estimate the maximum elastoplastic displacement demand of tilt-up wall accounting for the influence of flexible metal roof is presented.

A lower-rise tilt-up building usually includes a lateral force resisting system composed of tilt-up walls and metal roof. Subjected to the earthquake, this lateral force resisting system is modeled as a single degree of freedom system in the current practical structural design. This model may create great error if the roof is more flexible than the tilt-up wall.

In this research, the lateral force resisting system in a tilt-up building is modeled as a 2 degree of freedom dynamic system as the first mode of the roof is dominant over all other modes, and the tilt-up wall is modeled as a spring connected to a concentrated mass as well as the metal roof. The equivalent stiffness and mass of tilt-up wall and metal roof can be evaluated using static frame analysis. The concrete tilt-up wall with openings is modeled as a static frame simply supported at the ground. The metal roof can be modeled as a single-span beam simply supported at the tilt-up wall by both ends of roof. The idealized linear elastic 2 degree of freedom dynamic system, which model the lateral force resisting system in a tilt-up building, can be solved using mode-extraction and
spectral acceleration specified in Code NBCC. After that, combine these two modes, and the solutions (including the lateral seismic forces in wall and roof, the total displacement of roof relative to the ground, the displacement of wall) can be obtained using theoretical dynamic formula presented in Chapter 4.

Secondly, the basic understandings about the dynamic response of an inelastic 2 degree of freedom system, especially about the nonlinear displacement demand of inelastic tilt-up panel, were developed by performing a large amount of computer simulations using Program CANNY. The tests were intended to include all possible combinations of wall and roof used in practical construction of tilt-up buildings regarding their lateral stiffness and mass involved in seismic motions. The five earthquake records were used as the excitations to the testing specimen. Later, the nonlinear roof was added in the setups of tests, and the inelastic displacement of tilt-up wall is investigated accounting for the influence of roof yield.

Based on the insight of the test observations, a simple formula to calculate the total elastoplastic displacement demand of tilt-up wall accounting for the influence of elastic flexible roof was developed. This formula is presented in Chapter 4 as Eq. 4.3.2. The solutions of the equivalent linear elastic system are needed as the inputs to this formula. The accuracy of estimated total elastoplastic wall displacement demand was investigated by performing simulations. Of interest are the test data (wall displacement, roof displacement, roof force, wall force) at the time of maximum wall elastoplastic displacement demand. By contrasting the estimated results calculated by the formula and
the test data attained from computer-simulations using Program CANNY, the estimation by Eq. 4.3.2 is considered conservative and quite accurate for the practical structural design.

The conclusions about modeling the lateral force resisting system in a tilt-up building are summarized below:

1. The total mass on the roof and half mass of the transverse panels, if the transverse panels have openings, should be included as the node mass concentrated at the upper node as shown in Figure 2-1(b). Half of the total mass of in-plane tilt-up wall, parallel to the earthquake direction, should be used as the node mass concentrated at the lower node as shown in Figure 2-1(b).

2. The lateral stiffness of the tilt-up wall, corresponding to the first mode, is used as the stiffness of the lower spring as shown in Figure 2-1(b). The lateral stiffness of tilt-up panel is equal to the point force applied to the roof level of panel to cause a unit horizontal displacement.

3. The lateral stiffness of the metal roof, corresponding to the first mode, is used as the stiffness of the upper spring as shown in Figure 2-1(b). The lateral stiffness of the roof is calculated by the formula \( K_r = M_r \omega_r^2 \). \( M_r \) is calculated in Chapter 2. \( \omega_r \) is calculated according to the classical dynamic theory about a simply supported beam with uniformly distributed mass.

4. The equivalent concrete sectional flexural stiffness should be used as 25% of the gross flexural stiffness of concrete section accounting for the influence of concrete cracking.
CHAPTER 5  Summary, Conclusions and Design Suggestions  115

Through computer simulations, the conclusions about the nonlinear dynamic behavior of flexible tilt-up wall accounting for the influence of flexible roof is presented as follows:

1. In the nonlinear analysis of the idealized 2 degree of freedom lateral force resisting system in a tilt-up building, the mass ratio and the stiffness ratio between tilt-up walls and metal roofs are investigated. The mass ratio is the mass of roof (not including half of the total mass of transverse tilt-up panel) over the mass of wall, and the stiffness ratio is the equivalent lateral stiffness of the wall over the equivalent stiffness of the roof. Six typical roofs and three tilt-up panels used in practice and suitable for the nonlinear structural design were chosen to calculate these ratios. Through the calculations, the mass ratio of the roof over the wall ranges from 1.5 to 20, and the stiffness ratio of the wall over the roof ranges from 0.2 to 13 if the influence of concrete cracking is included. And if the wall stiffness is not reduced to account for cracking, the stiffness ratio ranges from 1 to 40.

2. The ductility of wall is highly sensitive to the stiffness ratio between the tilt-up wall and metal roof, which constitute the lateral force resisting system in a tilt-up building. Along with each fixed mass ratio levels (1, 3, and 9) between the wall and the roof, the maximum ductility of the in-elastic wall ranges from 1 to 45 as the stiffness ratio of the wall to roof varies from 3 to 25.

3. The ductility of wall is much less sensitive to the mass ratio than the stiffness ratio. The mass ratio of the roof over the wall ranges from 1 to 40 in the computer simulations accounting for the mass of the transverse tilt-up wall moved together with the roof in the seismic motion. Along with the variations of the mass ratios
of roof over wall, the range of maximum ductility of wall is presented in the
following table corresponding to each level of the stiffness ratio.

Table 5-1 Range of Wall Ductility

<table>
<thead>
<tr>
<th>Stiffness ratio</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range of Wall Ductility</td>
<td>1.5—3</td>
<td>2—5.5</td>
<td>3—8.5</td>
<td>4.5—18</td>
<td>8—35</td>
</tr>
</tbody>
</table>

With a given stiffness ratio, the ductility of wall fluctuates about a constant across
the range of the mass ratio between the wall and the roof. However from the
observations of wall ductility demand presented in Table 5-1, the mass ratio has
greater influence on the wall ductility with the increase of the stiffness ratio of
wall over roof.

4. Yield of tilt-up wall in the lateral force resisting system of a tilt-up building not
only reduces the seismic force in the tilt-up wall but also reduces the seismic force
in the metal roof. The forces in both wall and roof are approximately reduced to
the same degree although the roof remains linear elastic and only the wall yields
through earthquakes. The roof force ratio is calculated by dividing the maximum
roof force, attained when the elastic roof acts with the elastic wall, by the
maximum roof force, attained when the same elastic roof with initial properties
acts with the corresponding in-elastic wall. As the possible mass ratio of the roof
over the wall is between 1 and 40 and the possible stiffness ratio of the wall over
the roof is between 1 and 20, the roof force ratio is between 1 and 2 independent
of the five earthquake records. The distribution of the data points across the range
of the roof force ratio are not apparently related to the wall stiffness. As the wall
stiffness is 3668 kN/m, the roof force ratio is approximately between 1.5 and 2.
CHAPTER 5  Summary, Conclusions and Design Suggestions

As the wall stiffness are 1269 kN/m and 13367 kN/m, the roof force ratio is between 1 and 2, but most of the data is between 1.5 and 2.

5. Alternately, in the lateral force resisting system of a tilt-up building, yield of roof can reduce the dynamic response of tilt-up wall as well. Yield of roof can reduce the seismic force in tilt-up wall if the wall remains elastic. And also, yield of roof can reduce the elastoplastic displacement demand of tilt-up wall if the wall yields through the earthquake as well. For a specific test setup, the mass ratio of roof to wall is 5 and the stiffness ratio of wall to roof is 10, the ductility of the wall decreases approximately from 10 to 1 (1 means wall does not yield) while the ductility of the roof increases from 1 (1 means Roof does not yield) to 1.8. But the influence of the roof ductility on the wall ductility decays with the decrement of the stiffness ratio of the wall over the roof. This concept was confirmed by performing another specific simulation. In the case that the stiffness ratio is equal to 1 between wall and roof, the ductility of the wall decrease approximately from 2.3 to 1.7 while the ductility of the roof increases from 1 to 2.5.

6. The maximum forces do not occur at the same time in both wall and roof in the fully elastic tilt-up system, and the force of roof is not at the maximum but close to the maximum value when the in-elastic wall reaches the maximum elastoplastic displacement demand.

7. The ratio of maximum roof force attained in the corresponding linear system over the roof force at the time of maximum inelastic wall displacement is investigated. For the setup of simulation, the stiffness ratio and the mass ratio are between 1
and 20. The roof force ratio is between 1 and 4 independent of the three earthquake records of Vancouver Type. But most of the data are between 2 and 3.

8. At the instant of maximum wall displacement demand, the maximum displacement of in-elastic wall ($\Delta_w$) can be exactly calculated by subtracting the roof drift ($\Delta_r$) relative to the wall from the total roof displacement ($\Delta_{w+r}$) relative to the ground. The formula to estimate the maximum inelastic wall displacement demand was developed from this concept.

9. If the yield strength of wall in the nonlinear system is equal to half of the wall force in the corresponding fully linear elastic system, the roof force at the time of maximum inelastic wall displacement demand can be approximately equal to half of the maximum roof force in the fully elastic system. Since the roof remains linear elastic in both linear and nonlinear systems, the roof drift relative to the wall is linearly proportional to the roof force. Hence, the roof drift relative to the wall at the time of maximum inelastic wall displacement demand can be approximately equal to half of the maximum roof drift ($D_r$) in the fully elastic system.

10. The equal displacement principle to be in the current case was confirmed acceptable accuracy by computer simulations. Subjected to the excitation of earthquake loads, the total elastic roof displacement, when elastic roof is integrated with elastic wall as the lateral force resisting system in a tilt-up building, is approximately equal to the total roof displacement, when the same elastic roof is integrated with the corresponding in-elastic wall in a tilt-up building. As shown in the simulations, the former displacement is always a little larger than
the later one. So, it is conservative to use maximum total roof displacement \( D_{w_r} \) with elastic system to estimate the total roof displacement with the corresponding inelastic system in the formula of Eq. 4.3.2.

11. From Clause 8, 9 and 10, the maximum elastoplastic displacement demand \( \Delta_w \) of the inelastic wall can be conservatively expressed as \( D_{w_r} - D_r/R \). This formula is confirmed by performing computer simulations. The setups of simulations included the stiffness ratio of the wall over the roof from 1 to 20 and the mass ratio of the roof over the wall from 1 to 20. Three Vancouver Type earthquake records were used. The estimation of the elastoplastic wall displacement demand by the developed formula of Eq. 4.3.2 is accurate and conservative to be used in the practical structural design.

### 5.2 Further Research

The developed method to estimate the elastoplastic displacement of tilt-up wall as well as the conclusions of this research is based on the 2 degree of freedom dynamic model to simulate the lateral force resisting system in a single storey tilt-up building. However, this model is not properly correct if the stiffness center of the tilt-up panels is not located at the geometric center of the tilt-up building. A torsional moment is produced by the eccentricity. For such a case, the new model is set up as shown in Figure 5-1.

Consider the tilt-up building shown in Figure 2-1(a) subjected to a lateral earthquake excitation, and the roof is simply supported on the top of two tilt-up walls at its both ends. Suppose the two walls have different lateral stiffness, each of them is idealized as an equivalent spring, \( K_{w1} \) and \( K_{w2} \), with an equivalent point mass, \( M_{w1} \) and \( M_{w2} \). The metal
roof is idealized as a simply supported beam with the mass, m_r, evenly distributed along the beam span. Further research should be performed with this new model.

5.3 Design Suggestions

Based on this research four points are suggested in practical structural design of tilt-up wall with openings accounting for the influence of flexible metal roof.

1. Under severe earthquake excitations, either metal roof or tilt-up panel yields, generally it is impossible for both of them to yield.

2. According to the research by ESSA, TREMBLAY and ROGERS (2001), even though metal roof yields in earthquakes, it only yields in some specific small areas, not overall yield across the roof. It not practical to calculate the influence of roof yield, hence, the metal roof should be designed as a linear elastic component to resist lateral loads, especially seismic loads.

3. For the displacement-based concrete tilt-up panel design, the yield strength of panel is determined to be half of the maximum seismic force demand of the corresponding linear elastic panel. Or in other words, the force reduction factor (R) is 2 for the tilt-up panel. However, due to the influence of flexible roof, the tilt-up panel fails in too large displacement demand. This is the case for most of the typical tilt-up panels integrated with typical flexible roofs in tilt-up buildings. In order to reduce total wall displacement demand, the influence of wall over-strength should be included.

4. Increasing size of metal roof in its span direction not only increases the roof mass
involved in the seismic motion, but also increases the fundamental period of the system (Tilt-up wall integrated with Metal roof). Hence the increase of seismic force caused by increasing roof mass is counteracted by the decrease of design spectral acceleration due to increasing the fundamental period. To summarize, increasing size of roof actually basically does not increase the wall force. Therefore, if it is possible, an elastic structural design for the tilt-up wall panel is suggested in a tilt-up building.

Figure 5-1 Dynamic Model of a Tilt-up Building with Tilt-up Walls Having Different Stiffness
References


Motion Research, Proceedings of the NEH RF Conference and Workshop on
Appendix A

Calculation of Lateral Stiffness of Concrete Tilt-up Panel

Referred to Figure 2-2(b), the lateral bending stiffness of the tilt-up panel with two openings can be calculated by performing static framing analysis. Apply a lateral point load to the roof level of wall panel, and six intended deformations in this frame (2 translations at each beam level and 4 rotations at the joints) are shown in Figure 2-2(b). Assume the directions shown by arrows in the figure are positive. The total displacement at the top of wall ($u_5 + u_6$) can be found by solving the following matrix equations.

\[
\begin{bmatrix}
  k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} \\
  k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} \\
  k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} \\
  k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} \\
  k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} \\
  k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66}
\end{bmatrix}
\begin{bmatrix}
  u_1 \\
  u_2 \\
  u_3 \\
  u_4 \\
  u_5 \\
  u_6
\end{bmatrix}
= \begin{bmatrix}
  0 \\
  0 \\
  0 \\
  0 \\
  0 \\
  1
\end{bmatrix}
\]

A.1

\[
k_{11} = \frac{4EI}{c_1 H_2} + \frac{4EI}{b_1 L}
\]

A.2

\[
k_{12} = k_{21} = \frac{2EI}{b_1 L}
\]

A.3

\[
k_{13} = k_{31} = \frac{2EI}{c_1 H_2}
\]

A.4

\[
k_{14} = k_{41} = 0
\]

A.5

\[
k_{15} = k_{51} = \frac{-6EI}{c_1 H_2^2}
\]

A.6

\[
k_{16} = k_{61} = \frac{6EI}{c_1 H_2^2}
\]

A.7
\[ k_{22} = \frac{4EI_c2}{H_2} + \frac{4EI_b1}{L} \]  
A.8

\[ k_{23} = k_{32} = 0 \]  
A.9

\[ k_{24} = k_{42} = \frac{2EI_c2}{H_2} \]  
A.10

\[ k_{25} = k_{52} = -\frac{6EI_c2}{H_2} \]  
A.11

\[ k_{26} = k_{62} = \frac{6EI_c2}{H_2^2} \]  
A.12

\[ k_{33} = \frac{4EI_c1}{H_2} + \frac{4EI_b2}{L} + \frac{4EI_c1}{H_1} \]  
A.13

\[ k_{34} = k_{43} = \frac{2EI_b2}{L} \]  
A.14

\[ k_{35} = k_{53} = \frac{6EI_c1}{H_2^2} + \frac{6EI_c1}{H_1^2} \]  
A.15

\[ k_{36} = k_{63} = -\left( \frac{6EI_c1}{H_2} \right) \]  
A.16

\[ k_{44} = \frac{4EI_c2}{H_2} + \frac{4EI_b2}{L} + \frac{4EI_c2}{H_1} \]  
A.17

\[ k_{45} = k_{54} = \frac{6EI_c2}{H_2^2} + \frac{6EI_c2}{H_1^2} \]  
A.18

\[ k_{46} = k_{64} = -\left( \frac{6EI_c2}{H_2^2} \right) \]  
A.19

\[ k_{55} = \frac{12((EI_c1 + (EI_c2))}{H_1^2} + \frac{12((EI_c1 + (EI_c2))}{H_2^2} \]  
A.20

\[ k_{56} = k_{65} = \frac{-12((EI_c1 + (EI_c2))}{H_2^2} \]  
A.21

\[ k_{66} = \frac{12((EI_c1 + (EI_c2))}{H_2^2} \]  
A.22
The lateral bending stiffness of frame, \( k_{bend} \), can be calculated as follows:

\[
k_{bend} = \frac{1}{u5 + u6}
\]

The lateral shear stiffness of Pier 1 as shown in Figure 2-2(a), \( k_{pier1} \), is calculated as follows:

\[
k_{pier1} = \frac{G_c A_{pier1}}{1.2(H_1 + H_2)}
\]

, wherein \( A_{pier} \) is the area of pier cross-section indicated by hatch in Figure 2-2(a).

\( H_1 \) and \( H_2 \) are the height between the levels as shown in Figure 2-2(a).

The lateral shear stiffness of Pier 2 as shown in Figure 2-2(a), \( k_{pier2} \), is calculated as follows:

\[
k_{pier2} = \frac{G_c A_{pier2}}{1.2(H_1 + H_2)}
\]

The lateral shear stiffness of tilt-up wall, \( k_{shear} \), can be calculated as follows:

\[
k_{shear} = k_{pier1} + k_{pier2}
\]

The total lateral stiffness of wall is calculated as follows:

\[
K_w = \left( \frac{1}{k_{bend}} + \frac{1}{k_{shear}} \right)^{-1}
\]

Note: The effective flexural stiffness (EI) of concrete section should be used accounting for the influence of concrete cracking. See appendix B for more details.

For more accurate calculation of the lateral stiffness of wall, the influence of rigid extensions to model the joints in the panel should be included in the frame analysis.
Appendix B

Sample Calculation of Effective Concrete EI

B.1 Introduction

Concrete is constituted of complex materials. It is not isotropic in strength and stiffness. The flexural stiffness of concrete, $E_I$, is affected by many factors, such as degree of concrete, loading rate, loading history, concrete creep, shrinkage, cracks etc. Since the flexural stiffness is severely decreased due to the occurrence of concrete cracks, the effective concrete flexural stiffness, $E_{Ie}$, is used in the frame analysis of concrete. For the current case, the effective concrete flexural stiffness should be computed by performing a complete frame analysis of concrete tilt-up panel with openings. However consider simplifying the research and the adequate accuracy of sectional analysis, and thus, the effective flexural stiffness of concrete is determined only by performing the sectional analysis of concrete.

The curve of sectional moment versus curvature was developed using Program Response 2000. Two concrete sections, beam section and column section, were selected from the tilt-up panel with two openings used in Example 13.5 by Weiler (1995). The figure used in that example is copied here as Figure B-1. The influence of axial force and selected sectional yield moment of concrete are investigated in the calculations of the effective sectional flexural stiffness of concrete.

The effective flexural stiffness is evaluated graphically according to the equal area principal. Refer to Figure B-3(a), and draw a straight line across the data point of selected
yield strength on the curve and parallel to the horizontal coordinate axis. Start another line from the origin with an intended slope, which is the unknown effective sectional flexural stiffness of concrete. These two lines join together at a certain point. A mathematical equation can be set up by equating the area enclosed by those two lines and the horizontal coordinate axis to the area enclosed by the developed curve and the horizontal coordinate axis. The effective sectional stiffness of concrete can be found by solving this equation.

**B.2 Column Section with Axial Force**

Referred to Figure B-1, the size of concrete column section in the tilt-up panel is 190 mm by 900 mm. The reinforcement is placed evenly across the section in 5 rows, and for each row of reinforcement, 2 steel bars are placed at the both sides of concrete section. The size of reinforcement is M-20 for the top and bottom row, and M-15 is used for the middle rows. The degree of concrete is C25. As for steel, Yong’s modulus is 200000 Mpa and yield strength is 400 Mpa. No strain hardening is included in the analysis, but a light axial compressive force is exerted on the concrete column section.

The results are presented in Figure B-2 and Figure B-3. The influence of axial force is shown in Figure B-2 with 8 different values of axial force, but all the values are lower than the sectional moment causing the stress of 11% sectional compressive strength. It is observed that the sectional concrete flexural stiffness and moment capacity increase with the increase of axial force.
The influence of the selected yield moment is shown in Figure B-3(a) and Figure B-3(b). Figure B-3(a) presents the curve of sectional moment versus curvature. Three broken lines are built corresponding to the ratios of selected yield moment over maximum sectional moment. The slope of the broken lines is equal to the effective flexural stiffness. In Figure B-3(b), the variable of vertical coordinate axis is the ratio of the effective sectional flexural moment, $EI_c$, over the gross sectional flexural stiffness, $EI_g$, while the variable of horizontal coordinate axis is the ratio of the selected sectional yield moment over the maximum sectional moment. Through the observations of these two figures, the effective flexural stiffness decreases with the increase of the selected yield moment, and it is quite sensitive to the selection of yield moment.

The influence of the axial force is shown in Figure B-3(c). The variable of vertical coordinate axis is the ratio of the effective sectional flexural moment ($EI_c$) over the gross sectional flexural stiffness ($EI_g$) and the variable of horizontal coordinate axis is the ratio of the exerted force ($P$) on the concrete section over the sectional compressive strength ($A_gf_c$). The tilt-up wall is expected only to bear the gravity loads producing a stress lower than 10% of its sectional compressive strength since tilt-up wall is commonly used to resist lateral loads in the construction of lower-rise tilt-up building. It is observed that the effective sectional flexural stiffness increases linearly with the increase of the axial force exerted on the concrete section.
B.3 Beam Section without Axial Force

B.3.1 Distribution of Reinforcement

Referred to Figure B-1, the size of beam section is 190 mm by 1500 mm. The two concrete sections of beam element are distinguished by the different layout of reinforcement. For the first beam section, the rebar are evenly distributed across the concrete section in 5 rows, each row including 2 bars. The size of reinforcement is M-20 for the top and bottom rows, and M-15 is used for the other three rows. For the second beam section, the rebar is concentrated at the both ends of the section. 4 rebar of M-20 are placed at the bottom of the beam section and 2 bars of M-15 are placed at the top. The bending strengths of these two sections are approximately equated. The degree of concrete is C25. As for the steel, Yong's modulus is 200000 Mpa and yield strength is 400 MPa. No strain hardening of steel is included and no axial force is exerted on the beam section. Program Response 2000 is used to perform the sectional analysis.

The results are presented in Figure B-4. For clear observation, the two figures are shown as Plot (a) and Plot (b) with different scales. Along with approximately the same bending capacity, the curve for the case of evenly-distributed steel encloses smaller area with the coordinate axis than the curve for the case of concentrate-placed steel across the section. Therefore, the smaller sectional effective flexural stiffness is created by evenly placing the steel bars across the section than only putting the re-bars at the ends of the section.
B.3.2 Beam Section with Tension-Stiffness

The size of concrete section is 190 mm by 1500 mm. The reinforcement is evenly distributed in 5 rows across the section, each row including 2 bars. The size of reinforcement is M-20 for the top and bottom row, and M-15 is used for other rows. The degree of concrete is C25. As for the steel, Yong's modulus is 200000 Mpa and yield strength is 400 Mpa. No strain hardening of steel is included in the analysis. No axial load was exerted on the concrete section.

The results are presented in Figure B-5 by the two curves of sectional moment versus curvature. One of them includes concrete tension-stiffening and the other one excludes concrete tension-stiffening in the analysis. For more clear observation, Plot (a) and Plot (b) are presented in different scales. The curve for the case of no tension-stiffness is observed to overlap the curve for the case of tension-stiffness until the concrete beam section cracks. After that, the former curve is located below the later one until the moment is very close to the sectional moment capacity. Then, the two curves overlap again until the failure of the section. Since the curve in the case of including tension-stiffening encloses a larger area with the axis, it is concluded that the tension-stiffness increases the effective sectional flexural stiffness of concrete. However tension-stiffening does not raise the sectional moment capacity.
B.4 Conclusions

Because the gravity load exerted on the tilt-up wall is usually small, less than 10% of the sectional compressive strength of tilt-up panel, the influence of the axial force is advised conservatively to be neglected in the calculations of the sectional effective flexural stiffness of concrete. Hence, for the current research, the effective sectional flexural stiffness of concrete is recommended to be 0.25 times the gross sectional flexural stiffness in the static frame analysis of tilt-up panel with openings.
Figure B-1 Typical Tilt-up Panel Used in Example 13.5 by Weiler (1995)

Figure B-2 Sectional Moment versus Curvature with Influence of Axial Force
Figure B-3  Effective Sectional Flexural Stiffness of Concrete with Influence of Axial Force and Selected Yield Moment
Figure B-4  Sectional Moment versus Curvature with Influence of the Placement of Steel Bars across Concrete Section
Figure B-5 Sectional Moment versus Curvature with Influence of Tension stiffening
Clause 21.7.1.2 of the draft 2004 Canadian concrete code states that the requirements of Clause 21 shall apply to wall panels with openings when the maximum rotational demand on any part of the panel is greater than 0.02 radians. This example demonstrates how the calculations for rotational demand may be done. The example uses the wall panel given in Example 13.5 in Chapter 13 of the 1995 CPCA Handbook (by Gerry Weiler). The structure is assumed to be 100 ft by 200 ft in dimension, and the calculations are for the earthquake acting parallel to the 100 ft long walls. That is, the roof diaphragm is spanning 200 ft and the 100 ft long tilt-up walls resist the lateral forces due to the earthquake.

The dynamic response of the structure will be modeled as a 2 degree of freedom system with one mass and spring to represent the roof, and a second mass and spring to represent the wall panels.

Step 1: Calculate Mass and Stiffness of Tilt-up Wall Panel

(a) The mass of the wall panel that should be used for the equivalent model is half the total mass of the tilt-up wall panel.

For the panel shown above (Figure B-1, copied from Appendix B for convenience), this is:

\[ m_w = (9 \cdot 7.5 - 7.2 \cdot 3.0 - 7.2 \cdot 1.5) m^2 \cdot 0.19 m \cdot 2350 \frac{kN}{m^3} \cdot 0.5 = 78.35 kN = 7990 kg \]

The wall mass per panel width in the tilt-up building is:

\[ M_w = 2 \times 7990 kg = 15980 kg \]
(b) The stiffness of the spring representing the wall panel is equal to the in-plane stiffness of the tilt-up wall panel, that is, the force to cause a unit displacement at the top of the wall. Any linear frame analysis program can be used to do this calculation. The wall panel can be modeled as an equivalent frame with beams and columns located at the centroid of the concrete sections, and rigid extensions to model the joint region. To account for flexural cracking of the panel elements (beams and columns), the effective stiffness should be taken as 25% of the gross sectional stiffness \( I_e = 0.25 I_g \).

SAP2000 was used to do these calculations for the panel shown in Fig. B-1, and the result is

\[
 k_w = 8460 \frac{kN}{m}
\]

The wall stiffness per panel width in the tilt-up building is:

\[
 K_w = 2 \times 8460 \frac{kN}{m} = 16920 \frac{kN}{m}
\]

(c) As the mass and stiffness of the wall panel are known, the fundamental period of the wall can be determined from:

\[
 T_w = 2\pi \sqrt{\frac{m_w}{k_w}} = 2 \times 3.14 \times \sqrt{\frac{7990 \text{kg}}{8460000 \frac{N}{m}}} = 0.193 \text{s}
\]

**Step 2: Calculate Mass and Stiffness of Roof**

The diaphragm action of the roof is assumed to be similar to a simply supported deep beam with a uniformly distributed mass. Theoretically, this “beam” has an infinite number of modes. For simplicity, only the first mode is accounted for, and thus the roof is converted to an equivalent single degree of freedom system with the same period (frequency) as the first mode of the roof.

(a) Determine the mass of the roof:

For the current example, the span length of the roof \( L_r \) is 61 m, and the width of the roof \( W_r \) is 30.5 m. The mass of the roof is assumed to be 30 psf (20 psf dead load and 10 psf live load) which is equal to 146.5 kg/m\(^2\). Half the mass of the transverse panels, which move with the roof, must be added to the mass of the roof. The panels are 9 m long, and half the mass of each panel is 7990 kg.

Thus, the total mass of the roof is:

\[
 m_r = 61m \cdot 30.5m \cdot 146 \frac{kg}{m^2} + 2 \times 7990 \text{kg} \times \frac{61m}{9m} = 380400 \text{kg}
\]

The total mass of the roof per panel width is:
(b) Determine the stiffness of the equivalent single degree of freedom system representing the roof:

Determine the flexural deformation, $\Delta_m$, and shear deformation, $\Delta_s$, of the roof subjected to a uniformly distributed unit loading.

For the current example the result is:

$\Delta_m = 2.088 \times 10^{-6} m$

$\Delta_s = 1.979 \times 10^{-6} m$

The calculation of these deflections for the current roof is shown in Appendix C-a.

Determine the equivalent flexural stiffness of the roof accounting for shear deformation from the following equation:

\[
(EL)_{r-equ} = \frac{E \cdot I_r}{1 + \frac{\Delta_s}{\Delta_m}}
\]

where $E$ is Young's modulus for steel, and $I_r$ is the second moment of area of the roof chord angles. The calculation of $I_r$ for the current roof is shown in Appendix C-a.

For the current example:

\[
(EL)_{r-equ} = \frac{200000 MPa \times 4.306 \times 10^{11} mm^4}{1 + \frac{1.979 \times 10^{-6} m}{2.088 \times 10^{-6} m}} = 4.421 \times 10^{10} N \cdot m^2
\]

Determine the natural circular frequency of roof from:

\[
\omega_r = \frac{\pi^2}{L_r^2} \sqrt{\frac{(EL)_{r-equ}}{m_r}}
\]

For the current example:

\[
\omega_r = \frac{3.14^2}{(61 m)^2} \sqrt{\frac{4.421 \times 10^{10} N \cdot m^2}{6.24 \times 10^3 \frac{kg}{m}}} = 7.707 Hz
\]

Determine the equivalent lateral stiffness of the roof:

\[
k_r = \omega_r^2 \cdot m_r = (7.707 Hz)^2 \times 3.804 \times 10^5 kg = 19010 \frac{kN}{m}
\]

Determine the equivalent stiffness of the roof per panel width:
Appendix C  140

\[ K_r = \frac{k_r \times (9m)}{W_r} = 19010 \frac{kN}{m} \times \frac{9m}{30.5m} = 5.614 \times 10^3 \frac{kN}{m} \]

(c) The fundamental period of the roof can be determined as:

\[ T_r = \frac{2\pi}{\omega_r} = \frac{2 \times 3.14}{7.07 \text{Hz}} = 0.889s \]

**Step 3:** Conduct a linear dynamic (modal) analysis of the 2-DOF system and determine the elastic deflection of the roof and the elastic force in the roof.

These calculations can be done using any dynamic analysis software, or from first principles using software such as MathCAD. Appendix C-b provides a MathCAD sheet that can be used to do these calculations.

For the current example, the two mode shapes (normalized by the value at the wall) are:

\[ \phi_1 = \begin{bmatrix} \phi_{11} \\ \phi_{12} \end{bmatrix} = \begin{bmatrix} 1 \\ 0.026 \end{bmatrix} \quad \phi_2 = \begin{bmatrix} \phi_{21} \\ \phi_{22} \end{bmatrix} = \begin{bmatrix} -0.036 \\ 0.174 \end{bmatrix} \]

The equivalent masses relevant to the first and second modes are:

\[ M_1 = 1730150\text{kg} \quad M_2 = 16129\text{kg} \]

The equivalent stiffnesses relevant to the first and second modes are:

\[ K_1 = 64400 \frac{kN}{m} \quad K_2 = 22950 \frac{kN}{m} \]

The periods are:

\[ T_1 = 1.03s \quad T_2 = 0.167s \]

Using the design spectrum for Vancouver from the 2005 NBCC (for site class E) gives the following spectral values:

For \( T_1 = 1.03s \), \( S_1 = 0.618 \)
The displacement demand relevant to the first mode is
\[ D_1 = g \cdot S_1 \cdot \left( \frac{T_1}{2 \cdot \pi} \right)^2 = 9.81 \cdot \frac{N}{kg} \times 0.618 \times \left( \frac{1.03s}{2 \cdot \pi} \right)^2 = 0.163 \text{(m)} \]

For \( T_2 = 0.1746s \), \( S_2 = 1.786 \)

The displacement demand relevant to the first mode is
\[ D_2 = g \cdot S_2 \cdot \left( \frac{T_2}{2 \cdot \pi} \right)^2 = 9.81 \cdot \frac{N}{kg} \times 1.786 \times \left( \frac{0.1746s}{2 \cdot \pi} \right)^2 = 0.013 \text{(m)} \]

The factors corresponding to the first mode and the second mode (\( L_1 \) and \( L_2 \)) are calculated as follows:
\[
L_1 = \phi_1^T M_1 = \left( \phi_{w1} \quad \phi_{r1} \right) \begin{bmatrix} M_w & 0 \\ 0 & M_r \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \phi_{w1} M_w + \phi_{r1} M_r = 454730 \text{kg}
\]
\[
L_2 = \phi_2^T M_1 = \left( \phi_{w2} \quad \phi_{r2} \right) \begin{bmatrix} M_w & 0 \\ 0 & M_r \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \phi_{w2} M_w + \phi_{r2} M_r = 11890 \text{kg}
\]

The mode participation factors corresponding to the first mode and the second mode (\( \Gamma_1 \) and \( \Gamma_2 \)) are calculated as follows:
\[
\Gamma_1 = \frac{L_1}{M_1} = \frac{454730}{1730150} = 0.263
\]
\[
\Gamma_2 = \frac{L_2}{M_2} = \frac{11890}{16129} = 0.737
\]

The total elastic displacement of roof is:
\[ D_{w,e} = \sqrt{(\Gamma_1 D_1 \phi_{w1})^2 + (\Gamma_2 D_2 \phi_{w2})^2} = 0.167 \text{m} \]

The elastic roof drift relative to the wall is:
\[ D_r = \sqrt{(\Gamma_1 D_1 \phi_{r1} - \Gamma_1 D_1 \phi_{w1})^2 + (\Gamma_2 D_2 \phi_{r2} - \Gamma_2 D_2 \phi_{w2})^2} = 0.125 \text{m} \]

**Step 4: Estimate the Plastic Displacement Demand of Concrete Tilt-up Wall**

(a) Determine the wall over-strength factor. For the current sample wall, the yield strength of the wall (\( F_{yw} \)) is 271kN. This can be attained by performing push-over analysis for the wall frame.

(b) Calculate the seismic wall force demand:
\[ F_w = (M_w + M_r) \cdot S_1 \cdot g = (15980 + 112300) \text{kg} \times 0.618 \times 9.81 \frac{N}{kg} = 778 \text{kN} \]
The seismic force per wall panel \( (F_{ww}) \) is:
\[
F_{ww} = \frac{F_w}{2} = 389kN
\]

The wall over-strength factor is:
\[
\gamma_w = \frac{F_{yw}}{(F_{ww} / R) (389 / 2)kN} = 1.39
\]

(c) The plastic displacement demand of the wall can be given as follows:
\[
\Delta_{yw} = D_{w+r} - \frac{D_r}{(R / \gamma_w)} - \Delta_{yw} = 0.167 - \frac{0.125}{(2/1.39)} - 0.032 = 0.048m
\]

Step 5: Determine the In-elastic Rotational Demand on Concrete Tilt-up Wall

Estimate the rotational demand on the wall panel using a simple rigid-plastic model (i.e., neglect elastic deflections of the wall). If this rotation is less than the code allowable value, a detailed push-over analysis is not required.

For the current example, the estimated plastic rotation is calculated as following:
\[
\frac{0.048m}{3m - 0.9m/2} = 0.019 rad
\]

(See Fig. F-1).

In this case, the inelastic rotational demand on the wall panel is 0.019 rad smaller than the code allowable value, 0.02, without special ductile detailing.
Appendix C-a

Calculation of roof deflections

For the mechanical analysis, under the lateral loading the metal roof could be modeled as an I-beam comprised of edge chords as the equivalent flanges and corrugated sheets as the equivalent webs. Refer to CSSBI B 13-91 for more details.

The roof is assumed to have the following properties:
Width=100 feet
Length=200 feet
3x3x1/4 inch chord angle
Roof deck is 22 gauges (0.76mm) for normal shear area
Roof deck is 20 gauges (0.91mm) for critical shear area
Joist spacing is equal to 6 ft (1800mm)
1.5 inch (38mm) steel deck
Punching/welding spacing in normal shear areas is 18 inch button
Punching spacing in critical shear zones is 6 inch button
For the critical shear area account for 20% of total loading area
The ratio of span of deck unit to average length of deck sheet supplied is taken to be 3.

Determine the inertia of moment of metal roof

\[ I_r = 2 \left[ I_{ch} + A_{ch} \left( \frac{W_r}{2} \right)^2 \right] = 2 \left[ 518000 + 927 \left( \frac{100 \cdot 305}{2} \right)^2 \right] = 4.306 \times 10^{11} \text{mm}^4 \]

Determine the flexural deflection created by unit uniform distributed force.

\[ \Delta_m = \frac{5 \cdot w \cdot L_r^4}{384 \cdot E \cdot I_r} = \frac{5 \times 1 \cdot \frac{N}{m} \cdot (200 \text{ft})^4}{384 \times 200000 \text{MPa} \times 4.306 \times 10^{11} \text{mm}^4} = 2.088 \times 10^{-6} \text{m} \]

Determine the flexibility factor of metal roof. See Table 3(b), CSSBI B 13-91.

\[ F_{nor} = (69 + 247 \times \frac{1}{3}) = 151 \frac{\text{mm}}{N} \quad F_{cr} = (44 + 143 \times \frac{1}{3}) = 92 \frac{\text{mm}}{N} \]

Determine the average shear created by unit uniform distributed force.

\[ V_{nor} = 0.25 \cdot w \cdot \frac{0.8L_r}{W_r} = 0.25 \times 1 \cdot \frac{N}{m} \times \frac{0.8 \times 200 \text{ft}}{100 \text{ft}} = 0.4 \frac{N}{m} \]
\[ V_{cr} = 0.5 \cdot (0.5 \cdot w \cdot \frac{0.8L_r}{W_r} + 0.5 \cdot w \cdot \frac{L_r}{W_r}) = 0.45 \cdot w \cdot \frac{L_r}{W_r} = 0.9 \frac{N}{m} \]
Appendix C

Determine the shear deflection created by unit uniform distributed force.

\[
\Delta_s = \frac{V_{nor} \cdot (0.8L_e \cdot F_{nor})}{2 \times 10^6} + \frac{V_{cri} \cdot (0.2L_e \cdot F_{cri})}{2 \times 10^6} = \frac{0.4 \frac{N}{m} \times 0.8 \times 200 \text{ft} \times 151 \frac{mm}{N}}{2 \times 10^6}
\]

\[
= 1.979 \times 10^{-6} \text{m}
\]
Appendix C-b

Calculate the Modes of 2 Degree of Freedom System, Equivalent Mass and Equivalent Stiffness

Develop stiffness matrix for the system
\[
K = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}
\]

\[k_{11} = K_r + K_w = (5614 + 16920) \frac{kN}{m} = 22530 \frac{kN}{m}\]

\[k_{22} = K_r = 5614 \frac{kN}{m}\]

Develop mass matrix for the system
\[
M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}
\]

\[m_{11} = M_w = 15980 \text{kg} \quad m_{12} = m_{21} = 0 \quad m_{22} = M_r = 112300 \text{kg}\]

Determine the natural frequencies and periods of two modes
Solve the following equations with the assistant of math tools, say MathCAD.

\[
\det(K - \omega^2 \cdot M) = 0
\]

\[\omega_1 = 6.098 \text{Hz} \quad \omega_2 = 37.72 \text{Hz} \quad T_1 = \frac{2\pi}{\omega_1} = 1.03 \text{s} \quad T_2 = \frac{2\pi}{\omega_2} = 0.1666 \text{s}\]

Calculate the Modes

Assume \(\phi_1 = \phi_2 = 1\), substitute in equations

\[(k_{11} - m_{11} \cdot \omega_1^2) \cdot \phi_1 + k_{12} \phi_1 = 0\]

\[(k_{22} - m_{22} \cdot \omega_2^2) \cdot \phi_2 + k_{21} \phi_2 = 0\]

Then we have,

\[\phi_1 = \{3.908\} \quad \phi_2 = \{-0.036\}\]

Calculate the Equivalent Mass and Stiffness for the two Extracted Single Degree Systems

\[M_1 = \det(\phi_1^T \cdot M \cdot \phi_1) = \det\left(\begin{bmatrix} 3.908 \end{bmatrix} \cdot \begin{bmatrix} 15980 & 0 \\ 0 & 112300 \end{bmatrix} \cdot \begin{bmatrix} 3.908 \end{bmatrix}\right) = 173200 \text{kg}\]

\[M_2 = \det(\phi_2^T \cdot M \cdot \phi_2) = \det\left(\begin{bmatrix} -0.036 \end{bmatrix} \cdot \begin{bmatrix} 15980 & 0 \\ 0 & 112300 \end{bmatrix} \cdot \begin{bmatrix} -0.036 \end{bmatrix}\right) = 16130 \text{kg}\]

\[K_1 = \det(\phi_1^T \cdot K \cdot \phi_1) = \det\left(\begin{bmatrix} 1 \end{bmatrix} \cdot \begin{bmatrix} 22530 & -5614 \\ -5614 & 5614 \end{bmatrix} \cdot \begin{bmatrix} 1 \end{bmatrix}\right) = 64400 \frac{kN}{m}\]

\[K_2 = \det(\phi_2^T \cdot K \cdot \phi_2) = \det\left(\begin{bmatrix} -0.036 \end{bmatrix} \cdot \begin{bmatrix} 22530 & -5614 \\ -5614 & 5614 \end{bmatrix} \cdot \begin{bmatrix} -0.036 \end{bmatrix}\right) = 22950 \frac{kN}{m}\]
Appendix C-c

Notation:

$m_w$: Half mass of single tilt-up panel involved in the seismic motion.
$M_w$: Mass of wall panel per panel width in the tilt-up buildings.
$k_w$: In-plane stiffness of single tilt-up panel
$K_w$: In-plane stiffness of tilt-up wall panel per panel width in the tilt-up buildings.
$T_w$: Fundamental (in-plane) period of tilt-up wall panel.
$m_r$: Total mass of roof including added mass from transverse wall panels
$M_r$: Mass of metal roof per panel width
$k_r$: Equivalent lateral stiffness of metal roof
$K_r$: Equivalent lateral stiffness of metal roof per panel width
$L_r$: Span length of roof (perpendicular to the earthquake direction)
$W_r$: Width of roof (parallel to the earthquake direction)
$I_r$: Moment of inertia of metal roof
$I_{ch}$: Moment of Inertia of edge chord used in metal roof
$A_{ch}$: Area of cross section of edge chord used in metal roof
$\Delta_m$: Flexural deflection in the middle of metal roof
$\Delta_s$: Shear deflection in the middle of metal roof
$F_{nor}$: Flexibility factor of metal roof for the normal shear area
$F_{cri}$: Flexibility factor of metal roof for the high shear or critical area
$V_{nor}$: Average shear in metal roof for the normal shear area
$V_{cri}$: Average shear in metal roof for the high shear of critical area
$(EI)_{r-equ}$: Equivalent moment of inertia of metal roof accounting for the influence of shear strain
$\omega_r$: Natural circular frequency of metal roof
$T_r$: Natural period of metal roof
$\omega_1$: Natural circular frequency of first mode
$\omega_2$: Natural circular frequency of second mode
$T_1$: Natural period of first mode
$T_2$: Natural period of second mode
$\phi_1$: Mode shape of first mode
$\phi_2$: Mode shape of second mode
$M_1$: Equivalent mass relevant to first mode
$M_2$: Equivalent mass relevant to second mode
\[ K_1 \] : Equivalent stiffness relevant to first mode
\[ K_2 \] : Equivalent stiffness relevant to second mode
\[ D_1 \] : Displacement of the equivalent single degree of freedom system corresponding to first mode
\[ D_2 \] : Displacement of the equivalent single degree of freedom system corresponding to second mode
\[ F_{sw} \] : Seismic wall force per panel width
\[ F_w \] : Seismic force of wall panel
\[ F_{yw} \] : Yield strength of wall panel
\[ \gamma_w \] : Wall over-strength factor
\[ \Delta_{pw} \] : Plastic wall displacement demand
\[ D_{wer} \] : Total roof displacement demand
\[ D_r \] : Roof drift relative to wall
\[ R \] : Force reduction factor for tilt-up wall panel, equal to 2
Appendix C-d

Figure C-1 Plot of Wall Displacement versus In-plane Force

Push-over Analysis of a Tilt-up Wall with Openings (By SAP2000)
Appendix D

Confirmation of Example Calculation by SAP2000

S A P 2 0 0 0 (R)

Structural Analysis Programs

Student Version 7.40

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THE LICENSEE

Unauthorized use is in violation of Federal copyright laws

It is the responsibility of the user to verify all results produced by this program

26 Oct 2003  10:13:18
### DISPLACEMENT DEGREES OF FREEDOM

- **(A)** = Active DOF, equilibrium equation
- **(-)** = Restrained DOF, reaction computed
- **(+)** = Constrained DOF
- **(>)** = External substructure DOF
- **( )** = Null DOF

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### TOTAL ACCELERATED MASS AND LOCATION

**TOTAL MASS ACTIVATED BY ACCELERATION LOADS, IN GLOBAL COORDINATES**

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(*) NOTE: Dynamic load participation ratio excludes load applied to non-mass degrees of freedom
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**RESPONSE SPECTRUM MODAL AMPLITUDES**

IN RESPONSE-SPECTRUM LOCAL COORDINATES

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GLOBAL FORCE BALANCE

TOTAL FORCE AND MOMENT AT THE ORIGIN, IN GLOBAL COORDINATES

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<th>MX</th>
<th>MY</th>
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### Joint Displacements

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<th>U3</th>
<th>R1</th>
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### Joint Reactions

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MathCAD Spread Sheet for Calculating Roof Stiffness

Instructions: This spread sheet to evaluate the lateral stiffness of metal roof is only for the non-linear dynamic analysis of concrete tilt-up wall accounting for the influence of flexible roof. The roof is assumed to be simply supported by tilt-up wall at both ends. The deflection in the middle of roof is evaluated using the methods specified in CSSBI B13-91, DESIGN OF STEEL DECK DIAPHRAGM.

kN := 1000·N

Definition:

- \( A_{ch} \) ----- Sectional Area of Edge Chord of Steel Roof (mm\(^2\))
- \( I_{ch} \) ----- Sectional Moment of Inertia of Edge Chord of Steel Roof (mm\(^4\))
- \( \text{Len} \) ----- Length of Roof (m) (\(\text{Len}=\text{Len}_{\text{nor}}+\text{Len}_{\text{cri}}\))
- \( \text{Len}_{\text{nor}} \) ----- Length of Normal Shear Area in Roof (m)
- \( \text{Len}_{\text{cri}} \) ----- Total Length of Critical Shear Area in Roof (m) (Assume size and position be symmetrical to the normal shear area of roof)
- Width ----- Width of Roof (m)
- \( E \) ----- Young's Modules of Steel Roof (N/mm\(^2\))
- Dead ----- Dead Load (kg/m\(^2\))
- Live ----- Live Load (kg/m\(^2\))
- \( F_{\text{nor}} \) ----- Flexibility Factor for Normal Area of Roof (see CSSBI B13-91)(mm/N)
- \( F_{\text{cri}} \) ----- Flexibility Factor for Critical Area of Roof (see CSSBI B13-91)(mm/N)
- \( k_{\text{equ}} \) ----- Equivalent Lateral Stiffness of Roof
- \( \text{Wid}_w \) ----- Width of Wall per Piece
- \( m_w \) ----- the mass of a wall panel

Inputs:

- \( A_{ch} := 927\text{mm}^2 \)
- \( I_{ch} := 518000\text{mm}^4 \)
- \( \text{Len} := 200\text{ft} \)
- \( \text{Len}_{\text{nor}} := 160\text{ft} \)
- \( \text{Len}_{\text{cri}} := 40\text{ft} \)
- \( \text{Wid} := 100\text{ft} \)
- \( m_w := 15980\text{kg} \)
- \( \text{Dead} := 20 \text{lb/ft}^2 \)
- \( \text{Live} := 10 \text{lb/ft}^2 \)
- \( F_{\text{nor}} := 151\text{mm}^4/N \)
- \( F_{\text{cri}} := 92\text{mm}^4/N \)
- \( E := 200000\text{N/mm}^2 \)
- \( \text{Wid}_w := 9\text{m} \)
\[ l_{\text{roof}} := 2 \left[ \frac{1}{l_{\text{ch}}} + A_{\text{ch}} \left( \frac{\text{Wid}}{2} \right)^2 \right] \]

\[ l_{\text{roof}} = 4.306 \times 10^{11} \text{ mm}^4 \quad \text{M}_{\text{total}} := \text{Len} \cdot \text{Wid} \cdot (\text{Dead} + \text{Live}) + m_w \cdot 0.5 \cdot 2 \cdot \frac{\text{Len}}{\text{Wid}_w} \]

**Calculation of Equivalent Stiffness and mass per wall panel width**

\[ W := 1 \text{ N/m} \quad w_{\text{nor}} := \frac{W \cdot \text{Len}_{\text{nor}}}{4 \cdot \text{Wid}} \quad w_{\text{cri}} := \frac{1}{2} \cdot W \left( \frac{\text{Len}_{\text{nor}}}{2 \cdot \text{Wid}} + \frac{\text{Len}}{2 \cdot \text{Wid}} \right) \]

\[ \Delta_s := \frac{w_{\text{nor}} \cdot \text{Len}_{\text{nor}} \cdot F_{\text{nor}}}{2 \cdot 10^6} + \frac{w_{\text{cri}} \cdot \text{Len}_{\text{cri}} \cdot F_{\text{cri}}}{2 \cdot 10^6} \]

\[ \Delta_m = 2.088 \times 10^{-6} \text{ m} \quad \Delta_s = 1.978 \times 10^{-6} \text{ m} \]

\[ \text{EI} := E \cdot l_{\text{roof}} \quad \text{EI}_{\text{equ}} := \frac{\text{EI}}{1 + \frac{\Delta_s}{\Delta_m}} \quad \text{m}_r := \frac{\text{M}_{\text{total}}}{\text{Len}} \quad \omega := \frac{\pi^2}{\text{Len}^2} \sqrt{\frac{\text{EI}_{\text{equ}}}{\text{m}_r}} \quad \omega = 7.071 \text{ Hz} \]

\[ k_{\text{equ}} := \omega^2 \cdot \text{M}_{\text{total}} \frac{\text{Wid}_w}{\text{Wid}} \quad k_{\text{equ}} = 5.616 \times 10^3 \text{ kN/m} \]

\[ M_{\text{equ}} := \text{M}_{\text{total}} \frac{\text{Wid}_w}{\text{Wid}} \quad M_{\text{equ}} = 1.123 \times 10^5 \text{ kg} \]
Mathcad Spread Sheet for Calculating Roof Displacement Demand

\( M_{\text{wall}} \): Mass of Wall, account for half of total mass of wall only
\( M_{\text{roof}} \): Mass of Roof per panel width, account for total mass on roof
\( k_{\text{wall}} \): Lateral Stiffness of Wall, the applied force to form unit lateral displacement on the top of wall
\( k_{\text{roof}} \): Lateral Stiffness of Roof, see sample spread sheet

**Inputs:**

\[
M_{\text{wall}} := 15980 \text{kg} \quad M_{\text{roof}} := 112300 \text{kg} \quad k_{\text{wall}} := 16920 \frac{\text{kN}}{\text{m}} \quad k_{\text{roof}} := 5616 \frac{\text{kN}}{\text{m}} \quad \omega_1 := 1 \quad \omega_2 := 30
\]

Calculations of Natural Periods of Wall and Roof

\[
T_{\text{wall}} := 2\pi \sqrt{\frac{M_{\text{wall}}}{k_{\text{wall}}}} \quad T_{\text{wall}} = 0.19309 \text{ s}
\]
\[
T_{\text{roof}} := 2\pi \sqrt{\frac{M_{\text{roof}}}{k_{\text{roof}}}} \quad T_{\text{roof}} = 0.8885 \text{ s}
\]

Calculations of Modes of 2 Degree Dynamic system:

\[
k_{w1} := k_{\text{roof}} + k_{\text{wall}} \quad k_{w2} := -k_{\text{roof}} \quad k_{r1} := -k_{\text{roof}} \quad k_{r2} := k_{\text{roof}}
\]
\[
\omega_1 := \sqrt{\left(k_{w1} - M_{\text{wall}} \omega_1^2\right) - k_{w2} k_{r1} \omega_1^2} \text{ Hz} \quad \omega_1 = 6.09991 \text{ Hz}
\]
\[
\omega_2 := \sqrt{\left(k_{w1} - M_{\text{wall}} \omega_2^2\right) - k_{w2} k_{r1} \omega_2^2} \text{ Hz} \quad \omega_2 = 37.72351 \text{ Hz}
\]
\[
T_1 := \frac{2\pi}{\omega_1} \quad T_1 = 1.03005 \text{ s} \quad T_2 := \frac{2\pi}{\omega_2} \quad T_2 = 0.1666 \text{ s}
\]
\[
\phi_{w1} := 1 \quad \phi_{r1} := 1 \quad \phi_{r2} := 1
\]
\[
\phi_{w1} := \frac{\left(k_{w1} - M_{\text{wall}} \omega_1^2\right) \phi_{w1}}{-k_{w2}} \quad \phi_{r1} = 3.90694
\]
\[
\phi_{w2} := \frac{\left(k_{w1} - M_{\text{wall}} \omega_2^2\right) \phi_{w2}}{-k_{w2}} \quad \phi_{r2} = -0.03642
\]
Calculations of Equivalent Mass and Stiffness

\[
\phi_1 = \begin{pmatrix} \phi_{w,1} \\ \phi_{r,1} \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} \phi_{w,2} \\ \phi_{r,2} \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} 1 \\ -0.03642 \end{pmatrix}, \quad M := \begin{pmatrix} M_{\text{wall}} & 0 \\ 0 & M_{\text{roof}} \end{pmatrix}, \quad K := \begin{pmatrix} k_{w,1} & k_{w,2} \\ k_{r,1} & k_{r,2} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}
\]

\[
M_1 := \phi_1^T \cdot M \cdot \phi_1 = 1.73015 \times 10^6 \text{ kg} \quad M_2 := \phi_2^T \cdot M \cdot \phi_2 = 1.6129 \times 10^4 \text{ kg}
\]

\[
L_1 := \phi_1^T \cdot M \cdot \phi_1 = 4.5473 \times 10^4 \text{ kg} \quad L_2 := \phi_2^T \cdot M \cdot \phi_2 = 1.18898 \times 10^4 \text{ kg}
\]

\[
\Gamma_1 := \frac{L_1}{M_1} = 0.26283 \quad \Gamma_2 := \frac{L_2}{M_2} = 0.73717
\]

Calculations of Elastic displacement of Roof

Definition

\[
D_1 := \text{ Fully elastic displacement derived from first mode} \quad D_w := \text{ Elastic displacement of wall}
\]

\[
D_2 := \text{ Fully elastic displacement derived from second mode} \quad D_{tr} := \text{ Total Elastic displacement of Roof}
\]

\[
D_r := \text{ Elastic Roof drift relative to wall} \quad \Delta_w := \text{ Total elastoplastic displacement demand of wall}
\]

\[
\Delta_{iw} := \text{ Inelastic displacement demand of wall} \quad v_w := \text{ Wall over-strength factor}
\]

Assume F1 and F2 represent the seismic forces relevant to first mode and second mode

Inputs: Spectral Acceleration Factor and Yield Strength of Wall Panel:

\[
S_1 := 0.618 \quad S_2 := 0.846 \quad R := 2 \quad Y_w := 271 \text{ kN} \quad D_{yw} := 0.032 \text{ m}
\]

\[
D_1 := \frac{S_1 \cdot g}{\omega_1^2} = 0.16288 \text{ m} \quad D_2 := \frac{S_2 \cdot g}{\omega_2^2} = 0.04302 \text{ m}
\]

\[
D_w := \sqrt{\left(\Gamma_1 \cdot D_1 \cdot \phi_{w,1}\right)^2 + \left(\Gamma_2 \cdot D_2 \cdot \phi_{w,2}\right)^2} = 0.04302 \text{ m}
\]

\[
D_{tr} := \sqrt{\left(\Gamma_1 \cdot D_1 \cdot \phi_{r,1}\right)^2 + \left(\Gamma_2 \cdot D_2 \cdot \phi_{r,2}\right)^2} = 0.16725 \text{ m}
\]

\[
D_r := \sqrt{\left(\Gamma_1 \cdot D_1 \cdot \phi_{r,1} - \Gamma_1 \cdot D_1 \cdot \phi_{w,1}\right)^2 + \left(\Gamma_2 \cdot D_2 \cdot \phi_{r,2} - \Gamma_2 \cdot D_2 \cdot \phi_{w,2}\right)^2} = 0.12452 \text{ m}
\]
Estimation of inelastic displacement of tilt-up wall without over-strength

\[ \Delta_w := D_{tr} - \frac{D_r}{R} \]
\[ \Delta_{iw} := D_{tr} - \frac{D_r}{R} - D_{yw} \]
\[ \Delta_w = 0.10499 \text{ m} \]
\[ \Delta_{iw} = 0.07299 \text{ m} \]

Calculation of base shear (wall force)

\[ t := \begin{pmatrix} 1 \\ 1 \end{pmatrix} \]
\[ L_1 := \begin{pmatrix} \phi_1 & T \cdot M : t \end{pmatrix} \]
\[ M_{1x} := \frac{L_1^2}{M_1} \]
\[ M_{1x} = 1.19515 \times 10^5 \text{ kg} \]
\[ M_{1x} S_1 \cdot g = 724.32255 \text{ kN} \]
\[ M_{2x} := \frac{L_2^2}{M_2} \]
\[ M_{2x} = 8.76488 \times 10^3 \text{ kg} \]
\[ M_{2x} S_2 \cdot g = 72.71717 \text{ kN} \]
\[ M_{1x} + M_{2x} = 1.2828 \times 10^5 \text{ kg} \]
\[ M_{\text{wall}} + M_{\text{roof}} = 1.2828 \times 10^5 \text{ kg} \]
\[ \frac{M_{1x}}{M_{\text{wall}} + M_{\text{roof}}} = 0.93167 \]

Seismic Force of Wall Panel:
\[ F_w := \sqrt{(S_1 \cdot g \cdot M_{1x})^2 + (S_2 \cdot g \cdot M_{2x})^2} \cdot 0.5 \]
\[ F_w = 363.98178 \text{ kN} \]
\[ \nu_w := \frac{271 \text{ kN}}{F_w} \]
\[ \nu_w = 1.48909 \]

Calculated Seismic Force of Wall Panel by First Mode only:
\[ F_{1w} := S_1 \cdot g \cdot M_{1x} \cdot 0.5 \]
\[ F_{1w} = 362.16127 \text{ kN} \]
\[ \nu_{1w} := \frac{271 \text{ kN}}{F_{1w}} \]
\[ \nu_{1w} = 1.49657 \]

Calculation of Total Plastic Displacement Demand of Wall Including the Influence of Over-strength

\[ \Delta_{iw} := D_{tr} - \frac{D_r}{R} - D_{yw} \]
\[ \Delta_{iw} = 0.04254 \text{ m} \]