APPLICATION OF FINITE ELEMENT RELIABILITY ANALYSIS AND DECISION TREE ANALYSIS IN SEISMIC RETROFIT STRATEGY SELECTION

by

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B.Sc. in Civil Engineering, Bangladesh University of Engineering & Technology, 2003

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF APPLIED SCIENCE

In

THE FACULTY OF GRADUATE STUDIES

(Civil Engineering)

THE UNIVERSITY OF BRITISH COLUMBIA

July 2006

Abstract

Earthquakes were taking place from the very beginning of the earth's creation. However, with the expansion of the human population and the civilization, the destructive nature of earthquakes is felt more frequently and vibrantly at the present time than before. With time, building codes for the earthquake resistant design are changing. Building owners are facing the pressure from themselves (morally, financially) and from the government, municipalities etc. to upgrade their structures to meet the present code standard. But to upgrade a structure, the owner is often required to make a considerable investment. Before making a decision to invest, an owner looks at how much the loss can be reduced due to the upgrade of his/her structure. Hence, it is the responsibility of the engineer to provide necessary information to the owners regarding different retrofit options and what benefit these retrofit options will yield, so that owners can make a comparison between different options prior to making a decision.

There are uncertainties associated with the occurrence of future earthquakes and with the degree of damage of the structure given an earthquake. Therefore, with each outcome, there is probability associated.

In NBCC 2005 (National Building Code of Canada 2005), earthquake hazard data is provided for the earthquakes that have 2% chance of occurrence in 50 years for major cities of Canada. Utilizing this information with finite element reliability analysis, it is then possible to determine the probability of failure of the structure for a defined damage state.

In this thesis, finite element reliability analysis is used in conjunction with decision tree analysis, in order to provide a framework for presenting information to the owners. The

damage state is defined by the cost of damage. From finite element reliability analysis, the failure probabilities of the structure with and without different retrofit options are determined. The probabilities of occurrence of different events considered in the decision tree are thereby determined. All information (probability and cost) are presented in a decision tree, so that owners can make their decision by comparing of the expected cost of different retrofit options as well as the "do nothing" option. A comprehensive numerical example is provided in this thesis to explain the proposed methodology.

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Acknowledgement

I offer my enduring gratitude to my supervisor, Dr. Terje Haukaas for the technical support and advice that he extended to me during the course of this project. I thank Dr. Haukaas for enlarging my vision of engineering and providing coherent answers to my endless problems.

I would like to acknowledge my fellow college, Smitha D Koduru for sharing her knowledge of finite element reliability analysis and OpenSees.

Finally, I would like to thank my wife and my parents for the unconditional love, support and encouragement they have provided me in all my endeavors.

Chapter 1.0 Introduction

1.1 Overview of the Problem

Decision makers (building owners, government officials, law makers and politicians) are forced everyday to make decisions regarding retrofitting (re-strengthening) of existing structures that were not built according to the present code standard or that have deteriorated. A problem that engineers have to face when providing advice for retrofitting structures for earthquake loading is "uncertainty." There are uncertainties associated with the earthquake (in magnitude, occurrence time and location) and the behaviour of the structure. Another reality that engineers have to face is the need to convert engineering terms into an understable format for the decision makers, as most of them might not have an engineering background. An understable format might be in terms of damage costs or damage states. From the owners' point of view, the following questions are of importance when selecting retrofit strategy:

- How much should they spend on re-strengthening of the structure?
- How much benefit should they expect in return of their decision?
- Will there be any damage and if damaged, the type and extent?
- How much will be the damage cost?
- Who will finance the retrofit project?

Examples of such dilemma faced by the decision makers are in abundance in history. For example, The Olive View Hospital in Sylmar [15], California, was built according to the earthquake provisions of the building code in 1964. In 1971, magnitude 6.7 San Fernando earthquake, the structure partially collapsed. After that, the hospital was retrofitted and considerable amount of money was spent to make the hospital safer. The hospital was again subjected to the same level shaking in the 1994 Northridge earthquake. But this time the performance of the structure was excellent and no serious damage to the hospital was reported. Hence it is apparent that the performance of the structure under earthquake loading depends greatly on retrofit actions taken.

In order to guide the decision makers, engineers have to provide information about the performance of the structure with and without retrofit. While performing analysis of the structure, uncertainties have to be taken into account. This is achieved in this thesis by means of finite element reliability analysis.

Examples of retrofit strategy selection procedures proposed by different researchers are available in [5], [7], [21], [23] and [24]. In most cases, only the uncertainty in the occurrence of the earthquake event is considered, while uncertainties in modelling and behaviour of the structure are not.

It is evident that there is a need for developing methodologies to overcome the above shortcomings and to provide decision makers with data in an appealing format so that they can make retrofit decisions without extensive engineering knowledge.

1.2 Objective

The main objective of this thesis is to mix finite element reliability analysis with decision tree analysis to provide a framework for retrofit strategy selection.

1.3 Layout of the Thesis

In Chapter 2, fundamental rules associated with probability theory are discussed. An introduction to the first order reliability method (FORM) and finite element reliability analysis are also made in Chapter 2. Back ground for decision tree analysis, formulation of decision trees and decision making with decision trees under different criteria are discussed with examples in Chapter 3. A detailed discussion of the methodology developed in this thesis for the selection of the best retrofit strategy for a structure by the decision tree analysis in conjunction with finite element reliability analysis is provided in Chapter 4. In Chapter 5, a numerical example of the proposed methodology is provided. In the last chapter (Chapter 6) the findings of this thesis are presented and topics for future studies are suggested.

Chapter 2.0 Probability and Reliability Theories

2.1 Introduction

Uncertainties associated with an event can be expressed in terms of probabilities. A probability provides a numerical value that expresses the degree of likelihood associated with the occurrence of an event. Probability generates an analytical framework that helps to quantify the expectation regarding an event. The foundation for modern decision analysis, which guides the selection of the "best" possible alternative, is also laid from the same concept. In this chapter some basic rules of set theory and probability are discussed for the benefit of subsequent development. Also, the concept of reliability and finite element reliability analysis are introduced in the subsequent sections.

2.2 Set Theory

The elementary notions of the set and mathematical theory of probability are employed to define and model the different characteristics of a probabilistic problem. In this section and in the following sections the basic elements of the set and the probability theories are briefly discussed. The following terminologies are commonly employed in the set theory:

Sample Space- The set of all possibilities in a probabilistic problem is collectively called a sample space.

Sample Point- Each of the individual possibilities in a probabilistic problem is called a sample point.

Event- A subset of the sample space is defined as an event.

Special Events- The following events and notations are commonly employed in the set theory:

- An event with no sample point in a sample space is defined as an impossible event. This type of event is denoted by ϕ.
- An event containing all the sample points in a sample space is defined as a certain event. This type of event is denoted by S.
- The complementary of an event E is an event $\overline{E}$ that contains all the sample points of the sample space S except that are not in the event E.

2.2.1 The Venn Diagram

A "Venn diagram" provides a pictorial representation of a sample space and the events that the sample space contains. In a venn diagram, a sample space is represented by a rectangle, an event E is represented symbolically by a closed region within this rectangle and the part of the rectangle outside this closed region is the corresponding complementary event $\overline{E}$.

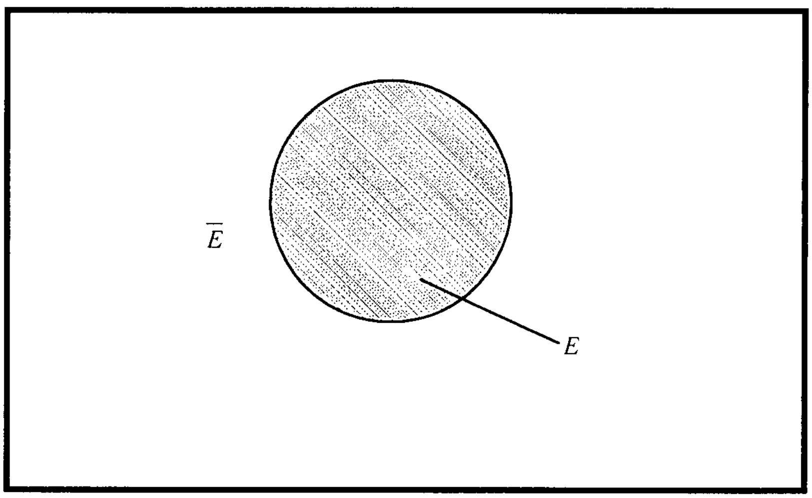

Figure 2. 1 A typical venn diagram.

2.2.2 Combinations of Events

There are basically two ways to combine or derive events from other events. These two ways are by union or intersection, which is described as follows:

Union- Lets consider two events E_1 and E_2. The union of these two events E_1 and E_2 is another event that represents the occurrence of the events E_1 or E_2, or both. The union operation is denoted by the symbol $\cup$ and the event of union of two events E_1 and E_2 is denoted by $E_1 \cup E_2$.

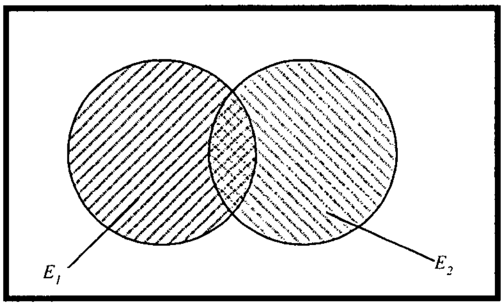

Figure 2. 2 The union of the events E_1 and E_2.

Intersection- Lets consider two events E_1 and E_2. The intersection of these two events E_1 and E_2 is another event that represents the joint occurrence of the events E_1 and E_2. The intersection operation is denoted by the symbol $\cap$ and the event of intersection of two events E_1 and E_2 is denoted by $E_1 \cap E_2$ or E_1E_2.

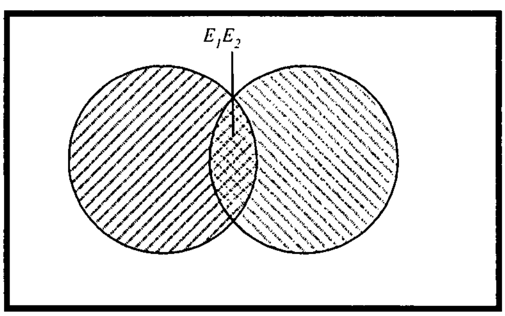

Figure 2. 3 The intersection of the events E_1 and E_2.

2.2.3 Mutually Exclusive Events

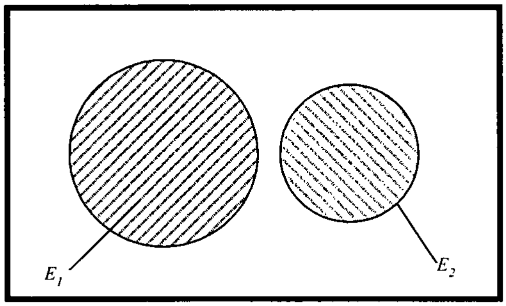

Figure 2. 4 The mutually exclusive events E_1 and E_2.

If the occurrence of an event is independent of the occurrence of another event then these two events are said to be mutually exclusive.

2.2.4 Collectively Exhaustive Events

Two or more events are said to be collectively exhaustive if the union of all these events constitute the underlying sample space.

2.3 Probability

The classical notion of probability expresses the long run frequency of an event occurring in many repeated random experiments.

$$P[Event] = \frac{Number\ of\ times\ event\ occurs}{Number\ of\ repeated\ random\ experiment} \qquad \text{.........................2.1}$$

Probabilities that conform to this notion are termed as "objective probabilities." Objective probabilities are determined from a series of similar events such as tossing a coin again and again or conducting series of experiments. These probabilities are basically the long run frequencies and more the data gathered the more precise the probability will be. In most practical situations, it is not possible to repeatedly generate the same event. This is common in structural engineering. In these cases probability is quantified differently – by "subjective probabilities," also known as "personal probabilities." This is in fact interpreted as the degree of belief of an individual.

2.3.1 Axioms of Probability

Like any other branches of mathematics, the theory of probability is based on axioms. These fundamental assumptions are:

1. For every event E in a sample space S, there is a probability associated with the occurrence of the event E and it is a non negative number.

$P(E) \geq 0$ 2.2

2. The probability of occurrence of a certain event S is

$P(S) = 1.0$ 2.3

3. The probability of occurrence of two mutually exclusive events E_1 and E_2 is

$P(E_1 \cup E_2) = P(E_1) + P(E_2)$ 2.4

2.3.2 Elementary Rules of Probability

There are some elementary rules that are derived from the axioms of the probability theory. These are required when determining the probability of occurrence of the events defined in terms of other events. These rules are briefly described in the following paragraphs.

Probability of Complement- The probability of the union of an event E and its complimentary event $\overline{E}$ is

$P(E \cup \overline{E}) = P(E) + P(\overline{E})$ 2.5

Since $E \cup \overline{E} = S$ (as the sample space S comprises of two mutually exclusive and collectively exhaustive events E and $\overline{E}$), according to Eq. (2.3)

$$P(E \cup \overline{E}) = P(S) = 1.0 \quad \text{.........} 2.6$$

Hence from Eq. (2.5)

$$P(\overline{E}) = 1 - P(E) \quad \text{.........} 2.7$$

Union Rule and Inclusion-Exclusion Rule- For two events E_1 and E_2 in Figure 2.2, the probability of occurrence of both the event E_1 and E_2 is

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 E_2) \quad \text{.........} 2.8$$

As shown in Figure 2.2, two events E_1 and E_2 have an overlapping area of E_1E_2. If the Eq. (2.4) is applied then the common area is counted twice. Hence the common area is deducted in Eq. (2.8).

Similarly for the E_1, $E_2 \ldots\ldots E_n$ events

$$P(E_1 \cup E_2 \cup \ldots\ldots E_n) = \sum_{i=1}^{n} P(E_i) - \sum_{i=1}^{n-i} \sum_{i<j}^{n} P(A_i \cap A_j) + \sum_{i=1}^{n-2} \sum_{i<j}^{n-2} \sum_{j<k}^{n} P(A_i \cap A_j \cap A_k) +$$

$$\ldots\ldots + (-1)^n P(A_1 \cap A_2 \cap \ldots\ldots \cap A_n) \quad \text{.........} 2.9$$

Conditional Probability Rule- If the probability of occurrence of an event depends on the occurrence (or non occurrence) of another event, then this type of associated probability is called the conditional probability.

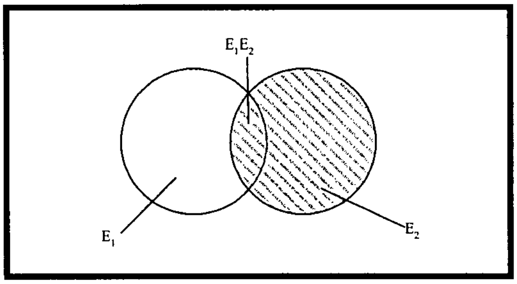

Figure 2. 5 The area the venn diagram showing the conditional probability of the event $E_1|E_2$.

The conditional probability of the event E_1 has occurred provided that the event E_2 has also occurred is given by

$$P(E_1 \mid E_2) = \frac{P(E_1E_2)}{P(E_2)} \qquad 2.10$$

(for all values of $P(E_2) \neq 0$)

Multiplication Rule- For any two events E_1 and E_2, the probability of joint occurrence of these two events is given by

$$P(E_1E_2) = P(E_1 \mid E_2)P(E_2) \qquad 2.11$$

If the events in consideration are statistically independent i.e. $P(E_1 \mid E_2) = P(E_1)$ or $P(E_2 \mid E_1) = P(E_2)$ then the Eq. (2.11) becomes

$$P(E_1E_2) = P(E_1)P(E_2) \qquad 2.12$$

Rule of Total Probability- If the occurrence of an event A depends on the occurrence of several other events E_i (where i =1… n) then the probability of occurrence of the event A is the expected probability i.e. the average probability weighted by those events E_i (provided that events are mutually exclusive and collectively exhaustive).

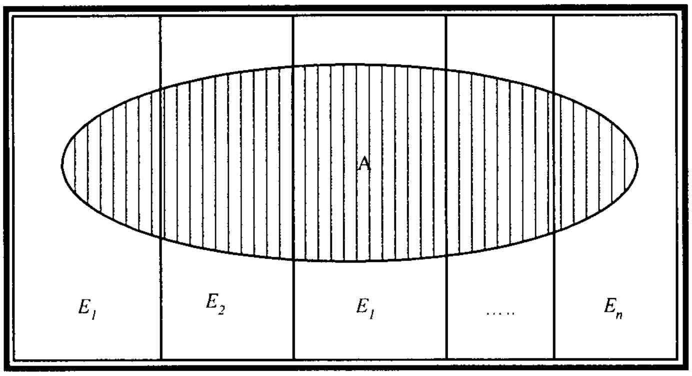

Figure 2. 6 A venn diagram with the events A and $E_1 ... E_n$.

The probability of occurrence of an event A is given by

$$P(A) = \sum_{i=1}^{n} P(A \mid E_i) P(E_i) \quad \text{.....2.13}$$

Baye's Theorem- This is the reverse sense of the rule of total probability. In this case probability of E_i is determined provided that the event A has occurred.

$$P(E_i \mid A) = \frac{P(A \mid E_i) P(E_i)}{P(A)} \quad \text{.....2.14}$$

2.3.3 Random Variables

The natural phenomenons are highly uncertain both in magnitude and time of occurrence. In the field of engineering, the possible outcomes of a natural phenomenon are quantified numerically, either naturally or artificially. In any case, an outcome or event may be identified through the value(s) of a function; such functions are called random variables. A random variable might be discrete or continuous in nature. The rule of describing the probability measures associated with all the values of a random variable is known as the probability distribution. A probability distribution is employed to express the probability values associated with a random variable. If X is a random variable, a cumulative distribution function, CDF is employed to describe its probability of occurrence. Symbolically it is expressed as follows

$F(x) \equiv P(X \leq x)$ 2.15

Discrete Random Variable- If a random variable is discrete in nature then its probability distribution is described by the probability mass function (PMF). If X is a discrete random variable then its PMF is a function expressing the probability of occurrence of X for all values of x.

$p(x_i) = P(X = x_i)$ 2.16

For a discrete random variable X with the PMF: $p_x(x_i) = P(X = x_i)$ the CDF of it is given by

$F(x) = P(X \leq x) = \sum_{all \cdot .x_i \leq x} P(X = x_i) = \sum_{all \cdot .x_i \leq x} p(x_i)$ 2.17

A PMF must have the following properties:

- $0 \leq p(x_i) \leq 1$
- $\sum_{all \cdot .x_i} p(x_i) \leq 1$

Continuous Random Variable- If X is a continuous random variable, the probability values of that random variable are associated with the intervals on the real line rather than a point. In case of a continuous random variable, the probability laws are described in terms of a probability density function (PDF). If *f(x)* is the PDF of a continuous random variable X, then the probability of occurrence of X in the interval a and b is given by

$P(a < X \leq b) = \int_a^b f(x)dx$ 2.18

Following the same process, the corresponding cumulative distribution function of that continuous random variable X is given by

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)dx \quad \text{..........} 2.19$$

The CDF of a continuous random variable X must have the following properties

- $F(-\infty) = 0$
- $F(+\infty) = 1.0$

The PDF of a continuous random variable is derived by differentiating the CDF of that random variable

$$f(x) = \frac{dFx(x)}{dx} \quad \text{..........} 2.20$$

The PDF of a continuous random variable X must have the following properties

- $0 \leq f(x)$
- $\int_{-\infty}^{\infty} f(x) = 1$

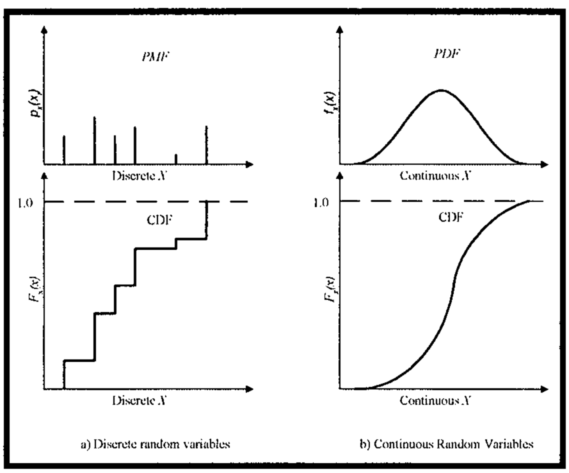

Figure 2. 7 The PMF/PDF and the CDF of a discrete and a continuous random variable.

2.3.4 Main Descriptors of a Random Variable

The random variables are frequently presented by the form of their distribution function and with some associated parameters. However in the practical situations, in most cases the form's of the distribution function is not readily available or not known accurately. Therefore in reality, the probabilistic characteristics of a random variable are described by the following descriptors or quantities.

Expected Mean- A random variable may vary within a wide range. Rather than dealing with the range, it is often necessary to know the central tendency of the random variable. Since different random variable may have different probabilities or probability densities, instead of just taking the average value of the random variable the "weighted average" of a random variable is of more importance. This is known as the "mean value" or the "expected value."

For a discrete random variable

$$\text{Mean} = \mu = E(X) = \sum_{all \cdot x_i} x_i p(x_i) \quad \text{......} 2.21$$

For s continuous random variable

$$\text{Mean} = E(X) = \int_{-\infty}^{\infty} x f(x) dx \quad \text{......} 2.22$$

Expected Variance and Standard Deviation- Besides the central value, the information of how widely or closely the random variable is spread from the central value are often necessary. Naturally, such a measure should be a function of deviation from the mean value. Only the numerical value is of more importance than whether the variables are above or below the mean. That's why the variance or square of the standard deviation is employed, which will provide an even function of the deviations.

For a discrete random variable, the variance of a random variable is computed by

Variance, *Var[X]* = (Standard Deviation)^2

$$\sigma_x^{\,2} = E[(x-\mu_x)^2] = \sum_{all\cdot.x_i}(x_i-\mu_x)^2\,p(x_i)\;\ldots\ldots 2.23$$

For a continuous random variable, the variance of a random variable is computed by

Var[X] = (Standard Deviation)^2

$$\sigma_x^{\,2} = E[(x-\mu_x)^2] = \int_{-\infty}^{\infty}(x-\mu_x)^2 f(x)dx\;\ldots\ldots 2.24$$

Coefficient of Variation- The standard deviation or variance of a random variable is often expressed with respect to the mean value of that random variable. This relative expression is called the "coefficient of variation". It is a non-dimensional measure of the dispersion or variability.

$$\text{Coefficient of Variation} = \delta_x = \frac{\sigma_x}{|\mu_x|}\;\ldots\ldots 2.25$$

Measure of Skewness- The symmetry or lack of symmetry on either side of the central value is also a descriptor of the distribution of a random variable and it is associated with the degree and the direction of asymmetry. If the distribution is symmetric than the skewness value will be zero other wise it will have a positive or negative value. If the values of a random variable are greater than the mean value then the skewness will have a positive value. In opposite case, it will have a negative value. The magnitude of skewness provides the degree of dispersion.

In case of a discrete random variable, the skewness is computed by

$$E[X-\mu_x]^3 = \sum_{all\cdot.x_i}(x_i-\mu_x)^3\,p(x_i)\;\ldots\ldots 2.26$$

In case of a continuous random variable, the skewness is computed by

$$E[X-\mu_x]^3 = \int_{-\infty}^{\infty}(x-\mu_x)^3 f(x)dx \quad \text{.......... } 2.27$$

The coefficient of skewness is a more widely applied non-dimensional measure of skewness.

$$\text{Coefficient of skewness} = \gamma_x = \frac{E[(x-\mu_x)^3]}{\sigma_x^{\ 3}} \quad \text{.......... } 2.28$$

2.3.5 Joint Probability Distribution (Bi-Variant)

The equations described in the above sections are for single random variables. However, in most of the physical phenomenon in the real world may depend on two or more random variables. In such cases the events in a sample space may be mapped into two or more dimensions of the real space. For instance, an event of interest might involve the runoff of a river $X = x$ provided that the rainfall is $Y = y$ or the runoff of a river is $X \leq x$ provided that the rainfall is $Y \leq y$. The above events are described by the following notations

$(X = x,\ Y = y) = [(X = x) \cap (Y = y)]$

$(X \leq x,\ Y \leq y) = [(X \leq x) \cap (Y \leq y)]$

There are probabilities of occurrence associated with the events represented by X and Y for any values of the pair x and y. The joint cumulative distribution function is employed to describe the probability of occurrence for all possible pairs of x and y, which is defined as

$$F(x,y) = P(X \leq x, Y \leq y) \quad \text{.......... } 2.29$$

The above CDF represents the probability of joint occurrence of an event when $X \leq x$ and $Y \leq y$. The joint probability function has to satisfy following properties

- $F(-\infty,-\infty)=0$; $F(\infty,\infty)=1.0$
- $F(-\infty,y)=0$; $F(\infty,y)=F(y)$; $F(x,-\infty)=0$; $F(x,\infty)=F(x)$
- $F(x,y)\geq 0$ and non decreasing function of *x* and *y*.

Discrete Random Variables- If *X* and *Y* are two discrete random variables then

Joint PMF: $p(x,y)=P(X=x,Y=y)$ 2.30

Joint CDF: $F(x,y)=\sum_{x_i\leq x,\,y_i\leq y} p(x_i,y_i)$ 2.31

Continuous Random Variables- If *X* and *Y* are two continuous random variables then

Joint PDF: $f(x,y)dxdy=P(x<X\leq x+dx,\,y<Y\leq y+dy)$ 2.32

Joint CDF: $F(x,y)=\int_{-\infty}^{x}\int_{-\infty}^{y} f(x,y)dxdy$ 2.33

For instance, the probability of occurrence of an event when $X\leq x$ and $Y\leq y$ is given by

$P(0<X\leq x,0<Y\leq y)=\int_{0}^{x}\int_{0}^{y} f(x,y)dxdy$, which represents the volume under the surface *f(x,y)* shown in Figure 2.8.

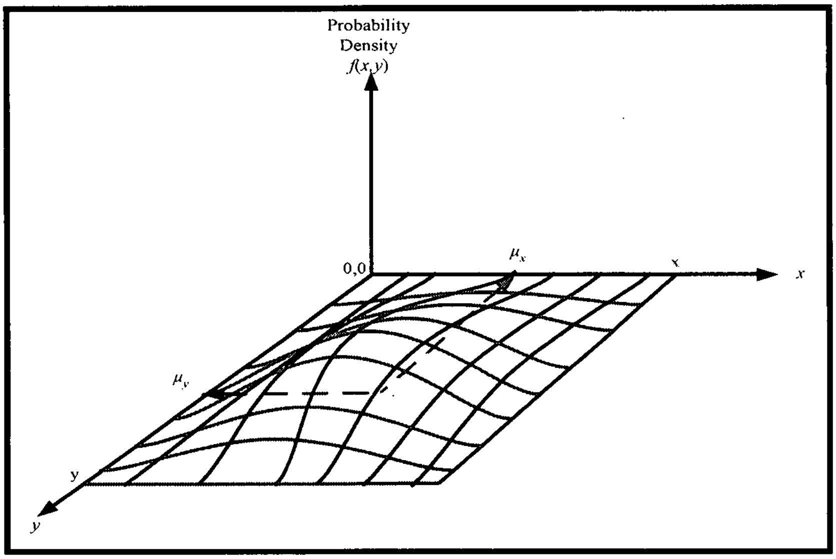

Figure 2. 8 The joint PDF of the continuous random variables *X* and *Y*.

The partial derivative of the joint cumulative distribution function provides the joint probability distribution function.

$$f(x,y)=\frac{\delta^2 F(x,y)}{\delta x\,\delta y} \quad \text{......2.34}$$

Conditional Probability Distribution Function- Similar to the case of Eq. (2.10), the probability of occurrence of $X = x$ or $X \leq x$ may depends on the occurrence of $Y = y$ or $Y \leq y$ or vice versa then the conditional probability distribution function is given by

$$f(x\mid y)=\frac{f(x,y)}{f(y)} \quad \text{......2.35}$$

(for all values of $f(y) \neq 0$)

From the multiplication rule, Eq. (2.35) is rearranged as follows

$$f(x,y) = f(x \mid y) f(y) = f(y \mid x) f(x) \qquad 2.36$$

Two continuous random variables are said to be statistically independent when their marginal PDF is equal to their conditional PDF, i.e.

$$f(x) = f(x \mid y) \text{ or } f(y) = f(y \mid x) \qquad 2.37$$

Therefore, according to the above equation for the statistical independent random variables Eq. (2.36) reduces to

$$f(x,y) = f(x) f(y) \qquad 2.38$$

Marginal Probability Distribution Function- The probability distribution function of an individual continuous random variable is obtained from its joint probability distribution function. From the rule of total probability

$$f(x) = \int_{-\infty}^{\infty} f(x \mid y) f(y) dy = \int_{-\infty}^{\infty} f(x,y) dy \qquad 2.39$$

Similarly

$$f(y) = \int_{-\infty}^{\infty} f(y \mid x) f(x) dx = \int_{-\infty}^{\infty} f(x,y) dx \qquad 2.40$$

2.3.6 Moments of Joint Probability Distribution

First Joint Moment- The first joint moment of the random variables X and Y is given by

$$E(XY) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy f(x,y) dx dy \qquad 2.41$$

If the random variable X and Y are statistically independent then Eq. (2.41) is written as

$$E(XY) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy f(x) f(y) dx dy = \int_{-\infty}^{\infty} x f(x) dx \int_{-\infty}^{\infty} y f(y) dy = E(X) E(Y) \qquad 2.42$$

Therefore the first joint moment of two random variables is the product of their mean.

First Joint Central Moment- The first central joint moment with respect to the mean μ_x and μ_y of the random variables X and Y is called the covariance of the random variables X and Y. This is expressed as follows

$$COV[XY] = E[(X-\mu_x)(Y-\mu_y)] = E(XY) - E(X)E(Y) \quad \text{..............................2.43}$$

The $COV[XY]$ provides the information regarding the linear inter-relationship of the random variables.

Correlation Coefficient- The correlation coefficient is the normalization of the $COV[XY]$ by the product of the standard deviation of the random variables. This is expressed as follows

$$\rho = \frac{COV[XY]}{\sigma_X \sigma_Y} \quad \text{..2.44}$$

The range of ρ lies in between -1 to +1 [3]. When X and Y are linearly related, a positive value of ρ means if X increases or decreases Y also increases or decreases; a negative value of ρ means if X increases or decreases Y also decreases or increase and when $\rho = 0$ there exists no correlation between the random variables. However, in case of a non-linear relationship between X and Y, the ρ might be zero even when high correlation exists between X and Y.

2.4 The Poisson's Distribution and First Occurrence of an Event

The real life events might occur at any time or space. For instance, an earthquake might occur at any time or at any space along the fault or the cracks can appear any where along the weld. The Poisson's process is employed to model the occurrences of such events. The Poisson's process is based on the following assumptions [3],

a. An event can occur randomly at any time or any point in the space.

b. The occurrence(s) of an event in a given time (or space) interval is independent of that in any other non-overlapping interval.

c. The probability of occurrence of an event in a small interval Δt is proportional to Δt, and can be given by $\lambda \Delta t$, where λ is the mean rate of occurrence of the event (assumed to be constant); and the probability of two or more occurrences of the event in Δt is negligible.

According to the above assumptions, if number of occurrence of event in time t is X then by Poisson's distribution, the probability of occurrence of that event is

$$P(X = x) = \frac{(\lambda t)^x}{x!} e^{-\lambda t} \quad \text{.........} 2.45$$

The mean occurrence rate λ is the average number of occurrence of an event per unit time. For instance, if an earthquake of certain magnitude has a return period of T then

$$\lambda = \frac{1}{T} \quad \text{.........} 2.46$$

The first occurrence time T has an exponential distribution if occurrence of an event follows the Poisson's process. $T > t$ means that no event occurs in time t and the probability of occurrence of no event is given by (Eq. (2.45))

$$P(T > t) = P(X = 0) = \frac{(\lambda t)^0}{0!} e^{-\lambda t} = e^{-\lambda t} \quad \text{.........} 2.47$$

Hence T is the first occurrence time in the Poisson's process. Since in the Poisson's process, it is assumed that the occurrence of an event in non-overlapping time interval is statistically independent, T also represents the time between two consecutive occurrences of the event. The CDF of T is given by

$$F(t) = P(T \le t) = 1 - T(T > t) = 1 - e^{-\lambda t} \quad \text{.........} 2.48$$

The PDF of T is

$$f(t) = \frac{dF}{dt} = \lambda e^{-\lambda t} \qquad \text{2.49}$$

The above concept is utilized in Chapter 4 to determine the probability of occurrence of an earthquake event that has a specified return period.

2.5 Data Analysis

From the previous sections it is evident that, if a distribution function of a random variable and its associated parameters are known then it is possible to determine the probability of occurrence of that event. Therefore, the first step is to determine the parameters (mean, standard deviation, variance etc). These parameters are determined from the observed data. In the practical world, the continuous data are collected from different stations (rain fall, flood level etc.), from material testing labs (crushing strength of concrete, yielding strength of steel, shear strength of soil etc.) and from many other sources. From these data by statistical analysis, it is possible to determine the distribution and its parameters that will best fit these data. The branch of statistics that deals with these sorts of problems is known as "Statistical Inference." It is not always possible to determine an exact distribution and its descriptors accurately due to the associated uncertainty (uncertainty due to inherent variability in nature and method of data collection). In practice, the real world is modeled as the random variables, with the probability distribution of those random variables and their associated parameter. The form of a probability distribution is determined either empirically or by physical consideration. The detailed descriptions of these methods are available in any basic statistical books. However in any case, the mean, μ and the standard deviation, σ of the observed data have to be determined. For a given set of sample data; there are different ways to compute the related parameters. In the following paragraphs, the method of moments applied to compute the mean, variance, coefficient of variation and correlation coefficient of a sample data are described. The idea of this method is that, the sample moments are employed to describe the corresponding moments of a random variable. For instance, a sample data set is shown in Table 2.1.

Table 2. 1 A sample data of the concrete's crushing strength (GPa).

Specimen Number, n	Strength (GPa), x_i
1	35.20
2	36.77
3	40.22
4	41.33
5	42.28
6	43.15
7	43.60
8	44.21
9	44.61
10	44.73
11	45.20
12	45.35
13	45.39
14	45.67
15	45.81
16	45.99
17	46.19
18	46.48
19	46.75
20	46.88

Specimen Number, n	Strength (GPa), x_i
21	49.12
22	49.36
23	49.99
24	50.23
25	50.65

Sample Mean- A sample mean is the average (weighted) of the sample size *n*.

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \text{2.50}$$

The mean value of the concrete's crushing strengths of Table 2.1 is 45.00 GPa.

Sample Variance- A sample variance is the average of dispersion of the sample data from the mean value of the sample.

$$s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 \qquad \text{2.51}$$

The above equation is a biased estimate of the variance as its expected value is not the true variance of a sample. This biasness is over come by dividing the sum of the square by *(n-1)* rather than *n*.

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \qquad \text{2.52}$$

Re-arraigning the previous equation, it is written as

$$s^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right) \qquad \text{2.53}$$

The variance of the sample data of the concrete's crushing strength of Table 2.1 is 14.44 GPa^2.

Coefficient of Variation- The ratio between the square root of the variance or the standard deviation and the mean of a sample data is called the coefficient of variation,

$$\delta = \frac{\sqrt{s^2}}{\bar{x}} \qquad 2.54$$

For the sample data of Table 2.1, the coefficient of variation is 0.084.

Sample Variance and Correlation Coefficient- In the section 2.3.6, the definitions and usefulness of the variance and the correlation coefficient are described. The sample variance is given by the following formula

$$C = \frac{1}{n-1}\left(\sum_{i=1}^{n} x_i y_i - n\overline{xy}\right) \qquad 2.55$$

Where n is number of samples, $\bar{x}$ and $\bar{y}$ are the average of the variables x_i and y_i associated with the sample.

The correlation coefficient between the two variables is given by

$$\rho = \frac{C}{S_x S_y} \qquad 2.56$$

Where C is the sample variance and the S_x and S_y are the standard deviation of the two variables (x_i and y_i) associated with the sample data in consideration.

For different specimens of the concrete; the concrete crushing strength and the young's modulus values are provided in Table 2.2.

Table 2.'2 A sample data of the concrete's crushing strength(GPa) and modulus of elasticity (GPa).

Specimen number, i	Concrete crushing strength, x_i (GPa)	Concrete young's modulus, y_i (GPa)
1	23.5	22100
2	24.8	23400
3	25.7	26030
4	26.1	24450
5	27.3	27420
6	28.6	24120
7	29.3	26900
8	30.1	27100
Average	$x_i = 26.925$	$y_i = 25190$
Standard deviation	$S_x = 2.302$	$S_y = 1952.85$

The sample variance of the concrete's crushing strength and modulus of elasticity is,

$C = 3305.43 \text{ GPa}^2$ 2.57

The correlation coefficient between the two variables is

$\rho = 0.735$ 2.58

It means a high correlation exists between the concrete's crushing strength and modulus of elasticity.

2.6 Link between the Real World Data and the Decision Making Process

The statistical inference method provides a link between the idealized probability model assumed or prescribed in a probabilistic analysis and the real world. The influence of the statistical inference in the real world decision making process is shown in Figure 2.9.

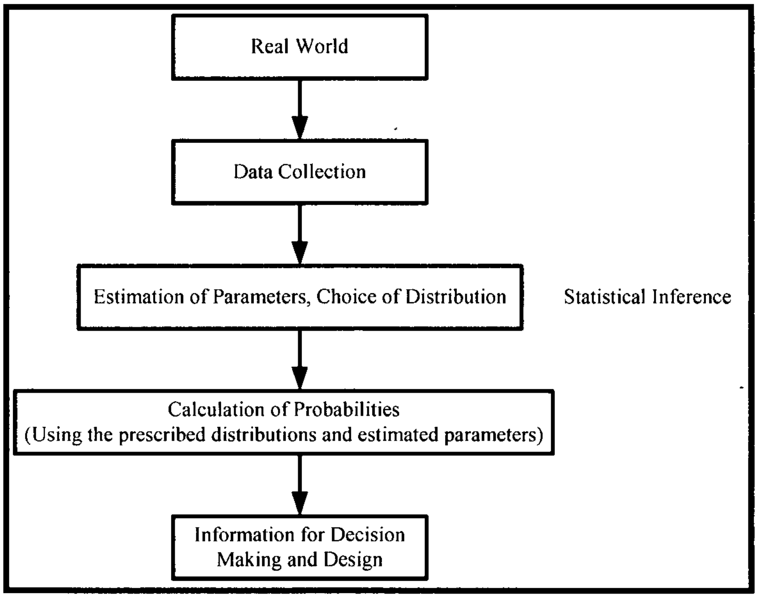

Figure 2. 9 The link between the decision making process and the real world data [3].

In general continuous data are collected from the real world. Then employing formulas previously described, the mean and the standard deviation of a sample data are calculated. The mean and the standard deviation are the main descriptors of a random variable. A number of common distributions have their parameters related to the mean and standard deviation of the sample data. For instance, in case of the normal distribution, the parameters μ and σ^2 are the mean and variance of the sample. Similarly different parameters of the different distributions are related to the mean and the variance of the sample. A list of major distributions and their parameters are listed in Appendix A. Thus by employing the statistical inference techniques, it is possible to determine the associated probability distributions of different input variables applied in finite element reliability analysis, which in turn will provide the failure probabilities of a structure. This probability value is then utilized as an input variable in the decision making process.

2.7 Structural Safety and Finite Element Reliability Analysis (FERA)

2.7.1 The Basic Reliability Problem

A frequency distribution of a structural element's strength (flexural, shear, axial, torsional, modulus of elasticity etc.) is shown in Figure 2.10. The distribution indicates that strengths around the mean occur very frequently. Conversely, very weak and very strong strengths occur rarely. Although the distribution in Figure 2.10 is taken as the normal distribution, which shows symmetry on the both sides of the mean value, in most practical cases the actual distribution might not be symmetric.

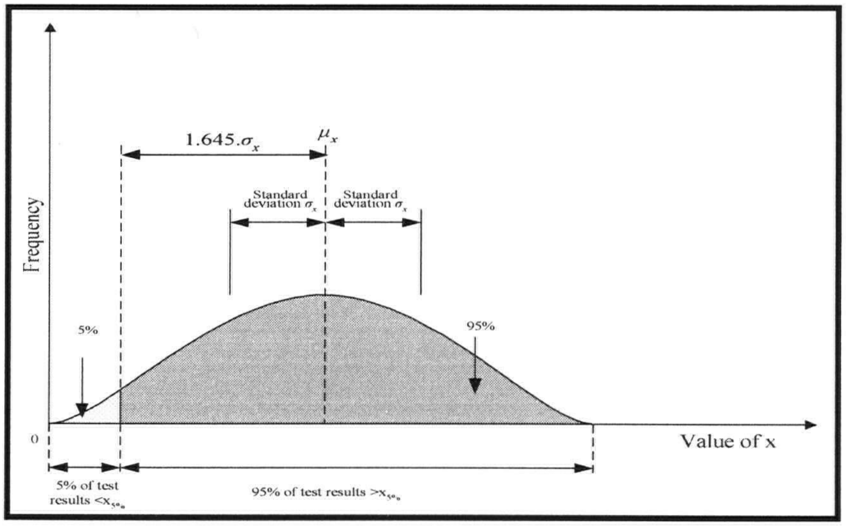

Figure 2. 10 A typical frequency distribution of the material property.

Similarly, in Figure 2.11 the probability distribution of a load variable is shown along with the distribution of the associated resistance. If the two extreme tail ends of the load and resistance curves do not intersect, or in other words if the resistance tail end values are much higher than the load extreme tail end values then the likelihood of the load exceeding the resistance is negligible. However, if the two extreme tail ends intersect, like in Figure 2.11, then that means that failure is likely. The shaded area represents the failure zone. The goal of any structural design is to reduce the shaded area as much as possible while considering economic and practical consequences.

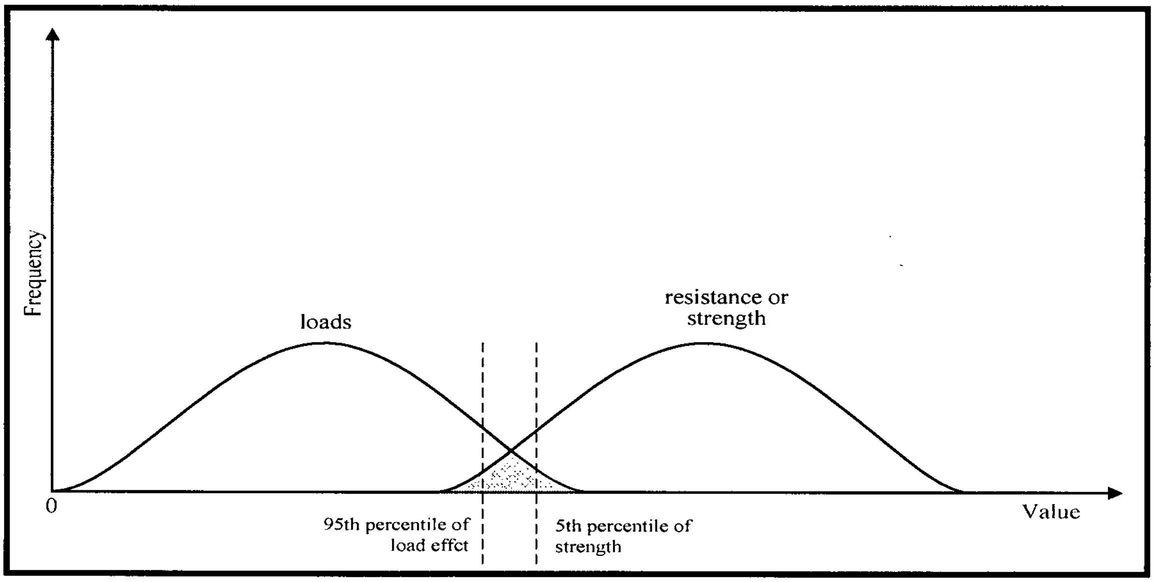

Figure 2. 11 The overlapping area of the load and resistance distribution depicting the failure zone.

The simplest safety measure is the ratio between the mean value of resistance and the mean value of load:

$$F_o = \frac{\mu_R}{\mu_S} \qquad \text{.......... 2.59}$$

However, as stated previously, a mean value does not provide information about the range or variability of the distribution. This problem is depicted in Figure 2.12.

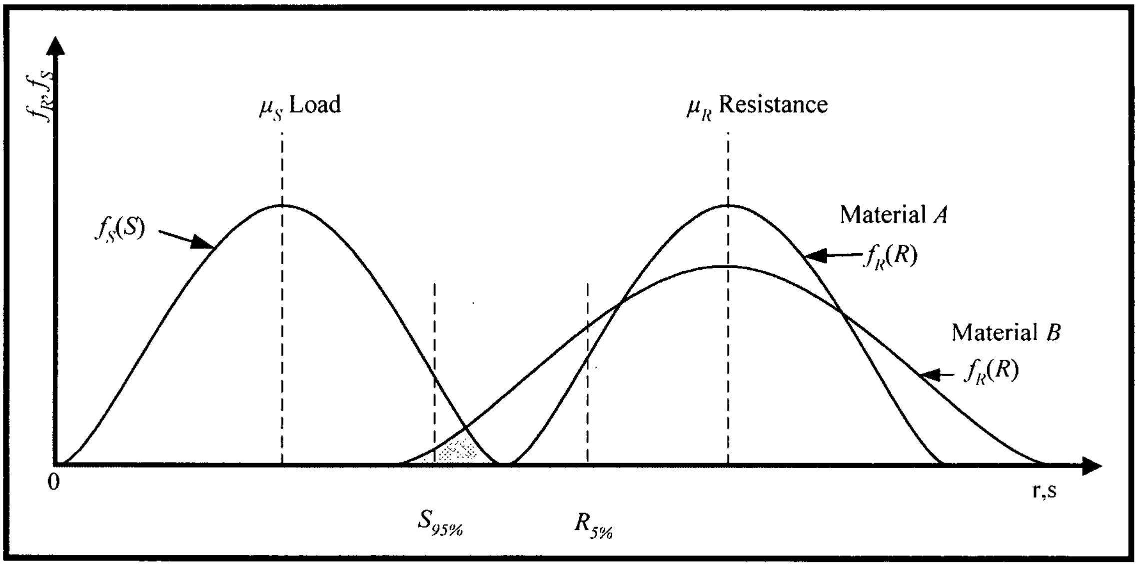

Figure 2. 12 The problem using the mean value as tail end value of the load and resistance distributions may or may not coincide.

The ratio of a certain percentile of resistance (S_P) and a certain percentile of load (R_P) is called the nominal safety factor, F_P:

$$F_p = \frac{R_{5\%}}{S_{95\%}} \quad \text{......} 2.60$$

This measure is a more accurate way of representing the overlapping area of the load and the resistance. However, the lack of information in the tail ends of the distributions makes the accuracy of this measure equally questionable.

In order to overcome the accuracy problem of the probability density functions, consider the "limit-state function" g:

$$g = R - S \quad \text{......} 2.61$$

All negative values of g indicate failure, while positive values represent the safe state. When both the load and resistance distributions are normal, g will also be a normally distributed variable with the following mean and standard deviation:

$$\mu_g = \mu_R - \mu_S \qquad 2.62$$

$$\sigma_g = \sqrt{\sigma_R^{\,2} + \sigma_S^{\,2}} \qquad 2.63$$

The significance of the mean value of the limit-state function is illustrated in Figure 2.13. It is observed that a high mean value indicates a safe structure, while a small or negative value is characteristic of an unsafe structure.

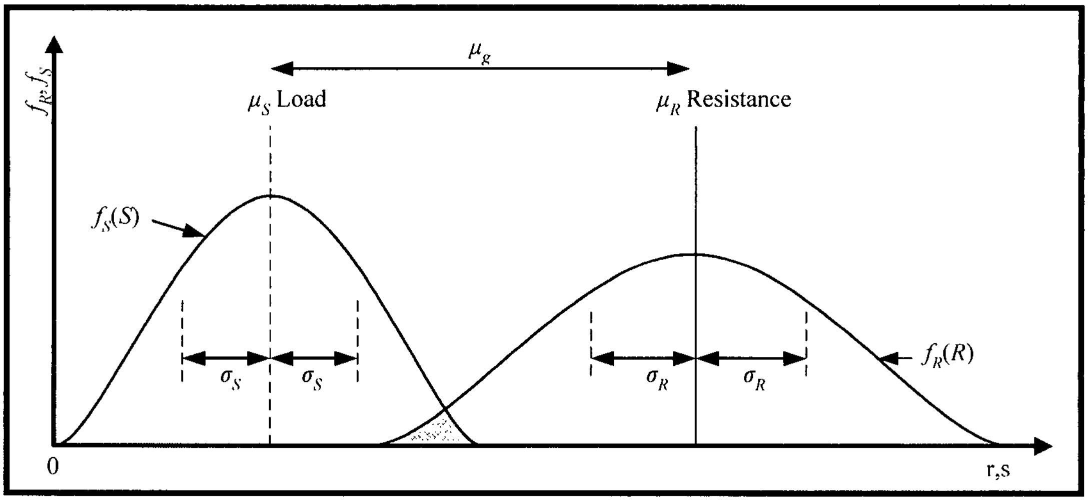

Figure 2. 13 The normally distributed load and resistance distribution.

The distribution of g is shown in Figure 2.14. The probability of failure is represented by the shaded area shown in Figure 2.14 and is given by the following integration

$$P_f = \int_{-\infty}^{0} f_g(g)\,dg \qquad 2.64$$

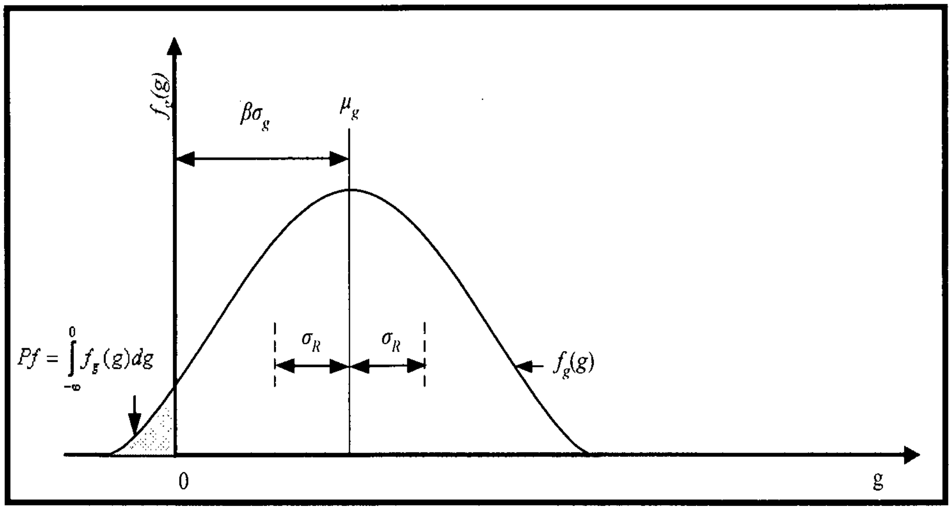

Figure 2. 14 The distribution of the limit-state function g.

The "reliability index" associated with the simple load-resistance reliability problem is defined as:

$$\beta = \frac{\mu_g}{\sigma_g} \qquad \text{2.65}$$

The probability of failure for the normal distribution is given by

$$P_f = P(g \leq 0) = \Phi(\frac{g - \mu_g}{\sigma_g}) \qquad \text{2.66}$$

In case of the standard normal distribution above equation reduces to

$$P_f = \Phi(-\frac{\mu_g}{\sigma_g}) = \Phi(-\beta) \qquad \text{2.67}$$

The procedure described above is valid for the case of two random variables that are normally distributed. More general cases are considered in the following sections.

2.7.2 Finite Element Reliability Analysis

Typically, there are more than two random variables involved in case of a structural analysis. Hence, the computation of the failure probability by Eq (2.65) is not possible in such cases. Besides the number of random variables involved in a structural analysis, each random variable may follow different type of distribution. In performance based engineering (introduced in Chapter 4), it is often necessary to determine the probability for various performance events. Therefore, in order to overcome the problems stated above the structural reliability methods are employed. The structural response quantities are utilized to specify the limit-state functions in structural reliability methods. The finite element method is a widely applied tool in structural engineering for deterministic analysis of the structures. The finite element method and advanced reliability methods such as "First Order Reliability Method" (FORM) are merged together to obtain probability estimates for the predefined performance criteria. The joint analysis is referred as "Finite Element Reliability Analysis" (FERA). The finite element code OpenSees developed by the "Pacific Earthquake Engineering Research" (PEER) center [20], is capable of performing such analysis. It is an open source software and freely downloadable from the web site http://opensees.berkeley.edu. It consists of objects necessary for defining the probability distributions for input variables and for reliability analysis. A detailed discussion regarding this topic is beyond the scope of this study and is available in [14]. To benefit the reader of this study, the key steps involved in the finite element reliability analysis are described briefly in the following sub-sections.

Uncertainty Modeling- The different parameters (load, dimension, strength etc.) employed to model and analysis a structure in a finite element analysis is selected deterministically. However, in FERA, these parameters are defined as the random variables in order to characterize their uncertain behavior. These parameters are introduced in a finite element reliability analysis by their probability distribution, mean, coefficient of variation and correlation coefficient.

Limit-state Functions- In FERA, the main objective is to find the probability of exceeding a performance criterion. This is achieved by means of the limit-state functions. The term "failure" is employed to define an event of not meeting the performance criteria.

The limit-state functions for the structural components are commonly denoted as $g(x)$, where x is the vector of basic random variables. Similarly to the previous section; $g \leq 0$ represents the failure domain, $g = 0$ represents the limit state and $g > 0$ represent the safe domain. In the space of uncorrelated standard normal variates y the performance function is denoted as $G(y)$ (which means $G(y(x)) = g(x)$ and $g(x(y)) = G(y)$). One of the main aspects of the finite element reliability methods is the inclusion of the response quantities from a finite element solution in a performance function. For instance, a performance function can have the following form

$g = threshold - response\ quantity$ 2.68

When the uncertain response quantity exceeds a certain threshold value, the performance function will have a negative value, which implies the failure domain. The response quantity of Eq. (2.68) can be the values of the stresses, stress resultants, strains, displacements etc obtained from a finite element solution. The threshold value indicates the limiting value of the performance criteria. OpenSees offers a framework to express performance functions analytically with respect to the response quantities and the threshold value.

General FORM Analysis- The primary concern in FERA is to estimate the probability of failure. In case of single performance function, the reliability problem is defined by the following multifold integral

$$P_f = \int \cdots \int_{g(x)\le 0} f(x)dx \quad \text{......} 2.69$$

In Eq. (2.69) P_f is the probability of failure, $\boldsymbol{x}$ is the vector of random finite element model parameters, $g(\boldsymbol{x})$ is the limit-state function and $f(\boldsymbol{x})$ is the joint PDF of $\boldsymbol{x}$. The integration of Eq. (2.69) is over the set of random variables $\boldsymbol{x}$, hence the FERA can be very large depending on the size of the random variables $\boldsymbol{x}$. A closed form solution of Eq. (2.69) is not available except for some special cases. For this reason, a number of methods have been developed for the purpose of solving the integral approximately. FORM is one of these methods. A brief description of FORM is provided in the following paragraphs.

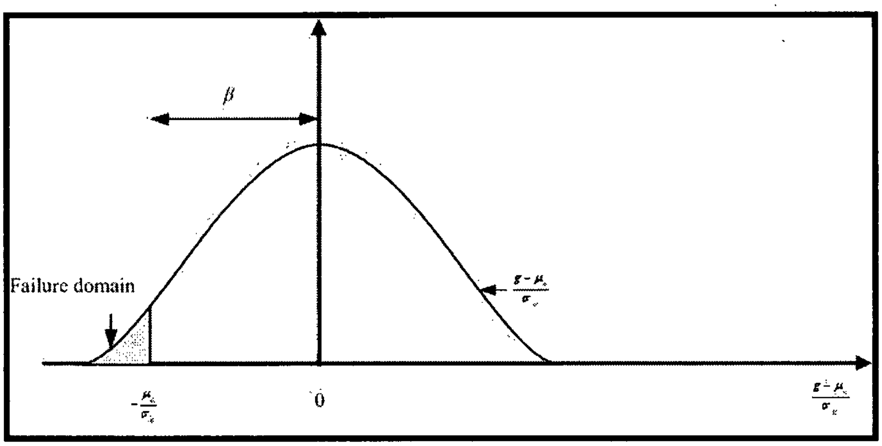

Figure 2. 15 The reliability index β incase of the standard normal distribution.

In a standard normal space, the reliability index β represents the distance between the mean (0) of that standard normal distribution and the boundary $g = 0$ of the standard function $\frac{g-\mu_x}{\sigma_x}$. In other words, the value of β represents how many standard deviations

$g = 0$ is away from the mean. Smaller the value of β, greater will be the failure probability and vise versa. The distance β is mentioned in the standard normal space instead of the original spaces of the random variable because in the original space different random variables may have different types of distribution. Besides the standard normal space has zero mean and unit variance. A plane, known as "limit-state surface", separates the failure domain and safe domain. From the elementary mathematics, it is known that the distance from a point to a plane is equal to the function value at the point divided by the norm of the gradient vector at that point. For instance, if a plane is defined by $G = 0$ in the standardized space then the distance from the origin, 0 to that plane is given by

$$\Delta = \frac{G(0)}{\| \nabla G \|} \qquad 2.70$$

In [9] it is mentioned that this Δ is also equal to the β. Hence, β is the minimum distance from the origin to the limit-state surface in the standardized space of random variables. This is illustrated in Figure 2.16 in case of a linear limit-state surface derived by two random variables.

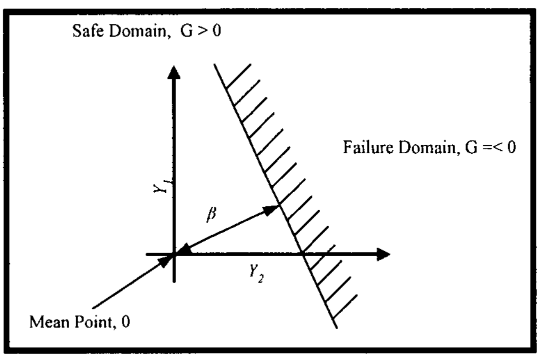

Figure 2. 16 The distance β from the origin to the linear limit-state surface in a standardized normal space.

The point from which β is minimum from the origin in the standard normal space is defined as the design point and is donated by y^*. The following relation (illustrated in Figure 2.17) is established in [9].

$$\beta = \| y^* \| = \alpha^T y \qquad 2.71$$

Where $\alpha = -\dfrac{\nabla G}{\| \nabla G \|}$.

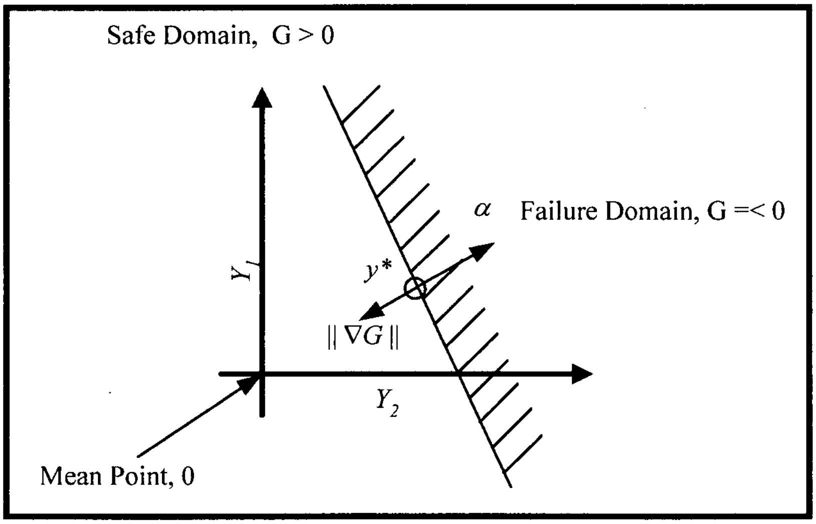

Figure 2.17 The design point on the (linear) limit-state surface.

Therefore in approximate solution of Eq. (2.69), the goal is to find the design point (y^*) and then reliability index β (Eq. (2.71)) in standard normal space. Finally, Eq. (2.67) is employed to determine the failure probability.

Since all the random variables are transformed to the standard normal space, the design point on the limit-state surface is the point with most probability density. In order to determine the distance between the origin and the limit-state surface, the limit state function is linearized at different points by the Taylor expansion on the limit state surface. In FORM, the limit state function is linearized at the design point to determine the distance from the origin to the limit state.

The linearization of the non-linear limit-state function by the Taylor expansion is illustrated in Figure 2.18.

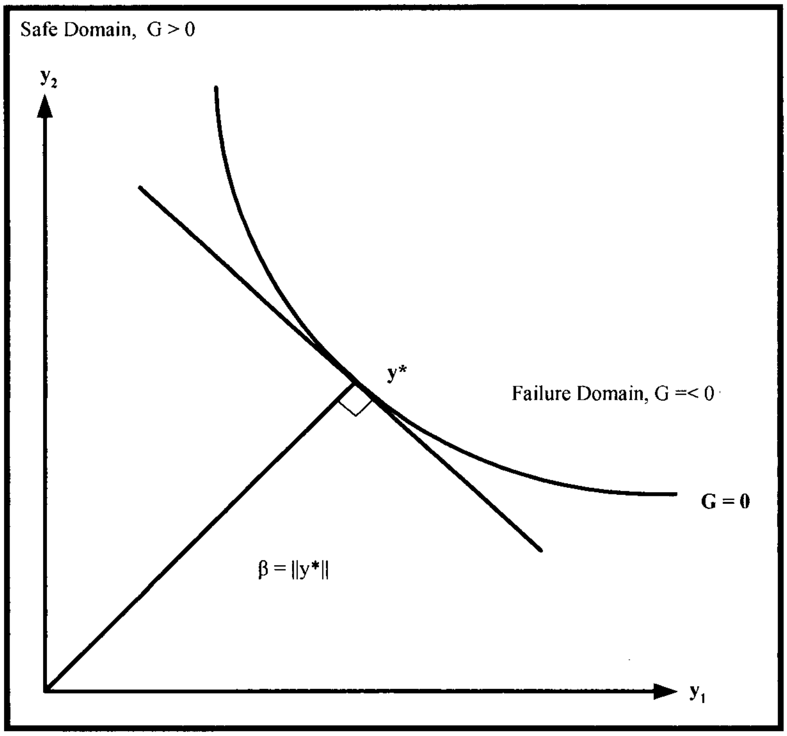

Figure 2. 18 The limit-state lines for linearized non-linear limit-state function by the Taylor expansion.

Different algorithms are available to find the design point on the limit-state surface. The detailed discussions regarding the implementation of FORM in determination of the design point are available in [12], [13], [14]. In this study, the iHLRF algorithm is employed. The key steps involved in the iHLRF algorithm are illustrated in Figure 2.19.

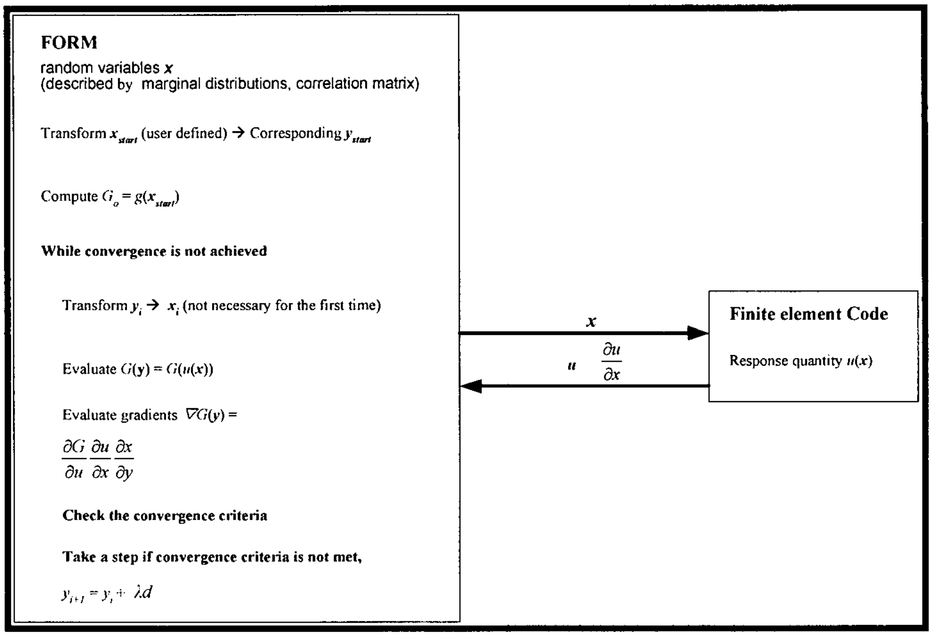

Figure 2. 19 A flow chart for the determination of the design point in FERA.

In the above figure, d represents the search direction vector and λ represents the step size. Two convergence criteria have to meet by the search algorithm. These are as follows:

1. The first check confirms that the design point is on the limit-state surface and it is ensured by the criteria $|\frac{G}{G_0}| \leq e_1$, where typically $e_1 = 10^{-3}$.
2. The second check confirms that at the design point (y^*) the gradient vector must point towards the origin in the standard normal space and it is ensured by the criteria $\| y - \alpha y \alpha^T \| \leq e_2$, where typically $e_2 = 10^{-3}$.

The input random variables $\boldsymbol{x}$ might be the normal random variables or the uncorrelated non-normal random variable or the correlated non-normal random variables. Hence, these random variables are transformed into the standard normal uncorrelated variables $\boldsymbol{y}$. At first step, the limit-state function (denoted by G_0) is evaluated with the initial values of $\boldsymbol{x}$.

The user typically defines these initial values of $\boldsymbol{x}$. These initial values of $\boldsymbol{x}$ are also employed in the finite element model of a structure to obtain the response quantities $u(\boldsymbol{x})$ and gradient vectors $\frac{\partial u}{\partial x}$. Then these values of $u(\boldsymbol{x})$ and $\frac{\partial u}{\partial x}$ are employed to evaluate the limit-state function $G(y)$ and its gradient $\nabla G(y)$ in the standard normal space. If the convergence criteria are no meet then a new set y values are generated and the same procedure is followed again. While searching for the design point, the finite element code has to be run again and again with the new set of random variables x and generates the response quantities $u(x)$ and gradient vectors $\delta u/\delta x$.

2.9 Conclusion

From the above discussions, it is evident that it is possible to model and analyze the uncertainties associated with the material and the structural behavior while performing the structural analysis by FERA. Hence, it is possible to analyze a structure probabilistically rather than in deterministic manner. Thus by employing state of the art FERA in OpenSees, the failure probability of a performance criterion for a structure is computed and then this probability values are employed in the engineering decision making process. In the next chapter, the applications of these probability values in the decision making process are discussed.

Chapter 3.0 Fundamentals of Decision Analysis

3.1 Introduction

Many factors have to be taken into consideration before making any engineering decision. As costs are associated with each decision, the consequence of each decision plays an important role in engineering decision making. Additionally, "uncertainty" is associated with the outcomes. So decision analysis should be performed before making any decision. Decision analysis provides a comparison between different decision alternatives, so that transparent and defendable decisions can be made.

For example, when selecting among different retrofit strategies, the engineer's duty is to recommend the best possible alternative(s) available on the market that will meet the owner(s) desired goal(s). A decision tree is a suitable means to present the results (in terms of money loss or gain) of alternative retrofit strategies. In this chapter, basic concepts regarding decision tree analysis are discussed.

3.2 Fundamentals of Decision Tree Analysis

Decision trees represent a graphical means of visualizing the decision problem. Different components of the decision tree are shown in Figure 3.1. The branches of a decision tree are connected in two types of forks. The first fork is called the "Decision Fork", from where decision branches are generated. The number of branches generated from the decision fork depends upon the number of actions considered by the decision maker. In a graphical representation, such as in Figure 3.1 the square or rectangle box is used to represent a decision fork. The second fork is called the "Chance Fork." Branches generated from this fork represent the events that can take place due to the decisions taken. Usually, it is represented by a circle in the graphical presentation; see Figure 3.1.

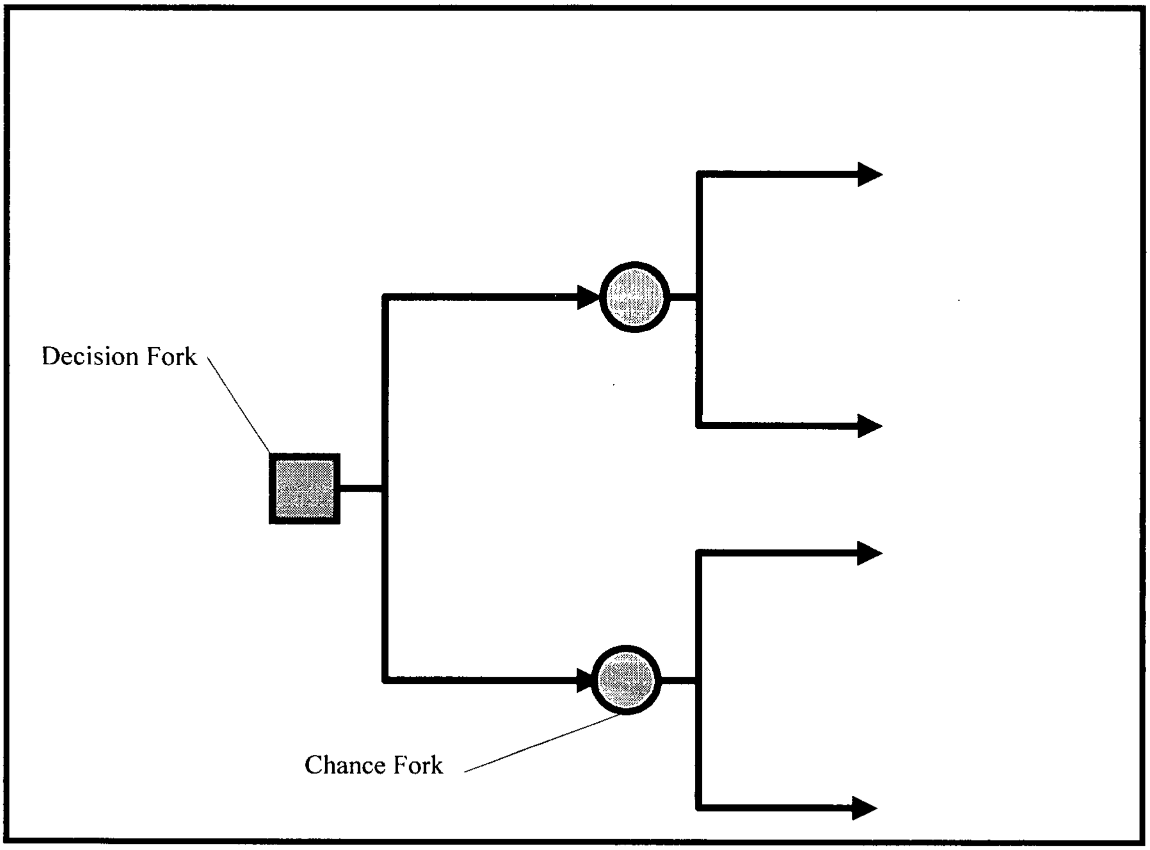

Figure 3. 1 Different components of a decision tree.

The decision tree proceeds chronologically from left to right; the later events/decisions following the earlier events/decisions. As shown in Figure 3.1, each branch of the decision fork leads to an event fork. There is uncertainty associated with each event. At the very end of each branch, there will be corresponding consequences and probabilities. The events considered at each chance fork must be mutually exclusive and collectively exhaustive, i.e., the sum of their probabilities must be equal to one.

As an example, let us assume that for a particular project there are two possible actions, a_1 and a_2, followed by two possible states of nature, θ_1 and θ_2. The probability of occurrence for each outcome of the state of nature (θ_1 and θ_2) is $P(\theta_1)$ and $P(\theta_2)$, respectively. The cost associated with each act-event pair is represented by u_{11}, u_{12}, u_{21} and u_{22}. The first number in the subscript represents the action number and the second number represents the event numbers. The decision tree starts with a decision fork at the left representing alternate courses of action followed by chance forks representing events that comprise the of outcome of nature. The generic decision tree for this project is shown in Figure 3.2.

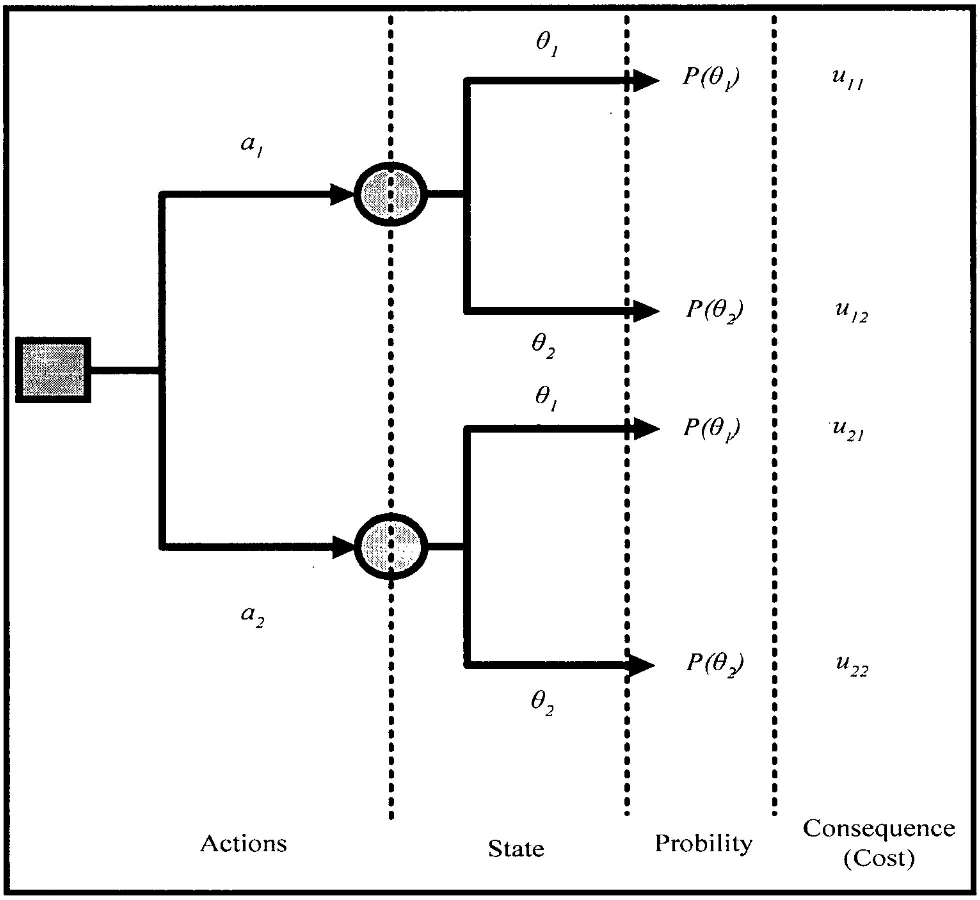

Figure 3. 2 A typical decision tree.

Example 3.1: A contractor has been hired for retrofitting a structure. A project engineer has been assigned by the contractor to run the project. The project engineer selects three designs suitable for the structure. Hence, he has to make a choice between three alternatives. From experience, he assumes three outcomes (unworkable, inefficient and satisfactory) for each decision (Design 1, Design 2 or Design 3) and the values of the probability of occurrence of each event and their consequence cost. The negative value of the consequence cost in case of an unworkable event indicates the amount of loss (including the fine, installation cost etc) that the contractor would face. In case of an inefficient event, it indicates the amount loss due to the repair, fine etc. The positive consequence cost of a satisfactory event indicates the amount of profit the contractor will receive. The arrangement of the engineer's decision tree is shown in Figure 3.3.

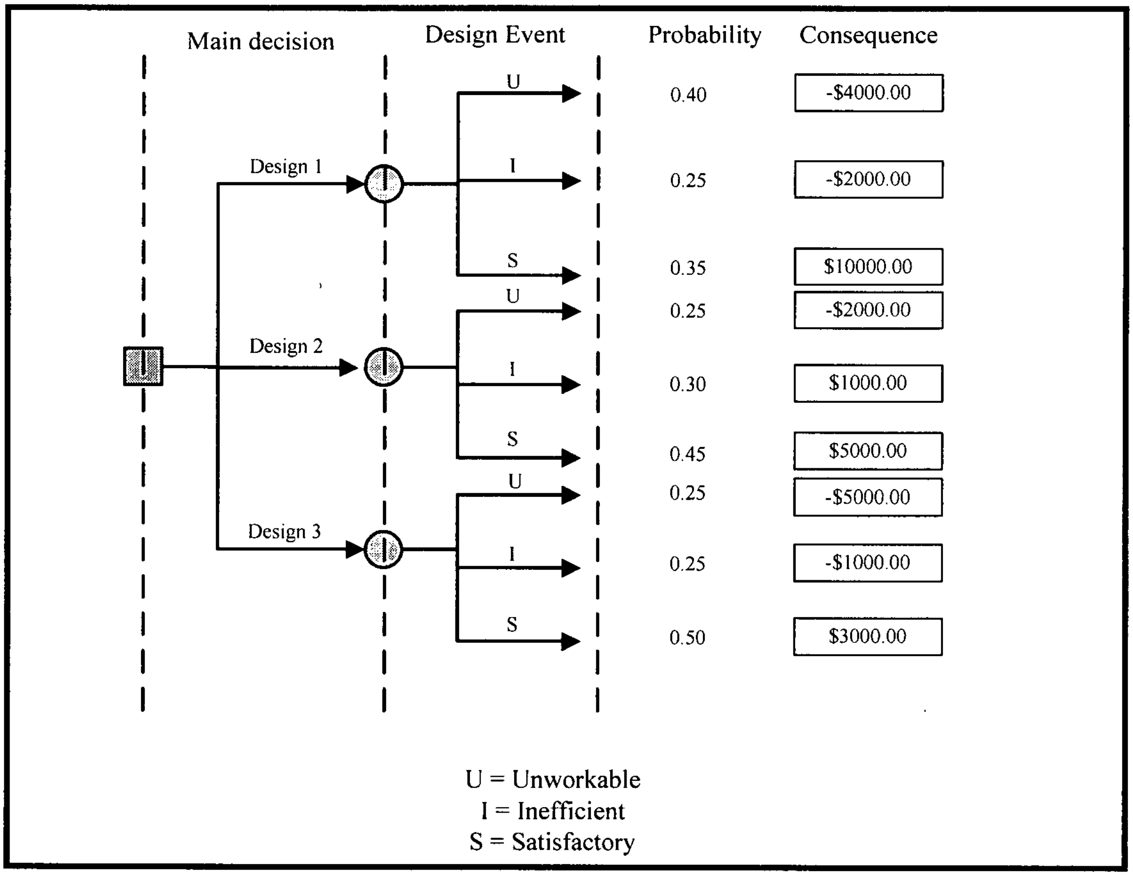

Figure 3. 3 The decision tree for Example 3.1.

3.3 Determination of the Expected Value

The expected cost of each event branch is calculated by multiplying the probability of occurrence of that event with the consequence cost of that event. The total expected cost of each decision branch is calculated by summing the expected cost of all the event branches on that decision branch. The total expected cost calculation of Example 3.1 is shown in Figure 3.4.

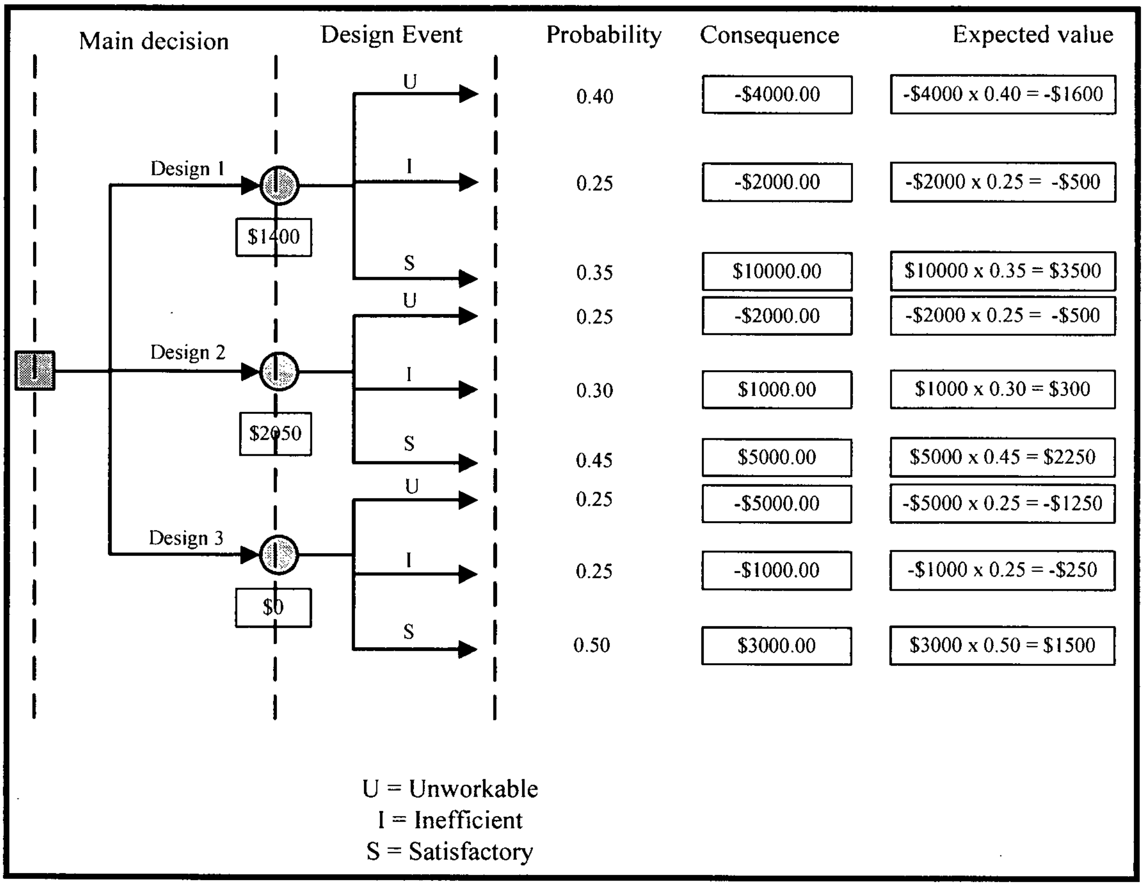

Figure 3. 4 Computation of the expected cost value in Example 3.1.

Engineers can make decisions from a decision tree depending on the selection criterion. The following criteria are commonly employed in a decision tree analysis.

Minimizing the Maximum Loss- This is based on the extremely pessimistic view on the outcomes of event. The goal of this criterion is to minimize the loss as much as possible. This criterion reflects the “risk averse” or “conservative attitude” of a person(s) who is not willing to gamble with his luck. In Example 3.1 according to this criterion, the project engineer would go with the “Design 2” option because if there is a loss; this option will yield a minimum loss of $2000.00.

Maximizing the Maximum Profit- This is based on the extremely optimistic view of the outcome of nature. The goal of this method is to maximize the profit as much as possible. This criterion reflects the “risk seekers” attitude of a person(s) who likes to play with his luck. In Example 3.1 according to this criterion, the project engineer would go with the “Design 1” option as it has a chance to yield a maximum profit of $10000.00.

Minimum Expected Cost- This is based on the attitude of a risk neutral person. That means a person who is not very much eager for profits but on the same time he is not very much afraid of taking risk. In Example 3.1 according to this criterion, the project engineer would go with the “Design 2” option, as it will yield a minimum expected cost of $2050.00. In this study, the minimum expected cost criterion is employed.

3.4 Conclusion

The decision tree has a wide application in the retrofit strategy selection process. This is because it enables engineers to present wide spectrum of cost related information in front of the decision makers (owners). In the next chapter a methodology is introduced, which utilizes both the finite element reliability analysis and decision tree analysis to provide useful information to the decision maker(s) in their effort to select best possible retrofit option.

Chapter 4.0 Methodology

4.1 Introduction

The trend of structural engineering is changing regarding earthquake design. The new trend is more oriented to different performance levels of the structure rather than merely life safety. In this thesis, performance based engineering is employed in conjunction with decision tree analysis, to determine whether retrofit is necessary for a structure or not. In the following paragraphs an introduction to performance-based engineering is presented followed by a description of the proposed methodology.

4.2 Performance-based Engineering

The performance of a structure during an earthquake greatly depends on the characteristics of the ground motion and the ability of the structure to dissipate energy through inelastic action. Inelastic response is affected by the global dynamic characteristics (e.g. period, damping etc.) as well as the material properties (e.g. yield strength, ductility capacity, etc.) and detailing of the members in the structure. In this thesis, the structure is modeled with non-linear elements to assess the degree of damage due to earthquake loading. Deterministic assessment of the response is impossible due to the presence uncertainties, including those associated with input parameters, analysis procedures, as well as in the ground motion characteristics.

In the past fifteen years, significant progresse has been made in the application of performance-based engineering to the seismic evaluation and design of structures [7]. In particular, significant advancement has been made in linking ground motion characteristics with building performance, and to describe building performance both qualitatively and quantitatively at different hazard levels. A detailed guideline and procedure for performance-based seismic evaluation of buildings is provided in [10] and [11]. These guidelines relate the damage of various structural elements to different levels of structural response. Probabilistic extensions to these procedures were first introduced in [16]. Probabilistic seismic demand along with information about capacities of elements can be used to determine the probability of exceeding certain performance levels. The PEER [20] centre has developed a framing equation [8] for economic loss assessment of structures at different performance levels. They follow the following sequence:

$$IM \rightarrow EDP \rightarrow DM \rightarrow DV \qquad 4.1$$

where *IM* represents intensity measure (magnitude of the earthquake etc.), *EDP* represents engineering damage parameters (displacement, stress of structure etc.), *DM* represents damage measures (amount of concrete spalled etc.) and *DV* represents decision variable (damage cost etc.). If there is an earthquake with a certain magnitude (*IM*) and the structural response for that *IM* is measured in terms of displacement (*EDP*) then if appropriate models are available, it is possible to determine the amount of damage (*DM*) incurred and hence the damage cost (*DV*). In the PEER framework, the sequence presented in Eq. (4.1) is intended to be carried out probabilistically. By using Eq. (4.1), it is possible to determine the probability of amage, and cost.

4.3 Proposed Methodology

In the following, the different steps associated with the proposed *FERA*-based decision tree approach are described. The methodology is intended to guide the decision maker when selecting between alternate retrofit strategies.

4.3.1 Structural Modelling

In this thesis, finite element models in OpenSees are used to model the structures. Displacement-based elements are used to model the beams and columns. The material properties (concrete compressive strength, strain at compressive strength, concrete crushing strength, strain at crushing strength, steel's yield strength, modulus of elasticity etc.) and structural dimensions (element size, steel area, clear cover etc.) are modeled as random variables in order to capture the uncertainties associated with them.

Most of the commercial software's capable of performing non-linear analysis (dynamic or static) are not capable of modeling the structure with material properties and structural dimension as random variable; hence uncertainties can not be accounted for in the analysis. In OpenSees, it is possible to perform non-linear analysis, while randomness in material properties and structural dimensions is taken in to consideration.

4.3.2 Ground Motion

In NBCC 2005 [17], spectral acceleration values for periods of 0.2s, 0.5s, 1.0s and 2.0s are provided for major cities of Canada. Detailed discussion of seismic hazard analysis used in NBCC 2005 can be found in [2]. These values are determined for earthquakes that have 2% chance of exceedence in 50 years. The values of spectral accelerations in NBCC 2005 reduce the requirement for performing site specific seismic hazard analysis. Using these spectral acceleration values, engineers can determine earthquake loading demand (lateral load, displacement etc.) for static analysis (push-over) of the structure. Spectral acceleration values (given in NBCC 2005) can also be used to simulate spectral acceleration compatible time histories for a particular site which then can be used in dynamic analysis. In the proposed method, spectral acceleration values provided in

NBCC 2005 are used to determine the target displacement (δ_t), for a particular structure at a specific site.

4.3.3 Analysis Procedure

Non-linear Dynamic Analysis (NDA) is the most advanced tool in determining the actual behaviour of the structure under earthquake loading. However, although Non-linear Static Analysis (NSA), also referred to as push-over analysis, is not able to provide the exact behaviour of the structure under earthquake loading, it provides engineers with information regarding failure mechanism and load path for the structure subjected to lateral loads. This method is popular among engineers because of its ability to generate the relationship between a global engineering demand parameter (e.g. displacement at the roof level of the building) and the lateral force. The idea behind non-linear static analysis is that the engineer gradually increases the lateral load on the structural model, while monitoring the response. This response could be the roof displacement and a plot is generated to show the total lateral load vs. roof displacement for that structure. This is referred to as a push-over curve. The applied lateral load can be distributed at different floor levels by following one of the methods described in FEMA 356 [11]. A typical push-over curve is shown in Figure 4.1.

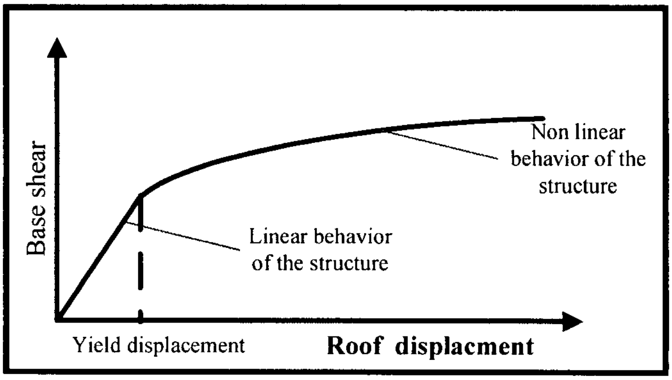

Figure 4. 1 A typical push-over curve.

The first part of the curve is linear, representing elastic behaviour of the structure. As the load increases, components of the structure begin to respond inelastically and damage increases. At a certain stage, there will be a large displacement increment due to a small increase in the load, which indicates that loss of structural integrity is near. The difference between the failure displacement (at collapse) and yield displacement (see Figure 4.1) represents the ductility of the structure. The more the difference, the more the structure will be able to dissipate earthquake energy. Non-linear static analysis does not require hundreds of earthquake time histories, hence the analysis is computationally efficient, and the results can be presented in a simple graph. In OpenSees it is possible to perform non-linear static analysis; this software is employed in this thesis to determine the structural performance.

4.3.4 Damage Indices and Damage Level

In earthquake engineering, various damage indicators are used to quantify damage of structures caused by seismic loading. Brief descriptions of a selection of damage indicators [19] employed in dynamic and non-linear analysis are given below:

a. Local damage indicator

- Inter-storey drift: Inter storey drift is the relative displacement of two neighbouring floors. It is normalized by the height of the storey in consideration.

$$ID = \frac{|\delta_{\max}|}{h} \quad \text{.......... 4.2}$$

where h is the storey height.

- Displacement ductility, μ: μ is defined as the ratio between maximum inter storey drift, ID and yield displacement, δ_y (displacement at which push-over curve transforms from linear state to non-linear state).

$$\mu = \frac{ID}{\delta_y} \quad \text{.......... 4.3}$$

- Normalized hysteric energy, NHE: Normalized hysteric energy is the summation of energy absorbed in each cycle under seismic loading scaled by two fold strain energy.

$$NHE = \frac{\sum_{1}^{N} \int V_u du}{V_y \delta_y} \quad \text{.......... 4.4}$$

N is the number of cycles. Integration represents the area under the push-over curve of each storey in each cycle and V_y is the yield force.

- Park / Ang damage indicator, DI: This represents the linear combination of normalized displacement and normalized hysteric energy.

$$DI = \frac{|\delta_{\max}|}{\delta_{ult}} + \beta \frac{HE}{V_y \delta_{ult}} \quad \text{.......... 4.5}$$

here, δ_{ult} is the maximum monotone displacement capacity, β is weighing factor and HE is the cumulative absorbed hysteric energy as described above.

b. Global displacement indicator

- Ductility, μ: It is the ratio between maximum relative displacement of the roof level with respect to the foundation of the structure and yield displacement of the structure.
- Normalized hysteric energy, I: The global normalized hysteric energy, I is the cumulative of hysteric energy of each storey normalized by the product of global yield displacement and global yield force which is the base shear at yield displacement.
- Park / Ang indicator: It is the weighted summation of local Park/Ang indicator of each storey. Weighing factor of each storey depends on the relationship between hysteric energy of that storey to the total hysteric energy of the structure.

Different limiting values of these damage indexes with respect to the different damage states are defined in [10], [11]. In [10], three damage states are defined to aid engineers in determining what constitutes an acceptable design. Brief descriptions of these "limit-states" are given below:

Immediate Occupancy- At this damage level, limited damage to the structure is expected after an earthquake. Vertical and lateral load resisting system would have the same pre-earthquake stiffness and there will be no life threatening damage. Minor repair of the structure may be required in the presence of tenants of residential complex and business can go on in commercial structure.

Life Safety- It is an intermediate state between total collapse and minor damage. Main load bearing system might have suffered severe damage but this has not resulted in large falling debris hazards, either within or outside the building. Injuries may occur during the earthquake but is not expected to be life threatening. Although structural damage is repairable but may not be economically feasible. Repair of the damage is required prior to make the structure functioning.

Collapse Prevention- At this stage of damage scale, structure is expected to be on the border of partial or total collapse. Although there will be sufficient degradation of stiffness of lateral load resisting system and permanent deformations of different elements of the structure, key components of the gravity load resisting system must continue to support the gravity loads. High level of injury and even fatalities are possible due to the collapse of different element or from falling debris. Structure may not be repairable from economic and engineering point of view. The structure would be uninhabitable and might collapse due to after shocks.

In order to guide the engineers to determine the damage state from a structural analysis, various limiting values of damage indicators are provided in [10] and [11]. Table 4.1 provides an example of these limits for concrete moment frame structures.

Table 4. 1 FEMA 273 drift limits for different structural performance level in case of the concrete moment frames [10].

	Immediate Occupancy	**Life Safety**	**Collapse Prevention**
Maximum Inter-storey drift (ID)	1%	2%	4%

As opposed to the approach described above, where the global displacement of the structure is employed to estimate the damage [10, 11], the methodology proposed herein defines damage by monitoring the actual strain in the members. This is facilitated by the fact that in OpenSees it is possible to monitor stress and strain values in the cross-sections of an element. As the load is increased during the push-over analysis, the stress and strain in the concrete or steel of particular cross-sections is monitored. Thus it is possible to determine the global displacement at which the concrete of a particular section reaches the crushing strength or the steel yields. An illustrative figure of this procedure is shown in Figure 4.2, where global displacement at which cracks starts to appear on the left-most column of the ground floor is determined from the push-over curve. It is possible to determine the global displacement for various scenarios (steel yields, concrete reaches its crushing strength of any element(s), etc.). Thus more refined and explicit information regarding the performance of the structure can be made available, which enables us to define damage states according to the owner(s) need. This feature of OpenSees is used when defining damage state in this thesis. Hence the owner(s) get information about specific damage cost and can choose to take remedial actions.

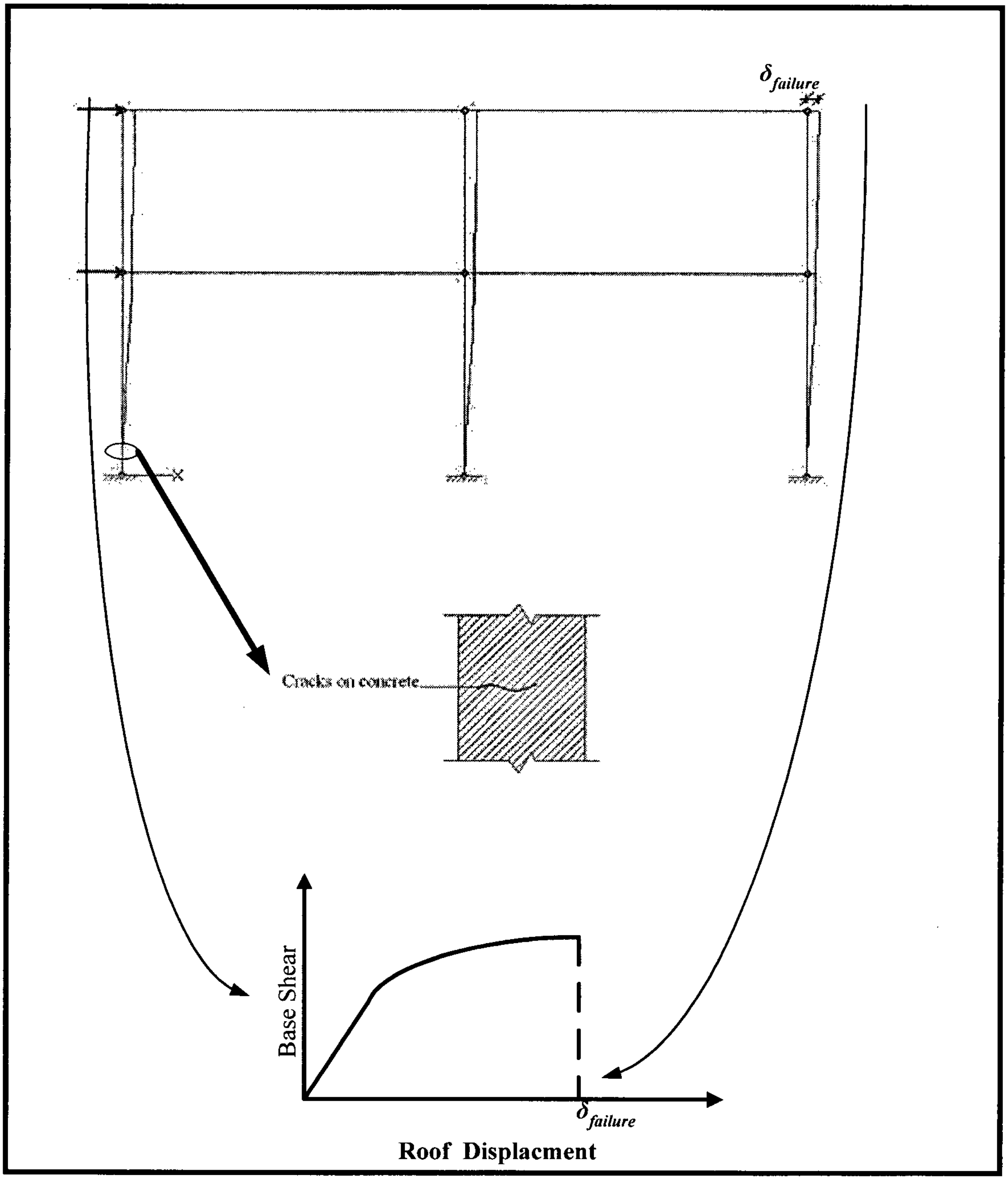

Figure 4. 2 The lateral load is gradually applied on the structure, the cracks starts to appear at some point; the loads and displacements are plotted on the push-over curve.

4.3.5 Computation of the Target Displacement by the Coefficient Method

The "target displacement," δ_t, is the maximum displacement a structure is expected to sustain under certain level of shaking. According to [10], the target displacement of a structure is given by,

$$\delta_t = C_0 C_1 C_2 C_3 S_a \frac{T_e^2}{4\pi^2} g \quad \text{.........} 4.6$$

where,

δ_t = Target displacement

C_0 = Modification factor used to relate spectral displacement with the roof displacement.

C_1 = Modification factor used to relate expected maximum inelastic displacement to the calculated displacement due to linear elastic response.

C_2 = Modification factor used to represent the effect of hysteresis shape on the maximum displacement response.

C_3 = Modification factor used to represent increased displacement caused by dynamic P-Δ effects.

T_e = Effective fundamental period.

S_a = Response spectrum acceleration at T_e.

In the following paragraphs, methods employed to compute different co-efficients of Eq. (4.6) are discussed briefly.

Computation of C_0- The value of C_0 is computed from Figure 3-2 in [10]. The value of C_0 depends on the number of a structure.

Computation of C_1- The value of C_1 depends on the effective fundamental period, T_e and the characteristic period of the response spectrum, T_0. T_0 is defined as the spectral period at which response spectrum transfers from constant acceleration segment to constant velocity segment. If $T_e \geq T_0$, then $C_1 = 1.0$ and for other cases C_1 is computed according to the section 3.3.3.3 of [10].

Computation of C_2- The value of C_2 can be obtained for different framing configuration and damage state of a structure by comparing T_e with T_o and from Table 3-1 of [10].

Computation of C_3- For the structures with positive post yield stiffness, the value of C_3 is 1.0 and for structures with negative post yield stiffness, C_3 is computed according to the section 3.3.3.3 of [10].

Computation of T_e- Effective fundamental period is defined by the following equation,

$$T_e = T_i \sqrt{\frac{K_i}{K_e}} \quad \text{..........} 4.7$$

where,

T_i = Elastic fundamental period of the structure in the direction under consideration determined by elastic dynamic analysis.

K_i = Elastic lateral stiffness of the structure in the direction under consideration.

K_e = Effective lateral stiffness of the structure in the direction under consideration.

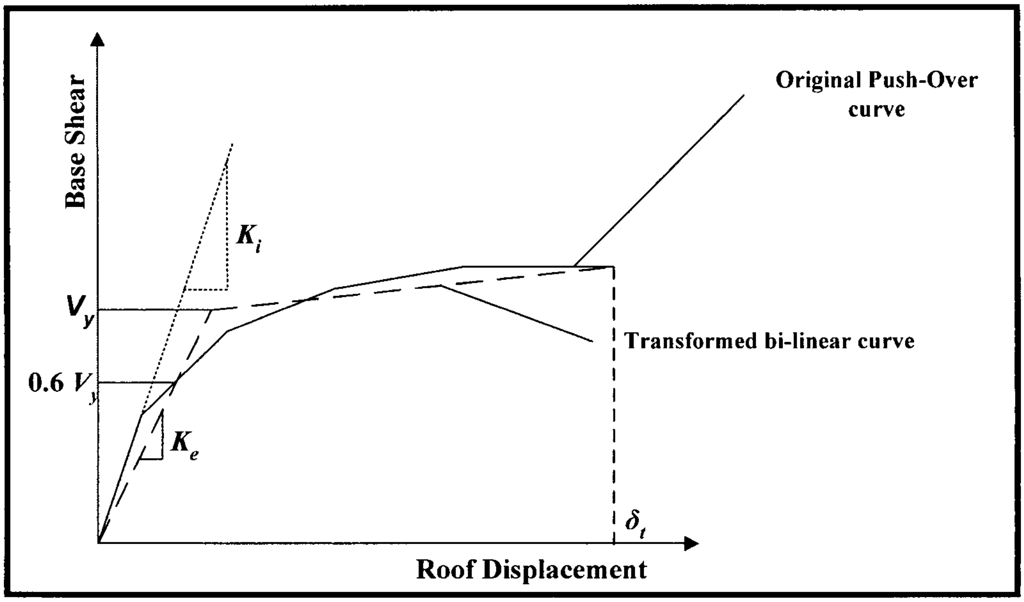

Figure 4. 3 The effective stiffness K_e, calculation [10].

The elastic fundamental period of a structure is determined by eigenvalue analysis of a mathematical model of that structure. The terms K_i and K_e are determined from the push-over curve generated by non-linear static analysis. In order to determine the effective stiffness K_e and effective yield strength V_y (which is used in the determination of K_e), the push-over curve of a structure is transformed into an idealized bilinear curve as shown in Figure 4.3. The initial slope of the bilinear curve is called K_e, which is taken as the secant stiffness calculated at a base shear force equal to the 60% of effective yield strength; see Figure 4.3.

Computation of S_a and δ_t- After computation of the effective fundamental period T_e, the spectral acceleration S_a can be taken from NBCC 2005 [17] for a particular site.

By following the procedures described above, all the variables of Eq. (4.7) can be determined and the value of the target displacement, δ_t is obtained.

4.3.6 Probability of Exceeding a Damage State

It is mentioned in section 2.8 that FERA is employed to determine the probability of exceedence of certain performance criterion. In order to perform FERA, the limit-state function similar to Eq.(2.68) needs to be defined. In Eq. (2.68), there are two terms- the threshold and the response quantity. In the proposed methodology, the target displacement, δ_t represents the term threshold. The term response quantity is defined by the global displacement at which the structure reaches certain damage state. This global displacement is denoted by $\delta_{failure}$ in the proposed methodology.

As described in the previous section, it is possible to determine the global displacement ($\delta_{failure}$) at which the concrete crack starts to appear on the left most column of the ground floor. If the limit-state function is defined as δ_t - $\delta_{failure}$ then by performing FERA it is possible to determine the probability of exceedence of the target displacement, δ_t when the concrete cracks appears on that ground floor column. Thus it is possible to define the damage states for different damage scenarios and determine the probability of occurrence of those damage states by FERA.

4.3.7 Conversion of Performance to Loss

The probability of occurrence of an event and the damage cost are combined together to determine the expected damage cost. In this study, the above procedure is followed for the structure with different retrofit options and without any retrofit to determine the expected damage cost in each case. However, these costs are the future damage costs and the decision is made in the present time; hence, these costs are discounted to the present value in order to compare with each other. Subsequently some assumptions are made such as an earthquake is a rare event and it is expected to occur not more than once in a lifetime of the structure and there will be no overlapping event and it can happen at any time in future. In case of the determination of the present value of cost, it is assumed that the cash flow is continuous. In a continuous cash flow, it is assumed that due to the inflation and interest; the present value of cash will increase exponentially as a function of the time, inflation and interest rate. Therefore, the future value C_f of a present value C_d is given by

$$C_f = C_d e^{it} \quad \text{......4.8}$$

i = Real interest rate (excluding inflation)

t = Time

Hence, in the present case, the future damage cost C_f in time t is discounted to the present value of C_d, by reversing Eq. (4.8).

$$C_d = C_f e^{-it} \quad \text{......4.9}$$

When the time of occurrence of the earthquake event in consideration is random then the expected present value of the future damage cost is

$$C_d = E[C_f e^{-it}] \quad \text{......4.10}$$

From the rule of total probability, the above equation is written as

$$C_d = C_f \int_0^{\infty} e^{-it} f(t)dt \quad \text{......4.11}$$

The occurrence of an earthquake is considered as Poisson's event due to its rare and independent random nature in time. If the earthquake has a mean rate of occurrence of λ then the time of first occurrence of such an event is given by (exponentially distributed, details are given on section 2.4)

$$f(t) = \lambda e^{-\lambda t} \quad \text{......4.12}$$

Putting the value $f(t)$ in Eq. (4.11), the following equation of the expected present value of a future damage cost is obtained

$$C_d = C_f \int_0^\infty e^{-it} \lambda e^{-\lambda t} dt = C_f \frac{\lambda}{\lambda + i} \quad \text{......} 4.13$$

Eq. (4.13) is generated from the two graphs shown in Figure 4.4. The first graph of Figure 4.4 is the plot of PDF of $f(t)$. The cash flow diagram is a useful tool for graphically representing the flow of cash with time. In a cash flow diagram, the upward arrows represent the positive cash flows (income, profit etc) and the downward arrows represent the negative cash flows. In the cash flow diagram of Figure 4.4 (b) the damage cost due to an earthquake event is considered over the life span of the structure.

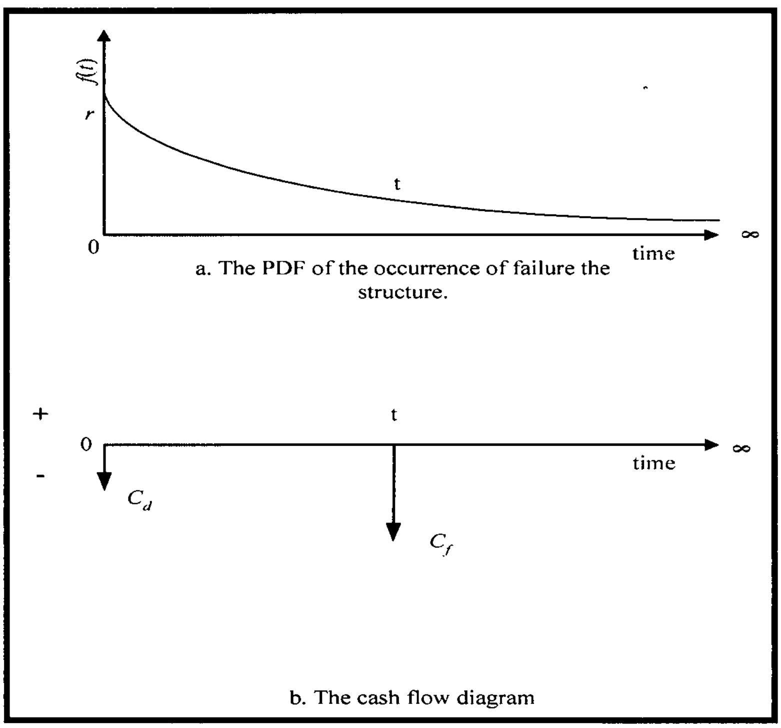

Figure 4. 4 The PDF of occurrence of the failure event and the cash flow diagram for a structure without retrofit.

When a structure is retrofitted, an earthquake might occur during the construction period and the probability of failure of that structure is same as without retrofit. However, after the construction of the retrofit system the probability of failure of the structure is reduced. Hence, the risks at both periods are considered. The cash flow diagram of Figure 4.5 (b) represents the cash flow scenario when the damaging earthquake happened during the construction phase of the retrofit system. The cash flow diagram of Figure 4.6 (b) is for the condition when the damaging earthquake happened after the construction of the retrofit system. If it is assumed that the construction cost is C_o and the construction time is t_c and the construction cost C_o is considered uniformly distributed over the time t_c then the present value of the construction cost is given by

$$C_{op} = \int_0^{t_c} \frac{C_o}{t_c} e^{-it}\, dt \qquad \text{4.14}$$

In most of the cases, the construction period will be very short compared to the life span of the structure. Hence, no discount is made to the future damage cost, as the value of cash will remain almost same in this short period of construction. If the damaging earthquake occurs after the installation of the retrofit system then the expected present value of the damage cost is

$$C_d = \int_{t_c}^{\infty} C_f e^{-it} \lambda e^{-\lambda(t-t_c)} dt \qquad \text{4.15}$$

In the above equation shifted exponential distribution ($\lambda e^{-\lambda(t-t_c)}$) of $f(t)$ is considered rather than exponential distribution of Eq. (4.12). This is because it is assumed that no earthquake event would occur during the period of t_c.

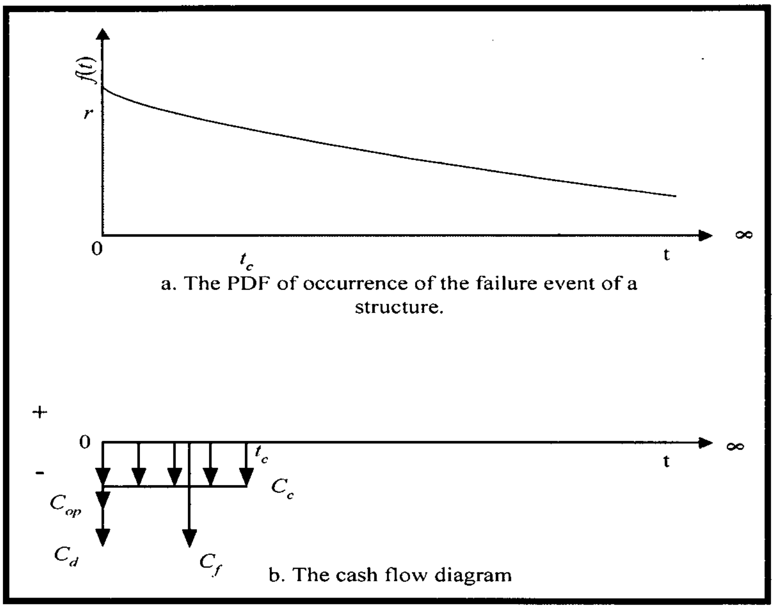

Figure 4. 5 The cash flow diagram and the PDF of occurrence of the failure event for a structure that suffers failure during the construction of the retrofit system.

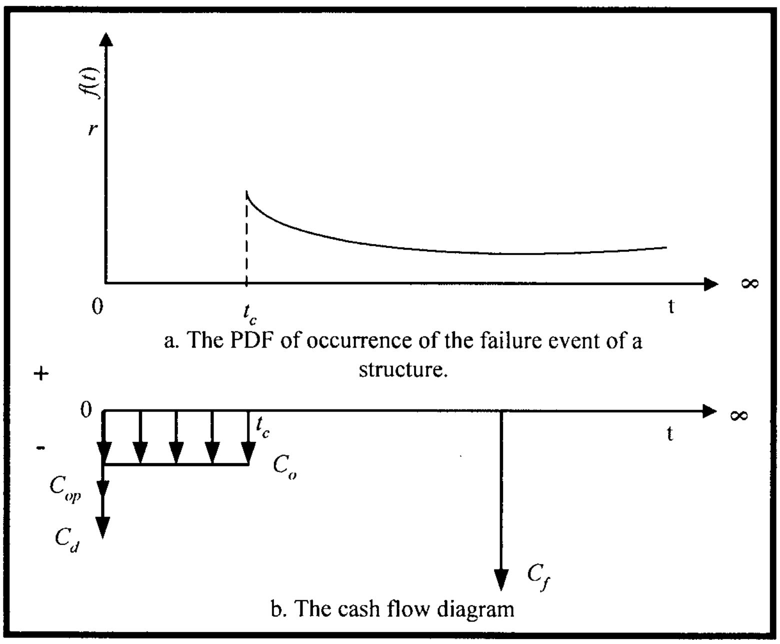

Figure 4. 6 The cash flow diagram and the PDF of occurrence of the failure event for a structure that suffers failure after the construction of the retrofit system.

4.3.8 Comparison of the Alternative Mitigation Measures

In this study, the decision making process is based on the minimization of the expected cost. For each case (with different retrofit options and without retrofit), FERA is performed and the expected present value of the damage is determined. While computing the total expected cost of the structure with retrofit, the present value of the construction cost of the retrofit system is added to the expected present value of the damage cost.

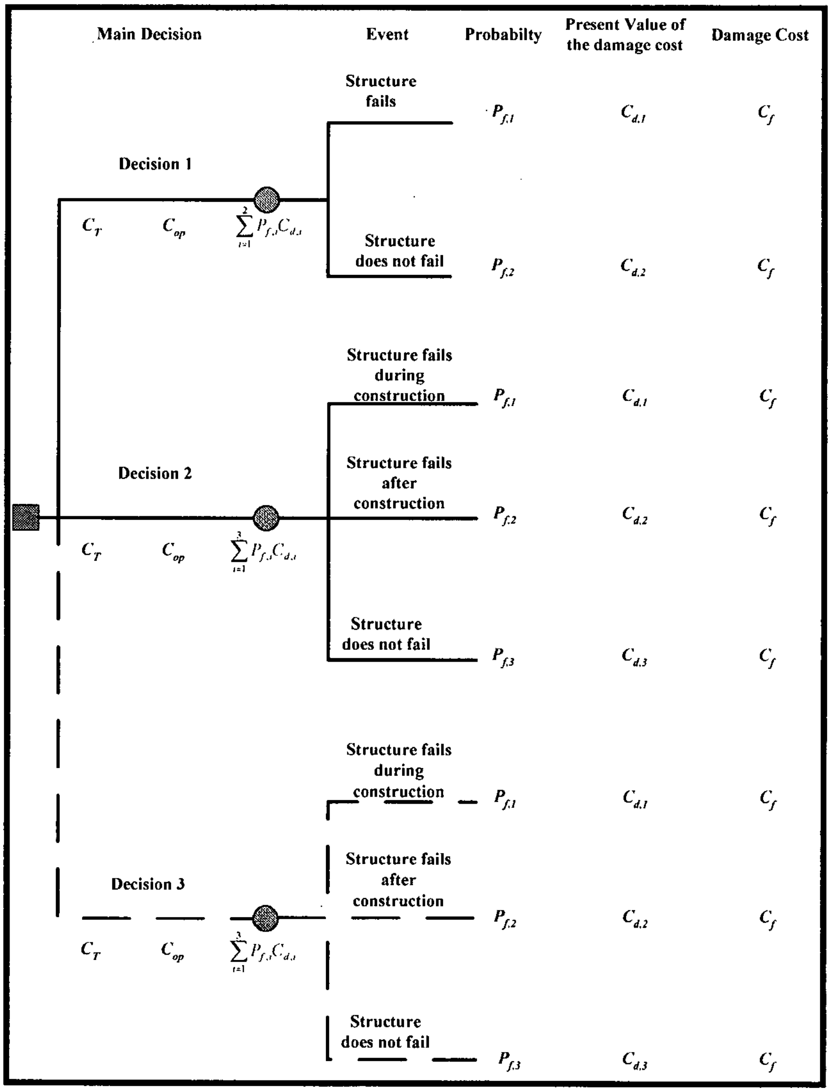

Figure 4. 7 The format of a decision tree employed for the purpose of selecting a retrofit option.

A typical decision tree employed in this study is shown in Figure 4.7 in which the different branches represent different decision alternatives. For instance, the first branch represents the costs of the original structure, where no retrofit is done. This branch is followed by an event fork which represents two states:

i. The failure event occurs during the life span of the structure, T_L.

ii. The failure event does not occur during the life span of the structure, T_L.

The probability of occurrence of a failure event depends on the occurrence of the eqarthquake event. The probability of occurrence of an earthquake event of certain level during the life span of the structure is given by

$$P(E) = \int_0^{T_L} \lambda e^{-\lambda t} dt \qquad 4.16$$

If the occurrence of an earthquake event is denoted by E then the probability of not occurrence of an earthquake event during the life span of the structure is given by

$$P(\overline{E}) = 1 - P(E) \qquad 4.17$$

Mathematically the conditional probability of failure is expressed as $P(f|E)$ (determined by FERA). With the occurrence of an earthquake event there is also a chance that there will be no earthquake ($P(\overline{E})$). Hence there will be a conditional probability ($P(f \mid \overline{E})$) of failure when there is no earthquake event. By the rules of total probability (section 2.3.2), the probability of occurrence of the failure event is determined by the following equation,

$$P_f = P(f \mid E)P(E) + P(f \mid \overline{E})P(\overline{E}) \qquad 4.18$$

Since the probability of failure in case of no earthquake event is 0, equation Eq. (4.18) reduces to

$$P_f = P(f \mid E)P(E) \quad \text{.......} 4.19$$

The conditional probability of faiure in case of the failuer state ($P(f|E)$) is computed by FERA and by putting the value of $P(f|E)$ and $P(E)$ of Eq. (4.16), the probabilify of occurrence of the failure state, $P_{f,1}$ of the first branch is computed.

Since the occurrence and not occurrence of a failure event with in the life span of a structure are two mutually exclusive and collectively exhaustive events, the probability of occurrence of the safe state, $P_{f,2}$ is obtained by subtracting $P_{f,1}$ from 1. The present value of the damage cost ($C_{d,1}$) of the failure state is computed by employing Eq. (4.13). In case of the safe state, the cost of failure is zero dollars. Hence, the expected present value of the damage cost is also zero dollars in this state. The expected present value of the damage cost of each state is multiplied by the probability of occurrence of that state and this value of the each state is summed ($\sum_{i=1}^{2} P_{f,i}C_{d,i}$) and put in the first chance node of the first branch. The first branch of the decision tree does not have any other cost associate with it. Hence the total cost of the first branch is,

$$C_T = \sum_{i=1}^{2} P_{f,i}C_{d,i} \quad \text{.......} 4.20$$

The subsequent branches are similar in structure. Each branch represents the structure with a different retrofit option. Each decision branch has a chance fork with three states:

i. The structure fails during the construction period.

ii. The structure fails after the construction period.

iii. The structure does not fail during the life span of the structure, T_L.

These type of branches have a construction cost (C_o) and construction time (t_c) associated with them. The construction cost is discounted to the present value (C_{op}) by employing

Eq. (4.14). In case of the first state of the second branch, the expected present value of the damage cost ($C_{d,1}$) would remain same as the future damage cost. The probability of occurrence of an earthquake event during the construction period is given by

$$P(E)=\int_{0}^{t_c}\lambda e^{-\lambda t}\,dt \quad \text{......} 4.21$$

The conditional probability of failure, $P(f|E)$ of the structure will be same as in case of the structure without any retrofit system. The probability of occurrence of the failure state during the construction of the retrofit system of the second branch, $P_{f,1}$ is computed by putting the value of $P(f|E)$ of the decision branch and the value of $P(E)$ of Eq. (4.21) in Eq. (4.19).

In case of failure state after the construction of the retrofit system of the second branch, the expected present value of the damage cost, $C_{d,2}$ is computed by employing Eq. (4.15). After the installation of the retrofit system, the conditional probability of occurrence of the failure event, $P(f|E)$ will be changed. The new $P(f|E)$ is computed by performing FERA in the structure with the retrofit system. The probability of occurrence of an earthquake event after the construction of the retrofit system is

$$P(E)=\int_{tc}^{T_L}\lambda e^{-\lambda t}\,dt \quad \text{......} 4.22$$

Therefore, the probability of occurrence of the failure state after the construction of the retrofit system of the second branch, $P_{f,2}$ is computed by putting the new value of $P(f|E)$ and the value of $P(E)$ of Eq. (4.22) in Eq. (4.19).

The probability of occurrence of the safe state of the second decision branch, $P_{f,3}$ is obtained by subtracting the probability of occurrence of the two failure states from 1. Since the safe state does not have any damage event, the expected present value of the damage cost is zero dollars. Similarly to the first decision branch, the expected present value of the damage cost of each state is multiplied by the probability of occurrence of that state and this value of the each state is summed ($\sum_{i=1}^{3} P_{f,i}C_{d,i}$) and put in the first chance node of the second branch. This value is added with the present value of the construction cost, C_{op} to obtain the total cost, C_T of the second decision branch.

$$C_T = C_{op} + \sum_{i=1}^{3} P_{f,i}C_{d,i} \quad \text{4.23}$$

4.3.9 Summary of the Methodology

The different steps of the proposed methodology are summarized below:

1. Create a sophisticated numerical model of the structure, e.g.; a finite element model in OpenSees
2. Determine the target displacement, δ_t by the "coefficient method" using information of the NBCC 2005.
3. Characterize the input parameters as random variables.
4. Define a limit-state function to define the failure state.
5. The probability of exceedence of that limit state is computed by finite element reliability analysis.
6. Perform similar analyses for structures with different retrofit options.
7. Compute the expected present value of the damage cost (C_d) and the present value of the construction cost (C_{op}) of different retrofit options.
8. All the values are organized in a decision tree.
9. Compute the total cost (C_T) of each branch.

4.4 Conclusion

The decision tree provides a structured framework, where it is possible to present all the cost information associated with alternate retrofit strategy. Thus decision maker(s) can make their decision easily by choosing the decision branch or the retrofit system with the minimum total expected cost by comparing different branches of a decision tree. Besides cost information, the probability values associated with each decision are also presented so that decision maker(s) can take their decision according to their risk taking attitude. In the following chapter, a numerical example of the proposed method is provided.

Chapter 5.0 Numerical Example

5.1 Introduction

The objective of this chapter is to present an example to demonstrate the proposed methodology (Chapter 4) in retrofit selection applications. In the following paragraphs structural modeling, reliability analysis, cost analysis, decision tree formulation and cost data are descried. In the numerical example presented in this chapter three options are considered. The first case considers the original structure without any retrofitting. In the second case, the structure with bracing installed in one bay is considered. In the last case the structure is analyzed with bracing in two bays.

5.2 Structural Modeling

The building under consideration is a two-storey reinforced concrete structure. A 3D view of the structure with the bracings installed in the shorter direction is shown in Figure 5.1. The structure is modeled with displacement-based beam-column elements with fibre-discretized cross-sections in OpenSees. The dimensions and applied loads of the considered exterior frame are shown in Figure 5.2 and the node and element numbers are shown in Figure 5.2. A typical bay width is 7.31m and the first and the second floor heights are 4.57m and 3.66m respectively. The exterior column size is 610x610mm and the interior columns size is 686x610mm with a beam depth is 457mm and the beam width is 610mm. The vertical loads are modeled as deterministic. The mean value, coefficient of variation, correlation coefficient (c.c.) and distribution type of different material properties and structural dimensions applied to model the structural uncertainty are listed in Table 5.1. In the present example, it is assumed that the decision maker's derived damage state of interest is defined by the cracking of the concrete in the bottom part of the element 1 (in Figure 5.2). The life span of the structure, T_L is considered to be 100 years.

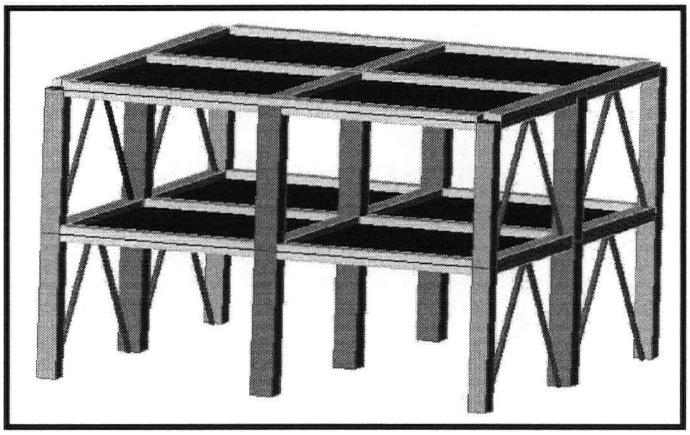

Figure 5. 1 A 3D view of the structure under consideration with the proposed bracings installed.

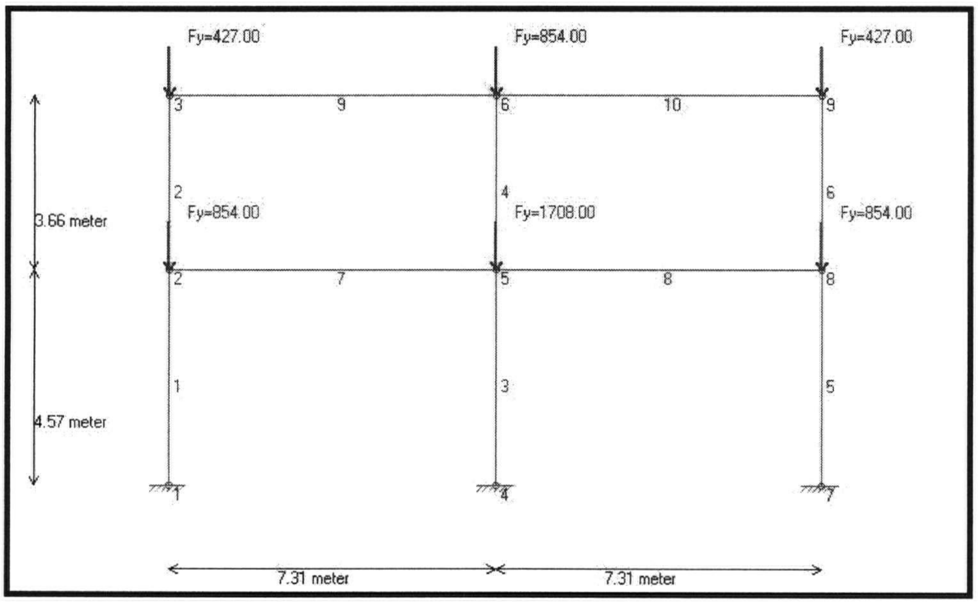

Figure 5. 2 The dimensions and applied vertical loads (in kN) in an exterior frame of the original structure.

Table 5. 1 The parameters of the random variables.

Material / Structure	Material Property	Mean Value	C.O.V.	c.c.	Distribution type
Cover Concrete	Compressive Strength, f'c	27850 (KPa)	0.10	0.3	Lognormal
	Strain at compressive Strength, epsco	0.002	0.10	0.3	Lognormal
	Strain at crushing Strength, epscu	0.006	0.10	0.3	Lognormal
Core Concrete	Compressive Strength, f'c	35854 (KPa)	0.10	0.3	Lognormal
	Strain at compressive Strength, epsco	0.005	0.10	0.3	Lognormal
	Crushing Strength, f'cu	32406.5 (KPa)	0.10	0.3	Lognormal
	Strain at crushing Strength, epscu	0.02	0.10	0.3	Lognormal
Steel	Yield Strength, fy	413700 (KPa)	0.10	0.3	Lognormal
	Initial Elastic Tangent, E	206850 (MPa)	0.05	0.3	Lognormal
Exterior columns	Height, h	610 (mm)	0.05	0.3	Lognormal

Material / Structure	Material Property	Mean Value	C.O.V.	c.c.	Distribution type
Exterior columns	Width, b	610 (mm)	0.05	0.3	Lognormal
	Clear Cover	50 (mm)	0.05	0.3	Lognormal
	Total steel area on side, As2	6000 (mm^2)	0.05	0.3	Lognormal
Interior columns	Height, h	686 (mm)	0.05	0.3	Lognormal
	Width, b	610 (mm)	0.05	0.3	Lognormal
	Clear Cover	50 (mm)	0.05	0.3	Lognormal
	Total steel area on side, As1	12000 (mm^2)	0.05	0.3	Lognormal
Beam	Height, h	457 (mm)	0.05	0.3	Lognormal
	Width, b	610 (mm)	0.05	0.3	Lognormal
	Total steel area on side, As3	4200 (mm^2)	0.05	0.3	Lognormal

5.3 Location and Site Specific Hazard Data

The location of the structure is Vancouver, British Columbia. The seismic hazard values for Vancouver, taken from [2], are listed in Table 5.2.

Table 5. 2 The median values of the spectral acceleration for Vancouver.

Location	**Median (50th percentile) values of spectral accelerations**			
	S_a (0.2s)	**S_a (0.5s)**	**S_a (1.0s)**	**S_a (2.0s)**
Vancouver	0.96	0.66	0.34	0.18

As mentioned previously, the above values are for ground motions that have 2% chance of exceedence in 50 year. Hence, the mean rate of occurrence, λ of earthquake is 0.000404 and the return period is 2475 years. The values of Table 5.2 will be applied to compute the target displacement, δ_t by the coefficient method.

5.4 Coefficients for Computation of Target Displacement

The following values of the various coefficients are employed while computing the target displacement according to Eq. (4.6): The value of C_0 is taken as 1.2, as the structure is a two-storey building (according section 4.3.5). The value of C_1 depends on the value of the characteristic period of the response spectrum, T_0, and the effective fundamental period, T_e. In the present case, the value of T_0 is 0.2s. For each case considered in this example, the effective fundamental period T_e is determined and compared with T_o to determine the value of C_1. Similar to the value of C_1, the value of C_2 is determined from [10] after computing the value of T_e. The value of C_3 depends on whether the structure has a positive post yield stiffness or negative post yield stiffness. Hence, the value of C_3 is determined after plotting the push-over curve for each of the three cases.

5.5 Expected Cost of the "No Retrofit" Decision

In case of "Decision 1" the original structure is analyzed without any retrofit measure. The goal is to determine the expected damage cost for the original structure for the specified damage state. For the computation of the target displacement, the push-over curve in Figure 5.3 is generated by employing the mean values of the input parameters.

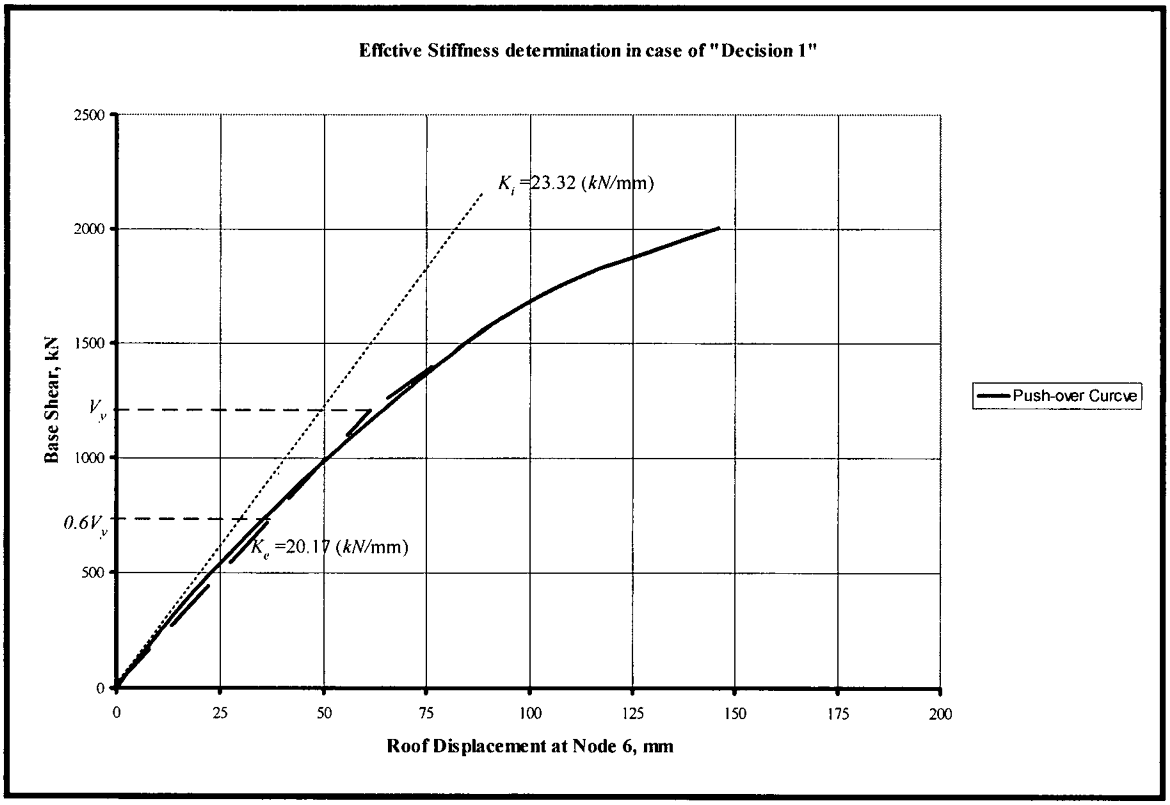

Figure 5. 3 The effective lateral stiffness determination in case of the "Decision 1."

From the graph in Figure 5.3, the value of the elastic lateral stiffness, K_i = 23.32 kN/mm and the value of effective lateral stiffness, K_e =20.17 kN/mm are determined. To determine the first natural frequency of vibration, the structure is idealized into a two-degree-of-freedom problem. The lateral displacement of each storey is one degree-of-freedom. The stiffness matrix and mass matrix is determined in a straightforward manner by the direct stiffness approach. Subsequently, the Mathcad® software is employed to determine the elastic fundamental period, T_i by eigenvalue analysis. The value of T_i is found to be 0.624s. From Eq. (4.7), the effective fundamental period is

$$T_e = 0.624\sqrt{\frac{23.32}{20.17}} = 0.67s \qquad 5.1$$

Since T_e (0.67s) is greater than T_o (0.2s), the value of C_1 is 1.0. Employing the values of T_e and T_o and from [10], the value of C_2 is found to be 1.2. From the graph of Figure 5.3, it is evident that the structure has a positive post yield stiffness. Hence, the value of C_3 is 1.0. Applying the value of T_e, the value of spectral acceleration, S_a is found to be 0.544g (from Table 5.2). Hence the value of the target displacement is

$$\delta_t = (1.2)(1.0)(1.2)(1.0)(0.544)(\frac{0.67^2}{4\pi^2})(9810) = 90mm \qquad 5.2$$

The limit-state function, employed for the evaluation of the probability of failure in FERA, is defined as: g = 90- $\delta_{failure}$. The $\delta_{failure}$ means the global displacement at which the concrete crack starts to appear in the bottom part of left most column (element 1) of the ground floor. There were a total of 104 random variables and after 5 iterations towards the design point in the FORM analysis the reliability index (β) is found to be 1.199. Hence the conditional probability of failure, $P(f|E)$ is 0.11527. In this example, it is assumed that the damage cost is $5,000,000.00 and the real interest rate is (i =) 4%. Therefore according to Eq. (4.13), the expected present value of the damage cost is

$$C_{d,1} = (\$5,000,000.00)\frac{0.000404}{0.000404 + 0.04} = \$49,995.05 \qquad 5.3$$

Incase of the no failure event, the expected present value of the damage cost is zero dollars. The probability of occurrence of an earthquake event with in the life span of the structure is (Eq. (4.16))

$$P(E) = \int_{0}^{100} (0.000404)e^{-0.000404t} dt = 0.04 \quad \text{.......... 5.4}$$

Hence, the probability of occurrence of the failure state is given by (Eq. (4.19))

$$P_{f,1} = (0.11527)(0.04) = 0.00461 \quad \text{.......... 5.5}$$

This result and the subsequent computations are summarized in Figure 5.8. The probability occurrence of the safe state, $P_{f,2}$ is (1-0.00461=) 0.99538. Hence, the total expected cost of the first branch is

$$\sum_{i=1}^{2} P_{f,i} C_{d,i} = (0.00461)(\$49{,}995.05) + (0.99538)(\$0.00) = \$230.48 \quad \text{.......... 5.6}$$

Since there is no other cost associated with the first decision, the total expected cost of the first decision (Decision 1) is (Eq. (4.20))

$$C_T = \$230.48 \quad \text{.......... 5.7}$$

5.6 Expected Cost of the Decision to Construct Bracings in One Bay

In case of the second decision "Decision 2," the steel bracings are installed in the first bay of each storey (shown in Figure 5.4). The truss element is employed to model the bracings in the structure. These truss elements have a mean area of 4354.8mm^2. Information employed to model the uncertainty of the truss element is provided in Table 5.3. The installation cost of the bracings is assumed to be \$5,000.00 and it is also assumed that it will take two years to install all the bracings.

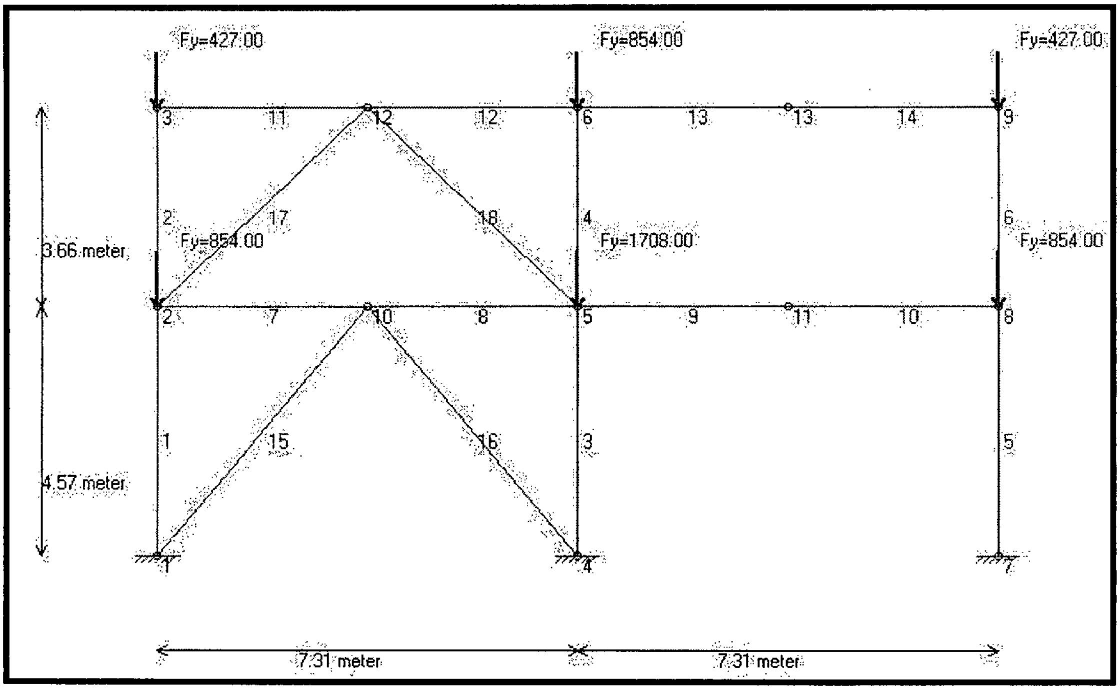

Figure 5. 4 A frame with bracings in one bay in case of the "Decision 2."

Table 5. 3 The Parameters of the bracings employed to model the uncertainty in case of the "Decision 2."

Structure	Material Property	Mean Value	C.O.V.	c.c.	Distribution type
Bracing	Yield Strength, fy	248220 (KPa)	0.10	0.3	Lognormal
	Initial Elastic Tangent, E	199955 (MPa)	0.05	0.3	Lognormal
	Area, As	4354.8 (mm^2)	0.05	0.3	Lognormal

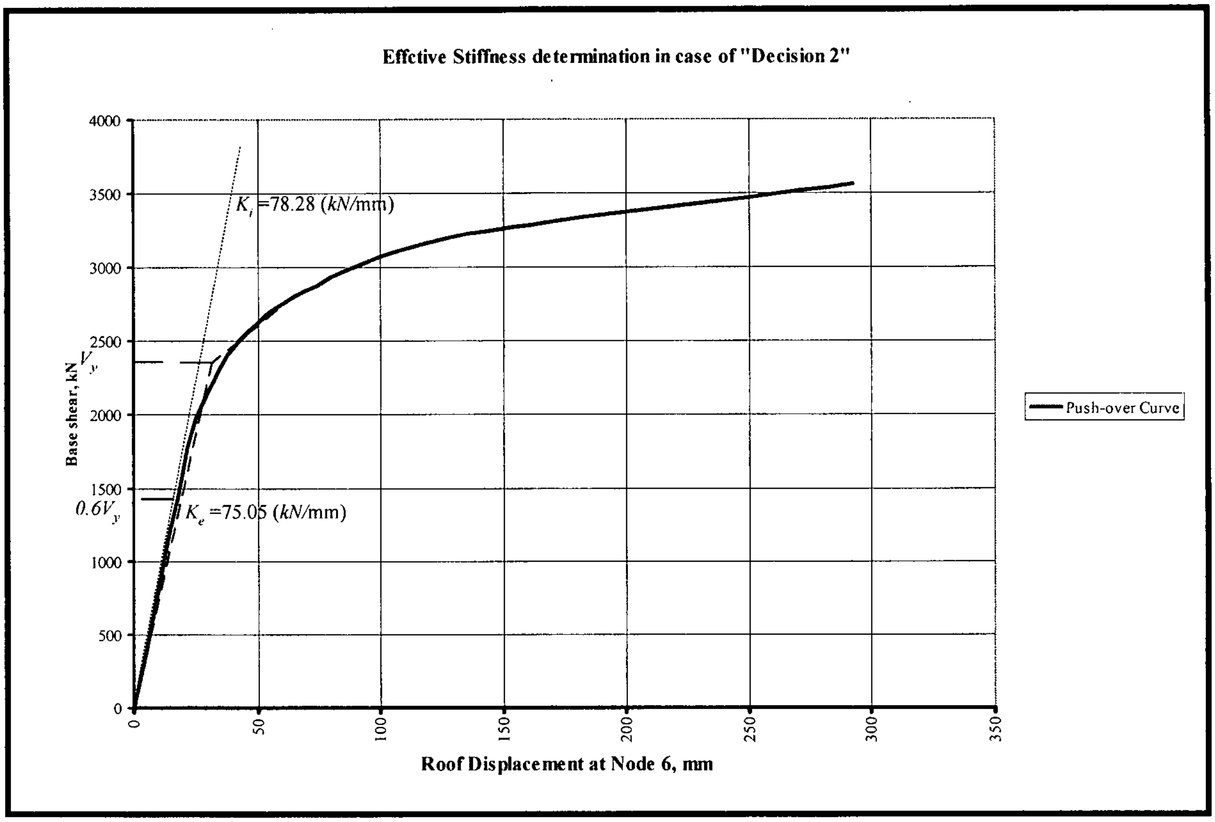

Figure 5. 5 The effective lateral stiffness determination in case of the "Decision 2."

From the graph of Figure 5.5, the value of elastic lateral stiffness, K_i = 78.28 kN/mm and the value of effective lateral stiffness, K_e = 75.05 kN/mm are determined. The value of T_i is found to be 0.478s. From Eq. (4.7), the effective fundamental period is

$$T_e = 0.478\sqrt{\frac{78.28}{75.05}} = 0.488s \qquad \text{5.8}$$

Since the value of T_e (0.488s) is greater than that of T_o (0.2s), the value of C_1 is 1.0. Applying the values of T_e and T_o and from [10], the value of C_2 is found to be 1.2. From the graph of Figure 5.5, it is clear that the structure has a positive post yield stiffness. Hence, the value of C_3 is 1.0. Employing the value of T_e, the value of spectral acceleration, S_a is found to be 0.672g (from Table 5.2). Therefore the value of the target displacement is

$$\delta_t = (1.2)(1.0)(1.2)(1.0)(0.672)(\frac{0.488^2}{4\pi^2})(9810) = 60.96mm \quad \text{.......... 5.9}$$

The limit-state function applied for the evaluation of the probability of failure is defined as: g = 60.96- $\delta_{failure}$. There were a total of 112 random variables and and after 7 iterations towards the design point in the FORM analysis, the reliability index (β) is found to be 1.2455. Hence the conditional probability of failure, $P(f|E)$ is 0.10648. The expected present value of the damage cost if earthquake occurs during the construction period of the retrofit system is will remain same as the future damage cost ($5,000,000.00). The probability of occurrence of an earthquake event during the construction of the retrofit system is (Eq. (4.21))

$$P(E) = \int_0^2 0.000404e^{-0.000404t} dt = 0.0008 \quad \text{.......... 5.10}$$

Hence, the probability of occurrence of the failure state during the construction period t_c is (Eq. (4.19)

$$P_{f,1} = (0.11527)(0.0008) = 0.000092 \quad \text{.......... 5.11}$$

The expected present value of the damage cost, $C_{d,2}$; if the earthquake occurs after the installation of the retrofit system is (Eq. (4.15))

$$C_{d,2} = \int_2^\infty (\$5,000,000.00)(0.000404)e^{-0.000404(t-2)}e^{-0.04t} dt = \$46,150.00 \quad \text{.......... 5.12}$$

The probability of occurrence of an earthquake event after the construction of the retrofit system is (Eq. (4.22))

$$P(E) = \int_2^{100} (0.000404)e^{-0.000404t} dt = 0.039 \quad \text{.......... 5.13}$$

Hence, the probability of occurrence of the failure state after the construction of the retrofit system is (Eq. (4.19)

$$P_{f,2} = (0.10648)(0.039) = 0.00415 \qquad (5.14)$$

The probability of occurrence of the no failure state of the second decision branch is (1-0.000092-0.00415=) 0.99576. Incase of the no failure event, the damage cost is zero dollars. The total expected present value of the damage cost of the second decision branch is

$$\sum_{i=1}^{3} P_{f,i}C_{d,i} = (0.000092)(\$5{,}000{,}000.00) + (0.00415)(\$46{,}150.00) + (0.99576)((\$0.00) = \$651.52 \qquad (5.15)$$

The present value of the construction cost is (Eq. (4.14))

$$C_{op} = \int_0^2 (\frac{\$5{,}000.0}{2})e^{-0.04t}dt = \$4805.00 \qquad (5.16)$$

The total cost of the "Decision 2" is

$$C_T = C_{op} + \sum_{i=1}^{3} P_{f,i}C_{d,i} = \$4805.00 + \$651.52 = \$5{,}456.52 \qquad (5.17)$$

5.7 Expected Cost of the Decision to Construct Bracings in Two Bays

In the third decision ("Decision 3"), the steel bracings are installed in both the bays of each storey. The size of the bracing is chosen 3226mm^2 (Figure 5.5). Information employed to model the uncertainty of the truss element is provided in Table 5.4. Installation cost is assumed to be \$10,000.00 and it is also assumed that it will take two years to install all the bracings.

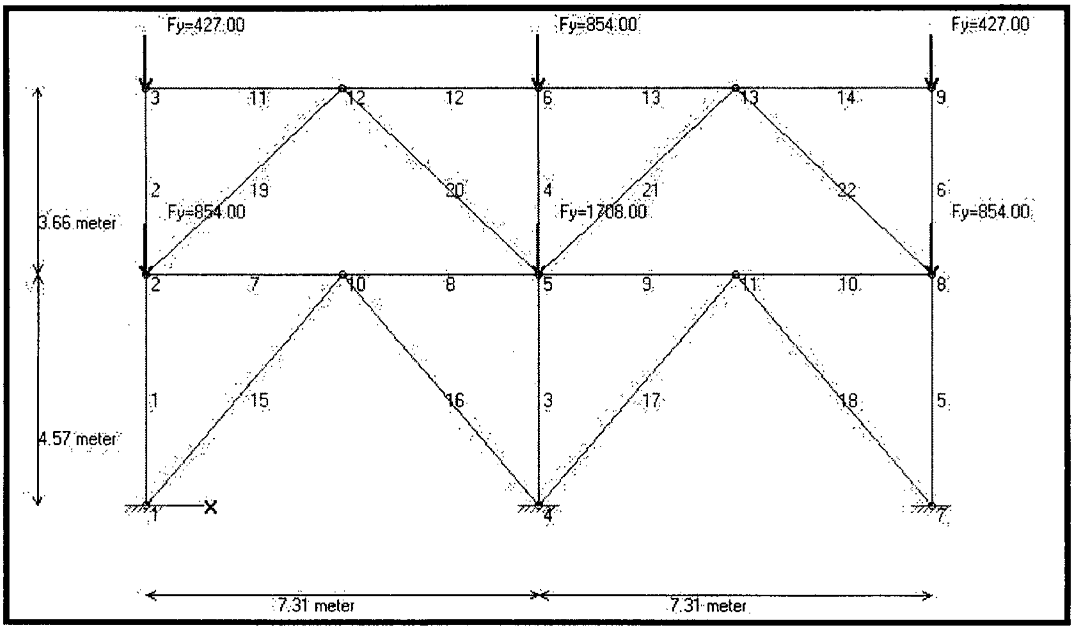

Figure 5.6 A Frame with bracings in two bays in case of the "Decision 3."

Table 5. 4 The Parameters of the bracings employed to model the uncertainty in case of the "Decision 3."

Structure	Material Property	Mean Value	C.O.V.	**c.c.**	Distribution type
Bracing	Yield Strength, fy	248220 (KPa)	0.10	0.3	Lognormal
	Initial Elastic Tangent, E	199955 (MPa)	0.05	0.3	Lognormal
	Area, As	3226 (mm^2)	0.05	0.3	Lognormal

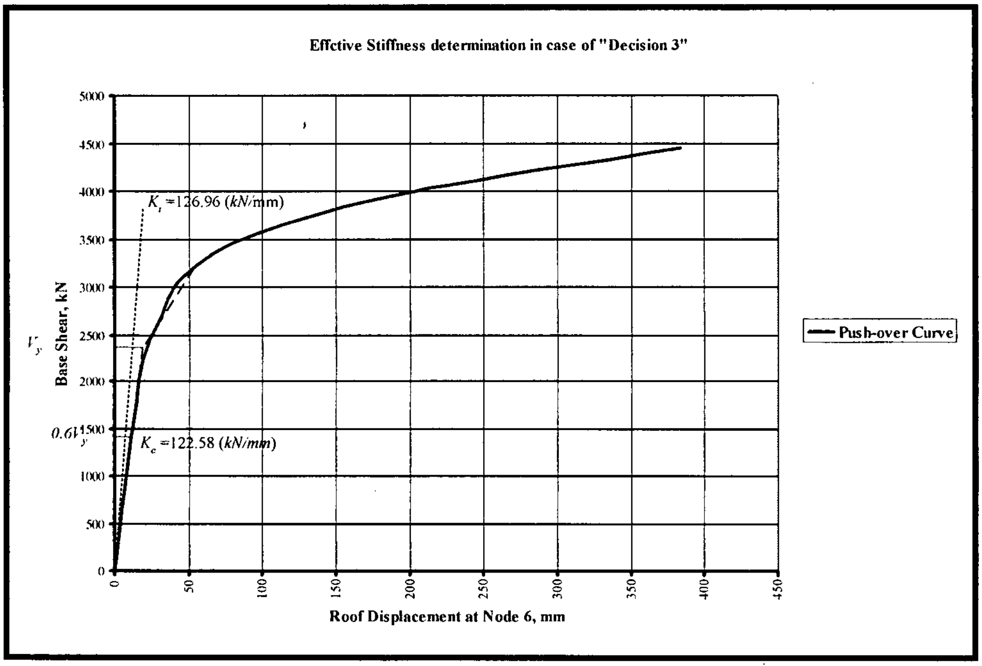

Figure 5. 7 The effective lateral stiffness determinations in case of the "Decision 3."

From the graph of Figure 5.7, the value of elastic lateral stiffness, K_i = 126.96 kN/mm and the value of effective lateral stiffness, K_e = 122.58 kN/mm are determined. The value of T_i is found to be 0.464s. From Eq. (4.7), the effective fundamental period is

$$T_e = 0.464\sqrt{\frac{126.96}{122.58}} = 0.472s \quad \text{.......... 5.18}$$

Since the value of T_e (0.472s) is greater than that of T_o (0.2s), the value of C_1 is 1.0. Employing the values of T_e and T_o and from [10], the value of C_2 is found to be 1.2. From the graph of Figure 5.7, it is clear that the structure has a positive post yield stiffness. Hence, the value of C_3 is 1.0. Employing the value of T_e, the value of spectral acceleration, S_a is found to be 0.688g (from Table 5.2). Therefore the value of the target displacement is

$$\delta_t = (1.2)(1.0)(1.2)(1.0)(0.688)(\frac{0.472^2}{4\pi^2})(9810) = 54.85mm \quad\text{5.19}$$

The limit-state function applied for the evaluation of the probability of failure is defined as: g = 54.85- $\delta_{failure}$. There were a total of 120 random variables and and after 13 iterations towards the design point in the FORM analysis, the reliability index (β) is found to be 2.6365. Hence the conditional probability of failure, $P(f|E)$ is 0.00419. The expected present value of the damage cost if earthquake occurs during the construction period of the retrofit system is will remain same as the future damage cost (\$5,000,000.00). The probability of occurrence of an earthquake event during the construction of the retrofit system is (Eq. (4.21))

$$P(E) = \int_0^2 0.000404e^{-0.000404t}dt = 0.0008 \quad\text{5.20}$$

Hence, the probability of occurrence of the failure state during the construction of the retrofit system is (Eq. (4.19)

$$P_{f,1} = (0.11527)(0.0008) = 0.000092 \quad\text{5.21}$$

The expected present value of the damage cost, $C_{d,2}$; if the earthquake occurs after the installation of the retrofit system is (Eq. (4.15))

$$C_{d,2} = \int_2^\infty (\$5,000,000.00)(0.000404)e^{-0.000404(t-2)}e^{-0.04t}dt = \$46,150.00 \quad\text{5.22}$$

The probability of occurrence of an earthquake event after the construction of the retrofit system is (Eq. (4.22))

$$P(E) = \int_2^{100} (0.000404)e^{-0.000404t}dt = 0.039 \quad\text{5.23}$$

Hence, the probability of occurrence of the failure state after the construction of the retrofit system is (Eq. (4.19)

$$P_{f,2} = (0.00419)(0.039) = 0.00016 \qquad 5.24$$

The probability of occurrence of the third state of the third decision branch is (1-0.000092-0.00016=) 0.99975. Incase of the third state, the damage cost is zero dollars. The total expected present value of the damage cost of the third decision branch is

$$\sum_{i=1}^{3} P_{f,i}C_{d,i} = (0.000092)(\$5{,}000{,}000.00) + (0.00016)(\$46{,}150.00) + (0.99975)((\$0.00) = \$467.38 \qquad 5.25$$

The present value of the construction cost is (Eq. (4.14))

$$C_{op} = \int_{0}^{2} (\frac{\$10{,}000.0}{2}) e^{-0.04t} dt = \$9{,}610.00 \qquad 5.26$$

The total cost of the "Decision 3" is

$$C_T = C_{op} + \sum_{i=1}^{3} P_{f,i}C_{d,i} = \$9.610.00 + \$467.38 = \$10{,}077.38 \qquad 5.27$$

5.8 Summary of Results in a Decision Tree

In the previous sections, the present values of different costs associated with each decision are calculated. These values are put in a decision tree format similar to that of Figure 4.7.

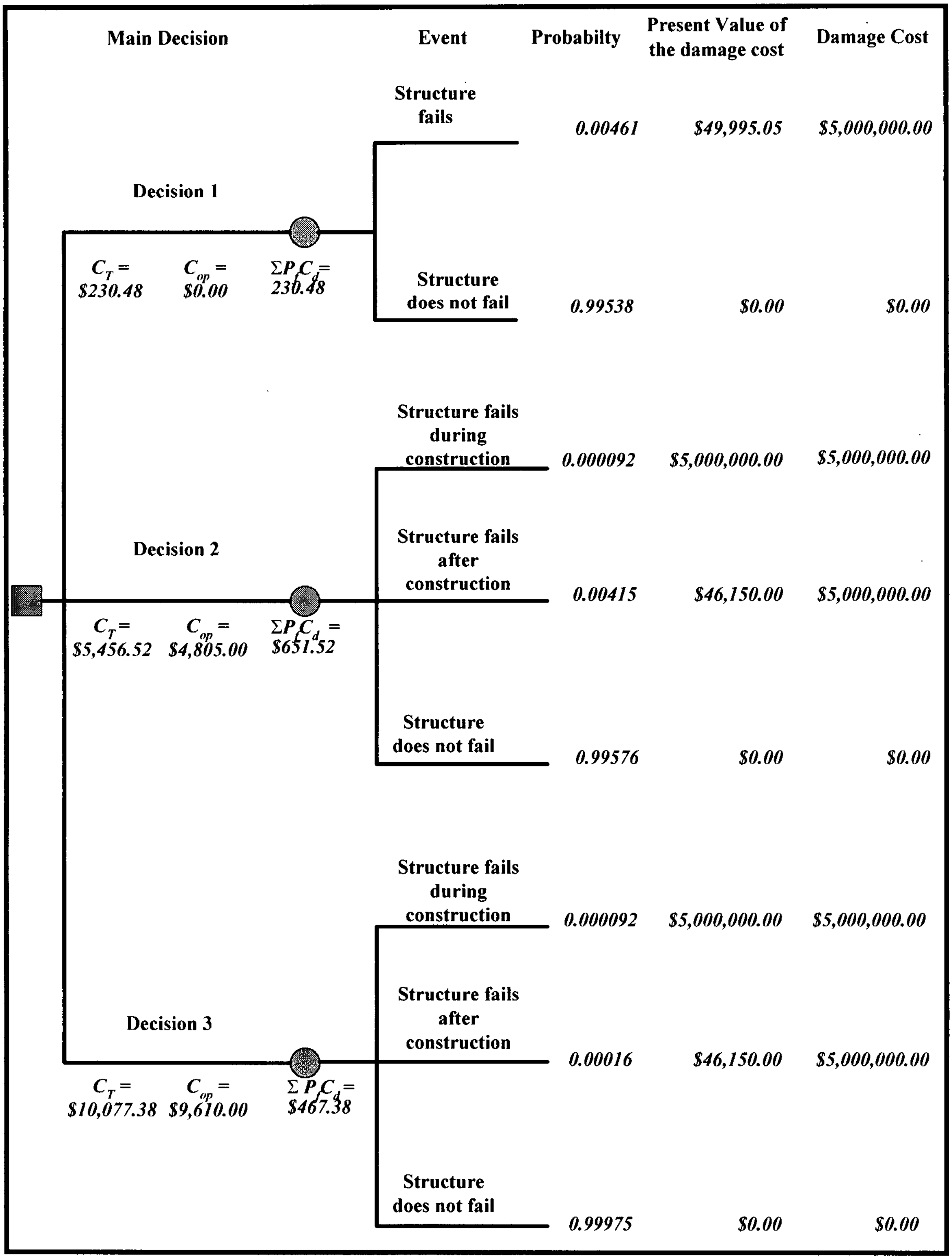

Figure 5. 8 A decision tree employed for the selection of the seismic retrofit strategy.

From the decision tree of Figure 5.8, it is evident that the "No retrofit" (Decision 1) decision is the most beneficial as it yields minimum total expected cost. However, there is an 11% chance that the failure event will occur if the damaging earthquake occurs during the life span of the structure and the expected damage cost will be $50,000.00. Although in case of "Decision 3" the installation cost is very high, the expected damage cost after the installation of the bracings in both of the bays is $46,150.00 and there is only 0.41% chance that it may happen if the damaging earthquake event takes place. Therefore, "Decision 3" might be a good option for the decision makers who cannot effort a big loss. Besides, the expected present value of the damage cost during the construction period is very high and it is strongly influenced by the total expected cost of the retrofit decisions. Since there is only 0.0092% chance that the damaging earthquake will occur during this short period and if the expected present value of the damage cost ($5,000,000.00) during this period is ignored then there will be a significant reduction in the expected present value of the damage cost (only (0.00016)($42,600.00) = $6.82) of the third decision (Decision 3) branch. There is also a factor need to be discussed, which is influencing the decision analysis. It is the significant reduction of the expected cost ($50,000.00) from the original damage cost of $5,000,000.00. This significant reduction is due to the very small value of λ (=0.000404) considered in this study. Since there is a very small chance that the damaging earthquake would occur during the life span of structure, the expected cost of the damage state is very small compared to the original damage cost.

5.9 Conclusion

From the numerical example presented in this chapter, the effectiveness of the proposed methodology is evident. Although only two retrofit options are considered in this example however, engineers can add more retrofit options for comparison. Another aspect of this method is to represent each decision in terms of money rather than the engineering parameters. From decision makers' point of view, by this method they can define their damage cost and then can compare the expected damage cost associated with their investments on the retrofit system. Overall, both the engineers and the decision makers can support their decision with logic on each step by adopting this method rather than making decision from experience.

Chapter 6.0 Conclusion

This thesis proposes a practical application of the finite element reliability analysis and decision tree analysis in the field of retrofit engineering. From the proposed methodology, the following conclusions have been drawn:

- The proposed methodology utilizes available information from the Canadian Code (NBCC 2005) which enables the practicing engineers to easily apply the proposed methodology while selecting between alternate retrofit strategies.
- In the proposed methodology, it is possible to define the damage states according to the need of the owner(s) which provides more flexibility to the owner(s) while choosing among different retrofit options.
- In this thesis, the state of the art finite element reliability analysis is employed to determine the failure probability of a structure which accounts for the uncertainties in the structural and material behaviour.
- The proposed method utilizes OpenSees for FERA. This is a free and upgradeable software which can be modified by the engineers according to their needs. New and updated elements and commands are available on the internet so that engineers can model and analysis different types of structures and define the damage state according to the client's need.
- The consequence of each retrofit option is presented in terms of monetary value. The probability of occurrence of different events is also provided with the expected cost of damage in a tree format. Therefore, owner(s) can compare

different loss scenario due to an earthquake event of the structure with and without different retrofit options.

6.1 Scope of Future Studies

The proposed methodology provides a link between the finite element reliability analysis and decision tree analysis. However, the finite element reliability analysis is itself an expanding subject. Hence with the advancement of the finite element reliability analysis, it is believed that the proposed methodology will also be modified. Few shortcomings of the proposed method were identified which are briefly discussed in the following paragraphs:

- If the limit-state definition gets more complicated i.e. if the failure state involves failure of more than one element or more than one component of an element (concrete failure, steel failure etc.) then system reliability analysis need to be performed. However, the system reliability analysis requires lots of computational time and the FORM analysis may not converge in some cases.
- The degradation of concrete with time is not taken into consideration. Concrete of building structures typically does not deteriorate as much as the concrete of bridge structures that are located in the regions with cold weather conditions. This is due to the salts applied for melting the ice in those regions. However, it is emphasized that if the building is for some reaons subjected to a corrosive environment (salt water, high sulfar in air, etc.) the degradation of concrete with time might be a critical factor that should be taken into account.

- The time dependent stresses (creep and shrinkage) are also not considered in the analysis. However, in case of the heavy concrete structures this might be an issue to consider.
- During the cost analysis, only the damage cost and cost of construction of the retrofit system are considered. However, the real world cost analysis will be more complicated; involving the cost functions generated from different sources such as the revenue earned, the insurance premium paid, the insurance recovery, the salvage value of the structure etc. In case of the damage cost, only the damage cost incurred in a damage state is considered. However, in some damage states, the loss from other sources such as the loss from the human casualties, the loss from the damage of the relevant properties, the loss due to the closure of the businesses etc need to be considered.

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Appendices

Appendix A

Distribution	PDF or PMF	Parameters	Relation to mean and variance
Binomial	$p(x) = \binom{n}{x} p^x (1-p)^{n-x}$ $x = 0,1,2,.........,n$	p	$E(X) = np$ $Var(X) = np(1-p)$
Geometric	$p(x) = p(1-p)^{x-1}$ $x = 0,1,2,........$	p	$E(X) = 1/p$ $Var(X) = (1-p)/p^2$
Poisson	$p(x) = \frac{(\lambda t)^x}{x!} e^{-\lambda t}$ $x = 0,1,2,.......$	λ	$E(X) = \lambda t$ $Var(X) = \lambda t$
Exponential	$f(x) = \lambda e^{-\lambda x}$ $x \geq 0$	p	$E(X) = 1/p$ $Var(X) = (1-p)/p^2$
Gamma	$f(x) = \frac{\lambda(\lambda t)^{k-1}}{\Gamma(k)} e^{-\lambda t}$ $x \geq 0$	λ, k	$E(X) = k/\lambda$ $Var(X) = k/\lambda^2$
Normal (Gaussian)	$f(x) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$ $-\infty < x < \infty$	μ, σ	$E(X) = \mu$ $Var(X) = \sigma^2$

Distribution	PDF or PMF	Parameters	Relation to mean and variance
Lognormal	$f(x)=\frac{1}{\sqrt{2\pi}\xi x}\exp\left[-\frac{1}{2}\left(\frac{\ln x-\lambda}{\xi}\right)^2\right]$ $x\geq 0$	λ, ζ	$E(X)=\exp\left(\lambda+\frac{1}{2}\xi^2\right)$ $Var(X)=E^2(X)[e^{\xi^2}-1]$
Rayleigh	$f(x)=\frac{x}{\alpha^2}\exp\left[-\frac{1}{2}\left(\frac{x}{\alpha}\right)^2\right]$ $x\geq 0$	α	$E(X)=\sqrt{\frac{\pi}{2}}\alpha$ $Var(X)=\left(2-\frac{\pi}{2}\right)\alpha^2$
Uniform	$f(x)=\frac{1}{b-a}$ $a<x<b$	a, b	$E(X)=\frac{a+b}{2}$ $Var(X)=\frac{1}{12}(b-a)^2$
Triangular	$f(x)=\frac{2}{b-a}\left(\frac{x-a}{u-a}\right)$ $a\leq x\leq u$ $f(x)=\frac{2}{b-a}\left(\frac{b-x}{b-u}\right)$ $u\leq x\leq b$	a, b, u	$E(X)=\frac{1}{3}(a+b+u)$ $\mathrm{Var}(X)=\frac{1}{12}(a^2+b^2+u^2$ $-ab-au-bu)$
Beta	$f(x)=\frac{1}{B(q,r)}\frac{(x-a)^{q-1}(b-x)^{r-1}}{(b-a)^{q+r-1}}$ $a\leq x\leq b$	a, b, q, r	$E(X)=a+\frac{q}{q+r}(b-a)$ $\mathrm{Var}(X)=\frac{qr}{(q+r)^2(q+r+1)}$