PROBABILISTIC CALIBRATION OF HIGHWAY GEOMETRIC DESIGN:
THEORETICAL ISSUES AND APPLICATIONS

by

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ABSTRACT

Road safety is an evolving area of research that underpins the continuous attempt to build safe and cost-efficient roads. Despite the considerable growth of the road safety literature, some concern remains regarding the safety level associated with standard geometric design models. The central concern is that the level of safety delivered by design standards is implicit and largely unknown. Accordingly, it remains difficult to form a judgment regarding the acceptability and the consistency of the safety level built in design standards. In order to account for the previous concern, road safety is quantified in the present practice by explicitly developed analytical tools or safety evaluation models. Those analytical tools, to some extent, guide the designer in investigating the safety consequences of different dimensioning scenarios for a highway design. However, in order to elicit and examine the implicit safety level in design standards, a quantifiable measure must be used to assess road safety. A parallel system is formulated for the assessment of safety levels in standard design outputs by tracing the propagation of uncertainty. Uncertainty can be incorporated into the design process through a probabilistic framework. Reliability theory, a subset of probability theory, offers a rational foundation for calculating the propagation of uncertainty throughout the design process. The main proposition that underlies the analysis presented in this thesis is that design safety level associated with standard design outputs should be consistent and close to a premeditated level. Several discussions are presented that suggest different methods of selecting a target design safety level. A general framework for calibrating standard design models is presented in accordance with the previous preposition. The thesis contains an application of the calibration framework to the standard design model of crest vertical curves. In order to study the effect of model uncertainty, represented by the combination of horizontal and vertical curves, a new sight distance model is formulated that enables the calculation of available sight distance in three-dimensional environment. The sight distance model is further augmented to the process of reliability analysis. The calibrated design charts are constructed in order to yield consistent design safety level.
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DEDICATION

To my father Samir
1 INTRODUCTION

This chapter contains three sections. The first section presents background information on the importance of road safety research and the current state of knowledge in road design. The second section discusses the statement of the research problem. The third section describes the thesis structure.

1.1 BACKGROUND

This first part of this section describes the state of knowledge regarding road safety and the practical significance of safety research. The second part discusses the significance of integrating the concept of risk into highway design and the usefulness of this approach in cost-effectiveness studies. The third part discusses general shortcomings of the current standardized geometric design. The fourth part introduces the concept of design safety and discusses its distinction from the current concept of safety.

1.1.1 The Importance of Road Safety Research

With the advent of the motorization age and the construction of high-speed highways, transportation practitioners realized that road collisions constitute an unavoidable liability to highway operation. The first recorded collision fatality in 1896 brought in light the importance of addressing collision risk, hence underlying the emergence of traffic safety as a discrete field of practice and research. The frequency and intensity of vehicle collisions were further amplified by the continuous increase in the number of operating vehicles as well as the public desire for high-mobility roads. Aside from the considerable cost that results from collision fatalities, the importance of road safety research is further signified by the relatively large capital cost of highway projects.
1.1.1.1 The Problem of Road Collisions

The world-wide impact of road collisions on human lives is generally large. The World Health Organization ranked road collisions as the leading cause of mortality due to injury, accounting for 22.8% of the global total (WHO, 2002). In the same report, traffic injuries were the ninth leading cause of disability-adjusted pre-mature death, accounting for 2.6% of the global total. Overall, the global incidence of road collision fatalities is estimated to be 1.2 million per year, placing road collisions at the eleventh leading cause of death in 2002, with the potential of climbing the list in the near future (WHO, 2002).

In recognition of the importance of road safety, the transportation authorities in Canada compile an annual statistics of road collisions. The total number of fatal collisions from 1985 to 2004 is 60,000, claiming approximately 65,000 victims, while the total number of injuries was 4.85 million within the same period (Transport Canada, 2004). The estimated annual cost of road collisions is $25 billion. It is noticeable however that the annual number of collision is steadily declining, a testimony to the effectiveness of the road safety improvement programs implemented in Canada. More locally, the annual financial cost of traffic collisions in British Columbia is relatively high due to the high estimates of fatalities cost. In 2004, the total number of road collision fatalities was 430 along with 78,000 injuries (Insurance Corporation of British Columbia, 2004). Based on a willing-to-pay economic model, fatality cost in British Columbia was estimated to be $4,170,000, injury cost was $97,000, and property-damage-only cost was $6,000 (Miller, 1992). The total toll amounted to $3.8 billion in 2004 (ICBC, 2004). It is worth noting that the decline in police reported collisions in British Columbia was a major factor in sustaining the annual decline in the national road collisions (Transport Canada, 2004).

In light of the previous statistics, the economic and societal consequences of road collisions are significant. In keeping with their domestic responsibilities, the highway authorities in Canada recognize the evolving need of a sound engineering approach to address road safety. The Canadian Council of Motors Transport Administrations (CCMTA) has embraced a set of actions that will strive to make roads in Canada one of the safest in the world by 2010. In order to realize this goal, the transportation authorities
in Canada adopted a package of initiatives that targets road safety. The initiatives cover several aspects that intersect with road safety operation, e.g. developing driver awareness programs, devising a new means of enforcement, and motivating road safety research.

1.1.1.2 Capital Investments in Highway Projects

The relatively large value of the capital investments allocated for highway projects brings in focus the cost-effectiveness of road design. With the longest worldwide road network, the Federal Highway Administration estimated that the United States will need to spend about $76 billion annually until 2020 to maintain the current conditions of the highway system, and $107 billion in order to efficiently improve the highway system (GAO, 2003). In British Columbia, the Sea-to-Sky highway improvement project is an example of the relatively large-scale investments allocated for highway projects. The estimated cost of the project is $600 million (2005 estimate), for a total length of 99 km. Based on the scale of the project, basic decisions during the geometric design phase, such as median width or roadside clearance width, can result in a considerable cost in the form of earth work or additional pavement materials. In pursuit of an efficient use of public funding and satisfying tax-payers as well as stakeholder, the decision maker is obliged to ensure the cost-effectiveness of the project.

Due to the evident inelasticity of highway geometric design toward capital investments, the designer is in a continuous struggle to cut construction cost, reduce the expected number of collisions, and justify expenditures to stakeholders. An engineering approach for road safety is based on the development of predictive tools that can assess the safety level of a proposed design and evaluate the implications of modifying any roadway or roadside features. Without the necessary engineering implements, the study of the cost-effectiveness of a proposed highway design can be decidedly challenging. The new approach proposed in this thesis involves a parallel measure for road safety that may prove particularly helpful in regulating and guiding the geometric design decisions.
1.1.2 The State of Knowledge in Road Safety

The current practice of road and traffic safety falls into two main approaches: reactive approach and proactive approach (De Leur & Sayed, 2003). The reactive approach to safety consists of the necessary assessment tools and statistical models for studying and analyzing the observed collision records of existing road segments and intersections. The main tasks undertaken through the reactive approach to safety are the identification and rectification of hazardous locations. The general objective of the reactive approach is to retrofit hazardous road-network locations in order to reduce the number of collisions through applying the adequate safety measures in a cost-effective fashion. The reactive approach is researched and developed to a satisfactory degree that is reflected by the relatively wide availability of statistical models and case studies that follow this approach to safety. Despite its level of development, the reactive approach is not instrumental to a road safety analysis that takes place at the planning or design stages.

The objective of using the proactive approach to safety is to provide the necessary assessment tools and statistical models for evaluating safety level that is associated with future designs. This package of engineering tools can be applied through the planning, the evaluation of alternatives, or the design phases of any project. The proactive approach to safety parallels the inductive scheme of engineering analysis, through which the engineer uses limited observations of safety performance obtained from current design cases in order to draw conclusions about similar or future cases. Due to its inductive nature, the proactive approach to safety requires a sizeable body of data and faces specific statistical issues related to model developing and application.

The proactive approach to road safety is embodied in the development of collision prediction models. The collision prediction models are statistical models that construct a relationship between a road safety indicator and a group of descriptive variables. A widely accepted road safety indicator is collision frequency (Hauer, 1995). The selection of the explanatory, or independent, variables need not necessarily be based on a causational relationship with collisions. In fact, the identification of the exact causes of road collisions is a challenging undertaking and often includes a certain degree of
uncertainty (Sayed & Navin, 1995). In practice, the significance of including an explanatory variable is largely statistical and follows the conventional rules of hypothesis testing. The main product of the proactive safety studies are collision prediction models that functions as statistical prediction models with the aim of assessing road safety level of a proposed highway design.

1.1.3 Historical Outlook on Road Safety Research

Unlike the majority of civil engineering disciplines, the development of safety research has not strictly followed the gradual scheme of knowledge accretion through experimental and theoretical development. The progress in safety research has been marked by times when the state of knowledge fell considerably short of delivering on some practical requirements. These occurrences redirected the progress of research in a corrective manner driven by the comments and suggestions raised by practitioners and researchers. This observed pattern of development draws mainly on the following examples.

In an inquest into the causes of the first recorded fatal collision in 1896, the coroner included the following remark in his report, “this should never happen again”. This minor comment reflects two major shortcomings of the road safety knowledge at that time. First, apparently, the coroner and the accompanying experts were surprised by the severity of the collision which contradicted what they had presumed regarding road operation: namely, that a collision should not lead to fatalities. Second, they believed that the occurrence of collisions is entirely preventable and hence, unrealistically, recommended a complete cessation of this occurrence in the future. It is currently believed that collision risk can be reduced, but cannot be eliminated (Hauer, 1998). Moreover, the occurrence of collisions is closely related to an integral characteristic of human behavior, which is fallibility. The presumption made that roads are, or can be, collision-free is analogous to believing that buildings are designed with a null probability of collapse. The latter would be an anomaly or an outdated belief within the realm of structural safety at that time.
Throughout many years in the twentieth century, the practice of road safety hovered around safety auditing and the use of road-side installations. The safety evaluation and safety assessment practice was subjective in the essence. In continuation, the highway geometric design standards were based on a set of holistic models that were supposed to address safety, driver comfort, cost-effectiveness, aesthetics, uncertainty in design parameters, etc. This resulted in widely accepted limiting design codes which provide limiting values of some design parameters, that once unsurpassed, yield safe and acceptable design output. This implicit and un-quantifiable safety level was, deservedly, subject to considerable criticism that questioned the fundamentals of the current geometric design practice (Hauer, 1988; Hauer, 1998). A remarkable example of the disparity between practical requirements and the state of knowledge is the conclusion provided in a report prepared for evaluating the safety levels of US roads, “the scientific engineering research necessary to answer these questions is quite limited, sometimes contradictory and often insufficient to establish firm and scientifically defensible relationships” (Hauer, 1988).

Another example of the shortcomings of the state of knowledge emerged during the safety assessment of Highway 407 project in Toronto, Ontario (Hauer, 1998). The project was reviewed by a separate committee in order to ensure its adherence to acceptable safety requirements. Unexpectedly, the committee activity was hindered by the lack of reliable safety models and concluded that, “The level of safety that materializes is largely unpremeditated and decisions about cost are made without reference to safety consequences”. Their recommendation was, “Road design should be more safety-conscious and more knowledge based” (Hauer, 1998). In a recent assessment of the road safety practice the following conclusion was reached, “... it becomes evident that explicit consideration of road safety issues and concerns is sadly lacking” (De Leur & Sayed, 2003). It is a humbling reality that the current design standards has not succeeded in delivering an acceptable or quantifiable safety level over almost one century of research and practice.
Road safety is one of a few civil engineering disciplines in that its critical literature commonly touches the fundamentals and basic concepts of the field. Haight (1988) posed a critical perspective against the entire practice of road safety improvement at that time, "Many of us have heard demands that we 'do something', but it is only recently that there have been suggestions that we should 'know what we are doing' before we begin to do it". The statement is a representative example of the previously described pattern of research development.

The literature contains several papers that provided critical discussions of some fundamental concepts of road safety that intersect with other disciplines. For example, Hauer (2004) provided a critical discussion of some of the basic concepts of statistical modeling in traffic safety. The literature contains several studies that questioned the practical benefits of safety improvement programs based on the principle of risk compensation (Farmer & Chambers, 1929; Farmer & Chambers, 1939; Koonstra, 1973).

### 1.1.4 Design Safety and Operation Safety

The widely accepted highway geometric design standards in North America are produced by AASHTO (American Association of State Highway and Transportation Officials) and TAC (Transportation Association of Canada). The process of highway geometric design includes the detailed spatial alignment and dimensioning of all the roadway and roadside features with the objective of providing safe, comfortable, and aesthetically pleasing driving conditions in a cost-effective fashion. The design models require information from design engineers in respect to the proposed road characteristics, the anticipated driver behavior, and the expected operating vehicles. The design models are based on an idealized mathematical description of the more complex driving conditions. This idealization of real functioning conditions is a core element in the process of civil engineering design that covers several disciplines besides highway design. The goal of idealizing the driving conditions is to provide time-saving and reliable design models for office engineers that help in maintaining a consistent and safe design outputs. The outputs
of highway geometric design are borderline geometric dimensions that ensure satisfactory driving conditions.

Highway geometric design has evolved since the early twentieth century in order to meet the requirements laid out by researchers and practitioners. On the course of development, there existed key requirements and principles that drove the geometric design practice to its current state of knowledge. For example, Leisch (1972) proposed the concept of dynamic design for safety, where the highway design addresses the requirements of drivers and vehicles under feasible range of operating conditions. Leisch (1977) proposed the integration of the concept of operating speed into the design standards as a more representative and realistic measure of mobility. Glennon and Harwood (1983) emphasized the necessity of considering driver's expectation in the dimensioning of road geometric elements. Lamm et al. (1988) contributed to the area of design consistency and the aesthetics of the driving environment. Jackson (1987) proposed an explicit and quantitative integration of safety into the design process.

Despite the evolving improvements in the practice of highway geometric design, several concerns remain regarding the state-of-the-art design standards. For example, the design models and the suggested design values for some parameters were often considered conservative and posed an obstacle to the cost-effectiveness of highway projects (Nusbaum, 1985). The outputs of the design models are boundary values with little or no investigation into the consequences of dimensioning roads beyond these limiting standards (Trietsch, 1987). For cases that are considerably different from standards, the current design practice does not offer a rational basis that assists the designer in exercise his judgment (Crowell, 1988). The safety level built into the design standards is implicit and largely unknown (Hauer, 1988). Navin (1990) raised key issues regarding the unavoidable randomness in all design variables and suggested integrating reliability theory into the design process. In fact, the latter issue is one of the main motivations behind the work presented in this thesis. While stepping through formulating a theoretical framework that accounts for uncertainty, it was found that the process involves a distinct source of risk that differs from collision risk. This source of risk influences the
confidence, or safety, level associated with the functionality of the design output within an acceptable range of operation. The term "design safety" refers to the previous safety level and will be used in this thesis as a distinction from the conventional interpretation of road safety - that is collision risk.

1.1.4.1 Sources of Uncertainty

All quantities, except physical and mathematical constants, used in engineering calculations involve some degree of uncertainty (Thoft-Christensen & Sorensen, 1982). The design process of highway projects progresses through different phases starting from reconnaissance and surveying works, and ending with detailed design. Each phase includes a group of engineering decisions that are taken under various degrees of uncertainties. Each source of uncertainty involved in the design process reduces the aggregate design safety level (ADSL) of the final design output. The potential sources of uncertainties in the field of civil engineering design are classified as: phenomenological, decisional, modeling, prediction, physical, statistical and human (Melchers, 1999).

Phenomenological uncertainty arises from novel or unorthodox designs that lie beyond the classical domain of design practice. Decision uncertainty arises from the process of judging the occurrence of an unfavorable design output, e.g. deficient sight distance. In some design tasks, the borderline between acceptable and unacceptable operating conditions is not clearly defined, e.g. collision frequency. Accordingly, the decision that specific operating conditions are unacceptable represents a distinct source of uncertainly. Modeling uncertainty arises from the idealization of actual operating conditions of an engineering system by a set of mathematical models. Prediction uncertainty depends on the completeness of the state of knowledge available to the designer that influences that design output. Physical uncertainty reflects the inherent randomness of the design parameters that can be reduced by additional information. For example, the randomness in perception and brake reaction time (PRT) is a physical uncertainty that is quantified by repeatedly observing its value for different drivers. Statistical uncertainty reflects the randomness of the statistical estimators, mean and higher moments. This is addressed by
treated these estimators as random variables. Human uncertainty accounts for the gross errors committed by the driver, which are unlikely to be offset by the road or the vehicle. Due to the practical limitation of breaking down collision sources and the unpredictable nature of this type of uncertainty, it is difficult at the current engineering knowledge to quantify this source of uncertainty in highway geometric design. The scope of the analysis in this thesis is limited to physical uncertainty.

1.1.4.2 Design Safety in the Current Design Practice

Although the uncertainty involved in highway geometric design is largely unavoidable, it can be quantified by means of reliability theory. Consequently, the calculated ADSL can be regulated by means of code-calibration in order to drive it closer to a premeditated and acceptable level. The integration of reliability theory into the code-calibration process is a state-of-the-art technique to address design uncertainty in several realms of civil engineering. In comparison, the current standard practice of highway geometric design does not account for uncertainty in design parameters through a scientifically defensible approach. As a result, some shortcomings can be observed in the current practice of code development:

1. The choice of the design values in the code stems largely from judgment rather than a technically defensible framework.

The choice of design values for various design parameters depends on the statistical distribution that describes their randomness. For example, the design value of Perception and Brake Reaction Time (PRT) is selected in AASHTO Green Book 2001 as 2.5 seconds. For it covers 98% of the drivers population, the design value is considered acceptable (Lerner, 1995). The value of PRT is used in the design of various highway elements: horizontal curves, vertical curves, and at-grade intersections. Although a common parameter, PRT is not consistently the most influential parameter regarding ADSL. For the case of a crest vertical curve, the most influential parameter, according to reliability theory, is the deceleration rate of the stopping vehicle. The percentile value of the design deceleration rate is 90th which is lower than PRT. The percentile value of
operating speed is commonly accepted as 85th, although operating speed is the second influential design parameter. It appears that the design values are selected at inconsistent percentile rankings and the difference cannot be explained by their relative influence on ADSL.

2. The values of ADSL are inconsistent and not close to a premeditated level.

The technical details of the calculation of ADSL values and the scatter of ADSL values regarding crest vertical curves are demonstrated in the coming chapters. Based on that analysis, it becomes evident that ADSL values obtained from the current design model possess considerable range of variation that spans approximately 60%. Theoretically, the operating conditions of the design outputs should pose a consistent risk level, otherwise the design models can be considered biased toward a particular group of design cases. In addition, the risk levels should be close to a target level that defines that boundary between underdesign and overdesign. It becomes necessary to calibrate the design models in order to yield design outputs of consistent and acceptable risk level as a means of ensuring cost-effectiveness and safety. Without a code-calibration process, the design safety associated with the design outputs will remain random and the overall confidence in the design output unpredictable. Moreover, the confidence in the highway geometric design is likely to be different from that associated with other engineering tasks in the same highway project, e.g. bridge structures, hydrologic facilities, pavement system, etc.

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1 ADSL can theoretically take values from 0% to 100%. The obtained values of ADSL ranged from approximately 40% to 100%.
Another issue that will be addressed is the feasibility of selecting conservative percentile values for design parameters. In spite of the relatively conservative percentile ranking of the input design parameters, ADSL can take significantly lower percentiles. For the case of crest vertical curves of 180 m length and 7% algebraic difference, ADSL is 63%. This is largely due to the difference between operating speed and design speed. Although some percentile rankings are conservative, the overall inconsistency in the percentile rankings and the absence of a technical basis for the selection process can considerably impact design outputs.

1.2 STATEMENT OF THE RESEARCH PROBLEM

It was demonstrated earlier that the design and the construction of highways involve relatively large capital investments and pose a costly risk to society through the occurrence of collisions. It was shown that the current design practice faces several challenges regarding the current method of handling uncertainties in the input design parameters. The shortcomings of the current highway geometric design are manifested by the inconsistent uncertainty level of numerous design cases analyzed in the course of this thesis. The code-calibration approach presented in this thesis aims at redressing the demonstrated shortcomings through regulating the design safety level associated with a feasible domain of design cases. The next section describes three problems addressed in the course of developing a framework for performing code-calibration of highway geometric design models. The main geometric element studied in this thesis is the design of crest vertical curves. The analytical approach applied to crest vertical curves can readily be applied to other geometric elements.

1.2.1 Developing an Improved Reliability Model for Available Sight Distance

1. Building an improved three-dimensional sight distance model

The currently available design model for crest vertical curves is developed in two-dimensional space and accordingly cannot be readily used for combined vertical and
horizontal curves. In addition, the design model does not consider vertical curvature in calculating the required stopping sight distance. The reliability models developed in the past for stopping sight distance do not consider the combined effect of vertical and horizontal curvatures. Due to the dependence of the current reliability methods on iteration and numerical differentiation, the three-dimensional sight distance models available in the literature cannot be readily integrated with available reliability models. The need for a more computationally effective three-dimensional sight distance model lead to the development of a new model.

*The first goal of this thesis is to develop an improved three-dimensional sight distance model.*

2. Building an improved sight distance reliability model

It is required to perform the code-calibration process within a feasible domain of design cases. This domain includes the combination of a horizontal curve and the designed vertical curve. There are practically unlimited configurations to combine the designed vertical curve with horizontally curved elements. As a result, only the most critical combination of horizontal and vertical curves is considered in the calibration process. Accordingly, the calibrated model can be used regardless of the coexisting horizontal curvature, although still operating in two-dimensional space.

After the most critical horizontal curvature is identified, the sight distance model can be integrated with an available reliability algorithm to ultimately calculate ADSL of the designed vertical curve.

*The second goal of this thesis is to develop a method to characterize the most critical combination of a horizontal and a vertical curve. Consequently, the reliability analysis can be conducted for this case by integrating the improved sight distance model with an available reliability algorithm.*
1.2.2 Conducting Code-Calibration for Crest Vertical Curve Design Model

The main premise that underlies the code-calibration process is that standard design models must yield safety level that is consistent and close to a premeditated level. The previous proposition is based on several discussions presented in the course of this thesis. The calibration procedure starts with selecting statistical distributions to represent the randomness in the design parameters. Using the selected distributions, the reliability model, developed according to the last objective, is applied to a feasible domain of designed crest vertical curves using AASHTO Green Book 2001 model. The distribution of ADSL before calibration is obtained and a hypothetical target ADSL is selected. The design equation is further modified with the aim of yielding ADSL values that are close to the selected target value. The previous procedure is applied for the two-dimensional and three-dimensional design models, where the former is applicable to crest vertical curves on tangents and the latter is applicable to crest vertical curve on horizontal curves. The before-and-after distributions of ADSL values are compared and the calibration process is evaluated.

The third goal of this thesis is to conduct a code-calibration process for the design models of crest vertical curves located on tangents and horizontal curves.

1.2.3 Demonstrating a Framework for Selecting Target Design Safety Levels

Although the mechanics of reliability analysis are adequately developed, the selection of a target ADSL is a strategic decision that requires several inputs beyond the realm of reliability theory. There is a tradeoff between cost-effectiveness and the expected safety level of a designed highway geometric element. The selected target ADSL can establish a balance in the previous tradeoff through minimizing construction and collision costs. However, several issues are needed to be addressed in order to link design safety to observable collisions on an operating highway element. In addition, the risk perception by the society can impose additional boundaries on construction and collision costs. A review of some models developed for estimating the acceptable risk level by individuals and society is presented.
The fourth goal of this thesis is proposing a framework for selecting target design safety level used in the code-calibration process.

1.3 THESIS STRUCTURE

The thesis is divided into five chapters. This first chapter provides an introduction to the thesis by presenting background information, a description of the research motivation, research objectives, and the structure of the thesis. The second chapter provides a theoretical discussion of the code-calibration process, a brief illustration of reliability theory, and a review of the highway geometric design literature that deals with sight distance analysis and reliability applications. The third chapter contains a detailed explanation of the developed three-dimensional sight distance model and the necessary conditions that characterize a critical combination of vertical and horizontal curves. The fourth chapter presents the code-calibration process for the design model of crest vertical curves that are located on tangents and horizontal curves. The fifth chapter contains research conclusions and suggestion for future research.
2 LITERATURE REVIEW

This chapter presents a review of subject areas related to the concept of risk-based highway geometric design. The first section presents background information that involves a theoretical discussion of some issues related to reliability theory and the concept of risk in current design practice. The second section provides a review of relevant research materials in the highway geometric design literature.

2.1 RISK IN HIGHWAY GEOMETRIC DESIGN

The emergence of probabilistic civil engineering design relies mainly on the imperfection of knowledge involved in the design process. Probability and reliability theories aim at providing a rational framework capable of quantifying and regulating the level of uncertainty in design. It is necessary to demonstrate the basic areas of reliability theory that lie within the scope of this thesis. The first part of this section provides background information on some topics related to probability theory. The second part presents a brief description of reliability theory and the various methods of analysis employed in this thesis. The third part discusses the general concept of integrating risk into the design process.

2.1.1 Probability Theory in Highway Geometric Design

Most of the design decisions in the field of civil engineering are not based on the unqualified presence or absence of knowledge, but rather the degree of knowledge. The two central steps toward the design of a civil engineering system are: system idealization and performance prediction. The ultimate goal is to limit the system performance within an acceptable operation frame during the feasible life of the engineering system. The design process does not traditionally involve the selection of ideal design alternatives that
entirely mitigate potential failures – otherwise the performance prediction would be a matter of common observation. Conceivably, all design decisions involve a specific level of incertitude which needs to be identified and regulated. It follows that a potential system failure is not entirely avoidable, but the consequences of system failure are minimized through the design process. The standardization of the design process, in order to account for the risk of failure, requires a rational assessment of the underlying uncertainties. Probability theory offers the quantitative tools that can be formulated and introduced as a continual element of risk-based design.

2.1.1.1 Stochastic Components in Highway Design

Highway geometric design is a multi-phased process, with each phase requiring a specific body of knowledge, expertise, and analysis in order to create a solid foundation for engineering decisions. Similar to other civil engineering disciplines, each design phase entails some assumptions, estimates, and predictions that contribute to an aggregate magnitude of uncertainty in the design process. Highway operation consists of three primary components: the driver, the vehicle, and the road. The designable components in the process of highway geometric design are the roadway and/or the roadside environment with the objective of delivering safe and comfortable operation. Unlike the majority of civil engineering disciplines, the human component is central to the safety and efficiency of the system. Human factors, such as driver behavior, expectations, aesthetic perception, mental workload, and potential errors, are essential factors that lend themselves to the design of all features of a highway system. Accordingly, risk-based highway geometric design can be viewed as the geometric delineation, alignment, and proportioning of roadway and roadside features in order to accommodate forgivable navigation or control errors that involve driver and/or vehicle. Evidently, the major stochastic parameters used in the design process are related to human factors and vehicle performance.

The driving mechanism can be divided into three primary tasks: control, guidance, and navigation (Lunenfeld & Alexander, 1990). The significance of human factors in the
design process is underscored in several driving situations: high cognitive or perceptive demand on drivers, erroneous information of road cues, or too little demand on drivers, Alexander & Lunenfeld, 1985). The complexity of introducing comprehensive human factor models to the design process bears mainly on two facts:

- The driving mechanism puts specific demand on drivers that entails complex cognitive, psychological, and perceptive processes. According to Kantowitz (1992), a thorough understanding and formulation of the gathering, processing, and interpretation of driving information is an area of ongoing research, and such comprehensive models are not present in the literature. During recent decades, road safety practitioners and researchers have become increasingly aware of the need for better understanding and consideration for human factors in the current design practice (NHSTA, 1994; Kanellaidis & Sakki, 1997).

- Highways are intended for public service, thus highway operation affects a relatively broad segment of society. Taoka (1989) emphasized the importance of individual differences across the driving population as an additional source of variability. Some aspects of individual variability are explained by age differences (Marmor, 1982; Fox, 1989). Other aspects are attributed to the natural diversity in driving performance.

It can be inferred that a major source of uncertainty in the design process bears on three driver-related issues: the need for a comprehensive understanding of human factors in the driving process, the individual differences among road users, and the anticipated number of road-users throughout the feasible life of a designed highway. In addition, traffic flow prediction brings to focus issues related to transportation planning, an area of research that involves different horizons of variability and uncertainty. The vehicle component in highway operation poses a different source of variability in terms of vehicle dimension, mechanical characteristics, and operational requirements. In sum, uncertainty in highway geometric design is a multifaceted input that requires a rational and scientifically defensible modeling framework. By means of probability theory, suitable statistical descriptors can be used to identify the randomness in design parameters. This
randomness cannot be entirely eliminated, due to practical restrictions; however the magnitude of uncertainty can be regulated in order to be restricted within an acceptable limit.

2.1.1.2 Role of Judgment in Highway Geometric Design

Engineering knowledge is the body of accumulated facts, theoretical constructs, and inferences obtained from theory and/or practice. Engineering judgment accounts for the synthesis of acquired engineering knowledge in order to reach decisions or draw conclusions in situations where theoretical implements or practical access to information is not adequate (Vick, 2002). Nelson and Stolterman (2003) characterized the role of judgment as the ability to acquire or project insight, through experience and reflection, into situations which are too complex, indefinable or intermediate to be solved by classical design tools. Engineering judgment underpins many decisions and is evidently prized and accepted by practitioners. Despite its favorable position in engineering practice, an exact understanding of a formalized function of engineering judgment is elusive and usually passed by with the development of analytical models (Vick, 2002). Similarly, judgment plays an important role in highway geometric design. In discussing designers' concerns about litigation, AASHTO Green Book 2001 emphasizes the value of judgment, “Designers need to remember that their skills, experience, and judgment are still valuable tools that should be applied to solving design problems”.

Highway designers are potentially faced with construction, rehabilitation, or maintenance projects in which the application of minimum standard requirements results in unacceptably high costs, major impacts on the adjacent environment, or poor safety performance. In novel design cases that entail special or unprecedented operational requirements, a strict adherence to design standards is not advisable according to AASHTO Green Book 2001. In these situations a designer is left to his own skill, expertise, and analytical devices in order to conceive a safe and cost-effective design solution. Moreover, the current standardized highway design offers limiting or boundary dimensions of highways. The magnitude of exceeding these standard limits is left to the
designer’s judgment, depending on any additional site requirements. This design methodology occasions the use of judgment in guiding the designer beyond minimum design requirements. It can be concluded that the role of judgment in the current practice of highway geometric design cannot be negated or supplemented by analytical tools. Due to the subjective nature of judgment, it poses a discrete source of uncertainty in the course of the design process. In formulating a rational framework that is capable of both identifying and regulating uncertainty, engineering judgment has to be considered in the development of a risk-based highway geometric design. An approach for achieving the previous objective is proposed in the course of this thesis.

2.1.1.3 Probability Interpretation in Highway Geometric Design

The broad objective of probabilistic design is to identify, quantify, and mathematically characterize uncertainties throughout the various design phases in order to manage and regulate the risk associated with design outputs. The analytical means of risk-based design bears significantly on reliability theory. As Ben-Haim (1994) stated, there are various types of mathematical models for characterizing uncertainty, namely probabilistic, fuzzy, and convex. Probabilistic uncertainty is the type used in the course of this thesis due to the availability of the necessary analytical tools ¹. Before delving into the mechanics of reliability analysis, it is necessary to provide a brief discussion of the interpretation of probability within the scope of highway geometric design.

¹ The selection of the most suitable type in highway geometric design is beyond the scope of this thesis, although it is likely a possible area of research.
In highway geometric design, the majority of input parameters regarding human factors and vehicle characteristics can be classified as stochastic processes. Harr (1987) defined stochastic processes as an observable experiment or phenomenon that yields outcomes that are too variable to be described by ordinary deterministic rules. Since its mathematics first appeared, the interpretation of probability within the realm of civil engineering has continuously been a controversial issue (Fishburn, 1964; Hasofer, 1984; Lind 1996). Salmon (1966) reports the following possible interpretations of probability:

- Classical (frequentist) probability is the ratio between favorable outcomes of an event that is equally likely to occur and all different outcomes. Prediction according to this interpretation implies that a future event depends on past observations of similar events, i.e. past observation can reliably predict the future despite the fact that the physical variability is not explained.

- Subjective probability is a mathematical scaling of the degree of belief associated with the outcome of an event. The application of subjective or Bayesian probability is not restricted to events amenable to repeated sampling or observations. This interpretation can be broadly defined to include antique or one-time occurrence, the validity of an uncertain proposition or hypothesis, and an uncertain state of nature. In addition, subjective probability can more comprehensibly employ Bayesian updating for amending reliability assessments in order to account for engineering judgment, additional information, or qualitative assessment of risk level (Lindley, 1972).

The main output of the risk and reliability analysis presented in this thesis is ADSL, which reflects the probability that a traveling vehicle will operate beyond the acceptable domain set out by design models. The latter is widely recognized as probability of failure in the realm of civil engineering. Petroski (1996) defined failure as any performance of an engineering system beyond a prescribed domain. However, the term failure is commonly perceived within the context of civil engineering as related to the physical collapse of a facility (Elishakoff, 2004). Navin (1990) used the term non-compliance to
supplant the term failure, in light of the absence of physical collapse within the context of highway and traffic engineering. For consistency, the term non-compliance is used in the course of this thesis, although the term failure can be correctly used.

Zeeger, et al. (1989) were the first to introduce a frequentist probabilistic definition of road safety. According to that definition, probability of non-compliance ($P_{nc}$) can be interpreted as the relative frequency of collision occurrence in reference to the total number of vehicles traversing a highway element. Despite its apparent validity, there are several concerns in regard to the applicability of the previous interpretation within the frame of risk-based design:

1. The occurrence of an unfavorable driving situation, represented by a high $P_{nc}$, may not necessarily lead to a collision, but rather evasive maneuvering or an insecure driving experience (Zheng, 1997). In addition, collisions are complex phenomena that involve the occurrence of numerous possible circumstances, some of which are unforeseeable at the design stage. The currently available body of knowledge does not permit an accurate and exhaustive association between specific driving circumstances and the occurrence of a collision. Moreover, $P_{nc}$ is calculated for design models that do not necessarily factor in all the variables that influence driving, but the design models rather rely on mathematical idealization of actual conditions. It follows that $P_{nc}$ is essentially representative of design safety, and as long as the link between the latter and operation safety is not investigated, an interpretation of $P_{nc}$ that involves collisions is largely speculative.

2. The currently available statistical distributions that can be used to describe the variability of input parameters are based on a limited sample of the population of drivers and vehicles. Hence, there is an unavoidable degree of approximation related to the used statistical distributions as a representative of the variability in driving conditions over all future designs.
3. Road collisions are generally random and rare events that, in more than 90% of the cases, involve human error (Wong & Nicholson 1992; Sayed & Navin 1995). These human errors can come in the form of slips, lapses, violation, error in anticipation or judgment, failure to read road information, and irresponsibility (Zheng, 1997). Human errors that result in collision involve gross mistakes in guidance or control that cannot be calculated a priori, but rather statistically associated with specific road features based on collision observations over a period of time. Accordingly, the association between $P_{nc}$ and collisions is unattainable without collision history.

The relationship between design safety and the observed system performance is a recent area of discussion in other fields in civil engineering, e.g. Lind (2005). Based on the previous discussion, it is commendable that $P_{nc}$ be interpreted as a subjective probability - rather than a frequentist probability. Therefore, it can be viewed as an indication of the overall factor of safety built into the design output that compensates for lack of knowledge or reflect a perceived risk.

2.1.2 Reliability Theory

Theory of reliability studies the probabilistic and mathematical models required to calculate the ability of a system to perform its stated purpose according to the anticipated operation conditions (Gertsbakh, 1989). Mayer (1926) was the first to propose the integration of probability theory into engineering design. As pioneered by Freudenthal (1954), the introduction of reliability analysis to the realm of civil engineering, and structural engineering in particular, was intended to replace the concept of factor of safety with a more rational framework for addressing uncertainty in design. Few decades after its first introduction, reliability theory became a widely accepted engineering discipline that addresses the probabilistic nature of engineering design within a sound analytical framework (Cornell, 1981). The conventional procedure of applying reliability methods is summarized as follows:
1. Defining the analytical model that predicts system performance,
2. Defining uncertainty in analysis inputs by means of probability distributions, and
3. Calculating the probability that the system performance is within an acceptable limit. In addition, the relative influence of inputs on the system performance is a secondary output of some reliability methods. It is expected that the soundness of reliability analysis depends on the accuracy of the input probabilistic information (Gertsbakh, 1989).

The basic elements of reliability analysis are an \( N \)-dimensional vector of input variables \( X = x_1, x_2, ..., x_n \) and limit state function or performance function \( G(X) \). The performance function \( G \) is constructed in that it yields positive outputs when the system performance is acceptable or safe, and yields negative values for unfavorable system performance. The performance function is conventionally written in terms of the difference between supply \( R \) and demand \( S \), as follows:

\[
G = R - S
\]  

2.1

In highway geometric design, supply represents the group of input parameters that are related to design requirements concerning safe and comfortable driving conditions. Demand represents driver or vehicle requirements that need to be accommodated. Demand and supply are uncertain, and their intersection represents conditions of probable non-compliance, as shown in Figure 2.1. The uncertainty regarding input variables can be characterized by assigning each input variable \( x \) a probability density function \( f(x) \). The reliability of a system, the probability of its being in the acceptable performance domain, can be obtained as:

\[
P_c = 1 - P_{nc} = \text{Prob}(G > 0) = 1 - \int_{G(x)>0} f(x)dx
\]  

2.2

where

\( P_c \) = probability of compliance or reliability

\( P_{nc} \) = probability of non-compliance or unreliability
According to Melchers (1999), the previous integration can be performed by:

1. Direct integration (applicable in limited number of special cases),
2. Numerical integration, such as Monte Carlo simulation, and
3. Transforming the integrand to a multi-normal joint probability density function, in order to avoid integration. First Order Second Moment (FOSM) and First Order Reliability Method (FORM) are prime examples of this method.

The performance function adopted in the reliability analysis presented in this thesis is not a closed-form function, but rather an iterative algorithm. Monte Carlo simulation was not selected due to the anticipated costly calculations and the need for investigating the relative influence of input variables – which is not offered by the latter method. FORM analysis was selected over FOSM due to the non-normality of some input parameters, the more accurate results offered by FORM, and the availability of an analytical platform, FERUM (Hawkaas & Kiureghian 2000), for the implementation of FORM analysis. FORM analysis attempts to facilitate the calculation of probabilities by means of
transforming the random variables $X$ to a standardized normal space $Y$ according to the following expression:

$$ p = F_x(x) = \Phi(y) $$

where

$p$ = some probability content associated with $X = x$

$F_x(x) =$ marginal cumulative distribution function of $X$

$\Phi(y) =$ cumulative distribution function for the standardized normal random variable $Y$

The reliability of a system is often expressed in terms of a safety, or reliability, index $\beta$, a unitless indicator that is directly proportional to the safety level. The reliability index $\beta$ can be calculated as follows:

$$ P_{nc} = \Phi(-\beta) $$

The prevalence of this expression in the literature of reliability analysis may be explained by two factors: 1) reliability index offers a more meaningful and interpretable numerical value in comparison to probability values which are normally of a low order of magnitude (Augusti, et al., 1984), 2) most reliability methods include probability transformation to a standardized normal space, hence it is convenient to express variability in the analysis output within the same space, and 3) Reliability index $\beta$ has a particular geometric significance that is central to FORM analysis. Figure 2.2 shows the relationship between $P_{nc}$ and reliability index.
Figure 2.2 The Relationship between Reliability Index and Logarithm the Probability of Non-Compliance.

Figure 2.3 The Relationship between Reliability Index and Probability of Non-Compliance.
2.1.3 The Concept of Risk-Based Design

In light of the foregoing discussions, it becomes evident that all engineering decisions taken during the design process involve a specific level of risk. The ultimate goal of a risk-conscious design is to manage and regulate the design safety level in order to minimize both potential of failure and construction cost (Lind, 1991). Despite the wide consideration for the probabilistic nature of the design process, reliability analysis is susceptible to vary according to initial assumptions regarding the probability distributions of design parameters as well as the acceptable risk level (Gertsbakh, 1989). Tichý (1991) and Carter (1997) pointed that risk regulation of standardized design is essentially an undertaking of the code developer rather than the office engineer. This assertion led to the emergence of design codes, or equivalent regulations, in order to delegate some design responsibilities to professional authorities (Lind, 1969). The following sections will briefly discuss some developmental aspects of design safety and acceptable risk in design.

2.1.3.1 Development of Risk-Based Design

In order to compensate for imperfectness in knowledge about system behavior, designers opt to maintain a specific magnitude of overdesign. Prior to the emergence of reliability methods, the magnitude of overdesign was characterized in terms of factor of safety. Factor of safety is analytically the ratio between the expected “supply” to the expended “demand”. Factor of safety concentrates design safety level in a single quantity, which is usually selected based on past experience and/or subjective assessment (Rao, 1992). The concept of factor of safety, which emerged essentially in applied mechanics, found feasible application in other areas: medicine (Butterfield, 1963; Jones, 1982; Taylor et al. 1982), and business (Solomon et al., 1983). In the realm of structural engineering, factor of safety was subject to numerous revisions and criticism regarding its analytical and practical validity (Freudenthal, 1956; Carter, 1997; Elishakoff, 1983 & 1999). Oden et al. (2003) stated that despite some early skepticism, e.g. Bolotin (1964) Cornell (1969), reliability methods are expected to be integrated into standardized design in the foreseeable future. One of the key steps in developing code-writing is the integration of
reliability methods into the formulation of design models as a rational framework for addressing uncertainty (Ellingwood, 1994). The ultimate objective of calibrating the design code is to eliminate the lack of equal probability of failure, in such it becomes close to a premeditated and acceptable level.

2.1.3.2 Acceptable Risk Level

The selection of an acceptable risk level, or probability of non-compliance, is a paramount step in standardized probabilistic design. Grandori (1998) noted that the majority of research in the field of reliability methods appears to be devoted to the analytical details of calculating probability rather than the selection of an acceptable value. In road safety, risk is expressed as the product of the probability of unfavorable outcome and the cost of consequences (Haight, 1986). Several studies, e.g. Keeney (1980a & 1980b), Vrijling et al. (1998) and Rackwitz (2002), considered the determination of acceptable risk as an area of research that involves a risk-benefit trade-off for individuals and society. Ditlevsen (1997) proposed a rational framework for selecting a target design safety level $\beta$, based on minimizing expected cost of failure, including probability of failure and societal cost of failure, and different types of construction cost. At the core of the last methods lies the relationship between the cost of system failure and $\beta$. Without investigating the link between design safety and operational safety and due to the aforementioned assumption regarding the subjective nature of design safety in the highway geometric design, a target design safety level ought to be selected based on a method that does not involve predicting number of collisions. Lind (1978) postulated that existing codes are optimal if the need for revision is not present. The novelty of the previous principle lies in the assumption that standardized design is in the long run self-optimizing. Roesset (2002) referred to a similar approach of calibrating some design models in which $\beta$ was selected based on past experience rather than optimization process. Similarly, in highway geometric design, past designs that are deemed optimal in respect to construction cost and safety performance can be used to estimate $\beta$, itself is considered a random variable. A relevant example of the self-optimization postulate is evident in the last release of AASHTO design guide,
wherein several concerns were raised regarding the cost-effectiveness of vertical curves design according to AASHTO. This was either attributed to an improved vehicle performance or to the rising cost of materials. The design model was revised in order to reduce the lengths of designed vertical curves without significant safety implications (Fambro et al., 1997).

2.2 PREVIOUS ANALYTICAL STUDIES OF SIGHT DISTANCE

The importance of considering sight distance availability in a safe and efficient operation of vehicles is recognized by most design manuals, e.g. AASHTO Green Book (2001) and TAC (1999). In order to keep with sight distance requirements, the roadway features should be dimensioned such that drivers are allowed a sufficient sight distance at any point on the road. In design practice, available sight distance is compared to some required sight distance – where the latter depends on the driving and controlling tasks, e.g. braking action, passing maneuver, complex driving decisions, and reading road cues.

The available sight distance depends on several road geometry parameters, such as cross section elements, roadside conditions, vertical alignments, and horizontal alignments. The analytical models of geometric design, although revised several times, have not significantly changed since AASHO 1954 design guide (Hassan et al., 1998). In most cases, sight distance calculations are undertaken in 2D projections, separating horizontal and vertical alignments. Several researchers have demonstrated the need for considering 3D sight distance in geometric design (Smith, 1994; Mannering, 2004). Smith (2004) noted that the 3D analysis represents “the weakest link in the overall design of highways”. This can be mainly attributed to the complexity of the computations required for 3D analysis.

The research addressing 3D highway design can generally be classified into three main categories: visual, quasi-analytical, and analytical techniques. Visualization tools for road environment have been available for a relatively long time (Tanton et al., 1986; Lanphair, 1996; Hassan et al., 2002). However, the use of 3D visualization is considered more
effective for the esthetic and driver perception research than for the alignment design (Bidulka et al., 2002). Sanchez (1994) proposed a quasi-analytical approach for calculating the 3D sight distance. The technique initially approximates the road surface to triangular elements using available CAD software (Inroads, 1990). A perspective view of the road from a selected driver location is then generated. The intersection between a line of sight and the roadway surface is evaluated by creating the surface profile under the line of sight. The calculated available sight distance (ASD) equals the traveled distance along the road, and the latter is obtained by summing the lengths of the elements along the driving course. Easa (1994 b) noted that this method requires relatively long analysis time.

The most flexible and efficient approach that can be included in standard design guides is the analytical approach. In the existing analytical practice, sight distance calculations are performed in 2D projections – separating the horizontal and vertical alignments. Several approaches are available for calculating the 2D sight distance. AASHTO Green Book 2001 provides simple formulas for calculating the 2D available stopping sight distance on horizontal curves, assuming that the point of obstruction is at \( S/2 \) and \( S < L \), where \( S \) is ASD and \( L \) is the horizontal curve length. For \( S > L \), ASD was studied by Olson et al (1984), and Waissi and Cleveland (1987). Taignidis (1998) provided a closed-form analytical approach for ASD on crest vertical curves. Lovell et al. (1999 & 2000 & 2001) introduced the concept of parametric representation of roadway alignments as a less computationally intensive and more flexible approach to calculate the 2D sight distance.

One of the earliest attempts for an analytical 3D analysis was proposed in the 60’s by Geissler (1968). However, a lack of computational resources at that time hindered the progress in this area. A 3D idealization of the roadway was presented by Chew (1989) for optimizing vertical and horizontal alignments simultaneously using a 3D model. In this model, the road profile is defined as a function in the 3D coordinate system, and the ground surface is idealized as a group of right-angle planar triangular elements. However, these approaches are limited in their ability to model roadway curvature and in their computational efficiency.
An analytical approach for calculating the 3D ASD using finite element representation of the road environment was proposed by Hassan et al. (1996). The approach consists of two stages: idealization and evaluation. The idealization is performed by tailoring a finite element mesh that fits the roadway/roadside features. Hassan et al. (1996) devised several elements that can be used to idealize the roadway surface: four-node, six-node, eight-node, and triangular elements. The four-node element is considered exact for single graded tangents and approximate for curved segments. The six-node element uses parabolic interpolation along the three-point sides and linear interpolation along the both other sides. The six-node element is exact for vertical curves, however it is approximate for combined horizontal and vertical curves. Hassan et al. (1996 & 1999) used 20-meter wide finite elements, and the evaluation is performed through incremental search algorithm. Each line of sight is checked for an intersection with all the finite elements, and then further, or closer, target points are selected. The evaluation of the 3D sight distance based on the finite element model was used for several applications: comparing required 3D sight distances and 2D sight distances (Hassan et al., 1997 a & 2000), modeling the available headlight sight distance on combined horizontal and sag vertical curves (Hassan et al., 1997 b & 1998), evaluating the available passing sight distance (Hassan et al., 1997 c & 1997 d), and proposing some general considerations for combining horizontal and vertical curves (Hassan et al., 1997 c).

2.3 PREVIOUS RELIABILITY-BASED RESEARCH

Moyer and Berry (1940) undertook the pioneering work of incorporating probabilistic methods into highway design by developing a method to determine the safe speed on highway curves. The method included calculating the margin of safety as the ratio between ball-bank reading recorded on a curve and a generally accepted safe value. Further, the operating speed was recognized as a random variable and hence selected based on specific percentile values – 85th percentile for design speed of 30 mph or less and 90th percentile for 35 mph.
Navin (1990) cited several concerns about the implicit and qualitative safety level in geometric design standards. In response, the concept of margin of safety based on standard design equations was introduced as a meaningful and quantitative measure of the safety level built into isolated highway components. The margin of safety was calculated based on assuming that variables are normally distributed and independent. The reliably method used was FOSM, in which obtained values of reliability index \( \beta \) represented the margin of safety. In this study, \( \beta \) values were calculated at desirable and minimum design requirements supplied by AASHTO (1984) and ITE Handbook (1982). The studied design models were those used to calculate stopping sight distance and passing sight distance, as well as the design of horizontal curves and crest vertical curves. As a recommendation for further research, a generic form of design equations was proposed as a means of addressing uncertainty in design:

\[
\phi \cdot P_H = S \cdot E \cdot T \cdot D \cdot e \cdot t \cdot d \cdot (P_{Dlv})
\]

where
- \( \phi \) = performance factor (or design safety parameter),
- \( S \) = highway system importance,
- \( E \) = exposure factor,
- \( T \) = traffic mix,
- \( D \) = driver mix,
- \( e \) = environmental factor,
- \( t \) = terrain factor, and
- \( d \) = desired design or construction standard.

The performance factor \( \phi \) is selected in order to make the highway supply parameter \( P_H \) large enough to maintain an acceptable safety margin against the driver/vehicle demand \( P_{Dlv} \). This study stands out in the design safety literature in its attempt to formulate a general framework for probabilistic design standards. However, more research was required in the area of code-calibration in order to lay down a rational framework that can
be adapted into the current standard design practice. Evidently, the recent releases of the major highway geometric design standards did not include the probabilistic methods proposed by Navin (1990).

Easa (1993) developed a probabilistic model based on FOSM reliability method for the design of the intergreen interval in signalized intersections. The main objective is to eliminate the zone where a driver faced with yellow signal fails to either stop or clear the intersection. In achieving so, the stopping sight distance is equated with intersection clearing distance. The variability in the input parameters was calculated based on some assumed values of the corresponding coefficient of variation. Two probabilities of non-compliance values were assumed and new design charts were reconstructed in order to obtain intergreen times. It is noteworthy that the developed probabilistic model was closed-form; hence the reconstruction of the calibrated design chart did not involve any numerical minimization – unlike the calibration process introduced in this thesis.

Faghri (1988), Easa (1994 a), and Easa (1999) studied the problem of sight distance at uncontrolled road intersections and road-railway grade crossings under different operational conditions. The main objective of intersection design is to permit an adequate sight distance for an approaching driver to safely stop prior to entering an intersection in order to preclude potential collision. The moments of the input variables were obtained from relevant studies or by assuming some values for the coefficient of variation and assigning specific percentiles for design values. The probability of non-compliance was calculated at difference values of available sight distance using FOSM reliability method.

Richl and Sayed (2006) studied the effect of median width along curved highway segments in order to understand the risk of sight distance restriction. If a median is potentially narrow, a driver may fail to stop within the available sight distance due to sight restriction caused by the median barrier. The probability distributions of input variables were all obtained from relevant studies. The reliability method used was Monte Carlo simulation. For two highway alignments, the probabilities of non-compliance were calculated for several scenarios of sight distance restriction. The obtained probabilities
were compared to the maximum obtainable probability of non-compliance for a horizontal curve design based on the same design speed of the two highway alignments.

It can be concluded that the application of reliability in the field of highway geometric design is not a new topic in the literature. Several studies in the literature attempted to calculate probabilities of non-compliance associated with the design outputs obtained from several design models. The issue of selecting a target risk level was not addressed either theoretically or through a case study. The single study that provided calibrated design charts was based on a closed-form probabilistic model, a case that is not encountered in all design models, including the analysis presented in this thesis. Several studies resorted to the assumption of arbitrary values for the coefficient of variation of input parameters. While these assumptions may appear to compensate for the lack of relevant studies, this approach cannot be employed in a standardized code-calibration process. In fact, the lack of studies that quantify uncertainty signifies the evolving need for further research toward a comprehensive risk-based design.

2.4 SUMMARY

This chapter presented a theoretical discussion of some issues related to probability theory and risk-based design. The interpretation of probability within the context of highway geometric design was discussed and it was suggested that subjective probability is a meaningful interpretation. The basics of reliability theory were laid out, in addition to definitions of some terms used in reliability analysis. A brief review of the historical development of risk-based design in civil engineering was also presented. The determination of acceptable risk was discussed and a method was proposed for code-calibration in the field of highway geometric design. Relevant studies in the literature of highway geometric design that involve applications of reliability theory were cited in the chapter. General comments and recommendations were drawn based on the literature review, with some recommendations implemented in the course of this thesis.
3 SIGHT DISTANCE MODEL

This chapter presents the analytical details of the probabilistic sight distance model used in the code-calibration process. The first section presents the mathematical details of the model. The second section discusses the results obtained from several case studies. In addition, a hypothesis is provided in order to determine critical situation of overlapping horizontal and vertical alignments.

3.1 SIGHT DISTANCE MODEL

The sight distance model presented in this chapter offers a new approach for calculating the 3D sight distance. The proposed model contains some core elements that are based on a formerly proposed 2D parametric idealization of the roadway to model 3D sight distances (Lovell, 1999). The usefulness of the new model is manifested by its relative time efficiency and the accuracy. Time efficiency is central to reliability analysis because of the relatively large number of iterations anticipated in the calibration process. The accuracy of the calculated sight distance is important for obtaining correct values for the numerical differentiation performed in FORM analysis.

Lovell (1999) introduced the parametric representation of roadway alignments as a less computationally intensive and a more flexible approach for the evaluation of the 2D ASD for any horizontal alignment configuration. Lovell et al. (2000 & 2001) proposed some enhancements in order to consider side obstruction with variable offsets and to account for the actual driven distance along the vehicle path. In the proposed sight distance model, a similar approach is adopted and extended for a 3D evaluation. The main improvements of the approach presented in this thesis over the finite element approach proposed by Hassan et al. (1996) are as follows:
1. The search process is performed on the roadway/roadside actual surface instead of a fitted surface,
2. The accuracy of the calculation, measured by the increment size, is significantly higher than Hassan et al. (1997) while maintaining satisfactory computational efficiency,
3. The search process is enhanced by a multi-step multi-direction search, and
4. By indexing all the road features to the centerline, it is possible to consider variable side slopes, side slope textures, variable lane widths, superelevation transition, and variable-width clearance zones. These advantages are a consequence of the parametric representation of the road environment.

Figure 3.1 illustrates the main difference between the parametric and finite element idealization of the road surface. In the proposed method, the roadway and the roadside surfaces are searched incrementally under the line of sight only, with smaller increments near the tangential point. This results in increased efficiency and accuracy of the search process. In the finite element method, the roadway/roadside idealization elements are considerably larger than the proposed method.

![Figure 3.1 The Road Surface Idealization According to the Proposed Technique (Left) and the Current Finite Element Model (Right).](image-url)
The following are the main assumptions and nomenclature used in the algorithm:

1. The origin of the Cartesian coordinate system $\mathbb{R}^3$ is set at the start point of the alignment centerline,

2. All formulations are undertaken in a parametric form, i.e. in terms of $t$, which is the length measured along the roadway centerline from the alignment start point to the point of interest,

3. $X'_n(t)$ is the Cartesian coordinates of a point at a centerline distance $t$ from the alignment start point, and lies on the $n^{th}$ roadway edge curve from the centerline of horizontal curve $i$ at a distance $w(t,n)$ of the alignment centerline. The distance is measured in the direction of the normal $N(t)$ pointing to the center of the horizontal projection of the horizontal curve on the $Z = 0$ plane, i.e. to the center of projection $C_i$,

4. The horizontal curves are considered positively deflected when the deflection angle $\Delta_i$ of the $i^{th}$ curve is counterclockwise. Within the analysis, it will be required to designate some terms based on the direction of the curve deflection. The function $signum(\Delta_i)$ returns $-1$ for negative $\Delta_i$ and $1$ for positive $\Delta_i$,

5. The alignment horizontal elements (tangents, curves and transition curves) are enumerated in ascending order according to the start stations of each element,

6. The start point of the vertical curve $VPC_i$ is located relative to the horizontal curve start point $PC_i$ at distance $A_i$. The vertical and horizontal curves enumerations are identical,

7. The line of sight equation is: $S(s) = X_o + sP$. Where $s$ is the distance along the line of sight, $P$ is a vector in the direction of the line of sight, and $X_o$ is the start point of the line of sight (driver eye coordinates), and

8. The superelevation $e(t)$ is used in absolute values and its sign is determined according to the curve deflection angle.
3.1.1 Formulation

The 2D part of this formulation is based on Lovell (1999) parametric approach. Define function \( D \) that determines the domain of the horizontal and vertical elements as \( D(E,i) = (t_0, t_f) \), where \( E \) is the element notation and \( i \) is the element enumeration. Assuming that the alignment starts by a tangent, the centerline equation for the first tangent is:

\[
X(t) = O + t \cdot \frac{P_i}{\|P_i\|} \quad \forall \quad t \in [t_0, t_f(D(T,1))], 0 \leq t < t_f(D(T,1))
\]

where

\[ O \] = coordinates of the alignment start point,
\[ P_i \] = vector in direction of the tangent that precedes the \( i^{th} \) horizontal curve, and
\[ t(D(E,i)) \] = inverse of the function \( D \).

Each vertical curve is defined by its first grade \( g_u \), second grade \( g_2 \), rate of vertical curvature \( k_v \), shift \( A \) from the start point of the \( i^{th} \) horizontal curve, \( t_{oi} \) is the station of the \( i^{th} \) vertical curve start point, and \( t_{fl} \) is the station of its end point. The elevation of any point is found as follows:

\[
Z(t) = \begin{cases} 
Z_{oi} + g_u(t - t_{f(i-1)}) & 0 \geq t - t_{oi} \\
Z_{oi} + g_u(t - t_{f(i-1)}) - \frac{a}(t-t_{oi})^2 & t_{fl} - t_{oi} \geq t - t_{oi} > 0 
\end{cases}
\]

\[
Z_{oi} = \sum_{x=1}^{n} \left\{ g_{i,x} \left( \frac{(f_i - f_x)}{2} + t_{ax} - t_{f(x-1)} \right) + \frac{(f_i - f_x)}{2} (g_{2,x} - g_{1,x}) \right\} ; \forall \quad t_{oi} \leq t < t_{fl} \quad 3.2
\]

The normal to the alignment centerline, pointing to the curve center or right to the direction of stations increase for tangents, can be obtained as follows:
\[ N_i = R_i \left( \text{signum}(\Delta_i) \cdot \frac{\pi}{2} \right) \cdot \frac{P_i}{\|P_i\|} \]

\[ P_i = P_1 + \sum_{x=1}^{x=i} \Delta_x \]

\[ R_i(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad 3.3 \]

where
\[ \theta = \text{rotation angle}. \]

\[ X_i(t) \] is the horizontal-projection coordinates of a point located on the centerline of the \( i^{th} \) circular curve, at distance \( t \) from the alignment start point. The latter can be calculated in terms of the curve radius \( r_i \), the horizontal curve center \( C_i \), the station of the horizontal curve start point \( t(PCI) \), \( N_i \), and \( \Delta_i \) as follows:

\[ X_i(t) = C_i + r_i N_i R \left( \frac{t-t(PCI)}{r_i} \cdot \text{signum}(\Delta_i) \right) \quad 3.4 \]

\[ t(X_i) = t(PCI) + r_i \cdot \text{Cos}^{-1} \left[ N_i^T \cdot \frac{C_i - X_i}{\|C_i - X_i\|} \right] \quad 3.5 \]

where
\[ t(X_i) = \text{inverse of the function } X_i(t). \]

It is required to formulate a family of curves, \( X_i^n \), parallel to the \( i^{th} \) horizontal curve. This family of curves is function of the offset \( w(t,n) \) from the centerline:

\[ X_i^n(t) = X_i(t) + w(t,n)N_i(t) \quad 3.6 \]
In addition, the inverse transformation is as follows:

\[
X_i(X_i^*) = C_i + r_i \cdot \frac{C_i - X_i^*}{\|C_i - X_i^*\|}
\]

Lovell et al. (1999 & 2000) provided a parametric representation for a clothoid spiral connecting a tangent to a curve and vice versa. For a spiral connecting TS and SC points at a distance \(w(t,l)\) from the center:

\[
X_i^1(t) = X(TS_i) + l_i \cdot R\left(\alpha_i^1\right) + \sum_{j=0}^{3} \frac{(-1)^j (\Delta_i^1)^{2j}}{(4j+1)!} \left(1 - \frac{t - t(TS_i)}{l_i}\right)^{4j+1} + \sum_{j=0}^{3} \frac{(-1)^j (\Delta_i^1)^{2j+1}}{(4j+3)!} \left(1 - \frac{t - t(TS_i)}{l_i}\right)^{4j+3}
\]

where

- \(TS_i\) = tangent spiral point benchmarked to the alignment start,
- \(\alpha_i^1\) = bearing of the tangent vector from the positive abscissa,
- \(\Delta_i^1\) = deflection of spiral segment near \(i^{th}\) curve start, and
- \(l_i^1\) = length of this spiral segment.

For an exit transition curve connecting CS and ST points:

\[
X_i^1(t) = X(TS_i) + l_i^1 \cdot S \cdot R\left(\pi - \alpha_i^1 - \Delta_i^1\right) + \sum_{j=0}^{3} \frac{(-1)^j (\Delta_i^1)^{2j}}{(4j+1)!} \left(1 - \frac{t - t(TS_i)}{l_i}\right)^{4j+1} + \sum_{j=0}^{3} \frac{(-1)^j (\Delta_i^1)^{2j+1}}{(4j+3)!} \left(1 - \frac{t - t(TS_i)}{l_i}\right)^{4j+3}
\]
3.1.2 Superelevation

A lateral offset is considered positive when it lies right of the centerline and negative on the other side. Therefore, the superelevation values can be used in absolute terms to calculate road surface elevation as follows:

\[
Z'(t) = Z(t) + w(t, n) \frac{e(t)}{100}
\]  \hspace{1cm} 3.10

\[
e(t) = \begin{cases} 
-e_c & t < T_i^1(1) \\
E\{t, e_i, \text{signum}(\Delta_i), e_c, \text{signum}(w(n)), T_i^1\} & T_i^1(1) < t < T_i^1(2) \\
e_i \cdot \text{signum}(\Delta_i) & T_i^1(2) < t
\end{cases}
\]  \hspace{1cm} 3.11

where

\[e_c = \text{absolute value of the crown section side slope},\]
\[e_i = \text{absolute value of the horizontal curve superelevation},\]
\[T_i^1 = \text{vector of the start and the end station of the superelevation transition segment at the curve start point, and}\]
\[T_i^2 = \text{similar vector for the transition segment at the horizontal curve end point.}\]

\(E(\cdot)\) is a superelevation transition function and is defined explicitly. For example, a linear superelevation transition is expressed as:

\[
E\{\cdot\} = -e_c + \text{signum}(w(t, n)) \cdot (t - T_i^1(1)) \cdot \frac{e_i + \text{signum}(w(t, n)) \cdot e_c}{(T_i^1)^T \begin{bmatrix} 1 & -1 \end{bmatrix}}
\]  \hspace{1cm} 3.12

Similarly, the elevations on the roadside features can be calculated knowing the side slopes besides the previous parameters. Assuming \(n\) roadside features with widths \(w_1, w_2, \ldots, w_n\) and side slopes \(e_1, e_2, \ldots, e_n\), the general equation takes the following form:
\[ ZW(t, w) = Z(t) + \sum_{j=1}^{n-1} w_j \cdot \frac{e_j}{100} + (w - \sum_{j=1}^{n-1} w_j) \cdot \frac{e_n}{100} \quad , \quad \sum_{j=1}^{n-1} w_j < w < \sum_{j=1}^{n} w_j \quad 3.13 \]

### 3.1.3 Sight Distance

In this section, it is required to formulate the equation of a line of sight connecting \( X_o \) and \( X_i(t) \), where the latter is the coordinate of the target point that has to be \( h \) above the road surface. Assume that the curve \( N = 2 \) is the centerline of the inner lane; where \( w(t, 2) \) equals half the lane width in addition to half the median width. The coordinates of \( X_i(t) \) can be calculated from Equations 3.6 and 3.13. The line of sight coordinates, parameterized by distance \( s \) along the vector connecting the driver eye location and the target point location, can be calculated as follows:

\[
S(s) = X_o + s \frac{(X_i(t) - X_o)}{\| (X_i(t) - X_o) \|} \quad 3.14
\]

\[
V(s) = ZW(t[S(s)]_i, \| S(s) - X_i(t[S(s)]) \|) \quad 3.15
\]

Equation 3.15 can be used to calculate the road surface profile under the line of sight. If elevations are obtained from a single roadside formation, e.g. road surface only, then Equation 3.15 is a smooth curve. If the line of sight swayed further from the road edge, the surface profile will be a composite curve characterized by the higher of the road surface profile and the side slope profile.

### 3.1.4 Obstruction Conditions

The next step is to examine the intersection between a line \( S(s) \) and a curve \( V(s) \). There are several approaches to test the intersection between a complex curve and a line. Assuming that the function \( Z(s) \) maps a distance \( s \) along the line of sight to its elevation,
it is possible to formulate a numerical searching algorithm for the first intersection condition as follows:

\[ I(X_r) = \min \left\{ s : (V(s) > Z(s)), \forall \ 0 < s < \|X_o - X_r\| \right\} \tag{3.16} \]

\( I(X_r) \) gives the closest intersection point to the driver eye location. The condition given in Equation 3.16 represents a line of sight obstruction by the road surface or the side slope(s). This condition can be tested mathematically by first simplifying Equation 3.16 to a closed-form equation in terms of distance \( s \). Second, the equation is differentiating for the highest point \( s^* \) which is compared to \( V(s^*) \). Alternatively, this condition can be evaluated numerically by any optimization tool, such as the optimization tools in Matlab.

Side slope values, roadside lateral distances, and road width can vary as long as these changes are linked to centerline stations. For a median barrier, the planar intersection between the line of sight and the barrier is obtained by solving the quadratic equation of the intersection between a line and a circle. The solution yields two values of \( s \), such that solution points lie on the median barrier and: \( \forall \ t(PC_i) + (X_o + s^* \cdot P) \leq t(PT_i), \ t > 0 \).

The factors of the quadratic equation are:

\[ a = 1, \ b = 2 \cdot (X_o - C_i)^T \cdot \frac{X_r - X_o}{\|X_r - X_o\|}, \ c = \|X_o - C_i\|^2 - r_i^2 \tag{3.17} \]

A line of sight obstruction occurs if its elevation is less than that of a median barrier at the above locations. Several median barriers can be included in the algorithm as long as
their locations are defined relative to the centerline. It was found that searching for an
unobstructed line of sight can converge slowly if the first estimate of the target point is
not properly selected. Furthermore, the optimization module in Matlab, for some cases of
initial target points, failed to converge to a solution of the minimization problem in
Equation 3.16. Processing time can be reduced significantly by choosing the first target
point based on approximating the roadside to a set of cones. Roadside surfaces (e.g. side
slope in cut sections) are complex surfaces. The side slope is approximated to a cone with
a vertex projection coinciding with the horizontal curve center. Knowing the average side
slope and the average edge elevation, this approximate cone can be established.

For any point $S(s)$ that lies on a line of sight that is unobstructed by the roadside cone,
the angle between a vector from cone vertex $V_i$ and this point should exceed
$\cot^{-1}(P_s/100)$, where $P_s$ is the side slope in percentage. $V_i$ coincides with $C_i$ and has a
$Z$ coordinate that is greater than the average side slope edge elevation by $\frac{rP_s}{100}$.

Neglecting the trivial case when $P_s < 0$, i.e. the upper cone pair reflected across the cone
vertex, the cone axis, $A_c$, is assumed to be pointing in the $-k$ direction. The cone surface
equation is written as:

$$A_c^T \left( \frac{X-V_i}{\|X-V_i\|} \right) = \cos \theta$$

This can be rewritten as:

$$(X-V_i)^T \cdot (AA^T - \cos^2 \theta \cdot I)(X-V_i) = 0$$

where

$$\cos \theta = \sin[\tan^{-1}\left( \frac{P_s}{100} \right)]$$

$I$ = the identity matrix.
Substituting by \( S(s) = X_o + sP = X \) it is possible to simplify Equation 3.19 to the quadratic form: \( a \cdot t^2 + b \cdot t + c = 0 \), in which:

\[
a = P^T \cdot (AA^T - \cos^2 \theta \cdot I) \cdot P,
\]

\[
b = 2 \cdot P^T \cdot (AA^T - \cos^2 \theta \cdot I) \cdot (X_o - V),
\]

and

\[
c = (X_o - V) \cdot (AA^T - \cos^2 \theta \cdot I) \cdot (X_o - V).
\]

A line of sight is obstructed if there is at least one real solution for the previous quadratic equation, i.e. \( b^2 - 4ac \geq 0 \).

### 3.1.5 Available Sight Distance

As mentioned in the formulation, the obstruction conditions are evaluated on the actual roadway and/or roadside surface. The search for line of sight obstruction can be performed as described in Equation 3.16 or by a slightly less effective, yet stable, search algorithm. The line of sight search algorithm is processed according to the flowchart presented in Figure 3.2. The algorithm was coded using Matlab and the search process is performed using two increments \( \Delta \) and \( \delta \). As illustrated in Figure 3.2, the target point is shifted by a value \( \Delta \) until the obstruction condition is reversed, e.g. starting by invisible target point until it is viewable. The process is repeated with a smaller and opposite-sign increment, \( \delta \). The values used in the program are 2.0 m and 0.1 m for \( \Delta \) and \( \delta \) respectively. The program code is provided in Appendix I.

### 3.1.6 Validation and Case Studies

The following numerical example is introduced to illustrate and verify the new algorithm. The base roadway segment is a simple horizontal curve, two tangents, and a vertical curve. The alignment parameters are presented in Table 3.1. The driver is located at the horizontal curve start point, which for the base case is 50 m before the vertical curve start point. The previous case is processed using the proposed algorithm in order to calculate the 3D available sight distance. The algorithm reports the tangential point for each line of sight, where the roadside and line of sight elevations can be calculated manually. It was
found that the ASD at this location is 167.626 m. For a tangential point at a distance 40.11 m from the driver eye, and at a centerline distance 277.626 m from the curve start point, and at an offset 8.57 m from the curve centerline, the roadside elevation is 1.76 m while the line of sight elevation is 1.77 m. The error is approximately 1 cm. The maximum error in all the runs undertaken within this research is 1.1 cm. The error can be readily reduced by decreasing the search algorithm increment $\delta$.

Figure 3.2 Flow Chart of the 3D Available Sight Distance Algorithm.
Table 3.1 Base Values for Roadway Elements.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number of lanes</td>
<td>2 lanes</td>
<td>Side slope</td>
<td>3:2</td>
</tr>
<tr>
<td>Lane width</td>
<td>3.7 m</td>
<td>Side slope toe offset</td>
<td>3.5 m</td>
</tr>
<tr>
<td>Clearance zone width</td>
<td>3.5 m</td>
<td>First longitudinal grade</td>
<td>4%</td>
</tr>
<tr>
<td>Clearance zone slope</td>
<td>4%</td>
<td>Second longitudinal grade</td>
<td>-3%</td>
</tr>
<tr>
<td>Driver centerline offset</td>
<td>1.95 m</td>
<td>k (rate of vertical curvature)</td>
<td>37</td>
</tr>
<tr>
<td>Horizontal curve radius</td>
<td>900 m</td>
<td>Driver eye height</td>
<td>1.15 m</td>
</tr>
<tr>
<td>Shift between horizontal curve</td>
<td>50 m</td>
<td>Object height</td>
<td>0.15 m</td>
</tr>
<tr>
<td>start and vertical curve start (A)</td>
<td>9.553E-5 '</td>
<td>Normal crown slope</td>
<td>2%</td>
</tr>
<tr>
<td>Horizontal curve deflection angle</td>
<td>50 m</td>
<td>Superelevation</td>
<td>6%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Superelevation transition length (in, out)</td>
<td>50 m</td>
</tr>
</tbody>
</table>

* Some of these parameters are changed throughout the analysis. These parameters are considered base values.

Furthermore, for validating the proposed algorithm compared to the available 2D models, the extreme cases of the vertical and horizontal curvatures are examined. The ASD created by only the vertical curvature of the roadway (VASD) is obtained by setting the horizontal curve radius to a very large value. The obtained ASD is 158.900 m while the 2D ASD obtained from AASHTO 2001 equation is 158.843 m with an error of 0.4%. The ASD caused by only the horizontal curvature of the roadway (HASD) is obtained by setting the vertical curvature to a flat grade. The obtained HASD is 213.995 m, while the 2D ASD using AASHTO 2001 equation yields an ASD of 214.102 m with an error 0.05%. The error is insignificant and the algorithm as set forth is considered accurate. For further validation of the developed algorithm, a program was written for a simple visualization of the roadway/roadside elements that can be factored in the sight distance model. The program allows for generating a perspective view of the road from the driver eye. The furthest visible point obtained from the analytical model is located on the road. Hence, the visibility of that point can be verified. Figure 3.3 show a sample visualization and verification. The program code is presented in Appendix II.
Sight distance profiles are developed for the alignment configurations presented in Table 3.1. The difference between 3D ASD and the shortest of HASD and VASD is denoted 2D/3D difference. 2D/3D difference is calculated as follows:

$$2D/3D \text{ difference} = (3D \text{ ASD} - \text{Min}\{HASD, VASD\})/3D \text{ ASD} \times 100$$

Negative 2D/3D difference represents the critical case when the 2D ASD models overestimate the actual sight distance permitted by the road environment. The difference between HASD and VASD is denoted 2D/2D difference and is positive for VASD > HASD. There are two main conclusions drawn from this case study:
1. The 3D ASD is generally close to the shortest of the two 2D sight distances as shown in Figure 3.4, and

2. 2D/3D difference is relatively high at the exit segments of the ASD profile and it is steadily low at the interior zone of the ASD profile, as shown in Figure 3.5. The interior zone is where the shorter 2D ASD flats at the minimum value.

The first observation led to examining the sensitivity of the 3D ASD toward the geometric feature that possesses the shortest 2D ASD (e.g. Horizontal curve radius when the shortest 2D ASD is HASD). Figure 3.6 shows the effect of the clearance zone grade on the available sight distance for different horizontal curve radii. It is observable that the shift between the horizontal curve and vertical curve start points is zero and the driver location is at the curves start point.

It can be inferred from Figure 3.6 that the sensitivity of ASD to horizontal curve radius is minimized by decreasing the clearance zone slope. This is consistent with the first observation, given the fact that for steeper clearance zone slopes, the shortest 2D ASD tends to be HASD. Moreover, for the shortest horizontal curve radius (200 m), the influence of the clearance zone slope is more pronounced than the largest curve (800 m). This is consistent with the first observation because, expectedly, the larger the horizontal curve, the more likely it is that the vertical curvature is controlling the sight distance availability.

The influence of changing the median barrier height is examined at different values of vertical curvature. The median barrier can obstruct the line of sight only through a horizontally curved segment. Intuitively, the effect of median barrier height is similar to the clearance zone slope value in that the higher the median barrier, the more influential the horizontal curve radius is. Figure 3.7 shows the effect of median barrier height on the available sight distance at variable vertical curvature.
FIGURE 3.4 3D Sight Distance Profile Accompanied by Elementary Profiles of The Vertical Curve only (VASD, a line of sight is obstructed by the roadway surface) and the Horizontal Curve only (HASD, a line of sight is obstructed by the side slope).
Figure 3.5 Flow 2D/3D Difference Profile for Two Widths of Clearance Zone at Different Horizontal Curve Radii.

Figure 3.6 Effect of the Clearance Zone Grade on the Available Sight Distance for Different Horizontal Curve Radii
The driver location is at the start point of the horizontal curve. The alignment configurations are identical to those in Table 3.1. As shown in Figure 3.7, the vertical curvature of the roadway has less influence on high median barriers. The previous set of observations shaped the main premises behind the choice of an analytical tool to determine the necessity of conducting a 3D analysis instead of using the 2D ASD values.

### 3.1.7 Critical Combination of Horizontal and Vertical Alignment

The presented algorithm can be effectively used in checking the safety standard of sight distance as provided by existing alignments. It was found that for some alignment configuration, the 3D sight distance may exceed the 2D sight distance obtained from AASHTO. From a safety point of view, the critical case occurs when 3D ASD is shorter than the 2D ASD within the interior zone of the sight distance profile. The overestimation of 3D ASD outside the interior zone is not taken into account in the following analysis because the ASD values are relatively long, see Figure 3.4. Accordingly, it is required to develop an analytical tool, or a mathematical condition, that can determine the situations at which there is a need for conducting a 3D ASD analysis. For the base case presented in Table 3.1, 2D/3D differences are calculated for a range of radii, as presented in Table 3.2. The relationship between the horizontal curve radius and the 2D/3D difference is shown in Figure 3.8.

The curve presented in Figure 3.8 passes two peaks: curvature peak (CP) and obstruction feature peak (OFP). CP is evidently located at the horizontal curve radius that makes the two 2D ASDs equal (VASD = HASD). OFP is located at the curve radius that features the transfer of the line of sight obstruction by one of the roadway elements, i.e. from the roadway surface or the clearance zone to the side slope.

The curve passes the origin because for extremely small horizontal curves the 3D ASD tends to HASD. The curve asymptotically tends to zero for horizontal curve radii longer than that of OFP, i.e. 3D ASD tends to VASD. It is beyond the scope of this thesis to derive the necessary equations to calculate the 2D/3D differences at CP and OFP.
Figure 3.7 Effect of Median Barrier Height on the Available Sight Distance at Variable Vertical Curvature.

Table 3.2 The Range of Variation of the Alignment Configuration Parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal curve radius (m)</td>
<td>300, 400, 500, 700, 900, 1100, and 1900.</td>
</tr>
<tr>
<td>Clearance zone width (m)</td>
<td>1.0, 3.5, 7.0, and 15.0</td>
</tr>
<tr>
<td>Side slope value (%)</td>
<td>50, 66, and 80</td>
</tr>
<tr>
<td>Vertical curvature (m/%)</td>
<td>25 and 37</td>
</tr>
<tr>
<td>Vertical curve – horizontal curve shift (m)</td>
<td>-50, 0, and 50. (The positive sign is in the direction of station increase)</td>
</tr>
</tbody>
</table>
Figure 3.8 The Relationship between Horizontal Curve Radius and the 2D/3D Difference.
However, CP can be calculated by solving for the horizontal curve radius that makes VASD and HASD equal. In fact, CP is more critical from safety point of view because the highest overestimation of the ASD value occurs at it. It can be inferred from Figure 3.8 that increasing the horizontal curve radius will result in increasing HASD with respect to VASD. Intuitively, increasing the vertical curvature would result in an opposite effect. The replacement of the horizontal curve radius axis in Figure 3.8 with the vertical curvature is expected to yield an inverted trend. Several alignment configurations were analyzed and the results shown in Figure 3.9 validate the anticipated curve trend. Additionally, it is shown that CP and OFP replaced their location relative to each others.

Figure 3.10 shows the 2D/3D difference and 2D/2D differences calculated for various alignment configurations and driver locations. The driver locations are distributed within and without the interior zone of each alignment configuration. It was found that for positive 2D/2D differences; almost all 2D/3D differences are negative. Accordingly, for positive 2D/2D differences there is evidently high necessity for conducting 3D analysis. For negative 2D/2D differences there is still proportion of the analysis points that requires 3D analysis. For this half-space, no conclusive rule can be stated in light of this analysis. However, for the points with negative 2D/2D, those located close to CP are more likely to possess negative 2D/3D differences. Accordingly, for the points far from CP that possess negative 2D/2D differences, there is evidently low necessity for conducting 3D analysis.

It is worthwhile to mention that there is a disagreement in the results of previous research on the difference between 2D and 3D sight distances. Sanchez (1994), for example, comparing the 3D sight distance and 2D sight distance for an interchange connector, reported, "... the difference between 2-D and 3-D SSDs is very small". In light of this research, this conclusion can not be generalized for all alignment configurations or driver locations. Driver locations outside the interior zone frequently show a relatively large difference between 2D and 3D sight distances. Moreover, the analysis points located within the interior zone that are close to CP may possess difference close to 10%.
Figure 3.9 The Relationship between Vertical Curvature and the 2D/3D Difference at Different Curve Radii R (m).
Figure 3.10 The Distribution of Analysis Points with Respect to 2D/3D Difference Signs.

Remarkably different from the previous conclusion, Hassan et al. (2000) stated that, “2D and 3D designs may differ significantly”. The significant difference between the 2D and 3D sight distances that Hassan et al. reported can be explained by the way the experiment was conducted, where the design parameter is varied (e.g. horizontal curve radius) until the 3D sight distance is equal to the required stopping sight distance. However, according to the previous analysis, if at the very driver location the sight distance is close to VASD, the horizontal curve radius can be changed significantly without impact on the ASD. As well, the vertical curvature can be reduced significantly without affecting the 3D ASD if the latter is the shorter of the two 2D sight distances. In light of this research, the results presented by Hassan et al. (2000) can be valid only for points close to CP.
3.2 SUMMARY

This chapter presented a new algorithm for evaluating 3D sight distance. The developed algorithm was based on a parametric representation of the roadway and roadside features. The algorithm is considered more efficient than the existing approaches and more flexible in modeling complicated roadside features. The algorithm was verified by several case studies. The influence of various configuration parameters on the 3D ASD was examined. It was found from these case studies that the 3D ASD is generally close to the shorter of the two 2D sight distances. The 3D ASD is evidently anticipated to be less than the 2D ASD if the 2D/2D difference value is positive - hence a 3D analysis is necessary. On the other hand, it is unlikely that the 3D analysis is necessary if the 2D/2D difference value is negative and the alignment configuration corresponds to a point far from the curvature peak (CP).

It was hypothesized that the difference between the 3D ASD and the shorter 2D sight distances is more pronounced at the alignment configurations with close 2D ASD values, i.e. close to CP. It is noteworthy that no mathematical proof was derived for the previous hypothesis. The location of CP and obstruction feature peak (OFP) points and the 2D/2D difference values at these points are important for an accurate assessment of the necessity of 3D analysis. A potential continuation of the analysis presented in this chapter is to derive the equations required to define these two points. This may be complicated and it is recommended that an empirical approach for defining CP and OFP is tried before a closed-form approach is sought.


4 CODE-CALIBRATION MODEL

This chapter presents an analytical model for regulating design risk level. The first section starts with presenting background information, and later discusses the set of concepts that underlies further analysis. The second section includes a general framework for calibrating standard geometric design models. The third section presents an application of the proposed calibration framework to the standard design model of crest vertical curves located on a tangent. Based on the sight distance model developed in chapter three, the fourth section presents a similar application to crest vertical curves on horizontal curves.

4.1 METHODOLOGY OF CODE-CALIBRATION

Let \( \beta_i \) be the target reliability index that represents an acceptable risk level, \( D_f \) be the feasible scope of input parameters that will be considered in the calibration process, and \( U_i \) be the scope of non-standard input parameters. In keeping with the proposition of risk consistency discussed in chapter two, a penalty function is used to quantify the difference between each \( \beta_i \) associated with design output \( i \) and \( \beta_i \). It is noteworthy that penalty function is a key element in the calibration process that can integrate socio-economic aspects of design. Penalty function can be formulated in order to differentiate between cases of overdesign and underdesign (Lind, 1977). For example, the following formulation takes into account construction and running costs of a highway element:

\[
p = P_i \left( \beta - \beta' \right) + \int_0^T \left( \beta - \beta' \right) e^{-rt} \, dt
\]

where

- \( T \) = expected number of years after which the geometric design may be reviewed,
$P_c$ = construction cost as a function of the overall design risk level,
$U_s$ = estimated annualized collision costs, and
$r$ = estimated continuous discount rate during the next $T$ years.

For demonstration purpose the penalty function used in the next analysis is assumed to be symmetrical as follows:

$$p = \sum_{i \in D} (\beta_i - \beta) ^ 2$$  \hspace{1cm} 4.2

In order to minimize the penalty value, a calibration factor $C_i$ is added to the standard design model in order to make risk level close to $\beta_i$. In order to avoid zero values, an exponential form is assumed as follows:

$$C_i = e^{A D_i + B U_i}$$  \hspace{1cm} 4.3

where $A$ and $B$ are vectors of constants multiplied by corresponding input parameters:

For simplicity, a single calibration factor is multiplied by the input parameter $x^* \in X$ that possesses the highest correlation with $\beta$. The calibration model is simply the following minimization problem:

$$C_i = Min \{p\} \quad ;\forall \; X = \{x_1, \ldots, x^* \cdot C_i, \ldots, x_n\}$$  \hspace{1cm} 4.4

The main steps of the calibration process are summarized as follows (Melchers, 1999):

1. Define the scope of the code, which is the range of input parameters that will be considered in the calibration process. Input parameters beyond this range are not necessarily expected to yield a consistent risk level,
2. Select design cases from the scope of the input parameters,
3. Select a target reliability index $\beta'$,
4. Conduct reliability analysis for the design cases to find the corresponding risk levels, and
5. Select calibration factors that will minimize the scatter of the reliability indices around the target value.

4.1.1 Calibration of 2D Crest Vertical Curve Design Model

This section presents an application of the previous calibration methodology to the design model of crest vertical curve located on a tangent. The 2D projection of the vertical curve is a sufficient space of calculation in absence of horizontal curvature. It is noteworthy that the reconstruction of the design charts presented in this section is for demonstrative purpose. The accuracy of the calibrated design charts is heavily contingent on the statistical distributions and the various assumptions included in the calibration process.

4.1.1.1 Source of Uncertainty in Standard Design of Crest Vertical Curves

The following sections discuss the selection of the statistical distributions that quantify uncertainty in the input parameters.

4.1.1.1.1 Perception Reaction Time:

The perception and reaction time (PRT) can be divided into five durations: latency, eye movement, fixation, recognition, and initial brake reaction (Hooper & McGee, 1983). The PRT values were found to range from 1.5s to 3.0 s depending on traffic density (Neuman, 1989). Other researchers investigated the relationship between PRT and design speed (McGee, 1989). The importance of considering the statistical distribution of PRT instead of the mean was raised by many researchers, e.g. Chang et al. (1985), Taoka (1989), and Schweitzer et al. (1995). Table 4.1 shows different reported values of the mean and standard deviation of PRT under unexpected objection conditions. The realism
of the test conditions for PRT studies, the unexpectedness of the objects placed on the road, the \textit{a priori} knowledge of the driver about test conduction, the stimulus that triggers braking, and the technique of marking brake initiation were questioned by several researchers (Hooper & McGee, 1983; Schweitzer et al., 1995; Green, 2000). In order to address these concerns, braking stimulus should be visual and the braking initiation should be measured at the onset of pressing the braking pedal, which counts the movement time from the gas pedal. The values from Lerner (1995) are considered in the current application due to their consistency with other studies and the acceptable unexpectedness of braking employed in this study.

**Table 4.1 Perception and Reaction Time Estimates from Different Studies**

<table>
<thead>
<tr>
<th>Study</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>No. of Samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sivak et al. (1982)</td>
<td>1.21</td>
<td>0.63</td>
<td>1644</td>
</tr>
<tr>
<td>Wortman et al. (1983)</td>
<td>1.3</td>
<td>0.6</td>
<td>839</td>
</tr>
<tr>
<td>Chang et al. (1985)</td>
<td>1.3</td>
<td>0.74</td>
<td>579</td>
</tr>
<tr>
<td>Olson et al. (1986)</td>
<td>1.1</td>
<td>0.15</td>
<td>49</td>
</tr>
<tr>
<td>Lerner (1995)</td>
<td>1.4</td>
<td>0.4</td>
<td>56</td>
</tr>
</tbody>
</table>

**4.1.1.1.2 Deceleration:**

The constant equivalent deceleration of drivers during braking replaced the old pavement friction coefficient for calculating SSD. Fitzpatrick et al. (2000) conducted a study for finding a design value of the deceleration rate. The measurements from vehicles without ABS, on wet pavement, and for expected objects were used. The deceleration rates for unexpected objects were reported as greater than the expected objects; hence, the later is adopted in AASHTO \textit{Green Book} 2001 and in this analysis.

**4.1.1.1.3 Driver Eye Height:**

Hammond (1971) proposed a statistical approach for calculating the distance from a reference point to the driver eye for a group of drivers. Olson (1986) employed this statistical approach for using the 95\textsuperscript{th} percentile value in the previous releases of
AASHTO. Fitzpatrick et al. (2000) measured object eye heights, taillights, and headlights for 1,318 vehicles and reported the mean and variance of the results. These findings were used to update AASHTO design values to count for the change in vehicle dimensions since the last study. These findings are the most recent and hence are used in the current application.

4.1.1.4 Operating Speed:
Operating speed is a driver choice that is entirely influenced by the driving conditions and the driver characteristics. Several studies investigated the difference between operating speed and inferred design speeds for highway geometric elements. The operating speed on crest vertical curves was found to be higher than the inferred design speed of 80 k/h and 95 k/h, while lower for design speed of 110 k/h (Messer et al., 1981; Jessen et al., 2001; Fitzpatrick et al., 2000). The 85th percentile speed and the corresponding mean square error, obtained from Fitzpatrick et al. (2000), are used in this analysis to calculate the mean and standard deviation of operating speed assuming a normal distribution. The approximation entailed by the assumption that the standard deviation of the 85th speed is equal to that of the population is hereby acknowledged.

4.1.1.2 Defining input parameters
Crest vertical curves are designed to provide adequate sight distance in order to allow the driver enough time for specific operational decisions. The length of this sight distance depends on the complexity of the driving decision. The current calibration model will consider only stopping sight distance and the analysis can similarly be applied to other sight distance requirements. The deterministic inputs are vertical curve length $L$ and algebraic difference $A$. The probabilistic input parameters are provided in Table 4.2. In order to implement the risk analysis, the minimum available sight distance is compared to the required stopping sight distance. The following SSD model is used:
\[ SSD = 0.278 \cdot V \cdot PRT + \frac{0.039 \cdot V^2}{a} \]

where

\( V \) = operating speed (k/h), and

\( a \) = the acceptable constant deceleration during stopping (m/s\(^2\)).

### Table 4.2 Distribution Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Distribution</th>
<th>Design Values</th>
<th>Percentile Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRT</td>
<td>1.5 s</td>
<td>0.4 s</td>
<td>Log Normal (^{a})</td>
<td>2.5 s(^d)</td>
<td>98.1</td>
</tr>
<tr>
<td>Driver Eye Height</td>
<td>1.14 m</td>
<td>0.055 m</td>
<td>Normal (^{b})</td>
<td>1.08 m</td>
<td>10.4</td>
</tr>
<tr>
<td>Driver Deceleration</td>
<td>4.2 m/s(^2)</td>
<td>0.6 m/s(^2)</td>
<td>Normal (^{b})</td>
<td>3.4 m/s(^2)</td>
<td>9.1</td>
</tr>
<tr>
<td>Object Height</td>
<td>-</td>
<td>-</td>
<td>Deterministic</td>
<td>0.6 m</td>
<td>-</td>
</tr>
<tr>
<td>Operating Speed (^{e})</td>
<td>105.1-149.69 A/L</td>
<td>5.57 k/h</td>
<td>Normal (^{c})</td>
<td>-</td>
<td>85</td>
</tr>
<tr>
<td>Operating Speed (^{f})</td>
<td>103.24 - 3676 / R</td>
<td>4.47 k/h</td>
<td>Normal (^{c})</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>


\(^{e}\) Crest vertical curves with k \(\leq\) 43.

\(^{f}\) Crest vertical curves with k \(\leq\) 43 combined with horizontal curves.

Several studies in the literature used design speed instead of operating speed in reliability model analysis (Navin, 1990; Easa, 1993 & 1999). However, it is more meaningful to use operating speed in formulating reliability models that involve highway design. This is due to the fact that drivers build their selection of a comfortable and safe driving speed based on their own interpretation of the specific conditions of the roadway and roadside environment. Accordingly, operating speed reflects some human factors and it should be taken into account along with other driver-dependent parameters, e.g. PRT and deceleration rate. The model uncertainty considered in the current application is caused by the variation of the roadway longitudinal grade during braking action (Taignidis, 1998 & 2001). The value of the entering grade \( g \) is taken as an input parameter that compensates for the lack of calculating the exact required stopping sight distance. The required SSD is calculated by means of summing braking distances over finite constant grades. The speed reduction during these increments is kept constant at 0.1 k/h. SSD is evaluated as follows:
SSD = 0.278 \cdot V \cdot PRT + \sum_{i=0}^{n} \frac{2V_j \cdot \Delta V}{25.92 \cdot \left( a + \frac{g_i \cdot 9.81}{100} \right)}

where \( V_j = V_i - i \cdot \Delta V \), \( \Delta V \) is the speed increment taken as 0.1 k/h, \( g_i \) is the longitudinal grade at the point on the vertical curve that corresponds to iteration \( i \), and \( n \) is the last iteration at which \( V_i \leq 0 \).

The minimum available sight distance (ASD) is calculated as follows:

\[
ASD^* = (200 \cdot k \cdot h_1)^{0.5} + (200 \cdot k \cdot h_2)^{0.5}
\]

\[
ASD = \begin{cases} 
 ASD^* & \text{if } L > ASD^* \\
 L + 200 \cdot \frac{k}{2L} \left( \sqrt{h_1} + \sqrt{h_2} \right)^2 & \text{if } L \leq ASD^* 
\end{cases}
\]

where \( h_1 \) is the driver eye height (m), \( h_2 \) is the object height (m), \( k \) is the rate of vertical curvature which equals \( L/A \).

The initial grade \( g_0 \) at which a vehicle starts braking is calculated as follows (Taignidis, 2001):

\[
x_M = \begin{cases} 
 L \cdot \sqrt{h_1} - \frac{L - ASD}{100 \cdot k \cdot \sqrt{h_1} \left( \sqrt{h_1} + \sqrt{h_2} \right)} & \text{if } L > ASD^* \\
 \frac{L - ASD}{100 \cdot k \cdot \sqrt{h_1} \left( \sqrt{h_1} + \sqrt{h_2} \right)} & \text{if } L \leq ASD^* 
\end{cases}
\]

\[
g_0 = \begin{cases} 
 g_2 & \text{if } x_M + 0.278 \cdot V \cdot PRT > L \\
 g - (x_M + 0.278 \cdot V \cdot PRT) \frac{L}{A} & \text{if } x_M + 0.278 \cdot V \cdot PRT \leq L 
\end{cases}
\]

It is important to note that the previous calculations imply that a potential object that triggers braking is located such that the object is viewed when the vehicle is at the point of minimum ASD. The design value of the vertical curve length is obtained as follows:
4.1.1.3 Target Reliability Index

The selection of $\beta_i$ is analogous to the determination of an acceptable quality of design for specific highway elements. The latter is an important decision that relies on the general policy of highway design. In other areas of civil engineering, several methods that are developed for the determination of $\beta_i$ involve minimizing the expected societal cost as well as construction cost (Ditlevsen, 1997; Vrijling et al., 1998).

Another approach for calculating $\beta_i$ value is proposed as an alternative to quantifying a relationship between $\beta$ and collision frequency. In this approach, for a relatively large group of sites, a multi-criteria assessment of the design quality can be undertaken. The criteria of evaluation may include: collision record, the ratio between actual construction cost and construction cost of the vertical curve had it been dimensioned at design lengths, an expert evaluation of the in situ driving conditions, and an expert assessment of the cost-effectiveness of design. The sites can be ranked according to the previous evaluation. A percentile ranking has to be selected in order to define the pool of sites that possess acceptable cost for society. The target reliability index $\beta_i$ can be calculated based on the average $\beta$ values that are inferred for the acceptable pool of sites.

The main shortcoming of the previous method is that, because it relies on existing vertical curves, it is difficult to study the cost-effectiveness of the expectedly small percentage of vertical curves that are dimensioned lower than standard requirements. The
main advantage of the previous method is that it offers a rational approach for integrating subjective risk assessment with some aspects of cost-effectiveness into standard highway design.

As a first step toward code-calibration, $\beta_i$ can be selected based on current design standards. In this respect, $\beta_i$ can readily be taken as the average of current design safety levels. One of the important issues related to the application code-calibration in highway design is whether $\beta_i$ is specific for each design speed. The exact definition of design speed and the methods used for its selection was the focus of many studies (Barnett, 1936; Leisch et al., 1977; McLean et al., 1976 & 1978 & 1979). It is evident that the difference between operating speed and design speed is recognized in the literature (McLean, 1978; Messer et al., 1981; Krammes et al., 1994; Fitzpatrick et al., 2000; Jessen et al., 2001). The definition of design speed evolved from being considered as a maximum safe speed to a selected design parameter used principally in dimensioning various highway features and reflects the general quality of highway design (Fitzpatrick et al., 2003). Based on the subjective interpretation of $P_{nc}$, $\beta_i$ is proportional to design speed and can be viewed, in part, as a probabilistic surrogate for design speed. Hence, $\beta_i$ can be calculated for each design speed as follows:

$$\beta_i = -\Phi^{-1}\left(\sum_{n \in D_{jV}, U_{jV}} \frac{P_{nci}}{n}\right)$$

4.10

where $D_{jV}$ and $U_{jV}$ are the feasible scope of input parameters at design speed $V$, and $P_{nci}$ is the probability of non-compliance at a design point $i$.

4.1.1.4 Pre-Calibration Distribution of Design Safety Levels

The performance function of the stopping sight distance is as follows:

$$g = ASD - SSD$$

4.11
The calibrated input parameter $x^*$, by which the calibration factor is multiplied, is selected to be SSD. The uncertainty in ASD arises from the variation of driver eye height, while the uncertainty in SSD arises from the variation of stopping deceleration, operating speed, varying grade, and PRT. Due to the non-normality of PRT, FORM analysis is used to calculate reliability indices. For simplicity, the input parameters are assumed to be statistically independent. A MATLAB program was written for an automated calculation of Equations 4.5 - 4.9. For 341 design cases, $\beta$ values are calculated and their distribution is presented in Figure 4.1. The effect of model uncertainty is summarized in Figure 4.2. It can be inferred that entering grade has a considerable effect on $\beta$ values. The small non-linearity in the curves represents the difference between FORM and FOSM analysis. Any increase in non-linearity indicates the inaccuracy of FOSM analysis in the current calibration model.

Figure 4.1 shows that 33% of the evaluated points possess reliability indices less than zero, i.e. $P_{nc} < 0.5$. This finding underscores the concern about the feasibility of selecting the standard design values, shown in Table 4.2, at relatively high percentile values. These relatively high risk values are essentially due to the large difference between the mean values of operating speed and design speed at approximately design speeds less than 90 k/h. This difference is further amplified by considering the effect of grade variation at vertical curves. In absolute terms, it is difficult to determine, based on the relatively simplified analysis in the current application, whether curves with high design speeds are over-designed, or those with low design speeds are under-designed. However, it can be stated that curves designed at high design speeds possess a disproportionately low design risk as compared to curves designed at low design speeds. For example, the average $P_{nc}$ of curves designed at 90 km/h is 5.9 times higher than curves designed at 70 km/h.

This standardized bias in risk level may result in an over-relaxation of design requirements at high design speeds. Therefore, the influence of design-related characteristics on driving conditions becomes more marginalized at high design speeds - leaving more space for confounding or unforeseeable effects.
Figure 4.1 Reliability Indices Scatter before Calibration of 2D Design Model.

Figure 4.2 The Relationship between Reliability Indices and the Inadequacy of Sight Distance at Various Design Speeds. Each Point on the Curves Represent a Value for Entering Grade Value. The Latter Ranges from 9% to -6%.
It can be hypothesized that the evident overdesign at high speeds can explain in part the random and inconclusive relationship between limited sight distance and collision rate. In the studies that dealt with safety implications of limited sight distance on crest vertical curves, (State of the Art Report, 1987; Urbanik et al., 1989), the criteria used for the definition of limited sight distance were based on AASHTO standard requirements. However, based on the previous hypothesis, they may not factually be limited.

4.1.1.5 Discussion of Results

Based on the method described in Equations 4.1-4.3, the calibration factor $C_i$ is calculated as follows:

$$C_i = e^{a_1+b_1+c_1V}$$

where $a, b, c$ are calibration coefficients.

Table 4.3 shows the results of applying the minimization process in Equation 4.4. Negative calibration coefficients are associated with input parameters that are directly proportional with pre-calibration $\beta$ values. The signs of the calibration factor are consistent with the previous argument. The absolute values of the calibration coefficients are proportional with the pre-calibration scatter in $\beta$ values. The results in Table 4.3 are consistent with the previous argument given that the pre-calibration variance of $\beta$ values is 0.004 at design speed 70 km/h while it is 0.0008 at design speed 100 km/h.

The sensitivity of the penalty function is presented in Table 4.3. The sensitivity is calculated by independently perturbing the input parameters by 1% and observing the percentage of change in the minimum value of the penalty function. As shown in Table 4.3, the penalty function is comparatively sensitive to all calibration coefficients. It can be concluded that the selection of the parameters in the calibration process was feasible.
### Table 4.3 Calibration Coefficients

<table>
<thead>
<tr>
<th>Design Speed (km/h)</th>
<th>Target Reliability</th>
<th>Calibration Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$a \left(10^2\right)$</td>
</tr>
<tr>
<td>70</td>
<td>-1.14</td>
<td>-3.23</td>
</tr>
<tr>
<td>75</td>
<td>-0.68</td>
<td>-2.80</td>
</tr>
<tr>
<td>80</td>
<td>-0.28</td>
<td>-2.23</td>
</tr>
<tr>
<td>85</td>
<td>0.17</td>
<td>-2.08</td>
</tr>
<tr>
<td>90</td>
<td>0.67</td>
<td>-2.00</td>
</tr>
<tr>
<td>95</td>
<td>1.19</td>
<td>-1.71</td>
</tr>
<tr>
<td>100</td>
<td>1.73</td>
<td>-1.58</td>
</tr>
<tr>
<td>Average Sensitivity Coefficients</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

*The average sensitivity factors for the given range of design speeds is benchmarked to the sensitivity of coefficient $a$ for comparison.

As a sample, the before-and-after distribution of $\beta$ values at design speed 70 km/h is presented in Figure 4.3 and Figure 4.4. The overall distribution of $\beta$ values after calibration is presented in Figure 4.5. Using the calibration factors shown in Table 4.3, the AASHTO design chart is reconstructed as shown in Figure 4.6 at an entering grade of 0%. Similar design charts can be produced for other values of entering grade. In order to compare design outputs before and after calibration, the vertical curve lengths obtained from both cases within the scope of input parameters are presented in Figure 4.6. It is noteworthy that the percentage of change in curve length due to calibration ranges from 36.5% to -24.4% with an average 1.4%. The insignificance of the change in the average curve length can be explained by selecting $\beta$, based on the average $\beta$ values before calibration. The vertical grouping of the points shown in Figure 4.7 is due to neglecting slope variation in calculating the pre-calibration curve lengths. The distribution of $\beta$ values for a complete list of design speeds is included in Appendix IV.
Figure 4.3 Sample Distribution of Reliability Indices before Calibration of 2D Design Model. Design Speed is 70 km/h.

Figure 4.4 Sample Distribution of Reliability Indices after Calibration of 2D Design Model. Design Speed is 70 km/h.
Figure 4.5 Reliability Indices Scatter after Calibration of 2D Design Model.

Figure 4.6 Reconstructed 2D Design Chart after Calibration. The Value of Entering Grade is 0% for all Design Speeds.
4.1.2 Calibration of 3D Crest Vertical Curve Design Model

The main objective of this section is to reconstruct the design model of crest vertical curve such that it accounts for the effect of horizontal curvature. Based on the hypothesis presented in chapter three, the most critical combination of horizontal and vertical curves can be obtained. The critical combination of horizontal and vertical curves provides the least available sight distance. The components of horizontal curvature needed for the composition of the most critical combination are: the horizontal curve radius, the dimensions of cross section elements, and the side slope value. Several design charts can be constructed for a feasible range of horizontal curvature components. For demonstration, the most critical combination will be calculated based on the base value presented in Table 3.1. The operating speed model is presented in Table 4.2, where the most significant input parameter is horizontal curve radius. The elimination of vertical curvature from the operating speed model is in part a result of the fact that horizontal curvature significantly controls driving behavior than vertical curvature. As a result of
superimposing horizontal curvature on the highway alignment, operating speeds are
closer to design speeds as compared to the presence of only vertical curvature.
Accordingly, the $\beta$ values presented in Table 4.3 are adopted in the calibration of the 3D
design model. The main difference between 2D and 3D calibration is that:

1. ASD is calculated based on the algorithm presented in chapter 3 and Appendix I.
The selection of horizontal curvature components is based on the hypothesis
presented in chapter three,

2. Operating speed is calculated according to Table 4.2 for a combined crest vertical
curve and a horizontal curve,

3. Without compromising accuracy, the convergence measures of FORM analysis
are relaxed to reach to a reliability index value in a time efficient way, and

4. The calculated 3D ASD is calculated to the nearest 10 cm. Accordingly, the step
length in calculating numerical differentiations is increased in order to capture
any variation in the limit state function.

Figure 4.8 and Figure 4.9 show the distribution of $\beta$ values before and after calibration
respectively. Figure 4.10 shows the reconstructed design charts for combined vertical and
horizontal curves.

Figure 4.8 Reliability Indices Scatter before Calibration of 3D Design Model.
Figure 4.9 Reliability Indices Scatter after Calibration of 3D Design Model.

Figure 4.10 Reconstructed 3D Design Chart after Calibration. The Value of Entering Grade is 0% for all Design Speeds.
For the 3D design model, the distribution of $\beta$ values for a complete list of design speeds is included in Appendix V. Figure 4.11 shows a comparison between the calibrated vertical curve lengths obtained from 2D and 3D design models. The decrease in operating speed due to the superimposition of horizontal curvature predominates over the corresponding reduction in ASD. It is manifested in Figure 4.11 that vertical curve lengths obtained from 2D calibrated design model are longer than those obtained from 3D design model.

![Figure 4.11 The Relationship between Calibrated Vertical Curve Lengths Obtained from 2D and 3D Design Models.](image)

**4.2 SUMMARY**

The analysis presented in this chapter involved a distinct safety level, namely design safety, or conversely design risk. The presence of this risk level stems from the uncertainty in the design model and the input parameters. Navin (1990) proposed a framework for calculating the probability of non-compliance to design requirements. The probability of non-compliance is used in this chapter as a measure of design safety. The main premise that underlies the analysis presented in this chapter is that the risk level
associated with standard design outputs has to be consistent and close to a premeditated level. The previous premise is addressed by means of calibrating the standard design models. The calculation of design risk is based on reliability analysis. Due to the reliance of reliability analysis on several initial assumptions, the code-calibration process is a responsibility of the code-developer. A theoretical discussion was presented in order to reach a pertinent interpretation of the probability of non-compliance within the context of highway geometric design.

A general method for code-calibration was presented. The method is based on multiplying some input parameters by calibration factors. The mathematical form of the calibration factors was constructed so that it compensates for the pre-calibration distribution of risk level. The degree of distribution of design risk level can be quantified by means of a penalty function. One form of the penalty function was proposed such that it is possible to integrate socio-economic factors into standardized design. In this form, the penalty is the present value of the estimated collision cost in addition to construction cost. The calibration method requires a target or acceptable risk level be determined. A general method for selecting target risk level was proposed. The method is based on evaluating the quality of design of a representative group of existing sites. The target risk level can be calculated as the average of a specific percentage of the representative group that are deemed as exhibiting an acceptable and cost-effective safety level. The method offers an approach for integrating subjective evaluation of risk into standard design. A preliminary method for selecting target risk levels was proposed and was adopted in the ensuing analysis. It was suggested that there is a unique target risk level for a selected design speed.

In principle, the proposed calibration method can be applied to all geometric design models. As a case study, the proposed calibration method was applied to the standard design model of crest vertical curves located on tangents as well as horizontal curves. The calculation of available sight distance for combined horizontal and vertical curves was performed according to the sight distance model presented in chapter three. The statistical distributions of the input parameters were taken from relevant studies in the
literature. The input parameters were assumed to be statistically independent, despite the possibility of correlation, e.g. perception reaction time and deceleration rate. The distribution of the pre-calibration probability of non-compliance is relatively wide, ranging from 7.7% to 98.6%. In addition, there is evidence that the higher group of design speeds exhibit disproportionately low risk level without any corresponding difference in collision rate. It is suggested that the curves at higher design speeds are over-designed – rather than curves at lower design speeds are under-designed. The previous suggestion cannot be emphasized solely based on the analysis presented in this chapter without empirical validation. However it may be in part supported by the absence of a conclusive relationship between available sight distance on crest vertical curves and both collisions and operating speed in the literature.

Based on the foregoing analysis, two main research areas are potentially important for code-calibration: acceptable risk and statistical distributions. The literature shows several studies about the determination of socially and individually acceptable public risk in hazardous engineering projects. However, further research is required in the area of road safety. The importance of quantifying the randomness in design parameters by means of statistical distributions should receive more focus. It is found that the majority of studies that involve field measurements of design parameters focus on single percentile value, e.g. mean or 85th percentile, rather than the entire statistical distribution.
5 SUMMARY AND CONCLUSIONS

5.1 SUMMARY

Road safety is a paramount area of study that stems its significance from the societal cost associated with road collisions as well as the relatively large investments allocated for highway projects. The importance of considering various sources of risk in highway design relies on two facts: the considerable cost for the society that results from road collisions, and the relatively large capital cost of highway projects. For example, the estimated annual cost of road collisions in Canada is $25 billion. In addition to monetary losses, road collision is the leading cause of mortalities due to injuries, accounting for 22.8% of the global total. Capital cost in highway projects is traditionally large due to the numerous engineering tasks needed for design and construction. As a result, cost effectiveness emerged as a fundamental means of assessing the economic feasibility of design concepts and construction methods. The integration of safety into standard highway geometric design is a widely reported issue of discussion in the literature. Based on foregoing discussions presented in the course of this thesis, it was found that several shortcomings still exist in the current practice that hinder a comprehensive formulation of a risk-conscious standard design. The main objective of this thesis is to provide theoretical concepts and constructs as well as illustrations for an analytical framework that accommodates further development and formulation of a risk-based highway geometric design.

The state of knowledge and practice in regard to road safety breaks down to proactive and reactive methods of safety assessment. Proactive methods emerged as an important approach that can effectively guide various design decisions in predicting road safety for future designs. Road safety prediction in the current practice is based on either expert assessment or collision prediction models. Some practical cases are presented in the
thesis in which designers faced considerable difficulty in delivering a quantified evaluation of proposed highway projects. This in part is due to the absence of relevant collision prediction models. However, the main reason is that safety level in the current design standards is largely implicit and unknown. This argument has been repeatedly reported in the literature. A parallel safety evaluation method was proposed in the literature in order to formulate a quantified safety evaluation. The alternative safety evaluation is based on calculating the propagation of various sources of uncertainty throughout design models. The aggregate level of uncertainty in design outputs, represented by the mathematical probability that a design case does not comply with design requirements, can act as an index of the general quality of the design.

A theoretical discussion was provided in order to differentiate the classical definition of safety from the alternative probabilistic safety. The term “design safety”, as a distinction from operational safety that conventionally involves collisions, was proposed to describe probabilistic safety. Several sources of uncertainty were discussed in the thesis. The levels of design safety in the current design standards were calculated based on selecting relevant statistical distributions for design inputs. The calculation of uncertainty propagation was implemented according to reliability theory. A brief introduction to the fundamentals of reliability theory was presented. Based on the results obtained from reliability analysis, it was evident that design safety level associated with standard design outputs is inconsistent. Moreover, the method for selecting design values for inputs was found to be largely based on adopting conservative percentile ranking. However, it was shown that this approach does not necessarily map into conservative design safety levels. It became evident that a sound and rational framework for addressing uncertainty and regulating design safety level is a potential area of development for highway geometric design. A discussion was presented for several studies in the literature of highway geometric design that included reliability analysis.

The main case study included in this thesis is the standard design of crest vertical curves. Crest vertical curves are designed according to sight distance requirements, comfort, and drainage. In most cases, sight distance availability is the controlling design requirement.
For a more accurate and realistic calculation of available sight distance, a new sight distance mode was developed in order to perform calculations in 3D environment. The 3D calculation of available sight distance is helpful in investigating the effect of superimposing horizontal curvature on crest vertical curves. Chapter three presented the mathematical details of the 3D sight distance model. For further analysis, it was important to calculate the case of combined vertical and horizontal alignment that creates the greatest restriction on the availability of sight distance. Based on analysis of the results obtained from the developed sight distance model, a hypothesis was proposed in attempt to determine the most critical combination of vertical and horizontal alignments.

The main proposition that underlies the analysis presented in this thesis is that design safety level obtained from design standards has to be consistent and close to a premeditated level. On order to realize this proposition, standard design codes have to be modified or calibrated. A theoretical and historical discussion of some aspects of code-calibration was presented in this thesis. A general framework for code-calibration was proposed. The framework can account for uncertainty in design model as well as input parameters. The code-calibration framework was applied to the standard design model of crest vertical curves. Two cases were considered: crest vertical curve located on a tangent and a horizontal curve. The latter case required the use of the sight distance model and the combination hypothesis presented in chapter three. The statistical distributions that describe the variability in input parameters were obtained from relevant studies in the literature. The main source of model uncertainty in the current standard design model arose from neglecting grade variability at crest vertical curves whilst calculating stopping sight distance. A program was written to calculate the exact stopping sight distance. The results of the application of code-calibration were presented in the thesis and further discussion was provided accordingly.

5.2 CONCLUSIONS

Based on the foregoing discussion, it becomes evident that the development of a risk-conscious design is one of the next steps in the course of developing the standards of
highway geometric design. The analysis presented in this thesis offers a rational and theoretically defensible framework for addressing uncertainty in the design process. Several conclusions are drawn from the analysis and discussions presented in this thesis:

1. Uncertainty in various stages of highway geometric design is unavoidable and is best addressed through a probabilistic framework.
2. It is evident that the current standard design does not consider uncertainty within the confines of a theoretically sound approach.
3. Reliability analysis involves some relevant assumptions and decisions that render it susceptible to inconsistency if entirely delegated to designers. The calibration of design safety is therefore a task that belongs to code developers and policy makers.
4. It was hypothesized in the course of this thesis that the most critical case of combined vertical and horizontal curves occurs when the individual available sight distances obtained from 2D analysis are equal. This hypothesis underlies the application of code-calibration for 3D standard design model.
5. Design safety is quantified in terms of the probability of non-compliance to design requirements. Within the context of highway geometric design, it is convenient to adopt the subjective interpretation of probability instead of the frequentist interpretation. This principle is useful in developing a better understanding of the code-calibration process.
6. The results obtained from code-calibration suggest that there is a degree of overdesign in the current standard design model of crest vertical curves. This suggestion is in part substantiated by relevant studies in the literature that failed to quantify a relationship between collisions and sight distance restriction at crest vertical curves.
7. The effect of reducing the available sight distance on a crest vertical curve due to superimposing a horizontal curve is eliminated by the corresponding reduction in operating speed. Accordingly, calibrated lengths of crest vertical curves were found to be longer in 2D design model than 3D design model.
5.3 RECOMMENDATION FOR FUTURE RESEARCH

The objective of this thesis was to provide a motivation for further development in some aspects of highway geometric design. The following are several areas of extension to the work undertaken in this thesis:

1. There was a challenge in finding relevant statistical distributions of design inputs in the literature. This in part is due to the lack of a general awareness of the importance of addressing uncertainty within design standards.

2. Operating speed represents a central element in the proposed calibration model. The operating speed models found in the literature are based on a simple association between observed speeds and main geometric features of highways. Further input variables may explain the variability between observed speeds and predicted speeds.

3. A hypothesis was presented in chapter three regarding the critical combination of vertical and horizontal alignments. It is hypothesized that the largest reduction of available sight distance occurs when the individual 2D available sight distances obtained for the horizontal and vertical curves are the same. The hypothesis can be proved, or refuted, by a relevant mathematical proof. It lies beyond the scope of this thesis to derive such a mathematical proof; however it remains a possible area of future research.

4. Three main methods were discussed regarding the selection of target reliability index – or in general target design safety level. One of the methods is based on investigating the relationship between design safety and observed collision. If the relationship is formulated, the code-calibration process can include useful elements of cost-benefit analysis. The second method for selecting target reliability index is based on collecting data regarding existing geometric features. It was beyond the scope of this thesis to follow this method of selecting target reliability index. However, an application of the latter method is a valuable addition to the process of code-calibration the standard models of highway geometric design.
5. The two applications presented in this thesis can be replicated for other geometric design models, e.g. passing sight distance, and horizontal curves. It is expected that through further applications, more improvements can be added to the calibration framework presented in this thesis.
6 REFRENCES


References


References


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References


APPENDIX I

SIGHT DISTANCE MATLAB CODE
clc

clear all

time =clock;

h1=1.149;h2=0.6;

[TL,TD]=textread('tan.txt','%f %f);
[R1,DE1,SP11,SP12]=textread('cur.txt','%f %f %f %f);
[g1,g2,k,A]=textread('ver.txt','%f %f %f %f');
[O(1,1),O(2,1),O(3,1)]=textread('FP.txt','First Point = %f %f %f');
[w,of,ssl,Nlanes,off,ecrown,mp]=textread('geom.txt','%f %f %f %f %f %f %f %f');

R90=[ 0 1 0
      -1 0 0
       0 0 1 ];

P1= [cos(TD*1E-5)
     sin(TD*1E-5)
      0 ];

N1=R90*P1;PC1=O+P1*TL;C1=PC1-sign(DE1)*R1*N1;tlPC1=TL;OS=0;
pnt=0;SSD=[0 0 0];srf=500;Last=300;

for to=100:5:100+xu
    pnt=pnt+1;
    SSD(pnt,1)=to;% Storing driver location coordinates
    trl=to+Last;
    Xo(3,1)=Z(to,g1,g2,PC1,A,k,TL); % elevation of driver eye
    Xo=Xi(to,TL,DE1,R1,N1,C1);tPC1=TL;theta=sign(DE1)*(to-tPC1)/R1;

    if to<tPC1
        theta=0;
    end
    Xoc=Xo-off*N(to,DE1,tPC1,R1,N1);
    Xoc(3,1)=Z(to,g1,g2,PC1,A,k,TL)+off*ei(to,SP11,SP12,ecrown,off,DE1,R1,tPC1)/100+h1;
    SSD(pnt,3)=Z(to,g1,g2,PC1,A,k,TL);tr=trl; iter=0; incr=-1;

while tr>to
    iter=iter+1;
    Xr=Xi(tr,TL,DE1,R1,N1,C1);theta=sign(DE1)*(tr-tPC1)/R1;
    if tr>tPC1 + R1*abs(DE1)/1E5;
        theta=DE1;
        break;
    end
end
end
Xrc=Xr-off*N(tr,DE1,tPC1,R1,N1);
Xrc(3,1)=Z(tr,g1,g2,PC1,A,k,TL)+off*ei(tr,SP11,SP12,ecrown,off,DE1,R1,tPC1)/100+h2;
ea=(Xrc(3,1)+Xoc(3,1))/2+ei((to+tr)/2,SP11,SP12,ecrown,off,DE1,R1,tPC1)*(w*Nlanes-off)/100+(eouti(t)/100*of*sign(of));

if tr>tPC1 + R1*abs(DE1)/1E5+50;
t=tPC1 + R1*abs(DE1)/1E5;Zinitial=Z(t,g1,g2,PC1,A,k,TL);
ea=Zinitial+ei(t,SP11,SP12,ecrown,of,DE1,R1,tPC1)*(w*Nlanes-off)/100+(eouti(t)/100*of*sign(of));
end

apex=ea+(R1-Nlanes*w-of)*ssl/100; Vi(3,1)=apex; Vi(2,1)=C1(2,1);Vi(1,1)=C1(1,1);
Ai=[0 0 -1];cosi=sin(atan(ssl/100));Pi=(Xrc-Xoc)/norm(Xrc-Xoc);
I=[1 0 0
   0 1 0
   0 0 1];
Mi=Ai'*Ai*cosi^2*I;deli=Xoc-Vi;coi=(deli'*Mi*deli);c2i=(Pi'*Mi*Pi);c1i=(Pi'*Mi*deli);
Clearance=-(c1i^2-coi*c2i);
if Clearance >0
    break
end
tr=tr+incr;
end

if tr-to>srf
    tr=tr+0.5;
    Xrc=Xr-off*N(tr,DE1,tPC1,R1,N1);
    Xrc(3,1)=Z(tr,g1,g2,PC1,A,k,TL)+off*ei(tr,SP11,SP12,ecrown,off,DE1,R1,tPC1)/100+h2;
end
s=norm(Xrc-Xoc);
while s >0 ;
s=s-norm(Xr-Xo)*0.0005;S=Si(s,Xrc,Xoc);
t=ii(S,TL,DE1,R1,N1,C1,O,P1);X=Xi(t,TL,DE1,R1,N1,C1);
aa=wi(t);eo=eouti(t);
if sign((X-S)*(C1-S))>0
    ein=ei(t,SP11,SP12,ecrown,off,DE1,R1,tPC1);
esl=eslope(t);Zinitial=Z(t,g1,g2,PC1,A,k,TL);
\[
\begin{align*}
Z_{\text{road}} &= Z_{\text{si}}(S, X, a, c_1, e, \text{in}, Z_{\text{initial}}, n, -\text{sign}(D_1) \cdot \text{of}, e, s) \\
\text{else} & \quad \text{ein} = e(t, S_{P11}, S_{P12}, e, \text{crown}, -\text{sign}(D_1) \cdot \text{of}, D_1, R_1, t_{PC1}) \\
& \quad e_{sl} = e_{slope}(t); Z_{\text{initial}} = Z(t, g_1, g_2, C_1, A, k, TL) \\
& \quad Z_{\text{road}} = Z_{\text{si}}(S, X, a, c_1, e, \text{in}, Z_{\text{initial}}, n, -\text{sign}(D_1) \cdot \text{of}, e, s) \\
\end{align*}
\]

end

Clearance = S(3, 1) - Z_{\text{road}};

if Clearance < 0
    break
end

end

if tr > tr_l
    tr = tr_l;
end

if s_{rf} < 500
    tr = to + s_{rf};
end

tr = to + 300; Clearance = -1;

if Clearance < 0
    incr = -2.5;
    while tr > to;
        X_r = X_i(tr, T_L, D_1, R_1, N_1, C_1);
        theta = sign(D_1) \cdot (tr - t_{PC1}) / R_1;
        if tr > t_{PC1} + R_1 \cdot abs(D_1) / 1E5; theta = D_1;
    end

    X_{rc} = X_r - off \cdot N(tr, D_1, t_{PC1}, R_1, N_1);

    X(r, 3, 1) = Z(tr, g_1, g_2, C_1, A, k, TL) + off \cdot e(t, S_{P11}, S_{P12}, e, \text{crown}, off, D_1, R_1, t_{PC1}) / 100 + h_2;

    Clearance = 1; s = \text{norm}(X_{rc} - X_{oc});
    while s > 0;
        s = s - \text{norm}(X_r - X_o) \cdot 0.0005; S = S_i(s, X_{rc}, X_{oc});
        t = t_i(S, T_L, D_1, R_1, N_1, C_1, O, P_1);
        X = X_i(t, T_L, D_1, R_1, N_1, C_1);
        a = \text{wi}(t);
        e_o = e_{outi}(t);
        if sign((X - S) \cdot (C_1 - S)) > 0
            \text{ein} = e(t, S_{P11}, S_{P12}, e, \text{crown}, off, D_1, R_1, t_{PC1}) \cdot e_{sl} = e_{slope}(t);
            Z_{\text{initial}} = Z(t, g_1, g_2, C_1, A, k, TL);
        \end{align*}
\]
Zroad=Zsi(S,X,aa,C1,eo,ein,Zinitial,Nlanes,sign(DE1)*off,of,esl);
else
  ein=ei(t,SP11,SP12,ecrown,-sign(DEl)*off,DE1,R1,tPC1);
esl=eslope(t);Zinitial=Z(t,g1,g2,PC1,A,k,TL);
Zroad=Zsi(S,X,aa,C1,eo,ein,Zinitial,Nlanes,-sign(DEl)*off,of,esl);
end
Clearance=S(3,1)-Zroad;

if Clearance <0
  wf=((S(1,1)-X(1,1))^2+(S(2,1)-X(2,1))^2)^0.5;
  DD=((S(1,1)-Xoc(1,1))^2+(S(2,1)-Xoc(2,1))^2)^0.5;
  break
end
if Clearance >0
  break
end
tr=tr+incr;
if tr>trl
  tr=tr-incr;
  disp(['The whole segment is viewable, greater than 300m, no constrained sight distance found!'])
srf=tr-to;OS=0;break
end
incr=0.07;
while Clearance>=0; % Now refining
  tr=tr+incr;
  if tr>trl ;
    tr=tr-incr;
    disp(['The whole segment is viewable, greater than 300m, no constrained sight distance found!'])
    srf=tr-to;OS=0;
    break
  end
  iter=iter+1;Xr=Xi(tr,TL,DE1,R1,N1,C1);theta=sign(DE1)*(tr-tPC1)/R1;
  if tr>tPC1 + R1*abs(DE1)/1E5
    theta=DE1;
  end
Xrc=Xr-off*N(tr,DE1,tPC1,R1,N1);
Xrc(3,1)=Z(tr,g1,g2,PC1,A,k,TL)+off*ei(tr,SP11,SP12,ecrown,off,DE1,R1,tPC1)/100+h2;
s=norm(Xrc-Xoc);

while s > 0;
    s=s-norm(Xrc-Xoc)*0.0005; S=Si(s,Xrc,Xoc); t=ti(S,TL,DE1,R1,N1,C1,0,P1);
    X=Xi(t,TL,DE1,R1,N1,C1); aa=wi(t); eo=eouti(t);
    if sign((X-S)'*(C1-S))>0
        ein=ei(t,SP11,SP12,ecrown,off,DE1,R1,tPC1);
        esl=eslope(t);
        Zinitial=Z(t,g1,g2,PC1,A,k,TL);
        Zroad=Zsi(S,X,aa,C1,eo,ein,Zinitial,Nlanes,sign(DE1)*off,of,esl);
    else
        ein=ei(t,SP11,SP12,ecrown,-sign(DE1)*off,DE1,R1,tPC1);
        esl=eslope(t);
        Zinitial=Z(t,g1,g2,PC1,A,k,TL);
        Zroad=Zsi(S,X,aa,C1,eo,ein,Zinitial,Nlanes,-sign(DE1)*off,of,esl);
    end
    Clearance=S(3,1)-Zroad;
    if Clearance<0
        wf=((S(1,1)-X(1,1))^2+(S(2,1)-X(2,1))^2)*0.5; DD=((S(1,1)-Xoc(1,1))^2+(S(2,1)-Xoc(2,1))^2)*0.5;
        break
    end
end

if Clearance<0
for td=tr:-0.2:to;
    Xr=Xi(td,TL,DE1,R1,N1,C1);
    Xrc=Xr-off*N(td,DE1,tPC1,R1,N1);
    Xoc=Xoc(3,1); Xoc(3,1)=0; PP=(Xrc-Xoc)/norm(Xrc-Xoc); a=(norm(PP))^2;
    b=2*(Xoc-C1)*PP; c=(norm(Xoc-C1))^2-R1^2;
    if b^2-4*a*c<0
        t1=(-b+(b^2-4*a*c)^0.5)/(2*a);
        if t1>0
            Z=Z(t,g1,g2,PC1,A,k,TL);
            Xrc(3,1)=Z(td,g1,g2,PC1,A,k,TL)+off*ei(td,SP11,SP12,ecrown,off,DE1,R1,tPC1)/100+h2;
            Xoc(3,1)=Zoc; Sii=Si(t1,Xrc,Xoc);
            if (Sii(3)-Zi)>mp

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break
end
end
t2=-(b-(b^2-4*a*c)^0.5)/(2*a);
if t2>0
  Zi=Z(t,g1,g2,PCI,A,k,TL);Sii=Si(t2,Xrc,Xoc);
  if(Sii(3)-Zi)>mp
    break
  end
end
else
  tr=td;break
end
end
disp(['Radius is: ',num2str(Rl),"')
disp(['Time to complete the analysis: ',num2str(etime(clock,time))," seconds'])
disp('')
disp(['Found an unobstructed line of sight with the following details:'])
disp('')
disp(['Coordinates of object: (' num2str(Xrc(1,1)) ',',num2str(Xrc(2,1)),',',num2str(Xrc(3,1)))'])
disp(['Object location (meterage): ',num2str(tr)])
disp('')
srf=tr-to;
disp(['Driver eye location (meterage): ',num2str(to)])
disp(['Distance from driver eye measured along the:'])
disp(['Orthometric projection on the centerline: ',num2str(tr)," m'])
disp(['Offset from centerline: ',num2str(off)," m'])
disp(['Elevation from road surface: ',num2str(Clearance)," m'])
disp(['Available sight distance: ',num2str((Rl-off)/Rl*(tr-to))," m'])
disp(['Clearance above ground at this point: ',num2str(Clearance)," m'])
disp(['At distance from driver eye: ',num2str(s)," m'])
disp(['At centerline coordinate: (' num2str(X(l,l)),V,num2str(X(2,l)),',',num2str(X(3,l)))'])

w1=Nlanes*aa; w2=wf-w1; w3=wf-wl-off;
if w2<0;
disp(['- Offset of the tangential point: ',num2str(wf)])
disp(['- Line of sight is obstructed by the: road surface'])
end
OS=1;
else if w2<of;
    disp(['- Offset of the tangential point :
         ',num2str(wf))
    disp(['- Line of sight is obstructed by the:
         clearance zone']])
    OS=2;
elseif w3>0
    disp(['- Offset of the tangential point :
         ',num2str(wf)])
    disp(['- Line of sight is obstructed by the:
         side slope'])
    OS=3;
end
    disp(['-Distance from driver eye :
         ',num2str(DD)])
    disp(['- Please Wait !'])
    break
    end
else
    incr=2.5;
    while tr>=to;
        Xr=Xi(tr,TL,DE1,R1,N1,C1);
        theta=sign(DE1)*(tr-tPC1)/R1;
        if tr>tPC1 + R1*abs(DE1)/lE5
            theta=DE1;
        end
        Xrc=Xr-off*N(tr,DE1,tPC1,R1,N1);
        Xrc(3,1)=Z(tr,g1,g2,PC1,A,k,TL)+off*ei(tr,SP11,SP12,ecrown,off,DE1,R1,tPC1)/100+h2;
        Clearance=1;

        s=norm(Xrc-Xoc);
        while s>0 ;
            s=s-norm(Xrc-Xoc)*0.0005;
            S=Si(s,Xrc,Xoc);
            t=ti(S,TL,DE1,R1,N1,C1,O,P1),X=Xi(t,TL,DE1,R1,N1,C1);
            wf=((S(1,1)-X(1,1))^2+(S(2,1)-X(2,1))^2)^0.5;
            DD=((S(1,1)-Xoc(1,1))^2+(S(2,1)-Xoc(2,1))^2)^0.5;
            aa=wi(t);eo^eouti(t);
            if sign((X-S)'*(C1-S))>0
                ein=ei(t,SP11,SP12,ecrown,off,DE1,R1,tPC1);
esl=eslope(t);Zinitial=Z(t,g1,g2,PC1,A,k,TL);
Zroad=Zsi(S,X,aa,C1,e1,Zinitial,Nlanes,sign(DE1)*off,of,esl);
else
  ein=e1(t,SP11,SP12,ecrown,-sign(DE1)*off,DEE1,R1,PC1);
esl=eslope(t);Zinitial=Z(t,g1,g2,PC1,A,k,TL);
Zroad=Zsi(S,X,aa,C1,e1,Zinitial,Nlanes,-sign(DE1)*off,of,esl);
end
Clearance=S(3,1)-Zroad;
if Clearance <0
  wf=((S(1,1)-X(1,1))^2+(S(2,1)-X(2,1))^2)^(1/2);
  DD=((S(1,1)-Xoc(1,1))^2+(S(2,1)-Xoc(2,1))^2)^(1/2);
  break
end
end
if Clearance <0
  break
end
tr=tr+incr;
if tr>trl
  tr=tr-incr;
  disp(['The whole segment is viewable, greater than 300m, no constrained sight distance found !'])
srf=tr-to;
  OS=0;
  break
end
end
incr=-0.080;
while Clearance<=0; % Now refining
  tr=tr+incr;
  if tr>trl
    tr=tr-incr;
    disp(['The whole segment is viewable, greater than 300m, no constrained sight distance found !'])
    srf=tr-to; OS=0;
    break
  end
  iter=iter+1;
end
\[ X_r = X_i(t, T_L, D E_1, R_1, N_1, C_1); \theta = \text{sign}(D E_1) \frac{t - t_{P C_1}}{R_1}; \]

\[ \text{if } t > t_{P C_1} + R_1 \frac{|D E_1|}{l_{E_5}} \]
\[ \theta = D E_1; \]
\[ \text{end} \]
\[ X_{rc} = X_r - \text{off} \times N(t, D E_1, t_{P C_1}, R_1, N_1); \]
\[ X_{rc}(3, 1) = Z(t, g_1, g_2, P C_1, A, k, T_L) + \text{off} \times e_i(t, S P_{11}, S P_{12}, e_{crown}, \text{off}, D E_1, R_1, t_{P C_1}) / 100 + h_2; \]

\[ s = \text{norm}(X_{rc} - X_{oc}); \]
\[ \text{while } s > 0; \]
\[ s = s - \text{norm}(X_{rc} - X_{oc}) \times 0.0005; \]
\[ S = \text{Si}(s, X_{rc}, X_{oc}); t = t_i(S, T_L, D E_1, R_1, N_1, C_1, O, P_1); \]
\[ X = X_i(t, T_L, D E_1, R_1, N_1, C_1); a = w_i(t); e_o = e_{out}(t); \]
\[ \text{if } \text{sign}((X - S)^*(C_1 - S)) > 0 \]
\[ e_i = e_i(t, S P_{11}, S P_{12}, e_{crown}, \text{off}, D E_1, R_1, t_{P C_1}); \]
\[ e_s = e_{slope}(t); Z_{initial} = Z(t, g_1, g_2, P C_1, A, k, T_L); \]
\[ Z_{road} = Z_{si}(S, X, a, C_1, e_o, e_i, Z_{initial}, N_{lanes}, \text{sign}(D E_1) \times \text{off}, e_s); \]
\[ \text{else} \]
\[ e_i = e_i(t, S P_{11}, S P_{12}, e_{crown}, -\text{sign}(D E_1) \times \text{off}, D E_1, R_1, t_{P C_1}); \]
\[ e_s = e_{slope}(t); Z_{initial} = Z(t, g_1, g_2, P C_1, A, k, T_L); \]
\[ Z_{road} = Z_{si}(S, X, a, C_1, e_o, e_i, Z_{initial}, N_{lanes}, -\text{sign}(D E_1) \times \text{off}, e_s); \]
\[ \text{end} \]
\[ \text{Clearance} = S(3, 1) - Z_{road}; \]
\[ \text{if } \text{Clearance} < 0 \]
\[ \text{lasts} = s; \text{lastS} = \text{norm}(X_{rc} - X_{oc}); \text{lastt} = t^*; \text{lastt} = t^*; \text{lasttr} = t_{P C_1}; \]
\[ \text{lastw} = ((S(1, 1) - X(1, 1))^2 + (S(2, 1) - X(2, 1))^2)^{0.5}; \]
\[ w_f = ((S(1, 1) - X(1, 1))^2 + (S(2, 1) - X(2, 1))^2)^{0.5}; \]
\[ D D = ((S(1, 1) - X_{oc}(1, 1))^2 + (S(2, 1) - X_{oc}(2, 1))^2)^{0.5}; \]
\[ \text{break} \]
\[ \text{end} \]
\[ \text{end} \]
\[ \text{disp}(['\text{Radius is: }', '\text{num2str(R1)}, ']') \]
\[ \text{disp}(['\text{Time to complete the analysis: }', '\text{num2str(etime(clock, time)), } ']') \]
\[ \text{disp}(['\text{Found an unobstructed line of sight with the following details: }']) \]
\[ \text{disp}(['\text{Coordinates of object: }', '\text{num2str(Xrc(1,1))}, ',' \text{num2str(Xrc(2,1))}, ',' \text{num2str(Xrc(3,1))}]]) \]
\[ \text{disp}(['\text{Object location (meterage): }', '\text{num2str(tr)}')) \]
disp(['

srf=tr-to;
disp(['Driver eye location (meterage): ',num2str(to)])
disp(['Distance from driver eye measured along the'])
disp(['Orthometric projection on the centerline: ',num2str(tr),']
disp(['Offset from centerline: ',num2str(off),']
disp(['Elevation from road surface: ',num2str(Clearance),']
disp(['Available sight distance: ',num2str((Rl-off)/Rl*(tr-to)),' m'])
disp(['Clearance above ground at this point: ',num2str(Clearance),']
disp(['At distance from driver eye: ',num2str(s),']
disp(['At centerline coordinate: (',num2str(X(l,l)),']

w1=Nlanes*a;w2=wf-w1;w3=wf-w1-off;
if w2<0;
disp(['- Offset of the tangential point: ',num2str(wf)])
disp(['- Line of sight is obstructed by the: road surface'])
OS=1;
elseif w2<of;
disp(['- Offset of the tangential point: ',num2str(wf)])
disp(['- Line of sight is obstructed by the: clearance zone'])
OS=2;
elseif w3>0

disp(['- Offset of the tangential point: ',num2str(wf)])
disp(['- Line of sight is obstructed by the: side slope'])
OS=3;
end
disp(['Distance from driver eye: ',num2str(DD)])
disp(['- Please Wait!'])
break
end

SSD(pnt,2)=(Rl-off)/R1*(tr-to); SSD(pnt,4)=OS;
DLOC(pnt,1)=Xoc(1,1); DLOC(pnt,2)=Xoc(2,1); DLOC(pnt,3)=Xoc(3,1); TLOC(pnt,1)=Xrc(1,1);
TLOC(pnt,2)=Xrc(2,1); TLOC(pnt,3)=Xrc(3,1);
save 'C:\TLOC.txt' TLOC -ascii; save 'C:\DLOC.txt' DLOC -ascii;
end
disp(['Done !'])
fname=['C:\A',num2str(Rl),'&',num2str(A),'.csv'];
csvwrite(fname,SSD)
end

function g=Zsi(S,X,aa,C1,ein,Zinitial,Nlanes,off,of,esl)
wl=Nlanes*aa/2+abs(off);
w2=((S(1,1)-X(1,1))^2+(S(2,1)-X(2,1))^2)^0.5-wl;
w3=((S(1,1)-X(1,1))^2+(S(2,1)-X(2,1))^2)^0.5-w1-of;
if w2<0;
  w2=0;w3=0;
w1=((S(1,1)-X(1,1))^2+(S(2,1)-X(2,1))^2)^0.5;
elseif w2<of;
  w3=0;
elseif w3>0
  w2=of;
w1=((S(1,1)-X(1,1))^2+(S(2,1)-X(2,1))^2)^0.5;
end

g=Zinitial+(ein/100*wl*sign(off)+(w2*eo/100)+(w3*esl/100));

function X = X(t);
tPC1=TL
theta=sign(DEl)*(t-tPC1)/Rl
X=C1+R1*N1*[ cos(theta) -sin(theta) 0
                      sin(theta) cos(theta) 0
                      0 0 0]};

function InvX = X(X,C1,R1);
X=C1-R1*(C1-X)/norm(C1-X);

function g = X(t,TL,DEl,Rl,Nl,C1);
tPC1=TL;
theta=sign(DEl)*(t-tPC1)/Rl;
g=C1-R1*[ cos(theta) -sin(theta) 0
          sin(theta) cos(theta) 0
          0 0 0]*N1;
APPENDIX II

VISUALIZATION MATLAB CODE
clear all
axes('Xcolor',[1 1 1],'Ycolor',[1 1 1],'Zcolor',[1 1 1])
time = clock;

h1=1.149;h2=0.6;[TL,TD]=textread('tan.txt','%f %f');
[R1,DE1,SP11,SP12]=textread('cur.txt','%f %f %f %f');
[g1,g2,k,A]=textread('ver.txt','%f %f %f %f');
[O(1,1),O(2,1),O(3,1)]=textread('FP.txt','First Point = %f %f %f');
[w,of,ssl,Nlanes,off,ecrown,mp]=textread('geom.txt','%f %f %f %f %f %f %f');

%Reading driver locations
[DLOC(:,1), DLOC(:,2), DLOC(:,3)]=textread('E:\karim\DLOC.txt','%f %f %f');
[TLOC(:,1), TLOC(:,2), TLOC(:,3)]=textread('E:\karim\TLOC.txt','%f %f %f');
R90=[ 0 1 0
     -1 0 0
     0 0 0 ];
P1=[cos(TD*1E-5)
    sin(TD*1E-5)
    0 ];
N1=R90*P1;PC1=O+P1*TL;C1=PC1-sign(DE1)*R1*N1;PC1=TL;
clc
disp(\'# GENERATING ROAD SURFACE\')
disp(['\'POINTS generation in progress ... \']
% Range of Generating the model
iter=0;
for t=-300:2:450;
    iter=iter+1;Xcl=Xi(t,TL,DE1,R1,N1,C1);
    Xcl(3,1)=Z(t,g1,g2,PC1,A,k,TL);off=w*Nlanes;
    Xright=Xcl-off*N(t,DE1,tPC1,R1,N1);
    Xright(3,1)=Z(t,g1,g2,PC1,A,k,TL)+off*ei(t,SP11,SP12,ecrown,off,DE1,R1,tPC1)/100;
    off=-w*Nlanes;Xleft=Xcl-off*N(t,DE1,tPC1,R1,N1);
    Xleft(3,1)=Z(t,g1,g2,PC1,A,k,TL)+off*ei(t,SP11,SP12,ecrown,off,DE1,R1,tPC1)/100;

    off=w*Nlanes+of;Xright1=Xcl-off*N(t,DE1,tPC1,R1,N1);
    aa=wi(t);eo=eouti(t);
    ein=ei(t,SP11,SP12,ecrown,off,DE1,R1,tPC1);esl=eslope(t);

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\[
\begin{align*}
Z_{\text{initial}} &= Z(t, g_1, g_2, PC_1, A, k, TL); \\
X_{\text{right1}}(3, 1) &= Z_{\text{si}}(X_{\text{right1}}, X_{cl}, aa, C_1, eo, ein, Z_{\text{initial}}, N_{\text{lanes}}, off, of, esl); \\
\text{off} &= -(w \cdot N_{\text{lanes}} + of); \\
X_{\text{left1}} &= X_{cl} - off \cdot N(t, DE_1, tPC_1, R_1, N_1); aa = wi(t); eo = eouti(t); \\
ein &= ei(t, SP_{11}, SP_{12}, ecrown, off, DE_1, R_1, tPC_1); esl = eslope(t); Z_{\text{initial}} = Z(t, g_1, g_2, PC_1, A, k, TL); \\
X_{\text{left1}}(3, 1) &= Z_{\text{si}}(X_{\text{right1}}, X_{cl}, aa, C_1, eo, ein, Z_{\text{initial}}, N_{\text{lanes}}, off, of, esl); \\
\text{off} &= -(w \cdot N_{\text{lanes}} + of + 10); \\
X_{\text{right2}} &= X_{cl} - off \cdot N(t, DE_1, tPC_1, R_1, N_1); aa = wi(t); eo = eouti(t); \\
ein &= ei(t, SP_{11}, SP_{12}, ecrown, off, DE_1, R_1, tPC_1); esl = eslope(t); Z_{\text{initial}} = Z(t, g_1, g_2, PC_1, A, k, TL); \\
X_{\text{left2}}(3, 1) &= Z_{\text{si}}(X_{\text{left2}}, X_{cl}, aa, C_1, eo, ein, Z_{\text{initial}}, N_{\text{lanes}}, off, of, esl); \\
\text{off} &= -(w \cdot N_{\text{lanes}} + of + 10); \\
X_{\text{left2}} &= X_{cl} - off \cdot N(t, DE_1, tPC_1, R_1, N_1); aa = wi(t); eo = eouti(t); \\
ein &= ei(t, SP_{11}, SP_{12}, ecrown, off, DE_1, R_1, tPC_1); esl = eslope(t); Z_{\text{initial}} = Z(t, g_1, g_2, PC_1, A, k, TL); \\
X_{\text{left2}}(3, 1) &= Z_{\text{si}}(X_{\text{left2}}, X_{cl}, aa, C_1, eo, ein, Z_{\text{initial}}, N_{\text{lanes}}, off, of, esl); \\
\end{align*}
\]

POINTS((iter-1) * 7 + 4, 1) = Xcl(1, 1); \\
POINTS((iter-1) * 7 + 4, 2) = Xcl(2, 1); \\
POINTS((iter-1) * 7 + 4, 3) = Xcl(3, 1); \\
POINTS((iter-1) * 7 + 5, 1) = Xright(1, 1); \\
POINTS((iter-1) * 7 + 5, 2) = Xright(2, 1); \\
POINTS((iter-1) * 7 + 5, 3) = Xright(3, 1); \\
POINTS((iter-1) * 7 + 6, 1) = Xright1(1, 1); \\
POINTS((iter-1) * 7 + 6, 2) = Xright1(2, 1); \\
POINTS((iter-1) * 7 + 6, 3) = Xright1(3, 1); \\
POINTS((iter-1) * 7 + 7, 1) = Xright2(1, 1); \\
POINTS((iter-1) * 7 + 7, 2) = Xright2(2, 1); \\
POINTS((iter-1) * 7 + 7, 3) = Xright2(3, 1); \\
POINTS((iter-1) * 7 + 8, 1) = Xleft(1, 1); \\
POINTS((iter-1) * 7 + 8, 2) = Xleft(2, 1); \\
POINTS((iter-1) * 7 + 8, 3) = Xleft(3, 1);
POINTS((iter-1)*7+2,1)=Xleft1(1,1);
POINTS((iter-1)*7+2,2)=Xleft1(2,1);
POINTS((iter-1)*7+2,3)=Xleft1(3,1);

POINTS((iter-1)*7+1,1)=Xleft2(1,1);
POINTS((iter-1)*7+1,2)=Xleft2(2,1);
POINTS((iter-1)*7+1,3)=Xleft2(3,1);
end

disp(['POINTS generated successfully!'])
disp(['TIME of generating points: ',num2str(etime(clock,time)),' seconds'])
disp([''])
disp(['SURFACE generation in progress ... '])
time=clock;

Npatches=(size(POINTS,1)/7-1)*6;patchrow=0;shiftcol=0;
for patchrow=1:1:Npatches/6
	%Four edges of each patch are patchid(1:6,1:2)
	for patchcol=1:1:6
		%Capturing first left point located at patchrow line
		PATCH(patchid,1).L1=[(POINTS((patchrow-1)*6+patchcol+shiftcol,1))
		(POINTS((patchrow-1)*6+patchcol+shiftcol,2))
		(POINTS((patchrow-1)*6+patchcol+shiftcol,3))];
	%Capturing first right point located at patchrow line
	PATCH(patchid,1).R1=[(POINTS((patchrow-1)*6+patchcol+1+shiftcol,1))
	(POINTS((patchrow-1)*6+patchcol+1+shiftcol,2))
	(POINTS((patchrow-1)*6+patchcol+1+shiftcol,3))];
	%Capturing second left point located at patchrow line+1
	PATCH(patchid,1).L2=[(POINTS((patchrow-1)*6+patchcol+7+shiftcol,1))
	(POINTS((patchrow-1)*6+patchcol+7+shiftcol,2))
	(POINTS((patchrow-1)*6+patchcol+7+shiftcol,3))];
	%Capturing second right point located at patchrow line+1
	PATCH(patchid,1).R2=[(POINTS((patchrow-1)*6+patchcol+8+shiftcol,1))
	(POINTS((patchrow-1)*6+patchcol+8+shiftcol,2))
	(POINTS((patchrow-1)*6+patchcol+8+shiftcol,3))];
(POINTS((patchrow-1)*6+patchcol+8+shiftcol,3));

end
shiftcol=1+shiftcol;
end
disp(['SURFACE generated successfully'])
disp(['TIME of generating surface: ',num2str(etime(clock,time)),' seconds'])
disp([''])
disp(['PATCHES generation in progress ...'])
for patchid=1:6:Npatches-5
v = [(PATCH(patchid,1).L1);(PATCH(patchid,1).R1);(PATCH(patchid,1).R2);(PATCH(patchid,1).L2)];
f = [1 2 3 4];fvc = [.2 .6 .3;4 .51 .4;0.7 0.46 0.4;0.4 0.56 0.4];
patch('Vertices',v,'Faces',f,'FaceVertexCData',fvc,...
    'FaceColor','flat','EdgeColor','flat',...
    'Marker','.','MarkerFaceColor','flat')
end
for patchid=2:6:Npatches-4
v = [(PATCH(patchid,1).L1);(PATCH(patchid,1).R1);(PATCH(patchid,1).R2);(PATCH(patchid,1).L2)];
f = [1 2 3 4];
fvc = [.8 .6 .3;0.7 0.55 0.4;1 1 1;0.7 0.55 0.4];
patch('Vertices',v,'Faces',f,'FaceVertexCData',fvc,...
    'FaceColor','flat','EdgeColor','flat',...
    'Marker','.','MarkerFaceColor','flat')
end
for patchid=3:6:Npatches-3
v = [(PATCH(patchid,1).L1);(PATCH(patchid,1).R1);(PATCH(patchid,1).R2);(PATCH(patchid,1).L2)];
f = [1 2 3 4];fvc = [.5 .5 .5;0.47 0.47 0.47;1 1 1;0.47 0.47 0.47];
patch('Vertices',v,'Faces',f,'FaceVertexCData',fvc,...
    'FaceColor','flat','EdgeColor','flat',...
    'Marker','.','MarkerFaceColor','flat')
end
for patchid=4:6:Npatches-2
v = [(PATCH(patchid,1).L1);(PATCH(patchid,1).R1);(PATCH(patchid,1).R2);(PATCH(patchid,1).L2)];
\texttt{f = [1 2 3 4]; fvc = [.5 .5 .5; 0.47 0.47 0.47; 1 1 1; 0.47 0.47 0.47];}
\texttt{patch('Vertices', v, 'Faces', f, 'FaceVertexCData', fvc, ...}
\texttt{FaceColor', 'flat', 'EdgeColor', 'flat', ...}
\texttt{Marker', ',', 'MarkerFaceColor', 'flat')}
\texttt{end}

\texttt{for patchid = 5:6:Npatches - 1}
\texttt{v = [(PATCH(patchid, 1).L1); (PATCH(patchid, 1).RT); (PATCH(patchid, 1).R2); (PATCH(patchid, 1).L2)];}
\texttt{f = [1 2 3 4]; fvc = [0.2 0.6 0.3; 0.4 0.51 0.4; 1 1 1; 0.4 0.56 0.4];}
\texttt{patch('Vertices', v, 'Faces', f, 'FaceVertexCData', fvc, ...}
\texttt{FaceColor', 'flat', 'EdgeColor', 'flat', ...}
\texttt{Marker', ',', 'MarkerFaceColor', 'flat')}
\texttt{end}

\texttt{for patchid = 6:6:Npatches}
\texttt{v = [(PATCH(patchid, 1).L1); (PATCH(patchid, 1).R1); (PATCH(patchid, 1).R2); (PATCH(patchid, 1).L2)];}
\texttt{f = [1 2 3 4]; fvc = [0.2 0.6 0.3; 0.4 0.51 0.4; 1 1 1; 0.4 0.56 0.4];}
\texttt{patch('Vertices', v, 'Faces', f, 'FaceVertexCData', fvc, ...}
\texttt{FaceColor', 'flat', 'EdgeColor', 'flat', ...}
\texttt{Marker', ',', 'MarkerFaceColor', 'flat')}
\texttt{end}

\texttt{for t = 0:1:tPC1 + R1 * abs(DE1) * 1E-5 + 200 - 10;}
\texttt{Xcl1 = Xi(t, TL, DE1, R1, N1, Cl); Xcl1(3, 1) = Z(t, g1, g2, PC1, A, k, TL);}
\texttt{Xcl2 = Xi(t + 1, TL, DE1, R1, N1, Cl); Xcl2(3, 1) = Z(t + 1, g1, g2, PC1, A, k, TL);}
\texttt{line(Xcl1, Xcl1 + [0 0 -mp]); line(Xcl1, Xcl2);}
\texttt{end}

\texttt{time = clock; camva = 140;}
\texttt{camzoom(1)}
\texttt{set(gca, 'DataAspectRatio', [1 1 1], 'PlotBoxAspectRatio', [1 1 1]);}
\texttt{disp(['PATCHES generated successfully !'])}
\texttt{disp(['TIME of generating patches: ', num2str(etime(clock, time)), ' seconds'])}
\texttt{disp([''])}
\texttt{disp(['Driver eye view generation in progress ... '])}

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campos([POINTS(4,1) POINTS(4,2) POINTS(4,3)+h1]);
camproj('perspective')
camtarget([POINTS(2*7+4,1) POINTS(2*7+4,2) POINTS(2*7+4,3)+h2]);
drawnow
set(gca,'DataAspectRatio',[1 1 1],'PlotBoxAspectRatio',[1 1 1]);
camproj('perspective')
time=clock;
camva=140;
for loc=1:1:size(DLOC(:,1))
campos([DLOC(loc,1) DLOC(loc,2) DLOC(loc,3)]);
camtarget([TLOC(loc,1) TLOC(loc,2) TLOC(loc,3)]);
text(TLOC(loc,1),TLOC(loc,2),TLOC(loc,3),'🎀','FontSize',10);
text(TLOC(loc,1),TLOC(loc,2),TLOC(loc,3),'📝','FontSize',8);
text(TLOC(loc,1),TLOC(loc,2),TLOC(loc,3)+0.4,['( ',num2str(fix(TLOC(loc,1))),'
','num2str(fix(TLOC(loc,2))),',num2str(fix(TLOC(loc,3))))'],'FontSize',7);
drawnow
end
disp(['PATCHES generated successfully !'])
disp(['TIME of generating patches: ',num2str(etime(clock,time)),' seconds'])
disp([''])
disp(['Driver eye view generation in progress ... '])

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APPENDIX III

MATLAB CODE OF USER DEFINED LIMIT STATE FUNCTION
function G = user_lsf(x);
    h1=x(1); h2=0.6; A=3; L=91.6; K=L/A; g1=9; g2=g1-A; ASD=(L+658/A)/2;

    if ASD<L
        ASD=(658*L/A)^0.5;
    end
    if L>=((200*K*h1)*0.5+(200*K*h2)*0.5);
        ASD=(200*K*h1)*0.5+(200*K*h2)*0.5; xM=L-ASD;
    else
        ASD=(L+200*K*(h1^0.5+h2^0.5)^2/L)/2;
        xM=L*h1^0.5/(h1^0.5+h2^0.5)-100*K*h1^0.5*(h1^0.5+h2^0.5)/L;
    end

    SSD=0; vF=x(4); xf=xM+0.278*x(4)*x(2);
    while vF>5
        if xf>L
            g=g2;
        else
            g=g1-xf/L*A;
        end
        vF=(vF^2-2*(x(3)+g*9.81/100)*3.6^2*0.005)^0.5; SSD=SSD+0.005; xf=xf+0.005;
    end

    SSD=0.278*x(4)*x(2)+SSD; G=ASD-SSD;
APPENDIX IV

COMPLETE BEFORE-AND-AFTER DISTRIBUTION OF RELIABILITY INDICES FOR 2D DESIGN MODEL OF CREST VERTICAL CURVES
Figure IV.1 Before-and-After Distribution of Reliability Indices for Design Speed of 75 km/h.
Figure IV.2 Before-and-After Distribution of Reliability Indices for Design Speed of 80 km/h.
Figure IV.3 Before-and-After Distribution of Reliability Indices for Design Speed of 85 km/h.
Figure IV.4 Before-and-After Distribution of Reliability Indices for Design Speed of 90 km/h.
Figure IV.5 Before-and-After Distribution of Reliability Indices for Design Speed of 95 km/h.
Figure IV.6 Before-and-After Distribution of Reliability Indices for Design Speed of 100 km/h.
Figure IV.7 Before-and-After Distribution of Reliability Indices for Design Speed of 105 km/h.
Figure IV.8 Before-and-After Distribution of Reliability Indices for Design Speed of 110 km/h.
APPENDIX V

COMPLETE BEFORE-AND-AFTER DISTRIBUTION OF RELIABILITY INDICES FOR 3D DESIGN MODEL OF CREST VERTICAL CURVES
Figure V.1 Before-and-After Distribution of Reliability Indices for Design Speed of 75 km/h.
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Figure V.3 Before-and-After Distribution of Reliability Indices for Design Speed of 85 km/h.
Figure V.4 Before-and-After Distribution of Reliability Indices for Design Speed of 90 km/h.
Figure V.5 Before-and-After Distribution of Reliability Indices for Design Speed of 95 km/h.
Figure V.6 Before-and-After Distribution of Reliability Indices for Design Speed of 100 km/h.
Figure V.7 Before-and-After Distribution of Reliability Indices for Design Speed of 105 km/h.
Figure V.8 Before-and-After Distribution of Reliability Indices for Design Speed of 110 km/h.