INVESTIGATION OF EFFECTIVE STIFFNESS OF HIGH-RISE
CONCRETE SHEAR WALLS

by

AARON DAVID KORCHINSKI
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ABSTRACT

High-rise concrete shear walls are expected to crack during an earthquake. To account for cracking when conducting linear seismic analysis an effective moment of inertia $I_e$; recommendations for $I_e$ vary widely. Expressions for the upper and lower bound effective stiffness were previously developed using bilinear curves with equal area-under-the-curve as the actual nonlinear force-displacement curves determined by numerically integrating a trilinear moment-curvature model.

In this thesis, the analysis tools required to study the transition between an uncracked and a severely cracked response are developed. These tools include a simplified force-displacement relationship that approximates the theoretical response better than numerical integration of a trilinear moment-curvature model. A hysteretic force-displacement model is also developed through using experimental data obtained from large scale wall specimen. The hysteretic model is implemented into OpenSees to allow for nonlinear time history analysis of high-rise concrete shear walls as a single-degree-of-freedom system that accounts for the effects of cracking. The maximum nonlinear displacement is used to find the effective elastic stiffness. The factor $\alpha$ quantifies the amount the initial stiffness of the wall needs to be reduced for an elastic system to have the same maximum displacement. A range of $R$ values are studied by scaling the ground motions so the elastic demand is $R$ times greater than the strength of the wall. This scaling method is verified as part of the thesis work. A suite of unmodified ground motions and a suite of modified spectrum matched ground motions are used in this study.

A set of generalized walls are developed to represent the range of wall shapes, reinforcement and axial load of typical high-rise concrete shear walls. A preliminary nonlinear time history analysis is conducted for nineteen generalized walls using two suites of ground motions at initial periods of 2 and 4 seconds with nineteen $R$ values ranging from 0.5 to 5.0. The results of the analysis suggest that determining effective stiffness from equal area bilinear curves has limited success. The trends in stiffness effective stiffness are remarkably similar considering the wide range of generalized walls studied. The initial period has greater influence on effective stiffness than the shape of the force-displacement curve. The effective stiffness dropped sharply for higher $R$ values for the 2 second initial period while remain constant for the 4 second initial period.
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1.0 Introduction

In Chapter 1 the current practice, existing recommendations and previous work conducted in studying the effective stiffness of concrete walls are discussed. The objectives, overview of the methodology used and organization of the thesis are also presented.

1.1 Background

Designers typically use commercial software packages such as ETABS to conduct a response spectrum analysis when designing the lateral force resisting system of high-rise buildings. This analysis is used to estimate the displacement demand (design displacement) of each storey. The 2005 National Building Code of Canada (NRCC, 2005) limits the amount of interstorey drift. The current Canadian concrete code A23.3-04 (CSA, 2004) uses the design displacement for calculations concerning wall ductility, punching shear of slabs and shear design of walls. Response spectrum analysis only provides a good estimate of displacement if cracking is accounted for.

The flexural rigidity of an uncracked wall is usually assumed to be \( E_cI_g \) where \( E_c \) is the concrete modulus of elasticity and \( I_g \) is the second moment of inertia of the gross concrete section. The influence of reinforcement on the second moment of inertia is often ignored for simplicity. During an earthquake, a wall will crack and the flexural rigidity decreases in the vicinity of the cracks. To account for cracking when conducting linear seismic analysis an effective moment of inertia \( I_e \) can be used; the flexural rigidity is reduced from \( E_cI_g \) to \( E_eI_e \).

The average reduction of flexural stiffness due to cracking can be expressed as \( \alpha = I_e/I_g \). The effective moment of inertia \( I_e \) is an average value accounting for the difference in stiffness between uncracked and cracked sections and the variation of cracking over the height of the wall (Ibrahim and Adebar, 2004). Recommendations for \( I_e \) vary widely and in some cases differ by a factor of three. The current approach is to use 0.7 \( I_e \) as suggested by the A23.3-94 commentary (CAC, 1995). This is based upon a simplification of the recommendation made by Paulay (1986).

The effective stiffness of a concrete wall can be expected to change as the wall becomes more cracked. The changing level of cracking prompted Ibrahim (2000) to bind the solution with
expressions for the upper and lower bound effective stiffness. The effective stiffness was determined by using the stiffness of the elastic portion of bilinear curves with the same area-under-the-curve as the actual nonlinear upper and lower bound force-displacement curves. The upper bound expression is shown by Equation 1.1 which is the same as the recommendation made by Paulay. The lower bound expression is shown by Equation 1.2.

\[
I_e = \left(0.6 + \frac{P}{f_c'A_g}\right)I_g \leq I_g \tag{1.1}
\]

\[
I_e = \left(0.2 + 2.5 \frac{P}{f_c'A_g}\right)I_g \leq 0.7I_g \tag{1.2}
\]

where  

- \( P \) = applied axial compression
- \( f_c' \) = specified concrete strength
- \( A_g \) = gross cross-sectional area of wall

The upper bound expression was put into A23.3-04 primarily because the 2005 National Building Code of Canada (NBCC) had increased earthquake demands but added \( R_o \) to reduce design force. This reduction does not however reduce displacement demands imposed by the increased earthquake demands. The 2005 NBCC increased interstorey drift allowances from 2.0% to 2.5%. The 25% increase is much less than the increase in displacement demands and serves as a way of tightening up on allowable displacement demands. Due to the uncertainty of using either the upper or lower bound, and not to err on the side of making all structures not work accordingly, A23.3-04 includes the upper-bound equation for \( I_e \). (Adebar, Mutrie and Devall, 2004)

### 1.2 Objectives of Thesis

The wide range in recommended values for the effective stiffness of concrete walls led to the development of the upper and lower bound equations by Ibrahim (2000). The actual response of a wall will be somewhere between these proposed bounds. The need for a simple model for conducting the nonlinear seismic analysis of high-rise concrete walls which transitions between the uncracked response and the severely cracked response is what led to the current study.
The objectives of the thesis are:

- Investigate whether using equal area bilinear approximations of the force-displacement response is the right approach in determining upper and lower bound effective stiffness values.
- Investigate the transition between uncracked response (upper bound) and the severely cracked response (lower bound) of high-rise concrete shear walls.
- Develop the tools to permit the nonlinear seismic analysis of high-rise concrete shear walls accounting for the transition between the uncracked and severely cracked response.

1.3 Overview of Methodology Used in Thesis

Sectional analysis is conducted using an appropriate tension stiffening model for high-rise concrete shear walls to determine moment-curvature response. The theoretical force-displacement response is found by numerical integration of the moment-curvature response. A simplified force-displacement relationship is developed that approximates the theoretical response. The approximation is much closer to the theoretical response than numerical integration of the Adebar and Ibrahim (2002) trilinear moment-curvature response.

A hysteretic force-displacement model is developed using experimental data obtained from large scale testing conducted by Adebar, Ibrahim and Bryson (2000). The simplified force-displacement relationship is used as the backbone response of the hysteretic model. The hysteretic model is implemented into OpenSees (PEER, 2005) to allow for nonlinear analysis of high-rise concrete shear walls that accounts for a transition between the uncracked and severely cracked response. The walls are modeled as a single-degree-of-freedom system using 3% mass proportional Rayleigh damping.

A suite of unmodified ground motions and a suite of ground motions modified by SYNTH (Naumoski, 2001) to match an altered version of the 2005 NBCC (NRCC, 2005) design spectrum for Vancouver, BC are used as part of this study. The period corresponding to the average maximum nonlinear displacement for each suite of ground motions is determined using the altered version of the 2005 NBCC design spectrum. The difference between this period and the initial period of the wall is used to calculate the stiffness reduction factor $\alpha$. The factor $\alpha$ quantifies the amount that the initial stiffness of the wall needs to be reduced for an elastic system to have the same displacement, or in other words, determine the effective elastic
stiffness. A range of $R$ values are studied by scaling the ground motions so the elastic demand is $R$ times greater than the strength of the wall. This scaling method is verified as part of the thesis work.

A set of generalized walls is developed by studying three different shaped walls, each with four different amounts of flange reinforcement, for axial loads ranging from 0 to $0.30f_yA_e$. Information required for the simplified force-displacement relationship is calculated and normalized by the wall strength. This is done for each wall with its range of reinforcement and axial load. The normalized results are used to determine a set of generalized walls which represent the range of wall shapes, reinforcement and axial load. This approach provides the widest range in force-displacement curves and minimizes the amount of nonlinear analyses required.

A preliminary nonlinear time history analysis is conducted for two suites of ground motions. Nineteen generalized walls are studied at initial periods of 2 and 4 seconds for each of the two suites of ground motions. The generalized walls are subsequently studied at nineteen $R$ values ranging from 0.5 to 5.0.

1.4 Organization of Thesis

This thesis is organized into six chapters and includes an electronic appendix. Chapter 2 describes the development of a simplified force-displacement relationship to approximate the theoretical force-displacement response of high-rise concrete shear walls. A hysteretic force-displacement model is developed using the experimental results of a large scale test specimen conducted by Adebar, Ibrahim and Bryson (2000). Chapter 3 describes how the hysteretic model is incorporated into OpenSees (PEER, 2005). Nonlinear time history analyses are conducted utilizing the hysteretic model. A procedure is developed to determine the effective stiffness of an elastic system based off the maximum nonlinear displacements. Chapter 4 describes a set of generalized walls developed to represent the full range of force-displacement curves for typical reinforced concrete shear walls. Chapter 5 presents the results of a preliminary nonlinear analysis conducted using the developed tools. The effective stiffness of each of the generalized walls is quantified as a reduction factor $\alpha$ for a range of typical $R$ values. Chapter 6 summarizes the accomplishments achieved through the current study and the conclusions it offers along with recommendations of future work.
An electronic appendix accompanies this thesis due to the computationally intense nature of the work conducted. The MATLAB scripts used to study moment-curvature and force-displacement relationships, OpenSees source files for the hysteretic model, Tcl input files to run analyses in OpenSees and the Microsoft Excel files used to post-process the data from the preliminary analysis are included. The organization and function of each of these files is thoroughly documented to allow interested parties to use and further refine the tools developed as part of this thesis. The raw data from Adebar, Ibrahim and Bryson (2000) used to develop the hysteretic model along with the raw data output from the preliminary analysis are also included. A more detailed description of the contents of the electronic appendix is found in Appendix E.
2.0 Simplified Force-Displacement Relationship and Hysteretic Model

The steps undertaken to develop a simplified force-displacement relationship are presented with data from three analysis walls. First, each analysis wall is described then the resulting data is used to illustrate subsequent steps. Second, the moment-curvature response of each wall must be determined. Discussion topics include the Adebar and Ibrahim trilinear model (2002), influence of tension stiffening, development of a moment-curvature program for theoretical response and the modified equal area trilinear model. Third, the actual force-displacement response must be determined before a simplified relationship can be created. Discussion topics include load distribution, description of a force-displacement program for theoretical and modeled response and the options considered for the simplified relationship. The complete simplified force-displacement relationship is presented.

The simplified force-displacement relationship is used to define the backbone of the force-displacement hysteretic model. The development of the hysteretic model through use of the experimental data from Adebar, Ibrahim and Bryson (2000) is described. The force-displacement hysteretic model is presented and compared with the experimental data.

2.1 Analysis Walls

In the interest of keeping the method as general as possible, three analysis walls are used to develop a simplified force-displacement relationship in order to accommodate different wall sizes and shapes. The walls presented are hypothetical 9m flanged and rectangular walls as seen in Figure 2.1 as well as a 1.625m flanged wall seen in Figure 2.2 which was tested at the University of British Columbia (UBC). Both the 9m flanged and rectangular walls represent components of the central seismic resisting core of a 30 storey high-rise concrete building. These walls have a reinforcement ratio $\rho$ of 0.01 for the flanges and end zones and 0.0027 for the webs. The reinforcement ratio $\rho$ is defined as the area of steel $A_s$ divided by the gross area of concrete $A_c$. The flange reinforcement satisfies the 0.0025 minimum as per A23.3-04 Clause 21.6.5.1 (CSA, 2004). The UBC Wall was included to provide a comparison between the simplified force-displacement relationship prediction and the actual experimental results from Adebar, Ibrahim and Bryson (2000). The flanges have a displacement ratio $\rho$ of 0.0065 for the flanges and 0.0026 for the web.
2.2 Moment-Curvature

The moment-curvature response of a member is required in order to obtain the force-displacement relationship. For a given load, a curvature diagram can be created from the moment diagram using the moment-curvature response. This is done in preparation to use the second-moment-area theorem.

2.2.1 Adebar and Ibrahim Trilinear Model

A trilinear moment-curvature response for reinforced concrete walls experiencing axial compression and cyclic bending was presented by Adebar and Ibrahim (2002). The concept of an upper-bound and a lower-bound moment-curvature response was introduced. The upper-bound represents an uncracked wall loaded monotonically until yielding while the lower-bound represents a wall reloaded after being severely cracked. This trilinear model is shown in Figure
2.3 and is defined by four parameters: the slope of the first linear segment assumed equal to the gross uncracked section stiffness \( E_c I_g \); the slope of the second linear segment assumed equal to the cracked section stiffness \( E_c I_{cr} \) with zero axial load; the intersection between the first two linear segments \( M_L \); and the nominal flexural capacity of the section \( M_N \).

Both \( E_c I_g \) and \( M_N \) are commonly calculated by designers. Through the use of a computer analysis program, by setting the axial load equal to zero and having zero concrete tension strength, the resulting moment-curvature response will have an initial slope equal to \( E_c I_{cr} \). The formulation for \( M_L \) as developed by Adebar and Ibrahim is shown in Equation 2.1.

\[
M_L = \beta f_{cr} S_g + P \left( \frac{S_g}{A_g} + 0.08 \ell_w \right)
\]  

(2.1)

where \( \beta = 1.5 \) for upper-bound response, 0 for lower-bound response

- \( f_{cr} \) = cracking strength of concrete
- \( S_g \) = gross section modulus related to tension face
- \( P \) = applied axial compression
- \( A_g \) = gross cross-sectional area of wall
- \( \ell_w \) = length of wall

In this thesis the Adebar and Ibrahim trilinear model will be used to determine the upper-bound response \((\beta = 1.5)\) unless noted otherwise. The upper-bound response provides the moment-curvature envelope which can then be used to develop the force-displacement envelope.
2.2.2 Influence of Tension Stiffening

Concrete alone behaves poorly in tension and requires reinforcement to maintain structural integrity. Once the concrete cracks the tension stresses are maintained across the crack solely by the reinforcement. However, at locations away from the cracks some tension stress is transferred back into the concrete through the bond between the two materials. The reinforcement experiences maximum strain at cracks and lower strain between the cracks. Concrete tension stress develops between the cracks and results in a greater cross-section than that of only the bare steel bar; this effect is known as tension stiffening (Bentz, 2000).

The computer program Response-2000 (Bentz, 2000) was used to generate moment-curvature data for the three analysis walls to provide a theoretical response to benchmark against the trilinear moment-curvature model presented by Adebar and Ibrahim (2002). For all three walls the Response-2000 results have segments that clearly match the slopes of the trilinear model. However, the Response-2000 results display significant rounding before capacity is reached as seen in Figure 2.4. This significant rounding can be explained by the difference in the methods used to account for tension stiffening by Response-2000 and the Adebar and Ibrahim trilinear model. The trilinear model was developed using the tension stiffening formulation shown in Equation 2.2.

\[
f_t = \frac{f_{cr}}{1 + \sqrt{500\varepsilon_c}}
\]  

(2.2)

where \(f_{cr}\) = concrete tension strength (MPa)  
\(\varepsilon_c\) = average concrete strain (mm/mm)

The above approach is an average-stress-average-strain relationship for a reinforced concrete member over a length that may include several cracks. At the cracks the local strain will be much larger than the average and for this reason a crack check must be performed. The capacity of the reinforced concrete member is limited by the capacity of the reinforcement at cracks.

Response-2000 also uses an average-stress-average-strain relationship to account for tension stiffening and performs the necessary crack check. The tension stiffening formulation developed by Bentz (2000) as part of Response-2000 is shown in Equations 2.3 and 2.4.
Figure 2.4 Moment-Curvature Response
where $f_{cr}$ = concrete tension strength (MPa)
$\varepsilon_c$ = average concrete strain (mm/mm)
$m$ = bond parameter (mm)
$A_c$ = area of concrete effectively bonded to bar (mm$^2$)
$d_b$ = diameter of bar in concrete stiffened area (mm)

This approach endeavours to include bond characteristics as it is suggested that tension stiffening is largely a bond phenomenon. Higher levels of tension stress in cracked concrete would be expected for a closely spaced arrangement of smaller diameter bars than a widely spaced arrangement of large bars. This approach agrees well with experimental data from different physical tests (Bentz, 2000); however, it should be noted that the test specimens were panel and shell elements that ranged from 70 to 285 mm thick.

A flanged wall section with a length of wall $\ell_w$ of 1625 mm was tested by Adebar, Ibrahim and Bryson (2000). This test was conducted to verify the accuracy of the Adebar and Ibrahim trilinear moment-curvature model. The experimental data and the trilinear model were in close agreement, suggesting that the tension stiffening method shown by Equation 2.2 is the most appropriate for reinforced concrete shear walls.

2.2.3 Moment-Curvature Program

The significantly rounded moment-curvature behaviours suggested by Response-2000 (Bentz, 2000) do not provide an adequate benchmark against the trilinear model. The rounding would significantly decrease the section stiffness and increase the expected displacements. In the interest of obtaining a proper comparison a MATLAB script $Mphi$ was created by using the tension stiffening method shown in Equation 2.2.

$Mphi$ calculates the moment-curvature response of a member using plane sections analysis to satisfy equilibrium of forces, strain compatibility and material stress-strain constitutive relationships. A linear strain distribution, or uniform curvature, is assumed and the material
constitutive relationships are specified. The strain distribution across the member can be expressed as a function of curvature and the strain at centroid \( \varepsilon_{cen} \). Through strain compatibility and the use of constitutive relationships for each material, the stress distribution resulting from the strain distribution of the assumed curvature can be obtained. With the stress distribution known, the axial force and bending moment capacities can be found satisfying equilibrium.

An iterative process is required to find the strain at centroid \( \varepsilon_{cen} \) for a given curvature and applied load. The difference between the given axial load \( P \) and the calculated axial force \( N \), \( P - N \), is then determined. The iterative solution for \( \varepsilon_{cen} \) is a function of strain and \( P - N \), concluding once \( P - N \) falls within a set tolerance. The curvature is now increased by a small increment and the iterative process is repeated starting with the solution of \( \varepsilon_{cen} \) for the previous curvature. Since the curvature increment is minimal, the change in strain will also be minimal. Starting the iterative process with the previous solution decreases computational effort; this is an important savings when generating several hundred moment-curvature points.

The iteration technique used by Mphi is based largely upon a subroutine written by Adebar (1990). A detailed explanation of this technique and corresponding computer code can be found in the electronic appendix. Details of the strain compatibility, constitutive and equilibrium relationships are also included.

### 2.2.4 Modified Equal Area Trilinear Model

The trilinear model developed by Adebar and Ibrahim (2002) performs well over a wide range of wall types and sizes. The expression for \( M_L \) was developed to be the best fit of data from several hundred walls. However, when focusing on three individual walls the error will become more evident. In the interest of not transferring these errors into further analysis an individually determined \( M_L \) was used for each wall.

The same basic premise as the Adebar and Ibrahim model is used in determining the modified \( M_L \); equal area underneath the trilinear and theoretical moment-curvature curves is assumed. The post cracking slope is also set as \( E_{cr} I_{cr} \) and the flat yield plateau commencing at \( M_N \) remains the same. It should be noted that \( M_N \) is taken from the output of Mphi while in a design
situation it could be found from the rectangular stress block simplification used by A23.3-04 (CSA, 2004). The initial slope was further refined by using the transformed moment of inertia $I_{tr}$ rather than the gross moment of inertia $I_g$. The transformed section modulus $S_{tr}$ is used to calculate the cracking moment $M_{cr}$ as seen in Equation 2.5. The transformed section properties include the stiffening effect of the steel reinforcement and are calculated using the parallel axis theorem.

\[
M_{cr} = S_{tr} \left( f_{cr} + \frac{P}{A_g} \right)
\]  

(2.5)

Use of the transformed properties allows the modified equal area trilinear model to have the same initial slope as the theoretical moment-curvature curve output from $Mphi$. The value calculated for $M_{cr}$ now indicates the exact point where the modified equal area trilinear model starts to deviate from the theoretical data. The same initial slope also makes the visual comparison between the modified equal area trilinear model and the theoretical data more intuitive.

The area underneath the theoretical moment-curvature curve was found by numerically integrating the output from $Mphi$ for each wall. Equations for the area underneath the modified equal area trilinear model were created in terms of $M_L$. An iterative process for $M_L$ was conducted so that the area underneath the modified trilinear model would equal the area underneath the theoretical curve. The modified trilinear model and the theoretical response can be seen for each wall in Figure 2.5. The difference between the $M_L$ calculated by the Adebar and Ibrahim trilinear model and the modified trilinear model was relatively small. The difference ranged from 3.2% to 16.3% as summarized in Table 2.1.

<table>
<thead>
<tr>
<th>Wall</th>
<th>$M_L$ (kN·m)</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flanged</td>
<td>303336.6</td>
<td>293638.3</td>
</tr>
<tr>
<td>Rectangular</td>
<td>107605.4</td>
<td>128594.5</td>
</tr>
<tr>
<td>UBC</td>
<td>1163.5</td>
<td>1210.5</td>
</tr>
</tbody>
</table>

Table 2.1 Theoretical and Equal Area ML Comparison
Figure 2.5 Modified Trilinear Moment-Curvature Response
2.3 Force-Displacement

For a given load, a curvature diagram can be created from the moment diagram using the moment-curvature response obtained previously. The second-moment-area theorem can now be used to determine the displacement at top of wall for the base shear resulting from the given load.

2.3.1 Load Distribution

The nominal flexural capacity of a cantilever wall has been determined as $M_N$. The amount of applied load that can be sustained prior to reaching $M_N$ depends on how the load is distributed over the height of the wall. When conducting seismic design, the inertial forces induced upon a structure by an earthquake become increasingly amplified over the height of the structure. To model this three load distributions were considered: first mode, linearly varying and a concentrated point load at the top of wall.

The expression for the first mode displacement distribution of a uniform cantilevered wall is defined by Equation 2.6 (Chopra, 2001). If the load distribution is assumed to have the same shape as the displacement distribution then the force distribution can be defined using the same expression.

$$f^*(x) = C_{1} \left[ \cosh \beta x - \cos \beta x - \frac{\cosh \beta L + \cos \beta L}{\sinh \beta L + \sin \beta L} (\sinh \beta x - \sin \beta x) \right]$$

where

- $L$ = height of wall
- $x$ = distance from base of wall
- $\beta L = 1.8751$

The linearly varying load distribution is in the shape of an inverted triangle having maximum load applied at top of wall. The concentrated point load at the top of wall distribution was included to use as a comparison with the experimental data obtained from the UBC wall which was loaded in this fashion.

The effects of load distribution on the force-displacement relationship are illustrated in. The initial stiffness has been normalized and the base shear $V_b$ has been divided by the nominal shear capacity $V_N$ to assist in comparison. The point load distribution is the least stiff, as one might
expect, since the load is concentrated the furthest away from the base of the wall. The first mode and linearly varying load distributions are quite similar with the first mode being only slightly less stiff. Due to the similarity, and far easier application, the linearly varying load distribution was chosen for all further analysis. This simplification is also commonly made in seismic design practice.

2.3.2 Force-Displacement Program
Calculation of the force-displacement relationship requires performance of many numerical integrations over the height of the wall which can be computed by the MATLAB script Fdisp. The wall height is assumed to be $8 \ell_w$, although the height is somewhat arbitrary as it only affects the magnitude of the results and not the shape of the relationship which is of primary interest in this thesis. The load acting along the wall is numerically integrated over many small increments to find shear and again integrated from shear to find moment. Now that the moment is known, the curvature along the height of the wall can be determined by either the theoretical or trilinear method. The theoretical method uses the calculated moment to linearly interpolate between the finely incremented $M_{phi}$ data points for the corresponding curvature. The trilinear method uses the calculated moment to linearly interpolate between the points of the equal area trilinear model for the corresponding curvature.
The curvature along the height of the wall, determined using either method, allows for the use of the second-moment-area theorem. The theorem states that the displacement $\Delta_{AB}$ of point $A$ from the tangent at point $B$ is equal to the second moment of area of the curvature diagram between points $A$ and $B$ taken about point $A$ as expressed in Equation 2.7.

$$\Delta_{AB} = \int_A^B \phi x \, dx \tag{2.7}$$

For a cantilever wall this simply means that the second moment of area of the curvature diagram is taken about the top of the wall as seen in Figure 2.7. The force-displacement relationship is found by repeating this process for a series of incrementally larger loads and plotting the displacement versus the base shear. The computer code for $F_{disp}$ can be found in the electronic appendix.

The force-displacement relationships from the theoretical and trilinear methods are shown in for the flanged wall. The moment-curvature response of the flanged wall has been repeated for ease of comparison. There are several important characteristics that are present for all three analysis walls so illustration of only one wall will suffice.

The trilinear method consistently produces higher force-displacement curves because the slope change of the trilinear method at $M_L$ is larger than $M_{cr}$ where the theoretical response becomes nonlinear. The trilinear model follows the initial slope $E_c I_c$ for longer and thus produces less curvature for a given moment than the theoretical curve which is now following a degrading slope. The post cracking slope of the theoretical force-displacement curve is steeper than the trilinear model. This is because the degrading slope of the theoretical moment-curvature curve is steeper than the post cracking slope of the trilinear moment-curvature model.
The difference between the theoretical and trilinear post cracking moment-curvature slopes causes the theoretical force-displacement curve to converge toward the trilinear model. The final displacement at $V_N$ is the same for both the trilinear and theoretical methods. This is expected since the area underneath both moment-curvature responses is the same, only the relative distribution of the area is different. Simply put, initially the trilinear moment-curvature underestimates curvature in the cracking region and overestimates curvature closer to $M_N$ with a net effect of almost no difference in displacement at $V_N$. 

Figure 2.8 Theoretical and Trilinear Force-Displacement Comparison (Flanged Wall)
2.3.3 Development of Simplified Force-Displacement Relationship

The goal is to develop a simple force-displacement relationship based off a trilinear moment-curvature model that is more representative of the theoretical behaviour. When examining there is an established initial slope before cracking and what appears to be a pronounced secondary slope that is approached before reaching the flat plateau at flexural capacity. An expression is required for the initial slope to begin transitioning towards the secondary slope once the section is cracked.

The modified Ramberg-Osgood function (Mattock, 1984), developed for the stress-strain relationship of prestressed reinforcing steel, uses a curve fitting factor to control the transition between two known slopes (Appendix A). While providing the transition between two slopes, the modified Ramberg-Osgood function has several shortcomings in a force-displacement application. First, there is no control as to at what point the function deviates from the first slope or converges to the second slope. Both of these key points should be included for the approximation to be an accurate representation. Second, the curve fitting factor only adjusts the degree of rounding, not the shape of the rounding. The modified Ramberg-Osgood function rounds uniformly between the two slopes while the actual force-displacement relationship rounding is largely skewed. Although proving ultimately unsuccessful in matching the shape of the force-displacement curve, the attempted use of the modified Ramberg-Osgood function provided valuable insight into what requirements the proposed solution must satisfy.

The use of a fourth order polynomial equation satisfies all the requirements necessary to adequately represent the force-displacement relationship. The general form of a fourth order polynomial is shown in Equation 2.8 and expresses the position at any point along the curve. The general form of the derivative is shown in Equation 2.9 and expresses the slope at any point along the curve.

\[ y = Ax^4 + Bx^3 + Cx^2 + Dx + E \]  
\[ \frac{dy}{dx} = 4Ax^3 + 3Bx^2 + 2Cx + D \]

The solution of the polynomial requires five pieces of information to determine the five constants expressed by upper case letters. The position of the start point, position of the end point, slope of the start point and the slope of the end point are used. The fifth piece of information is the position of a fit point that the curve must pass through between the start and
end points of the curve. The fit point provides the needed control not available when only using a third order polynomial. The use of a fit point serves the same purpose as the modified Ramberg-Osgood curve fitting factor but is far more effective for this force-displacement application.

### 2.4 Simplified Force-Displacement Relationship

The simplified force-displacement relationship is developed for a flexural member in terms of base shear and displacement at top of wall. The flexural properties of cracking moment $M_{cr}$ and nominal flexural capacity $M_N$ have corresponding base shears $V_{cr}$ and $V_N$ which serve as key points in the force-displacement relationship. The relationship is a piecewise continuous function consisting of three line segments seen in. The first segment is linear from the origin until cracking at $V_{cr}$, the next segment is a fourth order polynomial from $V_{cr}$ to $V_N$ and lastly a flat plateau from $V_N$ to ultimate capacity.

![Figure 2.9 Simplified Force-Displacement Relationship](image)

Cracking occurs at $V_{cr}$ calculated by Equation 2.10 for a linearly varying load distribution. The displacement at cracking $\Delta_{cr}$ is calculated by Equation 2.11 which can be derived by first principles.

\[
V_{cr} = \frac{3M_{cr}}{2L} \tag{2.10}
\]

\[
\Delta_{cr} = \frac{33}{120} \phi_{cr} L^2 \tag{2.11}
\]
The post cracking curve from $V_{cr}$ to $V_N$ is defined by a fourth order polynomial with local origin at $(\Delta_{cr}, V_{cr})$ effectively setting the start point position equal to zero. Using localized coordinates also eliminates the constant $E$ from the polynomial expression. The slope at the start point has been reduced to $0.9(V_{cr}/\Delta_{cr})$ through calibration from the three analysis walls. The end point position is $(\Delta_N - \Delta_{cr}, V_N - V_{cr})$ where $V_N$ is defined by Equation 2.12 and $\Delta_N$ is the displacement found using the second-moment-area theorem described previously in Section 2.3.2.

$$V_N = \frac{3M_N}{2L}$$

(2.12)

The end point slope is found by calculating the position of a tangent point $V_t = 0.95(V_N - V_{cr})$ and then using this point to approximate the slope. Through calibration from the three analysis walls, the best fit point was found to be $V_{fit} = 0.9(V_N - V_{cr})$. The difference between the theoretical and trilinear force-displacement curves at $0.9(V_N - V_{cr})$ and $0.95(V_N - V_{cr})$ in is minimal. Therefore the use of the trilinear model to calculate displacements with the second moment-area theorem is appropriate.

The force-displacement relationships for the three analysis walls are shown along with the theoretical and modified equal area trilinear methods. The approximation shown using the simplified force-displacement relationship has an initial slope of $0.9(V_{cr}/\Delta_{cr})$ after cracking, the second slope taken at a tangent point $0.95(V_N - V_{cr})$ and a fit point at $0.9(V_N - V_{cr})$. The simplified force-displacement relationship provides an estimate that follows the theoretical response more closely than integrating the equal area trilinear curve with the second-moment-area theorem.
Figure 2.10 Simplified Force-Displacement Relationship Comparison
2.5 Development of Force-Displacement Hysteretic Model

The development of the force-displacement hysteretic model is based on the experimental data obtained from the UBC wall by Adebar, Ibrahim and Bryson (2000). Testing consisted of loading the wall to a specified displacement in the “positive” direction, unloading and then reloading to the same specified displacement in the “negative” direction. This cycle was repeated four times for each incrementally larger specified displacement until the wall became severely damaged. The experimental force-displacement results can be seen in Figure 2.11.

![Figure 2.11 Experimental Results from Adebar, Ibrahim and Bryson](image)

The experimental data was gathered continuously over the four loading/unloading cycles for each specified displacement. To assist in development of a hysteretic model, the data was separated into positive loading, positive unloading, negative loading and negative unloading segments. The data segments were regrouped by the maximum positive/negative displacement experienced. This means that the last three cycles for a specified displacement are grouped with the first cycle of the next largest specified displacement. The respective loading and unloading paths for each maximum previous displacement grouping have minimal spread indicating that very little in-cycle degradation is occurring.
2.6 Hysteretic Force-Displacement Model

The simplified force-displacement relationship is used as the backbone response for the hysteretic force-displacement model. An additional point of interest $V_{cl}$, the point at which the cracks will theoretically close from the applied axial compression, is expressed in Equation 2.13. This point is similar to $V_{cr}$ except the tensile strength of the concrete is excluded.

$$V_{cl} = \frac{3}{2L} M_{cl} = \frac{3}{2L} \left( S \cdot \frac{P}{A_g} \right)$$

The hysteretic force-displacement model follows the backbone response when the wall is loaded less than $V_{cr}$ or beyond the previous maximum displacement. Once $V_{cr}$ is exceeded, the inelastic unloading paths return linearly from the maximum displacement to $V_{cl}$, as seen in Figure 2.12. Subsequent reloading then follows a linear path from $V_{cr}$ to the maximum previous displacement, where it rejoins the backbone. Once $V_N$ has been reached, the reloading paths no longer return to the curved section of the backbone; the reloading paths are now completely trilinear. The reloading paths radiate about the point $V_{cr}$ forming a series of degrading slopes.

![Figure 2.12 Hysteretic Force-Displacement Model for Small Displacements](image)

The loading lower bound has a slope parallel to the secondary slope determined by the simplified force-displacement relationship seen in. The unloading paths continue to return linearly from the maximum displacement to $V_{cl}$. The maximum displacements have exceeded the displacement at which the loading lower bound intercepts the yield plateau. All subsequent reloading now follows the lower bound regardless of maximum displacement.
The hysteretic force-displacement model was developed from full cyclic tests. In full cyclic tests, the wall is pushed from a maximum positive to a maximum negative displacement. The seismic response of concrete shear walls, however, does not typically involve full cycles. For the case of mid-cycle reloading, the reloading path follows the initial slope $k$ from the point where it leaves the unloading path as seen in Figure 2.14. The reloading path now intersects and follows a linear path from $V_{cr}$ to the maximum previous displacement. When the maximum displacement exceeds the loading lower bound intercept with the yield plateau, mid-cycle reloading will follow the initial slope $k$ until rejoining the lower bound. Unfortunately no experimental data is available to verify the presented mid-cycle reloading rule. The rule was determined as a rational way to return to the full cycle behaviour studied by the test.
The model is compared to the experimental data for small positive displacements having drift ratios ranging from 0.3% to 0.6% as shown in Figure 2.15. The experimental data and model predictions are shown by solid lines for loading and dashed lines for unloading. The model works extremely well for drifts of 0.3% and 0.4% for both the loading and unloading segments. For drifts of 0.5% and 0.6% the model works reasonably well. The second slope of the loading path agrees closely with the experimental data for displacements approaching the maximum previous displacement. Rounding of the experimental data is starting to occur in the range of $V_{cr}$. The area cut off by rounding is approximately equal to the area above the loading path due to residual displacements; the net effect is considered negligible.

The model is compared to the experimental data for large positive displacements having drift ratios ranging from 0.9% to 1.5% as shown in Figure 2.16. The loading path follows the lower bound for these displacements. The lower bound is conservative suggesting that the wall is less stiff for a drift of 0.9%. As the drift increases to 1.5% the lower bound becomes a more accurate representation. The additional area above the model from residual displacement continues to compensate for rounding. There is also an increasing amount of area above the model resulting from strain hardening. The model accounts for the increasing levels of energy dissipation from damage by allowing more area in the hysteretic loop. The experimental hysteretic loop is extremely pinched due to the axial compression of the wall closing the cracks; the model conservatively captures this phenomenon by always returning to $V_{cr}$. The hysteretic force-displacement model is compared with the experimental data for negative displacements having drift ratios ranging from 0.3% to 1.5% in Appendix B.

It is very difficult to predict residual displacements because to do so one must also predict the degree of which the initial slope $k$ softens. Residual displacements are not possible without a softening initial slope. A certain amount of applied lateral force, with resulting displacement, is required to counteract the axial compression that closes the cracks in the wall. When residual displacement occurs the displacement required to reach the point where cracks open increases. Thus the slope of the path must soften to reach the fixed point in space where cracks open. The residual displacements and softening slopes can be thought to effectively cancel each other out. By maintaining the initial slope $k$ and neglecting residual displacements, the model maintains a greater level of simplicity.
Figure 2.15 Hysteretic Model Shown with Small Displacement Experimental Data
Figure 2.16 Hysteretic Model Shown with Large Displacement Experimental Data
3.0 Nonlinear Analysis

The detailed hysteretic model presented in the previous chapter has been developed in order to find an effective stiffness relationship for concrete high-rise walls through nonlinear analysis. The current chapter provides a description of the computer program used and an explanation of how the hysteretic model was implemented. A pilot analysis was conducted to find the most appropriate simplified model to use in determining an effective stiffness. This analysis also explored the use of raw and altered ground motions. Several additional analysis considerations are presented. The chapter is concluded with a presentation of the full analysis details.

3.1 OpenSees

Computer program OpenSees (Open System for Earthquake Engineering Simulation) (PEER, 2005) was used to conduct the nonlinear dynamic analysis for the current study. OpenSees is an open-source program developed using the object-orientated C++ computer programming language. The object-oriented and open-source format allowed the previously described hysteretic model to be implemented relatively easily into a proven analysis program.

OpenSees is unlike most commercially available analysis software as it does not have a graphical user interface. It is run entirely by text commands that are entered in the command prompt window for the Windows operating system. There are modeling commands to specify the number of degrees of freedom, location of nodes, material properties, section properties, element properties and the location along with the type of loading. Analysis commands are used to specify if the analysis is static or dynamic, the integration algorithm, iteration algorithm and the method of solving linear systems of equations. Output commands record the element forces and the nodal displacements, velocities and accelerations. Most users perform numerical analysis in OpenSees by organizing the modeling, analysis and output commands in a single input file using the Tcl scripting language.

3.1.1 Material Model Framework

OpenSees (PEER, 2005) has several existing material models such as elastic and elastic-perfectly-plastic along with more specialized steel and concrete uniaxial material models. The basic framework for all uniaxial material models implemented into OpenSees involves strain as input and the resulting stress and tangent stiffness for the point on the stress-strain relationship as output. The strain input, referred to as a trial strain, is then used to calculate the trial stress and
trial tangent. Equilibrium is checked and the trial strain is modified iteratively until converging. This process is repeated for each time step when conducting a time history analysis.

The basic framework can also be used in force-displacement applications as was done in the current study. When using a single-degree-of-freedom system, setting the length \( L \) of a truss element equal to 1.0 results in equal strain \( \varepsilon \) and displacement \( \Delta \) as shown in Equation 3.1. Also setting area \( A \) equal to 1.0 results in equal stress-strain curve tangent (tangent modulus of elasticity) \( E \) and tangent stiffness \( k \) as shown in Equation 3.2.

\[
\varepsilon = \frac{\Delta}{L} = \frac{\Delta}{1} = \Delta 
\]

\[
k = \frac{EA}{L} = \frac{E \cdot 1}{1} = E
\]

### 3.1.2 Implementation of Hysteretic Material Model

The detailed hysteretic loading rules outlined in Section 2.5 were implemented as a new material model in OpenSees (PEER, 2005) named ADK hysteretic. Eleven input variables are required to define the shape of the force-displacement curve and location of the unloading point. The material model also utilizes four history variables, one state variable and three trial variables. All variables are summarized in Table 3.1 by variable type. Code Reference refers to how the variable is specified in the C++ computer code. The computer code is summarized in Table 3.2 and Table 3.3 and can be found in its entirety in the electronic appendix.

This material model requires four history variables to be stored and updated in order to produce the proper loading or unloading behaviour. The displacement and shear of the last converged time step are recorded as \( \Delta_{\text{prev}} \) and \( V_{\text{prev}} \) respectively. The maximum previous displacement in the positive and negative directions are recorded as \( \Delta_{\text{max} P} \) and \( \Delta_{\text{max} N} \) respectively. To assist in programming, a state variable (denoted \( \text{state} \)) is used to describe the behaviour of the material during the last converged time step. The eight possible values of \( \text{state} \) are the integers 1 to 4 inclusive and -1 to -4 inclusive. A positive \( \text{state} \) value indicates that the displacement of the last converged time step \( \Delta_{\text{prev}} \) was in the positive direction. When \( \text{state} = 1 \), both loading or unloading occur within the linear elastic region of the backbone curve ranging from the origin to \( V_{cr} \). When \( \text{state} = 2 \), virgin loading occurs along the nonlinear segment of the backbone curve.
between $V_{cr}$ and $V_N$ (nonlinear envelope) and then continues along the yield plateau from $V_N$. When $state = 3$, linear reloading occurs somewhere on the linear path between $V_{cr}$ and the previous maximum displacement in the positive direction. When $state = 4$, either mid-cycle reloading occurs or unloading continues from the previous displacement towards $V_{cl}$. The negative $state$ values (-1,-2,-3,-4) each represent the same type of behaviour, although in the negative direction, as their respective positive $state$ values. The behaviours quantified by the eight possible values of $state$ are illustrated in Figure 3.1 by a wall that has been loaded to three progressively larger displacements in both the positive and negative directions. One instance of mid-cycle reloading has occurred in both the positive and negative directions.

The hysteretic model algorithm begins by determining the change in displacement $dD$ between the trial displacement $\Delta$ and the displacement at the last converged time step $\Delta_{prev}$. A positive $dD$ indicates loading for a positive previous displacement and unloading for a negative previous displacement. Conversely, a negative $dD$ indicates unloading for a positive previous displacement and loading for a negative previous displacement. Once $dD$ is determined, the algorithm is dependent on the value of the state variable. Although the complexity within each
state varies, each state outputs a trial shear $V$, tangent stiffness $k$ and trial state variable $T_{state}$. Equilibrium is checked and trial displacement $\Delta$ is modified iteratively until converging. Once converged, the history variables $\Delta_{\text{prev}}$ and $V_{\text{prev}}$ are updated to equal the $\Delta$ and $V$ of the current iteration. If $\Delta$ is greater than $\Delta_{\text{max,P}}$ or less than $\Delta_{\text{max,N}}$ the respective history variable is updated. Lastly, the value of $T_{state}$ is used to update $state$ and the entire process is repeated for the next time step.

Determining the behaviour within each state is relatively straight forward, however, the change in displacement $dD$ can cause a transition between two states. For example, if $state = 1$ and the previous displacement is slightly less than $V_{cr}$, a positive $dD$ may be large enough to enter either $state = 2$ or $state = 3$. If the wall transitions to $state = 3$, it may reload linearly towards the nonlinear envelope, the yield plateau or along the lower bound. The identity of the new state, and the behaviour within that state, both depend on the previous maximum displacement in the positive direction $\Delta_{\text{max,P}}$. The algorithm within each state has a series of indicators, based on the values of the trial and history variables, to determine the behaviour within that state or if the wall in fact enters a new state. The algorithm is summarized for a positive change in relative displacement in Table 3.2 and for a negative change in relative displacement in Table 3.3.

The most complicated section of the algorithm concerns the possible mid-cycle loading that can occur in $state = 4$ and $state = -4$. For a given trial displacement $\Delta$, a unique trial shear $V$ can be determined on the full-cycle reloading path towards the maximum displacement in either the positive or negative direction. For mid-cycle reloading, this $V$ can be greater than or less than the shear calculated by following the initial stiffness $k_i$ over the change in displacement $dD$. If $V$ is less than the calculated shear, it means that the wall has reloaded along the initial stiffness and has rejoined the full-cycle reloading path. If $V$ is greater than the calculated shear, the wall is still reloading along the initial stiffness and has not rejoined the full-cycle reloading path. The two mid-cycle reloading options are illustrated in Figure 3.2 for $state = 4$. The labels for $\Delta_{ed}$, $\Delta_{cr}$ and $\Delta_N$, respectively, along the displacement axis have been omitted for clarity.
Table 3.1 ADK Hysteretic Variables

<table>
<thead>
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<th>Variable Type</th>
<th>Variable Name</th>
<th>Code Reference</th>
<th>Description</th>
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<td>input</td>
<td>( k_i )</td>
<td>ki</td>
<td>Initial stiffness of force-displacement curve</td>
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<tr>
<td></td>
<td>( V_{cl} )</td>
<td>Vcl</td>
<td>Shear at which cracks close from axial load</td>
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<td></td>
<td>( V_{cr} )</td>
<td>Vcr</td>
<td>Shear at which flexural cracks form</td>
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<td></td>
<td>( \Delta_N )</td>
<td>Dn</td>
<td>Displacement at flexural capacity</td>
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<td></td>
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<td></td>
<td>( D )</td>
<td>C4</td>
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<td>Displacement at current time step</td>
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<td></td>
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<td>Tstate</td>
<td>State of current time step</td>
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Figure 3.2 Mid-Cycle Reloading
Table 3.2 Algorithm Summary for Positive Change in Relative Displacement

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<tr>
<th>State</th>
<th>Indicator</th>
<th>Behaviour</th>
<th>Trial State</th>
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<tbody>
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<td>state&lt;1</td>
<td>D&lt;-D_c</td>
<td>unloading to -D_c</td>
<td>Tstate=-4</td>
</tr>
<tr>
<td></td>
<td>D&lt;0</td>
<td>unloading to origin</td>
<td>Tstate=-1</td>
</tr>
<tr>
<td></td>
<td>D&gt;0</td>
<td>reloading past origin</td>
<td>Tstate=1</td>
</tr>
<tr>
<td>state==1</td>
<td>D&lt;D_c</td>
<td>remains linear elastic</td>
<td>Tstate=1</td>
</tr>
<tr>
<td></td>
<td>D_prev==D_maxP</td>
<td>loads along nonlinear envelope</td>
<td>Tstate=2</td>
</tr>
<tr>
<td></td>
<td>D_maxP&lt;D_N</td>
<td>reloads towards nonlinear envelope</td>
<td>Tstate=3</td>
</tr>
<tr>
<td></td>
<td>D_maxP&lt;D_int</td>
<td>reloads towards yield plateau</td>
<td>Tstate=3</td>
</tr>
<tr>
<td></td>
<td>D_maxP&gt;D_int</td>
<td>reloads along lower bound</td>
<td>Tstate=3</td>
</tr>
<tr>
<td>state==2</td>
<td>D&lt;D_N</td>
<td>remains along nonlinear envelope</td>
<td>Tstate=2</td>
</tr>
<tr>
<td></td>
<td>D&gt;D_N</td>
<td>yields along plateau</td>
<td>Tstate=2</td>
</tr>
<tr>
<td></td>
<td>D&lt;D_N</td>
<td>reloads towards nonlinear envelope</td>
<td>Tstate=2</td>
</tr>
<tr>
<td></td>
<td>D&lt;D_maxP</td>
<td>does not reach nonlinear envelope</td>
<td>Tstate=3</td>
</tr>
<tr>
<td></td>
<td>D&lt;D_N</td>
<td>yields nonlinear envelope</td>
<td>Tstate=2</td>
</tr>
<tr>
<td></td>
<td>D&gt;D_N</td>
<td>yields along plateau</td>
<td>Tstate=2</td>
</tr>
<tr>
<td>state==3</td>
<td>D_maxP&lt;D_int</td>
<td>reloads towards yield plateau</td>
<td>Tstate=2</td>
</tr>
<tr>
<td></td>
<td>V&lt;V_N</td>
<td>does not reach yield plateau</td>
<td>Tstate=3</td>
</tr>
<tr>
<td></td>
<td>V==V_N</td>
<td>yields along plateau</td>
<td>Tstate=2</td>
</tr>
<tr>
<td></td>
<td>D_maxP&gt;D_int</td>
<td>reloads towards lower bound</td>
<td>Tstate=2</td>
</tr>
<tr>
<td></td>
<td>V&lt;V_N</td>
<td>does not reach lower bound</td>
<td>Tstate=3</td>
</tr>
<tr>
<td></td>
<td>V==V_N</td>
<td>yields along plateau</td>
<td>Tstate=2</td>
</tr>
<tr>
<td>state==4</td>
<td>D_maxP&lt;D_N</td>
<td>reloads towards nonlinear envelope</td>
<td>Tstate=2</td>
</tr>
<tr>
<td></td>
<td>D&lt;D_maxP</td>
<td>does not reach nonlinear envelope</td>
<td>Tstate=3</td>
</tr>
<tr>
<td></td>
<td>V&lt;(V_prev+dD_ki)</td>
<td>follows reloading path</td>
<td>Tstate=3</td>
</tr>
<tr>
<td></td>
<td>V&gt;(V_prev+dD_ki)</td>
<td>follows initial stiffness</td>
<td>Tstate=4</td>
</tr>
<tr>
<td></td>
<td>D&lt;D_N</td>
<td>reaches nonlinear envelope</td>
<td>Tstate=2</td>
</tr>
<tr>
<td></td>
<td>D&gt;N</td>
<td>yields along plateau</td>
<td>Tstate=2</td>
</tr>
<tr>
<td></td>
<td>D_maxP&lt;D_int</td>
<td>reloads towards yield plateau</td>
<td>Tstate=2</td>
</tr>
<tr>
<td></td>
<td>V&lt;(V_prev+dD_ki)</td>
<td>follows reloading path</td>
<td>Tstate=3</td>
</tr>
<tr>
<td></td>
<td>V&lt;V_N</td>
<td>does not reach yield plateau</td>
<td>Tstate=3</td>
</tr>
<tr>
<td></td>
<td>V==V_N</td>
<td>yields along plateau</td>
<td>Tstate=2</td>
</tr>
<tr>
<td></td>
<td>V&gt;(V_prev+dD_ki)</td>
<td>follows initial stiffness</td>
<td>Tstate=4</td>
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<tr>
<td></td>
<td>D_maxP&gt;D_int</td>
<td>reloads towards yield plateau</td>
<td>Tstate=2</td>
</tr>
<tr>
<td></td>
<td>V&lt;(V_prev+dD_ki)</td>
<td>follows lower bound</td>
<td>Tstate=3</td>
</tr>
<tr>
<td></td>
<td>V&lt;V_N</td>
<td>does not reach yield plateau</td>
<td>Tstate=3</td>
</tr>
<tr>
<td></td>
<td>V==V_N</td>
<td>yields along plateau</td>
<td>Tstate=2</td>
</tr>
<tr>
<td></td>
<td>V&gt;(V_prev+dD_ki)</td>
<td>follows initial stiffness</td>
<td>Tstate=4</td>
</tr>
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</table>
Table 3.3 Algorithm Summary for Negative Change in Relative Displacement

<table>
<thead>
<tr>
<th>State</th>
<th>Indicator</th>
<th>Behaviour</th>
<th>Trial State</th>
</tr>
</thead>
<tbody>
<tr>
<td>state≥1</td>
<td>D&gt;D₀</td>
<td>unloading to D₀</td>
<td>Tstate=4</td>
</tr>
<tr>
<td></td>
<td>D&gt;0</td>
<td>unloading to origin</td>
<td>Tstate=1</td>
</tr>
<tr>
<td></td>
<td>D&lt;0</td>
<td>reloading past origin</td>
<td>Tstate=-1</td>
</tr>
<tr>
<td>state=-1</td>
<td>D&gt;D₀</td>
<td>remains linear elastic</td>
<td>Tstate=-1</td>
</tr>
<tr>
<td></td>
<td>D&gt;maxN</td>
<td>loads along nonlinear envelope</td>
<td>Tstate=-2</td>
</tr>
<tr>
<td></td>
<td>D&gt;maxN</td>
<td>reloads towards nonlinear envelope</td>
<td>Tstate=-3</td>
</tr>
<tr>
<td></td>
<td>D&gt;int</td>
<td>reloads towards yield plateau</td>
<td>Tstate=-3</td>
</tr>
<tr>
<td></td>
<td>D&gt;int</td>
<td>reloads along lower bound</td>
<td>Tstate=-3</td>
</tr>
<tr>
<td>state=-2</td>
<td>D&gt;D₀</td>
<td>remains along nonlinear envelope</td>
<td>Tstate=-2</td>
</tr>
<tr>
<td></td>
<td>D&lt;0</td>
<td>yields along plateau</td>
<td>Tstate=-2</td>
</tr>
<tr>
<td></td>
<td>D&gt;maxN</td>
<td>reloads towards nonlinear envelope</td>
<td>Tstate=-3</td>
</tr>
<tr>
<td></td>
<td>D&gt;N</td>
<td>does not reach nonlinear envelope</td>
<td>Tstate=-3</td>
</tr>
<tr>
<td></td>
<td>D&gt;N</td>
<td>reaches nonlinear envelope</td>
<td>Tstate=-2</td>
</tr>
<tr>
<td></td>
<td>D&lt;N</td>
<td>yields along plateau</td>
<td>Tstate=-2</td>
</tr>
<tr>
<td>state=-3</td>
<td>D&gt;maxP</td>
<td>reloads towards yield plateau</td>
<td>Tstate=-3</td>
</tr>
<tr>
<td></td>
<td>V&gt;V₀</td>
<td>does not reach yield plateau</td>
<td>Tstate=-3</td>
</tr>
<tr>
<td></td>
<td>V=V₀</td>
<td>yields along plateau</td>
<td>Tstate=-2</td>
</tr>
<tr>
<td></td>
<td>V&lt;N</td>
<td>does not reach lower bound</td>
<td>Tstate=-3</td>
</tr>
<tr>
<td></td>
<td>V=N</td>
<td>yields along plateau</td>
<td>Tstate=-2</td>
</tr>
<tr>
<td>state=-4</td>
<td>D&gt;maxP</td>
<td>reloads towards yield plateau</td>
<td>Tstate=-3</td>
</tr>
<tr>
<td></td>
<td>V&lt;V₀</td>
<td>does not reach yield plateau</td>
<td>Tstate=-3</td>
</tr>
<tr>
<td></td>
<td>V=V₀</td>
<td>yields along plateau</td>
<td>Tstate=-2</td>
</tr>
<tr>
<td></td>
<td>V&gt;N</td>
<td>yields along plateau</td>
<td>Tstate=-2</td>
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<tr>
<td></td>
<td>V&lt;N</td>
<td>yields along plateau</td>
<td>Tstate=-2</td>
</tr>
</tbody>
</table>
3.2 Pilot Analysis

A detailed hysteretic model has been developed that can be used to predict the maximum displacement a cantilever wall will experience from a given earthquake. It is reasonable to expect that these displacements will increase with earthquakes of increasing magnitude. The objective of the pilot analysis is to determine how to relate the displacement predicted by the hysteretic model to an effective stiffness, producing the same displacement, of either an elastic or bilinear system. The use of either simplified system, as opposed to the detailed hysteretic model, will be of most practical interest to a design engineer. Three different material models were used to model the large-scale cantilever wall tested by Adebar, Ibrahim and Bryson (2000) as a single-degree-of-freedom system in OpenSees (PEER, 2005). The previously described hysteretic material model ADK hysteretic, the existing elastic material model ElasticMaterial, and the existing bilinear material model SteelOl were used. The analysis procedure, ground motions selection, analysis results and the implication of pilot results are discussed. A condensed version of this section has been published by Adebar, Korchinski and Haukaas (2007).

3.2.1 Analysis Procedure

For each ground motion, a single analysis was conducted using the hysteretic material model to predict the maximum nonlinear displacement of the cantilever wall. A series of analyses were then completed using the elastic and bilinear material models to predict the maximum displacement. The initial stiffness $k$ of the nonlinear material was also used as the starting stiffness value for the elastic and bilinear materials. The bilinear material however only follows the initial stiffness until reaching $V_y$, the shear at which the yield plateau develops in the nonlinear model, as illustrated in Figure 3.3.
The appropriate mass $m$ was determined in order to achieve an initial period of 2 and 4 seconds in the uncracked state where the initial stiffness $k$ is applicable to all three models. This mass was kept constant for all further analyses for each respective initial period. A series of elastic analyses were conducted over a range of stiffness from the initial stiffness of the wall down to 20% of the initial stiffness at intervals of 1%. A series of bilinear analyses, with $V_N$ constant, were conducted by modifying the slope of the elastic portion of the elastic-plastic response in a similar fashion as the elastic analysis. That is, the elastic stiffness was reduced from the initial stiffness of the wall down to 20% of the initial stiffness at intervals of 1%. Mass proportional Rayleigh damping of 5% was used for all analyses.

3.2.2 Ground Motions Selection

The four ground motions selected for the preliminary nonlinear analysis were taken from the suite of ground motions used for calibration of the displacement modification procedure included within FEMA 440 (ATC, 2005). The ground motion suite includes twenty ground motions for each of the site classes B, C, D and E of the National Earthquake Hazard Reduction Program (NEHRP) site classification system. Further information for the four selected ground motions is provided in Table 3.4. The rationale used in selecting these four ground motions was they all had a reasonably uniform increase in elastic spectral displacements in the 4 second period range. This period range was chosen to be representative of the longer periods experienced by high-rise buildings. The uniform increase was desired in order to facilitate the development of a clear relationship between nonlinearity and an elastic or bilinear system with lengthening period.
Table 3.4 Ground Motions Used in Preliminary Analysis

<table>
<thead>
<tr>
<th>Date</th>
<th>Earthquake</th>
<th>Magnitude (Ms)</th>
<th>Station name</th>
<th>PGA (cm/s²)</th>
<th>PGV (cm/s)</th>
<th>PGD (cm)</th>
<th>NEHRP Site class</th>
</tr>
</thead>
<tbody>
<tr>
<td>02/09/71</td>
<td>San Fernando</td>
<td>6.5</td>
<td>Pasadena, CIT Athenaeum</td>
<td>107.9</td>
<td>14.7</td>
<td>6.6</td>
<td>C</td>
</tr>
<tr>
<td>06/28/92</td>
<td>Landers</td>
<td>7.5</td>
<td>Yermo, Fire Station</td>
<td>240.3</td>
<td>57.5</td>
<td>37.5</td>
<td>C</td>
</tr>
<tr>
<td>10/17/89</td>
<td>Loma Prieta</td>
<td>7.1</td>
<td>Anderson Dam (downstream)</td>
<td>239.4</td>
<td>20.4</td>
<td>6.8</td>
<td>C</td>
</tr>
<tr>
<td>10/15/79</td>
<td>Imperial Valley</td>
<td>6.8</td>
<td>Calexico, Fire Station</td>
<td>269.6</td>
<td>21.1</td>
<td>6.5</td>
<td>D</td>
</tr>
</tbody>
</table>

The peak elastic demand of each ground motion was determined for a cantilever wall with stiffness $k$ for initial periods of 2 seconds and 4 seconds. The ratio between the peak elastic demand and the strength of the wall $V_N$ was calculated for each ground motion at each initial period. This ratio was used to determine the required scaling factor for the elastic demand on the cantilever wall, at each initial period, to be $R$ times the strength of the wall $V_N$. The ground motions were scaled to give $R = 1.5, 2.0, 2.5, 3.0, 4.0$ and $5.0$ at initial periods of 2 seconds and 4 seconds. Thus each ground motion was scaled to 12 different levels ($6R$ factors at 2 periods) resulting in a total of 48 scaled ground motions.

In addition, a companion set of altered ground motions were created using the computer program SYNTH (Naumoski, 2001). This program iteratively modifies the accelerogram until the acceleration spectrum matches the prescribed target spectrum. The target spectrum specified was an altered version of the Vancouver, BC accelerations determined by Open File 4459 (Adams and Halchuk, 2000) for development of the 2005 National Building Code of Canada design spectrum (NRCC, 2005). The design spectrum was altered to be more representative of longer periods. The 2005 National Building Code of Canada (NBCC) values of $S_a$ for 2 and 4 seconds were used, but the decrease between them was taken proportional to $1/T$ as opposed to linearly. The decrease proportional to $1/T$ was continued to $T = 6s$ when the decrease was set proportional to $1/T^2$ until $T = 10s$ as seen in Figure 3.4. The 2005 NBCC uses the $S_a$ for 4 seconds for all periods greater than 4 seconds. The alteration was done to reflect the levelling of spectral displacements for the longer periods. This levelling behaviour was observed in all four of the raw ground motions selected.
SYNTH calculates the acceleration spectrum for the raw accelerogram and determines the ratio between it and the target spectrum for a series of closely spaced periods specified by the user. This ratio is used to modify (increase or decrease) both the real and imaginary Fourier components of the accelerogram (Naumoski, 2001). An inverse Fourier transform is then performed on the modified components to create a new accelerogram. The entire process is repeated on each new accelerogram for a user specified number of iterations; this number was set at the recommended 10 iterations. SYNTH only changes the Fourier amplitude and does not change the Fourier phase spectrum because both the real and imaginary Fourier components are multiplied by the same ratio. This means that the strong motion duration and characteristics of the initial accelerogram are maintained in the modified accelerogram. When examining the two accelerograms they appear quite similar with the “peaks” and “valleys” occurring in the same places but with different amplitudes. SYNTH is able to produce a ground motion that very closely follows the prescribed target spectrum. It is however important to select initial ground motions whose acceleration spectra have similar shape to the target spectrum, primarily having peak accelerations occurring at similar periods. This limits the amount that the initial ground motion needs to be modified.

The modified ground motions were scaled to give six different $R$ values (1.5, 2.0, 2.5, 3.0, 4.0 and 5.0) at two different periods (2 and 4 seconds) resulting in an additional 48 ground motions. The scaling factors for each modified ground motion were determined in the same fashion as the raw ground motions described previously. The acceleration spectra for the modified ground-
motions generated by SYNTH are all quite similar to one another and closely follow the altered 2005 NBCC design spectrum.

The similarity of the modified acceleration spectra can be seen in Figure 3.5. In this figure, both the raw and modified ground motions have been scaled to \( R = 1.0 \) for initial periods of 2 seconds and 4 seconds. In all cases the scaling factor alters the magnitude of the response spectrum by one global amount and does not impact the shape of the response spectrum. For example, the response spectra for any other \( R \) value, scaled to the same initial period, would look the same except for the magnitude of the acceleration or displacement value. The relative difference between the responses of both the raw and modified ground motions would remain unchanged. Scaling to a different initial period, however, significantly changes the relative differences observed between all the ground motions. The raw response spectra have jagged “peaks” and “valleys” that make scaling at a particular period very susceptible to the local “high” or “low” occurring at that period. The modified response spectra are smoother and less sensitive to local variation when scaling at a particular period.
Figure 3.5 Response Spectra for Unmodified (U) and Modified (M) Ground Motions
3.2.3 Results from Preliminary Analysis

The maximum displacements from each elastic analysis are plotted versus period, which results in the well known elastic displacement spectrum. The maximum displacements for each bilinear analysis are also plotted versus the period calculated from the initial elastic range of the bilinear curve. The resulting bilinear displacement spectra are very different from the normally reported bilinear response calculated for a constant $R$ factor. In the current study, the strength of the bilinear elastic-perfectly-plastic model was held constant and only the initial stiffness was reduced. As the initial stiffness reduces, the corresponding elastic force demand reduces, and the required strength for a given $R$ factor reduces. The resulting spectrum from a bilinear analysis with constant strength is more erratic than the commonly produced inelastic spectra for constant $R$. Typical displacement spectra determined from the elastic analyses (E) and bilinear with constant strength analyses (B) are illustrated in Figure 3.6. The examples shown are for modified ground motions with $R=3.0$ based on an initial period of 4 seconds.

![Figure 3.6 Elastic and Bilinear Displacement Spectra](image)

The displacement spectra determined from the elastic analyses and bilinear with constant strength analyses by reducing the initial stiffness (increasing period) were normalized by the displacement predicted with the nonlinear model of the cantilever wall. The normalized spectra, or displacement ratio plots, for 2 second and 4 second initial periods are illustrated in Figure 3.7 and Figure 3.8 respectively. The point at which the line for a particular earthquake and analysis type crosses the displacement ratio of 1.0 indicates the period needed to get the same displacement estimate as the nonlinear model. The results are shown for the unmodified ground motions (left) and the modified ground motions (right), and for $R=1.5$ (top), $R=3.0$ (middle) and $R=5.0$ (bottom) based on the initial period. A complete presentation of the displacement ratio plots including $R=2.0$, $R=2.5$ and $R=4.0$ can be found in Appendix C.
Figure 3.7 Displacement Ratio Plots for 2 Second Initial Period
When examining the results presented in Figure 3.7 and Figure 3.8 the two main points of comparison are the use of unmodified or modified ground motions and the use of an elastic or bilinear system. The objective is to find the clearest relationship between the hysteretic model and a simplified model (elastic or bilinear). This relationship will determine how the effective stiffness changes from earthquakes of increasing magnitude.
The observed trends in the displacement ratio plots are usually better defined for modified ground motions than for unmodified ground motions. The modified displacement ratios generally increase more uniformly and have a more pronounced crossing of the displacement ratio = 1.0, thus establishing a relationship for effective stiffness based on equivalent displacement will be easier. The scaled displacement ratios are generally more erratic and have several crossings of the displacement ratio = 1.0 making any possible relationship less intuitive. It should be noted that the scaled displacement ratios for an initial period of 4 seconds are far more uniform for than an initial period of 2 seconds. The reason for this is the basis for unmodified ground motion selection was a uniform increase of the displacement spectrum in the 4 second region. The preliminary results show that using unmodified ground motions with similar behaviour for a certain period range does not guarantee similar behaviour for other period ranges. Modified ground motions, however, are created to approximate the same acceleration spectrum over the entire period range of the target spectrum specified by the user. Therefore, the relative differences in the displacement spectra are usually significantly less at any period range for the modified ground motions than the unmodified ground motions. A single modified ground motion can produce meaningful results for any initial period or range of initial periods. If studying the same range of initial periods with unmodified ground motions, individual ground motions would have to be individually selected for each period of interest.

The unmodified ground motions typically have displacement spectra with pronounced irregularities such as jagged “peaks” and “valleys” or flat “plateaus”. The period at which the displacement ratio =1.0, found by normalizing the spectrum by a single value, is quite sensitive to these irregularities. The normalizing displacement can be just small enough to cross before a peak or just large enough to miss the peak entirely. For example, the unmodified Yermo ground motions for an elastic system with a 2 second initial period, illustrated by the solid red lines on the left side of Figure 3.7, $R=5.0$ crosses before the plateau while the $R=3.0$ crosses after the plateau. The $R=3.0$ barely misses crossing before the plateau and crosses much later at a longer period (lower effective stiffness) than the larger $R=5.0$. A wall having significantly higher effective stiffness for a larger ground motion does not seem to be a reasonable result. The spectrum irregularities, which contribute to such results, are far less prevalent with modified ground motions. Irregularities still occur but they are smaller and more localized, thus have less influence on the effective stiffness.
Cleare trends in the displacement spectra are observed for elastic systems than for bilinear systems with constant strength. The bilinear displacement spectra, even when using modified ground motions, are generally quite erratic consisting of many irregularities. The "peaks" and "valleys" of the spectra are more pronounced and have larger relative differences between the maximum and minimum values. When scaling a ground motion to different intensities ($R=1.5, R=3.0, R=5.0$) the shapes of the spectra vary widely as seen in Figure 3.7 and Figure 3.8. The period at which the displacement ratio = 1.0 is quite sensitive to these irregularities and the wide variation of shapes.

The period at which the displacement ratio = 1.0, denoted effective period $T_e$, is used to determine the effective stiffness $k_e$ of the elastic and bilinear systems that would produce the same maximum displacement. The shift from the initial period $T_i$ to the effective period $T_e$ can be used to calculate $\alpha$ as shown by Equation 3.3. The effective stiffness $k_e$ of the system is determined by multiplying the initial stiffness $k_i$ by the stiffness reduction factor $\alpha$ as shown by Equation 3.4.

$$\alpha = \left(\frac{T_i}{T_e}\right)^2$$  \hspace{1cm} (3.3)

$$k_e = \alpha k_i$$ \hspace{1cm} (3.4)

Stiffness reduction factor $\alpha$ is calculated for all of the different $R$ values for each ground motion. The $R$ values quantify the number of times the elastic demand (at initial stiffness $k_i$) is greater than the strength of the wall $V_N$. The stiffness reduction $\alpha$ for each $R$ value is summarized in Figure 3.9 for the modified earthquakes, for both the elastic and bilinear systems, at $T_i$ of 2 and 4 seconds. If the displacement ratio for any analysis was greater than 1.0 at the initial period $T_i$, the stiffness reduction factor $\alpha$ was reported as 1.0.
Figure 3.9 Summary of Preliminary Analysis Results

The results in Figure 3.9 suggest a trend of stiffness reduction factor $\alpha = k_e / k_s$ decreasing gradually as the force reduction factor $R$ increases. For an uncracked wall with a period of 2 seconds there is a gradual decrease in the reduction factor from about 85% at $R=1.5$ to about 60% at $R=5.0$. For a taller, more flexible uncracked wall with a period of 4 seconds, the reduction factor is about 90% from $R=1.5$ to about $R=2.5$, and then gradually decreases to about 60% at $R=5.0$. The influence of the individual ground motions however remains very significant even when using modified ground motions. The trends for both initial periods are more pronounced for the elastic systems than the bilinear systems. The bilinear systems exhibit far more drastic changes between data points than the elastic systems.

The approach taken by Ibrahim (2000) to determine effective stiffness was to calculate the stiffness of a bilinear elastic-plastic force-displacement relationship with equal area-under-the-curve as the nonlinear relationship up to point that the nonlinear curve reaches the nominal capacity. This approach suggests that $\alpha$ should be taken as 80% which corresponds reasonably well with the average of the results shown in Figure 3.9. The 80% reduction in stiffness is typically high for ground motions scaled to less than $R=3.0$ and is typically low for ground motions scaled to greater than $R=3.0$.

3.2.4 Concluding Remarks of Pilot Analysis

Initially, the bilinear model was expected to be a better representation of the nonlinear hysteretic model than the elastic model since the bilinear model has a nonlinear component. The differences between the unloading rules of the nonlinear hysteretic and bilinear models however
have quite different effects on the maximum displacement. The hysteretic behaviour of both models for the Anderson ground motion scaled to $R=3.0$ is illustrated in Figure 3.10. For a small nonlinear event in the negative direction, the difference between the predicted displacements is minor. For a subsequent large nonlinear event in the positive direction the models remain comparable. The difference becomes apparent when going from a large nonlinear event in one direction to a large nonlinear event in the opposite direction. The nonlinear hysteretic model unloads at a much lower stiffness and returns more readily towards the origin where it can start loading in the negative direction. The bilinear model however must yield completely in the negative direction before it can start displacing in the negative direction. This has the potential to completely underestimate maximum displacements in one direction after experiencing a nonlinear event in the opposite direction. The significant pinching effect observed through physical testing (Adebar, Ibrahim and Bryson, 2000), and modeled by the nonlinear hysteretic model, has a fundamentally different behaviour and greater ability to transition between maximum displacements in opposite directions than the bilinear material model can facilitate.

![Figure 3.10 Hysteretic Response of Material Models](image)

The results of the pilot analysis suggest that the effective stiffness of the nonlinear hysteretic model is very sensitive to individual ground motions. This sensitivity can be best reduced, but not eliminated, through the use of modified ground motions and the use of an elastic system to determine an effective stiffness producing the same maximum displacement. From a design engineer standpoint, the use of modified ground motions scaled to the 2005 NBCC (NRCC,
2005) design spectrum will result in an effective stiffness relationship that is more useful for building design. The use of an elastic system with effective stiffness is also more compatible with current design practice.

3.3 Further Analysis Considerations

The pilot analysis has shown that the clearest effective stiffness relationship can be found with elastic systems and modified ground motions. Several additional analysis considerations need to be explored before conducting a full analysis using many ground motions. First, it needs to be confirmed that the “scaling ground motion magnitude for a fixed structure strength” approach used in the pilot analysis is equivalent to altering the structure strength for a given earthquake to achieve the desired $R$ value. This section also presents the concept of $R$ initial and $R$ effective and two possible methods of determining $\alpha$. The influence of trends in the nonlinear displacements predicted by the nonlinear hysteretic are discussed. Lastly, a more appropriate value for the Rayleigh damping coefficient is considered.

3.3.1 Scaling Structure vs. Scaling Ground Motions

When designing structures to resist earthquakes, designers typically adjust the properties of the structure (i.e., strength and stiffness) in order to meet the force and displacement demands imposed on the structure by the earthquake. The structure is expected to become damaged during the earthquake and behave nonlinearly. The degree to which the required strength of the structure to remain linear is greater than the strength at which nonlinearity begins is commonly referred to as the $R$ factor. Design codes such as the CSA A23.3-04 (CSA, 2004) limit the range of acceptable $R$ factors for different types of structural support systems and specify the necessary detailing of reinforcement for the system to perform safely.

The pilot study modeled the UBC wall for a range of $R$ factors by scaling the ground motions rather than altering the structure. In other words, the structure strength was fixed for a series of ground motions with increasing magnitude. This approach was taken since the model was already defined for the UBC wall and analyses for each $R$ factor only required adjusting the ground motion by a single scaling factor. This simpler approach was expected to produce the same results as the more conventional approach of varying the structure strength for a given ground motion to achieve the desired $R$ factor. To verify this, analyses were conducted scaling the structure for a fixed ground motion. The single scaling factor was applied to both the forces
and displacements of the four force-displacement pairs defining the UBC wall. Doing so ensured that the initial stiffness of the wall remained the same for each analysis. The fourth order polynomial defining the nonlinear portion of the force-displacement envelope required the same scaling factor to be applied differently to each of the constants of the polynomial expression. The general solution of a polynomial expression is shown in Equation 3.5; note that constant $E$ is zero since the expression is localized at $(d_{cr}, V_{cr})$. Scaling the resulting curve of a polynomial expression by a single factor $f$ requires multiplying each constant by $f$ raised to the appropriate exponent as shown in Equation 3.6.

$$y = Ax^4 + Bx^3 + Cx^2 + Dx + E$$  \hspace{1cm} (3.5)$$

$$y = (A*f)x^4 + (B*f^2)x^3 + (C*f^3)x^2 + (D*f^4)x + E$$  \hspace{1cm} (3.6)$$

The two methods of achieving a desired $R$ factor were conducted for $R=1.5, 2.0, 2.5, 3.0, 4.0$ and 5.0 for the four ground motions used in the pilot analysis. When scaling a ground motion for a set structure strength, the displacement spectra vary linearly according to the $R$ factor. When scaling a structure for a set ground motion, the force-displacement envelopes vary inversely according to the $R$ factor. The displacement spectra and force-displacement envelopes for the two respective methods are illustrated quantitatively in for the modified Anderson ground motion and the UBC wall.
The displacements predicted by the hysteretic model for each analysis method are shown in Figure 3.12 for the modified Anderson ground motion and the UBC wall for an initial period of 4 seconds. The $R$ factor that is plotted along the $x$-axis represents different things for each analysis method. For scaled ground motions for a set structure strength (GM) it represents ground motions of increasing magnitude. For scaled structures for a set ground motion (S) it represents structures with decreasing strength. The displacements for an elastic system having the same initial period and stiffness are also plotted for each scaled ground motion. The elastic displacements are increasing in a perfectly linear fashion as would be expected since the ground motion magnitudes are also increasing linearly. The displacements predicted by the hysteretic model with increasing ground motion magnitude increase somewhat linearly. Again, this is as
expected since ground motions of greater magnitude induce greater shear demands and greater
displacements. The displacements predicted by the hysteretic model with decreasing structure
strength are relatively constant, which is compatible with the concept of equal displacement.

![Figure 3.12 Predicted Displacements from Each Scaling Method](image)

The goal of either method is to relate the predicted nonlinear displacement of the hysteretic
model and the displacement of an elastic system. This relationship can then be used to find the
effective stiffness of a linear system that produces the same displacement as the hysteretic
model. In order to compare the relationships found through each method, the hysteretic
displacements from each method need to be normalized by the elastic displacements. For varying
ground motions, each nonlinear displacement is divided by the elastic displacement for that
ground motion. For varying structure strengths, the ground motion remains constant and all
nonlinear displacements are divided by the same elastic displacement. It should be noted that the
nonlinear and elastic displacements are the same for both methods when $R=1.0$ and the varied
parameter is multiplied by a factor of 1.0.

![Figure 3.13 Normalized Predicted Displacements](image)
The normalized results for each method are exactly the same as illustrated in. This implies that using either method will produce the same relationship between effective stiffness and increasing $R$. Values below 1.0 indicate that the displacement of an elastic system is greater than the nonlinear hysteretic system. Values above 1.0 however indicate that the displacement of an elastic system is less than the nonlinear hysteretic system. This means the stiffness of an elastic system must decrease in order to have the same displacement as the hysteretic system.

In conclusion, scaled ground motions for a given structure strength and scaled structure strengths for a given ground motion have the same relationship between increasing $R$ and effective stiffness. Scaling a ground motion for a given structure is a far more efficient approach when generating data to define this relationship for several structures using many ground motions. The parameters of the hysteretic model only need to be calculated once for each wall and the results can be calculated for the desired $R$ by multiplying each ground motion by a single factor that is unique to each analysis. Scaling a structure for a given ground motion, however, is a more physically meaningful and intuitive method. Since both methods have the same relationship, the first can be used to generate the results while the second method is useful in understanding how the results apply to real structures.

### 3.3.2 $R$ Initial vs. $R$ Effective

All previous discussion and reported results have been presented with an $R$ factor determined using the initial stiffness of the linear portion of the force-displacement envelope of the UBC wall. The $R$ factor is the number of times the force demand of an elastic system with the same initial stiffness exceeds the strength of the wall $V_N$. This $R$ factor calculated using the initial stiffness should more properly be thought of as $R$ initial. As the ground motions are scaled up (or structure strength is scaled down) the general trend is a decrease in effective stiffness for an elastic system having the same displacement as the predicted nonlinear displacement. Due to the lower effective stiffness, the resulting effective elastic demand is also lower. The difference between the effective elastic demand and the strength of the wall is lessened. Accordingly, the $R$ effective is less than the $R$ initial.

The relationship between stiffness reduction $\alpha$ and $R$ effective is of more practical interest to a design engineer. Current practice is to use $\alpha = I_e/I_s$ of 0.7 for linear seismic analysis to predict displacement demands as per the A23.3-94 commentary (CAC, 1995). Through the current
work, it is anticipated that a better understanding of $\alpha$ value will be achieved. A linear dynamic analysis conducted using the new $\alpha$ will predict a new elastic force demand. The value for $R$ effective can now be used to calculate the required strength for the structure. In design, the $R$ initial based off the elastic force demand at the structure’s initial stiffness is unused. In the current work, however, $R$ initial is a very convenient way to quantify the range over which the ground motions will be scaled up or structure strength scaled down. To develop a greater understanding of the difference between $R$ initial and $R$ effective the ground motion scaling approach was repeated for $R$ initial ranging from 1.0 to 5.0 at 0.25 intervals for the four ground motions used in the pilot analysis. The stiffness reduction $\alpha$ is plotted with both $R$ initial and $R$ effective for a 2 second initial period in and for a 4 second initial period in. The values of $\alpha$ remain unchanged for each ground motion at each initial period, only the spacing of the values along the x-axis change as a result of the method used to calculate $R$.

Both figures illustrate the two main differences between $R$ initial and $R$ effective. First, the $R$ effective values are lower than the $R$ initial values as explained previously. An $R$ initial value of 5.0 results in an $R$ effective ranging from approximately 3.2 to 4.0. The observed decrease between the two $R$ values varies more widely between ground motions than it does between initial periods. Second, the trend of decreasing $\alpha$ for increasing $R$ is more gradual for $R$ initial than for $R$ effective. When looking at the $R$ effective plots the line between two consecutive data points is sometimes vertical or even in the opposite direction as the general trend. These anomalies at first seem rather alarming but upon further inspection can be explained fairly easily. The data points are generated by increasing the ground motion magnitude by small uniform amounts. The sudden changes in the direction of the line are occurring when there is a relatively large decrease in $\alpha$ between two data points. The value of $R$ effective is sensitive to these changes since $\alpha$ is used to calculate $R$ effective. If $\alpha$ is no longer varying at a somewhat uniform rate then $R$ effective cannot be expected to vary uniformly either. The relatively large decreases in $\alpha$ between relatively close spaced data points can be attributed to the way in which the characteristics of a particular ground motion interact with a complex nonlinear hysteretic model.
Figure 3.14 R Value Comparison for 2 Second Initial Period
Figure 3.15 R Value Comparison for 4 Second Initial Period
3.3.3 Determination of Effective Period

In the pilot analysis and the further analysis refinement thus far, the effective stiffness has been determined by the period along the elastic displacement spectrum at which the elastic displacement matches that predicted by the nonlinear hysteretic model. Although the use of modified ground motions results in smoother spectra than raw ground motions, local irregularities are still present in the spectra. As discussed previously, the "peaks" and "valleys" can be troublesome when trying to match a displacement.

The modified ground motions have been made to match the same target spectrum. Referring back to Figure 3.5, one can visualize that the acceleration and displacement spectra of the four modified records are fluctuating around the target acceleration and displacement spectra. To eliminate the effect of these fluctuations the target displacement spectrum itself can be used to find the period at which the displacement equals that predicted by the nonlinear hysteretic model. Implementing this is quite simple since the target displacement spectrum increases linearly from 2 to 6 seconds and is then flat after 6 seconds.

The difference between using the elastic and design spectra to determine an effective period is illustrated in Figure 3.16 for the Anderson ground motion for an initial period of 2 seconds. The most notable differences between the two methods are for $R=3.75$ and $R=4.0$, where due to a local "peak", the entire elastic spectrum has larger displacements than the nonlinear hysteretic model. An effective period can however be found when using the design spectrum. Differences between the two methods can also be observed for $R=4.75$ and $R=5.0$ where a local "peak" results in shorter effective periods (higher effective stiffness).

When looking at the full range of $R$ values (initial) from 1.0 to 5.0 it is apparent that the elastic spectrum is fluctuating around the linearly increasing design spectrum. Considering these fluctuations, the elastic spectrum still produces effective periods reasonably close to those of the design spectrum with the exception of the few $R$ values previously mentioned. Use of the design spectrum to determine effective period does however offer the marked improvement of eliminating some scatter in the data from the influence of the irregularities in the elastic spectrum.
An additional benefit to using the design spectrum is that the elastic spectrum is no longer needed for the analysis procedure. Rather than storing the elastic spectrum for each ground motion, and searching for two points to interpolate between in order to find the effective period, the interpolation process can be reduced to two points common to all ground motions. This is a considerable saving in computation through reduced data storage and handling. The design spectrum displacement at the initial period can be used to determine the scaling factor needed to achieve each $R$ value used in the analysis. The scaling factor for all ground motions will now be the same for each $R$ value since all ground motions were altered to match the same target spectrum. Use of the elastic spectra meant that the displacements for each ground motion at the initial period were all slightly different. This resulted in unique scaling factors for each ground motion.

### 3.3.4 Influence of Relative Nonlinear Displacements

The parallel lines of the design spectra in Figure 3.16 make it easier to see that the general trend is increasing effective period (decreasing effective stiffness) for increasing values of $R$ initial. The one notable exception is for $R$ initial values ranging from 3.0 to 4.0 where the effective period is in fact decreasing. With the influence of the irregularities in the elastic spectrum removed through use of the design spectrum, the only other influence remaining on effective period is the trend of the displacements predicted by the nonlinear hysteretic model. The stiffness reduction $\alpha$ and $R$ initial are plotted along with the displacements predicted by the nonlinear hysteretic model for a given $R$ initial for a 2 second initial period in and for a 4 second initial period in. The $R$ initial values are used for clearer presentation as the data points are spaced evenly along the x-axis. The displacements predicted by a linear system, with the same initial stiffness, for increasing values of $R$ initial are shown by the black line. This line is increasing linearly because the ground motions are being scaled to achieve the desired $R$ initial value. As discussed previously in Section 3.3.1, scaling the ground motions up is equivalent to scaling the structure strength down to achieve a desired $R$ value.
Figure 3.16 Elastic and Design Spectrum Displacement Ratios
The behaviour of the $\alpha$ and $R$ initial plots on the left side of and is a direct result of the relative difference between the displacements of the nonlinear hysteretic model and the elastic model with the same initial stiffness. The relative difference refers to the vertical difference between a nonlinear data point and the black line that represents the linear displacement at a given value of $R$ initial. The amount that the relative slope between two consecutive data points changes also impacts the effective stiffness relationship.

For simplicity, when the linear displacement is greater than the nonlinear displacement, the stiffness reduction factor $\alpha$ is set equal to 1.0. In making this simplification the linear displacements over-predict what the nonlinear hysteretic model suggests. The relative difference between the models in these cases is however much less than the relative difference of the majority of data points where the nonlinear displacements are significantly higher than the linear displacements. Larger elastic displacements generally occur for small $R$ values where conceptually the structure is spending more time in the elastic range. For small $R$ values the idea of using the elastic stiffness of the structure to predict displacement seems reasonable.

The most peculiar effective stiffness relationships occur with the Anderson ground motion for both initial periods. The effective stiffness relationship for an initial period of 2 seconds is illustrated in the top left corner of. Following the displacement plot, located immediately to the right, data point by data point, the reason for this relationship becomes clear. Starting at $R=1.0$, the nonlinear displacements are larger resulting in a reduced effective stiffness. The slope between the displacement data points is slightly greater than the slope of the black line; the nonlinear displacements are increasing faster than the linear displacements thus resulting in the effective stiffness reducing further. At $R=2.0$, the slope between the displacement data points is approximately equal to the slope of the black line and the effective stiffness relationship flattens out periodically. By $R=3.0$ the data point slope is less than the black line and the effective stiffness actually increases to 1.0 when the nonlinear displacements cross over the black line at $R=3.75$. Starting at $R=4.0$, the relative difference between the linear and nonlinear displacements increases rapidly with a slope between data points much greater than the black line. This rapid increase characterizes the sharp drop in effective stiffness observed after $R=4.0$. In short, the rise in the effective stiffness relationship from $R=3.0$ to $R=4.0$ is a direct result of how the nonlinear displacements dip below the line representing the linear displacement.
calculated with the initial stiffness. A crossing of this line at such large $R$ values, occurring for both initial periods, is unique to the Anderson ground motion.

The most gradually decreasing effective stiffness relationships are observed with the Pasadena ground motion at an initial period of 2 seconds in. Examining the displacement plot, the slopes between data points are consistently greater than the slope of the linear displacements. The consistently greater, and relatively uniform, slopes between nonlinear displacement data points result in the smooth decrease of the effective stiffness relationship for increasing $R$ values. The effective stiffness relationship is slightly less gradual for the 4 second initial period in. Note that the large decrease in effective stiffness occurring at $R=3.5$ corresponds to the relatively higher slope between the displacement data points at $R=3.5$ and 3.75.

In summary, the shape of the effective stiffness relationship is dependent on the relative slope between the nonlinear displacement data points. If this relative slope is greater than the slope of the linear displacements, the effective stiffness decreases. Conversely, if this relative slope is less, the effective stiffness decreases. Any irregularities observed in the effective stiffness relationship, both large and small, result from the variability of the displacements predicted by the nonlinear hysteretic model. The nonlinear hysteretic model is thought to be a reasonable representation of how a high-rise concrete shear walls transition between different lateral load levels. The rate and direction of lateral load levels induced by each ground motion are highly variable. Although the modified ground motions have the same target spectrum, the variability of the strong motion characteristics remains.
Figure 3.17 Influence of Relative Displacements for 2 Second Initial Period
Figure 2.18 Influence of Relative Displacement for 4 Second Initial Period

- Linear
- Parabolic
- Circular
- Helical
3.3.5 Rayleigh Damping Coefficient

All analyses thus far has been conducted with 5% mass proportional Rayleigh damping. This damping value is commonly used for the dynamic analysis of linear systems. The hysteretic loops present in the nonlinear hysteretic model however provide hysteretic damping in addition to the specified Rayleigh damping. The mass of the single-degree-of-freedom (SDOF) system is constant regardless if the structure is behaving linearly or nonlinearly. For this reason, it was decided that 3% mass proportional Rayleigh damping would be more appropriate and would not overestimate the net damping experienced by the SDOF system.

The effects of reducing the Rayleigh damping coefficient from 5% to 3% can be seen in for the modified ground motions at each initial period. The respective results for each initial period are quite similar to one another. More variation can be observed between the individual ground motions than can be seen from changing the damping coefficient. The spread in data for each initial period also remains quite similar. Part of the reason for the minor difference in results between the two damping coefficients relates to how SYNTH (Naumoski, 2001) modifies the raw ground motions. SYNTH requires input of both the target spectrum and the damping coefficient so it can adjust the ground motion accordingly to match the target spectrum. The same target spectrum was used for both 3% and 5%, however, since the damping coefficients are different, the modified ground motions will also be slightly different. Since each modified ground motion is suited to its respective level of damping, having similar results from each analysis seems reasonable. The effects of damping would be expected to be more noticeable if unmodified ground motions were used.
3.3.6 Post Processing of Results

The values reported for the stiffness reduction factor $\alpha$ in Section 3.2 and thus far in Section 3.3 have been calculated individually for each ground motion at each $R$ value. Doing so illustrated that $\alpha$ can vary largely between ground motions for a given $R$ value. The variation has been attributed to how the strong motion characteristics of the ground motion interact with the hysteretic model to predict a maximum displacement. With some ground motions, however, the predicted displacements exceed those of the design spectrum used to find an effective period. The altered version of the 2005 NBCC design spectrum has displacements that increase linearly from 2 seconds to 6 seconds and then remain constant at the 6 second value. Nonlinear displacements larger than the 6 second displacement value cannot be used to interpolate an effective period and calculate a unique $\alpha$. The minimum value of $\alpha$ depends entirely upon the initial period of the structure. Using Equation 3.3 the minimum $\alpha$ can be calculated by substituting $T_e$ with 6 seconds. For a $T_i$ of 2 seconds the minimum value is 0.11 while for a $T_i$ of 4 seconds the minimum value is 0.44. The minimum value has only been reached for the Pomona ground motion with a $T_i$ of 4 seconds. The minimum value however could be reached
many times when conducting the analysis for a larger number of ground motions. The average
and standard deviation of $\alpha$ would be adversely affected by ground motions with minimum
values. The average response would be artificially high and have less standard deviation because
of the limit imposed by using an altered design spectrum that flattens at 6 seconds. Similarly,
stating that $\alpha=1.0$ for nonlinear displacements less than the elastic displacement at initial period
$T_i$, would result in a lower average response with less standard deviation. To avoid these issues,
the average and standard deviation can be calculated from the nonlinear displacements. The
average displacement and the average, plus or minus a standard deviation, can be used to
interpolate effective periods $T_e$ and calculate the respective $\alpha$ values.

3.4 Preliminary Analysis Details

The findings of the pilot analysis and the additional analysis considerations were used to
determine the procedures used in the preliminary analysis. For issues such as raw vs. altered
ground motions and $R$ initial vs. $R$ effective it was decided that a better comparison could be
made after conducting the analysis with both options. A brief summary of the key findings
implemented in the preliminary analysis is as follows:

- The stiffness reduction factor $\alpha$ defines the amount the stiffness of an elastic system is
  reduced in order to have the same maximum displacement predicted by the hysteretic
  model.
- Both altered and raw ground motions are used.
- The preliminary analysis is conducted for a series of structures with fixed strength;
  different $R$ initial values are achieved by scaling the ground motions.
- The stiffness reduction $\alpha$ is reported for both $R$ initial and $R$ effective.
- The effective period is determined from the design spectrum.
- The analysis is subject to 3% mass proportional Rayleigh damping.
- Post processing of nonlinear displacements is used to determine $\alpha$.

The OpenSees (PEER, 2005) input file, written in Tcl scripting language, has four major loops to
automate the preliminary analysis. The order of the analysis loop, from outside working in, is
wall type (described in Chapter 4), ground motion, initial period and $R$ value. For each of the
nineteen walls; forty ground motions; initial periods of 2 and 4 seconds; nineteen $R$ initial
values ranging from 0.5 to 5.0 at 0.25 increments are studied. This process is repeated for both
the modified and unmodified ground motions. The result is 3040 analyses per wall for a grand
total of 57,760 analyses. A detailed description of the OpenSees input file organization and inner
workings can be found in the electronic appendix.

The forty ground motions selected for the full analysis were taken from the suite of ground
motions used for calibration of the displacement modification procedure included within FEMA
440 (ATC, 2005). The twenty ground motions listed for each of NEHRP site classes B and C
were used since high-rise buildings are typically built on sites with firm soils. Arguably, site
class makes little difference when the ground motion is altered to the same target spectrum. In
the interest of providing more transparent results, the preliminary analysis was ultimately
conducted with both raw and altered ground motions. The raw ground motion properties are
listed in Table 3.5 for site class B and in Table 3.6 for site class C. The ground motions were
arbitrarily listed from 1 to 40 to facilitate the automation in the OpenSees input file.

The displacement spectra for the forty unmodified grounds motions scaled at 2 seconds and 4
seconds are illustrated in. The displacement spectra for the forty modified grounds motions are
illustrated in Figure 3.21. Both figures include the mean, mean plus/minus one standard
devation and the altered version of the 2005 NBCC design spectrum described previously. The
sensitivity of the unmodified ground motion spectra to the initial period is evident in. The
modified ground motion spectra follow the design spectrum reasonably well as seen in Figure
3.21. The modified ground motion spectra are far less sensitive to the initial period.

A strain hardening slope 2% of the initial stiffness was specified for the walls studied in the
preliminary analysis. This value corresponds well to the data from the UBC wall test and serves
as reasonable lower bound for the amount of strain hardening that could be expected for a typical
wall. A wall was analyzed with both 2% and 10% strain hardening to offer a comparison of how
strain hardening influences the maximum displacement experienced by the wall.
Figure 3.20 Unmodified Ground Motion Displacement Spectra
Figure 3.21 Modified Ground Motion Displacement Spectra
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<td>Station Code</td>
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<td>PGV (cm/s)</td>
<td>PGD (cm)</td>
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<td>Fun Valley</td>
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<td>6.1</td>
<td>Gilroy Gavilon college Phys Sci Bldg</td>
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<td>95.0</td>
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<td>Gilroy #6, San Ysidro Microwave site</td>
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<td>557.2</td>
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<td>06/28/92</td>
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<td>7.5</td>
<td>Yermo, Fire Station</td>
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<td>240.3</td>
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4.0 Walls Included in Nonlinear Analysis

The nonlinear analysis described in the previous chapter is conducted for nineteen walls. The current chapter describes the process by which these walls were selected. Hypothetical versions of the UBC wall test created by changing axial load and reinforcement amount were used to study changes in the shapes of the predicted force-displacement curves. This led to the development of generalized wall parameters. These parameters were used to define force-displacement curves for thirteen generalized walls. Three modified versions of the force-displacement curves were then developed for further study. Additional idealized force-displacement curves were included to provide two walls for comparison with FEMA 440 (ATC, 2005) and one wall to determine the influence of strain hardening on effective stiffness. The chapter concludes with a summary of the nineteen walls used in the nonlinear analysis.

4.1 UBC Wall Series

The shape of the force-displacement curve for the UBC wall test has been well established through the development of the simplified force-displacement relationship and the hysteretic force-displacement model. An additional seven hypothetical walls were studied in order to understand how the amount of reinforcement and level of axial load affects the shape of the force-displacement curve. The UBC test wall had five 10M bars in each flange and was subjected to an axial load of \(0.10f'_c A_s\). For the hypothetical walls, the axial load was varied from \(0.05f'_c A_s\) to \(0.25f'_c A_s\) for a wall with 10M flange reinforcement. The size of the flange reinforcement bars varied from 10M to 15M, 20M and 25M for a wall with \(0.10f'_c A_s\) axial load. The concrete geometry, location of reinforcement and the material properties of the hypothetical walls remained the same as the UBC wall. A summary of the parameters for the UBC wall series is presented in Table 4.1.

The theoretical moment-curvature response of each wall was approximated as a trilinear response with equal area under the curve using the methods previously described in Section 3.2. The Adebar and Ibrahim (2002) trilinear response was again calculated for comparison. All three moment-curvature responses for each wall in the UBC wall series are illustrated in. The Adebar and Ibrahim (A&I) prediction is generally quite good and is typically only slightly below the equal area (EA) trilinear representation of the theoretical response.
<table>
<thead>
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<th>Wall</th>
<th>Axial Load</th>
<th>Flange Reinforcement</th>
<th>ρf</th>
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<td>1</td>
<td>0.05$f_c$'$A_e$</td>
<td>10M</td>
<td>0.0065</td>
</tr>
<tr>
<td>2 (test)</td>
<td>0.10$f_c$'$A_e$</td>
<td>10M</td>
<td>0.0065</td>
</tr>
<tr>
<td>3</td>
<td>0.15$f_c$'$A_e$</td>
<td>10M</td>
<td>0.0065</td>
</tr>
<tr>
<td>4</td>
<td>0.20$f_c$'$A_e$</td>
<td>10M</td>
<td>0.0065</td>
</tr>
<tr>
<td>5</td>
<td>0.25$f_c$'$A_e$</td>
<td>10M</td>
<td>0.0065</td>
</tr>
<tr>
<td>6</td>
<td>0.10$f_c$'$A_e$</td>
<td>15M</td>
<td>0.0130</td>
</tr>
<tr>
<td>7</td>
<td>0.10$f_c$'$A_e$</td>
<td>20M</td>
<td>0.0194</td>
</tr>
<tr>
<td>8</td>
<td>0.10$f_c$'$A_e$</td>
<td>25M</td>
<td>0.0324</td>
</tr>
</tbody>
</table>

The equal area trilinear moment-curvature curve was used to calculate displacements required by the simplified force-displacement relationship as described previously in Section 2.4. A comparison of this simplified method and the theoretical force-displacement response can be seen in for a linearly varying load distribution. The simplified method produces a force-displacement response that follows the theoretical response closely. This comparison illustrates that the simplified method is applicable over a practical range of axial loads and reinforcement ratios.
Figure 4.1 UBC Wall Series Moment-Curvature
The simplified force-displacement relationship is used to define the loading upper bound curve for the hysteretic force-displacement relationship described previously in Section 2.6. The corresponding loading lower bound is defined as having a slope equal to the upper bound secondary slope. The upper and lower bounds are shown in the first row of for varying axial load and in the first row of for varying size of reinforcement. To better show the shape of the upper and lower bounds in a general sense, these curves are normalized by their respective maximum shear $V_N$ in the second row of each of these figures.
Figure 4.3 Effect of Axial Load on Upper and Lower Bound Force-Displacement Relationships
Figure 4.4 Effect of Reinforcement on Upper and Lower Bound Force-Displacement Relationships
The curves in the second row of the above figures no longer possess the same initial stiffness since each curve has been normalized by a different value. To return all curves to a common initial stiffness, the displacements of the shear normalized curves must be adjusted. In the third row of and, the shear normalized curves in the second row have had their displacements adjusted to yield curves with a common initial stiffness. The initial stiffness chosen for these final curves matches the curve for the UBC test wall with $0.10f'_cA_g$ axial load and 10M reinforcing bars. This wall is represented in both figures by a solid black line for ease of comparison.

Several observations can be made pertaining to the shear and initial stiffness normalized plots. First, there is a significant difference between the upper and lower bound curves for each wall. This may not be immediately obvious as the lower bound scale is twice that of the upper bound. The difference in scale helps illustrate a second point; the lower bound is far more sensitive to changes in axial load and reinforcement ratio than the upper bound. If both upper and lower bound were plotted to the same scale, the upper bound curves would be almost indistinguishable from one another.

The shear ratio at which the initial stiffness intercepts the lower bound, $V_L/V_N$, is most sensitive to level of axial load. Larger values of axial load result in higher values of $V_L/V_N$. The reinforcement ratio effects $V_L/V_N$ to a lesser extent; increasing reinforcement ratios results in lower values of $V_L/V_N$. Increasing reinforcement ratios also lowers the point at which nonlinearity begins on the upper bound. This point of nonlinearity remains almost unchanged with varying axial load and constant reinforcement ratio.

The slope of the lower bound $k_{LB}$ is almost entirely a function of the reinforcement ratio. Larger values of reinforcement ratios result in steeper lower bound slopes. By definition, the lower bound slope is the same as the secondary slope of the upper bound curve. The large variations in this common slope, however, cause relatively less difference between the upper bound curves when compared to the difference observed between the lower bound curves.
4.2 Determining Generalized Wall Parameters

The shapes of the shear and initial stiffness normalized force-displacement curves presented in the previous section can be summarized by two parameters; the ratio of \( V_L/V_N \) and the ratio of the lower bound stiffness to the initial stiffness \( k_{LB}/k_i \). A realistic range of these parameters is required in order to develop a broad series of normalized force-displacement curves. This broad series of curves can be used to conduct a generalized analysis to determine how the shape of the force-displacement curve influences effective stiffness. In addition to reinforcement and axial load discussed previously, wall geometry also dictates the shape of the force-displacement curves. Walls in high rise buildings are typically either rectangular or have flanged ends of various sizes. Walls of differing geometry, each with varying amounts of reinforcement, will be studied for a range of axial loads to determine a realistic range of \( V_L/V_N \) and \( k_{LB}/k_i \). This study will be conducted using the UBC wall geometry with 10M, 15M, 20M and 25M flange reinforcement and three additional walls. The additional walls are referred to according to their geometry: rectangular (R), small flange (SF) and big flange (BF). These three walls have a common length of 6.0m, web thickness of 0.5m, web reinforcement ratio \( \rho_w = 0.25\% \) and flange thickness (or concentrated end zone) of 0.6m as illustrated in. The flange width is 2.0m and 6.0m for the small and big flange walls respectively. The flange reinforcement ratio \( \rho_f \) ranges from 0.5\% to 2.0\% for the small and big flange walls and from 1.0\% to 4.0\% for the rectangular wall. For all four walls, for each flange reinforcement ratio \( \rho_f \), the axial load varies from 0 to \( 0.30f'_c' A_g \) at increments of 0.01\( f'_c' A_g \).

Determination of a realistic range for \( V_L/V_N \) and \( k_{LB}/k_i \) will be accomplished by calculating these ratios for each of the four wall geometries using the reinforcement amounts and axial loads described above. The following discussion, however, will be in terms of bending moment rather than shear force. Shear values are a direct function of the bending moment, load distribution and height of the wall; therefore the ratios of \( V_L/V_N \) and \( M_L/M_N \) are identical. The force-displacement properties needed to calculate \( k_{LB}/k_i \) are also a function of the moment-curvature properties of the wall. This function is however not as simple since the ratios of \( k_{LB}/k_i \) and \( E_c l_{cr}/E_c l_g \) are not identical. The linear property \( k_i \) is a direct function of the linear moment-
curvature slope $E_c I_s$, load distribution and height of wall. The displacement required to calculate $k_{LB}$, however, is a function of $M_L / M_N$, $E_c I_c / E_c I_s$, load distribution and height of the wall requiring integration with the second moment-area theorem described in Section 2.3.2. In summary, the force-displacement properties $V_L / V_N$ and $k_{LB} / k_i$ of interest can be related back to moment-curvature properties. It is these moment-curvature properties that will be studied in order to create a range of generalized force-displacement curves.

Figure 4.5 Walls Used to Determine Generalized Analysis Parameters

The nominal flexural capacity $M_N$ was calculated for each axial load and reinforcement amount using plane sections analysis with an equivalent rectangular stress block. To simplify calculation, several additional assumptions were made; all steel was lumped at the flange and web centroids,
all steel in tension flange yields, all steel in compression flange yields and all web steel yields in tension. This is by no means as accurate as finding the exact strain distribution iteratively, however, it does provide a reasonable approximation suitable for determining the wide range of possible $M_N$ values.

4.2.1 Lower Bound

The normalized force-displacement curves for the UBC wall series presented in the previous section indicate that axial load and the amount of reinforcement have the most significant effect on the lower bound curve. The lower bound curve was originally defined as deviating from the initial stiffness at the shear corresponding to the cracking moment $M_{cr}$. This definition was based on the experimental data for the UBC wall having an axial load of $0.10f_{ce}A_G$. A more generalized approach is to use the Adebar and Ibrahim (2002) expression for lower bound moment-curvature response $M'_L$ shown by Equation 2.1. This expression was developed by Ibrahim (2000) for a wide range of wall geometry, reinforcement amounts and axial loads. For the UBC wall the calculated values for $M'_L$ and $M_{cr}$ are 778kN·m and 861kN·m respectively while $M_N$ is 1612kN·m; the respective ratios of $M'_L$ and $M_{cr}$ to $M_N$ are 0.48 and 0.53. The negligible difference between the two ratios suggests that using $M'_L$ to define the lower bound is appropriate for the UBC wall. The use of $M'_L$ to define the starting point of the lower bound will be more applicable for a wider range of walls. The OpenSees (PEER, 2005) material model ADKhysteretic, however, has been programmed for the lower bound to deviate from the initial stiffness at $M_{cr}$, the same point at which nonlinearity begins in the upper bound curve. For the generalized analysis the calculated value of $M'_L$ will be used to define $M_{cr}$ in the OpenSees material model. This substitution will have little effect for walls with high $M'_L/M_N$ ratios because the values of $M'_L$ and $M_{cr}$ are quite similar for higher axial loads. For walls with low $M'_L/M_N$ ratios and low axial loads, $M'_L$ will be much lower than $M_{cr}$. The substitution of $M'_L$ for $M_{cr}$ will therefore cause the upper bound curve nonlinearity to start prematurely. This nonlinear curve, however, remains almost linear in the curve’s early stages for walls with low $M'_L/M_N$ ratios. The end result is that substituting the value of $M'_L$ for $M_{cr}$ in the OpenSees material model will have little effect on the shapes of the force-displacement curves for the generalized structures. This substitution is beneficial because the existing material model can be
used and the possible parameter of \((M_{cr} - M')/M_N\) does not need to be considered for the generalized structures.

An appropriate range of \(M'/M_N\) can be determined from Figure 4.6 which relates this ratio to axial load expressed as a portion of \(f_c'A_y\). Each wall has four bands of data resulting from the four levels of flange reinforcement used in each wall. The rightmost band of data for each wall has the lowest percentage of steel. The focus of the thesis is high rise concrete walls in which the axial load would typically be a minimum of \(0.05f_c'A_y\). With this in mind, an appropriate range of \(M'/M_N\) is from 0.2 to 0.8 with the majority of data points concentrated at ratios of 0.4, 0.5 and 0.6. The generalized analysis will be conducted for these five \(M'/M_N\) ratios.

![Figure 4.6 Appropriate Range for \(M'/M_N\)](image)

The ratio of \(k_{LB}/k_i\) can also be thought of in terms of displacement as \(\Delta_{LB}/\Delta_x\). The displacement at which the lower bound curve reaches the flexural capacity plateau \(V_N\) is \(\Delta_{LB}\). The displacement at which a projection of the initial stiffness \(k_i\) reaches \(V_N\) is \(\Delta_x\). As discussed previously, these displacements are a function of the moment-curvature properties with \(\Delta_{LB}\) requiring integration using the second-moment-area-theorem. A simple approximation for \(\Delta_{LB}/\Delta_x\) has been developed using the bending moment diagram for first mode load distribution. The lower portion of the bending moment diagram can be reasonably approximated.
by a triangle having the same maximum moment at the base and a height 75% of the wall height $h_w$. The displacement at top of wall for a linear wall with first mode load distribution is shown by Equation 4.1. The additional nonlinear displacement is found by idealizing the additional curvature due to cracking as a second triangle shown in Figure 4.7 and then integrating with the second-moment-area theorem. The final expression for $\Delta_{yLB}/\Delta_g$ is shown by Equation 4.1 for first mode load distribution. The complete derivation for this expression can be found in Appendix D.

\[ \Delta_g = 0.28h_w^2 \frac{M_N}{E_i I_g} \]  
\[ \frac{\Delta_{yLB}}{\Delta_g} = 1 + \left( \frac{M_N - M_L'}{M_N} \right)^2 \left( 1 - 0.33 \frac{M_L'}{M_N} \left( \frac{E_i I_g}{E_i I_{cr}} - 1 \right) \right) \]  
(4.1)
(4.2)

The expression for $\Delta_{yLB}/\Delta_g$ is presented as a function of $M_L'/M_N$ in Figure 4.8 for all four walls. The four bands of data for each wall result from the four levels of flange reinforcement used in each wall. The uppermost band of data for each wall has the lowest percentage of steel. It is clear that there is a wide variation in $\Delta_{yLB}/\Delta_g$ ratios for the five $M_L'/M_N$ ratios chosen to...
conduct the generalized analysis. Three $\Delta_{s,\text{LB}}/\Delta_s$ values have been selected for each of the $M_L'/M_N$ ratios 0.2, 0.4, 0.5 and 0.6 to account for this variation. These values are later summarized in Table 4.2. The result is a total of thirteen generalized walls to represent a realistic range of $V_L/V_N$ and $k_{LB}/k_j$. The thirteen walls are represented by the large dots in Figure 4.8.

![Figure 4.8 Appropriate range for $\Delta_{s,\text{LB}}/\Delta_s$](image)

### 4.2.2 Upper Bound

The lower bound force-displacement curve is now completely defined for the thirteen generalized walls; the upper bound force-displacement curve must now be defined. The UBC wall series moment-curvature diagrams shown in indicate that the Adebar and Ibrahim (2002) upper bound expression for $M_L''$ produces a trilinear curve with a very similar area underneath it as the theoretical curve. Using this expression for $M_L''$ to define the upper bound curve requires significantly less time than performing a detailed moment-curvature analysis and calculating an equal area trilinear curve for each axial load and produces very similar results. The secondary segments of the Adebar and Ibrahim upper bound and lower bound trilinear curves are parallel. The difference between the two curves as a ratio of flexural capacity can be defined as $(M_L''-M_L')/M_N$ which is also equivalent to $(V_L''-V_L')/V_N$. The displacement at which the upper bound reaches flexural capacity can be determined by finding the displacement along the lower bound at which the addition of the corresponding $V_L'/V_N$ ratio and the $(V_L''-V_L')/V_N$ ratio equals 1.0. The difference between the upper and lower bound curves is plotted as a function of...
\( M_L' / M_N \) in Figure 4.9. The uppermost band of data for each wall has the lowest percentage of steel. There is a wide variation of \( (M_L' - M_L') / M_N \) ratios for the five \( M_L' / M_N \) chosen ratios similar to that seen in Figure 4.8 for \( \Delta_{LB} / \Delta_s \). Three \( (M_L' - M_L') / M_N \) values have been selected for each of the \( M_L' / M_N \) ratios 0.2, 0.4, 0.5 and 0.6 to account for this variation, as summarized later in Table 4.2. The values of \( (M_L' - M_L') / M_N \) selected for the thirteen generalized walls are represented by the large dots in Figure 4.9.

![Figure 4.9 Appropriate Range for \( (M_L' - M_L') / M_N \)](image)

The upper bound curve end point position is now known and the end point slope, parallel to the lower bound stiffness \( k_{LB} \), is already defined. The fit point required for the fourth order polynomial used by the simplified force-displacement relationship can be taken at the ratio of 0.9(\( V_N - V_L' \)) / \( V_N \). This displacement can be approximated by taking a reduced portion (99%) of the displacement calculated by following the lower bound stiffness \( k_{LB} \) tangent from the end of the upper bound curve as shown in Equation 4.3.

\[
D_{fit} = D_N - 0.99 \frac{0.1(V_N - V_L') / V_N}{k_{LB}} \tag{4.3}
\]

The start point of the upper bound curve is taken to be \( V_L' \) instead of \( V_{cr} \) as discussed previously. The final piece of information required for the fourth order polynomial is the slope of the curve at the start point. This slope was adjusted as a portion of the initial slope \( k_i \) until the
curve displayed a smooth transition to $k_{LB}$ without having local maxima or minima. The slope reductions ranged from $1.0k_i$ to $0.6k_i$ as summarized later in Table 4.2.

The simplified force-displacement relationship was originally developed with $0.9k_i$ as the start point slope. Use of $0.9k_i$ performed well when all parameters of the simplified relationship were calculated for an individual wall. The range in initial stiffness reduction required to produce properly shaped curves is a product of the simplifications made for the generalized analysis. The values selected for $\Delta y_{LB}/\Delta_s$ in Figure 4.8 and $(M_L'-M_L)/M_N$ in Figure 4.9 for each of the $M_L'/M_N$ ratios used in the generalized analysis are reasonably distributed across the concentration of data points in each plot. The distribution of the data bands relative to one another is however slightly different when comparing these two plots. In the strictest sense, the values selected for $(M_L'-M_L)/M_N$ do not exactly coincide with those selected for $\Delta y_{LB}/\Delta_s$. The $(M_L'-M_L)/M_N$ values for each $\Delta y_{LB}/\Delta_s$ only provide a reasonable approximation of the location of where the upper bound curve reaches flexural capacity.

4.2.3 Unloading Point

The unloading point was originally defined as the shear corresponding to the moment where the axial load closes the cracks in the concrete $M_{cl}$ expressed in Equation 2.13. This definition is based on the experimental data for the UBC wall having an axial load of $0.10f_c'A_g$ and web reinforcement ratio $\rho_f=0.65\%$. The expression for $M_{cl}$ currently only considers the effects of axial load and not those of reinforcement. If a wall experiences a large thrust in one direction the concrete cracks and the steel yields in tension across the gap. If thrust back in the opposite direction, this steel must yield in compression before the gap can close and the cracked concrete can come into contact. A simple model to account for both axial load and amount of reinforcement is presented in Figure 4.10 and Equations 4.4 to 4.6.
\[ M_{cls} = \left( \frac{P - P_s}{A_g} \right) S - M_s \]  \hspace{2cm} (4.4)

\[ P_s = P_{sf} + P_{sw} = A_{sf} f_y + A_{sw} f_y \]  \hspace{2cm} (4.5)

\[ M_s = P_{sf} \frac{jd}{2} \]  \hspace{2cm} (4.6)

where \( jd \) = wall depth from compression face to tension steel centroid

\[ P \]

\[ M_{cls} \]

\[ jd/2 \]

\[ P_{sf} \]

\[ P_{sw} \]

Figure 4.10 Crack Closing Model

The value calculated for \( M_{cls} \) is lower than \( M_{cf} \) because a portion of the axial load \( P \) is required to yield the steel in the tension flange and the web that are now in compression in order to close the cracks. The moment of the steel in the tension flange must also be considered when summing the moments about the application of \( P \) at the gross section centroid. The crack closing point as a ratio of strength \( M_{cls}/M_N \) is plotted as a function of \( M_{L'}/M_N \) in Figure 4.11. All four walls exhibit similar near linear trends and can be approximated reasonably well from \( M_{L'}/M_N = 0.4 \) onward by a series of linearly increasing points shown by the large dots in this figure. For \( M_{L'}/M_N = 0.2 \) the data indicates that the axial load is not large enough to yield the reinforcing steel in compression and close the cracks before reaching the origin and loading in the opposite direction. The hysteretic model was not defined for such behaviour and as a result
the minimum value $M_{ch}/M_N = 0$ is used for $M_{L'}/M_N = 0.2$. This is not a significant concern as the most practical range of $M_{L'}/M_N$ for high-rise walls is from 0.5 to 0.6.

4.3 Generalized Force-Displacement Curves

The generalized wall parameters developed in the previous section describe the upper bound, lower bound and unloading point for thirteen generalized walls that represent a practical range of axial load, reinforcement amounts and wall geometries. The parameters are summarized in Table 4.2 in terms of shear normalized by shear at flexural capacity $V/V_N$ and in terms of displacement normalized by the displacement projected along the initial stiffness at flexural capacity $\Delta/\Delta_N$.

The upper and lower bounds of the thirteen generalized walls are illustrated together in for walls 1 to 6 and in for walls 7 to 13. The upper bounds and lower bounds for all thirteen walls are shown in and respectively. The horizontal scale of the upper bounds is double that of the lower bounds. This highlights, in a similar fashion as the UBC wall series, that the lower bounds are far more sensitive to changes in axial load, reinforcement and wall geometry than the upper bounds. The upper bounds for the wide range of the thirteen generalized structures are remarkably similar.
<table>
<thead>
<tr>
<th>Wall</th>
<th>$V_L/V_N$</th>
<th>$\Delta_{1LN}/\Delta_x$</th>
<th>$\Delta_{10N}/\Delta_x$</th>
<th>Slope reduction</th>
<th>$V_{so}/V_N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.20</td>
<td>3.00</td>
<td>0.29</td>
<td>2.00</td>
<td>0.90</td>
</tr>
<tr>
<td>2</td>
<td>0.20</td>
<td>5.00</td>
<td>0.38</td>
<td>2.70</td>
<td>0.90</td>
</tr>
<tr>
<td>3</td>
<td>0.20</td>
<td>8.00</td>
<td>0.46</td>
<td>3.50</td>
<td>0.85</td>
</tr>
<tr>
<td>4</td>
<td>0.40</td>
<td>2.00</td>
<td>0.15</td>
<td>1.60</td>
<td>0.90</td>
</tr>
<tr>
<td>5</td>
<td>0.40</td>
<td>4.00</td>
<td>0.23</td>
<td>2.60</td>
<td>0.70</td>
</tr>
<tr>
<td>6</td>
<td>0.40</td>
<td>6.00</td>
<td>0.30</td>
<td>3.20</td>
<td>0.60</td>
</tr>
<tr>
<td>7</td>
<td>0.50</td>
<td>2.00</td>
<td>0.10</td>
<td>1.70</td>
<td>0.80</td>
</tr>
<tr>
<td>8</td>
<td>0.50</td>
<td>3.50</td>
<td>0.18</td>
<td>2.40</td>
<td>0.65</td>
</tr>
<tr>
<td>9</td>
<td>0.50</td>
<td>5.00</td>
<td>0.24</td>
<td>2.80</td>
<td>0.50</td>
</tr>
<tr>
<td>10</td>
<td>0.60</td>
<td>2.00</td>
<td>0.10</td>
<td>1.65</td>
<td>0.80</td>
</tr>
<tr>
<td>11</td>
<td>0.60</td>
<td>3.00</td>
<td>0.15</td>
<td>2.10</td>
<td>0.70</td>
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<tr>
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<td>0.19</td>
<td>2.40</td>
<td>0.60</td>
</tr>
<tr>
<td>13</td>
<td>0.80</td>
<td>2.00</td>
<td>0.10</td>
<td>1.40</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Figure 4.12 Generalized Walls 1-6
Figure 4.13 Generalized Walls 7-13
4.4 Additional Force-Displacement Curves

Several additional force-displacement curves are considered to complement those of the thirteen generalized walls. The first group of additional curves are modified versions of generalized wall 3 used to study the effects of hypothetically changing the difference between the upper and
lower bound. The second group of additional curves are idealized curves such as elastic-perfectly-plastic and nonlinear-elastic that will offer comparison to the single-degree-of-freedom-systems studied as part of FEMA 440 (ATC, 2005).

4.4.1 Modified Versions of Generalized Curves

Development of the generalized wall parameters indicates that the upper bound curves are remarkably similar for widely varying lower bounds. Generalized walls 1, 2 and 3 all have $V_L'/V_N=0.2$ with $\Delta_{\gamma_{LB}}/\Delta_s$ values of 3.0, 5.0 and 8.0 respectively. Wall 1 has the steepest lower bound slope and the lowest $(V_L''-V_L')/V_N$ value of 0.29. Wall 3 has the gentlest lower bound slope and the highest $(V_L''-V_L')/V_N$ value of 0.46. The net result is similar upper bound curves for widely varying lower bound curves as shown previously by the top plot in. In the interest of forcing the hysteretic response of the wall towards the lower bound, three hypothetical versions of wall 3 were created. The value of $(V_L''-V_L')/V_N$ was reduced from 0.46 to 0.28 and 0.10 for walls 3A and 3B respectively. The lower bound $V_L'/V_N=0.2$ and $\Delta_{\gamma_{LB}}/\Delta_s=8.0$ remained constant so each wall maintained the same lower bound slope as wall 3. Wall 3C is the most extreme hypothetical version of wall 3; the difference between the upper and lower bound is minimal and the lower bound deviates from the initial stiffness at a very low value of $V/V_N$. For wall 3C $V_L'/V_N$ was reduced to 0.05 and $(V_L''-V_L')/V_N$ was reduced to 0.10. In order to maintain the same lower bound slope as wall 3; $\Delta_{\gamma_{LB}}/\Delta_s$ was increased from 8.0 to 9.31. The parameters for the modified versions of generalized wall 3 are summarized in Table 4.3. The upper and lower bound for the three modified walls are shown with wall 3 in Figure 4.16.

<table>
<thead>
<tr>
<th>Wall</th>
<th>$V_L'/V_N$</th>
<th>$\Delta_{\gamma_{LB}}/\Delta_s$</th>
<th>$(V_L''-V_L')/V_N$</th>
<th>$\Delta_{\gamma_{UB}}/\Delta_s$</th>
<th>Slope reduction</th>
<th>$V_{ub}/V_N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.20</td>
<td>8.00</td>
<td>0.46</td>
<td>3.50</td>
<td>0.85</td>
<td>0.00</td>
</tr>
<tr>
<td>3A</td>
<td>0.20</td>
<td>8.00</td>
<td>0.28</td>
<td>5.30</td>
<td>0.40</td>
<td>0.00</td>
</tr>
<tr>
<td>3B</td>
<td>0.20</td>
<td>8.00</td>
<td>0.10</td>
<td>7.00</td>
<td>0.20</td>
<td>0.00</td>
</tr>
<tr>
<td>3C</td>
<td>0.05</td>
<td>9.31</td>
<td>0.10</td>
<td>8.30</td>
<td>0.18</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 4.3 Modified Wall 3 Parameters
4.4.2 Idealized Force-Displacement Curves

The single-degree-of-freedom analyses conducted as part of FEMA 440 (ATC, 2005) included an elastic-perfectly-plastic system and a nonlinear-elastic system. Both models are well known and are relatively simple representations of nonlinear behaviour. These idealized force-displacement curves are included in the current study to allow for comparison with other studies.

A simplified version of ADKhysteretic, denoted ADKplateau, is used to imitate nonlinear-elastic behaviour. The nonlinear portion of ADKhysteretic described by the fourth order polynomial was removed and the linear segment along initial stiffness $k_i$ was extended to $V_N$. Loading is linear until $V_N$ where it becomes nonlinear and continues along the yield plateau. Thus far the nonlinear-elastic and ADKplateau models are identical. The difference lies in the unloading rules; the nonlinear-elastic model returns to origin along exactly the same path while the ADKplateau model unloads inelastically towards an unloading point $V_{ul}$. The two models are illustrated graphically in Figure 4.17. In the case of midcycle reloading for ADKplateau, the reloading path follows a slope parallel to $k_i$ until reaching the yield plateau and returning to the full cycle reloading path. Midcycle reloading has been illustrated for ADKplateau in Figure 4.17.
Four input variables are required by ADKplateau to define the shape of the force-displacement curve and location of the unloading point. The material model also utilizes two history variables, one state variable and three trial variables. The algorithms within each state are simplified versions of those used in ADKhysteretic. This simplification resulted in seven less input variables and two less history variables for ADKplateau. All variables are summarized in Table 4.4 by variable type. Code Reference refers to how the variable is specified in the C++ computer code found in the electronic appendix.

<table>
<thead>
<tr>
<th>Variable Type</th>
<th>Variable Name</th>
<th>Code Reference</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>input</td>
<td>$k_i$</td>
<td>ki</td>
<td>Initial stiffness of force-displacement curve</td>
</tr>
<tr>
<td></td>
<td>$V_{cr}$</td>
<td>Vcr</td>
<td>Shear at which cracks close from axial load</td>
</tr>
<tr>
<td></td>
<td>$V_n$</td>
<td>Vn</td>
<td>Shear at flexural capacity</td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td>beta</td>
<td>Strain hardening slope as portion of initial stiffness</td>
</tr>
<tr>
<td>history</td>
<td>$V_{prev}$</td>
<td>Vprev</td>
<td>Shear at last converged time step</td>
</tr>
<tr>
<td></td>
<td>$\Delta_{prev}$</td>
<td>Dprev</td>
<td>Displacement at last converged time step</td>
</tr>
<tr>
<td>state</td>
<td>state</td>
<td>state</td>
<td>State at last converged time step</td>
</tr>
<tr>
<td>trial</td>
<td>$V$</td>
<td>V</td>
<td>Shear at current time step</td>
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<td></td>
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<td>Displacement at current time step</td>
</tr>
<tr>
<td></td>
<td>$Tstate$</td>
<td>Tstate</td>
<td>State of current time step</td>
</tr>
</tbody>
</table>

The existing OpenSees (PEER, 2005) material model Steel01 was used to model elastic-perfectly-plastic force-displacement as done previously in Section 3.2.1. The input variables are summarized in Table 4.5 for this force-displacement application of the model. The computer
code for Steel01 has not been included in the electronic appendix as no changes were made to the model.

<table>
<thead>
<tr>
<th>Variable Type</th>
<th>Variable Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>input</td>
<td>$k_i$</td>
<td>Initial stiffness of force-displacement curve</td>
</tr>
<tr>
<td></td>
<td>$V_N$</td>
<td>Shear at flexural capacity</td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td>Strain hardening slope as portion of initial stiffness</td>
</tr>
</tbody>
</table>

### 4.5 Summary of Walls Used in Analysis

A total of nineteen walls are studied in the nonlinear analysis using three different OpenSees (PEER, 2005) material models; sixteen with ADKhysteretic, two with ADKplateau and one with Steel01. The nineteen walls have the same initial stiffness $k_i$ and shear strength at flexural capacity $V_N$. A strain hardening slope 2% of the initial stiffness $k_i$ was specified for all walls except one. This value corresponds well to the data from the UBC wall test and serves as reasonable lower bound for the amount of strain hardening that could be expected for a typical wall. A wall was analyzed with both 2% and 10% strain hardening to offer a comparison of how strain hardening influences the maximum displacement experienced by the wall.

The thirteen generalized walls in Section 4.3 and the three modified versions of the generalized walls in Section 4.4.1 are presented in terms of ratios $V/V_N$ and $\Delta/\Delta_s$. In order to perform the nonlinear analysis the ratios need to be converted into actual strength and displacement values. This is achieved by multiplying the ratios by and appropriate values of $V_N$ and $\Delta_s$, with $\Delta_s$ being the displacement at which a projection of $k_i$ reaches $V_N$. The strength and initial stiffness of the UBC wall, with linearly varying load distribution, are 206,516N and 10,133,000N/m respectively. For the purposes of converting the generalized wall ratios, these values are rounded to $V_N=200,000N$ and $k_i=10,000,000N/m$ which results in $\Delta_s=0.02m$. The rounded values of $V_N$ and $k_i$ are used to calculate the input parameters required by ADKhysteretic for walls 1 to 13 and 3A, 3B, 3C summarized in Table 4.6.
The rounded values of $V_n$ and $k_i$ are also used to define the input parameters required by $ADKplateau$ for walls 14 and 15; the closing point is $V_\alpha$, set as $0.75V_n$. Strain hardening is altered for $ADKplateau$ because this material model is more transparent than $ADKhysteretic$ and can demonstrate the effects of strain hardening on effective stiffness more clearly. The same values for $V_n$ and $k_i$ are used to define the input parameters required by $Steel01$ for wall 16. The input parameters for walls 14 to 16 are summarized in Table 4.6.
<table>
<thead>
<tr>
<th>Wall</th>
<th>Model</th>
<th>( k_i ) (N/m)</th>
<th>( V_c1 ) (N)</th>
<th>( V_c2 ) (N)</th>
<th>( D_n ) (m)</th>
<th>( V_n ) (N)</th>
<th>( C_1 ) (N/m^4)</th>
<th>( C_2 ) (N/m^3)</th>
<th>( C_3 ) (N/m^2)</th>
<th>( C_4 ) (N/m)</th>
<th>( D_{int} ) (m)</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ADK hysteretic</td>
<td>10000000</td>
<td>0</td>
<td>40000</td>
<td>0.040</td>
<td>200000</td>
<td>-32278306859</td>
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<td>9000000</td>
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</tr>
<tr>
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<td>40000</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.02</td>
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<tr>
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<td>150000</td>
<td>-</td>
<td>-</td>
<td>200000</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>-</td>
<td>0.10</td>
</tr>
<tr>
<td>16</td>
<td>Steel01</td>
<td>10000000</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>200000</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.02</td>
</tr>
</tbody>
</table>
5.0 Presentation of Preliminary Results

For each of the walls described in the previous chapter, the mean nonlinear displacement was calculated at each $R$ interval for both the suites of modified and unmodified ground motions. The standard deviations of the nonlinear displacements were also calculated. The mean and the mean plus one standard deviation displacements were used to calculate the mean stiffness reduction factor $\alpha(\mu)$ and the mean plus one standard deviation stiffness reduction factor $\alpha(\mu + \sigma)$. The results are presented separately for the generalized walls, modified versions of the generalized walls and the idealized walls. The raw data for all the walls is available in the electronic appendix.

5.1 Generalized Walls

The thirteen generalized walls described previously in Section 4.3 have been organized into groups of common $V_L/V_N$ values. The upper and lower bound force-displacement curves for the thirteen walls from Section 4.3 are presented again in Figure 5.1, grouped by colour for common $V_L/V_N$ values to allow for easier comparison of the results. The numbering of the walls is unchanged from Section 4.3. The lowest numbered wall within a colour group (common $V_L/V_N$ value) refers to the left most force-displacement curve. All figures within this section continue to be grouped by colour for common $V_L/V_N$ values.

The results for the 2 second initial period are shown in for the modified ground motions and in for the unmodified ground motions. Each data point is the mean ($\mu$) or mean plus one standard deviation ($\mu + \sigma$) value of $\alpha$ for forty ground motions. The $\alpha(\mu)$ values remain larger for $R$ effective values less than 2.5 but drop more rapidly afterward for the modified ground motions than the unmodified ground motions. The trend of the unmodified ground motions is much more gradual. The $\alpha(\mu + \sigma)$ values for the unmodified ground motions decrease far quicker and are significantly lower than those for the modified ground motions. This indicates that the gradual decrease of $\alpha(\mu)$ values for the unmodified ground motions has smoothed over an important trend in the data. The $\alpha(\mu + \sigma)$ values for the modified ground motions follow the $\alpha(\mu)$ values more closely.
Figure 5.1 Organization of Results for Walls 1-13
Figure 5.2 Modified Ground Motions for $T_j=2s$
Figure 5.3 Unmodified Ground Motions for $T_j=2s$
The results for the 4 second initial period are shown in for the modified ground motions and in for the unmodified ground motions. The $\alpha(\mu)$ values decrease slightly until they reach an $R$ effective value of 1.5 or 2 and are relatively level afterwards for both sets of ground motions. Comparing the $\alpha(\mu + \sigma)$ values in each figure indicates that the initial gradual decrease of the $\alpha(\mu)$ values is captured better for the modified ground motions than the unmodified ground motions. This levelling effect observed for the 4 second initial period is not present in the 2 second initial period at all. It is interesting however, when comparing the modified ground motion $\alpha(\mu)$ values for each initial period, the gradual decrease occurs over same range of initial $R$ effective values. The point at which the 2 second initial period $\alpha(\mu)$ values start to decrease quickly roughly coincides with the point at which the 4 second initial period $\alpha(\mu)$ values level out.

The gradual decrease observed in $\alpha(\mu)$ for low $R$ effective values at both initial periods suggests that an elastic system with the same initial stiffness does a relatively good job in predicting the nonlinear displacements for low $R$ values. This result makes physical sense since a gradual transition, rather than a sudden drop, between the initial stiffness and the effective stiffness should occur if the elastic capacity of the wall is only slightly exceeded. With increasing $R$ values, larger displacements and in turn lower effective stiffness values can be expected since both the 2 second and 4 second initial periods occur along the linearly increasing portion of the design spectrum. The 4 second initial period is however closer to the point at which the displacements in the design spectrum and the unmodified ground motion displacement spectra level off. The levelling of all displacement spectra is consistent with the concept of equal displacement. If the displacement remains constant then the effective stiffness to achieve this displacement will also remain constant.
Figure 5.4 Modified Ground Motions for $T = 4s$
Figure 5.5 Unmodified Ground Motions for $T_j=4s$
Another possible explanation to the levelling of $\alpha(\mu)$ for a 4 second initial period is that the ground motions used in this study are not rich enough in frequency content for this period range. The forty ground motions used have frequency content more suitable for shorter periods. The strong motion characteristics of a given ground motion are what dictates at which periods the largest response will occur. If the strong motion characteristics required for a longer period response are not present in the unmodified records they will also not be present in the modified records since SYNTH (Naumoski, 2001) only scales the existing frequency content. Referring back to Figure 3.21 it can be seen that modified ground motions do not follow the design spectrum nearly as well for longer periods than they do for shorter periods.

Several general observations can be made regardless of whether the ground motions were modified or unmodified or which initial period was used in the analysis. The resulting $\alpha(\mu)$ values for a given analysis are all fairly well banded considering the wide range of walls studied. More significant changes in $\alpha(\mu)$ are observed between initial periods than between wall 1 and 13 for a given initial period. The order in which the walls appear in the bands of data for each initial period is quite similar. Noticeable trends are observed within common $V_L/V_N$ values. The highest effective stiffness values occur for the walls with the largest difference between the upper and lower bound force-displacement curve. As the area between the upper and lower bounds decreases, for a given $V_L/V_N$ value, the effective stiffness decreases as well. As the area decreases between the upper and lower bounds so does the opportunity for hysteretic damping. This in turn increases nonlinear displacements which results in lower effective stiffness values. Wall 13 has the smallest difference between the upper and lower bound and consistently had the lowest $\alpha(\mu)$ values.

A method to determine effective stiffness proposed by Ibrahim (2000) was to use the stiffness of the elastic portion of bilinear curves with the equal area-under-the-curve as the actual nonlinear upper and lower bound force-displacement curves. Equal area bilinear curves were created for the upper and lower bound force-displacement curves for the generalized walls. The difference between the initial stiffness and that of the linear portion of the bilinear curves were used to calculate upper and lower bound $\alpha$ values. These $\alpha$ values were plotted with the modified ground motion $\alpha(\mu)$ values in Figure 5.6 and Figure 5.7. This method generally has limited success in predicting realistic bounds of effective stiffness. The limited success is related to the
well banded $\alpha(\mu)$ trends observed for the wide variation of walls studied. The relationship between the 2 and 4 second initial period $\alpha(\mu)$ values is similar for each of the 13 walls. Study of intermediate initial periods would determine if the $\alpha(\mu)$ relationships transition smoothly as the initial period is increased.

![Graphs showing data for different walls with labels and legends.](image)

Figure 5.6 Equal Area Comparison Walls 1-6
Figure 5.7 Equal Area Comparison Walls 7-13
5.2 Modified Versions of Generalized Walls

The modified versions of wall 3 described previously in Section 4.4.1 were created to force the hysteretic response toward the lower bound. The results for walls 3A, 3B and 3C are shown with wall 3 in Figure 5.8 for both initial periods using the modified ground motions. The 2 second initial period results are consistent with the trends observed in the generalized walls. The decreasing area between each of the successive modified walls decreases the amount of hysteretic damping available and results in larger nonlinear displacements and thus lower effective stiffness values. The wall 3C $\alpha(\mu)$ values do not start around 1.0 like the others since wall 3C only briefly follows the initial stiffness before being dominated by the lesser slope of the lower bound.

The 4 second initial period results of modified versions of the generalized walls are far more extreme than those observed for the generalized walls. Providing low $V_L/V_N$ values with only marginally higher upper bounds results in walls that spend little time at initial stiffness before cracking and then spend the majority of the time along their low secondary stiffness. This results in walls that quickly reach full yielding and have little capacity for hysteretic damping; this combination leads to large displacements even for small $R$ values especially considering the fact that $R$ is based on a projection of the initial stiffness reaching yield capacity. These modified walls quickly reached the displacements at which the generalized walls started to have level $\alpha(\mu)$ values. The extreme nature of wall 3C exacerbates the levelling phenomenon observed for an initial period of 4 seconds and larger $R$ effective values to a point where the $\alpha(\mu)$ values are actually increasing.

5.3 Idealized Walls

The three idealized walls described previously in Section 4.4.2 were included in this study to provide a basis of comparison for other studies. Walls 14 and 15 approximate nonlinear-elastic behaviour since the material model $ADKplateau$ unloading point $V_d$ was set only slightly below yield capacity $V_N$. Wall 16 uses an elastic-perfectly-plastic material model.
Figure 5.8 Modified Versions of Generalized Walls
Figure 5.9 Idealized Wall With Approximate Nonlinear-Elastic Behaviour
The results for walls 14 and 15 are shown in Figure 5.9 for both initial periods using the modified ground motions. The trends of each initial period are very similar to those observed for wall 13, which has the highest level of axial load and lowest stiffness reduction $\alpha(\mu)$ values of the generalized walls. Low $\alpha(\mu)$ values indicate that the maximum nonlinear-elastic displacements are larger than the displacement of an elastic system with the same initial stiffness. The same effect was found as part of the brief discussion of nonlinear-elastic behaviour included in FEMA 440 (ATC, 2005). FEMA 440 states that nonlinear-elastic behaviour is not often found in typical structural systems and that all systems exhibit some hysteretic damping. The results of the current study indicate walls with high axial load have little hysteretic damping and approach nonlinear-elastic behaviour.

Walls 14 and 15 were also used to investigate the effects of strain hardening. Wall 14 has less of a strain hardening slope than wall 15. This resulted in a consistently lower stiffness reduction $\alpha(\mu)$ for wall 14 at each initial period. The trend of $\alpha(\mu + \sigma)$ closely followed the trend of $\alpha(\mu)$ suggesting that increasing the strain hardening has little effect on the data other than shifting the stiffness reduction $\alpha(\mu)$ upwards. The upward shift in effective stiffness is relatively small compared to the larger changes observed between the generalized walls and especially large changes due to initial period.

The results for wall 16 are shown in Figure 5.10 for both initial periods using the modified ground motions. The $\alpha(\mu)$ values are greater than or equal to 1.0 indicating that the average elastic-perfectly-plastic displacements are consistently less than or equal to the average elastic displacements. Similar behaviour was observed for periods greater than 2 seconds for the El Centro ground motion in the description of elastoplastic idealization presented by Chopra (2001). An elastic-perfectly-plastic system does not prove to be a very good representation of the hysteretic response of high-rise concrete shear walls.
5.4 Summary of Results

The figures presented in this section thus far have consisted of data points that are the mean stiffness reduction $\alpha(\mu)$, or mean plus one standard deviation stiffness reduction $\alpha(\mu + \sigma)$, calculated from the nonlinear displacements of forty ground motions for nineteen $R$ values. The thirteen generalized structures represent a wide range of typical high-rise concrete shear walls. The values of $\alpha(\mu)$ and $\alpha(\mu + \sigma)$ are remarkably similar considering the wide range of walls from which they were derived. In the interest of determining the stiffness reduction of an “average wall”, the mean of the $\alpha(\mu)$ values for the thirteen walls was calculated at each interval of $R$. The stiffness reduction of an “average wall” is presented in for modified and unmodified ground motions for both initial periods.
In the interest of preliminary comparison, the definition of $\alpha = I_e/I_g$ and $\alpha = k_e/k_i$ will be considered equivalent. The value of $0.7I_g$ (or $\alpha = 0.7$) that is typically used in response spectrum analysis to account for cracking falls comfortably across the “average wall” data. The 4 second initial period values that flatten out for larger $R$ values could be approximated by $0.8I_g$ (or $\alpha = 0.8$). The simple average of the average technique suggests that current practice is reasonable and may even be conservative for longer period structures. Complete understanding of the flattening phenomenon observed for the 4 second initial period and a more thorough investigation of $\alpha(\mu)$ values for the generalized walls is required before a design recommendation can be made. The relationship between $k_e/k_i$ and $I_e/I_g$ should be derived by first principles before a direct comparison between the preliminary results ($\alpha$ values defined as $k_e/k_i$) and the existing recommendations is made.
6.0 Conclusions and Recommendations for Future Work

The force-displacement hysteretic model presented in Chapter 2 was developed using the experimental results of a large scale test specimen conducted by Adebar, Ibrahim and Bryson (2000) to specifically study high-rise concrete shear walls. The hysteretic model was incorporated into OpenSees (PEER, 2005) and a series of nonlinear time history analyses were conducted for two sets of ground motions, two initial periods, nineteen $R$ values and thirteen generalized walls as described in Chapter 3. The set of generalized walls presented in Chapter 4 represents the range of force-displacement curves for typical reinforced concrete shear walls. The results of the nonlinear analyses were used to determine the effective stiffness of an elastic system producing the same maximum displacement as the hysteretic model. The effective stiffness of each of the studied walls, quantified as a reduction factor $\alpha$ of the initial stiffness, were presented in Chapter 5. The accomplishments achieved through the current study and the conclusions it offers are summarized in this final chapter along with recommendations of future work.

6.1 Accomplishments

The first objective of the study was to investigate whether using equal area bilinear approximations of the force-displacement response is the right approach in determining upper and lower bound effective stiffness values. This method was found to have limited success for the wide range of generalized walls studied. The difference in area between the upper and lower bounds appears to be a more important consideration when conducting nonlinear time history analyses rather than the area of the upper and lower bounds alone.

The second objective was to investigate the transition between the uncracked response (upper bound) and the severely cracked response (lower bound) of high-rise concrete shear walls. This was accomplished by developing the force-displacement hysteretic model. The model is based off data obtained from the physical testing of a specimen specifically built to study high-rise concrete shear walls. Through using such system specific data, the model is uniquely tailored to the response of this structural system. The model allows for a transition between an uncracked wall and a severely cracked wall and is an integral part in understanding the seismic response of high-rise concrete walls.
The final objective of the current study was to develop the tools needed to permit the nonlinear seismic analysis of high-rise concrete shear walls accounting for the transition between the uncracked and severely cracked response. The major accomplishments achieved in order to meet this objective are listed below:

- Development of the MATLAB script Mphi to perform sectional analysis using an appropriate tension stiffening model for high-rise concrete walls.
- Development of simplified force-displacement approximation.
- Implementation of force-displacement hysteretic model into OpenSees (PEER, 2005).
- Development of a procedure to find effective stiffness of a linear system based off nonlinear displacement predicted by the hysteretic model.
- Development of a set of generalized walls to represent a full range of typical high-rise concrete shear walls.
- Performance of a preliminary nonlinear analysis using the developed tools.

Computer programs such as Response-2000 (Bentz, 2000) are available to perform sectional analysis of reinforced concrete members. The tension stiffening model used by Response-2000, however, is not as effective as other models in predicting the moment-curvature response of high-rise concrete walls. The MATLAB script Mphi uses an appropriate stiffening model for high-rise concrete walls and can produce detailed predictions of the moment-curvature response at user defined intervals.

The force-displacement response calculated using the Adebar and Ibrahim (2002) trilinear model alone over-predicts the theoretical response found by integrating the theoretical nonlinear moment-curvature response. The simplified force-displacement approximation uses the Adebar and Ibrahim trilinear model and several common calculated sectional properties to produce an estimate of the theoretical response. The approximation provides a good estimate of the theoretical force-displacement response and eliminates the need to perform a nonlinear moment-curvature analysis and subsequent numerical integration.

Implementation of the force-displacement hysteretic model into an analysis program such as OpenSees enables nonlinear time history analyses to be conducted. Existing hysteretic models available in OpenSees, or other commercially available software, do not have the capability to
model the behaviour specific to high-rise concrete shear walls in as much detail as the ADK hysteretic model.

The development of a procedure to find effective stiffness of an elastic system based off the nonlinear displacement predicted by the hysteretic model allows for the results of a complex analysis to be compared with a simple well known model. Matching nonlinear displacements along the design spectrum to find an equivalent elastic system is most useful to designers and has the most potential in developing future design recommendations.

The development of a set of generalized walls to represent a full range of typical high-rise concrete shear walls allows for a simple way to investigate how the relative shapes of the force-displacement curves affect nonlinear displacements. The use of generalized walls allows for the greatest understanding of the range of typical walls through conducting a minimal amount of nonlinear analyses.

Performance of a preliminary nonlinear analysis using the developed tools demonstrates their capabilities. The results provide an indication of how the parameters of initial period, $R$ value, and shape of the force-displacement curve influence the effective stiffness of walls. These preliminary results also indicate how the parameters of the study can be further refined to offer more insight into the response of high-rise concrete shear walls.

6.2 Conclusions
The most significant conclusion from the preliminary analysis is that the effective stiffness relationships of each wall are very similar considering the wide range of generalized walls that were studied. The variation in effective stiffness was relatively small for the 2 second and 4 second initial periods. Significant variation however was observed between the initial periods of 2 and 4 seconds. Both initial periods experienced very small stiffness reductions for low $R$ values, however, for larger $R$ values the effective stiffness for the 2 second initial period dropped quickly, while the 4 second initial period remained relatively constant. Wall 13 consistently had the lowest effective stiffness values for each initial period; this wall has the least amount of area between the upper and lower bound and is least capable of hysteretic damping. For walls with the same $V_L/V_N$ value, the effective stiffness was greatest for the wall with the largest area between the upper and lower bounds.
Determining effective stiffness using a bilinear curve with the same area-under-the-curve as the actual nonlinear upper and lower bound force-displacement curves had limited success in estimating the upper and lower bound effective stiffness. The effective stiffness of a structure is a function primarily of initial period and secondarily $V_L/V_N$ and difference in area between the upper and lower bound rather than the area of the upper and lower bounds alone.

The value of $0.7I_\epsilon$ typically used in response spectrum analysis to account for cracking appears to be a reasonable approach when compared to the generalized wall data. The 4 second initial period values that flatten out for larger $R$ values could be approximated by $0.8I_\epsilon$. Complete understanding of the flattening phenomenon observed for the 4 second initial period and a more thorough investigation of $a(\mu)$ values for the generalized walls is required before a design recommendation can be made.

6.3 Recommendations for Future Work

The greatest need for further work is in fully understanding the levelling of effective stiffness reduction $a(\mu)$ values observed for a 4 second initial period. Two possible explanations for this phenomenon have been suggested. First, the walls enter the equal displacement portion of the spectrum and the constant $a(\mu)$ values correspond to equal displacements. Second, the frequency content of the ground motions are not rich enough to adequately study this longer period range. The equal displacement concept can be explored by studying additional initial periods between 2 and 4 seconds along with several initial periods after 4 seconds. If the equal displacement concept is in effect the $a(\mu)$ values should transition gradually between 2 and 4 seconds. The frequency content possibility can be explored by using a different set of ground motions with richer frequency content for long periods. The ground motions used in the current study are for crustal earthquakes. If the same levelling effect is not observed when using a suite of subduction earthquakes then lack of frequency content would be the driving force behind the $a(\mu)$ levelling for a 4 second initial period.

The current study used the maximum nonlinear displacement predicted by the hysteretic model to find the stiffness reduction required for an elastic system to achieve the same displacement. The use of a single maximum value to describe the behaviour of the entire nonlinear time history
analysis may miss key information about the response of the wall. It would be very interesting to compare the complete time histories of all thirteen generalized walls for several different ground motions. This would provide a detailed picture of which portion of the force-displacement curves would be most visited by the walls for a given ground motion.

The preliminary results of the current study are presented as stiffness reduction factor $\alpha$ (or $k_s/k_i$) values while the existing recommendations are for effective rigidity $I_e/I_g$. Intuitively, the values of $k_s/k_i$ and $I_e/I_g$ should be approximately equal for a given wall. This relationship should be derived by first principles before a direct comparison between the preliminary results and the existing recommendations is made. Several of the existing recommendations allow for an increase in flexural rigidity for higher axial loads. The preliminary results obtained through nonlinear time history analysis, however, suggest that high axial load reduces the hysteretic damping capacity which increases nonlinear displacements and thus decreases effective stiffness, contrary to the recommendations. Consequently, the influence of axial load on effective stiffness needs to be examined in greater detail.

The greatest uncertainty of the hysteretic force-displacement model is in the unloading portion. The primary focus of the experimental work conducted by Adebar, Ibrahim and Bryson (2000) was to determine the loading portion of the moment-curvature response to verify the Adebar and Ibrahim (2002) trilinear model. Experimental work focused on the unloading moment-curvature response would allow for refinement of the presented hysteretic force-displacement model.
REFERENCES


APPENDIX A

Modified Ramberg-Osgood Function
The modified Ramberg-Osgood function was developed by Mattock (1984) for the stress-strain relationship of prestressed reinforcing steel. The function transitions between two slopes as shown below.

\[ f_p = E_p \varepsilon_{fp} \left[ A + \frac{1 - A}{(1 + (B \varepsilon_{fp})^C)^\frac{1}{C}} \right] \]

where  
- \( f_p \) = prestressed reinforcement stress  
- \( \varepsilon_{fp} \) = prestressed reinforcement strain  
- \( E_p \) = prestressed reinforcement modulus of elasticity  
- \( A \) = factor determined from second slope  
- \( B \) = intercept of second slope on stress axis  
- \( C \) = curve fitting factor (higher values cause more abrupt transition)
APPENDIX B

Hysteretic Model and Experimental Data
The hysteretic force-displacement model presented in Section 2.6 was developed using experimental data from Adebar, Ibrahim and Bryson (2000) obtained by testing a large scale wall specimen at the University of British Columbia. Testing consisted of loading the wall to a specified displacement in the “positive” direction, unloading and then reloading to the same specified displacement in the “negative” direction. This cycle was repeated four times for each incrementally larger specified displacement until the wall became severely damaged. The experimental force-displacement results are shown below.

The experimental data was gathered continuously over the four loading/unloading cycles for each specified displacement. To assist in development of a hysteretic model, the data was separated into positive loading, positive unloading, negative loading and negative unloading segments. The data segments were regrouped by the maximum positive/negative displacement experienced. This means that the last three cycles for a specified displacement are grouped with the first cycle of the next largest specified displacement.

The following diagrams display the hysteretic force-displacement model with the experimental data with drift ratios ranging from 0.3% to 1.5%. The model is first compared for displacements in the positive direction and then for displacements in the negative direction. The experimental data and model predictions are shown by solid lines for loading and dashed lines for unloading.
Previous Maximum $\Delta=-38$mm
$\Delta / h_w=0.3\%$

Previous Maximum $\Delta=-48$mm
$\Delta / h_w=0.4\%$

Previous Maximum $\Delta=-63$mm
$\Delta / h_w=0.5\%$

Previous Maximum $\Delta=-76$mm
$\Delta / h_w=0.6\%$
Previous Maximum $\Delta = 103\text{mm}$
$\Delta / h_w = 0.9\%$

Previous Maximum $\Delta = 138\text{mm}$
$\Delta / h_w = 1.1\%$

Previous Maximum $\Delta = 186\text{mm}$
$\Delta / h_w = 1.5\%$
APPENDIX C

Pilot Analysis Displacement Ratio Plots
The displacement spectra determined through the pilot analysis were obtained by conducting elastic analyses and bilinear with constant strength analyses by reducing the initial stiffness (increasing period) as described in Section 3.2. The displacement spectra were normalized by the displacement predicted with the hysteretic force-displacement model. The normalized spectra, or displacement ratio plots, are first presented for a 2 second initial period and then for a 4 second initial period. The point at which the line for a particular earthquake and analysis type crosses the displacement ratio of 1.0 indicates the period needed to get the same displacement estimate as the nonlinear model. The results are shown for the unmodified ground motions (left) and the modified ground motions (right). The $R$ values 1.5, 2.0, 2.5, 3.0, 4.0 and 5.0 are studied for each initial period.
APPENDIX D

Derivation of $\Delta_{yLB} / \Delta_g$ Expression
The displacement at top of wall for an uncracked wall with first mode load distribution is shown below. This expression can be derived from first principles.

\[ \Delta_s = 0.28h_w^2 \frac{M_N}{E_cI_g} \]

where

- \( h_w \) = height of wall
- \( M_N \) = nominal flexural capacity of wall
- \( E_c \) = concrete modulus of elasticity
- \( I_g \) = second moment of inertia of the gross concrete section

The bending moment diagram for a cantilever wall with first mode load distribution is shown in the figure below. The lower portion of the diagram can be reasonably approximated by a triangle having the same maximum moment at the base and a height 75% of the wall height \( h_w \). The displacement at top of wall for a linear wall with first mode load distribution is shown by the previous equation. The additional nonlinear displacement is found by idealizing the additional curvature due to cracking as a second triangle shown in the figure below and then integrating with the second-moment-area theorem.
Simplify expression for level arm of triangle used as part of second-moment-area theorem.

\[ h_w - 0.75h_w \left( \frac{M_N - M_L}{M_N} \right) \left( \frac{1}{3} \right) = h_w \left( 1 - 0.25 \left( \frac{M_N - M_L}{M_N} \right) \right) \]

Simplify expression for area of triangle used as part of second-moment-area theorem.

\[ \frac{1}{2} \left[ 0.75h_w \left( \frac{M_N - M_L}{M_N} \right) \left( M_N - M_L \right) \left( \frac{1}{E_c I_{cr}} - \frac{1}{E_c I_g} \right) \right] = \frac{1}{2} \left[ 0.75h_w \left( \frac{(M_N - M_L)^2}{M_N} \right) \left( \frac{1}{E_c I_{cr}} - \frac{1}{E_c I_g} \right) \right] \]

Formulate expression for \( \frac{\Delta_{sLB}}{\Delta_g} \) using second-moment-area theorem.

\[ \frac{\Delta_{sLB}}{\Delta_g} = \frac{0.28h_w^2 \frac{M_N}{E_c I_g} + \frac{1}{2} \left[ 0.75h_w \left( \frac{M_N - M_L}{M_N} \right) \left( \frac{1}{E_c I_{cr}} - \frac{1}{E_c I_g} \right) \right] h_w \left( 1 - 0.25 \left( \frac{M_N - M_L}{M_N} \right) \right)}{0.28h_w^2 \frac{M_N}{E_c I_g}} \]

Divide by denominator and factor out \( h_w^2 \).

\[ \frac{\Delta_{sLB}}{\Delta_g} = 1 + \frac{E_c I_g}{2(0.28)M_N} \left[ 0.75 \left( \frac{M_N - M_L}{M_N} \right)^2 \left( \frac{1}{E_c I_{cr}} - \frac{1}{E_c I_g} \right) \right] \left( 1 - 0.25 \left( \frac{M_N - M_L}{M_N} \right) \right) \]

Combine \( E_c I_g \) and \( \frac{1}{M_N} \) with first bracketed term to simplify expression.

\[ \frac{\Delta_{sLB}}{\Delta_g} = 1 + \frac{0.75 \left( \frac{M_N - M_L}{M_N} \right)^2 \left( \frac{E_c I_L}{E_c I_{cr}} - 1 \right) \left( 1 - 0.25 \left( \frac{M_N - M_L}{M_N} \right) \right)}{2(0.28)} \]
Expand last bracketed term.

\[
\frac{\Delta_{sLB}}{\Delta_g} = 1 + \frac{0.75}{2(0.28)} \left( \frac{M_N - M_L'}{M_N} \right)^2 \left( \frac{E_c I_g}{E_c I_{cr}} - 1 \right) \left( 1 - 0.25 \left( 1 - \frac{M_L'}{M_N} \right) \right)
\]

\[
\frac{\Delta_{sLB}}{\Delta_g} = 1 + \frac{0.75}{2(0.28)} \left( \frac{M_N - M_L'}{M_N} \right)^2 \left( \frac{E_c I_g}{E_c I_{cr}} - 1 \right) \left( 1 - 0.25 \frac{M_L'}{M_N} \right)
\]

Simplify last bracketed term.

\[
\frac{\Delta_{sLB}}{\Delta_g} = 1 + \frac{0.75}{2(0.28)} \left( \frac{M_N - M_L'}{M_N} \right)^2 \left( \frac{E_c I_g}{E_c I_{cr}} - 1 \right) \left( 0.75 - 0.25 \frac{M_L'}{M_N} \right)
\]

\[
\frac{\Delta_{sLB}}{\Delta_g} = 1 + \frac{0.75}{2(0.28)} \left( \frac{M_N - M_L'}{M_N} \right)^2 \left( \frac{E_c I_g}{E_c I_{cr}} - 1 \right) 0.25 \left( 3 - \frac{M_L'}{M_N} \right)
\]

Simplify expression.

\[
\frac{\Delta_{sLB}}{\Delta_g} = 1 + 0.33 \left( \frac{M_N - M_L'}{M_N} \right)^2 \left( \frac{E_c I_g}{E_c I_{cr}} - 1 \right) \left( 3 - \frac{M_L'}{M_N} \right)
\]

\[
\frac{\Delta_{sLB}}{\Delta_g} = 1 + \left( \frac{M_N - M_L'}{M_N} \right)^2 \left( 1 - 0.33 \frac{M_L'}{M_N} \right) \left( \frac{E_c I_g}{E_c I_{cr}} - 1 \right)
\]

(As shown in Equation 4.2)
APPENDIX E

Electronic Appendix Contents
An electronic appendix accompanies this thesis due to the computationally intense nature of the work conducted. The contents of each file folder included on the compact disk are listed below:

**E01 - Fdisp**

The *MATLAB* scripts *Fdisp-Theoretical.m* and *Fdisp-Trilinear.m* are used to numerically integrate over the wall height to calculate points along the force-displacement curve as described in Section 2.3.2. The script *Fdisp-Theoretical.m* interpolates between the theoretical moment-curvature points output from *Mphi* as part of the integration process. The script *Fdisp-Trilinear.m* interpolates between the trilinear points calculated from the Adebar Ibrahim model. Both scripts are set up to study the three analysis walls described in Section 2.1.

**E02 - Mphi**

The *MATLAB* script *Mphi.m* used to determine the moment-curvature response of a wall by sectional analysis is located in this folder. The function of this program is described in detail by the *Mphi program description.doc* Word file. The *MATLAB* script is set up to study the three walls, each with four levels of reinforcement, used to determine an appropriate range of generalized wall parameters.

**E03 - UBC Wall Data**

The raw data from the experimental test conducted by Adebar, Ibrahim and Bryson is included in *wall test results.xls*. The raw data has been separated into individual loading and unloading cycles in *load unload data.xls*.

**E04 - OpenSees**

The *OpenSees* source code for the material models implemented as part of the thesis are located in this folder. All of the implemented material models can be viewed using the Uniaxial Material Viewer. A compiled version of *OpenSees* included the implemented material models is also located in this folder.
E05 - Raw Ground Motions
The raw ground motions used as part of FEMA 440 are organized by site class in this folder. The time step for some of these ground motions have been adjusted so all ground motions have a common time step of 0.01 seconds. The time step adjusted ground motions are in individual text files and are also summarized in Excel files for each site class.

E06 - SYNTH
The computer program SYNTH was used to modify ground motions to match a modified version of the NBCC design spectrum described in Section 3.2.2. The way in which SYNTH modifies the ground motions is described briefly in Section 3.2.2 and in greater detail by the SYNTH user’s manual included in this folder. The input file SDRS.DAT specifies the period and pseudo acceleration spectral values of the altered NBCC design spectrum. The MATLAB script openseesgm.m takes the single column of accelerations output by SYNTH and transfers them into rows of five accelerations as required by OpenSees.

E07 - Spectra
The 3% Rayleigh damping displacement spectra for the modified and unmodified ground motions are included in this folder along with the modified NBCC design spectrum described in Section 3.2.2. The three Excel files include the data for each displacement spectrum and plots of each spectrum. The mean and mean plus one standard deviation displacement spectra for the modified and unmodified ground motions are also plotted. Three MATLAB scripts are included that strip the data from the Excel files and write it to individual files for each of the ground motions and for the design spectrum.

E08 - ADKhysPar
The MATLAB script ADKhysPar.m used to calculate the ADK hysteretic material model parameters for a given wall is located in this folder. This script uses the sectional properties of the wall and a trilinear moment-curvature relationship to numerically integrate over the wall height to find the three force-displacement points needed to define the 4th order polynomial constants. The numerical integration process is the same as MATLAB script Fdisp described in Section 2.3.2. The script ADKhysPar.m is set up to study the UBC wall variations described in Section 4.1. This script also calculates the transformed section properties of the UBC wall variations.
E09 - Generalized Walls

The *Excel* files used to determine the walls studied in the nonlinear analysis are located in this folder. The file *Interaction plots.xls* studies the UBC wall and three other walls, each with four levels of reinforcement, for a range of axial loads to determine an appropriate range of generalized wall parameters. The generalized wall parameters are used to create the force-displacement inputs for *ADKhystetric* material model in *Generalized Walls.xls*. This process is described in detail by *Generalized Walls EXAMPLE.xls*.

E10 - Nonlinear Analysis

The results of the nonlinear analysis for the 19 walls for both the modified and unmodified ground motions, and all *Tcl* files necessary to perform the analysis, are located in this folder. Instructions are given on how to the *OpenSees* analysis files are set up to perform the analysis by the *OpenSees Analysis Instructions.doc* Word file. The results of the nonlinear analysis are post processed in four *Excel* files. The post processing is described in detail by the *Combined Average EXAMPLE.xls* Excel file.

E11 - Thesis

An electronic copy of the thesis *Korchinski - Investigation of Effective Stiffness of High-rise Concrete Walls.pdf* is included complete with colour figures.