NON-LINEAR WAVE FORCES ON FLOATING BREAKWATERS

by

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Abstract

A non-linear numerical method of calculating wave forces on floating bodies is investigated. A computer program has been developed that will time step a given wave past a fixed two-dimensional rectangular breakwater. The effect of various input parameters on the accuracy of results is investigated, and optimal values of the parameters are determined. The required initial condition for the time stepping procedure has a specified incident wave attenuated to zero flow in the vicinity of the body. Incident waves predicted by both linear wave theory and Stokes' fifth order wave theory are tested. The decay length and group velocity of the incident wave train are varied. It is found that a decay length of approximately one wavelength is optimal, and that the group velocity of the developed flow is relatively independent of the initially specified group velocity. Different time step sizes, segment lengths, and time stepping equations are investigated. The tests conducted indicate that twenty-five time steps and ten segments per wave period are sufficient for accurate results. The Adams-Bashforth three step method is found to be the preferred time stepping technique. Forces on the fixed body and transmitted wave heights are obtained from the program. The results compare well with previously published results and clearly demonstrate the non-linearity of the method, with different force curves resulting from varying wave heights.

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NOMENCLATURE

a_{ij}, b_{ij}	=	matrix elements
A	=	immersed cross-sectional area of body
A_{ij}	=	matrix element
$[A]$	=	coefficient matrix
B	=	beam of breakwater
c_i	=	value of Green's identity numerically integrated on control surface
d	=	still water depth
D	=	draught of breakwater
$\underline{F}$	=	force vector acting on body
F_x	=	x-component of $\underline{F}$
F_z	=	z-component of $\underline{F}$
g	=	acceleration due to gravity
G	=	Green's function
H	=	incident wave height
H_t	=	transmitted wave height
i, j	=	segment indices
k	=	$2\pi/L$ = incident wave number
K_t	=	transmission coefficient
L_b	=	incident wave length
L_d	=	body dimension
L	=	decay length of incident wave
M	=	moment about x-z origin acting on body
n	=	direction normal to surface
n_x, n_z	=	direction cosines of $\underline{n}$
$\underline{n}$	=	unit normal vector directed outward from fluid region

$\underline{n}'$	= unit normal vector directed outward from the fluid region reflection in the seabed
N	= number of segments on $S_f + S_b$
N'	= number of segments on $S_f + S_b + S_c$
N_b	= number of segments on S_b
N_c	= number of segments on S_c
N_f	= number of segments on S_f
p	= pressure
r	= distance between $\underline{x}$ and $\underline{\xi}$
r'	= distance between $\underline{x}$ and $\underline{\xi}'$
S	= a closed surface
S_b	= immersed body surface
S_c	= control surface
S_f	= free surface
t	= time
T	= wave period
x	= horizontal coordinate measured in direction of wave propagation from midpoint of body beam
$\underline{x}$	= vector of point (x, z)
z	= vertical coordinate measured upwards from still water level
ΔS	= segment length
ΔS_b	= segment length on S_b
Δt	= time step
ϕ	= velocity potential
$\phi^{(w)}$	= velocity potential of incident wave
η	= free surface elevation above still water level
ω	= incident wave angular frequency
γ	= angle between $\underline{x} - \underline{\xi}$ and $\underline{n}$

γ' = angle between $\underline{x}-\underline{\xi}'$ and $\underline{n}'$
 ξ = x-component of $\underline{\xi}$
 $\underline{\xi}$ = vector of point (ξ, ζ)
 $\underline{\xi}'$ = vector of point $(\xi, -[\zeta+2d])$
 ζ = z-component of $\underline{\xi}$
 ψ_j = solution vector element
 $[\psi]$ = solution vector
 λ_i = coefficient vector element
 $[\lambda]$ = coefficient vector

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CHAPTER I: INTRODUCTION

1.1 Introduction

Wave-structure interaction is a topic of major importance in the design of marine structures. The forces generated by surface gravity waves on both fixed and floating structures are needed to predict the motions, stresses, and strains on a given structure. The purpose of this thesis is to examine one method of predicting the forces generated by finite amplitude water waves on a fixed breakwater.

Breakwaters are used to protect harbours and other coastal facilities. By interfering with an incident wave train, a breakwater reduces the wave heights within the protected area. One can construct a fixed structure such as a rubble mound breakwater, or alternately, place a floating breakwater and moor it with cables. The two alternatives are shown schematically in Figure 1.

In certain applications, the choice of a floating type of breakwater offers some advantages over the rubble mound type of breakwater. If one wants to effectively protect a specified marine area from a full range of long and short period waves, then a fixed rubble mound type of breakwater is usually necessary. However, if one is mainly concerned with waves of shorter periods, then a floating

breakwater is usually a viable alternative which is often cheaper, particularly in deeper water where a fixed rubble mound type of breakwater requires large amounts of material. In addition to the economic advantage, a floating breakwater interferes much less with sediment transport and marine life migration than does a rubble mound breakwater.

Considerations in designing a floating breakwater include the size, shape, construction, configuration, and connections between its units. The behaviour of the floating breakwater under the action of waves must be predicted. The forces generated by the waves on the breakwater, or exciting forces, can be predicted using experimental or analytical force results. Using the exciting forces with the system's dynamic properties (added mass and damping coefficients) and the mooring system properties, one can obtain predictions for the motions of the breakwater, the stresses and strains on the structure and mooring system, and the transmitted wave heights expressed as transmission coefficients. A flow chart of this analytical procedure is shown in Figure 2.

The general approach described above for solving the floating breakwater problem is not unique. Other solutions using different approaches exist and are briefly referred to in Section 1.2. The method used in this thesis to predict the exciting forces is set apart from other currently used techniques by the fact that it is a non-linear method. In addition, it must be stressed that the solution discussed

here is for a monochromatic wave approaching normal to the length of the breakwater. The relevance of this approach to actual field conditions must be considered when applying the results obtained here to a given design.

For purposes of this thesis, we restrict ourselves to a breakwater having a two-dimensional cross-section and investigate the exciting forces generated by a beam wave train. We use a time stepping procedure developed by Isaacson⁽¹⁾ which uses a Green's boundary integral identity combined with the usual governing equations and boundary conditions to time step an incident wave train past a fixed rectangular cross-section. More specifically, we investigate the limitations of the method, varying parameters such as the time step size, segment length, incident wave form, and time stepping equation. Force results on a fixed breakwater are obtained using the method and are compared to existing experimental results.

The forces are obtained from an analysis that assumes a motionless body. The body motions and actual transmission coefficients are not investigated. However, the method does provide a lower limit for the transmission coefficient, and hence an indication of the relative values of the transmission coefficient. The effectiveness of reducing wave heights could also be investigated for various breakwater cross-sections, but this is not done here.

1.2 Survey of Literature

Much has been written on the topic of floating breakwaters. Topics discussed have included the advantages and disadvantages (Joly⁽²⁾ and Adee⁽³⁾), case studies and operational experience (Adee⁽³⁾, and Miller⁽⁴⁾), experimental evaluation of performance (Brebner & Ofuya⁽⁵⁾, Hom-ma, Horikawa, & Mochizuki⁽⁶⁾, Skey⁽⁷⁾, Sutko⁽⁸⁾, and Harris & Webber⁽⁹⁾), and the theoretical prediction of performance (Mei & Black⁽¹⁰⁾, Longuet-Higgins⁽¹¹⁾, Nichols & Hirt⁽¹²⁾, and Ijima, Chou, & Yoshida⁽¹³⁾).

Recently, with the advent of the computer as an aid to methods involving lengthy computations, numerical methods have evolved that were previously impractical (if not impossible) due to the large volume of calculations. Numerical methods have been used by Mei & Black⁽¹⁰⁾, Nichols & Hirt⁽¹²⁾, Garrison⁽¹⁴⁾, and Fraser⁽¹⁵⁾, to obtain forces and motions of floating breakwaters. Boundary integral methods based on Green's identity have been used by Ijima & Chou⁽¹⁶⁾, and Ijima, Yoshida, & Yamamoto⁽¹⁷⁾. All of the methods referred to thus far are linear. The non-linear boundary integral method used in this thesis was developed by Isaacson⁽¹⁾. The results for the forces on a fixed breakwater obtained here are compared to those of Fraser⁽¹⁵⁾, and Vugts⁽¹⁹⁾.

1.3 General Description of Method

The method is based on the usual governing equations. Laplace's equation, the kinematic and dynamic free surface boundary conditions, and the seabed and body surface boundary conditions must all be satisfied. These equations are given in Chapter II. In order to consider a closed boundary over which the Green's identity used can be integrated, we choose vertical control surfaces some distance away from the body, and reflect about the seabed the bounding surface defined by the control surfaces, free surface, and body surface. The resulting closed surface is shown in Figure 3.

The problem is solved by first specifying an incident wave using a chosen wave theory in a manner that prescribes zero flow in the vicinity of the body. Using an appropriate wave theory, we specify the incident flow in terms of a wave height and period which define the initial flow potential and outward normal derivative of the potential along the entire closed boundary. The chosen wave theory also defines the free surface profile along the free surface on the incident side of the body. The control surfaces are placed sufficiently distant so that the scattered wave potential due to the interaction of the prescribed incident wave with the body does not have enough time to propagate to the control surfaces during the total flow development time chosen. Hence, we can prescribe values of the potential and outward normal derivative of the potential along the control

surfaces throughout the time stepping procedure, according to our chosen wave theory.

The free surface, body surface, and control surfaces are divided into an appropriate number of segments. Segment midpoint values of the potential and its outward normal derivative are assumed to be constant over each segment length. The kinematic and dynamic free surface boundary conditions, together with the Green's identity applied at each midpoint, are used to obtain new values for the potential and its normal derivative as well as the free surface elevation at the end of each time step. From the variation in potential values between adjacent segments, the fluid velocities can be approximated. Using Bernoulli's equation, the fluid velocities along the body surface give the pressures acting on each segment, and the pressures are numerically integrated to give the total forces acting on the body. We thus obtain the hydrodynamic forces acting on the body at each time step.

The time stepping procedure begins by specifying an incident wave flow for the first two or more time steps, and continues until sufficient time has elapsed for at least one complete wave to interact with the body. A full development of the theory and method is given in Chapters II and III.

1.4 Areas of Investigation

In this thesis, we examine various aspects of the time stepping procedure used. The effects of time step size and segment length on the accuracy of results is investigated. The time step size is treated in the dimensionless time step to wave period ratio, $\Delta t/T$, while the segment length is similarly treated as the segment length to wave length ratio, $\Delta S/L$. By varying the two parameters over an appropriate range for a given test case, we identify the desirable values of both $\Delta t/T$ and $\Delta S/L$ that will ensure good accuracy within the constraints of the method itself, and we verify the condition on the method stated by Isaacson, that $\Delta t/T \ll \Delta S/L$.

The global segment length parameter $\Delta S/L$ is limited both by the condition that $\Delta t/T \ll \Delta S/L$ and the need to have enough segments on the free surface to approximate the free surface profile. In addition, there must be enough segments on the body surface to allow reasonably accurate numerical integration of the total forces on the body from the pressure distribution defined by the velocities at each segment on the body surface. This minimum segment number requirement on the body surface, expressed as a body segment length to body dimension ratio, $\Delta S_b/L_b$, is determined.

The kinematic and dynamic free surface boundary conditions are used to obtain time stepping equations for the free surface elevation and the potential respectively. There are a number of techniques available that will

explicitly extrapolate values at $t+\Delta t$ from values at previous times given a defining equation for the time derivative of a given parameter. These numerical methods are described in Burden, Faires, & Reynolds⁽²⁰⁾. We test methods that require anywhere from two to five previous points to extrapolate a forward value, and examine and compare the results from each method.

Different wave theories can be used to describe the incident wave flow. In particular, a higher order theory such as Stokes' fifth order theory is desirable for steeper waves. The results obtained using linear and Stokes' fifth order theories for different wave steepnesses are compared, and the range of validity of each is verified.

When specifying an incident wave train, the necessity for zero flow in the vicinity of the body requires that a conventional harmonic wave train be decayed to zero flow near the body when specifying the precise incident wave form. A typical incident wave form to be used is shown in Figure 4. The length over which the decay of the incident wave occurs is examined. The decay length is treated as a multiple of the wavelength, and is designated L_d as shown in Figure 4. The effect of varying the different parameters mentioned above is investigated and discussed in Chapter IV.

After determining the constraints on the various parameters mentioned above, we obtain the forces on a fixed body and plot them as a function of frequency, all quantities being expressed in dimensionless form. The force

results are compared to previously published experimental results. Since the method is non-linear, force result curves are plotted for different values of H/d .

Calculating the exciting forces on the floating body by treating it as fixed also provides a lower limit on the transmission coefficient. By recording the wave heights on the side of the body opposite to the incident wave train, a transmission coefficient is obtained that provides a reliable lower limit for the actual coefficient value (which depends on the motions of the body). This transmission coefficient lower limit is compared to experimental results published by Nece & Richey⁽²¹⁾. Although a lower limit to the transmission coefficient is of little practical value, it does provide a relative indication of the effectiveness of various sections in reducing wave heights. The force and transmission coefficient results are presented and discussed in Chapter VI.

CHAPTER II: THEORETICAL DEVELOPMENT

2.1 Governing Equations

For our two-dimensional example, we have a fixed body of rectangular cross-section with beam B , draught D , and floating in water of uniform still water depth d . We attach an x - z coordinate system to the body, with x measured horizontally in the direction of wave propagation and z measured upward vertically from the still water level. The origin is located at still water level midway along the beam of the body. We let η denote the free surface elevation above still water level. The definition sketch corresponding to the above is shown in Figure 5.

The fluid is assumed to be non-viscous and incompressible, with the flow assumed to be irrotational. The fluid motion may thus be represented by the velocity potential ϕ . In the usual manner, it can be shown that ϕ satisfies the Laplace equation within the boundaries of the fluid,

$$\nabla^2 \phi = 0 \quad (1)$$

or, for our two-dimensional case,

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (1a)$$

The seabed is assumed to be horizontal and impermeable. Hence, no flow can occur normal to the seabed. This condition is expressed as

$$\frac{\partial \phi}{\partial z} = 0 \quad \text{at } z = -d \quad (2)$$

The body surface is also assumed to be impermeable, giving

$$\frac{\partial \phi}{\partial n} = 0 \quad \text{on } S_b. \quad (3)$$

Here, n denotes the direction normal to the surface.

The free surface of the fluid is governed by kinematic and dynamic free surface boundary conditions. The kinematic free surface boundary condition requires that the normal velocity of the free surface elevation correspond to the normal flow velocity, that is, that particles at the free surface remain attached to the free surface. This condition is usually expressed in two dimensions as

$$\frac{\partial \phi}{\partial z} = \frac{\partial \eta}{\partial t} + \frac{\partial \phi}{\partial x} \frac{\partial \eta}{\partial x} \quad \text{on } S_f \quad (4)$$

The form of (4) we use for our purposes is one used by Isaacson⁽²²⁾, given by

$$\frac{\partial \phi}{\partial n} = \frac{\partial \eta}{\partial t} n_z \quad \text{on } S_f \quad (4a)$$

Here, t denotes time and n_z is the direction cosine in the z direction of the vector $\underline{n}$, which we define as the unit outward normal to the surface. Similarly, n_x will denote the direction cosine of $\underline{n}$ in the x direction.

The dynamic free surface boundary condition requires that the pressure at the surface be uniform (atmospheric),

$$\frac{\partial \phi}{\partial t} + g\eta + (\nabla \phi)^2 = \text{constant} \quad \text{on } S_f \quad (5)$$

Here, g is the acceleration due to gravity, and the constant is the Bernoulli constant which we set equal to zero for convenience.

2.2 Green's Function Representation

The velocity potential ϕ at any point along a closed two-dimensional fluid boundary can be expressed as follows using a reduction of the second form of Green's theorem,

$$\phi(\underline{x}) = -\frac{1}{\pi} \int_S \left(G(\underline{x}, \underline{\xi}) \frac{\partial \phi}{\partial n}(\underline{\xi}) - \phi(\underline{\xi}) \frac{\partial G}{\partial n}(\underline{x}, \underline{\xi}) \right) dS \quad (6)$$

Here $\underline{x}$ is the point (x, z) being considered, $\underline{\xi}$ is a point (ξ, ζ) on the closed curve in the x - z plane over which the integration is being performed, dr is measured along the closed curve, and G is an appropriate Green's function. The origin of the (ξ, ζ) coordinates is the same as for the (x, z) coordinates.

The seabed boundary condition given by (2) allows us to reflect the points on the boundary defined by S_f, S_b , and S_c about the seabed and obtain a closed curve over which the Green's identity given by (6) can be integrated. This closed curve is shown in Figure 3.

The Green's function which accounts for the symmetry about the seabed is given by

$$G(\underline{x}, \underline{\xi}) = \ln(r) + \ln(r') \quad (7)$$

Here, r is the distance between $\underline{x}$ and $\underline{\xi}$ given by

$$r = |\underline{x} - \underline{\xi}| = \sqrt{(x - \xi)^2 + (z - \zeta)^2} \quad (8)$$

and r' is the distance between $\underline{x}$ and $\underline{\xi}' = (\xi, -[\zeta + 2d])$ (the reflection of $\underline{\xi}$ in the seabed) expressed as

$$r' = |\underline{x} - \underline{\xi}'| = \sqrt{(x - \xi)^2 + (z + \zeta + 2d)^2} \quad (9)$$

A definition sketch of the various coordinates and distances described here is shown in Figure 6. Referring to Figure 6, we obtain

$$\frac{\partial G}{\partial n}(\underline{x}, \underline{\xi}) = \frac{\cos \gamma}{r} + \frac{\cos \gamma'}{r'} \quad (10)$$

Here, γ is the angle between a vector from $\underline{x}$ to $\underline{\xi}$ and the vector $\underline{n}$, given by

$$\cos \gamma = \frac{\underline{n} \cdot (\underline{\xi} - \underline{x})}{r} \quad (11)$$

and γ' is the angle between a vector from $\underline{x}$ to $\underline{\xi}'$ and the vector $\underline{n}'$, given by

$$\cos \gamma' = \frac{\underline{n}' \cdot (\underline{\xi}' - \underline{x})}{r'} \quad (12)$$

2.3 Control Surface Conditions

As mentioned in Section 1.3, the vertical control surfaces to the right and left sides of the body cross-section are placed sufficiently distant to ensure that the scattered waves from the body do not reach the control surfaces during the entire time stepping procedure. If, as we shall assume, the incident wave train propagates from left to right, and the initial conditions specify zero flow in the vicinity of the body, then the right side control surface will have zero flow across it throughout the entire time stepping procedure. The left control surface will have flow conditions specified by the chosen incident wave theory,

$$\left. \begin{aligned} \phi &= \phi^{(w)} \\ \frac{\partial \phi}{\partial n} &= \frac{\partial \phi}{\partial n}^{(w)} \end{aligned} \right\} \text{on } S_c \text{ for all } t \quad (13)$$

Here, the superscript (w) denotes a value specified by a chosen wave theory at a given time, t .

Applying the conditions from (8) to the Green's identity given by (6), we obtain

$$\begin{aligned} \phi(\underline{x}) = & -\frac{1}{\pi} \int_{S_c} \left(G(\underline{x}, \underline{\xi}) \frac{\partial \phi}{\partial n}(\underline{\xi})^{(w)} - \phi(\underline{\xi})^{(w)} \frac{\partial G}{\partial n}(\underline{x}, \underline{\xi}) \right) dS \\ & - \frac{1}{\pi} \int_{S_f + S_b} \left(G(\underline{x}, \underline{\xi}) \frac{\partial \phi}{\partial n}(\underline{\xi}) - \phi(\underline{\xi}) \frac{\partial G}{\partial n}(\underline{x}, \underline{\xi}) \right) dS \end{aligned} \quad (14)$$

2.4 Forces on a Fixed Body

We wish to obtain the horizontal and vertical forces as well as the moment on a fixed body. From the potential distribution as a function of time and location in the fluid region, we can obtain the pressure distribution using the unsteady Bernoulli equation,

$$p = -\rho \left[\frac{\partial \phi}{\partial t} + \frac{1}{2} (\nabla \phi)^2 \right] \quad (15)$$

Integrating the pressure, as given by (15), over the body surface yields the forces on the body,

$$\underline{F} = \int_{S_b} p \underline{n} \, dS \quad (16)$$

$$M = \int_{S_b} p (\underline{r} \times \underline{n}) \, dS \quad (17)$$

The hydrostatic component of the pressure on the body is taken to be constant and is excluded from the above.

2.5 Summary of Time Stepping Procedure

To obtain the flow development, the time stepping procedure is implemented as follows. First, the closed boundary shown in Figure 3 is divided into segments in a manner that is symmetric about the seabed. On each side of the seabed we have N' segments, with N_b segments on the body surface, N_f segments on the free surface, and N_c segments on the control surfaces. A typical segment grid is shown in Figure 7. The incident wave form is specified at times t and $t-\Delta t$ by assigning values of ϕ , $\partial\phi/\partial n$, and η (according to our chosen wave theory) at the midpoint of each segment. The body surface boundary condition given by (3) then yields $\partial\phi/\partial n = 0$ on S_b .

The kinematic free surface boundary condition given by (4a) is used to obtain values of η at $t+\Delta t$ at the free surface segment midpoints. The extrapolated values can be obtained using a central difference approximation or another suitable method. Values of ϕ at $t+\Delta t$ at the free surface segment midpoints are obtained in a similar manner using the dynamic free surface boundary condition given by (5).

The Green's identity given by (9) is then applied at the free surface and body surface segment midpoints to yield N equations (where $N = N_f + N_b$) in N unknowns, that is, N_f values of $\partial\phi/\partial n$ at $t+\Delta t$ and N_b values of ϕ at $t+\Delta t$. The values of ϕ and $\partial\phi/\partial n$ at $t+\Delta t$ on S_c are known from the prescribed incident wave flow, as previously explained in Section 2.3. The N equations are solved on a computer and

hence we have all segment midpoint values for ϕ and $\partial\phi/\partial n$ at $t+\Delta t$.

With values of ϕ and $\partial\phi/\partial n$ at times $t-\Delta t$, t , and $t+\Delta t$ we obtain the pressure distribution at time t using a central difference approximation of (15). From the pressure distribution, we obtain the forces acting on the body at time t using a numerical integration of equations (16) and (17).

The values of ϕ , $\partial\phi/\partial n$, and η at time t become those at $t-\Delta t$, while those at $t+\Delta t$ become those at t , thus advancing the flow conditions one time step. The process for obtaining the required values at $t+\Delta t$ is repeated and in this manner we can develop the flow for as long as desired or constrained by accuracy or computer cost. The details of the complete procedure and its computer solution are presented and discussed in Chapter III. A user guide to the computer program developed for this thesis is given in Appendix I.

CHAPTER III: NUMERICAL PROCEDURE

3.1 Numerical Time Stepping Procedure

We begin with the closed boundary divided into segments as in Figure 7, with N_b body surface segments, N_f free surface segments, and N_c control surface segments. Recall that $N = N_f + N_b$, $N' = N + N_c$, and that all values of ϕ and $\partial\phi/\partial n$ at segment midpoints are assumed to be constant over the segment length S . Initial values of ϕ , and $\partial\phi/\partial n$ at times $t-\Delta t$ and t are assigned to all segments as defined by a chosen wave theory. There must be zero flow in the vicinity of the body and the wave propagation here is assumed to flow from left to right. A typical initial wave profile is shown in Figure 4.

As described in Section 2.5, we need to develop expressions from the kinematic and dynamic free surface boundary conditions which will give us extrapolated values of η and ϕ at $t+\Delta t$. We can rewrite (4a) and (5) using a central difference approximation to obtain

$$\eta_{t+\Delta t} = \eta_{t-\Delta t} + 2\Delta t \left(\frac{1}{n_z} \right) \left(\frac{\partial\phi}{\partial n} \right)_t \quad (18)$$

and

$$\phi_{t+\Delta t} = \phi_{t-\Delta t} - 2\Delta t \left[g\eta + \frac{1}{2}(\nabla\phi)^2 \right]_t \quad (19)$$

We note that to retain a non-zero Bernoulli constant in (5) would not affect our force results, and thus set it equal to zero for convenience.

We now need to have the Green's boundary integral identity as expressed by (11) in a form that can be numerically evaluated. Separating the boundary into segments, we get from (14)

$$\begin{aligned} \phi(\underline{x}_i) = & -\frac{1}{\pi} \sum_{j=1}^N \left(\int_{\Delta S_j} G(\underline{x}_i, \underline{\xi}) \frac{\partial \phi}{\partial n}(\underline{\xi}) dS - \int_{\Delta S_j} \phi(\underline{\xi}) \frac{\partial G}{\partial n}(\underline{x}_i, \underline{\xi}) dS \right) \\ & - \frac{1}{\pi} \sum_{j=N+1}^N \left(\int_{\Delta S_j} G(\underline{x}_i, \underline{\xi}) \frac{\partial \phi^{(w)}}{\partial n}(\underline{\xi}) dS - \int_{\Delta S_j} \phi^{(w)}(\underline{\xi}) \frac{\partial G}{\partial n}(\underline{x}_i, \underline{\xi}) dS \right) \\ & \text{for } i = 1, 2, \dots, N \end{aligned} \quad (20)$$

We assume the integrands to be constant over each segment, excepting when $i=j$, and represent the N equations generated by (20) in the matrix form,

$$[A] (\psi) = (\lambda) \quad (21)$$

where

$$\psi_j = \begin{cases} \phi_j & \text{for } j=1, 2, \dots, N_b \\ (\partial \phi / \partial n)_j & \text{for } j=N_b+1, N_b+2, \dots, N \end{cases} \quad (22)$$

$$A_{ij} = \begin{cases} a_{ij} & \text{for } j=1,2,\dots,N_b \\ b_{ij} & \text{for } j=N_b+1,N_b+2,\dots,N \end{cases} \quad (23)$$

$$\lambda_i = c_i - \sum_{j=N_b+1}^N a_{ij} \phi_j \quad (24)$$

In (23) and (24),

$$a_{ij} = - \frac{\Delta S_i}{\pi} \left(\frac{\partial G}{\partial n} \right)_{ij} \quad (25)$$

$$a_{ii} = 1 - \frac{\Delta S_i \cos \gamma_i}{2\pi(z_i+d)} \quad (26)$$

$$b_{ij} = \frac{\Delta S_i}{\pi} G_{ij} \quad (27)$$

$$b_{ii} = \{ \ln(\Delta S_i) + \ln(z_i+d) - 1 \} \quad (28)$$

$$c_i = \sum_{j=N+1}^{N'} \frac{\Delta S_j}{\pi} \left(G_{ij} \left(\frac{\partial \phi}{\partial n} \right)_i^{(w)} - \left(\frac{\partial G}{\partial n} \right)_{ij} \phi_j^{(w)} \right) \quad (29)$$

G and $(\partial G / \partial n)$ as given by equations (7) and (10) are numerically represented by

$$G_{ij} = \ln(\sqrt{(x_i-x_j)^2+(z_i-z_j)^2}) + \ln(\sqrt{(x_i-x_j)^2+(z_i+z_j+2d)^2}) \quad (30)$$

and

$$\left(\frac{\partial G}{\partial n}\right)_{ij} = \left[\frac{n_x(x_j - x_i) + n_z(z_j - z_i)}{(x_i - x_j)^2 + (z_i - z_j)^2} \right] + \left[\frac{n_x(x_j - x_i) + n_z([z_j + 2d] + z_i)}{(x_i - x_j)^2 + (z_i + z_j + 2d)^2} \right] \quad (31)$$

In (20), the ϕ_i from the left-hand side of (14) shows up in a_{ii} for $i=1,2,\dots,N_b$, and b_{ii} for $i=N_b+1,\dots,N$. The terms a_{ii} and b_{ii} are obtained by explicitly integrating the appropriate quantities in (20) over a segment length.

The matrix solution represented by (20) through (31), is applied at $t+\Delta t$. At $t+\Delta t$, the terms $(\partial\phi/\partial n)^{(w)}$ and $\phi^{(w)}$ (for $j=N+1,\dots,N'$) in (29) are obtained from the chosen incident wave theory and the ϕ_j in (24) are obtained from (19). The matrix equation represented by (20) is solved and yields $(\partial\phi/\partial n)$ at $t+\Delta t$ for $i=1,\dots,N_b$ and ϕ at $t+\Delta t$ for $i=N_b+1,\dots,N$.

With $\phi_{t+\Delta t}$ and $(\partial\phi/\partial n)_{t+\Delta t}$ known at all points on the body, the forces on the body can be calculated at time t using (15), (16), and (17), thus

$$F_x = \sum_{i=1}^{N_b} p_i \Delta S_i (n_x)_i \quad (32)$$

$$F_z = \sum_{i=1}^{N_b} p_i \Delta S_i (n_z)_i \quad (33)$$

$$M = \sum_{i=1}^{N_b} p_i \Delta S_i [x_i (n_z)_i - z_i (n_x)_i] \quad (34)$$

where

$$p = -\rho \left\{ \frac{(\phi_i)_{t+\Delta t} - (\phi_i)_{t-\Delta t}}{2\Delta t} + \frac{1}{2} \left[\left(\frac{\partial \phi}{\partial n_i} \right)^2 + \left(\frac{\phi_{i+1} - \phi_{i-1}}{2\Delta s} \right)^2 \right]_t \right\} \quad (35)$$

The time is now advanced one step, Δt . The $t+\Delta t$ terms become t terms, t becomes $t-\Delta t$, and the process is repeated as many times as needed for the desired flow development.

3.2 Numerical Procedure - Computer Solution

The time stepping procedure for flow development, given in Section 3.1, is readily programmed for a computer solution. The computer program developed for this thesis is written in FORTRAN IV. The program is approximately 1000 lines long and is compiled on the FORTRAN H compiler attached to an Amdahl 470 V/6 Model II computer. The compiled program requires approximately 10^6 bytes of storage space.

The flow chart shown in Figure 8 represents the basic structure of the program, which is driven by a main routine calling the relevant subroutines. For a typical example involving a total of 100 segments, a matrix of dimension $N=80$, and 100 time steps, the solution on the Amdahl 470 takes approximately 50 seconds CPU time, or about 0.5 seconds per time step. Considerations for program structure and input parameters are discussed in this section.

3.2.1 Segment Grid

When setting up the segment grid, the first consideration is choosing an appropriate segment length. It is clear that the segment length to wavelength ratio $\Delta S/L$ is the relevant parameter that defines the number of segments that will be required on the free surface. If one considers the cyclical motion of the fluid particles over a wave period, it is readily apparent that the time step Δt must be sufficiently small to ensure that the motion of the

particles is small compared to segment length, and hence that $\Delta t/T$ should be less than $\Delta S/L$ for good accuracy in the flow development. Thus, for a given $\Delta t/T$, $\Delta S/L$ must be small enough to yield good accuracy while being large enough to remain greater than the given $\Delta t/T$. The required ranges of $\Delta t/T$ and $\Delta S/L$ are investigated and discussed in Sections 4.4 and 4.5.

Our particular example is a rectangular cross-section. For this case, the segment lengths along the control surface and sides of the body will vary as the surface elevation at those locations changes. It is a straightforward procedure to scale the segment lengths along the control surface and body sides at each time step in order to account for the free surface elevation change. The scaling of the segments results in a small change in the location of the segment midpoints along the body sides and control surface. A refinement to the program could be incorporated that would interpolate values of ϕ and $\partial\phi/\partial n$ between segment midpoints, so that interpolated values could be assigned to new segment midpoint locations. Recalling that $\partial\phi/\partial n = 0$ on S_b , that we use the matrix solution for new values of ϕ on S_b , and that the control surface conditions are specified throughout the flow development using our chosen wave theory, we have no need of the refinement for our present situation.

The free surface segment lengths must also be adjusted at each time step, as the slopes of the segments

change with changing free surface elevation. For the rectangular body cross-section in this study, the x coordinates of the free surface remain the same throughout the time stepping procedure. However, if the program were written for a procedure that accounted for body motions, or if the geometry of the body in our procedure were altered so that the sides were not vertical at the waterline, then we would have to account at each time step for the change in horizontal free surface length to each side of the body. The change in horizontal free surface length would relocate the x coordinates of the free surface segment midpoints, which could be accounted for by using a refinement to the program as discussed in the preceding paragraph.

As the body dimensions are significantly smaller than the wavelength for waves of longer period, the segment length parameter $\Delta S/L$ found to be appropriate on the free surface may not be very useful on the body surface if only 1 or 2 segments are needed to meet the specified global $\Delta S/L$ requirement. Recalling that ϕ and $\partial\phi/\partial n$ (and hence p) are constant over each segment, a minimum number of segments are needed on the body surface to adequately describe the pressure distribution over the body surface. The moment acting on the body is most likely to be sensitive to this consideration. For purposes of the body segments we define a parameter $\Delta S_b/L_b$, where ΔS_b is the body surface segment length, and L_b is the dimension of the body along which the segment lies (for our example, B or D). The required range

for $\Delta S_b/L_b$ is investigated and discussed in Section 4.5.2.

3.2.2 Incident Wave Specification

When specifying the initial conditions within the fluid region, and hence the initial incident wave profile, we have a choice of several wave theories. For purposes of this thesis, we investigate the results from harmonic incident wave trains, such as those predicted by linear or higher order wave theories, which are modified to satisfy the still water condition by attenuating the flow predicted by the theory with a cosine modulation envelope, as shown in Figure 4. We can vary the length of the modulation envelope so that the number of wavelengths to full wave height is varied. The time stepping procedure can be run for a sufficient length of time to allow one or more waves to pass the body. The increase in forces from the first to the second and subsequent waves can thus be observed if it occurs. The major drawback of this approach is that the number of time steps required increases with the number of wavelengths that pass the body, thus significantly increasing the cost of the computer solution. The increase in cost is compounded by the fact that to be able to run more wavelengths past the body, a longer free surface and hence more segments are required to satisfy the condition that reflected waves not reach the control surface. This increases the size of the matrix to be solved at each time step, thus adding to the computer cost.

If the modulation envelope is treated as travelling with the group velocity c_g , then the number of wavelengths required for a full wave height to reach the body will be about twice the decay length, since the group velocity is about half the wave speed c . As a possible means of reducing the cost of the method, we investigate the effect of specifying a speed faster than c_g as the speed of the modulation envelope. Since there is no rigorously defined group velocity for Stokes' fifth order wave theory, we must specify a value for the group velocity when we use Stokes' fifth order wave theory for our incident wave. The effect of specifying a different value for c_g other than that predicted by linear wave theory and of varying the decay length is investigated and discussed in Sections 4.2 and 4.3 respectively.

3.2.3 Time Stepping Equation

As discussed briefly in Section 3.1, the time stepping equation used in extrapolating new values of η and ϕ on the free surface from the kinematic and dynamic free surface boundary conditions can be a central difference method which yields equations (18) and (19) or other methods of the same or higher order. For our purposes, the Adams-Bashforth multistep methods described in Burden, Faires, & Reynolds are useful, and we examine the effect of using these higher order time stepping methods on our results. The derivation of the relevant time stepping equations is described in

Appendix II and results using higher order time stepping equations are discussed in Section 5.1.

3.2.4 Matrix Solution

At each step of the time-stepping procedure, the $N \times N$ matrix generated by the Green's integral identity must be solved. Various packaged programs exist for matrix solution, each with its own advantages and disadvantages. In our case, the matrix is quite full and without any symmetry that can be exploited. The elements of the matrix can vary by several orders of magnitude, particularly across the columns. It is advantageous to use a matrix solution procedure that scales the columns to a closer order of magnitude before solving and then scales the solution accordingly.

The matrix solution is by far the most expensive component of the computer program in terms of computing time. It is therefore necessary that every effort be made to reduce the computing effort in solving the matrix. In our program, a direct matrix solution which first scales and decomposes the matrix is employed as the preferred solution technique. Rounding errors in the technique may result in a singular matrix being undetected, and hence the condition of the matrix is checked at each time step. If the preferred technique is unable to solve the matrix accurately, an iterative solution technique is called. For our particular method and computer solution, the computer time per step is

plotted as a function of N in Figure 9, which thus gives a relative indication of the increase in cost with matrix size.

CHAPTER IV: INPUT PARAMETER CONSIDERATIONS

4.1 Introduction

In this Chapter we investigate various input parameters used in the computer program developed from the method described in Chapter III. The effect of varying the group velocity, incident wave decay length, time step size, and segment length on both the free surface and the body surface is investigated. In each case we determine an optimal value for the parameter that will yield accurate results at minimum cost.

Unless otherwise stated in a given Section of this and the next Chapter, test runs referred to are for an incident wave prescribed by linear wave theory, of height $H = 1.0$ m , angular frequency $\omega = 1.5$ rad/sec , and decay length $L_d = 1.0$ wavelength , with one fully developed wave interacting with the body. The body is rectangular, with beam $B = 10.0$ m and draught $D = 2.5$ m , in water of depth $d = 20.0$ m . The time step ratio $\Delta t/T$ is 0.04 , the segment length ratio $\Delta S/L$ is 0.1 , and the body segment length ratio $\Delta S_b/L$ is 0.2 . The time stepping equation used is the central difference method given in (18) and (19). The user guide for the program, which describes all the necessary input parameters and options, is given in Appendix I.

4.2 Group Velocity

The incident wave train is specified according to a chosen wave theory. Throughout our study, we use either linear wave theory or Stokes' fifth order wave theory to assign initial values of ϕ , $\partial\phi/\partial n$, and η at the segment midpoints. We attenuate the incident wave as we go from the left control surface towards the body. The initial conditions are specified at two or more initial time steps, depending on the order of the time stepping procedure (see Appendix II). When using linear wave theory, we can apply the group velocity c_g as predicted by the theory to the initial motion of the modulation envelope. However, when using Stokes' fifth order wave theory, an explicit expression for the group velocity is not available. Hence, for Stokes' fifth order theory, we must assume a value for c_g that seems reasonable.

When using linear wave theory, one might expect that specifying the wave velocity c as the group velocity for the modulation envelope would cause a full wave height to reach the body sooner than if the normal group velocity $c_g \approx c/2$ were used. To investigate this possibility, the program developed for this thesis was run twice with the input values as defined at the beginning of this Chapter, changing only the initial group velocity of the modulation envelope between runs.

In the first case, the group velocity given by

$$c_g = \frac{c}{2} \left(1 + \frac{kd}{\sinh(kd)} \right)$$

was used as the initial group velocity of the modulation envelope. The forces on the body at each time step as well as the wave runup on each side of the body were recorded. The second test case used the wave velocity c as the initial group velocity of the modulation envelope, and corresponding results were recorded. The two sets of results were compared and found to be almost identical. The conclusion we draw is that the method itself with its physical constraints expressed by the boundary conditions and the Green's identity very strongly defines the flow development, and overrides the attempt (by setting the initial group velocity equal to the wave speed) to force a faster increase to full flow development in the vicinity of the body.

The conclusion stated above was further verified by using a five step time stepping procedure (described in Appendix II), and thus defining values of ϕ , $\partial\phi/\partial n$, and η at the first five time steps based on a group velocity $c_g = c$. All other input values were kept the same. The results for the first ten or so time steps differed significantly from previous results, but subsequent flow development yielded forces and wave runup values within 1% of the previous results at any given time step.

The results of this Section show that the flow development is relatively independent of the modulation envelope initial group velocity. For purposes of our

program, we set the group velocity equal to the value predicted by linear wave theory when using Stokes' fifth order wave theory.

4.3 Incident Wave Decay Length

When using linear or Stokes' fifth order wave theory to define our incident wave profiles, the decay length L_d of the initial wave profile (as shown in Figure 4) will determine the time to full flow development. By reducing the decay length we reduce the number of segments needed to define the free surface, and also shorten the time to full flow development, thus reducing the cost of running the program. Too short a decay length could excessively affect the accuracy of our results, while too long a decay length would unnecessarily make the cost of full flow development too expensive.

Using the input values given in Section 4.1, we now vary the decay length L_d from $L_d = 0.25$ up to $L_d = 1.75$. The maximum forces generated by the first fully developed wave crest to interact with the body are plotted as a function of decay length in Figure 10. In Figure 10, the forces are expressed as a percent variation from the value to which a given force converges.

The results of Figure 10 indicate that a decay length of about 0.75 and less yields somewhat unreliable results, while a decay length of 1.25 and greater does not affect results significantly. We will use $L_d = 1.0$ for our further investigations regarding numerical parameters, and will use $L_d = 1.25$ to obtain accurate results for plotting.

It is noted that for $L_d = 1.0$, it will take approximately two wave periods for a full wave height to

reach the body. This is due to the actual group velocity of the wave train being approximately $c/2$, as was seen in Section 4.2. As a result, for $L_d = 1.0$, the second wave crest to reach the body will be the first fully developed wave crest interacting with the body.

4.4 Time Step Size

The accuracy of our numerical solution is dependent on, among other factors, the size of the time step, which we express as a fraction of the wave period, by $\Delta t/T$. It is reasonable to expect that as the time step ratio $\Delta t/T$ is reduced (holding all other input parameters equal), the force results should converge to relatively uniform values.

We test the effect of varying $\Delta t/T$ by using the input parameters given in Section 4.1, and run the program changing only the value of $\Delta t/T$. Recalling from Section 3.2.1 that we expect that $\Delta t/T$ should be smaller than $\Delta S/L$ for accurate flow development, the largest value of $\Delta t/T$ that we test is $\Delta t/T = \Delta S/L = 0.1$. In addition, we test successively smaller values of $\Delta t/T$ recording the force and runup results at each time step. As for Figure 10, the maximum forces generated by the first fully developed wave to interact with the body are plotted in Figure 11 as a function of $\Delta t/T$.

The results of Figure 11 confirm our earlier discussion. For $\Delta t/T = \Delta S/L = 0.1$, the results diverged to the point where the program would not continue the flow development. This was expected for $\Delta t/T$ being in the same order of magnitude as $\Delta S/L$, since the particle motions for the flow development would be of the same order of magnitude as the segment lengths themselves. As we decrease $\Delta t/T$, the results quickly converge to uniform values. Little accuracy is gained by using $\Delta t/T < 0.05$, and for $\Delta t/T = 0.04$, the

results obtained are within 3% of the most accurate results. We conclude that if we use $\Delta t/T \leq 0.04$ for our subsequent investigations, we will have sufficient accuracy.

4.5 Segment Length

The importance of reducing the number of segments used is readily apparent when considering the cost of running the program. As discussed in Section 3.2.1, the segment length parameter $\Delta S/L$ is globally constrained by an upper and lower limit. In addition, the body segments must be small enough to have a sufficient number to accurately approximate the pressure distribution on the body surface at each time step. Both these considerations are investigated in this Section.

4.5.1 Global Segment Length

The segment length parameter $\Delta S/L$ is constrained in two ways. The first constraint is similar to that on $\Delta t/T$, that is, $\Delta S/L$ should be small enough to accurately approximate the free surface profile, and hence the flow development. The second constraint is the relationship of $\Delta S/L$ to $\Delta t/T$ (discussed in Section 3.2.1) that requires that $\Delta t/T \ll \Delta s/L$. Thus, there is an upper and a lower limit expected on $\Delta S/L$.

We test the effect of varying $\Delta S/L$ by again using the input values given in Section 4.1, with $\Delta t/T = 0.03$, and letting $\Delta S/L$ take on values ranging from 0.05 to 0.25. The maximum values for the forces generated by the first fully developed wave to interact with the body, the forces

expressed in dimensionless form, are plotted as a function of $\Delta S/L$ in Figure 12.

Our earlier discussion concerning upper and lower limits for $\Delta S/L$ is confirmed as the results remain relatively consistent when $\Delta S/L$ is in the vicinity of 0.1, and diverge as one increases or decreases $\Delta S/L$ beyond a limited region. As before, when $\Delta S/L$ is set to a value that is less than approximately twice the value of $\Delta t/T$, the results diverge to the point where the computer solution is not able to continue. In this set of tests, this was the case for $\Delta S/L = 0.05$. We conclude from Figure 12 that a value of 0.1 for $\Delta S/L$ will yield good results, and that $\Delta S/L$ should also be at least 2 to 3 times greater than $\Delta t/T$.

4.5.2 Body Segments

As discussed in Section 3.2.1, a global limit on $\Delta S/L$ may not provide enough body segments to accurately calculate the force results, particularly the moment M . Using the input values given in Section 4.1, with $\Delta t/T = 0.04$ and $\Delta S/L = 0.1$, we test different values of $\Delta S_b/L_b$ ranging from 0.1 to 0.25.

We should also check that the segment lengths on the body surface defined by $\Delta S_b/L_b$ fall within the global constraints on $\Delta S/L$ determined in Section 4.5.1. Along the horizontal segments defining the base of the body section,

we are concerned with horizontal particle motions, and thus the global limit should apply to the horizontal segments. For the vertical body segments, we are concerned with vertical particle motions, which can be considered to be smaller than the horizontal particle motions for a given time step by a factor of approximately $2H/L$ (allowing for a wave runup at the vertical body surface of as much as twice the wave height). The program whose user guide is given in Appendix I conducts this check and if necessary, resets the value of $\Delta S_b/L_b$ to fall within the global constraints.

Force results are plotted as for Figures 10 and 11, this time as a function of $\Delta S_b/L_b$, in Figure 13. We observe from Figure 13 that the moment is the result most affected by varying $\Delta S_b/L_b$. The sharp variation in results as one moves towards lower values of $\Delta S_b/L_b$ can be accounted for by the fact that for $\Delta S_b/L_b < 0.2$, $\Delta S_b/L < 2\Delta t/T$, which violates our previously verified condition that $\Delta S/L$ should be at least twice the value of $\Delta t/T$. We conclude that we should use as many segments as our global constraint on $\Delta S/L$ allows, and set $\Delta S_b/L_b = 0.2$ as a maximum value, giving a minimum of five segments along each body dimension.

CHAPTER V: OTHER CONSIDERATIONS

5.1 Time Stepping Equation

As discussed in Section 2.5, we use the kinematic and dynamic free surface boundary conditions to time step the values of η and ϕ on the free surface. The time stepping technique used to obtain (18) and (19) is a central difference method. For our purposes, we find that the Adams-Bashforth multistep methods described in Appendix II are also useful. We now investigate the effect on our results of using each of the methods listed.

Using the input values given in Section 4.1 with $\Delta t/T = 0.04$, and $\Delta S/L = 0.2$, we run our program using each of the time stepping methods given, that is, the central difference method, and the Adams-Bashforth two, three, four, and five step methods. We plot the results for the forces as a function of time for each method. The resulting plots are shown in Figure 14.

From our results of Figure 14, we see that the central difference method produces a slightly uneven plot, particularly for the first several time steps. The two, three, and four step methods give smoother results. The five step method produces highly erratic results which sawtooth about values coincident with the other results. The sawtooth results of the five step method are probably

caused by small perturbations from smooth results being magnified by the fitting of a fifth order polynomial to previous points when projecting forward at each time step for values at $t+\Delta t$. Anticipating that the same effect could occur for any of the higher order methods, we will use the three step method as the preferred method for subsequent work, and default to the two step method if the three step method leads to a divergent instability similar to the one observed here for the five step method.

5.2 Incident Wave Type

The wave theory chosen to describe the incident wave train depends on the wave steepness and H/d , as shown in Figure 15. The two theories employed as options in our program are linear wave theory and Stokes' fifth order wave theory. We seek to verify the range of validity for these theories by first running the program for a given wave condition falling within the normal range of linear wave theory, and comparing force results obtained using both linear and Stokes' fifth order theories for the incident waves. We then do the same for a wave within the normal range of Stokes' fifth order wave theory.

A wave with $\omega = 0.5$ rad/sec, $H = 1.0$ m, in water of depth $d = 20.0$ m, falls within the normal range of linear wave theory. A wave with $\omega = 2.0$ rad/sec, $H = 1.0$ m, in water of depth $d = 5.0$ m, falls within the normal range of Stokes' fifth order wave theory. We test both waves using linear and Stokes' fifth order wave theory. The vertical force results using each theory are compared for both waves. These results are shown in Figure 16.

The wave in the linear theory range produces virtually no difference between results using linear wave theory and results using Stokes' fifth order wave theory, while the wave in the Stokes' fifth order range produces a noticeable difference in the magnitude and phase of the results. We conclude that in order to obtain accurate results, it is important to use the appropriate wave theory

for a given incident wave.

CHAPTER VI: RESULTS

6.1 Exciting Forces

The primary purpose of this thesis was to explore the viability of the method described to predict the exciting wave forces on a fixed structure. Having investigated the range of the various numerical parameters and other considerations needed to obtain accurate results within the limitations of the method itself, we now proceed to generate force results over a range of ω . For all our results in this Chapter, we run the program described in Appendix I with $L_d = 1.25$, $\Delta t/T = 0.04$, $\Delta S/L = 0.1$, and $\Delta S_b/L_b = 0.2$. We use the Adams-Bashforth three-step method as our preferred time stepping technique, and use either linear or Stokes' fifth order wave theory, depending on the wave steepness, to describe our incident wave. In addition, care was taken to ensure that the magnitude of the the wave runup did not exceed the draught of the body, as this condition would render the flow development unsolvable.

We begin by using the program in a form modified to give the flow development without a body. A typical flow development for an incident wave prescribed by linear wave theory is shown in Figure 17. The initial wave form is shown at the top of Figure 17, with the free surface profile after 10, 20, 30, and 40 time steps shown below it.

Next, we compare results of force development as a function of time with linear results obtained from a finite element method used by Fraser. For an incident wave of height $H = 1.0$ m and $\omega = 1.5$ rad/sec on a body with $B = 10.0$ m and $D = 2.5$ m in water of depth $d = 20.0$ m, we produce the results shown in Figures 18 and 19. We see that the results obtained from our program, once fully developed, compare favorably with those obtained by Fraser both in magnitude and in phase. The notable difference between the results is that the results from our method are clearly non-linear, particularly for F_z and M .

We now use our program to generate force results over a range of frequencies. For each frequency tested, the maximum forces from the second fully developed wave to interact with the body are recorded. We compare our force results with Fraser's linear results and Vugts' experimental results. Fraser obtained his force results using a linear finite element method, and hence his results are not depth dependent. Vugts' results were obtained for deep water conditions. Since our method is non-linear, we obtain different force curves for different values of H/d . For a beam to draught ratio $B/D = 2.0$, force results generated by our program for $H/d = 0.05, 0.1$, and 0.2 are plotted in Figure 20, with both Fraser's and Vugts' results included for comparison. Similarly, force results for $B/D = 4.0$ and 8.0 are shown in Figures 21 and 22.

The results of Figures 20, 21, and 22 show that the

method used here has a fairly full range of applicability, and that the magnitude of most of the force results compares well to previous experimental and theoretical results. While the results for the horizontal force coincide closely with Fraser's linear results, our non-linear method produces curves for the vertical force and the moment that differ significantly in slope from the linear predictions.

6.2 Transmission Coefficient

An interesting byproduct of the method is the capability to monitor the transmission coefficient for the fixed case. Since the fixed case is used here only as an idealization to calculate exciting forces that can be used for a subsequent analysis of the actual response of a moored breakwater, the observed transmission coefficients are of little practical value, but rather, provide a lower limit for predicted transmission coefficients in an actual moored example. In our case of a fixed rectangular breakwater, we can also obtain from the transmission coefficient results an indication of preferred values of B/d for various ω . The transmission coefficient, K_t , obtained from our results is plotted as a function of ω for $B/D = 2.0$, 4.0 , and 8.0 in Figure 23, with $K_t = H_t/H$, where H_t is the transmitted wave height. Also plotted on Figure 23 are results from Nece and Richey for $B/D = 5.0$. The experimental results shown are for an incident wave of the same steepness for a given ω as for our results. The results obtained for the transmission coefficient were found to be relatively independent of depth over the range tested. As expected, our results for a fixed body give lower values for K_t than Nece and Richey's experimental results for a responding cable moored body, and thus appear to provide a reliable lower limit for K_t .

CHAPTER VII: CONCLUSIONS AND FURTHER STUDIES

7.1 Conclusions

We have examined the Green's identity flow development method developed by Isaacson, applying the method to a two-dimensional fixed rectangular cross-section. The most significant limitation of the method is the very large computing effort required to develop the flow for a few wave periods. We have determined the desirable range of various parameters to ensure accuracy of results.

The effect of specifying the group velocity as a value other than that predicted by linear wave theory was investigated. The only stage in the method where the group velocity is used is to specify the initial group velocity of the modulation envelope. It was discovered that the method itself as defined by the physical conditions and boundary integral strongly defined the group velocity of the flow development, and that specifying a different value for the group velocity affected the results of only the first few time steps.

For the decay length, a value of $L_d = 1.0$ was found to be adequate, with $L_d = 1.25$ preferred. Longer decay lengths resulted in excessive computing effort due to the increased number of segments and time steps necessary, while shorter decay lengths produced less accurate results.

The time step parameter $\Delta t/T$ of 0.04 or less was found to give good results. The global segment length parameter $\Delta S/L$ was constrained by both an upper and a lower bound. Higher values of $\Delta S/L$ did not provide enough segments to accurately approximate the free surface wave profile, while lower values of $\Delta S/L$ conflicted with the requirement that $\Delta S/L \ll \Delta t/T$. For accurate results, $\Delta S/L$ should be at least two or three times greater than $\Delta t/T$, and should not be greater than 0.1 .

The global value for $\Delta S/L$ was found not to be sufficient to describe the pressure distribution on the body surface. For the body surface, the parameter $\Delta S_b/L_b = 0.2$ was found to give enough segments on the body surface to allow fairly accurate calculation of the forces on the fixed body.

Different time stepping techniques were investigated, including a central difference method and the Adams-Bashforth multistep methods given in Appendix II. The higher order Adams-Bashforth methods seemed to frequently generate instabilities apparently due to the fitting of a higher order polynomial to small perturbations in previous time steps. Methods employing only two or three previous values were found to be satisfactory.

The importance of using a wave theory appropriate to the regime of a given wave was verified. In particular, linear wave theory and Stokes' fifth order wave theory were used, and the validity of each in their respective range of

applicability was confirmed.

A typical force-time plot generated by our method compared favorably both in magnitude and in phase to linear results obtained by Fraser. The flow development in the vicinity of the body as calculated by the present method was clearly non-linear, even when the initial incident wave flow parameters and the control surface parameters throughout the flow development were prescribed by linear wave theory.

The maximum dimensionless forces generated by incident waves of various frequencies as determined by our method compared well in magnitude to previous numerical and experimental results. We obtained different force curves for different values of H/d , and the slopes of many of our force curves differed significantly from previous theoretical linear and deep water experimental results, particularly for the vertical force and the moment. We conclude that the non-linear method used here has a significant potential for improved accuracy in the prediction of forces on floating bodies.

The method as presented here allows only for a relative comparison of the transmission coefficient K_t for different values of beam to draught ratio and frequency. The transmission coefficient for our fixed breakwater appeared to provide a valid lower limit for K_t . In order order to use the method to accurately predict transmission coefficients, the motions of the body must be determined, either by using the exciting forces obtained here together

with the hydrodynamic properties of the body to calculate the motions, as done by Fraser, or by refining the method to take motions into account at each time step.

7.2 Recommendations for Further Study

In this work we have investigated the application of the Green's identity flow development method to a fixed rectangular cross-section. The method described here could be extended in several ways. Adding different wave theories such as solitary wave theory or stream function wave theory to the options for the incident wave train would extend the range of applicability of the method. The method could also be extended to arbitrary cross-sections.

An extension of the method, incorporating body motions into the time stepping procedure, could be developed and applied to obtain body motions and stresses directly as well as accurate predictions of the transmission coefficient. By incorporating a slit from the body surface to a submerged body, a closed surface could be defined that would allow the method to be applied to submerged bodies. The present method has already been applied to floating three-dimensional bodies by Isaacson⁽²²⁾, though a correspondingly greater computer effort is required to maintain accuracy. With extensions and improvements to the method, a complete numerical analysis of various floating and submerged structures could be developed.

BIBLIOGRAPHY

- 1 Isaacson, Michael de St. Q.
"Steep Wave Effects on Large Offshore Structures",
Proc. Thirteenth Annual Offshore Technology
Conference, Vol. I, 1981, pp. 21-29.
- 2 Joly, J.
"On Floating Breakwaters", Royal Dublin Society
Scientific Proceedings, Vol. 10, 1905, pp.378-
383.
- 3 Adee, Bruce H.
"Operational Experience with Floating
Breakwaters", Marine Technology, Vol. 14, No. 4,
Oct. 1977, pp. 379-386
- 4 Miller, D. S.
"Floating Breakwaters", Civil Engineering, Vol.
44, No. 3, March 1974, pp.77-79.
- 5 Brebner, A. and Ofuya, A. O.
"Floating Breakwaters", Proc. Eleventh Conference
on Coastal Engineering, ASCE, Vol. II, 1968,
pp.1054-1094.
- 6 Hom-ma, M., Horikawa, K. and Mochizuki, H.
"An Experimental Study on Floating Breakwaters",
Coastal Engineering in Japan, Vol. 7, 1964, pp.
85-94.
- 7 Skey, S.
"Cylindrical Floating Breakwater", Dock and
Harbour Authority, Vol. 56, No. 658, August
1975, pp. 138-139.
- 8 Sutko, A. A.
"Floating Breakwaters - A Wave Tank Study",
Journal of Petroleum Technology, Vol. 27, March
1975, pp. 269-273.
- 9 Harris, A. J. and Webber, N. B.
"A Floating Breakwater", Proc. Eleventh
Conference on Coastal Engineering, ASCE, Vol. II,
1968, pp. 1049-1054.
- 10 Mei, Chiang C. and Black, Jared L.
"Scattering of Surface Waves by Rectangular
Obstacles in Waters of Finite Depth", Journal of
Fluid Mechanics, Vol. 38, 1969, pp.499-511.
- 11 Longuet-Higgins, M.
"The Mean Forces Exerted by Waves on Floating or

Submerged Bodies with Applications to Sand Bars and Wave Power Machines", Proc. Royal Soc. London A, Vol. 352, 1977, pp. 463-480.

- 12 Nichols, B. D. and Hirt, C. W.
 "Numerical Calculation of Wave Forces on Structures", Proc. Fifteenth Coastal Engineering Conference, Vol.III, 1975, pp. 2254-2270.
- 13 Ijima, T., Chou, C.R. and Yoshida, A.
 "Method of Analysis for Two-Dimensional Water Wave Problems", Proc. Fifteenth Coastal Engineering Conference, Vol. III, 1975, pp. 2717-2736.
- 14 Garrison, C. J.
 "Dynamic Response of Floating Bodies", Proc. Sixth Offshore Technology Conference, 1974.
- 15 Fraser, Glen A.
 "Dynamic Response of Moored Floating Breakwaters", M. Ap. Sc. Thesis, University of British Columbia, 1979.
- 16 Ijima, T. and Chou, C.R.
 "Analyses of Two-Dimensional Wave Problems by Means of Green's Identity Formula", Proc. JSCE, No. 252, Aug. 1976, pp. 57-71.
- 17 Ijima, T., Yoshida, A. and Yamamoto, T.
 "Two-Dimensional Motions of Moored and Floating Bodies with Arbitrary Cross-Section", Memoirs of the Faculty of Engineering, Kyushu University, Vol. 37, No. 3, Fukuoka, Japan, 1977, pp. 107-132.
- 18 Newman, J. N.
 "The Exciting Forces on Fixed Bodies in Waves", Journal of Ship Research, Vol. 6, No. 3, Dec. 1962, pp. 10-17.
- 19 Vugts, J. H.
 "The Hydrodynamic Coefficients for Swaying, Heaving, and Rolling Cylinders in a Free Surface", International Shipbuilding Progress, Vol. 15, No. 167, July 1968, pp. 251-276.
- 20 Burden, R. L., Faires, J. D., and Reynolds, A. C.,
 Numerical Analysis, Boston, Prindle Weber & Schmidt Inc., 1978.
- 21 Nece, R. E. and Richey, E. P.
 "Wave Transmission Tests of Floating Breakwaters for Oak Harbour", C. W. Harris Hydraulic Laboratory Technical Report 32, University of

Washington, Seattle, Washington, 1972.

22 Isaacson, M. de St. Q.

"Nonlinear Wave Forces on Large Offshore Structures", Coastal/Ocean Engineering Report, Department of Civil Engineering, University of British Columbia, 1981.

APPENDIX I

USER GUIDE FOR THE COMPUTER PROGRAM BWFIX

I.1 Introduction

This program was developed to time step a given incident wave past a fixed rectangular breakwater. The numerical method using Green's identity is described in Chapter 3 of this thesis. The required content and format for the input as well as the options and content for the output are given in detail below. The program is written in Fortran IV, and is stored in the file BWFIX. The compiled version of BWFIX is stored in BWFIX.C. Information concerning access to these files may be obtained by contacting:

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I.2 Running the Program

I.2.1 RUN COMMAND

To run the program the following command may be used:

```
$RUN BWFIX.C 5=datafile 6=-6 8=-8* 9=-9*
```

I.2.2 INPUT/OUTPUT DEVICES

The I/O devices are assigned as follows:

- UNIT 5 - Assigned to the datafile
- UNIT 6 - Assigned to the primary output file
- UNIT 8* - Assigned to the secondary output
file
- UNIT 9* - Assigned to the plot file

* optional - see LINE 1 under Required Input

I.3 Required Input

The input datafile is read on UNIT 5 and is specified in free format as follows:

LINE 1: NINIT,NDUMP,NPLOT,

NINIT = 0 for no initial segment grid
output

NINIT = 1 for output of initial segment
grid on UNIT 8

NDUMP = 0 for no intermediate value output

NDUMP = 1 for output of all intermediate
values of segment data (n , ϕ , etc.)
on UNIT 8

NPLOT = 0 for no plot data output

NPLOT = 1 for output of plot data on UNIT 9

Note: If NINIT and NDUMP = 0, UNIT 8 need not be
assigned in the \$RUN command.

If NPLOT = 0, UNIT 9 need not be assigned in the
\$RUN command.

LINE 2: NWVTP,NSTEP,NWAV,

NWVTP = 0 for incident wave given by linear

wave theory

NWVTP = 1 for incident wave given by
Stokes' fifth order wave theory

NSTEP = 1 for central difference time
stepping procedure

NSTEP = 2 for Adams-Bashforth two step time
stepping procedure

NSTEP = 3 for Adams-Bashforth three step
time stepping procedure

NSTEP = 4 for Adams-Bashforth four step
time stepping procedure

NSTEP = 5 for Adams-Bashforth five step
time stepping procedure

NWAV = number of fully developed waves to
interact with body

LINE 3: OMEGA,H,

OMEGA = angular frequency of incident wave
train

H = height of incident wave

LINE 4: D,B,BD,G,RHO,

D = still water depth

B = beam of breakwater

BD = draught of breakwater

G = acceleration due to gravity
RHO = density of water

LINE 5: DSL,DTT,DL,DSBLB,

DSL = global segment length to wavelength
ratio
DTT = time step to wave period ratio
DL = decay length of incident wave train
DSBLB = body segment length to body
dimension ratio

I.4 Output

The output from this program is as follows:

UNIT 6: Echos all the input values plus the following calculated values:

AK	=	wave number
C	=	wave speed
CG	=	initial group velocity of modulation envelope
DT	=	time step size
FSL	=	free surface length on each side of body
IT	=	total number of time steps
NBB	=	number of segments along body base
NBS	=	number of segments along each body side
NCS	=	number of segments along each control surface
NFS	=	number of segments along free surface at each side of body
NP	=	total number of segments
T	=	wave period
TTOT	=	total flow development time

In addition, the following values are printed at each time step:

$FHDIM = F / (.5 \rho g H A_k)$ - dimensionless
 horizontal force on breakwater
 $FVDIM = F / (.5 \rho g H B)$ - dimensionless
 vertical force on breakwater
 $FMDIM = M / (.5 \rho g H k B^3 / 12)$ - dimensionless
 moment about coordinate origin of
 breakwater
 $ETDIM =$ dimensionless free surface
 elevation at coordinate origin in
 absence of body
 $RNLDIM =$ dimensionless wave runup on left
 side of body
 $RNRDIM =$ dimensionless wave runup on right
 side of body

UNIT 8: If $NINIT = 1$, UNIT 8 stores the initial segment
 grid information before the time stepping
 procedure begins. The data stored for each
 segment is as follows:

$I =$ segment number
 $X(I) =$ x-coordinate at midpoint of segment
 I
 $Z(I) =$ z-coordinate at midpoint of segment
 I
 $XN(I) =$ x-direction cosine of $\underline{n}$ at midpoint

of segment I

$ZN(I) =$ z-direction cosine of $\underline{n}$ at midpoint
of segment I

$SLEN(I) =$ length of segment I

If $NDUMP = 1$, the information listed above (for $NINIT=1$) together with the following is stored at each time step:

$PH(I) =$ velocity potential at midpoint of
segment I

$PHN(I) =$ normal derivative of velocity
potential at midpoint of segment I

$ET(I) =$ free surface elevation of free
surface segment midpoint

UNIT 9: At each time step, UNIT 9 stores the forces, the theoretical free surface elevation at the origin without the body, and the wave runup at each side of the body. The results are stored in a format suitable for a plotting routine.

APPENDIX II

TIME STEPPING EQUATIONS

Given a first order differential equation of the form,

$$y' = f(x, t)$$

the following methods exist for approximate solutions:

Adams-Bashforth two step method:

$$y_{n+1} = y_n + \frac{\Delta t}{2} [3y'_n - y'_{n-1}]$$

Adams-Bashforth three step method:

$$y_{n+1} = y_n + \frac{\Delta t}{12} [23y'_n - 16y'_{n-1} + 5y'_{n-2}]$$

Adams-Bashforth four step method:

$$y_{n+1} = y_n + \frac{\Delta t}{24} [55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3}]$$

Adams-Bashforth five step method:

$$y_{n+1} = y_n + \frac{\Delta t}{720} [1901y'_n - 2774y'_{n-1} + 2616y'_{n-2} - 1274y'_{n-3} + 251y'_{n-4}]$$

For our given problem, the kinematic free surface boundary condition given by Equation (4a) in Chapter 2 gives us an expression for $\partial \eta / \partial t$,

$$\frac{\partial \eta}{\partial t} = \frac{1}{n_z} \frac{\partial \phi}{\partial n} \quad \text{on } S_f \quad (4a)$$

With known values of η_z and $\partial\phi/\partial n$ on the free surface at previous time steps, we obtain from the Adams-Bashforth two step method,

$$\eta_{t+\Delta t} = \eta_t + \frac{\Delta t}{2} \left\{ 3 \left(\frac{1}{\eta_z} \left(\frac{\partial\phi}{\partial n} \right) \right)_t - \left(\frac{1}{\eta_z} \left(\frac{\partial\phi}{\partial n} \right) \right)_{t-\Delta t} \right\}$$

Similarly, we can obtain corresponding time stepping equations for $\eta_{t+\Delta t}$ using the Adams-Bashforth three, four, and five step methods.

The dynamic free surface boundary condition given by Equation (5) in Chapter 2 gives us an expression for $\partial\phi/\partial t$,

$$\frac{\partial\phi}{\partial t} = - \left[g\eta + (\nabla\phi)^2 \right] \quad \text{on } S_f \quad (5)$$

For known values of η and $\nabla\phi$ at previous time steps, we easily obtain the time stepping formulae for $\phi_{t+\Delta t}$ as we did for $\eta_{t+\Delta t}$ above.

1(a): Rubble Mound Breakwater

1(b): Floating Breakwater

FIGURE 1: RUBBLE MOUND AND FLOATING BREAKWATERS

FIGURE 2: FLOW CHART FOR DYNAMIC ANALYSIS

FIGURE 3: CLOSED SURFACE FOR GREEN'S IDENTITY

FIGURE 4: TYPICAL INCIDENT WAVE PROFILE

FIGURE 5: DEFINITION SKETCH

FIGURE 6: GEOMETRICAL RELATIONSHIP BETWEEN $\underline{x}$, $\underline{\xi}$, AND $\underline{\xi}'$

FIGURE 7: SEGMENT GRID

FIGURE 8: FLOW CHART FOR COMPUTER PROGRAM

FIGURE 9: CPU SECONDS PER TIME STEP

FIGURE 10: EFFECT OF VARYING DECAY LENGTH

FIGURE 11: EFFECT OF VARYING TIME STEP SIZE

FIGURE 12: EFFECT OF VARYING GLOBAL SEGMENT LENGTH

FIGURE 13: EFFECT OF VARYING BODY SEGMENT LENGTH

FIGURE 14: EFFECT OF VARYING TIME STEPPING METHODS

FIGURE 15: WAVE THEORY RANGES OF VALIDITY

FIGURE 16: LINEAR AND STOKES' FIFTH ORDER WAVES

FIGURE 17; FLOW DEVELOPMENT WITHOUT BODY

FIGURE 18: TYPICAL HORIZONTAL FORCE DEVELOPMENT

FIGURE 19: TYPICAL VERTICAL FORCE AND MOMENT DEVELOPMENT

FIGURE 20: FORCE RESULTS FOR BEAM/DRAUGHT = 2.0

FIGURE 21: FORCE RESULTS FOR BEAM/DRAUGHT = 4.0

FIGURE 22: FORCE RESULTS FOR BEAM/DRAUGHT = 8.0

FIGURE 23: TRANSMISSION COEFFICIENT RESULTS