RELIABILITY OF SLENDER REINFORCED CONCRETE COLUMNS

by

RAJENDRA KUMAR BHOLA

B. Tech., Indian Institute of Technology, New Delhi, 1983

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF APPLIED SCIENCE

in

FACULTY OF GRADUATE STUDIES

Department of Civil Engineering

We accept this thesis as conforming

to the required standard

22

THE UNIVERSITY OF BRITISH COLUMBIA

January, 1985

Department of Civil Engineering

THE UNIVERSITY OF BRITISH COLUMBIA
2075 Wesbrook Place
Vancouver, Canada
V6T 1W5

Date: January, 1985

ABSTRACT

The effects of the variability in strength and loading on the reliability of slender, reinforced concrete columns are investigated using the Monte Carlo simulation technique. The columns are considered to be axially loaded with equal end eccentricities and no lateral load.

Variabilities in strength, axial load and eccentricity of axial loads are considered. A new procedure called the Implicit Uncorrelation Procedure has been developed to find the values of the failure function from the values of the basic variables named above.

The allowable axial load at various eccentricity levels corresponding to a probability of failure of one in one hundred thousand has been found for three different cross sections. Seven different slenderness ratios are considered for each cross section. The results are compared with those obtained by following the code procedures outlined in CAN3-A23.3-M77 and CSA-A23.3 (1984).

A change in the performance factor for moment magnification, ϕ_m , (as given in CSA-A23.3 (1984)) is recommended in order to obtain a more accurate and consistent level of reliability in the design of slender reinforced concrete columns.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ACKNOWLEDGEMENT

I wish to express my deepest gratitude to my supervisors, Professor N.D.Nathan, and Professor R.O.Foschi for their invaluable advice and guidance throughout the research and in the preparation of this thesis. Thanks are also due to all those who have rendered help in this project, directly or indirectly.

The financial support of the National Research Council of Canada in the form of a Research Assistantship is gratefully acknowledged.

1. INTRODUCTION

The actual strength of a reinforced concrete member differs from the nominal strength calculated by the design engineer due to variations in the material strengths and the geometry of the member, as well as the variabilities inherent in the equations used to compute the member strength. Similarly, designers use constant nominal values of loads in their calculation of forces, but the actual loads are variable. This variability in strength and loading is accounted for in one form or another in safety provisions of all existing building codes.

In this study the effect of these variables was investigated using the Monte Carlo technique for random simulation. Similar analyses have been applied to beams and columns by Allen(1970), Ellingwood(1977), Grant *et al* (1978) and Mirza and MacGregor(1982).

1.1 CODE METHOD

Cornell(1969) and Lind(1971) have shown that in order to achieve a consistent level of reliability in design, the code design criterion should be of the form:

$$\phi R \geq \lambda U$$

where:

R = design strength

U = nominal specified load

ϕ = strength reduction factor

λ = load factor

A summary of the actual procedure followed by the Code for the Design of Concrete Structures for Buildings (CSA-A23.3) is presented below.

1.1.1 METHOD OF CAN3-A23.3-M77

The factored design load (for only dead and live loads) is given by

$$U = \lambda_D D + \lambda_L L$$

where D and L are the nominal values of the dead and the live loads respectively and λ_D and λ_L are the corresponding load factors. The value for λ_D is taken as 1.4 while that for λ_L is taken as 1.7.

The short column strength of the column is reduced by a capacity reduction factor ϕ. The value of ϕ for pure bending is 0.90 and for axial compression or axial compression combined with bending it is 0.70. ϕ is linearly increased to 0.90 as the axial design load decreases from $0.10f_c' A_g$ to zero.

To account for the slenderness effect of the columns, the members are designed using a magnified moment M_c defined by:

$$M_c = \delta M_2$$

where M_2 is the larger design end moment on the member, based on U as described above, and is calculated from a conventional elastic frame analysis and δ is a moment magnification factor. M_c must be less than the reduced short column strength. δ is given by the relation:

$$\delta = \frac{C_m}{1 - P_u/\phi P_c} \geq 1.0$$

where ϕ varies as stated above, and

$$P_c = \frac{\pi^2 EI}{(kl_u)^2}$$

where kl_u is the effective length of the column.

EI is calculated as:

$$EI = \frac{0.2E_c I_g + E_{se} I_s}{1 + \beta_d}$$

following the notation of CAN3-A23.3-M77.

C_m is given by:

$$C_m = 0.6 + 0.4\frac{M_1}{M_2}$$

where M_1 and M_2 are the smaller and the larger design end moments respectively at the two ends of the member.

1.1.2 METHOD OF CSA A23.3(1984)

The total design load for the case when only the dead load and the live loads are acting is given by a relation similar to the one in CAN3-A23.3-M77 except that in this case λ_D is taken as 1.25 while λ_L has the value 1.50.

Material resistance factors are applied to the nominal strengths of concrete and reinforcing bars as shown in figure 1. The value for ϕ_c is 0.60 while that for ϕ_s is 0.85.

The slenderness effect is taken into account using a procedure identical to the one in CAN3-A23.3-M77 but

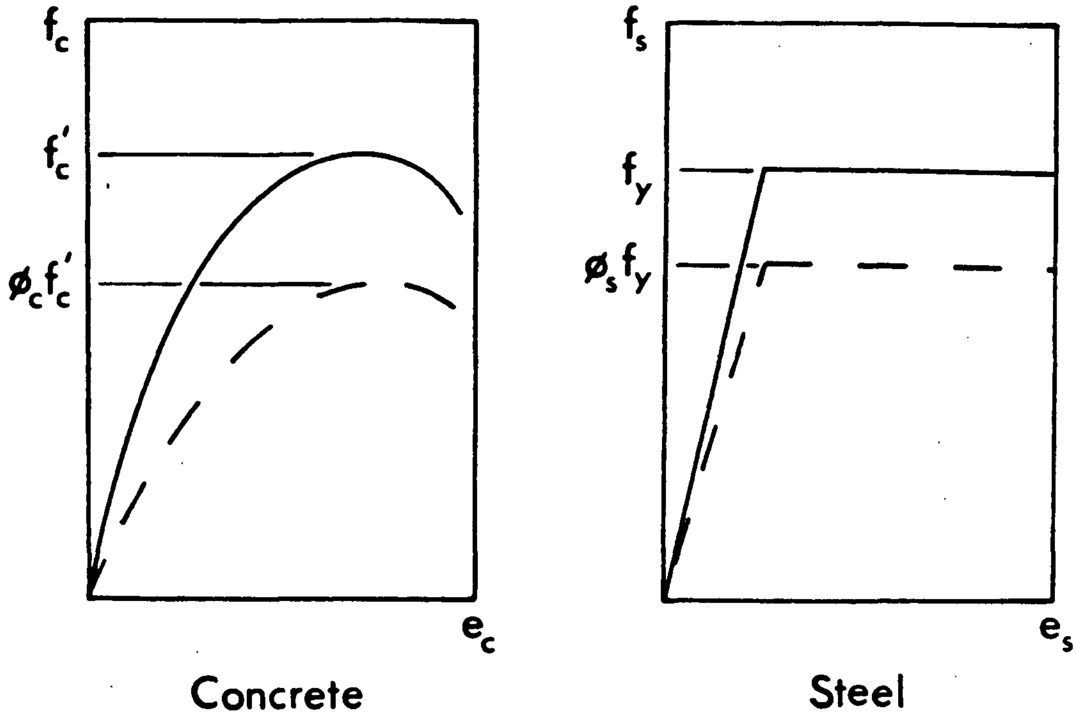

Figure 1. Material resistance factors

with the moment magnification factor defined as:

$$\delta = \frac{C_m}{1-P/\phi_m P_c} \geq 1.0$$

where $\phi_m = 0.65$.

2. MONTE CARLO SIMULATION

2.1 INTRODUCTION

If a relationship can be derived between the performance of a system and each variable affecting the performance, and if statistical properties of the distributions of all the variables are known, it is possible to use randomly selected values of the variables to calculate the variability of the system performance. This technique, shown schematically in figure 2 and called Monte Carlo simulation, was used in this study to determine the variability of the strength of reinforced concrete columns, because of the complexity of the strength relationships.

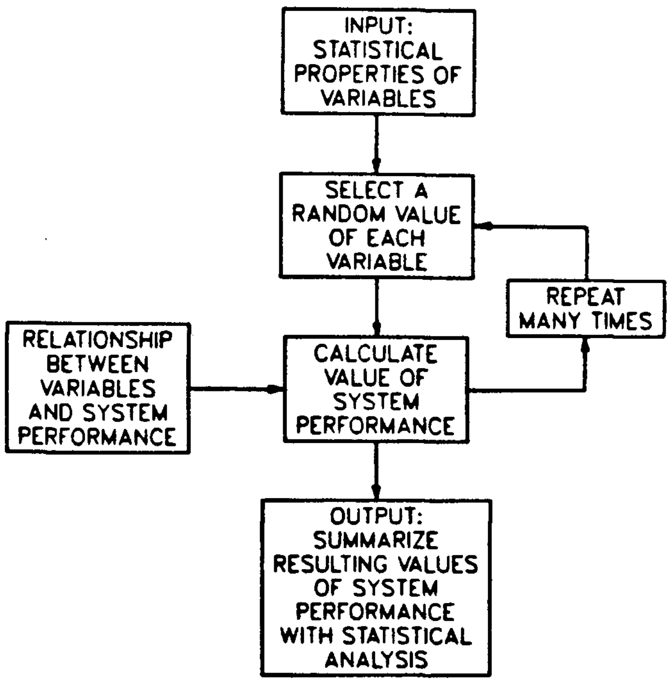

Figure 2. The Monte Carlo simulation technique

2.2 DESCRIPTION OF THE METHOD

The Monte Carlo method may be described as a means of solving problems numerically in mathematics, physics, engineering, and other sciences through sampling experiments. The problem may be posed in either probabalistic or deterministic form. In the probabalistic case the actual random variable or function appearing in the problem is simulated, whereas in the deterministic case an artificial random variable or function is first constructed and then simulated. The simulation process is computerised to follow the distribution properties of the variable. The method normally consists of the following steps:

1. simulation of the random variable function,
2. solution of the deterministic problem for a large number of realizations of the latter, and
3. statistical analysis of the results.

The present chapter and the two following it discuss each of these steps in detail.

2.3 VARIABILITY OF STRENGTH

In order to evaluate the variability of the strength of slender reinforced concrete members a knowledge of the variability of the parameters that affect the strength is necessary. These parameters are the concrete strength in compression and tension, the yield strength and the position of the reinforcement, and the dimensions of the cross section of the member. The variabilities of these parameters used in this study were based primarily on the data summarized by Mirza *et al* (1979) and Mirza and MacGregor (1979a,b).

Three major assumptions were made in determining the material strengths to be used in the derivation of the strength of the reinforced concrete columns.

1. The variability of the concrete properties and dimensions corresponds to average quality construction. This assumption is made so that the results may represent the average of Canadian construction practice. Similarly, the reinforcement was assumed to be drawn from a population representing all sources of reinforcement in Canada and the United States.
2. The material strengths were assumed to correspond to lower loading rates than those generally used in material tests or laboratories. The crushing strength of concrete was based on a 1-hour loading to failure, and the yield strength of steel was based on a so-called static loading rate. This is a conservative assumption since the strengths of concrete and steel tend to increase at high rates of loading.
3. Increase in the long-time strength of the concrete due to increased maturity of the concrete, as well as possible future corrosion of the reinforcement were ignored. Gardiner and Hatcher (1970) and Washa

and Wendt(1975) report that the mean strength of 20-25 year old concrete is expected to be 150-250% of the mean strength at 28 days. For this study however, this effect was disregarded and the concrete strength was related to the 28-day test cylender strength. This leads to a conservative estimate of member strength.

2.3.1 PROPERTIES OF CONCRETE

Under current design, production, testing, and quality-control procedures, the strength of concrete in a structure may differ from its specified design strength and may not be uniform throughout the structure. The major sources of variation in concrete strength are the variations in material properties and proportions of the concrete mix, the variations in mixing, transporting, placing and curing methods, the variations in testing procedures, and variations due to concrete being in a structure rather than in control specimens. The following data, as suggested by Mirza *et al*(1979), accounts for most of these effects.

Assuming a rate of loading corresponding to failure in a test lasting 1 hour, the mean compressive strength of concrete in a structure was taken as 23.36 MPa (3388 psi) for 27.6 MPa (4000 psi) concrete. The coefficient of variation for the cast-in-place concrete was taken as 0.175.

The mean value of the modulus of elasticity for the 27.6 MPa concrete was taken as 22476.91 MPa (3260 ksi) with a coefficient of variation of 0.12.

These properties were assumed to follow normal distributions.

2.3.2 PROPERTIES OF REINFORCEMENT

The sources of variation in the steel yield strength are the following:

1. Variation in the strength of the material itself.
2. Variation in the area of cross section of the bar.
3. Effect of the rate of loading.
4. Effect of bar diameter on the properties of the bars.
5. Effect of strain at which yield is defined.

The following data, as suggested by Mirza and MacGregor (1979b), accounts for the effect of most of these sources of variation.

The mean and the coefficient of variation for the static yield strength for the steel reinforcement were taken as 460.6 MPa (66.8 ksi) and 0.09, respectively, for grade 60 hot rolled bars. The yield strength was assumed to follow a Beta distribution. These values were assumed to be independent of bar size.

The mean value for the modulus of elasticity was taken as 201000 MPa (29200 ksi) with a coefficient of variation, 0.033. The probability distribution of the modulus of elasticity was considered to be normal.

The ratio of actual to nominal values of the area of cross section of the bars was assumed to follow a normal distribution truncated at 0.94 with a mean value of 0.99 and a coefficient of variation 0.024.

Since the steel in a concrete member must be some combination of whole bars, the area of steel actually provided may differ from that calculated. As suggested by Mirza and MacGregor(1979a), this effect has been considered by a modified lognormal distribution having a mean of 1.01, a coefficient of variation of 0.04, and a modification constant of 0.91 below which the modified lognormal distribution equals zero.

The variability of strength within a single bar is relatively small and is neglected in this study.

2.3.3 GEOMETRIC PROPERTIES

Geometric imperfections in reinforced concrete members are mainly caused by deviation from the specified values of the cross sectional shape and dimensions, the positioning of reinforcing bars, the horizontality and verticality of concrete lines, the alignments of columns and beams, and the grades and surfaces of the constructed structures. The data used to account for these effects in this study were as suggested by Mirza and MacGregor(1979a). It has been assumed that a normal distribution can be used to represent the distribution of the geometric imperfections of reinforced concrete members.

The mean deviation of the actual cross sectional dimensions from the specified dimension was taken as +1.52 mm (+0.06 in)[1]. The standard deviation of the cross sectional dimensions was taken as 6.35mm (0.25 in).

The location of vertical reinforcement in columns is affected by tolerances in the ties, forms, column alignment from floor to floor and care taken to center the reinforcement cage within the form. The mean deviation of concrete cover for steel bars was taken as +8.13 mm (+0.32 in)[2]. The standard deviation of the concrete cover for steel bars was taken as 4.32mm (0.17in).

2.4 VARIABILITY OF LOADS

In the analysis of safety it is necessary to deal with load effects such as moments, etc. rather than the loads themselves. It is therefore necessary to have a distribution of load effects. These are found by combining the

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[1] i.e. specified dimension + 1.52 mm.

[2] i.e. specified cover + 8.13 mm.

variability of the loads themselves with the variability introduced by the structural analysis. The latter component is small and can be ignored except in the case of dead load. The variability of the loads themselves is in turn the combined variability of the magnitude and the distribution of the load, and this influences the load effects on the columns. In the following, variability from both the components is considered.

2.4.1 VARIABILITY OF DEAD LOADS

Except in cases where the lower parts of the building have to be designed before the upper part is well defined, dead loads are known accurately in comparision with other loads. The ratio, R_D of actual to nominal dead load was represented by a normal distribution with the mean equal to 1.05 and the coefficient of variation equal to 0.07. This was based on studies of the variability of dead load effects in concrete structures resulting from variations in dimensions, densities, superimposed loads, and analysis (Ellingwood *et al* 1980). Allen(1975) assumed values of 1.0 and 0.07 whereas Lind *et al* (1978) used values of 1.0 and 0.05. Nowak and Lind(1979) assumed the values as 1.05 and 0.08 for site-cast concrete bridge structures.

2.4.2 VARIABILITY OF LIVE LOADS

The ratio, R_L of the maximum 30 year live load to the nominal load was assumed to have a mean of 0.70, and as suggested by Allen(1975) is independent of the tributary area. On the basis of load survey results of Mitchell and Woodgate(1971), the coefficient of variation of maximum live load is taken to be 0.30, and is independent of the tributary area. Nowak and Curtis(1980) suggest a gamma distribution for the live loads, while Allen(1975) does not specify the distribution. In the present study an extreme type I distribution was assumed for the maximum live load in 30 years.

2.4.3 LOAD COMBINATION

It is important to combine the different load effects properly so as to achieve a more realistic assessment of reliability. Load effects are usually random functions of time. When the design is resisting gravity loads, one possible load combination is the dead load (which would be constant in time) and the maximum live load (or the maximum occupancy load) in the lifetime of the structure. This combination of the loads has been considered in this study with the nominal values of the live and the dead loads equal to each other.

Define a dead load ratio factor, a as:

$$a = \frac{\text{Nominal dead load}}{\text{Total nominal load}}$$

where the total nominal load is the sum of the nominal values of the dead and the live loads. Hence for our case $a=1/2$.

Two more cases with $a=1/3$ and $a=2/3$ (i.e. with the nominal value of the live load equal to twice the nominal value of the dead load, and vice-versa) were also considered for one cross section in order to investigate the effect of this factor on the reliability.

The load effects of wind, snow and earthquake have not been considered.

No load factors have been applied.

The load combination procedure may be summarized as follows:

$$D + L = R_D\ D_N + R_L\ L_N$$

where

D = actual dead load

L = actual live load

D_N = nominal dead load

L_N = nominal live load

and R_D and R_L are as described previously.

Then

$$D + L = L_N\ \{R_D\ (\frac{D_N}{L_N}) + R_L\ \}$$

Now we have:

$$a = \frac{D_N}{D_N + L_N}$$

Rearranging

$$\frac{D_N}{L_N} = \frac{a}{1-a}$$

Hence, the load effect can finally be written as:

$$D + L = L_N \{R_D \left(\frac{a}{1-a} \right) + R_L \}$$

This formulation was used to simulate the load effect for this study.

3. STRENGTH MODEL

3.1 INTRODUCTION

This chapter describes a theoretical model for predicting the strength of slender reinforced columns. The model uses the stress strain behaviour for concrete as given by Desai and Krishnan (1964). The stress strain relationship for steel is assumed to be elastic-perfectly plastic.

The theory and assumptions in the model are described, along with a basis for the computer program used to obtain the strength of slender, reinforced concrete columns.

The organization of the program is similar to one developed by Nathan (1972) for reinforced and prestressed concrete, and verified for those materials by Alcock and Nathan(1977).

3.2 ASSUMPTIONS

The following assumptions are made :

1. Plane sections remain plane.
2. Material properties are constant along the length of the column.
3. Dimensions and error in placing of steel bars are constant along the length of the column.
4. If moment varies along a member, failure occurs at the cross section subjected to maximum moment, or by instability of the member.
5. Bending in only one plane is considered.

6. Effects of shrinkage are neglected.
7. No torsional or out-of-plane deformations are considered. Duration of load effects are not considered. Shear failures are not considered.
8. There is no slip between the concrete and the reinforcing steel.

3.3 CROSS SECTION BEHAVIOUR

The ultimate interaction diagram and the moment-curvature-axial load relationships for a cross-section are derived by using a simple step-by-step procedure to obtain axial load and moment capacities for a range of neutral axis depths and curvatures. Recall that the ultimate interaction diagram shows the limiting combinations of axial load and bending moment that a section can resist.

3.3.1 ULTIMATE INTERACTION DIAGRAM

For the reinforced concrete cross section such as the one shown in figure 3(a), the calculation begins by considering the top fibre at ultimate strain (failure strain for concrete). The neutral axis is then marched across the section. A typical location of the neutral axis produces a strain distribution across the section as shown in figure 3(b). The following procedure is then used to determine what combination of axial load and bending moment would produce this condition.

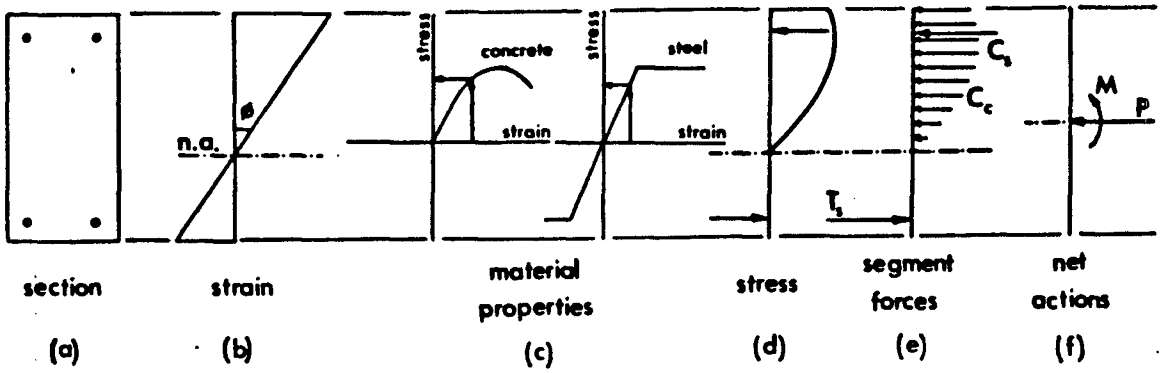

Figure 3. Cross section behaviour

The depth of the section is divided into a number of segments. The strain at the mid-height of each segment is evaluated. The appropriate stress-strain law is then used to find the stress for each segment as shown in figure 3(d). This stress is multiplied by the area of the segment to give the force acting on each segment, as shown in figure 3(e). The forces in the steel bars, C_s and T_s are found in a similar manner.

The forces for all the segments are added to give the required axial force, P. The forces in the segments are multiplied by the distance from the mid-height of the segment to the centroidal axis, and then added to give the bending moment, M, acting on the cross section.

At this stage, the following information is stored, before repeating the above calculation with increased neutral axis depth. The stored information is:

1. Curvature (input)
2. Net axial load (output)

3. Bending moment (output)

For a number of neutral axis locations, the procedure described above is repeated, till the neutral axis has marched across the section.

Hence we obtain the axial load-bending moment interaction diagram. The ultimate interaction diagram for a section of dimensions 500mm X 500mm with 2% steel is shown in figure 4. Any point inside the curve represents a combination of axial force and bending moment that the cross section can resist. Any point outside the curve represents failure.

3.3.2 CURVATURE CONTOURS

A number of values for curvature (30 in this case) are selected between zero and ϕ_{max}, where ϕ_{max} is the maximum curvature on the ultimate interaction diagram. The curvature contours are found for each of these curvature values. The procedure followed for finding the curvature contours is described below.

For a selection of curvature, the neutral axis is marched across the cross section. The net axial load and bending moment for each of these neutral axis locations is found by a procedure similar to the one described in the previous section. Figure 5 shows the relationship between axial force and bending moment for 30 curvature contours. Each line represents a single value of section curvature. Different points on a line of constant

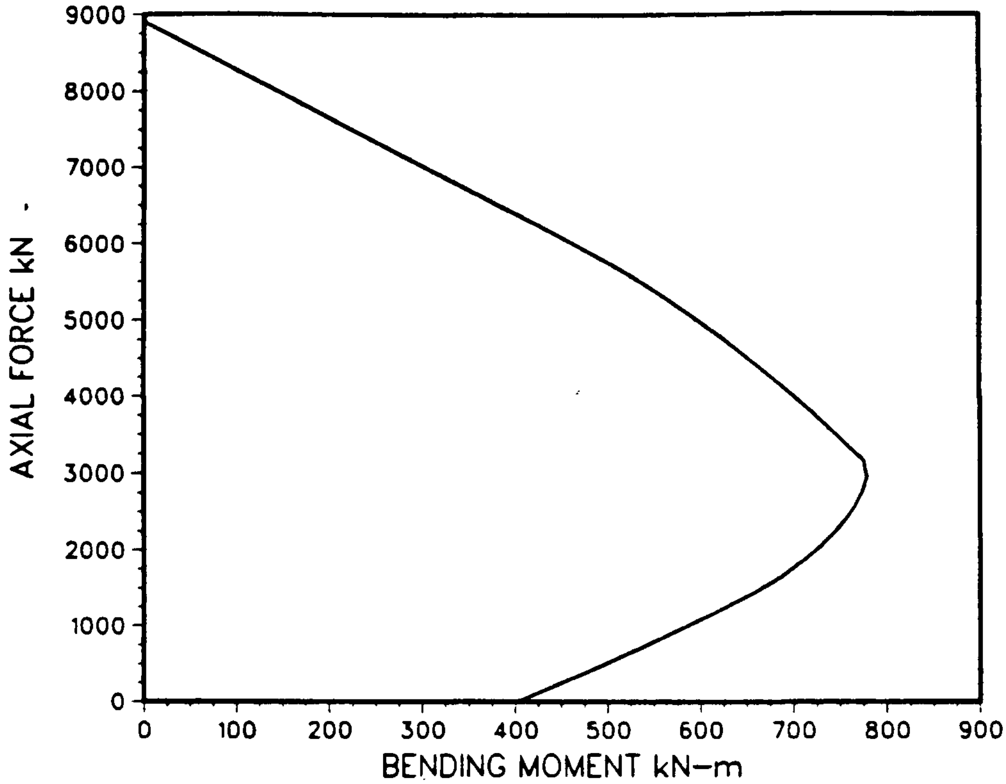

Figure 4. Ultimate interaction diagram

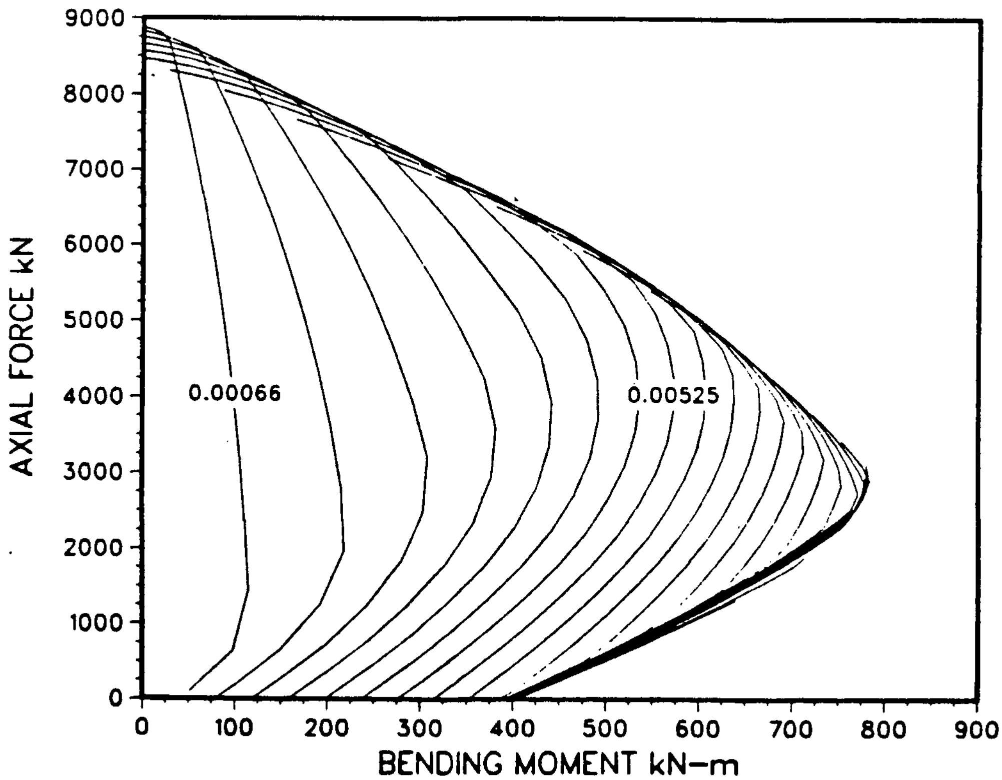

Figure 5. Curvature contours

curvature represent the combinations of axial load and moment required to produce that curvature for various neutral axis locations. Note that figure 4 is the envelope of all the points shown in figure 5.

3.3.3 MOMENT-CURVATURE CURVES

Figure 6 shows the moment-curvature relationships for several levels of axial loads,P. All the curves start at the origin but some have been shifted for clarity of presentation. P_0 is the axial compression strength of the column. These curves are computed from already stored data. A horizontal line on figure 5

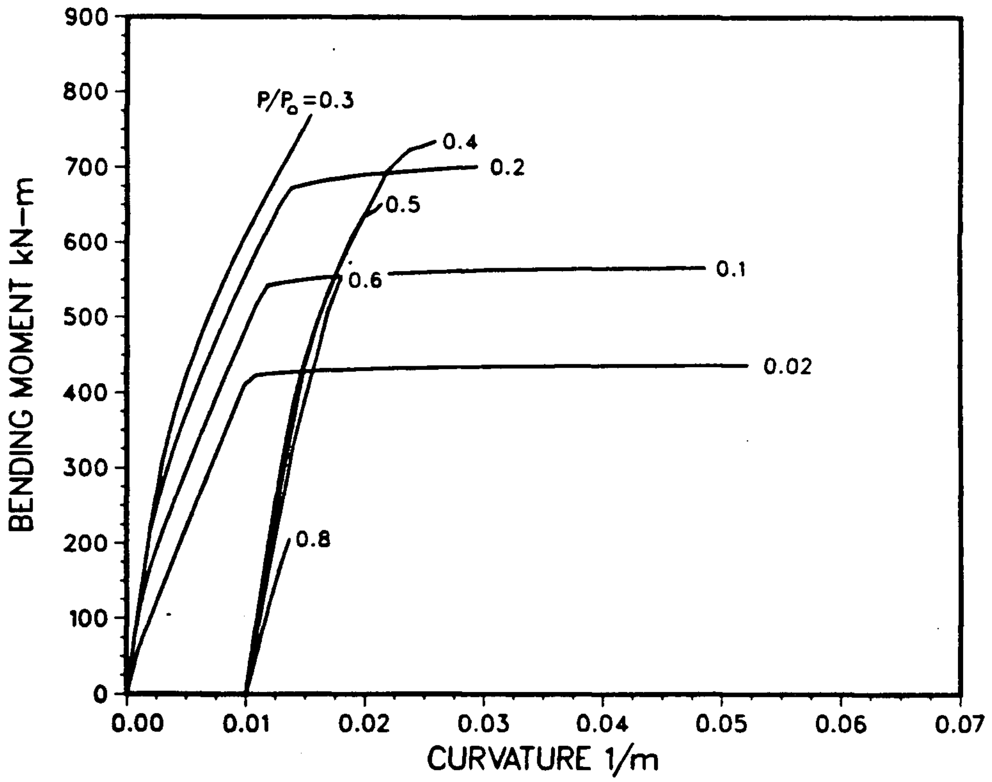

Figure 6. Moment-curvature-axial load relationships

represents a level of axial load. Each intersection of this horizontal line with a curvature contour provides a value of bending moment and curvature, which is plotted on figure 6. The final point for each moment-curvature curve represents the point on the ultimate interaction curve for that level of axial load. For high values of axial load, a falling branch of the moment-curvature curve exists, beyond the maximum moment, but this is not plotted as this information is not used in subsequent calculations.

3.4 SLENDER COLUMN BEHAVIOUR

This section describes the procedure used to establish the behaviour of columns of any length under the action of eccentric axial loads with equal end eccentricities and no lateral load as shown in figure 7.

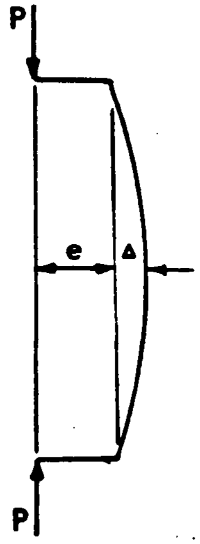

Figure 7. Column with axial load and equal end eccentricities

3.4.1 FAILURE MODES

In order to facilitate the understanding of the computer model, the behaviour of eccentrically loaded compression members will be reviewed in this section. Consider the possible behaviour of the member shown in figure 7, as the axial load P is increased to failure. If the end eccentricity for the axial load is e, then the bending moment at the ends is Pe, while the bending moment at the mid-span is $P(e+\Delta)$, where Δ is is the deflection at mid-span, as shown.

Figure 8 is an interaction diagram of axial load vs bending moment. The outer curved line is the ultimate interaction curve for the cross-section, and it represents material failure. Consider a load path for axial load and end moment, as axial load P is increased, represented by the line O-A, to P_1. The corresponding load path for **mid-span moment** is shown by the curved line O-B.

The horizontal distance between lines O-A and O-B represents the amount by which the initial mid-span moment, Pe has magnified to $P(e+\Delta)$. In this case the member fails at an axial load P_1, when the mid-span load path O-B intersects the material strength interaction diagram at the point B. This is described as **material failure** .

If the same member is loaded with a smaller eccentricity, the load path for end moments could be

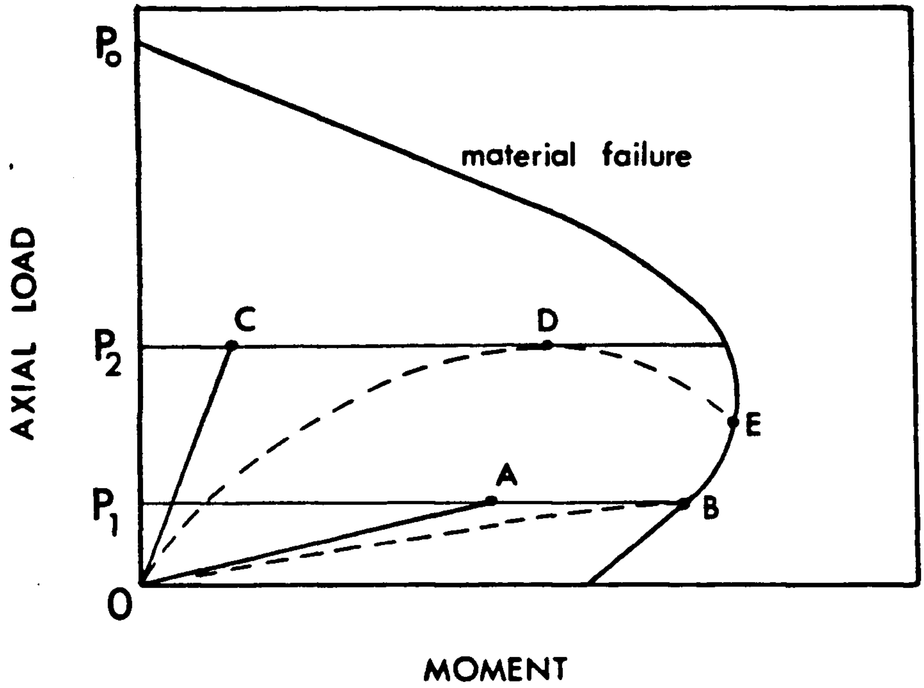

Figure 8. Behaviour of an eccentrically loaded column

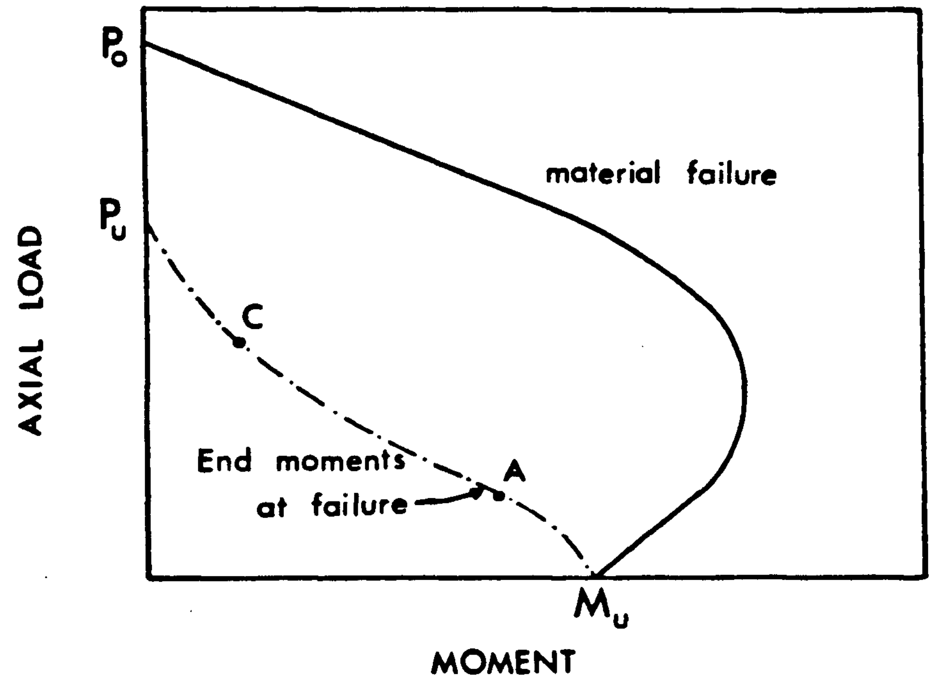

Figure 9. Interaction diagram for an eccentrically loaded column

shown by line O-C, and the load path for mid-span moments by line O-D. In this case an **instability failure** occurs when the axial load reaches a maximum value P_2. The mid-span moment at failure is shown by point D, which is well inside the material strength curve. If the member were loaded with a system under load control (for example, gravity loads) to load P_2, deformations would increase rapidly and a material failure would follow immediately. If the member were loaded under conditions of controlled displacement the load path shown by the extension of line O-D would be followed to eventual material failure at point E.

If this process is repeated many times for the same member using a range of eccentricities, figure 9 can be produced. The solid line is the ultimate interaction diagram. The chain-dotted line P_u-C-A-M_u is the locus of points such as A and C in figure 8, representing combinations of axial load and **end moment** (unmagnified moment) just causing failure.

3.4.2 CALCULATION PROCEDURE

A computer program for calculating points on the curves shown in figure 8 is described below.

The program can consider a column of any length made up of a number of segments of equal length. A method described by Galambos(1968) is used to develop column deflection curves for a given axial load, to determine the maximum end eccentricity, e, at which that load can be applied to that column.

As the column is symmetrically loaded, the slope is considered to be zero at mid-span. For the axial load under consideration, the moment at mid-span is initially set to the material failure moment for that load (a point on the ultimate interaction diagram). The corresponding mid-span deflection, $e+\Delta$, (from the line of axial load) is the failure moment divided by the axial load. To find the actual values of e and Δ it is necessary to calculate the deflected shape of the member. A column deflection curve is obtained by proceeding along the column, segment-by-segment from mid-span, calculating the deflection at each node.

Consider the calculations for a typical segment of length Δx, such as that shown in figure 10. If the deflection v_0 and slope v_0' are known at the starting node x_0, then the moment M_1, at the mid-point of the segment (point x_1) is approximately

$$M_1 = P(v_0+v_0' \cdot \Delta x/2)$$

The curvature, ϕ_1, at point x_1 can be obtained from the moment-curvature-axial load relationship (figure 5). The curvature is assumed to be constant along the segment. The deflections are assumed to be small such that $\phi=v''$.

The displacement, v_2, and slope, v_2', at the next node, x_2 are calculated from

$$v_2 = v_0 + v_0'(\Delta x) - \phi_1(\Delta x)^2/2$$

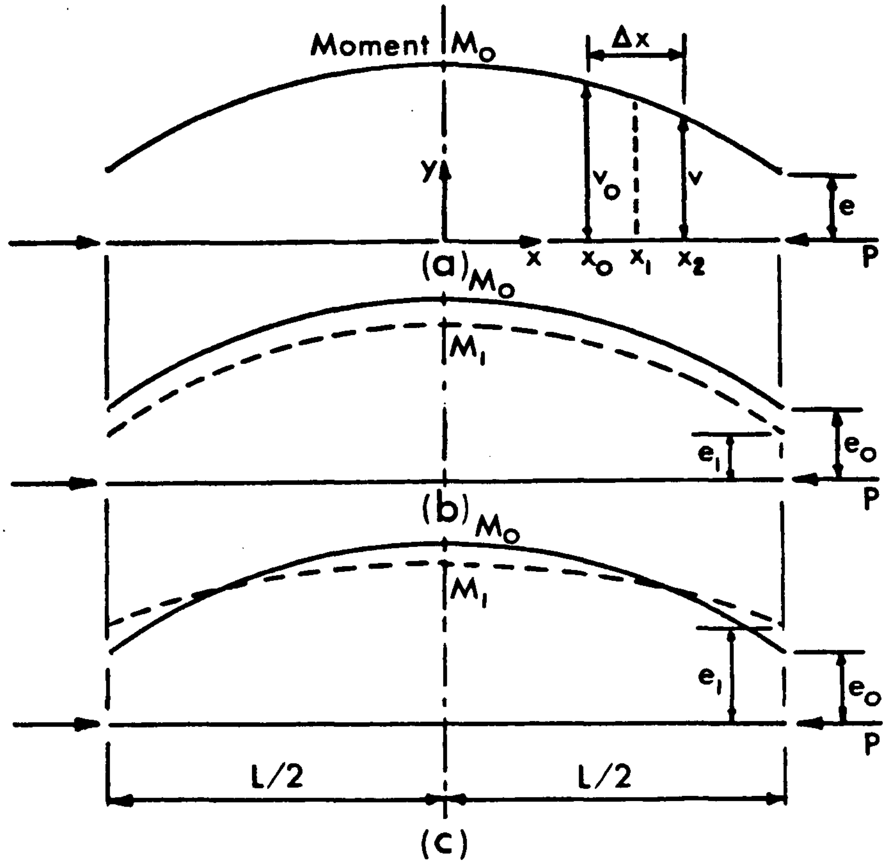

Figure 10. Column deflection curves

and:

$$v_2' = v_0' - \phi_1(\Delta x)$$

The moment M_2 at node x_2 is the product of P and v_2.

For the first calculation of the column deflection curve, (with the moment at mid-span equal to the failure moment at that load) the end eccentricity is calculated to be e_0. The calculation is then repeated for a lower mid-span moment. If the new end eccentricity, e_1, is less than e_0 (figure 10(b)), then this also represents material failure and this calculation is ignored. However, if e_1 is greater than e_0 (figure 10(c)) then this represents instability failure, and the calculation is repeated with smaller reductions in the mid-span moment till a peak value for the end eccentricity is observed. This peak value is then the end eccentricity at which instability failure occurs, and when multiplied by the axial load, it gives the end moment for instability failure.

3.5 PROGRAM INPUT

The following information is required as input to this computer model. Each of these items will be discussed in detail below.

1. Cross section properties:
   a. Cross section dimensions
   b. Properties of concrete
   c. Properties of reinforcement
2. Slenderness effect:
   a. Column length
   b. Segment length for column deflection curves
   c. Reduction ratio for mid-span moment

3.5.1 CROSS SECTION PROPERTIES

The program can be used to obtain column properties for any polygonal shape of cross section up to twenty corners. However, for the present study it has been used only for rectangular sections.

The program can consider various stress strain relationships for concrete as well as for steel.

However, for this study the stress strain relationship for concrete in compression is assumed to follow the modified Hognestad relationship, as presented by Desai and Krishnan (1964), by the following relation:

$$\frac{f}{f_0} = \frac{2(\epsilon/\epsilon_0)}{1+(\epsilon/\epsilon_0)^2}$$

Here f_0 and ϵ_0 are the peak stress and peak strain respectively. The compressive strength of concrete, f'_c is input, along with the modulus of elasticity, E_c . The peak strain is calculated by the following relation suggested by Desai and Krishnan(1964):

$$\epsilon_0 = \frac{2f'_c}{E_c}$$

After an ultimate strain of 0.0038 the concrete was assumed to have no strength even though it may still be confined by the column ties and capable of supporting some load. The strength of concrete in tension is neglected. The Youngs Modulus and the yield stress for the steel reinforcement are input. The reinforcement is assumed to follow the elastic-perfectly plastic stress-strain relationship. The elastic-perfectly plastic stress-strain relationship is somewhat conservative because the effect of strain hardening is neglected.

3.5.2 SLENDERNESS EFFECT

The program can handle a column of any length and slenderness ratio. The effect of the slenderness on the strength of the column is accounted for as described earlier.

Chen and Astuta(1976) have shown that a segment length of four times the radius of gyration gives sufficiently accurate results. For a rectangular section this corresponds to 1.16 times the section depth. A segment length equal to the section depth has been considered throughout this study.

The program also requires as input the rate at which the mid-span moment is to be reduced in the step-by-step procedure for the construction of the column deflection curves. A value of 0.05 times the maximum moment has been used.

3.6 PROGRAM OUTPUT

The output from this computer model typically consists of axial load-end moment interaction curves for several column lengths. The calculations described in the previous sections have been carried out for columns of several slenderness ratios, each at eight different levels of axial load between 0.02 to 0.60 times the maximum load. Figure 11 shows an interaction diagram showing the combination of axial load P and **end** moment Pe just producing failure for a typical 500mm X 500mm cross section column with 2% steel.

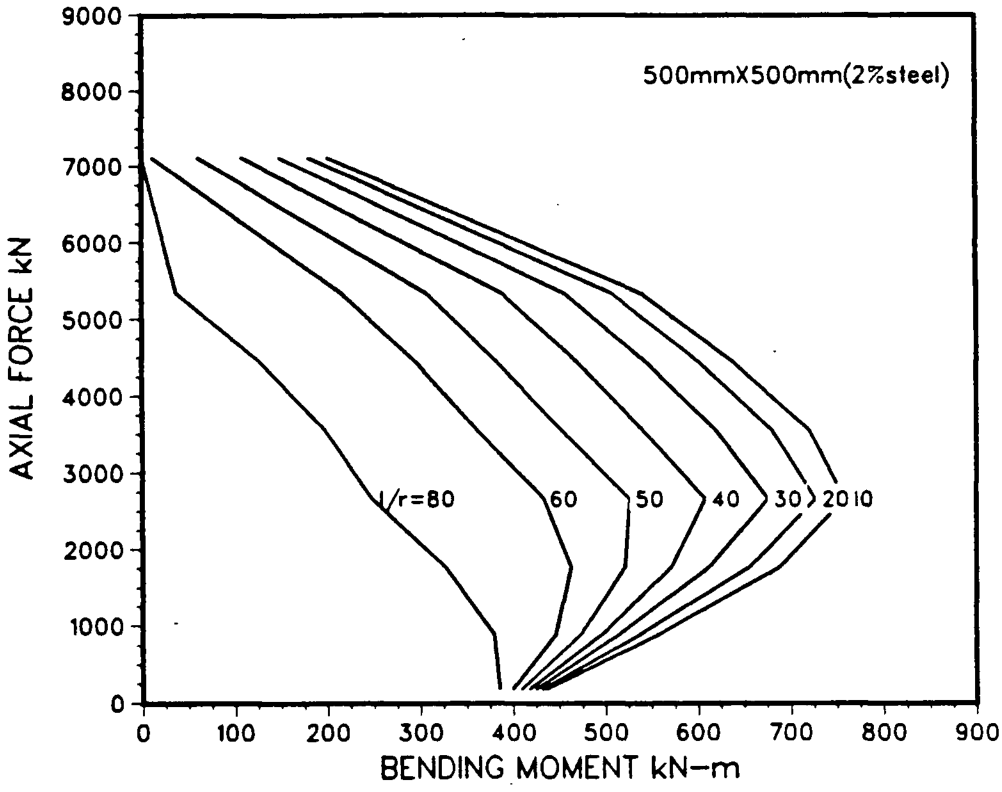

Figure 11. Strength interaction curve for slender columns

The outer line is the ultimate interaction diagram representing material strength (slenderness effects for a slenderness ratio of 10 are negligible). The inner curves correspond to the curve P_u-C-A-M_u in figure 9. For low slenderness ratios the inner curves are close to the ultimate interaction curve. As the slenderness ratio increases, the curves move inside the ultimate interaction diagram, indicating that behaviour under these loads is governed by instability failures.

These curves have been plotted directly from the computer output. Only eight levels of axial load were considered and linear interpolation was used to find the limiting bending moment for any intermediate axial load. This is considered to give sufficiently accurate results for the purpose of this study.

4. RELIABILITY EVALUATION

4.1 INTRODUCTION

This chapter describes the theory behind the method used to evaluate the reliability of the reinforced concrete columns, as well as the basis for the computer program used.

The evaluation of the reliability is done by finding the value of the reliability index for a number of values for the nominal axial load. Linear interpolation is then used to find the value of the allowable nominal axial load corresponding to the target reliability.

4.2 RELIABILITY INDEX CONCEPT

To facilitate a better understanding of the theoretical model for the reliability program, the safety index concept is briefly reviewed. The description presented below is similar to the one by Foschi(1979).

A design problem in structural analysis typically has a set of relevant variables $(X_1, X_2, \ldots, X_n)$, some of which are related to the resistance of the material, while others are related to the effect of the design loads. These variables must satisfy a design equation of the form

$$G(X_1, X_2, \ldots, X_n) \leq 0$$

In general, several such conditions could be considered, each of them defining the appropriate *limit states*.

In general, the variables X are random variables obeying some distribution function. Hence the design process is not deterministic, and the design conditions will not be satisfied for all values of the variables X. The design strategy usually followed is to specify a 'tolerated' range for which the variables should satisfy the design conditions. This gives a 'reliability' range, which can be expressed in terms of mathematical probabilities, provided the individual distributions of the design variables X are known. Veneziano (1974) describes some methods of doing this in detail.

Consider the simple case of only two design variables, R and U. R is the resistance for the problem, while U represents the load effect. R and U are called 'basic variables' of the design problem (Hasofer and Lind (1974)).

The failure criterion can be written as

$$U \geq R$$

where the equality would represent the boundary between survival and failure. If $\bar{U}$ and σ_U are the mean and standard deviation of U, and $\bar{R}$ and σ_R are the mean and standard deviation of R, we can define non-dimensional random variables x and y such that

$$x = \frac{(R - \bar{R})}{\sigma_R}$$

$$y = \frac{(U - \bar{U})}{\sigma_U}$$

Hence the failure criterion can be written as

$$y \geq \frac{(\bar{R}-\bar{U})}{\sigma_U} + x\left(\frac{\sigma_R}{\sigma_U}\right)$$

Note that this is the equation of a straight line with a slope σ_R/σ_U and an intercept $(\bar{R}-\bar{U})/\sigma_U$ on the y-axis as shown in figure 12.

If the variables R and U are not correlated, the points (x,y) representing a combination (R,U) will be distributed over the entire x-y plane. The failure criterion divides the plane into two parts. The upper part corresponds to combinations of R and U producing failure and the lower part corresponds to combinations representing survival.

The minimum distance between the origin O, corresponding to the mean values of R and U, and the boundary I-I, the 'failure surface' is given by the segment

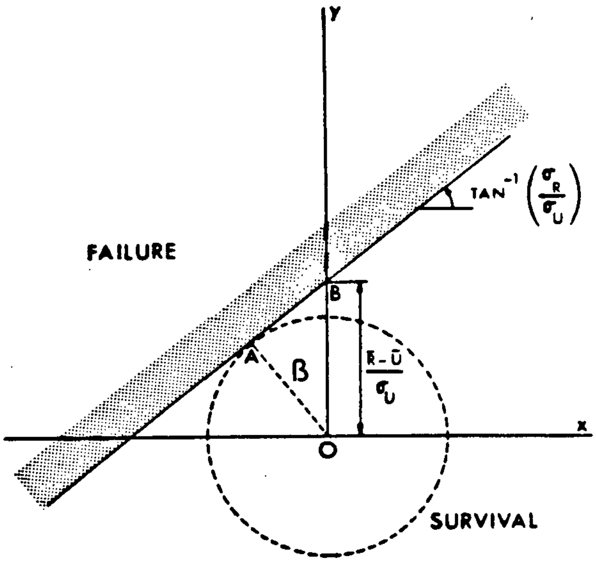

Figure 12. Definition of the reliability index

OA. This minimum distance, defined as the reliability index β, can be calculated from the figure as

$$\beta = \frac{(\bar{R} - \bar{U})}{\sqrt{(\sigma_R^2 + \sigma_U^2)}}$$

The point A is called the most likely failure point. Any combination of R and U below the failure surface (which is at a minimum distance of β from the mean point) **will not produce failure.** This is a reliability statement on the design problem and therefore, β may be used as a reliability measure, which can be related to the probability of failure, P_f . Even for the case when there are more than two basic variables and when the failure function, G, is non-linear, β is defined as the minimum distance from the mean point to the failure surface.

If we define:

$$Y = R-U$$

the failure criterion becomes the event $Y \leq 0$. Then

$$\bar{Y} = \bar{R}-\bar{U}$$

and

$$\sigma_Y = \sqrt{(\sigma_R^2 + \sigma_U^2)}$$

Hence, β can be written as

$$\beta = \bar{Y}/\sigma_Y$$

Hence, as shown in figure 13, the measure β becomes the number of standard deviations, σ_Y , that the mean $\bar{Y}$ is from the failure event Y=0. The probability of failure P_f is then given by $P(Y \leq 0)$, and corresponds to the area of the shaded portion of figure 13. If R and U are normally distributed, Y

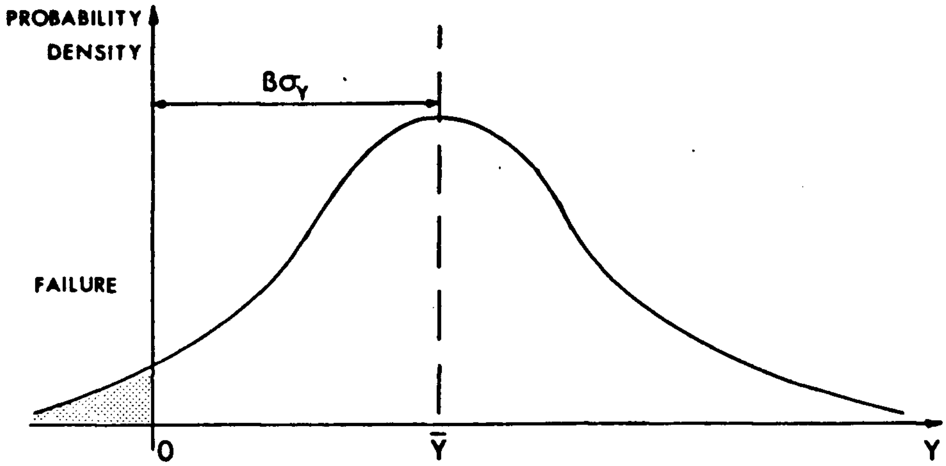

Figure 13. Reliability Index and the probability of failure

will be normally distributed and P_f can easily be obtained from the normal tables, knowing the value of β.

The relationship between β and P_f becomes very complex for the following cases:

1. G being a non-linear function
2. R and U being non-normal
3. R and U being correlated

Rackwitz and Fiessler(1978) suggest a transformation to the R and U distributions which minimizes the error in the calculation of β due to non-normal distributions. This transformation is the basis of the Rackwitz-Fiessler algorithm. This algorithm uses an iterative procedure on the variables to locate the most likely failure point, and hence to evaluate the reliability index.

If R and U are dependent on each other, then they have to be uncorrelated before using the Rackwitz-Fiessler algorithm.

For this study, a computer program based on the Rackwitz-Fiessler algorithm was used to evaluate the value of the reliability index, β from the values of the failure finction, G.

4.3 RELIABILITY CRITERION

The method used for evaluating the reliability of the reinforced concrete slender columns is analogous to the one described above.

The capacity of the column depends upon the eccentricity at which it is loaded, as well as upon the column properties. The interaction curves shown in figure 14 represent the column properties, and may be defined by the parameters P_0 and M_0. Hence, the capacity of the column may be written as

$$P_c = f(M,M_0,P_0)$$

where P_c is the axial force capacity of the column when a bending moment M is applied at the ends.

The failure function may be written as:

$$G = P_c - P_l$$

or:

$$G = f(M,M_0,P_0)-P_l$$

where P_l is the axial **load.** A negative value for the failure function represents failure. But for a given eccentricity, a higher axial load corresponds to a higher bending moment while a lower axial load corresponds to a lower bending moment. Thus, M and P_1 are correlated, or, P_c and P_1 are correlated.

This problem may be solved by the method described below.

4.3.1 IMPLICIT UNCORRELATION PROCEDURE

In this procedure the variables R and U are not explicitly uncorrelated. The procedure is presented here for solving the problem at hand. However, with some modifications the Implicit Uncorrelation Procedure can be generalized for any set of random correlated variables.

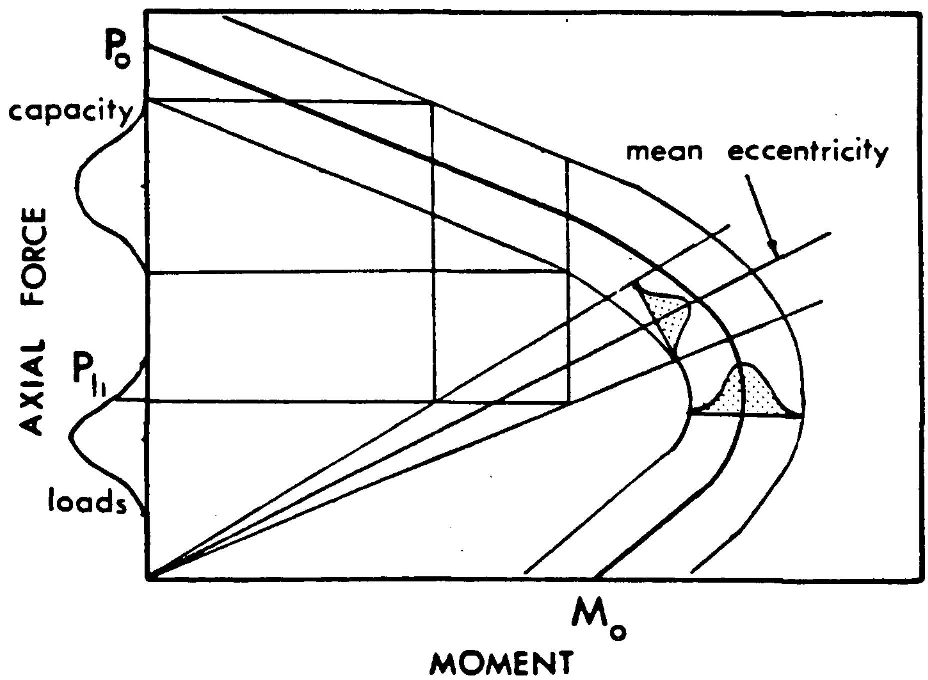

Figure 14. The Implicit Uncorrelation Procedure

Recall that in our case, axial load P_1 and bending moment M are correlated such that a high value of P_1 corresponds to a high value of M. Similarly, a low value of P_1 corresponds to a low value of M.

In the following it is assumed that we need to find the reliability index β for given mean values for axial load and eccentricity. The distribution of the axial load is known. The method for finding the standard deviation for eccentricity is described in the next section. The distribution of eccentricity is assumed to be normal.

The Implicit Uncorrelation Procedure is described below.

1. Simulate random values for axial load and for eccentricity for the given mean values and distributions.
2. Select one value of axial load to investigated (say P_{11} as shown in figure 14).
3. Multiply P_{11} by each of the simulated eccentricity values to get the corresponding range of bending moment values.
4. Using linear interpolation, find the capacity for each bending moment value for each interaction curve. This gives a distribution for capacities as shown in figure 14.
5. Subtract from each of the capacity values, the axial load value under investigation to get

values for the failure function G, which are stored in an array.

6. Repeat steps 2 through 5 till all the simulated axial load values have been considered.

The procedure described will lead to a set of values for the failure function G. Note that the failure function will not be distributed according to some standard distribution. These values are then used to evaluate the measure β, using the Rackwitz-Fiessler algorithm as outlined earlier in this chapter.

4.3.2 STANDARD DEVIATION FOR ECCENTRICITY

The uncertainity in the bending moment can mainly be due to two reasons.

1. Due to the exact magnitude of the loads being unknown.
2. Due to the exact distribution of the loads being unknown.

The variability of the loads for the first reason has been discussed in section 2.4. In the following it has been assumed that the distribution of the dead load is exactly known, and that it entails no uncertainity in the evaluation of the bending moment acting on the column.

A survey of different kinds of possible loading distributions on a beam reveal that it is reasonable to assume a variability of about 5% in evaluating the

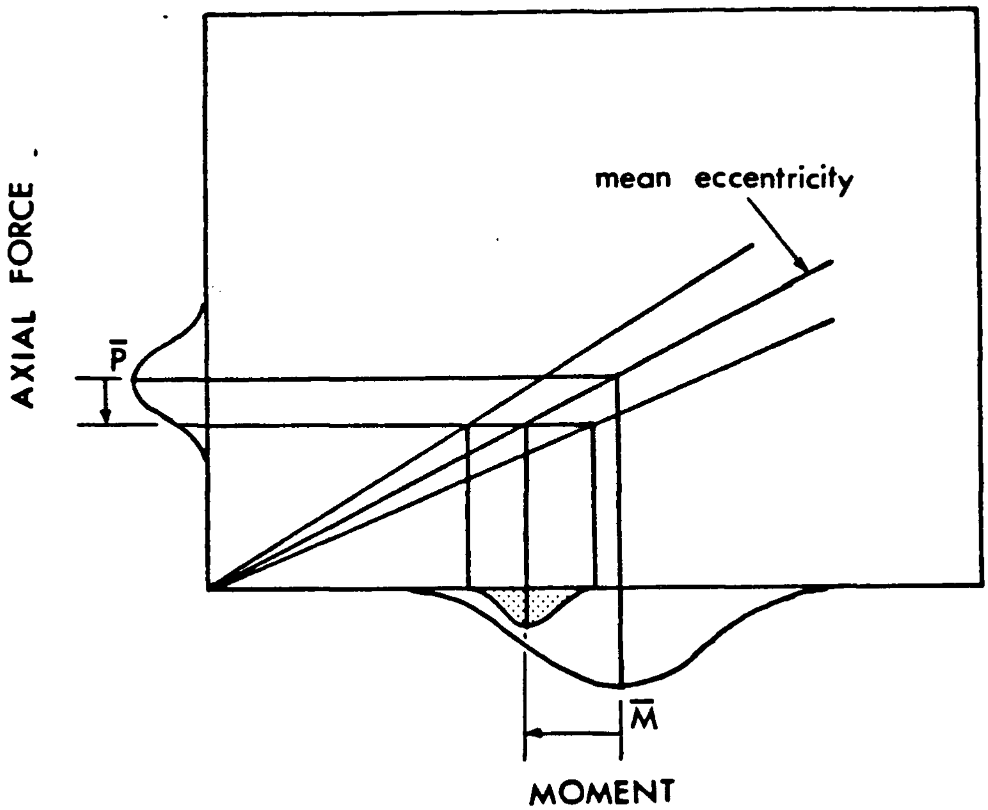

Figure 15. Standard deviation of eccentricity

bending moment when the exact force is known.

The method used for finding the standard deviation for the eccentricity is as follows.

1. Select the mean eccentricity for which the variability is to be investigated. (Figure 15).
2. Simulate an axial load distribution for some average axial load value. Hence, knowing the mean eccentricity the average value of the bending moment corresponding to a particular value of axial load is fixed.
3. Simulate values for the bending moment assuming a variability caused only by the *distribution* of

the load. Hence, calculate the set of corresponding eccentricity values.

4. Repeat steps 3 and 4 until all the simulated axial load values have been considered.

It was found that the variability of the eccentricity is independent of the nominal axial load.

Note that instead of using step 3 of the Implicit Uncorrelation Procedure, the values of the bending moments could had been simulated directly, without using the variability of the eccentricity. However, the simulation of the bending moments would be different for each level of the nominal axial load, whereas the simulation of the eccentricity values remains the same. Hence, this method leads to saving in the computer time due to the large number of nominal axial load levels at which the reliability is to be investigated.

4.4 SAMPLE SIZE

The procedure described above is capable of finding the value of the reliability index, β for a particular value of the nominal axial load. This procedure is to be repeated a number of times in order to find the nominal axial load corresponding to the target β for each cross section, and for each slenderness ratio. Hence, in order to save computer time it is necessary to determine the smallest possible sample sizes. Grant *et al* (1978) have suggested that a sample size of 200 column sections is adequate to represent the variability in strength. In the present study 250 column cross sections were simulated.

The axial load was simulated by 50 values while the sample size for the eccentricities was 20. This gives 1000 load points. The Code for the Design of Concrete Structures for Buildings (CSA A23.3) aims at a possibility of overloads of 1 in 1000, so a sample size of 1000 simulated load points seems to be adequate.

The above sample sizes will lead to 250,000 values for the failure function, G, which are to be placed in ascending order, and then used to evaluate the reliability index. In order to save computer time in this calculation only alternate ranked values of the failure function were considered after placing them in ascending order (a total of 125,000 values) and the result was compared with that obtained by using all the 250,000 values. A variation of about 5% was observed. Hence, the calculations for the reliability index were performed using all the 250,000 values of the failure function.

5. RESULTS

5.1 INTRODUCTION

The results obtained by the method described in the previous chapters are in the form of the value of the reliability index, β v/s the corresponding nominal axial load. The code aims at a probability of overloads for columns of one in a thousand, and a probability of understrength of one in a hundred. Now

Prob(failure) = Prob(overloads) X Prob(understrength)

Hence the the code aims at achieving a probability of failure of 1 in 100,000 for columns. The method for doing this is by specifying strength reduction factors and some load factors. The reliability index β corresponding to a probability of failure of 1 in 100,000 is 4.265.

5.2 RELIABILITY AND AXIAL LOAD

The first step in the analysis of the results is to obtain the value of the maximum allowable nominal axial load that would correspond to the target β of 4.265. Figure 16 shows the relation between β and the axial load for a 250mm X 250mm cross section with 2% steel, for a mean eccentricity of axial load of 50mm. Linear interpolation was used to obtain the axial load corresponding to the target β of 4.265. This maximum allowable nominal axial load corresponding to a β of 4.265 is referred to as the computer result in the following.

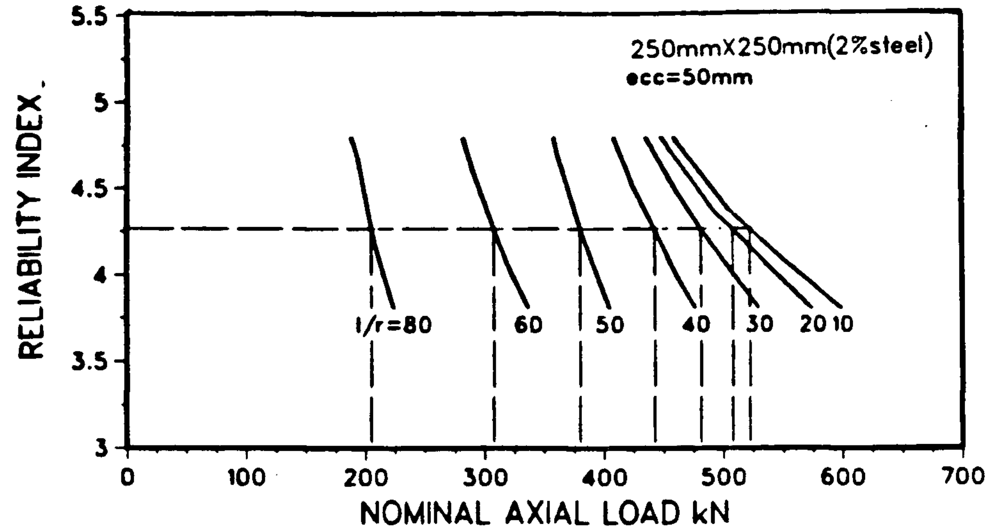

Figure 16. Reliability index β and maximum allowable nominal axial load

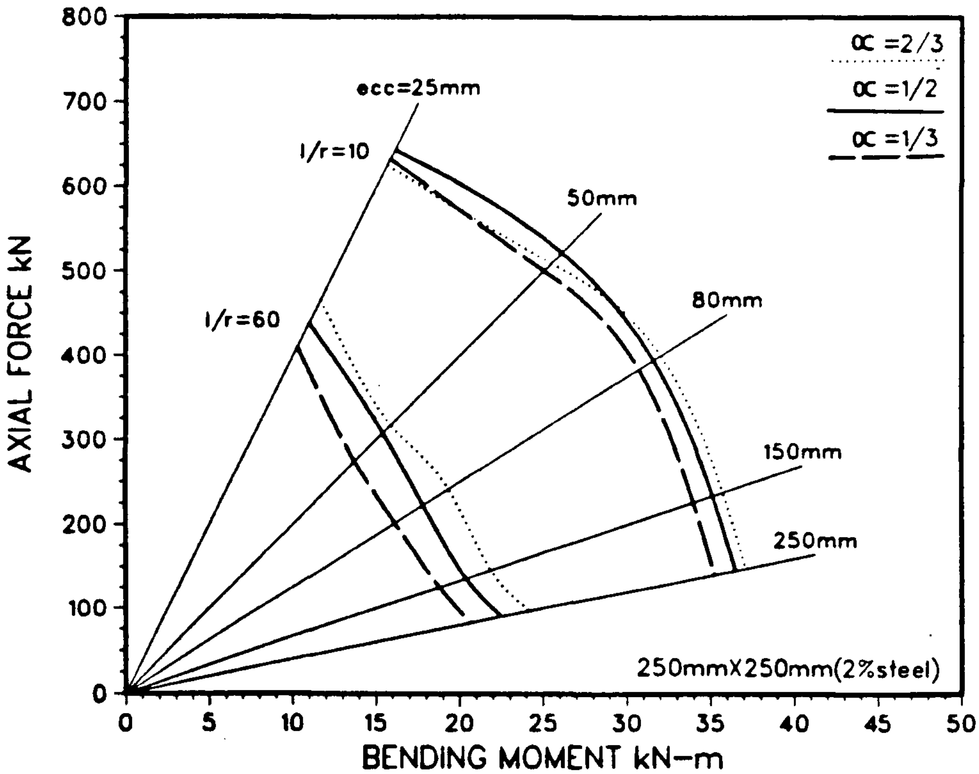

Figure 17. Computer result and the dead load ratio factor, a

5.3 RELIABILITY AND LOAD RATIO

As described earlier, the load effects of only the dead load and the live load were considered in this study. The dead load ratio factor a was defined in section 2.4.3. In this study a was primarily considered to have the value 1/2. However, to check the variation of the reliability with a, values of a equal to 1/3 and 2/3 were also considered for one cross section. The results are presented for slenderness ratio's of 10 and 60 in figure 17. Note that the value of a does not affect the results by more than about 7%.

5.4 RESULT FORMAT

In the present study it was assumed that the load factors λ as presented in the code adequately express the effect of variability of the load effects.

The computer results obtained by the present study were compared with those found by the code method. For the evaluation of the allowable axial load capacities by the code method, the strength of concrete was assumed to follow the modified Hognestad stress-strain relationship. This is more accurate than the rectangular stress block relationship and is allowed by the code.

The results are presented for the various specimen cross sections and slenderness ratios for different eccentricities of loading in tables 1 to 3 for CAN3-A23.3-M77 and in tables 4 to 6 for the new code. The results are in terms of the factor ζ where

Mean Ecc (mm)	Slenderness Ratio 10	20	30	40	50	60	80
25	0.87	0.91	0.99	1.12	1.28	1.15	1.38
50	0.91	0.96	1.01	1.06	1.07	1.06	1.09
80	0.95	0.94	0.93	0.95	0.95	1.01	0.94
150	0.93	0.94	0.93	0.92	0.92	0.94	0.92
250	1.10	1.06	0.99	0.92	0.91	0.90	0.89

Table 1. ζ_o for 250mm X 250mm column with 2% steel, γ=0.52

Mean Ecc (mm)	Slenderness Ratio 10	20	30	40	50	60	80
50	1.14	1.15	1.18	1.24	1.33	1.45	1.69
100	1.16	1.19	1.22	1.27	1.31	1.33	1.47
160	1.17	1.19	1.21	1.22	1.24	1.28	1.30
300	1.14	1.17	1.18	1.19	1.20	1.21	1.20
500	1.24	1.25	1.26	1.24	1.17	1.10	1.03

Table 2. ζ_o for 500mm X 500mm column with 2% steel, γ=0.76

Mean Ecc (mm)	Slenderness Ratio 10	20	30	40	50	60	80
50	1.06	1.05	1.03	1.01	1.00	1.04	1.15
100	1.05	1.05	1.04	1.04	1.06	1.09	1.20
160	1.06	1.07	1.06	1.09	1.09	1.12	1.19
300	1.03	1.03	1.03	1.04	1.06	1.12	1.21
500	1.03	1.04	1.05	1.06	1.09	1.10	1.12

Table 3. ζ_o for 500mm X 500mm column with 4% steel, γ=0.73

Mean Ecc	Slenderness Ratio						
(mm)	10	20	30	40	50	60	80
25	0.86	0.87	0.94	1.06	1.21	1.12	1.32
50	0.86	0.91	0.95	1.01	1.04	1.03	1.05
80	0.89	0.91	0.91	0.94	0.93	0.97	0.91
150	0.92	0.92	0.91	0.90	0.89	0.91	0.92
250	1.07	1.05	1.00	0.92	0.90	0.87	0.91

Table 4. ζ_n for 250mm X 250mm column with 2% steel, $\gamma=0.52$

Mean Ecc	Slenderness Ratio						
(mm)	10	20	30	40	50	60	80
50	1.11	1.10	1.11	1.16	1.25	1.36	1.59
100	1.08	1.11	1.13	1.18	1.22	1.24	1.35
160	1.07	1.10	1.11	1.12	1.13	1.15	1.17
300	0.97	0.98	0.99	1.00	1.01	1.02	1.03
500	0.96	0.97	0.97	0.98	0.95	0.91	0.90

Table 5. ζ_n for 500mm X 500mm column with 2% steel, $\gamma=0.76$

Mean Ecc	Slenderness Ratio						
(mm)	10	20	30	40	50	60	80
50	1.02	0.99	0.94	0.91	0.91	0.96	1.07
100	0.93	0.94	0.93	0.93	0.95	0.99	1.10
160	0.93	0.94	0.94	0.95	0.97	1.00	1.08
300	0.89	0.90	0.90	0.90	0.92	0.96	1.04
500	0.81	0.82	0.83	0.85	0.88	0.89	0.92

Table 6. ζ_n for 500mm X 500mm column with 4% steel, $\gamma=0.73$

$$\zeta = R_{actual} / R_{code}$$

where R_{actual} are the axial load capacities found by this study for a β of 4.265. R_{code} are the axial load capacities obtained by the code method. ζ_o refers to the computer results as compared with those obtained by following CAN3-A23.3-M77, while ζ_n refers to the computer results compared with those obtained by following CSA-A23.3(1984).

5.5 CODE EVALUATION

MacGregor *et al* (1970) surveyed 22000 columns in the late 1960s and found that nearly all of them had a slenderness ratio less than 30. Hence, in any attempt at code calibration columns with slenderness ratios of 30 or less must be considered important. However, it should also be realized that columns with higher slenderness ratios are the ones that usually create problems and as such their importance increases. In this study the columns were divided into two categories and each category was considered separately. The first category consists of columns with slenderness ratios of 30 or less. The second category consists of columns with slenderness ratios greater than 30.

Grant(1976) conducted a use study of column sizes and steel ratios of various buildings in Alberta. The results indicate that more than 50 percent of the column widths range from 16in.(406mm) to 24in.(610mm) and more than 50 percent of the columns have steel ratios between 0.005 and 0.015. Hence, out of the sections considered for this study, the 500mm X 500mm cross section with 2% steel would be one of the more widely used ones. Hence, a code modification which can lead to some improvement in the value of ζ for this particular cross section without adversely affecting the results for the other cross sections would be a practically acceptable proposal.

For an objective comparison between the proposal and the code procedure the following method was adopted.

The error ϵ is defined as:

$$\epsilon = \sqrt{\frac{\Sigma(\zeta - 1.0)^2}{N}}$$

where the summation is performed over all the values falling in a particular category.

Tables 4, 5 and 6 reveal that CSA-A23.3 (1984) leads to results that are close to the computer result for low slenderness, especially for the 500mm X 500mm cross section with 2% steel. However, with increasing slenderness, there is an increasing difference between the code and the computer results. Hence, it was felt that retaining the present ϕ_c and ϕ_s (i.e. 0.60 and 0.85 respectively), but with a different ϕ_m would be a realistic proposal for a more consistent code.

For this purpose, values of ϕ_m between 0.55 and 1.05 were considered at intervals of 0.05. The result is presented in figure 18 in terms of the factor ϵ. In figure 18 note that for the 500mm X 500mm (2% steel) cross section, the cumulative error ϵ is minimum for ϕ_m =1.0. However, at

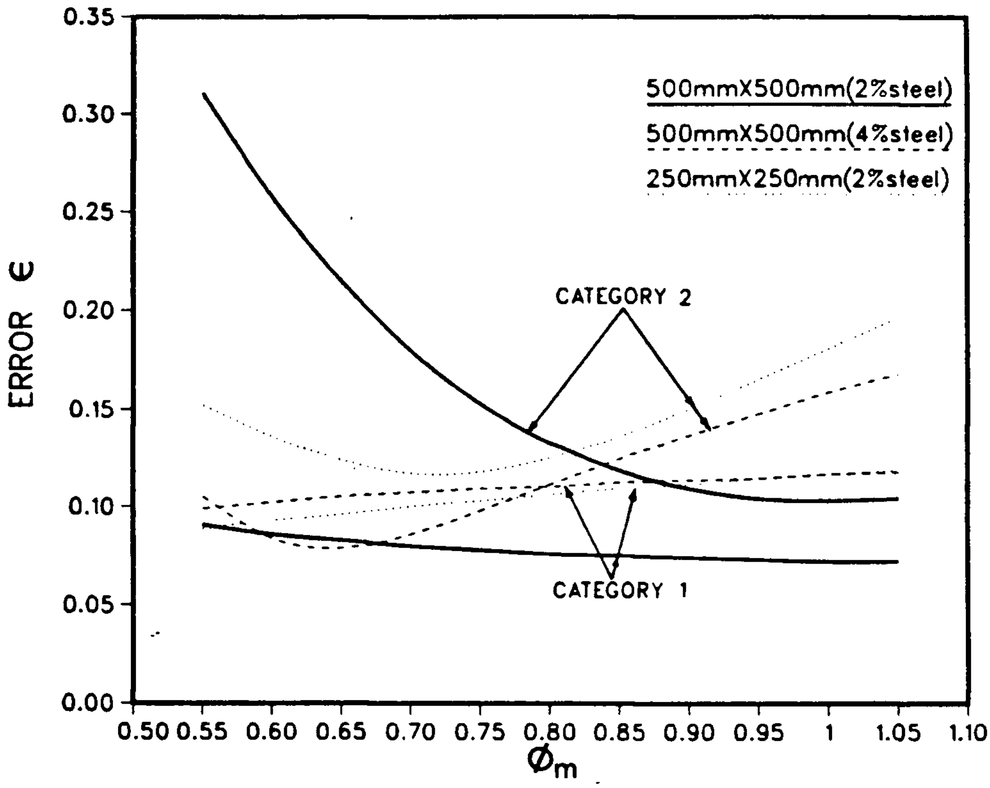

Figure 18. Variation of ϵ with ϕ_m

high values of ϕ_m some values of ζ became as low as 0.78. for this column (an error of up to 22% from the computer result). Such low values of β are incompatible with the code aim as they would lead to a very low reliability. It is proposed that for such common sections, the minimum value of ζ should not be less than 0.90, i.e. an error of more than 10% from the computer result is not allowed. With this condition, a value for ϕ_m of 0.70 was found to be optimum. However, this entails restricting the use of the moment magnification formula to columns with slenderness ratios of equal to or less than 60.

Mean Ecc	Slenderness Ratio						
(mm)	10	20	30	40	50	60	80
25	0.86	0.86	0.93	1.03	1.16	1.06	1.24
50	0.86	0.90	0.94	0.99	1.01	0.99	0.98
80	0.89	0.90	0.90	0.92	0.90	0.93	0.87
150	0.92	0.91	0.90	0.88	0.87	0.89	0.89
250	1.07	1.04	0.99	0.91	0.89	0.85	0.87

Table 7. ζ_{p1} for 250mm X 250mm column with 2% steel, γ=0.52

Mean Ecc	Slenderness Ratio						
(mm)	10	20	30	40	50	60	80
50	1.11	1.10	1.10	1.14	1.21	1.30	1.49
100	1.08	1.10	1.12	1.16	1.19	1.20	1.28
160	1.07	1.09	1.10	1.11	1.11	1.11	1.11
300	0.97	0.98	0.98	0.99	0.99	0.99	1.00
500	0.96	0.97	0.96	0.97	0.94	0.90	0.89

Table 8. ζ_{p1} for 500mm X 500mm column with 2% steel, γ=0.76

Mean Ecc	Slenderness Ratio						
(mm)	10	20	30	40	50	60	80
50	1.02	0.96	0.93	0.90	0.89	0.92	1.01
100	0.93	0.93	0.93	0.92	0.93	0.96	1.05
160	0.93	0.94	0.93	0.94	0.95	0.97	1.03
300	0.89	0.89	0.89	0.89	0.90	0.94	1.00
500	0.81	0.82	0.83	0.84	0.86	0.87	0.90

Table 9. ζ_{p1} for 500mm X 500mm column with 4% steel, γ=0.73

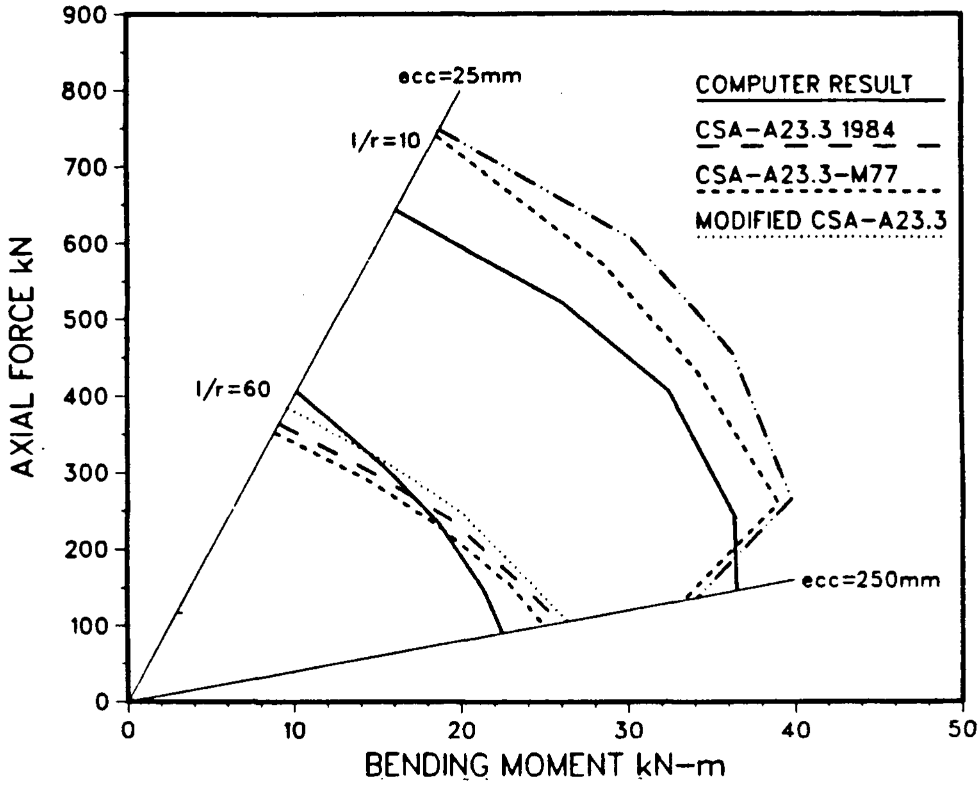

Figure 19. Results for the 250mm X 250mm column with 2% steel

The results for all the three sections in terms of the factor ζ (denoted in this case by the ζ_{p1}) are presented in Tables 7-9 for ϕ_c =0.60, ϕ_s =0.80 and ϕ_m =0.70. Note that this proposal does not have any significant adverse effect on the two less common cross sections (250mm X 250mm (2% steel) and 500mm X 500mm (4% steel)).

In addition to the above, note from Table 5 that CSA-A23.3 (1984) leads to a ζ less than 1.00 for high eccentricities while for low eccentricities the ζ is greater than 1.00. This is true even for the short column, which suggests a change in the material resistance factors, ϕ_c and ϕ_s . By increasing ϕ_c and decreasing ϕ_s this difference can

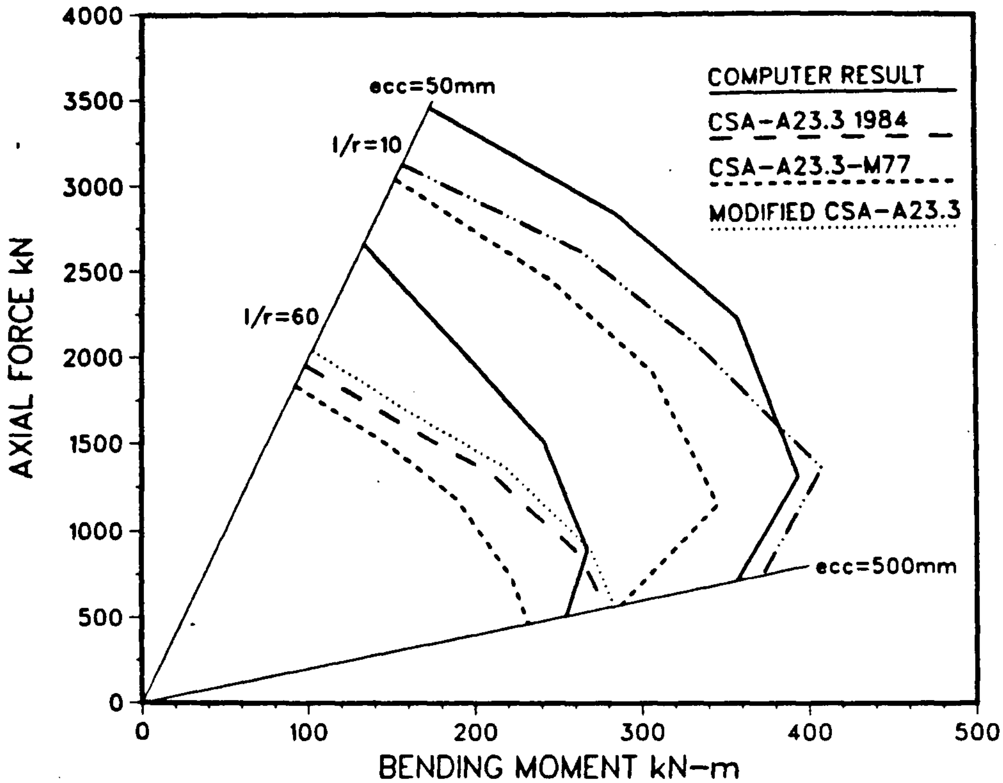

Figure 20. Results for the 500mm X 500mm column with 2% steel

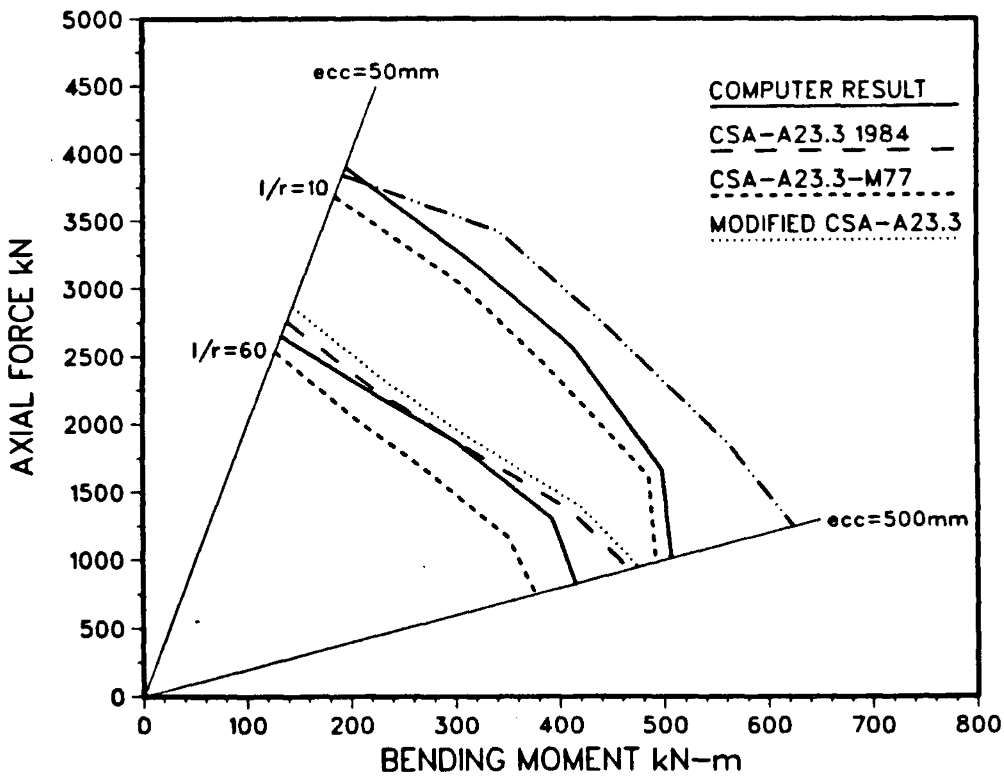

Figure 21. Results for the 500mm X 500mm column with 4% steel

Mean Ecc	Slenderness Ratio						
(mm)	10	20	30	40	50	60	80
25	0.86	0.86	0.90	1.01	1.14	1.05	1.24
50	0.83	0.87	0.91	0.97	1.00	0.98	0.99
80	0.86	0.88	0.88	0.90	0.89	0.94	0.87
150	0.92	0.92	0.91	0.89	0.88	0.89	0.88
250	1.09	1.05	0.99	0.92	0.92	0.90	0.85

Table 10. ζ_{p2} for 250mm X 250mm column with 2% steel, γ=0.52

Mean Ecc	Slenderness Ratio						
(mm)	10	20	30	40	50	60	80
50	1.11	1.10	1.07	1.11	1.18	1.29	1.49
100	1.04	1.07	1.09	1.13	1.17	1.18	1.28
160	1.04	1.06	1.07	1.08	1.09	1.11	1.12
300	0.96	0.98	0.99	1.00	1.00	1.01	1.02
500	1.00	1.01	1.00	1.00	0.97	0.94	0.89

Table 11. ζ_{p2} for 500mm X 500mm column with 2% steel, γ=0.76

Mean Ecc	Slenderness Ratio						
(mm)	10	20	30	40	50	60	80
50	1.00	0.97	0.92	0.89	0.88	0.92	1.01
100	0.92	0.92	0.91	0.91	0.93	0.95	1.05
160	0.92	0.93	0.93	0.93	0.95	0.97	1.03
300	0.89	0.89	0.89	0.89	0.91	0.94	1.02
500	0.83	0.84	0.85	0.86	0.89	0.90	0.93

Table 12. ζ_{p2} for 500mm X 500mm column with 4% steel, γ=0.73

be reduced. To this end the values of ζ, corresponding to ϕ_c =0.65, ϕ_s =0.80 and ϕ_m =0.70 (denoted in this case by ζ_{p2}), are presented for the three cross sections in Tables 10-12. Again, note that this leads to better results for the 500mm X 500mm (2% steel) cross section without adversly affecting the results for the other two sections.

Figures 19-21 show the results for the three cross sections for slenderness ratios of 10 and 60. The modified CSA-A23.3 result is the one with only the ϕ_m changed to 0.70. A comparison between the results from CSA-A23.3 (both 1977 and 1984 versions) and the proposed modified CSA-A23.3 is presented in Table 13 in terms of ϵ. In table 13 Proposal 1 is the one in which only ϕ_m is changed while for Proposal 2 all the three strength reduction factors are changed.

Section	Category	CSA-A23.3 M77	CSA-A23.3 (1984)	Proposal 1	Proposal 2
250mmX250mm	1	0.0698	0.0920	0.0998	0.1105
2% steel	2	0.1319	0.1151	0.1093	0.1054
500mmX500mm	1	0.1936	0.0822	0.0800	0.0594
2% steel	2	0.3063	0.2159	0.1801	0.1708
500mmX500mm	1	0.0472	0.1030	0.1066	0.1025
4% steel	2	0.1045	0.0791	0.0861	0.0809

Table 13. Comparison of results in terms of ϵ

6. CONCLUSIONS AND RECOMMENDATIONS

6.1 CONCLUSIONS

From table 13 we note that a modification in the code (CSA-A23.3 1984) to change the value of ϕ_m from 0.65 to 0.70 would reduce the root mean square error by as much as 16% for a column of common usage. A more drastic modification (i.e. to change ϕ_c , ϕ_s and ϕ_m to 0.65, 0.80 and 0.70 respectively) would lead to still more accurate results. However, a change in the material resistance factors would also affect members other than those with flexure combined with axial compression. Hence, a more detailed study is suggested to evaluate the consequences of such a change. Nevertheless, the first modification should be a practical suggestion.

It can also be noted in the results that as the columns become increasingly slender, the code results deviate more and more from the computer result. This is especially a cause of concern for large eccentricities of loading because the code results are non-conservative in such cases. Moreover, with the development of computer software, it is now possible to effectively predict the behaviour of slender columns. Hence, it is desirable and practical to restrict the use of the moment magnification formula to columns of slenderness ratios equal to or less than 60. Note that for a slenderness ratio of up to 60, the proposal to change ϕ_m to 0.70 leads to a lowest ζ of 0.90 for the commonly used section (Table 8).

6.2 RECOMMENDATIONS

It is recommended that subject to the following condition, the value of ϕ_m should be taken as 0.70 in the Code for the Design of Concrete Structures for Buildings (CSA-A23.3 1984). The condition to be satisfied is:

> The slenderness ratio of the column should not be greater than 60.

In case this condition cannot be satisfied, a rational procedure to evaluate the slenderness effect should be followed. A computer program similar to the one described in this thesis would be an acceptable rational procedure to this end.

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