

**THE VALUE OF ONE MONTH AHEAD INFLOW  
FORECASTING IN THE OPERATION OF A  
HYDROELECTRIC RESERVOIR**

by

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## ABSTRACT

The research assesses the value of forecast information in operating a hydro-electric project with a storage reservoir. The benefits are the increased hydro power production, when forecasts are available. The value of short term forecasts is determined by comparing results obtained with the use of one month ahead perfect predictions to those obtained without forecasts but a knowledge of the statistics of the possible flows. The benefits with perfect forecasts provide an upper limit to the benefits which could be obtained with actual less than perfect forecasts. The effects of generating capacity and flow patterns are also discussed.

The operation of a hypothetical but typical project is modelled using stochastic dynamic programming. A simple model of streamflow is formulated based on the historical statistics ( means and deviations).

The conclusions are: The inflow forecasts can improve the operational efficiency of the reservoir considerably because of the reduction in forecasting uncertainty. The maximum release constraints affect the additional expected values. The benefits from the forecasts increase as the discharge limits reduce. Flow predictions in the high flow season are most valuable when the runoff in that time period dominates the annual flow pattern. However flow predictions at other times of the year also have value.

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## Chapter 1

### INTRODUCTION

#### 1.1 STATEMENT OF THE PROBLEM

Achieving the most efficient operation of hydroelectric projects is a complex task. Firstly, there are uncontrollable and uncertain elements that affect the projects' operation, such as stream flow which is variable and difficult to forecast. The value of optimization techniques is dependent on the accuracy and the availability of information regarding the magnitude of future basin inflows. Secondly, there are multiple components ( reservoirs, canals, river diversions, power plants ... etc.) which must operate jointly. Thirdly, conditions such as electricity demand and regulations are continuously changing due to the inherent dynamic nature of society and technology. Fourthly, there exist many conflicting, interests and constraints that influence the management of the system.

Research efforts to improve the efficiency of hydro-electric power production have been mainly concentrated on mathematical models and solution techniques during the past few decades. Many different models and solution approaches, such as Linear Programming (LP), Deterministic Dynamic Programming (DDP), Linear Programmig-Dynamic Programming (LP-DP), Stochastic Dynamic Programming (SDP), State Increament Dynamic Programming (SIDP), and Markov Decision Process (MDP) have been studied by researchers.

Hydrologists are interested in building and studying stream flow forecasting models.

Their purpose is to provide more accurate predictions about inflows. Many models relating to this issue have been built. Less efforts have been made to study the role of stream flow forecasting models in the optimization of hydro-electric power plant operation, specifically the extent to which more accurate inflow forecasts result in an improvement in operating efficiency. But, now, more and more researchers have noted that to justify accurate information about inflows, the benefits and costs from using accurate runoff forecasts compared with using relatively inaccurate inflow information should be considered. In another words, the use of a complicated forecasting model must be based not only on the technical possibilities but also on the expected additional benefits it may provide. The expected additional returns give a clue as to how much money can be spent on research to find better stream flow forecasting models. It is well known that forecasting systems can be very costly, and the question that frequently arises is whether or not the benefits outweigh the cost. If we know the value of inflow information, it will help to make decisions about research proposals which aim at finding accurate inflow forecasting models and how much money one can spend on the project. When the budget for research and data gathering is higher than the additional benefits from the information, the work is difficult to justify. Otherwise, the research work is worth doing.

Unfortunately, the process of reservoir operation is complex, and the value of improved forecasting is very difficult to estimate. This is mainly because perfect predictions of stream flow are never available, especially for long term forecasting. There are inherent inaccuracies and these are difficult to quantify in a meaningful way. Stream flow, a stochastic process, is determined by many random factors which can not be foreseen.

Short term forecasting can be more accurate than long term prediction. In some cases, such as when winter flows result entirely from groundwater, short term forecasts of runoff can be almost completely accurate, so they may be termed as 'perfect forecasts'. Of course, 'short term', here, is a fuzzy word. It may refer to hourly, daily, weekly or

even monthly prediction.

In this study a hypothetical hydro project with realistic characteristics is used. Calculations are done on a monthly basis as it usual in power studies. The study first finds a long term discounted expected value for the output from the project when it is operated in an optimal way with no forecasts. Then various forecasts are assumed and their values assessed, with most effort concentrated on the one month ahead perfect forecast.

It is much simpler to determine the value of perfect information or a perfect forecast than of information with some inaccuracies. The value of perfect information gives a useful upper bound on the value of information or of a forecasting system. The aim of this thesis is to develop a methodology for operating a typical hydro electric plant with a storage reservoir in an optimal way and to assign a value to perfect short term forecasts.

## 1.2 METHODOLOGY

The approach adopted to achieve the stated objectives was to use a stochastic dynamic programming model to optimize the operation of a hypothetical hydro-electric power station and then to obtain the long term discounted expected benefit of its output. Next, the process was repeated assuming perfect forecasts one month ahead. For the purpose of calculating the additional benefits due to the use of one month ahead perfect inflow information, some modifications had to be made to the pure stochastic dynamic programming model. The benefit increment shown by the model reflected the reduction of uncertainty in the inflow prediction. Thus the difference between the energy production with the stochastic dynamic programming model and the modified one assuming perfect information measure the value of the perfect stream flow forecasts.

The major steps in the research are listed below:

- Build the dynamic programming model for stochastic and one month ahead perfect

stream flow prediction.

- Run the stochastic dynamic programming to get the one year optimal operating policy for the reservoir.
- Repeat the optimizing process using the value of various water levels obtained from the last run until the results stabilized in order to obtain the expected long term value of the output from the project— and the long term optimization policy for the hydro-electric project.
- Run the modified dynamic programming model and assume that perfect one month ahead forecasts were available. Again repeat until the results stabilized to find the value of operating with the accurate forecasts.
- Analyze the results and draw conclusions.

Details of the procedures are explained in the following chapters.

### 1.3 SOME OTHER PROBLEMS

Several aspects of the study are discussed here to define the problems encountered and the scope of the research.

#### 1.3.1 SELECTION OF OBJECTIVE FUNCTION

As mentioned above, water resource projects usually serve multiple purposes or multiple objectives. Single objective projects rarely exist in the real world. Fortunately, our purpose is to find the value of perfect stream flow information. Also the hydro-power plant which is studied is a hypothetical one, so there is no need to consider multiple objectives. This helps concentrate our interests on the main purpose. On the other

hand, multiple objective problems are more complicated than single objective. If two or more objectives are involved in the study, it is very difficult to judge the value of perfect information, since the trade-off among objectives affects the optimal solution. Although there are some techniques which deal with multiple objective optimization problems successfully, these techniques are usually very complicated and are difficult to use in large scale optimization problems. Therefore, using only one objective to study the value of perfect inflow forecasting simplifies the study.

The objective considered in this study is the energy production from the hypothetical hydro-power project. In order to measure the value of the perfect inflow and to make comparisons more meaningful, the energy production is transformed into monetary terms. However, the value of perfect information in this research refers only to the value in terms of increased energy production due to the reduction of uncertainty in predictions and the resulting increased operational efficiency of the reservoir.

### 1.3.2 THE WORK ENVIRONMENT OF THE HYDRO POWER PLANT

As described in Section 1.1, a hydro-electric project must work as a part of a whole system which contains multiple reservoirs, hydro power plants, canals ... etc., and it should be optimized under a given energy demand curve. Furthermore, in the real time operation of a hydro power station, the energy price varies with time which can cause complications in optimizing the operation of the reservoir. Such optimization problems require large scale systems analysis models and decomposition techniques will usually be needed.

However, as indicated above, this study is only for a hypothetical project. It is assumed that the project will operate independently to simplify the model and its calculation. That is, the hydro-electric power plant can be optimized to generate maximum energy at all times without considering the shape of the energy demand curve and without

dealing with the combined optimization of multiple reservoirs operation. This is quite realistic for a research project.

### **1.3.3 EFFECT OF THE MAXIMUM GENERATOR DISCHARGE ON THE OPERATION OF RESERVOIR**

The constraint of the maximum amount of water which can be released from a reservoir through the power plant can affect the operational efficiency of a hydro-electric plant. This is because the limitation of the maximum discharge influences the total amount of water spilled from the reservoir during the high flow period. The amount of spilled water from the reservoir is an indicator of energy loss. The smaller the amount of water spilled from a reservoir, the better the operational efficiency for the hydro power project is. If the upper bound of the release is too low, it causes a larger amount of water to be spilled than does the case which has a higher upper bound of release. This will, of course, reduce the operating benefits from the hydro electric plant. On the other extreme, if there is no upper bound to the release of water through the turbines, there will be no water being spilled over the spillway. It will produce maximum energy for a given reservoir size and no energy loss any time during the high flow period. It is the ideal case, from an energy production point of view.

However the maximum discharge constraint is determined by the capacity of the turbines installed in the power station. This study examines the effects of changing the upper bound of discharges by setting several release limits and then comparing the results.

### **1.3.4 OPERATIONAL POLICIES WITH VARIOUS FLOW PATTERNS**

Another interesting problem is the effect of different stream flow patterns on the operation policies of the hypothetical hydro power project. Here, a different flow pattern means that



for a given amount of annual inflow, the distribution of the mean monthly stream flows and their deviations are different. Finding out how the operation policies and the output changes with the flow patterns is of the research significance. In this study, two different flow distributions have been examined, one typical of the British Columbia interior and one typical of the B.C. Coast. The results will be shown in the related section.

#### 1.4 LITERATURE REVIEW

Stream flow forecasts and predictions play a fundamental role in the operation of hydro-power projects. The degree to which forecasts aid the operator in improving the operational efficiency is dependent on the quality of the forecasts. Many researchers have noted that it is worth deriving quantitative measures of the value of flow forecasts.

Joanna Mary Barnard(1989)[1] investigated the value of inflow forecasting models in the operation of a hydro-electric reservoir. She considered how conceptual hydrologic forecasts could be used in combination with optimizing techniques to improve the operational efficiency of a hydro-electric project. Her thesis used a stochastic dynamic programming model and a simulation model to study the problem. Barnard compared the role of three different inflow forecasting models: a naive forecast, a conceptual forecast and a perfect forecast (actually recorded inflows), in the operation of a hydro-power station. Some interesting results were found. She concluded that the accuracy of the forecast is more important in influencing the value of the conceptual forecast than the magnitude of the flows. However, since there is no way of determining when a forecast will be accurate, a policy must be developed to decide when forecasting could be of use in long term power production planning. The approach used in this study is somewhat similar but with a different emphasis.

Aris P. Georgakakos (1989)[2] evaluated the benefits of stream flow forecasting in

three specific systems: the Savannah River System in the state of Georgia, The High Aswan Dam, and the Equatorial Lake System. The approach taken was to simulate the performance of the systems under Extended Linear Quadratic Gaussian (ELQG) control with several stream flow models of varying forecasting power. The research concluded that probabilistic stream flow forecasting can considerably improve reservoir operation but the benefits are system specific.

Nabeel R.Mishalani and Richard N.Palmer (1988) [3] studied the benefits of forecasting to a water supply system. Questions relating operational losses to forecast period and accuracy were addressed. Some simple available forecasting techniques were assessed for their accuracy and applicability. The issues were addressed through the use of a simulation model, where the system was modelled as a single purpose reservoir supplying municipal and industrial water. Their conclusions were: (1) reservoir operation deteriorates markedly with the loss of forecast accuracy; (2) the optimal length of forecasting period is five months; (3) reservoir operation efficiency may be improved by as much as 88 percent if perfect predictive abilities are available; (4) the mean of the historic data is not recommended for predicting future flows because Markov methods are always superior; and (5) lag one autoregressive Markov schemes exhibit about a 9 percent improvement in operation over no forecasting.

Roman Krzysztofowicz (1983) [4] considered several fundamental questions about forecasting: How to optimally use categorical and probabilistic forecasts? What opportunity losses are expected to be incurred when forecast uncertainty is ignored? Why the classical contingency analysis is suboptimal? and what economic gains are to be expected from probabilistic forecasts. Analytic solutions were derived for the optimal and a nonoptimal (one that ignores forecast uncertainty) formulation of a single period quadratic decision problem with a categorical and probabilistic forecast of the state. He

concluded that: (1) Probabilistic forecasts can be at least as valuable as categorical forecasts, and categorical forecasts always have a nonnegative value if the decision maker accounts for the forecast uncertainty and employs the optimal (Bayesian) decision procedure. (2) No matter which forecasting method is used, an opportunity loss is always incurred by a decision maker who does not account for the forecast uncertainty. As a result, the actual value of a forecast may be negative. (3) A classical procedure of accounting for uncertainty of categorical forecasts by means of contingency analysis is suboptimal. It usually results in an opportunity loss and may also result in a negative actual value of the forecast. (4) Probabilistic forecasts are likely to be more valuable than categorical forecasts, even if used in suboptimal decision procedures. (5) The relative gains from probabilistic forecasts (over categorical forecasts) are likely to be greater for decision makers who employ suboptimal decision procedures (which ignores the categorical forecast uncertainty) than for those who already employ optimal decision procedures (which accounts for the categorical forecast uncertainty).

William W-G.Yeh et.al. (1982) [5] discussed the worth of inflow forecasts for reservoir operation. They used a simulation model to examine the benefits over a range of forecast accuracies and forecast periods of one month and longer. The conclusions obtained were that using the historical monthly means for estimates of streamflow rather than attempting any prediction, and releasing water on the basis of those estimates to generate hydro-power and minimize spill, produced amazingly good results which were only slightly worse than with reasonably good predictions. Thus, hydro-power benefit gains of several percent can be made either with good predictions on a month-to-month basis, or by using the historical means for estimates of stream flow and taking the uncertainty into account. Thus they suggested that until high confidence predictions can be established, use of the historical means is preferable.

Labadie, et.al. (1981) [6] investigated the worth of short term rainfall forecasting for

combined sewer overflow control. They addressed the question "what levels of forecast error can be tolerated before it is better to abandon adaptive ( anticipatory ) control policies utilizing forecast information in favor of simple reactive ( myopic ) control methods ?" Their study demonstrated that the expected forecast model errors are generally lower than the error threshold above which reactive policies become more attractive.

## 1.5 SUMMARY OF THE THESIS

The thesis begins with a short introduction to the problem at hand, including some related research issues, as well as describing some of the work which has been done by other researchers. In addition to this introductory chapter, the thesis consists of four other chapters as summarized below:

Chapter 2 establishes the Stochastic Dynamic Programming model used in this study. Next, it presents the modification used for analyzing the effects of the one month ahead perfect inflow forecast on the optimal operation of reservoir. Then, the solution techniques developed to solve the problem are discussed. The way to obtain the state transition probability matrices, both monthly and yearly, is also examined.

Chapter 3 describes the inflow estimating method used in this research and some related problems. The results of forecasting inflow are also shown in the chapter.

Chapter 4 presents the results of the optimization model and compares the results of the dynamic program for both stochastic and perfect inflow information. The analysis is then discussed.

Chapter 5 presents the conclusions obtained from the research and some suggestions for further study.

## Chapter 2

### THE STOCHASTIC DYNAMIC PROGRAMMING MODEL

The hydro-electric reservoir under consideration is modelled with stochastic dynamic programming (SDP) to optimize the operation of the reservoir along similar lines to Barnard (1989).

The solution technique is presented in the following sections. Since there are differences in reservoir operation between the pure stochastic stream flow situation and with the forecast inflows, the operation policies are decided in different ways. Therefore, they are modelled separately.

#### 2.1 STOCHASTIC DYNAMIC PROGRAMMING MODEL

Fig. 2.1 is a pictorial representation of the single reservoir system having inflow  $Q_t$  and making release decision  $D_t$  in each time period  $t$ ,  $t=1,2, \dots, T$ .

The model uses periods of time,  $t$ , as the stage variables. In this case months are used ( a time period of a month can be used without loss of generality ), so that  $t=1,2, \dots, 12$ .

The stream flow input,  $Q_t$ , to the reservoir is a stochastic variable. Its probability distribution can be any type which is suitable to the inflow historical records. Assume that, for a particular period of time  $t$ , a set of inflows  $\{Q_t^i\}$  and their corresponding probabilities,  $\{P[Q_t^i]\}$ ,  $i=1,2, \dots, I_t$ , are available.

The problem is that in each month,  $t$ , with the stochastic stream flow input  $Q_t$ , the amount of water released from the reservoir,  $D_t$ , has to be determined so as to maximize

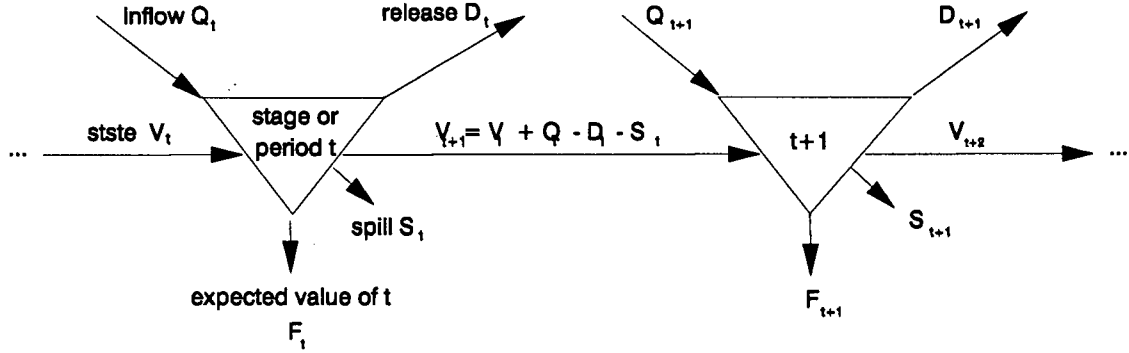


Figure 2.1: Sequential Reservoir Operation Process

the energy generation. Therefore, the decision variable in the model is the quantity of water to release during stage  $t$ . There are usually upper and lower bounds on the quantity of water which can be released from the reservoir. An upper bound might be due to physical size limitations of the stream below the reservoir in question or, more likely, the maximum capacity of the turbines; while a lower bound might be required to maintain navigation or ecological water balance. In this case, the quantity of water that can be discharged through the turbines provides the upper bound and zero flow the lower bound. For calculation convenience, the decision variable is discretized with  $K-1$  intervals between  $D_{min}$  and  $D_{max}$ .

$$D_k = \{D_t^k\}, k=1,2, \dots, K.$$

Where

$$D_1 = D_{min}$$

$$D_K = D_{max}$$

A release  $D_t^K$ , which is one of the set of all possible releases, and which satisfies the

system governing equation ( which will be described below ) is called a feasible release.

Let  $V_t$  be the state variable, which represents the volume of water in storage. The reservoir considered in this research has a predetermined maximum and minimum allowable volume, and they are represented by  $V_{max}$  and  $V_{min}$  . For calculating convenience, the state variables are discretized with the same unit of discretization as the decision variable  $D_k$ . There are  $N$  discrete units or states. That is

$$V_t = \{V_t^n\}$$

$$n = 1, 2, \dots, N.$$

where the minimum and maximum volumes are equivalent to  $V_1$  and  $V_N$  respectively.

That is

$$V_1 = V_{min}$$

$$V_N = V_{max}$$

With above variables definition, the stochastic dynamic programming model may be formulated as follows.

### 2.1.1 THE STATE TRANSITION EQUATION

As indicated in Fig.2.1, according to the principle of continuity (or mass balance), the governing equation of the reservoir state transition relationship is

$$V'_{t+1} = V_t + Q_t - D_t \quad (2.1)$$

subject to

$$V_{min} \leq V_t \leq V_{max}$$

$$D_{min} \leq D_t \leq D_{max}$$

which states that the storage volume at the beginning of month  $t+1$  is equal to the sum of the initial storage volume and stream flow input of month  $t$  minus the release made at the end of month  $t$ . If the final storage  $V_{t+1}$  is greater than  $V_{max}$  , in order to

keep it within the state constraints, the volume of spill,  $S_t$ , will be required.  $S_t$  is given by:

$$S_t = \begin{cases} V'_{t+1} - V_{max} & \text{when } V_{t+1} > V_{max} \\ 0 & \text{otherwise} \end{cases} \quad (2.2)$$

Thus the state transition equation is

$$V_{t+1} = V_t + Q_t - D_t - S_t \quad (2.3)$$

where:

$V_t$  = total volume of water in storage at time period  $t$ ;

$Q_t$  = total volume of inflow at time interval  $t$ ;

$D_t$  = volume of water released through the turbines at stage  $t$ ;

$S_t$  = volume of spill required to stay within constraints at stage  $t$ .

The decision process is to decide upon the release  $D_t$  to be made after observing the state  $V_t$  and  $V_{t+1}$  of the system, and maximize the objective function. The complexity of the stochastic process optimization is that each of the possible inflows must be analyzed for each state and discharge. Therefore several potential end states  $V_{t+1}$  exist corresponding to a initial state  $V_t$  and discharge  $D_t^k$ . Once each possible inflow has been examined, expected values of discharges, spills, and hence energy generation are calculated.

### 2.1.2 STAGE RETURN EQUATION

The stage return function represents the benefit or cost,  $R(d)$ , of the decision being made. The objective function of this study is maximum energy generation. Therefore, maximization of the objective function is needed during the decision process. The production of hydro-electric energy during any time period  $t$  is dependent on the installed plant capacity; the flow through the turbines; the average productive storage head; the number



of hours in the period; the plant factor; and a constant for converting the product of flow, head, and plant efficiency to kilowatt-hours of electrical energy, KWH. In order to calculate the energy generation, the mean volume should be computed first. Then it will be used to determine the average head. That is

$$H_{average,t} = f((V_t + V_{t+1})/2) \quad (2.4)$$

Then energy can be determined as a function of average head and discharge as following

$$E_t = \gamma \eta H_{average,t} Q_t T1 = PT1 \quad (2.5)$$

where

$P$  = power;

$E_t$  = energy generated in time period  $t$ ;

$\eta$  = efficiency factor;

$\gamma$  = density of water;

$H_{average,t}$  = average head of water above power house at  $t$ ;

$Q_t$  = flow of water through turbines during time period  $t$ ;

$T1$  = Number of hours in time period  $t$ .

The value of  $\eta$ , the efficiency factor, takes into account turbine efficiency, generation efficiency and other losses. Efficiency will actually vary with head, but can be considered constant if the range of head is relatively small. The value used in this study is 87% and would apply to  $\pm 20\%$  of the design head.

### 2.1.3 THE RECURSIVE EQUATION

In stochastic dynamic programming, the past is known but the future is uncertain. Thus the analysis has to proceed backwards in time.

The stochastic dynamic programming recursive equation determines the maximum or minimum of the objective function which is dependent upon the value of the stage return function and the discounted value of the state at the previous stage. It starts with known or assumed values of the possible states at the end of the year.

It is assumed that future returns are less valuable than present returns and hence, they are discounted by a discount factor  $\beta$ ,  $0 \leq \beta \leq 1$ ; where  $\beta$  is the monthly discount factor. The discount factor is applied to the second part of the expression.

$$F_t(V_k) = d_k^{Max} \left[ \sum_{i=1}^{i=I_t} (P_i(R_{ik} + \beta F_{t-1}(V_{ki}))) \right] \quad (2.6)$$

subject to:

$$V_{min} \leq V_k, V_{ki} \leq V_{max}$$

$$d_{min} \leq d_k \leq d_{max}$$

where

$P_i$  = Probability of inflow  $q_i$  in period  $t$ .

$F_t(V_k)$  = Expected value of state  $V_k$  with optimal operation of the reservoir with  $t$  remaining time periods;

$R_{ik}$  = Return from generating power from discharge  $d$  with the reservoir in state  $k$  ( at time  $t$  ) and inflow  $q_i$ ;

$F_{t-1}(V_{ki})$  = Expected value of being in state  $V_{ki}$  ( the state reacted from state  $k$  with inflow  $q_i$  and discharge  $d$  )

with  $t-1$  remaining time periods;

$d_k$  = Discharge. Various values are tried to find the maximum value

of  $F_t(V_k)$ .

#### 2.1.4 THE DYNAMIC PROGRAMMING MODEL FOR ONE MONTH AHEAD PERFECT INFLOW FORECAST

Assume that for a given time interval  $t$ , one month ahead perfect stream flow forecasts are available, or in other words, we know the stream flow input to the reservoir that will actually occur one month in advance. Problems are what is the value of this perfect inflow prediction, how to operate the reservoir based on the perfect flow information to improve the operational efficiency, and what are the extra benefits provided by the perfect forecast. Assume that all of the variable definitions are the same as above, and the one month ahead perfect forecasts are given in the form of a set of inflows  $[Q_i]_t$  and a set of probabilities  $[P_i]_t$  corresponding to the stream flow input,  $i=1,2, \dots, I_t$ . The recursive equation of this problem is:

$$F_t(V_k) = \sum_{i=1}^{i=I_t} [d_k^{Max} (R_{ik} + \beta F_{t-1}(V_{ki}))] P_i \quad (2.7)$$

where definitions as before.

The difference between eq. (2.6) and eq. (2.7) is, for a given reservoir state, equation (2.7) first finds the optimal release policy for every forecast flow and the corresponding expected value for state  $V_k$  given inflow  $q_i$ ; and then takes the expected value of the state with all the possible inflows. Equation (2.6) finds the expected value first, then takes the maximized expected value as the optimal state value.

The expected values calculated from eq. (2.7) should be higher than the results computed from eq. (2.6) since there is less uncertainty with the forecasts.

### 2.1.5 MATHEMATICAL STATEMENT FOR LONG TERM ANALYSIS

Long term reservoir operation is complicated unless the reservoir empties each year. This section formulates the problem when the reservoir does not empty each year.

Assume that  $n$  years operation will be analyzed,  $n=1,2, \dots$  and that it is possible to obtain the long term monthly state transition probability matrices in the form

$$P_{ij}^{nT+t}(d) = \text{Prob ( transition to state } j \text{ in time } t+1$$

from state  $i$  in month  $t$  of the  $n$ 'th year of

the process where the release was  $d$  )

and it is possible to obtain monthly transition rewards of the form

$$r_i^{nT+t}(d) = \text{expected immediate return when making the monthly}$$

transition from state  $i$  in month  $t$  to month  $t+1$ , in

the  $n$ 'th year of the process, when the release is  $d$ .

Assumption A: The expected immediate return is independent of the year of the process. That is

$$r_i^{nT+t}(d) = r_i^t(d) \forall n \quad (2.8)$$

Assumption B: The stream flow input to the reservoir is a stochastic process and is periodic in nature with a period of 12 months,  $T=12$ .

$$P_{ij}^{nT+t}(d) = P_{ij}^t \quad (2.9)$$

When the releases  $D$  becomes stationary, that is the set of releases remain the same for all years of the process  $n=1,2, \dots$  the steady long term operation condition is reached.  $D$  is the yearly release policy such that

$$D = (d_t, d_{t+1}, \dots, d_{t+T-1}) \quad (2.10)$$

where  $d_t$  is the vector of discharges in month  $t$  ( one discharge each state).

The monthly state transition probability matrices at time  $t$  will be

$$P^t(d) = \{P_{ij}^t\} = \begin{pmatrix} P_{11}^t(d_1) & P_{12}^t(d_1) & \cdots & P_{1N}^t(d_1) \\ P_{21}^t(d_2) & P_{22}^t(d_2) & \cdots & P_{2N}^t(d_2) \\ \cdots & \cdots & \cdots & \cdots \\ P_{N1}^t(d_N) & P_{N2}^t(d_N) & \cdots & P_{NN}^t(d_N) \end{pmatrix} \quad (2.11)$$

where  $t=1, 2, \dots, 12$ , and  $P_{ij}^t(d)$  is the probability of going from state  $i$  in time period  $t$  to state  $j$  in time period  $t+1$  given discharge  $d_i$ .

It will be shown that the yearly transition returns and probabilities are functions of the monthly transition returns and probabilities. It is convenient to use matrix notation to show the functional relationship between the yearly and monthly transition returns and probabilities.

The functional relationship between the yearly and monthly transition probabilities is

$$IP^1(D) = P^1(d_1)P^2(d_2) \cdots P^{12}(d_{12}) \quad (2.12)$$

where  $IP^1(D)$  is the yearly transition probability of year one.

The proof is quite simple: a yearly transition is comprised of 12 individual monthly transitions, hence, the yearly transition probability matrix is the product of the twelve monthly transition probability matrices. Generally, for month  $t$ , the yearly transition probability matrix is

$$IP^t(D) = P^t(d_t)P^{t+1}(d_{t+1}) \cdots P^{t+T-1}(d_{t+T-1}) \forall t, T+1=1 \quad (2.13)$$

and hence the long term transition probability is

$$\begin{aligned}
 IP(D) &= \begin{pmatrix} P^1(d_1)P^2(d_2)\cdots P^T(d_t) & \cdots & \cdots & 0 \\ \cdots & & \cdots & \cdots \\ \cdots & & \cdots & \cdots \\ 0 & & \cdots & \cdots & P^T(d_T)P^1(d_1)\cdots P^{T-1}(d_{T-1}) \end{pmatrix} \\
 &= \begin{pmatrix} IP^1(D) & \cdots & \cdots & 0 \\ 0 & IP^2(D) & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & IP^T(D) \end{pmatrix} \quad (2.14)
 \end{aligned}$$

where  $IP(D)$  represents long term state transition probability matrix.

The functional relationship between the yearly and monthly transition return,  $IR^t$ , of month  $t$  is

$$\begin{aligned}
 IR^t(D) &= R^t(d_t) + \beta P^t(d_t)R^{t+1}(d_{t+1}) + \cdots + \\
 &\quad \beta^{T-1} P^t(d_t)P^{t+1}(d_{t+1})\cdots \\
 &\quad P^{t+T-2}(d_{t+T-2})R^{t+T-1}(d_{t+T-1}) \forall t, T+1=1 \quad (2.15)
 \end{aligned}$$

and hence

$$\begin{aligned}
 IR(D) &= \begin{pmatrix} R^1(d_1) + \beta P^1(d_1)R^2(d_2) + \cdots + \\ \quad \beta^{T-1} P^1(d_1)P^2(d_2) \quad \cdots P^{T-1}(d_{T-1})R^T(d_T) \\ \quad \cdots \\ \quad \cdots \\ R^T(d_T) + \beta P^T(d_T)R^1(d_1) + \cdots + \\ \quad \beta^{T-1} P^T(d_T)P^1(d_1) \quad \cdots P^{T-2}(d_{T-2})R^{T-1}(d_{T-1}) \end{pmatrix} \quad (2.16)
 \end{aligned}$$

where  $IR(D)$  is the yearly transition return vector given release policy  $D$ .

The proof goes as follows: the yearly transition return is comprised of twelve monthly transition returns, each of which must be discounted and multiplied by the probability of obtaining these monthly returns.

Thus, we have a stochastic decision process where the expected immediate return vector  $IR(D)$  is defined by equation (2.16) and the transition probability matrix  $IP(D)$  is defined by equation (2.14).

The long-run expected discounted return vector  $F$  ( where

$$F = ( F_1, F_2, \dots, F_T ),$$

$$F^t = ( F_1^t, \dots, F_i^t, \dots, F_N^t ) ,$$

and

$F_i^t$  = long term expected discounted return when in state  $i$  at month  $t$  ) satisfies the following, for every feasible release policy  $D$  ( a feasible policy  $D$  is one which assigns a feasible release to each state).

$$F = IR(D) + \beta^T IP(D)F \quad (2.17)$$

and furthermore that the optimal long-run expected discounted return vector satisfies the functional equation

$$F^* = \overset{Max}{D} [IR(D) + \beta^T IP(D)F^*] \quad (2.18)$$

Equation (2.17) can be simplified as shown by the following derivations. Equation (2.11)

$$F = IR(D) + \beta^T IP(D)F$$

is equivalent to the following set of equations:

$$F^1 = R^1(d_1) + \beta P^1(d_1)F^2 \dots \dots \dots (1)$$

$$F^2 = R^2(d_2) + \beta P^2(d_2)F^3 \dots \dots \dots (2)$$

...

...

$$F^T = R^T(d_T) + \beta P^T(d_T)F^1, (T + 1 = 1) \dots \dots \dots (T) \quad (2.19)$$

Proof: for some t

$$F^t = IR^t(D) + \beta^t IP^t(D)F^t \quad (2.20)$$

remembering that

$$\begin{aligned} IR^t(D) &= R^t(d_t) + \beta P^t(d_t)R^{t+1}(d_{t+1}) + \dots \\ &\quad \beta^{T-1} P^t(d_t)P^{t+1}(d_{t+1}) \dots \\ &\quad + P^{t+T-2}(d_{t+T-2})R^{t+T-1}(d_{t+T-1}) \end{aligned} \quad (2.15)$$

and

$$IP^t(D) = P^t(d_t)P^{t+1}(d_{t+1}) \dots P^{t+T-1}(d_{t+T-1}) \quad (2.13)$$

so that equation (2.20) becomes

$$\begin{aligned} F^t &= R^t(d_t) + \beta P^t(d_{t+1})R^{t+1}(d_{t+1}) + \dots + \\ &\quad \beta^{T-1} P^t(d_t) \dots P^{t+T-1}(d_{t+T-2})R^{t+T-1}(d_{t+T-1}) \\ &\quad + \beta^T P^t(d_t) \dots P^{t+T-1}(d_{t+T-1})F^t \end{aligned} \quad (2.21)$$

Now assuming that equation (2.19) is correct



$$F^t = R^t(d_t) + \beta P^t(d_t)F^{t+1} \quad (2.19-t)$$

$$F^{t+1} = R^{t+1}(d_{t+1}) + \beta P^{t+1}(d_{t+1})F^{t+2} \quad (2.19-t+1)$$

...

...

$$F^{t+T-1} = R^{t+T-1}(d_{t+T-1}) + \beta P^{t+T-1}(d_{t+T-1})F^t \quad (2.19-t+T-1)$$

substitute the last equation (2.19 t+T-1) in the prior equation (2.19 t+T-2) and continue repeating this process until the final substitution for  $F^{t+1}$  in equation (2.13 t) has been made, in which case the following is obtained

$$\begin{aligned} F^t = & R^t(d_t) + \beta P^t(d_t)R^{t+1}(d_{t+1}) + \dots + \\ & \beta^{T-1} P^t(d_t) \dots P^{t+T-2}(d_{t+T-2}) R^{t+T-1}(d_{t+T-1}) \\ & + \beta^T P^t(d_t) \dots P^{t+T-1}(d_{t+T-1}) F^t \end{aligned} \quad (2.22)$$

Since equation (2.22) is identical to equation (2.21) the proof is complete. It is obvious that equations (2.19) are in a simpler form than those of equation (2.17), the most important simplification being that each equation in (2.19) is a function only of monthly release policy while each equation in (2.17) is a function of a yearly release policy. Thus, the results can be extended to the simpler case, that is

$$F^t = R^t(d_t) + \beta P^t(d_t)F^{t+1} \quad (2.23)$$

$$F^{t*} = \underset{d_t}{Max} [R^t(d_t) + \beta P^t(d_t)F^{t+1*}] \forall t \quad (2.24)$$

## 2.2 OPTIMAL OPERATION OVER ONE YEAR PERIOD

### 2.2.1 SOLUTION TECHNIQUE

The operation of the reservoir over one year period was optimized on a monthly inflow in conventional fashion with dynamic programming to find the optimal release policies  $\{D_t\}$ ,  $t=1,2,\dots,12$ , and to obtain the year end value of each state.

Fig.2.2 is the flow chart of the SDP problem and fig.2.3 presents the flow chart of the SDP with the one month ahead perfect forecast.

Solution of the dynamic programming model yields optimal one year operating policies in the form of release sequences, the monthly state transition probability matrices, the monthly reservoir state sequences, and the year end values of states.

The one year operation analysis was carried out starting with initial state and end with another state. The terminal state values should be estimated before beginning. After one year's operation a new group of terminal values will be produced. Obviously, this analysis will not lead to the steady optimal operating policies. So that the long term analysis is needed. This will be discussed in the next section.

### 2.2.2 CALCULATION OF STATE TRANSITION PROBABABILITY MATRICES

An important problem in the dynamic programming is to obtain the state transition probability matrices for each stage of the calculation. This is because the long term operation analysis is based on information from the state transition probability matrices and the associated operating policies.

The state transition probability  $P_{ij}$  is defined as: the probability that state  $i$ , at the beginning of stage  $t$ , changes to state  $j$ , at the end of stage  $t$ , in the course of operating the reservoir.

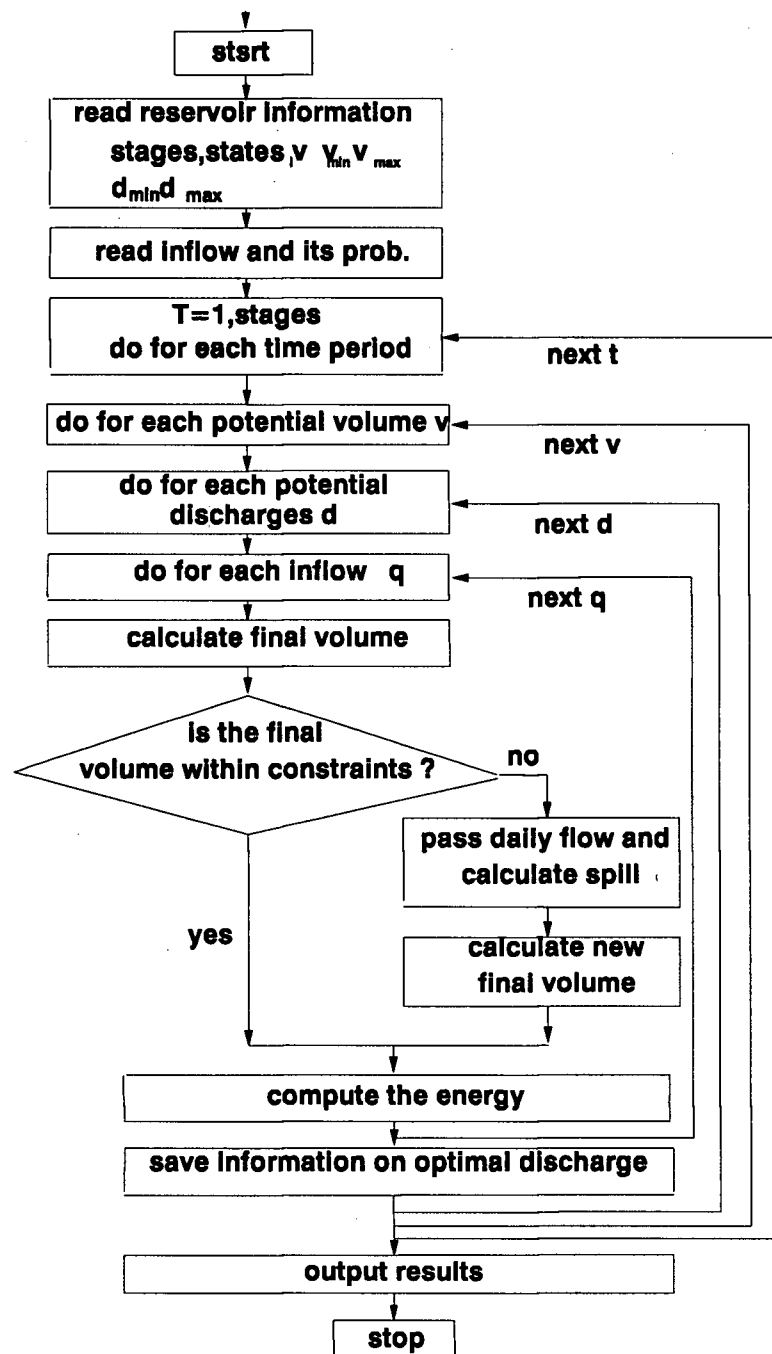


Figure 2.2: The Flow Chart of the SDP Problem

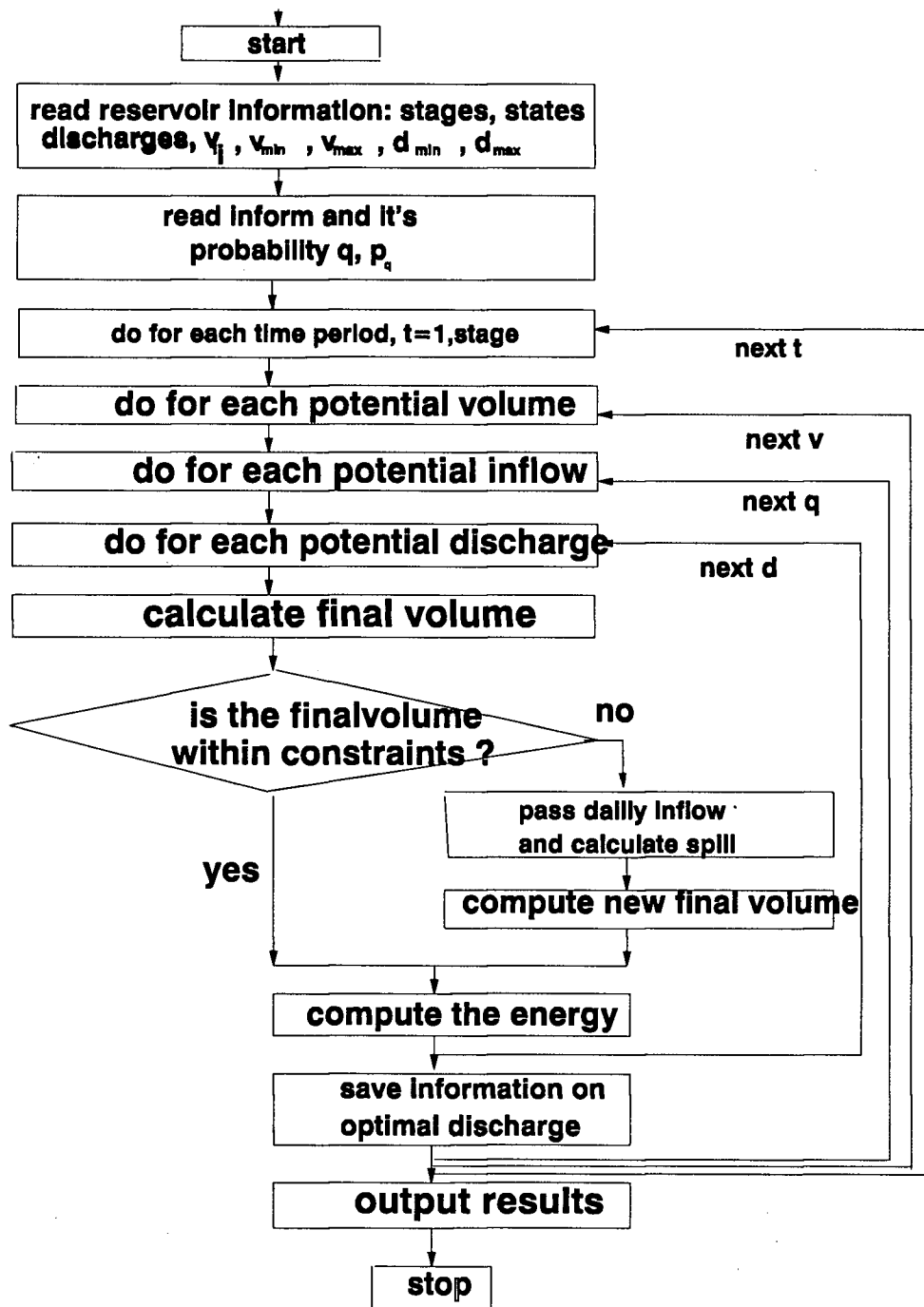


Figure 2.3: The Flow Chart of the Perfect Forecast Dynamic Program

$P_{ij}$	1	...	j	...	J
1	$P_{11}$	...	$P_{1j}$	...	$P_{1J}$
...	...	...	...	...	...
i	$P_{i1}$	...	$P_{ij}$	...	$P_{iJ}$
...	...	...	...	...	...
I	$P_{I1}$	...	$P_{Ij}$	...	$P_{IJ}$

According to the state transition equation

$$V_{t+1} = V_t + Q_t - D_t \quad (2.1)$$

it is known that, for a given initial state  $V_t$ , when release variable,  $D_t$ , has been decided, the probability of the state transitioning to  $V_{t+1}$  is equal to the probability of the input variable  $P_{Q_t}$ . When  $V_{t+1} \geq V_{max}$ , in order to keep the reservoir volume within the maximum allowable volume, the spill variable,  $S_t$ , is needed.

$$S_t = V_{t+1} - V_{max} \quad (2.2)$$

when

$$V_{t+1} \geq V_{max}$$

therefore

$$V'_{t+1} = V_t + Q_t - D_t - S_t \quad (2.3)$$

In this case, a problem is arises from the spill water  $S_t$ . That is, there may be several  $Q_t$  corresponding to  $V_{max}$ . In this situation, the transition probability  $P_{ij}$  from  $V_t$  to  $V_{max,t+1}$  is defined as:

$$P_{iJ} = \sum_{Q_t} P_{Q_t} \quad (2.25)$$

where  $Q_t \in \text{high flow}$ , when  $S_t \geq 0$

### 2.3 ANALYSIS OF LONG TERM RESERVIOR OPERATION

Assume that information from one year's operation is available. The next step is to get the long term reservoir operation policies. W.F.Caseltan and S.O.Russell developed an iteration technique to solve this problem (1974). However, a simpler iteration process is to use the information obtained from the previous year's calculation as the new input data, and then perform another round of calculation with the stochastic dynamic program model. This process is repeated until the operation policies stabilize. This means that if the reservoir's operation starting from water level A, after one year's operation the terminal values assigned to each state at the beginning of operation should be equal to the year end value computed from the iteration. Thus the terminal values calculated for one year's operation became the initial values for the next year's iteration. After several iterations, the terminal values reach a steady state, suggesting that the initial estimated values are no longer influencing the optimal operation policy decision and the optimal release policy (i.e. action) will be stationary; that is, the set of releases D will remain the same for all years (iterations)  $n = 0, 1, 2, \dots$  of the process.

Fig.(2.4) shows the iteration process to obtain the long term operating policies.

After one year's operation, 12 state transition probability matrices,  $\{P_{ij}\}^t$   $t=1, 2, \dots, 12$  were obtained. Based on these matrices, the yearly state transition probability matrix may be calculated by multiplying:

$$\{P_{ij}\} = \{P_{ij}\}^1 \{P_{ij}\}^2 \dots \{P_{ij}\}^{12} \quad (2.26)$$

Multiplying of the yearly state transition probability matrices yields the long term state transition probability matrix when the results of multiplying remain stationary.

$$\{P_{ij}\}_{opt} = \{P_{ij}\} \{P_{ij}\} \dots \{P_{ij}\} \quad (2.27)$$

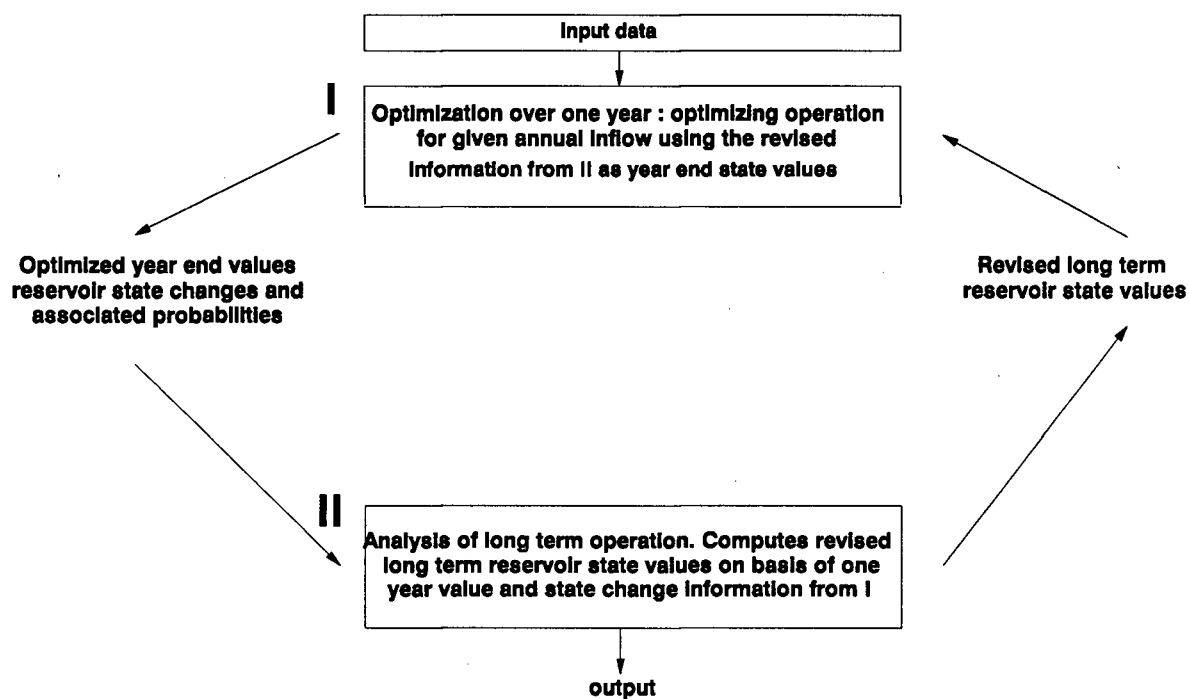


Figure 2.4: Schematic of Iterative Cycle Used in the Study

where  $\{P_{ij}\}_{opt}$  denotes the long term steady state transition probability matrix. Thus the long term optimal year end values of states will be:

$$F^* = \{P_{ij}\}_{opt} F \quad (2.28)$$

where  $F^*$  is the long term steady state terminal values while  $F$  is the one year's state terminal values.

## 2.4 MEASURING ADDITIONAL VALUES OF ONE MONTH AHEAD PERFECT FORECAST

Based on the information given from both the stochastic dynamic program and the optimization with the one month ahead perfect forecast, two different expected values for the reservoir states are obtained. Let  $F_s$  denotes the expected value obtained from the stochastic dynamic program and  $F_p$  represents the expected value of the one month ahead perfect forecast, then, the additional benefit due to using perfect information can be calculated by:

$$\nabla Value = F_p - F_s \quad (2.29)$$

for each state. Alternatively, the additional benefit may be expressed by the form of percentage:

$$\nabla Value = \frac{F_p - F_s}{F_s} \times \% \quad (2.30)$$



## Chapter 3

### INFLOW FORECAST

#### 3.1 PROBLEM DESCRIPTION

The streamflow forecast is a very important component of the dynamic programming model. The modelling of catchment behavior is quantitative whether reconstructing past precipitation-to-runoff behaviour or forecasting future runoff behaviour. However, all forecasts have some degree of uncertainty due to factors, such as rainfall, snowmelt, temperature, wind, solar radiation which are all random variables, which cannot be forecast exactly. Also catchment physical characteristics, such as size and shape, geology and soil type can not be accurately determined.

There are two basic types of forecasting used to analyze or to model catchment behaviour: stochastic and deterministic. Deterministic models can be further divided into two main categories. One uses mathematical descriptions of the relevant catchment processes to estimate discharges, and is termed a 'conceptual' model. The other way considers the whole catchment as a system of subsystems from which output occurs as responses to input. The latter concept features a system 'box', that is characterized by a unique response function,  $h(t)$ . The system approach concentrates on the operation performed by  $h(t)$  on an input,  $x(t)$ , to produce an output,  $y(t)$ , rather than considering the underlying physical reasoning. Many deterministic catchment models have been established, such as the UBC watershed model, the MIT model etc.

The statistical approach follows another line of thinking. It views streamflow as

a stochastic process. The purpose is to generate an inflow series which has the same statistical characteristics as the historical observations. Two basic techniques are used for streamflow generation. If the streamflow population can be described by a stationary stochastic process, whose parameters do not change over time, then a statistical model may be fitted to the streamflow. However the assumption that the process is stationary is not always plausible due to changes of runoff characteristics. An alternative scheme is to assume that precipitation is a stationary process and to predict the runoff sequence through an appropriate rainfall-runoff model of the river basin.

The model used in this research is a simple statistical one. Because a typical watershed is studied, it is impossible to generate inflow sequences by setting up a relationship between rainfall and runoff. The method provides probability distributions of monthly inflows using statistical data about the inflow pattern in the form of monthly means and standard deviations.

The forecast inflows take the form of a probability distribution for the monthly streamflows  $Q_t$  for each particular time interval  $t$ , (here a month).  $Q_t$  is assumed as a normally distributed random variable with mean  $\mu_{Q_t}$  and standard deviation  $\sigma_{Q_t}$ . In some cases the streamflows cannot be completely normally distributed because of the impossibility of negative flows. Thus an alternative treatment to avoid negative values is needed.  $Q_t$  is a continuous variables, as shown in Fig.3.1. However in order to keep the calculations within manageable limits, the variables need to be

discreted. In another words, the inflows are generated as a set of discrete values, each with an associated probability. The details of the approach are discussed in the next section.

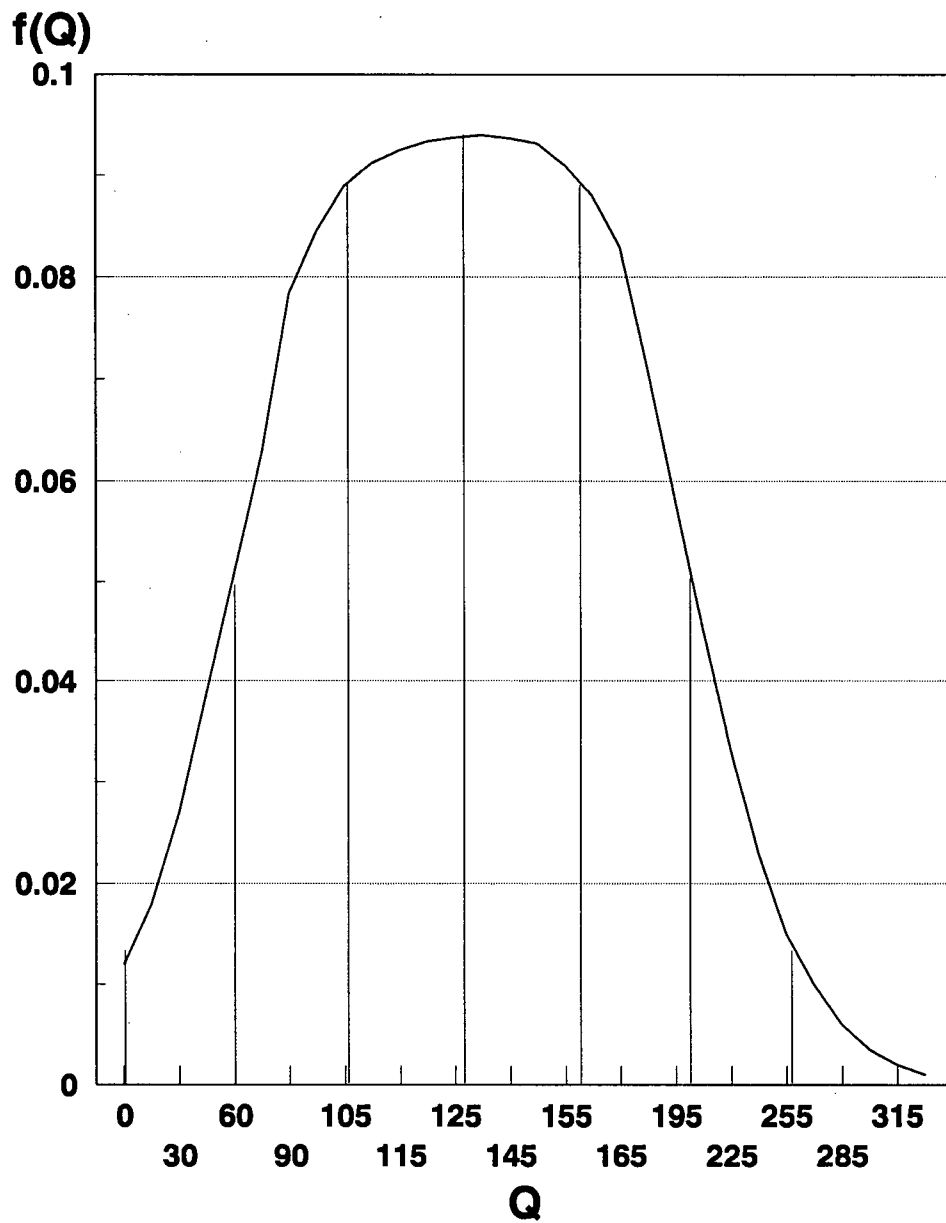


Figure 3.1: The Distribution of Inflow

## 3.2 THE INFLOW FORECASTING MODEL

### 3.2.1 THE INFLOW INTERVAL

For convenience of analysis, the forecast streamflow interval,  $\Delta Q$ , has the same unit as the discretized decision variable,  $\Delta D$ , and the state variable,  $\Delta V$ . This simplification makes the calculation of the state transition equation in the dynamic programming much easier and more accurate. In fact, the real world reservoir volume will change continuously all the time. However, if the interval value be chosen carefully, the operation optimization can be quite accurate.

### 3.2.2 LOWER AND UPPER VALUES

According to the characteristics of the normal distribution, the probability of variables occurring within the range of  $(\mu_Q - 3\sigma_Q, \mu_Q + 3\sigma_Q)$  is equal to 99.73%. Thus it includes almost all possible inflows which may occur during the time period and the lower value can be set as

$$Q_L = \mu_Q - 3\sigma_Q \quad (3.1)$$

subject to:  $Q_L \geq 0$

and the upper value will be

$$Q_U = \mu_Q + 3\sigma_Q \quad (3.2)$$

But the values of  $Q_L$  and  $Q_U$  usually do not meet the needs of inflow discretization. Thus the real lower and upper value used in this study are obtained by the following modification. For the lower value, firstly let

$$N_L = \frac{\mu_Q - 3\sigma_Q}{\Delta Q} \quad (3.3)$$

then, if  $N_L$  is not in integer, set  $N_L$  equal to the next lower integer. When  $N_L \leq 0$ , then set  $N_L = 0$ .

For the upper value, let

$$N_U = \frac{\mu_Q + 3\sigma_Q}{\Delta Q} \quad (3.4)$$

if  $N_U$  is not an integer, set  $N_U$  equal to the next higher integer. Thus the real lower value will be

$$Q'_L = N_L \times \Delta Q \quad (3.5)$$

the upper value will be

$$Q'_U = N_U \times \Delta Q \quad (3.6)$$

### 3.2.3 STREAM FLOWS AND THEIR PROBABILITIES

For each integer  $I$ ,  $N_L \leq I \leq N_U$ , let

$$\epsilon_1 = \frac{\mu_Q - I \times \Delta Q}{\sigma_Q} \quad (3.7)$$

$$\epsilon_2 = \frac{\mu_Q - I \times \Delta Q - \frac{\Delta Q}{2}}{\sigma_Q} \quad (3.8)$$

$$\epsilon_1 = \frac{\mu_Q - I \times \Delta Q + \frac{\Delta Q}{2}}{\sigma_Q} \quad (3.9)$$

$$P_1 = e^{-\frac{\epsilon_1^2}{2}} \quad (3.10)$$

$$P_2 = e^{-\frac{\epsilon_2^2}{2}} \quad (3.11)$$

$$P_3 = e^{-\frac{\epsilon_3^2}{2}} \quad (3.12)$$

Then the inflow,  $Q_I$  is

$$Q_I = I \times \Delta Q \quad (3.13)$$

and its probability,  $P(Q_I)$ , is

$$P_{prob}(Q_I) = \frac{2P_1 + P_2 + P_3}{4} \quad (3.14)$$

in order to ensure that the sum of the probabilities of all possible stream flows is equal to 1.0, the following modification for each probability is needed

$$P'_{prob}(Q_I) = \frac{P_{prob}(Q_I)}{\sum P_{prob}(Q_I)} \quad (3.15)$$

The calculation procedures are indicated by the flow chart.

For each month  $t$ , repeating the same calculation gives the set of streamflows and their probabilities of occurring.

The model described above is based on the assumption that the monthly inflows are seasonally independent. There is no serial correlation among streamflows. This is generally not the real situation in a particular watershed, since the flow in one period might give some hints of the likely flows in the following periods, especially in short term flow forecasting. In real world problems, any assumption regarding to time series

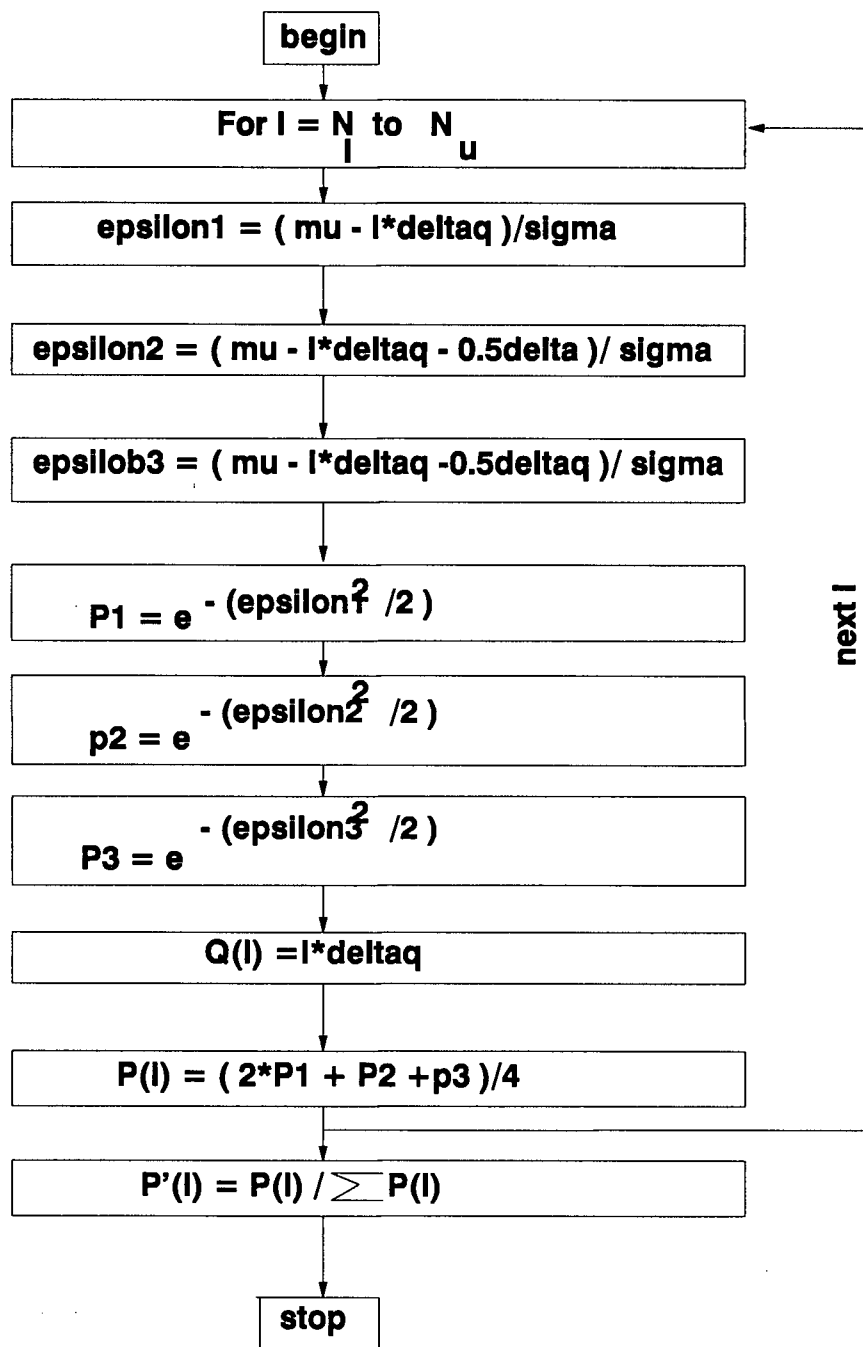


Figure 3.2: The Flow Chart of Inflow Forecasts

should be proved through statistical test. But in our test problem, the assumption was reasonable as the main interest lay in assessing the effects of stochastic forecasting on improving the operating efficiency of the reservoir, not on making accurate flow forecasts.

### 3.3 THE INFLOW STATISTICS

The research for this problem was carried out with a hypothetical hydroelectric project. For the purpose of convenience and comparison with other research, it was decided to use the data on reservoir size and inflows which had been used by Joanna Mary Barnard in her study (Barnard,1988). She described the watershed, the reservoir and its hydropower project and the inflow characteristics in detail. Here, the descriptions are not repeated except for the main characteristics of the flow which have significant influence on the research.

As already indicated in chapter I, two quite different flow patterns have been analyzed in this study. Table 3.1 presents the basic statistics describing the inflow sequence of flow pattern I. The mean annual runoff volume is  $1245.5 \text{ Mm}^3$  (million cubic meters).

Figure 3.2 shows the mean annual hydrograph being used.

The second flow pattern is presented below in table 3.2 and figure 3.3. Table 3.2 shows the distribution of mean monthly flows and their standard deviations during a one year period. Figure 3.3 is the corresponding mean annual flow hydrograph. It is named flow pattern II for comparing with flow pattern I.

Note that: % means both the mean monthly flows and the standard deviations are in % of the total annual flow in table 3.2.

Comparing flow pattern II with pattern I, it may be seen that, although the total annual flow for the two flow patterns is the same, the distribution of mean flows and their standard deviations are quite different. Table 3.3 shows the comparison of the two



Table 3.1: Flow Statistics of Flow Pattern I

Month	Min inflow $Mm^3$	Max inflow $Mm^3$	Mean inflow $Mm^3$	Standard deviation $Mm^3$
January	13.4	34.1	17.7	4.3
February	12.4	20.8	16.2	2.6
March	11.4	29.7	17.5	4.4
Aprile	20.5	79.1	44.0	16.6
May	99.3	262.1	182.0	45.0
June	238.8	491.2	334.2	67.1
July	182.5	373.0	273.0	57.6
August	108.2	294.2	155.2	38.1
September	55.2	178.1	91.0	31.5
October	34.4	95.9	54.7	14.0
November	21.1	64.9	37.1	11.9
December	14.8	41.5	22.9	5.5

Table 3.2: Inflow Statistics of Flow Pattern II

Month	mean Q		standard deviation $\sigma$	
	%	$Mm^3$	%	$Mm^3$
January	6.16	76.72	4.31	53.68
February	5.97	74.36	2.89	35.99
March	5.91	73.61	2.59	32.26
April	8.00	99.64	2.03	25.28
May	12.68	157.93	2.89	35.99
June	12.62	157.18	3.51	43.72
July	8.62	107.36	3.51	43.72
August	4.56	56.79	2.22	27.65
September	5.54	69.00	3.20	39.86
October	10.59	131.90	5.17	64.39
November	9.85	122.68	4.62	57.54
December	9.84	122.56	3.82	47.58

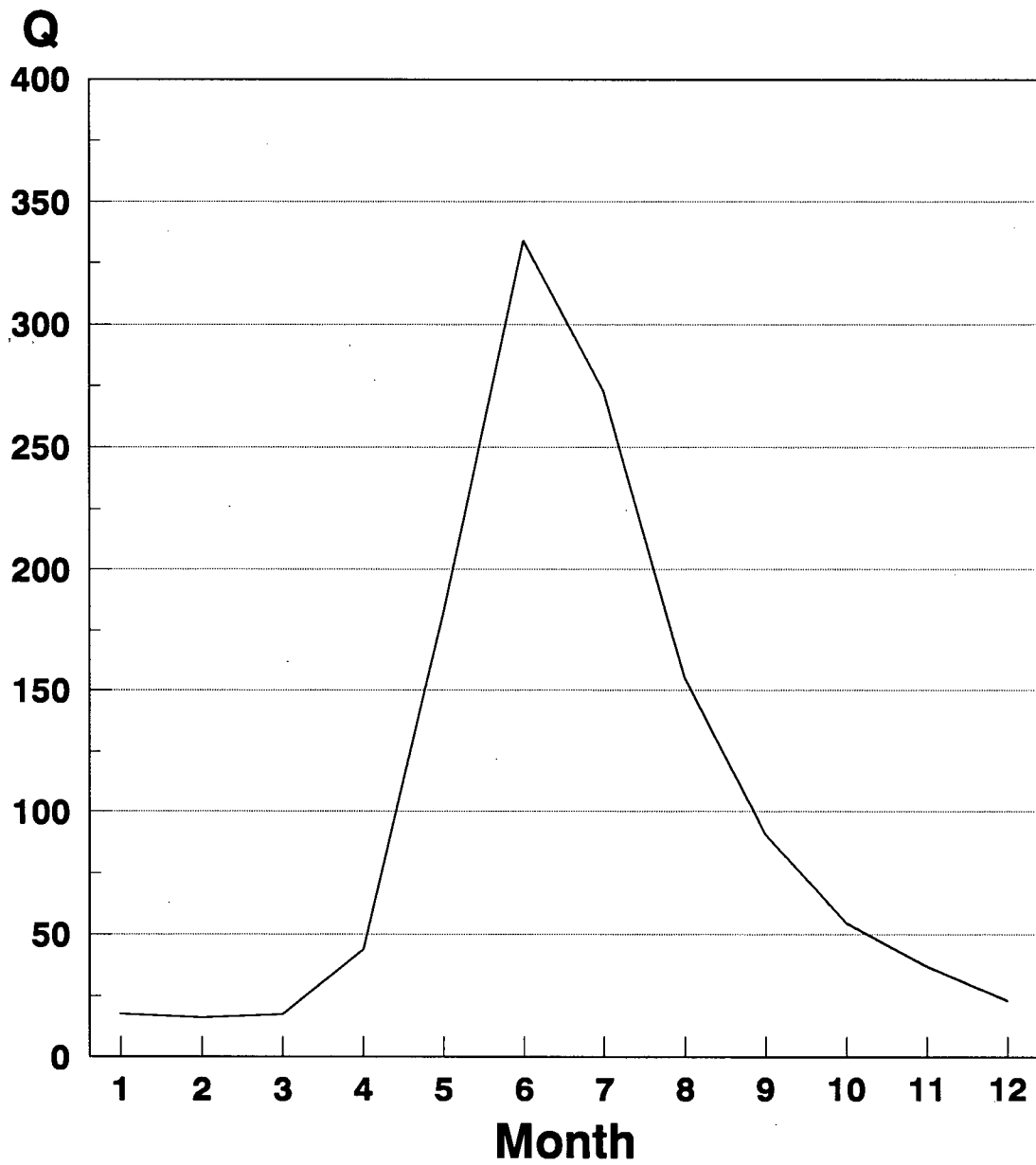


Figure 3.3: The Mean Annual Hydrograph

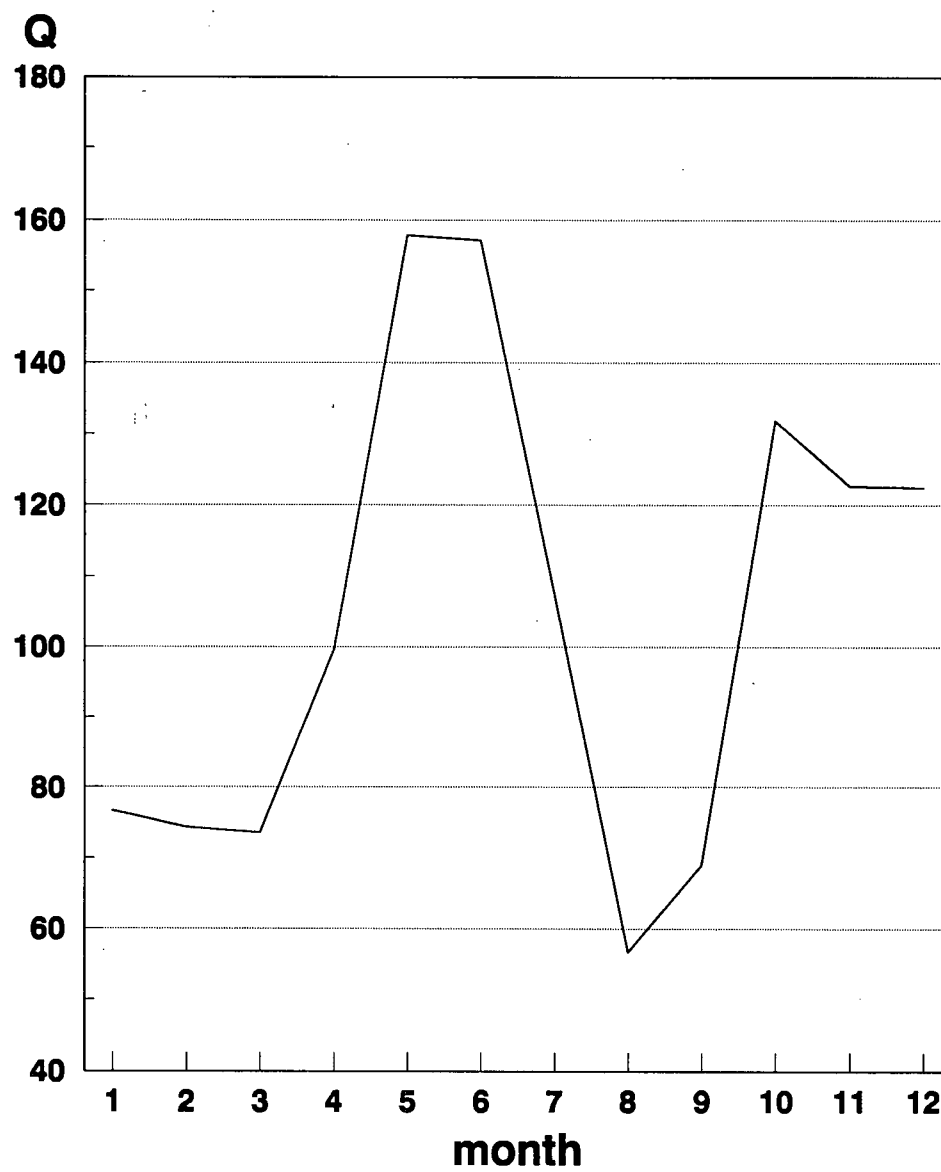


Figure 3.4: Mean Flow Hydrograph of Flow Pattern II

Table 3.3: Comparing Flow Characteristics

	Flow Pattern I			Flow Pattern II		
month	Q	$\sigma$	$\sigma/Q(\%)$	Q	$\sigma$	$\sigma/Q(\%)$
January	17.7	4.3	24.3	76.72	53.68	70.0
Feburary	16.2	2.6	16.0	74.36	35.99	48.4
March	17.5	4.4	25.1	73.61	32.26	43.8
April	44.0	16.6	37.7	99.64	25.28	25.37
May	182.0	45.0	24.7	157.93	35.99	22.8
June	334.2	67.1	20.1	157.18	43.72	27.8
July	273.0	57.6	21.1	107.36	43.72	40.7
August	155.2	38.1	24.5	55.79	27.68	48.7
September	91.0	31.5	34.6	69.0	39.86	57.8
October	54.7	14.0	25.6	131.9	64.4	48.8
November	37.1	11.9	32.1	122.68	57.54	46.9
December	22.9	5.5	24.0	122.56	47.58	38.8

inflows.

The peak flow of pattern I, which is equal to  $334.2 \text{ Mm}^3$  is much higher than the peak flow of flow pattern II, which is  $157.93 \text{ Mm}^3$ . There is only one peak flow period that happens from May to September for flow pattern I and its peak flow period is quite obvious and different from the low flow. The total flow during that period is 83.1% of the total annual flow. The differences between  $Q_{max}$  ( $334.2 \text{ Mm}^3$ ) and  $Q_{min}$  ( $16.2 \text{ Mm}^3$ ) is  $318 \text{ Mm}^3$ . But for flow pattern II, there are two peak flow periods. Its first peak flow period starts from May and ends with July. The second one is from October to December. The total peak flow is only 64% of the total annual flow, although the whole peak flow length is six months which is one month longer than the flow pattern I. The difference between  $Q_{max}$  ( $157.93 \text{ Mm}^3$ ) and  $Q_{min}$  ( $56.79 \text{ Mm}^3$ ) is only  $101.14 \text{ Mm}^3$ . That means that its variance of mean monthly inflow is smaller than pattern I.

On the other hand, the distribution of monthly deviations of flow pattern II shows that the monthly deviations in the low flow period are much greater compared with flow

pattern I. For example, in January, the deviation of flow pattern II is 70% of its monthly mean flow, while it is only 24.3% for pattern I at the same month. This means there is more dispersion of values in the set of low flows for pattern II.

The next chapter will show that these flow patterns have a significant influence on the optimization results of the reservoir operation.

### 3.4 THE OUTCOME OF INFLOW FORECAST

Using the method described in section 3.2, the inflow set was derived based on the information given in the section 3.3. A computer program has been developed to calculate the stream flows and their probabilities. Table 3.4 represents the results for flow pattern I. It can be seen that there are only a few of possible flows which can occur during the low flow period due to the small values of the monthly mean and deviation. For example, there are four possible flows in December of the forecasting year, and they are (0,0.005), (15,0.471), (30,0.516), (45,0.008). Their mean value is  $22.9 \text{ Mm}^3$  which is exactly equal to the input mean value. On the other hand, the range of possible flows in the high flow period is larger than during the low flow period. In June of the forecasting sequence, for example, the flow ranges from  $120 \text{ Mm}^3$  to  $540 \text{ Mm}^3$ , resulting in 29 discrete values. The mean value of the flows is equal to  $334.0 \text{ Mm}^3$ , slightly less than the input mean flow which is equal to  $334.2 \text{ Mm}^3$ .

Table 3.5 shows the flows and probabilities for stream flow pattern II. The generated inflow set shows some differences between input mean monthly flows and mean values of the generated flows. Because of the larger values of deviations and the relatively high values of mean inflow in the low flow period, the range of possible flows is wider than flow pattern I. In December of the forecasting year, for example, the flow ranges from 0 to  $270 \text{ Mm}^3$ , which is much wider than the same month's possible flow with pattern I.

Table 3.4: Inflows and Probabilities ( Flow Pattern I)

Q	Jan.	Feb.	March	Apr.	May	June
0	0.021	0.002	0.026	0.014		
15	0.780	0.970	0.783	0.086		
30	0.199	0.028	0.191	0.249		
45	0.000		0.000	0.343	0.001	
60				0.226	0.004	
75				0.071	0.008	
90				0.011	0.017	
105				0.001	0.031	
120					0.052	0.001
135					0.077	0.001
150					0.103	0.002
165					0.123	0.004
180					0.132	0.006
195					0.127	0.010
210					0.109	0.016
225					0.084	0.024
240					0.058	0.033
255					0.036	0.045
270					0.020	0.056
285					0.010	0.068
300					0.004	0.078
315					0.002	0.085
330					0.001	0.089

Fig.3.5 shows the probability distribution of the generated flows for pattern I in July.

Tables 3.6 and 3.7 show the means of the input data and the generated values.

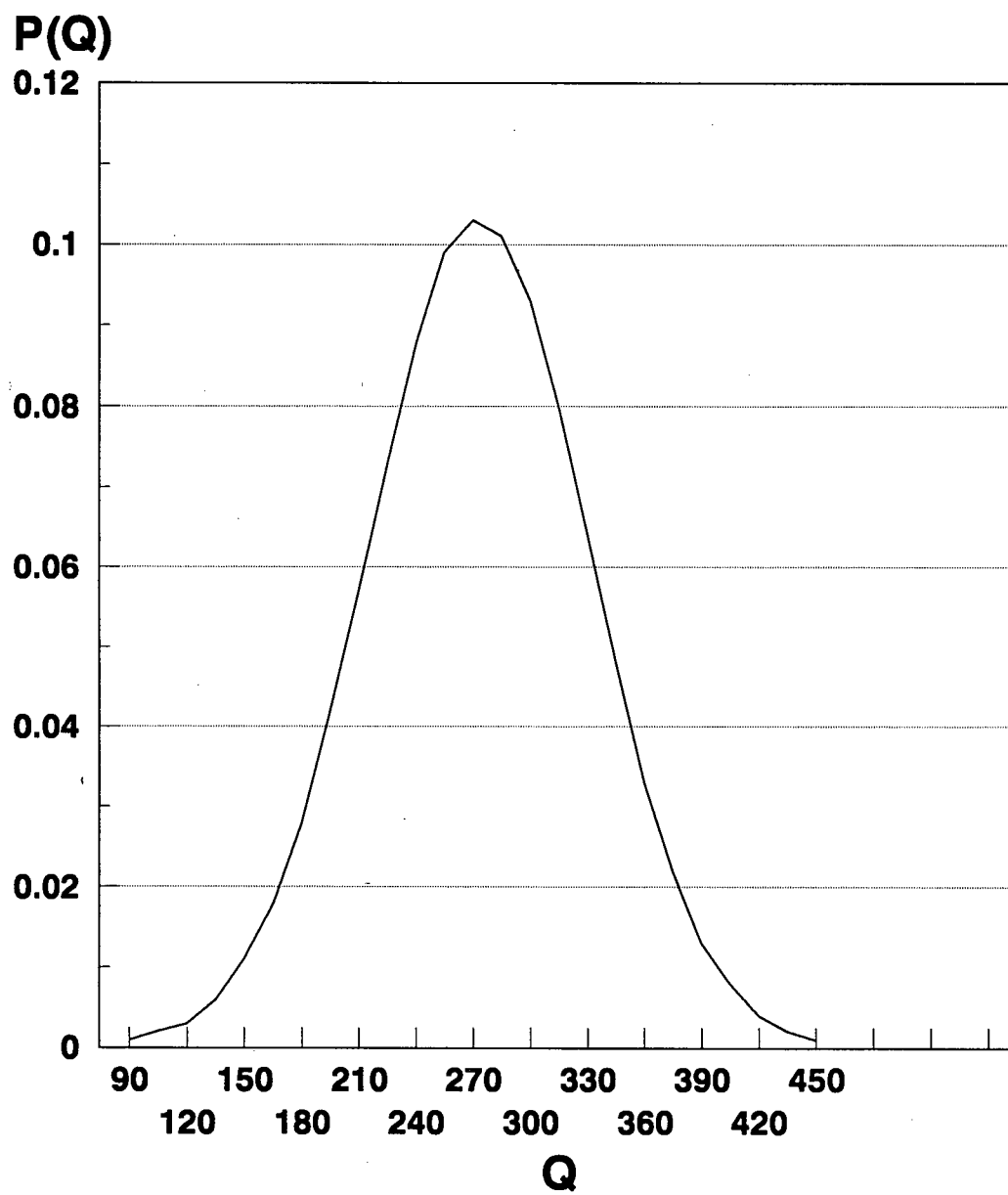


Figure 3.5: Probability Distribution of Generated Flow

Q	Jan.	Feb.	March	Apr.	May	June
345						0.088
360						0.083
375						0.074
390						0.063
405						0.051
420						0.040
435						0.029
450						0.020
465						0.013
480						0.009
495						0.005
510						0.003
525						0.002
540						0.001
Q	July	Aug.	Sept.	Oct.	Nov.	Dec.
0		0.002	0.003	0.000	0.008	0.005
15		0.005	0.011	0.012	0.110	0.471
30		0.013	0.030	0.103	0.395	0.516
45		0.029	0.067	0.324	0.382	0.008
60		0.056	0.117	0.375	0.099	
75		0.090	0.165	0.160	0.006	
90	0.001	0.126	0.187	0.025		
105	0.002	0.150	0.170	0.001		

Table 3.4 Results of Inflow Forecast ( Flow Pattern I), continued



Q	July	Aug.	Sept.	Oct.	Nov.	Dec.
120	0.003	0.154	0.124			
135	0.006	0.136	0.073			
150	0.011	0.103	0.034			
165	0.018	0.067	0.013			
180	0.028	0.038	0.004			
195	0.042	0.018	0.001			
210	0.057	0.007				
225	0.073	0.003				
240	0.088	0.001				
255	0.099					
270	0.103					
285	0.101					
300	0.093					
315	0.080					
330	0.064					
345	0.048					
360	0.033					
375	0.022					
390	0.013					
405	0.008					
420	0.004					
435	0.002					
450	0.001					

Table 3.4 Results of Inflow Forecast ( Flow Pattern I), continued

Table 3.5: Results of Inflow Forecast (Flow Pattern II)

Q	Jan.	Feb.	Mar.	Apr.	May	June
0	0.043	0.021	0.015			
15	0.061	0.044	0.037	0.001		0.001
30	0.081	0.079	0.076	0.006		0.002
45	0.099	0.120	0.126	0.025	0.001	0.005
60	0.112	0.154	0.169	0.071	0.004	0.012
75	0.118	0.166	0.184	0.147	0.012	0.024
90	0.114	0.152	0.162	0.216	0.029	0.042
105	0.103	0.117	0.116	0.227	0.057	0.067
120	0.085	0.076	0.067	0.170	0.096	0.095
135	0.066	0.042	0.032	0.091	0.135	0.120
150	0.047	0.019	0.012	0.035	0.161	0.134
165	0.031	0.007	0.004	0.009	0.162	0.134
180	0.019	0.002	0.001	0.002	0.137	0.119
195	0.011	0.001			0.098	0.094
210	0.006				0.059	0.066
225	0.003				0.030	0.042
240	0.001				0.013	0.023
255					0.005	0.012
270					0.001	0.005
285						0.002
300						0.001

Q	July	Aug.	Sept.	Oct.	Nov.	Dec.
0	0.007	0.028	0.035	0.012	0.011	0.006
15	0.015	0.072	0.062	0.018	0.018	0.012
30	0.029	0.137	0.096	0.027	0.029	0.023
45	0.050	0.196	0.128	0.039	0.042	0.039
60	0.077	0.213	0.149	0.051	0.058	0.060
75	0.104	0.174	0.151	0.064	0.075	0.084
90	0.126	0.107	0.134	0.076	0.089	0.106
105	0.136	0.050	0.103	0.086	0.100	0.121
120	0.131	0.017	0.069	0.093	0.105	0.125
135	0.112	0.005	0.040	0.094	0.103	0.118
150	0.086	0.001	0.020	0.090	0.094	0.101
165	0.058		0.009	0.083	0.080	0.078
180	0.035		0.003	0.071	0.064	0.054
195	0.019		0.001	0.058	0.048	0.035
210	0.009			0.045	0.033	0.020
225	0.004			0.033	0.022	0.010
240	0.001			0.023	0.013	0.005
255				0.015	0.008	0.002
270				0.010	0.004	0.001
285				0.006	0.002	
300				0.003	0.001	
315				0.002		
320				0.001		

Table 3.5 Results of Inflow Forecast (Flow Pattern II), continued

Table 3.6: Comparison of Mean Monthly Flow ( Flow Pattern I )

	Jan.	Feb.	Mar.	Apr.	May	June
I $\mu_Q$	17.7	16.2	17.5	44.0	182.0	334.2
O $\mu_Q$	17.7	16.2	17.5	44.2	181.9	334.0
$\Delta Q(\%)$	0.0	0.0	0.0	0.5	-0.05	-0.06
	July	Aug.	Sept.	Oct.	Nov.	Dec.
I $\mu_Q$	273.0	155.2	91.0	54.7	57.1	22.9
O $\mu_Q$	272.9	155.1	91.0	54.7	57.1	22.9
$\Delta Q(\%)$	-0.04	-0.06	0.0	0.0	0.0	0.0

Table 3.7: Comparison of Mean Monthly Flow ( Flow Pattern II )

	Jan.	Feb.	Mar.	Apr.	May	June
I $\mu_Q$	76.7	74.4	73.6	99.6	157.9	157.2
O $\mu_Q$	80.5	75.4	74.2	99.7	158.0	157.3
$\Delta Q(\%)$	4.9	1.4	0.8	0.06	0.04	0.08
	July	Aug.	Sept.	Oct.	Nov.	Dec.
I $\mu_Q$	107.4	55.8	69.0	131.9	122.7	122.6
O $\mu_Q$	107.7	57.6	71.6	134.1	124.3	118.7
$\Delta Q(\%)$	0.3	3.2	3.8	1.7	1.3	-3.1

## **Chapter 4**

### **RESULTS**

#### **4.1 RESERVOIR DESCRIPTION**

The hypothetical reservoir and hydroelectric power plant used in this study is the same as that used by and described by Joanna Mary Barnard (1989). The project's characteristics and the data used in the study are presented in this section.

##### **4.1.1 THE OBJECTIVE FUNCTION**

As mentioned before, for the purpose of simplification, the only objective of the hydro power project considered in this study is energy production. Other goals, such as flood control or water supply are not considered directly. However, goals of water supply and environmental protection can be included in the minimum discharge requirement, (that is the discharge must be greater than or equal to a predetermined minimum release in any time period), while flood control may be incorporated into the constraints of maximum reservoir volume and maximum release. The values chosen for use in comparing policies was the total year end values of each state over the long term optimal operation. These values are appropriate only because of the assumption that the reservoir is operated independently and therefore the energy is of the equal value at all times of the operating year. Energy generation is given in Gigawatt Hours (Gwh) of energy generated over the whole year. The percentage of potential increase in energy production from using the one month ahead perfect stream flow forecast compared to the stochastic stream flow

Table 4.1: Reservoir Constraints and Characteristics

Item	Value
Reservoir Design Volume, $Mm^3$	495
Minimum Volume, $Mm^3$	270
Maximum Volume, $Mm^3$	765
Minimum discharge, $Mm^3$	15
Maximum discharge, $Mm^3$	150, 180, 210
State Incremental Value, $Mm^3$	15
Discharge Incremental Value, $Mm^3$	15

can be transformed into dollars. The value of energy is assumed at 20 mills or 2 cents per KWH which is equivalent to \$20,000/Gwh.

#### 4.1.2 RESERVOIR VOLUME AND DISCHARGE

The minimum reservoir volume used in this study is 270  $Mm^3$  (Million cubic meters) while the maximum volume is 765  $Mm^3$  and the reservoir's live storage volume is 495  $Mm^3$ .

The minimum discharge is assumed to be 15  $Mm^3$  per month. In order to examine the effect of maximum discharge on the value of stream flow information, three alternative maximum discharges were used: 150  $Mm^3$ , 180  $Mm^3$ , and 210  $Mm^3$ . Table 4.1 is the summary of the reservoir constraints and its characteristics.

Both the state and decision variables, that is the reservoir volume and the release, were discretized with the same incremental value of 15  $Mm^3$ . The number of the reservoir states is 33. The state incremental value was chosen to obtain reasonable accuracy yet keep the number of states manageable. The reservoir elevation-volume curve is as shown in figure 4.1.

In the computer program for calculating energy production, it is convenient to have a mathematical equation to express the relationship between the elevation and the reservoir

volume. The best-fit mathematical expression is:

$$Elev = C_0 + C_1 * V + C_2 * V^2 \quad (4.1)$$

where

$$C_0 = 32.7308$$

$$C_1 = 0.078263$$

$$C_2 = -0.00001$$

#### 4.1.3 THE ADDITIONAL EXPECTED VALUES OF ONE MONTH AHEAD PERFECT FORECAST

To obtain the additional value of the one month ahead perfect stream flow forecast, the pure stochastic dynamic programming model was first used to optimize the reservoir's operation. The expected value of the power output with this type of operation provided the basis of comparison. The same optimization procedure was then used with the one month ahead perfect runoff forecasts and the expected value again computed. The results of the two sets of computations showed the differences between stochastic (i.e. with no forecasts) and operation with the forecast. Equation (2.29) and (2.30) given the definition of the additional benefit of the perfect stream flow forecast.

$$\Delta Value = F_p - F_s \quad (2.29)$$

or

$$\Delta Value = \frac{|F_p - F_s|}{F_s} \% \quad (2.30)$$

The values of the different optimized situations are shown in the following sections.

The following runs of the study models were performed

1. Stochastic forecast: for each of the maximum releases of 150  $Mm^3$ , 180  $Mm^3$ , and 210  $Mm^3$  with the two inflow patterns, the stochastic dynamic programming model was

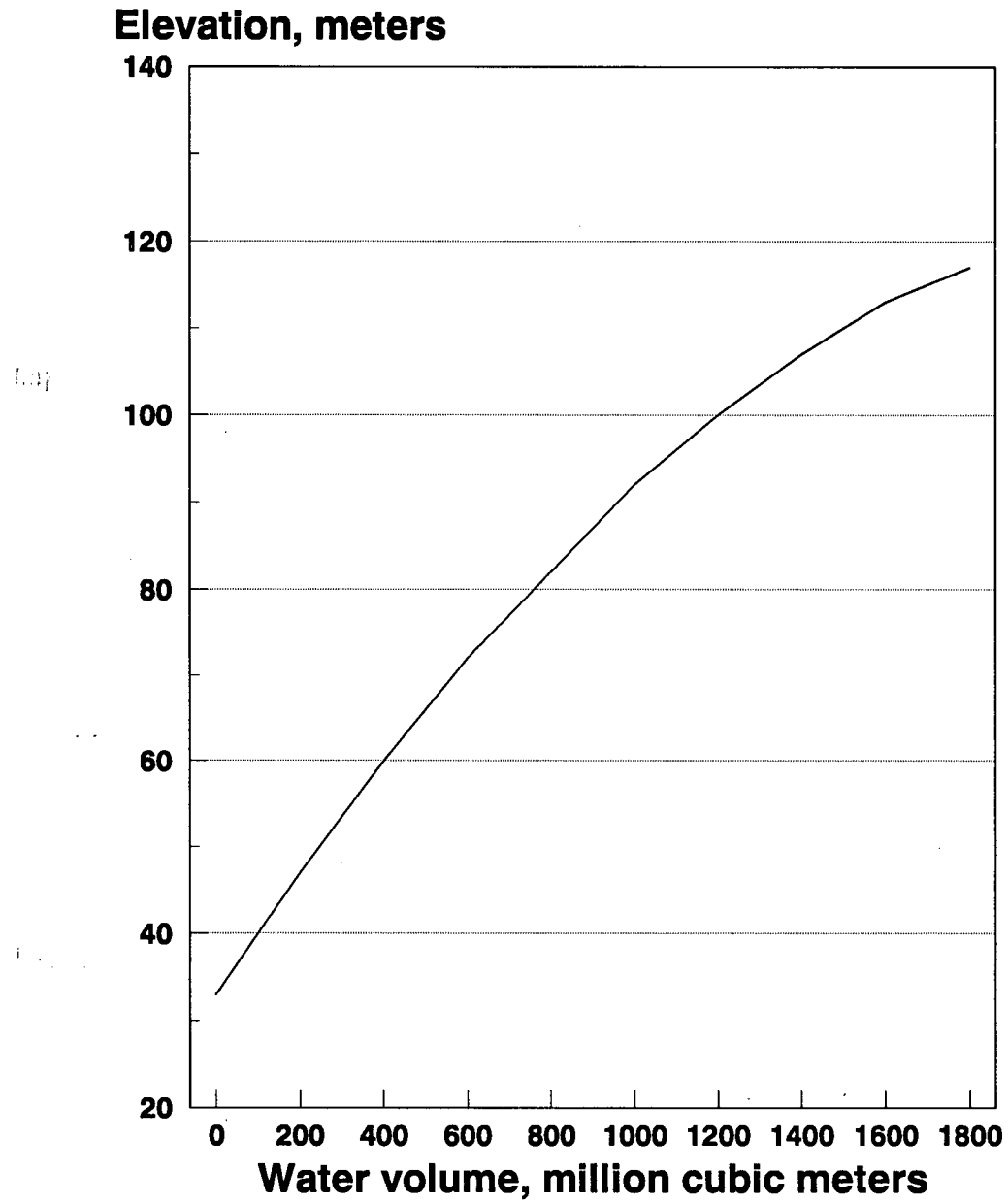


Figure 4.1: Reservoir Elevation-volume Curve



run to find the optimal reservoir operation policies and their values. Thus six operation policies were analyzed.

2. Perfect forecast: each of the cases corresponding to the stochastic dynamic programming were analyzed with the one month ahead perfect forecast dynamic programming model to make the comparison.

In real world problems, making accurate predictions for the peak flow period is of great interest because the total amount of flow in the high flow period has a significant influence on the reservoir operation. The dynamic programming model using a combination of stochastic and perfect short term forecasts was developed for the purpose of investigating the additional expected values of longer term perfect stream flow forecasts. That is, assuming that one ( or two, three,  $\dots$  ) months of one month ahead perfect forecasts were available, and using an appropriate combination of the stochastic and perfect forecast models to find the additional expected values. For flow pattern I, the analyses performed were:

one month perfect inflow prediction for June;

two months of perfect inflow predictions for June and July;

three months of perfect inflow predictions for June, July, and August;

four months of perfect inflow predictions for June, July, August and September.

For stream flow pattern II, optimization was performed for:

one month perfect inflow prediction for May;

two months of perfect inflow predictions for May and June;

three months of perfect inflow predictions for May, June and October;

four months of perfect inflow predictions for May, June, October and November.

For these cases the general maximum release constraint was  $D_{max}=180 \text{ Mm}^3/\text{month}$ .

Table 4.2: Average Reservoir Operation Processes with  $D_{max}=180 \text{ Mm}^3$ 

	1	2	3	4	5	6	7	8	9	10	11	12
	$\text{Mm}^3$											
Sto.	405	405	405	405	451	645	735	705	635	569	492	405
I	405	405	405	405	451	645	765	705	635	569	492	405
II	405	405	410	410	465	645	765	765	635	585	500	405
III	420	420	420	420	480	645	765	765	635	585	500	420
IV	420	420	435	435	480	645	765	765	635	600	500	420
Perf.	435	435	435	435	495	645	765	765	765	699	607	435

## 4.2 OPERATION RESULTS FOR FLOW PATTERN I

### 4.2.1 GENERAL MAXIMUM DISCHARGE ( $D_{max}=180 \text{ Mm}^3$ )

The long-term optimal operation policy for stochastic dynamic programming shows that on average the reservoir starts with a water volume of  $405 \text{ Mm}^3$  at the beginning of January and ends at the same volume at the end of the operating year. The maximum volume, which occurs at July, reached  $735 \text{ Mm}^3$ , that is two states less than full. Figure 4.2 and Table 4.2 show the average operating regime of the reservoir.

The table also contains the operating policies for the corresponding one month ahead perfect forecast and all the other runs. The average starting point for the one month ahead perfect forecast was  $435 \text{ Mm}^3$ , two states higher than with no forecasts. The water level reaches the allowable highest point, that is the volume of  $765 \text{ Mm}^3$ , in July and keeps level until September. At the end of the operating year, it returns to the starting water level. Fig.4.2 shows the average operation with all runs. The figure shows that the less uncertainty in stream flow prediction, the higher the average water level is and the more stages where the reservoir reaches the full state. These results suggest that accurate inflow predictions do improve the operating efficiency of the hydro-electric plant. Table 4.3 shows the year end expected values of the states. Table 4.4 shows the

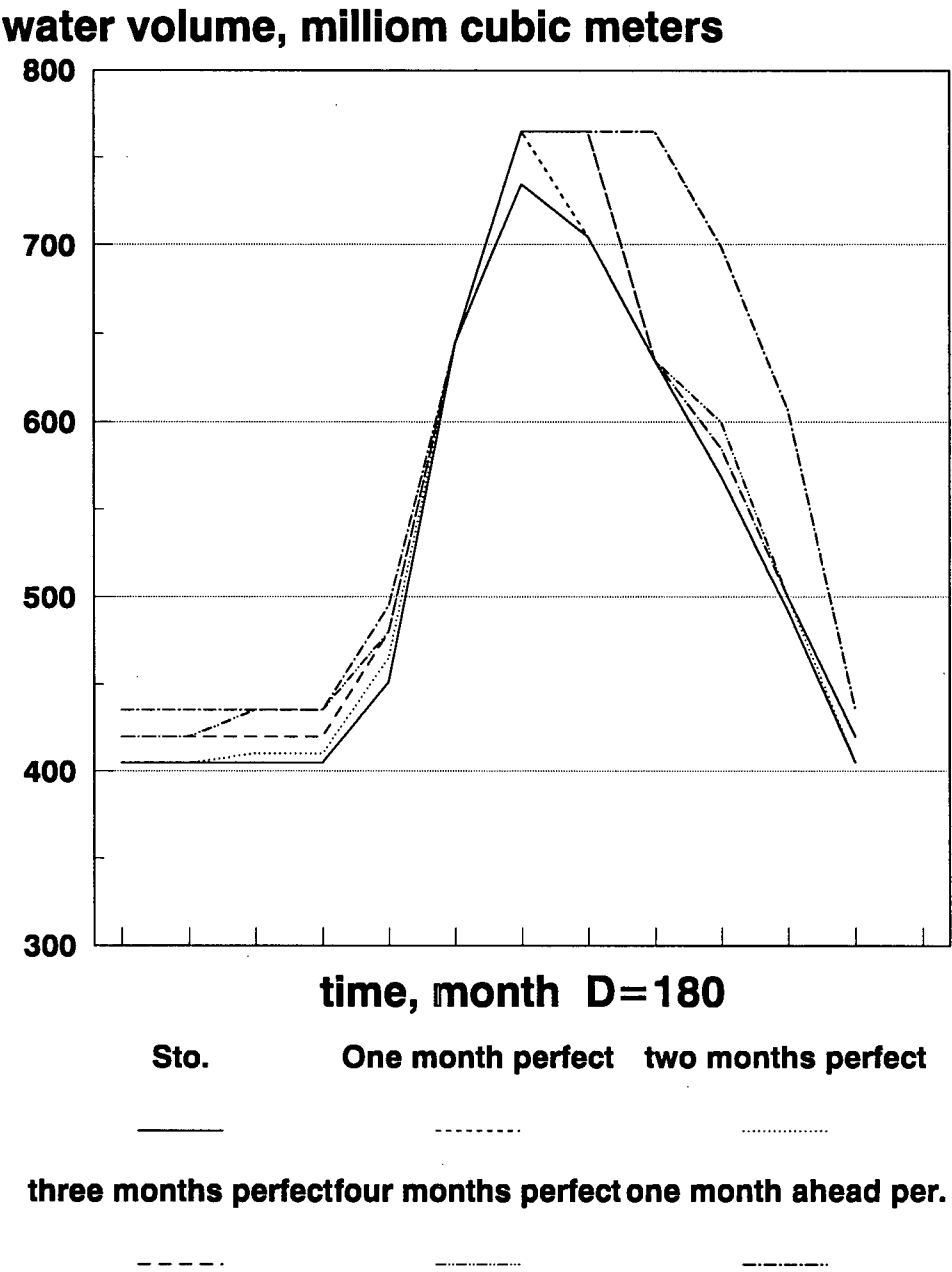


Figure 4.2: The Reservoir Operation Processes with  $D_{max}=180Mm^3$

Table 4.3: The Year End Values of States with  $D_{max}=180 \text{ } Mm^3$ 

state	Sto.	I	II	III	IV	Perf.
$Mm^3$	Gwh					
270	195.6	199.0	203.9	206.7	210.0	212.9
285	197.5	200.9	205.8	208.5	211.9	214.9
300	200.0	203.4	208.3	211.1	214.4	217.4
315	202.4	205.8	210.8	213.5	216.8	219.9
330	204.9	208.3	213.3	216.1	219.3	222.4
345	207.4	210.8	215.8	218.6	221.9	224.9
360	209.9	213.4	218.3	220.1	224.4	227.4
375	212.4	215.9	220.8	223.6	226.9	230.0
390	215.0	218.5	223.4	226.3	229.5	232.6
405	217.7	221.2	226.2	229.0	232.2	235.3
420	220.3	223.8	228.8	231.6	234.8	237.9
435	223.0	226.5	231.5	234.3	237.6	240.7
450	225.7	229.2	234.2	237.1	240.3	243.5
465	228.4	232.0	236.9	239.8	243.0	246.3
480	231.0	234.6	239.6	242.4	245.6	249.0
495	233.6	237.2	242.2	245.0	248.2	251.7
510	235.6	239.3	244.2	247.0	250.2	253.7
525	238.9	242.6	247.5	250.4	253.6	257.1
540	241.5	245.2	250.1	253.0	256.2	259.7
555	244.3	248.0	252.9	255.5	259.0	262.5
570	247.0	250.7	255.7	258.5	261.8	265.3
585	249.8	253.5	258.5	261.3	264.6	268.1
600	252.6	256.3	261.3	264.2	267.4	271.0
615	255.5	259.2	264.2	267.1	270.3	274.0
630	258.4	262.1	267.1	270.0	273.3	277.0
645	261.3	265.1	270.1	272.9	276.2	280.0
660	264.2	268.0	273.0	275.9	279.1	283.0
675	267.1	270.9	275.9	278.8	282.1	286.0
690	270.1	273.9	278.9	281.8	285.1	289.1
705	273.1	276.9	282.0	284.8	288.2	292.2
720	276.2	280.0	285.1	288.0	291.3	295.4
735	279.2	283.0	288.1	291.0	294.3	298.5
750	282.3	286.1	291.2	294.1	297.4	301.7
765	285.5	289.3	294.4	297.4	300.6	305.4

### The expected values of state, Gwh

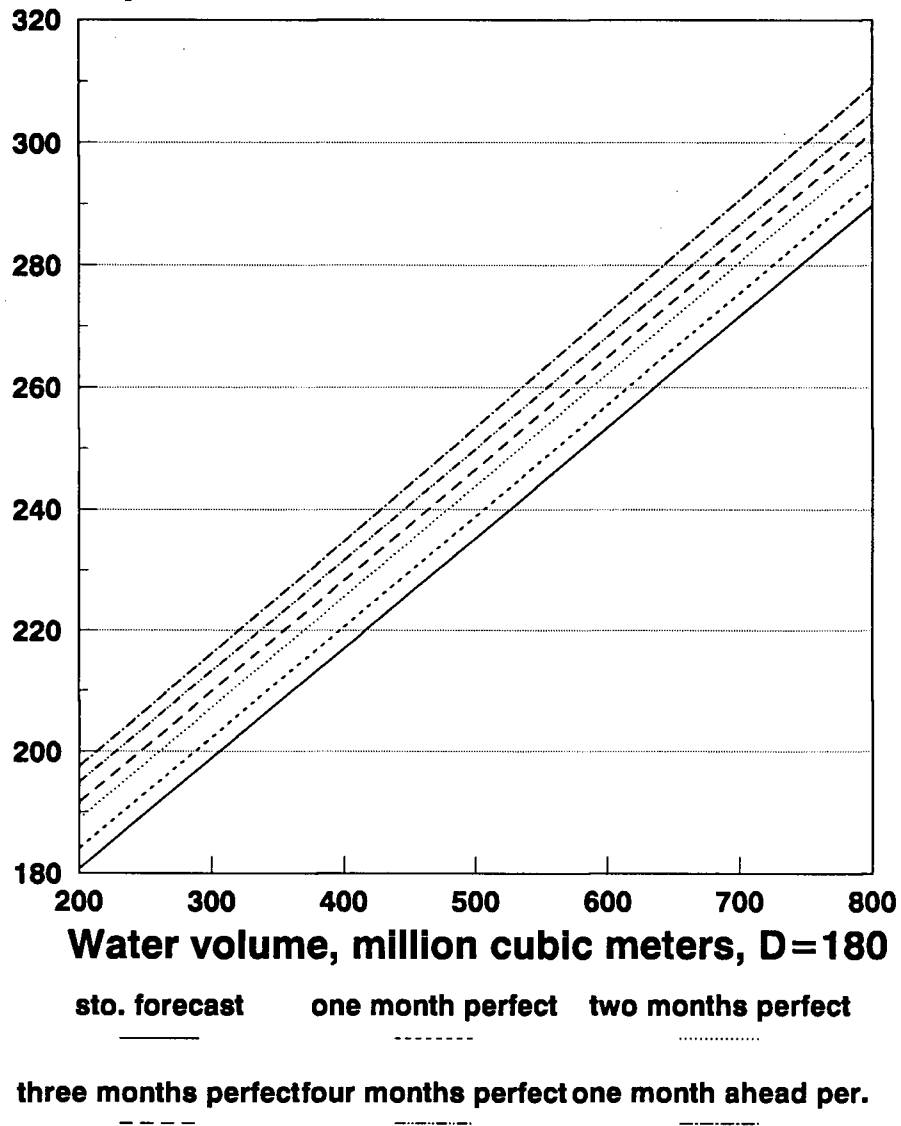


Figure 4.3: The Expected Values of States,  $D_{max} = 180 Mm^3$

Table 4.4: The Additional Values of States with  $D_{max}=180 \text{ Mm}^3$ 

	Stoch.	perfect		
state, $\text{Mm}^3$	Sto., Gwh	$\Delta V$ , Gwh	values, \$	%
270	195.6	17.3	334,000	8.8
285	197.5	17.4	348,000	8.8
300	200.0	17.4	348,000	8.7
315	202.4	17.5	350,000	8.6
330	204.9	17.5	350,000	8.5
345	207.4	17.5	350,000	8.4
360	209.9	17.5	350,000	8.3
375	212.4	17.6	352,000	8.3
390	215.0	17.6	352,000	8.2
405	217.7	17.6	352,000	8.1
420	220.3	17.6	352,000	8.0
435	223.0	17.7	354,000	7.9
450	225.7	17.8	356,000	7.9
465	228.4	17.9	358,000	7.8
480	231.0	18.0	360,000	7.8
495	233.6	18.1	362,000	7.8
510	235.6	18.1	362,000	7.8
525	238.9	18.2	364,000	7.7
540	241.5	18.2	364,000	7.7
555	244.3	18.2	364,000	7.6
570	247.0	18.3	366,000	7.6
585	249.8	18.3	366,000	7.5
600	252.6	18.4	368,000	7.5
615	255.5	18.5	370,000	7.4
630	258.4	18.6	372,000	7.3
645	261.3	18.7	374,000	7.3
660	264.2	18.8	376,000	7.2
675	267.1	18.9	378,000	7.2
690	270.1	19.0	380,000	7.1
705	273.1	19.1	382,000	7.1
720	276.2	19.2	384,000	7.0
735	279.2	19.3	386,000	7.0
750	282.3	19.4	388,000	6.9
765	285.5	19.5	390,000	6.8

expected values with the one month ahead perfect stream flow forecast compared to the stochastic forecast. The additional values lie between 16.8 Gwh for the minimum volume of  $270 \text{ Mm}^3$  to 19.6 Gwh for the maximum volume of  $765 \text{ Mm}^3$  ( see table 4.4 column 3 for details ). And the higher the water level is, the greater the additional expected value gains. When arranged as percentages, the additional values decrease with increasing reservoir state, from 8.6 percent of volume  $270 \text{ Mm}^3$  to 6.9 percent of volume  $765 \text{ Mm}^3$ . This is because the base value in calculating percentage incremental value of volume  $270 \text{ Mm}^3$  is equal to 195.6 Gwh, which is much lower than the base value of volume  $765 \text{ Mm}^3$ , which is equal to 285.5 Gwh.

Table 4.2 and 4.3 also show the results with one, two, three and four months of perfect forecasts by the dynamic program model. It can be concluded that along with the reduction in uncertainty, on average the reservoir water level will be higher and the values of the states will be greater, that is, the expected state values increase with the number of months which have perfect stream flow predictions in high flow season. Fig 4.3 shows the state's year end value increment curve. All of the expected values for part time perfect flow forecasting operations lie between the pure stochastic flow operation and the complete year with one month ahead flow forecasting optimization.

The outcomes with the peak stream flow period (i.e. four months ) perfect forecast indicated that the additional values of the state are very near to the state values with the whole year of one month ahead perfect forecasts. This suggests that perfect forecasts for the high flow period are more valuable than perfect forecasts for the low flow season. That is a reasonable result. First, from table 3.1, it is known that the amount of water input into the reservoir during that period is very high. The water volume and water level vary most in that time, and the energy production is obviously higher than in the low flow season. Thus accurate prediction of inflow during the high flow period has a significant influence in the operational efficiency of the reservoir. Second, from table 3.1,

Table 4.5: Reservoir Operation Processes for  $D_{max}=210 \text{ Mm}^3$ 

	1	2	3	4	5	6	7	8	9	10	11	12
	$\text{Mm}^3$											
Sto.	480	480	480	495	510	660	735	690	670	660	622	480
I	480	480	480	495	510	665	735	740	740	699	650	480
II	480	480	480	495	510	665	765	745	740	699	650	480
III	480	480	480	495	510	665	765	765	745	699	650	480
IV	480	480	480	495	510	665	765	765	765	699	650	480
Per.	495	495	495	495	510	743	765	765	765	765	650	495

it is seen that the flow variations during the high flow period is much greater than in the low flow season, which indicates the wide range of flow variance. Therefore perfect flow forecasts, which reduce uncertainty about the flows, should cause more improvement with high flows.

#### 4.2.2 RESULTS WITH MAXIMUM RELEASE $D_{max}=210 \text{ Mm}^3$

For the case where the maximum release equals  $210 \text{ Mm}^3$ , the reservoir volume starts on average at  $480 \text{ Mm}^3$  and ends with the same state except that with the one month ahead perfect forecast, which on average starts from water volume of  $495 \text{ Mm}^3$  and returns to the same volume at the end of operating year. For the case of stochastic operation and the one month perfect forecast during the high flow period, the highest average water levels during the operation are both  $735 \text{ Mm}^3$ . In the other months during the high flow period, the water levels with the one month perfect forecast are slightly higher than with the stochastic forecast. But in the other cases, the highest water level reaches the reservoir full state, that is  $V_{max} = 765 \text{ Mm}^3$ . Table 4.5 shows the differences among the different forecasting cases.

Comparing the corresponding water levels between having the maximum release limited to  $210 \text{ Mm}^3$  and equal to  $180 \text{ Mm}^3$ , it was found that the water level in the low



Table 4.6: Expected Values of States with  $D_{max}=210 \text{ Mm}^3$ 

state	Sto.	I	II	III	IV	Per.
$\text{Mm}^3$	Gwh					
270	202.3	205.1	208.3	210.2	214.7	218.1
285	205.6	208.4	211.6	213.5	218.6	221.4
300	208.4	211.2	214.4	216.3	221.4	224.3
315	212.5	215.4	218.5	220.4	225.5	228.5
330	215.0	217.9	221.0	223.0	227.9	231.0
345	217.9	220.8	224.0	225.9	229.6	234.0
360	221.0	223.9	227.1	229.0	232.7	237.1
375	223.1	226.0	229.2	231.1	235.2	239.3
390	226.3	229.2	232.4	234.3	237.0	242.5
405	228.9	231.9	235.0	236.9	240.5	245.2
420	232.2	235.2	238.3	240.3	244.3	248.2
435	233.6	236.6	239.8	241.7	248.0	250.1
450	236.7	239.7	242.9	244.8	250.1	253.2
465	239.8	242.8	246.0	247.9	253.3	256.4
480	242.2	245.3	248.4	250.3	255.7	258.8
495	244.6	247.7	250.8	252.7	257.3	261.3
510	247.3	250.4	253.5	255.4	259.6	264.0
525	250.4	253.5	256.6	258.7	262.6	267.2
540	252.6	255.7	258.8	260.9	264.3	269.5
555	255.8	259.0	262.0	263.9	268.8	272.7
570	258.5	261.7	264.8	266.7	271.7	275.4
585	261.1	264.3	267.4	269.3	275.4	278.1
600	263.4	266.6	269.7	271.7	277.6	280.4
615	266.2	269.4	272.5	274.3	280.5	283.2
630	268.2	271.5	274.5	276.5	283.2	285.3
645	270.5	273.8	276.7	278.9	285.6	287.7
660	273.3	276.6	279.5	281.6	287.5	290.6
675	276.4	279.7	282.7	284.8	289.3	293.8
690	279.4	282.7	285.8	287.6	291.4	296.8
705	282.4	285.8	288.8	290.6	293.4	299.8
720	285.6	289.0	292.0	293.9	297.5	303.1
735	287.7	290.1	294.2	296.0	300.1	305.3
750	291.9	294.1	298.4	301.0	303.2	309.6
765	295.1	298.5	301.6	303.6	307.3	312.9

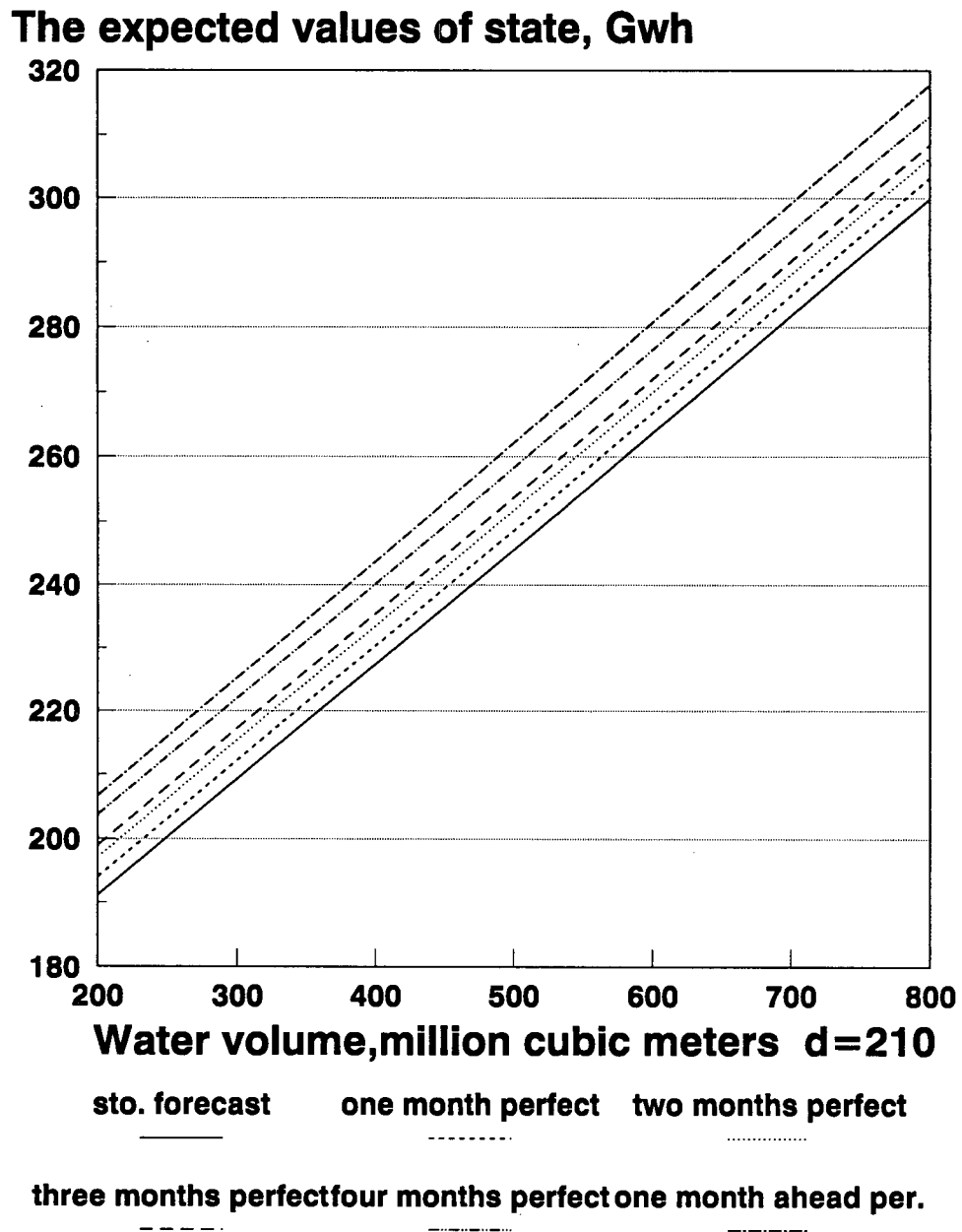


Figure 4.4: The Expected Values of States,  $D_{max} = 210 Mm^3$

Table 4.7: Additional Values of States  $D_{max}=210 \text{ Mm}^3$ 

state	Sto.	Perfect		
$\text{Mm}^3$	Gwh	$\Delta V$ Gwh	Value \$	%
270	202.3	15.8	316,000	7.8
285	205.6	15.8	316,000	7.7
300	208.4	15.9	318,000	7.6
315	212.5	16.0	320,000	7.5
330	215.0	16.1	322,000	7.4
345	217.9	16.1	322,000	7.4
360	221.0	16.1	322,000	7.3
375	223.1	16.2	324,000	7.3
390	226.3	16.2	324,000	7.2
405	228.9	16.3	326,000	7.1
420	232.2	16.4	328,000	7.1
435	233.6	16.5	330,000	7.1
450	236.7	16.5	330,000	7.0
465	239.8	16.6	332,000	6.9
480	242.2	16.6	332,000	6.9
495	244.6	16.7	332,000	6.8
510	247.3	16.7	334,000	6.8
525	250.4	16.8	336,000	6.7
540	252.6	16.9	338,000	6.7
555	253.8	16.9	338,000	6.6
570	258.6	16.9	338,000	6.5
585	261.1	17.0	340,000	6.5
600	263.4	17.0	340,000	6.5
615	266.2	17.0	340,000	6.4
630	268.2	17.1	342,000	6.4
645	270.5	17.2	344,000	6.4
660	273.3	17.3	346,000	6.3
675	276.4	17.4	348,000	6.3
690	279.4	17.4	348,000	6.2
705	282.4	17.4	348,000	6.1
720	285.6	17.5	350,000	6.1
735	287.7	17.6	352,000	6.1
750	291.9	17.7	354,000	6.0
765	295.1	17.8	356,000	6.0

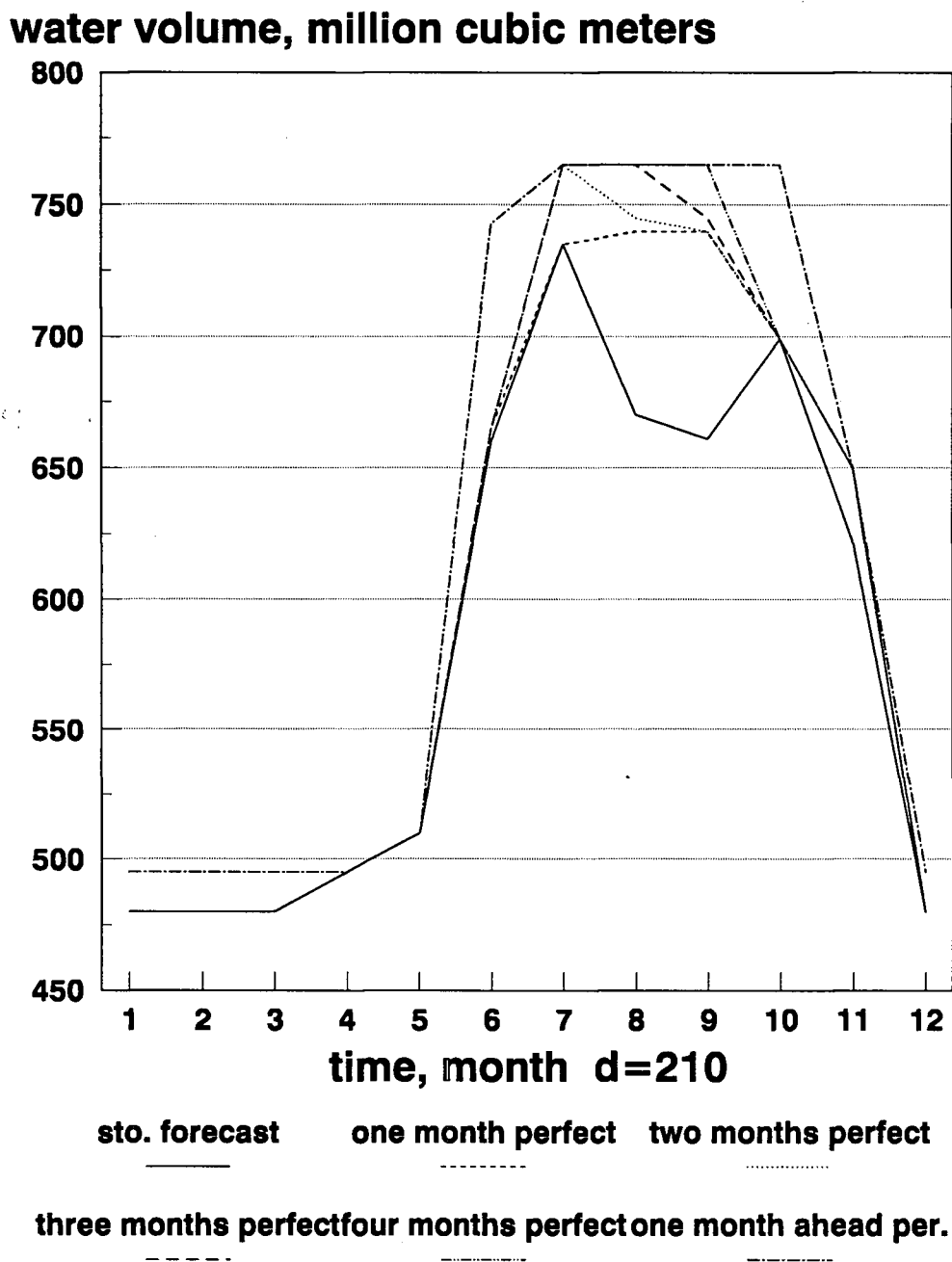


Figure 4.5: The Reservoir Operation Processes,  $D_{max} = 210Mm^3$

Table 4.8: Operation Processes of Flow Pattern I,  $D_{max} = 150 Mm^3$ 

	1	2	3	4	5	6	7	8	9	10	11	12
	$Mm^3$											
Sto.	360	360	360	389	488	660	715	660	564	516	495	360
I	360	360	360	389	488	665	720	665	564	516	495	360
II	360	360	360	389	488	665	735	665	564	516	495	360
III	360	360	360	389	488	665	750	665	580	535	495	360
IV	375	375	375	389	488	665	765	670	580	535	510	375
Pec.	390	390	390	420	541	665	765	670	645	550	540	390

flow period for the former case was higher than with the latter operation. For example, the average starting point for the former is  $480 Mm^3$ , but for the latter is  $405 Mm^3$ . The reason is that the ability to adjust high flows for the former is stronger than the latter. Because it allows the maximum release of  $210 Mm^3$ , the reservoir water level can be kept high to provide more energy production without the risk of reservoir spill.

However, when the maximum allowable release is equal to  $180 Mm^3$ ,  $30 Mm^3$  lower than  $210 Mm^3$ , its adjustable ability for flood flows is weaker. Because of the lower discharge capacity, the water level should keep lower before the high flow period. As a result, the energy production will be smaller than when the maximum allowable release is  $210 Mm^3$ . In the next section, it can be seen that the average starting reservoir volume for release limit of  $150 Mm^3$  case is even lower. Therefore, for the purpose of providing more energy production, a high release limit is recommended if possible.

Tables 4.6 and 4.7 show the state values. For the stochastic forecast operation (see column 2 of table 4.6), the state values range from 202.3 Gwh for the minimum reservoir volume of  $270 Mm^3$  to 295.1 Gwh for the maximum water volume of  $765 Mm^3$ . The state long term year end values with the one month ahead perfect forecast (see column 7, table 4.6) vary from 218.1 Gwh to 312.9 Gwh corresponding to the minimum and maximum

Table 4.9: The Year End Values of States  $D_{max} = 150Mm^3$ 

state	Sto.	I	II	III	IV	Per.
$Mm^3$	Gwh					
270	194.2	197.0	203.3	207.1	210.5	214.8
285	196.1	200.0	205.8	209.5	212.9	217.1
300	198.5	202.5	208.4	211.2	215.4	219.5
315	200.9	203.6	210.2	214.7	217.6	221.9
330	203.3	205.9	213.7	216.4	219.5	224.3
345	205.7	207.5	215.5	218.9	221.2	223.3
360	208.0	211.8	217.8	220.1	223.3	229.1
375	210.5	213.2	219.3	223.5	226.4	231.6
390	213.3	216.4	221.5	227.3	228.6	234.4
405	215.7	218.3	224.2	229.9	231.2	236.9
420	218.2	220.6	226.6	232.7	234.3	239.4
435	220.5	223.3	230.0	233.4	237.7	241.7
450	223.1	226.0	233.5	235.9	240.0	244.4
465	225.6	229.2	235.5	238.2	242.4	247.0
480	228.3	232.1	237.7	241.5	244.8	248.7
495	230.6	234.7	240.0	243.9	247.7	252.1
510	233.5	237.4	243.1	247.0	249.8	255.0
525	235.9	239.6	247.4	249.6	252.6	257.5
540	238.5	242.1	250.1	252.4	256.8	260.2
555	241.2	245.6	253.3	256.5	259.0	263.0
570	243.9	248.8	255.4	259.2	262.1	265.8
585	246.6	250.3	257.8	262.7	264.4	268.5
600	249.4	252.6	259.3	264.4	266.2	271.4
615	252.2	256.0	262.0	265.9	269.1	274.3
630	255.0	259.2	265.3	267.8	271.5	277.1
645	257.9	262.1	268.6	269.7	274.2	280.1
660	260.8	264.4	271.1	273.2	276.8	283.1
675	263.7	267.6	274.0	276.1	279.2	286.2
690	266.7	269.9	276.8	279.3	282.7	289.3
705	269.6	273.3	279.3	281.5	285.2	292.3
720	272.6	275.9	281.6	284.1	287.9	295.4
735	275.7	277.8	284.5	287.3	291.8	298.5
750	278.8	280.0	286.3	289.8	294.7	301.7
765	281.9	284.0	288.6	292.5	297.9	304.9

Table 4.10: The Additional Expected Values  $D_{max} = 150Mm^3$ 

state	Sto.	Perfect		
$Mm^3$	Gwh	$\Delta V$ Gwh	Value \$	%
270	194.2	20.6	412,000	10.6
285	196.1	21.0	420,000	10.7
300	198.5	21.0	420,000	10.6
315	200.9	21.0	420,000	10.5
330	203.3	21.0	420,000	10.3
345	205.7	21.0	420,000	10.2
360	208.0	21.1	422,000	10.1
375	210.8	21.1	422,000	10.0
390	213.3	21.1	422,000	9.9
405	215.7	21.2	424,000	9.8
420	218.2	21.2	424,000	9.7
435	220.5	21.2	424,000	9.6
450	223.1	21.3	426,000	9.5
465	225.6	21.4	428,000	9.5
480	228.3	21.4	428,000	9.4
495	230.6	21.5	430,000	9.3
510	233.5	21.5	430,000	9.2
525	235.9	21.6	432,000	9.2
540	238.5	21.7	434,000	9.1
555	241.2	21.8	436,000	9.0
570	243.9	21.9	438,000	9.0
585	246.6	21.9	438,000	8.9
600	249.4	22.0	440,000	8.8
615	252.2	22.1	442,000	8.8
630	255.0	22.1	442,000	8.7
645	257.9	22.2	444,000	8.6
660	260.8	22.3	446,000	8.6
675	263.7	22.5	450,000	8.5
690	266.7	22.6	452,000	8.5
705	269.6	22.7	454,000	8.4
720	272.6	22.8	456,000	8.4
735	275.7	22.8	456,000	8.3
750	278.8	22.9	458,000	8.2
765	281.9	23.0	460,000	8.2

reservoir volume. Comparing the two results it was found that the additional expected values with the perfect forecasts range from 7.8 % to 6.0 %.

#### 4.2.3 RESULTS WITH FLOW PATTERN I, $D_{max} = 150 Mm^3$

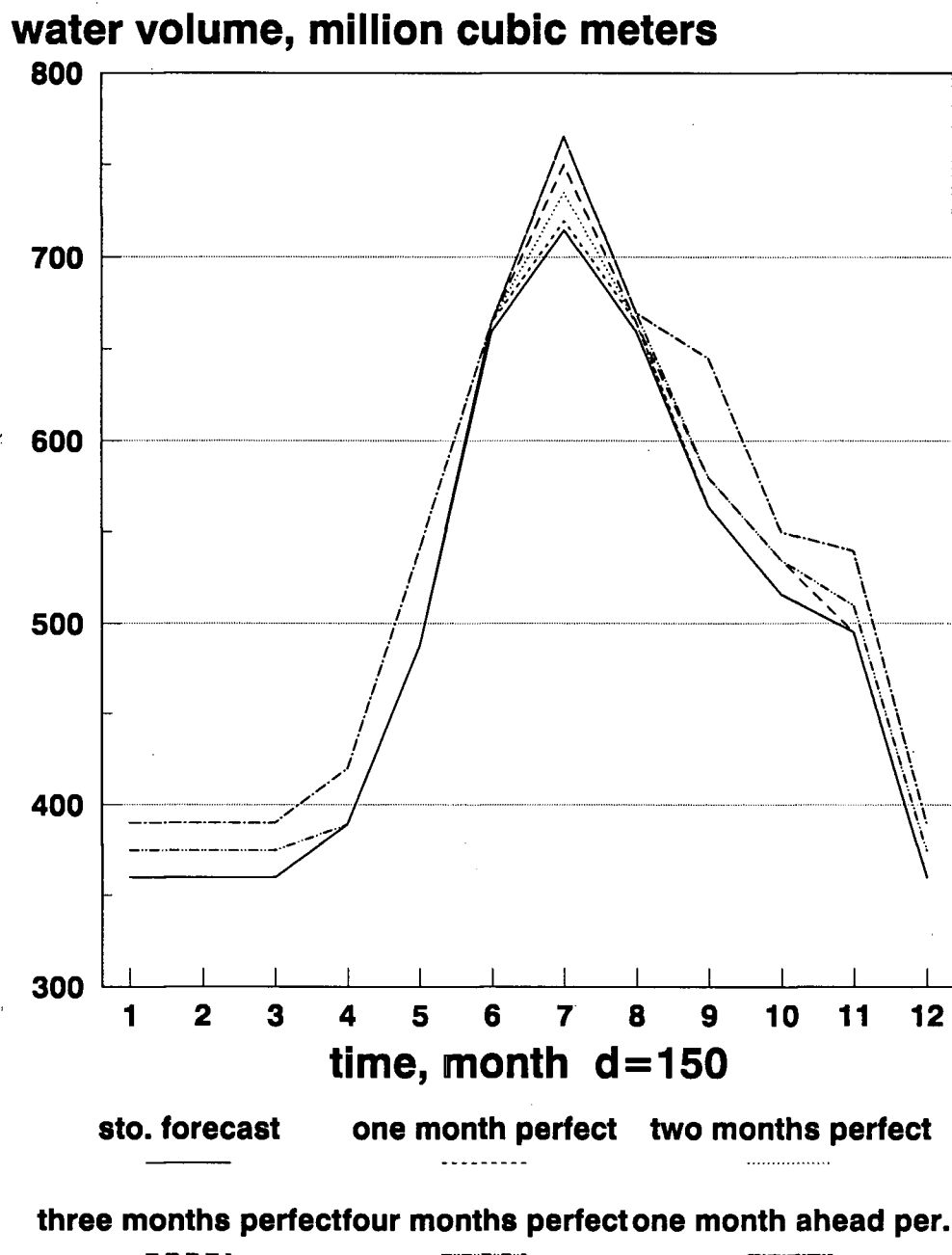
Tables 4.8, 4.9 and 4.10 tabulate the results of the production runs for maximum release limited to  $150 Mm^3$ . In the case with the stochastic flow forecast, the reservoir volume starts at an average of  $360 Mm^3$  at the beginning of January and ends at the same point at the end of December. The maximum volume of the year's operation is  $715 Mm^3$  (not reaching the reservoir full state). But with the perfect inflow forecast, the average starting point for the reservoir operation is  $390 Mm^3$  and the highest water level reaches the reservoir full state. The long term year end expected state values vary from 194.2 Gwh for water volume  $270 Mm^3$  to 281.9 Gwh for water level  $765 Mm^3$  in the case of the stochastic inflow forecast, and from 214.8 Gwh to 304.9 Gwh in the case of perfect flow prediction. The additional expected values due to the perfect forecast go from 20.6 Gwh to 23.0 Gwh, which are 10.6% to 8.2% improvements compared to the stochastic flow forecast.

#### 4.3 THE OPERATION RESULTS WITH FLOW PATTERN II

This section describes the results with the second general flow pattern. This pattern is typical of the B.C. Coast. As shown in section 3.3, although the mean annual flow is the same as flow pattern I, its distribution of mean monthly flows and standard deviations are quite different. The operation results demonstrated that the mean stream flow  $Q_i$ ,  $i=1,2, \dots, 12$  and their standard deviations  $D_{Q_i}$  have significant influence on the energy production.

Table 4.11 shows the operating pattern of the reservoir. With the stochastic dynamic



Figure 4.6: The Operation Processes,  $D_{max} = 150 Mm^3$

### The expected values of state, Gwh

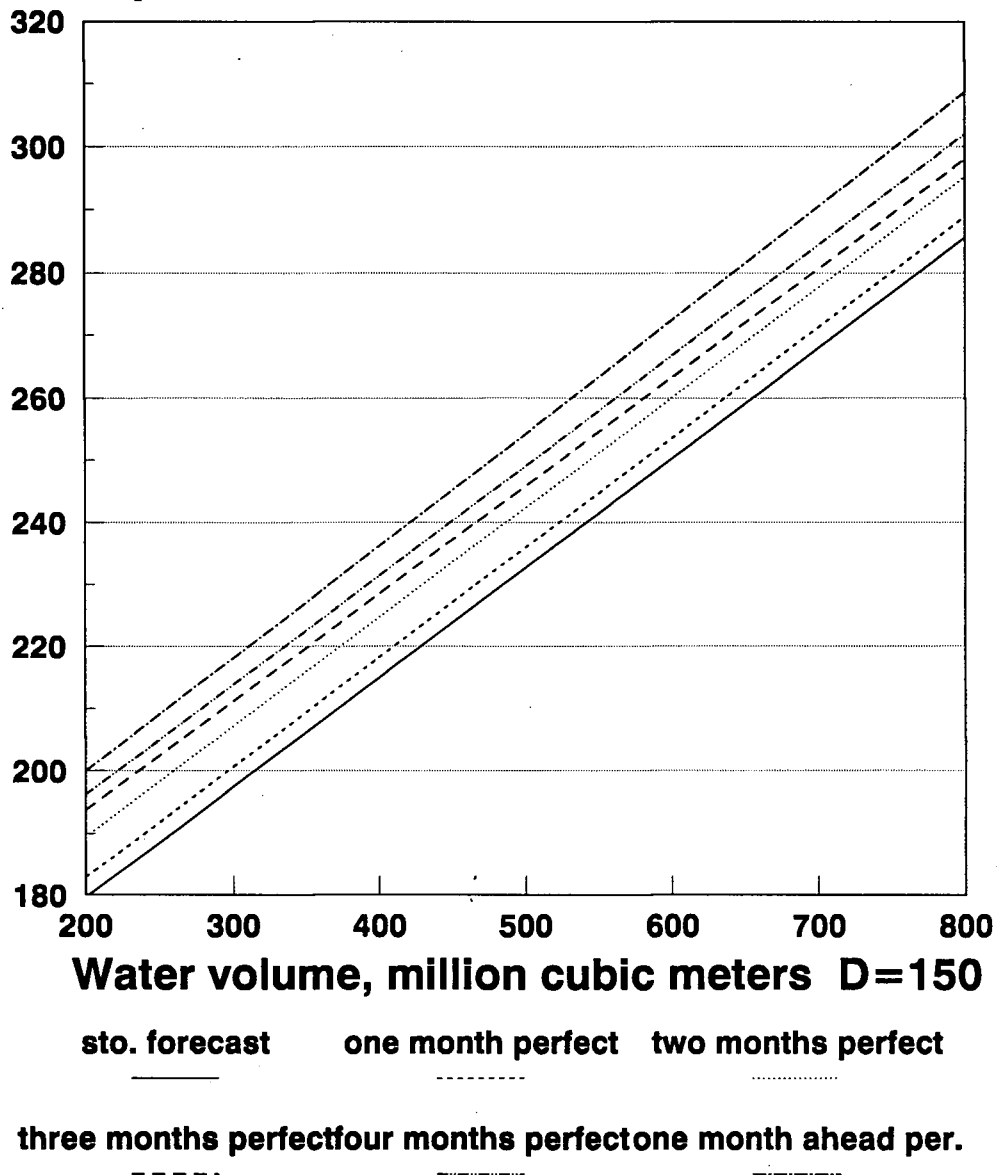
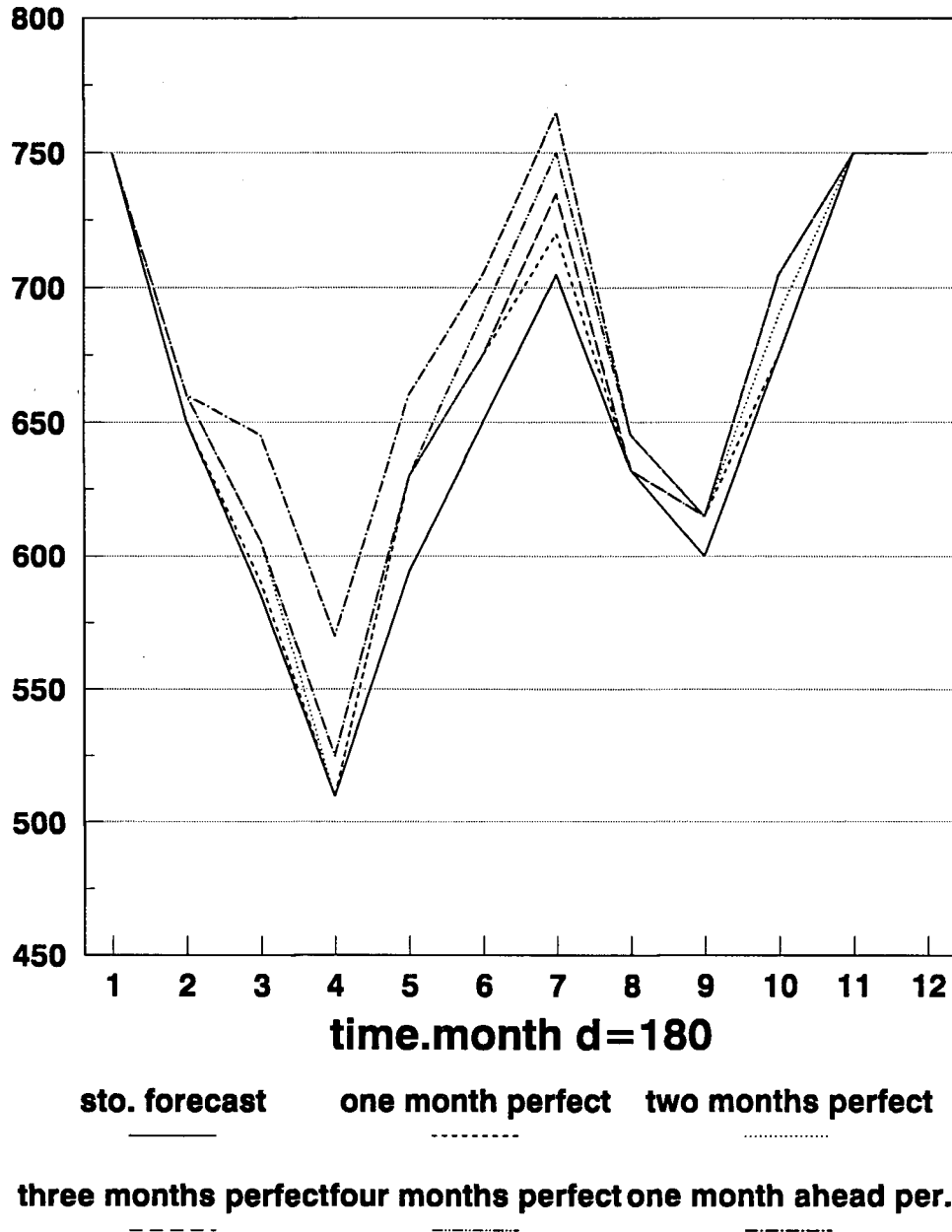


Figure 4.7: The Expected Values of States,  $D_{Max} = 150 Mm^3$

**water volume, million cubic meters**Figure 4.8: The Reservoir Operation Processes, Flow Pattern II,  $D_{max} = 180 Mm^3$

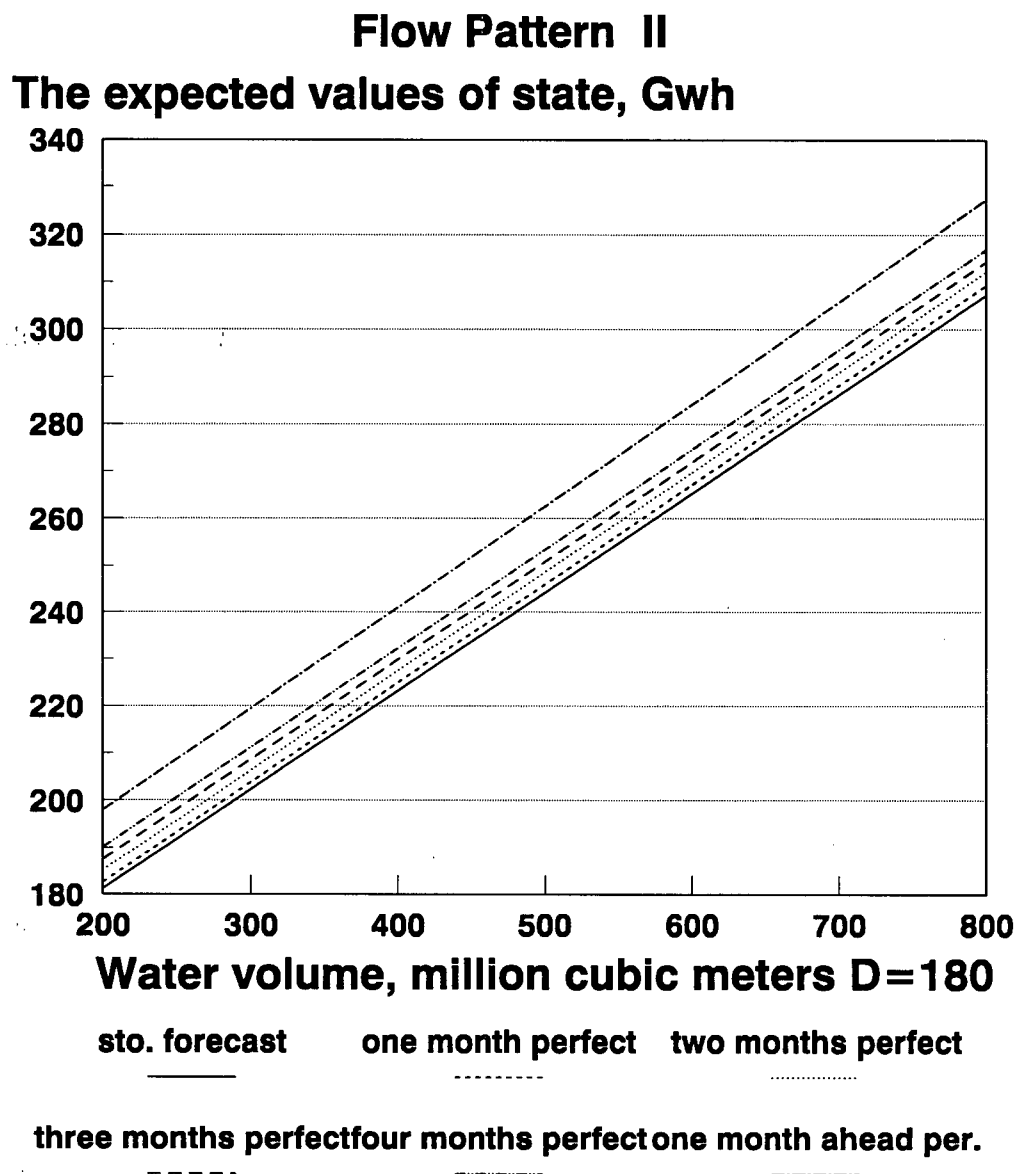


Figure 4.9: The Expected Values of States, Flow Pattern II,  $D_{max} = 180 Mm^3$

Table 4.11: Average Operation Pattern with General Flow Pattern II and  $D_{max} = 180 Mm^3$ 

	1	2	3	4	5	6	7	8	9	10	11	12
	$Mm^3$											
Sto.	750	650	585	510	594	650	705	632	600	675	750	750
I	750	650	590	510	630	675	720	632	615	675	750	750
II	750	660	605	510	630	675	735	632	615	690	750	750
III	750	660	605	525	630	675	735	632	615	705	750	750
IV	750	660	605	525	630	690	750	645	615	705	750	750
Per.	750	660	645	570	660	705	765	645	615	705	750	750

program, the average reservoir operation begins from volume  $750 Mm^3$ , which is near the reservoir full state. This is because inflows are high and range from  $131.7 Mm^3$  to  $122.56 Mm^3$  from October to December in the previous operating year. There are two peak flow periods during one year. The other peak flow starts from May and ends in July. Therefore there are two high water level periods in the reservoir operation process. In the first high flow period, that is from May to July, the highest water storage reaches  $705 Mm^3$  at July. And then it becomes lower until September, when the water volume reduces to  $600 Mm^3$ . At the end of December it returns to  $750 Mm^3$  again.

In the case with the one month ahead perfect forecast, the average starting water volume at the beginning of January is also  $750 Mm^3$ , and it ends with the same volume at the end of the operating year. The highest water volume is  $765 Mm^3$ , which is the reservoir full state, and that happens in July. It can be seen that the end states with the perfect forecast exceeded or were equal to the monthly end states of corresponding months in the stochastic forecast case due to reducing the uncertainty. Note that there are similar outcomes in the case of the operation with flow pattern I. This is a clear indication of the improvement in reservoir operational efficiency. In the other operation cases, that is with part time perfect stream flow forecasts, the month end states become

Table 4.12: The Year End Values of States  $D_{max} = 180Mm^3$ 

state	Sto.	I	II	III	IV	Per.
$Mm^3$	Gwh	Gwh	Gwh	Gwh	Gwh	Gwh
270	197.2	198.6	201.3	203.5	206.1	214.4
285	199.2	200.6	203.3	205.5	208.1	216.5
300	203.0	204.4	207.1	209.3	211.9	220.7
315	205.4	206.9	209.5	211.7	214.3	222.8
330	208.4	209.9	212.5	214.7	217.3	225.9
345	211.5	213.0	215.7	217.9	220.5	229.0
360	214.6	216.1	218.8	221.0	223.6	232.1
275	217.2	218.8	221.4	223.6	226.2	234.8
390	220.6	222.2	224.9	227.1	229.6	238.3
405	223.9	225.5	228.1	230.4	232.9	241.7
420	227.0	228.6	231.3	233.5	236.0	244.9
435	230.1	231.7	234.4	236.6	239.1	248.1
460	233.3	234.9	237.6	239.8	242.4	251.4
475	236.3	237.9	240.6	242.9	245.4	254.5
480	239.4	241.1	243.7	246.0	248.5	257.7
495	242.5	244.2	246.9	249.1	251.6	260.8
510	245.7	247.4	250.1	252.3	254.9	264.1
525	249.4	251.1	253.8	256.0	258.6	267.9
540	253.0	254.7	257.4	258.6	262.2	271.6
555	256.7	258.4	261.2	263.4	265.9	275.4
570	259.5	261.2	264.0	266.2	268.8	278.2
585	263.5	265.2	268.0	270.2	273.8	282.3
600	266.1	267.9	270.6	272.8	275.4	285.0
615	269.4	271.2	273.9	276.1	278.7	288.4
630	272.3	274.3	276.9	279.1	281.6	291.4
645	275.2	277.0	279.8	282.0	284.5	294.4
660	278.4	280.2	283.0	285.2	287.7	297.7
675	281.6	283.4	286.3	288.4	291.0	301.0
690	284.2	286.0	288.9	291.0	293.6	303.6
705	287.1	289.0	291.8	294.0	296.5	306.6
720	290.2	292.1	294.9	297.1	299.6	309.8
735	293.4	295.3	298.3	300.3	302.9	313.1
750	295.7	297.6	300.5	302.6	305.2	315.5
765	298.8	300.7	303.6	305.7	308.3	318.7

Table 4.13: The Additional Values of Forecasts with Flow Pattern II,  $D_{max} = 180 Mm^3$ 

state	Sto.	Perfect		
$Mm^3$	Gwh	$\Delta V$ (Gwh)	Value (\$)	%
270	197.2	17.2	344,000	8.7
285	199.2	17.3	346,000	8.7
300	202.3	17.4	348,000	8.6
315	205.4	17.4	348,000	8.5
330	208.4	17.5	350,000	8.4
345	211.5	17.5	350,000	8.3
360	214.6	17.5	350,000	8.2
375	217.7	17.6	352,000	8.1
390	220.6	17.7	354,000	8.0
405	223.9	17.8	356,000	7.9
420	227.0	17.9	358,000	7.9
435	230.1	18.0	360,000	7.8
450	233.3	18.1	362,000	7.8
465	236.3	18.2	364,000	7.7
480	239.4	18.3	366,000	7.6
495	242.5	18.3	366,000	7.6
510	245.7	18.4	368,000	7.5
525	249.4	18.5	370,000	7.4
540	253.0	18.6	372,000	7.4
555	256.7	18.7	374,000	7.3
570	259.5	18.7	374,000	7.2
585	263.5	18.8	376,000	7.1
600	266.1	18.9	378,000	7.1
615	269.4	19.0	380,000	7.1
630	272.3	19.1	382,000	7.0
645	275.5	19.2	384,000	7.0
660	278.9	19.3	386,000	6.9
675	281.6	19.4	388,000	6.9
690	284.2	19.4	388,000	6.8
705	287.5	19.5	390,000	6.8
720	296.2	19.6	392,000	6.7
735	293.4	19.7	394,000	6.7
750	295.7	19.8	396,000	6.6
765	298.8	19.9	398,000	6.6

higher as the number of months which have perfect inflow predictions increase (see table 4.11 for details).

Table 4.12 represents the year end expected values of the states for all operating runs. Table 4.13 shows the additional expected values with one month ahead perfect inflow forecasts compared to the results with pure stochastic stream flow prediction. From the second column of table 4.12, it can be seen that the state year end expected values, for stochastic flow forecast, range from 197.2 Gwh for 270  $Mm^3$  to 298.8 Gwh for 765  $Mm^3$ . Column 7 shows the state's year end expected values with perfect stream flow forecasting. The minimum and maximum expected values are 216.4 Gwh and 315.9 Gwh, which correspond to the lower and upper bounds of water volume respectively. Fig (4.9) plots the year end expected values against the reservoir states. This shows that there is a similar relationship between states and their year end values as with flow pattern I.



## Chapter 5

### CONCLUSIONS

This chapter presents the general conclusions obtained from the research. In Chapter I, several questions were posed:

- What is the value of one month ahead perfect inflow forecasts to reservoir operation?  
The answer could be used as a guide to whether or not it is worth making stream flow predictions.
- What benefits can be gained from varying term perfect stream flow forecasts in the high flow season?
- What is the effect of the maximum power plant capacity on the opportunities for improving the reservoir operating efficiency?
- How do different stream flow patterns affect the expected benefits which could be obtained by reducing the uncertainty about future flows with forecasting?

The results presented in chapter IV have already partially answered these questions. They are summarized in this chapter and the results of some further analyses and conclusions are presented.

#### 5.1 ADDITIONAL BENEFITS OF PERFECT INFORMATION

As stated in chapter I, the main purpose of this study is to find an upper limit to the value of flow forecasts by computing the expected value of perfect short term inflow

Table 5.1: Summary of Additional Benefits with Forecasts

	Flow Pattern I, \$			Flow Pattern II, \$
	$D_{max} = 150Mm^3$	$D_{max} = 180Mm^3$	$D_{max} = 210Mm^3$	$D_{max} = 180Mm^3$
I	76,000	70,000	62,000	38,000
II	196,000	170,000	124,000	96,000
III	242,000	226,000	162,000	138,000
IV	318,000	290,000	270,000	190,000
Per.	422,000	354,000	334,000	396,000

forecasts. The expected values of the perfect forecast is the value of the extra energy production possible with the perfect inflow predictions compared to the operation with pure stochastic flow estimates. The value of energy is assumed to be \$20,000/Gwh, so that the benefits can be evaluated approximately in dollar terms.

Table 5.1 summarizes the expected additional benefits with perfect inflow information, which correspond to the optimal long term operational policies of the hypothetical hydroelectric power project for each production run. It may be seen from the table that the perfect flow forecasts, both for one month ahead perfect forecasting and for part time perfect flow forecasting in the high flow season, improve the operational efficiency considerably by reducing the forecasting uncertainty. For example, when the allowable maximum release  $D_{max} = 180Mm^3$ , in the case of flow pattern I, the perfect forecasting could generate \$354,000 more than the stochastic forecast. The potential improvement due to perfect forecasting with flow pattern II could achieve \$396,000. The potential additional expected value of perfect inflow information for flow pattern I is slightly lower than with flow pattern II.

It is obvious that the effort of making perfect stream flow forecasting is worth doing when the cost of the research work is not greater than the extra expected benefits, that is \$354,000 for flow pattern I and \$396,000 for flow pattern II when the maximum allowable

discharge is equal to  $180 \text{ Mm}^3$ .

## 5.2 THE EFFECT OF MAXIMUM RELEASE ON THE POTENTIAL BENEFIT

This section discusses the role of the maximum allowable release (which depends on the generating capacity) in the improvement of reservoir operational efficiency.

Table 5.2 compares the percentage and the net potential additional expected state values with different maximum release capacities for flow pattern I. It is interesting to note, from both Table 5.1 and 5.2,

that the largest additional expected value among the three situations is when the maximum allowable release is limited to  $150 \text{ Mm}^3$ . The additional expected state values vary from 10.6 % of reservoir volume  $270 \text{ Mm}^3$  to 8.2 % of volume  $765 \text{ Mm}^3$ . When the maximum discharge is  $180 \text{ Mm}^3$ , The extra values lie between 8.6 % and 6.9 % . In the case of release limitation being  $210 \text{ Mm}^3$ , the potential expected benefits go from 7.8 % to 6.0 % respectively. That is, the additional expected values reduce when the discharge limitation increases. The results indicated that perfect inflow information is more valuable when the maximum allowable release is smaller for the same reservoir size.

The explanation of this phenomenon is as follows: The 'adjusting ability' of the reservoir is limited by the maximum release capacity. When  $D_{max}$  is small, the energy production is affected greatly by the release constraint and a lot of water may be spilled through spillway. Thus the perfect inflow prediction provides more improvement in reservoir operational efficiency. If  $D_{max}$  is large, the effect of improving operational efficiency by perfect inflow forecasting will be smaller relatively which suggests less additional expected values may be obtained. It can be concluded, from the above results, that if a hydroelectric power project has a strict limitation on discharges, it is more worthwhile

Table 5.2: Additional Expected Values of Different  $D_{max}$ 

state $Mm^3$	$D_{max} = 150Mm^3$		$D_{max} = 180Mm^3$		$D_{max} = 210Mm^3$	
	%	\$	%	\$	%	\$
270	10.6	412,000	8.8	334,000	7.8	316,000
285	10.7	420,000	8.8	348,000	7.7	316,000
300	10.6	420,000	8.7	348,000	7.6	318,000
315	10.5	420,000	8.6	350,000	7.5	320,000
330	10.3	420,000	8.5	350,000	7.4	322,000
345	10.2	420,000	8.4	350,000	7.4	322,000
360	10.1	422,000	8.3	350,000	7.3	322,000
375	10.0	422,000	8.3	352,000	7.3	324,000
390	9.9	422,000	8.2	352,000	7.2	324,000
405	9.8	424,000	8.1	352,000	7.1	326,000
420	9.7	424,000	8.0	352,000	7.1	328,000
435	9.6	424,000	7.9	354,000	7.1	330,000
450	9.5	426,000	7.9	356,000	7.0	330,000
465	9.5	428,000	7.8	358,000	6.9	332,000
480	9.4	428,000	7.8	360,000	6.9	332,000
495	9.3	430,000	7.8	362,000	6.8	334,000
510	9.2	430,000	7.8	362,000	6.8	334,000
525	9.2	432,000	7.7	364,000	6.7	336,000
540	9.1	434,000	7.7	364,000	6.7	338,000
555	9.0	436,000	7.6	364,000	6.6	338,000
570	9.0	438,000	7.6	366,000	6.5	338,000
585	8.9	438,000	7.5	366,000	6.5	340,000
600	8.8	440,000	7.5	378,000	6.5	340,000
615	8.8	442,000	7.4	370,000	6.4	340,000
630	8.7	442,000	7.3	372,000	6.4	342,000
645	8.6	444,000	7.3	374,000	6.4	344,000
660	8.6	446,000	7.2	376,000	6.3	346,000
675	8.5	450,000	7.2	378,000	6.3	348,000
690	8.5	452,000	7.1	380,000	6.2	348,000
705	8.4	454,000	7.1	382,000	6.1	348,000
720	8.4	456,000	7.0	384,000	6.1	350,000
735	8.3	456,000	7.0	386,000	6.1	352,000
750	8.2	458,000	6.9	388,000	6.0	354,000
765	8.2	460,000	6.8	390,000	6.0	356,000

to make inflow forecasts.

### 5.3 THE ROLE OF THE FLOW PATTERN IN THE RESERVOIR OPERATION

Comparing the results from the different flow patterns leads to an interesting conclusion. That is, the year end expected additional values with part time perfect inflow information depend on the pattern of mean flows and their deviations. Table 5.3 and 5.4 show the degree of improvement with part time perfect flow forecasts in the form of incremental percentages

of the additional values over the one month ahead perfect forecast.  $\Delta P$  represents the additional values with one month ahead perfect forecasting and thus may be viewed as the possible maximum improvement with perfect inflow information.  $\Delta i$ ,  $i=1,2, 3$  and  $4$ , is the additional values with one, two, three and four months of perfect forecasting. Thus  $\Delta i/\Delta P$  is the operational improvement of part time perfect inflow forecasts in terms of percentage. A comparison of the two tables shows that the additional values are about 20%, 47%, 63% and 80% of the possible maximum

improvements respectively for part time perfect forecasting with flow pattern I, whereas, the corresponding improvement are only about 9%, 23%, 35%, and 50% for flow pattern II. The reasons for these results are as follows. On the one hand, the total amount of inflow in the high flow period dominates the stream flow of the forecasting year with flow pattern I. It may be seen from chapter three that the stream flow in that period is about 83% of the total annual flow, so the energy production during that time is very high. Consequently, the improvement in reservoir operation efficiency due to the availability of part time perfect inflow prediction should also be great. But for the flow pattern II, the sum of the inflow in the two high flow periods is only about 64% of the total annual flow,

Table 5.3: The Additional Values with Flow Pattern I

State $Mm^3$	$(\Delta 1/\Delta P)$ %	$(\Delta 2/\Delta P)$ %	$(\Delta 3/\Delta P)$ %	$(\Delta 4/\Delta P)$ %	$\Delta P$ Gwh
270	19.7	48.0	64.2	83.2	17.3
285	19.5	47.7	63.8	82.8	17.4
300	19.5	47.7	63.8	82.8	17.4
315	19.4	48.0	63.4	82.3	17.5
330	19.4	48.0	64.0	82.3	17.5
345	19.4	48.0	64.0	82.9	17.5
360	20.0	48.0	64.0	82.9	17.5
375	19.9	47.7	63.6	82.4	17.6
390	19.9	47.7	64.2	82.4	17.6
405	19.9	48.3	64.2	82.4	17.6
420	19.9	48.3	64.2	82.9	17.6
435	19.8	48.0	63.8	82.5	17.7
450	19.7	47.8	64.0	82.0	17.8
465	20.1	47.5	63.7	81.6	17.9
480	20.0	47.8	63.3	81.1	18.0
495	19.9	47.5	63.0	80.6	18.1
510	20.4	47.5	63.0	81.2	18.1
525	20.3	47.3	63.2	80.8	18.2
540	20.3	47.3	63.2	80.8	18.2
555	20.3	47.3	63.2	81.3	18.2
570	20.2	47.5	62.8	80.9	18.3
585	20.2	47.5	62.8	80.9	18.3
600	20.1	47.3	63.0	80.4	18.4
615	20.0	47.0	62.7	80.5	18.5
630	19.9	46.8	62.4	80.1	18.6
645	20.3	47.1	62.0	79.7	18.7
660	20.2	46.8	62.2	79.3	18.8
675	20.1	46.6	61.9	79.4	18.9
690	20.0	46.3	61.6	78.9	19.0
705	19.9	46.6	61.3	79.1	19.1
720	19.8	46.4	61.5	78.6	19.2
735	19.7	46.1	61.1	78.2	19.3
750	19.6	45.9	60.8	77.8	19.4
765	19.5	45.6	61.0	77.4	19.5

Table 5.4: The Additional Values of Flow Pattern II

State $Mm^3$	$(\Delta 1/\Delta P)$ %	$(\Delta 2/\Delta P)$ %	$(\Delta 3/\Delta P)$ %	$(\Delta 4/\Delta P)$ %	$\Delta P$ Gwh
270	8.1	23.8	36.6	51.7	17.2
285	8.1	23.7	36.4	51.4	17.3
300	8.0	23.6	36.2	51.1	17.4
315	8.6	23.6	36.2	51.1	17.4
330	8.6	23.4	36.0	50.9	17.5
345	8.6	24.0	36.6	51.4	17.5
360	8.6	24.0	36.6	51.4	17.5
375	9.1	23.9	36.4	51.1	17.6
390	9.0	24.3	36.7	50.8	17.7
405	8.9	24.2	36.5	50.6	17.8
420	8.9	24.0	36.3	50.3	17.9
435	8.9	23.9	36.1	50.0	18.0
450	8.8	23.8	35.9	50.3	18.1
465	8.8	23.6	36.3	50.0	18.2
480	9.3	23.5	36.1	49.7	18.3
495	9.3	24.0	36.1	49.7	18.3
510	9.2	23.9	35.9	50.0	18.4
525	9.2	23.8	35.7	49.7	18.5
540	9.1	23.7	35.5	49.5	18.6
555	9.1	24.1	35.8	49.2	18.7
570	9.1	24.1	35.8	49.7	18.7
585	9.0	23.9	35.6	49.5	18.8
600	9.5	23.8	35.4	49.2	18.9
615	9.5	23.7	35.3	48.9	19.0
630	9.4	24.1	35.6	48.7	19.1
645	9.4	24.0	35.4	48.9	19.2
660	9.3	23.8	35.2	48.2	19.3
675	9.3	24.2	35.1	48.5	19.4
690	9.3	24.2	35.1	48.5	19.4
705	9.7	24.1	35.4	48.2	19.5
720	9.7	24.0	35.2	48.0	19.6
735	9.6	23.9	35.0	48.2	19.7
750	9.6	24.2	34.8	48.0	19.8
765	9.5	24.1	34.7	47.7	19.9

and thus has a relatively smaller influence on the improvement of reservoir operation than the former. The monthly standard deviation of the flow in the low flow season with flow pattern II, on the other hand, is much greater than flow pattern I. Therefore, its possible flows vary more widely during low flow periods, which means more uncertainty about the flow occurrence. For example, there are only 4 to 8 possible flow values with flow pattern I in the low flow season, whereas there are at least 12 possible flows in the low flow months with flow pattern II. As a result, there is only small additional value in reservoir operational efficiency improvement in the low flow periods with flow pattern II if perfect runoff information is available.

It can be concluded, from the above results, that more additional value may be obtained when the greater amount of stream flow occurs in the wet season. In other words, it is more valuable to make perfect inflow forecasts in the high flow period if the stream flow in that time dominates the total flow of the year as it does with flow pattern I. Where there is not much difference in the amount of water between the dry season and the high flow period as in flow pattern II for instance, it is better to get forecasts throughout the whole year.

Although there are some influences on the additional values of perfect forecasting owing to the different inflow patterns, that is flow pattern I and pattern II, the differences in the potential additional expected values between the two flow patterns is not very much as has been already pointed out in section 5.1.

#### 5.4 THE CONSEQUENCES OF PART TIME PERFECT FORECASTING

Making perfect stream flow forecasts for a whole year is a costly and difficult task usually. An alternative is to predict the inflows as accurately as possible during the high flow



period. This section examines the effects of part time perfect forecasting to the improvement of reservoir operational efficiency. Table 5.1, 5.3 and 5.4 tabulate the results of all operation runs. From these tables, it is clear that the perfect forecasts in the wet season have significant influence on the improvement in potential additional expected values. The more months which have perfect flow information in high flow season, the more improvement is possible. The potential for additional expected values with four months of perfect predictions may reach about 80% or 50% of the maximum possible improvement with flow pattern I and pattern II respectively, whereas the one, two or three month perfect forecasts have relatively smaller additional benefits, about 20%, 47% or 63% for flow pattern I and 9%, 23% or 35% for flow pattern II. Therefore, it is recommended to try to make as accurate flow predictions as possible for the whole wet season instead just part of the high flow period to get the best results.

## 5.5 RECOMMENDATIONS FOR FURTHER RESEARCH

From the experience gained in this study, the following further research is recommended.

- Find the additional expected values with two month ahead perfect forecasting. This is a more difficult issue to formulate into the optimization model than the model in the present study but it would be an interesting task both in theory and in practice.
- Examine the effects of different reservoir sizes on the improvement in expected values with one month ahead perfect flow forecasts. Reservoir size is usually one of the key elements in the control of its operation. Different reservoir sizes might cause different reactions to the perfect inflow forecasts.
- Study the role of one month ahead perfect forecasts in the operation of a system of reservoirs. Generally speaking, a hydroelectric power project is operated in a large

scale electric network, rather than as an individual project.

- Examine the effects of less than perfect forecasts in reservoir operation using a methodology similar to the perfect study.

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