OPTIMAL CULVERT SIZE SELECTION

by

PATRICK ALEXANDER NEUDORF

B.A.Sc., University of British Columbia, 1975

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF APPLIED SCIENCE

in

THE FACULTY OF GRADUATE STUDIES

The Department of Civil Engineering

We accept this thesis as conforming

to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

August, 1977

In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the Head of my Department or by his representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Department of Civil Engineering

The University of British Columbia
2075 Wesbrook Place
Vancouver, Canada
V6T 1W5

Date August 31, 1977

ABSTRACT

The hydraulic design criteria for culvert size selection currently employed by most highways departments, including British Columbia's, can lead to economically non-optimal culvert size choices. This thesis describes a method of economic analysis to determine the optimum sized culvert for any culvert site, taking into direct account the uncertainty of the data. The method is applied to a hypothetical culvert site, assuming different hydrologic and economic situations. The uncertainty in evaluating flood flows is taken into account, and methods of calculating the value of better information are presented. The hydrologic, hydraulic, and economic aspects of culvert selection and the problems and uncertainties in collecting data and making assumptions in each of these areas are discussed before the results are presented.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ACKNOWLEDGEMENT

The author wishes to thank Dr. S. O. Russell for his guidance in the research and preparation of this thesis. Thanks are also extended to Mr. Richard Brun, who prepared the figures, and to Mrs. Janet Bergeron, who typed the thesis. The author would also like to thank the National Research Council of Canada for their financial support during the past two years.

Chapter 1

INTRODUCTION

The British Columbia Department of Highways presently selects culvert sizes on the basis of two criteria (1):

(A) Culverts shall carry the 10-year flood with headwater depths equal to the diameter of the culvert.
(B) The culvert shall carry a 100-year flood (1.8 x 10-year) by surcharge without headwater damage and without loss through scour.
Either criterion may govern.

The first criterion appears to be rather arbitrary while the second criterion makes an attempt to weigh the cost of installing a larger pipe size against the savings from less frequent flood damage. The question is, "Why was the 100-year flood chosen?" These criteria can hardly be expected to result in selecting the optimal culvert size for all culvert sites in all circumstances. For instance, for culverts under low fills on low volume rural highways, designing for the 25-year flood may be appropriate. In contrast, the 500-year flood could be appropriate for a long culvert under a major highway where substantial damages to upstream or downstream property could result from flooding.

Another problem is, "What is the 10-year flood or 100-year flood?" There is often a great deal of uncertainty involved in evaluating flood flows for small watersheds. In addition to hydrologic uncertainty, culvert design is plagued by uncertainty in areas such as the hydraulic performance of culverts, debris clogging, what flow will cause washout, and estimating damage costs.

The United States Bureau of Public Roads (USBPR) has stated that 44% of the highway drainage dollar, or 15% of the highway construction dollar, is spent for culverts (2). An analysis of sixteen 1961 projects in British Columbia showed that 8.6% of the total cost was spent on culverts (1). Clearly, these questions warrant attention.

This thesis describes a method of economic analysis which can be used to determine the optimum culvert size for any culvert site, taking into direct account the uncertainty of the data. The method is applied to a hypothetical culvert site where a 100 ft culvert is to be placed on a 7% slope under a major rural two-lane highway. The roadway width, including shoulders, is 45 ft, and the highway embankments are sloped at 2:1. The roadway is 10 ft above the culvert invert at the entrance and 17 ft above the culvert invert at the exit. Reasonable flood frequency data, culvert costs, and flood damage costs were chosen. Only uncertainty in the flood frequency data was considered in the analysis, although uncertainty in other areas is discussed.

The idea of applying economic analysis to determine the optimum size culvert for a given site is not new. Pritchett (3) wrote a thesis entitled Application of the Principles of Engineering Economy to the Selection of Highway Culverts (1964), and this thesis is often mentioned in the literature. He concluded that substantial savings (15-20% in the four examples presented) would be realized by applying economic analysis. The purpose of the present thesis is to extend the analysis so that uncertainty in the data can be accounted for. The effect on the optimal decision of uncertainty in the flood frequency data is studied.

A very important question when faced with uncertainty is, "What is the value of better information?" Or in other words, "How much money, if any, should be spent on a data gathering program to reduce uncertainty?" This question is explored and possible solutions to the problem are presented. In addition, the sensitivity of the optimal decision to changes in the discount rate and the service life is studied as is the effect on the optimal decision of changing the damage costs.

The only type of culvert installation considered in the analysis is a single round corrugated metal pipe (CMP) with a vertical headwall and endwall. Different materials and shapes may be advantageous in some situations, but they are not considered here. Entrance improvement, which can result in a significant improvement in hydraulic efficiency, is discussed but not incorporated into the analysis. The structural engineering aspect of culvert design is not discussed.

Utility, rather than monetary value, could have been used as the basis for culvert selection. But since highway culverts are the responsibility of provincial governments, monetary value was chosen. Utility would be more appropriate for culverts on private land controlled by a firm or an individual with limited financial resources. In this case the individual or firm may be more averse to severe flood damage than the monetary value of the flood damage indicates. The following paragraph outlines the contents of the remaining chapters.

Chapter 2 illustrates the problem with a decision tree and outlines the formation and use of probability matrices and vectors which are used in the calculations. The next three chapters discuss various components of the decision tree. Chapter 3 discusses methods of evaluating flood flows and their inherent problems and presents the flood frequency distributions used in the analysis. Types of culvert flow are discussed in Chapter 4; Chapter 4 also includes short discussions of culvert entrance and exit improvement, the mechanics of a washout, and environmental considerations. Chapter 5 discusses the economic elements of the problem: the capital costs of culverts, flood damage costs, and how the capital cost is converted to an annual cost with emphasis on the question, "What is the correct discount rate?" The results are presented and discussed in Chapter 6, and conclusions are drawn in Chapter 7.

Chapter 2

METHOD OF SOLUTION

2.1 The Decision Tree

The culvert selection problem can be conveniently represented with a decision tree, as shown in Figure 2.1. Possible decisions (for example, culvert size) are shown as branches emanating from a decision point, represented by a square. Events which depend on chance or natural occurrence (for example, flood size) are shown as branches leading from a chance point, represented by a circle. Probabilities of chance events are also given on the branches. Figure 2.1 contains only one true chance point since a unique headwater level is assigned to each flood size for a given culvert diameter. A probable damage cost, calculated for each headwater level, is the final outcome at the end of each final branch of the decision tree.

The simple decision tree shown in Figure 2.1 is used in the analysis presented in Chapter 6. The decision tree could be complicated to include more decisions and more chance events. Figure 2.2 is a decision tree to which the type of entrance improvement has been added as a decision variable. Uncertainty in debris clogging, culvert hydraulics, and the headwater level which causes washout have also been added. There are only two debris clogging possibilities depicted along with their associated probabilities: either no debris clogging or complete debris clogging with no flow through the culvert. Intermediate degrees of debris clogging could be included. The same situation also applies to washout at a particular

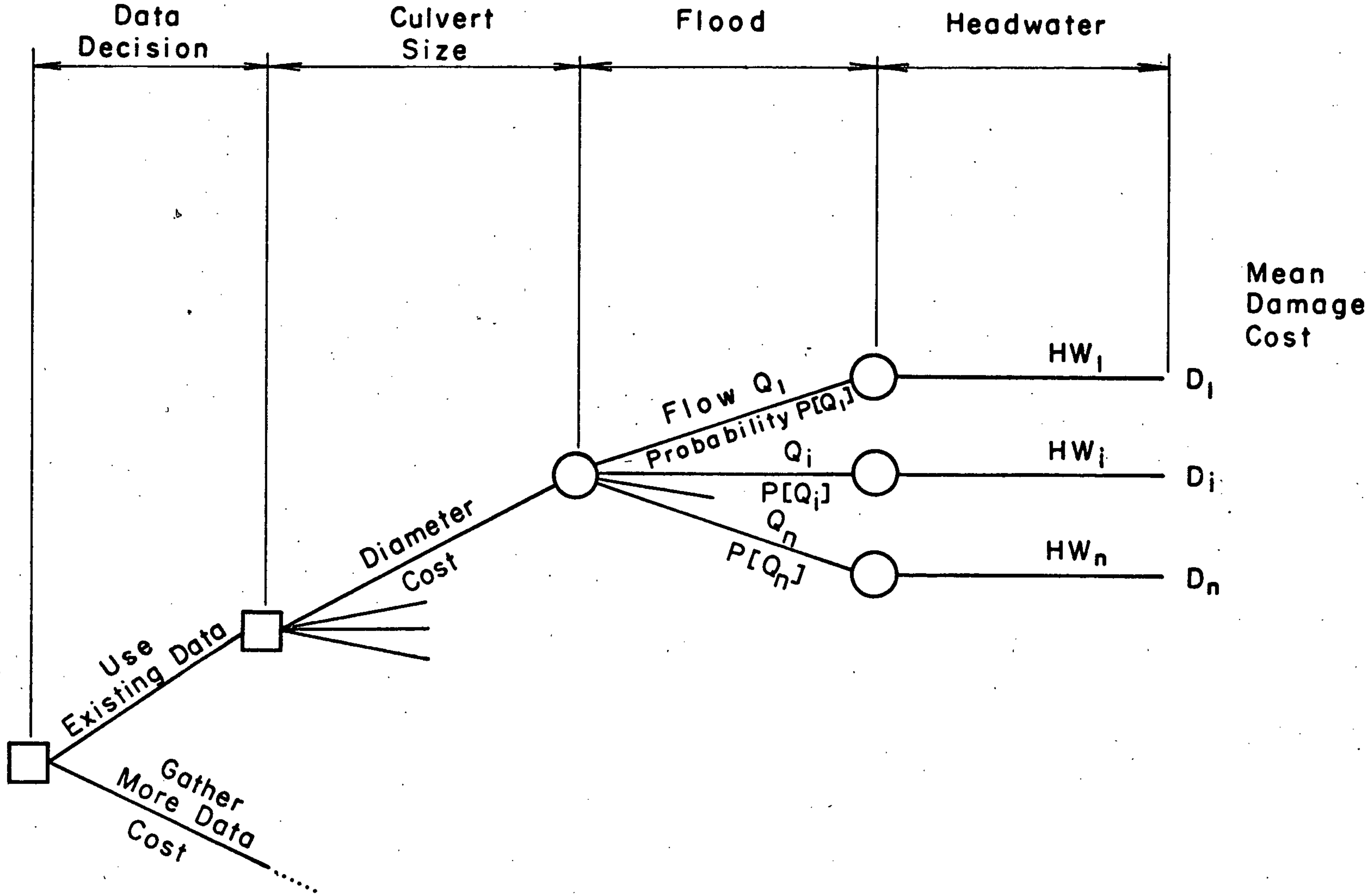


FIG. 2.1 DECISION TREE USED IN THE ANALYSIS.

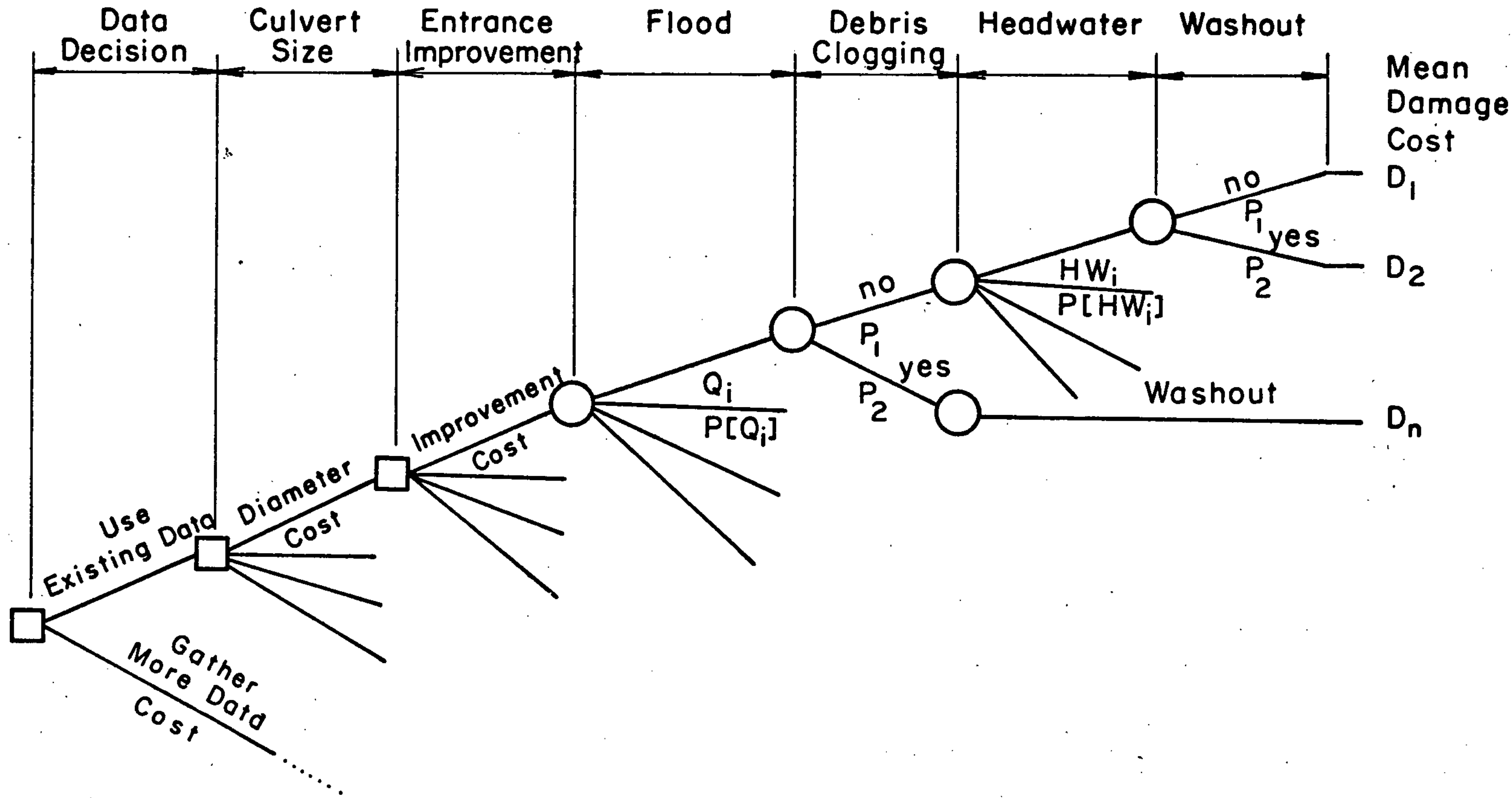


FIG. 2.2 MORE COMPLICATED DECISION TREE.

headwater level; intermediate degrees could also be included. These events are more fully discussed in Chapter 4.

2.2 Probability Matrices

Matrices and vectors (one-dimensional matrices) are very useful for handling decision tree information and calculations. The idea of representing a function bounded by upper and lower limits as a probability matrix was developed by Russell and Hershman (4) and subsequently used by Nyumbu (5) and Brox (6). It is a useful concept for dealing with uncertainty.

The formation of a probability matrix is illustrated for the hypothetical function Y = f(X), shown in Figure 2.3. For any value of X, the dependent variable Y is not known with certainty but lies somewhere between the upper and lower limits. The uncertainty about the true value of Y for a given value of X can be described by a probability density function.

In practice, the three curves of Figure 2.3 are unlikely to be known accurately, especially in cases where there is little data available. Determining the upper and lower bounds may be particularly difficult. However this does not necessarily decrease the usefulness of the method since the decision maker can increase the separation between the upper and lower limits as his uncertainty increases.

Likewise, the shape of the probability density function between the upper and lower bounds is unlikely to be known unless there is sufficient data to analyze. A truncated skew normal distribution, shown in Figure 2.4, was deemed appropriate. This variation of the normal distribution was developed by Ward for

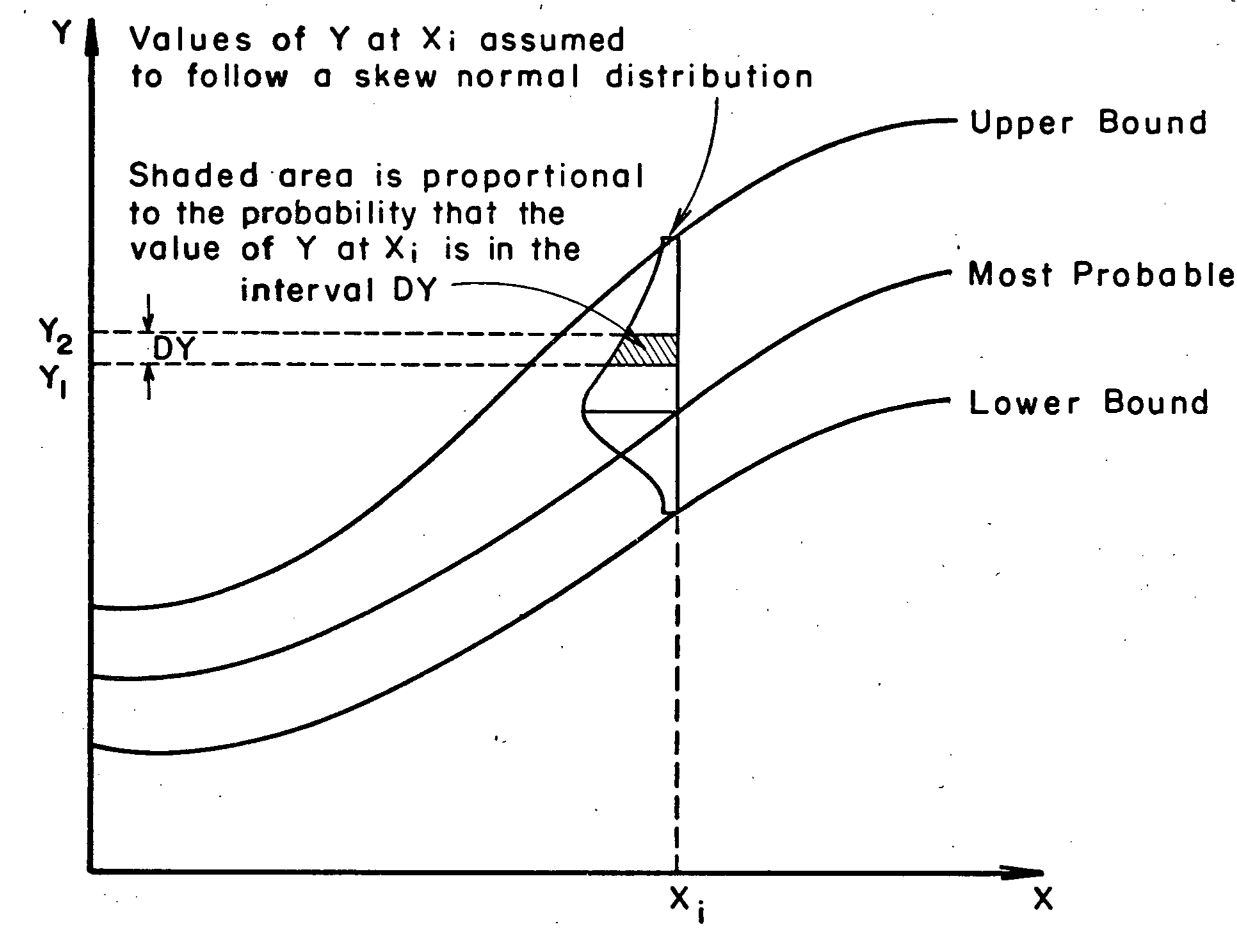


FIG. 2.3 HYPOTHETICAL FUNCTION Y = f(X).

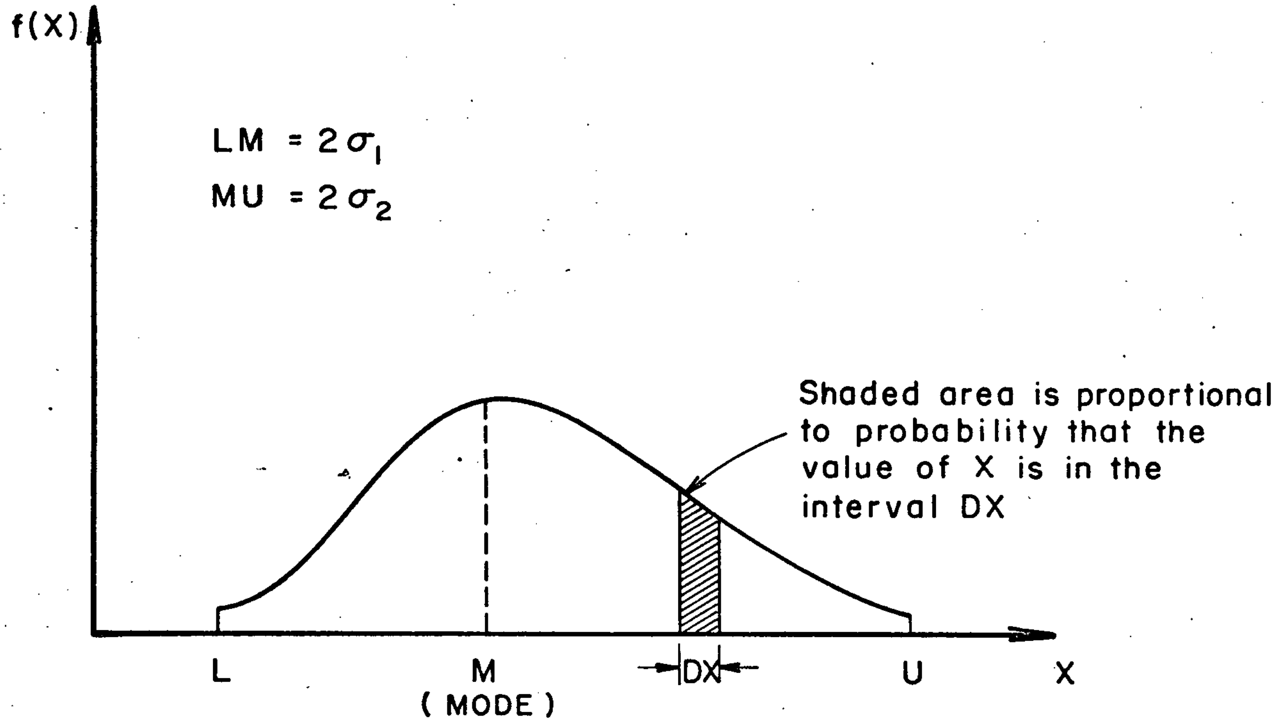


FIG. 2.4 TRUNCATED SKEW NORMAL DISTRIBUTION.

Hershman's thesis. The distribution is a composite made up from two truncated normal distributions. The bounds are two standard deviations from the mode. The density function is multiplied by $1 / (1 - .0456)$ to correct for the areas truncated at the ends of the distribution. In the case where the upper and lower bounds are equidistant from the mode, the density function reduces to a truncated normal distribution. Because this distribution is easy to work with and can handle cases in which the upper and lower bounds are not equidistant from the mode, it was considered a reasonable choice.

For any value of X the probability that Y is in the interval $DY = Y_2 - Y_1$ can be found by integrating the probability density function at X between Y_1 and Y_2 (see Figure 2.3). This is the basis for forming a probability matrix. The rows of the matrix represent discrete values of X and the columns represent Y intervals. An element of the matrix is the probability that the value of Y lies in certain interval DY for a specific value of X. The sum of the elements across any row necessarily equals 1.0. To simplify subsequent calculations the mid-points of all Y intervals are usually chosen to represent the columns. The discrete values of X are also commonly the mid-points of X intervals. In this way the information contained in the three continuous curves of Figure 2.3 is converted into discrete pieces.

Considering this, some judgement must be used in selecting the size of the matrix. If the intervals are too large accuracy will be lost. For example, given that DY is an interval above the mode at a specific X, the probability that the value of Y is in the lower half of the interval is greater than the probability that the value of Y is in the upper half of the interval. The value of the mean $u = \int_{Y_1}^{Y_2} Yp(Y)\,dY$, which is somewhat below the mid-point in this case, is the correct choice for a representative value of Y for the interval. Thus using the interval mid-points results in some inaccuracy. As the interval sizes (both X and Y) decrease, accuracy increases, but the number of computations involved in forming and utilizing the matrix increases. A computer program has been developed (Higgins 1975) for forming probability matrices, but still the matrices' sizes should not be excessively large as the uncertainty involved in plotting the three curves of Figure 2.3 usually does not justify large sized matrices.

The flood probability vector of Figure 2.1 was derived from a probability matrix. The derivation is discussed in Chapter 3. The uncertainty in culvert hydraulics could be described by a probability matrix with flow plotted on the X axis of Figure 2.3 and headwater plotted on the Y axis. A flood damage probability matrix could also be constructed from a graph similar to Figure 2.3 with headwater plotted on the X axis and damage cost plotted on the Y axis. Such a matrix is only necessary if a probability distribution of damage cost is to be calculated. Only a single mean damage cost curve is required to calculate the expected damage cost for each culvert diameter.

2.3 Calculations

The decision tree calculations for the existing data branch of Figure 2.1 are outlined below. The decision to gather more data will result in a new flood probability vector. This topic is more fully discussed in Chapters 3 and 6.

Steps:

1. Calculate flood probability vector for given flood frequency plot and flood interval vector (Chapter 3).
2. Choose culvert diameter (an annual investment charge for each culvert is calculated, see Chapter 5).
3. Calculate a headwater level for each flood interval and list the headwaters in a vector (Chapter 4).
4. Calculate annual damage cost for each headwater level from 3 and list the results in a vector (damage cost = headwater damage cost + washout cost if HW exceeds HW_{max}, see Chapter 5).
5. Calculate expected (or average) annual damage cost by multiplying each element in the flood probability vector by its corresponding element in the damage cost vector and summing the products; i.e., calculate the dot product of the two vectors.
6. Determine expected total annual cost by adding the annual investment charge and the expected annual damage cost.
7. Repeat steps 2 to 6 for all culvert diameters.
8. Plot results and choose culvert with minimum expected total annual cost (Chapter 6).

Costs are added on the basis of a statistical theorem which states that the expected value of the sum of two or more random variables is equal to the sum of the expected values of the individual random variables.

All the calculations are handled by three computer programs: the first calculates the flood probability vector, the second calculates the headwater vector for each culvert size, and the third program performs the remaining calculations utilizing the results of the first two programs.

Chapter 3

EVALUATION OF FLOOD FLOWS

3.1 Methods and Problems

There is a great deal of uncertainty associated with the evaluation of flood flows from small watersheds in British Columbia. One of the main problems is the lack of direct streamflow measurements for creeks on which culverts are to be located. Thus flood flows are normally evaluated indirectly. Precipitation-runoff relationships are commonly used. Hetherington's publication entitled The 25-Year Storm and Culvert Size - A Critical Appraisal (7) has a good discussion of the methods and problems of peak flow evaluation. Much of the discussion of this section is summarized from his paper.

In order to evaluate peak flows it is necessary to understand the meteorological and physical processes which produce them. There are many different ways in which a 25-year, 100-year, or any year peak flow could be generated. In coastal regions of British Columbia, major rain storms with durations of 12 to 36 hours or greater are the major cause of high peak runoff events. Rapid springtime melting of an above average winter snowpack is a probable cause of high peak flows in the Interior. In very small watersheds of a few hundred acres or less, high peak flows can also be generated by high intensity convective rainfalls (thundershowers) of duration less than 2 to 3 hours. Rain falling on snow can cause high runoff events for both coastal and interior watersheds. Also a flood with a relatively high return period can be generated when flow of lower return period is temporarily blocked by a debris jam.

Storm runoff water backed up behind the debris jam is released as a powerful surge when the dam collapses. The uncertainty about the conditions likely to cause high peak flows adds uncertainty to the indirect evaluation of peak flows.

Precipitation-runoff models are commonly used because some sort of precipitation data is usually available to apply to the watershed in question. However, meteorological stations are widely scattered throughout the province and mostly located at low elevations Most stations collect rainfall in standard, non-recording gauges; hence, data on short duration rainfall intensities is very limited. Many of the stations, particularly those with recording gauges, have a very short period of record which restricts the reliability of return period calculations.

Extrapolating precipitation data, horizontally as well as vertically, from observations taken at a single point is a difficult problem, particularly in mountainous terrain where precipitation patterns are complex. The orographic effects on precipitation can be very pronounced especially during major storms in areas where mountain slopes are exposed directly to rain-bearing winds, such as on the western slopes of Vancouver Island. The network of snow survey sites is also sparse, and the extrapolation of snow survey data is even more tenuous than for rainfall data.

The simplest rainfall-runoff models are empirical formulae relating peak flow to rainfall intensity and physiographic parameters of the watershed, such as drainage area or basin slope. The most popular formula is the so-called "rational formula" (Q = CIA) which is widely used by many agencies including the

British Columbia Department of Highways. All these formulae are deficient in that they do not recognize the complexity of the runoff process. Each formula contains an empirical constant, C, usually called the runoff coefficient, which is difficult to estimate for any watershed. C is a constant in the formula, but experience shows that its value varies widely from storm to storm (8). The already questionable reliability of these formulae decreases as the watershed area increases.

Models, such as the University of British Columbia Watershed Budget Model (9), are much more accurate in simulating the runoff process than simple formulae. These models also handle snowmelt and rain-on-snow conditions. Critical sequences of daily temperature as well as snowpack data are required to evaluate snowmelt runoff.

A key aspect of the U.B.C. Watershed Model is the division of the watershed into area-elevation bands to account for the elevation dependence of precipitation and temperature. In addition, other watershed characteristics such permeability and groundwater storage are frequently elevation dependent. Some period of streamflow record is helpful in evaluating the calibration parameters for the model. The reliability of the precipitation data, and not the limitations of model itself, is likely to impose the major limitation on the reliability of the computed peak flow values if the calibration parameters can be determined reasonably accurately.

Besides using precipitation-runoff models, peak flow data for large streams could be transposed to smaller streams on a simple discharge per unit area basis to estimate peak flows. The watersheds must have similar physiographic and climatic characteristics.

Even so, this approach is likely to underevaluate small stream peak flows because of differences in timing of runoff between large and small watersheds.

A survey of existing culvert installations can provide information on peak flows that is useful in predicting flows for other watersheds. Crest-stage gauges installed at culvert entrances and approach sections are very useful in this regard. The computed peak flow values along with the recorded precipitation data can be used to assess precipitation-runoff formulae and watershed models. If the record is long enough the return periods can also be estimated. Flows computed from discernable high-water marks are difficult to relate to a return period but still have some value in assessing existing installations.

3.2 Accounting for Uncertainty in Flood Flows

The uncertainty in evaluating flood flows is accounted for by placing upper and lower confidence limits, along with a most probable curve, on a flood frequency plot. The flood frequency distribution chosen for specifying the three curves was the Gumbel distribution. Other distributions, such as the log Pearson Type III, may be more appropriate and could be used equally well. Both the Gumbel and log Pearson Type III distributions consider only the annual floods, i.e., the maximum flood peak in each year. A partial duration series, which includes all independent flood events, differs substantially from an annual series at low return periods (less than about 5 years). Thus the partial duration series is the more appropriate choice if a culvert sustains damage at floods of a relatively low return period.

The most probable curve in the initial analysis was specified by setting $Q_{10} = 150$ cfs (i.e., the 10-year flood) and $Q_{100} = 220$ cfs. This line is labelled 1.0 in Figure 3.1. The lower and upper bounds were then simply specified as multiples of the most probable curve, such as a lower bound of 0.5 and an upper bound of 1.5 times the most probable curve. Thus the difference between the bounds increases as the return period increases. Actually the bounds need not be straight lines but could be any curves. For instance, if the hydrologist has very little confidence in predicting high return period floods, the bounds will diverge even more rapidly with increasing return period than the straight line bounds shown in Figure 3.1.

The flood probability vector, which can be plotted as a probability density function, is easily computed for a single line Gumbel plot by dividing the vertical axis into flood intervals and calculating the difference in the probabilities of the floods at the ends of each interval. The probability density functions for the most probable curve, 1.0, and two multiple curves alone, 1.2 and 1.5, are shown in Figure 3.2.

The information conveyed by specifying a most probable curve with upper and lower bounds can also be converted into a single flood probability vector and plotted as a probability density function or an equivalent single curve Gumbel plot. First, a probability matrix is formed from the Gumbel plot with its upper and lower bounds exactly the same as for any bounded function as outlined in Chapter 2. The horizontal scale of the Gumbel plot, which is linear with respect to the reduced variate b (the Gumbel equation is $P = e^{-e^{-b}}$

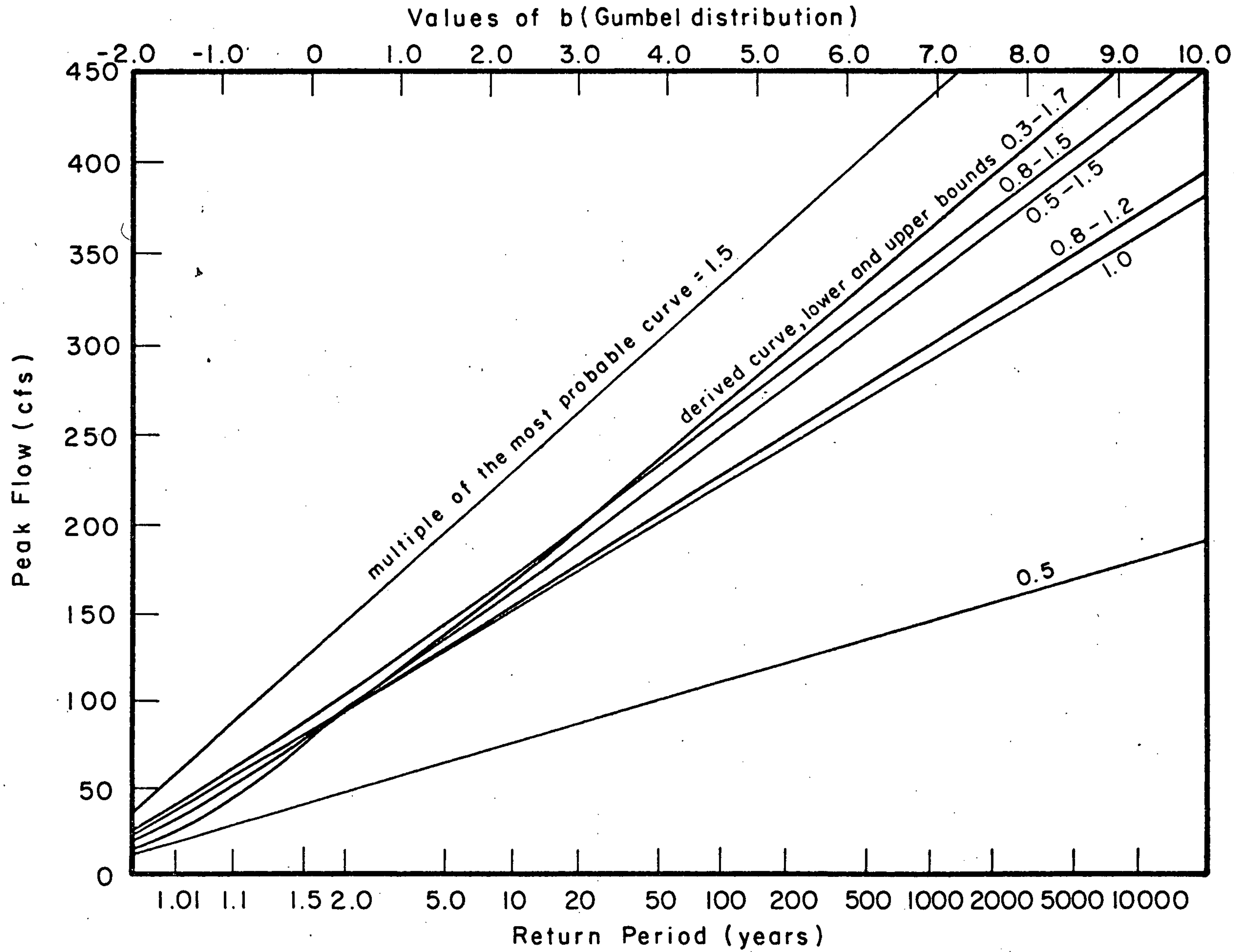


FIG. 3.1 FREQUENCY CURVES OF ANNUAL FLOODS (1.0 LINE : Q_{10}=150cfs , Q_{100}=220cfs.)

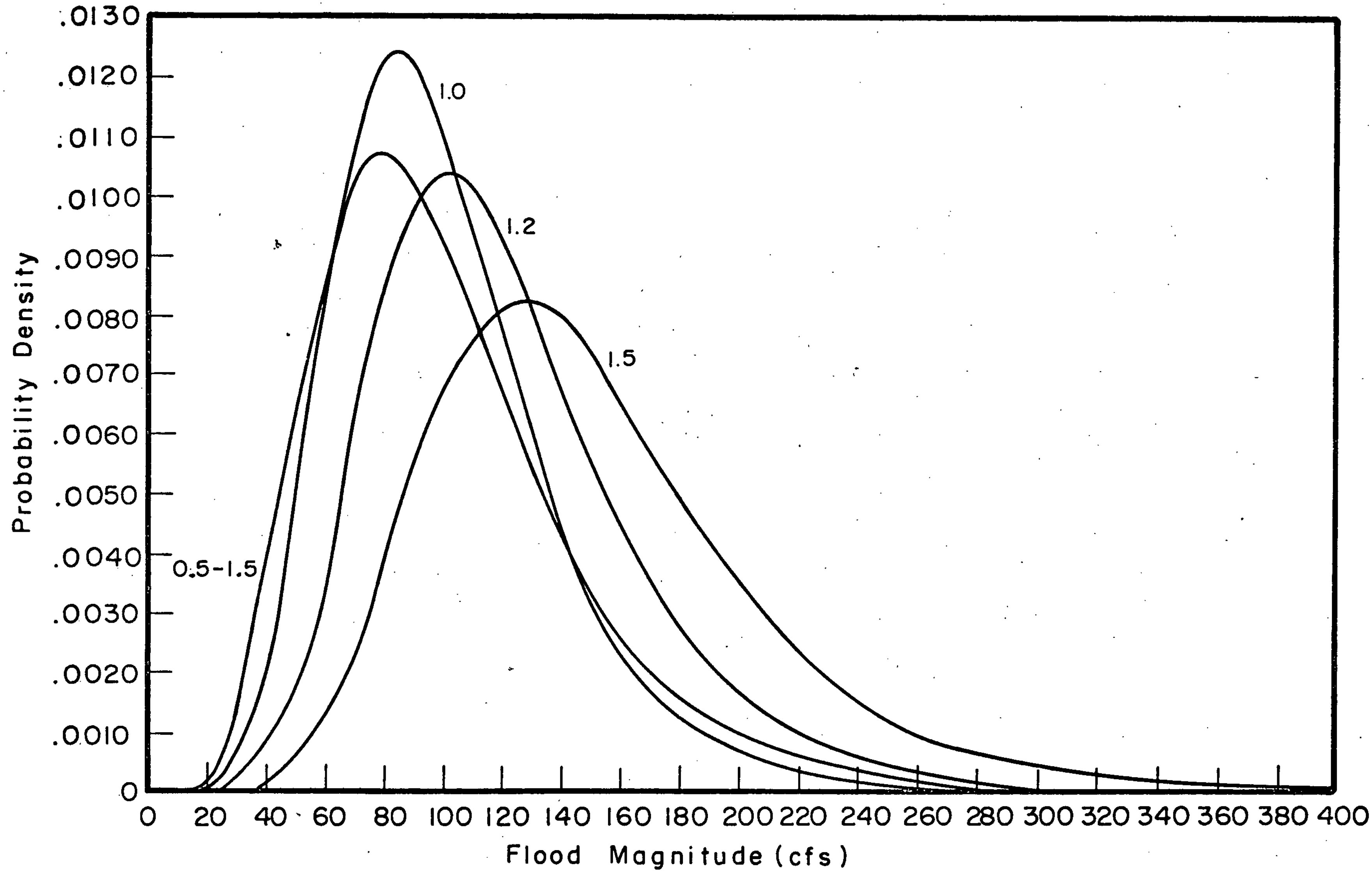


FIG. 3.2 FLOOD FREQUENCY DISTRIBUTIONS (1.0 CURVE: Q_{10} = 150 CFS, Q_{100} = 220 CFS).

where P is the probability of equalling or exceeding a flood of a given size), is divided up into equally sized b intervals over a suitably large range of b. The b intervals are in fact return period or probability intervals, for example, one representing the 37 to 45 year return periods, and these probabilities are calculated and temporarily stored in a vector (sum = 1.0). The rows of the probability matrix represent return period intervals, and each return period interval is in turn represented by the return period at the probability mid-point of the interval since only one point in each X interval is used in forming the matrix. The vertical scale of the Gumbel plot is divided into flood intervals, for example, 250-255 cfs, and the columns of the matrix represent these flood intervals. An element of the matrix then represents the probability that a flood of a given return period, say 40.6 years which is at the probability mid-point of the 37 to 45 year return period interval, lies within a certain range, say 250-255 cfs. The sum of the elements across any row, as usual, equals 1.0.

But if the elements of each row are multiplied by the probability of being in the corresponding flood interval, for example, the elements of the 37 to 45 year return period interval row are multiplied by (1/37)-(1/45), then the sum of all elements in the matrix will equal 1.0. An individual element of the matrix then represents the overall probability that a flood both lies within a certain range and belongs to a certain return period interval. The probability that a flood lies within a certain range, regardless of what return period interval it belongs to, is obtained by summing the elements of the respective flood interval column of the new matrix. Thus the information conveyed by a bounded Gumbel plot is converted into a single probability vector which can in turn be plotted as a probability density function or a single equivalent Gumbel curve.

Four bounded distributions, i.e., distributions derived from bounded Gumbel plots, along with three distributions derived from single lines were used in the initial analysis with the most probable curve, 1.0, specified by Q_{10} = 150 cfs and Q_{100} = 220 cfs (Q_{100}/Q_{10} = 1.47). Later, a different most probable curve with a Q_{100} to Q_{10} ratio equal to 1.8 was considered to see what effect steepening the Gumbel curve would have on the decision tree results. The 1.8 ratio is used by the British Columbia Department of Highways in their design criteria, although this ratio can vary considerably from watershed to watershed. For West Vancouver the Q_{100} to Q_{10} ratio is about 1.6 (10). The new most probable curve was specified by setting Q_{10} = 120 cfs and Q_{100} = 216 cfs. In this case, one bounded distribution, along with the most probable curve distribution alone, was used in the analysis.

Figure 3.1 shows the equivalent Gumbel plots of the four bounded distributions, as well as some single line Gumbel plots, all based on a 1.0 line with Q_{10} = 150 cfs and Q_{100} = 220 cfs. The curves derived from bounded distributions are labelled by the multiple factors of the lower and upper bounds, such as 0.5-1.5, while single lines are labelled with a single multiple factor, such as 1.5. This labelling system is used throughout the thesis. Figure 3.2 shows some of the probability density functions. Figure

3.3 shows Gumbel plots based on the new 1.0 line defined by Q_{10} = 120 cfs and Q_{100} = 216 cfs. Table 3.1 summarizes some of the information contained in Figures 3.1 and 3.3 by listing the effective floods of eleven return periods for the different distributions. The term effective flood is used to denote the flood derived by converting a bounded Gumbel plot into a single equivalent curve.

Looking at the results based on the 1.0 curve with Q_{10} = 150 cfs and Q_{100} = 220 cfs, for the symmetrically bounded Gumbel plots the bounded distributions are more unfavourable than the 1.0 distribution above a return period of about 2.3 years. The fact that they are more favourable below this return period has little significance since it is unlikely the design selected will sustain damage at floods below the 2.3-year return period. The 0.8-1.2 distribution differs surprisingly little from the 1.0 distribution. Increasing the steepness of the 1.0 line results in less difference between a bounded distribution and the 1.0 distribution; this can be seen by comparing the 0.5-1.5 and 1.0 curves in Figures 3.1 and 3.3.

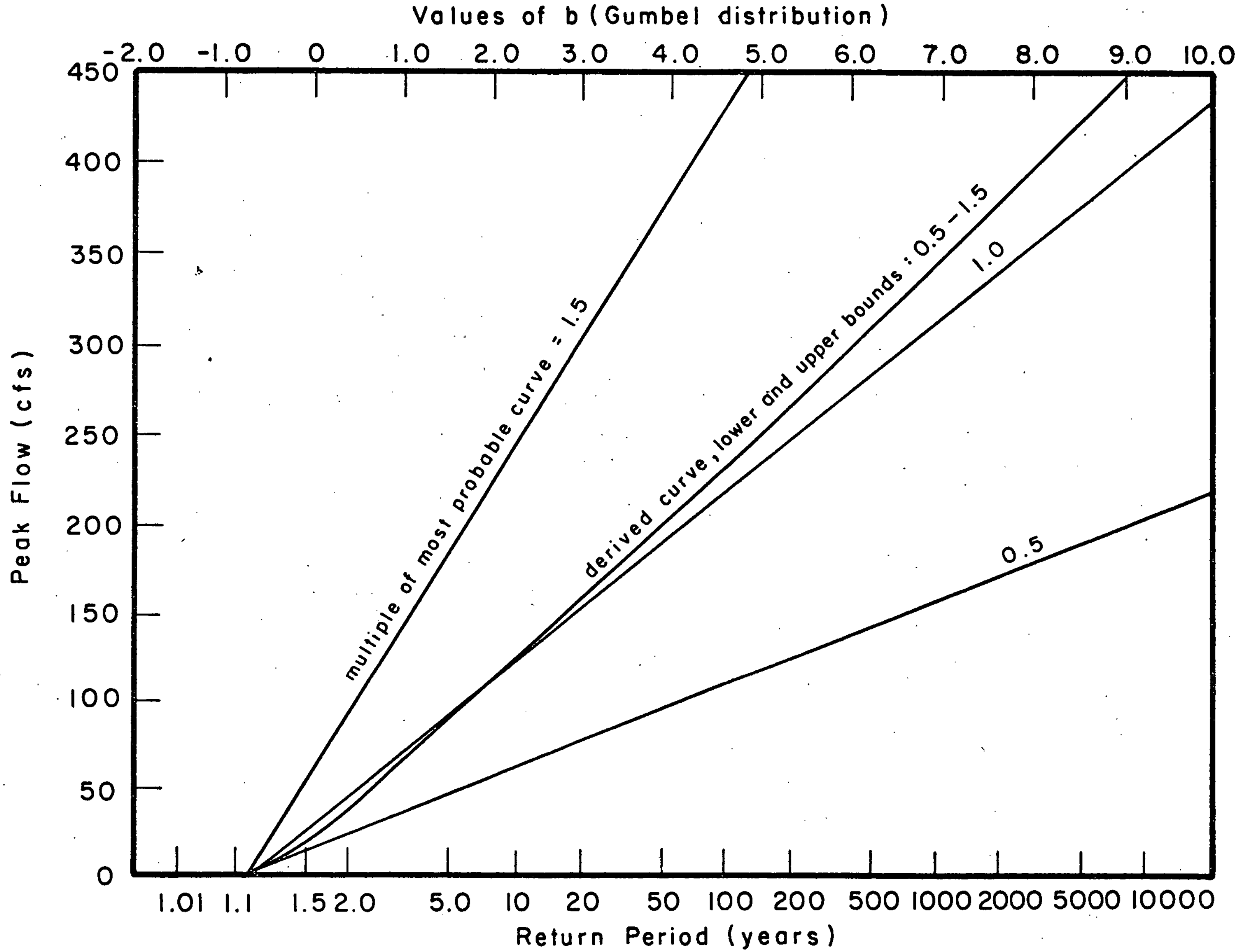


FIG. 3.3 FREQUENCY CURVES OF ANNUAL FLOODS (1.0 LINE: Q_{10}=120cfs, Q_{100}= 216cfs).

TABLE 3.1

COMPARISON OF EFFECTIVE FLOODS OF VARIOUS RETURN PERIODS FOR DIFFERENT DISTRIBUTIONS

I. 1.0 Flood Frequency Line Specified by Q_{10} = 150 cfs and Q_{100} = 220 cfs

Distribution	Return Period (yr)										
	1.1	2.0	5.0	10	20	50	100	200	500	1000	10000
1.0	57[1]	94	128	150	171	199	220	241	268	289	357
0.8-1.2	56	93	128	151	174	202	224	246	275	298	370
0.5-1.5	50	92	132	159	185	218	244	270	304	330	420
0.3-1.7	44	90	136	166	195	233	262	291	329	359	459
0.8-1.5[2]	60	103	143	170	197	231	257	283	317	343	433
1.2	68	113	153	180	206	239	264	289	322	346	429
1.5	85	141	191	225	257	299	330	361	402	433	536

II. 1.0 Flood Frequency Line Specified by Q_{10} = 120 cfs and Q_{100} = 216 cfs

Distribution	Return Period (yr)										
	1.1	2.0	5.0	10	20	50	100	200	500	1000	10000
1.0	0	43	89	120	149	187	216	244	282	310	404
0.5-1.5	0	41	89	122	155	198	232	265	311	345	463

[1] all effective flood values are in cfs

[2] mean value of this truncated skew normal distribution = 1.108 x most probable value

Chapter 4

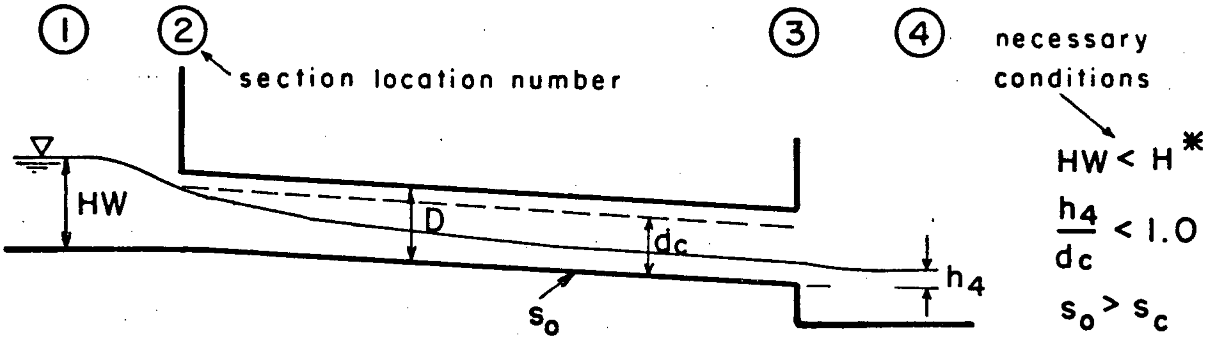
CULVERT HYDRAULICS

4.1 Types of Culvert Flow

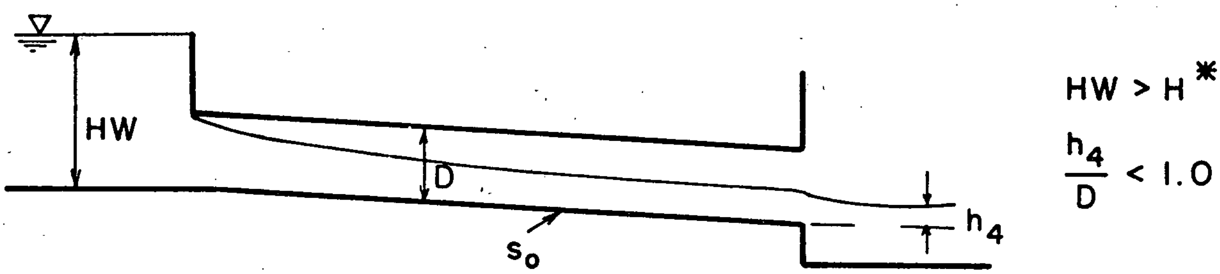
The relationship between the headwater depth and the discharge is greatly influenced by the type of flow through the culvert. The type of culvert flow occurring at a given discharge may be determined by many variables including the inlet geometry; the slope, size, and roughness of the culvert barrel; and the approach and tailwater conditions. For practical purposes culvert flow is commonly classified into six types. But by placing the culvert on a 7% slope and assuming the tailwater neither submerges the outlet nor reaches a subcritical depth causing backwater effects at any discharge, the number of possible flow types was reduced to three, shown in Figure 4.1. Both the 7% slope and the tailwater assumptions are reasonable in the mountainous and hilly terrain covering most of British Columbia.

The hydraulic computations were based on equations and tables compiled by R. W. Carter in 1957 (11). The equations for the three types of flow considered are given in the appendix. All computations were done by computer since some calculations required tedious iteration procedures. For example, calculation of the headwater depth requires a coefficient of discharge, but the coefficient of discharge is a function of the headwater level for flow types 1 and 2. The cross-sectional area of the headwater pool is assumed reasonably large so that the velocity head is negligible. In addition the volume of water stored in the headwater pool at any

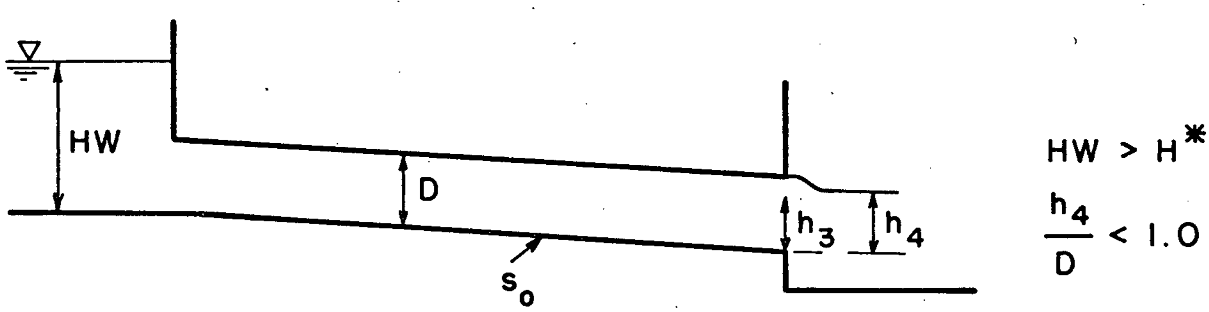
Type 1: Critical Depth at Inlet.



Type 2: Rapid Flow at Inlet.



Type 3: Full Flow Free Outfall.



NOTATION:

D = culvert dia (min. dia for CMP)
d_c = critical depth
h = piezometric head above culvert invert at downstream end
HW = depth of water in headwater pool
H^* = critical value for headwater depth (H^* = 1.5D used here)
s_c = critical slope for culvert barrel
s_o = bed slope of culvert barrel

FIG. 4.1 TYPES OF CULVERT FLOW.

headwater level is assumed small; so, effectively, at any time the discharge into the headwater pool equals the discharge through the culvert. The headwater-discharge curves for several culvert diameters are shown in Figure 4.2.

The entrance of an ordinary culvert will not be submerged if the headwater is less than a certain critical value, designated by H*, while the outlet is not submerged. The value of H* varies from 1.2 to 1.5 times the culvert diameter, D, depending on the entrance geometry, barrel characteristics, and approach conditions (12). Carter assumes H* = 1.5D, so this value was used in the calculations. Chow (12) states, "For a preliminary analysis, the upper limit H* = 1.5D may be used . . . because computations have shown that, where submergence was uncertain, greater accuracy could be obtained by assuming that the entrance was not submerged."

Type 1 flow results when the headwater is less than H*, the tailwater is lower than the critical depth, and the culvert slope is supercritical. Critical flow occurs at or near the culvert entrance, and the headwater depth depends only on the discharge, culvert size, and entrance geometry. Thus, this is an example of inlet control.

Type 2 flow is also an example of inlet control, but in this case with the entrance submerged. The inlet functions as an orifice with the flow entering the culvert contracting to a depth less than the diameter of the culvert barrel in a manner similar to the contraction of flow in the form of a jet under a sluice gate. In the case of a square-ended culvert set flush with a vertical head-wall and, indeed, with most culvert inlets, type 2 flow follows

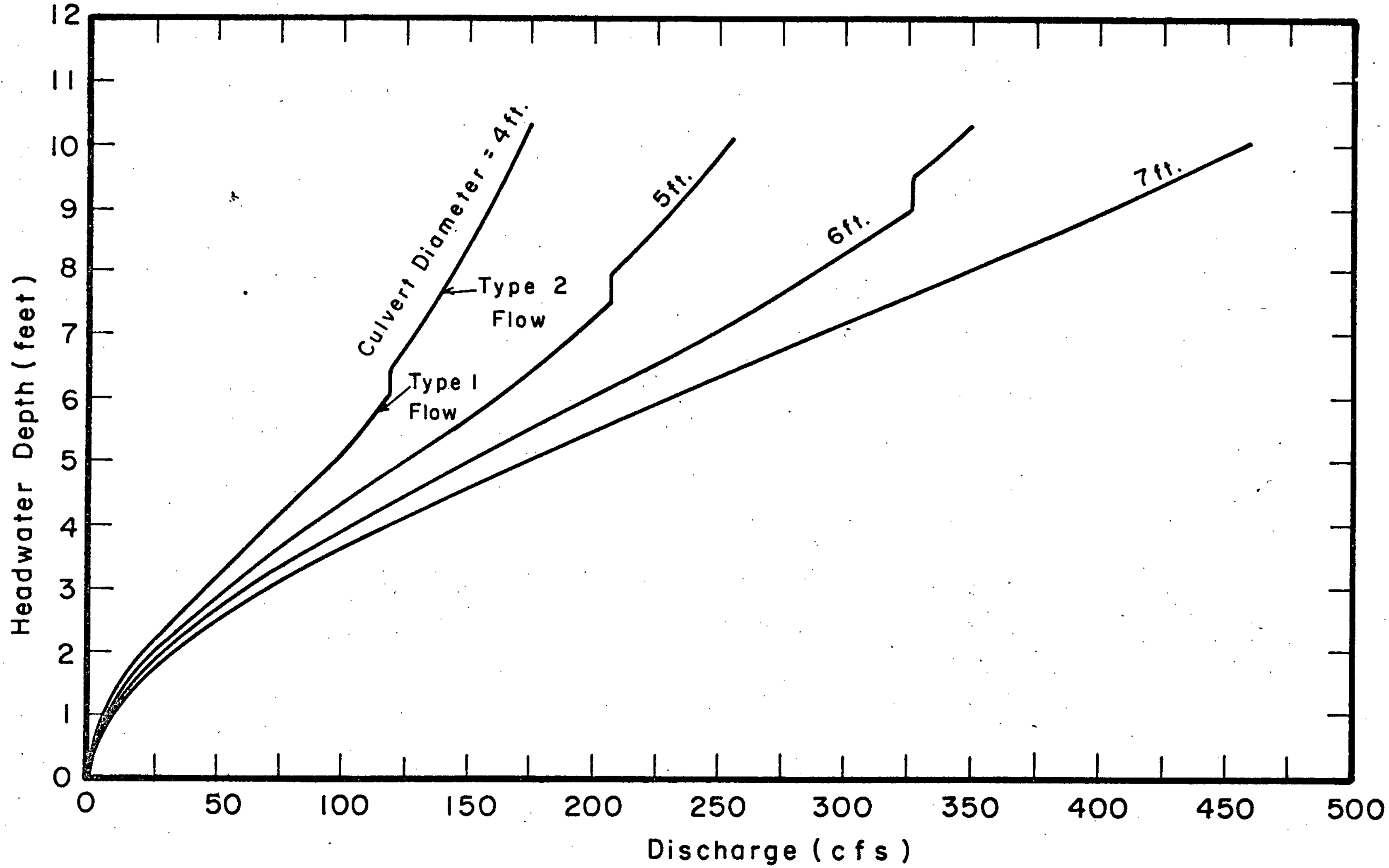


FIG. 4.2 HEADWATER-DISCHARGE CURVES.

type 1 flow as the headwater depth increases with increasing discharge. However at high submergences of the orifice the culvert may suddenly fill and type 3 flow occurs. Blaisdell (2) has found that the headpool level at which this occurs may be different each time the culvert fills, making an exact determination difficult. At this point there will be a sudden increase in flow through the culvert and a resulting decrease in the headpool level as the control changes from the orifice to the pipe.

A culvert is considered hydraulically short if the flow is type 2 and hydraulically long if the flow is type 3. Carter has prepared charts to roughly distinguish between these two flow types. The determination depends on many characteristics such as culvert diameter, length, and slope; entrance geometry; headwater level; entrance and outlet conditions; etc. In practice it turned out that, for all culvert diameters considered (3.5 to 7.0 ft) and over the headwater range of interest (up to 10 ft), in all submerged inlet cases the flow was type 2. Also, the 7% slope was a steep slope in all these cases although flow types 2 and 3 can occur on mild or steep slopes.

In type 3 flow the culvert barrel is under suction with the piezometric head at the outlet varying from a point below the centre to the top of the culvert. However, Neill (13) reports that the turbulent, aerated flow caused by the pipe corrugations may prevent the existence of sub-atmospheric pressures in the culvert and cause the culvert to flow partly full. This is a variation of type 3 flow and not type 2 flow.

4.2 Entrance and Exit Improvement

Entrance improvement should always be considered since it can increase the hydraulic efficiency of culverts and thus reduce the culvert size required. (An increase in hydraulic efficiency means that at a given flow the headwater surface can be lowered; or stated conversely, at a given headwater depth the flow accommodated can be increased.) Exit improvement may be required to prevent erosion problems.

The primary purposes of a headwall are to retain the fill and protect the embankment from erosion. Wingwalls can be used in addition to retain the fill and support the headwall. By retaining the fill behind the headwall, endwall, and wingwalls, savings can be realized by a reduction in the culvert length required. Where sufficient fall is available, culvert design can be improved by making the entrance into a sloping apron (14). The critical depth occurs on the apron, and the flow is accelerated along the apron and into the culvert. The sloping inlet has an appreciable effect as long as the culvert barrel does not flow full.

Rounding or tapering the inlet increases the hydraulic efficiency by increasing the coefficients of discharge for all flow types. A more spectacular increase in hydraulic efficiency can be obtained in some circumstances by employing special inlets, such as bell-mouth or hood inlets. This advantage applies only when the culvert entrance is submerged and mainly to culverts on steep slopes. The special inlet prevents inlet orifice control (type 2 flow) and causes the pipe to flow full (type 3 flow). Blaisdell (2) has found in experiments using a hood inlet that an intermediate flow type, slug and mixture flow, consisting of alternating slugs of full flow and air pockets, occurs before type 3 flow is established. As the inlet just becomes submerged, the additional head created by the short length of full conduit draws the headpool down admitting air to the culvert. The air flow decreases as discharge increases until the culvert flows completely full of water. There is very little increase in the headpool depth until the discharge is great enough to cause full flow.

Vortices at culvert inlets can adversely affect culvert performance, particularly during pipe control with low inlet submergences, and thus they can decrease the advantage of using special inlets. Vortices form over the inlet and admit air to the culvert through the vortex core. The air replaces water in the culvert and reduces the discharge. Vortices can reduce the culvert capacity to anywhere between that obtained with pipe control and that obtained with orifice control. On the other hand, surface vortices that do not have an air core may have little effect on the culvert capacity. Vortices can be inhibited by installing anti-vortex devices.

Plugging of culverts is considered by many to be one of the major problems associated with culverts (7). It can lead to major flood damage, even in cases of minor floods. Culverts should be designed to pass expected debris, keeping in mind that any debris jams that occur must be easily accessible by maintenance crews. Upstream debris racks are required in some locations. Plugging by ice forming inside the culvert can be a problem in British Columbia's Interior.

The outlet end of a culvert should be designed to avoid

(1) blockage by debris, (2) damage by flow undermining the culvert and embankment, and (3) erosion of the downstream channel. The greater roughness of corrugated metal pipe as compared to concrete pipe is an advantage in reducing outlet velocity. A stilling basin or energy dissipator of some sort may be required to reduce downstream erosion.

4.3 Mechanics of a Washout

An assumption is made in the analysis that the roadway will wash out as soon as the road is overtopped. It is further assumed that the washout results in the same damage to the roadway, no matter what flow caused the washout, and the culvert itself is not damaged in the process. These assumptions are not completely valid but were made to simplify the analysis.

The roadway is likely to withstand some overtopping, with minimal damage, before washing out. The washout mechanism may start with gravel being eroded at both the upstream and downstream embankments, eventually leading to the undermining and collapse of the road surface. Once the road surface collapses the flow rate over the road surface will increase dramatically, and the washout will proceed quickly. Given the uncertainties of the situation, it may be very difficult to estimate at what point a road will wash out.

A culvert is likely to sustain some damage during a washout, although a headwall and endwall may prevent it from being washed away. Scour under the culvert will mean that the culvert has to be lifted out and re-installed.

Highway embankments are not designed as dams. If ponding is allowed for in the design of a culvert, provision must be made so that seepage through the embankment will not lead to failure by piping or other means. Also the slopes of the embankments must not be so great that they collapse when saturated.

4.4 Environmental Considerations

Environmental considerations might be called intangibles in an economist's terms. It is difficult to place a monetary value on fish in a stream because they may be worth much more than their commercial value. If fish and other aquatic organisms are to be preserved in streams passing through culverts, economic analysis for culvert design may have to be supplemented by analysis of the effects of the proposed design on the organisms involved.

High flow velocities in culverts are common and may prevent fish from moving upstream. Reinforced concrete pipe, with its low roughness coefficient, is more of a problem than corrugated metal pipe. Baffles might be needed to reduce the velocity. Exit facilities, for example, 5 foot drops, often inhibit fish access to the culvert. One approach to the entire problem is to preserve the natural streambed by installing a sufficiently large arch structure, although it is bound to be much more expensive than a pipe culvert.

Chapter 5

ECONOMICS

5.1 Capital Cost

The approximate capital costs of installed culverts are shown in Table 5.1 and Figure 5.1. These costs are for 100 ft lengths of asbestos bonded, asphalt coated corrugated metal pipe (CMP) culverts, with vertical concrete headwalls and endwalls, as used in the analysis. The installed CMP costs are from the District of West Vancouver Drainage Survey by Dayton and Knight Ltd., Consulting Engineers (10). The installation cost is based on "average" conditions in West Vancouver and represents the cost of installing a culvert under an existing highway. Consequently the installation cost will be somewhat less for a new highway construction project, particularly under fills, as little or no excavation will be required The costs can only be taken as approximate because they depend to a large extent on the conditions at each culvert site. The transportation cost to the site is also a variable factor that must not be overlooked.

The cost levels used in the Dayton and Knight report are equivalent to an Engineering News-Record (ENR) Construction Cost Index of 2500 for 1975. The costs in Table 5.1 and Figure 5.1 have been adjusted to an ENR index of 3000 for 1977. The headwall-endwall set costs were calculated from California Division of Highways values presented in Pritchett's thesis (3) by multiplying by the ratio of the ENR index in 1977 to that in 1964 (3000/900). This method of updating costs is only approximate as the ENR index

TABLE 5.1

CAPITAL COSTS OF INSTALLED CULVERTS

Culvert Diameter (feet)	Pipe Cost* ($)	Headwall & Endwall Cost ($)	Total Cost ($)
3.0	5280	1270	6550
3.5	6300	1570	7870
4.0	7560	1870	9430
4.5	9120	2170	11290
5.0	10680	2470	13150
5.5	12480	2770	15250
6.0	14400	3080	17480
6.5	16560	3400	19960
7.0	19200	3700	22900

*for 100 ft length

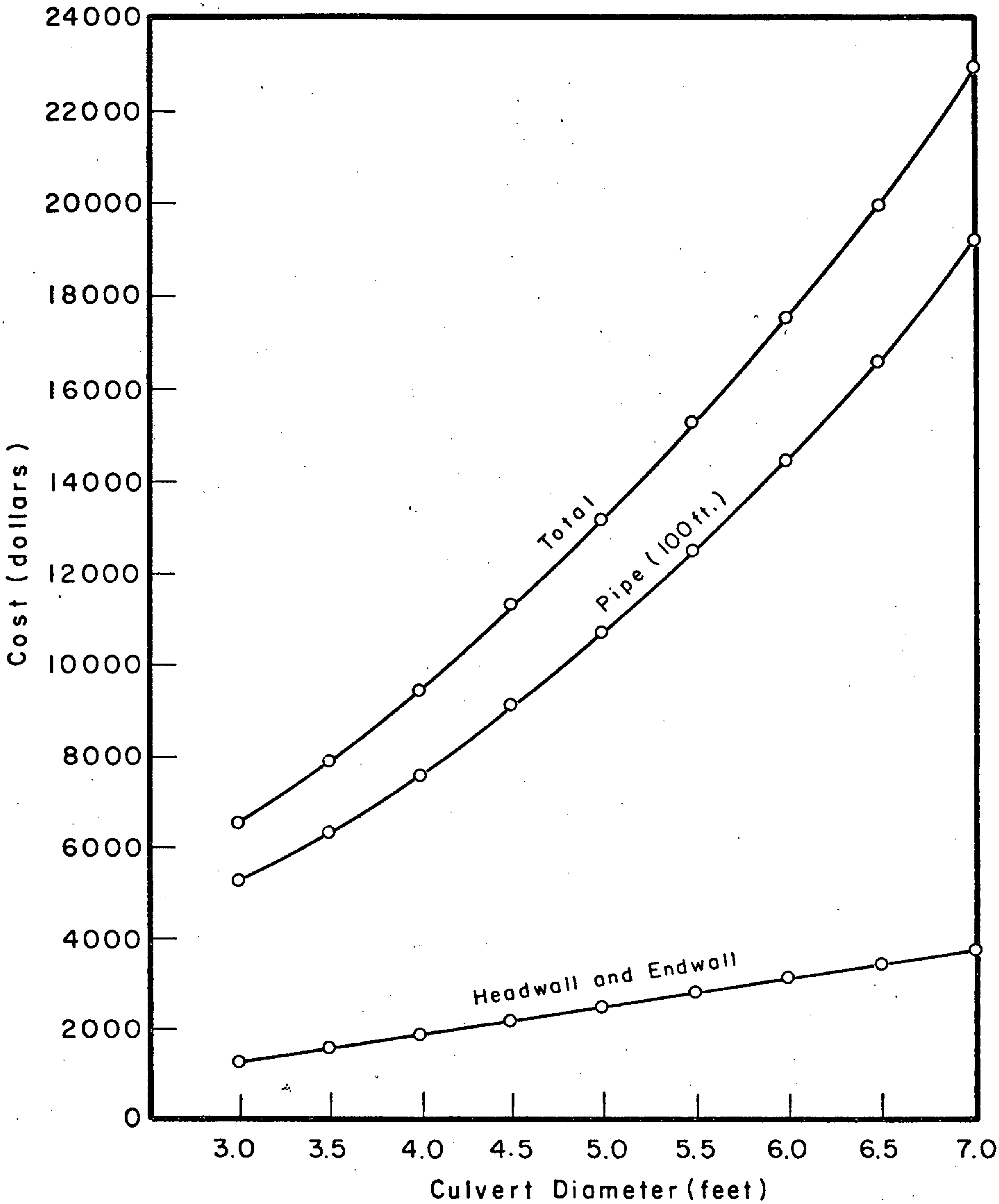


FIG. 5.1 CAPITAL COSTS OF INSTALLED CULVERTS.

represents the cost of a group of items consisting of fixed quantities of labour, cement, steel, and lumber, and not the cost of purchasing and installing culverts. There will also be disparities between California and British Columbia costs.

5.2 Flood Damage

The flood damage cost at a particular headwater level is the sum of two items: the headwater damage cost and the washout cost if the road washes out.

Headwater damage is the result of water backing up and flooding public or private property upstream of the culvert. Damage to the highway embankment, such as erosion of gravel caused by high headwater, is included under headwater damage. Upstream flooding is likely to be a problem only in populated areas where development encroaches on the stream, or in flood plains where substantial ponding can take place and inundate large areas of residential or agricultural land.

The headwater damage curve used in the analysis is shown in Figure 5.2. The shape of the curve was chosen arbitrarily with marginal flood damage first increasing then decreasing. A typical flood damage vs. depth curve for urban property is shown by James and Lee (15) as a combination of three straight lines with the first segment having the greatest slope and the final segment a slope of zero. The damage is assumed to be a function of headwater level only and not of culvert size. This may not be true in the case of damage to the highway embankment as velocity and turbulence around the culvert inlet at a given headwater will vary for different culvert diameters.

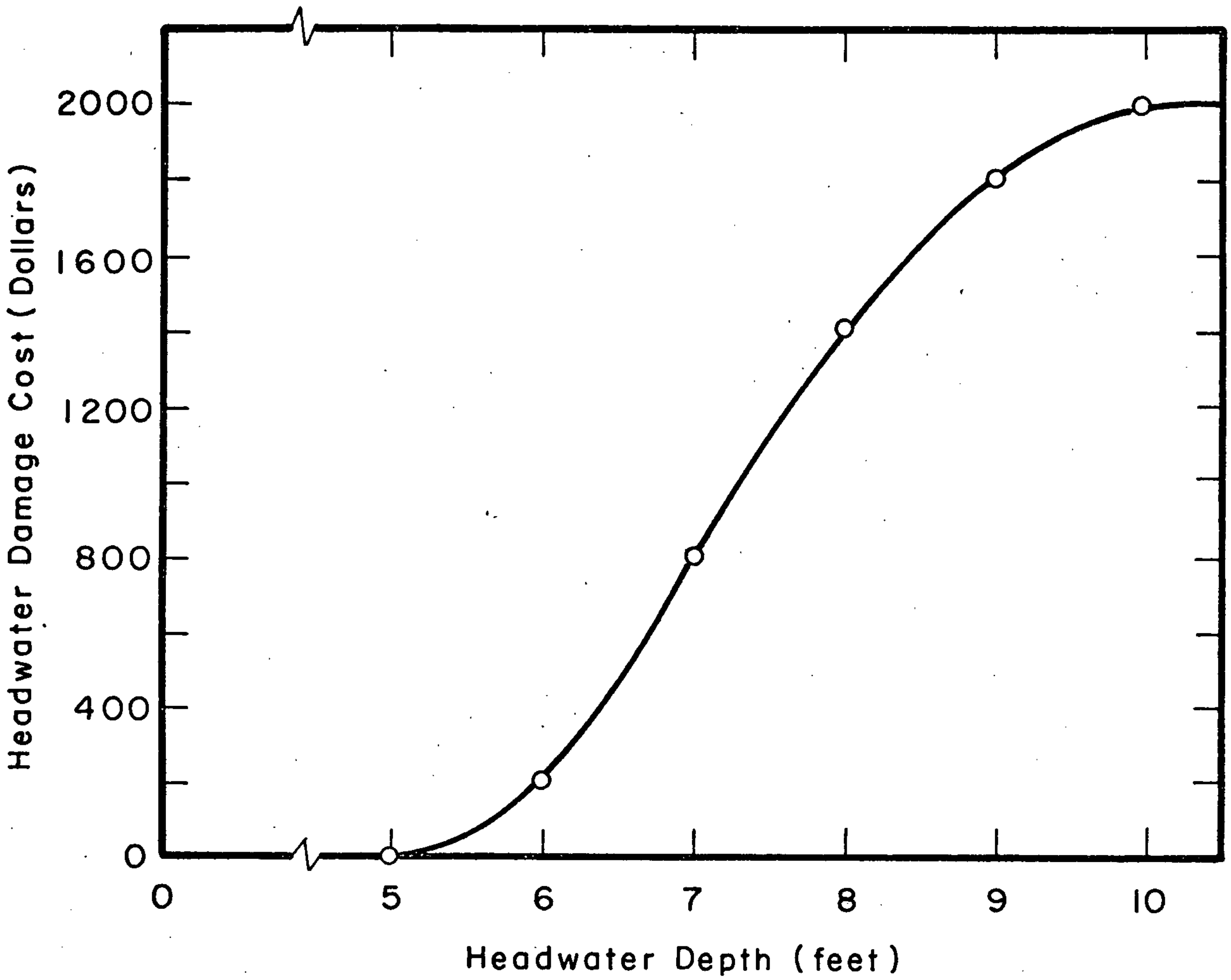


FIG. 5.2 HEADWATER DAMAGE FUNCTION.

The roadway is assumed to wash out if the headwater overtops the highway (i.e., exceeds 10 ft in this case). The washout is assumed to result in extensive damage to the roadway but no damage to the culvert and its headwall and endwall. The validity of these assumptions was discussed in Chapter 4. The washout cost is the sum of (1) the cost of repairing the highway, (2) expenses for flagmen, barricades, flares, and signing for traffic detours, and (3) the cost of interrupting traffic, which is borne by the road-users themselves. The repair cost will depend on the availability of labour, materials, and machinery, as well as the extent of damages.

The cost of interrupting traffic is more difficult to determine. It includes the increased motor vehicle operating cost for detour mileage, slowdowns, stops, and vehicle washing; the cost of increased travel time; and the cost of increased accident probability. These costs will vary from vehicle to vehicle, particularly between trucks and cars; therefore a weighted average must be used. The value of time lost for occupants of vehicles not on business is often evaluated at one-third the average wage.

The volume of traffic, time required to repair the road, and type of detour route available all influence the magnitude of the cost of interrupting traffic. If no detour is available on a major highway, the cost will be very high. Conversely, the cost will be low for minor highways.

A washout cost of \$15,000 is used in the initial analysis. The cost borne by the highways department for repairing the road and providing flagmen, barricades, etc. is assessed at \$5000, and the cost borne by the road-users at $10,000. The road-user cost is roughly calculated as the product of the average daily traffic (ADT), the time required to repair the road in days, and the average cost of delay per vehicle. The average daily traffic is the average 24-hour volume for a given year, counting both directions of travel. A typical ADT of 2500 for a major rural two-lane highway is assumed, and the time required to repair the highway is estimated at 2 days. The average cost of delay per vehicle, including both increases in operating cost and travel time, is set at $2.00 per vehicle. This low cost per vehicle implies a relatively minor detour.

It might be argued that road-user costs should not be included in the economic analysis since the highways department does not compensate motorists for the delay. However, looking at the problem from a broad social point of view, which a government should always do, these costs are real and must be included since highways are public entities and not privately owned.

Some mention of maintenance cost should be made, although it was not included in the analysis. Pritchett, in his thesis, assumes an equal average maintenance cost for pipe culverts from 18 to 96 in. on the basis that the larger culverts have a larger area of brush to clear at the entrance and exit of the pipe, but less sand and debris to clean out as compared to the smaller diameter culverts. Using this assumption, the culvert size decision will not be affected by the maintenance cost.

5.3 Annual Cost Comparison

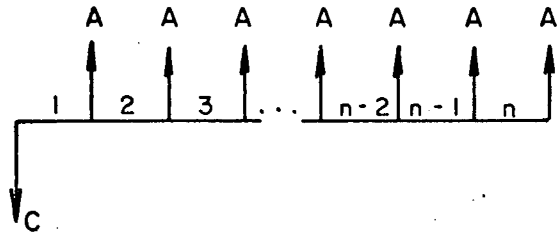
Before an economic analysis for choosing culvert size can be completed, the capital cost and expected annual damage cost, computed as outlined in Chapter 2, must be placed on a comparable basis so they can be added. The equivalent uniform annual cost method, in which the investment cost is converted to an annual cost, is used in this case. The present value method, which involves combining the investment cost and expected annual damage cost into a single present worth sum, could equally well be used and would yield the same result as the equivalent uniform annual cost method.

The factor to convert an investment cost into an equivalent annual cost is designated as the capital-recovery factor and may be computed from the expression $r(1 + r)^n/((1 + r)^n - 1)$, where r is the discount rate per annum and n is the estimated service life of the culvert or highway, whichever is shorter. The equation is for a series of n year-end payments, as shown in Figure 5.3, although the capital-recovery factor will not be significantly different for a series of n mid-year payments, as long as n is not too small.

The question of what is the correct discount rate to use in computing the capital-recovery factor is a matter of considerable debate. It is a very important question as a change of 1% in the discount rate (i.e., from 4% to 5% or from 7% to 6%) will often change the project selected. A low discount rate with a long service life will favour designs with a high capital cost since the annual investment charge will be lower than in the case where the discount rate is high or the service life is low.

The term discount rate is used to distinguish it from interest rate. Discount rate, r, as used here, is the real rate of interest as opposed to the money rate of interest, x. The discount rate can be computed as $r = (x - i)/(1 + i)$ or approximately $r = x - i$, where

Method 1: Uniform Series

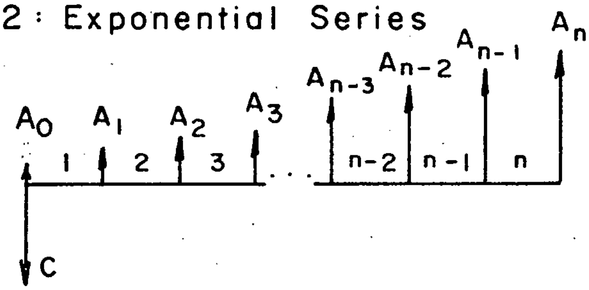


$$A = C \cdot CRF$$

$$CRF = \frac{r(1+r)^n}{(1+r)^n - 1} \quad \text{where } r = \frac{x-i}{1+i}$$

C = capital cost
A = equivalent annual cost <u>in base year dollars</u> (i.e. dollars at beginning of year 1)
CRF = capital-recovery factor
r = discount rate
x = money rate of interest
i = inflation rate

Method 2: Exponential Series



$$A_0 = C \cdot ECRF$$

$$ECRF = \frac{\left(\frac{1+i}{1+x}\right) - 1}{\left(\frac{1+i}{1+x}\right)\left[\left(\frac{1+i}{1+x}\right)^n - 1\right]}$$

A_x = annual cost <u>in dollars of year x</u>; $A_x = A_0 \, (1+i)^x$
ECRF = exponential series capital-recovery factor

N.B. A_0 is not included in the summation for calculating the ECRF, in conformity with the period-end step convention.

It can be easily proven that CRF = ECRF if r as defined above is used in calculating the CRF. Therefore the two methods are equivalent.

FIG. 5.3 CONVERTING CAPITAL COST TO ANNUAL COST.

i is the rate of inflation. This equation corrects the money rate of interest for the effect of inflation.

A discount rate of 4% was chosen for the initial analysis. This figure was based on an interest rate for risk free investment, such as government bonds, equal to about 10% and a rate of inflation equal to about 6%. In fact, both the money interest rate and the inflation rate are likely to fluctuate considerably over the service life of the culvert or highway. But fluctuations in the real interest rate are usually much smaller, as in the long run the money rate of interest adjusts to account for the inflation rate. As an example, interest rates on government savings bonds increased from about 5% in the early 1960s to 8 to 10% in the 1970s. But the calculated real interest rate held steady for 1965 to 1972 at a moderate level of 3% before it fell in 1973 (16).

An equivalent method of handling the problem of inflating costs is illustrated by the exponential series in Figure 5.3. Here the capital cost is converted to an exponential series of annual costs increasing at the rate of i per cent per annum, as opposed to a series of uniform annual costs. The expected annual damage cost is also assumed to increase exponentially at the rate of i per cent per year; therefore, the two series of annual costs can be added to determine the series of total annual costs for a given culvert diameter.

Actually, only the annual costs at the beginning of the base year need be computed since all annual costs increase at the same rate, i. Hence the culvert size decision can be made by comparing the total annual costs in the base year. The money rate of interest, x, is used to compute the annual investment charge at the beginning of the base year since the effect of inflation is taken into account directly. In fact, the annual investment charge computed at the beginning of the base year will be same for the exponential series method and the equivalent uniform annual cost method ($r = (x - i)/(1 + i)$); therefore the two methods are exactly equivalent.

The discussion of the exponential series is meant to point out the importance of taking the rate of inflation into account. It would be a serious error to calculate the capital-recovery factor for the equivalent uniform series method on the basis of the money rate of interest with its built-in inflation factor. This would amount to adding a uniform series, the annual capital cost, to an exponentially increasing series, the expected annual damage cost. If the equivalent uniform series method is applied, the money rate of interest must be corrected for the effect of inflation so that there will be two uniform series, both in base year dollars.

The foregoing discussion assumes that the expected annual damage cost increases at the same rate as inflation, or in other words, remains the same in real terms. Factors such as upstream land development and highway traffic growth will result in a real increase in the expected annual damage cost. Construction of alternate routes or switches to other modes of transportation (due to rapidly increasing gasoline prices, etc.) will result in a real decrease in the expected annual damage cost. It is often difficult to forecast these changes, particularly over a long period of time such as 20 or 30 years, but some attempt should be made.

It should be mentioned that annual cost calculations are valid regardless of the financing scheme employed to pay the capital cost, as long as the discount rate is appropriate for the circumstances (17).

A culvert service life of 30 years was used in the initial analysis. Actually this value is conservative as a properly installed, asbestos bonded, asphalt coated CMP can be expected to last much longer; particularly if in addition the invert is paved with asphalt or concrete to guard against sediment abrasion. Factors such as the corrosion potential at the proposed culvert site, the anticipated highway service life, and cost will influence the culvert material, material thickness, and type of protective treatment selected. For example, for temporary roadways such as logging roads, only simple galvanized CMP culverts would be justified. This decision could also be included in the decision tree of Figure 2.2 with different materials or protective coatings having different service lives. A further complication is introduced if culvert damage is anticipated when the roadway washes out since the service life of the culvert may be shortened or terminated by damage.

There may be a great deal of uncertainty in estimating the service life of a highway or culvert. In this regard it should be noted that if n is initially large, say 30 years, a large increase in n, say to 100 years, will only moderately change the capital-recovery factor. The difference in the capital-recovery factor with increasing n will decrease as the discount rate, r, increases.

Chapter 6

RESULTS

6.1 Annual Cost Curves for One Flood Frequency Distribution

The annual cost curves for the single line Gumbel plot defined by setting Q_{10} = 150 cfs and Q_{100} = 220 cfs are shown in Figure 6.1. The expected total cost curve (the word "expected" is often omitted for convenience) shows that the optimum culvert diameter is 5.0 ft with smaller diameter culverts becoming less competitive more rapidly than larger diameter culverts. The cost data from which Figure 6.1 was plotted, as well as some additional information, is given in Table 6.1.

The so-called marginal investment costs (MIC) listed in Table 6.1 are the differences in annual investment cost between given sized culverts and culverts of the next smaller size. Similarly marginal savings (MS) is the difference in expected total annual damage cost between a given sized culvert and the culvert of the next smaller size. The use of these marginal costs and savings is fully explained in the third section of this chapter.

Table 6.2 gives the annual probability of incurring some headwater damage and the annual probability of a washout for each culvert diameter, first using the Gumbel plot defined above and then using the Gumbel plot defined by Q_{10} = 120 cfs and Q_{100} = 216 cfs.

6.2 The Effect of Uncertainty and the Value of Better Information

The effect of uncertainty in the flood frequency plot with the most probable curve specified by Q_{10} = 150 cfs and Q_{100} = 220 cfs

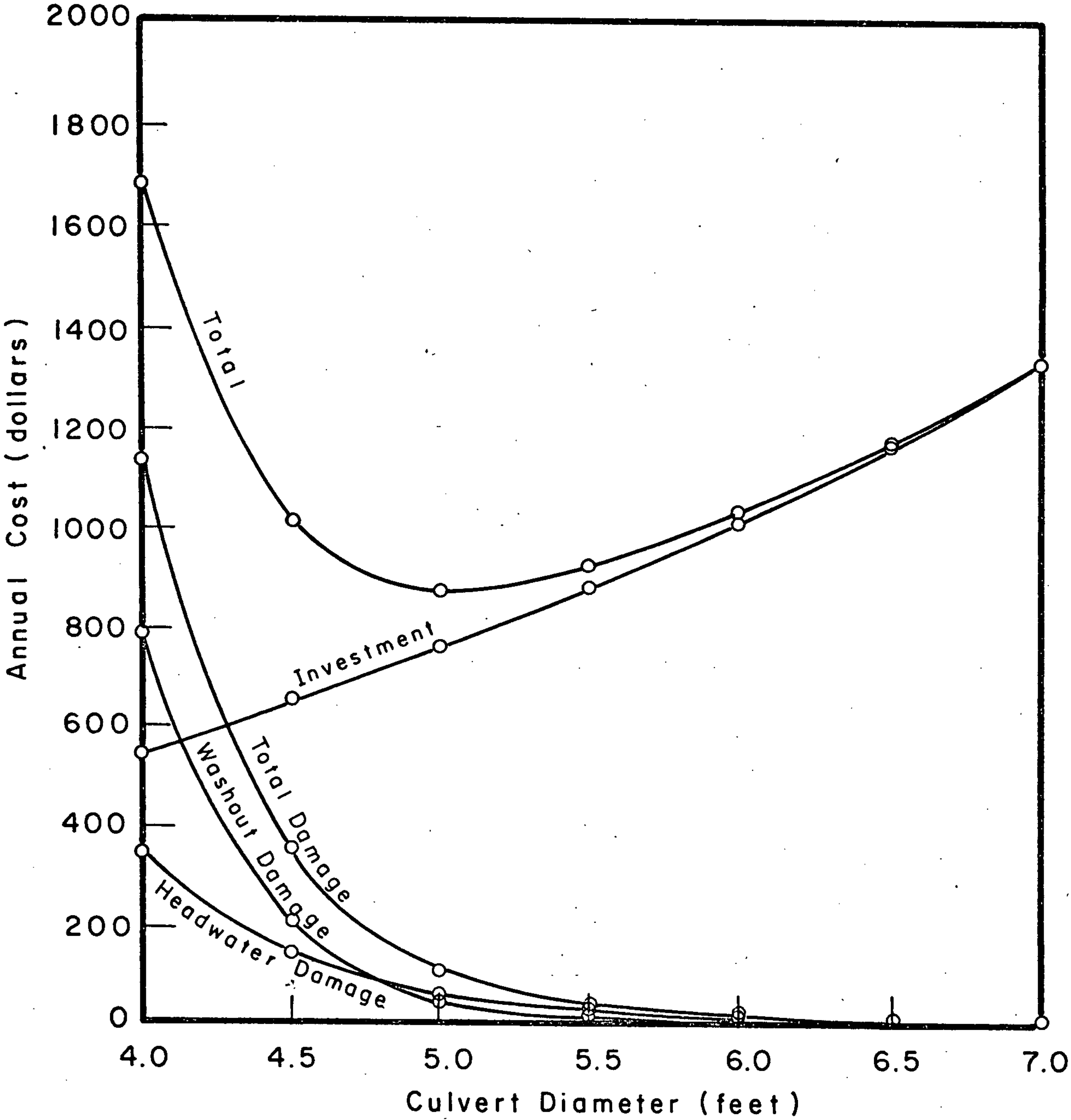


FIG. 6.1 ANNUAL COST CURVES FOR FLOOD FREQUENCY DISTRIBUTION DEFINED BY $Q_{10} = 150$ cfs AND $Q_{100} = 220$ cfs.

TABLE 6.1

ANNUAL COSTS FOR FLOOD FREQUENCY DISTRIBUTUION DEFINED BY Q_{10} = 150 CFS AND Q_{100} = 220 CFS

Culvert Diameter (ft)	Investment Cost[1] ($)	Marginal Investment Cost[2] ($/size)	Headwater Damage Cost ($)	Washout Cost ($)	Total Damage Cost ($)	Marginal Savings[3] ($/size)	Total Cost ($)	Increase in Total Cost from Optimum ($)	% Increase in Total Cost from Optimum
4.0	545	90	348	786	1134	1998	1679	810	93.2
4.5	653	108	146	209	356	778	1009	140	16.1
5.0	760	107	61	47	108	248	869	—	—
5.5	882	122	29	12	41	67	923	54	6.2
6.0	1011	129	15	3	18	23	1029	160	18.4
6.5	1154	143	8	0	9	9	1163	294	33.8
7.0	1324	170	5	0	5	4	1329	460	52.9

[1]based on r = 4% and n = 30 yr

[2]MIC = (annual investment cost of given culvert size) - (annual investment cost of next smaller culvert size)

[3]MS = (annual total damage cost of next smaller culvert size) - (annual total damage cost of given culvert size)

TABLE 6.2

PROBABILITIES OF INCURRING SOME HEADWATER DAMAGE AND

PROBABILITIES OF A WASHOUT

	Flood Frequency Distribution Specified by:			
	Q_{10} = 150 cfs, Q_{100} = 220 cfs		Q_{10} = 120 cfs, Q_{100} = 216 cfs	
Culvert Diameter (ft)	Probability HW $\geq$ 5 ft	Probability HW $\geq$ 10 ft	Probability HW $\geq$ 5 ft	Probability HW $\geq$ 10 ft
4.0	.48705	.05242	.17658	.03051
4.5	.33201	.01396	.12592	.01157
5.0	.21641	.00310	.08901	.00386
5.5	.13704	.00081	.06253	.00145
6.0	.10000	.00018	.04930	.00048
6.5	.06170	.00002	.03441	.00011
7.0	.04450	.00000	.02705	.00003

is shown in Figure 6.2. The curve labelling system is the same as in Chapter 3. The effect of uncertainty in changing the optimal decision from that of the most probable curve alone appears to be rather minimal. For a symmetric distribution (upper and lower bounds equidistant from the most probable curve), the bounds must be somewhat further apart than 0.5-1.5 before a switch to a 5.5 ft culvert is indicated. The expected total cost curve for the asymmetrically bounded distribution, 0.8-1.5, is very similar to that of the 0.3-1.7 distribution.

Figure 6.3 shows the results for the two distributions with the new most probable curve specified by Q_{10} = 120 cfs and Q_{100} = 216 cfs. As a result of the steeper most probable curve, the total costs of culvert diameters less than the optimum diameter increase less rapidly than in the cases shown in Figure 6.2, although again culverts smaller than the optimum become less competitive more rapidly than culverts larger than the optimum. The effect of uncertainty in changing the optimal decision is less with the new most probable curve, as can be seen by comparing the 0.5-1.5 and 1.0 curves of Figures 6.2 and 6.3.

Two methods of calculating the value of better information are discussed in the following paragraphs. The second method is the better of the two, and although this method was not actually used in the analysis, it warrants a full discussion.

The first method assumes that the most probable flood frequency curve is in fact the true curve. Then the true total cost curve is the 1.0 curve of Figure 6.2 or 6.3. The value of better information is simply the difference between the total costs on the

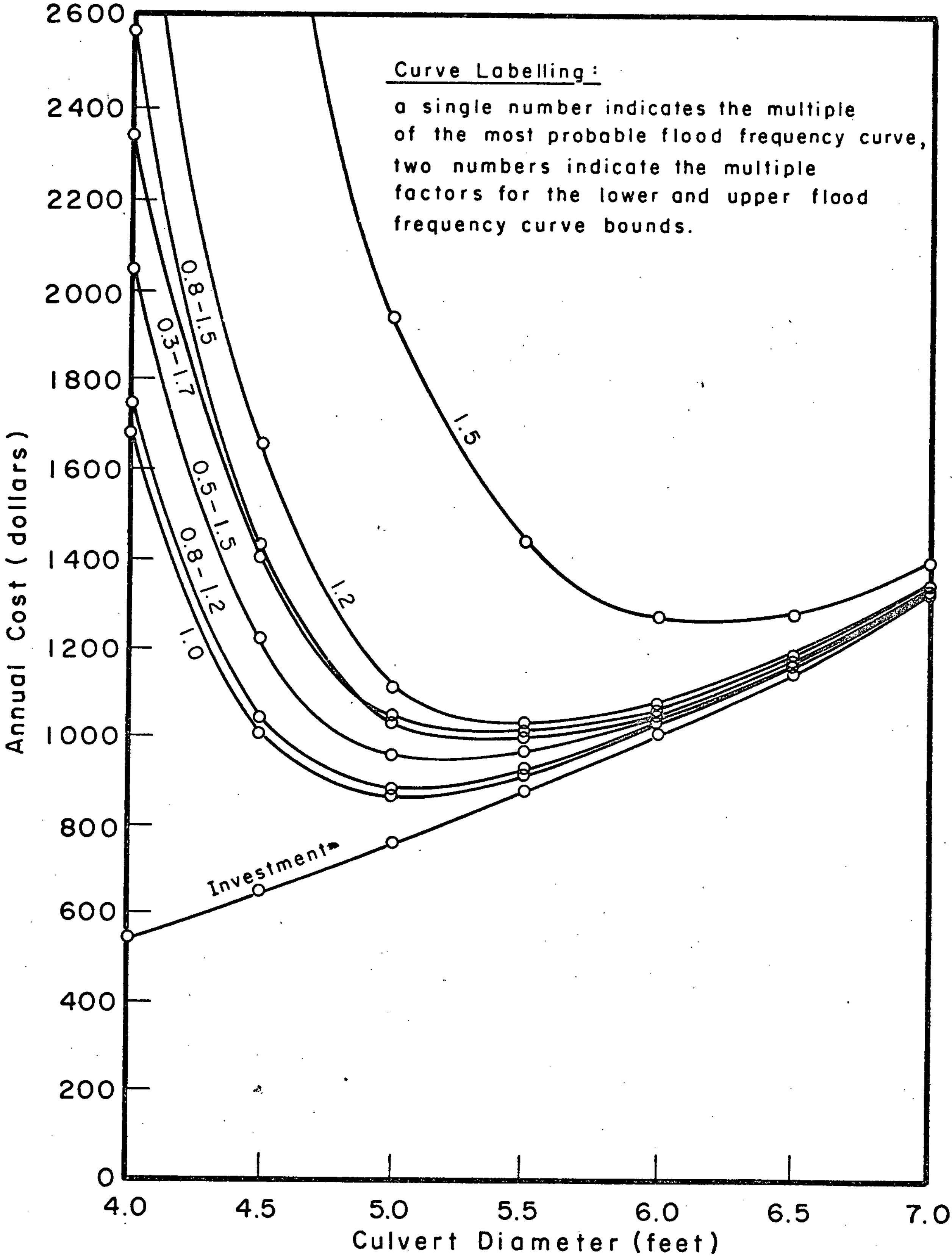


FIG.6.2 TOTAL ANNUAL COST CURVES FOR DIFFERENT FLOOD FREQUENCY DISTRIBUTIONS (1.0 CURVE: Q_{10} = 150 cfs AND Q_{100} = 220 cfs.)

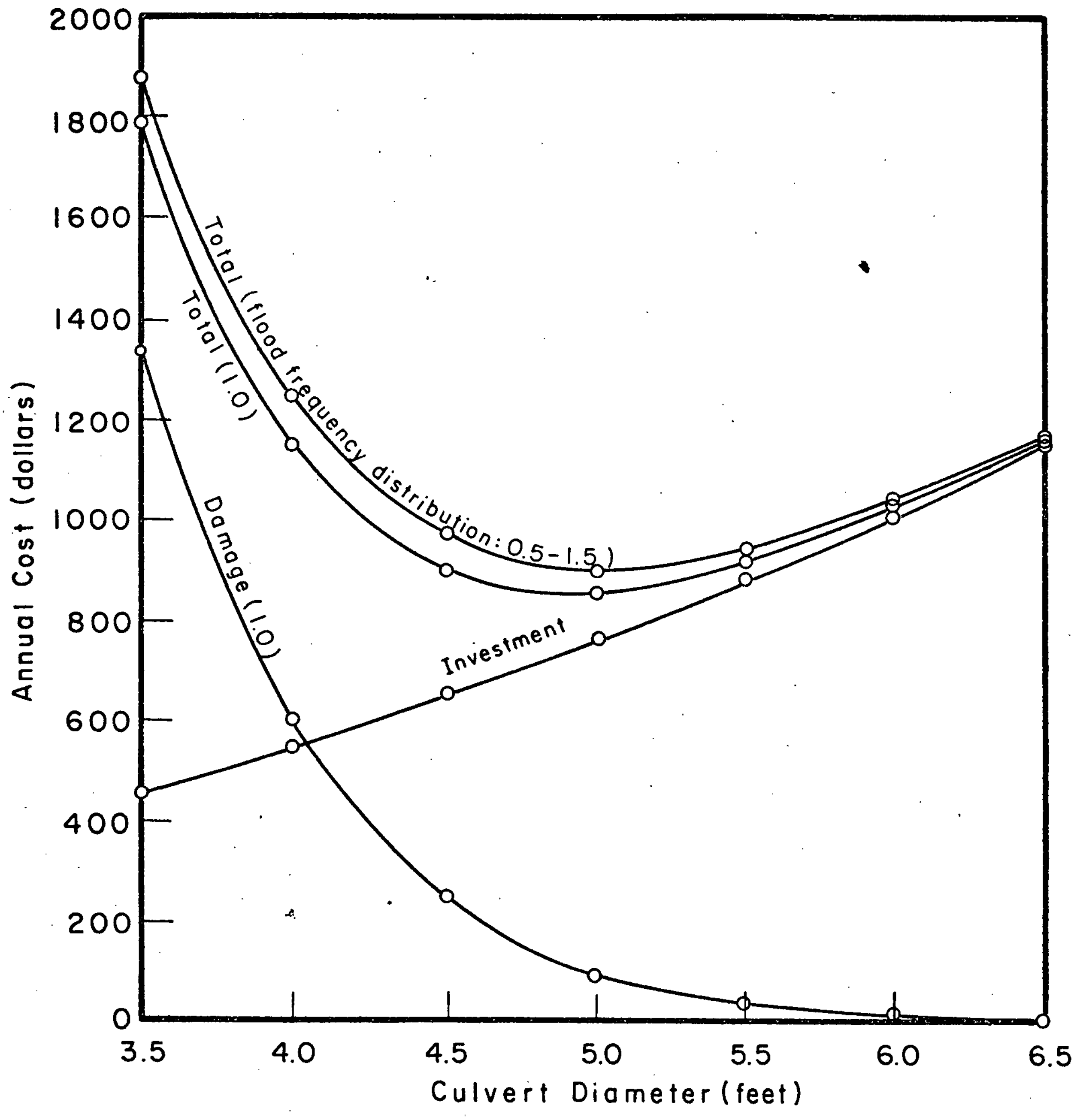


FIG. 6.3 ANNUAL COST CURVES (1.0 CURVE: Q_{10} = 120 cfs AND Q_{100} = 216 cfs).

1.0 curve of the culvert diameter chosen under uncertainty and the true optimum culvert diameter. The values of better information calculated in this manner for the most probable curve specified by Q_{10} = 150 cfs and Q_{100} = 220 cfs are given in Table 6.3. Using this method, better information only has a value if the optimum decision under uncertainty is different from the optimum decision of the most probable curve alone. If the percentage increase in total cost of the next larger size above the optimum is small, such as 0.52% for the 0.5-1.5 distribution, the decision maker will likely choose the larger size, changing the value of better information.

The method just discussed is fundamentally unsound because the true flood frequency curve is never known. In fact the value of better information may be substantial even if the optimum decision under uncertainty is the same as that of the most probable curve alone. For instance, taking the 0.8-1.2 distribution as an example, there is a chance that the true total cost curve is the 1.2 curve of Figure 6.2. Installing a 5.0 ft diameter culvert then results in a total cost of $84/yr more than the optimum for a 5.5 ft culvert. Similarly, if the 0.8 curve is the true curve the optimum culvert diameter will likely be 4.5 ft, and thus installing a 5.0 ft culvert results in a greater total cost than the optimum. These examples suggest a way to calculate the value of perfect information.

A number of total cost curves could be calculated for different multiples of the most probable curve between the upper and lower bounds. For example, if the bounds are 0.8 and 1.2, total cost curves could be calculated for the curves: 0.80, 0.85, 0.90, ... 1.20. A curve is then plotted of the total cost of the optimum culvert vs.

TABLE 6.3

THE EFFECT OF UNCERTAINTY IN CHANGING THE OPTIMAL DECISION AND THE VALUE OF BETTER INFORMATION

Flood Frequency Distribution[1]	Optimum Culvert Diameter (ft)	% Increase in Total Cost of Next Larger Size[2]	Value of Better Information[3] ($)	
			Annual Value	Present Value[4]
1.0	5.0	6.21	–	–
0.8-1.2	5.0	5.32	0	0
0.5-1.5	5.0	0.52	0	0
0.3-1.7	5.5	5.83	54	930
0.8-1.5	5.5	6.29	54	930
1.2	5.5	4.67	54	930
1.5	6.0	0.31	160	2770

[1] 1.0 curve: Q_{10} = 150 cfs, Q_{100} = 220 cfs

[2] using total annual cost curve for distribution being considered (see Figure 6.2)

[3] assuming better information results in the true curve being identified as the 1.0 curve

[4] Present Value = Annual Value / CRF
CRF = .05783; r = 4%, n = 30 yr
CRF = capital-recovery factor
r = discount rate
n = service life

the multiple of the most probable curve. The probabilities that the true curve lies within small intervals of multiples of the most probable curve (for example, 0.80-0.81, 0.81-0.82, ... 1.19-1.20) are then calculated from the truncated skew normal or normal distribution. The total cost of the optimum culvert at the midpoint of each interval is calculated from the previously constructed optimum cost curve and multiplied by the probability that the true curve is in that interval. The sum of these products over all intervals yields the expected total cost with perfect information. The value of perfect information is the difference between the expected total cost of the optimum culvert chosen under uncertainty and the expected total cost with perfect information.

It is interesting to note that the expected total costs with uncertainty of Figures 6.2 and 6.3 could also be calculated in a manner similar to that for the expected total cost with perfect information, rather than by reducing a bounded flood frequency plot to a single curve as outlined in Chapter 3. The only difference is that the cost of the culvert size being considered is used for all intervals instead of the cost of the optimum sized culvert.

In practice, no data gathering program will eliminate all uncertainty; so the value of perfect information fixes an uppermost limit to the value of better information. The value of better information in reducing the uncertainty limits from 0.5-1.5 to 0.8-1.2 might be estimated by subtracting the values of perfect information in the two cases. This is only an estimate because it cannot be known beforehand how the better information will change the uncertainty limits and the most probable curve.

After the new data is actually collected and a new total cost curve is drawn, the value of better information for a particular culvert site can be calculated by subtracting the total cost of the culvert size chosen after the data gathering from the total cost of the culvert size that would have been chosen before the data gathering, both these total costs being from the new curve. If this is done for a large number of culvert sites, such as along a proposed new highway route, then a fairly accurate monetary value of a data gathering program may result. The estimate of the value of the program made before it was instituted can then be compared to the calculated value of the program after it is completed to see how accurate the estimation procedure was.

The rough figures of Table 6.3 show that the value of data gathering can be substantial. Keeping in mind that these are for a single culvert site, it may be very worthwhile to install a network of precipitation gauges, or even install weirs and recording gauges in some streams, before selecting culvert sizes for a new highway.

6.3 Sensitivity of the Optimal Decision to Changes in the Discount Rate and the Service Life

The sensitivity of the optimal decision to changes in the discount rate and the service life was investigated by using marginal investment cost, MIC, and marginal savings, MS, curves. These curves are similar to an economist's marginal cost and marginal revenue curves that are used in analyzing a firm's revenue, cost, and profit picture. A firm seeking to maximize its profit produces to the point where marginal revenue (i.e., the revenue gained from the last unit of output) equals marginal cost (i.e., the cost of producing the last unit of output). Similarly, starting with a small culvert size, larger culvert sizes are selected until the point where the marginal investment cost of moving to the next larger size is greater than the marginal savings gained by moving to the next larger culvert size.

Figure 6.4 was constructed using the results for the case where the most probable curve is specified by Q_{10} = 150 cfs and Q_{100} = 220 cfs with uncertainty bounds of 0.5-1.5. There are four marginal investment cost curves representing different interest rates and service lifes, along with one marginal savings curve, shown in Figure 6.4. The optimal size culvert for a particular marginal savings, marginal cost curve combination is the first culvert size to the left of the intersection of the two curves.

Because there are only a limited number of culvert sizes available (4.0, 4.5, 5.0 ft, etc.) and because the marginal curves were constructed using incremental differences in costs and savings between culvert sizes rather than by taking instantaneous slopes on continuous curves, the intersection point does not indicate the optimum diameter. An intersection point near one of the fixed diameters, such as that for the r = 4%, n = 30 yr MIC curve which intersects the MS curve near a culvert diameter of 5.5 ft, indicates instead that the 5.5 ft culvert has nearly the same total cost as the 5.0 ft culvert. The difference between the MIC and MS curves at a particular culvert diameter is the difference in total cost between that culvert diameter and the next smaller culvert diameter. Thus it is relatively easy to see how competitive the optimum sized culvert is with culverts of smaller and larger size.

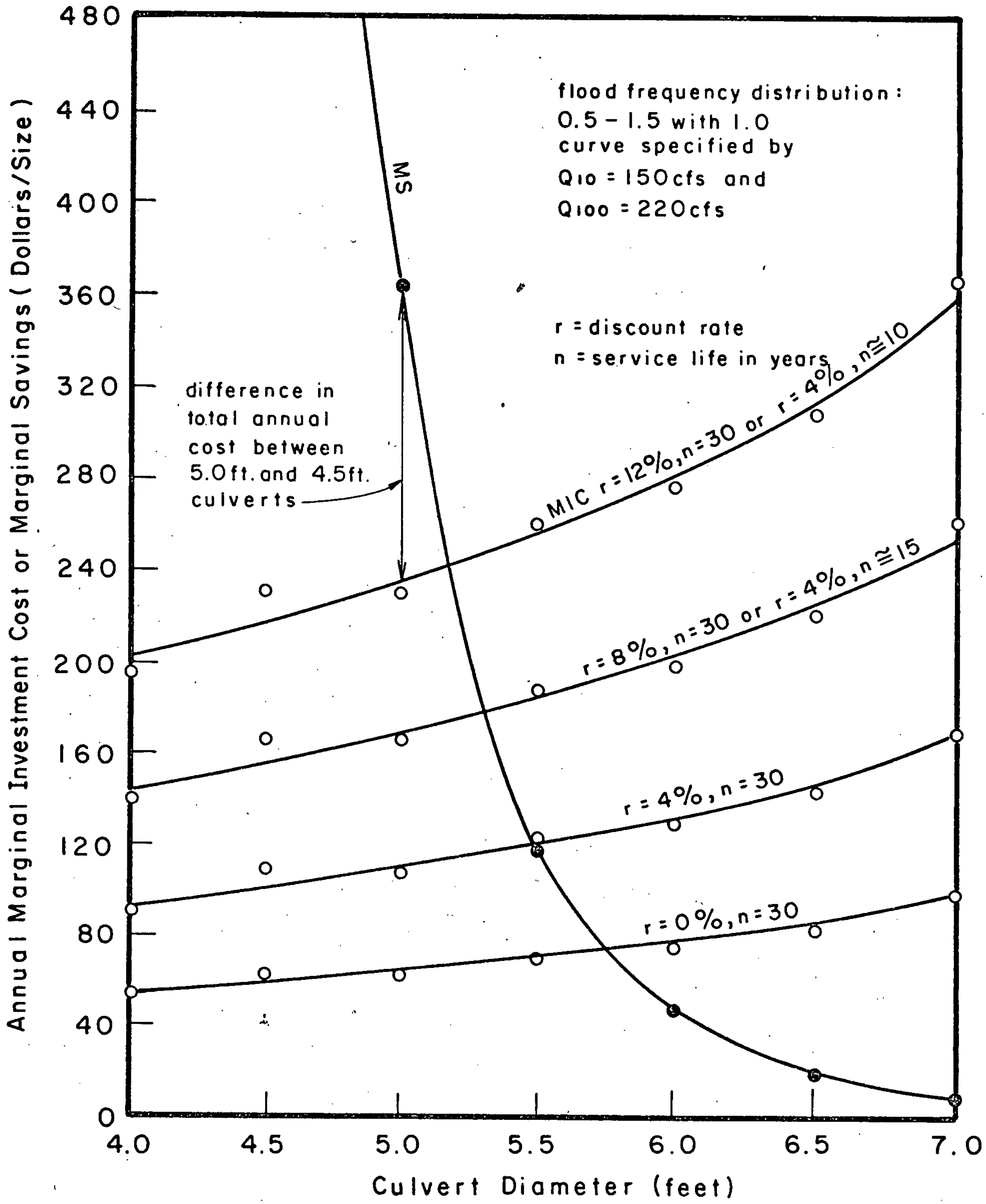


FIG. 6.4 SENSITIVITY OF THE OPTIMAL DECISION TO CHANGES IN THE DISCOUNT RATE AND THE SERVICE LIFE.

Looking at Figure 6.4, the optimal decision does not appear to be particularly sensitive to changes in the discount rate or to changes in the service life with the discount rate fixed at 4%. (Incidentally the MIC curve for $n = \infty$ and $r = 4\%$ lies about one quarter of the way between the $r = 0\%$, $n = 30$ yr curve and the $r = 4\%$, $n = 30$ yr curve, being closer to the lower curve.) If the MS curve were flatter then the optimal decision would be more sensitive to changes in the interest rate and the service life.

6.4 The Effect on the Optimal Decision of Changing the Damage Costs

Marginal savings and marginal investment cost curves were again used to determine the effect on the optimal decision of varying the washout cost and the headwater damage curve. Figure 6.5 shows the results for the most probable flood frequency curve specified by $Q_{10} = 150$ cfs and $Q_{100} = 220$ cfs with uncertainty limits of 0.5-1.5. The marginal cost curve represents the standard case with $r = 4\%$ and $n = 30$ yrs. Figure 6.6 shows the results for the most probable curve specified by $Q_{10} = 120$ cfs and $Q_{100} = 216$ cfs with the same uncertainty bounds as before. All the marginal savings curves are for the standard headwater damage curve, except one in each figure. The marginal savings curve for any washout cost and any multiple of the standard headwater damage curve could easily be plotted from the curves presented in Figures 6.5 or 6.6.

The greater spread of the MS curves of Figure 6.6 compared to Figure 6.5 indicates that the optimal decision will vary more with changing damage costs with the steeper most probable curve used in Figure 6.6. Table 6.4 summarizes some of the information of Figures

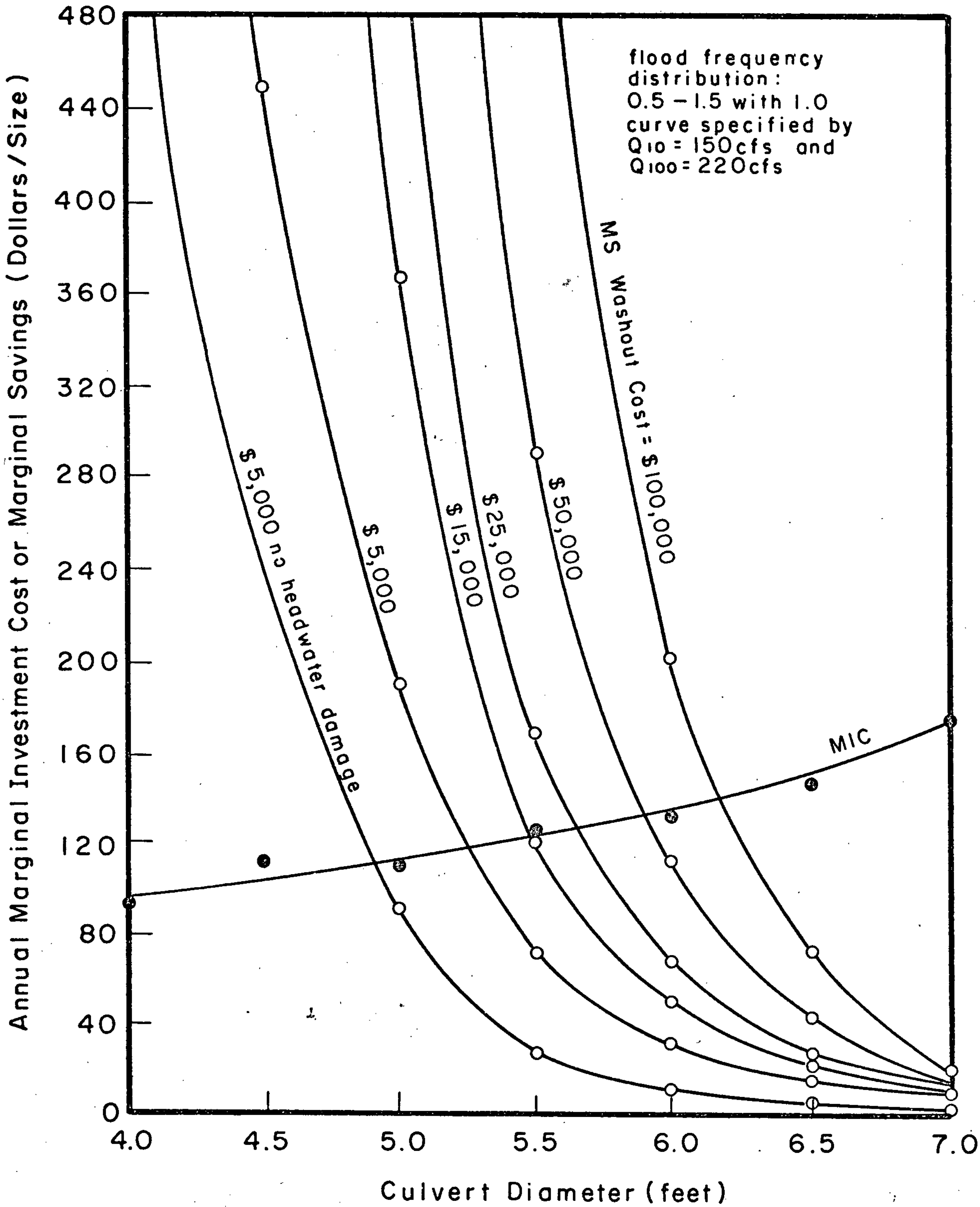


FIG. 6.5 THE EFFECT ON THE OPTIMAL DECISION OF CHANGING THE DAMAGE COSTS.

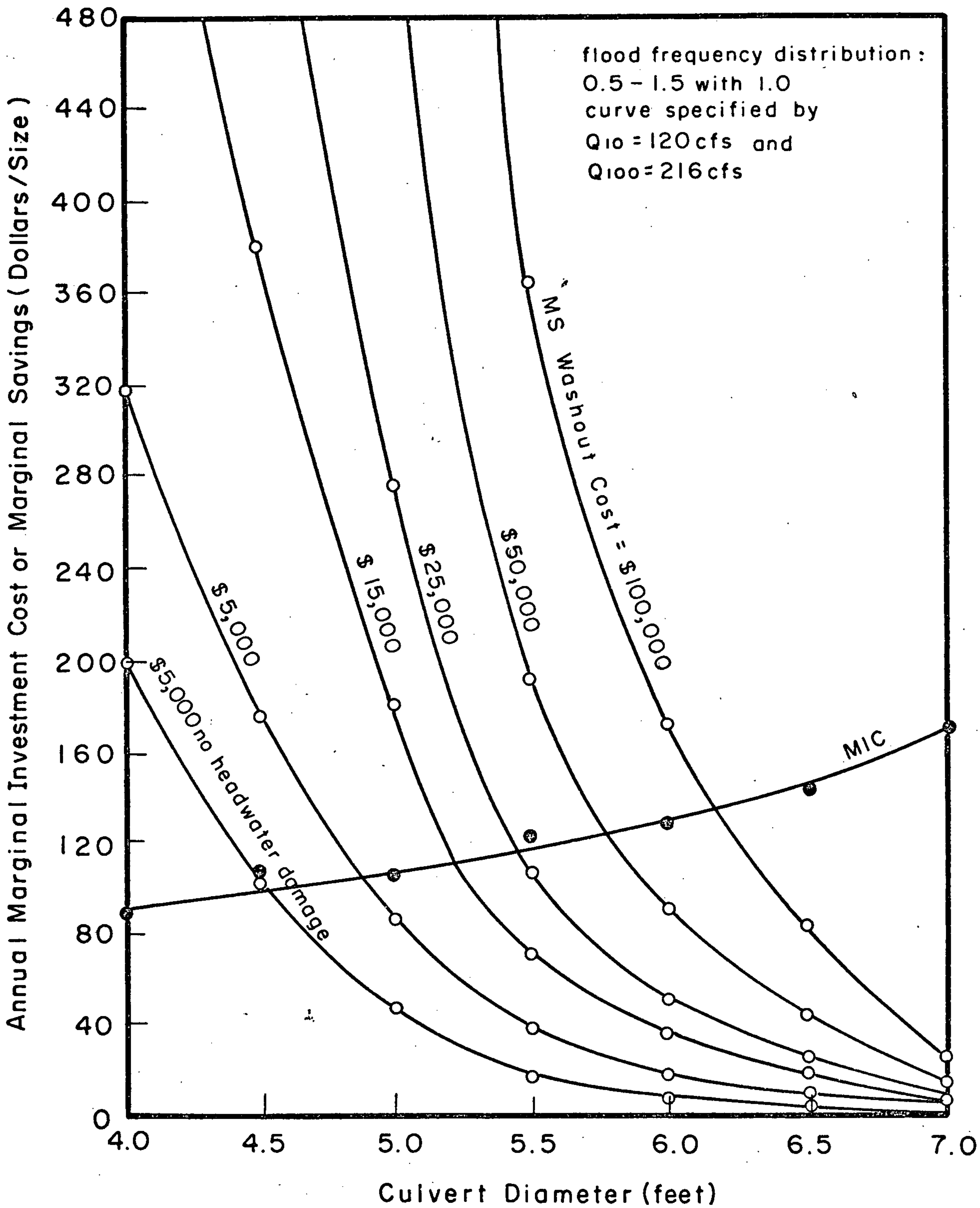


FIG. 6.6 THE EFFECT ON THE OPTIMAL DECISION OF CHANGING THE DAMAGE COSTS; NEW FLOOD FREQUENCY DISTRIBUTION.

6.5 and 6.6 by listing the optimal culvert diameter for each of the different damage costs in the two cases, along with the return periods for headwaters of 5.0 and 10.0 ft (the headwater at which headwater damage, if applicable, starts and the headwater causing washout) and the return period for the headwater depth equal to the diameter of culvert. Looking at the table, actually none of the optimum culverts meet the British Columbia Department of Highways' hydraulic design criteria (see introduction) since there is headwater damage at floods below the 100-year return period in all cases; however the 100-year flood headwater damage cost is very low in some cases.

Assuming there is no headwater damage (i.e., the only damage that can occur is a washout), in the case of the first flood frequency distribution, a 5.0 ft culvert is required to meet the British Columbia Department of Highways' hydraulic design criterion B, and a 5.5 ft culvert is required to meet criteria A and B. Thus the British Columbia Department of Highways would select a 5.5 ft diameter culvert, given that they use the derived single equivalent flood frequency curve. In the case of the second distribution, a 5.0 ft culvert would be chosen as it meets criteria A and B.

Table 6.5 shows the consequences of using the Highways Department's design criteria rather than the economic analysis method used in this thesis. No headwater damage is assumed in all cases. Substantial extra costs are incurred by using the Department of Highways' criteria if the washout cost is very low or very high. Low washout costs could reflect low volume rural highways while high washout costs can be incurred in cases where there is a substantial delay with moderate traffic volume or in cases where the traffic

TABLE 6.4

OPTIMUM CULVERT DIAMETERS AND RETURN PERIODS OF SIGNIFICANT HEADWATER LEVELS FOR DIFFERENT DAMAGE COSTS

I. Flood Frequency Distribution: 0.5-1.5 with 1.0 curve specified by Q_{10} = 150 cfs and Q_{100} = 220 cfs

Washout Cost ($)	Optimum Culvert Diameter (ft)	Return Period (yr)		
		HW ≥ 5.0 ft	HW ≥ 10.0 ft	HW ≥ D[1]
5000n[2]	4.5	3.0	45	2.1
5000	5.0	4.2	135	4.2
15000	5.0	4.2	135	4.2
25000	5.5	6.2	390	10.4
50000	5.5	6.2	390	10.4
100000	6.0	8.0	1300	31

II. Flood Frequency Distribution: 0.5-1.5 with 1.0 curve specified by Q_{10} = 120 cfs and Q_{100} = 216 cfs

Washout Cost ($)	Optimum Culvert Diameter (ft)	Return Period (yr)		
		HW ≥ 5.0 ft	HW ≥ 10.0 ft	HW ≥ D[1]
5000n[2]	4.0[3]	5.7	28	3.4
5000	4.5	7.8	64	5.7
15000	5.0	10.7	162	10.7
25000	5.0	10.7	162	10.7
50000	5.5	14.6	370	22
100000	6.0	18.1	900	52

[1] D = culvert diameter

[2] n = no headwater damage; otherwise standard headwater damage curve (Figure 5.2) is used

[3] The curves of Figure 6.6 indicate that the optimum culvert diameter is 4.5 ft, winning by a slight margin over the 4.0 ft culvert. But the MIC curve was drawn as a smooth curve which does not exactly pass through all the data points. Using the actual data points, the 4.0 ft culvert wins by a slight margin.

TABLE 6.5

COMPARISON OF ECONOMIC ANALYSIS WITH THE BRITISH COLUMBIA DEPARTMENT OF HIGHWAYS' DESIGN CRITERIA

I. Flood Frequency Distribution: 0.5-1.5 with 1.0 curve specified by Q_{10} = 150 cfs and Q_{100} = 220 cfs

Washout Cost[1] ($)	Optimum Culvert Diameter (ft)	Expected Total Annual Cost ($)	Expected Extra Annual Cost if Culvert Diameter Selected is 5.0 ft[2] ($)	Expected Extra Annual Cost if Culvert Diameter Selected is 5.5 ft[3] ($)
5000	4.5	778	19	117
15000	5.0	871	—	49
25000	5.0	945	—	1
50000	5.5	1009	121	—
100000	6.0	1088	412	48

II. Flood Frequency Distribution: 0.5-1.5 with 1.0 curve specified by Q_{10} = 120 cfs and Q_{100} = 216 cfs

Washout Cost[1] ($)	Optimum Culvert Diameter (ft)	Expected Total Annual Cost ($)	Expected Extra Annual Cost if Culvert Diameter Selected is 5.0 ft[3] ($)
5000	4.0	726	65
15000	5.0	853	—
25000	5.0	914	—
50000	5.5	1019	50
100000	6.0	1122	255

[1]no headwater damage assumed

[2]meets B.C. Dept. of Highways' criterion B only

[3]meets B.C. Dept. of Highways' criteria A and B (see Introduction for criteria)

volume alone is very high. The Highways Department's criteria are just not "right" for all roads under all conditions.

Figure 6.7 illustrates that the effect of uncertainty in changing the optimal decision is greater when the damage cost is greater, as the separation between the 1.0 and 0.5-1.5 MS curves increases with increasing damage cost. Consequently the value of better information is likely to be greater for high damage costs than for low damage costs.

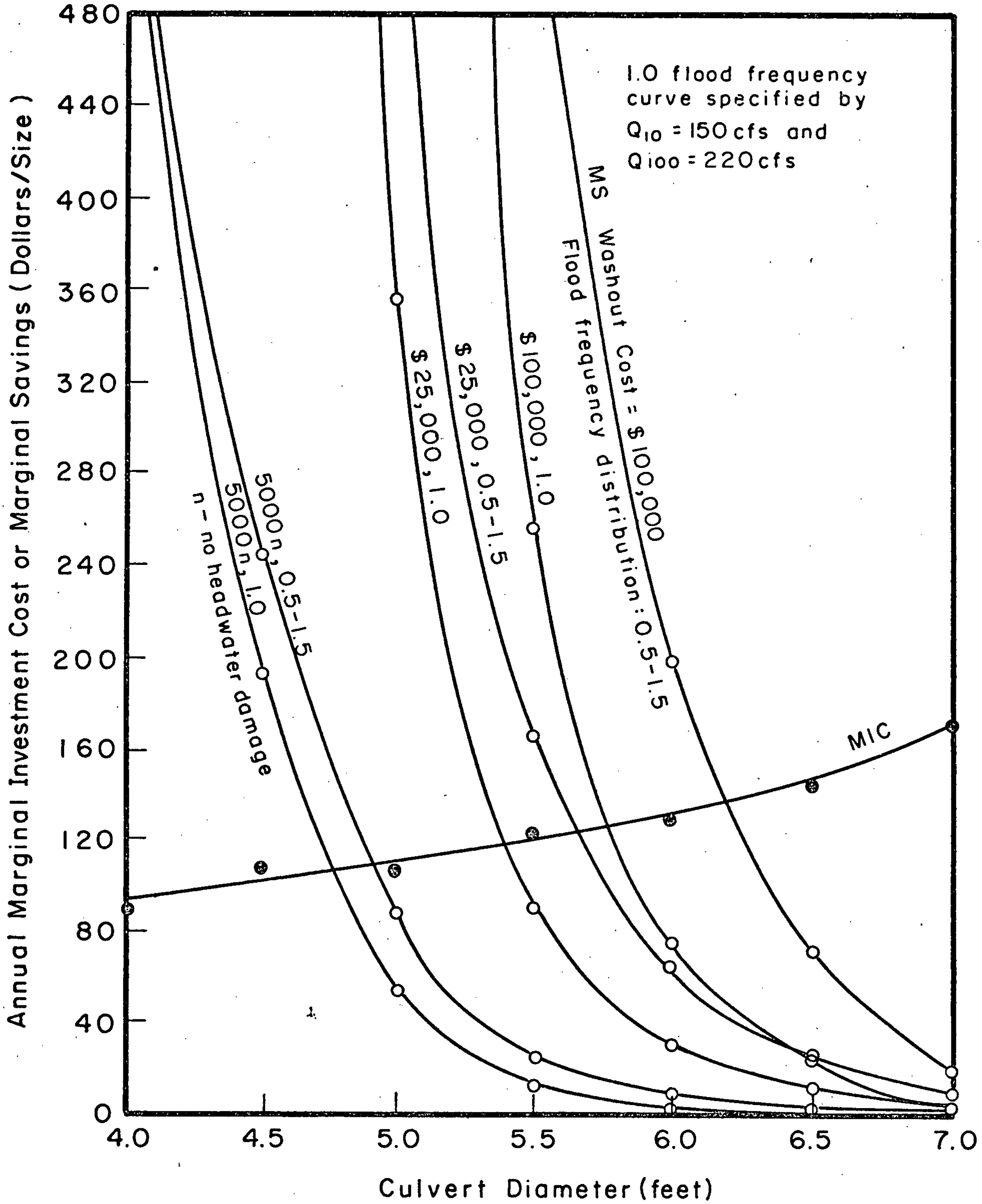


FIG. 6.7 THE EFFECT OF UNCERTAINTY IN CHANGING THE OPTIMAL DECISION AT DIFFERENT DAMAGE COSTS.

Chapter 7

CONCLUSION

This thesis has described a method of economic analysis to determine the optimum sized culvert for any culvert site. The method takes uncertainty into account and is capable of estimating the value of better information. Various aspects of the culvert selection problem: hydrologic, hydraulic, and economic were discussed, and the method was applied to a hypothetical culvert site, assuming different hydrologic and economic situations.

The potential advantages of employing economic analysis in culvert selection appear so great that one wonders why it has yet to be used. Linsley and Franzini (17) state, "The practical difficulty is that of estimating the probable damages from flows in excess of culvert capacity." This is very true, but research can solve the problem. It would not be difficult to conduct experiments to find out what causes a culvert to wash out. In addition to experiment, observations of culverts in the field operating during flood conditions and close inspections of culvert sites after washouts will lead to much improved damage cost estimates. Even if there is much uncertainty involved in estimating damage costs, this uncertainty could be accounted for in the economic analysis, and estimates of the value of better information in this area could be made.

Another argument that might be made is that the extra engineering cost involved in applying economic analysis to culvert selection will outweigh the savings from the program. This is very unlikely if all calculations are handled by computer. Although the initial cost of developing a good general program that is able to handle any situation may be high, it is bound to pay for itself in the long run. More input data is required for an economic analysis, but this data, for example, damage cost estimates, will be similar for many culvert sites. Initially the cost of obtaining data may be high, but it will decrease as a data bank is built up.

This thesis has considered only simplified, hypothetical cases, although several useful results were obtained. If further research is done, it would be worthwhile to consider real situations and to complicate the problem. The problems of debris clogging and estimating damage costs deserve more attention. Entrance improvement and different culvert materials and shapes should also be given consideration.

LIST OF REFERENCES

1. Nesbitt, M. C. (1963) Handbook of Culvert Hydraulics, Design, and Installation, B.C. Department of Highways, Materials Testing, Design, and Planning Branch, Victoria, B.C.

2. Blaisdell, F. W. (1966) "Hydraulic Efficiency in Culvert Design," Journal of the Highway Division, A.S.C.E., 92 (HW1): 11-22, Proc. Paper 4709.

3. Pritchett, Harold D. (1964) Application of the Principles of Engineering Economy to the Selection of Highway Culverts, Master's thesis, Stanford University, Department of Civil Eng.

4. Hershman, Stanley (1974) An Application of Decision Theory to Water Quality Management, Master's thesis, U.B.C., Department of Civil Eng.

5. Nyumbu, Inyambo L. (1976) The Effect of Uncertainty in Irrigation Development, Master's thesis, U.B.C., Department of Civil Eng.

6. Brox, Gunter H. (1976) Water Quality in the Lower Fraser River Basin: A Method to Estimate the Effect of Pollution on the Size of a Salmon Run, Master's thesis, U.B.C., Department of Civil Eng.

7. Hetherington, E. D. (1974) The 25-Year Storm and Culvert Size, Federal Department of the Environment, Canadian Forestry Service, Pacific Forest Research Centre, Victoria, B.C., Report BC-X-102.

8. Linsley, R. K., M. A. Kohler, and J. L. H. Paulhus (1958) Hydrology for Engineers, McGraw-Hill, New York.

9. Quick, M. C. and A. Pipes (1975) A Combined Snowmelt and Rainfall Runoff Model, unpublished leaflet, U.B.C., Department of Civil Eng.

10. --------. District of West Vancouver Drainage Survey, Dayton & Knight Ltd., Consulting Engineers, 1973.

11. Carter, R. W. (1957) Computation of Peak Discharge at Culverts, United States Geological Survey Circular 376.

12. Chow, V. T. (1959) Open-Channel Hydraulics, McGraw-Hill, New York.

13. Neill, C. R. (1962) Hydraulic Tests on Pipe Culverts, Research Council of Alberta, Alberta Highway Research Report 62-1.

14. Oglesby, C. H. and L. I. Hewes (1963) Highway Engineering, John Wiley & Sons, New York.

15. James, L. D. and R. R. Lee (1971) Economics of Water Resources Planning, McGraw-Hill, New York.

16. Samuelson, P. A. and A. Scott (1975) Economics, McGraw-Hill Ryerson Limited, Toronto.

17. Linsley, R. K. and J. B. Franzini (1972) Water-Resources Engineering, McGraw-Hill, New York.

APPENDIX

HEADWATER DEPTH CALCULATIONS

- see Reference 11 for additional information

Type 1 Flow: Critical Depth at Inlet

$$HW = (Q/cA_c)^2/(2g) + d_c - v_1^2/(2g) + h_{f_{1.2}}$$

where c is a function of (HW/D)

$v_1^2/(2g)$ and $h_{f_{1.2}}$ were assumed negligible.
The equation is solved by first calculating d_c ($Q/A_c = v_c$).

Type 5 Flow: Rapid Flow at Inlet

$$HW = (Q/cA_o)^2/(2g)$$

where c is a function of (HW/D)

Type 6 Flow: Full Flow Free Outfall

assuming $v_1^2/(2g)$ and $h_{f_{1.2}}$ are negligible

$$h_1 = (Q/cA_o)^2/(2g) + h_3 + h_{f_{2.3}}$$

where c is a constant for a particular inlet configuration

then $HW = h_1 - s_oL$

However, h_3 cannot be easily determined.

h_1 was in fact calculated from dimensionless ratio charts which are based on experiment, rather than from the above equation.

In addition to D, Q, and c; n, L, and s_o are required to calculate HW for type 6 flow.

Notation

Subscripts 1, 2, 3, and 4 denote location of section as shown in Figure 4.1.

A_o = area of culvert barrel

Notation (cont.)

A_c = area of flow at critical section

c = coefficient of discharge

D = culvert diameter (min. dia. for CMP)

d_c = critical depth

h = piezometric head above culvert invert at downstream end

h_f = head loss due to friction

HW = depth of water in headwater pool

L = length of culvert barrel

n = Manning's roughness coefficient

Q = discharge

s_o = bed slope of culvert barrel

v = velocity

v_c = critical velocity