EFFECTS OF STRESS PATH AND PRESTRAIN HISTORY ON THE UNDRAINED MONOTONIC
AND CYCLIC LOADING BEHAVIOUR OF SATURATED SAND

by

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ABSTRACT

Stress path and prestrain effects on the monotonic and cyclic undrained behaviour of a saturated sand are investigated in the triaxial test. It is shown that under identical consolidation stresses, the sand is contractive over a much larger range of relative densities in triaxial extension than in triaxial compression. The effective stress ratio at the initiation of contractive deformation (CSR) in extension is less than that in compression. During cyclic loading this feature makes the extension phase of the loading more damaging than the compression phase. The unique relationship between void ratio and undrained strength at PT state (which is similar to steady state line) noted in compression does not hold in extension. A separate relationship seems to emerge in extension for each initial void ratio and all such relationships lie to the left of the compression relationship, implying smaller PT state strengths in extension than in compression at equal void ratios.

A small prestrain history (stress state staying within CSR lines during prestraining) is shown not to alter the initially liquefiable character of sand. With no strain reversal on reloading, increasing prestrain strain level between CSR and PT lines makes the sand. Strain reversal on reloading, on the other hand, causes the sand to become more contractive with increasing prestrain history and may transform an initially dilative to contractive sand.
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NOTATIONS

CSR
critical effective stress ratio

D_r
relative density

D_{rc}
relative density after consolidation

D_{rcl}
relative density after reconsolidation

D_{ri}
initial relative density of specimen as prepared (under initial effective stress of 0.17 kgf/cm^2 (17 kPa))

D_{50}
effective grain size of soil sample; 50% by dry weight of sample is smaller than this grain size

e
void ratio
e_c
void ratio after consolidation
e_i
initial void ratio of specimen as prepared (under initial effective stress of 0.17 kgf/cm^2 (17 kPa))

ESP
effective stress path

K
bias relay constant

K_c
consolidation stress ratio = \sigma'_v/\sigma'_h or \sigma'_1/\sigma'_3

N
number of loading cycles

p'
= 1/2(\sigma'_v+\sigma'_h) or 1/2(\sigma'_1+\sigma'_3)

PT
phase transformation

q
= 1/2(\sigma_v-\sigma_h) or 1/2(\sigma_1-\sigma_3)

SSL
steady state line

S_{us}
undrained steady state strength

S_{pt}
undrained PT state strength

u
porewater pressure

u_b
back pressure

\Delta u
excess porewater pressure
\( \Delta u_a \) allowable \( \Delta u \) before ESP reaches CSR line
\( \varepsilon_{a,v} \) axial and volumetric strain
\( \phi' \) friction angle
\( \sigma_c \) total cell pressure
\( \sigma_c' \) effective confining pressure, \( \sigma_c - u_b \)
\( \sigma_d \) deviator stress
\( \sigma_{dcy} \) cyclic deviator stress
\( \sigma_{ds} \) static deviator stress
\( \sigma_h', \sigma_v' \) horizontal and vertical effective stresses
\( \sigma_m', \sigma_m' \) total and effective mean normal stresses
\( \sigma_{vc}' \) vertical effective consolidation stress
\( \sigma_l', \sigma_3' \) major and minor effective principal stresses
\( \sigma_{3c}' \) effective consolidation pressure in triaxial test
\( \tau_{cy} \) cyclic shear stress = \( \sigma_{dcy}/2 \)
\( \tau_s \) static shear stress = \( \sigma_{ds}/2 \)
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CHAPTER 1
INTRODUCTION

When saturated sand is subjected to rapid shearing it is generally considered undrained. Under such shearing the sand may develop large deformation, and for certain initial conditions may even flow like a frictional material. The undrained shearing could be due to cyclic earthquake loading or static loading which occurs over a very short period of time.

The concern for large deformation in saturated sand deposits associated with earthquake loading inspired this study of undrained behaviour in cyclic loading. The main emphasis here has been on the resistance of saturated sand to strain development little attention is paid as to the mechanism which is responsible for such strain development.

The development of large deformations under cyclic loading has been called liquefaction by many researchers (e.g., Finn et al., 1970; Ishihara, 1975; Lee and Seed, 1967; Seed and Lee, 1966). This is because these deformations are considered to develop when a condition of transient zero effective stress occurs in the sand at some stage of the undrained cyclic loading. Large deformation under undrained static loading which are associated with a soil mass losing a large percentage of its shearing resistance (strain softening response) has also been termed liquefaction (e.g. Castro, 1969; Casagrande, 1975; Castro, 1975).

It is now recognized that the large deformation could be the result of either a transient zero effective stress condition developed at a certain stage of the cyclic loading without any loss of shearing resistance, or a strain softening response developed under static or cyclic loading conditions (Castro, 1969; Seed, 1979). These two phenomena have been
termed cyclic mobility and liquefaction respectively.

Cyclic mobility is associated only with cyclic loading whereas liquefaction is associated with either cyclic or static loading (Castro, 1975; Casagrande, 1975; Seed, 1979). Various factors, most notably the level of static shear, affect liquefaction and cyclic mobility response differently (Castro, 1969; Castro and Poulos, 1975; Vaid and Chern, 1983). Thus a proper recognition of the mechanism of strain development (liquefaction or cyclic mobility) during cyclic loading is vital for a rational explanation of some of the conflicting ideas regarding undrained response of sand (Vaid and Chern, 1983).

Attempts have been made to predict the type of undrained monotonic and cyclic response of saturated sands from a knowledge of its initial state (void ratio, effecting confining pressure and static shear) and the superimposed shear loading (Castro et al, 1982 and Chern, 1985). The criteria for the occurrence of liquefaction in cyclic loading studies in the laboratory have been formulated on the tacit assumption that liquefaction will be triggered in the triaxial test on the compression side of loading. Similarly, the occurrence of liquefaction under monotonic loading has generally been confined to triaxial compression mode.

It has well been recognized that the undrained behaviour of saturated sand is a function of stress path (Bishop, 1971; El Sohby et al., 1973; Lade et al., 1976; Negussey, 1984), and triaxial compression is only one of many undrained loading path. For example, various soil elements along a potential slip surface through a slope are subjected to loading paths which approximate axial compression at the crest and axial extension at the toe. Thus, there is a continuous rotation of principal stresses until failure from zero at the crest to 90° at the toe. The possible effect of stress path on undrained response of sand has received little attention.

The first objective of this thesis is to investigate the effect of undrained stress path on liquefaction under monotonic loading using the
triaxial test. Since the triaxial test can simulate only two undrained stress paths (compression and extension), this study essentially involves investigation of differences in liquefaction under triaxial compression and extension conditions. If differences exist, their implication on the response of sand under cyclic loading with stress reversal is examined. Such loading invariably subjects the sand to both compressional and extensional modes of loading. Modifications to the criteria for liquefaction under cyclic loading will have to be made if compressional and extensional undrained response differ.

A sand deposit which has been shaken by a previous earthquake possesses a certain prestrain history. The effect of this prestrain history on the cyclic reloading resistance has been studied by some investigators (Finn et al, 1970; Bjerrum, 1973; Lee et al, 1973; Seed et al, 1977; Ishihara et al (1978 and 1982); and Suzuki et al, 1984). These studies have shown that small prestrain increases, whereas large prestrain decreases, the cyclic reloading resistance, despite small relative density increases associated with reconsolidation.

In most cases, however, there are no clear boundaries proposed between large and small prestraining. There is no information on whether prestrain was developed due to liquefaction or cyclic mobility. Furthermore, in many cases, the direction of reloading in relation to the sense of prestrain has not been explored. No study has been conducted to investigate the effect of prestrain on monotonic reloading. Whether or not contractive response will be eliminated by prestrain history has not been explored. Furthermore, the influence of consolidation stress ratio ($K_c$) on cyclic reloading resistance has not been clarified.

The second objective of this thesis is to conduct a systematic study on the effect of prestrain on a liquefiable sand under monotonic and cyclic reloading.
2.1 General Aspects of Undrained Behaviour of Sand

Undrained response of saturated sand is traditionally investigated separately under monotonic and cyclic loading conditions. Interest in monotonic loading has generally been related to undrained failure associated with flow slides. The characteristic feature of such behaviour is extremely large deformation under very small shear resistance. Interest in cyclic undrained loading behaviour has been related to the susceptibility of sand to accumulate undesirable deformation during earthquake shaking. Rapid increase in stresses which causes undrained loading condition could be a consequence of earthquake shaking, shock loading or even static loading.

2.1.1 Monotonic Loading Behaviour

The range of typical undrained triaxial compression behaviour of isotropically consolidated saturated sand under moderate confining pressure is shown in Fig. 2.1. The variation in stress-strain curves from type 1 to type 3 is associated with increasing relative density. Type 1 and 2 are strain softening types of stress-strain response - a behaviour associated with loss of shearing resistance after the occurrence of a peak. Sand showing such undrained behaviour is called contractive. In this thesis, the terms strain softening response and contractive response are used interchangeably. Type 1 response has been called liquefaction by Castro (1969), Casagrande (1975) and Seed (1979). It is a strain softening response with unlimited unidirectional strain and will be called herein
Fig. 2.1 Characteristic behaviour of saturated sand under undrained monotonic loading
true liquefaction. The characteristic feature of this type of response is continual deformation at constant void ratio, confining stress and shear resistance, called steady state. Such steady state response resembles flow of a fluid and is therefore also referred to as flow deformation (Poulos, 1971; Castro, 1975; Vaid and Chern, 1983). Type 2 response represents strain softening with limited unidirectional strain and has been called limited liquefaction (Castro, 1969).

The peak deviator stress in Fig. 2.1 indicates the initiation of strain softening behaviour. On the effective stress path (ESP), such as shown in Fig. 2.2, the effective stress ratio corresponding to the peak deviator stress has been called critical stress ratio (CSR) by Vaid and Chern (1983); Chern (1985). The arrow in Fig. 2.1 indicates the termination of strain softening response, i.e., the start of increase in shear resistance and decrease in pore pressure with further straining. The termination of strain softening is characterized by a sharp turnaround in the effective stress path (Fig. 2.2) previously identified as a phase transformation (PT) state by Ishihara et al. (1975). The PT state represents a discontinuity on the ESP. For a given sand the effective stress ratio at CSR and PT states were found to be constant and independent of void ratio and stress state prior to the commencement of undrained deformation (Chern, 1985). The ESP approaches the undrained failure envelope rather quickly with further straining after the PT state has been reached. For type 1 response, the terminal effective stress state stays on the PT line while steady state deformation continues indefinitely.

Type 3 response represents the strain hardening behaviour reflecting no loss of shear resistance. Sand showing such a behaviour is called dilative. The condition at the start of decrease in pore pressure after
Fig. 2.2 Undrained effective stress path for contractive and dilative response
reaching its maximum value has been called characteristic threshold (CT) by Luong (1980). CT occurs at the same effective stress ratio as the PT state for the strain softening type of response.

Predominant interest has so far been focused on the study of true liquefaction behaviour. Most of the understanding of this phenomena has come from monotonic loading of undrained conventional triaxial compression tests (Castro, 1969; Castro et al., 1982 and Casagrande, 1975). Such behaviour has been studied in relationship to the problem of flow slides. It has been shown by Castro (1969) that if sand undergoes true liquefaction, the effective confining stress and shear resistance during steady state deformation are uniquely related to void ratio, regardless of the initial consolidation stress conditions. This unique line within the three dimensional space of void ratio, shear and effective confining stress has been called the steady state line (Castro et al., 1982; Chern, 1985). Data from rounded to angular sands under a wide range of confining pressures and consolidation stress ratios have been presented in support of the steady state concept (Castro, 1969; Castro et al., 1982; Chern, 1985). Vaid and Chern (1983\(^1\),\(^2\)) have shown that the same concept also applies to limited liquefaction (type 2 response) when stress and void ratio considered are at the PT state. The corresponding line was referred to as the unique PT state line. A unique SSL is associated with true liquefaction (type 1 response) whereas a unique PT state line is associated with limited liquefaction. However, since these previous studies have been based largely on results derived from triaxial compression tests, the possible influence of alternative loading paths such as triaxial extension has not been explored.

Findings from a number of studies appear to indicate that the behaviour of sand subjected to triaxial extension loading paths could be very different from that under compression loading paths (Bishop, 1971; Miura
and Toki, 1982; Chang et al., 1982; Negussey, 1984; Chern, 1985). Miura and Toki (1982) showed that the degree of dilatancy in extension mode is very different from that under compression. For identical initial consolidation stress and void ratio states, the volumetric strain \( \varepsilon_v \) under drained loading in the extension mode was found to be much higher than that under the compression mode. These observations are especially significant in that for conventional triaxial tests, extension loading involves decreasing as opposed to increasing mean normal effective stress in compression loading. Similarly, very different effective stress paths were observed in undrained extension and compression tests. These differences were more profound for sands prepared by pluviation methods, and increased with decreasing density states. Fabric anisotropy inherent in loose pluviated specimens has been identified as the primary cause for the observed difference in their behaviour. An additional explanation based on the relative orientations of predominant slip planes in compression and extension modes has also been advanced to account for these phenomena.

Chang et al. (1982) showed that over a certain range of relative density, sand with identical initial consolidation state \( (e_c, \sigma_3^c, K_c) \) could be dilative (type 3 response) under compression loading path but contractive (type 1 or 2 response) under extension loading path in undrained condition. They also observed that the slopes of the PT state and failure line in extension were smaller than those in compression. The slope of the failure line in extension was shown to increase substantially with increasing relative density \( (D_r) \).

Negussey (1984) observed that for drained shearing under constant mean normal stress in the small strain range, the volumetric strain associated with extension loading paths is higher than for compression paths for
identical initial state. Volumetric strain occurring under drained loading would be a direct reflection of porewater pressure generation under undrained conditions. Consequently, the undrained response in compression and extension would be different. With a limited data support, Chern (1985) suggests that CSR in extension does not appear to be unique and may depend on the initial relative density of the sample prior to the application of consolidation stresses. The mobilized friction angle at CSR was found to be significantly smaller in extension than in compression. Furthermore, the extension side PT state line was not as well defined as the compression side one. While a unique compression side PT state line exists, the corresponding uniqueness of an extension side PT state line could not be established with certainty on the basis of limited data by Chern.

2.1.2 Cyclic Loading Behaviour

Initial interest in the undrained cyclic loading behaviour of sand was triggered by the extensive failures associated with saturated sand during the Niigata and Alaska earthquakes of 1964. Consideration centered predominantly on the response of saturated sand under level ground subjected to reversing shear stresses on horizontal planes (Seed and Lee, 1966). The stress conditions on such soil elements were simulated in the laboratory by undrained cyclic simple shear or cyclic triaxial tests on isotropically consolidated samples. The samples were subjected to constant amplitudes of cyclic shear stresses on horizontal plane in simple shear tests or constant pulsating deviator loads in the triaxial tests. Continued cyclic loading was found to result in the development of large strains and the sand was said to have liquefied. Cyclic shear or deviator stress amplitude which causes a specified level of strain in a fixed number
of stress cycles is called the resistance to liquefaction. Liquefaction in cyclic loading has thus been defined as a strain criterion with no attention paid to the mechanism of strain development.

Undrained cyclic loading of sand causes a progressive increase in pore water pressure and cyclic deformation in saturated sand with increasing number of cycles, regardless of its relative density. However, strain development may occur by two distinctly different mechanisms.

In the first type, at some stage during cyclic loading the sample undergoes unlimited deformation and thus true liquefaction. Castro (1969) has shown cases in which true liquefaction developed much in the same manner as that observed under monotonic loading (Fig. 2.3(a)). Unlimited deformation ensued once true liquefaction was triggered (Fig. 2.3(d)). Vaid and Chern (1983), and Chern (1985), however have shown cases of cyclic loading of sand wherein limited liquefaction developed in the same way as that observed in type 2 response under monotonic loading (Fig. 2.3(b)). It was shown that strain softening associated with limited liquefaction was also initiated at a constant critical value of effective stress ratio (CSR) regardless of the cyclic stress amplitude, void ratio or consolidation stress state of the sand. Following the arrest of limited liquefaction and the subsequent strain hardening until the peak amplitude of the imposed cyclic deviator stress on the compression side, large increase in pore pressure occurs during unloading and the effective stress state of the sample approaches zero but with very little change in deformation (Fig. 2.3(b,c)). Reloading in the extension region of the stress cycle causes the sample to undergo large deformation with the stress state moving along the undrained failure envelope. Subsequent unloading from the peak amplitude of deviator stress on the extension side once again brings
Fig. 2.3 Undrained cyclic loading behaviour of contractive sand — true liquefaction and limited liquefaction (After Chern, 1985)
the sand to a state of near zero effective stress with very little deformation. A further reloading into the compression region again causes the stress state to move along the failure envelope with development of large deformations. Repetitions of this loading and unloading process beyond a state of limited liquefaction result in a progressive increase of the cyclic deformation. This phenomenon of accumulating cyclic strain amplitude with cycles loading is known as cyclic mobility (Castro, 1969). The relationships between strain amplitude and number of loading cycles when liquefaction or limited liquefaction develops are shown schematically in Fig. 2.3(d).

In the second type of response, the sample develops a progressive increase in pore water pressure and cyclic deformation without strain softening. Vaid and Chern (1983) and Chern (1985) observed that such a sample undergoes very small deformation as long as its effective stress state remains below the phase transformation line (Fig. 2.4(a,b)). A significant amount of deformation was found to accumulate only when the stress state crossed the PT line during the loading phase. Unloading caused large increase in porewater pressure with little change in deformation. Further loading and unloading resulted in transient states of zero effective stress, and cyclic mobility as shown in Fig. 2.4(c).

In this thesis, the term liquefaction will be used only if sand deforms in a strain softening manner regardless of the nature of loading - monotonic or cyclic. This definition is consistent with that used by Castro (1969), except that it now encompasses limited liquefaction in addition to true liquefaction. The second type of response described above, in which the deformation developed during cyclic loading is not associated with strain softening during any stage of loading, will be
Fig. 2.4 Undrained cyclic loading behaviour of dilative sand - cyclic mobility (After Chern, 1985)
called cyclic mobility. This definition of cyclic mobility is after Castro (1969); Seed (1979).

Since a specified strain development during cyclic loading could be due to liquefaction, cyclic mobility or a combination of the two, the term "resistance to liquefaction" used to designate resistance to cyclic loading will herein be called resistance to strain development under cyclic loading. Since various factors affect liquefaction and cyclic mobility response differently (Castro, 1969; Castro et al., 1982; Vaid and Chern, 1983; Chern, 1985), it is imperative to know which phenomenon is responsible for strain development during cyclic loading. Only then a rational assessment can be made of the influence of various factors on the resistance to strain development.

2.1.3 Relationship Between Monotonic and Cyclic Loading

Castro (1969) and Castro et al. (1982) showed that the steady state line for true liquefaction is unique under monotonic and cyclic loading conditions. This implies that the undrained loading path (monotonic or cyclic) has no effect on the steady state line. Chern (1985) showed that the initiation of strain softening under monotonic and cyclic loading occurs at a unique value of effective stress ratio (CSR). Also the arrest of strain softening occurs at the unique ratio corresponding to the phase transformation line (PT). He presented a unified approach for determining the type of undrained response for both monotonic and cyclic loading conditions from a knowledge of the initial state \( (e_c, \sigma'_{3c}, K_c) \) of the sand and the superimposed shear loading.
2.2 **Effect of Prestrain History**

Prestrain history refers to any previous shearing (monotonic or cyclic). The previous shearing could be due to earthquake shaking, wave motion or other forms of static loading. In the laboratory prepared specimen, the prestrain history is generally imparted by monotonic or cyclic undrained loading which results in changes in both excess porewater pressure and strain magnitude. Depending on whether the area of interest is on the instant effect or delayed effect of prestraining, the sample could then be reloaded with or without reconsolidation to the original state of stress. In this study, all samples were reconsolidated before monotonic or cyclic reloading. It is thus a condition which represents a prestrain history event experienced by sand elements in the ground which might have been subjected to previous earthquake motions.

The effect of prestrain history on cyclic reloading behaviour of sand was first demonstrated by Finn, Bransby, and Pickering (1970). They showed, by means of triaxial and simple shear tests on conventional size samples of saturated sand, that the resistance to cyclic reloading was influenced by strain history to which the samples had previously been subjected. The cyclic reloading resistance decreased with large prestrain history ($\varepsilon_a > 5.0\%$) despite the increase in relative density of 10 to 15% after reconsolidation, whereas the resistance increased significantly with small prestrain history ($\varepsilon_a < 0.3\%$) with only 1.5 to 2.0% increase in relative density on reconsolidation. The mechanism of large prestrain development, liquefaction or cyclic mobility, is not clarified in their study. Similar results on the effect of small prestrain history have subsequently been presented by Bjerrum (1973), Lee and Focht (1973), and Seed, Mori and Chan (1977). The works of Bjerrum and Lee et al. were carried out on
undisturbed samples in the triaxial apparatus while those of Seed et al. were done in shaking table studies on large scale samples. Within the range of small prestrain, cyclic reloading resistance increases with increase in prestrain magnitude (Bjerrum, 1973; and Lee et al. 1973) Available results to date indicate that small prestrain increases while large prestrain reduces cyclic reloading resistance. However, no guidelines have been proposed as to what constitutes small prestrain and what constitutes large prestrain. The strain development mechanism causing reduced cyclic reloading resistance for samples with large prestrain history have not been identified. Furthermore, all prestrain histories were imparted using undrained cyclic loading, and on subsequent reconsolidation, specimens were reloaded under cyclic loading only.

Ishihara and Okada (1978) used the PT state to define the boundary between small and large prestrain. The sand was considered to possess a small prestrain history if its stress state remained below the PT line at all times and large prestrain if it was stressed beyond that line. The state of prestrain was therefore determined solely on the basis of stress rather than strain state. The prestrain history was developed by cyclic loading. Reconsolidated samples were then reloaded with one load cycle and the residual porewater pressure was determined at the end of each half cycle. They showed that the resistance to cyclic reloading was higher for prestrained as opposed to virgin samples. This was because of the reduced porewater pressure changes that occurred during cyclic reloading for samples with small prestrain. On the other hand, for a sample with large prestrain, porewater pressure changes on reloading were found to be a function of the directions of loading with respect to the direction of prestrain. Smaller porewater pressure is generated if the direction of
prestrain is the same as the reloading but the reverse is true if the
direction of prestrain is opposite to that of reloading. Thus, they
recognized the importance of the direction of prestrain in relation to the
direction of reloading. Subsequently, Ishihara and Okada (1982), showed
that the reloading behaviour was controlled by the magnitude and direction
of the large prestrain history to which the sample had been sheared under
low $\sigma_3'$ at the end of first loading. They suggested that complete
rearrangement of sand particles took place upon shearing under low $\sigma_3'$. The
concept of stress induced cross anisotropy was used to interpret the
observations.

Suzuki and Toki (1984) argued that the use of PT state to differenti­
te between small and large prestrain and thus increase or decrease in
cyclic reloading resistance was too crude. In their study, the prestrain
history was imparted by monotonic or cyclic loading and reconsolidated
samples were loaded by cyclic loading. They concluded that the resistance
to cyclic reloading became less than that of the virgin sample before the
previous prestrain history brought the specimen to the PT state. The
importance of the direction of loading was further clarified by looking at
porewater pressure generation during the compression and extension phase
separately in the first reloading cycle. In this regard, their observa­
tions were similar to those of Ishihara et al. Both Ishihara et al. and
Suzuki et al., however, confined their study to dilative sand only.

Previous investigations have not paid close attention to the process
of prestrain accumulation and as to whether the prestrain was a result of
liquefaction or cyclic mobility. Furthermore, the effect of prestrain on
contractive sand has not been investigated. Whether or not contractive
sand will remain contractive or susceptible to liquefaction after prestrain
history is of concern to geotechnical engineers. The mechanism of strain development resulting in reduced cyclic reloading resistance for sand with large prestrain history has not been investigated. Almost invariably, the previous investigations were conducted on isotropically consolidated sand where complete stress reversal is automatic upon cyclic reloading. The effect of prestrain on anisotropically consolidated sand has not been explored.
CHAPTER 3
EXPERIMENTATION

3.1 Material Tested

Tests were conducted on Ottawa sand, a natural sand processed by Ottawa Silica Company, Ottawa, Illinois. It is a uniform, medium sand with rounded particles which meets the ASTM designation C-109. The mineral composition is primarily quartz. The grain size distribution of the sand is shown in Fig. 3.1. The sand has particle sizes between 0.15 and 0.59 mm and $D_{50}=0.40$. The maximum and minimum void ratio in accordance with standard test method ASTM D2049, are 0.82 and 0.50 respectively. The undrained response of this sand under triaxial compression loading path has been studied extensively by Chern (1981, 1985) to which the undrained response in extension will be compared.

3.2 Testing Apparatus

All tests were conducted using the triaxial apparatus. A schematic layout of the testing apparatus is shown in Fig. 3.2. The testing system consists of a triaxial cell and a loading system. The loading system is capable of monotonic and cyclic loading under stress or strain controlled conditions, as well as monotonic consolidation under anisotropic stress conditions. The meaning of strain controlled cyclic loading in this study refers to constant strain-rate application of equal stress amplitude and does not refer to equal strain amplitude cyclic loading. The loading system could be regarded as consisting of two separate parts corresponding to the two different stages of testing. The consolidation system serves to consolidate the specimens and the axial loading system enables monotonic or
Fig. 3.1
Grain size distribution curve of Ottawa sand C-109
Fig. 3.2  Schematic layout of the testing apparatus and loading system
cyclic loading of the specimen during the shearing stage. A double acting water piston was used for smooth transfer from stress controlled anisotropic consolidation to subsequent strain controlled axial loading. The loading system can be changed from stress to strain or strain to stress controlled without changing stress conditions on the sample. The strain controlled system is essential to study the effect of prestraining for contractive samples. Only through this system can the desirable amounts of prestrain be applied past the peak deviator stress. Strain controlled loading has also been adopted to investigate the monotonic and cyclic undrained behaviour of sand. The undrained behaviour of sand has been shown to be unaffected by the mode of stress or strain controlled loading (Chang et al., 1982; Castro et al., 1982; Chern, 1985). Strain controlled testing also provides a better definition of steady and PT state strength. Under stress controlled loading, contractive sand deforms to steady state in a fraction of a second, thus making data acquisition a comparatively difficult task.

Before loading, the double acting water piston was first pressurized to an identical pressure in both the top and bottom chambers. If a stress controlled condition was required, regulated pressure was supplied to the water piston through reservoirs with valves A and B open. On the other hand, if a strain controlled condition was required, e.g., axial compression, the top side of the water piston was isolated from the air supply system by closing valve B. A constant rate of strain was then produced by feeding water to the piston at a constant rate from the water cylinder. To minimize compliance, complete saturation of the water piston and water cylinder was essential. These components were therefore assembled under water to facilitate full saturation.
Consolidation System

The consolidation system is capable of monotonic consolidation under anisotropic stress condition. Such loading is essential for simulating field stress conditions below sloping ground. The system has been described in full detail by Chern (1985).

This newly designed system enables the soil sample to be consolidated isotropically or anisotropically along any constant effective stress ratio path with proper adjustment of constants on the consolidation system (bias and ratio relays). Identical regulated pressure was supplied to the cell and bias and ratio relays which fed the top of the water piston. By increasing the supply pressure, any desired \( \sigma_3' \) could be obtained following a constant \( K_c \) stress path. After consolidation was completed, the pressure in the top chamber of the water piston was transferred to the axial loading system. With valves B and E closed, a pressure equalled to that in consolidation system was applied using the DC offset on the function generator with valve F opened. A smooth transfer was then made by opening valve B.

Axial Loading System

Both monotonic or cyclic loading under stress or strain control can be applied to the specimen with suitable valve close and open combinations. Strain controlled monotonic compression, as mentioned before, was achieved with valve B and C closed and feeding water to the top chamber of the piston. Similarly, axial extension loading was achieved with valve A and D closed and feeding water to the bottom chamber of the piston. The rate of strain for monotonic loading adopted in this investigation was 0.5% per minute.

Cyclic loading under stress controlled mode was applied by means of an electro-pneumatic (E/P) transducer driven by a function generator. A
sinusoidal wave form was used for cyclic loading with a low frequency of 0.1 Hz. An adjustable ratio relay between the water piston and E/P transducer was required because of the limited output pressure capacity of the transducer (21 to 245 kPa). The output pressure from the E/P transducer was magnified to the desired value by the ratio relay.

Strain controlled cyclic loading was done by feeding water in and out of the water cylinder to either the top or bottom chamber of the pressurized water piston. With valves C and B closed, water fed in and out of top chamber developed loading pulses. The direction of motion of the motor generator was controlled manually to obtain equal stress amplitude pulses under strain control. In this manner, very uniform pulses could be obtained. If flow deformation was triggered in compression, water was driven out from the water cylinder to any prescribed strain level under strain-rate controlled mode and if flow deformation was triggered in extension water was driven back to the water cylinder. Any prestrain magnitude past the peak deviator stress may be obtained in this manner.

3.3 Sample Preparation and Set-Up

The triaxial samples were 127 mm high and 63.5 mm diameter. Saturated test samples were formed by pluviating boiled sand in de-aired water which filled the sample cavity formed by a membrane lined split mold. While depositing sand, the tip of the pouring nozzle was always kept barely submerged in water. The pouring tip was traversed laterally during deposition in order to keep the sedimented sand surface approximately level at all times. All samples were formed loose in this manner. Higher initial densities, if required, were achieved by tappings on the base of the triaxial cell with a soft hammer while maintaining a gentle pressure on
the loading cap. Following sealing of the membrane with the top cap, a vacuum of 17 kPa was applied to the drainage line in order to provide a small confinement to the sample prior to dismantling the split mold. The initial relative density \(D_{r1}\) of the sample refers to this effective confining pressure \(\sigma'_{3c}\). The lowest \(D_{r1}\), prepared by this technique was approximately 30%. The detailed sample preparation technique has been described by Negussey (1984) and Chern (1985). This technique is believed to yield samples of uniform density throughout and virtually saturated with B value greater than 0.98 (Vaid and Finn, 1979).

The triaxial cell was assembled and the drainage line was connected to the volume change and porewater pressure measuring devices after centering on the loading platform. After the sample loading ram was connected to the loading piston rod, consolidation of sample was carried out. Anisotropic consolidation, if required, was achieved by raising the cell pressure and axial load simultaneously in a preset ratio using the consolidation system of the test apparatus. During the process of consolidation, the volume change, axial deformation and axial load were monitored at discrete levels of confining pressure on a strip chart recorder. When the consolidation pressure reached the desired value, the drainage line was kept open for a short period of time (about 5 minutes) until secondary consolidation, if any, was completed. Then the drainage line was closed and the sample was ready for undrained loading. During undrained shear loading, axial load, porewater pressure, cell pressure and axial deformation were monitored continuously and recorded on a strip chart recorder. The cyclic loading was applied at a frequency of 0.1 Hz.
3.4 Test Program

Monotonic and Cyclic Loading Tests

Four types of triaxial tests were performed and are listed as follows using the abbreviation adopted by Chern (1985).

1. IC-U - Isotropically Consolidated Undrained Monotonic Loading Tests.
2. AC-U - Anisotropically Consolidated Undrained Monotonic Loading Tests.
3. IC-U_{cy} - Isotropically Consolidated Undrained Cyclic Loading Tests.
4. AC-U_{cy} - Anisotropically Consolidated Undrained Cyclic Loading Tests.

Most monotonic loading tests were conducted in the extension mode with only a few in the compression mode. The extension loading was applied by reducing the axial stress while holding the cell pressure constant (stress path involving decreasing total mean normal stress \(\sigma_m\)).

Tests were conducted on samples with a wide range of relative density \(D_{rc} \text{= 35\% to 65\%}\), consolidation pressure \(\sigma_c' \text{= 2.0 to 25 kgf/cm}^2 \text{ (200 to 2500 kPa)}\) and anisotropic consolidation stress ratio \(K_c \text{ (1.0 to 2.0)}\).

Cyclic loading tests were conducted only on contractive samples with relative density \(D_{rc} \text{= 36.0 \pm 1.0\%}\), consolidation pressure \(\text{2.0 to 4.0 kgf/cm}^2 \text{ (200 to 400 kPa)}\) and anisotropic consolidation stress ratio \(K_c \text{ (1.0 to 1.25)}\).

Tests to Study Effect of Prestrain History

The initial state of the specimens was \(D_{rc} \text{= 36.0 \pm 1.0\%}, \sigma_c' \text{= 2.0 to 4.0 kgf/cm}^2 \text{ (200 to 400 kPa)}\) and \(K_c \text{= 1.0 to 1.25}, \) which will exhibit contractive response. The prestrain history was applied using cyclic loading \(\tau_{cy}/\sigma_{3c} \text{= 0.10 to 0.125 in order to represent previous seismic}\)
activities. In a few cases the strain history was applied by monotonic loading in order to investigate possible differences between the type of strain history. The magnitude of prestrain history ranged from a low of 0.1% to a high of 4.75% axial strain in compression or extension mode. The samples were then reconsolidated to the original state of effective stresses. Undrained monotonic (compression or extension) reloading or cyclic reloading was then performed to investigate the effects of prestrain history. More specific detail of the initial stress condition of virgin sample, the imposed prestrain history and mode of reloading will be described individually during the discussion of respective test results.
CHAPTER 4
EFFECTS OF STRESS PATH ON THE UNDRAINED MONOTONIC AND CYCLIC LOADING BEHAVIOUR OF SATURATED SAND

In this chapter, the undrained monotonic loading behaviour of isotropically and anisotropically sand will be discussed first followed by investigation of cyclic loading behaviour. Under conventional triaxial compression, \( \sigma_1 \) is the vertical principal stress (\( \sigma_v' \)) and \( \sigma_3 \) the horizontal principal stress (\( \sigma_H' \)). However, under triaxial extension loading, the vertical stress (\( \sigma_v' \)) is the minor principal stress while the horizontal stress (\( \sigma_H' \)) is the major principal stress. In this thesis, the data are presented in terms of \( \sigma_v' \) and \( \sigma_H' \) instead of \( \sigma_1 \) and \( \sigma_3 \). The deviator stress is defined as \( (\sigma_v' - \sigma_H') \). This enables a proper distinction to be made between compression and extension mode of loading. A positive \( (\sigma_v' - \sigma_H') \) represents compression loading whereas negative \( (\sigma_v' - \sigma_H') \) implies extension loading.

4.1 Monotonic Loading Behaviour

Monotonic extension tests were carried out on both isotropically and anisotropically consolidated samples. Anisotropic consolidation was performed with \( K_c \) values in compression, since this represents the in-situ stress state of soil elements. The initial state of the specimens tested covered a wide range of relative densities (\( D_{rc} = 35 \text{ to } 65\% \)), confining pressure (2.0 to 25.0 kgf/cm\(^2\) (200 to 2500 kPa)) and \( K_c \) values (1.0 to 2.0). Extension loading was applied by decreasing the axial stress \( \sigma_v \) while the radial stress \( \sigma_H \) was held constant. The behaviour of isotropically consolidated sand is considered first. The behaviour of anisotropically consolidated sand with compression \( K_c \) values when loaded in extension which involves principal stress rotation will be examined later in Section 4.1.2.
4.1.1 *Isotropically Consolidated Samples*

Typical undrained monotonic extension loading results for three $D_{rc}$ (35.8, 46.5 and 62.5%) at identical $\sigma'_3c$ of 2.0 kgf/cm$^2$ (200 kPa) are shown in Fig. 4.1(a, b, and c) respectively. Stress-strain relationships, excess pore water pressure developed and the effective stress paths in $q-p'$ stress space where $p' = \frac{\sigma'_v+\sigma'_H}{2}$ and $q = \frac{\sigma'_v-\sigma'_H}{2}$ are illustrated. These will be discussed under separate headings.

**Stress-Strain Relationship**

It may be noted that all samples ranging in density ($D_{rc}$) from 35.8 to 62.5% were contractive. As would be expected, the degree of strain softening (percent reduction in peak deviator stress) and the associated strain until PT state, reduces with increasing density. For samples with $D_{rc} = 35.8\%$, the minimum density achieved by the adopted pluviation method of sample preparation, the stress-strain relationship (Fig. 4.1a) is very similar to the unlimited flow deformation or steady state deformation (type 1 response) observed by Castro (1969) and Castro et al. (1982). With increasing $D_{rc}$, the response was typical of the limited flow type (type 2 response). A test done at $D_{rc} = 65\%$ showed very little contractive response. The contractive response would thus cease at $D_{rc}$ somewhat greater than 65%. In contrast, for $D_{rc}$ greater than approximately 43%, no contractive response was observed for compression loading (Chern, 1985). Contractive response in extension loading is therefore much more widespread. This is perhaps due to the inherent anisotropy of samples prepared by pluviation methods (Oda, 1972, 1976; Arthur et al., 1972).

For contractive response, the peak deviator stress was reached at a very low strain level, approximately 0.25%, which was significantly lower than 0.6% for compression. Moreover, the magnitude of the peak deviator
Fig. 4.1(a) Typical triaxial extension undrained behaviour of Ottawa sand with $D_{ri} = 32.5\%$
Fig. 4.1(b) Typical triaxial extension undrained behaviour of Ottawa sand with $D_{r1} = 43.5\%$
Test No: 27

$D_{rf} = 60 \%$, $D_{rc} = 62.5 \%$

$\sigma_{3c}' = 2.0 \text{ kgf/cm}^2$

Fig. 4.1(c) Typical triaxial extension undrained behaviour of Ottawa sand with $D_{rf} = 60\%$
stress was also significantly lower for identical end of consolidation conditions. Therefore, not only liquefaction will occur over a wider range of density but also trigger at a lower strain and shear stress level in extension than in compression.

Excess Porewater Pressure, $\Delta u$

The peak excess porewater pressure decreases with increasing $D_{rc}$ (Fig. 4.1). For $D_{rc} = 35.8\%$, $\Delta u$ reached a terminal value and had no tendency to decrease till the end of the test at 8 to 9\% strain. For a higher density, $\Delta u$ showed a peak which corresponded to the minimum deviator stress and started to decrease thereafter. However, it is more important to point out that for identical initial state ($D_{rc}$, $\sigma'_{3c}$, and $K_c$), $\Delta u$ at steady state was much higher in the extension mode than in the compression mode, even though extension loading involved decreasing as opposed to increasing the total mean normal stress ($\sigma_m$). This behaviour is consistent with the much higher volumetric strain observed in extension as opposed to compression in drained testing by Miura et al. (1982) and Negussey (1984).

Another interesting observation was that almost invariably, the excess porewater pressure was slightly negative at low axial strain levels, less than 0.1\%. Unfortunately, this observation is not clear from Fig. 4.1 because of the scale used. The magnitude of the negative $\Delta u$ increased with increasing relative density. This observation is more obvious for tests with higher confining pressure which will be shown later. The negative $\Delta u$ developed at low strain level is in line with the drained behaviour of Ottawa sand in the range of small strain observed by Negussey (1984). He showed that under an extension loading path with constant shear stress and decreasing $\sigma_m$, dilation was observed prior to contraction. The volumetric
swelling at low strain level can be identified very clearly from his results. The development of porewater pressure under undrained condition is a function of both changes in mean normal and shear stresses. The swelling tendency is due to a reduction in $\sigma_m$ at the start of the extension test which generated negative porewater pressure. The subsequent shear induced tendency for volumetric contraction caused the porewater pressure to increase under undrained conditions. In other words, the reduction in $\sigma_m$ component outweighs the shear induced volumetric contraction tendency at extremely low strain level and vice-versa at higher strain level.

**Effective Stress Path**

Because extension loading was carried out by reducing the vertical stress, the total stress path is inclined at minus 135° to the horizontal. For $D_{rc} = 35.8\%$ Fig. 4.1(a), the large $\Delta u$ pushed the effective stress path (ESP) close to the origin of stresses. The path ended at a point with no tendency to turn around as would be the case with development of true liquefaction. The turn around point (PT state) was more obvious for higher relative densities as shown in Fig. 4.1(b,c). Following the PT state, the ESP followed the failure line until the specimens necked at about 8-9% strain.

Typical undrained monotonic loading results at higher effective confining pressure are shown in Fig. 4.1(d). The initial state of the sample was $D_{rc} = 51.9\%$ and $\sigma_{3c}' = 20 \text{ kgf/cm}^2 (2000 \text{ kPa})$. Similar pattern of behaviour as noted for lower $\sigma_{3c}'$ was observed, i.e., contractive. As mentioned previously, the negative $\Delta u$ at low strain level was more obvious and persisted to a comparatively higher strain level (compare Fig. 4.1b and d).
Test no: 93
\( D_{r1} = 43.5\% \), \( D_{rc} = 52.9\% \)
\( \sigma_3^c = 20 \text{ kgf/cm}^2 \)

\[ P' = \frac{1}{2}(\sigma_H^c + \sigma_V^c) \text{ (kgf/cm}^2) \]

Fig. 4.1(d) Typical triaxial extension undrained behaviour of Ottawa sand with \( D_{r1} = 43.5\% \) under high confining pressure
Critical Stress Ratio (CSR)

Fig. 4.2 shows the critical stress ratio (CSR) states (peaks of deviator stress) for monotonic extension loading. The unique CSR line in compression observed by Chern (1985) is also included for comparison. Chern (1985) showed the CSR line in compression is independent of $\sigma_{3c}$, $K_c$ and $D_{rc}$ for contractive Ottawa sand. Similar triaxial compression tests were conducted and very good agreement was obtained with unique CSR line obtained by Chern. It may be seen in Fig. 4.2 that the CSR states in extension show a comparatively wider scatter and do not conform to a unique value. The mobilized $\phi$-angle ranges from 13° to 18° and increases with increasing initial relative density compared to 23.0° under triaxial compression. Thus the CSR line is not unique for extension loading.

However, it must be noted that CSR state in extension exists for sand over a significant range of density ($D_{ri} = 30$ to 63.5%) over which contractive response prevails. In contrast, the range of $D_{ri}$ over which contractive deformation occurs under compression loading is merely 30 to 38%. The lowest $D_{ri}$ prepared by pluviation technique was approximately 30%. Clearly contractive behaviour would occur for $D_{ri} < 30\%$. The wider range in $D_r$ may account for the observed non-uniqueness of CSR. However, this conclusion is not supported by the compression test results on an angular tailings sand (Chern, 1985), for which the compression CSR is unique over a wide range of initial density. For tailings, however, the densification was a consequence of consolidation, whereas, for Ottawa sand in this study, it was a consequence of vibration during sample preparation.

As indicated before, the slope of extension CSR line increases with increasing density (Fig. 4.2) and a careful study of the data indicates that the CSR appears to be a function of $D_{ri}$. The loci of CSR states for
Fig. 4.2  Effective stress conditions at the initiation of strain softening response (CSR) and start of dilation (PT state) in undrained compression and extension.
$D_{ri}$ of 32.5 ± 0.5% and 43.5±0.5% for sand over a range of $\sigma^1_{3c}$ (2.0 to 25.0 kgf/cm$^2$ (200 to 2500 kPa)) are shown in Fig. 4.2. Unique CSR line may be noted for specimens with the same $D_{ri}$. The corresponding $\phi$-angles for $D_{ri} = 32.5\%$ and 43.5% are, 13.5° and 16.0° respectively.

It is also important to point out that the slope of the CSR line in extension is much flatter than in compression. A lower CSR implies liquefaction will be triggered at a lower stress ratio. A difference in mobilized $\phi$-angle at CSR of close to 8.5° was observed between compression and extension. In fact, at a loose density $D_{ri} = 32.5\%$, the difference is as high as 9.5°. The difference in the slope of the CSR line may be due to inherent anisotropy in pluviated sand specimens. Arthur et al. (1972), Arulandan et al. (1984) and Oda (1972,1976) have shown that sand prepared by sedimentation methods possessed inherent anisotropy, which imparted to the sand a more stable structure in the direction of sedimentation, i.e., in the vertical or compression direction. Oda, observed that the number of points of individual grain contact is predominantly in the direction of deposition. Symes (1983) has also pointed out that the deformation characteristic of granular soil under drained conditions is a function of particle orientation and contact distribution or sand fabric. This deformation characteristic would be reflected in porewater pressure development under undrained condition.

For specimens prepared by the same technique, qualitatively, a certain $D_{ri}$ should correspond to a certain initial sand fabric. A certain degree of anisotropy reflects a certain sand fabric. Densification increases $D_{ri}$ and alters the initial sand fabric. El-Sohby et al. (1973), Negussey (1984) showed that proportional loading will not change the degree of anisotropy whereas vibratory densification does. They showed that the
degree of inherent anisotropy of a pluviated specimen decreases with increasing vibratory densification; i.e., it becomes more isotropic at a denser state. Since initial sand fabric is a function of $D_{ri}$, it can be argued that CSR is a function of the initial sand fabric. The highest degree of inherent anisotropy reflects the lowest CSR in extension and the CSR increases with decreasing inherent anisotropy (Fig. 4.2). This conclusion from extension loading can not however be extended to compression loading where a unique CSR line is observed. As mentioned before, relatively small amount of densification during sample preparation will cause the contractive behaviour to disappear in compression loading. Nevertheless, for the range of $D_{ri}$ (30 to 38%) where contractive response is observed in compression, the CSR line is essentially constant but not so in extension. The reason may be that the volumetric strain due to shear in compression loading mode is far less than in extension mode as observed by Negussey (1984). As a consequence, under undrained condition, the CSR is more stable in compression and less stable in extension. Indeed, CSR does not appear to be a unique parameter for a particular type of sand. Only a relatively stable nature of pluviated specimen under compression loading in triaxial condition may cause the CSR in compression to become not a function of initial sand fabric.

**Phase Transformation (PT) State**

Fig. 4.2 also shows the effective stress conditions at PT state in extension and compression. It may be noted that PT states in extension lie on a unique straight line regardless of $D_{ri}$ or initial sand fabric and consolidation stress level. Since the PT state is associated with large deformations, much of the initial fabric anisotropy will be destroyed dur-
ing the shearing process, resulting in a unique PT line in extension. Similarly the PT states in compression may be observed to lie on a unique line. The mobilized $\varphi$-angle corresponding to the PT state is approximately 29.5° for both compression and extension loading paths.

The PT state represents the minimum value of deviator stress after flow deformation of the limited liquefaction type. The value of $q$ at PT will be termed undrained PT state strength ($S_{pt}$).

The $\varphi$-angle for the undrained failure line is approximately 31.5° for low to medium high confining pressure 2 to 10 kgf/cm$^2$ (200 to 1,000 kPa), and is essentially identical for both extension and compression paths. Since the constant volume $\varphi$-angle is presumed to be a material property, it should not be affected by different stress paths.

PT State Line

Fig. 4.3 and 4.4 show respectively the relationships between void ratio after consolidation ($e_c$) and $S_{pt}$, and $e_c$ and $\sigma'_3$ at PT state. Since the effective stress conditions at PT state have been shown to lie on the constant stress ratio PT line through the origin, $S_{pt}$ and $\sigma'_3$ are uniquely related by

$$S_{pt} = \frac{\sigma'_3 \sin\varphi_{ss}}{1-\sin\varphi_{ss}}$$

where $\varphi_{ss} = \varphi$-angle corresponding to PT state. Therefore, Fig. 4.3 could be derived from Fig. 4.4 or vice-versa. The PT state data points in extension shown in Figs. 4.3 and 4.4 represent samples that have experienced a significant amount of strain softening resulting in axial strain until such state in excess of about 3%. This criterion of a minimum strain
until PT state is similar to that used by Chern (1985) in an attempt to investigate the uniqueness of the PT state line in compression. It may be noted that the PT state line in extension is not unique. The PT state appears to be a function of initial void ratio \( e_i \) much in the same manner as was noted for CSR. As shown in Figs. 4.3 and 4.4, three PT state lines may be constructed with three initial void ratios for contractive sand in extensional deformation. Different \( e_i \) corresponds to a different initial sand fabric which is unaltered by consolidation stresses. The PT state line in extension would, therefore, appear to be a function of initial void ratio.

The extension PT state line with looser \( e_i \) may be observed to be located above those with denser \( e_i \) (Figs. 4.3 and 4.4). This implies that \( S_{pt} \) is not a function of \( e_c \) alone as in the case under compression loading (Chern, 1985). Thus a complete knowledge of the initial sand state \( (e_c', \sigma_3^c', K_c) \) together with \( D'_{ri} \) would be essential when estimating \( S_{pt} \) in extension. A particular \( e_c \) could be achieved by consolidation of an undensified initially loose sand to a high \( \sigma_3^c \) or densification of a freshly deposited loose sand by vibration before consolidation. In Fig. 4.5, test specimens #21, 26, and 92 have different \( e_i \) and \( e_c \) but identical \( \sigma_3^c \) of 4.0 kgf/cm\(^2\) (400 kPa). The \( S_{pt} \) of denser sample #92, as expected, is higher than the \( S_{pt} \) of the looser samples. However, test specimens #26 and #36 have identical \( e_c \) but different initial void ratio and consolidation states, \( S_{pt} \) for the initially looser sample (#36) is much higher than that for the initially denser sample (#26).

One of the important aspects of investigating the PT state line in extension mode was to compare it with that under compression mode. Fig. 4.3 to 4.5 also show the PT state line for Ottawa sand in compression.
LEGEND:

- Mono $e_i = 0.715$, $D_t = 32.5\%$
- Cyclic $e_i = 0.701$, $D_t = 37.2\%$
- $e_i = 0.681$, $D_t = 43.5\%$
- $e_i = 0.655$ to $0.66$

Unique PT State Line in Compression (after Chern. 1985)

Fig. 4.3 Relationship between void ratio and PT state strength under undrained extension loading
LEGEND:

- ○ Monotonic  $e_i = 0.715  , D_r = 32.5\%$
- △ Cyclic  $e_i = 0.701  , D_r = 37.2\%$
- ★ $e_i = 0.681  , D_r = 43.5\%$
- ◊ $e_i = 0.655$ to $0.66$

Fig. 4.4 Relationship between void ratio and $\sigma^3$ at PT state under undrained extension loading
LEGEND:
# Test no.
△ Cyclic loading
○ Monotonic loading

Consolidation Curves
( $e_c$ vs $\sigma_3'c$ )

PT State Lines

$e_i = 0.715$
$e_i = 0.701$
$e_i = 0.681$

Fig. 4.5 Relationship between void ratio and $\sigma_3'$ at PT state under undrained loading or $\sigma_{3c}'$ at the end of consolidation
(Chern, 1985). A limited number of compression tests were conducted and very good agreement with the results of Chern was obtained. Since the PT state in compression exists over a much narrow range of $D_{ri}$ (30 to 38%) it is logical to compare the PT state line in extension over this density range. As shown in Figs. 4.3 and 4.4, the PT state line in compression is situated significantly to the right of those for extension. That is, for the same void ratio after consolidation, $S_{pt}$ in extension is much lower.

The main reason for this significant difference in $S_{pt}$ between compression and extension is the large difference in the amount of excess porewater pressure developed until PT state for identical initial conditions ($e_c$, $\sigma'_c$ and $K_c$). As illustrated by Equation (1), $S_{pt}$ is directly proportional to $\sigma'_3$ at PT state. Since $\sigma'_3$ at PT state equals to $\sigma'_3 - \Delta u$, $S_{pt}$ will also be different because the mobilized friction angle at PT state remains constant. The existence of inherent anisotropy, which accounts for the much higher excess porewater pressure in extension, contributes to the difference in PT state line between extension and compression modes of loading.

Since initial sand fabric accounts for the difference in behaviour between extension and compression, it is postulated that PT state line is a function of the initial fabric of the sand. However, Chern (1985) has shown that a unique PT state line in compression was obtained for specimens prepared either by, moist tamping or sedimentation. Specimens prepared by these two methods should result in different initial sand fabric. The influence of initial fabric would thus appear to be more profound in extension as opposed to compression loading of sand. The sense of inherent anisotropy causes to the specimen greater stability under compression loading.
It has been generally assumed that PT state line for a sand is unique and a function of \( e_c \) only. Been et al. (1985) recently introduced a state parameter concept to classify properties of sand with reference to a unique PT state line. This idea was inspired from the observation of a unique PT state line for a given sand. However, in the present investigation, PT state line has been shown to be a function of stress path. Not only is the PT state line not unique between compression and extension, but being a function of \( e_i \), it is also not unique in extension alone. Adopting PT state line in compression as a unique criteria may lead to unsafe design for other loading paths.

4.1.2 Anisotropically Consolidated Samples

Monotonic extension tests were also conducted on specimens with two series of \( D_{ri} \) (32.5% and 43.5%) consolidated anisotropically to \( K_c = 1.15, 1.5 \) and 2.0. Most of the tests were carried out at higher effective confining pressures (\( \sigma_{3c}' \)) of 10 and 20 kgf/cm\(^2\) (1000 and 2000 kPa).

Fig. 4.6(a,b) show typical behaviour of specimens for two \( K_c \) values (1.15 and 2.0) having identical \( \sigma_{3c}' \) of 10 kgf/cm\(^2\) (1000 kPa) and loose \( D_{ri} \) of 32.5%. Since the \( K_c \) values are different, the corresponding \( D_{rc} \) are also different. However, because of the flatness of the consolidation curves, \( D_{rc} \) for these two \( K_c \) value are very close to each other, being 39.5 and 41.0% for \( K_c = 1.15 \) and 2.0 respectively. This difference in \( D_{rc} \) is too small to affect the observation to any noticeable degree. Despite the stress-reversal, the behaviour is very similar to isotropically consolidated sand at the same \( D_{ri} \) and \( \sigma_{3c}' \). In both cases contractive response was observed and \( \Delta u \) was also high. Negative \( \Delta u \) was also observed at very small strain level. The magnitude of negative \( \Delta u \) was larger for higher \( K_c \) value. This is because unloading from a higher \( K_c \) value involves larger reduction in \( \sigma_m \). It appears that the peak deviator stress in extension is not a function of \( K_c \) value. This is contrary to the compression loading
Fig. 4.6(a) Typical triaxial extension undrained behaviour of anisotropically consolidated ($K_c = 1.15$) Ottawa sand with $D_{ri} = 32.5\%$.
Fig. 4.6(b) Typical triaxial extension undrained behaviour of anisotropically consolidated \((K_c = 2.0)\) Ottawa sand with \(D_{ri} = 32.5\%\).
behaviour where peak $\sigma_d$ increases with increasing $K_c$ at constant $\sigma_3'$. (Chern, 1985). Chern also found that under compressive loading, peak $\sigma_d$ was uniquely related to $\sigma_3'$ and independent of the individual values of $K_c$ and $\sigma_3'$. It appears that peak $\sigma_d$ in extension is uniquely related to the lateral consolidation stress $\sigma_1'$ and not $\sigma_3'$. The sand seems to lose the memory of the level of $\sigma_1'$ which it has been subjected to and appears to respond as if it was isotropically consolidated under $\sigma_3'$. Initial minor principle stress $\sigma_3'$ which becomes major principal stress $\sigma_1'$ at peak seems to be a better controlling parameter for $\sigma_d$ peak than $\sigma_1'$ in extension loading.

Typical behaviour of an initially denser sample is shown in Fig. 4.6(c). The initial state was $D_{ri} = 43.5\%, D_{rc} = 48.5\%, \sigma_3' = 10 \text{ kgf/cm}^2$ (1000 kPa) and $K_c = 1.50$. The behaviour is similar to that for initially loose samples but for the influences normally associated with increasing density.

Fig. 4.7 shows the CSR states for all anisotropically consolidated contractive sand. The CSR lines for $D_{ri} = 32.5\%$ and 43.5\% and $K_c = 1.0$ are also shown for reference. It may be noted that for a given $D_{ri}$, CSR for $K_c > 1.0$ agree very well with isotropically consolidated sand, despite stress reversal. Similarly the slope of PT line is the same for anisotropically and isotropically consolidated sand. It can therefore be said that both the CSR and PT lines in extension are independent of anisotropic consolidation on compression sides.

Consolidation and PT state lines for isotropically and anisotropically consolidated sand are shown in Fig. 4.8 for two initial relative densities. The compressibility of Ottawa sand is uniquely related to $\sigma_1'$ regardless of the individual values of $K_c$ and $\sigma_3'$ (Chern, 1985). Therefore, the
Test No: 99  
$D_{ri} = 43.5\%$, $K_c = 1.5$  
$\sigma^{'}_{3c} = 10.0\ \text{kgf/cm}^2$  
$D_{rc} = 48.5\%$

Fig. 4.6(c) Typical triaxial extension undrained behaviour of anisotropically consolidated ($K_c = 1.5$) Ottawa sand with $D_{ri} = 43.5\%$
Fig. 4.7 Effective stress conditions at the initiation of strain softening response (CSR) and start of dilation (PT state) for anisotropically consolidated sand with $D_{ri} = 32.5\%$ and $43.5\%$ in extension.
Fig. 4.8 Relationship between void ratio and $\sigma_{1c}$ at the end of consolidation and $\sigma_2$ at PT state for anisotropically consolidated sand with $D_{r1} = 32.5$ and 43.5% in extension.
consolidation curve \( e - \sigma'_{1c} \) applies regardless of the \( K_c \) value. It may be noted that for each of these relative density, PT state line is independent of \( K_c \) value. Again, the controlling factor still appears to be the \( D_{ri} \) and not the \( K_c \) value.

4.2 Cyclic Loading Behaviour

The previous section has shown that the monotonic loading behaviour of saturated sand in extension is very different from that in compression. For identical initial state \( (e_c, \sigma'_{3c}, K_c) \) the slope of the CSR line, the \( S_p \) or \( \sigma'_{3} \) at PT state in extension are significantly lower than that in compression. This section will investigate the implications of a weaker extension behaviour on cyclic loading response.

Cyclic loading tests were conducted only on contractive specimens using \( K_c = 1.0 \) to 1.25, \( \sigma'_{3c} = 2.0 \) to 4.0 kgf/cm\(^2\) (200 to 400 kPa) and \( D_{rc} = 36.0 \pm 1.0\% \). The cyclic stress ratio \( \tau_{dcy}/\sigma'_{3c} = 0.08 \) to 0.125 was used and \( D_{ri} \) of all the specimens was 32.5 \pm 0.5\%. With the range of initial conditions, contractive response will be expected upon shearing under monotonic condition, under both compressional and extensional modes.

Fig. 4.9 shows a typical cyclic loading effective stress path plotted in \( q-p' \) stress space for sand with an initial state of \( K_c = 1.0, D_{rc} = 37.5\% \) and \( \sigma'_{3c} = 4.0 \) kgf/cm\(^2\) (400 kPa). The residual porewater pressure accumulates with each cycle of loading. More residual porewater pressure is generated in the first and second cycles of loading than in the remaining cycles. In particular, the first cycle generated the highest residual porewater pressure. An important observation is that the extension half of the loading cycle generated higher \( \Delta u \) than the compression half. This observation is consistent with the monotonic behaviour where higher \( \Delta u \) was observed in extension than in compression when the same magnitude of shear
Test No: 90

$D_{ri} = 32.5\%$

$K_c = 1.0 \quad \sigma_{3c}' = 4.0 \text{ kgf/cm}^2$

$\alpha_{00}/2\sigma_{3c}' = 0.1$

$D_{rc} = 37.5\%$

Fig. 4.9 Undrained effective stress path during cyclic loading of contractive Ottawa sand
stress was applied. An approximately equal amount of \( \Delta u \) was generated for each subsequent loading cycle, except in the last two cycles. Therefore, in this region, the relationship between \( \Delta u \) and number of cycle can be represented by a straight line.

With continued cycling, the state of stress was drawn closer and closer to the CSR line. The stress state of isotropically consolidated sand reached the extension CSR first because of the flatter CSR line in extension. Large contractive deformations are therefore always triggered on the extension phase of the loading sequence in cyclic triaxial loading of isotropically consolidated samples. The effective stress ratio (CSR) at which the strain softening behaviour and flow deformation was triggered by cyclic loading was essentially identical to that under monotonic loading. Thus, regardless of the stress history, i.e., monotonic or cyclic, the slope of CSR on the extension phase is unique for a given \( D_{ri} \). The residual strain accumulated was extremely small, less than 0.3%, before the stress state was able to reach the CSR line.

For anisotropically consolidated sand, flow deformation can be triggered either on the compression or extension side depending on the relative values of \( K_c \) and cyclic stress ratio. Fig. 4.10(a) shows a typical cyclic loading ESP for conditions of no stress reversal and for which the low CSR in extension side will not be a factor. Flow deformation was triggered when the ESP reached the CSR line on the compression side, as was previously shown by Chern (1985). Fig. 4.10(b) shows a condition of stress reversal from an initial anisotropic state on the compression side which results in flow deformation on the compression side. In general under such situations the sand could undergo flow deformation on either the extension or compression side depending on both the level of \( K_c \) and cyclic
Fig. 4.10(a, b) Effective stress path during cyclic undrained loading of anisotropically consolidated contractive Ottawa sand.
stress ratio. This condition which will be investigated in the following section.

4.3 **Criterion for Liquefaction**

The criteria for liquefaction to occur simplified from Castro et al (1982) and Chern (1985) are as follows:

1) **Sand must be contractive under monotonic loading**

2) **The maximum shear stress (static or cyclic) must be greater than the undrained steady state shear strength, i.e.** \( \tau_c + \tau_s > S_{pt} \)

3) **There must be sufficient loading cycles to lead the ESP to the CSR lines.**

The assumption implicit in the above criteria is that the SSL and CSR are not affected by the stress path (compression or extension). The results on pluviated Ottawa sand presented previously have shown that it is not true. Hence, the above criteria need modification in recognition of the stress path effects on undrained behaviour which result in lower \( S_{pt} \) and CSR in extension than in compression for identical initial state of sand. In addition, the larger \( D_r \) over which pluviated sand is contractive in extension than in compression has a strong influence on its susceptibility to liquefaction if significant stress reversal is expected.

For Ottawa sand, \( D_{rc} \) alone and not its complete initial state has been shown (Chern, 1985) to give a very good prediction as to its susceptibility to contractive response. In assessing the first criterion alone, liquefaction could be triggered if sand is contractive either under compression or extension monotonic loading. Since sand is contractive over a much higher \( D_{rc} \) on extension side, liquefaction may occur on extension side at a much higher \( D_{rc} \). During cyclic loading with stress reversal liquefaction could be triggered either on compression or extension side. Under a given cyclic
stress ratio \( \frac{\tau_{cy}}{\sigma''_3} \), the condition under which liquefaction will be triggered on either side would be a function of stress reversal or \( \tau_s \). Thus, the amount of stress reversal plays a major role in assessing the second criterion which must be examined on both the compression and the extension sides.

The necessary condition for liquefaction to occur in compression is

\[ \tau_{cy} + \tau_s > S_{pt}^{(comp)} \]

and similarly, in extension would be

\[ |-\tau_{cy} + \tau_s| > S_{pt}^{(ext)} \]

For initially loose sand which is contractive in both compression and extension loading mode, the susceptibility to liquefaction depends on both the level and \( \tau_s \) and \( \tau_{cy} \). Since \( S_{pt}^{(ext)} \) is much lower than \( S_{pt}^{(comp)} \) for the same initial state, liquefaction will always occur in extension mode on isotropically consolidated sand (Fig. 4.9). With increasing \( \tau_s \), under certain \( \tau_{cy} \) value, the mechanism of strain development would change from liquefaction to cyclic mobility. This would occur when

\[ |-\tau_{cy} + \tau_s| < S_{pt}^{(ext)} \]

while \( (\tau_{cy} + \tau_s) \) is still less than \( S_{pt}^{(comp)} \), resulting in the nonsatisfaction of the second criterion. Further increase in \( \tau_s \) would cause liquefaction to be triggered on compression mode, when

\[ (\tau_s + \tau_{cy}) > S_{pt}^{(comp)} \]

Thus for a given \( \tau_{cy} \), the mechanism of strain development could change from liquefaction on the extension side, to cyclic mobility and finally to liquefaction on the compression side with increasing \( \tau_s \). However, for pluviated loose Ottawa sand, this scenario is unlikely to occur because \( S_{pt}^{(ext)} \) is extremely low, and the slightest stress reversal will invariably trigger liquefaction on the extension side. Thus, the stress region

\[ |-\tau_{cy} + \tau_s| < S_{pt}^{(ext)} \]

which eliminates liquefaction on the extension side is virtually nonexistent.

For initially denser pluviated sand which is contractive in extension and dilative in compression loading, increasing \( \tau_s \) could render the strain
development from liquefaction to cyclic mobility. When $\tau_s$ is low and
$|\tau_{cy} + \tau_s| > S_{pt}(ext)$, liquefaction then may be triggered on the extension
side. When $\tau_s$ is higher, such liquefaction cannot be triggered on the
compression side because sand is non-contractive in that loading mode. For
pluviated Ottawa sand, this situation would occur for specimens with $D_\text{ri}$
ranging from 38% to 50%. Furthermore, for denser sample ($D_\text{ri} > 50\%$)
liquefaction will not occur. In order to satisfy the third criterion,

enough loading cycles are required to move the effective stress state of
the sand to CSR line. All three criteria must be satisfied simultaneously
in order for liquefaction to occur.

The applicability of the above arguments is now investigated with
respect to the effect of $\tau_s$ on resistance of liquefaction on contractive
sand at a fixed $D_\text{rc}$ and $\sigma_{3\text{c}}'$. Comprehensive studies on this aspect on
pluviated Ottawa sand were carried out by Chern (1981, 1985). His results
for $D_\text{rc} = 36.0\%$ and $\sigma_{3\text{c}}' = 2.0 \text{ kgf/cm}^2$ (200 kPa) are reproduced in Fig.
4.11. Further cyclic loading tests at identical density and $\sigma_{3\text{c}}'$ using $K_c$
range of 1.0 to 1.25 were conducted. These results are also shown in Fig.
4.11 by data points and it may be noted that very consistent results were
obtained. The cyclic stress ratios used in Chern's and this study were
significantly higher than $S_{pt}$ in extension. Thus strain development, a
consequence of cyclic mobility as $\tau_s$ increases, did not occur. Liquefaction
was either triggered in compression or extension mode. With the range of
$\tau_{cy}$ applied to specimens, the maximum shear stress criterion as discussed
previously, was always satisfied in both the compression and extension
sides. There is a peak in the cyclic resistance curve which corresponds to
$K_c > 1.0$ (Fig. 4.11). With the initial state and imposed shear loading
used, the observed shape of this resistance curve with static shear could
4.11 Effect of $\tau_s$ on the cyclic stress ratio required to cause 5% strain in 10 loading cycles

Fig. 4.12 Schematic illustration of the effects of $\tau_s$ on the resistance of contractive Ottawa sand
be explained by the differences in CSR on compression and extension sides. The explanatory diagram in q-p' stress space is shown in Fig. 4.12. This diagram is constructed for pluviated Ottawa sand with D_{ri} = 32.5% for which \( \phi \) mobilized at CSR in extension and compression are 14.0 and 23.0° respectively. A lower extension CSR renders cyclic loading on isotropically consolidated sand (\( K_c = 1.0 \)) to reach the CSR on extension side first, and flow deformation is triggered. Increasing compression \( K_c \) values reduces the amount of stress reversal. More loading cycles of the same \( \tau_{cy} \) are then required for the ESP to reach the CSR on extension side which increases the resistance to liquefaction. When the \( K_c \) value is high enough, the ESP hits the CSR on the compression side first triggering flow deformation. It may be noted that to the left of the peak in the resistance curve (Fig. 4.11), liquefaction is triggered on extension side; whereas to the right of the peak, liquefaction is triggered on compression side. The peak cyclic resistance to liquefaction appears to fall on a \( K_c \) lines bisecting the angle between CSR states in compression and extension. Thus the stress space may be divided into two zones, A and B as shown in Fig. 4.12. In zone A increasing \( \tau_{s} \) always reduces the cyclic resistance whereas in zone B it increases the resistance. For pluviated Ottawa sand with an initial state of D_{ri} = 32.5%, \( \sigma^l_{3c} = 2.0 \text{ kgf/cm}^2 \) (200 kPa) and D_{rc} = 36.0±1.0%, the estimated \( K_c \) for maximum cyclic resistance is approximately 1.13.
5.1 Introduction

Tests were conducted on samples with initial state $D_{rc} = 36$ to 38.5%, $\sigma'_{3c} = 2.0$ to 6.0 kgf/cm$^2$ (200 to 600 kPa) and $K_c = 1.0$ to 1.25. For these initial states the sand would be contractive under monotonic loading in both compression and extension. $D_{ri}$ of all samples was 32.5%. For contractive sand flow deformation will be triggered when the ESP reaches the CSR line, either in compression or extension. In this study, small prestrain refers to strains which are induced by a shear stress history within the CSR lines, whereas large prestrain refers to strain states corresponding to a stress history beyond the CSR lines. This is different from the definition of Ishihara et al. (1978, 1982) who utilized the PT state to differentiate between small and large prestrain history. The effects of small prestrain history resulting from undrained cyclic loading will be investigated first. The effects of large prestrain history will be examined subsequently. All prestrain histories were developed using undrained monotonic or cyclic loading under strain-rate controlled conditions. This was essential for imparting the prescribed large prestrain histories to contractive specimens. After prestraining, the samples were reconsolidated to the original state of effective stresses and were then reloaded monotonically or cyclically under undrained condition.

The magnitude of prestrain refers to the axial strain developed under undrained loading of the virgin sample. The axial strain development associated with reconsolidation (which could be substantial in some cases)
was excluded because the focus was on strain development associated with shear loading. In the presentation of stress-strain data, in particular with large prestrain history, the end of reconsolidation was considered as the start of zero strain for undrained reloading responses.

5.2 Small Prestrain History

The prestrain was induced using cyclic stress ratio $\tau_{cy}/\sigma_{3c}'$ from 0.1 to 0.125. The magnitude of the induced prestrain ranged from axial strain of 0.05% to 0.2%. The associated excess porewater pressure expressed as a ratio $\Delta u/\Delta u_a$, where $\Delta u_a$ is the allowable excess porewater a sample could develop before reaching the CSR line (Fig. 5.1(a)), ranged from 50-95%.

The increase in relative density as a result of reconsolidation following prestrain history ranged from 1 to 2.0% for the imposed small prestrain levels. The cyclic stress ratio for cyclic reloading ranged from 0.1 to 0.135.

5.2.1 Monotonic Compression Reloading Behaviour

Fig. 5.1(a,b,c) show the stress-strain behaviour and ESP for small prestrain history and subsequent compression reloading after reconsolidation. Identical cyclic stress ratio of 0.125 was used to develop prestrain and was applied with or without initial static bias. The initial state of samples was $\sigma_{3c}' = 2.0 \text{ kgf/cm}^2$ (200 kPa), $D_{rc} = 36\pm1\%$ and $K_c = 1.0$ to 1.25. Thus complete, partial and no stress reversal prestrain histories were imparted to test samples #77, 75 and 76, respectively. The sense of prestrain for test samples #77 and 75 was extensional, whereas in test sample #76 it was compressional.

It may be noted in Fig. 5.1 that subsequent monotonic compression
Fig. 5.1(a) Undrained monotonic compression reloading behaviour of virgin contractive with small extension residual prestrain = 0.15%
Fig. 5.1(b) Undrained monotonic compression reloading behaviour of virgin contractive sand with small compression residual prestrain \( \varepsilon_{a} \) = 0.17%
Fig. 5.1(c) Undrained monotonic compression reloading behaviour of virgin contractive sand with small extension residual prestrain $E_d(\text{pre.}) = 0.12\%$
reloading still showed contractive strain softening behaviour similar to that for the virgin sample. Furthermore, the CSR was essentially identical to that for the virgin sand. Thus, a small prestrain history or prestraining within the CSR on contractive sand did not remove the contractive behaviour or the ability to flow on undrained reloading. However $S_{\text{pt}}$ for identical relative density ($D_{\text{rc}}$) was approximately 10% higher than that for virgin $S_{\text{pt}}$ even after proper accounting for a very small increase in density associated with prestraining. Some hardening (increase in CSR, $S_{\text{pt}}$ and $\sigma_d$), therefore appears to have occurred even at this small level of prestrain.

The sense of residual prestrain, whether extensional or compressional does not seem to be critical to the undrained reloading response as long as prestraining occurs within the CSR lines. For example, test samples #77 and 75 had residual strain in extension and test sample #76 had it in compression, but the subsequent behaviour on reloading was similar (Fig. 5.1). This is similar to the observation by Ishihara et al. (1975) who did not use CSR line as a limit for small prestrain criterion. In fact, Ishihara's conclusions were based on tests on dilative sand as opposed to contractive sand investigated in this study. Furthermore, the slopes of PT state and failure lines also appear to be unaffected by small prestrain history.

5.2.2 Monotonic Extension Reloading Behaviour

Monotonic undrained extension reloading behaviour was also investigated following essentially identical prestrain history, i.e., cyclic prestraining with small compressional or extensional residual prestrain. The cyclic stress ratio used during prestraining was 0.1. The initial state of samples was $\sigma_{3c} = 2.0$ to 6.0 kgf/cm$^2$ (200 to 600 kPa), $D_{\text{rc}} = 36$ to 38.5% and $K_c = 1.0$ to 1.25.
The extension side reloading behaviour with prestrain was also found to be similar to that of virgin sand. The peak deviator stress occurred at an average mobilized $\phi$ angle of 15.5°. This angle is slightly higher, by about 1.5 degrees, than that for virgin sand with similar relative density. Also, for the same $D_{rc}$, the undrained PT state strength ($S_{pt}$) of the prestrained sand was about 10% higher than that of the virgin sand. The slope of PT state and failure lines in extension also appear to be unaffected by small prestrain history.

The CSR, $S_{pt}$ and PT did not appear to be affected significantly by small prestrain history for either compression loading path or extension loading path. That is to say, if the virgin sand is susceptible to flow deformation, it remains susceptible to flow deformation despite small prestrain history. In practical terms, previous seismic activities which had not led to liquefaction, do not alter the susceptibility of a sand deposit to liquefaction.

5.2.3 CyclicReloading Behaviour

A limited number of undrained cyclic reloading tests were conducted on prestrained samples using $\sigma'_{3c} = 2.0$ to 4.0 kgf/cm$^2$ (200 to 400 kPa), $D_{rc} = 36 \pm 1.0\%$ and $K_c' = 1.15$ to 1.25. The cyclic stress ratio used for prestraining was 0.1 to 0.125 and the associated $\Delta u/\Delta u_a$ ranged from 50 to 90%. The magnitude of prestrain ranged from 0.1 to 0.2% axial strain. Since no stress reversal was allowed, all prestrain deformations were compressive.

Fig. 5.2 shows, for identical initial state, the residual excess pore-water pressure generated as a function of number of loading cycles (N) on virgin sample; test sample $#72$ with prestrain = 0.1% and test sample $#73$ prestrain = 0.2% under identical $\tau_{cy}$. The results show that the number of
All samples:

\[ D_{ri} = 32.5 \pm 0.5^\circ \],

\[ \sigma_{3c}' = 2.0 \text{ kgf/cm}^2, \quad K_c = 1.25 \]

\[ T_s = T_{cy} = 0.25 \text{ kgf/cm}^2 \]

Test no: 50

Virgin Sample

\[ \Delta U = 0.63 \text{ kgf/cm}^2 \text{ or } 50\% (\Delta U/\Delta U_a) \]

\[ \varepsilon_a (\text{pre.}) = 0.1\% \]

Test no: 72

\[ \Delta U = 1.04 \text{ kgf/cm}^2 \text{ or } 90\% (\Delta U/\Delta U_a) \]

\[ \varepsilon_a (\text{pre.}) = 0.2\% \]

Fig. 5.2  Porewater pressure changes for virgin contractive sand with and without small prestrain
loading cycles needed to generate the same $\Delta u$ increases with increasing prestrain. As shown previously, a small prestrain history does not erase the contractive behaviour of sand nor does it affect the CSR. The CSR is not a function of stress history (monotonic or cyclic), and is also unaffected by small prestrain history during cyclic reloading. However, the number of loading cycles to reach the CSR is much larger for samples with prestrain history than the virgin sample because of the reduced $\Delta u$ per loading cycle. In other words, the resistance to liquefaction is increased. This is consistent with the conclusions from Finn et al. (1970); Bjerrum (1973); Lee et al. (1975); and Seed (1977). However, their conclusions were based primarily on dilative sand as opposed to contractive sand investigated in this study. It may also be noted in Fig. 5.2 that there is a definite increase in resistance to liquefaction for higher prestrain magnitude provided that it is within the CSR boundaries. The number of loading cycles to trigger liquefaction increased from 14 for a virgin sample to 38 and 125 for prestrain magnitude of 0.1 and 0.2% respectively.

The above findings were based on tests in which prestrain was compressional. As shown previously, the CSR in extension and compression is not a function of the sense of small prestrain history. Therefore, it does not matter whether the prestrain is in the compression mode and liquefaction is triggered on extension mode or prestrain is on extension mode and liquefaction is triggered on compression mode, the resistance to liquefaction increases with prestrain below the CSR line.

The increase in the resistance to liquefaction caused by small prestraining appears to have resulted from better interlocking of the sand particles in the original structure. Small instabilities at contact points may have been eliminated without a general structural rearrangement taking
place. With a more stable structure as a result of the small prestrain history, the porewater pressure generation on subsequent undrained reloading decreases accordingly.

5.3 Large Prestrain History

Large prestraining refers to a strain history brought about by crossing the CSR boundary. The initial state of the samples was $\sigma_{3c}' = 2.0$ to $4.0$ kgf/cm$^2$ (200 to 400 kPa), $K_c = 1.0$ to 1.25 and $D_{rc} = 35.5$ to 37.5%. The prestrain history was imparted by cyclic stress ratio of $\tau_{cy}/\sigma_{3c}' = 0.1$ to 0.125 or by monotonic loading. The magnitude of prestrain over which full scale or localized flow deformation was simulated in either compression or extension mode ranged from 1.0 to 4.75%. The decrease in relative density on reconsolidation following prestraining ranged from 2 to 7.0%. A range of undrained reloading behaviour was studied simulating both reversal and non-reversal of strain direction in relation to the sense of prestrain. The sense of strain during reloading refers to strain with respect to the specimen configuration at the end of prestraining. Reloading without strain reversal will be investigated first, followed by reloading with strain reversal and cyclic reloading. Special attention will be paid to investigate the influence of prestraining on factors such as $S_{pt}$, CSR, $\Delta u$ and $\sigma_d$ peak during reloading.

5.3.1 Reloading Without Strain Reversal

Compression Reloading with Compression Prestrain

Cyclic stress ratio of 0.125 was applied for inducing prestrain to three identical samples with initial state of $\sigma_{3c}' = 2.0$ kgf/cm$^2$ (200 kPa), $D_{rc} = 36 \pm 0.5\%$ and $K_c = 1.25$. The increase in relative density on reconsolidation was 2.3 and 3.75% for 1, 2 and 4.75% prestrain respectively. Large
prestrain histories of 1 and 2% represent conditions beyond the CSR line but below the PT line, while 4.75% represents prestraining beyond the PT state.

The test results are shown in Fig. 5.3(a and b). It may be noted that virgin contractive sand with 1% prestrain still developed contractive strain softening response on monotonic compression reloading. It implies that the potential to develop flow deformation was not completely eliminated on account of this level of prestrain. However, the mobilized $\phi$-angle at CSR increased from 23° for virgin sand to 25.5° for the sand with 1% prestrain. This implies that the flow deformation will be triggered at a slightly higher stress ratio. Also, $S_{pt}$ increased from 0.6 to 0.9 kgf/cm$^2$ (60 to 90 kPa) and $\sigma_d$ peak increased from 0.8 to 1.1 kgf/cm$^2$ (80 to 110 kPa) for 1% prestrain history. When the prestrain magnitude was increased to 2%, the monotonic undrained reloading without strain reversal did not develop contractive response. Thus, potential to develop flow deformation associated with strain softening was eliminated. However, the slope of PT state line appears to be unaltered as a consequence of prestraining.

When the prestrain magnitude was increased to 4.75%, the specimen became extremely dilative on undrained reloading (Fig. 5.3b). The density change brought about by reconsolidation following the prestrain history was relatively small (3.75%). A virgin sample with the same initial state as the sample after 4.75% prestrain history, i.e. $\sigma'_3 = 2.0$ kgf/cm$^2$ (200 kPa), $D_{rc} = 39.75\%$ and $K_c = 1.25$ manifests contractive monotonic undrained loading behaviour. Such a change in behaviour cannot therefore be explained solely in terms of small change in density (Finn, et al., 1970; Toki et al., 1978; Ishihara et al., 1982) and appears to be largely a result of microscopic fabric changes.
Fig. 5.3(a) Undrained monotonic compression reloading behaviour of contractive sand with large compression prestrain (1 and 2%)
Fig. 5.3(b) Undrained monotonic compression reloading behaviour of contractive sand with large compression prestrain (4.75%)
Extension Reloading With Extension Prestrain

Cyclic stress ratio of 0.1 was applied for inducing prestrain to identical samples with initial state $\sigma_3' = 4.0 \text{ kgf/cm}^2 (400 \text{ kPa})$, $D_{rc} = 37.5 \pm 0.5\%$ and $K_c = 1.0$. A hydrostatic state of consolidation was chosen in order to develop prestrain on the extension side. Identical specimens were allowed to strain to 2 and 4% axial extension strain. Significantly higher porewater pressure developed during extension prestraining, and therefore the volume change associated with reconsolidation was comparatively higher than that with the same prestrain magnitude on compression side. The increase in relative density was 5 and 7%, thus the relative density after reconsolidation ($D_{rcl}$) were 42 and 44.5% for 2 and 4% prestrain histories respectively. During the prestraining process, the specimens underwent shear at very low effective confining stress of $0.2 \text{ kgf/cm}^2 (20 \text{ kPa})$ and $0.14 \text{ kgf/cm}^2 (14 \text{ kPa})$ for 2 and 4% prestraining, respectively.

Undrained reloading behaviour following large prestraining in extension but without strain reversal is shown in Fig. 5.4(a,b). The magnitude of negative porewater pressure at the initial stage of both tests is more than that for the virgin sample. However for 2% prestrain, positive porewater pressure started to develop at about 0.6% strain due to contractive shear induced volume change. The contractive strain softening response was again not completely destroyed by the prestraining process and the sample was still susceptible to flow deformation. However the mobilized $\phi$-angle at CSR increased from 14° for virgin sand to 22° for sand with prestrain. Prestraining also resulted in a dramatic increase in $S_{pt}$ from $0.2 \text{ kgf/cm}^2 (20 \text{ kPa})$ for virgin sand to $0.8 \text{ kgf/cm}^2 (80 \text{ kPa})$ for sand with prestrain.

For sand with 4% prestrain, which corresponds approximately to straining until the PT state, positive porewater pressure did not develop at any
Fig. 5.4(a) $\Delta u$ and $\sigma_d$ for undrained monotonic extension reloading of virgin contractive sand with large extension prestrain (2 and 4%)
Fig. 5.4(b) ESP for undrained monotonic extension reloading of virgin contractive sand with large extension prestrain (2 and 4%)
stage of reloading. The specimen started to dilate at the start of extension loading and the contractive strain softening response was completely erased. However, the slope of PT state appears to be unaltered. Again, the increase in relative density alone cannot account for this change in behaviour. As shown in the previous chapter, a virgin sample with an initial state identical to that after prestraining, \( \sigma'_{3c} = 4.0 \text{ kgf/cm}^2 \) (400 kPa), \( D_{rc} = 44.5\% \) and \( K_c = 1.0 \) is strongly contractive in monotonic undrained extension loading.

It is important to point out that prestraining beyond the CSR could change a contractive sand to a non-contractive or dilative sand without a significant increase in density. There is a definite trend which indicates that the larger the magnitude of prestrain, the less contractive the sand behaves on subsequent monotonic undrained loading. The associated increase in undrained PT state shear strength is disproportionate to the corresponding density increment. The prestrain magnitude necessary to erase completely the contractive behaviour of virgin sand is difficult to quantify as it is primarily a function of its initial state \( (D_{ri}, \sigma'_{3c}, K_c) \) and the mode of loading (compression or extension). However, from the results reported herein, there is a strong indication that this magnitude corresponds to the strain until PT state. Prestraining beyond the PT state appears to eliminate the contractive response provided no strain reversal occurs on reloading. On the other hand, if large prestrain is developed prior to reaching the PT line, only part of the potential to develop flow deformation is destroyed and the sand will still show contractive response upon reloading, though to a lesser degree.

Prestrain history has a hardening (increase in CSR, \( S_{pt} \) and \( \sigma_d \)) effect on sand if the direction of strain is unchanged during reloading. The
amount of hardening increased with increasing prestrain magnitude, and may thus be associated with microstructure changes. Hardening has also been shown to result when prestraining occurs entirely within the CSR lines. However, it appears that the hardening effect associated with small prestraining is a consequence of better interlocking as opposed to microstructure changes in the case of large prestrain history.

5.3.2 Reloading With Strain Reversal

Compression Reloading With Extension Prestrain

For the investigation of this phenomenon, the prestraining imparted was essentially identical to that of extension reloading with extension prestrain history. This consists of cyclic stress ratio of 0.1 applied to three identical samples with initial conditions of \( \sigma_{3c}^i = 2.0 \text{ kgf/cm}^2 \) (200 kPa), \( D_{rc} = 36 \pm 0.5\% \) and \( K_c = 1.0 \). Specimens were allowed to strain to 1, 2 and 4\% extension strain. The increase in relative density on reconsolidation was 3.75, 5.5 and 7.5\%, and thus \( D_{rcl} \) were 40, 41, and 43.5\% for 1, 2, and 4\% axial prestrain respectively. The prestraining paths and the undrained compression reloading behaviour of the sand for various extension prestrain magnitude on extension are shown in Fig. 5.5.

It may be noted that the peak deviator stress dropped from 1.3 kgf/cm\(^2\) (130 kPa) for the virgin sample to 0.93, 0.62, and 0.27 kgf/cm\(^2\) (93, 62 and 27 kPa), despite increases in \( D_r \) of 3.75, 5.5 and 7.5\% for 1, 2 and 4\% prestrain history, respectively. Development of higher excess \( \Delta u \) with increasing prestrain level led to the drastic reduction in \( \sigma_d \) peak. The strain level at \( \sigma_d \) peak was approximate constant at about 0.6\%. Therefore, the net strain is still extensional if reference is made to the virgin state. Sand with 1\% prestrain exhibited essentially no contractive
Fig. 5.5 Undrained monotonic compression reloading behaviour of virgin contractive sand with large extension prestrain (1%, 2% and 4%)
response; sand with 2% prestrain exhibited a very slight contractive response; and with 4% prestrain had a strong contractive response. Virgin sand with initial state of $\sigma^I_{3c} = 2.0$ kgf/cm$^2$ (200 kPa), $D_{rc} = 43.5\%$ and $K_c = 1.0$ which is identical to that of sand with 4% prestrain does not show contractive response in compression. Thus, the contractive undrained response is a consequence of large prestraining on the opposite side.

The mobilized $\phi$-angle at CSR was reduced from 23° for virgin loading to 18° and 13° for samples with 2 and 4% extension prestrain respectively, and $S_{pt}$ was reduced drastically from 0.5 to 0.31 and 0.13 kgf/cm$^2$ (50 to 31 and 13 kPa) despite increase in relative density. However, the slope of PT state of prestrained sand appears to be essentially identical to virgin sample.

**Extensional Reloading With Compression Prestrain**

For the investigation of this phenomenon prestrain history was imparted by monotonic loading on a sample with an initial state $\sigma^I_{3c} = 2.0$ kgf/cm$^2$ (200 kPa), $D_{rc} = 36.5\%$ an $K_c = 1.0$. The undrained behaviour where large compressional prestrain history was imparted by cyclic loading will be discussed later. The specimen was sheared to 4.5% axial compressive strain beyond the PT state under strain controlled loading and then unloaded back to hydrostatic condition. The strain accompanying unloading was 0.2% and the increase in relative density upon reconsolidation was 1.5%; thus, $D_{rcl}$ was 38%.

As shown in Fig. 5.6, the extensional undrained reloading response was softened (decrease in CSR, $S_{pt}$ and $\sigma_d$) as a consequence of a large compressional prestrain history. The already contractive extension undrained
Fig. 5.6 Undrained monotonic extension reloading behaviour of virgin contractive sand with large monotonic compression prestrain (4.3%)
behaviour of the pluviated sample became even more contractive. \( \sigma'_d \) peak was reduced from 0.6 kgf/cm\(^2\) (60 kPa) for a virgin sample to 0.3 kgf/cm\(^2\) (30 kPa). The mobilized \( \phi \)-angle at CSR was only 8.5° and \( S_{pt} \) was as low as 0.05 kgf/cm\(^2\) (5 kPa). The slope of PT state was not well defined but appeared to be unaltered. The excess porewater pressure associated with reloading was extremely high at the PT state (\( \Delta u/\sigma'_3c \) = 0.95).

Additional tests for different large prestrain levels were not carried out. However, similar to the compression reloading with large extension prestrain, a gradual reduction in \( \sigma'_d \) peak, CSR and \( S_{pt} \) with prestrain magnitude would likely occur if the prestrain levels were to vary between 0.5% to 4.3%, the strain range between CSR and PT state for the initial state under consideration.

As observed in Section 5.2, monotonic reloading behaviour is not significantly altered by small prestraining with or without strain reversal. However large prestraining without strain reversal hardens and with strain reversal softens the undrained response of sand. The amount of softening or hardening increases with the prestrain magnitude.

5.3.3 Cyclic Reloading Behaviour

Cyclic reloading with cyclic stress ratio of 0.125 was applied to a sample with 4.0% prestrain in compression imparted by cyclic loading using cyclic stress ratio of 0.125. The initial state of the sample was \( \sigma'_3c \) = 2.0 kgf/cm\(^2\) (200 kPa), \( D_{rc} \) = 36.0% and \( K_c \) = 1.08. The \( K_c \) value was chosen such that a significant amount of stress-reversal results. Relative density after reconsolidation, \( D_{rcl} \) was 3.5% higher at 39.5% compared to the 36% of the virgin sample. The cyclic undrained reloading behaviour as well as prestraining stress paths are shown in Fig. 5.7. Liquefaction was
Fig. 5.7 Undrained cyclic reloading behaviour of virgin contractive sand subjected to cyclic loading with large compression prestrain (4.0%)
triggered on the extension side involving strain reversal. The resistance to liquefaction because of this strain reversal was drastically reduced and the specimen liquefied in one cycle of reloading. The mobilized $\phi$ angle at CSR was reduced to 9.0° and the $S_{pt}$ was reduced to as low as 0.05 kgf/cm² (5 kPa). These reduced parameters values were essentially identical to those with 4% prestrain history induced by monotonic loading. It thus appears that the magnitude of prestrain rather than the prestraining method, i.e. monotonic or cyclic governs the reloading behaviour of contractive sand. The intuitive reason being that the mechanism of prestrain by both loading methods is identical, being associated with flow deformation triggered as a result of stress states moving beyond the CSR line. This is somewhat similar to the observation of Ishihara and Okada (1982) in that the reloading behaviour is controlled by the magnitude of prestrain accumulated at the end of prestrain history. Although their observation was based on results from testing of dilative sands, the same conclusion appears to hold for contractive sand.

The magnitude of prestrain accumulated governs both the cyclic and monotonic reloading behaviour. Results from the monotonic undrained reloading tests show that strain reversal is very critical to the subsequent undrained behaviour. Key parameters such as, CSR, $S_{pt}$, etc. are strongly influenced by the prestrain magnitude and strain reversal. The cyclic reloading resistance is lower, as expected, for significant amount of cyclic strain reversal at liquefaction.

Inferring from monotonic undrained reloading behaviour, specimen with large prestrain history when subjected to cyclic reloading without strain reversal would have increased resistance to strain development. This is a consequence of higher CSR and $S_{pt}$ for prestrained sand than for the sand
under virgin loading. Furthermore, when the prestrain development corresponds to a stress state beyond the PT line, the susceptibility to flow deformation would be eliminated and only cyclic mobility could occur upon cyclic reloading. That is to say, such a large prestrain history will change the strain development mechanism from liquefaction to cyclic mobility upon cyclic reloading without strain reversal.

So far, the reduced cyclic reloading resistance for a 4% prestrain history with significant strain reversal has been shown to be primarily a result of reduced CSR and $S_{pt}$ for loading with strain reversal at liquefaction. Other cyclic reloading tests for various prestrain magnitudes have not been carried out in this study. However, previous investigations (Suzuki and Toki 1981, 1984) have shown that the resistance to cyclic reloading for various prestrain histories of 0.8% and 3% in compression and extension, was respectively less than that for the virgin specimens. For the resistance to be lower than that of a virgin sample, different prestrain magnitudes, depending on the direction of residual prestrain, were required. This phenomenon was observed in dilative as well as contractive sand and appears to be a result of large prestrain and consequent changes in CSR.

Fig. 5.8 shows the effect of prestrain history on the slope of CSR lines. With increasing extension prestrain, the slope of the CSR line increases for extension reloading and decreases for compression reloading as compared with virgin CSR values. For a 2% extension prestrain magnitude, the CSR $\phi$-angle in compression is about $18^\circ$ which is still higher than the virgin CSR $\phi$- angle in extension ($\sim 14^\circ$). The CSR $\phi$-angle in extension is, however, increased from $14^\circ$ to $22^\circ$. Thus as a result of cyclic reloading with complete stress reversal (which exhibited strain
Fig. 5.8

Effect of prestrain on CSR

\[ \frac{1}{2} (\sigma_Y - \sigma_H) \ (\text{kgf/cm}^2) \]
reversal at liquefaction) and extension prestrain of 2%, flow deformation would then be triggered on the compression and not on the extension side. Therefore quantitatively, the resistance to cyclic reloading has increased despite strain reversal. On the other hand, for a prestrain of 2% in compression, the CSR in extension became lower than the already low $\phi$-angle of 14°. The resistance to cyclic reloading was thus reduced. However, for large prestrain magnitudes corresponding to stress states beyond PT line, the cyclic reloading resistance is lower than that of the virgin sand regardless of the direction of prestrain. This is in line with the observations of Ishihara and Okada (1978, 1982) who showed that the cyclic reloading resistance is lower than that for a virgin sample so long as the prestrain magnitude refers to a stress state beyond the PT line and especially with residual compression prestrain. This feature has also been observed by Suzuki and Toki (1984) on dilative sand and appears to be applicable to contractive sand as well.

5.4 **Effect of Prestrain History on PT State Line**

Undrained PT steady state strength ($S_{pt}$) has been shown to increase or decrease as a function of prestrain history. A collection of $S_{pt}$ upon reloading together with PT state lines in compression and extension for virgin sample having $D_w = 32.5\%$ are shown in Fig. 5.9.

With small prestrain and reloading with or without strain reversal, the $S_{pt}$ data essentially fall on the virgin PT state line. As pointed out before, the hardening effect due to such a small prestrain history increases the $S_{pt}$ slightly (~10%) above that for virgin sand as shown schematically in Fig. 5.10. The generation of $\Delta u$ during prestraining pushes the stress state toward the PT state line. On reconsolidation the
NOTE:
Dri of all Virgin Specimens are 32.5 ± 0.5%.

LEGEND:
- △ Comp. Small Prestrain
- □ Ext. Small Prestrain
- ▽ Comp. Large Prestrain W/O Strain Reversal
- △ Ext. Large Prestrain W/O Strain Reversal
- ○ Comp. Large Prestrain With Strain Reversal
- ○ Ext. Large Prestrain With Strain Reversal
- C Comp. prestrain
- E Ext. prestrain

Fig. 5.9 Effect of prestrain on PT state line
Fig. 5.10 Schematic explanation of the illusion that $S_{pt}$ with small prestrain is higher than virgin specimen but is to the left of virgin PT state line.
specimen follows a different consolidation curve and attains a different void ratio below $e_1$. Despite a comparatively smaller $\Delta u$ associated with reloading, PT state is reached to the left of the virgin PT state line but the magnitude of $\sigma'_3$ and hence $S_{pt}$ of the prestrained sand are higher than that of the virgin sand. Larger prestrain histories without strain reversal result in larger $\sigma'_3$ and hence larger $S_{pt}$ on reloading after prestraining as compared to the virgin sample.

For large prestrain history with strain reversal, $\sigma'_3$ and $S_{pt}$ on reloading are significantly lower than for virgin sand in spite of the increased density of the prestrained samples. The undrained PT state strength reduces with increasing prestrain magnitude. With 4% prestrain, $S_{pt}$ is as low as 0.1 kgf/cm$^2$ and 0.04 kgf/cm$^2$ (10 and 4 kPa) in compression and extensional reloading respectively. These strengths are so low that cyclic reloading with any noticeable strain reversal at liquefaction would trigger flow deformation.

As discussed in Chapter 4, the PT state line in extension is a function of initial sand fabric reflected in its $D_1$. It appears that PT state line in compression following prestraining is also a function of initial sand fabric. Large prestraining, which involves rearrangement of sand particles, changes the sand fabric as well as $\sigma'_3$ and $S_{pt}$ upon reloading. Thus a unique PT state line concept does not appear to be justified for sand specimen having histories of strain reversal and large prestrain, in compression or in extension. Unfortunately, changes in sand fabric is an intractable problem to investigate. Nevertheless, the sense and degree of anisotropy and their reflection on sand fabric can be assessed from indirect examination of response to hydrostatic compression as will be discussed subsequently.
5.5 **Possible Explanation of Observed Reloading Behaviour**

A large prestrain history may be postulated to result in a general rearrangement of sand particles. Observed changes in reloading behaviour appear to be primarily brought about by such microscopic fabric alterations. Reconsolidation data following prestraining, as shown in Fig. 5.11, give an indication of the sand fabric. Hydrostatic compression of an isotropic material would follow the reference relationship of $\varepsilon_v = 3\varepsilon_a$. Any deviation of measured test data from this reference relationship implies that the specimen possesses some degree of anisotropy. The consolidation responses shown in Fig. 5.11 indicate large changes in anisotropy from which definite trends can be established. That is, the degree of anisotropy is strongly influenced by the sense and magnitude of prestraining. Virgin consolidation curves show some degree of anisotropy for which $\varepsilon_v > 3\varepsilon_a$, which commonly occurs for loose pluviated sand. This implies that the specimen is more compressible in the lateral direction than in the axial direction. This observation was studied in detail by Oda (1972) who noted that the orientation of the principal axis of individual grains was predominantly horizontal in specimen prepared by the method of sedimentation under gravity.

The reconsolidation curves for specimens with large compression prestrain indicate that the axial strain is significantly smaller than the lateral strain and the ratio of $\varepsilon_v$ to $\varepsilon_a$ is in excess of 10. Upon large compressional prestraining under the associated low effective confining pressure, further particle rearrangement in a manner similar to the sedimentation process appears to develop. Thus the degree of anisotropy becomes more profound. The structure of the specimen gets progressively altered when strained beyond CSR. A new structure which is extremely
VOLUMETRIC STRAIN, $\varepsilon_v$ (%)

AXIAL STRAIN, $\varepsilon_a$ (%)

Fig. 5.11 Effect of prestrain on consolidation curves

LEGEND:
E: EXTENSION PRESTRAIN
C: COMPRESSION PRESTRAIN

Typical Virgin Consolidation Curves

Isotropic Line

$\varepsilon_h > \varepsilon_a$

$\varepsilon_a > \varepsilon_h$
strong in compression and weak in extension appears to be created and this bias increases with increasing compressive prestrain. According to Arthur (1972), a new structure could be formed by induced anisotropy. Ishihara et al. (1982) also formulated a schematic model that describes the rearrangement of sand particles upon large compression prestraining. Ishihara postulated that the grain-to-grain contact planes can be assumed to be in a predominantly horizontal direction and very few contact planes lie in the vertical direction. This model appears to offer a rational explanation of the observed behaviour.

The reconsolidation curves for samples with large extension prestrain histories indicate that the axial strain is larger than the lateral strain. The ratio of $\varepsilon_v$ to $\varepsilon_a$ ranges from approximately 1.4 and 1.0 for 2 and 4% extension prestraining respectively. This implies that the specimen is more compressible in the axial direction than in the lateral direction. Again, the bias increases with increased prestraining. With large extensional prestraining, the compressional reloading behaviour resembles the extension behaviour of a virgin sample. A sand structure similar to that formed by sedimentation but rotated by 90° appears to be created by the stress induced anisotropy. Ishihara postulated that the grain-to-grain contact planes are then predominantly in the vertical direction and very few contact planes in the horizontal direction.

It is important to point out that consolidation curves only give an indication of degree of anisotropy. The study of anisotropy is a whole new horizon, and in particular its interrelationship to changes in grain-to-grain contact as a consequence of straining. Separate comprehensive investigations will be required to study this phenomenon.
CHAPTER 6
SUMMARY AND CONCLUSIONS

With the objective of assessing the influence of stress path, undrained monotonic extension behaviour of a saturated sand has been studied under the triaxial condition and compared with the corresponding compression behaviour. The comparison was made over a range of relative density and confining pressure up to 25 kgf/cm$^2$ (2500 kPa) using both isotropically and anisotropically consolidated histories. Based on the test results, the following conclusions can be drawn.

1. The range of relative density over which contractive response manifested, was much higher in extension than in compression mode. For a certain range of relative density, the undrained response of sand could be dilative in compression but contractive in extension mode. Inherent anisotropy of pluviated sand is believed to be the main reason for the observed difference in behaviour.

2. The mobilized friction angle ($\phi$), corresponds to CSR (peak deviator stress) in extension, was not found to be constant in contrast to the unique value in compression. This angle remained sensibly constant for a given initial relative density ($D_{ri}$) and increased with increasing $D_{ri}$. Regardless of the $D_{ri}$, $\phi$ mobilized at CSR in extension was much smaller than that in compression. Both phase transformation (PT) and undrained failure lines, on the other hand, were identical for both compression and extension modes of loading.
3. The PT state line in extension was not unique. A different PT state line appeared to emerge for each $D_r^1$. Therefore, the PT state shear strength ($S_{pt}^1$) in extensional mode is not a function only of the void ratio after consolidation. It is also dependent on the initial relative density state $D_r^1$ prior to consolidation. Furthermore, regardless of $D_r^1$, PT state line lies significantly to the left of the unique PT state line in compression. Thus the $S_{pt}^1$ in extension is much lower than that in compression mode for the same $e_c$. Large differences in the amount of excess pore-water pressure at PT state between compression and extension mode were responsible for the observed difference in PT state lines. Since different stress paths may be involved under in-situ loading conditions, it would be inappropriate to adopt the unique compression PT state line under every situation.

4. The undrained extension response was found to be unaffected by the level of compression $K_c$ value. For a given $D_r^1$, CSR remained unchanged despite stress reversal during shear. Furthermore, the peak undrained strength was related uniquely to $\sigma_3^c$ in contrast to $\sigma_1^c$ for compression loading mode.

5. During cyclic loading of contractive sand, the slope of the CSR on the extension side was found unique for a given $D_r^1$ and equal to that under monotonic loading regardless of other initial state parameters or amplitude of cyclic stress. Because contractive undrained response manifested itself over a much higher relative density in triaxial extension, an initially denser sand could still be susceptible to liquefaction in extension. Since both $S_{pt}^1$ and CSR in extension were significantly lower than those in compression, the extension phase of a cyclic triaxial test on hydrostatically consolidated ($K_c = 1.0$) sand is much more damaging than compression
phase. Therefore, if stress-reversal is expected, it is imperative to check the criteria for flow deformation in extension.

Undrained monotonic and cyclic reloading behaviour of contractive sand subjected to different prestrain history ($\varepsilon_a$ of 0.1 to 4.75%) was also studied under triaxial conditions. The conclusions derived from this aspect of the study are summarized below.

1. Initially contractive sand with small prestrain below the CSR line was found to remain contractive on reloading. Thus, the previous seismic activities which did not liquefy sand do not remove the sand's susceptibility to liquefaction in the future. The parameters such as CSR and $S_{pt}$ on reloading remain essentially identical to virgin sample and are not affected by the sense of prestrain in relation to the direction of reloading. However, the amount of excess porewater pressure generated per loading cycle is reduced considerably when compared to that for virgin sand. Therefore, the resistance to liquefaction increases with small prestrain history, as more cycles of a given $\tau_{cy}$ are required to trigger liquefaction.

2. For initially contractive sand with large prestrain beyond CSR line, the undrained reloading behaviour is influenced by the direction of reloading with respect to the sense of prestrain. With no stress reversal, the contractive response diminishes and is ultimately eliminated with increasing prestrain. Large prestrain with no stress reversal on reloading could transform a contractive sand into a dilative sand without a significant change in relative density on reconsolidation following prestrain history. This conclusion was found valid separately for both the extension and compression modes.
For large prestrain with strain reversal during reloading, the parameters such as CSR, $S_{pt}$ and peak $\sigma_d$, etc. change dramatically. These parameters decrease with increasing magnitude of large prestrain. Large prestrain with strain reversal in reloading transforms a non-contractive sand into a contractive sand. The reloading resistance to liquefaction is drastically reduced especially where during prestrain PT state has been crossed.

For a given $\tau_{cy}$, the amount of strain reversal at liquefaction during cyclic loading is controlled by the $K_c$ value. Since strain reversal is critical to the cyclic reloading resistance of sand with large prestrain, the $K_c$ value plays an important role in cyclic reloading resistance.

3. The effect of density change brought about by prestraining was generally small. Therefore, it may be concluded that the effects of prestrain have been brought about mainly by microscopic fabric changes. The degree of anisotropy induced by large prestrain could be assessed from reconsolidation following prestrain.

4. The use of PT state to differentiate between small and large prestrain was not found to be adequate for virgin contractive sand. The liquefaction resistance on reloading can reduce with large prestrain well before PT state is reached. The amount of strain developed from CSR to PT state is significant (3 to 4.5% axial strain) for the initially contractive sand investigated in this study.
REFERENCES


