# INVESTIGATION OF THE DYNAMIC FRACTURE BEHAVIOR OF CONCRETE

#### by

#### NADARAJAH SOMASKANTHAN

B. Sc (Engineering), University of Moratuwa, Sri Lanka

# A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF APPLIED SCIENCE

in

## THE FACULTY OF GRADUATE STUDIES CIVIL ENGINEERING

We accept this thesis as conforming to the required standard

#### THE UNIVERSITY OF BRITISH COLUMBIA

September, 1989

## © NADARAJAH SOMASKANTHAN, 1989

In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the head of my department or by his or her representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Department of Civil Engineering

The University of British Columbia Vancouver, Canada

131 Date 9

DE-6 (2/88)

# ABSTRACT

Impact tests on plain and steel fiber-reinforced concrete beams, with varying length and notch depth, have been carried out using a drop weight impact machine and a fast data acquisition system. The rate dependant dynamic behavior as well as the relative performance of plain and fiber-reinforced concretes have been studied.

It was found that during the dynamic test data acquisition process, the use of analog filters give erroneous results, which was detected by the crack gauge employed in this study. When using the analog filter, the event time increased and the peak signal value decreased considerably, relative to the unfiltered or true values.

The inertial loads associated with the specimen during the impact event were evaluated on the basis of structural dynamic principles. The raw dynamic signals have been analyzed in the frequency-domain, and the dynamic characteristics of the system and specimen have been identified. Corrections for signal filtering and static load calibration have then been carried out.

In general, it was found that plain concrete is strain rate sensitive. The relationship between log of crack velocity and log of dynamic fracture toughness,  $K_{ID_c}$ , is linear with two distinct regions of applicability. The  $K_{ID_c}$  depend on notch depth as well as span length, indicating a geometrical dependency. The comparison based on span length showed that the shear effect will be predominant at shorter span.

Although the crack velocity is affected by the inclusion of fibers, the dynamic fracture toughness for plain concrete and fiber-reinforced concretes have been found to be the same at all strain rates. However, the effect of the fibers on the toughness of the fiber-reinforced materials may have been masked by the large variability in the data.

ii

# Table of Contents

-

ABSTRACT	ii
List of Tables	vii
List of Figures	x
List of Symbols	xi
Acknowledgement	xv
1 Introduction	1
1.1 General	1
1.2 Research Objective	2
1.3 Thesis Review	3
1.4 Literature Review	3
2 Theoretical Considerations	11
2.1 Introduction	11
2.2 Static Analysis	11
2.3 Relationship of Static and Dynamic Stress Intensity Factors	16
2.4 Dynamic Analysis	18
2.4.1 Introduction	18
2.4.2 Selection of Shape Functions For a Notched Beam	20
2.4.2.1 Method 1	21
2.4.2.2 Method 2	23

iii

		2.4.3	Derivation of Inertial Loads	27
	2.5	Deriva	tion of True Bending Load	29
	2.6	Dynan	nic Load Calibration	29
		2.6.1	Method 1	31
	,	2.6.2	Method 2	33
3	$\mathbf{Exp}$	erimer	ntal Aspects	35
	3.1	Introd	uction	35
	3.2	Materi	ials	35
	3.3	Specin	nen Preparation	36
		3.3.1	Fresh Concrete Preparation	36
		3.3.2	Specimen Casting and Compaction	36
		3.3.3	Specimen Curing	37
		3.3.4	Notching of Specimens	37
		3.3.5	Preparation of Accelerometer Base and Crack Gauge	37
	3.4	Testin	g Apparatus	39
		3.4.1	Impact Tests	39
			3.4.1.1 Tup	43
			3.4.1.2 Accelerometer	45
			3.4.1.3 Crack Gauge	45
		3.4.2	Data Acquisition System	48
		3.4.3	Static Tests	49
		3.4.4	Compression Tests	<b>5</b> 0
	3.5	Testin	ng Procedures	50
	·	3.5.1	Impact Tests	50
. •		3.5.2	Static Tests	52
• •		3.5.3	Compression Tests	52
	3.6	Testin	ng Program	52

iv

3.6.1       Impact Tests         3.6.2       Static Tests         3.6.3       Compression Tests         4       Data Analysis         4.1       Introduction         4.2       Frequency-Domain Analysis         4.3       Method of Analysis for Dynamic Testing         4.4       Concluding Remarks         5       Results and Discussion         5.1       Introduction         5.2       General Dynamic Fracture Behavior of Concrete         5.3       Effect of Notch Depth, Span and Drop Height on Dynamic Fracture         Parameters - Plain Concrete Specimens       .         5.3.1       Time to Fracture         5.3.2       Strain Rate         5.3.3       Crack Velocity         5.4       Effect of Notch Depth, Span and Drop Height on Dynamic Fracture         Parameters - Fiber-Reinforced Concrete Specimens         5.4.1       Crack Velocity         5.4.2       Dynamic Stress Intensity Factor         5.4.1       Crack Velocity         5.4.2       Dynamic Fracture Toughness         5.5       Summary of Results         6       Conclusions and Recommendations         6.1       Conclusions         6.2       Recommendations for Future Res	
3.6.1       Impact Tests         3.6.2       Static Tests         3.6.3       Compression Tests         4       Data Analysis         4.1       Introduction         4.2       Frequency-Domain Analysis         4.3       Method of Analysis for Dynamic Testing         4.4       Concluding Remarks         5       Results and Discussion         5.1       Introduction         5.2       General Dynamic Fracture Behavior of Concrete         5.3       Effect of Notch Depth, Span and Drop Height on Dynamic Fracture         Parameters - Plain Concrete Specimens       5.3.1         5.3.1       Time to Fracture         5.3.2       Strain Rate         5.3.3       Crack Velocity         5.4       Effect of Notch Depth, Span and Drop Height on Dynamic Fracture         Parameters - Fiber-Reinforced Concrete Specimens         5.4.1       Crack Velocity         5.4.2       Dynamic Fracture Toughness         5.4.3       Summary of Results         6       Conclusions and Recommendations         6.1       Conclusions         6.2       Recommendations for Future Research	
3.6.1       Impact Tests         3.6.2       Static Tests         3.6.3       Compression Tests         4       Data Analysis         4.1       Introduction         4.2       Frequency-Domain Analysis         4.3       Method of Analysis for Dynamic Testing         4.4       Concluding Remarks         5       Results and Discussion         5.1       Introduction         5.2       General Dynamic Fracture Behavior of Concrete         5.3       Effect of Notch Depth, Span and Drop Height on Dynamic Fracture         Parameters - Plain Concrete Specimens       5.3.1         5.3.1       Time to Fracture         5.3.2       Strain Rate         5.3.3       Crack Velocity         5.3.4       Dynamic Stress Intensity Factor         5.4       Effect of Notch Depth, Span and Drop Height on Dynamic Fracture         Parameters - Fiber-Reinforced Concrete Specimens       5.4.1         Crack Velocity       5.4.2         5.4.2       Dynamic Fracture Toughness         5.5       Summary of Results         6       Conclusions         6.1       Conclusions         6.2       Recommendations for Future Research	
3.6.1       Impact Tests         3.6.2       Static Tests         3.6.3       Compression Tests         4       Data Analysis         4.1       Introduction         4.2       Frequency-Domain Analysis         4.3       Method of Analysis for Dynamic Testing         4.4       Concluding Remarks         5       Results and Discussion         5.1       Introduction         5.2       General Dynamic Fracture Behavior of Concrete         5.3       Effect of Notch Depth, Span and Drop Height on Dynamic Fracture         Parameters - Plain Concrete Specimens       5.3.1         5.3.1       Time to Fracture         5.3.2       Strain Rate         5.3.3       Crack Velocity         5.4       Effect of Notch Depth, Span and Drop Height on Dynamic Fracture         Parameters - Fiber-Reinforced Concrete Specimens         5.4.1       Crack Velocity         5.4.2       Dynamic Fracture Toughness         5.5       Summary of Results         6       Conclusions and Recommendations         6.1       Conclusions         6.2       Recommendations for Future Research	
3.6.2       Static Tests         3.6.3       Compression Tests         4       Data Analysis         4.1       Introduction         4.2       Frequency-Domain Analysis         4.3       Method of Analysis for Dynamic Testing         4.4       Concluding Remarks         5       Results and Discussion         5.1       Introduction         5.2       General Dynamic Fracture Behavior of Concrete         5.3       Effect of Notch Depth, Span and Drop Height on Dynamic Fracture         Parameters - Plain Concrete Specimens       5.3.1         5.3.1       Time to Fracture         5.3.2       Strain Rate         5.3.3       Crack Velocity         5.3.4       Dynamic Stress Intensity Factor         5.4       Effect of Notch Depth, Span and Drop Height on Dynamic Fracture         Parameters - Fiber-Reinforced Concrete Specimens       5.4.1         Crack Velocity       5.4.2         Dynamic Fracture Toughness       5.5         Summary of Results       6         Conclusions       6.1         Conclusions       6.2         Recommendations for Future Research       6.2	52
3.6.3       Compression Tests         4       Data Analysis         4.1       Introduction         4.2       Frequency-Domain Analysis         4.3       Method of Analysis for Dynamic Testing         4.4       Concluding Remarks         5       Results and Discussion         5.1       Introduction         5.2       General Dynamic Fracture Behavior of Concrete         5.3       Effect of Notch Depth, Span and Drop Height on Dynamic Fracture         Parameters - Plain Concrete Specimens       5.3.1         5.3.1       Time to Fracture         5.3.2       Strain Rate         5.3.3       Crack Velocity         5.3.4       Dynamic Stress Intensity Factor         5.4       Effect of Notch Depth, Span and Drop Height on Dynamic Fracture         Parameters - Fiber-Reinforced Concrete Specimens       5.4.1         Crack Velocity       5.4.2         Dynamic Fracture Toughness       5.5         Summary of Results       5.5         6       Conclusions         6.1       Conclusions         6.2       Recommendations for Future Research	54
4 Data Analysis         4.1 Introduction         4.2 Frequency-Domain Analysis         4.3 Method of Analysis for Dynamic Testing         4.4 Concluding Remarks         5 Results and Discussion         5.1 Introduction         5.2 General Dynamic Fracture Behavior of Concrete         5.3 Effect of Notch Depth, Span and Drop Height on Dynamic Fracture         Parameters - Plain Concrete Specimens         5.3.1 Time to Fracture         5.3.2 Strain Rate         5.3.3 Crack Velocity         5.4 Dynamic Stress Intensity Factor         5.4 Effect of Notch Depth, Span and Drop Height on Dynamic Fracture         Parameters - Fiber-Reinforced Concrete Specimens         5.4.1 Crack Velocity         5.4.2 Dynamic Fracture Toughness         5.4.3 Summary of Results         6 Conclusions and Recommendations         6.1 Conclusions         6.2 Recommendations for Future Research	54
<ul> <li>4.1 Introduction</li> <li>4.2 Frequency-Domain Analysis</li> <li>4.3 Method of Analysis for Dynamic Testing</li> <li>4.4 Concluding Remarks</li> <li>5 Results and Discussion</li> <li>5.1 Introduction</li> <li>5.2 General Dynamic Fracture Behavior of Concrete</li> <li>5.3 Effect of Notch Depth, Span and Drop Height on Dynamic Fracture Parameters - Plain Concrete Specimens</li> <li>5.3.1 Time to Fracture</li> <li>5.3.2 Strain Rate</li> <li>5.3.3 Crack Velocity</li> <li>5.3.4 Dynamic Stress Intensity Factor</li> <li>5.4 Effect of Notch Depth, Span and Drop Height on Dynamic Fracture Parameters - Fiber-Reinforced Concrete Specimens</li> <li>5.4.1 Crack Velocity</li> <li>5.4.2 Dynamic Fracture Toughness</li> <li>5.5 Summary of Results</li> <li>6 Conclusions and Recommendations</li> <li>6.1 Conclusions</li> <li>6.2 Recommendations for Future Research</li> </ul>	56
<ul> <li>4.2 Frequency-Domain Analysis</li> <li>4.3 Method of Analysis for Dynamic Testing</li> <li>4.4 Concluding Remarks</li> <li>5 Results and Discussion</li> <li>5.1 Introduction</li> <li>5.2 General Dynamic Fracture Behavior of Concrete</li> <li>5.3 Effect of Notch Depth, Span and Drop Height on Dynamic Fracture Parameters - Plain Concrete Specimens</li> <li>5.3.1 Time to Fracture</li> <li>5.3.2 Strain Rate</li> <li>5.3.3 Crack Velocity</li> <li>5.3.4 Dynamic Stress Intensity Factor</li> <li>5.4 Effect of Notch Depth, Span and Drop Height on Dynamic Fracture Parameters - Fiber-Reinforced Concrete Specimens</li> <li>5.4.1 Crack Velocity</li> <li>5.4.2 Dynamic Fracture Toughness</li> <li>5.5 Summary of Results</li> <li>6 Conclusions and Recommendations</li> <li>6.1 Conclusions</li> <li>6.2 Recommendations for Future Research</li> </ul>	56
<ul> <li>4.3 Method of Analysis for Dynamic Testing</li> <li>4.4 Concluding Remarks</li> <li>5 Results and Discussion</li> <li>5.1 Introduction</li> <li>5.2 General Dynamic Fracture Behavior of Concrete</li> <li>5.3 Effect of Notch Depth, Span and Drop Height on Dynamic Fracture Parameters - Plain Concrete Specimens</li> <li>5.3.1 Time to Fracture</li> <li>5.3.2 Strain Rate</li> <li>5.3.3 Crack Velocity</li> <li>5.3.4 Dynamic Stress Intensity Factor</li> <li>5.4 Effect of Notch Depth, Span and Drop Height on Dynamic Fracture Parameters - Fiber-Reinforced Concrete Specimens</li> <li>5.4.1 Crack Velocity</li> <li>5.4.2 Dynamic Fracture Toughness</li> <li>5.5 Summary of Results</li> <li>6 Conclusions and Recommendations</li> <li>6.1 Conclusions</li> <li>6.2 Recommendations for Future Research</li> </ul>	61
<ul> <li>4.4 Concluding Remarks</li> <li>5 Results and Discussion</li> <li>5.1 Introduction</li> <li>5.2 General Dynamic Fracture Behavior of Concrete</li> <li>5.3 Effect of Notch Depth, Span and Drop Height on Dynamic Fracture Parameters - Plain Concrete Specimens</li> <li>5.3.1 Time to Fracture</li> <li>5.3.2 Strain Rate</li> <li>5.3.3 Crack Velocity</li> <li>5.3.4 Dynamic Stress Intensity Factor</li> <li>5.4 Effect of Notch Depth, Span and Drop Height on Dynamic Fracture Parameters - Fiber-Reinforced Concrete Specimens</li> <li>5.4.1 Crack Velocity</li> <li>5.4.2 Dynamic Fracture Toughness</li> <li>5.5 Summary of Results</li> <li>6 Conclusions and Recommendations</li> <li>6.1 Conclusions</li> <li>6.2 Recommendations for Future Research</li> </ul>	72
<ul> <li>5 Results and Discussion</li> <li>5.1 Introduction</li> <li>5.2 General Dynamic Fracture Behavior of Concrete</li> <li>5.3 Effect of Notch Depth, Span and Drop Height on Dynamic Fracture Parameters - Plain Concrete Specimens</li> <li>5.3.1 Time to Fracture</li> <li>5.3.2 Strain Rate</li> <li>5.3.3 Crack Velocity</li> <li>5.3.4 Dynamic Stress Intensity Factor</li> <li>5.4 Effect of Notch Depth, Span and Drop Height on Dynamic Fracture</li> <li>Parameters - Fiber-Reinforced Concrete Specimens</li> <li>5.4.1 Crack Velocity</li> <li>5.4.2 Dynamic Fracture Toughness</li> <li>5.5 Summary of Results</li> <li>6 Conclusions and Recommendations</li> <li>6.1 Conclusions</li> <li>6.2 Recommendations for Future Research</li> </ul>	82
<ul> <li>5 Results and Discussion</li> <li>5.1 Introduction</li> <li>5.2 General Dynamic Fracture Behavior of Concrete</li> <li>5.3 Effect of Notch Depth, Span and Drop Height on Dynamic Fracture Parameters - Plain Concrete Specimens</li> <li>5.3.1 Time to Fracture</li> <li>5.3.2 Strain Rate</li> <li>5.3.3 Crack Velocity</li> <li>5.3.4 Dynamic Stress Intensity Factor</li> <li>5.4 Effect of Notch Depth, Span and Drop Height on Dynamic Fracture Parameters - Fiber-Reinforced Concrete Specimens</li> <li>5.4.1 Crack Velocity</li> <li>5.4.2 Dynamic Fracture Toughness</li> <li>5.5 Summary of Results</li> <li>6 Conclusions and Recommendations</li> <li>6.1 Conclusions</li> <li>6.2 Recommendations for Future Research</li> </ul>	0.9
<ul> <li>5.1 Introduction</li> <li>5.2 General Dynamic Fracture Behavior of Concrete</li> <li>5.3 Effect of Notch Depth, Span and Drop Height on Dynamic Fracture Parameters - Plain Concrete Specimens</li> <li>5.3.1 Time to Fracture</li> <li>5.3.2 Strain Rate</li> <li>5.3.3 Crack Velocity</li> <li>5.3.4 Dynamic Stress Intensity Factor</li> <li>5.4 Effect of Notch Depth, Span and Drop Height on Dynamic Fracture Parameters - Fiber-Reinforced Concrete Specimens</li> <li>5.4.1 Crack Velocity</li> <li>5.4.2 Dynamic Fracture Toughness</li> <li>5.5 Summary of Results</li> <li>6.1 Conclusions and Recommendations</li> <li>6.2 Recommendations for Future Research</li> </ul>	83
<ul> <li>5.2 General Dynamic Fracture Behavior of Concrete</li></ul>	<b>შ</b> ა იე
<ul> <li>5.3 Effect of Notch Depth, Span and Drop Height on Dynamic Fracture Parameters - Plain Concrete Specimens</li></ul>	83
Farameters - Plain Concrete Specimens         5.3.1         Time to Fracture         5.3.2         Strain Rate         5.3.3         Crack Velocity         5.3.4         Dynamic Stress Intensity Factor         5.4         Effect of Notch Depth, Span and Drop Height on Dynamic Fracture         Parameters - Fiber-Reinforced Concrete Specimens         5.4.1       Crack Velocity         5.4.2       Dynamic Fracture Toughness         5.5       Summary of Results         6       Conclusions and Recommendations         6.1       Conclusions         6.2       Recommendations for Future Research	00
<ul> <li>5.3.1 Time to Fracture</li> <li>5.3.2 Strain Rate</li> <li>5.3.3 Crack Velocity</li> <li>5.3.4 Dynamic Stress Intensity Factor</li> <li>5.4 Effect of Notch Depth, Span and Drop Height on Dynamic Fracture Parameters - Fiber-Reinforced Concrete Specimens</li> <li>5.4.1 Crack Velocity</li> <li>5.4.2 Dynamic Fracture Toughness</li> <li>5.5 Summary of Results</li> <li>6 Conclusions and Recommendations</li> <li>6.1 Conclusions</li> <li>6.2 Recommendations for Future Research</li> </ul>	. 80
<ul> <li>5.3.2 Strain Rate</li> <li>5.3.3 Crack Velocity</li> <li>5.3.4 Dynamic Stress Intensity Factor</li> <li>5.4 Effect of Notch Depth, Span and Drop Height on Dynamic Fracture Parameters - Fiber-Reinforced Concrete Specimens</li> <li>5.4.1 Crack Velocity</li> <li>5.4.2 Dynamic Fracture Toughness</li> <li>5.5 Summary of Results</li> <li>6 Conclusions and Recommendations</li> <li>6.1 Conclusions</li> <li>6.2 Recommendations for Future Research</li> </ul>	80
<ul> <li>5.3.3 Crack Velocity</li> <li>5.3.4 Dynamic Stress Intensity Factor</li> <li>5.4 Effect of Notch Depth, Span and Drop Height on Dynamic Fracture Parameters - Fiber-Reinforced Concrete Specimens</li> <li>5.4.1 Crack Velocity</li> <li>5.4.2 Dynamic Fracture Toughness</li> <li>5.5 Summary of Results</li> <li>6 Conclusions and Recommendations</li> <li>6.1 Conclusions</li> <li>6.2 Recommendations for Future Research</li> </ul>	81 00
<ul> <li>5.3.4 Dynamic Stress Intensity Factor</li></ul>	90
<ul> <li>5.4 Effect of Notch Depth, Span and Drop Height on Dynamic Fracture Parameters - Fiber-Reinforced Concrete Specimens</li></ul>	92
Farameters - Fiber-Reinforced Concrete Specimens         5.4.1       Crack Velocity         5.4.2       Dynamic Fracture Toughness         5.5       Summary of Results         6       Conclusions and Recommendations         6.1       Conclusions         6.2       Recommendations for Future Research	00
5.4.1       Crack velocity         5.4.2       Dynamic Fracture Toughness         5.5       Summary of Results         6       Conclusions and Recommendations         6.1       Conclusions         6.2       Recommendations for Future Research	. 99
<ul> <li>5.4.2 Dynamic Fracture Tougnness</li></ul>	99
<ul> <li>5.5 Summary of Results</li> <li>6 Conclusions and Recommendations</li> <li>6.1 Conclusions</li> <li>6.2 Recommendations for Future Research</li> </ul>	100
<ul> <li>6 Conclusions and Recommendations</li> <li>6.1 Conclusions</li></ul>	104
<ul><li>6.1 Conclusions</li></ul>	109
6.2 Recommendations for Future Research	109
	111

# Bibliography

Appendices	119
A Test Results - Plain Concrete Specimens	119
B Test Results - Fiber-Reinforced Concrete Specimens.	122

# List of Tables

3.1	Impact Test Program - Plain Concrete Specimens
3.2	Impact Test Program - Fiber-Reinforced Concrete Specimens 53
3.3	Static Test Program - Plain Concrete Specimens
3.4	Static Test Program - Fiber-Reinforced Concrete Specimens 54
3.5	Compression Test Program - Plain and Fiber-Reinforced Concrete Spec-
	imens
5.1	Summary of Test Results of Plain Concrete Beams with 410 mm Span. 105
5.2	Summary of Test Results of Plain Concrete Beams with 720 mm Span. 106
5.3	Summary of Test Results of Fiber-Reinforced Concrete Beams with
	410 mm Span
5.4	Summary of Compression Test Results
A.1	Dynamic Test Results of Plain Concrete Specimens
A.2	Slow Test Results of Plain Concrete Specimens
B.1	Dynamic Test Results of Fiber-Reinforced Concrete Specimens 123
B.2	Slow Test Results of Fiber-Reinforced Concrete Specimens

# List of Figures

2.1	Stresses Near Crack Front	12
2.2	Basic Modes of Crack Surface Displacement	13
2.3	Notched, Three-Point Bending Test Specimen	15
2.4	Notched Beam Configuration	20
2.5	Model for Dynamic Load Calibration	<b>3</b> 0
3.1	Crack Gauge and Accelerometer Fixture	38
3.2	Impact Machine.	40
3.3	Impact Machine Components and Dimensions	41
3.4	Testing System	42
3.5	The Tup	43
3.6	Static Calibration Curve For Tup	44
3.7	Crack Gauge Configuration and Theoretical Output	46
3.8	Crack Gauge Set-up and Connection Details	47
3.9	Photo Cell Assembly	51
4.1	Unfiltered Crack Gauge Response	57
4.2	Unfiltered Crack Gauge Response and Filtered and Amplified Tup	
	Load Cell Response	58
4.3	Filtered Crack Gauge, Tup Load Cell and Accelerometer Outputs	60
4.4	Unfiltered Tup Load Cell Signal Versus Time for Specimen 4T4001.ASC	64
4.5	External Noise Variation Before the Impact Event	64
4.6	Tup Load Cell Signal Variation in Free Vibration Phase	65
4.7	FFT of the Tup Load Cell Signal	66

viii

4.8	FFT of the External Noise Signal Before the Impact Event	66
4.9	FFT of the Free Vibration Phase of the Hammer Assembly	67
4.10	Digital Low Pass Filter Characteristics	68
4.11	FFT of the Tup Load Cell Signal After Digital Filtering	69
4.12	Inverse FFT of the Tup Load Cell After Digital Filtering	69
4.13	FFT of Accelerometer Signal	70
4.14	FFT of Accelerometer Signal After Specimen Fracture	71
4.15	Unfiltered Tup Load Cell, Accelerometer and the Crack Gauge Signals	
	for Specimen 4T7003.ASC	73
4.16	Definitions of Significant Time Variables	73
4.17	Tup Load Cell Signal in Free Vibration Phase for Specimen 4T7003.ASC	75
4.18	FFT of Tup Load Cell Signal in the Free Vibration Phase	75
4.19	FFT of the Tup Load Cell Signal in Impact Event Phase	78
<b>4.2</b> 0	Filtered Time-Domain Signal of the Tup Load Cell	79
4.21	Tup Load, Inertial Load and Bending Load During the Impact Event	80
4.22	Energy Components of the Impact Event	81
5.1	Bending Load vs Time for Span = 410mm.	84
5.2	Bending Load vs Time for Span = 720mm	85
5.3	Average Time to Fracture Variation With Notch Depth and Span	86
5.4	Variation of Strain Rate with Drop Height	88
5.5	Variation of Strain Rate with Notch Depth and Span.	89
5.6	Variation of Crack Velocity with Drop Height	91
5.7	Variation of Kip with Time	92
5.8	Typical Variation of $dK_{1D}/dt$ with Time	94
5.9	Variation of $K_{\rm LD}$ with Crack Velocity	95
5 10	Variation of $K_{rp}$ with Notch Depth for Span - 410mm	06
5.10	Variation of $K_{}$ with Notch Depth for Span - 720mm	07
9.11	variation of $\Lambda_{ID_e}$ with Noted Depth for $\text{Span} = 120\text{mm}$	91

ix

					· .		
•		•					
5.12	Variation of Crack	s Speed with Fibe	er Content		••••	 99	
5.13	Variation of $K_{ID_c}$	with Drop Heigh	at for $a/B = 0.25$	••••	••••	 101	
5.14	Variation of $K_{ID_e}$	with Drop Heigh	t for $a/B = 0.35$		 	 102	
	· · ·						

# List of Symbols

x,y	: Distances in two perpendicular directions
r	: Radial distance
θ	: Angle from datum
Kı	: Stress intensity factor for mode I
$\sigma_x,  \sigma_y$	: Normal stresses
$ au_{xy}$	: Shear stress in $xy$ plane
ν	: Poisson ratio
€z	: Strain in $z$ direction
$ au_{xz},  au_{yz}$	: Shear stresses in $xz$ and $yz$ planes.
E	: Modulus of elasticity
Р	: Externally applied bending load
S	: Span of the beam
T .	: Thickness of the beam
В	: Depth of the beam
$\sigma_0$	: Far field applied stress
$F\left(\frac{a}{B}\right)$	: Function of crack length to depth
K <sub>ID</sub>	: Dynamic stress intensity factor for mode I
$f(V_{cr})$	: Function of crack velocity
$K_{IS}$	: Static stress intensity factor for mode I
М	: Bending moment
$\dot{U}(x,t)$	: Velocity of point at x
$\ddot{U}(x,t)$	: Acceleration of point at x
$\dot{U}_{0}$	: Velocity of beam center point
Ü <sub>0</sub>	: Acceleration of beam center point
$ar{m}_{ar{1}}$	: Mass per unit length
δU	: Virtual displacement of beam center point

xi

$\delta W_{inertia}$	: Virtual work of inertial forces
$P_{inertia}$	: Center-point inertial force
$P_{rot}$	: Equivalent center-point rotational inertia force
Presist	: Total inertial resistive force
$F_f$	: Force transmitted to ground
$P_{0}$	: Equivalent static force
$D_{d}$	: Overall dynamic amplification factor
$\Psi,  \alpha$	: Phase angles
$F_{fmax}$	: Maximum of load transmitted to ground
β	: Ratio of natural frequency of body to that of applied load
ξ	: Damping ratio
P <sub>max</sub>	: Maximum of applied load
D	: Dynamic amplification factor
$P_{tot}$	: Actual dynamic load
$P_{dyn}$	: Measured dynamic load
$C_{0}$	: Longitudinal wave speed
$C_{R}$	: Rayleigh wave speed
С	: Damping coefficient
$Q^{\dagger}$	: Stiffness of the body
m	: Mass of the body
<i>w</i>	: Displacement of the body
ŵ	: Velocity of the body
ŵ	: Acceleration of the body
g	: Acceleration of the gravity
l	: Length
U(x,t)	: Lateral displacement of beam

xii

$U_0, u_0$	: Lateral displacement of beam center
t	: Time
$\phi_T$	: Total shape function
$\phi_1, \phi_2$	: Shape functions
Ι	: Moment of inertia
Y(t)	: Time dependant function
$V_{cr}$	: Crack velocity
a	: Crack length
$A_n, b$	: Constants
$\omega_n$	: Natural circular frequency
$\omega$	: Excitation circular frequency
ρ	: Density
$\sigma_{f}$	: Fracture stress
σ	: Stress rate
k(V)	: Crack Velocity Dependant term
$\dot{\epsilon}$	: Strain rate
N	: Constant
$K_{IC}$	: Static critical stress intensity factor for
$K_{ID_{\epsilon}}$	: Dynamic fracture toughness for mode I
H	: Mass of the body
P(t)	: Time-varying force
λ	: Constant
$P_{bend}$	: True bending force
$C(\omega)$	: Fourier coefficient
$T_{0}$	: Event start time
$T_E$	: Event end time

mode I

$T_{P_{max}}$	: Time of maximum load
$t_{f}$	: Time to fracture
$t_t$	: Total event time
t <sub>cr</sub>	: Time of crack propagation
$f_n$	: Natural frequency
$T_n$	: Period of natural frequency
δ	: Logarithmic decrement
$P_{tot}max$	: Actual maximum dynamic load
$\Delta$	: Displacement
Ėŗ	: strain rate at failure

# Acknowledgement

The author is greatly indebted to his supervisor Professor R. J. Gray for his guidance, valuable suggestions and the encouragement throughout this research.

The author would also like to express his gratitude to Professors S. Mindess and A. Poursartip for reviewing the manuscript and making constructive criticisms.

Appreciation is extended to Ms. J. Wong, M. Nazar and C. Y. Cheng for their helpful participation in preparing and maintaining the instrumentation for the experiments. The help rendered by Mr. T. Srithar in all aspects of this research is acknowledged.

Finally, the research assistantship awarded by Department of Civil Engineering, University of British Columbia and the research grant provided by Canadian Natural Sciences and Engineering Research Council (NSERC) are gratefully acknowledged.

## Chapter 1

# Introduction

#### 1.1 General

Structures are subjected to different forms of loading during their life span, the most common of which are constant or so-called static loads. However, structures may also be subjected to loads which vary in magnitude, direction and time, that is, dynamic loads, such as wind loads, missile loads, earthquake loads, etc. Relationships describing structural and material response to static or quasi-static loads are well developed. On the other hand, the nature of the response to dynamic loading, its magnitude and variation with time, are complicated and not well understood. The inherent difficulty in assessing the effects of such loads is their short duration of action.

Dynamic loads may be classified as either single-cycle or multi-cycle. A moving mass striking a stationary object is an example of a single-cycle dynamic load, i.e. an impact load, and earthquake loading of a structure is an example of multi-cycle dynamic loading. In both cases it is evident that the load variation with time is rapid compared to that of quasi-static loading.

Impact loadings can be further divided into two categories: single-point and distributed impact loading. A structure hit by a missile is a single-point impact loading, whereas a shock wave loading is a distributed impact loading. When considering single-point impact loading, three major situations can arise due to the relative sizes of the impacting and impacted objects. These are:

1. a very large object struck by a small mass;

2. an impact involving comparable masses; and,

3. a small object struck by a large mass.

Dynamic loading of a structure is generally characterized by the rate at which strain or stress in the structural material increase, i.e. the strain or stress rate. Quasistatic loadings are associated with very low strain rates and impact loadings with higher strain rates. Most materials are strain rate sensitive, that is, their behavior and properties vary with changes in the rate at which they are strained.

Portland cement concrete is strain rate sensitive. Its inherent brittle nature and weakness under tensile loading conditions are generally overcome by reinforcing it with conventional steel reinforcement or with discrete, non-continuous fibers of various forms. Therefore, a better understanding of the behavior of plain concrete is necessary before meaningful analysis can be carried out for fiber-reinforced or conventionally reinforced concrete, to ascertain their behavior.

The fracture behavior of concrete subjected to impact loading can be characterized by the dynamic fracture toughness parameter. Especially, this approach is suitable to concrete due to its brittle nature where linear elastic fracture mechanics (LEFM) may be applicable. Comparison of the corresponding static fracture toughness to the dynamic fracture toughness is one way to analyze the strain rate effect.

## 1.2 Research Objective

The objective of the research project described in this thesis was to study the behavior of notched, portland cement concrete beams subjected to single-cycle dynamic or impact loading. The study concentrated on the variation of the dynamic fracture toughness parameter with varying notch depths and span lengths. Relative performance of steel fiber-reinforced concrete beams subjected to the same geometrical and loading conditions was also examined.

### 1.3 Thesis Review

Chapter 2 deals with the derivation of the equations necessary for the evaluation of the dynamic fracture toughness for mode I fracture conditions. The evaluation of inertial loads, the application of static calibration relationships to the calculation of dynamic loads, and the derivation of dynamic fracture toughness equations, are included. Chapter 3 deals with the experimental aspects of the impact tests and describes the important individual components of the test system. The analytical techniques used to treat the experimental data are described in Chapter 4; an example of the data reduction procedure is presented. In Chapter 5, the results of the experimental work, and their significance as to the variation of the dynamic fracture toughness with various geometrical and loading conditions, are presented and discussed. The conclusions reached and the suggestions for future study are reported in Chapter 6.

#### 1.4 Literature Review

Fracture mechanics concepts have been applied to Portland cement concrete by many researchers as the brittle nature of its behavior in tension is well suited to this approach. Mindess (1983) has given an extensive literature review on the applicability of LEFM to concrete.

LEFM was first applied to concrete by Kaplan (1961). He evaluated the critical strain energy release rate,  $G_{IC}$ , under flexural loading conditions. A specimen size dependency was noted and pre-peak, slow crack growth was postulated. However, there has been controversy over the use of LEFM for concretes. Shah and McGarry (1970) pointed out that, due to geometrical effects, valid fracture mechanics parameters cannot be obtained using LEFM. Swartz (1985) conducted many experiments on concrete flexure beams in order to obtain consistent values for the critical stress

intensity factor,  $K_{IC}$ , and  $G_{IC}$ . By using a dye penetration technique, he found that the crack tip process zone was very small, and hence concluded that LEFM is applicable to concrete. However, Shah (1988) has measured slow crack growth using special crack gauges and has concluded that to use LEFM and obtain specimen independent fracture toughness values, inclusion of the pre-peak slow crack growth is necessary.

Early investigators of strain rate effects on concrete, such as Watstein (1953), Green (1964), McNeely and Lash (1963), found that properties such as tensile strength, compressive strength and elastic modulus increase with strain rate. Thermally activated models have been used by Charles (1958) to predict the stress rate dependency of the fracture stress of brittle materials. He derived a relationship between the fracture stress,  $\sigma_f$ , and applied stress rate,  $\dot{\sigma}$ , as:

$$\sigma_f \propto \dot{\sigma}^{\frac{1}{N+1}}, \tag{1.1}$$

where N is a constant.

The effect of strain rate on fracture mechanics parameters has also been studied (Evans, 1974; Pollet and Burns, 1977; John and Shah, 1986; Banthia, 1987). Evans (1974), based on Eq. 1.1, proposed a relationship between crack velocity and stress intensity factor as:

$$V \propto K_I^N \tag{1.2}$$

where,

V = the crack velocity; and,

 $K_I$  = the stress intensity factor for mode I fracture.

The results of impact tests conducted by Gopalaratnam et. al. (1984) have suggested that N decreases at higher strain rates. Mindess (1985) reported a value of N = 30for crack velocities from  $10^{-6}$  to  $10^{-3}$  m/s based on slow crack growth studies. The

above observations are based on the evaluation of the stress intensity factor by static formulae.

Rate sensitive models have been proposed by Suaris and Shah (1984 and 1985) to predict high rate sensitivity in tension as compared to compression. Furthermore, Suaris and Shah (1984) performed variable strain rate tests (strain rates of  $0.67 \times 10^{-6}$  to 0.27) on mortar specimens under flexural loading conditions. It was observed that the increase in flexural strength with increasing strain rate was lower for materials with higher static flexure strengths.

The unavailability of closed form solutions for dynamic conditions is one of the contributing factors to the use of static stress intensity formulae for these cases. Dynamic loading conditions will require proper dynamic analysis. Analysis of the dynamic stress intensity factor around propagating cracks have been developed by Yoffe (1951) and Freund (1972). Yoffe (1952) concluded that crack branching would occur at a crack velocity of approximately  $0.6C_2$ , where  $C_2$  is the shear wave velocity. Freund (1972) derived a dynamic elastic solution for crack growth due to general loading and concluded that, for crack extension at constant rate :

$$K_{ID} = k(V) K_{IS} \tag{1.3}$$

where,

 $K_{ID}$  = the dynamic stress intensity factor;  $K_{IS}$  = the static stress intensity factor for mode I fracture; and, k(V) = a crack velocity dependent term.

Bienert and Kalthoff (1981) conducted wedge-loaded, double cantilever beam tests on transparent and opaque materials, and obtained  $K_{ID}$  values by observing the fringe patterns and applying the method of caustics. The crack velocities were measured by a high speed camera in their experiments, which enabled them to obtain the required

relationship. However, their additional work on instrumented impact testing of flexure specimens showed that the stress intensity factor continued to increase monotonically after the load had passed its peak and was decreasing. Therefore, they concluded that the results of the instrumented impact testing may be misleading if the associated load measurements are not corrected for inertia effects. They pointed out that the  $K_{ID}$  values obtained by the method of caustics would be more accurate than those from instrumented impact tests. However, methods based on fringe pattern observations are not applicable to concrete.

The relationship between crack velocity and dynamic stress intensity factor has been analyzed in great detail for materials such as resins and plastics (Kalthoff, 1985). It has been found, for these materials, that the relationship between  $K_{ID}$  and V shows a unique trend.  $K_{ID}$  seems to be a constant up to a particular crack velocity, where it then increases very rapidly. These data were obtained through two different testing techniques, the method of caustics and photoelasticity. Though Kalthoff's (1985) results showed a unique trend, the significant values of  $K_{ID}$  and crack velocity were found to depend on the type of specimen used. Dally et. al. (1985) suggested that the errors can be extremely large for the experimental procedures used in Kalthoff's work due to the three-dimensional nature of the stress state at the crack tip, and concluded that more test results are needed to verify the reported relationships.

The measurement of crack velocity is required to determine the relationship between  $K_{ID}$  and crack velocity, and many different techniques have been used in the past for crack growth measurements. Bhargava and Rehnstrom (1975) used high speed photography to study the crack growth in concrete beams subjected to explosive loading, and the same method was employed by Kinra and Kolsky (1977) on glass specimens. Huang et. al. (1978) adopted a ladder gage technique to study fast crack growth across interfaces in ceramics, and Lee et. al. (1981) made use of graphite rods to detect the progress of internal cracking in concrete. Mindess and Bentur (1985)

used high speed photography on specially illuminated concrete surfaces for the detection of crack growth. John and Shah (1986) mounted special brittle crack gages on concrete surfaces to detect continuous crack growth.

Many experimental methods for evaluating the response of engineering materials to single-cycle dynamic or impact loading have been developed, including:

1. free fall tests (Venzi et. al., 1970);

2. drop weight tests (Cotterell, 1962; Hibbert and Hannant, 1982);

3. explosive tests (Robin and Calderwood, 1978); and,

4. Hopkins split bar test (Kormeling, 1986).

Most of these methods can be used to determine the fracture mechanics properties of materials such as concrete, but instrumentation of the striking hammer and the specimen support is vital in order to obtain useful data. Instrumentation of the striking hammer is particularly important, since this provides the impact load-time history and thus allows many other parameters to be derived.

However, instrumented impact testing is not free from parasitic effects such as inertial loading of the specimen. Cotterell (1962) was the first to identify this phenomena and recognize that it is inherent in such a test system. He used elastic wave theory to account for the inertial oscillations and discontinuity in the load-time traces and concluded that at the beginning of the impact in Charpy tests, the response of the specimen is governed by the complex interaction of tensile and compressive stress wave propagation throughout the specimen. Subsequently, many others (Venzi et. al., 1970) have treated such inertial loading differently and have postulated that these loads may be taken into account by one of the following methods:

1. treating the Charpy specimen as an accelerating and decelerating body;

2. treating the striking hammer and specimen as an oscillating mass and spring system; or

3. treating the impact event as a wave propagation problem.

Sexton et. al. (1974) developed a model to account for inertial loadings of metallic specimens by considering rigid body accelerations. In addition, they related the magnitude of the inertial load discontinuity to the acoustic impedance of the striking hammer and the specimen, and the impact velocity, and found it in agreement with the theory drawn from elastic wave mechanism. They identified the types of loads contributing to the signal during an impact event as:

1. mechanical bending loads;

2. test system ringing; and,

3. inertial acceleration loads.

Server (1978) recommended that reliable data may only be obtained after three inertial oscillations, in which time the inertial effects would have died away. However, his recommendations, which are for metallic specimens, may not be applicable to concrete, since the failure time for concrete is much smaller (in the range of 0- $300 \mu$  seconds) than the inertial oscillation time postulated. The obvious aim, while testing for dynamic effects, is to obtain the true mechanical bending load, which can then be used to evaluate other required parameters. Therefore, it can be seen that non-accountability of other loads due to inertial accelerations and system ringing vibrations will introduce large errors.

There are other factors which are also identified as potential sources of error in dynamic testing. Wullaert (1970) pointed out that the dynamic load cell, data acquisition system and signal processing units used in such tests, are potential sources of

error and discussed methods to obtain reliable data. Methods for the dynamic calibration of load cells are also outlined by him. The importance of dynamic calibration has also been stressed by Hoover (1974), who proposed a practical calibration method for a load cell in a Charpy test system.

The system response time, which is the shortest possible time in which the data collected is a true representation of the actual signal (and there is no distortion of the actual signal), is another source of error. Ireland (1974) suggested methods for evaluating system response time, and concluded that if the time to fracture,  $t_f$ , is less than or equal to the response time,  $T_R$ , then the data collected has to be considered unreliable. Hoover (1974), used analog filtering of the dynamic load signal, and found that  $T_R$  and  $t_f$  increased, but the peak value of the signal decreased considerably. He also reported that the area under the dynamic load-time curve was not affected significantly. He concluded that higher response times are a result of analog filtering in the data acquisition process, and suggested that signals should not be filtered at this stage.

At the time when analytical procedures were being used to evaluate inertial load, other investigators were using sophisticated instrumentation and test system alterations to reduce such loads. Suaris and Shah (1982) introduced a rubber pad between the striking hammer and the test specimen at the contact point, and showed that such a modification reduced the inertial loads considerably. However, such modifications considerably reduce the strain rate, as well as the absorbed energy of the specimen, and thus the purpose of the test is defeated. Gopalaratnam et. al. (1984) instrumented the specimen support as well as the striking hammer in order to determine the actual bending load, and hence to evaluate the inertial load. Gopalaratnam and Shah (1986), based on previous experimental work, suggested guide lines for suitable test methods and specimen sizes, and concluded that inherent inertial load effects could be reduced significantly by: 1. reducing the impact velocity;

2. increasing the ratio of hammer mass to specimen mass; and,

3. increasing the ratio of specimen stiffness to that of the hammer-specimen contact zone.

In an impact test, the energy stored in the striking hammer is transferred to and absorbed by the test specimen into the following forms:

1. recoverable strain energy of the specimen and the testing machine;

2. vibration energy of the specimen;

3. kinetic energy of the specimen prior to fracture;

4. work required for fracture; and,

5. rotational kinetic energy of the specimen after fracture.

Lueth (1974) and Iyer and Miclot (1974) have discussed the above and suggested methods for taking them into account in the analysis. It is also possible that the local crushing of the test specimen at the point of contact would absorb considerable energy.

## Chapter 2

# **Theoretical Considerations**

## 2.1 Introduction

The analysis of dynamic fracture mechanics requires a sound understanding of basic fracture mechanics concepts as applied to static loading. The extension of static analysis to the dynamic condition requires proper knowledge of the forces acting on the specimen in the dynamic loading case. In particular, the inertial force, which is the additional force acting due to dynamic effects, has to be evaluated in order to apply the static formulae.

In this chapter, some basic fracture mechanics concepts are described and general equations which are used in fracture analysis are presented. The derivation of the dynamic fracture mechanics equations from the general static fracture mechanics equations, is also described. The equations required for proper dynamic analysis and evaluation of the inertial forces, which include the rotational inertia as well as the linear inertia, are derived for the notched beam case. Finally, the corrections necessary for the application of static calibration to the dynamic load signals are analyzed, and two methods are proposed for such corrections.

## 2.2 Static Analysis

For a two-dimensional problem, starting from the equilibrium equations, for an isotropic material, the governing compatibility equation is (Fig. 2.1):

$$\frac{\partial^4 \psi}{\partial x^4} + 2 \frac{\partial^4 \psi}{\partial x^2 \partial y^2} + \frac{\partial^4 \psi}{\partial y^4} = 0, \qquad (2.1)$$





Figure 2.1: Stresses Near Crack Front

which may be conveniently written as (Broek, 1982):

$$\nabla^2 \left( \nabla^2 \psi \right) = 0, \tag{2.2}$$

where:

$$\nabla^{2} = \frac{\partial^{2}}{\partial r^{2}} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial^{2}}{\partial \theta^{2}}, \qquad (2.3)$$

for the polar coordinate system (r and  $\theta$  are the distance and angle of the point under consideration from the origin).

The solution to Eq. 2.1, the biharmonic compatibility equation, is as follows (Broek, 1982):

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) \left[1 - \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right)\right] + \sigma_0 + O(r^{\frac{1}{2}}); \quad (2.4)$$

$$\sigma_{y} = \frac{K_{I}}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) \left[1 + \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right)\right] + O(r^{\frac{1}{2}}); \qquad (2.5)$$

and,

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{3\theta}{2}\right) + O(r^{\frac{1}{2}}), \qquad (2.6)$$

where  $\sigma_0$  is the far field applied stress, and  $O(r^{\frac{1}{2}})$  are higher order terms which do not contribute to the crack tip singularity stresses.

The near field stresses, omitting the higher order terms, may therefore be written as:

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta), \qquad (2.7)$$

where the notations are as in Fig. 2.1, and  $K_I$  is the stress intensity factor for mode I fracture (Fig. 2.2).

For an infinitely large plate, the parameter  $K_I$  is defined as:

$$K_I = \sigma_0 \sqrt{\pi a}.$$
 (2.8)

were 'a' is the half crack length at center. Therefore, the stresses at a point in the near field (Eqs. 2.4, 2.5, and 2.6) are given by the stress intensity factor and the position of the point relative to the crack tip.



Figure 2.2: Basic Modes of Crack Surface Displacement

It should be noted that Eqs. 2.4, 2.5 and 2.6 are the solutions for a two-dimensional

problem. If the stresses and strains are required for the three dimensional case, then,

1. for plane strain conditions:

$$\sigma_z = \nu(\sigma_x + \sigma_y),$$

 $\epsilon_z = 0;$ 

and

2. for plane stress conditions:

$$\sigma_z=0,$$

and

$$\tau_{xz}=\tau_{yz}=0.$$

The corresponding displacements for the two-dimensional case are given by:

$$v_{x} = \frac{K_{I}}{E} \left( \sqrt{\frac{r}{2\pi}} \right) \cos \left( \frac{\theta}{2} \right) \left[ 1 - 2\nu + \sin^{2} \left( \frac{\theta}{2} \right) \right], \qquad (2.9)$$

$$v_{y} = 2(1+\nu)\frac{K_{I}}{E}\left(\sqrt{\frac{r}{2\pi}}\right)\sin\left(\frac{\theta}{2}\right)\left[2-2\nu+\cos^{2}\left(\frac{\theta}{2}\right)\right],\qquad(2.10)$$

and

 $v_z = 0,$ 

where E is the elastic modulus,  $\nu$  is the Poisson ratio, and  $v_x$ ,  $v_y$  and  $v_z$  are displacements in x, y and z directions respectively.

Similar expressions can be derived for stresses and displacements under mode II and mode III fracture.

Linear elastic fracture mechanics (LEFM) assumes that there is negligible yielding ahead of the crack tip. That is, except a very small region surrounding the crack tip, the rest of the region behaves elastically. The solutions derived from Eq. 2.1 conform to this assumption.



Figure 2.3: Notched, Three-Point Bending Test Specimen

Eqs. 2.4, 2.5 and 2.6 are the solutions for stress application at an infinite boundary. However, when the boundaries are finite, some correction for boundary distances and conditions has to be made. There are many correction methods available, including (Tada et. al., 1973):

1. boundary collocation method;

2. Green's function method; and,

3. empirical formulation from finite element analysis.

Using the third method with a curve fitting solution, the stress and the stress intensity factor for a three-point bending test specimen with a crack length 'a', a depth 'B', and a span 'S' (Fig. 2.3), may be written as:

$$\sigma_0 = \frac{6PS}{4TB^2};\tag{2.11}$$

and,

$$K_I = \sigma_0 \sqrt{\pi a} \ F\left(\frac{a}{B}\right), \qquad (2.12)$$

where P is the center point bending load,  $\sigma_0$  is the bending stress, and T is the thickness of the specimen.  $F\left(\frac{a}{B}\right)$  is a geometrical correction factor which accounts

for the finite boundaries of the test configuration. For example,

for 
$$\frac{S}{B} = 4$$
,  
 $F\left(\frac{a}{B}\right) = 1.090 - 1.735 \left(\frac{a}{B}\right) + 8.20 \left(\frac{a}{B}\right)^2 - 14.18 \left(\frac{a}{B}\right)^3 + 14.57 \left(\frac{a}{B}\right)^4$ , (2.13)  
and for  $\frac{S}{B} = 8$ ,

$$F\left(\frac{a}{B}\right) = 1.107 - 2.120\left(\frac{a}{B}\right) + 7.71\left(\frac{a}{B}\right)^2 - 13.55\left(\frac{a}{B}\right)^3 + 14.25\left(\frac{a}{B}\right)^4.$$
 (2.14)

The critical value of  $K_I$  is that which occurs at the maximum bending load,  $P_{max}$ , and is termed the critical stress intensity factor,  $K_{IC}$ , for mode I crack opening.

The definition of  $K_I$  given in Eq. 2.12 is only applicable under static or quasistatic loading conditions, the implicit assumption being that the only force acting on the body is the quasi-static external load P. Body forces, such as the inertial forces that occur due to acceleration, as in the case of dynamic loading, are not present. If and when such forces do occur, the system will have to be reduced in some manner to one equivalent to that shown in Fig. 2.3, to make use of Eq. 2.12. The next section of this chapter discusses the justification for the use of static stress intensity factor formulae for the dynamic stress conditions for such a reduced system.

# 2.3 Relationship of Static and Dynamic Stress Intensity Factors

The relationships between the quasi-static stress intensity factor,  $K_{IS}$ , and the dynamic stress intensity factor,  $K_{ID}$ , has been examined by Freund (1972) and Rose (1976). This relationship is of the form:

$$K_{ID} = f(V_{cr}) K_{IS},$$
 (2.15)

where  $f(V_{cr})$  is a crack velocity function, which is given as:

$$f(V_{cr}) = \frac{1 - \frac{V_{cr}}{C_R}}{\left(1 - hV_{cr}\right)^{0.5}}.$$
 (2.16)

#### Chapter 2. Theoretical Considerations

In Eq. 2.16, h is a function of the wave speeds and the Poisson ratio of the material, and  $C_R$  is the Rayleigh wave speed.

For Poisson ratio,  $\nu = 0.25$  (Rose, 1976),

$$C_0 = \sqrt{\frac{E}{\rho}};$$
$$C_B = 0.58C_0;$$

and,

$$h=\frac{0.87}{C_R},$$

where  $\rho$  is the density of the material. For concrete, the longitudinal wave speed,  $C_0$ , is approximately equal to 3300 m/s (Banthia, 1987) and the crack velocity,  $V_{cr}$ , is between 100 and 250 m/s for the beams tested in this study (Section 5.1). Hence,  $f(V_{cr})$  in Eq. 2.16 becomes approximately equal to unity, and the dynamic stress intensity factor can be taken as equal to the static stress intensity factor.

By a similar argument, from the equations derived by Freund (1973), it can be shown that the relationship between  $K_{IS}$  and the static energy release rate,  $G_{IS}$ , is applicable to the dynamic case as well, i.e.:

$$G_{ID} = \frac{K_{ID}^2}{E'},$$
 (2.17)

where:

 $G_{ID}$  = the dynamic energy release rate;  $E' = E/(1 - \nu^2)$  for plane strain conditions; and, = E for plane stress conditions.

Therefore, if the true bending load is calculated as outlined in the next section, using the static formulae, the dynamic stress intensity factors can be calculated.

## 2.4 Dynamic Analysis

#### 2.4.1 Introduction

The basic equation of motion for a single degree of freedom (SDOF) body may be written as:

$$H\ddot{w} + C\dot{w} + Qw = P(t), \qquad (2.18)$$

where:

H =the mass of the body;

C = a damping constant;

Q = the stiffness of the body;

P(t) = an externally applied time-varying force;

w = the displacement of the body from the equilibrium position;

 $\dot{w}$  = the velocity of the body; and,

 $\ddot{w}$  = the acceleration of the body.

The first and the second time-dependant terms in Eq. 2.18 represent the inertial and the damping forces in the system, respectively. The inertial force, in fact, is the time derivation of the kinetic energy of the body.

The analysis of a beam specimen subjected to impact loading will therefore require proper evaluation of these dynamic forces to predict its equilibrium condition. The impact load can be treated as a time-dependant force, and the inertial forces acting on the beam can then be evaluated. To do so, however, some simplifying assumptions will have to be made in the analysis.

The total load measured during an impact test consists of the true mechanical loads, the inertial loads and the system vibration loads (Sexton et. al., 1974). As the aim of these experiments is to evaluate the true mechanical bending loads, and hence to determine the stress intensity factors, correct assessment of the inertial loads in the

#### Chapter 2. Theoretical Considerations

impact test is essential. The first identification of inertial loads in impact testing was by Cotterell (1962); since then, many others have proposed various interpretations for inertial loads evidenced in the load-time curves.

In an impact test in which the specimen mass is relatively small compared to the impacting mass, the inertial load component is small since it depends on the mass of the specimen. However, the system vibrations may be high. In the case of concrete beam testing, where the size and mass of the specimen are relatively large, the inertial load components can be as high as 50% of the total tup load (as will be shown in Chapter 4). Therefore, the inertial load evaluation becomes an important aspect in the dynamic impact testing of concrete.

In the analytical method adopted by Banthia (1987), suitable shape functions were assumed for the deflection of the impacted beam. In his case, where notched beams were not used, the assumption of the following sinusoidal or linear shape functions, may have been reasonable:

$$u(x,t) = u_0 \sin\left(\frac{\pi x}{l}\right), \qquad (2.19)$$

and

$$u(x,t) = u_0 \left(\frac{x}{l}\right), \qquad (2.20)$$

where:

x = the distance along the beam length; u(x,t) = the lateral displacement at x; l = the beam length;  $u_0$  = the displacement of beam center; and, t = time.

Eq. 2.19 was used for conventionally reinforced beams, and Eq. 2.20 was used for fiber-reinforced and plain concrete beams.

#### Chapter 2. Theoretical Considerations

The above equations may not be suitable for notched beams. Furthermore, Eqs. 2.19 and 2.20 do not consider the relevance of dynamic mode shapes. Since the problem at hand is of a dynamic nature, it is required that dynamic mode shapes must be established for the beam impact. It is also important to recognize that the crack length must be included in such relationships so that the inertial force calculations include components of crack length (i.e. notch sensitivity). However, the assumption of a SDOF is deemed to be still valid.

From the foregone discussion it is clear that selection of proper shape functions are the primary aim for inertial load evaluation. Some methods are now presented to obtain the total shape function, which is used to evaluate the inertial forces and hence the true mechanical bending load associated with an impact event.

## 2.4.2 Selection of Shape Functions For a Notched Beam



#### Figure 2.4: Notched Beam Configuration

The following are the important assumptions in the analysis to follow, which describes the methods for selecting the required shape functions and calculating inertial loads:

1. the beam is a one-dimensional continuum;
- 2. a generalized coordinate approach can be adopted, with the lateral deflection at the center point as the generalized coordinate;
- 3. calculation of the inertial load can be achieved by assuming a suitable total shape function incorporating the crack dimension;
- 4. incorporation of the instantaneous crack length will give the required crack length dependency of the inertial load; and,
- 5. removal of the inertial load from the center-point dynamic load, will reduce the system to the necessary form for analysis.

## 2.4.2.1 Method 1

The shape functions in this method are selected on the basis of static considerations. The total response (i.e. deflection) of the pre-cracked, three-point bend specimen, with a crack length 'a', is composed of two components:

1. the uncracked beam segment, which acts in an elastic manner; and

2. the cracked beam segments, which act as two identical rigid bodies.

Therefore, the total shape and displacements may be described as:

$$U(x,t) = U_0(t)\phi_T,$$
 (2.21)

where:

U(x,t) = the lateral displacement at x;  $U_0$  = the lateral displacement of the center point; and,  $\phi_T$  = the total shape function.

It is assumed that the total shape function  $\phi_T$  may be written as:

$$\phi_T = \left(\frac{B-a}{B}\right)\phi_1 + \left(\frac{a}{B}\right)\phi_2, \qquad (2.22)$$

where  $\phi_1$  and  $\phi_2$  are the shape functions associated with the uncracked and the cracked portions of the beam.

#### Evaluation of $\phi_1$

The evaluation of  $\phi_1$  is based on an exact static solution of the beam bending problem for a center-point load configuration. For a beam of length '2l' and a centerpoint loading of 'P', and measuring distances from one end of the beam:

$$U_0 = \frac{Pl^3}{6EI}.\tag{2.23}$$

The displacement U(x) may be written as:

$$U(x) = \frac{Plx}{6EI(2l)} \left[ 3l^2 - x^2 \right], \qquad (2.24)$$

where I is the moment of inertia. Substituting Eq. 2.23 into Eq. 2.24 and eliminating the EI terms, the result may be written as:

$$U(\mathbf{x}) = U_0 \left[ \frac{3}{2} \left( \frac{\mathbf{x}}{l} \right) - \frac{1}{2} \left( \frac{\mathbf{x}}{l} \right)^3 \right].$$
 (2.25)

From Eq. 2.25 it can be seen that the shape function  $\phi_1$  can now be written as:

$$\phi_1 = \left[\frac{3}{2}\left(\frac{x}{l}\right) - \frac{1}{2}\left(\frac{x}{l}\right)^3\right],\tag{2.26}$$

where Eqs. 2.25 and 2.26 are applicable for  $0 \le x \le l$ . Evaluation of  $\phi_2$ 

As the beam forms two equal halves at the crack plane when it fractures completely, causing a triangular displacement shape,  $\phi_2$  can be written as:

$$\phi_2 = \frac{x}{l}.\tag{2.27}$$

The derived shape functions, given by Eqs. 2.26 and 2.27, satisfy the necessary boundary conditions of the simply supported beam case: at x = 0,

$$\phi_1=0,$$

and

$$\phi_2=0;$$

and at x = l (the beam center point),

$$\phi_1=1,$$

and

$$\phi_2=1.$$

Substituting the above into Eq. 2.21, it follows that:

$$U(l) = \left(\frac{B-a}{B} + \frac{a}{B}\right)U_0 = U_0.$$

The moments at the ends also can be proved to be zero. Therefore, the obtained total shape function is satisfactory with respect to the boundary conditions. It can also be noted that the shape function given by Eq. 2.22 contains the element of crack length.

#### 2.4.2.2 Method 2

It is evident that the analysis of method 1 is inadequate in the sense that it does not deal with the dynamic mode shapes of the vibration problem. Method 1 is based purely on static considerations and thus forms a very simple analysis. This, however, is justifiable to some extent, as static procedures will be used for the stress intensity calculations.

Considering the impact problem as a total dynamic event, the relevant dynamic mode shapes may also be used to derive the total shape function. This second method

is therefore based on elasticity equations which satisfy the equilibrium and compatibility conditions.

The total shape function  $\phi_T$  is defined as in Eq. 2.22, and it is necessary to evaluate  $\phi_1$  and  $\phi_2$  as before.

The dynamic equilibrium of a distributed parametric system, considering elastic forces and inertial forces, may be written as (Clough and Penzien, 1982):

$$EI \frac{\partial^4 u}{\partial x^4} + m \frac{\partial^2 u}{\partial t^2} = 0, \qquad (2.28)$$

where:

m = the mass; and, u = the lateral displacement.

Eq. 2.28 is the basic equation for free vibrations, with the effects of damping and rotational inertia omitted. This equation can be rewritten as:

$$\frac{\partial^4 u}{\partial x^4} + \frac{m}{EI} \frac{\partial^2 u}{\partial t^2} = 0,$$

with a solution of the form:

$$u(x,t) = \phi(x)Y(t), \qquad (2.29)$$

where,  $\phi(x)$  is a constant shape function and Y(t) is a time dependant function. The solution to Eq. 2.29, is of the form

$$\frac{\partial^4 \phi}{\partial x^4}(x) - b^4 \phi(x) = 0, \qquad (2.30)$$

and

$$\ddot{Y}(t) + \lambda^2 Y(t) = 0,$$
 (2.31)

where  $\lambda$  is a constant and,

$$\lambda^2 = \frac{b^4 E I}{m},$$

and b is a constant which depends on the mode of vibration under consideration. The most general form of the solution for  $\phi(x)$ , for a beam problem, can be written as:

$$\phi(x) = A_1 \sin(bx) + A_2 \cos(bx) + A_3 \sinh(bx) + A_4 \cosh(bx), \quad (2.32)$$

where the constants  $A_n$  define the shape of the vibrating beam. These constants can be evaluated for specific cases based on the boundary conditions for the beam. The solution of Eq. 2.32 will yield the frequency equation necessary to calculate the value of 'b' for different modes of vibration. To obtain the values of  $A_n$  by the substitution of boundary conditions, the higher order derivatives of Eq.2.32 are necessary. Evaluation of  $\phi_1$ 

For the case of a simply supported, vibrating beam of length = 2l, the boundary conditions may be written as:

at 
$$x = 0$$
 and  $x = 2l$ ,

$$\phi_1(0) = \phi_1(2l) = 0; \tag{2.33}$$

and

$$M(0) = M(2l) = 0 = EI\phi'', \qquad (2.34)$$

where M is the bending moment. By substitution of Eqs. 2.33 and 2.34 into Eq. 2.32 and its higher order derivatives, the frequency equation may be written as:

$$sin(2bl) = 0.$$
 (2.35)

The solution of Eq. 2.35 may then be written as:

$$\phi_n = A_1 \sin \left(\frac{n\pi}{2l}\right) x. \tag{2.36}$$

Assuming the fundamental mode of vibration (n = 1), it can be shown that:

$$\phi_1(x) = A_1 \sin\left(\frac{\pi x}{2l}\right), \qquad (2.37)$$

where  $A_1$  is a constant and can be selected to suit the actual test conditions.

#### Evaluation of $\phi_2$

The selection of shape function for the vibration of the completely fractured half beam is based on the following points:

- 1. the broken half is assumed to be vibrating as a beam with one end pinned and other end free; and,
- 2. the free end is assumed to be the fractured end.

The following are the boundary conditions applicable to this case: at x = 0,

$$\phi_2(0)=0,$$

and

$$M(0) = 0 = EI\phi''(0). \tag{2.38}$$

At  $\mathbf{x} = l$ ,

$$\phi''(l) = 0 = \phi'''(l) = 0, \qquad (2.39)$$

where  $\phi''$  and  $\phi'''$  are the second and third derivatives with respect to x. Eq. 2.39 also indicates the moment and shear conditions. By substitution of the boundary conditions (Eqs. 2.38, 2.39) into the higher derivatives of Eq. 2.32, it is possible to obtain the frequency equation as:

$$sin (bl)cosh (bl) = cos (bl)sinh (bl),$$

which leads to:

$$tan (bl) = tanh (bl). \tag{2.40}$$

The approximate solution to the Eq.2.40 is available in the literature (see Harry, 1962) and may be written as:

$$b = \left(\frac{4n+1}{4}\right)\frac{\pi}{l}.$$
(2.41)

Assuming the fundamental mode of vibration (n = 1), choosing the amplitude to be unity at the free end, and substituting into Eq. 2.32, gives:

$$\phi_2(x) = -\frac{1}{\sqrt{2}} \left[ \sin\left(\frac{5\pi x}{4l}\right) + \frac{\sin\left(\frac{5\pi}{4}\right) \sinh\left(\frac{5\pi x}{4l}\right)}{\sinh\left(\frac{5\pi}{4}\right)} \right].$$
(2.42)

Combining Eqs. 2.37 and 2.42, the total shape function may be written as:

$$\phi_T = \frac{a}{B} \sin\left(\frac{\pi x}{2l}\right) + \frac{a-B}{\sqrt{2}B} \left[\sin\left(\frac{5\pi x}{4l}\right) + \frac{\sin\left(\frac{5\pi}{4}\right)\sinh\left(\frac{5\pi x}{4l}\right)}{\sinh\left(\frac{5\pi}{4}\right)}\right].$$
 (2.43)

The procedures described above give two methods to evaluate the necessary shape functions. The second method is more suitable for the dynamic analysis, because it takes into consideration the dynamic mode shapes, which in turn is a better representation of the actual dynamic force components involved.

## 2.4.3 Derivation of Inertial Loads

This section describes the method of evaluating the inertial loads from the shape functions derived in the previous section. The derivation is for a general case; by substituting suitable shape functions it is possible to obtain particular solutions.

Starting with Eq. 2.21 and differentiating with respect to time, the velocity and acceleration along the beam may be obtained as:

$$\dot{U}(x,t) = \dot{U}_0 \phi_T + V_{cr} U_0 \frac{\partial \phi_T}{\partial a}, \qquad (2.44)$$

and

$$\ddot{U}(x,t) = \ddot{U}_0 \phi_T + 2\dot{U}_0 V_{cr} \frac{\partial \phi_T}{\partial a}, \qquad (2.45)$$

where,

$$\dot{U}(x,t)$$
 = the velocity of any point x;  
 $\dot{U}_0$  = the velocity of the beam center point;

 $V_{cr}$  = the velocity of the crack;

$$U(x,t)$$
 = the acceleration at any point x; and,

$$U_0$$
 = the acceleration of the beam center point

Eqs. 2.44 and 2.45 represent the velocity and acceleration of any point in terms of the center-point velocity and acceleration, along with the crack length and velocity.

Now, using the principle of virtual work to determine the equivalent center-point inertial forces at dynamic equilibrium:

$$\delta W_{inertia} = 2 \int \bar{m} \ddot{U} \delta \left( U_0(t) \phi_T(x) \right) \, dx, \qquad (2.46)$$

and solving further,

$$\delta W_{inertia} = 2 \int \left( \bar{m} \ddot{U} \phi_T(x) \right) dx \, \delta U_0(t), \qquad (2.47)$$

where,

 $\delta W_{inertia}$  = the virtual work;  $\bar{m}$  = the mass per unit length; and,  $\delta U_0$  = the virtual displacement of the center point.

The equivalent center-point load,  $P_{inertia}$ , may now be calculated from:

$$P_{inertia} \delta U_0(t) = \delta W_{inertia},$$

which leads to an expression for  $P_{inertia}$  as:

$$P_{inertia} = 2 \int_0^l \left( \bar{m} \ddot{U} \phi_T(x) \right) dx. \qquad (2.48)$$

Now, substituting Eq. 2.45 into Eq. 2.48 and simplifying:

$$P_{inertia} = \left[ \int_0^l 2\bar{m}\phi_T^2 dx \right] \ddot{U}_0 + \left[ 4V_{cr}\bar{m} \int_0^l \phi_T \frac{\partial\phi_T}{\partial a} dx \right] \dot{U}_0.$$
(2.49)

The rotational inertia of the uncracked portion of the beam also will have to be considered in the analysis. It should be noted that at the initial formulation stage, the rotational inertia was omitted in order to reduce the complexity of the problem. In a similar fashion to the Eq. 2.46, the equivalent center-point load  $P_{rot}$  (due to rotational inertia) may be calculated as:

$$P_{rot}(t) = 2 \int_0^l \rho I \ddot{U}_0 \left(\frac{\partial \phi_T}{\partial x}\right)^2 dx + 4 \int_0^l \rho I U_0 V_{cr} \left(\frac{\partial \phi_T}{\partial x}\right) \left(\frac{\partial \phi_T}{\partial a}\right) dx.$$
(2.50)

From the Eqs. 2.49 and 2.50, it is now possible to calculate the total inertial resistive force,  $P_{resist}$ , as:

$$P_{resist} = P_{inertia} + P_{rot}.$$
 (2.51)

 $P_{resist}$  is the total inertial force resulting from the dynamic acceleration of the beam, which is assumed to be concentrated in the beam center.

## 2.5 Derivation of True Bending Load

Eq. 2.51 can be used to obtain the true mechanical bending force of  $P_{bend}$  as:

$$P_{bend} = P_{tot} - P_{resist}, \qquad (2.52)$$

where  $P_{tot}$  is the total dynamic force, including the inertial effects, acting on the beam.  $P_{tot}$  is the load used in Eq. 2.12 to derive the necessary stress intensity factors.

## 2.6 Dynamic Load Calibration

One of the major problems associated with dynamic testing is the proper calibration of the load-measurement system. These systems usually consist of a strain-gauged elastic element which deforms during the dynamic event. The resulting strain in the element and the strain gauges produces an analog signal which is a function of the applied load. Such systems are typically calibrated, i.e. the relationship between applied load

and strain gauge output is determined, under static or quasi-static loading conditions. However, the response of the element and the strain-gauges to dynamic loading may be significantly different from that to static loadings, and therefore the reduction of data obtained under dynamic loading conditions using a static calibration relationship may lead to significant errors in the analysis.



Figure 2.5: Model for Dynamic Load Calibration

Under dynamic loading conditions, as in an impact event, the strain gauges sense the inertial loads and the damping forces in the elastic element, as well as the elastic deformations (Eq. 2.18). Therefore, the load sensed by the strain gauges is much larger for the dynamic case in comparison to the static loading case. This load increase caused by the inertial effects of the elastic element is termed the dynamic amplification and may be characterized by the dynamic amplification factor, D. This

30

factor will depend on the mass of the elastic element and its damping characteristics. If only static calibration procedures are used, then the load measured during the dynamic event will have to be reduced due to the dynamic amplification effects of the measuring elastic element, in order to obtain the true dynamic load. The static calibration procedure is explained in Chapter 3.

The basic problem in such elastic element systems, is the evaluation of the mass and the stiffness of the system. There may be other components also adding stiffness to the system by other means, which are difficult to evaluate. In general, it is difficult to carry out dynamic calibrations, which may be affected by many external components. Therefore, evaluating the true dynamic forces by the static calibration process and correcting for the dynamic amplification, would be a better method. The next section presents two methods to evaluate the dynamic amplification factor, D.

#### **2.6.1** Method 1

It is assumed that the elastic element behaves as a single degree of freedom (SDOF) body (Fig. 2.5), and the applied force is sinusoidal in nature,  $P_0 sin\omega t$ , where  $P_0$  is the static force and  $\omega$  is the frequency of the load. The strain gauge is modelled as a ground and is located as shown in the figure. With  $P_0$  as the static load in case of a non-dynamic event, equations can be derived to calculate the total force transmitted to the ground (strain gauge) as a function of the system natural frequency and the applied load frequency. This will then lead us to the calculation of D, the dynamic amplification factor.

If the force reaching the ground is  $F_f$ , then (Fig. 2.5),

$$F_f = Qw + C\dot{w},\tag{2.53}$$

where

w = the distance from the equilibrium point;

31

Q = the stiffness up to point of the strain gauge; C = the damping coefficient; and,  $\dot{w}$  = the velocity.

The solution for a simple harmonic motion (Clough and Penzien, 1982) can be written as:

$$x = \frac{P_0}{Q} D_d \sin(\omega t - \Psi), \qquad (2.54)$$

where

 $D_d$  = the overall dynamic amplification factor; and  $\Psi$  = the phase angle.

Now, differentiating Eq. 2.54 and substituting into Eq. 2.53 gives:

 $F_f = P_0 D_d \sqrt{1 + \frac{C\omega^2}{Q}} \quad \sin(\omega t - \Psi_1), \qquad (2.55)$ 

where  $\Psi_1$  is the new phase angle. From Eq. 2.55, the maximum of  $F_f(t)$ ,  $F_{fmax}$ , is:

$$F_{fmax} = F_0 D_d \sqrt{1 + 2\xi \beta^2}, \qquad (2.56)$$

where

$$\beta = \frac{\omega}{\omega_n},$$

and

$$\xi = \frac{C\omega}{2Q\beta}.$$

 $\omega_n$  is the natural frequency of the body,  $\beta$  is the ratio of load frequency to the natural frequency of the beam, and  $\xi$  is the damping coefficient. Therefore, from Eq. 2.56, the dynamic amplification (D) sensed at the strain gauge can be deduced as:

$$D = \frac{F_{fmax}}{P_{max}},$$

which leads to:

$$D = \frac{\sqrt{1 + (2\xi\beta^2)}}{\sqrt{(1 - \beta^2)^2 + (2\xi\beta^2)^2}}.$$
 (2.57)

Eq. 2.57, which contains the damping coefficient, can further be reduced for the undamped ( $\xi = 0$ ) case, so that:

$$D = \frac{1}{1 - \beta^2}.$$
 (2.58)

From Eq. 2.58, it can be seen that, if the natural frequency of the structure,  $\omega_n$ , and the external force frequency,  $\omega$ , are almost equal, the D will be high, resulting in excessive signal output; this is the resonance condition.

## 2.6.2 Method 2

This method assumes that the impact load is a one-half sine wave pulse, and that the maximum response occurs in the impact event era. For the case under consideration, the impact event era response, including the transient component, may be written as:

$$\mathbf{x}(t) = \frac{P_0}{Q} \frac{1}{1 - \beta^2} \left( \sin \left( \omega t \right) - \beta \sin \left( \omega_n t \right) \right). \tag{2.59}$$

Eq. 2.59 is applicable for the vibrations of the undamped case. The magnitude of the dynamic response which results from the impact loading will depend on the impulse duration and the natural vibration of the body. The dynamic amplification factor, D, can be defined in this case as :

$$D=rac{x(t)}{\left(rac{P_0}{Q}
ight)}.$$

From Eq. 2.59, the maximum amplification factor for the fundamental mode of vibration, is:

$$D = \frac{2\beta}{1-\beta^2} \cos\left(\frac{\pi}{2\beta}\right).$$
 (2.60)

In Eq. 2.60, the term  $\beta$ , which is the ratio of the natural frequency of the body to that of the impact load, determines the amplification factor. The relationship between  $\beta$ and D for various types of impact loading are presented in a graphical form in the literature (see Clough and Penzien, 1982). The natural frequency of the body and the load frequencies are required for the calculation of  $\beta$ . The method of evaluating them are outlined in Chapter 4.

When the dynamic amplification factor has been determined by either of the methods suggested above, the actual dynamic load  $P_{tot}$  due to the impact event may be determined as:

$$P_{tot}(t) = \frac{P_{dyn}}{D},\tag{2.61}$$

where  $P_{dyn}$  is the output of the strain gauge signal converted into load units using the static calibration formula.

# Chapter 3

# **Experimental Aspects**

## 3.1 Introduction

The notched, plain and steel fibre reinforced concrete flexure specimens examined in this study were manufactured and tested using the materials, apparatus and procedures described in this chapter. The specimens were 100 mm by 100 mm in cross section and 410 mm or 720 mm long, allowing them to be tested over spans of 350mm and 720 mm respectively, with notch depths of 25, 35 or 50 mm.

## **3.2** Materials

The mix proportions of the concrete used in the specimens are coarse aggregate: fine aggregate: cement: water = 2.50: 2.25: 1.00: 0.45 by weight. CSA Type 10 normal Portland cement was used, with a maximum coarse aggregate size of 10 mm.

The fibers used are a brass-coated, cut wire manufactured by National-Standard Company. They are approximately 38 mm long with a diameter of 0.40 mm, giving an aspect ratio of about 95. Fiber contents of 2 and 4 percent by weight of the total weight of the concrete matrix were examined.

# **3.3** Specimen Preparation

## **3.3.1** Fresh Concrete Preparation

Generally, the procedures outlined in CSA A23.2-2C "Making Concrete Mixes in the Laboratory" and CSA A23.2-3C "Making and Curing Concrete Compression and Flexural Test Specimens" were followed. A pan-type mixer with a capacity of approximately 0.18 cubic meter was used to mix the fresh concrete. The mixing procedure for the plain concrete mixtures was as follows:

- 1. The coarse aggregate was first placed in the mixer with approximately 1 kg of water, and this material was mixed for about one minute.
- 2. The mixer was stopped, and the fine aggregate and the cement added.
- 3. The mixer was started again, and the remainder of the water added in a slow and uniform manner.
- 4. Once all the water had been added, the material was mixed for a period of 5 minutes.

Once the mixing was completed, a slump test was carried out in accordance with CSA A23.2-5C "Slump of Concrete".

Essentially the same procedure was followed for the fiber-reinforced mixes, with some of the fibers being added with the coarse aggregate, and the rest with the cement and fine aggregate.

## **3.3.2** Specimen Casting and Compaction

The flexure beams were cast in steel or plexiglass moulds which were clamped to a vibrating table. The moulds were filled in three layers, each layer being vibrated until full compaction was attained, i.e. until no air bubbles rose to the surface of

the fresh concrete; care was taken to avoid excessive vibration. The direction of the vibration was vertical and therefore parallel to the eventual loading direction for the test specimens.

Three cylindrical compression test specimens, nominally 100 mm in diameter and 200 mm in length, were also cast from each plain concrete and fibre reinforced concrete mix, generally in accordance with Sections 4 and 5 of CSA 23.2-3C.

## 3.3.3 Specimen Curing

After the removal from the vibrating table, the specimens were covered with polythene and left undisturbed for approximately 24 hours. They were then demoulded and placed in the laboratory moist curing room, where they remained until required.

## **3.3.4** Notching of Specimens

The flexure specimens were removed from the curing room after about 30 days and notched at the mid-span using a circular diamond saw. The desired notch depth (25, 35 or 50 mm) was set on the saw and the 3 mm wide notch cut as accurately as possible. The specimens were then returned to the curing room.

## 3.3.5 Preparation of Accelerometer Base and Crack Gauge

The specimens were removed from the curing room on the day before they were tested and the accelerometer base was attached and the crack gauge was prepared.

The accelerometer base, a circular piece of plastic containing a central threaded hole into which the accelerometer could be screwed, was fixed to the center of the bottom surface of the specimen, immediately next to the cut notch, using an epoxy adhesive; see Fig. 3.1.

The two circuit board strips for the crack gauge were fixed on one side of the



Figure 3.1: Crack Gauge and Accelerometer Fixture.

specimen using an epoxy adhesive, one on each side of the notch, as shown in Fig. 3.1. The conductive silver paint lines forming the crack gauge were then applied to the surface of the specimen, perpendicular to the notch and across the subsequent crack path. Care was taken to make the paint lines as thin as possible, but continuous between the appropriate points on the two circuit boards.

# **3.4** Testing Apparatus

## **3.4.1** Impact Tests

The equipment used to carry out impact tests is an important aspect of the experimental program. The impact event is completed in less than one millisecond for plain concretes and two milliseconds for fibre reinforced concretes, and the testing system must respond to such short duration events with precision and speed. The following points must therefore be taken into consideration in designing an appropriate system:

- 1. the testing machine should be able to deliver the necessary impact without losing excessive energy to components such as the supports, sliding guides, etc.;
- 2. the instrumentation should be able to capture sufficient data during the impact event that the whole event is effectively recorded; and,
- 3. the accuracy of the measuring instruments and the analog signal produced by the test components must represent the actual test situation.

The impact testing machine used in these experiments is a falling or drop weight machine built at the University of British Columbia. A photograph of the machine is shown in Fig. 3.2; the various elements of this machine are identified and its dimensions are shown in Fig. 3.3. The components of the testing system are shown schematically in Fig. 3.4.



Figure 3.2: Impact Machine.



Figure 3.3: Impact Machine Components and Dimensions

41



Figure 3.4: Testing System

#### 3.4.1.1 Tup



Figure 3.5: The Tup

The striking head, or tup, attached to the hammer assembly, is made of heat-treated high carbon steel. Two circular holes, 25 mm in diameter, are cut in the tup as shown schematically in Fig. 3.5. Strain gauges are fixed to the inside surface of the holes, and connected together to form a Wheatstone bridge circuit, as also shown schematically in Fig. 3.5. The important characteristics of the strain gauges are :

1. type : bonded,

2. resistance : 350.0  $\Omega \pm 0.3\%$ ,

3. gage Factor: 2.07  $\pm~0.5\%$  and

4. temperature coefficient :  $\pm 0.1\%$ .



The Wheatstone bridge output is balanced initially in a "no-load" condition.

Figure 3.6: Static Calibration Curve For Tup

The static calibration curve for the tup is shown in Fig. 3.6. This curve was obtained by loading and unloading the tup in an universal testing machine; the loading and unloading curves followed the same path. When a load is applied to the tup, it deforms and the strain gauge circuit becomes unbalanced and produces an output signal. The tup therefore acts as a load cell. During a test, the output signal from the strain gauge circuit is amplified and then collected by the data acquisition system for further processing.

## 3.4.1.2 Accelerometer

The accelerometer used was a piezoelectric sensor (Model 302A), manufactured by PCB Piezoelectronics, Inc., N. Y. The output from the accelerometer was sent to the data acquisition system, via a coaxial cable.

Some salient features of the accelerometer are:

- 1. resolution = 0.01g;
- 2. resonant frequency = 45 kHz;
- 3. maximum vibration / shock = 5000g; and,
- 4. load recovery  $< 10 \ \mu$  seconds.

### **3.4.1.3** Crack Gauge

The crack gauge employed to study crack propagation in the test specimen is shown schematically in Fig. 3.7. It essentially consists of lines of conductive silver paint (manufactured by G.C. Electronics) drawn perpendicular to the probable crack path as shown in Fig. 3.8. An electric current is passed through the paint lines; when a crack appears at the surface, the paint line breaks and the circuit voltage drops. The expected voltage drops are also shown in Fig. 3.7.

The crack gauge sensitivity depends on the thickness and width of the paint line; all necessary care was taken to draw the lines as thin as possible.

The crack gauge was powered externally by a 7 Volts DC power supply. The output voltage is fed to the data acquisition system without any signal amplification because the voltages were large enough. The voltage drop should be adequate enough so as to give a high resolution.









The discrete voltage drop obtained during the crack propagation gives an approximate crack speed which can be correlated with the tup load signal duration. More details are given in Chapters 4 and 5.

## 3.4.2 Data Acquisition System

The voltage signals obtained from the tup load cell, the accelerometer and the crack gauge are fed into the data acquisition system as shown schematically in Fig. 3.4. The tup load cell signal is amplified in the data acquisition unit, and the accelerometer has a built-in amplification unit. The data acquisition unit also has the ability of filtering the incoming analog signals. However, such filtering has a detrimental effect on these signals, as described and discussed in Chapter 4, and it was therefore decided to collect the raw signals and carry out digital filtering of the data to obtain meaningful results. The methods adopted for this procedure are also explained in Chapter 4.

The unfiltered analog signal is converted to a digital form, using an analog to digital (AD) conversion board, before being stored in the computer for processing. This conversion of signal format is necessary since the computer architecture can only handle digital wave forms. The AD board used in this series of experiments has a conversion speed of 1 MHz for one channel acquisition. When the data is acquired in n channels, the speed of the board reduces to 1/n MHz. Since three channels were used in these experiments, the time required to read one set of data is 3  $\mu$  seconds, which is adequate.

A scanning rate of  $15\mu$  seconds was actually used, which allowed data to be collected at intervals of  $15\mu$  second on all three channels. There is, however, a delay of  $1\mu$  second between each data point collected in any one instance. This is because of the computer internal architecture and its processing speed.

The number of data points collected is also dependant on the maximum addressable memory segment of the hardware, which is 64 K bytes. This allows 8192 data

points to be collected over a total time period of 81 millisecond, which is more than sufficient. The data collected before the impact event is used to identify external noise, and that collected after the impact event is used to identify dynamic characteristics of the hammer.

Some salient features of the AD board are:

1. 16-channel maximum input / output;

2. external instrument interface and scope driver software;

3. aggregate sampling rate up to 1MHz;

4. digital conversion with a 12 bit accuracy for a range of -10 V to +10 V; and,

5. memory buffer size 1 to 64 K (upgradable to 640 K).

The data collected from the AD board is written to the computer memory in a compressed mode, requiring a maximum of 64 K bytes. The data in the memory is then written to the mass storage media in an ASCII format for analysis by other programs. An IBM PC computer, with a random access memory (RAM) of 256 K is used for the data collection process. The software, Computerscope ISC-16, used for this purpose was purchased from RC Electronics, CA, USA.

### **3.4.3** Static Tests

The static, three-point flexural tests were carried in accordance with ASTM C 293-79 "Flexural Strength of Concrete (Using Simple Beam with Center-Point Loading)", using a closed-loop, electro-hydraulic testing machine operating under actuator stroke control. The center-point deflection of the specimen was measured with a LVDT transducer. The load applied was measured by a strain-gauged load cell connected to the testing machine. The signals from the load cell and the LVDT were sent to the X-Y plotter to produce a load-deflection curve.

# 3.4.4 Compression Tests

The compression tests were carried out in an Amsler compression testing machine using the maximum load range 50 kN (220,000 lbs).

# 3.5 Testing Procedures

#### **3.5.1** Impact Tests

The impact machine is controlled by an electro-pneumatic system. Components such as the breaks and locking pin are operated pneumatically while the hoist motor is operated by electrical means. The steps involved in carrying out an impact test are given below.

- 1. The hammer assembly is lifted by the hoist to the pre-determined height and the air breaks are applied.
- 2. The test specimen is mounted on the supports.
- 3. The accelerometer and the crack gauges are connected and checked for faults, and the tup load cell and accelerometer outputs are balanced to zero voltage.
- 4. The air breaks are released and the finer adjustments on the required drop height are carried out.
- 5. The photocell light required for triggering is checked and the computer is set to the external trigger mode, ready for data acquisition.
- 6. The hammer is dropped and as the specimen breaks, the air breaks are applied.
- 7. The hammer is hoisted back to the required height for the next test and the broken specimen is removed and new specimen is installed.

8. The memory contents of the computer, which stores the data, are saved on a mass storage media.

The impact event can be recorded only if the data acquisition system is triggered at the correct time. The triggering should start before the actual impact event has started and end well after the event is over. There are two broad classes of triggering mechanisms: external triggering and internal triggering. Internal triggering is not reliable for dynamic tests because the data acquisition system may be triggered well ahead of the actual impact event due to external noise levels and hence would miss the impact event data.



Figure 3.9: Photo Cell Assembly

In the impact tests carried out in this study, therefore, an external triggering mechanism was used. Since the tests are of short duration, it is not possible to trigger the data acquisition system manually, and hence a photo cell assembly, which is shown in Fig. 3.9 is used. The photo cell delivers a voltage of 2.5 volts when the hammer falls through and crosses its path. This voltage then activates the data

acquisition system.

## **3.5.2** Static Tests

The plain and fiber concrete specimens were tested with the crack gauge installed on these specimens as well. In addition to the load-deflection curves obtained from the X-Y plotter, the data acquisition system was used to collect the crack propagation and post-peak data during the tests. The crack gauge response, load cell signal and LVDT output were collected, at a scanning rate of 50 ms. For static tests, the data acquisition system was triggered by internal channel triggering, in which the triggering was accomplished whenever the voltage in the designated channel exceeded a particular value and was found satisfactory.

#### **3.5.3** Compression Tests

The compressive strength of the plain and fiber-reinforced concrete mixes was obtained by testing the control cylinders cast. The procedures adopted were according to CSA A23.2-9C "Compressive Strength of Cylindrical Concrete Specimens".

# 3.6 Testing Program

## **3.6.1** Impact Tests

The impact test program for the plain and fiber reinforced concrete mixes is summarized in Tables 3.1 and 3.2.

	Span	Drop Ht.	Notch Depth	No. of Beams
(	(mm)	(mm)	(mm)	
	420	150	25	3
			35	3
			50	3
				_
		250	25	3
			35	3
			50	3
	720	150	25	3
			35	3
			50	. 3
				•
	•	250	25	3
		÷	35	3
			50	3

Table 3.1: Impact Test Program - Plain Concrete Specimens

Table 3.2: Impact Test Program - Fiber-Reinforced Concrete Specimens.

Γ	Fiber	Drop Ht.	Notch Depth	No. of Beams
	Content	(mm)	(mm)	
	(%)			
$\left  \right $	2	150	25	3
	_		35	3
		250	25	3
			35	3
	4	150	25	3
		н. 	35	. 3
	:	250	25	3
			35	3

# 3.6.2 Static Tests

The static test program for the plain and fiber reinforced concrete mixes is summarized in Tables 3.3 and 3.4.

Span (mm)	Drop Ht. (mm)	Notch Depth (mm)	No. of Beams
<b>42</b> 0		25	3
		35	3
		50	3
720		25	3
		35	3
		50	3

Table 3.3: Static Test Program - Plain Concrete Specimens

Table 3.4: Static Test Program - Fiber-Reinforced Concrete Specimens

Fiber	Drop Ht.	Notch Depth	No. of Beams
Content	(mm)	(mm)	
(%)			
2		25	3
		35	3
		25	3
		35	3
4		25	3
		35	3
		25	3
		35	3

# **3.6.3** Compression Tests

The compression test program for plain and fiber reinforced concrete mixes is summarized in Table 3.5.

Table 3.5: Compression Test Program - Plain and Fiber-Reinforced Concrete Specimens

Batch	Beam Series	No. of
No.	ID.	Cylinders
		•
1	4T	3
2	5T	3
3	$6\mathrm{T}$	3
4	7T	3
5	$8\mathrm{T}$	3
6	9T	3
7	10T	3
8	$1\mathrm{TF}$	3
9	$2\mathrm{TF}$	3
10	$3\mathrm{TF}$	3
11	4TF	3
12	$5\mathrm{TF}$	3
13	$6\mathrm{TF}$	3

# Chapter 4

# Data Analysis

## 4.1 Introduction

Analysis of the raw data for a normal stress intensity calculation involves only simple steps to obtain the end result. However, in this study, the data reduction is neither straight forward nor easy. Starting with the output voltage from the load cell strain gauge circuit, the accelerometer, and the crack gauge, and obtaining the end results, such as stress intensity factors, strain rates and toughness parameters, involves frequency-domain as well as time-domain analysis. The following section on frequency-domain analysis presents some of the elements used in the analytical procedure, and the section on method of analysis describes, in a step-by-step manner, the procedure for obtaining the desired end results.

The experimental technique used in this study to obtain information on the cracking or fracture behavior of the specimens tested, is by using a crack gauge which is described in section 3.4.1.3. This technique does not produce a continuous record of crack extension with respect to time, but it gives a discrete crack extension-time record as shown in Fig. 4.1. The step drops in voltage, corresponding to fracture of the discrete, equally spaced paint lines making up the crack gauge, can be seen clearly in this figure. The data scan rate was  $15\mu$  seconds and no filtering or amplification have been used on the output signal. Since the power supply to the circuit was from an external DC source, the signal is free from any external noise. Voltages obtained from the crack gauge agree with those calculated theoretically (Fig. 3.7).

Fig. 4.2 shows the crack gauge signal and the tup load signal for a specimen,


Figure 4.1: Unfiltered Crack Gauge Response





where the tup load signal has been amplified as well as filtered through a low pass filter of 100 Hz. According to the filtered tup load cell signal, the peak load is reached approximately 1 ms after the start of the impact event, and the total event takes approximately 2.2 ms. However, the crack gauge signal indicates that the fracture of the specimen begins very soon after the impact event starts (0.03 ms) and requires only approximately 0.5 ms to complete.

In reality, crack propagation begins at the maximum load point, or a little earlier due to subcritical crack growth (John and Shah, 1986). In Fig. 4.2, the crack gauge signal indicates that the actual impact event is over well before the peak tup load signal is attained (i.e. the crack propagating point). Furthermore, the peak load shown by the filtered signal is far smaller than actual values (Figs. 4.2 and 4.4). Therefore, using the low pass filter for signal noise reduction at the analog wave form stage is detrimental and grossly misleading. Strain rates, crack propagation velocities and the peak loads calculated on the basis of such filtered signals and their time histories are therefore totally incorrect.

Fig. 4.3 shows the tup load, crack gauge signal and accelerometer signal filtered at the 100 Hz level at the analog wave form stage. The crack gauge signal shows a smooth voltage reduction pattern and not the required step voltage pattern. Furthermore, it can be observed that the crack gauge shows crack propagation begins when the tup load starts to increase. This too is incorrect.

The ability to check the overall system frequency response is another important aspect of using a crack gauge independent of the filtering and the amplification system. The system used may not be able to respond properly to the fast crack propagation event if its frequency response is inadequate. This phenomena has been observed by many researchers (Ireland, 1974; Hoover, 1974). Filtering at the analog stage not only creates a large phase shift in the signal, it also distorts the signal, which implies that the system frequency response is not adequate. In this study, the theoretically





calculated crack gauge response and the actually obtained response (in the unfiltered case) are the same for the scanning rate adopted. This indicates that the system frequency response in the unfiltered condition is adequate for the impact event under consideration.

Hence, using the data acquisition system at a scanning rate of  $15\mu$  seconds without any analog filtering is considered to give reliable, undistorted data. Numerical digital filtering can then be used to remove high frequency and other extraneous signals, i.e. noise. The purpose of the above discussion on the experimental technique is to emphasize the reliability of the data acquired in this study and the necessary precautions to be taken to identify and avoid sources of errors. This point is stressed at this stage because of the importance of collecting proper raw data, which is the fundamental important aspect of any experimental investigation.

It should be noted that the crack gauge used in this study was inexpensive and was accurate enough to give average crack propagation velocity. The reusability of the modules made it possible to carry out a large number of tests with minimal cost.

# 4.2 Frequency-Domain Analysis

Transient loading conditions, such as impact loading, and material and structural responses to them, are time-dependant variables, which should be studied using a time-domain analysis. However, these variables can be more conveniently studied using a frequency-domain analysis, since they are comprised of many periodic wave forms. The correct evaluation of such wave form characteristics is the prime aim of a time-domain to frequency-domain conversion procedure, such as the Fourier Transformation. In this procedure, the full characterization of wave forms of various frequencies is achieved by solving the Fourier transform pair for an aperiodic time

signal (as in impact loading). The aperiodic time signal is given as:

$$P(t) = \frac{1}{2\pi} \int_{\omega = -\infty}^{+\infty} C(w) exp(i\omega t) d\omega, \qquad (4.1)$$

and the harmonic amplitude function  $C(\omega)$  is given by:

$$C(\omega) = \int_{t=-\infty}^{+\infty} P(t) exp(-i\omega t) dt, \qquad (4.2)$$

where  $\omega$  is the frequency component and C is the Fourier coefficient.

For the evaluation of  $\omega$  and C, numerical, computer-oriented methods are required. One of the popular methods is to form the Discrete Fourier Transform (DFT) based on Eqs. 4.1 and 4.2, and then to solve these equations by the Fast Fourier Transform (FFT) algorithm (Clough and Penzien, 1982). This conversion procedure will yield the necessary frequencies and their amplitudes, which facilitate the frequency-domain analysis. For steady-state conditions, contributing frequencies may be few and limited, but for transient conditions, a spectra of frequencies will be obtained. Because impact loads are very short duration loads, the time-domain analysis will not be able to give useful information. In this case, a frequency-domain analysis procedure will yield the contributing frequencies as well as their relative importance.

One of many problems involved in dynamic load tests is the external noise in the test system and its interaction with the actual event signal. This noise consists of the electrical voltages created by external sources such as motors, power supply lines, amplifiers, computer components, etc. and may have a range of frequencies, from low to high. To obtain reliable data, therefore, one should eliminate such noise, which is not simple. Furthermore, a clear indication of the range of frequencies contributing towards an impact event signal is not known from previous studies.

Using the data collected from a "non event" experiment, it is possible to identify some components of the external noise which are present in the system throughout the tests (such as the power line signal at 60 Hz and the internal computer noise). However, other components of the external noise are only present during an actual

test and examination of the "non event" signals will not give any indication of these. Therefore, proper frequency-domain analysis provides a method for identifying the components of the noise and reducing their effects.

In this study, digital filtering has been adopted to remove as much of the external noise as possible. Digital filtering is a process by which unwanted frequencies are eliminated from a signal in the frequency-domain analysis by a mathematical process. In the mathematical algorithms, three types of filters are commonly employed: low pass filters, band pass filters and high pass filters. A low pass filter eliminates frequencies from the signal which are greater than the stipulated value, whereas a high pass filter eliminates frequencies less than the stipulated value. A band pass filter allows only the signals which lie within the specified band to pass and cuts the rest off. The filtered signal, when changed to the time domain by means of the inverse FFT, will be free of the particular frequencies which have been eliminated in the digital filtering process and will have reduced peak value.

As the impact event consists essentially of low frequencies, high frequency components of the signal are of no consequence and can be eliminated by the low pass filter.

The figures that follow explain some of the concepts outlined above. The frequencydomain analysis was carried out using the computer software VU-POINT Version 1.21 (produced by S-CUBED, A Division of Maxwell Laboratories, Inc., California, USA).

Fig.4.4 shows a typical tup cell load signal versus time curve. It can be seen that the signal consists predominantly of low frequency noise before it increases rapidly indicating the start of the impact event. After completion of the event, the signal represents the free vibration phase of the hammer assembly.

Fig. 4.5 shows the initial portion of the signal in Fig.4.4; the form of this signal is apparent and suggests that the external noise is periodic in nature.

The portion of the signal in Fig.4.1 after the impact event is reproduced in Fig.



**Time From Start of Data Acquisition** 







Figure 4.5: External Noise Variation Before the Impact Event



Figure 4.6: Tup Load Cell Signal Variation in Free Vibration Phase

4.6. The form of the signal indicates that the hammer is vibrating freely, and the signal can be used to determine the the natural frequency and the damping factors associated with these vibrations.

The Fast Fourier transform, FFT, of the total event signal (Fig. 4.4), including the the noise, is shown in Fig. 4.7, where the Y-axis is the Fourier coefficient (in volts/Hz) and the X-axis is the frequency in Hz. It can be seen that the major frequency contributions are within the range of 0 - 1000 Hz. This figure also suggests that frequencies above 2000 Hz are not significant.

Fig. 4.8 is the FFT of the initial portion (before the impact event) of the total signal. It can be seen that amplitude peaks occur at the frequencies around 60 and 180 Hz, which are the external power line frequencies and their multiples. This suggests that the line voltage is the prominent contributor to the external noise in this portion of the signal.

Since this noise signal will exist throughout the impact event, its effect will have to



Figure 4.7: FFT of the Tup Load Cell Signal





be minimized. The base line setting procedure, is one such method. In this method, the time-domain noise signal which varies periodically is selected within the required time bounds and its average amplitude is set to zero. By this, the total time-domain signal is affected and consequently the peak value of the tup load cell signal is reduced. Therefore, the effect of the external low frequency noise is taken into account to some extent.



Figure 4.9: FFT of the Free Vibration Phase of the Hammer Assembly

Fig. 4.9 is the FFT of the signal representing the free vibration of the hammer and its components after the impact event is complete. It is reasonable to assume that this signal is due to the natural frequency of the hammer, which, from the FFT may be assumed to be approximately 1500 Hz.

Fig. 4.10 shows an example of a digital filter, with the minimum transition levels specified. The actual cut off frequency has to be selected in advance of the digital filtering process, and the method used to select an appropriate frequency will be explained subsequently. As an example, in this figure the cutoff frequency has been



Figure 4.10: Digital Low Pass Filter Characteristics

assumed to be 1000 Hz, and the transition range from 1000 Hz to 0 Hz is 10 Hz.

The frequencies remaining in the signal after the low pass digital filtering process has been carried out are shown in Fig. 4.11. As can be seen, the resulting signal does not contain any frequencies above 1000 Hz, which is the cut off level.

Fig. 4.12 shows the inverse of the FFT of the filtered signal shown in Fig. 4.11; this is the time-domain signal after the filtering process has been implemented. It will be noted that the peak amplitude has dropped from 7.3 V to 3.4 V, and the curve is much smoother than before the signal was filtered.

Fig. 4.13 shows the FFT of the unfiltered signal from the accelerometer. The signal shows that the contributing frequencies are much different in nature than those in the tup load cell signal, because the characteristics of the built-in accelerometer amplifier are different from those of the amplifier used for the tup load signal. Since the tup load signals and the accelerometer signals are representative of the beam behavior, it is expected that the frequency components of the contributing signals



Figure 4.11: FFT of the Tup Load Cell Signal After Digital Filtering



Figure 4.12: Inverse FFT of the Tup Load Cell After Digital Filtering



Figure 4.13: FFT of Accelerometer Signal

will be the same. Therefore, it was decided to digitally filter the accelerometer signal in the same manner as the tup load signal as explained above. Unlike the tup load signal FFT, however, the FFT of the accelerometer is discontinuous and discrete. In theory, both signals should show the same FFT; this differences may be due to the characteristics of the different amplifiers used.

Fig. 4.14 shows the FFT of the accelerometer signal after the crack has propagated through the beam, i.e. after the tup load signal has returned to approximately zero. The peak of the FFT is the natural frequency of vibration of the broken half of the beam, which is approximately 6.8 kHz for this case. According to Fig. 4.9, the natural frequency of the hammer assembly is approximately 1.5 kHz. If the frequency of the total system vibration is close to either of these two frequencies, i.e. 1.5 kHz or 6.8 kHz, during an impact event, a resonance condition may occur. The resulting signal will be very large and may exceed the range of the measuring instrumentation. One should note, however, that the 6.8 kHz value is only for the free vibration of the broken half of the beam. During an impact event, when the crack propagates,



Figure 4.14: FFT of Accelerometer Signal After Specimen Fracture

the physical conditions change and so will the natural frequency of vibration of the beam. Therefore, it is difficult to predict the natural frequency of the beam during an impact event.

The use of digital filters and the associated mathematical techniques are explained in many texts (Blinchikoff and Zverev (1976), Hamming (1977)). The methods of sampling, discretization of continuous linear systems, and relevant sampling theorems are discussed in detail for general cases by Palm III (1983).

# 4.3 Method of Analysis for Dynamic Testing

The method adopted for analysis is described in the following sequence of steps. Each step is explained in detail, using the data for specimen 4T7003.ASC.

STEP 1

The unfiltered data from the tup load cell, the accelerometer and the crack gauge during an impact event were recorded as described in section 3.5.1. Fig. 4.15 shows the data from all three sources.

#### Notes on Step 1

The data was collected using the data acquisition system and stored on floppy disk for retrieval via the VU-POINT software for display. The sampling time was  $15\mu$  seconds. Note that the crack gauge response is a step function with time, and indicates that the crack started at the maximum load point and ended as the tup load fell to zero.

#### STEP 2

The event start time,  $T_0$ , event end time,  $T_E$ , maximum load time,  $T_{P_{max}}$ , the time to reach maximum load point, or to fracture initiation,  $t_f$ , the total event time,  $t_t$ , and the total crack propagation time,  $t_{cr}$ , were determined from the tup load signal as shown schematically in Fig. 4.16. The maximum load value,  $P_{max}$ , was noted, and the average crack propagation velocity calculated as described below.







Figure 4.16: Definitions of Significant Time Variables

#### Notes on Step 2

For most of the specimen tested, as in this example, the time shown by the crack gauge for completion of cracking, and the time for the tup load to fall from its maximum to zero were almost the same. The average crack velocity,  $V_{\rm cr}$ , is evaluated using the following equation:

$$V_{cr} = \frac{B-a}{t_{cr}},\tag{4.3}$$

where 'B' is the beam depth and 'a' is the initial notch depth. For the specimen under consideration (Fig. 4.15),

$$T_0 = 46.095 \text{ ms}, \quad T_{P_{max}} = 46.470 \text{ ms}, \quad T_E = 46.950 \text{ ms}, \text{ and } P_{max} = 8.12V.$$

Hence,

 $t_f = (46.470) - (46.095) = 0.375 \text{ ms},$  $t_t = (46.950) - (46.095) = 0.855 \text{ ms},$  $t_{cr} = (46.950) - (46.470) = 0.480 \text{ ms},$ 

and, from Eq. 4.3, for a = 25 mm and B = 100 mm,

$$V_{cr} = 157 \text{ m/s}.$$

#### STEP 3

The data for the period  $T_0$  to  $T_E$  was saved on the floppy disk for analysis at a later time.

#### STEP 4

The natural frequency of the hammer assembly  $(f_n)$  and the amplitudes of 3-4 successive cycles were determined from the data available after the impact event time (Fig. 4.17. The period of the fundamental frequency of the hammer,  $T_n$ , and the damping ratio,  $\xi$ , in the free vibration phase were then calculated as described below.



Figure 4.17: 'Tup Load Cell Signal in the Free Vibration Phase for Specimen 4T7003.ASC





#### Notes on Step 4

It is assumed that the hammer assembly is a SDOF structure, and that the signal variations after completion of the impact event are due to the free vibration of this assembly alone. The natural frequency of the hammer,  $f_n$ , is then determined from the FFT of this signal.

For this example,  $f_n = 1500$  Hz (Fig.4.18) and therefore:

$$T_n = \frac{1}{f_n} = 0.667$$
 ms.

 $\xi$  is calculated for the three successive cycles at times of 49.05 ms, 49.125 ms and 49.200 ms, where the corresponding amplitudes are 0.2044, 0.1854 and 0.1214 volts, respectively. The logarithmic decrement,  $\delta$ , is given (Clough and Penzien, 1982) as:

$$\delta = \frac{1}{n} \ln \frac{x_m}{x_{m+n}},\tag{4.4}$$

where  $x_m$  is the 'm' th amplitude of vibration and  $x_{m+n}$  is the 'm+n' th amplitude. Therefore,

$$\delta = \frac{1}{2} \ln \left( \frac{0.2044}{0.1214} \right) = 0.2604.$$

From the relationship between  $\xi$  and  $\delta$  (for low damping) which is (Clough and Penzien, 1982),

$$\xi = \frac{\delta}{2\pi},$$

 $\xi$  is obtained as  $\xi = 0.04$  (4%). The frequency ratio,  $\beta$ , is given by,

$$\beta = \frac{T_n}{T_l},$$

where  $T_l$  is the period of the sine wave of impact . As  $T_l = 2 t_t$ ,

$$\beta = \frac{T_l}{2 t_t} = \frac{0.667}{2(0.885)} = 0.377.$$

A value for the dynamic amplification factor, D, can now be obtained through the methods described previously in Chapter 2. Using the first method, Section 2.6.1,

the values of  $\xi$  and  $\beta$  are substituted into Eq. 2.57 to give D = 1.45. Using the second method, Section 2.6.2, D can be calculated from Eq. 2.60, giving a value of 1.55, or from the curves presented in Clough and Penzien (1982), giving the same value. As both methods yield approximately the same results it was decided to use the graphical technique in method 2 for the analysis of all the specimens.

#### STEP 5

The actual maximum dynamic load,  $(P_{tot})_{max}$ , acting on the specimen is given by (see sections 2.5, 2.6 and Eq. 2.61):

$$(P_{tot})_{max} = P_{max}/D.$$

For this example,

$$(P_{tot})_{max} = \frac{8.12}{1.55} = 5.30$$
 volts.

#### STEP 6

The FFT of the tup load signal (saved previously in Step 3) was obtained and a low pass filter was selected (by trial and error) and applied to the FFT so that the inverse FFT had the peak value equal to the value of  $(P_{tot})_{max}$  obtained in Step 5. The same filtering was then applied to the accelerometer signal.

#### Notes on Step 6

The selection of a value for the low pass filter is by trial and error. Initially a value is assumed, say 1500 Hz, and applied on the tup load signal. Then the FFT of the filtered signal is inverted so as to give the time-domain signal. The peak of this signal has to be equal to the value of  $(P_{tot})_{max}$  obtained in Step 5, and if it is not so, another value for the low pass filter is selected until the above condition is met.

Fig. 4.19 shows the FFT of the impact event signal produced by the tup load cell for specimen 4T7003.ASC. It can be seen that most of the contributing frequencies lie within the range of 0-1000 Hz. On this basis, a low pass digital filter set at 1000 Hz was selected, the same as that shown in Fig. 4.10. The filtering of the signal shown



Figure 4.19: FFT of the Tup Load Cell Signal in Impact Event Phase

in Fig. 4.19 yields a new load-time trace when inverted to the time-domain. This trace will be free of high frequency noise and corrected for hammer vibrational effects (i.e. the dynamic amplification). The effect of low frequency noise was reduced by by the setting the average of the low frequency signal to zero (i.e. the base line setting procedure) as explained previously. The maximum of this filtered trace,  $(P_{tot})_{max}$ , as shown in Fig 4.20, is 5.35 volts which is close enough to the expected value of 5.30 volts. This shows that the selected value of 1000 Hz for the low pass filter, for the example under consideration, is satisfactory.

#### STEP 7

The filtered time domain signal, the filtered accelerometer signal (as specified in Step 6) and the event time were saved on the diskette for further analysis.

#### STEP 8

The inertial load, true bending load, and  $K_{ID}$  were calculated for the event from the reduced data obtained in Step 7 by the program "ANALYSIS".





#### Notes on Step 8

The program "ANALYSIS" was written by the author and executes the methods previously described in Sections 2.3, 2.4 and 2.5 for calculating the inertial load,  $P_{inertia}$ , and true bending load,  $P_{bend}$ .



Figure 4.21: Tup Load, Inertial Load and Bending Load During the Impact Event

Fig. 4.21 shows the variation of the tup load, inertial load and the true bending load with time for the example under consideration. Having reduced the system to obtain true bending loads, the variation of  $K_{ID}$  with time can be calculated. STEP 9

From the data which was obtained in Step 8, the center point deflection, initiation energy, propagation energy, and strain rates were calculated.



Figure 4.22: Energy Components of the Impact Event

#### Notes on Step 9

The displacement,  $\Delta$ , of the center point was calculated by using the expression (Hibbert and Hannant, 1981):

$$\Delta = V_0 t - \frac{1}{m} \int \int P_{act} dt dt,$$

where  $V_0$  is the hammer velocity, m is the mass and  $P_{act}$  is the true bending load. The area under the bending load-displacement curve is comprised of the various energy components as shown schematically in Fig. 4.22. The area up to the maximum load point is the energy required for crack initiation, and the total area is the toughness. The area under the curve from the maximum load to the zero load level is the energy required for crack propagation. The areas were calculated using the trapezoidal rule. The strain rate at the crack tip at failure,  $\dot{\epsilon}_f$ , was calculated by (Banthia, 1987):

$$\dot{\epsilon}_f = \frac{6U_0 \ (B-a)}{L^2 \ t_f}.$$
(4.5)

The relevant results for the specimen under consideration are included with the rest of the results in Chapter 5.

### 4.4 Concluding Remarks

Steps 1-7 of the above analysis were carried out using the software VU-POINT, and Steps 8-9 using the program "ANALYSIS". The area calculations were by LOTUS 123 (product of LOTUS corporation).

From the preceding discussion, it is apparent that the process of data reduction for impact testing is not straight forward. Some of the main points which have emerged from this discussion are given below.

- 1. Two major functions are accomplished by using the digital filtering process:
  - (a) any high frequency noise is eliminated with no undesirable phase shifts or excessive signal attenuation occurs; and,
  - (b) by calculating the dynamic amplification factor, the static calibration procedure is adopted suitably for the dynamic load calculations.
- 2. Low frequency external signals, such as the line voltage (60 Hz) cannot be removed from the load signal because the contributing signals of the actual dynamic impact event are also of low frequency. Therefore, the elimination of the former will distort the actual signal and give erroneous results. The effects of low frequency noise can be reduced by the base line setting procedure.
- 3. The dynamic amplification factor (D) will depend on the event time  $(t_t)$  which in turn will depend on the notch depth and the hammer drop height.
- 4. By not filtering the actual analog signal, it is possible to identify the contributing frequencies and their relative importance. Therefore, it is desirable to avoid analog filtering and resort to digital filtering processes.

# Chapter 5

# **Results and Discussion**

## 5.1 Introduction

In this chapter the results of the experiments on plain and fiber reinforced concretes are presented and discussed. The experiments were carried out on plain concrete beams with notch depths of 25 mm, 35 mm or 50 mm, and span lengths of 410 mm or 720 mm. The fiber reinforced concrete specimens had 25 mm or 35 mm notch depths, with fiber contents of 2% or 4% by weight and a span length of 410 mm.

The results of the tests are summarized in Tables 5.1, 5.2 and 5.3, and presented in graphical form in Figs. 5.1 - 5.14.

# 5.2 General Dynamic Fracture Behavior of Concrete

Fig. 5.1 shows the variation of bending load with time during an impact event for two test specimens with a span of 410 mm. It can be seen that for a hammer drop height of 250 mm, the rate at which the bending load increases is greater than that for a drop height of 150 mm. As the applied stress is directly proportional to the bending load, the stress rate and hence the strain rate, will depend on the hammer drop height. It can also be seen from these figures that the time of crack propagation,  $t_{cr}$ , (as defined in Fig. 4.16) is smaller, and hence the crack velocity (Eq. 4.13) is higher, for the specimen tested with 250 mm drop height in comparison to that tested with a 150 mm drop height. For longer beams (720 mm span), as shown in Fig. 5.2, these differences are more pronounced.



Figure 5.1: Bending Load vs Time for Span = 410mm.



Figure 5.2: Bending Load vs Time for Span = 720mm.

# 5.3 Effect of Notch Depth, Span and Drop Height on Dynamic Fracture Parameters - Plain Concrete Specimens

5.3.1 Time to Fracture



Figure 5.3: Average Time to Fracture Variation With Notch Depth and Span

The variation of time to fracture,  $t_f$ , with notch depth, span length, and drop height is shown in Fig. 5.3. It is observable that for short beams (410 mm), the notch depth has little or no effect on time to fracture for both drop heights considered. On the other hand, for long beams (720 mm) the time to fracture decreases with notch depth. This may be due to the fact that shear effects may be larger at shorter spans, which

#### Chapter 5. Results and Discussion

may be masking any effect of notch depth. However, for longer spans, bending effect is predominant, and so a different behavior could be expected. Time to fracture is longer for long beams at small notch depths which is not intuitively evident. This may be primarily due to the large inertial resistance of the beam.

It can also be seen that, when the notch depth increases, the effect of the span becomes less significant. The  $t_f$  values for 720 mm and for 410 mm beams, converge to a constant value at greater notch depths. The  $t_f$  decreases as expected for higher notches because the beam becomes weaker at higher notch depths. Therefore, irrespective of the spans it could be expected that the beam would fail in a similar manner. The rate of decrement of  $t_f$  with respect to notch depth is larger for the longer span than the shorter one.

In general, for both spans, the  $t_f$  is longer for lower drop heights. This is because lower drop heights induce lower strain rates. At lower strain rates the stress rates are lower and hence the time to fracture is greater.

# 5.3.2 Strain Rate

Fig. 5.4 shows the variation of the strain rate with drop height. Static tests are considered to be zero drop height with zero strain rate. The strain rate increases with drop height. For the 720 mm span, it can be observed that a linear trend exists and the strain rate variability seems to be minimal. However, for the 410 mm span, there seems to be non-linear behavior at high strain rate. Furthermore, the shorter span has higher strain rates and the longer span has lower strain rates, because of the method of calculation of strain rates (see Eq. 4.5). It may be recalled that (Fig. 5.3) the longer span had higher  $t_f$  values when compared with the shorter span. Therefore, the strain rates induced on the longer span specimen would be smaller. It could also be observed that the rate of strain increase is greater for shorter span length. This may be attributed to the additional shear deformation of the shorter beams. Considering



Figure 5.4: Variation of Strain Rate with Drop Height.

Eq. 4.5, it can be inferred that when the beam displacements are higher, the strain rates too will be higher. The long beam, which is predominantly governed by bending will have small deformations even though its displacement would increase due to its span. The rate of increase of these deformations due to shear effects, may be much higher for short beams.



Figure 5.5: Variation of Strain Rate with Notch Depth and Span.

Fig. 5.5 shows the variation of strain rate with notch depth for the various span length and drop height considered in this study. For 720 mm span beam it is seen that the notch depth has little or no effect on strain rate for both drop heights considered.

#### Chapter 5. Results and Discussion

However, for the 410 mm span beam, the increased notch depth caused a decrease in strain rate. This behavior could be explained by considering Eq. 4.5. Since for short span beam (410 mm), the notch depth had no effect on  $t_f$  (Fig. 5.3), the reduction of the elastic ligament (B-a) causes a reduced strain rate. In the case of long beam (span of 720 mm), since  $t_f$  values also reduced with notch depth, the strain rate could be expected to remain almost constant for various notch depth.

It could also be observed from Fig. 5.5 that the effect of the drop height on the strain rate is somewhat consistent for the notch depth and span considered. The almost parallel best fit lines suggest that, the drop height has same effect on strain rate for all notch depths considered in this study.

The common method adopted in studying the strain rate behavior is by varying the drop height. It has been shown in Fig. 5.5 that, depending on the span, the different notch depth will also cause varied strain rate for the same drop height. Therefore, keeping the notch depth constant and varying the drop height to obtain variable strain rates may be reasonable due to the linear relationship of drop height and strain rate.

## 5.3.3 Crack Velocity

Fig. 5.6 shows the variation of crack velocity with drop height. The figure suggests that crack velocity seems to have a linear relationship for the 410 mm span and is much more variable in the case of 720 mm span. The crack speed increases with the drop height for both the span. The rate of velocity increment is almost the same up to a drop height of 150 mm and then the longer span rate reduces for higher drop heights when compared with the shorter span (420 mm).

The rate at which the crack velocity is increasing with the drop height has to be noted carefully. This could be used as a guideline when selecting the equipment required for data acquisition for high drop heights. When the crack speed is high,



Figure 5.6: Variation of Crack Velocity with Drop Height

then the system response time may not be adequate as discussed previously in Section 4.1. As it could be seen from this figure, at larger drop heights the crack speed would increase considerably and it would require higher data scan rates at the time of testing. This again has an effect on the data acquisition system capacity as well as the computer internal speed.





Figure 5.7: Variation of  $K_{ID}$  with Time

Fig. 5.7 shows the variation of dynamic stress intensity factor  $(K_{ID})$  with time for the 4 specimens shown in Figs. 5.1 and 5.2. These curves may be considered to be
the impact response curves (Kalthoff, 1985).

A discontinuity, or kink occurs, in these curves at the point where the specimen fractures; this behavior has been observed for most of the specimens tested. The  $K_{ID}$  value at this point,  $K_{ID_e}$ , is the dynamic fracture toughness of the material.

It can be recalled that in this analysis, the effects of crack velocity has been included in the calculation of  $K_{ID}$  (Section 2.4). The discontinuity in the curves in Fig. 5.7 is a result of the inclusion of these effects, and would not be seen if the calculations are carried out on the basis of crack length only as in many published results (John and Shah, 1986; Kalthoff, 1985).

Though not evident in Fig. 5.7, there are other important characteristics of the variation of  $K_{ID}$  with time. As shown in Fig. 5.8,  $dK_{ID}/dt$  curve seems decrease as the notched specimen approaches the crack instability point. This implies that closer to the crack instability point,  $K_{ID}$  remains almost constant. The initial sudden drops on the Fig. 5.7 is due to the calculation procedure of  $dK_{ID}/dt$  and has no significance to the behavior of the beam. Just after the crack starts to propagate,  $dK_{ID}/dt$  seems to increase very suddenly to large values and then smooths out. As mentioned previously, this sudden jump in  $dK_{ID}/dt$  curve is due to the inclusion of the crack velocity term in the analysis. The decreasing trend of  $dK_{ID}/dt$  after the crack has started to propagate shows that the  $K_{ID}$  may remain almost constant at this stage. This behavior is, perhaps due to the three point bend configuration. Here, the crack propagation energy is provided constantly until the beam fails completely.

Fig. 5.9 shows the variation of  $K_{ID}$  with crack velocity on a log-log scale plot. The data reported by John and Shah (1986) have also been included in this figure. In this study the strain rates and the crack velocities are high. The rate sensitive model proposed by Evans (1974) for brittle materials suggests that the relationship between log  $K_I$  and log V is linear. The data plotted seems to follow the model of Evans, but two predominantly distinct regions are observed. At high crack propagation velocities



Figure 5.8: Typical Variation of  $dK_{ID}/dt$  with Time



Figure 5.9: Variation of  $K_{ID}$  with Crack Velocity

#### Chapter 5. Results and Discussion

(greater than 100 m/s) the slope of the graph is very steep which suggests that the value of N (Eq. 1.2) is much higher when compared to the lower crack velocity region. John and Shah (1986) also observed the same type of behavior. However, it could be seen from Tables 5.1 and 5.2, the strain rates are almost an order higher than that reported by John and Shah (1986).

It could also be observed that the values of N are higher for 720 mm beam when compared with that of 410 mm beam, at low and high crack velocity regions. This behavior is expected because of the rate sensitivity of different span length.



Figure 5.10: Variation of  $K_{ID_c}$  with Notch Depth for Span = 410mm

Figs. 5.10 and 5.11 show the variation of  $K_{ID_e}$  with notch depth for static case





#### Chapter 5. Results and Discussion

and two drop heights of 150 and 250mm, for the two spans of 410 and 720mm. As expected, the  $K_{IC}$  for static test is almost constant for various notch depths. However, since concrete is a strain rate sensitive material,  $K_{ID_c}$  increases substantially with notch depth and drop height as well. The drop height effect on  $K_{ID_c}$  seems to be the same at all notch depths for the 420 mm and 720 mm beams. This may be explained by the fact that the strain rate is proportional to the drop height for a given notch depth. It can be observed in these figures that the variability of  $K_{ID_c}$  values are significant at high strain rate (i.e. 410 mm span and 50 mm notch depth).

From the Figs. 5.10 and 5.11 it can be inferred that the longer beam will have higher fracture toughness with lower strain rates, as opposed to shorter beam having lower fracture toughness at higher strain rate, for the same notch depth. Therefore, a general statement such as higher strain rate will cause higher stress intensities will need to be qualified with a specification of beam length and the corresponding notch depth which are the other parameters that influence the dynamic fracture toughness.

Generally, in static tests one would expect to obtain notch independent values (except for small span beams) of fracture toughness (as can be seen from the figures) and so it is possible to use the compliance method to obtain reasonably consistent values for  $K_{IC}$ . However, due to strain rate sensitivity of the notch depth, such compliance method of calculations to determine dynamic fracture toughness are not applicable for dynamic loading cases.

Therefore, from the foregone discussion it can be clearly seen that concrete is a strain rate sensitive material. Furthermore, its properties are affected by its span and the notch depths. This implies that the properties depend on the physical configuration unlike the static loading cases.

# 5.4 Effect of Notch Depth, Span and Drop Height on Dynamic Fracture Parameters - Fiber-Reinforced Concrete Specimens

From Tables 5.1, 5.2, 5.3 and 5.4 it can be seen that the time to fracture,  $t_f$ , and strain rate behavior are almost the same as described in Sections 5.3.1 and 5.3.2 for plain concrete specimens, and, therefore, they will not be discussed in this section.

### 5.4.1 Crack Velocity



Figure 5.12: Variation of Crack Speed with Fiber Content

#### Chapter 5. Results and Discussion

Fig. 5.12 shows the variation of crack velocity with the fiber content for various drop heights (strain rates) and a/B ratios (notch depth). It can be observed from this figure that for constant drop height and a/B ratio (i.e. constant strain rate), the crack velocity decreased with increasing fiber content. This effect is more predominant for higher drop heights. Furthermore, for constant drop height (i.e. strain rate), small differences between crack velocities are observed at all fiber contents. The velocities are slightly lower for a/B = 0.35 case when compared to a/B = 0.25. The velocity difference becomes a minimum at higher fiber contents. This behavior suggests that the effect of drop height on crack velocity is more predominant than notch depth.

It is also observed that for constant a/B ratios (i.e. notch depths) substantial difference exists between crack velocities at zero fiber contents. This difference decreases as fiber content increases, and is not significant at 4% fiber content.

In general, as the fiber content increases, crack velocity decreases and and converges to an approximately equal value for all the test conditions. This implies that the matrix cracking process is hindered by the inclusion of fibers, and hence the fracture properties are affected.

#### 5.4.2 Dynamic Fracture Toughness

Figs. 5.13 and 5.14 show the variation of the dynamic fracture toughness,  $K_{ID_e}$ , with fiber content and drop height. It could be observed from both graphs,  $K_{ID_e}$  increases with drop height for all fiber contents. However, the difference between  $K_{ID_e}$  values for a constant drop height and different fiber content is small. Furthermore, the rate of increment of  $K_{ID_e}$  with respect to drop height is almost same for all the fiber contents for a particular a/B ratio. This suggests that the drop height has the same effect on  $K_{ID_e}$  irrespective of the fiber content.

At lower drop height, the 2% fibers seems to give slightly higher  $K_{ID_c}$  values than 4% and 0% fibers, and at higher drop heights the 4% fiber seems to perform slightly







Figure 5.14: Variation of  $K_{ID_c}$  with Drop Height for a/B = 0.35

#### Chapter 5. Results and Discussion

better than the rest. Due to this phenomena at drop heights closer to 150 mm, there seems to be a transition in properties with respect to fiber contents. However, for a/B = 0.25, the 0% fiber specimens did not show this type of behavior.

The Figs. 5.13 and 5.14 which are for a/B ratios of 0.25 and 0.35 respectively, suggest that the  $K_{ID_e}$  values are higher for a/B = 0.35 than for a/B = 0.25. This suggests that the notch depth has considerable influence on  $K_{ID_e}$  at all drop heights (i.e. strain rate) and fiber contents.

The inconsistency observed with respect to effect of fiber content on  $K_{ID_e}$  may be due to the variability in test results, as shown by the error bars associated with the data points, which is very high. Therefore, it may be possible that such variation may be masking any real differences in the test data. Furthermore, there were no long span beams (i.e. 720 mm) cast with fibers; a comparison with span length is not possible.

Since it has been shown that plain concrete itself is strain rate dependent, the composite material of the fiber reinforced concrete should also be strain rate dependent. However, in general, in plain concrete, higher strain rates produce monotonic increase in fracture toughness (Figs. 5.10 and 5.11) value. But in the case of fiber reinforced concrete it is seen that the fibers have very little effect on  $K_{ID_c}$ .

### 5.5 Summary of Results

The results described in the previous section of this chapter are summarized as follows:

- 1. It was found that at the largest notch depth (50mm), the time to fracture was independent of span length or drop height. For the shorter span the time to fracture was not affected by notch depth, but for the longer span, it decreased with increasing notch depth.
- 2. Generally, the strain rate increased with drop height in a linear manner for the longer span, while a non-linear linear increase was observed for the shorter span beams. The strain rate was not affected by notch depth in the case of the longer span beams, but decreased in the case of the shorter span beams.
- 3. Crack velocities also increased with drop height; the rate of increase was more for the short span beams.
- 4. The variation of the dynamic stress intensity factor and its rate of change time, showed some interesting features such as kinks and transition regions which are significant. The inclusion of crack velocity terms was primarily responsible for this. Concrete is a rate sensitive material and the model proposed by Evans (1974) seems to be applicable. However, two distinct regions of applicability are observed.
- 5. The dynamic fracture toughness,  $K_{ID_e}$ , increased with notch depth as well as drop height.
- 6. The crack velocities in fiber-reinforced concrete decreased with notch depth and drop height so as to approach a constant value at higher notch depths. However, due to high variability in data, the effect of fibers were not evident when dynamic fracture toughness values are compared with plain concrete.

Drop	Notch	Time to		CrackV	/elocity	Str	ain	Kin	
Height	Depth	Fracture $(ms)$		(m	/s)	Rate $(/s)$		$\left(kNm^{-\frac{3}{2}}\right)$	
(mm)	(mm)	Mean	St.Dv	Mean	St.Dv	Mean	St.Dv	Mean	St.Dv
150	25	0.240	0.029	155	22.7	12.65	1.051	1659	26.6
	35	0.225	0.041	137	4.0	11.40	0.772	2364	678.1
	50	0.210	0.030	133	36.8	8.73	0.102	3574	200.5
						Į			
250	25	0.185	0.040	247	14.9	15.43	0.552	2494	772.8
l	35	0.180	0.020	197	26.4	12.19	0.887	3939	614.3
	50	0.165	0.020	140	39.4	11.74	1.774	5538	1573.5
								(	
							·		
Static	25			3.58 E-4	3.27 E-6			726	25.6
Test	35			5.36 E-4	1.04 E-5			764	43.1
	50			4.86 E-4	8.61 E-5			723	53.6

Table 5.1: Summary of Test Results of Plain Concrete Beams with 410 mm Span.

Drop	Notch	Time to		CrackVelocity		Strain		K <sub>IDe</sub>	
Height	Depth	Fractu	re $(ms)$	(m	/s)	. Rate $(/s)$		$\left(kNm^{-\frac{3}{2}}\right)$	
(mm)	(mm)	Mean	St.Dv	Mean	St.Dv	Mean	St.Dv	Mean	St.Dv
150	25	0.360	0.029	155	9.9	2.77	0.081	2447	562.3
	35	0.240	0.018	111	24.1	3.16	0.681	3192	558.5
	50	0.225	0.040	113	28.1	3.58	0.582	5947	267.6
•							ĺ		
250	25	0.270	0.063	180	12.2	3.92	0.665	4868	557.2
	35	0.225	0.090	182	4.8	4.14	0.859	5082	650.2
	50	0.180	0.030	175	21.9	4.22	0.226	8841	952.0
Static	25			5.38 E-4	1.54 E-4			692	31.6
Test	35	ļ		4.27 E-4	1.08 E-4			741	37.6
	50	·		2.61 E-4	3.89 E-5	<u> </u>		619	11.8

Table 5.2: Summary of Test Results of Plain Concrete Beams with 720 mm Span.

106

Drop	Notch	Time to		CrackVelocity		Strain		K <sub>IDc</sub>	
Height	Depth	Fractu	re ( <i>ms</i> )	(m	(s)	Rate $(/s)$		$\left(kNm^{-\frac{3}{2}}\right)$	
(mm)	(mm)	Mean	St.Dv	Mean	St.Dv	Mean	St.Dv	Mean	St.Dy
150	25	0.240	0.029	155	22.7	12.65	1.051	1659	26.6
	35	0.225	0.041	137	4.0	11.40	0.772	2364	678.1
	50	0.210	0.030	133	36.8	8.73	0.102	3574	200.5
250	25	0.185	0.040	247	14.9	15.43	0.552	2494	772.8
	35	0.180	0.020	197	26.4	12.19	0.887	3939	614.3
	50	0.165	0.020	140	39.4	11.74	1.774	5538	1573.5
Static	25			3.58 E-4	3.27 E-6			726	25.6
Test	35			5.36 E-4	1.04 E-5	[	ĺ	764	43.1
	50			4.86 E-4	8.61 E-5	l		723	53.6

Table 5.3: Summary of Test Results of Plain Concrete Beams with 410 mm Span.

107

Deres Carl	0				
Beam Series	Compressive				
ID ID	Streng	th (Mpa)			
	Mean	St.Dv			
4T	56.9	2.98			
5T	55.6	6.39			
6T	57.9	2.46			
7T	63.7	1.81			
8T	66.2	0.93			
9T	63.0	0.93			
10T	55.3	7.41			
1TF	44.0	0.41			
2TF	43.3	2.47			
3TF	46.8	2.46			
4TF	43.8	2.67			
5TF	41.5	2.98			
6TF	44.9	2.92			

Table 5.4: Summary of Compression Test Results.

## Chapter 6

## **Conclusions and Recommendations**

#### 6.1 Conclusions

In this study, the impact of plain and fiber reinforced concrete of notched beams have been studied and the following conclusions have been reached.

- 1. Analog filtering during the data acquisition period has been found to be detrimental. It not only attenuates the signal, but also shifts it and distorts the impact event times (phase shift) as shown by the crack gauge used in the study. Therefore, the measured time to fracture and the maximum load are incorrect which will lead to incorrect strain rate calculations as well as incorrect dynamic fracture toughness ( $K_{ID_c}$ ) values. To obviate the above problem the digital filtering technique was adopted in this study. The digital filtering process can be used to obtain the necessary signal which is free from external noise.
- 2. Frequency domain analysis of the impact signal is very useful in identifying various parameters and contributing frequencies, and their relative importance. The noise characteristics, various fundamental frequencies etc. have also been identified for the impact test system.
- 3. The corrections required for the adoption of static calibration curves for dynamically obtained loads have been recognized and two methods have been used. Frequency domain analysis have been carried out on all the test data to obtain meaningful results to study the fracture behavior of plain and fiber reinforced concretes.

- 4. A method to calculate the inertial loads have been developed based on dynamic mode shapes of the beam bending problem. The inclusion of crack lengths and the crack velocity makes the model more applicable to analyze the post peak region.
- 5. Significant shear effects may be the reason for the difference in behavior of long and short span beams. The relationship between span, time to fracture and strain rate suggests that at higher notch depths (higher a/B ratio) the beam behaves in a fashion closer to the pure bending case. This has been observed in time to fracture and strain rate variations.
- 6. Concrete is a strain rate sensitive material. The fracture toughness in dynamic case varied with the strain rate which was induced by various drop heights. The variation seems to follow the rate dependant model proposed by Evans (1974). However, there seems to be two distinct regions of applicability of the model.
- 7. Dynamic fracture toughness is affected by the notch depth as well as the span. This is mainly due to different strain rates induced at the notch tip in each case. Since concrete is rate sensitive this behavior is expected.
- 8. The inclusion of random steel fibers (aspect ratio = 92) also affected the crack velocity, thus it is expected to affect dynamic fracture properties of the composite. However, the fracture toughness was not significantly different from that of the plain concrete.
- 9. The high variability in the data for dynamic fracture toughness is masking the actual dynamic properties. Therefore, inconsistent results have been obtained.

## 6.2 Recommendations for Future Research

In this study, the dynamic stress intensity factor calculations have been achieved by evaluating the inertial loads and eliminating them from the measured load. However, a better method may be to include the inertial load term in the basic equilibrium equation and solve the equations by finite element methods. Inclusion of a theoretical model with moving crack tip will then give a better method of analysis.

## Bibliography

Banthia, N. P. (1987), "Impact Resistance of Concrete", Ph. D. Thesis, Department of Civil Engineering, University of British Columbia.

Bhargrava, J. and Rehnstrom, A. (1975), "High Speed Photography for Fracture Studies of Concrete", Cement and Concrete Research, Vol. 5, No. 3, pp. 239-248.

Bienert, J. and Kalthoff, J. F. (1981), "Experimental Determination of Dynamic stress Intensity Factor by Shadow Patterns", Experimental Evaluation of Stress Concentration and Intensity Factors, ed. G. C. Sih, Martinus Nijhoff Publishers, The Hauge, The Netherlands.

Blinchikoff, H. J. and Zverev, A. I. (1976), Filtering in Time and Frequency Domains, John Wiley and Sons, New York, USA.

**Broek, D. (1984)**, Elementary Engineering Fracture Mechanics, Martinus Nijhoff Publishers, The Hauge, The Netherlands.

Charles, R. J. (1958), "Dynamic Fatigue of Glass", Journal of Applied Physics, Vol. 29, No. 12, pp. 1657-1662.

Clough, R. W. and Penzien, J. (1982), Dynamics of Structures, McGraw-Hill Book Company, Singapore.

Cotterell, B. (1962), "Fracture Toughness and Charpy Impact Tests", British Welding Journal, Vol. 9, No.2, pp. 83-90.

112

Evans, A. G. (1974), "Slow Crack Growth in Brittle Material Under Dynamic Loading Condition", International Journal of Fracture, Vol. 10, No.2, pp. 251-259.

Freund, L. B. (1972), "Crack Propagation in an Elastic Solid Subjected to General Loading: Part I", Journal of Mechanics Physics and Solids, Vol. 20, No. 1, pp. 129-140.

Gopalaratnam, V. S. and Shah, S. P. (1986), "Properties of Steel Fiber Reinforced Concrete Subjected to Impact Loading", ACI Journal, Vol. 83, No 1, pp. 117-126..

Gopalaratnam, V. S., Shah, S. P and John, R. (1984), "A Modified Instrumented Charpy Test for Cement Based Composites", *Experimental Mechanics, Vol. 24, No. 2, pp. 102-111.* 

Green, H. (1964), "Impact Strength of Concrete", Proceedings of the Institution of Civil Engineers, Vol. 28, No. 4, pp. 383-396.

Hamming, R. W. (1977), Digital Filters, Prentice Hall, Englewood, New Jersy, USA.

Harris, C. M. and Crede, C. E. (1961), Shock and Vibration Handbook, Vol. 1, McGraw-Hill, Inc., USA.

Hibbert, A. P. and Hannant, D. G. (1982), "Toughness of Fiber Cement Composites", Composites, Vol. 13, No. 2, pp. 105-111.

Hibbert, A. P. and Hannant, D. J. (1981), "Impact Resistance of Fiber Concrete", Transport and Road Research Laboratory Supplementary Report 654, Crowthrone, U.K. Hoover, W. R. (1974), "Effects of Test System Response Time on Instrumented Charpy Impact Test Data", Instrumented Impact Testing, ASTM STP 563, American Society for Testing and Materials, Philadelphia, USA, pp. 203-214.

Huang, J. L., Virkar, A. V. and Huber, R. J. (1978), "Fast Crack Propagation Across Interfaces", Fracture Mechanics of Ceramics, ed. R. C. Bradt et. al., Martinus Nijhoff Publishers, The Hauge, The Netherlands, pp. 121-135.

Ireland, D. R. (1974), "Procedures and Problems Associated with Reliable Control of the Instrumented Impact Test", Instrumented Impact Testing, ASTM STP 563, American Society for Testing and Materials, Philadelphia, USA, pp. 3-29.

Iyer, K. R. and Miclot, R. B. (1974), "Instrumented Charpy Testing for Determination of the J-Integral", Instrumented Impact Testing, ASTM STP 563, American Society for Testing and Materials, Philadelphia, USA, pp. 146-165.

John, R. and Shah, S.P (1986), "Fracture of Concrete Subjected to Impact Loading", Cement, Concrete and Aggregates, Vol. 8, No.1, pp. 24-32.

Kalthoff, J. F. (1985), "On the Measurement of Dynamic Fracture Toughness - a Review of Recent Work", International Journal of Fracture, Vol. 27, No. 2, pp. 277-298.

Kaplan, M. F. (1961), "Crack Propagation and Fracture of Concrete", ACI Journal, Vol. 58, No. 11, pp. 591-610.

Kinra, V. K. and Kolsky, H. (1977), "Interaction Between Bending Fracture and Emitted Stress Waves", Engineering Fracture Mechanics, Vol. 9, No. 2, pp. 423-432. Kormeling, H. A. (1986), "Rate Theory and Impact Tensile Behavior of Plain Concrete", Fracture Toughness and Fracture Energy of Concrete, ed. F. H. Wittmann, Elsevier Science Publishers B. V., Amsterdam, pp. 467-476.

Lee, N. K., Mayfield, B. and Snell, C. (1981), "Progress of Internal Cracks in Concrete by Using Embedded Graphite Rods", Magazine of Concrete Research, Vol. 33, No. 116, pp. 180-183.

Lueth, R. C. (1974), "An analysis of Charpy Impact Testing as Applied to Cemented Carbide", Instrumented Impact Testing, ASTM STP 563, American Society for Testing and Materials, Philadelphia, USA, pp. 166-179.

McNeely, D. J. and Lash, S. D. (1963), "Tensile Strength of Concrete", ACI Journal, Vol. 60, No. 6, pp. 751-760.

Mindess, S. (1983), "The Applicability of Fracture Mechanics to Cement and Concrete : A Historical Review", Fracture Mechanics of Concrete, ed. F. H. Wittmann, Elsevier Science Publishers B. V., Amsterdam, The Netherlands, pp. 542-680.

Mindess, S. (1985), "The Cracking and Fracture of Concrete", Fracture Toughness and Fracture Energy of Concrete, ed. F. H. Wittmann, Elsevier Science Publishers B. V., Amsterdam, The Netherlands, pp. 629-699.

Mindess, S. (1985), "Rate of Loading Effects on the Fracture of Cementitious Materials", Application of Fracture Mechanics to Cementitious Composites, ed. S. P. Shah, Martinus Nijhoff Publishers, The Netherlands, pp. 617-636.

Mindess, S., Banthia, N. and Bentur, A. (1985), "The Impact Behavior of Concrete in Bending", Fracture Toughness and Fracture Energy of Concrete, ed. F. H. Wittmann, Elsevier Science Publishers B. V., Amsterdam, The Netherlands, pp. 443-452. Mindess, S. and Bentur, A. (1985), "A Preliminary Study of the Fracture of Concrete Beams Under Impact Loading, Using High Speed Photography", *Cement and Concrete Research, Vol. 15, No.3, pp. 474-484.* 

Palm III, W. J. (1983), Modeling Analysis and Control of Dynamic Systems, John Wiley and Sons, New York, USA.

Pollet, J. C. and Burns, S. J. (1977), "Thermally Activated Crack propagation Theory", International Journal of Fracture, Vol. 13, No.5, pp. 563-576.

Robin, P. J. and Calderwood, R. W. (1978), "Explosive Testing of Fiber Reinforced Concrete", Concrete, Vol. 12, No. 1, pp. 26-28.

Rose, L. R. F. (1976), "Recent Theoretical and Experimental Results on Fast Brittle Fracture", International Journal of Fracture, Vol. 12, No. 6, pp. 799-813.

Saxton, H. J., Ireland, D. R. and Server, W. L. (1974), "Analysis and Control of Inertial Effects During Instrumented Impact Testing", Instrumented Impact Testing, ASTM STP 563, American Society for Testing and Materials, Philadelphia, USA, pp. 50-73.

Server, W. L. (1978), "Impact Three Point Testing for Notched and Pre cracked Specimen", Journal of Testing and Evaluation, Vol. 6, No.1, pp. 29-34.

Shah, S. P. and McGarry, F. J. (1970), "Griffith Fracture Criterion and Concrete", Journal of Engineering Mechanics Division, ASCE, Vol. 97, No. 6, pp. 1663-1675.

Suaris, W. and Shah, S.P. (1982), "Strain Rate effects in Fiber Reinforced Concrete Subjected to Impact and Impulsive Loading", *Composites, Vol. 13,* No. 2, pp. 153-159. Suaris, W. and Shah, S. P. (1982), "Inertial Effects in Instrumented Impact Testing of Cementitious Composites", Cement Concrete and Aggregates, Vol. 3, No. 2, pp. 77-83.

Suaris, W. and Shah, S. P. (1983), "Properties of Concrete Subjected to Impact", Journal of Structural Division, ASCE, Vol. 109, No. 7, pp. 1727-1741.

Suaris, W. and Shah, S. P. (1984), "Rate Sensitive Damage Theory for Brittle Solids", Journal of Engineering Mechanics, ASCE, Vol.110, No. 6, pp. 985-997.

Suaris, W. and Shah, S. P. (1985), "Constitutive Model for Damage Loading of Concrete", Journal of Structural Engineering, ASCE, Vol. 111, No.3, pp. 563-576.

Swartz, S. E. (1986), "Fracture Toughness Testing of Concrete at Kansas State University : Is LEFM Acceptable", Fracture Toughness and Fracture Energy of Concrete, ed. F. H. Wittmann, Elsevier Science Publishers B. V., Amsterdam, The Netherlands, pp. 359-362.

Swartz, S. E. and Go, C. G. (1984), "Validity of Compliance Calibration to Cracked Concrete Beams in Bending", *Experimental Mechanics, Vol. 24, No. 2,* pp. 129-134.

Tada, H., Paris, P. C. and Irwin G. R. (1973), Stress Analysis of Crack Handbook, Del Research Corporation, Hellertown, Pennsylvania.

Venzi, S., Priest, A. H. and May, M. J. (1970), "Influence of Inertial Load in Instrumented Impact Tests", Impact Testing of Metals, ASTM STP 466, American Society for Testing and Materials, Philadelphia, USA, pp. 165-180. Watstein, D. (1953), "Effects of Straining Rate on the Compressive Strength and Elastic Properties of Concrete", ACI Journal, Vol. 49, No. 8, pp. 729-756.

Wullart, R. A. (1970), "Application of Charpy Impact Tests", Impact Testing of Metals, ASTM STP 466, American Society for Testing and Materials, Philadelphia, USA, pp. 148-164.

Yoffe, E. H. (1951), "The Moving Griffith Crack", Philosophical Magazine, Vol. 42, pp. 739-750.

## Appendix A

# Test Results - Plain Concrete Specimens

Specimen	Drop	Notch	Span	Strain	K <sub>1D</sub>	$V_{cr}$	Time to
	Height	Depth		Rate			Fracture
	(mm)	(mm)	(mm)	(/sec)	$(Knm^{-3/2})$	(m/s)	(msec)
4T4005	150	25	410	13.56	1658	180	0.245
5T4001				13.21	1628	160	0.195
5T4002				11.18	1651	125.0	0.265
5T4003		35		11.38	2373	131.0	0.180
5T4004				12.37	1531	139.0	0.285
5T4005				10.47	3189	140.0	0.21
4T4001		50		8.72	3675	89.28	0.225
8T4001			:	8.61	3294	175.0	0.165
4T4003				8.86	3753	108.0	0.240
6T4004	250	25	410	14.97	2134	250.0	0.195
6T4005				15.13	1796	227	0.225
6T4006				16.21	3578	263	0.135
7T4001		35		13.05	4781	216	0.165
7T4002				12.54	3705	160	0.205
7T4003				10.98	3332	216	0.165
4T4004		50		13.74	7033	175	0.165
9T4002				12.04	6218	160	0.150
6T4001				9.44	3363	85	0.195
4T7001	150	25	720	2.80	2561	166	0.375
4T7002				2.66	1092	142	0.330
4T7003				2.84	3688	157	0.375
4T7004		35		2.28	3981	144	$0.255^{+}$
5T7001				3.27	2766	88	0.255
5T7002				3.93	2829	100	$0.225^{-1}$
5T7003		50		4.38	6192	138	0.165
5T7004				3.33	5577	128	0.240
6T7001				3.02	6081	74	0.255
7T7001	250	25		4.10	4909	195	0.225
6T7003				3.92	5550	180	0.345
6T7004				3.73	4186	165	0.210
7T7002		35		4.23	4415	188	0.270
7T7003				3.13	5282	178	0.375
7T7004				5.07	4483	180	0.345
7T7001				4.33	4090	173	0.285
9T4002	:	50		4.53	7741	196	0.150
8T7001			1 	4.12	10065	145	0.180
8T7002				4.01	8744	185	0.225

Table A.1: Test Results of Plain Concrete Beam - Dynamic Tests.

Appendix A. Test Results - Plain Concrete Specimens

Specimen	Drop	Notch	Span	Strain	$K_{1D}$	V <sub>cr</sub>	Time to
	Height	Depth		Rate			Fracture
	(mm)	(mm)	(mm)	(/sec)	$(kNm^{-3/2})$	(m/s)	(ms)
					. ,		
8T4002	SLOW	25	410		704	3.54E-4	
8T4003	TESTS				714	3.62E-4	
8T4004					762	3.58E-4	
8T4005		35			807	5.42E-4	
8T4006					782	5.24E-4	
<b>9T</b> 4001					705	5.42E-4	
6T4002		50			589	6.08E-4	
6T4003					702	4.14E-4	
7T4006					704	4.39E-4	
9T7001		25	720		683	5.90E-4	
9T7002					658	7.13E-4	
9T7003					734	3.43E-4	
<b>9T7</b> 004		35			688	5.72E-4	
9T7005					764	3.12E-4	
10T7001					771	3.96E-4	
8T7003		50			633	3.16E-4	
8T7004					604	2.34E-4	
10T7002					615	2.33E-4	
							х - <sup>с</sup>

Table A.2: Slow Test Results of Plain Concrete Specimens.

## Appendix B

# Test Results - Fiber-Reinforced Concrete Specimens.

Specimen	Fiber	Drop	Notch	Span	Strain	$K_{1D}$	Vcr	Time to
	Cont	Height	Depth	•	Rate			Fracture
	%	(mm)	(mm)	(mm)	(/sec)	$(kNm^{-3/2})$	(m/s)	(ms)
•								
3TF40022	2	150	25	410	12.50	1695	117	0.195
3TF40023					11.27	2237	125	0.300
3TF40024			35		13.39	2299	150	0.195
3TF40025					17.83	2371	107	0.195
3TF40026					11.50	2588	92	0.225
3TF40027		250	25		12.77	3510	150	0.380
3TF40028					21.62	2368	144	0.180
3TF40029					16.67	2684	132	0.225
3TF400210			35		17.29	3144	160	0.195
4TF40021					17.59	3631	162	0.165
4TF40022					20.42	3606	171	0.150
	-							
1TF40041	4	150	25		11.55	2492	91	0.300
1TF40042					11.88	993	144	0.270
1TF40043					11.40	1744	125	0.225
1TF40044			35		18.10	2610	93	0.120
1TF40045					14.97	2083	87	0.165
1TF40046					15.35	899	120	0.165
1TF40047		250	25		20.25	3120	160	0.165
2TF40041					12.63	3120	91	0.300
1TF40048					19.45	2818	141	0.345
1TF40049			35		18.53	4425	92.8	0.195
2TF40042					15.91	4063	156	0.180
2TF40043					13.65	3068	150	0.195

Table B.1: Dynamic Test Results of Fiber-Reinforced Concrete Specimens.

Table B.2: Slow Test Results of Fiber-Reinforced Concrete Specimens.

Fiber	Drop	Notch	Span	Strain	$K_{1D}$	V <sub>cr</sub>	Time to
Cont	Height	Depth	-	Rate			Fracture
%	(mm)	(mm)	(mm)	(/sec)	$(kNm^{-3/2})$	(m/s)	(ms)
2	SLOW	25	410		644	1.03E-4	
	TESTS				768		
					781	1.06E-4	
	1	35			925	8.61E-5	
	. !				899	9.52E-5	
.	1	-			1053	9.21E-5	
	1	1		· ·			
4	. I	25			1117	1.16E-4	
		1			1126		
					1130	8.53E-5	
	1	35	· .		980	1.16E-4	
	1				961	8.63E-5	
		1			1058	8.21E-5	
	'	1 '					i
	Fiber Cont % 2	Fiber Cont %Drop Height (mm)2SLOW TESTS4	Fiber Cont MDrop Height (mm)Notch Depth (mm)2SLOW TESTS2535354253535	Fiber Cont MDrop Height (mm)Notch Depth (mm)Span (mm)2SLOW TESTS25410425354104253535	Fiber Cont MDrop Height (mm)Notch Depth (mm)Span Rate (mm)2SLOW TESTS254104253541042535410	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$

124